## A small note on using the left eigenvector of the one step transition matrix for a regular chain to determine the limiting distribution

In trying to solve problem 8.1, lecture notes, Math 504, I found that I need to determine ' w', which is the limiting distribution of the given markov chain. One way to determine the limiting distribution from P is by finding the normalized left eigenvector associated with eigenvalue 1 of the P matrix. This below shows how to do this for the P matrix given in the problem 8.1 and verify that it is the same as a row in the $P^N$ matrix where N is a large value

### First Setup the P matrix as shown in problem 8.1

```
pMat = {{p, q, 0}, {p, 0, q}, {0, p, q}};
pMat // MatrixForm
```

$$\begin{pmatrix} p & q & 0 \\ p & 0 & q \\ 0 & p & q \end{pmatrix}$$

### Find the eigenvalues and associated with the left eigenvectors. First transpose the matrix

```
(t = Transpose[pMat]) // MatrixForm
```

$$\begin{pmatrix} p & p & 0 \\ q & 0 & p \\ 0 & q & q \end{pmatrix}$$

### Now find the eigenvalues and vectors of the transposed matrix

```
({values, vec} = Eigensystem[t]);
```

### The eigenvalues are

```
values // MatrixForm
```

$$\begin{pmatrix} -\sqrt{p}\ \sqrt{q} \\ \sqrt{p}\ \sqrt{q} \\ p+q \end{pmatrix}$$

### And the corresponding eigenvectors

```
vec // MatrixForm
```

$$\begin{pmatrix} \frac{\sqrt{p}}{\sqrt{q}} & -\frac{\sqrt{p}+\sqrt{q}}{\sqrt{q}} & 1 \\ -\frac{\sqrt{p}}{\sqrt{q}} & -\frac{-\sqrt{p}+\sqrt{q}}{\sqrt{q}} & 1 \\ \frac{p^2}{q^2} & \frac{p}{q} & 1 \end{pmatrix}$$

### Now pick the eigenvector associated with eigenvalue 1. We see from the above that it is the 3 rd eigenvector (since p+q=1)

```
v = vec[[3]]
```

$$\left\{\frac{p^2}{q^2}, \frac{p}{q}, 1\right\}$$

### Normalize it. This is $\pi$ (the limiting distribution)

```
w = v / Length[v]
```

$$\left\{\frac{p^2}{3\,q^2}, \frac{p}{3\,q}, \frac{1}{3}\right\}$$

**Verify by comparing to row entry of $P^\infty$ Try some p and q values, say p = .5 and q = .5**

```
w /. {p → .5, q → .5}
```

$$\left\{0.3333333333, 0.3333333333, \frac{1}{3}\right\}$$

```
MatrixPower[pMat /. {p → .5, q → .5}, 100]
```

```
{{0.3333333333, 0.3333333333, 0.3333333333},
 {0.3333333333, 0.3333333333, 0.3333333333}, {0.3333333333, 0.3333333333, 0.3333333333}}
```

## References


1. Lecture notes, Mathematics 504. Professor B. Gearhart
2. Wikepedia http://en.wikipedia.org/wiki/Markov_chain