3

but $-1 = e^{-j\pi} \Rightarrow (-1)^k = e^{-jk\pi}$

$$\therefore G(e^{j\omega}) = \frac{1}{2}X(e^{j\omega/2}) + \frac{1}{2}\sum_k x(k)\, e^{-j(\frac{\omega}{2}+\pi)k}$$

$$= \frac{1}{2}X(e^{j\omega/2}) + \frac{1}{2}X(-e^{j\frac{\omega}{2}})$$

(d) $g(n) = \begin{cases} x(n/2) & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$

$$G(e^{j\omega}) = \sum_{\substack{n=-\infty \\ n \text{ even}}}^{\infty} x(n/2)\, e^{-j\omega n}$$, Let $\frac{n}{2} = k$

$$\Rightarrow G(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x(k)\, e^{-j\omega 2k} = X(e^{j2\omega})$$ ■

Q5. (1.25) 1 & 2

$$\mathcal{F}[x(n)] = X(e^{j\omega}) = \sum_n x(n)\, e^{-j\omega n}$$

(1) $x^*(n) \longleftrightarrow X^*(e^{-j\omega})$

$$\mathcal{F}[x^*(n)] = \sum_n x^*(n)\, e^{-j\omega n} = \left[\sum_n x(n)\, e^{j\omega n}\right]^*$$

$$= X^*(e^{-j\omega})$$

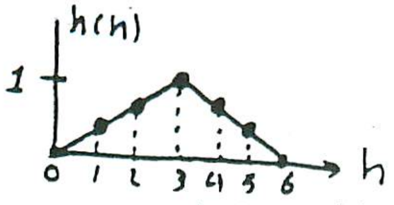
(2) $x^*(-n) \longleftrightarrow X^*(e^{j\omega})$

$$\mathcal{F}[x^*(-n)] = \sum_{n=-\infty}^{\infty} x^*(-n)\, e^{-j\omega n}$$ Let $n' = -n$

$$= \sum_{n'=-\infty}^{\infty} x^*(n')\, e^{j\omega n'}$$

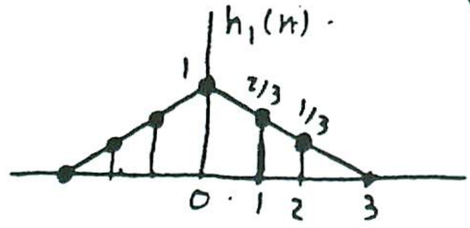
$$= \left[\sum_{n'} x(n')\, e^{-j\omega n'}\right]^* = X^*(e^{j\omega})$$ ■

Q.6



Magnitude & Phase of $H(e^{jw})$

Instead of working with $h(n)$, work with $h_1(n)$ which is simpler to use.



we know $h(n) = h_1(n-3)$

Then use $\mathcal{F}_s[x(n-n_0)] = X_s(e^{jw})\, e^{-jwn_0}$ to find $H(e^{jw})$

$$H_1(e^{jw}) = 1 + \frac{1}{3}\left(e^{j2w} + e^{-j2w}\right) + \frac{2}{3}\left(e^{jw} + e^{-jw}\right)$$

$$= 1 + \frac{2}{3}\cos 2w + \frac{4}{3}\cos w$$

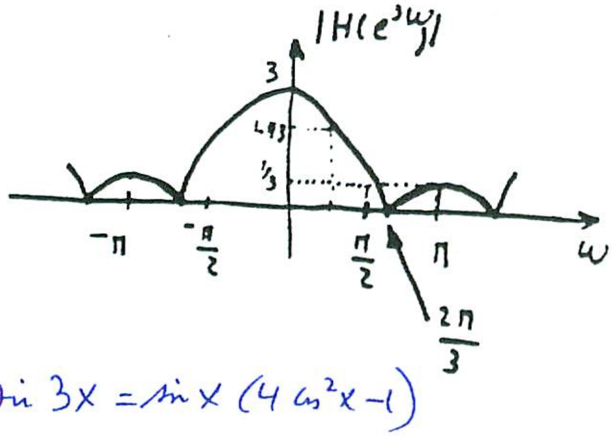
(here we have used $\mathcal{F}[\delta(t-a)] = e^{-jwa}$)

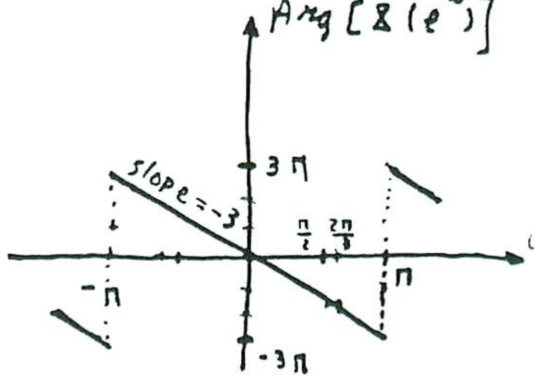
Now for $H(e^{jw})$ we have

$$H(e^{jw}) = e^{-j3w}\left[1 + \frac{2}{3}\cos 2w + \frac{4}{3}\cos w\right]$$

$$\therefore |H(e^{jw})| = 1 + \frac{2}{3}\cos 2w + \frac{4}{3}\cos w = \frac{1}{3}\left(\frac{\sin\frac{3w}{2}}{\sin w/2}\right)^2$$

& $\text{Arg}[H(e^{jw})] = -3w$





Use $\sin 3x = \sin x\,(4\cos^2 x - 1)$

Plot magnitude and phase.

EGRG 420

H.W. # 2 sol.

Q1 (1.12)

	Stable	Causal	Linear	S.I.
a) $T[x(n)] = g(n)\,x(n)$	yes if $\lvert g(n)\rvert < \infty$	yes	yes except when $g(n)=x(n)$	NO (yes if $g(n)$ is constant)
exam! e) $T[x(n)] = \sum_{k=n-n_0}^{n+n_0} x(k)$	yes	No	yes	yes
e) $T[x(n)] = e^{x(n)}$	yes	yes	No	yes

Q2 (1.19) show that:

$$\sum_n x(n)\,x^*(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\,X^*(e^{j\omega})\,d\omega$$

$$x(n) \triangleq \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\,e^{j\omega n}\,d\omega$$

hence $$x^*(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X^*(e^{j\omega})\,e^{-j\omega n}\,d\omega$$

$$\therefore \sum_n x(n)\,x^*(n) = \sum_n x(n)\,\frac{1}{2\pi}\int_{-\pi}^{\pi} X^*(e^{j\omega})\,e^{-j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X^*(e^{j\omega})\left(\sum_n x(n)\,e^{-j\omega n}\right)d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\,X^*(e^{j\omega})\,d\omega$$

QED

Q.3 (1.22)

Given $f(n)$ & $g(n)$ $\longrightarrow$ Real, Causal & Stable Sequences

$$f(n) \longleftrightarrow F(e^{j\omega})$$
$$g(n) \longleftrightarrow G(e^{j\omega})$$

Show that:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} F(e^{j\omega})\,G(e^{j\omega})\,d\omega = \left\{\frac{1}{2\pi}\int_{-\pi}^{\pi} G(e^{j\omega})\,d\omega\right\}\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi} F(e^{j\omega})\,d\omega\right\}$$

$$f(n) * g(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} F(e^{jw})\, G(e^{jw})\, e^{jwn}\, dw$$

$$\Rightarrow \quad f(n) * g(n)\Big|_{n=0} = \frac{1}{2\pi}\int_{-\pi}^{\pi} F(e^{jw})\, G(e^{jw})\, dw$$

$$\text{L.H.S.} = \sum_{k=-\infty}^{\infty} f(k)\, g(n-k)\Big|_{n=0} = \sum_{k} f(k)\, g(-k)$$

However $f(k)=0 \quad \forall k<0$ & $g(-k)=0 \quad \forall k>0$

$$\therefore \quad \text{L.H.S.} = f(0)\, g(0)$$

$$= \left\{\frac{1}{2\pi}\int_{-\pi}^{\pi} F(e^{jw})\, dw\right\}\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi} G(e^{jw})\, dw\right\}$$

Q4 (1.24) c & d

(c) $g(n) = x(2n)$

$$G(e^{jw}) = \sum_{n=-\infty}^{\infty} g(n)\, e^{-jwn}$$

$$= \sum_{n} x(2n)\, e^{-jwn}$$

$\hookrightarrow$ even terms only

$$G(e^{jw}) = \sum_{k} \underbrace{\frac{1+(-1)^k}{2}\, x(k)}_{\text{we only get the even terms}}\, e^{-jw\frac{k}{2}}$$

$$\therefore G(e^{jw}) = \frac{1}{2}\sum_{k} x(k)\, e^{-jw\frac{k}{2}} + \frac{1}{2}\sum_{k} (-1)^k x(k)\, e^{-jw\frac{k}{2}}$$