

iterative solution to differential equations diagrams

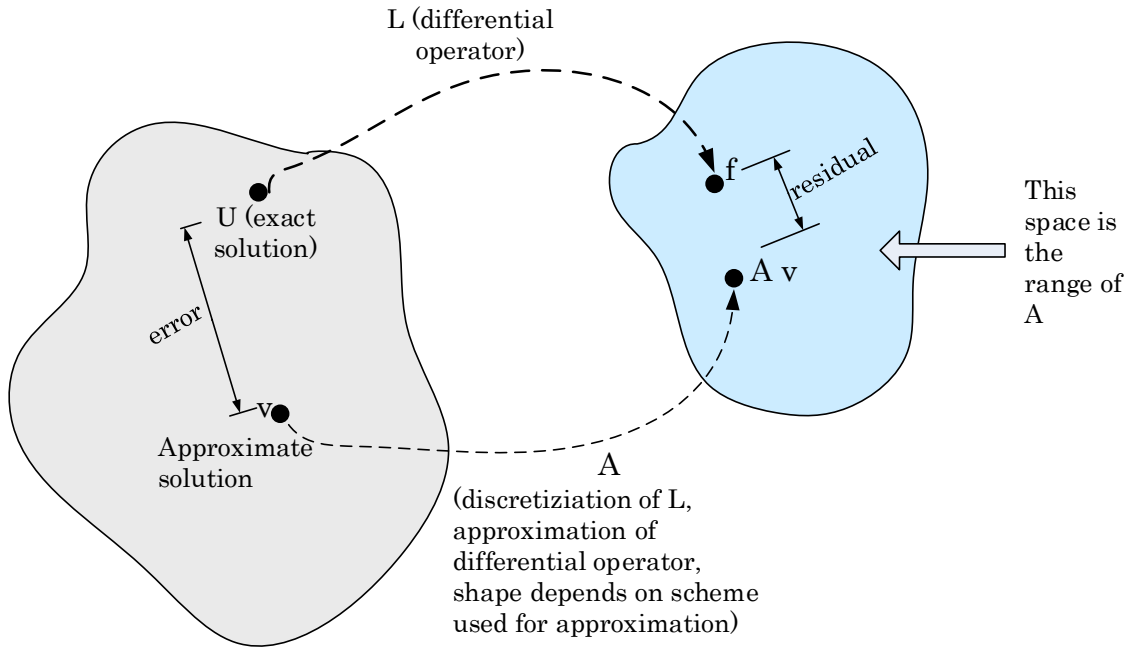
Nasser M. Abbasi

November 12, 2010

Compiled on January 29, 2024 at 2:49am

Diagrams to help illustrate some concepts in iterative solution to differential equations.

$$Lu=f \quad \text{Analog problem (hard)}$$



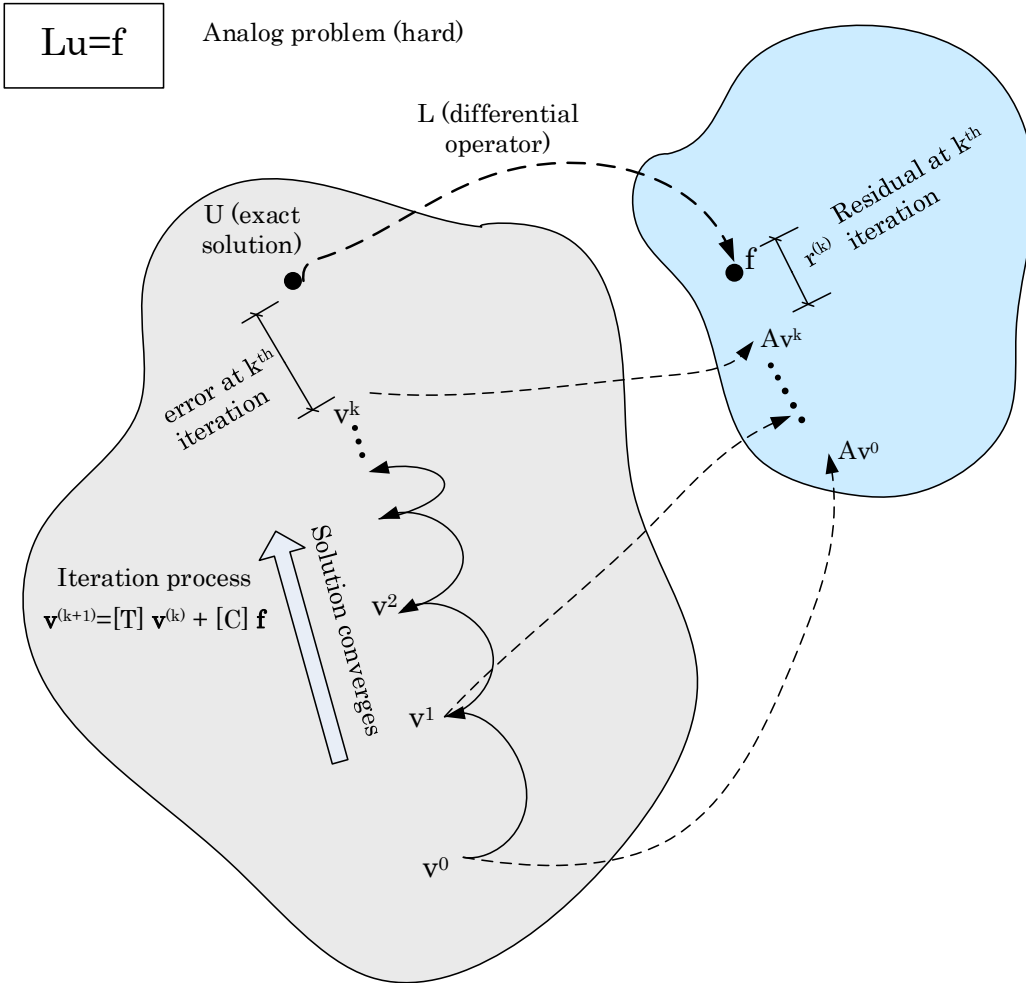
$$Av=f \quad \text{Discrete problem (easier)}$$

It is required to solve $Lu=f$.

In the above problem, the following are known (given): f, L

Algorithm:

1. A is constructed by approximating L using appropriate difference scheme.
2. The problem thus has been translated from $Lu=f$ to an algebraic problem $Av=f$, Now it is solved numerically for v .
3. $f-Av$ is the residual r , $u-v$ is the error e , and $(f-Av)/f$ is the relative residual
4. Note that $Ae=r$



It is required to solve $Lu=f$.
 In the above problem, the following are known (given): f, L
 Algorithm for iterative solution (also called relaxation)

A is not used at all. Only the point by point discretization approximation is used.
 LOOP until convergence
 LOOP over all grid points in solution space
 Update each point using point discretization scheme using Jacobian,
 Gauss-Seidel, or SOR etc... method
 END LOOP
 END LOOP

Good convergence test to use is to check that relative residual is less than some tolerance.