: EGEE 420 HW #1 SOLN.

$\boxed{1}\ \boxed{\text{Text } 1\text{-}2\ c}$

(C) $y(n) = h(n) * x(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$

for $n<0$; $y(n) = 0$ (No overlap)

" $n=0$, $y(0) = h(0)\,x(0) = -2$

" $n=1$; $y(1) = h(0)\,x(1) + h(1)\,x(0) = (-1)(-1) + 2\times 2 = 5$

" $n=2$; $y(2) = h(2)\,x(0) + h(1)\,x(1) = 1\times 2 + 2(-1) = 0$

" $n=3$; $y(3) = h(3)\,x(0) = 1\,(-1) = -1$

" $n \ge 4$; $y(n) = 0$ (No overlap)

Q.E.D.

$\boxed{2}\ \boxed{\text{Text } 1\text{-}3}$ $n_0 = 0$, $N = 4$

$$h(n) = \begin{cases} \alpha^n, & 0 \le n < 4 \\ 0, & \text{e.w.} \end{cases} \quad \& \quad x(n) = \begin{cases} \beta^n, & 0 \le n \\ 0, & n<0 \end{cases}$$

$y(n) = h(n) * x(n)$

i) <u>for $n<0$</u> , $y(n) = 0$ (No overlap)

ii) <u>for $0 \le n < 4$</u> ,
$$y(n) = \sum_{k=0}^{n} \alpha^{n-k}\beta^{k} = \alpha^n \sum_{k=0}^{n} (\beta/\alpha)^k$$
$$= \alpha^n \frac{1-(\beta/\alpha)^{n+1}}{1-(\beta/\alpha)} = \frac{\alpha^{n+1}-\beta^{n+1}}{\alpha-\beta} \quad \underline{\text{if } \alpha \ne \beta}$$
$$\text{and} = \alpha^n \sum_{k=0}^{n} 1^k = (n+1)\,\alpha^n \quad \underline{\text{if } \alpha = \beta}$$

iii) <u>for $n \ge 4$</u> ;
$$y(n) = \sum_{k=n-3}^{n} \alpha^{n-k}\beta^{k} = \alpha^n \sum_{k=n-3}^{n} \left(\frac{\beta}{\alpha}\right)^k$$

$k' = k-n+3 \Rightarrow$
$$= \alpha^n \sum_{k'=0}^{3} \left(\frac{\beta}{\alpha}\right)^{k'+n-3} = \alpha^n \left(\frac{\beta}{\alpha}\right)^{n-3} \sum_{k'=0}^{3} \left(\frac{\beta}{\alpha}\right)^{k'}$$
$$= \alpha^3 \beta^{n-3} \frac{1-\left(\frac{\beta}{\alpha}\right)^4}{1-\left(\frac{\beta}{\alpha}\right)} = \beta^{n-3}\,\frac{\alpha^4-\beta^4}{\alpha-\beta} \quad \underline{\text{if } \alpha \ne \beta}$$
$$\text{and} = \alpha^n \sum_{k'=0}^{3} 1^{k'} = 4\alpha^n \quad \underline{\text{if } \alpha = \beta}$$

$\therefore$
$$y(n) = \begin{cases} 0 & ; n<0 \\ \left.\begin{array}{ll} \dfrac{\alpha^{n+1}-\beta^{n+1}}{\alpha-\beta}, & \alpha \ne \beta \\ (n+1)\,\alpha^n ; & \alpha = \beta \end{array}\right\} & ; 0 \le n < 4 \\ \left.\begin{array}{ll} \beta^{n-3}\dfrac{\alpha^4-\beta^4}{\alpha-\beta} ; & \alpha \ne \beta \\ 4\alpha^n & \alpha = \beta \end{array}\right\} & n \ge 4. \end{cases}$$

Q.E.D

3 | Text 1-4

$e(n) = \alpha^n \quad \forall n$ and $x(n)$ and $y(n)$ are two arbitrary sequences

$$[e(n)x(n)] * [e(n)y(n)] = \sum_k \alpha^k x(k)\, \alpha^{n-k}\, y(n-k)$$

$$= \alpha^n \sum_k x(k)\, y(n-k) = e(n)[x(n) * y(n)] \checkmark$$

Q.E.D

Problem # 4

$$x(n) * (y(n) * w(n)) = x(n) * \left(\sum_l w(l)\, y(n-l)\right) = \sum_k x(k) \sum_l w(l)\, y(n-k-l)$$

$$= \sum_l \left[\sum_k x(k)\, y((n-l)-k)\right] w(l) = \sum_l \left(x(n-l) * y(n-l)\right) w(l)$$

$$= (x(n) * y(n)) * w(n) \checkmark$$

Q.E.D

Problem # 5

$y(n) - 2.5\, y(n-1) + y(n-2) = \delta(n) \quad , -\infty < n < \infty.$

Taking Z-transform of both sides $\Rightarrow$

$$Y(z) - 2.5\, Y(z)\, z^{-1} + Y(z)\, z^{-2} = 1$$

$$\therefore Y(z) = \frac{1}{1 - 2.5 z^{-1} + z^{-2}} = \frac{1}{(z^{-1}-2)(z^{-1}-.5)} = \frac{4/3}{1-2z^{-1}} + \frac{-1/3}{1-.5z^{-1}}$$

$\therefore$ Causal soln. : $\quad y(n) = \frac{4}{3}(2)^n u(n) - \frac{1}{3}(.5)^n u(n)$

Q.E.D

Problem # 6

$y(n) - y(n-1) + \frac{2}{9} y(n-2) = x(n) - x(n-1) \quad ; n \geq 0 \quad ; \quad x(n) = \delta(n) \quad , \quad y(-1) = 1, \; y(-2) = 0$

Substituting for $x(n)$ and $x(n-1)$ and taking one sided Z-T. $\Rightarrow$

$$Y(z) - [z^{-1}Y(z) + y(-1)] + \frac{2}{9}[z^{-2}Y(z) + z^{-1}y(-1) + y(-2)] = X(z) - [z^{-1}X(z) + x(-1)] = 1 - z^{-1}$$

$$\Rightarrow Y(z) = \frac{2 - \frac{11}{9} z^{-1}}{1 - z^{-1} + \frac{2}{9} z^{-2}} = \frac{5/3}{1 - \frac{1}{3} z^{-1}} + \frac{1/3}{1 - \frac{2}{3} z^{-1}}$$

$$\therefore y(n) = \frac{5}{3}\left(\frac{1}{3}\right)^n u(n) + \frac{1}{3}\left(\frac{2}{3}\right)^n u(n) = \frac{1}{3^{n+1}}(5 + 2^n)\, u(n).$$

Q.E.D

Problem # 7

$$X(z) = \frac{1}{(1-z^{-1})(1-.5z^{-1})} = \frac{2}{1-z^{-1}} + \frac{-1}{1-.5z^{-1}} \Rightarrow x(n) = [2 - (.5)^n]\, u(n)$$

Q.E.D

Problem # 8

$$|H(e^{j\omega})| \triangleq \left|\sum_n h(n) e^{-j\omega n}\right| \leq \sum_n |h(n)| \underbrace{|e^{-j\omega n}|}_{=1} = \sum_n |h(n)|$$

But for a stable system $\sum_n |h(n)| < \infty \quad \therefore |H(e^{j\omega})| < \infty \quad , 0 \leq \omega \leq 2\pi.$

Q.E.D