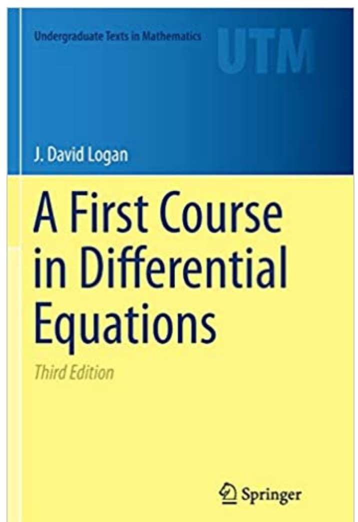
A Solution Manual For

A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.



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Contents

1 Chapter 1, First order differential equations. Section 1.1 First order equations. Exercises page 10

1.1 problem 1(a)

Internal problem ID [10331]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.1 First order equations. Exercises page 10
Problem number: 1(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' - \frac{2x}{t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(x(t),t)=2*x(t)/t,x(t), singsol=all)
```

$$x(t) = c_1 t^2$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 16

```
DSolve[x'[t]==2*x[t]/t,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 t^2$$
$$x(t) \to 0$$

1.2 problem 1(b)

Internal problem ID [10332]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.1 First order equations. Exercises page 10
Problem number: 1(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' + \frac{t}{x} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(x(t),t)=-t/x(t),x(t), singsol=all)
```

$$x(t) = \sqrt{-t^2 + c_1}$$
$$x(t) = -\sqrt{-t^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.086 (sec). Leaf size: 39

```
DSolve[x'[t]==-t/x[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\sqrt{-t^2 + 2c_1}$$
$$x(t) \to \sqrt{-t^2 + 2c_1}$$

1.3 problem 3

Internal problem ID [10333]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.1 First order equations. Exercises page 10
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' + x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(x(t),t)=-x(t)^2,x(t), singsol=all)
```

$$x(t) = \frac{1}{t + c_1}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 39

```
DSolve[x'[t]==-t/x[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\sqrt{-t^2 + 2c_1}$$
$$x(t) \to \sqrt{-t^2 + 2c_1}$$

1.4 problem 4

Internal problem ID [10334]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.1 First order equations. Exercises page 10
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 2x' + 2x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(x(t),t$2)+2*diff(x(t),t)+2*x(t)=0,x(t), singsol=all)
```

$$x(t) = c_1 e^{-t} \sin(t) + c_2 e^{-t} \cos(t)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 22

```
DSolve[x''[t]+2*x'[t]+2*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-t}(c_2 \cos(t) + c_1 \sin(t))$$

1.5 problem 5

Internal problem ID [10335]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.1 First order equations. Exercises page 10
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - e^{-x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(x(t),t)=exp(-x(t)),x(t), singsol=all)
```

$$x(t) = \ln(t + c_1)$$

✓ Solution by Mathematica

Time used: 0.255 (sec). Leaf size: 10

```
DSolve[x'[t]==Exp[-x[t]],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \log(t + c_1)$$

1.6 problem 6

Internal problem ID [10336]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.1 First order equations. Exercises page 10
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{x' + 2x - t^2 - 4t - 7 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve(diff(x(t),t)+2*x(t)=t^2+4*t+7,x(t), singsol=all)
```

$$x(t) = \frac{t^2}{2} + \frac{3t}{2} + \frac{11}{4} + \mathrm{e}^{-2t}c_1$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 25

```
DSolve[x'[t]+2*x[t]==t^2+4*t+7,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2}t(t+3) + c_1e^{-2t} + \frac{11}{4}$$

1.7 problem 7

Internal problem ID [10337]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.1 First order equations. Exercises page 10
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2x't - x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(2*t*diff(x(t),t)=x(t),x(t), singsol=all)
```

$$x(t) = c_1\sqrt{t}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 18

```
DSolve[2*t*x'[t]==x[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1\sqrt{t}$$
$$x(t) \to 0$$

1.8 problem 8

Internal problem ID [10338]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.1 First order equations. Exercises page 10
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2x'' - 6x = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(t^2*diff(x(t),t$2)-6*x(t)=0,x(t), singsol=all)
```

$$x(t) = c_1t^3 + \frac{c_2}{t^2}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 18

```
DSolve[t^2*x''[t]-6*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{c_2t^5 + c_1}{t^2}$$

1.9 problem 9

Internal problem ID [10339]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.1 First order equations. Exercises page 10
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2x'' - 5x' - 3x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(2*diff(x(t),t$2)-5*diff(x(t),t)-3*x(t)=0,x(t), singsol=all)
```

$$x(t) = c_1 e^{3t} + c_2 e^{-\frac{t}{2}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 24

```
DSolve[2*x''[t]-5*x'[t]-3*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{-t/2} + c_2 e^{3t}$$

2 Chapter 1, First order differential equations. Section 1.1.3 Geometric. Exercises page 15

2.1 problem 1

Internal problem ID [10340]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.1.3 Geometric. Exercises page 15
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - x\left(1 - \frac{x}{4}\right) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(diff(x(t),t)=x(t)*(1-x(t)/4),x(t), singsol=all)
```

$$x(t) = \frac{4}{1 + 4\,\mathrm{e}^{-t} c_1}$$

✓ Solution by Mathematica

Time used: 0.292 (sec). Leaf size: 32

```
DSolve[x'[t]==x[t]*(1-x[t]/4),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{4e^t}{e^t + e^{4c_1}}$$
$$x(t) \to 0$$
$$x(t) \to 4$$

2.2 problem 2

Internal problem ID [10341]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.1.3 Geometric. Exercises page 15
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{x' - x^2 - t^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
dsolve(diff(x(t),t)=x(t)^2+t^2,x(t), singsol=all)
```

$$x(t) = \frac{\left(-\operatorname{BesselJ}\left(-\frac{3}{4}, \frac{t^2}{2}\right) c_1 - \operatorname{BesselY}\left(-\frac{3}{4}, \frac{t^2}{2}\right)\right) t}{c_1 \operatorname{BesselJ}\left(\frac{1}{4}, \frac{t^2}{2}\right) + \operatorname{BesselY}\left(\frac{1}{4}, \frac{t^2}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.127 (sec). Leaf size: 93

```
DSolve[x'[t]==x[t]^2+t^2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{t\left(-\operatorname{BesselJ}\left(-\frac{3}{4}, \frac{t^2}{2}\right) + c_1 \operatorname{BesselJ}\left(\frac{3}{4}, \frac{t^2}{2}\right)\right)}{\operatorname{BesselJ}\left(\frac{1}{4}, \frac{t^2}{2}\right) + c_1 \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{t^2}{2}\right)}$$

$$x(t) \to \frac{t \operatorname{BesselJ}\left(\frac{3}{4}, \frac{t^2}{2}\right)}{\operatorname{BesselJ}\left(-\frac{1}{4}, \frac{t^2}{2}\right)}$$

3 Chapter 1, First order differential equations. Section 1.2 Antiderivatives. Exercises page 19

3.1 problem 1

Internal problem ID [10342]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.2 Antiderivatives. Exercises page 19
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - t\cos\left(t^2\right) = 0}$$

With initial conditions

$$[x(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve([diff(x(t),t)=t*cos(t^2),x(0) = 1],x(t), singsol=all)
```

$$x(t) = \frac{\sin\left(t^2\right)}{2} + 1$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 15

```
DSolve[{x'[t]==t*Cos[t^2],{x[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2}\left(\sin\left(t^2\right) + 2\right)$$

3.2 problem 2

Internal problem ID [10343]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.2 Antiderivatives. Exercises page 19
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - \frac{1+t}{\sqrt{t}} = 0}$$

With initial conditions

$$[x(1) = 4]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(x(t),t)=(1+t)/sqrt(t),x(1) = 4],x(t), singsol=all)
```

$$x(t) = \frac{2t^{\frac{3}{2}}}{3} + 2\sqrt{t} + \frac{4}{3}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 20

```
DSolve[{x'[t]==(1+t)/Sqrt[t],{x[1]==4}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{2}{3}\left(\sqrt{t}(t+3) + 2\right)$$

3.3 problem 3

Internal problem ID [10344]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.2 Antiderivatives. Exercises page 19
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{x'' + 3\sqrt{t} = 0}$$

With initial conditions

$$[x(1) = 4, x'(1) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve([diff(x(t),t$2)=-3*sqrt(t),x(1) = 4, D(x)(1) = 2],x(t), singsol=all)
```

$$x(t) = -\frac{4t^{\frac{5}{2}}}{5} + 4t + \frac{4}{5}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 19

```
DSolve[{x''[t]==-3*Sqrt[t],{x[1]==4,x'[1]==2}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{4}{5}\left(t^{5/2} - 5t - 1\right)$$

3.4 problem 4(a)

Internal problem ID [10345]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.2 Antiderivatives. Exercises page 19
Problem number: 4(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - t\,\mathrm{e}^{-2t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(x(t),t)=t*exp(-2*t),x(t), singsol=all)
```

$$x(t) = -\frac{(2t+1)\,\mathrm{e}^{-2t}}{4} + c_1$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 22

```
DSolve[x'[t]==t*Exp[-2*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{1}{4}e^{-2t}(2t+1) + c_1$$

3.5 problem 4(b)

Internal problem ID [10346]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.2 Antiderivatives. Exercises page 19
Problem number: 4(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - \frac{1}{t \ln(t)} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve(diff(x(t),t)=1/(t*ln(t)),x(t), singsol=all)
```

$$x(t) = \ln(\ln(t)) + c_1$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 11

```
DSolve[x'[t]==1/(t*Log[t]),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \log(\log(t)) + c_1$$

3.6 problem 4(c)

Internal problem ID [10347]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.2 Antiderivatives. Exercises page 19
Problem number: 4(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x'\sqrt{t} - \cos\left(\sqrt{t}\right) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(sqrt(t)*diff(x(t),t)=cos(sqrt(t)),x(t), singsol=all)
```

$$x(t) = 2\sin\left(\sqrt{t}\right) + c_1$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 16

```
DSolve[Sqrt[t]*x'[t]==Cos[Sqrt[t]],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 2\sin\left(\sqrt{t}\right) + c_1$$

3.7 problem 6

Internal problem ID [10348]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.2 Antiderivatives. Exercises page 19
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - \frac{e^{-t}}{\sqrt{t}} = 0}$$

With initial conditions

$$[x(1) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 20

```
dsolve([diff(x(t),t)=exp(-t)/sqrt(t),x(1) = 0],x(t), singsol=all)
```

$$x(t) = -\left(\operatorname{erf}(1) - \operatorname{erf}\left(\sqrt{t}\right)\right)\sqrt{\pi}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 22

```
DSolve[{x'[t]==Exp[-t]/Sqrt[t],{x[1]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{t}\right) - \operatorname{erf}(1)\right)$$

3.8 problem 7

Internal problem ID [10349]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.2 Antiderivatives. Exercises page 19
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x''t + x' - 1 = 0}$$

With initial conditions

$$[x(1) = 0, x'(1) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 9

```
dsolve([diff(t*diff(x(t),t),t)=1,x(1) = 0, D(x)(1) = 2],x(t), singsol=all)
```

$$x(t) = \ln(t) + t - 1$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 10

```
DSolve[{D[t*x'[t],t]==1,{x[1]==0,x'[1]==2}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to t + \log(t) - 1$$

4 Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26

4.1 problem 1(a)

Internal problem ID [10350]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 1(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - \sqrt{x} = 0}$$

With initial conditions

$$[x(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 11

```
dsolve([diff(x(t),t)=sqrt(x(t)),x(0) = 1],x(t), singsol=all)
```

$$x(t) = \frac{(t+2)^2}{4}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[{x'[t]==Sqrt[t],{x[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{2t^{3/2}}{3} + 1$$

4.2 problem 1(b)

Internal problem ID [10351]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 1(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - \mathrm{e}^{-2x} = 0}$$

With initial conditions

$$[x(0) = 1]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 13

```
dsolve([diff(x(t),t)=exp(-2*x(t)),x(0) = 1],x(t), singsol=all)
```

$$x(t) = \frac{\ln(2t + \mathrm{e}^2)}{2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 17

```
DSolve[{x'[t]==Exp[-2*x[t]],{x[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2}\log(2t + e^2)$$

4.3 problem 1(c)

Internal problem ID [10352]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 1(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - 1 - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 8

```
dsolve(diff(y(t),t)=1+y(t)^2,y(t), singsol=all)
```

$$y(t) = \tan(t + c_1)$$

✓ Solution by Mathematica

Time used: 0.141 (sec). Leaf size: 24

```
DSolve[y'[t]==1+y[t]^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \tan(t + c_1)$$
$$y(t) \to -i$$
$$y(t) \to i$$

4.4 problem 1(d)

Internal problem ID [10353]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 1(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{u' - \frac{1}{5-2u} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(u(t),t)=1/(5-2*u(t)),u(t), singsol=all)
```

$$u(t) = \frac{5}{2} - \frac{\sqrt{25-4t-4c_1}}{2}$$
$$u(t) = \frac{5}{2} + \frac{\sqrt{25-4t-4c_1}}{2}$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 49

```
DSolve[u'[t]==1/(5-2*u[t]),u[t],t,IncludeSingularSolutions -> True]
```

$$u(t) \to \frac{1}{2}\left(5 - \sqrt{-4t+25+4c_1}\right)$$
$$u(t) \to \frac{1}{2}\left(5 + \sqrt{-4t+25+4c_1}\right)$$

4.5 problem 1(e)

Internal problem ID [10354]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 1(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - ax - b = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(diff(x(t),t)=a*x(t)+b,x(t), singsol=all)
```

$$x(t) = -\frac{b}{a} + e^{at} c_1$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 30

```
DSolve[x'[t]==a*x[t]+b,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{b}{a} + c_1 e^{at}$$
$$x(t) \to -\frac{b}{a}$$

4.6 problem 1(f)

Internal problem ID [10355]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 1(f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{Q' - \frac{Q}{4+Q^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(diff(Q(t),t)=Q(t)/(4+Q(t)^2),Q(t), singsol=all)
```

$$Q(t) = \mathrm{e}^{-\frac{\mathrm{LambertW}\left(\frac{\mathrm{e}^{\frac{t}{2}+\frac{c_1}{2}}}{4}\right)}{2}+\frac{t}{4}+\frac{c_1}{4}}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 42

```
DSolve[Q'[t]==Q[t]/(4*Q[t]^2),Q[t],t,IncludeSingularSolutions -> True]
```

$$Q(t) \to -\frac{\sqrt{t+4c_1}}{\sqrt{2}}$$

$$Q(t) \to \frac{\sqrt{t+4c_1}}{\sqrt{2}}$$

4.7 problem 1(g)

Internal problem ID [10356]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 1(g).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - e^{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(x(t),t)=exp(x(t)^2),x(t), singsol=all)
```

$$t - \frac{\sqrt{\pi}\,\operatorname{erf}(x(t))}{2} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.364 (sec). Leaf size: 17

```
DSolve[x'[t]==Exp[x[t]^2],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \operatorname{erf}^{-1}\left(\frac{2(t + c_1)}{\sqrt{\pi}}\right)$$

4.8 problem 1(h)

Internal problem ID [10357]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 1(h).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - r(a - y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(t),t)=r*(a-y(t)),y(t), singsol=all)
```

$$y(t) = a + \mathrm{e}^{-tr} c_1$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 21

```
DSolve[y'[t]==r*(a-y[t]),y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to a + c_1 e^{-rt}$$
$$y(t) \to a$$

4.9 problem 4(a)

Internal problem ID [10358]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 4(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' - \frac{2x}{1+t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(x(t),t)=2*x(t)/(t+1),x(t), singsol=all)
```

$$x(t) = c_1(t+1)^2$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 18

```
DSolve[x'[t]==2*x[t]/(t+1),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1(t+1)^2$$
$$x(t) \to 0$$

4.10 problem 4(b)

Internal problem ID [10359]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 4(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\theta' - t\sqrt{t^2+1}\,\sec(\theta) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(theta(t),t)=t*sqrt(1+t^2)*sec(theta(t)),theta(t), singsol=all)
```

$$\theta(t) = \arcsin\left(\frac{(t^2+1)^{\frac{3}{2}}}{3} + c_1\right)$$

✓ Solution by Mathematica

Time used: 4.733 (sec). Leaf size: 89

```
DSolve[theta'[t]==t*Sqrt[1+t^2]*Sec[theta[t]],theta[t],t,IncludeSingularSolutions -> True]
```

$$\theta(t) \to \arcsin\left(\frac{1}{3}\left(\sqrt{t^2+1}t^2 + \sqrt{t^2+1} + 3c_1\right)\right)$$
$$\theta(t) \to \arcsin\left(\frac{1}{3}\left(\sqrt{t^2+1}t^2 + \sqrt{t^2+1} + 3c_1\right)\right)$$
$$\theta(t) \to \csc^{-1}\left(\frac{3}{(t^2+1)^{3/2}}\right)$$

4.11 problem 4(c)

Internal problem ID [10360]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 4(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(2u+1)\,u' - 1 - t = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve((2*u(t)+1)*diff(u(t),t)-(1+t)=0,u(t), singsol=all)
```

$$u(t) = -\frac{1}{2} - \frac{\sqrt{2t^2 + 4c_1 + 4t + 1}}{2}$$
$$u(t) = -\frac{1}{2} + \frac{\sqrt{2t^2 + 4c_1 + 4t + 1}}{2}$$

✓ Solution by Mathematica

Time used: 0.102 (sec). Leaf size: 55

```
DSolve[(2*u[t]+1)*u'[t]-(1+t)==0,u[t],t,IncludeSingularSolutions -> True]
```

$$u(t) \to \frac{1}{2}\left(-1 - \sqrt{2t(t+2) + 1 + 4c_1}\right)$$
$$u(t) \to \frac{1}{2}\left(-1 + \sqrt{2t(t+2) + 1 + 4c_1}\right)$$

4.12 problem 4(d)

Internal problem ID [10361]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 4(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{R' - (1+t)\left(1+R^2\right) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(R(t),t)=(t+1)*(1+R(t)^2),R(t), singsol=all)
```

$$R(t) = \tan\left(\frac{1}{2}t^2 + t + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.208 (sec). Leaf size: 31

```
DSolve[R'[t]==(t+1)*(1+R[t]^2),R[t],t,IncludeSingularSolutions -> True]
```

$$R(t) \to \tan\left(\frac{t^2}{2} + t + c_1\right)$$
$$R(t) \to -i$$
$$R(t) \to i$$

4.13 problem 4(e)

Internal problem ID [10362]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 4(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' + y + \frac{1}{y} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(t),t)+y(t)+1/y(t)=0,y(t), singsol=all)
```

$$y(t) = \sqrt{e^{-2t} c_1 - 1}$$
$$y(t) = -\sqrt{e^{-2t} c_1 - 1}$$

✓ Solution by Mathematica

Time used: 2.674 (sec). Leaf size: 57

```
DSolve[y'[t]+y[t]+1/y[t]==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to -\sqrt{-1 + e^{-2t+2c_1}}$$
$$y(t) \to \sqrt{-1 + e^{-2t+2c_1}}$$
$$y(t) \to -i$$
$$y(t) \to i$$

4.14 problem 4(f)

Internal problem ID [10363]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 4(f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1+t)\, x' + x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((1+t)*diff(x(t),t)+x(t)^2=0,x(t), singsol=all)
```

$$x(t) = \frac{1}{\ln(t+1) + c_1}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(1+t)*x'[t]+x[t]^2==0,y[t],t,IncludeSingularSolutions -> True]
```

Not solved

4.15 problem 5

Internal problem ID [10364]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \frac{1}{2y+1} = 0}$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve([diff(y(t),t)=1/(2*y(t)+1),y(0) = 1],y(t), singsol=all)
```

$$y(t) = -\frac{1}{2} + \frac{\sqrt{4t+9}}{2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[{y'[t]==1/(2*y[t]+1),{y[0]==1}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{1}{2}\left(\sqrt{4t+9} - 1\right)$$

4.16 problem 6

Internal problem ID [10365]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{x' - (4t - x)^2 = 0}$$

With initial conditions

$$[x(0) = 1]$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 28

```
dsolve([diff(x(t),t)=(4*t-x(t))^2,x(0) = 1],x(t), singsol=all)
```

$$x(t) = \frac{(4t-2)\,e^{4t} + 12t + 6}{3 + e^{4t}}$$

✓ Solution by Mathematica

Time used: 0.171 (sec). Leaf size: 21

```
DSolve[{x'[t]==(4*t-x[t])^2,{x[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 4t + \frac{12}{e^{4t} + 3} - 2$$

4.17 problem 7

Internal problem ID [10366]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' - 2tx^2 = 0}$$

With initial conditions

$$[x(0) = 1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 13

```
dsolve([diff(x(t),t)=2*t*x(t)^2,x(0) = 1],x(t), singsol=all)
```

$$x(t) = -\frac{1}{t^2 - 1}$$

✓ Solution by Mathematica

Time used: 0.125 (sec). Leaf size: 14

```
DSolve[{x'[t]==2*t*x[t]^2,{x[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{1 - t^2}$$

4.18 problem 8

Internal problem ID [10367]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' - t^2 \mathrm{e}^{-x} = 0}$$

With initial conditions

$$[x(0) = \ln(2)]$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 15

```
dsolve([diff(x(t),t)=t^2*exp(-x(t)),x(0) = ln(2)],x(t), singsol=all)
```

$$x(t) = -\ln(3) + \ln(t^3 + 6)$$

✓ Solution by Mathematica

Time used: 0.308 (sec). Leaf size: 15

```
DSolve[{x'[t]==t^2*Exp[-x[t]],{x[0]==Log[2]}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \log\left(\frac{1}{3}(t^3 + 6)\right)$$

4.19 problem 9

Internal problem ID [10368]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - x(4 + x) = 0}$$

With initial conditions

$$[x(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(x(t),t)=x(t)*(4+x(t)),x(0) = 1],x(t), singsol=all)
```

$$x(t) = \frac{4}{-1 + 5\,\mathrm{e}^{-4t}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 18

```
DSolve[{x'[t]==x[t]*(4+x[t]),{x[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{20}{e^{4t} - 5} - 4$$

4.20 problem 10(a)

Internal problem ID [10369]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 10(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' - e^{t+x} = 0}$$

With initial conditions

$$[x(0) = 0]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 13

```
dsolve([diff(x(t),t)=exp(t+x(t)),x(0) = 0],x(t), singsol=all)
```

$$x(t) = -\ln\left(-e^t + 2\right)$$

✓ Solution by Mathematica

Time used: 0.806 (sec). Leaf size: 15

```
DSolve[{x'[t]==Exp[t+x[t]],{x[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\log\left(2 - e^t\right)$$

4.21 problem 10(b)

Internal problem ID [10370]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 10(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{T' - 2at(T^2 - a^2) = 0}$$

With initial conditions

$$[T(0) = 0]$$

✓ Solution by Maple

Time used: 0.687 (sec). Leaf size: 31

```
dsolve([diff(T(t),t)=2*a*t*(T(t)^2-a^2),T(0) = 0],T(t), singsol=all)
```

$$T(t) = -\frac{a\left(e^{2t^2a^2} - 1\right)}{e^{2t^2a^2} + 1}$$

✓ Solution by Mathematica

Time used: 2.083 (sec). Leaf size: 16

```
DSolve[{T'[t]==2*a*t*(T[t]^2-a^2),{T[0]==0}},T[t],t,IncludeSingularSolutions -> True]
```

$$T(t) \to -a \tanh\left(a^2t^2\right)$$

4.22 problem 10(c)

Internal problem ID [10371]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 10(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - t^2 \tan(y) = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 5

```
dsolve([diff(y(t),t)=t^2*tan(y(t)),y(0) = 0],y(t), singsol=all)
```

$$y(t) = 0$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 6

```
DSolve[{y'[t]==t^2*Tan[y[t]],{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 0$$

4.23 problem 11

Internal problem ID [10372]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' - \frac{(4 + 2t)\, x}{\ln(x)} = 0}$$

With initial conditions

$$[x(0) = \mathrm{e}]$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 17

```
dsolve([diff(x(t),t)=(4+2*t)*x(t)/ln(x(t)),x(0) = exp(1)],x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{\sqrt{2t^2+8t+1}}$$

✓ Solution by Mathematica

Time used: 0.904 (sec). Leaf size: 19

```
DSolve[{x'[t]==(4+2*t)*x[t]/Log[x[t]],{x[0]==Exp[1]}},x[t],t,IncludeSingularSolutions -> True
```

$$x(t) \to e^{\sqrt{2t(t+4)+1}}$$

4.24 problem 12

Internal problem ID [10373]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{2ty^2}{t^2+1} = 0}$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(y(t),t)=2*t*y(t)^2/(1+t^2),y(0) = 0],y(t), singsol=all)
```

$$y(t) = 0$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 6

```
DSolve[{y'[t]==2*t*y[t]^2/(1+t^2),{y[0]==0}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to 0$$

4.25 problem 13

Internal problem ID [10374]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' - \frac{t^2}{1 - x^2} = 0}$$

With initial conditions

$$[x(1) = 1]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 122

```
dsolve([diff(x(t),t)=t^2/(1-x(t)^2),x(1) = 1],x(t), singsol=all)
```

$$x(t) = \frac{\left(-4 - 4t^3 + 4\sqrt{t^6 + 2t^3 - 3}\right)^{\frac{2}{3}} + 4}{2\left(-4 - 4t^3 + 4\sqrt{t^6 + 2t^3 - 3}\right)^{\frac{1}{3}}}$$

$$x(t) = -\frac{\left(1 + i\sqrt{3}\right)\left(-4 - 4t^3 + 4\sqrt{t^6 + 2t^3 - 3}\right)^{\frac{2}{3}} - 4i\sqrt{3} + 4}{4\left(-4 - 4t^3 + 4\sqrt{t^6 + 2t^3 - 3}\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 2.555 (sec). Leaf size: 188

```
DSolve[{x'[t]==t^2/(1-x[t]^2),{x[1]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{\sqrt[3]{-t^3+\sqrt{t^6+2t^3-3}-1}}{\sqrt[3]{2}}+\frac{\sqrt[3]{2}}{\sqrt[3]{-t^3+\sqrt{t^6+2t^3-3}-1}}$$

$$x(t) \to \frac{-i\sqrt[3]{2}\sqrt{3}\left(-t^3+\sqrt{t^6+2t^3-3}-1\right)^{2/3}-\sqrt[3]{2}\left(-t^3+\sqrt{t^6+2t^3-3}-1\right)^{2/3}+2i\sqrt{3}-2}{2\ 2^{2/3}\sqrt[3]{-t^3+\sqrt{t^6+2t^3-3}-1}}$$

4.26 problem 15

Internal problem ID [10375]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' - 6t(x-1)^{\frac{2}{3}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(x(t),t)=6*t*(x(t)-1)^(2/3),x(t), singsol=all)
```

$$c_1 + t^2 - (x(t)-1)^{\frac{1}{3}} = 0$$

✓ Solution by Mathematica

Time used: 0.195 (sec). Leaf size: 26

```
DSolve[x'[t]==6*t*(x[t]-1)^(2/3),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 1 + \frac{1}{27}(3t^2 + c_1)^3$$
$$x(t) \to 1$$

4.27 problem 21

Internal problem ID [10376]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{x' - \frac{4t^2 + 3x^2}{2xt} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(diff(x(t),t)=(4*t^2+3*x(t)^2)/(2*t*x(t)),x(t), singsol=all)
```

$$x(t) = \sqrt{c_1 t - 4}\, t$$
$$x(t) = -\sqrt{c_1 t - 4}\, t$$

✓ Solution by Mathematica

Time used: 0.286 (sec). Leaf size: 34

```
DSolve[x'[t]==(4*t^2+3*x[t]^2)/(2*t*x[t]),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -t\sqrt{-4 + c_1 t}$$
$$x(t) \to t\sqrt{-4 + c_1 t}$$

4.28 problem 23

Internal problem ID [10377]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{2\,\mathrm{e}^{2t}x + \mathrm{e}^{2t}x' - \mathrm{e}^{-t} = 0}$$

With initial conditions

$$[x(0) = 3]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve([diff(x(t)*exp(2*t),t)=exp(-t),x(0) = 3],x(t), singsol=all)
```

$$x(t) = -(\mathrm{e}^{-t} - 4)\,\mathrm{e}^{-2t}$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 18

```
DSolve[{D[x[t]*Exp[2*t],t]==Exp[-t],{x[0]==3}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-3t}(4e^t - 1)$$

4.29 problem 24

Internal problem ID [10378]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{\frac{x''t + x'}{t} + 2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(1/t*diff(t*diff(x(t),t),t)=-2,x(t), singsol=all)
```

$$x(t) = -\frac{t^2}{2} + \ln(t)\, c_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 20

```
DSolve[1/t*D[t*x'[t],t]==-2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{t^2}{2} + c_1 \log(t) + c_2$$

4.30 problem 26

Internal problem ID [10379]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y' - \frac{y^2 + 2yt}{t^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(t),t)=(y(t)^2+2*t*y(t))/t^2,y(t), singsol=all)
```

$$y(t) = \frac{t^2}{-t + c_1}$$

✓ Solution by Mathematica

Time used: 0.166 (sec). Leaf size: 22

```
DSolve[y'[t]==(y[t]^2+2*t*y[t])/t^2,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{t^2}{-t + c_1}$$
$$y(t) \to 0$$

4.31 problem 28

Internal problem ID [10380]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.3.1 Separable equations. Exercises page 26
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + y^2 e^{-t^2} = 0}$$

With initial conditions

$$\left[y(0) = \frac{1}{2}\right]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 16

```
dsolve([diff(y(t),t)=-y(t)^2*exp(-t^2),y(0) = 1/2],y(t), singsol=all)
```

$$y(t) = \frac{2}{4 + \sqrt{\pi}\,\mathrm{erf}(t)}$$

✓ Solution by Mathematica

Time used: 0.221 (sec). Leaf size: 19

```
DSolve[{y'[t]==-y[t]^2*Exp[-t^2],{y[0]==1/2}},y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{2}{\sqrt{\pi}\mathrm{erf}(t) + 4}$$

5 Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41

5.1 problem 1(a)

Internal problem ID [10381]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 1(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x' - 2t^3x + 6 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 56

```
dsolve(diff(x(t),t)=2*t^3*x(t)-6,x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{\frac{t^4}{2}} c_1 - \frac{3\,\mathrm{e}^{\frac{t^4}{4}} 128^{\frac{7}{8}} \left(2t^4 \,\mathrm{WhittakerM}\left(\frac{1}{8}, \frac{5}{8}, \frac{t^4}{2}\right) + 5\,\mathrm{WhittakerM}\left(\frac{9}{8}, \frac{5}{8}, \frac{t^4}{2}\right)\right)}{80t^3 \left(t^4\right)^{\frac{1}{8}}}$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 37

```
DSolve[x'[t]==2*t^3*x[t]-6,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2} e^{\frac{t^4}{2}} \left(3t \,\mathrm{ExpIntegralE}\left(\frac{3}{4}, \frac{t^4}{2}\right) + 2c_1\right)$$

5.2 problem 1(b)

Internal problem ID [10382]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 1(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\cos(t)\, x' - 2x \sin(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(cos(t)*diff(x(t),t)-2*x(t)*sin(x(t))=0,x(t), singsol=all)
```

$$\ln(\sec(t) + \tan(t)) - \left(\int^{x(t)} \frac{1}{2\_a \sin(\_a)} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 6.631 (sec). Leaf size: 40

```
DSolve[Cos[t]*x'[t]-2*x[t]*Sin[x[t]]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\csc(K[1])}{K[1]} dK[1]\&\right] \left[4\text{arctanh}\left(\tan\left(\frac{t}{2}\right)\right) + c_1\right]$$
$$x(t) \to 0$$

5.3 problem 1(c)

Internal problem ID [10383]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 1(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{x' - t + x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(x(t),t)=t-x(t)^2,x(t), singsol=all)
```

$$x(t) = \frac{c_1 \operatorname{AiryAi}(1, t) + \operatorname{AiryBi}(1, t)}{c_1 \operatorname{AiryAi}(t) + \operatorname{AiryBi}(t)}$$

✓ Solution by Mathematica

Time used: 0.157 (sec). Leaf size: 118

```
DSolve[x'[t]==t-x[t]^2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{i\sqrt{t}\left(\operatorname{BesselJ}\left(-\frac{2}{3}, \frac{2}{3}it^{3/2}\right) - c_1 \operatorname{BesselJ}\left(\frac{2}{3}, \frac{2}{3}it^{3/2}\right)\right)}{\operatorname{BesselJ}\left(\frac{1}{3}, \frac{2}{3}it^{3/2}\right) + c_1 \operatorname{BesselJ}\left(-\frac{1}{3}, \frac{2}{3}it^{3/2}\right)}$$

$$x(t) \to \frac{3 \operatorname{AiryAiPrime}(t) + \sqrt{3} \operatorname{AiryBiPrime}(t)}{3 \operatorname{AiryAi}(t) + \sqrt{3} \operatorname{AiryBi}(t)}$$

5.4 problem 1(d)

Internal problem ID [10384]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 1(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{7t^2x' - 3x + 2t = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(7*t^2*diff(x(t),t)=3*x(t)-2*t,x(t), singsol=all)
```

$$x(t) = \left(-\frac{2\,\mathrm{Ei}_1\left(-\frac{3}{7t}\right)}{7} + c_1\right)\mathrm{e}^{-\frac{3}{7t}}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 33

```
DSolve[7*t^2*x'[t]==3*x[t]-2*t,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{7}e^{-\frac{3}{7}/t}\left(2\,\mathrm{ExpIntegralEi}\left(\frac{3}{7t}\right) + 7c_1\right)$$

5.5 problem 1(e)

Internal problem ID [10385]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 1(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{x'x - 1 + xt = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(x(t)*diff(x(t),t)=1-t*x(t),x(t), singsol=all)
```

$$x(t) = -\frac{\left(2^{\frac{2}{3}}t^2 - 4\,\mathrm{RootOf}\left(\mathrm{AiryBi}\left(\_Z\right)2^{\frac{1}{3}}c_1 t + 2^{\frac{1}{3}}t\,\mathrm{AiryAi}\left(\_Z\right) - 2\,\mathrm{AiryBi}\left(1,\_Z\right)c_1 - 2\,\mathrm{AiryAi}\left(1,\_Z\right)\right)\right)2}{4}$$

✓ Solution by Mathematica

Time used: 0.238 (sec). Leaf size: 121

```
DSolve[x[t]*x'[t]==1-t*x[t],x[t],t,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{(-1)^{2/3}\sqrt[3]{2}t\,\mathrm{AiryAi}\left(-\frac{1}{2}\sqrt[3]{-\frac{1}{2}}(t^2+2x(t))\right) - 2\,\mathrm{AiryAiPrime}\left(-\frac{1}{2}\sqrt[3]{-\frac{1}{2}}(t^2+2x(t))\right)}{(-1)^{2/3}\sqrt[3]{2}t\,\mathrm{AiryBi}\left(-\frac{1}{2}\sqrt[3]{-\frac{1}{2}}(t^2+2x(t))\right) - 2\,\mathrm{AiryBiPrime}\left(-\frac{1}{2}\sqrt[3]{-\frac{1}{2}}(t^2+2x(t))\right)} + c_1 = 0, x(t)\right]$$

5.6 problem 1(f)

Internal problem ID [10386]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 1(f).
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{x'^2 + xt - \sqrt{1+t} = 0}$$

✗ Solution by Maple

```
dsolve(diff(x(t),t)^2+t*x(t)=sqrt(1+t),x(t), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x'[t]^2+t*x[t]==Sqrt[1+t],x[t],t,IncludeSingularSolutions -> True]
```

Not solved

5.7 problem 2(a)

Internal problem ID [10387]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 2(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x' + \frac{2x}{t} - t = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(x(t),t)=-(2/t)*x(t)+t,x(t), singsol=all)
```

$$x(t) = \frac{\frac{t^4}{4} + c_1}{t^2}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 19

```
DSolve[x'[t]==-(2/t)*x[t]+t,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{t^2}{4} + \frac{c_1}{t^2}$$

5.8 problem 2(b)

Internal problem ID [10388]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 2(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' + y - e^t = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(t),t)+y(t)=exp(t),y(t), singsol=all)
```

$$y(t) = \frac{e^t}{2} + e^{-t}c_1$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 21

```
DSolve[y'[t]+y[t]==Exp[t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{e^t}{2} + c_1 e^{-t}$$

5.9 problem 2(c)

Internal problem ID [10389]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 2(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x' + 2xt - \mathrm{e}^{-t^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(x(t),t)+2*t*x(t)=exp(-t^2),x(t), singsol=all)
```

$$x(t) = (t + c_1)\,\mathrm{e}^{-t^2}$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 17

```
DSolve[x'[t]+2*t*x[t]==Exp[-t^2],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-t^2}(t + c_1)$$

5.10 problem 2(d)

Internal problem ID [10390]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 2(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x't - t^2 + x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(t*diff(x(t),t)=-x(t)+t^2,x(t), singsol=all)
```

$$x(t) = \frac{\frac{t^3}{3} + c_1}{t}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 19

```
DSolve[t*x'[t]==-x[t]+t^2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{t^2}{3} + \frac{c_1}{t}$$

5.11 problem 2(e)

Internal problem ID [10391]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 2(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{\theta' + a\theta - e^{bt} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(theta(t),t)=-a*theta(t)+exp(b*t),theta(t), singsol=all)
```

$$\theta(t) = \left(\frac{e^{t(a+b)}}{a+b} + c_1\right) e^{-at}$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 31

```
DSolve[theta'[t]==-a*theta[t]+Exp[b*t],theta[t],t,IncludeSingularSolutions -> True]
```

$$\theta(t) \to \frac{e^{-at}\left(e^{t(a+b)} + c_1(a+b)\right)}{a+b}$$

5.12 problem 2(f)

Internal problem ID [10392]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 2(f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(t^2+1)\,x'+3xt-6t=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((t^2+1)*diff(x(t),t)=-3*t*x(t)+6*t,x(t), singsol=all)
```

$$x(t)=2+\frac{c_1}{(t^2+1)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 24

```
DSolve[(t^2+1)*x'[t]==-3*t*x[t]+6*t,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t)\to 2+\frac{c_1}{(t^2+1)^{3/2}}$$

$$x(t)\to 2$$

5.13 problem 3(a)

Internal problem ID [10393]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 3(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x' + \frac{5x}{t} - 1 - t = 0}$$

With initial conditions

$$[x(1) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve([diff(x(t),t)+(5/t)*x(t)=1+t,x(1) = 1],x(t), singsol=all)
```

$$x(t) = \frac{t^2}{7} + \frac{t}{6} + \frac{29}{42t^5}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 23

```
DSolve[{x'[t]+(5/t)*x[t]==1+t,{x[1]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{(6t+7)t^6 + 29}{42t^5}$$

5.14 problem 3(b)

Internal problem ID [10394]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 3(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' - \left(a + \frac{b}{t}\right) x = 0}$$

With initial conditions

$$[x(1) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 14

```
dsolve([diff(x(t),t)=(a+b/t)*x(t),x(1) = 1],x(t), singsol=all)
```

$$x(t) = t^b \mathrm{e}^{a(t-1)}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 16

```
DSolve[{x'[t]==(a+b/t)*x[t],{x[1]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{a(t-1)} t^b$$

5.15 problem 3(c)

Internal problem ID [10395]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 3(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{R' + \frac{R}{t} - \frac{2}{t^2+1} = 0}$$

With initial conditions

$$[R(1) = 3\ln(2)]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve([diff(R(t),t)+R(t)/t=2/(1+t^2),R(1) = 3*ln(2)],R(t), singsol=all)
```

$$R(t) = \frac{\ln(t^2+1) + 2\ln(2)}{t}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 17

```
DSolve[{R'[t]+R[t]/t==2/(1+t^2),{R[1]==Log[8]}},R[t],t,IncludeSingularSolutions -> True]
```

$$R(t) \to \frac{\log(4t^2+4)}{t}$$

5.16 problem 3(d)

Internal problem ID [10396]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 3(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{N' - N + 9\,\mathrm{e}^{-t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(N(t),t)=N(t)-9*exp(-t),N(t), singsol=all)
```

$$N(t) = \left(\frac{9\,\mathrm{e}^{-2t}}{2} + c_1\right)\mathrm{e}^{t}$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 32

```
DSolve[n'[t]==n[t]-9*exp[-t],n[t],t,IncludeSingularSolutions -> True]
```

$$n(t) \to e^{t}\left(\int_{1}^{t} -9e^{-K[1]}\exp(-K[1])dK[1] + c_1\right)$$

5.17 problem 3(e)

Internal problem ID [10397]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 3(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\cos(\theta)\, v' + v - 3 = 0}$$

With initial conditions

$$\left[v\left(\frac{\pi}{2}\right) = 1\right]$$

✗ Solution by Maple

```
dsolve([cos(theta)*diff(v(theta),theta)+v(theta)=3,v(1/2*Pi) = 1],v(theta), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{Cos[theta]*v'[theta]+v[theta]==3,{v[Pi/2]==1}},v[theta],theta,IncludeSingularSolution
```

$$\{\}$$

5.18 problem 3(f)

Internal problem ID [10398]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 3(f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{R' - \frac{R}{t} - t\,\mathrm{e}^{-t} = 0}$$

With initial conditions

$$[R(1) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve([diff(R(t),t)=R(t)/t+t*exp(-t),R(1) = 1],R(t), singsol=all)
```

$$R(t) = \left(-\mathrm{e}^{-t} + 1 + \mathrm{e}^{-1}\right) t$$

✓ Solution by Mathematica

Time used: 0.059 (sec). Leaf size: 18

```
DSolve[{R'[t]==R[t]/t+t*Exp[-t],{R[1]==1}},R[t],t,IncludeSingularSolutions -> True]
```

$$R(t) \to t\left(\sinh(t) - \cosh(t) + \frac{1}{e} + 1\right)$$

5.19 problem 4

Internal problem ID [10399]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' + ay - \sqrt{1+t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 60

```
dsolve(diff(y(t),t)+a*y(t)=sqrt(1+t),y(t), singsol=all)
```

$$y(t) = \left(2\,\mathrm{e}^{-a}\left(\frac{\sqrt{t+1}\,\mathrm{e}^{(t+1)a}}{2a} - \frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-a}\,\sqrt{t+1}\right)}{4a\sqrt{-a}}\right) + c_1\right)\mathrm{e}^{-at}$$

✓ Solution by Mathematica

Time used: 0.28 (sec). Leaf size: 39

```
DSolve[y'[t]+a*y[t]==Sqrt[1+t],y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to e^{-at}\left(-e^{-a}(t+1)^{3/2}\,\mathrm{ExpIntegralE}\left(-\frac{1}{2}, -a(t+1)\right) + c_1\right)$$

5.20 problem 5

Internal problem ID [10400]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' - 2xt = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(x(t),t)=2*t*x(t),x(t), singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{t^2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 18

```
DSolve[x'[t]==2*t*x[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{t^2}$$
$$x(t) \to 0$$

5.21 problem 6

Internal problem ID [10401]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x' + \frac{e^{-t} x}{t} - t = 0}$$

With initial conditions

$$[x(1) = 0]$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 23

```
dsolve([diff(x(t),t)+exp(-t)/t*x(t)=t,x(1) = 0],x(t), singsol=all)
```

$$x(t) = \left(\int_1^t \_z1 \, e^{-\operatorname{Ei}_1(\_z1)} d\_z1\right) e^{\operatorname{Ei}_1(t)}$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 31

```
DSolve[{x'[t]+Exp[-t]/t*x[t]==t,{x[1]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-\text{ExpIntegralEi}(-t)} \int_1^t e^{\text{ExpIntegralEi}(-K[1])} K[1] dK[1]$$

5.22 problem 7

Internal problem ID [10402]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x'' + x' - 3t = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(x(t),t$2)+diff(x(t),t)=3*t,x(t), singsol=all)
```

$$x(t) = -\mathrm{e}^{-t} c_1 + \frac{3t^2}{2} - 3t + c_2$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 25

```
DSolve[x''[t]+x'[t]==3*t,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{3}{2}(t-2)t - c_1 e^{-t} + c_2$$

5.23 problem 8

Internal problem ID [10403]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{x' - (t+x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve(diff(x(t),t)=(t+x(t))^2,x(t), singsol=all)
```

$$x(t) = -t - \tan(-t + c_1)$$

✓ Solution by Mathematica

Time used: 0.475 (sec). Leaf size: 14

```
DSolve[x'[t]==(t+x[t])^2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -t + \tan(t + c_1)$$

5.24 problem 9

Internal problem ID [10404]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - ax - b = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(x(t),t)=a*x(t)+b,x(t), singsol=all)
```

$$x(t) = -\frac{b}{a} + \mathrm{e}^{at} c_1$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 30

```
DSolve[x'[t]==a*x[t]+b,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{b}{a} + c_1 e^{at}$$
$$x(t) \to -\frac{b}{a}$$

5.25 problem 12

Internal problem ID [10405]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' + p(t)\,x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(x(t),t)+p(t)*x(t)=0,x(t), singsol=all)
```

$$x(t) = c_1 e^{\int -p(t)dt}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 27

```
DSolve[x'[t]+p[t]*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 \exp\left(\int_1^t -p(K[1])dK[1]\right)$$
$$x(t) \to 0$$

5.26 problem 15(a)

Internal problem ID [10406]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 15(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{x' - \frac{2x}{3t} - \frac{2t}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(x(t),t)=2/(3*t)*x(t)+2*t/x(t),x(t), singsol=all)
```

$$x(t) = \sqrt{t^{\frac{4}{3}}c_1 + 6t^2}$$
$$x(t) = -\sqrt{t^{\frac{4}{3}}c_1 + 6t^2}$$

✓ Solution by Mathematica

Time used: 3.706 (sec). Leaf size: 47

```
DSolve[x'[t]==2/(3*t)*x[t]+2*t/x[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\sqrt{6t^2 + c_1 t^{4/3}}$$
$$x(t) \to \sqrt{6t^2 + c_1 t^{4/3}}$$

5.27 problem 15(b)

Internal problem ID [10407]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 15(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Bernoulli]`

$$\boxed{x' - x(1 + x\,e^{t}) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(x(t),t)=x(t)*(1+x(t)*exp(t)),x(t), singsol=all)
```

$$x(t) = \frac{2}{2\,e^{-t}c_1 - e^{t}}$$

✓ Solution by Mathematica

Time used: 0.21 (sec). Leaf size: 27

```
DSolve[x'[t]==x[t]*(1+x[t]*Exp[t]),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{2e^{t}}{e^{2t} - 2c_1}$$

$$x(t) \to 0$$

5.28 problem 15(c)

Internal problem ID [10408]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 15(c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x' + \frac{x}{t} - \frac{1}{t x^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 75

```
dsolve(diff(x(t),t)=-1/t*x(t)+1/(t*x(t)^2),x(t), singsol=all)
```

$$x(t) = \frac{(t^3 + c_1)^{\frac{1}{3}}}{t}$$

$$x(t) = \frac{-\frac{(t^3+c_1)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\,(t^3+c_1)^{\frac{1}{3}}}{2}}{t}$$

$$x(t) = \frac{-\frac{(t^3+c_1)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\,(t^3+c_1)^{\frac{1}{3}}}{2}}{t}$$

✓ Solution by Mathematica

Time used: 0.29 (sec). Leaf size: 159

```
DSolve[x'[t]==-1/t*x[t]+1/(t*x[t]^2),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{\sqrt[3]{t^3 + e^{3c_1}}}{t}$$
$$x(t) \to -\frac{\sqrt[3]{-1}\sqrt[3]{t^3 + e^{3c_1}}}{t}$$
$$x(t) \to \frac{(-1)^{2/3}\sqrt[3]{t^3 + e^{3c_1}}}{t}$$
$$x(t) \to 1$$
$$x(t) \to -\sqrt[3]{-1}$$
$$x(t) \to (-1)^{2/3}$$
$$x(t) \to \frac{\sqrt[3]{t^3}}{t}$$
$$x(t) \to -\frac{\sqrt[3]{-1}\sqrt[3]{t^3}}{t}$$
$$x(t) \to \frac{(-1)^{2/3}\sqrt[3]{t^3}}{t}$$

5.29 problem 15(d)

Internal problem ID [10409]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 15(d).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y't^2 + 2yt - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(t^2*diff(y(t),t)+2*t*y(t)-y(t)^2=0,y(t), singsol=all)
```

$$y(t) = \frac{3t}{3c_1t^3 + 1}$$

✓ Solution by Mathematica

Time used: 0.161 (sec). Leaf size: 24

```
DSolve[t^2*y'[t]+2*t*y[t]-y[t]^2==0,y[t],t,IncludeSingularSolutions -> True]
```

$$y(t) \to \frac{3t}{1 + 3c_1t^3}$$
$$y(t) \to 0$$

5.30 problem 15(e)

Internal problem ID [10410]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 15(e).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x' - ax - bx^3 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 70

```
dsolve(diff(x(t),t)=a*x(t)+b*x(t)^3,x(t), singsol=all)
```

$$x(t) = \frac{\sqrt{(c_1 a\,\mathrm{e}^{-2at} - b)\,a}}{c_1 a\,\mathrm{e}^{-2at} - b}$$
$$x(t) = -\frac{\sqrt{(c_1 a\,\mathrm{e}^{-2at} - b)\,a}}{c_1 a\,\mathrm{e}^{-2at} - b}$$

✓ Solution by Mathematica

Time used: 1.781 (sec). Leaf size: 118

```
DSolve[x'[t]==a*x[t]+b*x[t]^3,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{i\sqrt{a}e^{a(t+c_1)}}{\sqrt{-1+be^{2a(t+c_1)}}}$$

$$x(t) \to \frac{i\sqrt{a}e^{a(t+c_1)}}{\sqrt{-1+be^{2a(t+c_1)}}}$$

$$x(t) \to 0$$

$$x(t) \to -\frac{i\sqrt{a}}{\sqrt{b}}$$

$$x(t) \to \frac{i\sqrt{a}}{\sqrt{b}}$$

5.31 problem 15(f)

Internal problem ID [10411]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 15(f).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{w' - wt - t^3w^3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(diff(w(t),t)=t*w(t)+t^3*w(t)^3,w(t), singsol=all)
```

$$w(t) = \frac{1}{\sqrt{\mathrm{e}^{-t^2}c_1 - t^2 + 1}}$$
$$w(t) = -\frac{1}{\sqrt{\mathrm{e}^{-t^2}c_1 - t^2 + 1}}$$

✓ Solution by Mathematica

Time used: 1.892 (sec). Leaf size: 80

```
DSolve[w'[t]==t*w[t]+t^3*w[t]^3,w[t],t,IncludeSingularSolutions -> True]
```

$$w(t) \to -\frac{ie^{\frac{t^2}{2}}}{\sqrt{e^{t^2}(t^2-1) - c_1}}$$
$$w(t) \to \frac{ie^{\frac{t^2}{2}}}{\sqrt{e^{t^2}(t^2-1) - c_1}}$$
$$w(t) \to 0$$

5.32 problem 16-b(i)

Internal problem ID [10412]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 16-b(i).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^3 + 3x'tx^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 88

```
dsolve(x(t)^3+3*t*x(t)^2*diff(x(t),t)=0,x(t), singsol=all)
```

$$x(t) = 0$$

$$x(t) = \frac{(-c_1 t^2)^{\frac{1}{3}}}{t}$$

$$x(t) = -\frac{(-c_1 t^2)^{\frac{1}{3}}}{2t} - \frac{i\sqrt{3}\,(-c_1 t^2)^{\frac{1}{3}}}{2t}$$

$$x(t) = -\frac{(-c_1 t^2)^{\frac{1}{3}}}{2t} + \frac{i\sqrt{3}\,(-c_1 t^2)^{\frac{1}{3}}}{2t}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 23

```
DSolve[x[t]^3+3*t*x[t]^2*x'[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 0$$

$$x(t) \to \frac{c_1}{\sqrt[3]{t}}$$

$$x(t) \to 0$$

5.33 problem 16-b(ii)

Internal problem ID [10413]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 16-b(ii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{t^3 + \frac{x}{t} + \left(x^2 + \ln\left(t\right)\right) x' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 415

```
dsolve(t^3+x(t)/t+(x(t)^2+ln(t))*diff(x(t),t)=0,x(t), singsol=all)
```

$$x(t) = \frac{\left(-3t^4-12c_1+\sqrt{64\ln(t)^3+9t^8+72t^4c_1+144c_1^2}\right)^{\frac{1}{3}}}{2} - \frac{2\ln(t)}{\left(-3t^4-12c_1+\sqrt{64\ln(t)^3+9t^8+72t^4c_1+144c_1^2}\right)^{\frac{1}{3}}}$$

$$x(t) = -\frac{\left(-3t^4-12c_1+\sqrt{64\ln(t)^3+9t^8+72t^4c_1+144c_1^2}\right)^{\frac{1}{3}}}{4} + \frac{\ln(t)}{\left(-3t^4-12c_1+\sqrt{64\ln(t)^3+9t^8+72t^4c_1+144c_1^2}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(-3t^4-12c_1+\sqrt{64\ln(t)^3+9t^8+72t^4c_1+144c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{2\ln(t)}{\left(-3t^4-12c_1+\sqrt{64\ln(t)^3+9t^8+72t^4c_1+144c_1^2}\right)^{\frac{1}{3}}}\right)}{2}$$

$$x(t) = -\frac{\left(-3t^4-12c_1+\sqrt{64\ln(t)^3+9t^8+72t^4c_1+144c_1^2}\right)^{\frac{1}{3}}}{4} + \frac{\ln(t)}{\left(-3t^4-12c_1+\sqrt{64\ln(t)^3+9t^8+72t^4c_1+144c_1^2}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(-3t^4-12c_1+\sqrt{64\ln(t)^3+9t^8+72t^4c_1+144c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{2\ln(t)}{\left(-3t^4-12c_1+\sqrt{64\ln(t)^3+9t^8+72t^4c_1+144c_1^2}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 1.749 (sec). Leaf size: 307

```
DSolve[t^3+x[t]/t+(x[t]^2+Log[t])*x'[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{-4\log(t) + \left(-3t^4 + \sqrt{64\log^3(t) + 9\left(t^4 - 4c_1\right)^2} + 12c_1\right)^{2/3}}{2\sqrt[3]{-3t^4 + \sqrt{64\log^3(t) + 9\left(t^4 - 4c_1\right)^2} + 12c_1}}$$

$$x(t) \to \frac{i\left(\sqrt{3} + i\right)\left(-3t^4 + \sqrt{64\log^3(t) + 9\left(t^4 - 4c_1\right)^2} + 12c_1\right)^{2/3} + \left(4 + 4i\sqrt{3}\right)\log(t)}{4\sqrt[3]{-3t^4 + \sqrt{64\log^3(t) + 9\left(t^4 - 4c_1\right)^2} + 12c_1}}$$

$$x(t) \to \frac{\left(-1 - i\sqrt{3}\right)\left(-3t^4 + \sqrt{64\log^3(t) + 9\left(t^4 - 4c_1\right)^2} + 12c_1\right)^{2/3} + \left(4 - 4i\sqrt{3}\right)\log(t)}{4\sqrt[3]{-3t^4 + \sqrt{64\log^3(t) + 9\left(t^4 - 4c_1\right)^2} + 12c_1}}$$

5.34 problem 16-b(iii)

Internal problem ID [10414]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 16-b(iii).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{x' + \frac{\sin(x) - x\sin(t)}{t\cos(x) + \cos(t)} = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 15

```
dsolve(diff(x(t),t)=- (sin(x(t))-x(t)*sin(t))/(t*cos(x(t))+cos(t)),x(t), singsol=all)
```

$$x(t)\cos(t) + t\sin(x(t)) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.181 (sec). Leaf size: 17

```
DSolve[x'[t]==- (Sin[x[t]]-x[t]*Sin[t])/(t*Cos[x[t]]+Cos[t]),x[t],t,IncludeSingularSolutions
```

$$\text{Solve}[t\sin(x(t)) + x(t)\cos(t) = c_1, x(t)]$$

5.35 problem 16-b(iv)

Internal problem ID [10415]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 16-b(iv).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x + 3x'tx^2 = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 35

```
dsolve(x(t)+3*t*x(t)^2*diff(x(t),t)=0,x(t), singsol=all)
```

$$x(t) = 0$$
$$x(t) = -\frac{\sqrt{-6\ln(t) + 9c_1}}{3}$$
$$x(t) = \frac{\sqrt{-6\ln(t) + 9c_1}}{3}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 51

```
DSolve[x[t]+3*t*x[t]^2*x'[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 0$$
$$x(t) \to -\sqrt{-\frac{2\log(t)}{3} + 2c_1}$$
$$x(t) \to \sqrt{-\frac{2\log(t)}{3} + 2c_1}$$
$$x(t) \to 0$$

5.36 problem 16-b(v)

Internal problem ID [10416]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 16-b(v).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2 - t^2x' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x(t)^2-t^2*diff(x(t),t)=0,x(t), singsol=all)
```

$$x(t) = \frac{t}{c_1t + 1}$$

✓ Solution by Mathematica

Time used: 0.122 (sec). Leaf size: 21

```
DSolve[x[t]^2-t^2*x'[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{t}{1 - c_1t}$$

$$x(t) \to 0$$

5.37 problem 16-b(vi)

Internal problem ID [10417]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 1, First order differential equations. Section 1.4.1. Integrating factors. Exercises page 41
Problem number: 16-b(vi).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{t \cot(x)\, x' + 2 = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 10

```
dsolve(t*cot(x(t))*diff(x(t),t)=-2,x(t), singsol=all)
```

$$x(t) = \arcsin\left(\frac{c_1}{t^2}\right)$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 14

```
DSolve[t*Cot[x[t]]*x'[t]==-2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \arcsin\left(\frac{e^{c_1}}{t^2}\right)$$

6 Chapter 2, Second order linear equations. Section 2.2.2 Real eigenvalues. Exercises page 90

6.1 problem 1(a)

Internal problem ID [10418]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.2.2 Real eigenvalues. Exercises page 90
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - 4x' + 4x = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 14

```
dsolve([diff(x(t),t$2)-4*diff(x(t),t)+4*x(t)=0,x(0) = 1, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = e^{2t}(-2t + 1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[{x''[t]-4*x'[t]+4*x[t]==0,{x[0]==1,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{2t}(1 - 2t)$$

6.2 problem 1(b)

Internal problem ID [10419]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.2.2 Real eigenvalues. Exercises page 90
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - 2x' = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([diff(x(t),t$2)-2*diff(x(t),t)=0,x(0) = 1, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = 1$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 6

```
DSolve[{x''[t]-2*x'[t]==0,{x[0]==1,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 1$$

6.3 problem 1(c)

Internal problem ID [10420]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.2.2 Real eigenvalues. Exercises page 90
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{\frac{x''}{2}+x'+\frac{x}{2}=0}$$

With initial conditions

$$[x(0)=1, x'(0)=0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 12

```
dsolve([1/2*diff(x(t),t$2)+diff(x(t),t)+1/2*x(t)=0,x(0) = 1, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t)=\mathrm{e}^{-t}(t+1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 14

```
DSolve[{1/2*x''[t]+x'[t]+1/2*x[t]==0,{x[0]==1,x'[0]==0}},x[t],t,IncludeSingularSolutions -> T
```

$$x(t)\to e^{-t}(t+1)$$

6.4 problem 1(d)

Internal problem ID [10421]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.2.2 Real eigenvalues. Exercises page 90
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 4x' + 3x = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(x(t),t$2)+4*diff(x(t),t)+3*x(t)=0,x(0) = 1, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{3\,\mathrm{e}^{-t}}{2} - \frac{\mathrm{e}^{-3t}}{2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[{x''[t]+4*x'[t]+3*x[t]==0,{x[0]==1,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-2t}(2\sinh(t) + \cosh(t))$$

6.5 problem 3(a)

Internal problem ID [10422]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.2.2 Real eigenvalues. Exercises page 90
Problem number: 3(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - 4x' + 4x = 0}$$

With initial conditions

$$[x(0) = -1, x'(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([diff(x(t),t$2)-4*diff(x(t),t)+4*x(t)=0,x(0) = -1, D(x)(0) = 2],x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{2t}(-1 + 4t)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[{x''[t]-4*x'[t]+4*x[t]==0,{x[0]==-1,x'[0]==2}},x[t],t,IncludeSingularSolutions -> True
```

$$x(t) \to e^{2t}(4t - 1)$$

6.6 problem 3(b)

Internal problem ID [10423]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.2.2 Real eigenvalues. Exercises page 90
Problem number: 3(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - 2x' = 0}$$

With initial conditions

$$[x(0) = -1, x'(0) = 2]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(x(t),t$2)-2*diff(x(t),t)=0,x(0) = -1, D(x)(0) = 2],x(t), singsol=all)
```

$$x(t) = -2 + e^{2t}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 12

```
DSolve[{x''[t]-2*x'[t]==0,{x[0]==-1,x'[0]==2}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{2t} - 2$$

6.7 problem 3(c)

Internal problem ID [10424]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.2.2 Real eigenvalues. Exercises page 90
Problem number: 3(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{\frac{x''}{2} + x' + \frac{x}{2} = 0}$$

With initial conditions

$$[x(0) = -1, x'(0) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([1/2*diff(x(t),t$2)+diff(x(t),t)+1/2*x(t)=0,x(0) = -1, D(x)(0) = 2],x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-t}(t-1)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 14

```
DSolve[{1/2*x''[t]+x'[t]+1/2*x[t]==0,{x[0]==-1,x'[0]==2}},x[t],t,IncludeSingularSolutions ->
```

$$x(t) \to e^{-t}(t-1)$$

6.8 problem 3(d)

Internal problem ID [10425]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.2.2 Real eigenvalues. Exercises page 90
Problem number: 3(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 4x' + 3x = 0}$$

With initial conditions

$$[x(0) = -1, x'(0) = 2]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve([diff(x(t),t$2)+4*diff(x(t),t)+3*x(t)=0,x(0) = -1, D(x)(0) = 2],x(t), singsol=all)
```

$$x(t) = -\frac{e^{-t}}{2} - \frac{e^{-3t}}{2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 14

```
DSolve[{x''[t]+4*x'[t]+3*x[t]==0,{x[0]==-1,x'[0]==2}},x[t],t,IncludeSingularSolutions -> True
```

$$x(t) \to -e^{-2t} \cosh(t)$$

7 Chapter 2, Second order linear equations. Section 2.2.3 Complex eigenvalues. Exercises page 94

7.1 problem 1(a)

Internal problem ID [10426]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.2.3 Complex eigenvalues. Exercises page 94
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + x' + 4x = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 31

```
dsolve([diff(x(t),t$2)+diff(x(t),t)+4*x(t)=0,x(0) = 1, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{e^{-\frac{t}{2}}\left(\sqrt{15}\sin\left(\frac{\sqrt{15}\,t}{2}\right) + 15\cos\left(\frac{\sqrt{15}\,t}{2}\right)\right)}{15}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 47

```
DSolve[{x''[t]+x'[t]+4*x[t]==0,{x[0]==1,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{15}e^{-t/2}\left(\sqrt{15}\sin\left(\frac{\sqrt{15}t}{2}\right) + 15\cos\left(\frac{\sqrt{15}t}{2}\right)\right)$$

7.2 problem 1(b)

Internal problem ID [10427]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.2.3 Complex eigenvalues. Exercises page 94
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - 4x' + 6x = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 29

```
dsolve([diff(x(t),t$2)-4*diff(x(t),t)+6*x(t)=0,x(0) = 1, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = -e^{2t}\left(\sqrt{2}\,\sin\left(\sqrt{2}\,t\right) - \cos\left(\sqrt{2}\,t\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 35

```
DSolve[{x''[t]-4*x'[t]+6*x[t]==0,{x[0]==1,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{2t}\left(\cos\left(\sqrt{2}t\right) - \sqrt{2}\sin\left(\sqrt{2}t\right)\right)$$

7.3 problem 1(c)

Internal problem ID [10428]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.2.3 Complex eigenvalues. Exercises page 94
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + 9x = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 8

```
dsolve([diff(x(t),t$2)+9*x(t)=0,x(0) = 1, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \cos(3t)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 9

```
DSolve[{x''[t]+9*x[t]==0,{x[0]==1,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \cos(3t)$$

7.4 problem 1(d)

Internal problem ID [10429]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.2.3 Complex eigenvalues. Exercises page 94
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - 12x = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 23

```
dsolve([diff(x(t),t$2)-12*x(t)=0,x(0) = 1, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{e^{2\sqrt{3}\,t}}{2} + \frac{e^{-2\sqrt{3}\,t}}{2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 14

```
DSolve[{x''[t]-12*x[t]==0,{x[0]==1,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \cosh\left(2\sqrt{3}t\right)$$

7.5 problem 1(e)

Internal problem ID [10430]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.2.3 Complex eigenvalues. Exercises page 94
Problem number: 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{2x'' + 3x' + 3x = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 31

```
dsolve([2*diff(x(t),t$2)+3*diff(x(t),t)+3*x(t)=0,x(0) = 1, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{e^{-\frac{3t}{4}}\left(\sqrt{15}\,\sin\left(\frac{\sqrt{15}\,t}{4}\right) + 5\cos\left(\frac{\sqrt{15}\,t}{4}\right)\right)}{5}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 42

```
DSolve[{x''[t]+3*x'[t]+3*x[t]==0,{x[0]==1,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-3t/2}\left(\sqrt{3}\sin\left(\frac{\sqrt{3}t}{2}\right) + \cos\left(\frac{\sqrt{3}t}{2}\right)\right)$$

7.6 problem 1(f)

Internal problem ID [10431]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.2.3 Complex eigenvalues. Exercises page 94
Problem number: 1(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{\frac{x''}{2}+\frac{5x'}{6}+\frac{2x}{9}=0}$$

With initial conditions

$$[x(0)=1, x'(0)=0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([1/2*diff(x(t),t$2)+5/6*diff(x(t),t)+2/9*x(t)=0,x(0) = 1, D(x)(0) = 0],x(t), singsol=a
```

$$x(t)=-\frac{e^{-\frac{4t}{3}}}{3}+\frac{4\,e^{-\frac{t}{3}}}{3}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 23

```
DSolve[{1/2*x''[t]+5/6*x'[t]+2/9*x[t]==0,{x[0]==1,x'[0]==0}},x[t],t,IncludeSingularSolutions
```

$$x(t)\to\frac{1}{3}e^{-4t/3}\left(4e^{t}-1\right)$$

8 Chapter 2, Second order linear equations. Section 2.2.4. Applications. Exercises page 99

8.1 problem 1

Internal problem ID [10432]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.2.4. Applications. Exercises page 99
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + x' + x = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 28

```
dsolve([diff(x(t),t$2)+diff(x(t),t)+x(t)=0,x(0) = 1, D(x)(0) = 1],x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-\frac{t}{2}} \left(\sqrt{3}\, \sin\left(\frac{\sqrt{3}\, t}{2}\right) + \cos\left(\frac{\sqrt{3}\, t}{2}\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 42

```
DSolve[{x''[t]+x'[t]+x[t]==0,{x[0]==1,x'[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-t/2} \left(\sqrt{3} \sin\left(\frac{\sqrt{3}t}{2}\right) + \cos\left(\frac{\sqrt{3}t}{2}\right)\right)$$

8.2 problem 2

Internal problem ID [10433]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.2.4. Applications. Exercises page 99
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + \frac{x'}{8} + x = 0}$$

With initial conditions

$$[x(0) = 2, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 31

```
dsolve([diff(x(t),t$2)+125/1000*diff(x(t),t)+x(t)=0,x(0) = 2, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{2\,\mathrm{e}^{-\frac{t}{16}}\left(\sqrt{255}\,\sin\left(\frac{\sqrt{255}\,t}{16}\right) + 255\cos\left(\frac{\sqrt{255}\,t}{16}\right)\right)}{255}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 47

```
DSolve[{x''[t]+125/1000*x'[t]+x[t]==0,{x[0]==2,x'[0]==0}},x[t],t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{2}{255}e^{-t/16}\left(\sqrt{255}\sin\left(\frac{\sqrt{255}t}{16}\right) + 255\cos\left(\frac{\sqrt{255}t}{16}\right)\right)$$

9 Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110

9.1 problem 1(a)

Internal problem ID [10434]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x' + x - 3t^3 + 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(x(t),t$2)+diff(x(t),t)+x(t)=3*t^3-1,x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right) c_2 + \mathrm{e}^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right) c_1 + 3t^3 - 9t^2 + 17$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 52

```
DSolve[x''[t]+x'[t]+x[t]==3*t^3-1,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 3(t-3)t^2 + e^{-t/2}\left(c_2 \cos\left(\frac{\sqrt{3}t}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}t}{2}\right)\right) + 17$$

9.2 problem 1(b)

Internal problem ID [10435]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x' + x - 3\cos(t) + 2\sin(t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(x(t),t$2)+diff(x(t),t)+x(t)=3*cos(t)-2*sin(t),x(t), singsol=all)
```

$$x(t) = e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\, t}{2}\right) c_2 + e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\, t}{2}\right) c_1 + 3\sin(t) + 2\cos(t)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 51

```
DSolve[x''[t]+x'[t]+x[t]==3*Cos[t]-2*Sin[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 3\sin(t) + 2\cos(t) + e^{-t/2}\left(c_2 \cos\left(\frac{\sqrt{3}t}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}t}{2}\right)\right)$$

9.3 problem 1(c)

Internal problem ID [10436]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' + x' + x - 12 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(x(t),t$2)+diff(x(t),t)+x(t)=12,x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right) c_2 + \mathrm{e}^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right) c_1 + 12$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 44

```
DSolve[x''[t]+x'[t]+x[t]==12,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 12 + e^{-t/2}\left(c_2 \cos\left(\frac{\sqrt{3}t}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}t}{2}\right)\right)$$

9.4 problem 1(d)

Internal problem ID [10437]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x' + x - t^2 e^{3t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(x(t),t$2)+diff(x(t),t)+x(t)=t^2*exp(3*t),x(t), singsol=all)
```

$$x(t) = e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\, t}{2}\right) c_2 + e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\, t}{2}\right) c_1 + \frac{(169t^2 - 182t + 72)\, e^{3t}}{2197}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 55

```
DSolve[x''[t]+x'[t]+x[t]==t^2*exp(3*t),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 3\exp\left((t-3)t^2 + 6\right) + e^{-t/2}\left(c_2 \cos\left(\frac{\sqrt{3}t}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}t}{2}\right)\right)$$

9.5 problem 1(e)

Internal problem ID [10438]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x' + x - 5\sin(7t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(x(t),t$2)+diff(x(t),t)+x(t)=5*sin(7*t),x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right) c_2 + \mathrm{e}^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right) c_1 - \frac{240\sin(7t)}{2353} - \frac{35\cos(7t)}{2353}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 60

```
DSolve[x''[t]+x'[t]+x[t]==5*Sin[7*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{5(48\sin(7t) + 7\cos(7t))}{2353} + e^{-t/2}\left(c_2 \cos\left(\frac{\sqrt{3}t}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}t}{2}\right)\right)$$

9.6 problem 1(f)

Internal problem ID [10439]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 1(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x' + x - e^{2t}\cos(t) - t^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 52

```
dsolve(diff(x(t),t$2)+diff(x(t),t)+x(t)=exp(2*t)*cos(t)+t^2,x(t), singsol=all)
```

$$x(t) = e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right) c_2 + e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right) c_1 + \frac{(6\cos(t) + 5\sin(t))\,e^{2t}}{61} + t^2 - 2t$$

✓ Solution by Mathematica

Time used: 0.901 (sec). Leaf size: 66

```
DSolve[x''[t]+x'[t]+x[t]==Exp[2*t]*Cos[t]+t^2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to (t-2)t + \frac{1}{61}e^{2t}(5\sin(t) + 6\cos(t)) + e^{-t/2}\left(c_2\cos\left(\frac{\sqrt{3}t}{2}\right) + c_1\sin\left(\frac{\sqrt{3}t}{2}\right)\right)$$

9.7 problem 1(g)

Internal problem ID [10440]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 1(g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x' + x - t\,\mathrm{e}^{-t} \sin(\pi t) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 108

```
dsolve(diff(x(t),t$2)+diff(x(t),t)+x(t)=t*exp(-t)*sin(Pi*t),x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right) c_2 + \mathrm{e}^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right) c_1 - \frac{\mathrm{e}^{-t}\left(\left(\pi^6 t + (-2t+3)\,\pi^4 + (2t-1)\,\pi^2 - 1 - t\right)\sin(\pi t) - \left((t-2)\,\pi^4 + (-t+4)\,\pi^2 + t\right)\cos(\pi t)\,\pi\right)}{\left(\pi^4 - \pi^2 + 1\right)^2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 117

```
DSolve[x''[t]+x'[t]+x[t]==t*Exp[-t]*Sin[Pi*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-t}\left(\frac{\left(t - \pi^2\left(2t + \pi^2\left(\left(\pi^2 - 2\right)t + 3\right) - 1\right) + 1\right)\sin(\pi t) + \pi\left(-\pi^2(t-4) + \pi^4(t-2) + t\right)\cos(\pi t)}{\left(1 - \pi^2 + \pi^4\right)^2} + e^{t/2}\left(c_2 \cos\left(\frac{\sqrt{3}t}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}t}{2}\right)\right)\right)$$

9.8 problem 1(h)

Internal problem ID [10441]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 1(h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x' + x - (t+2)\sin(\pi t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 106

```
dsolve(diff(x(t),t$2)+diff(x(t),t)+x(t)=(t+2)*sin(Pi*t),x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right) c_2 + \mathrm{e}^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right) c_1 + \frac{((-t-2)\,\pi^6 + (2t+7)\,\pi^4 + (-2t-5)\,\pi^2 + t + 1)\sin(\pi t) - \cos(\pi t)\,\pi((t+4)\,\pi^4 + (-t-6)\,\pi^2 + t + 2}{(\pi^4 - \pi^2 + 1)^2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 114

```
DSolve[x''[t]+x'[t]+x[t]==(t+2)*Sin[Pi*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{-(\pi^2-1)\,(t+\pi^4(t+2)-\pi^2(t+5))\sin(\pi t) + \sin(\pi t) - \pi(t+\pi^4(t+4)-\pi^2(t+6)+2)\cos(\pi t)}{(1-\pi^2+\pi^4)^2} + e^{-t/2}\left(c_2\cos\left(\frac{\sqrt{3}t}{2}\right) + c_1\sin\left(\frac{\sqrt{3}t}{2}\right)\right)$$

9.9 problem 1(i)

Internal problem ID [10442]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 1(i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' + x' + x - 4t - 5\,\mathrm{e}^{-t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(x(t),t$2)+diff(x(t),t)+x(t)=4*t+5*exp(-t),x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right) c_2 + \mathrm{e}^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right) c_1 + 4t - 4 + 5\,\mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.892 (sec). Leaf size: 54

```
DSolve[x''[t]+x'[t]+x[t]==4*t+5*Exp[-t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 4t + 5e^{-t} + e^{-t/2}\left(c_2 \cos\left(\frac{\sqrt{3}t}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}t}{2}\right)\right) - 4$$

9.10 problem 1(j)

Internal problem ID [10443]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 1(j).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x' + x - 5\sin(2t) - e^t t = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 50

```
dsolve(diff(x(t),t$2)+diff(x(t),t)+x(t)=5*sin(2*t)+t*exp(t),x(t), singsol=all)
```

$$x(t) = e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right) c_2 + e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right) c_1 - \frac{10\cos(2t)}{13} - \frac{15\sin(2t)}{13} + \frac{e^t(t-1)}{3}$$

✓ Solution by Mathematica

Time used: 1.831 (sec). Leaf size: 70

```
DSolve[x''[t]+x'[t]+x[t]==5*Sin[2*t]+t*Exp[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{3}e^t(t-1) - \frac{5}{13}(3\sin(2t) + 2\cos(2t)) + e^{-t/2}\left(c_2\cos\left(\frac{\sqrt{3}t}{2}\right) + c_1\sin\left(\frac{\sqrt{3}t}{2}\right)\right)$$

9.11 problem 1(k)

Internal problem ID [10444]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 1(k).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x' + x - t^3 - 1 + 4\cos(t)\, t = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
dsolve(diff(x(t),t$2)+diff(x(t),t)+x(t)=t^3+1-4*t*cos(t),x(t), singsol=all)
```

$$x(t) = e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\, t}{2}\right) c_2 + e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\, t}{2}\right) c_1 + (-4t + 8)\sin(t) + t^3 - 3t^2 - 4\cos(t) + 7$$

✓ Solution by Mathematica

Time used: 1.428 (sec). Leaf size: 62

```
DSolve[x''[t]+x'[t]+x[t]==t^3+1-4*t*Cos[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to (t-3)t^2 - 4(t-2)\sin(t) - 4\cos(t) + e^{-t/2}\left(c_2 \cos\left(\frac{\sqrt{3}t}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}t}{2}\right)\right) + 7$$

9.12 problem 1(L)

Internal problem ID [10445]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 1(L).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x' + x + 6 - 2\,\mathrm{e}^{2t} \sin(t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(x(t),t$2)+diff(x(t),t)+x(t)=-6+2*exp(2*t)*sin(t),x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}\,t}{2}\right) c_2 + \mathrm{e}^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\,t}{2}\right) c_1 - 6 + \frac{2(-5\cos(t) + 6\sin(t))\,\mathrm{e}^{2t}}{61}$$

✓ Solution by Mathematica

Time used: 0.56 (sec). Leaf size: 62

```
DSolve[x''[t]+x'[t]+x[t]==-6+2*Exp[2*t]*Sin[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{2}{61} e^{2t} (5\cos(t) - 6\sin(t)) + e^{-t/2} \left(c_2 \cos\left(\frac{\sqrt{3}t}{2}\right) + c_1 \sin\left(\frac{\sqrt{3}t}{2}\right)\right) - 6$$

9.13 problem 2(a)

Internal problem ID [10446]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 2(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 7x - t\,e^{3t} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve(diff(x(t),t$2)+7*x(t)=t*exp(3*t),x(t), singsol=all)
```

$$x(t) = \sin\left(\sqrt{7}\,t\right) c_2 + \cos\left(\sqrt{7}\,t\right) c_1 + \frac{(8t-3)\,e^{3t}}{128}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 42

```
DSolve[x''[t]+7*x[t]==t*Exp[3*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{128}e^{3t}(8t-3) + c_1\cos\left(\sqrt{7}t\right) + c_2\sin\left(\sqrt{7}t\right)$$

9.14 problem 2(b)

Internal problem ID [10447]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 2(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x'' - x' - 6 - e^{2t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(x(t),t$2)-diff(x(t),t)=6+exp(2*t),x(t), singsol=all)
```

$$x(t) = c_1 e^t + \frac{e^{2t}}{2} - 6t + c_2$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 26

```
DSolve[x''[t]-x'[t]==6+Exp[2*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -6t + \frac{e^{2t}}{2} + c_1 e^t + c_2$$

9.15 problem 2(c)

Internal problem ID [10448]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 2(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' + x - t^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(x(t),t$2)+x(t)=t^2,x(t), singsol=all)
```

$$x(t) = \sin(t) c_2 + c_1 \cos(t) + t^2 - 2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[x''[t]+x[t]==t^2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to t^2 + c_1 \cos(t) + c_2 \sin(t) - 2$$

9.16 problem 2(d)

Internal problem ID [10449]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 2(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' - 3x' - 4x - 2t^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(diff(x(t),t$2)-3*diff(x(t),t)-4*x(t)=2*t^2,x(t), singsol=all)
```

$$x(t) = c_2 e^{-t} + e^{4t} c_1 - \frac{t^2}{2} + \frac{3t}{4} - \frac{13}{16}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 35

```
DSolve[x''[t]-3*x'[t]-4*x[t]==2*t^2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{4}(3 - 2t)t + c_1 e^{-t} + c_2 e^{4t} - \frac{13}{16}$$

9.17 problem 2(e)

Internal problem ID [10450]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 2(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' + x - 9\,\mathrm{e}^{-t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(x(t),t$2)+x(t)=9*exp(-t),x(t), singsol=all)
```

$$x(t) = \sin(t)\, c_2 + c_1 \cos(t) + \frac{9\,\mathrm{e}^{-t}}{2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 25

```
DSolve[x''[t]+x[t]==9*Exp[-t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{9e^{-t}}{2} + c_1 \cos(t) + c_2 \sin(t)$$

9.18 problem 2(g)

Internal problem ID [10451]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 2(g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' - 4x - \cos(2t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(x(t),t$2)-4*x(t)=cos(2*t),x(t), singsol=all)
```

$$x(t) = c_2 e^{-2t} + c_1 e^{2t} - \frac{\cos(2t)}{8}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 30

```
DSolve[x''[t]-4*x[t]==Cos[2*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{1}{8}\cos(2t) + c_1 e^{2t} + c_2 e^{-2t}$$

9.19 problem 2(h)

Internal problem ID [10452]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 2(h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x' + 2x - \sin(2t)\, t = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 51

```
dsolve(diff(x(t),t$2)+diff(x(t),t)+2*x(t)=t*sin(2*t),x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{-\frac{t}{2}} \sin\left(\frac{\sqrt{7}\, t}{2}\right) c_2 + \mathrm{e}^{-\frac{t}{2}} \cos\left(\frac{\sqrt{7}\, t}{2}\right) c_1 + \frac{(-2t-1)\cos(2t)}{8} - \frac{\sin(2t)\,(t-2)}{4}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 67

```
DSolve[x''[t]+x'[t]+2*x[t]==t*Sin[2*t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -\frac{1}{4}(t-2)\sin(2t) - \frac{1}{8}(2t+1)\cos(2t) + e^{-t/2}\left(c_2 \cos\left(\frac{\sqrt{7}t}{2}\right) + c_1 \sin\left(\frac{\sqrt{7}t}{2}\right)\right)$$

9.20 problem 3

Internal problem ID [10453]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' - bx' + x - \sin(2t) = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 135

```
dsolve([diff(x(t),t$2)-b*diff(x(t),t)+x(t)=sin(2*t),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{\left(-\sqrt{b^2-4}\,b^2 - b^3 - 6\sqrt{b^2-4} + 4b\right) \mathrm{e}^{-\frac{\left(-b+\sqrt{b^2-4}\right)t}{2}} + \left(\sqrt{b^2-4}\,b^2 - b^3 + 6\sqrt{b^2-4} + 4b\right) \mathrm{e}^{\frac{\left(b+\sqrt{b^2-4}\right)t}{2}} + 2(b}{4b^4 - 7b^2 - 36}$$

✓ Solution by Mathematica

Time used: 0.268 (sec). Leaf size: 88

```
DSolve[{x''[t]-b*x'[t]+x[t]==Sin[2*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{2e^{\frac{bt}{2}}\left(\frac{(b^2+6)\sinh\left(\frac{1}{2}\sqrt{b^2-4t}\right)}{\sqrt{b^2-4}} - b\cosh\left(\frac{1}{2}\sqrt{b^2-4t}\right)\right) + 2b\cos(2t) - 3\sin(2t)}{4b^2+9}$$

9.21 problem 4

Internal problem ID [10454]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' - 3x' - 40x - 2\,\mathrm{e}^{-t} = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve([diff(x(t),t$2)-3*diff(x(t),t)-40*x(t)=2*exp(-t),x(0) = 0, D(x)(0) = 1],x(t), singsol=
```

$$x(t) = -\frac{\left(-22\,\mathrm{e}^{13t} + 13\,\mathrm{e}^{4t} + 9\right)\mathrm{e}^{-5t}}{234}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 30

```
DSolve[{x''[t]-3*x'[t]-40*x[t]==2*Exp[-t],{x[0]==0,x'[0]==1}},x[t],t,IncludeSingularSolutions
```

$$x(t) \to \frac{1}{234}e^{-5t}\left(-13e^{4t} + 22e^{13t} - 9\right)$$

9.22 problem 5

Internal problem ID [10455]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - 2x' - 4 = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve([diff(x(t),t$2)-2*diff(x(t),t)=4,x(0) = 1, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = e^{2t} - 2t$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 14

```
DSolve[{x''[t]-2*x'[t]==4,{x[0]==1,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{2t} - 2t$$

9.23 problem 6

Internal problem ID [10456]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.1 Nonhomogeneous Equations: Undetermined Coefficients. Exercises page 110
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 2x - \cos\left(\sqrt{2}\, t\right) = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve([diff(x(t),t$2)+2*x(t)=cos(sqrt(2)*t),x(0) = 0, D(x)(0) = 1],x(t), singsol=all)
```

$$x(t) = \frac{\sin\left(\sqrt{2}\, t\right)\sqrt{2}\,(t+2)}{4}$$

✓ Solution by Mathematica

Time used: 0.078 (sec). Leaf size: 25

```
DSolve[{x''[t]+2*x[t]==Cos[Sqrt[2]*t],{x[0]==0,x'[0]==1}},x[t],t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{(t+2)\sin\left(\sqrt{2}t\right)}{2\sqrt{2}}$$

10 Chapter 2, Second order linear equations. Section 2.3.2 Resonance Exercises page 114

10.1 problem 6

Internal problem ID [10457]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.2 Resonance Exercises page 114
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + \frac{x'}{100} + 4x - \cos(2t) = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 27

```
dsolve([diff(x(t),t$2)+1/100*diff(x(t),t)+4*x(t)=cos(2*t),x(0) = 0, D(x)(0) = 0],x(t), singso
```

$$x(t) = -\frac{20000\,\mathrm{e}^{-\frac{t}{200}}\sqrt{159999}\,\sin\left(\frac{\sqrt{159999}\,t}{200}\right)}{159999} + 50\sin(2t)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[{x''[t]+1/100**x'[t]+4*x[t]==Cos[2*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSoluti
```

Not solved

10.2 problem 7(a)

Internal problem ID [10458]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.2 Resonance Exercises page 114
Problem number: 7(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + w^2 x - \cos(\beta t) = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve([diff(x(t),t$2)+w^2*x(t)=cos(beta*t),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{\cos(tw) - \cos(\beta t)}{\beta^2 - w^2}$$

✓ Solution by Mathematica

Time used: 0.11 (sec). Leaf size: 28

```
DSolve[{x''[t]+w^2*x[t]==Cos[\[Beta]*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -
```

$$x(t) \rightarrow \frac{\cos(\beta t) - \cos(tw)}{w^2 - \beta^2}$$

10.3 problem 7(c)

Internal problem ID [10459]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.3.2 Resonance Exercises page 114
Problem number: 7(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 3025x - \cos(45t) = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 17

```
dsolve([diff(x(t),t$2)+(55)^2*x(t)=cos(45*t),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = -\frac{\cos(55t)}{1000} + \frac{\cos(45t)}{1000}$$

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 20

```
DSolve[{x''[t]+55^2*x[t]==Cos[45*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> Tr
```

$$x(t) \to \frac{\cos(45t) - \cos(55t)}{1000}$$

11 Chapter 2, Second order linear equations. Section 2.4.1 Cauchy-Euler equations. Exercises page 120

11.1 problem 1(a)

Internal problem ID [10460]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.1 Cauchy-Euler equations. Exercises page 120
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x'' + \frac{x}{t^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(x(t),t$2)=-1/t^2*x(t),x(t), singsol=all)
```

$$x(t) = c_1\sqrt{t}\,\sin\left(\frac{\sqrt{3}\,\ln(t)}{2}\right) + c_2\cos\left(\frac{\sqrt{3}\,\ln(t)}{2}\right)\sqrt{t}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 42

```
DSolve[x''[t]==-1/t^2*x[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \sqrt{t}\left(c_1\cos\left(\frac{1}{2}\sqrt{3}\log(t)\right) + c_2\sin\left(\frac{1}{2}\sqrt{3}\log(t)\right)\right)$$

11.2 problem 1(b)

Internal problem ID [10461]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.1 Cauchy-Euler equations. Exercises page 120
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x'' - \frac{4x}{t^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(x(t),t$2)=4/t^2*x(t),x(t), singsol=all)
```

$$x(t) = c_1 t^{\frac{1}{2}+\frac{\sqrt{17}}{2}} + c_2 t^{\frac{1}{2}-\frac{\sqrt{17}}{2}}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 34

```
DSolve[x''[t]==4/t^2*x[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to t^{\frac{1}{2}-\frac{\sqrt{17}}{2}}\left(c_2 t^{\sqrt{17}} + c_1\right)$$

11.3 problem 1(c)

Internal problem ID [10462]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.1 Cauchy-Euler equations. Exercises page 120
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{t^2x'' + 3x't + x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(t^2*diff(x(t),t$2)+3*t*diff(x(t),t)+x(t)=0,x(t), singsol=all)
```

$$x(t) = \frac{c_1}{t} + \frac{c_2 \ln(t)}{t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 17

```
DSolve[t^2*x''[t]+3*t*x'[t]+x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{c_2 \log(t) + c_1}{t}$$

11.4 problem 1(d)

Internal problem ID [10463]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.1 Cauchy-Euler equations. Exercises page 120
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$\boxed{tx'' + 4x' + \frac{2x}{t} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(t*diff(x(t),t$2)+4*diff(x(t),t)+2/t*x(t)=0,x(t), singsol=all)
```

$$x(t) = \frac{c_1}{t} + \frac{c_2}{t^2}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[t*x''[t]+4*x'[t]+2/t*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{c_2 t + c_1}{t^2}$$

11.5 problem 1(e)

Internal problem ID [10464]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.1 Cauchy-Euler equations. Exercises page 120
Problem number: 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2x'' - 7x't + 16x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(t^2*diff(x(t),t$2)-7*t*diff(x(t),t)+16*x(t)=0,x(t), singsol=all)
```

$$x(t) = t^4c_1 + c_2t^4 \ln(t)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 18

```
DSolve[t^2*x''[t]-7*t*x'[t]+16*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to t^4(4c_2 \log(t) + c_1)$$

11.6 problem 1(f)

Internal problem ID [10465]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.1 Cauchy-Euler equations. Exercises page 120
Problem number: 1(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$\boxed{t^2x'' + 3x't - 8x = 0}$$

With initial conditions

$$[x(1) = 0, x'(1) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve([t^2*diff(x(t),t$2)+3*t*diff(x(t),t)-8*x(t)=0,x(1) = 0, D(x)(1) = 2],x(t), singsol=all
```

$$x(t) = \frac{t^6 - 1}{3t^4}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 17

```
DSolve[{t^2*x''[t]+3*t*x'[t]-8*x[t]==0,{x[1]==0,x'[1]==2}},x[t],t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{t^6 - 1}{3t^4}$$

11.7 problem 1(g)

Internal problem ID [10466]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.1 Cauchy-Euler equations. Exercises page 120
Problem number: 1(g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{t^2x'' + x't = 0}$$

With initial conditions

$$[x(1) = 0, x'(1) = 2]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 8

```
dsolve([t^2*diff(x(t),t$2)+t*diff(x(t),t)=0,x(1) = 0, D(x)(1) = 2],x(t), singsol=all)
```

$$x(t) = 2\ln(t)$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 9

```
DSolve[{t^2*x''[t]+t*x'[t]==0,{x[1]==0,x'[1]==2}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 2\log(t)$$

11.8 problem 1(h)

Internal problem ID [10467]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.1 Cauchy-Euler equations. Exercises page 120
Problem number: 1(h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{t^2x'' - x't + 2x = 0}$$

With initial conditions

$$[x(1) = 0, x'(1) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve([t^2*diff(x(t),t$2)-t*diff(x(t),t)+2*x(t)=0,x(1) = 0, D(x)(1) = 1],x(t), singsol=all)
```

$$x(t) = t\sin(\ln(t))$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 10

```
DSolve[{t^2*x''[t]-t*x'[t]+2*x[t]==0,{x[1]==0,x'[1]==1}},x[t],t,IncludeSingularSolutions -> T
```

$$x(t) \to t\sin(\log(t))$$

11.9 problem 2

Internal problem ID [10468]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.1 Cauchy-Euler equations. Exercises page 120
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x'' + t^2 x' = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 1]$$

✓ Solution by Maple

Time used: 2.625 (sec). Leaf size: 65

```
dsolve([diff(x(t),t$2)+t^2*diff(x(t),t)=0,x(0) = 0, D(x)(0) = 1],x(t), singsol=all)
```

$$x(t) = \frac{\mathrm{e}^{-\frac{t^3}{3}} \sqrt{t}\left(4\,3^{\frac{5}{6}}(t^3)^{\frac{1}{6}} + 9\,\mathrm{WhittakerM}\left(\frac{1}{6}, \frac{2}{3}, \frac{t^3}{3}\right)\mathrm{e}^{\frac{t^3}{6}}\right) 3^{\frac{1}{6}} \left(\begin{cases} -\frac{1}{i\sqrt{3}-1} & t < 0 \\ \frac{1}{2} & 0 \leq t \end{cases}\right)}{6}$$

✓ Solution by Mathematica

Time used: 0.09 (sec). Leaf size: 32

```
DSolve[{x''[t]+t^2*x'[t]==0,{x[0]==0,x'[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{\mathrm{Gamma}\left(\frac{1}{3}\right)}{3^{2/3}} - \frac{1}{3} t\, \mathrm{ExpIntegralE}\left(\frac{2}{3}, \frac{t^3}{3}\right)$$

12 Chapter 2, Second order linear equations. Section 2.4.2 Variation of parameters. Exercises page 124

12.1 problem 1(a)

Internal problem ID [10469]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.2 Variation of parameters. Exercises page 124
Problem number: 1(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x - \tan(t) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(x(t),t$2)+x(t)=tan(t),x(t), singsol=all)
```

$$x(t) = \sin(t)\, c_2 + c_1 \cos(t) - \cos(t) \ln(\sec(t) + \tan(t))$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 22

```
DSolve[x''[t]+x[t]==Tan[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \cos(t)(-\operatorname{arctanh}(\sin(t)) + c_1) + c_2 \sin(t)$$

12.2 problem 1(b)

Internal problem ID [10470]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.2 Variation of parameters. Exercises page 124
Problem number: 1(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type **[[_2nd_order, _linear, _nonhomogeneous]]**

$$\boxed{x'' - x - e^t t = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(x(t),t$2)-x(t)=t*exp(t),x(t), singsol=all)
```

$$x(t) = c_2 e^{-t} + c_1 e^t + \frac{(t-1)\, e^t t}{4}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 33

```
DSolve[x''[t]-x[t]==t*Exp[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{8} e^t (2(t-1)t + 1 + 8c_1) + c_2 e^{-t}$$

12.3 problem 1(c)

Internal problem ID [10471]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.2 Variation of parameters. Exercises page 124
Problem number: 1(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' - x - \frac{1}{t} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(x(t),t$2)-x(t)=1/t,x(t), singsol=all)
```

$$x(t) = c_2 \mathrm{e}^{-t} + c_1 \mathrm{e}^{t} + \frac{\mathrm{Ei}_1(-t)\,\mathrm{e}^{-t}}{2} - \frac{\mathrm{Ei}_1(t)\,\mathrm{e}^{t}}{2}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 38

```
DSolve[x''[t]-x[t]==1/t,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2} e^{-t} \left(-\operatorname{ExpIntegralEi}(t) + e^{2t} (\operatorname{ExpIntegralEi}(-t) + 2c_1) + 2c_2\right)$$

12.4 problem 1(d)

Internal problem ID [10472]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.2 Variation of parameters. Exercises page 124
Problem number: 1(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{t^2x'' - 2x - t^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(t^2*diff(x(t),t$2)-2*x(t)=t^3,x(t), singsol=all)
```

$$x(t) = c_2t^2 + \frac{t^3}{4} + \frac{c_1}{t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 25

```
DSolve[t^2*x''[t]-2*x[t]==t^3,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{t^3}{4} + c_2t^2 + \frac{c_1}{t}$$

12.5 problem 1(e)

Internal problem ID [10473]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.2 Variation of parameters. Exercises page 124
Problem number: 1(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x - \frac{1}{1+t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(x(t),t$2)+x(t)=1/(1+t),x(t), singsol=all)
```

$$x(t) = \sin(t) c_2 + c_1 \cos(t) - \mathrm{Si}(t+1) \cos(t+1) + \mathrm{Ci}(t+1) \sin(t+1)$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 35

```
DSolve[x''[t]+x[t]==1/(1+t),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \mathrm{CosIntegral}(t+1) \sin(t+1) - \mathrm{Si}(t+1) \cos(t+1) + c_1 \cos(t) + c_2 \sin(t)$$

12.6 problem 1(f)

Internal problem ID [10474]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.2 Variation of parameters. Exercises page 124
Problem number: 1(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' - 2x' + x - \frac{e^t}{2t} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(x(t),t$2)-2*diff(x(t),t)+x(t)=1/(2*t)*exp(t),x(t), singsol=all)
```

$$x(t) = c_2 e^t + e^t t c_1 + \frac{e^t t(-1 + \ln(t))}{2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 29

```
DSolve[x''[t]-2*x'[t]+x[t]==1/(2*t)*Exp[t],x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2} e^t (t \log(t) + (-1 + 2c_2)t + 2c_1)$$

12.7 problem 2

Internal problem ID [10475]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.2 Variation of parameters. Exercises page 124
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{x'' + \frac{x'}{t} - a = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(x(t),t$2)+1/t*diff(x(t),t)=a,x(t), singsol=all)
```

$$x(t) = \frac{t^2 a}{4} + \ln(t) c_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 21

```
DSolve[x''[t]+1/t*x'[t]==a,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{at^2}{4} + c_1 \log(t) + c_2$$

12.8 problem 3

Internal problem ID [10476]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.2 Variation of parameters. Exercises page 124
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2x'' - 3x't + 3x - 4t^7 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(t^2*diff(x(t),t$2)-3*t*diff(x(t),t)+3*x(t)=4*t^7,x(t), singsol=all)
```

$$x(t) = \left(\frac{1}{6}t^6 + \frac{1}{2}c_1t^2 + c_2\right)t$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 23

```
DSolve[t^2*x''[t]-3*t*x'[t]+3*x[t]==4*t^7,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{t^7}{6} + c_2t^3 + c_1t$$

12.9 problem 7

Internal problem ID [10477]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.2 Variation of parameters. Exercises page 124
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' - x - \frac{\mathrm{e}^t}{1 + \mathrm{e}^t} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 39

```
dsolve(diff(x(t),t$2)-x(t)=exp(t)/(1+exp(t)),x(t), singsol=all)
```

$$x(t) = c_2\mathrm{e}^{-t} + c_1\mathrm{e}^t + \frac{(-\mathrm{e}^t + \mathrm{e}^{-t})\ln(1 + \mathrm{e}^t)}{2} + \frac{\mathrm{e}^t\ln(\mathrm{e}^t)}{2} - \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 46

```
DSolve[x''[t]-x[t]==Exp[t]/(1+Exp[t]),x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2}\left(-2e^t\left(\operatorname{arctanh}\left(2e^t + 1\right) - c_1\right) + e^{-t}\left(\log\left(e^t + 1\right) + 2c_2\right) - 1\right)$$

13 Chapter 2, Second order linear equations. Section 2.4.3 Reduction of order. Exercises page 125

13.1 problem 1

Internal problem ID [10478]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.3 Reduction of order. Exercises page 125
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x'' + x't + x = 0}$$

Given that one solution of the ode is

$$x_1 = \mathrm{e}^{-\frac{t^2}{2}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve([diff(x(t),t$2)+t*diff(x(t),t)+x(t)=0,exp(-t^2/2)],x(t), singsol=all)
```

$$x(t) = \operatorname{erf}\left(\frac{i\sqrt{2}\,t}{2}\right)\mathrm{e}^{-\frac{t^2}{2}}c_1 + c_2\mathrm{e}^{-\frac{t^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 34

```
DSolve[x''[t]+t*x'[t]+x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \sqrt{2}c_1 \operatorname{DawsonF}\left(\frac{t}{\sqrt{2}}\right) + c_2e^{-\frac{t^2}{2}}$$

13.2 problem 2

Internal problem ID [10479]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.3 Reduction of order. Exercises page 125
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{x'' - x't + x = 0}$$

Given that one solution of the ode is

$$x_1 = t$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve([diff(x(t),t$2)-t*diff(x(t),t)+x(t)=0,t],x(t), singsol=all)
```

$$x(t) = c_1 t + c_2\left(i\sqrt{2}\,\sqrt{\pi}\,\mathrm{e}^{\frac{t^2}{2}} - \pi\ \mathrm{erf}\left(\frac{i\sqrt{2}\,t}{2}\right)t\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 47

```
DSolve[x''[t]-t*x'[t]+x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{t\left(2c_1 - \sqrt{\pi}c_2\mathrm{erfi}\left(\frac{t}{\sqrt{2}}\right)\right)}{\sqrt{2}} + c_2 e^{\frac{t^2}{2}}$$

13.3 problem 4

Internal problem ID [10480]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.3 Reduction of order. Exercises page 125
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - 2ax' + a^2x = 0}$$

Given that one solution of the ode is

$$x_1 = \mathrm{e}^{at}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve([diff(x(t),t$2)-2*a*diff(x(t),t)+a^2*x(t)=0,exp(a*t)],x(t), singsol=all)
```

$$x(t) = \mathrm{e}^{at}c_1 + c_2\mathrm{e}^{at}t$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 18

```
DSolve[x''[t]-2*a*x'[t]+a^2*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{at}(c_2t + c_1)$$

13.4 problem 5

Internal problem ID [10481]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.3 Reduction of order. Exercises page 125
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' - \frac{(t+2)\,x'}{t} + \frac{(t+2)\,x}{t^2} = 0}$$

Given that one solution of the ode is

$$x_1 = t$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve([diff(x(t),t$2)-(t+2)/t*diff(x(t),t)+(t+2)/t^2*x(t)=0,t],x(t), singsol=all)
```

$$x(t) = c_1 t + c_2 e^t t$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 16

```
DSolve[x''[t]-(t+2)/t*x'[t]+(t+2)/t^2*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to t\left(c_2 e^t + c_1\right)$$

13.5 problem 6

Internal problem ID [10482]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.4.3 Reduction of order. Exercises page 125
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{t^2x'' + x't + \left(t^2 - \frac{1}{4}\right)x = 0}$$

Given that one solution of the ode is

$$x_1 = \frac{\cos(t)}{\sqrt{t}}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 19

```
dsolve([t^2*diff(x(t),t$2)+t*diff(x(t),t)+(t^2-1/4)*x(t)=0,cos(t)/sqrt(t)],x(t), singsol=all)
```

$$x(t) = \frac{c_1 \sin(t)}{\sqrt{t}} + \frac{c_2 \cos(t)}{\sqrt{t}}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 39

```
DSolve[t^2*x''[t]+t*x'[t]+(t^2-1/4)*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{e^{-it}(2c_1 - ic_2e^{2it})}{2\sqrt{t}}$$

14 Chapter 2, Second order linear equations. Section 2.5 Higher order equations. Exercises page 130

14.1 problem 1(a)

Internal problem ID [10483]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.5 Higher order equations. Exercises page 130
Problem number: 1(a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{x''' + x' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(x(t),t$3)+diff(x(t),t)=0,x(t), singsol=all)
```

$$x(t) = c_1 + \sin(t)\, c_2 + c_3 \cos(t)$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 19

```
DSolve[x'''[t]+x'[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to -c_2 \cos(t) + c_1 \sin(t) + c_3$$

14.2 problem 1(b)

Internal problem ID [10484]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.5 Higher order equations. Exercises page 130
Problem number: 1(b).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{x''' + x' - 1 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(x(t),t$3)+diff(x(t),t)=1,x(t), singsol=all)
```

$$x(t) = c_1 \sin(t) - c_2 \cos(t) + t + c_3$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 20

```
DSolve[x'''[t]+x'[t]==1,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to t - c_2 \cos(t) + c_1 \sin(t) + c_3$$

14.3 problem 1(c)

Internal problem ID [10485]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.5 Higher order equations. Exercises page 130
Problem number: 1(c).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{x''' + x'' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(diff(x(t),t$3)+diff(x(t),t$2)=0,x(t), singsol=all)
```

$$x(t) = c_1 + tc_2 + c_3 e^{-t}$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 20

```
DSolve[x'''[t]+x''[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{-t} + c_3 t + c_2$$

14.4 problem 1(d)

Internal problem ID [10486]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.5 Higher order equations. Exercises page 130
Problem number: 1(d).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{x''' - x' - 8x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 150

```
dsolve(diff(x(t),t$3)-diff(x(t),t)-8*x(t)=0,x(t), singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{\frac{\left(\left(108+3\sqrt{1293}\right)^{\frac{2}{3}}+3\right)t}{3\left(108+3\sqrt{1293}\right)^{\frac{1}{3}}}} - c_2 \mathrm{e}^{\frac{\left(-\frac{\left(108+3\sqrt{1293}\right)^{\frac{2}{3}}}{6}-\frac{1}{2}\right)t}{\left(108+3\sqrt{1293}\right)^{\frac{1}{3}}}} \sin\left(\frac{\left(\left(108+3\sqrt{1293}\right)^{\frac{2}{3}}\sqrt{3}-3\sqrt{3}\right)t}{6\left(108+3\sqrt{1293}\right)^{\frac{1}{3}}}\right) + c_3 \mathrm{e}^{\frac{\left(-\frac{\left(108+3\sqrt{1293}\right)^{\frac{2}{3}}}{6}-\frac{1}{2}\right)t}{\left(108+3\sqrt{1293}\right)^{\frac{1}{3}}}} \cos\left(\frac{\left(\left(108+3\sqrt{1293}\right)^{\frac{2}{3}}\sqrt{3}-3\sqrt{3}\right)t}{6\left(108+3\sqrt{1293}\right)^{\frac{1}{3}}}\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 69

```
DSolve[x'''[t]-x'[t]-8*x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_2 \exp\left(t\text{Root}\left[\#1^3 - \#1 - 8\&, 2\right]\right) + c_3 \exp\left(t\text{Root}\left[\#1^3 - \#1 - 8\&, 3\right]\right) + c_1 \exp\left(t\text{Root}\left[\#1^3 - \#1 - 8\&, 1\right]\right)$$

14.5 problem 1(e)

Internal problem ID [10487]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.5 Higher order equations. Exercises page 130
Problem number: 1(e).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{x''' + x'' - 2\,e^{t} - 3t^{2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(x(t),t$3)+diff(x(t),t$2)=2*exp(t)+3*t^2,x(t), singsol=all)
```

$$x(t) = \frac{t^{4}}{4} + 3t^{2} - t^{3} + e^{-t}c_{1} + e^{t} + tc_{2} + c_{3}$$

✓ Solution by Mathematica

Time used: 0.19 (sec). Leaf size: 37

```
DSolve[x'''[t]+x''[t]==2*Exp[t]+3*t^2,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{4}((t-4)t+12)t^{2} + e^{t} + c_{3}t + c_{1}e^{-t} + c_{2}$$

14.6 problem 1(f)

Internal problem ID [10488]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.5 Higher order equations. Exercises page 130
Problem number: 1(f).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{x''' - 8x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(x(t),t$3)-8*x(t)=0,x(t), singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{2t} + c_2 \mathrm{e}^{-t} \sin\left(\sqrt{3}\,t\right) + c_3 \mathrm{e}^{-t} \cos\left(\sqrt{3}\,t\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 49

```
DSolve[x'''[t]-x[t]==0,x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^t + e^{-t/2}\left(c_2 \cos\left(\frac{\sqrt{3}t}{2}\right) + c_3 \sin\left(\frac{\sqrt{3}t}{2}\right)\right)$$

14.7 problem 2

Internal problem ID [10489]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 2, Second order linear equations. Section 2.5 Higher order equations. Exercises page 130
Problem number: 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{x''' + x'' - x' - 4x = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = 0, x''(0) = -1]$$

✓ Solution by Maple

Time used: 0.516 (sec). Leaf size: 296

```
dsolve([diff(x(t),t$3)+diff(x(t),t$2)-diff(x(t),t)-4*x(t)=0,x(0) = 1, D(x)(0) = 0, (D@@2)(x)(
```

$$x(t) = \frac{\left(\left(\left(32\sqrt{113}+352\right)\left(388+36\sqrt{113}\right)^{\frac{1}{3}} + \left(-\sqrt{113}-25\right)\left(388+36\sqrt{113}\right)^{\frac{2}{3}} + 776\sqrt{113}+8136\right)\cos\left(\frac{\sqrt{3}\,(}{}\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 748

```
DSolve[{x'''[t]+x''[t]-x'[t]-4*x[t]==0,{x[0]==1,x'[0]==0,x''[0]==-1}},x[t],t,IncludeSingularS
```

$$x(t) \rightarrow \frac{\text{Root}\left[\#1^3+\#1^2-\#1-4\&,1\right]\exp\left(t\text{Root}\left[\#1^3+\#1^2-\#1-4\&,2\right]\right) - \text{Root}\left[\#1^3+\#1^2-\#1-4\&\right.}{}$$

15 Chapter 3, Laplace transform. Section 3.2.1 Initial value problems. Exercises page 156

15.1 problem 6(a)

Internal problem ID [10490]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.2.1 Initial value problems. Exercises page 156
Problem number: 6(a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{x' + 5x - \text{Heaviside}\,(t-2) = 0}$$

With initial conditions

$$[x(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve([diff(x(t),t)+5*x(t)=Heaviside(t-2),x(0) = 1],x(t), singsol=all)
```

$$x(t) = \frac{\text{Heaviside}\,(t-2)}{5} - \frac{\text{Heaviside}\,(t-2)\,\mathrm{e}^{-5t+10}}{5} + \mathrm{e}^{-5t}$$

✓ Solution by Mathematica

Time used: 0.053 (sec). Leaf size: 34

```
DSolve[{x'[t]+5*x[t]==UnitStep[t-2],{x[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \begin{cases} e^{-5t} & t \leq 2 \\ \frac{1}{5} - \frac{1}{5}e^{-5t}(-5+e^{10}) & \text{True} \end{cases}$$

15.2 problem 6(b)

Internal problem ID [10491]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.2.1 Initial value problems. Exercises page 156
Problem number: 6(b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{x' + x - \sin(2t) = 0}$$

With initial conditions

$$[x(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve([diff(x(t),t)+x(t)=sin(2*t),x(0) = 0],x(t), singsol=all)
```

$$x(t) = -\frac{2\cos(2t)}{5} + \frac{\sin(2t)}{5} + \frac{2\,\mathrm{e}^{-t}}{5}$$

✓ Solution by Mathematica

Time used: 0.096 (sec). Leaf size: 27

```
DSolve[{x'[t]+x[t]==Sin[2*t],{x[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{5}\left(2e^{-t} + \sin(2t) - 2\cos(2t)\right)$$

15.3 problem 6(c)

Internal problem ID [10492]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.2.1 Initial value problems. Exercises page 156
Problem number: 6(c).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - x' - 6x = 0}$$

With initial conditions

$$[x(0) = 2, x'(0) = -1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve([diff(x(t),t$2)-diff(x(t),t)-6*x(t)=0,x(0) = 2, D(x)(0) = -1],x(t), singsol=all)
```

$$x(t) = \frac{(3\,\mathrm{e}^{5t} + 7)\,\mathrm{e}^{-2t}}{5}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 23

```
DSolve[{x''[t]-x'[t]-6*x[t]==0,{x[0]==2,x'[0]==-1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{5}e^{-2t}\left(3e^{5t} + 7\right)$$

15.4 problem 6(d)

Internal problem ID [10493]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.2.1 Initial value problems. Exercises page 156
Problem number: 6(d).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - 2x' + 2x = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 1]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve([diff(x(t),t$2)-2*diff(x(t),t)+2*x(t)=0,x(0) = 0, D(x)(0) = 1],x(t), singsol=all)
```

$$x(t) = e^t \sin(t)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 11

```
DSolve[{x''[t]-2*x'[t]+2*x[t]==0,{x[0]==0,x'[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^t \sin(t)$$

15.5 problem 6(e)

Internal problem ID [10494]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.2.1 Initial value problems. Exercises page 156
Problem number: 6(e).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' - 2x' + 2x - \mathrm{e}^{-t} = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 1]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve([diff(x(t),t$2)-2*diff(x(t),t)+2*x(t)=exp(-t),x(0) = 0, D(x)(0) = 1],x(t), singsol=all
```

$$x(t) = \frac{\mathrm{e}^{-t}}{5} + \frac{(-\cos(t) + 7\sin(t))\,\mathrm{e}^{t}}{5}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 27

```
DSolve[{x''[t]-2*x'[t]+2*x[t]==Exp[-t],{x[0]==0,x'[0]==1}},x[t],t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{5}\left(e^{-t} - e^{t}(\cos(t) - 7\sin(t))\right)$$

15.6 problem 6(f)

Internal problem ID [10495]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.2.1 Initial value problems. Exercises page 156
Problem number: 6(f).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - x' = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 5

```
dsolve([diff(x(t),t$2)-diff(x(t),t)=0,x(0) = 1, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = 1$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 6

```
DSolve[{x''[t]-x'[t]==0,{x[0]==1,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to 1$$

15.7 problem 6(g)

Internal problem ID [10496]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.2.1 Initial value problems. Exercises page 156
Problem number: 6(g).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + \frac{2x'}{5} + 2x - 1 + \operatorname{Heaviside}(t-5) = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 58

```
dsolve([diff(x(t),t$2)+4/10*diff(x(t),t)+2*x(t)=1-Heaviside(t-5),x(0) = 0, D(x)(0) = 0],x(t),
```

$$x(t) = \frac{\operatorname{Heaviside}(t-5)\,\mathrm{e}^{-\frac{t}{5}+1}\left(\frac{\sin\left(-7+\frac{7t}{5}\right)}{7} + \cos\left(-7+\frac{7t}{5}\right)\right)}{2} + \frac{\left(-7\cos\left(\frac{7t}{5}\right) - \sin\left(\frac{7t}{5}\right)\right)\mathrm{e}^{-\frac{t}{5}}}{14} - \frac{\operatorname{Heaviside}(t-5)}{2} + \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 90

```
DSolve[{x''[t]+4/10*x'[t]+2*x[t]==1-UnitStep[t-5],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularS
```

$$x(t) \rightarrow \begin{cases} \frac{1}{14}e^{-t/5}\left((-7+7e\cos(7) - e\sin(7))\cos\left(\frac{7t}{5}\right) + (-1+e(\cos(7)+7\sin(7)))\sin\left(\frac{7t}{5}\right)\right) & t > 5 \\ \frac{1}{2} - \frac{1}{14}e^{-t/5}\left(7\cos\left(\frac{7t}{5}\right) + \sin\left(\frac{7t}{5}\right)\right) & \text{True} \end{cases}$$

15.8 problem 6(h)

Internal problem ID [10497]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.2.1 Initial value problems. Exercises page 156
Problem number: 6(h).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 9x - \sin(3t) = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(x(t),t$2)+9*x(t)=sin(3*t),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{\sin(3t)}{18} - \frac{\cos(3t)\,t}{6}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 21

```
DSolve[{x''[t]+9*x[t]==Sin[3*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{18}(\sin(3t) - 3t\cos(3t))$$

15.9 problem 6(i)

Internal problem ID [10498]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.2.1 Initial value problems. Exercises page 156
Problem number: 6(i).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{x'' - 2x - 1 = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 23

```
dsolve([diff(x(t),t$2)-2*x(t)=1,x(0) = 1, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{3\,e^{\sqrt{2}\,t}}{4} + \frac{3\,e^{-\sqrt{2}\,t}}{4} - \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 21

```
DSolve[{x''[t]-2*x[t]==1,{x[0]==1,x'[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2}\left(3\cosh\left(\sqrt{2}t\right) - 1\right)$$

15.10 problem 6(j)

Internal problem ID [10499]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.2.1 Initial value problems. Exercises page 156
Problem number: 6(j).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{x' - 2x - \text{Heaviside}\,(-1+t) = 0}$$

With initial conditions

$$[x(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve([diff(x(t),t)=2*x(t)+Heaviside(t-1),x(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{\text{Heaviside}\,(t-1)\,(-1+\mathrm{e}^{2t-2})}{2}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 25

```
DSolve[{x'[t]==2*x[t]+UnitStep[t-1],{x[0]==0}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \begin{cases} \frac{1}{2}(-1+e^{2t-2}) & t>1 \\ 0 & \text{True} \end{cases}$$

15.11 problem 11

Internal problem ID [10500]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.2.1 Initial value problems. Exercises page 156
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{x' + 4x - \cos(2t)\,\text{Heaviside}(2\pi - t) = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✗ Solution by Maple

```
dsolve([diff(x(t),t)+4*x(t)=cos(2*t)*Heaviside(2*Pi-t),x(0) = 0, D(x)(0) = 0],x(t), singsol=a
```

No solution found

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 28

```
DSolve[{x''[t]+4*x[t]==Cos[2*t]*UnitStep[2*Pi-t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSo
```

$$x(t) \to \begin{cases} \pi\cos(t)\sin(t) & t > 2\pi \\ \frac{1}{2}t\cos(t)\sin(t) & \text{True} \end{cases}$$

15.12 problem 12

Internal problem ID [10501]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.2.1 Initial value problems. Exercises page 156
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{x' - x + 2\,\mathrm{Heaviside}\,(-1+t) = 0}$$

With initial conditions

$$[x(0) = 1]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 20

```
dsolve([diff(x(t),t)=x(t)-2*Heaviside(t-1),x(0) = 1],x(t), singsol=all)
```

$$x(t) = \left(-2\,\mathrm{e}^{t-1} + 2\right)\mathrm{Heaviside}\,(t-1) + \mathrm{e}^{t}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 25

```
DSolve[{x'[t]==x[t]-2*UnitStep[t-1],{x[0]==1}},x[t],t,IncludeSingularSolutions -> True]
```

$$x(t) \to \begin{cases} e^{t} & t \leq 1 \\ 2 + (-2+e)e^{t-1} & \text{True} \end{cases}$$

15.13 problem 14

Internal problem ID [10502]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.2.1 Initial value problems. Exercises page 156
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{x' + x - \text{Heaviside}\,(-1+t) + \text{Heaviside}\,(t-2) = 0}$$

With initial conditions

$$[x(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve([diff(x(t),t)=-x(t)+Heaviside(t-1)-Heaviside(t-2),x(0) = 1],x(t), singsol=all)
```

$$x(t) = \text{Heaviside}\,(t-2)\,\mathrm{e}^{2-t} - \text{Heaviside}\,(t-2) - \text{Heaviside}\,(t-1)\,\mathrm{e}^{1-t} + \text{Heaviside}\,(t-1) + \mathrm{e}^{-t}$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 45

```
DSolve[{x'[t]==-x[t]+UnitStep[t-1]-UnitStep[t-2],{x[0]==1}},x[t],t,IncludeSingularSolutions -
```

$$x(t) \to \begin{cases} e^{-t} & t \leq 1 \\ e^{-t}(1+(-1+e)e) & t > 2 \\ 1-(-1+e)e^{-t} & \text{True} \end{cases}$$

15.14 problem 15

Internal problem ID [10503]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.2.1 Initial value problems. Exercises page 156
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + \pi^2 x - \pi^2 \,\mathrm{Heaviside}\,(1-t) = 0}$$

With initial conditions

$$[x(0) = 1, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve([diff(x(t),t$2)+Pi^2*x(t)=Pi^2*Heaviside(1-t),x(0) = 1, D(x)(0) = 0],x(t), singsol=all
```

$$x(t) = 1 + (-\cos(\pi t) - 1)\,\mathrm{Heaviside}\,(t-1)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 18

```
DSolve[{x''[t]+Pi^2*x[t]==Pi^2*UnitStep[1-t],{x[0]==1,x'[0]==0}},x[t],t,IncludeSingularSoluti
```

$$x(t) \to \begin{cases} 1 & t \leq 1 \\ -\cos(\pi t) & \text{True} \end{cases}$$

16 Chapter 3, Laplace transform. Section 3.3 The convolution property. Exercises page 162

16.1 problem 7

Internal problem ID [10504]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.3 The convolution property. Exercises page 162
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' - 4x - 1 + \text{Heaviside}\,(-1 + t) = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 48

```
dsolve([diff(x(t),t$2)-4*x(t)=1-Heaviside(t-1),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \frac{\mathrm{e}^{-2t}}{8} + \frac{\mathrm{e}^{2t}}{8} + \frac{\text{Heaviside}\,(t-1)}{4} - \frac{\text{Heaviside}\,(t-1)\,\mathrm{e}^{-2t+2}}{8} - \frac{1}{4} - \frac{\text{Heaviside}\,(t-1)\,\mathrm{e}^{2t-2}}{8}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 36

```
DSolve[{x''[t]-4*x[t]==1-UnitStep[t-1],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions ->
```

$$x(t) \to \begin{cases} \frac{\sinh^2(t)}{2} & t \leq 1 \\ \frac{1}{4}(\cosh(2t) - \cosh(2-2t)) & \text{True} \end{cases}$$

16.2 problem 8

Internal problem ID [10505]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.3 The convolution property. Exercises page 162
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x'' + 3x' + 2x - e^{-4t} = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve([diff(x(t),t$2)+3*diff(x(t),t)+2*x(t)=exp(-4*t),x(0) = 0, D(x)(0) = 0],x(t), singsol=a
```

$$x(t) = \frac{(e^{-3t} - 3\,e^{-t} + 2)\,e^{-t}}{6}$$

✓ Solution by Mathematica

Time used: 0.014 (sec). Leaf size: 28

```
DSolve[{x''[t]+3*x'[t]+2*x[t]==Exp[-4*t],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions
```

$$x(t) \to \frac{1}{6}e^{-4t}(e^t - 1)^2 (2e^t + 1)$$

17 Chapter 3, Laplace transform. Section 3.4 Impulsive sources. Exercises page 173

17.1 problem 2

Internal problem ID [10506]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.4 Impulsive sources. Exercises page 173
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{x' + 3x - (\delta(-1+t)) - \text{Heaviside}(t-4) = 0}$$

With initial conditions

$$[x(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve([diff(x(t),t)+3*x(t)=Dirac(t-1)+Heaviside(t-4),x(0) = 1],x(t), singsol=all)
```

$$x(t) = \text{Heaviside}(t-1)\,\mathrm{e}^{-3t+3} + \frac{\text{Heaviside}(t-4)}{3} - \frac{\text{Heaviside}(t-4)\,\mathrm{e}^{-3t+12}}{3} + \mathrm{e}^{-3t}$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 50

```
DSolve[{x'[t]+3*x[t]==DiracDelta[t-1]+UnitStep[t-4],{x[0]==1}},x[t],t,IncludeSingularSolution
```

$$x(t) \to \begin{cases} e^{-3t}(e^3\theta(t-1)+1) & t \leq 4 \\ \frac{1}{3} - \frac{1}{3}e^{-3t}(-3-3e^3+e^{12}) & \text{True} \end{cases}$$

17.2 problem 3

Internal problem ID [10507]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.4 Impulsive sources. Exercises page 173
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' - x - (\delta(t-5)) = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve([diff(x(t),t$2)-x(t)=Dirac(t-5),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = -\frac{\text{Heaviside}\,(t-5)\left(-\mathrm{e}^{-5+2t} + \mathrm{e}^{5}\right)\mathrm{e}^{-t}}{2}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 17

```
DSolve[{x''[t]-x[t]==DiracDelta[t-5],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> T
```

$$x(t) \to -\theta(t-5)\sinh(5-t)$$

17.3 problem 4

Internal problem ID [10508]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.4 Impulsive sources. Exercises page 173
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x - (\delta(t-2)) = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve([diff(x(t),t$2)+x(t)=Dirac(t-2),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = \text{Heaviside}\,(t-2)\sin\,(t-2)$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 17

```
DSolve[{x''[t]+x[t]==DiracDelta[t-2],{x[0]==0,x'[0]==0}},x[t],t,IncludeSingularSolutions -> T
```

$$x(t) \to -\theta(t-2)\sin(2-t)$$

17.4 problem 6

Internal problem ID [10509]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.4 Impulsive sources. Exercises page 173
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 4x - (\delta(t-2)) + \delta(t-5) = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve([diff(x(t),t$2)+4*x(t)=Dirac(t-2)-Dirac(t-5),x(0) = 0, D(x)(0) = 0],x(t), singsol=all)
```

$$x(t) = -\frac{\text{Heaviside}(t-5)\sin(2t-10)}{2} + \frac{\text{Heaviside}(t-2)\sin(2t-4)}{2}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 33

```
DSolve[{x''[t]+4*x[t]==DiracDelta[t-2]-DiracDelta[t-5],{x[0]==0,x'[0]==0}},x[t],t,IncludeSing
```

$$x(t) \to \frac{1}{2}(\theta(t-5)\sin(10-2t) - \theta(t-2)\sin(4-2t))$$

17.5 problem 7

Internal problem ID [10510]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.4 Impulsive sources. Exercises page 173
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + x - 3(\delta(-2\pi + t)) = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 1]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve([diff(x(t),t$2)+x(t)=3*Dirac(t-2*Pi),x(0) = 0, D(x)(0) = 1],x(t), singsol=all)
```

$$x(t) = \sin(t)(3\,\mathrm{Heaviside}(-2\pi + t) + 1)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 18

```
DSolve[{x''[t]+x[t]==3*DiracDelta[t-2*Pi],{x[0]==0,x'[0]==1}},x[t],t,IncludeSingularSolutions
```

$$x(t) \to 3\theta(t - 2\pi)\sin(t) + \sin(t)$$

17.6 problem 9

Internal problem ID [10511]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.4 Impulsive sources. Exercises page 173
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y' + y - (\delta(-1 + t)) = 0}$$

With initial conditions

$$[y(0) = 0, y'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve([diff(y(t),t$2)+diff(y(t),t)+y(t)=Dirac(t-1),y(0) = 0, D(y)(0) = 0],y(t), singsol=all)
```

$$y(t) = \frac{2\sqrt{3}\,\mathrm{Heaviside}\,(t-1)\,\mathrm{e}^{\frac{1}{2}-\frac{t}{2}}\sin\left(\frac{\sqrt{3}\,(t-1)}{2}\right)}{3}$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 40

```
DSolve[{y''[t]+y'[t]+y[t]==DiracDelta[t-1],{y[0]==0,y'[0]==0}},y[t],t,IncludeSingularSolution
```

$$y(t) \to \frac{2e^{\frac{1}{2}-\frac{t}{2}}\theta(t-1)\sin\left(\frac{1}{2}\sqrt{3}(t-1)\right)}{\sqrt{3}}$$

17.7 problem 10

Internal problem ID [10512]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 3, Laplace transform. Section 3.4 Impulsive sources. Exercises page 173
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x'' + 4x - \frac{(t-5)\,\text{Heaviside}\,(t-5)}{5} - \left(2 - \frac{t}{5}\right)\text{Heaviside}\,(t-10) = 0}$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve([diff(x(t),t$2)+4*x(t)=1/5*(t-5)*Heaviside(t-5)+(1-1/5*(t-5))*Heaviside(t-10),x(0) = 0
```

$$x(t) = \frac{\text{Heaviside}\,(t-10)\sin\,(2t-20)}{40} - \frac{\text{Heaviside}\,(t-5)\sin\,(2t-10)}{40} + \frac{(-2t+20)\,\text{Heaviside}\,(t-10)}{40} + \frac{(t-5)\,\text{Heaviside}\,(t-5)}{20}$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 55

```
DSolve[{x''[t]+4*x[t]==1/5*(t-5)*UnitStep[t-5]+(1-1/5*(t-5))*UnitStep[t-10],{x[0]==0,x'[0]==0
```

$$x(t) \to \begin{cases} \frac{1}{40}(2(t-5) + \sin(10-2t)) & 5 < t \leq 10 \\ \frac{1}{40}(\sin(10-2t) - \sin(20-2t) + 10) & t > 10 \end{cases}$$

18 Chapter 4, Linear Systems. Exercises page 190

18.1 problem 2(a)

Internal problem ID [10513]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 190
Problem number: 2(a).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -3y(t) \\
y'(t) &= 2x(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 48

```
dsolve([diff(x(t),t)=-3*y(t),diff(y(t),t)=2*x(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{\sqrt{6}\,(\cos(\sqrt{6}\,t)\,c_1 - \sin(\sqrt{6}\,t)\,c_2)}{2}$$

$$y(t) = c_1 \sin\left(\sqrt{6}\,t\right) + c_2 \cos\left(\sqrt{6}\,t\right)$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 69

```
DSolve[{x'[t]==-3*y[t],y'[t]==2*x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
x(t) &\to c_1 \cos\left(\sqrt{6}t\right) - \sqrt{\frac{3}{2}}c_2 \sin\left(\sqrt{6}t\right) \\
y(t) &\to c_2 \cos\left(\sqrt{6}t\right) + \sqrt{\frac{2}{3}}c_1 \sin\left(\sqrt{6}t\right)
\end{aligned}
$$

18.2 problem 2(b)

Internal problem ID [10514]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 190
Problem number: 2(b).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -2y(t)$$
$$y'(t) = -4x(t)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 52

```
dsolve([diff(x(t),t)=-2*y(t),diff(y(t),t)=-4*x(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{\sqrt{2}\left(c_1 e^{2\sqrt{2}\,t} - c_2 e^{-2\sqrt{2}\,t}\right)}{2}$$

$$y(t) = c_1 e^{2\sqrt{2}\,t} + c_2 e^{-2\sqrt{2}\,t}$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 70

```
DSolve[{x'[t]==-2*y[t],y'[t]==-4*x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 \cosh\left(2\sqrt{2}t\right) - \frac{c_2 \sinh\left(2\sqrt{2}t\right)}{\sqrt{2}}$$
$$y(t) \to c_2 \cosh\left(2\sqrt{2}t\right) - \sqrt{2}c_1 \sinh\left(2\sqrt{2}t\right)$$

18.3 problem 2(c)

Internal problem ID [10515]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 190
Problem number: 2(c).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -3x(t)$$
$$y'(t) = 2y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 20

```
dsolve([diff(x(t),t)=-3*x(t),diff(y(t),t)=2*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \mathrm{e}^{-3t}c_1$$

$$y(t) = c_2\mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 65

```
DSolve[{x'[t]==-3*x[t],y'[t]==3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{-3t}$$
$$y(t) \to c_2 e^{3t}$$
$$x(t) \to c_1 e^{-3t}$$
$$y(t) \to 0$$
$$x(t) \to 0$$
$$y(t) \to c_2 e^{3t}$$
$$x(t) \to 0$$
$$y(t) \to 0$$

18.4 problem 2(d)

Internal problem ID [10516]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 190
Problem number: 2(d).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 4y(t)$$
$$y'(t) = 2y(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 23

```
dsolve([diff(x(t),t)=4*y(t),diff(y(t),t)=2*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = 2c_2 e^{2t} + c_1$$

$$y(t) = c_2 e^{2t}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 65

```
DSolve[{x'[t]==4*x[t],y'[t]==2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{4t}$$
$$y(t) \to c_2 e^{2t}$$
$$x(t) \to c_1 e^{4t}$$
$$y(t) \to 0$$
$$x(t) \to 0$$
$$y(t) \to c_2 e^{2t}$$
$$x(t) \to 0$$
$$y(t) \to 0$$

18.5 problem 3(a)

Internal problem ID [10517]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 190
Problem number: 3(a).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t)$$
$$y'(t) = x(t) + 2y(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 24

```
dsolve([diff(x(t),t)=x(t),diff(y(t),t)=x(t)+2*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -c_2 e^t$$

$$y(t) = c_1 e^{2t} + c_2 e^t$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 32

```
DSolve[{x'[t]==x[t],y'[t]==x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^t$$
$$y(t) \to e^t\left((c_1 + c_2)e^t - c_1\right)$$

18.6 problem 3(b)

Internal problem ID [10518]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 190
Problem number: 3(b).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - y(t)$$
$$y'(t) = x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 33

```
dsolve([diff(x(t),t)=x(t)-y(t),diff(y(t),t)=x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = e^{t}(c_1 \cos(t) - \sin(t) c_2)$$

$$y(t) = e^{t}(c_1 \sin(t) + c_2 \cos(t))$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 39

```
DSolve[{x'[t]==x[t]-y[t],y'[t]==x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{t}(c_1 \cos(t) - c_2 \sin(t))$$
$$y(t) \to e^{t}(c_2 \cos(t) + c_1 \sin(t))$$

18.7 problem 3(c)

Internal problem ID [10519]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 190
Problem number: 3(c).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + 2y(t)$$
$$y'(t) = x(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=x(t)+2*y(t),diff(y(t),t)=x(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\mathrm{e}^{-t}c_1 + 2c_2\mathrm{e}^{2t}$$

$$y(t) = \mathrm{e}^{-t}c_1 + c_2\mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 65

```
DSolve[{x'[t]==x[t]+2*y[t],y'[t]==x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{3}e^{-t}\left(2(c_1 + c_2)e^{3t} + c_1 - 2c_2\right)$$
$$y(t) \to \frac{1}{3}e^{-t}\left((c_1 + c_2)e^{3t} - c_1 + 2c_2\right)$$

18.8 problem 3(d)

Internal problem ID [10520]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 190
Problem number: 3(d).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -x(t) - 2y(t)$$
$$y'(t) = 2x(t) - y(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 45

```
dsolve([diff(x(t),t)=-x(t)-2*y(t),diff(y(t),t)=2*x(t)-y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = e^{-t}(\cos(2t)\, c_1 - \sin(2t)\, c_2)$$

$$y(t) = e^{-t}(c_1 \sin(2t) + c_2 \cos(2t))$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 51

```
DSolve[{x'[t]==-x[t]-2*y[t],y'[t]==2*x[t]-y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tru
```

$$x(t) \to e^{-t}(c_1 \cos(2t) - c_2 \sin(2t))$$
$$y(t) \to e^{-t}(c_2 \cos(2t) + c_1 \sin(2t))$$

19 Chapter 4, Linear Systems. Exercises page 202

19.1 problem 1(a)

Internal problem ID [10521]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 202
Problem number: 1(a).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -2x(t) - 3y(t) \\
y'(t) &= -x(t) + 4y(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 95

```
dsolve([diff(x(t),t)=-2*x(t)-3*y(t),diff(y(t),t)=-x(t)+4*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -2c_1 e^{\left(1+2\sqrt{3}\right)t}\sqrt{3} + 2c_2 e^{-\left(-1+2\sqrt{3}\right)t}\sqrt{3} + 3c_1 e^{\left(1+2\sqrt{3}\right)t} + 3c_2 e^{-\left(-1+2\sqrt{3}\right)t}$$

$$y(t) = c_1 e^{\left(1+2\sqrt{3}\right)t} + c_2 e^{-\left(-1+2\sqrt{3}\right)t}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 94

```
DSolve[{x'[t]==-2*x[t]-3*y[t],y'[t]==-x[t]+4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$
\begin{aligned}
x(t) &\to \frac{1}{2}e^t\left(2c_1\cosh\left(2\sqrt{3}t\right) - \sqrt{3}(c_1+c_2)\sinh\left(2\sqrt{3}t\right)\right) \\
y(t) &\to \frac{1}{6}e^t\left(6c_2\cosh\left(2\sqrt{3}t\right) - \sqrt{3}(c_1-3c_2)\sinh\left(2\sqrt{3}t\right)\right)
\end{aligned}
$$

19.2 problem 1(b)

Internal problem ID [10522]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 202
Problem number: 1(b).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -3y(t)$$
$$y'(t) = -2x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=-3*y(t),diff(y(t),t)=-2*x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -c_1 e^{3t} + \frac{3c_2 e^{-2t}}{2}$$

$$y(t) = c_1 e^{3t} + c_2 e^{-2t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 72

```
DSolve[{x'[t]==-3*y[t],y'[t]==-2*x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{5} e^{-2t} \left((2c_1 - 3c_2) e^{5t} + 3(c_1 + c_2) \right)$$
$$y(t) \to \frac{1}{5} e^{-2t} \left((3c_2 - 2c_1) e^{5t} + 2(c_1 + c_2) \right)$$

19.3 problem 1(c)

Internal problem ID [10523]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 202
Problem number: 1(c).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -2x(t)$$
$$y'(t) = x(t)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 23

```
dsolve([diff(x(t),t)=-2*x(t),diff(y(t),t)=x(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -2c_2 e^{-2t}$$

$$y(t) = c_1 + c_2 e^{-2t}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 29

```
DSolve[{x'[t]==-2*x[t],y'[t]==x[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^{-2t}$$
$$y(t) \to c_1 e^{-t} \sinh(t) + c_2$$

19.4 problem 1(d)

Internal problem ID [10524]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 202
Problem number: 1(d).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -2x(t) - y(t)$$
$$y'(t) = -4y(t)$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 28

```
dsolve([diff(x(t),t)=-2*x(t)-y(t),diff(y(t),t)=-4*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{c_2 e^{-4t}}{2} + e^{-2t} c_1$$

$$y(t) = c_2 e^{-4t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 43

```
DSolve[{x'[t]==-2*x[t]-y[t],y'[t]==-4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to \frac{1}{2} e^{-4t} \left((2c_1 - c_2) e^{2t} + c_2 \right)$$
$$y(t) \to c_2 e^{-4t}$$

19.5 problem 1(e)

Internal problem ID [10525]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 202
Problem number: 1(e).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - 2y(t)$$
$$y'(t) = -2x(t) + 4y(t)$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 27

```
dsolve([diff(x(t),t)=x(t)-2*y(t),diff(y(t),t)=-2*x(t)+4*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{c_2 e^{5t}}{2} + 2c_1$$

$$y(t) = c_1 + c_2 e^{5t}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 59

```
DSolve[{x'[t]==x[t]-2*y[t],y'[t]==-2*x[t]+4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$x(t) \to \frac{1}{5}\left((c_1 - 2c_2)e^{5t} + 4c_1 + 2c_2\right)$$
$$y(t) \to \frac{1}{5}\left(-2(c_1 - 2c_2)e^{5t} + 2c_1 + c_2\right)$$

19.6 problem 1(f)

Internal problem ID [10526]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 202
Problem number: 1(f).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -6y(t)$$
$$y'(t) = 6y(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 23

```
dsolve([diff(x(t),t)=-6*y(t),diff(y(t),t)=6*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -c_2 e^{6t} + c_1$$

$$y(t) = c_2 e^{6t}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 30

```
DSolve[{x'[t]==-6*y[t],y'[t]==6*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to -c_2 e^{6t} + c_1 + c_2$$
$$y(t) \to c_2 e^{6t}$$

19.7 problem 3(a)

Internal problem ID [10527]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 202
Problem number: 3(a).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) + 3y(t)$$
$$y'(t) = -x(t) - 14$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 77

```
dsolve([diff(x(t),t)=2*x(t)+3*y(t),diff(y(t),t)=-x(t)-14],[x(t), y(t)], singsol=all)
```

$$x(t) = -14 + e^{t}\left(\sqrt{2}\,\sin\left(\sqrt{2}\,t\right) c_1 - \sqrt{2}\,\cos\left(\sqrt{2}\,t\right) c_2 - \sin\left(\sqrt{2}\,t\right) c_2 - \cos\left(\sqrt{2}\,t\right) c_1\right)$$

$$y(t) = \frac{28}{3} + e^{t}\left(\sin\left(\sqrt{2}\,t\right) c_2 + \cos\left(\sqrt{2}\,t\right) c_1\right)$$

✓ Solution by Mathematica

Time used: 0.152 (sec). Leaf size: 89

```
DSolve[{x'[t]==2*x[t]+3*y[t],y'[t]==-x[t]-14},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to c_1 e^t \cos\left(\sqrt{2}t\right) + \frac{(c_1 + 3c_2)e^t \sin\left(\sqrt{2}t\right)}{\sqrt{2}} - 14$$
$$y(t) \to c_2 e^t \cos\left(\sqrt{2}t\right) - \frac{(c_1 + c_2)e^t \sin\left(\sqrt{2}t\right)}{\sqrt{2}} + \frac{28}{3}$$

19.8 problem 3(b)

Internal problem ID [10528]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 202
Problem number: 3(b).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -3x(t) + 3y(t) \\
y'(t) &= x(t) + 2y(t) - 1
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 88

```
dsolve([diff(x(t),t)=-3*x(t)+3*y(t),diff(y(t),t)=x(t)+2*y(t)-1],[x(t), y(t)], singsol=all)
```

$$
x(t) = -\frac{e^{-\frac{(1+\sqrt{37})t}{2}} c_1\sqrt{37}}{2} + \frac{e^{\frac{(-1+\sqrt{37})t}{2}} c_2\sqrt{37}}{2} - \frac{5\,e^{-\frac{(1+\sqrt{37})t}{2}} c_1}{2} - \frac{5\,e^{\frac{(-1+\sqrt{37})t}{2}} c_2}{2} + \frac{1}{3}
$$

$$
y(t) = e^{\frac{(-1+\sqrt{37})t}{2}} c_2 + e^{-\frac{(1+\sqrt{37})t}{2}} c_1 + \frac{1}{3}
$$

✓ Solution by Mathematica

Time used: 0.409 (sec). Leaf size: 155

```
DSolve[{x'[t]==-3*x[t]+3*y[t],y'[t]==x[t]+2*y[t]-1},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$
\begin{aligned}
x(t) \to \frac{1}{222}e^{-\frac{1}{2}\left(1+\sqrt{37}\right)t}\Big(74e^{\frac{1}{2}\left(1+\sqrt{37}\right)t} - 3\left(\left(5\sqrt{37}-37\right)c_1 - 6\sqrt{37}c_2\right)e^{\sqrt{37}t} \\
+ 3\left(37+5\sqrt{37}\right)c_1 - 18\sqrt{37}c_2\Big)
\end{aligned}
$$

$$
y(t) \to \frac{1}{3} + \frac{1}{37}e^{-t/2}\left(37c_2\cosh\left(\frac{\sqrt{37}t}{2}\right) + \sqrt{37}(2c_1+5c_2)\sinh\left(\frac{\sqrt{37}t}{2}\right)\right)
$$

20 Chapter 4, Linear Systems. Exercises page 218

20.1 problem 2(a)

Internal problem ID [10529]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 218
Problem number: 2(a).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -x(t) + y(t) \\
y'(t) &= -3y(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 28

```
dsolve([diff(x(t),t)=-x(t)+y(t),diff(y(t),t)=-3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{c_2 e^{-3t}}{2} + e^{-t} c_1$$

$$y(t) = c_2 e^{-3t}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 43

```
DSolve[{x'[t]==-x[t]+y[t],y'[t]==-3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
x(t) &\to \frac{1}{2} e^{-3t} \left((2c_1 + c_2) e^{2t} - c_2 \right) \\
y(t) &\to c_2 e^{-3t}
\end{aligned}
$$

20.2 problem 2(b)

Internal problem ID [10530]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 218
Problem number: 2(b).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t)$$
$$y'(t) = 3x(t) - 4y(t)$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 24

```
dsolve([diff(x(t),t)=x(t),diff(y(t),t)=3*x(t)-4*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{5c_2 e^t}{3}$$

$$y(t) = e^{-4t}c_1 + c_2 e^t$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 148

```
DSolve[{x'[t]==x[t]+y[t],y'[t]==3*x[t]-4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True
```

$$x(t) \to \frac{1}{74}e^{-\frac{1}{2}\left(3+\sqrt{37}\right)t}\left(c_1\left(\left(37+5\sqrt{37}\right)e^{\sqrt{37}t}+37-5\sqrt{37}\right)+2\sqrt{37}c_2\left(e^{\sqrt{37}t}-1\right)\right)$$
$$y(t) \to \frac{1}{74}e^{-\frac{1}{2}\left(3+\sqrt{37}\right)t}\left(6\sqrt{37}c_1\left(e^{\sqrt{37}t}-1\right)+c_2\left(\left(37-5\sqrt{37}\right)e^{\sqrt{37}t}+37+5\sqrt{37}\right)\right)$$

20.3 problem 2(c)

Internal problem ID [10531]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 218
Problem number: 2(c).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -x(t) + y(t)\\
y'(t) &= x(t) - 2y(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 86

```
dsolve([diff(x(t),t)=-x(t)+y(t),diff(y(t),t)=x(t)-2*y(t)],[x(t), y(t)], singsol=all)
```

$$
x(t) = \frac{c_1 \mathrm{e}^{\frac{(\sqrt{5}-3)t}{2}}\sqrt{5}}{2} - \frac{c_2 \mathrm{e}^{-\frac{(\sqrt{5}+3)t}{2}}\sqrt{5}}{2} + \frac{c_1 \mathrm{e}^{\frac{(\sqrt{5}-3)t}{2}}}{2} + \frac{c_2 \mathrm{e}^{-\frac{(\sqrt{5}+3)t}{2}}}{2}
$$

$$
y(t) = c_1 \mathrm{e}^{\frac{(\sqrt{5}-3)t}{2}} + c_2 \mathrm{e}^{-\frac{(\sqrt{5}+3)t}{2}}
$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 143

```
DSolve[{x'[t]==-x[t]+y[t],y'[t]==x[t]-2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
x(t) &\to \frac{1}{10}e^{-\frac{1}{2}(3+\sqrt{5})t}\left(c_1\left(\left(5+\sqrt{5}\right)e^{\sqrt{5}t}+5-\sqrt{5}\right)+2\sqrt{5}c_2\left(e^{\sqrt{5}t}-1\right)\right)\\
y(t) &\to \frac{1}{10}e^{-\frac{1}{2}(3+\sqrt{5})t}\left(2\sqrt{5}c_1\left(e^{\sqrt{5}t}-1\right)+c_2\left(-\left(\sqrt{5}-5\right)e^{\sqrt{5}t}+5+\sqrt{5}\right)\right)
\end{aligned}
$$

20.4 problem 2(d)

Internal problem ID [10532]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 218
Problem number: 2(d).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= x(t) + y(t) \\
y'(t) &= -3x(t) + 3y(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 76

```
dsolve([diff(x(t),t)=x(t)+y(t),diff(y(t),t)=-3*x(t)+3*y(t)],[x(t), y(t)], singsol=all)
```

$$
x(t) = \frac{e^{2t}\left(\sin\left(\sqrt{2}\,t\right)\sqrt{2}\,c_2 - \cos\left(\sqrt{2}\,t\right)\sqrt{2}\,c_1 + c_1 \sin\left(\sqrt{2}\,t\right) + c_2 \cos\left(\sqrt{2}\,t\right)\right)}{3}
$$

$$
y(t) = e^{2t}\left(c_1 \sin\left(\sqrt{2}\,t\right) + c_2 \cos\left(\sqrt{2}\,t\right)\right)
$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 94

```
DSolve[{x'[t]==x[t]+y[t],y'[t]==-3*x[t]+3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tru
```

$$
\begin{aligned}
x(t) &\to \frac{1}{2}e^{2t}\left(2c_1 \cos\left(\sqrt{2}t\right) + \sqrt{2}(c_2 - c_1)\sin\left(\sqrt{2}t\right)\right) \\
y(t) &\to \frac{1}{2}e^{2t}\left(2c_2 \cos\left(\sqrt{2}t\right) + \sqrt{2}(c_2 - 3c_1)\sin\left(\sqrt{2}t\right)\right)
\end{aligned}
$$

20.5 problem 4

Internal problem ID [10533]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 218
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - 2y(t)$$
$$y'(t) = 3x(t) - 4y(t)$$

With initial conditions

$$[x(0) = 3, y(0) = 1]$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 34

```
dsolve([diff(x(t),t) = x(t)-2*y(t), diff(y(t),t) = 3*x(t)-4*y(t), x(0) = 3, y(0) = 1],[x(t),
```

$$x(t) = 7\,\mathrm{e}^{-t} - 4\,\mathrm{e}^{-2t}$$

$$y(t) = 7\,\mathrm{e}^{-t} - 6\,\mathrm{e}^{-2t}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 34

```
DSolve[{x'[t]==x[t]-2*y[t],y'[t]==3*x[t]-4*y[t]},{x[0]==3,y[0]==1},{x[t],y[t]},t,IncludeSingu
```

$$x(t) \to e^{-2t}(7e^t - 4)$$
$$y(t) \to e^{-2t}(7e^t - 6)$$

20.6 problem 5

Internal problem ID [10534]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 218
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 5x(t) - y(t)$$
$$y'(t) = 3x(t) + y(t)$$

With initial conditions

$$[x(0) = 2, y(0) = -1]$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 34

```
dsolve([diff(x(t),t) = 5*x(t)-y(t), diff(y(t),t) = 3*x(t)+y(t), x(0) = 2, y(0) = -1],[x(t), y
```

$$x(t) = \frac{7\,e^{4t}}{2} - \frac{3\,e^{2t}}{2}$$

$$y(t) = \frac{7\,e^{4t}}{2} - \frac{9\,e^{2t}}{2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 44

```
DSolve[{x'[t]==5*x[t]-y[t],y'[t]==3*x[t]+y[t]},{x[0]==2,y[0]==-1},{x[t],y[t]},t,IncludeSingul
```

$$x(t) \to \frac{1}{2}e^{2t}(7e^{2t} - 3)$$
$$y(t) \to \frac{1}{2}e^{2t}(7e^{2t} - 9)$$

21 Chapter 4, Linear Systems. Exercises page 225

21.1 problem 1(a)

Internal problem ID [10535]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 225
Problem number: 1(a).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -3x(t) + y(t)$$
$$y'(t) = -3y(t)$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 24

```
dsolve([diff(x(t),t)=-3*x(t)+y(t),diff(y(t),t)=-3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = (tc_2 + c_1)\,\mathrm{e}^{-3t}$$

$$y(t) = c_2\mathrm{e}^{-3t}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 29

```
DSolve[{x'[t]==-3*x[t]+y[t],y'[t]==-3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-3t}(c_2t + c_1)$$
$$y(t) \to c_2e^{-3t}$$

21.2 problem 1(b)

Internal problem ID [10536]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 225
Problem number: 1(b).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - y(t)$$
$$y'(t) = x(t) + 3y(t)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 32

```
dsolve([diff(x(t),t)=x(t)-y(t),diff(y(t),t)=x(t)+3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -e^{2t}(tc_2 + c_1 - c_2)$$

$$y(t) = e^{2t}(tc_2 + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 42

```
DSolve[{x'[t]==x[t]-y[t],y'[t]==x[t]+3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to -e^{2t}(c_1(t-1) + c_2 t)$$
$$y(t) \to e^{2t}((c_1 + c_2)t + c_2)$$

22 Chapter 4, Linear Systems. Exercises page 237

22.1 problem 4(a)

Internal problem ID [10537]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 237
Problem number: 4(a).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) + 2y(t)$$
$$y'(t) = 3x(t) + 2y(t)$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=x(t)+2*y(t),diff(y(t),t)=3*x(t)+2*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\mathrm{e}^{-t}c_1 + \frac{2c_2\mathrm{e}^{4t}}{3}$$

$$y(t) = \mathrm{e}^{-t}c_1 + c_2\mathrm{e}^{4t}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 68

```
DSolve[{x'[t]==x[t]+2*y[t],y'[t]==3*x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tr
```

$$x(t) \to \frac{1}{5}e^{-t}\left(2(c_1+c_2)e^{5t} + 3c_1 - 2c_2\right)$$
$$y(t) \to \frac{1}{5}e^{-t}\left(3(c_1+c_2)e^{5t} - 3c_1 + 2c_2\right)$$

22.2 problem 4(b)

Internal problem ID [10538]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 237
Problem number: 4(b).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -3x(t) + 4y(t)$$
$$y'(t) = -3y(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 25

```
dsolve([diff(x(t),t)=-3*x(t)+4*y(t),diff(y(t),t)=-3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = (4tc_2 + c_1)\,\mathrm{e}^{-3t}$$

$$y(t) = c_2\mathrm{e}^{-3t}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 30

```
DSolve[{x'[t]==-3*x[t]+4*y[t],y'[t]==-3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True]
```

$$x(t) \to e^{-3t}(4c_2t + c_1)$$
$$y(t) \to c_2e^{-3t}$$

22.3 problem 4(c)

Internal problem ID [10539]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 237
Problem number: 4(c).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) + 2y(t)$$
$$y'(t) = 6x(t) + 3y(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=2*x(t)+2*y(t),diff(y(t),t)=6*x(t)+3*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{2\,\mathrm{e}^{-t}c_1}{3} + \frac{c_2\mathrm{e}^{6t}}{2}$$

$$y(t) = \mathrm{e}^{-t}c_1 + c_2\mathrm{e}^{6t}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 74

```
DSolve[{x'[t]==2*x[t]+2*y[t],y'[t]==6*x[t]+3*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to \frac{1}{7}e^{-t}\left(c_1\left(3e^{7t}+4\right)+2c_2\left(e^{7t}-1\right)\right)$$
$$y(t) \to \frac{1}{7}e^{-t}\left(6c_1\left(e^{7t}-1\right)+c_2\left(4e^{7t}+3\right)\right)$$

22.4 problem 4(d)

Internal problem ID [10540]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 237
Problem number: 4(d).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = -5x(t) + 3y(t)$$
$$y'(t) = 2x(t) - 10y(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=-5*x(t)+3*y(t),diff(y(t),t)=2*x(t)-10*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{c_1 \mathrm{e}^{-11t}}{2} + 3c_2 \mathrm{e}^{-4t}$$

$$y(t) = c_1 \mathrm{e}^{-11t} + c_2 \mathrm{e}^{-4t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 69

```
DSolve[{x'[t]==-5*x[t]+3*y[t],y'[t]==2*x[t]-10*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -
```

$$x(t) \to \frac{1}{7}e^{-11t}\left(3(2c_1 + c_2)e^{7t} + c_1 - 3c_2\right)$$
$$y(t) \to \frac{1}{7}e^{-11t}\left(2c_1\left(e^{7t} - 1\right) + c_2\left(e^{7t} + 6\right)\right)$$

22.5 problem 4(e)

Internal problem ID [10541]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 237
Problem number: 4(e).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t)$$
$$y'(t) = 2y(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 20

```
dsolve([diff(x(t),t)=2*x(t)+0*y(t),diff(y(t),t)=0*x(t)+2*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = c_1 \mathrm{e}^{2t}$$

$$y(t) = c_2 \mathrm{e}^{2t}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 65

```
DSolve[{x'[t]==2*x[t]+0*y[t],y'[t]==0*x[t]+2*y[t]},{x[t],y[t]},t,IncludeSingularSolutions ->
```

$$x(t) \to c_1 e^{2t}$$
$$y(t) \to c_2 e^{2t}$$
$$x(t) \to c_1 e^{2t}$$
$$y(t) \to 0$$
$$x(t) \to 0$$
$$y(t) \to c_2 e^{2t}$$
$$x(t) \to 0$$
$$y(t) \to 0$$

22.6 problem 4(f)

Internal problem ID [10542]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 237
Problem number: 4(f).
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 3x(t) - 2y(t) \\
y'(t) &= 4x(t) - y(t)
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 54

```
dsolve([diff(x(t),t)=3*x(t)-2*y(t),diff(y(t),t)=4*x(t)-y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = \frac{\mathrm{e}^{t}(\cos(2t)\, c_1 + c_2 \cos(2t) + c_1 \sin(2t) - \sin(2t)\, c_2)}{2}$$

$$y(t) = \mathrm{e}^{t}(c_1 \sin(2t) + c_2 \cos(2t))$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 58

```
DSolve[{x'[t]==3*x[t]-2*y[t],y'[t]==4*x[t]-y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tr
```

$$
\begin{aligned}
x(t) &\to e^{t}(c_1 \cos(2t) + (c_1 - c_2)\sin(2t)) \\
y(t) &\to e^{t}(c_2 \cos(2t) + (2c_1 - c_2)\sin(2t))
\end{aligned}
$$

22.7 problem 4(g)

Internal problem ID [10543]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 237
Problem number: 4(g).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 5x(t) - 4y(t)$$
$$y'(t) = x(t) + y(t)$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 32

```
dsolve([diff(x(t),t)=5*x(t)-4*y(t),diff(y(t),t)=x(t)+y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = e^{3t}(2tc_2 + 2c_1 + c_2)$$

$$y(t) = e^{3t}(tc_2 + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 45

```
DSolve[{x'[t]==5*x[t]-4*y[t],y'[t]==x[t]+y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> True
```

$$x(t) \to e^{3t}(2c_1t - 4c_2t + c_1)$$
$$y(t) \to e^{3t}((c_1 - 2c_2)t + c_2)$$

22.8 problem 4(h)

Internal problem ID [10544]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 237
Problem number: 4(h).
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 9y(t)$$
$$y'(t) = -x(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 36

```
dsolve([diff(x(t),t)=0*x(t)+9*y(t),diff(y(t),t)=-x(t)+0*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -3c_1 \cos(3t) + 3c_2 \sin(3t)$$

$$y(t) = c_1 \sin(3t) + c_2 \cos(3t)$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 42

```
DSolve[{x'[t]==0*x[t]+9*y[t],y'[t]==-x[t]+0*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$x(t) \to c_1 \cos(3t) + 3c_2 \sin(3t)$$
$$y(t) \to c_2 \cos(3t) - \frac{1}{3}c_1 \sin(3t)$$

22.9 problem 5

Internal problem ID [10545]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 237
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 2x(t) + y(t)$$
$$y'(t) = -x(t)$$

With initial conditions

$$[x(0) = 1, y(0) = -1]$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 14

```
dsolve([diff(x(t),t) = 2*x(t)+y(t), diff(y(t),t) = -x(t), x(0) = 1, y(0) = -1],[x(t), y(t)],
```

$$x(t) = e^t$$

$$y(t) = -e^t$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 16

```
DSolve[{x'[t]==2*x[t]+y[t],y'[t]==-x[t]+0*y[t]},{x[0]==1,y[0]==-1},{x[t],y[t]},t,IncludeSingu
```

$$x(t) \to e^t$$
$$y(t) \to -e^t$$

22.10 problem 6

Internal problem ID [10546]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 237
Problem number: 6.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - 2y(t)$$
$$y'(t) = -2x(t) + 4y(t)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve([diff(x(t),t)=x(t)-2*y(t),diff(y(t),t)=-2*x(t)+4*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{c_2 e^{5t}}{2} + 2c_1$$

$$y(t) = c_1 + c_2 e^{5t}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 59

```
DSolve[{x'[t]==x[t]-2*y[t],y'[t]==-2*x[t]+4*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$x(t) \to \frac{1}{5}\left((c_1 - 2c_2)e^{5t} + 4c_1 + 2c_2\right)$$
$$y(t) \to \frac{1}{5}\left(-2(c_1 - 2c_2)e^{5t} + 2c_1 + c_2\right)$$

23 Chapter 4, Linear Systems. Exercises page 244

23.1 problem 3

Internal problem ID [10547]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 244
Problem number: 3.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = 3x(t) - y(t) + 1$$
$$y'(t) = x(t) + y(t) + 2$$

With initial conditions

$$[x(0) = 1, y(0) = 2]$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 32

```
dsolve([diff(x(t),t) = 3*x(t)-y(t)+1, diff(y(t),t) = x(t)+y(t)+2, x(0) = 1, y(0) = 2],[x(t),
```

$$x(t) = -\frac{3}{4} + e^{2t}\left(-\frac{3t}{2} + \frac{7}{4}\right)$$

$$y(t) = -\frac{5}{4} + e^{2t}\left(-\frac{3t}{2} + \frac{13}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 42

```
DSolve[{x'[t]==3*x[t]-y[t]+1,y'[t]==x[t]+y[t]+2},{x[0]==1,y[0]==2},{x[t],y[t]},t,IncludeSingu
```

$$x(t) \to \frac{1}{4}\left(e^{2t}(7 - 6t) - 3\right)$$
$$y(t) \to \frac{1}{4}\left(e^{2t}(13 - 6t) - 5\right)$$

23.2 problem 4

Internal problem ID [10548]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 244
Problem number: 4.
ODE order: 1.
ODE degree: 1.

Solve

$$\begin{aligned} x'(t) &= -5x(t) + 3y(t) + e^{-t} \\ y'(t) &= 2x(t) - 10y(t) \end{aligned}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 48

```
dsolve([diff(x(t),t)=-5*x(t)+3*y(t)+exp(-t),diff(y(t),t)=2*x(t)-10*y(t)],[x(t), y(t)], singso
```

$$x(t) = -\frac{e^{-11t}c_2}{2} + 3\,e^{-4t}c_1 + \frac{3\,e^{-t}}{10}$$

$$y(t) = e^{-11t}c_2 + e^{-4t}c_1 + \frac{e^{-t}}{15}$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 88

```
DSolve[{x'[t]==-5*x[t]+3*y[t]+Exp[-t],y'[t]==2*x[t]-10*y[t]},{x[t],y[t]},t,IncludeSingularSol
```

$$\begin{aligned} x(t) &\to \frac{1}{70}e^{-11t}\left(21e^{10t} + 30(2c_1 + c_2)e^{7t} + 10(c_1 - 3c_2)\right) \\ y(t) &\to \frac{1}{105}e^{-11t}\left(7e^{10t} + 15(2c_1 + c_2)e^{7t} - 30(c_1 - 3c_2)\right) \end{aligned}$$

23.3 problem 5

Internal problem ID [10549]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 244
Problem number: 5.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = y(t)$$
$$y'(t) = -x(t) + \cos(wt)$$

✓ Solution by Maple

Time used: 0.281 (sec). Leaf size: 71

```
dsolve([diff(x(t),t)=0*x(t)+y(t),diff(y(t),t)=-x(t)+cos(w*t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{\cos(t)\, c_2 w^2 - \sin(t)\, c_1 w^2 - c_2 \cos(t) + c_1 \sin(t) + \cos(tw)}{(w-1)(w+1)}$$

$$y(t) = \sin(t)\, c_2 + c_1 \cos(t) + \frac{w \sin(tw)}{w^2 - 1}$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 57

```
DSolve[{x'[t]==0*x[t]+y[t],y'[t]==-x[t]+Cos[w*t]},{x[t],y[t]},t,IncludeSingularSolutions -> T
```

$$x(t) \to -\frac{\cos(tw)}{w^2 - 1} + c_1 \cos(t) + c_2 \sin(t)$$
$$y(t) \to \frac{w \sin(tw)}{w^2 - 1} + c_2 \cos(t) - c_1 \sin(t)$$

23.4 problem 6

Internal problem ID [10550]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 244
Problem number: 6.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= 3x(t) + 2y(t) + 3 \\
y'(t) &= 7x(t) + 5y(t) + 2t
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 91

```
dsolve([diff(x(t),t)=3*x(t)+2*y(t)+3,diff(y(t),t)=7*x(t)+5*y(t)+2*t],[x(t), y(t)], singsol=al
```

$$
x(t) = \frac{\mathrm{e}^{\left(4+\sqrt{15}\right)t} c_2 \sqrt{15}}{7} - \frac{\mathrm{e}^{-\left(-4+\sqrt{15}\right)t} c_1 \sqrt{15}}{7} - \frac{\mathrm{e}^{\left(4+\sqrt{15}\right)t} c_2}{7} - \frac{\mathrm{e}^{-\left(-4+\sqrt{15}\right)t} c_1}{7} + 4t + 17
$$

$$
y(t) = \mathrm{e}^{\left(4+\sqrt{15}\right)t} c_2 + \mathrm{e}^{-\left(-4+\sqrt{15}\right)t} c_1 - 6t - 25
$$

✓ Solution by Mathematica

Time used: 1.637 (sec). Leaf size: 178

```
DSolve[{x'[t]==3*x[t]+2*y[t],y'[t]==7*x[t]+5*y[t]+2*t},{x[t],y[t]},t,IncludeSingularSolutions
```

$$
\begin{aligned}
x(t) \to \frac{1}{30} e^{-\left(\left(\sqrt{15}-4\right)t\right)} \Big(& 120 e^{\left(\sqrt{15}-4\right)t}(t+8) + \left(2\sqrt{15}c_2 - \left(\sqrt{15}-15\right)c_1\right) e^{2\sqrt{15}t} \\
& + \left(15+\sqrt{15}\right)c_1 - 2\sqrt{15}c_2 \Big)
\end{aligned}
$$

$$
\begin{aligned}
y(t) \to \frac{1}{30} e^{-\left(\left(\sqrt{15}-4\right)t\right)} \Big(& -60 e^{\left(\sqrt{15}-4\right)t}(3t+23) + \left(7\sqrt{15}c_1 + \left(15+\sqrt{15}\right)c_2\right) e^{2\sqrt{15}t} \\
& - 7\sqrt{15}c_1 - \left(\sqrt{15}-15\right)c_2 \Big)
\end{aligned}
$$

23.5 problem 7

Internal problem ID [10551]

Book: A First Course in Differential Equations by J. David Logan. Third Edition. Springer-Verlag, NY. 2015.
Section: Chapter 4, Linear Systems. Exercises page 244
Problem number: 7.
ODE order: 1.
ODE degree: 1.

Solve

$$x'(t) = x(t) - 3y(t)$$
$$y'(t) = 3x(t) + 7y(t)$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 35

```
dsolve([diff(x(t),t)=x(t)-3*y(t),diff(y(t),t)=3*x(t)+7*y(t)],[x(t), y(t)], singsol=all)
```

$$x(t) = -\frac{e^{4t}(3tc_2 + 3c_1 - c_2)}{3}$$

$$y(t) = e^{4t}(tc_2 + c_1)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 44

```
DSolve[{x'[t]==x[t]-3*y[t],y'[t]==3*x[t]+7*y[t]},{x[t],y[t]},t,IncludeSingularSolutions -> Tr
```

$$x(t) \to e^{4t}(-3c_1t - 3c_2t + c_1)$$
$$y(t) \to e^{4t}(3(c_1 + c_2)t + c_2)$$