## Chapter 7

## Pulse-Analog Modulation

### Problem 7.1

(a) The signal g(t) is

$$g(t) = 10\cos(20\pi t)\cos(200\pi t)$$

$$= 5[\cos(220\pi t) + \cos(180\pi t)]$$

The Fourier transform of g(t) is

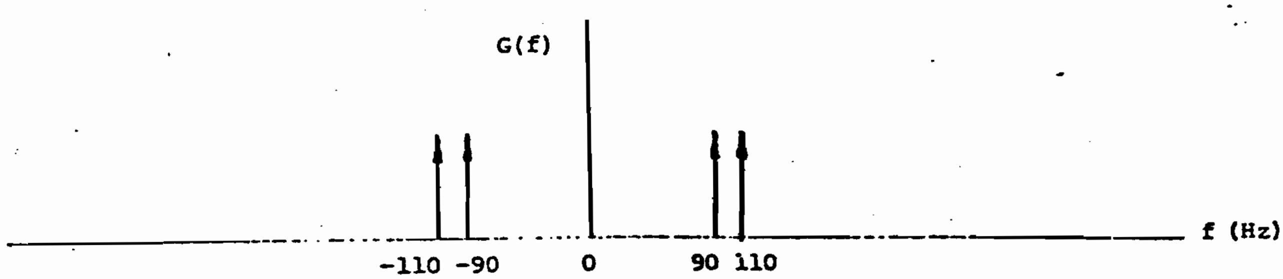
$$G(f) = 2.5[\delta(f-110) + \delta(f+110) + \delta(f-90) + \delta(f+90)]$$

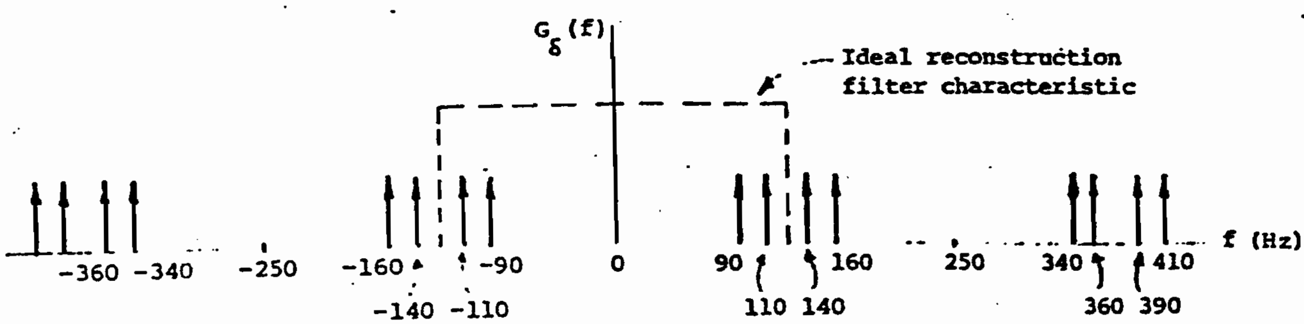
Hence, the spectrum of the sampled version of g(t), with a sampling period $T_s = 1/250$ s, is given by

$$G_s(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} G\left(f - \frac{n}{T_s}\right) = f_s \sum_{n=-\infty}^{+\infty} G(f - n f_s)$$

$$= 250 \times 2.5 \sum_{n=-\infty}^{\infty} [\delta(f-110-250n) + \delta(f+110-250n) + \delta(f-90-250n) + \delta(f+90-250n)]$$

(b) The spectra G(f) and $G_\delta(f)$ are illustrated below:



From this diagram, we deduce that in order to recover the original signal $g(t)$ from $g_\delta(t)$, we need to use a low-pass filter with a cutoff frequency that is greater than 110 Hz but less than 140 Hz.

(c) The highest frequency component of $g(t)$ is 110 Hz. The Nyquist rate of $g(t)$ is therefore 220 Hz.

(d) The signal $g(t)$ may be viewed as a band-pass signal occupying the frequency interval 90 to 110 Hz, that is,

$$f_u = 110 \qquad W = 110 - 90 = 20$$

$$f_s = \frac{2 f_u}{m}$$

$$m \leq \frac{f_u}{W} = \frac{110}{20} = 5.5 \Rightarrow m = 5$$

$$f_s = \frac{2 f_u}{m} = \frac{2 \times 110}{5} = 44 \text{ Hz}.$$

Problem 7.2

The spectrum of $g_1(t)$ is

$$G_1(f) = 5[\delta(f-50) + \delta(f+50)]$$

Hence, the spectrum of the sampled version of $g_1(t)$, using a sampling period $T_s = 1/75$ s, is

$$G_{1\delta}(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} G_1(f - \frac{n}{T_s})$$

$$= \cancel{3}75 \sum_{n=-\infty}^{\infty} [\delta(f-50-75n) + \delta(f+50-75n)] \tag{1}$$

Next, the spectrum of $g_2(t)$ is

$$G_2(f) = 5[\delta(f-25) + \delta(f+25)]$$

Hence, the spectrum of the sample version of $g_2(t)$, using a sampling period $T_s = 1/75$ s, is

$$G_{2\delta}(f) = 375 \sum_{n=-\infty}^{\infty} [\delta(f-25-75n) + \delta(f+25-75n)] \tag{2}$$

In the right-hand side of Eq. (2), substitute $n = \ell-1$ for the first term, and $n = m+1$ for the second term, and so rewrite this equation as follows:

$$G_{2\delta}(f) = 375 \sum_{\ell=-\infty}^{\infty} \delta(f+50-75\ell) + 375 \sum_{m=-\infty}^{\infty} \delta(f-50-75m)$$

$$= 375 \sum_{n=-\infty}^{\infty} [\delta(f-50-75n) + \delta(f+50-75n)] \qquad (3)$$

We thus find from Eqs. (1) and (3) that the spectra $G_{1\delta}(f)$ and $G_{2\delta}(f)$ are identical. That is, the sample versions of $g_1(t)$ and $g_2(t)$ are identical.

We note that the Nyquist rate of $g_1(t)$ is 100 Hz; hence, with a sampling rate of 75 Hz, the signal $g_1(t)$ is under-sampled by 25 Hz below the Nyquist rate. On the other hand, the Nyquist rate of $g_2(t)$ is 50 Hz; hence, the signal $g_2(t)$ is over-sampled by 25 Hz above the Nyquist rate. Thus, although $g_1(t)$ and $g_2(t)$ represent two sinusoidal waves of different frequencies, we find that by under-sampling $g_1(t)$ and over-sampling $g_2(t)$ appropriately, their sampled versions are identical.

Problem 7.3

Express the signal $g(t)$ as

$$g(t) = 10 \cos(60\pi t) \cos^2(160\pi t)$$

$$= 5 \cos(60\pi t)[1 + \cos(320\pi t)]$$

$$= 5 \cos(60\pi t) + 2.5 \cos(380\pi t) + 2.5 \cos(260\pi t)$$

The spectrum of $g(t)$ is

$$G(f) = 2.5[\delta(f-30) + \delta(f+30)] + 1.25[\delta(f-190) + \delta(f+190)] + 1.25[\delta(f-130) + \delta(f+130)]$$

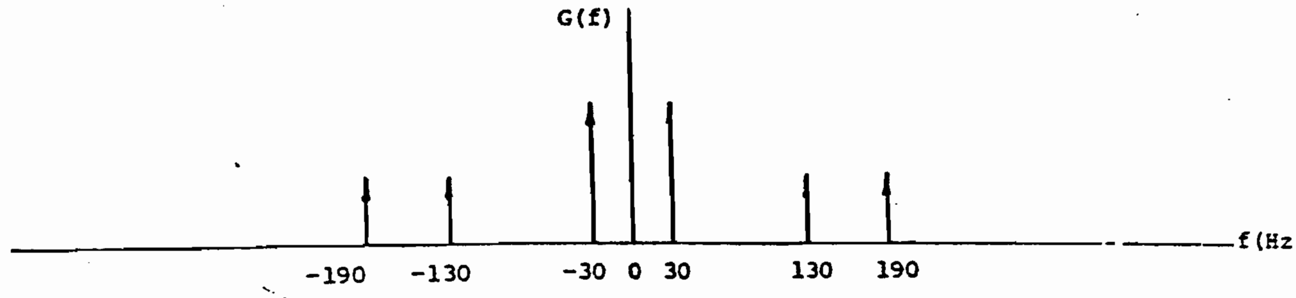
The corresponding spectrum of the sampled versionof $g(t)$, using a sampling rate of 400 Hz, is therefore

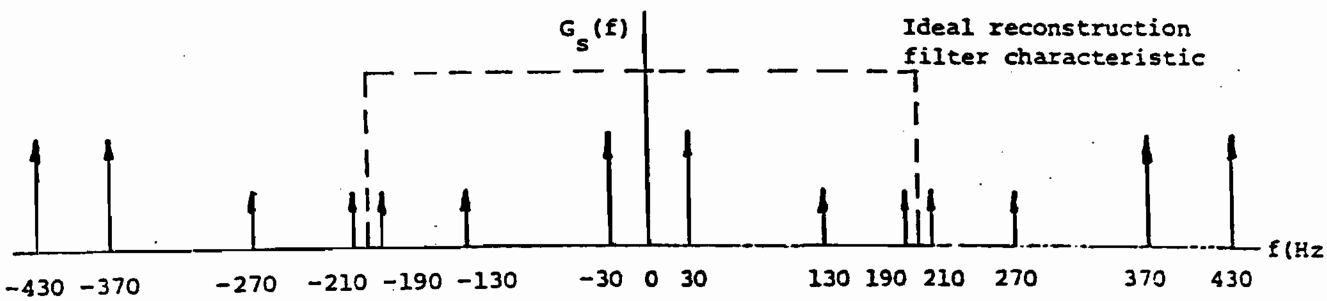
$$G_\delta(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} G(f - \frac{n}{T_s})$$

$$= 400 \sum_{n=-\infty}^{\infty} \left[ \begin{array}{l} 2.5[\delta(f-30-400n) + \delta(f+30-400n)] \\ + 1.25[\delta(f-190-400n) + \delta(f+190-400n)] \\ + 1.25[\delta(f-130-400n) + \delta(f+130-400n)] \end{array} \right]$$

The spectra $G(f)$ and $G_\delta(f)$ are illustrated below:

[redacted]




From this diagram, we deduce that in order to recover the original signal $g(t)$ from its sampled version, the low-pass reconstruction filter must have a cutoff frequency greater than 190 Hz but less than 210 Hz.

<u>Problem 7.4</u>

The signal at the sampling instants is:

$$g(nT) = \cos(2\pi f_1 nT + \frac{\pi}{2}) + A\cos(2\pi f_2 nT + \phi)$$

$$= 0\ , \qquad n = 0, 1, 2, \ldots$$

At $n = 0$,

$$\cos(\frac{\pi}{2}) + A\cos\phi = 0\ . \tag{1}$$

At $n = 1, 2, \ldots$, with $f_1 = 3.9$ kHz, $f_2 = 4.1$ kHz, and $T = 125$ µs, we have

$$\cos(0.975n\pi + \frac{\pi}{2}) + A\cos(1.025\,n\pi + \phi) = 0. \tag{2}$$

From (2) and $\cos(0.975n\pi + \frac{\pi}{2})$ being non-zero, $A$ must be non-zero. From (1) and $A$ being non-zero, $\phi$ must be $\pm\frac{\pi}{2}$. Equation (2) then becomes:

$$-\sin(0.975n\pi) \pm A\sin(1.025n\pi) = 0. \tag{3}$$

Since $\sin(\cdot)$ is odd symmetric about $n\pi$, $A$ equals 1 and the ambiguous sign in (3) is negative. Therefore, $\phi = \frac{\pi}{2}$.