

Finite difference approximation formulas

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May 8, 2012

1 Approximation to first derivative

These formulas below approximate u' at $x = x_j$ where j is the grid point number.

	formula	truncation error	Truncation error order	common name and common notation
1	$u'_j \approx \frac{1}{h}(u_{j+1} - u_j)$	$-u''_j \frac{h}{2} - u_j^{(3)} \frac{h^2}{3!} - \dots$	$O(h)$	one point forward D_+
2	$u'_j \approx \frac{1}{h}(u_j - u_{j-1})$	$u''_j \frac{h}{2} - u_j^{(3)} \frac{h^2}{3!} + \dots$	$O(h)$	one point backward D_-
3	$u'_j \approx \frac{1}{2h}(u_{j+1} - u_{j-1})$	$-u_j^{(3)} \frac{h^2}{6} - u_j^{(6)} \frac{h^5}{6!} - \dots$	$O(h^2)$	centered difference, $D_0 = \frac{D_+ + D_-}{2}$
4	$u'_j \approx \frac{1}{h}(\frac{3}{2}u_j - 2u_{j+1} + \frac{1}{2}u_{j+2})$	to do	$O(h^2)$	3 points forward difference
5	$u'_j \approx \frac{1}{6}(2u_{j+1} + 3u_j - 6u_{j-1} + u_{j-2})$	to do	$O(h^3)$	

2 Approximation to second derivative

These formulas below approximate u'' at $x = x_j$ where j is the grid point number. For approximation to u'' the accuracy of the approximation formula must be no less than 2.

	formula	truncation error	Truncation error order	common name
1	$u''_j \approx \frac{1}{h^2}(U_{j-1} - 2U_j + U_{j+1})$	$-u^{(4)} \frac{h^2}{12} - u^{(6)} \frac{h^4}{360} - \dots$	$O(h^2)$	3 points centered difference