HW 8
Electronic Communication Systems
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California State University, Fullerson

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Contents

1 Questions

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Drill 3.4)

$$v_2 = a_1 v_1(t) + a_2 v_1^2(t) \quad (1)$$

where, $v_1(t) = A_c \cos 2\pi f_c t + m(t) \quad (2)$

Subst. eq. (2) into eq (1)

$$v_2(t) = a_1\left[A_c \cos 2\pi f_c t + m(t)\right] + a_2\left[A_c \cos 2\pi f_c t + m(t)\right]^2$$

$$\Rightarrow v_2(t) = \underbrace{a_1 A_c\left[1 + \frac{2a_2}{a_1} m(t)\right]\cos 2\pi f_c t}_{\text{AM Signal}} + a_1 m(t) +$$

$$+ a_2 m^2(t) + a_2 A_c^2 \underbrace{\cos^2(2\pi f_c t)}_{\frac{1}{2}[1+\cos 4\pi f_c t]}$$

The signal at the output of bandpass filter is:

$$v_0(t) = a_1 A_M\left[1 + \frac{2a_2}{a_1} m(t)\right]\cos 2\pi f_c t$$

which is an AM wave.

2 Key solution

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Drill Prob. # 3.4) $v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$ (1) , $v_1(t) = A_c \cos(2\pi f_c t) + m(t)$ (2)

$$v_2(t) = a_1 A_c \cos 2\pi f_c t + a_1 m(t) + a_2 A_c^2 \cos^2 2\pi f_c t + a_2 m^2(t) + 2 a_2 A_c\, m(t) \cos 2\pi f_c t \quad (3)$$

Assume $M(f)$ has the following form:

fig#.

Set $g(t) \triangleq m^2(t) = m(t) \cdot m(t)$

$\Rightarrow G(f) = M(f) \otimes M(f)$ The spectrum of $g(t) = m^2(t)$ will extend from $-2w$ to $2w$ Hz, for example would be

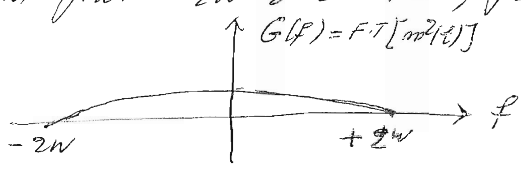

fig #2

Note: If you want to ~~g(t)~~ find $G(f)$, then you have to do $G(f) = \int_{-\infty}^{+\infty} M(\lambda) M(f-\lambda)\, d\lambda$. We are not interested to find the exact equation of $G(f)$, all we need to know is that the spectrum of $G(f) = F.T[m^2(t)]$ will extend from $-2W$ to $2W$ Hz.

Let us to take the F.T of eq (3) and plot it!

$$v_2(t) = a_1 A_c \cos 2\pi f_c t + a_1 m(t) + \frac{a_2 A_c^2}{2}\left[1 + \cos 4\pi f_c t\right] + a_2 m^2(t) + 2 a_2 A_c\, m(t) \cos 2\pi f_c t \quad (4)$$

$$\Rightarrow V_2(f) = \frac{a_1 A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + a_1 M(f) + \frac{a_2 A_c^2}{2}\delta(f) + \frac{a_2 A_c^2}{4}\left[\delta(f-2f_c) + \delta(f+2f_c)\right] + a_2 \underbrace{F.T[m^2(t)]}_{G(f)} + a_2 A_c\left[M(f-f_c) + M(f+f_c)\right] \quad (5)$$

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The plot of eq. (5) is shown in figure # 3.

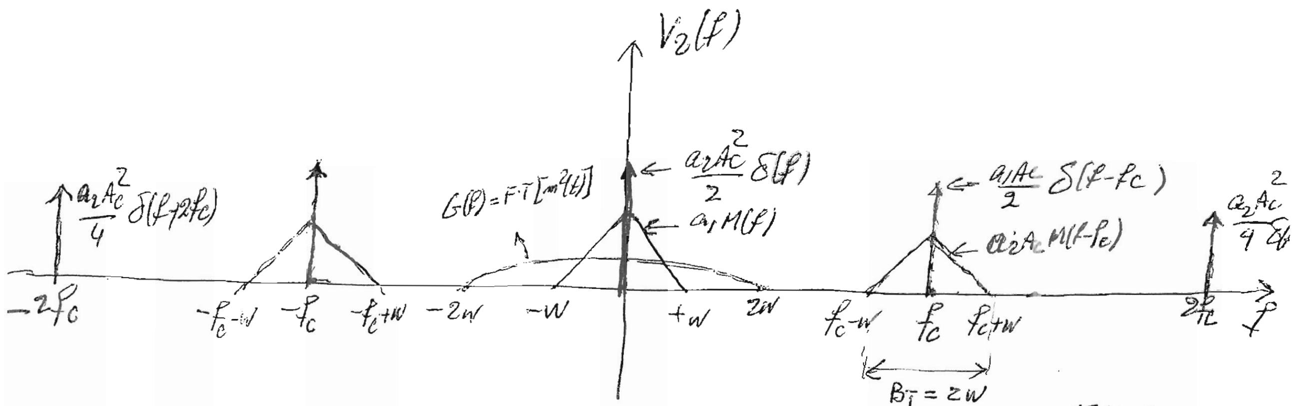

Fig # 3

Fig # 3 shows the spectral content of $v_2(t)$.

) To extract the desired AM signal, use eq (4) and identify the AM signal:

$$v_2(t) = \underbrace{a_1 A_c\left[1 + \frac{2a_2}{a_1} m(t)\right]\cos 2\pi f_c t}_{\text{Desired AM Signal}} + \underbrace{a_1 m(t) + a_2 m^2(t) + \frac{a_2 A_c^2}{2} + \frac{a_2 A_c^2}{2}\cos 4\pi f_c t}_{\text{undesired component}} \quad (6)$$

A Bandpass filter centered at f_c with total extend of $2W$ Hz, that is having a transfer function of:

$$H(f) = \text{rect}\left(\frac{f-f_c}{2W}\right) + \text{rec}\left(\frac{f+f_c}{2W}\right) \quad (7)$$

will pass the desired signal (AM signal) and eliminated the unwanted components.

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Using eq. (7) and figure #(3) we see that the required B.P.F must have a bandwidth of 2W Hz and centered at f_c, thus the cut-off frequencies of BPF are $f_c - W$ and $f_c + W$ Hz.

c) To avoide spectral overlapping of the desired signal (AM signal) with that of unwanted signals in $v_2(t)$, using figure # 3, we see that

1) $f_c - W \geq 2W \Rightarrow f_c \geq 3W$
2) $f_c + W \leq 2f_c \Rightarrow f_c \geq W$

Thus $f_c > 3W$

3.23

Assume $m(t)$ with spectrum of

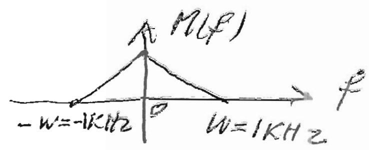

fig #1

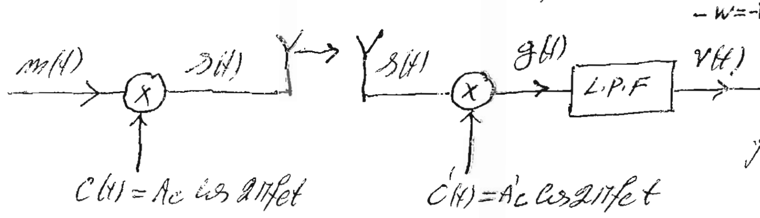

fig #(2) Coherent detection of DSB-SC.

$$s(t) = m(t)\,c(t) = A_c m(t) \cos 2\pi f_c t \Rightarrow S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] \quad (1)$$

$$g(t) = s(t)\cdot c'(t) = A_c A'_c m(t) \cos^2 2\pi f_c t = \frac{A_c A'_c}{2} m(t)\left[1 + \cos 4\pi f_c t\right] \quad (2)$$

$$G(f) = \frac{A_c A'_c}{2} M(f) + \frac{A_c A'_c}{4}\left[M(f-2f_c) + M(f+2f_c)\right] \quad (3)$$

a) For $f_c = 1.25$ KHz, the spectrum of $m(t)$, the spectrum of $s(t)$ and the spectrum of $v(t)$ (detector output) are:

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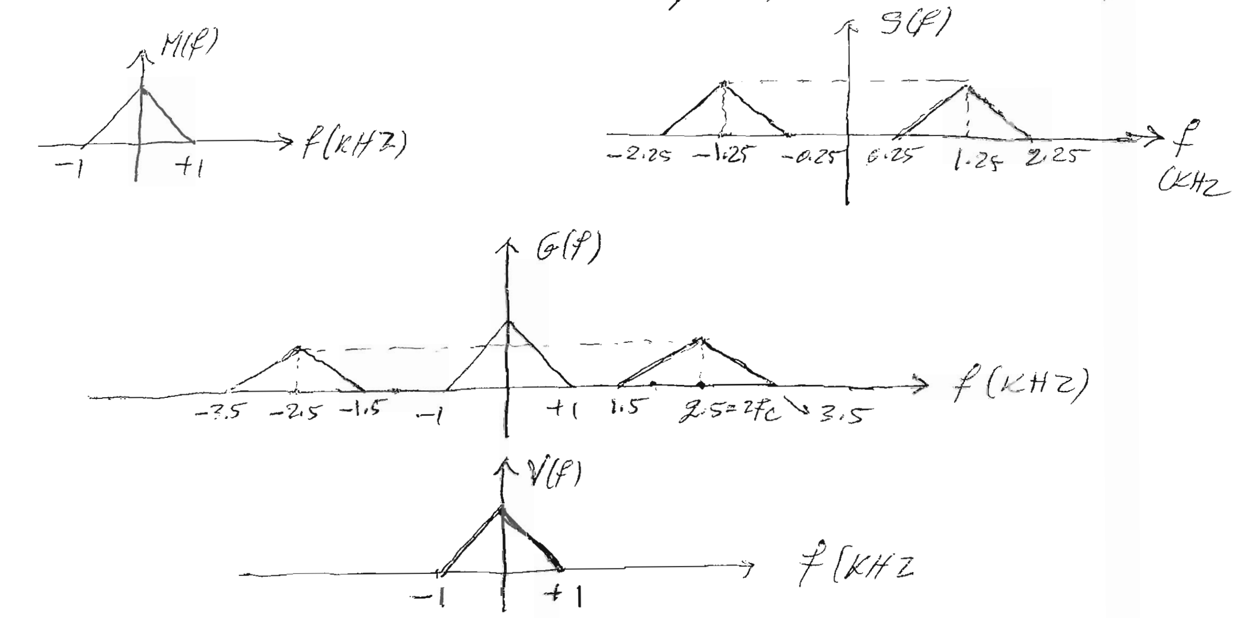
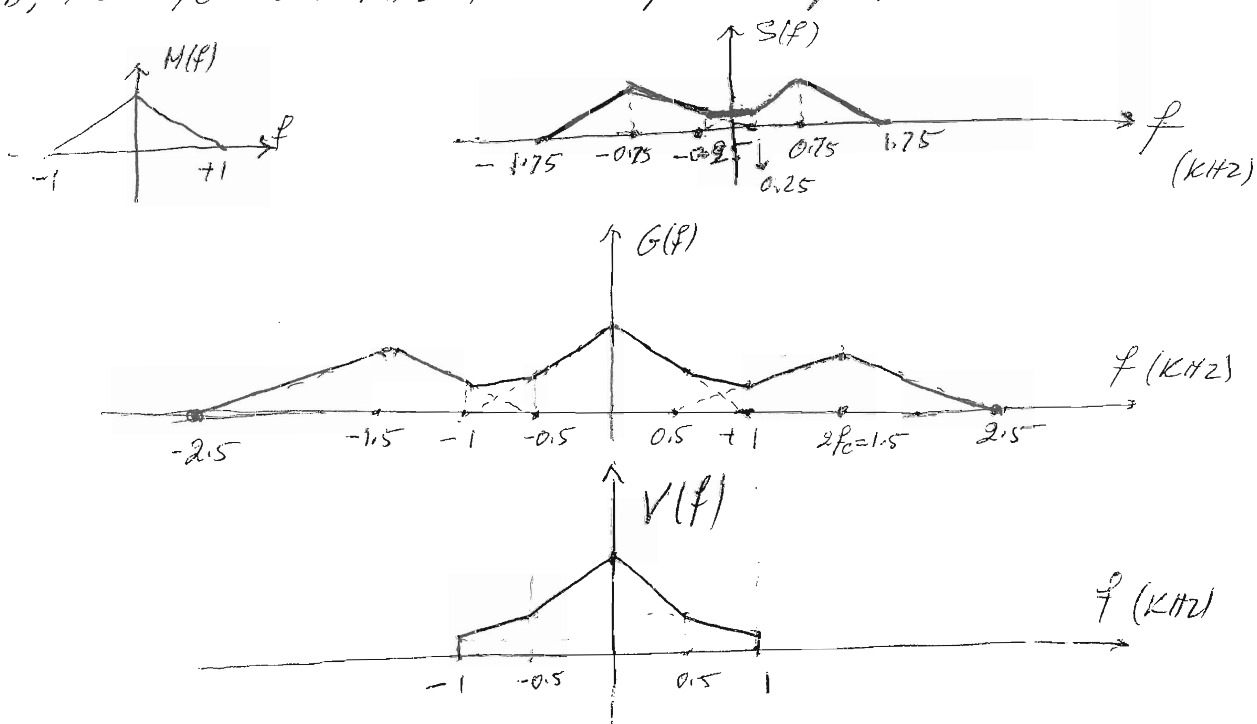


b) For $f_c = 0.75$ KHZ, the respective spectra are:



3 my graded HW

HW # 8

EE 443

Fall 2008
CSUF.

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18/20

Problem 5-1

AM broadcast transmitter is tested by feeding RF output into 50Ω load. Tone Modulation is used. Carrier frequency is 850 KHz and power output is 5000 W. The sinusoidal tone of 1000 Hz is set for 90% modulation.

a) Evaluate the FCC Power in dBK (dB over 1 KW) units.

b) Write an equation for the voltage that appears across the 50-Ω load, giving numerical values for all constants.

c) Sketch the spectrum of this voltage as it would appear on a calibrated spectrum analyzer.

d) What is the average power that is being dissipated in the dummy load?

e) What is peak envelope power?

Answer

a) $10 \log_{10}\left(\frac{5000}{1000}\right) = 6.9897 \sim \boxed{7 \text{ dbK}}$

b) $s(t) = A_c(1 + \mu \cos \omega_m t) \cos \omega_c t$ —— (1)

where ω_m is the tone frequency $2\pi(1000)$ rad/sec.
and ω_c is the Carrier frequency $2\pi(850{,}000)$ rad/sec.
$\mu = .9$. Need to find A_c;

Carrier Power $= \frac{A_c^2}{2}$ .. but this is normalized to 1 Ω.

hence $\boxed{P = \left(\frac{A_c^2}{2}\right)\frac{1}{R}}$ where $R = 50\ \Omega$.

so $P = \frac{A_c^2}{100} = A_c = \sqrt{100(P)}$, but $P = 5000$ Watt FCC

so $A_c = \sqrt{100(5000)} = \boxed{707.1 \text{ V}}$

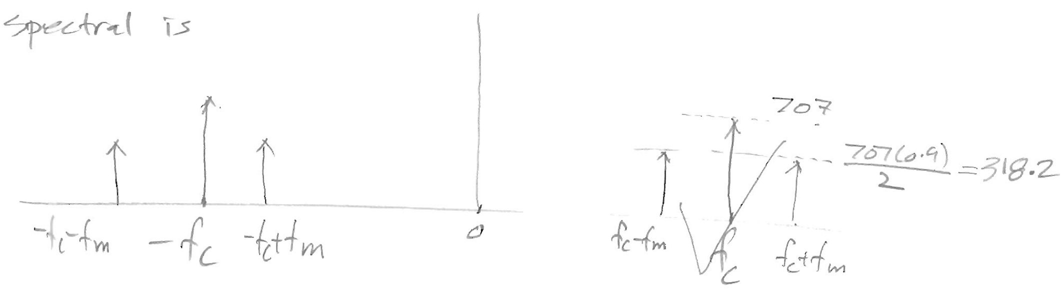
So Voltage equation is (1) given by

$$\boxed{s(t) = 707\,(1 + 0.9 \cos(2\pi \times 1000)t)\cos(2\pi \times 850{,}000)t}$$

→

$$s(t) = 707\cos 2\pi f_c t + \frac{707(0.9)}{2}\left[\cos(2\pi(f_c+f_m)t + \cos 2\pi(f_c-f_m)t\right]$$

hence spectral is



where $f_c = 850\,KHZ$
$f_m = 1\,KHZ$.

so $f_c+f_m = 851\,KHz$
$f_c-f_m = 849\,KHz$

(d) The Total Normalized average power $= \underbrace{\left(\frac{A_c^2}{2}\right)}_{\text{carrier}} + \underbrace{\left(\frac{A_c\mu}{2}\right)^2}_{\text{both sidebands.}}$

hence, we $R=50$, we obtain

$$P_{av.} \text{ in } R = \frac{1}{R}\left[\frac{A_c^2}{2} + \frac{A_c^2\mu^2}{4}\right]$$

$$= \frac{1}{50}\left[\frac{707^2}{2} + \frac{707^2(.9)^2}{4}\right] = \boxed{7{,}022.87 \text{ Watt}}$$

(e) $A_{max} = A_c(1+\mu)$

hence Peak Power (average) is $\frac{[A_c(1+\mu)]^2}{2} \times \frac{1}{R}$

$$= \frac{(707(1+0.9))^2}{2\times 50} = \boxed{18{,}044.5 \text{ Watt}}$$ this's not Normalized.

Normalized. PEP $= \frac{[707(1+.9)]^2}{2} = \boxed{902{,}227 \text{ watt}}$

5-3

AM transmitter modulated with $m(t) = 0.2 \sin W_1 t + 0.5 \cos W_2 t$

$f_1 = 500\,HZ$, $f_2 = 500\sqrt{2}\,HZ$. $A_c = 100$.

(a) Evaluate average power of the AM signal

(b) Evaluate Peak Envelope Power (PEP).

Answer

(a) average power (normalized) is given by

$$\frac{A_c^2}{2} + \left(\frac{A_c \mu}{2}\right)^2$$

$S(t) = A_c(1 + 0.2 \sin W_1 t + .5 \cos W_2 t) \cos W_c t$ ⇒ expands 2 sinusoid f.

⇒ another 2 sin. f.

$= A_c \cos W_c t + .2 A_c \sin W_1 t + .5 A_c \cos W_2 t$

x.

so normalized average power $\frac{A_c^2}{2} + \frac{(.2A_c)^2}{2} + \frac{(.5A_c)^2}{2}$

$$= \frac{100^2}{2} + \frac{20^2}{2} + \frac{50^2}{2} = \boxed{6{,}450 \text{ Watt}}$$ X See sol.

when using Load $R = 50\Omega$ given in problem 5-2, we obtain

$$\frac{6450}{50} = \boxed{129 \text{ Watt}}$$

(b) $A_{max} = A_c(1+\mu)$

hence $PEP = \frac{[100(1+.2)]^2}{2} + \frac{[100(1+.5)]^2}{2} =$

$= 7{,}200 + 11{,}250 = \boxed{18{,}450 \text{ Watt}}$

PEP over load of 50Ω, we obtain

$$PEP = \frac{[100(1.2)]^2}{2 \times 50} + \frac{[100(1.5)]^2}{2 \times 50} = \boxed{369 \text{ Watt}}$$

5-7

A DSB-SC signal is modulated by $m(t) = \cos\omega_1 t + 2\cos 2\omega_1 t$

where $f_1 = 500\text{Hz}$ and $A_c = 1$.

(a) write expression for DSB-SC signal and sketch picture of this waveform

(b) Evaluate and sketch the spectrum of this signal.

(c) Find the average (normalized) Power

d) Find PEP (normalized).

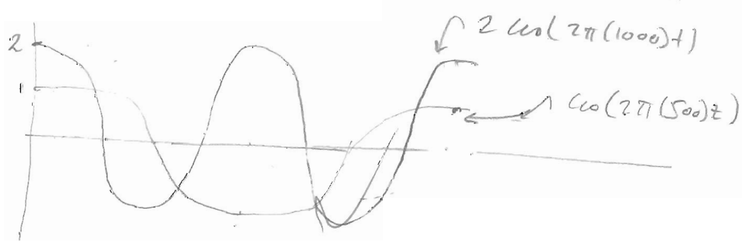
Answer

DSB-SC is double sided carrier suppressed.

a) $\boxed{s(t) = A_c \cos\omega_c t\; m(t)}$

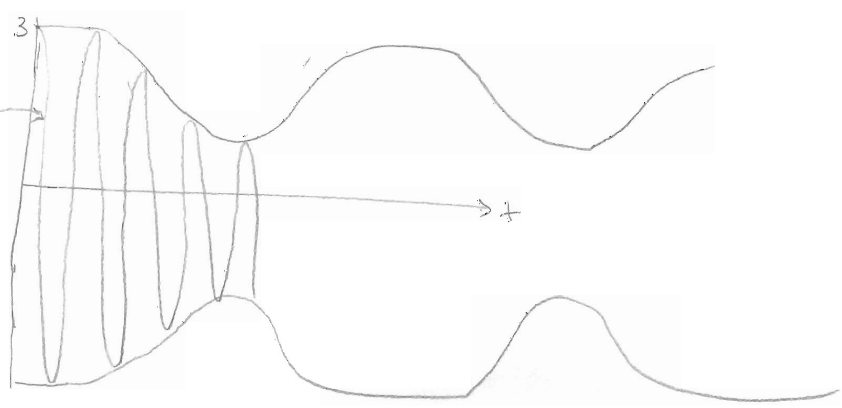
hence $s(t) = \cos 2\pi f_c t\,(\cos\omega_1 t + 2\cos 2\omega_1 t)$

$$= \cos 2\pi f_c t\,\underbrace{\left(\cos 2\pi(500)t + 2\cos(2\pi(1000)t)\right)}_{a(t)}$$



add these 2 signals ⇒ and multiply by carrier

$\cos 2\pi f_c t$



→

b) $s(t) = \cos 2\pi f_c \left(\cos 2\pi f_1 t + 2\cos 2\pi 2 f_1 t \right)$

$= \cos 2\pi f_c \cos 2\pi f_1 t + 2\cos 2\pi f_c \cos 4\pi f_1 t$

$= \frac{1}{2}\left(\cos 2\pi (f_c+f_1)t + \cos(2\pi(f_c-f_1)t) \right.$

$+ \left(\cos 2\pi (f_c+2f_1)t + \cos(2\pi(f_c-2f_1)t) \right)$ —— (1)

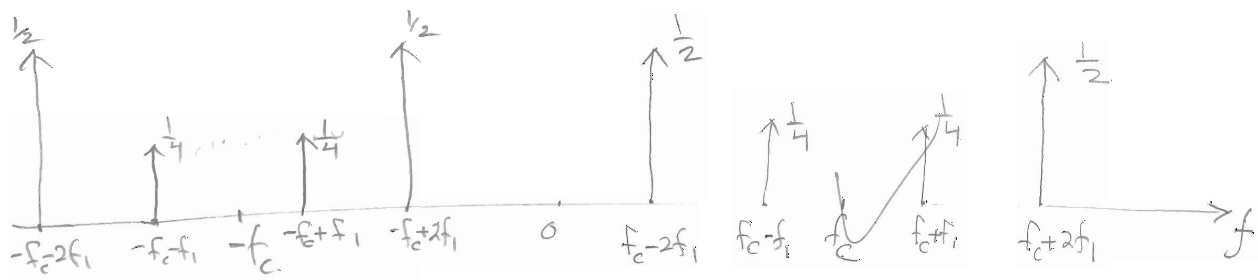
so $S(f) = \frac{1}{2}\cdot\left[\frac{1}{2}\left(\delta(f+f_c+f_1) + \delta(f-f_c-f_1)\right) \right.$

$\left. + \frac{1}{2}\left(\delta(f+f_c-f_1) + \delta(f-f_c+f_1)\right) \right]$

$+ \frac{1}{2}\left(\delta(f+f_c+2f_1) + \delta(f-f_c-2f_1)\right)$

$+ \frac{1}{2}\left(\delta(f+f_c-2f_1) + \delta(f-f_c+2f_1)\right)$

$= \frac{1}{4}\delta(f+f_c+f_1) + \frac{1}{4}\delta(f-f_c-f_1) + \frac{1}{4}\delta(f+f_c-f_1) + \frac{1}{4}\delta(f-f_c+f_1)$
$+ \frac{1}{2}\delta(f+f_c+2f_1) + \frac{1}{2}\delta(f-f_c-2f_1) + \frac{1}{2}\delta(f+f_c-2f_1) + \frac{1}{2}\delta(f-f_c+2f_1)$



(c) average power: From (1) using formula $\frac{\text{Amplitude}^2}{2}$ per tone:

$\frac{0.5^2}{2} + \frac{0.5^2}{2} + \frac{1^2}{2} + \frac{1^2}{2} = \frac{1}{8} + \frac{1}{8} + \frac{1}{2} + \frac{1}{2} = \boxed{1.25 \text{ Watt}}$

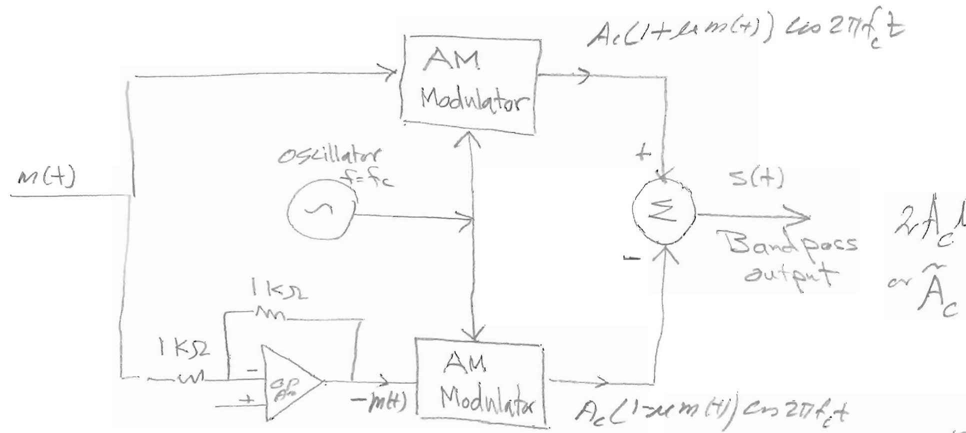
(d) $A_{max} = A_c + \mu$. $\mu = 2$ for the signal $\cos\omega_c t \cdot 2\cos 2\omega_1 t$

hence $A_{max} = 1+2 = \boxed{3}$

so $PEP = \frac{A_{max}^2}{2} = \frac{9}{2} = \boxed{4.5 \text{ watt}}$

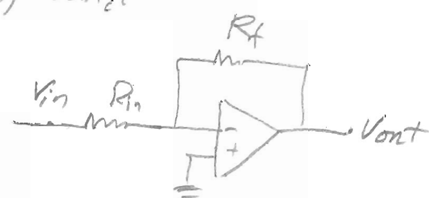
5-9 A DSB-SC signal can be generated from 2 AM signals. Using mathematics to describe signals at each point on figure, prove output is DSB-SC.

Answer



$2A_c\mu\, m(t)\cos 2\pi f_c t$ or $\tilde{A}_c\, m(t)\cos 2\pi f_c t$

op-Am acts as an inverting Amplifier



$$\boxed{V_{out} = -V_{in}\left(\frac{R_f}{R_{in}}\right)}$$

since $R_f = R_{in}$ in this problem, then $V_{out} = -V_{in}$.

hence $s(t) = A_c(1+\mu m(t))\cos 2\pi f_c t - [A_c(1-\mu m(t))\cos 2\pi f_c t]$

or

$$s(t) = \cancel{A_c\cos 2\pi f_c t} + A_c\mu m(t)\cos 2\pi f_c t - [\cancel{A_c\cos 2\pi f_c t} - A_c\mu\, m(t)\cos 2\pi f_c t]$$

$$= 2A_c\mu\, m(t)\cos 2\pi f_c t$$

Combine $2A_c\mu \Rightarrow \tilde{A}_c$ we obtain equation for DSB-SC.

$$\boxed{\tilde{A}_c\, m(t)\cos 2\pi f_c t}$$

hence the above circuit suppresses the carrier part.

8.50 From Text Book

Find spectral density $S_Z(f)$ if:

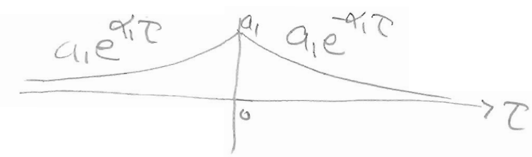
$$Z(t) = X(t) + Y(t)$$

where $X(t), Y(t)$ are independent zero-mean R.P. with

$$R_X(\tau) = a_1 e^{-\alpha_1|\tau|} \quad \text{and} \quad R_Y(\tau) = a_2 e^{-\alpha_2|\tau|}$$

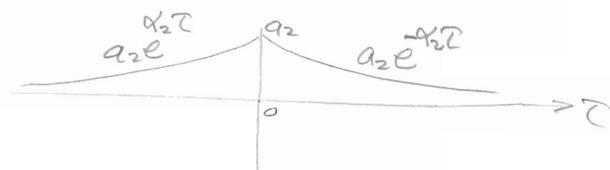
Answer

$R_X(\tau) =$



$\alpha_1 > 0$

$R_Y(\tau) =$



$\alpha_2 > 0$

The following are possible ways to solve this problem:

1. Find $R_Z(\tau)$ by adding $R_X(\tau) + R_Y(\tau)$. Then Find Fourier Transform of $R_Z(\tau)$, this gives $S_Z(f)$.
2. Find Fourier Transform of $R_X(\tau)$ and $R_Y(\tau)$. This gives $S_X(f)$ and $S_Y(f)$. Then due to Linearity of Fourier transform, add $S_X(f) + S_Y(f)$ to obtain $S_Z(f)$.

using method 1. First need to show that $R_Z(\tau) = R_X(\tau) + R_Y(\tau)$:

$$R_Z(\tau) = E\{(X(t)+Y(t))(X(t+\tau)+Y(t+\tau))\}$$

$$= E\{X(t)X(t+\tau) + X(t)Y(t+\tau) + Y(t)X(t+\tau) + Y(t)Y(t+\tau)\}$$

$$= R_X(\tau) + \underbrace{E(X(t)Y(t+\tau))}_{\text{since indep.}} + E(Y(t)X(t+\tau)) + R_Y(\tau)$$

$$\underbrace{E(X(t))}_{=0}\underbrace{E(Y(t+\tau))}_{=0} + \underbrace{E(Y(t))}_{=0}\underbrace{E(X(t+\tau))}_{=0}$$

so $\boxed{R_Z(\tau) = R_X(\tau) + R_Y(\tau)}$

hence $R_Z(\tau) = (a_1 e^{\alpha_1 \tau} + a_2 e^{\alpha_2 \tau})u(-\tau) + (a_1 e^{-\alpha_1 \tau} + a_2 e^{-\alpha_2 \tau})u(t)$ $\rightarrow$

$$\int_{-\infty} a_1 e \; e \; d\tau = a_1 \int_{-\infty} \qquad a\tau \hat{=} a_1 \frac{\;}{-j2\pi f+\alpha_1}\Big|_{-\infty}$$

$$= \frac{a_1}{-j2\pi f+\alpha_1}\,(1) = \boxed{\frac{a_1}{-j2\pi f+\alpha_1}}$$

and $$\int_{-\infty}^{0} a_2 e^{\alpha_2\tau} e^{-j2\pi f\tau}\, d\tau = \boxed{\frac{a_2}{-j2\pi f+\alpha_2}}$$

and $$\int_{0}^{\infty} a_1 e^{-\alpha_1\tau} e^{-j2\pi f\tau}\, d\tau = a_1 \int_{0}^{\infty} e^{\tau(-\alpha_1-j2\pi f)}\, d\tau$$

$$= a_1 \frac{\left[e^{\tau(-\alpha_1-j2\pi f)}\right]_0^{\infty}}{-\alpha_1-j2\pi f} = \frac{a_1}{-\alpha_1-j2\pi f}(-1) = \boxed{\frac{a_1}{\alpha_1+j2\pi f}}$$

and $$\int_{0}^{\infty} a_2 e^{-\alpha_2\tau} e^{-j2\pi f\tau}\, d\tau = \boxed{\frac{a_2}{\alpha_2+j2\pi f}}$$

hence $$\boxed{S_Z(f) = \frac{a_1}{\alpha_1-j2\pi f} + \frac{a_2}{\alpha_2-j2\pi f} + \frac{a_1}{\alpha_1+j2\pi f} + \frac{a_2}{\alpha_2+j2\pi f}}$$

This can be simplified more as follows.

$$S_Z(f) = \frac{a_1(\alpha_1+j2\pi f)+a_1(\alpha_1-j2\pi f)}{(\alpha_1-j2\pi f)(\alpha_1+j2\pi f)} + \frac{a_2(\alpha_2+j2\pi f)+a_2(\alpha_2-j2\pi f)}{(\alpha_2-j2\pi f)(\alpha_2+j2\pi f)}$$

$$= \boxed{\frac{2a_1\alpha_1}{\alpha^2+4\pi^2f^2} + \frac{2a_2\alpha_2}{\alpha_2^2+4\pi^2f^2}}$$