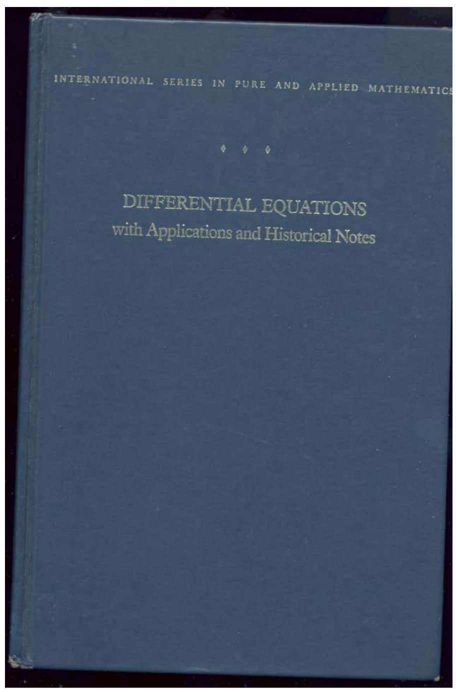
A Solution Manual For

Differential equations with applications and historial notes, George F. Simmons, 1971



Nasser M. Abbasi

March 3, 2024

Contents

1 Chapter 2, section 7, page 37

1.1 problem 1.a

Internal problem ID [3080]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 7, page 37
Problem number: 1.a.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{-y^2 + y'yx = -x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve((x^2-y(x)^2)+x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{-2\ln(x) + c_1}\, x$$
$$y(x) = -\sqrt{-2\ln(x) + c_1}\, x$$

✓ Solution by Mathematica

Time used: 0.186 (sec). Leaf size: 36

```
DSolve[(x^2-y[x]^2)+x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{-2\log(x) + c_1}$$
$$y(x) \to x\sqrt{-2\log(x) + c_1}$$

1.2 problem 1.b

Internal problem ID [3081]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 7, page 37
Problem number: 1.b.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y'x^2 - 2yx - 2y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x)-2*x*y(x)-2*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{-2x + c_1}$$

✓ Solution by Mathematica

Time used: 0.143 (sec). Leaf size: 22

```
DSolve[x^2*y'[x]-2*x*y[x]-2*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{-2x + c_1}$$

$$y(x) \to 0$$

1.3 problem 1.c

Internal problem ID [3082]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 7, page 37
Problem number: 1.c.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y'x^2 - 3(y^2 + x^2) \arctan\left(\frac{y}{x}\right) - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x^2*diff(y(x),x)=3*(x^2+y(x)^2)*arctan(y(x)/x)+x*y(x),y(x), singsol=all)
```

$$y(x) = \tan\left(c_1 x^3\right) x$$

✓ Solution by Mathematica

Time used: 0.179 (sec). Leaf size: 37

```
DSolve[x^2*y'[x]==3*(x^2+y[x]^2)*Arctan[y[x]/x]+x*y[x],y[x],x,IncludeSingularSolutions -> Tr
```

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{1}{\text{Arctan}(K[1])\left(K[1]^2+1\right)} dK[1] = 3\log(x) + c_1, y(x)\right]$$

1.4 problem 1.d

Internal problem ID [3083]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 7, page 37
Problem number: 1.d.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{x \sin\left(\frac{y}{x}\right) y' - \sin\left(\frac{y}{x}\right) y = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*sin(y(x)/x)*diff(y(x),x)=y(x)*sin(y(x)/x)+x,y(x), singsol=all)
```

$$y(x) = (\pi - \arccos(\ln(x) + c_1))\, x$$

✓ Solution by Mathematica

Time used: 0.435 (sec). Leaf size: 34

```
DSolve[x*Sin[y[x]/x]*y'[x]==y[x]*Sin[y[x]/x]+x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \arccos(-\log(x) - c_1)$$
$$y(x) \to x \arccos(-\log(x) - c_1)$$

1.5 problem 1.

Internal problem ID [3084]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 7, page 37
Problem number: 1..
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘]]`

$$\boxed{xy' - y - 2\,\mathrm{e}^{-\frac{y}{x}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)=y(x)+2*exp(- y(x)/x),y(x), singsol=all)
```

$$y(x) = \ln\left(\frac{2c_1x - 2}{x}\right)x$$

✓ Solution by Mathematica

Time used: 0.618 (sec). Leaf size: 16

```
DSolve[x*y'[x]==y[x]+2*Exp[- y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\log\left(-\frac{2}{x} + c_1\right)$$

1.6 problem 3.a

Internal problem ID [3085]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 7, page 37
Problem number: 3.a.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - (y + x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=(x+y(x))^2,y(x), singsol=all)
```

$$y(x) = -x - \tan(c_1 - x)$$

✓ Solution by Mathematica

Time used: 0.735 (sec). Leaf size: 14

```
DSolve[y'[x]==(x+y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + \tan(x + c_1)$$

1.7 problem 3.b

Internal problem ID [3086]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 7, page 37
Problem number: 3.b.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \sin(x - y + 1)^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)=sin(x-y(x)+1)^2,y(x), singsol=all)
```

$$y(x) = x + 1 + \arctan(c_1 - x)$$

✓ Solution by Mathematica

Time used: 0.344 (sec). Leaf size: 33

```
DSolve[y'[x]==Sin[x-y[x]+1]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[2y(x) - 2(\tan(-y(x) + x + 1) - \arctan(\tan(-y(x) + x + 1))) = c_1, y(x)]$$

1.8 problem 5.a

Internal problem ID [3087]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 7, page 37
Problem number: 5.a.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' - \frac{x+y+4}{x-y-6} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)=(x+y(x)+4)/(x-y(x)-6),y(x), singsol=all)
```

$$y(x) = -5 - \tan\left(\operatorname{RootOf}\left(2\_Z + \ln\left(\frac{1}{\cos(\_Z)^2}\right) + 2\ln(x-1) + 2c_1\right)\right)(x-1)$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 58

```
DSolve[y'[x]==(x+y[x]+4)/(x-y[x]-6),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\arctan\left(\frac{y(x)+x+4}{y(x)-x+6}\right) + \log\left(\frac{x^2+y(x)^2+10y(x)-2x+26}{2(x-1)^2}\right) + 2\log(x-1) + c_1 = 0, y(x)\right]$$

1.9 problem 5.b

Internal problem ID [3088]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 7, page 37
Problem number: 5.b.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y' - \frac{x+y+4}{x+y-6} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=(x+y(x)+4)/(x+y(x)-6),y(x), singsol=all)
```

$$y(x) = -x - 5\,\mathrm{LambertW}\left(-\frac{\mathrm{e}^{-\frac{2x}{5}} c_1 \mathrm{e}^{\frac{1}{5}}}{5}\right) + 1$$

✓ Solution by Mathematica

Time used: 4.019 (sec). Leaf size: 35

```
DSolve[y'[x]==(x+y[x]+4)/(x+y[x]-6),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -5W\left(-e^{-\frac{2x}{5}-1+c_1}\right) - x + 1$$
$$y(x) \to 1 - x$$

2 Chapter 2, section 8, page 41

2.1 problem 1

Internal problem ID [3089]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\left(x+\frac{2}{y}\right)y'+y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((x+2/y(x))*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-\operatorname{LambertW}\left(\frac{x\,\mathrm{e}^{\frac{c_1}{2}}}{2}\right)+\frac{c_1}{2}}$$

✓ Solution by Mathematica

Time used: 10.621 (sec). Leaf size: 58

```
DSolve[(x+2/y[x])*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\rightarrow\frac{2W\left(-\frac{1}{2}\sqrt{e^{c_1}x^2}\right)}{x}$$
$$y(x)\rightarrow\frac{2W\left(\frac{1}{2}\sqrt{e^{c_1}x^2}\right)}{x}$$
$$y(x)\rightarrow 0$$

2.2 problem 2

Internal problem ID [3090]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{\sin(x)\tan(y) + \cos(x)\sec(y)^2 y' = -1}$$

✗ Solution by Maple

```
dsolve((sin(x)*tan(y(x))+1)+(cos(x)*sec(y(x))^2)*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 2.318 (sec). Leaf size: 54

```
DSolve[(Sin[x]*Tan[y[x]]+1)+(Cos[x]*Sec[y[x]]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\arctan(\sin(x) + c_1 \cos(x))$$
$$y(x) \to -\frac{1}{2}\pi\sqrt{\cos^2(x)}\sec(x)$$
$$y(x) \to \frac{1}{2}\pi\sqrt{\cos^2(x)}\sec(x)$$

2.3 problem 3

Internal problem ID [3091]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{y + (x + y^3)\, y' = x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve((y(x)-x^3)+(x+y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$-\frac{x^4}{4} + y(x)\, x + \frac{y(x)^4}{4} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.173 (sec). Leaf size: 1210

```
DSolve[(y[x]-x^3)+(x+y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+\left(x^4+4c_1\right)^3}}-\frac{\sqrt[3]{3}\left(x^4+4c_1\right)}{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+\left(x^4+4c_1\right)^3}}}}+\sqrt{\frac{\cdots}{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4}\cdots}}}}}{\cdots}$$

$$y(x) \to \frac{\sqrt{\frac{6\sqrt{2}x}{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+\left(x^4+4c_1\right)^3}}-\frac{\sqrt[3]{3}\left(x^4+4c_1\right)}{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+\left(x^4+4c_1\right)^3}}}}}-\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+\left(x^4\cdots\right.}}}}{\cdots}$$

$$y(x) \to \frac{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+\left(x^4+4c_1\right)^3}}-\frac{\sqrt[3]{3}\left(x^4+4c_1\right)}{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+\left(x^4+4c_1\right)^3}}}}-\sqrt{-\frac{\cdots}{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4}\cdots}}}}}{\cdots}$$

$$y(x) \to \frac{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+\left(x^4+4c_1\right)^3}}-\frac{\sqrt[3]{3}\left(x^4+4c_1\right)}{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4+\left(x^4+4c_1\right)^3}}}}+\sqrt{-\frac{\cdots}{\sqrt{\sqrt[3]{9x^2+\sqrt{3}\sqrt{27x^4}\cdots}}}}}{\cdots}$$

2.4 problem 4

Internal problem ID [3092]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$\boxed{2y^2 - (4 - 2y + 4yx)\, y' = 4x - 5}$$

✗ Solution by Maple

```
dsolve((2*y(x)^2-4*x+5)=(4-2*y(x)+4*x*y(x))*diff(y(x),x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(2*y[x]^2-4*x+5)==(4-2*y[x]+4*x*y[x])*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.5 problem 5

Internal problem ID [3093]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y + y\cos(yx) + (x + x\cos(yx))\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve((y(x)+y(x)*cos(x*y(x)))+(x+x*cos(x*y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\pi}{x}$$
$$y(x) = \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 49

```
DSolve[(y[x]+y[x]*Cos[x*y[x]])+(x+x*Cos[x*y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{\pi}{x}$$
$$y(x) \to \frac{\pi}{x}$$
$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to -\frac{\pi}{x}$$
$$y(x) \to \frac{\pi}{x}$$

2.6 problem 6

Internal problem ID [3094]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\cos(x)\cos(y)^2 + 2\sin(x)\sin(y)\cos(y)\,y' = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 31

```
dsolve(cos(x)*cos(y(x))^2+(2*sin(x)*sin(y(x))*cos(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\pi}{2}$$
$$y(x) = \arccos\left(\sqrt{\sin(x)\,c_1}\right)$$
$$y(x) = \pi - \arccos\left(\sqrt{\sin(x)\,c_1}\right)$$

✓ Solution by Mathematica

Time used: 5.526 (sec). Leaf size: 73

```
DSolve[Cos[x]*Cos[y[x]]^2+(2*Sin[x]*Sin[y[x]]*Cos[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x) \to -\frac{\pi}{2}$$

$$y(x) \to \frac{\pi}{2}$$

$$y(x) \to -\arccos\left(-\frac{1}{4}c_1\sqrt{\sin(x)}\right)$$

$$y(x) \to \arccos\left(-\frac{1}{4}c_1\sqrt{\sin(x)}\right)$$

$$y(x) \to -\frac{\pi}{2}$$

$$y(x) \to \frac{\pi}{2}$$

2.7 problem 7

Internal problem ID [3095]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{(\sin(x)\sin(y) - x\,\mathrm{e}^{y})\,y' - \mathrm{e}^{y} - \cos(y)\cos(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((sin(x)*sin(y(x))-x*exp(y(x)))*diff(y(x),x)=exp(y(x))+cos(x)*cos(y(x)),y(x), singsol=
```

$$c_1 + \sin(x)\cos(y(x)) + x\,\mathrm{e}^{y(x)} = 0$$

✓ Solution by Mathematica

Time used: 0.626 (sec). Leaf size: 21

```
DSolve[(Sin[x]*Sin[y[x]]-x*Exp[y[x]])*y'[x]==Exp[y[x]]+Cos[x]*Cos[y[x]],y[x],x,IncludeSingul
```

$$\mathrm{Solve}\left[2\left(xe^{y(x)} + \sin(x)\cos(y(x))\right) = c_1, y(x)\right]$$

2.8 problem 8

Internal problem ID [3096]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$-\frac{\sin\left(\frac{x}{y}\right)}{y}+\frac{x\sin\left(\frac{x}{y}\right)y'}{y^2}=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(-1/y(x)*sin(x/y(x))+x/y(x)^2*sin(x/y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{x}{\pi-c_1}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 19

```
DSolve[-1/y[x]*Sin[x/y[x]]+x/y[x]^2*Sin[x/y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to c_1x$$
$$y(x)\to \text{ComplexInfinity}$$
$$y(x)\to \text{ComplexInfinity}$$

2.9 problem 9

Internal problem ID [3097]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y + (1 - x)\, y' = -1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve((1+y(x))+(1-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -1 + c_1(x - 1)$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 18

```
DSolve[(1+y[x])+(1-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 + c_1(x - 1)$$
$$y(x) \to -1$$

2.10 problem 10

Internal problem ID [3098]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{2y^3x + y\cos(x) + (3y^2x^2 + \sin(x))\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 375

```
dsolve((2*x*y(x)^3+y(x)*cos(x))+(3*x^2*y(x)^2+sin(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin\left(x\right)^3}-108c_1x\right)^{\frac{1}{3}}}{6x} - \frac{2\sin\left(x\right)}{x\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin\left(x\right)^3}-108c_1x\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin\left(x\right)^3}-108c_1x\right)^{\frac{1}{3}}}{12x} + \frac{\sin\left(x\right)}{x\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin\left(x\right)^3}-108c_1x\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}{6x}+\frac{2\sin(x)}{x\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin\left(x\right)^3}-108c_1x\right)^{\frac{1}{3}}}{12x} + \frac{\sin\left(x\right)}{x\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin\left(x\right)^3}-108c_1x\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}{6x}+\frac{2\sin(x)}{x\left(12\sqrt{3}\,\sqrt{27c_1^2x^2+4\sin(x)^3}-108c_1x\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 32.512 (sec). Leaf size: 339

```
DSolve[(2*x*y[x]^3+y[x]*Cos[x])+(3*x^2*y[x]^2+Sin[x])*y'[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{\sqrt[3]{9c_1x^4 + \sqrt{12x^6\sin^3(x) + 81c_1{}^2x^8}}}{\sqrt[3]{2}3^{2/3}x^2} - \frac{\sqrt[3]{\frac{2}{3}}\sin(x)}{\sqrt[3]{9c_1x^4 + \sqrt{12x^6\sin^3(x) + 81c_1{}^2x^8}}}$$

$$y(x) \to \frac{(1 + i\sqrt{3})\sin(x)}{2^{2/3}\sqrt[3]{27c_1x^4 + 3\sqrt{12x^6\sin^3(x) + 81c_1{}^2x^8}}} - \frac{(1 - i\sqrt{3})\sqrt[3]{27c_1x^4 + \sqrt{108x^6\sin^3(x) + 729c_1{}^2x^8}}}{6\sqrt[3]{2}x^2}$$

$$y(x) \to \frac{(1 - i\sqrt{3})\sin(x)}{2^{2/3}\sqrt[3]{27c_1x^4 + 3\sqrt{12x^6\sin^3(x) + 81c_1{}^2x^8}}} - \frac{(1 + i\sqrt{3})\sqrt[3]{27c_1x^4 + \sqrt{108x^6\sin^3(x) + 729c_1{}^2x^8}}}{6\sqrt[3]{2}x^2}$$

2.11 problem 11

Internal problem ID [3099]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 8, page 41
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, _Riccati]`

$$\boxed{-\frac{y}{1-y^2x^2}-\frac{xy'}{1-y^2x^2}=-1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(1=y(x)/(1-x^2*y(x)^2)+x/(1-x^2*y(x)^2)*diff(y(x),x),y(x), singsol=all)
```

$$y(x)=-\frac{e^{-2x}c_1+1}{x\left(e^{-2x}c_1-1\right)}$$

✓ Solution by Mathematica

Time used: 0.153 (sec). Leaf size: 18

```
DSolve[1==y[x]/(1-x^2*y[x]^2)+x/(1-x^2*y[x]^2)*y'[x],y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to\frac{\tanh(x+ic_1)}{x}$$

3 Chapter 2, section 10, page 47

3.1 problem 2(a)

Internal problem ID [3100]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 10, page 47
Problem number: 2(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{\left(3x^2 - y^2\right) y' - 2yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 402

```
dsolve((3*x^2-y(x)^2)*diff(y(x),x)-2*x*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}{6c_1} + \frac{2}{3c_1\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}} + \frac{1}{3c_1}$$

$$y(x) = -\frac{\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}{12c_1} - \frac{1}{3c_1\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}} + \frac{1}{3c_1} - \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}{6c_1} - \frac{2}{3c_1\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}{12c_1} - \frac{1}{3c_1\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}} + \frac{1}{3c_1} + \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}{6c_1} - \frac{2}{3c_1\left(12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1-108c_1^2x^2+8\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 60.175 (sec). Leaf size: 458

```
DSolve[(3*x^2-y[x]^2)*y'[x]-2*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(\frac{\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}e^{2c_1}}{\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - e^{c_1}\right)$$

$$y(x) \to \frac{i(\sqrt{3}+i)\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{6\sqrt[3]{2}} - \frac{i(\sqrt{3}-i)e^{2c_1}}{3\ 2^{2/3}\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

$$y(x) \to -\frac{i(\sqrt{3}-i)\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}}{6\sqrt[3]{2}} + \frac{i(\sqrt{3}+i)e^{2c_1}}{3\ 2^{2/3}\sqrt[3]{27e^{c_1}x^2+3\sqrt{81e^{2c_1}x^4-12e^{4c_1}x^2}-2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

3.2 problem 2(b)

Internal problem ID [3101]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 10, page 47
Problem number: 2(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel`

$$yx + \left(x^2 - yx\right) y' = 1$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve((x*y(x)-1)+(x^2-x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x - \sqrt{x^2 - 2\ln(x) + 2c_1}$$
$$y(x) = x + \sqrt{x^2 - 2\ln(x) + 2c_1}$$

✓ Solution by Mathematica

Time used: 0.46 (sec). Leaf size: 68

```
DSolve[(x*y[x]-1)+(x^2-x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \sqrt{-\frac{1}{x}}\sqrt{-x\left(x^2 - 2\log(x) + c_1\right)}$$
$$y(x) \to x + x\left(-\frac{1}{x}\right)^{3/2}\sqrt{-x\left(x^2 - 2\log(x) + c_1\right)}$$

3.3 problem 2(c)

Internal problem ID [3102]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 10, page 47
Problem number: 2(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{(x + 3y^4x^3)\, y' + y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 129

```
dsolve((x+3*x^3*y(x)^4)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-6xc_1\left(-x+\sqrt{12c_1^2+x^2}\right)}}{6xc_1}$$

$$y(x) = \frac{\sqrt{-6xc_1\left(-x+\sqrt{12c_1^2+x^2}\right)}}{6xc_1}$$

$$y(x) = -\frac{\sqrt{6}\,\sqrt{xc_1\left(x+\sqrt{12c_1^2+x^2}\right)}}{6xc_1}$$

$$y(x) = \frac{\sqrt{6}\,\sqrt{xc_1\left(x+\sqrt{12c_1^2+x^2}\right)}}{6xc_1}$$

✓ Solution by Mathematica

Time used: 10.044 (sec). Leaf size: 166

```
DSolve[(x+3*x^3*y[x]^4)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{c_1 - \frac{\sqrt{x^2(3+c_1{}^2x^2)}}{x^2}}}{\sqrt{3}}$$
$$y(x) \to \frac{\sqrt{c_1 - \frac{\sqrt{x^2(3+c_1{}^2x^2)}}{x^2}}}{\sqrt{3}}$$
$$y(x) \to -\frac{\sqrt{\frac{\sqrt{x^2(3+c_1{}^2x^2)}}{x^2} + c_1}}{\sqrt{3}}$$
$$y(x) \to \frac{\sqrt{\frac{\sqrt{x^2(3+c_1{}^2x^2)}}{x^2} + c_1}}{\sqrt{3}}$$
$$y(x) \to 0$$

3.4 problem 4(a)

Internal problem ID [3103]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 10, page 47
Problem number: 4(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\boxed{(x-1-y^2)\, y'-y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve((x-1-y(x)^2)*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{2}-\frac{\sqrt{c_1^2-4x+4}}{2}$$
$$y(x) = \frac{c_1}{2}+\frac{\sqrt{c_1^2-4x+4}}{2}$$

✓ Solution by Mathematica

Time used: 0.304 (sec). Leaf size: 56

```
DSolve[(x-1-y[x]^2)*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(c_1-\sqrt{-4x+4+c_1{}^2}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{-4x+4+c_1{}^2}+c_1\right)$$
$$y(x) \to 0$$

3.5 problem 4(b)

Internal problem ID [3104]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 10, page 47
Problem number: 4(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y - (x + y^3 x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(y(x)-(x+x*y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\left(\frac{1}{\operatorname{LambertW}(c_1 x^3)}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 4.377 (sec). Leaf size: 76

```
DSolve[y[x]-(x+x*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{W(e^{3c_1}x^3)}$$
$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{W(e^{3c_1}x^3)}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{W(e^{3c_1}x^3)}$$
$$y(x) \to 0$$

3.6 problem 4(c)

Internal problem ID [3105]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 10, page 47
Problem number: 4(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$\boxed{xy' - y^2x^3 - y = x^5}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x)=x^5+x^3*y(x)^2+y(x),y(x), singsol=all)
```

$$y(x) = \tan\left(\frac{x^4}{4} + c_1\right)x$$

✓ Solution by Mathematica

Time used: 0.212 (sec). Leaf size: 18

```
DSolve[x*y'[x]==x^5+x^3*y[x]^2+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\tan\left(\frac{x^4}{4} + c_1\right)$$

3.7 problem 4(d)

Internal problem ID [3106]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 10, page 47
Problem number: 4(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(y+x)\,y' - y = -x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve((y(x)+x)*diff(y(x),x)=(y(x)-x),y(x), singsol=all)
```

$$y(x) = \tan\left(\operatorname{RootOf}\left(2\_Z + \ln\left(\frac{1}{\cos(\_Z)^2}\right) + 2\ln(x) + 2c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 34

```
DSolve[(y[x]+x)*y'[x]==(y[x]-x),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{y(x)}{x}\right) + \frac{1}{2}\log\left(\frac{y(x)^2}{x^2} + 1\right) = -\log(x) + c_1, y(x)\right]$$

3.8 problem 4(e)

Internal problem ID [3107]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 10, page 47
Problem number: 4(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$\boxed{xy' - y - 9y^2 = x^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)=y(x)+x^2+9*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\tan(3x + 3c_1)\,x}{3}$$

✓ Solution by Mathematica

Time used: 0.285 (sec). Leaf size: 17

```
DSolve[x*y'[x]==y[x]+x^2+9*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}x\tan(3(x + c_1))$$

4 Chapter 2, section 11, page 49

4.1 problem 2(a)

Internal problem ID [3108]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 11, page 49
Problem number: 2(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' - 3y = x^4}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x)-3*y(x)=x^4,y(x), singsol=all)
```

$$y(x) = (x + c_1)\, x^3$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 13

```
DSolve[x*y'[x]-3*y[x]==x^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3(x + c_1)$$

4.2 problem 2(b)

Internal problem ID [3109]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 11, page 49
Problem number: 2(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y = \frac{1}{e^{2x} + 1}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+y(x)=1/(1+exp(2*x)),y(x), singsol=all)
```

$$y(x) = (\arctan(e^x) + c_1) e^{-x}$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]==1/(1+Exp[2*x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}(\arctan(e^x) + c_1)$$

4.3 problem 2(c)

Internal problem ID [3110]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 11, page 49
Problem number: 2(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2yx + (x^2+1)\,y' = \cot(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((1+x^2)*diff(y(x),x)+2*x*y(x)=cot(x),y(x), singsol=all)
```

$$y(x) = \frac{\ln(\sin(x)) + c_1}{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 19

```
DSolve[(1+x^2)*y'[x]+2*x*y[x]==Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log(\sin(x)) + c_1}{x^2+1}$$

4.4 problem 2(d)

Internal problem ID [3111]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 11, page 49
Problem number: 2(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + y = 2x\,\mathrm{e}^{-x} + x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)+y(x)=2*x*exp(-x)+x^2,y(x), singsol=all)
```

$$y(x) = x^2 - 2x + \mathrm{e}^{-x}x^2 + 2 + c_1\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.1 (sec). Leaf size: 29

```
DSolve[y'[x]+y[x]==2*x*Exp[-x]+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(x^2 + e^x\left(x^2 - 2x + 2\right) + c_1\right)$$

4.5 problem 2(e)

Internal problem ID [3112]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 11, page 49
Problem number: 2(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cot(x) = 2x\csc(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+y(x)*cot(x)=2*x*csc(x),y(x), singsol=all)
```

$$y(x) = \frac{x^2 + c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 14

```
DSolve[y'[x]+y[x]*Cot[x]==2*x*Csc[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x^2 + c_1)\csc(x)$$

4.6 problem 2(f)

Internal problem ID [3113]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, section 11, page 49
Problem number: 2(f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y - xy' = x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((2*y(x)-x^3)=x*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = (c_1 - x)\, x^2$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 15

```
DSolve[(2*y[x]-x^3)==x*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2(-x + c_1)$$

5 Chapter 2, End of chapter, page 61

5.1 problem 2

Internal problem ID [3114]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(-yx+1)\,y' - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve((1-x*y(x))*diff(y(x),x)=y(x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(-x\,\mathrm{e}^{-c_1}\right)-c_1}$$

✓ Solution by Mathematica

Time used: 2.155 (sec). Leaf size: 25

```
DSolve[(1-x*y[x])*y'[x]==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{W(-e^{-c_1}x)}{x}$$
$$y(x) \to 0$$

5.2 problem 3

Internal problem ID [3115]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{3y + (2y - 3x + 5)\, y' = -2x - 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve((2*x+3*y(x)+1)+(2*y(x)-3*x+5)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -1 - \tan\left(\mathrm{RootOf}\left(3\_Z + \ln\left(\frac{1}{\cos\left(\_Z\right)^2}\right) + 2\ln\left(x - 1\right) + 2c_1\right)\right)(x - 1)$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 68

```
DSolve[(2*x+3*y[x]+1)+(2*y[x]-3*x+5)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[54\arctan\left(\frac{3y(x) + 2x + 1}{2y(x) - 3x + 5}\right) + 18\log\left(\frac{4(x^2 + y(x)^2 + 2y(x) - 2x + 2)}{13(x - 1)^2}\right) + 36\log(x - 1) + 13c_1 = 0, y(x)\right]$$

5.3 problem 4

Internal problem ID [3116]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{xy' - \sqrt{y^2 + x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(x*diff(y(x),x)=sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{y(x)^2}{x^2} + \frac{y(x)\sqrt{x^2 + y(x)^2}}{x^2} + \ln\left(y(x) + \sqrt{x^2 + y(x)^2}\right) - 3\ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.313 (sec). Leaf size: 66

```
DSolve[x*y'[x]==Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\left(\frac{y(x)\left(\sqrt{\frac{y(x)^2}{x^2}+1}+\frac{y(x)}{x}\right)}{x} - \log\left(\sqrt{\frac{y(x)^2}{x^2}+1} - \frac{y(x)}{x}\right)\right) = \log(x) + c_1, y(x)\right]$$

5.4 problem 5

Internal problem ID [3117]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y^2 - (x^3 - yx)\,y' = 0$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 285

```
dsolve(y(x)^2=(x^3-x*y(x))*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = c_1\left(\frac{\left(-x^3+\sqrt{x^6-c_1^3}\right)^{\frac{1}{3}}}{x^3} + \frac{c_1}{x^3\left(-x^3+\sqrt{x^6-c_1^3}\right)^{\frac{1}{3}}}\right)x^2$$

$$y(x) = \frac{c_1\left(-\frac{2\left(-x^3+\sqrt{x^6-c_1^3}\right)^{\frac{1}{3}}}{x^3} - \frac{2c_1}{x^3\left(-x^3+\sqrt{x^6-c_1^3}\right)^{\frac{1}{3}}} - 2i\sqrt{3}\left(\frac{\left(-x^3+\sqrt{x^6-c_1^3}\right)^{\frac{1}{3}}}{x^3} - \frac{c_1}{x^3\left(-x^3+\sqrt{x^6-c_1^3}\right)^{\frac{1}{3}}}\right)\right)x^2}{4}$$

$$y(x) = \frac{c_1\left(-\frac{2\left(-x^3+\sqrt{x^6-c_1^3}\right)^{\frac{1}{3}}}{x^3} - \frac{2c_1}{x^3\left(-x^3+\sqrt{x^6-c_1^3}\right)^{\frac{1}{3}}} + 2i\sqrt{3}\left(\frac{\left(-x^3+\sqrt{x^6-c_1^3}\right)^{\frac{1}{3}}}{x^3} - \frac{c_1}{x^3\left(-x^3+\sqrt{x^6-c_1^3}\right)^{\frac{1}{3}}}\right)\right)x^2}{4}$$

✓ Solution by Mathematica

Time used: 60.13 (sec). Leaf size: 820

```
DSolve[y[x]^2==(x^3-x*y[x])*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 - \frac{\cdots}{9\sqrt[3]{x^{12}\left(-\cosh\left(\frac{3c_1}{4}\right)\right) - x^{12}\sinh\left(\frac{3c_1}{4}\right) + 2x^6\cosh\left(\frac{3c_1}{8}\right) + 2x^6\sinh\left(\frac{3c_1}{8}\right) + \sqrt{x^6\left(\cosh\left(\frac{15c \cdots}{16 \cdots}\right.\right.} \cdots}\ \frac{\cdots}{x^6\cosh\left(\frac{3c_1}{8}\right)+x^6\sinh\left(\frac{3c_1}{8}\right)-1}}$$

$$y(x) \to x^2 - \frac{\cdots}{9i\left(\sqrt{3}+i\right)\sqrt[3]{x^{12}\left(-\cosh\left(\frac{3c_1}{4}\right)\right) - x^{12}\sinh\left(\frac{3c_1}{4}\right) + 2x^6\cosh\left(\frac{3c_1}{8}\right) + 2x^6\sinh\left(\frac{3c_1}{8}\right) + \sqrt{x^6\left(\cos \cdots\right.} \cdots}\ \frac{\cdots}{x^6\cosh\left(\frac{3c_1}{8}\right)+x^6\sinh\left(\frac{3c_1}{8}\right) \cdots}}$$

$$y(x) \to x^2 - \frac{\cdots}{-\frac{\cdots}{9i\left(\sqrt{3}-i\right)\sqrt[3]{x^{12}\left(-\cosh\left(\frac{3c_1}{4}\right)\right) - x^{12}\sinh\left(\frac{3c_1}{4}\right) + 2x^6\cosh\left(\frac{3c_1}{8}\right) + 2x^6\sinh\left(\frac{3c_1}{8}\right) + \sqrt{x^6\left(\mathrm{co} \cdots\right.} \cdots}\ \frac{\cdots}{x^6\cosh\left(\frac{3c_1}{8}\right)+x^6\sinh\left(\frac{3c_1}{8}\right) \cdots}}}$$

5.5 problem 6

Internal problem ID [3118]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{y^3 + y - (y^2x^3 - x)\, y' = -x^2}$$

✗ Solution by Maple

```
dsolve((x^2+y(x)^3+y(x))=(x^3*y(x)^2-x)*diff(y(x),x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x^2+y[x]^3+y[x])==(x^3*y[x]^2-x)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

5.6 problem 8

Internal problem ID [3119]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + y = \cos(x)\, x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)+y(x)=x*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{\cos(x) + x\sin(x) + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 18

```
DSolve[x*y'[x]+y[x]==x*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\sin(x) + \cos(x) + c_1}{x}$$

5.7 problem 9

Internal problem ID [3120]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\left(yx - x^2\right) y' - y^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve((x*y(x)-x^2)*diff(y(x),x)=y(x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(-\frac{\mathrm{e}^{-c_1}}{x}\right)-c_1}$$

✓ Solution by Mathematica

Time used: 2.286 (sec). Leaf size: 25

```
DSolve[(x*y[x]-x^2)*y'[x]==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -xW\left(-\frac{e^{-c_1}}{x}\right)$$
$$y(x) \to 0$$

5.8 problem 10

Internal problem ID [3121]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{\left(\mathrm{e}^{x}-3y^{2}x^{2}\right)y'+\mathrm{e}^{x}y-2y^{3}x=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 347

```
dsolve((exp(x)-3*x^2*y(x)^2)*diff(y(x),x)+y(x)*exp(x)=2*x*y(x)^3,y(x), singsol=all)
```

$$y(x)=\frac{\left(108c_1x+12\sqrt{81c_1^2x^2-12\,\mathrm{e}^{3x}}\right)^{\frac{1}{3}}}{6x}+\frac{2\,\mathrm{e}^{x}}{x\left(108c_1x+12\sqrt{81c_1^2x^2-12\,\mathrm{e}^{3x}}\right)^{\frac{1}{3}}}$$

$$y(x)=-\frac{\left(108c_1x+12\sqrt{81c_1^2x^2-12\,\mathrm{e}^{3x}}\right)^{\frac{1}{3}}}{12x}-\frac{\mathrm{e}^{x}}{x\left(108c_1x+12\sqrt{81c_1^2x^2-12\,\mathrm{e}^{3x}}\right)^{\frac{1}{3}}}-\frac{i\sqrt{3}\left(\frac{\left(108c_1x+12\sqrt{81c_1^2x^2-12\,\mathrm{e}^{3x}}\right)^{\frac{1}{3}}}{6x}-\frac{2\,\mathrm{e}^{x}}{x\left(108c_1x+12\sqrt{81c_1^2x^2-12\,\mathrm{e}^{3x}}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x)=-\frac{\left(108c_1x+12\sqrt{81c_1^2x^2-12\,\mathrm{e}^{3x}}\right)^{\frac{1}{3}}}{12x}-\frac{\mathrm{e}^{x}}{x\left(108c_1x+12\sqrt{81c_1^2x^2-12\,\mathrm{e}^{3x}}\right)^{\frac{1}{3}}}+\frac{i\sqrt{3}\left(\frac{\left(108c_1x+12\sqrt{81c_1^2x^2-12\,\mathrm{e}^{3x}}\right)^{\frac{1}{3}}}{6x}-\frac{2\,\mathrm{e}^{x}}{x\left(108c_1x+12\sqrt{81c_1^2x^2-12\,\mathrm{e}^{3x}}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 48.841 (sec). Leaf size: 364

```
DSolve[(Exp[x]-3*x^2*y[x]^2)*y'[x]+y[x]*Exp[x]==2*x*y[x]^3,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{2\sqrt[3]{3}e^x x^2 + \sqrt[3]{2}\left(9c_1x^4 + \sqrt{-12e^{3x}x^6 + 81{c_1}^2x^8}\right)^{2/3}}{6^{2/3}x^2\sqrt[3]{9c_1x^4 + \sqrt{-12e^{3x}x^6 + 81{c_1}^2x^8}}}$$

$$y(x) \to \frac{i\left(\sqrt{3}+i\right)\sqrt[3]{9c_1x^4 + \sqrt{-12e^{3x}x^6 + 81{c_1}^2x^8}}}{2\sqrt[3]{2}3^{2/3}x^2} - \frac{\left(\sqrt{3}+3i\right)e^x}{2^{2/3}3^{5/6}\sqrt[3]{9c_1x^4 + \sqrt{-12e^{3x}x^6 + 81{c_1}^2x^8}}}$$

$$y(x) \to \frac{\left(-1-i\sqrt{3}\right)\sqrt[3]{9c_1x^4 + \sqrt{-12e^{3x}x^6 + 81{c_1}^2x^8}}}{2\sqrt[3]{2}3^{2/3}x^2} - \frac{\left(\sqrt{3}-3i\right)e^x}{2^{2/3}3^{5/6}\sqrt[3]{9c_1x^4 + \sqrt{-12e^{3x}x^6 + 81{c_1}^2x^8}}}$$

5.9 problem 12

Internal problem ID [3122]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{-xy' + y = -x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve((x^2+y(x))=x*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = (x + c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 11

```
DSolve[(x^2+y[x])==x*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(x + c_1)$$

5.10 problem 13

Internal problem ID [3123]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{xy' + y = \cos(x)\, x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x*diff(y(x),x)+y(x)=x^2*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{\sin(x)\, x^2 - 2\sin(x) + 2x\cos(x) + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.066 (sec). Leaf size: 25

```
DSolve[x*y'[x]+y[x]==x^2*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(x^2 - 2)\sin(x) + 2x\cos(x) + c_1}{x}$$

5.11 problem 14

Internal problem ID [3124]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{4y + (3x + 2y + 2)\, y' = -6x - 3}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve((6*x+4*y(x)+3)+(3*x+2*y(x)+2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{3x}{2} + \operatorname{LambertW}\left(\mathrm{e}^{-\frac{x}{2}} c_1\right)$$

✓ Solution by Mathematica

Time used: 4.333 (sec). Leaf size: 34

```
DSolve[(6*x+4*y[x]+3)+(3*x+2*y[x]+2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3x}{2} + W\left(-e^{-\frac{x}{2}-1+c_1}\right)$$
$$y(x) \to -\frac{3x}{2}$$

5.12 problem 15

Internal problem ID [3125]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _exact]`

$$\boxed{\cos (y+x)-x \sin (y+x)-x \sin (y+x) y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(cos(x+y(x))-x*sin(x+y(x))=x*sin(x+y(x))*diff(y(x),x),y(x), singsol=all)
```

$$y(x)=-x+\arccos \left(\frac{c_1}{x}\right)$$

✓ Solution by Mathematica

Time used: 10.102 (sec). Leaf size: 35

```
DSolve[Cos[x+y[x]]-x*Sin[x+y[x]]==x*Sin[x+y[x]]*y'[x],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\rightarrow -x-\arccos \left(-\frac{c_1}{x}\right)$$
$$y(x)\rightarrow -x+\arccos \left(-\frac{c_1}{x}\right)$$

5.13 problem 17

Internal problem ID [3126]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{y^2 e^{yx} + (e^{yx} + y\, e^{yx} x)\, y' = -\cos(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve((y(x)^2*exp(x*y(x))+cos(x))+(exp(x*y(x))+x*y(x)*exp(x*y(x)))*diff(y(x),x)=0,y(x), sin
```

$$y(x) = \frac{\operatorname{LambertW}(-x(c_1 + \sin(x)))}{x}$$

✓ Solution by Mathematica

Time used: 60.266 (sec). Leaf size: 19

```
DSolve[(y[x]^2*Exp[x*y[x]]+Cos[x])+(Exp[x*y[x]]+x*y[x]*Exp[x*y[x]])*y'[x]==0,y[x],x,IncludeS
```

$$y(x) \to \frac{W(x(-\sin(x) + c_1))}{x}$$

5.14 problem 18

Internal problem ID [3127]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _dAlembert]`

$$\boxed{y' \ln(-y+x) - \ln(-y+x) = 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)*ln(x-y(x))=1+ln(x-y(x)),y(x), singsol=all)
```

$$y(x) = -e^{\text{LambertW}\left((c_1-x)e^{-1}\right)+1} + x$$

✓ Solution by Mathematica

Time used: 0.127 (sec). Leaf size: 26

```
DSolve[y'[x]*Log[x-y[x]]==1+Log[x-y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[(x-y(x))(-\log(x-y(x))) - y(x) = c_1, y(x)]$$

5.15 problem 19

Internal problem ID [3128]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + 2yx = e^{-x^2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+2*x*y(x)=exp(-x^2),y(x), singsol=all)
```

$$y(x) = (x + c_1)\,e^{-x^2}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 17

```
DSolve[y'[x]+2*x*y[x]==Exp[-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x^2}(x + c_1)$$

5.16 problem 20

Internal problem ID [3129]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y^2 - 3yx - (x^2 - yx)\, y' = 2x^2}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 59

```
dsolve((y(x)^2-3*x*y(x)-2*x^2)=(x^2-x*y(x))*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^2 - \sqrt{2c_1^2 x^4 + 1}}{c_1 x}$$
$$y(x) = \frac{c_1 x^2 + \sqrt{2c_1^2 x^4 + 1}}{c_1 x}$$

✓ Solution by Mathematica

Time used: 0.701 (sec). Leaf size: 99

```
DSolve[(y[x]^2-3*x*y[x]-2*x^2)==(x^2-x*y[x])*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \frac{\sqrt{2x^4 + e^{2c_1}}}{x}$$
$$y(x) \to x + \frac{\sqrt{2x^4 + e^{2c_1}}}{x}$$
$$y(x) \to x - \frac{\sqrt{2}\sqrt{x^4}}{x}$$
$$y(x) \to \frac{\sqrt{2}\sqrt{x^4}}{x} + x$$

5.17 problem 21

Internal problem ID [3130]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2yx + (x^2 + 1)\, y' = 4x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((1+x^2)*diff(y(x),x)+2*x*y(x)=4*x^3,y(x), singsol=all)
```

$$y(x) = \frac{x^4 + c_1}{x^2 + 1}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 19

```
DSolve[(1+x^2)*y'[x]+2*x*y[x]==4*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4 + c_1}{x^2 + 1}$$

5.18 problem 22

Internal problem ID [3131]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{e^{x} \sin \left(y\right)-y \sin \left(y x\right)+\left(\cos \left(y\right) e^{x}-x \sin \left(y x\right)\right) y^{\prime}=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve((exp(x)*sin(y(x))-y(x)*sin(x*y(x)))+(exp(x)*cos(y(x))-x*sin(x*y(x)))*diff(y(x),x)=0,y
```

$$e^{x} \sin \left(y(x)\right)+\cos \left(y(x) x\right)+c_{1}=0$$

✓ Solution by Mathematica

Time used: 0.58 (sec). Leaf size: 19

```
DSolve[(Exp[x]*Sin[y[x]]-y[x]*Sin[x*y[x]])+(Exp[x]*Cos[y[x]]-x*Sin[x*y[x]])*y'[x]==0,y[x],x,
```

$$\text{Solve}\left[e^{x}\sin(y(x))+\cos(xy(x))=c_{1},y(x)\right]$$

5.19 problem 24

Internal problem ID [3132]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{\left(x\,\mathrm{e}^{y}+y-x^{2}\right)y'-2yx+\mathrm{e}^{y}=-x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve((x*exp(y(x))+y(x)-x^2)*diff(y(x),x)=(2*x*y(x) -exp(y(x))-x),y(x), singsol=all)
```

$$-y(x)\,x^{2}+x\,\mathrm{e}^{y(x)}+\frac{x^{2}}{2}+\frac{y(x)^{2}}{2}+c_{1}=0$$

✓ Solution by Mathematica

Time used: 0.315 (sec). Leaf size: 35

```
DSolve[(x*Exp[y[x]]+y[x]-x^2)*y'[x]==(2*x*y[x] -Exp[y[x]]-x),y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[x^{2}(-y(x))+\frac{x^{2}}{2}+xe^{y(x)}+\frac{y(x)^{2}}{2}=c_{1},y(x)\right]$$

5.20 problem 25

Internal problem ID [3133]

Book: Differential equations with applications and historial notes, George F. Simmons, 1971
Section: Chapter 2, End of chapter, page 61
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{-(x\,e^{x}-y\,e^{y})\,y' = -e^{x}(x+1)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve(exp(x)*(1+x)=(x*exp(x)-y(x)*exp(y(x)))*diff(y(x),x),y(x), singsol=all)
```

$$x\,e^{-y(x)+x} + \frac{y(x)^2}{2} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.307 (sec). Leaf size: 26

```
DSolve[Exp[x]*(1+x)==(x*Exp[x]-y[x]*Exp[y[x]])*y'[x],y[x],x,IncludeSingularSolutions -> True
```

$$\text{Solve}\left[-\frac{1}{2}y(x)^2 - xe^{x-y(x)} = c_1, y(x)\right]$$