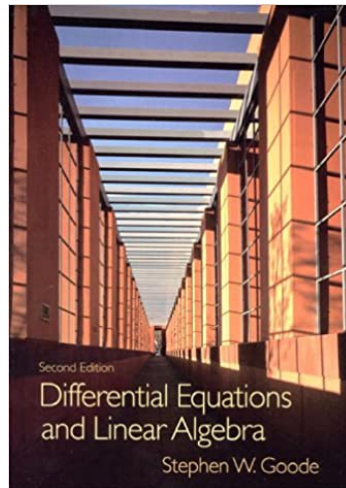**A Solution Manual For**

Differential equations and linear algebra, Stephen W. Goode, second edition, 2000

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1 1.4, page 36

1.1 problem 1

Internal problem ID [2035]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.4, page 36
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - 2yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=2*x*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{x^2}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 18

```
DSolve[y'[x]==2*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x^2}$$
$$y(x) \to 0$$

1.2   problem 2

Internal problem ID [2036]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.4, page 36
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - \frac{y^2}{x^2 + 1} = 0$$

✔ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)=y(x)^2/(x^2+1),y(x), singsol=all)
```

$$y(x) = -\frac{1}{\arctan(x) - c_1}$$

✔ Solution by Mathematica

Time used: 0.14 (sec). Leaf size: 19

```
DSolve[y'[x]==y[x]^2/(x^2+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\arctan(x) + c_1}$$

$$y(x) \to 0$$

1.3   problem 3

Internal problem ID [2037]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.4, page 36
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\mathrm{e}^{x+y}y' - 1 = 0$$

✓   Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(exp(x+y(x))*diff(y(x),x)-1=0,y(x), singsol=all)
```

$$y(x) = \ln\left(c_1 \mathrm{e}^x - 1\right) - x$$

✓   Solution by Mathematica

Time used: 0.086 (sec). Leaf size: 15

```
DSolve[Exp[x+y[x]]*y'[x]-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(\sinh(x) - \cosh(x) + c_1)$$

1.4 problem 4

Internal problem ID [2038]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.4, page 36
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - \frac{y}{x \ln(x)} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=y(x)/(x*ln(x)),y(x), singsol=all)
```

$$y(x) = \ln(x) c_1$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 15

```
DSolve[y'[x]==y[x]/(x*Log[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \log(x)$$
$$y(x) \to 0$$

1.5 problem 5

Internal problem ID [2039]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.4, page 36
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y - (-2 + x)\, y' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve(y(x)-(x-2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1(-2 + x)$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 16

```
DSolve[y[x]-(x-2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x - 2)$$
$$y(x) \to 0$$

1.6 problem 6

CAS Maple gives this as type `[_separable]`

$$ y' - \frac{2x(y-1)}{x^2+3} = 0 $$

✔ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)=(2*x*(y(x)-1))/(x^2+3),y(x), singsol=all)
```

$$ y(x) = 1 + \left(x^2 + 3\right) c_1 $$

✔ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 20

```
DSolve[y'[x]==(2*x*(y[x]-1))/(x^2+3),y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to 1 + c_1 \left(x^2 + 3\right) $$
$$ y(x) \to 1 $$

1.7 problem 7

Internal problem ID [2041]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.4, page 36
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y - y'x - 3 + 2y'x^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(y(x)-x*diff(y(x),x)=3-2*x^2*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{\left(-\frac{3}{x} + c_1\right)x}{2x - 1}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 24

```
DSolve[y[x]-x*y'[x]==3-2*x^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3 + c_1 x}{1 - 2x}$$
$$y(x) \to 3$$

1.8 problem 8

CAS Maple gives this as type [_separable]

$$y' - \frac{\cos(x-y)}{\sin(x)\sin(y)} + 1 = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)=(cos(x-y(x)))/(sin(x)*sin(y(x)))-1,y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{1}{\sin(x)c_1}\right)$$

✓ Solution by Mathematica

Time used: 5.614 (sec). Leaf size: 47

```
DSolve[y'[x]==(Cos[x-y[x]])/(Sin[x]*Sin[y[x]])-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(-\frac{1}{2}c_1\csc(x)\right)$$

$$y(x) \to \arccos\left(-\frac{1}{2}c_1\csc(x)\right)$$

$$y(x) \to -\frac{\pi}{2}$$

$$y(x) \to \frac{\pi}{2}$$

1.9 problem 9

Internal problem ID [2043]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.4, page 36
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \frac{x(-1+y^2)}{2(-2+x)(x-1)} = 0$$

✔ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=(x*( y(x)^2-1))/(2*(x-2)*(x-1)),y(x), singsol=all)
```

$$y(x) = -\tanh\left(\ln(-2+x) - \frac{\ln(x-1)}{2} + \frac{c_1}{2}\right)$$

✔ Solution by Mathematica

Time used: 0.767 (sec). Leaf size: 51

```
DSolve[y'[x]==(x*( y[x]^2-1))/(2*(x-2)*(x-1)),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x + e^{2c_1}(x-2)^2 - 1}{-x + e^{2c_1}(x-2)^2 + 1}$$

$$y(x) \to -1$$

$$y(x) \to 1$$

1.10 problem 10

Internal problem ID [2044]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.4, page 36
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$y' - \frac{x^2 y - 32}{-x^2 + 16} - 32 = 0$$

✔ Solution by Maple

Time used: 0.0 (sec). Leaf size: 77

```
dsolve(diff(y(x),x)=(x^2*y(x)-32)/(16-x^2) + 32,y(x), singsol=all)
```

$$y(x) = \left(32\,\mathrm{e}^x - 1440\,\mathrm{e}^{-4}\,\mathrm{Ei}_1\left(-x - 4\right) + \frac{128\,\mathrm{e}^x}{(x+4)^2} - \frac{1952\,\mathrm{e}^x}{x+4} + c_1 \right) \left(\frac{\mathrm{e}^{-x} x^2}{(x-4)^2} + \frac{8\,\mathrm{e}^{-x} x}{(x-4)^2} + \frac{16\,\mathrm{e}^{-x}}{(x-4)^2} \right)$$

✔ Solution by Mathematica

Time used: 0.196 (sec). Leaf size: 52

```
DSolve[y'[x]==(x^2*y[x]-32)/(16-x^2) + 32,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x-4}(32e^{x+4}((x-53)x - 224) + (x+4)^2\,(1440\,\mathrm{ExpIntegralEi}(x+4) + e^4 c_1))}{(x-4)^2}$$

1.11 problem 11

Internal problem ID [2045]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.4, page 36
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$(x-a)(x-b)\,y' - y + c = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve((x-a)*(x-b)*diff(y(x),x)-(y(x)-c)=0,y(x), singsol=all)
```

$$y(x) = c + (x-b)^{-\frac{1}{a-b}}(x-a)^{\frac{1}{a-b}}c_1$$

✓ Solution by Mathematica

Time used: 0.276 (sec). Leaf size: 41

```
DSolve[(x-a)*(x-b)*y'[x]-(y[x]-c)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c + c_1(x-b)^{\frac{1}{b-a}}(x-a)^{\frac{1}{a-b}}$$
$$y(x) \to c$$

1.12 problem 12

Internal problem ID [2046]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.4, page 36
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$(x^2 + 1) y' + y^2 + 1 = 0$$

With initial conditions

$$[y(0) = 1]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 11

```
dsolve([(x^2+1)*diff(y(x),x)+y(x)^2=-1,y(0) = 1],y(x), singsol=all)
```

$$y(x) = \cot\left(\arctan(x) + \frac{\pi}{4}\right)$$

✓ Solution by Mathematica

Time used: 0.23 (sec). Leaf size: 14

```
DSolve[{(x^2+1)*y'[x]+y[x]^2==-1,y[0]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cot\left(\arctan(x) + \frac{\pi}{4}\right)$$

1.13 problem 13

Internal problem ID [2047]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.4, page 36
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$(1 - x^2) y' + yx - ax = 0$$

With initial conditions

$$[y(0) = 2a]$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve([(1-x^2)*diff(y(x),x)+x*y(x)=a*x,y(0) = 2*a],y(x), singsol=all)
```

$$y(x) = a\left(1 - i\sqrt{x - 1}\,\sqrt{x + 1}\right)$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 21

```
DSolve[{(1-x^2)*y'[x]+x*y[x]==a*x,y[0]==2*a},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to a - ia\sqrt{x^2 - 1}$$

1.14   problem 14

Internal problem ID [2048]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.4, page 36
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - 1 + \frac{\sin\left(x + y\right)}{\cos\left(x\right)\sin\left(y\right)} = 0$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}\right]$$

✓ Solution by Maple

Time used: 0.469 (sec). Leaf size: 11

```
dsolve([diff(y(x),x)=1- (sin(x+y(x)))/(sin(y(x))*cos(x)),y(1/4*Pi) = 1/4*Pi],y(x), singsol=al
```

$$y(x) = \arccos\left(\frac{\sec\left(x\right)}{2}\right)$$

✓ Solution by Mathematica

Time used: 6.051 (sec). Leaf size: 10

```
DSolve[{y'[x]==1- Sin[x+y[x]]/(Sin[y[x]]*Cos[x]),y[Pi/4]==Pi/4},y[x],x,IncludeSingularSolutio
```

$$y(x) \to \sec^{-1}(2\cos(x))$$

1.15 problem 15

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.4, page 36
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - y^3 \sin(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)=y(x)^3*sin(x),y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{c_1 + 2\cos(x)}}$$

$$y(x) = -\frac{1}{\sqrt{c_1 + 2\cos(x)}}$$

✓ Solution by Mathematica

Time used: 0.175 (sec). Leaf size: 49

```
DSolve[y'[x]==y[x]^3*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{2}\sqrt{\cos(x) - c_1}}$$

$$y(x) \to \frac{1}{\sqrt{2}\sqrt{\cos(x) - c_1}}$$

$$y(x) \to 0$$

2 1.6, page 50

2.1   problem 1

Internal problem ID [2050]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.6, page 50
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - y - \mathrm{e}^{2x} = 0$$

✓   Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)-y(x)=exp(2*x),y(x), singsol=all)
```

$$y(x) = \left(\mathrm{e}^x + c_1\right) \mathrm{e}^x$$

✓   Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 15

```
DSolve[y'[x]-y[x]==Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x \left(e^x + c_1\right)$$

2.2   problem 2

Internal problem ID [2051]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.6, page 50
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y'x^2 - 4yx - x^7 \sin(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)-4*x*y(x)=x^7*sin(x),y(x), singsol=all)
```

$$y(x) = (\sin(x) - \cos(x) x + c_1) x^4$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 19

```
DSolve[x^2*y'[x]-4*x*y[x]==x^7*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^4 (\sin(x) - x \cos(x) + c_1)$$

2.3 problem 3

Internal problem ID [2052]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.6, page 50
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$y' + 2yx - 2x^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+2*x*y(x)=2*x^3,y(x), singsol=all)
```

$$y(x) = x^2 - 1 + \mathrm{e}^{-x^2} c_1$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 20

```
DSolve[y'[x]+2*x*y[x]==2*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + c_1 e^{-x^2} - 1$$

2.4 problem 4

Internal problem ID [2053]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.6, page 50
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$ y' + \frac{2xy}{x^2 + 1} - 4x = 0 $$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)+2*x/(1+x^2)*y(x)=4*x,y(x), singsol=all)
```

$$ y(x) = x^2 + 1 + \frac{c_1}{x^2 + 1} $$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 24

```
DSolve[y'[x]+2*x/(1+x^2)*y[x]==4*x,y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to \frac{x^4 + 2x^2 + c_1}{x^2 + 1} $$

2.5 problem 5

Internal problem ID [2054]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.6, page 50
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' + \frac{2xy}{x^2+1} - \frac{4}{(x^2+1)^2} = 0$$

✔ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)+2*x/(1+x^2)*y(x)=4/(1+x^2)^2,y(x), singsol=all)
```

$$y(x) = \frac{4\arctan(x) + c_1}{x^2+1}$$

✔ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 20

```
DSolve[y'[x]+2*x/(1+x^2)*y[x]==4/(1+x^2)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4\arctan(x) + c_1}{x^2+1}$$

2.6 problem 6

Internal problem ID [2055]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.6, page 50
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$2 \cos\left(x\right)^2 y' + y \sin\left(2x\right) - 4 \cos\left(x\right)^4 = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(2*cos(x)^2*diff(y(x),x)+y(x)*sin(2*x)=4*cos(x)^4,y(x), singsol=all)
```

$$y(x) = \left(2 \sin\left(x\right) + c_1\right) \cos\left(x\right)$$

✓ Solution by Mathematica
Time used: 0.054 (sec). Leaf size: 15

```
DSolve[2*Cos[x]^2*y'[x]+y[x]*Sin[2*x]==4*Cos[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \cos(x)(2\sin(x) + c_1)$$

2.7 problem 7

Internal problem ID [2056]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.6, page 50
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' + \frac{y}{x \ln (x)} - 9x^2 = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)+1/(x*ln(x))*y(x)=9*x^2,y(x), singsol=all)
```

$$y(x) = \frac{3x^3 \ln (x) - x^3 + c_1}{\ln (x)}$$

✓ Solution by Mathematica
Time used: 0.035 (sec). Leaf size: 24

```
DSolve[y'[x]+1/(x*Log[x])*y[x]==9*x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x^3 + \frac{-x^3 + c_1}{\log(x)}$$

2.8   problem 8

Internal problem ID [2057]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.6, page 50
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - y\tan(x) - 8\sin(x)^3 = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)-y(x)*tan(x)=8*sin(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{-\cos(2x) + \frac{\cos(4x)}{4} + c_1}{\cos(x)}$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 19

```
DSolve[y'[x]-y[x]*Tan[x]==8*Sin[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\sin^3(x)\tan(x) + c_1\sec(x)$$

2.9 problem 9

Internal problem ID [2058]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.6, page 50
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$ tx' + 2x - 4\,e^t = 0 $$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(t*diff(x(t),t)+2*x(t)=4*exp(t),x(t), singsol=all)
```

$$ x(t) = \frac{4(t-1)\,e^t + c_1}{t^2} $$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 20

```
DSolve[t*x'[t]+2*x[t]==4*Exp[t],x[t],t,IncludeSingularSolutions -> True]
```

$$ x(t) \to \frac{4e^t(t-1) + c_1}{t^2} $$

2.10   problem 10

Internal problem ID [2059]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.6, page 50
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - \sin\left(x\right)\left(y\sec\left(x\right) - 2\right) = 0$$

✓   Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=sin(x)*(y(x)*sec(x)-2),y(x), singsol=all)
```

$$y(x) = \frac{\frac{\cos(2x)}{2} + c_1}{\cos\left(x\right)}$$

✓   Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 20

```
DSolve[y'[x]==Sin[x]*(y[x]*Sec[x]-2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sec(x)(\cos(2x) + 2c_1)$$

2.11 problem 11

Internal problem ID [2060]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.6, page 50
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$1 - \sin(x)\, y - y' \cos(x) = 0$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 11

```
dsolve((1-y(x)*sin(x))-cos(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (\tan(x) + c_1) \cos(x)$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 13

```
DSolve[(1-y[x]*Sin[x])-Cos[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + c_1 \cos(x)$$

2.12 problem 12

Internal problem ID [2061]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.6, page 50
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - \frac{y}{x} - 2\ln(x)\,x^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)-y(x)/x=2*x^2*ln(x),y(x), singsol=all)
```

$$y(x) = \left(\ln(x)\,x^2 - \frac{x^2}{2} + c_1\right)x$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 23

```
DSolve[y'[x]-y[x]/x==2*x^2*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^3}{2} + x^3\log(x) + c_1 x$$

2.13 problem 13

Internal problem ID [2062]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.6, page 50
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$ y' + \alpha y - e^{\beta x} = 0 $$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)+alpha*y(x)=exp(beta*x),y(x), singsol=all)
```

$$ y(x) = \left(\frac{e^{x(\alpha+\beta)}}{\alpha + \beta} + c_1 \right) e^{-\alpha x} $$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 31

```
DSolve[y'[x]+\[Alpha]*y[x]==Exp[\[Beta]*x],y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to \frac{e^{\alpha(-x)} \left(e^{x(\alpha+\beta)} + c_1(\alpha + \beta) \right)}{\alpha + \beta} $$

2.14 problem 14

Internal problem ID [2063]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.6, page 50
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$y' + \frac{m}{x} - \ln(x) = 0$$

✔ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)+m/x=ln(x),y(x), singsol=all)
```

$$y(x) = x \ln(x) - x - m \ln(x) + c_1$$

✔ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 19

```
DSolve[y'[x]+m/x==Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x - m) \log(x) - x + c_1$$

3 1.8, page 68

3.1   problem 9

Internal problem ID [2064]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.8, page 68
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cla

$$(-y + 3x)\, y' - 3y = 0$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 17

```
dsolve((3*x-y(x))*diff(y(x),x)=3*y(x),y(x), singsol=all)
```

$$y(x) = e^{\mathrm{LambertW}\left(-3x\, e^{-3c_1}\right)+3c_1}$$

✓ Solution by Mathematica
Time used: 5.822 (sec). Leaf size: 25

```
DSolve[(3*x-y[x])*y'[x]==3*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{W\left(-3e^{-c_1}x\right)+c_1}$$
$$y(x) \to 0$$

3.2 problem 10

Internal problem ID [2065]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.8, page 68
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Riccati]`

$$
y' - \frac{(x+y)^2}{2x^2} = 0
$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=(x+y(x))^2/(2*x^2),y(x), singsol=all)
```

$$
y(x) = \tan\left(\frac{\ln(x)}{2} + \frac{c_1}{2}\right) x
$$

✓ Solution by Mathematica

Time used: 0.203 (sec). Leaf size: 17

```
DSolve[y'[x]==(x+y[x])^2/(2*x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to x \tan\left(\frac{\log(x)}{2} + c_1\right)
$$

3.3 problem 11

Internal problem ID [2066]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.8, page 68
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\sin\left(\frac{y}{x}\right)(y'x - y) - x\cos\left(\frac{y}{x}\right) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(sin(y(x)/x)*(x*diff(y(x),x)-y(x))=x*cos(y(x)/x),y(x), singsol=all)
```

$$y(x) = x \arccos\left(\frac{1}{c_1 x}\right)$$

✓ Solution by Mathematica

Time used: 24.563 (sec). Leaf size: 48

```
DSolve[Sin[y[x]/x]*(x*y'[x]-y[x])==x*Cos[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sec^{-1}\left(e^{c_1}x\right)$$
$$y(x) \to x\sec^{-1}\left(e^{c_1}x\right)$$
$$y(x) \to -\frac{\pi x}{2}$$
$$y(x) \to \frac{\pi x}{2}$$

3.4 problem 12

Internal problem ID [2067]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.8, page 68
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$y'x - \sqrt{16x^2 - y^2} - y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x)=sqrt(16*x^2-y(x)^2)+y(x),y(x), singsol=all)
```

$$- \arctan\left(\frac{y(x)}{\sqrt{16x^2 - y(x)^2}}\right) + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.384 (sec). Leaf size: 18

```
DSolve[x*y'[x]==Sqrt[16*x^2-y[x]^2]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -4x \cosh(i \log(x) + c_1)$$

3.5 problem 13

Internal problem ID [2068]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.8, page 68
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$ y'x - y - \sqrt{9x^2 + y^2} = 0 $$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(9*x^2+y(x)^2),y(x), singsol=all)
```

$$ \frac{y(x)}{x^2} + \frac{\sqrt{9x^2 + y(x)^2}}{x^2} - c_1 = 0 $$

✓ Solution by Mathematica

Time used: 0.351 (sec). Leaf size: 27

```
DSolve[x*y'[x]-y[x]==Sqrt[9*x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to \frac{9e^{c_1}x^2}{2} - \frac{e^{-c_1}}{2} $$

3.6   problem 14

Internal problem ID [2069]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.8, page 68
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$x\left(x^2 - y^2\right) - x\left(x^2 + y^2\right) y' = 0$$

✔   Solution by Maple
Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x*(x^2-y(x)^2)-x*(x^2+y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(\int^{-Z} \frac{-a^2 + 1}{-a^3 + -a^2 + -a - 1} d\_a + \ln\left(x\right) + c_1\right) x$$

✔   Solution by Mathematica
Time used: 0.126 (sec). Leaf size: 71

```
DSolve[x*(x^2-y[x]^2)-x*(x^2+y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{RootSum}\left[\#1^3 + \#1^2 + \#1 - 1\&, \frac{\#1^2 \log\left(\frac{y(x)}{x} - \#1\right) + \log\left(\frac{y(x)}{x} - \#1\right)}{3\#1^2 + 2\#1 + 1}\&\right] = \right.$$

$$\left. - \log(x) + c_1, y(x)\right]$$

3.7 problem 15

Internal problem ID [2070]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.8, page 68
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$y'x + \ln(x)\,y - y \ln(y) = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)+y(x)*ln(x)=y(x)*ln(y(x)),y(x), singsol=all)
```

$$y(x) = x\,\mathrm{e}^{-c_1 x}\mathrm{e}$$

✓ Solution by Mathematica

Time used: 0.235 (sec). Leaf size: 24

```
DSolve[x*y'[x]+y[x]*Log[x]==y[x]*Log[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to xe^{1+e^{c_1}x}$$
$$y(x) \to ex$$

3.8 problem 16

Internal problem ID [2071]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.8, page 68
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$ y' - \frac{y^2 + 2yx - 2x^2}{x^2 - yx + y^2} = 0 $$

✓ Solution by Maple

Time used: 0.579 (sec). Leaf size: 79

```
dsolve(diff(y(x),x)= (y(x)^2+2*x*y(x)-2*x^2)/(x^2-x*y(x)+y(x)^2),y(x), singsol=all)
```

$$ y(x) = -\frac{x\left(\text{RootOf}\left(2\_Z^6 + (9c_1 x^2 - 1)\_Z^4 - 6x^2 c_1 \_Z^2 + c_1 x^2\right)^2 - 1\right)}{\text{RootOf}\left(2\_Z^6 + (9c_1 x^2 - 1)\_Z^4 - 6x^2 c_1 \_Z^2 + c_1 x^2\right)^2} $$

Solution by Mathematica

Time used: 60.178 (sec). Leaf size: 372

```
DSolve[y'[x]== (y[x]^2+2*x*y[x]-2*x^2)/(x^2-x*y[x]+y[x]^2),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{\sqrt[3]{-54x^3 + 2\sqrt{729x^6 + (-9x^2 + 3e^{2c_1})^3}}}{3\sqrt[3]{2}}$$
$$- \frac{\sqrt[3]{2}(-3x^2 + e^{2c_1})}{\sqrt[3]{-54x^3 + 2\sqrt{729x^6 + (-9x^2 + 3e^{2c_1})^3}}} + x$$

$$y(x) \to \frac{\left(1 + i\sqrt{3}\right)\left(-3x^2 + e^{2c_1}\right)}{2^{2/3}\sqrt[3]{-54x^3 + 2\sqrt{729x^6 + (-9x^2 + 3e^{2c_1})^3}}}$$
$$+ \left(-\frac{1}{3}\right)^{2/3}\sqrt[3]{-9x^3 + \sqrt{3}\sqrt{27e^{2c_1}x^4 - 9e^{4c_1}x^2 + e^{6c_1}}} + x$$

$$y(x) \to -\frac{\left(1 + i\sqrt{3}\right)\sqrt[3]{-54x^3 + 2\sqrt{729x^6 + (-9x^2 + 3e^{2c_1})^3}}}{6\sqrt[3]{2}}$$
$$+ \frac{\left(1 - i\sqrt{3}\right)\left(-3x^2 + e^{2c_1}\right)}{2^{2/3}\sqrt[3]{-54x^3 + 2\sqrt{729x^6 + (-9x^2 + 3e^{2c_1})^3}}} + x$$

3.9 problem 17

Internal problem ID [2072]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.8, page 68
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A']]`

$$2xyy' - 2y^2 - x^2 e^{-\frac{y^2}{x^2}} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 26

```
dsolve(2*x*y(x)*diff(y(x),x)-(x^2*exp(-y(x)^2/x^2)+2*y(x)^2)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{\ln\left(\ln\left(x\right) + c_1\right)}\, x$$
$$y(x) = -\sqrt{\ln\left(\ln\left(x\right) + c_1\right)}\, x$$

✓ Solution by Mathematica

Time used: 2.171 (sec). Leaf size: 38

```
DSolve[2*x*y[x]*y'[x]-(x^2*Exp[-y[x]^2/x^2]+2*y[x]^2)==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -x\sqrt{\log(\log(x) + 2c_1)}$$
$$y(x) \to x\sqrt{\log(\log(x) + 2c_1)}$$

3.10   problem 18

Internal problem ID [2073]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.8, page 68
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Riccati]`

$$y'x^2 - y^2 - 3yx - x^2 = 0$$

✔ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x)=y(x)^2+3*x*y(x)+x^2,y(x), singsol=all)
```

$$y(x) = -\frac{x(\ln(x) + c_1 + 1)}{\ln(x) + c_1}$$

✔ Solution by Mathematica

Time used: 0.144 (sec). Leaf size: 25

```
DSolve[x^2*y'[x]==y[x]^2+3*x*y[x]+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(-1 - \frac{1}{\log(x) + c_1}\right)$$

$$y(x) \to -x$$

3.11 problem 19

Internal problem ID [2074]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.8, page 68
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$ yy' + x - \sqrt{x^2 + y^2} = 0 $$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 28

```
dsolve(y(x)*diff(y(x),x)=sqrt(x^2+y(x)^2)-x,y(x), singsol=all)
```

$$ -c_1 + \frac{\sqrt{x^2 + y(x)^2}}{y(x)^2} + \frac{x}{y(x)^2} = 0 $$

✓ Solution by Mathematica

Time used: 0.38 (sec). Leaf size: 57

```
DSolve[y[x]*y'[x]==Sqrt[x^2+y[x]^2]-x,y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to -e^{\frac{c_1}{2}} \sqrt{2x + e^{c_1}} $$
$$ y(x) \to e^{\frac{c_1}{2}} \sqrt{2x + e^{c_1}} $$
$$ y(x) \to 0 $$

3.12 problem 20

Internal problem ID [2075]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.8, page 68
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cla`

$$2x(2x + y)\, y' - y(4x - y) = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve(2*x*(y(x)+2*x)*diff(y(x),x)=y(x)*(4*x-y(x)),y(x), singsol=all)
```

$$y(x) = e^{\text{LambertW}\left(2\,e^{\frac{3c_1}{2}} x^{\frac{3}{2}}\right) - \frac{3c_1}{2} - \frac{3\ln(x)}{2}} x$$

✓ Solution by Mathematica

Time used: 5.13 (sec). Leaf size: 29

```
DSolve[2*x*(y[x]+2*x)*y'[x]==y[x]*(4*x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x}{W\left(2e^{-c_1} x^{3/2}\right)}$$

$$y(x) \to 0$$

3.13 problem 21

Internal problem ID [2076]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.8, page 68
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$y'x - \tan\left(\frac{y}{x}\right)x - y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 10

```
dsolve(x*diff(y(x),x)=x*tan(y(x)/x)+y(x),y(x), singsol=all)
```

$$y(x) = \arcsin(c_1 x) x$$

✓ Solution by Mathematica

Time used: 4.341 (sec). Leaf size: 19

```
DSolve[x*y'[x]==x*Tan[y[x]/x]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin(e^{c_1} x)$$
$$y(x) \to 0$$

3.14 problem 22

Internal problem ID [2077]

Book: Differential equations and linear algebra, Stephen W. Goode, second edition, 2000
Section: 1.8, page 68
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$ y' - \frac{\sqrt{x^2 + y^2}\, x + y^2}{yx} = 0 $$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=(x*sqrt(x^2+y(x)^2)+y(x)^2)/(x*y(x)),y(x), singsol=all)
```

$$ -\frac{\sqrt{x^2 + y(x)^2}}{x} + \ln(x) - c_1 = 0 $$

✓ Solution by Mathematica
Time used: 0.284 (sec). Leaf size: 48

```
DSolve[y'[x]==(x*Sqrt[x^2+y[x]^2]+y[x]^2)/(x*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to -x\sqrt{(\log(x) - 1 + c_1)(\log(x) + 1 + c_1)} $$
$$ y(x) \to x\sqrt{(\log(x) - 1 + c_1)(\log(x) + 1 + c_1)} $$