CALIFORNIA STATE UNIVERSITY FULLERTON™

Academic Integrity:

The Right Answer!

Academic integrity: Honesty in all academic endeavors is a core value at California State University, Fullerton. Whether using this bluebook, a scantron, other testing materials, or submitting essays and term papers, it is cheating if you attempt to gain an unfair academic advantage or assist others. A few examples are:

- Using unauthorized notes, materials or assistance during exams
- Using or copying the work of other students
- Submitting work that isn't your own
- Sharing answers to exam questions or class assignments
- Using the words or ideas of another without giving credit to the source; plagiarism

These and other forms of cheating not only dishonor our educational values but they also violate the trust that is crucial to intellectual and personal integrity. Consequently, cheating may result in severe disciplinary action including an "F" in the course and could lead to suspension from the University.

Let's make sure that grades accurately reflect what each student has actually learned.

Good luck on this examination!

EXAMINATION BOOK

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Name NASSER M. ABBASI

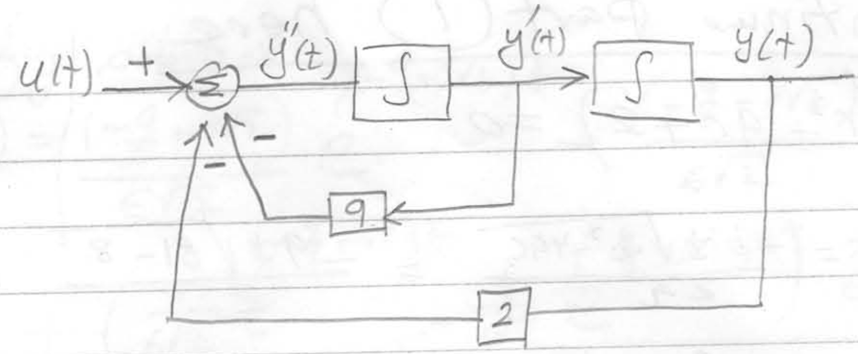
Subject EE 409

Class ______ **Section** ______

Instructor Dr Grewal **Date** 3/4/2010

Q1

(1)



$$y'' = -9y'(t) - 2y(t) + u(t)$$

(2) to find $h(t)$: solve the homogeneous D.E.

$$y''(t) + 9y'(t) + 2y(t) = 0$$

with IC $y(0) = 0,\ y'(0) = 1$.

hence $(D^2 + 9D + 2)\ y(t) = 0$

so char eq $\boxed{r^2 + 9r + 2 = 0}$ $\longrightarrow$ please see Next page.

Part (4) method. let $u = e^{jwt}$.

$\Rightarrow$ so $y(t) = H(jw)e^{jwt}$

now substitute into ODE, we obtain

$$\left(H(jw)e^{jwt}\right)'' + 9\left(H(jw)e^{jwt}\right)' + 2\left(H(jw)e^{jwt}\right) = e^{jwt}$$

$$\left(jw\,H(jw)e^{jwt}\right)' + 9\left(jw\,H(jw)e^{jwt}\right) + 2\,H(jw)e^{jwt} = e^{jwt}$$

$$(jw)^2 H(jw) + 9\,jw\,H(jw) + 2\,H(jw) = 1$$

$$H(jw)\left[-w^2 + 9jw + 2\right] = 1$$

so $\boxed{H(jw) = \dfrac{1}{2 - w^2 + 9jw}}$

continue part (1) here

$(r^2+9r+2)=0$

$$r=\frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-9\pm\sqrt{81-8}}{2} = -\frac{9}{2}\pm\sqrt{\frac{72}{4}}$$

$$= -\frac{9}{2}\pm\sqrt{18} = -\frac{9}{2}\pm 3\sqrt{2}$$

so $\boxed{r_1 = -\frac{9}{2}+3\sqrt{2}}$ and $\boxed{r_2 = -\frac{9}{2}-3\sqrt{2}}$

so $h(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$

$= C_1 e^{-\frac{9}{2}t} e^{3\sqrt{2}t} + C_2 e^{-\frac{9}{2}t} e^{-3\sqrt{2}t}$

now find c_1 and c_2

$h(0)=0 \Rightarrow \boxed{0 = C_1 + C_2}$

$h'(t) = C_1\left(-\frac{9}{2}+3\sqrt{2}\right)e^{\left(-\frac{9}{2}+3\sqrt{2}\right)t} + C_2\left(-\frac{9}{2}-3\sqrt{2}\right)e^{\left(-\frac{9}{2}-3\sqrt{2}\right)t}$

$h'(0)=1 \Rightarrow C_1\left(-\frac{9}{2}+3\sqrt{2}\right) + C_2\left(-\frac{9}{2}-3\sqrt{2}\right)$

or $\boxed{1 = C_1\left(-\frac{9}{2}+3\sqrt{2}\right) + C_2\left(-\frac{9}{2}-3\sqrt{2}\right)}$

but $C_1 = -C_2$ so the above is

$1 = -C_2\left(-\frac{9}{2}+3\sqrt{2}\right) + C_2\left(-\frac{9}{2}-3\sqrt{2}\right)$

$1 = \frac{9}{2}C_2 - 3\sqrt{2}\,C_2 - \frac{9}{2}C_2 - 3\sqrt{2}\,C_2$

$1 = -6\sqrt{2}\,C_2$ so $\boxed{C_2 = -\frac{1}{6\sqrt{2}}}$

so $\boxed{C_1 = \frac{1}{6\sqrt{2}}}$

So $h(t) = \left(\frac{1}{6\sqrt{2}} e^{\left(-\frac{9}{2}+3\sqrt{2}\right)t} - \frac{1}{6\sqrt{2}} e^{\left(-\frac{9}{2}-3\sqrt{2}\right)t}\right)\xi(t)$

(3) to verify solution.

$$h'(t) = \left(\frac{\left(-\frac{9}{2}+3\sqrt{2}\right)}{6\sqrt{2}} e^{\left(-\frac{9}{2}+3\sqrt{2}\right)t} - \frac{\left(-\frac{9}{2}-3\sqrt{2}\right)}{6\sqrt{2}} e^{\left(-\frac{9}{2}-3\sqrt{2}\right)t} \right) \xi(t)$$

$$+ \underbrace{\left(\frac{1}{6\sqrt{2}} e^{\left(-\frac{9}{2}+3\sqrt{2}\right)t} - \frac{1}{6\sqrt{2}} e^{\left(-\frac{9}{2}-3\sqrt{2}\right)t} \right)}_{=0} \delta(t)$$

$$\text{so } h'(t) = \left(\frac{\left(-\frac{9}{2}+3\sqrt{2}\right)}{6\sqrt{2}} e^{\left(-\frac{9}{2}+3\sqrt{2}\right)t} - \frac{\left(-\frac{9}{2}-3\sqrt{2}\right)}{6\sqrt{2}} e^{\left(-\frac{9}{2}-3\sqrt{2}\right)t} \right) \xi(t)$$

$$h''(t) = \left(\frac{\left(-\frac{9}{2}+3\sqrt{2}\right)^2}{6\sqrt{2}} e^{\left(-\frac{9}{2}+3\sqrt{2}\right)t} - \frac{\left(-\frac{9}{2}-3\sqrt{2}\right)^2}{6\sqrt{2}} e^{\left(-\frac{9}{2}-3\sqrt{2}\right)t} \right) \xi(t)$$

$$+ \left(\frac{\left(-\frac{9}{2}+3\sqrt{2}\right)}{6\sqrt{2}} e^{\left(-\frac{9}{2}+3\sqrt{2}\right)t} - \frac{\left(-\frac{9}{2}-3\sqrt{2}\right)}{6\sqrt{2}} e^{\left(-\frac{9}{2}-3\sqrt{2}\right)t} \right) \delta(t)$$

$$h''(t) = \frac{\left(-\frac{9}{2}+3\sqrt{2}\right)^2}{6\sqrt{2}} e^{\left(-\frac{9}{2}+3\sqrt{2}\right)t} - \frac{\left(-\frac{9}{2}-3\sqrt{2}\right)^2}{6\sqrt{2}} e^{\left(-\frac{9}{2}-3\sqrt{2}\right)t} \Bigg) \xi(t)$$

$$+ \underbrace{\frac{1}{6\sqrt{2}} \left(-\frac{9}{2}+3\sqrt{2} - \left(-\frac{9}{2}-3\sqrt{2}\right) \right)}_{=1 \text{ !! as we want}} \delta(t)$$

$$\text{so } h''(t) = \left[\frac{\left(-\frac{9}{2}+3\sqrt{2}\right)^2}{6\sqrt{2}} e^{\left(-\frac{9}{2}+3\sqrt{2}\right)t} - \frac{\left(-\frac{9}{2}-3\sqrt{2}\right)^2}{6\sqrt{2}} e^{\left(-\frac{9}{2}-3\sqrt{2}\right)t} \right] \xi(t)$$

$$+ \delta(t),$$

Plug into ODE $\Rightarrow$

$$LHS = h''(t) + 9\,h'(t) + 2h(t)$$

next $\longrightarrow$

$$LHS = \left[\frac{\left(-\frac{9}{2}+3\sqrt{2}\right)^2}{6\sqrt{2}} e^{\left(-\frac{9}{2}+3\sqrt{2}\right)t} - \frac{\left(-\frac{9}{2}-3\sqrt{2}\right)^2}{6\sqrt{2}} e^{\left(-\frac{9}{2}-3\sqrt{2}\right)t}\right]\zeta(t)$$

$$+\delta(t)$$

$$+9\left[\frac{\left(-\frac{9}{2}+3\sqrt{2}\right)}{6\sqrt{2}} e^{\left(-\frac{9}{2}+3\sqrt{2}\right)t} - \frac{\left(-\frac{9}{2}-3\sqrt{2}\right)}{6\sqrt{2}} e^{\left(-\frac{9}{2}-3\sqrt{2}\right)t}\right]\zeta(t)$$

$$+2\left[\frac{1}{6\sqrt{2}} e^{\left(-\frac{9}{2}+3\sqrt{2}\right)t} - \frac{1}{6\sqrt{2}} e^{\left(-\frac{9}{2}-3\sqrt{2}\right)t}\right]\zeta(t)$$

simplifying gives $LHS = \delta(t)$

hence verified.

since $h(t)$ is defined as the solution to the ODE when the input is $\delta(t)$, i.e. RHS is $\delta(t)$, then ~~we~~ verified it.

Part (4) Continue here from first page.

(4) Find $H(j\omega)$, $|H(j\omega)|$, $Arg(H(j\omega))$

from 1^{st} page, I found $H(j\omega)$

$$\boxed{H(j\omega) = \frac{-1}{2-\omega^2+9j\omega}}$$

From Nora

so $$|H(j\omega)|^2 = \frac{1}{(2-\omega^2)^2+(9\omega)^2}$$

so $$|H(j\omega)| = \sqrt{\frac{1}{(2-\omega^2)^2+(9\omega)^2}}$$

$$Arg(H(j\omega)) = -\tan^{-1}\left(\frac{9\omega}{2-\omega^2}\right)$$

$\longrightarrow$

Q 2

$$y_1(k) = A\,u_1(k) + B\,u_1(k-1) + C\,[u_1(k-2)]^2$$
$$y_2(k) = A\,u_2(k) + B\,u_2(k-1) + C\,[u_2(k-2)]^2$$

so

(1) $$\alpha y_1(k) + \beta y_2(k) = A\,[\alpha u_1(k) + \beta u_2(k)] + B\,[\alpha u_1(k-1) + \beta u_2(k-1)] + C\,[\alpha (u_1(k-2))^2 + \beta (u_2(k-2))^2]$$

different power

let $u_3 = \alpha u_1 + \beta u_2$.

so $$y_3(k) = A\,[\alpha u_1(k) + \beta u_2(k)] + B\,[\alpha u_1(k-1) + \beta u_2(k-1)] + C\,[(\alpha u_1(k-2) + \beta u_2(k-2))]^2$$

or (2) $$y_3(k) = A\,[\alpha u_1(k) + \beta u_2(k)] + B\,[\alpha u_1(k-1) + \beta u_2(k-1)] + C\,[\alpha^2 (u_1(k-2))^2 + \beta^2 (u_2(k-2))^2 + 2\alpha\beta\, u_1(k-2)\,u_2(k-2)]$$

extra term.

Compare (1) and (2) $\Rightarrow$ Not same.

$\Rightarrow$ Not linear.