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✥ Using unauthorized notes, materials or assistance during exams

✥ Using or copying the work of other students

✥ Submitting work that isn't your own

✥ Sharing answers to exam questions or class assignments

✥ Using the words or ideas of another without giving credit to the source; plagiarism

These and other forms of cheating not only dishonor our educational values but they also violate the trust that is crucial to intellectual and personal integrity. Consequently, cheating may result in severe disciplinary action including an "F" in the course and could lead to suspension from the University.

Let's make sure that grades accurately reflect what each student has actually learned.
Good luck on this examination!

EXAMINATION BOOK

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Subject Linear Systems

Class EE409 **Section** ______

Instructor Prof Grewal **Date** ______

① Solve $\left(1-\frac{S^{-2}}{9}\right) y(k) = \left(\frac{1}{3}\right)^{k}$ $\quad k \geq 0$

with IC = 0

find $y(k)$ and $h(k)$.

$y_h(k)$ is solution to $\left(1-\frac{S^{-2}}{9}\right) y(k) = 0$.

the char. equation is $r^2 - \frac{1}{9} = 0$

so roots are $\boxed{r = \pm \frac{1}{3}}$

hence $\boxed{y_h(k) = C_1 \left(\frac{1}{3}\right)^{k} + C_2 \left(-\frac{1}{3}\right)^{k}}$

$y_p(k)$ is solution due to forcing function $\left(\frac{1}{3}\right)^{k}$.
from table, solution is $y_p(k) = C_3 \left(\frac{1}{3}\right)^{k}$. but since this is double root, it becomes

$$\boxed{y_p(k) = C_3\, k \left(\frac{1}{3}\right)^{k}}$$

hence total solution is

$$y(k) = y_h(k) + y_p(k)$$

$$\boxed{y(k) = C_1 \left(\frac{1}{3}\right)^{k} + C_2 \left(-\frac{1}{3}\right)^{k} + C_3\, k \left(\frac{1}{3}\right)^{k}}$$

to find C_3, we plug $y_p(k)$ back into original difference equation

so $\left(1-\frac{S^{-2}}{9}\right) y_p(k) = \left(\frac{1}{3}\right)^{k}$

$$\left(1-\frac{S^{-2}}{9}\right) C_3\, k \left(\frac{1}{3}\right)^{k} = \left(\frac{1}{3}\right)^{k}$$

$$C_3\, k \left(\frac{1}{3}\right)^{k} - \frac{1}{9} C_3 (k-2) \left(\frac{1}{3}\right)^{k-2} = \left(\frac{1}{3}\right)^{k}$$ →

$$C_3 k \left(\tfrac{1}{3}\right)^k - \tfrac{1}{9} C_3 (k-2)\left(\tfrac{1}{3}\right)^{k-2} = \left(\tfrac{1}{3}\right)^k$$

$$C_3 k \left(\tfrac{1}{3}\right)^k - \tfrac{1}{9} C_3 \; k \left(\tfrac{1}{3}\right)^{k-2} + \tfrac{2}{9} C_3 \left(\tfrac{1}{3}\right)^{k-2} = \left(\tfrac{1}{3}\right)^k$$

$$C_3 k \left(\tfrac{1}{3}\right)^k - \tfrac{1}{9} C_3 k \left(\tfrac{1}{3}\right)^k \left(\tfrac{1}{3}\right)^{-2} + \tfrac{2}{9} C_3 \left(\tfrac{1}{3}\right)^k \left(\tfrac{1}{3}\right)^{-2} = \left(\tfrac{1}{3}\right)^k$$

Cancel $\left(\frac{1}{3}\right)^k$ since $\neq 0$ $\Rightarrow$

$$C_3 k - \tfrac{1}{9} C_3 k (9) + \tfrac{2}{9} C_3 \, 9 = 1$$

$$C_3 (k - k + 2) = 1$$

$$2 C_3 = 1$$

$$\boxed{C_3 = \tfrac{1}{2}}$$

so $y(k) = C_1 \left(\frac{1}{3}\right)^k + C_2 \left(-\frac{1}{3}\right)^k + \frac{1}{2} k \left(\frac{1}{3}\right)^k$

to find C_1, C_2 use initial conditions. i.e $\boxed{\begin{matrix} y(0) = 0 \\ y(1) = 0 \end{matrix}}$

asked instructor to verify meaning of IC.

$y(0) = 0$ so

$0 = C_1 + C_2$ — ①

and $y(1) = 0$ so

$0 = \frac{1}{3} C_1 - \frac{1}{3} C_2 + \frac{1}{2}\left(\frac{1}{3}\right) = \frac{1}{3} C_1 - \frac{1}{3} C_2 + \frac{1}{6}$ —

so 2 equation ①, ② to solve for C_1, C_2.

$0 = \frac{1}{3} C_1 + \frac{1}{3} C_2$ — ①

$0 = \frac{1}{3} C_1 - \frac{1}{3} C_2 + \frac{1}{6}$ — ②

add $\Rightarrow$ $0 = \frac{2}{3} C_1 + \frac{1}{6}$ so $\frac{2}{3} C_1 = -\frac{1}{6}$ $\Rightarrow$ $\boxed{C_1 = -\frac{3}{12}}$ =

so $C_2 = -C_1 = \frac{3}{12} = \frac{1}{4}$ $\Rightarrow$

hence $\boxed{y(k) = -\frac{1}{4}\left(\frac{1}{3}\right)^K + \frac{1}{4}\left(-\frac{1}{3}\right)^K + \frac{1}{2}k\left(\frac{1}{3}\right)^K}$ $K \geq 0$

zero otherwise

now to find $h(k)$.

let input be $\delta(k)$, hence

$$\left(1 - \frac{\delta^{-2}}{9}\right) h(k) = \delta(k) \qquad k \geq 0.$$

$$h(k) - \frac{1}{9}h(k-2) = \delta(k).$$

Solution for homogeneous part was already found to be

$$\boxed{h(k) = C_1\left(\frac{1}{3}\right)^k + C_2\left(-\frac{1}{3}\right)^k} \text{ ——— ①}$$

at $k=0$ difference equation becomes

$$h(0) - \frac{1}{9}\overset{=0}{h(-2)} = \delta(0) = 1$$

$$\boxed{h(0) = 1}$$

at $k=1$

$$h(1) - \frac{1}{9}\overset{=0}{h(-1)} = \delta(1) = 0$$

$$\boxed{h(1) = 0}$$

$h_k = LD\hat{h}_k$

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hence the equations to solve are (from ①)

$h(0) = 1 = C_1 + C_2$ (when $k=0$)

$h(1) = 0 = C_1\frac{1}{3} + C_2\left(-\frac{1}{3}\right)$

or

$$\left.\begin{array}{l} \frac{1}{3} = \frac{1}{3}C_1 + \frac{1}{3}C_2 \\ 0 = \frac{1}{3}C_1 - \frac{1}{3}C_2 \end{array}\right\} \Rightarrow \text{add} \Rightarrow \frac{1}{3} = \frac{2}{3}C_1 \Rightarrow \boxed{C_1 = \frac{1}{2}}$$

hence $C_2 = 1 - C_1 = 1 - \frac{1}{2} = \boxed{\frac{1}{2}}$

so $\boxed{\hat{h}(k) = \frac{1}{2}\left(\frac{1}{3}\right)^K + \frac{1}{2}\left(-\frac{1}{3}\right)^K}$ $K \geq 0$, zero otherwise

$\boxed{Q2}$

$$\dot{x}_1(t) = \frac{3}{4}x_1(t) + u_1(t)$$
$$\dot{x}_2(t) = \frac{1}{2}x_1(t) + \frac{1}{2}x_2(t) + u_2(t)$$
$$y(t) = x_1(t)$$

ⓐ Find A, B, C, D

ⓑ Find e^{At}

ⓒ Find matrix $(j\omega I - A)^{-1}$

ⓐ

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \underset{2\times 2}{\begin{pmatrix}\frac{3}{4} & 0\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}} \underset{2\times 1}{\begin{pmatrix}x_1\\ x_2\end{pmatrix}} + \underset{2\times 2}{\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}} \underset{2\times 1}{\begin{pmatrix}u_1\\ u_2\end{pmatrix}}$$

$$\underset{1\times 1}{y(t)} = \underset{1\times 2}{\begin{pmatrix}1 & 0\end{pmatrix}} \underset{2\times 1}{\begin{pmatrix}x_1\\ x_2\end{pmatrix}} + \underset{1\times 2}{\begin{pmatrix}0 & 0\end{pmatrix}} \underset{2\times 1}{\begin{pmatrix}u_1\\ u_2\end{pmatrix}}$$

so $A = \begin{pmatrix}\frac{3}{4} & 0\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}$ ✓

$$B = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$C = \begin{pmatrix}1 & 0\end{pmatrix}$$

$$D = \begin{pmatrix}0 & 0\end{pmatrix}$$

ⓑ to find e^{At}, first find eigenvalues of A

$$|A-\lambda I|=0$$

$$\begin{vmatrix} 3/4-\lambda & 0 \\ \frac{1}{2} & \frac{1}{2}-\lambda \end{vmatrix} = 0 \Rightarrow \left(\frac{3}{4}-\lambda\right)\left(\frac{1}{2}-\lambda\right) = 0$$

so $\boxed{\lambda_1 = \frac{3}{4} \ , \ \lambda_2 = \frac{1}{2}}$

so
$$\left.\begin{aligned} e^{\lambda_1 t} &= B_0 + B_1\lambda_1 \\ e^{\lambda_2 t} &= B_0 + B_1\lambda_2 \end{aligned}\right\} \Rightarrow \begin{aligned} e^{\frac{3}{4}t} &= B_0 + \frac{3}{4}B_1 \quad \text{—①} \\ e^{\frac{1}{2}t} &= B_0 + \frac{1}{2}B_1 \quad \text{—②} \end{aligned}$$

② − ① $\Rightarrow$

$$e^{\frac{1}{2}t} - e^{\frac{3}{4}t} = \frac{1}{2}B_1 - \frac{3}{4}B_1$$

$$e^{\frac{1}{2}t} - e^{\frac{3}{4}t} = -\frac{1}{4}B_1$$

so $\boxed{B_1 = 4\left(e^{\frac{3}{4}t} - e^{\frac{1}{2}t}\right)}$

so from ① we find B_0:

$$B_0 = e^{\frac{3}{4}t} - \frac{3}{4}\left(4\left(e^{\frac{3}{4}t} - e^{\frac{1}{2}t}\right)\right)$$

$$= e^{\frac{3}{4}t} - 3e^{\frac{3}{4}t} + 3e^{\frac{1}{2}t}$$

$$\boxed{B_0 = -2e^{\frac{3}{4}t} + 3e^{1/2 t}}$$

so $e^{At} = B_0 I + B_1 A$

$$= \left(-2e^{\frac{3}{4}t} + 3e^{\frac{t}{2}}\right)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + 4\left(e^{\frac{3t}{4}} - e^{\frac{t}{2}}\right)\begin{pmatrix} \frac{3}{4} & 0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$\longrightarrow$

$$e^{At} = \begin{pmatrix} -2e^{\frac{3}{4}t}+3e^{\frac{t}{2}} & 0 \\ 0 & -2e^{\frac{3}{4}t}+3e^{\frac{t}{2}} \end{pmatrix} + \begin{pmatrix} 3(e^{\frac{3}{4}t}-e^{\frac{t}{2}}) & 0 \\ 2(e^{\frac{3}{4}t}-e^{\frac{t}{2}}) & 2(e^{\frac{3}{4}t}-e^{\frac{1}{2}}) \end{pmatrix}$$

$$\boxed{e^{At} = \begin{pmatrix} e^{\frac{3}{4}t} & 0 \\ 2(e^{\frac{3}{4}t}-e^{\frac{t}{2}}) & e^{\frac{t}{2}} \end{pmatrix}}$$

✓

ⓒ

$$JωI - A = \begin{pmatrix} jω & 0 \\ 0 & jω \end{pmatrix} - \begin{pmatrix} \frac{3}{4} & 0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} jω-\frac{3}{4} & 0 \\ -\frac{1}{2} & jω-\frac{1}{2} \end{pmatrix}$$

$$|JωI-A| = \left(jω-\frac{3}{4}\right)\left(jω-\frac{1}{2}\right) = -ω^2-\frac{5}{4}jω+\frac{3}{8}$$

$$\boxed{|jωI-A| = \left(\frac{3}{8}-ω^2\right)+j\left(-\frac{5}{4}ω\right)}$$

✓

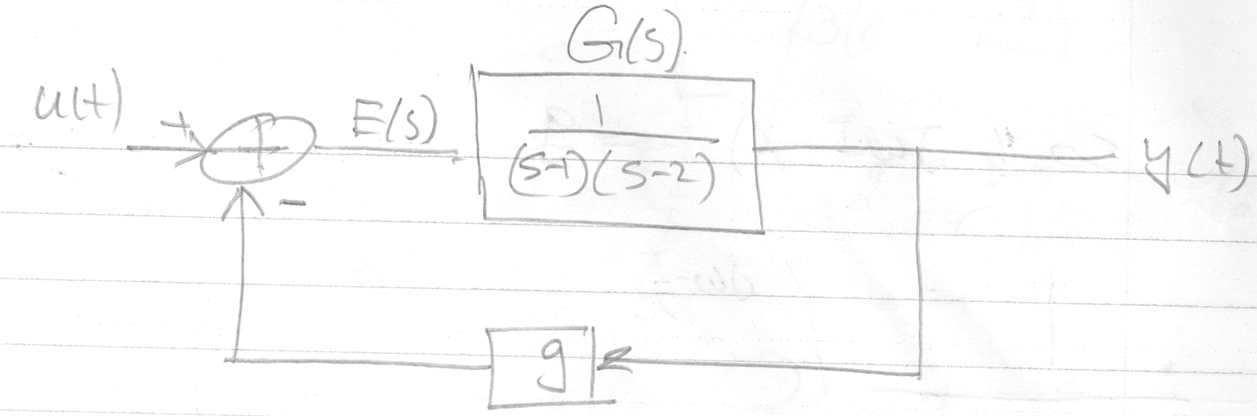
so $(J\omega I - A)^{-1} =$

$$\frac{1}{\left(\frac{3}{8}-\omega^2\right)+j\left(\frac{-5}{4}\omega\right)}\begin{pmatrix} j\omega-\frac{1}{2} & 0 \\ \frac{1}{2} & j\omega-\frac{3}{4}\end{pmatrix}$$

$\boxed{\text{or}}$ $$\frac{1}{\left(j\omega-\frac{3}{4}\right)\left(j\omega-\frac{1}{2}\right)}\begin{pmatrix} j\omega-\frac{1}{2} & 0 \\ \frac{1}{2} & j\omega-\frac{3}{4}\end{pmatrix}$$

$$= \boxed{\begin{pmatrix} \frac{1}{\left(j\omega-\frac{3}{4}\right)} & 0 \\ \frac{1/2}{\left(j\omega-\frac{3}{4}\right)\left(j\omega-\frac{1}{2}\right)} & \frac{1}{\left(j\omega-\frac{1}{2}\right)}\end{pmatrix}}$$

Q3



$$\boxed{\begin{aligned} E(s) &= u(s) - g\,y(s) \\ y(s) &= E(s)\,G(s) \end{aligned}}$$ —— ① —— ② where $G(s) = \frac{1}{(s-1)(s-2)}$

Sub ① into ② we obtain.

$$y(s) = [u(s) - g\,y(s)]\,G(s)$$

$$y(s) = u(s)\,G(s) - g\,y(s)\,G(s)$$

$$y(s)[1 + g\,G(s)] = u(s)\,G(s)$$

$$\boxed{\text{so } \frac{y(s)}{u(s)} = H(s) = \frac{G(s)}{1 + g\,G(s)}}$$ —— ③

for stability, we need to find the roots of the denominator of ③ and see for what values of g these roots are < 0.

Char. equation is $1 + g\,G(s) = 0$

$$1 + g\,\frac{1}{(s-1)(s-2)} = 0$$

or. $(S-1)(S-2) + g = 0$

hence $S^2 - 3S + (2+g) = 0$

hence $S = -\frac{b}{2} \pm \frac{1}{2}\sqrt{b^2-4ac} = \frac{3}{2} \pm \frac{1}{2}\sqrt{9-4(2+g)}$

or $S = \frac{3}{2} \pm \frac{1}{2}\sqrt{9-8-4g} = \frac{3}{2} \pm \frac{1}{2}\sqrt{1-4g}$

So
$$\boxed{\begin{aligned} S_1 &= \frac{3}{2} + \sqrt{\frac{1}{4}-g} \\ S_2 &= \frac{3}{2} - \sqrt{\frac{1}{4}-g} \end{aligned}}$$

both poles must be < 0 for stability. Consider each pole at a time.

$\underline{\underline{S_1}}$

this is NOT stable for any real g value.

If $\frac{1}{4}-g < 0$, then we have $Re(S_1) = \frac{3}{2} > 0$

and if $\frac{1}{4}-g > 0$, then we also have $Re(S_1) > \frac{3}{2}$

✓

So pole S_1 is always unstable!

So No value of g will make this system stable

Since both poles must be stable.

This can be also shown using Routh table;

problem!! ⟹ (at S^1 row)

S^2	1	$2+g$
S^1	-3	0
S^0	$-3(2+g)$	

↑ this column must all be > 0

Assume g is real. not possible to find such value.

⟹ we see that the first column can't be made all > 0 due to presence of "-3" term.