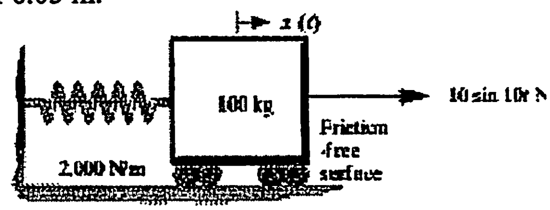
EGME 431 Handed 3/16/09 HW 2 SOLUTION

2.7 Consider the system in Figure P2.7, write the equation of motion and calculate the response assuming a) that the system is initially at rest, and b) that the system has an initial displacement of 0.05 m.



Solution: The equation of motion is

$$m\ddot{x} + kx = 10\sin 10t$$

Let us first determine the general solution for

$$\ddot{x} + \omega_n^2 x = f_0 \sin \omega t$$

Replacing the cosine function with a sine function in Eq. (2.4) and following the same argument, the general solution is:

$$x(t) = A_1 \sin \omega_n t + A_2 \cos \omega_n t + \frac{f_0}{\omega_n^2 - \omega^2} \sin \omega t$$

Using the initial conditions, $x(0) = x_0$ and $\dot{x}(0) = v_0$, a general expression for the response of a spring-mass system to a harmonic (sine) excitation is:

$$x(t) = \left(\frac{v_0}{\omega_n} - \frac{\omega}{\omega_n} \cdot \frac{f_0}{\omega_n^2 - \omega^2}\right) \sin \omega_n t + x_0 \cos \omega_n t + \frac{f_0}{\omega_n^2 - \omega^2} \sin \omega t$$

Given: k=2000 N/m, m=100 kg, ω=10 rad/s,

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{2000}{100}} = \sqrt{20} \text{ rad/s} = 4.472 \text{ rad/s} \qquad f_0 = \frac{F_0}{m} = \frac{10}{100} = 0.1 \text{ N/kg}$$

a) $x_0 = 0$ m, $v_0 = 0$ m/s

Using the general expression obtained above:

$$x(t) = \left(0 - \frac{10}{\sqrt{20}} \cdot \frac{0.1}{\sqrt{20}^2 - 10^2}\right) \sin \sqrt{20} t + 0 + \frac{0.1}{\sqrt{20}^2 - 10^2} \sin 10t$$

$$= 2.795 \times 10^{-3} \sin 4.472t - 1.25 \times 10^{-3} \sin 10t$$

b) $x_0 = 0.05$ m, $v_0 = 0$ m/s

$$x(t) = \left(0 - \frac{10}{\sqrt{20}} \cdot \frac{0.1}{\sqrt{20}^2 - 10^2}\right) \sin \sqrt{20} t + 0.05 \cos \sqrt{20} t + \frac{0.1}{\sqrt{20}^2 - 10^2} \sin 10t$$

$$= 0.002795 \sin 4.472t + 0.05 \cos 4.472t - 0.00125 \sin 10t$$

$$= 5.01 \times 10^{-2} \sin(4.472t + 1.515) - 1.25 \times 10^{-3} \sin 10t$$

2.10 Compute the initial conditions such that the response of:

$$m\ddot{x} + kx = F_0 \cos\omega t$$

oscillates at only one frequency (ω).

Solution: From Eq. (2.11):

$$x(t) = \frac{v_0}{\omega_n}\sin \omega_n t + (x_0 - \frac{f_0}{\omega_n^2 - \omega^2})\cos \omega_n t + \frac{f_0}{\omega_n^2 - \omega^2}\cos \omega t$$

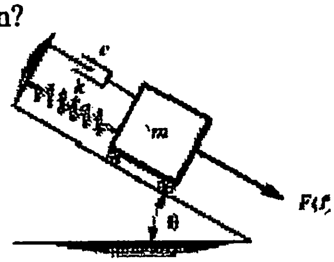
For the response of $m\ddot{x} + kx = F_0 \cos\omega t$ to have only one frequency content, namely, of the frequency of the forcing function, ω, the coefficients of the first two terms are set equal to zero. This yields that the initial conditions have to be

$$x_0 = \frac{f_0}{\omega_n^2 - \omega^2} \quad \text{and} \quad v_0 = 0$$

Then the solution becomes

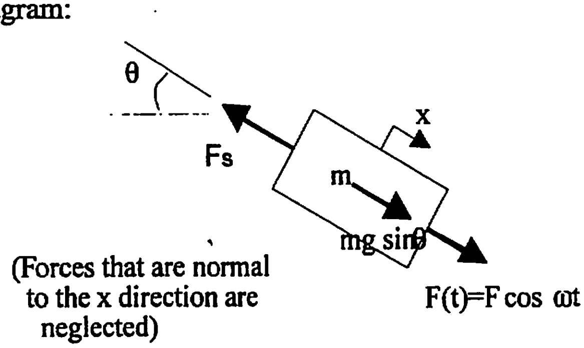
$$x(t) = \frac{f_0}{\omega_n^2 - \omega^2}\cos \omega t$$

2.29 Write the equation of motion for the system given in Figure P2.29 for the case that $F(t) = F\cos\omega t$ and the surface is friction free. Does the angle θ effect the magnitude of oscillation?



Solution:

Free body diagram:



Assuming x = 0 to be at the equilibrium:

$$\sum F_x = F + mg\sin\theta - F_s = m\ddot{x}$$

where $F_s = k(x + \frac{mg\sin\theta}{k})$ and $F(t) = F\cos\omega t$

Then the equation of motion is:

$$m\ddot{x} + kx = F\cos\omega t$$

Note that the equation of motion does not contain θ which means that the magnitude of the response is not affected by the angle of the incline.

2.46 Consider Example 2.4.1 for car 1 illustrated in Figure P2.46, if three passengers totaling 200 kg are riding in the car. Calculate the effect of the mass of the passengers on the deflection at 20, 80, 100, and 150 km/h. What is the effect of the added passenger mass on car 2?

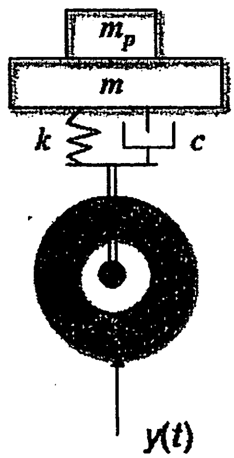


Figure P2.46 Model of a car suspension with the mass of the occupants, m_p, included.

Solution:

Add a mass of 200 kg to each car. From Example 2.4.1, the given values are: $m_1 = 1207$ kg, $m_2 = 1785$ kg, $k = 4\text{x}10^4$ N/m; $c = 2{,}000$ kg/s, $\omega_b = 0.29v$.

$$\text{Car 1: } \omega_1 = \sqrt{\frac{k}{m}} = \sqrt{\frac{4\times10^4}{1207}} = 5.76 \text{ rad/s}$$

$$\zeta_1 = \frac{c}{2\sqrt{km_1}} = \frac{2000}{2\sqrt{(4\times10^5)(1207)}} = 0.144$$

$$\text{Car 2: } \omega_2 = \sqrt{\frac{k}{m}} = \sqrt{\frac{4\times10^4}{1785}} = 4.73 \text{ rad/s}$$

$$\zeta_2 = \frac{c}{2\sqrt{km_2}} = \frac{2000}{2\sqrt{(4\times10^5)(1785)}} = 0.118$$

Using Equation (2.71): $X = Y\left[\dfrac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2}$ produces the following:

Speed (km/h)	ω_b (rad/s)	r_1	r_2	x_1 (cm)	x_2 (cm)
20	5.817	1.01	1.23	3.57	1.77
80	23.271	3.871	4.71	0.107	0.070
100	29.088	5.05	6.15	0.072	0.048
150	2.40	7.58	9.23	0.042	0.028

At lower speeds there is little effect from the passengers weight, but at higher speeds the added weight reduces the amplitude, particularly in the smaller car.

wrong
43.635

2.57 Consider a typical unbalanced machine problem as given in Figure P2.57 with a machine mass of 120 kg, a mount stiffness of 800 kN/m and a damping value of 500 kg/s. The out of balance force is measured to be 374 N at a running speed of 3000 rev/min. a) Determine the amplitude of motion due to the out of balance. b) If the out of balance mass is estimated to be 1% of the total mass, estimate the value of the *e*.

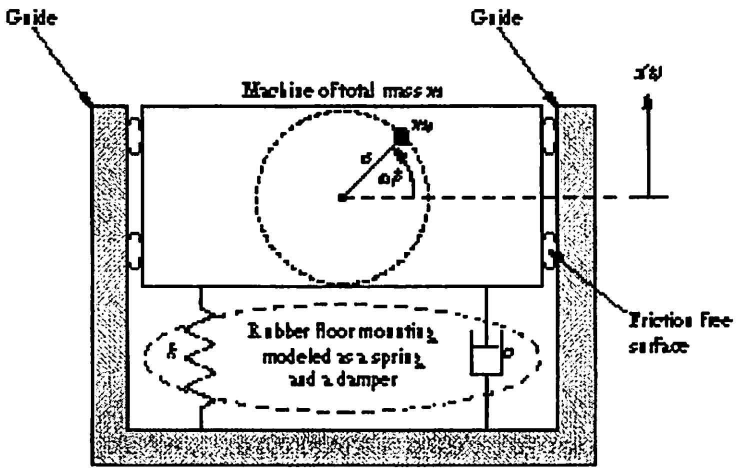


Figure P2.57 Typical unbalance machine problem.

Solution:

a) Using equation (2.84) with $m_0e = F_0/\omega_r^2$ yields:

$k := 800 \cdot 1000$ $\quad m := 120$ $\quad c := 500$ $\quad F0 := 374$

$\omega r := 100 \cdot \pi$ $\quad k = 8 \cdot 10^5$

$$\omega n := \sqrt{\frac{k}{m}} \qquad \zeta := \frac{c}{2 \cdot \sqrt{k \cdot m}}$$

$$r := \frac{\omega r}{\omega n} \qquad \omega n = 81.65 \qquad r = 3.848 \qquad \zeta = 0.026$$

$$X := \frac{F0}{\omega r^2 \cdot m} \cdot \frac{r^2}{\sqrt{\left(1 - r^2\right)^2 + (2 \cdot \zeta \cdot r)^2}} \qquad X = 3.386 \cdot 10^{-5}$$

b) Use the fact that $F_0 = m_0 e \omega_r^2$ to get

$$e := \frac{F0}{\omega r^2 \cdot (0.01 \cdot m)} \qquad e = 3.158 \cdot 10^{-3}$$

in meters.