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CALIFORNIA STATE UNIVERSITY FULLERTON™

Academic Integrity:

The Right Answer!

Academic integrity: Honesty in all academic endeavors is a core value at California State University, Fullerton. Whether using this bluebook, a scantron, other testing materials, or submitting essays and term papers, it is cheating if you attempt to gain an unfair academic advantage or assist others. A few examples are:

- Using unauthorized notes, materials or assistance during exams
- Using or copying the work of other students
- Submitting work that isn't your own
- Sharing answers to exam questions or class assignments
- Using the words or ideas of another without giving credit to the source; plagiarism

These and other forms of cheating not only dishonor our educational values but they also violate the trust that is crucial to intellectual and personal integrity. Consequently, cheating may result in severe disciplinary action including an "F" in the course and could lead to suspension from the University.

Let's make sure that grades accurately reflect what each student has actually learned.
Good luck on this examination!

EXAMINATION BOOK

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Subject Midterm Exam II

Class EE409 Section

Instructor Prof. Grewal Date 4/15/10

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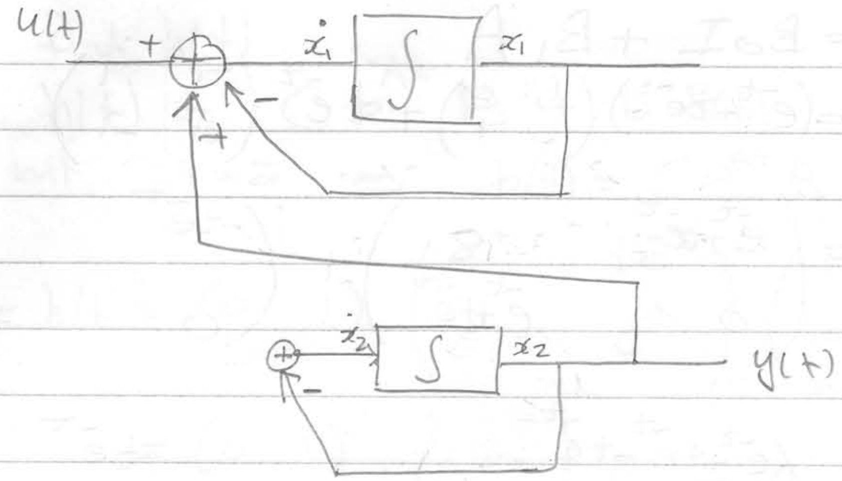
Q1



$-\lambda = 1$
$\lambda = -1$

(a)

$$\dot{x}_1 = u(t) - x_1 + x_2$$
$$\dot{x}_2 = -x_2$$
$$y(t) = x_2$$

so

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \overbrace{\begin{pmatrix} -1 & +1 \\ 0 & -1 \end{pmatrix}}^{A} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \overbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}^{B} u$$

$$y = \underbrace{\begin{pmatrix} 0 & 1 \end{pmatrix}}_{C} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \underbrace{[0]}_{D} u$$

(b) to find e^{At}, first find eigenvalues of A.

$$|A - \lambda I| = \begin{vmatrix} -1-\lambda & 1 \\ 0 & -1-\lambda \end{vmatrix} = (-1-\lambda)(-1-\lambda) = 0$$

so $(-1-\lambda)^2 = 0$ i.e roots are $-\lambda - 1 = 0$ or $\boxed{\lambda = -1}$ repeated roots

so $\boxed{\lambda_1 = -1,\ \lambda_2 = -1}$

so $e^{\lambda_1 t} = B_0 + B_1 \lambda_1$
derivative w.r.t. λ → and $t e^{\lambda_1 t} = B_1$

$\longrightarrow$ $e^{-t} = B_0 - B_1$, $t e^{-t} = B_1$ $\Big\}$ $B_0 = e^{-t} + t e^{-t}$, $B_1 = t e^{-t}$

so $\boxed{B_0 = e^{-t} + t e^{-t}, \quad B_1 = t e^{-t}}$

→

so $e^{At} = B_0 I + B_1 A$

$$= (e^{-t} + te^{-t})\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + te^{-t}\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} e^{-t}+te^{-t} & 0 \\ 0 & e^{-t}+te^{-t} \end{pmatrix} + \begin{pmatrix} -te^{-t} & te^{-t} \\ 0 & -te^{-t} \end{pmatrix}$$

$$= \begin{pmatrix} e^{-t}+te^{-t}-te^{-t} & te^{-t} \\ 0 & e^{-t}+te^{-t}-te^{-t} \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} e^{-t} & te^{-t} \\ 0 & e^{-t} \end{pmatrix}}$$

(c) Find $(JwI - A)^{-1}$ let $\Delta = (JwI - A)$

$$\Delta = Jw\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} Jw & 0 \\ 0 & Jw \end{pmatrix} - \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} Jw+1 & -1 \\ 0 & Jw+1 \end{pmatrix}$$

so $$\Delta^{-1} = \begin{pmatrix} Jw+1 & 1 \\ 0 & Jw+1 \end{pmatrix} \frac{1}{(Jw+1)^2}$$

$$= \boxed{\frac{\begin{pmatrix} Jw+1 & 1 \\ 0 & Jw+1 \end{pmatrix}}{-w^2 + 2Jw + 1}}$$

(d) find $h(t)$

$$h(t) = C e^{At} B + D\,\delta(t) \qquad t \geq 0.$$

but $[D] = \bar{0} \Rightarrow \quad h(t) = Ce^{At}B \quad t \geq 0.$

$$\text{so } h(t) = \underset{1\times 2}{\begin{pmatrix} 0 & 1 \end{pmatrix}} \underset{2\times 2}{\begin{pmatrix} e^{-t} & te^{-t} \\ 0 & e^{-t} \end{pmatrix}} \underset{2\times 1}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}$$

$$= \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-t} \\ 0 \end{pmatrix} = 0 \qquad \underline{\underline{t > 0}}$$

$$H(jw) = C(Ijw - A)^{-1} B$$

$$= \begin{pmatrix} 0 & 1 \end{pmatrix} \frac{\begin{pmatrix} Jw+1 & 1 \\ 0 & Jw+1 \end{pmatrix}}{2Jw - w^2 + 1} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} Jw+1 \\ 0 \end{pmatrix} \frac{1}{2Jw + w^2 + 1} = 0.$$

✓

Q2 $y''(t) + y(t) = e^t$ $y(0) = 1$, $y'(0) = 1$

using Laplace Method:

$$s^2 Y(s) - sy(0) - y'(0) + Y(s) = \frac{1}{s-1}$$

$$s^2 Y(s) - s - 1 + Y(s) = \frac{1}{s-1}$$

$$Y(s)\left[s^2+1\right] - s - 1 = \frac{1}{s-1}$$

$$Y(s) = \frac{s}{s^2+1} + \frac{1}{s^2+1} + \frac{1}{(s-1)(s^2+1)}$$

Consider this term for now:

$$\frac{1}{(s-1)(s^2+1)} = \frac{A}{(s-1)} + \frac{Bs+C}{s^2+1}$$

$$A = \lim_{s\to 1} \frac{1}{s^2+1} = \boxed{\frac{1}{2}}$$

So $$\frac{1}{(s-1)(s^2+1)} = \frac{\frac{1}{2}}{(s-1)} + \frac{Bs+C}{s^2+1}$$

So $$1 = \frac{1}{2}(s^2+1) + (Bs+C)(s-1)$$

$$1 = \frac{1}{2}s^2 + \frac{1}{2} + Bs^2 - Bs + Cs - C$$

$$1 = s^2\left[\frac{1}{2}+B\right] + s\left[C-B\right] + \frac{1}{2} - C$$

So $$\left.\begin{aligned} 1 &= \tfrac{1}{2} - C \\ 0 &= \tfrac{1}{2} + B \\ 0 &= C - B \end{aligned}\right\} \quad \boxed{\begin{aligned} C &= -\tfrac{1}{2} \\ B &= -\tfrac{1}{2} \end{aligned}}$$

So $$\frac{1}{(s-1)(s^2+1)} = \frac{\frac{1}{2}}{(s-1)} + \frac{-\frac{1}{2}s - \frac{1}{2}}{s^2+1}$$

$$= \boxed{\frac{\frac{1}{2}}{(s-1)} - \frac{1}{2}\frac{s}{s^2+1} - \frac{1}{2}\frac{1}{s^2+1}} \longrightarrow$$

so $\boxed{Y(s) = \frac{s}{s^2+1} + \frac{1}{s^2+1} + \frac{1}{2}\frac{1}{s-1} - \frac{1}{2}\frac{s}{s^2+1} - \frac{1}{2}\frac{1}{s^2+1}}$

$$\boxed{Y(s) = \frac{1}{2}\frac{1}{s^2+1} + \frac{1}{2}\frac{1}{s-1}}$$

now apply Laplace transform.

$$y(t) = \left(\frac{1}{2}\sin t + \frac{1}{2}e^{t}\right)\xi(t)$$

$$= \boxed{\frac{1}{2}\left(\sin t + e^{t}\right)\xi(t)}$$

instable !
due to this