statistics cheat sheet

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Contents

1 my first cheat sheet

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Chapter one, Probability Definitions:

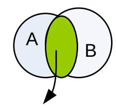
1) 2 events are disjoint if they have no outcomes in common. Written as $A \cap B = \emptyset$

2) 2 events are independent if occurrence of one does not give any indication of the occurrence of the other, in addition, we write $P(A \cap B) = P(A)\,P(B)$

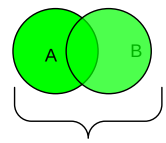
3) Sample space $\Omega = \{A, B, \ldots\}$ contains all possible events

Axiom of probability:
1. $P(\Omega)=1$
2. If $A \in \Omega$ then $P(A) \geq 0$
3. If A,B are disjoint then $P(A \cap B) = P(A)P(B)$

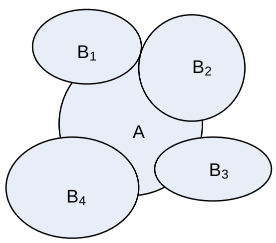
Learn how to proof things using all these axioms



$A \cap B \equiv$ A AND B
$P(A \cap B) = P(A|B)P(B)$
Multiplication law



$A \cup B \equiv$ A OR B
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

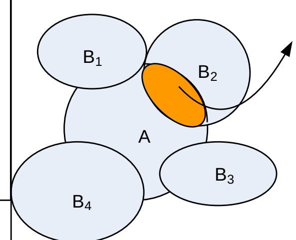


Suppose B_i are all disjoint, and that Bi's make up the whole Ω space. Then

$P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3) + \ldots$
$\quad = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3) + \ldots$

(this is called law of total probability)

Bayes Rule: This is really nothing more than the multiplication law, but used for the whole space. Assume that events B1,B2,B3,… are all disjoint, and they add up to $\cap$, and assume we have event A. Then we write



$P(A \cap B_2) = P(B_2|A)P(A)$
Hence
$P(B_2|A) = P(A \cap B_2)/P(A)$
But $P(A \cap B_2) = P(A|B_2)P(B_2)$
And P(A) is total probability of A, see law of total probability. Putting these together we get

$$P(B_2|A) = \frac{P(A|B_2)P(B_2)}{\sum P(A|B_i)P(B_i)}$$

Permutation and Combination: In Permutation the order we list things is important. Hence A,B would be different than B,A. Hence result of Permutations are ***larger*** than combination.
Let $\Omega = \{a,b,c,d\ldots\}$ be size n, how many way we can obtain r items out of n?

Permutation (order is important)
- With replacement: n^r
- Without replacement: $n(n-1)(n-2)\ldots(n-r+1)$ (this is what calculator does nPr. For n=4,r=2, nPr=12

Combination (Order is not important)
- $nCr = \dfrac{n!}{r!\,(n-r)!}$

For discrete:
$F(x) = P(X \leq x)$
$\quad = P(X<x) + F(x)$

$P(x) = F(k) - F(k-1)$
Also
$F(v) - F(u) = P(u < x \leq v)$

Example: $\Omega = \{a,b,c\}$.
Permut. With replace: {aa,ab,ac,bb,ba,bc,cc,ca,cb}
Permute No replace: $nP2 \rightarrow$ {ab,ac,ba,bc,ca,cb}
Combination: $nCr \rightarrow$ {ab,ac,bc}

Basic probability relations for CONTINUOUS random variable

$$P(a \leq X \leq b) = \int_a^b f(t)\,dt = \mathrm{CDF}(b) - \mathrm{CDF}(a)$$
$$P(X \leq a) = \int_{-\infty}^{a} f(t)\,dt$$

Note that for discrete random variable, we can talk about probability at a POINT, but in cont. case, the probability at a point is ZERO, so we need a range there.

Binomial coeff. $(a+b)^n = \sum_{k=0} nCk\, a^k\, b^{n-k}$

Capture/recapture method: This is a method to estimate population. Suppose we tag t animals. Then we capture m sample and find r are tagged, how is the sample size?

Example: n size population, t=3 tagged. M size sample taken out of population, has r=2 tagged.
n={o,o,x,o,x,x,o,o,o,o}
m={x,x,o,o,o}. Then

$$P(r=2) = \frac{\binom{t}{r}\binom{n-t}{m-r}}{\binom{n}{m}}$$

To estimate n above, we look for maximum likelihood of P(r) so we need the value of n which maximum the above P(r). This comes out to be largest integer not over m*t/r

Bernoulli: X 1 or 0 with probability p or (1-p)
Binomial: How many wins in n trial when probability of win is p?
$P(k) = nCk\ p^k\ (1-p)^{n-k}$ k=0,1,2,...
Geometric: How many trials until a win? (includes the winning trial)
$P(k) = (1-p)^{k-1}\, p$ k=1,2,3…
Negative binomial: How many triials to obtain r wins?
$P(k) = (k-1)C(r-1)\ p^r\ (1-p)^{k-r}$ k=1,2,.
Hypergeometeric: How many black balls drawn when removing m balls from urn with n balls total? P(k)=rCk * (n-r)C(m-k)/(nCm)

Poisson: Lambda: Rate of events. X number of events in given period of time. $P(k) = \text{lambda}^k\, e^{-\text{lambda}} / k!$

(multiply by TIME*lambda)

In a set of n objects contains n1,n2,n3 different subsets. The number of distinguishable permutations of the n objects is

$$\binom{n}{n_1,\ n_2,\ n_3} = \frac{n!}{n_1!\,n_2!\,n_3!}$$

Figure 1: statistics

2 second cheat sheet

problem: phone calls received at rate $\lambda = 2$ per hr. If person wants to take 10 min shower, what is probability a phone will ring during that time?

answer: first change to $\omega = \lambda \frac{10}{60} = 2\frac{10}{60} = .3333$, now we want $P(X \geq 1) = 1 - P(X \leq 1) = 1 - P(0)$

but $P(k) = \frac{\lambda^k}{k!}e^{-\lambda}$, but remember, we are using ω, so $P(k) = \frac{\omega^k}{k!}e^{-\omega}$ so $P(0) = \frac{.3333^0}{0!}e^{-.3333} = 0.777$

so $P(X \geq 1) = 1 - .777 = 0.283$, so 28% change the phone will ring.

How long can shower be if they wish probability of receiving no phone calls to be at most 0.5?

$P(0) = 0.5 = \frac{\omega 0}{0!}e^{-\omega} \rightarrow 0.5 = e^{-\omega}$ hence $\ln 0.5 = -\omega \rightarrow \omega = 0.693$, so $\lambda \frac{x}{60} = 0.693 \rightarrow x = 20.7$ minutes

To find quantile, say $\frac{1}{4}$, first find an expression for $F(x)$ as function of x, then solve for x in $F(x) = .25$

For median, solve for x in $F(x) = .5$

properties of CDF: 1. Show $F(x) \geq 0$ for all x. Do this by showing $F'(x) \geq 0$, and show limit $F(x) \rightarrow 1$ as $x \rightarrow \infty$ and limit $F(x) \rightarrow 0$ as $x \rightarrow -\infty$. And $\boxed{P(k_1 \leq T < k_2) = F(k_2) - F(k_1)}$

properties of pdf:

1. piecewise continuous
2. pdf$(x) \geq 0$
3. $\int_{-\infty}^{\infty} pdf(x) = 1$

remember $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$

The geometric distribution is the only discrete memoryless random distribution. It is a discrete analog of the exponential distribution. continuous

Some relations

$$\sum_{k=1}^{n} k = \frac{1}{2}n(n+1)$$

Geometric sum

$$\sum_{k=0}^{n} r^k = \frac{1-r^{n+1}}{1-r}$$

if $-1 < r < 1$, then

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}$$

if the sum is from 1 then

$$\sum_{k=1}^{n} r^k = \frac{r(1-r^{n+1})}{1-r}$$

if $-1 < r < 1$, then

$$\sum_{k=1}^{\infty} r^k = \frac{r}{1-r}$$

$$\Gamma(x) = \int_0^{\infty} u^{x-1} e^{-u} du$$

$$\Gamma(n) = (n-1)\,!$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

$$\int \ln(y)\, dy = -y + y \ln(y)$$

$$\int \frac{1}{y} dy = \ln(y)$$

And

$$\binom{n}{n_1 \quad n_2 \quad n_3} = \frac{n!}{n_1!\ n_2!\ n_3!}$$

If given joint density $f_{XY}(x,y)$ and asked to find conditional $P(X|Y) = \frac{f_{XY}(x,y)}{f_Y(y)}$ so need to find marginals. Marginal is found from $f_Y(y) = \int_x f_{XY}(x,y)\, dx$, and $f_X(x) = \int_y f_{XY}(x,y)\, dy$

To convert from x, y to polar, example: given $f(x,y) = c\sqrt{1-(x^2+y^2)}$ find c, where $x^2 + y^2 \leq 1$, then write

$$c \int_{\theta=-\pi}^{\theta=\pi} \int_{r=0}^{r=1} \sqrt{1-r^2} r dr d\theta$$

Use identity above.

law of total probablity: if we know $Y|X$ and X and want to know distribution of Y, then $f(Y) = \int_{-\infty}^{\infty} f_{Y|X}(y|x)\, f_X(x)\, dx$

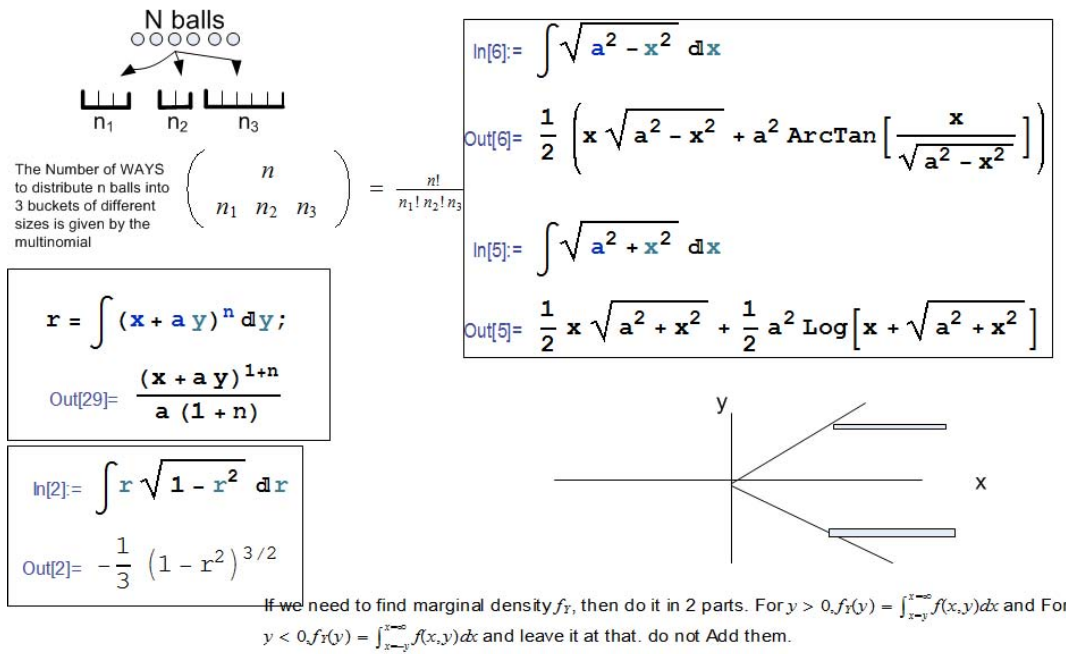


Figure 2: multi

$$Z = \frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \to N(0,1)$$

$$T = \frac{\bar{X}_n - \mu}{S_n/\sqrt{n}} \to T(n)$$

where S_n is *std* of the sample.

Note Var(sample) has chi square (n) distribution.

CI for T:

$$\Pr\left(-A < \frac{\bar{X}_n - \mu}{S_n/\sqrt{n}} < A\right) = 1 - \alpha$$

$$\Pr\left(\bar{X}_n - A\frac{S_n}{\sqrt{n}} < \mu < \bar{X}_n + A\frac{S_n}{\sqrt{n}}\right) = 1 - \alpha$$