

HW 4
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1 questions and hints

EE 443

HW # 4

page 1

Problem 1 Evaluate the transfer function of a linear system represented by the block diagram shown in Fig. P2.14.

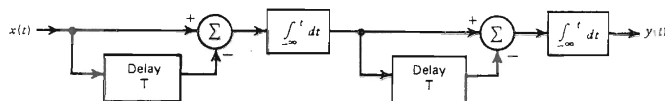


Figure P2.14

Problem 2.

- Determine the overall amplitude response of the cascade connection shown in P2.15, consisting of N identical stages, each with a time constant RC equal to τ_0 .
- Show that as N approaches infinity, the amplitude response of the cascade connection approaches the Gaussian function $\exp(-\frac{1}{2}f^2T^2)$, where for each value of N , the time constant τ_0 is selected so that

$$\tau_0^2 = \frac{T^2}{4\pi^2 N}$$

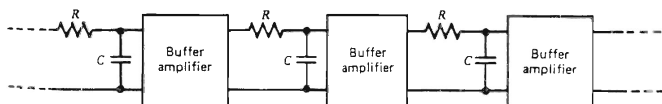


Figure P2.15

Problem 3. Determine the pre-envelope $g_p(t)$ corresponding to each of the following two signals:

(a) $g(t) = \sin(t)$

(b) $g(t) = [1 + k \cos(2\pi f_m t)] \cos(2\pi f_c t)$

(Hint use $\frac{\sin t}{t} \xleftrightarrow{H.T.} \frac{1 - \cos t}{t}$ See prob # 4)

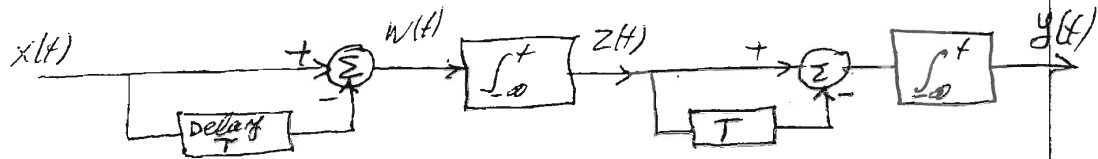
prob # 4) Verifying the following H.T.:

a) if $g(t) = \delta(t) \Rightarrow \hat{g}(t) = ?$

b) if $g(t) = \frac{\sin t}{t} \Rightarrow \hat{g}(t) = \frac{1 - \cos t}{t}$

prob # 5) Respond prob. 2.44 of your book

Prob # 1



Method # 1

Consider the first half the system. The second half is identical to the first half. $\Rightarrow$ If $H_1(f) = \frac{Z(f)}{X(f)}$, then $H(f) = \frac{Y(f)}{X(f)} = H_1(f) \times H_1(f) = H_1^2(f)$

• write time domain equations, then take their F.T.

$$w(t) = x(t) - x(t-T) \Rightarrow W(f) = X(f) [1 - e^{-j2\pi fT}]$$

$$z(t) = \int_{-\infty}^t w(t_1) dt_1 \Rightarrow Z(f) = \frac{W(f)}{j2\pi f} + \frac{1}{2} W(0) \delta(f) \quad (2)$$

Find $W(0)$, combine eq. (1) and (2) to find $H_1(f) = \frac{Z(f)}{X(f)}$

Method # 2

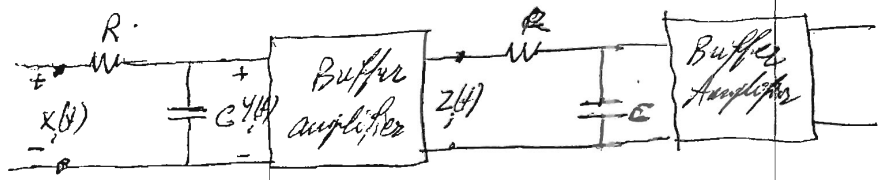
For first half find its impulse response $h_1(t)$. then $H_1(f) = F.T[h_1(t)]$.

to find $h_1(t)$: if $x(t) = \delta(t)$, then $z(t) = h_1(t)$

$$\text{That is } h_1(t) = \int_{-\infty}^t [\delta(t_1) - \delta(t_1 - T)] dt = ?$$

Prob # 2

a)



Note: Buffer Amplifier has unity gain $\Rightarrow z(t) = y(t)$
Thus the transfer function of the i th stage is?

$$H_i(f) = \frac{Z_i(f)}{X_i(f)} = \frac{Y_i(f)}{X_i(f)} = \frac{1}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j2\pi fRC}$$

prob #2 cont'd)

$$H_i(f) = \frac{1}{1 + j2\pi f \tau_0} \quad \text{where } \tau_0 \cong RC$$

- Find the overall $H(f)$, that is the transfer function of the cascade of N identical systems.
- Find the anglted response: that is $|H(f)| = ?$

$$b) \text{ Let } \tau_0^2 \cong \frac{\tau^2}{4\pi^2 N}$$

Use the definition of e number; that use:

$$\lim_{x \rightarrow \infty} \left(1 + \frac{\alpha}{x}\right)^{\pm \beta x} \cong \exp\left(\pm \frac{\alpha}{x} \beta x\right) = \exp(\alpha \beta)$$

$$\text{and find } \lim_{N \rightarrow \infty} |H(f)| = ?$$

prob #3

$$a) g(t) = \sin t, \quad \hat{g}(t) = ?$$

I) Time domain approach

$$\text{Use } \frac{\sin t}{t} \xrightarrow{H.T.} \frac{1 - \cos t}{t} \quad \text{see prob \#4}$$

$$\text{Thus } \frac{\sin \pi t}{\pi t} \xrightarrow{H.T.} \frac{1 - \cos \pi t}{\pi t}$$

$$\text{Use } g_+(t) = g(t) + j \hat{g}(t) \Rightarrow \begin{cases} \text{Ans } \hat{g}(t) = \sin(\frac{t}{2}) e^{j\frac{\pi t}{2}} \\ \text{Verify.} \end{cases}$$

II) Frequency domain approach:

Find $G(f)$, $G_+(f)$, $g_+(t)$ the find also

Complex envelope $\tilde{g}(t) = g_+(t) e^{-j2\pi f t}$ and

$$\text{Envelope } a(t) = |\tilde{g}(t)| = |g_+(t)| = ?$$

4) a) $g(t) = \delta(t)$, $\hat{g}(f) = ?$

You may use time or frequency domain approach.

b) $g(t) = \frac{\sin t}{t}$, $\hat{g}(f) = ?$ (Ans: $\hat{g}(f) = \frac{1}{f} (1 - \cos f)$)

Frequency domain approach: Remember

$$\text{rect}(t) \xleftrightarrow{\text{F.T.}} \text{sinc}(f)$$

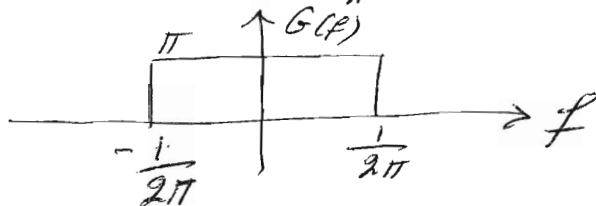
$$\text{sinc}(t) \xleftrightarrow{\text{F.T.}} \text{rect}(-f) = \text{rect}(f) \quad \text{Duality}$$

$$\text{Thus: } \text{sinc}(t) = \frac{\sin \pi t}{\pi t} \xleftrightarrow{\text{F.T.}} \text{rect}(f)$$

Using time scaling: If $x(t) \xleftrightarrow{\text{F.T.}} X(f)$
 then $x(at) \xleftrightarrow{\text{F.T.}} \frac{1}{|a|} X\left(\frac{f}{a}\right)$

in our case $a = \frac{1}{\pi}$:

$$g(t) = \frac{\sin t}{t} \xleftrightarrow{\text{F.T.}} \frac{1}{\frac{1}{\pi}} \text{rect}\left(\frac{f}{\frac{1}{\pi}}\right) = \pi \text{rect}\left(\frac{f}{\frac{1}{\pi}}\right) \Rightarrow$$



Now use:

$$\hat{G}(f) = -j \text{sgn}(f) G(f) = -j \pi \text{sgn}(f) \text{rect}\left(\frac{f}{\frac{1}{\pi}}\right)$$

continuous!

At some point you may use $\sin^2 x = \frac{1 - \cos 2x}{2}$

Prob. # 5) Use $R_g(\omega) = \bar{F}^T [S_g(f)]$

where $S_g(f) = \text{rect}\left(\frac{f}{4}\right) + \text{rect}\left(\frac{f}{2}\right)$

or assume 1

2 Problem 1

Solution Using transfer function cascading, then the overall transfer function for the system can be written as

$$H(f) = H_1(f) H_1(f) = [H_1(f)]^2 \quad (1)$$

Where

$$H_1(f) = \frac{Z(f)}{X(f)}$$

Where

$$\begin{aligned} Z(f) &= F \left\{ \int_{-\infty}^t w(\tau) d\tau \right\} \\ &= \frac{1}{j2\pi f} W(f) + \frac{W(0)}{2} \delta(f) \end{aligned} \quad (2)$$

Where

$$\begin{aligned} W(f) &= F \{x(t) - x(t - T)\} \\ &= X(f) - X(f) e^{-j2\pi f T} \\ &= X(f) [1 - e^{-j2\pi f T}] \end{aligned} \quad (3)$$

substitute (3) into (2) we obtain

$$\begin{aligned} Z(f) &= \frac{1}{j2\pi f} X(f) [1 - e^{-j2\pi f T}] + \frac{X(0) \overbrace{[1 - e^{-j2\pi 0 T}]}^{=0}}{2} \delta(f) \\ &= \frac{1}{j2\pi f} X(f) [1 - e^{-j2\pi f T}] \end{aligned} \quad (4)$$

Hence

$$\begin{aligned} H_1(f) &= \frac{Z(f)}{X(f)} \\ &= \frac{\frac{1}{j2\pi f} X(f) [1 - e^{-j2\pi f T}]}{X(f)} \end{aligned}$$

Hence

$$\boxed{H_1(f) = \frac{1}{j2\pi f} [1 - e^{-j2\pi f T}]}$$

Hence from (1)

$$\begin{aligned} H(f) &= \left(\frac{1}{j2\pi f} [1 - e^{-j2\pi f T}] \right)^2 \\ &= \frac{1}{-4\pi^2 f^2} [1 - e^{-j2\pi f T}]^2 \\ &= \frac{-1}{(2\pi f)^2} [1 - 2e^{-j2\pi f T} + e^{-j4\pi f T}] \end{aligned}$$

Hence

$$H(f) = \frac{1}{(2\pi f)^2} [2e^{-j2\pi fT} - e^{-j4\pi fT} - 1]$$

3 Problem 2

3.1 Part(a)

Transfer function for each stage is $H_i(f) = \frac{Y_i(f)}{X_i(f)} = \frac{1}{1+j2\pi fRC}$

Since $RC = \tau_0$, hence

$$H_i(f) = \frac{1}{1+j2\pi f\tau_0}$$

Then, for N stages, the overall transfer function is

$$H(f) = H_1(f) H_2(f) \cdots H_N(f)$$

Since they are identical stages, then the transfer function of each stage is the same, and the above becomes

$$H(f) = \left(\frac{1}{1+j2\pi f\tau_0} \right)^N$$

Hence the amplitude of the response is given by

$$\begin{aligned} |H(f)| &= \left(\frac{1}{|1+j2\pi f\tau_0|} \right)^N \\ &= \left(\frac{1}{\sqrt{1^2 + (2\pi f\tau_0)^2}} \right)^N \\ &= \left(\frac{1}{(1 + 4\pi^2 f^2 \tau_0^2)^{\frac{1}{2}}} \right)^N \\ &= \frac{1}{(1 + 4\pi^2 f^2 \tau_0^2)^{\frac{N}{2}}} \end{aligned}$$

Let $\tau_0^2 = \frac{\tau^2}{4\pi^2 N}$, the above becomes

$$|H(f)| = \frac{1}{\left(1 + \frac{f^2 \tau^2}{N} \right)^{\frac{N}{2}}} \quad (1)$$

3.2 Part (b)

Let $\alpha = f^2\tau^2, \beta = \frac{1}{2}$, then (1) becomes

$$|H(f)| = \frac{1}{\left(1 + \frac{\alpha}{N}\right)^{\beta N}}$$

But $\lim_{N \rightarrow \infty} \frac{1}{\left(1 + \frac{\alpha}{N}\right)^{\beta N}} = e^{-\alpha\beta}$, hence

$$\begin{aligned} |H(f)| &= \frac{1}{e^{\frac{f^2\tau^2}{2}}} \\ &= e^{-\frac{f^2\tau^2}{2}} \end{aligned}$$

Which is what we are asked to show.

4 Problem 3

4.1 Part(a)

(a) $g(t) = \text{sinc}(t)$

$$g_+(t) = g(t) + j\hat{g}(t) \tag{1}$$

Where $\hat{g}(t)$ is Hilbert transform of $g(t)$ defined as $\hat{g}(t) = g(t) \otimes \frac{1}{\pi t}$

$$\begin{aligned} \hat{G}(f) &= -j \text{sgn}(f) G(f) \\ &= -j \text{sgn}(f) \text{rect}(f) \end{aligned}$$

Now find the inverse Fourier transform.

I derive the above to answer problem 4 part (b). The answer is the following (please see problem 4 part(b) for the derivation

$$\hat{g}(t) = \frac{1}{\pi t} (1 - \cos \pi t)$$

In the above, I used $\text{sinc}(t) \equiv \frac{\sin \pi t}{\pi t}$. If one uses $\text{sinc}(t) \equiv \frac{\sin t}{t}$ then the answer becomes

$$\hat{g}(t) = \frac{1}{t} (1 - \cos t) \tag{2}$$

The problem statement seems to want us to use the second definition of $\text{sinc}(t)$, so I will continue the rest of the solution using (1).

Substitute (2) into (1) we obtain

$$\begin{aligned}
g_+(t) &= \text{sinc}(t) + j\frac{1}{t}(1 - \cos t) \\
&= \frac{\sin(t)}{t} + j\frac{1}{t}\left(1 - \frac{e^{jt} + e^{-jt}}{2}\right) \\
&= \frac{1}{t}\frac{e^{jt} - e^{-jt}}{2j} + \frac{1}{t}\left(j + \frac{e^{jt} + e^{-jt}}{2j}\right) \\
&= \frac{1}{t}\frac{e^{jt} - e^{-jt}}{2j} + \frac{j}{t} + \frac{1}{t}\frac{e^{jt} + e^{-jt}}{2j} \\
&= \frac{1}{t}\frac{e^{jt}}{2j} + \frac{j}{t} + \frac{1}{t}\frac{e^{jt}}{2j}
\end{aligned}$$

Hence

$$g_+(t) = \frac{1}{t}(j + e^{jt})$$

4.2 Part(b)

$$g(t) = [1 + k \cos 2\pi f_m t] \cos(2\pi f_c t)$$

$$g_+(t) = g(t) + j\hat{g}(t)$$

Where $\hat{g}(t)$ is Hilbert transform of $g(t)$ defined as $\hat{g}(t) = g(t) \otimes \frac{1}{\pi t}$.

$$G_+(f) = \begin{cases} 2G(f) & f > 0 \\ G(0) & f = 0 \\ 0 & f < 0 \end{cases}$$

But

$$G(f) = F[1 + k \cos 2\pi f_m t] \otimes F[\cos(2\pi f_c t)] \quad (1)$$

But

$$F[\cos(2\pi f_c t)] = \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$$

and

$$F[1 + k \cos 2\pi f_m t] = \delta(f) + \frac{k}{2}[\delta(f - f_m) + \delta(f + f_m)]$$

Hence (1) becomes

$$\begin{aligned}
G(f) &= \left\{ \delta(f) + \frac{k}{2}[\delta(f - f_m) + \delta(f + f_m)] \right\} \otimes \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)] \\
&= \delta(f) \otimes \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)] \\
&\quad + \frac{k}{2}[\delta(f - f_m) + \delta(f + f_m)] \otimes \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)] \\
&= \frac{1}{2}\delta(f) \otimes \delta(f - f_c) + \\
&\quad \frac{1}{2}\delta(f) \otimes \delta(f + f_c) + \\
&\quad \frac{k}{4}\delta(f - f_m) \otimes \delta(f - f_c) + \\
&\quad \frac{k}{4}\delta(f - f_m) \otimes \delta(f + f_c) + \\
&\quad \frac{k}{4}\delta(f + f_m) \otimes \delta(f - f_c) + \\
&\quad \frac{k}{4}\delta(f + f_m) \otimes \delta(f + f_c)
\end{aligned}$$

Hence

$$\begin{aligned}
G(f) &= \frac{1}{2}\delta(f + f_c) + \\
&\quad \frac{1}{2}\delta(f - f_c) + \\
&\quad \frac{k}{4}\delta(f - f_m + f_c) + \\
&\quad \frac{k}{4}\delta(f - f_m - f_c) + \\
&\quad \frac{k}{4}\delta(f + f_m + f_c) + \\
&\quad \frac{k}{4}\delta(f + f_m - f_c)
\end{aligned}$$

Hence for $f > 0$, $G_+(f) = 2G(f)$ and we obtain

$$G_+(f) = \delta(f - f_c) + \frac{k}{2}[\delta(f - f_m + f_c) + \delta(f - f_m - f_c) + \delta(f + f_m + f_c) + \delta(f + f_m - f_c)]$$

Then (since carrier frequency $f_c > f_m$), we could simplify the above, by keeping positive frequencies f

$$G_+(f) = \delta(f - f_c) + \frac{k}{2}[\delta(f - f_m - f_c) + \delta(f + f_m - f_c)]$$

or

$$G_+(f) = \delta(f - f_c) + \frac{k}{2}[\delta(f - (f_m + f_c)) + \delta(f - (f_c - f_m))]$$

Hence

$$\begin{aligned} g_+(t) &= e^{j2\pi f_c t} + \frac{k}{2}(e^{j2\pi(f_m + f_c)t} + e^{j2\pi(f_c - f_m)t}) \\ &= e^{j2\pi f_c t} + \frac{k}{2}(e^{j2\pi f_m t} e^{j2\pi f_c t} + e^{j2\pi f_c t} e^{-j2\pi f_m t}) \\ &= e^{j2\pi f_c t} \left[1 + \frac{k}{2}(e^{j2\pi f_m t} + e^{-j2\pi f_m t}) \right] \\ &= e^{j2\pi f_c t} \left[1 + \frac{k}{2}(2 \cos(2\pi f_m t)) \right] \\ &= e^{j2\pi f_c t} [1 + k \cos(2\pi f_m t)] \end{aligned}$$

5 Problem 4

5.1 Part(a)

$$g(t) = \delta(t)$$

$$\begin{aligned} \hat{g}(t) &= g(t) \otimes \frac{1}{\pi t} \\ &= \frac{1}{\pi} \int_{-\infty}^{\infty} \delta(\tau) \frac{1}{t - \tau} d\tau \\ &= \frac{1}{\pi} \int_{-\infty}^{\infty} \delta(\tau) \frac{1}{t} d\tau \\ &= \frac{1}{\pi t} \int_{-\infty}^{\infty} \delta(\tau) d\tau \\ &= \frac{1}{\pi t} \end{aligned}$$

5.2 Part(b)

And Since $\text{sgn}(f) = -1$ for $f < 0$ and $\text{sgn}(f) = 1$ for $f > 0$ then

$$\hat{G}(f) = -j \left[-\text{rect}\left(\frac{f + \frac{1}{4}}{\frac{1}{2}}\right) + \text{rect}\left(\frac{f - \frac{1}{4}}{\frac{1}{2}}\right) \right]$$

Hence

$$\hat{g}(t) = jF^{-1} \left[\text{rect}\left(\frac{f + \frac{1}{4}}{\frac{1}{2}}\right) - \text{rect}\left(\frac{f - \frac{1}{4}}{\frac{1}{2}}\right) \right] \quad (1)$$

But $F^{-1}\left(\text{rect}\left(\frac{f + \frac{1}{4}}{\frac{1}{2}}\right)\right) = \frac{1}{2} \text{sinc}\left(\frac{1}{2}t\right) e^{-j2\pi\frac{1}{4}t}$ and $F^{-1}\left(\text{rect}\left(\frac{f - \frac{1}{4}}{\frac{1}{2}}\right)\right) = \frac{1}{2} \text{sinc}\left(\frac{1}{2}t\right) e^{+j2\pi\frac{1}{4}t}$, hence (1) becomes

$$\begin{aligned} \hat{g}(t) &= j \left[\frac{1}{2} \text{sinc}\left(\frac{1}{2}t\right) e^{-j2\pi\frac{1}{4}t} - \frac{1}{2} \text{sinc}\left(\frac{1}{2}t\right) e^{+j2\pi\frac{1}{4}t} \right] \\ &= \frac{1}{2} \text{sinc}\left(\frac{1}{2}t\right) \left[j \left(e^{-j2\pi\frac{1}{4}t} - e^{j2\pi\frac{1}{4}t} \right) \right] \\ &= \frac{1}{2} \text{sinc}\left(\frac{1}{2}t\right) \left[\frac{e^{-j\frac{\pi}{2}t} - e^{j\frac{\pi}{2}t}}{-j} \right] \\ &= \text{sinc}\left(\frac{1}{2}t\right) \left[\frac{e^{j\frac{\pi}{2}t} - e^{-j\frac{\pi}{2}t}}{2j} \right] \\ &= \text{sinc}\left(\frac{1}{2}t\right) \left[\sin \frac{\pi}{2}t \right] \end{aligned}$$

But $\text{sinc}\left(\frac{1}{2}t\right) = \frac{\sin \frac{\pi t}{2}}{\frac{\pi t}{2}}$ hence

$$\begin{aligned} \hat{g}(t) &= \frac{\sin \frac{\pi t}{2}}{\frac{\pi t}{2}} \sin \frac{\pi}{2}t \\ &= \frac{2}{\pi t} \sin^2 \frac{\pi}{2}t \\ &= \frac{2}{\pi t} \left(\frac{1}{2} - \frac{1}{2} \cos \pi t \right) \\ &= \frac{1}{\pi t} (1 - \cos \pi t) \end{aligned}$$

6 problem 5

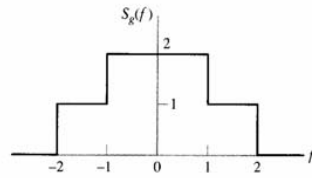


FIGURE 2.47
Problem 2.44

2.45 Consider the square wave $g(t)$ shown in Fig. 2.48. Find the power spectral density, average power, and autocorrelation function of this square wave. Does the wave have dc power? Explain your answer.

$g(t)$

Figure 1: the Problem statement

$$S_g(f) = \text{rect}\left(\frac{f}{4}\right) + \text{rect}\left(\frac{f}{2}\right)$$

$$R_g(\tau) = F^{-1}(S_g(f))$$

Hence

$$\begin{aligned} R_g(\tau) &= F^{-1}\left(\text{rect}\left(\frac{f}{4}\right) + \text{rect}\left(\frac{f}{2}\right)\right) \\ &= F^{-1}\left[\text{rect}\left(\frac{f}{4}\right)\right] + F^{-1}\left[\text{rect}\left(\frac{f}{2}\right)\right] \\ &= 4 \text{sinc}(4t) + 2 \text{sinc}(2t) \end{aligned}$$

```
In[7]:= Plot[4 Sinc[4 t] + 2 Sinc[2 t], {t, -4 Pi, 4 Pi},  
PlotRange -> All]
```

Out[7]=

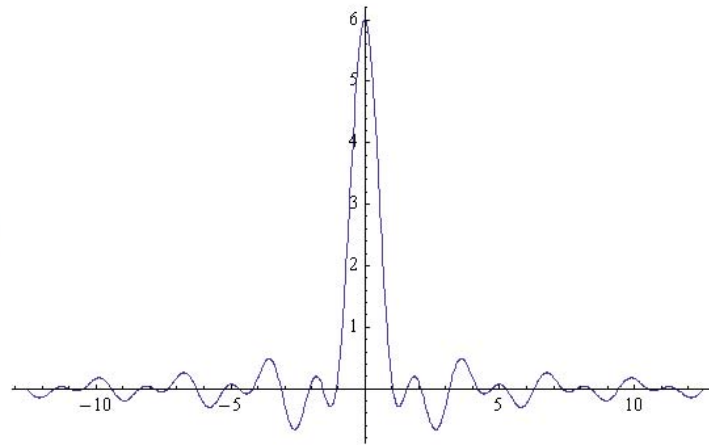


Figure 2: Plot for problem 5

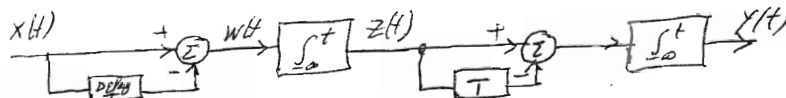
7 Key solution

EE 443

HW #4 Key

page

Problem 2.



The first integrator input is equal to $x(t) - x(t-T)$. The Fourier transform of this input signal is $[1 - \exp(-j2\pi fT)]X(f)$. The value of this transform is zero at $f=0$. It follows therefore that the Fourier transform of the first integrator output is equal to

$$*) \quad Z(f) = \frac{1}{j2\pi f} [1 - \exp(-j2\pi fT)]X(f)$$

That is: $w(t) = x(t) - x(t-T) \Rightarrow W(f) = X(f)[1 - \exp(-j2\pi fT)]$
 $Z(f) = \int_0^t w(t) dt \Rightarrow Z(f) = \frac{W(f)}{j2\pi f} + \frac{1}{2} W(0) \delta(f)$
 Since $W(0) = 0 \Rightarrow Z(f) = \frac{W(f)}{j2\pi f} \rightarrow (*)$

The transfer function of the first stage of the system of Fig. P2.5 is therefore equal to $H_1(f) = \frac{Z(f)}{X(f)}$

$$\Rightarrow H_1(f) = \frac{1}{j2\pi f} [1 - \exp(-j2\pi fT)] = \frac{\exp(-j\pi fT)}{j2\pi f} [e^{j\pi fT} - e^{-j\pi fT}] = \frac{\exp(-j\pi fT)}{j2\pi f} \cdot 2j \sin(\pi fT) = T \exp(-j\pi fT) \text{sinc}(fT)$$

The second stage of the system is identical to the first stage. The overall transfer function of the system is therefore:

$$H(f) = \frac{1}{(j2\pi f)^2} [1 - \exp(-j2\pi fT)]^2 = \frac{1 + \exp(-j4\pi fT) - 2\exp(-j2\pi fT)}{(j2\pi f)^2} = \frac{\exp(-j2\pi fT) [e^{j2\pi fT} + e^{-j2\pi fT} - 2]}{(j2\pi f)^2}$$

$$= \exp(-j2\pi fT) \left[\frac{\exp(j\pi fT) - \exp(-j\pi fT)}{j2\pi f} \right]^2$$

$$= \exp(-j2\pi fT) \left[\frac{\sin(\pi fT)}{\pi f} \right]^2$$

$$= T^2 \text{sinc}^2(fT) \exp(-j2\pi fT)$$

2^o method

The impulse response of the first half:

If $x(t) = \delta(t)$, then $z(t) = h_1(t)$, Thus:

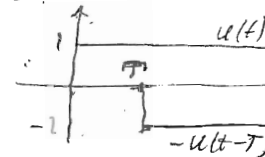
$$h_1(t) = \int_{-\infty}^t [\delta(t_1) - \delta(t_1 - T)] dt_1 = u(t) - u(t - T)$$

$$\Rightarrow h_1(t) = \text{rect}\left(\frac{t - T/2}{T}\right)$$

$-j\pi fT$

$$\Rightarrow H_1(f) = F.T[h_1(t)] = T \text{sinc}(fT) \exp(-j\pi fT)$$

$$H(f) = H_1^2(f) = T^2 \text{sinc}^2(fT) \exp(-j2\pi fT)$$



Problem 2.

(a) The transfer function of the i th stage of the system of Fig. P2.6 is

$$H_1(f) = \frac{1}{1+j2\pi fRC}$$

$$= \frac{1}{1+j2\pi f\tau_0}$$

where it is assumed that the buffer amplifier has a constant gain of one. The overall transfer function of the system is therefore

$$H(f) = \prod_{i=1}^N H_1(f)$$

$$= \frac{1}{(1+j2\pi f\tau_0)^N}$$

The corresponding amplitude response is

$$|H(f)| = \frac{1}{[1+(2\pi f\tau_0)^2]^{N/2}}$$

(b) Let

$$\tau_0^2 = \frac{T^2}{4\pi^2 N}$$

Then, we may rewrite the expression for the amplitude response as

$$|H(f)| = \left[1 + \frac{1}{N}(fT)^2 \right]^{-N/2}$$

In the limit, as N approaches infinity, we have

$$|H(f)| = \lim_{N \rightarrow \infty} \left[1 + \frac{1}{N}(fT)^2 \right]^{-N/2}$$

$$= \exp\left[-\frac{N}{2} \cdot \frac{1}{N}(fT)^2\right]$$

$$= \exp\left(-\frac{f^2 T^2}{2}\right)$$

Problem 3

(a) $g(t) = \text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$

We note that

$$\hat{g}(t) = \frac{1 - \cos(\pi t)}{\pi t}$$

Therefore,

$$\begin{aligned} g_+(t) &= g(t) + j\hat{g}(t) \\ &= \frac{\sin(\pi t)}{\pi t} + j \frac{1 - \cos(\pi t)}{\pi t} \end{aligned}$$

$$= \frac{j}{\pi t} [1 - \cos(\pi t) - j \sin(\pi t)]$$

$$= \frac{j}{\pi t} [1 - \exp(j\pi t)] = \frac{j}{\pi t} [e^{j\pi t} - 1] = \frac{j}{\pi t} \left[\frac{e^{j\pi t} - 1}{e^{j\pi/2} - 1} \right] e^{j\pi/2} = \frac{2 \sin(\pi/2)}{\pi t} e^{j\pi/2} = \text{sinc}(t/2) e^{j\pi/2}$$

(b) $g(t) = [1 + k \cos(2\pi f_m t)] \cos(2\pi f_c t)$

$$= \cos(2\pi f_c t) + \frac{k}{2} \cos[2\pi(f_c + f_m)t] + \frac{k}{2} \cos[2\pi(f_c - f_m)t]$$

Since the Hilbert transform of $\cos(2\pi f t)$ is equal to $\sin(2\pi f t)$, it follows that

$$\hat{g}(t) = \sin(2\pi f_c t) + \frac{k}{2} \sin[2\pi(f_c + f_m)t] + \frac{k}{2} \sin[2\pi(f_c - f_m)t]$$

where it is assumed that $f_c > f_m$. Therefore,

$$g_+(t) = \exp(j2\pi f_c t) + \frac{k}{2} \exp[j2\pi(f_c + f_m)t] + \frac{k}{2} \exp[j2\pi(f_c - f_m)t]$$

$$= [1 + \frac{k}{2} \exp(j2\pi f_m t) + \frac{k}{2} \exp(-j2\pi f_m t)] \exp(j2\pi f_c t)$$

$$= [1 + k \cos(2\pi f_m t)] \exp(j2\pi f_c t)$$

II^o method:

using freq. domain approach:

$$G(f) = \text{rect}(f) \quad \begin{array}{c} \uparrow G(f) \\ -1/2 \quad 1/2 \end{array} \quad f$$

$$G_+(f) = 2G(f) = 2 \text{rect}(f - 1/4) \quad \text{for } f \in [1/4, 3/4]$$

$$g_+(t) = 2 \cdot \frac{1}{2} \text{sinc}(t/2) e^{j2\pi \cdot 1/4 t}$$

$$\Rightarrow g_+(t) = \text{sinc}(t/2) e^{j2\pi \cdot 1/4 t} \quad f_c = 1/4$$

$$\tilde{g}(t) = g_+(t) e^{-j2\pi f_c t} = \text{sinc}(t/2)$$

$$a(t) = |\tilde{g}(t)| = |\text{sinc}(t/2)|$$

Problem 4

Prob #4 part b)

use domain approach:

$$g(t) = \frac{\sin t}{t}$$

The Hilbert transform of $\sin t/t$ is

$$\hat{g}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{g(\tau)}{t-\tau} d\tau$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin \tau}{\tau(t-\tau)} d\tau$$

$$= \frac{1}{\pi t} \int_{-\infty}^{\infty} \left(\frac{1}{\tau} + \frac{1}{t-\tau} \right) \sin \tau d\tau$$

$$= \frac{1}{\pi t} \int_{-\infty}^{\infty} \frac{\sin \tau}{\tau} d\tau + \frac{1}{\pi t} \int_{-\infty}^{\infty} \frac{\sin \tau}{t-\tau} d\tau$$

We note that

$$\int_{-\infty}^{\infty} \text{sinc}(t) dt = 1$$

Therefore,

$$\int_{-\infty}^{\infty} \frac{\sin \tau}{\tau} d\tau = \pi$$

$$\int_{-\infty}^{\infty} \frac{\sin \tau}{t-\tau} d\tau = \int_{-\infty}^{\infty} \frac{\sin(t-\tau)}{\tau} d\tau$$

$$= \sin t \int_{-\infty}^{\infty} \frac{\cos \tau}{\tau} d\tau - \cos t \int_{-\infty}^{\infty} \frac{\sin \tau}{\tau} d\tau$$

$$= -\pi \cos t \quad \text{it is odd function}$$

thus obtain

$$\hat{g}(t) = \frac{1}{t} (1 - \cos t)$$

$$\text{Thus } \hat{g}(t) = \frac{1}{t} (1 - \cos t)$$

$$a) \quad g(t) = \delta(t)$$

$$\hat{g}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\delta(\tau)}{t+\tau} d\tau = \frac{1}{\pi t} \int_{-\infty}^{\infty} \delta(\tau) d\tau = \frac{1}{\pi t}$$

Prob #4 part b

Freq. domain approach

Remember: $\text{rect}(t) \xrightarrow{F.T} \text{sinc}(f)$

using duality:

$$\text{sinc}(t) \xrightarrow{F.T} \text{rect}(-f) = \text{rect}(f)$$

Thus:

$$\text{sinc}(t) = \frac{\text{sinc}(t)}{\pi} \xleftrightarrow{F.T} \text{rect}(f)$$

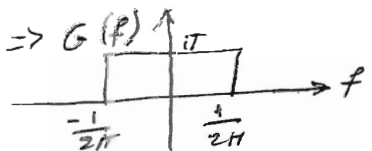
$$g(t) = \frac{\sin t}{t} = \frac{\text{sinc}(\pi t \cdot \frac{1}{\pi})}{(\pi t \cdot \frac{1}{\pi})} \xleftrightarrow{F.T} \frac{1}{(\frac{1}{\pi})} \text{rect}(f)$$

where time scaling is used.

That is if $g(t) \xrightarrow{F.T} G(f)$, then

$$g(at) \xrightarrow{F.T} \frac{1}{|a|} G\left(\frac{f}{a}\right), \text{ Theref.}$$

$$\text{in our case: } g(t) = \frac{\sin t}{t} \xrightarrow{F.T} \pi \text{rect}\left(\frac{f}{\pi}\right)$$



$$\hat{G}(f) = -j \text{sgn}(f) G(f) = -j \pi \text{sgn}(f) \left[\text{rect}\left(f - \frac{\pi}{4\pi}\right) + \text{rect}\left(f + \frac{\pi}{4\pi}\right) \right]$$

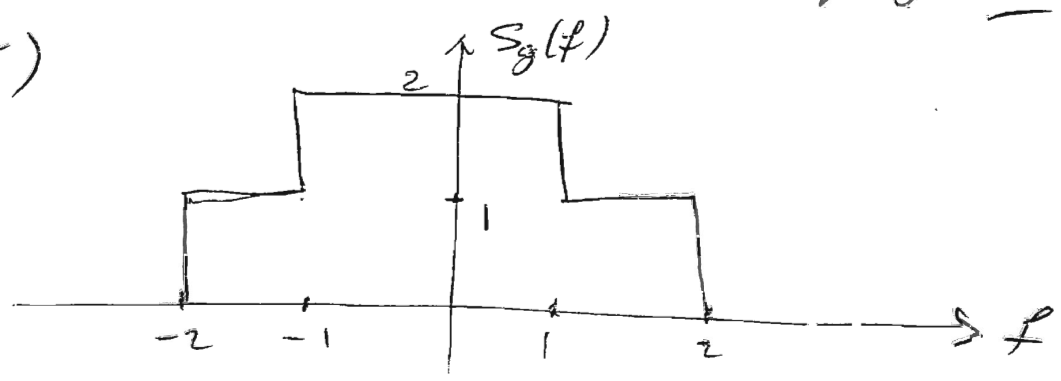
$$= -j \pi \left[\text{rect}\left(\frac{f - \frac{\pi}{4\pi}}{\frac{1}{2\pi}}\right) - \text{rect}\left(\frac{f + \frac{\pi}{4\pi}}{\frac{1}{2\pi}}\right) \right]$$

$$\Rightarrow \hat{g}(t) = -j \pi \left[\frac{1}{\pi} \text{sinc}\left(\frac{t}{\pi}\right) \right] \left[e^{j\frac{t}{2}} - e^{-j\frac{t}{2}} \right]$$

$$\hat{g}(t) = \text{sinc}\left(\frac{t}{\pi}\right) \sin\left(\frac{t}{2}\right)$$

$$= \frac{2}{t} \text{sinc}\left(\frac{t}{2}\right) = \frac{1}{t} (1 - \cos t)$$

prob # 5)



$$S_g(f) = \text{rect}\left(\frac{f}{4}\right) + \text{rect}\left(\frac{f}{2}\right)$$

$$R_g(\tau) = \mathcal{F}^{-1}[S_g(f)] = 4 \text{sinc}(4\tau) + 2 \text{sinc}(2\tau)$$

$$R_g(0) = P_{av} = 6 \text{ watts}$$

That is

$$R_g(0) = \int_{-\infty}^{+\infty} S_g(f) df = 6 \text{ W} \hat{=} P_{av}$$

8 my graded HW

⁴
HW~~3~~, EGEE 443. CSUF, Fall 2008

Nasser Abbasi

September 29, 2008

1 Problem 1

17.5
20

Problem 1 Evaluate the transfer function of a linear system represented by the block diagram shown in Fig. P2.14.

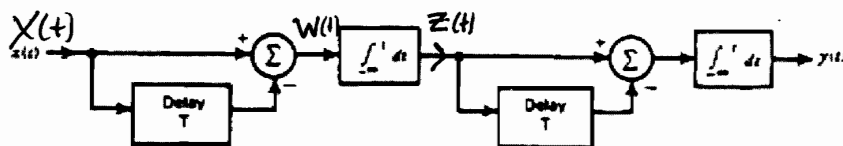


Figure P2.14

Solution Using transfer function cascading, then the overall transfer function for the system can be written as

$$H(f) = H_1(f) H_1(f) = [H_1(f)]^2 \quad (1)$$

Where

$$H_1(f) = \frac{Z(f)}{X(f)}$$

Where

$$\begin{aligned} Z(f) &= F \left\{ \int_{-\infty}^t w(\tau) d\tau \right\} \\ &= \frac{1}{j2\pi f} W(f) + \frac{W(0)}{2} \delta(f) \end{aligned} \quad (2)$$

Where

$$\begin{aligned} W(f) &= F \{ x(t) - x(t-T) \} \\ &= X(f) - X(f) e^{-j2\pi f T} \\ &= X(f) [1 - e^{-j2\pi f T}] \end{aligned} \quad (3)$$

substitute (3) into (2) we obtain

$$\begin{aligned} Z(f) &= \frac{1}{j2\pi f} X(f) [1 - e^{-j2\pi f T}] + \frac{X(0) \overbrace{[1 - e^{-j2\pi 0 T}]}^{=0}}{2} \delta(f) \\ &= \frac{1}{j2\pi f} X(f) [1 - e^{-j2\pi f T}] \end{aligned} \quad (4)$$

Hence

$$\begin{aligned} H_1(f) &= \frac{Z(f)}{X(f)} \\ &= \frac{\frac{1}{j2\pi f} X(f) [1 - e^{-j2\pi f T}]}{X(f)} \end{aligned}$$

Hence

$$H_1(f) = \frac{1}{j2\pi f} [1 - e^{-j2\pi fT}]$$

Hence from (1)

$$\begin{aligned} H(f) &= \left(\frac{1}{j2\pi f} [1 - e^{-j2\pi fT}] \right)^2 \\ &= \frac{1}{-4\pi^2 f^2} [1 - e^{-j2\pi fT}]^2 \\ &= \frac{-1}{(2\pi f)^2} [1 - 2e^{-j2\pi fT} + e^{-j4\pi fT}] \end{aligned}$$

Hence

$$H(f) = \frac{1}{(2\pi f)^2} [2e^{-j2\pi fT} - e^{-j4\pi fT} - 1]$$

✓ Simplify it,
see sol.

2 Problem 2

Problem 2.

- Determine the overall amplitude response of the cascade connection shown in P2.15, consisting of N identical stages, each with a time constant RC equal to τ_0 .
- Show that as N approaches infinity, the amplitude response of the cascade connection approaches the Gaussian function $\exp[-\frac{1}{2}f^2T^2]$, where for each value of N , the time constant τ_0 is selected so that

$$\tau_0^2 = \frac{T^2}{4\pi^2 N}$$

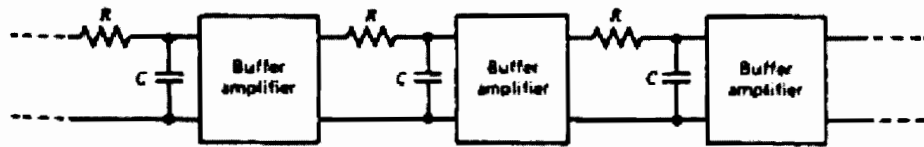


Figure P2.15

2.1 Part(a)

Transfer function for each stage is $H_i(f) = \frac{Y_i(f)}{X_i(f)} = \frac{1}{1+j2\pi fRC}$

Since $RC = \tau_0$, hence

$$H_i(f) = \frac{1}{1+j2\pi f\tau_0}$$

Then, for N stages, the overall transfer function is

$$H(f) = H_1(f) H_2(f) \cdots H_N(f)$$

Since they are identical stages, then the transfer function of each stage is the same, and the above becomes

$$H(f) = \left(\frac{1}{1+j2\pi f\tau_0} \right)^N$$

Hence the amplitude of the response is given by

$$\begin{aligned} |H(f)| &= \left(\frac{1}{|1+j2\pi f\tau_0|} \right)^N \\ &= \left(\frac{1}{\sqrt{1^2 + (2\pi f\tau_0)^2}} \right)^N \\ &= \left(\frac{1}{(1 + 4\pi^2 f^2 \tau_0^2)^{\frac{1}{2}}} \right)^N \\ &= \frac{1}{(1 + 4\pi^2 f^2 \tau_0^2)^{\frac{N}{2}}} \end{aligned}$$

Let $\tau_0^2 = \frac{\tau^2}{4\pi^2 N}$, the above becomes

$$|H(f)| = \frac{1}{\left(1 + \frac{f^2 \tau^2}{N}\right)^{\frac{N}{2}}} \quad (1)$$

2.2 Part (b)

Let $\alpha = f^2 \tau^2, \beta = \frac{1}{2}$, then (1) becomes

$$|H(f)| = \frac{1}{\left(1 + \frac{\alpha}{N}\right)^{\beta N}}$$

But $\lim_{N \rightarrow \infty} \frac{1}{\left(1 + \frac{\alpha}{N}\right)^{\beta N}} = e^{-\alpha\beta}$, hence

$$|H(f)| = \frac{1}{e^{\frac{f^2 \tau^2}{2}}}$$

Which is what we are asked to show.

3 Problem 3

Problem 3: Determine the pre-envelope $g_+(t)$ corresponding to each of the following two signals:

(a) $g(t) = \text{sinc}(t)$

(b) $g(t) = [1 + \cos(2\pi f_m t)] \cos(2\pi f_c t)$

(Hint use $\frac{\sin t}{t} \xrightarrow{\text{FT}} \frac{1 - \cos t}{t}$ See prob # 4)

(a) $g(t) = \text{sinc}(t)$

$$g_+(t) = g(t) + j\hat{g}(t) \quad (1)$$

Where $\hat{g}(t)$ is Hilbert transform of $g(t)$ defined as $\hat{g}(t) = g(t) \otimes \frac{1}{\pi t}$

$$\begin{aligned} \hat{G}(f) &= -j \text{sgn}(f) G(f) \\ &= -j \text{sgn}(f) \text{rect}(f) \end{aligned}$$

Now find the inverse Fourier transform.

I derive the above to answer problem 4 part (b). The answer is the following (please see problem 4 part(b) for the derivation

$$\hat{g}(t) = \frac{1}{\pi t} (1 - \cos \pi t)$$

In the above, I used $\text{sinc}(t) \equiv \frac{\sin \pi t}{\pi t}$. If one uses $\text{sinc}(t) \equiv \frac{\sin t}{t}$ then the answer becomes

$$\hat{g}(t) = \frac{1}{t} (1 - \cos t) \quad (2)$$

The problem statement seems to want us to use the second definition of $\text{sinc}(t)$, so I will continue the rest of the solution using (1).

Substitute (2) into (1) we obtain

$$\begin{aligned} g_+(t) &= \text{sinc}(t) + j \frac{1}{\pi t} (1 - \cos t) \\ &= \frac{\sin(t)}{t} + j \frac{1}{t} \left(1 - \frac{e^{jt} + e^{-jt}}{2} \right) \\ &= \frac{1}{t} \frac{e^{jt} - e^{-jt}}{2j} + \frac{1}{t} \left(j + \frac{e^{jt} + e^{-jt}}{2j} \right) \\ &= \frac{1}{t} \frac{e^{jt} - e^{-jt}}{2j} + \frac{j}{t} + \frac{1}{t} \frac{e^{jt} + e^{-jt}}{2j} \\ &= \frac{1}{t} \frac{e^{jt}}{2j} + \frac{j}{t} + \frac{1}{t} \frac{e^{jt}}{2j} \end{aligned}$$

Hence

$$g_+(t) = \frac{1}{t} (j + e^{jt})$$

See Sol.

3.1 Part(b)

$$g(t) = [1 + k \cos 2\pi f_m t] \cos(2\pi f_c t)$$

$$g_+(t) = g(t) + j\hat{g}(t)$$

Where $\hat{g}(t)$ is Hilbert transform of $g(t)$ defined as $\hat{g}(t) = g(t) \otimes \frac{1}{\pi t}$.

$$G_+(f) = \begin{cases} 2G(f) & f > 0 \\ G(0) & f = 0 \\ 0 & f < 0 \end{cases}$$

But

$$G(f) = F[1 + k \cos 2\pi f_m t] \otimes F[\cos(2\pi f_c t)] \quad (1)$$

But

$$F[\cos(2\pi f_c t)] = \frac{1}{2} [\delta(f - f_c) + \delta(f + f_c)]$$

and

$$F[1 + k \cos 2\pi f_m t] = \delta(f) + \frac{k}{2} [\delta(f - f_m) + \delta(f + f_m)]$$

Hence (1) becomes

$$\begin{aligned} G(f) &= \left\{ \delta(f) + \frac{k}{2} [\delta(f - f_m) + \delta(f + f_m)] \right\} \otimes \frac{1}{2} [\delta(f - f_c) + \delta(f + f_c)] \\ &= \delta(f) \otimes \frac{1}{2} [\delta(f - f_c) + \delta(f + f_c)] \\ &\quad + \frac{k}{2} [\delta(f - f_m) + \delta(f + f_m)] \otimes \frac{1}{2} [\delta(f - f_c) + \delta(f + f_c)] \\ &= \frac{1}{2} \delta(f) \otimes \delta(f - f_c) + \\ &\quad \frac{1}{2} \delta(f) \otimes \delta(f + f_c) + \\ &\quad \frac{k}{4} \delta(f - f_m) \otimes \delta(f - f_c) + \\ &\quad \frac{k}{4} \delta(f - f_m) \otimes \delta(f + f_c) + \\ &\quad \frac{k}{4} \delta(f + f_m) \otimes \delta(f - f_c) + \\ &\quad \frac{k}{4} \delta(f + f_m) \otimes \delta(f + f_c) \end{aligned}$$

Hence

$$\begin{aligned}
 G(f) = & \frac{1}{2} \delta(f + f_c) + \\
 & \frac{1}{2} \delta(f - f_c) + \\
 & \frac{k}{4} \delta(f - f_m + f_c) + \\
 & \frac{k}{4} \delta(f - f_m - f_c) + \\
 & \frac{k}{4} \delta(f + f_m + f_c) + \\
 & \frac{k}{4} \delta(f + f_m - f_c)
 \end{aligned}$$

Hence for $f > 0$, $G_+(f) = 2G(f)$ and we obtain

$$G_+(f) = \delta(f - f_c) + \frac{k}{2} [\delta(f - f_m + f_c) + \delta(f - f_m - f_c) + \delta(f + f_m + f_c) + \delta(f + f_m - f_c)]$$

Then (since carrier frequency $f_c > f_m$), we could simplify the above, by keeping positive frequencies f

$$G_+(f) = \delta(f - f_c) + \frac{k}{2} [\delta(f - f_m - f_c) + \delta(f + f_m - f_c)]$$

or

$$G_+(f) = \delta(f - f_c) + \frac{k}{2} [\delta(f - (f_m + f_c)) + \delta(f - (f_c - f_m))]$$

Hence

$$\begin{aligned}
 g_+(t) &= e^{j2\pi f_c t} + \frac{k}{2} (e^{j2\pi(f_m + f_c)t} + e^{j2\pi(f_c - f_m)t}) \\
 &= e^{j2\pi f_c t} + \frac{k}{2} (e^{j2\pi f_m t} e^{j2\pi f_c t} + e^{j2\pi f_c t} e^{-j2\pi f_m t}) \\
 &= e^{j2\pi f_c t} \left[1 + \frac{k}{2} (e^{j2\pi f_m t} + e^{-j2\pi f_m t}) \right] \\
 &= e^{j2\pi f_c t} \left[1 + \frac{k}{2} (2 \cos(2\pi f_m t)) \right] \\
 &= e^{j2\pi f_c t} [1 + k \cos(2\pi f_m t)]
 \end{aligned}$$

4 Problem 4

prob # 4) Verify the following H.T.:

a) if $g(t) = \delta(t) \Rightarrow \hat{g}(t) = ?$

b) if $g(t) = \frac{\sin t}{t} \Rightarrow \hat{g}(t) = \frac{1 - \cos t}{t}$

4.1 Part(a)

$$g(t) = \delta(t)$$

$$\begin{aligned}\hat{g}(t) &= g(t) \otimes \frac{1}{\pi t} \\ &= \frac{1}{\pi} \int_{-\infty}^{\infty} \delta(\tau) \frac{1}{t - \tau} d\tau \\ &= \frac{1}{\pi} \int_{-\infty}^{\infty} \delta(\tau) \frac{1}{t} d\tau \\ &= \frac{1}{\pi t} \int_{-\infty}^{\infty} \delta(\tau) d\tau \\ &= \frac{1}{\pi t} \end{aligned}$$

4.2 Part(b)

And Since $\text{sgn}(f) = -1$ for $f < 0$ and $\text{sgn}(f) = 1$ for $f > 0$ then

$$g(t) = \frac{\sin(\pi t \cdot \frac{1}{2})}{\pi t \cdot \frac{1}{2}} \Leftrightarrow \frac{1}{\pi t} \cdot \text{rect}(t)$$

$$\hat{G}(f) = -j \left[-\text{rect}\left(\frac{f + \frac{1}{4}}{\frac{1}{2}}\right) + \text{rect}\left(\frac{f - \frac{1}{4}}{\frac{1}{2}}\right) \right]$$

Hence

$$\hat{g}(t) = jF^{-1} \left[\text{rect}\left(\frac{f + \frac{1}{4}}{\frac{1}{2}}\right) - \text{rect}\left(\frac{f - \frac{1}{4}}{\frac{1}{2}}\right) \right] \quad (1)$$

But $F^{-1} \left(\text{rect}\left(\frac{f + \frac{1}{4}}{\frac{1}{2}}\right) \right) = \frac{1}{2} \text{sinc}\left(\frac{1}{2}t\right) e^{-j2\pi\frac{1}{4}t}$ and $F^{-1} \left(\text{rect}\left(\frac{f - \frac{1}{4}}{\frac{1}{2}}\right) \right) = \frac{1}{2} \text{sinc}\left(\frac{1}{2}t\right) e^{+j2\pi\frac{1}{4}t}$, hence (1) becomes

$$\begin{aligned} \hat{g}(t) &= j \left[\frac{1}{2} \text{sinc}\left(\frac{1}{2}t\right) e^{-j2\pi\frac{1}{4}t} - \frac{1}{2} \text{sinc}\left(\frac{1}{2}t\right) e^{+j2\pi\frac{1}{4}t} \right] \\ &= \frac{1}{2} \text{sinc}\left(\frac{1}{2}t\right) \left[j \left(e^{-j2\pi\frac{1}{4}t} - e^{j2\pi\frac{1}{4}t} \right) \right] \\ &= \frac{1}{2} \text{sinc}\left(\frac{1}{2}t\right) \left[\frac{e^{-j\frac{\pi}{2}t} - e^{j\frac{\pi}{2}t}}{-j} \right] \\ &= \text{sinc}\left(\frac{1}{2}t\right) \left[\frac{e^{j\frac{\pi}{2}t} - e^{-j\frac{\pi}{2}t}}{2j} \right] \\ &= \text{sinc}\left(\frac{1}{2}t\right) \left[\sin\left(\frac{\pi}{2}t\right) \right] \end{aligned}$$

But $\text{sinc}\left(\frac{1}{2}t\right) = \frac{\sin \frac{\pi t}{2}}{\frac{\pi t}{2}}$ hence

$$\begin{aligned} \hat{g}(t) &= \frac{\sin \frac{\pi t}{2}}{\frac{\pi t}{2}} \sin \frac{\pi}{2}t \\ &\neq \frac{2}{\pi t} \sin^2 \frac{\pi}{2}t \\ &= \frac{2}{\pi t} \left(\frac{1}{2} - \frac{1}{2} \cos \pi t \right) \\ &= \frac{1}{\pi t} (1 - \cos \pi t) \quad \text{see sol.} \end{aligned}$$

5 problem 5

prob #5) Respond prob 2.44 of your book

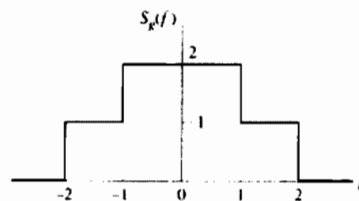


FIGURE 2.47
Problem 2.44

- 2.45 Consider the square wave $g(t)$ shown in Fig. 2.48. Find the power spectral density, average power, and autocorrelation function of this square wave. Does the wave have dc power? Explain your answer.

or

$$S_g(f) = \text{rect}\left(\frac{f}{4}\right) + \text{rect}\left(\frac{f}{2}\right)$$

$$R_g(\tau) = F^{-1}(S_g(f))$$

Hence

$$\begin{aligned} R_g(\tau) &= F^{-1}\left(\text{rect}\left(\frac{f}{4}\right) + \text{rect}\left(\frac{f}{2}\right)\right) \\ &= F^{-1}\left[\text{rect}\left(\frac{f}{4}\right)\right] + F^{-1}\left[\text{rect}\left(\frac{f}{2}\right)\right] \\ &= 4 \text{sinc}(4t) + 2 \text{sinc}(2t) \end{aligned}$$

$$R_g(0) = ?$$

In[7] = `Plot[4 Sinc[4 t] + 2 Sinc[2 t], {t, -4 Pi, 4 Pi},
PlotRange -> All]`

Out[7] =

