Missing Solutions for HW#5

EE 443 HW#5

1.15)

$$y(t) = \int_{t-T}^{t} x(\tau_1)\,d\tau_1 \qquad (1)$$

Method # 1

When $x(t) = \delta(t) \Rightarrow y(t) = h(t)$, Thus

$$h(t) = \int_{t-T}^{t} \delta(t)\,dt = u(t) - u(t-T) = rect\left(\frac{t-T/2}{T}\right)$$

Thus $H(f) = F.T[h(t)] = T\,Sinc(fT)\,e^{-j\pi fT}$

Method # 2

Differentiate eq (1):

$$\frac{dy(t)}{dt} = x(t) - x(t-T) \qquad (2)$$

Take F.T of eq (2)

$$Y(f) \cdot j2\pi f = X(f) - X(f)\,e^{-j2\pi fT}$$

$$Y(f) = \frac{X(f)}{j2\pi f}\left[1 - e^{-j2\pi fT}\right] = \frac{X(f)}{j2\pi f} \cdot e^{-j\pi fT}\left[e^{+j\pi fT} - e^{-j\pi fT}\right]$$

$$\Rightarrow Y(f) = X(f) \cdot T\,e^{-j\pi fT}\,Sinc(fT)$$

$$\frac{Y(f)}{X(f)} = T\,e^{-j\pi fT}\,Sinc(fT)$$

Problem 1.17

The autocorrelation function of X(t) is

$$R_X(\tau) = E[X(t+\tau)\, X(t)]$$

$$= A^2\, E[\cos(2\pi Ft + 2\pi F\tau - \theta)\cos(2\pi Ft - \theta)]$$

$$= \frac{A^2}{2} E[\cos(4\pi Ft + 2\pi F\tau - 2\theta) + \cos(2\pi F\tau)]$$

Averaging over Θ, and noting that Θ is uniformly distributed over 2π radians, we get

$$R_X(\tau) = \frac{A^2}{2} E[\cos(2\pi F\tau)]$$

$$= \frac{A^2}{2} \int_{-\infty}^{\infty} f_F(f)\cos(2\pi f\tau)\, df$$

Next, we note that $R_X(\tau)$ is related to the power spectral density by

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(f)\cos(2\pi f\tau)\, df$$

Therefore, comparing Eqs. (1) and (2), we deduce that the power spectral density of X(t) is

$$S_X(f) = \frac{A^2}{2} f_F(f)$$

When the frequency assumes a constant value, f_c (say), we have

$$f_F(f) = \frac{1}{2}\delta(f-f_c) + \frac{1}{2}\delta(f+f_c)$$

E12

Thus: $S_X(f) = \frac{A^2}{4}\left\{\delta(f-f_c) + \delta(f+f_c)\right\}$

EE 443 HW # 6

Key

8.35

$$S_X(f) = Tri(f) = \begin{cases} 1-|f|, & |f| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$R_X(\tau) = F^{-1}T[S_X(f)] = Sinc^2(\tau)$$

since $Tri(t) \xleftrightarrow{F.T} Sinc^2(f)$

8.32)

$$R_X(\tau) = \begin{cases} \sigma^2(1-|\tau|) & |\tau| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

using $Tri(t) \longleftrightarrow Sinc^2(f)$

$R_X(\tau) = \sigma^2 \, Tri(\tau)$ thus:

$$S_X(f) = F.T[R_X(\tau)] = \sigma^2 Sinc^2(f)$$

√ Problem 1.12

(a) The power spectral density consists of two components:

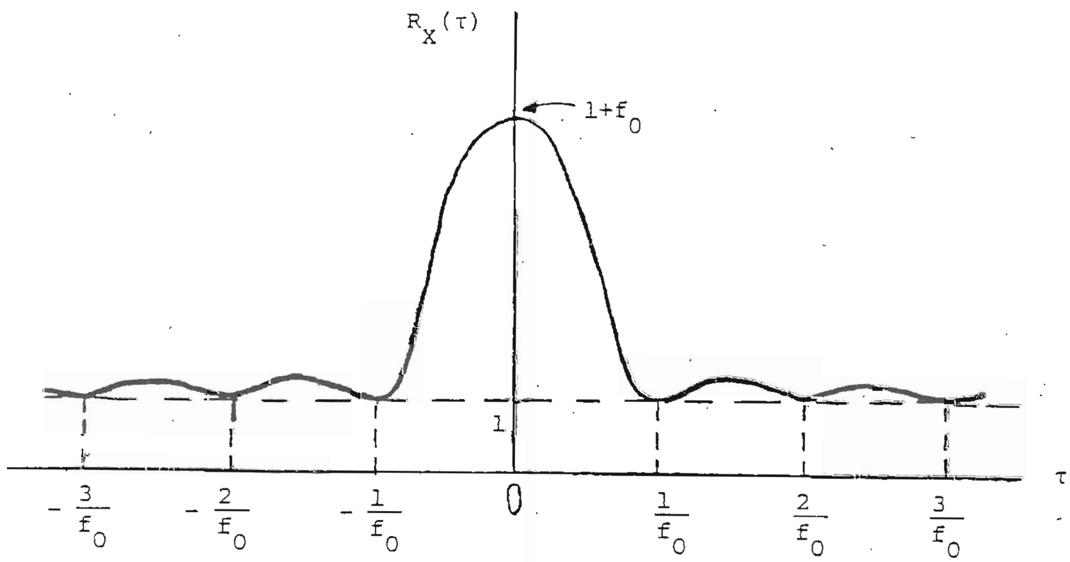
(1) A delta function $\delta(t)$ at the origin, whose inverse Fourier transform is one.

(2) A triangular component of unit amplitude and width $2f_0$, centered at the origin; the inverse Fourier transform of this component is $f_0 \, \text{sinc}^2(f_0\tau)$.

Therefore, the autocorrelation function of X(t) is

$$R_X(\tau) = 1 + f_0 \, \text{sinc}^2(f_0\tau)$$

which is sketched below:



$$= \cos[2\pi(t_1-t_2)]$$

(b) Since $R_X(\tau)$ contains a constant component of amplitude 1, it follows that the dc power contained in X(t) is 1.

(c) The mean-square value of X(t) is given by

$$E[X^2(t)] = R_X(0)$$

$$= 1 + f_0$$

The ac power contained in X(f) is therefore equal to f_0.

(d) If the sampling rate is f_0/n, where n is an integer, the samples are uncorrelated. They are not, however, statistically independent. They would be statistically independent if X(t) were a Gaussian process.