I) Amplitude Modulation

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a) • AM wave $s_{AM}(t) = A_c\,[1 + K_a\, m(t)]\cos w_c t$

• modulation index $\mu = \dfrac{A_{max} - A_{min}}{A_{max} + A_{min}}$, where A_{max} is the max. of envelope

b) DSB-SC $s(t) = A_c\, m(t) \cos w_c t$

c) SSB $s(t) = \dfrac{A_c}{2} m(t) \cos w_c t \mp \dfrac{A_c}{2} \hat{m}(t) \sin w_c t$

where (−) for USB and (+) for LSB

$\hat{m}(t) = H \cdot T\,[m(t)] = m(t) \circledast \dfrac{1}{\pi t}$ or

$\hat{M}(f) = -j\,\mathrm{Sgn}(f)\, M(f)$

II) PM wave;

$s(t) = A_c \cos(w_c t + K_p\, m(t))$

III) FM wave:

• $s(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t m(x)\,dx\right]$ (1)

• If $m(t)$ is a sine or cosine wave for example

if $m(t) = A_m \cos w_m t$ then eq (1) becomes single tone modulated signal:

• $s(t) = A_c \cos[2\pi f_c t + \beta \sin w_m t]$, where

• $\beta = \dfrac{\Delta f}{f_m} = \dfrac{K_f \cdot A_m}{f_m}$, β is modulation index

• $\Delta f = K_f A_m$ is the freq. deviation

• $f_i(t) = f_c + K_f\, m(t)$ inst. freq.

• $\theta_i(t) = 2\pi \int_0^t f_i(t_1)\, dt_1$ or $f_i(t) = \dfrac{1}{2\pi} \dfrac{d\theta_i(t)}{dt}$

$\theta_i(t)$ in the inst. phase.

IV) Narrow band noise $n(t)$: Note if $E\{n(t)\} = 0 \Rightarrow E\{n_I\} = E\{n_Q\} = 0$

• $n(t) = n_I(t) \cos w_c t - n_Q(t) \sin w_c t$

• $S_{N_I}(f) = S_{N_Q}(f) = [S_N(f - f_c) + S_N(f + f_c)]\, \mathrm{rect}\left(\dfrac{f}{2B}\right)$

where these are P.S.D of the narrowband noise and its in-phase and quadrature components

• The envelope of $n(t)$ is $a(t) = \sqrt{n_I^2 + n_Q^2}$

Table A11.1 Summary of Properties of the Fourier Transform

Property	Mathematical Description
1. Linearity	$ag_1(t) + bg_2(t) \rightleftharpoons aG_1(f) + bG_2(f)$ where a and b are constants
2. Time scaling	$g(at) \rightleftharpoons \frac{1}{\lvert a \rvert} G\left(\frac{f}{a}\right)$ where a is a constant
3. Duality	If $g(t) \rightleftharpoons G(f)$, then $G(t) \rightleftharpoons g(-f)$
4. Time shifting	$g(t - t_0) \rightleftharpoons G(f)\exp(-j2\pi f t_0)$
5. Frequency shifting	$\exp(j2\pi f_c t) g(t) \rightleftharpoons G(f - f_c)$
6. Area under $g(t)$	$\int_{-\infty}^{\infty} g(t)\, dt = G(0)$
7. Area under $G(f)$	$g(0) = \int_{-\infty}^{\infty} G(f)\, df$
8. Differentiation in the time domain	$\frac{d}{dt} g(t) \rightleftharpoons j2\pi f G(f)$
9. Integration in the time domain	$\int_{-\infty}^{t} g(\tau)\, d\tau \rightleftharpoons \frac{1}{j2\pi f} G(f) + \frac{G(0)}{2} \delta(f)$
10. Conjugate functions	If $g(t) \rightleftharpoons G(f)$, then $g^*(t) \rightleftharpoons G^*(-f)$
11. Multiplication in the time domain	$g_1(t) g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda) G_2(f - \lambda)\, d\lambda$
12. Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau) g_2(t - \tau)\, d\tau \rightleftharpoons G_1(f) G_2(f)$

Table A11.4 Trigonometric Identities

$$\exp(\pm j\theta) = \cos\theta \pm j\sin\theta$$
$$\cos\theta = \tfrac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$$
$$\sin\theta = \frac{1}{2j}[\exp(j\theta) - \exp(-j\theta)]$$
$$\sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta - \sin^2\theta = \cos(2\theta)$$
$$\cos^2\theta = \tfrac{1}{2}[1 + \cos(2\theta)]$$
$$\sin^2\theta = \tfrac{1}{2}[1 - \cos(2\theta)]$$
$$2\sin\theta\cos\theta = \sin(2\theta)$$
$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$
$$\sin\alpha\sin\beta = \tfrac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$