EGRG 420 H. W. #5

① 2-24

Causal LSI sys.

$$y(n) = y(n-1) + y(n-2) + x(n-1)$$

a) $H(z) = \dfrac{Y(z)}{X(z)} = ?$, Plot poles & zeros of $H(z)$, indicate R.C.

$$Y(z) = z^{-1}Y(z) + z^{-2}Y(z) + z^{-1}X(z)$$

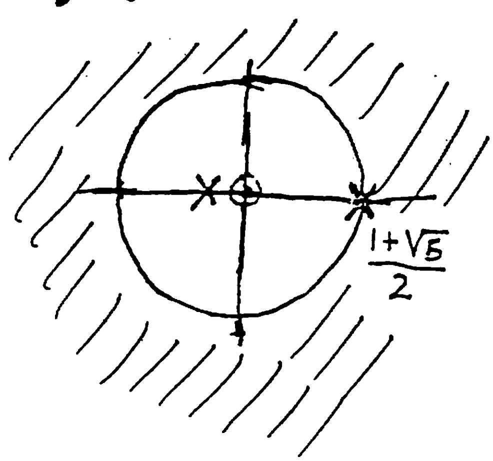
$$Y(z)\left[1 - z^{-1} - z^{-2}\right] = z^{-1}X(z)$$

$$H(z) = \frac{z^{-1}}{1 - z^{-1} - z^{-2}} = \frac{z^{-1}}{\left(1 - \frac{1+\sqrt{5}}{2}z^{-1}\right)\left(1 - \frac{1-\sqrt{5}}{2}z^{-1}\right)}$$

zero at $z = 0$

Poles at $\begin{cases} z = (1+\sqrt{5})/2 \\ z = (1-\sqrt{5})/2 \end{cases}$

zero at ∞



Causal sys. ⇒

$$R.C. = \left\{ |z| > \frac{1+\sqrt{5}}{2} \right\}$$

b) Unit-sample response = ?

$$H(z) = \frac{z^{-1}}{\left(1 - \frac{1+\sqrt{5}}{2}z^{-1}\right)\left(1 - \frac{1-\sqrt{5}}{2}z^{-1}\right)} = \left[\frac{A}{1 - \frac{1+\sqrt{5}}{2}z^{-1}} + \frac{B}{1 - \frac{1-\sqrt{5}}{2}z^{-1}}\right]$$

$$\Rightarrow \frac{z^{-1}}{\left(1-\frac{1+\sqrt5}{2}z^{-1}\right)\left(1-\frac{1-\sqrt5}{2}z^{-1}\right)} = \frac{A-\frac{1-\sqrt5}{2}Az^{-1}+B-\frac{1+\sqrt5}{2}Bz^{-1}}{\left(1-\frac{1+\sqrt5}{2}z^{-1}\right)\left(1-\frac{1-\sqrt5}{2}z^{-1}\right)}$$

$$\Rightarrow \begin{cases} A+B=0 \\ -\frac{1-\sqrt5}{2}A-\frac{1+\sqrt5}{2}B=1 \end{cases} \Rightarrow A=\frac{\sqrt5}{5}, \quad B=-A=\frac{-\sqrt5}{5}$$

$$H(z) = \frac{\sqrt5/5}{1-\frac{1+\sqrt5}{2}z^{-1}} - \frac{\sqrt5/5}{1-\frac{1-\sqrt5}{2}z^{-1}}$$

$$\boxed{h(n) = \frac{\sqrt5}{5}\left(\frac{1+\sqrt5}{2}\right)^n u(n) - \frac{\sqrt5}{5}\left(\frac{1-\sqrt5}{2}\right)^n u(n)}$$

Unstable

c) Find a stable (non-causal) $h(n)$.

stable system is:

$$h(n) = -\frac{\sqrt5}{5}\left(\frac{1+\sqrt5}{2}\right)^n u(-n-1) - \frac{\sqrt5}{5}\left(\frac{1-\sqrt5}{2}\right)^n u(n)$$

② 2-27 | Linear discrete-time SI sys.

$$y(n-1) - \frac{10}{3}y(n) + y(n+1) = x(n)$$

The sys. is stable. $h(n) = ?$

$$z^{-1}Y(z) - \frac{10}{3}Y(z) + zY(z) = X(z)$$

Hence $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{1}{z^{-1} - \frac{10}{3} + z} = \dfrac{z}{1 - \frac{10}{3}z + z^2}$.

$$\Rightarrow H(z) = \frac{z}{(z-\frac{1}{3})(z-3)}$$

stable sys. $\Rightarrow$ R.C. includes the unit circle.

Hence $z = \frac{1}{3}$ is the only pole enclosed by the unit circle.

For $\boxed{n \geq 0}$

$$h(n) = \text{Res}\left[\frac{z^n}{(z-\frac{1}{3})(z-3)}\right]\Bigg|_{z=\frac{1}{3}} = \frac{(1/3)^n}{-8/3} = \frac{-3}{8}\left(\frac{1}{3}\right)^n$$

For $\boxed{n < 0}$

$$h(n) = \text{Res}\left[\frac{p^{-n}}{(3-p)(\frac{1}{3}-p)}\right]\Bigg|_{p=\frac{1}{3}} = \frac{(\frac{1}{3})^{-n}}{8/3} = \frac{3}{8}\left(\frac{1}{3}\right)^{-n}$$

Hence $h(n) = -\frac{3}{8}(3)^n u(-n-1) - \frac{3}{8}(3^{-n})\, \underline{u}(n)$

3) $\mathcal{Z}^{-1}[X(z)Y(z^{-1})] = ?$

$$\mathcal{Z}^{-1}[X(z)\, Y(z^{-1})] = x(n) * y(-n)$$

4) $g(n) = a^n u(n) \qquad |a| < 1$

$$f(n) = \sum_{k=n}^{\infty} g(k)$$

a) Find $\dfrac{F(z)}{G(z)}$

b) R.C. of $f(n)$

$$f(n) = \sum_{k=n}^{\infty} g(k) = \sum_{k=-\infty}^{\infty} g(k)\, U(k-n) = g(n) * U(-n)$$

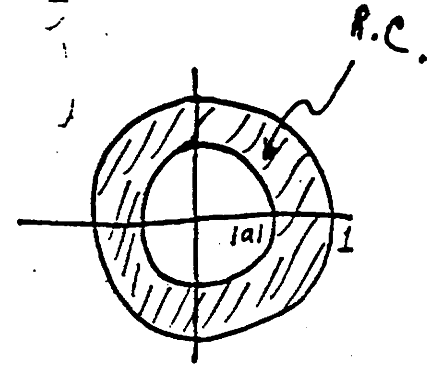
$$F(z) = G(z)\, U'(z)$$

$$\frac{F(z)}{G(z)} = U'(z) = \mathcal{Z}[U(-n)] = \frac{1}{1-z}$$

b) $$G(z) = \frac{1}{1-az^{-1}}$$

$G(z)$ has a R.H.S. pole at a

$U'(z)$ has a L.H.S. pole at 1



5) $$H(z) = \frac{a_3^* + a_2^* z^{-1} + a_1^* z^{-2} + z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}$$

Is this an all pass system?

$$|H(e^{jw})| = \frac{|a_3^* + a_2^* e^{-jw} + a_1^* e^{-2jw} + e^{-3jw}|}{|1 + a_1 e^{-jw} + a_2 e^{-2jw} + a_3 e^{-3jw}|}$$

$$= \frac{|e^{-3jw}| \cdot |1 + a_1^* e^{jw} + a_2^* e^{2jw} + a_3^* e^{3jw}|}{\underbrace{|1 + a_1 e^{-jw} + a_2 e^{-2jw} + a_3 e^{-3jw}|}_{P}}$$

$$|H(e^{jw})| = \frac{1 \cdot |P^*|}{|P|} = 1$$

$\therefore$ all pass system.