Note on how to calculate Discrete time Fourier transform for 2D data

Nasser M. Abbasi

July 8, 2025   Compiled on July 8, 2025 at 4:39pm

Given data

$$f(n,m)=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$$

To find its DFT, we compute the DFT of each column at a time, which generates a temporary matrix. Then compute the DFT of each row of the temporary matrix. This gives the DFT of the above.

The DFT of 1D is given by

$$F[s]=\frac{1}{n}\sum_{r=1}^{n} f[r]\,e^{\frac{2\pi}{n}i((r-1)(s-1))}$$

Hence for first column in the data, which is $\begin{pmatrix}1\\3\end{pmatrix}$ and using $n=2$ in this example (same number of rows as columns). Then the above becomes

$$\begin{aligned}F[s]&=\frac{1}{2}\sum_{r=1}^{2} f[r]\,e^{\pi i((r-1)(s-1))}\\&=\frac{1}{2}\left(f[1]+f[2]\,e^{\pi i(s-1)}\right)\\&=\frac{1}{2}\left(1+3e^{\pi i(s-1)}\right)\end{aligned}$$

Therefore

$$\begin{aligned}F[s=1]&=\frac{4}{2}=2\\F[s=2]&=\frac{1}{2}\left(1+3e^{\pi i}\right)=\frac{1}{2}(1-3)=-1\end{aligned}$$

Hence the first column of the temporary matrix in F space is $\begin{pmatrix}2\\-1\end{pmatrix}$. Now we find the DFT of the second column of the input which is $\begin{pmatrix}2\\4\end{pmatrix}$. we have (since $n=2$ in this example)

$$\begin{aligned}F[s]&=\frac{1}{2}\sum_{r=1}^{2} f[r]\,e^{\frac{2\pi}{2}i((r-1)(s-1))}\\&=\frac{1}{2}\left(f[1]+f[2]\,e^{\pi i(s-1)}\right)\\&=\frac{1}{2}\left(2+4e^{\pi i(s-1)}\right)\end{aligned}$$

Therefore

$$\begin{aligned}F[s=1]&=\frac{1}{2}(2+4)=3\\F[s=2]&=\frac{1}{2}\left(2+4e^{\pi i}\right)=\frac{1}{2}(2-4)=-1\end{aligned}$$

Hence the second column of the temporary matrix in F space is $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ which means after first pass, the temporary matrix in F space is now

$$\begin{pmatrix} 2 & 3 \\ -1 & -1 \end{pmatrix}$$

Now we apply DFT to each row of the above. This is the second pass. For the first row of the above, the DFT is

$$\begin{aligned} F[s] &= \frac{1}{2} \sum_{r=1}^{2} f[r] \, e^{\frac{2\pi}{2} i((r-1)(s-1))} \\ &= \frac{1}{2} \left(f[1] + f[2] \, e^{\pi i (s-1)} \right) \\ &= \frac{1}{2} \left(2 + 3e^{\pi i (s-1)} \right) \end{aligned}$$

Therefore the DFT of the first row becomes

$$\begin{aligned} F[s=1] &= \frac{1}{2}(2+3) = 2.5 \\ F[s=2] &= \frac{1}{2}\left(2+3e^{\pi i}\right) = \frac{1}{2}(2-3) = -0.5 \end{aligned}$$

Which is $\begin{pmatrix} 2.5 & -0.5 \end{pmatrix}$, and the DFT of the second is

$$\begin{aligned} F[s] &= \frac{1}{2} \sum_{r=1}^{2} f[r] \, e^{\frac{2\pi}{2} i((r-1)(s-1))} \\ &= \frac{1}{2} \left(f[1] + f[2] \, e^{\pi i (s-1)} \right) \\ &= \frac{1}{2} \left(-1 - e^{\pi i (s-1)} \right) \end{aligned}$$

which is

$$\begin{aligned} F[s=1] &= \frac{1}{2}(-1-1) = -1 \\ F[s=2] &= \frac{1}{2}\left(-1-e^{\pi i}\right) = \frac{1}{2}(-1+1) = 0 \end{aligned}$$

Which is $\begin{pmatrix} -2 & 1 \end{pmatrix}$. Therefore the final DFT is

$$\begin{pmatrix} 2.5 & -0.5 \\ -1 & 0 \end{pmatrix}$$