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Particular Solution guess

$F(x)$	y_p Guess
Ke^{bx}	Ae^{bx}
Kx^n	$A_n x^n + A_{n-1}x^{n-1} + \dots + A_0$
$\cos\omega x$	$A\cos\omega x + B\sin\omega x$
$Ke^{\alpha x}\cos\omega x$	$e^{\alpha x}(A\cos\omega x + B\sin\omega x)$

roots of char eq.	y_h
Distinct & Real	$Ae^{\lambda_1 x} + Be^{\lambda_2 x}$
double real	$Ae^{\lambda x} + Bxe^{\lambda_1 x}$
complex $\alpha \pm j\beta$	$e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

stiffness series

response spectrum: $\frac{|x(t)|_{max}}{\omega_n}$

$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$

$K = K_1 + K_2$

relations $\omega_n = \sqrt{\frac{K}{m}}$, $\xi = \frac{c}{c_r}$

$c_r = 2\sqrt{Km} = 2\omega_n m$

$\omega_d = \omega_n\sqrt{1-\xi^2}$ $\xi < 1$

$r = \frac{\omega}{\omega_n}$

Solutions $m\ddot{x} + Kx = 0$

$x = A\cos\omega_n t + B\sin\omega_n t$

$x = C\sin(\omega_n t + \phi)$

$C = \sqrt{A^2+B^2}$, $\phi = \tan^{-1}\frac{A}{B}$.

or $x = x_0\cos\omega_n t + \frac{v_0}{\omega_n}\sin\omega_n t$

$\lambda_{1,2} = -\frac{c}{2m} \pm \frac{1}{2m}\sqrt{c^2 - 4Km}$

$\lambda_{1,2}$ are distinct negative number if $c^2 - 4Km > 0$

$\lambda_{1,2}$ are complex if $c^2 - 4Km < 0$

$\lambda_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$

$\xi < 1$ complex $-\xi\omega_n \pm j\omega_d$

$\xi > 1$ overdamped $x(t) = e^{-\xi\omega_n t}\left(Ae^{-\omega_n\sqrt{\xi^2-1}\,t} + Be^{\dots}\right)$

$\xi = 1$ $x(t) = Ae^{-\xi\omega_n t} + Bte^{-\xi\omega_n t}$

$x(t) = Ae^{-\omega_n t} + Bte^{-\omega_n t}$

$m\ddot{x} + Kx = F(t) = F_0\sin\omega t$

$x(t) = A\cos\omega_n t + B\sin\omega_n t + \frac{F_0}{K}\frac{1}{1-r^2}\sin\omega t$

$A = x_0$, $B = \frac{v_0}{\omega_n} - \frac{F_0}{K}\frac{r}{1-r^2}$

$m\ddot{x} + c\dot{x} + Kx = 0 \Rightarrow \ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = 0$

$x(t) = e^{-\xi\omega_n t}(A\cos\omega_d t + B\sin\omega_d t)$ $\xi < 1$

→ complex conjugate roots

$\theta = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right)$

$m\ddot{x} + c\dot{x} + Kx = F_0\sin\omega t$

Steady State

$x(t) = e^{-\xi\omega_n t}(A\cos\omega_d t + B\sin\omega_d t) + \underbrace{\frac{F_0}{K}\frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}}_{R_d}\sin(\omega t - \theta)$ x_{ss}

convolution $x(t) = \int_0^t f(\tau)\,h(t-\tau)\,d\tau$

For small θ,

$r\theta = x$

$r\dot{\theta} = \dot{x}$

$T = \frac{1}{2}mv^2 + \frac{1}{2}J\dot{\theta}^2$

$U = \frac{1}{2}Kx^2 + mgL(1-\cos\theta)$

$\frac{d}{dt}\cos t = -\sin t$.

$\int\cos 3t = \frac{1}{3}\sin 3t$

$m\ddot{x} + c(\dot{x} - \dot{x}_g) + K(x - x_g) = 0$

$m\ddot{x} + c\dot{x} + Kx = c\dot{x}_g + Kx_g$ ← 2 particular Solutions

$x_g = u_g\sin\omega t$

$= c\,u_g\omega\cos\omega t + K u_g\sin\omega t$

$= F_0\sin(\omega t + \beta)$, $F_0 = u_g K\sqrt{1 - (2\xi r)^2}$

$\tan^{-1}\beta = 2\xi r$

$f(t)$ ← Forcing function

$h_u(t)$ ← impulse response undamped

$h_d(t)$ → impulse response damped

So $x(t) = \int_0^t f(t-\tau)\,h(\tau)\,d\tau$ or $x(t) = \int_0^t f(\tau)\,h(t-\tau)\,d\tau$

$h_d(t) = \frac{\hat{F}}{m\omega_d}e^{-\xi\omega_n t}\sin\omega_d t$, $h_u(t) = \frac{\hat{F}}{m\omega_n}\sin\omega_n t$

if impulse applied at t_1 and not at 0, then

$x(t) = \frac{\hat{F}}{m\omega_d}e^{-\xi\omega_n(t-t_1)}\sin\omega_d(t-t_1)\,\Phi(t-t_1)$

For Non conservative systems

$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{q}}\right) - \frac{\partial L}{\partial q} + \frac{\partial R_i}{\partial\dot{q}} = 0$

R_i = Rayleigh $= \frac{1}{2}c\dot{q}_i^2$

$\mathcal{L}(x'' + x' + x) = \dots$

$[s^2X(s) - sx(0) - x'(0)] + [sX(s) - x(0)] + X(s) = 0$

$\mathcal{L}[f(t)] = \int_0^\infty e^{-st}f(t)\,dt$

$\int x\cos ax = \frac{\cos ax}{a^2} + \frac{x\sin ax}{a}$

$\int\cos^2 ax = \frac{x}{2} + \frac{\sin 2ax}{4a}$

$\int\sin ax\cos ax = \frac{\sin^2 ax}{2a}$

$\int\sin px\cos qx = -\frac{\cos(p-q)x}{2(p-q)} - \frac{\cos(p+q)x}{2(p+q)}$

Lagrangian $L = T - U$

EQM $\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}} - \frac{\partial L}{\partial\theta} = Q_i$ generalized Force

Fourier Series of Function $g(t)$

$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos n\omega t + b_n\sin n\omega t$

$a_0 = \frac{1}{T/2}\int_0^T g(t)\,dt$

$a_n = \frac{1}{T/2}\int_0^T g(t)\cos n\omega t\,dt$

$b_n = \frac{1}{T/2}\int_0^T g(t)\sin n\omega t\,dt$

Where $\omega = 2\pi f = \frac{2\pi}{T}$

a_n a_t r θ m $\dot{\theta}$

$a_n = r\dot{\theta}^2$

$a_t = r\ddot{\theta}$

Force towards centre $= mr\dot{\theta}^2\sin\theta(t)$

Moment of Inertia

$I = mr^2$ thin cylinder

$I = \frac{mr^2}{2}$ solid.

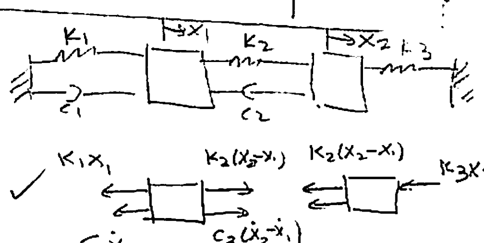
Solid disk $I = \frac{mr^2}{2}$

$I = mr^2$ Thin loop

Solid sphere $I = \frac{2mr^2}{5}$

$I = \frac{mL^2}{12}$

$J = \frac{mL^2}{3}$



$\sin 2A = 2\sin A\cos A$.

$\cos 2A = 2\cos^2 A - 1$

$\sin A\sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$

$\cos A\cos B = \frac{1}{2}(\cos(A-B) + \cos(A+B))$

$\sin A\cos B = \frac{1}{2}(\sin(A-B) + \sin(A+B))$

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

$\sin(A + 90) = \cos A$

$\cos(A + 90) = -\sin A$

To obtain generalized Forces use Virtual Work. $\delta W = F\,\delta q_i$ generalized Force

make virtual displacement to each generalized coordinate. For each external Force, find work done by it, make expression $\delta W = \bigcirc\,\delta\theta$, then this is Q_i

For example L θ F then $\delta W = F.\,\delta\theta L\cos\theta \Rightarrow \boxed{Q = FL\cos\theta}$

Kinematic equations

$d = v_i t + \frac{1}{2}at^2$

$v_f^2 = v_i^2 + 2ad$

$v_f = v_i + at$

$d = \frac{v_i + v_f}{2}t$

$t = \frac{2d}{v_i + v_f}$