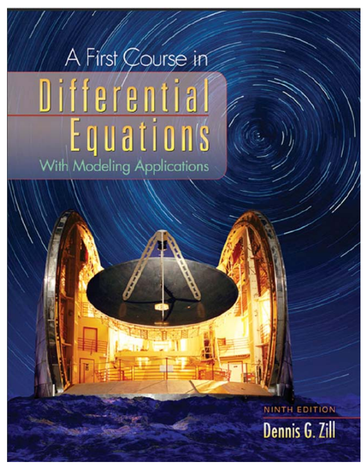
A Solution Manual For

A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.



Nasser M. Abbasi

October 12, 2023

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1 Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230

1.1 problem 15 (x=0)

Internal problem ID [4779]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 15 (x=0).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-25)\,y''+2y'x+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((x^2-25)*diff(y(x),x$2)+2*x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{1}{50}x^2+\frac{7}{15000}x^4\right)y(0)+\left(x+\frac{1}{50}x^3+\frac{13}{25000}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(x^2-25)*y''[x]+2*x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{13x^5}{25000}+\frac{x^3}{50}+x\right)+c_1\left(\frac{7x^4}{15000}+\frac{x^2}{50}+1\right)$$

1.2 problem 15 (x=1)

Internal problem ID [4780]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 15 (x=1).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-25)\, y''+2y'x+y=0}$$

With the expansion point for the power series method at $x=1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((x^2-25)*diff(y(x),x$2)+2*x*diff(y(x),x)+y(x)=0,y(x),type='series',x=1);
```

$$y(x)=\left(1+\frac{(x-1)^2}{48}+\frac{(x-1)^3}{864}+\frac{(x-1)^4}{1728}+\frac{29(x-1)^5}{414720}\right)y(1)+\left(x-1+\frac{(x-1)^2}{24}+\frac{5(x-1)^3}{216}+\frac{17(x-1)^4}{6912}+\frac{41(x-1)^5}{51840}\right)D(y)(1)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 87

```
AsymptoticDSolveValue[(x^2-25)*y''[x]+2*x*y'[x]+y[x]==0,y[x],{x,1,5}]
```

$$y(x)\to c_1\left(\frac{29(x-1)^5}{414720}+\frac{(x-1)^4}{1728}+\frac{1}{864}(x-1)^3+\frac{1}{48}(x-1)^2+1\right)+c_2\left(\frac{41(x-1)^5}{51840}+\frac{17(x-1)^4}{6912}+\frac{5}{216}(x-1)^3+\frac{1}{24}(x-1)^2+x-1\right)$$

1.3 problem 16 (x=0)

Internal problem ID [4781]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 16 (x=0).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-2x+10)\,y''+y'x-4y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve((x^2-2*x+10)*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{1}{5}x^2+\frac{1}{75}x^3+\frac{1}{750}x^4-\frac{13}{75000}x^5\right)y(0)+\left(x+\frac{1}{20}x^3+\frac{1}{200}x^4-\frac{13}{20000}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[(x^2-2*x+10)*y''[x]+x*y'[x]-4*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(-\frac{13x^5}{20000}+\frac{x^4}{200}+\frac{x^3}{20}+x\right)+c_1\left(-\frac{13x^5}{75000}+\frac{x^4}{750}+\frac{x^3}{75}+\frac{x^2}{5}+1\right)$$

1.4 problem 16 (x=1)

Internal problem ID [4782]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 16 (x=1).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x^2 - 2x + 10)\, y'' + y'x - 4y = 0$$

With the expansion point for the power series method at $x = 1$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((x^2-2*x+10)*diff(y(x),x$2)+x*diff(y(x),x)-4*y(x)=0,y(x),type='series',x=1);
```

$$y(x) = \left(1 + \frac{2(x-1)^2}{9} - \frac{2(x-1)^3}{243} + \frac{(x-1)^4}{4374} + \frac{22(x-1)^5}{98415}\right) y(1) + \left(x - 1 - \frac{(x-1)^2}{18} + \frac{14(x-1)^3}{243} - \frac{7(x-1)^4}{4374} - \frac{154(x-1)^5}{98415}\right) D(y)(1) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 87

```
AsymptoticDSolveValue[(x^2-2*x+10)*y''[x]+x*y'[x]-4*y[x]==0,y[x],{x,1,5}]
```

$$y(x) \to c_1\left(\frac{22(x-1)^5}{98415} + \frac{(x-1)^4}{4374} - \frac{2}{243}(x-1)^3 + \frac{2}{9}(x-1)^2 + 1\right) + c_2\left(-\frac{154(x-1)^5}{98415} - \frac{7(x-1)^4}{4374} + \frac{14}{243}(x-1)^3 - \frac{1}{18}(x-1)^2 + x - 1\right)$$

1.5 problem 17

Internal problem ID [4783]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^3}{6}\right) y(0) + \left(x + \frac{1}{12}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(\frac{x^4}{12} + x\right) + c_1 \left(\frac{x^3}{6} + 1\right)$$

1.6 problem 18

Internal problem ID [4784]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' + x^2 y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^4}{12}\right) y(0) + \left(x - \frac{1}{20}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(x - \frac{x^5}{20}\right) + c_1 \left(1 - \frac{x^4}{12}\right)$$

1.7 problem 19

Internal problem ID [4785]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{y'' - 2y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)-2*x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 - \frac{1}{8}x^4\right) y(0) + \left(x + \frac{1}{6}x^3 + \frac{1}{24}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]-2*x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{24} + \frac{x^3}{6} + x\right) + c_1\left(-\frac{x^4}{8} - \frac{x^2}{2} + 1\right)$$

1.8 problem 20

Internal problem ID [4786]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Hermite]`

$$\boxed{y'' - y'x + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve(diff(y(x),x$2)-x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(-x^2 + 1\right) y(0) + \left(x - \frac{1}{6}x^3 - \frac{1}{120}x^5\right) D(y)(0) + O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 33

```
AsymptoticDSolveValue[y''[x]-x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(1 - x^2\right) + c_2\left(-\frac{x^5}{120} - \frac{x^3}{6} + x\right)$$

1.9 problem 21

Internal problem ID [4787]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + x^2 y' + xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)+x^2*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^3}{6}\right) y(0) + \left(x - \frac{1}{6}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]+x^2*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2\left(x - \frac{x^4}{6}\right) + c_1\left(1 - \frac{x^3}{6}\right)$$

1.10 problem 22

Internal problem ID [4788]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + 2y'x + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+2*x*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - x^2 + \frac{1}{2}x^4\right) y(0) + \left(x - \frac{2}{3}x^3 + \frac{4}{15}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 40

```
AsymptoticDSolveValue[y''[x]+2*x*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{4x^5}{15} - \frac{2x^3}{3} + x\right) + c_1\left(\frac{x^4}{2} - x^2 + 1\right)$$

1.11 problem 23

Internal problem ID [4789]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(x-1)\,y''+y'=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((x-1)*diff(y(x),x$2)+diff(y(x),x)=0,y(x),type='series',x=0);
```

$$y(x)=y(0)+\left(x+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{4}x^4+\frac{1}{5}x^5\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 39

```
AsymptoticDSolveValue[(x-1)*y''[x]+y'[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{x^5}{5}+\frac{x^4}{4}+\frac{x^3}{3}+\frac{x^2}{2}+x\right)+c_1$$

1.12 problem 24

Internal problem ID [4790]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2+x)\, y'' + y'x - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve((x+2)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{4}x^2 - \frac{1}{24}x^3 + \frac{1}{480}x^5\right) y(0) + D(y)\,(0)\, x + O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[(x+2)*y''[x]+x*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{480} - \frac{x^3}{24} + \frac{x^2}{4} + 1\right) + c_2 x$$

1.13 problem 25

Internal problem ID [4791]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' - y'(x+1) - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve(diff(y(x),x$2)-(x+1)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{6}x^4+\frac{1}{15}x^5\right)y(0)+\left(x+\frac{1}{2}x^2+\frac{1}{2}x^3+\frac{1}{4}x^4+\frac{3}{20}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 70

```
AsymptoticDSolveValue[y''[x]-(x+1)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{15}+\frac{x^4}{6}+\frac{x^3}{6}+\frac{x^2}{2}+1\right)+c_2\left(\frac{3x^5}{20}+\frac{x^4}{4}+\frac{x^3}{2}+\frac{x^2}{2}+x\right)$$

1.14 problem 26

Internal problem ID [4792]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(x^2+1)\,y''-6y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
Order:=6;
dsolve((x^2+1)*diff(y(x),x$2)-6*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(x^4+3x^2+1\right)y(0)+\left(x^3+x\right)D(y)(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 25

```
AsymptoticDSolveValue[(x^2+1)*y''[x]-6*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(x^3+x\right)+c_1\left(x^4+3x^2+1\right)$$

1.15 problem 27

Internal problem ID [4793]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+2)\,y''+3y'x-y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve((x^2+2)*diff(y(x),x$2)+3*x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{1}{4}x^2-\frac{7}{96}x^4\right)y(0)+\left(x-\frac{1}{6}x^3+\frac{7}{120}x^5\right)D(y)(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[(x^2+2)*y''[x]+3*x*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{7x^5}{120}-\frac{x^3}{6}+x\right)+c_1\left(-\frac{7x^4}{96}+\frac{x^2}{4}+1\right)$$

1.16 problem 28

Internal problem ID [4794]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2-1)\,y''+y'x-y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve((x^2-1)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{2}x^2-\frac{1}{8}x^4\right)y(0)+D(y)\,(0)\,x+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 27

```
AsymptoticDSolveValue[(x^2-1)*y''[x]+x*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{x^4}{8}-\frac{x^2}{2}+1\right)+c_2x$$

1.17 problem 29

Internal problem ID [4795]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-1)\,y''-y'x+y=0}$$

With initial conditions

$$[y(0)=-2, y'(0)=6]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(x-1)*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(0) = -2, D(y)(0) = 6],y(x),type='series'
```

$$y(x)=-2+6x-x^2-\frac{1}{3}x^3-\frac{1}{12}x^4-\frac{1}{60}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{(x-1)*y''[x]-x*y'[x]+y[x]==0,{y[0]==-2,y'[0]==6}},y[x],{x,0,5}]
```

$$y(x)\to -\frac{x^5}{60}-\frac{x^4}{12}-\frac{x^3}{3}-x^2+6x-2$$

1.18 problem 30

Internal problem ID [4796]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x+1)\,y'' - (-x+2)\,y' + y = 0}$$

With initial conditions

$$[y(0) = 2, y'(0) = -1]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([(x+1)*diff(y(x),x$2)-(2-x)*diff(y(x),x)+y(x)=0,y(0) = 2, D(y)(0) = -1],y(x),type='ser
```

$$y(x) = 2 - x - 2x^2 - \frac{1}{3}x^3 + \frac{1}{2}x^4 - \frac{1}{30}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 34

```
AsymptoticDSolveValue[{(x+1)*y''[x]-(2-x)*y'[x]+y[x]==0,{y[0]==2,y'[0]==-1}},y[x],{x,0,5}]
```

$$y(x) \to -\frac{x^5}{30} + \frac{x^4}{2} - \frac{x^3}{3} - 2x^2 - x + 2$$

1.19 problem 31

Internal problem ID [4797]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 2y'x + 8y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = 0]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([diff(y(x),x$2)-2*x*diff(y(x),x)+8*y(x)=0,y(0) = 3, D(y)(0) = 0],y(x),type='series',x=
```

$$y(x) = 3 - 12x^2 + 4x^4 + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 22

```
AsymptoticDSolveValue[{y''[x]-2*x*y''[x]+8*y[x]==0,{y[0]==3,y'[0]==0}},y[x],{x,0,5}]
```

$$y(x) \to \frac{16x^5}{5} - 8x^3 - 12x^2 + 3$$

1.20 problem 32

Internal problem ID [4798]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(x^2+1)\,y''+2y'x=0}$$

With initial conditions

$$[y(0)=0, y'(0)=1]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([(x^2+1)*diff(y(x),x$2)+2*x*diff(y(x),x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='series',x
```

$$y(x)=x-\frac{1}{3}x^3+\frac{1}{5}x^5+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{(x^2+1)*y''[x]+2*x*y'[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x)\to\frac{x^5}{5}-\frac{x^3}{3}+x$$

1.21 problem 33

Internal problem ID [4799]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y \sin(x) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve(diff(y(x),x$2)+sin(x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{6}x^3 + \frac{1}{120}x^5\right) y(0) + \left(x - \frac{1}{12}x^4\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 35

```
AsymptoticDSolveValue[y''[x]+Sin[x]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(x - \frac{x^4}{12}\right) + c_1 \left(\frac{x^5}{120} - \frac{x^3}{6} + 1\right)$$

1.22 problem 34

Internal problem ID [4800]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 34.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + e^x y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve(diff(y(x),x$2)+exp(x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 - \frac{1}{6}x^3 - \frac{1}{120}x^5\right) y(0) + \left(x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{24}x^4 + \frac{1}{120}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[y''[x]+Exp[x]*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^5}{120} - \frac{x^3}{6} + \frac{x^2}{2} + 1\right) + c_2\left(\frac{x^5}{120} - \frac{x^4}{24} + \frac{x^3}{6} - \frac{x^2}{2} + x\right)$$

1.23 problem 39

Internal problem ID [4801]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.1.2 page 230
Problem number: 39.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' + y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4\right) y(0) + \left(x - \frac{1}{3}x^3 + \frac{1}{15}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \rightarrow c_2\left(\frac{x^5}{15} - \frac{x^3}{3} + x\right) + c_1\left(\frac{x^4}{8} - \frac{x^2}{2} + 1\right)$$

2 Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239

2.1 problem 1

Internal problem ID [4802]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$x^3y'' + 4x^2y' + 3y = 0$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^3*diff(y(x),x$2)+4*x^2*diff(y(x),x)+3*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 282

```
AsymptoticDSolveValue[x^3*y''[x]+4*x^2*y'[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to \frac{c_1 e^{-\frac{2i\sqrt{3}}{\sqrt{x}}}\left(-\frac{14315125825ix^{9/2}}{8796093022208\sqrt{3}} + \frac{8083075ix^{7/2}}{4294967296\sqrt{3}} - \frac{15015i\sqrt{3}x^{5/2}}{8388608} + \frac{385i\sqrt{3}x^{3/2}}{8192} + \frac{930483178625x^5}{844424930131968} - \frac{509233725x^4}{549755813888} + \frac{425425}{2684354} \right.}{x^{5/4}} + \frac{c_2 e^{\frac{2i\sqrt{3}}{\sqrt{x}}}\left(\frac{14315125825ix^{9/2}}{8796093022208\sqrt{3}} - \frac{8083075ix^{7/2}}{4294967296\sqrt{3}} + \frac{15015i\sqrt{3}x^{5/2}}{8388608} - \frac{385i\sqrt{3}x^{3/2}}{8192} + \frac{930483178625x^5}{844424930131968} - \frac{509233725x^4}{549755813888} + \frac{425425x^3}{26843545} \right.}{x^{5/4}}$$

2.2 problem 2

Internal problem ID [4803]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x+3)^2 y'' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 58

```
Order:=6;
dsolve(x*(x+3)^2*diff(y(x),x$2)-y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x \left(1 + \frac{1}{18}x - \frac{11}{972}x^2 + \frac{277}{104976}x^3 - \frac{12539}{18895680}x^4 + \frac{893821}{5101833600}x^5 + \mathrm{O}\left(x^6\right)\right) \\ & + c_2 \left(\ln(x)\left(\frac{1}{9}x + \frac{1}{162}x^2 - \frac{11}{8748}x^3 + \frac{277}{944784}x^4 - \frac{12539}{170061120}x^5 + \mathrm{O}\left(x^6\right)\right) \right. \\ & \left. + \left(1 - \frac{5}{108}x^2 + \frac{167}{26244}x^3 - \frac{13583}{11337408}x^4 + \frac{1327279}{5101833600}x^5 + \mathrm{O}\left(x^6\right)\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 87

```
AsymptoticDSolveValue[x*(x+3)^2*y''[x]-y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \to {} & c_1 \left(\frac{x(277x^3 - 1188x^2 + 5832x + 104976)\log(x)}{944784} \right. \\ & \left. + \frac{3037x^4 + 864x^3 - 174960x^2 + 6298560x + 11337408}{11337408}\right) \\ & + c_2 \left(-\frac{12539x^5}{18895680} + \frac{277x^4}{104976} - \frac{11x^3}{972} + \frac{x^2}{18} + x\right) \end{aligned}$$

2.3 problem 3

Internal problem ID [4804]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-9)^2 y'' + (x+3)\, y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((x^2-9)^2*diff(y(x),x$2)+(x+3)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{81}x^2 + \frac{1}{6561}x^3 - \frac{289}{708588}x^4 + \frac{304}{23914845}x^5\right) y(0) + \left(x - \frac{1}{54}x^2 - \frac{13}{2187}x^3 - \frac{131}{236196}x^4 - \frac{596}{1594323}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 70

```
AsymptoticDSolveValue[(x^2-9)^2*y''[x]+(x+3)*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{304x^5}{23914845} - \frac{289x^4}{708588} + \frac{x^3}{6561} - \frac{x^2}{81} + 1\right) + c_2\left(-\frac{596x^5}{1594323} - \frac{131x^4}{236196} - \frac{13x^3}{2187} - \frac{x^2}{54} + x\right)$$

2.4 problem 4

Internal problem ID [4805]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - \frac{y'}{x} + \frac{y}{(x-1)^3} = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 52

```
Order:=6;
dsolve(diff(y(x),x$2)-1/x*diff(y(x),x)+1/(x-1)^3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^2 \left(1 + \frac{1}{8}x^2 + \frac{1}{5}x^3 + \frac{49}{192}x^4 + \frac{423}{1400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(\ln(x)\left(-x^2 - \frac{1}{8}x^4 - \frac{1}{5}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(-2 - 2x^3 - \frac{45}{32}x^4 - \frac{34}{25}x^5 + \mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 71

```
AsymptoticDSolveValue[y''[x]-1/x*y'[x]+1/(x-1)^3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{1}{16}\left(x^2+8\right) x^2 \log(x) + \frac{1}{64}\left(-5x^4+64x^3-400x^2+64\right)\right) + c_2 \left(\frac{49x^6}{192} + \frac{x^5}{5} + \frac{x^4}{8} + x^2\right)$$

2.5 problem 5

Internal problem ID [4806]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^3+4x)\,y''-2y'x+6y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 60

```
Order:=6;
dsolve((x^3+4*x)*diff(y(x),x$2)-2*x*diff(y(x),x)+6*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned}y(x) &= c_1x\left(1-\frac{1}{2}x+\frac{1}{24}x^2+\frac{1}{48}x^3-\frac{1}{384}x^4-\frac{5}{2304}x^5+\mathrm{O}\left(x^6\right)\right)\\&\quad+c_2\left(\ln(x)\left(-\frac{3}{2}x+\frac{3}{4}x^2-\frac{1}{16}x^3-\frac{1}{32}x^4+\frac{1}{256}x^5+\mathrm{O}\left(x^6\right)\right)\right.\\&\quad\left.+\left(1+\frac{1}{2}x-\frac{7}{4}x^2+\frac{31}{96}x^3+\frac{1}{24}x^4-\frac{67}{3072}x^5+\mathrm{O}\left(x^6\right)\right)\right)\end{aligned}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 85

```
AsymptoticDSolveValue[(x^3+4*x)*y''[x]-2*x*y'[x]+6*y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned}y(x) \to\, & c_1\left(\frac{1}{96}(7x^4+37x^3-240x^2+192x+96)-\frac{1}{32}x(x^3+2x^2-24x+48)\log(x)\right)\\&+c_2\left(-\frac{x^5}{384}+\frac{x^4}{48}+\frac{x^3}{24}-\frac{x^2}{2}+x\right)\end{aligned}$$

2.6 problem 6

Internal problem ID [4807]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2(x-5)^2 y'' + 4y'x + (x^2-25)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 1179

```
Order:=6;
dsolve(x^2*(x-5)^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(x^2-25)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = x^{\frac{21}{50}}\left(c_1 x^{-\frac{\sqrt{2941}}{50}}\left(1+\frac{-1166-4\sqrt{2941}}{-3125+125\sqrt{2941}}x-\frac{9}{15625}\frac{879\sqrt{2941}-79709}{\left(-25+\sqrt{2941}\right)\left(-50+\sqrt{2941}\right)}x^2 + \frac{\frac{15291084\sqrt{2941}}{1953125}-\frac{906742764}{1953125}}{\left(-25+\sqrt{2941}\right)\left(-50+\sqrt{2941}\right)\left(-75+\sqrt{2941}\right)}x^3 - \frac{12}{244140625}\frac{-122814219551+2200649681\sqrt{2941}}{\left(-25+\sqrt{2941}\right)\left(-50+\sqrt{2941}\right)\left(-75+\sqrt{2941}\right)\left(-100+\sqrt{2941}\right)}x^4 + \frac{-\frac{10008934775328384}{152587890625}+\frac{181292058002304\sqrt{2941}}{152587890625}}{\left(-25+\sqrt{2941}\right)\left(-50+\sqrt{2941}\right)\left(-75+\sqrt{2941}\right)\left(-100+\sqrt{2941}\right)\left(-125+\sqrt{2941}\right)}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 x^{\frac{\sqrt{2941}}{50}}\left(1+\frac{1166-4\sqrt{2941}}{125\sqrt{2941}+3125}x+\frac{\frac{7911\sqrt{2941}}{15625}+\frac{717381}{15625}}{\left(\sqrt{2941}+25\right)\left(50+\sqrt{2941}\right)}x^2 + \frac{\frac{15291084\sqrt{2941}}{1953125}+\frac{906742764}{1953125}}{\left(\sqrt{2941}+25\right)\left(50+\sqrt{2941}\right)\left(\sqrt{2941}+75\right)}x^3 + \frac{\frac{1473770634612}{244140625}+\frac{26407796172\sqrt{2941}}{244140625}}{\left(\sqrt{2941}+25\right)\left(50+\sqrt{2941}\right)\left(\sqrt{2941}+75\right)\left(100+\sqrt{2941}\right)}x^4 + \frac{\frac{10008934775328384}{152587890625}+\frac{181292058002304\sqrt{2941}}{152587890625}}{\left(\sqrt{2941}+25\right)\left(50+\sqrt{2941}\right)\left(\sqrt{2941}+75\right)\left(100+\sqrt{2941}\right)\left(125+\sqrt{2941}\right)}x^5 + \mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 5384

```
AsymptoticDSolveValue[x^2*(x-5)^2*y''[x]+4*x*y'[x]+(x^2-25)*y[x]==0,y[x],{x,0,5}]
```

Too large to display

2.7 problem 7

Internal problem ID [4808]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+x-6)\,y''+(x+3)\,y'+(x-2)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
Order:=6;
dsolve((x^2+x-6)*diff(y(x),x$2)+(x+3)*diff(y(x),x)+(x-2)*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = &\left(1-\frac{1}{6}x^2-\frac{1}{108}x^3-\frac{17}{2592}x^4-\frac{7}{2160}x^5\right)y(0) \\ &+\left(x+\frac{1}{4}x^2+\frac{1}{36}x^3+\frac{23}{864}x^4+\frac{13}{1440}x^5\right)D(y)(0)+O\left(x^6\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 70

```
AsymptoticDSolveValue[(x^2+x-6)*y''[x]+(x+3)*y'[x]+(x-2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{7x^5}{2160}-\frac{17x^4}{2592}-\frac{x^3}{108}-\frac{x^2}{6}+1\right)+c_2\left(\frac{13x^5}{1440}+\frac{23x^4}{864}+\frac{x^3}{36}+\frac{x^2}{4}+x\right)$$

2.8 problem 8

Internal problem ID [4809]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x(x^2+1)^2 y''+y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 58

```
Order:=6;
dsolve(x*(x^2+1)^2*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x\left(1-\frac{1}{2}x+\frac{1}{12}x^2+\frac{23}{144}x^3-\frac{167}{2880}x^4-\frac{7993}{86400}x^5+\mathrm{O}\left(x^6\right)\right) \\ & + c_2\left(\ln(x)\left(-x+\frac{1}{2}x^2-\frac{1}{12}x^3-\frac{23}{144}x^4+\frac{167}{2880}x^5+\mathrm{O}\left(x^6\right)\right)\right. \\ & \left.+\left(1-\frac{3}{4}x^2+\frac{19}{36}x^3+\frac{85}{1728}x^4-\frac{21907}{86400}x^5+\mathrm{O}\left(x^6\right)\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 87

```
AsymptoticDSolveValue[x*(x^2+1)^2*y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$\begin{aligned} y(x) \to {} & c_1\left(\frac{361x^4+1056x^3-2160x^2+1728x+1728}{1728}\right. \\ & \left.-\frac{1}{144}x(23x^3+12x^2-72x+144)\log(x)\right)+c_2\left(-\frac{167x^5}{2880}+\frac{23x^4}{144}+\frac{x^3}{12}-\frac{x^2}{2}+x\right) \end{aligned}$$

2.9 problem 9

Internal problem ID [4810]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^3(x^2-25)(x-2)^2y''+3x(x-2)y'+7(5+x)y=0$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^3*(x^2-25)*(x-2)^2*diff(y(x),x$2)+3*x*(x-2)*diff(y(x),x)+7*(x+5)*y(x)=0,y(x),type='s
```

No solution found

✓ Solution by Mathematica

Time used: 0.088 (sec). Leaf size: 99

```
AsymptoticDSolveValue[x^3*(x^2-25)*(x-2)^2*y''[x]+3*x*(x-2)*y'[x]+7*(x+5)*y[x]==0,y[x],{x,0,5
```

$$y(x)\to c_2\left(-\frac{1337698720169782190618881x^5}{352638738432}+\frac{4284030153765326450 5x^4}{3265173504}-\frac{344729362309955x^3}{7558272}+\frac{3590248795x^2}{23328}-\frac{50309x}{108}+1\right)x^{35/6}+\frac{c_1e^{\frac{3}{50}/x}\left(-\frac{3790719800856046344847395276564 2999x^5}{5380840125000000000000000000}+\frac{274978743503260899898231806 01x^4}{797161500000000000000000}+\frac{1064989877173148278170 1x^3}{14762250000000000}+97\right.}{x^{1159/300}}$$

2.10 problem 10

Internal problem ID [4811]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(x^{3}-2x^{2}+3x\right)^{2} y''+x(x-3)^{2} y'-(x+1)\, y=0$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve((x^3-2*x^2+3*x)^2*diff(y(x),x$2)+x*(x-3)^2*diff(y(x),x)-(x+1)*y(x)=0,y(x),type='series
```

$$y(x) = \frac{c_2 x^{\frac{2}{3}}\left(1+\frac{1}{45}x+\frac{149}{3240}x^2+\frac{2701}{192456}x^3+\frac{236933}{121247280}x^4-\frac{67092967}{92754169200}x^5+\mathrm{O}\left(x^6\right)\right)+c_1\left(1+\frac{13}{9}x-\frac{5}{162}x^2+\frac{1591}{30618}x^3\right.}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 90

```
AsymptoticDSolveValue[(x^3-2*x^2+3*x)^2*y''[x]+x*(x-3)^2*y'[x]-(x+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\sqrt[3]{x}\left(-\frac{67092967x^5}{92754169200}+\frac{236933x^4}{121247280}+\frac{2701x^3}{192456}+\frac{149x^2}{3240}+\frac{x}{45}+1\right)+\frac{c_2\left(\frac{7435523x^5}{3224075400}+\frac{106583x^4}{5511240}+\frac{1591x^3}{30618}-\frac{5x^2}{162}+\frac{13x}{9}+1\right)}{\sqrt[3]{x}}$$

2.11 problem 11

Internal problem ID [4812]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-1)\,y''+5y'(x+1)+(x^2-x)\,y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve((x^2-1)*diff(y(x),x$2)+5*(x+1)*diff(y(x),x)+(x^2-x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1-\frac{1}{6}x^3-\frac{1}{8}x^4-\frac{3}{10}x^5\right)y(0)+\left(x+\frac{5}{2}x^2+5x^3+\frac{26}{3}x^4+\frac{1661}{120}x^5\right)D(y)\,(0)+O\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 61

```
AsymptoticDSolveValue[(x^2-1)*y''[x]+5*(x+1)*y'[x]+(x^2-x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{3x^5}{10}-\frac{x^4}{8}-\frac{x^3}{6}+1\right)+c_2\left(\frac{1661x^5}{120}+\frac{26x^4}{3}+5x^3+\frac{5x^2}{2}+x\right)$$

2.12 problem 12

Internal problem ID [4813]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (x+3)\,y' + 7x^2 y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 56

```
Order:=6;
dsolve(x*diff(y(x),x$2)+(x+3)*diff(y(x),x)+7*x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 - \frac{7}{15}x^3 + \frac{7}{120}x^4 - \frac{1}{150}x^5 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(\ln\left(x\right)\left(2x^2 - \frac{14}{15}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(-2 + 4x - 3x^2 + 4x^3 - 4x^4 + \frac{547}{225}x^5 + \mathrm{O}\left(x^6\right)\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 55

```
AsymptoticDSolveValue[x*y''[x]+(x+3)*y'[x]+7*x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{7x^4}{120} - \frac{7x^3}{15} + 1\right) + c_1\left(\frac{2x^4 - 2x^3 + 2x^2 - 2x + 1}{x^2} - \log(x)\right)$$

2.13 problem 13

Internal problem ID [4814]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + \left(\frac{5}{3}x + x^2\right)y' - \frac{y}{3} = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+(5/3*x+x^2)*diff(y(x),x)-1/3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{4}{3}}\left(1 - \frac{1}{7}x + \frac{1}{35}x^2 - \frac{1}{195}x^3 + \frac{1}{1248}x^4 - \frac{1}{9120}x^5 + \mathrm{O}\left(x^6\right)\right) + c_1\left(1 - 3x + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]+(5/3*x+x^2)*y'[x]-1/3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt[3]{x}\left(-\frac{x^5}{9120} + \frac{x^4}{1248} - \frac{x^3}{195} + \frac{x^2}{35} - \frac{x}{7} + 1\right) + \frac{c_2(1 - 3x)}{x}$$

2.14 problem 14

Internal problem ID [4815]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y' + 10y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+10*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x)\, c_2 + c_1)\left(1 - 10x + 25x^2 - \frac{250}{9}x^3 + \frac{625}{36}x^4 - \frac{125}{18}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(20x - 75x^2 + \frac{2750}{27}x^3 - \frac{15625}{216}x^4 + \frac{3425}{108}x^5 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 105

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+10*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{125x^5}{18} + \frac{625x^4}{36} - \frac{250x^3}{9} + 25x^2 - 10x + 1\right) + c_2\left(\frac{3425x^5}{108} - \frac{15625x^4}{216} + \frac{2750x^3}{27} - 75x^2 + \left(-\frac{125x^5}{18} + \frac{625x^4}{36} - \frac{250x^3}{9} + 25x^2 - 10x + 1\right)\log(x) + 20x\right)$$

2.15 problem 15

Internal problem ID [4816]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{2xy'' - y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{3}{2}} \left(1 - \frac{2}{5}x + \frac{2}{35}x^2 - \frac{4}{945}x^3 + \frac{2}{10395}x^4 - \frac{4}{675675}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 + 2x - 2x^2 + \frac{4}{9}x^3 - \frac{2}{45}x^4 + \frac{4}{1575}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 81

```
AsymptoticDSolveValue[2*x*y''[x]-y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{4x^5}{1575} - \frac{2x^4}{45} + \frac{4x^3}{9} - 2x^2 + 2x + 1\right) + c_1\left(-\frac{4x^5}{675675} + \frac{2x^4}{10395} - \frac{4x^3}{945} + \frac{2x^2}{35} - \frac{2x}{5} + 1\right) x^{3/2}$$

2.16 problem 16

Internal problem ID [4817]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2xy'' + 5y' + xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)+5*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2\left(1 - \frac{1}{14}x^2 + \frac{1}{616}x^4 + \mathrm{O}\left(x^6\right)\right)x^{\frac{3}{2}} + c_1\left(1 - \frac{1}{2}x^2 + \frac{1}{40}x^4 + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 47

```
AsymptoticDSolveValue[2*x*y''[x]+5*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4}{616} - \frac{x^2}{14} + 1\right) + \frac{c_2\left(\frac{x^4}{40} - \frac{x^2}{2} + 1\right)}{x^{3/2}}$$

2.17 problem 17

Internal problem ID [4818]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{4xy'' + \frac{y'}{2} + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(4*x*diff(y(x),x$2)+1/2*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{7}{8}} \left(1 - \frac{2}{15}x + \frac{2}{345}x^2 - \frac{4}{32085}x^3 + \frac{2}{1251315}x^4 - \frac{4}{294059025}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(1 - 2x + \frac{2}{9}x^2 - \frac{4}{459}x^3 + \frac{2}{11475}x^4 - \frac{4}{1893375}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 83

```
AsymptoticDSolveValue[4*x*y''[x]+1/2*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(-\frac{4x^5}{1893375} + \frac{2x^4}{11475} - \frac{4x^3}{459} + \frac{2x^2}{9} - 2x + 1\right) + c_1 x^{7/8} \left(-\frac{4x^5}{294059025} + \frac{2x^4}{1251315} - \frac{4x^3}{32085} + \frac{2x^2}{345} - \frac{2x}{15} + 1\right)$$

2.18 problem 18

Internal problem ID [4819]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' - y'x + y(x^2+1) = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+(x^2+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 - \frac{1}{6}x^2 + \frac{1}{168}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2x\left(1 - \frac{1}{10}x^2 + \frac{1}{360}x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 48

```
AsymptoticDSolveValue[2*x^2*y''[x]-x*y'[x]+(x^2+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1x\left(\frac{x^4}{360} - \frac{x^2}{10} + 1\right) + c_2\sqrt{x}\left(\frac{x^4}{168} - \frac{x^2}{6} + 1\right)$$

2.19 problem 19

Internal problem ID [4820]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{3xy'' + (-x+2)\,y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(3*x*diff(y(x),x$2)+(2-x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x^{\frac{1}{3}}\left(1+\frac{1}{3}x+\frac{1}{18}x^2+\frac{1}{162}x^3+\frac{1}{1944}x^4+\frac{1}{29160}x^5+\mathrm{O}\left(x^6\right)\right) \\ & + c_2\left(1+\frac{1}{2}x+\frac{1}{10}x^2+\frac{1}{80}x^3+\frac{1}{880}x^4+\frac{1}{12320}x^5+\mathrm{O}\left(x^6\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 85

```
AsymptoticDSolveValue[3*x*y''[x]+(2-x)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \sqrt[3]{x}\left(\frac{x^5}{29160}+\frac{x^4}{1944}+\frac{x^3}{162}+\frac{x^2}{18}+\frac{x}{3}+1\right)+c_2\left(\frac{x^5}{12320}+\frac{x^4}{880}+\frac{x^3}{80}+\frac{x^2}{10}+\frac{x}{2}+1\right)$$

2.20 problem 20

Internal problem ID [4821]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - \left(x - \frac{2}{9}\right) y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)-(x-2/9)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{3}} \left(1 + \frac{3}{2}x + \frac{9}{20}x^2 + \frac{9}{160}x^3 + \frac{27}{7040}x^4 + \frac{81}{492800}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 x^{\frac{2}{3}} \left(1 + \frac{3}{4}x + \frac{9}{56}x^2 + \frac{9}{560}x^3 + \frac{27}{29120}x^4 + \frac{81}{2329600}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 90

```
AsymptoticDSolveValue[x^2*y''[x]-(x-2/9)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \sqrt[3]{x} \left(\frac{81x^5}{492800} + \frac{27x^4}{7040} + \frac{9x^3}{160} + \frac{9x^2}{20} + \frac{3x}{2} + 1\right) + c_1 x^{2/3} \left(\frac{81x^5}{2329600} + \frac{27x^4}{29120} + \frac{9x^3}{560} + \frac{9x^2}{56} + \frac{3x}{4} + 1\right)$$

2.21 problem 21

Internal problem ID [4822]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 21.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{2xy'' - (3 + 2x)\, y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)-(3+2*x)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{5}{2}} \left(1 + \frac{4}{7}x + \frac{4}{21}x^2 + \frac{32}{693}x^3 + \frac{80}{9009}x^4 + \frac{64}{45045}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 + \frac{1}{3}x - \frac{1}{6}x^2 - \frac{1}{6}x^3 - \frac{5}{72}x^4 - \frac{7}{360}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 85

```
AsymptoticDSolveValue[2*x*y''[x]-(3+2*x)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(-\frac{7x^5}{360} - \frac{5x^4}{72} - \frac{x^3}{6} - \frac{x^2}{6} + \frac{x}{3} + 1\right) + c_1\left(\frac{64x^5}{45045} + \frac{80x^4}{9009} + \frac{32x^3}{693} + \frac{4x^2}{21} + \frac{4x}{7} + 1\right) x^{5/2}$$

2.22 problem 22

Internal problem ID [4823]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 22.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + y'x + \left(x^2 - \frac{4}{9}\right)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-4/9)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{4}{3}} \left(1 - \frac{3}{20}x^2 + \frac{9}{1280}x^4 + \mathrm{O}\left(x^6\right)\right) + c_1\left(1 - \frac{3}{4}x^2 + \frac{9}{128}x^4 + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 52

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-4/9)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x^{2/3}\left(\frac{9x^4}{1280} - \frac{3x^2}{20} + 1\right) + \frac{c_2\left(\frac{9x^4}{128} - \frac{3x^2}{4} + 1\right)}{x^{2/3}}$$

2.23 problem 23

Internal problem ID [4824]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2y'' + 9x^2y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(9*x^2*diff(y(x),x$2)+9*x^2*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^{\frac{1}{3}} \left(1 - \frac{1}{2}x + \frac{1}{5}x^2 - \frac{7}{120}x^3 + \frac{7}{528}x^4 - \frac{13}{5280}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2 x^{\frac{2}{3}} \left(1 - \frac{1}{2}x + \frac{5}{28}x^2 - \frac{1}{21}x^3 + \frac{11}{1092}x^4 - \frac{11}{6240}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 90

```
AsymptoticDSolveValue[9*x^2*y''[x]+9*x^2*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \sqrt[3]{x} \left(-\frac{13x^5}{5280} + \frac{7x^4}{528} - \frac{7x^3}{120} + \frac{x^2}{5} - \frac{x}{2} + 1\right) + c_1 x^{2/3} \left(-\frac{11x^5}{6240} + \frac{11x^4}{1092} - \frac{x^3}{21} + \frac{5x^2}{28} - \frac{x}{2} + 1\right)$$

2.24 problem 24

Internal problem ID [4825]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{2x^2y'' + 3y'x + (-1 + 2x)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(2*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+(2*x-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{3}{2}}\left(1 - \frac{2}{5}x + \frac{2}{35}x^2 - \frac{4}{945}x^3 + \frac{2}{10395}x^4 - \frac{4}{675675}x^5 + \mathrm{O}\left(x^6\right)\right) + c_1\left(1 + 2x - 2x^2 + \frac{4}{9}x^3 - \frac{2}{45}x^4 + \frac{4}{1575}x^5 +\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 84

```
AsymptoticDSolveValue[2*x^2*y''[x]+3*x*y'[x]+(2*x-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{4x^5}{675675} + \frac{2x^4}{10395} - \frac{4x^3}{945} + \frac{2x^2}{35} - \frac{2x}{5} + 1\right) + \frac{c_2\left(\frac{4x^5}{1575} - \frac{2x^4}{45} + \frac{4x^3}{9} - 2x^2 + 2x + 1\right)}{x}$$

2.25 problem 25

Internal problem ID [4826]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + 2y' - xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
Order:=6;
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)-x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 + \frac{1}{6}x^2 + \frac{1}{120}x^4 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 42

```
AsymptoticDSolveValue[x*y''[x]+2*y'[x]-x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^3}{24} + \frac{x}{2} + \frac{1}{x}\right) + c_2\left(\frac{x^4}{120} + \frac{x^2}{6} + 1\right)$$

2.26 problem 26

Internal problem ID [4827]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' + y'x + \left(x^2 - \frac{1}{4}\right)y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x \left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{x^{7/2}}{24} - \frac{x^{3/2}}{2} + \frac{1}{\sqrt{x}}\right) + c_2 \left(\frac{x^{9/2}}{120} - \frac{x^{5/2}}{6} + \sqrt{x}\right)$$

2.27 problem 27

Internal problem ID [4828]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 27.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{xy'' - y'x + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
Order:=6;
dsolve(x*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x \left(1 + \mathrm{O}\left(x^6\right)\right) + \left(-x + \mathrm{O}\left(x^6\right)\right) \ln\left(x\right) c_2 + \left(1 + x - \frac{1}{2}x^2 - \frac{1}{12}x^3 - \frac{1}{72}x^4 - \frac{1}{480}x^5 + \mathrm{O}\left(x^6\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 41

```
AsymptoticDSolveValue[x*y''[x]-x*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{1}{72}\left(-x^4 - 6x^3 - 36x^2 + 144x + 72\right) - x\log(x)\right) + c_2 x$$

2.28 problem 28

Internal problem ID [4829]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 28.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + \frac{3y'}{x} - 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
Order:=6;
dsolve(diff(y(x),x$2)+3/x*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1+\frac{1}{4}x^2+\frac{1}{48}x^4+\mathrm{O}\left(x^6\right)\right)x^2+c_2\left(\ln\left(x\right)\left(\left(-2\right)x^2-\frac{1}{2}x^4+\mathrm{O}\left(x^6\right)\right)+\left(-2+\frac{3}{8}x^4+\mathrm{O}\left(x^6\right)\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 57

```
AsymptoticDSolveValue[y''[x]+3/x*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^4}{48}+\frac{x^2}{4}+1\right)+c_1\left(\frac{1}{4}\left(x^2+4\right)\log(x)-\frac{5x^4+8x^2-16}{16x^2}\right)$$

2.29 problem 29

Internal problem ID [4830]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 29.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{xy'' + (1-x)\,y' - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x),x$2)+(1-x)*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x)\,c_2 + c_1)\left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(-x - \frac{3}{4}x^2 - \frac{11}{36}x^3 - \frac{25}{288}x^4 - \frac{137}{7200}x^5 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 107

```
AsymptoticDSolveValue[x*y''[x]+(1-x)*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right) + c_2\left(-\frac{137x^5}{7200} - \frac{25x^4}{288} - \frac{11x^3}{36} - \frac{3x^2}{4} + \left(\frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right)\log(x) - x\right)$$

2.30 problem 30

Internal problem ID [4831]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 30.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x)\, c_2 + c_1)\left(1 - x + \frac{1}{4}x^2 - \frac{1}{36}x^3 + \frac{1}{576}x^4 - \frac{1}{14400}x^5 + \mathrm{O}\left(x^6\right)\right) + \left(2x - \frac{3}{4}x^2 + \frac{11}{108}x^3 - \frac{25}{3456}x^4 + \frac{137}{432000}x^5 + \mathrm{O}\left(x^6\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 111

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^5}{14400} + \frac{x^4}{576} - \frac{x^3}{36} + \frac{x^2}{4} - x + 1\right) + c_2\left(\frac{137x^5}{432000} - \frac{25x^4}{3456} + \frac{11x^3}{108} - \frac{3x^2}{4} + \left(-\frac{x^5}{14400} + \frac{x^4}{576} - \frac{x^3}{36} + \frac{x^2}{4} - x + 1\right)\log(x) + 2x\right)$$

2.31 problem 31

Internal problem ID [4832]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' + (x-6)\,y' - 3y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
Order:=6;
dsolve(x*diff(y(x),x$2)+(x-6)*diff(y(x),x)-3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^7 \left(1 - \frac{1}{2}x + \frac{5}{36}x^2 - \frac{1}{36}x^3 + \frac{7}{1584}x^4 - \frac{7}{11880}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(3628800 - 1814400x + 362880x^2 - 30240x^3 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 63

```
AsymptoticDSolveValue[x*y''[x]+(x-6)*y'[x]-3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^3}{120} + \frac{x^2}{10} - \frac{x}{2} + 1\right) + c_2\left(\frac{7x^{11}}{1584} - \frac{x^{10}}{36} + \frac{5x^9}{36} - \frac{x^8}{2} + x^7\right)$$

2.32 problem 32

Internal problem ID [4833]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 32.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x(x-1)\,y''+3y'-2y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
Order:=6;
dsolve(x*(x-1)*diff(y(x),x$2)+3*diff(y(x),x)-2*y(x)=0,y(x),type='series',x=0);
```

$$\begin{aligned} y(x) = {} & c_1 x^4\left(1+2x+3x^2+4x^3+5x^4+6x^5+\mathrm{O}\left(x^6\right)\right) \\ & + c_2\left(-144-96x-48x^2+48x^4+96x^5+\mathrm{O}\left(x^6\right)\right) \end{aligned}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 55

```
AsymptoticDSolveValue[x*(x-1)*y''[x]+3*y'[x]-2*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_1\left(-\frac{x^4}{3}+\frac{x^2}{3}+\frac{2x}{3}+1\right)+c_2\left(5x^8+4x^7+3x^6+2x^5+x^4\right)$$

2.33 problem 33

Internal problem ID [4834]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 33.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^4y'' + \lambda y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^4*diff(y(x),x$2)+lambda*y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 50

```
AsymptoticDSolveValue[x^4*y''[x]+\[Lambda]*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x e^{\frac{i\sqrt{\lambda}}{x}} - \frac{i c_2 x e^{-\frac{i\sqrt{\lambda}}{x}}}{2\sqrt{\lambda}}$$

2.34 problem 36 (a)

Internal problem ID [4835]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 36 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{x^3y'' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^3*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 222

```
AsymptoticDSolveValue[x^3*y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 e^{-\frac{2i}{\sqrt{x}}} x^{3/4} \left(-\frac{468131288625 i x^{9/2}}{8796093022208} + \frac{66891825 i x^{7/2}}{4294967296} - \frac{72765 i x^{5/2}}{8388608} + \frac{105 i x^{3/2}}{8192} + \frac{33424574007825 x^5}{281474976710656} - \frac{14783093325 x^4}{549755813888} + \frac{2837835 x^3}{268435456} - \frac{4725 x^2}{524288} + \frac{15x}{512} - \frac{3i\sqrt{x}}{16} + 1 \right) + c_2 e^{\frac{2i}{\sqrt{x}}} x^{3/4} \left(\frac{468131288625 i x^{9/2}}{8796093022208} - \frac{66891825 i x^{7/2}}{4294967296} + \frac{72765 i x^{5/2}}{8388608} - \frac{105 i x^{3/2}}{8192} + \frac{33424574007825 x^5}{281474976710656} - \frac{14}{5} \right.$$

2.35 problem 36 (b)

Internal problem ID [4836]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.2 page 239
Problem number: 36 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{x^2y'' + (3x - 1)\,y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✗ Solution by Maple

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+(3*x-1)*diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

No solution found

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 43

```
AsymptoticDSolveValue[x^2*y''[x]+(3*x-1)*y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(120x^5 + 24x^4 + 6x^3 + 2x^2 + x + 1\right) + \frac{c_2e^{-1/x}}{x}$$

3 Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250

3.1 problem 1

Internal problem ID [4837]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(x^2 - \frac{1}{9}\right)y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1/9)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{2}{3}} \left(1 - \frac{3}{16}x^2 + \frac{9}{896}x^4 + \mathrm{O}\left(x^6\right)\right) + c_1\left(1 - \frac{3}{8}x^2 + \frac{9}{320}x^4 + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 52

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1/9)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt[3]{x}\left(\frac{9x^4}{896} - \frac{3x^2}{16} + 1\right) + \frac{c_2\left(\frac{9x^4}{320} - \frac{3x^2}{8} + 1\right)}{\sqrt[3]{x}}$$

3.2 problem 2

Internal problem ID [4838]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$\boxed{x^2y'' + y'x + (x^2 - 1)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(x^2-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^2\left(1 - \frac{1}{8}x^2 + \frac{1}{192}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln\left(x\right)\left(x^2 - \frac{1}{8}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-2 + \frac{3}{32}x^4 + \mathrm{O}\left(x^6\right)\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 58

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(x^2-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{192} - \frac{x^3}{8} + x\right) + c_1\left(\frac{1}{16}x(x^2 - 8)\log(x) - \frac{5x^4 - 16x^2 - 64}{64x}\right)$$

3.3 problem 3

Internal problem ID [4839]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4x^2y'' + 4y'x + (4x^2 - 25)\, y = 0$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2-25)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^5 \left(1 - \frac{1}{14}x^2 + \frac{1}{504}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 (2880 + 480x^2 + 120x^4 + \mathrm{O}\left(x^6\right))}{x^{\frac{5}{2}}}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 58

```
AsymptoticDSolveValue[4*x^2*y''[x]+4*x*y'[x]+(4*x^2-25)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{x^{3/2}}{24} + \frac{1}{x^{5/2}} + \frac{1}{6\sqrt{x}}\right) + c_2 \left(\frac{x^{13/2}}{504} - \frac{x^{9/2}}{14} + x^{5/2}\right)$$

3.4 problem 4

Internal problem ID [4840]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{16x^2y'' + 16y'x + (16x^2 - 1)\,y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
Order:=6;
dsolve(16*x^2*diff(y(x),x$2)+16*x*diff(y(x),x)+(16*x^2-1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2\sqrt{x}\left(1 - \frac{1}{5}x^2 + \frac{1}{90}x^4 + \mathrm{O}(x^6)\right) + c_1\left(1 - \frac{1}{3}x^2 + \frac{1}{42}x^4 + \mathrm{O}(x^6)\right)}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 52

```
AsymptoticDSolveValue[16*x^2*y''[x]+16*x*y'[x]+(16*x^2-1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt[4]{x}\left(\frac{x^4}{90} - \frac{x^2}{5} + 1\right) + \frac{c_2\left(\frac{x^4}{42} - \frac{x^2}{3} + 1\right)}{\sqrt[4]{x}}$$

3.5 problem 5

Internal problem ID [4841]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + y' + xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x)\, c_2 + c_1)\left(1 - \frac{1}{4}x^2 + \frac{1}{64}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(\frac{1}{4}x^2 - \frac{3}{128}x^4 + \mathrm{O}\left(x^6\right)\right)c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 60

```
AsymptoticDSolveValue[x*y''[x]+y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4}{64} - \frac{x^2}{4} + 1\right) + c_2\left(-\frac{3x^4}{128} + \frac{x^2}{4} + \left(\frac{x^4}{64} - \frac{x^2}{4} + 1\right)\log(x)\right)$$

3.6 problem 6

Internal problem ID [4842]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Bessel]`

$$\boxed{xy'' + y' + \left(x - \frac{4}{x}\right) y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(diff(x*diff(y(x),x),x)+(x-4/x)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^4 \left(1 - \frac{1}{12}x^2 + \frac{1}{384}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2(\ln\left(x\right)\left(9x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-144 - 36x^2 + \mathrm{O}\left(x^6\right)\right))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 52

```
AsymptoticDSolveValue[D[x*D[y[x],x],x]+(x-4/x)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{\left(x^2+8\right)^2}{64x^2} - \frac{1}{16}x^2 \log(x)\right) + c_2 \left(\frac{x^6}{384} - \frac{x^4}{12} + x^2\right)$$

3.7 problem 7

Internal problem ID [4843]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + (9x^2 - 4)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 47

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(9*x^2-4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x^4\left(1 - \frac{3}{4}x^2 + \frac{27}{128}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2(\ln(x)\left(729x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-144 - 324x^2 + \mathrm{O}\left(x^6\right)\right))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 54

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(9*x^2-4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{(9x^2+8)^2}{64x^2} - \frac{81}{16}x^2\log(x)\right) + c_2\left(\frac{27x^6}{128} - \frac{3x^4}{4} + x^2\right)$$

3.8 problem 8

Internal problem ID [4844]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(36x^2 - \frac{1}{4}\right)y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(36*x^2-1/4)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1x\left(1 - 6x^2 + \frac{54}{5}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 - 18x^2 + 54x^4 + \mathrm{O}\left(x^6\right)\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 52

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(36*x^2-1/4)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(54x^{7/2} - 18x^{3/2} + \frac{1}{\sqrt{x}}\right) + c_2\left(\frac{54x^{9/2}}{5} - 6x^{5/2} + \sqrt{x}\right)$$

3.9 problem 9

Internal problem ID [4845]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + \left(25x^2 - \frac{4}{9}\right)y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(25*x^2-4/9)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_2 x^{\frac{4}{3}}\left(1 - \frac{15}{4}x^2 + \frac{1125}{256}x^4 + \mathrm{O}\left(x^6\right)\right) + c_1\left(1 - \frac{75}{4}x^2 + \frac{5625}{128}x^4 + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 52

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(25*x^2-4/9)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 x^{2/3}\left(\frac{1125x^4}{256} - \frac{15x^2}{4} + 1\right) + \frac{c_2\left(\frac{5625x^4}{128} - \frac{75x^2}{4} + 1\right)}{x^{2/3}}$$

3.10 problem 10

Internal problem ID [4846]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + y'x + (2x^2 - 64)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+x*diff(y(x),x)+(2*x^2-64)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^8 \left(1 - \frac{1}{18}x^2 + \frac{1}{720}x^4 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2(-27360196043587190784000000 - 1954299717399085056000000x^2 - 81429154891628544000000x^4}{x^8}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 46

```
AsymptoticDSolveValue[x^2*y''[x]+x*y'[x]+(2*x^2-64)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^{12}}{720} - \frac{x^{10}}{18} + x^8\right) + c_1\left(\frac{1}{x^8} + \frac{1}{14x^6} + \frac{1}{336x^4}\right)$$

3.11 problem 13

Internal problem ID [4847]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + 2y' + 4y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 64

```
Order:=6;
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)+4*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1\left(1 - 2x + \frac{4}{3}x^2 - \frac{4}{9}x^3 + \frac{4}{45}x^4 - \frac{8}{675}x^5 + \mathrm{O}\left(x^6\right)\right)x + c_2\left(\ln\left(x\right)\left(\left(-4\right)x + 8x^2 - \frac{16}{3}x^3 + \frac{16}{9}x^4 - \frac{16}{45}x^5 + \mathrm{O}\left(\right.\right.\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 85

```
AsymptoticDSolveValue[x*y''[x]+2*y'[x]+4*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{4x^4}{45} - \frac{4x^3}{9} + \frac{4x^2}{3} - 2x + 1\right) + c_1\left(\frac{4}{9}\left(4x^3 - 12x^2 + 18x - 9\right)\log(x) - \frac{188x^4 - 480x^3 + 540x^2 - 108x - 27}{27x}\right)$$

3.12 problem 14

Internal problem ID [4848]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' + 3y' + xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 46

```
Order:=6;
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^2 \left(1 - \frac{1}{8}x^2 + \frac{1}{192}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(\ln\left(x\right)\left(x^2 - \frac{1}{8}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-2 + \frac{3}{32}x^4 + \mathrm{O}\left(x^6\right)\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 57

```
AsymptoticDSolveValue[x*y''[x]+3*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2 \left(\frac{x^4}{192} - \frac{x^2}{8} + 1\right) + c_1 \left(\frac{1}{16}\left(x^2 - 8\right)\log(x) - \frac{5x^4 - 16x^2 - 64}{64x^2}\right)$$

3.13 problem 15

Internal problem ID [4849]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' - y' + xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
Order:=6;
dsolve(x*diff(y(x),x$2)-diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^2 \left(1 - \frac{1}{8}x^2 + \frac{1}{192}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(\ln(x)\left(x^2 - \frac{1}{8}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-2 + \frac{3}{32}x^4 + \mathrm{O}\left(x^6\right)\right)\right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 59

```
AsymptoticDSolveValue[x*y''[x]-y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{1}{16}\left(x^2 - 8\right)x^2 \log(x) + \frac{1}{64}\left(-5x^4 + 16x^2 + 64\right)\right) + c_2\left(\frac{x^6}{192} - \frac{x^4}{8} + x^2\right)$$

3.14 problem 16

Internal problem ID [4850]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Lienard]`

$$\boxed{xy'' - 5y' + xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
Order:=6;
dsolve(x*diff(y(x),x$2)-5*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^6 \left(1 - \frac{1}{16}x^2 + \frac{1}{640}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(-86400 - 10800x^2 - 1350x^4 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 44

```
AsymptoticDSolveValue[x*y''[x]-5*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4}{64} + \frac{x^2}{8} + 1\right) + c_2\left(\frac{x^{10}}{640} - \frac{x^8}{16} + x^6\right)$$

3.15 problem 17

Internal problem ID [4851]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + (x^2 - 2)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+(x^2-2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 - \frac{1}{10}x^2 + \frac{1}{280}x^4 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(12 + 6x^2 - \frac{3}{2}x^4 + \mathrm{O}\left(x^6\right)\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 44

```
AsymptoticDSolveValue[x^2*y''[x]+(x^2-2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(-\frac{x^3}{8} + \frac{x}{2} + \frac{1}{x}\right) + c_2\left(\frac{x^6}{280} - \frac{x^4}{10} + x^2\right)$$

3.16 problem 18

Internal problem ID [4852]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 18.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + (16x^2 + 1)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 51

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)+(16*x^2+1)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left((\ln (x)\, c_2 + c_1) \left(1 - x^2 + \frac{1}{4}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(x^2 - \frac{3}{8}x^4 + \mathrm{O}\left(x^6\right)\right) c_2\right) \sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 69

```
AsymptoticDSolveValue[4*x^2*y''[x]+(16*x^2+1)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(\frac{x^4}{4} - x^2 + 1\right) + c_2\left(\sqrt{x}\left(x^2 - \frac{3x^4}{8}\right) + \sqrt{x}\left(\frac{x^4}{4} - x^2 + 1\right)\log(x)\right)$$

3.17 problem 19

Internal problem ID [4853]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 19.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + 3y' + yx^3 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
Order:=6;
dsolve(x*diff(y(x),x$2)+3*diff(y(x),x)+x^3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\left(1 - \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2\left(-2 + \frac{1}{4}x^4 + \mathrm{O}\left(x^6\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 30

```
AsymptoticDSolveValue[x*y''[x]+3*y'[x]+x^3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(1 - \frac{x^4}{24}\right) + c_1\left(\frac{1}{x^2} - \frac{x^2}{8}\right)$$

3.18 problem 20

Internal problem ID [4854]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 20.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{9x^2y'' + 9y'x + (x^6 - 36)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
Order:=6;
dsolve(9*x^2*diff(y(x),x$2)+9*x*diff(y(x),x)+(x^6-36)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1x^2\left(1 + \mathrm{O}\left(x^6\right)\right) + \frac{c_2(-144 + \mathrm{O}\left(x^6\right))}{x^2}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 16

```
AsymptoticDSolveValue[9*x^2*y''[x]+9*x*y'[x]+(x^6-36)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2x^2 + \frac{c_1}{x^2}$$

3.19 problem 22(a)

Internal problem ID [4855]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 22(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{y'' - x^2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
Order:=6;
dsolve(diff(y(x),x$2)-x^2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{x^4}{12}\right) y(0) + \left(x + \frac{1}{20}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 28

```
AsymptoticDSolveValue[y''[x]-x^2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{20} + x\right) + c_1\left(\frac{x^4}{12} + 1\right)$$

3.20 problem 22(b)

Internal problem ID [4856]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 22(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{xy'' + y' - 7yx^3 = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
Order:=6;
dsolve(x*diff(y(x),x$2)+diff(y(x),x)-7*x^3*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = (\ln(x)\, c_2 + c_1)\left(1 + \frac{7}{16}x^4 + \mathrm{O}\left(x^6\right)\right) + \left(-\frac{7}{32}x^4 + \mathrm{O}\left(x^6\right)\right) c_2$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 39

```
AsymptoticDSolveValue[x*y''[x]+y'[x]-7*x^3*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{7x^4}{16} + 1\right) + c_2\left(\left(\frac{7x^4}{16} + 1\right)\log(x) - \frac{7x^4}{32}\right)$$

3.21 problem 23

Internal problem ID [4857]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4\right) y(0) + \left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{120} - \frac{x^3}{6} + x\right) + c_1\left(\frac{x^4}{24} - \frac{x^2}{2} + 1\right)$$

3.22 problem 24

Internal problem ID [4858]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 24.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 4y'x + (x^2 + 2)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
Order:=6;
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(x^2+2)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \mathrm{O}\left(x^6\right)\right)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 40

```
AsymptoticDSolveValue[x^2*y''[x]+4*x*y'[x]+(x^2+2)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^3}{120} - \frac{x}{6} + \frac{1}{x}\right) + c_1\left(\frac{x^2}{24} + \frac{1}{x^2} - \frac{1}{2}\right)$$

3.23 problem 25

Internal problem ID [4859]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 25.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{16x^2y'' + 32y'x + (x^4 - 12)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
Order:=6;
dsolve(16*x^2*diff(y(x),x$2)+32*x*diff(y(x),x)+(x^4-12)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \frac{c_1 x^2 \left(1 - \frac{1}{384}x^4 + \mathrm{O}\left(x^6\right)\right) + c_2 \left(-2 + \frac{1}{64}x^4 + \mathrm{O}\left(x^6\right)\right)}{x^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 40

```
AsymptoticDSolveValue[16*x^2*y''[x]+32*x*y'[x]+(x^4-12)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(\frac{1}{x^{3/2}} - \frac{x^{5/2}}{128}\right) + c_2 \left(\sqrt{x} - \frac{x^{9/2}}{384}\right)$$

3.24 problem 26

Internal problem ID [4860]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Exercises. 6.3.1 page 250
Problem number: 26.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' - 4y'x + (16x^4 + 3)\, y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 31

```
Order:=6;
dsolve(4*x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(16*x^4+3)*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \sqrt{x}\left(x\left(1 - \frac{1}{5}x^4 + \mathrm{O}\left(x^6\right)\right)c_1 + \left(1 - \frac{1}{3}x^4 + \mathrm{O}\left(x^6\right)\right)c_2\right)$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 40

```
AsymptoticDSolveValue[4*x^2*y''[x]-4*x*y'[x]+(16*x^4+3)*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\sqrt{x} - \frac{x^{9/2}}{3}\right) + c_2\left(x^{3/2} - \frac{x^{11/2}}{5}\right)$$

4 Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Chapter 6 review exercises. page 253

4.1 problem 9

Internal problem ID [4861]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Chapter 6 review exercises. page 253
Problem number: 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F(`

$$\boxed{2xy'' + y' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
Order:=6;
dsolve(2*x*diff(y(x),x$2)+diff(y(x),x)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1\sqrt{x}\left(1 - \frac{1}{3}x + \frac{1}{30}x^2 - \frac{1}{630}x^3 + \frac{1}{22680}x^4 - \frac{1}{1247400}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(1 - x + \frac{1}{6}x^2 - \frac{1}{90}x^3 + \frac{1}{2520}x^4 - \frac{1}{113400}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 83

```
AsymptoticDSolveValue[2*x*y''[x]+y'[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\sqrt{x}\left(-\frac{x^5}{1247400} + \frac{x^4}{22680} - \frac{x^3}{630} + \frac{x^2}{30} - \frac{x}{3} + 1\right) + c_2\left(-\frac{x^5}{113400} + \frac{x^4}{2520} - \frac{x^3}{90} + \frac{x^2}{6} - x + 1\right)$$

4.2 problem 10

Internal problem ID [4862]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Chapter 6 review exercises. page 253
Problem number: 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{y'' - y'x - y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
Order:=6;
dsolve(diff(y(x),x$2)-x*diff(y(x),x)-y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 + \frac{1}{2}x^2 + \frac{1}{8}x^4\right) y(0) + \left(x + \frac{1}{3}x^3 + \frac{1}{15}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 42

```
AsymptoticDSolveValue[y''[x]-x*y'[x]-y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_2\left(\frac{x^5}{15} + \frac{x^3}{3} + x\right) + c_1\left(\frac{x^4}{8} + \frac{x^2}{2} + 1\right)$$

4.3 problem 11

Internal problem ID [4863]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Chapter 6 review exercises. page 253
Problem number: 11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(x-1)\,y''+3y=0}$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve((x-1)*diff(y(x),x$2)+3*y(x)=0,y(x),type='series',x=0);
```

$$y(x)=\left(1+\frac{3}{2}x^2+\frac{1}{2}x^3+\frac{5}{8}x^4+\frac{9}{20}x^5\right)y(0)+\left(x+\frac{1}{2}x^3+\frac{1}{4}x^4+\frac{9}{40}x^5\right)D(y)\,(0)+O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[(x-1)*y''[x]+3*y[x]==0,y[x],{x,0,5}]
```

$$y(x)\to c_2\left(\frac{9x^5}{40}+\frac{x^4}{4}+\frac{x^3}{2}+x\right)+c_1\left(\frac{9x^5}{20}+\frac{5x^4}{8}+\frac{x^3}{2}+\frac{3x^2}{2}+1\right)$$

4.4 problem 12

Internal problem ID [4864]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Chapter 6 review exercises. page 253
Problem number: 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - x^2 y' + xy = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
Order:=6;
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)+x*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(1 - \frac{x^3}{6}\right) y(0) + D(y)(0)\, x + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 20

```
AsymptoticDSolveValue[y''[x]-x^2*y'[x]+x*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1 \left(1 - \frac{x^3}{6}\right) + c_2 x$$

4.5 problem 13

Internal problem ID [4865]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Chapter 6 review exercises. page 253
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' - (2+x)\,y' + 2y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
Order:=6;
dsolve(x*diff(y(x),x$2)-(x+2)*diff(y(x),x)+2*y(x)=0,y(x),type='series',x=0);
```

$$y(x) = c_1 x^3 \left(1 + \frac{1}{4}x + \frac{1}{20}x^2 + \frac{1}{120}x^3 + \frac{1}{840}x^4 + \frac{1}{6720}x^5 + \mathrm{O}\left(x^6\right)\right) + c_2\left(12 + 12x + 6x^2 + 2x^3 + \frac{1}{2}x^4 + \frac{1}{10}x^5 + \mathrm{O}\left(x^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 66

```
AsymptoticDSolveValue[x*y''[x]-(x+2)*y'[x]+2*y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(\frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1\right) + c_2\left(\frac{x^7}{840} + \frac{x^6}{120} + \frac{x^5}{20} + \frac{x^4}{4} + x^3\right)$$

4.6 problem 14

Internal problem ID [4866]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Chapter 6 review exercises. page 253
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\cos(x) y'' + y = 0}$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
Order:=6;
dsolve(cos(x)*diff(y(x),x$2)+y(x)=0,y(x),type='series',x=0);
```

$$y(x) = \left(-\frac{x^2}{2} + 1\right) y(0) + \left(x - \frac{1}{6}x^3 - \frac{1}{60}x^5\right) D(y)(0) + O(x^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 35

```
AsymptoticDSolveValue[Cos[x]*y''[x]+y[x]==0,y[x],{x,0,5}]
```

$$y(x) \to c_1\left(1 - \frac{x^2}{2}\right) + c_2\left(-\frac{x^5}{60} - \frac{x^3}{6} + x\right)$$

4.7 problem 15

Internal problem ID [4867]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Chapter 6 review exercises. page 253
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y'x + 2y = 0}$$

With initial conditions

$$[y(0) = 3, y'(0) = -2]$$

With the expansion point for the power series method at $x = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
Order:=6;
dsolve([diff(y(x),x$2)+x*diff(y(x),x)+2*y(x)=0,y(0) = 3, D(y)(0) = -2],y(x),type='series',x=0
```

$$y(x) = 3 - 2x - 3x^2 + x^3 + x^4 - \frac{1}{4}x^5 + \mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 26

```
AsymptoticDSolveValue[{y''[x]+x*y'[x]+2*y[x]==0,{y[0]==3,y'[0]==-2}},y[x],{x,0,5}]
```

$$y(x) \to -\frac{x^5}{4} + x^4 + x^3 - 3x^2 - 2x + 3$$

4.8 problem 16

Internal problem ID [4868]

Book: A FIRST COURSE IN DIFFERENTIAL EQUATIONS with Modeling Applications. Dennis G. Zill. 9th edition. Brooks/Cole. CA, USA.
Section: Chapter 6. SERIES SOLUTIONS OF LINEAR EQUATIONS. Chapter 6 review exercises. page 253
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$\boxed{(2+x)\,y''+3y=0}$$

With initial conditions

$$[y(0)=0, y'(0)=1]$$

With the expansion point for the power series method at $x=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
Order:=6;
dsolve([(x+2)*diff(y(x),x$2)+3*y(x)=0,y(0) = 0, D(y)(0) = 1],y(x),type='series',x=0);
```

$$y(x)=x-\frac{1}{4}x^3+\frac{1}{16}x^4+\mathrm{O}\left(x^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 19

```
AsymptoticDSolveValue[{(x+2)*y''[x]+3*y[x]==0,{y[0]==0,y'[0]==1}},y[x],{x,0,5}]
```

$$y(x)\to\frac{x^4}{16}-\frac{x^3}{4}+x$$