10.13

Nasser Abbasi

8/8

a) $\int_0^1 e^x \, dx = [e^x]_0^1 = \boxed{e^1 - e^0}$

b) $\int_0^{2\pi} \sin^4(8x)\, dx = \cancel{\left[32 \sin^3(8x) \cos(8x)\right]_0^{2\pi}}$

$$= \left[-\frac{1}{32} \sin^3(8x) \cos(8x) - \frac{3}{64} \cos(8x) \sin(8x) + \frac{3}{8} x \right]_0^{2\pi}$$

$$= \left[-\frac{1}{32} \sin^3(16\pi) \cos(16\pi) - \frac{3}{64} \cos(16\pi) \sin(16\pi) + \frac{3}{8}(2\pi) \right]$$

$$- \left[-\frac{1}{32} \sin^3(0) \cos(0) - \frac{3}{64} \cos(0) \sin(0) + 0 \right]$$

but $\sin(n\pi) = 0$, $\cos(n\pi) = 1$ when n even.

$\sin(0) = 0$, $\cos(0) = 1$.

so $\int_0^{2\pi} \sin^4(8x)\, dx = \left[0 - 0 + \frac{3}{4}\pi\right] - [0] = \boxed{\frac{3}{4}\pi}$ exact answer

c) $\int_0^1 \sqrt{x}\, dx = \int_0^1 x^{\frac{1}{2}}\, dx = \left[\frac{2}{3} x^{\frac{3}{2}}\right]_0^1$ [illegible] ok

$$= \frac{2}{3}\left[[1]^{3/2} - [0]^{3/2}\right] = \boxed{\frac{2}{3}}$$

4/4

d) $\int_0^1 (1-x^2)^{1/2}\, dx = \left[\frac{1}{2} x \sqrt{1-x^2} + \frac{1}{2} \arcsin(x)\right]_0^1$

$$= \left[\frac{1}{2}\sqrt{1-1^2} + \frac{1}{2}\arcsin(1)\right] - \left[0 + \frac{1}{2}\arcsin(0)\right]$$

$$= \frac{1}{2}\left(\frac{\pi}{2}\right) - [0] = \boxed{\frac{\pi}{4}}$$

Integral is zero

[illegible] $\frac{D}{16}(x)$ = From Table

~~Can continue to find P_{10} using eq 10.15, but instead use matalab legendre function.~~

n=6

$$(6+1)\, P_{6+1}(x) = (12+1)\, x\, P_6(x) - 6\, P_5(x)$$

$$7\, P_7 = 13x\left[\frac{1}{16}\left(231x^6 - 315x^4 + 105x^2 - 5\right)\right] - 6\left[\frac{1}{8}\left(63x^5 - 70x^3 + 15x\right)\right]$$

$$P_7 = \frac{1}{16}\left(3003x^7 - 4851x^5 + 2205x^3 - \frac{245}{16}x\right)$$

n= 7

$$(7+1)\, P_{7+1} = (15)\, x\, P_7 - 7\, P_6$$

$$= 15x\left[\frac{1}{16}\left(3003x^7 - 4851x^5 + 2205x^3 - \frac{245}{16}x\right)\right] - 7\left[\frac{1}{16}\left(231x^6 - 315x^4 + 105x^2 - 5\right)\right]$$

$$P_8 = \frac{1}{128}\left[45045x^8 - 72765x^6 + 33075x^4 - 3675x^2 - 21021x^7 + 28665x^5 - 9555x^3 + 455x\right]$$

n=8

$$9\, P_9 = 17x\, P_8 - 8\, P_7$$

$${}_9 = \left[\frac{85085x^9}{128} - \frac{476399}{384}x^7 + \frac{96971x^5}{128} - \frac{67865}{384}x^3 - \frac{119119}{384}x^8 + \frac{54145}{128}x^6 - \frac{54145}{384}x^4 + \frac{7735}{1152}x^2 + \frac{215}{18}x\right]$$

⟶

$n = 9$

$$10 P_{10} = 19 x P_9 - 9 P_8$$

$$P_{10} = \frac{323323}{256} x^{10} - \frac{2566949}{960} x^8 + \frac{1248667}{640} x^6 - \frac{109123}{192} x^4$$

$$- \frac{2263261}{3840} x^9 + \frac{152243}{160} x^9 + \frac{152243}{160} x^7 - \frac{180271}{384} x^5$$

$$+ \frac{23023}{288} x^3 + \frac{119119}{2304} x^2 - \frac{819}{256} x$$

(f) ?

Problem 10.13
Nasser Abbasi

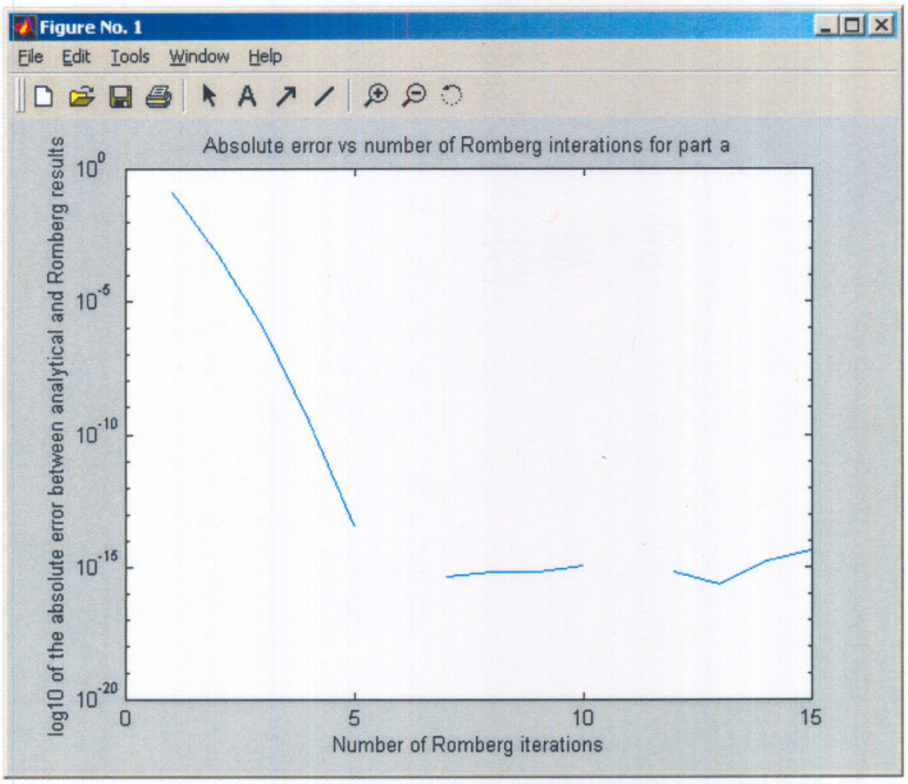
Output
I run Romberg integration for max of 15 iterations.

Part (a)
Exact answer for this part is exp(1)-1.

```
» nma_problem_10_13


 program to solve problem 10.13
 Nasser Abbasi

enter problem part [a,b,c,d,e,f]:'a'
Roomberg diagonal values for part a, Exact solution = 1.7182818284590
Row 1,  diagonal element =   1.8591409142295
Row 2,  diagonal element =   1.7188611518766
Row 3,  diagonal element =   1.7182826879248
Row 4,  diagonal element =   1.7182818287945
Row 5,  diagonal element =   1.7182818284591
Row 6,  diagonal element =   1.7182818284590
Row 7,  diagonal element =   1.7182818284590
Row 8,  diagonal element =   1.7182818284590
Row 9,  diagonal element =   1.7182818284590
Row 10,  diagonal element =   1.7182818284590
Row 11,  diagonal element =   1.7182818284590
Row 12,  diagonal element =   1.7182818284590
Row 13,  diagonal element =   1.7182818284590
Row 14,  diagonal element =   1.7182818284590
Row 15,  diagonal element =   1.7182818284591
»
```

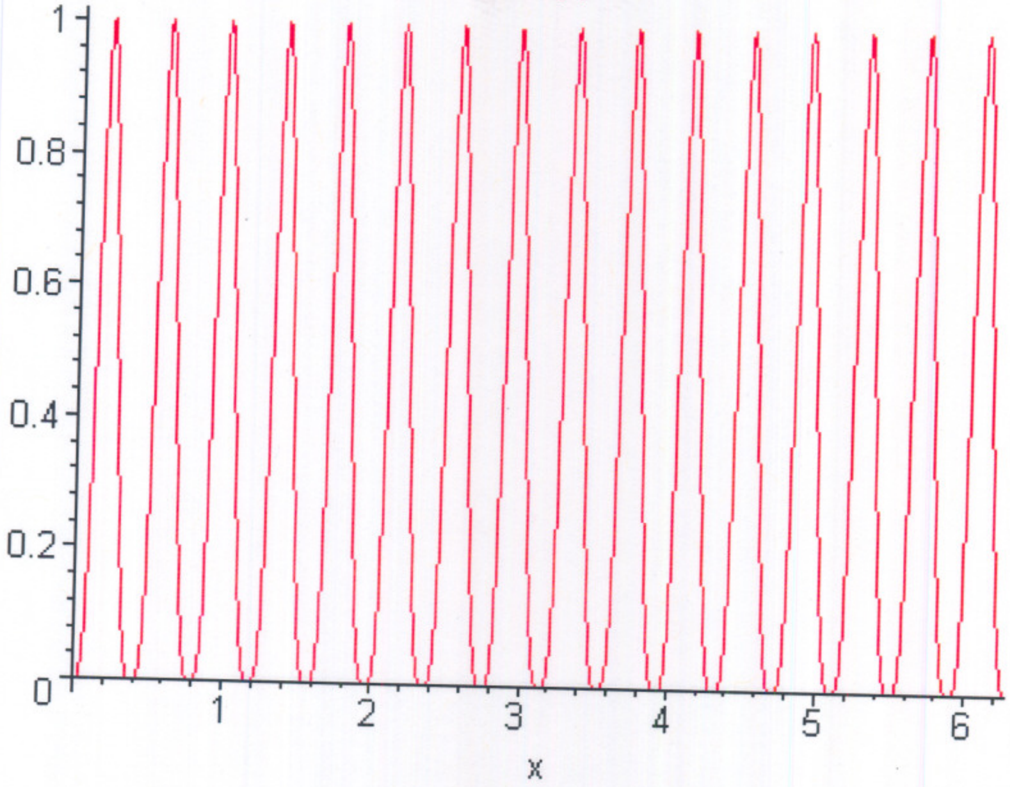


✓

The above shows the absErr is becoming less and less but after iteration 5, no more improvement is seen, and we see the absErr increasing very little when iterations become large (14 and 15).

Part (b)
The exact answer to this is ¾ PI.

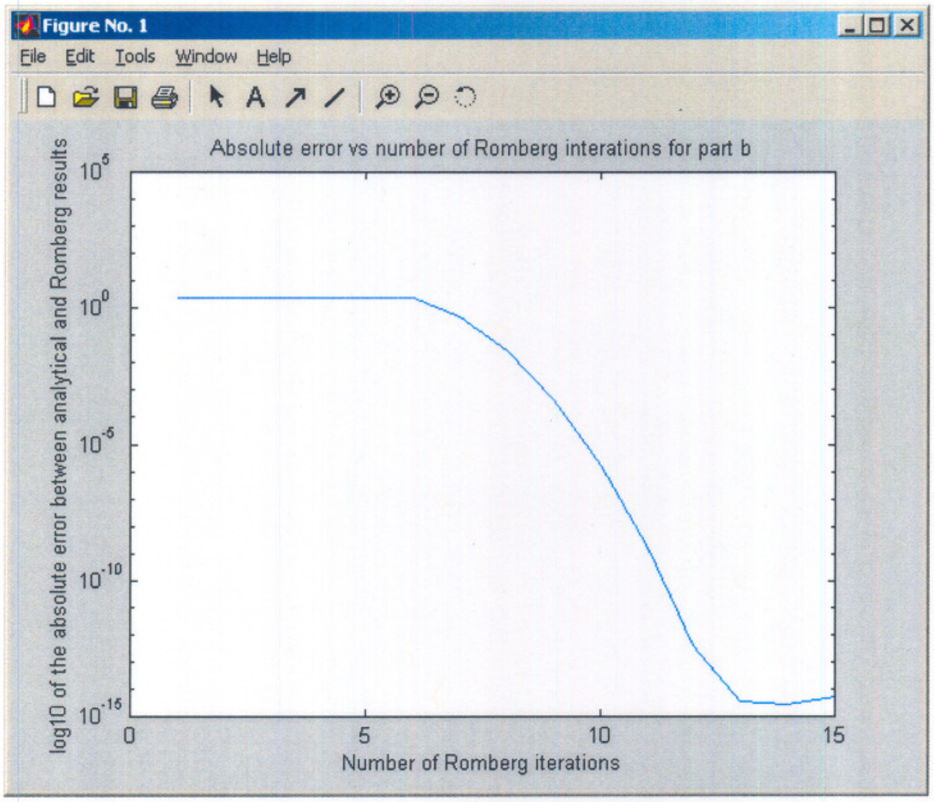
```
>plot( (sin(8*x))^4,x=0..2*Pi);
```



```
» nma_problem_10_13

  program to solve problem 10.13
  Nasser Abbasi

enter problem part [a,b,c,d,e,f]:'b'
Roomberg diagonal values for part b, Exact solution = 2.3561944901923
Row 1,  diagonal element =   0.0000000000000
Row 2,  diagonal element =   0.0000000000000
Row 3,  diagonal element =   0.0000000000000
Row 4,  diagonal element =   0.0000000000000
Row 5,  diagonal element =   0.0000000000000
Row 6,  diagonal element =   4.5612183751723
Row 7,  diagonal element =   1.9013430443026
Row 8,  diagonal element =   2.3827353809869
Row 9,  diagonal element =   2.3557875435276
Row 10,  diagonal element =   2.3561960721608
Row 11,  diagonal element =   2.3561944886493
Row 12,  diagonal element =   2.3561944901927
Row 13,  diagonal element =   2.3561944901923
Row 14,  diagonal element =   2.3561944901923
Row 15,  diagonal element =   2.3561944901923
```



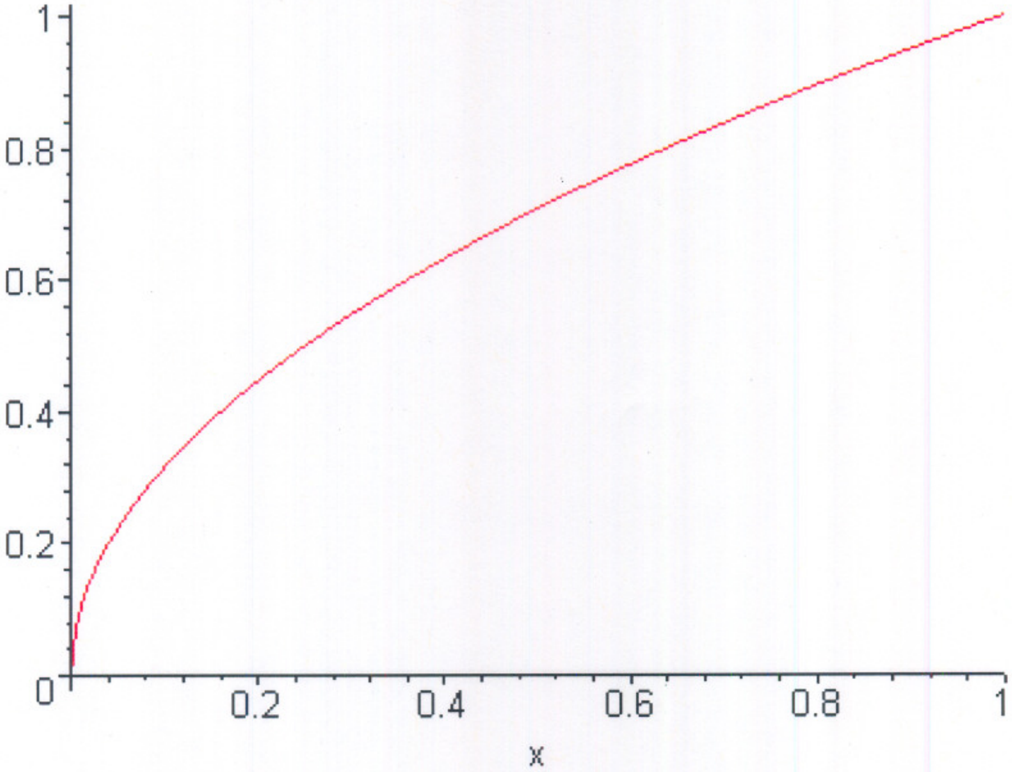
✓

In this, Romberg was actually giving zero as the answer, until iteration 6. So Romberg was way off in this one for the first 6-8 iterations. Then it got closer to the exact answer as the number of iterations increased. But again notice at iteration 15, the absError actually increased a little.

Notice where the grid points fall. For the first few iterations they fall at the zeros.

Part (C)
Exact solution here is 2/3

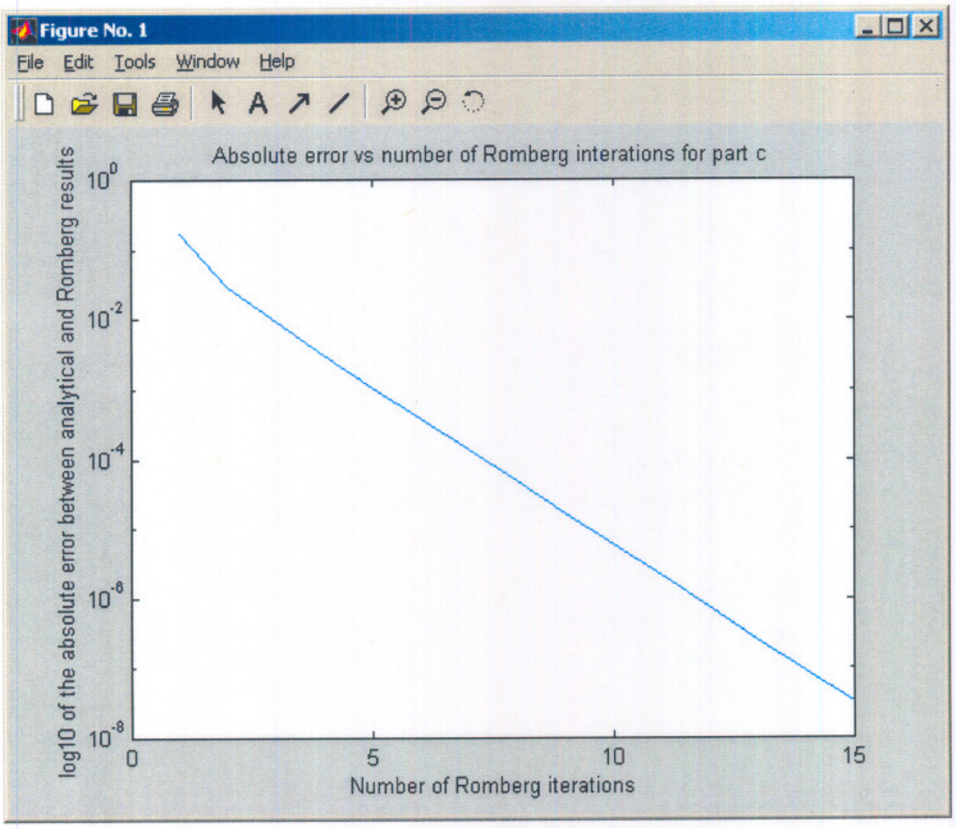
```
> plot( sqrt(x),x=0..1);
```



```
» nma_problem_10_13


  program to solve problem 10.13
  Nasser Abbasi

enter problem part [a,b,c,d,e,f]:'c'
Roomberg diagonal values for part c, Exact solution = 0.6666666666667
Row 1,  diagonal element =   0.5000000000000
Row 2,  diagonal element =   0.6380711874577
Row 3,  diagonal element =   0.6577566032816
Row 4,  diagonal element =   0.6636075691123
Row 5,  diagonal element =   0.6655928651295
Row 6,  diagonal element =   0.6662876990338
Row 7,  diagonal element =   0.6665327411999
Row 8,  diagonal element =   0.6666193221483
Row 9,  diagonal element =   0.6666499283187
Row 10,  diagonal element =   0.6666607488083
Row 11,  diagonal element =   0.6666645743914
Row 12,  diagonal element =   0.6666659269360
Row 13,  diagonal element =   0.6666664051324
Row 14,  diagonal element =   0.6666665742003
Row 15,  diagonal element =   0.6666666339749
»
```

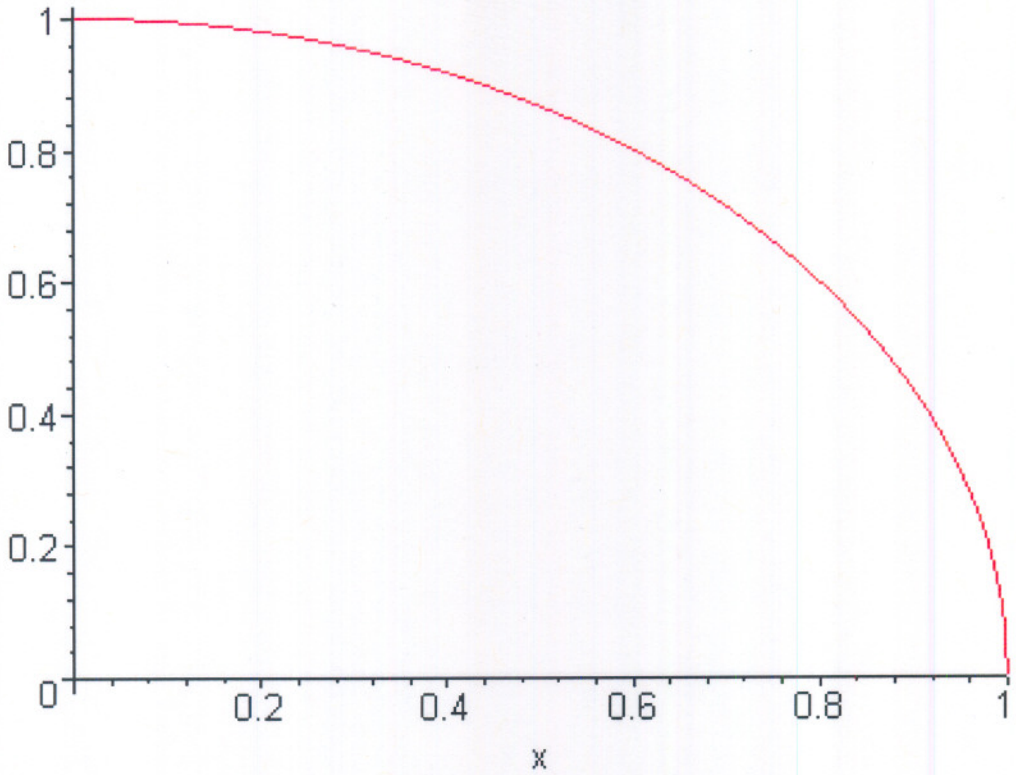


This part showed the abs errors always decreases as iteration is increased. This is because the absError is still not large enough for round off error (for double, need to reach 10^-14 or more for round off to kick in).

✓

Part (d)
Exact answer for this part is PI/4

```
> plot( sqrt(1-x^2),x=0..1);
```

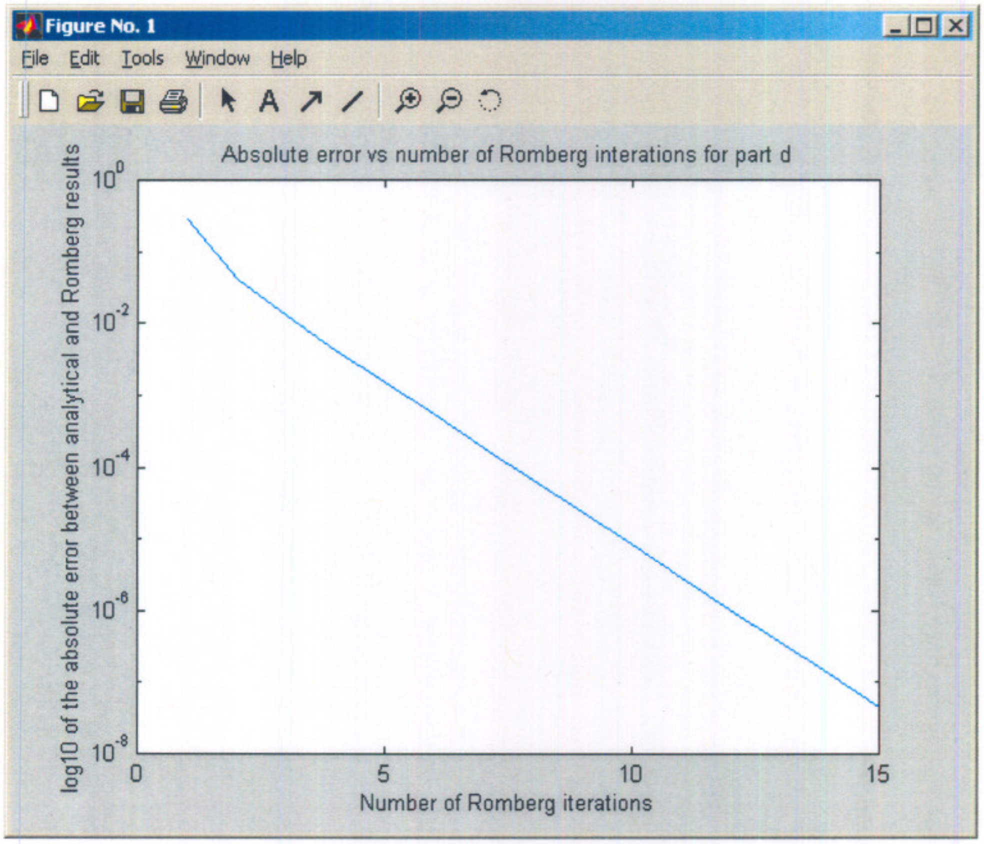


```
» nma_problem_10_13

  program to solve problem 10.13
  Nasser Abbasi

enter problem part [a,b,c,d,e,f]:'d'
Roomberg diagonal values for part d, Exact solution = 0.7853981633974
Row 1,  diagonal element =   0.5000000000000
Row 2,  diagonal element =   0.7440169358563
Row 3,  diagonal element =   0.7726909122621
Row 4,  diagonal element =   0.7810545410576
Row 5,  diagonal element =   0.7838765458406
Row 6,  diagonal element =   0.7848616873345
Row 7,  diagonal element =   0.7852086696293
Row 8,  diagonal element =   0.7853311914173
Row 9,  diagonal element =   0.7853744888423
Row 10,  diagonal element =   0.7853897937591
Row 11,  diagonal element =   0.7853952043810
Row 12,  diagonal element =   0.7853971172439
Row 13,  diagonal element =   0.7853977935293
Row 14,  diagonal element =   0.7853980326298
Row 15,  diagonal element =   0.7853981171642
```

✓



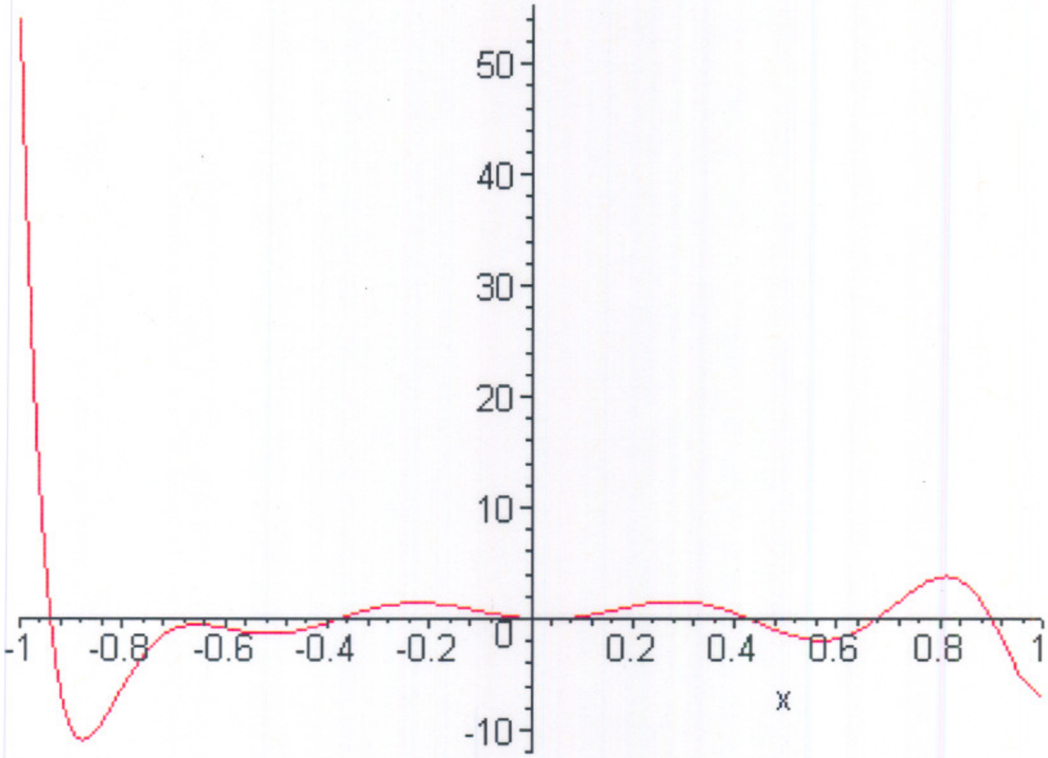
This part is similar to part c. It showed the abs errors always decreases as iteration is increased. This is because the absError is still not large enough for round off error (for double, need to reach 10^-14 or more for round off to kick in).

Use Maple to help me find P10 using equation 10.15 in book.

```
> P10 := %;
```

$$P10 := \frac{323323}{256}x^{10} - \frac{2566949}{960}x^{8} + \frac{1248667}{640}x^{6} - \frac{109123}{192}x^{4} - \frac{2263261}{3840}x^{9} + \frac{152243}{160}x^{7} - \frac{180271}{384}x^{5} + \frac{23023}{288}x^{3} + \frac{119119}{2304}x^{2} - \frac{819}{256}x$$

```
> plot(P10,x=-1..1);
```



The exact integral is 0 of P10 from −1..1

```
> int(P10,x=-1..1);
>
```

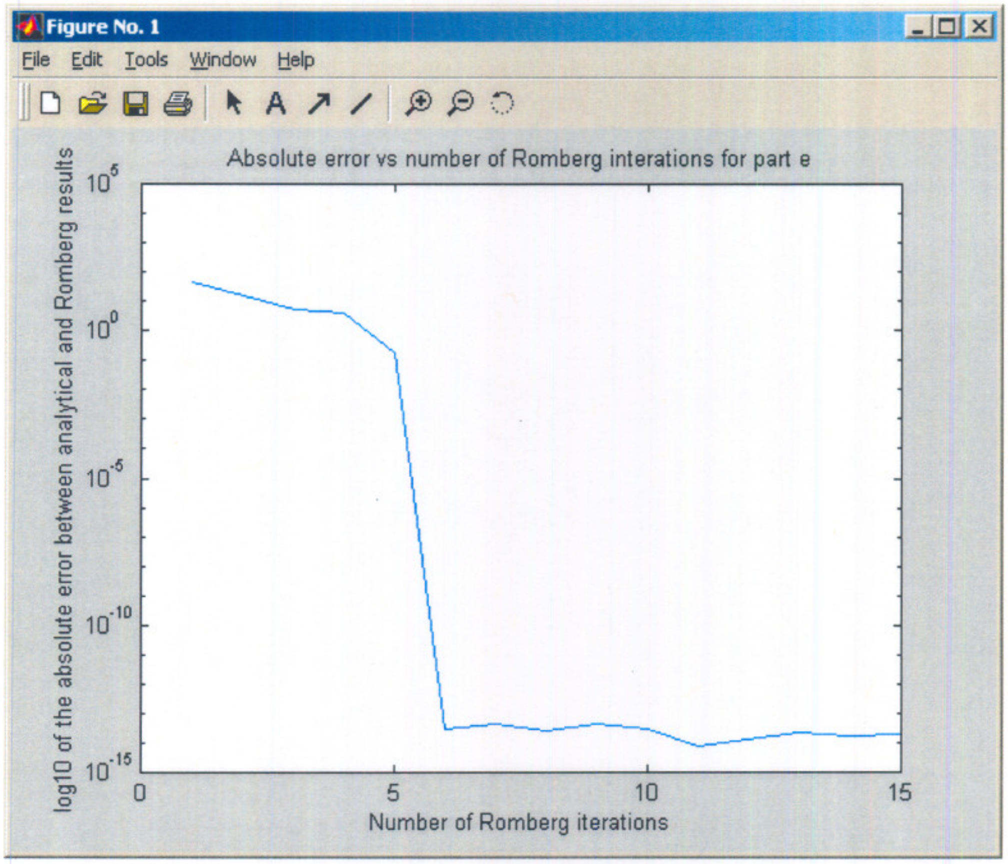
0

← See

```
» nma_problem_10_13


 program to solve problem 10.13
 Nasser Abbasi

enter problem part [a,b,c,d,e,f]:'e'
Roomberg diagonal values for part e, Exact solution = 0.0000000000000
Row 1,  diagonal element =  46.9388888888892
Row 2,  diagonal element =  15.6462962962964
Row 3,  diagonal element =   5.4198629195602
Row 4,  diagonal element =   4.2572827826606
Row 5,  diagonal element =   0.1868756612142
Row 6,  diagonal element =   0.0000000000000
Row 7,  diagonal element =   0.0000000000000
Row 8,  diagonal element =   0.0000000000000
Row 9,  diagonal element =   0.0000000000000
Row 10,  diagonal element =   0.0000000000000
Row 11,  diagonal element =   0.0000000000000
Row 12,  diagonal element =   0.0000000000000
Row 13,  diagonal element =   0.0000000000000
Row 14,  diagonal element =   0.0000000000000
Row 15,  diagonal element =   0.0000000000000
»
```



✓

This showed that the abs error decreased as number of iterations reached 6 very quickly, then abs error remained unchanged as number of iterations is increased more, although at about iteration 12 I see small increase.

```
» nma_problem_10_13


 program to solve problem 10.13
 Nasser Abbasi

enter problem part [a,b,c,d,e,f]:'f'
Roomberg diagonal values for part f, Exact solution =59.7690001151112
Row 1,  diagonal element =  2972.6500308642235
Row 2,  diagonal element =  990.8833436214079
Row 3,  diagonal element =  464.9019967094017
Row 4,  diagonal element =  234.6787608206562
Row 5,  diagonal element =  137.6807502193288
Row 6,  diagonal element =  69.6008330666823
Row 7,  diagonal element =  59.9362358893052
Row 8,  diagonal element =  59.7693433957478
Row 9,  diagonal element =  59.7690001928081
Row 10,  diagonal element =  59.7690001151129
Row 11,  diagonal element =  59.7690001151115
Row 12,  diagonal element =  59.7690001151116
Row 13,  diagonal element =  59.7690001151116
Row 14,  diagonal element =  59.7690001151115
Row 15,  diagonal element =  59.7690001151115
»
```