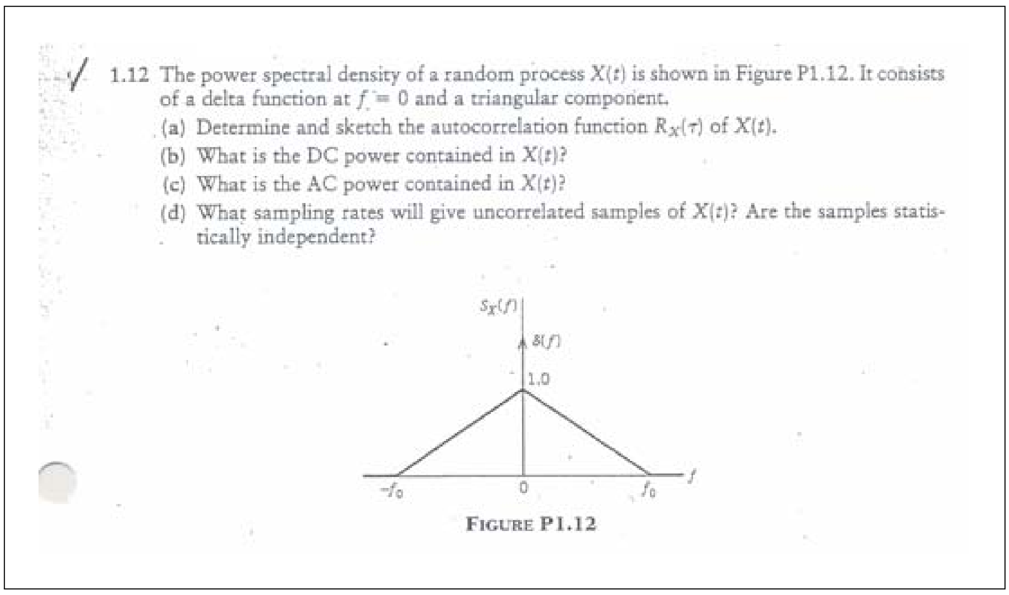
HW 5
Electronic Communication Systems
Fall 2008
California State University, Fullerson

Nasser M. Abbasi

Fall 2008 Compiled on May 29, 2019 at 11:41pm

Contents

1 Problem 1


1.12 The power spectral density of a random process $X(t)$ is shown in Figure P1.12. It consists of a delta function at $f = 0$ and a triangular component.

(a) Determine and sketch the autocorrelation function $R_X(\tau)$ of $X(t)$.

(b) What is the DC power contained in $X(t)$?

(c) What is the AC power contained in $X(t)$?

(d) What sampling rates will give uncorrelated samples of $X(t)$? Are the samples statistically independent?

FIGURE P1.12

Figure 1: the Problem statement

1.1 Part(a)

Assuming stationary process,

$$R_x(\tau) \Leftrightarrow S_x(f)$$

But $S_x(f) = \delta(f) + tri\left(\frac{f}{2f_0}\right)$, hence

$$\begin{aligned} R_x(\tau) &= F^{-1}\left(\delta(f) + tri\left(\frac{f}{2f_0}\right)\right) \\ &= \int_{-\infty}^{\infty}\left[\delta(f) + tri\left(\frac{f}{2f_0}\right)\right] e^{j2\pi f\tau} df \end{aligned}$$

But $F^{-1}\left(tri\left(\frac{f}{2f_0}\right)\right) = f_0 \frac{\sin^2(f_0\pi\tau)}{f_0^2\pi^2\tau^2}$, and $F^{-1}(\delta(f)) = 1$, hence the above becomes

Hence

$$R_x(\tau) = \overbrace{1}^{\text{dc part}} + \overbrace{f_0 \operatorname{sinc}^2(f_0\tau)}^{\text{AC part}}$$

1.2 Part(b)

$$P_x(0) = 1 + f_0$$

Hence DC power in $X(t)$ is given 1 watt.

1.3 Part(c)

The AC power is f_0 watt.

1.4 Part(d)

Since $R_x(\tau) = 1 + f_0 \operatorname{sinc}^2(f_0\tau)$, we need to make this zero. But this has no real root as solution (assuming $f_0 \geq 0$)

To obtain a solution, I will only consider the AC part.

Hence we need to solve for τ in

$$R_x(\tau) = f_0 \operatorname{sinc}^2(f_0\tau) = 0$$

i.e. the AC part only.

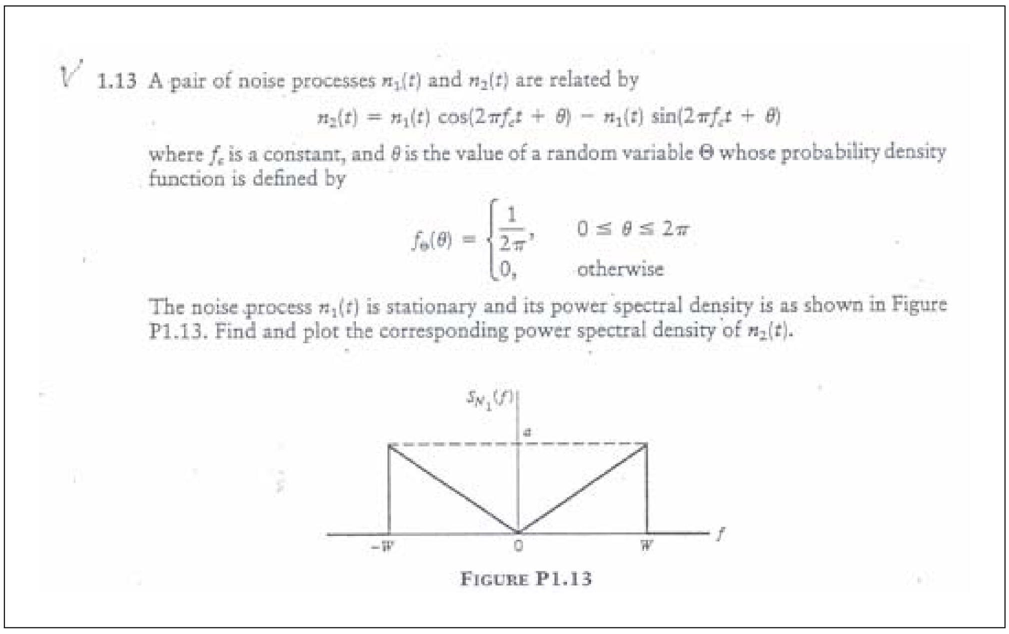
This is zero when $\text{sinc}^2(f_0\tau) = 0$ or when $\sin(\pi f_0 \tau) = 0$ or when

$\pi f_0 \tau = k\pi,\ k = \pm 1, \pm 2, \cdots.$

Hence when

$$\tau = \pm\frac{1}{f_0}, \pm\frac{2}{f_0}, \cdots$$

2 Problem 2



1.13 A pair of noise processes $n_1(t)$ and $n_2(t)$ are related by

$$n_2(t) = n_1(t)\cos(2\pi f_c t + \theta) - n_1(t)\sin(2\pi f_c t + \theta)$$

where f_c is a constant, and θ is the value of a random variable Θ whose probability density function is defined by

$$f_\Theta(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \le \theta \le 2\pi \\ 0, & \text{otherwise} \end{cases}$$

The noise process $n_1(t)$ is stationary and its power spectral density is as shown in Figure P1.13. Find and plot the corresponding power spectral density of $n_2(t)$.

FIGURE P1.13

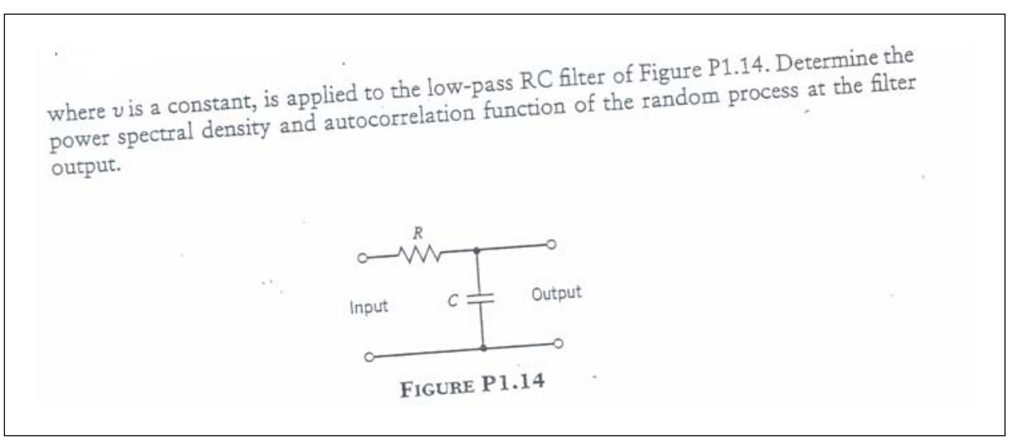
Figure 2: the Problem statement

(see graded HW for solution)

3 Problem 3

A random telegraph signal $X(t)$ charaterized by the autocorrelation function

$$R_X(\tau) - e^{-2\nu|tau|}$$



where υ is a constant, is applied to the low-pass RC filter of Figure P1.14. Determine the power spectral density and autocorrelation function of the random process at the filter output.

FIGURE P1.14

Figure 3: the Problem statement

Let $S_y(f)$ be the psd of the output, then

$$S_y(f) = S_x(f)\,|H(f)|^2$$

But

$$
\begin{aligned}
S_x(f) &= \mathcal{F}\left(R_x(\tau)\right) \\
&= \int_{-\infty}^{0} e^{2v\tau} e^{-j2\pi f\tau} d\tau + \int_{0}^{\infty} e^{-2v\tau} e^{-j2\pi f\tau} d\tau \\
&= \int_{-\infty}^{0} e^{\tau(2v-j2\pi f)} d\tau + \int_{0}^{\infty} e^{\tau(-2v-j2\pi f)} d\tau \\
&= \frac{\left[e^{\tau(2v-j2\pi f)}\right]_{-\infty}^{0}}{2v-j2\pi f} + \frac{\left[e^{\tau(-2v-j2\pi f)}\right]_{0}^{\infty}}{-2v-j2\pi f} \\
&= \frac{1}{2v-j2\pi f} + \frac{-1}{-2v-j2\pi f} \\
&= \frac{1}{2v-j2\pi f} + \frac{1}{2v+j2\pi f} \\
&= \frac{4v}{4v^2+4\pi^2 f^2}
\end{aligned}
$$

Now we need to find $H(f)$. Using voltage divider $H(f) = \frac{Y(f)}{X(f)} = \frac{\frac{1}{j2\pi fC}}{R+\frac{1}{j2\pi fC}}$ hence

$$
H(f) = \frac{1}{j2\pi fRC+1}
$$

Hence

$$
|H(f)| = \frac{1}{\sqrt{1+(2\pi fRC)^2}}
$$

Then

$$
\begin{aligned}
S_y(f) &= S_x(f)\,|H(f)|^2 \\
&= \left(\frac{4v}{4v^2+4\pi^2 f^2}\right)\left(\frac{1}{1+(2\pi fRC)^2}\right) \\
&= \frac{4v}{(4v^2+4\pi^2 f^2)(1+4\pi^2 f^2 R^2 C^2)} \\
&= \frac{4v}{4v^2+4v^2(2\pi fRC)^2+4\pi^2 f^2+4\pi^2 f^2(2\pi fRC)^2} \\
&= \frac{4v}{4v^2+16v^2\pi^2 f^2 R^2 C^2+4\pi^2 f^2+16\pi^2 f^2\pi^2 f^2 R^2 C^2} \\
&= \frac{v}{v^2+4v^2\pi^2 f^2 R^2 C^2+\pi^2 f^2+4\pi^4 f^4 R^2 C^2}
\end{aligned}
$$

Now, $R_y(\tau)$ is the inverse Fourier transform of the above.

4 Problem 4

1.15 A *running integrator* is defined by

$$y(t) = \int_{t-T}^{t} x(\tau)\, d\tau$$

where $x(t)$ is the input, $y(t)$ is the output, and T is the integration period. Both $x(t)$ and $y(t)$ are sample functions of stationary processes $X(t)$ and $Y(t)$, respectively. Show that the power spectral density of the integrator output is related to that of the integrator input as

$$S_Y(f) = T^2 \operatorname{sinc}^2(fT) S_X(f)$$

Figure 4: the Problem statement

(see graded HW for solution)

5 Key solution

Missing Solutions for HW#5 page 1

EE 443 HW#5

1.15)

1.15)

$$y(t) = \int_{t-T}^{t} x(\tau_1)\, d\tau_1 \qquad (1)$$

Method # 1

When $x(t) = \delta(t) \Rightarrow y(t) = h(t)$, Thus

$$h(t) = \int_{t-T}^{t} \delta(t)\, dt = u(t) - u(t-T) = \operatorname{rect}\left(\frac{t - T/2}{T}\right)$$

Thus $H(f) = F.T[h(t)] = T \operatorname{Sinc}(fT)\, e^{-j\pi f T}$

Method # 2

Differentiate eq (1) :

$$\frac{dy(t)}{dt} = x(t) - x(t-T) \qquad (2)$$

Take F.T of eq (2)

$$Y(f) \cdot j2\pi f = X(f) - X(f)\, e^{-j2\pi f T}$$

$$Y(f) = \frac{X(f)}{j2\pi f}\left[1 - e^{-j2\pi f T}\right] = \frac{X(f)}{j2\pi f} \cdot e^{-j\pi f T}\left[e^{+j\pi f T} - e^{-j\pi}\right.$$

$$\Rightarrow Y(f) = X(f) \cdot T\, e^{j\pi f T} \operatorname{Sinc}(fT)$$

$$Y(f) = T\, e^{-j\pi f T} \operatorname{Sinc}(fT)$$

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Problem 1.17

The autocorrelation function of X(t) is

$$R_X(\tau) = E[X(t+\tau)\, X(t)]$$

$$= A^2\, E[\cos(2\pi Ft + 2\pi F\tau - \Theta)\cos(2\pi Ft - \Theta)]$$

$$= \frac{A^2}{2} E[\cos(4\pi Ft + 2\pi F\tau - 2\Theta) + \cos(2\pi F\tau)]$$

Averaging over Θ, and noting that Θ is uniformly distributed over 2π radians, we get

$$R_X(\tau) = \frac{A^2}{2} E[\cos(2\pi F\tau)]$$

$$= \frac{A^2}{2} \int_{-\infty}^{\infty} f_F(f) \cos(2\pi f\tau)\, df$$

Next, we note that $R_X(\tau)$ is related to the power spectral density by

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(f) \cos(2\pi f\tau)\, df$$

Therefore, comparing Eqs. (1) and (2), we deduce that the power spectral density of X(t) is

$$S_X(f) = \frac{A^2}{2} f_F(f)$$

When the frequency assumes a constant value, f_c (say), we have

$$f_F(f) = \frac{1}{2}\delta(f-f_c) + \frac{1}{2}\delta(f+f_c)$$

E12

Thus: $S_X(f) = \frac{A^2}{4}\{\delta(f-f_c) + \delta(f+f_c)\}$

EE 443 HW # 6

Key

8.35

$$S_x(f) = Tri(f) = \begin{cases} 1-|f|, & |f| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$R_x(\tau) = F^{-1}_T[S_x(f)] = \text{sinc}^2(\tau)$$

since $Tri(t) \overset{F.T}{\longleftrightarrow} \text{sinc}^2(f)$

8.32)

$$R_x(\tau) = \begin{cases} \sigma^2(1-|\tau|) & |\tau| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

using $Tri(t) \longleftrightarrow \text{sinc}^2(f)$

$R_x(\tau) = \sigma^2\, Tri(\tau)$ thus ;

$$S_x(f) = F.T[R_x(\tau)] = \sigma^2 \text{sinc}^2(f)$$

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Problem 1.12

(a) The power spectral density consists of two components:

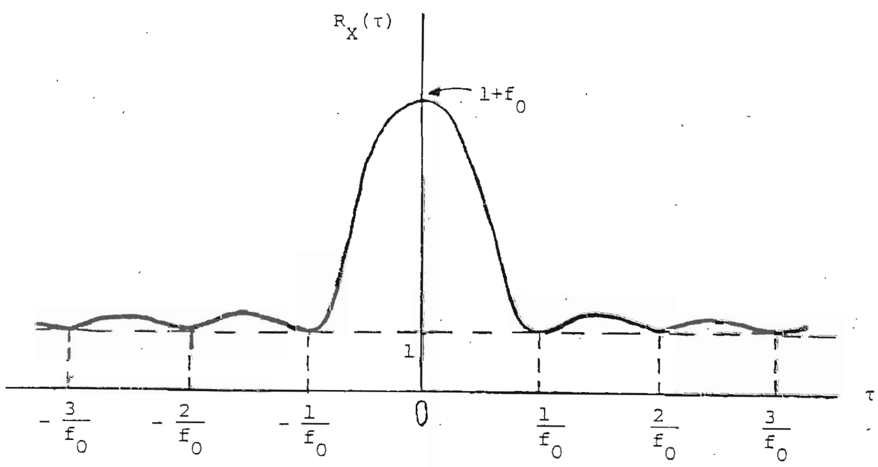
(1) A delta function $\delta(t)$ at the origin, whose inverse Fourier transform is one.

(2) A triangular component of unit amplitude and width $2f_0$, centered at the origin; the inverse Fourier transform of this component is $f_0 \operatorname{sinc}^2(f_0\tau)$.

Therefore, the autocorrelation function of X(t) is

$$R_X(\tau) = 1 + f_0 \operatorname{sinc}^2(f_0\tau)$$

which is sketched below:



$= \cos[2\pi(t_1-t_2)]$

(b) Since $R_X(\tau)$ contains a constant component of amplitude 1, it follows that the dc power contained in X(t) is 1.

(c) The mean-square value of X(t) is given by

$$E[X^2(t)] = R_X(0)$$
$$= 1 + f_0$$

The ac power contained in X(f) is therefore equal to f_0.

(d) If the sampling rate is f_0/n, where n is an integer, the samples are uncorrelated. They are not, however, statistically independent. They would be statistically independent if X(t) were a Gaussian process.

6 my graded HW

HW5, EGEE 443. CSUF, Fall 2008

Nasser Abbasi

October 16, 2008

14.8/20 14/20

1 Problem 1

✓ 1.12 The power spectral density of a random process $X(t)$ is shown in Figure P1.12. It consists of a delta function at $f = 0$ and a triangular component.

(a) Determine and sketch the autocorrelation function $R_X(\tau)$ of $X(t)$.

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(c) What is the AC power contained in $X(t)$?

(d) What sampling rates will give uncorrelated samples of $X(t)$? Are the samples statistically independent?

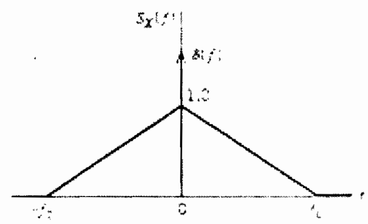


FIGURE P1.12

Solution

1.1 Part(a)

Assuming stationary process,

$$R_x(\tau) \Leftrightarrow S_x(f)$$

$tri\left(\frac{f}{f_0}\right)$

But $S_x(f) = \delta(f) + tri\left(\frac{f}{2f_0}\right)$, hence

$$R_x(\tau) = F^{-1}\left(\delta(f) + tri\left(\frac{f}{2f_0}\right)\right)$$

$$= \int_{-\infty}^{\infty} \left[\delta(f) + tri\left(\frac{f}{2f_0}\right)\right] e^{j2\pi f\tau} df$$

-0.5

But $F^{-1}\left(tri\left(\frac{f}{2f_0}\right)\right) = f_0 \frac{\sin^2(f_0\pi\tau)}{f_0^2\pi^2\tau^2}$, and $F^{-1}(\delta(f)) = 1$, hence the above becomes

Hence

$$\boxed{R_x(\tau) = \overbrace{1}^{\text{dc part}} + \overbrace{f_0 \operatorname{sinc}^2(f_0\tau)}^{\text{AC part}}}$$

1.2 Part(b)

$$P_x(0) = 1 + f_0$$

Hence DC power in $X(t)$ is given $\boxed{\text{1 watt.}}$

1.3 Part(c)

The AC power is $\boxed{f_0 \text{ watt}}$.

1.4 Part(d)

Since $R_x(\tau) = 1 + f_0 \operatorname{sinc}^2(f_0\tau)$, we need to make this zero. But this has no real root as solution (assuming $f_0 \geq 0$)

To obtain a solution, I will only consider the AC part.

Hence we need to solve for τ in

$$R_x(\tau) = f_0 \operatorname{sinc}^2(f_0\tau) = 0$$

i.e. the AC part only.

This is zero when $\operatorname{sinc}^2(f_0\tau) = 0$ or when $\sin(\pi f_0\tau) = 0$ or when

$\pi f_0\tau = k\pi$, $k = \pm 1, \pm 2, \cdots$.

Hence when

$$\tau = \pm\frac{1}{f_0}, \pm\frac{2}{f_0}, \cdots$$

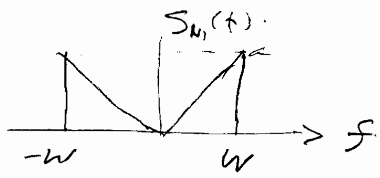
→ $\tau = \pm \frac{k}{f_0}$

-0.5

~~sampling~~

are the samples statistically independent?

\# 2



4

$$R_{n_2}(\tau) = E\{ N_2(t) \quad N_2(t+\tau) \}$$

$$= E\{ (N_1(t)\cos(w_c t+\theta) - N_1(t)\sin(w_c t+\theta))$$
$$(N_1(t+\tau)\cos(w_c t+w_c\tau+\theta) - N_1(t+\tau)\sin(w_c t+w_c\tau+\theta)) \}$$

$$= E\{ N_1(t)\, N_1(t+\tau)\cos(w_c t+\theta)\cos(w_c t+w_c\tau+\theta)$$
$$- N_1(t)\, N_1(t+\tau)\sin(w_c t+\theta)\cos(w_c t+w_c\tau+\theta)$$
$$- N_1(t)\, N_1(t+\tau)\cos(w_c t+\theta)\sin(w_c t+w_c\tau+\theta)$$
$$+ N_1(t)\, N_1(t+\tau)\sin(w_c t+\theta)\sin(w_c t+w_c\tau+\theta) \}$$

$$= E\{ N_1(t) N_1(t+\tau) \} \; E\{ \cos(w_c t+\theta)\cos(w_c t+w_c\tau+\theta)$$
$$- \sin(w_c t+\theta)\cos(w_c t+w_c\tau+\theta)$$
$$- \cos(w_c t+\theta)\sin(w_c t+w_c\tau+\theta)$$
$$+ \sin(w_c t+\theta)\sin(w_c t+w_c\tau+\theta) \}$$

$$R_{N_2}(\tau) = R_{N_1}(\tau) \; E\{ \quad " \quad \}$$

using $\cos A \cos B = \frac{1}{2}(\cos(A-B) + \cos(A+B))$
$\sin A \cos B = \frac{1}{2}(\sin(A-B) + \sin(A+B))$ the above becomes
$\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$

$$= R_{N_1}(\tau)\, E\{ \tfrac{1}{2}(\cos(w_c t+\theta - w_c t - w_c\tau - \theta) + \cos(w_c t+\theta+w_c t+w_c\tau+\theta))$$
$$- \tfrac{1}{2}(\sin(w_c t+\theta - w_c t - w_c\tau - \theta) + \sin(w_c t+\theta+w_c t+w_c\tau+\theta))$$
$$- \tfrac{1}{2}(\sin(w_c t+w_c\tau+\theta - w_c t - \theta) + \sin(w_c t+\theta+w_c t+w_c\tau+\theta))$$
$$+ \tfrac{1}{2}(\cos(w_c t+\theta - w_c t - w_c\tau - \theta) - \cos(w_c t+\theta+w_c t+w_c\tau+\theta)) \} \longrightarrow$$

$$= R_{N_1}(\tau)\; E\Big\{ \tfrac{1}{2}\big(\cos(-\omega_c\tau) + \cos(2\omega_c t + 2\theta + \omega_c\tau)\big) \qquad 2$$

$$-\tfrac{1}{2}\big(\sin(-\omega_c\tau) + \sin(2\omega_c t + 2\theta + \omega_c\tau)\big)$$

$$-\tfrac{1}{2}\big(\sin(\omega_c\tau) + \sin(2\omega_c t + 2\theta + \omega_c\tau)\big)$$

$$+\tfrac{1}{2}\big(\cos(-\omega_c\tau) - \cos(2\omega_c t + 2\theta + \omega_c\tau)\big)\Big\}$$

let $\alpha = 2\omega_c t + 2\theta + \omega_c\tau$

$$= R_{N_1}(\tau)\; E\Big\{ \tfrac{1}{2}\big(\cos(\omega_c\tau) + \cos\alpha\big) - \tfrac{1}{2}\big(-\sin(\omega_c\tau) + \sin\alpha\big)$$

$$-\tfrac{1}{2}\big(\sin(\omega_c\tau) + \sin\alpha\big) + \tfrac{1}{2}\big(\cos\omega_c\tau - \cos\alpha\big)\Big\}$$

$$= R_N(\tau)\; E\Big\{ \tfrac{1}{2}\cos\omega_c\tau + \tfrac{1}{2}\cos\alpha + \tfrac{1}{2}\sin(\omega_c\tau) - \tfrac{1}{2}\sin\alpha$$

$$-\tfrac{1}{2}\sin\omega_c\tau - \tfrac{1}{2}\sin\alpha + \tfrac{1}{2}\cos\omega_c\tau - \tfrac{1}{2}\cos\alpha\Big\}$$

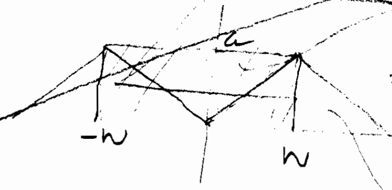
$$= R_N(\tau)\; E\{\cos\omega_c\tau - \sin\alpha\}$$

$$= R_N(\tau)\Big(E\underbrace{(\cos\omega_c\tau)}_{\text{constant}} - \underbrace{E\big(\sin(2\omega_c\tau + 2\theta + \omega_c\tau)\big)}_{=0}\Big)$$

so $\boxed{R_{N_2}(\tau) = \cos(\omega_c\tau)\, R_{N_1}(\tau)}$

~~Now we find $R_{N_1}(T)$ as inverse Fourier Transform of $S_{N_1}(f)$~~

~~$S_{N_1}(f)$ can be written as 2 triangular functions times a rect function as follows~~



$$= a\,\mathrm{Rect}\left(\frac{f}{2w}\right)\left(\mathrm{tri}\left(\frac{f+w}{2w}\right) + \mathrm{tri}\left(\frac{f-w}{2w}\right)\right)$$

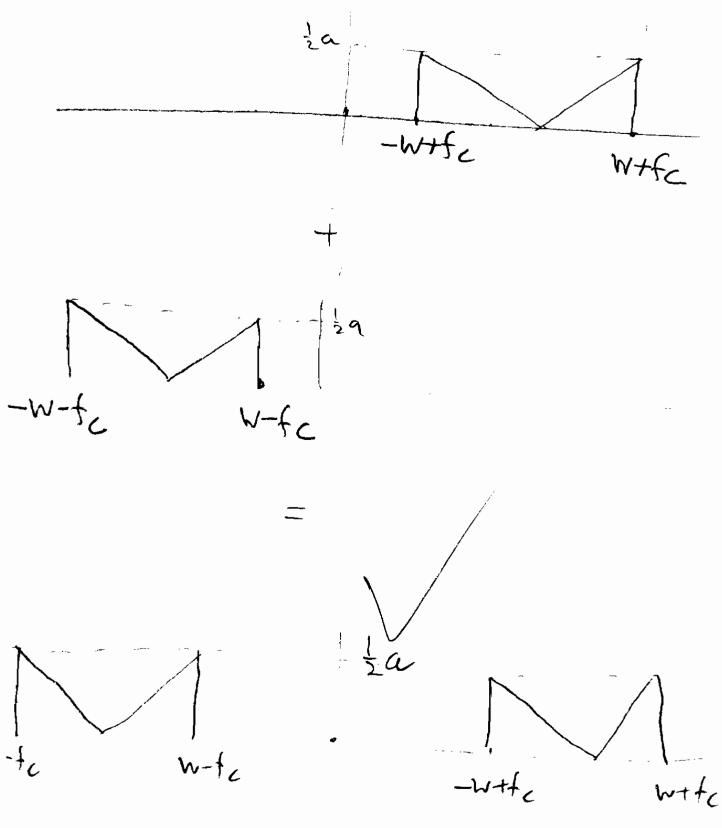
so $S_{N_2}(f) = F.T.\left[\cos(w_c\tau)\, R_{N_1}(\tau)\right]$

$$= F.T.\left[\cos(w_c\tau)\right] \otimes F.T.\left[R_{N_1}(\tau)\right]$$

But $F.T.\left[R_{N_1}(\tau)\right]$ is given as its $S_{N_1}(f)$.

and $F.T.\left[\cos(w_c\tau)\right] = \frac{1}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$

so $S_{N_2}(f)$ is $\boxed{\frac{1}{2}\left[S_{N_1}(f-f_c) + S_{N_1}(f+f_c)\right]}$



$S_{N_2}(f)$

3 Problem 3

1.14 A *random telegraph signal* $X(t)$, characterized by the autocorrelation function

$$R_X(\tau) = \exp(-2v|\tau|)$$

where v is a constant, is applied to the low-pass RC filter of Figure P1.14. Determine the power spectral density and autocorrelation function of the random process at the filter output.

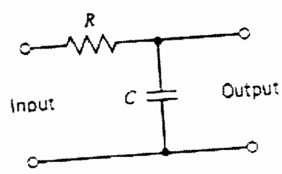


FIGURE P1.14

Let $S_y(f)$ be the psd of the output, then

$$S_y(f) = S_x(f)\,|H(f)|^2$$

But

$$\begin{aligned}
S_x(f) &= \mathcal{F}(R_x(\tau)) \\
&= \int_{-\infty}^{0} e^{2v\tau} e^{-j2\pi f\tau} d\tau + \int_{0}^{\infty} e^{-2v\tau} e^{-j2\pi f\tau} d\tau \\
&= \int_{-\infty}^{0} e^{\tau(2v-j2\pi f)} d\tau + \int_{0}^{\infty} e^{\tau(-2v-j2\pi f)} d\tau \\
&= \frac{\left[e^{\tau(2v-j2\pi f)}\right]_{-\infty}^{0}}{2v-j2\pi f} + \frac{\left[e^{\tau(-2v-j2\pi f)}\right]_{0}^{\infty}}{-2v-j2\pi f} \\
&= \frac{1}{2v-j2\pi f} + \frac{-1}{-2v-j2\pi f} \\
&= \frac{1}{2v-j2\pi f} + \frac{1}{2v+j2\pi f} \\
&= \frac{4v}{4v^2+4\pi^2 f^2}
\end{aligned}$$

Now we need to find $H(f)$. Using voltage divider $H(f) = \frac{Y(f)}{X(f)} = \frac{\frac{1}{j2\pi fC}}{R+\frac{1}{j2\pi fC}}$

hence

$$H(f) = \frac{1}{j2\pi f RC + 1}$$

Hence

$$|H(f)| = \frac{1}{\sqrt{1 + (2\pi f RC)^2}}$$

Then

$$\begin{aligned}
S_y(f) &= S_x(f)\,|H(f)|^2 \\
&= \left(\frac{4v}{4v^2 + 4\pi^2 f^2}\right)\left(\frac{1}{1 + (2\pi f RC)^2}\right) \\
&= \frac{4v}{(4v^2 + 4\pi^2 f^2)(1 + 4\pi^2 f^2 R^2 C^2)} \\
&= \frac{4v}{4v^2 + 4v^2(2\pi f RC)^2 + 4\pi^2 f^2 + 4\pi^2 f^2 (2\pi f RC)^2} \\
&= \frac{4v}{4v^2 + 16v^2\pi^2 f^2 R^2 C^2 + 4\pi^2 f^2 + 16\pi^2 f^2 \pi^2 f^2 R^2 C^2} \\
&= \frac{v}{v^2 + 4v^2\pi^2 f^2 R^2 C^2 + \pi^2 f^2 + 4\pi^4 f^4 R^2 C^2}
\end{aligned}$$

Now, $R_y(\tau)$ is the inverse Fourier transform of the above.

P.S. is there a trick to find $F.T.^{-1}$ of above fast ?

See sol. →

4 Problem 4

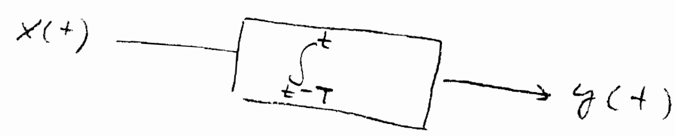
1.15 A *running integrator* is defined by

$$y(t) = \int_{t-T}^{t} x(\tau)\, d\tau$$

where $x(t)$ is the input, $y(t)$ is the output, and T is the integration period. Both $x(t)$ and $y(t)$ are sample functions of stationary processes $X(t)$ and $Y(t)$, respectively. Show that the power spectral density of the integrator output is related to that of the integrator input as

$$S_Y(f) = T^2 \operatorname{sinc}^2(fT) S_X(f)$$

$$y(t) = \int_{t-T}^{t} x(\tau)\, d\tau .$$



$$S_Y(f) = S_X(f)\,|H(f)|^2 .$$

Now

$$R_y(\tau) = E\{y(t)\ y(t+\tau)\}$$

$$= E\left\{\int_{t-T}^{t} x(t_1)\,dt_1 \int_{t+\tau-T}^{t+\tau} x(t_2)\,dt_2\right\}$$

$$= \int_{t-T}^{t} \int_{t+\tau-T}^{t+\tau} \underbrace{E\{x(t_1)\,x(t_2)\}}_{\text{autocorrelation of } x(t)}\, dt_1\, dt_2$$

$$= \int_{t-T}^{t} \int_{t+\tau-T}^{t+\tau} R_x(\tau)\, dt_1 dt_2$$

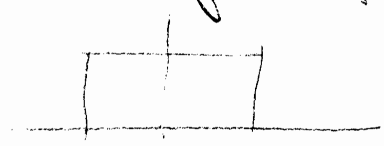
$$R_y(\tau) = R_x(\tau) \int_{t-T}^{t} \int_{t+\tau-T}^{t+\tau} dt_1 dt_2 \quad \Rightarrow R_y(\tau) = R_x(\tau)\, T^2$$

So taking F.T of each side, we obtain

$$\boxed{S_Y(f) = T^2\, S_X(f).}$$

P.S I don't know how to get the sinc function in there!

I know that Running integrator is L.P.F.



X −3

and its F.T. is sinc. fuction, but I did not know if I could use this fact in solving this problem.

Remember :

$$y(t) = x(t) \circledast h(t)$$

when $x(t) = \delta(t) \Rightarrow y(t) = \delta(t) \circledast h(t) = h(t)$ → impulse response

$$\therefore \quad h(t) = y(t) = \int_{t-T}^{t} \delta(\tau)\, d\tau$$

See sol.