Generating state space in controllable form from differential equations

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Contents

This note shows examples of how to generate states space A, B, C, D from differential equations. The state space will be in the controllable form.

Every transfer function which is proper is realizable. Which means the transfer function $G(s) = \frac{N(s)}{D(s)}$ has its numerator polynomial $N(s)$ of at most the same order as the numerator $D(s)$. Therefore $G(s) = \frac{s^2}{s^2+s+1}$ is proper but $G(s) = \frac{s^3}{s^2+s+1}$ is not. To use this method, we start by writing

$$G(s) = k + \tilde{G}(s)$$

Where $\tilde{G}(s)$ is strict proper transfer function. A strict proper transfer function is one which has $N(s)$ polynomial of order at most one less than $D(s)$. If $G(s)$ was already a strict proper transfer function, then k above will be zero.

Converting a proper $G(s)$ to strict proper is done using long division. Then the result of the division is moved directly to A, B, C, D in some specific manner. If $G(s)$ was already strict proper then of course the long division is not needed.

The following two examples illustrate this method.

1 Example 1

$$y''(t) + 3y'(t) + 2y(t) = u(t)$$

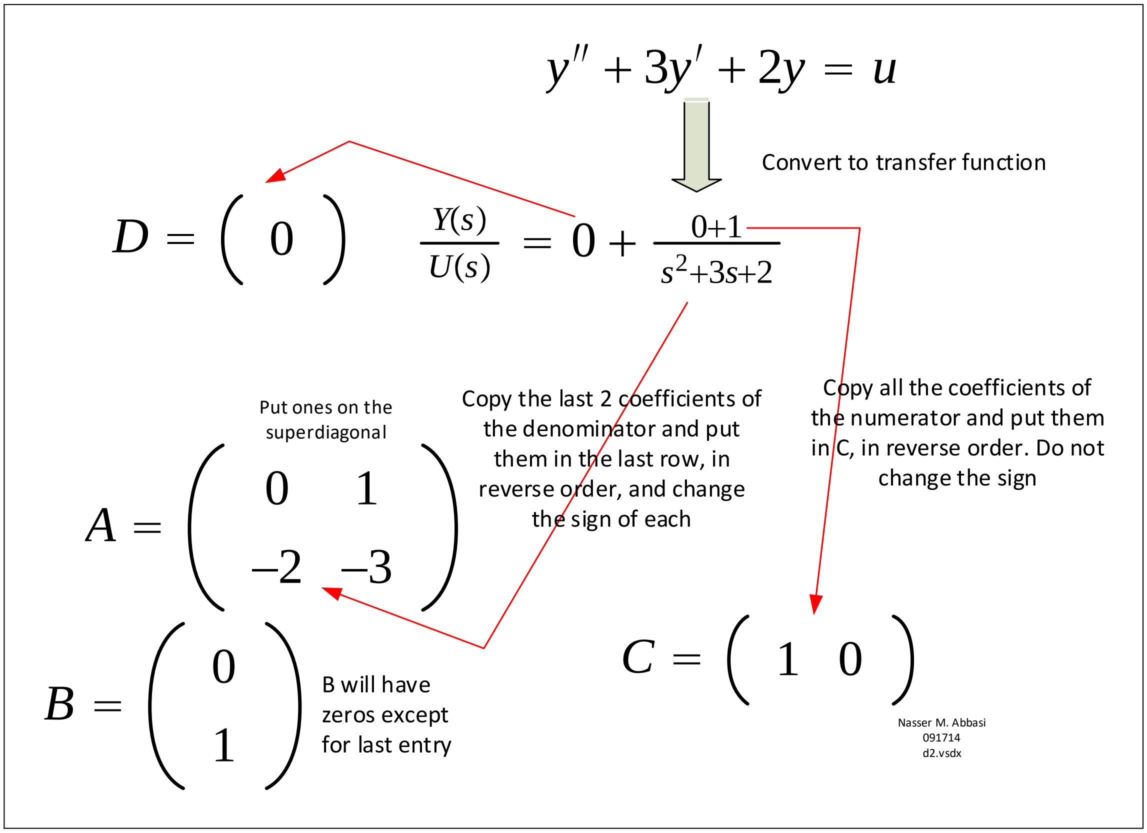

Figure 1: Example one

2 Example 2

$$y'''(t) + 6y''(t) - 2y'(t) - 7y(t) = 4u'''(t) + 3u''(t) + 2u'(t) + 4u(t)$$

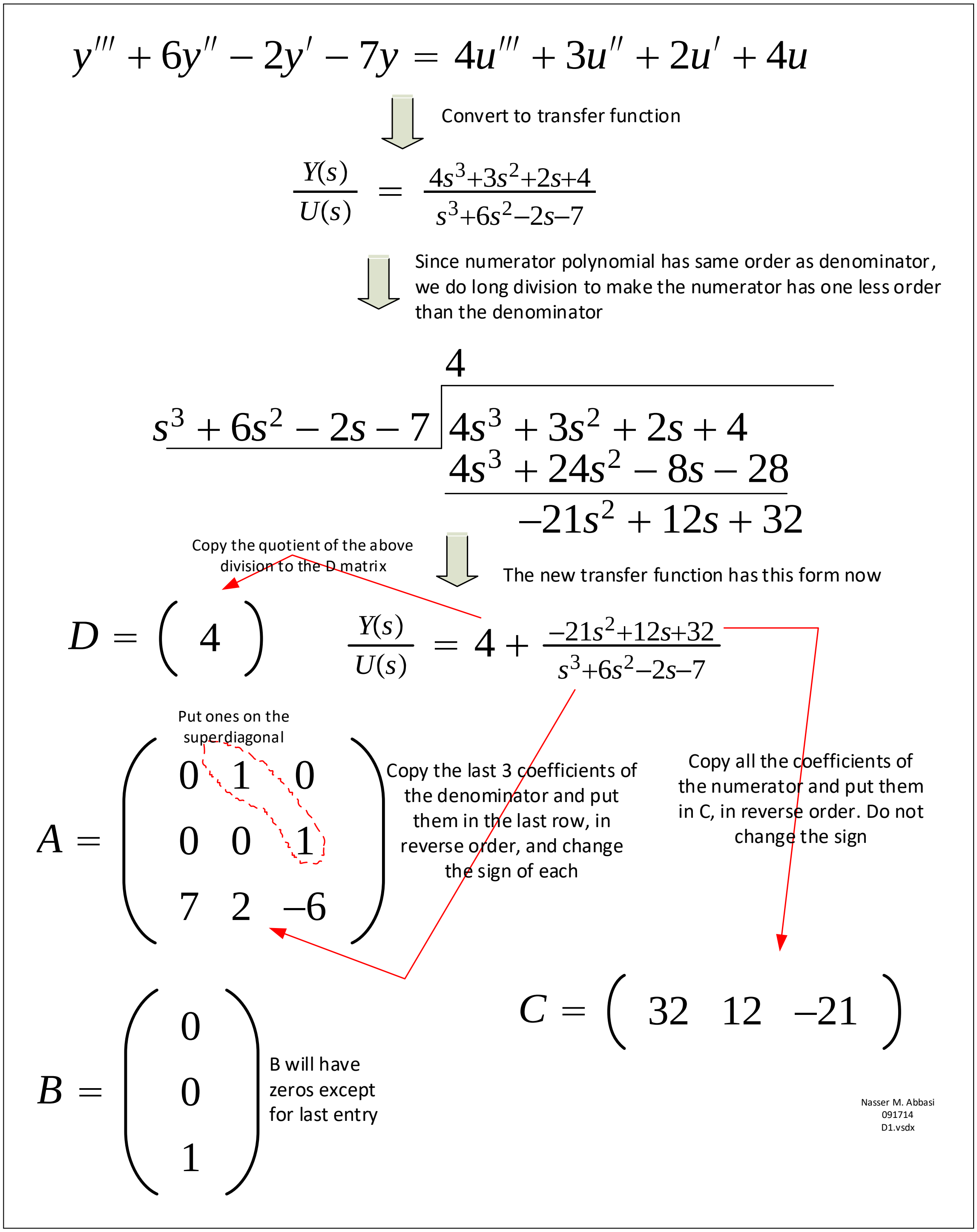

Figure 2: Example two

3 References

1. Lecture notes, ECE 717 Linear systems, Fall 2014, University of Wisconsin, Madison by Professor B. Ross Barmish

2. Linear system theory and design, Chi-Tsong Chen.