

# Generating state space in controllable form from differential equations

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July 2, 2015

Compiled on July 8, 2025 at 4:47pm

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This note shows examples of how to generate states space  $A, B, C, D$  from differential equations. The state space will be in the controllable form.

Every transfer function which is proper is realizable. Which means the transfer function  $G(s) = \frac{N(s)}{D(s)}$  has its numerator polynomial  $N(s)$  of at most the same order as the denominator  $D(s)$ . Therefore  $G(s) = \frac{s^2}{s^2+s+1}$  is proper but  $G(s) = \frac{s^3}{s^2+s+1}$  is not. To use this method, we start by writing

$$G(s) = k + \tilde{G}(s)$$

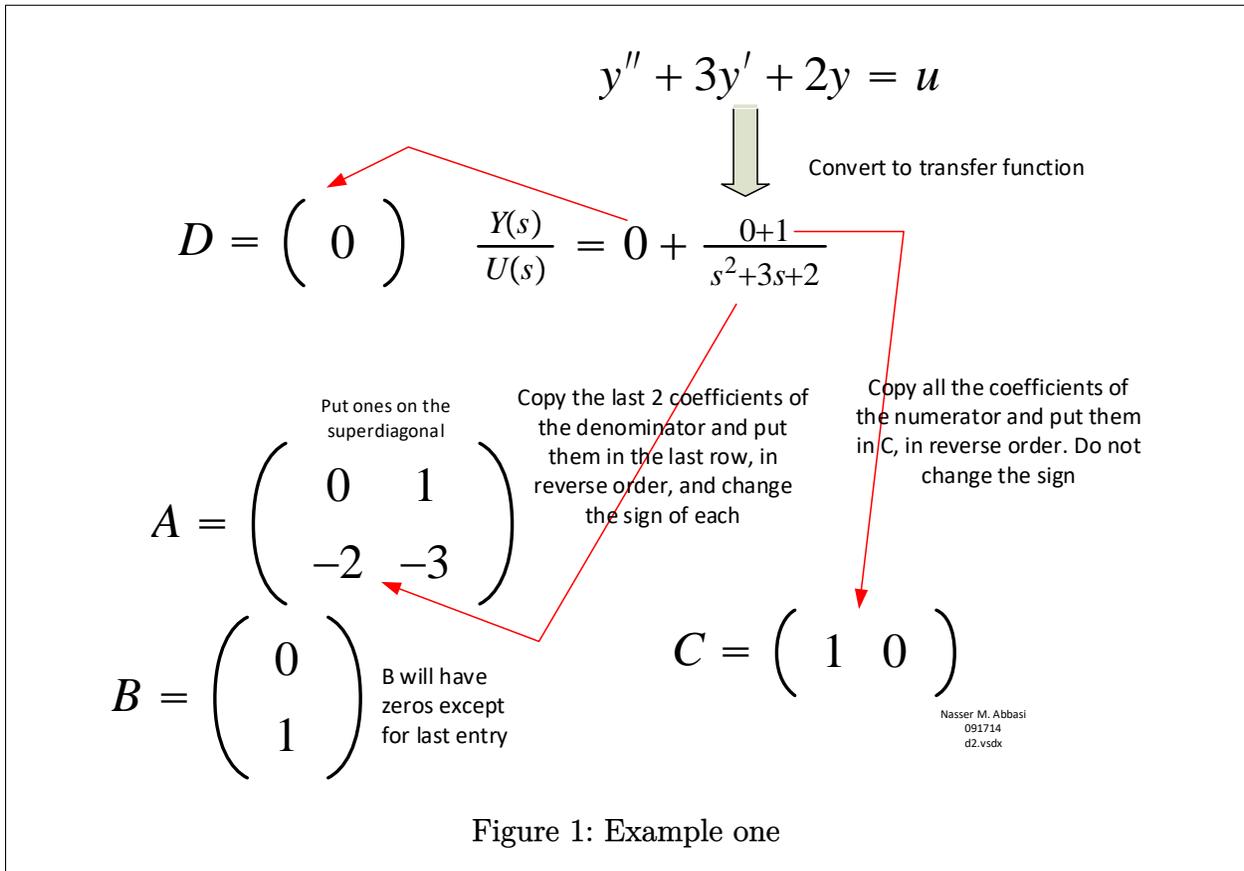
Where  $\tilde{G}(s)$  is strict proper transfer function. A strict proper transfer function is one which has  $N(s)$  polynomial of order at most one less than  $D(s)$ . If  $G(s)$  was already a strict proper transfer function, then  $k$  above will be zero.

Converting a proper  $G(s)$  to strict proper is done using long division. Then the result of the division is moved directly to  $A, B, C, D$  in some specific manner. If  $G(s)$  was already strict proper then of course the long division is not needed.

The following two examples illustrate this method.

# 1 Example 1

$$y''(t) + 3y'(t) + 2y(t) = u(t)$$



# 2 Example 2

$$y'''(t) + 6y''(t) - 2y'(t) - 7y(t) = 4u'''(t) + 3u''(t) + 2u'(t) + 4u(t)$$

$$y''' + 6y'' - 2y' - 7y = 4u''' + 3u'' + 2u' + 4u$$

Convert to transfer function

$$\frac{Y(s)}{U(s)} = \frac{4s^3 + 3s^2 + 2s + 4}{s^3 + 6s^2 - 2s - 7}$$

Since numerator polynomial has same order as denominator, we do long division to make the numerator has one less order than the denominator

$$\begin{array}{r}
 4 \\
 \overline{s^3 + 6s^2 - 2s - 7 \phantom{+ 0} \bigg| 4s^3 + 3s^2 + 2s + 4} \\
 \underline{4s^3 + 24s^2 - 8s - 28} \\
 -21s^2 + 12s + 32
 \end{array}$$

Copy the quotient of the above division to the D matrix

The new transfer function has this form now

$$D = \begin{pmatrix} 4 \end{pmatrix} \quad \frac{Y(s)}{U(s)} = 4 + \frac{-21s^2 + 12s + 32}{s^3 + 6s^2 - 2s - 7}$$

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 7 & 2 & -6 \end{pmatrix}$$

Put ones on the superdiagonal

Copy the last 3 coefficients of the denominator and put them in the last row, in reverse order, and change the sign of each

Copy all the coefficients of the numerator and put them in C, in reverse order. Do not change the sign

$$B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

B will have zeros except for last entry

$$C = \begin{pmatrix} 32 & 12 & -21 \end{pmatrix}$$

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Figure 2: Example two

### 3 References

1. Lecture notes, ECE 717 Linear systems, Fall 2014, University of Wisconsin, Madison by Professor B. Ross Barmish
2. Linear system theory and design, Chi-Tsong Chen.