

HW 8
Electronic Communication Systems
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0.1 Questions

EE 443

Sept 3

HW # 8

page 4

Drill 3.4)

$$V_2 = a_1 V_1(t) + a_2 V_1^2(t) \quad (1)$$

$$\text{where, } V_1(t) = A_c \cos 2\pi f_c t + m(t) \quad (2)$$

Subst. eq. (2) into eq (1)

$$\begin{aligned} V_2(t) &= a_1 [A_c \cos 2\pi f_c t + m(t)] + a_2 [A_c \cos 2\pi f_c t + m(t)]^2 \\ \Rightarrow V_2(t) &= a_1 A_c \underbrace{\left[1 + \frac{2a_2}{a_1} m(t)\right] \cos 2\pi f_c t}_{\text{AM Signal}} + a_1 m(t) + \\ &\quad + a_2 m^2(t) + a_2 A_c^2 \underbrace{\cos^2(2\pi f_c t)}_{\frac{1}{2}[1 + \cos 4\pi f_c t]} \end{aligned}$$

The signal at the output of bandpass filter is:

$$V_o(t) = a_1 A_c \left[1 + \frac{2a_2}{a_1} m(t)\right] \cos 2\pi f_c t$$

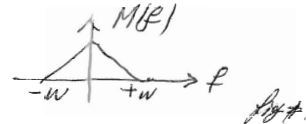
which is an AM wave.

0.2 Key solution

EE 443 HW #8 Chapt (3) Page 1
Drill Prob. #3, 4) $U_2(t) = a_1 U_1(t) + a_2 U_1^2(t)$ (1), $U_1(t) = A_c \cos(2\pi f_c t) + m(t)$ (2)

$$U_2(t) = a_1 A_c \cos(2\pi f_c t) + a_1 m(t) + a_2 A_c^2 \cos^2(2\pi f_c t) + a_2 m^2(t) + 2 a_2 A_c m(t) \cos(2\pi f_c t) \quad (3)$$

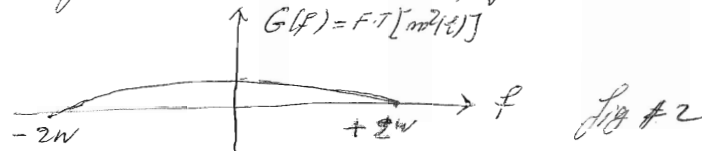
Assume $M(f)$ has the following form:



Let $g(t) \triangleq m^2(t) = m(t) \cdot m(t)$

$\Rightarrow G(f) = M(f) \otimes M(f)$ The spectrum of $g(t) = m^2(t)$

Will extend from $-2W$ to $2W$ Hz, for example would be



Note: If you want to ~~find~~ find $G(f)$, then you have

to do $G(f) = \int_{-\infty}^{+\infty} M(\lambda) M(f-\lambda) d\lambda$. We are not interested to find the exact equation of $G(f)$, all we need to know is that the spectrum of $G(f) = F.T[m^2(t)]$ will extend from $-2W$ to $2W$ Hz.

Let us to take the F.T of eq(3) and plot it!

$$U_2(t) = a_1 A_c \cos(2\pi f_c t) + a_1 m(t) + \frac{a_2 A_c^2}{2} [1 + \cos(4\pi f_c t)] + a_2 m^2(t) + 2 a_2 A_c m(t) \cos(2\pi f_c t) \quad (4)$$

$$\Rightarrow V_2(f) = \frac{a_1 A_c}{2} [\delta(f-f_c) + \delta(f+f_c)] + a_1 M(f) + \frac{a_2 A_c^2}{2} \delta(f) + \frac{a_2 A_c^2}{4} [\delta(f-2f_c) + \delta(f+2f_c)] + a_2 \underbrace{F.T[m^2(t)]}_{G(f)} + a_2 A_c [M(f-f_c) + M(f+f_c)] \quad (5)$$

The plot of eq. (5) is shown in figure # 3.

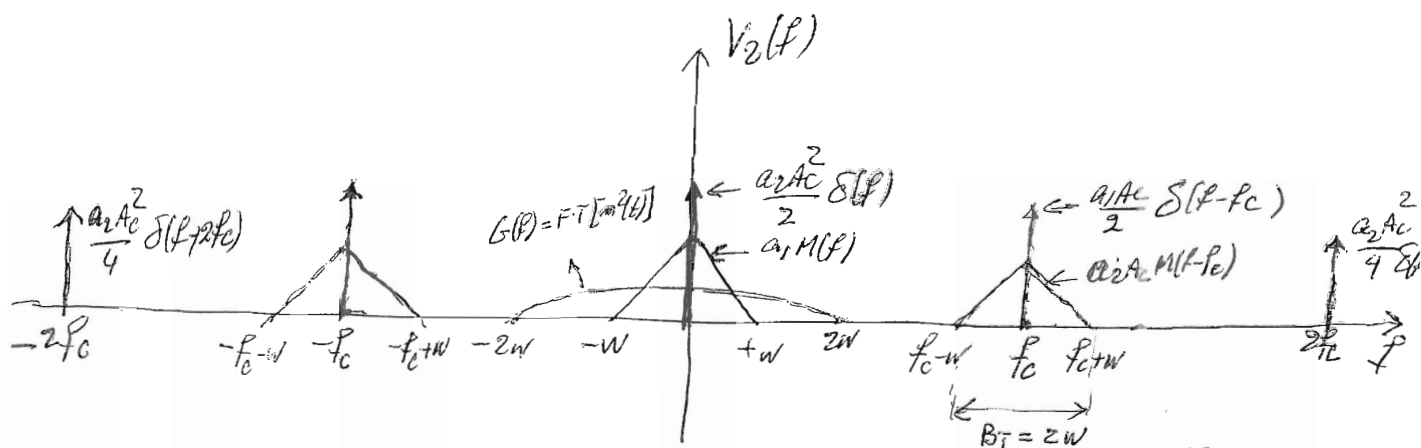


Fig # 3

Fig # 3 shows the spectral content of $V_2(t)$.

1) To extract the desired AM signal, use eq (4) and identify the AM signal:

$$V_2(t) = \underbrace{a_1 A_c \left[1 + \frac{2a_2}{a_1} m(t) \right] \cos 2\pi f_c t}_{\text{Desired AM signal}} + \underbrace{\left(a_1 m(t) + a_2 m^2(t) + \frac{a_2 A_c^2}{2} + \frac{a_2 A_c^2}{2} \cos 4\pi f_c t \right)}_{\text{Undesired component}} \quad (6)$$

A Bandpass filter centered at f_c with total extend of $2W/2$, That is having a transfer function of :

$$H(f) = \text{rect}\left(\frac{f-f_c}{2W}\right) + \text{rec}\left(\frac{f+f_c}{2W}\right) \quad (7)$$

will pass the desired signal (AM signal) and eliminated the unwanted components.

Using eq. (7) and figure # (3) we see that the required B.P.F must have a bandwidth of $2W$ Hz and centered at f_c , thus the cut-off frequencies of BPF are $f_c - W$ and $f_c + W$ Hz.

c) To avoid spectral overlapping of the desired signal (AM signal) with that of unwanted signals in $V_d(t)$, using figure # 3, we see that

$$\left. \begin{array}{l} 1) f_c - W \geq 2W \Rightarrow f_c \geq 3W \\ 2) f_c + W \leq 2f_c \Rightarrow f_c \geq W \end{array} \right\} \text{Thus } f_c \geq 3W$$

3.23

Assume $m(t)$ with spectrum of

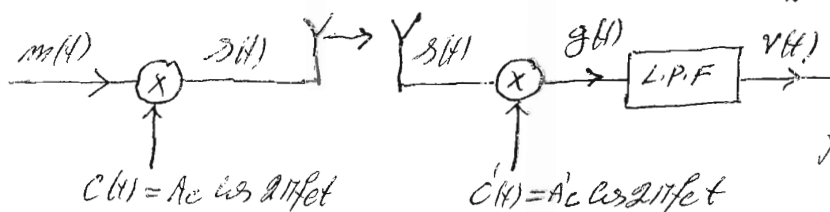
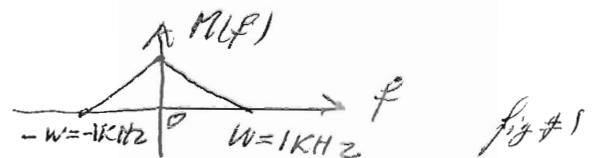


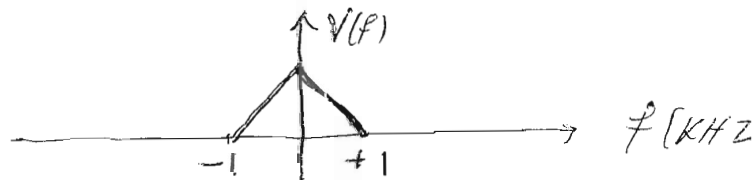
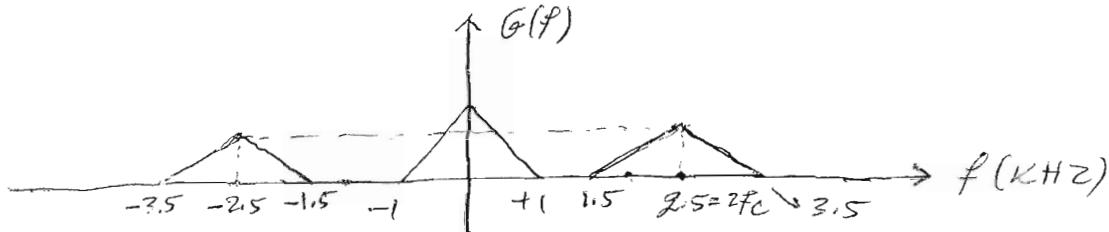
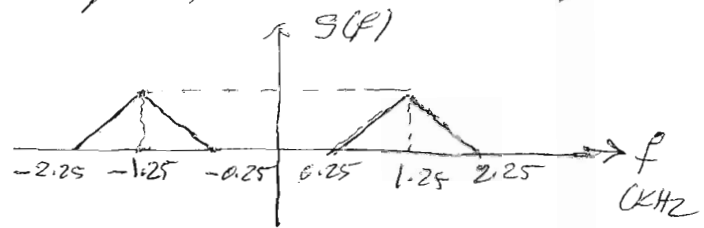
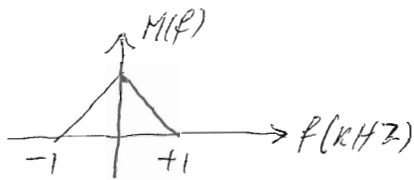
fig # (2) Coherent detection of DSB-SC.

$$s(t) = m(t) c(t) = A_c m(t) \cos 2\pi f_c t \Rightarrow S(f) = \frac{A_c}{2} [M(f-f_c) + M(f+f_c)] \quad (1)$$

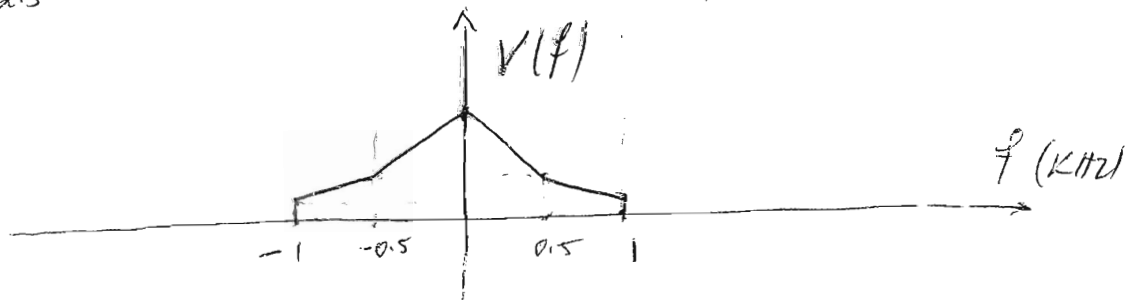
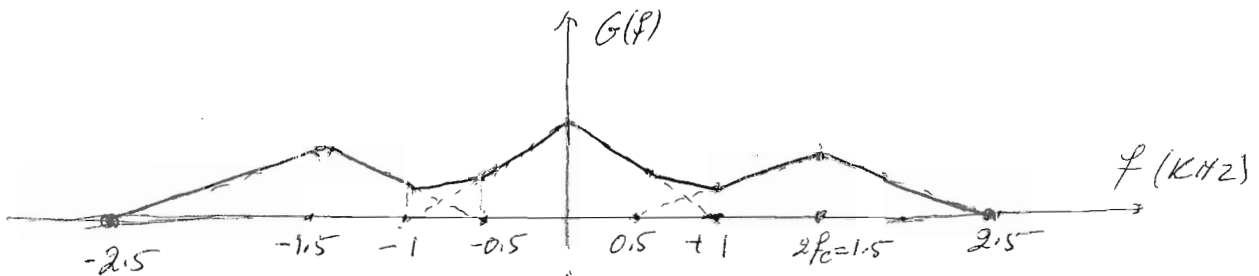
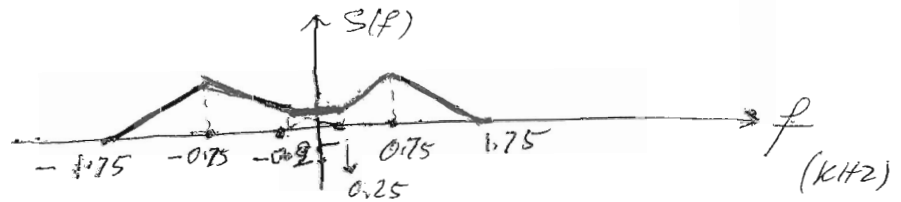
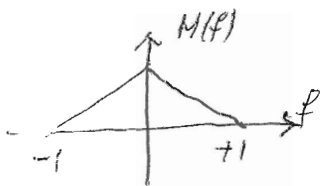
$$g(t) = s(t) \cdot c'(t) = A_c A_c' m(t) \cos^2 2\pi f_c t = \frac{A_c A_c'}{2} m(t) [1 + \cos 4\pi f_c t] \quad (2)$$

$$G(f) = \frac{A_c A_c'}{2} M(f) + \frac{A_c A_c'}{4} [M(f-2f_c) + M(f+2f_c)] \quad (3)$$

a) For $f_c = 1.25 \text{ kHz}$, the spectrum of $m(t)$, the spectrum of $s(t)$ and the spectrum of $v(t)$ (detector output) are:



b) For $f_c = 0.75$ kHz, the respective spectra are :



0.3 my graded HW

HW# 8

EE 443

Fall 2008
CSUF.

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Problem 5-1

18
20

AM broadcast transmitter is tested by feeding RF output into $50\text{-}\Omega$ load. Tone Modulation is used. Carrier frequency is 850 kHz and power output is 5000 W . The sinusoidal tone of 1000 Hz is set for 90% modulation.

- Evaluate the FCC Power in dBK (dB over 1 kW) units.
- Write an equation for the voltage that appears across the $50\text{-}\Omega$ load, giving numerical values for all constants.
- Sketch the spectrum of this voltage as it would appear on a calibrated spectrum analyzer.
- What is the average power that is being dissipated in the dummy load?
- What is peak envelope power?

Answer

$$a) 10 \log_{10} \left(\frac{5000}{1000} \right) = 6.9897 \sim \boxed{7 \text{ dBK}}$$

$$b) s(t) = A_c (1 + \mu \cos \omega_m t) \cos \omega_c t \quad \text{--- (1)}$$

where ω_m is the tone frequency $2\pi(1000)\text{ rad/sec}$.

and ω_c is the carrier frequency $2\pi(850,000)\text{ rad/sec}$.

$\mu = 0.9$. Need to find A_c ;

Carrier Power = $\frac{A_c^2}{2}$ but this is normalized to $1\text{ }\Omega$.

$$\text{hence } \boxed{P = \left(\frac{A_c^2}{2} \right) \frac{1}{R}} \text{ where } R = 50\text{ }\Omega.$$

$$\text{So } P = \frac{A_c^2}{100} = A_c = \sqrt{100(P)} \quad \text{but } P = 5000\text{ Watt} \quad \text{FCC}$$

$$\text{So } A_c = \sqrt{100(5000)} = \boxed{707.1 \text{ V}}$$

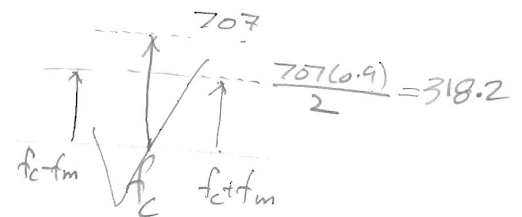
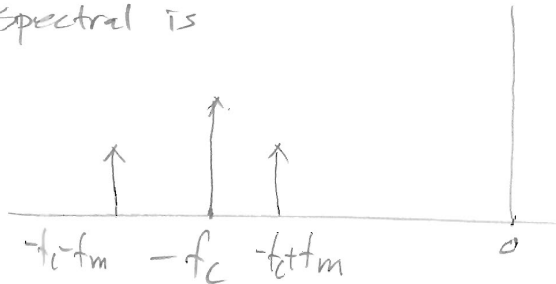
So voltage equation is (1) given by

$$\boxed{s(t) = 707 (1 + 0.9 \cos(2\pi \times 1000)t) \cos(2\pi \times 850,000)t}$$



$$s(t) = 707 \cos 2\pi f_c t + \frac{707(0.9)}{2} [\cos(2\pi(f_c + f_m)t) + \cos 2\pi(f_c - f_m)t]$$

hence spectral is



where $f_c = 850 \text{ KHz}$

$f_m = 1 \text{ KHz}$

so $f_c + f_m = 851 \text{ KHz}$

$f_c - f_m = 849 \text{ KHz}$

(d)

The Total Normalized average power = $\left(\frac{A_c^2}{2}\right) + \left(\frac{A_c \mu}{2}\right)^2$

hence, we $R=50$, we obtain

$$P_{av. \text{ in } R} = \frac{1}{R} \left[\frac{A_c^2}{2} + \frac{A_c^2 \mu^2}{4} \right]$$

$$= \frac{1}{50} \left[\frac{707^2}{2} + \frac{707^2 (0.9)^2}{4} \right] = \boxed{7022.87 \text{ Watt}}$$

© $A_{max} = A_c(1+\mu)$

hence Peak Power (average) is $\frac{[A_c(1+\mu)]^2}{2} \times \frac{1}{R}$

$$= \frac{(707(1+0.9))^2}{2 \times 50} = \boxed{18,044.5 \text{ Watt}}$$

Normalized PEP = $\frac{[707(1+0.9)]^2}{2} = \boxed{902,227 \text{ watt}}$

this is not Normalized.

5-3

AM transmitter modulated with $m(t) = 0.2 \sin \omega_1 t + 0.5 \cos \omega_2 t$

$f_1 = 500 \text{ Hz}$, $f_2 = 500\sqrt{2} \text{ Hz}$. $A_c = 100$.

(a) Evaluate average power of the AM signal

(b) Evaluate Peak Envelope Power (PEP).

Answer

(a) average power (normalized) is given by

$$\frac{A_c^2}{2} + \left(\frac{A_c \mu}{2}\right)^2$$

$$\begin{aligned} s(t) &= A_c(1 + \underbrace{0.2 \sin \omega_1 t + 0.5 \cos \omega_2 t}_{\text{expands } \Rightarrow \text{ sinusoid } f.}) \cos \omega_c t \Rightarrow \text{another } \Rightarrow \text{ sin. } f. \\ &= A_c \cos \omega_c t + \underbrace{.2 A_c \sin \omega_1 t + .5 A_c \cos \omega_2 t}_{x}. \end{aligned}$$

so normalized average power

$$\begin{aligned} &\frac{A_c^2}{2} + \frac{(.2 A_c)^2}{2} + \frac{(.5 A_c)^2}{2} \\ &= \frac{100^2}{2} + \frac{20^2}{2} + \frac{50^2}{2} = \boxed{6,450 \text{ Watt}} \quad \times \text{ See sol.} \end{aligned}$$

When using Load $R = 50 \Omega$ given in problem 5-2, we obtain

$$\frac{6450}{50} = \boxed{129 \text{ Watt}} \quad \checkmark$$

(b) $A_{\max} = A_c(1 + \mu)$

$$\text{here } \text{PEP} = \frac{[100(1+.2)]^2}{2} + \frac{[100(1+.5)]^2}{2} =$$

$$= 7,200 + 11,250 = \boxed{18,450 \text{ Watt}} \quad \checkmark$$

PEP over load of 50Ω , we obtain

$$\text{PEP} = \frac{[100(1.2)]^2}{2 \times 50} + \frac{[100(1.5)]^2}{2 \times 50} = \boxed{369 \text{ Watt}} \quad \checkmark$$

5-7

A DSB-SC signal is modulated by $m(t) = \cos \omega_1 t + 2 \cos 2\omega_1 t$

Where $f_1 = 500 \text{ Hz}$ and $A_c = 1$.

- (a) write expression for DSB-SC signal and sketch picture of this waveform
- (b) Evaluate and sketch the spectrum of this signal.
- (c) Find the average (normalized) power
- (d) Find PEP (normalized).

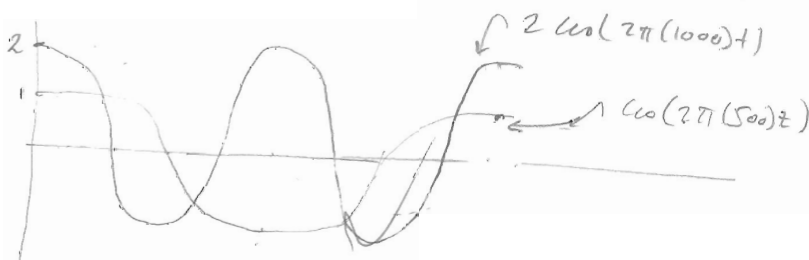
Answer

DSB-SC is double sided carrier suppressed.

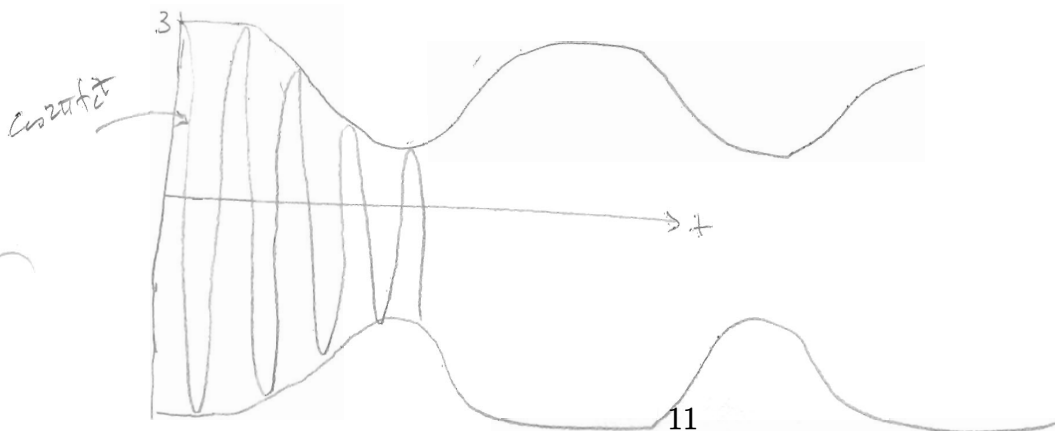
a) $s(t) = A_c \cos \omega_c t m(t)$

hence $s(t) = \cos 2\pi f_c t (\cos \omega_1 t + 2 \cos 2\omega_1 t)$

$$= \cos 2\pi f_c t (\underbrace{\cos 2\pi(500)t + 2 \cos(2\pi(1000)t)}_{a(t)})$$

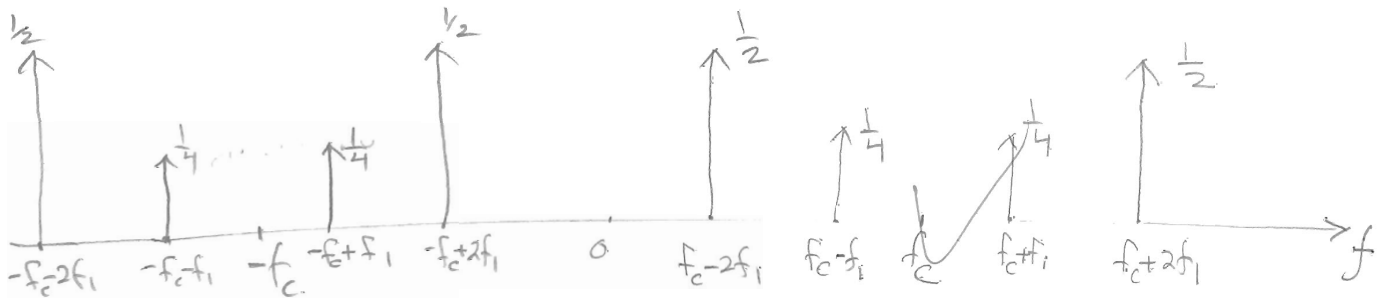


add these
2 signals $\Rightarrow$
and multiply by carrier



$$\begin{aligned}
 b) \quad s(t) &= \cos 2\pi f_c (\cos 2\pi f_1 t + 2 \cos 2\pi 2f_1 t) \\
 &= \cos 2\pi f_c \cos 2\pi f_1 t + 2 \cos 2\pi f_c \cos 4\pi f_1 t \\
 &= \frac{1}{2} (\cos 2\pi (f_c + f_1) t + \cos 2\pi (f_c - f_1) t) \\
 &\quad + (\cos 2\pi (f_c + 2f_1) t + \cos 2\pi (f_c - 2f_1) t) \quad \text{--- (1)}
 \end{aligned}$$

$$\begin{aligned}
 \therefore S(f) &= \frac{1}{2} \left[\frac{1}{2} (\delta(f + f_c + f_1) + \delta(f - f_c - f_1)) \right. \\
 &\quad \left. + \frac{1}{2} (\delta(f + f_c - f_1) + \delta(f - f_c + f_1)) \right] \\
 &\quad + \frac{1}{2} (\delta(f + f_c + 2f_1) + \delta(f - f_c - 2f_1)) \\
 &\quad + \frac{1}{2} (\delta(f + f_c - 2f_1) + \delta(f - f_c + 2f_1)) \\
 &= \frac{1}{4} \delta(f + f_c + f_1) + \frac{1}{4} \delta(f - f_c - f_1) + \frac{1}{4} \delta(f + f_c - f_1) + \frac{1}{4} \delta(f - f_c + f_1) \\
 &\quad + \frac{1}{2} \delta(f + f_c + 2f_1) + \frac{1}{2} \delta(f - f_c - 2f_1) + \frac{1}{2} \delta(f + f_c - 2f_1) + \frac{1}{2} \delta(f - f_c + 2f_1)
 \end{aligned}$$



c) Average power: From (1) using Formula $\frac{\text{Amplitude}^2}{2}$ per tone:

$$\frac{0.5^2}{2} + \frac{0.5^2}{2} + \frac{1^2}{2} + \frac{1^2}{2} = \frac{1}{8} + \frac{1}{8} + \frac{1}{2} + \frac{1}{2} = 1.25 \text{ Watt}$$

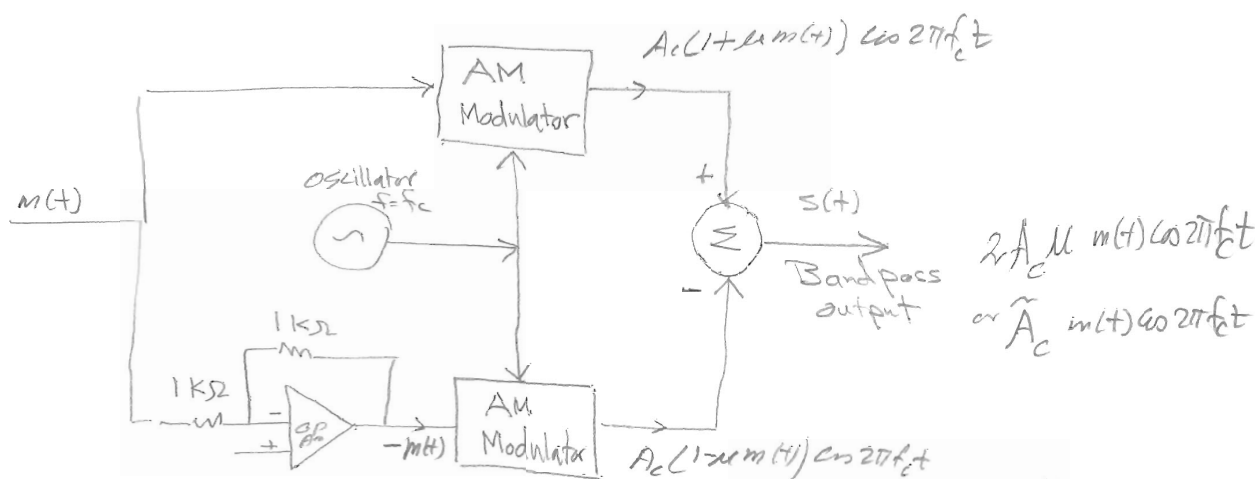
d) $A_{\max} = A_c + m$. $m=2$ for the signal $\cos 2\pi f_1 t + 2 \cos 4\pi f_1 t$

hence $A_{\max} = 1 + 2 = 3$

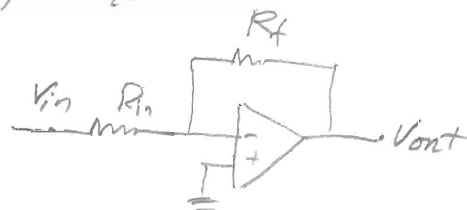
$\therefore \text{PEP} = \frac{A_{\max}^2}{2} = \frac{12}{2} = 4.5 \text{ watt}$

5-9 A DSB-SC signal can be generated from 2 AM signals. Using mathematics to describe signals at each point in figure, prove output is DSB-SC.

Answer



Q2 AM acts as an inverting Amplifier



$$V_{out} = -V_{in} \left(\frac{R_f}{R_{in}} \right)$$

Since $R_f = R_{in}$ in this problem, then $V_{out} = -V_{in}$.

$$\text{hence } S(t) = A_c(1 + \mu m(t))\cos(2\pi f_c t) - [A_c(1 - \mu m(t))\cos(2\pi f_c t)]$$

or

$$\begin{aligned} S(t) &= A_c \cos(2\pi f_c t) + A_c \mu m(t) \cos(2\pi f_c t) - [A_c \cos(2\pi f_c t) - A_c \mu m(t) \cos(2\pi f_c t)] \\ &= 2A_c \mu m(t) \cos(2\pi f_c t) \end{aligned}$$

Combine $2A_c \mu \Rightarrow \tilde{A}_c$

$$\tilde{A}_c m(t) \cos(2\pi f_c t)$$

we obtain equation for DSB-SC

hence the above circuit suppresses the Carrier part.

8.50 From Text Book

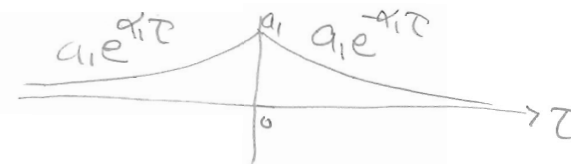
Find spectral density $S_Z(f)$ if.

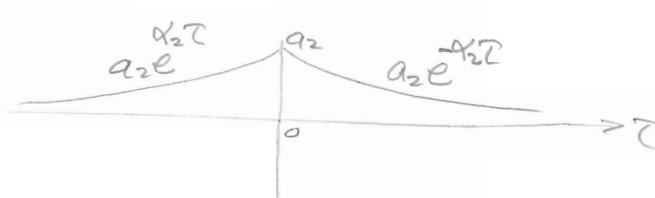
$$Z(t) = X(t) + Y(t)$$

where $X(t), Y(t)$ are independent zero-mean R.P. with

$$R_X(\tau) = a_1 e^{-\alpha_1 |\tau|} \quad \text{and} \quad R_Y(\tau) = a_2 e^{-\alpha_2 |\tau|}$$

Answer

$$R_X(\tau) = \begin{array}{c} a_1 e^{\alpha_1 \tau} \\ \text{---} \end{array} \begin{array}{c} a_1 e^{-\alpha_1 \tau} \end{array} \quad \alpha_1 > 0$$


$$R_Y(\tau) = \begin{array}{c} a_2 e^{\alpha_2 \tau} \\ \text{---} \end{array} \begin{array}{c} a_2 e^{-\alpha_2 \tau} \end{array} \quad \alpha_2 > 0$$


The following are possible ways to solve this problem:

- ① Find $R_Z(\tau)$ by adding $R_X(\tau) + R_Y(\tau)$. Find Fourier Transform of $R_Z(\tau)$, this gives $S_Z(f)$.
- ② Find Fourier Transform of $R_X(\tau)$ and $R_Y(\tau)$. This gives $S_X(f)$ and $S_Y(f)$. Then due to Linearity of Fourier transform, add $S_X(f) + S_Y(f)$ to obtain $S_Z(f)$.

using method ①. First need to show that $R_Z(\tau) = R_X(\tau) + R_Y(\tau)$:

$$\begin{aligned} R_Z(\tau) &= E\{(X(t) + Y(t))(X(t+\tau) + Y(t+\tau))\} \\ &= E\{X(t)X(t+\tau) + X(t)Y(t+\tau) + Y(t)X(t+\tau) + Y(t)Y(t+\tau)\} \\ &= R_X(\tau) + E(X(t)Y(t+\tau)) + E(Y(t)X(t+\tau)) + R_Y(\tau) \\ &\quad \downarrow \text{since indep.} \quad \downarrow \\ &\quad E(X(t))E(Y(t+\tau)) + E(Y(t))E(X(t+\tau)) \\ &\quad = 0 \quad = 0 \quad = 0 \quad = 0 \end{aligned}$$

so $R_Z(\tau) = R_X(\tau) + R_Y(\tau)$

hence $R_Z(\tau) = (a_1 e^{\alpha_1 \tau} + a_2 e^{\alpha_2 \tau}) u(-\tau) + (a_1 e^{-\alpha_1 \tau} + a_2 e^{-\alpha_2 \tau}) u(\tau) \rightarrow$

$$\lim_{\omega \rightarrow \infty} \int_{-\infty}^{\infty} u_1 e^{j\omega \tau} d\tau = a_1 \int_{-\infty}^{\infty} e^{j\omega \tau} d\tau$$

$$a_1 \int_{-\infty}^{\infty} e^{j\omega \tau} d\tau = \frac{1}{-j2\pi f + \alpha_1} \Big|_{-\infty}^{\infty}$$

$$= \frac{a_1}{-j2\pi f + \alpha_1} (1) = \boxed{\frac{a_1}{-j2\pi f + \alpha_1}}$$

$$\text{and } \int_{-\infty}^{\infty} a_2 e^{\alpha_2 \tau} e^{-j2\pi f \tau} d\tau = \boxed{\frac{a_2}{-j2\pi f + \alpha_2}}$$

$$\text{and } \int_0^{\infty} a_1 e^{-\alpha_1 \tau} e^{-j2\pi f \tau} d\tau = a_1 \int_0^{\infty} e^{\tau(-\alpha_1 - j2\pi f)} d\tau$$

$$= a_1 \frac{[e^{\tau(-\alpha_1 - j2\pi f)}]_0^{\infty}}{-\alpha_1 - j2\pi f} = \frac{a_1}{-\alpha_1 - j2\pi f} (-1) = \boxed{\frac{a_1}{\alpha_1 + j2\pi f}}$$

$$\text{and } \int_0^{\infty} a_2 e^{-\alpha_2 \tau} e^{-j2\pi f \tau} d\tau = \boxed{\frac{a_2}{\alpha_2 + j2\pi f}}$$

$$\text{hence } \boxed{S_z(f) = \frac{a_1}{\alpha_1 - j2\pi f} + \frac{a_2}{\alpha_2 - j2\pi f} + \frac{a_1}{\alpha_1 + j2\pi f} + \frac{a_2}{\alpha_2 + j2\pi f}}$$

This can be simplified more as follows.

$$S_z(f) = \frac{a_1(\alpha_1 + j2\pi f) + a_1(\alpha_1 - j2\pi f)}{(\alpha_1 - j2\pi f)(\alpha_1 + j2\pi f)} + \frac{a_2(\alpha_2 + j2\pi f) + a_2(\alpha_2 - j2\pi f)}{(\alpha_2 - j2\pi f)(\alpha_2 + j2\pi f)}$$

$$= \boxed{\frac{2a_1\alpha_1}{\alpha_1^2 + 4\pi^2 f^2} + \frac{2a_2\alpha_2}{\alpha_2^2 + 4\pi^2 f^2}}$$