EE 443 Hw #8 chapt (3) page 1

Drill Prob. # 3.4) $V_2(t) = a_1 V_1(t) + a_2 V_1^2(t)$ (1) , $V_1(t) = A_c \cos(2\pi f_c t) + m(t)$ (2)

$$V_2(t) = a_1 A_c \cos 2\pi f_c t + a_1 m(t) + a_2 A_c^2 \cos^2 2\pi f_c t + a_2 m^2(t) + 2 a_2 A_c m(t) \cos 2\pi f_c t \quad (3)$$

Assume $M(f)$ has the following form:

fig #.

Set $g(t) \triangleq m^2(t) = m(t) \cdot m(t)$

$\Rightarrow G(f) = M(f) \circledast M(f)$ The spectrum of $g(t) = m^2(t)$ will extend from $-2w$ to $2w$ HZ, for example would be

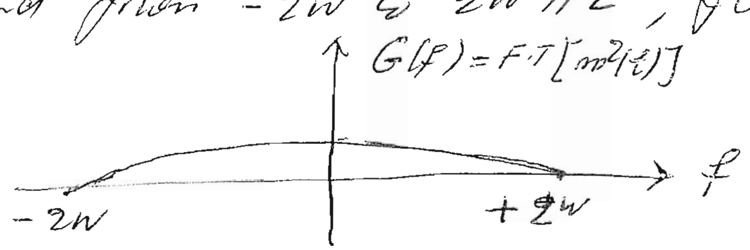


fig #2

Note: If you want to ~~g(t)~~ find $G(f)$, then you have to do $G(f) = \int_{-\infty}^{+\infty} M(\lambda) M(f-\lambda) d\lambda$. We are not interested to find the exact equation of $G(f)$, all we need to know is that the spectrum of $G(f) = F.T[m^2(t)]$ will extend from $-2W$ to $2W$ Hz.

Let us to take the F.T of eq (3) and plot it!

$$V_2(t) = a_1 A_c \cos 2\pi f_c t + a_1 m(t) + \frac{a_2 A_c^2}{2}\left[1 + \cos 4\pi f_c t\right] + a_2 m^2(t) + 2 a_2 A_c m(t) \cos 2\pi f_c t \quad (4)$$

$$\Rightarrow V_2(f) = \frac{a_1 A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + a_1 M(f) + \frac{a_2 A_c^2}{2}\delta(f) + \frac{a_2 A_c^2}{4}\left[\delta(f-2f_c) + \delta(f+2f_c)\right] + a_2 \underbrace{F.T\left[m^2(t)\right]}_{G(f)} + a_2 A_c\left[M(f-f_c) + M(f+f_c)\right] \quad (5)$$

The plot of eq.(5) is shown in figure # 3.

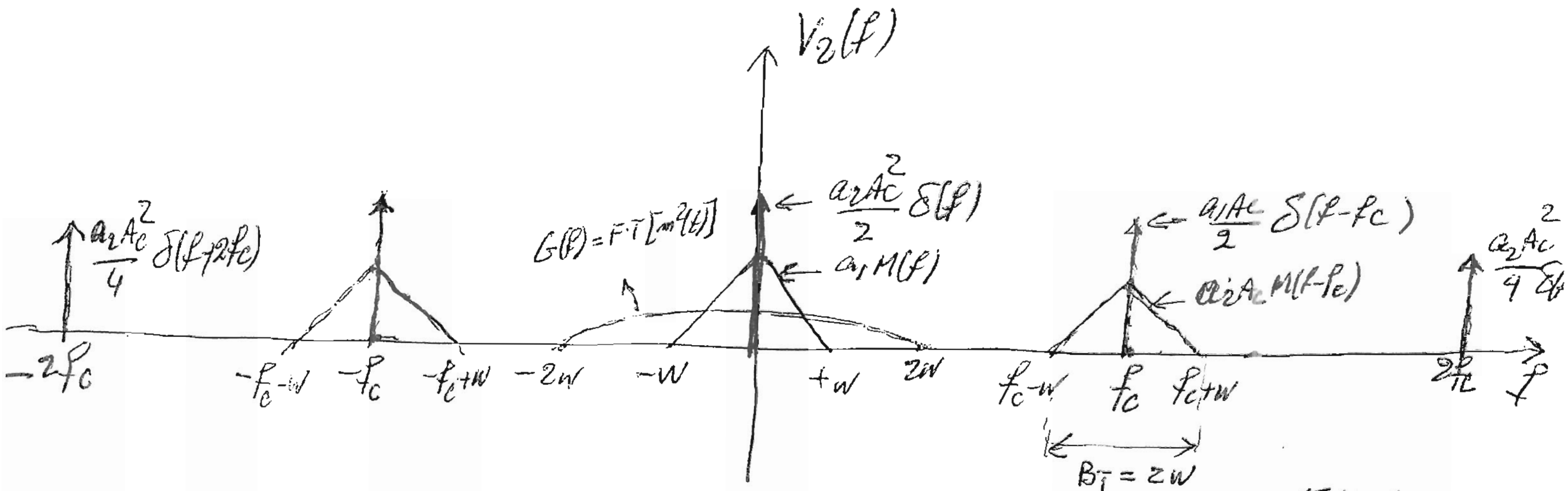


Fig # 3

Fig # 3 shows the spectral content of $v_2(t)$.

-) To extract the desired AM signal, use eq (4) and identify the AM signal:

$$v_2(t) = \underbrace{a_1 A_c\left[1+\frac{2a_2}{a_1}m(t)\right]\cos 2\pi f_c t}_{\text{Desired AM Signal}} + \underbrace{\left(a_1 m(t) + a_2 m^2(t) + \frac{a_2 A_c^2}{2} + \frac{a_2 A_c^2}{2}\cos 4\pi f_c t\right)}_{\text{undesired Component}} \quad (6)$$

A Bandpass filter centered at f_c with total extend of $2W$ Hz, That is having a transfer function of:

$$H(f) = \text{rect}\left(\frac{f-f_c}{2W}\right) + \text{rec}\left(\frac{f+f_c}{2W}\right) \qquad (7)$$

will pass the desired signal (AM signal) and eliminated the unwanted components.

Using eq. (7) and figure #(3) we see that the required B.P.F must have a bandwidth of 2W Hz and centered at f_c, thus the cut-off frequencies of BPF are $f_c - W$ and $f_c + W$ Hz.

c) To avoide spectral overlapping of the desired signal (AM signal) with that of unwanted signals in $v_2(t)$, using figure # 3, we see that

1) $f_c - W \geq 2W \Rightarrow f_c \geq 3W$
2) $f_c + W \leq 2f_c \Rightarrow f_c \geq W$

$\Bigg\}$ Thus $f_c > 3W$

3.23

Assume $m(t)$ with spectrum of

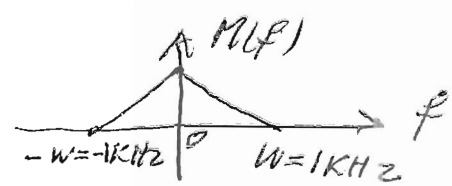


fig #1

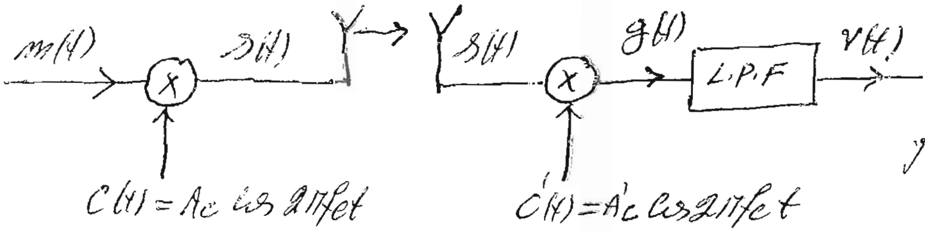


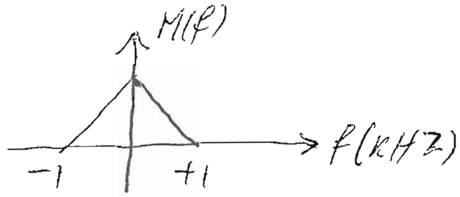
fig #(2) Coherent detection of DSB-SC.

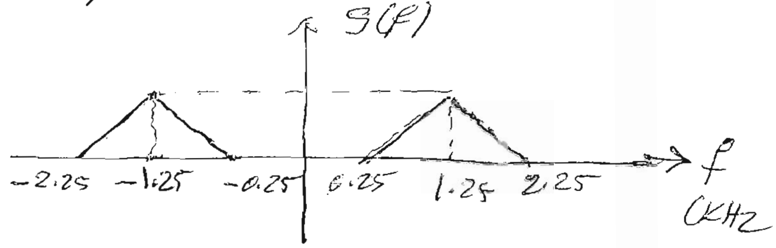
$$s(t) = m(t)\,c(t) = A_c\, m(t) \cos 2\pi f_c t \Rightarrow S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] \quad (1)$$

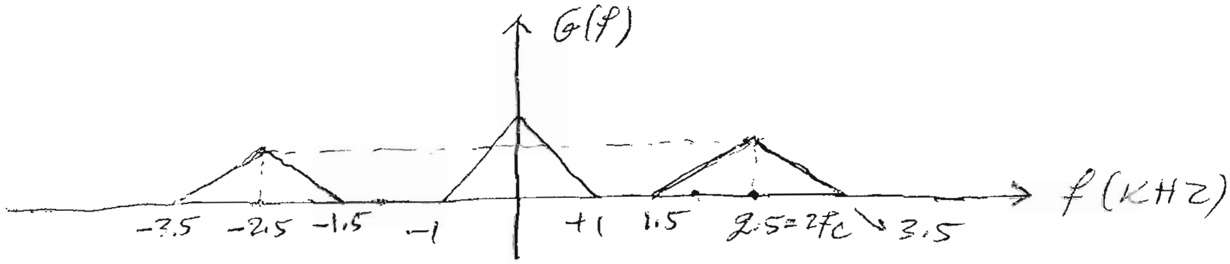
$$g(t) = s(t)\cdot c'(t) = A_c A'_c\, m(t) \cos^2 2\pi f_c t = \frac{A_c A'_c}{2} m(t)\left[1 + \cos 4\pi f_c t\right] \quad (2)$$

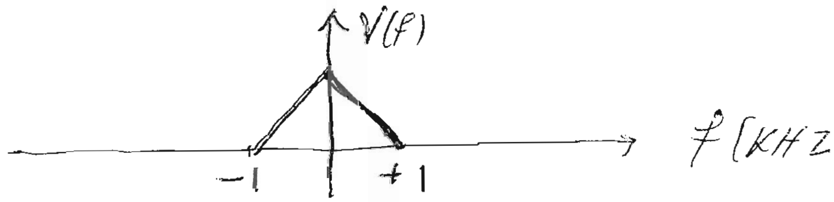
$$G(f) = \frac{A_c A'_c}{2} M(f) + \frac{A_c A'_c}{4}\left[M(f-2f_c) + M(f+2f_c)\right] \quad (3)$$

a) For $f_c = 1.25$ KHz, the spectrum of $m(t)$, the spectrum of $s(t)$ and the spectrum of $v(t)$ (detector output) are:









b) For $f_c = 0.75$ KHZ, the respective spectra are :