

Computer algebra independent integration tests

Summer 2022 edition

0-Independent-test-suites/5-Hearn-Problems

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [284]. This is test number [5].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.0.1 (February 17, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.0.1.debian on Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem via sagemath 9.6.
5. Fricas 1.3.7 (June 30, 2021) based on based on ecl 21.2.1 on Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem via sagemath 9.6.
6. Giac/Xcas 1.9.0-7 (April 2022) on on Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem. Direct testing using C++ API.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem via sagemath 9.6.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Mathics 4.0 via sagemath 9.6.

Maxima, Fricas, Mathics are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems. Mathics was called using its own interface in Sagemath as in this example

```
from sage.interfaces.mathics import mathics
res = mathics('Integrate[Sin[x]/(3 + Cos[x])^2,x]')
```

Sympy was called directly from Python. Giac was also called directly via its C++ interface.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (284)	0.00 (0)
Maple	99.30 (282)	0.70 (2)
Fricas	98.94 (281)	1.06 (3)
Rubi	98.24 (279)	1.76 (5)
Mupad	95.07 (270)	4.93 (14)
Giac	94.37 (268)	5.63 (16)
Sympy	89.08 (253)	10.92 (31)
Maxima	88.73 (252)	11.27 (32)
Mathics	87.32 (248)	12.68 (36)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

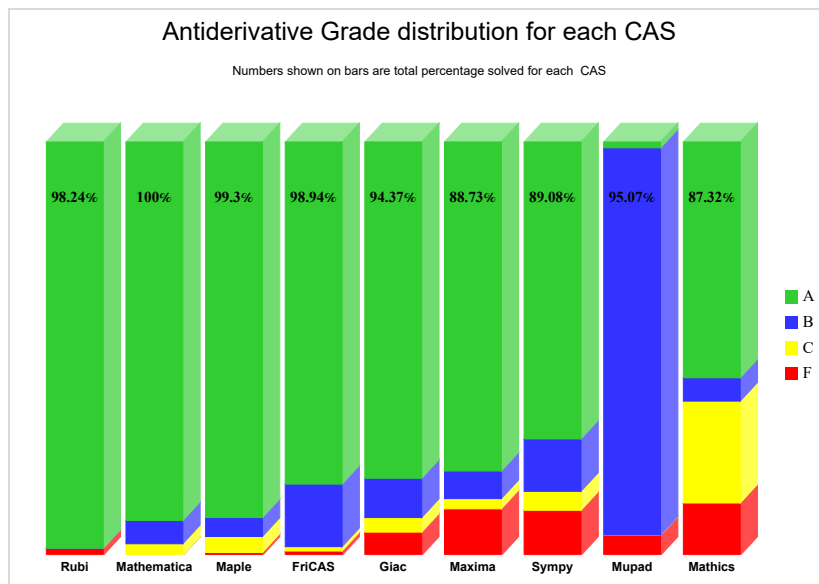
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

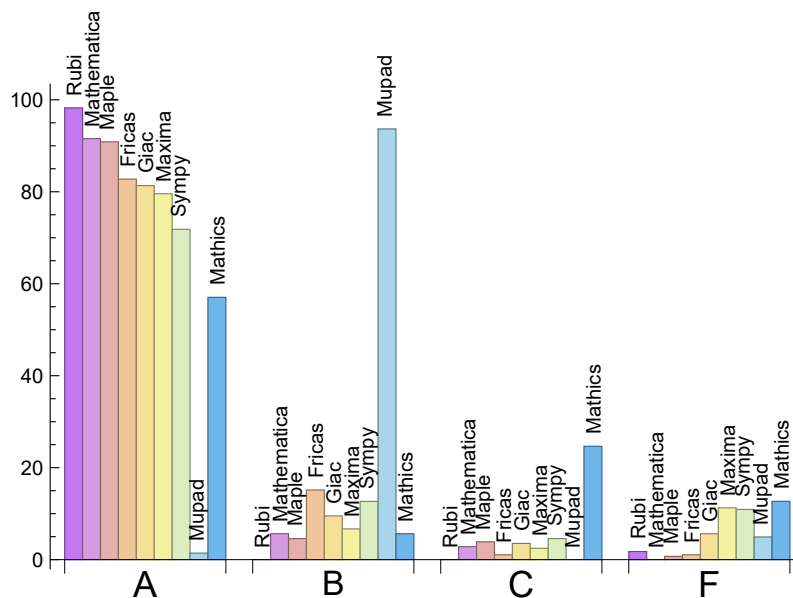
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.24	0.00	0.00	1.76
Mathematica	91.55	5.63	2.82	0.00
Maple	90.85	4.58	3.87	0.70
Fricas	82.75	15.14	1.06	1.06
Giac	81.34	9.51	3.52	5.63
Maxima	79.58	6.69	2.46	11.27
Sympy	71.83	12.68	4.58	10.92
Mathics	57.04	5.63	24.65	12.68
Mupad	N/A	93.66	0.00	4.93

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	5	100.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	2	100.00 %	0.00 %	0.00 %
Fricas	3	100.00 %	0.00 %	0.00 %
Giac	16	100.00 %	0.00 %	0.00 %
Maxima	32	68.75 %	0.00 %	31.25 %
Sympy	31	87.10 %	9.68 %	3.23 %
Mupad	14	100.00 %	0.00 %	0.00 %
Mathics	36	0.00 %	16.67 %	83.33 %

Table 1.4: Failure statistics for each CAS

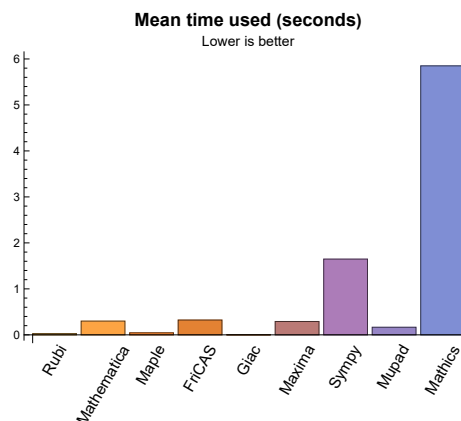
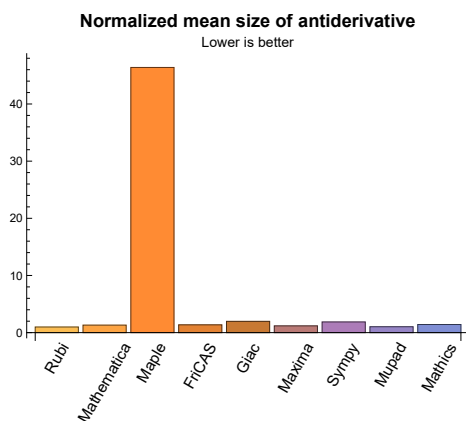
1.3 Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.03	39.11	0.99	22.00	1.00
Mathematica	0.30	65.89	1.31	23.00	1.00
Maple	0.04	4305.99	46.39	22.00	0.92
Maxima	0.29	35.11	1.18	19.00	0.88
Fricas	0.32	49.68	1.36	22.00	1.00
Sympy	1.65	86.70	1.87	20.00	1.00
Giac	0.00	134.41	1.97	26.00	1.10
Mupad	0.17	35.62	1.03	18.00	0.86
Mathics	5.85	57.35	1.42	21.00	0.97

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.



1.4 list of integrals that has no closed form antiderivative

{75, 145, 170, 273}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {145}

Fricas {}

Sympy {}

Giac {}

Mupad {}

Mathics {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {279, 281}

Mathics {56, 60, 61, 64, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 122, 123, 124, 126, 134, 135, 136, 137, 138, 139, 140, 141, 144, 150, 151, 152, 156, 158, 161, 165, 166, 173, 175, 178, 179, 180, 181, 183, 184, 185, 186, 190, 191, 194, 195, 202, 203, 205, 206, 209, 210, 220, 224, 248, 275}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for **Rubi**, **Mathematica** and **Mathics**. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima `integrate` was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

For Giac, the call `taille(anti_derivative,RAND_MAX)`; is used to find leaf size.

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

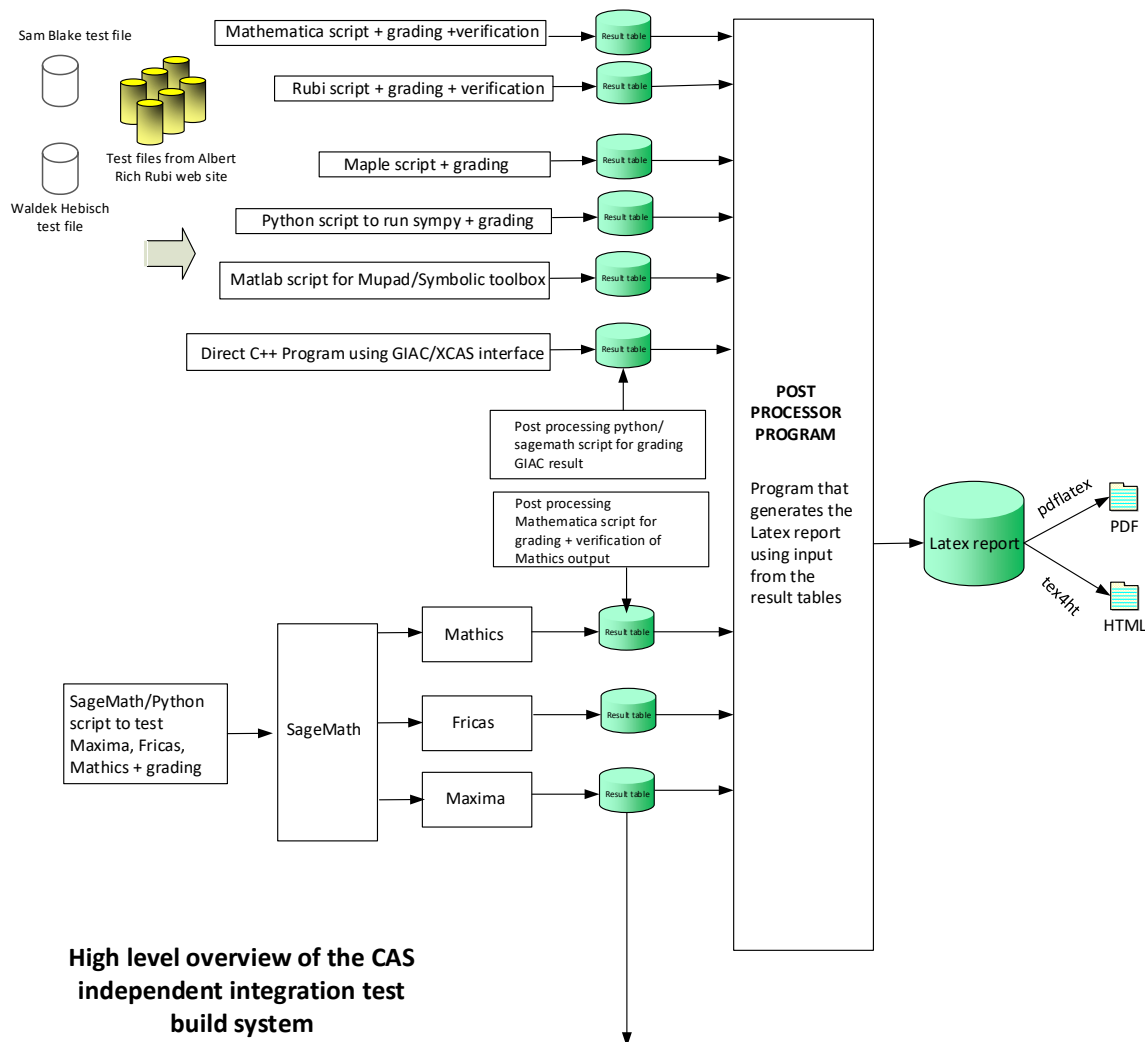
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand,the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

The following fields present only in Rubi Table file

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,...}" which is list of the rules used by Rubi

Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 280, 282, 283 }

B grade: { }

C grade: { }

F grade: { 169, 278, 279, 281, 284 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 47, 48, 49, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 192, 193, 195, 196, 198, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 280, 282, 283, 284 }

B grade: { 52, 81, 82, 108, 111, 120, 121, 190, 194, 199, 202, 211, 235, 236, 256, 260 }

C grade: { 20, 44, 45, 51, 197, 278, 279, 281 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 281, 283, 284 }

B grade: { 32, 60, 61, 110, 124, 126, 204, 237, 242, 256, 257, 278, 280 }

C grade: { 49, 51, 136, 137, 138, 139, 140, 141, 203, 279, 282 }

F grade: { 86, 251 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 41, 46, 47, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 106, 107, 108, 109, 110, 113, 114, 115, 116, 117, 118, 119, 120, 121, 124, 125, 126, 127, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 162, 164, 165, 167, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 198, 199, 200, 201, 202, 205, 206, 207, 208, 209, 210, 212, 215, 216, 217, 218, 219, 220, 221, 223, 224, 227, 228, 229, 230, 231, 232, 233, 234, 237, 238, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 283 }

B grade: { 32, 81, 82, 111, 112, 128, 129, 130, 146, 191, 194, 195, 204, 222, 225, 226, 241, 256, 280 }

C grade: { 102, 103, 104, 105, 166, 168, 197 }

F grade: { 8, 39, 40, 42, 43, 44, 45, 49, 51, 86, 122, 123, 147, 160, 161, 163, 176, 196, 203, 211, 213, 214, 235, 236, 239, 251, 257, 278, 279, 281, 282, 284 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 38, 43, 45, 46, 47, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 83, 84, 85, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 196, 198, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 248, 249, 250, 251, 252, 253, 255, 258, 259, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 276, 277, 279, 282, 283 }

B grade: { 36, 37, 39, 40, 41, 42, 44, 48, 49, 78, 79, 80, 81, 82, 90, 103, 110, 111, 112, 127, 133, 194, 195, 199, 204, 220, 222, 227, 228, 235, 236, 244, 245, 246, 247, 254, 256, 260, 268, 269, 278, 280, 284 }

C grade: { 128, 129, 197 }

F grade: { 86, 257, 281 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37, 38, 40, 42, 43, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 74, 75, 76, 77, 78, 79, 83, 84, 85, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 130, 132, 134, 135, 136, 137, 144, 145, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 181, 182, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 199, 200, 201, 202, 205, 206, 209, 210, 215, 216, 217, 218, 219, 220, 221, 223, 225, 226, 227, 230, 231, 232, 240, 241, 243, 244, 245, 248, 250, 252, 253, 255, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 273, 274, 275, 276, 277, 282, 283, 284 }

B grade: { 7, 8, 13, 32, 39, 41, 44, 80, 81, 82, 90, 103, 104, 131, 133, 142, 143, 178, 179, 180, 183, 184, 189, 204, 222, 228, 229, 233, 234, 237, 239, 246, 247, 254, 256, 272 }

C grade: { 9, 14, 31, 50, 70, 72, 73, 138, 139, 140, 141, 203, 224 }

F grade: { 12, 86, 128, 129, 146, 147, 160, 162, 163, 176, 196, 197, 198, 207, 208, 211, 212, 213, 214, 235, 236, 238, 242, 249, 251, 257, 265, 278, 279, 280, 281 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 223, 224, 227, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 244, 248, 249, 250, 251, 252, 253, 254, 255, 258, 259, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 276, 277, 282, 283, 284 }

B grade: { 23, 32, 79, 81, 110, 112, 127, 176, 178, 179, 195, 198, 199, 222, 225, 226, 228, 235, 236, 245, 246, 247, 256, 260, 268, 269, 280 }

C grade: { 134, 135, 136, 137, 138, 139, 140, 141, 161, 166 }

F grade: { 56, 63, 86, 128, 129, 160, 162, 163, 197, 200, 201, 203, 257, 278, 279, 281 }

2.1.8 Mupad

A grade: { 75, 145, 170, 273 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 274, 275, 276, 277, 280, 282, 283, 284 }

C grade: { }

F grade: { 102, 103, 104, 105, 128, 129, 163, 169, 203, 249, 257, 278, 279, 281 }

2.1.9 Mathics

A grade: { 1, 2, 3, 4, 5, 6, 7, 10, 11, 13, 15, 16, 17, 18, 19, 20, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 36, 37, 38, 41, 43, 46, 51, 52, 53, 54, 55, 57, 58, 59, 62, 63, 65, 66, 67, 68, 69, 71, 74, 76, 77, 78, 79, 80, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 106, 118, 119, 120, 121, 125, 127, 130, 131, 132, 133, 142, 143, 148, 149, 153, 154, 155, 159, 164, 167, 168, 169, 171, 172, 173, 174, 177, 181, 182, 185, 186, 190, 192, 193, 195, 199, 200, 201, 202, 215, 216, 217, 218, 219, 221, 222, 223, 225, 226, 227, 229, 230, 231, 232, 233, 234, 237, 239, 240, 241, 243, 244, 245, 250, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 274, 276, 277, 280, 282, 283, 284 }

B grade: { 8, 39, 44, 81, 82, 178, 179, 180, 183, 184, 189, 204, 228, 246, 247, 256 }

C grade: { 9, 14, 22, 23, 31, 35, 40, 42, 45, 47, 48, 49, 50, 56, 60, 61, 64, 70, 72, 73, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 122, 123, 124, 126, 134, 135, 136, 137, 138, 139, 140, 141, 144, 150, 151, 152, 156, 157, 158, 161, 165, 166, 175, 188, 191, 194, 203, 205, 206, 209, 210, 220, 224, 248, 275 }

F grade: { 12, 21, 75, 86, 128, 129, 145, 146, 147, 160, 162, 163, 170, 176, 187, 196, 197, 198, 207, 208, 211, 212, 213, 214, 235, 236, 238, 242, 249, 251, 257, 265, 273, 278, 279, 281 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	14	13	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.62	0.88	0.81	0.75
time (sec)	N/A	0.001	0.000	0.004	0.281	0.310	0.024	0.000	0.021	1.618

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	17	22	15	15
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.77	1.00	0.68	0.68
time (sec)	N/A	0.007	0.001	0.018	0.266	0.337	0.028	0.000	0.039	1.640

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	15	22	15	15
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.68	1.00	0.68	0.68
time (sec)	N/A	0.003	0.001	0.022	0.257	0.322	0.027	0.000	0.029	1.639

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	3	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.50	1.00	1.00
time (sec)	N/A	0.000	0.000	0.003	0.281	0.330	0.025	0.000	0.009	1.480

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	27	32	46	29	27	22	44
N.S.	1	1.00	0.67	0.75	0.89	1.28	0.81	0.75	0.61	1.22
time (sec)	N/A	0.008	0.008	0.041	0.261	0.326	0.059	0.001	0.118	1.847

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	24	33	24	32	26	28
N.S.	1	1.00	0.88	0.78	0.75	1.03	0.75	1.00	0.81	0.88
time (sec)	N/A	0.008	0.008	0.035	0.268	0.333	0.079	0.000	0.034	1.757

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	43	40	34	144	35	40	34
N.S.	1	1.00	0.85	1.08	1.00	0.85	3.60	0.88	1.00	0.85
time (sec)	N/A	0.018	0.012	0.043	0.266	0.327	0.548	0.001	0.250	4.071

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	35	0	120	124	38	46	125
N.S.	1	1.00	1.12	1.03	0.00	3.53	3.65	1.12	1.35	3.68
time (sec)	N/A	0.019	0.008	0.115	0.000	0.348	0.116	0.001	0.198	4.026

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	26	16	14	23
N.S.	1	1.00	1.00	0.94	0.88	0.88	1.62	1.00	0.88	1.44
time (sec)	N/A	0.004	0.005	0.024	0.340	0.321	0.087	0.000	0.038	1.798

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	17	14	14	22	21	14	14
N.S.	1	1.00	0.84	0.89	0.74	0.74	1.16	1.11	0.74	0.74
time (sec)	N/A	0.007	0.004	0.212	0.352	0.320	0.051	0.001	0.031	1.678

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	35	36	39	71	41	45	49	49
N.S.	1	1.00	0.71	0.73	0.80	1.45	0.84	0.92	1.00	1.00
time (sec)	N/A	0.018	0.017	0.043	0.346	0.417	0.091	0.002	0.126	1.996

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	69	78	81	0	73	87	0
N.S.	1	1.00	0.91	1.01	1.15	1.19	0.00	1.07	1.28	0.00
time (sec)	N/A	0.033	0.021	0.077	0.255	0.466	0.000	0.001	0.568	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	34	44	43	32	121	40	256	33
N.S.	1	1.00	0.72	0.94	0.91	0.68	2.57	0.85	5.45	0.70
time (sec)	N/A	0.013	0.008	0.048	0.268	0.321	0.369	0.001	0.304	3.513

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	30	41	40	30	393	46	191	49
N.S.	1	1.00	0.75	1.02	1.00	0.75	9.82	1.15	4.78	1.22
time (sec)	N/A	0.012	0.010	0.061	0.341	0.328	0.684	0.001	0.175	6.580

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	19	19	19	24	25	19
N.S.	1	1.00	1.00	0.74	0.70	0.70	0.70	0.89	0.93	0.70
time (sec)	N/A	0.011	0.003	0.030	0.342	0.328	0.065	0.001	0.046	1.689

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	40	35	34	34	41	47	46	34
N.S.	1	1.00	0.98	0.85	0.83	0.83	1.00	1.15	1.12	0.83
time (sec)	N/A	0.015	0.007	0.028	0.347	0.336	0.068	0.001	0.206	1.870

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	34	32	31	40	31	41	33	37
N.S.	1	1.00	0.79	0.74	0.72	0.93	0.72	0.95	0.77	0.86
time (sec)	N/A	0.083	0.014	0.038	0.362	0.327	0.073	0.001	0.043	1.856

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	64	52	72	100	73	106	33	51
N.S.	1	1.00	0.75	0.61	0.85	1.18	0.86	1.25	0.39	0.60
time (sec)	N/A	0.026	0.013	0.000	0.364	0.328	0.074	0.001	0.002	2.082

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	64	52	72	100	73	106	33	51
N.S.	1	1.00	0.75	0.61	0.85	1.18	0.86	1.25	0.39	0.60
time (sec)	N/A	0.026	0.007	0.023	0.349	0.340	0.076	0.001	0.205	2.062

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	73	54	53	53	70	67	47	53
N.S.	1	1.00	1.09	0.81	0.79	0.79	1.04	1.00	0.70	0.79
time (sec)	N/A	0.023	0.038	0.018	0.350	0.330	0.106	0.001	0.179	2.121

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	17	19	18	20	20	16	18	0
N.S.	1	1.00	0.94	1.06	1.00	1.11	1.11	0.89	1.00	0.00
time (sec)	N/A	0.003	0.016	0.023	0.303	0.342	0.028	0.000	0.220	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	36	42	53	201	83	94	213
N.S.	1	1.00	0.85	0.92	1.08	1.36	5.15	2.13	2.41	5.46
time (sec)	N/A	0.008	0.027	0.020	0.260	0.330	0.242	0.002	0.392	2.457

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	73	68	96	597	154	192	496
N.S.	1	1.00	0.95	1.22	1.13	1.60	9.95	2.57	3.20	8.27
time (sec)	N/A	0.014	0.036	0.026	0.260	0.334	0.399	0.002	0.601	4.459

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	10	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.00	1.00	1.00
time (sec)	N/A	0.001	0.001	0.021	0.284	0.328	0.028	0.000	0.026	1.608

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	13	10	9	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.08	0.83	0.75	1.00	1.00
time (sec)	N/A	0.001	0.002	0.022	0.271	0.316	0.067	0.000	0.118	1.693

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	17	14	19	18	17
N.S.	1	1.00	1.00	1.06	1.00	0.94	0.78	1.06	1.00	0.94
time (sec)	N/A	0.006	0.002	0.023	0.253	0.327	0.049	0.000	0.035	1.685

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	29	29	26	34	29	28
N.S.	1	1.00	1.00	0.97	0.94	0.94	0.84	1.10	0.94	0.90
time (sec)	N/A	0.010	0.003	0.023	0.278	0.345	0.055	0.000	0.127	1.801

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	16	10	21	15	17
N.S.	1	1.00	1.00	1.06	1.00	0.89	0.56	1.17	0.83	0.94
time (sec)	N/A	0.002	0.003	0.023	0.254	0.320	0.070	0.000	0.134	1.726

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	26	19	37	25	30
N.S.	1	1.00	1.00	1.04	1.00	0.93	0.68	1.32	0.89	1.07
time (sec)	N/A	0.009	0.003	0.025	0.265	0.326	0.098	0.000	0.054	1.837

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	45	63	37	52	45	49
N.S.	1	1.00	0.83	1.02	1.07	1.50	0.88	1.24	1.07	1.17
time (sec)	N/A	0.015	0.026	0.026	0.280	0.413	0.147	0.000	0.188	2.098

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	20	11	10	20
N.S.	1	1.00	1.00	1.10	1.00	1.00	2.00	1.10	1.00	2.00
time (sec)	N/A	0.002	0.002	0.032	0.350	0.317	0.056	0.000	0.037	1.973

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	22	21	18	15	23	10	18
N.S.	1	1.00	1.00	2.20	2.10	1.80	1.50	2.30	1.00	1.80
time (sec)	N/A	0.002	0.002	0.024	0.338	0.331	0.061	0.000	0.159	1.726

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	66	58	66	63	78	107	72	53
N.S.	1	1.00	0.85	0.74	0.85	0.81	1.00	1.37	0.92	0.68
time (sec)	N/A	0.028	0.014	0.026	0.369	0.325	0.162	0.001	0.277	2.202

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	65	54	56	68	71	91	72	49
N.S.	1	1.00	0.88	0.73	0.76	0.92	0.96	1.23	0.97	0.66
time (sec)	N/A	0.021	0.012	0.029	0.363	0.322	0.156	0.001	0.356	2.145

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	89	92	97	300	20	127	101	26
N.S.	1	1.00	0.77	0.80	0.84	2.61	0.17	1.10	0.88	0.23
time (sec)	N/A	0.041	0.017	0.024	0.364	0.335	0.079	0.001	0.315	1.842

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	32	34	56	46	55	20	31
N.S.	1	1.00	1.23	0.91	0.97	1.60	1.31	1.57	0.57	0.89
time (sec)	N/A	0.009	0.012	0.026	0.370	0.332	0.158	0.001	0.158	1.930

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	33	41	60	48	67	18	32
N.S.	1	1.00	1.23	0.94	1.17	1.71	1.37	1.91	0.51	0.91
time (sec)	N/A	0.009	0.010	0.030	0.369	0.334	0.162	0.001	0.175	1.916

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	120	93	151	161	151	176	45	83
N.S.	1	1.00	0.70	0.54	0.88	0.94	0.88	1.03	0.26	0.49
time (sec)	N/A	0.072	0.033	0.024	0.365	0.327	0.204	0.004	0.111	2.587

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	68	56	0	132	146	108	93	107
N.S.	1	1.00	0.93	0.77	0.00	1.81	2.00	1.48	1.27	1.47
time (sec)	N/A	0.049	0.036	0.033	0.000	0.339	0.241	0.005	0.224	3.239

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	68	56	0	132	24	108	93	33
N.S.	1	1.00	0.93	0.77	0.00	1.81	0.33	1.48	1.27	0.45
time (sec)	N/A	0.012	0.018	0.029	0.000	0.346	0.192	0.004	0.135	1.935

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	83	54	75	91	158	95	67	69
N.S.	1	1.00	1.15	0.75	1.04	1.26	2.19	1.32	0.93	0.96
time (sec)	N/A	0.046	0.021	0.023	0.346	0.331	0.191	0.000	0.098	2.857

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	60	0	123	24	155	98	33
N.S.	1	1.00	1.00	0.90	0.00	1.84	0.36	2.31	1.46	0.49
time (sec)	N/A	0.038	0.022	0.034	0.000	0.334	0.199	0.005	0.231	1.947

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	60	0	87	92	78	117	63
N.S.	1	1.00	1.00	0.90	0.00	1.30	1.37	1.16	1.75	0.94
time (sec)	N/A	0.008	0.013	0.029	0.000	0.348	0.112	0.003	0.197	2.707

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	F	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	91	251	0	298	994	412	61	443
N.S.	1	1.00	0.46	1.28	0.00	1.52	5.07	2.10	0.31	2.26
time (sec)	N/A	0.100	0.038	0.061	0.000	0.360	0.644	0.086	0.225	17.416

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	F	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	91	253	0	268	24	410	132	33
N.S.	1	1.00	0.46	1.29	0.00	1.37	0.12	2.09	0.67	0.17
time (sec)	N/A	0.092	0.049	0.061	0.000	0.330	0.315	0.085	0.110	2.075

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	66	65	65	83	89	88	65
N.S.	1	1.00	1.03	0.90	0.89	0.89	1.14	1.22	1.21	0.89
time (sec)	N/A	0.066	0.010	0.040	0.342	0.325	0.136	0.000	0.091	2.232

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	122	111	112	176	14	182	140	21
N.S.	1	1.00	0.88	0.80	0.81	1.28	0.10	1.32	1.01	0.15
time (sec)	N/A	0.128	0.025	0.083	0.346	0.419	0.297	0.002	0.210	1.980

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	115	95	107	279	14	173	135	21
N.S.	1	1.00	0.83	0.69	0.78	2.02	0.10	1.25	0.98	0.15
time (sec)	N/A	0.227	0.016	0.098	0.354	0.337	0.142	0.002	0.129	1.877

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	209	22	0	1043	14	337	288	21
N.S.	1	1.00	0.62	0.06	0.00	3.08	0.04	0.99	0.85	0.06
time (sec)	N/A	0.192	0.005	0.027	0.000	0.345	1.129	0.004	0.304	2.790

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	98	61	88	116	44	131	45	46
N.S.	1	1.00	1.01	0.63	0.91	1.20	0.45	1.35	0.46	0.47
time (sec)	N/A	0.032	0.015	0.038	0.346	0.344	85.044	0.000	0.165	60.599

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	42	30	0	220	165	297	53	124
N.S.	1	1.00	0.15	0.11	0.00	0.80	0.60	1.08	0.19	0.45
time (sec)	N/A	0.176	0.009	0.016	0.000	0.335	0.108	0.005	0.100	3.249

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	260	41	40	40	46	51	52	40
N.S.	1	1.00	5.31	0.84	0.82	0.82	0.94	1.04	1.06	0.82
time (sec)	N/A	0.025	0.073	0.027	0.358	0.321	0.088	0.001	0.219	1.953

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	5	7	6	6
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.62	0.88	0.75	0.75
time (sec)	N/A	0.001	0.001	0.003	0.270	0.325	0.038	0.000	0.023	1.560

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	16	9	11
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.94	0.53	0.65
time (sec)	N/A	0.002	0.001	0.000	0.266	0.325	0.045	0.000	0.034	1.640

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	16	9	11
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.94	0.53	0.65
time (sec)	N/A	0.004	0.002	0.003	0.279	0.322	0.046	0.000	0.032	1.595

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	19	19	26	25	56	0	32	36
N.S.	1	1.00	0.73	0.73	1.00	0.96	2.15	0.00	1.23	1.38
time (sec)	N/A	0.006	0.006	0.014	0.267	0.337	0.219	0.000	0.239	2.092

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	12	15	15	16	12	12
N.S.	1	1.00	1.00	1.07	0.80	1.00	1.00	1.07	0.80	0.80
time (sec)	N/A	0.002	0.001	0.000	0.265	0.332	0.047	0.000	0.030	1.612

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	104	71	103	133	132	71	71
N.S.	1	1.00	1.00	0.82	0.56	0.81	1.05	1.04	0.56	0.56
time (sec)	N/A	0.083	0.003	0.007	0.267	0.361	0.176	0.002	0.177	2.242

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75	0.75
time (sec)	N/A	0.007	0.002	0.007	0.260	0.327	0.058	0.000	0.060	1.600

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	9	3	2	2	3	2	2
N.S.	1	1.00	1.00	4.50	1.50	1.00	1.00	1.50	1.00	1.00
time (sec)	N/A	0.001	0.002	0.004	0.291	0.309	0.232	0.000	0.004	1.667

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	11	5	4	3	5	4	4
N.S.	1	1.00	1.00	2.75	1.25	1.00	0.75	1.25	1.00	1.00
time (sec)	N/A	0.002	0.005	0.010	0.277	0.390	0.246	0.000	0.016	1.693

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	3	4	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	1.00	1.33	1.00	1.00
time (sec)	N/A	0.007	0.004	0.000	0.268	0.322	0.049	0.000	0.120	1.514

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	15	6	19	14	0	17	17
N.S.	1	1.00	1.00	0.88	0.35	1.12	0.82	0.00	1.00	1.00
time (sec)	N/A	0.020	0.018	0.007	0.290	0.325	0.333	0.000	0.030	1.844

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	15	11	22	21
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.25	0.92	1.83	1.75
time (sec)	N/A	0.010	0.003	0.013	0.261	0.341	0.443	0.000	0.146	1.957

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	27	25	25	22	27	21	22
N.S.	1	1.00	1.00	0.96	0.89	0.89	0.79	0.96	0.75	0.79
time (sec)	N/A	0.007	0.003	0.006	0.266	0.351	0.060	0.001	0.141	1.959

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	48	47	47	44	52	47	44
N.S.	1	1.00	0.98	0.89	0.87	0.87	0.81	0.96	0.87	0.81
time (sec)	N/A	0.017	0.008	0.020	0.296	0.337	0.070	0.001	0.170	2.079

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	30	38	34	24	37	35	41
N.S.	1	1.00	0.93	1.03	1.31	1.17	0.83	1.28	1.21	1.41
time (sec)	N/A	0.008	0.009	0.025	0.274	0.333	0.121	0.001	0.232	2.170

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	53	44	39	42	65	66	38
N.S.	1	1.00	1.00	1.15	0.96	0.85	0.91	1.41	1.43	0.83
time (sec)	N/A	0.015	0.011	0.015	0.263	0.339	0.090	0.001	0.189	2.195

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	82	57	49	54	104	75	49
N.S.	1	1.00	1.00	1.39	0.97	0.83	0.92	1.76	1.27	0.83
time (sec)	N/A	0.021	0.013	0.013	0.271	0.335	0.097	0.001	0.182	2.361

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	23	23	36	31	23	32
N.S.	1	1.00	1.00	1.04	1.00	1.00	1.57	1.35	1.00	1.39
time (sec)	N/A	0.005	0.002	0.017	0.346	0.324	0.079	0.001	0.066	2.123

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	29	28	23	31	28	51	32
N.S.	1	1.00	0.96	1.07	1.04	0.85	1.15	1.04	1.89	1.19
time (sec)	N/A	0.014	0.003	0.013	0.267	0.317	0.079	0.001	0.041	1.959

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	37	36	36	53	50	65	45
N.S.	1	1.00	1.00	0.84	0.82	0.82	1.20	1.14	1.48	1.02
time (sec)	N/A	0.015	0.002	0.018	0.344	0.327	0.094	0.002	0.036	2.199

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	63	61	73	53
N.S.	1	1.00	1.00	0.83	0.81	0.81	1.17	1.13	1.35	0.98
time (sec)	N/A	0.016	0.002	0.021	0.349	0.333	0.100	0.002	0.128	2.282

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	31	31	29	37	25	32
N.S.	1	1.00	1.00	1.28	1.24	1.24	1.16	1.48	1.00	1.28
time (sec)	N/A	0.006	0.003	0.014	0.281	0.403	0.082	0.001	0.074	2.046

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	N/A	A	A	A	A	A	A	A	A	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	0	0	0	0	0	0	0	-1	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.12	0.00
time (sec)	N/A	0.004	0.025	0.025	0.000	0.341	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	3	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	0.75	1.00	1.00
time (sec)	N/A	0.001	0.002	0.013	0.264	0.336	0.026	0.000	0.021	1.670

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	2	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.001	0.002	0.013	0.267	0.333	0.026	0.000	0.027	1.518

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	3	11	5	5	5	5
N.S.	1	1.00	1.00	1.20	0.60	2.20	1.00	1.00	1.00	1.00
time (sec)	N/A	0.001	0.002	0.011	0.285	0.341	0.029	0.001	0.029	1.643

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	16	3	20	13	3
N.S.	1	1.00	1.00	1.33	1.00	5.33	1.00	6.67	4.33	1.00
time (sec)	N/A	0.001	0.002	0.016	0.262	0.328	0.032	0.002	0.197	1.582

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	26	25	37	75	30	27	37
N.S.	1	1.00	1.29	1.24	1.19	1.76	3.57	1.43	1.29	1.76
time (sec)	N/A	0.015	0.030	0.029	0.352	0.339	0.174	0.002	0.246	2.634

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	33	7	15	17	15	30	11	15
N.S.	1	1.00	11.00	2.33	5.00	5.67	5.00	10.00	3.67	5.00
time (sec)	N/A	0.002	0.004	0.017	0.278	0.338	0.051	0.001	0.129	1.825

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	17	9	15	19	15	11	5	15
N.S.	1	1.00	3.40	1.80	3.00	3.80	3.00	2.20	1.00	3.00
time (sec)	N/A	0.001	0.004	0.020	0.268	0.336	0.051	0.001	0.045	1.830

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	13	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.93	0.71	0.71
time (sec)	N/A	0.003	0.002	0.000	0.301	0.344	0.028	0.000	0.032	1.749

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	17	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.85	0.80	0.80
time (sec)	N/A	0.010	0.002	0.022	0.257	0.326	0.153	0.001	0.158	1.847

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	11	11	8	11	10	10
N.S.	1	1.00	1.15	0.85	0.85	0.85	0.62	0.85	0.77	0.77
time (sec)	N/A	0.004	0.002	0.000	0.253	0.328	0.034	0.000	0.034	1.703

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0	35	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.005	0.031	0.017	0.000	0.336	0.000	0.000	0.315	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	18	17	18	15	15
N.S.	1	1.00	1.00	0.84	0.79	0.95	0.89	0.95	0.79	0.79
time (sec)	N/A	0.013	0.003	0.027	0.258	0.339	0.229	0.001	0.045	2.016

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	12	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.86	0.71	0.71
time (sec)	N/A	0.004	0.002	0.000	0.252	0.329	0.028	0.000	0.026	1.747

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	15	11	9	10	8	11	9	9
N.S.	1	1.00	1.36	1.00	0.82	0.91	0.73	1.00	0.82	0.82
time (sec)	N/A	0.004	0.002	0.013	0.253	0.329	0.030	0.000	0.031	1.753

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	7	5	6	2	2
N.S.	1	1.00	1.00	1.50	1.00	3.50	2.50	3.00	1.00	1.00
time (sec)	N/A	0.003	0.002	0.039	0.273	0.323	0.028	0.000	0.023	1.772

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	7	11	10	20	8	6	6
N.S.	1	1.00	1.00	0.47	0.73	0.67	1.33	0.53	0.40	0.40
time (sec)	N/A	0.005	0.006	0.028	0.264	0.351	0.142	0.000	0.028	1.949

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	8	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.00	1.00	1.00
time (sec)	N/A	0.005	0.002	0.011	0.258	0.331	0.071	0.000	0.021	1.815

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	15	18	15	15	17	15	15	17
N.S.	1	1.00	0.88	1.06	0.88	0.88	1.00	0.88	0.88	1.00
time (sec)	N/A	0.012	0.011	0.000	0.248	0.329	0.102	0.000	0.028	1.846

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	19	19	36	28	19	20
N.S.	1	1.00	1.00	1.00	0.76	0.76	1.44	1.12	0.76	0.80
time (sec)	N/A	0.008	0.008	0.000	0.274	0.322	0.105	0.001	0.055	2.036

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	37	26	29	56	35	28	28
N.S.	1	1.00	0.71	0.90	0.63	0.71	1.37	0.85	0.68	0.68
time (sec)	N/A	0.019	0.026	0.017	0.256	0.321	0.155	0.001	0.155	2.209

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	23	23	23	39	32	25	25
N.S.	1	1.00	0.94	0.70	0.70	0.70	1.18	0.97	0.76	0.76
time (sec)	N/A	0.013	0.007	0.023	0.253	0.324	0.154	0.000	0.066	2.106

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	7	7	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.005	0.002	0.019	0.270	0.313	0.070	0.000	0.019	1.787

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	17	14	14	17	16	14	16
N.S.	1	1.00	0.88	1.06	0.88	0.88	1.06	1.00	0.88	1.00
time (sec)	N/A	0.013	0.010	0.020	0.273	0.322	0.102	0.000	0.026	1.917

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	19	19	36	26	19	20
N.S.	1	1.00	1.00	1.00	0.76	0.76	1.44	1.04	0.76	0.80
time (sec)	N/A	0.009	0.008	0.022	0.275	0.346	0.105	0.000	0.151	1.999

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	37	26	29	56	35	28	28
N.S.	1	1.00	0.71	0.90	0.63	0.71	1.37	0.85	0.68	0.68
time (sec)	N/A	0.019	0.026	0.034	0.257	0.348	0.154	0.000	0.065	2.131

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	23	23	25	39	30	25	24
N.S.	1	1.00	0.94	0.70	0.70	0.76	1.18	0.91	0.76	0.73
time (sec)	N/A	0.014	0.007	0.032	0.253	0.322	0.154	0.000	0.169	2.090

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	C	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	13	2	2	2	-1	2
N.S.	1	1.00	1.00	1.50	6.50	1.00	1.00	1.00	-0.50	1.00
time (sec)	N/A	0.007	0.009	0.012	0.300	0.323	0.314	0.001	0.000	1.940

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	C	B	B	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	13	11	12	2	-1	13
N.S.	1	1.00	1.00	1.50	6.50	5.50	6.00	1.00	-0.50	6.50
time (sec)	N/A	0.007	0.002	0.033	0.288	0.424	0.463	0.001	0.000	2.231

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	C	A	B	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	15	20	17	11	-1	20
N.S.	1	1.00	1.00	1.10	1.50	2.00	1.70	1.10	-0.10	2.00
time (sec)	N/A	0.014	0.002	0.036	0.289	0.317	0.720	0.002	0.000	2.664

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	C	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	17	17	10	16	-1	11
N.S.	1	1.00	1.00	0.80	1.13	1.13	0.67	1.07	-0.07	0.73
time (sec)	N/A	0.022	0.009	0.031	0.294	0.327	0.575	0.001	0.000	2.283

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	20	18	12	16	16	10
N.S.	1	1.00	1.00	1.42	1.67	1.50	1.00	1.33	1.33	0.83
time (sec)	N/A	0.004	0.003	0.015	0.280	0.344	0.046	0.001	0.021	1.676

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	22	12	11	11	14	10	11	21
N.S.	1	1.00	2.00	1.09	1.00	1.00	1.27	0.91	1.00	1.91
time (sec)	N/A	0.002	0.007	0.019	0.264	0.325	0.064	0.000	0.021	1.691

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	10	10	12	9	10	20
N.S.	1	1.00	2.10	1.10	1.00	1.00	1.20	0.90	1.00	2.00
time (sec)	N/A	0.002	0.007	0.021	0.248	0.311	0.063	0.000	0.022	1.729

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	11	18	19	12	16	26
N.S.	1	1.00	1.00	1.42	0.92	1.50	1.58	1.00	1.33	2.17
time (sec)	N/A	0.002	0.007	0.012	0.266	0.322	0.064	0.002	0.234	2.218

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	19	26	11	27	29	57	28	37
N.S.	1	1.00	1.73	2.36	1.00	2.45	2.64	5.18	2.55	3.36
time (sec)	N/A	0.002	0.007	0.014	0.263	0.324	0.172	0.011	0.190	2.709

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	B	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	38	21	26	30	17	17	12	26
N.S.	1	1.00	3.17	1.75	2.17	2.50	1.42	1.42	1.00	2.17
time (sec)	N/A	0.002	0.013	0.033	0.258	0.341	0.276	0.002	0.117	1.962

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	19	26	28	34	48	11	43
N.S.	1	1.00	1.00	1.73	2.36	2.55	3.09	4.36	1.00	3.91
time (sec)	N/A	0.003	0.002	0.027	0.268	0.337	0.314	0.005	0.024	2.367

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	24	23	46	24	18	32
N.S.	1	1.00	0.92	1.08	0.96	0.92	1.84	0.96	0.72	1.28
time (sec)	N/A	0.006	0.020	0.027	0.256	0.324	0.097	0.001	0.209	2.240

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	22	22	22	37	29	24	34
N.S.	1	1.00	1.07	0.81	0.81	0.81	1.37	1.07	0.89	1.26
time (sec)	N/A	0.006	0.009	0.038	0.267	0.325	0.128	0.001	0.147	2.378

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	22	22	46	23	18	32
N.S.	1	1.00	0.92	1.08	0.88	0.88	1.84	0.92	0.72	1.28
time (sec)	N/A	0.005	0.014	0.031	0.285	0.318	0.095	0.001	0.177	2.575

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	22	22	21	36	22	24	34
N.S.	1	1.00	1.00	0.85	0.85	0.81	1.38	0.85	0.92	1.31
time (sec)	N/A	0.006	0.006	0.036	0.276	0.327	0.145	0.001	0.025	2.286

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	18	58	12	10	76
N.S.	1	1.00	1.00	1.10	1.00	1.80	5.80	1.20	1.00	7.60
time (sec)	N/A	0.005	0.003	0.027	0.274	0.311	0.541	0.001	0.145	2.447

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	9	9	3	9	4	4
N.S.	1	1.00	0.67	0.56	1.00	1.00	0.33	1.00	0.44	0.44
time (sec)	N/A	0.005	0.003	0.016	0.274	0.310	0.099	0.000	0.171	1.696

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	10	10	7	11	6	8
N.S.	1	1.00	0.67	0.75	0.83	0.83	0.58	0.92	0.50	0.67
time (sec)	N/A	0.006	0.008	0.018	0.257	0.302	0.170	0.001	0.166	1.743

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	23	11	15	18	8	11	10	10
N.S.	1	1.00	2.30	1.10	1.50	1.80	0.80	1.10	1.00	1.00
time (sec)	N/A	0.004	0.007	0.020	0.254	0.299	0.185	0.001	0.024	1.762

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	11	15	17	8	11	10	10
N.S.	1	1.00	2.27	1.00	1.36	1.55	0.73	1.00	0.91	0.91
time (sec)	N/A	0.005	0.008	0.032	0.304	0.303	0.191	0.003	0.034	1.830

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	0	148	141	57	45	176
N.S.	1	1.00	1.00	0.98	0.00	3.70	3.52	1.42	1.12	4.40
time (sec)	N/A	0.026	0.026	0.048	0.000	0.323	2.958	0.004	0.383	6.884

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	43	0	287	1872	69	58	477
N.S.	1	1.00	0.94	0.91	0.00	6.11	39.83	1.47	1.23	10.15
time (sec)	N/A	0.037	0.043	0.094	0.000	0.340	65.066	0.005	0.260	242.835

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	47	158	117	54	105	71	52	69
N.S.	1	1.00	0.64	2.16	1.60	0.74	1.44	0.97	0.71	0.95
time (sec)	N/A	0.027	0.093	0.038	0.292	0.320	0.209	0.004	0.137	3.306

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	12	20	13	9	9
N.S.	1	1.00	1.00	0.80	0.73	0.80	1.33	0.87	0.60	0.60
time (sec)	N/A	0.005	0.006	0.042	0.258	0.311	0.143	0.000	0.025	2.099

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	47	158	113	54	105	71	52	69
N.S.	1	1.00	0.64	2.16	1.55	0.74	1.44	0.97	0.71	0.95
time (sec)	N/A	0.026	0.085	0.042	0.274	0.314	0.208	0.003	0.103	3.303

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	22	14	31	14	34	21	12
N.S.	1	1.00	1.00	1.57	1.00	2.21	1.00	2.43	1.50	0.86
time (sec)	N/A	0.006	0.003	0.017	0.287	0.307	0.046	0.002	0.168	1.896

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	C	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	101	138	491	182	0	0	-1	0
N.S.	1	1.00	0.97	1.33	4.72	1.75	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.140	0.115	0.054	0.399	0.323	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	C	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	133	237	777	212	0	0	-1	0
N.S.	1	1.00	0.87	1.55	5.08	1.39	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.255	0.233	0.045	0.632	0.325	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	20	107	21	19	27	13	17
N.S.	1	1.00	1.00	1.33	7.13	1.40	1.27	1.80	0.87	1.13
time (sec)	N/A	0.010	0.012	0.013	0.343	0.313	0.075	0.006	0.022	2.062

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	13	26	14	13	11
N.S.	1	1.00	1.00	0.80	0.73	0.87	1.73	0.93	0.87	0.73
time (sec)	N/A	0.005	0.008	0.057	0.263	0.311	0.144	0.001	0.044	2.115

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	14	19	10	19	14	13	18	10
N.S.	1	1.00	0.58	0.79	0.42	0.79	0.58	0.54	0.75	0.42
time (sec)	N/A	0.016	0.005	0.046	0.257	0.302	0.033	0.000	0.044	1.958

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	6	15	9	18	12	7	6	8
N.S.	1	1.00	0.86	2.14	1.29	2.57	1.71	1.00	0.86	1.14
time (sec)	N/A	0.014	0.007	0.028	0.272	0.301	0.033	0.001	0.064	1.886

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	22	33	25	22	104	381	22	88
N.S.	1	1.00	0.69	1.03	0.78	0.69	3.25	11.91	0.69	2.75
time (sec)	N/A	0.007	0.016	0.033	0.267	0.315	0.295	0.003	0.024	2.832

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	32	24	20	107	383	20	87
N.S.	1	1.00	0.65	1.03	0.77	0.65	3.45	12.35	0.65	2.81
time (sec)	N/A	0.005	0.014	0.031	0.268	0.315	0.294	0.007	0.017	2.730

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	A	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	50	58	60	60	308	1369	57	249
N.S.	1	1.00	0.60	0.69	0.71	0.71	3.67	16.30	0.68	2.96
time (sec)	N/A	0.032	0.037	0.042	0.274	0.316	0.509	0.004	0.285	3.643

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	A	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	49	56	58	58	304	1373	55	245
N.S.	1	1.00	0.59	0.67	0.70	0.70	3.66	16.54	0.66	2.95
time (sec)	N/A	0.029	0.035	0.039	0.288	0.306	0.502	0.004	0.213	3.633

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	C	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	94	102	107	115	665	3343	133	549
N.S.	1	1.00	0.58	0.63	0.66	0.71	4.10	20.64	0.82	3.39
time (sec)	N/A	0.113	0.061	0.057	0.316	0.311	1.058	0.005	0.392	4.971

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	C	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	93	100	105	111	668	3351	132	547
N.S.	1	1.00	0.58	0.62	0.65	0.69	4.15	20.81	0.82	3.40
time (sec)	N/A	0.109	0.053	0.057	0.273	0.315	1.045	0.005	0.348	5.046

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	C	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	169	166	186	203	1355	6997	231	1128
N.S.	1	1.00	0.65	0.64	0.71	0.78	5.19	26.81	0.89	4.32
time (sec)	N/A	0.273	0.105	0.074	0.292	0.310	2.196	0.008	0.627	9.866

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	C	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	168	164	184	202	1352	6997	232	1126
N.S.	1	1.00	0.65	0.63	0.71	0.78	5.20	26.91	0.89	4.33
time (sec)	N/A	0.266	0.086	0.073	0.282	0.320	2.157	0.008	0.640	14.776

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	17	114	18	14	20
N.S.	1	1.00	1.00	0.80	0.76	0.68	4.56	0.72	0.56	0.80
time (sec)	N/A	0.018	0.011	0.063	0.258	0.319	1.260	0.001	0.166	6.831

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	25	116	26	22	22
N.S.	1	1.00	1.00	0.77	0.73	0.83	3.87	0.87	0.73	0.73
time (sec)	N/A	0.019	0.011	0.061	0.254	0.321	1.249	0.001	0.276	3.781

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	55	64	55	59	100	103	67	63
N.S.	1	1.00	0.65	0.75	0.65	0.69	1.18	1.21	0.79	0.74
time (sec)	N/A	0.041	0.052	0.067	0.255	0.317	0.280	0.001	0.230	2.976

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	N/A	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	240	0	0	0	-1	0
N.S.	1	0.00	0.00	0.00	17.14	0.00	0.00	0.00	-0.07	0.00
time (sec)	N/A	0.297	0.580	0.262	0.258	0.320	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	41	16	0	33	10	0
N.S.	1	1.00	1.00	0.92	3.42	1.33	0.00	2.75	0.83	0.00
time (sec)	N/A	0.024	0.010	0.069	0.252	0.318	0.000	0.008	0.175	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	76	107	0	312	0	114	133	0
N.S.	1	1.00	0.99	1.39	0.00	4.05	0.00	1.48	1.73	0.00
time (sec)	N/A	0.065	0.175	0.153	0.000	0.342	0.000	0.008	0.485	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	12	13	15	16	13	13
N.S.	1	1.00	1.00	0.82	0.71	0.76	0.88	0.94	0.76	0.76
time (sec)	N/A	0.002	0.003	0.023	0.253	0.319	0.128	0.000	0.133	1.946

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	10	13	15	15	13	13
N.S.	1	1.00	1.00	0.82	0.59	0.76	0.88	0.88	0.76	0.76
time (sec)	N/A	0.002	0.003	0.018	0.265	0.319	0.126	0.000	0.148	1.981

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	3	2	2	2	2	2	4
N.S.	1	1.00	1.00	1.00	0.67	0.67	0.67	0.67	0.67	1.33
time (sec)	N/A	0.001	0.000	0.007	0.256	0.292	0.018	0.000	0.007	1.685

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	8	7	8	16
N.S.	1	1.00	1.00	1.12	1.00	1.00	1.00	0.88	1.00	2.00
time (sec)	N/A	0.002	0.000	0.012	0.252	0.336	0.043	0.000	0.155	1.807

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	8	8	7	7	8	16
N.S.	1	1.00	1.00	1.00	0.89	0.89	0.78	0.78	0.89	1.78
time (sec)	N/A	0.001	0.004	0.008	0.255	0.291	0.031	0.000	0.027	1.972

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	9	4	4	3	4	4	4
N.S.	1	1.00	1.00	2.25	1.00	1.00	0.75	1.00	1.00	1.00
time (sec)	N/A	0.007	0.009	0.013	0.299	0.306	0.330	0.000	0.014	1.976

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	38	29	23	22	15	26	22	25
N.S.	1	1.00	1.58	1.21	0.96	0.92	0.62	1.08	0.92	1.04
time (sec)	N/A	0.011	0.018	0.014	0.257	0.313	0.062	0.001	0.093	1.914

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	9	10	12
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.75	0.83	1.00
time (sec)	N/A	0.013	0.012	0.012	0.269	0.311	0.045	0.000	0.045	1.768

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	14	12	14	13	14	20
N.S.	1	1.00	1.00	1.15	1.08	0.92	1.08	1.00	1.08	1.54
time (sec)	N/A	0.005	0.006	0.010	0.262	0.303	0.044	0.000	0.052	1.796

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	22	23	85	26	26	21	32
N.S.	1	1.00	1.00	0.71	0.74	2.74	0.84	0.84	0.68	1.03
time (sec)	N/A	0.019	0.026	0.020	0.354	0.336	0.090	0.001	0.228	2.106

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	14	19	13	13	19	14	13	27
N.S.	1	1.00	0.67	0.90	0.62	0.62	0.90	0.67	0.62	1.29
time (sec)	N/A	0.006	0.009	0.010	0.265	0.306	0.048	0.000	0.027	1.949

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	102	143	101	101	102	111	101	102
N.S.	1	1.00	0.63	0.88	0.62	0.62	0.63	0.68	0.62	0.63
time (sec)	N/A	0.149	0.068	0.035	0.264	0.303	0.061	0.000	0.362	2.341

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	0	18	0	0	18	0
N.S.	1	1.00	1.00	1.06	0.00	1.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.014	0.015	0.024	0.000	0.310	0.000	0.000	0.204	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	0	14	31	263	14	40
N.S.	1	1.00	1.00	1.07	0.00	1.00	2.21	18.79	1.00	2.86
time (sec)	N/A	0.010	0.012	0.022	0.000	0.311	0.262	0.004	0.167	2.056

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	21	10	19	0	0	19	0
N.S.	1	1.00	1.00	1.24	0.59	1.12	0.00	0.00	1.12	0.00
time (sec)	N/A	0.012	0.013	0.029	0.310	0.300	0.000	0.000	0.131	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	43	79	0	54	0	0	-1	0
N.S.	1	1.00	0.67	1.23	0.00	0.84	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.063	0.131	0.040	0.000	0.313	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	16	16	12	15	15	16
N.S.	1	1.00	1.00	1.00	1.00	1.00	0.75	0.94	0.94	1.00
time (sec)	N/A	0.020	0.048	0.027	0.314	0.297	0.062	0.001	0.196	1.900

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	15	11	11	23
N.S.	1	1.00	1.00	0.92	0.85	0.85	1.15	0.85	0.85	1.77
time (sec)	N/A	0.005	0.010	0.015	0.258	0.303	0.049	0.000	0.026	1.834

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	C	A	A	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	7	8	11	7	7
N.S.	1	1.00	1.00	0.73	0.82	0.64	0.73	1.00	0.64	0.64
time (sec)	N/A	0.002	0.011	0.016	0.268	0.310	0.083	0.000	0.017	1.881

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	5	7	6	7
N.S.	1	1.00	1.00	0.78	0.67	0.67	0.56	0.78	0.67	0.78
time (sec)	N/A	0.004	0.008	0.007	0.274	0.321	0.040	0.000	0.018	1.692

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	17	25	17	17	14	20	17	17
N.S.	1	1.00	0.63	0.93	0.63	0.63	0.52	0.74	0.63	0.63
time (sec)	N/A	0.067	0.009	0.022	0.296	0.299	0.049	0.001	0.158	1.795

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	F	A	A	A	A	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	25	23	22	36	31	35	-1	25
N.S.	1	0.00	1.00	0.92	0.88	1.44	1.24	1.40	-0.04	1.00
time (sec)	N/A	0.659	0.113	0.046	0.319	0.311	0.122	0.013	0.000	2.209

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	N/A	A	A	A	A	A	A	A	A	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0	-1	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08	0.00
time (sec)	N/A	0.031	0.016	0.010	0.000	0.309	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	10	10	8	9	10	11
N.S.	1	1.00	1.00	1.09	0.91	0.91	0.73	0.82	0.91	1.00
time (sec)	N/A	0.010	0.005	0.015	0.283	0.309	0.973	0.000	0.026	2.639

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	17	21	15	15	17	14	28	17
N.S.	1	1.00	0.77	0.95	0.68	0.68	0.77	0.64	1.27	0.77
time (sec)	N/A	0.033	0.009	0.014	0.284	0.304	1.649	0.001	0.219	3.423

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	17	28	11	11	10	12	11	12
N.S.	1	1.00	0.74	1.22	0.48	0.48	0.43	0.52	0.48	0.52
time (sec)	N/A	0.007	0.005	0.010	0.251	0.306	0.048	0.000	0.042	1.695

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	19	12	12	12	19	12	12
N.S.	1	1.00	1.00	1.19	0.75	0.75	0.75	1.19	0.75	0.75
time (sec)	N/A	0.002	0.000	0.017	0.342	0.293	0.026	0.000	0.030	1.677

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	43	43	58	89	1166	70	49	1019
N.S.	1	1.00	0.75	0.75	1.02	1.56	20.46	1.23	0.86	17.88
time (sec)	N/A	0.019	0.017	0.036	0.334	0.340	2.004	0.002	0.094	14.134

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	95	140	0	300	0	299	88	0
N.S.	1	1.00	0.82	1.21	0.00	2.59	0.00	2.58	0.76	0.00
time (sec)	N/A	0.045	0.219	0.039	0.000	0.324	0.000	0.021	0.169	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	21	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	1.31	0.75	0.75
time (sec)	N/A	0.001	0.007	0.026	0.253	0.298	0.027	0.000	0.025	1.639

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	26	26	30	202	99	25	96
N.S.	1	1.00	1.00	0.76	0.76	0.88	5.94	2.91	0.74	2.82
time (sec)	N/A	0.006	0.014	0.035	0.250	0.301	0.556	0.001	0.030	3.803

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	38	41	42	666	145	37	184
N.S.	1	1.00	0.66	0.72	0.77	0.79	12.57	2.74	0.70	3.47
time (sec)	N/A	0.009	0.018	0.041	0.289	0.301	0.848	0.001	0.148	7.661

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	42	73	68	42	27	74
N.S.	1	1.00	1.00	0.80	1.20	2.09	1.94	1.20	0.77	2.11
time (sec)	N/A	0.007	0.020	0.031	0.348	0.321	0.685	0.001	0.146	2.900

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	37	47	93	44	58	31	38
N.S.	1	1.00	1.00	0.95	1.21	2.38	1.13	1.49	0.79	0.97
time (sec)	N/A	0.007	0.048	0.032	0.335	0.313	0.889	0.002	0.054	2.857

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	13	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.93	0.86	0.86
time (sec)	N/A	0.001	0.006	0.026	0.297	0.291	0.027	0.000	0.018	1.664

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	23	26	26	19	162	36	25	73
N.S.	1	1.00	0.72	0.81	0.81	0.59	5.06	1.12	0.78	2.28
time (sec)	N/A	0.005	0.012	0.026	0.251	0.300	0.547	0.001	0.029	3.343

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	37	41	31	600	61	37	160
N.S.	1	1.00	0.69	0.73	0.80	0.61	11.76	1.20	0.73	3.14
time (sec)	N/A	0.008	0.019	0.027	0.260	0.305	0.852	0.001	0.038	7.174

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	32	56	24	27	17	16
N.S.	1	1.00	1.00	0.78	1.39	2.43	1.04	1.17	0.74	0.70
time (sec)	N/A	0.005	0.015	0.026	0.339	0.320	0.480	0.001	0.144	2.240

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	40	60	93	44	65	33	40
N.S.	1	1.00	1.00	0.98	1.46	2.27	1.07	1.59	0.80	0.98
time (sec)	N/A	0.008	0.043	0.031	0.338	0.323	1.132	0.001	0.159	3.127

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	24	25	21	25	24	22	21	0
N.S.	1	1.00	1.04	1.09	0.91	1.09	1.04	0.96	0.91	0.00
time (sec)	N/A	0.003	0.019	0.029	0.256	0.316	0.030	0.000	0.251	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	38	43	45	58	216	95	94	226
N.S.	1	1.00	0.79	0.90	0.94	1.21	4.50	1.98	1.96	4.71
time (sec)	N/A	0.010	0.026	0.031	0.263	0.302	0.248	0.002	0.448	2.584

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	37	52	37	230	72	43	96
N.S.	1	1.00	0.87	0.67	0.95	0.67	4.18	1.31	0.78	1.75
time (sec)	N/A	0.020	0.023	0.050	0.393	0.343	0.428	0.000	0.162	3.846

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	38	11	14	14	2	15	10	2
N.S.	1	1.00	3.17	0.92	1.17	1.17	0.17	1.25	0.83	0.17
time (sec)	N/A	0.001	0.002	0.053	0.253	0.311	0.063	0.001	0.197	1.639

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	37	50	71	34	119	51	30	83
N.S.	1	1.00	0.86	1.16	1.65	0.79	2.77	1.19	0.70	1.93
time (sec)	N/A	0.004	0.035	0.034	0.254	0.326	1.564	0.002	0.217	3.607

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	22	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	1.00	0.73	0.73
time (sec)	N/A	0.007	0.011	0.013	0.264	0.317	0.104	0.001	0.245	1.792

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	18	8	14	11	11
N.S.	1	1.00	1.00	0.92	0.85	1.38	0.62	1.08	0.85	0.85
time (sec)	N/A	0.002	0.002	0.049	0.255	0.310	0.305	0.001	0.348	1.870

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	B	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	20	7	16	18	19	7	16	21
N.S.	1	1.00	2.50	0.88	2.00	2.25	2.38	0.88	2.00	2.62
time (sec)	N/A	0.002	0.060	0.132	0.342	0.312	0.419	0.002	0.312	2.054

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	25	25	8	35	10	6
N.S.	1	1.00	1.00	0.79	1.79	1.79	0.57	2.50	0.71	0.43
time (sec)	N/A	0.004	0.012	0.095	0.260	0.306	0.414	0.001	0.186	2.027

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	26	14	0	22	0	26	18	0
N.S.	1	1.00	1.44	0.78	0.00	1.22	0.00	1.44	1.00	0.00
time (sec)	N/A	0.011	0.048	0.075	0.000	0.304	0.000	0.003	0.359	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	C	C	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	27	23	17	55	0	0	32	0
N.S.	1	1.00	0.90	0.77	0.57	1.83	0.00	0.00	1.07	0.00
time (sec)	N/A	0.014	0.046	0.139	0.356	0.317	0.000	0.000	0.538	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	14	17	0	48	14	0
N.S.	1	1.00	1.00	1.12	0.82	1.00	0.00	2.82	0.82	0.00
time (sec)	N/A	0.006	0.121	0.038	0.347	0.303	0.000	0.004	0.151	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	12	3	2	14	2	14	2	2
N.S.	1	1.00	6.00	1.50	1.00	7.00	1.00	7.00	1.00	1.00
time (sec)	N/A	0.001	0.011	0.041	0.384	0.306	0.062	0.001	0.028	1.646

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	30	15	14	14	17	0	14	14
N.S.	1	1.00	1.50	0.75	0.70	0.70	0.85	0.00	0.70	0.70
time (sec)	N/A	0.600	0.016	0.051	0.272	0.302	0.255	0.000	0.345	1.816

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	30	20	19	0	20	20
N.S.	1	1.00	1.00	0.88	1.25	0.83	0.79	0.00	0.83	0.83
time (sec)	N/A	0.198	13.736	0.070	0.391	0.319	0.308	0.000	0.310	1.983

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	71	24	31	29	8	33	23	9
N.S.	1	1.00	2.63	0.89	1.15	1.07	0.30	1.22	0.85	0.33
time (sec)	N/A	0.005	0.006	0.057	0.264	0.309	0.083	0.002	0.687	1.663

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	25	0	216	51	0	-1	33
N.S.	1	1.00	1.00	0.30	0.00	2.63	0.62	0.00	-0.01	0.40
time (sec)	N/A	0.047	0.067	0.026	0.000	0.331	1.728	0.000	0.000	3.536

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	39	40	29	48	19	169	40
N.S.	1	1.00	1.00	3.25	3.33	2.42	4.00	1.58	14.08	3.33
time (sec)	N/A	0.033	0.012	0.113	0.352	0.298	1.209	0.001	0.061	50.647

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	33	36	85	66	42	32	54
N.S.	1	1.00	1.00	0.82	0.90	2.12	1.65	1.05	0.80	1.35
time (sec)	N/A	0.008	0.007	0.036	0.262	0.334	0.501	0.002	0.479	2.339

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	66	57	41	42	46	40	42
N.S.	1	1.00	1.00	1.43	1.24	0.89	0.91	1.00	0.87	0.91
time (sec)	N/A	0.022	0.025	0.041	0.346	0.299	0.545	0.001	0.665	2.566

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	45	52	40	52	0	42	51	0
N.S.	1	1.00	1.22	1.41	1.08	1.41	0.00	1.14	1.38	0.00
time (sec)	N/A	0.011	0.079	0.116	0.377	0.324	0.000	0.009	0.105	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	66	74	77	97	0	54	72	0
N.S.	1	1.00	1.08	1.21	1.26	1.59	0.00	0.89	1.18	0.00
time (sec)	N/A	0.015	0.106	0.129	0.367	0.308	0.000	0.011	0.246	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	29	22	19	37
N.S.	1	1.00	1.00	0.87	0.83	0.83	1.26	0.96	0.83	1.61
time (sec)	N/A	0.003	0.003	0.040	0.258	0.297	0.228	0.000	0.289	2.131

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	36	18	24	48
N.S.	1	1.00	1.00	0.89	0.86	0.86	1.29	0.64	0.86	1.71
time (sec)	N/A	0.003	0.003	0.043	0.251	0.314	0.303	0.001	0.272	2.128

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	F(-2)	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	150	47	0	155	0	72	50	0
N.S.	1	1.00	2.68	0.84	0.00	2.77	0.00	1.29	0.89	0.00
time (sec)	N/A	0.026	0.177	0.026	0.000	0.331	0.000	0.010	0.979	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	86	70	68	94	0	96	67	0
N.S.	1	1.00	1.06	0.86	0.84	1.16	0.00	1.19	0.83	0.00
time (sec)	N/A	0.019	0.183	0.117	0.255	0.323	0.000	0.010	0.237	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	53	56	0	58	0	43	54	0
N.S.	1	1.00	1.20	1.27	0.00	1.32	0.00	0.98	1.23	0.00
time (sec)	N/A	0.026	0.075	0.028	0.000	0.308	0.000	0.012	0.400	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	72	78	0	106	0	43	72	0
N.S.	1	1.00	1.06	1.15	0.00	1.56	0.00	0.63	1.06	0.00
time (sec)	N/A	0.032	0.111	0.028	0.000	0.318	0.000	0.012	0.442	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	22	10	14	11	11
N.S.	1	1.00	1.00	0.92	0.85	1.69	0.77	1.08	0.85	0.85
time (sec)	N/A	0.011	0.005	0.039	0.262	0.307	0.101	0.010	0.089	1.963

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	11	17	19	11	13
N.S.	1	1.00	1.00	0.84	0.79	0.58	0.89	1.00	0.58	0.68
time (sec)	N/A	0.004	0.002	0.008	0.259	0.297	0.051	0.001	0.165	1.741

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	21	20	12	10	12	12	12
N.S.	1	1.00	1.00	1.75	1.67	1.00	0.83	1.00	1.00	1.00
time (sec)	N/A	0.006	0.020	0.025	0.259	0.340	0.072	0.001	0.083	1.946

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	10	10	7	8	10	11
N.S.	1	1.00	1.00	1.00	0.91	0.91	0.64	0.73	0.91	1.00
time (sec)	N/A	0.016	0.007	0.013	0.292	0.307	0.073	0.002	0.216	1.737

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	22	20	26	22	22
N.S.	1	1.00	1.00	0.89	0.85	0.81	0.74	0.96	0.81	0.81
time (sec)	N/A	0.012	0.004	0.026	0.318	0.298	0.069	0.000	0.154	1.840

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	21	32	20	50	167	46	15	89
N.S.	1	1.00	0.64	0.97	0.61	1.52	5.06	1.39	0.45	2.70
time (sec)	N/A	0.002	0.046	0.033	0.254	0.297	2.903	0.005	0.299	5.344

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	10	8	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.78	1.11	0.89	1.00
time (sec)	N/A	0.001	0.002	0.031	0.269	0.330	0.045	0.000	0.197	1.767

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	28	28	39	52	13	28
N.S.	1	1.00	1.00	0.74	1.47	1.47	2.05	2.74	0.68	1.47
time (sec)	N/A	0.004	0.020	0.072	0.349	0.304	0.116	0.001	0.075	2.072

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	12	11	14	15	23	12	12
N.S.	1	1.00	0.84	0.63	0.58	0.74	0.79	1.21	0.63	0.63
time (sec)	N/A	0.002	0.008	0.012	0.262	0.321	0.516	0.000	0.024	2.099

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	112	89	86	104	153	139	119	87
N.S.	1	1.00	1.27	1.01	0.98	1.18	1.74	1.58	1.35	0.99
time (sec)	N/A	0.027	0.063	0.039	0.344	0.326	0.920	0.005	0.181	3.732

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	34	9	7	17	9	9
N.S.	1	1.00	1.00	1.11	3.78	1.00	0.78	1.89	1.00	1.00
time (sec)	N/A	0.007	0.004	0.016	0.259	0.299	0.077	0.000	0.021	1.779

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	34	9	7	18	9	9
N.S.	1	1.00	1.00	1.11	3.78	1.00	0.78	2.00	1.00	1.00
time (sec)	N/A	0.007	0.003	0.015	0.256	0.294	0.075	0.000	0.026	1.775

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	26	7	15	7	7
N.S.	1	1.00	1.00	0.89	0.78	2.89	0.78	1.67	0.78	0.78
time (sec)	N/A	0.002	0.005	0.014	0.268	0.302	0.067	0.001	0.186	1.838

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	16	13	26	22	27	13	29
N.S.	1	1.00	1.00	1.33	1.08	2.17	1.83	2.25	1.08	2.42
time (sec)	N/A	0.021	0.068	0.032	0.249	0.313	0.168	0.016	0.321	2.096

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	32	75	27	22	26
N.S.	1	1.00	1.00	0.82	0.79	1.14	2.68	0.96	0.79	0.93
time (sec)	N/A	0.016	0.015	0.051	0.268	0.316	0.351	0.001	0.332	2.777

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	13	15	19	14	15
N.S.	1	1.00	1.00	0.76	0.71	0.62	0.71	0.90	0.67	0.71
time (sec)	N/A	0.005	0.003	0.000	0.343	0.306	0.093	0.001	0.019	1.746

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	13	15	21	15	15
N.S.	1	1.00	1.00	0.76	0.71	0.62	0.71	1.00	0.71	0.71
time (sec)	N/A	0.005	0.003	0.034	0.356	0.304	0.095	0.002	0.048	1.737

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	23	22	19	26	22	41	26
N.S.	1	1.00	0.96	1.00	0.96	0.83	1.13	0.96	1.78	1.13
time (sec)	N/A	0.012	0.003	0.012	0.256	0.301	0.075	0.000	0.144	1.827

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	28	32	17	14	14
N.S.	1	1.00	1.00	0.83	0.78	1.56	1.78	0.94	0.78	0.78
time (sec)	N/A	0.011	0.015	0.066	0.258	0.322	0.142	0.001	0.026	2.025

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	28	32	18	14	14
N.S.	1	1.00	1.00	0.83	0.78	1.56	1.78	1.00	0.78	0.78
time (sec)	N/A	0.008	0.010	0.047	0.267	0.311	0.144	0.001	0.024	2.019

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	F	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	40	17	0	24	0	29	16	0
N.S.	1	1.00	3.33	1.42	0.00	2.00	0.00	2.42	1.33	0.00
time (sec)	N/A	0.004	0.009	0.062	0.000	0.296	0.000	0.003	0.139	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	F	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	42	23	0	26	0	46	18	0
N.S.	1	1.00	3.00	1.64	0.00	1.86	0.00	3.29	1.29	0.00
time (sec)	N/A	0.006	0.008	0.087	0.000	0.310	0.000	0.003	0.136	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	16	22	9	10	36	18	10	12
N.S.	1	1.00	1.33	1.83	0.75	0.83	3.00	1.50	0.83	1.00
time (sec)	N/A	0.005	0.006	0.043	0.385	0.309	0.373	0.002	0.145	2.365

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	22	20	18	0	29	12	0
N.S.	1	1.00	1.29	1.57	1.43	1.29	0.00	2.07	0.86	0.00
time (sec)	N/A	0.006	0.007	0.052	0.350	0.302	0.000	0.002	0.033	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	0	13	63	43	21	30
N.S.	1	1.00	0.81	0.67	0.00	0.62	3.00	2.05	1.00	1.43
time (sec)	N/A	0.005	0.063	0.025	0.000	0.297	0.225	0.003	0.193	2.262

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	31	18	18	20	22	18	18
N.S.	1	1.00	0.92	1.29	0.75	0.75	0.83	0.92	0.75	0.75
time (sec)	N/A	0.007	0.010	0.039	0.272	0.298	0.063	0.001	0.106	1.769

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	9	33	16	7	19	8	8
N.S.	1	1.00	1.50	0.75	2.75	1.33	0.58	1.58	0.67	0.67
time (sec)	N/A	0.003	0.060	0.079	0.264	0.293	0.423	0.002	0.037	2.032

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	33	92	24	24	0	39	24	0
N.S.	1	1.00	1.03	2.88	0.75	0.75	0.00	1.22	0.75	0.00
time (sec)	N/A	0.008	0.020	0.033	0.266	0.297	0.000	0.001	0.030	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	26	32	31	41	26	26
N.S.	1	1.00	1.00	0.82	0.79	0.97	0.94	1.24	0.79	0.79
time (sec)	N/A	0.006	0.009	0.017	0.424	0.313	0.065	0.001	0.060	1.968

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	11	3	4	3	3
N.S.	1	1.00	1.00	1.33	1.00	3.67	1.00	1.33	1.00	1.00
time (sec)	N/A	0.002	0.002	0.004	0.271	0.316	0.029	0.001	0.025	1.618

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	16	16	48	19	52	10	12
N.S.	1	1.00	1.50	1.33	1.33	4.00	1.58	4.33	0.83	1.00
time (sec)	N/A	0.007	0.003	0.001	0.360	0.323	0.034	0.002	0.028	1.879

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	18	7	10	3	9
N.S.	1	1.00	1.00	1.33	1.00	6.00	2.33	3.33	1.00	3.00
time (sec)	N/A	0.002	0.002	0.009	0.283	0.309	0.060	0.001	0.011	1.965

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	18	12	11	3	12
N.S.	1	1.00	1.00	1.33	1.00	6.00	4.00	3.67	1.00	4.00
time (sec)	N/A	0.002	0.003	0.008	0.271	0.317	0.154	0.001	0.137	2.047

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	8	7	8	16
N.S.	1	1.00	1.00	1.12	1.00	1.00	1.00	0.88	1.00	2.00
time (sec)	N/A	0.002	0.000	0.012	0.271	0.313	0.042	0.000	0.165	1.693

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	29	32	10	10	0	11	-1	0
N.S.	1	1.00	0.59	0.65	0.20	0.20	0.00	0.22	-0.02	0.00
time (sec)	N/A	0.010	0.006	0.013	0.358	0.311	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	17	13	12	12	12	17	10	12
N.S.	1	1.00	1.06	0.81	0.75	0.75	0.75	1.06	0.62	0.75
time (sec)	N/A	0.005	0.002	0.028	0.357	0.317	0.036	0.000	0.145	1.732

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	36	0	0	30	0	51	32	0
N.S.	1	1.00	0.90	0.00	0.00	0.75	0.00	1.28	0.80	0.00
time (sec)	N/A	0.014	0.007	0.003	0.000	0.321	0.000	0.005	0.042	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	67	63	58	55	109	71	69	72
N.S.	1	1.00	0.52	0.49	0.45	0.43	0.85	0.55	0.54	0.56
time (sec)	N/A	0.121	0.119	0.041	0.280	0.313	0.723	0.001	0.278	3.075

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	18	20	17	17	27	18	17	21
N.S.	1	1.00	0.60	0.67	0.57	0.57	0.90	0.60	0.57	0.70
time (sec)	N/A	0.022	0.018	0.026	0.265	0.329	0.185	0.000	0.171	1.886

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	7	17	17	9	7	19
N.S.	1	1.00	0.82	0.73	0.64	1.55	1.55	0.82	0.64	1.73
time (sec)	N/A	0.001	0.001	0.038	0.273	0.307	0.049	0.000	0.066	1.774

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	33	32	32	41	43	46	32
N.S.	1	1.00	1.00	0.82	0.80	0.80	1.02	1.08	1.15	0.80
time (sec)	N/A	0.013	0.007	0.036	0.341	0.315	0.065	0.001	0.115	1.880

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	23	22	17	17	15	24	6	17
N.S.	1	1.00	2.88	2.75	2.12	2.12	1.88	3.00	0.75	2.12
time (sec)	N/A	0.002	0.003	0.034	0.252	0.301	0.047	0.000	0.067	1.707

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	194	394	0	0	0	0	-1	0
N.S.	1	1.00	0.85	1.74	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.125	0.185	0.043	0.000	0.305	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	30	26	25	24	37	29	51	30
N.S.	1	1.00	1.25	1.08	1.04	1.00	1.54	1.21	2.12	1.25
time (sec)	N/A	0.006	0.005	0.011	0.255	0.303	0.681	0.000	0.443	2.478

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	71	54	57	69	73	93	70	49
N.S.	1	1.00	0.91	0.69	0.73	0.88	0.94	1.19	0.90	0.63
time (sec)	N/A	0.029	0.016	0.027	0.340	0.308	0.150	0.001	0.272	2.130

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	12	3	2	14	2	14	2	2
N.S.	1	1.00	6.00	1.50	1.00	7.00	1.00	7.00	1.00	1.00
time (sec)	N/A	0.001	0.001	0.028	0.390	0.314	0.061	0.001	0.002	1.592

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	31	21	20	25	24	33	20	20
N.S.	1	1.00	1.15	0.78	0.74	0.93	0.89	1.22	0.74	0.74
time (sec)	N/A	0.002	0.020	0.049	0.353	0.312	0.083	0.001	0.048	1.824

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	16	8	10	10	16
N.S.	1	1.00	1.00	1.10	1.00	1.60	0.80	1.00	1.00	1.60
time (sec)	N/A	0.003	0.003	0.028	0.249	0.302	0.037	0.000	0.031	1.639

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	12	15	14	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.75	0.94	0.88	0.88
time (sec)	N/A	0.003	0.001	0.003	0.338	0.317	0.060	0.000	0.220	1.716

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	34	33	24	32	59	24	33
N.S.	1	1.00	0.72	0.85	0.82	0.60	0.80	1.48	0.60	0.82
time (sec)	N/A	0.015	0.009	0.004	0.337	0.315	0.111	0.003	0.029	1.907

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	29	14	13	29	0	14	9	0
N.S.	1	1.00	1.38	0.67	0.62	1.38	0.00	0.67	0.43	0.00
time (sec)	N/A	0.076	0.022	0.105	0.264	0.318	0.000	0.002	0.810	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	19	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	1.36	0.71	0.71
time (sec)	N/A	0.004	0.002	0.018	0.264	0.318	0.030	0.001	0.002	1.667

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	18	14	16	16	16
N.S.	1	1.00	1.00	0.94	0.89	1.00	0.78	0.89	0.89	0.89
time (sec)	N/A	0.009	0.003	0.043	0.266	0.317	0.059	0.000	0.152	1.730

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	20	7	6	16	7	21	6	6
N.S.	1	1.00	2.00	0.70	0.60	1.60	0.70	2.10	0.60	0.60
time (sec)	N/A	0.001	0.011	0.047	0.348	0.318	0.069	0.001	0.038	1.649

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	12	5	4	14	3	14	4	4
N.S.	1	1.00	2.00	0.83	0.67	2.33	0.50	2.33	0.67	0.67
time (sec)	N/A	0.001	0.011	0.047	0.370	0.321	0.066	0.001	0.029	1.624

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	16	16	22	23	16	16
N.S.	1	1.00	1.00	0.81	0.76	0.76	1.05	1.10	0.76	0.76
time (sec)	N/A	0.011	0.007	0.159	0.344	0.311	0.056	0.000	0.142	1.716

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	42	47	73	46	52	45	50
N.S.	1	1.00	0.96	0.79	0.89	1.38	0.87	0.98	0.85	0.94
time (sec)	N/A	0.019	0.015	0.020	0.345	0.312	0.088	0.001	0.061	2.016

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	46	41	41	762	46	46	41
N.S.	1	1.00	1.00	0.94	0.84	0.84	15.55	0.94	0.94	0.84
time (sec)	N/A	0.049	0.025	0.044	0.264	0.333	55.713	0.001	0.210	65.470

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	N/A	A	A	A	A	A	A	A	A	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07	0.00
time (sec)	N/A	0.003	2.635	0.055	0.000	0.314	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	28	27	44	26	71	27	37
N.S.	1	1.00	1.00	1.00	0.96	1.57	0.93	2.54	0.96	1.32
time (sec)	N/A	0.133	33.446	0.190	0.290	0.317	0.173	0.012	0.338	1.966

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	136	128	160	145	1277	127	140	422
N.S.	1	1.00	0.68	0.64	0.80	0.73	6.42	0.64	0.70	2.12
time (sec)	N/A	0.069	0.153	0.177	0.284	0.324	135.784	0.003	1.111	164.248

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	15	20	15	21	15	16
N.S.	1	1.00	1.00	0.89	0.83	1.11	0.83	1.17	0.83	0.89
time (sec)	N/A	0.004	0.007	0.013	0.266	0.319	0.151	0.001	0.155	1.787

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	22	21	37	36	50	21	34
N.S.	1	1.00	1.00	0.92	0.88	1.54	1.50	2.08	0.88	1.42
time (sec)	N/A	0.005	0.022	0.016	0.325	0.302	0.236	0.004	0.066	2.072

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	F	C	B	F	B	F	F	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	5137	1197351	0	179	0	0	-1	0
N.S.	1	0.00	54.65	12737.78	0.00	1.90	0.00	0.00	-0.01	0.00
time (sec)	N/A	1.122	16.339	1.343	0.000	0.373	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	F	C	C	F	A	F	F	F	F(-2)
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	142	0	630	354	0	223	0	0	-1	0
N.S.	1	0.00	4.44	2.49	0.00	1.57	0.00	0.00	-0.01	0.00
time (sec)	N/A	1.940	3.476	0.339	0.000	0.367	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	31	1088	171	162	0	98	172	17
N.S.	1	1.00	1.48	51.81	8.14	7.71	0.00	4.67	8.19	0.81
time (sec)	N/A	0.171	0.567	0.199	0.345	0.302	0.000	0.014	2.373	261.497

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	F	C	A	F	F	F	F	F	F(-2)
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	4030	0	3168	4640	0	0	0	0	-1	0
N.S.	1	0.00	0.79	1.15	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.019	8.015	0.868	0.000	0.340	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	181	1356	0	269	330	465	265	212
N.S.	1	1.00	0.55	4.11	0.00	0.82	1.00	1.41	0.80	0.64
time (sec)	N/A	0.572	0.094	0.135	0.000	0.313	9.481	0.016	0.767	14.515

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	5	5	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	1.25	1.25	1.00	1.00
time (sec)	N/A	0.007	0.002	0.031	0.267	0.329	0.378	0.000	0.020	1.901

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	F	A	A	F	B	A	A	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	71	0	71	61	0	137	76	113	103	60
N.S.	1	0.00	1.00	0.86	0.00	1.93	1.07	1.59	1.45	0.85
time (sec)	N/A	0.468	0.025	0.037	0.000	0.321	0.117	0.014	0.280	2.377

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [282] had the largest ratio of [107]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	0	1.00	6	0.000
2	A	3	2	1.00	13	0.154
3	A	2	1	1.00	10	0.100
4	A	1	1	1.00	3	0.333
5	A	2	1	1.00	11	0.091
6	A	2	1	1.00	14	0.071
7	A	2	1	1.00	20	0.050
8	A	2	2	1.00	12	0.167
9	A	3	3	1.00	13	0.231
10	A	2	2	1.00	10	0.200
11	A	6	5	1.00	13	0.385
12	A	2	1	1.00	23	0.043
13	A	4	3	1.00	20	0.150
14	A	3	2	1.00	22	0.091
15	A	5	4	1.00	14	0.286
16	A	6	6	1.00	9	0.667
17	A	3	2	1.00	16	0.125
18	A	9	6	1.00	7	0.857
19	A	9	6	1.00	11	0.546
20	A	9	5	1.00	10	0.500
21	A	1	1	1.00	7	0.143
22	A	2	1	1.00	9	0.111
23	A	2	1	1.00	11	0.091
24	A	1	1	1.00	7	0.143
25	A	1	1	1.00	7	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	1	1.00	9	0.111
27	A	2	1	1.00	11	0.091
28	A	3	3	1.00	11	0.273
29	A	2	1	1.00	11	0.091
30	A	2	1	1.00	11	0.091
31	A	1	1	1.00	9	0.111
32	A	1	1	1.00	11	0.091
33	A	6	6	1.00	9	0.667
34	A	6	6	1.00	7	0.857
35	A	6	6	1.00	11	0.546
36	A	3	3	1.00	7	0.429
37	A	3	3	1.00	9	0.333
38	A	9	6	1.00	9	0.667
39	A	3	3	1.00	12	0.250
40	A	3	3	1.00	12	0.250
41	A	3	2	1.00	12	0.167
42	A	3	2	1.00	12	0.167
43	A	3	2	1.00	12	0.167
44	A	9	5	1.00	10	0.500
45	A	9	5	1.00	12	0.417
46	A	10	6	1.00	7	0.857
47	A	10	6	1.00	7	0.857
48	A	10	6	1.00	7	0.857
49	A	19	6	1.00	7	0.857
50	A	13	10	1.00	7	1.429
51	A	19	6	1.00	12	0.500
52	A	7	7	1.00	11	0.636
53	A	1	1	1.00	2	0.500
54	A	1	1	1.00	4	0.250
55	A	1	1	1.00	6	0.167
56	A	1	1	1.00	6	0.167
57	A	2	2	1.00	4	0.500
58	A	11	2	1.00	8	0.250
59	A	2	2	1.00	8	0.250
60	A	1	1	1.00	4	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	2	2	1.00	6	0.333
62	A	2	2	1.00	8	0.250
63	A	3	3	1.00	8	0.375
64	A	2	2	1.00	8	0.250
65	A	2	1	1.00	8	0.125
66	A	4	4	1.00	10	0.400
67	A	2	2	1.00	10	0.200
68	A	3	2	1.00	8	0.250
69	A	3	2	1.00	10	0.200
70	A	3	3	1.00	8	0.375
71	A	3	3	1.00	10	0.300
72	A	4	3	1.00	12	0.250
73	A	4	3	1.00	12	0.250
74	A	3	3	1.00	10	0.300
75	A	0	0	0.00	0	0.000
76	A	1	1	1.00	2	0.500
77	A	1	1	1.00	2	0.500
78	A	1	1	1.00	2	0.500
79	A	1	1	1.00	2	0.500
80	A	2	2	1.00	6	0.333
81	A	1	1	1.00	2	0.500
82	A	1	1	1.00	2	0.500
83	A	2	2	1.00	4	0.500
84	A	3	3	1.00	8	0.375
85	A	2	1	1.00	4	0.250
86	A	1	1	1.00	4	0.250
87	A	3	2	1.00	11	0.182
88	A	2	2	1.00	4	0.500
89	A	2	1	1.00	4	0.250
90	A	2	2	1.00	4	0.500
91	A	1	1	1.00	7	0.143
92	A	2	2	1.00	4	0.500
93	A	3	2	1.00	6	0.333
94	A	2	2	1.00	6	0.333
95	A	4	4	1.00	8	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.00	6	0.500
97	A	2	2	1.00	4	0.500
98	A	3	2	1.00	6	0.333
99	A	2	2	1.00	6	0.333
100	A	4	4	1.00	8	0.500
101	A	3	3	1.00	6	0.500
102	A	1	1	1.00	6	0.167
103	A	1	1	1.00	6	0.167
104	A	2	2	1.00	6	0.333
105	A	3	2	1.00	8	0.250
106	A	2	2	1.00	4	0.500
107	A	1	1	1.00	6	0.167
108	A	1	1	1.00	6	0.167
109	A	1	1	1.00	6	0.167
110	A	1	1	1.00	6	0.167
111	A	1	1	1.00	6	0.167
112	A	1	1	1.00	6	0.167
113	A	2	2	1.00	8	0.250
114	A	2	1	1.00	8	0.125
115	A	2	2	1.00	8	0.250
116	A	2	1	1.00	8	0.125
117	A	2	2	1.00	8	0.250
118	A	1	1	1.00	6	0.167
119	A	1	1	1.00	8	0.125
120	A	1	1	1.00	6	0.167
121	A	1	1	1.00	8	0.125
122	A	3	3	1.00	8	0.375
123	A	3	3	1.00	10	0.300
124	A	4	4	1.00	12	0.333
125	A	1	1	1.00	7	0.143
126	A	4	4	1.00	12	0.333
127	A	2	2	1.00	4	0.500
128	A	17	8	1.00	8	1.000
129	A	34	9	1.00	8	1.125
130	A	3	3	1.00	6	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	1	1	1.00	9	0.111
132	A	3	3	1.00	9	0.333
133	A	3	2	1.00	9	0.222
134	A	1	1	1.00	6	0.167
135	A	1	1	1.00	6	0.167
136	A	4	3	1.00	7	0.429
137	A	4	3	1.00	7	0.429
138	A	11	5	1.00	9	0.556
139	A	11	5	1.00	9	0.556
140	A	25	5	1.00	9	0.556
141	A	25	5	1.00	9	0.556
142	A	5	2	1.00	11	0.182
143	A	5	2	1.00	11	0.182
144	A	6	4	1.00	10	0.400
145	A	0	0	0.00	0	0.000
146	A	4	3	1.00	9	0.333
147	A	5	5	1.00	15	0.333
148	A	1	1	1.00	3	0.333
149	A	1	1	1.00	3	0.333
150	A	1	1	1.00	3	0.333
151	A	1	1	1.00	3	0.333
152	A	1	1	1.00	5	0.200
153	A	1	1	1.00	9	0.111
154	A	4	4	1.00	11	0.364
155	A	3	2	1.00	13	0.154
156	A	2	2	1.00	9	0.222
157	A	2	2	1.00	18	0.111
158	A	2	2	1.00	7	0.286
159	A	21	2	1.00	7	0.286
160	A	2	2	1.00	9	0.222
161	A	2	2	1.00	7	0.286
162	A	2	2	1.00	7	0.286
163	A	5	3	1.00	12	0.250
164	A	1	1	1.00	14	0.071
165	A	1	1	1.00	7	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	1	1	1.00	5	0.200
167	A	1	1	1.00	7	0.143
168	A	7	3	1.00	12	0.250
169	F	0	0	N/A	0	N/A
170	A	0	0	0.00	0	0.000
171	A	2	3	1.00	6	0.500
172	A	5	5	1.00	7	0.714
173	A	3	3	1.00	10	0.300
174	A	1	0	1.00	13	0.000
175	A	4	4	1.00	12	0.333
176	A	5	4	1.00	19	0.210
177	A	1	1	1.00	9	0.111
178	A	2	1	1.00	11	0.091
179	A	2	1	1.00	13	0.077
180	A	3	3	1.00	13	0.231
181	A	3	3	1.00	13	0.231
182	A	1	1	1.00	9	0.111
183	A	2	1	1.00	11	0.091
184	A	2	1	1.00	13	0.077
185	A	2	2	1.00	13	0.154
186	A	3	3	1.00	13	0.231
187	A	1	1	1.00	11	0.091
188	A	2	1	1.00	13	0.077
189	A	6	6	1.00	18	0.333
190	A	2	2	1.00	9	0.222
191	A	4	3	1.00	13	0.231
192	A	3	3	1.00	6	0.500
193	A	1	1	1.00	13	0.077
194	A	2	2	1.00	13	0.154
195	A	3	3	1.00	13	0.231
196	A	3	3	1.00	14	0.214
197	A	3	3	1.00	18	0.167
198	A	1	1	1.00	20	0.050
199	A	1	1	1.00	9	0.111
200	A	3	2	1.00	65	0.031

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	5	4	1.00	68	0.059
202	A	5	2	1.00	21	0.095
203	A	6	6	1.00	21	0.286
204	A	4	3	1.00	23	0.130
205	A	2	2	1.00	16	0.125
206	A	3	3	1.00	25	0.120
207	A	2	2	1.00	24	0.083
208	A	2	2	1.00	29	0.069
209	A	1	1	1.00	18	0.056
210	A	1	1	1.00	23	0.043
211	A	3	3	1.00	24	0.125
212	A	3	3	1.00	22	0.136
213	A	3	3	1.00	26	0.115
214	A	3	3	1.00	31	0.097
215	A	3	3	1.00	16	0.188
216	A	1	1	1.00	8	0.125
217	A	2	2	1.00	6	0.333
218	A	1	1	1.00	20	0.050
219	A	3	2	1.00	13	0.154
220	A	2	2	1.00	13	0.154
221	A	3	3	1.00	9	0.333
222	A	2	2	1.00	13	0.154
223	A	2	1	1.00	11	0.091
224	A	5	5	1.00	13	0.385
225	A	2	2	1.00	4	0.500
226	A	2	2	1.00	4	0.500
227	A	1	1	1.00	4	0.250
228	A	1	1	1.00	28	0.036
229	A	4	2	1.00	11	0.182
230	A	3	3	1.00	4	0.750
231	A	3	3	1.00	4	0.750
232	A	3	3	1.00	8	0.375
233	A	3	2	1.00	7	0.286
234	A	3	2	1.00	7	0.286
235	A	1	1	1.00	8	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	1	1	1.00	10	0.100
237	A	1	1	1.00	8	0.125
238	A	1	1	1.00	10	0.100
239	A	3	3	1.00	17	0.176
240	A	4	3	1.00	13	0.231
241	A	2	2	1.00	11	0.182
242	A	4	3	1.00	13	0.231
243	A	3	3	1.00	8	0.375
244	A	1	1	1.00	2	0.500
245	A	3	2	1.00	4	0.500
246	A	1	1	1.00	2	0.500
247	A	1	1	1.00	2	0.500
248	A	1	1	1.00	3	0.333
249	A	4	3	1.00	12	0.250
250	A	2	1	1.00	13	0.077
251	A	3	3	1.00	14	0.214
252	A	14	8	1.00	12	0.667
253	A	4	3	1.00	7	0.429
254	A	1	1	1.00	5	0.200
255	A	6	6	1.00	9	0.667
256	A	2	2	1.00	9	0.222
257	A	10	3	1.00	15	0.200
258	A	6	2	1.00	11	0.182
259	A	6	6	1.00	7	0.857
260	A	1	1	1.00	9	0.111
261	A	2	2	1.00	9	0.222
262	A	2	1	1.00	7	0.143
263	A	2	2	1.00	2	1.000
264	A	4	3	1.00	6	0.500
265	A	4	2	1.00	17	0.118
266	A	2	2	1.00	4	0.500
267	A	2	1	1.00	19	0.053
268	A	1	1	1.00	11	0.091
269	A	1	1	1.00	9	0.111
270	A	2	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	3	2	1.00	24	0.083
272	A	2	1	1.00	29	0.034
273	A	0	0	0.00	0	0.000
274	A	9	7	1.00	54	0.130
275	A	4	3	1.00	21	0.143
276	A	1	1	1.00	2	0.500
277	A	1	1	1.00	4	0.250
278	F	0	0	N/A	0	N/A
279	F	0	0	N/A	0	N/A
280	A	1	1	1.00	85	0.012
281	F	0	0	N/A	0	N/A
282	A	20	8	1.00	107	0.075
283	A	2	2	1.00	7	0.286
284	F	0	0	N/A	0	N/A

Chapter 3

Listing of integrals

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3.4	$\int \frac{1}{x} dx$	105
3.5	$\int \frac{(1+x)^3}{(-1+x)^4} dx$	108
3.6	$\int \frac{1}{(-1+x)x(1+x)^2} dx$	111
3.7	$\int \frac{b+ax}{(-p+x)(-q+x)} dx$	114
3.8	$\int \frac{1}{c+bx+ax^2} dx$	117
3.9	$\int \frac{b+ax}{1+x^2} dx$	121
3.10	$\int \frac{1}{3-2x+x^2} dx$	124
3.11	$\int \frac{1}{(-1+x)^2(1+x)^2} dx$	127
3.12	$\int \frac{x}{(-a+x)(-b+x)(-c+x)} dx$	131
3.13	$\int \frac{x}{(a^2+x^2)(b^2+x^2)} dx$	134
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3.15	$\int \frac{x}{(-1+x)(1+x^2)} dx$	142
3.16	$\int \frac{x}{1+x^3} dx$	145
3.17	$\int \frac{x^3}{(-1+x)^2(1+x^3)} dx$	149
3.18	$\int \frac{1}{1+x^4} dx$	152
3.19	$\int \frac{x^2}{1+x^4} dx$	156
3.20	$\int \frac{1}{1+x^2+x^4} dx$	160
3.21	$\int (a + bx)^p dx$	164
3.22	$\int x(a + bx)^p dx$	167
3.23	$\int x^2(a + bx)^p dx$	171
3.24	$\int \frac{1}{a+bx} dx$	175
3.25	$\int \frac{1}{(a+bx)^2} dx$	178

3.26	$\int \frac{x}{a+bx} dx$	181
3.27	$\int \frac{x^2}{a+bx} dx$	184
3.28	$\int \frac{1}{x(a+bx)} dx$	187
3.29	$\int \frac{1}{x^2(a+bx)} dx$	190
3.30	$\int \frac{1}{x^2(a+bx)^2} dx$	193
3.31	$\int \frac{1}{c^2+x^2} dx$	196
3.32	$\int \frac{1}{c^2-x^2} dx$	199
3.33	$\int \frac{1}{-1+2x^3} dx$	202
3.34	$\int \frac{1}{-2+x^3} dx$	207
3.35	$\int \frac{1}{-b+ax^3} dx$	212
3.36	$\int \frac{1}{-2+x^4} dx$	217
3.37	$\int \frac{1}{-1+5x^4} dx$	221
3.38	$\int \frac{1}{7+3x^4} dx$	225
3.39	$\int \frac{1}{-1+3x^2+x^4} dx$	230
3.40	$\int \frac{1}{-1-3x^2+x^4} dx$	234
3.41	$\int \frac{1}{1-3x^2+x^4} dx$	238
3.42	$\int \frac{1}{1-4x^2+x^4} dx$	242
3.43	$\int \frac{1}{1+4x^2+x^4} dx$	246
3.44	$\int \frac{1}{2+x^2+x^4} dx$	250
3.45	$\int \frac{1}{2-x^2+x^4} dx$	256
3.46	$\int \frac{1}{-1+x^6} dx$	261
3.47	$\int \frac{1}{-2+x^6} dx$	265
3.48	$\int \frac{1}{2+x^6} dx$	270
3.49	$\int \frac{1}{1+x^8} dx$	275
3.50	$\int \frac{1}{-1+x^8} dx$	281
3.51	$\int \frac{1}{1-x^4+x^8} dx$	286
3.52	$\int \frac{x^7}{1+x^{12}} dx$	291
3.53	$\int \log(x) dx$	296
3.54	$\int x \log(x) dx$	299
3.55	$\int x^2 \log(x) dx$	302
3.56	$\int x^p \log(x) dx$	305
3.57	$\int \log^2(x) dx$	308
3.58	$\int x^9 \log^{11}(x) dx$	311
3.59	$\int \frac{\log^2(x)}{x} dx$	315
3.60	$\int \frac{1}{\log(x)} dx$	318
3.61	$\int \frac{1}{\log(1+x)} dx$	321
3.62	$\int \frac{1}{x \log(x)} dx$	324
3.63	$\int \frac{1}{x^2 \log^2(x)} dx$	327
3.64	$\int \frac{\log^p(x)}{x} dx$	330
3.65	$\int (b+ax) \log(x) dx$	333
3.66	$\int (b+ax)^2 \log(x) dx$	336

3.67	$\int \frac{\log(x)}{(b+ax)^2} dx$	340
3.68	$\int x \log(b+ax) dx$	343
3.69	$\int x^2 \log(b+ax) dx$	346
3.70	$\int \log(a^2+x^2) dx$	350
3.71	$\int x \log(a^2+x^2) dx$	353
3.72	$\int x^2 \log(a^2+x^2) dx$	357
3.73	$\int x^4 \log(a^2+x^2) dx$	361
3.74	$\int \log(-a^2+x^2) dx$	365
3.75	$\int \log(\log(\log(\log(x)))) dx$	368
3.76	$\int \sin(x) dx$	371
3.77	$\int \cos(x) dx$	374
3.78	$\int \tan(x) dx$	377
3.79	$\int \cot(x) dx$	380
3.80	$\int \frac{1}{(1+\tan(x))^2} dx$	383
3.81	$\int \sec(x) dx$	387
3.82	$\int \csc(x) dx$	390
3.83	$\int \sin^2(x) dx$	393
3.84	$\int x^3 \sin(x^2) dx$	396
3.85	$\int \sin^3(x) dx$	400
3.86	$\int \sin^p(x) dx$	403
3.87	$\int \cos(x) (1+\sin^2(x))^2 dx$	406
3.88	$\int \cos^2(x) dx$	409
3.89	$\int \cos^3(x) dx$	412
3.90	$\int \sec^2(x) dx$	415
3.91	$\int \sin(x) \sin(2x) dx$	418
3.92	$\int x \sin(x) dx$	421
3.93	$\int x^2 \sin(x) dx$	424
3.94	$\int x \sin^2(x) dx$	427
3.95	$\int x^2 \sin^2(x) dx$	430
3.96	$\int x \sin^3(x) dx$	434
3.97	$\int x \cos(x) dx$	438
3.98	$\int x^2 \cos(x) dx$	441
3.99	$\int x \cos^2(x) dx$	444
3.100	$\int x^2 \cos^2(x) dx$	447
3.101	$\int x \cos^3(x) dx$	451
3.102	$\int \frac{\sin(x)}{x} dx$	455
3.103	$\int \frac{\cos(x)}{x} dx$	458
3.104	$\int \frac{\sin(x)}{x^2} dx$	461
3.105	$\int \frac{\sin^2(x)}{x} dx$	464
3.106	$\int \tan^3(x) dx$	467
3.107	$\int \sin(a+bx) dx$	470
3.108	$\int \cos(a+bx) dx$	473
3.109	$\int \tan(a+bx) dx$	476

3.110	$\int \cot(a + bx) dx$	479
3.111	$\int \csc(a + bx) dx$	482
3.112	$\int \sec(a + bx) dx$	485
3.113	$\int \sin^2(a + bx) dx$	488
3.114	$\int \sin^3(a + bx) dx$	491
3.115	$\int \cos^2(a + bx) dx$	494
3.116	$\int \cos^3(a + bx) dx$	497
3.117	$\int \sec^2(a + bx) dx$	500
3.118	$\int \frac{1}{1+\cos(x)} dx$	503
3.119	$\int \frac{1}{1-\cos(x)} dx$	506
3.120	$\int \frac{1}{1+\sin(x)} dx$	509
3.121	$\int \frac{1}{1-\sin(x)} dx$	512
3.122	$\int \frac{1}{a+b\sin(x)} dx$	515
3.123	$\int \frac{1}{a+\cos(x)+b\sin(x)} dx$	519
3.124	$\int x^2 \sin^2(a + bx) dx$	524
3.125	$\int \cos(x) \cos(2x) dx$	528
3.126	$\int x^2 \cos^2(a + bx) dx$	531
3.127	$\int \cot^3(x) dx$	535
3.128	$\int x^3 \tan^4(x) dx$	538
3.129	$\int x^3 \tan^6(x) dx$	543
3.130	$\int x \tan^2(x) dx$	549
3.131	$\int \cos(3x) \sin(2x) dx$	552
3.132	$\int \cos^2(x) \sin^2(x) dx$	555
3.133	$\int \csc^2(x) \sec^2(x) dx$	558
3.134	$\int d^x \sin(x) dx$	561
3.135	$\int d^x \cos(x) dx$	564
3.136	$\int d^x x \sin(x) dx$	567
3.137	$\int d^x x \cos(x) dx$	572
3.138	$\int d^x x^2 \sin(x) dx$	577
3.139	$\int d^x x^2 \cos(x) dx$	584
3.140	$\int d^x x^3 \sin(x) dx$	591
3.141	$\int d^x x^3 \cos(x) dx$	602
3.142	$\int \sin(x) \sin(2x) \sin(3x) dx$	613
3.143	$\int \cos(x) \cos(2x) \cos(3x) dx$	616
3.144	$\int x^2 \sin^3(kx) dx$	619
3.145	$\int x \cos(k \csc(x)) \cot(x) \csc(x) dx$	623
3.146	$\int \cot\left(\frac{x}{2}\right) \cot(x) dx$	626
3.147	$\int \frac{\sin(ax)}{(b+c\sin(ax))^2} dx$	630
3.148	$\int \sin(\log(x)) dx$	635
3.149	$\int \cos(\log(x)) dx$	638
3.150	$\int e^x dx$	641
3.151	$\int a^x dx$	644
3.152	$\int e^{ax} dx$	647

3.153	$\int \frac{e^{ax}}{x} dx$	650
3.154	$\int \frac{1}{a+be^{mx}} dx$	653
3.155	$\int \frac{e^{2x}}{1+e^x} dx$	657
3.156	$\int e^{2x+ax} dx$	660
3.157	$\int \frac{1}{be^{-mx}+ae^{mx}} dx$	663
3.158	$\int e^{ax} x dx$	667
3.159	$\int e^x x^{20} dx$	670
3.160	$\int a^x b^{-x} dx$	675
3.161	$\int a^x b^x dx$	678
3.162	$\int \frac{a^x}{x^2} dx$	682
3.163	$\int \frac{a^x x}{(1+bx)^2} dx$	685
3.164	$\int \frac{e^{ax} x}{(1+ax)^2} dx$	689
3.165	$\int k^{x^2} x dx$	692
3.166	$\int e^{x^2} dx$	695
3.167	$\int e^{x^2} x dx$	698
3.168	$\int \frac{e^{\frac{1}{x}}(1+x)}{x^4} dx$	701
3.169	$\int \frac{e^{1-e^{x^2}} x + 2x^2 (x+2x^3)}{(1-e^{x^2} x)^2} dx$	705
3.170	$\int e^{e^{e^x}} dx$	708
3.171	$\int e^x \log(x) dx$	711
3.172	$\int e^x x \log(x) dx$	714
3.173	$\int e^{2x} \log(e^x) dx$	717
3.174	$\int (2x + \sqrt{2} x^2) dx$	720
3.175	$\int \frac{\log(x)}{\sqrt{b+ax}} dx$	723
3.176	$\int \sqrt{a+bx} \sqrt{c+dx} dx$	729
3.177	$\int \sqrt{a+bx} dx$	733
3.178	$\int x \sqrt{a+bx} dx$	736
3.179	$\int x^2 \sqrt{a+bx} dx$	739
3.180	$\int \frac{\sqrt{a+bx}}{x} dx$	743
3.181	$\int \frac{\sqrt{a+bx}}{x^2} dx$	747
3.182	$\int \frac{1}{\sqrt{a+bx}} dx$	751
3.183	$\int \frac{x}{\sqrt{a+bx}} dx$	754
3.184	$\int \frac{x^2}{\sqrt{a+bx}} dx$	757
3.185	$\int \frac{1}{x\sqrt{a+bx}} dx$	761
3.186	$\int \frac{1}{x^2\sqrt{a+bx}} dx$	765
3.187	$\int (a+bx)^{p/2} dx$	769
3.188	$\int x(a+bx)^{p/2} dx$	772
3.189	$\int \tan^{-1} \left(\frac{-\sqrt{2}+2x}{\sqrt{2}} \right) dx$	776

3.190	$\int \frac{1}{\sqrt{-1+x^2}} dx$	781
3.191	$\int \sqrt{x} \sqrt{1+x} dx$	784
3.192	$\int \sin(\sqrt{x}) dx$	788
3.193	$\int \frac{x}{(1-x^2)^{9/8}} dx$	791
3.194	$\int \frac{x}{\sqrt{1-x^4}} dx$	794
3.195	$\int \frac{1}{x\sqrt{1+x^4}} dx$	797
3.196	$\int \frac{x}{\sqrt{1+x^2+x^4}} dx$	801
3.197	$\int \frac{1}{x\sqrt{-1+x^2-x^4}} dx$	805
3.198	$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}} dx$	809
3.199	$\int \frac{1}{\sqrt{1+x^2}} dx$	812
3.200	$\int \frac{\sqrt{x} \sqrt{1+x} + \sqrt{x} \sqrt{2+x} + \sqrt{1+x} \sqrt{2+x}}{2\sqrt{x} \sqrt{1+x} \sqrt{2+x}} dx$	815
3.201	$\int \frac{-2\sqrt{1+x^3} + 5x^4\sqrt{1+x^3} - 3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3} \sqrt{1-2x+x^5}} dx$	818
3.202	$\int \left(\frac{10}{\sqrt{-4+x^2}} + \frac{1}{\sqrt{-1+x^2}} \right) dx$	822
3.203	$\int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{x^2} dx$	825
3.204	$\int \frac{x^2}{2(1+x^3+\sqrt{1+x^3})} dx$	829
3.205	$\int \frac{1}{\sqrt{-\alpha^2 + 2hr^2}} dr$	833
3.206	$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}} dr$	837
3.207	$\int \frac{1}{r\sqrt{-\alpha^2 - 2kr + 2hr^2}} dr$	841
3.208	$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 - 2kr + 2hr^2}} dr$	845
3.209	$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2}} dr$	849
3.210	$\int \frac{r}{\sqrt{-\alpha^2 - \epsilon^2 + 2er^2}} dr$	852
3.211	$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}} dr$	855
3.212	$\int \frac{r}{\sqrt{-\alpha^2 - 2kr + 2er^2}} dr$	859
3.213	$\int \frac{1}{r\sqrt{-\alpha^2 + 2hr^2 - 2kr^4}} dr$	863
3.214	$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}} dr$	867
3.215	$\int a \cos(5+3x) \sin^2(5+3x) dx$	871
3.216	$\int \frac{\log(x^2)}{x^3} dx$	874
3.217	$\int x \sin(a+x) dx$	877
3.218	$\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)} dx$	880
3.219	$\int \frac{x^3}{b+ax^2} dx$	883

3.220	$\int \frac{\sqrt{x}}{(1+x)^{7/2}} dx$	886
3.221	$\int \frac{1}{x(1+x)} dx$	890
3.222	$\int \frac{1}{\sqrt{x}(-1+2x)} dx$	893
3.223	$\int \sqrt{x}(1+x^2) dx$	896
3.224	$\int \frac{\sqrt[3]{-a+x}}{x} dx$	899
3.225	$\int x \sinh(x) dx$	904
3.226	$\int x \cosh(x) dx$	907
3.227	$\int \tanh(2x) dx$	910
3.228	$\int \frac{-1+i\epsilon \sinh(x)}{ia-x+i\epsilon \cosh(x)} dx$	913
3.229	$\int \cos^2(x) \sin(3+2x) dx$	916
3.230	$\int x \tan^{-1}(x) dx$	919
3.231	$\int x \cot^{-1}(x) dx$	922
3.232	$\int x \log(a+x^2) dx$	925
3.233	$\int \cos(x) \sin(a+x) dx$	929
3.234	$\int \cos(a+x) \sin(x) dx$	932
3.235	$\int \sqrt{1+\sin(x)} dx$	935
3.236	$\int \sqrt{1-\sin(x)} dx$	938
3.237	$\int \sqrt{1+\cos(x)} dx$	941
3.238	$\int \sqrt{1-\cos(x)} dx$	944
3.239	$\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}} dx$	947
3.240	$\int \frac{1}{1-\sqrt{1+x}} dx$	950
3.241	$\int \frac{x}{\sqrt{36+x^4}} dx$	953
3.242	$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}} dx$	956
3.243	$\int \log(2+3x^2) dx$	960
3.244	$\int \cot(x) dx$	964
3.245	$\int \cot^4(x) dx$	967
3.246	$\int \tanh(x) dx$	970
3.247	$\int \coth(x) dx$	973
3.248	$\int b^x dx$	976
3.249	$\int \sqrt{2+\frac{1}{x^4}+x^4} dx$	979
3.250	$\int \frac{1+2x}{2+3x} dx$	983
3.251	$\int x \log(x+\sqrt{1+x^2}) dx$	986
3.252	$\int x(1+e^x \sin(x))^2 dx$	989
3.253	$\int e^x x \cos(x) dx$	994
3.254	$\int \frac{1}{(-3+x)^4} dx$	998
3.255	$\int \frac{x}{-1+x^3} dx$	1001
3.256	$\int \frac{x}{-1+x^4} dx$	1005
3.257	$\int \frac{(1+x^3) \log(x)}{2+x^4} dx$	1008
3.258	$\int (\log(x) + \log(1+x) + \log(2+x)) dx$	1012

3.259	$\int \frac{1}{5+x^3} dx$	1015
3.260	$\int \frac{1}{\sqrt{1+x^2}} dx$	1020
3.261	$\int \sqrt{3+x^2} dx$	1023
3.262	$\int \frac{x}{(1+x)^2} dx$	1026
3.263	$\int \sin^{-1}(x) dx$	1029
3.264	$\int x^2 \sin^{-1}(x) dx$	1032
3.265	$\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)} dx$	1036
3.266	$\int \cos^2(x) dx$	1039
3.267	$\int \frac{-2-3x+5x^2}{(-2+x)x^2} dx$	1042
3.268	$\int \frac{1}{\sqrt{9+4x^2}} dx$	1045
3.269	$\int \frac{1}{\sqrt{4+x^2}} dx$	1048
3.270	$\int \frac{1}{10-12x+9x^2} dx$	1051
3.271	$\int \frac{1}{x^4-2x^5+2x^6-2x^7+x^8} dx$	1054
3.272	$\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)} dx$	1057
3.273	$\int \frac{1}{(2-\log(1+x^2))^5} dx$	1061
3.274	$\int \left(\frac{e^{x^2}}{x} + 2e^{x^2}x \log(x) + \frac{-2+\log(x)}{(x+\log^2(x))^2} + \frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)} \right) dx$	1064
3.275	$\int e^{\frac{x}{2}+xz} x^4 \sin^4(\pi z) dz$	1068
3.276	$\int \operatorname{erf}(x) dx$	1073
3.277	$\int \operatorname{erf}(a+x) dx$	1076
3.278	$\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2 \sqrt{1+2x^2+4x^3+x^4}} dx$	1079
3.279	$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}} dy$	1083
3.280	$\int \frac{x(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2})}{(4-5x^2+x^4)(1+\sqrt{-4+x^2}+\sqrt{-1+x^2})} dx$	1087
3.281	$\int \left(\sqrt{9-4\sqrt{2}} x - \sqrt{2} \sqrt{1+4x+2x^2+x^4} \right) dx$	1091
3.282	$\int \frac{e^{-\frac{x}{y}} \left(\pi^2 (-3mc^8+4mc^9+24mc^6x-48mc^7x-144mc^5x^2-24mc^2x^3+176mc^3x^3+3x^4+12mcx^4)+12mc^3\pi^2(3mc-12mc^2-8x) \right)}{384x^2} dx$	
3.283	$\int \sec(x) \sin(2x) dx$	1108
3.284	$\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}} dx$	1111

3.1 $\int (1 + x + x^2) dx$

Optimal. Leaf size=16

$$x + \frac{x^2}{2} + \frac{x^3}{3}$$

[Out] $x+1/2*x^2+1/3*x^3$

Rubi [A]

time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$,

Rules used = {}

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Antiderivative was successfully verified.

[In] $\text{Int}[1 + x + x^2, x]$

[Out] $x + x^2/2 + x^3/3$

Rubi steps

$$\int (1 + x + x^2) dx = x + \frac{x^2}{2} + \frac{x^3}{3}$$

Mathematica [A]

time = 0.00, size = 16, normalized size = 1.00

$$x + \frac{x^2}{2} + \frac{x^3}{3}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[1 + x + x^2, x]$

[Out] $x + x^2/2 + x^3/3$

Mathics [A]

time = 1.62, size = 12, normalized size = 0.75

$$x \left(1 + \frac{x}{2} + \frac{x^2}{3} \right)$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1 + x + x^2,x]')`

[Out] `x (1 + x / 2 + x ^ 2 / 3)`

Maple [A]

time = 0.00, size = 13, normalized size = 0.81

method	result	size
gospers	$x + \frac{1}{2}x^2 + \frac{1}{3}x^3$	13
default	$x + \frac{1}{2}x^2 + \frac{1}{3}x^3$	13
norman	$x + \frac{1}{2}x^2 + \frac{1}{3}x^3$	13
risch	$x + \frac{1}{2}x^2 + \frac{1}{3}x^3$	13

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2+x+1,x,method=_RETURNVERBOSE)`

[Out] `x+1/2*x^2+1/3*x^3`

Maxima [A]

time = 0.28, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2+x+1,x, algorithm="maxima")`

[Out] `1/3*x^3 + 1/2*x^2 + x`

Fricas [A]

time = 0.31, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2+x+1,x, algorithm="fricas")`

[Out] `1/3*x^3 + 1/2*x^2 + x`

Sympy [A]

time = 0.02, size = 10, normalized size = 0.62

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2+x+1,x)

[Out] x**3/3 + x**2/2 + x

Giac [A]

time = 0.00, size = 14, normalized size = 0.88

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2+x+1,x)

[Out] 1/3*x^3 + 1/2*x^2 + x

Mupad [B]

time = 0.02, size = 13, normalized size = 0.81

$$\frac{x(2x^2 + 3x + 6)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x + x^2 + 1,x)

[Out] (x*(3*x + 2*x^2 + 6))/6

3.2 $\int x^2 (x + 2x^2)^2 dx$

Optimal. Leaf size=22

$$\frac{x^5}{5} + \frac{2x^6}{3} + \frac{4x^7}{7}$$

[Out] 1/5*x^5+2/3*x^6+4/7*x^7

Rubi [A]

time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {661, 45}

$$\frac{4x^7}{7} + \frac{2x^6}{3} + \frac{x^5}{5}$$

Antiderivative was successfully verified.

[In] Int[x^2*(x + 2*x^2)^2,x]

[Out] x^5/5 + (2*x^6)/3 + (4*x^7)/7

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 661

```
Int[((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist
[1/e^p, Int[(e*x)^(m + p)*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, m}, x] &&
IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int x^2 (x + 2x^2)^2 dx &= \int x^4 (1 + 2x)^2 dx \\ &= \int (x^4 + 4x^5 + 4x^6) dx \\ &= \frac{x^5}{5} + \frac{2x^6}{3} + \frac{4x^7}{7} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 22, normalized size = 1.00

$$\frac{x^5}{5} + \frac{2x^6}{3} + \frac{4x^7}{7}$$

Antiderivative was successfully verified.

`[In] Integrate[x^2*(x + 2*x^2)^2,x]``[Out] x^5/5 + (2*x^6)/3 + (4*x^7)/7`**Mathics [A]**

time = 1.64, size = 15, normalized size = 0.68

$$\frac{x^5(21 + 70x + 60x^2)}{105}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^2*(2*x^2 + x)^2,x]')``[Out] x ^ 5 (21 + 70 x + 60 x ^ 2) / 105`**Maple [A]**

time = 0.02, size = 17, normalized size = 0.77

method	result	size
gospers	$\frac{x^5(60x^2+70x+21)}{105}$	16
default	$\frac{1}{5}x^5 + \frac{2}{3}x^6 + \frac{4}{7}x^7$	17
norman	$\frac{1}{5}x^5 + \frac{2}{3}x^6 + \frac{4}{7}x^7$	17
risch	$\frac{1}{5}x^5 + \frac{2}{3}x^6 + \frac{4}{7}x^7$	17

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2*(2*x^2+x)^2,x,method=_RETURNVERBOSE)``[Out] 1/5*x^5+2/3*x^6+4/7*x^7`**Maxima [A]**

time = 0.27, size = 16, normalized size = 0.73

$$\frac{4}{7}x^7 + \frac{2}{3}x^6 + \frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2*(2*x^2+x)^2,x, algorithm="maxima")`

[Out] $4/7*x^7 + 2/3*x^6 + 1/5*x^5$

Fricas [A]

time = 0.34, size = 16, normalized size = 0.73

$$\frac{4}{7}x^7 + \frac{2}{3}x^6 + \frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(2*x^2+x)^2,x, algorithm="fricas")`

[Out] $4/7*x^7 + 2/3*x^6 + 1/5*x^5$

Sympy [A]

time = 0.03, size = 17, normalized size = 0.77

$$\frac{4x^7}{7} + \frac{2x^6}{3} + \frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(2*x**2+x)**2,x)`

[Out] $4*x**7/7 + 2*x**6/3 + x**5/5$

Giac [A]

time = 0.00, size = 22, normalized size = 1.00

$$\frac{4}{7}x^7 + \frac{2}{3}x^6 + \frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(2*x^2+x)^2,x)`

[Out] $4/7*x^7 + 2/3*x^6 + 1/5*x^5$

Mupad [B]

time = 0.04, size = 15, normalized size = 0.68

$$\frac{x^5 (60x^2 + 70x + 21)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(x + 2*x^2)^2,x)`

[Out] $(x^5*(70*x + 60*x^2 + 21))/105$

3.3 $\int x(1 + 2x + x^2) dx$

Optimal. Leaf size=22

$$\frac{x^2}{2} + \frac{2x^3}{3} + \frac{x^4}{4}$$

[Out] 1/2*x^2+2/3*x^3+1/4*x^4

Rubi [A]

time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {14}

$$\frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[x*(1 + 2*x + x^2),x]

[Out] x^2/2 + (2*x^3)/3 + x^4/4

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int x(1 + 2x + x^2) dx &= \int (x + 2x^2 + x^3) dx \\ &= \frac{x^2}{2} + \frac{2x^3}{3} + \frac{x^4}{4} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 22, normalized size = 1.00

$$\frac{x^2}{2} + \frac{2x^3}{3} + \frac{x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[x*(1 + 2*x + x^2),x]

[Out] x^2/2 + (2*x^3)/3 + x^4/4

Mathics [A]

time = 1.64, size = 15, normalized size = 0.68

$$\frac{x^2(6 + 8x + 3x^2)}{12}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x*(x^2 + 2*x + 1),x]')`[Out] `x ^ 2 (6 + 8 x + 3 x ^ 2) / 12`**Maple [A]**

time = 0.02, size = 17, normalized size = 0.77

method	result	size
gospers	$\frac{x^2(3x^2+8x+6)}{12}$	16
default	$\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{1}{4}x^4$	17
norman	$\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{1}{4}x^4$	17
risch	$\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{1}{4}x^4$	17

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(x^2+2*x+1),x,method=_RETURNVERBOSE)`[Out] `1/2*x^2+2/3*x^3+1/4*x^4`**Maxima [A]**

time = 0.26, size = 16, normalized size = 0.73

$$\frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x^2+2*x+1),x, algorithm="maxima")`[Out] `1/4*x^4 + 2/3*x^3 + 1/2*x^2`**Fricas [A]**

time = 0.32, size = 16, normalized size = 0.73

$$\frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x^2+2*x+1),x, algorithm="fricas")`

[Out] $1/4*x^4 + 2/3*x^3 + 1/2*x^2$

Sympy [A]

time = 0.03, size = 15, normalized size = 0.68

$$\frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x**2+2*x+1),x)`

[Out] $x**4/4 + 2*x**3/3 + x**2/2$

Giac [A]

time = 0.00, size = 22, normalized size = 1.00

$$\frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x^2+2*x+1),x)`

[Out] $1/4*x^4 + 2/3*x^3 + 1/2*x^2$

Mupad [B]

time = 0.03, size = 15, normalized size = 0.68

$$\frac{x^2 (3x^2 + 8x + 6)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(2*x + x^2 + 1),x)`

[Out] $(x^2*(8*x + 3*x^2 + 6))/12$

3.4 $\int \frac{1}{x} dx$

Optimal. Leaf size=2

$$\log(x)$$

[Out] ln(x)

Rubi [A]

time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {29}

$$\log(x)$$

Antiderivative was successfully verified.

[In] Int[x⁽⁻¹⁾, x]

[Out] Log[x]

Rule 29

Int[(x_)⁽⁻¹⁾, x_Symbol] :> Simp[Log[x], x]

Rubi steps

$$\int \frac{1}{x} dx = \log(x)$$

Mathematica [A]

time = 0.00, size = 2, normalized size = 1.00

$$\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[x⁽⁻¹⁾, x]

[Out] Log[x]

Mathics [A]

time = 1.48, size = 2, normalized size = 1.00

$$\text{Log}[x]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/x,x]')`

[Out] `Log[x]`

Maple [A]

time = 0.00, size = 3, normalized size = 1.50

method	result	size
default	$\ln(x)$	3
norman	$\ln(x)$	3
risch	$\ln(x)$	3

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x,x,method=_RETURNVERBOSE)`

[Out] `ln(x)`

Maxima [A]

time = 0.28, size = 2, normalized size = 1.00

$\log(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x,x, algorithm="maxima")`

[Out] `log(x)`

Fricas [A]

time = 0.33, size = 2, normalized size = 1.00

$\log(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x,x, algorithm="fricas")`

[Out] `log(x)`

Sympy [A]

time = 0.03, size = 2, normalized size = 1.00

$\log(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x,x)`

[Out] `log(x)`

Giac [A]

time = 0.00, size = 3, normalized size = 1.50

$$\ln |x|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x,x)`

[Out] `log(abs(x))`

Mupad [B]

time = 0.01, size = 2, normalized size = 1.00

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x,x)`

[Out] `log(x)`

$$3.5 \quad \int \frac{(1+x)^3}{(-1+x)^4} dx$$

Optimal. Leaf size=36

$$\frac{8}{3(1-x)^3} - \frac{6}{(1-x)^2} + \frac{6}{1-x} + \log(1-x)$$

[Out] 8/3/(1-x)^3-6/(1-x)^2+6/(1-x)+ln(1-x)

Rubi [A]

time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{6}{1-x} - \frac{6}{(1-x)^2} + \frac{8}{3(1-x)^3} + \log(1-x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)^3/(-1 + x)^4,x]

[Out] 8/(3*(1 - x)^3) - 6/(1 - x)^2 + 6/(1 - x) + Log[1 - x]

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int [ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{(1+x)^3}{(-1+x)^4} dx &= \int \left(\frac{8}{(-1+x)^4} + \frac{12}{(-1+x)^3} + \frac{6}{(-1+x)^2} + \frac{1}{-1+x} \right) dx \\ &= \frac{8}{3(1-x)^3} - \frac{6}{(1-x)^2} + \frac{6}{1-x} + \log(1-x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 24, normalized size = 0.67

$$-\frac{2(4-9x+9x^2)}{3(-1+x)^3} + \log(-1+x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)^3/(-1 + x)^4,x]

[Out] (-2*(4 - 9*x + 9*x^2))/(3*(-1 + x)^3) + Log[-1 + x]

Mathics [A]

time = 1.85, size = 44, normalized size = 1.22

$$\frac{-\frac{8}{3} + 6x - 6x^2 + \text{Log}[-1 + x](-1 + 3x - 3x^2 + x^3)}{-1 + 3x - 3x^2 + x^3}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(x + 1)^3/(x - 1)^4,x]')

[Out] (-8 / 3 + 6 x - 6 x ^ 2 + Log[-1 + x] (-1 + 3 x - 3 x ^ 2 + x ^ 3)) / (-1 + 3 x - 3 x ^ 2 + x ^ 3)

Maple [A]

time = 0.04, size = 27, normalized size = 0.75

method	result	size
norman	$\frac{-6x^2+6x-\frac{8}{3}}{(-1+x)^3} + \ln(-1+x)$	22
risch	$\frac{-6x^2+6x-\frac{8}{3}}{(-1+x)^3} + \ln(-1+x)$	22
default	$\ln(-1+x) - \frac{6}{-1+x} - \frac{6}{(-1+x)^2} - \frac{8}{3(-1+x)^3}$	27
meijerg	$\frac{x(x^2-3x+3)}{3(1-x)^3} + \frac{x^2(3-x)}{2(1-x)^3} + \frac{x^3}{(1-x)^3} + \frac{x(22x^2-30x+12)}{12(1-x)^3} + \ln(1-x)$	74

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)^3/(-1+x)^4,x,method=_RETURNVERBOSE)

[Out] ln(-1+x)-6/(-1+x)-6/(-1+x)^2-8/3/(-1+x)^3

Maxima [A]

time = 0.26, size = 32, normalized size = 0.89

$$-\frac{2(9x^2 - 9x + 4)}{3(x^3 - 3x^2 + 3x - 1)} + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^3/(-1+x)^4,x, algorithm="maxima")

[Out] -2/3*(9*x^2 - 9*x + 4)/(x^3 - 3*x^2 + 3*x - 1) + log(x - 1)

Fricas [A]

time = 0.33, size = 46, normalized size = 1.28

$$-\frac{18x^2 - 3(x^3 - 3x^2 + 3x - 1)\log(x - 1) - 18x + 8}{3(x^3 - 3x^2 + 3x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^3/(-1+x)^4,x, algorithm="fricas")

[Out] $-1/3*(18*x^2 - 3*(x^3 - 3*x^2 + 3*x - 1)*\log(x - 1) - 18*x + 8)/(x^3 - 3*x^2 + 3*x - 1)$

Sympy [A]

time = 0.06, size = 29, normalized size = 0.81

$$\frac{-18x^2 + 18x - 8}{3x^3 - 9x^2 + 9x - 3} + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)**3/(-1+x)**4,x)

[Out] $(-18*x**2 + 18*x - 8)/(3*x**3 - 9*x**2 + 9*x - 3) + \log(x - 1)$

Giac [A]

time = 0.00, size = 27, normalized size = 0.75

$$\frac{\frac{1}{6}(-36x^2 + 36x - 16)}{(x - 1)^3} + \ln|x - 1|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^3/(-1+x)^4,x)

[Out] $-2/3*(9*x^2 - 9*x + 4)/(x - 1)^3 + \log(\text{abs}(x - 1))$

Mupad [B]

time = 0.12, size = 22, normalized size = 0.61

$$\ln(x - 1) - \frac{6x^2 - 6x + \frac{8}{3}}{(x - 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 1)^3/(x - 1)^4,x)

[Out] $\log(x - 1) - (6*x^2 - 6*x + 8/3)/(x - 1)^3$

3.6 $\int \frac{1}{(-1+x)x(1+x)^2} dx$

Optimal. Leaf size=32

$$-\frac{1}{2(1+x)} + \frac{1}{4} \log(1-x) - \log(x) + \frac{3}{4} \log(1+x)$$

[Out] -1/2/(1+x)+1/4*ln(1-x)-ln(x)+3/4*ln(1+x)

Rubi [A]

time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {84}

$$-\frac{1}{2(x+1)} + \frac{1}{4} \log(1-x) - \log(x) + \frac{3}{4} \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[1/((-1 + x)*x*(1 + x)^2), x]

[Out] -1/2*1/(1 + x) + Log[1 - x]/4 - Log[x] + (3*Log[1 + x])/4

Rule 84

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{1}{(-1+x)x(1+x)^2} dx &= \int \left(\frac{1}{4(-1+x)} - \frac{1}{x} + \frac{1}{2(1+x)^2} + \frac{3}{4(1+x)} \right) dx \\ &= -\frac{1}{2(1+x)} + \frac{1}{4} \log(1-x) - \log(x) + \frac{3}{4} \log(1+x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 28, normalized size = 0.88

$$\frac{1}{4} \left(-\frac{2}{1+x} + \log(1-x) - 4 \log(x) + 3 \log(1+x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((-1 + x)*x*(1 + x)^2), x]

[Out] $(-2/(1+x) + \text{Log}[1-x] - 4*\text{Log}[x] + 3*\text{Log}[1+x])/4$

Mathics [A]

time = 1.76, size = 28, normalized size = 0.88

$$\frac{-2 + (1+x)(\text{Log}[-1+x] - 4\text{Log}[x] + 3\text{Log}[1+x])}{4 + 4x}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(x*(x-1)*(x+1)^2),x]')`

[Out] $(-2 + (1+x)(\text{Log}[-1+x] - 4\text{Log}[x] + 3\text{Log}[1+x])) / (4(1+x))$

Maple [A]

time = 0.04, size = 25, normalized size = 0.78

method	result	size
default	$-\ln(x) + \frac{\ln(-1+x)}{4} - \frac{1}{2(1+x)} + \frac{3\ln(1+x)}{4}$	25
norman	$-\ln(x) + \frac{\ln(-1+x)}{4} - \frac{1}{2(1+x)} + \frac{3\ln(1+x)}{4}$	25
risch	$-\ln(x) + \frac{\ln(-1+x)}{4} - \frac{1}{2(1+x)} + \frac{3\ln(1+x)}{4}$	25

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-1+x)/x/(1+x)^2,x,method=_RETURNVERBOSE)`

[Out] $-\ln(x) + 1/4*\ln(-1+x) - 1/2/(1+x) + 3/4*\ln(1+x)$

Maxima [A]

time = 0.27, size = 24, normalized size = 0.75

$$-\frac{1}{2(x+1)} + \frac{3}{4} \log(x+1) + \frac{1}{4} \log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1+x)/x/(1+x)^2,x, algorithm="maxima")`

[Out] $-1/2/(x+1) + 3/4*\log(x+1) + 1/4*\log(x-1) - \log(x)$

Fricas [A]

time = 0.33, size = 33, normalized size = 1.03

$$\frac{3(x+1)\log(x+1) + (x+1)\log(x-1) - 4(x+1)\log(x) - 2}{4(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1+x)/x/(1+x)^2,x, algorithm="fricas")`

[Out] $1/4*(3*(x + 1)*\log(x + 1) + (x + 1)*\log(x - 1) - 4*(x + 1)*\log(x) - 2)/(x + 1)$

Sympy [A]

time = 0.08, size = 24, normalized size = 0.75

$$-\log(x) + \frac{\log(x-1)}{4} + \frac{3\log(x+1)}{4} - \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1+x)/x/(1+x)**2,x)`

[Out] $-\log(x) + \log(x - 1)/4 + 3*\log(x + 1)/4 - 1/(2*x + 2)$

Giac [A]

time = 0.00, size = 32, normalized size = 1.00

$$-\ln|x| + \frac{\ln|x-1|}{4} + \frac{3}{4}\ln|x+1| - \frac{\frac{1}{4} \cdot 2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1+x)/x/(1+x)^2,x)`

[Out] $-1/2/(x + 1) + 3/4*\log(\text{abs}(x + 1)) + 1/4*\log(\text{abs}(x - 1)) - \log(\text{abs}(x))$

Mupad [B]

time = 0.03, size = 26, normalized size = 0.81

$$\frac{\ln(x-1)}{4} + \frac{3\ln(x+1)}{4} - \ln(x) - \frac{1}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(x-1)*(x+1)^2),x)`

[Out] $\log(x - 1)/4 + (3*\log(x + 1))/4 - \log(x) - 1/(2*(x + 1))$

3.7 $\int \frac{b+ax}{(-p+x)(-q+x)} dx$

Optimal. Leaf size=40

$$\frac{(b+ap)\log(p-x)}{p-q} - \frac{(b+aq)\log(q-x)}{p-q}$$

[Out] (a*p+b)*ln(p-x)/(p-q)-(a*q+b)*ln(q-x)/(p-q)

Rubi [A]

time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {78}

$$\frac{(ap+b)\log(p-x)}{p-q} - \frac{(aq+b)\log(q-x)}{p-q}$$

Antiderivative was successfully verified.

[In] Int[(b + a*x)/((-p + x)*(-q + x)),x]

[Out] ((b + a*p)*Log[p - x])/(p - q) - ((b + a*q)*Log[q - x])/(p - q)

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rubi steps

$$\begin{aligned} \int \frac{b+ax}{(-p+x)(-q+x)} dx &= \int \left(\frac{-b-ap}{(p-q)(p-x)} + \frac{b+aq}{(p-q)(q-x)} \right) dx \\ &= \frac{(b+ap)\log(p-x)}{p-q} - \frac{(b+aq)\log(q-x)}{p-q} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 34, normalized size = 0.85

$$\frac{(b+ap)\log(-p+x) - (b+aq)\log(-q+x)}{p-q}$$

Antiderivative was successfully verified.

[In] Integrate[(b + a*x)/((-p + x)*(-q + x)),x]

[Out] ((b + a*p)*Log[-p + x] - (b + a*q)*Log[-q + x])/(p - q)

Mathics [A]

time = 4.07, size = 34, normalized size = 0.85

$$\frac{-\operatorname{Log}[-q+x](aq+b) + \operatorname{Log}[-p+x](ap+b)}{p-q}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(a*x + b)/((x - p)*(x - q)),x]')

[Out] (-Log[-q + x] (a q + b) + Log[-p + x] (a p + b)) / (p - q)

Maple [A]

time = 0.04, size = 43, normalized size = 1.08

method	result	size
norman	$\frac{(ap+b)\ln(p-x)}{p-q} - \frac{(aq+b)\ln(q-x)}{p-q}$	41
default	$\frac{(-aq-b)\ln(q-x)}{p-q} + \frac{(ap+b)\ln(p-x)}{p-q}$	43
risch	$-\frac{\ln(-q+x)aq}{p-q} - \frac{\ln(-q+x)b}{p-q} + \frac{\ln(p-x)ap}{p-q} + \frac{\ln(p-x)b}{p-q}$	66

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+b)/(-p+x)/(-q+x),x,method=_RETURNVERBOSE)

[Out] (-a*q-b)/(p-q)*ln(q-x)+(a*p+b)*ln(p-x)/(p-q)

Maxima [A]

time = 0.27, size = 40, normalized size = 1.00

$$\frac{(ap+b)\log(-p+x)}{p-q} - \frac{(aq+b)\log(-q+x)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+b)/(-p+x)/(-q+x),x, algorithm="maxima")

[Out] (a*p + b)*log(-p + x)/(p - q) - (a*q + b)*log(-q + x)/(p - q)

Fricas [A]

time = 0.33, size = 34, normalized size = 0.85

$$\frac{(ap+b)\log(-p+x) - (aq+b)\log(-q+x)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+b)/(-p+x)/(-q+x),x, algorithm="fricas")

[Out] ((a*p + b)*log(-p + x) - (a*q + b)*log(-q + x))/(p - q)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(26) = 52

time = 0.55, size = 144, normalized size = 3.60

$$\frac{(ap + b) \log\left(x + \frac{-2apq - bp - bq - \frac{p^2(ap+b)}{p-q} + \frac{2pq(ap+b)}{p-q} - \frac{q^2(ap+b)}{p-q}}{ap+aq+2b}\right)}{p - q} - \frac{(aq + b) \log\left(x + \frac{-2apq - bp - bq + \frac{p^2(aq+b)}{p-q} - \frac{2pq(aq+b)}{p-q} + \frac{q^2(aq+b)}{p-q}}{ap+aq+2b}\right)}{p - q}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+b)/(-p+x)/(-q+x),x)

[Out] (a*p + b)*log(x + (-2*a*p*q - b*p - b*q - p**2*(a*p + b)/(p - q) + 2*p*q*(a*p + b)/(p - q) - q**2*(a*p + b)/(p - q))/(a*p + a*q + 2*b))/(p - q) - (a*q + b)*log(x + (-2*a*p*q - b*p - b*q + p**2*(a*q + b)/(p - q) - 2*p*q*(a*q + b)/(p - q) + q**2*(a*q + b)/(p - q))/(a*p + a*q + 2*b))/(p - q)

Giac [A]

time = 0.00, size = 35, normalized size = 0.88

$$\frac{(aq + b) \ln |x - q|}{-p + q} + \frac{(ap + b) \ln |x - p|}{p - q}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+b)/(-p+x)/(-q+x),x)

[Out] (a*p + b)*log(abs(-p + x))/(p - q) - (a*q + b)*log(abs(-q + x))/(p - q)

Mupad [B]

time = 0.25, size = 40, normalized size = 1.00

$$\frac{\ln(x - p) (b + a p)}{p - q} - \frac{\ln(x - q) (b + a q)}{p - q}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + a*x)/((p - x)*(q - x)),x)

[Out] (log(x - p)*(b + a*p))/(p - q) - (log(x - q)*(b + a*q))/(p - q)

3.8 $\int \frac{1}{c+bx+ax^2} dx$

Optimal. Leaf size=34

$$-\frac{2 \tanh^{-1} \left(\frac{b+2ax}{\sqrt{b^2-4ac}} \right)}{\sqrt{b^2-4ac}}$$

[Out] $-2*\operatorname{arctanh}((2*a*x+b)/(-4*a*c+b^2)^{(1/2)})/(-4*a*c+b^2)^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {632, 212}

$$-\frac{2 \tanh^{-1} \left(\frac{2ax+b}{\sqrt{b^2-4ac}} \right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + b*x + a*x^2)^{-1}, x]$

[Out] $(-2*\operatorname{ArcTanh}[(b + 2*a*x)/\operatorname{Sqrt}[b^2 - 4*a*c]])/\operatorname{Sqrt}[b^2 - 4*a*c]$

Rule 212

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 632

$\operatorname{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{c+bx+ax^2} dx &= - \left(2 \operatorname{Subst} \left(\int \frac{1}{b^2-4ac-x^2} dx, x, b+2ax \right) \right) \\ &= - \frac{2 \tanh^{-1} \left(\frac{b+2ax}{\sqrt{b^2-4ac}} \right)}{\sqrt{b^2-4ac}} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 38, normalized size = 1.12

$$\frac{2 \tan^{-1} \left(\frac{b+2ax}{\sqrt{-b^2+4ac}} \right)}{\sqrt{-b^2+4ac}}$$

Antiderivative was successfully verified.

`[In] Integrate[(c + b*x + a*x^2)^(-1),x]``[Out] (2*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]`**Mathics [B]** Leaf count is larger than twice the leaf count of optimal. 135 vs. 2(34) = 68.

time = 4.03, size = 125, normalized size = 3.68

$$\left(\text{Log} \left[\frac{2a \left(2c \sqrt{-\frac{1}{4ac-b^2}} + x \right) + b - b^2 \sqrt{-\frac{1}{4ac-b^2}}}{a} \right] - \text{Log} \left[\frac{-2a \left(2c \sqrt{-\frac{1}{4ac-b^2}} - x \right) + b + b^2 \sqrt{-\frac{1}{4ac-b^2}}}{a} \right] \right) \sqrt{-\frac{1}{4ac-b^2}}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(a*x^2 + b*x + c),x]')`
`[Out] (Log[(2 a (2 c Sqrt[-1 / (4 a c - b ^ 2)] + x) + b - b ^ 2 Sqrt[-1 / (4 a c - b ^ 2)]) / a] - Log[(-2 a (2 c Sqrt[-1 / (4 a c - b ^ 2)] - x) + b + b ^ 2 Sqrt[-1 / (4 a c - b ^ 2)]) / a]) Sqrt[-1 / (4 a c - b ^ 2)]`
Maple [A]

time = 0.12, size = 35, normalized size = 1.03

method	result	size
default	$\frac{2 \arctan \left(\frac{2ax+b}{\sqrt{4ac-b^2}} \right)}{\sqrt{4ac-b^2}}$	35
risch	$-\frac{\ln \left(2ax + \sqrt{-4ac+b^2} + b \right)}{\sqrt{-4ac+b^2}} + \frac{\ln \left(-2ax + \sqrt{-4ac+b^2} - b \right)}{\sqrt{-4ac+b^2}}$	61

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(a*x^2+b*x+c),x,method=_RETURNVERBOSE)``[Out] 2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))`**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^2+b*x+c),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details)

Fricas [A]

time = 0.35, size = 120, normalized size = 3.53

$$\left[\frac{\log\left(\frac{2a^2x^2+2abx+b^2-2ac-\sqrt{b^2-4ac}(2ax+b)}{ax^2+bx+c}\right)}{\sqrt{b^2-4ac}}, -\frac{2\sqrt{-b^2+4ac}\arctan\left(-\frac{\sqrt{-b^2+4ac}(2ax+b)}{b^2-4ac}\right)}{b^2-4ac} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^2+b*x+c),x, algorithm="fricas")

[Out] [log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(34) = 68

time = 0.12, size = 124, normalized size = 3.65

$$-\sqrt{-\frac{1}{4ac-b^2}} \log\left(x + \frac{-4ac\sqrt{-\frac{1}{4ac-b^2}} + b^2\sqrt{-\frac{1}{4ac-b^2}} + b}{2a}\right) + \sqrt{-\frac{1}{4ac-b^2}} \log\left(x + \frac{4ac\sqrt{-\frac{1}{4ac-b^2}} - b^2\sqrt{-\frac{1}{4ac-b^2}} + b}{2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x**2+b*x+c),x)

[Out] -sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*a)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*a))

Giac [A]

time = 0.00, size = 38, normalized size = 1.12

$$\frac{2\arctan\left(\frac{2ax+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^2+b*x+c),x)

[Out] 2*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)

Mupad [B]

time = 0.20, size = 46, normalized size = 1.35

$$\frac{2 \operatorname{atan}\left(\frac{b}{\sqrt{4ac - b^2}} + \frac{2ax}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c + b*x + a*x^2),x)

[Out] (2*atan(b/(4*a*c - b^2)^(1/2) + (2*a*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)^(1/2)

3.9 $\int \frac{b+ax}{1+x^2} dx$

Optimal. Leaf size=16

$$b \tan^{-1}(x) + \frac{1}{2}a \log(1+x^2)$$

[Out] b*arctan(x)+1/2*a*ln(x^2+1)

Rubi [A]

time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {649, 209, 266}

$$\frac{1}{2}a \log(x^2 + 1) + b \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(b + a*x)/(1 + x^2), x]

[Out] b*ArcTan[x] + (a*Log[1 + x^2])/2

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 649

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[(-a)*c]

Rubi steps

$$\begin{aligned} \int \frac{b+ax}{1+x^2} dx &= a \int \frac{x}{1+x^2} dx + b \int \frac{1}{1+x^2} dx \\ &= b \tan^{-1}(x) + \frac{1}{2}a \log(1+x^2) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 16, normalized size = 1.00

$$b \tan^{-1}(x) + \frac{1}{2}a \log(1 + x^2)$$

Antiderivative was successfully verified.

[In] Integrate[(b + a*x)/(1 + x^2),x]

[Out] b*ArcTan[x] + (a*Log[1 + x^2])/2

Mathics [C] Result contains complex when optimal does not.

time = 1.80, size = 23, normalized size = 1.44

$$\frac{\text{Log}[-I + x](a - Ib)}{2} + \frac{\text{Log}[I + x](a + Ib)}{2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(a*x + b)/(1 + x^2),x]')

[Out] Log[-I + x] (a - I b) / 2 + Log[I + x] (a + I b) / 2

Maple [A]

time = 0.02, size = 15, normalized size = 0.94

method	result	size
default	$b \arctan(x) + \frac{a \ln(x^2+1)}{2}$	15
meijerg	$b \arctan(x) + \frac{a \ln(x^2+1)}{2}$	15
risch	$b \arctan(x) + \frac{a \ln(x^2+1)}{2}$	15

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+b)/(x^2+1),x,method=_RETURNVERBOSE)

[Out] b*arctan(x)+1/2*a*ln(x^2+1)

Maxima [A]

time = 0.34, size = 14, normalized size = 0.88

$$b \arctan(x) + \frac{1}{2}a \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+b)/(x^2+1),x, algorithm="maxima")

[Out] b*arctan(x) + 1/2*a*log(x^2 + 1)

Fricas [A]

time = 0.32, size = 14, normalized size = 0.88

$$b \arctan(x) + \frac{1}{2} a \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+b)/(x^2+1),x, algorithm="fricas")

[Out] b*arctan(x) + 1/2*a*log(x^2 + 1)

Sympy [C] Result contains complex when optimal does not.

time = 0.09, size = 26, normalized size = 1.62

$$\left(\frac{a}{2} - \frac{ib}{2}\right) \log(x - i) + \left(\frac{a}{2} + \frac{ib}{2}\right) \log(x + i)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+b)/(x**2+1),x)

[Out] (a/2 - I*b/2)*log(x - I) + (a/2 + I*b/2)*log(x + I)

Giac [A]

time = 0.00, size = 16, normalized size = 1.00

$$\frac{1}{2} a \ln(x^2 + 1) + b \arctan x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+b)/(x^2+1),x)

[Out] b*arctan(x) + 1/2*a*log(x^2 + 1)

Mupad [B]

time = 0.04, size = 14, normalized size = 0.88

$$\frac{a \ln(x^2 + 1)}{2} + b \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + a*x)/(x^2 + 1),x)

[Out] (a*log(x^2 + 1))/2 + b*atan(x)

3.10 $\int \frac{1}{3-2x+x^2} dx$

Optimal. Leaf size=19

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{\sqrt{2}}$$

[Out] -1/2*arctan(1/2*(1-x)*2^(1/2))*2^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {632, 210}

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(3 - 2*x + x^2)^(-1), x]

[Out] -(ArcTan[(1 - x)/Sqrt[2]]/Sqrt[2])

Rule 210

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{3-2x+x^2} dx &= -\left(2\text{Subst}\left(\int \frac{1}{-8-x^2} dx, x, -2+2x\right)\right) \\ &= -\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 16, normalized size = 0.84

$$\frac{\tan^{-1}\left(\frac{-1+x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(3 - 2*x + x^2)^(-1),x]``[Out] ArcTan[(-1 + x)/Sqrt[2]]/Sqrt[2]`**Mathics [A]**

time = 1.68, size = 14, normalized size = 0.74

$$\frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2}(-1+x)}{2}\right]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^2 - 2*x + 3),x]')``[Out] Sqrt[2] ArcTan[Sqrt[2] (-1 + x) / 2] / 2`**Maple [A]**

time = 0.21, size = 17, normalized size = 0.89

method	result	size
risch	$\frac{\sqrt{2} \arctan\left(\frac{(-1+x)\sqrt{2}}{2}\right)}{2}$	15
default	$\frac{\sqrt{2} \arctan\left(\frac{(2x-2)\sqrt{2}}{4}\right)}{2}$	17

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(x^2-2*x+3),x,method=_RETURNVERBOSE)``[Out] 1/2*2^(1/2)*arctan(1/4*(2*x-2)*2^(1/2))`**Maxima [A]**

time = 0.35, size = 14, normalized size = 0.74

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} (x - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-2*x+3),x, algorithm="maxima")

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))

Fricas [A]

time = 0.32, size = 14, normalized size = 0.74

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} (x - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-2*x+3),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))

Sympy [A]

time = 0.05, size = 22, normalized size = 1.16

$$\frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} x}{2} - \frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2-2*x+3),x)

[Out] sqrt(2)*atan(sqrt(2)*x/2 - sqrt(2)/2)/2

Giac [A]

time = 0.00, size = 21, normalized size = 1.11

$$\frac{2 \arctan\left(\frac{x-1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-2*x+3),x)

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))

Mupad [B]

time = 0.03, size = 14, normalized size = 0.74

$$\frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} (x-1)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2 - 2*x + 3),x)

[Out] (2^(1/2)*atan((2^(1/2)*(x - 1))/2))/2

$$3.11 \quad \int \frac{1}{(-1+x)^2(1+x^2)^2} dx$$

Optimal. Leaf size=49

$$\frac{1}{4(1-x)} - \frac{1}{4(1+x^2)} + \frac{1}{4} \tan^{-1}(x) - \frac{1}{2} \log(1-x) + \frac{1}{4} \log(1+x^2)$$

[Out] 1/4/(1-x)-1/4/(x^2+1)+1/4*arctan(x)-1/2*ln(1-x)+1/4*ln(x^2+1)

Rubi [A]

time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {755, 815, 649, 209, 266}

$$-\frac{1}{4(x^2+1)} + \frac{1}{4} \log(x^2+1) + \frac{1}{4(1-x)} - \frac{1}{2} \log(1-x) + \frac{1}{4} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[1/((-1 + x)^2*(1 + x^2)^2), x]

[Out] 1/(4*(1 - x)) - 1/(4*(1 + x^2)) + ArcTan[x]/4 - Log[1 - x]/2 + Log[1 + x^2]/4

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 649

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[(-a)*c]

Rule 755

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(-d + e*x)^(m + 1)*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c

$x^2)^{(p+1)}, x], x] /; \text{FreeQ}\{a, c, d, e, m\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0]$
 $\&\& \text{LtQ}[p, -1] \&\& \text{IntQuadraticQ}[a, 0, c, d, e, m, p, x]$

Rule 815

$\text{Int}[(((d_.) + (e_.)*(x_.))^{(m_.)*((f_.) + (g_.)*(x_.)))/((a_.) + (c_.)*(x_.)^2),$
 $x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],$
 $x] /; \text{FreeQ}\{a, c, d, e, f, g\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{IntegerQ}[m]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(-1+x)^2(1+x^2)^2} dx &= -\frac{1}{4(1+x^2)} - \frac{1}{4} \int \frac{-4+2x}{(-1+x)^2(1+x^2)} dx \\ &= -\frac{1}{4(1+x^2)} - \frac{1}{4} \int \left(-\frac{1}{(-1+x)^2} + \frac{2}{-1+x} + \frac{-1-2x}{1+x^2} \right) dx \\ &= \frac{1}{4(1-x)} - \frac{1}{4(1+x^2)} - \frac{1}{2} \log(1-x) - \frac{1}{4} \int \frac{-1-2x}{1+x^2} dx \\ &= \frac{1}{4(1-x)} - \frac{1}{4(1+x^2)} - \frac{1}{2} \log(1-x) + \frac{1}{4} \int \frac{1}{1+x^2} dx + \frac{1}{2} \int \frac{x}{1+x^2} dx \\ &= \frac{1}{4(1-x)} - \frac{1}{4(1+x^2)} + \frac{1}{4} \tan^{-1}(x) - \frac{1}{2} \log(1-x) + \frac{1}{4} \log(1+x^2) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 35, normalized size = 0.71

$$\frac{1}{4} \left(\frac{1}{1-x} - \frac{1}{1+x^2} + \tan^{-1}(x) - 2 \log(-1+x) + \log(1+x^2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((-1+x)^2*(1+x^2)^2),x]

[Out] ((1-x)^(-1) - (1+x^2)^(-1) + ArcTan[x] - 2*Log[-1+x] + Log[1+x^2])/4

Mathics [A]

time = 2.00, size = 49, normalized size = 1.00

$$\frac{-\frac{x(1+x)}{4} + \frac{(-1+x-x^2+x^3)(\text{ArcTan}[x]+\text{Log}[1+x^2]-2\text{Log}[-1+x])}{4}}{-1+x-x^2+x^3}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/((x-1)*(x^2+1))^2,x]')`

[Out] $(-x(1+x)/4 + (-1+x-x^2+x^3)(\text{ArcTan}[x] + \text{Log}[1+x^2] - 2\text{Log}[-1+x])/4) / (-1+x-x^2+x^3)$

Maple [A]

time = 0.04, size = 36, normalized size = 0.73

method	result	size
default	$-\frac{1}{4(-1+x)} - \frac{\ln(-1+x)}{2} - \frac{1}{4(x^2+1)} + \frac{\ln(x^2+1)}{4} + \frac{\arctan(x)}{4}$	36
risch	$\frac{-\frac{1}{4}x^2 - \frac{1}{4}x}{(x^2+1)(-1+x)} - \frac{\ln(-1+x)}{2} + \frac{\ln(x^2+1)}{4} + \frac{\arctan(x)}{4}$	42

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-1+x)^2/(x^2+1)^2,x,method=_RETURNVERBOSE)`

[Out] $-1/4/(-1+x) - 1/2*\ln(-1+x) - 1/4/(x^2+1) + 1/4*\ln(x^2+1) + 1/4*\arctan(x)$

Maxima [A]

time = 0.35, size = 39, normalized size = 0.80

$$-\frac{x^2+x}{4(x^3-x^2+x-1)} + \frac{1}{4}\arctan(x) + \frac{1}{4}\log(x^2+1) - \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1+x)^2/(x^2+1)^2,x, algorithm="maxima")`

[Out] $-1/4*(x^2+x)/(x^3-x^2+x-1) + 1/4*\arctan(x) + 1/4*\log(x^2+1) - 1/2*\log(x-1)$

Fricas [A]

time = 0.42, size = 71, normalized size = 1.45

$$\frac{x^2 - (x^3 - x^2 + x - 1)\arctan(x) - (x^3 - x^2 + x - 1)\log(x^2 + 1) + 2(x^3 - x^2 + x - 1)\log(x - 1) + x}{4(x^3 - x^2 + x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1+x)^2/(x^2+1)^2,x, algorithm="fricas")`

[Out] $-1/4*(x^2 - (x^3 - x^2 + x - 1)*\arctan(x) - (x^3 - x^2 + x - 1)*\log(x^2 + 1) + 2*(x^3 - x^2 + x - 1)*\log(x - 1) + x)/(x^3 - x^2 + x - 1)$

Sympy [A]

time = 0.09, size = 41, normalized size = 0.84

$$\frac{-x^2-x}{4x^3-4x^2+4x-4} - \frac{\log(x-1)}{2} + \frac{\log(x^2+1)}{4} + \frac{\text{atan}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+x)**2/(x**2+1)**2,x)

[Out] $(-x^2 - x)/(4x^3 - 4x^2 + 4x - 4) - \log(x - 1)/2 + \log(x^2 + 1)/4 + \operatorname{atan}(x)/4$

Giac [A]

time = 0.00, size = 45, normalized size = 0.92

$$-\frac{\ln|x-1|}{2} + \frac{\ln(x^2+1)}{4} + \frac{\arctan x}{4} + \frac{-x^2-x}{4(x^3-x^2+x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+x)^2/(x^2+1)^2,x)

[Out] $-1/4*(x^2 + x)/(x^3 - x^2 + x - 1) + 1/4*\arctan(x) + 1/4*\log(x^2 + 1) - 1/2*\log(\operatorname{abs}(x - 1))$

Mupad [B]

time = 0.13, size = 49, normalized size = 1.00

$$-\frac{\ln(x-1)}{2} - \frac{\frac{x^2}{4} + \frac{x}{4}}{x^3 - x^2 + x - 1} + \ln(x-i) \left(\frac{1}{4} - \frac{1}{8}i\right) + \ln(x+1i) \left(\frac{1}{4} + \frac{1}{8}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^2 + 1)^2*(x - 1)^2),x)

[Out] $\log(x - 1i)*(1/4 - 1i/8) - \log(x - 1)/2 + \log(x + 1i)*(1/4 + 1i/8) - (x/4 + x^2/4)/(x - x^2 + x^3 - 1)$

$$3.12 \quad \int \frac{x}{(-a+x)(-b+x)(-c+x)} dx$$

Optimal. Leaf size=68

$$\frac{a \log(a-x)}{(a-b)(a-c)} - \frac{b \log(b-x)}{(a-b)(b-c)} + \frac{c \log(c-x)}{(a-c)(b-c)}$$

[Out] a*ln(a-x)/(a-b)/(a-c)-b*ln(b-x)/(a-b)/(b-c)+c*ln(c-x)/(a-c)/(b-c)

Rubi [A]

time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {153}

$$\frac{a \log(a-x)}{(a-b)(a-c)} - \frac{b \log(b-x)}{(a-b)(b-c)} + \frac{c \log(c-x)}{(a-c)(b-c)}$$

Antiderivative was successfully verified.

[In] Int[x/((-a + x)*(-b + x)*(-c + x)),x]

[Out] (a*Log[a - x])/((a - b)*(a - c)) - (b*Log[b - x])/((a - b)*(b - c)) + (c*Log[c - x])/((a - c)*(b - c))

Rule 153

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))

Rubi steps

$$\begin{aligned} \int \frac{x}{(-a+x)(-b+x)(-c+x)} dx &= \int \left(-\frac{a}{(a-b)(a-c)(a-x)} + \frac{b}{(a-b)(b-c)(b-x)} + \frac{c}{(a-c)(-b+c)(c-x)} \right) dx \\ &= \frac{a \log(a-x)}{(a-b)(a-c)} - \frac{b \log(b-x)}{(a-b)(b-c)} + \frac{c \log(c-x)}{(a-c)(b-c)} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 62, normalized size = 0.91

$$\frac{a(b-c) \log(-a+x) + b(-a+c) \log(-b+x) + (a-b)c \log(-c+x)}{(a-b)(a-c)(b-c)}$$

Antiderivative was successfully verified.

[In] Integrate[x/((-a + x)*(-b + x)*(-c + x)),x]

[Out] (a*(b - c)*Log[-a + x] + b*(-a + c)*Log[-b + x] + (a - b)*c*Log[-c + x])/((a - b)*(a - c)*(b - c))

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x/((x-a)*(x-b)*(x-c)),x]')

[Out] Timed out

Maple [A]

time = 0.08, size = 69, normalized size = 1.01

method	result	size
default	$\frac{a \ln(a-x)}{(a-b)(a-c)} - \frac{b \ln(b-x)}{(a-b)(b-c)} + \frac{c \ln(c-x)}{(a-c)(b-c)}$	69
norman	$\frac{c \ln(c-x)}{ab-ac-bc+c^2} + \frac{a \ln(a-x)}{(a-b)(a-c)} - \frac{b \ln(b-x)}{(a-b)(b-c)}$	72
risch	$\frac{c \ln(c-x)}{ab-ac-bc+c^2} - \frac{b \ln(-b+x)}{ab-ac-b^2+bc} + \frac{a \ln(-a+x)}{a^2-ab-ac+bc}$	79

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-a+x)/(-b+x)/(-c+x),x,method=_RETURNVERBOSE)

[Out] a*ln(a-x)/(a-b)/(a-c)-b*ln(b-x)/(a-b)/(b-c)+c*ln(c-x)/(a-c)/(b-c)

Maxima [A]

time = 0.26, size = 78, normalized size = 1.15

$$\frac{a \log(-a + x)}{a^2 - ab - (a - b)c} - \frac{b \log(-b + x)}{ab - b^2 - (a - b)c} + \frac{c \log(-c + x)}{ab - (a + b)c + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-a+x)/(-b+x)/(-c+x),x, algorithm="maxima")

[Out] a*log(-a + x)/(a^2 - a*b - (a - b)*c) - b*log(-b + x)/(a*b - b^2 - (a - b)*c) + c*log(-c + x)/(a*b - (a + b)*c + c^2)

Fricas [A]

time = 0.47, size = 81, normalized size = 1.19

$$\frac{(a - b)c \log(-c + x) + (ab - ac) \log(-a + x) - (ab - bc) \log(-b + x)}{a^2b - ab^2 + (a - b)c^2 - (a^2 - b^2)c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-a+x)/(-b+x)/(-c+x),x, algorithm="fricas")

[Out] ((a - b)*c*log(-c + x) + (a*b - a*c)*log(-a + x) - (a*b - b*c)*log(-b + x)) / (a^2*b - a*b^2 + (a - b)*c^2 - (a^2 - b^2)*c)

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-a+x)/(-b+x)/(-c+x),x)

[Out] Timed out

Giac [A]

time = 0.00, size = 73, normalized size = 1.07

$$\frac{c \ln |x - c|}{ab - ac - bc + c^2} + \frac{b \ln |x - b|}{-ab + ac + b^2 - bc} + \frac{a \ln |x - a|}{a^2 - ab - ac + bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-a+x)/(-b+x)/(-c+x),x)

[Out] a*log(abs(-a + x))/(a^2 - a*b - a*c + b*c) - b*log(abs(-b + x))/(a*b - b^2 - a*c + b*c) + c*log(abs(-c + x))/(a*b - a*c - b*c + c^2)

Mupad [B]

time = 0.57, size = 87, normalized size = 1.28

$$\ln(x - a) \left(\frac{b}{(a - b)(b - c)} - \frac{c}{(a - c)(b - c)} \right) - \frac{b \ln(x - b)}{(a - b)(b - c)} + \frac{c \ln(x - c)}{(a - c)(b - c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x/((a - x)*(b - x)*(c - x)),x)

[Out] log(x - a)*(b/((a - b)*(b - c)) - c/((a - c)*(b - c))) - (b*log(x - b))/((a - b)*(b - c)) + (c*log(x - c))/((a - c)*(b - c))

3.13 $\int \frac{x}{(a^2+x^2)(b^2+x^2)} dx$

Optimal. Leaf size=47

$$-\frac{\log(a^2+x^2)}{2(a^2-b^2)} + \frac{\log(b^2+x^2)}{2(a^2-b^2)}$$

[Out] $-1/2*\ln(a^2+x^2)/(a^2-b^2)+1/2*\ln(b^2+x^2)/(a^2-b^2)$

Rubi [A]

time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {455, 36, 31}

$$\frac{\log(b^2+x^2)}{2(a^2-b^2)} - \frac{\log(a^2+x^2)}{2(a^2-b^2)}$$

Antiderivative was successfully verified.

[In] Int[x/((a^2 + x^2)*(b^2 + x^2)),x]

[Out] $-1/2*\text{Log}[a^2 + x^2]/(a^2 - b^2) + \text{Log}[b^2 + x^2]/(2*(a^2 - b^2))$

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 455

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a^2 + x^2)(b^2 + x^2)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(a^2 + x)(b^2 + x)} dx, x, x^2 \right) \\
&= -\frac{\text{Subst} \left(\int \frac{1}{a^2+x} dx, x, x^2 \right)}{2(a^2 - b^2)} + \frac{\text{Subst} \left(\int \frac{1}{b^2+x} dx, x, x^2 \right)}{2(a^2 - b^2)} \\
&= -\frac{\log(a^2 + x^2)}{2(a^2 - b^2)} + \frac{\log(b^2 + x^2)}{2(a^2 - b^2)}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 34, normalized size = 0.72

$$\frac{-\log(a^2 + x^2) + \log(b^2 + x^2)}{2(a^2 - b^2)}$$

Antiderivative was successfully verified.

`[In] Integrate[x/((a^2 + x^2)*(b^2 + x^2)),x]``[Out] (-Log[a^2 + x^2] + Log[b^2 + x^2])/(2*(a^2 - b^2))`**Mathics [A]**

time = 3.51, size = 33, normalized size = 0.70

$$\frac{\text{Log}[b^2 + x^2] - \text{Log}[a^2 + x^2]}{2(a + b)(a - b)}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x/((x^2+a^2)*(x^2+b^2)),x]')``[Out] (Log[b ^ 2 + x ^ 2] - Log[a ^ 2 + x ^ 2]) / (2 (a + b) (a - b))`**Maple [A]**

time = 0.05, size = 44, normalized size = 0.94

method	result	size
default	$-\frac{\ln(a^2+x^2)}{2(a^2-b^2)} + \frac{\ln(b^2+x^2)}{2a^2-2b^2}$	44
norman	$-\frac{\ln(a^2+x^2)}{2(a^2-b^2)} + \frac{\ln(b^2+x^2)}{2a^2-2b^2}$	44
risch	$-\frac{\ln(-a^2-x^2)}{2(a^2-b^2)} + \frac{\ln(b^2+x^2)}{2a^2-2b^2}$	48

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x/(a^2+x^2)/(b^2+x^2),x,method=_RETURNVERBOSE)`

[Out] $-1/2*\ln(a^2+x^2)/(a^2-b^2)+1/2*\ln(b^2+x^2)/(a^2-b^2)$

Maxima [A]

time = 0.27, size = 43, normalized size = 0.91

$$-\frac{\log(a^2 + x^2)}{2(a^2 - b^2)} + \frac{\log(b^2 + x^2)}{2(a^2 - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a^2+x^2)/(b^2+x^2),x, algorithm="maxima")`

[Out] $-1/2*\log(a^2 + x^2)/(a^2 - b^2) + 1/2*\log(b^2 + x^2)/(a^2 - b^2)$

Fricas [A]

time = 0.32, size = 32, normalized size = 0.68

$$-\frac{\log(a^2 + x^2) - \log(b^2 + x^2)}{2(a^2 - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a^2+x^2)/(b^2+x^2),x, algorithm="fricas")`

[Out] $-1/2*(\log(a^2 + x^2) - \log(b^2 + x^2))/(a^2 - b^2)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(36) = 72

time = 0.37, size = 121, normalized size = 2.57

$$\frac{\log\left(-\frac{a^4}{2(a-b)(a+b)} + \frac{a^2b^2}{(a-b)(a+b)} + \frac{a^2}{2} - \frac{b^4}{2(a-b)(a+b)} + \frac{b^2}{2} + x^2\right)}{2(a-b)(a+b)} - \frac{\log\left(\frac{a^4}{2(a-b)(a+b)} - \frac{a^2b^2}{(a-b)(a+b)} + \frac{a^2}{2} + \frac{b^4}{2(a-b)(a+b)} + \frac{b^2}{2} + x^2\right)}{2(a-b)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a**2+x**2)/(b**2+x**2),x)`

[Out] $\log(-a**4/(2*(a - b)*(a + b)) + a**2*b**2/((a - b)*(a + b)) + a**2/2 - b**4/(2*(a - b)*(a + b)) + b**2/2 + x**2)/(2*(a - b)*(a + b)) - \log(a**4/(2*(a - b)*(a + b)) - a**2*b**2/((a - b)*(a + b)) + a**2/2 + b**4/(2*(a - b)*(a + b)) + b**2/2 + x**2)/(2*(a - b)*(a + b))$

Giac [A]

time = 0.00, size = 40, normalized size = 0.85

$$\frac{\ln(x^2+b^2)}{a^2-b^2} + \frac{\ln(x^2+a^2)}{-a^2+b^2}$$

2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a^2+x^2)/(b^2+x^2),x)

[Out] -1/2*log(a^2 + x^2)/(a^2 - b^2) + 1/2*log(b^2 + x^2)/(a^2 - b^2)

Mupad [B]

time = 0.30, size = 256, normalized size = 5.45

$$\frac{\operatorname{atan}\left(\frac{\left(\frac{x^2(8a^2+8b^2)+16a^2b^2}{2(a^2-b^2)}-4x^2\right)1i-\left(\frac{x^2(8a^2+8b^2)+16a^2b^2}{2(a^2-b^2)}+4x^2\right)1i}{\frac{x^2(8a^2+8b^2)+16a^2b^2}{2(a^2-b^2)}-4x^2+\frac{x^2(8a^2+8b^2)+16a^2b^2}{2(a^2-b^2)}+4x^2}\right)}{a^2-b^2}1i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((a^2 + x^2)*(b^2 + x^2)),x)

[Out] (atan((((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 - b^2)) - 4*x^2)*1i)/(2*(a^2 - b^2)) - (((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 - b^2)) + 4*x^2)*1i)/(2*(a^2 - b^2)))/(((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 - b^2)) - 4*x^2)/(2*(a^2 - b^2)) + ((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 - b^2)) + 4*x^2)/(2*(a^2 - b^2))))*1i)/(a^2 - b^2)

$$3.14 \quad \int \frac{x^2}{(a^2+x^2)(b^2+x^2)} dx$$

Optimal. Leaf size=40

$$\frac{a \tan^{-1}\left(\frac{x}{a}\right)}{a^2 - b^2} - \frac{b \tan^{-1}\left(\frac{x}{b}\right)}{a^2 - b^2}$$

[Out] $a \cdot \arctan(x/a)/(a^2-b^2) - b \cdot \arctan(x/b)/(a^2-b^2)$

Rubi [A]

time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {492, 209}

$$\frac{a \tan^{-1}\left(\frac{x}{a}\right)}{a^2 - b^2} - \frac{b \tan^{-1}\left(\frac{x}{b}\right)}{a^2 - b^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a^2 + x^2)*(b^2 + x^2)),x]

[Out] (a*ArcTan[x/a]/(a^2 - b^2) - (b*ArcTan[x/b]))/(a^2 - b^2)

Rule 209

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 492

Int[((e_)*(x_)^(m_))/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2*n - 1]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(a^2+x^2)(b^2+x^2)} dx &= \frac{a^2 \int \frac{1}{a^2+x^2} dx}{a^2 - b^2} - \frac{b^2 \int \frac{1}{b^2+x^2} dx}{a^2 - b^2} \\ &= \frac{a \tan^{-1}\left(\frac{x}{a}\right)}{a^2 - b^2} - \frac{b \tan^{-1}\left(\frac{x}{b}\right)}{a^2 - b^2} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 30, normalized size = 0.75

$$\frac{a \tan^{-1}\left(\frac{x}{a}\right) - b \tan^{-1}\left(\frac{x}{b}\right)}{a^2 - b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a^2 + x^2)*(b^2 + x^2)),x]

[Out] (a*ArcTan[x/a] - b*ArcTan[x/b])/(a^2 - b^2)

Mathics [C] Result contains complex when optimal does not.

time = 6.58, size = 49, normalized size = 1.22

$$\frac{\frac{I}{2}(-a \operatorname{Log}[-Ia + x] + a \operatorname{Log}[Ia + x] - b \operatorname{Log}[Ib + x] + b \operatorname{Log}[-Ib + x])}{(a + b)(a - b)}}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^2/((x^2+a^2)*(x^2+b^2)),x]')

[Out] I / 2 (-a Log[-I a + x] + a Log[I a + x] - b Log[I b + x] + b Log[-I b + x]) / ((a + b) (a - b))

Maple [A]

time = 0.06, size = 41, normalized size = 1.02

method	result	size
default	$\frac{a \arctan\left(\frac{x}{a}\right)}{a^2 - b^2} - \frac{b \arctan\left(\frac{x}{b}\right)}{a^2 - b^2}$	41
risch	$\frac{a \arctan\left(\frac{x}{a}\right)}{a^2 - b^2} - \frac{b \arctan\left(\frac{x}{b}\right)}{a^2 - b^2}$	41

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a^2+x^2)/(b^2+x^2),x,method=_RETURNVERBOSE)

[Out] a*arctan(x/a)/(a^2-b^2)-b*arctan(x/b)/(a^2-b^2)

Maxima [A]

time = 0.34, size = 40, normalized size = 1.00

$$\frac{a \arctan\left(\frac{x}{a}\right)}{a^2 - b^2} - \frac{b \arctan\left(\frac{x}{b}\right)}{a^2 - b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a^2+x^2)/(b^2+x^2),x, algorithm="maxima")

[Out] a*arctan(x/a)/(a^2 - b^2) - b*arctan(x/b)/(a^2 - b^2)

Fricas [A]

time = 0.33, size = 30, normalized size = 0.75

$$\frac{a \arctan\left(\frac{x}{a}\right) - b \arctan\left(\frac{x}{b}\right)}{a^2 - b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a^2+x^2)/(b^2+x^2),x, algorithm="fricas")**[Out]** (a*arctan(x/a) - b*arctan(x/b))/(a^2 - b^2)**Sympy [C]** Result contains complex when optimal does not.

time = 0.68, size = 393, normalized size = 9.82

$$\frac{ia \log\left(\frac{-\frac{2ax^2}{(a-b)(a+b)} + \frac{2bx^2}{(a-b)(a+b)} - \frac{2ax^2}{(a-b)(a+b)} + \frac{2bx^2}{(a-b)(a+b)} + x\right)}{2(a-b)(a+b)} + \frac{ia \log\left(\frac{-\frac{2ax^2}{(a-b)(a+b)} - \frac{2bx^2}{(a-b)(a+b)} + \frac{2ax^2}{(a-b)(a+b)} - \frac{2bx^2}{(a-b)(a+b)} + x\right)}{2(a-b)(a+b)} - \frac{ib \log\left(\frac{-\frac{2ax^2}{(a-b)(a+b)} + \frac{2bx^2}{(a-b)(a+b)} - \frac{2ax^2}{(a-b)(a+b)} + \frac{2bx^2}{(a-b)(a+b)} + x\right)}{2(a-b)(a+b)} + \frac{ib \log\left(\frac{-\frac{2ax^2}{(a-b)(a+b)} - \frac{2bx^2}{(a-b)(a+b)} + \frac{2ax^2}{(a-b)(a+b)} - \frac{2bx^2}{(a-b)(a+b)} + x\right)}{2(a-b)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(a**2+x**2)/(b**2+x**2),x)

[Out] $-I*a*\log(-2*I*a**7/((a-b)**3*(a+b)**3) + 4*I*a**5*b**2/((a-b)**3*(a+b)**3) - 2*I*a**3*b**4/((a-b)**3*(a+b)**3) + I*a**3/((a-b)*(a+b)) + I*a*b**2/((a-b)*(a+b)) + x)/(2*(a-b)*(a+b)) + I*a*\log(2*I*a**7/((a-b)**3*(a+b)**3) - 4*I*a**5*b**2/((a-b)**3*(a+b)**3) + 2*I*a**3*b**4/((a-b)**3*(a+b)**3) - I*a**3/((a-b)*(a+b)) - I*a*b**2/((a-b)*(a+b)) + x)/(2*(a-b)*(a+b)) - I*b*\log(-2*I*a**4*b**3/((a-b)**3*(a+b)**3) + 4*I*a**2*b**5/((a-b)**3*(a+b)**3) + I*a**2*b/((a-b)*(a+b)) - 2*I*b**7/((a-b)**3*(a+b)**3) + I*b**3/((a-b)*(a+b)) + x)/(2*(a-b)*(a+b)) + I*b*\log(2*I*a**4*b**3/((a-b)**3*(a+b)**3) - 4*I*a**2*b**5/((a-b)**3*(a+b)**3) - I*a**2*b/((a-b)*(a+b)) + 2*I*b**7/((a-b)**3*(a+b)**3) - I*b**3/((a-b)*(a+b)) + x)/(2*(a-b)*(a+b))$

Giac [A]

time = 0.00, size = 46, normalized size = 1.15

$$-\frac{2b^2 \arctan\left(\frac{x}{b}\right)}{2(a^2 - b^2)b} + \frac{2a^2 \arctan\left(\frac{x}{a}\right)}{2(a^2 - b^2)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a^2+x^2)/(b^2+x^2),x)**[Out]** a*arctan(x/a)/(a^2 - b^2) - b*arctan(x/b)/(a^2 - b^2)**Mupad [B]**

time = 0.18, size = 191, normalized size = 4.78

$$\frac{a \operatorname{atan}\left(\frac{x(2a^4+2b^4) - \frac{a^2 x(8a^6-8a^4b^2-8a^2b^4+8b^6)}{(2a^2-2b^2)^2}}{ab^2(2a^2-2b^2)}\right)}{a^2 - b^2} - \frac{b \operatorname{atan}\left(\frac{x(2a^4+2b^4) - \frac{b^2 x(8a^6-8a^4b^2-8a^2b^4+8b^6)}{(2a^2-2b^2)^2}}{a^2 b(2a^2-2b^2)}\right)}{a^2 - b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2/((a^2 + x^2)*(b^2 + x^2)),x)$

[Out] $-(a*\text{atan}((x*(2*a^4 + 2*b^4) - (a^2*x*(8*a^6 + 8*b^6 - 8*a^2*b^4 - 8*a^4*b^2)))/(2*a^2 - 2*b^2)^2)/(a*b^2*(2*a^2 - 2*b^2)))/(a^2 - b^2) - (b*\text{atan}((x*(2*a^4 + 2*b^4) - (b^2*x*(8*a^6 + 8*b^6 - 8*a^2*b^4 - 8*a^4*b^2)))/(2*a^2 - 2*b^2)^2)/(a^2*b*(2*a^2 - 2*b^2)))/(a^2 - b^2)$

3.15 $\int \frac{x}{(-1+x)(1+x^2)} dx$

Optimal. Leaf size=27

$$\frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \log(1-x) - \frac{1}{4} \log(1+x^2)$$

[Out] 1/2*arctan(x)+1/2*ln(1-x)-1/4*ln(x^2+1)

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {815, 649, 209, 266}

$$-\frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(1 - x) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x/((-1 + x)*(1 + x^2)),x]

[Out] ArcTan[x]/2 + Log[1 - x]/2 - Log[1 + x^2]/4

Rule 209

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 649

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[(-a)*c]

Rule 815

Int[(((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)))/((a_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{x}{(-1+x)(1+x^2)} dx &= \int \left(\frac{1}{2(-1+x)} + \frac{1-x}{2(1+x^2)} \right) dx \\
&= \frac{1}{2} \log(1-x) + \frac{1}{2} \int \frac{1-x}{1+x^2} dx \\
&= \frac{1}{2} \log(1-x) + \frac{1}{2} \int \frac{1}{1+x^2} dx - \frac{1}{2} \int \frac{x}{1+x^2} dx \\
&= \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \log(1-x) - \frac{1}{4} \log(1+x^2)
\end{aligned}$$

Mathematica [A]

time = 0.00, size = 27, normalized size = 1.00

$$\frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \log(1-x) - \frac{1}{4} \log(1+x^2)$$

Antiderivative was successfully verified.

`[In] Integrate[x/((-1 + x)*(1 + x^2)),x]``[Out] ArcTan[x]/2 + Log[1 - x]/2 - Log[1 + x^2]/4`**Mathics [A]**

time = 1.69, size = 19, normalized size = 0.70

$$-\frac{\text{Log}[1+x^2]}{4} + \frac{\text{ArcTan}[x]}{2} + \frac{\text{Log}[-1+x]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x/((x-1)*(x^2+1)),x]')``[Out] -Log[1 + x ^ 2] / 4 + ArcTan[x] / 2 + Log[-1 + x] / 2`**Maple [A]**

time = 0.03, size = 20, normalized size = 0.74

method	result	size
default	$\frac{\ln(-1+x)}{2} - \frac{\ln(x^2+1)}{4} + \frac{\arctan(x)}{2}$	20
risch	$\frac{\ln(-1+x)}{2} - \frac{\ln(x^2+1)}{4} + \frac{\arctan(x)}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x/(-1+x)/(x^2+1),x,method=_RETURNVERBOSE)``[Out] 1/2*ln(-1+x)-1/4*ln(x^2+1)+1/2*arctan(x)`

Maxima [A]

time = 0.34, size = 19, normalized size = 0.70

$$\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-1+x)/(x^2+1),x, algorithm="maxima")

[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x - 1)

Fricas [A]

time = 0.33, size = 19, normalized size = 0.70

$$\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-1+x)/(x^2+1),x, algorithm="fricas")

[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x - 1)

Sympy [A]

time = 0.06, size = 19, normalized size = 0.70

$$\frac{\log(x - 1)}{2} - \frac{\log(x^2 + 1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-1+x)/(x**2+1),x)

[Out] log(x - 1)/2 - log(x**2 + 1)/4 + atan(x)/2

Giac [A]

time = 0.00, size = 24, normalized size = 0.89

$$\frac{\ln|x - 1|}{2} - \frac{\ln(x^2 + 1)}{4} + \frac{\arctan x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-1+x)/(x^2+1),x)

[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x - 1))

Mupad [B]

time = 0.05, size = 25, normalized size = 0.93

$$\frac{\ln(x - 1)}{2} + \ln(x - i) \left(-\frac{1}{4} - \frac{1}{4}i \right) + \ln(x + i) \left(-\frac{1}{4} + \frac{1}{4}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((x^2 + 1)*(x - 1)),x)

[Out] log(x - 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)

3.16 $\int \frac{x}{1+x^3} dx$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1+x) + \frac{1}{6}\log(1-x+x^2)$$

[Out] $-1/3*\ln(1+x)+1/6*\ln(x^2-x+1)-1/3*\arctan(1/3*(1-2*x)*3^{(1/2)})*3^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {298, 31, 648, 632, 210, 642}

$$\frac{1}{6}\log(x^2-x+1) - \frac{1}{3}\log(x+1) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x/(1 + x^3), x]

[Out] $-(\text{ArcTan}[(1 - 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3]) - \text{Log}[1 + x]/3 + \text{Log}[1 - x + x^2]/6$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] :> Simp[(-Rt[-a, 2]*Rt[-b, 2])⁽⁻¹⁾)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

Rule 298

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])⁽⁻¹⁾, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

`x] && NeQ[b^2 - 4*a*c, 0]`

Rule 642

`Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]`

Rule 648

`Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rubi steps

$$\begin{aligned} \int \frac{x}{1+x^3} dx &= -\left(\frac{1}{3} \int \frac{1}{1+x} dx\right) + \frac{1}{3} \int \frac{1+x}{1-x+x^2} dx \\ &= -\frac{1}{3} \log(1+x) + \frac{1}{6} \int \frac{-1+2x}{1-x+x^2} dx + \frac{1}{2} \int \frac{1}{1-x+x^2} dx \\ &= -\frac{1}{3} \log(1+x) + \frac{1}{6} \log(1-x+x^2) - \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x\right) \\ &= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3} \log(1+x) + \frac{1}{6} \log(1-x+x^2) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 40, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3} \log(1+x) + \frac{1}{6} \log(1-x+x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 + x^3), x]

[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6

Mathics [A]

time = 1.87, size = 34, normalized size = 0.83

$$-\frac{\text{Log}[1+x]}{3} + \frac{\text{Log}[1-x+x^2]}{6} + \frac{\sqrt{3} \text{ArcTan}\left[\frac{\sqrt{3}(-1+2x)}{3}\right]}{3}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x/(1+x^3),x]')`

[Out] $-\text{Log}[1+x] / 3 + \text{Log}[1-x+x^2] / 6 + \text{Sqrt}[3] \text{ArcTan}[\text{Sqrt}[3] (-1+2x) / 3] / 3$

Maple [A]

time = 0.03, size = 35, normalized size = 0.85

method	result	size
default	$-\frac{\ln(1+x)}{3} + \frac{\ln(x^2-x+1)}{6} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	35
risch	$\frac{\ln(4x^2-4x+4)}{6} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3} - \frac{\ln(1+x)}{3}$	37
meijerg	$-\frac{x^2 \ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}} + \frac{x^2 \ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}} + \frac{x^2 \sqrt{3} \arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{2}{3}}}$	80

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(x^3+1),x,method=_RETURNVERBOSE)`

[Out] $-1/3*\ln(1+x)+1/6*\ln(x^2-x+1)+1/3*3^{(1/2)}*\arctan(1/3*(2*x-1)*3^{(1/2)})$

Maxima [A]

time = 0.35, size = 34, normalized size = 0.83

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x-1)\right) + \frac{1}{6} \log(x^2-x+1) - \frac{1}{3} \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^3+1),x, algorithm="maxima")`

[Out] $1/3*\text{sqrt}(3)*\arctan(1/3*\text{sqrt}(3)*(2*x-1)) + 1/6*\log(x^2-x+1) - 1/3*\log(x+1)$

Fricas [A]

time = 0.34, size = 34, normalized size = 0.83

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x-1)\right) + \frac{1}{6} \log(x^2-x+1) - \frac{1}{3} \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^3+1),x, algorithm="fricas")`

[Out] $\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{6}\log(x^2-x+1) - \frac{1}{3}\log(x+1)$

Sympy [A]

time = 0.07, size = 41, normalized size = 1.00

$$-\frac{\log(x+1)}{3} + \frac{\log(x^2-x+1)}{6} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x**3+1),x)`

[Out] $-\log(x+1)/3 + \log(x^2-x+1)/6 + \sqrt{3}\operatorname{atan}(2\sqrt{3}x/3 - \sqrt{3}/3)/3$

Giac [A]

time = 0.00, size = 47, normalized size = 1.15

$$\frac{\ln(x^2-x+1)}{6} - \frac{1}{3}\sqrt{3} \arctan\left(-\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) - \frac{\ln|x+1|}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^3+1),x)`

[Out] $-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{6}\log(x^2-x+1) - \frac{1}{3}\log(\operatorname{abs}(x+1))$

Mupad [B]

time = 0.21, size = 46, normalized size = 1.12

$$-\frac{\ln(x+1)}{3} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3} \operatorname{li}}{2}\right) \left(-\frac{1}{6} + \frac{\sqrt{3} \operatorname{li}}{6}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2}\right) \left(\frac{1}{6} + \frac{\sqrt{3} \operatorname{li}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(x^3+1),x)`

[Out] $\log(x + (3^{1/2} \operatorname{li})/2 - 1/2) * ((3^{1/2} \operatorname{li})/6 + 1/6) - \log(x - (3^{1/2} \operatorname{li})/2 - 1/2) * ((3^{1/2} \operatorname{li})/6 - 1/6) - \log(x+1)/3$

$$3.17 \quad \int \frac{x^3}{(-1+x)^2(1+x^3)} dx$$

Optimal. Leaf size=43

$$\frac{1}{2(1-x)} + \frac{3}{4} \log(1-x) - \frac{1}{12} \log(1+x) - \frac{1}{3} \log(1-x+x^2)$$

[Out] 1/2/(1-x)+3/4*ln(1-x)-1/12*ln(1+x)-1/3*ln(x^2-x+1)

Rubi [A]

time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6857, 642}

$$-\frac{1}{3} \log(x^2 - x + 1) + \frac{1}{2(1-x)} + \frac{3}{4} \log(1-x) - \frac{1}{12} \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[x^3/((-1 + x)^2*(1 + x^3)),x]

[Out] 1/(2*(1 - x)) + (3*Log[1 - x])/4 - Log[1 + x]/12 - Log[1 - x + x^2]/3

Rule 642

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]

Rule 6857

Int[(u_)/((a_) + (b_.)*(x_)^n), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(-1+x)^2(1+x^3)} dx &= \int \left(\frac{1}{2(-1+x)^2} + \frac{3}{4(-1+x)} - \frac{1}{12(1+x)} + \frac{1-2x}{3(1-x+x^2)} \right) dx \\ &= \frac{1}{2(1-x)} + \frac{3}{4} \log(1-x) - \frac{1}{12} \log(1+x) + \frac{1}{3} \int \frac{1-2x}{1-x+x^2} dx \\ &= \frac{1}{2(1-x)} + \frac{3}{4} \log(1-x) - \frac{1}{12} \log(1+x) - \frac{1}{3} \log(1-x+x^2) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 34, normalized size = 0.79

$$\frac{1}{12} \left(-\frac{6}{-1+x} + 9 \log(-1+x) - \log(1+x) - 4 \log((-1+x)^2 + x) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[x^3/((-1 + x)^2*(1 + x^3)),x]``[Out] (-6/(-1 + x) + 9*Log[-1 + x] - Log[1 + x] - 4*Log[(-1 + x)^2 + x])/12`**Mathics [A]**

time = 1.86, size = 37, normalized size = 0.86

$$\frac{-6 + (-1 + x) (-4 \operatorname{Log}[1 - x + x^2] - \operatorname{Log}[1 + x] + 9 \operatorname{Log}[-1 + x])}{-12 + 12x}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^3/((x-1)^2*(x^3+1)),x]')``[Out] (-6 + (-1 + x) (-4 Log[1 - x + x ^ 2] - Log[1 + x] + 9 Log[-1 + x])) / (12 (-1 + x))`**Maple [A]**

time = 0.04, size = 32, normalized size = 0.74

method	result	size
default	$-\frac{1}{2(-1+x)} + \frac{3 \ln(-1+x)}{4} - \frac{\ln(1+x)}{12} - \frac{\ln(x^2-x+1)}{3}$	32
norman	$-\frac{1}{2(-1+x)} + \frac{3 \ln(-1+x)}{4} - \frac{\ln(1+x)}{12} - \frac{\ln(x^2-x+1)}{3}$	32
risch	$-\frac{1}{2(-1+x)} + \frac{3 \ln(-1+x)}{4} - \frac{\ln(1+x)}{12} - \frac{\ln(x^2-x+1)}{3}$	32

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^3/(-1+x)^2/(x^3+1),x,method=_RETURNVERBOSE)``[Out] -1/2/(-1+x)+3/4*ln(-1+x)-1/12*ln(1+x)-1/3*ln(x^2-x+1)`**Maxima [A]**

time = 0.36, size = 31, normalized size = 0.72

$$-\frac{1}{2(x-1)} - \frac{1}{3} \log(x^2 - x + 1) - \frac{1}{12} \log(x + 1) + \frac{3}{4} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^3/(-1+x)^2/(x^3+1),x, algorithm="maxima")`

[Out] $-1/2/(x - 1) - 1/3*\log(x^2 - x + 1) - 1/12*\log(x + 1) + 3/4*\log(x - 1)$

Fricas [A]

time = 0.33, size = 40, normalized size = 0.93

$$\frac{4(x-1)\log(x^2-x+1) + (x-1)\log(x+1) - 9(x-1)\log(x-1) + 6}{12(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(-1+x)^2/(x^3+1),x, algorithm="fricas")`

[Out] $-1/12*(4*(x - 1)*\log(x^2 - x + 1) + (x - 1)*\log(x + 1) - 9*(x - 1)*\log(x - 1) + 6)/(x - 1)$

Sympy [A]

time = 0.07, size = 31, normalized size = 0.72

$$\frac{3\log(x-1)}{4} - \frac{\log(x+1)}{12} - \frac{\log(x^2-x+1)}{3} - \frac{1}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(-1+x)**2/(x**3+1),x)`

[Out] $3*\log(x - 1)/4 - \log(x + 1)/12 - \log(x^2 - x + 1)/3 - 1/(2*x - 2)$

Giac [A]

time = 0.00, size = 41, normalized size = 0.95

$$-\frac{\ln|x+1|}{12} - \frac{\ln(x^2-x+1)}{3} + \frac{3}{4}\ln|x-1| - \frac{\frac{1}{4} \cdot 2}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(-1+x)^2/(x^3+1),x)`

[Out] $-1/2/(x - 1) - 1/3*\log(x^2 - x + 1) - 1/12*\log(\text{abs}(x + 1)) + 3/4*\log(\text{abs}(x - 1))$

Mupad [B]

time = 0.04, size = 33, normalized size = 0.77

$$\frac{3\ln(x-1)}{4} - \frac{\ln(x+1)}{12} - \frac{\ln(x^2-x+1)}{3} - \frac{1}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/((x^3 + 1)*(x - 1)^2),x)`

[Out] $(3*\log(x - 1))/4 - \log(x + 1)/12 - \log(x^2 - x + 1)/3 - 1/(2*(x - 1))$

3.18 $\int \frac{1}{1+x^4} dx$

Optimal. Leaf size=85

$$-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

[Out] 1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)-1/8*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2+x*2^(1/2))*2^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {217, 1179, 642, 1176, 631, 210}

$$-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}} + \frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}} - \frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)^(-1), x]

[Out] -1/2*ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) - Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 631

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{1+x^4} dx &= \frac{1}{2} \int \frac{1-x^2}{1+x^4} dx + \frac{1}{2} \int \frac{1+x^2}{1+x^4} dx \\ &= \frac{1}{4} \int \frac{1}{1-\sqrt{2}x+x^2} dx + \frac{1}{4} \int \frac{1}{1+\sqrt{2}x+x^2} dx - \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2} dx}{4\sqrt{2}} - \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2} dx}{4\sqrt{2}} \\ &= -\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1-\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1+\sqrt{2}x\right)}{2\sqrt{2}} \\ &= -\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 64, normalized size = 0.75

$$\frac{-2 \tan^{-1}\left(1-\sqrt{2}x\right) + 2 \tan^{-1}\left(1+\sqrt{2}x\right) - \log\left(1-\sqrt{2}x+x^2\right) + \log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^4)^(-1), x]

[Out] $(-2*\text{ArcTan}[1 - \text{Sqrt}[2]*x] + 2*\text{ArcTan}[1 + \text{Sqrt}[2]*x] - \text{Log}[1 - \text{Sqrt}[2]*x + x^2] + \text{Log}[1 + \text{Sqrt}[2]*x + x^2])/(4*\text{Sqrt}[2])$

Mathics [A]

time = 2.08, size = 51, normalized size = 0.60

$$\frac{\sqrt{2} \left(\text{Log} \left[1 + \sqrt{2} x + x^2 \right] - \text{Log} \left[1 - \sqrt{2} x + x^2 \right] + 2 \text{ArcTan} \left[-1 + \sqrt{2} x \right] + 2 \text{ArcTan} \left[1 + \sqrt{2} x \right] \right)}{8}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(1+x^4),x]')`

[Out] $\text{Sqrt}[2] (\text{Log}[1 + \text{Sqrt}[2] x + x^2] - \text{Log}[1 - \text{Sqrt}[2] x + x^2] + 2 \text{ArcTan}[-1 + \text{Sqrt}[2] x] + 2 \text{ArcTan}[1 + \text{Sqrt}[2] x]) / 8$

Maple [A]

time = 0.00, size = 52, normalized size = 0.61

method	result
risch	$\frac{\sum_{-R=\text{RootOf}(-Z^4+1)} \frac{\ln(-R+x)}{-R^3}}{4}$
default	$\frac{\sqrt{2} \left(\ln \left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}} \right) + 2 \arctan(1+x\sqrt{2}) + 2 \arctan(-1+x\sqrt{2}) \right)}{8}$
meijerg	$-\frac{x\sqrt{2} \ln(1-\sqrt{2}(x^4)^{\frac{1}{4}}+\sqrt{x^4})}{8(x^4)^{\frac{1}{4}}} + \frac{x\sqrt{2} \arctan\left(\frac{\sqrt{2}(x^4)^{\frac{1}{4}}}{2-\sqrt{2}(x^4)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{1}{4}}} + \frac{x\sqrt{2} \ln(1+\sqrt{2}(x^4)^{\frac{1}{4}}+\sqrt{x^4})}{8(x^4)^{\frac{1}{4}}} + \frac{x\sqrt{2}}{8(x^4)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4+1),x,method=_RETURNVERBOSE)`

[Out] $1/8*2^{(1/2)}*(\ln((1+x^2+x*2^{(1/2)})/(1+x^2-x*2^{(1/2)}))+2*\arctan(1+x*2^{(1/2)})+2*\arctan(-1+x*2^{(1/2)}))$

Maxima [A]

time = 0.36, size = 72, normalized size = 0.85

$$\frac{1}{4} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} (2x + \sqrt{2}) \right) + \frac{1}{4} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} (2x - \sqrt{2}) \right) + \frac{1}{8} \sqrt{2} \log(x^2 + \sqrt{2}x + 1) - \frac{1}{8} \sqrt{2} \log(x^2 - \sqrt{2}x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4+1),x, algorithm="maxima")`

[Out] $1/4*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*(2*x + \text{sqrt}(2))) + 1/4*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*(2*x - \text{sqrt}(2))) + 1/8*\text{sqrt}(2)*\log(x^2 + \text{sqrt}(2)*x + 1) - 1/8*\text{sqrt}(2)*\log(x^2 - \text{sqrt}(2)*x + 1)$

Fricas [A]

time = 0.33, size = 100, normalized size = 1.18

$$-\frac{1}{2}\sqrt{2} \arctan\left(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 + \sqrt{2}x + 1} - 1\right) - \frac{1}{2}\sqrt{2} \arctan\left(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 - \sqrt{2}x + 1} + 1\right) + \frac{1}{8}\sqrt{2} \log\left(4x^2 + 4\sqrt{2}x + 4\right) - \frac{1}{8}\sqrt{2} \log\left(4x^2 - 4\sqrt{2}x + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+1),x, algorithm="fricas")

[Out] -1/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 1/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 1/8*sqrt(2)*log(4*x^2 + 4*sqrt(2)*x + 4) - 1/8*sqrt(2)*log(4*x^2 - 4*sqrt(2)*x + 4)

Sympy [A]

time = 0.07, size = 73, normalized size = 0.86

$$-\frac{\sqrt{2} \log\left(x^2 - \sqrt{2}x + 1\right)}{8} + \frac{\sqrt{2} \log\left(x^2 + \sqrt{2}x + 1\right)}{8} + \frac{\sqrt{2} \operatorname{atan}\left(\sqrt{2}x - 1\right)}{4} + \frac{\sqrt{2} \operatorname{atan}\left(\sqrt{2}x + 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**4+1),x)

[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8 + sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4

Giac [A]

time = 0.00, size = 106, normalized size = 1.25

$$-\frac{1}{8}\sqrt{2} \ln\left(x^2 - \sqrt{2}x + 1\right) + \frac{1}{4}\sqrt{2} \arctan\left(\frac{x - \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}\right) + \frac{1}{8}\sqrt{2} \ln\left(x^2 + \sqrt{2}x + 1\right) + \frac{1}{4}\sqrt{2} \arctan\left(\frac{x + \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+1),x)

[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1)

Mupad [B]

time = 0.00, size = 33, normalized size = 0.39

$$\sqrt{2} \operatorname{atan}\left(\sqrt{2}x \left(\frac{1}{2} - \frac{1}{2}i\right)\right) \left(\frac{1}{4} + \frac{1}{4}i\right) + \sqrt{2} \operatorname{atan}\left(\sqrt{2}x \left(\frac{1}{2} + \frac{1}{2}i\right)\right) \left(\frac{1}{4} - \frac{1}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4 + 1),x)

[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 + 1i/4) + 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(1/4 - 1i/4)

3.19 $\int \frac{x^2}{1+x^4} dx$

Optimal. Leaf size=85

$$-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

[Out] 1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/8*ln(1+x^2+x*2^(1/2))*2^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$,

Rules used = {303, 1176, 631, 210, 1179, 642}

$$\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}} - \frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}} - \frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(1 + x^4),x]

[Out] -1/2*ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) + Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 303

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 631

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{1+x^4} dx &= -\left(\frac{1}{2} \int \frac{1-x^2}{1+x^4} dx\right) + \frac{1}{2} \int \frac{1+x^2}{1+x^4} dx \\ &= \frac{1}{4} \int \frac{1}{1-\sqrt{2}x+x^2} dx + \frac{1}{4} \int \frac{1}{1+\sqrt{2}x+x^2} dx + \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2} dx}{4\sqrt{2}} + \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2} dx}{4\sqrt{2}} \\ &= \frac{\log(1-\sqrt{2}x+x^2)}{4\sqrt{2}} - \frac{\log(1+\sqrt{2}x+x^2)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1-\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1+\sqrt{2}x\right)}{2\sqrt{2}} \\ &= -\frac{\tan^{-1}(1-\sqrt{2}x)}{2\sqrt{2}} + \frac{\tan^{-1}(1+\sqrt{2}x)}{2\sqrt{2}} + \frac{\log(1-\sqrt{2}x+x^2)}{4\sqrt{2}} - \frac{\log(1+\sqrt{2}x+x^2)}{4\sqrt{2}} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 64, normalized size = 0.75

$$\frac{-2 \tan^{-1}(1-\sqrt{2}x) + 2 \tan^{-1}(1+\sqrt{2}x) + \log(1-\sqrt{2}x+x^2) - \log(1+\sqrt{2}x+x^2)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(1 + x^4), x]

[Out] $(-2*\text{ArcTan}[1 - \text{Sqrt}[2]*x] + 2*\text{ArcTan}[1 + \text{Sqrt}[2]*x] + \text{Log}[1 - \text{Sqrt}[2]*x + x^2] - \text{Log}[1 + \text{Sqrt}[2]*x + x^2])/(4*\text{Sqrt}[2])$

Mathics [A]

time = 2.06, size = 51, normalized size = 0.60

$$\frac{\sqrt{2} \left(\text{Log} \left[1 - \sqrt{2} x + x^2 \right] - \text{Log} \left[1 + \sqrt{2} x + x^2 \right] + 2\text{ArcTan} \left[-1 + \sqrt{2} x \right] + 2\text{ArcTan} \left[1 + \sqrt{2} x \right] \right)}{8}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x^2/(1+x^4),x]')`

[Out] $\text{Sqrt}[2] (\text{Log}[1 - \text{Sqrt}[2] x + x^2] - \text{Log}[1 + \text{Sqrt}[2] x + x^2] + 2 \text{ArcTan}[-1 + \text{Sqrt}[2] x] + 2 \text{ArcTan}[1 + \text{Sqrt}[2] x]) / 8$

Maple [A]

time = 0.02, size = 52, normalized size = 0.61

method	result
risch	$\frac{\left(\sum_{-R=\text{RootOf}(-Z^4+1)} \frac{\ln\left(-\frac{R+x}{-R}\right)}{4} \right)}{4}$
default	$\frac{\sqrt{2} \left(\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right) + 2 \arctan(1+x\sqrt{2}) + 2 \arctan(-1+x\sqrt{2}) \right)}{8}$
meijerg	$\frac{x^3 \sqrt{2} \ln\left(1 - \sqrt{2} (x^4)^{\frac{1}{4}} + \sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}} + \frac{x^3 \sqrt{2} \arctan\left(\frac{\sqrt{2} (x^4)^{\frac{1}{4}}}{2 - \sqrt{2} (x^4)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{3}{4}}} - \frac{x^3 \sqrt{2} \ln\left(1 + \sqrt{2} (x^4)^{\frac{1}{4}} + \sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}} + \frac{x^3 \sqrt{2} \arctan\left(\frac{\sqrt{2} (x^4)^{\frac{1}{4}}}{2 + \sqrt{2} (x^4)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{3}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(x^4+1),x,method=_RETURNVERBOSE)`

[Out] $1/8*2^{(1/2)}*(\ln((1+x^2-x*2^{(1/2)})/(1+x^2+x*2^{(1/2)}))+2*\arctan(1+x*2^{(1/2)})+2*\arctan(-1+x*2^{(1/2)}))$

Maxima [A]

time = 0.35, size = 72, normalized size = 0.85

$$\frac{1}{4} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} (2x + \sqrt{2})\right) + \frac{1}{4} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} (2x - \sqrt{2})\right) - \frac{1}{8} \sqrt{2} \log(x^2 + \sqrt{2}x + 1) + \frac{1}{8} \sqrt{2} \log(x^2 - \sqrt{2}x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(x^4+1),x, algorithm="maxima")`

[Out] $1/4*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*(2*x + \text{sqrt}(2))) + 1/4*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*(2*x - \text{sqrt}(2))) - 1/8*\text{sqrt}(2)*\log(x^2 + \text{sqrt}(2)*x + 1) + 1/8*\text{sqrt}(2)*\log(x^2 - \text{sqrt}(2)*x + 1)$

Fricas [A]

time = 0.34, size = 100, normalized size = 1.18

$$-\frac{1}{2}\sqrt{2} \arctan\left(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 + \sqrt{2}x + 1} - 1\right) - \frac{1}{2}\sqrt{2} \arctan\left(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 - \sqrt{2}x + 1} + 1\right) - \frac{1}{8}\sqrt{2} \log(4x^2 + 4\sqrt{2}x + 4) + \frac{1}{8}\sqrt{2} \log(4x^2 - 4\sqrt{2}x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^4+1),x, algorithm="fricas")

[Out] -1/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 1/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) - 1/8*sqrt(2)*log(4*x^2 + 4*sqrt(2)*x + 4) + 1/8*sqrt(2)*log(4*x^2 - 4*sqrt(2)*x + 4)

Sympy [A]

time = 0.08, size = 73, normalized size = 0.86

$$\frac{\sqrt{2} \log(x^2 - \sqrt{2}x + 1)}{8} - \frac{\sqrt{2} \log(x^2 + \sqrt{2}x + 1)}{8} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}x - 1)}{4} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}x + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(x**4+1),x)

[Out] sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 - sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8 + sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4

Giac [A]

time = 0.00, size = 106, normalized size = 1.25

$$\frac{1}{8}\sqrt{2} \ln(x^2 - \sqrt{2}x + 1) + \frac{1}{4}\sqrt{2} \arctan\left(\frac{x - \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}\right) - \frac{1}{8}\sqrt{2} \ln(x^2 + \sqrt{2}x + 1) + \frac{1}{4}\sqrt{2} \arctan\left(\frac{x + \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^4+1),x)

[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1)

Mupad [B]

time = 0.21, size = 33, normalized size = 0.39

$$\sqrt{2} \operatorname{atan}\left(\sqrt{2}x \left(\frac{1}{2} - \frac{1}{2}i\right)\right) \left(\frac{1}{4} - \frac{1}{4}i\right) + \sqrt{2} \operatorname{atan}\left(\sqrt{2}x \left(\frac{1}{2} + \frac{1}{2}i\right)\right) \left(\frac{1}{4} + \frac{1}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^4 + 1),x)

[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 - 1i/4) + 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(1/4 + 1i/4)

3.20 $\int \frac{1}{1+x^2+x^4} dx$

Optimal. Leaf size=67

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}\log(1-x+x^2) + \frac{1}{4}\log(1+x+x^2)$$

[Out] $-1/4*\ln(x^2-x+1)+1/4*\ln(x^2+x+1)-1/6*\arctan(1/3*(1-2*x)*3^{(1/2)})*3^{(1/2)}+1/6*\arctan(1/3*(1+2*x)*3^{(1/2)})*3^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1108, 648, 632, 210, 642}

$$-\frac{1}{4}\log(x^2-x+1) + \frac{1}{4}\log(x^2+x+1) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + x^2 + x^4)^{-1}, x]$

[Out] $-1/2*\text{ArcTan}[(1 - 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3] + \text{ArcTan}[(1 + 2*x)/\text{Sqrt}[3]]/(2*\text{Sqrt}[3]) - \text{Log}[1 - x + x^2]/4 + \text{Log}[1 + x + x^2]/4$

Rule 210

$\text{Int}[(a_. + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1}*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 632

$\text{Int}[(a_. + (b_.)*(x_) + (c_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[(d_. + (e_.)*(x_))/((a_. + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
+ x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{1+x^2+x^4} dx &= \frac{1}{2} \int \frac{1-x}{1-x+x^2} dx + \frac{1}{2} \int \frac{1+x}{1+x+x^2} dx \\ &= \frac{1}{4} \int \frac{1}{1-x+x^2} dx - \frac{1}{4} \int \frac{-1+2x}{1-x+x^2} dx + \frac{1}{4} \int \frac{1}{1+x+x^2} dx + \frac{1}{4} \int \frac{1+2x}{1+x+x^2} dx \\ &= -\frac{1}{4} \log(1-x+x^2) + \frac{1}{4} \log(1+x+x^2) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x \right) - \frac{1}{2} \text{S} \\ &= -\frac{\tan^{-1} \left(\frac{1-2x}{\sqrt{3}} \right)}{2\sqrt{3}} + \frac{\tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)}{2\sqrt{3}} - \frac{1}{4} \log(1-x+x^2) + \frac{1}{4} \log(1+x+x^2) \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.04, size = 73, normalized size = 1.09

$$\frac{i \left(\sqrt{1-i\sqrt{3}} \tan^{-1} \left(\frac{1}{2} (-i+\sqrt{3}) x \right) - \sqrt{1+i\sqrt{3}} \tan^{-1} \left(\frac{1}{2} (i+\sqrt{3}) x \right) \right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^4)^(-1), x]
```

```
[Out] (I*(Sqrt[1 - I*Sqrt[3]]*ArcTan[(-I + Sqrt[3])*x]/2] - Sqrt[1 + I*Sqrt[3]]*
ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[6]
```

Mathics [A]

time = 2.12, size = 53, normalized size = 0.79

$$-\frac{\text{Log}[1-x+x^2]}{4} + \frac{\sqrt{3} \text{ArcTan} \left[\frac{\sqrt{3}(-1+2x)}{3} \right]}{6} + \frac{\sqrt{3} \text{ArcTan} \left[\frac{\sqrt{3}(1+2x)}{3} \right]}{6} + \frac{\text{Log}[1+x+x^2]}{4}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(1+x^2+x^4),x]')`

[Out] $-\text{Log}[1 - x + x^2] / 4 + \text{Sqrt}[3] \text{ArcTan}[\text{Sqrt}[3] (-1 + 2x) / 3] / 6 + \text{Sqrt}[3] \text{ArcTan}[\text{Sqrt}[3] (1 + 2x) / 3] / 6 + \text{Log}[1 + x + x^2] / 4$

Maple [A]

time = 0.02, size = 54, normalized size = 0.81

method	result	size
default	$\frac{\ln(x^2+x+1)}{4} + \frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6} - \frac{\ln(x^2-x+1)}{4} + \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$	54
risch	$-\frac{\ln(4x^2-4x+4)}{4} + \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6} + \frac{\ln(4x^2+4x+4)}{4}$	60

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4+x^2+1),x,method=_RETURNVERBOSE)`

[Out] $1/4*\ln(x^2+x+1)+1/6*\arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/4*\ln(x^2-x+1)+1/6*3^(1/2)*\arctan(1/3*(2*x-1)*3^(1/2))$

Maxima [A]

time = 0.35, size = 53, normalized size = 0.79

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}\log(x^2+x+1) - \frac{1}{4}\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4+x^2+1),x, algorithm="maxima")`

[Out] $1/6*\text{sqrt}(3)*\arctan(1/3*\text{sqrt}(3)*(2*x + 1)) + 1/6*\text{sqrt}(3)*\arctan(1/3*\text{sqrt}(3)*(2*x - 1)) + 1/4*\log(x^2 + x + 1) - 1/4*\log(x^2 - x + 1)$

Fricas [A]

time = 0.33, size = 53, normalized size = 0.79

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}\log(x^2+x+1) - \frac{1}{4}\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4+x^2+1),x, algorithm="fricas")`

[Out] $1/6*\text{sqrt}(3)*\arctan(1/3*\text{sqrt}(3)*(2*x + 1)) + 1/6*\text{sqrt}(3)*\arctan(1/3*\text{sqrt}(3)*(2*x - 1)) + 1/4*\log(x^2 + x + 1) - 1/4*\log(x^2 - x + 1)$

Sympy [A]

time = 0.11, size = 70, normalized size = 1.04

$$-\frac{\log(x^2 - x + 1)}{4} + \frac{\log(x^2 + x + 1)}{4} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x - \sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x + \sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**4+x**2+1),x)**[Out]** -log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6**Giac [A]**

time = 0.00, size = 67, normalized size = 1.00

$$-\frac{\ln(x^2 - x + 1)}{4} + \frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\ln(x^2 + x + 1)}{4} + \frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+x^2+1),x)**[Out]** 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)**Mupad [B]**

time = 0.18, size = 47, normalized size = 0.70

$$\operatorname{atanh}\left(\frac{2x}{-1 + \sqrt{3} \operatorname{li}}\right) \left(-\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{6}\right) + \operatorname{atanh}\left(\frac{2x}{1 + \sqrt{3} \operatorname{li}}\right) \left(\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2 + x^4 + 1),x)**[Out]** atanh((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) + atanh((2*x)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/6 + 1/2)

3.21 $\int (a + bx)^p dx$

Optimal. Leaf size=18

$$\frac{(a + bx)^{1+p}}{b(1 + p)}$$

[Out] (b*x+a)^(1+p)/b/(1+p)

Rubi [A]

time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{(a + bx)^{p+1}}{b(p + 1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^p, x]

[Out] (a + b*x)^(1 + p)/(b*(1 + p))

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\int (a + bx)^p dx = \frac{(a + bx)^{1+p}}{b(1 + p)}$$

Mathematica [A]

time = 0.02, size = 17, normalized size = 0.94

$$\frac{(a + bx)^{1+p}}{b + bp}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^p, x]

[Out] (a + b*x)^(1 + p)/(b + b*p)

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(a+b*x)^p,x]')`

[Out] caught exception:

Maple [A]

time = 0.02, size = 19, normalized size = 1.06

method	result	size
gosper	$\frac{(bx+a)^{1+p}}{b(1+p)}$	19
default	$\frac{(bx+a)^{1+p}}{b(1+p)}$	19
risch	$\frac{(bx+a)(bx+a)^p}{b(1+p)}$	22
norman	$\frac{x e^{p \ln(bx+a)}}{1+p} + \frac{a e^{p \ln(bx+a)}}{b(1+p)}$	37

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)^p,x,method=_RETURNVERBOSE)`

[Out] $(b*x+a)^{(1+p)}/b/(1+p)$

Maxima [A]

time = 0.30, size = 18, normalized size = 1.00

$$\frac{(bx+a)^{p+1}}{b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^p,x, algorithm="maxima")`

[Out] $(b*x + a)^{(p + 1)}/(b*(p + 1))$

Fricas [A]

time = 0.34, size = 20, normalized size = 1.11

$$\frac{(bx+a)(bx+a)^p}{bp+b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^p,x, algorithm="fricas")`

[Out] $(b*x + a)*(b*x + a)^p/(b*p + b)$

Sympy [A]

time = 0.03, size = 20, normalized size = 1.11

$$\frac{\begin{cases} \frac{(a+bx)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(a+bx) & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**p,x)**[Out]** Piecewise(((a + b*x)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x), True))/b**Giac [A]**

time = 0.00, size = 16, normalized size = 0.89

$$\frac{(bx + a)^{p+1}}{b(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^p,x)**[Out]** (b*x + a)^(p + 1)/(b*(p + 1))**Mupad [B]**

time = 0.22, size = 18, normalized size = 1.00

$$\frac{(a + bx)^{p+1}}{b(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^p,x)**[Out]** (a + b*x)^(p + 1)/(b*(p + 1))

3.22 $\int x(a + bx)^p dx$

Optimal. Leaf size=39

$$-\frac{a(a + bx)^{1+p}}{b^2(1 + p)} + \frac{(a + bx)^{2+p}}{b^2(2 + p)}$$

[Out] $-a*(b*x+a)^{(1+p)}/b^2/(1+p)+(b*x+a)^{(2+p)}/b^2/(2+p)$

Rubi [A]

time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$\frac{(a + bx)^{p+2}}{b^2(p + 2)} - \frac{a(a + bx)^{p+1}}{b^2(p + 1)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*(a + b*x)^p, x]$

[Out] $-((a*(a + b*x)^{(1 + p)})/(b^2*(1 + p))) + (a + b*x)^{(2 + p)}/(b^2*(2 + p))$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) || \text{LtQ}[9*m + 5*(n + 1), 0] || \text{GtQ}[m + n + 2, 0])$

Rubi steps

$$\begin{aligned} \int x(a + bx)^p dx &= \int \left(-\frac{a(a + bx)^p}{b} + \frac{(a + bx)^{1+p}}{b} \right) dx \\ &= -\frac{a(a + bx)^{1+p}}{b^2(1 + p)} + \frac{(a + bx)^{2+p}}{b^2(2 + p)} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 33, normalized size = 0.85

$$\frac{(a + bx)^{1+p}(-a + b(1 + p)x)}{b^2(1 + p)(2 + p)}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[x*(a + b*x)^p, x]$

[Out] $((a + b*x)^{(1 + p)}*(-a + b*(1 + p)*x))/(b^2*(1 + p)*(2 + p))$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.46, size = 213, normalized size = 5.46

Piecewise $\left[\left\{ \left\{ \frac{x^2 a^p}{2}, b=0 \right\}, \left\{ \frac{a + a \operatorname{Log} \left[\frac{a}{b} + x \right] + b x \operatorname{Log} \left[\frac{a}{b} + x \right]}{b^2 (a + b x)}, p=-2 \right\}, \left\{ \frac{-a \operatorname{Log} \left[\frac{a}{b} + x \right] + b x}{b^2}, p=-1 \right\} \right], -\frac{a^2 (a + b x)^p}{2b^2 + 3b^2 p + b^2 p^2} + \frac{a b p x (a + b x)^p}{2b^2 + 3b^2 p + b^2 p^2} + \frac{b^2 x^2 (a + b x)^p}{2b^2 + 3b^2 p + b^2 p^2} + \frac{b^2 p x^2 (a + b x)^p}{2b^2 + 3b^2 p + b^2 p^2} \right]$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x*(a+b*x)^p,x]')`

[Out] `Piecewise[{{x ^ 2 a ^ p / 2, b == 0}, {(a + a Log[a / b + x] + b x Log[a / b + x]) / (b ^ 2 (a + b x)), p == -2}, {(-a Log[a / b + x] + b x) / b ^ 2, p == -1}}, -a ^ 2 (a + b x) ^ p / (2 b ^ 2 + 3 b ^ 2 p + b ^ 2 p ^ 2) + a b p x (a + b x) ^ p / (2 b ^ 2 + 3 b ^ 2 p + b ^ 2 p ^ 2) + b ^ 2 x ^ 2 (a + b x) ^ p / (2 b ^ 2 + 3 b ^ 2 p + b ^ 2 p ^ 2) + b ^ 2 p x ^ 2 (a + b x) ^ p / (2 b ^ 2 + 3 b ^ 2 p + b ^ 2 p ^ 2)]`

Maple [A]

time = 0.02, size = 36, normalized size = 0.92

method	result	size
gospers	$-\frac{(bx+a)^{1+p}(-xpb-bx+a)}{b^2(p^2+3p+2)}$	36
risch	$-\frac{(-x^2b^2p-xapb-x^2b^2+a^2)(bx+a)^p}{b^2(2+p)(1+p)}$	50
norman	$\frac{x^2 e^{p \ln(bx+a)}}{2+p} + \frac{pax e^{p \ln(bx+a)}}{b(p^2+3p+2)} - \frac{a^2 e^{p \ln(bx+a)}}{b^2(p^2+3p+2)}$	73

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x+a)^p,x,method=_RETURNVERBOSE)`

[Out] $-(b*x+a)^{(1+p)}*(-b*p*x-b*x+a)/b^2/(p^2+3*p+2)$

Maxima [A]

time = 0.26, size = 42, normalized size = 1.08

$$\frac{(b^2(p+1)x^2 + abpx - a^2)(bx + a)^p}{(p^2 + 3p + 2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x+a)^p,x, algorithm="maxima")`

[Out] $(b^2*(p + 1)*x^2 + a*b*p*x - a^2)*(b*x + a)^p/((p^2 + 3*p + 2)*b^2)$

Fricas [A]

time = 0.33, size = 53, normalized size = 1.36

$$\frac{(abpx + (b^2p + b^2)x^2 - a^2)(bx + a)^p}{b^2p^2 + 3b^2p + 2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^p,x, algorithm="fricas")

[Out] (a*b*p*x + (b^2*p + b^2)*x^2 - a^2)*(b*x + a)^p/(b^2*p^2 + 3*b^2*p + 2*b^2)

Sympy [A]

time = 0.24, size = 201, normalized size = 5.15

$$\begin{cases} \frac{a^p x^2}{2} & \text{for } b = 0 \\ \frac{a \log\left(\frac{a}{b} + x\right)}{ab^2 + b^3 x} + \frac{a}{ab^2 + b^3 x} + \frac{bx \log\left(\frac{a}{b} + x\right)}{ab^2 + b^3 x} & \text{for } p = -2 \\ -\frac{a \log\left(\frac{a}{b} + x\right)}{b^2} + \frac{x}{b} & \text{for } p = -1 \\ -\frac{a^2(a+bx)^p}{b^2 p^2 + 3b^2 p + 2b^2} + \frac{abpx(a+bx)^p}{b^2 p^2 + 3b^2 p + 2b^2} + \frac{b^2 px^2(a+bx)^p}{b^2 p^2 + 3b^2 p + 2b^2} + \frac{b^2 x^2(a+bx)^p}{b^2 p^2 + 3b^2 p + 2b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)**p,x)

[Out] Piecewise((a**p*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) + a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(p, -2)), (-a*log(a/b + x)/b**2 + x/b, Eq(p, -1)), (-a**2*(a + b*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2) + a*b*p*x*(a + b*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2) + b**2*p*x**2*(a + b*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2) + b**2*x**2*(a + b*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2), True))

Giac [A]

time = 0.00, size = 83, normalized size = 2.13

$$\frac{-a^2 e^{p \ln(a+bx)} + abpx e^{p \ln(a+bx)} + b^2 px^2 e^{p \ln(a+bx)} + b^2 x^2 e^{p \ln(a+bx)}}{b^2 p^2 + 3b^2 p + 2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^p,x)

[Out] ((b*x + a)^p*b^2*p*x^2 + (b*x + a)^p*a*b*p*x + (b*x + a)^p*b^2*x^2 - (b*x + a)^p*a^2)/(b^2*p^2 + 3*b^2*p + 2*b^2)

Mupad [B]

time = 0.39, size = 94, normalized size = 2.41

$$\begin{cases} -\frac{a \ln(a+bx) - bx}{b^2} & \text{if } p = -1 \\ \frac{\ln(a+bx) + \frac{a}{a+bx}}{b^2} & \text{if } p = -2 \\ \frac{2 \left(\frac{(a+bx)^{p+2}}{2p+4} - \frac{a(a+bx)^{p+1}}{2p+2} \right)}{b^2} & \text{if } p \neq -1 \wedge p \neq -2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^p,x)
```

```
[Out] piecewise(p == -1, -(a*log(a + b*x) - b*x)/b^2, p == -2, (log(a + b*x) + a/
(a + b*x))/b^2, p >= -1 & p <= -2, (2*((a + b*x)^(p + 2)/(2*p + 4) - (a*(a
+ b*x)^(p + 1))/(2*p + 2)))/b^2)
```

3.23 $\int x^2(a + bx)^p dx$

Optimal. Leaf size=60

$$\frac{a^2(a + bx)^{1+p}}{b^3(1 + p)} - \frac{2a(a + bx)^{2+p}}{b^3(2 + p)} + \frac{(a + bx)^{3+p}}{b^3(3 + p)}$$

[Out] $a^2*(b*x+a)^{(1+p)}/b^3/(1+p)-2*a*(b*x+a)^{(2+p)}/b^3/(2+p)+(b*x+a)^{(3+p)}/b^3/(3+p)$

Rubi [A]

time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^2(a + bx)^{p+1}}{b^3(p + 1)} - \frac{2a(a + bx)^{p+2}}{b^3(p + 2)} + \frac{(a + bx)^{p+3}}{b^3(p + 3)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*(a + b*x)^p, x]$

[Out] $(a^2*(a + b*x)^{(1 + p)})/(b^3*(1 + p)) - (2*a*(a + b*x)^{(2 + p)})/(b^3*(2 + p)) + (a + b*x)^{(3 + p)}/(b^3*(3 + p))$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_. + (d_.)*(x_.))^{(n_.)}, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) || \text{LtQ}[9*m + 5*(n + 1), 0] || \text{GtQ}[m + n + 2, 0])$

Rubi steps

$$\begin{aligned} \int x^2(a + bx)^p dx &= \int \left(\frac{a^2(a + bx)^p}{b^2} - \frac{2a(a + bx)^{1+p}}{b^2} + \frac{(a + bx)^{2+p}}{b^2} \right) dx \\ &= \frac{a^2(a + bx)^{1+p}}{b^3(1 + p)} - \frac{2a(a + bx)^{2+p}}{b^3(2 + p)} + \frac{(a + bx)^{3+p}}{b^3(3 + p)} \end{aligned}$$

Mathematica [A]

time = 0.04, size = 57, normalized size = 0.95

$$\frac{(a + bx)^{1+p} (2a^2 - 2ab(1 + p)x + b^2(2 + 3p + p^2)x^2)}{b^3(1 + p)(2 + p)(3 + p)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x)^p,x]

[Out] $((a + b*x)^{(1 + p)}*(2*a^2 - 2*a*b*(1 + p)*x + b^2*(2 + 3*p + p^2)*x^2))/(b^3*(1 + p)*(2 + p)*(3 + p))$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 4.46, size = 496, normalized size = 8.27

Piecewise[{{(x^3 a^p / 3, b == 0), {(a^2 (3 + 2 Log[(a + b x) / b]) / (b^3 (a^2 + 2 a b x + b^2 x^2)), p == -3}, {(-2 a^2 (1 + Log[(a + b x) / b]) - 2 a b x Log[(a + b x) / b] + b^2 x^2) / (b^3 (a + b x)), p == -2}, {(a^2 Log[a / b + x] - a b x + b^2 x^2 / 2) / b^3, p == -1}}, 2 a^3 (a + b x)^p / (6 b^3 + 11 b^3 p + 6 b^3 p^2 + b^3 p^3) - 2 a^2 b p x (a + b x)^p / (6 b^3 + 11 b^3 p + 6 b^3 p^2 + b^3 p^3) + a b^2 p x^2 (a + b x)^p / (6 b^3 + 11 b^3 p + 6 b^3 p^2 + b^3 p^3) + a b^2 p^2 x^2 (a + b x)^p / (6 b^3 + 11 b^3 p + 6 b^3 p^2 + b^3 p^3) + 2 b^3 x^3 (a + b x)^p / (6 b^3 + 11 b^3 p + 6 b^3 p^2 + b^3 p^3) + 3 b^3 p x^3 (a + b x)^p / (6 b^3 + 11 b^3 p + 6 b^3 p^2 + b^3 p^3) + b^3 p^2 x^3 (a + b x)^p / (6 b^3 + 11 b^3 p + 6 b^3 p^2 + b^3 p^3)}

Antiderivative was successfully verified.

[In] mathics('Integrate[x^2*(a+b*x)^p,x]')

[Out] Piecewise[{{x^3 a^p / 3, b == 0}, {(a^2 (3 + 2 Log[(a + b x) / b]) / (b^3 (a^2 + 2 a b x + b^2 x^2)), p == -3}, {(-2 a^2 (1 + Log[(a + b x) / b]) - 2 a b x Log[(a + b x) / b] + b^2 x^2) / (b^3 (a + b x)), p == -2}, {(a^2 Log[a / b + x] - a b x + b^2 x^2 / 2) / b^3, p == -1}}, 2 a^3 (a + b x)^p / (6 b^3 + 11 b^3 p + 6 b^3 p^2 + b^3 p^3) - 2 a^2 b p x (a + b x)^p / (6 b^3 + 11 b^3 p + 6 b^3 p^2 + b^3 p^3) + a b^2 p x^2 (a + b x)^p / (6 b^3 + 11 b^3 p + 6 b^3 p^2 + b^3 p^3) + a b^2 p^2 x^2 (a + b x)^p / (6 b^3 + 11 b^3 p + 6 b^3 p^2 + b^3 p^3) + 2 b^3 x^3 (a + b x)^p / (6 b^3 + 11 b^3 p + 6 b^3 p^2 + b^3 p^3) + 3 b^3 p x^3 (a + b x)^p / (6 b^3 + 11 b^3 p + 6 b^3 p^2 + b^3 p^3) + b^3 p^2 x^3 (a + b x)^p / (6 b^3 + 11 b^3 p + 6 b^3 p^2 + b^3 p^3)}

Maple [A]

time = 0.03, size = 73, normalized size = 1.22

method	result	size
gosper	$\frac{(bx+a)^{1+p}(b^2p^2x^2+3x^2b^2p-2xapb+2x^2b^2-2axb+2a^2)}{b^3(p^3+6p^2+11p+6)}$	73
risch	$\frac{(b^3p^2x^3+ab^2p^2x^2+3b^3px^3+x^2apb^2+2x^3b^3-2a^2pxb+2a^3)(bx+a)^p}{(2+p)(3+p)(1+p)b^3}$	88
norman	$\frac{x^3e^{p \ln(bx+a)}}{3+p} + \frac{apx^2e^{p \ln(bx+a)}}{b(p^2+5p+6)} + \frac{2a^3e^{p \ln(bx+a)}}{b^3(p^3+6p^2+11p+6)} - \frac{2pa^2xe^{p \ln(bx+a)}}{b^2(p^3+6p^2+11p+6)}$	114

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x+a)^p,x,method=_RETURNVERBOSE)

[Out] $(b*x+a)^{(1+p)}*(b^2*p^2*x^2+3*b^2*p*x^2-2*a*b*p*x+2*b^2*x^2-2*a*b*x+2*a^2)/b^3/(p^3+6*p^2+11*p+6)$

Maxima [A]

time = 0.26, size = 68, normalized size = 1.13

$$\frac{((p^2 + 3p + 2)b^3x^3 + (p^2 + p)ab^2x^2 - 2a^2bpx + 2a^3)(bx + a)^p}{(p^3 + 6p^2 + 11p + 6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2*(b*x+a)^p,x, algorithm="maxima")`

`[Out] ((p^2 + 3*p + 2)*b^3*x^3 + (p^2 + p)*a*b^2*x^2 - 2*a^2*b*p*x + 2*a^3)*(b*x + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3)`

Fricas [A]

time = 0.33, size = 96, normalized size = 1.60

$$\frac{(2a^2bpx - (b^3p^2 + 3b^3p + 2b^3)x^3 - 2a^3 - (ab^2p^2 + ab^2p)x^2)(bx + a)^p}{b^3p^3 + 6b^3p^2 + 11b^3p + 6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2*(b*x+a)^p,x, algorithm="fricas")`

`[Out] -(2*a^2*b*p*x - (b^3*p^2 + 3*b^3*p + 2*b^3)*x^3 - 2*a^3 - (a*b^2*p^2 + a*b^2*p)*x^2)*(b*x + a)^p/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)`

Sympy [A]

time = 0.40, size = 597, normalized size = 9.95

$$\left\{ \begin{array}{ll} \frac{a^p x^3}{3} & \text{for } b = 0 \\ \frac{2a^2 \log\left(\frac{a}{b} + x\right)}{2a^2 b^3 + 4ab^3 x + 2b^3 x^2} + \frac{3a^2}{2a^2 b^3 + 4ab^3 x + 2b^3 x^2} + \frac{4abx \log\left(\frac{a}{b} + x\right)}{2a^2 b^3 + 4ab^3 x + 2b^3 x^2} + \frac{4abx}{2a^2 b^3 + 4ab^3 x + 2b^3 x^2} + \frac{2b^2 x^2 \log\left(\frac{a}{b} + x\right)}{2a^2 b^3 + 4ab^3 x + 2b^3 x^2} & \text{for } p = -3 \\ \frac{2a^2 \log\left(\frac{a}{b} + x\right)}{ab^3 + b^4 x} - \frac{2a^2}{ab^3 + b^4 x} - \frac{2abx \log\left(\frac{a}{b} + x\right)}{ab^3 + b^4 x} + \frac{b^2 x^2}{ab^3 + b^4 x} & \text{for } p = -2 \\ \frac{a^2 \log\left(\frac{a}{b} + x\right)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b} & \text{for } p = -1 \\ \frac{2a^3(a+bx)^p}{b^3 p^3 + 6b^3 p^2 + 11b^3 p + 6b^3} - \frac{2a^2 b p x (a+bx)^p}{b^3 p^3 + 6b^3 p^2 + 11b^3 p + 6b^3} + \frac{ab^2 p^2 x^2 (a+bx)^p}{b^3 p^3 + 6b^3 p^2 + 11b^3 p + 6b^3} + \frac{ab^2 p x^2 (a+bx)^p}{b^3 p^3 + 6b^3 p^2 + 11b^3 p + 6b^3} + \frac{b^3 p^2 x^3 (a+bx)^p}{b^3 p^3 + 6b^3 p^2 + 11b^3 p + 6b^3} + \frac{3b^3 p x^3 (a+bx)^p}{b^3 p^3 + 6b^3 p^2 + 11b^3 p + 6b^3} + \frac{2b^3 x^3 (a+bx)^p}{b^3 p^3 + 6b^3 p^2 + 11b^3 p + 6b^3} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x**2*(b*x+a)**p,x)`

`[Out] Piecewise((a**p*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(p, -3)), (-2*a**2*log(a/b + x)/(a*b**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3 + b**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(p, -2)), (a**2*log(a/b + x)/b**3 - a*x/b**2 + x**2/(2*b), Eq(p, -1)), (2*a**3*(a + b*x)**p/(b**3*p**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) - 2*a**2*b*p*x*(a + b*x)**p/(b**3*p**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) + a*b**2*p**2*x**2*(a + b*x)**p/(b**3*p**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3)`

```
p**2 + 11*b**3*p + 6*b**3) + a*b**2*p*x**2*(a + b*x)**p/(b**3*p**3 + 6*b**3
*p**2 + 11*b**3*p + 6*b**3) + b**3*p**2*x**3*(a + b*x)**p/(b**3*p**3 + 6*b
**3*p**2 + 11*b**3*p + 6*b**3) + 3*b**3*p*x**3*(a + b*x)**p/(b**3*p**3 + 6*b
**3*p**2 + 11*b**3*p + 6*b**3) + 2*b**3*x**3*(a + b*x)**p/(b**3*p**3 + 6*b
**3*p**2 + 11*b**3*p + 6*b**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(60) = 120.

time = 0.00, size = 154, normalized size = 2.57

$$\frac{2a^3e^{p\ln(a+bx)} - 2a^2bpxe^{p\ln(a+bx)} + ab^2p^2x^2e^{p\ln(a+bx)} + ab^2px^2e^{p\ln(a+bx)} + b^3p^2x^3e^{p\ln(a+bx)} + 3b^3px^3e^{p\ln(a+bx)} + 2b^3x^3e^{p\ln(a+bx)}}{b^3p^3 + 6b^3p^2 + 11b^3p + 6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^p,x)
```

```
[Out] ((b*x + a)^p*b^3*p^2*x^3 + (b*x + a)^p*a*b^2*p^2*x^2 + 3*(b*x + a)^p*b^3*p*
x^3 + (b*x + a)^p*a*b^2*p*x^2 + 2*(b*x + a)^p*b^3*x^3 - 2*(b*x + a)^p*a^2*b
*p*x + 2*(b*x + a)^p*a^3)/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

Mupad [B]

time = 0.60, size = 192, normalized size = 3.20

$$\left\{ \begin{array}{ll} \frac{2a^2 \ln(a+bx) + b^2 x^2 - 2abx}{2b^3} & \text{if } p = -1 \\ \frac{x}{b^2} - \frac{a^2}{b^3(a+bx)} - \frac{2a \ln(a+bx)}{b^3} & \text{if } p = -2 \\ \frac{\ln(a+bx) + \frac{2a}{a+bx} - \frac{a^2}{2(a+bx)^2}}{b^3} & \text{if } p = -3 \\ \frac{2(a+bx)^{p+1} (8a^2 - 8abpx - 8abx + 4b^2p^2x^2 + 12b^2px^2 + 8b^2x^2)}{b^3(8p^3 + 48p^2 + 88p + 48)} & \text{if } p \neq -1 \wedge p \neq -2 \wedge p \neq -3 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^p,x)
```

```
[Out] piecewise(p == -1, (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3), p == -
2, x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*log(a + b*x))/b^3, p == -3, (log(a +
b*x) + (2*a)/(a + b*x) - a^2/(2*(a + b*x)^2))/b^3, p ~= -1 & p ~= -2 & p ~=
-3, (2*(a + b*x)^(p + 1)*(8*a^2 + 8*b^2*x^2 + 12*b^2*p*x^2 - 8*a*b*x + 4*b
^2*p^2*x^2 - 8*a*b*p*x))/(b^3*(88*p + 48*p^2 + 8*p^3 + 48)))
```

3.24 $\int \frac{1}{a+bx} dx$

Optimal. Leaf size=10

$$\frac{\log(a + bx)}{b}$$

[Out] ln(b*x+a)/b

Rubi [A]

time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {31}

$$\frac{\log(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(-1),x]

[Out] Log[a + b*x]/b

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\int \frac{1}{a + bx} dx = \frac{\log(a + bx)}{b}$$

Mathematica [A]

time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(-1),x]

[Out] Log[a + b*x]/b

Mathics [A]

time = 1.61, size = 10, normalized size = 1.00

$$\frac{\text{Log}[a + bx]}{b}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(a+b*x),x]')`

[Out] `Log[a + b x] / b`

Maple [A]

time = 0.02, size = 11, normalized size = 1.10

method	result	size
default	$\frac{\ln(bx+a)}{b}$	11
norman	$\frac{\ln(bx+a)}{b}$	11
risch	$\frac{\ln(bx+a)}{b}$	11

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x+a),x,method=_RETURNVERBOSE)`

[Out] `ln(b*x+a)/b`

Maxima [A]

time = 0.28, size = 10, normalized size = 1.00

$$\frac{\log(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x+a),x, algorithm="maxima")`

[Out] `log(b*x + a)/b`

Fricas [A]

time = 0.33, size = 10, normalized size = 1.00

$$\frac{\log(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x+a),x, algorithm="fricas")`

[Out] `log(b*x + a)/b`

Sympy [A]

time = 0.03, size = 7, normalized size = 0.70

$$\frac{\log(a + bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a),x)

[Out] log(a + b*x)/b

Giac [A]

time = 0.00, size = 10, normalized size = 1.00

$$\frac{\ln |xb + a|}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a),x)

[Out] log(abs(b*x + a))/b

Mupad [B]

time = 0.03, size = 10, normalized size = 1.00

$$\frac{\ln (a + b x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*x),x)

[Out] log(a + b*x)/b

3.25 $\int \frac{1}{(a+bx)^2} dx$

Optimal. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

[Out] -1/b/(b*x+a)

Rubi [A]

time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$,

Rules used = {32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(-2),x]

[Out] -(1/(b*(a + b*x)))

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{b(a+bx)}$$

Mathematica [A]

time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(-2),x]

[Out] -(1/(b*(a + b*x)))

Mathics [A]

time = 1.69, size = 12, normalized size = 1.00

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(a+b*x)^2,x]')`

[Out] $-1 / (b (a + b x))$

Maple [A]

time = 0.02, size = 13, normalized size = 1.08

method	result	size
gospers	$-\frac{1}{b(bx+a)}$	13
default	$-\frac{1}{b(bx+a)}$	13
norman	$\frac{x}{a(bx+a)}$	13
risch	$-\frac{1}{b(bx+a)}$	13

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x+a)^2,x,method=_RETURNVERBOSE)`

[Out] $-1/b/(b*x+a)$

Maxima [A]

time = 0.27, size = 12, normalized size = 1.00

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x+a)^2,x, algorithm="maxima")`

[Out] $-1/((b*x + a)*b)$

Fricas [A]

time = 0.32, size = 13, normalized size = 1.08

$$-\frac{1}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x+a)^2,x, algorithm="fricas")`

[Out] $-1/(b^2*x + a*b)$

Sympy [A]

time = 0.07, size = 10, normalized size = 0.83

$$-\frac{1}{ab + b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x+a)**2,x)`

[Out] `-1/(a*b + b**2*x)`

Giac [A]

time = 0.00, size = 9, normalized size = 0.75

$$-\frac{1}{b(xb + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x+a)^2,x)`

[Out] `-1/((b*x + a)*b)`

Mupad [B]

time = 0.12, size = 12, normalized size = 1.00

$$-\frac{1}{b(a + bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b*x)^2,x)`

[Out] `-1/(b*(a + b*x))`

3.26 $\int \frac{x}{a+bx} dx$

Optimal. Leaf size=18

$$\frac{x}{b} - \frac{a \log(a + bx)}{b^2}$$

[Out] x/b-a*ln(b*x+a)/b^2

Rubi [A]

time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$\frac{x}{b} - \frac{a \log(a + bx)}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x),x]

[Out] x/b - (a*Log[a + b*x])/b^2

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{a+bx} dx &= \int \left(\frac{1}{b} - \frac{a}{b(a+bx)} \right) dx \\ &= \frac{x}{b} - \frac{a \log(a+bx)}{b^2} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 18, normalized size = 1.00

$$\frac{x}{b} - \frac{a \log(a + bx)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*x),x]

[Out] $x/b - (a*\text{Log}[a + b*x])/b^2$

Mathics [A]

time = 1.69, size = 17, normalized size = 0.94

$$\frac{-a\text{Log}[a + bx] + bx}{b^2}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x/(a+b*x),x]')`

[Out] $(-a \text{Log}[a + b x] + b x) / b^2$

Maple [A]

time = 0.02, size = 19, normalized size = 1.06

method	result	size
default	$\frac{x}{b} - \frac{a \ln(bx+a)}{b^2}$	19
norman	$\frac{x}{b} - \frac{a \ln(bx+a)}{b^2}$	19
risch	$\frac{x}{b} - \frac{a \ln(bx+a)}{b^2}$	19

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(b*x+a),x,method=_RETURNVERBOSE)`

[Out] $x/b - a*\ln(b*x+a)/b^2$

Maxima [A]

time = 0.25, size = 18, normalized size = 1.00

$$\frac{x}{b} - \frac{a \log(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x+a),x, algorithm="maxima")`

[Out] $x/b - a*\log(b*x + a)/b^2$

Fricas [A]

time = 0.33, size = 17, normalized size = 0.94

$$\frac{bx - a \log(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x+a),x, algorithm="fricas")`

[Out] $(b*x - a*\log(b*x + a))/b^2$

Sympy [A]

time = 0.05, size = 14, normalized size = 0.78

$$-\frac{a \log(a + bx)}{b^2} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x+a),x)`

[Out] $-a*\log(a + b*x)/b**2 + x/b$

Giac [A]

time = 0.00, size = 19, normalized size = 1.06

$$\frac{x}{b} - \frac{a \ln |xb + a|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x+a),x)`

[Out] $x/b - a*\log(\text{abs}(b*x + a))/b^2$

Mupad [B]

time = 0.03, size = 18, normalized size = 1.00

$$-\frac{a \ln(a + bx) - bx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a + b*x),x)`

[Out] $-(a*\log(a + b*x) - b*x)/b^2$

3.27 $\int \frac{x^2}{a+bx} dx$

Optimal. Leaf size=31

$$-\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \log(a+bx)}{b^3}$$

[Out] $-a*x/b^2+1/2*x^2/b+a^2*\ln(b*x+a)/b^3$

Rubi [A]

time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^2 \log(a+bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b*x), x]

[Out] $-((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3$

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int [ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{x^2}{a+bx} dx &= \int \left(-\frac{a}{b^2} + \frac{x}{b} + \frac{a^2}{b^2(a+bx)} \right) dx \\ &= -\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \log(a+bx)}{b^3} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 31, normalized size = 1.00

$$-\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b*x), x]

[Out] $-\frac{(a*x)}{b^2} + \frac{x^2}{2*b} + \frac{(a^2*\text{Log}[a + b*x])}{b^3}$

Mathics [A]

time = 1.80, size = 28, normalized size = 0.90

$$\frac{a^2 \text{Log}[a + bx] - abx + \frac{b^2 x^2}{2}}{b^3}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x^2/(a+b*x),x]')`

[Out] $(a^2 \text{Log}[a + b x] - a b x + \frac{b^2 x^2}{2}) / b^3$

Maple [A]

time = 0.02, size = 30, normalized size = 0.97

method	result	size
default	$-\frac{\frac{1}{2}x^2b+ax}{b^2} + \frac{a^2 \ln(bx+a)}{b^3}$	30
norman	$-\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \ln(bx+a)}{b^3}$	30
risch	$-\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \ln(bx+a)}{b^3}$	30

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x+a),x,method=_RETURNVERBOSE)`

[Out] $-1/b^2*(-1/2*x^2*b+a*x)+a^2*\ln(b*x+a)/b^3$

Maxima [A]

time = 0.28, size = 29, normalized size = 0.94

$$\frac{a^2 \log(bx + a)}{b^3} + \frac{bx^2 - 2ax}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x+a),x, algorithm="maxima")`

[Out] $a^2*\log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2$

Fricas [A]

time = 0.35, size = 29, normalized size = 0.94

$$\frac{b^2 x^2 - 2 abx + 2 a^2 \log(bx + a)}{2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x+a),x, algorithm="fricas")`

[Out] $1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*\log(b*x + a))/b^3$

Sympy [A]

time = 0.06, size = 26, normalized size = 0.84

$$\frac{a^2 \log(a + bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x+a),x)`

[Out] $a**2*\log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)$

Giac [A]

time = 0.00, size = 34, normalized size = 1.10

$$\frac{\frac{1}{2}x^2b - xa}{b^2} + \frac{a^2 \ln|xb + a|}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x+a),x)`

[Out] $a^2*\log(\text{abs}(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2$

Mupad [B]

time = 0.13, size = 29, normalized size = 0.94

$$\frac{2a^2 \ln(a + bx) + b^2 x^2 - 2abx}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a + b*x),x)`

[Out] $(2*a^2*\log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3)$

3.28 $\int \frac{1}{x(a+bx)} dx$

Optimal. Leaf size=18

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

[Out] ln(x)/a-ln(b*x+a)/a

Rubi [A]

time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x)),x]

[Out] Log[x]/a - Log[a + b*x]/a

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{x(a+bx)} dx &= \frac{\int \frac{1}{x} dx}{a} - \frac{b \int \frac{1}{a+bx} dx}{a} \\ &= \frac{\log(x)}{a} - \frac{\log(a+bx)}{a} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 18, normalized size = 1.00

$$\frac{\log(x)}{a} - \frac{\log(a + bx)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x)),x]

[Out] Log[x]/a - Log[a + b*x]/a

Mathics [A]

time = 1.73, size = 17, normalized size = 0.94

$$\frac{\text{Log}[x] - \text{Log}\left[\frac{a}{b} + x\right]}{a}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x*(a+b*x)),x]')

[Out] (Log[x] - Log[a / b + x]) / a

Maple [A]

time = 0.02, size = 19, normalized size = 1.06

method	result	size
default	$\frac{\ln(x)}{a} - \frac{\ln(bx+a)}{a}$	19
norman	$\frac{\ln(x)}{a} - \frac{\ln(bx+a)}{a}$	19
risch	$\frac{\ln(-x)}{a} - \frac{\ln(bx+a)}{a}$	21

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x+a),x,method=_RETURNVERBOSE)

[Out] ln(x)/a-ln(b*x+a)/a

Maxima [A]

time = 0.25, size = 18, normalized size = 1.00

$$-\frac{\log(bx + a)}{a} + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x+a),x, algorithm="maxima")

[Out] -log(b*x + a)/a + log(x)/a

Fricas [A]

time = 0.32, size = 16, normalized size = 0.89

$$-\frac{\log(bx + a) - \log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/x/(b*x+a),x, algorithm="fricas")``[Out] -(log(b*x + a) - log(x))/a`**Sympy [A]**

time = 0.07, size = 10, normalized size = 0.56

$$\frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/x/(b*x+a),x)``[Out] (log(x) - log(a/b + x))/a`**Giac [A]**

time = 0.00, size = 21, normalized size = 1.17

$$\frac{\ln|x|}{a} - \frac{b \ln|xb + a|}{ba}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/x/(b*x+a),x)``[Out] -log(abs(b*x + a))/a + log(abs(x))/a`**Mupad [B]**

time = 0.13, size = 15, normalized size = 0.83

$$-\frac{2 \operatorname{atanh}\left(\frac{2bx}{a} + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(x*(a + b*x)),x)``[Out] -(2*atanh((2*b*x)/a + 1))/a`

3.29 $\int \frac{1}{x^2(a+bx)} dx$

Optimal. Leaf size=28

$$-\frac{1}{ax} - \frac{b \log(x)}{a^2} + \frac{b \log(a+bx)}{a^2}$$

[Out] $-1/a/x - b*\ln(x)/a^2 + b*\ln(b*x+a)/a^2$

Rubi [A]

time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{b \log(x)}{a^2} + \frac{b \log(a+bx)}{a^2} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^2*(a + b*x)),x]`

[Out] $-(1/(a*x)) - (b*\text{Log}[x])/a^2 + (b*\text{Log}[a + b*x])/a^2$

Rule 46

`Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])`

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2(a+bx)} dx &= \int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)} \right) dx \\ &= -\frac{1}{ax} - \frac{b \log(x)}{a^2} + \frac{b \log(a+bx)}{a^2} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 28, normalized size = 1.00

$$-\frac{1}{ax} - \frac{b \log(x)}{a^2} + \frac{b \log(a+bx)}{a^2}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^2*(a + b*x)),x]`

[Out] $-(1/(a*x)) - (b*\text{Log}[x])/a^2 + (b*\text{Log}[a + b*x])/a^2$

Mathics [A]

time = 1.84, size = 30, normalized size = 1.07

$$\frac{-a - bx (\text{Log}[x] - \text{Log}[\frac{a+bx}{b}])}{a^2x}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(x^2*(a+b*x)),x]')`

[Out] $(-a - b x (\text{Log}[x] - \text{Log}[(a + b x) / b])) / (a^2 x)$

Maple [A]

time = 0.02, size = 29, normalized size = 1.04

method	result	size
default	$-\frac{1}{ax} - \frac{b \ln(x)}{a^2} + \frac{b \ln(bx+a)}{a^2}$	29
norman	$-\frac{1}{ax} - \frac{b \ln(x)}{a^2} + \frac{b \ln(bx+a)}{a^2}$	29
risch	$-\frac{1}{ax} + \frac{b \ln(-bx-a)}{a^2} - \frac{b \ln(x)}{a^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(b*x+a),x,method=_RETURNVERBOSE)`

[Out] $-1/a/x - b*\ln(x)/a^2 + b*\ln(b*x+a)/a^2$

Maxima [A]

time = 0.27, size = 28, normalized size = 1.00

$$\frac{b \log (bx + a)}{a^2} - \frac{b \log (x)}{a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x+a),x, algorithm="maxima")`

[Out] $b*\log(b*x + a)/a^2 - b*\log(x)/a^2 - 1/(a*x)$

Fricas [A]

time = 0.33, size = 26, normalized size = 0.93

$$\frac{bx \log (bx + a) - bx \log (x) - a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x+a),x, algorithm="fricas")`

[Out] $(b*x*\log(b*x + a) - b*x*\log(x) - a)/(a^2*x)$

Sympy [A]

time = 0.10, size = 19, normalized size = 0.68

$$-\frac{1}{ax} + \frac{b(-\log(x) + \log(\frac{a}{b} + x))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(b*x+a),x)`

[Out] $-1/(a*x) + b*(-\log(x) + \log(a/b + x))/a**2$

Giac [A]

time = 0.00, size = 37, normalized size = 1.32

$$\frac{b^2 \ln |xb + a|}{ba^2} - \frac{b \ln |x|}{a^2} - \frac{a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x+a),x)`

[Out] $b*\log(\text{abs}(b*x + a))/a^2 - b*\log(\text{abs}(x))/a^2 - 1/(a*x)$

Mupad [B]

time = 0.05, size = 25, normalized size = 0.89

$$\frac{2b \operatorname{atanh}\left(\frac{2bx}{a} + 1\right)}{a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(a + b*x)),x)`

[Out] $(2*b*\operatorname{atanh}((2*b*x)/a + 1))/a^2 - 1/(a*x)$

$$3.30 \quad \int \frac{1}{x^2(a+bx)^2} dx$$

Optimal. Leaf size=42

$$-\frac{1}{a^2x} - \frac{b}{a^2(a+bx)} - \frac{2b \log(x)}{a^3} + \frac{2b \log(a+bx)}{a^3}$$

[Out] $-1/a^2/x - b/a^2/(b*x+a) - 2*b*\ln(x)/a^3 + 2*b*\ln(b*x+a)/a^3$

Rubi [A]

time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{2b \log(x)}{a^3} + \frac{2b \log(a+bx)}{a^3} - \frac{b}{a^2(a+bx)} - \frac{1}{a^2x}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*x)^2), x]

[Out] $-(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^3$

Rule 46

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2(a+bx)^2} dx &= \int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)} \right) dx \\ &= -\frac{1}{a^2x} - \frac{b}{a^2(a+bx)} - \frac{2b \log(x)}{a^3} + \frac{2b \log(a+bx)}{a^3} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 35, normalized size = 0.83

$$-\frac{a \left(\frac{1}{x} + \frac{b}{a+bx} \right) + 2b \log(x) - 2b \log(a+bx)}{a^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a + b*x)^2),x]

[Out] $-\left(\frac{a}{x} + \frac{b}{a + bx}\right) + \frac{2b \log[x] - 2b \log[a + bx]}{a^3}$

Mathics [A]

time = 2.10, size = 49, normalized size = 1.17

$$\frac{a(-a - 2bx) - 2bx(a + bx)(\log[x] - \log\left[\frac{a+bx}{b}\right])}{a^3x(a + bx)}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x*(a+b*x))^2,x]')

[Out] $(a(-a - 2bx) - 2bx(a + bx)(\log[x] - \log[(a + bx)/b])) / (a^3x(a + bx))$

Maple [A]

time = 0.03, size = 43, normalized size = 1.02

method	result	size
default	$-\frac{1}{a^2x} - \frac{b}{a^2(bx+a)} - \frac{2b \ln(x)}{a^3} + \frac{2b \ln(bx+a)}{a^3}$	43
risch	$\frac{-\frac{2bx}{a^2} - \frac{1}{a}}{x(bx+a)} - \frac{2b \ln(x)}{a^3} + \frac{2b \ln(-bx-a)}{a^3}$	49
norman	$\frac{\frac{2b^2x^2}{a^3} - \frac{1}{a}}{x(bx+a)} - \frac{2b \ln(x)}{a^3} + \frac{2b \ln(bx+a)}{a^3}$	50

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] $-1/a^2/x - b/a^2/(b*x+a) - 2*b*\ln(x)/a^3 + 2*b*\ln(b*x+a)/a^3$

Maxima [A]

time = 0.28, size = 45, normalized size = 1.07

$$-\frac{2bx + a}{a^2bx^2 + a^3x} + \frac{2b \log(bx + a)}{a^3} - \frac{2b \log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="maxima")

[Out] $-(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*\log(b*x + a)/a^3 - 2*b*\log(x)/a^3$

Fricas [A]

time = 0.41, size = 63, normalized size = 1.50

$$-\frac{2abx + a^2 - 2(b^2x^2 + abx) \log(bx + a) + 2(b^2x^2 + abx) \log(x)}{a^3bx^2 + a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="fricas")

[Out] $-(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*\log(b*x + a) + 2*(b^2*x^2 + a*b*x)*\log(x))/(a^3*b*x^2 + a^4*x)$

Sympy [A]

time = 0.15, size = 37, normalized size = 0.88

$$\frac{-a - 2bx}{a^3x + a^2bx^2} + \frac{2b(-\log(x) + \log(\frac{a}{b} + x))}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x+a)**2,x)

[Out] $(-a - 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-\log(x) + \log(a/b + x))/a**3$

Giac [A]

time = 0.00, size = 52, normalized size = 1.24

$$-\frac{2b \ln|x|}{a^3} + \frac{2b^2 \ln|xb + a|}{ba^3} - \frac{2xb + a}{a^2(x^2b + xa)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x+a)^2,x)

[Out] $2*b*\log(\text{abs}(b*x + a))/a^3 - 2*b*\log(\text{abs}(x))/a^3 - (2*b*x + a)/((b*x^2 + a*x)*a^2)$

Mupad [B]

time = 0.19, size = 45, normalized size = 1.07

$$\frac{2b \ln\left(\frac{a+bx}{x}\right)}{a^3} - \frac{1}{ax(a+bx)} - \frac{2b}{a^2(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + b*x)^2),x)

[Out] $(2*b*\log((a + b*x)/x))/a^3 - 1/(a*x*(a + b*x)) - (2*b)/(a^2*(a + b*x))$

3.31

$$\int \frac{1}{c^2+x^2} dx$$

Optimal. Leaf size=10

$$\frac{\tan^{-1}\left(\frac{x}{c}\right)}{c}$$

[Out] arctan(x/c)/c

Rubi [A]

time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {209}

$$\frac{\tan^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(c^2 + x^2)^(-1),x]

[Out] ArcTan[x/c]/c

Rule 209

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\int \frac{1}{c^2+x^2} dx = \frac{\tan^{-1}\left(\frac{x}{c}\right)}{c}$$

Mathematica [A]

time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(c^2 + x^2)^(-1),x]

[Out] ArcTan[x/c]/c

Mathics [C] Result contains complex when optimal does not.

time = 1.97, size = 20, normalized size = 2.00

$$\frac{\frac{1}{2} (\text{Log}[Ic + x] - \text{Log}[-Ic + x])}{c}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(c^2+x^2),x]')`

[Out] `I / 2 (Log[I c + x] - Log[-I c + x]) / c`

Maple [A]

time = 0.03, size = 11, normalized size = 1.10

method	result	size
default	$\frac{\arctan\left(\frac{x}{c}\right)}{c}$	11
risch	$\frac{\arctan\left(\frac{x}{c}\right)}{c}$	11

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c^2+x^2),x,method=_RETURNVERBOSE)`

[Out] `arctan(x/c)/c`

Maxima [A]

time = 0.35, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c^2+x^2),x, algorithm="maxima")`

[Out] `arctan(x/c)/c`

Fricas [A]

time = 0.32, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c^2+x^2),x, algorithm="fricas")`

[Out] `arctan(x/c)/c`

Sympy [C] Result contains complex when optimal does not.

time = 0.06, size = 20, normalized size = 2.00

$$\frac{-\frac{i \log(-ic+x)}{2} + \frac{i \log(ic+x)}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c**2+x**2),x)

[Out] (-I*log(-I*c + x)/2 + I*log(I*c + x)/2)/c

Giac [A]

time = 0.00, size = 11, normalized size = 1.10

$$\frac{2 \arctan\left(\frac{x}{c}\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c^2+x^2),x)

[Out] arctan(x/c)/c

Mupad [B]

time = 0.04, size = 10, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c^2 + x^2),x)

[Out] atan(x/c)/c

3.32

$$\int \frac{1}{c^2 - x^2} dx$$

Optimal. Leaf size=10

$$\frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

[Out] arctanh(x/c)/c

Rubi [A]

time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {212}

$$\frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(c^2 - x^2)^(-1),x]

[Out] ArcTanh[x/c]/c

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{1}{c^2 - x^2} dx = \frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

Mathematica [A]

time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(c^2 - x^2)^(-1),x]

[Out] ArcTanh[x/c]/c

Mathics [A]

time = 1.73, size = 18, normalized size = 1.80

$$\frac{\text{Log}[c + x] - \text{Log}[-c + x]}{2c}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(c^2-x^2),x]')`

[Out] `(Log[c + x] - Log[-c + x]) / (2 c)`

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

time = 0.02, size = 22, normalized size = 2.20

method	result	size
default	$\frac{\ln(c+x)}{2c} - \frac{\ln(c-x)}{2c}$	22
norman	$\frac{\ln(c+x)}{2c} - \frac{\ln(c-x)}{2c}$	22
risch	$-\frac{\ln(-c+x)}{2c} + \frac{\ln(c+x)}{2c}$	22

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c^2-x^2),x,method=_RETURNVERBOSE)`

[Out] `1/2/c*ln(c+x)-1/2/c*ln(c-x)`

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

time = 0.34, size = 21, normalized size = 2.10

$$\frac{\log(c + x)}{2c} - \frac{\log(-c + x)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c^2-x^2),x, algorithm="maxima")`

[Out] `1/2*log(c + x)/c - 1/2*log(-c + x)/c`

Fricas [A]

time = 0.33, size = 18, normalized size = 1.80

$$\frac{\log(c + x) - \log(-c + x)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c^2-x^2),x, algorithm="fricas")`

[Out] $1/2*(\log(c + x) - \log(-c + x))/c$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(5) = 10$

time = 0.06, size = 15, normalized size = 1.50

$$-\frac{\frac{\log(-c+x)}{2} - \frac{\log(c+x)}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c**2-x**2),x)`

[Out] $-(\log(-c + x)/2 - \log(c + x)/2)/c$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.
time = 0.00, size = 23, normalized size = 2.30

$$-\frac{\ln|x-c|}{2c} + \frac{\ln|x+c|}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c^2-x^2),x)`

[Out] $1/2*\log(\text{abs}(c + x))/c - 1/2*\log(\text{abs}(-c + x))/c$

Mupad [B]

time = 0.16, size = 10, normalized size = 1.00

$$\frac{\text{atanh}\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c^2 - x^2),x)`

[Out] $\text{atanh}(x/c)/c$

3.33 $\int \frac{1}{-1+2x^3} dx$

Optimal. Leaf size=78

$$-\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{2}x}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}} - \frac{\log\left(1+\sqrt[3]{2}x+2^{2/3}x^2\right)}{6\sqrt[3]{2}}$$

[Out] $1/6*\ln(1-2^{(1/3)*x})*2^{(2/3)}-1/12*\ln(1+2^{(1/3)*x}+2^{(2/3)*x^2})*2^{(2/3)}-1/6*\arctan(1/3*(1+2*2^{(1/3)*x})*3^{(1/2)})*2^{(2/3)}*3^{(1/2)}$

Rubi [A]

time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {206, 31, 648, 631, 210, 642}

$$-\frac{\log\left(2^{2/3}x^2 + \sqrt[3]{2}x + 1\right)}{6\sqrt[3]{2}} + \frac{\log\left(1 - \sqrt[3]{2}x\right)}{3\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{2}x+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + 2*x^3)^(-1), x]

[Out] $-(\text{ArcTan}[(1 + 2*2^{(1/3)*x})/\text{Sqrt}[3]]/(2^{(1/3)}*\text{Sqrt}[3])) + \text{Log}[1 - 2^{(1/3)*x}]/(3*2^{(1/3)}) - \text{Log}[1 + 2^{(1/3)*x} + 2^{(2/3)*x^2}]/(6*2^{(1/3)})$

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{-1+2x^3} dx &= \frac{1}{3} \int \frac{1}{-1+\sqrt[3]{2}x} dx + \frac{1}{3} \int \frac{-2-\sqrt[3]{2}x}{1+\sqrt[3]{2}x+2^{2/3}x^2} dx \\
 &= \frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}} - \frac{1}{2} \int \frac{1}{1+\sqrt[3]{2}x+2^{2/3}x^2} dx - \frac{\int \frac{\sqrt[3]{2}+2^{2/3}x}{1+\sqrt[3]{2}x+2^{2/3}x^2} dx}{6\sqrt[3]{2}} \\
 &= \frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}} - \frac{\log\left(1+\sqrt[3]{2}x+2^{2/3}x^2\right)}{6\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2\sqrt[3]{2}x\right)}{\sqrt[3]{2}} \\
 &= -\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{2}x}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}} - \frac{\log\left(1+\sqrt[3]{2}x+2^{2/3}x^2\right)}{6\sqrt[3]{2}}
 \end{aligned}$$

Mathematica [A]

time = 0.01, size = 66, normalized size = 0.85

$$\frac{2\sqrt{3} \tan^{-1}\left(\frac{1+2\sqrt[3]{2}x}{\sqrt{3}}\right) - 2 \log\left(1-\sqrt[3]{2}x\right) + \log\left(1+\sqrt[3]{2}x+2^{2/3}x^2\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x^3)^(-1), x]
```

[Out] $-1/6*(2*\text{Sqrt}[3]*\text{ArcTan}[(1 + 2*2^{(1/3)}*x)/\text{Sqrt}[3]] - 2*\text{Log}[1 - 2^{(1/3)}*x] + \text{Log}[1 + 2^{(1/3)}*x + 2^{(2/3)}*x^2])/2^{(1/3)}$

Mathics [A]

time = 2.20, size = 53, normalized size = 0.68

$$\frac{2^{\frac{2}{3}} \left(-2\sqrt{3} \text{ArcTan} \left[\frac{\sqrt{3} (1+2^{\frac{1}{3}}x)}{3} \right] - \text{Log} \left[\frac{2^{\frac{1}{3}}}{2} + \frac{2^{\frac{2}{3}}x}{2} + x^2 \right] + 2\text{Log} \left[-\frac{2^{\frac{2}{3}}}{2} + x \right] \right)}{12}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(2*x^3-1),x]')`

[Out] $2^{(2/3)} (-2 \text{Sqrt}[3] \text{ArcTan}[\text{Sqrt}[3] (1 + 2^{(1/3)}x) / 3] - \text{Log}[2^{(1/3)} / 2 + 2^{(2/3)}x / 2 + x^2] + 2 \text{Log}[-2^{(2/3)} / 2 + x]) / 12$

Maple [A]

time = 0.03, size = 58, normalized size = 0.74

method	result	size
risch	$\frac{\sum_{-R=\text{RootOf}(2Z^3-1)} \frac{\ln(-R+x)}{-R^2}}{6}$	24
default	$\frac{2^{\frac{2}{3}} \ln\left(x - \frac{2^{\frac{2}{3}}}{2}\right)}{6} - \frac{2^{\frac{2}{3}} \ln\left(x^2 + \frac{2^{\frac{2}{3}}x}{2} + \frac{2^{\frac{1}{3}}}{2}\right)}{12} - \frac{\arctan\left(\frac{(1+2^{\frac{1}{3}}x)\sqrt{3}}{3}\right)}{6} 2^{\frac{2}{3}}\sqrt{3}$	58
meijerg	$\frac{2^{\frac{2}{3}}x \left(\ln\left(1 - 2^{\frac{1}{3}}(x^3)^{\frac{1}{3}}\right) - \frac{\ln\left(1 + 2^{\frac{1}{3}}(x^3)^{\frac{1}{3}} + 2^{\frac{2}{3}}(x^3)^{\frac{2}{3}}\right)}{2} - \sqrt{3} \arctan\left(\frac{\sqrt{3} 2^{\frac{1}{3}}(x^3)^{\frac{1}{3}}}{2 + 2^{\frac{1}{3}}(x^3)^{\frac{1}{3}}}\right) \right)}{6(x^3)^{\frac{1}{3}}}$	82

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*x^3-1),x,method=_RETURNVERBOSE)`

[Out] $1/6*2^{(2/3)}*\ln(x-1/2*2^{(2/3)})-1/12*2^{(2/3)}*\ln(x^2+1/2*2^{(2/3)}*x+1/2*2^{(1/3)})-1/6*\arctan(1/3*(1+2*2^{(1/3)}*x)*3^{(1/2)})*2^{(2/3)}*3^{(1/2)}$

Maxima [A]

time = 0.37, size = 66, normalized size = 0.85

$$-\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}(2\cdot 2^{\frac{2}{3}}x+2^{\frac{1}{3}})\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}x^2+2^{\frac{1}{3}}x+1\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\frac{1}{2}\cdot 2^{\frac{2}{3}}(2^{\frac{1}{3}}x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^3-1),x, algorithm="maxima")

[Out] $-1/6*\sqrt{3}*2^{2/3}*\arctan(1/6*\sqrt{3}*2^{2/3}*(2*2^{2/3}*x + 2^{1/3})) - 1/12*2^{2/3}*\log(2^{2/3}*x^2 + 2^{1/3}*x + 1) + 1/6*2^{2/3}*\log(1/2*2^{2/3}*(2^{1/3}*x - 1))$

Fricas [A]

time = 0.33, size = 63, normalized size = 0.81

$-\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\left(2\cdot 2^{\frac{2}{3}}x + 2^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2x^2 + 2^{\frac{2}{3}}x + 2^{\frac{1}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(2x - 2^{\frac{2}{3}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^3-1),x, algorithm="fricas")

[Out] $-1/6*\sqrt{6}*2^{1/6}*\arctan(1/6*\sqrt{6}*2^{1/6}*(2*2^{2/3}*x + 2^{1/3})) - 1/12*2^{2/3}*\log(2*x^2 + 2^{2/3}*x + 2^{1/3}) + 1/6*2^{2/3}*\log(2*x - 2^{2/3})$

Sympy [A]

time = 0.16, size = 78, normalized size = 1.00

$\frac{2^{\frac{2}{3}}\log\left(x - \frac{2^{\frac{2}{3}}}{2}\right)}{6} - \frac{2^{\frac{2}{3}}\log\left(x^2 + \frac{2^{\frac{2}{3}}x}{2} + \frac{\sqrt[3]{2}}{2}\right)}{12} - \frac{2^{\frac{2}{3}}\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt[3]{2}\sqrt{3}x + \sqrt{3}}{3}\right)}{6}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x**3-1),x)

[Out] $2^{2/3}*\log(x - 2^{2/3}/2)/6 - 2^{2/3}*\log(x^2 + 2^{2/3}*x/2 + 2^{1/3})/12 - 2^{2/3}*\sqrt{3}*\operatorname{atan}(2*2^{1/3}*\sqrt{3}*x/3 + \sqrt{3}/3)/6$

Giac [A]

time = 0.00, size = 107, normalized size = 1.37

$-\frac{1}{12}\cdot 4^{\frac{1}{3}}\ln\left(x^2 + \left(\frac{1}{2}\right)^{\frac{1}{3}}x + \left(\frac{1}{2}\right)^{\frac{1}{3}}\left(\frac{1}{2}\right)^{\frac{1}{3}}\right) - \frac{1}{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{x + \left(\frac{1}{2}\right)^{\frac{1}{3}}}{\frac{1}{2}\sqrt{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}}\right) + \frac{1}{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}\ln\left|x - \left(\frac{1}{2}\right)^{\frac{1}{3}}\right|$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^3-1),x)

[Out] $-1/3*\sqrt{3}*(1/2)^{1/3}*\arctan(2/3*\sqrt{3}*(1/2)^{2/3}*(2*x + (1/2)^{1/3})) - 1/12*4^{1/3}*\log(x^2 + (1/2)^{1/3}*x + (1/2)^{2/3}) + 1/3*(1/2)^{1/3}*\log(\operatorname{abs}(x - (1/2)^{1/3}))$

Mupad [B]

time = 0.28, size = 72, normalized size = 0.92

$\frac{2^{2/3}\ln\left(x - \frac{2^{2/3}}{2}\right)}{6} + \frac{2^{2/3}\ln\left(x - \frac{2^{2/3}(-1+\sqrt{3}\operatorname{li})}{4}\right)(-1+\sqrt{3}\operatorname{li})}{12} - \frac{2^{2/3}\ln\left(x + \frac{2^{2/3}(1+\sqrt{3}\operatorname{li})}{4}\right)(1+\sqrt{3}\operatorname{li})}{12}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*x^3 - 1),x)`

[Out] $(2^{2/3} \log(x - 2^{2/3}/2))/6 + (2^{2/3} \log(x - (2^{2/3} \cdot (3^{1/2} \cdot 1i - 1))/4) \cdot (3^{1/2} \cdot 1i - 1))/12 - (2^{2/3} \log(x + (2^{2/3} \cdot (3^{1/2} \cdot 1i + 1))/4) \cdot (3^{1/2} \cdot 1i + 1))/12$

3.34 $\int \frac{1}{-2+x^3} dx$

Optimal. Leaf size=74

$$-\frac{\tan^{-1}\left(\frac{1+2^{2/3}x}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(\sqrt[3]{2}-x\right)}{3 \cdot 2^{2/3}} - \frac{\log\left(2^{2/3} + \sqrt[3]{2}x + x^2\right)}{6 \cdot 2^{2/3}}$$

[Out] 1/6*ln(2^(1/3)-x)*2^(1/3)-1/12*ln(2^(2/3)+2^(1/3)*x+x^2)*2^(1/3)-1/6*arctan(1/3*(1+2^(2/3)*x)*3^(1/2))*2^(1/3)*3^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {206, 31, 648, 631, 210, 642}

$$-\frac{\log\left(x^2 + \sqrt[3]{2}x + 2^{2/3}\right)}{6 \cdot 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-x\right)}{3 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}x+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-2 + x^3)^(-1), x]

[Out] -(ArcTan[(1 + 2^(2/3)*x)/Sqrt[3]]/(2^(2/3)*Sqrt[3])) + Log[2^(1/3) - x]/(3*2^(2/3)) - Log[2^(2/3) + 2^(1/3)*x + x^2]/(6*2^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(n_+1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{-2 + x^3} dx &= \int \frac{1}{-\sqrt[3]{2} + x} dx + \int \frac{-2\sqrt[3]{2} - x}{2^{2/3} + \sqrt[3]{2} x + x^2} dx \\ &= \frac{\log(\sqrt[3]{2} - x)}{3 \cdot 2^{2/3}} - \frac{\int \frac{\sqrt[3]{2} + 2x}{2^{2/3} + \sqrt[3]{2} x + x^2} dx}{6 \cdot 2^{2/3}} - \frac{\int \frac{1}{2^{2/3} + \sqrt[3]{2} x + x^2} dx}{2\sqrt[3]{2}} \\ &= \frac{\log(\sqrt[3]{2} - x)}{3 \cdot 2^{2/3}} - \frac{\log(2^{2/3} + \sqrt[3]{2} x + x^2)}{6 \cdot 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3 - x^2} dx, x, 1 + 2^{2/3}x\right)}{2^{2/3}} \\ &= -\frac{\tan^{-1}\left(\frac{1+2^{2/3}x}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(\sqrt[3]{2} - x)}{3 \cdot 2^{2/3}} - \frac{\log(2^{2/3} + \sqrt[3]{2} x + x^2)}{6 \cdot 2^{2/3}} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 65, normalized size = 0.88

$$-\frac{2\sqrt{3} \tan^{-1}\left(\frac{1+2^{2/3}x}{\sqrt{3}}\right) - 2 \log(2 - 2^{2/3}x) + \log(2 + 2^{2/3}x + \sqrt[3]{2} x^2)}{6 \cdot 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^3)^(-1), x]
```

[Out] $-1/6*(2*\text{Sqrt}[3]*\text{ArcTan}[(1 + 2^{(2/3)}*x)/\text{Sqrt}[3]] - 2*\text{Log}[2 - 2^{(2/3)}*x] + \text{Log}[2 + 2^{(2/3)}*x + 2^{(1/3)}*x^2])/2^{(2/3)}$

Mathics [A]

time = 2.14, size = 49, normalized size = 0.66

$$\frac{2^{\frac{1}{3}} \left(-2\sqrt{3} \text{ArcTan} \left[\frac{\sqrt{3} (1+2^{\frac{2}{3}}x)}{3} \right] - \text{Log} \left[2^{\frac{2}{3}} + 2^{\frac{1}{3}}x + x^2 \right] + 2\text{Log} \left[-2^{\frac{1}{3}} + x \right] \right)}{12}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(x^3-2),x]')`

[Out] $2^{(1/3)}(-2\text{Sqrt}[3]\text{ArcTan}[\text{Sqrt}[3](1+2^{(2/3)}x)/3] - \text{Log}[2^{(2/3)} + 2^{(1/3)}x + x^2] + 2\text{Log}[-2^{(1/3)} + x])/12$

Maple [A]

time = 0.03, size = 54, normalized size = 0.73

method	result	size
risch	$\frac{\sum_{-R=\text{RootOf}(-Z^3-2)} \frac{\ln(-R+x)}{-R^2}}{3}$	22
default	$\frac{2^{\frac{1}{3}} \ln(x-2^{\frac{1}{3}})}{6} - \frac{\ln(2^{\frac{2}{3}}+2^{\frac{1}{3}}x+x^2)2^{\frac{1}{3}}}{12} - \frac{\arctan\left(\frac{(1+2^{\frac{2}{3}}x)\sqrt{3}}{3}\right)2^{\frac{1}{3}}\sqrt{3}}{6}$	54
meijerg	$\frac{2^{\frac{1}{3}}x \left(\ln\left(1 - \frac{2^{\frac{2}{3}}(x^3)^{\frac{1}{3}}}{2}\right) - \frac{\ln\left(1 + \frac{2^{\frac{2}{3}}(x^3)^{\frac{1}{3}}}{2} + \frac{2^{\frac{1}{3}}(x^3)^{\frac{2}{3}}}{2}\right)}{2} - \sqrt{3} \arctan\left(\frac{\sqrt{3} 2^{\frac{2}{3}}(x^3)^{\frac{1}{3}}}{4+2^{\frac{2}{3}}(x^3)^{\frac{1}{3}}}\right) \right)}{6(x^3)^{\frac{1}{3}}}$	84

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3-2),x,method=_RETURNVERBOSE)`

[Out] $1/6*2^{(1/3)}*\ln(x-2^{(1/3)})-1/12*\ln(2^{(2/3)}+2^{(1/3)}*x+x^2)*2^{(1/3)}-1/6*\arctan(1/3*(1+2^{(2/3)}*x)*3^{(1/2)})*2^{(1/3)}*3^{(1/2)}$

Maxima [A]

time = 0.36, size = 56, normalized size = 0.76

$$-\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2x+2^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(x^2+2^{\frac{1}{3}}x+2^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(x-2^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-2),x, algorithm="maxima")

[Out] $-1/6*\sqrt{3}*2^{(1/3)}*\arctan(1/6*\sqrt{3}*2^{(2/3)}*(2*x + 2^{(1/3)})) - 1/12*2^{(1/3)}*\log(x^2 + 2^{(1/3)}*x + 2^{(2/3)}) + 1/6*2^{(1/3)}*\log(x - 2^{(1/3)})$

Fricas [A]

time = 0.32, size = 68, normalized size = 0.92

$-\frac{1}{6} \cdot 4^{\frac{1}{6}} \sqrt{3} \arctan\left(\frac{1}{6} \cdot 4^{\frac{1}{6}} \left(4^{\frac{2}{3}} \sqrt{3} x + 4^{\frac{1}{3}} \sqrt{3}\right)\right) - \frac{1}{24} \cdot 4^{\frac{2}{3}} \log\left(2x^2 + 4^{\frac{2}{3}}x + 2 \cdot 4^{\frac{1}{3}}\right) + \frac{1}{12} \cdot 4^{\frac{2}{3}} \log\left(2x - 4^{\frac{2}{3}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-2),x, algorithm="fricas")

[Out] $-1/6*4^{(1/6)}*\sqrt{3}*\arctan(1/6*4^{(1/6)}*(4^{(2/3)}*\sqrt{3}*x + 4^{(1/3)}*\sqrt{3}))) - 1/24*4^{(2/3)}*\log(2*x^2 + 4^{(2/3)}*x + 2*4^{(1/3)}) + 1/12*4^{(2/3)}*\log(2*x - 4^{(2/3)})$

Sympy [A]

time = 0.16, size = 71, normalized size = 0.96

$$\frac{\sqrt[3]{2} \log\left(x - \sqrt[3]{2}\right)}{6} - \frac{\sqrt[3]{2} \log\left(x^2 + \sqrt[3]{2} x + 2^{\frac{2}{3}}\right)}{12} - \frac{\sqrt[3]{2} \sqrt{3} \operatorname{atan}\left(\frac{2^{\frac{2}{3}} \sqrt{3} x + \sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**3-2),x)

[Out] $2^{(1/3)}*\log(x - 2^{(1/3)})/6 - 2^{(1/3)}*\log(x**2 + 2^{(1/3)}*x + 2^{(2/3)})/12 - 2^{(1/3)}*\sqrt{3}*\operatorname{atan}(2^{(2/3)}*\sqrt{3}*x/3 + \sqrt{3}/3)/6$

Giac [A]

time = 0.00, size = 91, normalized size = 1.23

$-\frac{1}{12} \cdot 2^{\frac{1}{3}} \ln\left(x^2 + 2^{\frac{1}{3}}x + 2^{\frac{1}{3}} \cdot 2^{\frac{1}{3}}\right) - \frac{1}{6} \cdot 2^{\frac{1}{3}} \sqrt{3} \arctan\left(\frac{x + \frac{2^{\frac{1}{3}}}{2}}{\frac{1}{2} \sqrt{3} \cdot 2^{\frac{1}{3}}}\right) + \frac{1}{6} \cdot 2^{\frac{1}{3}} \ln\left|x - 2^{\frac{1}{3}}\right|$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-2),x)

[Out] $-1/6*\sqrt{3}*2^{(1/3)}*\arctan(1/6*\sqrt{3}*2^{(2/3)}*(2*x + 2^{(1/3)})) - 1/12*2^{(1/3)}*\log(x^2 + 2^{(1/3)}*x + 2^{(2/3)}) + 1/6*2^{(1/3)}*\log(\operatorname{abs}(x - 2^{(1/3)}))$

Mupad [B]

time = 0.36, size = 72, normalized size = 0.97

$$\frac{2^{1/3} \ln(x - 2^{1/3})}{6} + \frac{2^{1/3} \ln\left(x - \frac{2^{1/3}(-1 + \sqrt{3} \operatorname{li})}{2}\right) (-1 + \sqrt{3} \operatorname{li})}{12} - \frac{2^{1/3} \ln\left(x + \frac{2^{1/3}(1 + \sqrt{3} \operatorname{li})}{2}\right) (1 + \sqrt{3} \operatorname{li})}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 - 2),x)
```

```
[Out] (2^(1/3)*log(x - 2^(1/3)))/6 + (2^(1/3)*log(x - (2^(1/3)*(3^(1/2)*1i - 1)))/  
2*(3^(1/2)*1i - 1))/12 - (2^(1/3)*log(x + (2^(1/3)*(3^(1/2)*1i + 1)))/2*(3  
^(1/2)*1i + 1))/12
```

3.35 $\int \frac{1}{-b+ax^3} dx$

Optimal. Leaf size=115

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}+2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}} + \frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{\log\left(b^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}$$

[Out] $\frac{1}{3}\ln(b^{1/3}-a^{1/3}x)/a^{1/3}/b^{2/3}-\frac{1}{6}\ln(b^{2/3}+a^{1/3}b^{1/3}x+a^{2/3}x^2)/a^{1/3}/b^{2/3}-\frac{1}{3}\arctan(1/3*(b^{1/3}+2*a^{1/3}*x)/b^{1/3}*3^{1/2})/a^{1/3}/b^{2/3}*3^{1/2}$

Rubi [A]

time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {206, 31, 648, 631, 210, 642}

$$-\frac{\log\left(a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{6\sqrt[3]{a}b^{2/3}} + \frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{a}x+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(-b + a*x^3)^(-1), x]

[Out] $-\frac{\text{ArcTan}\left[\frac{b^{1/3}+2a^{1/3}x}{\sqrt{3}b^{1/3}}\right]}{\sqrt{3}a^{1/3}b^{2/3}} + \frac{\text{Log}\left[\frac{b^{1/3}-a^{1/3}x}{3a^{1/3}b^{2/3}}\right]}{3a^{1/3}b^{2/3}} - \frac{\text{Log}\left[\frac{b^{2/3}+a^{1/3}b^{1/3}x+a^{2/3}x^2}{6a^{1/3}b^{2/3}}\right]}{6a^{1/3}b^{2/3}}$

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & (LtQ[a, 0] || LtQ[b, 0])

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-b + ax^3} dx &= \frac{\int \frac{1}{-\sqrt[3]{b} + \sqrt[3]{a} x} dx}{3b^{2/3}} + \frac{\int \frac{-2\sqrt[3]{b} - \sqrt[3]{a} x}{b^{2/3} + \sqrt[3]{a} \sqrt[3]{b} x + a^{2/3} x^2} dx}{3b^{2/3}} \\
&= \frac{\log(\sqrt[3]{b} - \sqrt[3]{a} x)}{3\sqrt[3]{a} b^{2/3}} - \frac{\int \frac{\sqrt[3]{a} \sqrt[3]{b} + 2a^{2/3} x}{b^{2/3} + \sqrt[3]{a} \sqrt[3]{b} x + a^{2/3} x^2} dx}{6\sqrt[3]{a} b^{2/3}} - \frac{\int \frac{1}{b^{2/3} + \sqrt[3]{a} \sqrt[3]{b} x + a^{2/3} x^2} dx}{2\sqrt[3]{b}} \\
&= \frac{\log(\sqrt[3]{b} - \sqrt[3]{a} x)}{3\sqrt[3]{a} b^{2/3}} - \frac{\log(b^{2/3} + \sqrt[3]{a} \sqrt[3]{b} x + a^{2/3} x^2)}{6\sqrt[3]{a} b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 + \frac{2\sqrt[3]{a} x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a} b^{2/3}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{b} + 2\sqrt[3]{a} x}{\sqrt{3} \sqrt[3]{b}}\right)}{\sqrt{3} \sqrt[3]{a} b^{2/3}} + \frac{\log(\sqrt[3]{b} - \sqrt[3]{a} x)}{3\sqrt[3]{a} b^{2/3}} - \frac{\log(b^{2/3} + \sqrt[3]{a} \sqrt[3]{b} x + a^{2/3} x^2)}{6\sqrt[3]{a} b^{2/3}}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 89, normalized size = 0.77

$$\frac{2\sqrt{3} \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{a} x}{\sqrt[3]{b}}}{\sqrt{3}}\right) - 2\log(\sqrt[3]{b} - \sqrt[3]{a} x) + \log(b^{2/3} + \sqrt[3]{a} \sqrt[3]{b} x + a^{2/3} x^2)}{6\sqrt[3]{a} b^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(-b + a*x^3)^(-1),x]

[Out] $-\frac{1}{6} \cdot (2 \cdot \sqrt{3} \cdot \text{ArcTan}[\frac{1 + (2 \cdot a^{1/3} \cdot x)}{b^{1/3}}] / \sqrt{3}) - 2 \cdot \text{Log}[b^{1/3} - a^{1/3} \cdot x] + \text{Log}[b^{2/3} + a^{1/3} \cdot b^{1/3} \cdot x + a^{2/3} \cdot x^2] / (a^{1/3} \cdot b^{2/3})$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.84, size = 26, normalized size = 0.23

$$\text{RootSum}[-1 + 27ab^2\#1^3\&, \text{Log}[-3b\#1 + x]\#1\&]$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(a*x^3-b),x]')

[Out] $\text{RootSum}[-1 + 27 a b^2 \#1^3\&, \text{Log}[-3 b \#1 + x] \#1\&]$

Maple [A]

time = 0.02, size = 92, normalized size = 0.80

method	result	size
risch	$\frac{\sum_{-R=\text{RootOf}(aZ^3-b)} \frac{\ln(-R+x)}{-R^2}}{3a}$	29
default	$\frac{\ln\left(x - \left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{\ln\left(x^2 + \left(\frac{b}{a}\right)^{\frac{1}{3}}x + \left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}+1\right)}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$	92

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x^3-b),x,method=_RETURNVERBOSE)

[Out] $\frac{1}{3} \cdot a / (b/a)^{2/3} \cdot \ln(x - (b/a)^{1/3}) - \frac{1}{6} \cdot a / (b/a)^{2/3} \cdot \ln(x^2 + (b/a)^{1/3} \cdot x + (b/a)^{2/3}) - \frac{1}{3} \cdot a / (b/a)^{2/3} \cdot 3^{1/2} \cdot \arctan(1/3 \cdot 3^{1/2} \cdot (2 / (b/a)^{1/3}) \cdot x + 1)$

Maxima [A]

time = 0.36, size = 97, normalized size = 0.84

$$-\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x + \left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2 + x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{\log\left(x - \left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^3-b),x, algorithm="maxima")

[Out] $-1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + (b/a)^{(1/3)})/(b/a)^{(1/3)})/(a*(b/a)^{(2/3)}) - 1/6*\log(x^2 + x*(b/a)^{(1/3)} + (b/a)^{(2/3)})/(a*(b/a)^{(2/3)}) + 1/3*\log(x - (b/a)^{(1/3)})/(a*(b/a)^{(2/3)})$

Fricas [A]

time = 0.34, size = 300, normalized size = 2.61

$$\left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}} \log\left(\frac{2abx^2 - 3(ab^2)^{\frac{1}{3}}x + (ab^2)^{\frac{2}{3}}}{(ab^2)^{\frac{1}{3}}}\right) - (ab^2)^{\frac{1}{3}} \log(abx^2 + (ab^2)^{\frac{1}{3}}x + (ab^2)^{\frac{2}{3}}) + 2(ab^2)^{\frac{1}{3}} \log(abx - (ab^2)^{\frac{1}{3}})}{6ab^2} - \frac{6\sqrt{\frac{1}{3}}ab\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}} \arctan\left(\frac{\sqrt{\frac{1}{3}}(2(ab^2)^{\frac{1}{3}}x + (ab^2)^{\frac{1}{3}})\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}}{(ab^2)^{\frac{1}{3}}}\right) + (ab^2)^{\frac{1}{3}} \log(abx^2 + (ab^2)^{\frac{1}{3}}x + (ab^2)^{\frac{2}{3}}) - 2(ab^2)^{\frac{1}{3}} \log(abx - (ab^2)^{\frac{1}{3}})}{6ab^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^3-b),x, algorithm="fricas")

[Out] $[1/6*(3*\sqrt{1/3}*a*b*\sqrt{-(a*b^2)^{(1/3)}/a})*\log((2*a*b*x^3 - 3*(a*b^2)^{(1/3)}*b*x + b^2 - 3*\sqrt{1/3}*(2*a*b*x^2 - (a*b^2)^{(2/3)}*x - (a*b^2)^{(1/3)}*b)*\sqrt{-(a*b^2)^{(1/3)}/a})/(a*x^3 - b) - (a*b^2)^{(2/3)}*\log(a*b*x^2 + (a*b^2)^{(2/3)}*x + (a*b^2)^{(1/3)}*b) + 2*(a*b^2)^{(2/3)}*\log(a*b*x - (a*b^2)^{(2/3)})/(a*b^2), -1/6*(6*\sqrt{1/3}*a*b*\sqrt{(a*b^2)^{(1/3)}/a})*\arctan(\sqrt{1/3}*(2*(a*b^2)^{(2/3)}*x + (a*b^2)^{(1/3)}*b)*\sqrt{(a*b^2)^{(1/3)}/a}/b^2) + (a*b^2)^{(2/3)}*\log(a*b*x^2 + (a*b^2)^{(2/3)}*x + (a*b^2)^{(1/3)}*b) - 2*(a*b^2)^{(2/3)}*\log(a*b*x - (a*b^2)^{(2/3)})/(a*b^2)]$

Sympy [A]

time = 0.08, size = 20, normalized size = 0.17

$$\text{RootSum}(27t^3ab^2 - 1, (t \mapsto t \log(-3tb + x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x**3-b),x)

[Out] RootSum(27*_t**3*a*b**2 - 1, Lambda(_t, _t*log(-3*_t*b + x)))

Giac [A]

time = 0.00, size = 127, normalized size = 1.10

$$\frac{(a^2b)^{\frac{1}{3}} \ln\left(x^2 + \left(\frac{b}{a}\right)^{\frac{1}{3}}x + \left(\frac{b}{a}\right)^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{6ab} - \frac{(a^2b)^{\frac{1}{3}} \arctan\left(\frac{x + \left(\frac{b}{a}\right)^{\frac{1}{3}}}{\frac{1}{2}\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab} + \frac{\left(\frac{b}{a}\right)^{\frac{1}{3}} \ln\left|x - \left(\frac{b}{a}\right)^{\frac{1}{3}}\right|}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x^3-b),x)

[Out] $\frac{1}{3} \cdot \left(\frac{b}{a}\right)^{1/3} \cdot \log\left(\frac{\text{abs}\left(x - \left(\frac{b}{a}\right)^{1/3}\right)}{b}\right) - \frac{1}{3} \cdot \sqrt{3} \cdot \left(a^2 b\right)^{1/3} \cdot \arctan\left(\frac{1}{3} \cdot \sqrt{3} \cdot \frac{2x + \left(\frac{b}{a}\right)^{1/3}}{\left(\frac{b}{a}\right)^{1/3}}\right) - \frac{1}{6} \cdot \left(a^2 b\right)^{1/3} \cdot \log\left(\frac{x^2 + x \cdot \left(\frac{b}{a}\right)^{1/3} + \left(\frac{b}{a}\right)^{2/3}}{a \cdot b}\right)$

Mupad [B]

time = 0.31, size = 101, normalized size = 0.88

$$\frac{\ln\left(\frac{a^{1/3}x - b^{1/3}}{3a^{1/3}b^{2/3}}\right)}{3a^{1/3}b^{2/3}} + \frac{\ln\left(3a^2x - \frac{3a^{5/3}b^{1/3}(-1+\sqrt{3}i)}{2}\right)(-1+\sqrt{3}i)}{6a^{1/3}b^{2/3}} - \frac{\ln\left(3a^2x + \frac{3a^{5/3}b^{1/3}(1+\sqrt{3}i)}{2}\right)(1+\sqrt{3}i)}{6a^{1/3}b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(b - a*x^3),x)

[Out] $\log\left(\frac{a^{1/3}x - b^{1/3}}{3a^{1/3}b^{2/3}}\right) + \frac{\log\left(3a^2x - \frac{3a^{5/3}b^{1/3}(-1+\sqrt{3}i)}{2}\right)(-1+\sqrt{3}i)}{6a^{1/3}b^{2/3}} - \frac{\log\left(3a^2x + \frac{3a^{5/3}b^{1/3}(1+\sqrt{3}i)}{2}\right)(1+\sqrt{3}i)}{6a^{1/3}b^{2/3}}$

3.36 $\int \frac{1}{-2+x^4} dx$

Optimal. Leaf size=35

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2 \cdot 2^{3/4}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2 \cdot 2^{3/4}}$$

[Out] $-1/4 \cdot \arctan(1/2 \cdot x \cdot 2^{(3/4)}) \cdot 2^{(1/4)} - 1/4 \cdot \operatorname{arctanh}(1/2 \cdot x \cdot 2^{(3/4)}) \cdot 2^{(1/4)}$

Rubi [A]

time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {218, 212, 209}

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2 \cdot 2^{3/4}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2 \cdot 2^{3/4}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(-2 + x^4)^{-1}, x]$

[Out] $-1/2 \cdot \operatorname{ArcTan}[x/2^{(1/4)}]/2^{(3/4)} - \operatorname{ArcTanh}[x/2^{(1/4)}]/(2 \cdot 2^{(3/4)})$

Rule 209

$\operatorname{Int}[(a + (b \cdot x^2)^{-1}), x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2] \cdot \operatorname{Rt}[b, 2])) \cdot \operatorname{ArcTan}[\operatorname{Rt}[b, 2] \cdot (x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{PosQ}[a/b] \ \&\& \ (\operatorname{GtQ}[a, 0] \ || \ \operatorname{GtQ}[b, 0])$

Rule 212

$\operatorname{Int}[(a + (b \cdot x^2)^{-1}), x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2] \cdot \operatorname{Rt}[-b, 2])) \cdot \operatorname{ArcTanh}[\operatorname{Rt}[-b, 2] \cdot (x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 218

$\operatorname{Int}[(a + (b \cdot x^4)^{-1}), x_Symbol] \rightarrow \operatorname{With}[\{r = \operatorname{Numerator}[\operatorname{Rt}[-a/b, 2]], s = \operatorname{Denominator}[\operatorname{Rt}[-a/b, 2]]\}, \operatorname{Dist}[r/(2 \cdot a), \operatorname{Int}[1/(r - s \cdot x^2), x], x] + \operatorname{Dist}[r/(2 \cdot a), \operatorname{Int}[1/(r + s \cdot x^2), x], x]] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \ !\operatorname{GtQ}[a/b, 0]$

Rubi steps

$$\int \frac{1}{-2+x^4} dx = -\frac{\int \frac{1}{\sqrt{2}-x^2} dx}{2\sqrt{2}} - \frac{\int \frac{1}{\sqrt{2}+x^2} dx}{2\sqrt{2}}$$

$$= -\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2 \cdot 2^{3/4}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2 \cdot 2^{3/4}}$$

Mathematica [A]

time = 0.01, size = 43, normalized size = 1.23

$$\frac{2 \tan^{-1}\left(\frac{x}{\sqrt[4]{2}}\right) - \log(2 - 2^{3/4}x) + \log(2 + 2^{3/4}x)}{4 \cdot 2^{3/4}}$$

Antiderivative was successfully verified.

`[In] Integrate[(-2 + x^4)^(-1), x]``[Out] -1/4*(2*ArcTan[x/2^(1/4)] - Log[2 - 2^(3/4)*x] + Log[2 + 2^(3/4)*x])/2^(3/4)`**Mathics [A]**

time = 1.93, size = 31, normalized size = 0.89

$$\frac{2^{1/4} \left(\log\left[-2^{1/4} + x\right] - 2 \operatorname{ArcTan}\left[\frac{2^{3/4}x}{2}\right] - \log\left[2^{1/4} + x\right] \right)}{8}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^4-2), x]')``[Out] 2^(1/4) (Log[-2^(1/4) + x] - 2 ArcTan[2^(3/4) x / 2] - Log[2^(1/4) + x]) / 8`**Maple [A]**

time = 0.03, size = 32, normalized size = 0.91

method	result	size
risch	$\frac{\sum_{-R=\text{RootOf}(-Z^4-2)} \frac{\ln(-R+x)}{-R^3}}{4}$	22
default	$\frac{2^{1/4} \left(\ln\left(\frac{x+2^{1/4}}{x-2^{1/4}}\right) + 2 \arctan\left(\frac{x 2^{3/4}}{2}\right) \right)}{8}$	32

meijerg	$\frac{2^{\frac{1}{4}}x \left(\ln \left(1 - \frac{2^{\frac{3}{4}}(x^4)^{\frac{1}{4}}}{2} \right) - \ln \left(1 + \frac{2^{\frac{3}{4}}(x^4)^{\frac{1}{4}}}{2} \right) - 2 \arctan \left(\frac{2^{\frac{3}{4}}(x^4)^{\frac{1}{4}}}{2} \right) \right)}{8(x^4)^{\frac{1}{4}}}$	54
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4-2),x,method=_RETURNVERBOSE)`

[Out] `-1/8*2^(1/4)*(ln((x+2^(1/4))/(x-2^(1/4)))+2*arctan(1/2*x*2^(3/4)))`

Maxima [A]

time = 0.37, size = 34, normalized size = 0.97

$$-\frac{1}{4} \cdot 2^{\frac{1}{4}} \arctan \left(\frac{1}{2} \cdot 2^{\frac{3}{4}} x \right) + \frac{1}{8} \cdot 2^{\frac{1}{4}} \log \left(\frac{x - 2^{\frac{1}{4}}}{x + 2^{\frac{1}{4}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4-2),x, algorithm="maxima")`

[Out] `-1/4*2^(1/4)*arctan(1/2*2^(3/4)*x) + 1/8*2^(1/4)*log((x - 2^(1/4))/(x + 2^(1/4)))`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.

time = 0.33, size = 56, normalized size = 1.60

$$\frac{1}{8} \cdot 8^{\frac{3}{4}} \arctan \left(-\frac{1}{2} \cdot 8^{\frac{1}{4}} x + \frac{1}{2} \cdot 8^{\frac{1}{4}} \sqrt{x^2 + \sqrt{2}} \right) - \frac{1}{32} \cdot 8^{\frac{3}{4}} \log \left(4x + 8^{\frac{3}{4}} \right) + \frac{1}{32} \cdot 8^{\frac{3}{4}} \log \left(4x - 8^{\frac{3}{4}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4-2),x, algorithm="fricas")`

[Out] `1/8*8^(3/4)*arctan(-1/2*8^(1/4)*x + 1/2*8^(1/4)*sqrt(x^2 + sqrt(2))) - 1/32*8^(3/4)*log(4*x + 8^(3/4)) + 1/32*8^(3/4)*log(4*x - 8^(3/4))`

Sympy [A]

time = 0.16, size = 46, normalized size = 1.31

$$\frac{\sqrt[4]{2} \log \left(x - \sqrt[4]{2} \right)}{8} - \frac{\sqrt[4]{2} \log \left(x + \sqrt[4]{2} \right)}{8} - \frac{\sqrt[4]{2} \operatorname{atan} \left(\frac{2^{\frac{3}{4}} x}{2} \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**4-2),x)`

[Out] `2**(1/4)*log(x - 2**(1/4))/8 - 2**(1/4)*log(x + 2**(1/4))/8 - 2**(1/4)*atan(2**(3/4)*x/2)/4`

Giac [A]

time = 0.00, size = 55, normalized size = 1.57

$$\frac{1}{8} \cdot 2^{\frac{1}{4}} \ln \left| x - 2^{\frac{1}{4}} \right| - \frac{1}{8} \cdot 2^{\frac{1}{4}} \ln \left| x + 2^{\frac{1}{4}} \right| - \frac{1}{4} \cdot 2^{\frac{1}{4}} \arctan \left(\frac{x}{2^{\frac{1}{4}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4-2),x)

[Out] $-1/4 \cdot 2^{1/4} \cdot \arctan(1/2 \cdot 2^{3/4} \cdot x) - 1/8 \cdot 2^{1/4} \cdot \log(\text{abs}(x + 2^{1/4})) + 1/8 \cdot 2^{1/4} \cdot \log(\text{abs}(x - 2^{1/4}))$

Mupad [B]

time = 0.16, size = 20, normalized size = 0.57

$$\frac{2^{1/4} \left(\text{atan} \left(\frac{2^{3/4} x}{2} \right) + \text{atanh} \left(\frac{2^{3/4} x}{2} \right) \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4 - 2),x)

[Out] $-(2^{1/4} \cdot (\text{atan}((2^{3/4} \cdot x)/2) + \text{atanh}((2^{3/4} \cdot x)/2)))/4$

3.37 $\int \frac{1}{-1+5x^4} dx$

Optimal. Leaf size=35

$$-\frac{\tan^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}} - \frac{\tanh^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}$$

[Out] $-1/10*\arctan(5^{(1/4)*x})*5^{(3/4)}-1/10*\operatorname{arctanh}(5^{(1/4)*x})*5^{(3/4)}$

Rubi [A]

time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {218, 212, 209}

$$-\frac{\tan^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}} - \frac{\tanh^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(-1 + 5*x^4)^{-1}, x]$

[Out] $-1/2*\operatorname{ArcTan}[5^{(1/4)*x}]/5^{(1/4)} - \operatorname{ArcTanh}[5^{(1/4)*x}]/(2*5^{(1/4)})$

Rule 209

$\operatorname{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))*\operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{PosQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{GtQ}[b, 0])$

Rule 212

$\operatorname{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 218

$\operatorname{Int}[(a_ + (b_.)*(x_)^4)^{-1}, x_Symbol] \rightarrow \operatorname{With}\{r = \operatorname{Numerator}[\operatorname{Rt}[-a/b, 2]], s = \operatorname{Denominator}[\operatorname{Rt}[-a/b, 2]]\}, \operatorname{Dist}[r/(2*a), \operatorname{Int}[1/(r - s*x^2), x], x] + \operatorname{Dist}[r/(2*a), \operatorname{Int}[1/(r + s*x^2), x], x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \ !\operatorname{GtQ}[a/b, 0]$

Rubi steps

$$\int \frac{1}{-1+5x^4} dx = -\left(\frac{1}{2} \int \frac{1}{1-\sqrt[4]{5}x^2} dx\right) - \frac{1}{2} \int \frac{1}{1+\sqrt[4]{5}x^2} dx$$

$$= -\frac{\tan^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}} - \frac{\tanh^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}$$

Mathematica [A]

time = 0.01, size = 43, normalized size = 1.23

$$-\frac{2 \tan^{-1}\left(\sqrt[4]{5}x\right) - \log\left(1 - \sqrt[4]{5}x\right) + \log\left(1 + \sqrt[4]{5}x\right)}{4\sqrt[4]{5}}$$

Antiderivative was successfully verified.

`[In] Integrate[(-1 + 5*x^4)^(-1), x]``[Out] -1/4*(2*ArcTan[5^(1/4)*x] - Log[1 - 5^(1/4)*x] + Log[1 + 5^(1/4)*x])/5^(1/4)`**Mathics [A]**

time = 1.92, size = 32, normalized size = 0.91

$$\frac{5^{\frac{3}{4}} \left(\log\left[-\frac{5^{\frac{3}{4}}}{5} + x\right] - 2 \operatorname{ArcTan}\left[5^{\frac{1}{4}}x\right] - \log\left[\frac{5^{\frac{3}{4}}}{5} + x\right] \right)}{20}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(5*x^4-1), x]')``[Out] 5^(3/4) (Log[-5^(3/4)/5 + x] - 2 ArcTan[5^(1/4)x] - Log[5^(3/4)/5 + x]) / 20`**Maple [A]**

time = 0.03, size = 33, normalized size = 0.94

method	result	size
risch	$\frac{\sum_{R=\text{RootOf}(5Z^4-1)} \frac{\ln(-R+x)}{-R^3}}{20}$	24
default	$-\frac{5^{\frac{3}{4}} \left(\ln\left(\frac{x+\frac{5^{\frac{3}{4}}}{5}}{x-\frac{5^{\frac{3}{4}}}{5}}\right) + 2 \arctan\left(5^{\frac{1}{4}}x\right) \right)}{20}$	33

meijerg	$\frac{5^{\frac{3}{4}}x \left(\ln \left(1 - 5^{\frac{1}{4}}(x^4)^{\frac{1}{4}} \right) - \ln \left(1 + 5^{\frac{1}{4}}(x^4)^{\frac{1}{4}} \right) - 2 \arctan \left(5^{\frac{1}{4}}(x^4)^{\frac{1}{4}} \right) \right)}{20(x^4)^{\frac{1}{4}}}$	52
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5*x^4-1),x,method=_RETURNVERBOSE)`

[Out] $-1/20 \cdot 5^{3/4} \cdot (\ln((x+1/5 \cdot 5^{3/4})/(x-1/5 \cdot 5^{3/4}))) + 2 \cdot \arctan(5^{1/4} \cdot x)$

Maxima [A]

time = 0.37, size = 41, normalized size = 1.17

$$-\frac{1}{10} \cdot 5^{\frac{3}{4}} \arctan \left(5^{\frac{1}{4}} x \right) + \frac{1}{20} \cdot 5^{\frac{3}{4}} \log \left(\frac{\sqrt{5} x - 5^{\frac{1}{4}}}{\sqrt{5} x + 5^{\frac{1}{4}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5*x^4-1),x, algorithm="maxima")`

[Out] $-1/10 \cdot 5^{3/4} \cdot \arctan(5^{1/4} \cdot x) + 1/20 \cdot 5^{3/4} \cdot \log((\sqrt{5} \cdot x - 5^{1/4})/(\sqrt{5} \cdot x + 5^{1/4}))$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(23) = 46.

time = 0.33, size = 60, normalized size = 1.71

$$\frac{1}{5} \cdot 5^{\frac{3}{4}} \arctan \left(-5^{\frac{1}{4}} x + \frac{1}{5} \cdot 5^{\frac{1}{4}} \sqrt{25 x^2 + 5 \sqrt{5}} \right) - \frac{1}{20} \cdot 5^{\frac{3}{4}} \log \left(5 x + 5^{\frac{3}{4}} \right) + \frac{1}{20} \cdot 5^{\frac{3}{4}} \log \left(5 x - 5^{\frac{3}{4}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5*x^4-1),x, algorithm="fricas")`

[Out] $1/5 \cdot 5^{3/4} \cdot \arctan(-5^{1/4} \cdot x + 1/5 \cdot 5^{1/4} \cdot \sqrt{25 \cdot x^2 + 5 \cdot \sqrt{5}}) - 1/20 \cdot 5^{3/4} \cdot \log(5 \cdot x + 5^{3/4}) + 1/20 \cdot 5^{3/4} \cdot \log(5 \cdot x - 5^{3/4})$

Sympy [A]

time = 0.16, size = 48, normalized size = 1.37

$$\frac{5^{\frac{3}{4}} \log \left(x - \frac{5^{\frac{3}{4}}}{5} \right)}{20} - \frac{5^{\frac{3}{4}} \log \left(x + \frac{5^{\frac{3}{4}}}{5} \right)}{20} - \frac{5^{\frac{3}{4}} \operatorname{atan} \left(\sqrt[4]{5} x \right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5*x**4-1),x)`

[Out] $5^{3/4} \cdot \log(x - 5^{3/4}/5)/20 - 5^{3/4} \cdot \log(x + 5^{3/4}/5)/20 - 5^{3/4} \cdot \operatorname{atan}(5^{1/4} \cdot x)/10$

Giac [A]

time = 0.00, size = 67, normalized size = 1.91

$$\frac{1}{20} \cdot \left(5^{\frac{1}{4}}\right)^3 \ln \left| x - \left(\frac{1}{5}\right)^{\frac{1}{4}} \right| - \frac{1}{20} \cdot \left(5^{\frac{1}{4}}\right)^3 \ln \left| x + \left(\frac{1}{5}\right)^{\frac{1}{4}} \right| - \frac{1}{10} \cdot \left(5^{\frac{1}{4}}\right)^3 \arctan \left(\frac{x}{\left(\frac{1}{5}\right)^{\frac{1}{4}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^4-1),x)**[Out]** -1/10*5^(3/4)*arctan(5*(1/5)^(3/4)*x) - 1/20*5^(3/4)*log(abs(x + (1/5)^(1/4))) + 1/20*5^(3/4)*log(abs(x - (1/5)^(1/4)))**Mupad [B]**

time = 0.18, size = 18, normalized size = 0.51

$$\frac{5^{3/4} (\operatorname{atan}(5^{1/4} x) + \operatorname{atanh}(5^{1/4} x))}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^4 - 1),x)**[Out]** -(5^(3/4)*(atan(5^(1/4)*x) + atanh(5^(1/4)*x)))/10

3.38 $\int \frac{1}{7+3x^4} dx$

Optimal. Leaf size=171

$$\frac{\tan^{-1}\left(1 - \sqrt[4]{\frac{3}{7}} \sqrt{2} x\right)}{2\sqrt{2} \sqrt[4]{3} 7^{3/4}} + \frac{\tan^{-1}\left(1 + \sqrt[4]{\frac{3}{7}} \sqrt{2} x\right)}{2\sqrt{2} \sqrt[4]{3} 7^{3/4}} - \frac{\log\left(\sqrt{21} - \sqrt{2} 3^{3/4} \sqrt[4]{7} x + 3x^2\right)}{4\sqrt{2} \sqrt[4]{3} 7^{3/4}} + \frac{\log\left(\sqrt{21} + \sqrt{2} 3^{3/4} \sqrt[4]{7} x + 3x^2\right)}{4\sqrt{2} \sqrt[4]{3} 7^{3/4}}$$

[Out] 1/84*arctan(-1+1/7*3^(1/4)*7^(3/4)*x*2^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)+1/84*arctan(1+1/7*3^(1/4)*7^(3/4)*x*2^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)-1/168*ln(3*x^2-3^(3/4)*7^(1/4)*x*2^(1/2)+21^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)+1/168*ln(3*x^2+3^(3/4)*7^(1/4)*x*2^(1/2)+21^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)

Rubi [A]

time = 0.07, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {217, 1179, 642, 1176, 631, 210}

$$\frac{\log\left(3x^2 - \sqrt{2} 3^{3/4} \sqrt[4]{7} x + \sqrt{21}\right)}{4\sqrt{2} \sqrt[4]{3} 7^{3/4}} + \frac{\log\left(3x^2 + \sqrt{2} 3^{3/4} \sqrt[4]{7} x + \sqrt{21}\right)}{4\sqrt{2} \sqrt[4]{3} 7^{3/4}} - \frac{\tan^{-1}\left(1 - \sqrt[4]{\frac{3}{7}} \sqrt{2} x\right)}{2\sqrt{2} \sqrt[4]{3} 7^{3/4}} + \frac{\tan^{-1}\left(\sqrt[4]{\frac{3}{7}} \sqrt{2} x + 1\right)}{2\sqrt{2} \sqrt[4]{3} 7^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(7 + 3*x^4)^(-1), x]

[Out] -1/2*ArcTan[1 - (3/7)^(1/4)*Sqrt[2]*x]/(Sqrt[2]*3^(1/4)*7^(3/4)) + ArcTan[1 + (3/7)^(1/4)*Sqrt[2]*x]/(2*Sqrt[2]*3^(1/4)*7^(3/4)) - Log[Sqrt[21] - Sqrt[2]*3^(3/4)*7^(1/4)*x + 3*x^2]/(4*Sqrt[2]*3^(1/4)*7^(3/4)) + Log[Sqrt[21] + Sqrt[2]*3^(3/4)*7^(1/4)*x + 3*x^2]/(4*Sqrt[2]*3^(1/4)*7^(3/4))

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 631


```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{7+3x^4} dx &= \int \frac{\sqrt{7}-\sqrt{3}x^2}{7+3x^4} dx + \int \frac{\sqrt{7}+\sqrt{3}x^2}{7+3x^4} dx \\ &= -\frac{\int \frac{\sqrt{2}\sqrt[4]{\frac{7}{3}}+2x}{-\sqrt{\frac{7}{3}}-\sqrt{2}\sqrt[4]{\frac{7}{3}}x-x^2} dx}{4\sqrt{2}\sqrt[4]{3}7^{3/4}} - \frac{\int \frac{\sqrt{2}\sqrt[4]{\frac{7}{3}}-2x}{-\sqrt{\frac{7}{3}}+\sqrt{2}\sqrt[4]{\frac{7}{3}}x-x^2} dx}{4\sqrt{2}\sqrt[4]{3}7^{3/4}} + \frac{\int \frac{1}{\sqrt{\frac{7}{3}}-\sqrt{2}\sqrt[4]{\frac{7}{3}}x+x^2} dx}{4\sqrt{21}} + \dots \\ &= -\frac{\log\left(\sqrt{21}-\sqrt{2}3^{3/4}\sqrt[4]{7}x+3x^2\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}} + \frac{\log\left(\sqrt{21}+\sqrt{2}3^{3/4}\sqrt[4]{7}x+3x^2\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\frac{7}{3}}-\sqrt{2}\sqrt[4]{\frac{7}{3}}x+x^2} dx\right)}{4\sqrt{21}} \\ &= -\frac{\tan^{-1}\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)}{2\sqrt{2}\sqrt[4]{3}7^{3/4}} + \frac{\tan^{-1}\left(1+\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)}{2\sqrt{2}\sqrt[4]{3}7^{3/4}} - \frac{\log\left(\sqrt{21}-\sqrt{2}3^{3/4}\sqrt[4]{7}x+3x^2\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 120, normalized size = 0.70

$$\frac{-2 \tan^{-1} \left(1 - \sqrt[4]{\frac{3}{7}} \sqrt{2} x \right) + 2 \tan^{-1} \left(1 + \sqrt[4]{\frac{3}{7}} \sqrt{2} x \right) - \log \left(7 - \sqrt{2} \sqrt[4]{3} 7^{3/4} x + \sqrt{21} x^2 \right) + \log \left(7 + \sqrt{2} \sqrt[4]{3} 7^{3/4} x + \sqrt{21} x^2 \right)}{4 \sqrt{2} \sqrt[4]{3} 7^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(7 + 3*x^4)^(-1), x]

[Out] (-2*ArcTan[1 - (3/7)^(1/4)*Sqrt[2]*x] + 2*ArcTan[1 + (3/7)^(1/4)*Sqrt[2]*x] - Log[7 - Sqrt[2]*3^(1/4)*7^(3/4)*x + Sqrt[21]*x^2] + Log[7 + Sqrt[2]*3^(1/4)*7^(3/4)*x + Sqrt[21]*x^2])/(4*Sqrt[2]*3^(1/4)*7^(3/4))

Mathics [A]

time = 2.59, size = 83, normalized size = 0.49

$$\frac{\sqrt{2} 189^{1/4} \left(\text{Log} \left[\frac{\sqrt{21}}{3} + \frac{\sqrt{2} 189^{1/4} x}{3} + x^2 \right] - \text{Log} \left[\frac{\sqrt{21}}{3} - \frac{\sqrt{2} 189^{1/4} x}{3} + x^2 \right] + 2 \text{ArcTan} \left[-1 + \frac{\sqrt{2} 3^{1/4} 7^{3/4} x}{7} \right] + 2 \text{ArcTan} \left[1 + \frac{\sqrt{2} 3^{1/4} 7^{3/4} x}{7} \right] \right)}{168}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(3*x^4+7), x]')

[Out] Sqrt[2] 189^(1/4) (Log[Sqrt[21] / 3 + Sqrt[2] 189^(1/4) x / 3 + x^2] - Log[Sqrt[21] / 3 - Sqrt[2] 189^(1/4) x / 3 + x^2] + 2 ArcTan[-1 + Sqrt[2] 3^(1/4) 7^(3/4) x / 7] + 2 ArcTan[1 + Sqrt[2] 3^(1/4) 7^(3/4) x / 7]) / 168

Maple [A]

time = 0.02, size = 93, normalized size = 0.54

method	result
risch	$\frac{\sum_{-R=\text{RootOf}(3Z^4+7)} \frac{\ln(-R+x)}{-R^3}}{12}$
default	$\frac{\sqrt{3} 21^{1/4} \sqrt{2} \left(\ln \left(\frac{x^2 + \sqrt{3} 21^{1/4} x \sqrt{2} + \sqrt{21}}{x^2 - \sqrt{3} 21^{1/4} x \sqrt{2} + \sqrt{21}} \right) + 2 \arctan \left(\frac{\sqrt{2} \sqrt{3} 21^{3/4} x + 1}{21} \right) + 2 \arctan \left(\frac{\sqrt{2} \sqrt{3} 21^{3/4} x - 1}{21} \right) \right)}{168}$
meijerg	$1029^{3/4} \left(-\frac{x \sqrt{2} \ln \left(1 - \frac{\sqrt{2} 3^{1/4} 7^{3/4} (x^4)^{1/4} + \sqrt{3} \sqrt{7} \sqrt{x^4}}{7} \right)}{2 (x^4)^{1/4}} + \frac{x \sqrt{2} \arctan \left(\frac{\sqrt{2} 3^{1/4} 7^{3/4} (x^4)^{1/4}}{14 - \sqrt{2} 3^{1/4} 7^{3/4} (x^4)^{1/4}} \right)}{(x^4)^{1/4}} + \frac{x \sqrt{2} \ln \left(1 + \frac{\sqrt{2} 3^{1/4} 7^{3/4} (x^4)^{1/4}}{7} \right)}{2 (x^4)^{1/4}} \right)$

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Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x^4+7),x,method=_RETURNVERBOSE)

[Out] $\frac{1}{168} \cdot 3^{\frac{1}{2}} \cdot 21^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot (\ln((x^2 + \frac{1}{3} \cdot 3^{\frac{1}{2}} \cdot 21^{\frac{1}{4}}) \cdot x \cdot 2^{\frac{1}{2}} + \frac{1}{3} \cdot 21^{\frac{1}{2}})) / (x^2 - \frac{1}{3} \cdot 3^{\frac{1}{2}} \cdot 21^{\frac{1}{4}} \cdot x \cdot 2^{\frac{1}{2}} + \frac{1}{3} \cdot 21^{\frac{1}{2}})) + 2 \cdot \arctan(1/21 \cdot 2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} \cdot 21^{\frac{3}{4}} \cdot x + 1) + 2 \cdot \arctan(1/21 \cdot 2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} \cdot 21^{\frac{3}{4}} \cdot x - 1)$

Maxima [A]

time = 0.37, size = 151, normalized size = 0.88

$$\frac{1}{84} \cdot 7^{\frac{1}{4}} \cdot 3^{\frac{3}{4}} \sqrt{2} \arctan\left(\frac{1}{42} \cdot 7^{\frac{3}{4}} \cdot 3^{\frac{3}{4}} \sqrt{2} (2\sqrt{3}x + 7^{\frac{1}{4}} \cdot 3^{\frac{1}{4}} \sqrt{2})\right) + \frac{1}{84} \cdot 7^{\frac{1}{4}} \cdot 3^{\frac{3}{4}} \sqrt{2} \arctan\left(\frac{1}{42} \cdot 7^{\frac{3}{4}} \cdot 3^{\frac{3}{4}} \sqrt{2} (2\sqrt{3}x - 7^{\frac{1}{4}} \cdot 3^{\frac{1}{4}} \sqrt{2})\right) + \frac{1}{168} \cdot 7^{\frac{1}{4}} \cdot 3^{\frac{3}{4}} \sqrt{2} \log(\sqrt{3}x^2 + 7^{\frac{1}{4}} \cdot 3^{\frac{1}{4}} \sqrt{2}x + \sqrt{7}) - \frac{1}{168} \cdot 7^{\frac{1}{4}} \cdot 3^{\frac{3}{4}} \sqrt{2} \log(\sqrt{3}x^2 - 7^{\frac{1}{4}} \cdot 3^{\frac{1}{4}} \sqrt{2}x + \sqrt{7})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^4+7),x, algorithm="maxima")

[Out] $\frac{1}{84} \cdot 7^{\frac{1}{4}} \cdot 3^{\frac{3}{4}} \cdot \sqrt{2} \cdot \arctan(1/42 \cdot 7^{\frac{3}{4}} \cdot 3^{\frac{3}{4}} \cdot \sqrt{2}) \cdot (2 \cdot \sqrt{3} \cdot x + 7^{\frac{1}{4}} \cdot 3^{\frac{1}{4}} \cdot \sqrt{2}) + \frac{1}{84} \cdot 7^{\frac{1}{4}} \cdot 3^{\frac{3}{4}} \cdot \sqrt{2} \cdot \arctan(1/42 \cdot 7^{\frac{3}{4}} \cdot 3^{\frac{3}{4}} \cdot \sqrt{2}) \cdot (2 \cdot \sqrt{3} \cdot x - 7^{\frac{1}{4}} \cdot 3^{\frac{1}{4}} \cdot \sqrt{2}) + \frac{1}{168} \cdot 7^{\frac{1}{4}} \cdot 3^{\frac{3}{4}} \cdot \sqrt{2} \cdot \log(\sqrt{3} \cdot x^2 + 7^{\frac{1}{4}} \cdot 3^{\frac{1}{4}} \cdot \sqrt{2} \cdot x + \sqrt{7}) - \frac{1}{168} \cdot 7^{\frac{1}{4}} \cdot 3^{\frac{3}{4}} \cdot \sqrt{2} \cdot \log(\sqrt{3} \cdot x^2 - 7^{\frac{1}{4}} \cdot 3^{\frac{1}{4}} \cdot \sqrt{2} \cdot x + \sqrt{7})$

Fricas [A]

time = 0.33, size = 161, normalized size = 0.94

$$-\frac{1}{2058} \cdot 1029^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{1}{147} \cdot 1029^{\frac{3}{4}} \sqrt{2} \sqrt{1029^{\frac{1}{4}} \sqrt{2} x + 147 \sqrt{21}} - \frac{1}{7} \cdot 1029^{\frac{1}{4}} \sqrt{2} x - 1\right) - \frac{1}{2058} \cdot 1029^{\frac{1}{4}} \sqrt{2} \arctan\left(-\frac{1}{7} \cdot 1029^{\frac{3}{4}} \sqrt{2} x + \frac{1}{2058} \cdot 1029^{\frac{1}{4}} \sqrt{-588 \cdot 1029^{\frac{3}{4}} \sqrt{2} x + 86436 x^2 + 28812 \sqrt{21}} + 1\right) + \frac{1}{8232} \cdot 1029^{\frac{1}{4}} \sqrt{2} \log(588 \cdot 1029^{\frac{3}{4}} \sqrt{2} x + 86436 x^2 + 28812 \sqrt{21}) - \frac{1}{8232} \cdot 1029^{\frac{1}{4}} \sqrt{2} \log(-588 \cdot 1029^{\frac{3}{4}} \sqrt{2} x + 86436 x^2 + 28812 \sqrt{21})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^4+7),x, algorithm="fricas")

[Out] $-1/2058 \cdot 1029^{\frac{3}{4}} \cdot \sqrt{2} \cdot \arctan(1/147 \cdot 1029^{\frac{1}{4}} \cdot \sqrt{3} \cdot \sqrt{2}) \cdot \sqrt{1029^{\frac{3}{4}} \cdot \sqrt{2} \cdot x + 147 \cdot x^2 + 49 \cdot \sqrt{21}} - 1/7 \cdot 1029^{\frac{1}{4}} \cdot \sqrt{2} \cdot x - 1 - 1/2058 \cdot 1029^{\frac{3}{4}} \cdot \sqrt{2} \cdot \arctan(-1/7 \cdot 1029^{\frac{1}{4}} \cdot \sqrt{2} \cdot x + 1/2058 \cdot 1029^{\frac{1}{4}} \cdot \sqrt{2}) \cdot \sqrt{-588 \cdot 1029^{\frac{3}{4}} \cdot \sqrt{2} \cdot x + 86436 \cdot x^2 + 28812 \cdot \sqrt{21}} + 1 + 1/8232 \cdot 1029^{\frac{3}{4}} \cdot \sqrt{2} \cdot \log(588 \cdot 1029^{\frac{3}{4}} \cdot \sqrt{2} \cdot x + 86436 \cdot x^2 + 28812 \cdot \sqrt{21}) - 1/8232 \cdot 1029^{\frac{3}{4}} \cdot \sqrt{2} \cdot \log(-588 \cdot 1029^{\frac{3}{4}} \cdot \sqrt{2} \cdot x + 86436 \cdot x^2 + 28812 \cdot \sqrt{21})$

Sympy [A]

time = 0.20, size = 151, normalized size = 0.88

$$-\frac{\sqrt[4]{189} \sqrt{2} \log\left(x^2 - \frac{\sqrt[4]{189} \sqrt{2} x + \sqrt[4]{21}}{3}\right)}{168} + \frac{\sqrt[4]{189} \sqrt{2} \log\left(x^2 + \frac{\sqrt[4]{189} \sqrt{2} x + \sqrt[4]{21}}{3}\right)}{168} + \frac{\sqrt{2} \cdot 3^{\frac{3}{4}} \cdot \sqrt[4]{7} \operatorname{atan}\left(\frac{\sqrt{2} \cdot \sqrt[4]{3} \cdot 7^{\frac{3}{4}} x - 1}{7}\right)}{84} + \frac{\sqrt{2} \cdot 3^{\frac{3}{4}} \cdot \sqrt[4]{7} \operatorname{atan}\left(\frac{\sqrt{2} \cdot \sqrt[4]{3} \cdot 7^{\frac{3}{4}} x + 1}{7}\right)}{84}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x**4+7),x)

[Out] $-189^{**\frac{1}{4}} \cdot \sqrt{2} \cdot \log(x^{**2} - 189^{**\frac{1}{4}} \cdot \sqrt{2} \cdot x/3 + \sqrt{21}/3)/168 + 189^{**\frac{1}{4}} \cdot \sqrt{2} \cdot \log(x^{**2} + 189^{**\frac{1}{4}} \cdot \sqrt{2} \cdot x/3 + \sqrt{21}/3)/168 + \sqrt{2} \cdot 3^{\frac{3}{4}} \cdot \sqrt[4]{7} \cdot \operatorname{atan}\left(\frac{\sqrt{2} \cdot \sqrt[4]{3} \cdot 7^{\frac{3}{4}} x - 1}{7}\right) + \sqrt{2} \cdot 3^{\frac{3}{4}} \cdot \sqrt[4]{7} \cdot \operatorname{atan}\left(\frac{\sqrt{2} \cdot \sqrt[4]{3} \cdot 7^{\frac{3}{4}} x + 1}{7}\right)$

$t(2)*3**(3/4)*7**(1/4)*atan(sqrt(2)*3**(1/4)*7**(3/4)*x/7 - 1)/84 + sqrt(2)*3**(3/4)*7**(1/4)*atan(sqrt(2)*3**(1/4)*7**(3/4)*x/7 + 1)/84$

Giac [A]

time = 0.00, size = 176, normalized size = 1.03

$$-\frac{1}{168} \cdot 756^{\frac{1}{4}} \ln \left(x^2 - \left(\frac{7}{3}\right)^{\frac{1}{4}} \sqrt{2} x + \left(\frac{7}{3}\right)^{\frac{1}{4}} \left(\frac{7}{3}\right)^{\frac{1}{4}} \right) + \frac{1}{84} \cdot 756^{\frac{1}{4}} \arctan \left(\frac{x - \frac{1}{2} \sqrt{2} \left(\frac{7}{3}\right)^{\frac{1}{4}}}{\frac{1}{2} \sqrt{2} \left(\frac{7}{3}\right)^{\frac{1}{4}}} \right) + \frac{1}{168} \cdot 756^{\frac{1}{4}} \ln \left(x^2 + \left(\frac{7}{3}\right)^{\frac{1}{4}} \sqrt{2} x + \left(\frac{7}{3}\right)^{\frac{1}{4}} \left(\frac{7}{3}\right)^{\frac{1}{4}} \right) + \frac{1}{84} \cdot 756^{\frac{1}{4}} \arctan \left(\frac{x + \frac{1}{2} \sqrt{2} \left(\frac{7}{3}\right)^{\frac{1}{4}}}{\frac{1}{2} \sqrt{2} \left(\frac{7}{3}\right)^{\frac{1}{4}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^4+7),x)

[Out] 1/84*756^(1/4)*arctan(3/14*(7/3)^(3/4)*sqrt(2)*(2*x + (7/3)^(1/4)*sqrt(2))) + 1/84*756^(1/4)*arctan(3/14*(7/3)^(3/4)*sqrt(2)*(2*x - (7/3)^(1/4)*sqrt(2))) + 1/168*756^(1/4)*log(x^2 + (7/3)^(1/4)*sqrt(2)*x + sqrt(7/3)) - 1/168*756^(1/4)*log(x^2 - (7/3)^(1/4)*sqrt(2)*x + sqrt(7/3))

Mupad [B]

time = 0.11, size = 45, normalized size = 0.26

$$\sqrt{2} \cdot 189^{1/4} \operatorname{atan} \left(\sqrt{2} \cdot 189^{3/4} x \left(\frac{1}{126} - \frac{1}{126} i \right) \right) \left(\frac{1}{84} + \frac{1}{84} i \right) + \sqrt{2} \cdot 189^{1/4} \operatorname{atan} \left(\sqrt{2} \cdot 189^{3/4} x \left(\frac{1}{126} + \frac{1}{126} i \right) \right) \left(\frac{1}{84} - \frac{1}{84} i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x^4 + 7),x)

[Out] 2^(1/2)*189^(1/4)*atan(2^(1/2)*189^(3/4)*x*(1/126 - 1i/126))*(1/84 + 1i/84) + 2^(1/2)*189^(1/4)*atan(2^(1/2)*189^(3/4)*x*(1/126 + 1i/126))*(1/84 - 1i/84)

$$3.39 \quad \int \frac{1}{-1+3x^2+x^4} dx$$

Optimal. Leaf size=73

$$-\sqrt{\frac{2}{13(3+\sqrt{13})}} \tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}} x\right) - \sqrt{\frac{1}{26}(3+\sqrt{13})} \tanh^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{13}}} x\right)$$

[Out] $-1/13*\arctan(x*2^{(1/2)/(3+13^{(1/2)})}^{(1/2)})*26^{(1/2)/(3+13^{(1/2)})}^{(1/2)}-1/26*\operatorname{arctanh}(x*2^{(1/2)/(-3+13^{(1/2)})}^{(1/2)})*(78+26*13^{(1/2)})^{(1/2)}$

Rubi [A]

time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1107, 213, 209}

$$-\sqrt{\frac{2}{13(3+\sqrt{13})}} \tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}} x\right) - \sqrt{\frac{1}{26}(3+\sqrt{13})} \tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}} x\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-1 + 3*x^2 + x^4)^{-1}, x]$

[Out] $-(\text{Sqrt}[2/(13*(3 + \text{Sqrt}[13]))]*\text{ArcTan}[\text{Sqrt}[2/(3 + \text{Sqrt}[13])]*x]) - \text{Sqrt}[(3 + \text{Sqrt}[13])/26]*\text{ArcTanh}[\text{Sqrt}[2/(-3 + \text{Sqrt}[13])]*x]$

Rule 209

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))*\text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{GtQ}[b, 0])$

Rule 213

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[b, 2])^{-1})*\text{ArcTanh}[\text{Rt}[b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{GtQ}[b, 0])$

Rule 1107

$\text{Int}[(a_ + (b_)*(x_)^2 + (c_)*(x_)^4)^{-1}, x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[c/q, \text{Int}[1/(b/2 - q/2 + c*x^2), x], x] - \text{Dist}[c/q, \text{Int}[1/(b/2 + q/2 + c*x^2), x], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{PosQ}[b^2 - 4*a*c]$

Rubi steps

$$\int \frac{1}{-1 + 3x^2 + x^4} dx = \frac{\int \frac{1}{\frac{3}{2} - \frac{\sqrt{13}}{2} + x^2} dx}{\sqrt{13}} - \frac{\int \frac{1}{\frac{3}{2} + \frac{\sqrt{13}}{2} + x^2} dx}{\sqrt{13}}$$

$$= -\sqrt{\frac{2}{13(3 + \sqrt{13})}} \tan^{-1}\left(\sqrt{\frac{2}{3 + \sqrt{13}}} x\right) - \sqrt{\frac{1}{26}(3 + \sqrt{13})} \tanh^{-1}\left(\sqrt{\frac{2}{-3 - \sqrt{13}}} x\right)$$

Mathematica [A]

time = 0.04, size = 68, normalized size = 0.93

$$\frac{\sqrt{-3 + \sqrt{13}} \tan^{-1}\left(\sqrt{\frac{2}{3 + \sqrt{13}}} x\right) + \sqrt{3 + \sqrt{13}} \tanh^{-1}\left(\sqrt{\frac{2}{-3 + \sqrt{13}}} x\right)}{\sqrt{26}}$$

Antiderivative was successfully verified.

`[In] Integrate[(-1 + 3*x^2 + x^4)^(-1), x]``[Out] -((Sqrt[-3 + Sqrt[13]]*ArcTan[Sqrt[2/(3 + Sqrt[13])]]*x) + Sqrt[3 + Sqrt[13]]*ArcTanh[Sqrt[2/(-3 + Sqrt[13])]]*x))/Sqrt[26])`**Mathics [B]** Leaf count is larger than twice the leaf count of optimal. 158 vs. 2(73) = 146.

time = 3.24, size = 107, normalized size = 1.47

$$\frac{\sqrt{26} \left(-2\sqrt{-3 + \sqrt{13}} \operatorname{ArcTan}\left[\frac{2\sqrt{2}x}{\sqrt{-3 + \sqrt{13}}(3 + \sqrt{13})}\right] - \sqrt{3 + \sqrt{13}} \operatorname{Log}\left[\frac{-3\sqrt{2}\sqrt{3 + \sqrt{13}}}{4} + \frac{\sqrt{26}\sqrt{3 + \sqrt{13}}}{4} + x\right] + \sqrt{3 + \sqrt{13}} \operatorname{Log}\left[-\frac{\sqrt{26}\sqrt{3 + \sqrt{13}}}{4} + \frac{3\sqrt{2}\sqrt{3 + \sqrt{13}}}{4} + x\right] \right)}{52}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^4+3*x^2-1), x]')``[Out] Sqrt[26] (-2 Sqrt[-3 + Sqrt[13]] ArcTan[2 Sqrt[2] x / (Sqrt[-3 + Sqrt[13]] (3 + Sqrt[13]))] - Sqrt[3 + Sqrt[13]] Log[-3 Sqrt[2] Sqrt[3 + Sqrt[13]] / 4 + Sqrt[26] Sqrt[3 + Sqrt[13]] / 4 + x] + Sqrt[3 + Sqrt[13]] Log[-Sqrt[26] Sqrt[3 + Sqrt[13]] / 4 + 3 Sqrt[2] Sqrt[3 + Sqrt[13]] / 4 + x]) / 52`**Maple [A]**

time = 0.03, size = 56, normalized size = 0.77

method	result	size
risch	$\frac{\sum_{-R=\text{RootOf}(-Z^4+3-Z^2-1)} \frac{\ln(-R+x)}{2-R^3+3-R}}{2}$	35

default	$-\frac{2\sqrt{13} \operatorname{arctanh}\left(\frac{2x}{\sqrt{-6+2\sqrt{13}}}\right)}{13\sqrt{-6+2\sqrt{13}}} - \frac{2\sqrt{13} \operatorname{arctan}\left(\frac{2x}{\sqrt{6+2\sqrt{13}}}\right)}{13\sqrt{6+2\sqrt{13}}}$	56
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4+3*x^2-1),x,method=_RETURNVERBOSE)`

[Out] $-2/13 \cdot 13^{1/2} / (-6+2 \cdot 13^{1/2})^{1/2} \cdot \operatorname{arctanh}(2x / (-6+2 \cdot 13^{1/2})^{1/2}) - 2/13 \cdot 13^{1/2} / (6+2 \cdot 13^{1/2})^{1/2} \cdot \operatorname{arctan}(2x / (6+2 \cdot 13^{1/2})^{1/2})$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4+3*x^2-1),x, algorithm="maxima")`

[Out] `integrate(1/(x^4 + 3*x^2 - 1), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(50) = 100$.

time = 0.34, size = 132, normalized size = 1.81

$$\frac{1}{13} \sqrt{26} \sqrt{\sqrt{13}-3} \operatorname{arctan}\left(\frac{1}{52} \sqrt{26} \sqrt{13} \sqrt{2x^2+\sqrt{13}+3} \sqrt{\sqrt{13}-3} - \frac{1}{26} \sqrt{26} \sqrt{13} x \sqrt{\sqrt{13}-3}\right) + \frac{1}{52} \sqrt{26} \sqrt{\sqrt{13}+3} \log\left(\sqrt{26}(3\sqrt{13}-13)\sqrt{\sqrt{13}+3}+52x\right) - \frac{1}{52} \sqrt{26} \sqrt{\sqrt{13}+3} \log\left(-\sqrt{26}(3\sqrt{13}-13)\sqrt{\sqrt{13}+3}+52x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4+3*x^2-1),x, algorithm="fricas")`

[Out] $1/13 \cdot \sqrt{26} \cdot \sqrt{\sqrt{13}-3} \cdot \operatorname{arctan}(1/52 \cdot \sqrt{26} \cdot \sqrt{13} \cdot \sqrt{2x^2+\sqrt{13}+3} \cdot \sqrt{\sqrt{13}-3} - 1/26 \cdot \sqrt{26} \cdot \sqrt{13} \cdot x \cdot \sqrt{\sqrt{13}-3}) + 1/52 \cdot \sqrt{26} \cdot \sqrt{\sqrt{13}+3} \cdot \log(\sqrt{26} \cdot (3 \cdot \sqrt{13}-13) \cdot \sqrt{\sqrt{13}+3} + 52x) - 1/52 \cdot \sqrt{26} \cdot \sqrt{\sqrt{13}+3} \cdot \log(-\sqrt{26} \cdot (3 \cdot \sqrt{13}-13) \cdot \sqrt{\sqrt{13}+3} + 52x)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(68) = 136$

time = 0.24, size = 146, normalized size = 2.00

$$\sqrt{\frac{3}{104} + \frac{\sqrt{13}}{104}} \log\left(x - 22\sqrt{\frac{3}{104} + \frac{\sqrt{13}}{104}} + 312\left(\frac{3}{104} + \frac{\sqrt{13}}{104}\right)^{\frac{3}{2}}\right) - \sqrt{\frac{3}{104} + \frac{\sqrt{13}}{104}} \log\left(x - 312\left(\frac{3}{104} + \frac{\sqrt{13}}{104}\right)^{\frac{3}{2}} + 22\sqrt{\frac{3}{104} + \frac{\sqrt{13}}{104}}\right) - 2\sqrt{\frac{3}{104} + \frac{\sqrt{13}}{104}} \operatorname{atan}\left(\frac{2\sqrt{2}x}{3\sqrt{-3+\sqrt{13}} + \sqrt{13}\sqrt{-3+\sqrt{13}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**4+3*x**2-1),x)`

[Out] $\sqrt{3/104 + \sqrt{13}/104} \cdot \log(x - 22 \cdot \sqrt{3/104 + \sqrt{13}/104}) + 312 \cdot (3/104 + \sqrt{13}/104) \cdot (3/2) - \sqrt{3/104 + \sqrt{13}/104} \cdot \log(x - 312 \cdot (3/104 + \sqrt{13}/104) \cdot (3/2) + 22 \cdot \sqrt{3/104 + \sqrt{13}/104}) - 2 \cdot \sqrt{-3/104 + \sqrt{13}/104} \cdot \operatorname{atan}(2 \cdot \sqrt{2} \cdot x / (3 \cdot \sqrt{-3 + \sqrt{13}}) + \sqrt{13} \cdot \sqrt{-3 + \sqrt{13}}))$

Giac [A]

time = 0.00, size = 108, normalized size = 1.48

$$\frac{1}{52} \sqrt{26(\sqrt{13}+3)} \ln \left| x - \sqrt{\frac{-3+\sqrt{13}}{2}} \right| - \frac{1}{52} \sqrt{26(\sqrt{13}+3)} \ln \left| x + \sqrt{\frac{-3+\sqrt{13}}{2}} \right| - \frac{1}{26} \sqrt{26(\sqrt{13}-3)} \operatorname{arctan} \left(\frac{x}{\sqrt{\frac{-3-\sqrt{13}}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(1/(x^4+3x^2-1),x)$

[Out] $-1/26 \cdot \sqrt{26 \cdot \sqrt{13} - 78} \cdot \operatorname{arctan}(x/\sqrt{1/2 \cdot \sqrt{13} + 3/2}) - 1/52 \cdot \sqrt{26 \cdot \sqrt{13} + 78} \cdot \log(\operatorname{abs}(x + \sqrt{1/2 \cdot \sqrt{13} - 3/2})) + 1/52 \cdot \sqrt{26 \cdot \sqrt{13} + 78} \cdot \log(\operatorname{abs}(x - \sqrt{1/2 \cdot \sqrt{13} - 3/2}))$

Mupad [B]

time = 0.22, size = 93, normalized size = 1.27

$$\frac{\sqrt{26} \operatorname{atanh} \left(\frac{\sqrt{26} x}{2\sqrt{\sqrt{13}+3}} + \frac{3\sqrt{13}\sqrt{26} x}{26\sqrt{\sqrt{13}+3}} \right) \sqrt{\sqrt{13}+3}}{26} - \frac{\sqrt{26} \operatorname{atanh} \left(\frac{\sqrt{26} x}{2\sqrt{3-\sqrt{13}}} - \frac{3\sqrt{13}\sqrt{26} x}{26\sqrt{3-\sqrt{13}}} \right) \sqrt{3-\sqrt{13}}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}(1/(3x^2 + x^4 - 1),x)$

[Out] $-(26^{1/2} \cdot \operatorname{atanh}((26^{1/2} \cdot x)/(2 \cdot (13^{1/2} + 3)^{1/2})) + (3 \cdot 13^{1/2} \cdot 26^{1/2} \cdot x)/(26 \cdot (13^{1/2} + 3)^{1/2})) \cdot (13^{1/2} + 3)^{1/2} / 26 - (26^{1/2} \cdot \operatorname{atanh}((26^{1/2} \cdot x)/(2 \cdot (3 - 13^{1/2})^{1/2})) - (3 \cdot 13^{1/2} \cdot 26^{1/2} \cdot x)/(26 \cdot (3 - 13^{1/2})^{1/2})) \cdot (3 - 13^{1/2})^{1/2} / 26$

$$3.40 \quad \int \frac{1}{-1-3x^2+x^4} dx$$

Optimal. Leaf size=73

$$-\sqrt{\frac{1}{26} (3 + \sqrt{13})} \tan^{-1} \left(\sqrt{\frac{2}{-3 + \sqrt{13}}} x \right) - \sqrt{\frac{2}{13 (3 + \sqrt{13})}} \tanh^{-1} \left(\sqrt{\frac{2}{3 + \sqrt{13}}} x \right)$$

[Out] $-1/13*\operatorname{arctanh}(x*2^{(1/2)}/(3+13^{(1/2)})^{(1/2)})*26^{(1/2)}/(3+13^{(1/2)})^{(1/2)}-1/26*\operatorname{arctan}(x*2^{(1/2)}/(-3+13^{(1/2)})^{(1/2)})*(78+26*13^{(1/2)})^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1107, 213, 209}

$$-\sqrt{\frac{1}{26} (3 + \sqrt{13})} \tan^{-1} \left(\sqrt{\frac{2}{\sqrt{13} - 3}} x \right) - \sqrt{\frac{2}{13 (3 + \sqrt{13})}} \tanh^{-1} \left(\sqrt{\frac{2}{3 + \sqrt{13}}} x \right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(-1 - 3*x^2 + x^4)^{-1}, x]$

[Out] $-(\operatorname{Sqrt}[(3 + \operatorname{Sqrt}[13])/26]*\operatorname{ArcTan}[\operatorname{Sqrt}[2/(-3 + \operatorname{Sqrt}[13])]*x]) - \operatorname{Sqrt}[2/(13*(3 + \operatorname{Sqrt}[13]))]*\operatorname{ArcTanh}[\operatorname{Sqrt}[2/(3 + \operatorname{Sqrt}[13])]*x]$

Rule 209

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))*\operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{GtQ}[b, 0])$

Rule 213

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2])^{-1})*\operatorname{ArcTanh}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{GtQ}[b, 0])$

Rule 1107

$\operatorname{Int}[(a_ + (b_)*(x_)^2 + (c_)*(x_)^4)^{-1}, x_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[b^2 - 4*a*c, 2]\}, \operatorname{Dist}[c/q, \operatorname{Int}[1/(b/2 - q/2 + c*x^2), x], x] - \operatorname{Dist}[c/q, \operatorname{Int}[1/(b/2 + q/2 + c*x^2), x], x] /; \operatorname{FreeQ}\{a, b, c\}, x] \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \&\& \operatorname{PosQ}[b^2 - 4*a*c]$

Rubi steps

$$\int \frac{1}{-1 - 3x^2 + x^4} dx = \frac{\int \frac{1}{-\frac{3}{2} - \frac{\sqrt{13}}{2} + x^2} dx}{\sqrt{13}} - \frac{\int \frac{1}{-\frac{3}{2} + \frac{\sqrt{13}}{2} + x^2} dx}{\sqrt{13}}$$

$$= -\sqrt{\frac{1}{26} (3 + \sqrt{13})} \tan^{-1} \left(\sqrt{\frac{2}{-3 + \sqrt{13}}} x \right) - \sqrt{\frac{2}{13 (3 + \sqrt{13})}} \tanh^{-1} \left(\sqrt{\frac{2}{3 + \sqrt{13}}} x \right)$$

Mathematica [A]

time = 0.02, size = 68, normalized size = 0.93

$$\frac{\sqrt{3 + \sqrt{13}} \tan^{-1} \left(\sqrt{\frac{2}{-3 + \sqrt{13}}} x \right) + \sqrt{-3 + \sqrt{13}} \tanh^{-1} \left(\sqrt{\frac{2}{3 + \sqrt{13}}} x \right)}{\sqrt{26}}$$

Antiderivative was successfully verified.

`[In] Integrate[(-1 - 3*x^2 + x^4)^(-1), x]``[Out] -((Sqrt[3 + Sqrt[13]]*ArcTan[Sqrt[2/(-3 + Sqrt[13])]]*x) + Sqrt[-3 + Sqrt[13]]*ArcTanh[Sqrt[2/(3 + Sqrt[13])]]*x))/Sqrt[26])`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.94, size = 33, normalized size = 0.45

$$\text{RootSum}[-1 + 156\#1^2 + 2704\#1^4 \&, \text{Log}[x - 312\#1^3 - 22\#1] \#1 \&]$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^4-3*x^2-1), x]')``[Out] RootSum[-1 + 156 #1 ^ 2 + 2704 #1 ^ 4&, Log[x - 312 #1 ^ 3 - 22 #1] #1&]`**Maple [A]**

time = 0.03, size = 56, normalized size = 0.77

method	result	size
risch	$\frac{\left(\sum_{R=\text{RootOf}(_Z^4-3_Z^2-1)} \frac{\ln(_R+x)}{2_R^3-3_R} \right)}{2}$	35

default	$-\frac{2\sqrt{13} \operatorname{arctanh}\left(\frac{2x}{\sqrt{6+2\sqrt{13}}}\right)}{13\sqrt{6+2\sqrt{13}}} - \frac{2\sqrt{13} \operatorname{arctan}\left(\frac{2x}{\sqrt{-6+2\sqrt{13}}}\right)}{13\sqrt{-6+2\sqrt{13}}}$	56
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4-3*x^2-1),x,method=_RETURNVERBOSE)`

[Out] $-2/13*13^{(1/2)}/(6+2*13^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2*x/(6+2*13^{(1/2)})^{(1/2)})-2/13*13^{(1/2)}/(-6+2*13^{(1/2)})^{(1/2)}*\operatorname{arctan}(2*x/(-6+2*13^{(1/2)})^{(1/2)})$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4-3*x^2-1),x, algorithm="maxima")`

[Out] `integrate(1/(x^4 - 3*x^2 - 1), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(50) = 100.

time = 0.35, size = 132, normalized size = 1.81

$$\frac{1}{13}\sqrt{26}\sqrt{\sqrt{13}+3}\operatorname{arctan}\left(\frac{1}{52}\sqrt{26}\sqrt{13}\sqrt{2x^2+\sqrt{13}-3}\sqrt{\sqrt{13}+3}-\frac{1}{26}\sqrt{26}\sqrt{13}x\sqrt{\sqrt{13}+3}\right)-\frac{1}{52}\sqrt{26}\sqrt{\sqrt{13}-3}\log\left(\sqrt{26}(3\sqrt{13}+13)\sqrt{\sqrt{13}-3}+52x\right)+\frac{1}{52}\sqrt{26}\sqrt{\sqrt{13}-3}\log\left(-\sqrt{26}(3\sqrt{13}+13)\sqrt{\sqrt{13}-3}+52x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4-3*x^2-1),x, algorithm="fricas")`

[Out] $1/13*\operatorname{sqrt}(26)*\operatorname{sqrt}(\operatorname{sqrt}(13)+3)*\operatorname{arctan}(1/52*\operatorname{sqrt}(26)*\operatorname{sqrt}(13)*\operatorname{sqrt}(2)*\operatorname{sqrt}(2*x^2+\operatorname{sqrt}(13)-3)*\operatorname{sqrt}(\operatorname{sqrt}(13)+3)-1/26*\operatorname{sqrt}(26)*\operatorname{sqrt}(13)*x*\operatorname{sqrt}(\operatorname{sqrt}(13)+3))-1/52*\operatorname{sqrt}(26)*\operatorname{sqrt}(\operatorname{sqrt}(13)-3)*\log(\operatorname{sqrt}(26)*(3*\operatorname{sqrt}(13)+13)*\operatorname{sqrt}(\operatorname{sqrt}(13)-3)+52*x)+1/52*\operatorname{sqrt}(26)*\operatorname{sqrt}(\operatorname{sqrt}(13)-3)*\log(-\operatorname{sqrt}(26)*(3*\operatorname{sqrt}(13)+13)*\operatorname{sqrt}(\operatorname{sqrt}(13)-3)+52*x)$

Sympy [A]

time = 0.19, size = 24, normalized size = 0.33

$$\operatorname{RootSum}(2704t^4 + 156t^2 - 1, (t \mapsto t \log(-312t^3 - 22t + x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**4-3*x**2-1),x)`

[Out] `RootSum(2704*_t**4 + 156*_t**2 - 1, Lambda(_t, _t*log(-312*_t**3 - 22*_t + x)))`

Giac [A]

time = 0.00, size = 108, normalized size = 1.48

$$\frac{1}{52}\sqrt{26(\sqrt{13}-3)} \ln \left| x - \sqrt{\frac{3+\sqrt{13}}{2}} \right| - \frac{1}{52}\sqrt{26(\sqrt{13}-3)} \ln \left| x + \sqrt{\frac{3+\sqrt{13}}{2}} \right| - \frac{1}{26}\sqrt{26(\sqrt{13}+3)} \arctan \left(\frac{x}{\sqrt{\frac{3-\sqrt{13}}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(x^4-3*x^2-1),x)`

`[Out] -1/26*sqrt(26*sqrt(13) + 78)*arctan(x/sqrt(1/2*sqrt(13) - 3/2)) - 1/52*sqrt(26*sqrt(13) - 78)*log(abs(x + sqrt(1/2*sqrt(13) + 3/2))) + 1/52*sqrt(26*sqrt(13) - 78)*log(abs(x - sqrt(1/2*sqrt(13) + 3/2)))`

Mupad [B]

time = 0.13, size = 93, normalized size = 1.27

$$\frac{\sqrt{26} \operatorname{atanh} \left(\frac{\sqrt{26} x}{2\sqrt{\sqrt{13}-3}} - \frac{3\sqrt{13}\sqrt{26} x}{26\sqrt{\sqrt{13}-3}} \right) \sqrt{\sqrt{13}-3}}{26} - \frac{\sqrt{26} \operatorname{atanh} \left(\frac{\sqrt{26} x}{2\sqrt{-\sqrt{13}-3}} + \frac{3\sqrt{13}\sqrt{26} x}{26\sqrt{-\sqrt{13}-3}} \right) \sqrt{-\sqrt{13}-3}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(-1/(3*x^2 - x^4 + 1),x)`

`[Out] - (26^(1/2)*atanh((26^(1/2)*x)/(2*(13^(1/2) - 3)^(1/2)) - (3*13^(1/2)*26^(1/2)*x)/(26*(13^(1/2) - 3)^(1/2)))*(13^(1/2) - 3)^(1/2))/26 - (26^(1/2)*atanh((26^(1/2)*x)/(2*(-13^(1/2) - 3)^(1/2)) + (3*13^(1/2)*26^(1/2)*x)/(26*(-13^(1/2) - 3)^(1/2)))*(-13^(1/2) - 3)^(1/2))/26`

$$3.41 \quad \int \frac{1}{1-3x^2+x^4} dx$$

Optimal. Leaf size=72

$$-\sqrt{\frac{2}{5(3+\sqrt{5})}} \tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}} x\right) + \sqrt{\frac{1}{10}(3+\sqrt{5})} \tanh^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})} x\right)$$

[Out] $-1/5*\operatorname{arctanh}(x*2^{(1/2)/(3+5^{(1/2)})}^{(1/2)})*10^{(1/2)/(3+5^{(1/2)})}^{(1/2)}+\operatorname{arctanh}(x*(1/2+1/2*5^{(1/2)}))^{(1/2+1/10*5^{(1/2)})}$

Rubi [A]

time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1107, 213}

$$\sqrt{\frac{1}{10}(3+\sqrt{5})} \tanh^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})} x\right) - \sqrt{\frac{2}{5(3+\sqrt{5})}} \tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}} x\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(1 - 3*x^2 + x^4)^{-1}, x]$

[Out] $-(\operatorname{Sqrt}[2/(5*(3 + \operatorname{Sqrt}[5]))]*\operatorname{ArcTanh}[\operatorname{Sqrt}[2/(3 + \operatorname{Sqrt}[5])]*x]) + \operatorname{Sqrt}[(3 + \operatorname{Sqrt}[5])/10]*\operatorname{ArcTanh}[\operatorname{Sqrt}[(3 + \operatorname{Sqrt}[5])/2]*x]$

Rule 213

$\operatorname{Int}[(a_+ + (b_-)*(x_-)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2])^{-1})*\operatorname{ArcTanh}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1107

$\operatorname{Int}[(a_+ + (b_-)*(x_-)^2 + (c_-)*(x_-)^4)^{-1}, x_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[b^2 - 4*a*c, 2]\}, \operatorname{Dist}[c/q, \operatorname{Int}[1/(b/2 - q/2 + c*x^2), x], x] - \operatorname{Dist}[c/q, \operatorname{Int}[1/(b/2 + q/2 + c*x^2), x], x] /;$ FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{1}{1-3x^2+x^4} dx &= \frac{\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2} dx}{\sqrt{5}} - \frac{\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2} dx}{\sqrt{5}} \\ &= -\sqrt{\frac{2}{5(3+\sqrt{5})}} \tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}} x\right) + \sqrt{\frac{1}{10}(3+\sqrt{5})} \tanh^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})} x\right) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 83, normalized size = 1.15

$$\frac{1}{20} \left(- \left((5 + \sqrt{5}) \log(-1 + \sqrt{5} - 2x) \right) - (-5 + \sqrt{5}) \log(1 + \sqrt{5} - 2x) + (5 + \sqrt{5}) \log(-1 + \sqrt{5} + 2x) + (-5 + \sqrt{5}) \log(1 + \sqrt{5} + 2x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 3*x^2 + x^4)^(-1), x]

[Out] $-(5 + \sqrt{5}) \operatorname{Log}[-1 + \sqrt{5} - 2x] - (-5 + \sqrt{5}) \operatorname{Log}[1 + \sqrt{5} - 2x] + (5 + \sqrt{5}) \operatorname{Log}[-1 + \sqrt{5} + 2x] + (-5 + \sqrt{5}) \operatorname{Log}[1 + \sqrt{5} + 2x] / 20$

Mathics [A]

time = 2.86, size = 69, normalized size = 0.96

$$-\frac{(5 + \sqrt{5}) \operatorname{Log}\left[\frac{1}{2} - \frac{\sqrt{5}}{2} + x\right]}{20} - \frac{(5 - \sqrt{5}) \operatorname{Log}\left[\frac{1}{2} + \frac{\sqrt{5}}{2} + x\right]}{20} + \frac{(5 + \sqrt{5}) \operatorname{Log}\left[-\frac{1}{2} + \frac{\sqrt{5}}{2} + x\right]}{20} + \frac{(5 - \sqrt{5}) \operatorname{Log}\left[-\frac{1}{2} - \frac{\sqrt{5}}{2} + x\right]}{20}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x^4-3*x^2+1), x]')

[Out] $-(5 + \sqrt{5}) \operatorname{Log}[1/2 - \sqrt{5}/2 + x] / 20 - (5 - \sqrt{5}) \operatorname{Log}[1/2 + \sqrt{5}/2 + x] / 20 + (5 + \sqrt{5}) \operatorname{Log}[-1/2 + \sqrt{5}/2 + x] / 20 + (5 - \sqrt{5}) \operatorname{Log}[-1/2 - \sqrt{5}/2 + x] / 20$

Maple [A]

time = 0.02, size = 54, normalized size = 0.75

method	result
default	$-\frac{\ln(x^2+x-1)}{4} + \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)}{10} + \frac{\ln(x^2-x-1)}{4} + \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)}{10}$
risch	$\frac{\ln(2x-1+\sqrt{5})}{4} + \frac{\ln(2x-1+\sqrt{5})\sqrt{5}}{20} + \frac{\ln(2x-1-\sqrt{5})}{4} - \frac{\ln(2x-1-\sqrt{5})\sqrt{5}}{20} + \frac{\ln(2x+\sqrt{5}+1)\sqrt{5}}{20} - \frac{\ln(2x+\sqrt{5}+1)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4-3*x^2+1), x, method=_RETURNVERBOSE)

[Out] $-1/4 \ln(x^2+x-1) + 1/10 \cdot 5^{(1/2)} \cdot \operatorname{arctanh}(1/5 \cdot (1+2x) \cdot 5^{(1/2)}) + 1/4 \ln(x^2-x-1) + 1/10 \cdot 5^{(1/2)} \cdot \operatorname{arctanh}(1/5 \cdot (2x-1) \cdot 5^{(1/2)})$

Maxima [A]

time = 0.35, size = 75, normalized size = 1.04

$$-\frac{1}{20} \sqrt{5} \log\left(\frac{2x - \sqrt{5} + 1}{2x + \sqrt{5} + 1}\right) - \frac{1}{20} \sqrt{5} \log\left(\frac{2x - \sqrt{5} - 1}{2x + \sqrt{5} - 1}\right) - \frac{1}{4} \log(x^2 + x - 1) + \frac{1}{4} \log(x^2 - x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4-3*x^2+1),x, algorithm="maxima")

[Out] $-1/20*\sqrt{5}*\log((2*x - \sqrt{5} + 1)/(2*x + \sqrt{5} + 1)) - 1/20*\sqrt{5}*\log((2*x - \sqrt{5} - 1)/(2*x + \sqrt{5} - 1)) - 1/4*\log(x^2 + x - 1) + 1/4*\log(x^2 - x - 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(42) = 84.

time = 0.33, size = 91, normalized size = 1.26

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^2 + \sqrt{5}(2x+1) + 2x+3}{x^2+x-1}\right) + \frac{1}{20}\sqrt{5}\log\left(\frac{2x^2 + \sqrt{5}(2x-1) - 2x+3}{x^2-x-1}\right) - \frac{1}{4}\log(x^2+x-1) + \frac{1}{4}\log(x^2-x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4-3*x^2+1),x, algorithm="fricas")

[Out] $1/20*\sqrt{5}*\log((2*x^2 + \sqrt{5}*(2*x + 1) + 2*x + 3)/(x^2 + x - 1)) + 1/20*\sqrt{5}*\log((2*x^2 + \sqrt{5}*(2*x - 1) - 2*x + 3)/(x^2 - x - 1)) - 1/4*\log(x^2 + x - 1) + 1/4*\log(x^2 - x - 1)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(58) = 116

time = 0.19, size = 158, normalized size = 2.19

$$\left(\frac{\sqrt{5}}{20} + \frac{1}{4}\right)\log\left(x - \frac{7}{2} - \frac{7\sqrt{5}}{10} + 120\left(\frac{\sqrt{5}}{20} + \frac{1}{4}\right)^3\right) + \left(\frac{1}{4} - \frac{\sqrt{5}}{20}\right)\log\left(x - \frac{7}{2} + 120\left(\frac{1}{4} - \frac{\sqrt{5}}{20}\right)^3 + \frac{7\sqrt{5}}{10}\right) + \left(-\frac{1}{4} + \frac{\sqrt{5}}{20}\right)\log\left(x - \frac{7\sqrt{5}}{10} + 120\left(-\frac{1}{4} + \frac{\sqrt{5}}{20}\right)^3 + \frac{7}{2}\right) + \left(-\frac{1}{4} - \frac{\sqrt{5}}{20}\right)\log\left(x + 120\left(-\frac{1}{4} - \frac{\sqrt{5}}{20}\right)^3 + \frac{7\sqrt{5}}{10} + \frac{7}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**4-3*x**2+1),x)

[Out] $(\sqrt{5}/20 + 1/4)*\log(x - 7/2 - 7*\sqrt{5}/10 + 120*(\sqrt{5}/20 + 1/4)**3) + (1/4 - \sqrt{5}/20)*\log(x - 7/2 + 120*(1/4 - \sqrt{5}/20)**3 + 7*\sqrt{5}/10) + (-1/4 + \sqrt{5}/20)*\log(x - 7*\sqrt{5}/10 + 120*(-1/4 + \sqrt{5}/20)**3 + 7/2) + (-1/4 - \sqrt{5}/20)*\log(x + 120*(-1/4 - \sqrt{5}/20)**3 + 7*\sqrt{5}/10 + 7/2)$

Giac [A]

time = 0.00, size = 95, normalized size = 1.32

$$\frac{\ln|x^2 - x - 1|}{4} - \frac{\ln\left|\frac{2x-1-\sqrt{5}}{2x-1+\sqrt{5}}\right|}{4\sqrt{5}} - \frac{\ln|x^2 + x - 1|}{4} - \frac{\ln\left|\frac{2x+1-\sqrt{5}}{2x+1+\sqrt{5}}\right|}{4\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4-3*x^2+1),x)

[Out] $-1/20*\sqrt{5}*\log(\text{abs}((2*x - \sqrt{5}) + 1)/(2*x + \sqrt{5} + 1))) - 1/20*\sqrt{5}*\log(\text{abs}((2*x - \sqrt{5}) - 1)/(2*x + \sqrt{5} - 1))) - 1/4*\log(\text{abs}(x^2 + x - 1)) + 1/4*\log(\text{abs}(x^2 - x - 1))$

Mupad [B]

time = 0.10, size = 67, normalized size = 0.93

$$\operatorname{atanh}\left(\frac{4x}{\sqrt{5}-3} - \frac{2\sqrt{5}x}{\sqrt{5}-3}\right)\left(\frac{\sqrt{5}}{10} - \frac{1}{2}\right) + \operatorname{atanh}\left(\frac{4x}{\sqrt{5}+3} + \frac{2\sqrt{5}x}{\sqrt{5}+3}\right)\left(\frac{\sqrt{5}}{10} + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^4 - 3*x^2 + 1), x)$

[Out] $\operatorname{atanh}((4*x)/(5^{(1/2)} - 3) - (2*5^{(1/2)*x})/(5^{(1/2)} - 3))*(5^{(1/2)}/10 - 1/2) + \operatorname{atanh}((4*x)/(5^{(1/2)} + 3) + (2*5^{(1/2)*x})/(5^{(1/2)} + 3))*(5^{(1/2)}/10 + 1/2)$

$$3.42 \quad \int \frac{1}{1-4x^2+x^4} dx$$

Optimal. Leaf size=67

$$\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}(2-\sqrt{3})} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}(2+\sqrt{3})}$$

[Out] $1/2*\operatorname{arctanh}(x/(1/2*6^{(1/2)}-1/2*2^{(1/2)}))/(3/2*2^{(1/2)}-1/2*6^{(1/2)})-1/2*\operatorname{arctanh}(x/(1/2*6^{(1/2)}+1/2*2^{(1/2)}))/(3/2*2^{(1/2)}+1/2*6^{(1/2)})$

Rubi [A]

time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1107, 213}

$$\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}(2-\sqrt{3})} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}(2+\sqrt{3})}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(1 - 4*x^2 + x^4)^{-1}, x]$

[Out] $\operatorname{ArcTanh}[x/\operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]]]/(2*\operatorname{Sqrt}[3*(2 - \operatorname{Sqrt}[3])]) - \operatorname{ArcTanh}[x/\operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]]]/(2*\operatorname{Sqrt}[3*(2 + \operatorname{Sqrt}[3])])$

Rule 213

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2])^{-1})*\operatorname{ArcTanh}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /;$ $\operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{LtQ}[a, 0] \ || \ \operatorname{GtQ}[b, 0])$

Rule 1107

$\operatorname{Int}[(a_ + (b_)*(x_)^2 + (c_)*(x_)^4)^{-1}, x_Symbol] := \operatorname{With}\{q = \operatorname{Rt}[b^2 - 4*a*c, 2]\}, \operatorname{Dist}[c/q, \operatorname{Int}[1/(b/2 - q/2 + c*x^2), x], x] - \operatorname{Dist}[c/q, \operatorname{Int}[1/(b/2 + q/2 + c*x^2), x], x] /;$ $\operatorname{FreeQ}\{a, b, c, x\} \ \&\& \ \operatorname{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \operatorname{PosQ}[b^2 - 4*a*c]$

Rubi steps

$$\int \frac{1}{1-4x^2+x^4} dx = \frac{\int \frac{1}{-2-\sqrt{3}+x^2} dx}{2\sqrt{3}} - \frac{\int \frac{1}{-2+\sqrt{3}+x^2} dx}{2\sqrt{3}}$$

$$= \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}(2-\sqrt{3})} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}(2+\sqrt{3})}$$

Mathematica [A]

time = 0.02, size = 67, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}(2-\sqrt{3})} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}(2+\sqrt{3})}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTanh[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTanh[x/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.95, size = 33, normalized size = 0.49

$$\text{RootSum}\left[1 - 192\#1^2 + 2304\#1^4 \&, \text{Log}\left[x - 28\#1 + 384\#1^3\right] \#1\&\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^4-4*x^2+1),x]')
```

```
[Out] RootSum[1 - 192 #1 ^ 2 + 2304 #1 ^ 4&, Log[x - 28 #1 + 384 #1 ^ 3] #1&]
```

Maple [A]

time = 0.03, size = 60, normalized size = 0.90

method	result	size
risch	$\frac{\left(\sum_{-R=\text{RootOf}(-Z^4-4Z^2+1)} \frac{\ln\left(-\frac{R+x}{R^3-2R}\right)}{4}\right)}{4}$	33

default	$\frac{\sqrt{3} \operatorname{arctanh}\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}-3\sqrt{2}} - \frac{\sqrt{3} \operatorname{arctanh}\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3(\sqrt{6}+\sqrt{2})}$	60
---------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4-4*x^2+1),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{3}3^{(1/2)}/(6^{(1/2)}-2^{(1/2)})*\operatorname{arctanh}(2*x/(6^{(1/2)}-2^{(1/2)}))-1/3*3^{(1/2)}/(6^{(1/2)}+2^{(1/2)})*\operatorname{arctanh}(2*x/(6^{(1/2)}+2^{(1/2)}))$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4-4*x^2+1),x, algorithm="maxima")`

[Out] `integrate(1/(x^4 - 4*x^2 + 1), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(55) = 110.

time = 0.33, size = 123, normalized size = 1.84

$$-\frac{1}{12}\sqrt{3}\sqrt{\sqrt{3}+2}\log(\sqrt{\sqrt{3}+2}(\sqrt{3}-2)+x)+\frac{1}{12}\sqrt{3}\sqrt{\sqrt{3}+2}\log(-\sqrt{\sqrt{3}+2}(\sqrt{3}-2)+x)-\frac{1}{12}\sqrt{3}\sqrt{-\sqrt{3}+2}\log((\sqrt{3}+2)\sqrt{-\sqrt{3}+2}+x)+\frac{1}{12}\sqrt{3}\sqrt{-\sqrt{3}+2}\log(-(\sqrt{3}+2)\sqrt{-\sqrt{3}+2}+x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4-4*x^2+1),x, algorithm="fricas")`

[Out] $-1/12*\operatorname{sqrt}(3)*\operatorname{sqrt}(\operatorname{sqrt}(3)+2)*\log(\operatorname{sqrt}(\operatorname{sqrt}(3)+2)*(\operatorname{sqrt}(3)-2)+x)+1/12*\operatorname{sqrt}(3)*\operatorname{sqrt}(\operatorname{sqrt}(3)+2)*\log(-\operatorname{sqrt}(\operatorname{sqrt}(3)+2)*(\operatorname{sqrt}(3)-2)+x)-1/12*\operatorname{sqrt}(3)*\operatorname{sqrt}(-\operatorname{sqrt}(3)+2)*\log((\operatorname{sqrt}(3)+2)*\operatorname{sqrt}(-\operatorname{sqrt}(3)+2)+x)+1/12*\operatorname{sqrt}(3)*\operatorname{sqrt}(-\operatorname{sqrt}(3)+2)*\log(-(\operatorname{sqrt}(3)+2)*\operatorname{sqrt}(-\operatorname{sqrt}(3)+2)+x)$

Sympy [A]

time = 0.20, size = 24, normalized size = 0.36

$$\operatorname{RootSum}(2304t^4 - 192t^2 + 1, (t \mapsto t \log(384t^3 - 28t + x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**4-4*x**2+1),x)`

[Out] `RootSum(2304*_t**4 - 192*_t**2 + 1, Lambda(_t, _t*log(384*_t**3 - 28*_t + x)))`

Giac [A]

time = 0.00, size = 155, normalized size = 2.31

$$\frac{1}{24} (3\sqrt{2} - \sqrt{6}) \ln \left| -\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} + x \right| + \frac{1}{24} (-3\sqrt{2} + \sqrt{6}) \ln \left| x + \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} \right| - \frac{1}{24} (3\sqrt{2} + \sqrt{6}) \ln \left| -\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} + x \right| - \frac{1}{24} (-3\sqrt{2} - \sqrt{6}) \ln \left| x + \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} \right|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4-4*x^2+1),x)

[Out] 1/24*(sqrt(6) - 3*sqrt(2))*log(abs(x + 1/2*sqrt(6) + 1/2*sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*log(abs(x + 1/2*sqrt(6) - 1/2*sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*log(abs(x - 1/2*sqrt(6) + 1/2*sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*log(abs(x - 1/2*sqrt(6) - 1/2*sqrt(2)))

Mupad [B]

time = 0.23, size = 98, normalized size = 1.46

$$\operatorname{atanh} \left(\frac{5\sqrt{2}x}{\sqrt{2}\sqrt{6}+4} + \frac{3\sqrt{6}x}{\sqrt{2}\sqrt{6}+4} \right) \left(\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{12} \right) - \operatorname{atanh} \left(\frac{5\sqrt{2}x}{\sqrt{2}\sqrt{6}-4} - \frac{3\sqrt{6}x}{\sqrt{2}\sqrt{6}-4} \right) \left(\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{12} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4 - 4*x^2 + 1),x)

[Out] atanh((5*2^(1/2)*x)/(2^(1/2)*6^(1/2) + 4) + (3*6^(1/2)*x)/(2^(1/2)*6^(1/2) + 4))*(2^(1/2)/4 + 6^(1/2)/12) - atanh((5*2^(1/2)*x)/(2^(1/2)*6^(1/2) - 4) - (3*6^(1/2)*x)/(2^(1/2)*6^(1/2) - 4))*(2^(1/2)/4 - 6^(1/2)/12)

$$3.43 \quad \int \frac{1}{1+4x^2+x^4} dx$$

Optimal. Leaf size=67

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}(2-\sqrt{3})} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}(2+\sqrt{3})}$$

[Out] $\frac{1}{2} \arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{2} \arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)$

Rubi [A]

time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1107, 209}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}(2-\sqrt{3})} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}(2+\sqrt{3})}$$

Antiderivative was successfully verified.

[In] Int[(1 + 4*x^2 + x^4)^(-1), x]

[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[x/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])

Rule 209

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1107

Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\int \frac{1}{1+4x^2+x^4} dx = \frac{\int \frac{1}{2-\sqrt{3}+x^2} dx}{2\sqrt{3}} - \frac{\int \frac{1}{2+\sqrt{3}+x^2} dx}{2\sqrt{3}}$$

$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}(2-\sqrt{3})} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}(2+\sqrt{3})}$$

Mathematica [A]

time = 0.01, size = 67, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}(2-\sqrt{3})} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}(2+\sqrt{3})}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[x/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Mathics [A]

time = 2.71, size = 63, normalized size = 0.94

$$\frac{\sqrt{3} \left(-\sqrt{2-\sqrt{3}} \operatorname{ArcTan} \left[\frac{x}{(2+\sqrt{3})\sqrt{2-\sqrt{3}}} \right] + \sqrt{2+\sqrt{3}} \operatorname{ArcTan} \left[\frac{x}{\sqrt{2+\sqrt{3}}(2-\sqrt{3})} \right] \right)}{6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^4+4*x^2+1),x]')
```

```
[Out] Sqrt[3] (-Sqrt[2 - Sqrt[3]] ArcTan[x / ((2 + Sqrt[3]) Sqrt[2 - Sqrt[3]])] + Sqrt[2 + Sqrt[3]] ArcTan[x / (Sqrt[2 + Sqrt[3]] (2 - Sqrt[3]))]) / 6
```

Maple [A]

time = 0.03, size = 60, normalized size = 0.90

method	result	size
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risch	$\frac{\left(\sum_{-R=\text{RootOf}(-Z^4+4Z^2+1)} \frac{\ln\left(-\frac{R+x}{R^3+2R}\right)}{-R^3+2R} \right)}{4}$	33
default	$-\frac{\sqrt{3} \arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3(\sqrt{6}+\sqrt{2})} + \frac{\sqrt{3} \arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}-3\sqrt{2}}$	60

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4+4*x^2+1),x,method=_RETURNVERBOSE)`

[Out] $-1/3*3^{(1/2)}/(6^{(1/2)}+2^{(1/2)})*\arctan(2*x/(6^{(1/2)}+2^{(1/2)}))+1/3*3^{(1/2)}/(6^{(1/2)}-2^{(1/2)})*\arctan(2*x/(6^{(1/2)}-2^{(1/2)}))$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4+4*x^2+1),x, algorithm="maxima")`

[Out] `integrate(1/(x^4 + 4*x^2 + 1), x)`

Fricas [A]

time = 0.35, size = 87, normalized size = 1.30

$-\frac{1}{3}\sqrt{3}\sqrt{\sqrt{3}+2}\arctan\left(-\left(x-\sqrt{x^2-\sqrt{3}+2}\right)\sqrt{\sqrt{3}+2}\right)+\frac{1}{3}\sqrt{3}\sqrt{-\sqrt{3}+2}\arctan\left(-x\sqrt{-\sqrt{3}+2}+\sqrt{x^2+\sqrt{3}+2}\sqrt{-\sqrt{3}+2}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4+4*x^2+1),x, algorithm="fricas")`

[Out] $-1/3*\text{sqrt}(3)*\text{sqrt}(\text{sqrt}(3)+2)*\arctan(-(x-\text{sqrt}(x^2-\text{sqrt}(3)+2))*\text{sqrt}(\text{sqrt}(3)+2))+1/3*\text{sqrt}(3)*\text{sqrt}(-\text{sqrt}(3)+2)*\arctan(-x*\text{sqrt}(-\text{sqrt}(3)+2)+\text{sqrt}(x^2+\text{sqrt}(3)+2)*\text{sqrt}(-\text{sqrt}(3)+2))$

Sympy [A]

time = 0.11, size = 92, normalized size = 1.37

$-2\sqrt{\frac{1}{24}-\frac{\sqrt{3}}{48}}\text{atan}\left(\frac{x}{\sqrt{3}\sqrt{2-\sqrt{3}}+2\sqrt{2-\sqrt{3}}}\right)-2\sqrt{\frac{\sqrt{3}}{48}+\frac{1}{24}}\text{atan}\left(\frac{x}{-2\sqrt{\sqrt{3}+2}+\sqrt{3}\sqrt{\sqrt{3}+2}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**4+4*x**2+1),x)`

[Out] $-2\sqrt{1/24 - \sqrt{3}/48} \operatorname{atan}\left(\frac{x}{\sqrt{3}\sqrt{2 - \sqrt{3}}}\right) + 2\sqrt{2 - \sqrt{3}} \operatorname{atan}\left(\frac{x}{\sqrt{3}\sqrt{2 - \sqrt{3}}}\right) - 2\sqrt{\sqrt{3}/48 + 1/24} \operatorname{atan}\left(\frac{x}{-2\sqrt{\sqrt{3} + 2} + \sqrt{3}}\right) + 2\sqrt{\sqrt{3} + 2} \operatorname{atan}\left(\frac{x}{-2\sqrt{\sqrt{3} + 2} + \sqrt{3}}\right)$

Giac [A]

time = 0.00, size = 78, normalized size = 1.16

$$\frac{1}{12} \left(3\sqrt{2} + \sqrt{6}\right) \arctan\left(\frac{x}{\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}}\right) - \frac{1}{12} \left(3\sqrt{2} - \sqrt{6}\right) \arctan\left(\frac{x}{\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4+4*x^2+1),x)`

[Out] $1/12*(\sqrt{6} - 3*\sqrt{2})*\arctan(2*x/(\sqrt{6} + \sqrt{2})) + 1/12*(\sqrt{6} + 3*\sqrt{2})*\arctan(2*x/(\sqrt{6} - \sqrt{2}))$

Mupad [B]

time = 0.20, size = 117, normalized size = 1.75

$$2 \operatorname{atanh}\left(\frac{24x\sqrt{\frac{\sqrt{3}}{48} - \frac{1}{24}}}{2\sqrt{3} - 4} - \frac{16\sqrt{3}x\sqrt{\frac{\sqrt{3}}{48} - \frac{1}{24}}}{2\sqrt{3} - 4}\right) \sqrt{\frac{\sqrt{3}}{48} - \frac{1}{24}} - 2 \operatorname{atanh}\left(\frac{24x\sqrt{\frac{\sqrt{3}}{48} - \frac{1}{24}}}{2\sqrt{3} + 4} + \frac{16\sqrt{3}x\sqrt{\frac{\sqrt{3}}{48} - \frac{1}{24}}}{2\sqrt{3} + 4}\right) \sqrt{\frac{\sqrt{3}}{48} - \frac{1}{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(4*x^2 + x^4 + 1),x)`

[Out] $2*\operatorname{atanh}\left(\frac{(24*x*(3^{(1/2)}/48 - 1/24)^{(1/2)})}{(2*3^{(1/2)} - 4)} - \frac{(16*3^{(1/2)}*x*(3^{(1/2)}/48 - 1/24)^{(1/2)})}{(2*3^{(1/2)} - 4)}\right) + 2*\operatorname{atanh}\left(\frac{(24*x*(-3^{(1/2)}/48 - 1/24)^{(1/2)})}{(2*3^{(1/2)} + 4)} + \frac{(16*3^{(1/2)}*x*(-3^{(1/2)}/48 - 1/24)^{(1/2)})}{(2*3^{(1/2)} + 4)}\right)$

3.44 $\int \frac{1}{2+x^2+x^4} dx$

Optimal. Leaf size=196

$$-\frac{1}{2}\sqrt{\frac{1}{14}(-1+2\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right) + \frac{1}{2}\sqrt{\frac{1}{14}(-1+2\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{1+2\sqrt{2}}}\right)$$

[Out] $-1/28*\arctan((-2*x+(-1+2*2^{(1/2)})^{(1/2)})/(1+2*2^{(1/2)})^{(1/2)})*(-14+28*2^{(1/2)})^{(1/2)}+1/28*\arctan((2*x+(-1+2*2^{(1/2)})^{(1/2)})/(1+2*2^{(1/2)})^{(1/2)})*(-14+28*2^{(1/2)})^{(1/2)}-1/4*\ln(x^2+2^{(1/2)}-x*(-1+2*2^{(1/2)})^{(1/2)})/(-2+4*2^{(1/2)})^{(1/2)}+1/4*\ln(x^2+2^{(1/2)}+x*(-1+2*2^{(1/2)})^{(1/2)})/(-2+4*2^{(1/2)})^{(1/2)}$

Rubi [A]

time = 0.10, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1108, 648, 632, 210, 642}

$$-\frac{\log\left(x^2 - \sqrt{2\sqrt{2}-1}x + \sqrt{2}\right)}{4\sqrt{2(2\sqrt{2}-1)}} + \frac{\log\left(x^2 + \sqrt{2\sqrt{2}-1}x + \sqrt{2}\right)}{4\sqrt{2(2\sqrt{2}-1)}} - \frac{1}{2}\sqrt{\frac{1}{14}(2\sqrt{2}-1)} \tan^{-1}\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{1+2\sqrt{2}}}\right) + \frac{1}{2}\sqrt{\frac{1}{14}(2\sqrt{2}-1)} \tan^{-1}\left(\frac{2x + \sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2 + x^2 + x^4)^(-1), x]

[Out] $-1/2*(\text{Sqrt}[(-1 + 2*\text{Sqrt}[2])/14]*\text{ArcTan}[(\text{Sqrt}[-1 + 2*\text{Sqrt}[2]] - 2*x)/\text{Sqrt}[1 + 2*\text{Sqrt}[2]]) + (\text{Sqrt}[(-1 + 2*\text{Sqrt}[2])/14]*\text{ArcTan}[(\text{Sqrt}[-1 + 2*\text{Sqrt}[2]] + 2*x)/\text{Sqrt}[1 + 2*\text{Sqrt}[2]])/2 - \text{Log}[\text{Sqrt}[2] - \text{Sqrt}[-1 + 2*\text{Sqrt}[2]]*x + x^2]/(4*\text{Sqrt}[2*(-1 + 2*\text{Sqrt}[2])]) + \text{Log}[\text{Sqrt}[2] + \text{Sqrt}[-1 + 2*\text{Sqrt}[2]]*x + x^2]/(4*\text{Sqrt}[2*(-1 + 2*\text{Sqrt}[2])])$

Rule 210

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)]*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d},

e}], x] && EqQ[2*c*d - b*e, 0]

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{2+x^2+x^4} dx &= \frac{\int \frac{\sqrt{-1+2\sqrt{2}}-x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2} dx}{2\sqrt{2}(-1+2\sqrt{2})} + \frac{\int \frac{\sqrt{-1+2\sqrt{2}}+x}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2} dx}{2\sqrt{2}(-1+2\sqrt{2})} \\ &= \frac{\int \frac{1}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2} dx}{4\sqrt{2}} + \frac{\int \frac{1}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2} dx}{4\sqrt{2}} - \frac{\int \frac{-\sqrt{-1+2\sqrt{2}}}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2} dx}{4\sqrt{2}(-1+2\sqrt{2})} \\ &= -\frac{\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2}(-1+2\sqrt{2})} + \frac{\log\left(\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2}(-1+2\sqrt{2})} - \frac{\text{Subst}\left(\frac{1}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2}, \sqrt{-1+2\sqrt{2}}-x\right)}{4\sqrt{2}(-1+2\sqrt{2})} \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2}(1+2\sqrt{2})} + \frac{\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2}(1+2\sqrt{2})} - \frac{\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2}(-1+2\sqrt{2})} \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.04, size = 91, normalized size = 0.46

$$-\frac{i \tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}(1-i\sqrt{7})}}\right)}{\sqrt{\frac{7}{2}(1-i\sqrt{7})}} + \frac{i \tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}(1+i\sqrt{7})}}\right)}{\sqrt{\frac{7}{2}(1+i\sqrt{7})}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x^2 + x^4)^(-1),x]

[Out] ((-I)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/Sqrt[(7*(1 - I*Sqrt[7]))/2] + (I*ArcTan[x/Sqrt[(1 + I*Sqrt[7])/2]])/Sqrt[(7*(1 + I*Sqrt[7]))/2]

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 595 vs. 2(196) = 392.
time = 17.42, size = 443, normalized size = 2.26

$\sqrt{\frac{7}{2}(1-i\sqrt{7})} \left[-\frac{i \tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}(1-i\sqrt{7})}}\right)}{\sqrt{\frac{7}{2}(1-i\sqrt{7})}} + \frac{i \tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}(1+i\sqrt{7})}}\right)}{\sqrt{\frac{7}{2}(1+i\sqrt{7})}} \right]$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x^4+x^2+2),x]')

[Out] Sqrt[14] (-Sqrt[1 + 2 Sqrt[2]] Log[-11 / 28 - 33 Sqrt[9 + 4 Sqrt[2]] / 28 + 11 Sqrt[2] Sqrt[9 + 4 Sqrt[2]] / 28 + 83 Sqrt[2] / 28 - (-16 Sqrt[7] Sqrt[1 + 2 Sqrt[2]] + 3 Sqrt[14] Sqrt[25 + 22 Sqrt[2]] + 5 Sqrt[14] Sqrt[1 + 2 Sqrt[2]]) x / 28 + x^2] + 2 Sqrt[1 - 2 Sqrt[9 + 4 Sqrt[2]] + 6 Sqrt[2]] ArcTan[(-8 Sqrt[2] Sqrt[1 + 2 Sqrt[2]] + 3 Sqrt[25 + 22 Sqrt[2]] + 5 Sqrt[1 + 2 Sqrt[2]] + 4 Sqrt[14] x) / (Sqrt[57 - 18 Sqrt[9 + 4 Sqrt[2]] - 8 Sqrt[2] Sqrt[9 + 4 Sqrt[2]] + 58 Sqrt[2]] + 7 Sqrt[1 - 2 Sqrt[9 + 4 Sqrt[2]] + 6 Sqrt[2]])] + 2 Sqrt[1 - 2 Sqrt[9 + 4 Sqrt[2]] + 6 Sqrt[2]] ArcTan[(-5 Sqrt[1 + 2 Sqrt[2]] - 3 Sqrt[25 + 22 Sqrt[2]] + 8 Sqrt[2] Sqrt[1 + 2 Sqrt[2]] + 4 Sqrt[14] x) / (Sqrt[57 - 18 Sqrt[9 + 4 Sqrt[2]] - 8 Sqrt[2] Sqrt[9 + 4 Sqrt[2]] + 58 Sqrt[2]] + 7 Sqrt[1 - 2 Sqrt[9 + 4 Sqrt[2]] + 6 Sqrt[2]])] + Sqrt[1 + 2 Sqrt[2]] Log[-11 / 28 - 33 Sqrt[9 + 4 Sqrt[2]] / 28 + 11 Sqrt[2] Sqrt[9 + 4 Sqrt[2]] / 28 + 83 Sqrt[2] / 28 + (-16 Sqrt[7] Sqrt[1 + 2 Sqrt[2]] + 3 Sqrt[14] Sqrt[25 + 22 Sqrt[2]] + 5 Sqrt[14] Sqrt[1 + 2 Sqrt[2]]) x / 28 + x^2) / 56

Maple [A]

time = 0.06, size = 251, normalized size = 1.28

method	result
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risch	$\frac{\sum_{R=\text{RootOf}(-Z^4+Z^2+2)} \frac{\ln(-R+x)}{2R^3+R}}{2}$
default	$\frac{\left(\sqrt{-1+2\sqrt{2}}\sqrt{2}+4\sqrt{-1+2\sqrt{2}}\right)\ln\left(x^2+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)}{56} + \frac{\left(7\sqrt{2}-\frac{\left(\sqrt{-1+2\sqrt{2}}\sqrt{2}\right)}{\sqrt{2}}\right)}{56}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4+x^2+2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{56} \left((-1+2\sqrt{2})^{1/2} \right)^{1/2} \cdot 2^{1/2} + 4 \left((-1+2\sqrt{2})^{1/2} \right)^{1/2} \cdot \ln(x^2+2^{1/2}+x \cdot (-1+2\sqrt{2})^{1/2}) + \frac{1}{14} \left(7 \cdot 2^{1/2} - \frac{1}{2} \left((-1+2\sqrt{2})^{1/2} \right)^{1/2} \cdot 2^{1/2} + 4 \left((-1+2\sqrt{2})^{1/2} \right)^{1/2} \cdot \left((-1+2\sqrt{2})^{1/2} \right)^{1/2} \right) / \left(1+2 \cdot 2^{1/2} \right)^{1/2} \cdot \arctan\left(\frac{2 \cdot x + \left((-1+2\sqrt{2})^{1/2} \right)^{1/2}}{\left(1+2 \cdot 2^{1/2} \right)^{1/2}}\right) - \frac{1}{56} \left((-1+2\sqrt{2})^{1/2} \right)^{1/2} \cdot 2^{1/2} + 4 \left((-1+2\sqrt{2})^{1/2} \right)^{1/2} \cdot \ln(x^2+2^{1/2}-x \cdot \left((-1+2\sqrt{2})^{1/2} \right)^{1/2}) - \frac{1}{14} \left(-7 \cdot 2^{1/2} + \frac{1}{2} \left((-1+2\sqrt{2})^{1/2} \right)^{1/2} \cdot 2^{1/2} + 4 \left((-1+2\sqrt{2})^{1/2} \right)^{1/2} \cdot \left((-1+2\sqrt{2})^{1/2} \right)^{1/2} \right) / \left(1+2 \cdot 2^{1/2} \right)^{1/2} \cdot \arctan\left(\frac{2 \cdot x - \left((-1+2\sqrt{2})^{1/2} \right)^{1/2}}{\left(1+2 \cdot 2^{1/2} \right)^{1/2}}\right)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4+x^2+2),x, algorithm="maxima")`

[Out] `integrate(1/(x^4 + x^2 + 2), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(135) = 270.

time = 0.36, size = 298, normalized size = 1.52

$$\frac{1}{56} \sqrt{7} \sqrt{-4\sqrt{2}+8} \arctan\left(\frac{\frac{1}{56} \sqrt{7} \sqrt{-4\sqrt{2}+8}}{\sqrt{-4\sqrt{2}+8}} + \frac{1}{56} \sqrt{7} \sqrt{-4\sqrt{2}+8} \sqrt{-4\sqrt{2}+8}}{\sqrt{-4\sqrt{2}+8}} - \frac{1}{56} \sqrt{7} \sqrt{-4\sqrt{2}+8}\right) - \frac{1}{56} \sqrt{7} \sqrt{-4\sqrt{2}+8} \arctan\left(\frac{\frac{1}{56} \sqrt{7} \sqrt{-4\sqrt{2}+8}}{\sqrt{-4\sqrt{2}+8}} + \frac{1}{56} \sqrt{7} \sqrt{-4\sqrt{2}+8} \sqrt{-4\sqrt{2}+8}}{\sqrt{-4\sqrt{2}+8}} + \frac{1}{56} \sqrt{7} \sqrt{-4\sqrt{2}+8}\right) + \frac{1}{56} \sqrt{7} \sqrt{-4\sqrt{2}+8} \arctan\left(\frac{\frac{1}{56} \sqrt{7} \sqrt{-4\sqrt{2}+8}}{\sqrt{-4\sqrt{2}+8}} - \frac{1}{56} \sqrt{7} \sqrt{-4\sqrt{2}+8} \sqrt{-4\sqrt{2}+8}}{\sqrt{-4\sqrt{2}+8}} - \frac{1}{56} \sqrt{7} \sqrt{-4\sqrt{2}+8}\right) - \frac{1}{56} \sqrt{7} \sqrt{-4\sqrt{2}+8} \arctan\left(\frac{\frac{1}{56} \sqrt{7} \sqrt{-4\sqrt{2}+8}}{\sqrt{-4\sqrt{2}+8}} - \frac{1}{56} \sqrt{7} \sqrt{-4\sqrt{2}+8} \sqrt{-4\sqrt{2}+8}}{\sqrt{-4\sqrt{2}+8}} + \frac{1}{56} \sqrt{7} \sqrt{-4\sqrt{2}+8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4+x^2+2),x, algorithm="fricas")`

[Out] $-1/56 \cdot 8^{1/4} \cdot \sqrt{7} \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16} \cdot \arctan\left(\frac{-1/56 \cdot 8^{3/4} \cdot \sqrt{7} \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16}}{\sqrt{-4\sqrt{2}+16}} + \frac{1/112 \cdot 8^{3/4} \cdot \sqrt{7} \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16}}{\sqrt{-4\sqrt{2}+16}} + \frac{4 \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16}}{\sqrt{-4\sqrt{2}+16}} + \frac{4 \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16}}{\sqrt{-4\sqrt{2}+16}}\right) + \frac{1}{112} \cdot 8^{3/4} \cdot \sqrt{7} \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16} \cdot \arctan\left(\frac{1/112 \cdot 8^{3/4} \cdot \sqrt{7} \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16}}{\sqrt{-4\sqrt{2}+16}} + \frac{4 \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16}}{\sqrt{-4\sqrt{2}+16}} + \frac{4 \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16}}{\sqrt{-4\sqrt{2}+16}}\right) - \frac{1}{7} \cdot \sqrt{7} \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16} \cdot \arctan\left(\frac{-1/56 \cdot 8^{3/4} \cdot \sqrt{7} \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16}}{\sqrt{-4\sqrt{2}+16}} - \frac{1/112 \cdot 8^{3/4} \cdot \sqrt{7} \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16}}{\sqrt{-4\sqrt{2}+16}} - \frac{4 \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16}}{\sqrt{-4\sqrt{2}+16}} - \frac{4 \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16}}{\sqrt{-4\sqrt{2}+16}}\right) - \frac{1}{7} \cdot \sqrt{7} \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16} \cdot \arctan\left(\frac{-1/56 \cdot 8^{3/4} \cdot \sqrt{7} \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16}}{\sqrt{-4\sqrt{2}+16}} - \frac{1/112 \cdot 8^{3/4} \cdot \sqrt{7} \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16}}{\sqrt{-4\sqrt{2}+16}} - \frac{4 \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16}}{\sqrt{-4\sqrt{2}+16}} - \frac{4 \cdot \sqrt{2} \cdot \sqrt{-4\sqrt{2}+16}}{\sqrt{-4\sqrt{2}+16}}\right)$

2) + 16)*arctan(-1/56*8^(3/4)*sqrt(7)*sqrt(2)*x*sqrt(-4*sqrt(2) + 16) + 1/12*8^(3/4)*sqrt(7)*sqrt(2)*sqrt(4*x^2 - 8^(1/4)*x*sqrt(-4*sqrt(2) + 16) + 4*sqrt(2))*sqrt(-4*sqrt(2) + 16) + 1/7*sqrt(7)*(2*sqrt(2) - 1)) + 1/224*8^(1/4)*(sqrt(2) + 4)*sqrt(-4*sqrt(2) + 16)*log(16*x^2 + 4*8^(1/4)*x*sqrt(-4*sqrt(2) + 16) + 16*sqrt(2)) - 1/224*8^(1/4)*(sqrt(2) + 4)*sqrt(-4*sqrt(2) + 16)*log(16*x^2 - 4*8^(1/4)*x*sqrt(-4*sqrt(2) + 16) + 16*sqrt(2))

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 994 vs. 2(158) = 316

time = 0.64, size = 994, normalized size = 5.07

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**4+x**2+2),x)

[Out] sqrt(1/224 + sqrt(2)/112)*log(x**2 + x*(-4*sqrt(7)*sqrt(1 + 2*sqrt(2)))/7 + 5*sqrt(14)*sqrt(1 + 2*sqrt(2)))/28 + 3*sqrt(14)*sqrt(1 + 2*sqrt(2))*sqrt(4*sqrt(2) + 9)/28 - 33*sqrt(4*sqrt(2) + 9)/28 - 11/28 + 11*sqrt(2)*sqrt(4*sqrt(2) + 9)/28 + 83*sqrt(2)/28 - sqrt(1/224 + sqrt(2)/112)*log(x**2 + x*(-3*sqrt(14)*sqrt(1 + 2*sqrt(2)))*sqrt(4*sqrt(2) + 9)/28 - 5*sqrt(14)*sqrt(1 + 2*sqrt(2)))/28 + 4*sqrt(7)*sqrt(1 + 2*sqrt(2))/7 - 33*sqrt(4*sqrt(2) + 9)/28 - 11/28 + 11*sqrt(2)*sqrt(4*sqrt(2) + 9)/28 + 83*sqrt(2)/28 + 2*sqrt(-sqrt(4*sqrt(2) + 9)/112 + 1/224 + 3*sqrt(2)/112)*atan(4*sqrt(14)*x/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2))) - 8*sqrt(2)*sqrt(1 + 2*sqrt(2))/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2))) + 5*sqrt(1 + 2*sqrt(2))/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2))) + 3*sqrt(1 + 2*sqrt(2))*sqrt(4*sqrt(2) + 9)/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2))) + 2*sqrt(-sqrt(4*sqrt(2) + 9)/112 + 1/224 + 3*sqrt(2)/112)*atan(4*sqrt(14)*x/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2))) - 3*sqrt(1 + 2*sqrt(2))*sqrt(4*sqrt(2) + 9)/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2))) - 5*sqrt(1 + 2*sqrt(2))/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2))) + 8*sqrt(2)*sqrt(1 + 2*sqrt(2))/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)))

Giac [A]

time = 0.09, size = 412, normalized size = 2.10

$$\frac{(-2^{\frac{1}{2}}\sqrt{2(-\sqrt{2}+4)}\sqrt{7}-2^{\frac{1}{2}}\sqrt{2(\sqrt{2}+4)})\ln\left(x^2-2\cdot 2^{\frac{1}{2}}\sqrt{\frac{1-\sqrt{2}}{2}}x+2^{\frac{1}{2}}\cdot 2^{\frac{1}{2}}\right)}{32\sqrt{7}}+\frac{(-2^{\frac{1}{2}}\sqrt{2(-\sqrt{2}+4)}+2^{\frac{1}{2}}\sqrt{2(\sqrt{2}+4)}\sqrt{7})\operatorname{arctan}\left(\frac{-\sqrt{\frac{1-\sqrt{2}}{2}}x^{\frac{1}{2}}}{\sqrt{1+\frac{\sqrt{2}}{2}}x^{\frac{1}{2}}}\right)}{16\sqrt{7}}+\frac{(2^{\frac{1}{2}}\sqrt{2(-\sqrt{2}+4)}\sqrt{7}+2^{\frac{1}{2}}\sqrt{2(\sqrt{2}+4)})\ln\left(x^2+2\cdot 2^{\frac{1}{2}}\sqrt{\frac{1-\sqrt{2}}{2}}x+2^{\frac{1}{2}}\cdot 2^{\frac{1}{2}}\right)}{32\sqrt{7}}+\frac{(2^{\frac{1}{2}}\sqrt{2(-\sqrt{2}+4)}-2^{\frac{1}{2}}\sqrt{2(\sqrt{2}+4)}\sqrt{7})\operatorname{arctan}\left(\frac{\sqrt{\frac{1-\sqrt{2}}{2}}x^{\frac{1}{2}}}{\sqrt{1+\frac{\sqrt{2}}{2}}x^{\frac{1}{2}}}\right)}{16\sqrt{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+x^2+2),x)

[Out] $-1/112\sqrt{7}(\sqrt{7}2^{1/4}\sqrt{2\sqrt{2}+8} - 2^{1/4}\sqrt{-2\sqrt{2}+8})\arctan(-2\cdot 2^{3/4}\sqrt{1/2}(x + 2^{1/4}\sqrt{-1/8\sqrt{2}+1/2}))/\sqrt{\sqrt{2}+4} + 1/112\sqrt{7}(\sqrt{7}2^{1/4}\sqrt{2\sqrt{2}+8} - 2^{1/4}\sqrt{-2\sqrt{2}+8})\arctan(2\cdot 2^{3/4}\sqrt{1/2}(x - 2^{1/4}\sqrt{-1/8\sqrt{2}+1/2}))/\sqrt{\sqrt{2}+4} + 1/224\sqrt{7}(\sqrt{7}2^{1/4}\sqrt{-2\sqrt{2}+8} + 2^{1/4}\sqrt{2\sqrt{2}+8})\log(x^2 + 2\cdot 2^{1/4}x\sqrt{-1/8\sqrt{2}+1/2} + \sqrt{2}) - 1/224\sqrt{7}(\sqrt{7}2^{1/4}\sqrt{-2\sqrt{2}+8} + 2^{1/4}\sqrt{2\sqrt{2}+8})\log(x^2 - 2\cdot 2^{1/4}x\sqrt{-1/8\sqrt{2}+1/2} + \sqrt{2})$

Mupad [B]

time = 0.22, size = 61, normalized size = 0.31

$$\frac{\operatorname{atan}\left(\frac{\sqrt{7} x \sqrt{7 - \sqrt{7} 7i}}{14}\right) \sqrt{7 - \sqrt{7} 7i} \operatorname{li} - \sqrt{7} \operatorname{atan}\left(\frac{x \sqrt{1 + \sqrt{7} 1i}}{2}\right) \sqrt{1 + \sqrt{7} 1i} \operatorname{li}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2 + x^4 + 2),x)

[Out] $(\operatorname{atan}((7^{1/2})x(7 - 7^{1/2}7i)^{1/2})/14)*(7 - 7^{1/2}7i)^{1/2}1i)/14 - (7^{1/2})\operatorname{atan}((x(7^{1/2}1i + 1)^{1/2})/2)*(7^{1/2}1i + 1)^{1/2}1i)/14$

3.45 $\int \frac{1}{2-x^2+x^4} dx$

Optimal. Leaf size=196

$$-\frac{1}{2} \sqrt{\frac{1}{14} (1+2\sqrt{2})} \tan^{-1} \left(\frac{\sqrt{1+2\sqrt{2}} - 2x}{\sqrt{-1+2\sqrt{2}}} \right) + \frac{1}{2} \sqrt{\frac{1}{14} (1+2\sqrt{2})} \tan^{-1} \left(\frac{\sqrt{1+2\sqrt{2}} + 2x}{\sqrt{-1+2\sqrt{2}}} \right) - \dots$$

[Out] $-1/28*\arctan((-2*x+(1+2*2^{(1/2)})^{(1/2)})/(-1+2*2^{(1/2)})^{(1/2)})*(14+28*2^{(1/2)})^{(1/2)}+1/28*\arctan((2*x+(1+2*2^{(1/2)})^{(1/2)})/(-1+2*2^{(1/2)})^{(1/2)})*(14+28*2^{(1/2)})^{(1/2)}-1/4*\ln(x^2+2^{(1/2)}-x*(1+2*2^{(1/2)})^{(1/2)})/(2+4*2^{(1/2)})^{(1/2)}+1/4*\ln(x^2+2^{(1/2)}+x*(1+2*2^{(1/2)})^{(1/2)})/(2+4*2^{(1/2)})^{(1/2)}$

Rubi [A]

time = 0.09, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1108, 648, 632, 210, 642}

$$-\frac{\log(x^2 - \sqrt{1+2\sqrt{2}}x + \sqrt{2})}{4\sqrt{2}(1+2\sqrt{2})} + \frac{\log(x^2 + \sqrt{1+2\sqrt{2}}x + \sqrt{2})}{4\sqrt{2}(1+2\sqrt{2})} - \frac{1}{2} \sqrt{\frac{1}{14}(1+2\sqrt{2})} \tan^{-1} \left(\frac{\sqrt{1+2\sqrt{2}} - 2x}{\sqrt{2\sqrt{2}-1}} \right) + \frac{1}{2} \sqrt{\frac{1}{14}(1+2\sqrt{2})} \tan^{-1} \left(\frac{2x + \sqrt{1+2\sqrt{2}}}{\sqrt{2\sqrt{2}-1}} \right)$$

Antiderivative was successfully verified.

[In] Int[(2 - x^2 + x^4)^(-1), x]

[Out] $-1/2*(\text{Sqrt}[(1 + 2*\text{Sqrt}[2])/14]*\text{ArcTan}[(\text{Sqrt}[1 + 2*\text{Sqrt}[2]] - 2*x)/\text{Sqrt}[-1 + 2*\text{Sqrt}[2]]) + (\text{Sqrt}[(1 + 2*\text{Sqrt}[2])/14]*\text{ArcTan}[(\text{Sqrt}[1 + 2*\text{Sqrt}[2]] + 2*x)/\text{Sqrt}[-1 + 2*\text{Sqrt}[2]])/2 - \text{Log}[\text{Sqrt}[2] - \text{Sqrt}[1 + 2*\text{Sqrt}[2]]*x + x^2]/(4*\text{Sqrt}[2*(1 + 2*\text{Sqrt}[2])]) + \text{Log}[\text{Sqrt}[2] + \text{Sqrt}[1 + 2*\text{Sqrt}[2]]*x + x^2]/(4*\text{Sqrt}[2*(1 + 2*\text{Sqrt}[2])])$

Rule 210

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)]*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x])/b], x] /; FreeQ[{a, b, c, d}, x]

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{2 - x^2 + x^4} dx &= \frac{\int \frac{\sqrt{1+2\sqrt{2}} - x}{\sqrt{2} - \sqrt{1+2\sqrt{2}} x + x^2} dx}{2\sqrt{2}(1+2\sqrt{2})} + \frac{\int \frac{\sqrt{1+2\sqrt{2}} + x}{\sqrt{2} + \sqrt{1+2\sqrt{2}} x + x^2} dx}{2\sqrt{2}(1+2\sqrt{2})} \\ &= \frac{\int \frac{1}{\sqrt{2} - \sqrt{1+2\sqrt{2}} x + x^2} dx}{4\sqrt{2}} + \frac{\int \frac{1}{\sqrt{2} + \sqrt{1+2\sqrt{2}} x + x^2} dx}{4\sqrt{2}} - \frac{\int \frac{-\sqrt{1+2\sqrt{2}} + 2x}{\sqrt{2} - \sqrt{1+2\sqrt{2}} x + x^2} dx}{4\sqrt{2}(1+2\sqrt{2})} \\ &= -\frac{\log\left(\sqrt{2} - \sqrt{1+2\sqrt{2}} x + x^2\right)}{4\sqrt{2}(1+2\sqrt{2})} + \frac{\log\left(\sqrt{2} + \sqrt{1+2\sqrt{2}} x + x^2\right)}{4\sqrt{2}(1+2\sqrt{2})} - \frac{\text{Subst}\left(\int \frac{1}{1-u^2} du, \sqrt{2} - \sqrt{1+2\sqrt{2}} x + x^2, u\right)}{4\sqrt{2}(1+2\sqrt{2})} \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{1+2\sqrt{2}} - 2x}{\sqrt{-1+2\sqrt{2}}}\right)}{2\sqrt{2}(-1+2\sqrt{2})} + \frac{\tan^{-1}\left(\frac{\sqrt{1+2\sqrt{2}} + 2x}{\sqrt{-1+2\sqrt{2}}}\right)}{2\sqrt{2}(-1+2\sqrt{2})} - \frac{\log\left(\sqrt{2} - \sqrt{1+2\sqrt{2}} x + x^2\right)}{4\sqrt{2}(1+2\sqrt{2})} \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.05, size = 91, normalized size = 0.46

$$-\frac{i \tan^{-1} \left(\frac{x}{\sqrt{\frac{1}{2}(-1 - i\sqrt{7})}} \right)}{\sqrt{\frac{7}{2}(-1 - i\sqrt{7})}} + \frac{i \tan^{-1} \left(\frac{x}{\sqrt{\frac{1}{2}(-1 + i\sqrt{7})}} \right)}{\sqrt{\frac{7}{2}(-1 + i\sqrt{7})}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - x^2 + x^4)^(-1),x]

[Out] ((-I)*ArcTan[x/Sqrt[(-1 - I*Sqrt[7])/2]])/Sqrt[(7*(-1 - I*Sqrt[7]))/2] + (I*ArcTan[x/Sqrt[(-1 + I*Sqrt[7])/2]])/Sqrt[(7*(-1 + I*Sqrt[7]))/2]

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.07, size = 33, normalized size = 0.17

$$\text{RootSum}[1 + 28\#1^2 + 1568\#1^4 \&, \text{Log}[x - 112\#1^3 + 6\#1] \#1 \&]$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x^4-x^2+2),x]')

[Out] RootSum[1 + 28 #1 ^ 2 + 1568 #1 ^ 4&, Log[x - 112 #1 ^ 3 + 6 #1] #1&]

Maple [A]

time = 0.06, size = 253, normalized size = 1.29

method	result
risch	$\frac{\sum_{R=\text{RootOf}(_Z^4-_Z^2+2)} \frac{\ln(-R+x)}{2R^3-R}}{2}$
default	$\frac{\left(-\sqrt{1+2\sqrt{2}}\sqrt{2}+4\sqrt{1+2\sqrt{2}}\right) \ln\left(x^2+\sqrt{2}+x\sqrt{1+2\sqrt{2}}\right)}{56} + \frac{\left(7\sqrt{2}-\frac{\left(-\sqrt{1+2\sqrt{2}}\sqrt{2}+4\sqrt{1+2\sqrt{2}}\right)\sqrt{2}}{7}\right)}{56}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4-x^2+2),x,method=_RETURNVERBOSE)

[Out] 1/56*(-(1+2*2^(1/2))^(1/2)*2^(1/2)+4*(1+2*2^(1/2))^(1/2))*ln(x^2+2^(1/2)+x*(1+2*2^(1/2))^(1/2))+1/14*(7*2^(1/2)-1/2*(-(1+2*2^(1/2))^(1/2)*2^(1/2)+4*(1+2*2^(1/2))^(1/2))*(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2)*arctan((2*x+(1

Giac [A]

time = 0.08, size = 410, normalized size = 2.09

$$\frac{\left(2^{1/2}\sqrt{-\sqrt{x+4}}-2^{1/2}\sqrt{\sqrt{x+4}}\sqrt{x}\right)\ln\left(x^2-2\cdot 2^{1/2}\sqrt{\frac{\sqrt{x+4}}{x}}x+2^{1/2}\cdot 2^{1/2}\right)}{32\sqrt{7}}+\frac{\left(2^{1/2}\sqrt{-\sqrt{x+4}}\sqrt{x}+2^{1/2}\sqrt{\sqrt{x+4}}\sqrt{x}\right)\operatorname{arctan}\left(\frac{-\sqrt{\frac{\sqrt{x+4}}{x}}x}{\sqrt{\frac{1-\sqrt{x+4}}{x}}x}\right)}{16\sqrt{7}}+\frac{\left(-2^{1/2}\sqrt{-\sqrt{x+4}}+2^{1/2}\sqrt{\sqrt{x+4}}\sqrt{x}\right)\ln\left(x^2+2\cdot 2^{1/2}\sqrt{\frac{\sqrt{x+4}}{x}}x+2^{1/2}\cdot 2^{1/2}\right)}{32\sqrt{7}}+\frac{\left(-2^{1/2}\sqrt{-\sqrt{x+4}}\sqrt{x}-2^{1/2}\sqrt{\sqrt{x+4}}\sqrt{x}\right)\operatorname{arctan}\left(\frac{-\sqrt{\frac{\sqrt{x+4}}{x}}x}{\sqrt{\frac{1-\sqrt{x+4}}{x}}x}\right)}{16\sqrt{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4-x^2+2),x)

[Out] $-1/112*\sqrt{7}*(\sqrt{7}*2^{(1/4)}*\sqrt{-2*\sqrt{2}+8}+2^{(1/4)}*\sqrt{2*\sqrt{2}+8})*\operatorname{arctan}(-1/4*2^{(3/4)}*(2^{(1/4)}*\sqrt{1/2}*\sqrt{\sqrt{2}+4}+2*x)/\sqrt{-1/8*\sqrt{2}+1/2})+1/112*\sqrt{7}*(\sqrt{7}*2^{(1/4)}*\sqrt{-2*\sqrt{2}+8}+2^{(1/4)}*\sqrt{2*\sqrt{2}+8})*\operatorname{arctan}(-1/4*2^{(3/4)}*(2^{(1/4)}*\sqrt{1/2}*\sqrt{\sqrt{2}+4}-2*x)/\sqrt{-1/8*\sqrt{2}+1/2})+1/224*\sqrt{7}*(\sqrt{7}*2^{(1/4)}*\sqrt{2*\sqrt{2}+8}-2^{(1/4)}*\sqrt{-2*\sqrt{2}+8})*\log(2^{(1/4)}*\sqrt{1/2}*x*\sqrt{\sqrt{2}+4}+x^2+\sqrt{2})-1/224*\sqrt{7}*(\sqrt{7}*2^{(1/4)}*\sqrt{2*\sqrt{2}+8}-2^{(1/4)}*\sqrt{-2*\sqrt{2}+8})*\log(-2^{(1/4)}*\sqrt{1/2}*x*\sqrt{\sqrt{2}+4}+x^2+\sqrt{2})$

Mupad [B]

time = 0.11, size = 132, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{x\sqrt{-7-\sqrt{7}}\operatorname{li}\operatorname{li}}{4\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\operatorname{li}\right)}+\frac{\sqrt{7}x\sqrt{-7-\sqrt{7}}\operatorname{li}}{28\left(\frac{1}{2}+\frac{\sqrt{7}}{2}\operatorname{li}\right)}\right)\sqrt{-7-\sqrt{7}}\operatorname{li}\operatorname{li}}{14}+\frac{\sqrt{7}\operatorname{atan}\left(\frac{x\sqrt{-1+\sqrt{7}}\operatorname{li}}{4\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\operatorname{li}\right)}-\frac{\sqrt{7}x\sqrt{-1+\sqrt{7}}\operatorname{li}}{4\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\operatorname{li}\right)}\right)\sqrt{-1+\sqrt{7}}\operatorname{li}\operatorname{li}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4 - x^2 + 2),x)

[Out] $(\operatorname{atan}((x*(-7^{(1/2)}*7i-7)^{(1/2)}*1i)/(4*((7^{(1/2)}*1i)/2+1/2))+7^{(1/2)}*x*(-7^{(1/2)}*7i-7)^{(1/2)})/(28*((7^{(1/2)}*1i)/2+1/2)))*(-7^{(1/2)}*7i-7)^{(1/2)}*1i/14+(7^{(1/2)}*\operatorname{atan}((x*(7^{(1/2)}*1i-1)^{(1/2)})/(4*((7^{(1/2)}*1i)/2-1/2))-(7^{(1/2)}*x*(7^{(1/2)}*1i-1)^{(1/2)}*1i)/(4*((7^{(1/2)}*1i)/2-1/2)))*7^{(1/2)}*1i-1)^{(1/2)}*1i/14$

3.46 $\int \frac{1}{-1+x^6} dx$

Optimal. Leaf size=73

$$\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{3} \tanh^{-1}(x) + \frac{1}{12} \log(1-x+x^2) - \frac{1}{12} \log(1+x+x^2)$$

[Out] $-1/3*\operatorname{arctanh}(x)+1/12*\ln(x^2-x+1)-1/12*\ln(x^2+x+1)+1/6*\operatorname{arctan}(1/3*(1-2*x)*3^{(1/2)})*3^{(1/2)}-1/6*\operatorname{arctan}(1/3*(1+2*x)*3^{(1/2)})*3^{(1/2)}$

Rubi [A]

time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {216, 648, 632, 210, 642, 212}

$$\frac{1}{12} \log(x^2 - x + 1) - \frac{1}{12} \log(x^2 + x + 1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{3} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^6)^(-1), x]

[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTanh[x]/3 + Log[1 - x + x^2]/12 - Log[1 + x + x^2]/12

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 216

Int[((a_) + (b_.)*(x_)^(n_))(-1), x_Symbol] := Module[{r = Numerator[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{-1+x^6} dx &= -\left(\frac{1}{3} \int \frac{1-\frac{x}{2}}{1-x+x^2} dx\right) - \frac{1}{3} \int \frac{1+\frac{x}{2}}{1+x+x^2} dx - \frac{1}{3} \int \frac{1}{1-x^2} dx \\ &= -\frac{1}{3} \tanh^{-1}(x) + \frac{1}{12} \int \frac{-1+2x}{1-x+x^2} dx - \frac{1}{12} \int \frac{1+2x}{1+x+x^2} dx - \frac{1}{4} \int \frac{1}{1-x+x^2} dx - \frac{1}{4} \int \frac{1}{1+x+x^2} dx \\ &= -\frac{1}{3} \tanh^{-1}(x) + \frac{1}{12} \log(1-x+x^2) - \frac{1}{12} \log(1+x+x^2) + \frac{1}{2} \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, \right. \\ &= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{3} \tanh^{-1}(x) + \frac{1}{12} \log(1-x+x^2) - \frac{1}{12} \log(1+x+x^2) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 75, normalized size = 1.03

$$\frac{1}{12} \left(-2\sqrt{3} \tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\sqrt{3} \tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) + 2\log(1-x) - 2\log(1+x) + \log(1-x+x^2) - \log(1+x+x^2) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)^(-1), x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])/12
```

Mathics [A]

time = 2.23, size = 65, normalized size = 0.89

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{\sqrt{3}(-1+2x)}{3}\right]}{6} - \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{\sqrt{3}(1+2x)}{3}\right]}{6} - \frac{\operatorname{Log}[1+x]}{6} - \frac{\operatorname{Log}[1+x+x^2]}{12} + \frac{\operatorname{Log}[1-x+x^2]}{12} + \frac{\operatorname{Log}[-1+x]}{6}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^6-1),x]')`

```
[Out] -Sqrt[3] ArcTan[Sqrt[3] (-1 + 2 x) / 3] / 6 - Sqrt[3] ArcTan[Sqrt[3] (1 + 2
x) / 3] / 6 - Log[1 + x] / 6 - Log[1 + x + x ^ 2] / 12 + Log[1 - x + x ^ 2
] / 12 + Log[-1 + x] / 6
```

Maple [A]

time = 0.04, size = 66, normalized size = 0.90

method	result
risch	$-\frac{\ln(1+x)}{6} + \frac{\ln(-1+x)}{6} - \frac{\ln(x^2+x+1)}{12} - \frac{\sqrt{3} \arctan\left(\frac{2\sqrt{3}(x+\frac{1}{2})}{3}\right)}{6} + \frac{\ln(x^2-x+1)}{12} - \frac{\sqrt{3} \arctan\left(\frac{2(x-\frac{1}{2})\sqrt{3}}{3}\right)}{6}$
default	$\frac{\ln(-1+x)}{6} - \frac{\ln(x^2+x+1)}{12} - \frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6} - \frac{\ln(1+x)}{6} + \frac{\ln(x^2-x+1)}{12} - \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$
meijerg	$\frac{x \left(\ln\left(1-(x^6)^{\frac{1}{6}}\right) - \ln\left(1+(x^6)^{\frac{1}{6}}\right) + \frac{\ln\left(1-(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2} - \sqrt{3} \arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{2-(x^6)^{\frac{1}{6}}}\right) - \frac{\ln\left(1+(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2} - \sqrt{3} \arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{2+(x^6)^{\frac{1}{6}}}\right) \right)}{6(x^6)^{\frac{1}{6}}}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(x^6-1),x,method=_RETURNVERBOSE)`

```
[Out] 1/6*ln(-1+x)-1/12*ln(x^2+x+1)-1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/6*ln(1+x)+1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]

time = 0.34, size = 65, normalized size = 0.89

$$-\frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{12} \log(x^2+x+1) + \frac{1}{12} \log(x^2-x+1) - \frac{1}{6} \log(x+1) + \frac{1}{6} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(x^6-1),x, algorithm="maxima")`

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1
) + 1/6*log(x - 1)
```

Fricas [A]

time = 0.32, size = 65, normalized size = 0.89

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{12}\log(x^2+x+1) + \frac{1}{12}\log(x^2-x+1) - \frac{1}{6}\log(x+1) + \frac{1}{6}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-1),x, algorithm="fricas")

[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1) + 1/6*log(x - 1)

Sympy [A]

time = 0.14, size = 83, normalized size = 1.14

$$\frac{\log(x-1)}{6} - \frac{\log(x+1)}{6} + \frac{\log(x^2-x+1)}{12} - \frac{\log(x^2+x+1)}{12} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x-\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x+\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**6-1),x)

[Out] log(x - 1)/6 - log(x + 1)/6 + log(x**2 - x + 1)/12 - log(x**2 + x + 1)/12 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6

Giac [A]

time = 0.00, size = 89, normalized size = 1.22

$$\frac{\ln|x-1|}{6} - \frac{\ln|x+1|}{6} + \frac{\ln(x^2-x+1)}{12} - \frac{1}{6}\sqrt{3}\arctan\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) - \frac{\ln(x^2+x+1)}{12} - \frac{1}{6}\sqrt{3}\arctan\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-1),x)

[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x + 1)) + 1/6*log(abs(x - 1))

Mupad [B]

time = 0.09, size = 88, normalized size = 1.21

$$-\frac{\operatorname{atanh}(x)}{3} - \operatorname{atan}\left(\frac{x\operatorname{li}}{1+\sqrt{3}\operatorname{li}} + \frac{\sqrt{3}x}{1+\sqrt{3}\operatorname{li}}\right)\left(\frac{\sqrt{3}}{6} + \frac{1}{6}i\right) - \operatorname{atan}\left(\frac{x\operatorname{li}}{-1+\sqrt{3}\operatorname{li}} - \frac{\sqrt{3}x}{-1+\sqrt{3}\operatorname{li}}\right)\left(\frac{\sqrt{3}}{6} - \frac{1}{6}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6 - 1),x)

[Out] - atanh(x)/3 - atan((x*1i)/(3^(1/2)*1i + 1) + (3^(1/2)*x)/(3^(1/2)*1i + 1)) * (3^(1/2)/6 + 1i/6) - atan((x*1i)/(3^(1/2)*1i - 1) - (3^(1/2)*x)/(3^(1/2)*1i - 1)) * (3^(1/2)/6 - 1i/6)

3.47 $\int \frac{1}{-2+x^6} dx$

Optimal. Leaf size=138

$$\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2^{5/6}x}{\sqrt{3}}\right)}{2 \cdot 2^{5/6}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2^{5/6}x}{\sqrt{3}}\right)}{2 \cdot 2^{5/6}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} + \frac{\log\left(\sqrt[3]{2} - \sqrt[6]{2}x + x^2\right)}{12 \cdot 2^{5/6}} - \frac{\log\left(\sqrt[3]{2} + \sqrt[6]{2}x\right)}{12 \cdot 2^{5/6}}$$

[Out] $-1/6*\operatorname{arctanh}(1/2*x*2^{(5/6)})*2^{(1/6)}+1/24*\ln(2^{(1/3)}-2^{(1/6)}*x+x^2)*2^{(1/6)}-1/24*\ln(2^{(1/3)}+2^{(1/6)}*x+x^2)*2^{(1/6)}-1/12*\operatorname{arctan}(-1/3*3^{(1/2)}+1/3*2^{(5/6)}*x*3^{(1/2)})*2^{(1/6)}*3^{(1/2)}-1/12*\operatorname{arctan}(1/3*3^{(1/2)}+1/3*2^{(5/6)}*x*3^{(1/2)})*2^{(1/6)}*3^{(1/2)}$

Rubi [A]

time = 0.13, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {216, 648, 632, 210, 642, 212}

$$\frac{\log\left(x^2 - \sqrt[6]{2}x + \sqrt[3]{2}\right)}{12 \cdot 2^{5/6}} - \frac{\log\left(x^2 + \sqrt[6]{2}x + \sqrt[3]{2}\right)}{12 \cdot 2^{5/6}} + \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2^{5/6}x}{\sqrt{3}}\right)}{2 \cdot 2^{5/6}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2^{5/6}x}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{2 \cdot 2^{5/6}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(-2 + x^6)^{-1}, x]$

[Out] $\operatorname{ArcTan}[1/\operatorname{Sqrt}[3] - (2^{(5/6)}*x)/\operatorname{Sqrt}[3]]/(2*2^{(5/6)}*\operatorname{Sqrt}[3]) - \operatorname{ArcTan}[1/\operatorname{Sqrt}[3] + (2^{(5/6)}*x)/\operatorname{Sqrt}[3]]/(2*2^{(5/6)}*\operatorname{Sqrt}[3]) - \operatorname{ArcTanh}[x/2^{(1/6)}]/(3*2^{(5/6)}) + \operatorname{Log}[2^{(1/3)} - 2^{(1/6)}*x + x^2]/(12*2^{(5/6)}) - \operatorname{Log}[2^{(1/3)} + 2^{(1/6)}*x + x^2]/(12*2^{(5/6)})$

Rule 210

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1})*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 212

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 216

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^{n_+})^{-1}, x_Symbol] \rightarrow \operatorname{Module}[\{r = \operatorname{Numerator}[\operatorname{Rt}[-a/b, n]], s = \operatorname{Denominator}[\operatorname{Rt}[-a/b, n]], k, u\}, \operatorname{Simp}[u = \operatorname{Int}[(r - s*\operatorname{Cos}[(2*k*Pi)/n])*x]/(r^2 - 2*r*s*\operatorname{Cos}[(2*k*Pi)/n])*x + s^2*x^2), x] + \operatorname{Int}[(r + s*\operatorname{Cos}[(2*k*Pi)/n])*x]/(r^2 - 2*r*s*\operatorname{Cos}[(2*k*Pi)/n])*x + s^2*x^2), x]$

$*k\pi)/n]*x)/(r^2 + 2*r*s*\text{Cos}[(2*k\pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*\text{Int}[1/(r^2 - s^2*x^2), x] + \text{Dist}[2*(r/(a*n)), \text{Sum}[u, \{k, 1, (n - 2)/4\}], x], x]] /; \text{FreeQ}\{a, b\}, x] \&\& \text{IGtQ}[(n - 2)/4, 0] \&\& \text{NegQ}[a/b]$

Rule 632

$\text{Int}[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2]^{-1}, x_Symbol] := \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[(d_.) + (e_.)*(x_.)]/[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2], x_Symbol] := \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 648

$\text{Int}[(d_.) + (e_.)*(x_.)]/[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2], x_Symbol] := \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rubi steps

$$\begin{aligned} \int \frac{1}{-2+x^6} dx &= -\frac{\int \frac{\sqrt[6]{2}-\frac{x}{2}}{\sqrt[3]{2}-\sqrt[6]{2}xx^2} dx}{3 \cdot 2^{5/6}} - \frac{\int \frac{\sqrt[6]{2}+\frac{x}{2}}{\sqrt[3]{2}+\sqrt[6]{2}xx^2} dx}{3 \cdot 2^{5/6}} - \frac{\int \frac{1}{\sqrt[3]{2}-x^2} dx}{3 \cdot 2^{2/3}} \\ &= -\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} + \frac{\int \frac{-\sqrt[6]{2}+2x}{\sqrt[3]{2}-\sqrt[6]{2}xx^2} dx}{12 \cdot 2^{5/6}} - \frac{\int \frac{\sqrt[6]{2}+2x}{\sqrt[3]{2}+\sqrt[6]{2}xx^2} dx}{12 \cdot 2^{5/6}} - \frac{\int \frac{1}{\sqrt[3]{2}-\sqrt[6]{2}xx^2} dx}{4 \cdot 2^{2/3}} - \int \frac{1}{-3-x^2} dx \\ &= -\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}xx^2\right)}{12 \cdot 2^{5/6}} - \frac{\log\left(\sqrt[3]{2}+\sqrt[6]{2}xx^2\right)}{12 \cdot 2^{5/6}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2} dx\right)}{2} \\ &= \frac{\tan^{-1}\left(\frac{1-2^{5/6}x}{\sqrt{3}}\right)}{2 \cdot 2^{5/6}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2^{5/6}x}{\sqrt{3}}\right)}{2 \cdot 2^{5/6}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}xx^2\right)}{12 \cdot 2^{5/6}} - \frac{\log\left(\sqrt[3]{2}+\sqrt[6]{2}xx^2\right)}{12 \cdot 2^{5/6}} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 122, normalized size = 0.88

$$\frac{2\sqrt{3} \tan^{-1}\left(\frac{-1+2^{5/6}x}{\sqrt{3}}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{1+2^{5/6}x}{\sqrt{3}}\right) - 2\log(2-2^{5/6}x) + 2\log(2+2^{5/6}x) - \log(2-2^{5/6}x+2^{2/3}x^2) + \log(2+2^{5/6}x+2^{2/3}x^2)}{12 \cdot 2^{5/6}}$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + x^6)^(-1), x]

[Out] $-1/12*(2*\sqrt{3}*\text{ArcTan}[(-1 + 2^{(5/6)}*x)/\sqrt{3}]] + 2*\sqrt{3}*\text{ArcTan}[(1 + 2^{(5/6)}*x)/\sqrt{3}] - 2*\text{Log}[2 - 2^{(5/6)}*x] + 2*\text{Log}[2 + 2^{(5/6)}*x] - \text{Log}[2 - 2^{(5/6)}*x + 2^{(2/3)}*x^2] + \text{Log}[2 + 2^{(5/6)}*x + 2^{(2/3)}*x^2])/2^{(5/6)}$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.98, size = 21, normalized size = 0.15

$$\text{RootSum}[-1 + 1492992\#1^6 \&, \text{Log}[x - 12\#1] \#1 \&]$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x^6-2), x]')

[Out] RootSum[-1 + 1492992 #1 ^ 6&, Log[x - 12 #1] #1&]

Maple [A]

time = 0.08, size = 111, normalized size = 0.80

method	result
risch	$\frac{\sum_{R=\text{RootOf}(_Z^6-2)} \frac{\ln(-R+x)}{-R^5}}{6}$
default	$-\frac{\ln(2^{\frac{1}{3}}+2^{\frac{1}{6}}x+x^2)2^{\frac{1}{6}}}{24} - \frac{\arctan\left(\frac{\sqrt{3}}{3} + \frac{2^{\frac{5}{6}}x\sqrt{3}}{3}\right)2^{\frac{1}{6}}\sqrt{3}}{12} - \frac{2^{\frac{1}{6}}\ln(x+2^{\frac{1}{6}})}{12} + \frac{\ln(2^{\frac{1}{3}}-2^{\frac{1}{6}}x+x^2)2^{\frac{1}{6}}}{24} - \frac{\arctan\left(-\frac{\sqrt{3}}{3} + \frac{2^{\frac{5}{6}}x\sqrt{3}}{3}\right)2^{\frac{1}{6}}\sqrt{3}}{12} + \frac{2^{\frac{1}{6}}\ln(x-2^{\frac{1}{6}})}{12}$
meijerg	$2^{\frac{1}{6}}x \left(\ln\left(1 - \frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2}\right) - \ln\left(1 + \frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2}\right) \right) + \frac{\ln\left(1 - \frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2} + \frac{2^{\frac{2}{3}}(x^6)^{\frac{1}{3}}}{2}\right)}{2} - \sqrt{3} \arctan\left(\frac{\sqrt{3}}{4-2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}\right) - \frac{\ln\left(1 + \frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2} + \frac{2^{\frac{2}{3}}(x^6)^{\frac{1}{3}}}{2}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6-2), x, method=_RETURNVERBOSE)

[Out] $-1/24*\ln(2^{(1/3)}+2^{(1/6)}*x+x^2)*2^{(1/6)}-1/12*\arctan(1/3*3^{(1/2)}+1/3*2^{(5/6)}*x*3^{(1/2)})*2^{(1/6)}*3^{(1/2)}-1/12*2^{(1/6)}*\ln(x+2^{(1/6)})+1/24*\ln(2^{(1/3)}-2^{(1/6)}*x+x^2)*2^{(1/6)}-1/12*\arctan(-1/3*3^{(1/2)}+1/3*2^{(5/6)}*x*3^{(1/2)})*2^{(1/6)}*3^{(1/2)}+1/12*2^{(1/6)}*\ln(x-2^{(1/6)})$

Maxima [A]

time = 0.35, size = 112, normalized size = 0.81

$$-\frac{1}{12}\sqrt{3}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{1}{6}}(2x+2^{\frac{1}{6}})\right) - \frac{1}{12}\sqrt{3}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{1}{6}}(2x-2^{\frac{1}{6}})\right) - \frac{1}{24}\cdot 2^{\frac{1}{6}}\log(x^2+2^{\frac{1}{3}}x+2^{\frac{1}{6}}) + \frac{1}{24}\cdot 2^{\frac{1}{6}}\log(x^2-2^{\frac{1}{3}}x+2^{\frac{1}{6}}) - \frac{1}{12}\cdot 2^{\frac{1}{6}}\log(x+2^{\frac{1}{6}}) + \frac{1}{12}\cdot 2^{\frac{1}{6}}\log(x-2^{\frac{1}{6}})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-2),x, algorithm="maxima")

[Out] $-1/12*\sqrt{3}*2^{(1/6)}*\arctan(1/6*\sqrt{3}*2^{(5/6)}*(2*x + 2^{(1/6)})) - 1/12*\sqrt{3}*2^{(1/6)}*\arctan(1/6*\sqrt{3}*2^{(5/6)}*(2*x - 2^{(1/6)})) - 1/24*2^{(1/6)}*\log(x^2 + 2^{(1/6)}*x + 2^{(1/3)}) + 1/24*2^{(1/6)}*\log(x^2 - 2^{(1/6)}*x + 2^{(1/3)}) - 1/12*2^{(1/6)}*\log(x + 2^{(1/6)}) + 1/12*2^{(1/6)}*\log(x - 2^{(1/6)})$

Fricas [A]

time = 0.42, size = 176, normalized size = 1.28

$$\frac{1}{96} \cdot 32^{1/6} \sqrt{3} \arctan\left(-\frac{1}{3} \cdot 32^{1/6} \sqrt{3} x + \frac{1}{12} \cdot 32^{1/6} \sqrt{3} \sqrt{16x^2 + 32^{2/3}x + 8 \cdot 4^{2/3}} - \frac{1}{3} \sqrt{3}\right) + \frac{1}{96} \cdot 32^{1/6} \sqrt{3} \arctan\left(-\frac{1}{3} \cdot 32^{1/6} \sqrt{3} x + \frac{1}{12} \cdot 32^{1/6} \sqrt{3} \sqrt{16x^2 - 32^{2/3}x + 8 \cdot 4^{2/3}} + \frac{1}{3} \sqrt{3}\right) - \frac{1}{384} \cdot 32^{5/6} \log(1024x^2 + 64 \cdot 32^{5/6}x + 512 \cdot 4^{2/3}) + \frac{1}{384} \cdot 32^{5/6} \log(1024x^2 - 64 \cdot 32^{5/6}x + 512 \cdot 4^{2/3}) - \frac{1}{192} \cdot 32^{5/6} \log(16x + 32^{5/6}) + \frac{1}{192} \cdot 32^{5/6} \log(16x - 32^{5/6})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-2),x, algorithm="fricas")

[Out] $1/96*32^{(5/6)}*\sqrt{3}*\arctan(-1/3*32^{(1/6)}*\sqrt{3}*x + 1/12*32^{(1/6)}*\sqrt{3}*\sqrt{16*x^2 + 32^{(5/6)}*x + 8*4^{(2/3)}}) - 1/3*\sqrt{3} + 1/96*32^{(5/6)}*\sqrt{3}*\arctan(-1/3*32^{(1/6)}*\sqrt{3}*x + 1/12*32^{(1/6)}*\sqrt{3}*\sqrt{16*x^2 - 32^{(5/6)}*x + 8*4^{(2/3)}}) + 1/3*\sqrt{3} - 1/384*32^{(5/6)}*\log(1024*x^2 + 64*32^{(5/6)}*x + 512*4^{(2/3)}) + 1/384*32^{(5/6)}*\log(1024*x^2 - 64*32^{(5/6)}*x + 512*4^{(2/3)}) - 1/192*32^{(5/6)}*\log(16*x + 32^{(5/6)}) + 1/192*32^{(5/6)}*\log(16*x - 32^{(5/6)})$

Sympy [A]

time = 0.30, size = 14, normalized size = 0.10

$$\text{RootSum}\left(1492992t^6 - 1, (t \mapsto t \log(-12t + x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**6-2),x)

[Out] RootSum(1492992*_t**6 - 1, Lambda(_t, _t*log(-12*_t + x))

Giac [A]

time = 0.00, size = 182, normalized size = 1.32

$$\frac{1}{12} \cdot 2^{5/6} \ln|x - 2^{1/6}| - \frac{1}{12} \cdot 2^{5/6} \ln|x + 2^{1/6}| + \frac{1}{24} \cdot 2^{5/6} \ln(x^2 - 2^{1/3}x + 2^{1/3}) - \frac{1}{12} \cdot 2^{5/6} \sqrt{3} \arctan\left(\frac{x - 2^{1/6}}{\frac{1}{3}\sqrt{3} \cdot 2^{1/6}}\right) - \frac{1}{24} \cdot 2^{5/6} \ln(x^2 + 2^{1/3}x + 2^{1/3}) - \frac{1}{12} \cdot 2^{5/6} \sqrt{3} \arctan\left(\frac{x + 2^{1/6}}{\frac{1}{3}\sqrt{3} \cdot 2^{1/6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-2),x)

[Out] $-1/12*\sqrt{3}*2^{(1/6)}*\arctan(1/6*\sqrt{3}*2^{(5/6)}*(2*x + 2^{(1/6)})) - 1/12*\sqrt{3}*2^{(1/6)}*\arctan(1/6*\sqrt{3}*2^{(5/6)}*(2*x - 2^{(1/6)})) - 1/24*2^{(1/6)}*\log(x^2 + 2^{(1/6)}*x + 2^{(1/3)}) + 1/24*2^{(1/6)}*\log(x^2 - 2^{(1/6)}*x + 2^{(1/3)}) - 1/12*2^{(1/6)}*\log(\text{abs}(x + 2^{(1/6)})) + 1/12*2^{(1/6)}*\log(\text{abs}(x - 2^{(1/6)}))$

Mupad [B]

time = 0.21, size = 140, normalized size = 1.01

$$-\frac{2^{1/6} \operatorname{atanh}\left(\frac{2^{5/6} x}{2}\right)}{6} + \frac{2^{1/6} \operatorname{atan}\left(\frac{2^{1/6} x i}{2\left(-\frac{2^{1/3}}{2} + \frac{2^{1/3}}{2} \sqrt{3} i\right)} - \frac{2^{1/6} \sqrt{3} x}{2\left(-\frac{2^{1/3}}{2} + \frac{2^{1/3}}{2} \sqrt{3} i\right)}\right) (1 + \sqrt{3} i) i}{12} + \frac{2^{1/6} \operatorname{atan}\left(\frac{2^{1/6} x i}{2\left(\frac{2^{1/3}}{2} + \frac{2^{1/3}}{2} \sqrt{3} i\right)} + \frac{2^{1/6} \sqrt{3} x}{2\left(\frac{2^{1/3}}{2} + \frac{2^{1/3}}{2} \sqrt{3} i\right)}\right) (-1 + \sqrt{3} i) i}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^6 - 2),x)`

[Out] $(2^{1/6} \operatorname{atan}((2^{1/6} x i) / (2 * ((2^{1/3} * 3^{1/2} * i) / 2 - 2^{1/3} / 2))) - (2^{1/6} * 3^{1/2} * x) / (2 * ((2^{1/3} * 3^{1/2} * i) / 2 - 2^{1/3} / 2))) * (3^{1/2} * i + 1) * i) / 12 - (2^{1/6} \operatorname{atanh}((2^{5/6} x) / 2)) / 6 + (2^{1/6} \operatorname{atan}((2^{1/6} x i) / (2 * ((2^{1/3} * 3^{1/2} * i) / 2 + 2^{1/3} / 2))) + (2^{1/6} * 3^{1/2} * x) / (2 * ((2^{1/3} * 3^{1/2} * i) / 2 + 2^{1/3} / 2))) * (3^{1/2} * i - 1) * i) / 12$

3.48 $\int \frac{1}{2+x^6} dx$

Optimal. Leaf size=138

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} - \frac{\tan^{-1}\left(\sqrt{3} - 2^{5/6}x\right)}{6 \cdot 2^{5/6}} + \frac{\tan^{-1}\left(\sqrt{3} + 2^{5/6}x\right)}{6 \cdot 2^{5/6}} - \frac{\log\left(\sqrt[3]{2} - \sqrt[6]{2} \sqrt{3} x + x^2\right)}{4 \cdot 2^{5/6} \sqrt{3}} + \frac{\log\left(\sqrt[3]{2} + \sqrt[6]{2} \sqrt{3} x + x^2\right)}{4 \cdot 2^{5/6} \sqrt{3}}$$

[Out] 1/6*arctan(1/2*x*2^(5/6))*2^(1/6)+1/12*arctan(x*2^(5/6)-3^(1/2))*2^(1/6)+1/12*arctan(x*2^(5/6)+3^(1/2))*2^(1/6)-1/24*ln(2^(1/3)+x^2-2^(1/6)*x*3^(1/2))*2^(1/6)*3^(1/2)+1/24*ln(2^(1/3)+x^2+2^(1/6)*x*3^(1/2))*2^(1/6)*3^(1/2)

Rubi [A]

time = 0.23, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {215, 648, 632, 210, 642, 209}

$$-\frac{\log\left(x^2 - \sqrt[6]{2} \sqrt{3} x + \sqrt[3]{2}\right)}{4 \cdot 2^{5/6} \sqrt{3}} + \frac{\log\left(x^2 + \sqrt[6]{2} \sqrt{3} x + \sqrt[3]{2}\right)}{4 \cdot 2^{5/6} \sqrt{3}} + \frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} - \frac{\tan^{-1}\left(\sqrt{3} - 2^{5/6}x\right)}{6 \cdot 2^{5/6}} + \frac{\tan^{-1}\left(2^{5/6}x + \sqrt{3}\right)}{6 \cdot 2^{5/6}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x^6)^(-1), x]

[Out] ArcTan[x/2^(1/6)]/(3*2^(5/6)) - ArcTan[Sqrt[3] - 2^(5/6)*x]/(6*2^(5/6)) + ArcTan[Sqrt[3] + 2^(5/6)*x]/(6*2^(5/6)) - Log[2^(1/3) - 2^(1/6)*Sqrt[3]*x + x^2]/(4*2^(5/6)*Sqrt[3]) + Log[2^(1/3) + 2^(1/6)*Sqrt[3]*x + x^2]/(4*2^(5/6)*Sqrt[3])

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 215

Int[((a_) + (b_.)*(x_)^(n_))(-1), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x], k, u, v]

x^2), x]; $2*(r^2/(a*n))*\text{Int}[1/(r^2 + s^2*x^2), x] + \text{Dist}[2*(r/(a*n)), \text{Sum}[u, \{k, 1, (n - 2)/4\}], x, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{IGtQ}[(n - 2)/4, 0] \ \&\& \ \text{PosQ}[a/b]$

Rule 632

$\text{Int}[(a_. + (b_.)*(x_) + (c_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[(d_. + (e_.)*(x_))/((a_. + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 648

$\text{Int}[(d_. + (e_.)*(x_))/((a_. + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rubi steps

$$\begin{aligned} \int \frac{1}{2+x^6} dx &= \frac{\int \frac{\sqrt[6]{2} - \frac{\sqrt{3}x}{2}}{\sqrt[3]{2} - \sqrt[6]{2}\sqrt{3}x+x^2} dx}{3 \cdot 2^{5/6}} + \frac{\int \frac{\sqrt[6]{2} + \frac{\sqrt{3}x}{2}}{\sqrt[3]{2} + \sqrt[6]{2}\sqrt{3}x+x^2} dx}{3 \cdot 2^{5/6}} + \frac{\int \frac{1}{\sqrt[3]{2}+x^2} dx}{3 \cdot 2^{2/3}} \\ &= \frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} + \frac{\int \frac{1}{\sqrt[3]{2} - \sqrt[6]{2}\sqrt{3}x+x^2} dx}{12 \cdot 2^{2/3}} + \frac{\int \frac{1}{\sqrt[3]{2} + \sqrt[6]{2}\sqrt{3}x+x^2} dx}{12 \cdot 2^{2/3}} - \frac{\int \frac{-\sqrt[6]{2}\sqrt{3}+2x}{\sqrt[3]{2} - \sqrt[6]{2}\sqrt{3}x+x^2} dx}{4 \cdot 2^{5/6}\sqrt{3}} \\ &= \frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} - \frac{\log\left(\sqrt[3]{2} - \sqrt[6]{2}\sqrt{3}x+x^2\right)}{4 \cdot 2^{5/6}\sqrt{3}} + \frac{\log\left(\sqrt[3]{2} + \sqrt[6]{2}\sqrt{3}x+x^2\right)}{4 \cdot 2^{5/6}\sqrt{3}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2} - \sqrt[6]{2}\sqrt{3}x+x^2} dx\right)}{4 \cdot 2^{5/6}\sqrt{3}} \\ &= \frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} - \frac{\tan^{-1}\left(\sqrt{3} - 2^{5/6}x\right)}{6 \cdot 2^{5/6}} + \frac{\tan^{-1}\left(\sqrt{3} + 2^{5/6}x\right)}{6 \cdot 2^{5/6}} - \frac{\log\left(\sqrt[3]{2} - \sqrt[6]{2}\sqrt{3}x+x^2\right)}{4 \cdot 2^{5/6}\sqrt{3}} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 115, normalized size = 0.83

$$\frac{4 \tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right) - 2 \tan^{-1}\left(\sqrt{3} - 2^{5/6}x\right) + 2 \tan^{-1}\left(\sqrt{3} + 2^{5/6}x\right) - \sqrt{3} \log\left(2 - 2^{5/6}\sqrt{3}x + 2^{2/3}x^2\right) + \sqrt{3} \log\left(2 + 2^{5/6}\sqrt{3}x + 2^{2/3}x^2\right)}{12 \cdot 2^{5/6}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x^6)^(-1),x]

[Out] (4*ArcTan[x/2^(1/6)] - 2*ArcTan[Sqrt[3] - 2^(5/6)*x] + 2*ArcTan[Sqrt[3] + 2^(5/6)*x] - Sqrt[3]*Log[2 - 2^(5/6)*Sqrt[3]*x + 2^(2/3)*x^2] + Sqrt[3]*Log[2 + 2^(5/6)*Sqrt[3]*x + 2^(2/3)*x^2])/(12*2^(5/6))

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.88, size = 21, normalized size = 0.15

$$\text{RootSum}\left[1 + 1492992\#1^6 \&, \text{Log}[x + 12\#1] \#1\&\right]$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x^6+2),x]')

[Out] RootSum[1 + 1492992 #1 ^ 6&, Log[x + 12 #1] #1&]

Maple [A]

time = 0.10, size = 95, normalized size = 0.69

method	result
risch	$\frac{\sum_{-R=\text{RootOf}(-Z^6+2)} \frac{\ln(-R+x)}{-R^5}}{6}$
default	$\frac{\arctan\left(\frac{x2^{\frac{5}{6}}}{2}\right)2^{\frac{1}{6}}}{6} + \frac{\arctan\left(x2^{\frac{5}{6}}-\sqrt{3}\right)2^{\frac{1}{6}}}{12} + \frac{\arctan\left(x2^{\frac{5}{6}}+\sqrt{3}\right)2^{\frac{1}{6}}}{12} - \frac{\ln\left(2^{\frac{1}{3}}+x^2-2^{\frac{1}{6}}x\sqrt{3}\right)2^{\frac{1}{6}}\sqrt{3}}{24} + \frac{\ln\left(2^{\frac{1}{3}}+x^2+2^{\frac{1}{6}}x\sqrt{3}\right)2^{\frac{1}{6}}\sqrt{3}}{24}$
meijerg	$2^{\frac{1}{6}} \left(-\frac{x\sqrt{3} \ln\left(1 - \frac{\sqrt{3}}{2} \frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2} + \frac{2^{\frac{2}{3}}(x^6)^{\frac{1}{3}}}{2}\right)}{2(x^6)^{\frac{1}{6}}} + \frac{x \arctan\left(\frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{4 - \sqrt{3} \frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2}}\right)}{(x^6)^{\frac{1}{6}}} + \frac{2x \arctan\left(\frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2}\right)}{(x^6)^{\frac{1}{6}}} + \frac{x\sqrt{3} \ln\left(1 + \frac{\sqrt{3}}{2} \frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2}\right)}{2(x^6)^{\frac{1}{6}}} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6+2),x,method=_RETURNVERBOSE)

[Out] 1/6*arctan(1/2*x*2^(5/6))*2^(1/6)+1/12*arctan(x*2^(5/6)-3^(1/2))*2^(1/6)+1/12*arctan(x*2^(5/6)+3^(1/2))*2^(1/6)-1/24*ln(2^(1/3)+x^2-2^(1/6)*x*3^(1/2))*2^(1/6)*3^(1/2)+1/24*ln(2^(1/3)+x^2+2^(1/6)*x*3^(1/2))*2^(1/6)*3^(1/2)

Maxima [A]

time = 0.35, size = 107, normalized size = 0.78

$$\frac{1}{24} \sqrt{3} 2^{\frac{1}{6}} \log\left(x^2 + \sqrt{3} 2^{\frac{1}{6}} x + 2^{\frac{1}{3}}\right) - \frac{1}{24} \sqrt{3} 2^{\frac{1}{6}} \log\left(x^2 - \sqrt{3} 2^{\frac{1}{6}} x + 2^{\frac{1}{3}}\right) + \frac{1}{12} \cdot 2^{\frac{1}{6}} \arctan\left(\frac{1}{2} \cdot 2^{\frac{5}{6}} (2x + \sqrt{3} 2^{\frac{1}{6}})\right) + \frac{1}{12} \cdot 2^{\frac{1}{6}} \arctan\left(\frac{1}{2} \cdot 2^{\frac{5}{6}} (2x - \sqrt{3} 2^{\frac{1}{6}})\right) + \frac{1}{6} \cdot 2^{\frac{1}{6}} \arctan\left(\frac{1}{2} \cdot 2^{\frac{5}{6}} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6+2),x, algorithm="maxima")

[Out] $\frac{1}{24}\sqrt{3} \cdot 2^{1/6} \cdot \log(x^2 + \sqrt{3} \cdot 2^{1/6} \cdot x + 2^{1/3}) - \frac{1}{24}\sqrt{3} \cdot 2^{1/6} \cdot \log(x^2 - \sqrt{3} \cdot 2^{1/6} \cdot x + 2^{1/3}) + \frac{1}{12} \cdot 2^{1/6} \cdot \arctan\left(\frac{1}{2} \cdot 2^{5/6} \cdot (2x + \sqrt{3} \cdot 2^{1/6})\right) + \frac{1}{12} \cdot 2^{1/6} \cdot \arctan\left(\frac{1}{2} \cdot 2^{5/6} \cdot (2x - \sqrt{3} \cdot 2^{1/6})\right) + \frac{1}{6} \cdot 2^{1/6} \cdot \arctan\left(\frac{1}{2} \cdot 2^{5/6} \cdot x\right)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(94) = 188.

time = 0.34, size = 279, normalized size = 2.02

$\frac{1}{12} \sqrt{3} \log(4x \sqrt{3} \sqrt{x^2 + 2\sqrt{3}x + 2}) + \frac{1}{12} \sqrt{3} \log(4x \sqrt{3} \sqrt{x^2 - 2\sqrt{3}x + 2}) - \frac{1}{12} \sqrt{3} \log(-4x \sqrt{3} \sqrt{x^2 + 2\sqrt{3}x + 2}) - \frac{1}{12} \sqrt{3} \log(-4x \sqrt{3} \sqrt{x^2 - 2\sqrt{3}x + 2}) - \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{2} \sqrt{3} \sqrt{x^2 + 2\sqrt{3}x + 2}\right) + \frac{1}{12} \sqrt{3} \arctan\left(-\frac{1}{2} \sqrt{3} \sqrt{x^2 - 2\sqrt{3}x + 2}\right) - \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{2} \sqrt{3} \sqrt{x^2 + 2\sqrt{3}x + 2}\right) - \frac{1}{12} \sqrt{3} \arctan\left(-\frac{1}{2} \sqrt{3} \sqrt{x^2 - 2\sqrt{3}x + 2}\right) - \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{2} \sqrt{3} \sqrt{x^2 + 2\sqrt{3}x + 2}\right) - \frac{1}{12} \sqrt{3} \arctan\left(-\frac{1}{2} \sqrt{3} \sqrt{x^2 - 2\sqrt{3}x + 2}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6+2),x, algorithm="fricas")

[Out] $\frac{1}{768} \cdot 32^{5/6} \cdot \sqrt{3} \cdot \log(64 \cdot 32^{5/6} \cdot \sqrt{3} \cdot x + 1024 \cdot x^2 + 512 \cdot 4^{2/3}) + \frac{1}{768} \cdot 32^{5/6} \cdot \sqrt{3} \cdot \log(16 \cdot 32^{5/6} \cdot \sqrt{3} \cdot x + 256 \cdot x^2 + 128 \cdot 4^{2/3}) - \frac{1}{768} \cdot 32^{5/6} \cdot \sqrt{3} \cdot \log(-16 \cdot 32^{5/6} \cdot \sqrt{3} \cdot x + 256 \cdot x^2 + 128 \cdot 4^{2/3}) - \frac{1}{768} \cdot 32^{5/6} \cdot \sqrt{3} \cdot \log(-64 \cdot 32^{5/6} \cdot \sqrt{3} \cdot x + 1024 \cdot x^2 + 512 \cdot 4^{2/3}) - \frac{1}{48} \cdot 32^{5/6} \cdot \sqrt{3} \cdot \arctan\left(\frac{1}{4} \cdot 32^{1/6} \cdot \sqrt{3} \cdot \sqrt{2x^2 + 4^{2/3}}\right) - \frac{1}{2} \cdot 32^{1/6} \cdot x - \frac{1}{96} \cdot 32^{5/6} \cdot \arctan(-32^{1/6} \cdot x + \frac{1}{4} \cdot 32^{1/6} \cdot \sqrt{32^{5/6} \cdot \sqrt{3} \cdot x + 16 \cdot x^2 + 8 \cdot 4^{2/3}}) - \sqrt{3} - \frac{1}{192} \cdot 32^{5/6} \cdot \arctan(-32^{1/6} \cdot x + \frac{1}{16} \cdot 32^{1/6} \cdot \sqrt{-16 \cdot 32^{5/6} \cdot \sqrt{3} \cdot x + 256 \cdot x^2 + 128 \cdot 4^{2/3}}) + \sqrt{3} - \frac{1}{192} \cdot 32^{5/6} \cdot \arctan(-32^{1/6} \cdot x + \frac{1}{32} \cdot 32^{1/6} \cdot \sqrt{-64 \cdot 32^{5/6} \cdot \sqrt{3} \cdot x + 1024 \cdot x^2 + 512 \cdot 4^{2/3}}) + \sqrt{3}$

Sympy [A]

time = 0.14, size = 14, normalized size = 0.10

$$\text{RootSum}(1492992t^6 + 1, (t \mapsto t \log(12t + x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**6+2),x)

[Out] RootSum(1492992*_t**6 + 1, Lambda(_t, _t*log(12*_t + x)))

Giac [A]

time = 0.00, size = 173, normalized size = 1.25

$$-\frac{1}{24} \cdot 2^{5/6} \sqrt{3} \ln(x^2 - 2^{5/6} \sqrt{3} x + 2^{5/6} \cdot 2^{5/6}) + \frac{1}{12} \cdot 2^{5/6} \arctan\left(\frac{x - \frac{1}{2} \sqrt{3} \cdot 2^{5/6}}{2^{5/6}}\right) + \frac{1}{6} \cdot 2^{5/6} \arctan\left(\frac{x}{2^{5/6}}\right) + \frac{1}{24} \cdot 2^{5/6} \sqrt{3} \ln(x^2 + 2^{5/6} \sqrt{3} x + 2^{5/6} \cdot 2^{5/6}) + \frac{1}{12} \cdot 2^{5/6} \arctan\left(\frac{x + \frac{1}{2} \sqrt{3} \cdot 2^{5/6}}{2^{5/6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6+2),x)

[Out] $\frac{1}{24}\sqrt{3}2^{1/6}\log(x^2 + \sqrt{3}2^{1/6}x + 2^{1/3}) - \frac{1}{24}\sqrt{3}2^{1/6}\log(x^2 - \sqrt{3}2^{1/6}x + 2^{1/3}) + \frac{1}{12}2^{1/6}\arctan\left(\frac{1/2 \cdot 2^{5/6}(2x + \sqrt{3}2^{1/6})}{1}\right) + \frac{1}{12}2^{1/6}\arctan\left(\frac{1/2 \cdot 2^{5/6}(2x - \sqrt{3}2^{1/6})}{1}\right) + \frac{1}{6}2^{1/6}\arctan\left(\frac{1/2 \cdot 2^{5/6}x}{1}\right)$

Mupad [B]

time = 0.13, size = 135, normalized size = 0.98

$$\frac{2^{1/6} \operatorname{atan}\left(\frac{2^{5/6}x}{2}\right)}{6} + \frac{2^{1/6} \operatorname{atan}\left(\frac{\frac{2^{1/6}x}{2\left(-\frac{2^{1/3}}{2} + \frac{2^{1/3}\sqrt{3}}{2}i\right)} + \frac{2^{1/6}\sqrt{3}xi}{2\left(-\frac{2^{1/3}}{2} + \frac{2^{1/3}\sqrt{3}}{2}i\right)}}{\sqrt{3}-i}\right) \operatorname{li}}{12} + \frac{2^{1/6} \operatorname{atan}\left(\frac{\frac{2^{1/6}x}{2\left(\frac{2^{1/3}}{2} + \frac{2^{1/3}\sqrt{3}}{2}i\right)} - \frac{2^{1/6}\sqrt{3}xi}{2\left(\frac{2^{1/3}}{2} + \frac{2^{1/3}\sqrt{3}}{2}i\right)}}{\sqrt{3}+i}\right) \operatorname{li}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}(1/(x^6 + 2), x)$

[Out] $(2^{1/6}\operatorname{atan}((2^{5/6}x)/2))/6 + (2^{1/6}\operatorname{atan}((2^{1/6}x)/(2*((2^{1/3})3^{1/2}*1i)/2 - 2^{1/3}/2)) + (2^{1/6}3^{1/2}x*1i)/(2*((2^{1/3})3^{1/2}*1i)/2 - 2^{1/3}/2)))*(3^{1/2} - 1i)*1i)/12 + (2^{1/6}\operatorname{atan}((2^{1/6}x)/(2*((2^{1/3})3^{1/2}*1i)/2 + 2^{1/3}/2)) - (2^{1/6}3^{1/2}x*1i)/(2*((2^{1/3})3^{1/2}*1i)/2 + 2^{1/3}/2)))*(3^{1/2} + 1i)*1i)/12$

3.49 $\int \frac{1}{1+x^8} dx$

Optimal. Leaf size=339

$$\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2(2-\sqrt{2})}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2(2+\sqrt{2})}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2(2-\sqrt{2})}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2(2+\sqrt{2})}}$$

[Out] $-1/16*\ln(1+x^2-x*(2-2^{(1/2)})^{(1/2)})*(2-2^{(1/2)})^{(1/2)}+1/16*\ln(1+x^2+x*(2-2^{(1/2)})^{(1/2)})*(2-2^{(1/2)})^{(1/2)}-1/4*\arctan((-2*x+(2-2^{(1/2)})^{(1/2)})/(2+2^{(1/2)})^{(1/2)})/(4-2*2^{(1/2)})^{(1/2)}+1/4*\arctan((2*x+(2-2^{(1/2)})^{(1/2)})/(2+2^{(1/2)})^{(1/2)})/(4-2*2^{(1/2)})^{(1/2)}-1/16*\ln(1+x^2-x*(2+2^{(1/2)})^{(1/2)})*(2+2^{(1/2)})^{(1/2)}+1/16*\ln(1+x^2+x*(2+2^{(1/2)})^{(1/2)})*(2+2^{(1/2)})^{(1/2)}-1/4*\arctan((-2*x+(2+2^{(1/2)})^{(1/2)})/(2-2^{(1/2)})^{(1/2)})/(4+2*2^{(1/2)})^{(1/2)}+1/4*\arctan((2*x+(2+2^{(1/2)})^{(1/2)})/(2-2^{(1/2)})^{(1/2)})/(4+2*2^{(1/2)})^{(1/2)}$

Rubi [A]

time = 0.19, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {219, 1183, 648, 632, 210, 642}

$$\frac{-\frac{1}{16}\sqrt{2-\sqrt{2}}\log(x^2-\sqrt{2-\sqrt{2}}x+1)+\frac{1}{16}\sqrt{2-\sqrt{2}}\log(x^2+\sqrt{2-\sqrt{2}}x+1)-\frac{1}{16}\sqrt{2+\sqrt{2}}\log(x^2-\sqrt{2+\sqrt{2}}x+1)+\frac{1}{16}\sqrt{2+\sqrt{2}}\log(x^2+\sqrt{2+\sqrt{2}}x+1)}{4\sqrt{2(2-\sqrt{2})}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2(2+\sqrt{2})}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2(2-\sqrt{2})}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2(2+\sqrt{2})}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2(2-\sqrt{2})}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^8)^(-1), x]

[Out] $-1/4*\text{ArcTan}[(\text{Sqrt}[2 - \text{Sqrt}[2]] - 2*x)/\text{Sqrt}[2 + \text{Sqrt}[2]]]/\text{Sqrt}[2*(2 - \text{Sqrt}[2])] - \text{ArcTan}[(\text{Sqrt}[2 + \text{Sqrt}[2]] - 2*x)/\text{Sqrt}[2 - \text{Sqrt}[2]]]/(4*\text{Sqrt}[2*(2 + \text{Sqrt}[2])]) + \text{ArcTan}[(\text{Sqrt}[2 - \text{Sqrt}[2]] + 2*x)/\text{Sqrt}[2 + \text{Sqrt}[2]]]/(4*\text{Sqrt}[2*(2 - \text{Sqrt}[2])]) + \text{ArcTan}[(\text{Sqrt}[2 + \text{Sqrt}[2]] + 2*x)/\text{Sqrt}[2 - \text{Sqrt}[2]]]/(4*\text{Sqrt}[2*(2 + \text{Sqrt}[2])]) - (\text{Sqrt}[2 - \text{Sqrt}[2]]*\text{Log}[1 - \text{Sqrt}[2 - \text{Sqrt}[2]]*x + x^2])/16 + (\text{Sqrt}[2 - \text{Sqrt}[2]]*\text{Log}[1 + \text{Sqrt}[2 - \text{Sqrt}[2]]*x + x^2])/16 - (\text{Sqrt}[2 + \text{Sqrt}[2]]*\text{Log}[1 - \text{Sqrt}[2 + \text{Sqrt}[2]]*x + x^2])/16 + (\text{Sqrt}[2 + \text{Sqrt}[2]]*\text{Log}[1 + \text{Sqrt}[2 + \text{Sqrt}[2]]*x + x^2])/16$

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 219

```
Int[((a_) + (b_)*(x_)^(n_))^(n_ - 1), x_Symbol] := With[{r = Numerator[Rt[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[r/(2*Sqrt[2]*a), Int[(Sqrt[2]*r - s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Dist[r/(2*Sqrt[2]*a), Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

Rule 632

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_ - 1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+x^8} dx &= \frac{\int \frac{\sqrt{2-x^2}}{1-\sqrt{2-x^2}x^2+x^4} dx}{2\sqrt{2}} + \frac{\int \frac{\sqrt{2+x^2}}{1+\sqrt{2+x^2}x^2+x^4} dx}{2\sqrt{2}} \\
&= \frac{\int \frac{\sqrt{2(2-\sqrt{2})} - (-1+\sqrt{2})x}{1-\sqrt{2-\sqrt{2}}x+x^2} dx}{4\sqrt{2(2-\sqrt{2})}} + \frac{\int \frac{\sqrt{2(2-\sqrt{2})} + (-1+\sqrt{2})x}{1+\sqrt{2-\sqrt{2}}x+x^2} dx}{4\sqrt{2(2-\sqrt{2})}} + \frac{\int \frac{\sqrt{2(2+\sqrt{2})}}{1-\sqrt{2+\sqrt{2}}x+x^2} dx}{4\sqrt{2(2+\sqrt{2})}} \\
&= \frac{1}{8} \sqrt{\frac{1}{2}(3-2\sqrt{2})} \int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2} dx + \frac{1}{8} \sqrt{\frac{1}{2}(3-2\sqrt{2})} \int \frac{1}{1+\sqrt{2+\sqrt{2}}x+x^2} dx \\
&= -\frac{1}{16} \sqrt{2-\sqrt{2}} \log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right) + \frac{1}{16} \sqrt{2-\sqrt{2}} \log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right) \\
&= -\frac{1}{8} \sqrt{2+\sqrt{2}} \tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{8} \sqrt{2-\sqrt{2}} \tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{8}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 209, normalized size = 0.62

$$\frac{1}{4} \tan^{-1}\left(\sec\left(\frac{\pi}{8}\right)\left(x - \sin\left(\frac{\pi}{8}\right)\right)\right) \cos\left(\frac{\pi}{8}\right) + \frac{1}{4} \tan^{-1}\left(\sec\left(\frac{\pi}{8}\right)\left(x + \sin\left(\frac{\pi}{8}\right)\right)\right) \cos\left(\frac{\pi}{8}\right) - \frac{1}{8} \cos\left(\frac{\pi}{8}\right) \log\left(1+x^2-2x\cos\left(\frac{\pi}{8}\right)\right) + \frac{1}{8} \cos\left(\frac{\pi}{8}\right) \log\left(1+x^2+2x\cos\left(\frac{\pi}{8}\right)\right) + \frac{1}{4} \tan^{-1}\left(\left(x - \cos\left(\frac{\pi}{8}\right)\right) \csc\left(\frac{\pi}{8}\right)\right) \sin\left(\frac{\pi}{8}\right) + \frac{1}{4} \tan^{-1}\left(\left(x + \cos\left(\frac{\pi}{8}\right)\right) \csc\left(\frac{\pi}{8}\right)\right) \sin\left(\frac{\pi}{8}\right) - \frac{1}{8} \log\left(1+x^2-2x\sin\left(\frac{\pi}{8}\right)\right) \sin\left(\frac{\pi}{8}\right) + \frac{1}{8} \log\left(1+x^2+2x\sin\left(\frac{\pi}{8}\right)\right) \sin\left(\frac{\pi}{8}\right)$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + x^8)^(-1), x]`

```
[Out] (ArcTan[Sec[Pi/8]*(x - Sin[Pi/8])]*Cos[Pi/8])/4 + (ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])]*Cos[Pi/8])/4 - (Cos[Pi/8]*Log[1 + x^2 - 2*x*Cos[Pi/8]])/8 + (Cos[Pi/8]*Log[1 + x^2 + 2*x*Cos[Pi/8]])/8 + (ArcTan[(x - Cos[Pi/8])*Csc[Pi/8]]*Sin[Pi/8])/4 + (ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]]*Sin[Pi/8])/4 - (Log[1 + x^2 - 2*x*Sin[Pi/8]]*Sin[Pi/8])/8 + (Log[1 + x^2 + 2*x*Sin[Pi/8]]*Sin[Pi/8])/8
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.79, size = 21, normalized size = 0.06

$$\text{RootSum}\left[1 + 16777216\#1^8 \&, \text{Log}[x + 8\#1] \#1\&\right]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(x^8+1),x]')`

[Out] `RootSum[1 + 16777216 #1 ^ 8&, Log[x + 8 #1] #1&]`

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 22, normalized size = 0.06

method	result
default	$\frac{\left(\sum_{-R=\text{RootOf}(-Z^8+1)} \frac{\ln(-R+x)}{-R^7} \right)}{8}$
risch	$\left(\sum_{-R=\text{RootOf}(-Z^8+1)} \frac{\ln(-R+x)}{-R^7} \right)$
meijerg	$-\frac{x \cos\left(\frac{\pi}{8}\right) \ln\left(1-2 \cos\left(\frac{\pi}{8}\right) (x^8)^{\frac{1}{8}} + (x^8)^{\frac{1}{4}}\right)}{8(x^8)^{\frac{1}{8}}} + \frac{x \sin\left(\frac{\pi}{8}\right) \arctan\left(\frac{\sin\left(\frac{\pi}{8}\right) (x^8)^{\frac{1}{8}}}{1-\cos\left(\frac{\pi}{8}\right) (x^8)^{\frac{1}{8}}}\right)}{4(x^8)^{\frac{1}{8}}} - \frac{x \cos\left(\frac{3\pi}{8}\right) \ln\left(1-2 \cos\left(\frac{3\pi}{8}\right) (x^8)^{\frac{1}{8}} + (x^8)^{\frac{1}{4}}\right)}{8(x^8)^{\frac{1}{8}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^8+1),x,method=_RETURNVERBOSE)`

[Out] `1/8*sum(1/_R^7*ln(-_R+x),_R=RootOf(_Z^8+1))`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^8+1),x, algorithm="maxima")`

[Out] `integrate(1/(x^8 + 1), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. 2(245) = 490.

time = 0.34, size = 1043, normalized size = 3.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^8+1),x, algorithm="fricas")`

[Out] `-1/16*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sqrt(2)*x - sqrt(2)*sqrt(4*x^2 + 2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 4) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(`

$2) \cdot \sqrt{-\sqrt{2} + 2}) \cdot \arctan(-2 \cdot \sqrt{2} \cdot x - \sqrt{2} \cdot \sqrt{4x^2 - 2\sqrt{2}}) \cdot x \cdot \sqrt{\sqrt{2} + 2} + 2 \cdot \sqrt{2} \cdot x \cdot \sqrt{-\sqrt{2} + 2} + 4 - \sqrt{\sqrt{2} + 2} + \sqrt{-\sqrt{2} + 2}) / (\sqrt{\sqrt{2} + 2} + \sqrt{-\sqrt{2} + 2})) + 1/16 \cdot (\sqrt{2} \cdot \sqrt{\sqrt{2} + 2} - \sqrt{2} \cdot \sqrt{-\sqrt{2} + 2}) \cdot \arctan((2 \cdot \sqrt{2} \cdot x - \sqrt{2} \cdot \sqrt{4x^2 + 2\sqrt{2}}) \cdot x \cdot \sqrt{\sqrt{2} + 2} + 2 \cdot \sqrt{2} \cdot x \cdot \sqrt{-\sqrt{2} + 2} + 4 + \sqrt{\sqrt{2} + 2} + \sqrt{-\sqrt{2} + 2}) / (\sqrt{\sqrt{2} + 2} - \sqrt{-\sqrt{2} + 2})) + 1/16 \cdot (\sqrt{2} \cdot \sqrt{\sqrt{2} + 2} - \sqrt{2} \cdot \sqrt{-\sqrt{2} + 2}) \cdot \arctan((2 \cdot \sqrt{2} \cdot x - \sqrt{2} \cdot \sqrt{4x^2 - 2\sqrt{2}}) \cdot x \cdot \sqrt{\sqrt{2} + 2} - 2 \cdot \sqrt{2} \cdot x \cdot \sqrt{-\sqrt{2} + 2} + 4 - \sqrt{\sqrt{2} + 2} - \sqrt{-\sqrt{2} + 2}) / (\sqrt{\sqrt{2} + 2} - \sqrt{-\sqrt{2} + 2})) + 1/64 \cdot (\sqrt{2} \cdot \sqrt{\sqrt{2} + 2} + \sqrt{2} \cdot \sqrt{-\sqrt{2} + 2}) \cdot \log(4x^2 + 2\sqrt{2} \cdot x \cdot \sqrt{\sqrt{2} + 2} + 2\sqrt{2} \cdot x \cdot \sqrt{-\sqrt{2} + 2} + 4) + 1/64 \cdot (\sqrt{2} \cdot \sqrt{\sqrt{2} + 2} - \sqrt{2} \cdot \sqrt{-\sqrt{2} + 2}) \cdot \log(4x^2 + 2\sqrt{2} \cdot x \cdot \sqrt{\sqrt{2} + 2} - 2\sqrt{2} \cdot x \cdot \sqrt{-\sqrt{2} + 2} + 4) - 1/64 \cdot (\sqrt{2} \cdot \sqrt{\sqrt{2} + 2} - \sqrt{2} \cdot \sqrt{-\sqrt{2} + 2}) \cdot \log(4x^2 - 2\sqrt{2} \cdot x \cdot \sqrt{\sqrt{2} + 2} + 2) + 2\sqrt{2} \cdot x \cdot \sqrt{-\sqrt{2} + 2} + 4) - 1/64 \cdot (\sqrt{2} \cdot \sqrt{\sqrt{2} + 2} + \sqrt{2} \cdot \sqrt{-\sqrt{2} + 2}) \cdot \log(4x^2 - 2\sqrt{2} \cdot x \cdot \sqrt{\sqrt{2} + 2} - 2\sqrt{2} \cdot x \cdot \sqrt{-\sqrt{2} + 2} + 4) - 1/8 \cdot \sqrt{\sqrt{2} + 2} \cdot \arctan(-2x - 2\sqrt{2} \cdot \sqrt{x^2 + x \cdot \sqrt{-\sqrt{2} + 2} + 1} + \sqrt{-\sqrt{2} + 2}) / \sqrt{\sqrt{2} + 2}) - 1/8 \cdot \sqrt{\sqrt{2} + 2} \cdot \arctan(-2x - 2\sqrt{2} \cdot \sqrt{x^2 - x \cdot \sqrt{-\sqrt{2} + 2} + 1} - \sqrt{-\sqrt{2} + 2}) / \sqrt{\sqrt{2} + 2}) - 1/8 \cdot \sqrt{-\sqrt{2} + 2} \cdot \arctan(-2x - 2\sqrt{2} \cdot \sqrt{x^2 + x \cdot \sqrt{\sqrt{2} + 2} + 1} + \sqrt{\sqrt{2} + 2}) / \sqrt{-\sqrt{2} + 2}) - 1/8 \cdot \sqrt{-\sqrt{2} + 2} \cdot \arctan(-2x - 2\sqrt{2} \cdot \sqrt{x^2 - x \cdot \sqrt{\sqrt{2} + 2} + 1} - \sqrt{\sqrt{2} + 2}) / \sqrt{-\sqrt{2} + 2}) + 1/32 \cdot \sqrt{\sqrt{2} + 2} \cdot \log(x^2 + x \cdot \sqrt{\sqrt{2} + 2} + 1) - 1/32 \cdot \sqrt{\sqrt{2} + 2} \cdot \log(x^2 - x \cdot \sqrt{\sqrt{2} + 2} + 1) + 1/32 \cdot \sqrt{-\sqrt{2} + 2} \cdot \log(x^2 + x \cdot \sqrt{-\sqrt{2} + 2} + 1) - 1/32 \cdot \sqrt{-\sqrt{2} + 2} \cdot \log(x^2 - x \cdot \sqrt{-\sqrt{2} + 2} + 1) + 1)$

Sympy [A]

time = 1.13, size = 14, normalized size = 0.04

$$\text{RootSum}(16777216t^8 + 1, (t \mapsto t \log(8t + x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**8+1),x)

[Out] RootSum(16777216*_t**8 + 1, Lambda(_t, _t*log(8*_t + x)))

Giac [A]

time = 0.00, size = 337, normalized size = 0.99

$$\frac{1}{16} \sqrt{\sqrt{2} + 2} \ln(x^2 - \sqrt{2 + \sqrt{2}} x + 1) + \frac{1}{8} \sqrt{-\sqrt{2} + 2} \arctan\left(\frac{x - \sqrt{2 + \sqrt{2}}}{\sqrt{2 + \sqrt{2}}}\right) - \frac{1}{16} \sqrt{-\sqrt{2} + 2} \ln(x^2 - \sqrt{2 - \sqrt{2}} x + 1) + \frac{1}{8} \sqrt{\sqrt{2} + 2} \arctan\left(\frac{x - \sqrt{2 - \sqrt{2}}}{\sqrt{2 - \sqrt{2}}}\right) + \frac{1}{16} \sqrt{-\sqrt{2} + 2} \ln(x^2 + \sqrt{2 - \sqrt{2}} x + 1) + \frac{1}{8} \sqrt{\sqrt{2} + 2} \arctan\left(\frac{x + \sqrt{2 - \sqrt{2}}}{\sqrt{2 - \sqrt{2}}}\right) + \frac{1}{16} \sqrt{\sqrt{2} + 2} \ln(x^2 + \sqrt{2 + \sqrt{2}} x + 1) + \frac{1}{8} \sqrt{-\sqrt{2} + 2} \arctan\left(\frac{x + \sqrt{2 + \sqrt{2}}}{\sqrt{2 + \sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8+1),x)

[Out] 1/8*sqrt(sqrt(2) + 2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 1/8*sqrt(sqrt(2) + 2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/16*sqrt(sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1/16*sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/16*sqrt(-sqrt(2) + 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/16*sqrt(-sqrt(2) + 2)*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1)

Mupad [B]

time = 0.30, size = 288, normalized size = 0.85

$$\operatorname{atan}\left(\frac{x\sqrt{-\sqrt{2}-2}i}{\sqrt{2-\sqrt{2}}\sqrt{-\sqrt{2}-2}+\sqrt{2}}-\frac{x\sqrt{2-\sqrt{2}}i}{\sqrt{2-\sqrt{2}}\sqrt{-\sqrt{2}-2}+\sqrt{2}}\right)\left(\frac{\sqrt{-\sqrt{2}-2}i}{4}-\frac{\sqrt{2-\sqrt{2}}i}{8}\right)-\operatorname{atan}\left(\frac{x\sqrt{\sqrt{2}-2}i}{\sqrt{2}+\sqrt{\sqrt{2}-2}\sqrt{\sqrt{2}+2}}+\frac{x\sqrt{\sqrt{2}+2}i}{\sqrt{2}+\sqrt{\sqrt{2}-2}\sqrt{\sqrt{2}+2}}\right)\left(\frac{\sqrt{\sqrt{2}-2}i}{4}+\frac{\sqrt{\sqrt{2}+2}i}{8}\right)+\operatorname{atan}\left(\frac{-\sqrt{2}x\sqrt{\sqrt{2}+2}}{\sqrt{2}+\sqrt{\sqrt{2}-2}}+\sqrt{\sqrt{2}+2}\left(\frac{1}{4}+\frac{i}{8}\right)\right)\left(\frac{\sqrt{2}i}{16}-\frac{i}{8}\right)\sqrt{\sqrt{2}+2}-\operatorname{atan}\left(x\sqrt{\sqrt{2}+2}\left(\frac{1}{4}-\frac{i}{8}\right)+\frac{\sqrt{2}x\sqrt{\sqrt{2}+2}i}{4}\right)\left(\frac{\sqrt{2}}{16}-\frac{1}{8}+\frac{i}{16}\right)\sqrt{\sqrt{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^8 + 1),x)

[Out] atan((x*(- 2^(1/2) - 2)^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1/2) - 2)^(1/2) + 2^(1/2)) - (x*(2 - 2^(1/2))^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1/2) - 2)^(1/2) + 2^(1/2)))*(((- 2^(1/2) - 2)^(1/2)*1i)/8 - ((2 - 2^(1/2))^(1/2)*1i)/8) - atan((x*(2^(1/2) - 2)^(1/2)*1i)/(2^(1/2) + (2^(1/2) - 2)^(1/2)*(2^(1/2) + 2)^(1/2)) + (x*(2^(1/2) + 2)^(1/2)*1i)/(2^(1/2) + (2^(1/2) - 2)^(1/2)*(2^(1/2) + 2)^(1/2)))*((2^(1/2) - 2)^(1/2)*1i)/8 + ((2^(1/2) + 2)^(1/2)*1i)/8) + atan(x*(2^(1/2) + 2)^(1/2)*(1/2 + 1i/2) - (2^(1/2)*x*(2^(1/2) + 2)^(1/2))/2)*((2^(1/2)*1i)/16 - (1/16 + 1i/16))*(2^(1/2) + 2)^(1/2)*2i - atan(x*(2^(1/2) + 2)^(1/2)*(1/2 - 1i/2) + (2^(1/2)*x*(2^(1/2) + 2)^(1/2)*1i)/2)*((2^(1/2)/16 - (1/16 - 1i/16))*(2^(1/2) + 2)^(1/2)*2i

3.50 $\int \frac{1}{-1+x^8} dx$

Optimal. Leaf size=97

$$-\frac{1}{4} \tan^{-1}(x) + \frac{\tan^{-1}(1 - \sqrt{2}x)}{4\sqrt{2}} - \frac{\tan^{-1}(1 + \sqrt{2}x)}{4\sqrt{2}} - \frac{1}{4} \tanh^{-1}(x) + \frac{\log(1 - \sqrt{2}x + x^2)}{8\sqrt{2}} - \frac{\log(1 + \sqrt{2}x + x^2)}{8\sqrt{2}}$$

[Out] -1/4*arctan(x)-1/4*arctanh(x)-1/8*arctan(-1+x*2^(1/2))*2^(1/2)-1/8*arctan(1+x*2^(1/2))*2^(1/2)+1/16*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/16*ln(1+x^2+x*2^(1/2))*2^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.429$, Rules used = {220, 218, 212, 209, 217, 1179, 642, 1176, 631, 210}

$$\frac{\log(x^2 - \sqrt{2}x + 1)}{8\sqrt{2}} - \frac{\log(x^2 + \sqrt{2}x + 1)}{8\sqrt{2}} - \frac{1}{4} \tan^{-1}(x) + \frac{\tan^{-1}(1 - \sqrt{2}x)}{4\sqrt{2}} - \frac{\tan^{-1}(\sqrt{2}x + 1)}{4\sqrt{2}} - \frac{1}{4} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^8)^(-1), x]

[Out] -1/4*ArcTan[x] + ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) - ArcTan[1 + Sqrt[2]*x]/(4*Sqrt[2]) - ArcTanh[x]/4 + Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2]) - Log[1 + Sqrt[2]*x + x^2]/(8*Sqrt[2])

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217


```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(n_)^(-1), x_Symbol] := With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] && !GtQ[a/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-1+x^8} dx &= -\left(\frac{1}{2} \int \frac{1}{1-x^4} dx\right) - \frac{1}{2} \int \frac{1}{1+x^4} dx \\
&= -\left(\frac{1}{4} \int \frac{1}{1-x^2} dx\right) - \frac{1}{4} \int \frac{1}{1+x^2} dx - \frac{1}{4} \int \frac{1-x^2}{1+x^4} dx - \frac{1}{4} \int \frac{1+x^2}{1+x^4} dx \\
&= -\frac{1}{4} \tan^{-1}(x) - \frac{1}{4} \tanh^{-1}(x) - \frac{1}{8} \int \frac{1}{1-\sqrt{2}x+x^2} dx - \frac{1}{8} \int \frac{1}{1+\sqrt{2}x+x^2} dx + \frac{\int \frac{\sqrt{2}}{-1-\sqrt{2}x+x^2} dx}{8\sqrt{2}} \\
&= -\frac{1}{4} \tan^{-1}(x) - \frac{1}{4} \tanh^{-1}(x) + \frac{\log(1-\sqrt{2}x+x^2)}{8\sqrt{2}} - \frac{\log(1+\sqrt{2}x+x^2)}{8\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}}{-1-\sqrt{2}x+x^2} dx\right)}{8\sqrt{2}} \\
&= -\frac{1}{4} \tan^{-1}(x) + \frac{\tan^{-1}(1-\sqrt{2}x)}{4\sqrt{2}} - \frac{\tan^{-1}(1+\sqrt{2}x)}{4\sqrt{2}} - \frac{1}{4} \tanh^{-1}(x) + \frac{\log(1-\sqrt{2}x+x^2)}{8\sqrt{2}} - \frac{\log(1+\sqrt{2}x+x^2)}{8\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}}{-1-\sqrt{2}x+x^2} dx\right)}{8\sqrt{2}}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 98, normalized size = 1.01

$$\frac{1}{16} \left(-4 \tan^{-1}(x) + 2\sqrt{2} \tan^{-1}(1-\sqrt{2}x) - 2\sqrt{2} \tan^{-1}(1+\sqrt{2}x) + 2 \log(1-x) - 2 \log(1+x) + \sqrt{2} \log(1-\sqrt{2}x+x^2) - \sqrt{2} \log(1+\sqrt{2}x+x^2) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(-1 + x^8)^(-1), x]`

```
[Out] (-4*ArcTan[x] + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] + 2*Log[1 - x] - 2*Log[1 + x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/16
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 60.60, size = 46, normalized size = 0.47

$$\text{RootSum}[1 + 4096\#1^4 \&, \text{Log}[x - 8\#1] \#1\&] - \frac{\text{Log}[1+x]}{8} - \frac{I}{8} \text{Log}[I+x] + \frac{I}{8} \text{Log}[-I+x] + \frac{\text{Log}[-1+x]}{8}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^8-1), x]')`

```
[Out] RootSum[1 + 4096 #1 ^ 4&, Log[x - 8 #1] #1&] - Log[1 + x] / 8 - I / 8 Log[I + x] + I / 8 Log[-I + x] + Log[-1 + x] / 8
```

Maple [A]

time = 0.04, size = 61, normalized size = 0.63

method	result
risch	$-\frac{\ln(1+x)}{8} + \frac{\left(\sum_{R=\text{RootOf}(-Z^4+1)} -R \ln(-R+x) \right)}{8} + \frac{\ln(-1+x)}{8} - \frac{\arctan(x)}{4}$
default	$-\frac{\arctan(x)}{4} - \frac{\operatorname{arctanh}(x)}{4} - \frac{\sqrt{2} \left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right) + 2\arctan(1+x\sqrt{2}) + 2\arctan(-1+x\sqrt{2}) \right)}{16}$
meijerg	$x \left(\ln\left(1-(x^8)^{\frac{1}{8}}\right) - \ln\left(1+(x^8)^{\frac{1}{8}}\right) + \frac{\sqrt{2} \ln\left(1-\sqrt{2}(x^8)^{\frac{1}{8}}+(x^8)^{\frac{1}{4}}\right)}{2} - \sqrt{2} \arctan\left(\frac{\sqrt{2}(x^8)^{\frac{1}{8}}}{2-\sqrt{2}(x^8)^{\frac{1}{8}}}\right) - 2\arctan\left((x^8)^{\frac{1}{8}}\right) - \frac{\sqrt{2}}{8(x^8)^{\frac{1}{8}}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arctan(x)-1/4*arctanh(x)-1/16*2^(1/2)*(ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))+2*arctan(1+x*2^(1/2))+2*arctan(-1+x*2^(1/2)))
```

Maxima [A]

time = 0.35, size = 88, normalized size = 0.91

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x+\sqrt{2})\right) - \frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x-\sqrt{2})\right) - \frac{1}{16}\sqrt{2}\log(x^2+\sqrt{2}x+1) + \frac{1}{16}\sqrt{2}\log(x^2-\sqrt{2}x+1) - \frac{1}{4}\arctan(x) - \frac{1}{8}\log(x+1) + \frac{1}{8}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-1),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*arctan(x) - 1/8*log(x + 1) + 1/8*log(x - 1)
```

Fricas [A]

time = 0.34, size = 116, normalized size = 1.20

$$\frac{1}{4}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right) + \frac{1}{4}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right) - \frac{1}{16}\sqrt{2}\log(4x^2+4\sqrt{2}x+4) + \frac{1}{16}\sqrt{2}\log(4x^2-4\sqrt{2}x+4) - \frac{1}{4}\arctan(x) - \frac{1}{8}\log(x+1) + \frac{1}{8}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) + 1/4*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) - 1/16*sqrt(2)*log(4*x^2 + 4*sqrt(2)*x + 4) + 1/16*sqrt(2)*log(4*x^2 - 4*sqrt(2)*x + 4) - 1/4*arctan(x) - 1/8*log(x + 1) + 1/8*log(x - 1)
```

Sympy [C] Result contains complex when optimal does not.

time = 85.04, size = 44, normalized size = 0.45

$$\frac{\log(x-1)}{8} - \frac{\log(x+1)}{8} + \frac{i \log(x-i)}{8} - \frac{i \log(x+i)}{8} + \text{RootSum}(4096t^4 + 1, (t \mapsto t \log(-8t + x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**8-1),x)

[Out] log(x - 1)/8 - log(x + 1)/8 + I*log(x - I)/8 - I*log(x + I)/8 + RootSum(4096*_t**4 + 1, Lambda(_t, _t*log(-8*_t + x)))

Giac [A]

time = 0.00, size = 131, normalized size = 1.35

$$\frac{\ln|x-1|}{8} - \frac{\ln|x+1|}{8} + \frac{1}{16}\sqrt{2} \ln(x^2 - \sqrt{2}x + 1) - \frac{1}{8}\sqrt{2} \arctan\left(\frac{x - \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}\right) - \frac{\arctan x}{4} - \frac{1}{16}\sqrt{2} \ln(x^2 + \sqrt{2}x + 1) - \frac{1}{8}\sqrt{2} \arctan\left(\frac{x + \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8-1),x)

[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*arctan(x) - 1/8*log(abs(x + 1)) + 1/8*log(abs(x - 1))

Mupad [B]

time = 0.16, size = 45, normalized size = 0.46

$$\frac{\text{atan}(x \text{ i}) \text{ i}}{4} - \frac{\text{atan}(x)}{4} + \sqrt{2} \text{atan}\left(\sqrt{2} x \left(\frac{1}{2} - \frac{1}{2} \text{i}\right)\right) \left(-\frac{1}{8} - \frac{1}{8} \text{i}\right) + \sqrt{2} \text{atan}\left(\sqrt{2} x \left(\frac{1}{2} + \frac{1}{2} \text{i}\right)\right) \left(-\frac{1}{8} + \frac{1}{8} \text{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^8 - 1),x)

[Out] (atan(x*i)*i)/4 - atan(x)/4 - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/8 + 1i/8) - 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(1/8 - 1i/8)

3.51 $\int \frac{1}{1-x^4+x^8} dx$

Optimal. Leaf size=275

$$\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

```
[Out] -1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)-1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1360, 1183, 648, 632, 210, 642}

$$\frac{\log(x^2 - \sqrt{2-\sqrt{3}}x + 1)}{4\sqrt{6}} + \frac{\log(x^2 + \sqrt{2-\sqrt{3}}x + 1)}{4\sqrt{6}} - \frac{\log(x^2 - \sqrt{2+\sqrt{3}}x + 1)}{4\sqrt{6}} + \frac{\log(x^2 + \sqrt{2+\sqrt{3}}x + 1)}{4\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4 + x^8)^(-1), x]

```
[Out] -1/2*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/Sqrt[6] - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

$x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[\frac{(d) + (e) \cdot x}{(a) + (b) \cdot x + (c) \cdot x^2}, x_{\text{Symbol}}] \rightarrow \text{Simp}[d \cdot (\text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]/b), x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

Rule 648

$\text{Int}[\frac{(d) + (e) \cdot x}{(a) + (b) \cdot x + (c) \cdot x^2}, x_{\text{Symbol}}] \rightarrow \text{Dist}[\frac{2 \cdot c \cdot d - b \cdot e}{2 \cdot c}, \text{Int}[1/(a + b \cdot x + c \cdot x^2), x], x] + \text{Dist}[e/(2 \cdot c), \text{Int}[(b + 2 \cdot c \cdot x)/(a + b \cdot x + c \cdot x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2 \cdot c \cdot d - b \cdot e, 0] \&\& \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4 \cdot a \cdot c]$

Rule 1183

$\text{Int}[\frac{(d) + (e) \cdot x^2}{(a) + (b) \cdot x^2 + (c) \cdot x^4}, x_{\text{Symbol}}] \rightarrow \text{With}\{q = \text{Rt}[a/c, 2]\}, \text{With}\{r = \text{Rt}[2 \cdot q - b/c, 2]\}, \text{Dist}[1/(2 \cdot c \cdot q \cdot r), \text{Int}[(d \cdot r - (d - e \cdot q) \cdot x)/(q - r \cdot x + x^2), x], x] + \text{Dist}[1/(2 \cdot c \cdot q \cdot r), \text{Int}[(d \cdot r + (d - e \cdot q) \cdot x)/(q + r \cdot x + x^2), x], x]] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \&\& \text{NeQ}[c \cdot d^2 - b \cdot d \cdot e + a \cdot e^2, 0] \&\& \text{NegQ}[b^2 - 4 \cdot a \cdot c]$

Rule 1360

$\text{Int}[\frac{(a) + (b) \cdot x^{(n)} + (c) \cdot x^{(2n)}}{x}, x_{\text{Symbol}}] \rightarrow \text{With}\{q = \text{Rt}[a/c, 2]\}, \text{With}\{r = \text{Rt}[2 \cdot q - b/c, 2]\}, \text{Dist}[1/(2 \cdot c \cdot q \cdot r), \text{Int}[(r - x^{(n/2)})/(q - r \cdot x^{(n/2)} + x^n), x], x] + \text{Dist}[1/(2 \cdot c \cdot q \cdot r), \text{Int}[(r + x^{(n/2)})/(q + r \cdot x^{(n/2)} + x^n), x], x]] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{EqQ}[n2, 2 \cdot n] \&\& \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \&\& \text{IGtQ}[n/2, 0] \&\& \text{NegQ}[b^2 - 4 \cdot a \cdot c]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{1-x^4+x^8} dx &= \frac{\int \frac{\sqrt{3}-x^2}{1-\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x^2}{1+\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} \\
&= \frac{\int \frac{\sqrt{3(2-\sqrt{3})} - (-1+\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3(2-\sqrt{3})}} + \frac{\int \frac{\sqrt{3(2-\sqrt{3})} + (-1+\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3(2-\sqrt{3})}} + \frac{\int \frac{\sqrt{3(2+\sqrt{3})} - (-1-\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3(2+\sqrt{3})}} \\
&= -\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{6}} - \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{6}} \\
&= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.01, size = 42, normalized size = 0.15

$$\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&, \frac{\log(x-\#1)}{-\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4 + x^8)^(-1), x]

[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) &]/4

Mathics [A]

time = 3.25, size = 124, normalized size = 0.45

$$\sqrt{6}\left(\text{Log}\left[1+\sqrt{6}x+3x^2+\sqrt{6}x^3+x^4\right]-\text{Log}\left[1-\sqrt{6}x+3x^2-\sqrt{6}x^3+x^4\right]+2\text{ArcTan}\left[-3+2\sqrt{6}x-4x^2+\sqrt{6}x^3\right]+2\text{ArcTan}\left[-\frac{1}{3}+\frac{\sqrt{6}x}{3}\right]+2\text{ArcTan}\left[\frac{1}{3}+\frac{\sqrt{6}x}{3}\right]+2\text{ArcTan}\left[3+2\sqrt{6}x+4x^2+\sqrt{6}x^3\right]\right)$$

24

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x^8-x^4+1),x]')

[Out] Sqrt[6] (Log[1 + Sqrt[6] x + 3 x ^ 2 + Sqrt[6] x ^ 3 + x ^ 4] - Log[1 - Sqrt[6] x + 3 x ^ 2 - Sqrt[6] x ^ 3 + x ^ 4] + 2 ArcTan[-3 + 2 Sqrt[6] x - 4 x ^ 2 + Sqrt[6] x ^ 3] + 2 ArcTan[-1 / 3 + Sqrt[6] x / 3] + 2 ArcTan[1 / 3 + Sqrt[6] x / 3] + 2 ArcTan[3 + 2 Sqrt[6] x + 4 x ^ 2 + Sqrt[6] x ^ 3]) / 24

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 30, normalized size = 0.11

method	result	size
default	$\frac{\sum_{-R=\text{RootOf}(9-Z^4+1)} -R \ln(3-R^2+3-Rx+x^2)}{4}$	30
risch	$\frac{\sum_{-R=\text{RootOf}(9-Z^4+1)} -R \ln(3-R^2+3-Rx+x^2)}{4}$	30

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^8-x^4+1),x,method=_RETURNVERBOSE)

[Out] 1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8-x^4+1),x, algorithm="maxima")

[Out] integrate(1/(x^8 - x^4 + 1), x)

Fricas [A]

time = 0.34, size = 220, normalized size = 0.80

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(\frac{\sqrt{3}\sqrt{2}(x^3-x)+x^2-\sqrt{3}\sqrt{2}\sqrt{(x^3+x)+3x^2+1}(\sqrt{3}\sqrt{2}x-2)}{3x^2-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(\frac{\sqrt{3}\sqrt{2}(x^3-x)-x^2-\sqrt{3}\sqrt{2}\sqrt{(x^3+x)+3x^2+1}(\sqrt{3}\sqrt{2}x+2)}{3x^2-2}\right)+\frac{1}{24}\sqrt{3}\sqrt{2}\log(36x^4+36\sqrt{3}\sqrt{2}(x^3+x)+108x^2+36)-\frac{1}{24}\sqrt{3}\sqrt{2}\log(36x^4-36\sqrt{3}\sqrt{2}(x^3+x)+108x^2+36)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8-x^4+1),x, algorithm="fricas")

[Out] -1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2 - 2)) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqrt(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*x^2 - 2)) + 1/24*sqrt(3)*sqrt(2)*log(36*x^4 + 36*sqrt(3)*sqrt(2)*(x^3 + x) + 108

$$*x^2 + 36) - 1/24*\sqrt{3}*\sqrt{2}*\log(36*x^4 - 36*\sqrt{3}*\sqrt{2}*(x^3 + x) + 108*x^2 + 36)$$

Sympy [A]

time = 0.11, size = 165, normalized size = 0.60

$$\frac{\sqrt{6} \cdot \left(2 \operatorname{atan} \left(\frac{\sqrt{6}x - \frac{1}{3}}{\frac{2x^2}{3} - \frac{2}{3}i} \right) + 2 \operatorname{atan} \left(\frac{\sqrt{6}x^3 - 4x^2 + 2\sqrt{6}x - 3}{\frac{2x^2}{3} - \frac{2}{3}i} \right) \right)}{24} + \frac{\sqrt{6} \cdot \left(2 \operatorname{atan} \left(\frac{\sqrt{6}x + \frac{1}{3}}{\frac{2x^2}{3} + \frac{2}{3}i} \right) + 2 \operatorname{atan} \left(\frac{\sqrt{6}x^3 + 4x^2 + 2\sqrt{6}x + 3}{\frac{2x^2}{3} + \frac{2}{3}i} \right) \right)}{24} - \frac{\sqrt{6} \log \left(x^4 - \sqrt{6}x^3 + 3x^2 - \sqrt{6}x + 1 \right)}{24} + \frac{\sqrt{6} \log \left(x^4 + \sqrt{6}x^3 + 3x^2 + \sqrt{6}x + 1 \right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**8-x**4+1),x)

[Out] $\sqrt{6}*(2*\operatorname{atan}(\sqrt{6}*x/3 - 1/3) + 2*\operatorname{atan}(\sqrt{6}*x**3 - 4*x**2 + 2*\sqrt{6}*(6)*x - 3))/24 + \sqrt{6}*(2*\operatorname{atan}(\sqrt{6}*x/3 + 1/3) + 2*\operatorname{atan}(\sqrt{6}*x**3 + 4*x**2 + 2*\sqrt{6}*x + 3))/24 - \sqrt{6}*\log(x**4 - \sqrt{6}*x**3 + 3*x**2 - \sqrt{6}*x + 1)/24 + \sqrt{6}*\log(x**4 + \sqrt{6}*x**3 + 3*x**2 + \sqrt{6}*x + 1)/24$

Giac [A]

time = 0.00, size = 297, normalized size = 1.08

$$-\frac{1}{24}\sqrt{6}\ln\left(x^2 + \frac{1}{2}(-\sqrt{2} - \sqrt{6})x + 1\right) + \frac{1}{12}\sqrt{6}\arctan\left(\frac{x - \frac{\sqrt{6} + \sqrt{2}}{2}}{\frac{\sqrt{6} + \sqrt{2}}{2}}\right) - \frac{1}{24}\sqrt{6}\ln\left(x^2 + \frac{1}{2}(\sqrt{2} - \sqrt{6})x + 1\right) + \frac{1}{12}\sqrt{6}\arctan\left(\frac{x - \frac{\sqrt{6} - \sqrt{2}}{2}}{\frac{\sqrt{6} - \sqrt{2}}{2}}\right) + \frac{1}{24}\sqrt{6}\ln\left(x^2 + \frac{1}{2}(-\sqrt{2} + \sqrt{6})x + 1\right) + \frac{1}{12}\sqrt{6}\arctan\left(\frac{x - \frac{\sqrt{6} - \sqrt{2}}{2}}{\frac{\sqrt{6} - \sqrt{2}}{2}}\right) - \frac{1}{24}\sqrt{6}\ln\left(x^2 + \frac{1}{2}(\sqrt{2} + \sqrt{6})x + 1\right) + \frac{1}{12}\sqrt{6}\arctan\left(\frac{x - \frac{\sqrt{6} + \sqrt{2}}{2}}{\frac{\sqrt{6} + \sqrt{2}}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8-x^4+1),x)

[Out] $1/12*\sqrt{6}*\arctan((4*x + \sqrt{6} - \sqrt{2})/(\sqrt{6} + \sqrt{2})) + 1/12*\sqrt{6}*\arctan((4*x - \sqrt{6} + \sqrt{2})/(\sqrt{6} + \sqrt{2})) + 1/12*\sqrt{6}*\arctan((4*x + \sqrt{6} + \sqrt{2})/(\sqrt{6} - \sqrt{2})) + 1/12*\sqrt{6}*\arctan((4*x - \sqrt{6} - \sqrt{2})/(\sqrt{6} - \sqrt{2})) + 1/24*\sqrt{6}*\log(x^2 + 1/2*x*(\sqrt{6} + \sqrt{2}) + 1) - 1/24*\sqrt{6}*\log(x^2 - 1/2*x*(\sqrt{6} + \sqrt{2}) + 1) + 1/24*\sqrt{6}*\log(x^2 + 1/2*x*(\sqrt{6} - \sqrt{2}) + 1) - 1/24*\sqrt{6}*\log(x^2 - 1/2*x*(\sqrt{6} - \sqrt{2}) + 1)$

Mupad [B]

time = 0.10, size = 53, normalized size = 0.19

$$\sqrt{6} \operatorname{atan} \left(\frac{\sqrt{6} x \left(\frac{1}{3} + \frac{1}{3}i \right)}{\frac{2x^2}{3} - \frac{2}{3}i} \right) \left(-\frac{1}{12} - \frac{1}{12}i \right) + \sqrt{6} \operatorname{atan} \left(\frac{\sqrt{6} x \left(\frac{1}{3} - \frac{1}{3}i \right)}{\frac{2x^2}{3} + \frac{2}{3}i} \right) \left(-\frac{1}{12} + \frac{1}{12}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^8 - x^4 + 1),x)

[Out] $-6^{(1/2)}*\operatorname{atan}((6^{(1/2)}*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i/12) - 6^{(1/2)}*\operatorname{atan}((6^{(1/2)}*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1i/12)$

3.52 $\int \frac{x^7}{1+x^{12}} dx$

Optimal. Leaf size=49

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{12} \log(1+x^4) + \frac{1}{24} \log(1-x^4+x^8)$$

[Out] $-1/12*\ln(x^4+1)+1/24*\ln(x^8-x^4+1)-1/12*\arctan(1/3*(-2*x^4+1)*3^{(1/2)})*3^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {281, 298, 31, 648, 632, 210, 642}

$$-\frac{1}{12} \log(x^4+1) - \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{24} \log(x^8-x^4+1)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^7/(1+x^{12}),x]$

[Out] $-1/4*\text{ArcTan}[(1-2*x^4)/\text{Sqrt}[3]]/\text{Sqrt}[3] - \text{Log}[1+x^4]/12 + \text{Log}[1-x^4+x^8]/24$

Rule 31

$\text{Int}[(a_ + (b_)*(x_))^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] \text{ ; FreeQ}\{a, b\}, x]$

Rule 210

$\text{Int}[(a_ + (b_)*(x_)^2)^{(-1)}, x_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{(-1)}*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] \text{ ; FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])]$

Rule 281

$\text{Int}[(x_)^{(m_)}*(a_ + (b_)*(x_)^{(n_))^{(p_)}, x_Symbol] \rightarrow \text{With}\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{(m+1)/k-1}*(a + b*x^{(n/k)})^p, x], x, x^k], x] \text{ ; } k \neq 1] \text{ ; FreeQ}\{a, b, p\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 298

$\text{Int}[(x_)/((a_ + (b_)*(x_)^3), x_Symbol] \rightarrow \text{Dist}[-(3*\text{Rt}[a, 3]*\text{Rt}[b, 3])^{(-1)}, \text{Int}[1/(\text{Rt}[a, 3] + \text{Rt}[b, 3]*x), x], x] + \text{Dist}[1/(3*\text{Rt}[a, 3]*\text{Rt}[b, 3]), I$

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{x^7}{1+x^{12}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x}{1+x^3} dx, x, x^4 \right) \\ &= -\left(\frac{1}{12} \text{Subst} \left(\int \frac{1}{1+x} dx, x, x^4 \right) \right) + \frac{1}{12} \text{Subst} \left(\int \frac{1+x}{1-x+x^2} dx, x, x^4 \right) \\ &= -\frac{1}{12} \log(1+x^4) + \frac{1}{24} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^4 \right) + \frac{1}{8} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^4 \right) \\ &= -\frac{1}{12} \log(1+x^4) + \frac{1}{24} \log(1-x^4+x^8) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^4 \right) \\ &= -\frac{\tan^{-1} \left(\frac{1-2x^4}{\sqrt{3}} \right)}{4\sqrt{3}} - \frac{1}{12} \log(1+x^4) + \frac{1}{24} \log(1-x^4+x^8) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 260 vs. 2(49) = 98.

time = 0.07, size = 260, normalized size = 5.31

$$\frac{1}{24} \left(2\sqrt{3} \tan^{-1} \left(\frac{1+\sqrt{3}-2\sqrt{2}x}{1-\sqrt{3}} \right) - 2\sqrt{3} \tan^{-1} \left(\frac{1-\sqrt{3}+2\sqrt{2}x}{1+\sqrt{3}} \right) + 2\sqrt{3} \tan^{-1} \left(\frac{-1+\sqrt{3}+2\sqrt{2}x}{1+\sqrt{3}} \right) - 2\sqrt{3} \tan^{-1} \left(\frac{1+\sqrt{3}+2\sqrt{2}x}{-1-\sqrt{3}} \right) - 2 \log(1-\sqrt{2}x+x^2) - 2 \log(1+\sqrt{2}x+x^2) + \log(2+\sqrt{2}x-\sqrt{6}x+2x^2) + \log(2+\sqrt{2}(-1+\sqrt{3})x+2x^2) + \log(2-(\sqrt{2}+\sqrt{6})x+2x^2) + \log(2+(\sqrt{2}+\sqrt{6})x+2x^2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(1 + x^12),x]

[Out] (2*Sqrt[3]*ArcTan[(1 + Sqrt[3] - 2*Sqrt[2]*x)/(1 - Sqrt[3])] - 2*Sqrt[3]*ArcTan[(1 - Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] + 2*Sqrt[3]*ArcTan[(-1 + Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] - 2*Sqrt[3]*ArcTan[(1 + Sqrt[3] + 2*Sqrt[2]*x)/(-1 + Sqrt[3])] - 2*Log[1 - Sqrt[2]*x + x^2] - 2*Log[1 + Sqrt[2]*x + x^2] + Log[2 + Sqrt[2]*x - Sqrt[6]*x + 2*x^2] + Log[2 + Sqrt[2]*(-1 + Sqrt[3])*x + 2*x^2] + Log[2 - (Sqrt[2] + Sqrt[6])*x + 2*x^2] + Log[2 + (Sqrt[2] + Sqrt[6])*x + 2*x^2])/24

Mathics [A]

time = 1.95, size = 40, normalized size = 0.82

$$-\frac{\text{Log}[1 + x^4]}{12} + \frac{\text{Log}[1 - x^4 + x^8]}{24} + \frac{\sqrt{3} \text{ArcTan}\left[\frac{\sqrt{3}(-1+2x^4)}{3}\right]}{12}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^7/(x^12+1),x]')

[Out] -Log[1 + x ^ 4] / 12 + Log[1 - x ^ 4 + x ^ 8] / 24 + Sqrt[3] ArcTan[Sqrt[3] (-1 + 2 x ^ 4) / 3] / 12

Maple [A]

time = 0.03, size = 41, normalized size = 0.84

method	result	size
risch	$-\frac{\ln(x^4+1)}{12} + \frac{\ln(x^8-x^4+1)}{24} + \frac{\sqrt{3} \arctan\left(\frac{2(x^4-\frac{1}{2})\sqrt{3}}{3}\right)}{12}$	39
default	$-\frac{\ln(x^4+1)}{12} + \frac{\ln(x^8-x^4+1)}{24} + \frac{\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	41
meijerg	$-\frac{x^8 \ln\left(1+(x^{12})^{\frac{1}{3}}\right)}{12(x^{12})^{\frac{2}{3}}} + \frac{x^8 \ln\left(1-(x^{12})^{\frac{1}{3}}+(x^{12})^{\frac{2}{3}}\right)}{24(x^{12})^{\frac{2}{3}}} + \frac{x^8 \sqrt{3} \arctan\left(\frac{\sqrt{3}(x^{12})^{\frac{1}{3}}}{2-(x^{12})^{\frac{1}{3}}}\right)}{12(x^{12})^{\frac{2}{3}}}$	80

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(x^12+1),x,method=_RETURNVERBOSE)

[Out] -1/12*ln(x^4+1)+1/24*ln(x^8-x^4+1)+1/12*arctan(1/3*(2*x^4-1)*3^(1/2))*3^(1/2)

Maxima [A]

time = 0.36, size = 40, normalized size = 0.82

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right) + \frac{1}{24} \log(x^8 - x^4 + 1) - \frac{1}{12} \log(x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^7/(x^12+1),x, algorithm="maxima")`

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/12*log(x^4 + 1)
```

Fricas [A]

time = 0.32, size = 40, normalized size = 0.82

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right) + \frac{1}{24} \log(x^8 - x^4 + 1) - \frac{1}{12} \log(x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^7/(x^12+1),x, algorithm="fricas")`

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/12*log(x^4 + 1)
```

Sympy [A]

time = 0.09, size = 46, normalized size = 0.94

$$-\frac{\log(x^4 + 1)}{12} + \frac{\log(x^8 - x^4 + 1)}{24} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x**7/(x**12+1),x)`

```
[Out] -log(x**4 + 1)/12 + log(x**8 - x**4 + 1)/24 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12
```

Giac [A]

time = 0.00, size = 51, normalized size = 1.04

$$\frac{\frac{\ln(x^8 - x^4 + 1)}{6} + \frac{1}{3} \sqrt{3} \arctan\left(\frac{2(x^4 - \frac{1}{2})}{\sqrt{3}}\right) - \frac{\ln(x^4 + 1)}{3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^7/(x^12+1),x)`

[Out] $\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4 - 1)\right) + \frac{1}{24}\log(x^8 - x^4 + 1) - \frac{1}{12}\log(x^4 + 1)$

Mupad [B]

time = 0.22, size = 52, normalized size = 1.06

$$-\frac{\ln(x^4 + 1)}{12} - \ln\left(x^4 - \frac{\sqrt{3} \operatorname{li}}{2} - \frac{1}{2}\right) \left(-\frac{1}{24} + \frac{\sqrt{3} \operatorname{li}}{24}\right) + \ln\left(x^4 + \frac{\sqrt{3} \operatorname{li}}{2} - \frac{1}{2}\right) \left(\frac{1}{24} + \frac{\sqrt{3} \operatorname{li}}{24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}(x^7/(x^{12} + 1), x)$

[Out] $\log\left(\frac{3^{1/2} \operatorname{li}}{2} + x^4 - \frac{1}{2}\right) \left(\frac{3^{1/2} \operatorname{li}}{24} + \frac{1}{24}\right) - \log\left(x^4 - \frac{3^{1/2} \operatorname{li}}{2} - \frac{1}{2}\right) \left(\frac{3^{1/2} \operatorname{li}}{24} - \frac{1}{24}\right) - \log(x^4 + 1)/12$

3.53 $\int \log(x) dx$

Optimal. Leaf size=8

$$-x + x \log(x)$$

[Out] $-x+x*\ln(x)$

Rubi [A]

time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2332}

$$x \log(x) - x$$

Antiderivative was successfully verified.

[In] Int[Log[x],x]

[Out] $-x + x*\text{Log}[x]$

Rule 2332

Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]

Rubi steps

$$\int \log(x) dx = -x + x \log(x)$$

Mathematica [A]

time = 0.00, size = 8, normalized size = 1.00

$$-x + x \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[Log[x],x]

[Out] $-x + x*\text{Log}[x]$

Mathics [A]

time = 1.56, size = 6, normalized size = 0.75

$$x(-1 + \text{Log}[x])$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Log[x],x]')
```

```
[Out] x (-1 + Log[x])
```

Maple [A]

time = 0.00, size = 9, normalized size = 1.12

method	result	size
lookup	$-x + x \ln(x)$	9
default	$-x + x \ln(x)$	9
norman	$-x + x \ln(x)$	9
risch	$-x + x \ln(x)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] -x+x*ln(x)
```

Maxima [A]

time = 0.27, size = 8, normalized size = 1.00

$$x \log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="maxima")
```

```
[Out] x*log(x) - x
```

Fricas [A]

time = 0.33, size = 8, normalized size = 1.00

$$x \log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="fricas")
```

```
[Out] x*log(x) - x
```

Sympy [A]

time = 0.04, size = 5, normalized size = 0.62

$$x \log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(x),x)

[Out] x*log(x) - x

Giac [A]

time = 0.00, size = 7, normalized size = 0.88

$$x \ln x - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x),x)

[Out] x*log(x) - x

Mupad [B]

time = 0.02, size = 6, normalized size = 0.75

$$x (\ln(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(x),x)

[Out] x*(log(x) - 1)

3.54 $\int x \log(x) dx$

Optimal. Leaf size=17

$$-\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

[Out] $-1/4*x^2+1/2*x^2*\ln(x)$

Rubi [A]

time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2341}

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] `Int[x*Log[x],x]`

[Out] $-1/4*x^2 + (x^2*\text{Log}[x])/2$

Rule 2341

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_)^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]`

Rubi steps

$$\int x \log(x) dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

Mathematica [A]

time = 0.00, size = 17, normalized size = 1.00

$$-\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

Antiderivative was successfully verified.

[In] `Integrate[x*Log[x],x]`

[Out] $-1/4*x^2 + (x^2*\text{Log}[x])/2$

Mathics [A]

time = 1.64, size = 11, normalized size = 0.65

$$\frac{x^2(-1 + 2\text{Log}[x])}{4}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x*Log[x],x]')``[Out] x ^ 2 (-1 + 2 Log[x]) / 4`**Maple [A]**

time = 0.00, size = 14, normalized size = 0.82

method	result	size
default	$-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}$	14
norman	$-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}$	14
risch	$-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*ln(x),x,method=_RETURNVERBOSE)``[Out] -1/4*x^2+1/2*x^2*ln(x)`**Maxima [A]**

time = 0.27, size = 13, normalized size = 0.76

$$\frac{1}{2}x^2 \log(x) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*log(x),x, algorithm="maxima")``[Out] 1/2*x^2*log(x) - 1/4*x^2`**Fricas [A]**

time = 0.32, size = 13, normalized size = 0.76

$$\frac{1}{2}x^2 \log(x) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*log(x),x, algorithm="fricas")``[Out] 1/2*x^2*log(x) - 1/4*x^2`

Sympy [A]

time = 0.05, size = 12, normalized size = 0.71

$$\frac{x^2 \log(x)}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*ln(x),x)**[Out]** x**2*log(x)/2 - x**2/4**Giac [A]**

time = 0.00, size = 16, normalized size = 0.94

$$-\frac{x^2}{4} + \frac{1}{2}x^2 \ln x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*log(x),x)**[Out]** 1/2*x^2*log(x) - 1/4*x^2**Mupad [B]**

time = 0.03, size = 9, normalized size = 0.53

$$\frac{x^2 (\ln(x) - \frac{1}{2})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*log(x),x)**[Out]** (x^2*(log(x) - 1/2))/2

3.55 $\int x^2 \log(x) dx$

Optimal. Leaf size=17

$$-\frac{x^3}{9} + \frac{1}{3}x^3 \log(x)$$

[Out] $-1/9*x^3+1/3*x^3*\ln(x)$

Rubi [A]

time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2341}

$$\frac{1}{3}x^3 \log(x) - \frac{x^3}{9}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*\text{Log}[x],x]$

[Out] $-1/9*x^3 + (x^3*\text{Log}[x])/3$

Rule 2341

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*((d_.)*(x_.))^{(m_.)}, x_Symbol] :>$
 $\text{Simp}[(d*x)^{(m+1)}*((a + b*\text{Log}[c*x^n])/(d*(m+1))), x] - \text{Simp}[b*n*((d*x)^{(m+1)})/(d*(m+1)^2), x] /;$ FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\int x^2 \log(x) dx = -\frac{x^3}{9} + \frac{1}{3}x^3 \log(x)$$

Mathematica [A]

time = 0.00, size = 17, normalized size = 1.00

$$-\frac{x^3}{9} + \frac{1}{3}x^3 \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[x^2*\text{Log}[x],x]$

[Out] $-1/9*x^3 + (x^3*\text{Log}[x])/3$

Mathics [A]

time = 1.60, size = 11, normalized size = 0.65

$$\frac{x^3(-1 + 3\text{Log}[x])}{9}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^2*Log[x],x]')``[Out] x ^ 3 (-1 + 3 Log[x]) / 9`**Maple [A]**

time = 0.00, size = 14, normalized size = 0.82

method	result	size
default	$-\frac{x^3}{9} + \frac{x^3 \ln(x)}{3}$	14
norman	$-\frac{x^3}{9} + \frac{x^3 \ln(x)}{3}$	14
risch	$-\frac{x^3}{9} + \frac{x^3 \ln(x)}{3}$	14

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2*ln(x),x,method=_RETURNVERBOSE)``[Out] -1/9*x^3+1/3*x^3*ln(x)`**Maxima [A]**

time = 0.28, size = 13, normalized size = 0.76

$$\frac{1}{3}x^3 \log(x) - \frac{1}{9}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2*log(x),x, algorithm="maxima")``[Out] 1/3*x^3*log(x) - 1/9*x^3`**Fricas [A]**

time = 0.32, size = 13, normalized size = 0.76

$$\frac{1}{3}x^3 \log(x) - \frac{1}{9}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2*log(x),x, algorithm="fricas")``[Out] 1/3*x^3*log(x) - 1/9*x^3`

Sympy [A]

time = 0.05, size = 12, normalized size = 0.71

$$\frac{x^3 \log(x)}{3} - \frac{x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*ln(x),x)

[Out] x**3*log(x)/3 - x**3/9

Giac [A]

time = 0.00, size = 16, normalized size = 0.94

$$-\frac{x^3}{9} + \frac{1}{3}x^3 \ln x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*log(x),x)

[Out] 1/3*x^3*log(x) - 1/9*x^3

Mupad [B]

time = 0.03, size = 9, normalized size = 0.53

$$\frac{x^3 \left(\ln(x) - \frac{1}{3} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*log(x),x)

[Out] (x^3*(log(x) - 1/3))/3

3.56 $\int x^p \log(x) dx$

Optimal. Leaf size=26

$$-\frac{x^{1+p}}{(1+p)^2} + \frac{x^{1+p} \log(x)}{1+p}$$

[Out] $-x^{(1+p)}/(1+p)^2+x^{(1+p)}*\ln(x)/(1+p)$

Rubi [A]

time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2341}

$$\frac{x^{p+1} \log(x)}{p+1} - \frac{x^{p+1}}{(p+1)^2}$$

Antiderivative was successfully verified.

[In] Int[x^p*Log[x],x]

[Out] $-(x^{(1+p)}/(1+p)^2) + (x^{(1+p)}*Log[x])/(1+p)$

Rule 2341

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_)^(m_.), x_Symbol] :> Simp[(d*x)^(m+1)*((a+b*Log[c*x^n])/(d*(m+1))), x] - Simp[b*n*((d*x)^(m+1))/(d*(m+1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\int x^p \log(x) dx = -\frac{x^{1+p}}{(1+p)^2} + \frac{x^{1+p} \log(x)}{1+p}$$

Mathematica [A]

time = 0.01, size = 19, normalized size = 0.73

$$\frac{x^{1+p}(-1 + (1+p) \log(x))}{(1+p)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^p*Log[x],x]

[Out] $(x^{(1+p)}*(-1 + (1+p)*Log[x]))/(1+p)^2$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.09, size = 36, normalized size = 1.38

$$\text{Piecewise} \left[\left\{ \left\{ \frac{(-1 + p \text{Log}[x] + \text{Log}[x]) x^{1+p}}{1 + 2p + p^2}, p \neq -1 \right\} \right\}, \frac{\text{Log}[x]^2}{2} \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[x^p*Log[x],x]')`

[Out] `Piecewise[{{(-1 + p Log[x] + Log[x]) x ^ (1 + p) / (1 + 2 p + p ^ 2), p != -1}}, Log[x] ^ 2 / 2]`

Maple [A]

time = 0.01, size = 19, normalized size = 0.73

method	result	size
risch	$\frac{x(\ln(x)p + \ln(x) - 1)x^p}{(1+p)^2}$	19
norman	$\frac{x \ln(x) e^{\ln(x)p}}{1+p} - \frac{x e^{\ln(x)p}}{p^2 + 2p + 1}$	34

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^p*ln(x),x,method=_RETURNVERBOSE)`

[Out] `x*(ln(x)*p+ln(x)-1)/(1+p)^2*x^p`

Maxima [A]

time = 0.27, size = 26, normalized size = 1.00

$$\frac{x^{p+1} \log(x)}{p+1} - \frac{x^{p+1}}{(p+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^p*log(x),x, algorithm="maxima")`

[Out] `x^(p + 1)*log(x)/(p + 1) - x^(p + 1)/(p + 1)^2`

Fricas [A]

time = 0.34, size = 25, normalized size = 0.96

$$\frac{((p+1)x \log(x) - x)x^p}{p^2 + 2p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^p*log(x),x, algorithm="fricas")`

[Out] $((p + 1)*x*\log(x) - x)*x^p/(p^2 + 2*p + 1)$

Sympy [A]

time = 0.22, size = 56, normalized size = 2.15

$$\begin{cases} \frac{pxx^p \log(x)}{p^2+2p+1} + \frac{xx^p \log(x)}{p^2+2p+1} - \frac{xx^p}{p^2+2p+1} & \text{for } p \neq -1 \\ \frac{\log(x)^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**p*ln(x),x)`

[Out] `Piecewise((p*x*x**p*log(x)/(p**2 + 2*p + 1) + x*x**p*log(x)/(p**2 + 2*p + 1) - x*x**p/(p**2 + 2*p + 1), Ne(p, -1)), (log(x)**2/2, True))`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^p*log(x),x)`

[Out] Could not integrate

Mupad [B]

time = 0.24, size = 32, normalized size = 1.23

$$\begin{cases} \frac{\ln(x)^2}{2} & \text{if } p = -1 \\ \frac{x^{p+1} (\ln(x)(p+1)-1)}{(p+1)^2} & \text{if } p \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^p*log(x),x)`

[Out] `piecewise(p == -1, log(x)^2/2, p ~= -1, (x^(p + 1)*(log(x)*(p + 1) - 1))/(p + 1)^2)`

3.57 $\int \log^2(x) dx$

Optimal. Leaf size=15

$$2x - 2x \log(x) + x \log^2(x)$$

[Out] 2*x-2*x*ln(x)+x*ln(x)^2

Rubi [A]

time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$,

Rules used = {2333, 2332}

$$2x + x \log^2(x) - 2x \log(x)$$

Antiderivative was successfully verified.

[In] Int[Log[x]^2,x]

[Out] 2*x - 2*x*Log[x] + x*Log[x]^2

Rule 2332

Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]

Rule 2333

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]

Rubi steps

$$\begin{aligned} \int \log^2(x) dx &= x \log^2(x) - 2 \int \log(x) dx \\ &= 2x - 2x \log(x) + x \log^2(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 15, normalized size = 1.00

$$2x - 2x \log(x) + x \log^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[Log[x]^2,x]

[Out] $2*x - 2*x*\text{Log}[x] + x*\text{Log}[x]^2$

Mathics [A]

time = 1.61, size = 12, normalized size = 0.80

$$x (2 + \text{Log}[x]^2 - 2\text{Log}[x])$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(Log[x])^2,x]')`

[Out] $x (2 + \text{Log}[x]^2 - 2 \text{Log}[x])$

Maple [A]

time = 0.00, size = 16, normalized size = 1.07

method	result	size
default	$2x - 2x \ln(x) + x \ln(x)^2$	16
norman	$2x - 2x \ln(x) + x \ln(x)^2$	16
risch	$2x - 2x \ln(x) + x \ln(x)^2$	16

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(x)^2,x,method=_RETURNVERBOSE)`

[Out] $2*x-2*x*\ln(x)+x*\ln(x)^2$

Maxima [A]

time = 0.26, size = 12, normalized size = 0.80

$$(\log(x)^2 - 2 \log(x) + 2)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x)^2,x, algorithm="maxima")`

[Out] $(\log(x)^2 - 2*\log(x) + 2)*x$

Fricas [A]

time = 0.33, size = 15, normalized size = 1.00

$$x \log(x)^2 - 2x \log(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x)^2,x, algorithm="fricas")`

[Out] $x*\log(x)^2 - 2*x*\log(x) + 2*x$

Sympy [A]

time = 0.05, size = 15, normalized size = 1.00

$$x \log(x)^2 - 2x \log(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(x)**2,x)

[Out] x*log(x)**2 - 2*x*log(x) + 2*x

Giac [A]

time = 0.00, size = 16, normalized size = 1.07

$$x \ln^2 x - 2x \ln x + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)^2,x)

[Out] x*log(x)^2 - 2*x*log(x) + 2*x

Mupad [B]

time = 0.03, size = 12, normalized size = 0.80

$$x (\ln(x)^2 - 2 \ln(x) + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(x)^2,x)

[Out] x*(log(x)^2 - 2*log(x) + 2)

3.58 $\int x^9 \log^{11}(x) dx$

Optimal. Leaf size=127

$$-\frac{6237x^{10}}{156250000} + \frac{6237x^{10} \log(x)}{15625000} - \frac{6237x^{10} \log^2(x)}{3125000} + \frac{2079x^{10} \log^3(x)}{312500} - \frac{2079x^{10} \log^4(x)}{125000} + \frac{2079x^{10} \log^5(x)}{62500} - \frac{693x^{10}}{125000} + \frac{99x^{10} \log^6(x)}{1250000} - \frac{99x^{10} \log^7(x)}{12500000} + \frac{11x^{10} \log^8(x)}{100000000} - \frac{11x^{10} \log^9(x)}{1000000000} + \frac{11x^{10} \log^{10}(x)}{10000000000} - \frac{11x^{10} \log^{11}(x)}{100000000000}$$

[Out] $-6237/156250000*x^{10}+6237/15625000*x^{10}*\ln(x)-6237/3125000*x^{10}*\ln(x)^2+2079/312500*x^{10}*\ln(x)^3-2079/125000*x^{10}*\ln(x)^4+2079/62500*x^{10}*\ln(x)^5-693/125000*x^{10}*\ln(x)^6+99/1250000*x^{10}*\ln(x)^7-99/10000000*x^{10}*\ln(x)^8+11/100000000*x^{10}*\ln(x)^9-11/1000000000*x^{10}*\ln(x)^{10}+1/10000000000*x^{10}*\ln(x)^{11}$

Rubi [A]

time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$,

Rules used = {2342, 2341}

$$-\frac{6237x^{10}}{156250000} + \frac{1}{10}x^{10}\log^{11}(x) - \frac{11}{100}x^{10}\log^{10}(x) + \frac{11}{100}x^{10}\log^9(x) - \frac{99x^{10}\log^8(x)}{1000} + \frac{99x^{10}\log^7(x)}{1250} - \frac{693x^{10}\log^6(x)}{12500} + \frac{2079x^{10}\log^5(x)}{62500} - \frac{2079x^{10}\log^4(x)}{125000} + \frac{2079x^{10}\log^3(x)}{312500} - \frac{6237x^{10}\log^2(x)}{3125000} + \frac{6237x^{10}\log(x)}{15625000}$$

Antiderivative was successfully verified.

[In] Int[x^9*Log[x]^11,x]

[Out] $(-6237*x^{10})/156250000 + (6237*x^{10}*\text{Log}[x])/15625000 - (6237*x^{10}*\text{Log}[x]^2)/3125000 + (2079*x^{10}*\text{Log}[x]^3)/312500 - (2079*x^{10}*\text{Log}[x]^4)/125000 + (2079*x^{10}*\text{Log}[x]^5)/62500 - (693*x^{10}*\text{Log}[x]^6)/12500 + (99*x^{10}*\text{Log}[x]^7)/125000 - (99*x^{10}*\text{Log}[x]^8)/1000 + (11*x^{10}*\text{Log}[x]^9)/100 - (11*x^{10}*\text{Log}[x]^10)/100 + (x^{10}*\text{Log}[x]^11)/10$

Rule 2341

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] > Simp[(d*x)^(m+1)*((a + b*Log[c*x^n])/(d*(m+1))), x] - Simp[b*n*((d*x)^(m+1))/(d*(m+1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 2342

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] > Simp[(d*x)^(m+1)*((a + b*Log[c*x^n])^p/(d*(m+1))), x] - Dist[b*n*(p/(m+1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p-1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int x^9 \log^{11}(x) dx &= \frac{1}{10} x^{10} \log^{11}(x) - \frac{11}{10} \int x^9 \log^{10}(x) dx \\
&= -\frac{11}{100} x^{10} \log^{10}(x) + \frac{1}{10} x^{10} \log^{11}(x) + \frac{11}{10} \int x^9 \log^9(x) dx \\
&= \frac{11}{100} x^{10} \log^9(x) - \frac{11}{100} x^{10} \log^{10}(x) + \frac{1}{10} x^{10} \log^{11}(x) - \frac{99}{100} \int x^9 \log^8(x) dx \\
&= -\frac{99x^{10} \log^8(x)}{1000} + \frac{11}{100} x^{10} \log^9(x) - \frac{11}{100} x^{10} \log^{10}(x) + \frac{1}{10} x^{10} \log^{11}(x) + \frac{99}{125} \int x^9 \log^7(x) dx \\
&= \frac{99x^{10} \log^7(x)}{1250} - \frac{99x^{10} \log^8(x)}{1000} + \frac{11}{100} x^{10} \log^9(x) - \frac{11}{100} x^{10} \log^{10}(x) + \frac{1}{10} x^{10} \log^{11}(x) - \frac{6}{125} \int x^9 \log^6(x) dx \\
&= -\frac{693x^{10} \log^6(x)}{12500} + \frac{99x^{10} \log^7(x)}{1250} - \frac{99x^{10} \log^8(x)}{1000} + \frac{11}{100} x^{10} \log^9(x) - \frac{11}{100} x^{10} \log^{10}(x) + \frac{1}{10} x^{10} \log^{11}(x) - \frac{6}{125} \int x^9 \log^5(x) dx \\
&= \frac{2079x^{10} \log^5(x)}{62500} - \frac{693x^{10} \log^6(x)}{12500} + \frac{99x^{10} \log^7(x)}{1250} - \frac{99x^{10} \log^8(x)}{1000} + \frac{11}{100} x^{10} \log^9(x) - \frac{1}{10} x^{10} \log^{10}(x) + \frac{1}{10} x^{10} \log^{11}(x) - \frac{6}{125} \int x^9 \log^4(x) dx \\
&= -\frac{2079x^{10} \log^4(x)}{125000} + \frac{2079x^{10} \log^5(x)}{62500} - \frac{693x^{10} \log^6(x)}{12500} + \frac{99x^{10} \log^7(x)}{1250} - \frac{99x^{10} \log^8(x)}{1000} + \frac{11}{100} x^{10} \log^9(x) - \frac{1}{10} x^{10} \log^{10}(x) + \frac{1}{10} x^{10} \log^{11}(x) - \frac{6}{125} \int x^9 \log^3(x) dx \\
&= \frac{2079x^{10} \log^3(x)}{312500} - \frac{2079x^{10} \log^4(x)}{125000} + \frac{2079x^{10} \log^5(x)}{62500} - \frac{693x^{10} \log^6(x)}{12500} + \frac{99x^{10} \log^7(x)}{1250} - \frac{99x^{10} \log^8(x)}{1000} + \frac{11}{100} x^{10} \log^9(x) - \frac{1}{10} x^{10} \log^{10}(x) + \frac{1}{10} x^{10} \log^{11}(x) - \frac{6}{125} \int x^9 \log^2(x) dx \\
&= -\frac{6237x^{10} \log^2(x)}{3125000} + \frac{2079x^{10} \log^3(x)}{312500} - \frac{2079x^{10} \log^4(x)}{125000} + \frac{2079x^{10} \log^5(x)}{62500} - \frac{693x^{10} \log^6(x)}{12500} + \frac{99x^{10} \log^7(x)}{1250} - \frac{99x^{10} \log^8(x)}{1000} + \frac{11}{100} x^{10} \log^9(x) - \frac{1}{10} x^{10} \log^{10}(x) + \frac{1}{10} x^{10} \log^{11}(x) - \frac{6}{125} \int x^9 \log(x) dx \\
&= -\frac{6237x^{10}}{156250000} + \frac{6237x^{10} \log(x)}{15625000} - \frac{6237x^{10} \log^2(x)}{3125000} + \frac{2079x^{10} \log^3(x)}{312500} - \frac{2079x^{10} \log^4(x)}{125000}
\end{aligned}$$

Mathematica [A]

time = 0.00, size = 127, normalized size = 1.00

$$-\frac{6237x^{10}}{156250000} + \frac{6237x^{10} \log(x)}{15625000} - \frac{6237x^{10} \log^2(x)}{3125000} + \frac{2079x^{10} \log^3(x)}{312500} - \frac{2079x^{10} \log^4(x)}{125000} + \frac{2079x^{10} \log^5(x)}{62500} - \frac{693x^{10} \log^6(x)}{12500} + \frac{99x^{10} \log^7(x)}{1250} - \frac{99x^{10} \log^8(x)}{1000} + \frac{11}{100} x^{10} \log^9(x) - \frac{11}{100} x^{10} \log^{10}(x) + \frac{1}{10} x^{10} \log^{11}(x)$$

Antiderivative was successfully verified.

[In] Integrate[x^9*Log[x]^11,x]

[Out] $(-6237*x^{10})/156250000 + (6237*x^{10}*Log[x])/15625000 - (6237*x^{10}*Log[x]^2)/3125000 + (2079*x^{10}*Log[x]^3)/312500 - (2079*x^{10}*Log[x]^4)/125000 + (2079*x^{10}*Log[x]^5)/62500 - (693*x^{10}*Log[x]^6)/12500 + (99*x^{10}*Log[x]^7)/12500 - (99*x^{10}*Log[x]^8)/1000 + (11*x^{10}*Log[x]^9)/100 - (11*x^{10}*Log[x]^10)/100 + (x^{10}*Log[x]^11)/10$

Mathics [A]

time = 2.24, size = 71, normalized size = 0.56

$$x^{10} (-6237 - 17187500 \text{Log}[x]^{10} - 15468750 \text{Log}[x]^9 - 8662500 \text{Log}[x]^8 - 2598750 \text{Log}[x]^7 - 311850 \text{Log}[x]^6 + 62370 \text{Log}[x] + 1039500 \text{Log}[x]^3 + 5197500 \text{Log}[x]^5 + 12375000 \text{Log}[x]^7 + 15625000 \text{Log}[x]^{11} + 17187500 \text{Log}[x]^9) / 156250000$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^9*Log[x]^11,x]')

[Out] $x^{10} (-6237 - 17187500 \operatorname{Log}[x]^6 - 2598750 \operatorname{Log}[x]^4 - 311850 \operatorname{Log}[x]^2 + 62370 \operatorname{Log}[x] + 1039500 \operatorname{Log}[x]^3 + 5197500 \operatorname{Log}[x]^5 + 12375000 \operatorname{Log}[x]^7 + 15625000 \operatorname{Log}[x]^9) / 156250000$

Maple [A]

time = 0.01, size = 104, normalized size = 0.82

method	result
default	$-\frac{6237x^{10}}{156250000} + \frac{6237x^{10} \ln(x)}{15625000} - \frac{6237x^{10} \ln(x)^2}{3125000} + \frac{2079x^{10} \ln(x)^3}{312500} - \frac{2079x^{10} \ln(x)^4}{125000} + \frac{2079x^{10} \ln(x)^5}{62500} - \frac{693x^{10} \ln(x)^6}{12500} + \frac{99x^{10} \ln(x)^7}{125000} - \frac{99x^{10} \ln(x)^8}{100000} + \frac{11x^{10} \ln(x)^9}{100000} - \frac{11x^{10} \ln(x)^{10}}{1000000} + \frac{11x^{10} \ln(x)^{11}}{10000000}$
risch	$-\frac{6237x^{10}}{156250000} + \frac{6237x^{10} \ln(x)}{15625000} - \frac{6237x^{10} \ln(x)^2}{3125000} + \frac{2079x^{10} \ln(x)^3}{312500} - \frac{2079x^{10} \ln(x)^4}{125000} + \frac{2079x^{10} \ln(x)^5}{62500} - \frac{693x^{10} \ln(x)^6}{12500} + \frac{99x^{10} \ln(x)^7}{125000} - \frac{99x^{10} \ln(x)^8}{100000} + \frac{11x^{10} \ln(x)^9}{100000} - \frac{11x^{10} \ln(x)^{10}}{1000000} + \frac{11x^{10} \ln(x)^{11}}{10000000}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9*ln(x)^11,x,method=_RETURNVERBOSE)

[Out] $-6237/156250000*x^{10}+6237/15625000*x^{10}*\ln(x)-6237/3125000*x^{10}*\ln(x)^2+2079/312500*x^{10}*\ln(x)^3-2079/125000*x^{10}*\ln(x)^4+2079/62500*x^{10}*\ln(x)^5-693/12500*x^{10}*\ln(x)^6+99/1250*x^{10}*\ln(x)^7-99/1000*x^{10}*\ln(x)^8+11/100*x^{10}*\ln(x)^9-11/100*x^{10}*\ln(x)^{10}+1/10*x^{10}*\ln(x)^{11}$

Maxima [A]

time = 0.27, size = 71, normalized size = 0.56

$\frac{1}{156250000} (15625000 \log(x)^{11} - 17187500 \log(x)^{10} + 17187500 \log(x)^9 - 15468750 \log(x)^8 + 12375000 \log(x)^7 - 8662500 \log(x)^6 + 5197500 \log(x)^5 - 2598750 \log(x)^4 + 1039500 \log(x)^3 - 311850 \log(x)^2 + 62370 \log(x) - 6237) x^{10}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*log(x)^11,x, algorithm="maxima")

[Out] $1/156250000*(15625000*\log(x)^{11} - 17187500*\log(x)^{10} + 17187500*\log(x)^9 - 15468750*\log(x)^8 + 12375000*\log(x)^7 - 8662500*\log(x)^6 + 5197500*\log(x)^5 - 2598750*\log(x)^4 + 1039500*\log(x)^3 - 311850*\log(x)^2 + 62370*\log(x) - 6237)*x^{10}$

Fricas [A]

time = 0.36, size = 103, normalized size = 0.81

$\frac{1}{10} x^{10} \log(x)^{11} - \frac{11}{100} x^{10} \log(x)^{10} + \frac{11}{100} x^{10} \log(x)^9 - \frac{99}{1000} x^{10} \log(x)^8 + \frac{99}{1250} x^{10} \log(x)^7 - \frac{693}{12500} x^{10} \log(x)^6 + \frac{2079}{62500} x^{10} \log(x)^5 - \frac{2079}{125000} x^{10} \log(x)^4 + \frac{2079}{312500} x^{10} \log(x)^3 - \frac{6237}{3125000} x^{10} \log(x)^2 + \frac{6237}{15625000} x^{10} \log(x) - \frac{6237}{156250000} x^{10}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*log(x)^11,x, algorithm="fricas")

[Out] $1/10*x^{10}*\log(x)^{11} - 11/100*x^{10}*\log(x)^{10} + 11/100*x^{10}*\log(x)^9 - 99/1000*x^{10}*\log(x)^8 + 99/1250*x^{10}*\log(x)^7 - 693/12500*x^{10}*\log(x)^6 + 2079/62500*x^{10}*\log(x)^5 - 2079/125000*x^{10}*\log(x)^4 + 2079/312500*x^{10}*\log(x)^3 - 6237/3125000*x^{10}*\log(x)^2 + 6237/15625000*x^{10}*\log(x) - 6237/156250000*x^{10}$

Sympy [A]

time = 0.18, size = 133, normalized size = 1.05

$$\frac{x^{10} \log(x)^{11}}{10} - \frac{11x^{10} \log(x)^{10}}{100} + \frac{11x^{10} \log(x)^9}{100} - \frac{99x^{10} \log(x)^8}{1000} + \frac{99x^{10} \log(x)^7}{1250} - \frac{693x^{10} \log(x)^6}{12500} + \frac{2079x^{10} \log(x)^5}{62500} - \frac{2079x^{10} \log(x)^4}{125000} + \frac{2079x^{10} \log(x)^3}{312500} - \frac{6237x^{10} \log(x)^2}{3125000} + \frac{6237x^{10} \log(x)}{15625000} - \frac{6237x^{10}}{156250000}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9*ln(x)**11,x)`

[Out] $x^{10}*\log(x)^{11}/10 - 11*x^{10}*\log(x)^{10}/100 + 11*x^{10}*\log(x)^9/100 - 99*x^{10}*\log(x)^8/1000 + 99*x^{10}*\log(x)^7/1250 - 693*x^{10}*\log(x)^6/12500 + 2079*x^{10}*\log(x)^5/62500 - 2079*x^{10}*\log(x)^4/125000 + 2079*x^{10}*\log(x)^3/312500 - 6237*x^{10}*\log(x)^2/3125000 + 6237*x^{10}*\log(x)/15625000 - 6237*x^{10}/156250000$

Giac [A]

time = 0.00, size = 132, normalized size = 1.04

$$\frac{1}{10}x^{10}\ln^{11}x - \frac{11}{100}x^{10}\ln^{10}x + \frac{11}{100}x^{10}\ln^9x - \frac{99}{1000}x^{10}\ln^8x + \frac{99}{1250}x^{10}\ln^7x - \frac{693}{12500}x^{10}\ln^6x + \frac{2079}{62500}x^{10}\ln^5x - \frac{2079}{125000}x^{10}\ln^4x + \frac{2079}{312500}x^{10}\ln^3x - \frac{6237}{3125000}x^{10}\ln^2x - \frac{6237}{156250000}x^{10} + \frac{6237}{15625000}x^{10}\ln x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9*log(x)^11,x)`

[Out] $1/10*x^{10}*\log(x)^{11} - 11/100*x^{10}*\log(x)^{10} + 11/100*x^{10}*\log(x)^9 - 99/1000*x^{10}*\log(x)^8 + 99/1250*x^{10}*\log(x)^7 - 693/12500*x^{10}*\log(x)^6 + 2079/62500*x^{10}*\log(x)^5 - 2079/125000*x^{10}*\log(x)^4 + 2079/312500*x^{10}*\log(x)^3 - 6237/3125000*x^{10}*\log(x)^2 + 6237/15625000*x^{10}*\log(x) - 6237/156250000*x^{10}$

Mupad [B]

time = 0.18, size = 71, normalized size = 0.56

$$\frac{6237x^{10} \left(\frac{15625000 \ln(x)^{11}}{6237} - \frac{1562500 \ln(x)^{10}}{567} + \frac{1562500 \ln(x)^9}{567} - \frac{156250 \ln(x)^8}{63} + \frac{125000 \ln(x)^7}{63} - \frac{12500 \ln(x)^6}{9} + \frac{2500 \ln(x)^5}{3} - \frac{1250 \ln(x)^4}{3} + \frac{500 \ln(x)^3}{3} - 50 \ln(x)^2 + 10 \ln(x) - 1 \right)}{156250000}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9*log(x)^11,x)`

[Out] $(6237*x^{10}*(10*\log(x) - 50*\log(x)^2 + (500*\log(x)^3)/3 - (1250*\log(x)^4)/3 + (2500*\log(x)^5)/3 - (12500*\log(x)^6)/9 + (125000*\log(x)^7)/63 - (156250*\log(x)^8)/63 + (1562500*\log(x)^9)/567 - (1562500*\log(x)^{10})/567 + (15625000*\log(x)^{11})/6237 - 1)/156250000$

$$3.59 \quad \int \frac{\log^2(x)}{x} dx$$

Optimal. Leaf size=8

$$\frac{\log^3(x)}{3}$$

[Out] 1/3*ln(x)^3

Rubi [A]

time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2339, 30}

$$\frac{\log^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[Log[x]^2/x,x]

[Out] Log[x]^3/3

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2339

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{\log^2(x)}{x} dx &= \text{Subst} \left(\int x^2 dx, x, \log(x) \right) \\ &= \frac{\log^3(x)}{3} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 8, normalized size = 1.00

$$\frac{\log^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[Log[x]^2/x,x]

[Out] Log[x]^3/3

Mathics [A]

time = 1.60, size = 6, normalized size = 0.75

$$\frac{\text{Log}[x]^3}{3}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Log[x]^2/x,x]')

[Out] Log[x] ^ 3 / 3

Maple [A]

time = 0.01, size = 7, normalized size = 0.88

method	result	size
derivativdivides	$\frac{\ln(x)^3}{3}$	7
default	$\frac{\ln(x)^3}{3}$	7
norman	$\frac{\ln(x)^3}{3}$	7
risch	$\frac{\ln(x)^3}{3}$	7

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(x)^2/x,x,method=_RETURNVERBOSE)

[Out] 1/3*ln(x)^3

Maxima [A]

time = 0.26, size = 6, normalized size = 0.75

$$\frac{1}{3} \log(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)^2/x,x, algorithm="maxima")

[Out] 1/3*log(x)^3

Fricas [A]

time = 0.33, size = 6, normalized size = 0.75

$$\frac{1}{3} \log(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)^2/x,x, algorithm="fricas")

[Out] 1/3*log(x)^3

Sympy [A]

time = 0.06, size = 5, normalized size = 0.62

$$\frac{\log(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(x)**2/x,x)

[Out] log(x)**3/3

Giac [A]

time = 0.00, size = 7, normalized size = 0.88

$$\frac{\ln^3 x}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)^2/x,x)

[Out] 1/3*log(x)^3

Mupad [B]

time = 0.06, size = 6, normalized size = 0.75

$$\frac{\ln(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(x)^2/x,x)

[Out] log(x)^3/3

$$3.60 \quad \int \frac{1}{\log(x)} dx$$

Optimal. Leaf size=2

$\text{li}(x)$

[Out] Li(x)

Rubi [A]

time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2335}

$\text{li}(x)$

Antiderivative was successfully verified.

[In] Int[Log[x]^(-1), x]

[Out] LogIntegral[x]

Rule 2335

Int[Log[(c_.)*(x_)]^(-1), x_Symbol] := Simp[LogIntegral[c*x]/c, x] /; FreeQ[c, x]

Rubi steps

$$\int \frac{1}{\log(x)} dx = \text{li}(x)$$

Mathematica [A]

time = 0.00, size = 2, normalized size = 1.00

$\text{li}(x)$

Antiderivative was successfully verified.

[In] Integrate[Log[x]^(-1), x]

[Out] LogIntegral[x]

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

time = 1.67, size = 2, normalized size = 1.00

$\text{li}[x]$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/Log[x],x]')
```

```
[Out] li[x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8 vs. $2(2) = 4$.
time = 0.00, size = 9, normalized size = 4.50

method	result	size
default	$-\text{expIntegral}(1, -\ln(x))$	9
risch	$-\text{expIntegral}(1, -\ln(x))$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] -Ei(1,-ln(x))
```

Maxima [A]

time = 0.29, size = 3, normalized size = 1.50

$$\text{Ei}(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="maxima")
```

```
[Out] Ei(log(x))
```

Fricas [A]

time = 0.31, size = 2, normalized size = 1.00

$$\text{log_integral}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="fricas")
```

```
[Out] log_integral(x)
```

Sympy [A]

time = 0.23, size = 2, normalized size = 1.00

$$\text{li}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(x),x)
```

[Out] $\text{li}(x)$

Giac [A]

time = 0.00, size = 3, normalized size = 1.50

$\text{Ei}(\ln x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/log(x),x)`

[Out] $\text{Ei}(\log(x))$

Mupad [B]

time = 0.00, size = 2, normalized size = 1.00

$\text{logint}(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/log(x),x)`

[Out] $\text{logint}(x)$

3.61 $\int \frac{1}{\log(1+x)} dx$

Optimal. Leaf size=4

$$\text{li}(1+x)$$

[Out] Li(1+x)

Rubi [A]

time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2436, 2335}

$$\text{li}(x+1)$$

Antiderivative was successfully verified.

[In] Int[Log[1 + x]^(-1), x]

[Out] LogIntegral[1 + x]

Rule 2335

Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ[c, x]

Rule 2436

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^p, x_Symbol] :> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\log(1+x)} dx &= \text{Subst} \left(\int \frac{1}{\log(x)} dx, x, 1+x \right) \\ &= \text{li}(1+x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 4, normalized size = 1.00

$$\text{li}(1+x)$$

Antiderivative was successfully verified.

[In] Integrate[Log[1 + x]^(-1), x]

[Out] LogIntegral[1 + x]

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

time = 1.69, size = 4, normalized size = 1.00

$$\text{li}[1 + x]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/Log[x+1],x]')

[Out] li[1 + x]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(4) = 8$.
time = 0.01, size = 11, normalized size = 2.75

method	result	size
derivativedivides	$-\text{expIntegral}(1, -\ln(1+x))$	11
default	$-\text{expIntegral}(1, -\ln(1+x))$	11
risch	$-\text{expIntegral}(1, -\ln(1+x))$	11

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/ln(1+x),x,method=_RETURNVERBOSE)

[Out] -Ei(1,-ln(1+x))

Maxima [A]

time = 0.28, size = 5, normalized size = 1.25

$$\text{Ei}(\log(x + 1))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(1+x),x, algorithm="maxima")

[Out] Ei(log(x + 1))

Fricas [A]

time = 0.39, size = 4, normalized size = 1.00

$$\text{log_integral}(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(1+x),x, algorithm="fricas")

[Out] log_integral(x + 1)

Sympy [A]

time = 0.25, size = 3, normalized size = 0.75

$$\operatorname{li}(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/ln(1+x),x)

[Out] li(x + 1)

Giac [A]

time = 0.00, size = 5, normalized size = 1.25

$$\operatorname{Ei}(\ln(x + 1))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(1+x),x)

[Out] Ei(log(x + 1))

Mupad [B]

time = 0.02, size = 4, normalized size = 1.00

$$\operatorname{logint}(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/log(x + 1),x)

[Out] logint(x + 1)

$$3.62 \quad \int \frac{1}{x \log(x)} dx$$

Optimal. Leaf size=3

$\log(\log(x))$

[Out] $\ln(\ln(x))$

Rubi [A]

time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2339, 29}

$\log(\log(x))$

Antiderivative was successfully verified.

[In] `Int[1/(x*Log[x]),x]`

[Out] `Log[Log[x]]`

Rule 29

`Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]`

Rule 2339

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] := Dist[1/(b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]`

Rubi steps

$$\int \frac{1}{x \log(x)} dx = \text{Subst} \left(\int \frac{1}{x} dx, x, \log(x) \right) \\ = \log(\log(x))$$

Mathematica [A]

time = 0.00, size = 3, normalized size = 1.00

$\log(\log(x))$

Antiderivative was successfully verified.

[In] `Integrate[1/(x*Log[x]),x]`

[Out] $\text{Log}[\text{Log}[x]]$

Mathics [A]

time = 1.51, size = 3, normalized size = 1.00

$\text{Log}[\text{Log}[x]]$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(x*Log[x]),x]')`

[Out] $\text{Log}[\text{Log}[x]]$

Maple [A]

time = 0.00, size = 4, normalized size = 1.33

method	result	size
derivativedivides	$\ln(\ln(x))$	4
default	$\ln(\ln(x))$	4
norman	$\ln(\ln(x))$	4
risch	$\ln(\ln(x))$	4

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/ln(x),x,method=_RETURNVERBOSE)`

[Out] $\ln(\ln(x))$

Maxima [A]

time = 0.27, size = 3, normalized size = 1.00

$\log(\log(x))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/log(x),x, algorithm="maxima")`

[Out] $\log(\log(x))$

Fricas [A]

time = 0.32, size = 3, normalized size = 1.00

$\log(\log(x))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/log(x),x, algorithm="fricas")`

[Out] $\log(\log(x))$

Sympy [A]

time = 0.05, size = 3, normalized size = 1.00

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/ln(x),x)

[Out] log(log(x))

Giac [A]

time = 0.00, size = 4, normalized size = 1.33

$$\ln|\ln x|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(x),x)

[Out] log(abs(log(x)))

Mupad [B]

time = 0.12, size = 3, normalized size = 1.00

$$\ln(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*log(x)),x)

[Out] log(log(x))

3.63 $\int \frac{1}{x^2 \log^2(x)} dx$

Optimal. Leaf size=17

$$-\text{Ei}(-\log(x)) - \frac{1}{x \log(x)}$$

[Out] -Ei(-ln(x))-1/x/ln(x)

Rubi [A]

time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2343, 2346, 2209}

$$-\text{Ei}(-\log(x)) - \frac{1}{x \log(x)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Log[x]^2),x]

[Out] -ExpIntegralEi[-Log[x]] - 1/(x*Log[x])

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Sim
p[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_)^(m_.), x_Symbol
] := Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] := Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \log^2(x)} dx &= -\frac{1}{x \log(x)} - \int \frac{1}{x^2 \log(x)} dx \\
&= -\frac{1}{x \log(x)} - \text{Subst} \left(\int \frac{e^{-x}}{x} dx, x, \log(x) \right) \\
&= -\text{Ei}(-\log(x)) - \frac{1}{x \log(x)}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 17, normalized size = 1.00

$$-\text{Ei}(-\log(x)) - \frac{1}{x \log(x)}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x^2*Log[x]^2),x]``[Out] -ExpIntegralEi[-Log[x]] - 1/(x*Log[x])`**Mathics [A]**

time = 1.84, size = 17, normalized size = 1.00

$$-\frac{1}{x \text{Log}[x]} - \text{ExpIntegralEi}[-\text{Log}[x]]$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x*Log[x])^2,x]')``[Out] -1 / (x Log[x]) - ExpIntegralEi[-Log[x]]`**Maple [A]**

time = 0.01, size = 15, normalized size = 0.88

method	result	size
default	$-\frac{1}{x \ln(x)} + \text{expIntegral}(1, \ln(x))$	15
risch	$-\frac{1}{x \ln(x)} + \text{expIntegral}(1, \ln(x))$	15

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/x^2/ln(x)^2,x,method=_RETURNVERBOSE)``[Out] -1/x/ln(x)+Ei(1,ln(x))`

Maxima [A]

time = 0.29, size = 6, normalized size = 0.35

$$-\Gamma(-1, \log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(x)^2,x, algorithm="maxima")

[Out] -gamma(-1, log(x))

Fricas [A]

time = 0.32, size = 19, normalized size = 1.12

$$-\frac{x \log(x) \log_integral\left(\frac{1}{x}\right) + 1}{x \log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(x)^2,x, algorithm="fricas")

[Out] -(x*log(x)*log_integral(1/x) + 1)/(x*log(x))

Sympy [A]

time = 0.33, size = 14, normalized size = 0.82

$$-Ei(-\log(x)) - \frac{1}{x \log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/ln(x)**2,x)

[Out] -Ei(-log(x)) - 1/(x*log(x))

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(x)^2,x)

[Out] Could not integrate

Mupad [B]

time = 0.03, size = 17, normalized size = 1.00

$$-ei(-\ln(x)) - \frac{1}{x \ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*log(x)^2),x)

[Out] -ei(-log(x)) - 1/(x*log(x))

3.64 $\int \frac{\log^p(x)}{x} dx$

Optimal. Leaf size=12

$$\frac{\log^{1+p}(x)}{1+p}$$

[Out] $\ln(x)^{(1+p)}/(1+p)$

Rubi [A]

time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2339, 30}

$$\frac{\log^{p+1}(x)}{p+1}$$

Antiderivative was successfully verified.

[In] Int[Log[x]^p/x,x]

[Out] Log[x]^(1 + p)/(1 + p)

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2339

Int[((a_) + Log[(c_)*(x_)^(n_)])*(b_)^(p_)/(x_), x_Symbol] := Dist[1/(b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{\log^p(x)}{x} dx &= \text{Subst} \left(\int x^p dx, x, \log(x) \right) \\ &= \frac{\log^{1+p}(x)}{1+p} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 12, normalized size = 1.00

$$\frac{\log^{1+p}(x)}{1+p}$$

Antiderivative was successfully verified.

[In] Integrate[Log[x]^p/x,x]

[Out] Log[x]^(1 + p)/(1 + p)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.96, size = 21, normalized size = 1.75

$$\text{Piecewise} \left[\left\{ \left\{ \frac{\text{Log}[x]^{1+p}}{1+p}, p \neq -1 \right\} \right\}, \text{Log}[\text{Log}[x]] \right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(Log[x])^p/x,x]')

[Out] Piecewise[{{Log[x]^(1 + p)/(1 + p), p != -1}}, Log[Log[x]]]

Maple [A]

time = 0.01, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$\frac{\ln(x)^{1+p}}{1+p}$	13
default	$\frac{\ln(x)^{1+p}}{1+p}$	13
risch	$\frac{\ln(x) \ln(x)^p}{1+p}$	13
norman	$\frac{\ln(x) e^{p \ln(\ln(x))}}{1+p}$	15

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(x)^p/x,x,method=_RETURNVERBOSE)

[Out] ln(x)^(1+p)/(1+p)

Maxima [A]

time = 0.26, size = 12, normalized size = 1.00

$$\frac{\log(x)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)^p/x,x, algorithm="maxima")

[Out] log(x)^(p + 1)/(p + 1)

Fricas [A]

time = 0.34, size = 12, normalized size = 1.00

$$\frac{\log(x)^p \log(x)}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(log(x)^p/x,x, algorithm="fricas")``[Out] log(x)^p*log(x)/(p + 1)`**Sympy [A]**

time = 0.44, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\log(x)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(\log(x)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(ln(x)**p/x,x)``[Out] Piecewise((log(x)**(p + 1)/(p + 1), Ne(p, -1)), (log(log(x)), True))`**Giac [A]**

time = 0.00, size = 11, normalized size = 0.92

$$\frac{(\ln x)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(log(x)^p/x,x)``[Out] log(x)^(p + 1)/(p + 1)`**Mupad [B]**

time = 0.15, size = 22, normalized size = 1.83

$$\begin{cases} \ln(\ln(x)) & \text{if } p = -1 \\ \frac{\ln(x)^{p+1}}{p+1} & \text{if } p \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(log(x)^p/x,x)``[Out] piecewise(p == -1, log(log(x)), p ~= -1, log(x)^(p + 1)/(p + 1))`

3.65 $\int (b + ax) \log(x) dx$

Optimal. Leaf size=28

$$-bx - \frac{ax^2}{4} + bx \log(x) + \frac{1}{2}ax^2 \log(x)$$

[Out] $-b*x-1/4*a*x^2+b*x*\ln(x)+1/2*a*x^2*\ln(x)$

Rubi [A]

time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2350}

$$-\frac{ax^2}{4} + \frac{1}{2}ax^2 \log(x) - bx + bx \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b + a*x)*\text{Log}[x], x]$

[Out] $-(b*x) - (a*x^2)/4 + b*x*\text{Log}[x] + (a*x^2*\text{Log}[x])/2$

Rule 2350

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^(n_.)]*(b_.)]*((d_) + (e_.)*(x_)^(r_.))^(q_.), x_Symbol] := \text{With}[\{u = \text{IntHide}[(d + e*x^r)^q, x]\}, \text{Dist}[a + b*\text{Log}[c*x^n], u, x] - \text{Dist}[b*n, \text{Int}[\text{SimplifyIntegrand}[u/x, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, n, r\}, x] \&\& \text{IGtQ}[q, 0]$

Rubi steps

$$\begin{aligned} \int (b + ax) \log(x) dx &= \frac{1}{2} (2bx + ax^2) \log(x) - \int \left(b + \frac{ax}{2}\right) dx \\ &= -bx - \frac{ax^2}{4} + \frac{1}{2} (2bx + ax^2) \log(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 28, normalized size = 1.00

$$-bx - \frac{ax^2}{4} + bx \log(x) + \frac{1}{2}ax^2 \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(b + a*x)*\text{Log}[x], x]$

[Out] $-(b*x) - (a*x^2)/4 + b*x*\text{Log}[x] + (a*x^2*\text{Log}[x])/2$

Mathics [A]

time = 1.96, size = 22, normalized size = 0.79

$$\frac{x(-ax - 4b + 2\text{Log}[x](ax + 2b))}{4}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[Log[x]*(a*x+b),x]')`

[Out] $x(-a x - 4 b + 2 \text{Log}[x](a x + 2 b)) / 4$

Maple [A]

time = 0.01, size = 27, normalized size = 0.96

method	result	size
norman	$-bx - \frac{ax^2}{4} + bx \ln(x) + \frac{ax^2 \ln(x)}{2}$	25
risch	$(\frac{1}{2}ax^2 + bx) \ln(x) - \frac{ax^2}{4} - bx$	25
default	$a\left(-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}\right) + b(-x + x \ln(x))$	27

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x+b)*ln(x),x,method=_RETURNVERBOSE)`

[Out] $a*(-1/4*x^2+1/2*x^2*\ln(x))+b*(-x+x*\ln(x))$

Maxima [A]

time = 0.27, size = 25, normalized size = 0.89

$$-\frac{1}{4}ax^2 - bx + \frac{1}{2}(ax^2 + 2bx) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x+b)*log(x),x, algorithm="maxima")`

[Out] $-1/4*a*x^2 - b*x + 1/2*(a*x^2 + 2*b*x)*\log(x)$

Fricas [A]

time = 0.35, size = 25, normalized size = 0.89

$$-\frac{1}{4}ax^2 - bx + \frac{1}{2}(ax^2 + 2bx) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x+b)*log(x),x, algorithm="fricas")`

[Out] $-1/4*a*x^2 - b*x + 1/2*(a*x^2 + 2*b*x)*\log(x)$

Sympy [A]

time = 0.06, size = 22, normalized size = 0.79

$$-\frac{ax^2}{4} - bx + \left(\frac{ax^2}{2} + bx\right) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x+b)*ln(x),x)`

[Out] $-a*x**2/4 - b*x + (a*x**2/2 + b*x)*\log(x)$

Giac [A]

time = 0.00, size = 27, normalized size = 0.96

$$-\frac{1}{4}ax^2 - bx + \frac{1}{2}ax^2 \ln x + bx \ln x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x+b)*log(x),x)`

[Out] $1/2*a*x^2*\log(x) - 1/4*a*x^2 + b*x*\log(x) - b*x$

Mupad [B]

time = 0.14, size = 21, normalized size = 0.75

$$-\frac{x(4b + ax - 4b \ln(x) - 2ax \ln(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(x)*(b + a*x),x)`

[Out] $-(x*(4*b + a*x - 4*b*\log(x) - 2*a*x*\log(x)))/4$

3.66 $\int (b + ax)^2 \log(x) dx$

Optimal. Leaf size=54

$$-b^2x - \frac{1}{2}abx^2 - \frac{a^2x^3}{9} - \frac{b^3 \log(x)}{3a} + \frac{(b + ax)^3 \log(x)}{3a}$$

[Out] $-b^2x - 1/2*a*b*x^2 - 1/9*a^2*x^3 - 1/3*b^3*\ln(x)/a + 1/3*(a*x+b)^3*\ln(x)/a$

Rubi [A]

time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {32, 2350, 12, 45}

$$-\frac{a^2x^3}{9} - \frac{b^3 \log(x)}{3a} - \frac{1}{2}abx^2 + \frac{\log(x)(ax + b)^3}{3a} - b^2x$$

Antiderivative was successfully verified.

[In] Int[(b + a*x)^2*Log[x], x]

[Out] $-(b^2*x) - (a*b*x^2)/2 - (a^2*x^3)/9 - (b^3*\text{Log}[x])/(3*a) + ((b + a*x)^3*\text{Log}[x])/(3*a)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2350

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.) + (e_.)*(x_)^(r_.))^(q_.), x_Symbol] := With[{u = IntHide[(d + e*x^r)^q, x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned}
\int (b+ax)^2 \log(x) dx &= \frac{(b+ax)^3 \log(x)}{3a} - \int \frac{(b+ax)^3}{3ax} dx \\
&= \frac{(b+ax)^3 \log(x)}{3a} - \frac{\int \frac{(b+ax)^3}{x} dx}{3a} \\
&= \frac{(b+ax)^3 \log(x)}{3a} - \frac{\int \left(3ab^2 + \frac{b^3}{x} + 3a^2bx + a^3x^2\right) dx}{3a} \\
&= -b^2x - \frac{1}{2}abx^2 - \frac{a^2x^3}{9} - \frac{b^3 \log(x)}{3a} + \frac{(b+ax)^3 \log(x)}{3a}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 53, normalized size = 0.98

$$-b^2x - \frac{1}{2}abx^2 - \frac{a^2x^3}{9} + b^2x \log(x) + abx^2 \log(x) + \frac{1}{3}a^2x^3 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(b + a*x)^2*Log[x],x]

[Out] -(b^2*x) - (a*b*x^2)/2 - (a^2*x^3)/9 + b^2*x*Log[x] + a*b*x^2*Log[x] + (a^2*x^3*Log[x])/3

Mathics [A]

time = 2.08, size = 44, normalized size = 0.81

$$\frac{x(-2a^2x^2 - 9abx - 18b^2 + 6\text{Log}[x](a^2x^2 + 3abx + 3b^2))}{18}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(a*x+b)^2*Log[x],x]')

[Out] x(-2 a ^ 2 x ^ 2 - 9 a b x - 18 b ^ 2 + 6 Log[x] (a ^ 2 x ^ 2 + 3 a b x + 3 b ^ 2)) / 18

Maple [A]

time = 0.02, size = 48, normalized size = 0.89

method	result	size
risch	$-b^2x - \frac{abx^2}{2} - \frac{a^2x^3}{9} - \frac{b^3 \ln(x)}{3a} + \frac{(ax+b)^3 \ln(x)}{3a}$	47
default	$a^2 \left(-\frac{x^3}{9} + \frac{x^3 \ln(x)}{3}\right) + 2ab \left(-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}\right) + b^2(-x + x \ln(x))$	48

norman	$b^2x \ln(x) + abx^2 \ln(x) - \frac{a^2x^3}{9} - b^2x - \frac{abx^2}{2} + \frac{a^2x^3 \ln(x)}{3}$	48
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x+b)^2*ln(x),x,method=_RETURNVERBOSE)`

[Out] $a^2*(-1/9*x^3+1/3*x^3*\ln(x))+2*a*b*(-1/4*x^2+1/2*x^2*\ln(x))+b^2*(-x+x*\ln(x))$

Maxima [A]

time = 0.30, size = 47, normalized size = 0.87

$$-\frac{1}{9}a^2x^3 - \frac{1}{2}abx^2 - b^2x + \frac{1}{3}(a^2x^3 + 3abx^2 + 3b^2x) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x+b)^2*log(x),x, algorithm="maxima")`

[Out] $-1/9*a^2*x^3 - 1/2*a*b*x^2 - b^2*x + 1/3*(a^2*x^3 + 3*a*b*x^2 + 3*b^2*x)*\log(x)$

Fricas [A]

time = 0.34, size = 47, normalized size = 0.87

$$-\frac{1}{9}a^2x^3 - \frac{1}{2}abx^2 - b^2x + \frac{1}{3}(a^2x^3 + 3abx^2 + 3b^2x) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x+b)^2*log(x),x, algorithm="fricas")`

[Out] $-1/9*a^2*x^3 - 1/2*a*b*x^2 - b^2*x + 1/3*(a^2*x^3 + 3*a*b*x^2 + 3*b^2*x)*\log(x)$

Sympy [A]

time = 0.07, size = 44, normalized size = 0.81

$$-\frac{a^2x^3}{9} - \frac{abx^2}{2} - b^2x + \left(\frac{a^2x^3}{3} + abx^2 + b^2x\right) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x+b)**2*ln(x),x)`

[Out] $-a**2*x**3/9 - a*b*x**2/2 - b**2*x + (a**2*x**3/3 + a*b*x**2 + b**2*x)*\log(x)$

Giac [A]

time = 0.00, size = 52, normalized size = 0.96

$$-\frac{1}{9}a^2x^3 - \frac{1}{2}abx^2 - b^2x + \frac{1}{3}a^2x^3 \ln x + b^2x \ln x + abx^2 \ln x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+b)^2*log(x),x)

[Out] $\frac{1}{3}a^2x^3\log(x) - \frac{1}{9}a^2x^3 + abx^2\log(x) - \frac{1}{2}abx^2 + b^2x\log(x) - b^2x$

Mupad [B]

time = 0.17, size = 47, normalized size = 0.87

$$b^2 x \ln(x) - \frac{a^2 x^3}{9} - b^2 x + \frac{a^2 x^3 \ln(x)}{3} - \frac{abx^2}{2} + abx^2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(x)*(b + a*x)^2,x)

[Out] $b^2x\log(x) - \frac{a^2x^3}{9} - b^2x + \frac{a^2x^3\log(x)}{3} - \frac{abx^2}{2} + abx^2\log(x)$

$$3.67 \quad \int \frac{\log(x)}{(b+ax)^2} dx$$

Optimal. Leaf size=29

$$\frac{x \log(x)}{b(b+ax)} - \frac{\log(b+ax)}{ab}$$

[Out] x*ln(x)/b/(a*x+b)-ln(a*x+b)/a/b

Rubi [A]

time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2351, 31}

$$\frac{x \log(x)}{b(ax+b)} - \frac{\log(ax+b)}{ab}$$

Antiderivative was successfully verified.

[In] Int[Log[x]/(b + a*x)^2,x]

[Out] (x*Log[x])/(b*(b + a*x)) - Log[b + a*x]/(a*b)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2351

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x_Symbol] := Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x] && EqQ[r*(q + 1) + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{\log(x)}{(b+ax)^2} dx &= \frac{x \log(x)}{b(b+ax)} - \frac{\int \frac{1}{b+ax} dx}{b} \\ &= \frac{x \log(x)}{b(b+ax)} - \frac{\log(b+ax)}{ab} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 27, normalized size = 0.93

$$\frac{\frac{x \log(x)}{b+ax} - \frac{\log(b+ax)}{a}}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Log[x]/(b + a*x)^2,x]

[Out] ((x*Log[x])/(b + a*x) - Log[b + a*x]/a)/b

Mathics [A]

time = 2.17, size = 41, normalized size = 1.41

$$\frac{-b \operatorname{Log}[x] + (ax + b) (\operatorname{Log}[x] - \operatorname{Log}\left[\frac{ax+b}{a}\right])}{ab(ax + b)}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Log[x]/(a*x+b)^2,x]')

[Out] (-b Log[x] + (a x + b) (Log[x] - Log[(a x + b) / a])) / (a b (a x + b))

Maple [A]

time = 0.02, size = 30, normalized size = 1.03

method	result	size
default	$\frac{x \ln(x)}{b(ax+b)} - \frac{\ln(ax+b)}{ab}$	30
norman	$\frac{x \ln(x)}{b(ax+b)} - \frac{\ln(ax+b)}{ab}$	30
risch	$-\frac{\ln(x)}{a(ax+b)} - \frac{\ln(ax+b)}{ab} + \frac{\ln(-x)}{ba}$	41

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(x)/(a*x+b)^2,x,method=_RETURNVERBOSE)

[Out] x*ln(x)/b/(a*x+b)-ln(a*x+b)/a/b

Maxima [A]

time = 0.27, size = 38, normalized size = 1.31

$$-\frac{\frac{\log(ax+b)}{b} - \frac{\log(x)}{b}}{a} - \frac{\log(x)}{(ax+b)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)/(a*x+b)^2,x, algorithm="maxima")

[Out] -(log(a*x + b)/b - log(x)/b)/a - log(x)/((a*x + b)*a)

Fricas [A]

time = 0.33, size = 34, normalized size = 1.17

$$\frac{ax \log(x) - (ax + b) \log(ax + b)}{a^2bx + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)/(a*x+b)^2,x, algorithm="fricas")

[Out] (a*x*log(x) - (a*x + b)*log(a*x + b))/(a^2*b*x + a*b^2)

Sympy [A]

time = 0.12, size = 24, normalized size = 0.83

$$-\frac{\log(x)}{a^2x + ab} + \frac{\log(x) - \log\left(x + \frac{b}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(x)/(a*x+b)**2,x)

[Out] -log(x)/(a**2*x + a*b) + (log(x) - log(x + b/a))/(a*b)

Giac [A]

time = 0.00, size = 37, normalized size = 1.28

$$\frac{\ln|x|}{ab} - \frac{a \ln|xa + b|}{a^2b} - \frac{\ln x}{a(xa + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)/(a*x+b)^2,x)

[Out] -log(x)/((a*x + b)*a) - log(abs(a*x + b))/(a*b) + log(abs(x))/(a*b)

Mupad [B]

time = 0.23, size = 35, normalized size = 1.21

$$\frac{x^2 \ln(x)}{b(ax^2 + bx)} - \frac{\ln(b + ax)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(x)/(b + a*x)^2,x)

[Out] (x^2*log(x))/(b*(b*x + a*x^2)) - log(b + a*x)/(a*b)

3.68 $\int x \log(b + ax) dx$

Optimal. Leaf size=46

$$\frac{bx}{2a} - \frac{x^2}{4} - \frac{b^2 \log(b + ax)}{2a^2} + \frac{1}{2}x^2 \log(b + ax)$$

[Out] $1/2*b*x/a - 1/4*x^2 - 1/2*b^2*\ln(a*x+b)/a^2 + 1/2*x^2*\ln(a*x+b)$

Rubi [A]

time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2442, 45}

$$-\frac{b^2 \log(ax + b)}{2a^2} + \frac{1}{2}x^2 \log(ax + b) + \frac{bx}{2a} - \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Int[x*Log[b + a*x],x]

[Out] (b*x)/(2*a) - x^2/4 - (b^2*Log[b + a*x])/(2*a^2) + (x^2*Log[b + a*x])/2

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2442

Int[((a_.) + Log[(c_.)*(d_.) + (e_.)*(x_)^(n_.)]*(b_.))*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]

Rubi steps

$$\begin{aligned} \int x \log(b + ax) dx &= \frac{1}{2}x^2 \log(b + ax) - \frac{1}{2}a \int \frac{x^2}{b + ax} dx \\ &= \frac{1}{2}x^2 \log(b + ax) - \frac{1}{2}a \int \left(-\frac{b}{a^2} + \frac{x}{a} + \frac{b^2}{a^2(b + ax)} \right) dx \\ &= \frac{bx}{2a} - \frac{x^2}{4} - \frac{b^2 \log(b + ax)}{2a^2} + \frac{1}{2}x^2 \log(b + ax) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 46, normalized size = 1.00

$$\frac{bx}{2a} - \frac{x^2}{4} - \frac{b^2 \log(b+ax)}{2a^2} + \frac{1}{2}x^2 \log(b+ax)$$

Antiderivative was successfully verified.

[In] Integrate[x*Log[b + a*x],x]

[Out] (b*x)/(2*a) - x^2/4 - (b^2*Log[b + a*x])/(2*a^2) + (x^2*Log[b + a*x])/2

Mathics [A]

time = 2.19, size = 38, normalized size = 0.83

$$\frac{bx}{2a} - \frac{b^2 \text{Log}[ax+b]}{2a^2} - \frac{x^2}{4} + \frac{x^2 \text{Log}[ax+b]}{2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x*Log[a*x+b],x]')

[Out] b x / (2 a) - b ^ 2 Log[a x + b] / (2 a ^ 2) - x ^ 2 / 4 + x ^ 2 Log[a x + b] / 2

Maple [A]

time = 0.02, size = 53, normalized size = 1.15

method	result	size
norman	$\frac{bx}{2a} - \frac{x^2}{4} - \frac{b^2 \ln(ax+b)}{2a^2} + \frac{x^2 \ln(ax+b)}{2}$	39
risch	$\frac{bx}{2a} - \frac{x^2}{4} - \frac{b^2 \ln(ax+b)}{2a^2} + \frac{x^2 \ln(ax+b)}{2}$	39
derivativedivides	$\frac{-b(\ln(ax+b)(ax+b)-ax-b) + \frac{(ax+b)^2 \ln(ax+b)}{2} - \frac{(ax+b)^2}{4}}{a^2}$	53
default	$\frac{-b(\ln(ax+b)(ax+b)-ax-b) + \frac{(ax+b)^2 \ln(ax+b)}{2} - \frac{(ax+b)^2}{4}}{a^2}$	53

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*ln(a*x+b),x,method=_RETURNVERBOSE)

[Out] 1/a^2*(-b*(ln(a*x+b)*(a*x+b)-a*x-b)+1/2*(a*x+b)^2*ln(a*x+b)-1/4*(a*x+b)^2)

Maxima [A]

time = 0.26, size = 44, normalized size = 0.96

$$\frac{1}{2}x^2 \log(ax+b) - \frac{1}{4}a \left(\frac{2b^2 \log(ax+b)}{a^3} + \frac{ax^2 - 2bx}{a^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*log(a*x+b),x, algorithm="maxima")

[Out] $1/2*x^2*log(a*x + b) - 1/4*a*(2*b^2*log(a*x + b)/a^3 + (a*x^2 - 2*b*x)/a^2)$

Fricas [A]

time = 0.34, size = 39, normalized size = 0.85

$$-\frac{a^2x^2 - 2abx - 2(a^2x^2 - b^2)\log(ax + b)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*log(a*x+b),x, algorithm="fricas")

[Out] $-1/4*(a^2*x^2 - 2*a*b*x - 2*(a^2*x^2 - b^2)*log(a*x + b))/a^2$

Sympy [A]

time = 0.09, size = 42, normalized size = 0.91

$$-a \left(\frac{x^2}{4a} - \frac{bx}{2a^2} + \frac{b^2 \log(ax + b)}{2a^3} \right) + \frac{x^2 \log(ax + b)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*ln(a*x+b),x)

[Out] $-a*(x**2/(4*a) - b*x/(2*a**2) + b**2*log(a*x + b)/(2*a**3)) + x**2*log(a*x + b)/2$

Giac [A]

time = 0.00, size = 65, normalized size = 1.41

$$\frac{b(ax + b)}{a^2} - \frac{(ax + b)^2}{4a^2} + \frac{(ax + b)^2 \ln(ax + b)}{2a^2} - \frac{b(ax + b) \ln(ax + b)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*log(a*x+b),x)

[Out] $1/2*(a*x + b)^2*log(a*x + b)/a^2 - (a*x + b)*b*log(a*x + b)/a^2 - 1/4*(a*x + b)^2/a^2 + (a*x + b)*b/a^2$

Mupad [B]

time = 0.19, size = 66, normalized size = 1.43

$$\begin{cases} \frac{x^2 (\ln(ax) - \frac{1}{2})}{2} & \text{if } b = 0 \\ \frac{\ln(b+ax) \left(x^2 - \frac{b^2}{a^2}\right)}{2} - \frac{b^2 \left(\frac{a^2x^2}{2b^2} - \frac{ax}{b}\right)}{2a^2} & \text{if } b \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*log(b + a*x),x)

[Out] $piecewise(b == 0, (x^2*(\log(a*x) - 1/2))/2, b \neq 0, (\log(b + a*x)*(x^2 - b^2/a^2))/2 - (b^2*((a^2*x^2)/(2*b^2) - (a*x)/b))/(2*a^2))$

3.69 $\int x^2 \log(b + ax) dx$

Optimal. Leaf size=59

$$-\frac{b^2x}{3a^2} + \frac{bx^2}{6a} - \frac{x^3}{9} + \frac{b^3 \log(b + ax)}{3a^3} + \frac{1}{3}x^3 \log(b + ax)$$

[Out] $-1/3*b^2*x/a^2+1/6*b*x^2/a-1/9*x^3+1/3*b^3*\ln(a*x+b)/a^3+1/3*x^3*\ln(a*x+b)$

Rubi [A]

time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2442, 45}

$$\frac{b^3 \log(ax + b)}{3a^3} - \frac{b^2x}{3a^2} + \frac{1}{3}x^3 \log(ax + b) + \frac{bx^2}{6a} - \frac{x^3}{9}$$

Antiderivative was successfully verified.

[In] Int[x^2*Log[b + a*x],x]

[Out] $-1/3*(b^2*x)/a^2 + (b*x^2)/(6*a) - x^3/9 + (b^3*Log[b + a*x])/(3*a^3) + (x^3*Log[b + a*x])/3$

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2442

Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_))^(q_.), x_Symbol] := Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]

Rubi steps

$$\begin{aligned} \int x^2 \log(b + ax) dx &= \frac{1}{3}x^3 \log(b + ax) - \frac{1}{3}a \int \frac{x^3}{b + ax} dx \\ &= \frac{1}{3}x^3 \log(b + ax) - \frac{1}{3}a \int \left(\frac{b^2}{a^3} - \frac{bx}{a^2} + \frac{x^2}{a} - \frac{b^3}{a^3(b + ax)} \right) dx \\ &= -\frac{b^2x}{3a^2} + \frac{bx^2}{6a} - \frac{x^3}{9} + \frac{b^3 \log(b + ax)}{3a^3} + \frac{1}{3}x^3 \log(b + ax) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 59, normalized size = 1.00

$$-\frac{b^2x}{3a^2} + \frac{bx^2}{6a} - \frac{x^3}{9} + \frac{b^3 \log(b+ax)}{3a^3} + \frac{1}{3}x^3 \log(b+ax)$$

Antiderivative was successfully verified.

`[In] Integrate[x^2*Log[b + a*x],x]`

```
[Out] -1/3*(b^2*x)/a^2 + (b*x^2)/(6*a) - x^3/9 + (b^3*Log[b + a*x])/(3*a^3) + (x^3*Log[b + a*x])/3
```

Mathics [A]

time = 2.36, size = 49, normalized size = 0.83

$$\frac{bx^2}{6a} - \frac{b^2x}{3a^2} + \frac{b^3 \text{Log}[ax+b]}{3a^3} - \frac{x^3}{9} + \frac{x^3 \text{Log}[ax+b]}{3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^2*Log[a*x+b],x]')`

```
[Out] b x ^ 2 / (6 a) - b ^ 2 x / (3 a ^ 2) + b ^ 3 Log[a x + b] / (3 a ^ 3) - x ^ 3 / 9 + x ^ 3 Log[a x + b] / 3
```

Maple [A]

time = 0.01, size = 82, normalized size = 1.39

method	result	size
norman	$-\frac{b^2x}{3a^2} + \frac{bx^2}{6a} - \frac{x^3}{9} + \frac{b^3 \ln(ax+b)}{3a^3} + \frac{x^3 \ln(ax+b)}{3}$	50
risch	$-\frac{b^2x}{3a^2} + \frac{bx^2}{6a} - \frac{x^3}{9} + \frac{b^3 \ln(ax+b)}{3a^3} + \frac{x^3 \ln(ax+b)}{3}$	50
derivativedivides	$\frac{b^2(\ln(ax+b)(ax+b)-ax-b)-2b\left(\frac{(ax+b)^2 \ln(ax+b)}{2} - \frac{(ax+b)^2}{4}\right) + \frac{(ax+b)^3 \ln(ax+b)}{3} - \frac{(ax+b)^3}{9}}{a^3}$	82
default	$\frac{b^2(\ln(ax+b)(ax+b)-ax-b)-2b\left(\frac{(ax+b)^2 \ln(ax+b)}{2} - \frac{(ax+b)^2}{4}\right) + \frac{(ax+b)^3 \ln(ax+b)}{3} - \frac{(ax+b)^3}{9}}{a^3}$	82

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2*ln(a*x+b),x,method=_RETURNVERBOSE)`

```
[Out] 1/a^3*(b^2*(ln(a*x+b)*(a*x+b)-a*x-b)-2*b*(1/2*(a*x+b)^2*ln(a*x+b)-1/4*(a*x+b)^2)+1/3*(a*x+b)^3*ln(a*x+b)-1/9*(a*x+b)^3)
```

Maxima [A]

time = 0.27, size = 57, normalized size = 0.97

$$\frac{1}{3}x^3 \log(ax+b) + \frac{1}{18}a \left(\frac{6b^3 \log(ax+b)}{a^4} - \frac{2a^2x^3 - 3abx^2 + 6b^2x}{a^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*log(a*x+b),x, algorithm="maxima")

[Out] $1/3*x^3*log(a*x + b) + 1/18*a*(6*b^3*log(a*x + b)/a^4 - (2*a^2*x^3 - 3*a*b*x^2 + 6*b^2*x)/a^3)$

Fricas [A]

time = 0.33, size = 49, normalized size = 0.83

$$-\frac{2a^3x^3 - 3a^2bx^2 + 6ab^2x - 6(a^3x^3 + b^3)\log(ax + b)}{18a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*log(a*x+b),x, algorithm="fricas")

[Out] $-1/18*(2*a^3*x^3 - 3*a^2*b*x^2 + 6*a*b^2*x - 6*(a^3*x^3 + b^3)*log(a*x + b))/a^3$

Sympy [A]

time = 0.10, size = 54, normalized size = 0.92

$$-a \left(\frac{x^3}{9a} - \frac{bx^2}{6a^2} + \frac{b^2x}{3a^3} - \frac{b^3 \log(ax + b)}{3a^4} \right) + \frac{x^3 \log(ax + b)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*ln(a*x+b),x)

[Out] $-a*(x**3/(9*a) - b*x**2/(6*a**2) + b**2*x/(3*a**3) - b**3*log(a*x + b)/(3*a**4)) + x**3*log(a*x + b)/3$

Giac [A]

time = 0.00, size = 104, normalized size = 1.76

$$-\frac{b^2(ax + b)}{a^3} - \frac{(ax + b)^3}{9a^3} + \frac{b(ax + b)^2}{2a^3} + \frac{(ax + b)^3 \ln(ax + b)}{3a^3} - \frac{b(ax + b)^2 \ln(ax + b)}{a^3} + \frac{b^2(ax + b) \ln(ax + b)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*log(a*x+b),x)

[Out] $1/3*(a*x + b)^3*log(a*x + b)/a^3 - (a*x + b)^2*b*log(a*x + b)/a^3 + (a*x + b)*b^2*log(a*x + b)/a^3 - 1/9*(a*x + b)^3/a^3 + 1/2*(a*x + b)^2*b/a^3 - (a*x + b)*b^2/a^3$

Mupad [B]

time = 0.18, size = 75, normalized size = 1.27

$$\begin{cases} \frac{x^3 (\ln(ax) - \frac{1}{3})}{3} & \text{if } b = 0 \\ \frac{\ln(b+ax) \left(x^3 + \frac{b^3}{a^3} \right)}{3} - \frac{b^3 \left(\frac{a^3 x^3}{3b^3} - \frac{a^2 x^2}{2b^2} + \frac{ax}{b} \right)}{3a^3} & \text{if } b \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(b + a*x),x)
```

```
[Out] piecewise(b == 0, (x^3*(log(a*x) - 1/3))/3, b != 0, (log(b + a*x)*(x^3 + b^3/a^3))/3 - (b^3*(- (a^2*x^2)/(2*b^2) + (a^3*x^3)/(3*b^3) + (a*x)/b))/(3*a^3))
```

3.70 $\int \log(a^2 + x^2) dx$

Optimal. Leaf size=23

$$-2x + 2a \tan^{-1}\left(\frac{x}{a}\right) + x \log(a^2 + x^2)$$

[Out] $-2*x+2*a*\arctan(x/a)+x*\ln(a^2+x^2)$

Rubi [A]

time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2498, 327, 209}

$$x \log(a^2 + x^2) + 2a \tan^{-1}\left(\frac{x}{a}\right) - 2x$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Log}[a^2 + x^2], x]$

[Out] $-2*x + 2*a*\text{ArcTan}[x/a] + x*\text{Log}[a^2 + x^2]$

Rule 209

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))*\text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 327

$\text{Int}[(c_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}), x_Symbol] \rightarrow \text{Simp}[c^{(n-1)}*(c*x)^{(m-n+1)}*((a + b*x^n)^{(p+1)}/(b*(m+n*p+1))), x] - \text{Dist}[a*c^n*((m-n+1)/(b*(m+n*p+1))), \text{Int}[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n-1] \ \&\& \ \text{NeQ}[m+n*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 2498

$\text{Int}[\text{Log}[(c_)*((d_ + (e_)*(x_)^{(n_)})^{(p_)}), x_Symbol] \rightarrow \text{Simp}[x*\text{Log}[c*(d + e*x^n)^p], x] - \text{Dist}[e*n*p, \text{Int}[x^n/(d + e*x^n), x], x] /; \text{FreeQ}\{c, d, e, n, p\}, x]$

Rubi steps

$$\begin{aligned} \int \log(a^2 + x^2) dx &= x \log(a^2 + x^2) - 2 \int \frac{x^2}{a^2 + x^2} dx \\ &= -2x + x \log(a^2 + x^2) + (2a^2) \int \frac{1}{a^2 + x^2} dx \\ &= -2x + 2a \tan^{-1}\left(\frac{x}{a}\right) + x \log(a^2 + x^2) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 23, normalized size = 1.00

$$-2x + 2a \tan^{-1}\left(\frac{x}{a}\right) + x \log(a^2 + x^2)$$

Antiderivative was successfully verified.

`[In] Integrate[Log[a^2 + x^2], x]``[Out] -2*x + 2*a*ArcTan[x/a] + x*Log[a^2 + x^2]`**Mathics [C]** Result contains complex when optimal does not.

time = 2.12, size = 32, normalized size = 1.39

$$-Ia (\text{Log}[-Ia + x] - \text{Log}[Ia + x]) - 2x + x \text{Log}[a^2 + x^2]$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Log[x^2+a^2], x]')``[Out] -I a (Log[-I a + x] - Log[I a + x]) - 2 x + x Log[a ^ 2 + x ^ 2]`**Maple [A]**

time = 0.02, size = 24, normalized size = 1.04

method	result	size
default	$-2x + 2a \arctan\left(\frac{x}{a}\right) + x \ln(a^2 + x^2)$	24
risch	$-2x + 2a \arctan\left(\frac{x}{a}\right) + x \ln(a^2 + x^2)$	24

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(ln(a^2+x^2), x, method=_RETURNVERBOSE)``[Out] -2*x+2*a*arctan(x/a)+x*ln(a^2+x^2)`**Maxima [A]**

time = 0.35, size = 23, normalized size = 1.00

$$2a \arctan\left(\frac{x}{a}\right) + x \log(a^2 + x^2) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a^2+x^2),x, algorithm="maxima")

[Out] 2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x

Fricas [A]

time = 0.32, size = 23, normalized size = 1.00

$$2 a \arctan \left(\frac{x}{a} \right) + x \log \left(a^2 + x^2 \right) - 2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a^2+x^2),x, algorithm="fricas")

[Out] 2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x

Sympy [C] Result contains complex when optimal does not.

time = 0.08, size = 36, normalized size = 1.57

$$-2a \left(\frac{i \log(-ia + x)}{2} - \frac{i \log(ia + x)}{2} \right) + x \log(a^2 + x^2) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(a**2+x**2),x)

[Out] -2*a*(I*log(-I*a + x)/2 - I*log(I*a + x)/2) + x*log(a**2 + x**2) - 2*x

Giac [A]

time = 0.00, size = 31, normalized size = 1.35

$$x \ln(a^2 + x^2) - 2 \left(x - \frac{2a^2 \arctan\left(\frac{x}{a}\right)}{2a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a^2+x^2),x)

[Out] 2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x

Mupad [B]

time = 0.07, size = 23, normalized size = 1.00

$$x \ln(a^2 + x^2) - 2x + 2a \operatorname{atan}\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(a^2 + x^2),x)

[Out] x*log(a^2 + x^2) - 2*x + 2*a*atan(x/a)

3.71 $\int x \log(a^2 + x^2) dx$

Optimal. Leaf size=27

$$-\frac{x^2}{2} + \frac{1}{2}(a^2 + x^2) \log(a^2 + x^2)$$

[Out] $-1/2*x^2+1/2*(a^2+x^2)*\ln(a^2+x^2)$

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2504, 2436, 2332}

$$\frac{1}{2}(a^2 + x^2) \log(a^2 + x^2) - \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Log}[a^2 + x^2], x]$

[Out] $-1/2*x^2 + ((a^2 + x^2)*\text{Log}[a^2 + x^2])/2$

Rule 2332

$\text{Int}[\text{Log}[(c_)*(x_)^{(n_)}], x_Symbol] \rightarrow \text{Simp}[x*\text{Log}[c*x^n], x] - \text{Simp}[n*x, x] /; \text{FreeQ}[\{c, n\}, x]$

Rule 2436

$\text{Int}[(a_ + \text{Log}[(c_)*((d_ + (e_)*(x_)^{(n_)})*(b_))^{(p_)}], x_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}[\{a, b, c, d, e, n, p\}, x]$

Rule 2504

$\text{Int}[(a_ + \text{Log}[(c_)*((d_ + (e_)*(x_)^{(n_)}))^{(p_)}]*(b_))^{(q_)}*(x_)^{(m_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*\text{Log}[c*(d + e*x)^p])^q, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, e, m, n, p, q\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]] \&\& (\text{GtQ}[(m + 1)/n, 0] \|\| \text{IGtQ}[q, 0]) \&\& !(\text{EqQ}[q, 1] \&\& \text{ILtQ}[n, 0] \&\& \text{IGtQ}[m, 0])$

Rubi steps

$$\begin{aligned} \int x \log(a^2 + x^2) dx &= \frac{1}{2} \text{Subst} \left(\int \log(a^2 + x) dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \log(x) dx, x, a^2 + x^2 \right) \\ &= -\frac{x^2}{2} + \frac{1}{2} (a^2 + x^2) \log(a^2 + x^2) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 26, normalized size = 0.96

$$\frac{1}{2} (-x^2 + (a^2 + x^2) \log(a^2 + x^2))$$

Antiderivative was successfully verified.

`[In] Integrate[x*Log[a^2 + x^2], x]``[Out] (-x^2 + (a^2 + x^2)*Log[a^2 + x^2])/2`**Mathics [A]**

time = 1.96, size = 32, normalized size = 1.19

$$\frac{a^2 \text{Log}[a^2 + x^2]}{2} - \frac{x^2}{2} + \frac{x^2 \text{Log}[a^2 + x^2]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x*Log[x^2+a^2], x]')``[Out] a ^ 2 Log[a ^ 2 + x ^ 2] / 2 - x ^ 2 / 2 + x ^ 2 Log[a ^ 2 + x ^ 2] / 2`**Maple [A]**

time = 0.01, size = 29, normalized size = 1.07

method	result	size
derivativdivides	$\frac{(a^2+x^2) \ln(a^2+x^2)}{2} - \frac{a^2}{2} - \frac{x^2}{2}$	29
default	$\frac{(a^2+x^2) \ln(a^2+x^2)}{2} - \frac{a^2}{2} - \frac{x^2}{2}$	29
norman	$-\frac{x^2}{2} + \frac{x^2 \ln(a^2+x^2)}{2} + \frac{\ln(a^2+x^2)a^2}{2}$	33
risch	$-\frac{x^2}{2} + \frac{x^2 \ln(a^2+x^2)}{2} + \frac{\ln(a^2+x^2)a^2}{2}$	33

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*ln(a^2+x^2), x, method=_RETURNVERBOSE)`

[Out] $1/2*(a^2+x^2)*\ln(a^2+x^2)-1/2*a^2-1/2*x^2$

Maxima [A]

time = 0.27, size = 28, normalized size = 1.04

$$-\frac{1}{2}a^2 - \frac{1}{2}x^2 + \frac{1}{2}(a^2 + x^2) \log(a^2 + x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(a^2+x^2),x, algorithm="maxima")`

[Out] $-1/2*a^2 - 1/2*x^2 + 1/2*(a^2 + x^2)*\log(a^2 + x^2)$

Fricas [A]

time = 0.32, size = 23, normalized size = 0.85

$$-\frac{1}{2}x^2 + \frac{1}{2}(a^2 + x^2) \log(a^2 + x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(a^2+x^2),x, algorithm="fricas")`

[Out] $-1/2*x^2 + 1/2*(a^2 + x^2)*\log(a^2 + x^2)$

Sympy [A]

time = 0.08, size = 31, normalized size = 1.15

$$\frac{a^2 \log(a^2 + x^2)}{2} + \frac{x^2 \log(a^2 + x^2)}{2} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*ln(a**2+x**2),x)`

[Out] $a**2*\log(a**2 + x**2)/2 + x**2*\log(a**2 + x**2)/2 - x**2/2$

Giac [A]

time = 0.00, size = 28, normalized size = 1.04

$$\frac{-a^2 - x^2 + (a^2 + x^2) \ln(a^2 + x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(a^2+x^2),x)`

[Out] $-1/2*a^2 - 1/2*x^2 + 1/2*(a^2 + x^2)*\log(a^2 + x^2)$

Mupad [B]

time = 0.04, size = 51, normalized size = 1.89

$$\frac{a^2 \ln(x - \sqrt{-a^2})}{2} + \frac{x^2 \ln(a^2 + x^2)}{2} - \frac{x^2}{2} + \frac{a^2 \ln(x + \sqrt{-a^2})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(a^2 + x^2),x)
```

```
[Out] (a^2*log(x - (-a^2)^(1/2)))/2 + (x^2*log(a^2 + x^2))/2 - x^2/2 + (a^2*log(x  
+ (-a^2)^(1/2)))/2
```

3.72 $\int x^2 \log(a^2 + x^2) dx$

Optimal. Leaf size=44

$$\frac{2a^2x}{3} - \frac{2x^3}{9} - \frac{2}{3}a^3 \tan^{-1}\left(\frac{x}{a}\right) + \frac{1}{3}x^3 \log(a^2 + x^2)$$

[Out] $2/3*a^2*x-2/9*x^3-2/3*a^3*\arctan(x/a)+1/3*x^3*\ln(a^2+x^2)$

Rubi [A]

time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2505, 308, 209}

$$-\frac{2}{3}a^3 \tan^{-1}\left(\frac{x}{a}\right) + \frac{1}{3}x^3 \log(a^2 + x^2) + \frac{2a^2x}{3} - \frac{2x^3}{9}$$

Antiderivative was successfully verified.

[In] Int[x^2*Log[a^2 + x^2],x]

[Out] $(2*a^2*x)/3 - (2*x^3)/9 - (2*a^3*\text{ArcTan}[x/a])/3 + (x^3*\text{Log}[a^2 + x^2])/3$

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 308

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rule 2505

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_)^(m_.)), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m + 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int x^2 \log(a^2 + x^2) dx &= \frac{1}{3} x^3 \log(a^2 + x^2) - \frac{2}{3} \int \frac{x^4}{a^2 + x^2} dx \\
&= \frac{1}{3} x^3 \log(a^2 + x^2) - \frac{2}{3} \int \left(-a^2 + x^2 + \frac{a^4}{a^2 + x^2} \right) dx \\
&= \frac{2a^2 x}{3} - \frac{2x^3}{9} + \frac{1}{3} x^3 \log(a^2 + x^2) - \frac{1}{3} (2a^4) \int \frac{1}{a^2 + x^2} dx \\
&= \frac{2a^2 x}{3} - \frac{2x^3}{9} - \frac{2}{3} a^3 \tan^{-1} \left(\frac{x}{a} \right) + \frac{1}{3} x^3 \log(a^2 + x^2)
\end{aligned}$$

Mathematica [A]

time = 0.00, size = 44, normalized size = 1.00

$$\frac{2a^2x}{3} - \frac{2x^3}{9} - \frac{2}{3}a^3 \tan^{-1} \left(\frac{x}{a} \right) + \frac{1}{3}x^3 \log(a^2 + x^2)$$

Antiderivative was successfully verified.

`[In] Integrate[x^2*Log[a^2 + x^2], x]``[Out] (2*a^2*x)/3 - (2*x^3)/9 - (2*a^3*ArcTan[x/a])/3 + (x^3*Log[a^2 + x^2])/3`**Mathics [C]** Result contains complex when optimal does not.

time = 2.20, size = 45, normalized size = 1.02

$$\frac{I}{3} a^3 (\text{Log}[-Ia + x] - \text{Log}[Ia + x]) + \frac{2a^2x}{3} - \frac{2x^3}{9} + \frac{x^3 \text{Log}[a^2 + x^2]}{3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^2*Log[x^2+a^2], x]')``[Out] I / 3 a ^ 3 (Log[-I a + x] - Log[I a + x]) + 2 a ^ 2 x / 3 - 2 x ^ 3 / 9 + x ^ 3 Log[a ^ 2 + x ^ 2] / 3`**Maple [A]**

time = 0.02, size = 37, normalized size = 0.84

method	result	size
default	$\frac{2a^2x}{3} - \frac{2x^3}{9} - \frac{2a^3 \arctan(\frac{x}{a})}{3} + \frac{x^3 \ln(a^2+x^2)}{3}$	37
risch	$\frac{2a^2x}{3} - \frac{2x^3}{9} - \frac{2a^3 \arctan(\frac{x}{a})}{3} + \frac{x^3 \ln(a^2+x^2)}{3}$	37

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2*ln(a^2+x^2), x, method=_RETURNVERBOSE)`

[Out] $2/3*a^2*x-2/9*x^3-2/3*a^3*\arctan(x/a)+1/3*x^3*\ln(a^2+x^2)$

Maxima [A]

time = 0.34, size = 36, normalized size = 0.82

$$-\frac{2}{3}a^3 \arctan\left(\frac{x}{a}\right) + \frac{1}{3}x^3 \log(a^2 + x^2) + \frac{2}{3}a^2x - \frac{2}{9}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*log(a^2+x^2),x, algorithm="maxima")`

[Out] $-2/3*a^3*\arctan(x/a) + 1/3*x^3*\log(a^2 + x^2) + 2/3*a^2*x - 2/9*x^3$

Fricas [A]

time = 0.33, size = 36, normalized size = 0.82

$$-\frac{2}{3}a^3 \arctan\left(\frac{x}{a}\right) + \frac{1}{3}x^3 \log(a^2 + x^2) + \frac{2}{3}a^2x - \frac{2}{9}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*log(a^2+x^2),x, algorithm="fricas")`

[Out] $-2/3*a^3*\arctan(x/a) + 1/3*x^3*\log(a^2 + x^2) + 2/3*a^2*x - 2/9*x^3$

Sympy [C] Result contains complex when optimal does not.

time = 0.09, size = 53, normalized size = 1.20

$$-2a^3 \left(-\frac{i \log(-ia + x)}{6} + \frac{i \log(ia + x)}{6} \right) + \frac{2a^2x}{3} + \frac{x^3 \log(a^2 + x^2)}{3} - \frac{2x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*ln(a**2+x**2),x)`

[Out] $-2*a**3*(-I*\log(-I*a + x)/6 + I*\log(I*a + x)/6) + 2*a**2*x/3 + x**3*\log(a**2 + x**2)/3 - 2*x**3/9$

Giac [A]

time = 0.00, size = 50, normalized size = 1.14

$$\frac{1}{3}x^3 \ln(a^2 + x^2) - 2 \left(\frac{3x^3 - 9xa^2}{27} + \frac{2a^4 \arctan\left(\frac{x}{a}\right)}{2 \cdot 3a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*log(a^2+x^2),x)`

[Out] $-2/3*a^3*\arctan(x/a) + 1/3*x^3*\log(a^2 + x^2) + 2/3*a^2*x - 2/9*x^3$

Mupad [B]

time = 0.04, size = 65, normalized size = 1.48

$$\frac{2a^2x}{3} - \frac{\ln(x - \sqrt{-a^2}) (-a^2)^{3/2}}{3} + \frac{x^3 \ln(a^2 + x^2)}{3} + \frac{\ln(x + \sqrt{-a^2}) (-a^2)^{3/2}}{3} - \frac{2x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2*log(a^2 + x^2),x)`

```
[Out] (2*a^2*x)/3 - (log(x - (-a^2)^(1/2))*(-a^2)^(3/2))/3 + (x^3*log(a^2 + x^2))
/3 + (log(x + (-a^2)^(1/2))*(-a^2)^(3/2))/3 - (2*x^3)/9
```

3.73 $\int x^4 \log(a^2 + x^2) dx$

Optimal. Leaf size=54

$$-\frac{2a^4x}{5} + \frac{2a^2x^3}{15} - \frac{2x^5}{25} + \frac{2}{5}a^5 \tan^{-1}\left(\frac{x}{a}\right) + \frac{1}{5}x^5 \log(a^2 + x^2)$$

[Out] $-2/5*a^4*x+2/15*a^2*x^3-2/25*x^5+2/5*a^5*\arctan(x/a)+1/5*x^5*\ln(a^2+x^2)$

Rubi [A]

time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2505, 308, 209}

$$\frac{2}{5}a^5 \tan^{-1}\left(\frac{x}{a}\right) - \frac{2a^4x}{5} + \frac{2a^2x^3}{15} + \frac{1}{5}x^5 \log(a^2 + x^2) - \frac{2x^5}{25}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^4*\text{Log}[a^2 + x^2], x]$

[Out] $(-2*a^4*x)/5 + (2*a^2*x^3)/15 - (2*x^5)/25 + (2*a^5*\text{ArcTan}[x/a])/5 + (x^5*\text{Log}[a^2 + x^2])/5$

Rule 209

$\text{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))*\text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 308

$\text{Int}[(x_.)^{(m_.)}/((a_.) + (b_.)*(x_.)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{PolynomialDivide}[x^m, a + b*x^n, x], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, 2*n - 1]$

Rule 2505

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.)^{(n_.)})^{(p_.)}]]*(b_.)*((f_.)*(x_.)^{(m_.)}), x_Symbol] \rightarrow \text{Simp}[(f*x)^{m+1}*((a + b*\text{Log}[c*(d + e*x^n)^p])/(f*(m+1))), x] - \text{Dist}[b*e*n*(p/(f*(m+1))), \text{Int}[x^{n-1}*((f*x)^{m+1}/(d + e*x^n)), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p\}, x \ \&\& \ \text{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned}
\int x^4 \log(a^2 + x^2) dx &= \frac{1}{5} x^5 \log(a^2 + x^2) - \frac{2}{5} \int \frac{x^6}{a^2 + x^2} dx \\
&= \frac{1}{5} x^5 \log(a^2 + x^2) - \frac{2}{5} \int \left(a^4 - a^2 x^2 + x^4 - \frac{a^6}{a^2 + x^2} \right) dx \\
&= -\frac{2a^4 x}{5} + \frac{2a^2 x^3}{15} - \frac{2x^5}{25} + \frac{1}{5} x^5 \log(a^2 + x^2) + \frac{1}{5} (2a^6) \int \frac{1}{a^2 + x^2} dx \\
&= -\frac{2a^4 x}{5} + \frac{2a^2 x^3}{15} - \frac{2x^5}{25} + \frac{2}{5} a^5 \tan^{-1} \left(\frac{x}{a} \right) + \frac{1}{5} x^5 \log(a^2 + x^2)
\end{aligned}$$

Mathematica [A]

time = 0.00, size = 54, normalized size = 1.00

$$-\frac{2a^4 x}{5} + \frac{2a^2 x^3}{15} - \frac{2x^5}{25} + \frac{2}{5} a^5 \tan^{-1} \left(\frac{x}{a} \right) + \frac{1}{5} x^5 \log(a^2 + x^2)$$

Antiderivative was successfully verified.

`[In] Integrate[x^4*Log[a^2 + x^2], x]``[Out] (-2*a^4*x)/5 + (2*a^2*x^3)/15 - (2*x^5)/25 + (2*a^5*ArcTan[x/a])/5 + (x^5*Log[a^2 + x^2])/5`**Mathics [C]** Result contains complex when optimal does not.

time = 2.28, size = 53, normalized size = 0.98

$$\frac{I}{5} a^5 (\text{Log}[Ia + x] - \text{Log}[-Ia + x]) - \frac{2a^4 x}{5} + \frac{2a^2 x^3}{15} - \frac{2x^5}{25} + \frac{x^5 \text{Log}[a^2 + x^2]}{5}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^4*Log[x^2+a^2], x]')``[Out] I / 5 a ^ 5 (Log[I a + x] - Log[-I a + x]) - 2 a ^ 4 x / 5 + 2 a ^ 2 x ^ 3 / 15 - 2 x ^ 5 / 25 + x ^ 5 Log[a ^ 2 + x ^ 2] / 5`**Maple [A]**

time = 0.02, size = 45, normalized size = 0.83

method	result	size
default	$-\frac{2a^4 x}{5} + \frac{2a^2 x^3}{15} - \frac{2x^5}{25} + \frac{2a^5 \arctan\left(\frac{x}{a}\right)}{5} + \frac{x^5 \ln(a^2 + x^2)}{5}$	45
risch	$-\frac{2a^4 x}{5} + \frac{2a^2 x^3}{15} - \frac{2x^5}{25} + \frac{2a^5 \arctan\left(\frac{x}{a}\right)}{5} + \frac{x^5 \ln(a^2 + x^2)}{5}$	45

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*ln(a^2+x^2),x,method=_RETURNVERBOSE)`

[Out] $-2/5*a^4*x+2/15*a^2*x^3-2/25*x^5+2/5*a^5*\arctan(x/a)+1/5*x^5*\ln(a^2+x^2)$

Maxima [A]

time = 0.35, size = 44, normalized size = 0.81

$$\frac{2}{5}a^5 \arctan\left(\frac{x}{a}\right) + \frac{1}{5}x^5 \log(a^2 + x^2) - \frac{2}{5}a^4x + \frac{2}{15}a^2x^3 - \frac{2}{25}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*log(a^2+x^2),x, algorithm="maxima")`

[Out] $2/5*a^5*\arctan(x/a) + 1/5*x^5*\log(a^2 + x^2) - 2/5*a^4*x + 2/15*a^2*x^3 - 2/25*x^5$

Fricas [A]

time = 0.33, size = 44, normalized size = 0.81

$$\frac{2}{5}a^5 \arctan\left(\frac{x}{a}\right) + \frac{1}{5}x^5 \log(a^2 + x^2) - \frac{2}{5}a^4x + \frac{2}{15}a^2x^3 - \frac{2}{25}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*log(a^2+x^2),x, algorithm="fricas")`

[Out] $2/5*a^5*\arctan(x/a) + 1/5*x^5*\log(a^2 + x^2) - 2/5*a^4*x + 2/15*a^2*x^3 - 2/25*x^5$

Sympy [C] Result contains complex when optimal does not.

time = 0.10, size = 63, normalized size = 1.17

$$-2a^5 \left(\frac{i \log(-ia + x)}{10} - \frac{i \log(ia + x)}{10} \right) - \frac{2a^4x}{5} + \frac{2a^2x^3}{15} + \frac{x^5 \log(a^2 + x^2)}{5} - \frac{2x^5}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*ln(a**2+x**2),x)`

[Out] $-2*a**5*(I*log(-I*a + x)/10 - I*log(I*a + x)/10) - 2*a**4*x/5 + 2*a**2*x**3/15 + x**5*log(a**2 + x**2)/5 - 2*x**5/25$

Giac [A]

time = 0.00, size = 61, normalized size = 1.13

$$\frac{1}{5}x^5 \ln(a^2 + x^2) - 2 \left(\frac{125x^5 - \frac{625}{3}x^3a^2 + 625xa^4}{3125} - \frac{2a^6 \arctan\left(\frac{x}{a}\right)}{2 \cdot 5a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*log(a^2+x^2),x)

[Out] $\frac{2}{5}a^5\arctan(x/a) + \frac{1}{5}x^5\log(a^2 + x^2) - \frac{2}{5}a^4x + \frac{2}{15}a^2x^3 - \frac{2}{25}x^5$

Mupad [B]

time = 0.13, size = 73, normalized size = 1.35

$$\frac{x^5 \ln(a^2 + x^2)}{5} - \frac{2a^4x}{5} - \frac{\ln(x - \sqrt{-a^2}) (-a^2)^{5/2}}{5} + \frac{\ln(x + \sqrt{-a^2}) (-a^2)^{5/2}}{5} - \frac{2x^5}{25} + \frac{2a^2x^3}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*log(a^2 + x^2),x)

[Out] $\frac{(x^5\log(a^2 + x^2))}{5} - \frac{(2a^4x)}{5} - \frac{(\log(x - (-a^2)^{(1/2)}))*(-a^2)^{(5/2)})}{5} + \frac{(\log(x + (-a^2)^{(1/2)}))*(-a^2)^{(5/2)})}{5} - \frac{(2x^5)}{25} + \frac{(2a^2x^3)}{15}$

3.74 $\int \log(-a^2 + x^2) dx$

Optimal. Leaf size=25

$$-2x + 2a \tanh^{-1}\left(\frac{x}{a}\right) + x \log(-a^2 + x^2)$$

[Out] $-2*x+2*a*\operatorname{arctanh}(x/a)+x*\ln(-a^2+x^2)$

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2498, 327, 213}

$$x \log(x^2 - a^2) + 2a \tanh^{-1}\left(\frac{x}{a}\right) - 2x$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Log}[-a^2 + x^2], x]$

[Out] $-2*x + 2*a*\operatorname{ArcTanh}[x/a] + x*\operatorname{Log}[-a^2 + x^2]$

Rule 213

$\operatorname{Int}[(a + b \cdot x^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a, 2] \cdot \operatorname{Rt}[b, 2])^{-1}] \cdot \operatorname{ArcTanh}[\operatorname{Rt}[b, 2] \cdot (x/\operatorname{Rt}[-a, 2])], x] /;$ $\operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{LtQ}[a, 0] \ || \ \operatorname{GtQ}[b, 0])$

Rule 327

$\operatorname{Int}[(c \cdot x)^m \cdot (a + b \cdot x^n)^p, x_Symbol] \rightarrow \operatorname{Simp}[c^{n-1} \cdot (c \cdot x)^{m-n+1} \cdot (a + b \cdot x^n)^{p+1} / (b \cdot (m + n \cdot p + 1))], x] - \operatorname{Dist}[a \cdot c^n \cdot (m - n + 1) / (b \cdot (m + n \cdot p + 1)), \operatorname{Int}[(c \cdot x)^{m-n} \cdot (a + b \cdot x^n)^p, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, p, x\} \ \&\& \ \operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{GtQ}[m, n - 1] \ \&\& \ \operatorname{NeQ}[m + n \cdot p + 1, 0] \ \&\& \ \operatorname{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 2498

$\operatorname{Int}[\operatorname{Log}[(c \cdot x)^d + (e \cdot x^n)^p], x_Symbol] \rightarrow \operatorname{Simp}[x \cdot \operatorname{Log}[c \cdot (d + e \cdot x^n)^p], x] - \operatorname{Dist}[e \cdot n \cdot p, \operatorname{Int}[x^n / (d + e \cdot x^n), x], x] /;$ $\operatorname{FreeQ}\{c, d, e, n, p, x\}$

Rubi steps

$$\begin{aligned}
\int \log(-a^2 + x^2) dx &= x \log(-a^2 + x^2) - 2 \int \frac{x^2}{-a^2 + x^2} dx \\
&= -2x + x \log(-a^2 + x^2) - (2a^2) \int \frac{1}{-a^2 + x^2} dx \\
&= -2x + 2a \tanh^{-1}\left(\frac{x}{a}\right) + x \log(-a^2 + x^2)
\end{aligned}$$

Mathematica [A]

time = 0.00, size = 25, normalized size = 1.00

$$-2x + 2a \tanh^{-1}\left(\frac{x}{a}\right) + x \log(-a^2 + x^2)$$

Antiderivative was successfully verified.

`[In] Integrate[Log[-a^2 + x^2],x]``[Out] -2*x + 2*a*ArcTanh[x/a] + x*Log[-a^2 + x^2]`**Mathics [A]**

time = 2.05, size = 32, normalized size = 1.28

$$-a (\text{Log}[-a + x] - \text{Log}[a + x]) - 2x + x \text{Log}[-a^2 + x^2]$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Log[x^2-a^2],x]')``[Out] -a (Log[-a + x] - Log[a + x]) - 2 x + x Log[-a ^ 2 + x ^ 2]`**Maple [A]**

time = 0.01, size = 32, normalized size = 1.28

method	result	size
default	$x \ln(-a^2 + x^2) - 2x + a \ln(a + x) - a \ln(a - x)$	32
risch	$x \ln(-a^2 + x^2) - 2x + a \ln(a + x) - a \ln(-a + x)$	32

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(ln(-a^2+x^2),x,method=_RETURNVERBOSE)``[Out] x*ln(-a^2+x^2)-2*x+a*ln(a+x)-a*ln(a-x)`**Maxima [A]**

time = 0.28, size = 31, normalized size = 1.24

$$x \log(-a^2 + x^2) + a \log(a + x) - a \log(-a + x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(-a^2+x^2),x, algorithm="maxima")

[Out] x*log(-a^2 + x^2) + a*log(a + x) - a*log(-a + x) - 2*x

Fricas [A]

time = 0.40, size = 31, normalized size = 1.24

$$x \log(-a^2 + x^2) + a \log(a + x) - a \log(-a + x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(-a^2+x^2),x, algorithm="fricas")

[Out] x*log(-a^2 + x^2) + a*log(a + x) - a*log(-a + x) - 2*x

Sympy [A]

time = 0.08, size = 29, normalized size = 1.16

$$-2a \left(\frac{\log(-a + x)}{2} - \frac{\log(a + x)}{2} \right) + x \log(-a^2 + x^2) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(-a**2+x**2),x)

[Out] -2*a*(log(-a + x)/2 - log(a + x)/2) + x*log(-a**2 + x**2) - 2*x

Giac [A]

time = 0.00, size = 37, normalized size = 1.48

$$x \ln(-a^2 + x^2) + 2 \left(-\frac{1}{2}a \ln|x - a| + \frac{1}{2}a \ln|x + a| - x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(-a^2+x^2),x)

[Out] x*log(-a^2 + x^2) + a*log(abs(a + x)) - a*log(abs(-a + x)) - 2*x

Mupad [B]

time = 0.07, size = 25, normalized size = 1.00

$$x \ln(x^2 - a^2) - 2x + 2a \operatorname{atanh}\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(x^2 - a^2),x)

[Out] x*log(x^2 - a^2) - 2*x + 2*a*atanh(x/a)

3.75 $\int \log(\log(\log(\log(x)))) dx$

Optimal. Leaf size=8

Int(log(log(log(log(x))))), x)

[Out] CannotIntegrate(ln(ln(ln(ln(x))))), x)

Rubi [A]

time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \log(\log(\log(\log(x)))) dx$$

Verification is not applicable to the result.

[In] Int[Log[Log[Log[Log[x]]]], x]

[Out] Defer[Int][Log[Log[Log[Log[x]]]], x]

Rubi steps

$$\int \log(\log(\log(\log(x)))) dx = \int \log(\log(\log(\log(x)))) dx$$

Mathematica [A]

time = 0.03, size = 0, normalized size = 0.00

$$\int \log(\log(\log(\log(x)))) dx$$

Verification is not applicable to the result.

[In] Integrate[Log[Log[Log[Log[x]]]], x]

[Out] Integrate[Log[Log[Log[Log[x]]]], x]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Log[Log[Log[Log[x]]]], x]')

[Out] cought exception: maximum recursion depth exceeded in comparison

Maple [A]

time = 0.02, size = 0, normalized size = 0.00

$$\int \ln(\ln(\ln(\ln(x)))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(ln(ln(ln(x))))),x)

[Out] int(ln(ln(ln(ln(x))))),x)

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(log(log(log(x))))),x, algorithm="maxima")

[Out] x*log(log(log(log(x)))) - integrate(1/(log(x)*log(log(x))*log(log(log(x))))), x)

Fricas [A]

time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(log(log(log(x))))),x, algorithm="fricas")

[Out] integral(log(log(log(log(x))))), x)

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$x \log(\log(\log(\log(x)))) - \int \frac{1}{\log(x) \log(\log(x)) \log(\log(\log(x)))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(ln(ln(ln(x))))),x)

[Out] x*log(log(log(log(x)))) - Integral(1/(log(x)*log(log(x))*log(log(log(x))))), x)

Giac [A] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(log(log(log(x))))),x)

[Out] Could not integrate

Mupad [A]

time = 0.00, size = -1, normalized size = -0.12

$$\int \ln(\ln(\ln(\ln(x)))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(log(log(log(x))))),x)

[Out] int(log(log(log(log(x))))), x)

3.76 $\int \sin(x) dx$

Optimal. Leaf size=4

$$-\cos(x)$$

[Out] $-\cos(x)$

Rubi [A]

time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2718}

$$-\cos(x)$$

Antiderivative was successfully verified.

[In] `Int[Sin[x],x]`

[Out] `-Cos[x]`

Rule 2718

`Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\int \sin(x) dx = -\cos(x)$$

Mathematica [A]

time = 0.00, size = 4, normalized size = 1.00

$$-\cos(x)$$

Antiderivative was successfully verified.

[In] `Integrate[Sin[x],x]`

[Out] `-Cos[x]`

Mathics [A]

time = 1.67, size = 4, normalized size = 1.00

$$-\text{Cos}[x]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[Sin[x],x]')`

[Out] `-Cos[x]`

Maple [A]

time = 0.01, size = 5, normalized size = 1.25

method	result	size
lookup	$-\cos(x)$	5
default	$-\cos(x)$	5
risch	$-\cos(x)$	5
norman	$-\frac{2}{1+\tan^2(\frac{x}{2})}$	13
meijerg	$\sqrt{\pi} \left(\frac{1}{\sqrt{\pi}} - \frac{\cos(x)}{\sqrt{\pi}} \right)$	16

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x),x,method=_RETURNVERBOSE)`

[Out] `-cos(x)`

Maxima [A]

time = 0.26, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x),x, algorithm="maxima")`

[Out] `-cos(x)`

Fricas [A]

time = 0.34, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x),x, algorithm="fricas")`

[Out] `-cos(x)`

Sympy [A]

time = 0.03, size = 3, normalized size = 0.75

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x)
```

```
[Out] -cos(x)
```

Giac [A]

time = 0.00, size = 3, normalized size = 0.75

$$-\cos x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x)
```

```
[Out] -cos(x)
```

Mupad [B]

time = 0.02, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x),x)
```

```
[Out] -cos(x)
```

3.77 $\int \cos(x) dx$

Optimal. Leaf size=2

$$\sin(x)$$

[Out] $\sin(x)$

Rubi [A]

time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2717}

$$\sin(x)$$

Antiderivative was successfully verified.

[In] `Int[Cos[x],x]`

[Out] `Sin[x]`

Rule 2717

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rubi steps

$$\int \cos(x) dx = \sin(x)$$

Mathematica [A]

time = 0.00, size = 2, normalized size = 1.00

$$\sin(x)$$

Antiderivative was successfully verified.

[In] `Integrate[Cos[x],x]`

[Out] `Sin[x]`

Mathics [A]

time = 1.52, size = 2, normalized size = 1.00

$$\text{Sin}[x]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Cos[x],x]')
```

```
[Out] Sin[x]
```

Maple [A]

time = 0.01, size = 3, normalized size = 1.50

method	result	size
lookup	$\sin(x)$	3
default	$\sin(x)$	3
meijerg	$\sin(x)$	3
risch	$\sin(x)$	3
norman	$\frac{2 \tan(\frac{x}{2})}{1 + \tan^2(\frac{x}{2})}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] sin(x)
```

Maxima [A]

time = 0.27, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="maxima")
```

```
[Out] sin(x)
```

Fricas [A]

time = 0.33, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="fricas")
```

```
[Out] sin(x)
```

Sympy [A]

time = 0.03, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x)
```

```
[Out] sin(x)
```

Giac [A]

time = 0.00, size = 2, normalized size = 1.00

$$\sin x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x)
```

```
[Out] sin(x)
```

Mupad [B]

time = 0.03, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x),x)
```

```
[Out] sin(x)
```

3.78 $\int \tan(x) dx$

Optimal. Leaf size=5

$$-\log(\cos(x))$$

[Out] $-\ln(\cos(x))$

Rubi [A]

time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556}

$$-\log(\cos(x))$$

Antiderivative was successfully verified.

[In] `Int[Tan[x],x]`

[Out] `-Log[Cos[x]]`

Rule 3556

`Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\int \tan(x) dx = -\log(\cos(x))$$

Mathematica [A]

time = 0.00, size = 5, normalized size = 1.00

$$-\log(\cos(x))$$

Antiderivative was successfully verified.

[In] `Integrate[Tan[x],x]`

[Out] `-Log[Cos[x]]`

Mathics [A]

time = 1.64, size = 5, normalized size = 1.00

$$-\text{Log}[\text{Cos}[x]]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[Tan[x],x]')`

[Out] `-Log[Cos[x]]`

Maple [A]

time = 0.01, size = 6, normalized size = 1.20

method	result	size
lookup	$-\ln(\cos(x))$	6
default	$-\ln(\cos(x))$	6
derivativedivides	$\frac{\ln(1+\tan^2(x))}{2}$	10
norman	$\frac{\ln(1+\tan^2(x))}{2}$	10
risch	$ix - \ln(e^{2ix} + 1)$	16

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(x),x,method=_RETURNVERBOSE)`

[Out] `-ln(cos(x))`

Maxima [A]

time = 0.28, size = 3, normalized size = 0.60

$$\log(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x),x, algorithm="maxima")`

[Out] `log(sec(x))`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.
time = 0.34, size = 11, normalized size = 2.20

$$-\frac{1}{2} \log\left(\frac{1}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x),x, algorithm="fricas")`

[Out] `-1/2*log(1/(tan(x)^2 + 1))`

Sympy [A]

time = 0.03, size = 5, normalized size = 1.00

$$-\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x)
```

```
[Out] -log(cos(x))
```

Giac [A]

time = 0.00, size = 5, normalized size = 1.00

$$-\ln |\cos x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x)
```

```
[Out] -log(abs(cos(x)))
```

Mupad [B]

time = 0.03, size = 5, normalized size = 1.00

$$-\ln (\cos (x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x),x)
```

```
[Out] -log(cos(x))
```

3.79 $\int \cot(x) dx$

Optimal. Leaf size=3

$$\log(\sin(x))$$

[Out] $\ln(\sin(x))$

Rubi [A]

time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556}

$$\log(\sin(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[x], x]$

[Out] $\text{Log}[\text{Sin}[x]]$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\int \cot(x) dx = \log(\sin(x))$$

Mathematica [A]

time = 0.00, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Cot}[x], x]$

[Out] $\text{Log}[\text{Sin}[x]]$

Mathics [A]

time = 1.58, size = 3, normalized size = 1.00

$$\text{Log}[\text{Sin}[x]]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/Tan[x],x]')
```

```
[Out] Log[Sin[x]]
```

Maple [A]

time = 0.02, size = 4, normalized size = 1.33

method	result	size
lookup	$\ln(\sin(x))$	4
default	$\ln(\sin(x))$	4
derivativedivides	$-\frac{\ln(1+\tan^2(x))}{2} + \ln(\tan(x))$	14
norman	$-\frac{\ln(1+\tan^2(x))}{2} + \ln(\tan(x))$	14
risch	$-ix + \ln(e^{2ix} - 1)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(sin(x))
```

Maxima [A]

time = 0.26, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x),x, algorithm="maxima")
```

```
[Out] log(sin(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(3) = 6$.
time = 0.33, size = 16, normalized size = 5.33

$$\frac{1}{2} \log\left(\frac{\tan(x)^2}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(tan(x)^2/(tan(x)^2 + 1))
```

Sympy [A]

time = 0.03, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/tan(x),x)`

[Out] `log(sin(x))`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(3) = 6$.
time = 0.00, size = 20, normalized size = 6.67

$$\frac{\ln(\tan^2 x)}{2} - \frac{\ln(\tan^2 x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/tan(x),x)`

[Out] `-1/2*log(tan(x)^2 + 1) + 1/2*log(tan(x)^2)`

Mupad [B]

time = 0.20, size = 13, normalized size = 4.33

$$\ln(\tan(x)) - \frac{\ln(\tan(x)^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/tan(x),x)`

[Out] `log(tan(x)) - log(tan(x)^2 + 1)/2`

$$3.80 \quad \int \frac{1}{(1+\tan(x))^2} dx$$

Optimal. Leaf size=21

$$\frac{1}{2} \log(\cos(x) + \sin(x)) - \frac{1}{2(1 + \tan(x))}$$

[Out] 1/2*ln(cos(x)+sin(x))-1/2/(1+tan(x))

Rubi [A]

time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3564, 3611}

$$\frac{1}{2} \log(\sin(x) + \cos(x)) - \frac{1}{2(\tan(x) + 1)}$$

Antiderivative was successfully verified.

[In] Int[(1 + Tan[x])^(-2),x]

[Out] Log[Cos[x] + Sin[x]]/2 - 1/(2*(1 + Tan[x]))

Rule 3564

Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[b*((a + b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]

Rule 3611

Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/((a_) + (b_)*tan[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1 + \tan(x))^2} dx &= -\frac{1}{2(1 + \tan(x))} + \frac{1}{2} \int \frac{1 - \tan(x)}{1 + \tan(x)} dx \\ &= \frac{1}{2} \log(\cos(x) + \sin(x)) - \frac{1}{2(1 + \tan(x))} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 27, normalized size = 1.29

$$\frac{\log(\cos(x) + \sin(x)) + \tan(x) + \log(\cos(x) + \sin(x)) \tan(x)}{2 + 2 \tan(x)}$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + Tan[x])^(-2), x]``[Out] (Log[Cos[x] + Sin[x]] + Tan[x] + Log[Cos[x] + Sin[x]]*Tan[x])/(2 + 2*Tan[x])`**Mathics [A]**

time = 2.63, size = 37, normalized size = 1.76

$$\frac{-2 - \text{Log}\left[\frac{1}{\text{Cos}[x]^2}\right] (1 + \text{Tan}[x]) + 2\text{Log}[1 + \text{Tan}[x]] + 2\text{Log}[1 + \text{Tan}[x]] \text{Tan}[x]}{4 + 4\text{Tan}[x]}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(1+Tan[x])^2, x]')``[Out] (-2 - Log[1 / Cos[x]^2] (1 + Tan[x]) + 2 Log[1 + Tan[x]] + 2 Log[1 + Tan[x]] Tan[x]) / (4 (1 + Tan[x]))`**Maple [A]**

time = 0.03, size = 26, normalized size = 1.24

method	result	size
derivativedivides	$-\frac{1}{2(\tan(x)+1)} + \frac{\ln(\tan(x)+1)}{2} - \frac{\ln(1+\tan^2(x))}{4}$	26
default	$-\frac{1}{2(\tan(x)+1)} + \frac{\ln(\tan(x)+1)}{2} - \frac{\ln(1+\tan^2(x))}{4}$	26
norman	$-\frac{1}{2(\tan(x)+1)} + \frac{\ln(\tan(x)+1)}{2} - \frac{\ln(1+\tan^2(x))}{4}$	26
risch	$-\frac{ix}{2} - \frac{1}{2(e^{2ix}+i)} + \frac{\ln(e^{2ix}+i)}{2}$	29

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(tan(x)+1)^2, x, method=_RETURNVERBOSE)``[Out] -1/2/(tan(x)+1)+1/2*ln(tan(x)+1)-1/4*ln(1+tan(x)^2)`**Maxima [A]**

time = 0.35, size = 25, normalized size = 1.19

$$-\frac{1}{2(\tan(x) + 1)} - \frac{1}{4} \log(\tan(x)^2 + 1) + \frac{1}{2} \log(\tan(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+tan(x))^2,x, algorithm="maxima")

[Out] -1/2/(tan(x) + 1) - 1/4*log(tan(x)^2 + 1) + 1/2*log(tan(x) + 1)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

time = 0.34, size = 37, normalized size = 1.76

$$\frac{(\tan(x) + 1) \log\left(\frac{\tan(x)^2 + 2 \tan(x) + 1}{\tan(x)^2 + 1}\right) + \tan(x) - 1}{4(\tan(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+tan(x))^2,x, algorithm="fricas")

[Out] 1/4*((tan(x) + 1)*log((tan(x)^2 + 2*tan(x) + 1)/(tan(x)^2 + 1)) + tan(x) - 1)/(tan(x) + 1)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(17) = 34

time = 0.17, size = 75, normalized size = 3.57

$$\frac{2 \log(\tan(x) + 1) \tan(x)}{4 \tan(x) + 4} + \frac{2 \log(\tan(x) + 1)}{4 \tan(x) + 4} - \frac{\log(\tan^2(x) + 1) \tan(x)}{4 \tan(x) + 4} - \frac{\log(\tan^2(x) + 1)}{4 \tan(x) + 4} - \frac{2}{4 \tan(x) + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+tan(x))**2,x)

[Out] 2*log(tan(x) + 1)*tan(x)/(4*tan(x) + 4) + 2*log(tan(x) + 1)/(4*tan(x) + 4) - log(tan(x)**2 + 1)*tan(x)/(4*tan(x) + 4) - log(tan(x)**2 + 1)/(4*tan(x) + 4) - 2/(4*tan(x) + 4)

Giac [A]

time = 0.00, size = 30, normalized size = 1.43

$$-\frac{\ln(\tan^2 x + 1)}{4} + \frac{\ln|\tan x + 1|}{2} - \frac{1}{2(\tan x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+tan(x))^2,x)

[Out] -1/2/(tan(x) + 1) - 1/4*log(tan(x)^2 + 1) + 1/2*log(abs(tan(x) + 1))

Mupad [B]

time = 0.25, size = 27, normalized size = 1.29

$$\frac{\ln(\tan(x) + 1)}{2} - \frac{\ln(\tan(x)^2 + 1)}{4} - \frac{1}{2(\tan(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(tan(x) + 1)^2,x)`

[Out] `log(tan(x) + 1)/2 - log(tan(x)^2 + 1)/4 - 1/(2*(tan(x) + 1))`

3.81 $\int \sec(x) dx$

Optimal. Leaf size=3

$$\tanh^{-1}(\sin(x))$$

[Out] arctanh(sin(x))

Rubi [A]

time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3855}

$$\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Sec[x],x]

[Out] ArcTanh[Sin[x]]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \sec(x) dx = \tanh^{-1}(\sin(x))$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 33 vs. $2(3) = 6$. time = 0.00, size = 33, normalized size = 11.00

$$-\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \log\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x],x]

[Out] -Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 19 vs. $2(3) = 6$. time = 1.83, size = 15, normalized size = 5.00

$$-\frac{\text{Log}[-1 + \text{Sin}[x]]}{2} + \frac{\text{Log}[1 + \text{Sin}[x]]}{2}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/Cos[x],x]')`

[Out] `-Log[-1 + Sin[x]] / 2 + Log[1 + Sin[x]] / 2`

Maple [A]

time = 0.02, size = 7, normalized size = 2.33

method	result	size
default	$\ln(\sec(x) + \tan(x))$	7
norman	$-\ln(\tan(\frac{x}{2}) - 1) + \ln(1 + \tan(\frac{x}{2}))$	18
risch	$\ln(e^{ix} + i) - \ln(e^{ix} - i)$	22

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/cos(x),x,method=_RETURNVERBOSE)`

[Out] `ln(sec(x)+tan(x))`

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.

time = 0.28, size = 15, normalized size = 5.00

$$\frac{1}{2} \log(\sin(x) + 1) - \frac{1}{2} \log(\sin(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/cos(x),x, algorithm="maxima")`

[Out] `1/2*log(sin(x) + 1) - 1/2*log(sin(x) - 1)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(3) = 6$.
time = 0.34, size = 17, normalized size = 5.67

$$\frac{1}{2} \log(\sin(x) + 1) - \frac{1}{2} \log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/cos(x),x, algorithm="fricas")`

[Out] `1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)`

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$
time = 0.05, size = 15, normalized size = 5.00

$$-\frac{\log(\sin(x) - 1)}{2} + \frac{\log(\sin(x) + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/cos(x),x)`

[Out] `-log(sin(x) - 1)/2 + log(sin(x) + 1)/2`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(3) = 6.
time = 0.00, size = 30, normalized size = 10.00

$$2 \left(-\frac{\ln \left| \sin x + \frac{1}{\sin x} - 2 \right|}{8} + \frac{\ln \left| \sin x + \frac{1}{\sin x} + 2 \right|}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/cos(x),x)`

[Out] `1/4*log(abs(1/sin(x) + sin(x) + 2)) - 1/4*log(abs(1/sin(x) + sin(x) - 2))`

Mupad [B]

time = 0.13, size = 11, normalized size = 3.67

$$\ln \left(\frac{1}{\cos(x)} \right) + \ln(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/cos(x),x)`

[Out] `log(1/cos(x)) + log(sin(x) + 1)`

3.82 $\int \csc(x) dx$

Optimal. Leaf size=5

$$-\tanh^{-1}(\cos(x))$$

[Out] `-arctanh(cos(x))`

Rubi [A]

time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3855}

$$-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] `Int[Csc[x],x]`

[Out] `-ArcTanh[Cos[x]]`

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\int \csc(x) dx = -\tanh^{-1}(\cos(x))$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 17 vs. $2(5) = 10$. time = 0.00, size = 17, normalized size = 3.40

$$-\log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] `Integrate[Csc[x],x]`

[Out] `-Log[Cos[x/2]] + Log[Sin[x/2]]`

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 19 vs. $2(5) = 10$. time = 1.83, size = 15, normalized size = 3.00

$$-\frac{\text{Log}[1 + \text{Cos}[x]]}{2} + \frac{\text{Log}[-1 + \text{Cos}[x]]}{2}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/Sin[x],x]')`

[Out] `-Log[1 + Cos[x]] / 2 + Log[-1 + Cos[x]] / 2`

Maple [A]

time = 0.02, size = 9, normalized size = 1.80

method	result	size
norman	$\ln\left(\tan\left(\frac{x}{2}\right)\right)$	6
default	$\ln(\csc(x) - \cot(x))$	9
risch	$-\ln(1 + e^{ix}) + \ln(e^{ix} - 1)$	20

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/sin(x),x,method=_RETURNVERBOSE)`

[Out] `ln(csc(x)-cot(x))`

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

time = 0.27, size = 15, normalized size = 3.00

$$-\frac{1}{2} \log(\cos(x) + 1) + \frac{1}{2} \log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sin(x),x, algorithm="maxima")`

[Out] `-1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(5) = 10$.

time = 0.34, size = 19, normalized size = 3.80

$$-\frac{1}{2} \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + \frac{1}{2} \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sin(x),x, algorithm="fricas")`

[Out] `-1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)`

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$

time = 0.05, size = 15, normalized size = 3.00

$$\frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sin(x),x)`

[Out] `log(cos(x) - 1)/2 - log(cos(x) + 1)/2`

Giac [A]

time = 0.00, size = 11, normalized size = 2.20

$$\frac{2}{2} \ln \left| \tan \left(\frac{x}{2} \right) \right|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sin(x),x)`

[Out] `log(abs(tan(1/2*x)))`

Mupad [B]

time = 0.04, size = 5, normalized size = 1.00

$$\ln \left(\tan \left(\frac{x}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/sin(x),x)`

[Out] `log(tan(x/2))`

3.83 $\int \sin^2(x) dx$

Optimal. Leaf size=14

$$\frac{x}{2} - \frac{1}{2} \cos(x) \sin(x)$$

[Out] 1/2*x-1/2*cos(x)*sin(x)

Rubi [A]

time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2715, 8}

$$\frac{x}{2} - \frac{1}{2} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^2,x]

[Out] x/2 - (Cos[x]*Sin[x])/2

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*SIN[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \sin^2(x) dx &= -\frac{1}{2} \cos(x) \sin(x) + \frac{\int 1 dx}{2} \\ &= \frac{x}{2} - \frac{1}{2} \cos(x) \sin(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2} - \frac{1}{4} \sin(2x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^2,x]

[Out] x/2 - Sin[2*x]/4

Mathics [A]

time = 1.75, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin(2x)}{4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Sin[x]^2,x]')

[Out] x / 2 - Sin[2 x] / 4

Maple [A]

time = 0.00, size = 11, normalized size = 0.79

method	result	size
default	$\frac{x}{2} - \frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2} - \frac{\sin(2x)}{4}$	11
meijerg	$\frac{\sqrt{\pi} \left(\frac{2x}{\sqrt{\pi}} - \frac{\sin(2x)}{\sqrt{\pi}} \right)}{4}$	22
norman	$\frac{\tan^3\left(\frac{x}{2}\right) + x\left(\tan^2\left(\frac{x}{2}\right) + \frac{x}{2} + \frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2} - \tan\left(\frac{x}{2}\right)\right)}{\left(1 + \tan^2\left(\frac{x}{2}\right)\right)^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*x-1/2*cos(x)*sin(x)

Maxima [A]

time = 0.30, size = 10, normalized size = 0.71

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2,x, algorithm="maxima")

[Out] 1/2*x - 1/4*sin(2*x)

Fricas [A]

time = 0.34, size = 10, normalized size = 0.71

$$-\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2,x, algorithm="fricas")

[Out] -1/2*cos(x)*sin(x) + 1/2*x

Sympy [A]

time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**2,x)

[Out] x/2 - sin(x)*cos(x)/2

Giac [A]

time = 0.00, size = 13, normalized size = 0.93

$$\frac{x}{2} - \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2,x)

[Out] 1/2*x - 1/4*sin(2*x)

Mupad [B]

time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^2,x)

[Out] x/2 - sin(2*x)/4

3.84 $\int x^3 \sin(x^2) dx$

Optimal. Leaf size=20

$$-\frac{1}{2}x^2 \cos(x^2) + \frac{\sin(x^2)}{2}$$

[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)

Rubi [A]

time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3460, 3377, 2717}

$$\frac{\sin(x^2)}{2} - \frac{1}{2}x^2 \cos(x^2)$$

Antiderivative was successfully verified.

[In] Int[x^3*Sin[x^2],x]

[Out] -1/2*(x^2*Cos[x^2]) + Sin[x^2]/2

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 3377

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-c + d*x)^m*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3460

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned}
 \int x^3 \sin(x^2) dx &= \frac{1}{2} \text{Subst} \left(\int x \sin(x) dx, x, x^2 \right) \\
 &= -\frac{1}{2} x^2 \cos(x^2) + \frac{1}{2} \text{Subst} \left(\int \cos(x) dx, x, x^2 \right) \\
 &= -\frac{1}{2} x^2 \cos(x^2) + \frac{\sin(x^2)}{2}
 \end{aligned}$$

Mathematica [A]

time = 0.00, size = 20, normalized size = 1.00

$$-\frac{1}{2} x^2 \cos(x^2) + \frac{\sin(x^2)}{2}$$

Antiderivative was successfully verified.

`[In] Integrate[x^3*Sin[x^2],x]``[Out] -1/2*(x^2*Cos[x^2]) + Sin[x^2]/2`**Mathics [A]**

time = 1.85, size = 16, normalized size = 0.80

$$-\frac{x^2 \text{Cos}[x^2]}{2} + \frac{\text{Sin}[x^2]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^3*Sin[x^2],x]')``[Out] -x ^ 2 Cos[x ^ 2] / 2 + Sin[x ^ 2] / 2`**Maple [A]**

time = 0.02, size = 17, normalized size = 0.85

method	result	size
derivativedivides	$-\frac{x^2 \cos(x^2)}{2} + \frac{\sin(x^2)}{2}$	17
default	$-\frac{x^2 \cos(x^2)}{2} + \frac{\sin(x^2)}{2}$	17
risch	$-\frac{x^2 \cos(x^2)}{2} + \frac{\sin(x^2)}{2}$	17
meijerg	$\sqrt{\pi} \left(-\frac{x^2 \cos(x^2)}{2\sqrt{\pi}} + \frac{\sin(x^2)}{2\sqrt{\pi}} \right)$	27

norman	$\frac{-\frac{x^2}{2} + \frac{x^2 \left(\tan^2 \left(\frac{x^2}{2} \right) \right)}{2} + \tan \left(\frac{x^2}{2} \right)}{1 + \tan^2 \left(\frac{x^2}{2} \right)}$	39
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*sin(x^2),x,method=_RETURNVERBOSE)`

[Out] `-1/2*x^2*cos(x^2)+1/2*sin(x^2)`

Maxima [A]

time = 0.26, size = 16, normalized size = 0.80

$$-\frac{1}{2}x^2 \cos(x^2) + \frac{1}{2} \sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*sin(x^2),x, algorithm="maxima")`

[Out] `-1/2*x^2*cos(x^2) + 1/2*sin(x^2)`

Fricas [A]

time = 0.33, size = 16, normalized size = 0.80

$$-\frac{1}{2}x^2 \cos(x^2) + \frac{1}{2} \sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*sin(x^2),x, algorithm="fricas")`

[Out] `-1/2*x^2*cos(x^2) + 1/2*sin(x^2)`

Sympy [A]

time = 0.15, size = 15, normalized size = 0.75

$$-\frac{x^2 \cos(x^2)}{2} + \frac{\sin(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*sin(x**2),x)`

[Out] `-x**2*cos(x**2)/2 + sin(x**2)/2`

Giac [A]

time = 0.00, size = 17, normalized size = 0.85

$$\frac{-x^2 \cos(x^2) + \sin(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x)
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Mupad [B]

time = 0.16, size = 16, normalized size = 0.80

$$\frac{\sin(x^2)}{2} - \frac{x^2 \cos(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x)
```

```
[Out] sin(x^2)/2 - (x^2*cos(x^2))/2
```

3.85 $\int \sin^3(x) dx$

Optimal. Leaf size=13

$$-\cos(x) + \frac{\cos^3(x)}{3}$$

[Out] `-cos(x)+1/3*cos(x)^3`

Rubi [A]

time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2713}

$$\frac{\cos^3(x)}{3} - \cos(x)$$

Antiderivative was successfully verified.

[In] `Int[Sin[x]^3,x]`

[Out] `-Cos[x] + Cos[x]^3/3`

Rule 2713

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rubi steps

$$\begin{aligned} \int \sin^3(x) dx &= -\text{Subst} \left(\int (1 - x^2) dx, x, \cos(x) \right) \\ &= -\cos(x) + \frac{\cos^3(x)}{3} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 15, normalized size = 1.15

$$-\frac{3 \cos(x)}{4} + \frac{1}{12} \cos(3x)$$

Antiderivative was successfully verified.

[In] `Integrate[Sin[x]^3,x]`

[Out] `(-3*Cos[x])/4 + Cos[3*x]/12`

Mathics [A]

time = 1.70, size = 10, normalized size = 0.77

$$\frac{\cos [x] (-5 + \cos [2x])}{6}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sin[x]^3,x]')``[Out] Cos[x] (-5 + Cos[2 x]) / 6`**Maple** [A]

time = 0.00, size = 11, normalized size = 0.85

method	result	size
default	$-\frac{(2+\sin^2(x))\cos(x)}{3}$	11
risch	$-\frac{3\cos(x)}{4} + \frac{\cos(3x)}{12}$	12
norman	$\frac{-4(\tan^2(\frac{x}{2})) - \frac{4}{3}}{(1+\tan^2(\frac{x}{2}))^3}$	22

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sin(x)^3,x,method=_RETURNVERBOSE)``[Out] -1/3*(2+sin(x)^2)*cos(x)`**Maxima** [A]

time = 0.25, size = 11, normalized size = 0.85

$$\frac{1}{3} \cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(x)^3,x, algorithm="maxima")``[Out] 1/3*cos(x)^3 - cos(x)`**Fricas** [A]

time = 0.33, size = 11, normalized size = 0.85

$$\frac{1}{3} \cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(x)^3,x, algorithm="fricas")``[Out] 1/3*cos(x)^3 - cos(x)`

Sympy [A]

time = 0.03, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**3,x)

[Out] cos(x)**3/3 - cos(x)

Giac [A]

time = 0.00, size = 11, normalized size = 0.85

$$-\cos x + \frac{\cos^3 x}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3,x)

[Out] 1/3*cos(x)^3 - cos(x)

Mupad [B]

time = 0.03, size = 10, normalized size = 0.77

$$\frac{\cos(x) (\cos(x)^2 - 3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^3,x)

[Out] (cos(x)*(cos(x)^2 - 3))/3

3.86 $\int \sin^p(x) dx$

Optimal. Leaf size=44

$$\frac{\cos(x) {}_2F_1\left(\frac{1}{2}, \frac{1+p}{2}; \frac{3+p}{2}; \sin^2(x)\right) \sin^{1+p}(x)}{(1+p)\sqrt{\cos^2(x)}}$$

[Out] $\cos(x) \cdot \text{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{2} + \frac{1}{2}p\right], \left[\frac{3}{2} + \frac{1}{2}p\right], \sin(x)^2\right) \cdot \sin(x)^{(1+p)} / (1+p) / (\cos(x)^2)^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2722}

$$\frac{\cos(x) \sin^{p+1}(x) {}_2F_1\left(\frac{1}{2}, \frac{p+1}{2}; \frac{p+3}{2}; \sin^2(x)\right)}{(p+1)\sqrt{\cos^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^p,x]

[Out] $(\text{Cos}[x] \cdot \text{Hypergeometric2F1}\left[\frac{1}{2}, (1+p)/2, (3+p)/2, \text{Sin}[x]^2\right] \cdot \text{Sin}[x]^{(1+p)}) / ((1+p) \cdot \text{Sqrt}[\text{Cos}[x]^2])$

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2, x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int \sin^p(x) dx = \frac{\cos(x) {}_2F_1\left(\frac{1}{2}, \frac{1+p}{2}; \frac{3+p}{2}; \sin^2(x)\right) \sin^{1+p}(x)}{(1+p)\sqrt{\cos^2(x)}}$$

Mathematica [A]

time = 0.03, size = 44, normalized size = 1.00

$$-\cos(x) {}_2F_1\left(\frac{1}{2}, \frac{1-p}{2}; \frac{3}{2}; \cos^2(x)\right) \sin^{1+p}(x) \sin^2(x)^{\frac{1}{2}(-1-p)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^p,x]

[Out] $-(\text{Cos}[x] \cdot \text{Hypergeometric2F1}[1/2, (1 - p)/2, 3/2, \text{Cos}[x]^2] \cdot \text{Sin}[x]^{(1 + p)} \cdot (\text{Sin}[x]^2)^{((-1 - p)/2)})$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sin[x]^p,x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [F]

time = 0.02, size = 0, normalized size = 0.00

$$\int \sin^p(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^p,x)

[Out] int(sin(x)^p,x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^p,x, algorithm="maxima")

[Out] integrate(sin(x)^p, x)

Fricas [F]

time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^p,x, algorithm="fricas")

[Out] integral(sin(x)^p, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^p(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**p,x)

[Out] Integral(sin(x)**p, x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^p,x)

[Out] Could not integrate

Mupad [B]

time = 0.32, size = 35, normalized size = 0.80

$$\frac{\cos(x) \sin(x)^{p+1} {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - \frac{p}{2}; \frac{3}{2}; \cos(x)^2\right)}{(\sin(x)^2)^{\frac{p}{2} + \frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^p,x)

[Out] -(cos(x)*sin(x)^(p + 1)*hypergeom([1/2, 1/2 - p/2], 3/2, cos(x)^2))/(sin(x)^2)^(p/2 + 1/2)

3.87 $\int \cos(x) (1 + \sin^2(x))^2 dx$

Optimal. Leaf size=19

$$\sin(x) + \frac{2 \sin^3(x)}{3} + \frac{\sin^5(x)}{5}$$

[Out] $\sin(x) + 2/3 * \sin(x)^3 + 1/5 * \sin(x)^5$

Rubi [A]

time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3269, 200}

$$\frac{\sin^5(x)}{5} + \frac{2 \sin^3(x)}{3} + \sin(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[x] * (1 + \text{Sin}[x]^2)^2, x]$

[Out] $\text{Sin}[x] + (2 * \text{Sin}[x]^3) / 3 + \text{Sin}[x]^5 / 5$

Rule 200

$\text{Int}[(a + (b \cdot x)^n)^p, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b \cdot x^n)^p, x], x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$

Rule 3269

$\text{Int}[\cos[(e \cdot x) + (f \cdot x)^m] \cdot ((a \cdot x) + (b \cdot x) \cdot \sin[(e \cdot x) + (f \cdot x)^m])^{p-1}, x_Symbol] \rightarrow \text{With}\{\text{ff} = \text{FreeFactors}[\text{Sin}[e + f \cdot x], x]\}, \text{Dist}[\text{ff}/f, \text{Subst}[\text{Int}[(1 - \text{ff}^2 \cdot x^2)^{(m-1)/2} \cdot (a + b \cdot \text{ff}^2 \cdot x^2)^p, x], x, \text{Sin}[e + f \cdot x] / \text{ff}], x] /; \text{FreeQ}\{a, b, e, f, p\}, x\} \ \&\& \ \text{IntegerQ}[(m-1)/2]$

Rubi steps

$$\begin{aligned} \int \cos(x) (1 + \sin^2(x))^2 dx &= \text{Subst} \left(\int (1 + x^2)^2 dx, x, \sin(x) \right) \\ &= \text{Subst} \left(\int (1 + 2x^2 + x^4) dx, x, \sin(x) \right) \\ &= \sin(x) + \frac{2 \sin^3(x)}{3} + \frac{\sin^5(x)}{5} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 19, normalized size = 1.00

$$\sin(x) + \frac{2 \sin^3(x)}{3} + \frac{\sin^5(x)}{5}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*(1 + Sin[x]^2)^2,x]

[Out] Sin[x] + (2*Sin[x]^3)/3 + Sin[x]^5/5

Mathics [A]

time = 2.02, size = 15, normalized size = 0.79

$$\sin [x] + \frac{\sin [x]^5}{5} + \frac{2\sin [x]^3}{3}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(Sin[x]^2+1)^2*Cos[x],x]')

[Out] Sin[x] + Sin[x] ^ 5 / 5 + 2 Sin[x] ^ 3 / 3

Maple [A]

time = 0.03, size = 16, normalized size = 0.84

method	result	size
derivativedivides	$\sin (x) + \frac{2(\sin ^3(x))}{3} + \frac{(\sin ^5(x))}{5}$	16
default	$\sin (x) + \frac{2(\sin ^3(x))}{3} + \frac{(\sin ^5(x))}{5}$	16
risch	$\frac{13 \sin (x)}{8} + \frac{\sin (5x)}{80} - \frac{11 \sin (3x)}{48}$	18

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*(1+sin(x)^2)^2,x,method=_RETURNVERBOSE)

[Out] sin(x)+2/3*sin(x)^3+1/5*sin(x)^5

Maxima [A]

time = 0.26, size = 15, normalized size = 0.79

$$\frac{1}{5} \sin (x)^5 + \frac{2}{3} \sin (x)^3 + \sin (x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(1+sin(x)^2)^2,x, algorithm="maxima")

[Out] 1/5*sin(x)^5 + 2/3*sin(x)^3 + sin(x)

Fricas [A]

time = 0.34, size = 18, normalized size = 0.95

$$\frac{1}{15} (3 \cos (x)^4 - 16 \cos (x)^2 + 28) \sin (x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(1+sin(x)^2)^2,x, algorithm="fricas")`

[Out] `1/15*(3*cos(x)^4 - 16*cos(x)^2 + 28)*sin(x)`

Sympy [A]

time = 0.23, size = 17, normalized size = 0.89

$$\frac{\sin^5(x)}{5} + \frac{2\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(1+sin(x)**2)**2,x)`

[Out] `sin(x)**5/5 + 2*sin(x)**3/3 + sin(x)`

Giac [A]

time = 0.00, size = 18, normalized size = 0.95

$$\frac{\sin^5 x}{5} + \frac{2}{3} \sin^3 x + \sin x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(1+sin(x)^2)^2,x)`

[Out] `1/5*sin(x)^5 + 2/3*sin(x)^3 + sin(x)`

Mupad [B]

time = 0.04, size = 15, normalized size = 0.79

$$\frac{\sin(x)^5}{5} + \frac{2\sin(x)^3}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*(sin(x)^2 + 1)^2,x)`

[Out] `sin(x) + (2*sin(x)^3)/3 + sin(x)^5/5`

3.88 $\int \cos^2(x) dx$

Optimal. Leaf size=14

$$\frac{x}{2} + \frac{1}{2} \cos(x) \sin(x)$$

[Out] 1/2*x+1/2*cos(x)*sin(x)

Rubi [A]

time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2715, 8}

$$\frac{x}{2} + \frac{1}{2} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2,x]

[Out] x/2 + (Cos[x]*Sin[x])/2

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*SIn[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIn[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos^2(x) dx &= \frac{1}{2} \cos(x) \sin(x) + \frac{\int 1 dx}{2} \\ &= \frac{x}{2} + \frac{1}{2} \cos(x) \sin(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2} + \frac{1}{4} \sin(2x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^2,x]

[Out] x/2 + Sin[2*x]/4

Mathics [A]

time = 1.75, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin[2x]}{4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Cos[x]^2,x]')

[Out] x / 2 + Sin[2 x] / 4

Maple [A]

time = 0.00, size = 11, normalized size = 0.79

method	result	size
default	$\frac{x}{2} + \frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2} + \frac{\sin(2x)}{4}$	11
norman	$\frac{x(\tan^2(\frac{x}{2})) + \frac{x}{2} - (\tan^3(\frac{x}{2})) + \frac{x(\tan^4(\frac{x}{2}))}{2} + \tan(\frac{x}{2})}{(1+\tan^2(\frac{x}{2}))^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*x+1/2*cos(x)*sin(x)

Maxima [A]

time = 0.25, size = 10, normalized size = 0.71

$$\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2,x, algorithm="maxima")

[Out] 1/2*x + 1/4*sin(2*x)

Fricas [A]

time = 0.33, size = 10, normalized size = 0.71

$$\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2,x, algorithm="fricas")

[Out] 1/2*cos(x)*sin(x) + 1/2*x

Sympy [A]

time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(x) \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**2,x)

[Out] x/2 + sin(x)*cos(x)/2

Giac [A]

time = 0.00, size = 12, normalized size = 0.86

$$\frac{x}{2} + \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2,x)

[Out] 1/2*x + 1/4*sin(2*x)

Mupad [B]

time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2,x)

[Out] x/2 + sin(2*x)/4

3.89 $\int \cos^3(x) dx$

Optimal. Leaf size=11

$$\sin(x) - \frac{\sin^3(x)}{3}$$

[Out] `sin(x)-1/3*sin(x)^3`

Rubi [A]

time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2713}

$$\sin(x) - \frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

[In] `Int[Cos[x]^3,x]`

[Out] `Sin[x] - Sin[x]^3/3`

Rule 2713

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rubi steps

$$\begin{aligned} \int \cos^3(x) dx &= -\text{Subst} \left(\int (1 - x^2) dx, x, -\sin(x) \right) \\ &= \sin(x) - \frac{\sin^3(x)}{3} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 15, normalized size = 1.36

$$\frac{3 \sin(x)}{4} + \frac{1}{12} \sin(3x)$$

Antiderivative was successfully verified.

[In] `Integrate[Cos[x]^3,x]`

[Out] `(3*Sin[x])/4 + Sin[3*x]/12`

Mathics [A]

time = 1.75, size = 9, normalized size = 0.82

$$\sin[x] - \frac{\sin[x]^3}{3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Cos[x]^3,x]')``[Out] Sin[x] - Sin[x]^3 / 3`**Maple [A]**

time = 0.01, size = 11, normalized size = 1.00

method	result	size
default	$\frac{(2+\cos^2(x)) \sin(x)}{3}$	11
risch	$\frac{3 \sin(x)}{4} + \frac{\sin(3x)}{12}$	12

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x)^3,x,method=_RETURNVERBOSE)``[Out] 1/3*(2+cos(x)^2)*sin(x)`**Maxima [A]**

time = 0.25, size = 9, normalized size = 0.82

$$-\frac{1}{3} \sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)^3,x, algorithm="maxima")``[Out] -1/3*sin(x)^3 + sin(x)`**Fricas [A]**

time = 0.33, size = 10, normalized size = 0.91

$$\frac{1}{3} (\cos(x)^2 + 2) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)^3,x, algorithm="fricas")``[Out] 1/3*(cos(x)^2 + 2)*sin(x)`

Sympy [A]

time = 0.03, size = 8, normalized size = 0.73

$$-\frac{\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**3,x)

[Out] -sin(x)**3/3 + sin(x)

Giac [A]

time = 0.00, size = 11, normalized size = 1.00

$$\sin x - \frac{\sin^3 x}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3,x)

[Out] -1/3*sin(x)^3 + sin(x)

Mupad [B]

time = 0.03, size = 9, normalized size = 0.82

$$\sin(x) - \frac{\sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^3,x)

[Out] sin(x) - sin(x)^3/3

3.90 $\int \sec^2(x) dx$

Optimal. Leaf size=2

$\tan(x)$

[Out] $\tan(x)$

Rubi [A]

time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3852, 8}

$\tan(x)$

Antiderivative was successfully verified.

[In] `Int[Sec[x]^2,x]`

[Out] `Tan[x]`

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 3852

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rubi steps

$$\int \sec^2(x) dx = -\text{Subst}\left(\int 1 dx, x, -\tan(x)\right) \\ = \tan(x)$$

Mathematica [A]

time = 0.00, size = 2, normalized size = 1.00

$\tan(x)$

Antiderivative was successfully verified.

[In] `Integrate[Sec[x]^2,x]`

[Out] Tan[x]

Mathics [A]

time = 1.77, size = 2, normalized size = 1.00

Tan[x]

Antiderivative was successfully verified.

[In] mathics('Integrate[1/Cos[x]^2,x]')

[Out] Tan[x]

Maple [A]

time = 0.04, size = 3, normalized size = 1.50

method	result	size
default	$\tan(x)$	3
risch	$\frac{2i}{e^{2ix}+1}$	13
norman	$-\frac{2 \tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1}$	17

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(x)^2,x,method=_RETURNVERBOSE)

[Out] tan(x)

Maxima [A]

time = 0.27, size = 2, normalized size = 1.00

tan(x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(x)^2,x, algorithm="maxima")

[Out] tan(x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7 vs. $2(2) = 4$.
time = 0.32, size = 7, normalized size = 3.50

$\frac{\sin(x)}{\cos(x)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(x)^2,x, algorithm="fricas")

[Out] sin(x)/cos(x)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 5 vs. $2(2) = 4$
 time = 0.03, size = 5, normalized size = 2.50

$$\frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/cos(x)**2,x)`

[Out] `sin(x)/cos(x)`

Giac [A]

time = 0.00, size = 6, normalized size = 3.00

$$\frac{2}{2} \tan x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/cos(x)^2,x)`

[Out] `tan(x)`

Mupad [B]

time = 0.02, size = 2, normalized size = 1.00

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/cos(x)^2,x)`

[Out] `tan(x)`

3.91 $\int \sin(x) \sin(2x) dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2} - \frac{1}{6} \sin(3x)$$

[Out] 1/2*sin(x)-1/6*sin(3*x)

Rubi [A]

time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4367}

$$\frac{\sin(x)}{2} - \frac{1}{6} \sin(3x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]*Sin[2*x],x]

[Out] Sin[x]/2 - Sin[3*x]/6

Rule 4367

Int[sin[(a_.) + (b_.)*(x_.)]*sin[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\int \sin(x) \sin(2x) dx = \frac{\sin(x)}{2} - \frac{1}{6} \sin(3x)$$

Mathematica [A]

time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sin(x)}{2} - \frac{1}{6} \sin(3x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]*Sin[2*x],x]

[Out] Sin[x]/2 - Sin[3*x]/6

Mathics [A]

time = 1.95, size = 6, normalized size = 0.40

$$\frac{2\text{Sin}[x]^3}{3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sin[x]*Sin[2*x],x]')``[Out] 2 Sin[x] ^ 3 / 3`**Maple [A]**

time = 0.03, size = 7, normalized size = 0.47

method	result	size
derivativdivides	$\frac{2(\sin^3(x))}{3}$	7
default	$\frac{2(\sin^3(x))}{3}$	7
risch	$\frac{\sin(x)}{2} - \frac{\sin(3x)}{6}$	12
norman	$\frac{-\frac{2 \tan(x) (\tan^2(\frac{x}{2}))}{3} + \frac{4 (\tan^2(x) \tan(\frac{x}{2}))}{3} + \frac{2 \tan(x)}{3} - \frac{4 \tan(\frac{x}{2})}{3}}{(1 + \tan^2(\frac{x}{2}))(1 + \tan^2(x))}$	51

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sin(x)*sin(2*x),x,method=_RETURNVERBOSE)``[Out] 2/3*sin(x)^3`**Maxima [A]**

time = 0.26, size = 11, normalized size = 0.73

$$-\frac{1}{6} \sin(3x) + \frac{1}{2} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(x)*sin(2*x),x, algorithm="maxima")``[Out] -1/6*sin(3*x) + 1/2*sin(x)`**Fricas [A]**

time = 0.35, size = 10, normalized size = 0.67

$$-\frac{2}{3} (\cos(x)^2 - 1) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(2*x),x, algorithm="fricas")

[Out] $-2/3*(\cos(x)^2 - 1)*\sin(x)$

Sympy [A]

time = 0.14, size = 20, normalized size = 1.33

$$-\frac{2 \sin(x) \cos(2x)}{3} + \frac{\sin(2x) \cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(2*x),x)

[Out] $-2*\sin(x)*\cos(2*x)/3 + \sin(2*x)*\cos(x)/3$

Giac [A]

time = 0.00, size = 8, normalized size = 0.53

$$\frac{2}{3} \sin^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(2*x),x)

[Out] $2/3*\sin(x)^3$

Mupad [B]

time = 0.03, size = 6, normalized size = 0.40

$$\frac{2 \sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)*sin(x),x)

[Out] $(2*\sin(x)^3)/3$

3.92 $\int x \sin(x) dx$

Optimal. Leaf size=8

$$-x \cos(x) + \sin(x)$$

[Out] `-x*cos(x)+sin(x)`

Rubi [A]

time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3377, 2717}

$$\sin(x) - x \cos(x)$$

Antiderivative was successfully verified.

[In] `Int[x*Sin[x],x]`

[Out] `-(x*Cos[x]) + Sin[x]`

Rule 2717

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 3377

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-`
`(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co`
`s[e + f*x], x], x] /;` `FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rubi steps

$$\begin{aligned} \int x \sin(x) dx &= -x \cos(x) + \int \cos(x) dx \\ &= -x \cos(x) + \sin(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 8, normalized size = 1.00

$$-x \cos(x) + \sin(x)$$

Antiderivative was successfully verified.

[In] `Integrate[x*Sin[x],x]`

[Out] $-(x*\text{Cos}[x]) + \text{Sin}[x]$

Mathics [A]

time = 1.82, size = 8, normalized size = 1.00

$$-x\text{Cos}[x] + \text{Sin}[x]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x*Sin[x],x]')`

[Out] $-x \text{Cos}[x] + \text{Sin}[x]$

Maple [A]

time = 0.01, size = 9, normalized size = 1.12

method	result	size
default	$-x \cos(x) + \sin(x)$	9
risch	$-x \cos(x) + \sin(x)$	9
meijerg	$2\sqrt{\pi} \left(-\frac{x \cos(x)}{2\sqrt{\pi}} + \frac{\sin(x)}{2\sqrt{\pi}} \right)$	22
norman	$\frac{x(\tan^2(\frac{x}{2})-x+2\tan(\frac{x}{2}))}{1+\tan^2(\frac{x}{2})}$	30

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*sin(x),x,method=_RETURNVERBOSE)`

[Out] $-x*\cos(x)+\sin(x)$

Maxima [A]

time = 0.26, size = 8, normalized size = 1.00

$$-x \cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x),x, algorithm="maxima")`

[Out] $-x*\cos(x) + \sin(x)$

Fricas [A]

time = 0.33, size = 8, normalized size = 1.00

$$-x \cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x),x, algorithm="fricas")`

[Out] $-x*\cos(x) + \sin(x)$

Sympy [A]

time = 0.07, size = 7, normalized size = 0.88

$$-x \cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x),x)`

[Out] $-x*\cos(x) + \sin(x)$

Giac [A]

time = 0.00, size = 8, normalized size = 1.00

$$-x \cos x + \sin x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x),x)`

[Out] $-x*\cos(x) + \sin(x)$

Mupad [B]

time = 0.02, size = 8, normalized size = 1.00

$$\sin(x) - x \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*sin(x),x)`

[Out] $\sin(x) - x*\cos(x)$

3.93 $\int x^2 \sin(x) dx$

Optimal. Leaf size=17

$$2 \cos(x) - x^2 \cos(x) + 2x \sin(x)$$

[Out] 2*cos(x)-x^2*cos(x)+2*x*sin(x)

Rubi [A]

time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,

Rules used = {3377, 2718}

$$x^2(-\cos(x)) + 2x \sin(x) + 2 \cos(x)$$

Antiderivative was successfully verified.

[In] Int[x^2*Sin[x],x]

[Out] 2*Cos[x] - x^2*Cos[x] + 2*x*Sin[x]

Rule 2718

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3377

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rubi steps

$$\begin{aligned} \int x^2 \sin(x) dx &= -x^2 \cos(x) + 2 \int x \cos(x) dx \\ &= -x^2 \cos(x) + 2x \sin(x) - 2 \int \sin(x) dx \\ &= 2 \cos(x) - x^2 \cos(x) + 2x \sin(x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 15, normalized size = 0.88

$$-((-2 + x^2) \cos(x)) + 2x \sin(x)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sin[x],x]

[Out] -((-2 + x^2)*Cos[x]) + 2*x*Sin[x]

Mathics [A]

time = 1.85, size = 17, normalized size = 1.00

$$2x\sin[x] - x^2\cos[x] + 2\cos[x]$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^2*Sin[x],x]')

[Out] 2 x Sin[x] - x ^ 2 Cos[x] + 2 Cos[x]

Maple [A]

time = 0.00, size = 18, normalized size = 1.06

method	result	size
risch	$(-x^2 + 2)\cos(x) + 2x\sin(x)$	17
default	$2\cos(x) - x^2\cos(x) + 2x\sin(x)$	18
meijerg	$4\sqrt{\pi} \left(-\frac{1}{2\sqrt{\pi}} + \frac{(-\frac{x^2}{2}+1)\cos(x)}{2\sqrt{\pi}} + \frac{x\sin(x)}{2\sqrt{\pi}} \right)$	34
norman	$\frac{x^2(\tan^2(\frac{x}{2})) - x^2 + 4x\tan(\frac{x}{2}) + 4}{1 + \tan^2(\frac{x}{2})}$	36

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*sin(x),x,method=_RETURNVERBOSE)

[Out] 2*cos(x)-x^2*cos(x)+2*x*sin(x)

Maxima [A]

time = 0.25, size = 15, normalized size = 0.88

$$-(x^2 - 2)\cos(x) + 2x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sin(x),x, algorithm="maxima")

[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)

Fricas [A]

time = 0.33, size = 15, normalized size = 0.88

$$-(x^2 - 2)\cos(x) + 2x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sin(x),x, algorithm="fricas")

[Out] $-(x^2 - 2)*\cos(x) + 2*x*\sin(x)$

Sympy [A]

time = 0.10, size = 17, normalized size = 1.00

$$-x^2 \cos(x) + 2x \sin(x) + 2 \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*sin(x),x)

[Out] $-x**2*\cos(x) + 2*x*\sin(x) + 2*\cos(x)$

Giac [A]

time = 0.00, size = 15, normalized size = 0.88

$$(-x^2 + 2) \cos x + 2x \sin x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sin(x),x)

[Out] $-(x^2 - 2)*\cos(x) + 2*x*\sin(x)$

Mupad [B]

time = 0.03, size = 15, normalized size = 0.88

$$2x \sin(x) - \cos(x) (x^2 - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*sin(x),x)

[Out] $2*x*\sin(x) - \cos(x)*(x^2 - 2)$

3.94 $\int x \sin^2(x) dx$

Optimal. Leaf size=25

$$\frac{x^2}{4} - \frac{1}{2}x \cos(x) \sin(x) + \frac{\sin^2(x)}{4}$$

[Out] 1/4*x^2-1/2*x*cos(x)*sin(x)+1/4*sin(x)^2

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3391, 30}

$$\frac{x^2}{4} + \frac{\sin^2(x)}{4} - \frac{1}{2}x \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[x*Sin[x]^2,x]

[Out] x^2/4 - (x*Cos[x]*Sin[x])/2 + Sin[x]^2/4

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3391

Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c + d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int x \sin^2(x) dx &= -\frac{1}{2}x \cos(x) \sin(x) + \frac{\sin^2(x)}{4} + \frac{\int x dx}{2} \\ &= \frac{x^2}{4} - \frac{1}{2}x \cos(x) \sin(x) + \frac{\sin^2(x)}{4} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^2}{4} - \frac{1}{8} \cos(2x) - \frac{1}{4}x \sin(2x)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sin[x]^2,x]

[Out] $x^2/4 - \text{Cos}[2*x]/8 - (x*\text{Sin}[2*x])/4$

Mathics [A]

time = 2.04, size = 20, normalized size = 0.80

$$-\frac{1}{8} - \frac{x \text{Sin}[2x]}{4} + \frac{x^2}{4} - \frac{\text{Cos}[2x]}{8}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x*Sin[x]^2,x]')

[Out] $-1 / 8 - x \text{Sin}[2 x] / 4 + x^2 / 4 - \text{Cos}[2 x] / 8$

Maple [A]

time = 0.00, size = 25, normalized size = 1.00

method	result	size
risch	$\frac{x^2}{4} - \frac{\cos(2x)}{8} - \frac{x \sin(2x)}{4}$	20
default	$x \left(\frac{x}{2} - \frac{\cos(x) \sin(x)}{2} \right) - \frac{x^2}{4} + \frac{(\sin^2(x))}{4}$	25
meijerg	$\frac{\sqrt{\pi} \left(\frac{2x^2+1}{2\sqrt{\pi}} - \frac{\cos(2x)}{2\sqrt{\pi}} - \frac{x \sin(2x)}{\sqrt{\pi}} \right)}{4}$	38
norman	$\frac{x(\tan^3(\frac{x}{2})) + \tan^2(\frac{x}{2}) + \frac{x^2}{4} - x \tan(\frac{x}{2}) + \frac{x^2(\tan^2(\frac{x}{2}))}{2} + \frac{x^2(\tan^4(\frac{x}{2}))}{4}}{(1+\tan^2(\frac{x}{2}))^2}$	61

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(x)^2,x,method=_RETURNVERBOSE)

[Out] $x*(1/2*x-1/2*\cos(x)*\sin(x))-1/4*x^2+1/4*\sin(x)^2$

Maxima [A]

time = 0.27, size = 19, normalized size = 0.76

$$\frac{1}{4} x^2 - \frac{1}{4} x \sin(2x) - \frac{1}{8} \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)^2,x, algorithm="maxima")

[Out] $1/4*x^2 - 1/4*x*\sin(2*x) - 1/8*\cos(2*x)$

Fricas [A]

time = 0.32, size = 19, normalized size = 0.76

$$-\frac{1}{2}x \cos(x) \sin(x) + \frac{1}{4}x^2 - \frac{1}{4}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*sin(x)^2,x, algorithm="fricas")``[Out] -1/2*x*cos(x)*sin(x) + 1/4*x^2 - 1/4*cos(x)^2`**Sympy [A]**

time = 0.11, size = 36, normalized size = 1.44

$$\frac{x^2 \sin^2(x)}{4} + \frac{x^2 \cos^2(x)}{4} - \frac{x \sin(x) \cos(x)}{2} + \frac{\sin^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*sin(x)**2,x)``[Out] x**2*sin(x)**2/4 + x**2*cos(x)**2/4 - x*sin(x)*cos(x)/2 + sin(x)**2/4`**Giac [A]**

time = 0.00, size = 28, normalized size = 1.12

$$-\frac{\cos(2x)}{8} - \frac{2}{8}x \sin(2x) + \frac{1}{2} \cdot \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*sin(x)^2,x)``[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)`**Mupad [B]**

time = 0.06, size = 19, normalized size = 0.76

$$\frac{\sin(x)^2}{4} - \frac{x \sin(2x)}{4} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*sin(x)^2,x)``[Out] sin(x)^2/4 - (x*sin(2*x))/4 + x^2/4`

3.95 $\int x^2 \sin^2(x) dx$

Optimal. Leaf size=41

$$-\frac{x}{4} + \frac{x^3}{6} + \frac{1}{4} \cos(x) \sin(x) - \frac{1}{2} x^2 \cos(x) \sin(x) + \frac{1}{2} x \sin^2(x)$$

[Out] $-1/4*x+1/6*x^3+1/4*\cos(x)*\sin(x)-1/2*x^2*\cos(x)*\sin(x)+1/2*x*\sin(x)^2$

Rubi [A]

time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3392, 30, 2715, 8}

$$\frac{x^3}{6} - \frac{1}{2} x^2 \sin(x) \cos(x) - \frac{x}{4} + \frac{1}{2} x \sin^2(x) + \frac{1}{4} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] `Int[x^2*Sin[x]^2,x]`

[Out] $-1/4*x + x^3/6 + (\text{Cos}[x]*\text{Sin}[x])/4 - (x^2*\text{Cos}[x]*\text{Sin}[x])/2 + (x*\text{Sin}[x]^2)/2$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

Rule 2715

`Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 3392

`Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]`

Rubi steps

$$\begin{aligned}
\int x^2 \sin^2(x) dx &= -\frac{1}{2}x^2 \cos(x) \sin(x) + \frac{1}{2}x \sin^2(x) + \frac{\int x^2 dx}{2} - \frac{1}{2} \int \sin^2(x) dx \\
&= \frac{x^3}{6} + \frac{1}{4} \cos(x) \sin(x) - \frac{1}{2}x^2 \cos(x) \sin(x) + \frac{1}{2}x \sin^2(x) - \frac{\int 1 dx}{4} \\
&= -\frac{x}{4} + \frac{x^3}{6} + \frac{1}{4} \cos(x) \sin(x) - \frac{1}{2}x^2 \cos(x) \sin(x) + \frac{1}{2}x \sin^2(x)
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 29, normalized size = 0.71

$$\frac{1}{24} (4x^3 - 6x \cos(2x) + (3 - 6x^2) \sin(2x))$$

Antiderivative was successfully verified.

`[In] Integrate[x^2*Sin[x]^2,x]``[Out] (4*x^3 - 6*x*Cos[2*x] + (3 - 6*x^2)*Sin[2*x])/24`**Mathics [A]**

time = 2.21, size = 28, normalized size = 0.68

$$-\frac{x \cos[2x]}{4} - \frac{x^2 \sin[2x]}{4} + \frac{x^3}{6} + \frac{\sin[2x]}{8}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^2*Sin[x]^2,x]')``[Out] -x Cos[2 x] / 4 - x ^ 2 Sin[2 x] / 4 + x ^ 3 / 6 + Sin[2 x] / 8`**Maple [A]**

time = 0.02, size = 37, normalized size = 0.90

method	result	size
meijerg	$\frac{x^5 \operatorname{hypergeom}\left(\left[1, \frac{5}{2}\right], \left[\frac{3}{2}, 2, \frac{7}{2}\right], -x^2\right)}{5}$	19
risch	$\frac{x^3}{6} - \frac{x \cos(2x)}{4} - \frac{(2x^2-1) \sin(2x)}{8}$	27
default	$x^2 \left(\frac{x}{2} - \frac{\cos(x) \sin(x)}{2} \right) - \frac{x(\cos^2(x))}{2} + \frac{\cos(x) \sin(x)}{4} + \frac{x}{4} - \frac{x^3}{3}$	37
norman	$\frac{x^2 \left(\tan^3\left(\frac{x}{2}\right) - \frac{x}{4} + \frac{x^3}{6} - \frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{2} + \frac{3x \left(\tan^2\left(\frac{x}{2}\right)\right)}{2} - \frac{x \left(\tan^4\left(\frac{x}{2}\right)\right)}{4} - x^2 \tan\left(\frac{x}{2}\right) + \frac{x^3 \left(\tan^2\left(\frac{x}{2}\right)\right)}{3} + \frac{x^3 \left(\tan^4\left(\frac{x}{2}\right)\right)}{6} + \frac{\tan\left(\frac{x}{2}\right)}{2} \right)}{\left(1 + \tan^2\left(\frac{x}{2}\right)\right)^2}$	94

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*sin(x)^2,x,method=_RETURNVERBOSE)`

[Out] $x^2*(1/2*x-1/2*\cos(x)*\sin(x))-1/2*x*\cos(x)^2+1/4*\cos(x)*\sin(x)+1/4*x-1/3*x^3$

Maxima [A]

time = 0.26, size = 26, normalized size = 0.63

$$\frac{1}{6}x^3 - \frac{1}{4}x \cos(2x) - \frac{1}{8}(2x^2 - 1) \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*sin(x)^2,x, algorithm="maxima")`

[Out] $1/6*x^3 - 1/4*x*\cos(2*x) - 1/8*(2*x^2 - 1)*\sin(2*x)$

Fricas [A]

time = 0.32, size = 29, normalized size = 0.71

$$\frac{1}{6}x^3 - \frac{1}{2}x \cos(x)^2 - \frac{1}{4}(2x^2 - 1) \cos(x) \sin(x) + \frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*sin(x)^2,x, algorithm="fricas")`

[Out] $1/6*x^3 - 1/2*x*\cos(x)^2 - 1/4*(2*x^2 - 1)*\cos(x)*\sin(x) + 1/4*x$

Sympy [A]

time = 0.15, size = 56, normalized size = 1.37

$$\frac{x^3 \sin^2(x)}{6} + \frac{x^3 \cos^2(x)}{6} - \frac{x^2 \sin(x) \cos(x)}{2} + \frac{x \sin^2(x)}{4} - \frac{x \cos^2(x)}{4} + \frac{\sin(x) \cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*sin(x)**2,x)`

[Out] $x**3*\sin(x)**2/6 + x**3*\cos(x)**2/6 - x**2*\sin(x)*\cos(x)/2 + x*\sin(x)**2/4 - x*\cos(x)**2/4 + \sin(x)*\cos(x)/4$

Giac [A]

time = 0.00, size = 35, normalized size = 0.85

$$-\frac{4}{16}x \cos(2x) - \frac{1}{16}(4x^2 - 2) \sin(2x) + \frac{1}{2} \cdot \frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*sin(x)^2,x)`

[Out] `1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)`

Mupad [B]

time = 0.16, size = 28, normalized size = 0.68

$$\frac{\sin(2x)}{8} - \frac{x \cos(2x)}{4} - \frac{x^2 \sin(2x)}{4} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*sin(x)^2,x)`

[Out] `sin(2*x)/8 - (x*cos(2*x))/4 - (x^2*sin(2*x))/4 + x^3/6`

3.96 $\int x \sin^3(x) dx$

Optimal. Leaf size=33

$$-\frac{2}{3}x \cos(x) + \frac{2 \sin(x)}{3} - \frac{1}{3}x \cos(x) \sin^2(x) + \frac{\sin^3(x)}{9}$$

[Out] $-2/3*x*\cos(x)+2/3*\sin(x)-1/3*x*\cos(x)*\sin(x)^2+1/9*\sin(x)^3$

Rubi [A]

time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3391, 3377, 2717}

$$\frac{\sin^3(x)}{9} + \frac{2 \sin(x)}{3} - \frac{2}{3}x \cos(x) - \frac{1}{3}x \sin^2(x) \cos(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Sin}[x]^3,x]$

[Out] $(-2*x*\text{Cos}[x])/3 + (2*\text{Sin}[x])/3 - (x*\text{Cos}[x]*\text{Sin}[x]^2)/3 + \text{Sin}[x]^3/9$

Rule 2717

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \text{ :> } \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$
 $\text{FreeQ}[\{c, d\}, x]$

Rule 3377

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \text{ :> } \text{Simp}[($
 $-(c + d*x)^m*(\text{Cos}[e + f*x]/f), x] + \text{Dist}[d*(m/f), \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}$
 $[e + f*x], x], x] /;$ $\text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

Rule 3391

$\text{Int}[((c_.) + (d_.)*(x_.))*(b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}, x_Symbol] \text{ :> } \text{Simp}[d*((b*\text{Sin}[e + f*x])^n/(f^2*n^2)), x] + (\text{Dist}[b^2*((n-1)/n), \text{Int}[(c$
 $+ d*x)*(b*\text{Sin}[e + f*x])^{(n-2)}, x], x] - \text{Simp}[b*(c + d*x)*\text{Cos}[e + f*x]*((b$
 $*\text{Sin}[e + f*x])^{(n-1)}/(f*n)), x]) /;$ $\text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[n, 1]$
 $]$

Rubi steps

$$\begin{aligned}
\int x \sin^3(x) dx &= -\frac{1}{3}x \cos(x) \sin^2(x) + \frac{\sin^3(x)}{9} + \frac{2}{3} \int x \sin(x) dx \\
&= -\frac{2}{3}x \cos(x) - \frac{1}{3}x \cos(x) \sin^2(x) + \frac{\sin^3(x)}{9} + \frac{2}{3} \int \cos(x) dx \\
&= -\frac{2}{3}x \cos(x) + \frac{2 \sin(x)}{3} - \frac{1}{3}x \cos(x) \sin^2(x) + \frac{\sin^3(x)}{9}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 31, normalized size = 0.94

$$-\frac{3}{4}x \cos(x) + \frac{1}{12}x \cos(3x) + \frac{3 \sin(x)}{4} - \frac{1}{36} \sin(3x)$$

Antiderivative was successfully verified.

`[In] Integrate[x*Sin[x]^3,x]``[Out] (-3*x*Cos[x])/4 + (x*Cos[3*x])/12 + (3*Sin[x])/4 - Sin[3*x]/36`**Mathics [A]**

time = 2.11, size = 25, normalized size = 0.76

$$-x \cos[x] + \frac{x \cos[x]^3}{3} - \frac{\cos[x]^2 \sin[x]}{9} + \frac{7 \sin[x]}{9}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x*Sin[x]^3,x]')``[Out] -x Cos[x] + x Cos[x]^3 / 3 - Cos[x]^2 Sin[x] / 9 + 7 Sin[x] / 9`**Maple [A]**

time = 0.02, size = 23, normalized size = 0.70

method	result	size
default	$-\frac{x(2+\sin^2(x)) \cos(x)}{3} + \frac{(\sin^3(x))}{9} + \frac{2 \sin(x)}{3}$	23
risch	$-\frac{3x \cos(x)}{4} + \frac{3 \sin(x)}{4} + \frac{x \cos(3x)}{12} - \frac{\sin(3x)}{36}$	24
norman	$\frac{-\frac{2x}{3} + \frac{32 \left(\tan^3\left(\frac{x}{2}\right)\right)}{9} + \frac{4 \left(\tan^5\left(\frac{x}{2}\right)\right)}{3} - 2x \left(\tan^2\left(\frac{x}{2}\right)\right) + 2x \left(\tan^4\left(\frac{x}{2}\right)\right) + \frac{2x \left(\tan^6\left(\frac{x}{2}\right)\right)}{3} + \frac{4 \tan\left(\frac{x}{2}\right)}{3}}{\left(1 + \tan^2\left(\frac{x}{2}\right)\right)^3}$	65

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*sin(x)^3,x,method=_RETURNVERBOSE)`

[Out] $-1/3*x*(2+\sin(x)^2)*\cos(x)+1/9*\sin(x)^3+2/3*\sin(x)$

Maxima [A]

time = 0.25, size = 23, normalized size = 0.70

$$\frac{1}{12}x \cos(3x) - \frac{3}{4}x \cos(x) - \frac{1}{36} \sin(3x) + \frac{3}{4} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x)^3,x, algorithm="maxima")`

[Out] $1/12*x*\cos(3*x) - 3/4*x*\cos(x) - 1/36*\sin(3*x) + 3/4*\sin(x)$

Fricas [A]

time = 0.32, size = 23, normalized size = 0.70

$$\frac{1}{3}x \cos(x)^3 - x \cos(x) - \frac{1}{9}(\cos(x)^2 - 7) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x)^3,x, algorithm="fricas")`

[Out] $1/3*x*\cos(x)^3 - x*\cos(x) - 1/9*(\cos(x)^2 - 7)*\sin(x)$

Sympy [A]

time = 0.15, size = 39, normalized size = 1.18

$$-x \sin^2(x) \cos(x) - \frac{2x \cos^3(x)}{3} + \frac{7 \sin^3(x)}{9} + \frac{2 \sin(x) \cos^2(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x)**3,x)`

[Out] $-x*\sin(x)**2*\cos(x) - 2*x*\cos(x)**3/3 + 7*\sin(x)**3/9 + 2*\sin(x)*\cos(x)**2/3$

Giac [A]

time = 0.00, size = 32, normalized size = 0.97

$$-\frac{3}{4}x \cos x + \frac{3}{4} \sin x + \frac{3}{36}x \cos(3x) - \frac{\sin(3x)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x)^3,x)`

[Out] $1/12*x*\cos(3*x) - 3/4*x*\cos(x) - 1/36*\sin(3*x) + 3/4*\sin(x)$

Mupad [B]

time = 0.07, size = 25, normalized size = 0.76

$$\frac{x \cos(x)^3}{3} - \frac{\sin(x) \cos(x)^2}{9} - x \cos(x) + \frac{7 \sin(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*sin(x)^3,x)`

[Out] `(7*sin(x))/9 + (x*cos(x)^3)/3 - (cos(x)^2*sin(x))/9 - x*cos(x)`

3.97 $\int x \cos(x) dx$

Optimal. Leaf size=7

$$\cos(x) + x \sin(x)$$

[Out] $\cos(x) + x \sin(x)$

Rubi [A]

time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3377, 2718}

$$x \sin(x) + \cos(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x \cos[x], x]$

[Out] $\cos[x] + x \sin[x]$

Rule 2718

$\text{Int}[\sin[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow \text{Simp}[-\cos[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3377

$\text{Int}[(c_. + (d_.)(x_.))^{(m_.)} \sin[(e_.) + (f_.)(x_.)], x_Symbol] \rightarrow \text{Simp}[-(c + d*x)^m * (\cos[e + f*x]/f), x] + \text{Dist}[d*(m/f), \text{Int}[(c + d*x)^{(m-1)} * \cos[e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

Rubi steps

$$\begin{aligned} \int x \cos(x) dx &= x \sin(x) - \int \sin(x) dx \\ &= \cos(x) + x \sin(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 7, normalized size = 1.00

$$\cos(x) + x \sin(x)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[x \cos[x], x]$

[Out] $\text{Cos}[x] + x*\text{Sin}[x]$

Mathics [A]

time = 1.79, size = 7, normalized size = 1.00

$$x\text{Sin}[x] + \text{Cos}[x]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x*Cos[x],x]')`

[Out] $x \text{Sin}[x] + \text{Cos}[x]$

Maple [A]

time = 0.02, size = 8, normalized size = 1.14

method	result	size
default	$\cos(x) + x \sin(x)$	8
risch	$\cos(x) + x \sin(x)$	8
norman	$\frac{2x \tan(\frac{x}{2}) + 2}{1 + \tan^2(\frac{x}{2})}$	21
meijerg	$2\sqrt{\pi} \left(-\frac{1}{2\sqrt{\pi}} + \frac{\cos(x)}{2\sqrt{\pi}} + \frac{x \sin(x)}{2\sqrt{\pi}} \right)$	27

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cos(x),x,method=_RETURNVERBOSE)`

[Out] $\cos(x) + x*\sin(x)$

Maxima [A]

time = 0.27, size = 7, normalized size = 1.00

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x),x, algorithm="maxima")`

[Out] $x*\sin(x) + \cos(x)$

Fricas [A]

time = 0.31, size = 7, normalized size = 1.00

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x),x, algorithm="fricas")`

[Out] $x*\sin(x) + \cos(x)$

Sympy [A]

time = 0.07, size = 7, normalized size = 1.00

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x),x)`

[Out] $x*\sin(x) + \cos(x)$

Giac [A]

time = 0.00, size = 7, normalized size = 1.00

$$\cos x + x \sin x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x),x)`

[Out] $x*\sin(x) + \cos(x)$

Mupad [B]

time = 0.02, size = 7, normalized size = 1.00

$$\cos(x) + x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cos(x),x)`

[Out] $\cos(x) + x*\sin(x)$

3.98 $\int x^2 \cos(x) dx$

Optimal. Leaf size=16

$$2x \cos(x) - 2 \sin(x) + x^2 \sin(x)$$

[Out] 2*x*cos(x)-2*sin(x)+x^2*sin(x)

Rubi [A]

time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3377, 2717}

$$x^2 \sin(x) - 2 \sin(x) + 2x \cos(x)$$

Antiderivative was successfully verified.

[In] Int[x^2*Cos[x],x]

[Out] 2*x*Cos[x] - 2*Sin[x] + x^2*Sin[x]

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 3377

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rubi steps

$$\begin{aligned} \int x^2 \cos(x) dx &= x^2 \sin(x) - 2 \int x \sin(x) dx \\ &= 2x \cos(x) + x^2 \sin(x) - 2 \int \cos(x) dx \\ &= 2x \cos(x) - 2 \sin(x) + x^2 \sin(x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 14, normalized size = 0.88

$$2x \cos(x) + (-2 + x^2) \sin(x)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Cos[x],x]

[Out] 2*x*Cos[x] + (-2 + x^2)*Sin[x]

Mathics [A]

time = 1.92, size = 16, normalized size = 1.00

$$2x\cos[x] + x^2\sin[x] - 2\sin[x]$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^2*Cos[x],x]')

[Out] 2 x Cos[x] + x ^ 2 Sin[x] - 2 Sin[x]

Maple [A]

time = 0.02, size = 17, normalized size = 1.06

method	result	size
risch	$2x \cos(x) + (x^2 - 2) \sin(x)$	15
default	$2x \cos(x) - 2 \sin(x) + x^2 \sin(x)$	17
meijerg	$4\sqrt{\pi} \left(\frac{x \cos(x)}{2\sqrt{\pi}} - \frac{(-\frac{3x^2}{2} + 3) \sin(x)}{6\sqrt{\pi}} \right)$	29
norman	$\frac{2x - 2x \tan^2(\frac{x}{2}) + 2x^2 \tan(\frac{x}{2}) - 4 \tan(\frac{x}{2})}{1 + \tan^2(\frac{x}{2})}$	40

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cos(x),x,method=_RETURNVERBOSE)

[Out] 2*x*cos(x)-2*sin(x)+x^2*sin(x)

Maxima [A]

time = 0.27, size = 14, normalized size = 0.88

$$2x \cos(x) + (x^2 - 2) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(x),x, algorithm="maxima")

[Out] 2*x*cos(x) + (x^2 - 2)*sin(x)

Fricas [A]

time = 0.32, size = 14, normalized size = 0.88

$$2x \cos(x) + (x^2 - 2) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x²*cos(x),x, algorithm="fricas")

[Out] 2*x*cos(x) + (x² - 2)*sin(x)

Sympy [A]

time = 0.10, size = 17, normalized size = 1.06

$$x^2 \sin(x) + 2x \cos(x) - 2 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*cos(x),x)

[Out] x**2*sin(x) + 2*x*cos(x) - 2*sin(x)

Giac [A]

time = 0.00, size = 16, normalized size = 1.00

$$2x \cos x - (-x^2 + 2) \sin x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x²*cos(x),x)

[Out] 2*x*cos(x) + (x² - 2)*sin(x)

Mupad [B]

time = 0.03, size = 14, normalized size = 0.88

$$\sin(x) (x^2 - 2) + 2x \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x²*cos(x),x)

[Out] sin(x)*(x² - 2) + 2*x*cos(x)

3.99 $\int x \cos^2(x) dx$

Optimal. Leaf size=25

$$\frac{x^2}{4} + \frac{\cos^2(x)}{4} + \frac{1}{2}x \cos(x) \sin(x)$$

[Out] 1/4*x^2+1/4*cos(x)^2+1/2*x*cos(x)*sin(x)

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3391, 30}

$$\frac{x^2}{4} + \frac{\cos^2(x)}{4} + \frac{1}{2}x \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[x*Cos[x]^2,x]

[Out] x^2/4 + Cos[x]^2/4 + (x*Cos[x]*Sin[x])/2

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3391

Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[d*((b*SIN[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c + d*x)*(b*SIN[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b*SIN[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int x \cos^2(x) dx &= \frac{\cos^2(x)}{4} + \frac{1}{2}x \cos(x) \sin(x) + \frac{\int x dx}{2} \\ &= \frac{x^2}{4} + \frac{\cos^2(x)}{4} + \frac{1}{2}x \cos(x) \sin(x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^2}{4} + \frac{1}{8} \cos(2x) + \frac{1}{4}x \sin(2x)$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[x]^2,x]

[Out] x^2/4 + Cos[2*x]/8 + (x*Sin[2*x])/4

Mathics [A]

time = 2.00, size = 20, normalized size = 0.80

$$-\frac{1}{8} + \frac{x \sin[2x]}{4} + \frac{x^2}{4} + \frac{\cos[2x]}{8}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x*Cos[x]^2,x]')

[Out] -1 / 8 + x Sin[2 x] / 4 + x ^ 2 / 4 + Cos[2 x] / 8

Maple [A]

time = 0.02, size = 25, normalized size = 1.00

method	result	size
risch	$\frac{x^2}{4} + \frac{\cos(2x)}{8} + \frac{x \sin(2x)}{4}$	20
default	$x \left(\frac{x}{2} + \frac{\cos(x) \sin(x)}{2} \right) - \frac{x^2}{4} - \frac{(\sin^2(x))}{4}$	25
norman	$\frac{x \tan(\frac{x}{2}) - (\tan^2(\frac{x}{2})) + \frac{x^2}{4} - x(\tan^3(\frac{x}{2})) + \frac{x^2(\tan^2(\frac{x}{2}))}{2} + \frac{x^2(\tan^4(\frac{x}{2}))}{4}}{(1 + \tan^2(\frac{x}{2}))^2}$	63

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(x)^2,x,method=_RETURNVERBOSE)

[Out] x*(1/2*x+1/2*cos(x)*sin(x))-1/4*x^2-1/4*sin(x)^2

Maxima [A]

time = 0.28, size = 19, normalized size = 0.76

$$\frac{1}{4} x^2 + \frac{1}{4} x \sin(2x) + \frac{1}{8} \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x)^2,x, algorithm="maxima")

[Out] 1/4*x^2 + 1/4*x*sin(2*x) + 1/8*cos(2*x)

Fricas [A]

time = 0.35, size = 19, normalized size = 0.76

$$\frac{1}{2} x \cos(x) \sin(x) + \frac{1}{4} x^2 + \frac{1}{4} \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x)^2,x, algorithm="fricas")`

[Out] `1/2*x*cos(x)*sin(x) + 1/4*x^2 + 1/4*cos(x)^2`

Sympy [A]

time = 0.11, size = 36, normalized size = 1.44

$$\frac{x^2 \sin^2(x)}{4} + \frac{x^2 \cos^2(x)}{4} + \frac{x \sin(x) \cos(x)}{2} - \frac{\sin^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x)**2,x)`

[Out] `x**2*sin(x)**2/4 + x**2*cos(x)**2/4 + x*sin(x)*cos(x)/2 - sin(x)**2/4`

Giac [A]

time = 0.00, size = 26, normalized size = 1.04

$$\frac{\cos(2x)}{8} + \frac{2}{8}x \sin(2x) + \frac{1}{2} \cdot \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x)^2,x)`

[Out] `1/4*x^2 + 1/4*x*sin(2*x) + 1/8*cos(2*x)`

Mupad [B]

time = 0.15, size = 19, normalized size = 0.76

$$\frac{x \sin(2x)}{4} - \frac{\sin(x)^2}{4} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cos(x)^2,x)`

[Out] `(x*sin(2*x))/4 - sin(x)^2/4 + x^2/4`

3.100 $\int x^2 \cos^2(x) dx$

Optimal. Leaf size=41

$$-\frac{x}{4} + \frac{x^3}{6} + \frac{1}{2}x \cos^2(x) - \frac{1}{4} \cos(x) \sin(x) + \frac{1}{2}x^2 \cos(x) \sin(x)$$

[Out] $-1/4*x+1/6*x^3+1/2*x*\cos(x)^2-1/4*\cos(x)*\sin(x)+1/2*x^2*\cos(x)*\sin(x)$

Rubi [A]

time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3392, 30, 2715, 8}

$$\frac{x^3}{6} + \frac{1}{2}x^2 \sin(x) \cos(x) - \frac{x}{4} + \frac{1}{2}x \cos^2(x) - \frac{1}{4} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[x^2*Cos[x]^2,x]

[Out] $-1/4*x + x^3/6 + (x*\cos[x]^2)/2 - (\cos[x]*\sin[x])/4 + (x^2*\cos[x]*\sin[x])/2$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3392

Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rubi steps

$$\begin{aligned}
\int x^2 \cos^2(x) dx &= \frac{1}{2}x \cos^2(x) + \frac{1}{2}x^2 \cos(x) \sin(x) + \frac{\int x^2 dx}{2} - \frac{1}{2} \int \cos^2(x) dx \\
&= \frac{x^3}{6} + \frac{1}{2}x \cos^2(x) - \frac{1}{4} \cos(x) \sin(x) + \frac{1}{2}x^2 \cos(x) \sin(x) - \frac{\int 1 dx}{4} \\
&= -\frac{x}{4} + \frac{x^3}{6} + \frac{1}{2}x \cos^2(x) - \frac{1}{4} \cos(x) \sin(x) + \frac{1}{2}x^2 \cos(x) \sin(x)
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 29, normalized size = 0.71

$$\frac{1}{24} (4x^3 + 6x \cos(2x) + (-3 + 6x^2) \sin(2x))$$

Antiderivative was successfully verified.

`[In] Integrate[x^2*Cos[x]^2,x]``[Out] (4*x^3 + 6*x*Cos[2*x] + (-3 + 6*x^2)*Sin[2*x])/24`**Mathics [A]**

time = 2.13, size = 28, normalized size = 0.68

$$\frac{x \cos[2x]}{4} + \frac{x^2 \sin[2x]}{4} + \frac{x^3}{6} - \frac{\sin[2x]}{8}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^2*Cos[x]^2,x]')``[Out] x Cos[2 x] / 4 + x ^ 2 Sin[2 x] / 4 + x ^ 3 / 6 - Sin[2 x] / 8`**Maple [A]**

time = 0.03, size = 37, normalized size = 0.90

method	result	size
risch	$\frac{x^3}{6} + \frac{x \cos(2x)}{4} + \frac{(2x^2-1) \sin(2x)}{8}$	27
default	$x^2 \left(\frac{x}{2} + \frac{\cos(x) \sin(x)}{2} \right) + \frac{x \cos^2(x)}{2} - \frac{\cos(x) \sin(x)}{4} - \frac{x}{4} - \frac{x^3}{3}$	37
norman	$\frac{x^2 \tan(\frac{x}{2}) + \frac{x}{4} + \frac{x^3}{6} + \frac{(\tan^3(\frac{x}{2}))}{2} - \frac{3x(\tan^2(\frac{x}{2}))}{2} + \frac{x(\tan^4(\frac{x}{2}))}{4} - x^2(\tan^3(\frac{x}{2})) + \frac{x^3(\tan^2(\frac{x}{2}))}{3} + \frac{x^3(\tan^4(\frac{x}{2}))}{6} - \frac{\tan(\frac{x}{2})}{2}}{(1+\tan^2(\frac{x}{2}))^2}$	94

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cos(x)^2,x,method=_RETURNVERBOSE)`

[Out] $x^2*(1/2*x+1/2*\cos(x)*\sin(x))+1/2*x*\cos(x)^2-1/4*\cos(x)*\sin(x)-1/4*x-1/3*x^3$

Maxima [A]

time = 0.26, size = 26, normalized size = 0.63

$$\frac{1}{6}x^3 + \frac{1}{4}x \cos(2x) + \frac{1}{8}(2x^2 - 1) \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(x)^2,x, algorithm="maxima")`

[Out] $1/6*x^3 + 1/4*x*\cos(2*x) + 1/8*(2*x^2 - 1)*\sin(2*x)$

Fricas [A]

time = 0.35, size = 29, normalized size = 0.71

$$\frac{1}{6}x^3 + \frac{1}{2}x \cos(x)^2 + \frac{1}{4}(2x^2 - 1) \cos(x) \sin(x) - \frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(x)^2,x, algorithm="fricas")`

[Out] $1/6*x^3 + 1/2*x*\cos(x)^2 + 1/4*(2*x^2 - 1)*\cos(x)*\sin(x) - 1/4*x$

Sympy [A]

time = 0.15, size = 56, normalized size = 1.37

$$\frac{x^3 \sin^2(x)}{6} + \frac{x^3 \cos^2(x)}{6} + \frac{x^2 \sin(x) \cos(x)}{2} - \frac{x \sin^2(x)}{4} + \frac{x \cos^2(x)}{4} - \frac{\sin(x) \cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*cos(x)**2,x)`

[Out] $x**3*\sin(x)**2/6 + x**3*\cos(x)**2/6 + x**2*\sin(x)*\cos(x)/2 - x*\sin(x)**2/4 + x*\cos(x)**2/4 - \sin(x)*\cos(x)/4$

Giac [A]

time = 0.00, size = 35, normalized size = 0.85

$$\frac{4}{16}x \cos(2x) - \frac{1}{16}(-4x^2 + 2) \sin(2x) + \frac{1}{2} \cdot \frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(x)^2,x)`

[Out] $1/6*x^3 + 1/4*x*\cos(2*x) + 1/8*(2*x^2 - 1)*\sin(2*x)$

Mupad [B]

time = 0.06, size = 28, normalized size = 0.68

$$\frac{x \cos(2x)}{4} - \frac{\sin(2x)}{8} + \frac{x^2 \sin(2x)}{4} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cos(x)^2,x)`

[Out] `(x*cos(2*x))/4 - sin(2*x)/8 + (x^2*sin(2*x))/4 + x^3/6`

3.101 $\int x \cos^3(x) dx$

Optimal. Leaf size=33

$$\frac{2 \cos(x)}{3} + \frac{\cos^3(x)}{9} + \frac{2}{3}x \sin(x) + \frac{1}{3}x \cos^2(x) \sin(x)$$

[Out] 2/3*cos(x)+1/9*cos(x)^3+2/3*x*sin(x)+1/3*x*cos(x)^2*sin(x)

Rubi [A]

time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3391, 3377, 2718}

$$\frac{2}{3}x \sin(x) + \frac{\cos^3(x)}{9} + \frac{2 \cos(x)}{3} + \frac{1}{3}x \sin(x) \cos^2(x)$$

Antiderivative was successfully verified.

[In] Int[x*Cos[x]^3,x]

[Out] (2*Cos[x])/3 + Cos[x]^3/9 + (2*x*Sin[x])/3 + (x*Cos[x]^2*Sin[x])/3

Rule 2718

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3377

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3391

Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c + d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned}
\int x \cos^3(x) dx &= \frac{\cos^3(x)}{9} + \frac{1}{3}x \cos^2(x) \sin(x) + \frac{2}{3} \int x \cos(x) dx \\
&= \frac{\cos^3(x)}{9} + \frac{2}{3}x \sin(x) + \frac{1}{3}x \cos^2(x) \sin(x) - \frac{2}{3} \int \sin(x) dx \\
&= \frac{2 \cos(x)}{3} + \frac{\cos^3(x)}{9} + \frac{2}{3}x \sin(x) + \frac{1}{3}x \cos^2(x) \sin(x)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 31, normalized size = 0.94

$$\frac{3 \cos(x)}{4} + \frac{1}{36} \cos(3x) + \frac{3}{4}x \sin(x) + \frac{1}{12}x \sin(3x)$$

Antiderivative was successfully verified.

`[In] Integrate[x*Cos[x]^3,x]``[Out] (3*Cos[x])/4 + Cos[3*x]/36 + (3*x*Sin[x])/4 + (x*Sin[3*x])/12`**Mathics [A]**

time = 2.09, size = 24, normalized size = 0.73

$$-\frac{x \sin[x]^3}{3} + x \sin[x] - \frac{\cos[x] \sin[x]^2}{9} + \frac{7 \cos[x]}{9}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x*Cos[x]^3,x]')``[Out] -x Sin[x]^3 / 3 + x Sin[x] - Cos[x] Sin[x]^2 / 9 + 7 Cos[x] / 9`**Maple [A]**

time = 0.03, size = 23, normalized size = 0.70

method	result	size
default	$\frac{x(2+\cos^2(x)) \sin(x)}{3} + \frac{(\cos^3(x))}{9} + \frac{2 \cos(x)}{3}$	23
risch	$\frac{3 \cos(x)}{4} + \frac{3x \sin(x)}{4} + \frac{\cos(3x)}{36} + \frac{x \sin(3x)}{12}$	24
norman	$\frac{-\frac{2(\tan^4(\frac{x}{2}))}{3} - \frac{8(\tan^6(\frac{x}{2}))}{9} + 2x \tan(\frac{x}{2}) + \frac{4x(\tan^3(\frac{x}{2}))}{3} + 2x(\tan^5(\frac{x}{2})) + \frac{2}{3}}{(1+\tan^2(\frac{x}{2}))^3}$	55

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*cos(x)^3,x,method=_RETURNVERBOSE)`

[Out] $1/3*x*(2+\cos(x)^2)*\sin(x)+1/9*\cos(x)^3+2/3*\cos(x)$

Maxima [A]

time = 0.25, size = 23, normalized size = 0.70

$$\frac{1}{12}x \sin(3x) + \frac{3}{4}x \sin(x) + \frac{1}{36} \cos(3x) + \frac{3}{4} \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x)^3,x, algorithm="maxima")`

[Out] $1/12*x*\sin(3*x) + 3/4*x*\sin(x) + 1/36*\cos(3*x) + 3/4*\cos(x)$

Fricas [A]

time = 0.32, size = 25, normalized size = 0.76

$$\frac{1}{9} \cos(x)^3 + \frac{1}{3} (x \cos(x)^2 + 2x) \sin(x) + \frac{2}{3} \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x)^3,x, algorithm="fricas")`

[Out] $1/9*\cos(x)^3 + 1/3*(x*\cos(x)^2 + 2*x)*\sin(x) + 2/3*\cos(x)$

Sympy [A]

time = 0.15, size = 39, normalized size = 1.18

$$\frac{2x \sin^3(x)}{3} + x \sin(x) \cos^2(x) + \frac{2 \sin^2(x) \cos(x)}{3} + \frac{7 \cos^3(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x)**3,x)`

[Out] $2*x*\sin(x)**3/3 + x*\sin(x)*\cos(x)**2 + 2*\sin(x)**2*\cos(x)/3 + 7*\cos(x)**3/9$

Giac [A]

time = 0.00, size = 30, normalized size = 0.91

$$\frac{3}{4} \cos x + \frac{3}{4} x \sin x + \frac{\cos(3x)}{36} + \frac{3}{36} x \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x)^3,x)`

[Out] $1/12*x*\sin(3*x) + 3/4*x*\sin(x) + 1/36*\cos(3*x) + 3/4*\cos(x)$

Mupad [B]

time = 0.17, size = 25, normalized size = 0.76

$$\frac{\cos(x)^3}{9} + \frac{x \sin(x) \cos(x)^2}{3} + \frac{2 \cos(x)}{3} + \frac{2 x \sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^3,x)
```

```
[Out] (2*cos(x))/3 + cos(x)^3/9 + (2*x*sin(x))/3 + (x*cos(x)^2*sin(x))/3
```

3.102 $\int \frac{\sin(x)}{x} dx$

Optimal. Leaf size=2

Si(x)

[Out] Si(x)

Rubi [A]

time = 0.01, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3380}

Si(x)

Antiderivative was successfully verified.

[In] Int[Sin[x]/x,x]

[Out] SinIntegral[x]

Rule 3380

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rubi steps

$$\int \frac{\sin(x)}{x} dx = \text{Si}(x)$$

Mathematica [A]

time = 0.01, size = 2, normalized size = 1.00

Si(x)

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/x,x]

[Out] SinIntegral[x]

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

time = 1.94, size = 2, normalized size = 1.00

Si[x]

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[Sin[x]/x,x]')`

[Out] `Si[x]`

Maple [A]

time = 0.01, size = 3, normalized size = 1.50

method	result	size
default	$\text{sinIntegral}(x)$	3
meijerg	$\text{sinIntegral}(x)$	3
risch	$-\frac{\pi \operatorname{csgn}(x)}{2} + \text{sinIntegral}(x)$	9

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/x,x,method=_RETURNVERBOSE)`

[Out] `Si(x)`

Maxima [C] Result contains complex when optimal does not.

time = 0.30, size = 13, normalized size = 6.50

$$-\frac{1}{2}i \operatorname{Ei}(ix) + \frac{1}{2}i \operatorname{Ei}(-ix)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/x,x, algorithm="maxima")`

[Out] `-1/2*I*Ei(I*x) + 1/2*I*Ei(-I*x)`

Fricas [A]

time = 0.32, size = 2, normalized size = 1.00

$\text{Si}(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/x,x, algorithm="fricas")`

[Out] `sin_integral(x)`

Sympy [A]

time = 0.31, size = 2, normalized size = 1.00

$\text{Si}(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/x,x)

[Out] Si(x)

Giac [A]

time = 0.00, size = 2, normalized size = 1.00

$\text{Si}(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/x,x)

[Out] sin_integral(x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.50

$\text{sinint}(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/x,x)

[Out] sinint(x)

3.103 $\int \frac{\cos(x)}{x} dx$

Optimal. Leaf size=2

$\text{Ci}(x)$

[Out] $\text{Ci}(x)$

Rubi [A]

time = 0.01, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3383}

$\text{Ci}(x)$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[x]/x, x]$

[Out] $\text{CosIntegral}[x]$

Rule 3383

$\text{Int}[\sin[(e_.) + (f_.)(x_)]/((c_.) + (d_.)(x_)), x_Symbol] \rightarrow \text{Simp}[\text{CosIntegral}[e - \text{Pi}/2 + f*x]/d, x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{EqQ}[d*(e - \text{Pi}/2) - c*f, 0]$

Rubi steps

$$\int \frac{\cos(x)}{x} dx = \text{Ci}(x)$$

Mathematica [A]

time = 0.00, size = 2, normalized size = 1.00

$\text{Ci}(x)$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Cos}[x]/x, x]$

[Out] $\text{CosIntegral}[x]$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

time = 2.23, size = 13, normalized size = 6.50

$$\text{Ci}[x] - \text{Log}[x] + \frac{\text{Log}[x^2]}{2}$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[Cos[x]/x,x]')`

[Out] `Ci[x] - Log[x] + Log[x ^ 2] / 2`

Maple [A]

time = 0.03, size = 3, normalized size = 1.50

method	result	size
default	$\text{cosineIntegral}(x)$	3
risch	$\text{cosineIntegral}(x) - \frac{i\pi \text{csgn}(ix)\text{csgn}(x)}{2} + \frac{i\pi \text{csgn}(ix)}{2}$	24
meijerg	$\frac{\sqrt{\pi} \left(\frac{2\gamma+2\ln(x)}{\sqrt{\pi}} - \frac{2\gamma}{\sqrt{\pi}} - \frac{2\ln(2)}{\sqrt{\pi}} - \frac{2\ln(\frac{x}{2})}{\sqrt{\pi}} + \frac{2\text{cosineIntegral}(x)}{\sqrt{\pi}} \right)}{2}$	48

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)/x,x,method=_RETURNVERBOSE)`

[Out] `Ci(x)`

Maxima [C] Result contains complex when optimal does not.

time = 0.29, size = 13, normalized size = 6.50

$$\frac{1}{2} \text{Ei}(ix) + \frac{1}{2} \text{Ei}(-ix)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/x,x, algorithm="maxima")`

[Out] `1/2*Ei(I*x) + 1/2*Ei(-I*x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(2) = 4.

time = 0.42, size = 11, normalized size = 5.50

$$\frac{1}{2} \text{Ci}(-x) + \frac{1}{2} \text{Ci}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/x,x, algorithm="fricas")`

[Out] `1/2*cos_integral(-x) + 1/2*cos_integral(x)`

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(2) = 4

time = 0.46, size = 12, normalized size = 6.00

$$-\log(x) + \frac{\log(x^2)}{2} + \text{Ci}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x,x)
```

```
[Out] -log(x) + log(x**2)/2 + Ci(x)
```

Giac [A]

time = 0.00, size = 2, normalized size = 1.00

$$\text{Ci}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x,x)
```

```
[Out] cos_integral(x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.50

$$\text{cosint}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/x,x)
```

```
[Out] cosint(x)
```

3.104 $\int \frac{\sin(x)}{x^2} dx$

Optimal. Leaf size=10

$$\text{Ci}(x) - \frac{\sin(x)}{x}$$

[Out] Ci(x)-sin(x)/x

Rubi [A]

time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3378, 3383}

$$\text{Ci}(x) - \frac{\sin(x)}{x}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/x^2,x]

[Out] CosIntegral[x] - Sin[x]/x

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(c
+ d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sin(x)}{x^2} dx &= -\frac{\sin(x)}{x} + \int \frac{\cos(x)}{x} dx \\ &= \text{Ci}(x) - \frac{\sin(x)}{x} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 10, normalized size = 1.00

$$\text{Ci}(x) - \frac{\sin(x)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/x^2,x]

[Out] CosIntegral[x] - Sin[x]/x

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

time = 2.66, size = 20, normalized size = 2.00

$$-\frac{\text{Sin}[x]}{x} + \text{Ci}[x] - \text{Log}[x] + \frac{\text{Log}[x^2]}{2}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sin[x]/x^2,x]')

[Out] -Sin[x] / x + Ci[x] - Log[x] + Log[x ^ 2] / 2

Maple [A]

time = 0.04, size = 11, normalized size = 1.10

method	result	size
default	$\text{cosineIntegral}(x) - \frac{\sin(x)}{x}$	11
risch	$\text{cosineIntegral}(x) - \frac{i\pi \text{csgn}(ix)\text{csgn}(x)}{2} + \frac{i\pi \text{csgn}(ix)}{2} - \frac{\sin(x)}{x}$	31
meijerg	$\frac{\sqrt{\pi} \left(\frac{4\gamma - 4 + 4\ln(x)}{\sqrt{\pi}} + \frac{4}{\sqrt{\pi}} - \frac{4\gamma}{\sqrt{\pi}} - \frac{4\ln(2)}{\sqrt{\pi}} - \frac{4\ln\left(\frac{x}{2}\right)}{\sqrt{\pi}} - \frac{4\sin(x)}{\sqrt{\pi}x} + \frac{4\text{cosineIntegral}(x)}{\sqrt{\pi}} \right)}{4}$	65

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/x^2,x,method=_RETURNVERBOSE)

[Out] Ci(x)-sin(x)/x

Maxima [C] Result contains complex when optimal does not.

time = 0.29, size = 15, normalized size = 1.50

$$\frac{1}{2} \Gamma(-1, ix) + \frac{1}{2} \Gamma(-1, -ix)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/x^2,x, algorithm="maxima")

[Out] 1/2*gamma(-1, I*x) + 1/2*gamma(-1, -I*x)

Fricas [A]

time = 0.32, size = 20, normalized size = 2.00

$$\frac{x \text{Ci}(-x) + x \text{Ci}(x) - 2 \sin(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/x^2,x, algorithm="fricas")

[Out] 1/2*(x*cos_integral(-x) + x*cos_integral(x) - 2*sin(x))/x

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$

time = 0.72, size = 17, normalized size = 1.70

$$-\log(x) + \frac{\log(x^2)}{2} + \text{Ci}(x) - \frac{\sin(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/x**2,x)

[Out] -log(x) + log(x**2)/2 + Ci(x) - sin(x)/x

Giac [A]

time = 0.00, size = 11, normalized size = 1.10

$$\frac{x\text{Ci}(x) - \sin x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/x^2,x)

[Out] (x*cos_integral(x) - sin(x))/x

Mupad [F]

time = 0.00, size = -1, normalized size = -0.10

$$\text{cosint}(x) - \frac{\sin(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/x^2,x)

[Out] cosint(x) - sin(x)/x

3.105 $\int \frac{\sin^2(x)}{x} dx$

Optimal. Leaf size=15

$$-\frac{\text{Ci}(2x)}{2} + \frac{\log(x)}{2}$$

[Out] -1/2*Ci(2*x)+1/2*ln(x)

Rubi [A]

time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3393, 3383}

$$\frac{\log(x)}{2} - \frac{\text{Ci}(2x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^2/x,x]

[Out] -1/2*CosIntegral[2*x] + Log[x]/2

Rule 3383

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rule 3393

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rubi steps

$$\begin{aligned} \int \frac{\sin^2(x)}{x} dx &= \int \left(\frac{1}{2x} - \frac{\cos(2x)}{2x} \right) dx \\ &= \frac{\log(x)}{2} - \frac{1}{2} \int \frac{\cos(2x)}{x} dx \\ &= -\frac{\text{Ci}(2x)}{2} + \frac{\log(x)}{2} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 15, normalized size = 1.00

$$-\frac{\text{Ci}(2x)}{2} + \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^2/x,x]

[Out] -1/2*CosIntegral[2*x] + Log[x]/2

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

time = 2.28, size = 11, normalized size = 0.73

$$-\frac{\text{Ci}[2x]}{2} + \frac{\text{Log}[x]}{2}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sin[x]^2/x,x]')

[Out] -Ci[2 x] / 2 + Log[x] / 2

Maple [A]

time = 0.03, size = 12, normalized size = 0.80

method	result	size
default	$-\frac{\text{cosineIntegral}(2x)}{2} + \frac{\ln(x)}{2}$	12
risch	$-\frac{\text{cosineIntegral}(2x)}{2} + \frac{i\pi \text{csgn}(ix)\text{csgn}(x)}{4} - \frac{i\pi \text{csgn}(ix)}{4} + \frac{\ln(x)}{2}$	32
meijerg	$\frac{\sqrt{\pi} \left(\frac{2\gamma}{\sqrt{\pi}} + \frac{2\ln(2)}{\sqrt{\pi}} + \frac{2\ln(x)}{\sqrt{\pi}} - \frac{2\text{cosineIntegral}(2x)}{\sqrt{\pi}} \right)}{4}$	36

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^2/x,x,method=_RETURNVERBOSE)

[Out] -1/2*Ci(2*x)+1/2*ln(x)

Maxima [C] Result contains complex when optimal does not.

time = 0.29, size = 17, normalized size = 1.13

$$-\frac{1}{4} \text{Ei}(2ix) - \frac{1}{4} \text{Ei}(-2ix) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/x,x, algorithm="maxima")

[Out] -1/4*Ei(2*I*x) - 1/4*Ei(-2*I*x) + 1/2*log(x)

Fricas [A]

time = 0.33, size = 17, normalized size = 1.13

$$-\frac{1}{4} \text{Ci}(2x) - \frac{1}{4} \text{Ci}(-2x) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/x,x, algorithm="fricas")

[Out] -1/4*cos_integral(2*x) - 1/4*cos_integral(-2*x) + 1/2*log(x)

Sympy [A]

time = 0.58, size = 10, normalized size = 0.67

$$\frac{\log(x)}{2} - \frac{\text{Ci}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**2/x,x)

[Out] log(x)/2 - Ci(2*x)/2

Giac [A]

time = 0.00, size = 16, normalized size = 1.07

$$-\frac{1}{2}\text{Ci}(2x) + \frac{1}{2}\ln x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/x,x)

[Out] -1/2*cos_integral(2*x) + 1/2*log(x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.07

$$\frac{\ln(x)}{2} - \frac{\text{cosint}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^2/x,x)

[Out] log(x)/2 - cosint(2*x)/2

3.106 $\int \tan^3(x) dx$

Optimal. Leaf size=12

$$\log(\cos(x)) + \frac{\tan^2(x)}{2}$$

[Out] $\ln(\cos(x))+1/2*\tan(x)^2$

Rubi [A]

time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 3556}

$$\frac{\tan^2(x)}{2} + \log(\cos(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Tan}[x]^3, x]$

[Out] $\text{Log}[\text{Cos}[x]] + \text{Tan}[x]^2/2$

Rule 3554

$\text{Int}[(b \cdot \tan[(c \cdot x) + (d \cdot x)])^n, x_Symbol] \rightarrow \text{Simp}[b \cdot ((b \cdot \tan[c + d \cdot x])^{n-1} / (d \cdot (n-1))), x] - \text{Dist}[b^2, \text{Int}[(b \cdot \tan[c + d \cdot x])^{n-2}, x], x] /;$ $\text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{GtQ}[n, 1]$

Rule 3556

$\text{Int}[\tan[(c \cdot x) + (d \cdot x)], x_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d \cdot x], x]]/d, x] /;$ $\text{FreeQ}\{c, d\}, x$

Rubi steps

$$\begin{aligned} \int \tan^3(x) dx &= \frac{\tan^2(x)}{2} - \int \tan(x) dx \\ &= \log(\cos(x)) + \frac{\tan^2(x)}{2} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 12, normalized size = 1.00

$$\log(\cos(x)) + \frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[x]^3,x]

[Out] Log[Cos[x]] + Sec[x]^2/2

Mathics [A]

time = 1.68, size = 10, normalized size = 0.83

$$\text{Log}[\text{Cos}[x]] + \frac{1}{2\text{Cos}[x]^2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Tan[x]^3,x]')

[Out] Log[Cos[x]] + 1 / (2 Cos[x]^2)

Maple [A]

time = 0.02, size = 17, normalized size = 1.42

method	result	size
derivativedivides	$\frac{\tan^2(x)}{2} - \frac{\ln(1+\tan^2(x))}{2}$	17
default	$\frac{\tan^2(x)}{2} - \frac{\ln(1+\tan^2(x))}{2}$	17
norman	$\frac{\tan^2(x)}{2} - \frac{\ln(1+\tan^2(x))}{2}$	17
risch	$-ix + \frac{2e^{2ix}}{(e^{2ix}+1)^2} + \ln(e^{2ix} + 1)$	30

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)^3,x,method=_RETURNVERBOSE)

[Out] 1/2*tan(x)^2-1/2*ln(1+tan(x)^2)

Maxima [A]

time = 0.28, size = 20, normalized size = 1.67

$$-\frac{1}{2(\sin(x)^2 - 1)} + \frac{1}{2} \log(\sin(x)^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)^3,x, algorithm="maxima")

[Out] -1/2/(sin(x)^2 - 1) + 1/2*log(sin(x)^2 - 1)

Fricas [A]

time = 0.34, size = 18, normalized size = 1.50

$$\frac{1}{2} \tan(x)^2 + \frac{1}{2} \log\left(\frac{1}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)^3,x, algorithm="fricas")

[Out] 1/2*tan(x)^2 + 1/2*log(1/(tan(x)^2 + 1))

Sympy [A]

time = 0.05, size = 12, normalized size = 1.00

$$\log(\cos(x)) + \frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)**3,x)

[Out] log(cos(x)) + 1/(2*cos(x)**2)

Giac [A]

time = 0.00, size = 16, normalized size = 1.33

$$\frac{\tan^2 x - \ln(\tan^2 x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)^3,x)

[Out] 1/2*tan(x)^2 - 1/2*log(tan(x)^2 + 1)

Mupad [B]

time = 0.02, size = 16, normalized size = 1.33

$$\ln(\cos(x)) - \frac{\cos(x)^2 - 1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)^3,x)

[Out] log(cos(x)) - (cos(x)^2 - 1)/(2*cos(x)^2)

3.107 $\int \sin(a + bx) dx$

Optimal. Leaf size=11

$$-\frac{\cos(a + bx)}{b}$$

[Out] $-\cos(b*x+a)/b$

Rubi [A]

time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2718}

$$-\frac{\cos(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b*x], x]

[Out] $-(\text{Cos}[a + b*x])/b$

Rule 2718

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \sin(a + bx) dx = -\frac{\cos(a + bx)}{b}$$

Mathematica [A]

time = 0.01, size = 22, normalized size = 2.00

$$-\frac{\cos(a) \cos(bx)}{b} + \frac{\sin(a) \sin(bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b*x], x]

[Out] $-(\text{Cos}[a] * \text{Cos}[b*x])/b + (\text{Sin}[a] * \text{Sin}[b*x])/b$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.69, size = 21, normalized size = 1.91

$$\text{Piecewise} \left[\left[\left\{ \left\{ -\frac{\text{Cos}[a + bx]}{b}, b \neq 0 \right\} \right\}, x \text{Sin}[a] \right] \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[Sin[a+b*x],x]')`

[Out] `Piecewise[{{-Cos[a + b x] / b, b != 0}}, x Sin[a]]`

Maple [A]

time = 0.02, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$-\frac{\cos(bx+a)}{b}$	12
default	$-\frac{\cos(bx+a)}{b}$	12
risch	$-\frac{\cos(bx+a)}{b}$	12
norman	$\frac{2\left(\tan^2\left(\frac{bx}{2} + \frac{a}{2}\right)\right)}{b\left(1 + \tan^2\left(\frac{bx}{2} + \frac{a}{2}\right)\right)}$	32
meijerg	$\frac{\sin(a)\sin(bx)}{b} + \frac{\cos(a)\sqrt{\pi}}{b} \left(\frac{1}{\sqrt{\pi}} - \frac{\cos(bx)}{\sqrt{\pi}} \right)$	34

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(b*x+a),x,method=_RETURNVERBOSE)`

[Out] `-cos(b*x+a)/b`

Maxima [A]

time = 0.26, size = 11, normalized size = 1.00

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(b*x+a),x, algorithm="maxima")`

[Out] `-cos(b*x + a)/b`

Fricas [A]

time = 0.32, size = 11, normalized size = 1.00

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(b*x+a),x, algorithm="fricas")`

[Out] `-cos(b*x + a)/b`

Sympy [A]

time = 0.06, size = 14, normalized size = 1.27

$$\begin{cases} -\frac{\cos(a+bx)}{b} & \text{for } b \neq 0 \\ x \sin(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a),x)**[Out]** Piecewise((-cos(a + b*x)/b, Ne(b, 0)), (x*sin(a), True))**Giac [A]**

time = 0.00, size = 10, normalized size = 0.91

$$-\frac{\cos(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a),x)**[Out]** -cos(b*x + a)/b**Mupad [B]**

time = 0.02, size = 11, normalized size = 1.00

$$-\frac{\cos(a + bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a + b*x),x)**[Out]** -cos(a + b*x)/b

3.108 $\int \cos(a + bx) dx$

Optimal. Leaf size=10

$$\frac{\sin(a + bx)}{b}$$

[Out] sin(b*x+a)/b

Rubi [A]

time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2717}

$$\frac{\sin(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x],x]

[Out] Sin[a + b*x]/b

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rubi steps

$$\int \cos(a + bx) dx = \frac{\sin(a + bx)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

time = 0.01, size = 21, normalized size = 2.10

$$\frac{\cos(bx) \sin(a)}{b} + \frac{\cos(a) \sin(bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b*x],x]

[Out] (Cos[b*x]*Sin[a])/b + (Cos[a]*Sin[b*x])/b

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.73, size = 20, normalized size = 2.00

$$\text{Piecewise} \left[\left\{ \left\{ \frac{\text{Sin}[a + bx]}{b}, b \neq 0 \right\} \right\}, x \text{Cos}[a] \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[Cos[a+b*x],x]')`

[Out] `Piecewise[{{Sin[a + b x] / b, b != 0}}, x Cos[a]]`

Maple [A]

time = 0.02, size = 11, normalized size = 1.10

method	result	size
derivativedivides	$\frac{\sin(bx+a)}{b}$	11
default	$\frac{\sin(bx+a)}{b}$	11
risch	$\frac{\sin(bx+a)}{b}$	11
norman	$\frac{2 \tan\left(\frac{bx}{2} + \frac{a}{2}\right)}{b \left(1 + \tan^2\left(\frac{bx}{2} + \frac{a}{2}\right)\right)}$	30
meijerg	$\frac{\cos(a) \sin(bx)}{b} - \frac{\sin(a) \sqrt{\pi} \left(\frac{1}{\sqrt{\pi}} - \frac{\cos(bx)}{\sqrt{\pi}} \right)}{b}$	35

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(b*x+a),x,method=_RETURNVERBOSE)`

[Out] `sin(b*x+a)/b`

Maxima [A]

time = 0.25, size = 10, normalized size = 1.00

$$\frac{\sin(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a),x, algorithm="maxima")`

[Out] `sin(b*x + a)/b`

Fricas [A]

time = 0.31, size = 10, normalized size = 1.00

$$\frac{\sin(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a),x, algorithm="fricas")

[Out] sin(b*x + a)/b

Sympy [A]

time = 0.06, size = 12, normalized size = 1.20

$$\begin{cases} \frac{\sin(a+bx)}{b} & \text{for } b \neq 0 \\ x \cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a),x)

[Out] Piecewise((sin(a + b*x)/b, Ne(b, 0)), (x*cos(a), True))

Giac [A]

time = 0.00, size = 9, normalized size = 0.90

$$\frac{\sin(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a),x)

[Out] sin(b*x + a)/b

Mupad [B]

time = 0.02, size = 10, normalized size = 1.00

$$\frac{\sin(a + bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b*x),x)

[Out] sin(a + b*x)/b

3.109 $\int \tan(a + bx) dx$

Optimal. Leaf size=12

$$-\frac{\log(\cos(a + bx))}{b}$$

[Out] $-\ln(\cos(b*x+a))/b$

Rubi [A]

time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3556}

$$-\frac{\log(\cos(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] `Int[Tan[a + b*x], x]`

[Out] $-(\text{Log}[\text{Cos}[a + b*x]])/b$

Rule 3556

`Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d *x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\int \tan(a + bx) dx = -\frac{\log(\cos(a + bx))}{b}$$

Mathematica [A]

time = 0.01, size = 12, normalized size = 1.00

$$-\frac{\log(\cos(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] `Integrate[Tan[a + b*x], x]`

[Out] $-(\text{Log}[\text{Cos}[a + b*x]])/b$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.22, size = 26, normalized size = 2.17

$$\text{Piecewise} \left[\left\{ \left\{ \frac{\text{Log}[1 + \text{Tan}[a + bx]^2]}{2b}, b \neq 0 \right\} \right\}, x \text{Tan}[a] \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[Tan[a+b*x],x]')`

[Out] `Piecewise[{{Log[1 + Tan[a + b x] ^ 2] / (2 b), b != 0}}, x Tan[a]]`

Maple [A]

time = 0.01, size = 17, normalized size = 1.42

method	result	size
derivativedivides	$\frac{\ln(1+\tan^2(bx+a))}{2b}$	17
default	$\frac{\ln(1+\tan^2(bx+a))}{2b}$	17
norman	$\frac{\ln(1+\tan^2(bx+a))}{2b}$	17
risch	$ix + \frac{2ia}{b} - \frac{\ln(e^{2i(bx+a)}+1)}{b}$	30

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(b*x+a),x,method=_RETURNVERBOSE)`

[Out] `1/2/b*ln(1+tan(b*x+a)^2)`

Maxima [A]

time = 0.27, size = 11, normalized size = 0.92

$$\frac{\log(\sec(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(b*x+a),x, algorithm="maxima")`

[Out] `log(sec(b*x + a))/b`

Fricas [A]

time = 0.32, size = 18, normalized size = 1.50

$$-\frac{\log\left(\frac{1}{\tan^2(bx+a)+1}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(b*x+a),x, algorithm="fricas")`

[Out] `-1/2*log(1/(tan(b*x + a)^2 + 1))/b`

Sympy [A]

time = 0.06, size = 19, normalized size = 1.58

$$\begin{cases} \frac{\log(\tan^2(a+bx)+1)}{2b} & \text{for } b \neq 0 \\ x \tan(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a),x)

[Out] Piecewise((log(tan(a + b*x)**2 + 1)/(2*b), Ne(b, 0)), (x*tan(a), True))

Giac [A]

time = 0.00, size = 12, normalized size = 1.00

$$-\frac{\ln |\cos(bx + a)|}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a),x)

[Out] -log(abs(cos(b*x + a)))/b

Mupad [B]

time = 0.23, size = 16, normalized size = 1.33

$$\frac{\ln(\tan(a + bx)^2 + 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(a + b*x),x)

[Out] log(tan(a + b*x)^2 + 1)/(2*b)

3.110 $\int \cot(a + bx) dx$

Optimal. Leaf size=11

$$\frac{\log(\sin(a + bx))}{b}$$

[Out] ln(sin(b*x+a))/b

Rubi [A]

time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3556}

$$\frac{\log(\sin(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Cot[a + b*x],x]

[Out] Log[Sin[a + b*x]]/b

Rule 3556

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \cot(a + bx) dx = \frac{\log(\sin(a + bx))}{b}$$

Mathematica [A]

time = 0.01, size = 19, normalized size = 1.73

$$\frac{\log(\cos(a + bx)) + \log(\tan(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[a + b*x],x]

[Out] (Log[Cos[a + b*x]] + Log[Tan[a + b*x]])/b

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.71, size = 37, normalized size = 3.36

$$\text{Piecewise} \left[\left\{ \left\{ \frac{\text{Log}[\text{Tan}[a + bx]] - \frac{\text{Log}[1 + \text{Tan}[a + bx]^2]}{2}}{b}, b \neq 0 \right\} \right\}, \frac{x}{\text{Tan}[a]} \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[1/Tan[a+b*x],x]')`

[Out] `Piecewise[{{(Log[Tan[a + b x]] - Log[1 + Tan[a + b x]^2] / 2) / b, b != 0}}, x / Tan[a]]`

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

time = 0.01, size = 26, normalized size = 2.36

method	result	size
derivativedivides	$-\frac{\frac{\ln(1+\tan^2(bx+a))}{2} + \ln(\tan(bx+a))}{b}$	26
default	$-\frac{\frac{\ln(1+\tan^2(bx+a))}{2} + \ln(\tan(bx+a))}{b}$	26
norman	$\frac{\ln(\tan(bx+a))}{b} - \frac{\ln(1+\tan^2(bx+a))}{2b}$	29
risch	$-ix - \frac{2ia}{b} + \frac{\ln(e^{2i(bx+a)} - 1)}{b}$	29

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/tan(b*x+a),x,method=_RETURNVERBOSE)`

[Out] `1/b*(-1/2*ln(1+tan(b*x+a)^2)+ln(tan(b*x+a)))`

Maxima [A]

time = 0.26, size = 11, normalized size = 1.00

$$\frac{\log(\sin(bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/tan(b*x+a),x, algorithm="maxima")`

[Out] `log(sin(b*x + a))/b`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

time = 0.32, size = 27, normalized size = 2.45

$$\frac{\log\left(\frac{\tan(bx+a)^2}{\tan(bx+a)^2+1}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(b*x+a),x, algorithm="fricas")

[Out] 1/2*log(tan(b*x + a)^2/(tan(b*x + a)^2 + 1))/b

Sympy [A]

time = 0.17, size = 29, normalized size = 2.64

$$\begin{cases} -\frac{\log(\tan^2(a+bx)+1)}{2b} + \frac{\log(\tan(a+bx))}{b} & \text{for } b \neq 0 \\ \frac{x}{\tan(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(b*x+a),x)

[Out] Piecewise((-log(tan(a + b*x)**2 + 1)/(2*b) + log(tan(a + b*x))/b, Ne(b, 0)), (x/tan(a), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(11) = 22.
time = 0.01, size = 57, normalized size = 5.18

$$\frac{2 \left(\frac{\ln\left(\frac{|1-\cos(a+bx)|}{|1+\cos(a+bx)|}\right)}{4} - \frac{\ln\left|\frac{1-\cos(a+bx)}{1+\cos(a+bx)}+1\right|}{2} \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(b*x+a),x)

[Out] 1/2*(log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 2*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b

Mupad [B]

time = 0.19, size = 28, normalized size = 2.55

$$\frac{\ln(\tan(a + bx))}{b} - \frac{\ln(\tan(a + bx)^2 + 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/tan(a + b*x),x)

[Out] log(tan(a + b*x))/b - log(tan(a + b*x)^2 + 1)/(2*b)

3.111 $\int \csc(a + bx) dx$

Optimal. Leaf size=12

$$-\frac{\tanh^{-1}(\cos(a + bx))}{b}$$

[Out] -arctanh(cos(b*x+a))/b

Rubi [A]

time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3855}

$$-\frac{\tanh^{-1}(\cos(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b*x],x]

[Out] -(ArcTanh[Cos[a + b*x]]/b)

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \csc(a + bx) dx = -\frac{\tanh^{-1}(\cos(a + bx))}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 38 vs. 2(12) = 24.

time = 0.01, size = 38, normalized size = 3.17

$$-\frac{\log\left(\cos\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b} + \frac{\log\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[a + b*x],x]

[Out] -(Log[Cos[a/2 + (b*x)/2]]/b) + Log[Sin[a/2 + (b*x)/2]]/b

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.96, size = 26, normalized size = 2.17

$$\text{Piecewise} \left[\left\{ \left\{ \frac{\text{Log} \left[\text{Tan} \left[\frac{a}{2} + \frac{bx}{2} \right] \right]}{b}, b \neq 0 \right\} \right\}, \frac{x}{\text{Sin}[a]} \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[1/Sin[a+b*x],x]')`

[Out] `Piecewise[{{Log[Tan[a / 2 + b x / 2]] / b, b != 0}}, x / Sin[a]]`

Maple [A]

time = 0.03, size = 21, normalized size = 1.75

method	result	size
norman	$\frac{\ln\left(\tan\left(\frac{bx}{2} + \frac{a}{2}\right)\right)}{b}$	15
derivativedivides	$\frac{\ln(\csc(bx+a) - \cot(bx+a))}{b}$	21
default	$\frac{\ln(\csc(bx+a) - \cot(bx+a))}{b}$	21
risch	$-\frac{\ln(e^{i(bx+a)} + 1)}{b} + \frac{\ln(e^{i(bx+a)} - 1)}{b}$	35

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/sin(b*x+a),x,method=_RETURNVERBOSE)`

[Out] `1/b*ln(csc(b*x+a)-cot(b*x+a))`

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

time = 0.26, size = 26, normalized size = 2.17

$$-\frac{\log(\cos(bx+a)+1) - \log(\cos(bx+a)-1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sin(b*x+a),x, algorithm="maxima")`

[Out] `-1/2*(log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(12) = 24$.

time = 0.34, size = 30, normalized size = 2.50

$$-\frac{\log\left(\frac{1}{2} \cos(bx+a) + \frac{1}{2}\right) - \log\left(-\frac{1}{2} \cos(bx+a) + \frac{1}{2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sin(b*x+a),x, algorithm="fricas")`

[Out] $-1/2*(\log(1/2*\cos(b*x + a) + 1/2) - \log(-1/2*\cos(b*x + a) + 1/2))/b$

Sympy [A]

time = 0.28, size = 17, normalized size = 1.42

$$\begin{cases} \frac{\log\left(\tan\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b} & \text{for } b \neq 0 \\ \frac{x}{\sin(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sin(b*x+a),x)`

[Out] `Piecewise((log(tan(a/2 + b*x/2))/b, Ne(b, 0)), (x/sin(a), True))`

Giac [A]

time = 0.00, size = 17, normalized size = 1.42

$$\frac{2 \ln \left| \tan \left(\frac{bx+a}{2} \right) \right|}{b \cdot 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sin(b*x+a),x)`

[Out] $\log(\text{abs}(\tan(1/2*b*x + 1/2*a)))/b$

Mupad [B]

time = 0.12, size = 12, normalized size = 1.00

$$-\frac{\text{atanh}(\cos(a + bx))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/sin(a + b*x),x)`

[Out] $-\text{atanh}(\cos(a + b*x))/b$

3.112 $\int \sec(a + bx) dx$

Optimal. Leaf size=11

$$\frac{\tanh^{-1}(\sin(a + bx))}{b}$$

[Out] arctanh(sin(b*x+a))/b

Rubi [A]

time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3855}

$$\frac{\tanh^{-1}(\sin(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b*x],x]

[Out] ArcTanh[Sin[a + b*x]]/b

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \sec(a + bx) dx = \frac{\tanh^{-1}(\sin(a + bx))}{b}$$

Mathematica [A]

time = 0.00, size = 11, normalized size = 1.00

$$\frac{\tanh^{-1}(\sin(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b*x],x]

[Out] ArcTanh[Sin[a + b*x]]/b

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.37, size = 43, normalized size = 3.91

$$\text{Piecewise} \left[\left\{ \left\{ \frac{\text{Log} \left[1 + \text{Tan} \left[\frac{a}{2} + \frac{bx}{2} \right] \right] - \text{Log} \left[-1 + \text{Tan} \left[\frac{a}{2} + \frac{bx}{2} \right] \right]}{b}, b \neq 0 \right\} \right\}, \frac{x}{\text{Cos}[a]} \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[1/Cos[a+b*x],x]')`

[Out] `Piecewise[{{(Log[1 + Tan[a / 2 + b x / 2]] - Log[-1 + Tan[a / 2 + b x / 2]]) / b, b != 0}}, x / Cos[a]]`

Maple [A]

time = 0.03, size = 19, normalized size = 1.73

method	result	size
derivativedivides	$\frac{\ln(\sec(bx+a)+\tan(bx+a))}{b}$	19
default	$\frac{\ln(\sec(bx+a)+\tan(bx+a))}{b}$	19
norman	$\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b} - \frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}$	35
risch	$\frac{\ln(e^{i(bx+a)}+i)}{b} - \frac{\ln(e^{i(bx+a)}-i)}{b}$	37

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/cos(b*x+a),x,method=_RETURNVERBOSE)`

[Out] `1/b*ln(sec(b*x+a)+tan(b*x+a))`

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(11) = 22.

time = 0.27, size = 26, normalized size = 2.36

$$\frac{\log(\sin(bx+a)+1) - \log(\sin(bx+a)-1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/cos(b*x+a),x, algorithm="maxima")`

[Out] `1/2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(11) = 22.

time = 0.34, size = 28, normalized size = 2.55

$$\frac{\log(\sin(bx+a)+1) - \log(-\sin(bx+a)+1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(b*x+a),x, algorithm="fricas")

[Out] 1/2*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b

Sympy [A]

time = 0.31, size = 34, normalized size = 3.09

$$\begin{cases} -\frac{\log\left(\tan\left(\frac{a}{2} + \frac{bx}{2}\right) - 1\right)}{b} + \frac{\log\left(\tan\left(\frac{a}{2} + \frac{bx}{2}\right) + 1\right)}{b} & \text{for } b \neq 0 \\ \frac{x}{\cos(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(b*x+a),x)

[Out] Piecewise((-log(tan(a/2 + b*x/2) - 1)/b + log(tan(a/2 + b*x/2) + 1)/b, Ne(b, 0)), (x/cos(a), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(11) = 22.

time = 0.00, size = 48, normalized size = 4.36

$$\frac{2 \left(-\frac{\ln\left|\sin(a+bx) + \frac{1}{\sin(a+bx)} - 2\right|}{8} + \frac{\ln\left|\sin(a+bx) + \frac{1}{\sin(a+bx)} + 2\right|}{8} \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(b*x+a),x)

[Out] 1/4*(log(abs(1/sin(b*x + a) + sin(b*x + a) + 2)) - log(abs(1/sin(b*x + a) + sin(b*x + a) - 2)))/b

Mupad [B]

time = 0.02, size = 11, normalized size = 1.00

$$\frac{\operatorname{atanh}(\sin(a + bx))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(a + b*x),x)

[Out] atanh(sin(a + b*x))/b

3.113 $\int \sin^2(a + bx) dx$

Optimal. Leaf size=25

$$\frac{x}{2} - \frac{\cos(a + bx) \sin(a + bx)}{2b}$$

[Out] 1/2*x-1/2*cos(b*x+a)*sin(b*x+a)/b

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2715, 8}

$$\frac{x}{2} - \frac{\sin(a + bx) \cos(a + bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b*x]^2,x]

[Out] x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*SIN[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \sin^2(a + bx) dx &= -\frac{\cos(a + bx) \sin(a + bx)}{2b} + \frac{\int 1 dx}{2} \\ &= \frac{x}{2} - \frac{\cos(a + bx) \sin(a + bx)}{2b} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 23, normalized size = 0.92

$$-\frac{-2(a + bx) + \sin(2(a + bx))}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b*x]^2,x]

[Out] $-1/4*(-2*(a + b*x) + \text{Sin}[2*(a + b*x)])/b$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.24, size = 32, normalized size = 1.28

$$\text{Piecewise} \left[\left\{ \left\{ \frac{\frac{bx}{2} - \frac{\text{Sin}[2a+2bx]}{4}}{b}, b \neq 0 \right\} \right\}, x \text{Sin}[a]^2 \right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sin[a+b*x]^2,x]')

[Out] Piecewise[{{(b x / 2 - Sin[2 a + 2 b x] / 4) / b, b != 0}}, x Sin[a]^2]

Maple [A]

time = 0.03, size = 27, normalized size = 1.08

method	result	size
risch	$\frac{x}{2} - \frac{\sin(2bx+2a)}{4b}$	19
derivativedivides	$-\frac{\cos(bx+a)\sin(bx+a) + \frac{bx}{2} + \frac{a}{2}}{b}$	27
default	$-\frac{\cos(bx+a)\sin(bx+a) + \frac{bx}{2} + \frac{a}{2}}{b}$	27
norman	$\frac{\tan^3\left(\frac{bx}{2} + \frac{a}{2}\right) + x\left(\tan^2\left(\frac{bx}{2} + \frac{a}{2}\right)\right) + \frac{x}{2} - \frac{\tan\left(\frac{bx}{2} + \frac{a}{2}\right)}{b} + x\left(\tan^4\left(\frac{bx}{2} + \frac{a}{2}\right)\right)}{\left(1 + \tan^2\left(\frac{bx}{2} + \frac{a}{2}\right)\right)^2}$	77

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] $1/b*(-1/2*\cos(b*x+a)*\sin(b*x+a)+1/2*b*x+1/2*a)$

Maxima [A]

time = 0.26, size = 24, normalized size = 0.96

$$\frac{2bx + 2a - \sin(2bx + 2a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)^2,x, algorithm="maxima")

[Out] $1/4*(2*b*x + 2*a - \sin(2*b*x + 2*a))/b$

Fricas [A]

time = 0.32, size = 23, normalized size = 0.92

$$\frac{bx - \cos(bx + a) \sin(bx + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(b*x+a)^2,x, algorithm="fricas")``[Out] 1/2*(b*x - cos(b*x + a)*sin(b*x + a))/b`**Sympy [A]**

time = 0.10, size = 46, normalized size = 1.84

$$\begin{cases} \frac{x \sin^2(a+bx)}{2} + \frac{x \cos^2(a+bx)}{2} - \frac{\sin(a+bx) \cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x \sin^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(b*x+a)**2,x)``[Out] Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*sin(a)**2, True))`**Giac [A]**

time = 0.00, size = 24, normalized size = 0.96

$$\frac{x}{2} - \frac{4b \sin(2a + 2bx)}{(4b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(b*x+a)^2,x)``[Out] 1/2*x - 1/4*sin(2*b*x + 2*a)/b`**Mupad [B]**

time = 0.21, size = 18, normalized size = 0.72

$$\frac{x}{2} - \frac{\sin(2a + 2bx)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sin(a + b*x)^2,x)``[Out] x/2 - sin(2*a + 2*b*x)/(4*b)`

3.114 $\int \sin^3(a + bx) dx$

Optimal. Leaf size=27

$$-\frac{\cos(a + bx)}{b} + \frac{\cos^3(a + bx)}{3b}$$

[Out] $-\cos(b*x+a)/b+1/3*\cos(b*x+a)^3/b$

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2713}

$$\frac{\cos^3(a + bx)}{3b} - \frac{\cos(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b*x]^3,x]

[Out] $-(\text{Cos}[a + b*x]/b) + \text{Cos}[a + b*x]^3/(3*b)$

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned} \int \sin^3(a + bx) dx &= -\frac{\text{Subst}\left(\int (1 - x^2) dx, x, \cos(a + bx)\right)}{b} \\ &= -\frac{\cos(a + bx)}{b} + \frac{\cos^3(a + bx)}{3b} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 29, normalized size = 1.07

$$-\frac{3 \cos(a + bx)}{4b} + \frac{\cos(3(a + bx))}{12b}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b*x]^3,x]

[Out] $(-3*\text{Cos}[a + b*x])/(4*b) + \text{Cos}[3*(a + b*x)]/(12*b)$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.38, size = 34, normalized size = 1.26

$$\text{Piecewise} \left[\left\{ \left\{ \frac{\text{Cos}[a + bx] (-5 + \text{Cos}[2a + 2bx])}{6b}, b \neq 0 \right\} \right\}, x \text{Sin}[a]^3 \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[Sin[a+b*x]^3,x]')`

[Out] `Piecewise[{{Cos[a + b x] (-5 + Cos[2 a + 2 b x]) / (6 b), b != 0}}, x Sin[a]^3]`

Maple [A]

time = 0.04, size = 22, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{(2+\sin^2(bx+a)) \cos(bx+a)}{3b}$	22
default	$-\frac{(2+\sin^2(bx+a)) \cos(bx+a)}{3b}$	22
risch	$-\frac{3 \cos(bx+a)}{4b} + \frac{\cos(3bx+3a)}{12b}$	27
norman	$-\frac{4 \left(\tan^2 \left(\frac{bx}{2} + \frac{a}{2} \right) \right)}{b} - \frac{4}{3b \left(1 + \tan^2 \left(\frac{bx}{2} + \frac{a}{2} \right) \right)^3}$	39

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(b*x+a)^3,x,method=_RETURNVERBOSE)`

[Out] `-1/3/b*(2+sin(b*x+a)^2)*cos(b*x+a)`

Maxima [A]

time = 0.27, size = 22, normalized size = 0.81

$$\frac{\cos(bx + a)^3 - 3 \cos(bx + a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(b*x+a)^3,x, algorithm="maxima")`

[Out] `1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b`

Fricas [A]

time = 0.32, size = 22, normalized size = 0.81

$$\frac{\cos(bx + a)^3 - 3 \cos(bx + a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)^3,x, algorithm="fricas")

[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b

Sympy [A]

time = 0.13, size = 37, normalized size = 1.37

$$\begin{cases} -\frac{\sin^2(a+bx)\cos(a+bx)}{b} - \frac{2\cos^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x \sin^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)**3,x)

[Out] Piecewise((-sin(a + b*x)**2*cos(a + b*x)/b - 2*cos(a + b*x)**3/(3*b), Ne(b, 0)), (x*sin(a)**3, True))

Giac [A]

time = 0.00, size = 29, normalized size = 1.07

$$-\frac{\cos(bx+a)}{b} + \frac{\frac{1}{3}\left(\frac{\cos(bx+a)b}{b}\right)^3}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)^3,x)

[Out] 1/3*cos(b*x + a)^3/b - cos(b*x + a)/b

Mupad [B]

time = 0.15, size = 24, normalized size = 0.89

$$-\frac{3 \cos(a + bx) - \cos(a + bx)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a + b*x)^3,x)

[Out] -(3*cos(a + b*x) - cos(a + b*x)^3)/(3*b)

3.115 $\int \cos^2(a + bx) dx$

Optimal. Leaf size=25

$$\frac{x}{2} + \frac{\cos(a + bx) \sin(a + bx)}{2b}$$

[Out] 1/2*x+1/2*cos(b*x+a)*sin(b*x+a)/b

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2715, 8}

$$\frac{\sin(a + bx) \cos(a + bx)}{2b} + \frac{x}{2}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]^2,x]

[Out] x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*Sine[c + d*x])^(n - 1)/(d*n), x] + Dist[b^2*((n - 1)/n), Int[(b*Sine[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos^2(a + bx) dx &= \frac{\cos(a + bx) \sin(a + bx)}{2b} + \frac{\int 1 dx}{2} \\ &= \frac{x}{2} + \frac{\cos(a + bx) \sin(a + bx)}{2b} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 23, normalized size = 0.92

$$\frac{2(a + bx) + \sin(2(a + bx))}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b*x]^2,x]

[Out] (2*(a + b*x) + Sin[2*(a + b*x)])/(4*b)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.58, size = 32, normalized size = 1.28

$$\text{Piecewise} \left[\left\{ \left\{ \frac{bx}{2} + \frac{\text{Sin}[2a+2bx]}{4}, b \neq 0 \right\} \right\}, x \text{Cos}[a]^2 \right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Cos[a+b*x]^2,x]')

[Out] Piecewise[{{(b x / 2 + Sin[2 a + 2 b x] / 4) / b, b != 0}}, x Cos[a]^2]

Maple [A]

time = 0.03, size = 27, normalized size = 1.08

method	result	size
risch	$\frac{x}{2} + \frac{\sin(2bx+2a)}{4b}$	19
derivativedivides	$\frac{\frac{\cos(bx+a)\sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2}}{b}$	27
default	$\frac{\frac{\cos(bx+a)\sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2}}{b}$	27
norman	$\frac{\frac{\tan\left(\frac{bx}{2} + \frac{a}{2}\right)}{b} + x \left(\tan^2\left(\frac{bx}{2} + \frac{a}{2}\right) \right) + \frac{x}{2} - \frac{\tan^3\left(\frac{bx}{2} + \frac{a}{2}\right)}{b} + x \left(\tan^4\left(\frac{bx}{2} + \frac{a}{2}\right) \right)}{\left(1 + \tan^2\left(\frac{bx}{2} + \frac{a}{2}\right)\right)^2}$	77

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)

Maxima [A]

time = 0.29, size = 22, normalized size = 0.88

$$\frac{2bx + 2a + \sin(2bx + 2a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2,x, algorithm="maxima")

[Out] 1/4*(2*b*x + 2*a + sin(2*b*x + 2*a))/b

Fricas [A]

time = 0.32, size = 22, normalized size = 0.88

$$\frac{bx + \cos(bx + a) \sin(bx + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(b*x+a)^2,x, algorithm="fricas")``[Out] 1/2*(b*x + cos(b*x + a)*sin(b*x + a))/b`**Sympy [A]**

time = 0.09, size = 46, normalized size = 1.84

$$\begin{cases} \frac{x \sin^2(a+bx)}{2} + \frac{x \cos^2(a+bx)}{2} + \frac{\sin(a+bx) \cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x \cos^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(b*x+a)**2,x)``[Out] Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 + sin(a + b*x)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*cos(a)**2, True))`**Giac [A]**

time = 0.00, size = 23, normalized size = 0.92

$$\frac{x}{2} + \frac{4b \sin(2a + 2bx)}{(4b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(b*x+a)^2,x)``[Out] 1/2*x + 1/4*sin(2*b*x + 2*a)/b`**Mupad [B]**

time = 0.18, size = 18, normalized size = 0.72

$$\frac{x}{2} + \frac{\sin(2a + 2bx)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(a + b*x)^2,x)``[Out] x/2 + sin(2*a + 2*b*x)/(4*b)`

3.116 $\int \cos^3(a + bx) dx$

Optimal. Leaf size=26

$$\frac{\sin(a + bx)}{b} - \frac{\sin^3(a + bx)}{3b}$$

[Out] sin(b*x+a)/b-1/3*sin(b*x+a)^3/b

Rubi [A]

time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2713}

$$\frac{\sin(a + bx)}{b} - \frac{\sin^3(a + bx)}{3b}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]^3,x]

[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned} \int \cos^3(a + bx) dx &= -\frac{\text{Subst}\left(\int (1 - x^2) dx, x, -\sin(a + bx)\right)}{b} \\ &= \frac{\sin(a + bx)}{b} - \frac{\sin^3(a + bx)}{3b} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 26, normalized size = 1.00

$$\frac{\sin(a + bx)}{b} - \frac{\sin^3(a + bx)}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b*x]^3,x]

[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.29, size = 34, normalized size = 1.31

$$\text{Piecewise} \left[\left\{ \left\{ \frac{(5 + \text{Cos}[2a + 2bx]) \text{Sin}[a + bx]}{6b}, b \neq 0 \right\} \right\}, x \text{Cos}[a]^3 \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[Cos[a+b*x]^3,x]')`

[Out] `Piecewise[{{(5 + Cos[2 a + 2 b x]) Sin[a + b x] / (6 b), b != 0}}, x Cos[a]^3]`

Maple [A]

time = 0.04, size = 22, normalized size = 0.85

method	result	size
derivativedivides	$\frac{(2+\cos^2(bx+a)) \sin(bx+a)}{3b}$	22
default	$\frac{(2+\cos^2(bx+a)) \sin(bx+a)}{3b}$	22
risch	$\frac{3 \sin(bx+a)}{4b} + \frac{\sin(3bx+3a)}{12b}$	27

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(b*x+a)^3,x,method=_RETURNVERBOSE)`

[Out] `1/3/b*(2+cos(b*x+a)^2)*sin(b*x+a)`

Maxima [A]

time = 0.28, size = 22, normalized size = 0.85

$$-\frac{\sin(bx+a)^3 - 3 \sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)^3,x, algorithm="maxima")`

[Out] `-1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b`

Fricas [A]

time = 0.33, size = 21, normalized size = 0.81

$$\frac{(\cos(bx+a)^2 + 2) \sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3,x, algorithm="fricas")

[Out] 1/3*(cos(b*x + a)^2 + 2)*sin(b*x + a)/b

Sympy [A]

time = 0.14, size = 36, normalized size = 1.38

$$\begin{cases} \frac{2 \sin^3(a+bx)}{3b} + \frac{\sin(a+bx) \cos^2(a+bx)}{b} & \text{for } b \neq 0 \\ x \cos^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**3,x)

[Out] Piecewise((2*sin(a + b*x)**3/(3*b) + sin(a + b*x)*cos(a + b*x)**2/b, Ne(b, 0)), (x*cos(a)**3, True))

Giac [A]

time = 0.00, size = 22, normalized size = 0.85

$$\frac{\sin(bx + a) - \frac{\sin^3(bx+a)}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3,x)

[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b

Mupad [B]

time = 0.03, size = 24, normalized size = 0.92

$$\frac{3 \sin(a + bx) - \sin(a + bx)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b*x)^3,x)

[Out] (3*sin(a + b*x) - sin(a + b*x)^3)/(3*b)

3.117 $\int \sec^2(a + bx) dx$

Optimal. Leaf size=10

$$\frac{\tan(a + bx)}{b}$$

[Out] $\tan(b*x+a)/b$

Rubi [A]

time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3852, 8}

$$\frac{\tan(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[a + b*x]^2, x]$

[Out] $\text{Tan}[a + b*x]/b$

Rule 8

$\text{Int}[a_, x_Symbol] \text{ :> Simp}[a*x, x] \text{ /; FreeQ}[a, x]$

Rule 3852

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \text{ :> Dist}[-d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] \text{ /; FreeQ}\{c, d\}, x] \ \&\& \ \text{IGtQ}[n/2, 0]$

Rubi steps

$$\begin{aligned} \int \sec^2(a + bx) dx &= -\frac{\text{Subst}(\int 1 dx, x, -\tan(a + bx))}{b} \\ &= \frac{\tan(a + bx)}{b} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tan(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b*x]^2,x]

[Out] Tan[a + b*x]/b

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.45, size = 76, normalized size = 7.60

$$\text{Piecewise} \left[\left\{ \left\{ \text{DirectedInfinity}[x], b == 0 \mid a == -\frac{\text{Pi}}{2} - bx \ \&\& \ a == -\frac{\text{Pi}}{2} \mid a == -\frac{\text{Pi}}{2} - bx \right\}, \left\{ \frac{x}{\text{Cos}[a]^2}, b == 0 \right\} \right\}, \frac{-2 \text{Tan} \left[\frac{a}{2} + \frac{bx}{2} \right]}{-b + b \text{Tan} \left[\frac{a}{2} + \frac{bx}{2} \right]^2} \right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/Cos[a+b*x]^2,x]')

[Out] Piecewise[{{DirectedInfinity[x], b == 0 || a == -Pi / 2 - b x && a == -Pi / 2 || a == -Pi / 2 - b x}, {x / Cos[a]^2, b == 0}}, -2 Tan[a / 2 + b x / 2] / (-b + b Tan[a / 2 + b x / 2]^2)]

Maple [A]

time = 0.03, size = 11, normalized size = 1.10

method	result	size
derivativedivides	$\frac{\tan(bx+a)}{b}$	11
default	$\frac{\tan(bx+a)}{b}$	11
risch	$\frac{2i}{b(e^{2i(bx+a)}+1)}$	20
norman	$-\frac{2 \tan\left(\frac{bx}{2} + \frac{a}{2}\right)}{b\left(\tan^2\left(\frac{bx}{2} + \frac{a}{2}\right) - 1\right)}$	30

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] tan(b*x+a)/b

Maxima [A]

time = 0.27, size = 10, normalized size = 1.00

$$\frac{\tan(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(b*x+a)^2,x, algorithm="maxima")

[Out] tan(b*x + a)/b

Fricas [A]

time = 0.31, size = 18, normalized size = 1.80

$$\frac{\sin(bx + a)}{b \cos(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/cos(b*x+a)^2,x, algorithm="fricas")``[Out] sin(b*x + a)/(b*cos(b*x + a))`**Sympy [A]**

time = 0.54, size = 58, normalized size = 5.80

$$\begin{cases} \tilde{\infty}x & \text{for } (a = -\frac{\pi}{2} \vee a = -bx - \frac{\pi}{2}) \wedge (a = -bx - \frac{\pi}{2} \vee b = 0) \\ \frac{x}{\cos^2(a)} & \text{for } b = 0 \\ -\frac{2 \tan\left(\frac{a}{2} + \frac{bx}{2}\right)}{b \tan^2\left(\frac{a}{2} + \frac{bx}{2}\right) - b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/cos(b*x+a)**2,x)``[Out] Piecewise((zoo*x, (Eq(b, 0) | Eq(a, -b*x - pi/2)) & (Eq(a, -pi/2) | Eq(a, -b*x - pi/2))), (x/cos(a)**2, Eq(b, 0)), (-2*tan(a/2 + b*x/2)/(b*tan(a/2 + b*x/2)**2 - b), True))`**Giac [A]**

time = 0.00, size = 12, normalized size = 1.20

$$\frac{2 \tan(a + bx)}{b \cdot 2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/cos(b*x+a)^2,x)``[Out] tan(b*x + a)/b`**Mupad [B]**

time = 0.14, size = 10, normalized size = 1.00

$$\frac{\tan(a + bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/cos(a + b*x)^2,x)``[Out] tan(a + b*x)/b`

$$3.118 \quad \int \frac{1}{1+\cos(x)} dx$$

Optimal. Leaf size=9

$$\frac{\sin(x)}{1 + \cos(x)}$$

[Out] sin(x)/(1+cos(x))

Rubi [A]

time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2727}

$$\frac{\sin(x)}{\cos(x) + 1}$$

Antiderivative was successfully verified.

[In] Int[(1 + Cos[x])^(-1), x]

[Out] Sin[x]/(1 + Cos[x])

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \frac{1}{1 + \cos(x)} dx = \frac{\sin(x)}{1 + \cos(x)}$$

Mathematica [A]

time = 0.00, size = 6, normalized size = 0.67

$$\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Cos[x])^(-1), x]

[Out] Tan[x/2]

Mathics [A]

time = 1.70, size = 4, normalized size = 0.44

$$\text{Tan} \left[\frac{x}{2} \right]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(1+Cos[x]),x]')`[Out] `Tan[x / 2]`**Maple [A]**

time = 0.02, size = 5, normalized size = 0.56

method	result	size
default	$\tan\left(\frac{x}{2}\right)$	5
norman	$\tan\left(\frac{x}{2}\right)$	5
risch	$\frac{2i}{1+e^{ix}}$	13

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1+cos(x)),x,method=_RETURNVERBOSE)`[Out] `tan(1/2*x)`**Maxima [A]**

time = 0.27, size = 9, normalized size = 1.00

$$\frac{\sin(x)}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+cos(x)),x, algorithm="maxima")`[Out] `sin(x)/(cos(x) + 1)`**Fricas [A]**

time = 0.31, size = 9, normalized size = 1.00

$$\frac{\sin(x)}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+cos(x)),x, algorithm="fricas")`[Out] `sin(x)/(cos(x) + 1)`

Sympy [A]

time = 0.10, size = 3, normalized size = 0.33

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+cos(x)),x)

[Out] tan(x/2)

Giac [A]

time = 0.00, size = 9, normalized size = 1.00

$$\frac{2}{2} \tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+cos(x)),x)

[Out] tan(1/2*x)

Mupad [B]

time = 0.17, size = 4, normalized size = 0.44

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x) + 1),x)

[Out] tan(x/2)

$$3.119 \quad \int \frac{1}{1-\cos(x)} dx$$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

[Out] `-sin(x)/(1-cos(x))`

Rubi [A]

time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2727}

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

[In] `Int[(1 - Cos[x])^(-1), x]`

[Out] `-(Sin[x]/(1 - Cos[x]))`

Rule 2727

`Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

Rubi steps

$$\int \frac{1}{1-\cos(x)} dx = -\frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A]

time = 0.01, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] `Integrate[(1 - Cos[x])^(-1), x]`

[Out] `-Cot[x/2]`

Mathics [A]

time = 1.74, size = 8, normalized size = 0.67

$$-\frac{1}{\tan\left[\frac{x}{2}\right]}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(1-Cos[x]),x]')``[Out] -1 / Tan[x / 2]`**Maple [A]**

time = 0.02, size = 9, normalized size = 0.75

method	result	size
default	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
norman	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
risch	$-\frac{2i}{e^{ix}-1}$	13

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(1-cos(x)),x,method=_RETURNVERBOSE)``[Out] -1/tan(1/2*x)`**Maxima [A]**

time = 0.26, size = 10, normalized size = 0.83

$$-\frac{\cos(x) + 1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(1-cos(x)),x, algorithm="maxima")``[Out] -(cos(x) + 1)/sin(x)`**Fricas [A]**

time = 0.30, size = 10, normalized size = 0.83

$$-\frac{\cos(x) + 1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(1-cos(x)),x, algorithm="fricas")`

[Out] $-(\cos(x) + 1)/\sin(x)$

Sympy [A]

time = 0.17, size = 7, normalized size = 0.58

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-cos(x)),x)`

[Out] $-1/\tan(x/2)$

Giac [A]

time = 0.00, size = 11, normalized size = 0.92

$$-\frac{2}{2 \tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-cos(x)),x)`

[Out] $-1/\tan(1/2*x)$

Mupad [B]

time = 0.17, size = 6, normalized size = 0.50

$$-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(cos(x) - 1),x)`

[Out] $-\cot(x/2)$

$$3.120 \quad \int \frac{1}{1+\sin(x)} dx$$

Optimal. Leaf size=10

$$-\frac{\cos(x)}{1+\sin(x)}$$

[Out] $-\cos(x)/(1+\sin(x))$

Rubi [A]

time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2727}

$$-\frac{\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + \text{Sin}[x])^{-1}, x]$

[Out] $-(\text{Cos}[x]/(1 + \text{Sin}[x]))$

Rule 2727

$\text{Int}[(a_ + (b_.) * \text{sin}[(c_.) + (d_.) * (x_)])^{-1}, x_Symbol] \rightarrow \text{Simp}[-\text{Cos}[c + d * x] / (d * (b + a * \text{Sin}[c + d * x])), x] /;$ FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \frac{1}{1+\sin(x)} dx = -\frac{\cos(x)}{1+\sin(x)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

time = 0.01, size = 23, normalized size = 2.30

$$\frac{2 \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(1 + \text{Sin}[x])^{-1}, x]$

[Out] $(2 * \text{Sin}[x/2]) / (\text{Cos}[x/2] + \text{Sin}[x/2])$

Mathics [A]

time = 1.76, size = 10, normalized size = 1.00

$$\frac{-2}{1 + \operatorname{Tan}\left[\frac{x}{2}\right]}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(1+Sin[x]),x]')``[Out] -2 / (1 + Tan[x / 2])`**Maple [A]**

time = 0.02, size = 11, normalized size = 1.10

method	result	size
default	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)}$	11
norman	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)}$	11
risch	$-\frac{2}{e^{ix}+i}$	13

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(sin(x)+1),x,method=_RETURNVERBOSE)``[Out] -2/(1+tan(1/2*x))`**Maxima [A]**

time = 0.25, size = 15, normalized size = 1.50

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(1+sin(x)),x, algorithm="maxima")``[Out] -2/(sin(x)/(cos(x) + 1) + 1)`**Fricas [A]**

time = 0.30, size = 18, normalized size = 1.80

$$\frac{\cos(x) - \sin(x) + 1}{\cos(x) + \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(1+sin(x)),x, algorithm="fricas")`

[Out] $-(\cos(x) - \sin(x) + 1)/(\cos(x) + \sin(x) + 1)$

Sympy [A]

time = 0.19, size = 8, normalized size = 0.80

$$-\frac{2}{\tan\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+sin(x)),x)`

[Out] $-2/(\tan(x/2) + 1)$

Giac [A]

time = 0.00, size = 11, normalized size = 1.10

$$-\frac{2}{\tan\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+sin(x)),x)`

[Out] $-2/(\tan(1/2*x) + 1)$

Mupad [B]

time = 0.02, size = 10, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(sin(x) + 1),x)`

[Out] $-2/(\tan(x/2) + 1)$

$$3.121 \quad \int \frac{1}{1-\sin(x)} dx$$

Optimal. Leaf size=11

$$\frac{\cos(x)}{1-\sin(x)}$$

[Out] cos(x)/(1-sin(x))

Rubi [A]

time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2727}

$$\frac{\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sin[x])^(-1),x]

[Out] Cos[x]/(1 - Sin[x])

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \frac{1}{1-\sin(x)} dx = \frac{\cos(x)}{1-\sin(x)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

time = 0.01, size = 25, normalized size = 2.27

$$\frac{2 \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sin[x])^(-1),x]

[Out] (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])

Mathics [A]

time = 1.83, size = 10, normalized size = 0.91

$$\frac{-2}{-1 + \operatorname{Tan}\left[\frac{x}{2}\right]}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(1-Sin[x]),x]')``[Out] -2 / (-1 + Tan[x / 2])`**Maple [A]**

time = 0.03, size = 11, normalized size = 1.00

method	result	size
default	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	11
norman	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	11
risch	$\frac{2}{e^{ix}-i}$	13

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(1-sin(x)),x,method=_RETURNVERBOSE)``[Out] -2/(tan(1/2*x)-1)`**Maxima [A]**

time = 0.30, size = 15, normalized size = 1.36

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(1-sin(x)),x, algorithm="maxima")``[Out] -2/(sin(x)/(cos(x) + 1) - 1)`**Fricas [A]**

time = 0.30, size = 17, normalized size = 1.55

$$\frac{\cos(x) + \sin(x) + 1}{\cos(x) - \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(1-sin(x)),x, algorithm="fricas")`

[Out] $(\cos(x) + \sin(x) + 1)/(\cos(x) - \sin(x) + 1)$

Sympy [A]

time = 0.19, size = 8, normalized size = 0.73

$$-\frac{2}{\tan\left(\frac{x}{2}\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-sin(x)),x)`

[Out] $-2/(\tan(x/2) - 1)$

Giac [A]

time = 0.00, size = 11, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-sin(x)),x)`

[Out] $-2/(\tan(1/2*x) - 1)$

Mupad [B]

time = 0.03, size = 10, normalized size = 0.91

$$-\frac{2}{\tan\left(\frac{x}{2}\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(sin(x) - 1),x)`

[Out] $-2/(\tan(x/2) - 1)$

3.122 $\int \frac{1}{a+b \sin(x)} dx$

Optimal. Leaf size=40

$$\frac{2 \tan^{-1} \left(\frac{b+a \tan(\frac{x}{2})}{\sqrt{a^2 - b^2}} \right)}{\sqrt{a^2 - b^2}}$$

[Out] 2*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2739, 632, 210}

$$\frac{2 \tan^{-1} \left(\frac{a \tan(\frac{x}{2})+b}{\sqrt{a^2 - b^2}} \right)}{\sqrt{a^2 - b^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[x])^(-1),x]

[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a + b \sin(x)} dx &= 2 \text{Subst} \left(\int \frac{1}{a + 2bx + ax^2} dx, x, \tan\left(\frac{x}{2}\right) \right) \\
&= - \left(4 \text{Subst} \left(\int \frac{1}{-4(a^2 - b^2) - x^2} dx, x, 2b + 2a \tan\left(\frac{x}{2}\right) \right) \right) \\
&= \frac{2 \tan^{-1} \left(\frac{b + a \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2}} \right)}{\sqrt{a^2 - b^2}}
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 40, normalized size = 1.00

$$\frac{2 \tan^{-1} \left(\frac{b + a \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2}} \right)}{\sqrt{a^2 - b^2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + b*Sin[x])^(-1),x]``[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 6.88, size = 176, normalized size = 4.40

$$\text{Piecewise} \left[\left\{ \left\{ \text{DirectedInfinity} \left[\text{Log} \left[\text{Tan} \left[\frac{x}{2} \right] \right] \right], a == 0 \&\& b == 0 \right\}, \left\{ \frac{2\sqrt{b^2}}{b(b \text{Tan} \left[\frac{x}{2} \right] - \sqrt{b^2})}, a == -\sqrt{b^2} \right\}, \left\{ \frac{-2\sqrt{b^2}}{b(b \text{Tan} \left[\frac{x}{2} \right] + \sqrt{b^2})}, a == \sqrt{b^2} \right\}, \left\{ \frac{\text{Log} \left[\text{Tan} \left[\frac{x}{2} \right] \right]}{b}, a == 0 \right\} \right\}, -\frac{\text{Log} \left[\frac{\sqrt{-a^2 + b^2} + \frac{b}{a} + \text{Tan} \left[\frac{x}{2} \right]}{\sqrt{-a^2 + b^2}} \right]}{\sqrt{-a^2 + b^2}} + \frac{\text{Log} \left[\frac{-\sqrt{-a^2 + b^2} + \frac{b}{a} + \text{Tan} \left[\frac{x}{2} \right]}{\sqrt{-a^2 + b^2}} \right]}{\sqrt{-a^2 + b^2}} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(a+b*Sin[x]),x]')`

```
[Out] Piecewise[{{DirectedInfinity[Log[Tan[x / 2]]], a == 0 && b == 0}, {2 Sqrt[b
^ 2] / (b (b Tan[x / 2] - Sqrt[b ^ 2])), a == -Sqrt[b ^ 2]}, {-2 Sqrt[b ^
2] / (b (b Tan[x / 2] + Sqrt[b ^ 2])), a == Sqrt[b ^ 2]}, {Log[Tan[x / 2]]
/ b, a == 0}}, -Log[Sqrt[-a ^ 2 + b ^ 2] / a + b / a + Tan[x / 2]] / Sqrt[-
a ^ 2 + b ^ 2] + Log[-Sqrt[-a ^ 2 + b ^ 2] / a + b / a + Tan[x / 2]] / Sqrt
[-a ^ 2 + b ^ 2]]
```

Maple [A]

time = 0.05, size = 39, normalized size = 0.98

method	result	size
--------	--------	------

default	$\frac{2 \arctan\left(\frac{2a \tan\left(\frac{x}{2}\right) + 2b}{2\sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2}}$	39
risch	$-\frac{\ln\left(\frac{e^{ix} + ia\sqrt{-a^2 + b^2} - a^2 + b^2}{b\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}} + \frac{\ln\left(\frac{e^{ix} + ia\sqrt{-a^2 + b^2} + a^2 - b^2}{b\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}}$	119

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*sin(x)),x,method=_RETURNVERBOSE)`

[Out] $2/(a^2-b^2)^{(1/2)}*\arctan(1/2*(2*a*\tan(1/2*x)+2*b)/(a^2-b^2)^{(1/2)})$

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*sin(x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more details)

Fricas [A]

time = 0.32, size = 148, normalized size = 3.70

$$\left[-\frac{\sqrt{-a^2 + b^2} \log\left(\frac{(2a^2 - b^2)\cos(x)^2 - 2ab\sin(x) - a^2 - b^2 + (a\cos(x)\sin(x) + b\cos(x))\sqrt{-a^2 + b^2}}{b^2\cos(x)^2 - 2ab\sin(x) - a^2 - b^2}\right)}{2(a^2 - b^2)}, -\frac{\arctan\left(\frac{-a\sin(x) + b}{\sqrt{a^2 - b^2}\cos(x)}\right)}{\sqrt{a^2 - b^2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*sin(x)),x, algorithm="fricas")`

[Out] $[-1/2*\sqrt{-a^2 + b^2}*\log(((2*a^2 - b^2)*\cos(x)^2 - 2*a*b*\sin(x) - a^2 - b^2 + 2*(a*\cos(x)*\sin(x) + b*\cos(x))*\sqrt{-a^2 + b^2}))/((b^2*\cos(x)^2 - 2*a*b*\sin(x) - a^2 - b^2)))/(a^2 - b^2), -\arctan(-(a*\sin(x) + b)/(\sqrt{a^2 - b^2}*\cos(x)))/\sqrt{a^2 - b^2}]$

Sympy [A]

time = 2.96, size = 141, normalized size = 3.52

$$\left\{ \begin{array}{ll} \infty \log\left(\tan\left(\frac{x}{2}\right)\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b} & \text{for } a = 0 \\ \frac{2\sqrt{b^2}}{b^2 \tan\left(\frac{x}{2}\right) - b\sqrt{b^2}} & \text{for } a = -\sqrt{b^2} \\ -\frac{2\sqrt{b^2}}{b^2 \tan\left(\frac{x}{2}\right) + b\sqrt{b^2}} & \text{for } a = \sqrt{b^2} \\ \frac{\log\left(\tan\left(\frac{x}{2}\right) + \frac{b}{a} - \frac{\sqrt{-a^2 + b^2}}{a}\right)}{\sqrt{-a^2 + b^2}} - \frac{\log\left(\tan\left(\frac{x}{2}\right) + \frac{b}{a} + \frac{\sqrt{-a^2 + b^2}}{a}\right)}{\sqrt{-a^2 + b^2}} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sin(x)),x)

[Out] Piecewise((zoo*log(tan(x/2))), Eq(a, 0) & Eq(b, 0)), (log(tan(x/2))/b, Eq(a, 0)), (2*sqrt(b**2)/(b**2*tan(x/2) - b*sqrt(b**2)), Eq(a, -sqrt(b**2))), (-2*sqrt(b**2)/(b**2*tan(x/2) + b*sqrt(b**2)), Eq(a, sqrt(b**2))), (log(tan(x/2) + b/a - sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2) - log(tan(x/2) + b/a + sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2), True))

Giac [A]

time = 0.00, size = 57, normalized size = 1.42

$$\frac{2 \cdot 2 \left(\arctan\left(\frac{a \tan\left(\frac{x}{2}\right) + b}{\sqrt{a^2 - b^2}}\right) + \pi \operatorname{sign}(a) \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \right)}{2\sqrt{a^2 - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sin(x)),x)

[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/sqrt(a^2 - b^2)

Mupad [B]

time = 0.38, size = 45, normalized size = 1.12

$$\frac{2 \operatorname{atan}\left(\frac{b}{\sqrt{a^2 - b^2}} + \frac{a \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*sin(x)),x)

[Out] (2*atan(b/(a^2 - b^2)^(1/2) + (a*tan(x/2))/(a^2 - b^2)^(1/2)))/(a^2 - b^2)^(1/2)

$$3.123 \quad \int \frac{1}{a + \cos(x) + b \sin(x)} dx$$

Optimal. Leaf size=47

$$-\frac{2 \tanh^{-1} \left(\frac{b - (1-a) \tan\left(\frac{x}{2}\right)}{\sqrt{1 - a^2 + b^2}} \right)}{\sqrt{1 - a^2 + b^2}}$$

[Out] -2*arctanh((b-(1-a)*tan(1/2*x))/(-a^2+b^2+1)^(1/2))/(-a^2+b^2+1)^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3203, 632, 212}

$$-\frac{2 \tanh^{-1} \left(\frac{b - (1-a) \tan\left(\frac{x}{2}\right)}{\sqrt{-a^2 + b^2 + 1}} \right)}{\sqrt{-a^2 + b^2 + 1}}$$

Antiderivative was successfully verified.

[In] Int[(a + Cos[x] + b*Sin[x])^(-1),x]

[Out] (-2*ArcTanh[(b - (1 - a)*Tan[x/2])/Sqrt[1 - a^2 + b^2]]/Sqrt[1 - a^2 + b^2])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3203

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{a + \cos(x) + b \sin(x)} dx &= 2 \text{Subst} \left(\int \frac{1}{1 + a + 2bx + (-1 + a)x^2} dx, x, \tan\left(\frac{x}{2}\right) \right) \\
&= - \left(4 \text{Subst} \left(\int \frac{1}{4(1 - a^2 + b^2) - x^2} dx, x, 2b + 2(-1 + a) \tan\left(\frac{x}{2}\right) \right) \right) \\
&= - \frac{2 \tanh^{-1} \left(\frac{b - (1 - a) \tan\left(\frac{x}{2}\right)}{\sqrt{1 - a^2 + b^2}} \right)}{\sqrt{1 - a^2 + b^2}}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 44, normalized size = 0.94

$$\frac{2 \tan^{-1} \left(\frac{b + (-1 + a) \tan\left(\frac{x}{2}\right)}{\sqrt{-1 + a^2 - b^2}} \right)}{\sqrt{-1 + a^2 - b^2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + Cos[x] + b*Sin[x])^(-1),x]``[Out] (2*ArcTan[(b + (-1 + a)*Tan[x/2])/Sqrt[-1 + a^2 - b^2]])/Sqrt[-1 + a^2 - b^2]`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 242.83, size = 477, normalized size = 10.15

$$\text{Piecewise}\left[\left\{\left\{\frac{\log\left[\frac{b + \tan\left[\frac{x}{2}\right]}{1 + b \tan\left[\frac{x}{2}\right]}\right]}{b}, a == 1\right\}, \left\{\frac{(32 + 24 b^2 \sqrt{1 + b^2} + 40 b^2 + 2 b^4 \sqrt{1 + b^2} + 10 b^4 + 32 \sqrt{1 + b^2}) \tan\left[\frac{x}{2}\right] + 48 b^2 \tan\left[\frac{x}{2}\right] - 20 b^3 - 12 b^3 \sqrt{1 + b^2} + 6 b^4 \sqrt{1 + b^2} + b^6 \tan\left[\frac{x}{2}\right] + 32 \sqrt{1 + b^2} \tan\left[\frac{x}{2}\right] + 32 \tan\left[\frac{x}{2}\right]}{(-16 b - 16 b \sqrt{1 + b^2} + 32 b^2 \sqrt{1 + b^2}) \tan\left[\frac{x}{2}\right] + 48 b^2 \tan\left[\frac{x}{2}\right] - 20 b^3 + 12 b^3 \sqrt{1 + b^2} - 6 b^4 \sqrt{1 + b^2} \tan\left[\frac{x}{2}\right] + 18 b^4 \tan\left[\frac{x}{2}\right] - 5 b^5 + b^5 \sqrt{1 + b^2} + b^6 \tan\left[\frac{x}{2}\right] - 32 \sqrt{1 + b^2}}\right\}, \left\{\frac{(32 - 24 b^2 \sqrt{1 + b^2} + 40 b^2 - 2 b^4 \sqrt{1 + b^2} + 10 b^4 - 32 \sqrt{1 + b^2}) \tan\left[\frac{x}{2}\right] + 48 b^2 \tan\left[\frac{x}{2}\right] - 20 b^3 + 12 b^3 \sqrt{1 + b^2} - 6 b^4 \sqrt{1 + b^2} \tan\left[\frac{x}{2}\right] + 18 b^4 \tan\left[\frac{x}{2}\right] - 5 b^5 + b^5 \sqrt{1 + b^2} + b^6 \tan\left[\frac{x}{2}\right] - 32 \sqrt{1 + b^2}}{(-16 b + 16 b \sqrt{1 + b^2}) \tan\left[\frac{x}{2}\right] + 48 b^2 \tan\left[\frac{x}{2}\right] - 20 b^3 + 12 b^3 \sqrt{1 + b^2} - 6 b^4 \sqrt{1 + b^2} \tan\left[\frac{x}{2}\right] + 18 b^4 \tan\left[\frac{x}{2}\right] - 5 b^5 + b^5 \sqrt{1 + b^2} + b^6 \tan\left[\frac{x}{2}\right] - 32 \sqrt{1 + b^2}}\right\}\right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(a+b*Sin[x]+Cos[x]),x]')`
`[Out] Piecewise[{{Log[1 / b + Tan[x / 2]] / b, a == 1}, {(32 + 24 b ^ 2 Sqrt[1 + b ^ 2] + 40 b ^ 2 + 2 b ^ 4 Sqrt[1 + b ^ 2] + 10 b ^ 4 + 32 Sqrt[1 + b ^ 2]) / (-16 b - 16 b Sqrt[1 + b ^ 2] + 32 b ^ 2 Sqrt[1 + b ^ 2] Tan[x / 2] + 48 b ^ 2 Tan[x / 2] - 20 b ^ 3 - 12 b ^ 3 Sqrt[1 + b ^ 2] + 6 b ^ 4 Sqrt[1 + b ^ 2] Tan[x / 2] + 18 b ^ 4 Tan[x / 2] - 5 b ^ 5 - b ^ 5 Sqrt[1 + b ^ 2] + b ^ 6 Tan[x / 2] + 32 Sqrt[1 + b ^ 2] Tan[x / 2] + 32 Tan[x / 2]), a == - Sqrt[1 + b ^ 2]}, {(32 - 24 b ^ 2 Sqrt[1 + b ^ 2] + 40 b ^ 2 - 2 b ^ 4 Sqrt[1 + b ^ 2] + 10 b ^ 4 - 32 Sqrt[1 + b ^ 2]) / (-16 b + 16 b Sqrt[1 + b ^ 2] - 32 b ^ 2 Sqrt[1 + b ^ 2] Tan[x / 2] + 48 b ^ 2 Tan[x / 2] - 20 b ^ 3 + 12 b ^ 3 Sqrt[1 + b ^ 2] - 6 b ^ 4 Sqrt[1 + b ^ 2] Tan[x / 2] + 18 b ^ 4 Tan[x / 2] - 5 b ^ 5 + b ^ 5 Sqrt[1 + b ^ 2] + b ^ 6 Tan[x / 2] - 32 Sqrt[1 +`

$b^2 \tan[x/2] + 32 \tan[x/2])$, $a = \sqrt{1 + b^2}$, $-\log[b / (-1 + a) + \tan[x/2] + \sqrt{1 - a^2 + b^2} / (-1 + a)] / \sqrt{1 - a^2 + b^2} + \log[b / (-1 + a) + \tan[x/2] - \sqrt{1 - a^2 + b^2} / (-1 + a)] / \sqrt{1 - a^2 + b^2}$

Maple [A]

time = 0.09, size = 43, normalized size = 0.91

method	result
default	$\frac{2 \arctan\left(\frac{2(a-1)\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2-1}}\right)}{\sqrt{a^2-b^2-1}}$
risch	$-\frac{\ln\left(\frac{e^{ix} + iab\sqrt{-a^2+b^2+1} + ia^2 - ib^2 - a^2b + b^3 + a\sqrt{-a^2+b^2+1} - i + b}{(b^2+1)\sqrt{-a^2+b^2+1}}\right)}{\sqrt{-a^2+b^2+1}} + \frac{\ln\left(\frac{e^{ix} + iab\sqrt{-a^2+b^2+1} - ia^2 + ib^2}{(b^2+1)\sqrt{-a^2+b^2+1}}\right)}{\sqrt{-a^2+b^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)`

[Out] $2/(a^2-b^2-1)^{1/2} \arctan(1/2*(2*(a-1)*\tan(1/2*x)+2*b)/(a^2-b^2-1)^{1/2})$

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+cos(x)+b*sin(x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(b^2-a^2+1>0)', see 'assume?' for more details)

Fricas [A]

time = 0.34, size = 287, normalized size = 6.11

$$\left[\frac{\sqrt{-a^2+b^2+1} \log\left(\frac{-b^4+(a^2+3)b^2-(2a^2b^2-b^4-2a^2+1)\cos(x)^2-a^2+(ab^2+a)\cos(x)+2(ab^3+ab-(b^2-2a^2-1)b)\cos(x)\sin(x)-2(2ab\cos(x)^2-ab+(b^2+8)\cos(x)-(b^2-(ab^2-a)\cos(x)+1)\sin(x))\sqrt{-a^2+b^2+1}+2}{(b^2-1)\cos(x)^2-a^2-b^2-2a\cos(x)-2(ab+b\cos(x))\sin(x)}\right)}{2(a^2-b^2-1)}, \arctan\left(\frac{-(ab\sin(x)+b^2+a\cos(x)+1)\sqrt{a^2-b^2-1}}{(b^2-(a^2-1)b)\cos(x)+(a^2-b^2-1)\sin(x)}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+cos(x)+b*sin(x)),x, algorithm="fricas")`

[Out] $[-1/2*\sqrt{-a^2+b^2+1}*\log(-(b^4+(a^2+3)*b^2-(2*a^2*b^2-b^4-2*a^2+1)*\cos(x)^2-a^2+2*(a*b^2+a)*\cos(x)+2*(a*b^3+a*b-(b^3-($

$$2*a^2 - 1)*b)*\cos(x))*\sin(x) - 2*(2*a*b*\cos(x)^2 - a*b + (b^3 + b)*\cos(x) - (b^2 - (a*b^2 - a)*\cos(x) + 1)*\sin(x))*\sqrt{-a^2 + b^2 + 1} + 2)/((b^2 - 1)*\cos(x)^2 - a^2 - b^2 - 2*a*\cos(x) - 2*(a*b + b*\cos(x))*\sin(x)))/(a^2 - b^2 - 1), \arctan(-(a*b*\sin(x) + b^2 + a*\cos(x) + 1)*\sqrt{a^2 - b^2 - 1})/((b^3 - (a^2 - 1)*b)*\cos(x) + (a^2 - b^2 - 1)*\sin(x)))/\sqrt{a^2 - b^2 - 1}]$$

Sympy [A]

time = 65.07, size = 1872, normalized size = 39.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+cos(x)+b*sin(x)),x)

[Out] Piecewise((log(tan(x/2) + 1/b)/b, Eq(a, 1)), (2*b**4*sqrt(b**2 + 1)/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**5 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) - 12*b**3*sqrt(b**2 + 1) - 20*b**3 + 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) - 16*b*sqrt(b**2 + 1) - 16*b + 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 10*b**4/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**5 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) - 12*b**3*sqrt(b**2 + 1) - 20*b**3 + 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) - 16*b*sqrt(b**2 + 1) - 16*b + 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 24*b**2*sqrt(b**2 + 1)/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**5 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) - 12*b**3*sqrt(b**2 + 1) - 20*b**3 + 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) - 16*b*sqrt(b**2 + 1) - 16*b + 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 40*b**2/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**5 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) - 12*b**3*sqrt(b**2 + 1) - 20*b**3 + 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) - 16*b*sqrt(b**2 + 1) - 16*b + 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 32*sqrt(b**2 + 1)/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**5 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) - 12*b**3*sqrt(b**2 + 1) - 20*b**3 + 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) - 16*b*sqrt(b**2 + 1) - 16*b + 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 32/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**5 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) - 12*b**3*sqrt(b**2 + 1) - 20*b**3 + 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) - 16*b*sqrt(b**2 + 1) - 16*b + 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)), Eq(a, -sqrt(b**2 + 1))), (-2*b**4*sqrt(b**2 + 1)/(b**6*tan(x/2) + b**5*sqrt(b**2 + 1) - 5*b**5 - 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) + 12*b**3*sqrt(b**2 + 1) - 20*b**3 - 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) + 16*b*sqrt(b**2 + 1) - 16*b - 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 10*b**4/(b**6*tan(x/2) + b**5*sqrt(b**2 + 1) - 5*b**5 - 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) + 12*b**3*sqrt(b**2 + 1) - 20*b**3 - 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) + 16*b*sqrt(b**2 + 1) - 16*b - 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) - 24*b**2*sqrt(b**2 + 1)/(b**6*tan(x/2) + b**5*sqrt(b**2 + 1) - 5*b**5 - 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) + 12*b*

```

*3*sqrt(b**2 + 1) - 20*b**3 - 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan
(x/2) + 16*b*sqrt(b**2 + 1) - 16*b - 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/
2)) + 40*b**2/(b**6*tan(x/2) + b**5*sqrt(b**2 + 1) - 5*b**5 - 6*b**4*sqrt(b
**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) + 12*b**3*sqrt(b**2 + 1) - 20*b**3 - 3
2*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) + 16*b*sqrt(b**2 + 1) - 1
6*b - 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) - 32*sqrt(b**2 + 1)/(b**6*t
an(x/2) + b**5*sqrt(b**2 + 1) - 5*b**5 - 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 1
8*b**4*tan(x/2) + 12*b**3*sqrt(b**2 + 1) - 20*b**3 - 32*b**2*sqrt(b**2 + 1)
*tan(x/2) + 48*b**2*tan(x/2) + 16*b*sqrt(b**2 + 1) - 16*b - 32*sqrt(b**2 +
1)*tan(x/2) + 32*tan(x/2)) + 32/(b**6*tan(x/2) + b**5*sqrt(b**2 + 1) - 5*b*
*5 - 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) + 12*b**3*sqrt(b**2
+ 1) - 20*b**3 - 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) + 16*b*
sqrt(b**2 + 1) - 16*b - 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)), Eq(a, sq
rt(b**2 + 1))), (log(b/(a - 1) + tan(x/2) - sqrt(-a**2 + b**2 + 1)/(a - 1))
/sqrt(-a**2 + b**2 + 1) - log(b/(a - 1) + tan(x/2) + sqrt(-a**2 + b**2 + 1)
/(a - 1))/sqrt(-a**2 + b**2 + 1), True))

```

Giac [A]

time = 0.00, size = 69, normalized size = 1.47

$$\frac{2 \cdot 2 \left(\arctan \left(\frac{a \tan\left(\frac{x}{2}\right) + b - \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2 - 1}} \right) + \pi \operatorname{sign}(2a - 2) \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \right)}{2\sqrt{a^2 - b^2 - 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+cos(x)+b*sin(x)),x)

[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*a - 2) + arctan((a*tan(1/2*x) + b - tan(1/2*x))/sqrt(a^2 - b^2 - 1)))/sqrt(a^2 - b^2 - 1)

Mupad [B]

time = 0.26, size = 58, normalized size = 1.23

$$\begin{cases} \frac{\ln(b \tan\left(\frac{x}{2}\right) + 1)}{b} & \text{if } a = 1 \\ \frac{2 \operatorname{atan}\left(\frac{b + \tan\left(\frac{x}{2}\right)(a-1)}{\sqrt{a^2 - b^2 - 1}}\right)}{\sqrt{a^2 - b^2 - 1}} & \text{if } a \neq 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + cos(x) + b*sin(x)),x)

[Out] piecewise(a == 1, log(b*tan(x/2) + 1)/b, a ~= 1, (2*atan((b + tan(x/2)*(a - 1))/(a^2 - b^2 - 1)^(1/2)))/(a^2 - b^2 - 1)^(1/2))

3.124 $\int x^2 \sin^2(a + bx) dx$

Optimal. Leaf size=73

$$-\frac{x}{4b^2} + \frac{x^3}{6} + \frac{\cos(a + bx) \sin(a + bx)}{4b^3} - \frac{x^2 \cos(a + bx) \sin(a + bx)}{2b} + \frac{x \sin^2(a + bx)}{2b^2}$$

[Out] $-1/4*x/b^2+1/6*x^3+1/4*\cos(b*x+a)*\sin(b*x+a)/b^3-1/2*x^2*\cos(b*x+a)*\sin(b*x+a)/b+1/2*x*\sin(b*x+a)^2/b^2$

Rubi [A]

time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,

Rules used = {3392, 30, 2715, 8}

$$\frac{\sin(a + bx) \cos(a + bx)}{4b^3} + \frac{x \sin^2(a + bx)}{2b^2} - \frac{x^2 \sin(a + bx) \cos(a + bx)}{2b} - \frac{x}{4b^2} + \frac{x^3}{6}$$

Antiderivative was successfully verified.

[In] `Int[x^2*Sin[a + b*x]^2,x]`

[Out] $-1/4*x/b^2 + x^3/6 + (\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(4*b^3) - (x^2*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/(2*b) + (x*\text{Sin}[a + b*x]^2)/(2*b^2)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

Rule 2715

`Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 3392

`Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2), x] + (Dist[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n), x]) /;`

FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rubi steps

$$\begin{aligned} \int x^2 \sin^2(a + bx) dx &= -\frac{x^2 \cos(a + bx) \sin(a + bx)}{2b} + \frac{x \sin^2(a + bx)}{2b^2} + \frac{\int x^2 dx}{2} - \frac{\int \sin^2(a + bx) dx}{2b^2} \\ &= \frac{x^3}{6} + \frac{\cos(a + bx) \sin(a + bx)}{4b^3} - \frac{x^2 \cos(a + bx) \sin(a + bx)}{2b} + \frac{x \sin^2(a + bx)}{2b^2} - \frac{\int 1 dx}{4b^2} \\ &= -\frac{x}{4b^2} + \frac{x^3}{6} + \frac{\cos(a + bx) \sin(a + bx)}{4b^3} - \frac{x^2 \cos(a + bx) \sin(a + bx)}{2b} + \frac{x \sin^2(a + bx)}{2b^2} \end{aligned}$$

Mathematica [A]

time = 0.09, size = 47, normalized size = 0.64

$$\frac{4b^3x^3 - 6bx \cos(2(a + bx)) + (3 - 6b^2x^2) \sin(2(a + bx))}{24b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sin[a + b*x]^2,x]

[Out] (4*b^3*x^3 - 6*b*x*Cos[2*(a + b*x)] + (3 - 6*b^2*x^2)*Sin[2*(a + b*x)])/(24*b^3)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 3.31, size = 69, normalized size = 0.95

$$\text{Piecewise} \left[\left[\left\{ \frac{-\frac{bx \cos[2a+2bx]}{4} - \frac{b^2x^2 \sin[2a+2bx]}{4} + \frac{b^3x^3}{6} + \frac{\sin[2a+2bx]}{8}}{b^3}, b \neq 0 \right\} \right], \frac{x^3 \sin[a]^2}{3} \right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x^2*Sin[a+b*x]^2,x]')

[Out] Piecewise[{{(-b x Cos[2 a + 2 b x] / 4 - b ^ 2 x ^ 2 Sin[2 a + 2 b x] / 4 + b ^ 3 x ^ 3 / 6 + Sin[2 a + 2 b x] / 8) / b ^ 3, b != 0}}, x ^ 3 Sin[a] ^ 2 / 3]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(63) = 126.

time = 0.04, size = 158, normalized size = 2.16

method	result
--------	--------

risch	$\frac{x^3}{6} - \frac{x \cos(2bx+2a)}{4b^2} - \frac{(2x^2b^2-1) \sin(2bx+2a)}{8b^3}$
derivativdivides	$\frac{a^2 \left(-\frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right) - 2a \left((bx+a) \left(-\frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right) - \frac{(bx+a)^2}{4} + \frac{\sin^2(bx+a)}{4} \right) + (bx+a)^2}{b^3}$
default	$\frac{a^2 \left(-\frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right) - 2a \left((bx+a) \left(-\frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right) - \frac{(bx+a)^2}{4} + \frac{\sin^2(bx+a)}{4} \right) + (bx+a)^2}{b^3}$
norman	$\frac{\frac{x^2 \left(\tan^3 \left(\frac{bx}{2} + \frac{a}{2} \right) \right)}{b} + \frac{x^3}{6} + \frac{\tan \left(\frac{bx}{2} + \frac{a}{2} \right)}{2b^3} - \frac{\tan^3 \left(\frac{bx}{2} + \frac{a}{2} \right)}{2b^3} - \frac{x}{4b^2} + \frac{x^3 \left(\tan^2 \left(\frac{bx}{2} + \frac{a}{2} \right) \right)}{3} + \frac{x^3 \left(\tan^4 \left(\frac{bx}{2} + \frac{a}{2} \right) \right)}{6} + \frac{3x \left(\tan^2 \left(\frac{bx}{2} + \frac{a}{2} \right) \right)}{2b^2} - x}{\left(1 + \tan^2 \left(\frac{bx}{2} + \frac{a}{2} \right) \right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{b^3} * (a^2 * (-1/2 * \cos(b*x+a) * \sin(b*x+a) + 1/2 * b*x + 1/2 * a) - 2 * a * ((b*x+a) * (-1/2 * \cos(b*x+a) * \sin(b*x+a) + 1/2 * b*x + 1/2 * a) - 1/4 * (b*x+a)^2 + 1/4 * \sin(b*x+a)^2) + (b*x+a)^2 * (-1/2 * \cos(b*x+a) * \sin(b*x+a) + 1/2 * b*x + 1/2 * a) - 1/2 * (b*x+a) * \cos(b*x+a)^2 + 1/4 * \cos(b*x+a) * \sin(b*x+a) + 1/4 * b*x + 1/4 * a - 1/3 * (b*x+a)^3)$

Maxima [A]

time = 0.29, size = 117, normalized size = 1.60

$$\frac{4(bx+a)^3 + 6(2bx+2a - \sin(2bx+2a))a^2 - 6(2(bx+a)^2 - 2(bx+a)\sin(2bx+2a) - \cos(2bx+2a))a - 6(bx+a)\cos(2bx+2a) - 3(2(bx+a)^2 - 1)\sin(2bx+2a)}{24b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*sin(b*x+a)^2,x, algorithm="maxima")`

[Out] $\frac{1}{24} * (4 * (b*x + a)^3 + 6 * (2 * b*x + 2 * a - \sin(2 * b*x + 2 * a)) * a^2 - 6 * (2 * (b*x + a)^2 - 2 * (b*x + a) * \sin(2 * b*x + 2 * a) - \cos(2 * b*x + 2 * a)) * a - 6 * (b*x + a) * \cos(2 * b*x + 2 * a) - 3 * (2 * (b*x + a)^2 - 1) * \sin(2 * b*x + 2 * a)) / b^3$

Fricas [A]

time = 0.32, size = 54, normalized size = 0.74

$$\frac{2b^3x^3 - 6bx \cos(bx+a)^2 - 3(2b^2x^2 - 1) \cos(bx+a) \sin(bx+a) + 3bx}{12b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*sin(b*x+a)^2,x, algorithm="fricas")`

[Out] $\frac{1}{12} * (2 * b^3 * x^3 - 6 * b * x * \cos(b * x + a)^2 - 3 * (2 * b^2 * x^2 - 1) * \cos(b * x + a) * \sin(b * x + a) + 3 * b * x) / b^3$

Sympy [A]

time = 0.21, size = 105, normalized size = 1.44

$$\begin{cases} \frac{x^3 \sin^2(a+bx)}{6} + \frac{x^3 \cos^2(a+bx)}{6} - \frac{x^2 \sin(a+bx) \cos(a+bx)}{2b} + \frac{x \sin^2(a+bx)}{4b^2} - \frac{x \cos^2(a+bx)}{4b^2} + \frac{\sin(a+bx) \cos(a+bx)}{4b^3} & \text{for } b \neq 0 \\ \frac{x^3 \sin^2(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*sin(b*x+a)**2,x)

[Out] Piecewise((x**3*sin(a + b*x)**2/6 + x**3*cos(a + b*x)**2/6 - x**2*sin(a + b*x)*cos(a + b*x)/(2*b) + x*sin(a + b*x)**2/(4*b**2) - x*cos(a + b*x)**2/(4*b**2) + sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne(b, 0)), (x**3*sin(a)**2/3, True))

Giac [A]

time = 0.00, size = 71, normalized size = 0.97

$$-\frac{16 \cdot 4x b b^3 \cos(2a + 2bx)}{(-16b^3)^2} - \frac{16(4x^2 b^2 - 2)b^3 \sin(2a + 2bx)}{(-16b^3)^2} + \frac{1}{2} \cdot \frac{1}{3} x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sin(b*x+a)^2,x)

[Out] 1/6*x^3 - 1/4*x*cos(2*b*x + 2*a)/b^2 - 1/8*(2*b^2*x^2 - 1)*sin(2*b*x + 2*a)/b^3

Mupad [B]

time = 0.14, size = 52, normalized size = 0.71

$$\frac{x^3}{6} + \frac{\sin(2a + 2bx)}{8b^3} - \frac{x \cos(2a + 2bx)}{4b^2} - \frac{x^2 \sin(2a + 2bx)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*sin(a + b*x)^2,x)

[Out] x^3/6 + sin(2*a + 2*b*x)/(8*b^3) - (x*cos(2*a + 2*b*x))/(4*b^2) - (x^2*sin(2*a + 2*b*x))/(4*b)

3.125 $\int \cos(x) \cos(2x) dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2} + \frac{1}{6} \sin(3x)$$

[Out] 1/2*sin(x)+1/6*sin(3*x)

Rubi [A]

time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4368}

$$\frac{\sin(x)}{2} + \frac{1}{6} \sin(3x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cos[2*x],x]

[Out] Sin[x]/2 + Sin[3*x]/6

Rule 4368

Int[cos[(a_.) + (b_.)*(x_.)]*cos[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\int \cos(x) \cos(2x) dx = \frac{\sin(x)}{2} + \frac{1}{6} \sin(3x)$$

Mathematica [A]

time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sin(x)}{2} + \frac{1}{6} \sin(3x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cos[2*x],x]

[Out] Sin[x]/2 + Sin[3*x]/6

Mathics [A]

time = 2.10, size = 9, normalized size = 0.60

$$\sin [x] - \frac{2\sin [x]^3}{3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Cos[x]*Cos[2*x],x]')``[Out] Sin[x] - 2 Sin[x]^3 / 3`**Maple [A]**

time = 0.04, size = 12, normalized size = 0.80

method	result	size
default	$\frac{\sin(x)}{2} + \frac{\sin(3x)}{6}$	12
risch	$\frac{\sin(x)}{2} + \frac{\sin(3x)}{6}$	12
norman	$\frac{-\frac{4 \tan(x) \left(\tan^2\left(\frac{x}{2}\right)\right)}{3} + \frac{2 \left(\tan^2(x)\right) \tan\left(\frac{x}{2}\right)}{3} + \frac{4 \tan(x)}{3} - \frac{2 \tan\left(\frac{x}{2}\right)}{3}}{\left(1 + \tan^2\left(\frac{x}{2}\right)\right) \left(1 + \tan^2(x)\right)}$	51

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x)*cos(2*x),x,method=_RETURNVERBOSE)``[Out] 1/2*sin(x)+1/6*sin(3*x)`**Maxima [A]**

time = 0.26, size = 11, normalized size = 0.73

$$\frac{1}{6} \sin(3x) + \frac{1}{2} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)*cos(2*x),x, algorithm="maxima")``[Out] 1/6*sin(3*x) + 1/2*sin(x)`**Fricas [A]**

time = 0.31, size = 12, normalized size = 0.80

$$\frac{1}{3} (2 \cos(x)^2 + 1) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)*cos(2*x),x, algorithm="fricas")`

[Out] $1/3*(2*\cos(x)^2 + 1)*\sin(x)$

Sympy [A]

time = 0.14, size = 20, normalized size = 1.33

$$-\frac{\sin(x)\cos(2x)}{3} + \frac{2\sin(2x)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(2*x),x)`

[Out] $-\sin(x)*\cos(2*x)/3 + 2*\sin(2*x)*\cos(x)/3$

Giac [A]

time = 0.00, size = 13, normalized size = 0.87

$$\frac{\sin x}{2} + \frac{\sin(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(2*x),x)`

[Out] $1/6*\sin(3*x) + 1/2*\sin(x)$

Mupad [B]

time = 0.03, size = 9, normalized size = 0.60

$$\sin(x) - \frac{2\sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(2*x)*cos(x),x)`

[Out] $\sin(x) - (2*\sin(x)^3)/3$

3.126 $\int x^2 \cos^2(a + bx) dx$

Optimal. Leaf size=73

$$-\frac{x}{4b^2} + \frac{x^3}{6} + \frac{x \cos^2(a + bx)}{2b^2} - \frac{\cos(a + bx) \sin(a + bx)}{4b^3} + \frac{x^2 \cos(a + bx) \sin(a + bx)}{2b}$$

[Out] $-1/4*x/b^2+1/6*x^3+1/2*x*\cos(b*x+a)^2/b^2-1/4*\cos(b*x+a)*\sin(b*x+a)/b^3+1/2*x^2*\cos(b*x+a)*\sin(b*x+a)/b$

Rubi [A]

time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3392, 30, 2715, 8}

$$-\frac{\sin(a + bx) \cos(a + bx)}{4b^3} + \frac{x \cos^2(a + bx)}{2b^2} + \frac{x^2 \sin(a + bx) \cos(a + bx)}{2b} - \frac{x}{4b^2} + \frac{x^3}{6}$$

Antiderivative was successfully verified.

[In] Int[x^2*Cos[a + b*x]^2,x]

[Out] $-1/4*x/b^2 + x^3/6 + (x*\text{Cos}[a + b*x]^2)/(2*b^2) - (\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/ (4*b^3) + (x^2*\text{Cos}[a + b*x]*\text{Sin}[a + b*x])/ (2*b)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3392

Int[((c_) + (d_)*(x_))^(m_)*((b_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2), x] + (Dist[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n), x]) /;

FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rubi steps

$$\begin{aligned} \int x^2 \cos^2(a + bx) dx &= \frac{x \cos^2(a + bx)}{2b^2} + \frac{x^2 \cos(a + bx) \sin(a + bx)}{2b} + \frac{\int x^2 dx}{2} - \frac{\int \cos^2(a + bx) dx}{2b^2} \\ &= \frac{x^3}{6} + \frac{x \cos^2(a + bx)}{2b^2} - \frac{\cos(a + bx) \sin(a + bx)}{4b^3} + \frac{x^2 \cos(a + bx) \sin(a + bx)}{2b} - \frac{\int 1 dx}{4b^2} \\ &= -\frac{x}{4b^2} + \frac{x^3}{6} + \frac{x \cos^2(a + bx)}{2b^2} - \frac{\cos(a + bx) \sin(a + bx)}{4b^3} + \frac{x^2 \cos(a + bx) \sin(a + bx)}{2b} \end{aligned}$$

Mathematica [A]

time = 0.08, size = 47, normalized size = 0.64

$$\frac{4b^3 x^3 + 6bx \cos(2(a + bx)) + (-3 + 6b^2 x^2) \sin(2(a + bx))}{24b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Cos[a + b*x]^2,x]

[Out] (4*b^3*x^3 + 6*b*x*Cos[2*(a + b*x)] + (-3 + 6*b^2*x^2)*Sin[2*(a + b*x)])/(24*b^3)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 3.30, size = 69, normalized size = 0.95

$$\text{Piecewise} \left[\left[\left\{ \left\{ \frac{bx \text{Cos}[2a+2bx]}{4} + \frac{b^2 x^2 \text{Sin}[2a+2bx]}{4} + \frac{b^3 x^3}{6} - \frac{\text{Sin}[2a+2bx]}{8}, b \neq 0 \right\} \right\}, \frac{x^3 \text{Cos}[a]^2}{3} \right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x^2*Cos[a+b*x]^2,x]')

[Out] Piecewise[{{(b x Cos[2 a + 2 b x] / 4 + b ^ 2 x ^ 2 Sin[2 a + 2 b x] / 4 + b ^ 3 x ^ 3 / 6 - Sin[2 a + 2 b x] / 8) / b ^ 3, b != 0}}, x ^ 3 Cos[a] ^ 2 / 3]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(63) = 126.

time = 0.04, size = 158, normalized size = 2.16

method	result
--------	--------

risch	$\frac{x^3}{6} + \frac{x \cos(2bx+2a)}{4b^2} + \frac{(2x^2b^2-1) \sin(2bx+2a)}{8b^3}$
derivativdivides	$\frac{a^2 \left(\frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right) - 2a \left((bx+a) \left(\frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right) - \frac{(bx+a)^2}{4} - \frac{(\sin^2(bx+a))}{4} \right) + (bx+a)^2 \left(\frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right)}{b^3}$
default	$\frac{a^2 \left(\frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right) - 2a \left((bx+a) \left(\frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right) - \frac{(bx+a)^2}{4} - \frac{(\sin^2(bx+a))}{4} \right) + (bx+a)^2 \left(\frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right)}{b^3}$
norman	$\frac{\frac{x^2 \tan\left(\frac{bx}{2} + \frac{a}{2}\right)}{b} + \frac{x^3}{6} - \frac{\tan\left(\frac{bx}{2} + \frac{a}{2}\right)}{2b^3} + \frac{\tan^3\left(\frac{bx}{2} + \frac{a}{2}\right)}{2b^3} + \frac{x}{4b^2} + \frac{x^3 \left(\tan^2\left(\frac{bx}{2} + \frac{a}{2}\right)\right)}{3} + \frac{x^3 \left(\tan^4\left(\frac{bx}{2} + \frac{a}{2}\right)\right)}{6} - \frac{3x \left(\tan^2\left(\frac{bx}{2} + \frac{a}{2}\right)\right)}{2b^2} + \frac{x \left(\tan^4\left(\frac{bx}{2} + \frac{a}{2}\right)\right)}{2b^2}}{\left(1 + \tan^2\left(\frac{bx}{2} + \frac{a}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cos(b*x+a)^2,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{b^3} \left(a^2 \left(\frac{1}{2} \cos(bx+a) \sin(bx+a) + \frac{1}{2} bx + \frac{1}{2} a \right) - 2a \left((bx+a) \left(\frac{1}{2} \cos(bx+a) \sin(bx+a) + \frac{1}{2} bx + \frac{1}{2} a \right) - \frac{1}{4} (bx+a)^2 - \frac{1}{4} \sin^2(bx+a) \right) + (bx+a)^2 \left(\frac{1}{2} \cos(bx+a) \sin(bx+a) + \frac{1}{2} bx + \frac{1}{2} a \right) - \frac{1}{4} bx - \frac{1}{4} a - \frac{1}{3} (bx+a)^3 \right)$

Maxima [A]

time = 0.27, size = 113, normalized size = 1.55

$$\frac{4(bx+a)^3 + 6(2bx+2a + \sin(2bx+2a))a^2 - 6(2(bx+a)^2 + 2(bx+a) \sin(2bx+2a) + \cos(2bx+2a))a + 6(bx+a) \cos(2bx+2a) + 3(2(bx+a)^2 - 1) \sin(2bx+2a)}{24b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(b*x+a)^2,x, algorithm="maxima")`

[Out] $\frac{1}{24} \left(4(bx+a)^3 + 6(2bx+2a + \sin(2bx+2a))a^2 - 6(2(bx+a)^2 + 2(bx+a) \sin(2bx+2a) + \cos(2bx+2a))a + 6(bx+a) \cos(2bx+2a) + 3(2(bx+a)^2 - 1) \sin(2bx+2a) \right) / b^3$

Fricas [A]

time = 0.31, size = 54, normalized size = 0.74

$$\frac{2b^3x^3 + 6bx \cos(bx+a)^2 + 3(2b^2x^2 - 1) \cos(bx+a) \sin(bx+a) - 3bx}{12b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(b*x+a)^2,x, algorithm="fricas")`

[Out] $\frac{1}{12} \left(2b^3x^3 + 6bx \cos(bx+a)^2 + 3(2b^2x^2 - 1) \cos(bx+a) \sin(bx+a) - 3bx \right) / b^3$

Sympy [A]

time = 0.21, size = 105, normalized size = 1.44

$$\begin{cases} \frac{x^3 \sin^2(a+bx)}{6} + \frac{x^3 \cos^2(a+bx)}{6} + \frac{x^2 \sin(a+bx) \cos(a+bx)}{2b} - \frac{x \sin^2(a+bx)}{4b^2} + \frac{x \cos^2(a+bx)}{4b^2} - \frac{\sin(a+bx) \cos(a+bx)}{4b^3} & \text{for } b \neq 0 \\ \frac{x^3 \cos^2(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*cos(b*x+a)**2,x)

[Out] Piecewise((x**3*sin(a + b*x)**2/6 + x**3*cos(a + b*x)**2/6 + x**2*sin(a + b*x)*cos(a + b*x)/(2*b) - x*sin(a + b*x)**2/(4*b**2) + x*cos(a + b*x)**2/(4*b**2) - sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne(b, 0)), (x**3*cos(a)**2/3, True))

Giac [A]

time = 0.00, size = 71, normalized size = 0.97

$$\frac{16 \cdot 4x b b^3 \cos(2a + 2bx)}{(-16b^3)^2} - \frac{16(-4x^2 b^2 + 2)b^3 \sin(2a + 2bx)}{(-16b^3)^2} + \frac{1}{2} \cdot \frac{1}{3} x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(b*x+a)^2,x)

[Out] 1/6*x^3 + 1/4*x*cos(2*b*x + 2*a)/b^2 + 1/8*(2*b^2*x^2 - 1)*sin(2*b*x + 2*a)/b^3

Mupad [B]

time = 0.10, size = 52, normalized size = 0.71

$$\frac{x^3}{6} - \frac{\sin(2a + 2bx)}{8b^3} + \frac{x \cos(2a + 2bx)}{4b^2} + \frac{x^2 \sin(2a + 2bx)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cos(a + b*x)^2,x)

[Out] x^3/6 - sin(2*a + 2*b*x)/(8*b^3) + (x*cos(2*a + 2*b*x))/(4*b^2) + (x^2*sin(2*a + 2*b*x))/(4*b)

3.127 $\int \cot^3(x) dx$

Optimal. Leaf size=14

$$-\frac{1}{2} \cot^2(x) - \log(\sin(x))$$

[Out] -1/2*cot(x)^2-ln(sin(x))

Rubi [A]

time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 3556}

$$-\frac{1}{2} \cot^2(x) - \log(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cot[x]^3,x]

[Out] -1/2*Cot[x]^2 - Log[Sin[x]]

Rule 3554

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[b*((b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3556

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \cot^3(x) dx &= -\frac{1}{2} \cot^2(x) - \int \cot(x) dx \\ &= -\frac{1}{2} \cot^2(x) - \log(\sin(x)) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2} \csc^2(x) - \log(\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]^3,x]

[Out] $-1/2*\text{Csc}[x]^2 - \text{Log}[\text{Sin}[x]]$

Mathics [A]

time = 1.90, size = 12, normalized size = 0.86

$$-\text{Log}[\text{Sin}[x]] - \frac{1}{2\text{Sin}[x]^2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/Tan[x]^3,x]')

[Out] $-\text{Log}[\text{Sin}[x]] - 1 / (2 \text{Sin}[x]^2)$

Maple [A]

time = 0.02, size = 22, normalized size = 1.57

method	result	size
derivativedivides	$\frac{\ln(1+\tan^2(x))}{2} - \frac{1}{2\tan(x)^2} - \ln(\tan(x))$	22
default	$\frac{\ln(1+\tan^2(x))}{2} - \frac{1}{2\tan(x)^2} - \ln(\tan(x))$	22
norman	$\frac{\ln(1+\tan^2(x))}{2} - \frac{1}{2\tan(x)^2} - \ln(\tan(x))$	22
risch	$ix + \frac{2e^{2ix}}{(e^{2ix}-1)^2} - \ln(e^{2ix}-1)$	32

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/tan(x)^3,x,method=_RETURNVERBOSE)

[Out] $1/2*\ln(1+\tan(x)^2)-1/2/\tan(x)^2-\ln(\tan(x))$

Maxima [A]

time = 0.29, size = 14, normalized size = 1.00

$$-\frac{1}{2\sin(x)^2} - \frac{1}{2}\log(\sin(x)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(x)^3,x, algorithm="maxima")

[Out] $-1/2/\sin(x)^2 - 1/2*\log(\sin(x)^2)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

time = 0.31, size = 31, normalized size = 2.21

$$\frac{\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)\tan(x)^2 + \tan(x)^2 + 1}{2\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(x)^3,x, algorithm="fricas")

[Out] -1/2*(log(tan(x)^2/(tan(x)^2 + 1))*tan(x)^2 + tan(x)^2 + 1)/tan(x)^2

Sympy [A]

time = 0.05, size = 14, normalized size = 1.00

$$-\log(\sin(x)) - \frac{1}{2\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(x)**3,x)

[Out] -log(sin(x)) - 1/(2*sin(x)**2)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

time = 0.00, size = 34, normalized size = 2.43

$$\frac{\ln(\tan^2 x + 1)}{2} - \frac{\ln(\tan^2 x)}{2} + \frac{\tan^2 x - 1}{2\tan^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(x)^3,x)

[Out] 1/2*(tan(x)^2 - 1)/tan(x)^2 + 1/2*log(tan(x)^2 + 1) - 1/2*log(tan(x)^2)

Mupad [B]

time = 0.17, size = 21, normalized size = 1.50

$$\frac{\ln(\tan(x)^2 + 1)}{2} - \ln(\tan(x)) - \frac{1}{2\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/tan(x)^3,x)

[Out] log(tan(x)^2 + 1)/2 - log(tan(x)) - 1/(2*tan(x)^2)

3.128 $\int x^3 \tan^4(x) dx$

Optimal. Leaf size=104

$$-\frac{x^2}{2} + \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2 \log(1 + e^{2ix}) + \log(\cos(x)) + 4ix \operatorname{Li}_2(-e^{2ix}) - 2\operatorname{Li}_3(-e^{2ix}) + x \tan(x) - x^3 \tan(x) - \frac{1}{2}x^2 \tan^2(x)$$

[Out] $-1/2*x^2+4/3*I*x^3+1/4*x^4-4*x^2*\ln(1+\exp(2*I*x))+\ln(\cos(x))+4*I*x*\operatorname{polylog}(2,-\exp(2*I*x))-2*\operatorname{polylog}(3,-\exp(2*I*x))+x*\tan(x)-x^3*\tan(x)-1/2*x^2*\tan(x)^2+1/3*x^3*\tan(x)^3$

Rubi [A]

time = 0.14, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3801, 3556, 30, 3800, 2221, 2611, 2320, 6724}

$$4ix \operatorname{Li}_2(-e^{2ix}) - 2\operatorname{Li}_3(-e^{2ix}) + \frac{x^4}{4} + \frac{4ix^3}{3} + \frac{1}{3}x^3 \tan^3(x) - x^3 \tan(x) - \frac{x^2}{2} - 4x^2 \log(1 + e^{2ix}) - \frac{1}{2}x^2 \tan^2(x) + x \tan(x) + \log(\cos(x))$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3*\operatorname{Tan}[x]^4, x]$

[Out] $-1/2*x^2 + ((4*I)/3)*x^3 + x^4/4 - 4*x^2*\operatorname{Log}[1 + E^((2*I)*x)] + \operatorname{Log}[\operatorname{Cos}[x]] + (4*I)*x*\operatorname{PolyLog}[2, -E^((2*I)*x)] - 2*\operatorname{PolyLog}[3, -E^((2*I)*x)] + x*\operatorname{Tan}[x] - x^3*\operatorname{Tan}[x] - (x^2*\operatorname{Tan}[x]^2)/2 + (x^3*\operatorname{Tan}[x]^3)/3$

Rule 30

$\operatorname{Int}[(x_)^m, x_Symbol] \rightarrow \operatorname{Simp}[x^{(m+1)}/(m+1), x] /; \operatorname{FreeQ}[m, x] \ \&\& \ \operatorname{NeQ}[m, -1]$

Rule 2221

$\operatorname{Int}[(((F_)^((g_)*(e_) + (f_)*(x_))))^{(n_)*((c_) + (d_)*(x_))^{(m_)} / ((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_))))^{(n_)}), x_Symbol] \rightarrow \operatorname{Simp}[((c + d*x)^m/(b*f*g*n*\operatorname{Log}[F]))*\operatorname{Log}[1 + b*((F^(g*(e + f*x)))^n/a)], x] - \operatorname{Dist}[d*(m/(b*f*g*n*\operatorname{Log}[F])), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \ \&\& \ \operatorname{IGtQ}[m, 0]$

Rule 2320

$\operatorname{Int}[u, x_Symbol] \rightarrow \operatorname{With}\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \operatorname{FunctionOfExponentialQ}[u, x] \ \&\& \ \operatorname{!MatchQ}[u, (w_)*((a_)*(v_)^n)^m] /; \operatorname{FreeQ}\{a, m, n\}, x] \ \&\& \ \operatorname{IntegerQ}[m*n] \ \&\& \ \operatorname{!MatchQ}[u, E^((c_)*((a_) + (b_)*x))* (F_)[v_] /; \operatorname{FreeQ}\{a, b, c\}, x] \ \&\& \ \operatorname{InverseFunctionQ}[F[x]]$

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
+ f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] := Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^3 \tan^4(x) dx &= \frac{1}{3}x^3 \tan^3(x) - \int x^3 \tan^2(x) dx - \int x^2 \tan^3(x) dx \\
&= -x^3 \tan(x) - \frac{1}{2}x^2 \tan^2(x) + \frac{1}{3}x^3 \tan^3(x) + 3 \int x^2 \tan(x) dx + \int x^3 dx + \int x^2 \tan(x) dx \\
&= \frac{4ix^3}{3} + \frac{x^4}{4} + x \tan(x) - x^3 \tan(x) - \frac{1}{2}x^2 \tan^2(x) + \frac{1}{3}x^3 \tan^3(x) - 2i \int \frac{e^{2ix}x^2}{1+e^{2ix}} dx - 6i \\
&= -\frac{x^2}{2} + \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2 \log(1+e^{2ix}) + \log(\cos(x)) + x \tan(x) - x^3 \tan(x) - \frac{1}{2}x^2 \tan^2(x) \\
&= -\frac{x^2}{2} + \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2 \log(1+e^{2ix}) + \log(\cos(x)) + 4ix \operatorname{Li}_2(-e^{2ix}) + x \tan(x) - x^3 \tan(x) \\
&= -\frac{x^2}{2} + \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2 \log(1+e^{2ix}) + \log(\cos(x)) + 4ix \operatorname{Li}_2(-e^{2ix}) + x \tan(x) - x^3 \tan(x) \\
&= -\frac{x^2}{2} + \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2 \log(1+e^{2ix}) + \log(\cos(x)) + 4ix \operatorname{Li}_2(-e^{2ix}) - 2\operatorname{Li}_3(-e^{2ix}) + \dots
\end{aligned}$$

Mathematica [A]

time = 0.11, size = 101, normalized size = 0.97

$$\frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2 \log(1+e^{2ix}) + \log(\cos(x)) + 4ix \operatorname{Li}_2(-e^{2ix}) - 2\operatorname{Li}_3(-e^{2ix}) - \frac{1}{2}x^2 \sec^2(x) + x \tan(x) - \frac{4}{3}x^3 \tan(x) + \frac{1}{3}x^3 \sec^2(x) \tan(x)$$

Antiderivative was successfully verified.

`[In] Integrate[x^3*Tan[x]^4,x]`

```
[Out] ((4*I)/3)*x^3 + x^4/4 - 4*x^2*Log[1 + E^((2*I)*x)] + Log[Cos[x]] + (4*I)*x*
PolyLog[2, -E^((2*I)*x)] - 2*PolyLog[3, -E^((2*I)*x)] - (x^2*Sec[x]^2)/2 +
x*Tan[x] - (4*x^3*Tan[x])/3 + (x^3*Sec[x]^2*Tan[x])/3
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[x^3*Tan[x]^4,x]')`

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]

time = 0.05, size = 138, normalized size = 1.33

method	result
risch	$\frac{x^4}{4} - \frac{2ix(6x^2e^{4ix} + 6x^2e^{2ix} - 3e^{4ix} - 3ixe^{4ix} + 4x^2 - 6e^{2ix} - 3ixe^{2ix} - 3)}{3(e^{2ix} + 1)^3} - 2\ln(e^{ix}) + \ln(e^{2ix} + 1) + \frac{8ix^3}{3} - 4x^2\ln(e$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*tan(x)^4,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{4}x^4 - \frac{2}{3}I*x*(6*x^2*\exp(4*I*x) + 6*x^2*\exp(2*I*x) - 3*\exp(4*I*x) - 3*I*x*\exp(4*I*x) + 4*x^2 - 6*\exp(2*I*x) - 3*I*x*\exp(2*I*x) - 3)/(\exp(2*I*x) + 1)^3 - 2*\ln(\exp(I*x)) + \ln(\exp(2*I*x) + 1) + 8/3*I*x^3 - 4*x^2*\ln(\exp(2*I*x) + 1) + 4*I*x*\text{polylog}(2, -\exp(2*I*x)) - 2*\text{polylog}(3, -\exp(2*I*x))$

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(80) = 160$.
time = 0.40, size = 491, normalized size = 4.72

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*tan(x)^4,x, algorithm="maxima")`

[Out] $-(3*I*x^4 + 12*(4*x^2 + (4*x^2 - 1)*\cos(6*x) + 3*(4*x^2 - 1)*\cos(4*x) + 3*(4*x^2 - 1)*\cos(2*x) - (-4*I*x^2 + I)*\sin(6*x) - 3*(-4*I*x^2 + I)*\sin(4*x) - 3*(-4*I*x^2 + I)*\sin(2*x) - 1)*\arctan2(\sin(2*x), \cos(2*x) + 1) + (3*I*x^4 - 32*x^3 + 24*x)*\cos(6*x) - 3*(-3*I*x^4 + 16*x^3 + 8*I*x^2 - 16*x)*\cos(4*x) - 3*(-3*I*x^4 + 16*x^3 + 8*I*x^2 - 8*x)*\cos(2*x) - 48*(x*\cos(6*x) + 3*x*\cos(4*x) + 3*x*\cos(2*x) + I*x*\sin(6*x) + 3*I*x*\sin(4*x) + 3*I*x*\sin(2*x) + x)*\text{dilog}(-e^{(2*I*x)}) - 6*(4*I*x^2 + (4*I*x^2 - I)*\cos(6*x) + 3*(4*I*x^2 - I)*\cos(4*x) + 3*(4*I*x^2 - I)*\cos(2*x) - (4*x^2 - 1)*\sin(6*x) - 3*(4*x^2 - 1)*\sin(4*x) - 3*(4*x^2 - 1)*\sin(2*x) - I)*\log(\cos(2*x)^2 + \sin(2*x)^2 + 2*\cos(2*x) + 1) - 24*(I*\cos(6*x) + 3*I*\cos(4*x) + 3*I*\cos(2*x) - \sin(6*x) - 3*\sin(4*x) - 3*\sin(2*x) + I)*\text{polylog}(3, -e^{(2*I*x)}) - (3*x^4 + 32*I*x^3 - 24*I*x)*\sin(6*x) - 3*(3*x^4 + 16*I*x^3 - 8*x^2 - 16*I*x)*\sin(4*x) - 3*(3*x^4 + 16*I*x^3 - 8*x^2 - 8*I*x)*\sin(2*x))/(-12*I*\cos(6*x) - 36*I*\cos(4*x) - 36*I*\cos(2*x) + 12*\sin(6*x) + 36*\sin(4*x) + 36*\sin(2*x) - 12*I)$

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 182, normalized size = 1.75

$$\frac{1}{3}x^3 \tan(x)^3 + \frac{1}{4}x^4 - \frac{1}{2}x^2 \tan(x)^2 - \frac{1}{2}x^2 - 2ix \text{Li}\left(\frac{2(i \tan(x) - 1)}{\tan(x)^2 + 1} + 1\right) + 2ix \text{Li}\left(\frac{2(-i \tan(x) - 1)}{\tan(x)^2 + 1} + 1\right) - \frac{1}{2}(4x^2 - 1) \log\left(-\frac{2(i \tan(x) - 1)}{\tan(x)^2 + 1}\right) - \frac{1}{2}(4x^2 - 1) \log\left(-\frac{2(-i \tan(x) - 1)}{\tan(x)^2 + 1}\right) - (x^2 - x) \tan(x) - \text{polylog}\left(3, \frac{\tan(x)^2 + 2i \tan(x) - 1}{\tan(x)^2 + 1}\right) - \text{polylog}\left(3, \frac{\tan(x)^2 - 2i \tan(x) - 1}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*tan(x)^4,x, algorithm="fricas")`

[Out] $\frac{1}{3}x^3 \tan(x)^3 + \frac{1}{4}x^4 - \frac{1}{2}x^2 \tan(x)^2 - \frac{1}{2}x^2 - 2Ix \operatorname{dilog}(2(I \tan(x) - 1)/(\tan(x)^2 + 1) + 1) + 2Ix \operatorname{dilog}(2(-I \tan(x) - 1)/(\tan(x)^2 + 1) + 1) - \frac{1}{2}(4x^2 - 1) \log(-2(I \tan(x) - 1)/(\tan(x)^2 + 1)) - \frac{1}{2}(4x^2 - 1) \log(-2(-I \tan(x) - 1)/(\tan(x)^2 + 1)) - (x^3 - x) \tan(x) - \operatorname{polylog}(3, (\tan(x)^2 + 2I \tan(x) - 1)/(\tan(x)^2 + 1)) - \operatorname{polylog}(3, (\tan(x)^2 - 2I \tan(x) - 1)/(\tan(x)^2 + 1))$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \tan^4(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*tan(x)**4,x)`

[Out] `Integral(x**3*tan(x)**4, x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*tan(x)^4,x)`

[Out] `Could not integrate`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \tan(x)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*tan(x)^4,x)`

[Out] `int(x^3*tan(x)^4, x)`

3.129 $\int x^3 \tan^6(x) dx$

Optimal. Leaf size=153

$$\frac{19x^2}{20} - \frac{23ix^3}{15} - \frac{x^4}{4} + \frac{23}{5}x^2 \log(1 + e^{2ix}) - 2\log(\cos(x)) - \frac{23}{5}ix\text{Li}_2(-e^{2ix}) + \frac{23}{10}\text{Li}_3(-e^{2ix}) - \frac{19}{10}x \tan(x) + x^3 \tan(x)$$

[Out] 19/20*x^2-23/15*I*x^3-1/4*x^4+23/5*x^2*ln(1+exp(2*I*x))-2*ln(cos(x))-23/5*I*x*polylog(2,-exp(2*I*x))+23/10*polylog(3,-exp(2*I*x))-19/10*x*tan(x)+x^3*tan(x)-1/20*tan(x)^2+4/5*x^2*tan(x)^2+1/10*x*tan(x)^3-1/3*x^3*tan(x)^3-3/20*x^2*tan(x)^4+1/5*x^3*tan(x)^5

Rubi [A]

time = 0.25, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.125$, Rules used = {3801, 3554, 3556, 30, 3800, 2221, 2611, 2320, 6724}

$$-\frac{23}{5}ix\text{Li}_2(-e^{2ix}) + \frac{23}{10}\text{Li}_3(-e^{2ix}) - \frac{x^4}{4} - \frac{23ix^3}{15} + \frac{1}{5}x^3 \tan^5(x) - \frac{1}{3}x^3 \tan^3(x) + x^3 \tan(x) + \frac{19x^2}{20} + \frac{23}{5}x^2 \log(1 + e^{2ix}) - \frac{3}{20}x^2 \tan^4(x) + \frac{4}{5}x^2 \tan^2(x) + \frac{1}{10}x \tan^3(x) - \frac{\tan^2(x)}{20} - \frac{19}{10}x \tan(x) - 2\log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[x^3*Tan[x]^6,x]

[Out] (19*x^2)/20 - ((23*I)/15)*x^3 - x^4/4 + (23*x^2*Log[1 + E^((2*I)*x)])/5 - 2*Log[Cos[x]] - ((23*I)/5)*x*PolyLog[2, -E^((2*I)*x)] + (23*PolyLog[3, -E^((2*I)*x)])/10 - (19*x*Tan[x])/10 + x^3*Tan[x] - Tan[x]^2/20 + (4*x^2*Tan[x]^2)/5 + (x*Tan[x]^3)/10 - (x^3*Tan[x]^3)/3 - (3*x^2*Tan[x]^4)/20 + (x^3*Tan[x]^5)/5

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2221

Int[(((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2320

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_)*(b_)*x))

$(F_)[v_]$ /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]

Rule 2611

Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))] * ((f_) + (g_) * (x_)^(m_)), x_Symbol] := Simp[(-f + g*x)^m * (PolyLog[2, (-e)*(F^(c*(a + b*x)))^n] / (b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m - 1) * PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 3554

Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[b*((b*Tan[c + d*x])^(n - 1) / (d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3556

Int[tan[(c_) + (d_)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3800

Int[((c_) + (d_)*(x_))^(m_)*tan[(e_) + (f_)*(x_)], x_Symbol] := Simp[I * ((c + d*x)^(m + 1) / (d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m * (E^(2*I*(e + f*x)) / (1 + E^(2*I*(e + f*x))))], x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 3801

Int[((c_) + (d_)*(x_))^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(c + d*x)^m * ((b*Tan[e + f*x])^(n - 1) / (f*(n - 1))), x] + (-Dist[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1) * (b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m * (b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 6724

Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)] / ((d_) + (e_)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p] / (e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned}
\int x^3 \tan^6(x) dx &= \frac{1}{5}x^3 \tan^5(x) - \frac{3}{5} \int x^2 \tan^5(x) dx - \int x^3 \tan^4(x) dx \\
&= -\frac{1}{3}x^3 \tan^3(x) - \frac{3}{20}x^2 \tan^4(x) + \frac{1}{5}x^3 \tan^5(x) + \frac{3}{10} \int x \tan^4(x) dx + \frac{3}{5} \int x^2 \tan^3(x) dx + \frac{1}{10} \int x^3 \tan^2(x) dx \\
&= x^3 \tan(x) + \frac{4}{5}x^2 \tan^2(x) + \frac{1}{10}x \tan^3(x) - \frac{1}{3}x^3 \tan^3(x) - \frac{3}{20}x^2 \tan^4(x) + \frac{1}{5}x^3 \tan^5(x) - \frac{1}{10}x^3 \tan^6(x) \\
&= -\frac{23ix^3}{15} - \frac{x^4}{4} - \frac{19}{10}x \tan(x) + x^3 \tan(x) - \frac{\tan^2(x)}{20} + \frac{4}{5}x^2 \tan^2(x) + \frac{1}{10}x \tan^3(x) - \frac{1}{3}x^3 \tan^3(x) \\
&= \frac{19x^2}{20} - \frac{23ix^3}{15} - \frac{x^4}{4} + \frac{23}{5}x^2 \log(1 + e^{2ix}) - 2 \log(\cos(x)) - \frac{19}{10}x \tan(x) + x^3 \tan(x) - \frac{\tan^2(x)}{20} \\
&= \frac{19x^2}{20} - \frac{23ix^3}{15} - \frac{x^4}{4} + \frac{23}{5}x^2 \log(1 + e^{2ix}) - 2 \log(\cos(x)) - \frac{23}{5}ix \operatorname{Li}_2(-e^{2ix}) - \frac{19}{10}x \tan(x) \\
&= \frac{19x^2}{20} - \frac{23ix^3}{15} - \frac{x^4}{4} + \frac{23}{5}x^2 \log(1 + e^{2ix}) - 2 \log(\cos(x)) - \frac{23}{5}ix \operatorname{Li}_2(-e^{2ix}) - \frac{19}{10}x \tan(x) \\
&= \frac{19x^2}{20} - \frac{23ix^3}{15} - \frac{x^4}{4} + \frac{23}{5}x^2 \log(1 + e^{2ix}) - 2 \log(\cos(x)) - \frac{23}{5}ix \operatorname{Li}_2(-e^{2ix}) + \frac{23}{10} \operatorname{Li}_3(-e^{2ix})
\end{aligned}$$

Mathematica [A]

time = 0.23, size = 133, normalized size = 0.87

$$\frac{1}{60}(-92ix^3 - 15x^4 + 276x^2 \log(1 + e^{2ix}) - 120 \log(\cos(x)) - 276ix \operatorname{Li}_2(-e^{2ix}) + 138 \operatorname{Li}_3(-e^{2ix}) - 3 \sec^2(x) + 66ix^2 \sec^2(x) - 9x^2 \sec^4(x) - 120ix \tan(x) + 92x^3 \tan(x) + 6ix \sec^2(x) \tan(x) - 44ix^3 \sec^2(x) \tan(x) + 12x^3 \sec^4(x) \tan(x))$$

Antiderivative was successfully verified.

`[In] Integrate[x^3*Tan[x]^6,x]`

```
[Out] ((-92*I)*x^3 - 15*x^4 + 276*x^2*Log[1 + E^((2*I)*x)] - 120*Log[Cos[x]] - (276*I)*x*PolyLog[2, -E^((2*I)*x)] + 138*PolyLog[3, -E^((2*I)*x)] - 3*Sec[x]^2 + 66*x^2*Sec[x]^2 - 9*x^2*Sec[x]^4 - 120*x*Tan[x] + 92*x^3*Tan[x] + 6*x*Sec[x]^2*Tan[x] - 44*x^3*Sec[x]^2*Tan[x] + 12*x^3*Sec[x]^4*Tan[x])/60
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[x^3*Tan[x]^6,x]')`

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python object
```


Maple [A]

time = 0.04, size = 237, normalized size = 1.55

method	result
risch	$-\frac{x^4}{4} + \frac{i(9ie^{6ix} + 90x^3e^{8ix} - 162ix^2e^{4ix} - 66ix^2e^{8ix} + 180x^3e^{6ix} - 66xe^{8ix} + 3ie^{2ix} + 3ie^{8ix} + 280x^3e^{4ix} - 246xe^{6ix} - 66ix^2e^{2ix} + 9ie^{4ix})}{15(e^{2ix} + 1)^5}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^3*tan(x)^6,x,method=_RETURNVERBOSE)`

```
[Out] -1/4*x^4+1/15*I*(9*I*exp(6*I*x)+90*x^3*exp(8*I*x)-162*I*x^2*exp(4*I*x)-66*I*x^2*exp(8*I*x)+180*x^3*exp(6*I*x)-66*x*exp(8*I*x)+3*I*exp(2*I*x)+3*I*exp(8*I*x)+280*x^3*exp(4*I*x)-246*x*exp(6*I*x)-66*I*x^2*exp(2*I*x)+9*I*exp(4*I*x)+140*x^3*exp(2*I*x)-354*x*exp(4*I*x)-162*I*x^2*exp(6*I*x)+46*x^3-234*x*exp(2*I*x)-60*x)/(exp(2*I*x)+1)^5+4*ln(exp(I*x))-2*ln(exp(2*I*x)+1)-46/15*I*x^3+23/5*x^2*ln(exp(2*I*x)+1)-23/5*I*x*polylog(2,-exp(2*I*x))+23/10*polylog(3,-exp(2*I*x))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 777 vs. 2(113) = 226.

time = 0.63, size = 777, normalized size = 5.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^3*tan(x)^6,x, algorithm="maxima")`

```
[Out] (15*I*x^4 + 12*(23*x^2 + (23*x^2 - 10)*cos(10*x) + 5*(23*x^2 - 10)*cos(8*x) + 10*(23*x^2 - 10)*cos(6*x) + 10*(23*x^2 - 10)*cos(4*x) + 5*(23*x^2 - 10)*cos(2*x) - (-23*I*x^2 + 10*I)*sin(10*x) - 5*(-23*I*x^2 + 10*I)*sin(8*x) - 10*(-23*I*x^2 + 10*I)*sin(6*x) - 10*(-23*I*x^2 + 10*I)*sin(4*x) - 5*(-23*I*x^2 + 10*I)*sin(2*x) - 10)*arctan2(sin(2*x), cos(2*x) + 1) + (15*I*x^4 - 184*x^3 + 240*x)*cos(10*x) + (75*I*x^4 - 560*x^3 - 264*I*x^2 + 936*x + 12*I)*cos(8*x) - 2*(-75*I*x^4 + 560*x^3 + 324*I*x^2 - 708*x - 18*I)*cos(6*x) - 6*(-25*I*x^4 + 120*x^3 + 108*I*x^2 - 164*x - 6*I)*cos(4*x) - 3*(-25*I*x^4 + 120*x^3 + 88*I*x^2 - 88*x - 4*I)*cos(2*x) - 276*(x*cos(10*x) + 5*x*cos(8*x) + 10*x*cos(6*x) + 10*x*cos(4*x) + 5*x*cos(2*x) + I*x*sin(10*x) + 5*I*x*sin(8*x) + 10*I*x*sin(6*x) + 10*I*x*sin(4*x) + 5*I*x*sin(2*x) + x)*dilog(-e^(2*I*x)) - 6*(23*I*x^2 + (23*I*x^2 - 10*I)*cos(10*x) + 5*(23*I*x^2 - 10*I)*cos(8*x) + 10*(23*I*x^2 - 10*I)*cos(6*x) + 10*(23*I*x^2 - 10*I)*cos(4*x) + 5*(23*I*x^2 - 10*I)*cos(2*x) - (23*x^2 - 10)*sin(10*x) - 5*(23*x^2 - 10)*sin(8*x) - 10*(23*x^2 - 10)*sin(6*x) - 10*(23*x^2 - 10)*sin(4*x) - 5*(23*x^2 - 10)*sin(2*x) - 10*I)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - 138*(I*cos(10*x) + 5*I*cos(8*x) + 10*I*cos(6*x) + 10*I*cos(4*x) + 5*I*cos(2*x) - sin(10*x) - 5*sin(8*x) - 10*sin(6*x) - 10*sin(4*x) - 5*sin(2*x) + I)*polylog(3, -e^(2*I*x)) - (15*x^4 + 184*I*x^3 - 240*I*x)*sin(10*x) - (75*x^4 + 560*I
```

```
*x^3 - 264*x^2 - 936*I*x + 12)*sin(8*x) - 2*(75*x^4 + 560*I*x^3 - 324*x^2 -
  708*I*x + 18)*sin(6*x) - 6*(25*x^4 + 120*I*x^3 - 108*x^2 - 164*I*x + 6)*si
n(4*x) - 3*(25*x^4 + 120*I*x^3 - 88*x^2 - 88*I*x + 4)*sin(2*x))/(-60*I*cos(
10*x) - 300*I*cos(8*x) - 600*I*cos(6*x) - 600*I*cos(4*x) - 300*I*cos(2*x) +
  60*sin(10*x) + 300*sin(8*x) + 600*sin(6*x) + 600*sin(4*x) + 300*sin(2*x) -
  60*I)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 212, normalized size = 1.39

$$\frac{1}{5}x^3 \tan(x)^5 - \frac{3}{20}x^2 \tan(x)^4 - \frac{1}{4}x^4 - \frac{1}{30}(10x^3 - 3x) \tan(x)^3 + \frac{1}{20}(16x^2 - 1) \tan(x)^2 + \frac{19}{20}x^2 + \frac{23}{10}x \operatorname{Li}\left(\frac{2(1 - \tan(x) - 1)}{\tan(x)^2 + 1}\right) - \frac{23}{10}x \operatorname{Li}\left(\frac{2(-1 - \tan(x) - 1)}{\tan(x)^2 + 1}\right) + \frac{1}{10}(23x^2 - 10) \log\left(\frac{-2(1 - \tan(x) - 1)}{\tan(x)^2 + 1}\right) + \frac{1}{10}(10x^2 - 19x) \tan(x) + \frac{23}{20} \operatorname{polylog}\left(3, \frac{\tan(x)^2 + 2 \tan(x) - 1}{\tan(x)^2 + 1}\right) + \frac{23}{20} \operatorname{polylog}\left(3, \frac{\tan(x)^2 - 2 \tan(x) - 1}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^6,x, algorithm="fricas")
```

```
[Out] 1/5*x^3*tan(x)^5 - 3/20*x^2*tan(x)^4 - 1/4*x^4 - 1/30*(10*x^3 - 3*x)*tan(x)
^3 + 1/20*(16*x^2 - 1)*tan(x)^2 + 19/20*x^2 + 23/10*I*x*dilog(2*(I*tan(x) -
  1)/(tan(x)^2 + 1) + 1) - 23/10*I*x*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1)
  + 1) + 1/10*(23*x^2 - 10)*log(-2*(I*tan(x) - 1)/(tan(x)^2 + 1)) + 1/10*(23*
  x^2 - 10)*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) + 1/10*(10*x^3 - 19*x)*tan
  (x) + 23/20*polylog(3, (tan(x)^2 + 2*I*tan(x) - 1)/(tan(x)^2 + 1)) + 23/20*
  polylog(3, (tan(x)^2 - 2*I*tan(x) - 1)/(tan(x)^2 + 1))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \tan^6(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*tan(x)**6,x)
```

```
[Out] Integral(x**3*tan(x)**6, x)
```

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^6,x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \tan(x)^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(x)^6,x)
```

```
[Out] int(x^3*tan(x)^6, x)
```

3.130 $\int x \tan^2(x) dx$

Optimal. Leaf size=15

$$-\frac{x^2}{2} + \log(\cos(x)) + x \tan(x)$$

[Out] -1/2*x^2+ln(cos(x))+x*tan(x)

Rubi [A]

time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3801, 3556, 30}

$$-\frac{x^2}{2} + x \tan(x) + \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[x*Tan[x]^2,x]

[Out] -1/2*x^2 + Log[Cos[x]] + x*Tan[x]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3556

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3801

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Dist[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rubi steps

$$\begin{aligned} \int x \tan^2(x) dx &= x \tan(x) - \int x dx - \int \tan(x) dx \\ &= -\frac{x^2}{2} + \log(\cos(x)) + x \tan(x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 15, normalized size = 1.00

$$-\frac{x^2}{2} + \log(\cos(x)) + x \tan(x)$$

Antiderivative was successfully verified.

`[In] Integrate[x*Tan[x]^2,x]``[Out] -1/2*x^2 + Log[Cos[x]] + x*Tan[x]`**Mathics [A]**

time = 2.06, size = 17, normalized size = 1.13

$$x \text{Tan}[x] - \frac{x^2}{2} - \frac{\text{Log}\left[\frac{1}{\text{Cos}[x]^2}\right]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x*Tan[x]^2,x]')``[Out] x Tan[x] - x ^ 2 / 2 - Log[1 / Cos[x] ^ 2] / 2`**Maple [A]**

time = 0.01, size = 20, normalized size = 1.33

method	result	size
norman	$x \tan(x) - \frac{x^2}{2} - \frac{\ln(1+\tan^2(x))}{2}$	20
risch	$-\frac{x^2}{2} - 2ix + \frac{2ix}{e^{2ix}+1} + \ln(e^{2ix} + 1)$	32

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*tan(x)^2,x,method=_RETURNVERBOSE)``[Out] x*tan(x)-1/2*x^2-1/2*ln(1+tan(x)^2)`**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(13) = 26.

time = 0.34, size = 107, normalized size = 7.13

$$\frac{-x^2 \cos(2x)^2 + x^2 \sin(2x)^2 + 2x^2 \cos(2x) + x^2 - (\cos(2x)^2 + \sin(2x)^2 + 2 \cos(2x) + 1) \log(\cos(2x)^2 + \sin(2x)^2 + 2 \cos(2x) + 1) - 4x \sin(2x)}{2(\cos(2x)^2 + \sin(2x)^2 + 2 \cos(2x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*tan(x)^2,x, algorithm="maxima")`

[Out] $-1/2*(x^2*\cos(2*x)^2 + x^2*\sin(2*x)^2 + 2*x^2*\cos(2*x) + x^2 - (\cos(2*x)^2 + \sin(2*x)^2 + 2*\cos(2*x) + 1)*\log(\cos(2*x)^2 + \sin(2*x)^2 + 2*\cos(2*x) + 1) - 4*x*\sin(2*x))/(\cos(2*x)^2 + \sin(2*x)^2 + 2*\cos(2*x) + 1)$

Fricas [A]

time = 0.31, size = 21, normalized size = 1.40

$$-\frac{1}{2}x^2 + x \tan(x) + \frac{1}{2} \log\left(\frac{1}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(x)^2,x, algorithm="fricas")`

[Out] $-1/2*x^2 + x*\tan(x) + 1/2*\log(1/(\tan(x)^2 + 1))$

Sympy [A]

time = 0.08, size = 19, normalized size = 1.27

$$-\frac{x^2}{2} + x \tan(x) - \frac{\log(\tan^2(x) + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(x)**2,x)`

[Out] $-x**2/2 + x*\tan(x) - \log(\tan(x)**2 + 1)/2$

Giac [A]

time = 0.01, size = 27, normalized size = 1.80

$$-\frac{1}{2}x^2 + x \tan x + \frac{1}{2} \ln\left(\frac{4}{\tan^2 x + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(x)^2,x)`

[Out] $-1/2*x^2 + x*\tan(x) + 1/2*\log(4/(\tan(x)^2 + 1))$

Mupad [B]

time = 0.02, size = 13, normalized size = 0.87

$$\ln(\cos(x)) + x \tan(x) - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*tan(x)^2,x)`

[Out] $\log(\cos(x)) + x*\tan(x) - x^2/2$

3.131 $\int \cos(3x) \sin(2x) dx$

Optimal. Leaf size=15

$$\frac{\cos(x)}{2} - \frac{1}{10} \cos(5x)$$

[Out] 1/2*cos(x)-1/10*cos(5*x)

Rubi [A]

time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4369}

$$\frac{\cos(x)}{2} - \frac{1}{10} \cos(5x)$$

Antiderivative was successfully verified.

[In] Int[Cos[3*x]*Sin[2*x],x]

[Out] Cos[x]/2 - Cos[5*x]/10

Rule 4369

Int[cos[(c_.) + (d_.)*(x_.)]*sin[(a_.) + (b_.)*(x_.)], x_Symbol] := Simp[-Cos[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\int \cos(3x) \sin(2x) dx = \frac{\cos(x)}{2} - \frac{1}{10} \cos(5x)$$

Mathematica [A]

time = 0.01, size = 15, normalized size = 1.00

$$\frac{\cos(x)}{2} - \frac{1}{10} \cos(5x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[3*x]*Sin[2*x],x]

[Out] Cos[x]/2 - Cos[5*x]/10

Mathics [A]

time = 2.11, size = 11, normalized size = 0.73

$$-\frac{\cos[5x]}{10} + \frac{\cos[x]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sin[2*x]*Cos[3*x],x]')``[Out] -Cos[5 x] / 10 + Cos[x] / 2`**Maple [A]**

time = 0.06, size = 12, normalized size = 0.80

method	result	size
default	$\frac{\cos(x)}{2} - \frac{\cos(5x)}{10}$	12
risch	$\frac{\cos(x)}{2} - \frac{\cos(5x)}{10}$	12
norman	$-\frac{4(\tan^2(x)) - 4(\tan^2(\frac{3x}{2}))}{5} + \frac{12 \tan(x) \tan(\frac{3x}{2})}{5(1+\tan^2(\frac{3x}{2}))(1+\tan^2(x))}$	43

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(3*x)*sin(2*x),x,method=_RETURNVERBOSE)``[Out] 1/2*cos(x)-1/10*cos(5*x)`**Maxima [A]**

time = 0.26, size = 11, normalized size = 0.73

$$-\frac{1}{10} \cos(5x) + \frac{1}{2} \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(3*x)*sin(2*x),x, algorithm="maxima")``[Out] -1/10*cos(5*x) + 1/2*cos(x)`**Fricas [A]**

time = 0.31, size = 13, normalized size = 0.87

$$-\frac{8}{5} \cos(x)^5 + 2 \cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(3*x)*sin(2*x),x, algorithm="fricas")``[Out] -8/5*cos(x)^5 + 2*cos(x)^3`

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$
time = 0.14, size = 26, normalized size = 1.73

$$\frac{3 \sin(2x) \sin(3x)}{5} + \frac{2 \cos(2x) \cos(3x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(3*x)*sin(2*x),x)`

[Out] `3*sin(2*x)*sin(3*x)/5 + 2*cos(2*x)*cos(3*x)/5`

Giac [A]

time = 0.00, size = 14, normalized size = 0.93

$$-\frac{\cos(5x)}{10} + \frac{\cos x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(3*x)*sin(2*x),x)`

[Out] `-1/10*cos(5*x) + 1/2*cos(x)`

Mupad [B]

time = 0.04, size = 13, normalized size = 0.87

$$2 \cos(x)^3 - \frac{8 \cos(x)^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(3*x)*sin(2*x),x)`

[Out] `2*cos(x)^3 - (8*cos(x)^5)/5`

3.132 $\int \cos^2(x) \sin^2(x) dx$

Optimal. Leaf size=24

$$\frac{x}{8} + \frac{1}{8} \cos(x) \sin(x) - \frac{1}{4} \cos^3(x) \sin(x)$$

[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)

Rubi [A]

time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2648, 2715, 8}

$$\frac{x}{8} - \frac{1}{4} \sin(x) \cos^3(x) + \frac{1}{8} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2*Sin[x]^2,x]

[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2648

Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^n_]*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m - 1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^n_, x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
\int \cos^2(x) \sin^2(x) dx &= -\frac{1}{4} \cos^3(x) \sin(x) + \frac{1}{4} \int \cos^2(x) dx \\
&= \frac{1}{8} \cos(x) \sin(x) - \frac{1}{4} \cos^3(x) \sin(x) + \frac{\int 1 dx}{8} \\
&= \frac{x}{8} + \frac{1}{8} \cos(x) \sin(x) - \frac{1}{4} \cos^3(x) \sin(x)
\end{aligned}$$

Mathematica [A]

time = 0.00, size = 14, normalized size = 0.58

$$\frac{x}{8} - \frac{1}{32} \sin(4x)$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[x]^2*Sin[x]^2,x]``[Out] x/8 - Sin[4*x]/32`**Mathics [A]**

time = 1.96, size = 10, normalized size = 0.42

$$\frac{x}{8} - \frac{\text{Sin}[4x]}{32}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sin[x]^2*Cos[x]^2,x]')``[Out] x / 8 - Sin[4 x] / 32`**Maple [A]**

time = 0.05, size = 19, normalized size = 0.79

method	result	size
risch	$\frac{x}{8} - \frac{\sin(4x)}{32}$	11
default	$\frac{x}{8} + \frac{\cos(x) \sin(x)}{8} - \frac{(\cos^3(x) \sin(x))}{4}$	19
norman	$\frac{x}{8} + \frac{7(\tan^3(\frac{x}{2}))}{4} - \frac{7(\tan^5(\frac{x}{2}))}{4} + \frac{(\tan^7(\frac{x}{2}))}{4} + \frac{x(\tan^2(\frac{x}{2}))}{2} + \frac{3x(\tan^4(\frac{x}{2}))}{4} + \frac{x(\tan^6(\frac{x}{2}))}{2} + \frac{x(\tan^8(\frac{x}{2}))}{8} - \frac{\tan(\frac{x}{2})}{4}$ $(1+\tan^2(\frac{x}{2}))^4$	82

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x)^2*sin(x)^2,x,method=_RETURNVERBOSE)``[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)`

Maxima [A]

time = 0.26, size = 10, normalized size = 0.42

$$\frac{1}{8}x - \frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")``[Out] 1/8*x - 1/32*sin(4*x)`**Fricas [A]**

time = 0.30, size = 19, normalized size = 0.79

$$-\frac{1}{8}(2\cos(x)^3 - \cos(x))\sin(x) + \frac{1}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")``[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x`**Sympy [A]**

time = 0.03, size = 14, normalized size = 0.58

$$\frac{x}{8} - \frac{\sin(2x)\cos(2x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)**2*sin(x)**2,x)``[Out] x/8 - sin(2*x)*cos(2*x)/16`**Giac [A]**

time = 0.00, size = 13, normalized size = 0.54

$$\frac{x}{8} - \frac{\sin(4x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)^2*sin(x)^2,x)``[Out] 1/8*x - 1/32*sin(4*x)`**Mupad [B]**

time = 0.04, size = 18, normalized size = 0.75

$$\frac{\cos(x)\sin(x)^3}{4} - \frac{\cos(x)\sin(x)}{8} + \frac{x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x)^2*sin(x)^2,x)``[Out] x/8 - (cos(x)*sin(x))/8 + (cos(x)*sin(x)^3)/4`

3.133 $\int \csc^2(x) \sec^2(x) dx$

Optimal. Leaf size=7

$$-\cot(x) + \tan(x)$$

[Out] $-\cot(x) + \tan(x)$

Rubi [A]

time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2700, 14}

$$\tan(x) - \cot(x)$$

Antiderivative was successfully verified.

[In] `Int[Csc[x]^2*Sec[x]^2,x]`

[Out] `-Cot[x] + Tan[x]`

Rule 14

`Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Rule 2700

`Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]`

Rubi steps

$$\begin{aligned} \int \csc^2(x) \sec^2(x) dx &= \text{Subst} \left(\int \frac{1+x^2}{x^2} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \left(1 + \frac{1}{x^2} \right) dx, x, \tan(x) \right) \\ &= -\cot(x) + \tan(x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 6, normalized size = 0.86

$$-2 \cot(2x)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^2*Sec[x]^2,x]

[Out] -2*Cot[2*x]

Mathics [A]

time = 1.89, size = 8, normalized size = 1.14

$$\frac{-2}{\tan[2x]}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(Sin[x]^2*Cos[x]^2),x]')

[Out] -2 / Tan[2 x]

Maple [A]

time = 0.03, size = 15, normalized size = 2.14

method	result	size
default	$\frac{1}{\cos(x)\sin(x)} - 2 \cot(x)$	15
risch	$-\frac{4i}{(e^{2ix}+1)(e^{2ix}-1)}$	22
norman	$\frac{\frac{1}{2}-3(\tan^2(\frac{x}{2}))+\frac{(\tan^4(\frac{x}{2}))}{2}}{(\tan^2(\frac{x}{2})-1)\tan(\frac{x}{2})}$	36

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(x)^2/sin(x)^2,x,method=_RETURNVERBOSE)

[Out] 1/cos(x)/sin(x)-2*cot(x)

Maxima [A]

time = 0.27, size = 9, normalized size = 1.29

$$-\frac{1}{\tan(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="maxima")

[Out] -1/tan(x) + tan(x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(7) = 14$.

time = 0.30, size = 18, normalized size = 2.57

$$\frac{2 \cos(x)^2 - 1}{\cos(x) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/cos(x)^2/sin(x)^2,x, algorithm="fricas")`

[Out] $-(2*\cos(x)^2 - 1)/(\cos(x)*\sin(x))$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(5) = 10

time = 0.03, size = 12, normalized size = 1.71

$$-\frac{2 \cos(2x)}{\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/cos(x)**2/sin(x)**2,x)`

[Out] $-2*\cos(2*x)/\sin(2*x)$

Giac [A]

time = 0.00, size = 7, normalized size = 1.00

$$\tan x - \frac{1}{\tan x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/cos(x)^2/sin(x)^2,x)`

[Out] $-1/\tan(x) + \tan(x)$

Mupad [B]

time = 0.06, size = 6, normalized size = 0.86

$$-2 \cot(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cos(x)^2*sin(x)^2),x)`

[Out] $-2*\cot(2*x)$

3.134 $\int d^x \sin(x) dx$

Optimal. Leaf size=32

$$-\frac{d^x \cos(x)}{1 + \log^2(d)} + \frac{d^x \log(d) \sin(x)}{1 + \log^2(d)}$$

[Out] $-d^x \cos(x)/(1+\ln(d)^2)+d^x \ln(d) \sin(x)/(1+\ln(d)^2)$

Rubi [A]

time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4517}

$$\frac{d^x \log(d) \sin(x)}{\log^2(d) + 1} - \frac{d^x \cos(x)}{\log^2(d) + 1}$$

Antiderivative was successfully verified.

[In] Int[d^x*Sin[x],x]

[Out] $-((d^x \cos[x])/(1 + \text{Log}[d]^2)) + (d^x \text{Log}[d] \sin[x])/(1 + \text{Log}[d]^2)$

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
  Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int d^x \sin(x) dx = -\frac{d^x \cos(x)}{1 + \log^2(d)} + \frac{d^x \log(d) \sin(x)}{1 + \log^2(d)}$$

Mathematica [A]

time = 0.02, size = 22, normalized size = 0.69

$$\frac{d^x (-\cos(x) + \log(d) \sin(x))}{1 + \log^2(d)}$$

Antiderivative was successfully verified.

[In] Integrate[d^x*Sin[x],x]

[Out] $(d^x * (-\text{Cos}[x] + \text{Log}[d] \sin[x]))/(1 + \text{Log}[d]^2)$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.83, size = 88, normalized size = 2.75

$$\text{Piecewise}\left[\left\{\left\{\frac{(-Ix\text{Cos}[x] + x\text{Sin}[x] - \text{Cos}[x])E^{-Ix}}{2}, d==E^{-I}\right\}, \left\{\frac{(Ix\text{Cos}[x] + x\text{Sin}[x] - \text{Cos}[x])E^{Ix}}{2}, d==E^I\right\}\right\}, -\frac{\text{Cos}[x]d^x}{1 + \text{Log}[d]^2} + \frac{\text{Log}[d]d^x\text{Sin}[x]}{1 + \text{Log}[d]^2}\right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[d^x*Sin[x],x]')`

[Out] `Piecewise[{{(-I x Cos[x] + x Sin[x] - Cos[x]) E ^ (-I x) / 2, d == E ^ (-I)}, {(I x Cos[x] + x Sin[x] - Cos[x]) E ^ (I x) / 2, d == E ^ I}}, -Cos[x] d ^ x / (1 + Log[d] ^ 2) + Log[d] d ^ x Sin[x] / (1 + Log[d] ^ 2)]`

Maple [A]

time = 0.03, size = 33, normalized size = 1.03

method	result	size
risch	$-\frac{d^x \cos(x)}{1+\ln(d)^2} + \frac{d^x \ln(d) \sin(x)}{1+\ln(d)^2}$	33
norman	$\frac{e^{x \ln(d)} \left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\ln(d)^2} - \frac{e^{x \ln(d)}}{1+\ln(d)^2} + \frac{2 \ln(d) e^{x \ln(d)} \tan\left(\frac{x}{2}\right)}{1+\ln(d)^2}$	69

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(d^x*sin(x),x,method=_RETURNVERBOSE)`

[Out] `-d^x*cos(x)/(1+ln(d)^2)+d^x*ln(d)*sin(x)/(1+ln(d)^2)`

Maxima [A]

time = 0.27, size = 25, normalized size = 0.78

$$\frac{d^x \log(d) \sin(x) - d^x \cos(x)}{\log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d^x*sin(x),x, algorithm="maxima")`

[Out] `(d^x*log(d)*sin(x) - d^x*cos(x))/(log(d)^2 + 1)`

Fricas [A]

time = 0.31, size = 22, normalized size = 0.69

$$\frac{(\log(d) \sin(x) - \cos(x))d^x}{\log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d^x*sin(x),x, algorithm="fricas")
 [Out] (log(d)*sin(x) - cos(x))*d^x/(log(d)^2 + 1)

Sympy [A]

time = 0.29, size = 104, normalized size = 3.25

$$\begin{cases} \frac{xe^{-ix} \sin(x)}{2} - \frac{ixe^{-ix} \cos(x)}{2} - \frac{e^{-ix} \cos(x)}{2} & \text{for } d = e^{-i} \\ \frac{xe^{ix} \sin(x)}{2} + \frac{ixe^{ix} \cos(x)}{2} - \frac{e^{ix} \cos(x)}{2} & \text{for } d = e^i \\ \frac{d^x \log(d) \sin(x)}{\log(d)^2 + 1} - \frac{d^x \cos(x)}{\log(d)^2 + 1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d**x*sin(x),x)

[Out] Piecewise((x*exp(-I*x)*sin(x)/2 - I*x*exp(-I*x)*cos(x)/2 - exp(-I*x)*cos(x)/2, Eq(d, exp(-I))), (x*exp(I*x)*sin(x)/2 + I*x*exp(I*x)*cos(x)/2 - exp(I*x)*cos(x)/2, Eq(d, exp(I))), (d**x*log(d)*sin(x)/(log(d)**2 + 1) - d**x*cos(x)/(log(d)**2 + 1), True))

Giac [C] Result contains complex when optimal does not.

time = 0.00, size = 381, normalized size = 11.91

$$e^{ix} \left(-\frac{2(-2\operatorname{sgn}(d) + 2x + 4) \cos\left(\frac{\operatorname{atan2}(d-2x, 1)}{2}\right)}{(4\ln|d|)^2 + (2\operatorname{sgn}(d) - 2x - 4)^2} - \frac{4 \cdot 2\ln|d| \sin\left(\frac{\operatorname{atan2}(d-2x, 1)}{2}\right)}{(4\ln|d|)^2 + (2\operatorname{sgn}(d) - 2x - 4)^2} \right) + e^{-ix} \left(\frac{2(-2\operatorname{sgn}(d) + 2x - 4) \cos\left(\frac{\operatorname{atan2}(d-2x, 1)}{2}\right)}{(4\ln|d|)^2 + (2\operatorname{sgn}(d) - 2x + 4)^2} - \frac{4 \cdot 2\ln|d| \sin\left(\frac{\operatorname{atan2}(d-2x, 1)}{2}\right)}{(4\ln|d|)^2 + (2\operatorname{sgn}(d) - 2x + 4)^2} \right) + \frac{1}{2} e^{ix} \left(\frac{2e^{i\operatorname{atan2}(d-2x, 1)}}{(4\ln|d| + x - 2\operatorname{sgn}(d) - x - 2) + 4} + \frac{2e^{-i\operatorname{atan2}(d-2x, 1)}}{(4\ln|d| - x - 2\operatorname{sgn}(d) + x - 2) - 4} \right) + \frac{1}{2} e^{-ix} \left(\frac{2e^{i\operatorname{atan2}(d-2x, 1)}}{(4\ln|d| + x - 2\operatorname{sgn}(d) - x - 2) - 4} + \frac{2e^{-i\operatorname{atan2}(d-2x, 1)}}{(4\ln|d| - x - 2\operatorname{sgn}(d) + x - 2) + 4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d^x*sin(x),x)

[Out] abs(d)^x*((pi - pi*sgn(d) - 2)*cos(1/2*pi*x*sgn(d) - 1/2*pi*x + x)/((pi - pi*sgn(d) - 2)^2 + 4*log(abs(d))^2) + 2*log(abs(d))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x + x)/((pi - pi*sgn(d) - 2)^2 + 4*log(abs(d))^2)) - abs(d)^x*((pi - pi*sgn(d) + 2)*cos(1/2*pi*x*sgn(d) - 1/2*pi*x - x)/((pi - pi*sgn(d) + 2)^2 + 4*log(abs(d))^2) + 2*log(abs(d))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x)/((pi - pi*sgn(d) + 2)^2 + 4*log(abs(d))^2)) - abs(d)^x*(-I*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) + 4*I) - I*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(2*I*pi - 2*I*pi*sgn(d) + 4*log(abs(d)) - 4*I)) - abs(d)^x*(I*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x - I*x)/(-2*I*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) - 4*I) + I*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x + I*x)/(2*I*pi - 2*I*pi*sgn(d) + 4*log(abs(d)) + 4*I))

Mupad [B]

time = 0.02, size = 22, normalized size = 0.69

$$\frac{d^x (\cos(x) - \ln(d) \sin(x))}{\ln(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(d^x*sin(x),x)

[Out] -(d^x*(cos(x) - log(d)*sin(x)))/(log(d)^2 + 1)

3.135 $\int d^x \cos(x) dx$

Optimal. Leaf size=31

$$\frac{d^x \cos(x) \log(d)}{1 + \log^2(d)} + \frac{d^x \sin(x)}{1 + \log^2(d)}$$

[Out] $d^x \cos(x) \ln(d) / (1 + \ln(d)^2) + d^x \sin(x) / (1 + \ln(d)^2)$

Rubi [A]

time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4518}

$$\frac{d^x \sin(x)}{\log^2(d) + 1} + \frac{d^x \log(d) \cos(x)}{\log^2(d) + 1}$$

Antiderivative was successfully verified.

[In] Int[d^x*Cos[x],x]

[Out] $(d^x \cos(x) \log(d)) / (1 + \log(d)^2) + (d^x \sin(x)) / (1 + \log(d)^2)$

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_.)]*(F_)^((c_.)*((a_.) + (b_.)*(x_.))), x_Symbol] :=
  Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int d^x \cos(x) dx = \frac{d^x \cos(x) \log(d)}{1 + \log^2(d)} + \frac{d^x \sin(x)}{1 + \log^2(d)}$$

Mathematica [A]

time = 0.01, size = 20, normalized size = 0.65

$$\frac{d^x (\cos(x) \log(d) + \sin(x))}{1 + \log^2(d)}$$

Antiderivative was successfully verified.

[In] Integrate[d^x*Cos[x],x]

[Out] $(d^x * (\cos(x) \log(d) + \sin(x))) / (1 + \log(d)^2)$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.73, size = 87, normalized size = 2.81

$$\text{Piecewise}\left[\left\{\left\{\frac{(I x \sin [x]+x \cos [x]+I \cos [x]) E^{-I x}}{2}, d==E^{-I}\right\},\left\{\frac{(-I x \sin [x]+x \cos [x]-I \cos [x]) E^{I x}}{2}, d==E^I\right\}\right\}, \frac{\cos [x] \log [d] d^x}{1+\log [d]^2}+\frac{d^x \sin [x]}{1+\log [d]^2}\right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[d^x*cos[x],x]')`

[Out] `Piecewise[{{(I x Sin[x] + x Cos[x] + I Cos[x]) E ^ (-I x) / 2, d == E ^ (-I)}}, {(-I x Sin[x] + x Cos[x] - I Cos[x]) E ^ (I x) / 2, d == E ^ I}}, Cos[x] Log[d] d ^ x / (1 + Log[d] ^ 2) + d ^ x Sin[x] / (1 + Log[d] ^ 2)]`

Maple [A]

time = 0.03, size = 32, normalized size = 1.03

method	result	size
risch	$\frac{d^x \cos(x) \ln(d)}{1+\ln(d)^2} + \frac{d^x \sin(x)}{1+\ln(d)^2}$	32
norman	$\frac{\frac{\ln(d)e^x \ln(d)}{1+\ln(d)^2} + \frac{2e^x \ln(d) \tan\left(\frac{x}{2}\right)}{1+\ln(d)^2} - \frac{\ln(d)e^x \ln(d) \left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\ln(d)^2}}{1+\tan^2\left(\frac{x}{2}\right)}$	71

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(d^x*cos(x),x,method=_RETURNVERBOSE)`

[Out] `d^x*cos(x)*ln(d)/(1+ln(d)^2)+d^x*sin(x)/(1+ln(d)^2)`

Maxima [A]

time = 0.27, size = 24, normalized size = 0.77

$$\frac{d^x \cos(x) \log(d) + d^x \sin(x)}{\log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d^x*cos(x),x, algorithm="maxima")`

[Out] `(d^x*cos(x)*log(d) + d^x*sin(x))/(log(d)^2 + 1)`

Fricas [A]

time = 0.32, size = 20, normalized size = 0.65

$$\frac{(\cos(x) \log(d) + \sin(x))d^x}{\log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d^x*cos(x),x, algorithm="fricas")

[Out] (cos(x)*log(d) + sin(x))*d^x/(log(d)^2 + 1)

Sympy [A]

time = 0.29, size = 107, normalized size = 3.45

$$\begin{cases} \frac{ixe^{-ix} \sin(x)}{2} + \frac{xe^{-ix} \cos(x)}{2} + \frac{ie^{-ix} \cos(x)}{2} & \text{for } d = e^{-i} \\ -\frac{ixe^{ix} \sin(x)}{2} + \frac{xe^{ix} \cos(x)}{2} - \frac{ie^{ix} \cos(x)}{2} & \text{for } d = e^i \\ \frac{d^x \log(d) \cos(x)}{\log(d)^2 + 1} + \frac{d^x \sin(x)}{\log(d)^2 + 1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d**x*cos(x),x)

[Out] Piecewise((I*x*exp(-I*x)*sin(x)/2 + x*exp(-I*x)*cos(x)/2 + I*exp(-I*x)*cos(x)/2, Eq(d, exp(-I))), (-I*x*exp(I*x)*sin(x)/2 + x*exp(I*x)*cos(x)/2 - I*exp(I*x)*cos(x)/2, Eq(d, exp(I))), (d**x*log(d)*cos(x)/(log(d)**2 + 1) + d**x*sin(x)/(log(d)**2 + 1), True))

Giac [C] Result contains complex when optimal does not.

time = 0.01, size = 383, normalized size = 12.35

$$e^{hd} \left(\frac{4 \cdot 2 \ln |d| \cdot \cos\left(\frac{\operatorname{atan2}(d, -2x)}{2}\right) - 2(-2\operatorname{sign}(d) + 2x - 4) \sin\left(\frac{\operatorname{atan2}(d, -2x)}{2}\right)}{(4 \ln |d|)^2 + (2\operatorname{sign}(d) - 2x + 4)^2} \right) + e^{hd} \left(\frac{4 \cdot 2 \ln |d| \cdot \cos\left(\frac{\operatorname{atan2}(d, -2x)}{2}\right) - 2(-2\operatorname{sign}(d) + 2x + 4) \sin\left(\frac{\operatorname{atan2}(d, -2x)}{2}\right)}{(4 \ln |d|)^2 + (2\operatorname{sign}(d) - 2x - 4)^2} \right) + \frac{e^{hd} \left(-\frac{2i \cos(\operatorname{atan2}(d, -2x))}{4 \ln |d| + 2\operatorname{sign}(d) - 2x - 4} + \frac{2i \cos(\operatorname{atan2}(d, -2x))}{4 \ln |d| + 2\operatorname{sign}(d) - 2x + 4} \right)}{2} + \frac{e^{hd} \left(-\frac{2i \cos(\operatorname{atan2}(d, -2x))}{4 \ln |d| + 2\operatorname{sign}(d) - 2x - 4} + \frac{2i \cos(\operatorname{atan2}(d, -2x))}{4 \ln |d| + 2\operatorname{sign}(d) - 2x + 4} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d^x*cos(x),x)

[Out] abs(d)^x*(2*cos(1/2*pi*x*sgn(d) - 1/2*pi*x + x)*log(abs(d))/((pi - pi*sgn(d) - 2)^2 + 4*log(abs(d))^2) - (pi - pi*sgn(d) - 2)*sin(1/2*pi*x*sgn(d) - 1/2*pi*x + x)/((pi - pi*sgn(d) - 2)^2 + 4*log(abs(d))^2)) + abs(d)^x*(2*cos(1/2*pi*x*sgn(d) - 1/2*pi*x - x)*log(abs(d))/((pi - pi*sgn(d) + 2)^2 + 4*log(abs(d))^2) - (pi - pi*sgn(d) + 2)*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x)/((pi - pi*sgn(d) + 2)^2 + 4*log(abs(d))^2)) + I*abs(d)^x*(I*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) + 4*I) - I*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(2*I*pi - 2*I*pi*sgn(d) + 4*log(abs(d)) - 4*I)) + I*abs(d)^x*(I*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x - I*x)/(-2*I*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) - 4*I) - I*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x + I*x)/(2*I*pi - 2*I*pi*sgn(d) + 4*log(abs(d)) + 4*I))

Mupad [B]

time = 0.02, size = 20, normalized size = 0.65

$$\frac{d^x (\sin(x) + \ln(d) \cos(x))}{\ln(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(d^x*cos(x),x)

[Out] (d^x*(sin(x) + log(d)*cos(x)))/(log(d)^2 + 1)

3.136 $\int d^x x \sin(x) dx$

Optimal. Leaf size=84

$$\frac{2d^x \cos(x) \log(d)}{(1 + \log^2(d))^2} - \frac{d^x x \cos(x)}{1 + \log^2(d)} + \frac{d^x \sin(x)}{(1 + \log^2(d))^2} - \frac{d^x \log^2(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x \log(d) \sin(x)}{1 + \log^2(d)}$$

[Out] $2*d^x*\cos(x)*\ln(d)/(1+\ln(d)^2)^2-d^x*x*\cos(x)/(1+\ln(d)^2)+d^x*\sin(x)/(1+\ln(d)^2)^2-d^x*\ln(d)^2*\sin(x)/(1+\ln(d)^2)^2+d^x*x*\ln(d)*\sin(x)/(1+\ln(d)^2)$

Rubi [A]

time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$,

Rules used = {4517, 4553, 4518}

$$\frac{x d^x \log(d) \sin(x)}{\log^2(d) + 1} - \frac{d^x \log^2(d) \sin(x)}{(\log^2(d) + 1)^2} + \frac{d^x \sin(x)}{(\log^2(d) + 1)^2} - \frac{x d^x \cos(x)}{\log^2(d) + 1} + \frac{2 d^x \log(d) \cos(x)}{(\log^2(d) + 1)^2}$$

Antiderivative was successfully verified.

[In] Int[d^x*x*Sin[x],x]

[Out] $(2*d^x*\cos[x]*\log[d])/(1 + \log[d]^2)^2 - (d^x*x*\cos[x])/(1 + \log[d]^2) + (d^x*\sin[x])/(1 + \log[d]^2)^2 - (d^x*\log[d]^2*\sin[x])/(1 + \log[d]^2)^2 + (d^x*x*\log[d]*\sin[x])/(1 + \log[d]^2)$

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :=
  Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :=
  Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_)^(m_.)*Sin[(d_.) + (e_.)*
(x_)^(n_.)], x_Symbol] := Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int d^x x \sin(x) dx &= -\frac{d^x x \cos(x)}{1 + \log^2(d)} + \frac{d^x x \log(d) \sin(x)}{1 + \log^2(d)} - \int \left(-\frac{d^x \cos(x)}{1 + \log^2(d)} + \frac{d^x \log(d) \sin(x)}{1 + \log^2(d)} \right) dx \\
&= -\frac{d^x x \cos(x)}{1 + \log^2(d)} + \frac{d^x x \log(d) \sin(x)}{1 + \log^2(d)} + \frac{\int d^x \cos(x) dx}{1 + \log^2(d)} - \frac{\log(d) \int d^x \sin(x) dx}{1 + \log^2(d)} \\
&= \frac{2d^x \cos(x) \log(d)}{(1 + \log^2(d))^2} - \frac{d^x x \cos(x)}{1 + \log^2(d)} + \frac{d^x \sin(x)}{(1 + \log^2(d))^2} - \frac{d^x \log^2(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x \log(d) \sin(x)}{1 + \log^2(d)}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 50, normalized size = 0.60

$$\frac{d^x (-\cos(x) (x - 2 \log(d) + x \log^2(d)) + (1 + x \log(d) - \log^2(d) + x \log^3(d)) \sin(x))}{(1 + \log^2(d))^2}$$

Antiderivative was successfully verified.

`[In] Integrate[d^x*x*Sin[x],x]`

```
[Out] (d^x*(-(Cos[x]*(x - 2*Log[d] + x*Log[d]^2)) + (1 + x*Log[d] - Log[d]^2 + x*Log[d]^3)*Sin[x]))/(1 + Log[d]^2)^2
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 3.64, size = 249, normalized size = 2.96

$$\text{Piecewise}\left[\left\{\left\{\frac{-x \cos[x] + x^2 \sin[x] - x^2 \cos[x] + x^2 \sin[x] + x \cos[x] E^{-x}}{4}, \{(-x \cos[x] - x^2 \sin[x] + x^2 \cos[x] - x^2 \sin[x] - x \cos[x]) E^{-x}\}\right\}, \frac{x \cos[x] d^x \log[d]^2}{1 + \log[d]^2 + 2 \log[d]^2} - \frac{x \cos[x] d^x}{1 + \log[d]^2 + 2 \log[d]^2} + \frac{x \log[d] d^x \sin[x]}{1 + \log[d]^2 + 2 \log[d]^2} + \frac{x^2 \log[d]^2 \sin[x]}{1 + \log[d]^2 + 2 \log[d]^2} - \frac{d^x \log[d]^2 \sin[x]}{1 + \log[d]^2 + 2 \log[d]^2} + \frac{2 \cos[x] \log[d] d^x}{1 + \log[d]^2 + 2 \log[d]^2} + \frac{d^x \sin[x]}{1 + \log[d]^2 + 2 \log[d]^2}\right\}\right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[x*d^x*Sin[x],x]')`

```
[Out] Piecewise[{{(-x Cos[x] + I x Sin[x] - I x ^ 2 Cos[x] + x ^ 2 Sin[x] + I Cos[x]) E ^ (-I x) / 4, d == E ^ (-I)}, {(-x Cos[x] - I x Sin[x] + I x ^ 2 Cos[x] + x ^ 2 Sin[x] - I Cos[x]) E ^ (I x) / 4, d == E ^ I}}, -x Cos[x] d ^ x Log[d] ^ 2 / (1 + Log[d] ^ 4 + 2 Log[d] ^ 2) - x Cos[x] d ^ x / (1 + Log[d] ^ 4 + 2 Log[d] ^ 2) + x Log[d] d ^ x Sin[x] / (1 + Log[d] ^ 4 + 2 Log[d] ^ 2) + x d ^ x Log[d] ^ 3 Sin[x] / (1 + Log[d] ^ 4 + 2 Log[d] ^ 2) - d ^ x Log[d] ^ 2 Sin[x] / (1 + Log[d] ^ 4 + 2 Log[d] ^ 2) + 2 Cos[x] Log[d] d ^ x / (1 + Log[d] ^ 4 + 2 Log[d] ^ 2) + d ^ x Sin[x] / (1 + Log[d] ^ 4 + 2 Log[d] ^ 2)}
```

Maple [C] Result contains complex when optimal does not.

time = 0.04, size = 58, normalized size = 0.69

method	result	size
risch	$-\frac{i(-1+x \ln(d)+ix)d^x e^{ix}}{2(\ln(d)+i)^2} + \frac{i(-1+x \ln(d)-ix)d^x e^{-ix}}{2(\ln(d)-i)^2}$	58
norman	$\frac{x e^{x \ln(d)} \left(\tan^2\left(\frac{x}{2}\right) \right) + \frac{2 \ln(d) e^{x \ln(d)}}{(1+\ln(d))^2} - \frac{x e^{x \ln(d)}}{1+\ln(d)^2} - \frac{2 \ln(d) e^{x \ln(d)} \left(\tan^2\left(\frac{x}{2}\right) \right)}{(1+\ln(d))^2} - \frac{2(\ln(d)^2-1) e^{x \ln(d)} \tan\left(\frac{x}{2}\right)}{(1+\ln(d))^2} + \frac{2 \ln(d) x e^{x \ln(d)} \tan\left(\frac{x}{2}\right)}{1+\ln(d)^2}}{1+\tan^2\left(\frac{x}{2}\right)}$	137

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(d^x*x*sin(x),x,method=_RETURNVERBOSE)`

[Out]
$$-1/2*I*(-1+x*\ln(d)+I*x)*d^x/(\ln(d)+I)^2*\exp(I*x)+1/2*I*(-1+x*\ln(d)-I*x)*d^x/(\ln(d)-I)^2*\exp(-I*x)$$

Maxima [A]

time = 0.27, size = 60, normalized size = 0.71

$$\frac{((\log(d)^2 + 1)x - 2 \log(d))d^x \cos(x) - ((\log(d)^3 + \log(d))x - \log(d)^2 + 1)d^x \sin(x)}{\log(d)^4 + 2 \log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d^x*x*sin(x),x, algorithm="maxima")`

[Out]
$$-(((\log(d)^2 + 1)*x - 2*\log(d))*d^x*\cos(x) - ((\log(d)^3 + \log(d))*x - \log(d)^2 + 1)*d^x*\sin(x))/(\log(d)^4 + 2*\log(d)^2 + 1)$$

Fricas [A]

time = 0.32, size = 60, normalized size = 0.71

$$\frac{(x \cos(x) \log(d)^2 + x \cos(x) - 2 \cos(x) \log(d) - (x \log(d)^3 + x \log(d) - \log(d)^2 + 1) \sin(x))d^x}{\log(d)^4 + 2 \log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d^x*x*sin(x),x, algorithm="fricas")`

[Out]
$$-(x*\cos(x)*\log(d)^2 + x*\cos(x) - 2*\cos(x)*\log(d) - (x*\log(d)^3 + x*\log(d) - \log(d)^2 + 1)*\sin(x))*d^x/(\log(d)^4 + 2*\log(d)^2 + 1)$$

Sympy [A]

time = 0.51, size = 308, normalized size = 3.67

$$\begin{cases} \frac{x^2 e^{-ix} \sin(x)}{4} - \frac{ix^2 e^{-ix} \cos(x)}{4} + \frac{ix e^{-ix} \sin(x)}{4} - \frac{x e^{-ix} \cos(x)}{4} + \frac{ie^{-ix} \cos(x)}{4} & \text{for } d = e^{-i} \\ \frac{x^2 e^{ix} \sin(x)}{4} + \frac{ix^2 e^{ix} \cos(x)}{4} - \frac{ix e^{ix} \sin(x)}{4} - \frac{x e^{ix} \cos(x)}{4} - \frac{ie^{ix} \cos(x)}{4} & \text{for } d = e^i \\ \frac{d^x x \log(d)^3 \sin(x)}{\log(d)^4 + 2 \log(d)^2 + 1} - \frac{d^x x \log(d)^2 \cos(x)}{\log(d)^4 + 2 \log(d)^2 + 1} + \frac{d^x x \log(d) \sin(x)}{\log(d)^4 + 2 \log(d)^2 + 1} - \frac{d^x x \cos(x)}{\log(d)^4 + 2 \log(d)^2 + 1} - \frac{d^x \log(d)^2 \sin(x)}{\log(d)^4 + 2 \log(d)^2 + 1} + \frac{2d^x \log(d) \cos(x)}{\log(d)^4 + 2 \log(d)^2 + 1} + \frac{d^x \sin(x)}{\log(d)^4 + 2 \log(d)^2 + 1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d**x*x*sin(x),x)

[Out] Piecewise((x**2*exp(-I*x)*sin(x)/4 - I*x**2*exp(-I*x)*cos(x)/4 + I*x*exp(-I*x)*sin(x)/4 - x*exp(-I*x)*cos(x)/4 + I*exp(-I*x)*cos(x)/4, Eq(d, exp(-I))), (x**2*exp(I*x)*sin(x)/4 + I*x**2*exp(I*x)*cos(x)/4 - I*x*exp(I*x)*sin(x)/4 - x*exp(I*x)*cos(x)/4 - I*exp(I*x)*cos(x)/4, Eq(d, exp(I))), (d**x*x*log(d)**3*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*x*log(d)**2*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) + d**x*x*log(d)*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*x*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*log(d)**2*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) + 2*d**x*log(d)*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) + d**x*sin(x)/(log(d)**4 + 2*log(d)**2 + 1), True))

Giac [C] Result contains complex when optimal does not.

time = 0.00, size = 1369, normalized size = 16.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d^x*x*sin(x),x)

[Out] $\frac{1}{2} * (((2\pi + \pi^2 \operatorname{sgn}(d) - \pi^2 + 2 \log(\operatorname{abs}(d))^2 - 2\pi \operatorname{sgn}(d) - 2) * (\pi * x * \operatorname{sgn}(d) - \pi * x + 2 * x) / ((2\pi + \pi^2 \operatorname{sgn}(d) - \pi^2 + 2 \log(\operatorname{abs}(d))^2 - 2\pi \operatorname{sgn}(d) - 2)^2 + 4 * (\pi * \log(\operatorname{abs}(d)) * \operatorname{sgn}(d) - \pi * \log(\operatorname{abs}(d)) + 2 * \log(\operatorname{abs}(d)))^2) - 4 * (\pi * \log(\operatorname{abs}(d)) * \operatorname{sgn}(d) - \pi * \log(\operatorname{abs}(d)) + 2 * \log(\operatorname{abs}(d))) * (x * \log(\operatorname{abs}(d)) - 1) / ((2\pi + \pi^2 \operatorname{sgn}(d) - \pi^2 + 2 \log(\operatorname{abs}(d))^2 - 2\pi \operatorname{sgn}(d) - 2)^2 + 4 * (\pi * \log(\operatorname{abs}(d)) * \operatorname{sgn}(d) - \pi * \log(\operatorname{abs}(d)) + 2 * \log(\operatorname{abs}(d)))^2)) * \cos(1/2 * \pi * x * \operatorname{sgn}(d) - 1/2 * \pi * x + x) + 2 * ((\pi * x * \operatorname{sgn}(d) - \pi * x + 2 * x) * (\pi * \log(\operatorname{abs}(d)) * \operatorname{sgn}(d) - \pi * \log(\operatorname{abs}(d)) + 2 * \log(\operatorname{abs}(d))) / ((2\pi + \pi^2 \operatorname{sgn}(d) - \pi^2 + 2 \log(\operatorname{abs}(d))^2 - 2\pi \operatorname{sgn}(d) - 2)^2 + 4 * (\pi * \log(\operatorname{abs}(d)) * \operatorname{sgn}(d) - \pi * \log(\operatorname{abs}(d)) + 2 * \log(\operatorname{abs}(d)))^2) + (2\pi + \pi^2 \operatorname{sgn}(d) - \pi^2 + 2 \log(\operatorname{abs}(d))^2 - 2\pi \operatorname{sgn}(d) - 2) * (x * \log(\operatorname{abs}(d)) - 1) / ((2\pi + \pi^2 \operatorname{sgn}(d) - \pi^2 + 2 \log(\operatorname{abs}(d))^2 - 2\pi \operatorname{sgn}(d) - 2)^2 + 4 * (\pi * \log(\operatorname{abs}(d)) * \operatorname{sgn}(d) - \pi * \log(\operatorname{abs}(d)) + 2 * \log(\operatorname{abs}(d)))^2) * \sin(1/2 * \pi * x * \operatorname{sgn}(d) - 1/2 * \pi * x + x) * \operatorname{abs}(d)^x + 1/2 * (((2\pi - \pi^2 \operatorname{sgn}(d) + \pi^2 - 2 \log(\operatorname{abs}(d))^2 - 2\pi \operatorname{sgn}(d) + 2) * (\pi * x * \operatorname{sgn}(d) - \pi * x - 2 * x) / ((2\pi - \pi^2 \operatorname{sgn}(d) + \pi^2 - 2 \log(\operatorname{abs}(d))^2 - 2\pi \operatorname{sgn}(d) + 2)^2 + 4 * (\pi * \log(\operatorname{abs}(d)) * \operatorname{sgn}(d) - \pi * \log(\operatorname{abs}(d)) - 2 * \log(\operatorname{abs}(d)))^2) + 4 * (\pi * \log(\operatorname{abs}(d)) * \operatorname{sgn}(d) - \pi * \log(\operatorname{abs}(d)) - 2 * \log(\operatorname{abs}(d))) * (x * \log(\operatorname{abs}(d)) - 1) / ((2\pi - \pi^2 \operatorname{sgn}(d) + \pi^2 - 2 \log(\operatorname{abs}(d))^2 - 2\pi \operatorname{sgn}(d) + 2)^2 + 4 * (\pi * \log(\operatorname{abs}(d)) * \operatorname{sgn}(d) - \pi * \log(\operatorname{abs}(d)) - 2 * \log(\operatorname{abs}(d)))^2) * \cos(1/2 * \pi * x * \operatorname{sgn}(d) - 1/2 * \pi * x - x) - 2 * ((\pi * x * \operatorname{sgn}(d) - \pi * x - 2 * x) * (\pi * \log(\operatorname{abs}(d)) * \operatorname{sgn}(d) - \pi * \log(\operatorname{abs}(d)) - 2 * \log(\operatorname{abs}(d))) / ((2\pi - \pi^2 \operatorname{sgn}(d) + \pi^2 - 2 \log(\operatorname{abs}(d))^2 - 2\pi \operatorname{sgn}(d) + 2)^2 + 4 * (\pi * \log(\operatorname{abs}(d)) * \operatorname{sgn}(d) - \pi * \log(\operatorname{abs}(d)) - 2 * \log(\operatorname{abs}(d)))^2) - (2\pi - \pi^2 \operatorname{sgn}(d) + \pi^2 - 2 \log(\operatorname{abs}(d))^2 - 2\pi \operatorname{sgn}(d) + 2) * (x * \log(\operatorname{abs}(d)) - 1) / ((2\pi - \pi^2 \operatorname{sgn}(d) + \pi^2 - 2 \log(\operatorname{abs}(d))^2 - 2\pi \operatorname{sgn}(d) + 2)^2 + 4 * (\pi * \log(\operatorname{abs}(d)) * \operatorname{sgn}(d) - \pi * \log(\operatorname{abs}(d)) - 2 * \log(\operatorname{abs}(d)))^2) * \sin(1/2 * \pi * x * \operatorname{sgn}(d) - 1/2 * \pi * x - x) * \operatorname{abs}(d)^x - 1/4 * ((\pi * x * \operatorname{sgn}(d) - \pi * x - 2 * I * x * \log(\operatorname{abs}(d)) + 2 * x + 2 * I) * e^{(1/2 * I * \pi * x * \operatorname{sgn}(d) - 1/2 * I * \pi * x + I * x) /$

$$\begin{aligned}
& (2\pi + \pi^2 \operatorname{sgn}(d) + 2I\pi \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - \pi^2 - 2I\pi \log(\operatorname{abs}(d)) \\
& + 2\log(\operatorname{abs}(d))^2 - 2\pi \operatorname{sgn}(d) + 4I\log(\operatorname{abs}(d)) - 2) - (\pi x \operatorname{sgn}(d) - \pi \\
& *x + 2I x \log(\operatorname{abs}(d)) + 2x - 2I) e^{(-1/2 I \pi x \operatorname{sgn}(d) + 1/2 I \pi x - I x)} \\
& / (2\pi + \pi^2 \operatorname{sgn}(d) - 2I\pi \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - \pi^2 + 2I\pi \log(\operatorname{abs}(d)) \\
& + 2\log(\operatorname{abs}(d))^2 - 2\pi \operatorname{sgn}(d) - 4I\log(\operatorname{abs}(d)) - 2) * \operatorname{abs}(d)^x - 1/4 * \\
& ((\pi x \operatorname{sgn}(d) - \pi x - 2I x \log(\operatorname{abs}(d)) - 2x + 2I) e^{(1/2 I \pi x \operatorname{sgn}(d)} \\
& - 1/2 I \pi x - I x) / (2\pi - \pi^2 \operatorname{sgn}(d) - 2I\pi \log(\operatorname{abs}(d)) \operatorname{sgn}(d) + \pi^2 \\
& + 2I\pi \log(\operatorname{abs}(d)) - 2\log(\operatorname{abs}(d))^2 - 2\pi \operatorname{sgn}(d) + 4I\log(\operatorname{abs}(d)) + 2) \\
& - (\pi x \operatorname{sgn}(d) - \pi x + 2I x \log(\operatorname{abs}(d)) - 2x - 2I) e^{(-1/2 I \pi x \operatorname{sgn}(d)} \\
& + 1/2 I \pi x + I x) / (2\pi - \pi^2 \operatorname{sgn}(d) + 2I\pi \log(\operatorname{abs}(d)) \operatorname{sgn}(d) + \pi \\
& ^2 - 2I\pi \log(\operatorname{abs}(d)) - 2\log(\operatorname{abs}(d))^2 - 2\pi \operatorname{sgn}(d) - 4I\log(\operatorname{abs}(d)) + \\
& 2) * \operatorname{abs}(d)^x
\end{aligned}$$

Mupad [B]

time = 0.28, size = 57, normalized size = 0.68

$$\frac{d^x (\sin(x) + 2 \ln(d) \cos(x) - \ln(d)^2 \sin(x) - x \cos(x) + x \ln(d) \sin(x) - x \ln(d)^2 \cos(x) + x \ln(d)^3 \sin(x))}{(\ln(d)^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(d^x*x*sin(x),x)

[Out] (d^x*(sin(x) + 2*log(d)*cos(x) - log(d)^2*sin(x) - x*cos(x) + x*log(d)*sin(x) - x*log(d)^2*cos(x) + x*log(d)^3*sin(x)))/(log(d)^2 + 1)^2

3.137 $\int d^x x \cos(x) dx$

Optimal. Leaf size=83

$$\frac{d^x \cos(x)}{(1 + \log^2(d))^2} - \frac{d^x \cos(x) \log^2(d)}{(1 + \log^2(d))^2} + \frac{d^x x \cos(x) \log(d)}{1 + \log^2(d)} - \frac{2d^x \log(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x \sin(x)}{1 + \log^2(d)}$$

[Out] $d^x \cos(x)/(1+\ln(d)^2)^2 - d^x \cos(x) \ln(d)^2/(1+\ln(d)^2)^2 + d^x x \cos(x) \ln(d)/(1+\ln(d)^2) - 2 d^x \ln(d) \sin(x)/(1+\ln(d)^2)^2 + d^x x \sin(x)/(1+\ln(d)^2)$

Rubi [A]

time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$,

Rules used = {4518, 4554, 4517}

$$\frac{x d^x \sin(x)}{\log^2(d) + 1} - \frac{2 d^x \log(d) \sin(x)}{(\log^2(d) + 1)^2} + \frac{x d^x \log(d) \cos(x)}{\log^2(d) + 1} - \frac{d^x \log^2(d) \cos(x)}{(\log^2(d) + 1)^2} + \frac{d^x \cos(x)}{(\log^2(d) + 1)^2}$$

Antiderivative was successfully verified.

[In] Int[d^x*x*Cos[x],x]

[Out] $(d^x \cos[x])/(1 + \log[d]^2)^2 - (d^x \cos[x] \log[d]^2)/(1 + \log[d]^2)^2 + (d^x x \cos[x] \log[d])/(1 + \log[d]^2) - (2 d^x \log[d] \sin[x])/(1 + \log[d]^2)^2 + (d^x x \sin[x])/(1 + \log[d]^2)$

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :=
  Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :=
  Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] := Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int d^x x \cos(x) dx &= \frac{d^x x \cos(x) \log(d)}{1 + \log^2(d)} + \frac{d^x x \sin(x)}{1 + \log^2(d)} - \int \left(\frac{d^x \cos(x) \log(d)}{1 + \log^2(d)} + \frac{d^x \sin(x)}{1 + \log^2(d)} \right) dx \\
&= \frac{d^x x \cos(x) \log(d)}{1 + \log^2(d)} + \frac{d^x x \sin(x)}{1 + \log^2(d)} - \frac{\int d^x \sin(x) dx}{1 + \log^2(d)} - \frac{\log(d) \int d^x \cos(x) dx}{1 + \log^2(d)} \\
&= \frac{d^x \cos(x)}{(1 + \log^2(d))^2} - \frac{d^x \cos(x) \log^2(d)}{(1 + \log^2(d))^2} + \frac{d^x x \cos(x) \log(d)}{1 + \log^2(d)} - \frac{2d^x \log(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x \sin(x)}{1 + \log^2(d)}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 49, normalized size = 0.59

$$\frac{d^x (\cos(x) (1 + x \log(d) - \log^2(d) + x \log^3(d)) + (x - 2 \log(d) + x \log^2(d)) \sin(x))}{(1 + \log^2(d))^2}$$

Antiderivative was successfully verified.

`[In] Integrate[d^x*x*Cos[x], x]`

```
[Out] (d^x*(Cos[x]*(1 + x*Log[d] - Log[d]^2 + x*Log[d]^3) + (x - 2*Log[d] + x*Log[d]^2)*Sin[x]))/(1 + Log[d]^2)^2
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 3.63, size = 245, normalized size = 2.95

```
Piecewise[{{(I x Cos[x] + x Sin[x] + I x^2 Cos[x] - I Sin[x]) E^{-I x} / 4, d == E^{-I}}, {(-I x Cos[x] + x Sin[x] - I x^2 Cos[x] + I Sin[x]) E^{I x} / 4, d == E^{I}}, x Cos[x] Log[d] d^x / (1 + Log[d]^4 + 2 Log[d]^2) + x Cos[x] d^x Log[d]^3 / (1 + Log[d]^4 + 2 Log[d]^2) + x d^x Log[d]^2 Sin[x] / (1 + Log[d]^4 + 2 Log[d]^2) + x d^x Sin[x] / (1 + Log[d]^4 + 2 Log[d]^2) - 2 Log[d] d^x Sin[x] / (1 + Log[d]^4 + 2 Log[d]^2) - Cos[x] d^x Log[d]^2 / (1 + Log[d]^4 + 2 Log[d]^2) + Cos[x] d^x / (1 + Log[d]^4 + 2 Log[d]^2)}
```

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[x*d^x*Cos[x], x]')`

```
[Out] Piecewise[{{(I x Cos[x] + x Sin[x] + I x^2 Sin[x] + x^2 Cos[x] - I Sin[x]) E^{-I x} / 4, d == E^{-I}}, {(-I x Cos[x] + x Sin[x] - I x^2 Sin[x] + x^2 Cos[x] + I Sin[x]) E^{I x} / 4, d == E^{I}}, x Cos[x] Log[d] d^x / (1 + Log[d]^4 + 2 Log[d]^2) + x Cos[x] d^x Log[d]^3 / (1 + Log[d]^4 + 2 Log[d]^2) + x d^x Log[d]^2 Sin[x] / (1 + Log[d]^4 + 2 Log[d]^2) + x d^x Sin[x] / (1 + Log[d]^4 + 2 Log[d]^2) - 2 Log[d] d^x Sin[x] / (1 + Log[d]^4 + 2 Log[d]^2) - Cos[x] d^x Log[d]^2 / (1 + Log[d]^4 + 2 Log[d]^2) + Cos[x] d^x / (1 + Log[d]^4 + 2 Log[d]^2)}
```

Maple [C] Result contains complex when optimal does not.

time = 0.04, size = 56, normalized size = 0.67

method	result
risch	$\frac{(-1+x \ln(d)+ix)d^x e^{ix}}{2(\ln(d)+i)^2} + \frac{(-1+x \ln(d)-ix)d^x e^{-ix}}{2(\ln(d)-i)^2}$
norman	$\frac{\frac{\ln(d)x e^x \ln(d)}{1+\ln(d)^2} + \frac{(\ln(d)^2-1)e^x \ln(d) \left(\tan^2\left(\frac{x}{2}\right)\right)}{(1+\ln(d)^2)^2} - \frac{(\ln(d)^2-1)e^x \ln(d)}{(1+\ln(d)^2)^2} - \frac{4 \ln(d)e^x \ln(d) \tan\left(\frac{x}{2}\right)}{(1+\ln(d)^2)^2} + \frac{2x e^x \ln(d) \tan\left(\frac{x}{2}\right)}{1+\ln(d)^2} - \frac{\ln(d)x e^x \ln(d) \left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\ln(d)^2}}{1+\tan^2\left(\frac{x}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(d^x*x*cos(x),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{2}*(-1+x*\ln(d)+I*x)*d^x/(\ln(d)+I)^2*\exp(I*x)+\frac{1}{2}*(-1+x*\ln(d)-I*x)*d^x/(\ln(d)-I)^2*\exp(-I*x)$

Maxima [A]

time = 0.29, size = 58, normalized size = 0.70

$$\frac{((\log(d)^3 + \log(d))x - \log(d)^2 + 1)d^x \cos(x) + ((\log(d)^2 + 1)x - 2 \log(d))d^x \sin(x)}{\log(d)^4 + 2 \log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d^x*x*cos(x),x, algorithm="maxima")`

[Out] $((\log(d)^3 + \log(d))*x - \log(d)^2 + 1)*d^x*\cos(x) + ((\log(d)^2 + 1)*x - 2*\log(d))*d^x*\sin(x)/(\log(d)^4 + 2*\log(d)^2 + 1)$

Fricas [A]

time = 0.31, size = 58, normalized size = 0.70

$$\frac{(x \cos(x) \log(d)^3 + x \cos(x) \log(d) - \cos(x) \log(d)^2 + (x \log(d)^2 + x - 2 \log(d)) \sin(x) + \cos(x))d^x}{\log(d)^4 + 2 \log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d^x*x*cos(x),x, algorithm="fricas")`

[Out] $(x*\cos(x)*\log(d)^3 + x*\cos(x)*\log(d) - \cos(x)*\log(d)^2 + (x*\log(d)^2 + x - 2*\log(d))*\sin(x) + \cos(x))*d^x/(\log(d)^4 + 2*\log(d)^2 + 1)$

Sympy [A]

time = 0.50, size = 304, normalized size = 3.66

$$\begin{cases} \frac{ix^2e^{-ix}\sin(x)}{4} + \frac{x^2e^{-ix}\cos(x)}{4} + \frac{xe^{-ix}\sin(x)}{4} + \frac{ixe^{-ix}\cos(x)}{4} + \frac{e^{-ix}\cos(x)}{4} & \text{for } d = e^{-i} \\ -\frac{ix^2e^{ix}\sin(x)}{4} + \frac{x^2e^{ix}\cos(x)}{4} + \frac{xe^{ix}\sin(x)}{4} - \frac{ixe^{ix}\cos(x)}{4} + \frac{e^{ix}\cos(x)}{4} & \text{for } d = e^i \\ \frac{d^x x \log(d)^3 \cos(x)}{\log(d)^4 + 2 \log(d)^2 + 1} + \frac{d^x x \log(d)^2 \sin(x)}{\log(d)^4 + 2 \log(d)^2 + 1} + \frac{d^x x \log(d) \cos(x)}{\log(d)^4 + 2 \log(d)^2 + 1} + \frac{d^x x \sin(x)}{\log(d)^4 + 2 \log(d)^2 + 1} - \frac{d^x \log(d)^2 \cos(x)}{\log(d)^4 + 2 \log(d)^2 + 1} - \frac{2d^x \log(d) \sin(x)}{\log(d)^4 + 2 \log(d)^2 + 1} + \frac{d^x \cos(x)}{\log(d)^4 + 2 \log(d)^2 + 1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d**x*x*cos(x),x)

[Out] Piecewise((I*x**2*exp(-I*x)*sin(x)/4 + x**2*exp(-I*x)*cos(x)/4 + x*exp(-I*x)*sin(x)/4 + I*x*exp(-I*x)*cos(x)/4 + exp(-I*x)*cos(x)/4, Eq(d, exp(-I))), (-I*x**2*exp(I*x)*sin(x)/4 + x**2*exp(I*x)*cos(x)/4 + x*exp(I*x)*sin(x)/4 - I*x*exp(I*x)*cos(x)/4 + exp(I*x)*cos(x)/4, Eq(d, exp(I))), (d**x*x*log(d)*3*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) + d**x*x*log(d)**2*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) + d**x*x*log(d)*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) + d**x*x*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*log(d)**2*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) - 2*d**x*log(d)*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) + d**x*cos(x)/(log(d)**4 + 2*log(d)**2 + 1), True))

Giac [C] Result contains complex when optimal does not.

time = 0.00, size = 1373, normalized size = 16.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d^x*x*cos(x),x)

[Out]
$$\frac{1}{2} \cdot (2 \cdot ((\pi \cdot x \cdot \operatorname{sgn}(d) - \pi \cdot x + 2 \cdot x) \cdot (\pi \cdot \log(\operatorname{abs}(d)) \cdot \operatorname{sgn}(d) - \pi \cdot \log(\operatorname{abs}(d)) + 2 \cdot \log(\operatorname{abs}(d)))) / (((2 \cdot \pi + \pi^2 \cdot \operatorname{sgn}(d) - \pi^2 + 2 \cdot \log(\operatorname{abs}(d)))^2 - 2 \cdot \pi \cdot \operatorname{sgn}(d) - 2)^2 + 4 \cdot (\pi \cdot \log(\operatorname{abs}(d)) \cdot \operatorname{sgn}(d) - \pi \cdot \log(\operatorname{abs}(d)) + 2 \cdot \log(\operatorname{abs}(d)))^2) + (2 \cdot \pi + \pi^2 \cdot \operatorname{sgn}(d) - \pi^2 + 2 \cdot \log(\operatorname{abs}(d)))^2 - 2 \cdot \pi \cdot \operatorname{sgn}(d) - 2) \cdot (x \cdot \log(\operatorname{abs}(d)) - 1) / (((2 \cdot \pi + \pi^2 \cdot \operatorname{sgn}(d) - \pi^2 + 2 \cdot \log(\operatorname{abs}(d)))^2 - 2 \cdot \pi \cdot \operatorname{sgn}(d) - 2)^2 + 4 \cdot (\pi \cdot \log(\operatorname{abs}(d)) \cdot \operatorname{sgn}(d) - \pi \cdot \log(\operatorname{abs}(d)) + 2 \cdot \log(\operatorname{abs}(d)))^2)) \cdot \cos(1/2 \cdot \pi \cdot x \cdot \operatorname{sgn}(d) - 1/2 \cdot \pi \cdot x + x) - ((2 \cdot \pi + \pi^2 \cdot \operatorname{sgn}(d) - \pi^2 + 2 \cdot \log(\operatorname{abs}(d)))^2 - 2 \cdot \pi \cdot \operatorname{sgn}(d) - 2) \cdot (\pi \cdot x \cdot \operatorname{sgn}(d) - \pi \cdot x + 2 \cdot x) / (((2 \cdot \pi + \pi^2 \cdot \operatorname{sgn}(d) - \pi^2 + 2 \cdot \log(\operatorname{abs}(d)))^2 - 2 \cdot \pi \cdot \operatorname{sgn}(d) - 2)^2 + 4 \cdot (\pi \cdot \log(\operatorname{abs}(d)) \cdot \operatorname{sgn}(d) - \pi \cdot \log(\operatorname{abs}(d)) + 2 \cdot \log(\operatorname{abs}(d)))^2) \cdot (x \cdot \log(\operatorname{abs}(d)) - 1) / (((2 \cdot \pi + \pi^2 \cdot \operatorname{sgn}(d) - \pi^2 + 2 \cdot \log(\operatorname{abs}(d)))^2 - 2 \cdot \pi \cdot \operatorname{sgn}(d) - 2)^2 + 4 \cdot (\pi \cdot \log(\operatorname{abs}(d)) \cdot \operatorname{sgn}(d) - \pi \cdot \log(\operatorname{abs}(d)) + 2 \cdot \log(\operatorname{abs}(d)))^2)) \cdot \sin(1/2 \cdot \pi \cdot x \cdot \operatorname{sgn}(d) - 1/2 \cdot \pi \cdot x + x) \cdot \operatorname{abs}(d)^x + 1/2 \cdot (2 \cdot ((\pi \cdot x \cdot \operatorname{sgn}(d) - \pi \cdot x - 2 \cdot x) \cdot (\pi \cdot \log(\operatorname{abs}(d)) \cdot \operatorname{sgn}(d) - \pi \cdot \log(\operatorname{abs}(d)) - 2 \cdot \log(\operatorname{abs}(d)))) / (((2 \cdot \pi - \pi^2 \cdot \operatorname{sgn}(d) + \pi^2 - 2 \cdot \log(\operatorname{abs}(d)))^2 - 2 \cdot \pi \cdot \operatorname{sgn}(d) + 2)^2 + 4 \cdot (\pi \cdot \log(\operatorname{abs}(d)) \cdot \operatorname{sgn}(d) - \pi \cdot \log(\operatorname{abs}(d)) - 2 \cdot \log(\operatorname{abs}(d)))^2) - (2 \cdot \pi - \pi^2 \cdot \operatorname{sgn}(d) + \pi^2 - 2 \cdot \log(\operatorname{abs}(d)))^2 - 2 \cdot \pi \cdot \operatorname{sgn}(d) + 2) \cdot (x \cdot \log(\operatorname{abs}(d)) - 1) / (((2 \cdot \pi - \pi^2 \cdot \operatorname{sgn}(d) + \pi^2 - 2 \cdot \log(\operatorname{abs}(d)))^2 - 2 \cdot \pi \cdot \operatorname{sgn}(d) + 2)^2 + 4 \cdot (\pi \cdot \log(\operatorname{abs}(d)) \cdot \operatorname{sgn}(d) - \pi \cdot \log(\operatorname{abs}(d)) - 2 \cdot \log(\operatorname{abs}(d)))^2)) \cdot \cos(1/2 \cdot \pi \cdot x \cdot \operatorname{sgn}(d) - 1/2 \cdot \pi \cdot x - x) + ((2 \cdot \pi - \pi^2 \cdot \operatorname{sgn}(d) + \pi^2 - 2 \cdot \log(\operatorname{abs}(d)))^2 - 2 \cdot \pi \cdot \operatorname{sgn}(d) + 2) \cdot (\pi \cdot x \cdot \operatorname{sgn}(d) - \pi \cdot x - 2 \cdot x) / (((2 \cdot \pi - \pi^2 \cdot \operatorname{sgn}(d) + \pi^2 - 2 \cdot \log(\operatorname{abs}(d)))^2 - 2 \cdot \pi \cdot \operatorname{sgn}(d) + 2)^2 + 4 \cdot (\pi \cdot \log(\operatorname{abs}(d)) \cdot \operatorname{sgn}(d) - \pi \cdot \log(\operatorname{abs}(d)) - 2 \cdot \log(\operatorname{abs}(d)))^2) + 4 \cdot (\pi \cdot \log(\operatorname{abs}(d)) \cdot \operatorname{sgn}(d) - \pi \cdot \log(\operatorname{abs}(d)) - 2 \cdot \log(\operatorname{abs}(d)))^2) \cdot (x \cdot \log(\operatorname{abs}(d)) - 1) / (((2 \cdot \pi - \pi^2 \cdot \operatorname{sgn}(d) + \pi^2 - 2 \cdot \log(\operatorname{abs}(d)))^2 - 2 \cdot \pi \cdot \operatorname{sgn}(d) + 2)^2 + 4 \cdot (\pi \cdot \log(\operatorname{abs}(d)) \cdot \operatorname{sgn}(d) - \pi \cdot \log(\operatorname{abs}(d)) - 2 \cdot \log(\operatorname{abs}(d)))^2)) \cdot \sin(1/2 \cdot \pi \cdot x \cdot \operatorname{sgn}(d) - 1/2 \cdot \pi \cdot x - x) \cdot \operatorname{abs}(d)^x + 1/4 \cdot (-I \cdot (\pi \cdot x \cdot \operatorname{sgn}(d) - \pi \cdot x - 2 \cdot I \cdot x \cdot \log(\operatorname{abs}(d)) + 2 \cdot x + 2 \cdot I) \cdot e^{(1/2 \cdot I \cdot \pi \cdot x \cdot \operatorname{sgn}(d) - 1/2 \cdot I \cdot \pi \cdot x + I$$

```

x)/(2*pi + pi^2*sgn(d) + 2*I*pi*log(abs(d))*sgn(d) - pi^2 - 2*I*pi*log(abs(
d)) + 2*log(abs(d))^2 - 2*pi*sgn(d) + 4*I*log(abs(d)) - 2) - I*(pi*x*sgn(d)
- pi*x + 2*I*x*log(abs(d)) + 2*x - 2*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x
- I*x)/(2*pi + pi^2*sgn(d) - 2*I*pi*log(abs(d))*sgn(d) - pi^2 + 2*I*pi*log
(abs(d)) + 2*log(abs(d))^2 - 2*pi*sgn(d) - 4*I*log(abs(d)) - 2))*abs(d)^x -
1/4*(-I*(pi*x*sgn(d) - pi*x - 2*I*x*log(abs(d)) - 2*x + 2*I)*e^(1/2*I*pi*x
*sgn(d) - 1/2*I*pi*x - I*x)/(2*pi - pi^2*sgn(d) - 2*I*pi*log(abs(d))*sgn(d)
+ pi^2 + 2*I*pi*log(abs(d)) - 2*log(abs(d))^2 - 2*pi*sgn(d) + 4*I*log(abs(
d)) + 2) - I*(pi*x*sgn(d) - pi*x + 2*I*x*log(abs(d)) - 2*x - 2*I)*e^(-1/2*I
*pi*x*sgn(d) + 1/2*I*pi*x + I*x)/(2*pi - pi^2*sgn(d) + 2*I*pi*log(abs(d))*s
gn(d) + pi^2 - 2*I*pi*log(abs(d)) - 2*log(abs(d))^2 - 2*pi*sgn(d) - 4*I*log
(abs(d)) + 2))*abs(d)^x

```

Mupad [B]

time = 0.21, size = 55, normalized size = 0.66

$$\frac{d^x (\cos(x) - 2 \ln(d) \sin(x) - \ln(d)^2 \cos(x) + x \sin(x) + x \ln(d) \cos(x) + x \ln(d)^3 \cos(x) + x \ln(d)^2 \sin(x))}{(\ln(d)^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(d^x*x*cos(x),x)

[Out] (d^x*(cos(x) - 2*log(d)*sin(x) - log(d)^2*cos(x) + x*sin(x) + x*log(d)*cos(x) + x*log(d)^3*cos(x) + x*log(d)^2*sin(x)))/(log(d)^2 + 1)^2

3.138 $\int d^x x^2 \sin(x) dx$

Optimal. Leaf size=162

$$\frac{2d^x \cos(x)}{(1 + \log^2(d))^3} - \frac{6d^x \cos(x) \log^2(d)}{(1 + \log^2(d))^3} + \frac{4d^x x \cos(x) \log(d)}{(1 + \log^2(d))^2} - \frac{d^x x^2 \cos(x)}{1 + \log^2(d)} - \frac{6d^x \log(d) \sin(x)}{(1 + \log^2(d))^3} + \frac{2d^x \log^3(d) \sin(x)}{(1 + \log^2(d))^3} +$$

[Out] $2*d^x*\cos(x)/(1+\ln(d)^2)^3-6*d^x*\cos(x)*\ln(d)^2/(1+\ln(d)^2)^3+4*d^x*x*\cos(x)*\ln(d)/(1+\ln(d)^2)^2-d^x*x^2*\cos(x)/(1+\ln(d)^2)-6*d^x*\ln(d)*\sin(x)/(1+\ln(d)^2)^3+2*d^x*\ln(d)^3*\sin(x)/(1+\ln(d)^2)^3+2*d^x*x*\sin(x)/(1+\ln(d)^2)^2-2*d^x*x*\ln(d)^2*\sin(x)/(1+\ln(d)^2)^2+d^x*x^2*\ln(d)*\sin(x)/(1+\ln(d)^2)$

Rubi [A]

time = 0.11, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {4517, 4553, 14, 4518, 4554}

$$\frac{x^2 d^x \log(d) \sin(x)}{\log^2(d)+1} - \frac{x^2 d^x \cos(x)}{\log^2(d)+1} - \frac{2x d^x \log^2(d) \sin(x)}{(\log^2(d)+1)^2} + \frac{2x d^x \sin(x)}{(\log^2(d)+1)^2} - \frac{6d^x \log(d) \sin(x)}{(\log^2(d)+1)^3} + \frac{2d^x \log^3(d) \sin(x)}{(\log^2(d)+1)^3} + \frac{4x d^x \log(d) \cos(x)}{(\log^2(d)+1)^2} - \frac{6d^x \log^2(d) \cos(x)}{(\log^2(d)+1)^3} + \frac{2d^x \cos(x)}{(\log^2(d)+1)^3}$$

Antiderivative was successfully verified.

[In] Int[d^x*x^2*Sin[x],x]

[Out] $(2*d^x*\cos[x])/(1 + \log[d]^2)^3 - (6*d^x*\cos[x]*\log[d]^2)/(1 + \log[d]^2)^3 + (4*d^x*x*\cos[x]*\log[d])/(1 + \log[d]^2)^2 - (d^x*x^2*\cos[x])/(1 + \log[d]^2) - (6*d^x*\log[d]*\sin[x])/(1 + \log[d]^2)^3 + (2*d^x*\log[d]^3*\sin[x])/(1 + \log[d]^2)^3 + (2*d^x*x*\sin[x])/(1 + \log[d]^2)^2 - (2*d^x*x*\log[d]^2*\sin[x])/(1 + \log[d]^2)^2 + (d^x*x^2*\log[d]*\sin[x])/(1 + \log[d]^2)$

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 4517

Int[(F_)^((c_)*((a_) + (b_)*(x_)))*Sin[(d_) + (e_)*(x_)], x_Symbol] := Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]

Rule 4518

Int[Cos[(d_) + (e_)*(x_)]*(F_)^((c_)*((a_) + (b_)*(x_))), x_Symbol] := Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F

reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]

Rule 4553

```
Int[(F_)^((c_)*((a_) + (b_)*(x_)))*((f_)*(x_))^(m_)*Sin[(d_) + (e_)*
(x_)]^(n_), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_) + (e_)*(x_)]^(n_)*(F_)^((c_)*((a_) + (b_)*(x_)))*((f_)*
(x_))^(m_), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
 \int d^x x^2 \sin(x) dx &= -\frac{d^x x^2 \cos(x)}{1 + \log^2(d)} + \frac{d^x x^2 \log(d) \sin(x)}{1 + \log^2(d)} - 2 \int x \left(-\frac{d^x \cos(x)}{1 + \log^2(d)} + \frac{d^x \log(d) \sin(x)}{1 + \log^2(d)} \right) dx \\
 &= -\frac{d^x x^2 \cos(x)}{1 + \log^2(d)} + \frac{d^x x^2 \log(d) \sin(x)}{1 + \log^2(d)} - 2 \int \left(-\frac{d^x x \cos(x)}{1 + \log^2(d)} + \frac{d^x x \log(d) \sin(x)}{1 + \log^2(d)} \right) dx \\
 &= -\frac{d^x x^2 \cos(x)}{1 + \log^2(d)} + \frac{d^x x^2 \log(d) \sin(x)}{1 + \log^2(d)} + \frac{2 \int d^x x \cos(x) dx}{1 + \log^2(d)} - \frac{(2 \log(d)) \int d^x x \sin(x) dx}{1 + \log^2(d)} \\
 &= \frac{4d^x x \cos(x) \log(d)}{(1 + \log^2(d))^2} - \frac{d^x x^2 \cos(x)}{1 + \log^2(d)} + \frac{2d^x x \sin(x)}{(1 + \log^2(d))^2} - \frac{2d^x x \log^2(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x^2 \log(d)}{1 + \log^2(d)} \\
 &= \frac{4d^x x \cos(x) \log(d)}{(1 + \log^2(d))^2} - \frac{d^x x^2 \cos(x)}{1 + \log^2(d)} + \frac{2d^x x \sin(x)}{(1 + \log^2(d))^2} - \frac{2d^x x \log^2(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x^2 \log(d)}{1 + \log^2(d)} \\
 &= \frac{2d^x \cos(x)}{(1 + \log^2(d))^3} - \frac{2d^x \cos(x) \log^2(d)}{(1 + \log^2(d))^3} + \frac{4d^x x \cos(x) \log(d)}{(1 + \log^2(d))^2} - \frac{d^x x^2 \cos(x)}{1 + \log^2(d)} - \frac{2d^x \log(d) \sin(x)}{(1 + \log^2(d))^2}
 \end{aligned}$$

Mathematica [A]

time = 0.06, size = 94, normalized size = 0.58

$$\frac{d^x (-\cos(x) (-2 + x^2 - 4x \log(d) + 2(3 + x^2) \log^2(d) - 4x \log^3(d) + x^2 \log^4(d)) + (2x + (-6 + x^2) \log(d) + 2(1 + x^2) \log^3(d) - 2x \log^4(d) + x^2 \log^5(d)) \sin(x))}{(1 + \log^2(d))^3}$$

Antiderivative was successfully verified.

[In] Integrate[d^x*x^2*Sin[x], x]

```
[Out] (d^x*(-(Cos[x]*(-2 + x^2 - 4*x*Log[d] + 2*(3 + x^2)*Log[d]^2 - 4*x*Log[d]^3 + x^2*Log[d]^4)) + (2*x + (-6 + x^2)*Log[d] + 2*(1 + x^2)*Log[d]^3 - 2*x*Log[d]^4 + x^2*Log[d]^5)*Sin[x]))/(1 + Log[d]^2)^3
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 4.97, size = 549, normalized size = 3.39

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2*d^x*Sin[x],x]')
```

```
[Out] Piecewise[{{(I / 4 x Cos[x] + x Sin[x] / 4 - x ^ 2 Cos[x] / 4 + I / 4 x ^ 2 Sin[x] - I / 6 x ^ 3 Cos[x] + x ^ 3 Sin[x] / 6 + Cos[x] / 4) E ^ (-I x), d == E ^ (-I)}, {((-I / 4) x Cos[x] + x Sin[x] / 4 - x ^ 2 Cos[x] / 4 - I / 4 x ^ 2 Sin[x] + I / 6 x ^ 3 Cos[x] + x ^ 3 Sin[x] / 6 + Cos[x] / 4) E ^ (I x), d == E ^ I}}, -2 x d ^ x Log[d] ^ 4 Sin[x] / (1 + Log[d] ^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) + 2 x d ^ x Sin[x] / (1 + Log[d] ^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) + 4 x Cos[x] Log[d] d ^ x / (1 + Log[d] ^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) + 4 x Cos[x] d ^ x Log[d] ^ 3 / (1 + Log[d] ^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) - 2 x ^ 2 Cos[x] d ^ x Log[d] ^ 2 / (1 + Log[d] ^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) - x ^ 2 Cos[x] d ^ x Log[d] ^ 4 / (1 + Log[d] ^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) - x ^ 2 Cos[x] d ^ x / (1 + Log[d] ^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) + 2 x ^ 2 d ^ x Log[d] ^ 3 Sin[x] / (1 + Log[d] ^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) + x ^ 2 Log[d] d ^ x Sin[x] / (1 + Log[d] ^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) + x ^ 2 d ^ x Log[d] ^ 5 Sin[x] / (1 + Log[d] ^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) - 6 Cos[x] d ^ x Log[d] ^ 2 / (1 + Log[d] ^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) - 6 Log[d] d ^ x Sin[x] / (1 + Log[d] ^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) + 2 Cos[x] d ^ x / (1 + Log[d] ^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) + 2 d ^ x Log[d] ^ 3 Sin[x] / (1 + Log[d] ^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4)]
```

Maple [C] Result contains complex when optimal does not.

time = 0.06, size = 102, normalized size = 0.63

method	result
risch	$\frac{i\left(2+\ln(d)^2x^2+2i\ln(d)x^2-x^2-2x\ln(d)-2ix\right)d^xe^{ix}}{2(\ln(d)+i)^3} + \frac{i\left(2-2x\ln(d)+2ix+\ln(d)^2x^2-2i\ln(d)x^2-x^2\right)d^xe^{-ix}}{2(\ln(d)-i)^3}$
norman	$\frac{x^2e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\ln(d)^2} - \frac{x^2e^{x\ln(d)}}{1+\ln(d)^2} - \frac{2\left(3\ln(d)^2-1\right)e^{x\ln(d)}}{\left(1+\ln(d)^2\right)^3} + \frac{4\ln(d)x e^{x\ln(d)}}{\left(1+\ln(d)^2\right)^2} + \frac{2\left(3\ln(d)^2-1\right)e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{\left(1+\ln(d)^2\right)^3} - \frac{4\ln(d)x e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{\left(1+\ln(d)^2\right)^2} + \frac{2e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\tan^2\left(\frac{x}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^2*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(2+ln(d)^2*x^2+2*I*ln(d)*x^2-x^2-2*x*ln(d)-2*I*x)*d^x/(ln(d)+I)^3*exp(I*x)+1/2*I*(2-2*x*ln(d)+2*I*x+ln(d)^2*x^2-2*I*ln(d)*x^2-x^2)*d^x/(ln(d)-I)^3*exp(-I*x)
```

Maxima [A]

time = 0.32, size = 107, normalized size = 0.66

$$\frac{((\log(d)^4 + 2 \log(d)^2 + 1)x^2 - 4(\log(d)^3 + \log(d))x + 6 \log(d)^2 - 2)d^x \cos(x) - ((\log(d)^5 + 2 \log(d)^3 + \log(d))x^2 + 2 \log(d)^3 - 2(\log(d)^4 - 1)x - 6 \log(d))d^x \sin(x)}{\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*sin(x),x, algorithm="maxima")
```

```
[Out] -(((log(d)^4 + 2*log(d)^2 + 1)*x^2 - 4*(log(d)^3 + log(d))*x + 6*log(d)^2 - 2)*d^x*cos(x) - ((log(d)^5 + 2*log(d)^3 + log(d))*x^2 + 2*log(d)^3 - 2*(log(d)^4 - 1)*x - 6*log(d))*d^x*sin(x))/(log(d)^6 + 3*log(d)^4 + 3*log(d)^2 + 1)
```

Fricas [A]

time = 0.31, size = 115, normalized size = 0.71

$$\frac{(x^2 \cos(x) \log(d)^4 - 4x \cos(x) \log(d)^3 + 2(x^2 + 3) \cos(x) \log(d)^2 - 4x \cos(x) \log(d) + (x^2 - 2) \cos(x) - (x^2 \log(d)^5 - 2x \log(d)^4 + 2(x^2 + 1) \log(d)^3 + (x^2 - 6) \log(d) + 2x) \sin(x))d^x}{\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^2*cos(x)*log(d)^4 - 4*x*cos(x)*log(d)^3 + 2*(x^2 + 3)*cos(x)*log(d)^2 - 4*x*cos(x)*log(d) + (x^2 - 2)*cos(x) - (x^2*log(d)^5 - 2*x*log(d)^4 + 2*(x^2 + 1)*log(d)^3 + (x^2 - 6)*log(d) + 2*x)*sin(x))*d^x/(log(d)^6 + 3*log(d)^4 + 3*log(d)^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.

time = 1.06, size = 665, normalized size = 4.10

$$\frac{(x^2 \cos(x) \log(d)^4 - 4x \cos(x) \log(d)^3 + 2(x^2 + 3) \cos(x) \log(d)^2 - 4x \cos(x) \log(d) + (x^2 - 2) \cos(x) - (x^2 \log(d)^5 - 2x \log(d)^4 + 2(x^2 + 1) \log(d)^3 + (x^2 - 6) \log(d) + 2x) \sin(x))d^x}{\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x**2*sin(x),x)
```

```
[Out] Piecewise((x**3*exp(-I*x)*sin(x)/6 - I*x**3*exp(-I*x)*cos(x)/6 + I*x**2*exp(-I*x)*sin(x)/4 - x**2*exp(-I*x)*cos(x)/4 + x*exp(-I*x)*sin(x)/4 + I*x*exp(-I*x)*cos(x)/4 + exp(-I*x)*cos(x)/4, Eq(d, exp(-I))), (x**3*exp(I*x)*sin(x)/6 + I*x**3*exp(I*x)*cos(x)/6 - I*x**2*exp(I*x)*sin(x)/4 - x**2*exp(I*x)*cos(x)/4 + x*exp(I*x)*sin(x)/4 - I*x*exp(I*x)*cos(x)/4 + exp(I*x)*cos(x)/4, Eq(d, exp(I))), (d**x*x**2*log(d)**5*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - d**x*x**2*log(d)**4*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*x**2*log(d)**3*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 2*d**x*x**2*log(d)**2*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*x**2*log(d)**1*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 2*d**x*x**2*log(d)**0*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1), otherwise))
```

```

)**2 + 1) - 2*d**x*x**2*log(d)**2*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)
)**2 + 1) + d**x*x**2*log(d)*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2
+ 1) - d**x*x**2*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 2*d**
x*x*log(d)**4*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 4*d**x*x
*log(d)**3*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 4*d**x*x*lo
g(d)*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*x*sin(x)/(
log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*log(d)**3*sin(x)/(log(d)
)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**x*log(d)**2*cos(x)/(log(d)**6
+ 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**x*log(d)*sin(x)/(log(d)**6 + 3*log(
d)**4 + 3*log(d)**2 + 1) + 2*d**x*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)
)**2 + 1), True))

```

Giac [C] Result contains complex when optimal does not.

time = 0.00, size = 3343, normalized size = 20.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*sin(x),x)
```

```

[Out] -1/2*(((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(ab
s(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)*(pi^2
*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d) + 2*pi*x^2 -
2*x^2 - 4*x*log(abs(d)) + 4)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn
(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2
- 3*pi*sgn(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*
log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))
^2) - 2*(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^2*log(abs(d))
- pi*x*sgn(d) + pi*x - 2*x)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)
) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(ab
s(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(a
bs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)^2 +
(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 - 6*pi*lo
g(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d)
) - 1/2*pi*x + x) - (2*(3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi
^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sg
n(d) - 2)*(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^2*log(abs(
d)) - pi*x*sgn(d) + pi*x - 2*x)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sg
n(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2
- 3*pi*sgn(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) +
2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)
))^2) + (pi^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d)
+ 2*pi*x^2 - 2*x^2 - 4*x*log(abs(d)) + 4)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi
^2*log(abs(d)) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(
d)) - 6*log(abs(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^
3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sg

```

$$\begin{aligned}
& n(d) - 2)^2 + (3\pi^2 \log(\text{abs}(d)) \text{sgn}(d) - 3\pi^2 \log(\text{abs}(d)) + 2 \log(\text{abs}(d)))^3 - 6\pi \log(\text{abs}(d)) \text{sgn}(d) + 6\pi \log(\text{abs}(d)) - 6 \log(\text{abs}(d)))^2) \sin(\\
& 1/2 \pi x \text{sgn}(d) - 1/2 \pi x + x) \text{abs}(d)^x + 1/2 * (((3\pi - \pi^3 \text{sgn}(d) + 3\pi \\
& \log(\text{abs}(d))^2 \text{sgn}(d) + \pi^3 - 3\pi \log(\text{abs}(d))^2 - 3\pi^2 \text{sgn}(d) + 3\pi^2 \\
& - 6 \log(\text{abs}(d))^2 - 3\pi \text{sgn}(d) + 2) * (\pi^2 x^2 \text{sgn}(d) - \pi^2 x^2 + 2x^2 \log(\text{abs}(d))^2 + 2\pi x^2 \text{sgn}(d) - 2\pi x^2 - 2x^2 - 4x \log(\text{abs}(d)) + 4) / ((\\
& 3\pi - \pi^3 \text{sgn}(d) + 3\pi \log(\text{abs}(d))^2 \text{sgn}(d) + \pi^3 - 3\pi \log(\text{abs}(d))^2 \\
& - 3\pi^2 \text{sgn}(d) + 3\pi^2 - 6 \log(\text{abs}(d))^2 - 3\pi \text{sgn}(d) + 2)^2 + (3\pi^2 \log(\text{abs}(d)) \text{sgn}(d) - 3\pi^2 \log(\text{abs}(d)) + 2 \log(\text{abs}(d))^3 + 6\pi \log(\text{abs}(d)) \\
& * \text{sgn}(d) - 6\pi \log(\text{abs}(d)) - 6 \log(\text{abs}(d)))^2) - 2 * (\pi x^2 \log(\text{abs}(d)) \text{sgn}(\\
& d) - \pi x^2 \log(\text{abs}(d)) - 2x^2 \log(\text{abs}(d)) - \pi x \text{sgn}(d) + \pi x + 2x) * (3\pi \\
& \log(\text{abs}(d)) \text{sgn}(d) - 3\pi^2 \log(\text{abs}(d)) + 2 \log(\text{abs}(d))^3 + 6\pi \log(\text{abs}(d)) \text{sgn}(d) - 6\pi \log(\text{abs}(d)) - 6 \log(\text{abs}(d))) / ((3\pi - \pi^3 \text{sgn}(d) + 3\pi \\
& \log(\text{abs}(d))^2 \text{sgn}(d) + \pi^3 - 3\pi \log(\text{abs}(d))^2 - 3\pi^2 \text{sgn}(d) + 3\pi^2 \\
& - 6 \log(\text{abs}(d))^2 - 3\pi \text{sgn}(d) + 2)^2 + (3\pi^2 \log(\text{abs}(d)) \text{sgn}(d) - 3\pi \\
& \log(\text{abs}(d)) + 2 \log(\text{abs}(d))^3 + 6\pi \log(\text{abs}(d)) \text{sgn}(d) - 6\pi \log(\text{abs}(d)) \\
& - 6 \log(\text{abs}(d)))^2) * \cos(1/2 \pi x \text{sgn}(d) - 1/2 \pi x - x) - (2 * (3\pi - \pi \\
& \log(\text{abs}(d)) \text{sgn}(d) + 3\pi \log(\text{abs}(d))^2 \text{sgn}(d) + \pi^3 - 3\pi \log(\text{abs}(d))^2 - 3\pi^2 \\
& \text{sgn}(d) + 3\pi^2 - 6 \log(\text{abs}(d))^2 - 3\pi \text{sgn}(d) + 2) * (\pi x^2 \log(\text{abs}(d)) \text{sgn}(\\
& d) - \pi x^2 \log(\text{abs}(d)) - 2x^2 \log(\text{abs}(d)) - \pi x \text{sgn}(d) + \pi x + 2x) / \\
& (((3\pi - \pi^3 \text{sgn}(d) + 3\pi \log(\text{abs}(d))^2 \text{sgn}(d) + \pi^3 - 3\pi \log(\text{abs}(d))^2 \\
& - 3\pi^2 \text{sgn}(d) + 3\pi^2 - 6 \log(\text{abs}(d))^2 - 3\pi \text{sgn}(d) + 2)^2 + (3\pi^2 \\
& \log(\text{abs}(d)) \text{sgn}(d) - 3\pi^2 \log(\text{abs}(d)) + 2 \log(\text{abs}(d))^3 + 6\pi \log(\text{abs}(d)) \\
& * \text{sgn}(d) - 6\pi \log(\text{abs}(d)) - 6 \log(\text{abs}(d)))^2) + (\pi^2 x^2 \text{sgn}(d) - \pi^2 x^2 \\
& x^2 + 2x^2 \log(\text{abs}(d))^2 + 2\pi x^2 \text{sgn}(d) - 2\pi x^2 - 2x^2 - 4x \log(\text{abs}(d)) \\
& + 4) * (3\pi^2 \log(\text{abs}(d)) \text{sgn}(d) - 3\pi^2 \log(\text{abs}(d)) + 2 \log(\text{abs}(d))^3 \\
& + 6\pi \log(\text{abs}(d)) \text{sgn}(d) - 6\pi \log(\text{abs}(d)) - 6 \log(\text{abs}(d))) / ((3\pi - \pi \\
& \log(\text{abs}(d)) \text{sgn}(d) + 3\pi \log(\text{abs}(d))^2 \text{sgn}(d) + \pi^3 - 3\pi \log(\text{abs}(d))^2 - 3\pi^2 \\
& \text{sgn}(d) + 3\pi^2 - 6 \log(\text{abs}(d))^2 - 3\pi \text{sgn}(d) + 2)^2 + (3\pi^2 \log(\text{abs}(d)) \\
& * \text{sgn}(d) - 3\pi^2 \log(\text{abs}(d)) + 2 \log(\text{abs}(d))^3 + 6\pi \log(\text{abs}(d)) \text{sgn}(d) - \\
& 6\pi \log(\text{abs}(d)) - 6 \log(\text{abs}(d)))^2) * \sin(1/2 \pi x \text{sgn}(d) - 1/2 \pi x - x) \\
& * \text{abs}(d)^x - 2 * ((-I \pi^2 x^2 \text{sgn}(d) + 2\pi x^2 \log(\text{abs}(d)) \text{sgn}(d) + I \pi^2 x^2 \\
& - 2\pi x^2 \log(\text{abs}(d)) - 2I x^2 \log(\text{abs}(d))^2 + 2I \pi x^2 \text{sgn}(d) - 2I \\
& \pi x^2 + 4x^2 \log(\text{abs}(d)) - 2\pi x \text{sgn}(d) + 2\pi x + 2I x^2 + 4I x \log(\text{abs}(d)) - 4x - 4I) * e^{(1/2 I \pi x \text{sgn}(d) - 1/2 I \pi x + I x) / (24 I \pi - 8 \\
& I \pi^3 \text{sgn}(d) + 24 \pi^2 \log(\text{abs}(d)) \text{sgn}(d) + 24 I \pi \log(\text{abs}(d))^2 \text{sgn}(d) + \\
& 8 I \pi^3 - 24 \pi^2 \log(\text{abs}(d)) - 24 I \pi \log(\text{abs}(d))^2 + 16 \log(\text{abs}(d))^3 \\
& + 24 I \pi^2 \text{sgn}(d) - 48 \pi \log(\text{abs}(d)) \text{sgn}(d) - 24 I \pi^2 + 48 \pi \log(\text{abs}(d)) \\
&) + 48 I \log(\text{abs}(d))^2 - 24 I \pi \text{sgn}(d) - 48 \log(\text{abs}(d)) - 16 I) + (-I \pi^2 x^2 \text{sgn}(d) - 2\pi x^2 \log(\text{abs}(d)) \text{sgn}(d) + I \pi^2 x^2 + 2\pi x^2 \log(\text{abs}(d)) \\
& - 2I x^2 \log(\text{abs}(d))^2 + 2I \pi x^2 \text{sgn}(d) - 2I \pi x^2 - 4x^2 \log(\text{abs}(d)) \\
& + 2\pi x \text{sgn}(d) - 2\pi x + 2I x^2 + 4I x \log(\text{abs}(d)) + 4x - 4I) * e^{(-1/2 I \pi x \text{sgn}(d) + 1/2 I \pi x - I x) / (-24 I \pi + 8 I \pi^3 \text{sgn}(d) + 24 \pi \\
& \log(\text{abs}(d)) \text{sgn}(d) - 24 I \pi \log(\text{abs}(d))^2 \text{sgn}(d) - 8 I \pi^3 - 24 \pi^2 \log(\text{abs}(d)) + 24 I \pi \log(\text{abs}(d))^2 + 16 \log(\text{abs}(d))^3 - 24 I \pi^2 \text{sgn}(d) -
\end{aligned}$$

$$\begin{aligned}
& 48\pi \log(\operatorname{abs}(d)) \operatorname{sgn}(d) + 24I\pi^2 + 48\pi \log(\operatorname{abs}(d)) - 48I \log(\operatorname{abs}(d))^2 \\
& + 24I\pi \operatorname{sgn}(d) - 48 \log(\operatorname{abs}(d)) + 16I) \operatorname{abs}(d)^x - 2((I\pi^2 x^2 \operatorname{sgn}(d) - 2\pi x^2 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - I\pi^2 x^2 + 2\pi x^2 \log(\operatorname{abs}(d)) + 2I x^2 \log(\operatorname{abs}(d))^2 + 2I\pi x^2 \operatorname{sgn}(d) - 2I\pi x^2 + 4x^2 \log(\operatorname{abs}(d)) + 2\pi x \operatorname{sgn}(d) - 2\pi x - 2I x^2 - 4I x \log(\operatorname{abs}(d)) - 4x + 4I) e^{(1/2 I \pi x \operatorname{sgn}(d) - 1/2 I \pi x - I x)} / (24I\pi - 8I\pi^3 \operatorname{sgn}(d) + 24\pi^2 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) + 24I\pi \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) + 8I\pi^3 - 24\pi^2 \log(\operatorname{abs}(d)) - 24I\pi \log(\operatorname{abs}(d))^2 + 16 \log(\operatorname{abs}(d))^3 - 24I\pi^2 \operatorname{sgn}(d) + 48\pi \log(\operatorname{abs}(d)) \operatorname{sgn}(d) + 24I\pi^2 - 48\pi \log(\operatorname{abs}(d)) - 48I \log(\operatorname{abs}(d))^2 - 24I\pi \operatorname{sgn}(d) - 48 \log(\operatorname{abs}(d)) + 16I) + (I\pi^2 x^2 \operatorname{sgn}(d) + 2\pi x^2 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - I\pi^2 x^2 - 2\pi x^2 \log(\operatorname{abs}(d)) + 2I x^2 \log(\operatorname{abs}(d))^2 + 2I\pi x^2 \operatorname{sgn}(d) - 2I\pi x^2 - 4x^2 \log(\operatorname{abs}(d)) - 2\pi x \operatorname{sgn}(d) + 2\pi x - 2I x^2 - 4I x \log(\operatorname{abs}(d)) + 4x + 4I) e^{(-1/2 I \pi x \operatorname{sgn}(d) + 1/2 I \pi x + I x)} / (-24I\pi + 8I\pi^3 \operatorname{sgn}(d) + 24\pi^2 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - 24I\pi \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) - 8I\pi^3 - 24\pi^2 \log(\operatorname{abs}(d)) + 24I\pi \log(\operatorname{abs}(d))^2 + 16 \log(\operatorname{abs}(d))^3 + 24I\pi^2 \operatorname{sgn}(d) + 48\pi \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - 24I\pi^2 - 48\pi \log(\operatorname{abs}(d)) + 48I \log(\operatorname{abs}(d))^2 + 24I\pi \operatorname{sgn}(d) - 48 \log(\operatorname{abs}(d)) - 16I) \operatorname{abs}(d)^x
\end{aligned}$$

Mupad [B]

time = 0.39, size = 133, normalized size = 0.82

$$\frac{d^x (2 \cos(x) - x^2 \cos(x) + 2x \sin(x)) + d^x \ln(d)^3 (2 \sin(x) + 2x^2 \sin(x) + 4x \cos(x)) - d^x \ln(d)^2 (6 \cos(x) + 2x^2 \cos(x)) + d^x \ln(d) (x^2 \sin(x) - 6 \sin(x) + 4x \cos(x)) - d^x \ln(d)^4 (x^2 \cos(x) + 2x \sin(x)) + d^x x^2 \ln(d)^5 \sin(x)}{(\ln(d)^2 + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(d^x*x^2*sin(x),x)`

[Out] $(d^x(2\cos(x) - x^2\cos(x) + 2x\sin(x)) + d^x\log(d)^3(2\sin(x) + 2x^2\sin(x) + 4x\cos(x)) - d^x\log(d)^2(6\cos(x) + 2x^2\cos(x)) + d^x\log(d)(x^2\sin(x) - 6\sin(x) + 4x\cos(x)) - d^x\log(d)^4(x^2\cos(x) + 2x\sin(x)) + d^x x^2 \log(d)^5 \sin(x)) / (\log(d)^2 + 1)^3$

3.139 $\int d^x x^2 \cos(x) dx$

Optimal. Leaf size=161

$$-\frac{6d^x \cos(x) \log(d)}{(1 + \log^2(d))^3} + \frac{2d^x \cos(x) \log^3(d)}{(1 + \log^2(d))^3} + \frac{2d^x x \cos(x)}{(1 + \log^2(d))^2} - \frac{2d^x x \cos(x) \log^2(d)}{(1 + \log^2(d))^2} + \frac{d^x x^2 \cos(x) \log(d)}{1 + \log^2(d)} - \frac{2d^x \sin(x)}{(1 + \log^2(d))^3}$$

[Out] $-6*d^x*\cos(x)*\ln(d)/(1+\ln(d)^2)^3+2*d^x*\cos(x)*\ln(d)^3/(1+\ln(d)^2)^3+2*d^x*x*\cos(x)/(1+\ln(d)^2)^2-2*d^x*x*\cos(x)*\ln(d)^2/(1+\ln(d)^2)^2+d^x*x^2*\cos(x)*\ln(d)/(1+\ln(d)^2)-2*d^x*\sin(x)/(1+\ln(d)^2)^3+6*d^x*\ln(d)^2*\sin(x)/(1+\ln(d)^2)^3-4*d^x*x*\ln(d)*\sin(x)/(1+\ln(d)^2)^2+d^x*x^2*\sin(x)/(1+\ln(d)^2)$

Rubi [A]

time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {4518, 4554, 14, 4517, 4553}

$$\frac{x^2 d^x \sin(x)}{\log^2(d)+1} + \frac{x^2 d^x \log(d) \cos(x)}{\log^2(d)+1} - \frac{4x d^x \log(d) \sin(x)}{(\log^2(d)+1)^2} + \frac{6d^x \log^2(d) \sin(x)}{(\log^2(d)+1)^3} - \frac{2d^x \sin(x)}{(\log^2(d)+1)^3} - \frac{2x d^x \log^2(d) \cos(x)}{(\log^2(d)+1)^2} + \frac{2x d^x \cos(x)}{(\log^2(d)+1)^2} - \frac{6d^x \log(d) \cos(x)}{(\log^2(d)+1)^3} + \frac{2d^x \log^3(d) \cos(x)}{(\log^2(d)+1)^3}$$

Antiderivative was successfully verified.

[In] Int[d^x*x^2*Cos[x], x]

[Out] $(-6*d^x*\cos[x]*\log[d])/(1 + \log[d]^2)^3 + (2*d^x*\cos[x]*\log[d]^3)/(1 + \log[d]^2)^3 + (2*d^x*x*\cos[x])/(1 + \log[d]^2)^2 - (2*d^x*x*\cos[x]*\log[d]^2)/(1 + \log[d]^2)^2 + (d^x*x^2*\cos[x]*\log[d])/(1 + \log[d]^2) - (2*d^x*\sin[x])/(1 + \log[d]^2)^3 + (6*d^x*\log[d]^2*\sin[x])/(1 + \log[d]^2)^3 - (4*d^x*x*\log[d]*\sin[x])/(1 + \log[d]^2)^2 + (d^x*x^2*\sin[x])/(1 + \log[d]^2)$

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 4517

Int[(F_)^((c_)*((a_)+(b_)*(x_)))*Sin[(d_)+(e_)*(x_)], x_Symbol] := Simp[b*c*Log[F]*F^(c*(a+b*x))*(Sin[d+e*x]/(e^2+b^2*c^2*Log[F]^2)), x] - Simp[e*F^(c*(a+b*x))*(Cos[d+e*x]/(e^2+b^2*c^2*Log[F]^2)), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2+b^2*c^2*Log[F]^2, 0]

Rule 4518

Int[Cos[(d_)+(e_)*(x_)]*(F_)^((c_)*((a_)+(b_)*(x_))), x_Symbol] := Simp[b*c*Log[F]*F^(c*(a+b*x))*(Cos[d+e*x]/(e^2+b^2*c^2*Log[F]^2)), x] + Simp[e*F^(c*(a+b*x))*(Sin[d+e*x]/(e^2+b^2*c^2*Log[F]^2)), x] /; F

reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]

Rule 4553

Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]

Rule 4554

Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int d^x x^2 \cos(x) dx &= \frac{d^x x^2 \cos(x) \log(d)}{1 + \log^2(d)} + \frac{d^x x^2 \sin(x)}{1 + \log^2(d)} - 2 \int x \left(\frac{d^x \cos(x) \log(d)}{1 + \log^2(d)} + \frac{d^x \sin(x)}{1 + \log^2(d)} \right) dx \\
 &= \frac{d^x x^2 \cos(x) \log(d)}{1 + \log^2(d)} + \frac{d^x x^2 \sin(x)}{1 + \log^2(d)} - 2 \int \left(\frac{d^x x \cos(x) \log(d)}{1 + \log^2(d)} + \frac{d^x x \sin(x)}{1 + \log^2(d)} \right) dx \\
 &= \frac{d^x x^2 \cos(x) \log(d)}{1 + \log^2(d)} + \frac{d^x x^2 \sin(x)}{1 + \log^2(d)} - \frac{2 \int d^x x \sin(x) dx}{1 + \log^2(d)} - \frac{(2 \log(d)) \int d^x x \cos(x) dx}{1 + \log^2(d)} \\
 &= \frac{2d^x x \cos(x)}{(1 + \log^2(d))^2} - \frac{2d^x x \cos(x) \log^2(d)}{(1 + \log^2(d))^2} + \frac{d^x x^2 \cos(x) \log(d)}{1 + \log^2(d)} - \frac{4d^x x \log(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x^2 \sin(x)}{1 + \log^2(d)} \\
 &= \frac{2d^x x \cos(x)}{(1 + \log^2(d))^2} - \frac{2d^x x \cos(x) \log^2(d)}{(1 + \log^2(d))^2} + \frac{d^x x^2 \cos(x) \log(d)}{1 + \log^2(d)} - \frac{4d^x x \log(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x^2 \sin(x)}{1 + \log^2(d)} \\
 &= -\frac{2d^x \cos(x) \log(d)}{(1 + \log^2(d))^3} + \frac{2d^x \cos(x) \log^3(d)}{(1 + \log^2(d))^3} + \frac{2d^x x \cos(x)}{(1 + \log^2(d))^2} - \frac{2d^x x \cos(x) \log^2(d)}{(1 + \log^2(d))^2} + \frac{d^x x^2 \sin(x)}{1 + \log^2(d)}
 \end{aligned}$$

Mathematica [A]

time = 0.05, size = 93, normalized size = 0.58

$$\frac{d^x (\cos(x) (2x + (-6 + x^2) \log(d) + 2(1 + x^2) \log^3(d) - 2x \log^4(d) + x^2 \log^5(d)) + (-2 + x^2 - 4x \log(d) + 2(3 + x^2) \log^2(d) - 4x \log^3(d) + x^2 \log^4(d)) \sin(x))}{(1 + \log^2(d))^3}$$

Antiderivative was successfully verified.

[In] Integrate[d^x*x^2*Cos[x], x]


```
[Out] (d^x*(Cos[x]*(2*x + (-6 + x^2)*Log[d] + 2*(1 + x^2)*Log[d]^3 - 2*x*Log[d]^4
+ x^2*Log[d]^5) + (-2 + x^2 - 4*x*Log[d] + 2*(3 + x^2)*Log[d]^2 - 4*x*Log[
d]^3 + x^2*Log[d]^4)*Sin[x]))/(1 + Log[d]^2)^3
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 5.05, size = 547, normalized size = 3.40

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2*d^x*Cos[x],x]')
```

```
[Out] Piecewise[{{((-I / 4) x Sin[x] + x Cos[x] / 4 + I / 4 x ^ 2 Cos[x] + x ^ 2
Sin[x] / 4 + I / 6 x ^ 3 Sin[x] + x ^ 3 Cos[x] / 6 - Sin[x] / 4) E ^ (-I x)
, d == E ^ (-I)}, {(I / 4 x Sin[x] + x Cos[x] / 4 - I / 4 x ^ 2 Cos[x] + x
^ 2 Sin[x] / 4 - I / 6 x ^ 3 Sin[x] + x ^ 3 Cos[x] / 6 - Sin[x] / 4) E ^ (I
x), d == E ^ I}}, -4 x Log[d] d ^ x Sin[x] / (1 + Log[d] ^ 6 + 3 Log[d] ^
2 + 3 Log[d] ^ 4) - 4 x d ^ x Log[d] ^ 3 Sin[x] / (1 + Log[d] ^ 6 + 3 Log[d]
^ 2 + 3 Log[d] ^ 4) - 2 x Cos[x] d ^ x Log[d] ^ 4 / (1 + Log[d] ^ 6 + 3 L
og[d] ^ 2 + 3 Log[d] ^ 4) + 2 x Cos[x] d ^ x / (1 + Log[d] ^ 6 + 3 Log[d] ^
2 + 3 Log[d] ^ 4) + 2 x ^ 2 Cos[x] d ^ x Log[d] ^ 3 / (1 + Log[d] ^ 6 + 3
Log[d] ^ 2 + 3 Log[d] ^ 4) + 2 x ^ 2 d ^ x Log[d] ^ 2 Sin[x] / (1 + Log[d]
^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) + x ^ 2 Cos[x] Log[d] d ^ x / (1 + Log[d]
^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) + x ^ 2 Cos[x] d ^ x Log[d] ^ 5 / (1 +
Log[d] ^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) + x ^ 2 d ^ x Log[d] ^ 4 Sin[x]
/ (1 + Log[d] ^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) + x ^ 2 d ^ x Sin[x] / (1
+ Log[d] ^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) - 6 Cos[x] Log[d] d ^ x / (1 +
Log[d] ^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) - 2 d ^ x Sin[x] / (1 + Log[d] ^
6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) + 2 Cos[x] d ^ x Log[d] ^ 3 / (1 + Log[d]
^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4) + 6 d ^ x Log[d] ^ 2 Sin[x] / (1 + Log[d]
^ 6 + 3 Log[d] ^ 2 + 3 Log[d] ^ 4)]
```

Maple [C] Result contains complex when optimal does not.

time = 0.06, size = 100, normalized size = 0.62

method	result
risch	$\frac{(2+\ln(d))^2 x^2 + 2i \ln(d) x^2 - x^2 - 2x \ln(d) - 2ix}{2(\ln(d)+i)^3} d^x e^{ix} + \frac{(2-2x \ln(d) + 2ix + \ln(d))^2 x^2 - 2i \ln(d) x^2 - x^2}{2(\ln(d)-i)^3} d^x e^{-ix}$
norman	$\frac{\ln(d) x^2 e^{x \ln(d)}}{1+\ln(d)^2} + \frac{2x^2 e^{x \ln(d)} \tan\left(\frac{x}{2}\right)}{1+\ln(d)^2} - \frac{2(\ln(d)^2-1) x e^{x \ln(d)}}{(1+\ln(d)^2)^2} + \frac{4(3 \ln(d)^2-1) e^{x \ln(d)} \tan\left(\frac{x}{2}\right)}{(1+\ln(d)^2)^3} + \frac{2 \ln(d) (\ln(d)^2-3) e^{x \ln(d)}}{(1+\ln(d)^2)^3} - \frac{8 \ln(d) x e^{x \ln(d)} \tan\left(\frac{x}{2}\right)}{(1+\ln(d)^2)^2} + \frac{1}{1+\tan^2\left(\frac{x}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^2*cos(x),x,method=_RETURNVERBOSE)
```

[Out] $\frac{1}{2}*(2+\ln(d)^2*x^2+2*I*\ln(d)*x^2-x^2-2*x*\ln(d)-2*I*x)*d^x/(\ln(d)+I)^3*\exp(I*x)+\frac{1}{2}*(2-2*x*\ln(d)+2*I*x+\ln(d)^2*x^2-2*I*\ln(d)*x^2-x^2)*d^x/(\ln(d)-I)^3*\exp(-I*x)$

Maxima [A]

time = 0.27, size = 105, normalized size = 0.65

$$\frac{((\log(d)^5 + 2 \log(d)^3 + \log(d))x^2 + 2 \log(d)^3 - 2(\log(d)^4 - 1)x - 6 \log(d))d^x \cos(x) + ((\log(d)^4 + 2 \log(d)^2 + 1)x^2 - 4(\log(d)^3 + \log(d))x + 6 \log(d)^2 - 2)d^x \sin(x)}{\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d^x*x^2*cos(x),x, algorithm="maxima")`

[Out] $((\log(d)^5 + 2*\log(d)^3 + \log(d))*x^2 + 2*\log(d)^3 - 2*(\log(d)^4 - 1)*x - 6*\log(d))*d^x*\cos(x) + ((\log(d)^4 + 2*\log(d)^2 + 1)*x^2 - 4*(\log(d)^3 + \log(d))*x + 6*\log(d)^2 - 2)*d^x*\sin(x)/(\log(d)^6 + 3*\log(d)^4 + 3*\log(d)^2 + 1)$

Fricas [A]

time = 0.32, size = 111, normalized size = 0.69

$$\frac{(x^2 \cos(x) \log(d)^5 - 2x \cos(x) \log(d)^4 + 2(x^2 + 1) \cos(x) \log(d)^3 + (x^2 - 6) \cos(x) \log(d) + 2x \cos(x) + (x^2 \log(d)^4 - 4x \log(d)^3 + 2(x^2 + 3) \log(d)^2 + x^2 - 4x \log(d) - 2) \sin(x))d^x}{\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d^x*x^2*cos(x),x, algorithm="fricas")`

[Out] $(x^2*\cos(x)*\log(d)^5 - 2*x*\cos(x)*\log(d)^4 + 2*(x^2 + 1)*\cos(x)*\log(d)^3 + (x^2 - 6)*\cos(x)*\log(d) + 2*x*\cos(x) + (x^2*\log(d)^4 - 4*x*\log(d)^3 + 2*(x^2 + 3)*\log(d)^2 + x^2 - 4*x*\log(d) - 2)*\sin(x))*d^x/(\log(d)^6 + 3*\log(d)^4 + 3*\log(d)^2 + 1)$

Sympy [C] Result contains complex when optimal does not.

time = 1.05, size = 668, normalized size = 4.15

$$\frac{d^{x^2} \cos(x) \log(d)^5 - 2x d^{x^2} \cos(x) \log(d)^4 + 2(x^2 + 1) d^{x^2} \cos(x) \log(d)^3 + (x^2 - 6) d^{x^2} \cos(x) \log(d) + 2x d^{x^2} \cos(x) + (x^2 \log(d)^4 - 4x \log(d)^3 + 2(x^2 + 3) \log(d)^2 + x^2 - 4x \log(d) - 2) d^{x^2} \sin(x)}{\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d**x*x**2*cos(x),x)`

[Out] $\text{Piecewise}((I*x**3*\exp(-I*x)*\sin(x)/6 + x**3*\exp(-I*x)*\cos(x)/6 + x**2*\exp(-I*x)*\sin(x)/4 + I*x**2*\exp(-I*x)*\cos(x)/4 - I*x*\exp(-I*x)*\sin(x)/4 + x*\exp(-I*x)*\cos(x)/4 - I*\exp(-I*x)*\cos(x)/4, \text{Eq}(d, \exp(-I))), (-I*x**3*\exp(I*x)*\sin(x)/6 + x**3*\exp(I*x)*\cos(x)/6 + x**2*\exp(I*x)*\sin(x)/4 - I*x**2*\exp(I*x)*\cos(x)/4 + I*x*\exp(I*x)*\sin(x)/4 + x*\exp(I*x)*\cos(x)/4 + I*\exp(I*x)*\cos(x)/4, \text{Eq}(d, \exp(I))), (d**x*x**2*\log(d)**5*\cos(x)/(\log(d)**6 + 3*\log(d)**4 + 3*\log(d)**2 + 1) + d**x*x**2*\log(d)**4*\sin(x)/(\log(d)**6 + 3*\log(d)**4 + 3*\log(d)**2 + 1) + 2*d**x*x**2*\log(d)**3*\cos(x)/(\log(d)**6 + 3*\log(d)**4 + 3*$

```
log(d)**2 + 1) + 2*d**x**2*log(d)**2*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*
log(d)**2 + 1) + d**x**2*log(d)*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)
)**2 + 1) + d**x**2*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) -
2*d**x*log(d)**4*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 4*d
**x*log(d)**3*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 4*d**
*x*log(d)*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*cos
(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*log(d)**3*cos(x)/(
log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 6*d**x*log(d)**2*sin(x)/(log(d)
)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**x*log(d)*cos(x)/(log(d)**6 + 3
*log(d)**4 + 3*log(d)**2 + 1) - 2*d**x*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*
log(d)**2 + 1), True))
```

Giac [C] Result contains complex when optimal does not.

time = 0.00, size = 3351, normalized size = 20.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*cos(x),x)
```

```
[Out] 1/2*((2*(3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(a
bs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)*(pi*
x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^2*log(abs(d)) - pi*x*sgn(
d) + pi*x - 2*x)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 -
3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d)
- 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3
- 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2) + (pi^2*x
^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d) + 2*pi*x^2 - 2
*x^2 - 4*x*log(abs(d)) + 4)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d))
+ 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs
(d))))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(ab
s(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)^2 + (
3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 - 6*pi*log
(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2)*cos(1/2*pi*x*sgn(d)
- 1/2*pi*x + x) + ((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3
- 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(
d) - 2)*(pi^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d)
+ 2*pi*x^2 - 2*x^2 - 4*x*log(abs(d)) + 4)/((3*pi - pi^3*sgn(d) + 3*pi*log(
abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*l
og(abs(d))^2 - 3*pi*sgn(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log
(abs(d)) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6
*log(abs(d)))^2) - 2*(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^
2*log(abs(d)) - pi*x*sgn(d) + pi*x - 2*x)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi
^2*log(abs(d)) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)
)) - 6*log(abs(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3
- 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn
```

$$\begin{aligned}
& (d - 2)^2 + (3\pi^2 \log(\text{abs}(d)) \text{sgn}(d) - 3\pi^2 \log(\text{abs}(d)) + 2 \log(\text{abs}(d)))^3 - 6\pi \log(\text{abs}(d)) \text{sgn}(d) + 6\pi \log(\text{abs}(d)) - 6 \log(\text{abs}(d)))^2) \sin(1/2 \pi x \text{sgn}(d) - 1/2 \pi x + x) \text{abs}(d)^x + 1/2 * ((2 * (3\pi - \pi^3 \text{sgn}(d) + 3\pi \log(\text{abs}(d))^2 \text{sgn}(d) + \pi^3 - 3\pi \log(\text{abs}(d))^2 - 3\pi^2 \text{sgn}(d) + 3\pi^2 - 6 \log(\text{abs}(d))^2 - 3\pi \text{sgn}(d) + 2) * (\pi x^2 \log(\text{abs}(d)) \text{sgn}(d) - \pi x^2 \log(\text{abs}(d)) - 2x^2 \log(\text{abs}(d)) - \pi x \text{sgn}(d) + \pi x + 2x) / ((3\pi - \pi^3 \text{sgn}(d) + 3\pi \log(\text{abs}(d))^2 \text{sgn}(d) + \pi^3 - 3\pi \log(\text{abs}(d))^2 - 3\pi^2 \text{sgn}(d) + 3\pi^2 - 6 \log(\text{abs}(d))^2 - 3\pi \text{sgn}(d) + 2))^2 + (3\pi^2 \log(\text{abs}(d)) \text{sgn}(d) - 3\pi^2 \log(\text{abs}(d)) + 2 \log(\text{abs}(d))^3 + 6\pi \log(\text{abs}(d)) \text{sgn}(d) - 6\pi \log(\text{abs}(d)) - 6 \log(\text{abs}(d)))^2) + (\pi^2 x^2 \text{sgn}(d) - \pi^2 x^2 + 2x^2 \log(\text{abs}(d))^2 + 2\pi x^2 \text{sgn}(d) - 2\pi x^2 - 2x^2 - 4x \log(\text{abs}(d)) + 4) * (3\pi^2 \log(\text{abs}(d)) \text{sgn}(d) - 3\pi^2 \log(\text{abs}(d)) + 2 \log(\text{abs}(d))^3 + 6\pi \log(\text{abs}(d)) \text{sgn}(d) - 6\pi \log(\text{abs}(d)) - 6 \log(\text{abs}(d))) / ((3\pi - \pi^3 \text{sgn}(d) + 3\pi \log(\text{abs}(d))^2 \text{sgn}(d) + \pi^3 - 3\pi \log(\text{abs}(d))^2 - 3\pi^2 \text{sgn}(d) + 3\pi^2 - 6 \log(\text{abs}(d))^2 - 3\pi \text{sgn}(d) + 2))^2 + (3\pi^2 \log(\text{abs}(d)) \text{sgn}(d) - 3\pi^2 \log(\text{abs}(d)) + 2 \log(\text{abs}(d))^3 + 6\pi \log(\text{abs}(d)) \text{sgn}(d) - 6\pi \log(\text{abs}(d)) - 6 \log(\text{abs}(d)))^2) * \cos(1/2 \pi x \text{sgn}(d) - 1/2 \pi x - x) + ((3\pi - \pi^3 \text{sgn}(d) + 3\pi \log(\text{abs}(d))^2 \text{sgn}(d) + \pi^3 - 3\pi \log(\text{abs}(d))^2 - 3\pi^2 \text{sgn}(d) + 3\pi^2 - 6 \log(\text{abs}(d))^2 - 3\pi \text{sgn}(d) + 2) * (\pi^2 x^2 \text{sgn}(d) - \pi^2 x^2 + 2x^2 \log(\text{abs}(d))^2 + 2\pi x^2 \text{sgn}(d) - 2\pi x^2 - 2x^2 - 4x \log(\text{abs}(d)) + 4) / ((3\pi - \pi^3 \text{sgn}(d) + 3\pi \log(\text{abs}(d))^2 \text{sgn}(d) + \pi^3 - 3\pi \log(\text{abs}(d))^2 - 3\pi^2 \text{sgn}(d) + 3\pi^2 - 6 \log(\text{abs}(d))^2 - 3\pi \text{sgn}(d) + 2))^2 + (3\pi^2 \log(\text{abs}(d)) \text{sgn}(d) - 3\pi^2 \log(\text{abs}(d)) + 2 \log(\text{abs}(d))^3 + 6\pi \log(\text{abs}(d)) \text{sgn}(d) - 6\pi \log(\text{abs}(d)) - 6 \log(\text{abs}(d)))^2) - 2 * (\pi x^2 \log(\text{abs}(d)) \text{sgn}(d) - \pi x^2 \log(\text{abs}(d)) - 2x^2 \log(\text{abs}(d)) - \pi x \text{sgn}(d) + \pi x + 2x) * (3\pi^2 \log(\text{abs}(d)) \text{sgn}(d) - 3\pi^2 \log(\text{abs}(d)) + 2 \log(\text{abs}(d))^3 + 6\pi \log(\text{abs}(d)) \text{sgn}(d) - 6\pi \log(\text{abs}(d)) - 6 \log(\text{abs}(d))) / ((3\pi - \pi^3 \text{sgn}(d) + 3\pi \log(\text{abs}(d))^2 \text{sgn}(d) + \pi^3 - 3\pi \log(\text{abs}(d))^2 - 3\pi^2 \text{sgn}(d) + 3\pi^2 - 6 \log(\text{abs}(d))^2 - 3\pi \text{sgn}(d) + 2))^2 + (3\pi^2 \log(\text{abs}(d)) \text{sgn}(d) - 3\pi^2 \log(\text{abs}(d)) + 2 \log(\text{abs}(d))^3 + 6\pi \log(\text{abs}(d)) \text{sgn}(d) - 6\pi \log(\text{abs}(d)) - 6 \log(\text{abs}(d)))^2) * \sin(1/2 \pi x \text{sgn}(d) - 1/2 \pi x - x) * \text{abs}(d)^x - 2 * (-I * (I \pi^2 x^2 \text{sgn}(d) - 2\pi x^2 \log(\text{abs}(d)) \text{sgn}(d) - I \pi^2 x^2 + 2\pi x^2 \log(\text{abs}(d)) + 2I x^2 \log(\text{abs}(d))^2 - 2I \pi x^2 \text{sgn}(d) + 2I \pi x^2 - 4x^2 \log(\text{abs}(d)) + 2\pi x \text{sgn}(d) - 2\pi x - 2I x^2 - 4I x \log(\text{abs}(d)) + 4x + 4I) * e^{(1/2 I \pi x \text{sgn}(d) - 1/2 I \pi x + I x)} / (24 I \pi - 8 I \pi^3 \text{sgn}(d) + 24 \pi^2 \log(\text{abs}(d)) \text{sgn}(d) + 24 I \pi \log(\text{abs}(d))^2 \text{sgn}(d) + 8 I \pi^3 - 24 \pi^2 \log(\text{abs}(d)) - 24 I \pi \log(\text{abs}(d))^2 + 16 \log(\text{abs}(d))^3 + 24 I \pi^2 \text{sgn}(d) - 48 \pi \log(\text{abs}(d)) \text{sgn}(d) - 24 I \pi^2 + 48 \pi \log(\text{abs}(d)) + 48 I \log(\text{abs}(d))^2 - 24 I \pi \text{sgn}(d) - 48 \log(\text{abs}(d)) - 16 I) + I * (I \pi^2 x^2 \text{sgn}(d) + 2\pi x^2 \log(\text{abs}(d)) \text{sgn}(d) - I \pi^2 x^2 - 2\pi x^2 \log(\text{abs}(d)) + 2I x^2 \log(\text{abs}(d))^2 - 2I \pi x^2 \text{sgn}(d) + 2I \pi x^2 + 4x^2 \log(\text{abs}(d)) - 2\pi x \text{sgn}(d) + 2\pi x - 2I x^2 - 4I x \log(\text{abs}(d)) - 4x + 4I) * e^{(-1/2 I \pi x \text{sgn}(d) + 1/2 I \pi x - I x)} / (-24 I \pi + 8 I \pi^3 \text{sgn}(d) + 24 \pi^2 \log(\text{abs}(d)) \text{sgn}(d) - 24 I \pi \log(\text{abs}(d))^2 \text{sgn}(d) - 8 I \pi^3 - 24 \pi^2 \log(\text{abs}(d)) + 24 I \pi \log(\text{abs}(d))^2 + 16 \log(\text{abs}(d))^3 - 24 I \pi^2 \text{sgn}(d)
\end{aligned}$$

$$\begin{aligned}
& - 48\pi \log(\text{abs}(d)) \text{sgn}(d) + 24I\pi^2 + 48\pi \log(\text{abs}(d)) - 48I \log(\text{abs}(d))^2 + 24I\pi \text{sgn}(d) - 48 \log(\text{abs}(d)) + 16I) \text{abs}(d)^x - 2(-I(I\pi^2 x^2 \text{sgn}(d) - 2\pi x^2 \log(\text{abs}(d)) \text{sgn}(d) - I\pi^2 x^2 + 2\pi x^2 \log(\text{abs}(d)) + 2Ix^2 \log(\text{abs}(d))^2 + 2I\pi x^2 \text{sgn}(d) - 2I\pi x^2 + 4x^2 \log(\text{abs}(d))) + 2\pi x \text{sgn}(d) - 2\pi x - 2Ix^2 - 4Ix \log(\text{abs}(d)) - 4x + 4I) e^{(1/2I\pi x \text{sgn}(d) - 1/2I\pi x - Ix)/(24I\pi - 8I\pi^3 \text{sgn}(d) + 24\pi^2 \log(\text{abs}(d)) \text{sgn}(d) + 24I\pi \log(\text{abs}(d))^2 \text{sgn}(d) + 8I\pi^3 - 24\pi^2 \log(\text{abs}(d)) - 24I\pi \log(\text{abs}(d))^2 + 16 \log(\text{abs}(d))^3 - 24I\pi^2 \text{sgn}(d) + 48\pi i \log(\text{abs}(d)) \text{sgn}(d) + 24I\pi^2 - 48\pi \log(\text{abs}(d)) - 48I \log(\text{abs}(d))^2 - 24I\pi \text{sgn}(d) - 48 \log(\text{abs}(d)) + 16I) + I(I\pi^2 x^2 \text{sgn}(d) + 2\pi x^2 \log(\text{abs}(d)) \text{sgn}(d) - I\pi^2 x^2 - 2\pi x^2 \log(\text{abs}(d)) + 2Ix^2 \log(\text{abs}(d))^2 + 2I\pi x^2 \text{sgn}(d) - 2I\pi x^2 - 4x^2 \log(\text{abs}(d)) - 2\pi x \text{sgn}(d) + 2\pi x - 2Ix^2 - 4Ix \log(\text{abs}(d)) + 4x + 4I) e^{(-1/2I\pi x \text{sgn}(d) + 1/2I\pi x + Ix)/(-24I\pi + 8I\pi^3 \text{sgn}(d) + 24\pi^2 \log(\text{abs}(d)) \text{sgn}(d) - 24I\pi \log(\text{abs}(d))^2 \text{sgn}(d) - 8I\pi^3 - 24\pi^2 \log(\text{abs}(d)) + 24I\pi \log(\text{abs}(d))^2 + 16 \log(\text{abs}(d))^3 + 24I\pi^2 \text{sgn}(d) + 48\pi \log(\text{abs}(d)) \text{sgn}(d) - 24I\pi^2 - 48\pi \log(\text{abs}(d)) + 48I \log(\text{abs}(d))^2 + 24I\pi \text{sgn}(d) - 48 \log(\text{abs}(d)) - 16I) \text{abs}(d)^x
\end{aligned}$$

Mupad [B]

time = 0.35, size = 132, normalized size = 0.82

$$\frac{d^x (x^2 \sin(x) - 2 \sin(x) + 2x \cos(x)) + d^x \ln(d)^3 (2 \cos(x) + 2x^2 \cos(x) - 4x \sin(x)) + d^x \ln(d)^2 (6 \sin(x) + 2x^2 \sin(x)) - d^x \ln(d) (6 \cos(x) - x^2 \cos(x) + 4x \sin(x)) + d^x \ln(d)^4 (x^2 \sin(x) - 2x \cos(x)) + d^x x^2 \ln(d)^5 \cos(x)}{(\ln(d)^2 + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(d^x x^2 \cos(x), x)$

[Out] $(d^x (x^2 \sin(x) - 2 \sin(x) + 2x \cos(x)) + d^x \log(d)^3 (2 \cos(x) + 2x^2 \cos(x) - 4x \sin(x)) + d^x \log(d)^2 (6 \sin(x) + 2x^2 \sin(x)) - d^x \log(d) (6 \cos(x) - x^2 \cos(x) + 4x \sin(x)) + d^x \log(d)^4 (x^2 \sin(x) - 2x \cos(x)) + d^x x^2 \log(d)^5 \cos(x)) / (\log(d)^2 + 1)^3$

3.140 $\int d^x x^3 \sin(x) dx$

Optimal. Leaf size=261

$$-\frac{24d^x \cos(x) \log(d)}{(1 + \log^2(d))^4} + \frac{24d^x \cos(x) \log^3(d)}{(1 + \log^2(d))^4} + \frac{6d^x x \cos(x)}{(1 + \log^2(d))^3} - \frac{18d^x x \cos(x) \log^2(d)}{(1 + \log^2(d))^3} + \frac{6d^x x^2 \cos(x) \log(d)}{(1 + \log^2(d))^2} - \frac{d^x x^3}{1 + \log^2(d)}$$

[Out] $-24*d^x*\cos(x)*\ln(d)/(1+\ln(d)^2)^4+24*d^x*\cos(x)*\ln(d)^3/(1+\ln(d)^2)^4+6*d^x*x*\cos(x)/(1+\ln(d)^2)^3-18*d^x*x*\cos(x)*\ln(d)^2/(1+\ln(d)^2)^3+6*d^x*x^2*\cos(x)*\ln(d)/(1+\ln(d)^2)^2-d^x*x^3*\cos(x)/(1+\ln(d)^2)-6*d^x*\sin(x)/(1+\ln(d)^2)^4+36*d^x*\ln(d)^2*\sin(x)/(1+\ln(d)^2)^4-6*d^x*\ln(d)^4*\sin(x)/(1+\ln(d)^2)^4-18*d^x*x*\ln(d)*\sin(x)/(1+\ln(d)^2)^3+6*d^x*x*\ln(d)^3*\sin(x)/(1+\ln(d)^2)^3+3*d^x*x^2*\sin(x)/(1+\ln(d)^2)^2-3*d^x*x^2*\ln(d)^2*\sin(x)/(1+\ln(d)^2)^2+d^x*x^3*\ln(d)*\sin(x)/(1+\ln(d)^2)$

Rubi [A]

time = 0.27, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {4517, 4553, 14, 4518, 4554}

$$\frac{x^3 d^x \log(d) \sin(x)}{\log^2(d)+1} - \frac{x^3 d^x \cos(x)}{\log^2(d)+1} + \frac{3x^2 d^x \sin(x)}{(\log^2(d)+1)^2} - \frac{3x^2 d^x \log^2(d) \sin(x)}{(\log^2(d)+1)^2} + \frac{6x^2 d^x \log(d) \cos(x)}{(\log^2(d)+1)^2} - \frac{18x^2 d^x \log^2(d) \sin(x)}{(\log^2(d)+1)^2} + \frac{36d^x \log^2(d) \sin(x)}{(\log^2(d)+1)^2} - \frac{6d^x \sin(x)}{(\log^2(d)+1)^4} - \frac{6d^x \log^4(d) \sin(x)}{(\log^2(d)+1)^4} + \frac{6x d^x \log^3(d) \sin(x)}{(\log^2(d)+1)^3} - \frac{18x d^x \log^2(d) \cos(x)}{(\log^2(d)+1)^3} + \frac{6x d^x \cos(x)}{(\log^2(d)+1)^3} - \frac{24d^x \log(d) \cos(x)}{(\log^2(d)+1)^4} + \frac{24d^x \log^2(d) \cos(x)}{(\log^2(d)+1)^4}$$

Antiderivative was successfully verified.

[In] Int[d^x*x^3*Sin[x],x]

[Out] $(-24*d^x*\cos[x]*\log[d])/(1 + \log[d]^2)^4 + (24*d^x*\cos[x]*\log[d]^3)/(1 + \log[d]^2)^4 + (6*d^x*x*\cos[x])/(1 + \log[d]^2)^3 - (18*d^x*x*\cos[x]*\log[d]^2)/(1 + \log[d]^2)^3 + (6*d^x*x^2*\cos[x]*\log[d])/(1 + \log[d]^2)^2 - (d^x*x^3*\cos[x])/(1 + \log[d]^2) - (6*d^x*\sin[x])/(1 + \log[d]^2)^4 + (36*d^x*\log[d]^2*\sin[x])/(1 + \log[d]^2)^4 - (6*d^x*\log[d]^4*\sin[x])/(1 + \log[d]^2)^4 - (18*d^x*x*\log[d]*\sin[x])/(1 + \log[d]^2)^3 + (6*d^x*x*\log[d]^3*\sin[x])/(1 + \log[d]^2)^3 + (3*d^x*x^2*\sin[x])/(1 + \log[d]^2)^2 - (3*d^x*x^2*\log[d]^2*\sin[x])/(1 + \log[d]^2)^2 + (d^x*x^3*\log[d]*\sin[x])/(1 + \log[d]^2)$

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 4517

Int[(F_)^((c_)*((a_) + (b_)*(x_)))*Sin[(d_) + (e_)*(x_)], x_Symbol] := Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_.)]*(F_)^((c_.)*((a_.) + (b_.)*(x_.))), x_Symbol] :=
  Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_.)))*((f_.)*(x_.))^(m_.)*Sin[(d_.) + (e_.)*
(x_.)]^(n_.), x_Symbol] := Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_.)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_.)))*((f_.)*
(x_.))^(m_.), x_Symbol] := Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int d^x x^3 \sin(x) dx &= -\frac{d^x x^3 \cos(x)}{1 + \log^2(d)} + \frac{d^x x^3 \log(d) \sin(x)}{1 + \log^2(d)} - 3 \int x^2 \left(-\frac{d^x \cos(x)}{1 + \log^2(d)} + \frac{d^x \log(d) \sin(x)}{1 + \log^2(d)} \right) dx \\
&= -\frac{d^x x^3 \cos(x)}{1 + \log^2(d)} + \frac{d^x x^3 \log(d) \sin(x)}{1 + \log^2(d)} - 3 \int \left(-\frac{d^x x^2 \cos(x)}{1 + \log^2(d)} + \frac{d^x x^2 \log(d) \sin(x)}{1 + \log^2(d)} \right) dx \\
&= -\frac{d^x x^3 \cos(x)}{1 + \log^2(d)} + \frac{d^x x^3 \log(d) \sin(x)}{1 + \log^2(d)} + \frac{3 \int d^x x^2 \cos(x) dx}{1 + \log^2(d)} - \frac{(3 \log(d)) \int d^x x^2 \sin(x) dx}{1 + \log^2(d)} \\
&= \frac{6d^x x^2 \cos(x) \log(d)}{(1 + \log^2(d))^2} - \frac{d^x x^3 \cos(x)}{1 + \log^2(d)} + \frac{3d^x x^2 \sin(x)}{(1 + \log^2(d))^2} - \frac{3d^x x^2 \log^2(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x^3 \log(d)}{1 + \log^2(d)} \\
&= \frac{6d^x x^2 \cos(x) \log(d)}{(1 + \log^2(d))^2} - \frac{d^x x^3 \cos(x)}{1 + \log^2(d)} + \frac{3d^x x^2 \sin(x)}{(1 + \log^2(d))^2} - \frac{3d^x x^2 \log^2(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x^3 \log(d)}{1 + \log^2(d)} \\
&= \frac{6d^x x^2 \cos(x) \log(d)}{(1 + \log^2(d))^2} - \frac{d^x x^3 \cos(x)}{1 + \log^2(d)} + \frac{3d^x x^2 \sin(x)}{(1 + \log^2(d))^2} - \frac{3d^x x^2 \log^2(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x^3 \log(d)}{1 + \log^2(d)} \\
&= \frac{6d^x x \cos(x)}{(1 + \log^2(d))^3} - \frac{6d^x x \cos(x) \log^2(d)}{(1 + \log^2(d))^3} + \frac{6d^x x^2 \cos(x) \log(d)}{(1 + \log^2(d))^2} - \frac{d^x x^3 \cos(x)}{1 + \log^2(d)} - \frac{6d^x x \log(d)}{(1 + \log^2(d))^2} \\
&= \frac{6d^x x \cos(x)}{(1 + \log^2(d))^3} - \frac{6d^x x \cos(x) \log^2(d)}{(1 + \log^2(d))^3} + \frac{6d^x x^2 \cos(x) \log(d)}{(1 + \log^2(d))^2} - \frac{d^x x^3 \cos(x)}{1 + \log^2(d)} - \frac{6d^x x \log(d)}{(1 + \log^2(d))^2} \\
&= -\frac{12d^x \cos(x) \log(d)}{(1 + \log^2(d))^4} + \frac{12d^x \cos(x) \log^3(d)}{(1 + \log^2(d))^4} + \frac{6d^x x \cos(x)}{(1 + \log^2(d))^3} - \frac{6d^x x \cos(x) \log^2(d)}{(1 + \log^2(d))^3} + \frac{6d^x x \log(d)}{(1 + \log^2(d))^2}
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 169, normalized size = 0.65

$$\frac{d^x (-\cos(x) (x(-6+x^2) - 6(-4+x^2)\log(d) + 3x(4+x^2)\log^2(d) - 12(2+x^2)\log^3(d) + 3x(6+x^2)\log^4(d) - 6x^2\log^5(d) + x^3\log^6(d)) + (3(-2+x^2) + x(-18+x^2)\log(d) + 3(12+x^2)\log^2(d) + 3x(-4+x^2)\log^3(d) - 3(2+x^2)\log^4(d) + 3x(2+x^2)\log^5(d) - 3x^2\log^6(d) + x^3\log^7(d))\sin(x)}{(1 + \log^2(d))^4}$$

Antiderivative was successfully verified.

`[In] Integrate[d^x*x^3*Sin[x],x]`

```

[Out] (d^x*(-(Cos[x]*(x*(-6 + x^2) - 6*(-4 + x^2)*Log[d] + 3*x*(4 + x^2)*Log[d]^2 - 12*(2 + x^2)*Log[d]^3 + 3*x*(6 + x^2)*Log[d]^4 - 6*x^2*Log[d]^5 + x^3*Log[d]^6)) + (3*(-2 + x^2) + x*(-18 + x^2)*Log[d] + 3*(12 + x^2)*Log[d]^2 + 3*x*(-4 + x^2)*Log[d]^3 - 3*(2 + x^2)*Log[d]^4 + 3*x*(2 + x^2)*Log[d]^5 - 3*x^2*Log[d]^6 + x^3*Log[d]^7)*Sin[x]))/(1 + Log[d]^2)^4

```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 9.87, size = 1128, normalized size = 4.32

result too large to display

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x^3*d^x*Sin[x],x]')

[Out] Piecewise[{{(-3 I x Sin[x] + 3 x Cos[x] + 3 I x ^ 2 Cos[x] + 3 x ^ 2 Sin[x] - 2 x ^ 3 Cos[x] + 2 I x ^ 3 Sin[x] - I x ^ 4 Cos[x] + x ^ 4 Sin[x] - 3 Sin[x]) E ^ (-I x) / 8, d == E ^ (-I)}, {(3 I x Sin[x] + 3 x Cos[x] - 3 I x ^ 2 Cos[x] + 3 x ^ 2 Sin[x] - 2 x ^ 3 Cos[x] - 2 I x ^ 3 Sin[x] + I x ^ 4 Cos[x] + x ^ 4 Sin[x] - 3 Sin[x]) E ^ (I x) / 8, d == E ^ I}}, -18 x Cos[x] d ^ x Log[d] ^ 4 / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 18 x Log[d] d ^ x Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 12 x Cos[x] d ^ x Log[d] ^ 2 / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 12 x d ^ x Log[d] ^ 3 Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 6 x Cos[x] d ^ x / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 6 x d ^ x Log[d] ^ 5 Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 3 x ^ 2 d ^ x Log[d] ^ 4 Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 3 x ^ 2 d ^ x Log[d] ^ 6 Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 3 x ^ 2 d ^ x Log[d] ^ 2 Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 3 x ^ 2 d ^ x Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 6 x ^ 2 Cos[x] Log[d] d ^ x / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 6 x ^ 2 Cos[x] d ^ x Log[d] ^ 5 / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 12 x ^ 2 Cos[x] d ^ x Log[d] ^ 3 / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 3 x ^ 3 Cos[x] d ^ x Log[d] ^ 2 / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 3 x ^ 3 Cos[x] d ^ x Log[d] ^ 4 / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - x ^ 3 Cos[x] d ^ x Log[d] ^ 6 / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - x ^ 3 Cos[x] d ^ x / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 3 x ^ 3 d ^ x Log[d] ^ 3 Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 3 x ^ 3 d ^ x Log[d] ^ 5 Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + x ^ 3 Log[d] d ^ x Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + x ^ 3 d ^ x Log[d] ^ 7 Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 24 Cos[x] Log[d] d ^ x / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 6 d ^ x Log[d] ^ 4 Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 6 d ^ x Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 24 Cos[x] d ^ x Log[d] ^ 3 / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 36 d ^ x Log[d] ^ 2 Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4)]

Maple [C] Result contains complex when optimal does not.

time = 0.07, size = 166, normalized size = 0.64

method	result
risch	$\frac{i(-6+\ln(d)^3 x^3+3i \ln(d)^2 x^3-3 \ln(d) x^3-i x^3+6 x \ln(d)+6 i x-3 \ln(d)^2 x^2-6 i \ln(d) x^2+3 x^2) d^x e^{i x}}{2(\ln(d)+i)^4} + \frac{i(-6+6 x \ln(d)-6 i x-3 \ln(d)^2 x^2)}{2(\ln(d)+i)^4}$
norman	$\frac{x^3 e^{x \ln(d)} \left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\ln(d)^2} - \frac{x^3 e^{x \ln(d)}}{1+\ln(d)^2} + \frac{6 \ln(d) x^2 e^{x \ln(d)}}{\ln(d)^4+2 \ln(d)^2+1} + \frac{2 \ln(d) x^3 e^{x \ln(d)} \tan\left(\frac{x}{2}\right)}{1+\ln(d)^2} - \frac{6(\ln(d)^2-1) x^2 e^{x \ln(d)} \tan\left(\frac{x}{2}\right)}{\ln(d)^4+2 \ln(d)^2+1} - \frac{6(3 \ln(d)^2-1) x e^{x \ln(d)}}{(1+\ln(d)^2)(\ln(d)^4+2 \ln(d)^2+1)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(d^x*x^3*sin(x),x,method=_RETURNVERBOSE)`

[Out]
$$-1/2*I*(-6+\ln(d)^3*x^3+3*I*\ln(d)^2*x^3-3*\ln(d)*x^3-I*x^3+6*x*\ln(d)+6*I*x-3*\ln(d)^2*x^2-6*I*\ln(d)*x^2+3*x^2)*d^x/(\ln(d)+I)^4*\exp(I*x)+1/2*I*(-6+6*x*\ln(d)-6*I*x-3*\ln(d)^2*x^2+6*I*\ln(d)*x^2+3*x^2+\ln(d)^3*x^3-3*I*\ln(d)^2*x^3-3*\ln(d)*x^3+I*x^3)*d^x/(\ln(d)-I)^4*\exp(-I*x)$$

Maxima [A]

time = 0.29, size = 186, normalized size = 0.71

$$\frac{((\log(d)^6+3 \log(d)^4+3 \log(d)^2+1)x^3-6(\log(d)^5+2 \log(d)^3+\log(d))x^2-24 \log(d)^3+6(3 \log(d)^4+2 \log(d)^2-1)x+24 \log(d))d^x \cos(x)-((\log(d)^7+3 \log(d)^5+3 \log(d)^3+\log(d))x^3-6 \log(d)^4-3(\log(d)^6+\log(d)^4-\log(d)^2-1)x^2+6(\log(d)^5-2 \log(d)^3-3 \log(d))x+36 \log(d)^2-6)d^x \sin(x)}{\log(d)^8+4 \log(d)^6+6 \log(d)^4+4 \log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d^x*x^3*sin(x),x, algorithm="maxima")`

[Out]
$$-\left(\left(\log(d)^6+3 \log(d)^4+3 \log(d)^2+1\right)x^3-6\left(\log(d)^5+2 \log(d)^3+\log(d)\right)x^2-24 \log(d)^3+6\left(3 \log(d)^4+2 \log(d)^2-1\right)x+24 \log(d)\right)d^x \cos(x)-\left(\left(\log(d)^7+3 \log(d)^5+3 \log(d)^3+\log(d)\right)x^3-6 \log(d)^4-3\left(\log(d)^6+\log(d)^4-\log(d)^2-1\right)x^2+6\left(\log(d)^5-2 \log(d)^3-3 \log(d)\right)x+36 \log(d)^2-6\right)d^x \sin(x)\left/\left(\log(d)^8+4 \log(d)^6+6 \log(d)^4+4 \log(d)^2+1\right)\right.$$

Fricas [A]

time = 0.31, size = 203, normalized size = 0.78

$$\frac{(x^3 \cos(x) \log(d)^6-6 x^2 \cos(x) \log(d)^5+3(x^3+6 x) \cos(x) \log(d)^4-12(x^2+2) \cos(x) \log(d)^3+3(x^3+4 x) \cos(x) \log(d)^2-6(x^2-4) \cos(x) \log(d)+(x^3-6 x) \cos(x)-\left(x^3 \log(d)^7-3 x^2 \log(d)^6+3(x^3+2 x) \log(d)^5-3(x^2+2) \log(d)^4+3(x^3-4 x) \log(d)^3+3(x^2+12) \log(d)^2+3 x^2+(x^3-18 x) \log(d)-6\right) \sin(x)) d^x}{\log(d)^8+4 \log(d)^6+6 \log(d)^4+4 \log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d^x*x^3*sin(x),x, algorithm="fricas")`

[Out]
$$-(x^3 \cos(x) \log(d)^6-6 x^2 \cos(x) \log(d)^5+3(x^3+6 x) \cos(x) \log(d)^4-12(x^2+2) \cos(x) \log(d)^3+3(x^3+4 x) \cos(x) \log(d)^2-6(x^2-4) \cos(x) \log(d)+(x^3-6 x) \cos(x)-\left(x^3 \log(d)^7-3 x^2 \log(d)^6+3(x^3+2 x) \log(d)^5-3(x^2+2) \log(d)^4+3(x^3-4 x) \log(d)^3+3(x^2+12) \log(d)^2+3 x^2+(x^3-18 x) \log(d)-6\right) \sin(x)) d^x / (\log(d)^8+4 \log(d)^6+6 \log(d)^4+4 \log(d)^2+1)$$

Sympy [C] Result contains complex when optimal does not.

time = 2.20, size = 1355, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d**x*x**3*sin(x),x)

[Out] Piecewise((x**4*exp(-I*x)*sin(x)/8 - I*x**4*exp(-I*x)*cos(x)/8 + I*x**3*exp(-I*x)*sin(x)/4 - x**3*exp(-I*x)*cos(x)/4 + 3*x**2*exp(-I*x)*sin(x)/8 + 3*I*x**2*exp(-I*x)*cos(x)/8 - 3*I*x*exp(-I*x)*sin(x)/8 + 3*x*exp(-I*x)*cos(x)/8 - 3*I*exp(-I*x)*cos(x)/8, Eq(d, exp(-I))), (x**4*exp(I*x)*sin(x)/8 + I*x**4*exp(I*x)*cos(x)/8 - I*x**3*exp(I*x)*sin(x)/4 - x**3*exp(I*x)*cos(x)/4 + 3*x**2*exp(I*x)*sin(x)/8 - 3*I*x**2*exp(I*x)*cos(x)/8 + 3*I*x*exp(I*x)*sin(x)/8 + 3*x*exp(I*x)*cos(x)/8 + 3*I*exp(I*x)*cos(x)/8, Eq(d, exp(I))), (d**x*x**3*log(d)**7*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - d**x*x**3*log(d)**6*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**5*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**3*log(d)**4*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**3*log(d)**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + d**x*x**3*log(d)*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - d**x*x**3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**6*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 6*d**x*x**2*log(d)**5*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**4*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 12*d**x*x**2*log(d)**3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**2*log(d)**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 6*d**x*x**2*log(d)*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 6*d**x*x*log(d)**5*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 18*d**x*x*log(d)**4*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x*log(d)**3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x*log(d)**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 18*d**x*x*log(d)*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 6*d**x*x*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*d**x*log(d)**4*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 24*d**x*log(d)**3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 36*d**x*log(d)**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 24*d**x*log(d)*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*d**x*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1), True))

Giac [C] Result contains complex when optimal does not.

time = 0.01, size = 6997, normalized size = 26.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d^x*x^3*sin(x),x)

[Out]
$$\frac{1}{2} \left(\left((4\pi + \pi^4 \operatorname{sgn}(d) - 6\pi^2 \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) - \pi^4 + 6\pi^2 \log(\operatorname{abs}(d))^2 - 2\log(\operatorname{abs}(d))^4 - 4\pi^3 \operatorname{sgn}(d) + 12\pi \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) + 4\pi^3 - 12\pi \log(\operatorname{abs}(d))^2 + 6\pi^2 \operatorname{sgn}(d) - 6\pi^2 + 12\log(\operatorname{abs}(d))^2 - 4\pi \operatorname{sgn}(d) - 2 \right) \left(\pi^3 x^3 \operatorname{sgn}(d) - 3\pi x^3 \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) - \pi^3 x^3 + 3\pi x^3 \log(\operatorname{abs}(d))^2 - 3\pi^2 x^3 \operatorname{sgn}(d) + 3\pi^2 x^3 - 6x^3 \log(\operatorname{abs}(d))^2 + 3\pi x^3 \operatorname{sgn}(d) + 6\pi x^2 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - 3\pi x^3 - 6\pi x^2 \log(\operatorname{abs}(d)) + 2x^3 + 12x^2 \log(\operatorname{abs}(d)) - 6\pi x \operatorname{sgn}(d) + 6\pi x - 12x \right) \right) / \left((4\pi + \pi^4 \operatorname{sgn}(d) - 6\pi^2 \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) - \pi^4 + 6\pi^2 \log(\operatorname{abs}(d))^2 - 2\log(\operatorname{abs}(d))^4 - 4\pi^3 \operatorname{sgn}(d) + 12\pi \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) + 4\pi^3 - 12\pi \log(\operatorname{abs}(d))^2 + 6\pi^2 \operatorname{sgn}(d) - 6\pi^2 + 12\log(\operatorname{abs}(d))^2 - 4\pi \operatorname{sgn}(d) - 2 \right)^2 + 16 \left(\pi^3 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - \pi \log(\operatorname{abs}(d))^3 \operatorname{sgn}(d) - \pi^3 \log(\operatorname{abs}(d)) + \pi \log(\operatorname{abs}(d))^3 - 3\pi^2 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) + 3\pi^2 \log(\operatorname{abs}(d)) - 2\log(\operatorname{abs}(d))^3 + 3\pi \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - 3\pi \log(\operatorname{abs}(d)) + 2\log(\operatorname{abs}(d)) \right)^2 + 4 \left(3\pi^2 x^3 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - 3\pi^2 x^3 \log(\operatorname{abs}(d)) + 2x^3 \log(\operatorname{abs}(d))^3 - 6\pi x^3 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) + 6\pi x^3 \log(\operatorname{abs}(d)) - 3\pi^2 x^2 \operatorname{sgn}(d) + 3\pi^2 x^2 - 6x^3 \log(\operatorname{abs}(d)) - 6x^2 \log(\operatorname{abs}(d))^2 + 6\pi x^2 \operatorname{sgn}(d) - 6\pi x^2 + 6x^2 + 12x \log(\operatorname{abs}(d)) - 12 \right) \left(\pi^3 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - \pi \log(\operatorname{abs}(d))^3 \operatorname{sgn}(d) - \pi^3 \log(\operatorname{abs}(d)) + \pi \log(\operatorname{abs}(d))^3 - 3\pi^2 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) + 3\pi^2 \log(\operatorname{abs}(d)) - 2\log(\operatorname{abs}(d))^3 + 3\pi \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - 3\pi \log(\operatorname{abs}(d)) + 2\log(\operatorname{abs}(d)) \right) / \left((4\pi + \pi^4 \operatorname{sgn}(d) - 6\pi^2 \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) - \pi^4 + 6\pi^2 \log(\operatorname{abs}(d))^2 - 2\log(\operatorname{abs}(d))^4 - 4\pi^3 \operatorname{sgn}(d) + 12\pi \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) + 4\pi^3 - 12\pi \log(\operatorname{abs}(d))^2 + 6\pi^2 \operatorname{sgn}(d) - 6\pi^2 + 12\log(\operatorname{abs}(d))^2 - 4\pi \operatorname{sgn}(d) - 2 \right)^2 + 16 \left(\pi^3 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - \pi \log(\operatorname{abs}(d))^3 \operatorname{sgn}(d) - \pi^3 \log(\operatorname{abs}(d)) + \pi \log(\operatorname{abs}(d))^3 - 3\pi^2 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) + 3\pi^2 \log(\operatorname{abs}(d)) - 2\log(\operatorname{abs}(d))^3 + 3\pi \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - 3\pi \log(\operatorname{abs}(d)) + 2\log(\operatorname{abs}(d)) \right)^2 \right) \cos\left(\frac{1}{2}\pi x \operatorname{sgn}(d) - \frac{1}{2}\pi x + x\right) - \left((4\pi + \pi^4 \operatorname{sgn}(d) - 6\pi^2 \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) - \pi^4 + 6\pi^2 \log(\operatorname{abs}(d))^2 - 2\log(\operatorname{abs}(d))^4 - 4\pi^3 \operatorname{sgn}(d) + 12\pi \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) + 4\pi^3 - 12\pi \log(\operatorname{abs}(d))^2 + 6\pi^2 \operatorname{sgn}(d) - 6\pi^2 + 12\log(\operatorname{abs}(d))^2 - 4\pi \operatorname{sgn}(d) - 2 \right) \left(3\pi^2 x^3 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - 3\pi^2 x^3 \log(\operatorname{abs}(d)) + 2x^3 \log(\operatorname{abs}(d))^3 - 6\pi x^3 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) + 6\pi x^3 \log(\operatorname{abs}(d)) - 3\pi^2 x^2 \operatorname{sgn}(d) + 3\pi^2 x^2 - 6x^3 \log(\operatorname{abs}(d)) - 6x^2 \log(\operatorname{abs}(d))^2 + 6\pi x^2 \operatorname{sgn}(d) - 6\pi x^2 + 6x^2 + 12x \log(\operatorname{abs}(d)) - 12 \right) / \left((4\pi + \pi^4 \operatorname{sgn}(d) - 6\pi^2 \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) - \pi^4 + 6\pi^2 \log(\operatorname{abs}(d))^2 - 2\log(\operatorname{abs}(d))^4 - 4\pi^3 \operatorname{sgn}(d) + 12\pi \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) + 4\pi^3 - 12\pi \log(\operatorname{abs}(d))^2 + 6\pi^2 \operatorname{sgn}(d) - 6\pi^2 + 12\log(\operatorname{abs}(d))^2 - 4\pi \operatorname{sgn}(d) - 2 \right)^2 + 16 \left(\pi^3 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - \pi \log(\operatorname{abs}(d))^3 \operatorname{sgn}(d) - \pi^3 \log(\operatorname{abs}(d)) + \pi \log(\operatorname{abs}(d))^3 - 3\pi^2 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) + 3\pi^2 \log(\operatorname{abs}(d)) - 2\log(\operatorname{abs}(d))^3 + 3\pi \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - 3\pi \log(\operatorname{abs}(d)) + 2\log(\operatorname{abs}(d)) \right)^2 \right)$$

$$\begin{aligned}
& \text{abs}(d)) * \text{sgn}(d) - 6 * \pi * x^3 * \log(\text{abs}(d)) - 3 * \pi^2 * x^2 * \text{sgn}(d) + 3 * \pi^2 * x^2 - 6 * \\
& x^3 * \log(\text{abs}(d)) - 6 * x^2 * \log(\text{abs}(d))^2 - 6 * \pi * x^2 * \text{sgn}(d) + 6 * \pi * x^2 + 6 * x^2 \\
& + 12 * x * \log(\text{abs}(d)) - 12) / ((4 * \pi - \pi^4 * \text{sgn}(d) + 6 * \pi^2 * \log(\text{abs}(d))^2 * \text{sgn}(d) \\
& + \pi^4 - 6 * \pi^2 * \log(\text{abs}(d))^2 + 2 * \log(\text{abs}(d))^4 - 4 * \pi^3 * \text{sgn}(d) + 12 * \pi * \log \\
& (\text{abs}(d))^2 * \text{sgn}(d) + 4 * \pi^3 - 12 * \pi * \log(\text{abs}(d))^2 - 6 * \pi^2 * \text{sgn}(d) + 6 * \pi^2 \\
& - 12 * \log(\text{abs}(d))^2 - 4 * \pi * \text{sgn}(d) + 2)^2 + 16 * (\pi^3 * \log(\text{abs}(d)) * \text{sgn}(d) - \pi * \\
& \log(\text{abs}(d))^3 * \text{sgn}(d) - \pi^3 * \log(\text{abs}(d)) + \pi * \log(\text{abs}(d))^3 + 3 * \pi^2 * \log(\text{abs} \\
& (d)) * \text{sgn}(d) - 3 * \pi^2 * \log(\text{abs}(d)) + 2 * \log(\text{abs}(d))^3 + 3 * \pi * \log(\text{abs}(d)) * \text{sgn}(d) \\
&) - 3 * \pi * \log(\text{abs}(d)) - 2 * \log(\text{abs}(d)))^2) + 4 * (\pi^3 * x^3 * \text{sgn}(d) - 3 * \pi * x^3 * \log \\
& (\text{abs}(d))^2 * \text{sgn}(d) - \pi^3 * x^3 + 3 * \pi * x^3 * \log(\text{abs}(d))^2 + 3 * \pi^2 * x^3 * \text{sgn}(d) \\
& - 3 * \pi^2 * x^3 + 6 * x^3 * \log(\text{abs}(d))^2 + 3 * \pi * x^3 * \text{sgn}(d) + 6 * \pi * x^2 * \log(\text{abs}(d)) \\
& * \text{sgn}(d) - 3 * \pi * x^3 - 6 * \pi * x^2 * \log(\text{abs}(d)) - 2 * x^3 - 12 * x^2 * \log(\text{abs}(d)) - 6 * \\
& \pi * x * \text{sgn}(d) + 6 * \pi * x + 12 * x) * (\pi^3 * \log(\text{abs}(d)) * \text{sgn}(d) - \pi * \log(\text{abs}(d))^3 * \text{sgn} \\
& (d) - \pi^3 * \log(\text{abs}(d)) + \pi * \log(\text{abs}(d))^3 + 3 * \pi^2 * \log(\text{abs}(d)) * \text{sgn}(d) - 3 * \\
& \pi^2 * \log(\text{abs}(d)) + 2 * \log(\text{abs}(d))^3 + 3 * \pi * \log(\text{abs}(d)) * \text{sgn}(d) - 3 * \pi * \log(\text{abs} \\
& (d)) - 2 * \log(\text{abs}(d))) / ((4 * \pi - \pi^4 * \text{sgn}(d) + 6 * \pi^2 * \log(\text{abs}(d))^2 * \text{sgn}(d) + \\
& \pi^4 - 6 * \pi^2 * \log(\text{abs}(d))^2 + 2 * \log(\text{abs}(d))^4 - 4 * \pi^3 * \text{sgn}(d) + 12 * \pi * \log(\text{abs} \\
& (d))^2 * \text{sgn}(d) + 4 * \pi^3 - 12 * \pi * \log(\text{abs}(d))^2 - 6 * \pi^2 * \text{sgn}(d) + 6 * \pi^2 - 1 \\
& 2 * \log(\text{abs}(d))^2 - 4 * \pi * \text{sgn}(d) + 2)^2 + 16 * (\pi^3 * \log(\text{abs}(d)) * \text{sgn}(d) - \pi * \log \\
& (\text{abs}(d))^3 * \text{sgn}(d) - \pi^3 * \log(\text{abs}(d)) + \pi * \log(\text{abs}(d))^3 + 3 * \pi^2 * \log(\text{abs}(d) \\
&) * \text{sgn}(d) - 3 * \pi^2 * \log(\text{abs}(d)) + 2 * \log(\text{abs}(d))^3 + 3 * \pi * \log(\text{abs}(d)) * \text{sgn}(d) - \\
& 3 * \pi * \log(\text{abs}(d)) - 2 * \log(\text{abs}(d)))^2) * \sin(1/2 * \pi * x * \text{sgn}(d) - 1/2 * \pi * x - x) \\
& * \text{abs}(d)^x - 1/4 * ((\pi^3 * x^3 * \text{sgn}(d) + 3 * I * \pi^2 * x^3 * \log(\text{abs}(d)) * \text{sgn}(d) - 3 * \pi * \\
& x^3 * \log(\text{abs}(d))^2 * \text{sgn}(d) - \pi^3 * x^3 - 3 * I * \pi^2 * x^3 * \log(\text{abs}(d)) + 3 * \pi * x^3 * \log \\
& (\text{abs}(d))^2 + 2 * I * x^3 * \log(\text{abs}(d))^3 - 3 * \pi^2 * x^3 * \text{sgn}(d) - 6 * I * \pi * x^3 * \log(\text{abs} \\
& (d)) * \text{sgn}(d) + 3 * \pi^2 * x^3 + 6 * I * \pi * x^3 * \log(\text{abs}(d)) - 6 * x^3 * \log(\text{abs}(d))^2 - \\
& 3 * I * \pi^2 * x^2 * \text{sgn}(d) + 3 * \pi * x^3 * \text{sgn}(d) + 6 * \pi * x^2 * \log(\text{abs}(d)) * \text{sgn}(d) + 3 * I * \\
& \pi^2 * x^2 - 3 * \pi * x^3 - 6 * \pi * x^2 * \log(\text{abs}(d)) - 6 * I * x^3 * \log(\text{abs}(d)) - 6 * I * x^2 * \\
& \log(\text{abs}(d))^2 + 6 * I * \pi * x^2 * \text{sgn}(d) - 6 * I * \pi * x^2 + 2 * x^3 + 12 * x^2 * \log(\text{abs}(d)) \\
& - 6 * \pi * x * \text{sgn}(d) + 6 * \pi * x + 6 * I * x^2 + 12 * I * x * \log(\text{abs}(d)) - 12 * x - 12 * I) * e^{(\\
& 1/2 * I * \pi * x * \text{sgn}(d) - 1/2 * I * \pi * x + I * x) / (4 * \pi + \pi^4 * \text{sgn}(d) + 4 * I * \pi^3 * \log(\text{abs} \\
& (d)) * \text{sgn}(d) - 6 * \pi^2 * \log(\text{abs}(d))^2 * \text{sgn}(d) - 4 * I * \pi * \log(\text{abs}(d))^3 * \text{sgn}(d) - \\
& \pi^4 - 4 * I * \pi^3 * \log(\text{abs}(d)) + 6 * \pi^2 * \log(\text{abs}(d))^2 + 4 * I * \pi * \log(\text{abs}(d))^3 - \\
& 2 * \log(\text{abs}(d))^4 - 4 * \pi^3 * \text{sgn}(d) - 12 * I * \pi^2 * \log(\text{abs}(d)) * \text{sgn}(d) + 12 * \pi * \log \\
& (\text{abs}(d))^2 * \text{sgn}(d) + 4 * \pi^3 + 12 * I * \pi^2 * \log(\text{abs}(d)) - 12 * \pi * \log(\text{abs}(d))^2 - \\
& 8 * I * \log(\text{abs}(d))^3 + 6 * \pi^2 * \text{sgn}(d) + 12 * I * \pi * \log(\text{abs}(d)) * \text{sgn}(d) - 6 * \pi^2 - 1 \\
& 2 * I * \pi * \log(\text{abs}(d)) + 12 * \log(\text{abs}(d))^2 - 4 * \pi * \text{sgn}(d) + 8 * I * \log(\text{abs}(d)) - 2) \\
& - (\pi^3 * x^3 * \text{sgn}(d) - 3 * I * \pi^2 * x^3 * \log(\text{abs}(d)) * \text{sgn}(d) - 3 * \pi * x^3 * \log(\text{abs}(d)) \\
& ^2 * \text{sgn}(d) - \pi^3 * x^3 + 3 * I * \pi^2 * x^3 * \log(\text{abs}(d)) + 3 * \pi * x^3 * \log(\text{abs}(d))^2 - \\
& 2 * I * x^3 * \log(\text{abs}(d))^3 - 3 * \pi^2 * x^3 * \text{sgn}(d) + 6 * I * \pi * x^3 * \log(\text{abs}(d)) * \text{sgn}(d) + \\
& 3 * \pi^2 * x^3 - 6 * I * \pi * x^3 * \log(\text{abs}(d)) - 6 * x^3 * \log(\text{abs}(d))^2 + 3 * I * \pi^2 * x^2 * \text{sgn} \\
& (d) + 3 * \pi * x^3 * \text{sgn}(d) + 6 * \pi * x^2 * \log(\text{abs}(d)) * \text{sgn}(d) - 3 * I * \pi^2 * x^2 - 3 * \pi \\
& * x^3 - 6 * \pi * x^2 * \log(\text{abs}(d)) + 6 * I * x^3 * \log(\text{abs}(d)) + 6 * I * x^2 * \log(\text{abs}(d))^2 - \\
& 6 * I * \pi * x^2 * \text{sgn}(d) + 6 * I * \pi * x^2 + 2 * x^3 + 12 * x^2 * \log(\text{abs}(d)) - 6 * \pi * x * \text{sgn}(d) \\
&) + 6 * \pi * x - 6 * I * x^2 - 12 * I * x * \log(\text{abs}(d)) - 12 * x + 12 * I) * e^{(-1/2 * I * \pi * x * \text{sgn}
\end{aligned}$$

$$\begin{aligned}
& (d) + 1/2*I*pi*x - I*x)/(4*pi + pi^4*sgn(d) - 4*I*pi^3*log(abs(d))*sgn(d) - \\
& 6*pi^2*log(abs(d))^2*sgn(d) + 4*I*pi*log(abs(d))^3*sgn(d) - pi^4 + 4*I*pi^3 \\
& 3*log(abs(d)) + 6*pi^2*log(abs(d))^2 - 4*I*pi*log(abs(d))^3 - 2*log(abs(d)) \\
& ^4 - 4*pi^3*sgn(d) + 12*I*pi^2*log(abs(d))*sgn(d) + 12*pi*log(abs(d))^2*sgn \\
& (d) + 4*pi^3 - 12*I*pi^2*log(abs(d)) - 12*pi*log(abs(d))^2 + 8*I*log(abs(d) \\
&)^3 + 6*pi^2*sgn(d) - 12*I*pi*log(abs(d))*sgn(d) - 6*pi^2 + 12*I*pi*log(abs \\
& (d)) + 12*log(abs(d))^2 - 4*pi*sgn(d) - 8*I*log(abs(d)) - 2))*abs(d)^x - 1/ \\
& 4*((pi^3*x^3*sgn(d) + 3*I*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi*x^3*log(abs(d) \\
&)^2*sgn(d) - pi^3*x^3 - 3*I*pi^2*x^3*log(abs(d)) + 3*pi*x^3*log(abs(d))^2 + \\
& 2*I*x^3*log(abs(d))^3 + 3*pi^2*x^3*sgn(d) + 6*I*pi*x^3*log(abs(d))*sgn(d) \\
& - 3*pi^2*x^3 - 6*I*pi*x^3*log(abs(d)) + 6*x^3*log(abs(d))^2 - 3*I*pi^2*x^2* \\
& sgn(d) + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) + 3*I*pi^2*x^2 - 3*p \\
& i*x^3 - 6*pi*x^2*log(abs(d)) - 6*I*x^3*log(abs(d)) - 6*I*x^2*log(abs(d))^2 \\
& - 6*I*pi*x^2*sgn(d) + 6*I*pi*x^2 - 2*x^3 - 12*x^2*log(abs(d)) - 6*pi*x*sgn \\
& (d) + 6*pi*x + 6*I*x^2 + 12*I*x*log(abs(d)) + 12*x - 12*I)*e^(1/2*I*pi*x*sgn \\
& (d) - 1/2*I*pi*x - I*x)/(4*pi - pi^4*sgn(d) - 4*I*pi^3*log(abs(d))*sgn(d) + \\
& 6*pi^2*log(abs(d))^2*sgn(d) + 4*I*pi*log(abs(d))^3*sgn(d) + pi^4 + 4*I*pi^3 \\
& 3*log(abs(d)) - 6*pi^2*log(abs(d))^2 - 4*I*pi*log(abs(d))^3 + 2*log(abs(d)) \\
& ^4 - 4*pi^3*sgn(d) - 12*I*pi^2*log(abs(d))*sgn(d) + 12*pi*log(abs(d))^2*sgn \\
& (d) + 4*pi^3 + 12*I*pi^2*log(abs(d)) - 12*pi*log(abs(d))^2 - 8*I*log(abs(d) \\
&)^3 - 6*pi^2*sgn(d) - 12*I*pi*log(abs(d))*sgn(d) + 6*pi^2 + 12*I*pi*log(abs \\
& (d)) - 12*log(abs(d))^2 - 4*pi*sgn(d) + 8*I*log(abs(d)) + 2) - (pi^3*x^3*sg \\
& n(d) - 3*I*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi*x^3*log(abs(d))^2*sgn(d) - pi \\
& ^3*x^3 + 3*I*pi^2*x^3*log(abs(d)) + 3*pi*x^3*log(abs(d))^2 - 2*I*x^3*log(ab \\
& s(d))^3 + 3*pi^2*x^3*sgn(d) - 6*I*pi*x^3*log(abs(d))*sgn(d) - 3*pi^2*x^3 + \\
& 6*I*pi*x^3*log(abs(d)) + 6*x^3*log(abs(d))^2 + 3*I*pi^2*x^2*sgn(d) + 3*pi*x \\
& ^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) - 3*I*pi^2*x^2 - 3*pi*x^3 - 6*pi*x^ \\
& 2*log(abs(d)) + 6*I*x^3*log(abs(d)) + 6*I*x^2*log(abs(d))^2 + 6*I*pi*x^2*sg \\
& n(d) - 6*I*pi*x^2 - 2*x^3 - 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x - 6 \\
& *I*x^2 - 12*I*x*log(abs(d)) + 12*x + 12*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi \\
& *x + I*x)/(4*pi - pi^4*sgn(d) + 4*I*pi^3*log(abs(d))*sgn(d) + 6*pi^2*log(ab \\
& s(d))^2*sgn(d) - 4*I*pi*log(abs(d))^3*sgn(d) + pi^4 - 4*I*pi^3*log(abs(d)) \\
& - 6*pi^2*log(abs(d))^2 + 4*I*pi*log(abs(d))^3 + 2*log(abs(d))^4 - 4*pi^3*sg \\
& n(d) + 12*I*pi^2*log(abs(d))*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - \\
& 12*I*pi^2*log(abs(d)) - 12*pi*log(abs(d))^2 + 8*I*log(abs(d))^3 - 6*pi^2*s \\
& gn(d) + 12*I*pi*log(abs(d))*sgn(d) + 6*pi^2 - 12*I*pi*log(abs(d)) - 12*log \\
& (abs(d))^2 - 4*pi*sgn(d) - 8*I*log(abs(d)) + 2))*abs(d)^x
\end{aligned}$$

Mupad [B]

time = 0.63, size = 231, normalized size = 0.89

$$\frac{d^2 \ln(x) + 2 \ln(x) - 3 \ln^2(x) - 6 \ln(x) - d^2 \ln(d) (6 \ln^2(x) + 3 \ln^2(x) + 6 \ln(x) + d^2 \ln(d)^2) + d^2 \ln(d)^2 (6 \ln(x) + 3 \ln^2(x) + 3 \ln(x) \ln(d)) - d^2 \ln(d)^3 (24 \ln(x) + 12 \ln^2(x) + 3 \ln^2(x) \ln(d) - 12 \ln(x) \ln(d)^2) - d^2 \ln(d)^4 (36 \ln(x) - 3 \ln^2(x) + 3 \ln^2(x) \ln(d) - 12 \ln(x) \ln(d)^2) + d^2 \ln(d)^5 (2 \ln^2(x) + 3 \ln^2(x) \ln(d) + d^2 \ln(d) (24 \ln(x) - 6 \ln^2(x) - d^2 \ln(d)^2) + 18 \ln(x) \ln(d)^2) - d^2 \ln(d)^6 \ln(x)}{(d^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(d^x*x^3*sin(x),x)

```
[Out] -(d^x*(6*sin(x) + x^3*cos(x) - 3*x^2*sin(x) - 6*x*cos(x)) - d^x*log(d)^5*(6
*x^2*cos(x) + 3*x^3*sin(x) + 6*x*sin(x)) + d^x*log(d)^4*(6*sin(x) + 3*x^3*c
os(x) + 3*x^2*sin(x) + 18*x*cos(x)) - d^x*log(d)^3*(24*cos(x) + 12*x^2*cos(
x) + 3*x^3*sin(x) - 12*x*sin(x)) - d^x*log(d)^2*(36*sin(x) - 3*x^3*cos(x) +
3*x^2*sin(x) - 12*x*cos(x)) + d^x*log(d)^6*(x^3*cos(x) + 3*x^2*sin(x)) + d
^x*log(d)*(24*cos(x) - 6*x^2*cos(x) - x^3*sin(x) + 18*x*sin(x)) - d^x*x^3*l
og(d)^7*sin(x))/(log(d)^2 + 1)^4
```


3.141 $\int d^x x^3 \cos(x) dx$

Optimal. Leaf size=260

$$-\frac{6d^x \cos(x)}{(1 + \log^2(d))^4} + \frac{36d^x \cos(x) \log^2(d)}{(1 + \log^2(d))^4} - \frac{6d^x \cos(x) \log^4(d)}{(1 + \log^2(d))^4} - \frac{18d^x x \cos(x) \log(d)}{(1 + \log^2(d))^3} + \frac{6d^x x \cos(x) \log^3(d)}{(1 + \log^2(d))^3} + \frac{3d^x x^2 \cos(x) \log^2(d)}{(1 + \log^2(d))^2} - \frac{6d^x x^2 \cos(x) \log^4(d)}{(1 + \log^2(d))^2} - \frac{18d^x x^2 x \cos(x) \log(d)}{(1 + \log^2(d))^2} + \frac{6d^x x^2 x \cos(x) \log^3(d)}{(1 + \log^2(d))^2} + \frac{3d^x x^3 \cos(x) \log^2(d)}{(1 + \log^2(d))}$$

[Out] $-6*d^x*\cos(x)/(1+\ln(d)^2)^4+36*d^x*\cos(x)*\ln(d)^2/(1+\ln(d)^2)^4-6*d^x*\cos(x)*\ln(d)^4/(1+\ln(d)^2)^4-18*d^x*x*\cos(x)*\ln(d)/(1+\ln(d)^2)^3+6*d^x*x*\cos(x)*\ln(d)^3/(1+\ln(d)^2)^3+3*d^x*x^2*\cos(x)/(1+\ln(d)^2)^2-3*d^x*x^2*\cos(x)*\ln(d)^2/(1+\ln(d)^2)^2+d^x*x^3*\cos(x)*\ln(d)/(1+\ln(d)^2)+24*d^x*\ln(d)*\sin(x)/(1+\ln(d)^2)^4-24*d^x*\ln(d)^3*\sin(x)/(1+\ln(d)^2)^4-6*d^x*x*\sin(x)/(1+\ln(d)^2)^3+18*d^x*x*\ln(d)^2*\sin(x)/(1+\ln(d)^2)^3-6*d^x*x^2*\ln(d)*\sin(x)/(1+\ln(d)^2)^2+d^x*x^3*\sin(x)/(1+\ln(d)^2)$

Rubi [A]

time = 0.27, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {4518, 4554, 14, 4517, 4553}

$$\frac{x^2 d^x \sin(x)}{\log^2(d)+1} + \frac{x^2 d^x \log(d) \cos(x)}{\log^2(d)+1} - \frac{6x^2 d^x \log(d) \sin(x)}{(\log^2(d)+1)^2} - \frac{3x^2 d^x \log^2(d) \cos(x)}{(\log^2(d)+1)^2} + \frac{3x^2 d^x \cos(x)}{(\log^2(d)+1)^2} + \frac{18x d^x \log^2(d) \sin(x)}{(\log^2(d)+1)^3} - \frac{6x d^x \sin(x)}{(\log^2(d)+1)^3} + \frac{24d^x \log(d) \sin(x)}{(\log^2(d)+1)^4} - \frac{24d^x \log^2(d) \sin(x)}{(\log^2(d)+1)^4} - \frac{18x d^x \log(d) \cos(x)}{(\log^2(d)+1)^4} + \frac{36d^x \log^2(d) \cos(x)}{(\log^2(d)+1)^4} - \frac{6d^x \cos(x)}{(\log^2(d)+1)^4} - \frac{6d^x \log^4(d) \cos(x)}{(\log^2(d)+1)^4} + \frac{6x d^x \log^3(d) \cos(x)}{(\log^2(d)+1)^3}$$

Antiderivative was successfully verified.

[In] Int[d^x*x^3*Cos[x], x]

[Out] $(-6*d^x*\text{Cos}[x])/(1 + \text{Log}[d]^2)^4 + (36*d^x*\text{Cos}[x]*\text{Log}[d]^2)/(1 + \text{Log}[d]^2)^4 - (6*d^x*\text{Cos}[x]*\text{Log}[d]^4)/(1 + \text{Log}[d]^2)^4 - (18*d^x*x*\text{Cos}[x]*\text{Log}[d])/(1 + \text{Log}[d]^2)^3 + (6*d^x*x*\text{Cos}[x]*\text{Log}[d]^3)/(1 + \text{Log}[d]^2)^3 + (3*d^x*x^2*\text{Cos}[x])/(1 + \text{Log}[d]^2)^2 - (3*d^x*x^2*\text{Cos}[x]*\text{Log}[d]^2)/(1 + \text{Log}[d]^2)^2 + (d^x*x^3*\text{Cos}[x]*\text{Log}[d])/(1 + \text{Log}[d]^2) + (24*d^x*\text{Log}[d]*\text{Sin}[x])/(1 + \text{Log}[d]^2)^4 - (24*d^x*\text{Log}[d]^3*\text{Sin}[x])/(1 + \text{Log}[d]^2)^4 - (6*d^x*x*\text{Sin}[x])/(1 + \text{Log}[d]^2)^3 + (18*d^x*x*\text{Log}[d]^2*\text{Sin}[x])/(1 + \text{Log}[d]^2)^3 - (6*d^x*x^2*\text{Log}[d]*\text{Sin}[x])/(1 + \text{Log}[d]^2)^2 + (d^x*x^3*\text{Sin}[x])/(1 + \text{Log}[d]^2)$

Rule 14

```
Int[(u)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] := Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
  Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int d^x x^3 \cos(x) dx &= \frac{d^x x^3 \cos(x) \log(d)}{1 + \log^2(d)} + \frac{d^x x^3 \sin(x)}{1 + \log^2(d)} - 3 \int x^2 \left(\frac{d^x \cos(x) \log(d)}{1 + \log^2(d)} + \frac{d^x \sin(x)}{1 + \log^2(d)} \right) dx \\
&= \frac{d^x x^3 \cos(x) \log(d)}{1 + \log^2(d)} + \frac{d^x x^3 \sin(x)}{1 + \log^2(d)} - 3 \int \left(\frac{d^x x^2 \cos(x) \log(d)}{1 + \log^2(d)} + \frac{d^x x^2 \sin(x)}{1 + \log^2(d)} \right) dx \\
&= \frac{d^x x^3 \cos(x) \log(d)}{1 + \log^2(d)} + \frac{d^x x^3 \sin(x)}{1 + \log^2(d)} - \frac{3 \int d^x x^2 \sin(x) dx}{1 + \log^2(d)} - \frac{(3 \log(d)) \int d^x x^2 \cos(x) dx}{1 + \log^2(d)} \\
&= \frac{3d^x x^2 \cos(x)}{(1 + \log^2(d))^2} - \frac{3d^x x^2 \cos(x) \log^2(d)}{(1 + \log^2(d))^2} + \frac{d^x x^3 \cos(x) \log(d)}{1 + \log^2(d)} - \frac{6d^x x^2 \log(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x^3 \sin(x)}{1 + \log^2(d)} \\
&= \frac{3d^x x^2 \cos(x)}{(1 + \log^2(d))^2} - \frac{3d^x x^2 \cos(x) \log^2(d)}{(1 + \log^2(d))^2} + \frac{d^x x^3 \cos(x) \log(d)}{1 + \log^2(d)} - \frac{6d^x x^2 \log(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x^3 \sin(x)}{1 + \log^2(d)} \\
&= \frac{3d^x x^2 \cos(x)}{(1 + \log^2(d))^2} - \frac{3d^x x^2 \cos(x) \log^2(d)}{(1 + \log^2(d))^2} + \frac{d^x x^3 \cos(x) \log(d)}{1 + \log^2(d)} - \frac{6d^x x^2 \log(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x^3 \sin(x)}{1 + \log^2(d)} \\
&= -\frac{6d^x x \cos(x) \log(d)}{(1 + \log^2(d))^3} + \frac{6d^x x \cos(x) \log^3(d)}{(1 + \log^2(d))^3} + \frac{3d^x x^2 \cos(x)}{(1 + \log^2(d))^2} - \frac{3d^x x^2 \cos(x) \log^2(d)}{(1 + \log^2(d))^2} + \frac{d^x x^3 \sin(x)}{1 + \log^2(d)} \\
&= -\frac{6d^x x \cos(x) \log(d)}{(1 + \log^2(d))^3} + \frac{6d^x x \cos(x) \log^3(d)}{(1 + \log^2(d))^3} + \frac{3d^x x^2 \cos(x)}{(1 + \log^2(d))^2} - \frac{3d^x x^2 \cos(x) \log^2(d)}{(1 + \log^2(d))^2} + \frac{d^x x^3 \sin(x)}{1 + \log^2(d)} \\
&= -\frac{6d^x \cos(x)}{(1 + \log^2(d))^4} + \frac{12d^x \cos(x) \log^2(d)}{(1 + \log^2(d))^4} - \frac{6d^x \cos(x) \log^4(d)}{(1 + \log^2(d))^4} - \frac{6d^x x \cos(x) \log(d)}{(1 + \log^2(d))^3} + \frac{6d^x x^2 \sin(x)}{(1 + \log^2(d))^2}
\end{aligned}$$

Mathematica [A]

time = 0.09, size = 168, normalized size = 0.65

$$\frac{d^x (\cos(x) (3(-2+x^2) + x(-18+x^2)\log(d) + 3(12+x^2)\log^2(d) + 3x(-4+x^2)\log^3(d) - 3(2+x^2)\log^4(d) + 3x(2+x^2)\log^5(d) - 3x^2\log^6(d) + x^3\log^7(d)) + (x(-6+x^2) - 6(-4+x^2)\log(d) + 3x(4+x^2)\log^2(d) - 12(2+x^2)\log^3(d) + 3x(6+x^2)\log^4(d) - 6x^2\log^5(d) + x^3\log^6(d)) \sin(x)}{(1 + \log^2(d))^4}$$

Antiderivative was successfully verified.

[In] Integrate[d^x*x^3*Cos[x],x]

[Out] (d^x*(Cos[x]*(3*(-2 + x^2) + x*(-18 + x^2)*Log[d] + 3*(12 + x^2)*Log[d]^2 + 3*x*(-4 + x^2)*Log[d]^3 - 3*(2 + x^2)*Log[d]^4 + 3*x*(2 + x^2)*Log[d]^5 - 3*x^2*Log[d]^6 + x^3*Log[d]^7) + (x*(-6 + x^2) - 6*(-4 + x^2)*Log[d] + 3*x*(4 + x^2)*Log[d]^2 - 12*(2 + x^2)*Log[d]^3 + 3*x*(6 + x^2)*Log[d]^4 - 6*x^2*Log[d]^5 + x^3*Log[d]^6)*Sin[x]))/(1 + Log[d]^2)^4

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 14.78, size = 1126, normalized size = 4.33

result too large to display

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[x^3*d^x*Cos[x],x]')`

[Out] `Piecewise[{{(-3 x Sin[x] - 3 I x Cos[x] - 3 I x ^ 2 Sin[x] + 3 x ^ 2 Cos[x] + 2 I x ^ 3 Cos[x] + 2 x ^ 3 Sin[x] + I x ^ 4 Sin[x] + x ^ 4 Cos[x] + 3 I Sin[x]) E ^ (-I x) / 8, d == E ^ (-I)}, {(-3 x Sin[x] + 3 I x Cos[x] + 3 I x ^ 2 Sin[x] + 3 x ^ 2 Cos[x] - 2 I x ^ 3 Cos[x] + 2 x ^ 3 Sin[x] - I x ^ 4 Sin[x] + x ^ 4 Cos[x] - 3 I Sin[x]) E ^ (I x) / 8, d == E ^ I}}, -18 x Cos[x] Log[d] d ^ x / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 12 x Cos[x] d ^ x Log[d] ^ 3 / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 6 x d ^ x Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 6 x Cos[x] d ^ x Log[d] ^ 5 / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 12 x d ^ x Log[d] ^ 2 Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 18 x d ^ x Log[d] ^ 4 Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 12 x ^ 2 d ^ x Log[d] ^ 3 Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 6 x ^ 2 Log[d] d ^ x Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 6 x ^ 2 d ^ x Log[d] ^ 5 Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 3 x ^ 2 Cos[x] d ^ x Log[d] ^ 4 / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 3 x ^ 2 Cos[x] d ^ x Log[d] ^ 6 / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 3 x ^ 2 Cos[x] d ^ x / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 3 x ^ 3 Cos[x] d ^ x Log[d] ^ 3 / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 3 x ^ 3 Cos[x] d ^ x Log[d] ^ 5 / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 3 x ^ 3 d ^ x Log[d] ^ 2 Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 3 x ^ 3 d ^ x Log[d] ^ 4 Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + x ^ 3 Cos[x] Log[d] d ^ x / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + x ^ 3 Cos[x] d ^ x Log[d] ^ 7 / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + x ^ 3 d ^ x Log[d] ^ 6 Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + x ^ 3 d ^ x Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 24 d ^ x Log[d] ^ 3 Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 6 Cos[x] d ^ x Log[d] ^ 4 / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) - 6 Cos[x] d ^ x / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 24 Log[d] d ^ x Sin[x] / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4) + 36 Cos[x] d ^ x Log[d] ^ 2 / (1 + Log[d] ^ 8 + 4 Log[d] ^ 2 + 4 Log[d] ^ 6 + 6 Log[d] ^ 4)}`

Maple [C] Result contains complex when optimal does not.
time = 0.07, size = 164, normalized size = 0.63

method	result
risch	$\frac{(-6+\ln(d)^3 x^3+3i \ln(d)^2 x^3-3 \ln(d) x^3-i x^3+6 x \ln(d)+6 i x-3 \ln(d)^2 x^2-6 i \ln(d) x^2+3 x^2) d^x e^{i x}}{2(\ln(d)+i)^4} + \frac{(-6+6 x \ln(d)-6 i x-3 \ln(d)^2 x^2+3 x^2)}{2(\ln(d)+i)^4}$
norman	$\frac{\ln(d) x^3 e^{x \ln(d)}}{1+\ln(d)^2} + \frac{2 x^3 e^{x \ln(d)} \tan\left(\frac{x}{2}\right)}{1+\ln(d)^2} - \frac{3(\ln(d)^2-1) x^2 e^{x \ln(d)}}{\ln(d)^4+2 \ln(d)^2+1} - \frac{6(\ln(d)^4-6 \ln(d)^2+1) e^{x \ln(d)}}{(\ln(d)^6+3 \ln(d)^4+3 \ln(d)^2+1)(1+\ln(d)^2)} - \frac{\ln(d) x^3 e^{x \ln(d)} \left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\ln(d)^2} + \frac{3(\ln(d)^2-1) x^2 e^{x \ln(d)}}{\ln(d)^4+2 \ln(d)^2+1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^3*cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-6+ln(d)^3*x^3+3*I*ln(d)^2*x^3-3*ln(d)*x^3-I*x^3+6*x*ln(d)+6*I*x-3*ln(d)^2*x^2-6*I*ln(d)*x^2+3*x^2)*d^x/(ln(d)+I)^4*exp(I*x)+1/2*(-6+6*x*ln(d)-6*I*x-3*ln(d)^2*x^2+6*I*ln(d)*x^2+3*x^2+ln(d)^3*x^3-3*I*ln(d)^2*x^3-3*ln(d)*x^3+I*x^3)*d^x/(ln(d)-I)^4*exp(-I*x)
```

Maxima [A]

time = 0.28, size = 184, normalized size = 0.71

$$\frac{((\log(d)^7 + 3 \log(d)^5 + 3 \log(d)^3 + \log(d))x^3 - 6 \log(d)^4 - 3(\log(d)^6 + \log(d)^4 - \log(d)^2 - 1)x^2 + 6(\log(d)^5 - 2 \log(d)^3 - 3 \log(d))x + 36 \log(d)^2 - 6)d^x \cos(x) + ((\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1)x^3 - 6(\log(d)^5 + 2 \log(d)^3 + \log(d))x^2 - 24 \log(d)^3 + 6(3 \log(d)^4 + 2 \log(d)^2 - 1)x + 24 \log(d))d^x \sin(x)}{\log(d)^8 + 4 \log(d)^6 + 6 \log(d)^4 + 4 \log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*cos(x),x, algorithm="maxima")
```

```
[Out] (((log(d)^7 + 3*log(d)^5 + 3*log(d)^3 + log(d))*x^3 - 6*log(d)^4 - 3*(log(d)^6 + log(d)^4 - log(d)^2 - 1)*x^2 + 6*(log(d)^5 - 2*log(d)^3 - 3*log(d))*x + 36*log(d)^2 - 6)*d^x*cos(x) + ((log(d)^6 + 3*log(d)^4 + 3*log(d)^2 + 1)*x^3 - 6*(log(d)^5 + 2*log(d)^3 + log(d))*x^2 - 24*log(d)^3 + 6*(3*log(d)^4 + 2*log(d)^2 - 1)*x + 24*log(d))*d^x*sin(x))/(log(d)^8 + 4*log(d)^6 + 6*log(d)^4 + 4*log(d)^2 + 1)
```

Fricas [A]

time = 0.32, size = 202, normalized size = 0.78

$$\frac{(x^3 \cos(x) \log(d)^7 - 3x^2 \cos(x) \log(d)^6 + 3(x^3 + 2x) \cos(x) \log(d)^5 - 3(x^2 + 2) \cos(x) \log(d)^4 + 3(x^3 - 4x) \cos(x) \log(d)^3 + 3(x^2 - 12) \cos(x) \log(d)^2 + (x^3 - 18x) \cos(x) \log(d) + 3(x^2 - 2) \cos(x) + (x^3 \log(d)^6 - 6x^2 \log(d)^5 + 3(x^3 + 6x) \log(d)^4 - 12(x^2 + 2) \log(d)^3 + x^3 + 3(x^2 + 4x) \log(d)^2 - 6(x^2 - 4) \log(d) - 6x) \sin(x)) d^x}{\log(d)^8 + 4 \log(d)^6 + 6 \log(d)^4 + 4 \log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*cos(x),x, algorithm="fricas")
```

```
[Out] (x^3*cos(x)*log(d)^7 - 3*x^2*cos(x)*log(d)^6 + 3*(x^3 + 2*x)*cos(x)*log(d)^5 - 3*(x^2 + 2)*cos(x)*log(d)^4 + 3*(x^3 - 4*x)*cos(x)*log(d)^3 + 3*(x^2 + 12)*cos(x)*log(d)^2 + (x^3 - 18*x)*cos(x)*log(d) + 3*(x^2 - 2)*cos(x) + (x^3*log(d)^6 - 6*x^2*log(d)^5 + 3*(x^3 + 6*x)*log(d)^4 - 12*(x^2 + 2)*log(d)^3 + x^3 + 3*(x^2 + 4*x)*log(d)^2 - 6*(x^2 - 4)*log(d) - 6*x)*sin(x))*d^x/(log(d)^8 + 4*log(d)^6 + 6*log(d)^4 + 4*log(d)^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.

time = 2.16, size = 1352, normalized size = 5.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d**x*x**3*cos(x),x)

[Out] Piecewise((I*x**4*exp(-I*x)*sin(x)/8 + x**4*exp(-I*x)*cos(x)/8 + x**3*exp(-I*x)*sin(x)/4 + I*x**3*exp(-I*x)*cos(x)/4 - 3*I*x**2*exp(-I*x)*sin(x)/8 + 3*x**2*exp(-I*x)*cos(x)/8 - 3*x*exp(-I*x)*sin(x)/8 - 3*I*x*exp(-I*x)*cos(x)/8 - 3*exp(-I*x)*cos(x)/8, Eq(d, exp(-I))), (-I*x**4*exp(I*x)*sin(x)/8 + x**4*exp(I*x)*cos(x)/8 + x**3*exp(I*x)*sin(x)/4 - I*x**3*exp(I*x)*cos(x)/4 + 3*I*x**2*exp(I*x)*sin(x)/8 + 3*x**2*exp(I*x)*cos(x)/8 - 3*x*exp(I*x)*sin(x)/8 + 3*I*x*exp(I*x)*cos(x)/8 - 3*exp(I*x)*cos(x)/8, Eq(d, exp(I))), (d**x*x**3*log(d)**7*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + d**x*x**3*log(d)**6*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**5*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**4*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + d**x*x**3*log(d)*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + d**x*x**3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**6*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*d**x*x**2*log(d)**5*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**4*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x**2*log(d)**3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**2*log(d)**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*d**x*x**2*log(d)*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 6*d**x*x*log(d)**5*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 18*d**x*x*log(d)**4*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x*log(d)**3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 12*d**x*x*log(d)**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 18*d**x*x*log(d)*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*d**x*x*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*d**x*log(d)**4*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 24*d**x*log(d)**3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 36*d**x*log(d)**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 24*d**x*log(d)*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*d**x*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1), True))

Giac [C] Result contains complex when optimal does not.

time = 0.01, size = 6997, normalized size = 26.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d^x*x^3*cos(x),x)

[Out]
$$\begin{aligned} & -1/2*((4\pi + \pi^4 \operatorname{sgn}(d) - 6\pi^2 \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) - \pi^4 + 6\pi^2 \log(\operatorname{abs}(d))^2 - 2\log(\operatorname{abs}(d))^4 - 4\pi^3 \operatorname{sgn}(d) + 12\pi \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) \\ & + 4\pi^3 - 12\pi \log(\operatorname{abs}(d))^2 + 6\pi^2 \operatorname{sgn}(d) - 6\pi^2 + 12\log(\operatorname{abs}(d))^2 - 4\pi \operatorname{sgn}(d) - 2)*(3\pi^2 x^3 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - 3\pi^2 x^3 \log(\operatorname{abs}(d)) \\ & + 2x^3 \log(\operatorname{abs}(d))^3 - 6\pi x^3 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) + 6\pi x^3 \log(\operatorname{abs}(d)) - 3\pi^2 x^2 \operatorname{sgn}(d) + 3\pi^2 x^2 - 6x^3 \log(\operatorname{abs}(d)) - 6x^2 \log(\operatorname{abs}(d))^2 \\ & + 6\pi x^2 \operatorname{sgn}(d) - 6\pi x^2 + 6x^2 + 12x \log(\operatorname{abs}(d)) - 12)/((4\pi + \pi^4 \operatorname{sgn}(d) - 6\pi^2 \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) - \pi^4 + 6\pi^2 \log(\operatorname{abs}(d))^2 - 2\log(\operatorname{abs}(d))^4 - 4\pi^3 \operatorname{sgn}(d) + 12\pi \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) + 4\pi^3 - 12\pi \log(\operatorname{abs}(d))^2 + 6\pi^2 \operatorname{sgn}(d) - 6\pi^2 + 12\log(\operatorname{abs}(d))^2 - 4\pi \operatorname{sgn}(d) - 2)^2 + 16(\pi^3 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - \pi \log(\operatorname{abs}(d))^3 \operatorname{sgn}(d) - \pi^3 \log(\operatorname{abs}(d)) + \pi \log(\operatorname{abs}(d))^3 - 3\pi^2 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) + 3\pi^2 \log(\operatorname{abs}(d)) - 2\log(\operatorname{abs}(d))^3 + 3\pi \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - 3\pi \log(\operatorname{abs}(d)) + 2\log(\operatorname{abs}(d)))^2) - 4(\pi^3 x^3 \operatorname{sgn}(d) - 3\pi x^3 \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) - \pi^3 x^3 + 3\pi x^3 \log(\operatorname{abs}(d))^2 - 3\pi^2 x^3 \operatorname{sgn}(d) + 3\pi^2 x^3 - 6x^3 \log(\operatorname{abs}(d))^2 + 3\pi x^3 \operatorname{sgn}(d) + 6\pi x^2 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - 3\pi x^3 - 6\pi x^2 \log(\operatorname{abs}(d)) + 2x^3 + 12x^2 \log(\operatorname{abs}(d)) - 6\pi x \operatorname{sgn}(d) + 6\pi x - 12x)(\pi^3 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - \pi \log(\operatorname{abs}(d))^3 \operatorname{sgn}(d) - \pi^3 \log(\operatorname{abs}(d)) + \pi \log(\operatorname{abs}(d))^3 - 3\pi^2 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) + 3\pi^2 \log(\operatorname{abs}(d)) - 2\log(\operatorname{abs}(d))^3 + 3\pi \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - 3\pi \log(\operatorname{abs}(d)) + 2\log(\operatorname{abs}(d))) / ((4\pi + \pi^4 \operatorname{sgn}(d) - 6\pi^2 \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) - \pi^4 + 6\pi^2 \log(\operatorname{abs}(d))^2 - 2\log(\operatorname{abs}(d))^4 - 4\pi^3 \operatorname{sgn}(d) + 12\pi \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) + 4\pi^3 - 12\pi \log(\operatorname{abs}(d))^2 + 6\pi^2 \operatorname{sgn}(d) - 6\pi^2 + 12\log(\operatorname{abs}(d))^2 - 4\pi \operatorname{sgn}(d) - 2)^2 + 16(\pi^3 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - \pi \log(\operatorname{abs}(d))^3 \operatorname{sgn}(d) - \pi^3 \log(\operatorname{abs}(d)) + \pi \log(\operatorname{abs}(d))^3 - 3\pi^2 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) + 3\pi^2 \log(\operatorname{abs}(d)) - 2\log(\operatorname{abs}(d))^3 + 3\pi \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - 3\pi \log(\operatorname{abs}(d)) + 2\log(\operatorname{abs}(d)))^2) \\ & * \cos(1/2\pi x \operatorname{sgn}(d) - 1/2\pi x + x) + ((4\pi + \pi^4 \operatorname{sgn}(d) - 6\pi^2 \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) - \pi^4 + 6\pi^2 \log(\operatorname{abs}(d))^2 - 2\log(\operatorname{abs}(d))^4 - 4\pi^3 \operatorname{sgn}(d) + 12\pi \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) + 4\pi^3 - 12\pi \log(\operatorname{abs}(d))^2 + 6\pi^2 \operatorname{sgn}(d) - 6\pi^2 + 12\log(\operatorname{abs}(d))^2 - 4\pi \operatorname{sgn}(d) - 2)(\pi^3 x^3 \operatorname{sgn}(d) - 3\pi x^3 \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) - \pi^3 x^3 + 3\pi x^3 \log(\operatorname{abs}(d))^2 - 3\pi^2 x^3 \operatorname{sgn}(d) + 3\pi^2 x^3 - 6x^3 \log(\operatorname{abs}(d))^2 + 3\pi x^3 \operatorname{sgn}(d) + 6\pi x^2 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - 3\pi x^3 - 6\pi x^2 \log(\operatorname{abs}(d)) + 2x^3 + 12x^2 \log(\operatorname{abs}(d)) - 6\pi x \operatorname{sgn}(d) + 6\pi x - 12x) / ((4\pi + \pi^4 \operatorname{sgn}(d) - 6\pi^2 \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) - \pi^4 + 6\pi^2 \log(\operatorname{abs}(d))^2 - 2\log(\operatorname{abs}(d))^4 - 4\pi^3 \operatorname{sgn}(d) + 12\pi \log(\operatorname{abs}(d))^2 \operatorname{sgn}(d) + 4\pi^3 - 12\pi \log(\operatorname{abs}(d))^2 + 6\pi^2 \operatorname{sgn}(d) - 6\pi^2 + 12\log(\operatorname{abs}(d))^2 - 4\pi \operatorname{sgn}(d) - 2)^2 + 16(\pi^3 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - \pi \log(\operatorname{abs}(d))^3 \operatorname{sgn}(d) - \pi^3 \log(\operatorname{abs}(d)) + \pi \log(\operatorname{abs}(d))^3 - 3\pi^2 \log(\operatorname{abs}(d)) \operatorname{sgn}(d) + 3\pi^2 \log(\operatorname{abs}(d)) - 2\log(\operatorname{abs}(d))^3 + 3\pi \log(\operatorname{abs}(d)) \operatorname{sgn}(d) - 3\pi \log(\operatorname{abs}(d)) + 2\log(\operatorname{abs}(d)))^2) \end{aligned}$$

$$\begin{aligned}
& 2*x^3*sgn(d) - 3*pi^2*x^3 + 6*x^3*log(abs(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) - 3*pi*x^3 - 6*pi*x^2*log(abs(d)) - 2*x^3 - 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x + 12*x)/((4*pi - pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + pi^4 - 6*pi^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 12*log(abs(d))^2 - 4*pi*sgn(d) + 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) - 2*log(abs(d)))^2) - 4*(3*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi^2*x^3*log(abs(d)) + 2*x^3*log(abs(d))^3 + 6*pi*x^3*log(abs(d))*sgn(d) - 6*pi*x^3*log(abs(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x^3*log(abs(d)) - 6*x^2*log(abs(d))^2 - 6*pi*x^2*sgn(d) + 6*pi*x^2 + 6*x^2 + 12*x*log(abs(d)) - 12)*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) - 2*log(abs(d)))/((4*pi - pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + pi^4 - 6*pi^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 12*log(abs(d))^2 - 4*pi*sgn(d) + 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) - 2*log(abs(d)))^2)*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x)*abs(d)^x + 1/4*(-I*(pi^3*x^3*sgn(d) + 3*I*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi*x^3*log(abs(d))^2*sgn(d) - pi^3*x^3 - 3*I*pi^2*x^3*log(abs(d)) + 3*pi*x^3*log(abs(d))^2 + 2*I*x^3*log(abs(d))^3 - 3*pi^2*x^3*sgn(d) - 6*I*pi*x^3*log(abs(d))*sgn(d) + 3*pi^2*x^3 + 6*I*pi*x^3*log(abs(d)) - 6*x^3*log(abs(d))^2 - 3*I*pi^2*x^2*sgn(d) + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) + 3*I*pi^2*x^2 - 3*pi*x^3 - 6*pi*x^2*log(abs(d)) - 6*I*x^3*log(abs(d)) - 6*I*x^2*log(abs(d))^2 + 6*I*pi*x^2*sgn(d) - 6*I*pi*x^2 + 2*x^3 + 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x + 6*I*x^2 + 12*I*x*log(abs(d)) - 12*x - 12*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x + I*x)/(4*pi + pi^4*sgn(d) + 4*I*pi^3*log(abs(d))*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - 4*I*pi*log(abs(d))^3*sgn(d) - pi^4 - 4*I*pi^3*log(abs(d)) + 6*pi^2*log(abs(d))^2 + 4*I*pi*log(abs(d))^3 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) - 12*I*pi^2*log(abs(d))*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 + 12*I*pi^2*log(abs(d)) - 12*pi*log(abs(d))^2 - 8*I*log(abs(d))^3 + 6*pi^2*sgn(d) + 12*I*pi*log(abs(d))*sgn(d) - 6*pi^2 - 12*I*pi*log(abs(d)) + 12*log(abs(d))^2 - 4*pi*sgn(d) + 8*I*log(abs(d)) - 2) - I*(pi^3*x^3*sgn(d) - 3*I*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi*x^3*log(abs(d))^2*sgn(d) - pi^3*x^3 + 3*I*pi^2*x^3*log(abs(d)) + 3*pi*x^3*log(abs(d))^2 - 2*I*x^3*log(abs(d))^3 - 3*pi^2*x^3*sgn(d) + 6*I*pi*x^3*log(abs(d))*sgn(d) + 3*pi^2*x^3 - 6*I*pi*x^3*log(abs(d)) - 6*x^3*log(abs(d))^2 + 3*I*pi^2*x^2*sgn(d) + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) - 3*I*pi^2*x^2 - 3*pi*x^3 - 6*pi*x^2*log(abs(d)) + 6*I*x^3*log(abs(d)) + 6*I*x^2*log(abs(d))^2 - 6*I*pi*x^2*sgn(d) + 6*I*pi*x^2 + 2*x^3 + 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x - 6*I*x^2 - 12*I*x*log(abs(d)) - 12*x + 12*I)*e^(-1/2*I*pi
\end{aligned}$$

```

*x*sgn(d) + 1/2*I*pi*x - I*x)/(4*pi + pi^4*sgn(d) - 4*I*pi^3*log(abs(d))*sgn
n(d) - 6*pi^2*log(abs(d))^2*sgn(d) + 4*I*pi*log(abs(d))^3*sgn(d) - pi^4 + 4
*I*pi^3*log(abs(d)) + 6*pi^2*log(abs(d))^2 - 4*I*pi*log(abs(d))^3 - 2*log(a
bs(d))^4 - 4*pi^3*sgn(d) + 12*I*pi^2*log(abs(d))*sgn(d) + 12*pi*log(abs(d))
^2*sgn(d) + 4*pi^3 - 12*I*pi^2*log(abs(d)) - 12*pi*log(abs(d))^2 + 8*I*log(
abs(d))^3 + 6*pi^2*sgn(d) - 12*I*pi*log(abs(d))*sgn(d) - 6*pi^2 + 12*I*pi*l
og(abs(d)) + 12*log(abs(d))^2 - 4*pi*sgn(d) - 8*I*log(abs(d)) - 2))*abs(d)^
x - 1/4*(-I*(pi^3*x^3*sgn(d) + 3*I*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi*x^3*l
og(abs(d))^2*sgn(d) - pi^3*x^3 - 3*I*pi^2*x^3*log(abs(d)) + 3*pi*x^3*log(ab
s(d))^2 + 2*I*x^3*log(abs(d))^3 + 3*pi^2*x^3*sgn(d) + 6*I*pi*x^3*log(abs(d)
)*sgn(d) - 3*pi^2*x^3 - 6*I*pi*x^3*log(abs(d)) + 6*x^3*log(abs(d))^2 - 3*I*
pi^2*x^2*sgn(d) + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) + 3*I*pi^2*
x^2 - 3*pi*x^3 - 6*pi*x^2*log(abs(d)) - 6*I*x^3*log(abs(d)) - 6*I*x^2*log(a
bs(d))^2 - 6*I*pi*x^2*sgn(d) + 6*I*pi*x^2 - 2*x^3 - 12*x^2*log(abs(d)) - 6*
pi*x*sgn(d) + 6*pi*x + 6*I*x^2 + 12*I*x*log(abs(d)) + 12*x - 12*I)*e^(1/2*I
*pi*x*sgn(d) - 1/2*I*pi*x - I*x)/(4*pi - pi^4*sgn(d) - 4*I*pi^3*log(abs(d))
*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + 4*I*pi*log(abs(d))^3*sgn(d) + pi^4
+ 4*I*pi^3*log(abs(d)) - 6*pi^2*log(abs(d))^2 - 4*I*pi*log(abs(d))^3 + 2*lo
g(abs(d))^4 - 4*pi^3*sgn(d) - 12*I*pi^2*log(abs(d))*sgn(d) + 12*pi*log(abs(
d))^2*sgn(d) + 4*pi^3 + 12*I*pi^2*log(abs(d)) - 12*pi*log(abs(d))^2 - 8*I*l
og(abs(d))^3 - 6*pi^2*sgn(d) - 12*I*pi*log(abs(d))*sgn(d) + 6*pi^2 + 12*I*p
i*log(abs(d)) - 12*log(abs(d))^2 - 4*pi*sgn(d) + 8*I*log(abs(d)) + 2) - I*(
pi^3*x^3*sgn(d) - 3*I*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi*x^3*log(abs(d))^2*
sgn(d) - pi^3*x^3 + 3*I*pi^2*x^3*log(abs(d)) + 3*pi*x^3*log(abs(d))^2 - 2*I
*x^3*log(abs(d))^3 + 3*pi^2*x^3*sgn(d) - 6*I*pi*x^3*log(abs(d))*sgn(d) - 3*
pi^2*x^3 + 6*I*pi*x^3*log(abs(d)) + 6*x^3*log(abs(d))^2 + 3*I*pi^2*x^2*sgn(
d) + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) - 3*I*pi^2*x^2 - 3*pi*x^
3 - 6*pi*x^2*log(abs(d)) + 6*I*x^3*log(abs(d)) + 6*I*x^2*log(abs(d))^2 + 6*
I*pi*x^2*sgn(d) - 6*I*pi*x^2 - 2*x^3 - 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) +
6*pi*x - 6*I*x^2 - 12*I*x*log(abs(d)) + 12*x + 12*I)*e^(-1/2*I*pi*x*sgn(d)
+ 1/2*I*pi*x + I*x)/(4*pi - pi^4*sgn(d) + 4*I*pi^3*log(abs(d))*sgn(d) + 6*
pi^2*log(abs(d))^2*sgn(d) - 4*I*pi*log(abs(d))^3*sgn(d) + pi^4 - 4*I*pi^3*l
og(abs(d)) - 6*pi^2*log(abs(d))^2 + 4*I*pi*log(abs(d))^3 + 2*log(abs(d))^4
- 4*pi^3*sgn(d) + 12*I*pi^2*log(abs(d))*sgn(d) + 12*pi*log(abs(d))^2*sgn(d)
+ 4*pi^3 - 12*I*pi^2*log(abs(d)) - 12*pi*log(abs(d))^2 + 8*I*log(abs(d))^3
- 6*pi^2*sgn(d) + 12*I*pi*log(abs(d))*sgn(d) + 6*pi^2 - 12*I*pi*log(abs(d)
) - 12*log(abs(d))^2 - 4*pi*sgn(d) - 8*I*log(abs(d)) + 2))*abs(d)^x

```

Mupad [B]

time = 0.64, size = 232, normalized size = 0.89

$\frac{d^2 \ln(\cos(x) - 3x^2 \cos(x) - x^2 \sin(x) + 6x \sin(x)) - d^2 \ln(0.2^2 \cos(x) - 6x^2 \sin(x) + 6x \cos(x)) + d^2 \ln(8 \cos(x) + 3x^2 \cos(x) - 3x^2 \sin(x) - 18x \sin(x)) + d^2 \ln(24 \cos(x) - 3x^2 \sin(x) + 12x^2 \sin(x) + 12x \cos(x)) - d^2 \ln(8 \cos(x) + 3x^2 \cos(x) + 12x^2 \sin(x) + 12x \sin(x)) + d^2 \ln(13^2 \cos(x) - x^2 \sin(x)) - d^2 \ln(24 \sin(x) + x^2 \cos(x) - 4x^2 \sin(x) - 18x \cos(x)) - d^2 \ln(8 \cos(x) - 3x^2 \sin(x) - 18x \cos(x)) - d^2 \ln(8 \cos(x) - 3x^2 \sin(x) - 18x \cos(x))}{(\sin(x)^2 + 1)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(d^x*x^3*cos(x),x)

```
[Out] -(d^x*(6*cos(x) - 3*x^2*cos(x) - x^3*sin(x) + 6*x*sin(x)) - d^x*log(d)^5*(3
*x^3*cos(x) - 6*x^2*sin(x) + 6*x*cos(x)) + d^x*log(d)^4*(6*cos(x) + 3*x^2*c
os(x) - 3*x^3*sin(x) - 18*x*sin(x)) + d^x*log(d)^3*(24*sin(x) - 3*x^3*cos(x)
) + 12*x^2*sin(x) + 12*x*cos(x)) - d^x*log(d)^2*(36*cos(x) + 3*x^2*cos(x) +
3*x^3*sin(x) + 12*x*sin(x)) + d^x*log(d)^6*(3*x^2*cos(x) - x^3*sin(x)) - d
^x*log(d)*(24*sin(x) + x^3*cos(x) - 6*x^2*sin(x) - 18*x*cos(x)) - d^x*x^3*l
og(d)^7*cos(x))/(log(d)^2 + 1)^4
```

3.142 $\int \sin(x) \sin(2x) \sin(3x) dx$

Optimal. Leaf size=25

$$-\frac{1}{8} \cos(2x) - \frac{1}{16} \cos(4x) + \frac{1}{24} \cos(6x)$$

[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)

Rubi [A]

time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4440, 2718}

$$-\frac{1}{8} \cos(2x) - \frac{1}{16} \cos(4x) + \frac{1}{24} \cos(6x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]*Sin[2*x]*Sin[3*x],x]

[Out] -1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24

Rule 2718

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4440

Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.) + (f_.)*(x_)]^(r_.), x_Symbol] := Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H, sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]

Rubi steps

$$\begin{aligned} \int \sin(x) \sin(2x) \sin(3x) dx &= \int \left(\frac{1}{4} \sin(2x) + \frac{1}{4} \sin(4x) - \frac{1}{4} \sin(6x) \right) dx \\ &= \frac{1}{4} \int \sin(2x) dx + \frac{1}{4} \int \sin(4x) dx - \frac{1}{4} \int \sin(6x) dx \\ &= -\frac{1}{8} \cos(2x) - \frac{1}{16} \cos(4x) + \frac{1}{24} \cos(6x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{8} \cos(2x) - \frac{1}{16} \cos(4x) + \frac{1}{24} \cos(6x)$$

Antiderivative was successfully verified.

`[In] Integrate[Sin[x]*Sin[2*x]*Sin[3*x],x]``[Out] -1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24`**Mathics [A]**

time = 6.83, size = 20, normalized size = 0.80

$$\frac{5}{48} - \frac{\text{Cos}[2x]}{8} - \frac{\text{Cos}[4x]}{16} + \frac{\text{Cos}[6x]}{24}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sin[x]*Sin[2*x]*Sin[3*x],x]')``[Out] 5 / 48 - Cos[2 x] / 8 - Cos[4 x] / 16 + Cos[6 x] / 24`**Maple [A]**

time = 0.06, size = 20, normalized size = 0.80

method	result	size
default	$-\frac{\cos(2x)}{8} - \frac{\cos(4x)}{16} + \frac{\cos(6x)}{24}$	20
risch	$-\frac{\cos(2x)}{8} - \frac{\cos(4x)}{16} + \frac{\cos(6x)}{24}$	20

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sin(x)*sin(2*x)*sin(3*x),x,method=_RETURNVERBOSE)``[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)`**Maxima [A]**

time = 0.26, size = 19, normalized size = 0.76

$$\frac{1}{24} \cos(6x) - \frac{1}{16} \cos(4x) - \frac{1}{8} \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")``[Out] 1/24*cos(6*x) - 1/16*cos(4*x) - 1/8*cos(2*x)`

Fricas [A]

time = 0.32, size = 17, normalized size = 0.68

$$\frac{4}{3} \cos(x)^6 - \frac{5}{2} \cos(x)^4 + \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")``[Out] 4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2`**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(19) = 38

time = 1.26, size = 114, normalized size = 4.56

$$\frac{x \sin(x) \sin(2x) \sin(3x)}{4} + \frac{x \sin(x) \cos(2x) \cos(3x)}{4} + \frac{x \sin(2x) \cos(x) \cos(3x)}{4} - \frac{x \sin(3x) \cos(x) \cos(2x)}{4} - \frac{5 \sin(x) \sin(2x) \cos(3x)}{24} - \frac{\sin(2x) \sin(3x) \cos(x)}{8} - \frac{\cos(x) \cos(2x) \cos(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(x)*sin(2*x)*sin(3*x),x)`
`[Out] x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*cos(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - 5*sin(x)*sin(2*x)*cos(3*x)/24 - sin(2*x)*sin(3*x)*cos(x)/8 - cos(x)*cos(2*x)*cos(3*x)/6`
Giac [A]

time = 0.00, size = 18, normalized size = 0.72

$$-\frac{4}{3} \sin^6 x + \frac{3}{2} \sin^4 x$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(x)*sin(2*x)*sin(3*x),x)``[Out] -4/3*sin(x)^6 + 3/2*sin(x)^4`**Mupad [B]**

time = 0.17, size = 14, normalized size = 0.56

$$-\frac{\sin(x)^4 (8 \sin(x)^2 - 9)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sin(2*x)*sin(3*x)*sin(x),x)``[Out] -(sin(x)^4*(8*sin(x)^2 - 9))/6`

3.143 $\int \cos(x) \cos(2x) \cos(3x) dx$

Optimal. Leaf size=30

$$\frac{x}{4} + \frac{1}{8} \sin(2x) + \frac{1}{16} \sin(4x) + \frac{1}{24} \sin(6x)$$

[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)

Rubi [A]

time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4440, 2717}

$$\frac{x}{4} + \frac{1}{8} \sin(2x) + \frac{1}{16} \sin(4x) + \frac{1}{24} \sin(6x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cos[2*x]*Cos[3*x],x]

[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 4440

Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.) + (f_.)*(x_)]^(r_.), x_Symbol] := Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H, sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]

Rubi steps

$$\begin{aligned} \int \cos(x) \cos(2x) \cos(3x) dx &= \int \left(\frac{1}{4} + \frac{1}{4} \cos(2x) + \frac{1}{4} \cos(4x) + \frac{1}{4} \cos(6x) \right) dx \\ &= \frac{x}{4} + \frac{1}{4} \int \cos(2x) dx + \frac{1}{4} \int \cos(4x) dx + \frac{1}{4} \int \cos(6x) dx \\ &= \frac{x}{4} + \frac{1}{8} \sin(2x) + \frac{1}{16} \sin(4x) + \frac{1}{24} \sin(6x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 30, normalized size = 1.00

$$\frac{x}{4} + \frac{1}{8} \sin(2x) + \frac{1}{16} \sin(4x) + \frac{1}{24} \sin(6x)$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]``[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24`**Mathics [A]**

time = 3.78, size = 22, normalized size = 0.73

$$\frac{x}{4} + \frac{\text{Sin}[6x]}{24} + \frac{\text{Sin}[4x]}{16} + \frac{\text{Sin}[2x]}{8}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]')``[Out] x / 4 + Sin[6 x] / 24 + Sin[4 x] / 16 + Sin[2 x] / 8`**Maple [A]**

time = 0.06, size = 23, normalized size = 0.77

method	result	size
default	$\frac{x}{4} + \frac{\sin(2x)}{8} + \frac{\sin(4x)}{16} + \frac{\sin(6x)}{24}$	23
risch	$\frac{x}{4} + \frac{\sin(2x)}{8} + \frac{\sin(4x)}{16} + \frac{\sin(6x)}{24}$	23

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x)*cos(2*x)*cos(3*x),x,method=_RETURNVERBOSE)``[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)`**Maxima [A]**

time = 0.25, size = 22, normalized size = 0.73

$$\frac{1}{4}x + \frac{1}{24} \sin(6x) + \frac{1}{16} \sin(4x) + \frac{1}{8} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="maxima")``[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)`

Fricas [A]

time = 0.32, size = 25, normalized size = 0.83

$$\frac{1}{12} (16 \cos(x)^5 - 10 \cos(x)^3 + 3 \cos(x)) \sin(x) + \frac{1}{4} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="fricas")**[Out]** 1/12*(16*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*sin(x) + 1/4*x**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(22) = 44

time = 1.25, size = 116, normalized size = 3.87

$$-\frac{x \sin(x) \sin(2x) \cos(3x)}{4} + \frac{x \sin(x) \sin(3x) \cos(2x)}{4} + \frac{x \sin(2x) \sin(3x) \cos(x)}{4} + \frac{x \cos(x) \cos(2x) \cos(3x)}{4} + \frac{3 \sin(x) \sin(2x) \sin(3x)}{8} + \frac{\sin(x) \cos(2x) \cos(3x)}{3} + \frac{5 \sin(2x) \cos(x) \cos(3x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(2*x)*cos(3*x),x)**[Out]** -x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*sin(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 + 3*sin(x)*sin(2*x)*sin(3*x)/8 + sin(x)*cos(2*x)*cos(3*x)/3 + 5*sin(2*x)*cos(x)*cos(3*x)/24**Giac [A]**

time = 0.00, size = 26, normalized size = 0.87

$$\frac{x}{4} + \frac{\sin(2x)}{8} + \frac{\sin(4x)}{16} + \frac{\sin(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(2*x)*cos(3*x),x)**[Out]** 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)**Mupad [B]**

time = 0.28, size = 22, normalized size = 0.73

$$\frac{x}{4} + \frac{\sin(2x)}{8} + \frac{\sin(4x)}{16} + \frac{\sin(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)*cos(3*x)*cos(x),x)**[Out]** x/4 + sin(2*x)/8 + sin(4*x)/16 + sin(6*x)/24

3.144 $\int x^2 \sin^3(kx) dx$

Optimal. Leaf size=85

$$\frac{14 \cos(kx)}{9k^3} - \frac{2x^2 \cos(kx)}{3k} - \frac{2 \cos^3(kx)}{27k^3} + \frac{4x \sin(kx)}{3k^2} - \frac{x^2 \cos(kx) \sin^2(kx)}{3k} + \frac{2x \sin^3(kx)}{9k^2}$$

[Out] $14/9*\cos(k*x)/k^3-2/3*x^2*\cos(k*x)/k-2/27*\cos(k*x)^3/k^3+4/3*x*\sin(k*x)/k^2-1/3*x^2*\cos(k*x)*\sin(k*x)^2/k+2/9*x*\sin(k*x)^3/k^2$

Rubi [A]

time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3392, 3377, 2718, 2713}

$$-\frac{2 \cos^3(kx)}{27k^3} + \frac{14 \cos(kx)}{9k^3} + \frac{2x \sin^3(kx)}{9k^2} + \frac{4x \sin(kx)}{3k^2} - \frac{2x^2 \cos(kx)}{3k} - \frac{x^2 \sin^2(kx) \cos(kx)}{3k}$$

Antiderivative was successfully verified.

[In] `Int[x^2*Sin[k*x]^3,x]`

[Out] $(14*\text{Cos}[k*x])/(9*k^3) - (2*x^2*\text{Cos}[k*x])/(3*k) - (2*\text{Cos}[k*x]^3)/(27*k^3) + (4*x*\text{Sin}[k*x])/(3*k^2) - (x^2*\text{Cos}[k*x]*\text{Sin}[k*x]^2)/(3*k) + (2*x*\text{Sin}[k*x]^3)/(9*k^2)$

Rule 2713

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rule 2718

`Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rule 3377

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rule 3392

`Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d`

```

^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

```

Rubi steps

$$\begin{aligned}
\int x^2 \sin^3(kx) dx &= -\frac{x^2 \cos(kx) \sin^2(kx)}{3k} + \frac{2x \sin^3(kx)}{9k^2} + \frac{2}{3} \int x^2 \sin(kx) dx - \frac{2 \int \sin^3(kx) dx}{9k^2} \\
&= -\frac{2x^2 \cos(kx)}{3k} - \frac{x^2 \cos(kx) \sin^2(kx)}{3k} + \frac{2x \sin^3(kx)}{9k^2} + \frac{2 \text{Subst}(\int (1 - x^2) dx, x, \cos(kx))}{9k^3} \\
&= \frac{2 \cos(kx)}{9k^3} - \frac{2x^2 \cos(kx)}{3k} - \frac{2 \cos^3(kx)}{27k^3} + \frac{4x \sin(kx)}{3k^2} - \frac{x^2 \cos(kx) \sin^2(kx)}{3k} + \frac{2x \sin^3(kx)}{9k^2} \\
&= \frac{14 \cos(kx)}{9k^3} - \frac{2x^2 \cos(kx)}{3k} - \frac{2 \cos^3(kx)}{27k^3} + \frac{4x \sin(kx)}{3k^2} - \frac{x^2 \cos(kx) \sin^2(kx)}{3k} + \frac{2x \sin^3(kx)}{9k^2}
\end{aligned}$$

Mathematica [A]

time = 0.05, size = 55, normalized size = 0.65

$$\frac{-81(-2 + k^2 x^2) \cos(kx) + (-2 + 9k^2 x^2) \cos(3kx) - 6kx(-27 \sin(kx) + \sin(3kx))}{108k^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[k*x]^3,x]
```

```
[Out] (-81*(-2 + k^2*x^2)*Cos[k*x] + (-2 + 9*k^2*x^2)*Cos[3*k*x] - 6*k*x*(-27*Sin[k*x] + Sin[3*k*x]))/(108*k^3)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.98, size = 63, normalized size = 0.74

$$\text{Piecewise} \left[\left\{ \left\{ \frac{6kx(6 + \sin[kx]^2) \sin[kx] + 9k^2 x^2 \cos[kx] (-3 + \cos[kx]^2) + 2 \cos[kx] (20 + \sin[kx]^2)}{27k^3}, k \neq 0 \right\} \right\}, 0 \right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sin[k*x]^3*x^2,x]')
```

```
[Out] Piecewise[{{(6 k x (6 + Sin[k x]^2) Sin[k x] + 9 k^2 x^2 Cos[k x] (-3 + Cos[k x]^2) + 2 Cos[k x] (20 + Sin[k x]^2)) / (27 k^3), k != 0)}, 0]
```

Maple [A]

time = 0.07, size = 64, normalized size = 0.75

method	result
risch	$-\frac{3(x^2k^2-2)\cos(kx)}{4k^3} + \frac{3x\sin(kx)}{2k^2} + \frac{(9x^2k^2-2)\cos(3kx)}{108k^3} - \frac{x\sin(3kx)}{18k^2}$
derivativedivides	$\frac{-\frac{k^2x^2(2+\sin^2(kx))\cos(kx)}{3} + \frac{4\cos(kx)}{3} + \frac{4kx\sin(kx)}{k^3} + \frac{2kx(\sin^3(kx))}{9} + \frac{2(2+\sin^2(kx))\cos(kx)}{27}}{k^3}$
default	$-\frac{k^2x^2(2+\sin^2(kx))\cos(kx)}{3} + \frac{4\cos(kx)}{3} + \frac{4kx\sin(kx)}{k^3} + \frac{2kx(\sin^3(kx))}{9} + \frac{2(2+\sin^2(kx))\cos(kx)}{27}$
norman	$\frac{-\frac{2x^2}{3k} + \frac{80}{27k^3} + \frac{8x\tan\left(\frac{kx}{2}\right)}{3k^2} + \frac{64x\left(\tan^3\left(\frac{kx}{2}\right)\right)}{9k^2} + \frac{8x\left(\tan^5\left(\frac{kx}{2}\right)\right)}{3k^2} - \frac{2x^2\left(\tan^2\left(\frac{kx}{2}\right)\right)}{k} + \frac{2x^2\left(\tan^4\left(\frac{kx}{2}\right)\right)}{k} + \frac{2x^2\left(\tan^6\left(\frac{kx}{2}\right)\right)}{3k} + \frac{8\left(\tan^4\left(\frac{kx}{2}\right)\right)}{3k}}{\left(1+\tan^2\left(\frac{kx}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*sin(k*x)^3,x,method=_RETURNVERBOSE)`

[Out] $1/k^3*(-1/3*k^2*x^2*(2+\sin(k*x)^2)*\cos(k*x)+4/3*\cos(k*x)+4/3*k*x*\sin(k*x)+2/9*k*x*\sin(k*x)^3+2/27*(2+\sin(k*x)^2)*\cos(k*x))$

Maxima [A]

time = 0.26, size = 55, normalized size = 0.65

$$\frac{6kx\sin(3kx) - 162kx\sin(kx) - (9k^2x^2 - 2)\cos(3kx) + 81(k^2x^2 - 2)\cos(kx)}{108k^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*sin(k*x)^3,x, algorithm="maxima")`

[Out] $-1/108*(6*k*x*\sin(3*k*x) - 162*k*x*\sin(k*x) - (9*k^2*x^2 - 2)*\cos(3*k*x) + 81*(k^2*x^2 - 2)*\cos(k*x))/k^3$

Fricas [A]

time = 0.32, size = 59, normalized size = 0.69

$$\frac{(9k^2x^2 - 2)\cos(kx)^3 - 3(9k^2x^2 - 14)\cos(kx) - 6(kx\cos(kx))^2 - 7kx\sin(kx)}{27k^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*sin(k*x)^3,x, algorithm="fricas")`

[Out] $1/27*((9*k^2*x^2 - 2)*\cos(k*x)^3 - 3*(9*k^2*x^2 - 14)*\cos(k*x) - 6*(k*x*\cos(k*x))^2 - 7*k*x*\sin(k*x))/k^3$

Sympy [A]

time = 0.28, size = 100, normalized size = 1.18

$$\begin{cases} -\frac{x^2\sin^2(kx)\cos(kx)}{k} - \frac{2x^2\cos^3(kx)}{3k} + \frac{14x\sin^3(kx)}{9k^2} + \frac{4x\sin(kx)\cos^2(kx)}{3k^2} + \frac{14\sin^2(kx)\cos(kx)}{9k^3} + \frac{40\cos^3(kx)}{27k^3} & \text{for } k \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*sin(k*x)**3,x)

[Out] Piecewise((-x**2*sin(k*x)**2*cos(k*x)/k - 2*x**2*cos(k*x)**3/(3*k) + 14*x*sin(k*x)**3/(9*k**2) + 4*x*sin(k*x)*cos(k*x)**2/(3*k**2) + 14*sin(k*x)**2*cos(k*x)/(9*k**3) + 40*cos(k*x)**3/(27*k**3), Ne(k, 0)), (0, True))

Giac [A]

time = 0.00, size = 103, normalized size = 1.21

$$\frac{4(-3x^2k^2 + 6)k^3 \cos(kx)}{(-4k^3)^2} + \frac{4 \cdot 6xkk^3 \sin(kx)}{(-4k^3)^2} + \frac{108(9x^2k^2 - 2)k^3 \cos(3kx)}{(-108k^3)^2} - \frac{108 \cdot 6xkk^3 \sin(3kx)}{(-108k^3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sin(k*x)^3,x)

[Out] -1/18*x*sin(3*k*x)/k^2 + 3/2*x*sin(k*x)/k^2 + 1/108*(9*k^2*x^2 - 2)*cos(3*k*x)/k^3 - 3/4*(k^2*x^2 - 2)*cos(k*x)/k^3

Mupad [B]

time = 0.23, size = 67, normalized size = 0.79

$$\frac{\frac{14 \cos(kx)}{9} - \frac{2 \cos(kx)^3}{27} + k \left(\frac{14x \sin(kx)}{9} - \frac{2x \cos(kx)^2 \sin(kx)}{9} \right) + k^2 \left(\frac{x^2 \cos(kx)^3}{3} - x^2 \cos(kx) \right)}{k^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*sin(k*x)^3,x)

[Out] ((14*cos(k*x))/9 - (2*cos(k*x)^3)/27 + k*((14*x*sin(k*x))/9 - (2*x*cos(k*x))^2*sin(k*x))/9) + k^2*((x^2*cos(k*x)^3)/3 - x^2*cos(k*x))/k^3

3.145 $\int x \cos(k \csc(x)) \cot(x) \csc(x) dx$

Optimal. Leaf size=14

$$\text{Int}(x \cos(k \csc(x)) \cot(x) \csc(x), x)$$

[Out] CannotIntegrate(x*cos(k*csc(x))*cot(x)*csc(x), x)

Rubi [A]

time = 0.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x \cos(k \csc(x)) \cot(x) \csc(x) dx$$

Verification is not applicable to the result.

[In] Int[x*Cos[k*Csc[x]]*Cot[x]*Csc[x], x]

[Out] Defer[Int][x*Cos[k*Csc[x]]*Cot[x]*Csc[x], x]

Rubi steps

$$\int x \cos(k \csc(x)) \cot(x) \csc(x) dx = \int x \cos(k \csc(x)) \cot(x) \csc(x) dx$$

Mathematica [A]

time = 0.58, size = 0, normalized size = 0.00

$$\int x \cos(k \csc(x)) \cot(x) \csc(x) dx$$

Verification is not applicable to the result.

[In] Integrate[x*Cos[k*Csc[x]]*Cot[x]*Csc[x], x]

[Out] Integrate[x*Cos[k*Csc[x]]*Cot[x]*Csc[x], x]

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x*Cos[k/Sin[x]]*Cos[x]/Sin[x]^2, x]')

[Out] Timed out

Maple [A]

time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x \cos(x) \cos\left(\frac{k}{\sin(x)}\right)}{\sin(x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cos(x)*cos(k/sin(x))/sin(x)^2,x)`

[Out] `int(x*cos(x)*cos(k/sin(x))/sin(x)^2,x)`

Maxima [A] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(13) = 26$.

time = 0.26, size = 240, normalized size = 17.14

$$\frac{\left(xe^{\left(\frac{4k\cos(2x)\cos(x)}{\cos(2x)^2+\sin(2x)^2-2\cos(2x)+1} + \frac{4k\sin(2x)\sin(x)}{\cos(2x)^2+\sin(2x)^2-2\cos(2x)+1}\right)} + xe^{\left(\frac{4k\cos(x)}{\cos(2x)^2+\sin(2x)^2-2\cos(2x)+1}\right)}\right)e^{\left(-\frac{2k\cos(2x)\cos(x)}{\cos(2x)^2+\sin(2x)^2-2\cos(2x)+1} - \frac{2k\sin(2x)\sin(x)}{\cos(2x)^2+\sin(2x)^2-2\cos(2x)+1} - \frac{2k\cos(x)}{\cos(2x)^2+\sin(2x)^2-2\cos(2x)+1}\right)} \sin\left(\frac{2(k\cos(x)\sin(2x)-k\cos(2x)\sin(x)+k\sin(x))}{\cos(2x)^2+\sin(2x)^2-2\cos(2x)+1}\right)}$$

2k

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="maxima")`

[Out] `-1/2*(x*e^(4*k*cos(2*x)*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)) + 4*k*sin(2*x)*sin(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)) + x*e^(4*k*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1))*e^(-2*k*cos(2*x)*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) - 2*k*sin(2*x)*sin(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) - 2*k*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1))*sin(2*(k*cos(x)*sin(2*x) - k*cos(2*x)*sin(x) + k*sin(x))/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1))/k`

Fricas [A]

time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="fricas")`

[Out] `integral(-x*cos(x)*cos(k/sin(x))/(cos(x)^2 - 1), x)`

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cos(x) \cos\left(\frac{k}{\sin(x)}\right)}{\sin^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x)*cos(k/sin(x))/sin(x)**2,x)

[Out] Integral(x*cos(x)*cos(k/sin(x))/sin(x)**2, x)

Giac [A] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x)

[Out] Could not integrate

Mupad [A]

time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x \cos\left(\frac{k}{\sin(x)}\right) \cos(x)}{\sin(x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*cos(k/sin(x))*cos(x))/sin(x)^2,x)

[Out] int((x*cos(k/sin(x))*cos(x))/sin(x)^2, x)

3.146 $\int \cot\left(\frac{x}{2}\right) \cot(x) dx$

Optimal. Leaf size=12

$$-x - \cot\left(\frac{x}{2}\right)$$

[Out] -x-cot(1/2*x)

Rubi [A]

time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {12, 464, 209}

$$-x - \cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[Cot[x/2]*Cot[x],x]

[Out] -x - Cot[x/2]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 464

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[c*(e*x)^(m+1)*((a+b*x^n)^(p+1)/(a*e*(m+1))), x] + Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \cot\left(\frac{x}{2}\right) \cot(x) dx &= 2\text{Subst}\left(\int \frac{1-x^2}{2x^2(1+x^2)} dx, x, \tan\left(\frac{x}{2}\right)\right) \\
&= \text{Subst}\left(\int \frac{1-x^2}{x^2(1+x^2)} dx, x, \tan\left(\frac{x}{2}\right)\right) \\
&= -\cot\left(\frac{x}{2}\right) - 2\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \tan\left(\frac{x}{2}\right)\right) \\
&= -x - \cot\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 12, normalized size = 1.00

$$-x - \cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

`[In] Integrate[Cot[x/2]*Cot[x], x]``[Out] -x - Cot[x/2]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Cos[x]/(Sin[x]*Tan[x/2]), x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.07, size = 11, normalized size = 0.92

method	result	size
default	$-x - \cot\left(\frac{x}{2}\right)$	11
norman	$\frac{-1-x \tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)}$	17
risch	$-x - \frac{2i}{e^{ix}-1}$	17

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x)/sin(x)/tan(1/2*x), x, method=_RETURNVERBOSE)`

[Out] $-x - \cot(1/2*x)$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(10) = 20$.

time = 0.25, size = 41, normalized size = 3.42

$$-\frac{x \cos(x)^2 + x \sin(x)^2 - 2x \cos(x) + x + 2 \sin(x)}{\cos(x)^2 + \sin(x)^2 - 2 \cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/sin(x)/tan(1/2*x),x, algorithm="maxima")`

[Out] $-(x*\cos(x)^2 + x*\sin(x)^2 - 2*x*\cos(x) + x + 2*\sin(x))/(\cos(x)^2 + \sin(x)^2 - 2*\cos(x) + 1)$

Fricas [A]

time = 0.32, size = 16, normalized size = 1.33

$$-\frac{x \tan\left(\frac{1}{2}x\right) + 1}{\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/sin(x)/tan(1/2*x),x, algorithm="fricas")`

[Out] $-(x*\tan(1/2*x) + 1)/\tan(1/2*x)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\sin(x) \tan\left(\frac{x}{2}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/sin(x)/tan(1/2*x),x)`

[Out] `Integral(cos(x)/(sin(x)*tan(x/2)), x)`

Giac [A]

time = 0.01, size = 33, normalized size = 2.75

$$2 \cdot 2 \left(\frac{\tan\left(\frac{x}{2 \cdot 2}\right)}{8} - \frac{1}{8 \tan\left(\frac{x}{2 \cdot 2}\right)} - \frac{x}{2 \cdot 2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/sin(x)/tan(1/2*x),x)`

[Out] $-x - 1/2/\tan(1/4*x) + 1/2*\tan(1/4*x)$

Mupad [B]

time = 0.18, size = 10, normalized size = 0.83

$$-x - \cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)/(tan(x/2)*sin(x)),x)`

[Out] $-x - \cot(x/2)$

$$3.147 \quad \int \frac{\sin(ax)}{(b+c \sin(ax))^2} dx$$

Optimal. Leaf size=77

$$-\frac{2c \tan^{-1}\left(\frac{c+b \tan\left(\frac{ax}{2}\right)}{\sqrt{b^2-c^2}}\right)}{a(b^2-c^2)^{3/2}} - \frac{b \cos(ax)}{a(b^2-c^2)(b+c \sin(ax))}$$

[Out] $-2*c*\arctan((c+b*\tan(1/2*a*x))/(b^2-c^2)^{(1/2)})/a/(b^2-c^2)^{(3/2)}-b*\cos(a*x)/a/(b^2-c^2)/(b+c*\sin(a*x))$

Rubi [A]

time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2833, 12, 2739, 632, 210}

$$-\frac{2c \tan^{-1}\left(\frac{b \tan\left(\frac{ax}{2}\right)+c}{\sqrt{b^2-c^2}}\right)}{a(b^2-c^2)^{3/2}} - \frac{b \cos(ax)}{a(b^2-c^2)(c \sin(ax)+b)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sin}[a*x]/(b + c*\text{Sin}[a*x])^2, x]$

[Out] $(-2*c*\text{ArcTan}[(c + b*\text{Tan}[(a*x)/2])/ \text{Sqrt}[b^2 - c^2]])/(a*(b^2 - c^2)^{(3/2)}) - (b*\text{Cos}[a*x])/(a*(b^2 - c^2)*(b + c*\text{Sin}[a*x]))$

Rule 12

$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

Rule 210

$\text{Int}[((a_*) + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 632

$\text{Int}[(a_*) + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 2739

$\text{Int}[(a_*) + (b_)*\sin[(c_*) + (d_)*(x_)])^{-1}, x_Symbol] \rightarrow \text{With}\{e = \text{FreeFactors}[\text{Tan}[(c + d*x)/2], x]\}, \text{Dist}[2*(e/d), \text{Subst}[\text{Int}[1/(a + 2*b*e*x + a*$

e^{2*x^2}), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2833

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(-b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned}
 \int \frac{\sin(ax)}{(b + c \sin(ax))^2} dx &= -\frac{b \cos(ax)}{a(b^2 - c^2)(b + c \sin(ax))} + \frac{\int \frac{c}{b + c \sin(ax)} dx}{-b^2 + c^2} \\
 &= -\frac{b \cos(ax)}{a(b^2 - c^2)(b + c \sin(ax))} - \frac{c \int \frac{1}{b + c \sin(ax)} dx}{b^2 - c^2} \\
 &= -\frac{b \cos(ax)}{a(b^2 - c^2)(b + c \sin(ax))} - \frac{(2c) \text{Subst} \left(\int \frac{1}{b + 2cx + bx^2} dx, x, \tan \left(\frac{ax}{2} \right) \right)}{a(b^2 - c^2)} \\
 &= -\frac{b \cos(ax)}{a(b^2 - c^2)(b + c \sin(ax))} + \frac{(4c) \text{Subst} \left(\int \frac{1}{-4(b^2 - c^2) - x^2} dx, x, 2c + 2b \tan \left(\frac{ax}{2} \right) \right)}{a(b^2 - c^2)} \\
 &= -\frac{2c \tan^{-1} \left(\frac{c + b \tan \left(\frac{ax}{2} \right)}{\sqrt{b^2 - c^2}} \right)}{a(b^2 - c^2)^{3/2}} - \frac{b \cos(ax)}{a(b^2 - c^2)(b + c \sin(ax))}
 \end{aligned}$$

Mathematica [A]

time = 0.18, size = 76, normalized size = 0.99

$$-\frac{2c \tan^{-1} \left(\frac{c + b \tan \left(\frac{ax}{2} \right)}{\sqrt{b^2 - c^2}} \right)}{(b^2 - c^2)^{3/2}} + \frac{b \cos(ax)}{(b - c)(b + c)(b + c \sin(ax))}$$

a

Antiderivative was successfully verified.

[In] Integrate[Sin[a*x]/(b + c*Sin[a*x])^2,x]

[Out] -(((2*c*ArcTan[(c + b*Tan[(a*x)/2])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(3/2) + (b*Cos[a*x])/((b - c)*(b + c)*(b + c*Sin[a*x])))/a)

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Sin[a*x]/(b+c*Sin[a*x])^2,x]')``[Out] Timed out`**Maple [A]**

time = 0.15, size = 107, normalized size = 1.39

method	result
derivativdivides	$\frac{-8c \tan\left(\frac{ax}{2}\right) - 8b}{(4b^2 - 4c^2)\left(b \tan^2\left(\frac{ax}{2}\right) + 2c \tan\left(\frac{ax}{2}\right) + b\right)} - \frac{8c \arctan\left(\frac{2b \tan\left(\frac{ax}{2}\right) + 2c}{2\sqrt{b^2 - c^2}}\right)}{(4b^2 - 4c^2)\sqrt{b^2 - c^2}}$
default	$\frac{-8c \tan\left(\frac{ax}{2}\right) - 8b}{(4b^2 - 4c^2)\left(b \tan^2\left(\frac{ax}{2}\right) + 2c \tan\left(\frac{ax}{2}\right) + b\right)} - \frac{8c \arctan\left(\frac{2b \tan\left(\frac{ax}{2}\right) + 2c}{2\sqrt{b^2 - c^2}}\right)}{(4b^2 - 4c^2)\sqrt{b^2 - c^2}}$
risch	$\frac{2ib(c - ib e^{iax})}{c(-b^2 + c^2)a(c e^{2iax} - c + 2ib e^{iax})} - \frac{ic \ln\left(e^{iax} + \frac{i(b\sqrt{b^2 - c^2} + b^2 - c^2)}{\sqrt{b^2 - c^2}c}\right)}{\sqrt{b^2 - c^2}(b+c)(b-c)a} + \frac{ic \ln\left(e^{iax} + \frac{i(b\sqrt{b^2 - c^2} - b^2 + c^2)}{\sqrt{b^2 - c^2}c}\right)}{\sqrt{b^2 - c^2}(b+c)(b-c)a}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sin(a*x)/(b+c*sin(a*x))^2,x,method=_RETURNVERBOSE)`

```
[Out] 1/a*(4*(-2*c*tan(1/2*a*x)-2*b)/(4*b^2-4*c^2)/(b*tan(1/2*a*x)^2+2*c*tan(1/2*
a*x)+b)-8*c/(4*b^2-4*c^2)/(b^2-c^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*a*x)+2*c)
/(b^2-c^2)^(1/2)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(a*x)/(b+c*sin(a*x))^2,x, algorithm="maxima")`

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see 'assume?' for
more de
```

Fricas [A]

time = 0.34, size = 312, normalized size = 4.05

$$\left[\frac{(c^2 \sin(ax) + bc)\sqrt{-b^2 + c^2} \log\left(\frac{(2b^2 - c^2) \cos(ax)^2 - 2bc \sin(ax) - b^2 - c^2 + 2(b \cos(ax) \sin(ax) + c \cos(ax))\sqrt{-b^2 + c^2}}{c^2 \cos(ax)^2 - 2bc \sin(ax) - b^2 - c^2}\right) - 2(b^3 - bc^2) \cos(ax) (c^2 \sin(ax) + bc)\sqrt{b^2 - c^2} \arctan\left(\frac{-b \sin(ax) + c}{\sqrt{b^2 - c^2} \cos(ax)}\right) - (b^3 - bc^2) \cos(ax)}{2(ab^5 - 2ab^3c^2 + abc^4 + (ab^4c - 2ab^2c^3 + ac^5) \sin(ax))}, \frac{(c^2 \sin(ax) + bc)\sqrt{b^2 - c^2} \arctan\left(\frac{-b \sin(ax) + c}{\sqrt{b^2 - c^2} \cos(ax)}\right) - (b^3 - bc^2) \cos(ax)}{ab^5 - 2ab^3c^2 + abc^4 + (ab^4c - 2ab^2c^3 + ac^5) \sin(ax)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(a*x)/(b+c*sin(a*x))^2,x, algorithm="fricas")`

```
[Out] [1/2*((c^2*sin(a*x) + b*c)*sqrt(-b^2 + c^2)*log(((2*b^2 - c^2)*cos(a*x))^2 -
2*b*c*sin(a*x) - b^2 - c^2 + 2*(b*cos(a*x)*sin(a*x) + c*cos(a*x))*sqrt(-b^
2 + c^2))/(c^2*cos(a*x)^2 - 2*b*c*sin(a*x) - b^2 - c^2)) - 2*(b^3 - b*c^2)*
cos(a*x))/(a*b^5 - 2*a*b^3*c^2 + a*b*c^4 + (a*b^4*c - 2*a*b^2*c^3 + a*c^5)*
sin(a*x)), ((c^2*sin(a*x) + b*c)*sqrt(b^2 - c^2)*arctan(-(b*sin(a*x) + c)/(
sqrt(b^2 - c^2)*cos(a*x))) - (b^3 - b*c^2)*cos(a*x))/(a*b^5 - 2*a*b^3*c^2 +
a*b*c^4 + (a*b^4*c - 2*a*b^2*c^3 + a*c^5)*sin(a*x))]
```

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(a*x)/(b+c*sin(a*x))^2,x)``[Out] Timed out`**Giac [A]**

time = 0.01, size = 114, normalized size = 1.48

$$\frac{2 \left(\frac{\tan\left(\frac{1}{2}ax\right)c+b}{(c^2-b^2)(\tan^2\left(\frac{1}{2}ax\right)b+2\tan\left(\frac{1}{2}ax\right)c+b)} + \frac{2c \left(\arctan\left(\frac{b \tan\left(\frac{1}{2}ax\right)+c}{\sqrt{b^2-c^2}}\right) + \pi \operatorname{sign}(b) \lfloor \frac{ax}{2\pi} + \frac{1}{2} \rfloor \right)}{2(c^2-b^2)\sqrt{b^2-c^2}} \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(a*x)/(b+c*sin(a*x))^2,x)`

```
[Out] -2*((pi*floor(1/2*a*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*a*x) + c)/sqrt(b
^2 - c^2)))*c/(b^2 - c^2)^(3/2) + (c*tan(1/2*a*x) + b)/((b*tan(1/2*a*x)^2 +
2*c*tan(1/2*a*x) + b)*(b^2 - c^2)))/a
```

Mupad [B]

time = 0.49, size = 133, normalized size = 1.73

$$-\frac{\frac{2b}{b^2-c^2} + \frac{2c \tan\left(\frac{ax}{2}\right)}{b^2-c^2}}{a \left(b \tan\left(\frac{ax}{2}\right)^2 + 2c \tan\left(\frac{ax}{2}\right) + b \right)} - \frac{2c \operatorname{atan}\left(\frac{\left(\frac{2c^2}{(b+c)^{3/2}(b-c)^{3/2}} + \frac{2bc \tan\left(\frac{ax}{2}\right)}{(b+c)^{3/2}(b-c)^{3/2}}\right)(b^2-c^2)}{2c}\right)}{a(b+c)^{3/2}(b-c)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(a*x)/(b + c*sin(a*x))^2,x)`

[Out]
$$- \left(\frac{2b}{b^2 - c^2} + \frac{2c \tan\left(\frac{ax}{2}\right)}{b^2 - c^2} \right) / \left(a \left(b + 2c \tan\left(\frac{ax}{2}\right) + b \tan\left(\frac{ax}{2}\right)^2 \right) - \frac{2c \operatorname{atan}\left(\frac{2c^2}{(b+c)^{3/2}(b-c)^{3/2}}\right) + (2bc \tan\left(\frac{ax}{2}\right) / ((b+c)^{3/2}(b-c)^{3/2})) * (b^2 - c^2)}{2c} \right) / \left(a (b+c)^{3/2} (b-c)^{3/2} \right)$$

3.148 $\int \sin(\log(x)) dx$

Optimal. Leaf size=17

$$-\frac{1}{2}x \cos(\log(x)) + \frac{1}{2}x \sin(\log(x))$$

[Out] -1/2*x*cos(ln(x))+1/2*x*sin(ln(x))

Rubi [A]

time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4563}

$$\frac{1}{2}x \sin(\log(x)) - \frac{1}{2}x \cos(\log(x))$$

Antiderivative was successfully verified.

[In] Int[Sin[Log[x]],x]

[Out] -1/2*(x*Cos[Log[x]]) + (x*Sin[Log[x]])/2

Rule 4563

Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] - Simp[b*d*n*x*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b^2*d^2*n^2 + 1, 0]

Rubi steps

$$\int \sin(\log(x)) dx = -\frac{1}{2}x \cos(\log(x)) + \frac{1}{2}x \sin(\log(x))$$

Mathematica [A]

time = 0.00, size = 17, normalized size = 1.00

$$-\frac{1}{2}x \cos(\log(x)) + \frac{1}{2}x \sin(\log(x))$$

Antiderivative was successfully verified.

[In] Integrate[Sin[Log[x]],x]

[Out] -1/2*(x*Cos[Log[x]]) + (x*Sin[Log[x]])/2

Mathics [A]

time = 1.95, size = 13, normalized size = 0.76

$$-\frac{\sqrt{2} x \cos\left[\frac{\text{Pi}}{4} + \text{Log}[x]\right]}{2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sin[Log[x]],x]')
```

```
[Out] -Sqrt[2] x Cos[Pi / 4 + Log[x]] / 2
```

Maple [A]

time = 0.02, size = 14, normalized size = 0.82

method	result	size
lookup	$-\frac{x \cos(\ln(x))}{2} + \frac{x \sin(\ln(x))}{2}$	14
default	$-\frac{x \cos(\ln(x))}{2} + \frac{x \sin(\ln(x))}{2}$	14
risch	$\left(-\frac{1}{4} - \frac{i}{4}\right) x x^i + \left(-\frac{1}{4} + \frac{i}{4}\right) x x^{-i}$	22
norman	$\frac{x \tan\left(\frac{\ln(x)}{2}\right) - \frac{x}{2} + \frac{x \left(\tan^2\left(\frac{\ln(x)}{2}\right)\right)}{2}}{1 + \tan^2\left(\frac{\ln(x)}{2}\right)}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Maxima [A]

time = 0.25, size = 12, normalized size = 0.71

$$-\frac{1}{2} x (\cos(\log(x)) - \sin(\log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x)),x, algorithm="maxima")
```

```
[Out] -1/2*x*(cos(log(x)) - sin(log(x)))
```

Fricas [A]

time = 0.32, size = 13, normalized size = 0.76

$$-\frac{1}{2} x \cos(\log(x)) + \frac{1}{2} x \sin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(log(x)),x, algorithm="fricas")

[Out] $-1/2*x*\cos(\log(x)) + 1/2*x*\sin(\log(x))$

Sympy [A]

time = 0.13, size = 15, normalized size = 0.88

$$\frac{x \sin(\log(x))}{2} - \frac{x \cos(\log(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(ln(x)),x)

[Out] $x*\sin(\log(x))/2 - x*\cos(\log(x))/2$

Giac [A]

time = 0.00, size = 16, normalized size = 0.94

$$\frac{1}{2}x \sin(\ln x) - \frac{1}{2}x \cos(\ln x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(log(x)),x)

[Out] $-1/2*x*\cos(\log(x)) + 1/2*x*\sin(\log(x))$

Mupad [B]

time = 0.13, size = 13, normalized size = 0.76

$$-\frac{\sqrt{2} x \cos\left(\frac{\pi}{4} + \ln(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(log(x)),x)

[Out] $-(2^{(1/2)}*x*\cos(\pi/4 + \log(x)))/2$

3.149 $\int \cos(\log(x)) dx$

Optimal. Leaf size=17

$$\frac{1}{2}x \cos(\log(x)) + \frac{1}{2}x \sin(\log(x))$$

[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))

Rubi [A]

time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4564}

$$\frac{1}{2}x \sin(\log(x)) + \frac{1}{2}x \cos(\log(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[Log[x]],x]

[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2

Rule 4564

Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] + Simp[b*d*n*x*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b^2*d^2*n^2 + 1, 0]

Rubi steps

$$\int \cos(\log(x)) dx = \frac{1}{2}x \cos(\log(x)) + \frac{1}{2}x \sin(\log(x))$$

Mathematica [A]

time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2}x \cos(\log(x)) + \frac{1}{2}x \sin(\log(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[Log[x]],x]

[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2

Mathics [A]

time = 1.98, size = 13, normalized size = 0.76

$$\frac{\sqrt{2} x \sin \left[\frac{\text{Pi}}{4} + \text{Log}[x] \right]}{2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Cos[Log[x]],x]')
```

```
[Out] Sqrt[2] x Sin[Pi / 4 + Log[x]] / 2
```

Maple [A]

time = 0.02, size = 14, normalized size = 0.82

method	result	size
lookup	$\frac{x \cos(\ln(x))}{2} + \frac{x \sin(\ln(x))}{2}$	14
default	$\frac{x \cos(\ln(x))}{2} + \frac{x \sin(\ln(x))}{2}$	14
risch	$\left(\frac{1}{4} - \frac{i}{4}\right) x x^i + \left(\frac{1}{4} + \frac{i}{4}\right) x x^{-i}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Maxima [A]

time = 0.26, size = 10, normalized size = 0.59

$$\frac{1}{2} x (\cos(\log(x)) + \sin(\log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x*(cos(log(x)) + sin(log(x)))
```

Fricas [A]

time = 0.32, size = 13, normalized size = 0.76

$$\frac{1}{2} x \cos(\log(x)) + \frac{1}{2} x \sin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A]

time = 0.13, size = 15, normalized size = 0.88

$$\frac{x \sin(\log(x))}{2} + \frac{x \cos(\log(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(ln(x)),x)

[Out] x*sin(log(x))/2 + x*cos(log(x))/2

Giac [A]

time = 0.00, size = 15, normalized size = 0.88

$$\frac{1}{2}x \sin(\ln x) + \frac{1}{2}x \cos(\ln x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(log(x)),x)

[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))

Mupad [B]

time = 0.15, size = 13, normalized size = 0.76

$$\frac{\sqrt{2} x \sin\left(\frac{\pi}{4} + \ln(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(log(x)),x)

[Out] (2^(1/2)*x*sin(pi/4 + log(x)))/2

3.150 $\int e^x dx$

Optimal. Leaf size=3

$$e^x$$

[Out] exp(x)

Rubi [A]

time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2225}

$$e^x$$

Antiderivative was successfully verified.

[In] Int[E^x,x]

[Out] E^x

Rule 2225

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\int e^x dx = e^x$$

Mathematica [A]

time = 0.00, size = 3, normalized size = 1.00

$$e^x$$

Antiderivative was successfully verified.

[In] Integrate[E^x,x]

[Out] E^x

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.68, size = 4, normalized size = 1.33

ConditionalExpression[E, {True}]

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[E^x,x]')`

[Out] `ConditionalExpression[E, {True}]`

Maple [A]

time = 0.01, size = 3, normalized size = 1.00

method	result	size
gospers	e^x	3
lookup	e^x	3
derivativeldivides	e^x	3
default	e^x	3
norman	e^x	3
risch	e^x	3
meijerg	$-1 + e^x$	5

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x),x,method=_RETURNVERBOSE)`

[Out] `exp(x)`

Maxima [A]

time = 0.26, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x),x, algorithm="maxima")`

[Out] `e^x`

Fricas [A]

time = 0.29, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x),x, algorithm="fricas")`

[Out] `e^x`

Sympy [A]

time = 0.02, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x)
```

```
[Out] exp(x)
```

Giac [A]

time = 0.00, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x)
```

```
[Out] e^x
```

Mupad [B]

time = 0.01, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x),x)
```

```
[Out] exp(x)
```

3.151 $\int a^x dx$

Optimal. Leaf size=8

$$\frac{a^x}{\log(a)}$$

[Out] $a^x/\ln(a)$

Rubi [A]

time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2225}

$$\frac{a^x}{\log(a)}$$

Antiderivative was successfully verified.

[In] Int[a^x,x]

[Out] a^x/Log[a]

Rule 2225

Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\int a^x dx = \frac{a^x}{\log(a)}$$

Mathematica [A]

time = 0.00, size = 8, normalized size = 1.00

$$\frac{a^x}{\log(a)}$$

Antiderivative was successfully verified.

[In] Integrate[a^x,x]

[Out] a^x/Log[a]

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.81, size = 16, normalized size = 2.00

$$\text{Piecewise} \left[\left\{ \left\{ \frac{a^x}{\text{Log}[a]}, \text{Log}[a] \neq 0 \right\} \right\}, x \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[a^x,x]')`

[Out] `Piecewise[{{a ^ x / Log[a], Log[a] != 0}}, x]`

Maple [A]

time = 0.01, size = 9, normalized size = 1.12

method	result	size
gospers	$\frac{a^x}{\ln(a)}$	9
derivativedivides	$\frac{a^x}{\ln(a)}$	9
default	$\frac{a^x}{\ln(a)}$	9
risch	$\frac{a^x}{\ln(a)}$	9
norman	$\frac{e^{x \ln(a)}}{\ln(a)}$	11
meijerg	$-\frac{1-e^{x \ln(a)}}{\ln(a)}$	16

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a^x,x,method=_RETURNVERBOSE)`

[Out] `a^x/ln(a)`

Maxima [A]

time = 0.25, size = 8, normalized size = 1.00

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a^x,x, algorithm="maxima")`

[Out] `a^x/log(a)`

Fricas [A]

time = 0.34, size = 8, normalized size = 1.00

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a^x,x, algorithm="fricas")`

[Out] `a^x/log(a)`

Sympy [A]

time = 0.04, size = 8, normalized size = 1.00

$$\begin{cases} \frac{a^x}{\log(a)} & \text{for } \log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a**x,x)

[Out] Piecewise((a**x/log(a), Ne(log(a), 0)), (x, True))

Giac [A]

time = 0.00, size = 7, normalized size = 0.88

$$\frac{a^x}{\ln a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x,x)

[Out] a^x/log(a)

Mupad [B]

time = 0.16, size = 8, normalized size = 1.00

$$\frac{a^x}{\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a^x,x)

[Out] a^x/log(a)

3.152 $\int e^{ax} dx$

Optimal. Leaf size=9

$$\frac{e^{ax}}{a}$$

[Out] exp(a*x)/a

Rubi [A]

time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2225}

$$\frac{e^{ax}}{a}$$

Antiderivative was successfully verified.

[In] Int[E^(a*x), x]

[Out] E^(a*x)/a

Rule 2225

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\int e^{ax} dx = \frac{e^{ax}}{a}$$

Mathematica [A]

time = 0.00, size = 9, normalized size = 1.00

$$\frac{e^{ax}}{a}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a*x), x]

[Out] E^(a*x)/a

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.97, size = 16, normalized size = 1.78

$$\text{Piecewise} \left[\left\{ \left\{ \frac{E^{ax}}{a}, a \neq 0 \right\} \right\}, x \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[E^(a*x),x]')`

[Out] `Piecewise[{{E ^ (a x) / a, a != 0}}, x]`

Maple [A]

time = 0.01, size = 9, normalized size = 1.00

method	result	size
gospers	$\frac{e^{ax}}{a}$	9
derivativdivides	$\frac{e^{ax}}{a}$	9
default	$\frac{e^{ax}}{a}$	9
norman	$\frac{e^{ax}}{a}$	9
risch	$\frac{e^{ax}}{a}$	9
meijerg	$-\frac{1-e^{ax}}{a}$	14

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(a*x),x,method=_RETURNVERBOSE)`

[Out] `exp(a*x)/a`

Maxima [A]

time = 0.25, size = 8, normalized size = 0.89

$$\frac{e^{(ax)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(a*x),x, algorithm="maxima")`

[Out] `e^(a*x)/a`

Fricas [A]

time = 0.29, size = 8, normalized size = 0.89

$$\frac{e^{(ax)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(a*x),x, algorithm="fricas")`

[Out] `e^(a*x)/a`

Sympy [A]

time = 0.03, size = 7, normalized size = 0.78

$$\begin{cases} \frac{e^{ax}}{a} & \text{for } a \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(a*x),x)

[Out] Piecewise((exp(a*x)/a, Ne(a, 0)), (x, True))

Giac [A]

time = 0.00, size = 7, normalized size = 0.78

$$\frac{e^{ax}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(a*x),x)

[Out] e^(a*x)/a

Mupad [B]

time = 0.03, size = 8, normalized size = 0.89

$$\frac{e^{ax}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(a*x),x)

[Out] exp(a*x)/a

3.153 $\int \frac{e^{ax}}{x} dx$

Optimal. Leaf size=4

$Ei(ax)$

[Out] $Ei(a*x)$

Rubi [A]

time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2209}

$Ei(ax)$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{(a*x)}/x,x]$

[Out] $\text{ExpIntegralEi}[a*x]$

Rule 2209

$\text{Int}[(F_)^((g_)*((e_) + (f_)*(x_)))/((c_) + (d_)*(x_)), x_Symbol] \rightarrow \text{Simp}[(F^{(g*(e - c*(f/d))})/d)*\text{ExpIntegralEi}[f*g*(c + d*x)*(Log[F]/d)], x] /; \text{FreeQ}\{F, c, d, e, f, g, x\} \&\& \text{!TrueQ}\{UseGamma\}$

Rubi steps

$$\int \frac{e^{ax}}{x} dx = Ei(ax)$$

Mathematica [A]

time = 0.01, size = 4, normalized size = 1.00

$Ei(ax)$

Antiderivative was successfully verified.

[In] $\text{Integrate}[E^{(a*x)}/x,x]$

[Out] $\text{ExpIntegralEi}[a*x]$

Mathics [A]

time = 1.98, size = 4, normalized size = 1.00

$\text{ExpIntegralEi}[ax]$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[E^(a*x)/x,x]')
```

```
[Out] ExpIntegralEi[a x]
```

Maple [A]

time = 0.01, size = 9, normalized size = 2.25

method	result	size
derivativides	$-\expIntegral(1, -ax)$	9
default	$-\expIntegral(1, -ax)$	9
risch	$-\expIntegral(1, -ax)$	9
meijerg	$\ln(x) + \ln(-a) - \ln(-ax) - \expIntegral(1, -ax)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -Ei(1,-a*x)
```

Maxima [A]

time = 0.30, size = 4, normalized size = 1.00

$$Ei(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)/x,x, algorithm="maxima")
```

```
[Out] Ei(a*x)
```

Fricas [A]

time = 0.31, size = 4, normalized size = 1.00

$$Ei(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)/x,x, algorithm="fricas")
```

```
[Out] Ei(a*x)
```

Sympy [A]

time = 0.33, size = 3, normalized size = 0.75

$$Ei(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(a*x)/x,x)

[Out] Ei(a*x)

Giac [A]

time = 0.00, size = 4, normalized size = 1.00

$Ei(ax)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(a*x)/x,x)

[Out] Ei(a*x)

Mupad [B]

time = 0.01, size = 4, normalized size = 1.00

$ei(ax)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(a*x)/x,x)

[Out] ei(a*x)

3.154 $\int \frac{1}{a+be^{mx}} dx$

Optimal. Leaf size=24

$$\frac{x}{a} - \frac{\log(a + be^{mx})}{am}$$

[Out] x/a-ln(a+b*exp(m*x))/a/m

Rubi [A]

time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2320, 36, 29, 31}

$$\frac{x}{a} - \frac{\log(a + be^{mx})}{am}$$

Antiderivative was successfully verified.

[In] Int[(a + b*E^(m*x))^(-1), x]

[Out] x/a - Log[a + b*E^(m*x)]/(a*m)

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 2320

Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\begin{aligned} \int \frac{1}{a + be^{mx}} dx &= \frac{\text{Subst} \left(\int \frac{1}{x(a+bx)} dx, x, e^{mx} \right)}{m} \\ &= \frac{\text{Subst} \left(\int \frac{1}{x} dx, x, e^{mx} \right)}{am} - \frac{b \text{Subst} \left(\int \frac{1}{a+bx} dx, x, e^{mx} \right)}{am} \\ &= \frac{x}{a} - \frac{\log(a + be^{mx})}{am} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 38, normalized size = 1.58

$$\frac{\log(e^{mx})}{am} - \frac{\log(a^2m + abe^{mx}m)}{am}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + b*E^(m*x))^(-1), x]``[Out] Log[E^(m*x)]/(a*m) - Log[a^2*m + a*b*E^(m*x)*m]/(a*m)`**Mathics [A]**

time = 1.91, size = 25, normalized size = 1.04

$$\frac{mx - \text{Log} \left[\frac{a}{b} + E^{mx} \right]}{am}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(a+b*E^(m*x)), x]')``[Out] (m x - Log[a / b + E ^ (m x)]) / (a m)`**Maple [A]**

time = 0.01, size = 29, normalized size = 1.21

method	result	size
norman	$\frac{x}{a} - \frac{\ln(a+be^{mx})}{am}$	24
risch	$\frac{x}{a} - \frac{\ln(e^{mx} + \frac{a}{b})}{am}$	26
derivativedivides	$\frac{-\frac{\ln(a+be^{mx})}{a} + \frac{\ln(e^{mx})}{a}}{m}$	29
default	$\frac{-\frac{\ln(a+be^{mx})}{a} + \frac{\ln(e^{mx})}{a}}{m}$	29

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*exp(m*x)),x,method=_RETURNVERBOSE)`

[Out] `1/m*(-1/a*ln(a+b*exp(m*x))+1/a*ln(exp(m*x)))`

Maxima [A]

time = 0.26, size = 23, normalized size = 0.96

$$\frac{x}{a} - \frac{\log(b e^{mx} + a)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*exp(m*x)),x, algorithm="maxima")`

[Out] `x/a - log(b*e^(m*x) + a)/(a*m)`

Fricas [A]

time = 0.31, size = 22, normalized size = 0.92

$$\frac{mx - \log(b e^{mx} + a)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*exp(m*x)),x, algorithm="fricas")`

[Out] `(m*x - log(b*e^(m*x) + a))/(a*m)`

Sympy [A]

time = 0.06, size = 15, normalized size = 0.62

$$\frac{x}{a} - \frac{\log\left(\frac{a}{b} + e^{mx}\right)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*exp(m*x)),x)`

[Out] `x/a - log(a/b + exp(m*x))/(a*m)`

Giac [A]

time = 0.00, size = 26, normalized size = 1.08

$$\frac{mx}{a} - \frac{b \ln|e^{mx} b + a|}{ba}$$

$$m$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*exp(m*x)),x)

[Out] (m*x/a - log(abs(b*e^(m*x) + a))/a)/m

Mupad [B]

time = 0.09, size = 22, normalized size = 0.92

$$\frac{\ln(a + b e^{m x}) - m x}{a m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*exp(m*x)),x)

[Out] -(log(a + b*exp(m*x)) - m*x)/(a*m)

3.155 $\int \frac{e^{2x}}{1+e^x} dx$

Optimal. Leaf size=12

$$e^x - \log(1 + e^x)$$

[Out] exp(x)-ln(1+exp(x))

Rubi [A]

time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2280, 45}

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

[In] Int[E^(2*x)/(1 + E^x),x]

[Out] E^x - Log[1 + E^x]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2280

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Dist[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/Den
ominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{e^{2x}}{1+e^x} dx &= \text{Subst} \left(\int \frac{x}{1+x} dx, x, e^x \right) \\ &= \text{Subst} \left(\int \left(1 + \frac{1}{-1-x} \right) dx, x, e^x \right) \\ &= e^x - \log(1 + e^x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 12, normalized size = 1.00

$$e^x - \log(1 + e^x)$$

Antiderivative was successfully verified.

[In] Integrate[E^(2*x)/(1 + E^x),x]

[Out] E^x - Log[1 + E^x]

Mathics [A]

time = 1.77, size = 12, normalized size = 1.00

$$E^x - \text{Log}[1 + E^x]$$

Antiderivative was successfully verified.

[In] mathics('Integrate[E^(2*x)/(1+E^x),x]')

[Out] E ^ x - Log[1 + E ^ x]

Maple [A]

time = 0.01, size = 11, normalized size = 0.92

method	result	size
default	$e^x - \ln(1 + e^x)$	11
norman	$e^x - \ln(1 + e^x)$	11
risch	$e^x - \ln(1 + e^x)$	11

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(2*x)/(1+exp(x)),x,method=_RETURNVERBOSE)

[Out] exp(x)-ln(1+exp(x))

Maxima [A]

time = 0.27, size = 10, normalized size = 0.83

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="maxima")

[Out] e^x - log(e^x + 1)

Fricas [A]

time = 0.31, size = 10, normalized size = 0.83

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(2*x)/(1+exp(x)),x, algorithm="fricas")`

[Out] $e^x - \log(e^x + 1)$

Sympy [A]

time = 0.05, size = 8, normalized size = 0.67

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(2*x)/(1+exp(x)),x)`

[Out] $\exp(x) - \log(\exp(x) + 1)$

Giac [A]

time = 0.00, size = 9, normalized size = 0.75

$$e^x - \ln(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(2*x)/(1+exp(x)),x)`

[Out] $e^x - \log(e^x + 1)$

Mupad [B]

time = 0.04, size = 10, normalized size = 0.83

$$e^x - \ln(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(2*x)/(exp(x) + 1),x)`

[Out] $\exp(x) - \log(\exp(x) + 1)$

3.156 $\int e^{2x+ax} dx$

Optimal. Leaf size=13

$$\frac{e^{(2+a)x}}{2+a}$$

[Out] exp((2+a)*x)/(2+a)

Rubi [A]

time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2259, 2225}

$$\frac{e^{(a+2)x}}{a+2}$$

Antiderivative was successfully verified.

[In] Int[E^(2*x + a*x), x]

[Out] E^((2 + a)*x)/(2 + a)

Rule 2225

Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 2259

Int[(u_)*(F_)^((a_) + (b_)*(v_)), x_Symbol] := Int[u*F^(a + b*NormalizePowerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && PowerOfLinearQ[v, x] && !PowerOfLinearMatchQ[v, x]

Rubi steps

$$\begin{aligned} \int e^{2x+ax} dx &= \int e^{(2+a)x} dx \\ &= \frac{e^{(2+a)x}}{2+a} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 13, normalized size = 1.00

$$\frac{e^{(2+a)x}}{2+a}$$

Antiderivative was successfully verified.

[In] Integrate[E^(2*x + a*x),x]

[Out] E^((2 + a)*x)/(2 + a)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.80, size = 20, normalized size = 1.54

$$\text{Piecewise} \left[\left\{ \left\{ \frac{E^{x(2+a)}}{2+a}, a \neq -2 \right\} \right\}, x \right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[E^(2*x)*E^(a*x),x]')

[Out] Piecewise[{{E ^ (x (2 + a)) / (2 + a), a != -2}}, x]

Maple [A]

time = 0.01, size = 15, normalized size = 1.15

method	result	size
risch	$\frac{e^{(2+a)x}}{2+a}$	13
gospers	$\frac{e^{ax+2x}}{2+a}$	15
derivativedivides	$\frac{e^{ax+2x}}{2+a}$	15
default	$\frac{e^{ax+2x}}{2+a}$	15
norman	$\frac{e^{ax+2x}}{2+a}$	15
meijerg	$\frac{1-e^{-x(-a-2)}}{-a-2}$	22

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(a*x+2*x),x,method=_RETURNVERBOSE)

[Out] 1/(2+a)*exp(a*x+2*x)

Maxima [A]

time = 0.26, size = 14, normalized size = 1.08

$$\frac{e^{(ax+2x)}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(a*x+2*x),x, algorithm="maxima")

[Out] $e^{(a+2)x}/(a+2)$

Fricas [A]

time = 0.30, size = 12, normalized size = 0.92

$$\frac{e^{(a+2)x}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(a*x+2*x),x, algorithm="fricas")`

[Out] $e^{(a+2)x}/(a+2)$

Sympy [A]

time = 0.04, size = 14, normalized size = 1.08

$$\begin{cases} \frac{e^{ax+2x}}{a+2} & \text{for } a \neq -2 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(a*x+2*x),x)`

[Out] `Piecewise((exp(a*x + 2*x)/(a + 2), Ne(a, -2)), (x, True))`

Giac [A]

time = 0.00, size = 13, normalized size = 1.00

$$\frac{e^{ax+2x}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(a*x+2*x),x)`

[Out] $e^{(a+2)x}/(a+2)$

Mupad [B]

time = 0.05, size = 14, normalized size = 1.08

$$\frac{e^{2x+ax}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(2*x + a*x),x)`

[Out] $\exp(2x + ax)/(a + 2)$

$$3.157 \quad \int \frac{1}{be^{-mx} + ae^{mx}} dx$$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{a} e^{mx}}{\sqrt{b}}\right)}{\sqrt{a} \sqrt{b} m}$$

[Out] arctan(exp(m*x)*a^(1/2)/b^(1/2))/m/a^(1/2)/b^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2320, 211}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a} e^{mx}}{\sqrt{b}}\right)}{\sqrt{a} \sqrt{b} m}$$

Antiderivative was successfully verified.

[In] Int[(b/E^(m*x) + a*E^(m*x))^(-1), x]

[Out] ArcTan[(Sqrt[a]*E^(m*x))/Sqrt[b]]/(Sqrt[a]*Sqrt[b]*m)

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2320

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\begin{aligned} \int \frac{1}{be^{-mx} + ae^{mx}} dx &= \frac{\text{Subst}\left(\int \frac{1}{b+ax^2} dx, x, e^{mx}\right)}{m} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a} e^{mx}}{\sqrt{b}}\right)}{\sqrt{a} \sqrt{b} m} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 31, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a} e^{mx}}{\sqrt{b}}\right)}{\sqrt{a} \sqrt{b} m}$$

Antiderivative was successfully verified.

[In] Integrate[(b/E^(m*x) + a*E^(m*x))^(-1), x]

[Out] ArcTan[(Sqrt[a]*E^(m*x))/Sqrt[b]]/(Sqrt[a]*Sqrt[b]*m)

Mathics [C] Result contains complex when optimal does not.

time = 2.11, size = 32, normalized size = 1.03

$$\frac{\text{RootSum}\left[1 + 4ab\#1^2 \&, \text{Log}\left[2b\#1 + E^{mx}\right] \#1 \&\right]}{m}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(a*E^(m*x)+b*E^(-m*x)), x]')

[Out] RootSum[1 + 4 a b #1 ^ 2&, Log[2 b #1 + E ^ (m x)] #1&] / m

Maple [A]

time = 0.02, size = 22, normalized size = 0.71

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{a e^{mx}}{\sqrt{ab}}\right)}{m \sqrt{ab}}$	22
default	$\frac{\arctan\left(\frac{a e^{mx}}{\sqrt{ab}}\right)}{m \sqrt{ab}}$	22
risch	$-\frac{\ln\left(e^{mx} - \frac{b}{\sqrt{-ab}}\right)}{2\sqrt{-ab} m} + \frac{\ln\left(e^{mx} + \frac{b}{\sqrt{-ab}}\right)}{2\sqrt{-ab} m}$	53

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b/exp(m*x)+a*exp(m*x)), x, method=_RETURNVERBOSE)

[Out] 1/m/(a*b)^(1/2)*arctan(a*exp(m*x)/(a*b)^(1/2))

Maxima [A]

time = 0.35, size = 23, normalized size = 0.74

$$-\frac{\arctan\left(\frac{be^{-mx}}{\sqrt{ab}}\right)}{\sqrt{ab} m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/exp(m*x)+a*exp(m*x)),x, algorithm="maxima")

[Out] $-\arctan(b \cdot e^{-m \cdot x} / \sqrt{a \cdot b}) / (\sqrt{a \cdot b} \cdot m)$

Fricas [A]

time = 0.34, size = 85, normalized size = 2.74

$$\left[-\frac{\sqrt{-ab} \log\left(\frac{ae^{2mx}-2\sqrt{-ab}e^{mx}-b}{ae^{2mx}+b}\right)}{2abm}, -\frac{\sqrt{ab} \arctan\left(\frac{\sqrt{ab}e^{-mx}}{a}\right)}{abm} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/exp(m*x)+a*exp(m*x)),x, algorithm="fricas")

[Out] $[-1/2 \cdot \sqrt{-a \cdot b} \cdot \log((a \cdot e^{2 \cdot m \cdot x} - 2 \cdot \sqrt{-a \cdot b} \cdot e^{m \cdot x} - b) / (a \cdot e^{2 \cdot m \cdot x} + b)) / (a \cdot b \cdot m), -\sqrt{a \cdot b} \cdot \arctan(\sqrt{a \cdot b} \cdot e^{-m \cdot x} / a) / (a \cdot b \cdot m)]$

Sympy [A]

time = 0.09, size = 26, normalized size = 0.84

$$\frac{\text{RootSum}(4z^2ab + 1, (i \mapsto i \log(-2ia + e^{-mx})))}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/exp(m*x)+a*exp(m*x)),x)

[Out] $\text{RootSum}(4 \cdot z^2 \cdot a \cdot b + 1, \text{Lambda}(i, i \cdot \log(-2 \cdot i \cdot a + \exp(-m \cdot x)))) / m$

Giac [A]

time = 0.00, size = 26, normalized size = 0.84

$$\frac{\arctan\left(\frac{ae^{mx}}{\sqrt{ab}}\right)}{\sqrt{ab} m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b/exp(m*x)+a*exp(m*x)),x)

[Out] $\arctan(a \cdot e^{m \cdot x} / \sqrt{a \cdot b}) / (\sqrt{a \cdot b} \cdot m)$

Mupad [B]

time = 0.23, size = 21, normalized size = 0.68

$$\frac{\text{atan}\left(\frac{a e^{m x}}{\sqrt{a b}}\right)}{m \sqrt{a b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*exp(m*x) + b*exp(-m*x)),x)
```

```
[Out] atan((a*exp(m*x))/(a*b)^(1/2))/(m*(a*b)^(1/2))
```

3.158 $\int e^{ax} x dx$

Optimal. Leaf size=21

$$-\frac{e^{ax}}{a^2} + \frac{e^{ax}x}{a}$$

[Out] $-\exp(a*x)/a^2+\exp(a*x)*x/a$

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2207, 2225}

$$\frac{xe^{ax}}{a} - \frac{e^{ax}}{a^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{(a*x)*x}, x]$

[Out] $-(E^{(a*x)}/a^2) + (E^{(a*x)*x})/a$

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int e^{ax} x dx &= \frac{e^{ax} x}{a} - \frac{\int e^{ax} dx}{a} \\ &= -\frac{e^{ax}}{a^2} + \frac{e^{ax} x}{a} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 14, normalized size = 0.67

$$\frac{e^{ax}(-1 + ax)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(a*x)*x,x]

[Out] (E^(a*x)*(-1 + a*x))/a^2

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.95, size = 27, normalized size = 1.29

$$\text{Piecewise} \left[\left\{ \left\{ \frac{(-1 + ax) E^{ax}}{a^2}, a^2 \neq 0 \right\} \right\}, \frac{x^2}{2} \right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x*E^(a*x),x]')

[Out] Piecewise[{{(-1 + a x) E ^ (a x) / a ^ 2, a ^ 2 != 0}}, x ^ 2 / 2]

Maple [A]

time = 0.01, size = 19, normalized size = 0.90

method	result	size
gosper	$\frac{(ax-1)e^{ax}}{a^2}$	14
risch	$\frac{(ax-1)e^{ax}}{a^2}$	14
derivativdivides	$\frac{a e^{ax} x - e^{ax}}{a^2}$	19
default	$\frac{a e^{ax} x - e^{ax}}{a^2}$	19
meijerg	$\frac{1 - \frac{(-2ax+2)e^{ax}}{2}}{a^2}$	19
norman	$-\frac{e^{ax}}{a^2} + \frac{e^{ax}x}{a}$	20

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(a*x)*x,x,method=_RETURNVERBOSE)

[Out] 1/a^2*(a*exp(a*x)*x-exp(a*x))

Maxima [A]

time = 0.26, size = 13, normalized size = 0.62

$$\frac{(ax - 1)e^{(ax)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(a*x)*x,x, algorithm="maxima")

[Out] (a*x - 1)*e^(a*x)/a^2

Fricas [A]

time = 0.31, size = 13, normalized size = 0.62

$$\frac{(ax - 1)e^{(ax)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(a*x)*x,x, algorithm="fricas")``[Out] (a*x - 1)*e^(a*x)/a^2`**Sympy [A]**

time = 0.05, size = 19, normalized size = 0.90

$$\begin{cases} \frac{(ax-1)e^{ax}}{a^2} & \text{for } a^2 \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(a*x)*x,x)``[Out] Piecewise(((a*x - 1)*exp(a*x)/a**2, Ne(a**2, 0)), (x**2/2, True))`**Giac [A]**

time = 0.00, size = 14, normalized size = 0.67

$$\frac{(xa - 1)e^{ax}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(a*x)*x,x)``[Out] (a*x - 1)*e^(a*x)/a^2`**Mupad [B]**

time = 0.03, size = 13, normalized size = 0.62

$$\frac{e^{ax} (ax - 1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*exp(a*x),x)``[Out] (exp(a*x)*(a*x - 1))/a^2`

3.159 $\int e^x x^{20} dx$

Optimal. Leaf size=163

2432902008176640000e^x-2432902008176640000e^xx+1216451004088320000e^xx²-405483668029440000e^xx³

```
[Out] 2432902008176640000*exp(x)-2432902008176640000*exp(x)*x+1216451004088320000
*exp(x)*x^2-405483668029440000*exp(x)*x^3+101370917007360000*exp(x)*x^4-202
74183401472000*exp(x)*x^5+3379030566912000*exp(x)*x^6-482718652416000*exp(x
)*x^7+60339831552000*exp(x)*x^8-6704425728000*exp(x)*x^9+670442572800*exp(x
)*x^10-60949324800*exp(x)*x^11+5079110400*exp(x)*x^12-390700800*exp(x)*x^13
+27907200*exp(x)*x^14-1860480*exp(x)*x^15+116280*exp(x)*x^16-6840*exp(x)*x^
17+380*exp(x)*x^18-20*exp(x)*x^19+exp(x)*x^20
```

Rubi [A]

time = 0.15, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2207, 2225}

1/20 - 36/20 + 360/20 - 6480/20 + 138240/20 - 186480/20 + 2790720/20 - 39070080/20 + 507911040/20 - 6094932480/20 + 67044257280/20 - 670442572800/20 + 6033983152000/20 - 40718624160000/20 + 3379030566912000/20 - 20274183401472000/20 + 101370917007360000/20 - 405483668029440000/20 + 1216451004088320000/20 - 2432902008176640000/20 + 2432902008176640000/20

Antiderivative was successfully verified.

[In] Int[E^x*x^20, x]

```
[Out] 2432902008176640000*E^x - 2432902008176640000*E^x*x + 1216451004088320000*E
^x*x^2 - 405483668029440000*E^x*x^3 + 101370917007360000*E^x*x^4 - 20274183
401472000*E^x*x^5 + 3379030566912000*E^x*x^6 - 482718652416000*E^x*x^7 + 60
339831552000*E^x*x^8 - 6704425728000*E^x*x^9 + 670442572800*E^x*x^10 - 6094
9324800*E^x*x^11 + 5079110400*E^x*x^12 - 390700800*E^x*x^13 + 27907200*E^x*
x^14 - 1860480*E^x*x^15 + 116280*E^x*x^16 - 6840*E^x*x^17 + 380*E^x*x^18 -
20*E^x*x^19 + E^x*x^20
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m]
] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

Mathematica [A]

time = 0.07, size = 102, normalized size = 0.63

e^x (2432902008176640000 - 2432902008176640000*x + 1216451004088320000*x^2 - 405483668029440000*x^3 + 101370917007360000*x^4 - 20274183401472000*x^5 + 3379030566912000*x^6 - 482718652416000*x^7 + 60339831552000*x^8 - 670442572800*x^9 + 670442572800*x^10 - 60949324800*x^11 + 5079110400*x^12 - 390700800*x^13 + 27907200*x^14 - 1860480*x^15 + 116280*x^16 - 6840*x^17 + 380*x^18 - 20*x^19 + x^20)

Antiderivative was successfully verified.

[In] Integrate[E^x*x^20,x]

[Out] E^x*(2432902008176640000 - 2432902008176640000*x + 1216451004088320000*x^2 - 405483668029440000*x^3 + 101370917007360000*x^4 - 20274183401472000*x^5 + 3379030566912000*x^6 - 482718652416000*x^7 + 60339831552000*x^8 - 670442572800*x^9 + 670442572800*x^10 - 60949324800*x^11 + 5079110400*x^12 - 390700800*x^13 + 27907200*x^14 - 1860480*x^15 + 116280*x^16 - 6840*x^17 + 380*x^18 - 20*x^19 + x^20)

Mathics [A]

time = 2.34, size = 102, normalized size = 0.63

(2432902008176640000 - 2432902008176640000*x + 1216451004088320000*x^2 - 405483668029440000*x^3 + 101370917007360000*x^4 - 20274183401472000*x^5 + 3379030566912000*x^6 - 482718652416000*x^7 + 60339831552000*x^8 - 670442572800*x^9 + 670442572800*x^10 - 60949324800*x^11 + 5079110400*x^12 - 390700800*x^13 + 27907200*x^14 - 1860480*x^15 + 116280*x^16 - 6840*x^17 + 380*x^18 - 20*x^19 + x^20) E^x

Antiderivative was successfully verified.

[In] mathics('Integrate[x^20*E^x,x]')

[Out] (2432902008176640000 - 2432902008176640000 x + 1216451004088320000 x ^ 2 - 405483668029440000 x ^ 3 + 101370917007360000 x ^ 4 - 20274183401472000 x ^ 5 + 3379030566912000 x ^ 6 - 482718652416000 x ^ 7 + 60339831552000 x ^ 8 - 6704425728000 x ^ 9 + 670442572800 x ^ 10 - 60949324800 x ^ 11 + 5079110400 x ^ 12 - 390700800 x ^ 13 + 27907200 x ^ 14 - 1860480 x ^ 15 + 116280 x ^ 16 - 6840 x ^ 17 + 380 x ^ 18 - 20 x ^ 19 + x ^ 20) E ^ x

Maple [A]

time = 0.04, size = 143, normalized size = 0.88

method	result
gosper	$(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + \dots)$
risch	$(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + \dots)$
meijerg	$-2432902008176640000 + \frac{(21x^{20} - 420x^{19} + 7980x^{18} - 143640x^{17} + 2441880x^{16} - 39070080x^{15} + 586051200x^{14} - 820471680x^{13} + \dots)}{e^x}$
default	$2432902008176640000 e^x - 2432902008176640000 e^x x + 1216451004088320000 e^x x^2 - 405483668029440000 e^x x^3 + \dots$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)*x^20,x,method=_RETURNVERBOSE)

[Out] 2432902008176640000*exp(x)-2432902008176640000*exp(x)*x+1216451004088320000*exp(x)*x^2-405483668029440000*exp(x)*x^3+101370917007360000*exp(x)*x^4-20274183401472000*exp(x)*x^5+3379030566912000*exp(x)*x^6-482718652416000*exp(x)*x^7+60339831552000*exp(x)*x^8-670442572800*exp(x)*x^9+670442572800*exp(x)*x^10-60949324800*exp(x)*x^11+5079110400*exp(x)*x^12-390700800*exp(x)*x^13+27907200*exp(x)*x^14-1860480*exp(x)*x^15+116280*exp(x)*x^16-6840*exp(x)*x^17+380*exp(x)*x^18-20*exp(x)*x^19+exp(x)*x^20

74183401472000*exp(x)*x^5+3379030566912000*exp(x)*x^6-482718652416000*exp(x)
)*x^7+60339831552000*exp(x)*x^8-6704425728000*exp(x)*x^9+670442572800*exp(x)
)*x^10-60949324800*exp(x)*x^11+5079110400*exp(x)*x^12-390700800*exp(x)*x^13
 +27907200*exp(x)*x^14-1860480*exp(x)*x^15+116280*exp(x)*x^16-6840*exp(x)*x^17
 +380*exp(x)*x^18-20*exp(x)*x^19+exp(x)*x^20

Maxima [A]

time = 0.26, size = 101, normalized size = 0.62

(x²⁰ - 20x¹⁹ + 380x¹⁸ - 6840x¹⁷ + 116280x¹⁶ - 1860480x¹⁵ + 27907200x¹⁴ - 390700800x¹³ + 5079110400x¹² - 60949324800x¹¹ + 670442572800x¹⁰ - 6704425728000x⁹ + 60339831552000x⁸ - 482718652416000x⁷ + 3379030566912000x⁶ - 20274183401472000x⁵ + 101370917007360000x⁴ - 405483668029440000x³ + 1216451004088320000x² - 2432902008176640000x + 2432902008176640000)e^x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*x^20,x, algorithm="maxima")

[Out] (x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^15 + 27907
 200*x^14 - 390700800*x^13 + 5079110400*x^12 - 60949324800*x^11 + 6704425728
 00*x^10 - 6704425728000*x^9 + 60339831552000*x^8 - 482718652416000*x^7 + 33
 79030566912000*x^6 - 20274183401472000*x^5 + 101370917007360000*x^4 - 40548
 3668029440000*x^3 + 1216451004088320000*x^2 - 2432902008176640000*x + 24329
 02008176640000)*e^x

Fricas [A]

time = 0.30, size = 101, normalized size = 0.62

(x²⁰ - 20x¹⁹ + 380x¹⁸ - 6840x¹⁷ + 116280x¹⁶ - 1860480x¹⁵ + 27907200x¹⁴ - 390700800x¹³ + 5079110400x¹² - 60949324800x¹¹ + 670442572800x¹⁰ - 6704425728000x⁹ + 60339831552000x⁸ - 482718652416000x⁷ + 3379030566912000x⁶ - 20274183401472000x⁵ + 101370917007360000x⁴ - 405483668029440000x³ + 1216451004088320000x² - 2432902008176640000x + 2432902008176640000)e^x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*x^20,x, algorithm="fricas")

[Out] (x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^15 + 27907
 200*x^14 - 390700800*x^13 + 5079110400*x^12 - 60949324800*x^11 + 6704425728
 00*x^10 - 6704425728000*x^9 + 60339831552000*x^8 - 482718652416000*x^7 + 33
 79030566912000*x^6 - 20274183401472000*x^5 + 101370917007360000*x^4 - 40548
 3668029440000*x^3 + 1216451004088320000*x^2 - 2432902008176640000*x + 24329
 02008176640000)*e^x

Sympy [A]

time = 0.06, size = 102, normalized size = 0.63

(x²⁰ - 20x¹⁹ + 380x¹⁸ - 6840x¹⁷ + 116280x¹⁶ - 1860480x¹⁵ + 27907200x¹⁴ - 390700800x¹³ + 5079110400x¹² - 60949324800x¹¹ + 670442572800x¹⁰ - 6704425728000x⁹ + 60339831552000x⁸ - 482718652416000x⁷ + 3379030566912000x⁶ - 20274183401472000x⁵ + 101370917007360000x⁴ - 405483668029440000x³ + 1216451004088320000x² - 2432902008176640000x + 2432902008176640000)e^x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*x**20,x)

[Out] (x**20 - 20*x**19 + 380*x**18 - 6840*x**17 + 116280*x**16 - 1860480*x**15 +
 27907200*x**14 - 390700800*x**13 + 5079110400*x**12 - 60949324800*x**11 +

670442572800*x**10 - 6704425728000*x**9 + 60339831552000*x**8 - 482718652416000*x**7 + 3379030566912000*x**6 - 20274183401472000*x**5 + 101370917007360000*x**4 - 405483668029440000*x**3 + 1216451004088320000*x**2 - 2432902008176640000*x + 2432902008176640000)*exp(x)

Giac [A]

time = 0.00, size = 111, normalized size = 0.68

$(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 5079110400x^{12} - 60949324800x^{11} + 670442572800x^{10} - 6704425728000x^9 + 60339831552000x^8 - 482718652416000x^7 + 3379030566912000x^6 - 20274183401472000x^5 + 101370917007360000x^4 - 405483668029440000x^3 + 1216451004088320000x^2 - 2432902008176640000x + 2432902008176640000)e^x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*x^20,x)

[Out] (x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^15 + 27907200*x^14 - 390700800*x^13 + 5079110400*x^12 - 60949324800*x^11 + 670442572800*x^10 - 6704425728000*x^9 + 60339831552000*x^8 - 482718652416000*x^7 + 3379030566912000*x^6 - 20274183401472000*x^5 + 101370917007360000*x^4 - 405483668029440000*x^3 + 1216451004088320000*x^2 - 2432902008176640000*x + 2432902008176640000)*e^x

Mupad [B]

time = 0.36, size = 101, normalized size = 0.62

$(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 5079110400x^{12} - 60949324800x^{11} + 670442572800x^{10} - 6704425728000x^9 + 60339831552000x^8 - 482718652416000x^7 + 3379030566912000x^6 - 20274183401472000x^5 + 101370917007360000x^4 - 405483668029440000x^3 + 1216451004088320000x^2 - 2432902008176640000x + 2432902008176640000)e^x$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^20*exp(x),x)

[Out] exp(x)*(1216451004088320000*x^2 - 2432902008176640000*x - 405483668029440000*x^3 + 101370917007360000*x^4 - 20274183401472000*x^5 + 3379030566912000*x^6 - 482718652416000*x^7 + 60339831552000*x^8 - 6704425728000*x^9 + 670442572800*x^10 - 60949324800*x^11 + 5079110400*x^12 - 390700800*x^13 + 27907200*x^14 - 1860480*x^15 + 116280*x^16 - 6840*x^17 + 380*x^18 - 20*x^19 + x^20 + 2432902008176640000)

3.160 $\int a^x b^{-x} dx$

Optimal. Leaf size=18

$$\frac{a^x b^{-x}}{\log(a) - \log(b)}$$

[Out] $a^x/(b^x)/(\ln(a)-\ln(b))$

Rubi [A]

time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2325, 2225}

$$\frac{a^x b^{-x}}{\log(a) - \log(b)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[a^x/b^x, x]$

[Out] $a^x/(b^x*(\text{Log}[a] - \text{Log}[b]))$

Rule 2225

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))^{(n_.)}, x_Symbol] := \text{Simp}[(F^{(c*(a + b*x)))^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

Rule 2325

$\text{Int}[(u_)*(F_)^{(v_)*(G_)^{(w_)}, x_Symbol] := \text{With}\{z = v*\text{Log}[F] + w*\text{Log}[G]\}, \text{Int}[u*\text{NormalizeIntegrand}[E^z, x], x] /; \text{BinomialQ}[z, x] \|\| (\text{PolynomialQ}[z, x] \&\& \text{LeQ}[\text{Exponent}[z, x], 2]) /; \text{FreeQ}\{F, G\}, x]$

Rubi steps

$$\begin{aligned} \int a^x b^{-x} dx &= \int e^{x(\log(a) - \log(b))} dx \\ &= \frac{a^x b^{-x}}{\log(a) - \log(b)} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 18, normalized size = 1.00

$$\frac{a^x b^{-x}}{\log(a) - \log(b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^x/b^x,x]
```

```
[Out] a^x/(b^x*(Log[a] - Log[b]))
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: Invalid NaN comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[a^x/b^x,x]')
```

```
[Out] cought exception: Invalid NaN comparison
```

Maple [A]

time = 0.02, size = 19, normalized size = 1.06

method	result	size
gospers	$\frac{a^x b^{-x}}{\ln(a) - \ln(b)}$	19
risch	$\frac{a^x b^{-x}}{\ln(a) - \ln(b)}$	19
norman	$\frac{e^{x \ln(a)} e^{-x \ln(b)}}{\ln(a) - \ln(b)}$	23
meijerg	$-\frac{1 - e^{x \ln(a) \left(1 - \frac{\ln(b)}{\ln(a)}\right)}}{\ln(a) \left(1 - \frac{\ln(b)}{\ln(a)}\right)}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x/(b^x),x,method=_RETURNVERBOSE)
```

```
[Out] a^x/(b^x)/(ln(a)-ln(b))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/(b^x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-log(b)/log(a)>0)', see 'assume?' f
or more
```

Fricas [A]

time = 0.31, size = 18, normalized size = 1.00

$$\frac{a^x}{b^x(\log(a) - \log(b))}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(a^x/(b^x),x, algorithm="fricas")``[Out] a^x/(b^x*(log(a) - log(b)))`**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(a**x/(b**x),x)``[Out] Exception raised: TypeError`**Giac [F] N/A**

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(a^x/(b^x),x)``[Out] Could not integrate`**Mupad [B]**

time = 0.20, size = 18, normalized size = 1.00

$$\frac{a^x}{b^x(\ln(a) - \ln(b))}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(a^x/b^x,x)``[Out] a^x/(b^x*(log(a) - log(b)))`

3.161 $\int a^x b^x dx$

Optimal. Leaf size=14

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

[Out] $a^x b^x / (\ln(a) + \ln(b))$

Rubi [A]

time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2325, 2225}

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[a^x b^x, x]$

[Out] $(a^x b^x) / (\text{Log}[a] + \text{Log}[b])$

Rule 2225

$\text{Int}[(F_)^((c_.) * ((a_.) + (b_.) * (x_)))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x))})^n / (b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

Rule 2325

$\text{Int}[(u_.) * (F_)^{(v_)} * (G_)^{(w_)}, x_Symbol] \rightarrow \text{With}\{z = v*\text{Log}[F] + w*\text{Log}[G]\}, \text{Int}[u*\text{NormalizeIntegrand}[E^z, x], x] /; \text{BinomialQ}[z, x] \parallel (\text{PolynomialQ}[z, x] \&\& \text{LeQ}[\text{Exponent}[z, x], 2]) /; \text{FreeQ}\{F, G\}, x]$

Rubi steps

$$\begin{aligned} \int a^x b^x dx &= \int e^{x(\log(a) + \log(b))} dx \\ &= \frac{a^x b^x}{\log(a) + \log(b)} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 14, normalized size = 1.00

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

Antiderivative was successfully verified.

[In] Integrate[a^x*b^x,x]

[Out] (a^x*b^x)/(Log[a] + Log[b])

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.06, size = 40, normalized size = 2.86

$$\text{Piecewise} \left[\left\{ \left\{ \frac{a^x b^x}{\text{Log}[a] + \text{Log}[b]}, a \neq \frac{1}{b} \right\} \right\}, \frac{\left(\frac{1}{b}\right)^x b^x}{\text{Log}\left[\frac{1}{b}\right] + \text{Log}[b]} \right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[a^x*b^x,x]')

[Out] Piecewise[{{a ^ x b ^ x / (Log[a] + Log[b]), a != 1 / b}}, (1 / b) ^ x b ^ x / (Log[1 / b] + Log[b])]

Maple [A]

time = 0.02, size = 15, normalized size = 1.07

method	result	size
gospers	$\frac{a^x b^x}{\ln(a)+\ln(b)}$	15
risch	$\frac{a^x b^x}{\ln(a)+\ln(b)}$	15
norman	$\frac{e^{x \ln(a)} e^{x \ln(b)}}{\ln(a)+\ln(b)}$	19
meijerg	$-\frac{1-e^{x \ln(b) \left(1+\frac{\ln(a)}{\ln(b)}\right)}}{\ln(b) \left(1+\frac{\ln(a)}{\ln(b)}\right)}$	36

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a^x*b^x,x,method=_RETURNVERBOSE)

[Out] a^x*b^x/(ln(a)+ln(b))

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x*b^x,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(log(b)/log(a)>0)', see 'assume?' for more

Fricas [A]

time = 0.31, size = 14, normalized size = 1.00

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x*b^x,x, algorithm="fricas")

[Out] a^x*b^x/(log(a) + log(b))

Sympy [A]

time = 0.26, size = 31, normalized size = 2.21

$$\begin{cases} \frac{a^x b^x}{\log(a) + \log(b)} & \text{for } a \neq \frac{1}{b} \\ \frac{b^x \left(\frac{1}{b}\right)^x}{\log\left(\frac{1}{b}\right) + \log(b)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a**x*b**x,x)

[Out] Piecewise((a**x*b**x/(log(a) + log(b)), Ne(a, 1/b)), (b**x*(1/b)**x/(log(1/b) + log(b)), True))

Giac [C] Result contains complex when optimal does not.

time = 0.00, size = 263, normalized size = 18.79

$$e^{(\ln|a|+\ln|b|)x} \left(\frac{2(2\ln|a|+2\ln|b|)\cos\left(\left(\frac{1}{2}\pi(1-\operatorname{sgn}(a))+\frac{1}{2}\pi(1-\operatorname{sgn}(b))\right)x\right)}{(2\ln|a|+2\ln|b|)^2+(-\pi\operatorname{sign}(a)-\pi\operatorname{sign}(b)+2\pi)^2} - \frac{2(\pi\operatorname{sign}(a)+\pi\operatorname{sign}(b)-2\pi)\sin\left(\left(\frac{1}{2}\pi(1-\operatorname{sgn}(a))+\frac{1}{2}\pi(1-\operatorname{sgn}(b))\right)x\right)}{(2\ln|a|+2\ln|b|)^2+(-\pi\operatorname{sign}(a)-\pi\operatorname{sign}(b)+2\pi)^2} \right) + \frac{e^{(\ln|a|+\ln|b|)x} \left(\frac{2\pi\left(\frac{1}{2}\pi(1-\operatorname{sgn}(a))+\frac{1}{2}\pi(1-\operatorname{sgn}(b))\right)x}{2\ln|a|+2\ln|b|-\pi\operatorname{sign}(a)-\pi\operatorname{sign}(b)+\pi/2} - \frac{2\pi\left(\frac{1}{2}\pi(1-\operatorname{sgn}(a))+\frac{1}{2}\pi(1-\operatorname{sgn}(b))\right)x}{2\ln|a|+2\ln|b|+\pi\operatorname{sign}(a)+\pi\operatorname{sign}(b)-\pi/2} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x*b^x,x)

[Out] 2*e^(x*(log(abs(a)) + log(abs(b))))*(2*(log(abs(a)) + log(abs(b)))*cos(-1/2*pi*x*sgn(a) - 1/2*pi*x*sgn(b) + pi*x)/((2*pi - pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) + log(abs(b)))^2) + (2*pi - pi*sgn(a) - pi*sgn(b))*sin(-1/2*pi*x*sgn(a) - 1/2*pi*x*sgn(b) + pi*x)/((2*pi - pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) + log(abs(b)))^2)) - e^(x*(log(abs(a)) + log(abs(b))))*(e^(1/2*I*pi*x*sgn(a) + 1/2*I*pi*x*sgn(b) - I*pi*x)/(-2*I*pi + I*pi*sgn(a) + I*pi*sgn(b) + 2*log(abs(a)) + 2*log(abs(b))) - e^(-1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b) + I*pi*x)/(2*I*pi - I*pi*sgn(a) - I*pi*sgn(b) + 2*log(abs(a)) + 2*log(abs(b))))

Mupad [B]

time = 0.17, size = 14, normalized size = 1.00

$$\frac{a^x b^x}{\ln(a) + \ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a^x*b^x,x)`

[Out] `(a^x*b^x)/(log(a) + log(b))`

3.162 $\int \frac{a^x}{x^2} dx$

Optimal. Leaf size=17

$$-\frac{a^x}{x} + \text{Ei}(x \log(a)) \log(a)$$

[Out] `-a^x/x+Ei(x*ln(a))*ln(a)`

Rubi [A]

time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2208, 2209}

$$\log(a) \text{Ei}(x \log(a)) - \frac{a^x}{x}$$

Antiderivative was successfully verified.

[In] `Int[a^x/x^2,x]`

[Out] `-(a^x/x) + ExpIntegralEi[x*Log[a]]*Log[a]`

Rule 2208

`Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1))), x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

Rule 2209

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

Rubi steps

$$\begin{aligned} \int \frac{a^x}{x^2} dx &= -\frac{a^x}{x} + \log(a) \int \frac{a^x}{x} dx \\ &= -\frac{a^x}{x} + \text{Ei}(x \log(a)) \log(a) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 17, normalized size = 1.00

$$-\frac{a^x}{x} + \text{Ei}(x \log(a)) \log(a)$$

Antiderivative was successfully verified.

[In] Integrate[a^x/x^2,x]

[Out] $-(a^x/x) + \text{ExpIntegralEi}[x*\text{Log}[a]]*\text{Log}[a]$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[a^x/x^2,x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.03, size = 21, normalized size = 1.24

method	result
risch	$-\frac{a^x}{x} - \ln(a) \text{expIntegral}(1, -x \ln(a))$
meijerg	$-\ln(a) \left(\frac{1}{x \ln(a)} + 1 - \ln(x) - i\pi - \ln(\ln(a)) - \frac{2+2x \ln(a)}{2x \ln(a)} + \frac{e^x \ln(a)}{x \ln(a)} + \ln(-x \ln(a)) + \text{expIntegral}(1, -x \ln(a)) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a^x/x^2,x,method=_RETURNVERBOSE)

[Out] $-a^x/x - \ln(a)*\text{Ei}(1, -x*\ln(a))$

Maxima [A]

time = 0.31, size = 10, normalized size = 0.59

$$\Gamma(-1, -x \log(a)) \log(a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x/x^2,x, algorithm="maxima")

[Out] $\text{gamma}(-1, -x*\log(a))*\log(a)$

Fricas [A]

time = 0.30, size = 19, normalized size = 1.12

$$\frac{x \text{Ei}(x \log(a)) \log(a) - a^x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x/x^2,x, algorithm="fricas")

[Out] (x*Ei(x*log(a))*log(a) - a^x)/x

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^x}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a**x/x**2,x)

[Out] Integral(a**x/x**2, x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x/x^2,x)

[Out] Could not integrate

Mupad [B]

time = 0.13, size = 19, normalized size = 1.12

$$-\ln(a) \operatorname{expint}(-x \ln(a)) - \frac{a^x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a^x/x^2,x)

[Out] - log(a)*expint(-x*log(a)) - a^x/x

3.163 $\int \frac{a^x x}{(1+bx)^2} dx$

Optimal. Leaf size=64

$$\frac{a^x}{b^2(1+bx)} + \frac{a^{-1/b} \operatorname{Ei}\left(\frac{(1+bx)\log(a)}{b}\right)}{b^2} - \frac{a^{-1/b} \operatorname{Ei}\left(\frac{(1+bx)\log(a)}{b}\right) \log(a)}{b^3}$$

[Out] $a^x/b^2/(b*x+1)+\operatorname{Ei}((b*x+1)*\ln(a)/b)/(a^{(1/b)})/b^2-\operatorname{Ei}((b*x+1)*\ln(a)/b)*\ln(a)/(a^{(1/b)})/b^3$

Rubi [A]

time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2230, 2208, 2209}

$$-\frac{a^{-1/b} \log(a) \operatorname{Ei}\left(\frac{(bx+1)\log(a)}{b}\right)}{b^3} + \frac{a^{-1/b} \operatorname{Ei}\left(\frac{(bx+1)\log(a)}{b}\right)}{b^2} + \frac{a^x}{b^2(bx+1)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a^x*x)/(1+b*x)^2,x]$

[Out] $a^x/(b^2*(1+b*x)) + \operatorname{ExpIntegralEi}(((1+b*x)*\operatorname{Log}[a])/b)/(a^{b^{-1}}*b^2) - (\operatorname{ExpIntegralEi}(((1+b*x)*\operatorname{Log}[a])/b)*\operatorname{Log}[a])/(a^{b^{-1}}*b^3)$

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.)+(f_.)*(x_))))^(n_.)*((c_.)+(d_.)*(x_))^(m_), x_Symbol] := Simp[(c+d*x)^(m+1)*((b*F^(g*(e+f*x)))^n/(d*(m+1))), x] - Dist[f*g*n*(Log[F]/(d*(m+1))), Int[(c+d*x)^(m+1)*(b*F^(g*(e+f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.)+(f_.)*(x_)))/((c_.)+(d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e-c*(f/d)))/d)*ExpIntegralEi[f*g*(c+d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2230

```
Int[(F_)^((c_.)*(v_))* (u_)^(m_.)*(w_), x_Symbol] := Int[ExpandIntegrand[F^(c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && IntegerQ[m] && !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned}
\int \frac{a^x x}{(1+bx)^2} dx &= \int \left(-\frac{a^x}{b(1+bx)^2} + \frac{a^x}{b(1+bx)} \right) dx \\
&= -\frac{\int \frac{a^x}{(1+bx)^2} dx}{b} + \frac{\int \frac{a^x}{1+bx} dx}{b} \\
&= \frac{a^x}{b^2(1+bx)} + \frac{a^{-1/b} \text{Ei} \left(\frac{(1+bx) \log(a)}{b} \right)}{b^2} - \frac{\log(a) \int \frac{a^x}{1+bx} dx}{b^2} \\
&= \frac{a^x}{b^2(1+bx)} + \frac{a^{-1/b} \text{Ei} \left(\frac{(1+bx) \log(a)}{b} \right)}{b^2} - \frac{a^{-1/b} \text{Ei} \left(\frac{(1+bx) \log(a)}{b} \right) \log(a)}{b^3}
\end{aligned}$$

Mathematica [A]

time = 0.13, size = 43, normalized size = 0.67

$$\frac{\frac{a^x b}{1+bx} + a^{-1/b} \text{Ei} \left(\frac{(1+bx) \log(a)}{b} \right) (b - \log(a))}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^x*x)/(1 + b*x)^2,x]
```

```
[Out] ((a^x*b)/(1 + b*x) + (ExpIntegralEi[((1 + b*x)*Log[a])/b]*(b - Log[a]))/a^b^(-1))/b^3
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x*a^x/(1+b*x)^2,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python object
```

Maple [A]

time = 0.04, size = 79, normalized size = 1.23

method	result	size
--------	--------	------

risch	$-\frac{a^{-\frac{1}{b}} \exp\text{Integral}\left(1, -x \ln(a) - \frac{\ln(a)}{b}\right)}{b^2} + \frac{\ln(a)a^x}{b^3 \left(x \ln(a) + \frac{\ln(a)}{b}\right)} + \frac{\ln(a)a^{-\frac{1}{b}} \exp\text{Integral}\left(1, -x \ln(a) - \frac{\ln(a)}{b}\right)}{b^3}$	79
-------	---	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a^x*x/(b*x+1)^2,x,method=_RETURNVERBOSE)`

[Out] $-1/b^2*a^{(-1/b)}*Ei(1, -x*\ln(a)-\ln(a)/b)+\ln(a)/b^3*a^x/(x*\ln(a)+\ln(a)/b)+\ln(a)/b^3*a^{(-1/b)}*Ei(1, -x*\ln(a)-\ln(a)/b)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a^x*x/(b*x+1)^2,x, algorithm="maxima")`

[Out] $a^x*x/(b^2*x^2*\log(a) + 2*b*x*\log(a) + \log(a)) + \text{integrate}((b*x - 1)*a^x/(b^3*x^3*\log(a) + 3*b^2*x^2*\log(a) + 3*b*x*\log(a) + \log(a)), x)$

Fricas [A]

time = 0.31, size = 54, normalized size = 0.84

$$\frac{a^x b + \frac{(b^2 x - (b x + 1) \log(a) + b) Ei\left(\frac{(b x + 1) \log(a)}{b}\right)}{a^{\left(\frac{1}{b}\right)}}}{b^4 x + b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a^x*x/(b*x+1)^2,x, algorithm="fricas")`

[Out] $(a^x*b + (b^2*x - (b*x + 1)*\log(a) + b)*Ei((b*x + 1)*\log(a)/b)/a^{(1/b)})/(b^4*x + b^3)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^x x}{(b x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a**x*x/(b*x+1)**2,x)`

[Out] `Integral(a**x*x/(b*x + 1)**2, x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a^x*x/(b*x+1)^2,x)`

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a^x x}{(bx + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^x*x)/(b*x + 1)^2,x)`

[Out] `int((a^x*x)/(b*x + 1)^2, x)`

$$3.164 \quad \int \frac{e^{ax} x}{(1+ax)^2} dx$$

Optimal. Leaf size=16

$$\frac{e^{ax}}{a^2(1+ax)}$$

[Out] exp(a*x)/a^2/(a*x+1)

Rubi [A]

time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2228}

$$\frac{e^{ax}}{a^2(ax+1)}$$

Antiderivative was successfully verified.

[In] Int[(E^(a*x)*x)/(1 + a*x)^2,x]

[Out] E^(a*x)/(a^2*(1 + a*x))

Rule 2228

Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] := With[{b = Coefficient[v, x, 1], d = Coefficient[u, x, 0], e = Coefficient[u, x, 1], f = Coefficient[w, x, 0], g = Coefficient[w, x, 1]}, Simp[g*u^(m + 1)*(F^(c*v)/(b*c*e*Log[F])), x] /; EqQ[e*g*(m + 1) - b*c*(e*f - d*g)*Log[F], 0]] /; FreeQ[{F, c, m}, x] && LinearQ[{u, v, w}, x]

Rubi steps

$$\int \frac{e^{ax} x}{(1+ax)^2} dx = \frac{e^{ax}}{a^2(1+ax)}$$

Mathematica [A]

time = 0.05, size = 16, normalized size = 1.00

$$\frac{e^{ax}}{a^2(1+ax)}$$

Antiderivative was successfully verified.

[In] Integrate[(E^(a*x)*x)/(1 + a*x)^2,x]

[Out] E^(a*x)/(a^2*(1 + a*x))

Mathics [A]

time = 1.90, size = 16, normalized size = 1.00

$$\frac{E^{ax}}{a^2(1+ax)}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x*E^(a*x)/(1+a*x)^2,x]')`[Out] `E^(a x) / (a ^ 2 (1 + a x))`**Maple [A]**

time = 0.03, size = 16, normalized size = 1.00

method	result	size
gospers	$\frac{e^{ax}}{a^2(ax+1)}$	16
derivativdivides	$\frac{e^{ax}}{a^2(ax+1)}$	16
default	$\frac{e^{ax}}{a^2(ax+1)}$	16
norman	$\frac{e^{ax}}{a^2(ax+1)}$	16
risch	$\frac{e^{ax}}{a^2(ax+1)}$	16

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(a*x)*x/(a*x+1)^2,x,method=_RETURNVERBOSE)`[Out] `exp(a*x)/a^2/(a*x+1)`**Maxima [A]**

time = 0.31, size = 16, normalized size = 1.00

$$\frac{e^{(ax)}}{a^3x + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(a*x)*x/(a*x+1)^2,x, algorithm="maxima")`[Out] `e^(a*x)/(a^3*x + a^2)`**Fricas [A]**

time = 0.30, size = 16, normalized size = 1.00

$$\frac{e^{(ax)}}{a^3x + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(a*x)*x/(a*x+1)^2,x, algorithm="fricas")

[Out] e^(a*x)/(a^3*x + a^2)

Sympy [A]

time = 0.06, size = 12, normalized size = 0.75

$$\frac{e^{ax}}{a^3x + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(a*x)*x/(a*x+1)**2,x)

[Out] exp(a*x)/(a**3*x + a**2)

Giac [A]

time = 0.00, size = 15, normalized size = 0.94

$$\frac{e^{ax}}{a^3x + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(a*x)*x/(a*x+1)^2,x)

[Out] e^(a*x)/(a^3*x + a^2)

Mupad [B]

time = 0.20, size = 15, normalized size = 0.94

$$\frac{e^{ax}}{a^2 (ax + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*exp(a*x))/(a*x + 1)^2,x)

[Out] exp(a*x)/(a^2*(a*x + 1))

3.165 $\int k^{x^2} x dx$

Optimal. Leaf size=13

$$\frac{k^{x^2}}{2 \log(k)}$$

[Out] $1/2*k^{(x^2)}/\ln(k)$

Rubi [A]

time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2240}

$$\frac{k^{x^2}}{2 \log(k)}$$

Antiderivative was successfully verified.

[In] Int[k^x^2*x,x]

[Out] $k^x^2/(2*\text{Log}[k])$

Rule 2240

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n *Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]

Rubi steps

$$\int k^{x^2} x dx = \frac{k^{x^2}}{2 \log(k)}$$

Mathematica [A]

time = 0.01, size = 13, normalized size = 1.00

$$\frac{k^{x^2}}{2 \log(k)}$$

Antiderivative was successfully verified.

[In] Integrate[k^x^2*x,x]

[Out] $k^x^2/(2*\text{Log}[k])$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.83, size = 23, normalized size = 1.77

$$\text{Piecewise} \left[\left\{ \left\{ \frac{k^{x^2}}{2 \text{Log}[k]}, \text{Log}[k] \neq 0 \right\} \right\}, \frac{x^2}{2} \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[x*k^(x^2),x]')`

[Out] `Piecewise[{{k ^ x ^ 2 / (2 Log[k]), Log[k] != 0}}, x ^ 2 / 2]`

Maple [A]

time = 0.02, size = 12, normalized size = 0.92

method	result	size
gospers	$\frac{k^{x^2}}{2 \ln(k)}$	12
derivativdivides	$\frac{k^{x^2}}{2 \ln(k)}$	12
default	$\frac{k^{x^2}}{2 \ln(k)}$	12
risch	$\frac{k^{x^2}}{2 \ln(k)}$	12
norman	$\frac{e^{\ln(k)x^2}}{2 \ln(k)}$	14
meijerg	$-\frac{1-e^{\ln(k)x^2}}{2 \ln(k)}$	18

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(k^(x^2)*x,x,method=_RETURNVERBOSE)`

[Out] `1/2*k^(x^2)/ln(k)`

Maxima [A]

time = 0.26, size = 11, normalized size = 0.85

$$\frac{k^{(x^2)}}{2 \log(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(k^(x^2)*x,x, algorithm="maxima")`

[Out] `1/2*k^(x^2)/log(k)`

Fricas [A]

time = 0.30, size = 11, normalized size = 0.85

$$\frac{k^{(x^2)}}{2 \log(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(k^(x^2)*x,x, algorithm="fricas")``[Out] 1/2*k^(x^2)/log(k)`**Sympy [A]**

time = 0.05, size = 15, normalized size = 1.15

$$\begin{cases} \frac{k^{x^2}}{2 \log(k)} & \text{for } \log(k) \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(k**(x**2)*x,x)``[Out] Piecewise((k**(x**2)/(2*log(k)), Ne(log(k), 0)), (x**2/2, True))`**Giac [A]**

time = 0.00, size = 11, normalized size = 0.85

$$\frac{k^{x^2}}{2 \ln k}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(k^(x^2)*x,x)``[Out] 1/2*k^(x^2)/log(k)`**Mupad [B]**

time = 0.03, size = 11, normalized size = 0.85

$$\frac{k^{x^2}}{2 \ln(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(k^(x^2)*x,x)``[Out] k^(x^2)/(2*log(k))`

3.166 $\int e^{x^2} dx$

Optimal. Leaf size=11

$$\frac{1}{2}\sqrt{\pi} \operatorname{erfi}(x)$$

[Out] 1/2*erfi(x)*Pi^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2235}

$$\frac{1}{2}\sqrt{\pi} \operatorname{erfi}(x)$$

Antiderivative was successfully verified.

[In] Int[E^x^2,x]

[Out] (Sqrt[Pi]*Erfi[x])/2

Rule 2235

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)) ^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rubi steps

$$\int e^{x^2} dx = \frac{1}{2}\sqrt{\pi} \operatorname{erfi}(x)$$

Mathematica [A]

time = 0.01, size = 11, normalized size = 1.00

$$\frac{1}{2}\sqrt{\pi} \operatorname{erfi}(x)$$

Antiderivative was successfully verified.

[In] Integrate[E^x^2,x]

[Out] (Sqrt[Pi]*Erfi[x])/2

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

time = 1.88, size = 7, normalized size = 0.64

$$\frac{\sqrt{\text{Pi}} \operatorname{erfi}[x]}{2}$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[E^(x^2),x]')`

[Out] `Sqrt[Pi] erfi[x] / 2`

Maple [A]

time = 0.02, size = 8, normalized size = 0.73

method	result	size
default	$\frac{\operatorname{erfi}(x)\sqrt{\pi}}{2}$	8
meijerg	$\frac{\operatorname{erfi}(x)\sqrt{\pi}}{2}$	8
risch	$\frac{\operatorname{erfi}(x)\sqrt{\pi}}{2}$	8

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x^2),x,method=_RETURNVERBOSE)`

[Out] `1/2*erfi(x)*Pi^(1/2)`

Maxima [C] Result contains complex when optimal does not.

time = 0.27, size = 9, normalized size = 0.82

$$-\frac{1}{2}i\sqrt{\pi}\operatorname{erf}(ix)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2),x, algorithm="maxima")`

[Out] `-1/2*I*sqrt(pi)*erf(I*x)`

Fricas [A]

time = 0.31, size = 7, normalized size = 0.64

$$\frac{1}{2}\sqrt{\pi}\operatorname{erfi}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2),x, algorithm="fricas")`

[Out] $1/2*\sqrt{\pi}*\operatorname{erfi}(x)$

Sympy [A]

time = 0.08, size = 8, normalized size = 0.73

$$\frac{\sqrt{\pi} \operatorname{erfi}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x**2),x)`

[Out] $\sqrt{\pi}*\operatorname{erfi}(x)/2$

Giac [C] Result contains complex when optimal does not.

time = 0.00, size = 11, normalized size = 1.00

$$-\frac{\sqrt{\pi} \operatorname{erf}(-ix)}{i \cdot 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2),x)`

[Out] $1/2*I*\sqrt{\pi}*\operatorname{erf}(-I*x)$

Mupad [B]

time = 0.02, size = 7, normalized size = 0.64

$$\frac{\sqrt{\pi} \operatorname{erfi}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x^2),x)`

[Out] $(\pi^{1/2}*\operatorname{erfi}(x))/2$

3.167 $\int e^{x^2} x dx$

Optimal. Leaf size=9

$$\frac{e^{x^2}}{2}$$

[Out] 1/2*exp(x^2)

Rubi [A]

time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2240}

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Int[E^x^2*x,x]

[Out] E^x^2/2

Rule 2240

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n *Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]

Rubi steps

$$\int e^{x^2} x dx = \frac{e^{x^2}}{2}$$

Mathematica [A]

time = 0.01, size = 9, normalized size = 1.00

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Integrate[E^x^2*x,x]

[Out] E^x^2/2

Mathics [A]

time = 1.69, size = 7, normalized size = 0.78

$$\frac{E^{x^2}}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x*E^(x^2),x]')``[Out] E ^ x ^ 2 / 2`**Maple [A]**

time = 0.01, size = 7, normalized size = 0.78

method	result	size
gospers	$\frac{e^{x^2}}{2}$	7
derivativedivides	$\frac{e^{x^2}}{2}$	7
default	$\frac{e^{x^2}}{2}$	7
norman	$\frac{e^{x^2}}{2}$	7
risch	$\frac{e^{x^2}}{2}$	7
meijerg	$-\frac{1}{2} + \frac{e^{x^2}}{2}$	9

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(exp(x^2)*x,x,method=_RETURNVERBOSE)``[Out] 1/2*exp(x^2)`**Maxima [A]**

time = 0.27, size = 6, normalized size = 0.67

$$\frac{1}{2} e^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x^2)*x,x, algorithm="maxima")``[Out] 1/2*e^(x^2)`**Fricas [A]**

time = 0.32, size = 6, normalized size = 0.67

$$\frac{1}{2} e^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2)*x,x, algorithm="fricas")`

[Out] `1/2*e^(x^2)`

Sympy [A]

time = 0.04, size = 5, normalized size = 0.56

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x**2)*x,x)`

[Out] `exp(x**2)/2`

Giac [A]

time = 0.00, size = 7, normalized size = 0.78

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2)*x,x)`

[Out] `1/2*e^(x^2)`

Mupad [B]

time = 0.02, size = 6, normalized size = 0.67

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*exp(x^2),x)`

[Out] `exp(x^2)/2`

3.168

$$\int \frac{e^{\frac{1}{x}}(1+x)}{x^4} dx$$

Optimal. Leaf size=27

$$-e^{\frac{1}{x}} - \frac{e^{\frac{1}{x}}}{x^2} + \frac{e^{\frac{1}{x}}}{x}$$

[Out] $-\exp(1/x) - \exp(1/x)/x^2 + \exp(1/x)/x$

Rubi [A]

time = 0.07, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6874, 2243, 2240}

$$-\frac{e^{\frac{1}{x}}}{x^2} - e^{\frac{1}{x}} + \frac{e^{\frac{1}{x}}}{x}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(E^x^{-1}*(1 + x))/x^4, x]$

[Out] $-E^x^{-1} - E^x^{-1}/x^2 + E^x^{-1}/x$

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n *Log[F])), x]
/; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*Log[F])), x]
- Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b*(c + d*x)^n), x], x]
/; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*(m + 1)/n] && LtQ[0, (m + 1)/n, 5]
&& IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])
```

Rule 6874

```
Int[u_, x_Symbol]
:> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{1}{x}}(1+x)}{x^4} dx &= \int \left(\frac{e^{\frac{1}{x}}}{x^4} + \frac{e^{\frac{1}{x}}}{x^3} \right) dx \\
&= \int \frac{e^{\frac{1}{x}}}{x^4} dx + \int \frac{e^{\frac{1}{x}}}{x^3} dx \\
&= -\frac{e^{\frac{1}{x}}}{x^2} - \frac{e^{\frac{1}{x}}}{x} - 2 \int \frac{e^{\frac{1}{x}}}{x^3} dx - \int \frac{e^{\frac{1}{x}}}{x^2} dx \\
&= e^{\frac{1}{x}} - \frac{e^{\frac{1}{x}}}{x^2} + \frac{e^{\frac{1}{x}}}{x} + 2 \int \frac{e^{\frac{1}{x}}}{x^2} dx \\
&= -e^{\frac{1}{x}} - \frac{e^{\frac{1}{x}}}{x^2} + \frac{e^{\frac{1}{x}}}{x}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 17, normalized size = 0.63

$$\frac{e^{\frac{1}{x}}(-1+x-x^2)}{x^2}$$

Antiderivative was successfully verified.

`[In] Integrate[(E^x^(-1)*(1 + x))/x^4,x]``[Out] (E^x^(-1)*(-1 + x - x^2))/x^2`**Mathics [A]**

time = 1.80, size = 17, normalized size = 0.63

$$\frac{(-1+x-x^2)E^{\frac{1}{x}}}{x^2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(x+1)*E^(1/x)/x^4,x]')``[Out] (-1 + x - x ^ 2) E ^ (1 / x) / x ^ 2`**Maple [A]**

time = 0.02, size = 25, normalized size = 0.93

method	result	size
gosper	$-\frac{(x^2-x+1)e^{\frac{1}{x}}}{x^2}$	18
risch	$-\frac{(x^2-x+1)e^{\frac{1}{x}}}{x^2}$	18

derivativedivides	$-e^{\frac{1}{x}} - \frac{e^{\frac{1}{x}}}{x^2} + \frac{e^{\frac{1}{x}}}{x}$	25
default	$-e^{\frac{1}{x}} - \frac{e^{\frac{1}{x}}}{x^2} + \frac{e^{\frac{1}{x}}}{x}$	25
norman	$\frac{x^2 e^{\frac{1}{x}} - x e^{\frac{1}{x}} - x^3 e^{\frac{1}{x}}}{x^3}$	30
meijerg	$1 - \frac{\left(\frac{3}{x^2} - \frac{6}{x} + 6\right) e^{\frac{1}{x}}}{3} + \frac{\left(2 - \frac{2}{x}\right) e^{\frac{1}{x}}}{2}$	34

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(1/x)*(1+x)/x^4,x,method=_RETURNVERBOSE)`

[Out] `-exp(1/x)-exp(1/x)/x^2+exp(1/x)/x`

Maxima [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.30, size = 17, normalized size = 0.63

$$-\Gamma\left(3, -\frac{1}{x}\right) + \Gamma\left(2, -\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(1/x)*(1+x)/x^4,x, algorithm="maxima")`

[Out] `-gamma(3, -1/x) + gamma(2, -1/x)`

Fricas [A]

time = 0.30, size = 17, normalized size = 0.63

$$-\frac{(x^2 - x + 1)e^{\frac{1}{x}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(1/x)*(1+x)/x^4,x, algorithm="fricas")`

[Out] `-(x^2 - x + 1)*e^(1/x)/x^2`

Sympy [A]

time = 0.05, size = 14, normalized size = 0.52

$$\frac{(-x^2 + x - 1)e^{\frac{1}{x}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(1/x)*(1+x)/x**4,x)`

[Out] `(-x**2 + x - 1)*exp(1/x)/x**2`

Giac [A]

time = 0.00, size = 20, normalized size = 0.74

$$-\left(\frac{1}{x}\right)^2 e^{\frac{1}{x}} + \frac{e^{\frac{1}{x}}}{x} - e^{\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1/x)*(1+x)/x^4,x)**[Out]** -e^(1/x) + e^(1/x)/x - e^(1/x)/x^2**Mupad [B]**

time = 0.16, size = 17, normalized size = 0.63

$$-\frac{e^{1/x} (x^2 - x + 1)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(1/x)*(x + 1))/x^4,x)**[Out]** -(exp(1/x)*(x^2 - x + 1))/x^2

$$3.169 \quad \int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2} dx$$

Optimal. Leaf size=25

$$-\frac{e^{1-e^{x^2}x}}{-1+e^{x^2}x}$$

[Out] $-\exp(1-\exp(x^2)*x)/(-1+\exp(x^2)*x)$

Rubi [F]

time = 0.66, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2} dx$$

Verification is not applicable to the result.

[In] $\text{Int}[(E^{(1 - E^{x^2}x + 2x^2)}*(x + 2x^3))/(1 - E^{x^2}x)^2, x]$

[Out] $\text{Defer}[\text{Int}[(E^{(1 - E^{x^2}x + 2x^2)}*x)/(-1 + E^{x^2}x)^2, x] + 2*\text{Defer}[\text{Int}[(E^{(1 - E^{x^2}x + 2x^2)}*x^3)/(-1 + E^{x^2}x)^2, x]$

Rubi steps

$$\begin{aligned} \int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2} dx &= \int \frac{e^{1-e^{x^2}x+2x^2}x(1+2x^2)}{(1-e^{x^2}x)^2} dx \\ &= \int \left(\frac{e^{1-e^{x^2}x+2x^2}x}{(-1+e^{x^2}x)^2} + \frac{2e^{1-e^{x^2}x+2x^2}x^3}{(-1+e^{x^2}x)^2} \right) dx \\ &= 2 \int \frac{e^{1-e^{x^2}x+2x^2}x^3}{(-1+e^{x^2}x)^2} dx + \int \frac{e^{1-e^{x^2}x+2x^2}x}{(-1+e^{x^2}x)^2} dx \end{aligned}$$

Mathematica [A]

time = 0.11, size = 25, normalized size = 1.00

$$-\frac{e^{1-e^{x^2}x}}{-1+e^{x^2}x}$$

Antiderivative was successfully verified.

[In] Integrate[(E^(1 - E^x^2*x + 2*x^2))*(x + 2*x^3))/(1 - E^x^2*x)^2,x]

[Out] -(E^(1 - E^x^2*x)/(-1 + E^x^2*x))

Mathics [A]

time = 2.21, size = 25, normalized size = 1.00

$$-\frac{E^{1-xE^{x^2}}}{-1 + xE^{x^2}}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(2*x^3+x)*(E^(x^2))^2*E^(1-x*E^(x^2)))/(1-x*E^(x^2))^2,x
,')

[Out] -E^(1 - x E^x^2) / (-1 + x E^x^2)

Maple [A]

time = 0.05, size = 23, normalized size = 0.92

method	result	size
risch	$-\frac{e^{1-e^{x^2}}x}{-1+e^{x^2}}$	23

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x,method=_RETURNVERB
OSE)

[Out] -exp(1-exp(x^2)*x)/(-1+exp(x^2)*x)

Maxima [A]

time = 0.32, size = 22, normalized size = 0.88

$$-\frac{e^{(-xe^{(x^2)}+1)}}{xe^{(x^2)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x, algorithm="maxima")

[Out] -e^(-x*e^(x^2) + 1)/(x*e^(x^2) - 1)

Fricas [A]

time = 0.31, size = 36, normalized size = 1.44

$$-\frac{e^{(2x^2-xe^{(x^2)}+1)}}{xe^{(3x^2)}-e^{(2x^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x, algorithm="fricas")

[Out] $-e^{(2x^2 - xe^{x^2}) + 1} / (xe^{(3x^2)} - e^{(2x^2)})$

Sympy [A]

time = 0.12, size = 31, normalized size = 1.24

$$-\frac{e^{2x^2 - xe^{x^2} + 1}}{xe^{3x^2} - e^{2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1-exp(x**2)*x+2*x**2)*(2*x**3+x)/(1-exp(x**2)*x)**2,x)

[Out] $-\exp(2x^{**2} - x*\exp(x^{**2}) + 1) / (x*\exp(3x^{**2}) - \exp(2x^{**2}))$

Giac [A]

time = 0.01, size = 35, normalized size = 1.40

$$-\frac{ee^{2x^2 - xe^{x^2}}}{x(e^{x^2})^3 - (e^{x^2})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x)

[Out] $-e^{(-e^{(x^2)}*x + 2*x^2 + 1)} / (e^{(3*x^2)}*x - e^{(2*x^2)})$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{e^{2x^2 - xe^{x^2} + 1} (2x^3 + x)}{(xe^{x^2} - 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(2*x^2 - x*exp(x^2) + 1)*(x + 2*x^3))/(x*exp(x^2) - 1)^2,x)

[Out] int((exp(2*x^2 - x*exp(x^2) + 1)*(x + 2*x^3))/(x*exp(x^2) - 1)^2, x)

$$3.170 \quad \int e^{e^{e^{e^x}}} dx$$

Optimal. Leaf size=12

$$\text{Int} \left(e^{e^{e^{e^x}}}, x \right)$$

[Out] CannotIntegrate(exp(exp(exp(exp(x))))), x)

Rubi [A]

time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$,

Rules used = {}

$$\int e^{e^{e^{e^x}}} dx$$

Verification is not applicable to the result.

[In] Int[E^E^E^E^x,x]

[Out] Defer[Subst][Defer[Int][E^E^E^x/x, x], x, E^x]

Rubi steps

$$\int e^{e^{e^{e^x}}} dx = \text{Subst} \left(\int \frac{e^{e^{e^x}}}{x} dx, x, e^x \right)$$

Mathematica [A]

time = 0.02, size = 0, normalized size = 0.00

$$\int e^{e^{e^{e^x}}} dx$$

Verification is not applicable to the result.

[In] Integrate[E^E^E^E^x,x]

[Out] Integrate[E^E^E^E^x, x]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in ___instancecheck___

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[E^(E^(E^(E^x))),x]')`

[Out] `cought exception: maximum recursion depth exceeded in __instancecheck__`

Maple [A]

time = 0.01, size = 0, normalized size = 0.00

$$\int e^{e^{e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(exp(exp(exp(x))))),x)`

[Out] `int(exp(exp(exp(exp(x))))),x)`

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(exp(exp(exp(x))))),x, algorithm="maxima")`

[Out] `integrate(e^(e^(e^(e^x))), x)`

Fricas [A]

time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(exp(exp(exp(x))))),x, algorithm="fricas")`

[Out] `integral(e^(e^(e^(e^x))), x)`

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int e^{e^{e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(exp(exp(exp(x))))),x)`

[Out] `Integral(exp(exp(exp(exp(x))))), x)`

Giac [A] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(exp(exp(x))))),x)
```

```
[Out] Could not integrate
```

Mupad [A]

```
time = 0.00, size = -1, normalized size = -0.08
```

$$\int e^{e^{e^{e^x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(exp(exp(exp(x))))),x)
```

```
[Out] int(exp(exp(exp(exp(x))))), x)
```

3.171 $\int e^x \log(x) dx$

Optimal. Leaf size=11

$$-\text{Ei}(x) + e^x \log(x)$$

[Out] $-\text{Ei}(x) + \exp(x) * \ln(x)$

Rubi [A]

time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2225, 2634, 2209}

$$e^x \log(x) - \text{Ei}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^x * \text{Log}[x], x]$

[Out] $-\text{ExpIntegralEi}[x] + E^x * \text{Log}[x]$

Rule 2209

$\text{Int}[(F_)^\wedge((g_.) * ((e_.) + (f_.) * (x_))) / ((c_.) + (d_.) * (x_)), x_Symbol] \text{ :> Simp}[(F^\wedge(g * (e - c * (f/d))) / d) * \text{ExpIntegralEi}[f * g * (c + d * x) * (\text{Log}[F] / d)], x] \text{ /; FreeQ}\{F, c, d, e, f, g\}, x] \&\amp; \text{!TrueQ}\{\$UseGamma\}$

Rule 2225

$\text{Int}[(F_)^\wedge((c_.) * ((a_.) + (b_.) * (x_)))^\wedge(n_), x_Symbol] \text{ :> Simp}[(F^\wedge(c * (a + b * x)))^\wedge n / (b * c * n * \text{Log}[F]), x] \text{ /; FreeQ}\{F, a, b, c, n\}, x]$

Rule 2634

$\text{Int}[\text{Log}[u] * (v_), x_Symbol] \text{ :> With}\{w = \text{IntHide}[v, x]\}, \text{Dist}[\text{Log}[u], w, x] - \text{Int}[\text{SimplifyIntegrand}[w * (D[u, x] / u), x], x] \text{ /; InverseFunctionFreeQ}[w, x] \text{ /; InverseFunctionFreeQ}[u, x]$

Rubi steps

$$\begin{aligned} \int e^x \log(x) dx &= e^x \log(x) - \int \frac{e^x}{x} dx \\ &= -\text{Ei}(x) + e^x \log(x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 11, normalized size = 1.00

$$-\text{Ei}(x) + e^x \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[E^x*Log[x],x]

[Out] -ExpIntegralEi[x] + E^x*Log[x]

Mathics [A]

time = 2.64, size = 11, normalized size = 1.00

$$-\text{ExpIntegralEi}[x] + \text{Log}[x] E^x$$

Antiderivative was successfully verified.

[In] mathics('Integrate[E^x*Log[x],x]')

[Out] -ExpIntegralEi[x] + Log[x] E ^ x

Maple [A]

time = 0.02, size = 12, normalized size = 1.09

method	result	size
risch	$e^x \ln(x) + \text{expIntegral}(1, -x)$	12

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)*ln(x),x,method=_RETURNVERBOSE)

[Out] exp(x)*ln(x)+Ei(1,-x)

Maxima [A]

time = 0.28, size = 10, normalized size = 0.91

$$e^x \log(x) - \text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*log(x),x, algorithm="maxima")

[Out] e^x*log(x) - Ei(x)

Fricas [A]

time = 0.31, size = 10, normalized size = 0.91

$$e^x \log(x) - \text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*log(x),x, algorithm="fricas")

[Out] e^x*log(x) - Ei(x)

Sympy [A]

time = 0.97, size = 8, normalized size = 0.73

$$e^x \log(x) - \text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*ln(x),x)

[Out] exp(x)*log(x) - Ei(x)

Giac [A]

time = 0.00, size = 9, normalized size = 0.82

$$e^x \ln x - \text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*log(x),x)

[Out] e^x*log(x) - Ei(x)

Mupad [B]

time = 0.03, size = 10, normalized size = 0.91

$$e^x \ln(x) - \text{ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)*log(x),x)

[Out] exp(x)*log(x) - ei(x)

3.172 $\int e^x x \log(x) dx$

Optimal. Leaf size=22

$$-e^x + \text{Ei}(x) - e^x \log(x) + e^x x \log(x)$$

[Out] $-\exp(x)+\text{Ei}(x)-\exp(x)*\ln(x)+\exp(x)*x*\ln(x)$

Rubi [A]

time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {2207, 2225, 2634, 2230, 2209}

$$\text{Ei}(x) - e^x - e^x \log(x) + e^x x \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^x*x*\text{Log}[x], x]$

[Out] $-E^x + \text{ExpIntegralEi}[x] - E^x*\text{Log}[x] + E^x*x*\text{Log}[x]$

Rule 2207

$\text{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))}^{(n_*)}*((c_*) + (d_*)*(x_*))^{(m_*)}, x_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*((b*F^{(g*(e + f*x)))^n/(f*g*n*\text{Log}[F])), x] - \text{Dist}[d*(m/(f*g*n*\text{Log}[F])), \text{Int}[(c + d*x)^{(m-1)}*(b*F^{(g*(e + f*x)))^n}, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0] \&\& \text{IntegerQ}[2*m] \&\& !\text{TrueQ}\{\$UseGamma\}$

Rule 2209

$\text{Int}[(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))}/((c_*) + (d_*)*(x_*)), x_Symbol] \rightarrow \text{Simp}[(F^{(g*(e - c*(f/d))})/d)*\text{ExpIntegralEi}[f*g*(c + d*x)*(Log[F]/d)], x] /; \text{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\text{TrueQ}\{\$UseGamma\}$

Rule 2225

$\text{Int}[(F_*)^{((c_*)*((a_*) + (b_*)*(x_*)))}^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x)))^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

Rule 2230

$\text{Int}[(F_*)^{((c_*)*(v_*))}*(u_*)^{(m_*)}*(w_*), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[F^{(c*\text{ExpandToSum}[v, x])}, w*\text{NormalizePowerOfLinear}[u, x]^m, x], x] /; \text{FreeQ}\{F, c\}, x] \&\& \text{PolynomialQ}[w, x] \&\& \text{LinearQ}[v, x] \&\& \text{PowerOfLinearQ}[u, x] \&\& \text{IntegerQ}[m] \&\& !\text{TrueQ}\{\$UseGamma\}$

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] := With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
- Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x]
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned} \int e^x x \log(x) dx &= -e^x \log(x) + e^x x \log(x) - \int \frac{e^x(-1+x)}{x} dx \\ &= -e^x \log(x) + e^x x \log(x) - \int \left(e^x - \frac{e^x}{x} \right) dx \\ &= -e^x \log(x) + e^x x \log(x) - \int e^x dx + \int \frac{e^x}{x} dx \\ &= -e^x + \text{Ei}(x) - e^x \log(x) + e^x x \log(x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 17, normalized size = 0.77

$$-e^x + \text{Ei}(x) + e^x(-1+x) \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x*Log[x],x]
```

```
[Out] -E^x + ExpIntegralEi[x] + E^x*(-1+x)*Log[x]
```

Mathics [A]

time = 3.42, size = 17, normalized size = 0.77

$$\text{ExpIntegralEi}[x] - E^x + \text{Log}[x](-1+x) E^x$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x*E^x*Log[x],x]')
```

```
[Out] ExpIntegralEi[x] - E^x + Log[x](-1+x) E^x
```

Maple [A]

time = 0.01, size = 21, normalized size = 0.95

method	result	size
risch	$(-1+x)e^x \ln(x) - \text{expIntegral}(1, -x) - e^x$	21

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)*x*ln(x),x,method=_RETURNVERBOSE)`

[Out] $(-1+x)*\exp(x)*\ln(x)-\text{Ei}(1,-x)-\exp(x)$

Maxima [A]

time = 0.28, size = 15, normalized size = 0.68

$$(x-1)e^x \log(x) + \text{Ei}(x) - e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x*log(x),x, algorithm="maxima")`

[Out] $(x-1)*e^x*\log(x) + \text{Ei}(x) - e^x$

Fricas [A]

time = 0.30, size = 15, normalized size = 0.68

$$(x-1)e^x \log(x) + \text{Ei}(x) - e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x*log(x),x, algorithm="fricas")`

[Out] $(x-1)*e^x*\log(x) + \text{Ei}(x) - e^x$

Sympy [A]

time = 1.65, size = 17, normalized size = 0.77

$$(xe^x - e^x) \log(x) - e^x + \text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x*ln(x),x)`

[Out] $(x*\exp(x) - \exp(x))*\log(x) - \exp(x) + \text{Ei}(x)$

Giac [A]

time = 0.00, size = 14, normalized size = 0.64

$$\text{Ei}(x) - e^x + (x-1)e^x \ln x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x*log(x),x)`

[Out] $e^x*(x-1)*\log(x) - e^x + \text{Ei}(x)$

Mupad [B]

time = 0.22, size = 28, normalized size = 1.27

$$\text{ei}(x) - \frac{x e^x + x e^x \ln(x) - x^2 e^x \ln(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*exp(x)*log(x),x)`

[Out] $\text{ei}(x) - (x*\exp(x) + x*\exp(x)*\log(x) - x^2*\exp(x)*\log(x))/x$

3.173 $\int e^{2x} \log(e^x) dx$

Optimal. Leaf size=23

$$-\frac{e^{2x}}{4} + \frac{1}{2}e^{2x} \log(e^x)$$

[Out] -1/4*exp(2*x)+1/2*exp(2*x)*ln(exp(x))

Rubi [A]

time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2225, 2634, 12}

$$\frac{1}{2}e^{2x} \log(e^x) - \frac{e^{2x}}{4}$$

Antiderivative was successfully verified.

[In] Int[E^(2*x)*Log[E^x],x]

[Out] -1/4*E^(2*x) + (E^(2*x)*Log[E^x])/2

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2225

Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 2634

Int[Log[u]*(v_), x_Symbol] := With[{w = IntHide[v, x]}, Dist[Log[u], w, x] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x]] /; InverseFunctionFreeQ[u, x]

Rubi steps

$$\begin{aligned} \int e^{2x} \log(e^x) dx &= \frac{1}{2}e^{2x} \log(e^x) - \int \frac{e^{2x}}{2} dx \\ &= \frac{1}{2}e^{2x} \log(e^x) - \frac{1}{2} \int e^{2x} dx \\ &= -\frac{e^{2x}}{4} + \frac{1}{2}e^{2x} \log(e^x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 17, normalized size = 0.74

$$\frac{1}{4}e^{2x}(-1 + 2\log(e^x))$$

Antiderivative was successfully verified.

[In] Integrate[E^(2*x)*Log[E^x],x]

[Out] (E^(2*x)*(-1 + 2*Log[E^x]))/4

Mathics [A]

time = 1.69, size = 12, normalized size = 0.52

$$\frac{(-1 + 2x) E^{2x}}{4}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[E^(2*x)*Log[E^x],x]')

[Out] (-1 + 2 x) E ^ (2 x) / 4

Maple [A]

time = 0.01, size = 28, normalized size = 1.22

method	result	size
norman	$-\frac{e^{2x}}{4} + \frac{e^{2x} \ln(e^x)}{2}$	17
risch	$-\frac{e^{2x}}{4} + \frac{e^{2x} \ln(e^x)}{2}$	17
default	$\frac{e^{2x} x}{2} - \frac{e^{2x}}{4} + \frac{e^{2x} (\ln(e^x) - x)}{2}$	28

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(2*x)*ln(exp(x)),x,method=_RETURNVERBOSE)

[Out] 1/2*exp(2*x)*x-1/4*exp(2*x)+1/2*exp(2*x)*(ln(exp(x))-x)

Maxima [A]

time = 0.25, size = 11, normalized size = 0.48

$$\frac{1}{4}(2x - 1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)*log(exp(x)),x, algorithm="maxima")

[Out] 1/4*(2*x - 1)*e^(2*x)

Fricas [A]

time = 0.31, size = 11, normalized size = 0.48

$$\frac{1}{4} (2x - 1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)*log(exp(x)),x, algorithm="fricas")

[Out] 1/4*(2*x - 1)*e^(2*x)

Sympy [A]

time = 0.05, size = 10, normalized size = 0.43

$$\frac{(2x - 1)e^{2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)*ln(exp(x)),x)

[Out] (2*x - 1)*exp(2*x)/4

Giac [A]

time = 0.00, size = 12, normalized size = 0.52

$$\frac{1}{4} (2x - 1)e^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)*log(exp(x)),x)

[Out] 1/4*e^(2*x)*(2*x - 1)

Mupad [B]

time = 0.04, size = 11, normalized size = 0.48

$$\frac{e^{2x} (2x - 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(exp(x))*exp(2*x),x)

[Out] (exp(2*x)*(2*x - 1))/4

3.174 $\int (2x + \sqrt{2} x^2) dx$

Optimal. Leaf size=16

$$x^2 + \frac{\sqrt{2} x^3}{3}$$

[Out] $x^2 + 1/3 * x^3 * 2^{(1/2)}$

Rubi [A]

time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{\sqrt{2} x^3}{3} + x^2$$

Antiderivative was successfully verified.

[In] Int[2*x + Sqrt[2]*x^2,x]

[Out] $x^2 + (\text{Sqrt}[2]*x^3)/3$

Rubi steps

$$\int (2x + \sqrt{2} x^2) dx = x^2 + \frac{\sqrt{2} x^3}{3}$$

Mathematica [A]

time = 0.00, size = 16, normalized size = 1.00

$$x^2 + \frac{\sqrt{2} x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[2*x + Sqrt[2]*x^2,x]

[Out] $x^2 + (\text{Sqrt}[2]*x^3)/3$

Mathics [A]

time = 1.68, size = 12, normalized size = 0.75

$$\frac{x^2 (3 + \sqrt{2} x)}{3}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[Sqrt[2]*x^2 + 2*x,x]')`

[Out] $x^2 (3 + \text{Sqrt}[2] x) / 3$

Maple [A]

time = 0.02, size = 19, normalized size = 1.19

method	result	size
norman	$x^2 + \frac{x^3\sqrt{2}}{3}$	13
risch	$x^2 + \frac{x^3\sqrt{2}}{3}$	13
default	$\sqrt{2} \left(\frac{x^3}{3} + \frac{x^2\sqrt{2}}{2} \right)$	19
gosper	$\frac{x^2(2x+3\sqrt{2})(x\sqrt{2}+2)}{6x+6\sqrt{2}}$	29

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(2*x+x^2*2^(1/2),x,method=_RETURNVERBOSE)`

[Out] $2^{(1/2)}*(1/3*x^3+1/2*x^2*2^{(1/2)})$

Maxima [A]

time = 0.34, size = 12, normalized size = 0.75

$$\frac{1}{3} \sqrt{2} x^3 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*x+x^2*2^(1/2),x, algorithm="maxima")`

[Out] $1/3*\text{sqrt}(2)*x^3 + x^2$

Fricas [A]

time = 0.29, size = 12, normalized size = 0.75

$$\frac{1}{3} \sqrt{2} x^3 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*x+x^2*2^(1/2),x, algorithm="fricas")`

[Out] $1/3*\text{sqrt}(2)*x^3 + x^2$

Sympy [A]

time = 0.03, size = 12, normalized size = 0.75

$$\frac{\sqrt{2} x^3}{3} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*x+x**2*2**(1/2),x)`

[Out] `sqrt(2)*x**3/3 + x**2`

Giac [A]

time = 0.00, size = 19, normalized size = 1.19

$$\frac{2}{2}x^2 + \frac{1}{3}\sqrt{2}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*x+x^2*2^(1/2),x)`

[Out] `1/3*sqrt(2)*x^3 + x^2`

Mupad [B]

time = 0.03, size = 12, normalized size = 0.75

$$\frac{x^2 \left(\sqrt{2} x + 3 \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(2*x + 2^(1/2)*x^2,x)`

[Out] `(x^2*(2^(1/2)*x + 3))/3`

$$3.175 \quad \int \frac{\log(x)}{\sqrt{b+ax}} dx$$

Optimal. Leaf size=57

$$-\frac{4\sqrt{b+ax}}{a} + \frac{4\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)}{a} + \frac{2\sqrt{b+ax} \log(x)}{a}$$

[Out] 4*arctanh((a*x+b)^(1/2)/b^(1/2))*b^(1/2)/a-4*(a*x+b)^(1/2)/a+2*ln(x)*(a*x+b)^(1/2)/a

Rubi [A]

time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2356, 52, 65, 214}

$$-\frac{4\sqrt{ax+b}}{a} + \frac{2\log(x)\sqrt{ax+b}}{a} + \frac{4\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[Log[x]/Sqrt[b + a*x],x]

[Out] (-4*Sqrt[b + a*x])/a + (4*Sqrt[b]*ArcTanh[Sqrt[b + a*x]/Sqrt[b]])/a + (2*Sqrt[b + a*x]*Log[x])/a

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2356

Int[((a_) + Log[(c_)*(x_)^(n_)])*(b_)^(p_)*((d_) + (e_)*(x_))^(q_), x_Symbol] := Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x] - Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q, -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] && !IGtQ[q, 0]) || (EqQ[p, 2] && NeQ[q, 1]))

Rubi steps

$$\begin{aligned} \int \frac{\log(x)}{\sqrt{b+ax}} dx &= \frac{2\sqrt{b+ax} \log(x)}{a} - \frac{2 \int \frac{\sqrt{b+ax}}{x} dx}{a} \\ &= -\frac{4\sqrt{b+ax}}{a} + \frac{2\sqrt{b+ax} \log(x)}{a} - \frac{(2b) \int \frac{1}{x\sqrt{b+ax}} dx}{a} \\ &= -\frac{4\sqrt{b+ax}}{a} + \frac{2\sqrt{b+ax} \log(x)}{a} - \frac{(4b)\text{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^2}{a}} dx, x, \sqrt{b+ax}\right)}{a^2} \\ &= -\frac{4\sqrt{b+ax}}{a} + \frac{4\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)}{a} + \frac{2\sqrt{b+ax} \log(x)}{a} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 43, normalized size = 0.75

$$\frac{4\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right) + 2\sqrt{b+ax}(-2 + \log(x))}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Log[x]/Sqrt[b + a*x], x]

[Out] (4*Sqrt[b]*ArcTanh[Sqrt[b + a*x]/Sqrt[b]] + 2*Sqrt[b + a*x]*(-2 + Log[x]))/a

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 14.13, size = 1019, normalized size = 17.88

result too large to display

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Log[x]/Sqrt[a*x+b],x]')
```

```
[Out] Piecewise[{{2 (a (-2 + Log[-a x / (a x + b)] - Log[b / (a x + b)]) Sqrt[(a x + b) / a] + 2 Sqrt[a] Sqrt[b] ArcCoth[Sqrt[b] / (Sqrt[a] Sqrt[(a x + b) / a])]) / a ^ (3 / 2), Abs[b / a + x] < 1 && Abs[a / (a x + b)] < 1 && Abs[b / (a x + b)] > 1}, {2 (a (-2 - I Pi + Log[a x / (a x + b)] - Log[b / (a x + b)]) Sqrt[(a x + b) / a] + 2 Sqrt[a] Sqrt[b] ArcTanh[Sqrt[b] / (Sqrt[a] Sqrt[(a x + b) / a])]) / a ^ (3 / 2), Abs[b / a + x] < 1 && Abs[a / (a x + b)] < 1}, {2 (a (-2 + I Pi + Log[-a x / (a x + b)] + Log[b / a] - Log[b / (a x + b)]) Sqrt[(a x + b) / a] + 2 Sqrt[a] Sqrt[b] ArcCoth[Sqrt[b] / (Sqrt[a] Sqrt[(a x + b) / a])]) / a ^ (3 / 2), Abs[b / a + x] < 1 && Abs[b / (a x + b)] > 1}, {2 (a (-2 + Log[a x / (a x + b)] + Log[b / a] - Log[b / (a x + b)]) Sqrt[(a x + b) / a] + 2 Sqrt[a] Sqrt[b] ArcTanh[Sqrt[b] / (Sqrt[a] Sqrt[(a x + b) / a])]) / a ^ (3 / 2), Abs[b / a + x] < 1}, {2 (a (-2 + I Pi + Log[-a x / (a x + b)] + Log[b / a] - Log[b / (a x + b)]) Sqrt[(a x + b) / a] + 2 Sqrt[a] Sqrt[b] ArcCoth[Sqrt[b] / (Sqrt[a] Sqrt[(a x + b) / a])]) / a ^ (3 / 2), Abs[a / (a x + b)] < 1 && Abs[b / (a x + b)] > 1}, {2 (a (-2 + Log[a x / (a x + b)] + Log[b / a] - Log[b / (a x + b)]) Sqrt[(a x + b) / a] + 2 Sqrt[a] Sqrt[b] ArcTanh[Sqrt[b] / (Sqrt[a] Sqrt[(a x + b) / a])]) / a ^ (3 / 2), Abs[a / (a x + b)] < 1}, {(a (I Pi meijerg[{{1}, {3 / 2}}, {{1 / 2}, {0}}, b / a + x] + I Pi meijerg[{{3 / 2, 1}, {}], {}, {1 / 2, 0}}, b / a + x] - 4 Sqrt[(a x + b) / a] - 2 Log[b / (a x + b)] Sqrt[(a x + b) / a] + 2 Log[-a x / (a x + b)] Sqrt[(a x + b) / a] + Log[b / a] meijerg[{{1}, {3 / 2}}, {{1 / 2}, {0}}, b / a + x] + Log[b / a] meijerg[{{3 / 2, 1}, {}], {}, {1 / 2, 0}}, b / a + x) + 4 Sqrt[a] Sqrt[b] ArcCoth[Sqrt[b] / (Sqrt[a] Sqrt[(a x + b) / a])]) / a ^ (3 / 2), Abs[b / (a x + b)] > 1}}, -4 Sqrt[b / a + x] / Sqrt[a] - 2 Log[b / (a (b / a + x))] Sqrt[b / a + x] / Sqrt[a] + 2 Log[1 - b / (a (b / a + x))] Sqrt[b / a + x] / Sqrt[a] + Log[b / a] meijerg[{{1}, {3 / 2}}, {{1 / 2}, {0}}, b / a + x] / Sqrt[a] + Log[b / a] meijerg[{{3 / 2, 1}, {}], {}, {1 / 2, 0}}, b / a + x] / Sqrt[a] - 2 I Pi Sqrt[b / a + x] / Sqrt[a] + I Pi meijerg[{{1}, {3 / 2}}, {{1 / 2}, {0}}, b / a + x] / Sqrt[a] + I Pi meijerg[{{3 / 2, 1}, {}], {}, {1 / 2, 0}}, b / a + x] / Sqrt[a] + 4 Sqrt[b] ArcTanh[Sqrt[b] / (Sqrt[a] Sqrt[b / a + x])] / a]
```

Maple [A]

time = 0.04, size = 43, normalized size = 0.75

method	result	size
derivativedivides	$\frac{2\sqrt{ax+b} \ln(x) - 4\sqrt{ax+b} + 4\sqrt{b} \operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$	43

default	$\frac{2\sqrt{ax+b} \ln(x) - 4\sqrt{ax+b} + 4\sqrt{b} \operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$	43
---------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(x)/(a*x+b)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $2/a*((a*x+b)^{(1/2)}*\ln(x)-2*(a*x+b)^{(1/2)}+2*b^{(1/2)}*\operatorname{arctanh}((a*x+b)^{(1/2)}/b^{(1/2)}))$

Maxima [A]

time = 0.33, size = 58, normalized size = 1.02

$$\frac{2\left(\sqrt{ax+b} \log(x) - \sqrt{b} \log\left(\frac{\sqrt{ax+b} - \sqrt{b}}{\sqrt{ax+b} + \sqrt{b}}\right) - 2\sqrt{ax+b}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x)/(a*x+b)^(1/2),x, algorithm="maxima")`

[Out] $2*(\operatorname{sqrt}(a*x + b)*\log(x) - \operatorname{sqrt}(b)*\log((\operatorname{sqrt}(a*x + b) - \operatorname{sqrt}(b))/(\operatorname{sqrt}(a*x + b) + \operatorname{sqrt}(b)))) - 2*\operatorname{sqrt}(a*x + b))/a$

Fricas [A]

time = 0.34, size = 89, normalized size = 1.56

$$\left[\frac{2\left(\sqrt{ax+b}(\log(x)-2) + \sqrt{b} \log\left(\frac{ax+2\sqrt{ax+b}\sqrt{b}+2b}{x}\right)\right)}{a}, \frac{2\left(\sqrt{ax+b}(\log(x)-2) - 2\sqrt{-b} \operatorname{arctan}\left(\frac{\sqrt{ax+b}\sqrt{-b}}{b}\right)\right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x)/(a*x+b)^(1/2),x, algorithm="fricas")`

[Out] $[2*(\operatorname{sqrt}(a*x + b)*(\log(x) - 2) + \operatorname{sqrt}(b)*\log((a*x + 2*\operatorname{sqrt}(a*x + b)*\operatorname{sqrt}(b) + 2*b)/x))/a, 2*(\operatorname{sqrt}(a*x + b)*(\log(x) - 2) - 2*\operatorname{sqrt}(-b)*\operatorname{arctan}(\operatorname{sqrt}(a*x + b)*\operatorname{sqrt}(-b)/b))/a]$

Sympy [A]

time = 2.00, size = 1166, normalized size = 20.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(x)/(a*x+b)**(1/2),x)`

[Out] $\operatorname{Piecewise}((4*\operatorname{sqrt}(b)*\operatorname{acoth}(\operatorname{sqrt}(b)/(\operatorname{sqrt}(a)*\operatorname{sqrt}(x + b/a))))/a - 2*\operatorname{sqrt}(x + b/a)*\log(b/(a*(x + b/a)))/\operatorname{sqrt}(a) + 2*\operatorname{sqrt}(x + b/a)*\log(-1 + b/(a*(x + b/a)))$

```

))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a), (Abs(x + b/a) < 1) & (1/Abs(x + b/a)
< 1) & (Abs(b/(a*(x + b/a))) > 1)), (4*sqrt(b)*atanh(sqrt(b)/(sqrt(a)*sqrt(
x + b/a)))/a - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/
a)*log(1 - b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) - 2*I*pi*sqrt
(x + b/a)/sqrt(a), (Abs(x + b/a) < 1) & (1/Abs(x + b/a) < 1)), (4*sqrt(b)*a
coth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*log(b/a)/sqrt(a)
- 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(-1 + b
/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) + 2*I*pi*sqrt(x + b/a)/sq
rt(a), (Abs(x + b/a) < 1) & (Abs(b/(a*(x + b/a))) > 1)), (4*sqrt(b)*atanh(s
qrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*log(b/a)/sqrt(a) - 2*sq
rt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(1 - b/(a*(x
+ b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a), Abs(x + b/a) < 1), (4*sqrt(b)*a
coth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*log(b/a)/sqrt(a)
- 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(-1 + b
/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) + 2*I*pi*sqrt(x + b/a)/sq
rt(a), (1/Abs(x + b/a) < 1) & (Abs(b/(a*(x + b/a))) > 1)), (4*sqrt(b)*atanh(
sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*log(b/a)/sqrt(a) - 2*
sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(1 - b/(a*(
x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a), 1/Abs(x + b/a) < 1), (4*sqrt(
b)*acoth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a - 2*sqrt(x + b/a)*log(b/(a*(x +
b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(-1 + b/(a*(x + b/a)))/sqrt(a) - 4*sq
rt(x + b/a)/sqrt(a) + meijerg(((1,), (3/2,)), ((1/2,), (0,)), x + b/a)*log(b
/a)/sqrt(a) + I*pi*meijerg(((1,), (3/2,)), ((1/2,), (0,)), x + b/a)/sqrt(a)
+ meijerg(((3/2, 1), ()), ((), (1/2, 0)), x + b/a)*log(b/a)/sqrt(a) + I*pi
*meijerg(((3/2, 1), ()), ((), (1/2, 0)), x + b/a)/sqrt(a), Abs(b/(a*(x + b/
a))) > 1), (4*sqrt(b)*atanh(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a - 2*sqrt(x +
b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(1 - b/(a*(x + b/a)
))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) - 2*I*pi*sqrt(x + b/a)/sqrt(a) + meije
rg(((1,), (3/2,)), ((1/2,), (0,)), x + b/a)*log(b/a)/sqrt(a) + I*pi*meijerg
(((1,), (3/2,)), ((1/2,), (0,)), x + b/a)/sqrt(a) + meijerg(((3/2, 1), ()),
((), (1/2, 0)), x + b/a)*log(b/a)/sqrt(a) + I*pi*meijerg(((3/2, 1), ()), (
), (1/2, 0)), x + b/a)/sqrt(a), True))

```

Giac [A]

time = 0.00, size = 70, normalized size = 1.23

$$\frac{2 \left(\sqrt{ax+b} \ln\left(\frac{ax+b-b}{a}\right) + 2 \left(-\sqrt{ax+b} - \frac{2b \arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{2\sqrt{-b}} \right) \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)/(a*x+b)^(1/2),x)

[Out] -2*(2*b*arctan(sqrt(a*x + b)/sqrt(-b))/sqrt(-b) - sqrt(a*x + b)*log(x) + 2*sqrt(a*x + b))/a

Mupad [B]

time = 0.09, size = 49, normalized size = 0.86

$$\frac{2\sqrt{b} \ln\left(\frac{2b+ax+2\sqrt{b}\sqrt{b+ax}}{x}\right)}{a} + \frac{2(\ln(x)-2)\sqrt{b+ax}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(x)/(b + a*x)^(1/2),x)

[Out] (2*b^(1/2)*log((2*b + a*x + 2*b^(1/2)*(b + a*x)^(1/2))/x))/a + (2*(log(x) - 2)*(b + a*x)^(1/2))/a

3.176 $\int \sqrt{a + bx} \sqrt{c + dx} dx$

Optimal. Leaf size=116

$$\frac{(bc - ad)\sqrt{a + bx} \sqrt{c + dx}}{4bd} + \frac{(a + bx)^{3/2}\sqrt{c + dx}}{2b} - \frac{(bc - ad)^2 \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a + bx}}{\sqrt{b}\sqrt{c + dx}}\right)}{4b^{3/2}d^{3/2}}$$

[Out] $-1/4*(-a*d+b*c)^2*\operatorname{arctanh}(d^{1/2}*(b*x+a)^{1/2}/b^{1/2}/(d*x+c)^{1/2})/b^{3/2}/d^{3/2}+1/2*(b*x+a)^{3/2}*(d*x+c)^{1/2}/b+1/4*(-a*d+b*c)*(b*x+a)^{1/2}*(d*x+c)^{1/2}/b/d$

Rubi [A]

time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$-\frac{(bc - ad)^2 \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a + bx}}{\sqrt{b}\sqrt{c + dx}}\right)}{4b^{3/2}d^{3/2}} + \frac{\sqrt{a + bx} \sqrt{c + dx} (bc - ad)}{4bd} + \frac{(a + bx)^{3/2}\sqrt{c + dx}}{2b}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + b*x]*Sqrt[c + d*x],x]`

[Out] $((b*c - a*d)*\operatorname{Sqrt}[a + b*x]*\operatorname{Sqrt}[c + d*x])/(4*b*d) + ((a + b*x)^{3/2}*\operatorname{Sqrt}[c + d*x])/(2*b) - ((b*c - a*d)^2*\operatorname{ArcTanh}[(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a + b*x])]/(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x]))/(4*b^{3/2}*d^{3/2})$

Rule 52

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*(b*c - a*d)/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 65

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 212


```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned} \int \sqrt{a+bx} \sqrt{c+dx} \, dx &= \frac{(a+bx)^{3/2} \sqrt{c+dx}}{2b} + \frac{(bc-ad) \int \frac{\sqrt{a+bx}}{\sqrt{c+dx}} \, dx}{4b} \\ &= \frac{(bc-ad) \sqrt{a+bx} \sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2} \sqrt{c+dx}}{2b} - \frac{(bc-ad)^2 \int \frac{1}{\sqrt{a+bx} \sqrt{c+dx}} \, dx}{8bd} \\ &= \frac{(bc-ad) \sqrt{a+bx} \sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2} \sqrt{c+dx}}{2b} - \frac{(bc-ad)^2 \operatorname{Subst} \left(\int \frac{1}{\sqrt{1-dx}} \, dx \right)}{8bd} \\ &= \frac{(bc-ad) \sqrt{a+bx} \sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2} \sqrt{c+dx}}{2b} - \frac{(bc-ad)^2 \operatorname{Subst} \left(\int \frac{1}{1-dx} \, dx \right)}{4b^2 d} \\ &= \frac{(bc-ad) \sqrt{a+bx} \sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2} \sqrt{c+dx}}{2b} - \frac{(bc-ad)^2 \tanh^{-1} \left(\frac{\sqrt{d}}{\sqrt{b}} \right)}{4b^{3/2} d^{3/2}} \end{aligned}$$

Mathematica [A]

time = 0.22, size = 95, normalized size = 0.82

$$\frac{\sqrt{a+bx} \sqrt{c+dx} (ad + b(c + 2dx))}{4bd} - \frac{(bc - ad)^2 \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d} \sqrt{a+bx}} \right)}{4b^{3/2} d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[c + d*x], x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(a*d + b*(c + 2*d*x)))/(4*b*d) - ((b*c - a*d)^
2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])]/(4*b^(3/2)*d^(3
/2))
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[a+b*x]*Sqrt[c+d*x],x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]

time = 0.04, size = 140, normalized size = 1.21

method	result
default	$\frac{\sqrt{bx+a} (dx+c)^{\frac{3}{2}}}{2d} - \frac{(-ad+bc) \left(\frac{\sqrt{dx+c}}{b} \sqrt{bx+a} - \frac{(ad-bc) \sqrt{(bx+a)(dx+c)} \ln\left(\frac{\frac{1}{2}ad + \frac{1}{2}bc + bdx + \sqrt{bd}x^2}{\sqrt{bd}} + \sqrt{bd}x^2\right)}{2b\sqrt{dx+c} \sqrt{bx+a} \sqrt{bd}} \right)}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)-1/4*(-a*d+b*c)/d*(1/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2))*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see 'assume?' for more detail
```

Fricas [A]

time = 0.32, size = 300, normalized size = 2.59

$$\left[\frac{(b^2c^2 - 2abcd + a^2d^2)\sqrt{bd} \log\left(\frac{8b^2d^2x^2 + b^2c^2 + 6abcd + a^2d^2 - 4(2bdx + bc + ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c} + 8(b^2cd + abd^2)x}{10b^2d^2}\right) + 4(2b^2d^2x + b^2cd + abd^2)\sqrt{bx+a}\sqrt{dx+c}}{8b^2d^2}, \frac{(b^2c^2 - 2abcd + a^2d^2)\sqrt{-bd} \operatorname{arctan}\left(\frac{2bdc + bcad\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2(b^2d^2x + b^2cd + abd^2)\sqrt{-bd}}\right) + 2(2b^2d^2x + b^2cd + abd^2)\sqrt{bx+a}\sqrt{dx+c}}{8b^2d^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")

[Out] [1/16*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x + b^2*c*d + a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2), 1/8*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(2*b^2*d^2*x + b^2*c*d + a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2)]

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx} \sqrt{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/2),x)

[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(90) = 180.

time = 0.02, size = 299, normalized size = 2.58

$$\frac{2 \left(\frac{\frac{1}{2} \sqrt{a+bx} \sqrt{c+dx}}{2d} - \frac{\frac{1}{2} \sqrt{-a+bx} \sqrt{c+dx}}{2d} \right) \sqrt{a+bx} \sqrt{-abd+b^2c+bd(a+bx)} + \frac{2(-2a^2d+2abd+2b^2c^2) \sqrt{-abd+b^2c+bd(a+bx)} - \sqrt{bd} \sqrt{a+bx}}{16d\sqrt{bd}}}{b} + \frac{2 \operatorname{arctan} \left(\frac{\frac{1}{2} \sqrt{a+bx} \sqrt{-abd+b^2c+bd(a+bx)} + \frac{2(-a^2-d) \sqrt{-abd+b^2c+bd(a+bx)} - \sqrt{bd} \sqrt{a+bx}}{\sqrt{bd}} \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x)

[Out] -1/4*(4*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*a*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*abs(b)/b^2)/b

Mupad [B]

time = 0.17, size = 88, normalized size = 0.76

$$\left(\frac{x}{2} + \frac{ad+bc}{4bd} \right) \sqrt{a+bx} \sqrt{c+dx} - \frac{\ln \left(ad+bc+2bdx+2\sqrt{b}\sqrt{d}\sqrt{a+bx}\sqrt{c+dx} \right) (ad-bc)^2}{8b^{3/2}d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^(1/2)*(c + d*x)^(1/2),x)

[Out] (x/2 + (a*d + b*c)/(4*b*d))*a + b*x)^(1/2)*(c + d*x)^(1/2) - (log(a*d + b*c + 2*b*d*x + 2*b^(1/2)*d^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2))*(a*d - b*c)^2)/(8*b^(3/2)*d^(3/2))

3.177 $\int \sqrt{a + bx} \, dx$

Optimal. Leaf size=16

$$\frac{2(a + bx)^{3/2}}{3b}$$

[Out] $2/3*(b*x+a)^{(3/2)}/b$

Rubi [A]

time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{2(a + bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x], x]

[Out] $(2*(a + b*x)^{(3/2)})/(3*b)$

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\int \sqrt{a + bx} \, dx = \frac{2(a + bx)^{3/2}}{3b}$$

Mathematica [A]

time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a + bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x], x]

[Out] $(2*(a + b*x)^{(3/2)})/(3*b)$

Mathics [A]

time = 1.64, size = 12, normalized size = 0.75

$$\frac{2(a + bx)^{\frac{3}{2}}}{3b}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[Sqrt[a+b*x],x]')`

[Out] $2(a + bx)^{3/2} / (3b)$

Maple [A]

time = 0.03, size = 13, normalized size = 0.81

method	result	size
gospers	$\frac{2(bx+a)^{3/2}}{3b}$	13
derivativdivides	$\frac{2(bx+a)^{3/2}}{3b}$	13
default	$\frac{2(bx+a)^{3/2}}{3b}$	13
trager	$\frac{2(bx+a)^{3/2}}{3b}$	13
risch	$\frac{2(bx+a)^{3/2}}{3b}$	13

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $2/3*(b*x+a)^{3/2}/b$

Maxima [A]

time = 0.25, size = 12, normalized size = 0.75

$$\frac{2(bx + a)^{3/2}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^(1/2),x, algorithm="maxima")`

[Out] $2/3*(b*x + a)^{3/2}/b$

Fricas [A]

time = 0.30, size = 12, normalized size = 0.75

$$\frac{2(bx + a)^{3/2}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^(1/2),x, algorithm="fricas")`

[Out] $2/3*(b*x + a)^{3/2}/b$

Sympy [A]

time = 0.03, size = 12, normalized size = 0.75

$$\frac{2(a + bx)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**(1/2),x)

[Out] 2*(a + b*x)**(3/2)/(3*b)

Giac [A]

time = 0.00, size = 21, normalized size = 1.31

$$\frac{\sqrt{a + bx} (a + bx)}{\frac{3}{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/2),x)

[Out] 2/3*(b*x + a)^(3/2)/b

Mupad [B]

time = 0.03, size = 12, normalized size = 0.75

$$\frac{2(a + bx)^{3/2}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^(1/2),x)

[Out] (2*(a + b*x)^(3/2))/(3*b)

3.178 $\int x \sqrt{a + bx} dx$

Optimal. Leaf size=34

$$-\frac{2a(a+bx)^{3/2}}{3b^2} + \frac{2(a+bx)^{5/2}}{5b^2}$$

[Out] $-2/3*a*(b*x+a)^{(3/2)}/b^2+2/5*(b*x+a)^{(5/2)}/b^2$

Rubi [A]

time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{2(a+bx)^{5/2}}{5b^2} - \frac{2a(a+bx)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[a + b*x],x]

[Out] $(-2*a*(a + b*x)^{(3/2)})/(3*b^2) + (2*(a + b*x)^{(5/2)})/(5*b^2)$

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rubi steps

$$\begin{aligned} \int x \sqrt{a + bx} dx &= \int \left(-\frac{a\sqrt{a + bx}}{b} + \frac{(a + bx)^{3/2}}{b} \right) dx \\ &= -\frac{2a(a + bx)^{3/2}}{3b^2} + \frac{2(a + bx)^{5/2}}{5b^2} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 34, normalized size = 1.00

$$\frac{2\sqrt{a + bx} (-2a^2 + abx + 3b^2x^2)}{15b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[a + b*x],x]

[Out] $(2\sqrt{a + bx} * (-2a^2 + a * bx + 3b^2 * x^2)) / (15b^2)$

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 108 vs. $2(34) = 68$.
time = 3.80, size = 96, normalized size = 2.82

$$\frac{2\sqrt{a} \left(2a^3 \left(1 - \sqrt{\frac{a + bx}{a}} \right) + a^2bx \left(2 - \sqrt{\frac{a + bx}{a}} \right) + 4ab^2x^2 \sqrt{\frac{a + bx}{a}} + 3b^3x^3 \sqrt{\frac{a + bx}{a}} \right)}{15b^2(a + bx)}$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[x*Sqrt[a+bx],x]')`

[Out] $2 \sqrt{a} (2 a^3 (1 - \sqrt{(a + b x) / a}) + a^2 b x (2 - \sqrt{(a + b x) / a}) + 4 a b^2 x^2 \sqrt{(a + b x) / a} + 3 b^3 x^3 \sqrt{(a + b x) / a}) / (15 b^2 (a + b x))$

Maple [A]

time = 0.04, size = 26, normalized size = 0.76

method	result	size
gospers	$-\frac{2(bx+a)^{\frac{3}{2}}(-3bx+2a)}{15b^2}$	21
derivativedivides	$\frac{2(bx+a)^{\frac{5}{2}} - \frac{2a(bx+a)^{\frac{3}{2}}}{3}}{5b^2}$	26
default	$\frac{2(bx+a)^{\frac{5}{2}} - \frac{2a(bx+a)^{\frac{3}{2}}}{3}}{5b^2}$	26
trager	$-\frac{2(-3x^2b^2 - abx + 2a^2)\sqrt{bx+a}}{15b^2}$	32
risch	$-\frac{2(-3x^2b^2 - abx + 2a^2)\sqrt{bx+a}}{15b^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $2/b^2*(1/5*(b*x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))$

Maxima [A]

time = 0.25, size = 26, normalized size = 0.76

$$\frac{2(bx+a)^{\frac{5}{2}}}{5b^2} - \frac{2(bx+a)^{\frac{3}{2}}a}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x+a)^(1/2),x, algorithm="maxima")`

[Out] $2/5*(b*x + a)^(5/2)/b^2 - 2/3*(b*x + a)^(3/2)*a/b^2$

Fricas [A]

time = 0.30, size = 30, normalized size = 0.88

$$\frac{2(3b^2x^2 + abx - 2a^2)\sqrt{bx + a}}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^(1/2),x, algorithm="fricas")

[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(31) = 62

time = 0.56, size = 202, normalized size = 5.94

$$-\frac{4a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x} + \frac{4a^{\frac{9}{2}}}{15a^2b^2+15ab^3x} - \frac{2a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x} + \frac{4a^{\frac{7}{2}}bx}{15a^2b^2+15ab^3x} + \frac{8a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x} + \frac{6a^{\frac{3}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)**(1/2),x)

[Out] $-4a^{9/2}\sqrt{1+b*x/a}/(15a^{**2}b^{**2}+15a*b^{**3}*x)+4a^{9/2}/(15a^{**2}b^{**2}+15a*b^{**3}*x)-2a^{7/2}b*x*\sqrt{1+b*x/a}/(15a^{**2}b^{**2}+15a*b^{**3}*x)+4a^{7/2}b*x/(15a^{**2}b^{**2}+15a*b^{**3}*x)+8a^{5/2}b^2*x^2*\sqrt{1+b*x/a}/(15a^{**2}b^{**2}+15a*b^{**3}*x)+6a^{3/2}b^3*x^3*\sqrt{1+b*x/a}/(15a^{**2}b^{**2}+15a*b^{**3}*x)$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(26) = 52. time = 0.00, size = 99, normalized size = 2.91

$$\frac{2b\left(\frac{1}{5}\sqrt{a+bx}^{(a+bx)^2-\frac{2}{3}}\sqrt{a+bx}^{(a+bx)a}\sqrt{a+bx}^{a^2}\right)}{b^2} + \frac{2a\left(\frac{1}{3}\sqrt{a+bx}^{(a+bx)-a}\sqrt{a+bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^(1/2),x)

[Out] $2/15*(5*((b*x + a)^{(3/2)} - 3*\sqrt{b*x + a})*a)*a/b + (3*(b*x + a)^{(5/2)} - 10*(b*x + a)^{(3/2})*a + 15*\sqrt{b*x + a})*a^2)/b)$

Mupad [B]

time = 0.03, size = 25, normalized size = 0.74

$$-\frac{10a(a+bx)^{3/2} - 6(a+bx)^{5/2}}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + b*x)^(1/2),x)

[Out] $-(10*a*(a + b*x)^{(3/2)} - 6*(a + b*x)^{(5/2)})/(15*b^2)$

3.179 $\int x^2 \sqrt{a + bx} dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{3/2}}{3b^3} - \frac{4a(a+bx)^{5/2}}{5b^3} + \frac{2(a+bx)^{7/2}}{7b^3}$$

[Out] $2/3*a^2*(b*x+a)^{(3/2)}/b^3-4/5*a*(b*x+a)^{(5/2)}/b^3+2/7*(b*x+a)^{(7/2)}/b^3$

Rubi [A]

time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{2a^2(a+bx)^{3/2}}{3b^3} + \frac{2(a+bx)^{7/2}}{7b^3} - \frac{4a(a+bx)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

[In] `Int[x^2*Sqrt[a + b*x], x]`

[Out] $(2*a^2*(a + b*x)^{(3/2)})/(3*b^3) - (4*a*(a + b*x)^{(5/2)})/(5*b^3) + (2*(a + b*x)^{(7/2)})/(7*b^3)$

Rule 45

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]`

Rubi steps

$$\begin{aligned} \int x^2 \sqrt{a + bx} dx &= \int \left(\frac{a^2 \sqrt{a + bx}}{b^2} - \frac{2a(a + bx)^{3/2}}{b^2} + \frac{(a + bx)^{5/2}}{b^2} \right) dx \\ &= \frac{2a^2(a + bx)^{3/2}}{3b^3} - \frac{4a(a + bx)^{5/2}}{5b^3} + \frac{2(a + bx)^{7/2}}{7b^3} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 35, normalized size = 0.66

$$\frac{2(a + bx)^{3/2} (8a^2 - 12abx + 15b^2x^2)}{105b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*sqrt[a + b*x],x]

[Out] (2*(a + b*x)^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3)

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 200 vs. 2(53) = 106.
time = 7.66, size = 184, normalized size = 3.47

$$\frac{2\sqrt{a} \left(8a^6 \left(-1 + \sqrt{\frac{a+bx}{a}} \right) + 4a^5bx \left(-6 + 5\sqrt{\frac{a+bx}{a}} \right) + 3a^4b^2x^2 \left(-8 + 5\sqrt{\frac{a+bx}{a}} \right) + 10a^2b^3x^3 (2a + 5bx) \sqrt{\frac{a+bx}{a}} - 8a^3b^3x^3 + 48ab^5x^5 \sqrt{\frac{a+bx}{a}} + 15b^6x^6 \sqrt{\frac{a+bx}{a}} \right)}{105b^3 (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x^2*sqrt[a+b*x],x]')

[Out] 2 Sqrt[a] (8 a ^ 6 (-1 + Sqrt[(a + b x) / a]) + 4 a ^ 5 b x (-6 + 5 Sqrt[(a + b x) / a]) + 3 a ^ 4 b ^ 2 x ^ 2 (-8 + 5 Sqrt[(a + b x) / a]) + 10 a ^ 2 b ^ 3 x ^ 3 (2 a + 5 b x) Sqrt[(a + b x) / a] - 8 a ^ 3 b ^ 3 x ^ 3 + 48 a b ^ 5 x ^ 5 Sqrt[(a + b x) / a] + 15 b ^ 6 x ^ 6 Sqrt[(a + b x) / a]) / (1 05 b ^ 3 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))

Maple [A]

time = 0.04, size = 38, normalized size = 0.72

method	result	size
gospers	$\frac{2(bx+a)^{\frac{3}{2}}(15x^2b^2-12abx+8a^2)}{105b^3}$	32
derivativedivides	$\frac{2(bx+a)^{\frac{7}{2}}}{7} - \frac{4a(bx+a)^{\frac{5}{2}}}{5b^3} + \frac{2a^2(bx+a)^{\frac{3}{2}}}{3}$	38
default	$\frac{2(bx+a)^{\frac{7}{2}}}{7} - \frac{4a(bx+a)^{\frac{5}{2}}}{5b^3} + \frac{2a^2(bx+a)^{\frac{3}{2}}}{3}$	38
trager	$\frac{2(15b^3x^3+3ab^2x^2-4a^2bx+8a^3)\sqrt{bx+a}}{105b^3}$	43
risch	$\frac{2(15b^3x^3+3ab^2x^2-4a^2bx+8a^3)\sqrt{bx+a}}{105b^3}$	43

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/b^3*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2))

Maxima [A]

time = 0.29, size = 41, normalized size = 0.77

$$\frac{2(bx+a)^{\frac{7}{2}}}{7b^3} - \frac{4(bx+a)^{\frac{5}{2}}a}{5b^3} + \frac{2(bx+a)^{\frac{3}{2}}a^2}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="maxima")

[Out] $2/7*(b*x + a)^{(7/2)}/b^3 - 4/5*(b*x + a)^{(5/2)}*a/b^3 + 2/3*(b*x + a)^{(3/2)}*a^2/b^3$

Fricas [A]

time = 0.30, size = 42, normalized size = 0.79

$$\frac{2(15b^3x^3 + 3ab^2x^2 - 4a^2bx + 8a^3)\sqrt{bx+a}}{105b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x+a)^(1/2),x, algorithm="fricas")`

[Out] $2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*\text{sqrt}(b*x + a)/b^3$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(49) = 98$

time = 0.85, size = 666, normalized size = 12.57

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x+a)**(1/2),x)`

[Out] $16*a^{**}(23/2)*\text{sqrt}(1 + b*x/a)/(105*a^{**}8*b^{**}3 + 315*a^{**}7*b^{**}4*x + 315*a^{**}6*b^{**}5*x^{**}2 + 105*a^{**}5*b^{**}6*x^{**}3) - 16*a^{**}(23/2)/(105*a^{**}8*b^{**}3 + 315*a^{**}7*b^{**}4*x + 315*a^{**}6*b^{**}5*x^{**}2 + 105*a^{**}5*b^{**}6*x^{**}3) + 40*a^{**}(21/2)*b*x*\text{sqrt}(1 + b*x/a)/(105*a^{**}8*b^{**}3 + 315*a^{**}7*b^{**}4*x + 315*a^{**}6*b^{**}5*x^{**}2 + 105*a^{**}5*b^{**}6*x^{**}3) - 48*a^{**}(21/2)*b*x/(105*a^{**}8*b^{**}3 + 315*a^{**}7*b^{**}4*x + 315*a^{**}6*b^{**}5*x^{**}2 + 105*a^{**}5*b^{**}6*x^{**}3) + 30*a^{**}(19/2)*b^{**}2*x^{**}2*\text{sqrt}(1 + b*x/a)/(105*a^{**}8*b^{**}3 + 315*a^{**}7*b^{**}4*x + 315*a^{**}6*b^{**}5*x^{**}2 + 105*a^{**}5*b^{**}6*x^{**}3) - 48*a^{**}(19/2)*b^{**}2*x^{**}2/(105*a^{**}8*b^{**}3 + 315*a^{**}7*b^{**}4*x + 315*a^{**}6*b^{**}5*x^{**}2 + 105*a^{**}5*b^{**}6*x^{**}3) + 40*a^{**}(17/2)*b^{**}3*x^{**}3*\text{sqrt}(1 + b*x/a)/(105*a^{**}8*b^{**}3 + 315*a^{**}7*b^{**}4*x + 315*a^{**}6*b^{**}5*x^{**}2 + 105*a^{**}5*b^{**}6*x^{**}3) - 16*a^{**}(17/2)*b^{**}3*x^{**}3/(105*a^{**}8*b^{**}3 + 315*a^{**}7*b^{**}4*x + 315*a^{**}6*b^{**}5*x^{**}2 + 105*a^{**}5*b^{**}6*x^{**}3) + 100*a^{**}(15/2)*b^{**}4*x^{**}4*\text{sqrt}(1 + b*x/a)/(105*a^{**}8*b^{**}3 + 315*a^{**}7*b^{**}4*x + 315*a^{**}6*b^{**}5*x^{**}2 + 105*a^{**}5*b^{**}6*x^{**}3) + 96*a^{**}(13/2)*b^{**}5*x^{**}5*\text{sqrt}(1 + b*x/a)/(105*a^{**}8*b^{**}3 + 315*a^{**}7*b^{**}4*x + 315*a^{**}6*b^{**}5*x^{**}2 + 105*a^{**}5*b^{**}6*x^{**}3) + 30*a^{**}(11/2)*b^{**}6*x^{**}6*\text{sqrt}(1 + b*x/a)/(105*a^{**}8*b^{**}3 + 315*a^{**}7*b^{**}4*x + 315*a^{**}6*b^{**}5*x^{**}2 + 105*a^{**}5*b^{**}6*x^{**}3)$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(41) = 82$.
time = 0.00, size = 145, normalized size = 2.74

$$\frac{2b\left(\frac{1}{7}\sqrt{a+bx}^{(a+bx)^3} - \frac{2}{3}\sqrt{a+bx}^{(a+bx)^2}a + \sqrt{a+bx}^{(a+bx)}a^2 - \sqrt{a+bx}a^3\right)}{b^3} + \frac{2a\left(\frac{1}{5}\sqrt{a+bx}^{(a+bx)^2} - \frac{2}{3}\sqrt{a+bx}^{(a+bx)}a + \sqrt{a+bx}a^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)^(1/2),x)

[Out] $\frac{2}{105} \cdot (7 \cdot (3 \cdot (b \cdot x + a)^{5/2} - 10 \cdot (b \cdot x + a)^{3/2} \cdot a + 15 \cdot \sqrt{b \cdot x + a} \cdot a^2) \cdot a/b^2 + 3 \cdot (5 \cdot (b \cdot x + a)^{7/2} - 21 \cdot (b \cdot x + a)^{5/2} \cdot a + 35 \cdot (b \cdot x + a)^{3/2} \cdot a^2 - 35 \cdot \sqrt{b \cdot x + a} \cdot a^3)/b^2)/b$

Mupad [B]

time = 0.15, size = 37, normalized size = 0.70

$$\frac{30(a+bx)^{7/2} - 84a(a+bx)^{5/2} + 70a^2(a+bx)^{3/2}}{105b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + b*x)^(1/2),x)

[Out] $(30 \cdot (a + b \cdot x)^{7/2} - 84 \cdot a \cdot (a + b \cdot x)^{5/2} + 70 \cdot a^2 \cdot (a + b \cdot x)^{3/2}) / (105 \cdot b^3)$

$$3.180 \quad \int \frac{\sqrt{a+bx}}{x} dx$$

Optimal. Leaf size=35

$$2\sqrt{a+bx} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

[Out] $-2*\operatorname{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)})*a^{(1/2)}+2*(b*x+a)^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 65, 214}

$$2\sqrt{a+bx} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + b*x]/x,x]`

[Out] `2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]`

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[
m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{x} dx &= 2\sqrt{a+bx} + a \int \frac{1}{x\sqrt{a+bx}} dx \\
&= 2\sqrt{a+bx} + \frac{(2a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+bx}\right)}{b} \\
&= 2\sqrt{a+bx} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 35, normalized size = 1.00

$$2\sqrt{a+bx} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

`[In] Integrate[Sqrt[a + b*x]/x,x]``[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]`**Mathics [B]** Leaf count is larger than twice the leaf count of optimal. 94 vs. 2(35) = 70.
time = 2.90, size = 74, normalized size = 2.11

$$\frac{2\left(-\sqrt{a} \operatorname{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right](a+bx) + a\sqrt{b}\sqrt{x}\sqrt{\frac{a+bx}{bx}} + b^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{\frac{a+bx}{bx}}\right)}{a+bx}$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Sqrt[a+b*x]/x,x]')``[Out] 2 (-Sqrt[a] ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] (a + b x) + a Sqrt[b] Sqrt[x] Sqrt[(a + b x) / (b x)] + b ^ (3 / 2) x ^ (3 / 2) Sqrt[(a + b x) / (b x)]) / (a + b x)`**Maple [A]**

time = 0.03, size = 28, normalized size = 0.80

method	result	size
derivativedivides	$-2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) \sqrt{a} + 2\sqrt{bx+a}$	28

default	$-2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) \sqrt{a} + 2\sqrt{bx+a}$	28
---------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)^(1/2)/x,x,method=_RETURNVERBOSE)`

[Out] $-2*\operatorname{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)})*a^{(1/2)}+2*(b*x+a)^{(1/2)}$

Maxima [A]

time = 0.35, size = 42, normalized size = 1.20

$$\sqrt{a} \log\left(\frac{\sqrt{bx+a} - \sqrt{a}}{\sqrt{bx+a} + \sqrt{a}}\right) + 2\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^(1/2)/x,x, algorithm="maxima")`

[Out] $\sqrt{a}*\log((\sqrt{b*x+a} - \sqrt{a})/(\sqrt{b*x+a} + \sqrt{a})) + 2*\sqrt{b*x+a}$

Fricas [A]

time = 0.32, size = 73, normalized size = 2.09

$$\left[\sqrt{a} \log\left(\frac{bx - 2\sqrt{bx+a}\sqrt{a} + 2a}{x}\right) + 2\sqrt{bx+a}, 2\sqrt{-a} \arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + 2\sqrt{bx+a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^(1/2)/x,x, algorithm="fricas")`

[Out] $[\sqrt{a}*\log((b*x - 2*\sqrt{b*x+a}*\sqrt{a} + 2*a)/x) + 2*\sqrt{b*x+a}, 2*\sqrt{-a}*\arctan(\sqrt{b*x+a}*\sqrt{-a}/a) + 2*\sqrt{b*x+a}]$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs.

$2(31) = 62$

time = 0.69, size = 68, normalized size = 1.94

$$-2\sqrt{a} \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) + \frac{2a}{\sqrt{b}\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{2\sqrt{b}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)**(1/2)/x,x)`

[Out] $-2*\sqrt{a}*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*\sqrt{x})) + 2*a/(\sqrt{b}*\sqrt{x}*\sqrt{a/(b*x)+1}) + 2*\sqrt{b}*\sqrt{x}/\sqrt{a/(b*x)+1}$

Giac [A]

time = 0.00, size = 42, normalized size = 1.20

$$2\sqrt{a+bx} + \frac{4a \arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/2)/x,x)**[Out]** 2*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)**Mupad [B]**

time = 0.15, size = 27, normalized size = 0.77

$$2\sqrt{a+bx} - 2\sqrt{a} \operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^(1/2)/x,x)**[Out]** 2*(a + b*x)^(1/2) - 2*a^(1/2)*atanh((a + b*x)^(1/2)/a^(1/2))

$$3.181 \quad \int \frac{\sqrt{a+bx}}{x^2} dx$$

Optimal. Leaf size=39

$$-\frac{\sqrt{a+bx}}{x} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

[Out] $-b \operatorname{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)})/a^{(1/2)} - (b*x+a)^{(1/2)}/x$

Rubi [A]

time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {43, 65, 214}

$$-\frac{\sqrt{a+bx}}{x} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x]/x^2,x]

[Out] $-(\text{Sqrt}[a + b*x]/x) - (b*\text{ArcTanh}[\text{Sqrt}[a + b*x]/\text{Sqrt}[a]])/\text{Sqrt}[a]$

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{x^2} dx &= -\frac{\sqrt{a+bx}}{x} + \frac{1}{2}b \int \frac{1}{x\sqrt{a+bx}} dx \\
&= -\frac{\sqrt{a+bx}}{x} + \text{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+bx} \right) \\
&= -\frac{\sqrt{a+bx}}{x} - \frac{b \tanh^{-1} \left(\frac{\sqrt{a+bx}}{\sqrt{a}} \right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A]

time = 0.05, size = 39, normalized size = 1.00

$$-\frac{\sqrt{a+bx}}{x} - \frac{b \tanh^{-1} \left(\frac{\sqrt{a+bx}}{\sqrt{a}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

`[In] Integrate[Sqrt[a + b*x]/x^2,x]``[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]`**Mathics [A]**

time = 2.86, size = 38, normalized size = 0.97

$$-\frac{b \text{ArcSinh} \left[\frac{\sqrt{a}}{\sqrt{b} \sqrt{x}} \right]}{\sqrt{a}} - \frac{\sqrt{b} \sqrt{1 + \frac{a}{bx}}}{\sqrt{x}}$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Sqrt[a+b*x]/x^2,x]')``[Out] -b ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] / Sqrt[a] - Sqrt[b] Sqrt[1 + a / (b x)] / Sqrt[x]`**Maple [A]**

time = 0.03, size = 37, normalized size = 0.95

method	result	size
risch	$-\frac{b \operatorname{arctanh} \left(\frac{\sqrt{bx+a}}{\sqrt{a}} \right)}{\sqrt{a}} - \frac{\sqrt{bx+a}}{x}$	32

derivativedivides	$2b \left(-\frac{\sqrt{bx+a}}{2bx} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}} \right)$	37
default	$2b \left(-\frac{\sqrt{bx+a}}{2bx} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}} \right)$	37

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)`

[Out] $2*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*\operatorname{arctanh}((b*x+a)^(1/2)/a^(1/2))/a^(1/2))$

Maxima [A]

time = 0.34, size = 47, normalized size = 1.21

$$\frac{b \log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{bx+a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^(1/2)/x^2,x, algorithm="maxima")`

[Out] $1/2*b*\log((\operatorname{sqrt}(b*x+a)-\operatorname{sqrt}(a))/(\operatorname{sqrt}(b*x+a)+\operatorname{sqrt}(a)))/\operatorname{sqrt}(a)-\operatorname{sqrt}(b*x+a)/x$

Fricas [A]

time = 0.31, size = 93, normalized size = 2.38

$$\left[\frac{\sqrt{a} b x \log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) - 2\sqrt{bx+a} a}{2ax}, \frac{\sqrt{-a} b x \operatorname{arctan}\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) - \sqrt{bx+a} a}{ax} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^(1/2)/x^2,x, algorithm="fricas")`

[Out] $[1/2*(\operatorname{sqrt}(a)*b*x*\log((b*x-2*\operatorname{sqrt}(b*x+a)*\operatorname{sqrt}(a)+2*a)/x)-2*\operatorname{sqrt}(b*x+a)*a)/(a*x), (\operatorname{sqrt}(-a)*b*x*\operatorname{arctan}(\operatorname{sqrt}(b*x+a)*\operatorname{sqrt}(-a)/a)-\operatorname{sqrt}(b*x+a)*a)/(a*x)]$

Sympy [A]

time = 0.89, size = 44, normalized size = 1.13

$$-\frac{\sqrt{b} \sqrt{\frac{a}{bx}+1}}{\sqrt{x}} - \frac{b \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**(1/2)/x**2,x)

[Out] $-\sqrt{b}\sqrt{a/(b*x) + 1}/\sqrt{x} - b*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}\sqrt{x}))/\sqrt{a}$

Giac [A]

time = 0.00, size = 58, normalized size = 1.49

$$\frac{\frac{\sqrt{a+bx} b^2}{-a-bx+a} + \frac{2b^2 \arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{2\sqrt{-a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/2)/x^2,x)

[Out] $(b^2*\arctan(\sqrt{b*x + a}/\sqrt{-a}))/\sqrt{-a} - \sqrt{b*x + a}*b/x/b$

Mupad [B]

time = 0.05, size = 31, normalized size = 0.79

$$-\frac{\sqrt{a+bx}}{x} - \frac{b \operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^(1/2)/x^2,x)

[Out] $-(a + b*x)^{(1/2)}/x - (b*\operatorname{atanh}((a + b*x)^{(1/2)}/a^{(1/2)}))/a^{(1/2)}$

$$3.182 \quad \int \frac{1}{\sqrt{a+bx}} dx$$

Optimal. Leaf size=14

$$\frac{2\sqrt{a+bx}}{b}$$

[Out] 2*(b*x+a)^(1/2)/b

Rubi [A]

time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*x],x]

[Out] (2*Sqrt[a + b*x])/b

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}} dx = \frac{2\sqrt{a+bx}}{b}$$

Mathematica [A]

time = 0.01, size = 14, normalized size = 1.00

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b*x],x]

[Out] (2*Sqrt[a + b*x])/b

Mathics [A]

time = 1.66, size = 12, normalized size = 0.86

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/Sqrt[a+b*x],x]')``[Out] 2 Sqrt[a + b x] / b`**Maple [A]**

time = 0.03, size = 13, normalized size = 0.93

method	result	size
gospers	$\frac{2\sqrt{bx+a}}{b}$	13
derivativdivides	$\frac{2\sqrt{bx+a}}{b}$	13
default	$\frac{2\sqrt{bx+a}}{b}$	13
trager	$\frac{2\sqrt{bx+a}}{b}$	13
risch	$\frac{2\sqrt{bx+a}}{b}$	13

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)``[Out] 2*(b*x+a)^(1/2)/b`**Maxima [A]**

time = 0.30, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(b*x+a)^(1/2),x, algorithm="maxima")``[Out] 2*sqrt(b*x + a)/b`**Fricas [A]**

time = 0.29, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(b*x + a)/b

Sympy [A]

time = 0.03, size = 10, normalized size = 0.71

$$\frac{2\sqrt{a + bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**(1/2),x)

[Out] 2*sqrt(a + b*x)/b

Giac [A]

time = 0.00, size = 13, normalized size = 0.93

$$\frac{2\sqrt{a + bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(1/2),x)

[Out] 2*sqrt(b*x + a)/b

Mupad [B]

time = 0.02, size = 12, normalized size = 0.86

$$\frac{2\sqrt{a + bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*x)^(1/2),x)

[Out] (2*(a + b*x)^(1/2))/b

$$3.183 \quad \int \frac{x}{\sqrt{a+bx}} dx$$

Optimal. Leaf size=32

$$-\frac{2a\sqrt{a+bx}}{b^2} + \frac{2(a+bx)^{3/2}}{3b^2}$$

[Out] $2/3*(b*x+a)^{(3/2)}/b^2-2*a*(b*x+a)^{(1/2)}/b^2$

Rubi [A]

time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{2(a+bx)^{3/2}}{3b^2} - \frac{2a\sqrt{a+bx}}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a + b*x], x]

[Out] $(-2*a*\text{Sqrt}[a + b*x])/b^2 + (2*(a + b*x)^{(3/2)})/(3*b^2)$

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{a+bx}} dx &= \int \left(-\frac{a}{b\sqrt{a+bx}} + \frac{\sqrt{a+bx}}{b} \right) dx \\ &= -\frac{2a\sqrt{a+bx}}{b^2} + \frac{2(a+bx)^{3/2}}{3b^2} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 23, normalized size = 0.72

$$\frac{2(-2a+bx)\sqrt{a+bx}}{3b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[a + b*x], x]

[Out] $(2*(-2*a + b*x)*\text{Sqrt}[a + b*x])/(3*b^2)$

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 83 vs. $2(32) = 64$.
time = 3.34, size = 73, normalized size = 2.28

$$\frac{2\sqrt{a} \left(2a^2 \left(1 - \sqrt{\frac{a+bx}{a}} \right) + abx \left(2 - \sqrt{\frac{a+bx}{a}} \right) + b^2x^2 \sqrt{\frac{a+bx}{a}} \right)}{3b^2(a+bx)}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x/Sqrt[a+b*x], x]')

[Out] $2 \text{Sqrt}[a] (2 a^2 (1 - \text{Sqrt}[(a + b x) / a]) + a b x (2 - \text{Sqrt}[(a + b x) / a]) + b^2 x^2 \text{Sqrt}[(a + b x) / a]) / (3 b^2 (a + b x))$

Maple [A]

time = 0.03, size = 26, normalized size = 0.81

method	result	size
gospers	$-\frac{2\sqrt{bx+a}(-bx+2a)}{3b^2}$	21
trager	$-\frac{2\sqrt{bx+a}(-bx+2a)}{3b^2}$	21
risch	$-\frac{2\sqrt{bx+a}(-bx+2a)}{3b^2}$	21
derivativedivides	$\frac{2(bx+a)^{\frac{3}{2}}}{3} - \frac{2a\sqrt{bx+a}}{b^2}$	26
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3} - \frac{2a\sqrt{bx+a}}{b^2}$	26

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x+a)^(1/2), x, method=_RETURNVERBOSE)

[Out] $2/b^2*(1/3*(b*x+a)^(3/2)-a*(b*x+a)^(1/2))$

Maxima [A]

time = 0.25, size = 26, normalized size = 0.81

$$\frac{2(bx+a)^{\frac{3}{2}}}{3b^2} - \frac{2\sqrt{bx+a}a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x+a)^(1/2), x, algorithm="maxima")

[Out] $2/3*(b*x + a)^{(3/2)}/b^2 - 2*\sqrt{b*x + a}*a/b^2$

Fricas [A]

time = 0.30, size = 19, normalized size = 0.59

$$\frac{2\sqrt{bx+a}(bx-2a)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x+a)^(1/2),x, algorithm="fricas")`

[Out] $2/3*\sqrt{b*x + a}*(b*x - 2*a)/b^2$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(29) = 58$

time = 0.55, size = 162, normalized size = 5.06

$$-\frac{4a^{\frac{7}{2}}\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x} + \frac{4a^{\frac{7}{2}}}{3a^2b^2+3ab^3x} - \frac{2a^{\frac{5}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x} + \frac{4a^{\frac{5}{2}}bx}{3a^2b^2+3ab^3x} + \frac{2a^{\frac{3}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x+a)**(1/2),x)`

[Out] $-4*a^{(7/2)}*\sqrt{1 + b*x/a}/(3*a^{**2}*b^{**2} + 3*a*b^{**3}*x) + 4*a^{(7/2)}/(3*a^{**2}*b^{**2} + 3*a*b^{**3}*x) - 2*a^{(5/2)}*b*x*\sqrt{1 + b*x/a}/(3*a^{**2}*b^{**2} + 3*a*b^{**3}*x) + 4*a^{(5/2)}*b*x/(3*a^{**2}*b^{**2} + 3*a*b^{**3}*x) + 2*a^{(3/2)}*b^{**2}*x^{**2}*\sqrt{1 + b*x/a}/(3*a^{**2}*b^{**2} + 3*a*b^{**3}*x)$

Giac [A]

time = 0.00, size = 36, normalized size = 1.12

$$\frac{2\left(\frac{1}{3}\sqrt{a+bx}(a+bx) - a\sqrt{a+bx}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x+a)^(1/2),x)`

[Out] $2/3*((b*x + a)^{(3/2)} - 3*\sqrt{b*x + a}*a)/b^2$

Mupad [B]

time = 0.03, size = 25, normalized size = 0.78

$$-\frac{6a\sqrt{a+bx} - 2(a+bx)^{3/2}}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a + b*x)^(1/2),x)`

[Out] $-(6*a*(a + b*x)^{(1/2)} - 2*(a + b*x)^{(3/2)})/(3*b^2)$

$$3.184 \quad \int \frac{x^2}{\sqrt{a+bx}} dx$$

Optimal. Leaf size=51

$$\frac{2a^2\sqrt{a+bx}}{b^3} - \frac{4a(a+bx)^{3/2}}{3b^3} + \frac{2(a+bx)^{5/2}}{5b^3}$$

[Out] $-4/3*a*(b*x+a)^{(3/2)}/b^3+2/5*(b*x+a)^{(5/2)}/b^3+2*a^2*(b*x+a)^{(1/2)}/b^3$

Rubi [A]

time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{2a^2\sqrt{a+bx}}{b^3} + \frac{2(a+bx)^{5/2}}{5b^3} - \frac{4a(a+bx)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[a + b*x], x]

[Out] $(2*a^2*sqrt[a + b*x])/b^3 - (4*a*(a + b*x)^{(3/2)})/(3*b^3) + (2*(a + b*x)^{(5/2)})/(5*b^3)$

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{a+bx}} dx &= \int \left(\frac{a^2}{b^2\sqrt{a+bx}} - \frac{2a\sqrt{a+bx}}{b^2} + \frac{(a+bx)^{3/2}}{b^2} \right) dx \\ &= \frac{2a^2\sqrt{a+bx}}{b^3} - \frac{4a(a+bx)^{3/2}}{3b^3} + \frac{2(a+bx)^{5/2}}{5b^3} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 35, normalized size = 0.69

$$\frac{2\sqrt{a+bx} (8a^2 - 4abx + 3b^2x^2)}{15b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[a + b*x],x]

[Out] (2*sqrt[a + b*x]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3)

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 174 vs. 2(51) = 102.
time = 7.17, size = 160, normalized size = 3.14

$$\frac{2\sqrt{a} \left(8a^5 \left(-1 + \sqrt{\frac{a+bx}{a}} \right) + 4a^4bx \left(-6 + 5\sqrt{\frac{a+bx}{a}} \right) + 3a^3b^2x^2 \left(-8 + 5\sqrt{\frac{a+bx}{a}} \right) + \frac{5b^3x^3(a+bx)^2}{\sqrt{\frac{a+bx}{a}}} - 8a^2b^3x^3 + 3b^5x^5\sqrt{\frac{a+bx}{a}} \right)}{15b^3(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x^2/Sqrt[a+b*x],x]')

[Out] 2 Sqrt[a] (8 a ^ 5 (-1 + Sqrt[(a + b x) / a]) + 4 a ^ 4 b x (-6 + 5 Sqrt[(a + b x) / a]) + 3 a ^ 3 b ^ 2 x ^ 2 (-8 + 5 Sqrt[(a + b x) / a]) + 5 b ^ 3 x ^ 3 (a + b x) ^ 2 / Sqrt[(a + b x) / a] - 8 a ^ 2 b ^ 3 x ^ 3 + 3 b ^ 5 x ^ 5 Sqrt[(a + b x) / a]) / (15 b ^ 3 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))

Maple [A]

time = 0.03, size = 37, normalized size = 0.73

method	result	size
gospers	$\frac{2\sqrt{bx+a}}{15b^3} (3x^2b^2-4abx+8a^2)$	32
trager	$\frac{2\sqrt{bx+a}}{15b^3} (3x^2b^2-4abx+8a^2)$	32
risch	$\frac{2\sqrt{bx+a}}{15b^3} (3x^2b^2-4abx+8a^2)$	32
derivativdivides	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5} - \frac{4a(bx+a)^{\frac{3}{2}}}{3} + 2a^2\sqrt{bx+a}}{b^3}$	37
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5} - \frac{4a(bx+a)^{\frac{3}{2}}}{3} + 2a^2\sqrt{bx+a}}{b^3}$	37

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/b^3*(1/5*(b*x+a)^(5/2)-2/3*a*(b*x+a)^(3/2)+a^2*(b*x+a)^(1/2))

Maxima [A]

time = 0.26, size = 41, normalized size = 0.80

$$\frac{2(bx+a)^{\frac{5}{2}}}{5b^3} - \frac{4(bx+a)^{\frac{3}{2}}a}{3b^3} + \frac{2\sqrt{bx+a}a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="maxima")

[Out] $2/5*(b*x + a)^{(5/2)}/b^3 - 4/3*(b*x + a)^{(3/2)}*a/b^3 + 2*\sqrt{b*x + a}*a^2/b^3$

Fricas [A]

time = 0.31, size = 31, normalized size = 0.61

$$\frac{2(3b^2x^2 - 4abx + 8a^2)\sqrt{bx + a}}{15b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="fricas")

[Out] $2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*\sqrt{b*x + a}/b^3$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(48) = 96

time = 0.85, size = 600, normalized size = 11.76

$$\frac{2\sqrt{bx+a}}{15b^3} \left(3b^2x^2 - 4abx + 8a^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x+a)**(1/2),x)

[Out] $16*a^{**}(21/2)*\sqrt{1 + b*x/a}/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3}) - 16*a^{**}(21/2)/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3}) + 40*a^{**}(19/2)*b*x*\sqrt{1 + b*x/a}/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3}) - 48*a^{**}(19/2)*b*x/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3}) + 30*a^{**}(17/2)*b^{**2}*x^{**2}*\sqrt{1 + b*x/a}/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3}) - 48*a^{**}(17/2)*b^{**2}*x^{**2}/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3}) + 10*a^{**}(15/2)*b^{**3}*x^{**3}*\sqrt{1 + b*x/a}/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3}) - 16*a^{**}(15/2)*b^{**3}*x^{**3}/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3}) + 10*a^{**}(13/2)*b^{**4}*x^{**4}*\sqrt{1 + b*x/a}/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3}) + 6*a^{**}(11/2)*b^{**5}*x^{**5}*\sqrt{1 + b*x/a}/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3})$

Giac [A]

time = 0.00, size = 61, normalized size = 1.20

$$\frac{2\left(\frac{1}{5}\sqrt{a+bx}(a+bx)^2 - \frac{2}{3}\sqrt{a+bx}(a+bx)a + \sqrt{a+bx}a^2\right)}{bb^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x+a)^(1/2),x)`

[Out] $2/15*(3*(b*x + a)^{(5/2)} - 10*(b*x + a)^{(3/2)}*a + 15*\sqrt{b*x + a}*a^2)/b^3$

Mupad [B]

time = 0.04, size = 37, normalized size = 0.73

$$\frac{6(a + bx)^{5/2} - 20a(a + bx)^{3/2} + 30a^2\sqrt{a + bx}}{15b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a + b*x)^(1/2),x)`

[Out] $(6*(a + b*x)^{(5/2)} - 20*a*(a + b*x)^{(3/2)} + 30*a^2*(a + b*x)^{(1/2)})/(15*b^3)$
)

$$3.185 \quad \int \frac{1}{x\sqrt{a+bx}} dx$$

Optimal. Leaf size=23

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

[Out] $-2*\operatorname{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)})/a^{(1/2)}$

Rubi [A]

time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {65, 214}

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] `Int[1/(x*Sqrt[a + b*x]),x]`

[Out] `(-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]`

Rule 65

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 214

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{a+bx}} dx &= \frac{2\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+bx}\right)}{b} \\ &= -\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 23, normalized size = 1.00

$$-\frac{2 \tanh^{-1} \left(\frac{\sqrt{a+bx}}{\sqrt{a}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x*Sqrt[a + b*x]),x]``[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]`**Mathics [A]**

time = 2.24, size = 16, normalized size = 0.70

$$\frac{-2 \operatorname{ArcSinh} \left[\frac{\sqrt{a}}{\sqrt{b} \sqrt{x}} \right]}{\sqrt{a}}$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x*Sqrt[a+b*x]),x]')``[Out] -2 ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] / Sqrt[a]`**Maple [A]**

time = 0.03, size = 18, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh} \left(\frac{\sqrt{bx+a}}{\sqrt{a}} \right)}{\sqrt{a}}$	18
default	$-\frac{2 \operatorname{arctanh} \left(\frac{\sqrt{bx+a}}{\sqrt{a}} \right)}{\sqrt{a}}$	18

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)``[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)`**Maxima [A]**

time = 0.34, size = 32, normalized size = 1.39

$$\frac{\log \left(\frac{\sqrt{bx+a} - \sqrt{a}}{\sqrt{bx+a} + \sqrt{a}} \right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")

[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)

Fricas [A]

time = 0.32, size = 56, normalized size = 2.43

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a+2a}}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")

[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a)/a]

Sympy [A]

time = 0.48, size = 24, normalized size = 1.04

$$-\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x+a)**(1/2),x)

[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)

Giac [A]

time = 0.00, size = 27, normalized size = 1.17

$$\frac{2\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x+a)^(1/2),x)

[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)

Mupad [B]

time = 0.14, size = 17, normalized size = 0.74

$$-\frac{2\operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(1/2)),x)
```

```
[Out] -(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

$$3.186 \quad \int \frac{1}{x^2 \sqrt{a + bx}} dx$$

Optimal. Leaf size=41

$$-\frac{\sqrt{a+bx}}{ax} + \frac{b \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

[Out] b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)-(b*x+a)^(1/2)/a/x

Rubi [A]

time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {44, 65, 214}

$$\frac{b \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\sqrt{a+bx}}{ax}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[a + b*x]),x]

[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)

Rule 44

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && LtQ[n, 0]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 214

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \sqrt{a+bx}} dx &= -\frac{\sqrt{a+bx}}{ax} - \frac{b \int \frac{1}{x \sqrt{a+bx}} dx}{2a} \\
&= -\frac{\sqrt{a+bx}}{ax} - \frac{\text{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+bx} \right)}{a} \\
&= -\frac{\sqrt{a+bx}}{ax} + \frac{b \tanh^{-1} \left(\frac{\sqrt{a+bx}}{\sqrt{a}} \right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 41, normalized size = 1.00

$$-\frac{\sqrt{a+bx}}{ax} + \frac{b \tanh^{-1} \left(\frac{\sqrt{a+bx}}{\sqrt{a}} \right)}{a^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x^2*sqrt[a + b*x]),x]``[Out] -(sqrt[a + b*x]/(a*x)) + (b*ArcTanh[sqrt[a + b*x]/sqrt[a]])/a^(3/2)`**Mathics [A]**

time = 3.13, size = 40, normalized size = 0.98

$$-\frac{\sqrt{b} \sqrt{1 + \frac{a}{bx}}}{a\sqrt{x}} + \frac{b \text{ArcSinh} \left[\frac{\sqrt{a}}{\sqrt{b} \sqrt{x}} \right]}{a^{3/2}}$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x^2*sqrt[a+b*x]),x]')``[Out] -sqrt[b] Sqrt[1 + a / (b x)] / (a Sqrt[x]) + b ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] / a ^ (3 / 2)`**Maple [A]**

time = 0.03, size = 40, normalized size = 0.98

method	result	size
risch	$\frac{b \operatorname{arctanh} \left(\frac{\sqrt{bx+a}}{\sqrt{a}} \right)}{a^{3/2}} - \frac{\sqrt{bx+a}}{ax}$	34

derivativedivides	$2b \left(-\frac{\sqrt{bx+a}}{2abx} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	40
default	$2b \left(-\frac{\sqrt{bx+a}}{2abx} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	40

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $2*b*(-1/2*(b*x+a)^(1/2)/a/b/x+1/2/a^(3/2)*\operatorname{arctanh}((b*x+a)^(1/2)/a^(1/2)))$

Maxima [A]

time = 0.34, size = 60, normalized size = 1.46

$$-\frac{\sqrt{bx+a} b}{(bx+a)a-a^2} - \frac{b \log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="maxima")`

[Out] $-\operatorname{sqrt}(b*x+a)*b/((b*x+a)*a-a^2) - 1/2*b*\log((\operatorname{sqrt}(b*x+a) - \operatorname{sqrt}(a))/(\operatorname{sqrt}(b*x+a) + \operatorname{sqrt}(a)))/a^(3/2)$

Fricas [A]

time = 0.32, size = 93, normalized size = 2.27

$$\left[\frac{\sqrt{a} b x \log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) - 2\sqrt{bx+a} a}{2a^2 x}, -\frac{\sqrt{-a} b x \operatorname{arctan}\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + \sqrt{bx+a} a}{a^2 x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="fricas")`

[Out] $[1/2*(\operatorname{sqrt}(a)*b*x*\log((b*x+2*\operatorname{sqrt}(b*x+a)*\operatorname{sqrt}(a)+2*a)/x) - 2*\operatorname{sqrt}(b*x+a)*a)/(a^2*x), -(\operatorname{sqrt}(-a)*b*x*\operatorname{arctan}(\operatorname{sqrt}(b*x+a)*\operatorname{sqrt}(-a)/a) + \operatorname{sqrt}(b*x+a)*a)/(a^2*x)]$

Sympy [A]

time = 1.13, size = 44, normalized size = 1.07

$$-\frac{\sqrt{b} \sqrt{\frac{a}{bx}+1}}{a\sqrt{x}} + \frac{b \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(b*x+a)**(1/2),x)`

[Out] $-\sqrt{b}\sqrt{a/(b*x) + 1}/(a*\sqrt{x}) + b*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*\sqrt{x}))/a**(3/2)$

Giac [A]

time = 0.00, size = 65, normalized size = 1.59

$$\frac{2 \left(-\frac{\sqrt{a+bx} b^2}{2a(a+bx-a)} - \frac{b^2 \arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{a \cdot 2\sqrt{-a}} \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(b*x+a)^(1/2),x)`

[Out] $-(b^2*\arctan(\sqrt{b*x + a}/\sqrt{-a}))/(\sqrt{-a}*a) + \sqrt{b*x + a}*b/(a*x))/b$

Mupad [B]

time = 0.16, size = 33, normalized size = 0.80

$$\frac{b \operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\sqrt{a+bx}}{a x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(a + b*x)^(1/2)),x)`

[Out] $(b*\operatorname{atanh}((a + b*x)^(1/2)/a^(1/2)))/a^(3/2) - (a + b*x)^(1/2)/(a*x)$

3.187 $\int (a + bx)^{p/2} dx$

Optimal. Leaf size=23

$$\frac{2(a + bx)^{\frac{2+p}{2}}}{b(2 + p)}$$

[Out] $2*(b*x+a)^{(1+1/2*p)}/b/(2+p)$

Rubi [A]

time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {32}

$$\frac{2(a + bx)^{\frac{p+2}{2}}}{b(p + 2)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x)^{(p/2)}, x]$

[Out] $(2*(a + b*x)^{((2 + p)/2)})/(b*(2 + p))$

Rule 32

$\text{Int}[(a_.) + (b_.)*(x_.))^{(m_.)}, x_Symbol] \text{ :> Simp}[(a + b*x)^{(m + 1)}/(b*(m + 1)), x] \text{ /; FreeQ}\{a, b, m\}, x \ \&\& \ \text{NeQ}\{m, -1\}$

Rubi steps

$$\int (a + bx)^{p/2} dx = \frac{2(a + bx)^{\frac{2+p}{2}}}{b(2 + p)}$$

Mathematica [A]

time = 0.02, size = 24, normalized size = 1.04

$$\frac{2(a + bx)^{1+\frac{p}{2}}}{2b + bp}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(a + b*x)^{(p/2)}, x]$

[Out] $(2*(a + b*x)^{(1 + p/2)})/(2*b + b*p)$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Sqrt[a+b*x]^p,x]')``[Out] cought exception:`**Maple [A]**

time = 0.03, size = 25, normalized size = 1.09

method	result	size
gospers	$\frac{2(bx+a)(bx+a)^{\frac{p}{2}}}{b(2+p)}$	25
risch	$\frac{2(bx+a)(bx+a)^{\frac{p}{2}}}{b(2+p)}$	25

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(((b*x+a)^(1/2))^p,x,method=_RETURNVERBOSE)``[Out] 2*(b*x+a)*((b*x+a)^(1/2))^p/b/(2+p)`**Maxima [A]**

time = 0.26, size = 21, normalized size = 0.91

$$\frac{2(bx+a)^{\frac{1}{2}p+1}}{b(p+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(((b*x+a)^(1/2))^p,x, algorithm="maxima")``[Out] 2*(b*x + a)^(1/2*p + 1)/(b*(p + 2))`**Fricas [A]**

time = 0.32, size = 25, normalized size = 1.09

$$\frac{2(bx+a)\sqrt{bx+a}^p}{bp+2b}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(((b*x+a)^(1/2))^p,x, algorithm="fricas")``[Out] 2*(b*x + a)*sqrt(b*x + a)^p/(b*p + 2*b)`

Sympy [A]

time = 0.03, size = 24, normalized size = 1.04

$$\frac{\begin{cases} \frac{(a+bx)^{\frac{p}{2}+1}}{\frac{p}{2}+1} & \text{for } p \neq -2 \\ \log(a+bx) & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(((b*x+a)**(1/2))**p,x)``[Out] Piecewise(((a + b*x)**(p/2 + 1)/(p/2 + 1), Ne(p, -2)), (log(a + b*x), True)))/b`**Giac [A]**

time = 0.00, size = 22, normalized size = 0.96

$$\frac{(bx+a)^{\frac{p}{2}+1}}{b\left(\frac{p}{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(((b*x+a)^(1/2))^p,x)``[Out] 2*(b*x + a)^(1/2*p + 1)/(b*(p + 2))`**Mupad [B]**

time = 0.25, size = 21, normalized size = 0.91

$$\frac{2(a+bx)^{\frac{p}{2}+1}}{b(p+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((a + b*x)^(p/2),x)``[Out] (2*(a + b*x)^(p/2 + 1))/(b*(p + 2))`

3.188 $\int x(a + bx)^{p/2} dx$

Optimal. Leaf size=48

$$-\frac{2a(a + bx)^{\frac{2+p}{2}}}{b^2(2 + p)} + \frac{2(a + bx)^{\frac{4+p}{2}}}{b^2(4 + p)}$$

[Out] $-2*a*(b*x+a)^{(1+1/2*p)}/b^2/(2+p)+2*(b*x+a)^{(2+1/2*p)}/b^2/(4+p)$

Rubi [A]

time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{2(a + bx)^{\frac{p+4}{2}}}{b^2(p + 4)} - \frac{2a(a + bx)^{\frac{p+2}{2}}}{b^2(p + 2)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*(a + b*x)^{(p/2)}, x]$

[Out] $(-2*a*(a + b*x)^{((2 + p)/2)})/(b^2*(2 + p)) + (2*(a + b*x)^{((4 + p)/2)})/(b^2*(4 + p))$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int x(a + bx)^{p/2} dx &= \int \left(\frac{(a + bx)^{1+\frac{p}{2}}}{b} - \frac{a(a + bx)^{p/2}}{b} \right) dx \\ &= -\frac{2a(a + bx)^{\frac{2+p}{2}}}{b^2(2 + p)} + \frac{2(a + bx)^{\frac{4+p}{2}}}{b^2(4 + p)} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 38, normalized size = 0.79

$$\frac{2(a + bx)^{1+\frac{p}{2}}(-2a + b(2 + p)x)}{b^2(2 + p)(4 + p)}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x)^(p/2), x]

[Out] (2*(a + b*x)^(1 + p/2)*(-2*a + b*(2 + p)*x))/(b^2*(2 + p)*(4 + p))

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.58, size = 226, normalized size = 4.71

Piecewise $\left[\left\{ \left\{ \frac{x^2 a^{\frac{p}{2}}}{2}, b=0 \right\}, \left\{ \frac{a + a \operatorname{Log} \left[\frac{a}{b} + x \right] + b x \operatorname{Log} \left[\frac{a}{b} + x \right]}{b^2 (a + b x)}, p=-4 \right\}, \left\{ \frac{-a \operatorname{Log} \left[\frac{a}{b} + x \right] + b x}{b^2}, p=-2 \right\} \right], \frac{-4 a^2 (a + b x)^{\frac{p}{2}}}{8 b^2 + 6 b^2 p + b^2 p^2} + \frac{2 a b p x (a + b x)^{\frac{p}{2}}}{8 b^2 + 6 b^2 p + b^2 p^2} + \frac{4 b^2 x^2 (a + b x)^{\frac{p}{2}}}{8 b^2 + 6 b^2 p + b^2 p^2} + \frac{2 b^2 p x^2 (a + b x)^{\frac{p}{2}}}{8 b^2 + 6 b^2 p + b^2 p^2} \right]$

Antiderivative was successfully verified.

[In] mathics('Integrate[x*sqrt[a+b*x]^p,x]')

[Out] Piecewise[{{x^2 a^(p/2)/2, b==0}, {(a + a Log[a/b + x] + b x Log[a/b + x]) / (b^2 (a + b x)), p == -4}, {(-a Log[a/b + x] + b x) / b^2, p == -2}}, -4 a^2 (a + b x)^(p/2) / (8 b^2 + 6 b^2 p + b^2 p^2) + 2 a b p x (a + b x)^(p/2) / (8 b^2 + 6 b^2 p + b^2 p^2) + 4 b^2 x^2 (a + b x)^(p/2) / (8 b^2 + 6 b^2 p + b^2 p^2) + 2 b^2 p x^2 (a + b x)^(p/2) / (8 b^2 + 6 b^2 p + b^2 p^2)]

Maple [A]

time = 0.03, size = 43, normalized size = 0.90

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{p}{2}}(-xpb-2bx+2a)(bx+a)}{b^2(p^2+6p+8)}$	43
risch	$-\frac{2(-x^2b^2p-xapb-2x^2b^2+2a^2)(bx+a)^{\frac{p}{2}}}{b^2(4+p)(2+p)}$	54

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*((b*x+a)^(1/2))^p,x,method=_RETURNVERBOSE)

[Out] -2*((b*x+a)^(1/2))^p*(-b*p*x-2*b*x+2*a)*(b*x+a)/b^2/(p^2+6*p+8)

Maxima [A]

time = 0.26, size = 45, normalized size = 0.94

$$\frac{2(b^2(p+2)x^2 + abpx - 2a^2)(bx+a)^{\frac{1}{2}p}}{(p^2 + 6p + 8)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x+a)^(1/2))^p,x, algorithm="maxima")

[Out] 2*(b^2*(p + 2)*x^2 + a*b*p*x - 2*a^2)*(b*x + a)^(1/2*p)/((p^2 + 6*p + 8)*b^2)

Fricas [A]

time = 0.30, size = 58, normalized size = 1.21

$$\frac{2(abpx + (b^2p + 2b^2)x^2 - 2a^2)\sqrt{bx + a}^p}{b^2p^2 + 6b^2p + 8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x+a)^(1/2))^p,x, algorithm="fricas")

[Out] 2*(a*b*p*x + (b^2*p + 2*b^2)*x^2 - 2*a^2)*sqrt(b*x + a)^p/(b^2*p^2 + 6*b^2*p + 8*b^2)

Sympy [A]

time = 0.25, size = 216, normalized size = 4.50

$$\begin{cases} \frac{a^{\frac{p}{2}}x^2}{2} & \text{for } b = 0 \\ \frac{a \log\left(\frac{a}{b} + x\right)}{ab^2 + b^3x} + \frac{a}{ab^2 + b^3x} + \frac{bx \log\left(\frac{a}{b} + x\right)}{ab^2 + b^3x} & \text{for } p = -4 \\ -\frac{a \log\left(\frac{a}{b} + x\right)}{b^2} + \frac{x}{b} & \text{for } p = -2 \\ -\frac{4a^2(a+bx)^{\frac{p}{2}}}{b^2p^2 + 6b^2p + 8b^2} + \frac{2abpx(a+bx)^{\frac{p}{2}}}{b^2p^2 + 6b^2p + 8b^2} + \frac{2b^2px^2(a+bx)^{\frac{p}{2}}}{b^2p^2 + 6b^2p + 8b^2} + \frac{4b^2x^2(a+bx)^{\frac{p}{2}}}{b^2p^2 + 6b^2p + 8b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x+a)**(1/2))**p,x)

[Out] Piecewise((a**(p/2)*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) + a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(p, -4)), (-a*log(a/b + x)/b**2 + x/b, Eq(p, -2)), (-4*a**2*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2) + 2*a*b*p*x*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2) + 2*b**2*p*x**2*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2) + 4*b**2*x**2*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2), True))

Giac [A]

time = 0.00, size = 95, normalized size = 1.98

$$\frac{-4a^2e^{\frac{1}{2}p\ln(a+bx)} + 2abpxe^{\frac{1}{2}p\ln(a+bx)} + 2b^2px^2e^{\frac{1}{2}p\ln(a+bx)} + 4b^2x^2e^{\frac{1}{2}p\ln(a+bx)}}{b^2p^2 + 6b^2p + 8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x+a)^(1/2))^p,x)

[Out] 2*((b*x + a)^(1/2*p)*b^2*p*x^2 + (b*x + a)^(1/2*p)*a*b*p*x + 2*(b*x + a)^(1/2*p)*b^2*x^2 - 2*(b*x + a)^(1/2*p)*a^2)/(b^2*p^2 + 6*b^2*p + 8*b^2)

Mupad [B]

time = 0.45, size = 94, normalized size = 1.96

$$\left\{ \begin{array}{ll} -\frac{a \ln(a+bx)-bx}{b^2} & \text{if } p = -2 \\ \frac{\ln(a+bx)+\frac{a}{a+bx}}{b^2} & \text{if } p = -4 \\ \frac{2 \left(\frac{(a+bx)^{\frac{p}{2}+2}}{p+4} - \frac{a(a+bx)^{\frac{p}{2}+1}}{p+2} \right)}{b^2} & \text{if } p \neq -2 \wedge p \neq -4 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a + b*x)^(p/2),x)`

[Out] `piecewise(p == -2, -(a*log(a + b*x) - b*x)/b^2, p == -4, (log(a + b*x) + a/(a + b*x))/b^2, p ~= -2 & p ~= -4, (2*((a + b*x)^(p/2 + 2)/(p + 4) - (a*(a + b*x)^(p/2 + 1))/(p + 2)))/b^2)`

$$3.189 \quad \int \tan^{-1} \left(\frac{-\sqrt{2} + 2x}{\sqrt{2}} \right) dx$$

Optimal. Leaf size=55

$$\frac{\tan^{-1}(1 - \sqrt{2}x)}{\sqrt{2}} - x \tan^{-1}(1 - \sqrt{2}x) - \frac{\log(1 - \sqrt{2}x + x^2)}{2\sqrt{2}}$$

[Out] $x \arctan(-1 + x \cdot 2^{(1/2)}) - 1/2 \arctan(-1 + x \cdot 2^{(1/2)}) \cdot 2^{(1/2)} - 1/4 \ln(1 + x^2 - x \cdot 2^{(1/2)}) \cdot 2^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5311, 12, 648, 631, 210, 642}

$$-\frac{\log(x^2 - \sqrt{2}x + 1)}{2\sqrt{2}} - x \tan^{-1}(1 - \sqrt{2}x) + \frac{\tan^{-1}(1 - \sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[(-Sqrt[2] + 2*x)/Sqrt[2]],x]

[Out] ArcTan[1 - Sqrt[2]*x]/Sqrt[2] - x*ArcTan[1 - Sqrt[2]*x] - Log[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 631

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] := Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \tan^{-1}\left(\frac{-\sqrt{2} + 2x}{\sqrt{2}}\right) dx &= -x \tan^{-1}(1 - \sqrt{2}x) - \int \frac{x}{\sqrt{2}(1 - \sqrt{2}x + x^2)} dx \\
&= -x \tan^{-1}(1 - \sqrt{2}x) - \frac{\int \frac{x}{1 - \sqrt{2}x + x^2} dx}{\sqrt{2}} \\
&= -x \tan^{-1}(1 - \sqrt{2}x) - \frac{1}{2} \int \frac{1}{1 - \sqrt{2}x + x^2} dx - \frac{\int \frac{-\sqrt{2} + 2x}{1 - \sqrt{2}x + x^2} dx}{2\sqrt{2}} \\
&= -x \tan^{-1}(1 - \sqrt{2}x) - \frac{\log(1 - \sqrt{2}x + x^2)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1 - x^2} dx, x, 1 - \sqrt{2}x\right)}{\sqrt{2}} \\
&= \frac{\tan^{-1}(1 - \sqrt{2}x)}{\sqrt{2}} - x \tan^{-1}(1 - \sqrt{2}x) - \frac{\log(1 - \sqrt{2}x + x^2)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 48, normalized size = 0.87

$$\frac{1}{4} \left(2 \left(\sqrt{2} - 2x \right) \tan^{-1} \left(1 - \sqrt{2}x \right) - \sqrt{2} \log \left(1 - \sqrt{2}x + x^2 \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(-Sqrt[2] + 2*x)/Sqrt[2]], x]
```


[Out] $(2*(\text{Sqrt}[2] - 2*x)*\text{ArcTan}[1 - \text{Sqrt}[2]*x] - \text{Sqrt}[2]*\text{Log}[1 - \text{Sqrt}[2]*x + x^2])/4$

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 121 vs. 2(55) = 110.
time = 3.85, size = 96, normalized size = 1.75

$$\frac{\frac{x \text{Log}\left[\frac{1-\sqrt{2}x+x^2}{2}\right] + x \text{ArcTan}\left[-1+\sqrt{2}x\right](2+x^2) - \frac{3\sqrt{2}x^2 \text{ArcTan}\left[-1+\sqrt{2}x\right]}{2} - \frac{\sqrt{2} \text{ArcTan}\left[-1+\sqrt{2}x\right]}{2} - \frac{\sqrt{2} \text{Log}\left[\frac{1-\sqrt{2}x+x^2}{4}\right](1+x^2)}{4}}{1-\sqrt{2}x+x^2}}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[ArcTan[(-Sqrt[2]+2*x)/Sqrt[2]],x]')`

[Out] $(x \text{Log}[1 - \text{Sqrt}[2] x + x^2] / 2 + x \text{ArcTan}[-1 + \text{Sqrt}[2] x] (2 + x^2) - 3 \text{Sqrt}[2] x^2 \text{ArcTan}[-1 + \text{Sqrt}[2] x] / 2 - \text{Sqrt}[2] \text{ArcTan}[-1 + \text{Sqrt}[2] x] / 2 - \text{Sqrt}[2] \text{Log}[1 - \text{Sqrt}[2] x + x^2] (1 + x^2) / 4) / (1 - \text{Sqrt}[2] x + x^2)$

Maple [A]

time = 0.05, size = 37, normalized size = 0.67

method	result
derivativdivides	$\frac{\sqrt{2} \left((-1+x\sqrt{2}) \arctan(-1+x\sqrt{2}) - \frac{\ln\left(\frac{(-1+x\sqrt{2})^2+1}{2}\right)}{2} \right)}{2}$
default	$\frac{\sqrt{2} \left((-1+x\sqrt{2}) \arctan(-1+x\sqrt{2}) - \frac{\ln\left(\frac{(-1+x\sqrt{2})^2+1}{2}\right)}{2} \right)}{2}$
risch	$\frac{ix \ln\left(1 + \frac{i(-2x+\sqrt{2})\sqrt{2}}{2}\right)}{2} - \frac{ix \ln\left(1 - \frac{i(-2x+\sqrt{2})\sqrt{2}}{2}\right)}{2} - \frac{\sqrt{2} \ln(4-4x\sqrt{2}+4x^2)}{4} - \frac{\sqrt{2} \arctan(-1+x\sqrt{2})}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x,method=_RETURNVERBOSE)`

[Out] $1/2*2^{(1/2)}*((-1+x*2^{(1/2)})*\arctan(-1+x*2^{(1/2)})-1/2*\ln((-1+x*2^{(1/2)})^2+1))$

Maxima [A]

time = 0.39, size = 52, normalized size = 0.95

$$\frac{1}{4} \sqrt{2} \left(\sqrt{2} (2x - \sqrt{2}) \arctan\left(\frac{1}{2} \sqrt{2} (2x - \sqrt{2})\right) - \log\left(\frac{1}{2} (2x - \sqrt{2})^2 + 1\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x, algorithm="maxima")

[Out] 1/4*sqrt(2)*(sqrt(2)*(2*x - sqrt(2))*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - log(1/2*(2*x - sqrt(2))^2 + 1))

Fricas [A]

time = 0.34, size = 37, normalized size = 0.67

$$\frac{1}{2} \left(2x - \sqrt{2} \right) \arctan \left(\sqrt{2} x - 1 \right) - \frac{1}{4} \sqrt{2} \log \left(x^2 - \sqrt{2} x + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x, algorithm="fricas")

[Out] 1/2*(2*x - sqrt(2))*arctan(sqrt(2)*x - 1) - 1/4*sqrt(2)*log(x^2 - sqrt(2)*x + 1)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(48) = 96

time = 0.43, size = 230, normalized size = 4.18

$$\frac{4x^3 \operatorname{atan}(\sqrt{2}x-1)}{4x^2-4\sqrt{2}x+4} - \frac{\sqrt{2}x^2 \log(x^2-\sqrt{2}x+1)}{4x^2-4\sqrt{2}x+4} - \frac{6\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}x-1)}{4x^2-4\sqrt{2}x+4} + \frac{2x \log(x^2-\sqrt{2}x+1)}{4x^2-4\sqrt{2}x+4} + \frac{8x \operatorname{atan}(\sqrt{2}x-1)}{4x^2-4\sqrt{2}x+4} - \frac{\sqrt{2} \log(x^2-\sqrt{2}x+1)}{4x^2-4\sqrt{2}x+4} - \frac{2\sqrt{2} \operatorname{atan}(\sqrt{2}x-1)}{4x^2-4\sqrt{2}x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(1/2*(2*x-2**(1/2))*2**(1/2)),x)

[Out] 4*x**3*atan(sqrt(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4) - sqrt(2)*x**2*log(x**2 - sqrt(2)*x + 1)/(4*x**2 - 4*sqrt(2)*x + 4) - 6*sqrt(2)*x**2*atan(sqrt(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4) + 2*x*log(x**2 - sqrt(2)*x + 1)/(4*x**2 - 4*sqrt(2)*x + 4) + 8*x*atan(sqrt(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4) - sqrt(2)*log(x**2 - sqrt(2)*x + 1)/(4*x**2 - 4*sqrt(2)*x + 4) - 2*sqrt(2)*atan(sqrt(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4)

Giac [A]

time = 0.00, size = 72, normalized size = 1.31

$$\frac{\frac{1}{2} \left(2x - \sqrt{2} \right) \sqrt{2} \arctan \left(\frac{1}{2} \left(2x - \sqrt{2} \right) \sqrt{2} \right) - \frac{\ln \left(\left(\frac{1}{2} \left(2x - \sqrt{2} \right) \sqrt{2} \right)^2 + 1 \right)}{2}}{\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x)

[Out] 1/4*sqrt(2)*(sqrt(2)*(2*x - sqrt(2))*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - log(1/2*(2*x - sqrt(2))^2 + 1))

Mupad [B]

time = 0.16, size = 43, normalized size = 0.78

$$\operatorname{atan}\left(\frac{\sqrt{2}(2x - \sqrt{2})}{2}\right)\left(x - \frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2} \ln\left(\left(2x - \sqrt{2}\right)^2 + 2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(atan((2^(1/2)*(2*x - 2^(1/2)))/2),x)`

[Out] `atan((2^(1/2)*(2*x - 2^(1/2)))/2)*(x - 2^(1/2)/2) - (2^(1/2)*log((2*x - 2^(1/2))^2 + 2))/4`

$$3.190 \quad \int \frac{1}{\sqrt{-1+x^2}} dx$$

Optimal. Leaf size=12

$$\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

[Out] arctanh(x/(x^2-1)^(1/2))

Rubi [A]

time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {223, 212}

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-1 + x^2], x]

[Out] ArcTanh[x/Sqrt[-1 + x^2]]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 223

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-1+x^2}} dx &= \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{-1+x^2}} \right) \\ &= \tanh^{-1} \left(\frac{x}{\sqrt{-1+x^2}} \right) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 38 vs. 2(12) = 24.

time = 0.00, size = 38, normalized size = 3.17

$$-\frac{1}{2} \log \left(1 - \frac{x}{\sqrt{-1+x^2}} \right) + \frac{1}{2} \log \left(1 + \frac{x}{\sqrt{-1+x^2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-1 + x^2],x]

[Out] $-1/2*\text{Log}[1 - x/\text{Sqrt}[-1 + x^2]] + \text{Log}[1 + x/\text{Sqrt}[-1 + x^2]]/2$

Mathics [A]

time = 1.64, size = 2, normalized size = 0.17

$\text{ArcCosh}[x]$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/Sqrt[x^2-1],x]')

[Out] ArcCosh[x]

Maple [A]

time = 0.05, size = 11, normalized size = 0.92

method	result	size
default	$\ln(x + \sqrt{x^2 - 1})$	11
trager	$\ln(x + \sqrt{x^2 - 1})$	11
meijerg	$\frac{\sqrt{-\text{signum}(x^2 - 1)} \arcsin(x)}{\sqrt{\text{signum}(x^2 - 1)}}$	22

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)

[Out] $\ln(x + (x^2 - 1)^{1/2})$

Maxima [A]

time = 0.25, size = 14, normalized size = 1.17

$\log(2x + 2\sqrt{x^2 - 1})$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-1)^(1/2),x, algorithm="maxima")

[Out] $\log(2*x + 2*\text{sqrt}(x^2 - 1))$

Fricas [A]

time = 0.31, size = 14, normalized size = 1.17

$-\log(-x + \sqrt{x^2 - 1})$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2-1)^(1/2),x, algorithm="fricas")`

[Out] `-log(-x + sqrt(x^2 - 1))`

Sympy [A]

time = 0.06, size = 2, normalized size = 0.17

$$\operatorname{acosh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2-1)**(1/2),x)`

[Out] `acosh(x)`

Giac [A]

time = 0.00, size = 15, normalized size = 1.25

$$-\ln \left| \sqrt{x^2 - 1} - x \right|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2-1)^(1/2),x)`

[Out] `-log(abs(-x + sqrt(x^2 - 1)))`

Mupad [B]

time = 0.20, size = 10, normalized size = 0.83

$$\ln \left(x + \sqrt{x^2 - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2 - 1)^(1/2),x)`

[Out] `log(x + (x^2 - 1)^(1/2))`

3.191 $\int \sqrt{x} \sqrt{1+x} dx$

Optimal. Leaf size=43

$$\frac{1}{4}\sqrt{x}\sqrt{1+x} + \frac{1}{2}x^{3/2}\sqrt{1+x} - \frac{1}{4}\sinh^{-1}(\sqrt{x})$$

[Out] $-1/4*\operatorname{arcsinh}(x^{(1/2)})+1/2*x^{(3/2)}*(1+x)^{(1/2)}+1/4*x^{(1/2)}*(1+x)^{(1/2)}$

Rubi [A]

time = 0.00, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 56, 221}

$$\frac{1}{2}\sqrt{x+1}x^{3/2} + \frac{1}{4}\sqrt{x+1}\sqrt{x} - \frac{1}{4}\sinh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[x]*Sqrt[1+x],x]`

[Out] $(\operatorname{Sqrt}[x]*\operatorname{Sqrt}[1+x])/4 + (x^{(3/2)}*\operatorname{Sqrt}[1+x])/2 - \operatorname{ArcSinh}[\operatorname{Sqrt}[x]]/4$

Rule 52

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 56

`Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]`

Rule 221

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]`

Rubi steps

$$\begin{aligned}
\int \sqrt{x} \sqrt{1+x} \, dx &= \frac{1}{2} x^{3/2} \sqrt{1+x} + \frac{1}{4} \int \frac{\sqrt{x}}{\sqrt{1+x}} \, dx \\
&= \frac{1}{4} \sqrt{x} \sqrt{1+x} + \frac{1}{2} x^{3/2} \sqrt{1+x} - \frac{1}{8} \int \frac{1}{\sqrt{x} \sqrt{1+x}} \, dx \\
&= \frac{1}{4} \sqrt{x} \sqrt{1+x} + \frac{1}{2} x^{3/2} \sqrt{1+x} - \frac{1}{4} \text{Subst} \left(\int \frac{1}{\sqrt{1+x^2}} \, dx, x, \sqrt{x} \right) \\
&= \frac{1}{4} \sqrt{x} \sqrt{1+x} + \frac{1}{2} x^{3/2} \sqrt{1+x} - \frac{1}{4} \sinh^{-1}(\sqrt{x})
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 37, normalized size = 0.86

$$\frac{1}{4} \left(\sqrt{x} \sqrt{1+x} (1+2x) - \tanh^{-1} \left(\sqrt{\frac{x}{1+x}} \right) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[Sqrt[x]*Sqrt[1+x],x]``[Out] (Sqrt[x]*Sqrt[1+x]*(1+2*x) - ArcTanh[Sqrt[x/(1+x)]])/4`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 3.61, size = 83, normalized size = 1.93

$$\text{Piecewise} \left[\left\{ \left\{ \frac{\sqrt{x} \sqrt{1+x}}{4} + \frac{x^{3/2} \sqrt{1+x}}{2} - \frac{\text{ArcCosh}[\sqrt{1+x}]}{4}, \text{Abs}[1+x] > 1 \right\} \right\}, \frac{I(1+x)^{5/2}}{2\sqrt{-x}} - \frac{I\sqrt{1+x}}{4\sqrt{-x}} + \frac{I \text{ArcSin}[\sqrt{1+x}]}{4} + \frac{I 3(1+x)^{3/2}}{4\sqrt{-x}} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Sqrt[x+1]*Sqrt[x],x]')`

```
[Out] Piecewise[{{Sqrt[x] Sqrt[1+x] / 4 + x^(3/2) Sqrt[1+x] / 2 - ArcCosh[Sqrt[1+x]] / 4, Abs[1+x] > 1}}, -I(1+x)^(5/2) / (2 Sqrt[-x]) - I Sqrt[1+x] / (4 Sqrt[-x]) + I ArcSin[Sqrt[1+x]] / 4 + I 3(1+x)^(3/2) / (4 Sqrt[-x])}]
```

Maple [A]

time = 0.03, size = 50, normalized size = 1.16

method	result	size
meijerg	$-\frac{\sqrt{\pi} \sqrt{x}^{(3+6x)} \sqrt{1+x}}{6} + \frac{\sqrt{\pi} \operatorname{arcsinh}(\sqrt{x})}{2}$	34

risch	$\frac{(1+2x)\sqrt{x}\sqrt{1+x}}{4} - \frac{\sqrt{x(1+x)}\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{8\sqrt{1+x}\sqrt{x}}$	45
default	$\frac{\sqrt{x}(1+x)^{\frac{3}{2}}}{2} - \frac{\sqrt{x}\sqrt{1+x}}{4} - \frac{\sqrt{x(1+x)}\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{8\sqrt{1+x}\sqrt{x}}$	50

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{2}x^{1/2}(1+x)^{3/2} - \frac{1}{4}x^{1/2}(1+x)^{1/2} - \frac{1}{8}(x(1+x))^{1/2}/(1+x)^{1/2}/x^{1/2} * \ln(x+1/2+(x^2+x)^{1/2})$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(27) = 54$.

time = 0.25, size = 71, normalized size = 1.65

$$\frac{\frac{(x+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}} + \frac{\sqrt{x+1}}{\sqrt{x}}}{4\left(\frac{(x+1)^2}{x^2} - \frac{2(x+1)}{x} + 1\right)} - \frac{1}{8} \log\left(\frac{\sqrt{x+1}}{\sqrt{x}} + 1\right) + \frac{1}{8} \log\left(\frac{\sqrt{x+1}}{\sqrt{x}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)*(1+x)^(1/2),x, algorithm="maxima")`

[Out] $\frac{1}{4}((x+1)^{3/2}/x^{3/2} + \sqrt{x+1}/\sqrt{x})/((x+1)^2/x^2 - 2(x+1)/x + 1) - \frac{1}{8}\log(\sqrt{x+1}/\sqrt{x} + 1) + \frac{1}{8}\log(\sqrt{x+1}/\sqrt{x} - 1)$

Fricas [A]

time = 0.33, size = 34, normalized size = 0.79

$$\frac{1}{4}(2x+1)\sqrt{x+1}\sqrt{x} + \frac{1}{8}\log\left(2\sqrt{x+1}\sqrt{x} - 2x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)*(1+x)^(1/2),x, algorithm="fricas")`

[Out] $\frac{1}{4}(2x+1)\sqrt{x+1}\sqrt{x} + \frac{1}{8}\log(2\sqrt{x+1}\sqrt{x} - 2x - 1)$

Sympy [A]

time = 1.56, size = 119, normalized size = 2.77

$$\left\{ \begin{array}{l} -\frac{\operatorname{acosh}\left(\sqrt{x+1}\right)}{4} + \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{x}} - \frac{3(x+1)^{\frac{3}{2}}}{4\sqrt{x}} + \frac{\sqrt{x+1}}{4\sqrt{x}} \quad \text{for } |x+1| > 1 \\ \frac{i \operatorname{asin}\left(\sqrt{x+1}\right)}{4} - \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{-x}} + \frac{3i(x+1)^{\frac{3}{2}}}{4\sqrt{-x}} - \frac{i\sqrt{x+1}}{4\sqrt{-x}} \quad \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/2)*(1+x)**(1/2),x)

[Out] Piecewise((-acosh(sqrt(x + 1))/4 + (x + 1)**(5/2)/(2*sqrt(x)) - 3*(x + 1)**(3/2)/(4*sqrt(x)) + sqrt(x + 1)/(4*sqrt(x)), Abs(x + 1) > 1), (I*asin(sqrt(x + 1))/4 - I*(x + 1)**(5/2)/(2*sqrt(-x)) + 3*I*(x + 1)**(3/2)/(4*sqrt(-x)) - I*sqrt(x + 1)/(4*sqrt(-x)), True))

Giac [A]

time = 0.00, size = 51, normalized size = 1.19

$$2 \left(2 \left(\frac{1}{8} \sqrt{x} \sqrt{x} + \frac{1}{16} \right) \sqrt{x} \sqrt{x+1} + \frac{\ln(\sqrt{x+1} - \sqrt{x})}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*(1+x)^(1/2),x)

[Out] 1/4*(2*x + 1)*sqrt(x + 1)*sqrt(x) + 1/4*log(sqrt(x + 1) - sqrt(x))

Mupad [B]

time = 0.22, size = 30, normalized size = 0.70

$$\sqrt{x} \left(\frac{x}{2} + \frac{1}{4} \right) \sqrt{x+1} - \frac{\ln\left(x + \sqrt{x} \sqrt{x+1} + \frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)*(x + 1)^(1/2),x)

[Out] x^(1/2)*(x/2 + 1/4)*(x + 1)^(1/2) - log(x + x^(1/2)*(x + 1)^(1/2) + 1/2)/8

3.192 $\int \sin(\sqrt{x}) dx$

Optimal. Leaf size=22

$$-2\sqrt{x} \cos(\sqrt{x}) + 2 \sin(\sqrt{x})$$

[Out] 2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3442, 3377, 2717}

$$2 \sin(\sqrt{x}) - 2\sqrt{x} \cos(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Sin[Sqrt[x]],x]

[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3377

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3442

Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_Symbol] := Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[1/n]

Rubi steps

$$\begin{aligned} \int \sin(\sqrt{x}) dx &= 2 \text{Subst} \left(\int x \sin(x) dx, x, \sqrt{x} \right) \\ &= -2\sqrt{x} \cos(\sqrt{x}) + 2 \text{Subst} \left(\int \cos(x) dx, x, \sqrt{x} \right) \\ &= -2\sqrt{x} \cos(\sqrt{x}) + 2 \sin(\sqrt{x}) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 22, normalized size = 1.00

$$-2\sqrt{x} \cos(\sqrt{x}) + 2 \sin(\sqrt{x})$$

Antiderivative was successfully verified.

`[In] Integrate[Sin[Sqrt[x]],x]``[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]`**Mathics [A]**

time = 1.79, size = 16, normalized size = 0.73

$$-2\sqrt{x} \text{Cos}[\sqrt{x}] + 2\text{Sin}[\sqrt{x}]$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sin[Sqrt[x]],x]')``[Out] -2 Sqrt[x] Cos[Sqrt[x]] + 2 Sin[Sqrt[x]]`**Maple [A]**

time = 0.01, size = 17, normalized size = 0.77

method	result	size
derivativedivides	$2 \sin(\sqrt{x}) - 2 \cos(\sqrt{x}) \sqrt{x}$	17
default	$2 \sin(\sqrt{x}) - 2 \cos(\sqrt{x}) \sqrt{x}$	17
meijerg	$4\sqrt{\pi} \left(-\frac{\sqrt{x} \cos(\sqrt{x})}{2\sqrt{\pi}} + \frac{\sin(\sqrt{x})}{2\sqrt{\pi}} \right)$	28

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sin(x^(1/2)),x,method=_RETURNVERBOSE)``[Out] 2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)`**Maxima [A]**

time = 0.26, size = 16, normalized size = 0.73

$$-2\sqrt{x} \cos(\sqrt{x}) + 2 \sin(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(x^(1/2)),x, algorithm="maxima")``[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))`

Fricas [A]

time = 0.32, size = 16, normalized size = 0.73

$$-2\sqrt{x}\cos(\sqrt{x}) + 2\sin(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x^(1/2)),x, algorithm="fricas")

[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))

Sympy [A]

time = 0.10, size = 20, normalized size = 0.91

$$-2\sqrt{x}\cos(\sqrt{x}) + 2\sin(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x**(1/2)),x)

[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))

Giac [A]

time = 0.00, size = 22, normalized size = 1.00

$$2(-\sqrt{x}\cos(\sqrt{x}) + \sin(\sqrt{x}))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x^(1/2)),x)

[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))

Mupad [B]

time = 0.25, size = 16, normalized size = 0.73

$$2\sin(\sqrt{x}) - 2\sqrt{x}\cos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x^(1/2)),x)

[Out] 2*sin(x^(1/2)) - 2*x^(1/2)*cos(x^(1/2))

$$3.193 \quad \int \frac{x}{(1-x^2)^{9/8}} dx$$

Optimal. Leaf size=13

$$\frac{4}{\sqrt[8]{1-x^2}}$$

[Out] 4/(-x^2+1)^(1/8)

Rubi [A]

time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$\frac{4}{\sqrt[8]{1-x^2}}$$

Antiderivative was successfully verified.

[In] Int[x/(1 - x^2)^(9/8),x]

[Out] 4/(1 - x^2)^(1/8)

Rule 267

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int \frac{x}{(1-x^2)^{9/8}} dx = \frac{4}{\sqrt[8]{1-x^2}}$$

Mathematica [A]

time = 0.00, size = 13, normalized size = 1.00

$$\frac{4}{\sqrt[8]{1-x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 - x^2)^(9/8),x]

[Out] 4/(1 - x^2)^(1/8)

Mathics [A]

time = 1.87, size = 11, normalized size = 0.85

$$\frac{4}{(1-x^2)^{\frac{1}{8}}}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x*Sqrt[1 - x^2]^(-9/4),x]')`[Out] `4 / (1 - x ^ 2) ^ (1 / 8)`**Maple [A]**

time = 0.05, size = 12, normalized size = 0.92

method	result	size
derivativdivides	$\frac{4}{(-x^2+1)^{\frac{1}{8}}}$	12
default	$\frac{4}{(-x^2+1)^{\frac{1}{8}}}$	12
risch	$\frac{4}{(-x^2+1)^{\frac{1}{8}}}$	12
meijerg	$\frac{x^2 \operatorname{hypergeom}\left(\left[1, \frac{9}{8}\right], [2], x^2\right)}{2}$	15
gosper	$-\frac{4(-1+x)(1+x)}{(-x^2+1)^{\frac{9}{8}}}$	18
trager	$-\frac{4(-x^2+1)^{\frac{7}{8}}}{x^2-1}$	19

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(-x^2+1)^(9/8),x,method=_RETURNVERBOSE)`[Out] `4/(-x^2+1)^(1/8)`**Maxima [A]**

time = 0.26, size = 11, normalized size = 0.85

$$\frac{4}{(-x^2+1)^{\frac{1}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-x^2+1)^(9/8),x, algorithm="maxima")`[Out] `4/(-x^2 + 1)^(1/8)`**Fricas [A]**

time = 0.31, size = 18, normalized size = 1.38

$$-\frac{4(-x^2+1)^{\frac{7}{8}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-x^2+1)^(9/8),x, algorithm="fricas")`

[Out] $-4*(-x^2 + 1)^{(7/8)}/(x^2 - 1)$

Sympy [A]

time = 0.30, size = 8, normalized size = 0.62

$$\frac{4}{\sqrt[8]{1-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-x**2+1)**(9/8),x)`

[Out] $4/(1 - x**2)**(1/8)$

Giac [A]

time = 0.00, size = 14, normalized size = 1.08

$$\frac{4}{(-x^2 + 1)^{\frac{1}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-x^2+1)^(9/8),x)`

[Out] $4/(-x^2 + 1)^{(1/8)}$

Mupad [B]

time = 0.35, size = 11, normalized size = 0.85

$$\frac{4}{(1-x^2)^{1/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(1-x^2)^(9/8),x)`

[Out] $4/(1-x^2)^{(1/8)}$

$$3.194 \quad \int \frac{x}{\sqrt{1-x^4}} dx$$

Optimal. Leaf size=8

$$\frac{1}{2} \sin^{-1}(x^2)$$

[Out] 1/2*arcsin(x^2)

Rubi [A]

time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {281, 222}

$$\frac{1}{2} \sin^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[1 - x^4],x]

[Out] ArcSin[x^2]/2

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 281

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{1-x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1-x^2}} dx, x, x^2 \right) \\ &= \frac{1}{2} \sin^{-1}(x^2) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16. time = 0.06, size = 20, normalized size = 2.50

$$-\frac{1}{2} \tan^{-1} \left(\frac{\sqrt{1-x^4}}{x^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[1 - x^4],x]

[Out] -1/2*ArcTan[Sqrt[1 - x^4]/x^2]

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.05, size = 21, normalized size = 2.62

$$\text{Piecewise} \left[\left\{ \left\{ \left(-\frac{I}{2} \right) \text{ArcCosh} [x^2], \text{Abs} [x^4] > 1 \right\} \right\}, \frac{\text{ArcSin} [x^2]}{2} \right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x/Sqrt[1 - x^4],x]')

[Out] Piecewise[{{(-I / 2) ArcCosh[x ^ 2], Abs[x ^ 4] > 1}}, ArcSin[x ^ 2] / 2]

Maple [A]

time = 0.13, size = 7, normalized size = 0.88

method	result	size
default	$\frac{\arcsin(x^2)}{2}$	7
meijerg	$\frac{\arcsin(x^2)}{2}$	7
elliptic	$\frac{\arcsin(x^2)}{2}$	7
trager	$\frac{\text{RootOf}(-Z^2+1) \ln(\text{RootOf}(-Z^2+1) \sqrt{-x^4+1} + x^2)}{2}$	30

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*arcsin(x^2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

time = 0.34, size = 16, normalized size = 2.00

$$-\frac{1}{2} \arctan \left(\frac{\sqrt{-x^4+1}}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+1)^(1/2),x, algorithm="maxima")

[Out] -1/2*arctan(sqrt(-x^4 + 1)/x^2)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(6) = 12.
time = 0.31, size = 18, normalized size = 2.25

$$-\arctan\left(\frac{\sqrt{-x^4+1}-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+1)^(1/2),x, algorithm="fricas")

[Out] -arctan((sqrt(-x^4 + 1) - 1)/x^2)

Sympy [A]

time = 0.42, size = 19, normalized size = 2.38

$$\begin{cases} -\frac{i \operatorname{acosh}(x^2)}{2} & \text{for } |x^4| > 1 \\ \frac{\operatorname{asin}(x^2)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x**4+1)**(1/2),x)

[Out] Piecewise((-I*acosh(x**2)/2, Abs(x**4) > 1), (asin(x**2)/2, True))

Giac [A]

time = 0.00, size = 7, normalized size = 0.88

$$\frac{\arcsin(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+1)^(1/2),x)

[Out] 1/2*arcsin(x^2)

Mupad [B]

time = 0.31, size = 16, normalized size = 2.00

$$\frac{\operatorname{atan}\left(\frac{x^2}{\sqrt{1-x^4}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(1 - x^4)^(1/2),x)

[Out] atan(x^2/(1 - x^4)^(1/2))/2

$$3.195 \quad \int \frac{1}{x \sqrt{1+x^4}} dx$$

Optimal. Leaf size=14

$$-\frac{1}{2} \tanh^{-1}(\sqrt{1+x^4})$$

[Out] -1/2*arctanh((x^4+1)^(1/2))

Rubi [A]

time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {272, 65, 213}

$$-\frac{1}{2} \tanh^{-1}(\sqrt{x^4+1})$$

Antiderivative was successfully verified.

[In] Int[1/(x*sqrt[1 + x^4]),x]

[Out] -1/2*ArcTanh[Sqrt[1 + x^4]]

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{1+x^4}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x\sqrt{1+x}} dx, x, x^4 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1+x^4} \right) \\ &= -\frac{1}{2} \tanh^{-1} \left(\sqrt{1+x^4} \right)\end{aligned}$$

Mathematica [A]

time = 0.01, size = 14, normalized size = 1.00

$$-\frac{1}{2} \tanh^{-1} \left(\sqrt{1+x^4} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x*Sqrt[1 + x^4]),x]``[Out] -1/2*ArcTanh[Sqrt[1 + x^4]]`**Mathics [A]**

time = 2.03, size = 6, normalized size = 0.43

$$-\frac{\text{ArcSinh} \left[\frac{1}{x^2} \right]}{2}$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x*Sqrt[1 + x^4]),x]')``[Out] -ArcSinh[1 / x ^ 2] / 2`**Maple [A]**

time = 0.10, size = 11, normalized size = 0.79

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	11
elliptic	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	11
trager	$\frac{\ln\left(\frac{-1+\sqrt{x^4+1}}{x^2}\right)}{2}$	17

meijerg	$\frac{(4\ln(x)-2\ln(2))\sqrt{\pi}-2\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{x^4+1}}{2}\right)}{4\sqrt{\pi}}$	37
---------	---	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `-1/2*arctanh(1/(x^4+1)^(1/2))`

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.

time = 0.26, size = 25, normalized size = 1.79

$$-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(x^4+1)^(1/2),x,algorithm="maxima")`

[Out] `-1/4*log(sqrt(x^4+1)+1)+1/4*log(sqrt(x^4+1)-1)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.

time = 0.31, size = 25, normalized size = 1.79

$$-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(x^4+1)^(1/2),x,algorithm="fricas")`

[Out] `-1/4*log(sqrt(x^4+1)+1)+1/4*log(sqrt(x^4+1)-1)`

Sympy [A]

time = 0.41, size = 8, normalized size = 0.57

$$\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(x**4+1)**(1/2),x)`

[Out] `-asinh(x**(-2))/2`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.
time = 0.00, size = 35, normalized size = 2.50

$$\frac{\frac{\ln\left(\sqrt{x^4+1}-1\right)}{2}}{2}-\frac{\ln\left(\sqrt{x^4+1}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1)^(1/2),x)
```

```
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

Mupad [B]

time = 0.19, size = 10, normalized size = 0.71

$$-\frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^4 + 1)^(1/2)),x)
```

```
[Out] -atanh((x^4 + 1)^(1/2))/2
```

$$3.196 \quad \int \frac{x}{\sqrt{1+x^2+x^4}} dx$$

Optimal. Leaf size=18

$$\frac{1}{2} \sinh^{-1} \left(\frac{1+2x^2}{\sqrt{3}} \right)$$

[Out] 1/2*arcsinh(1/3*(2*x^2+1)*3^(1/2))

Rubi [A]

time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1121, 633, 221}

$$\frac{1}{2} \sinh^{-1} \left(\frac{2x^2+1}{\sqrt{3}} \right)$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[1 + x^2 + x^4],x]

[Out] ArcSinh[(1 + 2*x^2)/Sqrt[3]]/2

Rule 221

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 633

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*(-4*(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 1121

Int[(x_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rubi steps

$$\int \frac{x}{\sqrt{1+x^2+x^4}} dx = \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1+x+x^2}} dx, x, x^2 \right)$$

$$= \frac{\text{Subst} \left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}} dx, x, 1+2x^2 \right)}{2\sqrt{3}}$$

$$= \frac{1}{2} \sinh^{-1} \left(\frac{1+2x^2}{\sqrt{3}} \right)$$

Mathematica [A]

time = 0.05, size = 26, normalized size = 1.44

$$-\frac{1}{2} \log \left(-1 - 2x^2 + 2\sqrt{1+x^2+x^4} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[x/Sqrt[1 + x^2 + x^4], x]``[Out] -1/2*Log[-1 - 2*x^2 + 2*Sqrt[1 + x^2 + x^4]]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[x/Sqrt[1 + x^2 + x^4], x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.08, size = 14, normalized size = 0.78

method	result	size
default	$\frac{\text{arcsinh} \left(\frac{2\sqrt{3} \left(x^2 + \frac{1}{2} \right)}{3} \right)}{2}$	14
elliptic	$\frac{\text{arcsinh} \left(\frac{2\sqrt{3} \left(x^2 + \frac{1}{2} \right)}{3} \right)}{2}$	14

trager	$-\frac{\ln\left(-2x^2+2\sqrt{x^4+x^2+1}-1\right)}{2}$	23
--------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `1/2*arcsinh(2/3*3^(1/2)*(x^2+1/2))`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^4+x^2+1)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x/sqrt(x^4 + x^2 + 1), x)`

Fricas [A]

time = 0.30, size = 22, normalized size = 1.22

$$-\frac{1}{2} \log\left(-2x^2 + 2\sqrt{x^4 + x^2 + 1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^4+x^2+1)^(1/2),x, algorithm="fricas")`

[Out] `-1/2*log(-2*x^2 + 2*sqrt(x^4 + x^2 + 1) - 1)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x^2 - x + 1)(x^2 + x + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x**4+x**2+1)**(1/2),x)`

[Out] `Integral(x/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)`

Giac [A]

time = 0.00, size = 26, normalized size = 1.44

$$-\frac{\ln\left(2\left(\sqrt{x^4+x^2+1}-x^2\right)-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^4+x^2+1)^(1/2),x)

[Out] -1/2*log(-2*x^2 + 2*sqrt(x^4 + x^2 + 1) - 1)

Mupad [B]

time = 0.36, size = 18, normalized size = 1.00

$$\frac{\ln\left(\sqrt{x^4 + x^2 + 1} + x^2 + \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^2 + x^4 + 1)^(1/2),x)

[Out] log((x^2 + x^4 + 1)^(1/2) + x^2 + 1/2)/2

$$3.197 \quad \int \frac{1}{x \sqrt{-1 + x^2 - x^4}} dx$$

Optimal. Leaf size=30

$$-\frac{1}{2} \tan^{-1} \left(\frac{2 - x^2}{2\sqrt{-1 + x^2 - x^4}} \right)$$

[Out] -1/2*arctan(1/2*(-x^2+2)/(-x^4+x^2-1)^(1/2))

Rubi [A]

time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1128, 738, 210}

$$-\frac{1}{2} \tan^{-1} \left(\frac{2 - x^2}{2\sqrt{-x^4 + x^2 - 1}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[-1 + x^2 - x^4]),x]

[Out] -1/2*ArcTan[(2 - x^2)/(2*Sqrt[-1 + x^2 - x^4])]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 738

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1128

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{-1+x^2-x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{-1+x-x^2}} dx, x, x^2 \right) \\ &= -\text{Subst} \left(\int \frac{1}{-4-x^2} dx, x, \frac{-2+x^2}{\sqrt{-1+x^2-x^4}} \right) \\ &= \frac{1}{2} \tan^{-1} \left(\frac{-2+x^2}{2\sqrt{-1+x^2-x^4}} \right) \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.05, size = 27, normalized size = 0.90

$$-i \tanh^{-1} \left(x^2 + i\sqrt{-1+x^2-x^4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[-1 + x^2 - x^4]),x]

[Out] (-I)*ArcTanh[x^2 + I*Sqrt[-1 + x^2 - x^4]]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x*Sqrt[x^2 - 1 - x^4]),x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [A]

time = 0.14, size = 23, normalized size = 0.77

method	result	size
default	$\frac{\arctan\left(\frac{x^2-2}{2\sqrt{-x^4+x^2-1}}\right)}{2}$	23
elliptic	$\frac{\arctan\left(\frac{x^2-2}{2\sqrt{-x^4+x^2-1}}\right)}{2}$	23
trager	$\frac{\text{RootOf}(-Z^2+1) \ln\left(\frac{-\text{RootOf}(-Z^2+1)x^2+2\sqrt{-x^4+x^2-1}+2\text{RootOf}(-Z^2+1)}{x^2}\right)}{2}$	48

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(-x^4+x^2-1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `1/2*arctan(1/2*(x^2-2)/(-x^4+x^2-1)^(1/2))`

Maxima [C] Result contains complex when optimal does not.

time = 0.36, size = 17, normalized size = 0.57

$$-\frac{1}{2}i \operatorname{arsinh} \left(-\frac{1}{3} \sqrt{3} + \frac{2 \sqrt{3}}{3 x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-x^4+x^2-1)^(1/2),x, algorithm="maxima")`

[Out] `-1/2*I*arcsinh(-1/3*sqrt(3) + 2/3*sqrt(3)/x^2)`

Fricas [C] Result contains complex when optimal does not.

time = 0.32, size = 55, normalized size = 1.83

$$\frac{1}{4}i \log \left(\frac{x^2 + 2i \sqrt{-x^4 + x^2 - 1} - 2}{2 x^2} \right) - \frac{1}{4}i \log \left(\frac{x^2 - 2i \sqrt{-x^4 + x^2 - 1} - 2}{2 x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-x^4+x^2-1)^(1/2),x, algorithm="fricas")`

[Out] `1/4*I*log(1/2*(x^2 + 2*I*sqrt(-x^4 + x^2 - 1) - 2)/x^2) - 1/4*I*log(1/2*(x^2 - 2*I*sqrt(-x^4 + x^2 - 1) - 2)/x^2)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{-x^4 + x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-x**4+x**2-1)**(1/2),x)`

[Out] `Integral(1/(x*sqrt(-x**4 + x**2 - 1)), x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-x^4+x^2-1)^(1/2),x)`

[Out] Could not integrate

Mupad [B]

time = 0.54, size = 32, normalized size = 1.07

$$\frac{\ln\left(\frac{1}{x^2}\right) 1i}{2} + \frac{\ln\left(x^2 - 2 + \sqrt{-x^4 + x^2 - 1}\right) 2i}{2} 1i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(x^2 - x^4 - 1)^(1/2)),x)`

[Out] `(log(1/x^2)*1i)/2 + (log((x^2 - x^4 - 1)^(1/2)*2i + x^2 - 2)*1i)/2`

$$3.198 \quad \int \frac{1+x}{(1-x)^2 \sqrt{1+x^2}} dx$$

Optimal. Leaf size=17

$$\frac{\sqrt{1+x^2}}{1-x}$$

[Out] $(x^2+1)^{(1/2)}/(1-x)$

Rubi [A]

time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {817}

$$\frac{\sqrt{x^2+1}}{1-x}$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/((1 - x)^2*Sqrt[1 + x^2]),x]

[Out] Sqrt[1 + x^2]/(1 - x)

Rule 817

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 + a*e^2)), x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && EqQ[c*d*f + a*e*g, 0]

Rubi steps

$$\int \frac{1+x}{(1-x)^2 \sqrt{1+x^2}} dx = \frac{\sqrt{1+x^2}}{1-x}$$

Mathematica [A]

time = 0.12, size = 17, normalized size = 1.00

$$\frac{\sqrt{1+x^2}}{1-x}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)/((1 - x)^2*Sqrt[1 + x^2]),x]

[Out] $\text{Sqrt}[1 + x^2]/(1 - x)$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(1 + x)/((1 - x)^2*Sqrt[1 + x^2]),x]')`

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.04, size = 19, normalized size = 1.12

method	result	size
gospers	$-\frac{\sqrt{x^2 + 1}}{-1+x}$	15
trager	$-\frac{\sqrt{x^2 + 1}}{-1+x}$	15
risch	$-\frac{\sqrt{x^2 + 1}}{-1+x}$	15
default	$-\frac{\sqrt{(-1+x)^2 + 2x}}{-1+x}$	19

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+x)/(1-x)^2/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/(-1+x)*((-1+x)^2+2*x)^(1/2)$

Maxima [A]

time = 0.35, size = 14, normalized size = 0.82

$$-\frac{\sqrt{x^2 + 1}}{x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)/(1-x)^2/(x^2+1)^(1/2),x, algorithm="maxima")`

[Out] $-\text{sqrt}(x^2 + 1)/(x - 1)$

Fricas [A]

time = 0.30, size = 17, normalized size = 1.00

$$\frac{x + \sqrt{x^2 + 1} - 1}{x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(1-x)^2/(x^2+1)^(1/2),x, algorithm="fricas")

[Out] -(x + sqrt(x^2 + 1) - 1)/(x - 1)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + 1}{(x - 1)^2 \sqrt{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(1-x)**2/(x**2+1)**(1/2),x)

[Out] Integral((x + 1)/((x - 1)**2*sqrt(x**2 + 1)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(14) = 28.
time = 0.00, size = 48, normalized size = 2.82

$$\frac{2(\sqrt{x^2 + 1} - x - 1)}{-\left(\sqrt{x^2 + 1} - x\right)^2 - 2\left(\sqrt{x^2 + 1} - x\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(1-x)^2/(x^2+1)^(1/2),x)

[Out] 2*(x - sqrt(x^2 + 1) + 1)/((x - sqrt(x^2 + 1))^2 - 2*x + 2*sqrt(x^2 + 1) - 1)

Mupad [B]

time = 0.15, size = 14, normalized size = 0.82

$$-\frac{\sqrt{x^2 + 1}}{x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 1)/((x^2 + 1)^(1/2)*(x - 1)^2),x)

[Out] -(x^2 + 1)^(1/2)/(x - 1)

$$3.199 \quad \int \frac{1}{\sqrt{1+x^2}} dx$$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

[Out] arcsinh(x)

Rubi [A]

time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {221}

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 + x^2], x]

[Out] ArcSinh[x]

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}} dx = \sinh^{-1}(x)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 12 vs. 2(2) = 4. time = 0.01, size = 12, normalized size = 6.00

$$\tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 + x^2], x]

[Out] ArcTanh[x/Sqrt[1 + x^2]]

Mathics [A]

time = 1.65, size = 2, normalized size = 1.00

$$\text{ArcSinh}[x]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/Sqrt[1 + x^2],x]')
```

```
[Out] ArcSinh[x]
```

Maple [A]

time = 0.04, size = 3, normalized size = 1.50

method	result	size
default	$\operatorname{arcsinh}(x)$	3
meijerg	$\operatorname{arcsinh}(x)$	3
trager	$\ln(x + \sqrt{x^2 + 1})$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsinh(x)
```

Maxima [A]

time = 0.38, size = 2, normalized size = 1.00

$$\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(2) = 4$.
time = 0.31, size = 14, normalized size = 7.00

$$-\log(-x + \sqrt{x^2 + 1})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 1))
```

Sympy [A]

time = 0.06, size = 2, normalized size = 1.00

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+1)**(1/2),x)

[Out] asinh(x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(2) = 4$.
time = 0.00, size = 14, normalized size = 7.00

$$-\ln\left(\sqrt{x^2+1}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)^(1/2),x)

[Out] -log(-x + sqrt(x^2 + 1))

Mupad [B]

time = 0.03, size = 2, normalized size = 1.00

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2 + 1)^(1/2),x)

[Out] asinh(x)

$$3.200 \quad \int \frac{\sqrt{x} \sqrt{1+x} + \sqrt{x} \sqrt{2+x} + \sqrt{1+x} \sqrt{2+x}}{2\sqrt{x} \sqrt{1+x} \sqrt{2+x}} dx$$

Optimal. Leaf size=20

$$\sqrt{x} + \sqrt{1+x} + \sqrt{2+x}$$

[Out] $x^{(1/2)} + (1+x)^{(1/2)} + (2+x)^{(1/2)}$

Rubi [A]

time = 0.60, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 65, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {12, 6820}

$$\sqrt{x} + \sqrt{x+1} + \sqrt{x+2}$$

Antiderivative was successfully verified.

[In] `Int[(Sqrt[x]*Sqrt[1 + x] + Sqrt[x]*Sqrt[2 + x] + Sqrt[1 + x]*Sqrt[2 + x])/(2*Sqrt[x]*Sqrt[1 + x]*Sqrt[2 + x]),x]`

[Out] `Sqrt[x] + Sqrt[1 + x] + Sqrt[2 + x]`

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

Rule 6820

`Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]`

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x} \sqrt{1+x} + \sqrt{x} \sqrt{2+x} + \sqrt{1+x} \sqrt{2+x}}{2\sqrt{x} \sqrt{1+x} \sqrt{2+x}} dx &= \frac{1}{2} \int \frac{\sqrt{x} \sqrt{1+x} + \sqrt{x} \sqrt{2+x} + \sqrt{1+x} \sqrt{2+x}}{\sqrt{x} \sqrt{1+x} \sqrt{2+x}} \\ &= \frac{1}{2} \int \left(\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{1+x}} + \frac{1}{\sqrt{2+x}} \right) dx \\ &= \sqrt{x} + \sqrt{1+x} + \sqrt{2+x} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 30, normalized size = 1.50

$$\frac{1}{2} \left(2\sqrt{x} + 2\sqrt{1+x} + 2\sqrt{2+x} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*Sqrt[1 + x] + Sqrt[x]*Sqrt[2 + x] + Sqrt[1 + x]*Sqrt[2 + x])/(2*Sqrt[x]*Sqrt[1 + x]*Sqrt[2 + x]),x]
```

```
[Out] (2*Sqrt[x] + 2*Sqrt[1 + x] + 2*Sqrt[2 + x])/2
```

Mathics [A]

time = 1.82, size = 14, normalized size = 0.70

$$\sqrt{x} + \sqrt{1+x} + \sqrt{2+x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(Sqrt[x]*Sqrt[1 + x] + Sqrt[x]*Sqrt[2 + x] + Sqrt[1 + x]*Sqrt[2 + x])/(2*Sqrt[x]*Sqrt[1 + x]*Sqrt[2 + x]),x]')
```

```
[Out] Sqrt[x] + Sqrt[1 + x] + Sqrt[2 + x]
```

Maple [A]

time = 0.05, size = 15, normalized size = 0.75

method	result	size
default	$\sqrt{x} + \sqrt{1+x} + \sqrt{2+x}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)+(1+x)^(1/2)+(2+x)^(1/2)
```

Maxima [A]

time = 0.27, size = 14, normalized size = 0.70

$$\sqrt{x+2} + \sqrt{x+1} + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x + 2) + sqrt(x + 1) + sqrt(x)
```

Fricas [A]

time = 0.30, size = 14, normalized size = 0.70

$$\sqrt{x+2} + \sqrt{x+1} + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x, algorithm="fricas")

[Out] sqrt(x + 2) + sqrt(x + 1) + sqrt(x)

Sympy [A]

time = 0.26, size = 17, normalized size = 0.85

$$\sqrt{x} + \sqrt{x+1} + \sqrt{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2*(x**(1/2)*(1+x)**(1/2)+x**(1/2)*(2+x)**(1/2)+(1+x)**(1/2)*(2+x)**(1/2))/x**(1/2)/(1+x)**(1/2)/(2+x)**(1/2),x)

[Out] sqrt(x) + sqrt(x + 1) + sqrt(x + 2)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x)

[Out] Could not integrate

Mupad [B]

time = 0.34, size = 14, normalized size = 0.70

$$\sqrt{x+1} + \sqrt{x+2} + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^(1/2)*(x + 1)^(1/2))/2 + (x^(1/2)*(x + 2)^(1/2))/2 + ((x + 1)^(1/2)*(x + 2)^(1/2))/2)/(x^(1/2)*(x + 1)^(1/2)*(x + 2)^(1/2)),x)

[Out] (x + 1)^(1/2) + (x + 2)^(1/2) + x^(1/2)

$$3.201 \quad \int \frac{-2\sqrt{1+x^3} + 5x^4\sqrt{1+x^3} - 3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}} dx$$

Optimal. Leaf size=24

$$-\sqrt{1+x^3} + \sqrt{1-2x+x^5}$$

[Out] $-(x^3+1)^{(1/2)}+(x^5-2*x+1)^{(1/2)}$

Rubi [A]

time = 0.20, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 68, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {12, 6820, 267, 2124}

$$\sqrt{x^5 - 2x + 1} - \sqrt{x^3 + 1}$$

Antiderivative was successfully verified.

[In] `Int[(-2*Sqrt[1 + x^3] + 5*x^4*Sqrt[1 + x^3] - 3*x^2*Sqrt[1 - 2*x + x^5])/(2*Sqrt[1 + x^3]*Sqrt[1 - 2*x + x^5]),x]`

[Out] `-Sqrt[1 + x^3] + Sqrt[1 - 2*x + x^5]`

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

Rule 267

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]`

Rule 2124

`Int[(Pm_)*(Qn_)^(p_), x_Symbol] := With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n]), x] + Dist[Simplify[Pm - Coeff[Pm, x, m]*(D[Qn, x]/(n*Coeff[Qn, x, n])], Int[Qn^p, x], x] /; EqQ[m, n - 1] && EqQ[D[Simplify[Pm - (Coeff[Pm, x, m]/(n*Coeff[Qn, x, n])]*D[Qn, x]], x], 0]] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && NeQ[p, -1]`

Rule 6820

`Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]`

Rubi steps

$$\begin{aligned}
\int \frac{-2\sqrt{1+x^3} + 5x^4\sqrt{1+x^3} - 3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}} dx &= \frac{1}{2} \int \frac{-2\sqrt{1+x^3} + 5x^4\sqrt{1+x^3} - 3x^2\sqrt{1-2x+x^5}}{\sqrt{1+x^3}\sqrt{1-2x+x^5}} \\
&= \frac{1}{2} \int \left(-\frac{3x^2}{\sqrt{1+x^3}} - \frac{2}{\sqrt{1-2x+x^5}} + \frac{5x^4}{\sqrt{1-2x+x^5}} \right) dx \\
&= -\left(\frac{3}{2} \int \frac{x^2}{\sqrt{1+x^3}} dx \right) + \frac{5}{2} \int \frac{x^4}{\sqrt{1-2x+x^5}} dx \\
&= -\sqrt{1+x^3} + \sqrt{1-2x+x^5}
\end{aligned}$$

Mathematica [A]

time = 13.74, size = 24, normalized size = 1.00

$$-\sqrt{1+x^3} + \sqrt{1-2x+x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*Sqrt[1 + x^3] + 5*x^4*Sqrt[1 + x^3] - 3*x^2*Sqrt[1 - 2*x + x^5])/(2*Sqrt[1 + x^3]*Sqrt[1 - 2*x + x^5]),x]
```

```
[Out] -Sqrt[1 + x^3] + Sqrt[1 - 2*x + x^5]
```

Mathics [A]

time = 1.98, size = 20, normalized size = 0.83

$$\sqrt{1-2x+x^5} - \sqrt{1+x^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(-2*Sqrt[1 + x^3] + 5*x^4*Sqrt[1 + x^3] - 3*x^2*Sqrt[1 - 2*x + x^5])/(2*Sqrt[1 + x^3]*Sqrt[1 - 2*x + x^5]),x]')
```

```
[Out] Sqrt[1 - 2 x + x ^ 5] - Sqrt[1 + x ^ 3]
```

Maple [A]

time = 0.07, size = 21, normalized size = 0.88

method	result	size
default	$-\sqrt{x^3+1} + \sqrt{x^5-2x+1}$	21
elliptic	$-\sqrt{x^3+1} + \frac{(-1+x)(x^4+x^3+x^2+x-1)}{\sqrt{x^5-2x+1}}$	37

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2))/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-(x^3+1)^{1/2}+(x^5-2x+1)^{1/2}$

Maxima [A]

time = 0.39, size = 30, normalized size = 1.25

$$\sqrt{x^4 + x^3 + x^2 + x - 1} \sqrt{x - 1} - \sqrt{x^3 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2))/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x, algorithm="maxima")`

[Out] $\sqrt{x^4 + x^3 + x^2 + x - 1} \sqrt{x - 1} - \sqrt{x^3 + 1}$

Fricas [A]

time = 0.32, size = 20, normalized size = 0.83

$$\sqrt{x^5 - 2x + 1} - \sqrt{x^3 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2))/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x, algorithm="fricas")`

[Out] $\sqrt{x^5 - 2x + 1} - \sqrt{x^3 + 1}$

Sympy [A]

time = 0.31, size = 19, normalized size = 0.79

$$-\sqrt{x^3 + 1} + \sqrt{x^5 - 2x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/2*(-2*(x**3+1)**(1/2)+5*x**4*(x**3+1)**(1/2)-3*x**2*(x**5-2*x+1)**(1/2))/(x**3+1)**(1/2)/(x**5-2*x+1)**(1/2),x)`

[Out] $-\sqrt{x^3 + 1} + \sqrt{x^5 - 2x + 1}$

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2))/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x)`

[Out] Could not integrate

Mupad [B]

time = 0.31, size = 20, normalized size = 0.83

$$\sqrt{x^5 - 2x + 1} - \sqrt{x^3 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-((x^3 + 1)^(1/2) + (3*x^2*(x^5 - 2*x + 1)^(1/2))/2 - (5*x^4*(x^3 + 1)^(1/2))/2)/((x^3 + 1)^(1/2)*(x^5 - 2*x + 1)^(1/2)),x)`

[Out] `(x^5 - 2*x + 1)^(1/2) - (x^3 + 1)^(1/2)`

$$3.202 \quad \int \left(\frac{10}{\sqrt{-4+x^2}} + \frac{1}{\sqrt{-1+x^2}} \right) dx$$

Optimal. Leaf size=27

$$10 \tanh^{-1} \left(\frac{x}{\sqrt{-4+x^2}} \right) + \tanh^{-1} \left(\frac{x}{\sqrt{-1+x^2}} \right)$$

[Out] 10*arctanh(x/(x^2-4)^(1/2))+arctanh(x/(x^2-1)^(1/2))

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {223, 212}

$$10 \tanh^{-1} \left(\frac{x}{\sqrt{x^2-4}} \right) + \tanh^{-1} \left(\frac{x}{\sqrt{x^2-1}} \right)$$

Antiderivative was successfully verified.

[In] Int[10/Sqrt[-4 + x^2] + 1/Sqrt[-1 + x^2], x]

[Out] 10*ArcTanh[x/Sqrt[-4 + x^2]] + ArcTanh[x/Sqrt[-1 + x^2]]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 223

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \left(\frac{10}{\sqrt{-4+x^2}} + \frac{1}{\sqrt{-1+x^2}} \right) dx &= 10 \int \frac{1}{\sqrt{-4+x^2}} dx + \int \frac{1}{\sqrt{-1+x^2}} dx \\ &= 10 \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{-4+x^2}} \right) + \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{-1+x^2}} \right) \\ &= 10 \tanh^{-1} \left(\frac{x}{\sqrt{-4+x^2}} \right) + \tanh^{-1} \left(\frac{x}{\sqrt{-1+x^2}} \right) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 71 vs. 2(27) = 54.

time = 0.01, size = 71, normalized size = 2.63

$$-5 \log \left(1 - \frac{x}{\sqrt{-4+x^2}} \right) + 5 \log \left(1 + \frac{x}{\sqrt{-4+x^2}} \right) - \frac{1}{2} \log \left(1 - \frac{x}{\sqrt{-1+x^2}} \right) + \frac{1}{2} \log \left(1 + \frac{x}{\sqrt{-1+x^2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[10/Sqrt[-4 + x^2] + 1/Sqrt[-1 + x^2], x]

[Out] -5*Log[1 - x/Sqrt[-4 + x^2]] + 5*Log[1 + x/Sqrt[-4 + x^2]] - Log[1 - x/Sqrt[-1 + x^2]]/2 + Log[1 + x/Sqrt[-1 + x^2]]/2

Mathics [A]

time = 1.66, size = 9, normalized size = 0.33

$$\text{ArcCosh}[x] + 10 \text{ArcCosh}\left[\frac{x}{2}\right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/Sqrt[x^2-1]+10/Sqrt[x^2-4], x]')

[Out] ArcCosh[x] + 10 ArcCosh[x / 2]

Maple [A]

time = 0.06, size = 24, normalized size = 0.89

method	result	size
default	$\ln(x + \sqrt{x^2 - 1}) + 10 \ln(x + \sqrt{x^2 - 4})$	24
meijerg	$\frac{10 \sqrt{-\text{signum}(-1 + \frac{x^2}{4})} \arcsin(\frac{x}{2})}{\sqrt{\text{signum}(-1 + \frac{x^2}{4})}} + \frac{\sqrt{-\text{signum}(x^2 - 1)} \arcsin(x)}{\sqrt{\text{signum}(x^2 - 1)}}$	51

Verification of antiderivative is not currently implemented for this CAS.

[In] int(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2), x, method=_RETURNVERBOSE)

[Out] ln(x+(x^2-1)^(1/2))+10*ln(x+(x^2-4)^(1/2))

Maxima [A]

time = 0.26, size = 31, normalized size = 1.15

$$\log(2x + 2\sqrt{x^2 - 1}) + 10 \log(2x + 2\sqrt{x^2 - 4})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2), x, algorithm="maxima")

[Out] $\log(2x + 2\sqrt{x^2 - 1}) + 10\log(2x + 2\sqrt{x^2 - 4})$

Fricas [A]

time = 0.31, size = 29, normalized size = 1.07

$$-\log\left(-x + \sqrt{x^2 - 1}\right) - 10\log\left(-x + \sqrt{x^2 - 4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x, algorithm="fricas")`

[Out] $-\log(-x + \sqrt{x^2 - 1}) - 10\log(-x + \sqrt{x^2 - 4})$

Sympy [A]

time = 0.08, size = 8, normalized size = 0.30

$$10\operatorname{acosh}\left(\frac{x}{2}\right) + \operatorname{acosh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(10/(x**2-4)**(1/2)+1/(x**2-1)**(1/2),x)`

[Out] $10\operatorname{acosh}(x/2) + \operatorname{acosh}(x)$

Giac [A]

time = 0.00, size = 33, normalized size = 1.22

$$-10\ln\left|\sqrt{x^2 - 4} - x\right| - \ln\left|\sqrt{x^2 - 1} - x\right|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x)`

[Out] $-\log(\operatorname{abs}(-x + \sqrt{x^2 - 1})) - 10\log(\operatorname{abs}(-x + \sqrt{x^2 - 4}))$

Mupad [B]

time = 0.69, size = 23, normalized size = 0.85

$$\ln\left(x + \sqrt{x^2 - 1}\right) + 10\ln\left(x + \sqrt{x^2 - 4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2 - 1)^(1/2) + 10/(x^2 - 4)^(1/2),x)`

[Out] $\log(x + (x^2 - 1)^{1/2}) + 10\log(x + (x^2 - 4)^{1/2})$

$$3.203 \quad \int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{x} dx$$

Optimal. Leaf size=82

$$2\sqrt{x + \sqrt{a^2 + x^2}} - 2\sqrt{a} \tan^{-1} \left(\frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a}} \right) - 2\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a}} \right)$$

[Out] $-2*\arctan((x+(a^2+x^2)^{(1/2)})^{(1/2)}/a^{(1/2)})*a^{(1/2)}-2*\arctanh((x+(a^2+x^2)^{(1/2)})^{(1/2)}/a^{(1/2)})*a^{(1/2)}+2*(x+(a^2+x^2)^{(1/2)})^{(1/2)}$

Rubi [A]

time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2144, 470, 335, 218, 212, 209}

$$2\sqrt{\sqrt{a^2 + x^2} + x} - 2\sqrt{a} \tan^{-1} \left(\frac{\sqrt{\sqrt{a^2 + x^2} + x}}{\sqrt{a}} \right) - 2\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{\sqrt{a^2 + x^2} + x}}{\sqrt{a}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]

[Out] $2*\text{Sqrt}[x + \text{Sqrt}[a^2 + x^2]] - 2*\text{Sqrt}[a]*\text{ArcTan}[\text{Sqrt}[x + \text{Sqrt}[a^2 + x^2]]/\text{Sqrt}[a]] - 2*\text{Sqrt}[a]*\text{ArcTanh}[\text{Sqrt}[x + \text{Sqrt}[a^2 + x^2]]/\text{Sqrt}[a]]$

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 218

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 335

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n)]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 470

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p + 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rule 2144

Int[((g_) + (h_)*(x_))^(m_)*((e_)*(x_) + (f_)*Sqrt[(a_) + (c_)*(x_)^2])^(n_), x_Symbol] := Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m - 2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{x} dx &= \text{Subst} \left(\int \frac{a^2 + x^2}{\sqrt{x} (-a^2 + x^2)} dx, x, x + \sqrt{a^2 + x^2} \right) \\
 &= 2\sqrt{x + \sqrt{a^2 + x^2}} + (2a^2) \text{Subst} \left(\int \frac{1}{\sqrt{x} (-a^2 + x^2)} dx, x, x + \sqrt{a^2 + x^2} \right) \\
 &= 2\sqrt{x + \sqrt{a^2 + x^2}} + (4a^2) \text{Subst} \left(\int \frac{1}{-a^2 + x^4} dx, x, \sqrt{x + \sqrt{a^2 + x^2}} \right) \\
 &= 2\sqrt{x + \sqrt{a^2 + x^2}} - (2a) \text{Subst} \left(\int \frac{1}{a - x^2} dx, x, \sqrt{x + \sqrt{a^2 + x^2}} \right) - (2a) \text{Subst} \left(\int \frac{1}{a + x^2} dx, x, \sqrt{x + \sqrt{a^2 + x^2}} \right) \\
 &= 2\sqrt{x + \sqrt{a^2 + x^2}} - 2\sqrt{a} \tan^{-1} \left(\frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a}} \right) - 2\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a}} \right)
 \end{aligned}$$

Mathematica [A]

time = 0.07, size = 82, normalized size = 1.00

$$2\sqrt{x + \sqrt{a^2 + x^2}} - 2\sqrt{a} \tan^{-1} \left(\frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a}} \right) - 2\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]

[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 3.54, size = 33, normalized size = 0.40

$$\frac{\sqrt{2} \sqrt{x} \Gamma\left[-\frac{1}{4}\right] \operatorname{hyper}\left[\left\{-\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}\right\}, \left\{\frac{1}{2}, \frac{3}{4}\right\}, \frac{a^2 \exp_{\text{polar}}[i\pi]}{x^2}\right]}{2 \Gamma\left[\frac{3}{4}\right]}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sqrt[x+Sqrt[x^2+a^2]]/x,x]')

[Out] -Sqrt[2] Sqrt[x] Gamma[-1 / 4] hyper[{-1 / 4, -1 / 4, 1 / 4}, {1 / 2, 3 / 4}, a ^ 2 exp_polar[I Pi] / x ^ 2] / (2 Gamma[3 / 4])

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.

time = 0.03, size = 25, normalized size = 0.30

method	result	size
meijerg	$2\sqrt{2} \sqrt{x} \operatorname{hypergeom}\left(\left[-\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}\right], \left[\frac{1}{2}, \frac{3}{4}\right], -\frac{a^2}{x^2}\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(a^2+x^2)^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)

[Out] 2*2^(1/2)*x^(1/2)*hypergeom([-1/4,-1/4,1/4],[1/2,3/4],-a^2/x^2)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)

Fricas [A]

time = 0.33, size = 216, normalized size = 2.63

$$\left[-2\sqrt{a} \arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) + \sqrt{a} \log\left(\frac{a^2 + \sqrt{a^2+x^2}a - (a-x)\sqrt{a} + \sqrt{a^2+x^2}\sqrt{a}}{x}\sqrt{x+\sqrt{a^2+x^2}}\right) + 2\sqrt{x+\sqrt{a^2+x^2}} + 2\sqrt{-a} \arctan\left(\frac{\sqrt{-a}\sqrt{x+\sqrt{a^2+x^2}}}{a}\right) + \sqrt{-a} \log\left(\frac{a^2 - \sqrt{a^2+x^2}a + (\sqrt{-a}(a+x) - \sqrt{a^2+x^2}\sqrt{-a})\sqrt{x+\sqrt{a^2+x^2}}}{x}\right) + 2\sqrt{x+\sqrt{a^2+x^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="fricas")

[Out] [-2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) + sqrt(a)*log((a^2 + sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x + sqrt(a^2 + x^2)))/x) + 2*sqrt(x + sqrt(a^2 + x^2)), 2*sqrt(-a)*arctan(sqrt(-a)*sqrt(x + sqrt(a^2 + x^2))/a) + sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a + (sqrt(-a)*(a + x) - sqrt(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x) + 2*sqrt(x + sqrt(a^2 + x^2))]

Sympy [C] Result contains complex when optimal does not.

time = 1.73, size = 51, normalized size = 0.62

$$\frac{\sqrt{x} \Gamma^2\left(-\frac{1}{4}\right) \Gamma\left(\frac{1}{4}\right) {}_3F_2\left(\begin{matrix} -\frac{1}{4}, -\frac{1}{4}, \frac{1}{4} \\ \frac{1}{2}, \frac{3}{4} \end{matrix} \middle| \frac{a^2 e^{i\pi}}{x^2}\right)}{8\pi \Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)

[Out] sqrt(x)*gamma(-1/4)**2*gamma(1/4)*hyper((-1/4, -1/4, 1/4), (1/2, 3/4), a**2*exp_polar(I*pi)/x**2)/(8*pi*gamma(3/4))

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + (a^2 + x^2)^(1/2))^(1/2)/x,x)

[Out] int((x + (a^2 + x^2)^(1/2))^(1/2)/x, x)

$$3.204 \quad \int \frac{3x^2}{2\left(1+x^3+\sqrt{1+x^3}\right)} dx$$

Optimal. Leaf size=12

$$\log\left(1+\sqrt{1+x^3}\right)$$

[Out] ln(1+(x^3+1)^(1/2))

Rubi [A]

time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {12, 2186, 31}

$$\log\left(\sqrt{x^3+1}+1\right)$$

Antiderivative was successfully verified.

[In] Int[(3*x^2)/(2*(1 + x^3 + Sqrt[1 + x^3])),x]

[Out] Log[1 + Sqrt[1 + x^3]]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2186

Int[(x_)^(m_)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] := Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a + b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[(m + 1)/n]

Rubi steps

$$\begin{aligned}
\int \frac{3x^2}{2(1+x^3+\sqrt{1+x^3})} dx &= \frac{3}{2} \int \frac{x^2}{1+x^3+\sqrt{1+x^3}} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+x+\sqrt{1+x}} dx, x, x^3 \right) \\
&= \text{Subst} \left(\int \frac{1}{1+x} dx, x, \sqrt{1+x^3} \right) \\
&= \log(1+\sqrt{1+x^3})
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 12, normalized size = 1.00

$$\log(1+\sqrt{1+x^3})$$

Antiderivative was successfully verified.

[In] Integrate[(3*x^2)/(2*(1 + x^3 + Sqrt[1 + x^3])),x]

[Out] Log[1 + Sqrt[1 + x^3]]

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 52 vs. $2(12) = 24$.
time = 50.65, size = 40, normalized size = 3.33

$$-\frac{\text{Log}[1+x^3]}{4} + \frac{\text{Log}[3]}{2} + \frac{\text{Log}[1+x^3+\sqrt{1+x^3}]}{2} + \frac{\text{Log}[1+\sqrt{1+x^3}]}{2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(3*x^2)/(2*(1 + x^3 + Sqrt[1 + x^3])),x]')

[Out] -Log[1 + x ^ 3] / 4 + Log[3] / 2 + Log[1 + x ^ 3 + Sqrt[1 + x ^ 3]] / 2 + L
og[1 + Sqrt[1 + x ^ 3]] / 2

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs.
 $2(10) = 20$.

time = 0.11, size = 39, normalized size = 3.25

method	result	size
trager	$\frac{\ln(-x^3-2\sqrt{x^3+1}-2)}{2}$	20
default	$-\frac{\ln(1+x)}{2} + \frac{3\ln(x)}{2} - \frac{\ln(x^2-x+1)}{2} + \frac{\ln(x^3+1)}{2} + \text{arctanh}(\sqrt{x^3+1})$	39

elliptic	$\frac{(1+\sqrt{x^3+1})\sqrt{x^3+1}\left(\frac{3\ln(x)}{2}+\operatorname{arctanh}(\sqrt{x^3+1})\right)}{1+x^3+\sqrt{x^3+1}}$	45
----------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x,method=_RETURNVERBOSE)`

[Out] $-1/2*\ln(1+x)+3/2*\ln(x)-1/2*\ln(x^2-x+1)+1/2*\ln(x^3+1)+\operatorname{arctanh}((x^3+1)^(1/2))$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(10) = 20.

time = 0.35, size = 40, normalized size = 3.33

$$-\frac{1}{2} \log(x^2 - x + 1) + \log\left(\frac{x^3 + \sqrt{x^2 - x + 1} \sqrt{x + 1} + 1}{\sqrt{x + 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x, algorithm="maxima")`

[Out] $-1/2*\log(x^2 - x + 1) + \log((x^3 + \operatorname{sqrt}(x^2 - x + 1)*\operatorname{sqrt}(x + 1) + 1)/\operatorname{sqrt}(x + 1))$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(10) = 20.

time = 0.30, size = 29, normalized size = 2.42

$$\frac{3}{2} \log(x) + \frac{1}{2} \log(\sqrt{x^3+1} + 1) - \frac{1}{2} \log(\sqrt{x^3+1} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x, algorithm="fricas")`

[Out] $3/2*\log(x) + 1/2*\log(\operatorname{sqrt}(x^3 + 1) + 1) - 1/2*\log(\operatorname{sqrt}(x^3 + 1) - 1)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(10) = 20

time = 1.21, size = 48, normalized size = 4.00

$$-\frac{\log(2\sqrt{x^3+1})}{2} + \frac{\log(2\sqrt{x^3+1} + 2)}{2} + \frac{\log(3x^3 + 3\sqrt{x^3+1} + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(3/2*x**2/(1+x**3+(x**3+1)**(1/2)),x)`

[Out] $-\log(2*\operatorname{sqrt}(x**3 + 1))/2 + \log(2*\operatorname{sqrt}(x**3 + 1) + 2)/2 + \log(3*x**3 + 3*\operatorname{sqrt}(x**3 + 1) + 3)/2$

Giac [A]

time = 0.00, size = 19, normalized size = 1.58

$$\frac{2 \cdot 3 \ln \left(\sqrt{x^3 + 1} + 1 \right)}{2 \cdot 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x)

[Out] log(sqrt(x^3 + 1) + 1)

Mupad [B]

time = 0.06, size = 169, normalized size = 14.08

$$\frac{3 \ln(x)}{2} + \frac{3 \left(\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2} \right) \sqrt{\frac{x - \frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}{-\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}} \sqrt{\frac{x+1}{\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}} \sqrt{\frac{\frac{1}{2} - x + \frac{\sqrt{3} \operatorname{li}}{2}}{\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}} \Pi \left(\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}, \operatorname{asin} \left(\sqrt{\frac{x+1}{\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}} \right) \right) \left| -\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2} \right.}{\sqrt{x^3 + \left(-\left(-\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2} \right) \left(\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2} \right) - 1 \right) x - \left(-\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2} \right) \left(\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2} \right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2)/(2*((x^3 + 1)^(1/2) + x^3 + 1)),x)

[Out] (3*log(x))/2 + (3*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))/(x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)

$$3.205 \quad \int \frac{1}{\sqrt{-\alpha^2 + 2hr^2}} dr$$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{h}r}{\sqrt{-\alpha^2 + 2hr^2}}\right)}{\sqrt{2}\sqrt{h}}$$

[Out] $1/2*\operatorname{arctanh}(r*2^{(1/2)}*h^{(1/2)}/(2*h*r^2-\alpha^2)^{(1/2)})*2^{(1/2)}/h^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{h}r}{\sqrt{2hr^2 - \alpha^2}}\right)}{\sqrt{2}\sqrt{h}}$$

Antiderivative was successfully verified.

[In] `Int[1/Sqrt[-alpha^2 + 2*h*r^2],r]`

[Out] `ArcTanh[(Sqrt[2]*Sqrt[h]*r)/Sqrt[-alpha^2 + 2*h*r^2]]/(Sqrt[2]*Sqrt[h])`

Rule 212

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 223

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-\alpha^2 + 2hr^2}} dr &= \text{Subst} \left(\int \frac{1}{1 - 2hr^2} dr, r, \frac{r}{\sqrt{-\alpha^2 + 2hr^2}} \right) \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{h}r}{\sqrt{-\alpha^2 + 2hr^2}}\right)}{\sqrt{2}\sqrt{h}} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 40, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{h}r}{\sqrt{-\alpha^2+2hr^2}}\right)}{\sqrt{2}\sqrt{h}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/Sqrt[-alpha^2 + 2*h*r^2],r]``[Out] ArcTanh[(Sqrt[2]*Sqrt[h]*r)/Sqrt[-alpha^2 + 2*h*r^2]]/(Sqrt[2]*Sqrt[h])`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.34, size = 54, normalized size = 1.35

$$\text{Piecewise}\left[\left[\left[\frac{\sqrt{2}\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{h}r}{\alpha}\right]}{2\sqrt{h}}, \text{Abs}\left[\frac{hr^2}{\alpha^2}\right] > \frac{1}{2}\right]\right], -\frac{I\sqrt{2}\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{h}r}{\alpha}\right]}{2\sqrt{h}}\right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/Sqrt[2*h*r^2-alpha^2],r]')``[Out] Piecewise[{{Sqrt[2] ArcCosh[Sqrt[2] Sqrt[h] r / alpha] / (2 Sqrt[h]), Abs[h r ^ 2 / alpha ^ 2] > 1 / 2}}, -I Sqrt[2] ArcSin[Sqrt[2] Sqrt[h] r / alpha] / (2 Sqrt[h])]`**Maple [A]**

time = 0.04, size = 33, normalized size = 0.82

method	result	size
default	$\frac{\ln\left(\sqrt{h}r\sqrt{2}+\sqrt{2hr^2-\alpha^2}\right)\sqrt{2}}{2\sqrt{h}}$	33

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(2*h*r^2-alpha^2)^(1/2),r,method=_RETURNVERBOSE)``[Out] 1/2*ln(h^(1/2)*r*2^(1/2)+(2*h*r^2-alpha^2)^(1/2))*2^(1/2)/h^(1/2)`**Maxima [A]**

time = 0.26, size = 36, normalized size = 0.90

$$\frac{\sqrt{2}\log\left(4hr+2\sqrt{2}\sqrt{2hr^2-\alpha^2}\sqrt{h}\right)}{2\sqrt{h}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*h*r^2-alpha^2)^(1/2),r, algorithm="maxima")

[Out] 1/2*sqrt(2)*log(4*h*r + 2*sqrt(2)*sqrt(2*h*r^2 - alpha^2)*sqrt(h))/sqrt(h)

Fricas [A]

time = 0.33, size = 85, normalized size = 2.12

$$\left[\frac{\sqrt{2} \log\left(4hr^2 + 2\sqrt{2}\sqrt{2hr^2 - \alpha^2}\sqrt{h}r - \alpha^2\right)}{4\sqrt{h}}, -\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{h}} \arctan\left(\frac{\sqrt{2}hr\sqrt{-\frac{1}{h}}}{\sqrt{2hr^2 - \alpha^2}}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*h*r^2-alpha^2)^(1/2),r, algorithm="fricas")

[Out] [1/4*sqrt(2)*log(4*h*r^2 + 2*sqrt(2)*sqrt(2*h*r^2 - alpha^2)*sqrt(h)*r - alpha^2)/sqrt(h), -1/2*sqrt(2)*sqrt(-1/h)*arctan(sqrt(2)*h*r*sqrt(-1/h)/sqrt(2*h*r^2 - alpha^2)]]

Sympy [A]

time = 0.50, size = 66, normalized size = 1.65

$$\left\{ \begin{array}{l} \frac{\sqrt{2} \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{h}r}{\alpha}\right)}{2\sqrt{h}} \quad \text{for } \left|\frac{hr^2}{\alpha^2}\right| > \frac{1}{2} \\ -\frac{\sqrt{2} i \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{h}r}{\alpha}\right)}{2\sqrt{h}} \quad \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*h*r**2-alpha**2)**(1/2),r)

[Out] Piecewise((sqrt(2)*acosh(sqrt(2)*sqrt(h)*r/alpha)/(2*sqrt(h)), Abs(h*r**2/alpha**2) > 1/2), (-sqrt(2)*I*asin(sqrt(2)*sqrt(h)*r/alpha)/(2*sqrt(h)), True))

Giac [A]

time = 0.00, size = 42, normalized size = 1.05

$$-\frac{\ln\left|\sqrt{-\alpha^2 + 2hr^2} - \sqrt{2h}r\right|}{\sqrt{2}\sqrt{h}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*h*r^2-alpha^2)^(1/2),r)

[Out] -1/2*sqrt(2)*log(abs(-sqrt(2)*sqrt(h)*r + sqrt(2*h*r^2 - alpha^2)))/sqrt(h)

Mupad [B]

time = 0.48, size = 32, normalized size = 0.80

$$\frac{\sqrt{2} \ln\left(\sqrt{2hr^2 - \alpha^2} + \sqrt{2} \sqrt{h} r\right)}{2\sqrt{h}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*h*r^2 - alpha^2)^(1/2),r)

[Out] (2^(1/2)*log((2*h*r^2 - alpha^2)^(1/2) + 2^(1/2)*h^(1/2)*r))/(2*h^(1/2))

$$3.206 \quad \int \frac{1}{r \sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}} dr$$

Optimal. Leaf size=46

$$\frac{\tan^{-1} \left(\frac{\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}}{\sqrt{\alpha^2 + \epsilon^2}} \right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

[Out] arctan((2*h*r^2-alpha^2-epsilon^2)^(1/2)/(alpha^2+epsilon^2)^(1/2))/(alpha^2+epsilon^2)^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {272, 65, 211}

$$\frac{\tan^{-1} \left(\frac{\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}}{\sqrt{\alpha^2 + \epsilon^2}} \right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]),r]

[Out] ArcTan[Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]/Sqrt[alpha^2 + epsilon^2]]/Sqrt[alpha^2 + epsilon^2]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}} dr &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}} dr, r, r^2 \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{-\frac{\alpha^2 - \epsilon^2}{2h} + \frac{r^2}{2h}} dr, r, \sqrt{-\alpha^2 - \epsilon^2 + 2hr^2} \right)}{2h} \\
&= \frac{\tan^{-1} \left(\frac{\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}}{\sqrt{\alpha^2 + \epsilon^2}} \right)}{\sqrt{\alpha^2 + \epsilon^2}}
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 46, normalized size = 1.00

$$\frac{\tan^{-1} \left(\frac{\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}}{\sqrt{\alpha^2 + \epsilon^2}} \right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]),r]``[Out] ArcTan[Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]/Sqrt[alpha^2 + epsilon^2]]/Sqrt[alpha^2 + epsilon^2]`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.57, size = 42, normalized size = 0.91

$$\frac{\text{ArcSinh} \left[\frac{\sqrt{2} \sqrt{\text{polar_lift}[-\alpha^2 - \epsilon^2]}}{2\sqrt{h} r} \right]}{\sqrt{\text{polar_lift}[-\alpha^2 - \epsilon^2]}}$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(r*Sqrt[2*h*r^2-alpha^2-epsilon^2]),r]')``[Out] -ArcSinh[Sqrt[2] Sqrt[polar_lift[-alpha ^ 2 - epsilon ^ 2]] / (2 Sqrt[h] r)] / Sqrt[polar_lift[-alpha ^ 2 - epsilon ^ 2]]`**Maple [A]**

time = 0.04, size = 66, normalized size = 1.43

method	result	size
--------	--------	------

default	$-\frac{\ln\left(\frac{-2\alpha^2-2\epsilon^2+2\sqrt{-\alpha^2-\epsilon^2}\sqrt{2hr^2-\alpha^2-\epsilon^2}}{r}\right)}{\sqrt{-\alpha^2-\epsilon^2}}$	66
---------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r,method=_RETURNVERBOSE)`

[Out] $-1/(-\alpha^2-\epsilon^2)^{(1/2)}*\ln((-2*\alpha^2-2*\epsilon^2+2*(-\alpha^2-\epsilon^2)^{(1/2)}*(2*h*r^2-\alpha^2-\epsilon^2)^{(1/2)})/r)$

Maxima [A]

time = 0.35, size = 57, normalized size = 1.24

$$-\frac{\arcsin\left(\frac{\sqrt{2}\alpha^2}{2\sqrt{(\alpha^2+\epsilon^2)hr}}+\frac{\sqrt{2}\epsilon^2}{2\sqrt{(\alpha^2+\epsilon^2)hr}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="maxima")`

[Out] $-\arcsin(1/2*\sqrt{2}*\alpha^2/(\sqrt{(\alpha^2+\epsilon^2)*h}*r)+1/2*\sqrt{2}*\epsilon^2/(\sqrt{(\alpha^2+\epsilon^2)*h}*r))/\sqrt{\alpha^2+\epsilon^2}$

Fricas [A]

time = 0.30, size = 41, normalized size = 0.89

$$-\frac{\arctan\left(\frac{\sqrt{\alpha^2+\epsilon^2}}{\sqrt{2hr^2-\alpha^2-\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="fricas")`

[Out] $-\arctan(\sqrt{\alpha^2+\epsilon^2}/\sqrt{2*h*r^2-\alpha^2-\epsilon^2})/\sqrt{\alpha^2+\epsilon^2}$

Sympy [A]

time = 0.55, size = 42, normalized size = 0.91

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{\operatorname{polar_lift}(-\alpha^2-\epsilon^2)}}{2\sqrt{hr}}\right)}{\sqrt{\operatorname{polar_lift}(-\alpha^2-\epsilon^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/r/(2*h*r**2-alpha**2-epsilon**2)**(1/2),r)

[Out] -asinh(sqrt(2)*sqrt(polar_lift(-alpha**2 - epsilon**2))/(2*sqrt(h)*r))/sqrt(polar_lift(-alpha**2 - epsilon**2))

Giac [A]

time = 0.00, size = 46, normalized size = 1.00

$$\frac{(5.0 \times 10^{-13}) \cdot (2.0 \times 10^{24}) h \cdot 2.0 \cdot 0.5 \arctan\left(\frac{\sqrt{-(1.0 \times 10^{24}) \alpha^2 + (2.0 \times 10^{24}) h r^2 - 1.0}}{\sqrt{(1.0 \times 10^{24}) \alpha^2 + 1.0}}\right)}{h \sqrt{(1.0 \times 10^{24}) \alpha^2 + 1.0}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r)

[Out] 1.0000000000000000e12*arctan(sqrt(2.0000000000000000e24*h*r^2 - 1.0000000000000000e24*alpha^2 - 1.0000000000000000))/sqrt(1.0000000000000000e24*alpha^2 + 1.0000000000000000))/sqrt(1.0000000000000000e24*alpha^2 + 1.0000000000000000)

Mupad [B]

time = 0.66, size = 40, normalized size = 0.87

$$\frac{\operatorname{atan}\left(\frac{\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}}{\sqrt{\alpha^2 + \epsilon^2}}\right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(r*(2*h*r^2 - alpha^2 - epsilon^2)^(1/2)),r)

[Out] atan((2*h*r^2 - alpha^2 - epsilon^2)^(1/2)/(alpha^2 + epsilon^2)^(1/2))/(alpha^2 + epsilon^2)^(1/2)

$$3.207 \quad \int \frac{1}{r \sqrt{-\alpha^2 - 2kr + 2hr^2}} dr$$

Optimal. Leaf size=37

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+kr}{\alpha\sqrt{-\alpha^2-2kr+2hr^2}}\right)}{\alpha}$$

[Out] -arctan((alpha^2+k*r)/alpha/(2*h*r^2-alpha^2-2*k*r)^(1/2))/alpha

Rubi [A]

time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {738, 210}

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+kr}{\alpha\sqrt{-\alpha^2+2hr^2-2kr}}\right)}{\alpha}$$

Antiderivative was successfully verified.

[In] Int[1/(r*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2]),r]

[Out] -(ArcTan[(alpha^2 + k*r)/(alpha*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2])]/alpha)

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 738

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{r \sqrt{-\alpha^2 - 2kr + 2hr^2}} dr &= - \left(2 \text{Subst} \left(\int \frac{1}{-4\alpha^2 - r^2} dr, r, \frac{-2\alpha^2 - 2kr}{\sqrt{-\alpha^2 - 2kr + 2hr^2}} \right) \right) \\ &= - \frac{\tan^{-1}\left(\frac{\alpha^2+kr}{\alpha\sqrt{-\alpha^2-2kr+2hr^2}}\right)}{\alpha} \end{aligned}$$

Mathematica [A]

time = 0.08, size = 45, normalized size = 1.22

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{2} \sqrt{h} r - \sqrt{-\alpha^2 - 2kr + 2hr^2}}{\alpha} \right)}{\alpha}$$

Antiderivative was successfully verified.

[In] Integrate[1/(r*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2]),r]

[Out] (-2*ArcTan[(Sqrt[2]*Sqrt[h]*r - Sqrt[-alpha^2 - 2*k*r + 2*h*r^2])/alpha])/alpha

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(r*Sqrt[2*h*r^2-alpha^2-2*k*r]),r]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.12, size = 52, normalized size = 1.41

method	result	size
default	$\frac{\ln \left(\frac{-2\alpha^2 - 2kr + 2\sqrt{-\alpha^2} \sqrt{2hr^2 - \alpha^2 - 2kr}}{r} \right)}{\sqrt{-\alpha^2}}$	52

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r,method=_RETURNVERBOSE)

[Out] -1/(-alpha^2)^(1/2)*ln((-2*alpha^2-2*k*r+2*(-alpha^2)^(1/2)*(2*h*r^2-alpha^2-2*k*r)^(1/2))/r)

Maxima [A]

time = 0.38, size = 40, normalized size = 1.08

$$\frac{\arcsin \left(\frac{k}{\sqrt{2\alpha^2 h + k^2}} + \frac{\alpha^2}{\sqrt{2\alpha^2 h + k^2} r} \right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="maxima")

[Out] -arcsin(k/sqrt(2*alpha^2*h + k^2) + alpha^2/(sqrt(2*alpha^2*h + k^2)*r))/alpha

Fricas [A]

time = 0.32, size = 52, normalized size = 1.41

$$\frac{\arctan\left(\frac{\sqrt{2hr^2 - \alpha^2 - 2kr}(\alpha^2 + kr)}{2\alpha hr^2 - \alpha^3 - 2\alpha kr}\right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="fricas")

[Out] -arctan(sqrt(2*h*r^2 - alpha^2 - 2*k*r)*(alpha^2 + k*r)/(2*alpha*h*r^2 - alpha^3 - 2*alpha*k*r))/alpha

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{r\sqrt{-\alpha^2 + 2hr^2 - 2kr}} dr$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/r/(2*h*r**2-alpha**2-2*k*r)**(1/2),r)

[Out] Integral(1/(r*sqrt(-alpha**2 + 2*h*r**2 - 2*k*r)), r)

Giac [A]

time = 0.01, size = 42, normalized size = 1.14

$$\frac{\frac{1}{2} \cdot 4 \arctan\left(\frac{\sqrt{-\alpha^2 + 2hr^2 - 2kr} - \sqrt{2h}r}{\alpha}\right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r)

[Out] 2*arctan(-(sqrt(2)*sqrt(h)*r - sqrt(2*h*r^2 - alpha^2 - 2*k*r))/alpha)/alpha

Mupad [B]

time = 0.10, size = 51, normalized size = 1.38

$$\frac{\ln\left(\frac{\sqrt{-\alpha^2} \sqrt{-\alpha^2 + 2hr^2 - 2kr}}{r} - \frac{\alpha^2}{r} - k\right)}{\sqrt{-\alpha^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*h*r^2 - 2*k*r - alpha^2)^(1/2)),r)
```

```
[Out] -log(((alpha^2)^(1/2)*(2*h*r^2 - 2*k*r - alpha^2)^(1/2))/r - alpha^2/r - k)/alpha^2^(1/2)
```

$$3.208 \quad \int \frac{1}{r \sqrt{-\alpha^2 - \epsilon^2 - 2kr + 2hr^2}} dr$$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\alpha^2 + \epsilon^2 + kr}{\sqrt{\alpha^2 + \epsilon^2} \sqrt{-\alpha^2 - \epsilon^2 - 2kr + 2hr^2}}\right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

[Out] $-\arctan\left(\frac{\alpha^2 + \epsilon^2 + kr}{\sqrt{\alpha^2 + \epsilon^2} \sqrt{-\alpha^2 - \epsilon^2 - 2kr + 2hr^2}}\right) / \sqrt{\alpha^2 + \epsilon^2}$

Rubi [A]

time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {738, 210}

$$-\frac{\tan^{-1}\left(\frac{\alpha^2 + \epsilon^2 + kr}{\sqrt{\alpha^2 + \epsilon^2} \sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr}}\right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

Antiderivative was successfully verified.

[In] `Int[1/(r*Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2]),r]`

[Out] `-(ArcTan[(alpha^2 + epsilon^2 + k*r)/(Sqrt[alpha^2 + epsilon^2]*Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2])]/Sqrt[alpha^2 + epsilon^2])`

Rule 210

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 738

`Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]`

Rubi steps

$$\int \frac{1}{r \sqrt{-\alpha^2 - \epsilon^2 - 2kr + 2hr^2}} dr = - \left(2 \text{Subst} \left(\int \frac{1}{4(-\alpha^2 - \epsilon^2) - r^2} dr, r, \frac{2(-\alpha^2 - \epsilon^2) - 2kr}{\sqrt{-\alpha^2 - \epsilon^2 - 2kr + 2hr^2}} \right) \right) = - \frac{\tan^{-1}\left(\frac{\alpha^2 + \epsilon^2 + kr}{\sqrt{\alpha^2 + \epsilon^2} \sqrt{-\alpha^2 - \epsilon^2 - 2kr + 2hr^2}}\right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

Mathematica [A]

time = 0.11, size = 66, normalized size = 1.08

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{2} \sqrt{h} r - \sqrt{-\alpha^2 - \epsilon^2 - 2kr + 2hr^2}}{\sqrt{\alpha^2 + \epsilon^2}} \right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2]),r]`

```
[Out] (-2*ArcTan[(Sqrt[2]*Sqrt[h]*r - Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2])
/Sqrt[alpha^2 + epsilon^2]])/Sqrt[alpha^2 + epsilon^2]
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

caught exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(r*Sqrt[2*h*r^2-alpha^2-epsilon^2-2*k*r]),r]')`

```
[Out] caught exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]

time = 0.13, size = 74, normalized size = 1.21

method	result	size
default	$\frac{\ln \left(\frac{-2\alpha^2 - 2\epsilon^2 - 2kr + 2 \sqrt{-\alpha^2 - \epsilon^2} \sqrt{2hr^2 - \alpha^2 - \epsilon^2 - 2kr}}{r} \right)}{\sqrt{-\alpha^2 - \epsilon^2}}$	74

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r,method=_RETURNVERBOSE)`

```
[Out] -1/(-alpha^2-epsilon^2)^(1/2)*ln((-2*alpha^2-2*epsilon^2-2*k*r+2*(-alpha^2-
epsilon^2)^(1/2)*(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2))/r)
```

Maxima [A]

time = 0.37, size = 77, normalized size = 1.26

$$\frac{\arcsin \left(\frac{k}{\sqrt{2(\alpha^2 + \epsilon^2)h + k^2}} + \frac{\alpha^2}{\sqrt{2(\alpha^2 + \epsilon^2)h + k^2}r} + \frac{\epsilon^2}{\sqrt{2(\alpha^2 + \epsilon^2)h + k^2}r} \right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r, algorithm="maxima")

[Out] -arcsin(k/sqrt(2*(alpha^2 + epsilon^2)*h + k^2) + alpha^2/(sqrt(2*(alpha^2 + epsilon^2)*h + k^2)*r) + epsilon^2/(sqrt(2*(alpha^2 + epsilon^2)*h + k^2)*r))/sqrt(alpha^2 + epsilon^2)

Fricas [A]

time = 0.31, size = 97, normalized size = 1.59

$$-\frac{\arctan\left(-\frac{\sqrt{2hr^2 - \alpha^2 - \epsilon^2 - 2kr}(\alpha^2 + \epsilon^2 + kr)\sqrt{\alpha^2 + \epsilon^2}}{\alpha^4 + 2\alpha^2\epsilon^2 + \epsilon^4 - 2(\alpha^2 + \epsilon^2)hr^2 + 2(\alpha^2 + \epsilon^2)kr}\right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r, algorithm="fricas")

[Out] -arctan(-sqrt(2*h*r^2 - alpha^2 - epsilon^2 - 2*k*r)*(alpha^2 + epsilon^2 + k*r)*sqrt(alpha^2 + epsilon^2)/(alpha^4 + 2*alpha^2*epsilon^2 + epsilon^4 - 2*(alpha^2 + epsilon^2)*h*r^2 + 2*(alpha^2 + epsilon^2)*k*r))/sqrt(alpha^2 + epsilon^2)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr}} dr$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/r/(2*h*r**2-alpha**2-epsilon**2-2*k*r)**(1/2),r)

[Out] Integral(1/(r*sqrt(-alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r)), r)

Giac [A]

time = 0.01, size = 54, normalized size = 0.89

$$\frac{(2.0 \times 10^{12}) \arctan\left(\frac{\sqrt{-(1.0 \times 10^{24})\alpha^2 + (2.0 \times 10^{24})hr^2 - (2.0 \times 10^{24})kr - 1.0} - \sqrt{(2.0 \times 10^{24})hr}}{\sqrt{(1.0 \times 10^{24})\alpha^2 + 1.0}}\right)}{\sqrt{(1.0 \times 10^{24})\alpha^2 + 1.0}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r)

[Out] 2.0000000000000000e12*arctan((-1.41421356237309e12*sqrt(h)*r + sqrt(2.0000000000000000e24*h*r^2 - 1.0000000000000000e24*alpha^2 - 2.0000000000000000e24*k*r -

$$3.209 \quad \int \frac{r}{\sqrt{-\alpha^2 + 2er^2}} dr$$

Optimal. Leaf size=23

$$\frac{\sqrt{-\alpha^2 + 2er^2}}{2e}$$

[Out] 1/2*(2*e*r^2-alpha^2)^(1/2)/e

Rubi [A]

time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {267}

$$\frac{\sqrt{2er^2 - \alpha^2}}{2e}$$

Antiderivative was successfully verified.

[In] Int[r/Sqrt[-alpha^2 + 2*e*r^2],r]

[Out] Sqrt[-alpha^2 + 2*e*r^2]/(2*e)

Rule 267

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2}} dr = \frac{\sqrt{-\alpha^2 + 2er^2}}{2e}$$

Mathematica [A]

time = 0.00, size = 23, normalized size = 1.00

$$\frac{\sqrt{-\alpha^2 + 2er^2}}{2e}$$

Antiderivative was successfully verified.

[In] Integrate[r/Sqrt[-alpha^2 + 2*e*r^2],r]

[Out] Sqrt[-alpha^2 + 2*e*r^2]/(2*e)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.

time = 2.13, size = 37, normalized size = 1.61

$$\text{Piecewise} \left[\left\{ \left\{ \frac{\sqrt{-\alpha^2 + 2er^2}}{2e}, e \neq 0 \right\} \right\}, \frac{r^2}{2\sqrt{-\alpha^2}} \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[r/Sqrt[2*e*r^2-alpha^2],r]')`

[Out] `Piecewise[{{Sqrt[-alpha ^ 2 + 2 e r ^ 2] / (2 e), e != 0}}, r ^ 2 / (2 Sqrt [-alpha ^ 2])]`

Maple [A]

time = 0.04, size = 20, normalized size = 0.87

method	result	size
gospers	$\frac{\sqrt{2er^2 - \alpha^2}}{2e}$	20
derivativedivides	$\frac{\sqrt{2er^2 - \alpha^2}}{2e}$	20
default	$\frac{\sqrt{2er^2 - \alpha^2}}{2e}$	20
trager	$\frac{\sqrt{2er^2 - \alpha^2}}{2e}$	20
risch	$-\frac{-2er^2 + \alpha^2}{2e\sqrt{2er^2 - \alpha^2}}$	30

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(r/(2*e*r^2-alpha^2)^(1/2),r,method=_RETURNVERBOSE)`

[Out] `1/2*(2*e*r^2-alpha^2)^(1/2)/e`

Maxima [A]

time = 0.26, size = 19, normalized size = 0.83

$$\frac{1}{2} \sqrt{2r^2e - \alpha^2} e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*e*r^2-alpha^2)^(1/2),r, algorithm="maxima")`

[Out] `1/2*sqrt(2*r^2*e - alpha^2)*e^(-1)`

Fricas [A]

time = 0.30, size = 19, normalized size = 0.83

$$\frac{1}{2} \sqrt{2r^2e - \alpha^2} e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(r/(2*e*r^2-alpha^2)^(1/2),r, algorithm="fricas")

[Out] 1/2*sqrt(2*r^2*e - alpha^2)*e^(-1)

Sympy [A]

time = 0.23, size = 29, normalized size = 1.26

$$\begin{cases} \frac{\sqrt{-\alpha^2 + 2er^2}}{2e} & \text{for } e \neq 0 \\ \frac{r^2}{2\sqrt{-\alpha^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(r/(2*e*r**2-alpha**2)**(1/2),r)

[Out] Piecewise((sqrt(-alpha**2 + 2*e*r**2)/(2*e), Ne(e, 0)), (r**2/(2*sqrt(-alpha**2)), True))

Giac [A]

time = 0.00, size = 22, normalized size = 0.96

$$\frac{\sqrt{-\alpha^2 + 2r^2e}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(r/(2*e*r^2-alpha^2)^(1/2),r)

[Out] 1/2*sqrt(2*e*r^2 - alpha^2)/e

Mupad [B]

time = 0.29, size = 19, normalized size = 0.83

$$\frac{\sqrt{2er^2 - \alpha^2}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(r/(2*e*r^2 - alpha^2)^(1/2),r)

[Out] (2*e*r^2 - alpha^2)^(1/2)/(2*e)

$$3.210 \quad \int \frac{r}{\sqrt{-\alpha^2 - \epsilon^2 + 2er^2}} dr$$

Optimal. Leaf size=28

$$\frac{\sqrt{-\alpha^2 - \epsilon^2 + 2er^2}}{2e}$$

[Out] $1/2*(2*e*r^2-\alpha^2-\epsilon^2)^{(1/2)}/e$

Rubi [A]

time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {267}

$$\frac{\sqrt{-\alpha^2 + 2er^2 - \epsilon^2}}{2e}$$

Antiderivative was successfully verified.

[In] Int[r/Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2],r]

[Out] Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2]/(2*e)

Rule 267

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int \frac{r}{\sqrt{-\alpha^2 - \epsilon^2 + 2er^2}} dr = \frac{\sqrt{-\alpha^2 - \epsilon^2 + 2er^2}}{2e}$$

Mathematica [A]

time = 0.00, size = 28, normalized size = 1.00

$$\frac{\sqrt{-\alpha^2 - \epsilon^2 + 2er^2}}{2e}$$

Antiderivative was successfully verified.

[In] Integrate[r/Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2],r]

[Out] Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2]/(2*e)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.

time = 2.13, size = 48, normalized size = 1.71

$$\text{Piecewise} \left[\left\{ \left\{ \frac{\sqrt{-\alpha^2 + 2er^2 - \epsilon^2}}{2e}, e \neq 0 \right\} \right\}, \frac{r^2}{2\sqrt{-\alpha^2 - \epsilon^2}} \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[r/Sqrt[2*e*r^2-alpha^2-epsilon^2],r]')`

[Out] `Piecewise[{{Sqrt[-alpha ^ 2 + 2 e r ^ 2 - epsilon ^ 2] / (2 e), e != 0}}, r ^ 2 / (2 Sqrt[-alpha ^ 2 - epsilon ^ 2])]`

Maple [A]

time = 0.04, size = 25, normalized size = 0.89

method	result	size
gospers	$\frac{\sqrt{2er^2 - \alpha^2 - \epsilon^2}}{2e}$	25
derivativdivides	$\frac{\sqrt{2er^2 - \alpha^2 - \epsilon^2}}{2e}$	25
default	$\frac{\sqrt{2er^2 - \alpha^2 - \epsilon^2}}{2e}$	25
trager	$\frac{\sqrt{2er^2 - \alpha^2 - \epsilon^2}}{2e}$	25
risch	$-\frac{-2er^2 + \alpha^2 + \epsilon^2}{2e\sqrt{2er^2 - \alpha^2 - \epsilon^2}}$	38

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r,method=_RETURNVERBOSE)`

[Out] `1/2*(2*e*r^2-alpha^2-epsilon^2)^(1/2)/e`

Maxima [A]

time = 0.25, size = 24, normalized size = 0.86

$$\frac{1}{2} \sqrt{2r^2e - \alpha^2 - \epsilon^2} e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="maxima")`

[Out] `1/2*sqrt(2*r^2*e - alpha^2 - epsilon^2)*e^(-1)`

Fricas [A]

time = 0.31, size = 24, normalized size = 0.86

$$\frac{1}{2} \sqrt{2r^2e - \alpha^2 - \epsilon^2} e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="fricas")`

[Out] $\frac{1}{2}\sqrt{2r^2e - \alpha^2 - \epsilon^2}e^{-1}$

Sympy [A]

time = 0.30, size = 36, normalized size = 1.29

$$\begin{cases} \frac{\sqrt{-\alpha^2 + 2er^2 - \epsilon^2}}{2e} & \text{for } e \neq 0 \\ \frac{r^2}{2\sqrt{-\alpha^2 - \epsilon^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*e*r**2-alpha**2-epsilon**2)**(1/2),r)`

[Out] `Piecewise((sqrt(-alpha**2 + 2*e*r**2 - epsilon**2)/(2*e), Ne(e, 0)), (r**2/(2*sqrt(-alpha**2 - epsilon**2)), True))`

Giac [A]

time = 0.00, size = 18, normalized size = 0.64

$$(1.83939720586 \times 10^{-13}) \sqrt{-(1.0 \times 10^{24}) \alpha^2 + (2.0 \times 10^{24}) r^2 \cdot 2.71828182846 - 1.0}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r)`

[Out] $(1.83939720586000e-13)\sqrt{-1.00000000000000e24*\alpha^2 + 5.43656365692000e24*r^2 - 1.000000000000000}$

Mupad [B]

time = 0.27, size = 24, normalized size = 0.86

$$\frac{\sqrt{-\alpha^2 - \epsilon^2 + 2er^2}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(r/(2*e*r^2 - alpha^2 - epsilon^2)^(1/2),r)`

[Out] $(2*e*r^2 - \alpha^2 - \epsilon^2)^{(1/2)}/(2*e)$

$$3.211 \quad \int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}} dr$$

Optimal. Leaf size=56

$$-\frac{\tan^{-1}\left(\frac{e^{-2kr^2}}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2 + 2er^2 - 2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}$$

[Out] $-1/4*\arctan(1/2*(-2*k*r^2+e)*2^{(1/2)}/k^{(1/2)/(-2*k*r^4+2*e*r^2-\alpha^2)^{(1/2)})*2^{(1/2)}/k^{(1/2)}$

Rubi [A]

time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1121, 635, 210}

$$-\frac{\tan^{-1}\left(\frac{e^{-2kr^2}}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2 + 2er^2 - 2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}$$

Antiderivative was successfully verified.

[In] `Int[r/Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4],r]`

[Out] $-1/2*\text{ArcTan}[(e - 2*k*r^2)/(\text{Sqrt}[2]*\text{Sqrt}[k]*\text{Sqrt}[-\alpha^2 + 2*e*r^2 - 2*k*r^4])]/(\text{Sqrt}[2]*\text{Sqrt}[k])$

Rule 210

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])`

Rule 635

`Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 1121

`Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]`

Rubi steps

$$\begin{aligned}
\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}} dr &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{-\alpha^2 + 2er - 2kr^2}} dr, r, r^2 \right) \\
&= \text{Subst} \left(\int \frac{1}{-8k - r^2} dr, r, \frac{2(e - 2kr^2)}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}} \right) \\
&= -\frac{\tan^{-1} \left(\frac{e - 2kr^2}{\sqrt{2} \sqrt{k} \sqrt{-\alpha^2 + 2er^2 - 2kr^4}} \right)}{2\sqrt{2} \sqrt{k}}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 150 vs. 2(56) = 112.

time = 0.18, size = 150, normalized size = 2.68

$$\frac{2\sqrt{-k} \tan^{-1} \left(\frac{\sqrt{k} (2\sqrt{-k} r^2 - \sqrt{2} \sqrt{-\alpha^2 + 2er^2 - 2kr^4})}{e} \right) + \sqrt{k} \log \left(e^2 + 4ekr^2 - 2k(\alpha^2 + 4kr^4 + 2\sqrt{2} \sqrt{-k} r^2 \sqrt{-\alpha^2 + 2er^2 - 2kr^4}) \right)}{4\sqrt{2} \sqrt{-k^2}}$$

Antiderivative was successfully verified.

[In] Integrate[r/Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4], r]

[Out] $-1/4*(2*\text{Sqrt}[-k]*\text{ArcTan}[(\text{Sqrt}[k]*(2*\text{Sqrt}[-k]*r^2 - \text{Sqrt}[2]*\text{Sqrt}[-\alpha^2 + 2*e*r^2 - 2*k*r^4]))/e] + \text{Sqrt}[k]*\text{Log}[e^2 + 4*e*k*r^2 - 2*k*(\alpha^2 + 4*k*r^4 + 2*\text{Sqrt}[2]*\text{Sqrt}[-k]*r^2*\text{Sqrt}[-\alpha^2 + 2*e*r^2 - 2*k*r^4])])]/(\text{Sqrt}[2]*\text{Sqrt}[-k^2])$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[r/Sqrt[2*e*r^2-alpha^2-2*k*r^4], r]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.03, size = 47, normalized size = 0.84

method	result	size
default	$\frac{\sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{k} (r^2 - \frac{e}{2k})}{\sqrt{-2k r^4 + 2e r^2 - \alpha^2}} \right)}{4\sqrt{k}}$	47

elliptic	$\frac{\sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{k} (r^2 - \frac{e}{2k})}{\sqrt{-2kr^4 + 2er^2 - \alpha^2}}\right)}{4\sqrt{k}}$	47
----------	--	----

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r,method=_RETURNVERBOSE)
[Out] 1/4*2^(1/2)/k^(1/2)*arctan(2^(1/2)*k^(1/2)*(r^2-1/2*e/k)/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*alpha^2*k-%e^2>0)', see 'assume?'
for mo
```

Fricas [A]

time = 0.33, size = 155, normalized size = 2.77

$$\left[\frac{\sqrt{2} \sqrt{-k} \log\left(8k^2r^4 - 8kr^2e + 2\alpha^2k - 2\sqrt{2} \sqrt{-2kr^4 + 2r^2e - \alpha^2} (2kr^2 - e) \sqrt{-k} + e^2\right)}{8k}, -\frac{\sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{-2kr^4 + 2r^2e - \alpha^2} (2kr^2 - e) \sqrt{k}}{2(2k^2r^4 - 2kr^2e + \alpha^2k)}\right)}{4\sqrt{k}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r, algorithm="fricas")
[Out] [-1/8*sqrt(2)*sqrt(-k)*log(8*k^2*r^4 - 8*k*r^2*e + 2*alpha^2*k - 2*sqrt(2)*
sqrt(-2*k*r^4 + 2*r^2*e - alpha^2)*(2*k*r^2 - e)*sqrt(-k) + e^2)/k, -1/4*sq
rt(2)*arctan(1/2*sqrt(2)*sqrt(-2*k*r^4 + 2*r^2*e - alpha^2)*(2*k*r^2 - e)*s
qrt(k)/(2*k^2*r^4 - 2*k*r^2*e + alpha^2*k))/sqrt(k)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}} dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*k*r**4+2*e*r**2-alpha**2)**(1/2),r)
```


[Out] Integral(r/sqrt(-alpha**2 + 2*e*r**2 - 2*k*r**4), r)

Giac [A]

time = 0.01, size = 72, normalized size = 1.29

$$-\frac{\ln \left| -\sqrt{2} \sqrt{-k} \left(\sqrt{-\alpha^2 - 2kr^4 + 2r^2e} - \sqrt{-2k} r^2 \right) + e \right|}{2\sqrt{2} \sqrt{-k}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r)

[Out] -1/4*sqrt(2)*log(abs(sqrt(2)*(sqrt(2)*sqrt(-k)*r^2 - sqrt(-2*k*r^4 + 2*e*r^2 - alpha^2))*sqrt(-k) + e))/sqrt(-k)

Mupad [B]

time = 0.98, size = 50, normalized size = 0.89

$$\frac{\sqrt{2} \ln \left(\sqrt{-\alpha^2 - 2kr^4 + 2er^2} + \frac{\sqrt{2} (e - 2kr^2)}{2\sqrt{-k}} \right)}{4\sqrt{-k}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(r/(2*e*r^2 - 2*k*r^4 - alpha^2)^(1/2),r)

[Out] (2^(1/2)*log((2*e*r^2 - 2*k*r^4 - alpha^2)^(1/2) + (2^(1/2)*(e - 2*k*r^2))/(2*(-k)^(1/2))))/(4*(-k)^(1/2))

$$3.212 \quad \int \frac{r}{\sqrt{-\alpha^2 - 2kr + 2er^2}} dr$$

Optimal. Leaf size=81

$$\frac{\sqrt{-\alpha^2 - 2kr + 2er^2}}{2e} - \frac{k \tanh^{-1} \left(\frac{k-2er}{\sqrt{2} \sqrt{e} \sqrt{-\alpha^2 - 2kr + 2er^2}} \right)}{2\sqrt{2} e^{3/2}}$$

[Out] $-1/4*k*\operatorname{arctanh}(1/2*(-2*e*r+k)*2^{(1/2)}/e^{(1/2)}/(2*e*r^2-\alpha^2-2*k*r)^{(1/2)})/e^{(3/2)*2^{(1/2)}+1/2*(2*e*r^2-\alpha^2-2*k*r)^{(1/2)}/e}$

Rubi [A]

time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {654, 635, 212}

$$\frac{\sqrt{-\alpha^2 + 2er^2 - 2kr}}{2e} - \frac{k \tanh^{-1} \left(\frac{k-2er}{\sqrt{2} \sqrt{e} \sqrt{-\alpha^2 + 2er^2 - 2kr}} \right)}{2\sqrt{2} e^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[r/Sqrt[-alpha^2 - 2*k*r + 2*e*r^2],r]`

[Out] `Sqrt[-alpha^2 - 2*k*r + 2*e*r^2]/(2*e) - (k*ArcTanh[(k - 2*e*r)/(Sqrt[2]*Sqrt[e]*Sqrt[-alpha^2 - 2*k*r + 2*e*r^2])])/(2*Sqrt[2]*e^(3/2))`

Rule 212

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 635

`Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 654

`Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]`

Rubi steps

$$\begin{aligned}
\int \frac{r}{\sqrt{-\alpha^2 - 2kr + 2er^2}} dr &= \frac{\sqrt{-\alpha^2 - 2kr + 2er^2}}{2e} + \frac{k \int \frac{1}{\sqrt{-\alpha^2 - 2kr + 2er^2}} dr}{2e} \\
&= \frac{\sqrt{-\alpha^2 - 2kr + 2er^2}}{2e} + \frac{k \text{Subst} \left(\int \frac{1}{8e-r^2} dr, r, \frac{-2k+4er}{\sqrt{-\alpha^2 - 2kr + 2er^2}} \right)}{e} \\
&= \frac{\sqrt{-\alpha^2 - 2kr + 2er^2}}{2e} - \frac{k \tanh^{-1} \left(\frac{k-2er}{\sqrt{2} \sqrt{e} \sqrt{-\alpha^2 - 2kr + 2er^2}} \right)}{2\sqrt{2} e^{3/2}}
\end{aligned}$$

Mathematica [A]

time = 0.18, size = 86, normalized size = 1.06

$$\frac{2\sqrt{e} \sqrt{-\alpha^2 + 2r(-k + er)} - \sqrt{2} k \log \left(-e \left(k - 2er + \sqrt{2} \sqrt{e} \sqrt{-\alpha^2 - 2kr + 2er^2} \right) \right)}{4e^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[r/Sqrt[-alpha^2 - 2*k*r + 2*e*r^2], r]`

```
[Out] (2*Sqrt[e]*Sqrt[-alpha^2 + 2*r*(-k + e*r)] - Sqrt[2]*k*Log[-(e*(k - 2*e*r + Sqrt[2]*Sqrt[e]*Sqrt[-alpha^2 - 2*k*r + 2*e*r^2]))])/(4*e^(3/2))
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[r/Sqrt[2*e*r^2-alpha^2-2*k*r], r]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.12, size = 70, normalized size = 0.86

method	result	size
default	$ \frac{\sqrt{2e r^2 - \alpha^2 - 2kr}}{2e} + \frac{k \ln \left(\frac{(2er-k)\sqrt{2}}{2\sqrt{e}} + \sqrt{2e r^2 - \alpha^2 - 2kr} \right) \sqrt{2}}{4e^{3/2}} $	70

risch	$-\frac{-2er^2 + \alpha^2 + 2kr}{2e\sqrt{2er^2 - \alpha^2 - 2kr}} + \frac{k \ln\left(\frac{(2er-k)\sqrt{2} + \sqrt{2er^2 - \alpha^2 - 2kr}}{2\sqrt{e}}\right)\sqrt{2}}{4e^{\frac{3}{2}}}$	84
-------	---	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r,method=_RETURNVERBOSE)`

[Out] $\frac{1}{2}*(2*e*r^2-alpha^2-2*k*r)^(1/2)/e+1/4*k/e^(3/2)*\ln(1/2*(2*e*r-k)*2^(1/2)/e^(1/2)+(2*e*r^2-alpha^2-2*k*r)^(1/2))*2^(1/2)$

Maxima [A]

time = 0.26, size = 68, normalized size = 0.84

$$\frac{1}{4}\sqrt{2}ke^{(-\frac{3}{2})}\log\left(4re+2\sqrt{2}\sqrt{2r^2e-\alpha^2-2kr}e^{\frac{1}{2}}-2k\right)+\frac{1}{2}\sqrt{2r^2e-\alpha^2-2kr}e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="maxima")`

[Out] $\frac{1}{4}*\sqrt{2}*k*e^{(-3/2)}*\log(4*r*e+2*\sqrt{2}*\sqrt{2*r^2*e-\alpha^2-2*k*r})*e^{(1/2)}-2*k)+\frac{1}{2}*\sqrt{2*r^2*e-\alpha^2-2*k*r}*e^{(-1)}$

Fricas [A]

time = 0.32, size = 94, normalized size = 1.16

$$\frac{1}{8}\left(\sqrt{2}ke^{\frac{1}{2}}\log\left(8r^2e^2+2\sqrt{2}\sqrt{2r^2e-\alpha^2-2kr}(2re-k)e^{\frac{1}{2}}+k^2-2(\alpha^2+4kr)e\right)+4\sqrt{2r^2e-\alpha^2-2kr}e\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="fricas")`

[Out] $\frac{1}{8}*(\sqrt{2}*k*e^{(1/2)}*\log(8*r^2*e^2+2*\sqrt{2}*\sqrt{2*r^2*e-\alpha^2-2*k*r})*(2*r*e-k)*e^{(1/2)}+k^2-2*(\alpha^2+4*k*r)*e)+4*\sqrt{2*r^2*e-\alpha^2-2*k*r}*e)*e^{(-2)}$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr}} dr$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*e*r**2-alpha**2-2*k*r)**(1/2),r)`

[Out] `Integral(r/sqrt(-alpha**2 + 2*e*r**2 - 2*k*r), r)`

Giac [A]

time = 0.01, size = 96, normalized size = 1.19

$$\frac{2\sqrt{-\alpha^2 - 2kr + 2r^2e}}{4e} - \frac{2k \ln \left| \sqrt{2} \left(\sqrt{-\alpha^2 - 2kr + 2r^2e} - \sqrt{2e} r \right) e^{\frac{1}{2}} + k \right|}{4\sqrt{2} e^{\frac{1}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r)**[Out]** -1/4*sqrt(2)*k*log(abs(-sqrt(2)*(sqrt(2)*sqrt(e)*r - sqrt(2*e*r^2 - alpha^2 - 2*k*r))*sqrt(e) + k))/e^(3/2) + 1/2*sqrt(2*e*r^2 - alpha^2 - 2*k*r)/e**Mupad [B]**

time = 0.24, size = 67, normalized size = 0.83

$$\frac{\sqrt{-\alpha^2 + 2er^2 - 2kr}}{2e} + \frac{\sqrt{2} k \ln \left(\sqrt{-\alpha^2 + 2er^2 - 2kr} - \frac{\sqrt{2} (k-2er)}{2\sqrt{e}} \right)}{4e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(r/(2*e*r^2 - 2*k*r - alpha^2)^(1/2),r)**[Out]** (2*e*r^2 - 2*k*r - alpha^2)^(1/2)/(2*e) + (2^(1/2)*k*log((2*e*r^2 - 2*k*r - alpha^2)^(1/2) - (2^(1/2)*(k - 2*e*r))/(2*e^(1/2))))/(4*e^(3/2))

$$3.213 \quad \int \frac{1}{r \sqrt{-\alpha^2 + 2hr^2 - 2kr^4}} dr$$

Optimal. Leaf size=44

$$\frac{\tan^{-1}\left(\frac{\alpha^2 - hr^2}{\alpha \sqrt{-\alpha^2 + 2hr^2 - 2kr^4}}\right)}{2\alpha}$$

[Out] -1/2*arctan((-h*r^2+alpha^2)/alpha/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2))/alpha

Rubi [A]

time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1128, 738, 210}

$$\frac{\tan^{-1}\left(\frac{\alpha^2 - hr^2}{\alpha \sqrt{-\alpha^2 + 2hr^2 - 2kr^4}}\right)}{2\alpha}$$

Antiderivative was successfully verified.

[In] Int[1/(r*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4]),r]

[Out] -1/2*ArcTan[(alpha^2 - h*r^2)/(alpha*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4])/alpha]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 738

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1128

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{1}{r\sqrt{-\alpha^2 + 2hr^2 - 2kr^4}} dr &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{r\sqrt{-\alpha^2 + 2hr - 2kr^2}} dr, r, r^2 \right) \\
&= -\text{Subst} \left(\int \frac{1}{-4\alpha^2 - r^2} dr, r, \frac{2(-\alpha^2 + hr^2)}{\sqrt{-\alpha^2 + 2hr^2 - 2kr^4}} \right) \\
&= \frac{\tan^{-1} \left(\frac{-\alpha^2 + hr^2}{\alpha\sqrt{-\alpha^2 + 2hr^2 - 2kr^4}} \right)}{2\alpha}
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 53, normalized size = 1.20

$$\frac{\tan^{-1} \left(\frac{\sqrt{2} \sqrt{-k} r^2}{\alpha} - \frac{\sqrt{-\alpha^2 + 2hr^2 - 2kr^4}}{\alpha} \right)}{\alpha}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(r*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4]),r]``[Out] -(ArcTan[(Sqrt[2]*Sqrt[-k]*r^2)/alpha - Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4]/alpha])/alpha)`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(r*Sqrt[2*h*r^2-alpha^2-2*k*r^4]),r]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [A]**

time = 0.03, size = 56, normalized size = 1.27

method	result	size
default	$-\frac{\ln \left(\frac{-2\alpha^2 + 2hr^2 + 2\sqrt{-\alpha^2} \sqrt{-2kr^4 + 2hr^2 - \alpha^2}}{r^2} \right)}{2\sqrt{-\alpha^2}}$	56

elliptic	$-\frac{\ln\left(\frac{-2\alpha^2+2hr^2+2\sqrt{-\alpha^2}\sqrt{-2kr^4+2hr^2-\alpha^2}}{r^2}\right)}{2\sqrt{-\alpha^2}}$	56
----------	---	----

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-alpha^2)^(1/2)*ln((-2*alpha^2+2*h*r^2+2*(-alpha^2)^(1/2)*(-2*k*r^4+2*h*r^2-alpha^2)^(1/2))/r^2)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*alpha^2*k-h^2>0)', see 'assume?'
for mor
```

Fricas [A]

time = 0.31, size = 58, normalized size = 1.32

$$-\frac{\arctan\left(\frac{\sqrt{-2kr^4+2hr^2-\alpha^2}(hr^2-\alpha^2)}{2\alpha kr^4-2\alpha hr^2+\alpha^3}\right)}{2\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r, algorithm="fricas")
```

```
[Out] -1/2*arctan(sqrt(-2*k*r^4+2*h*r^2-alpha^2)*(h*r^2-alpha^2)/(2*alpha*k
*r^4-2*alpha*h*r^2+alpha^3))/alpha
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}} dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r**4+2*h*r**2-alpha**2)**(1/2),r)
```

```
[Out] Integral(1/(r*sqrt(-alpha**2+2*h*r**2-2*k*r**4)), r)
```


Giac [A]

time = 0.01, size = 43, normalized size = 0.98

$$\frac{\arctan\left(\frac{\sqrt{-\alpha^2 + 2hr^2 - 2kr^4} - \sqrt{-2kr^2}}{\alpha}\right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r)

[Out] arctan(-(sqrt(2)*sqrt(-k)*r^2 - sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2))/alpha)/alpha

Mupad [B]

time = 0.40, size = 54, normalized size = 1.23

$$\frac{\ln\left(\frac{1}{r^2}\right) + \ln\left(hr^2 - \alpha^2 + \sqrt{-\alpha^2} \sqrt{-\alpha^2 - 2kr^4 + 2hr^2}\right)}{2\sqrt{-\alpha^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(r*(2*h*r^2 - 2*k*r^4 - alpha^2)^(1/2)),r)

[Out] -(log(1/r^2) + log(hr^2 - alpha^2 + (-alpha^2)^(1/2)*(2*h*r^2 - 2*k*r^4 - alpha^2)^(1/2)))/(2*(-alpha^2)^(1/2))

$$3.214 \quad \int \frac{1}{r \sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}} dr$$

Optimal. Leaf size=68

$$\frac{\tan^{-1} \left(\frac{\alpha^2 + \epsilon^2 - hr^2}{\sqrt{\alpha^2 + \epsilon^2} \sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}} \right)}{2\sqrt{\alpha^2 + \epsilon^2}}$$

[Out] $-1/2*\arctan((-h*r^2+\alpha^2+\epsilon^2)/(\alpha^2+\epsilon^2)^{(1/2)/(-2*k*r^4+2*h*r^2-\alpha^2-\epsilon^2)^{(1/2)})/(\alpha^2+\epsilon^2)^{(1/2)}$

Rubi [A]

time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1128, 738, 210}

$$\frac{\tan^{-1} \left(\frac{\alpha^2 + \epsilon^2 - hr^2}{\sqrt{\alpha^2 + \epsilon^2} \sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}} \right)}{2\sqrt{\alpha^2 + \epsilon^2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(r*\text{Sqrt}[-\alpha^2 - \epsilon^2 + 2*h*r^2 - 2*k*r^4]),r]$

[Out] $-1/2*\text{ArcTan}[(\alpha^2 + \epsilon^2 - h*r^2)/(\text{Sqrt}[\alpha^2 + \epsilon^2]*\text{Sqrt}[-\alpha^2 - \epsilon^2 + 2*h*r^2 - 2*k*r^4])]/\text{Sqrt}[\alpha^2 + \epsilon^2]$

Rule 210

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 738

$\text{Int}[1/(((d_ + (e_)*(x_))*\text{Sqrt}[(a_ + (b_)*(x_ + (c_)*(x_)^2)]), x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0]$

Rule 1128

$\text{Int}[(x_)^{(m_)*((a_ + (b_)*(x_)^2 + (c_)*(x_)^4)^{(p_)}), x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{((m-1)/2)*(a + b*x + c*x^2)^p}, x], x, x^2], x] /; \text{FreeQ}\{a, b, c, p\}, x \ \&\& \ \text{IntegerQ}[(m-1)/2]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}} dr &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr - 2kr^2}} dr, r, r^2 \right) \\
&= -\text{Subst} \left(\int \frac{1}{4(-\alpha^2 - \epsilon^2) - r^2} dr, r, \frac{2(-\alpha^2 - \epsilon^2 + hr^2)}{\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}} \right) \\
&= \frac{\tan^{-1} \left(\frac{-\alpha^2 - \epsilon^2 + hr^2}{\sqrt{\alpha^2 + \epsilon^2} \sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}} \right)}{2\sqrt{\alpha^2 + \epsilon^2}}
\end{aligned}$$

Mathematica [A]

time = 0.11, size = 72, normalized size = 1.06

$$-\frac{\tan^{-1} \left(\frac{\sqrt{2} \sqrt{-k} r^2 - \sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}}{\sqrt{\alpha^2 + \epsilon^2}} \right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4]),r]``[Out] -(ArcTan[(Sqrt[2]*Sqrt[-k]*r^2 - Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4])/Sqrt[alpha^2 + epsilon^2]]/Sqrt[alpha^2 + epsilon^2])`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(r*Sqrt[2*h*r^2-alpha^2-epsilon^2-2*k*r^4]),r]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.03, size = 78, normalized size = 1.15

method	result	size
default	$-\frac{\ln \left(\frac{-2\alpha^2 - 2\epsilon^2 + 2hr^2 + 2\sqrt{-\alpha^2 - \epsilon^2} \sqrt{-2kr^4 + 2hr^2 - \alpha^2 - \epsilon^2}}{r^2} \right)}{2\sqrt{-\alpha^2 - \epsilon^2}}$	78
elliptic	$-\frac{\ln \left(\frac{-2\alpha^2 - 2\epsilon^2 + 2hr^2 + 2\sqrt{-\alpha^2 - \epsilon^2} \sqrt{-2kr^4 + 2hr^2 - \alpha^2 - \epsilon^2}}{r^2} \right)}{2\sqrt{-\alpha^2 - \epsilon^2}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r,method=_RETURNVERBOSE)
[Out] -1/2/(-alpha^2-epsilon^2)^(1/2)*ln((-2*alpha^2-2*epsilon^2+2*h*r^2+2*(-alpha^2-epsilon^2)^(1/2)*(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2))/r^2)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(2*epsilon^2*k+2*alpha^2*k>0)', see 'assume
```

Fricas [A]

time = 0.32, size = 106, normalized size = 1.56

$$\frac{\arctan\left(\frac{\sqrt{-2kr^4 + 2hr^2 - \alpha^2 - \epsilon^2} (hr^2 - \alpha^2 - \epsilon^2) \sqrt{\alpha^2 + \epsilon^2}}{2(\alpha^2 + \epsilon^2)kr^4 + \alpha^4 + 2\alpha^2\epsilon^2 + \epsilon^4 - 2(\alpha^2 + \epsilon^2)hr^2}\right)}{2\sqrt{\alpha^2 + \epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="fricas")
```

```
[Out] -1/2*arctan(sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2 - epsilon^2)*(h*r^2 - alpha^2 - epsilon^2)*sqrt(alpha^2 + epsilon^2)/(2*(alpha^2 + epsilon^2)*k*r^4 + alpha^4 + 2*alpha^2*epsilon^2 + epsilon^4 - 2*(alpha^2 + epsilon^2)*h*r^2))/sqrt(alpha^2 + epsilon^2)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}} dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r**4+2*h*r**2-alpha**2-epsilon**2)**(1/2),r)
```

[Out] Integral(1/(r*sqrt(-alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r**4)), r)

Giac [A]

time = 0.01, size = 43, normalized size = 0.63

$$\frac{\arctan\left(\frac{\sqrt{-\alpha^2 + 2hr^2 - 2kr^4} - \sqrt{-2k}r}{\alpha}\right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r)

[Out] arctan(-(sqrt(2)*sqrt(-k)*r^2 - sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2))/alpha)/alpha

Mupad [B]

time = 0.44, size = 72, normalized size = 1.06

$$\frac{\ln\left(h - \frac{\alpha^2 + \epsilon^2}{r^2} + \frac{\sqrt{-\alpha^2 - \epsilon^2} \sqrt{-\alpha^2 - \epsilon^2 - 2kr^4 + 2hr^2}}{r^2}\right)}{2\sqrt{-\alpha^2 - \epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(r*(2*h*r^2 - 2*k*r^4 - alpha^2 - epsilon^2)^(1/2)),r)

[Out] -log(h - (alpha^2 + epsilon^2)/r^2 + ((- alpha^2 - epsilon^2)^(1/2)*(2*h*r^2 - 2*k*r^4 - alpha^2 - epsilon^2)^(1/2))/r^2)/(2*(- alpha^2 - epsilon^2)^(1/2))

3.215 $\int a \cos(5 + 3x) \sin^2(5 + 3x) dx$

Optimal. Leaf size=13

$$\frac{1}{9}a \sin^3(5 + 3x)$$

[Out] 1/9*a*sin(5+3*x)^3

Rubi [A]

time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {12, 2644, 30}

$$\frac{1}{9}a \sin^3(3x + 5)$$

Antiderivative was successfully verified.

[In] Int[a*Cos[5 + 3*x]*Sin[5 + 3*x]^2,x]

[Out] (a*Sin[5 + 3*x]^3)/9

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2644

Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && LtQ[0, m, n])

Rubi steps

$$\begin{aligned} \int a \cos(5 + 3x) \sin^2(5 + 3x) dx &= a \int \cos(5 + 3x) \sin^2(5 + 3x) dx \\ &= \frac{1}{3}a \text{Subst} \left(\int x^2 dx, x, \sin(5 + 3x) \right) \\ &= \frac{1}{9}a \sin^3(5 + 3x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{9}a \sin^3(5 + 3x)$$

Antiderivative was successfully verified.

[In] Integrate[a*cos[5 + 3*x]*sin[5 + 3*x]^2,x]

[Out] (a*sin[5 + 3*x]^3)/9

Mathics [A]

time = 1.96, size = 11, normalized size = 0.85

$$\frac{a \sin [5 + 3x]^3}{9}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[a*sin[3*x+5]^2*cos[3*x+5],x]')

[Out] a Sin[5 + 3 x] ^ 3 / 9

Maple [A]

time = 0.04, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$\frac{a(\sin^3(5+3x))}{9}$	12
default	$\frac{a(\sin^3(5+3x))}{9}$	12
risch	$\frac{a \sin(5+3x)}{12} - \frac{a \sin(15+9x)}{36}$	20
norman	$\frac{8a(\tan^3(\frac{5}{2} + \frac{3x}{2}))}{9(1+\tan^2(\frac{5}{2} + \frac{3x}{2}))^3}$	24

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a*cos(5+3*x)*sin(5+3*x)^2,x,method=_RETURNVERBOSE)

[Out] 1/9*a*sin(5+3*x)^3

Maxima [A]

time = 0.26, size = 11, normalized size = 0.85

$$\frac{1}{9}a \sin(3x + 5)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*cos(5+3*x)*sin(5+3*x)^2,x, algorithm="maxima")

[Out] $1/9*a*\sin(3*x + 5)^3$

Fricas [A]

time = 0.31, size = 22, normalized size = 1.69

$$-\frac{1}{9} (a \cos(3x + 5)^2 - a) \sin(3x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a*cos(5+3*x)*sin(5+3*x)^2,x, algorithm="fricas")`

[Out] $-1/9*(a*\cos(3*x + 5)^2 - a)*\sin(3*x + 5)$

Sympy [A]

time = 0.10, size = 10, normalized size = 0.77

$$\frac{a \sin^3(3x + 5)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a*cos(5+3*x)*sin(5+3*x)**2,x)`

[Out] $a*\sin(3*x + 5)**3/9$

Giac [A]

time = 0.01, size = 14, normalized size = 1.08

$$\frac{a \sin^3(5 + 3x)}{3 \cdot 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a*cos(5+3*x)*sin(5+3*x)^2,x)`

[Out] $1/9*a*\sin(3*x + 5)^3$

Mupad [B]

time = 0.09, size = 11, normalized size = 0.85

$$\frac{a \sin(3x + 5)^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a*cos(3*x + 5)*sin(3*x + 5)^2,x)`

[Out] $(a*\sin(3*x + 5)^3)/9$

$$3.216 \quad \int \frac{\log(x^2)}{x^3} dx$$

Optimal. Leaf size=19

$$-\frac{1}{2x^2} - \frac{\log(x^2)}{2x^2}$$

[Out] $-1/2/x^2-1/2*\ln(x^2)/x^2$

Rubi [A]

time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2341}

$$-\frac{1}{2x^2} - \frac{\log(x^2)}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[Log[x^2]/x^3,x]

[Out] $-1/2*1/x^2 - \text{Log}[x^2]/(2*x^2)$

Rule 2341

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_)^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{\log(x^2)}{x^3} dx = -\frac{1}{2x^2} - \frac{\log(x^2)}{2x^2}$$

Mathematica [A]

time = 0.00, size = 19, normalized size = 1.00

$$-\frac{1}{2x^2} - \frac{\log(x^2)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Log[x^2]/x^3,x]

[Out] $-1/2*1/x^2 - \text{Log}[x^2]/(2*x^2)$

Mathics [A]

time = 1.74, size = 13, normalized size = 0.68

$$\frac{-1 - \operatorname{Log}[x^2]}{2x^2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Log[x^2]/x^3,x]')``[Out] (-1 - Log[x ^ 2]) / (2 x ^ 2)`**Maple [A]**

time = 0.01, size = 16, normalized size = 0.84

method	result	size
norman	$-\frac{1}{2} - \frac{\ln(x^2)}{2x^2}$	13
default	$-\frac{1}{2x^2} - \frac{\ln(x^2)}{2x^2}$	16
risch	$-\frac{1}{2x^2} - \frac{\ln(x^2)}{2x^2}$	16

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(ln(x^2)/x^3,x,method=_RETURNVERBOSE)``[Out] -1/2/x^2-1/2*ln(x^2)/x^2`**Maxima [A]**

time = 0.26, size = 15, normalized size = 0.79

$$-\frac{\log(x^2)}{2x^2} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(log(x^2)/x^3,x, algorithm="maxima")``[Out] -1/2*log(x^2)/x^2 - 1/2/x^2`**Fricas [A]**

time = 0.30, size = 11, normalized size = 0.58

$$-\frac{\log(x^2) + 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(log(x^2)/x^3,x, algorithm="fricas")`

[Out] $-1/2*(\log(x^2) + 1)/x^2$

Sympy [A]

time = 0.05, size = 17, normalized size = 0.89

$$-\frac{\log(x^2)}{2x^2} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(x**2)/x**3,x)`

[Out] $-\log(x**2)/(2*x**2) - 1/(2*x**2)$

Giac [A]

time = 0.00, size = 19, normalized size = 1.00

$$\frac{-\frac{\ln(x^2)}{x^2} - \frac{1}{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x^2)/x^3,x)`

[Out] $-1/2*\log(x^2)/x^2 - 1/2/x^2$

Mupad [B]

time = 0.16, size = 11, normalized size = 0.58

$$-\frac{\ln(x^2) + 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(x^2)/x^3,x)`

[Out] $-(\log(x^2) + 1)/(2*x^2)$

3.217 $\int x \sin(a + x) dx$

Optimal. Leaf size=12

$$-x \cos(a + x) + \sin(a + x)$$

[Out] `-x*cos(a+x)+sin(a+x)`

Rubi [A]

time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3377, 2717}

$$\sin(a + x) - x \cos(a + x)$$

Antiderivative was successfully verified.

[In] `Int[x*Sin[a + x],x]`

[Out] `-(x*Cos[a + x]) + Sin[a + x]`

Rule 2717

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 3377

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-c + d*x)^m*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /;`
`FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rubi steps

$$\begin{aligned} \int x \sin(a + x) dx &= -x \cos(a + x) + \int \cos(a + x) dx \\ &= -x \cos(a + x) + \sin(a + x) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 12, normalized size = 1.00

$$-x \cos(a + x) + \sin(a + x)$$

Antiderivative was successfully verified.

[In] `Integrate[x*Sin[a + x],x]`

[Out] $-(x*\text{Cos}[a + x]) + \text{Sin}[a + x]$

Mathics [A]

time = 1.95, size = 12, normalized size = 1.00

$$-x\text{Cos}[a + x] + \text{Sin}[a + x]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x*Sin[x+a],x]')`

[Out] $-x \text{Cos}[a + x] + \text{Sin}[a + x]$

Maple [A]

time = 0.02, size = 21, normalized size = 1.75

method	result	size
risch	$-x \cos(a + x) + \sin(a + x)$	13
derivativedivides	$a \cos(a + x) + \sin(a + x) - (a + x) \cos(a + x)$	21
default	$a \cos(a + x) + \sin(a + x) - (a + x) \cos(a + x)$	21
norman	$\frac{x(\tan^2(\frac{a+x}{2})) - x + 2 \tan(\frac{a+x}{2})}{1 + \tan^2(\frac{a+x}{2})}$	42
meijerg	$2 \sin(a) \sqrt{\pi} \left(-\frac{1}{2\sqrt{\pi}} + \frac{\cos(x)}{2\sqrt{\pi}} + \frac{x \sin(x)}{2\sqrt{\pi}} \right) + 2 \cos(a) \sqrt{\pi} \left(-\frac{x \cos(x)}{2\sqrt{\pi}} + \frac{\sin(x)}{2\sqrt{\pi}} \right)$	53

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*sin(a+x),x,method=_RETURNVERBOSE)`

[Out] $a*\cos(a+x)+\sin(a+x)-(a+x)*\cos(a+x)$

Maxima [A]

time = 0.26, size = 20, normalized size = 1.67

$$-(a + x) \cos(a + x) + a \cos(a + x) + \sin(a + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(a+x),x, algorithm="maxima")`

[Out] $-(a + x)*\cos(a + x) + a*\cos(a + x) + \sin(a + x)$

Fricas [A]

time = 0.34, size = 12, normalized size = 1.00

$$-x \cos(a + x) + \sin(a + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(a+x),x, algorithm="fricas")`

[Out] `-x*cos(a + x) + sin(a + x)`

Sympy [A]

time = 0.07, size = 10, normalized size = 0.83

$$-x \cos(a + x) + \sin(a + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(a+x),x)`

[Out] `-x*cos(a + x) + sin(a + x)`

Giac [A]

time = 0.00, size = 12, normalized size = 1.00

$$-x \cos(a + x) + \sin(a + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(a+x),x)`

[Out] `-x*cos(a + x) + sin(a + x)`

Mupad [B]

time = 0.08, size = 12, normalized size = 1.00

$$\sin(a + x) - x \cos(a + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*sin(a + x),x)`

[Out] `sin(a + x) - x*cos(a + x)`

$$3.218 \quad \int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)} dx$$

Optimal. Leaf size=11

$$\frac{e^{-x}x}{\log(x)}$$

[Out] x/exp(x)/ln(x)

Rubi [A]

time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2232}

$$\frac{e^{-x}x}{\log(x)}$$

Antiderivative was successfully verified.

[In] Int[(-1 + (1 - x)*Log[x])/(E^x*Log[x]^2), x]

[Out] x/(E^x*Log[x])

Rule 2232

Int[Log[(d_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((e_) + Log[(d_.)*(x_)]*(h_.)*((f_.) + (g_.)*(x_))), x_Symbol] :> Simp[e*x*F^(c*(a + b*x))*(Log[d*x]^(n + 1)/(n + 1)), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, n}, x] && EqQ[e - f*h*(n + 1), 0] && EqQ[g*h*(n + 1) - b*c*e*Log[F], 0] && NeQ[n, -1]

Rubi steps

$$\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)} dx = \frac{e^{-x}x}{\log(x)}$$

Mathematica [A]

time = 0.01, size = 11, normalized size = 1.00

$$\frac{e^{-x}x}{\log(x)}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + (1 - x)*Log[x])/(E^x*Log[x]^2), x]

[Out] $x/(E^x \cdot \text{Log}[x])$

Mathics [A]

time = 1.74, size = 11, normalized size = 1.00

$$\frac{x E^{-x}}{\text{Log}[x]}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(Log[x]*(1-x)-1)/(E^x*Log[x]^2),x]')`

[Out] $x E^{-x} / \text{Log}[x]$

Maple [A]

time = 0.01, size = 11, normalized size = 1.00

method	result	size
norman	$\frac{x e^{-x}}{\ln(x)}$	11
risch	$\frac{x e^{-x}}{\ln(x)}$	11

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-1+(1-x)*ln(x))/exp(x)/ln(x)^2,x,method=_RETURNVERBOSE)`

[Out] $x/\exp(x)/\ln(x)$

Maxima [A]

time = 0.29, size = 10, normalized size = 0.91

$$\frac{x e^{(-x)}}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+(1-x)*log(x))/exp(x)/log(x)^2,x, algorithm="maxima")`

[Out] $x * e^{(-x)} / \log(x)$

Fricas [A]

time = 0.31, size = 10, normalized size = 0.91

$$\frac{x e^{(-x)}}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+(1-x)*log(x))/exp(x)/log(x)^2,x, algorithm="fricas")`

[Out] $x e^{-x} / \log(x)$

Sympy [A]

time = 0.07, size = 7, normalized size = 0.64

$$\frac{x e^{-x}}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+(1-x)*ln(x))/exp(x)/ln(x)**2,x)`

[Out] $x \exp(-x) / \log(x)$

Giac [A]

time = 0.00, size = 8, normalized size = 0.73

$$\frac{x e^{-x}}{\ln x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+(1-x)*log(x))/exp(x)/log(x)^2,x)`

[Out] $x / (e^x \log(x))$

Mupad [B]

time = 0.22, size = 10, normalized size = 0.91

$$\frac{x e^{-x}}{\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(exp(-x)*(log(x)*(x - 1) + 1))/log(x)^2,x)`

[Out] $(x \exp(-x)) / \log(x)$

3.219 $\int \frac{x^3}{b+ax^2} dx$

Optimal. Leaf size=27

$$\frac{x^2}{2a} - \frac{b \log(b + ax^2)}{2a^2}$$

[Out] 1/2*x^2/a-1/2*b*ln(a*x^2+b)/a^2

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{x^2}{2a} - \frac{b \log(ax^2 + b)}{2a^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/(b + a*x^2),x]

[Out] x^2/(2*a) - (b*Log[b + a*x^2])/(2*a^2)

Rule 45

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{b+ax^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x}{b+ax} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{a} - \frac{b}{a(b+ax)} \right) dx, x, x^2 \right) \\ &= \frac{x^2}{2a} - \frac{b \log(b+ax^2)}{2a^2} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 27, normalized size = 1.00

$$\frac{x^2}{2a} - \frac{b \log(b + ax^2)}{2a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(b + a*x^2),x]

[Out] x^2/(2*a) - (b*Log[b + a*x^2])/(2*a^2)

Mathics [A]

time = 1.84, size = 22, normalized size = 0.81

$$\frac{ax^2 - b \operatorname{Log}[ax^2 + b]}{2a^2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^3/(a*x^2+b),x]')

[Out] (a x ^ 2 - b Log[a x ^ 2 + b]) / (2 a ^ 2)

Maple [A]

time = 0.03, size = 24, normalized size = 0.89

method	result	size
default	$\frac{x^2}{2a} - \frac{b \ln(ax^2+b)}{2a^2}$	24
norman	$\frac{x^2}{2a} - \frac{b \ln(ax^2+b)}{2a^2}$	24
risch	$\frac{x^2}{2a} - \frac{b \ln(ax^2+b)}{2a^2}$	24

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a*x^2+b),x,method=_RETURNVERBOSE)

[Out] 1/2*x^2/a-1/2*b*ln(a*x^2+b)/a^2

Maxima [A]

time = 0.32, size = 23, normalized size = 0.85

$$\frac{x^2}{2a} - \frac{b \log(ax^2 + b)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a*x^2+b),x, algorithm="maxima")

[Out] 1/2*x^2/a - 1/2*b*log(a*x^2 + b)/a^2

Fricas [A]

time = 0.30, size = 22, normalized size = 0.81

$$\frac{ax^2 - b \log(ax^2 + b)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a*x^2+b),x, algorithm="fricas")**[Out]** 1/2*(a*x^2 - b*log(a*x^2 + b))/a^2**Sympy [A]**

time = 0.07, size = 20, normalized size = 0.74

$$\frac{x^2}{2a} - \frac{b \log(ax^2 + b)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(a*x**2+b),x)**[Out]** x**2/(2*a) - b*log(a*x**2 + b)/(2*a**2)**Giac [A]**

time = 0.00, size = 26, normalized size = 0.96

$$\frac{\frac{x^2}{a} - \frac{b \ln|x^2 a + b|}{a^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a*x^2+b),x)**[Out]** 1/2*x^2/a - 1/2*b*log(abs(a*x^2 + b))/a^2**Mupad [B]**

time = 0.15, size = 22, normalized size = 0.81

$$-\frac{b \ln(ax^2 + b) - ax^2}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b + a*x^2),x)**[Out]** -(b*log(b + a*x^2) - a*x^2)/(2*a^2)

$$3.220 \quad \int \frac{\sqrt{x}}{(1+x)^{7/2}} dx$$

Optimal. Leaf size=33

$$\frac{2x^{3/2}}{5(1+x)^{5/2}} + \frac{4x^{3/2}}{15(1+x)^{3/2}}$$

[Out] $2/5*x^{(3/2)}/(1+x)^{(5/2)}+4/15*x^{(3/2)}/(1+x)^{(3/2)}$

Rubi [A]

time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {47, 37}

$$\frac{4x^{3/2}}{15(x+1)^{3/2}} + \frac{2x^{3/2}}{5(x+1)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(1+x)^(7/2),x]

[Out] $(2*x^{(3/2)})/(5*(1+x)^{(5/2)}) + (4*x^{(3/2)})/(15*(1+x)^{(3/2)})$

Rule 37

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(Simplify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + n + 2] && !IntegerQ[m] && !IntegerQ[n] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n])) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x}}{(1+x)^{7/2}} dx &= \frac{2x^{3/2}}{5(1+x)^{5/2}} + \frac{2}{5} \int \frac{\sqrt{x}}{(1+x)^{5/2}} dx \\ &= \frac{2x^{3/2}}{5(1+x)^{5/2}} + \frac{4x^{3/2}}{15(1+x)^{3/2}} \end{aligned}$$

Mathematica [A]

time = 0.05, size = 21, normalized size = 0.64

$$\frac{2x^{3/2}(5+2x)}{15(1+x)^{5/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[Sqrt[x]/(1+x)^(7/2),x]``[Out] (2*x^(3/2)*(5+2*x))/(15*(1+x)^(5/2))`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.

time = 5.34, size = 89, normalized size = 2.70

$$\text{Piecewise} \left[\left[\left[\frac{2Ix(5+2x)\sqrt{-\frac{x}{1+x}}}{15(1+2x+x^2)}, \text{Abs}[1+x] > 1 \right] \right], -\frac{2\sqrt{1-\frac{1}{1+x}}}{5(1+x)^2} + \frac{2\sqrt{1-\frac{1}{1+x}}}{15(1+x)} + \frac{4\sqrt{1-\frac{1}{1+x}}}{15} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[x^(1/2)*(x+1)^(-7/2),x]')`
`[Out] Piecewise[{{2 I / 15 x (5 + 2 x) Sqrt[-x / (1 + x)] / (1 + 2 x + x ^ 2), 1 / Abs[1 + x] > 1}}, -2 Sqrt[1 - 1 / (1 + x)] / (5 (1 + x) ^ 2) + 2 Sqrt[1 - 1 / (1 + x)] / (15 (1 + x)) + 4 Sqrt[1 - 1 / (1 + x)] / 15]`
Maple [A]

time = 0.03, size = 32, normalized size = 0.97

method	result	size
gospers	$\frac{2x^{\frac{3}{2}}(5+2x)}{15(1+x)^{\frac{5}{2}}}$	16
meijerg	$\frac{2x^{\frac{3}{2}}(5+2x)}{15(1+x)^{\frac{5}{2}}}$	16
risch	$\frac{2x^{\frac{3}{2}}(5+2x)}{15(1+x)^{\frac{5}{2}}}$	16
default	$-\frac{2\sqrt{x}}{5(1+x)^{\frac{5}{2}}} + \frac{2\sqrt{x}}{15(1+x)^{\frac{3}{2}}} + \frac{4\sqrt{x}}{15\sqrt{1+x}}$	32

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^(1/2)/(1+x)^(7/2),x,method=_RETURNVERBOSE)``[Out] -2/5/(1+x)^(5/2)*x^(1/2)+2/15/(1+x)^(3/2)*x^(1/2)+4/15/(1+x)^(1/2)*x^(1/2)`

Maxima [A]

time = 0.25, size = 20, normalized size = 0.61

$$\frac{2x^{\frac{5}{2}} \left(\frac{5(x+1)}{x} - 3 \right)}{15(x+1)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(1+x)^(7/2),x, algorithm="maxima")

[Out] 2/15*x^(5/2)*(5*(x + 1)/x - 3)/(x + 1)^(5/2)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(21) = 42.

time = 0.30, size = 50, normalized size = 1.52

$$\frac{2 \left(2x^3 + (2x^2 + 5x)\sqrt{x+1}\sqrt{x} + 6x^2 + 6x + 2 \right)}{15(x^3 + 3x^2 + 3x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(1+x)^(7/2),x, algorithm="fricas")

[Out] 2/15*(2*x^3 + (2*x^2 + 5*x)*sqrt(x + 1)*sqrt(x) + 6*x^2 + 6*x + 2)/(x^3 + 3*x^2 + 3*x + 1)

Sympy [A]

time = 2.90, size = 167, normalized size = 5.06

$$\begin{cases} \frac{4i\sqrt{-1 + \frac{1}{x+1}}(x+1)^2}{-15x+15(x+1)^2-15} - \frac{2i\sqrt{-1 + \frac{1}{x+1}}(x+1)}{-15x+15(x+1)^2-15} - \frac{8i\sqrt{-1 + \frac{1}{x+1}}}{-15x+15(x+1)^2-15} + \frac{6i\sqrt{-1 + \frac{1}{x+1}}}{(x+1)(-15x+15(x+1)^2-15)} & \text{for } \frac{1}{|x+1}| > 1 \\ \frac{4\sqrt{1 - \frac{1}{x+1}}}{15} + \frac{2\sqrt{1 - \frac{1}{x+1}}}{15(x+1)} - \frac{2\sqrt{1 - \frac{1}{x+1}}}{5(x+1)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/2)/(1+x)**(7/2),x)

[Out] Piecewise((4*I*sqrt(-1 + 1/(x + 1))*(x + 1)**2/(-15*x + 15*(x + 1)**2 - 15) - 2*I*sqrt(-1 + 1/(x + 1))*(x + 1)/(-15*x + 15*(x + 1)**2 - 15) - 8*I*sqrt(-1 + 1/(x + 1))/(-15*x + 15*(x + 1)**2 - 15) + 6*I*sqrt(-1 + 1/(x + 1))/((x + 1)*(-15*x + 15*(x + 1)**2 - 15)), 1/Abs(x + 1) > 1), (4*sqrt(1 - 1/(x + 1))/15 + 2*sqrt(1 - 1/(x + 1))/(15*(x + 1)) - 2*sqrt(1 - 1/(x + 1))/(5*(x + 1)**2), True))

Giac [A]

time = 0.00, size = 46, normalized size = 1.39

$$\frac{4 \left(\frac{1}{15} \sqrt{x} \sqrt{x} + \frac{1}{6} \right) \sqrt{x} \sqrt{x} \sqrt{x} \sqrt{x+1}}{(x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(1+x)^(7/2), x)**[Out]** 2/15*(2*x + 5)*x^(3/2)/(x + 1)^(5/2)**Mupad [B]**

time = 0.30, size = 15, normalized size = 0.45

$$\frac{2 x^{3/2} (2 x + 5)}{15 (x + 1)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x + 1)^(7/2), x)**[Out]** (2*x^(3/2)*(2*x + 5))/(15*(x + 1)^(5/2))

$$3.221 \quad \int \frac{1}{x(1+x)} dx$$

Optimal. Leaf size=9

$$\log(x) - \log(1+x)$$

[Out] ln(x)-ln(1+x)

Rubi [A]

time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {36, 29, 31}

$$\log(x) - \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 + x)),x]

[Out] Log[x] - Log[1 + x]

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{x(1+x)} dx &= \int \frac{1}{x} dx - \int \frac{1}{1+x} dx \\ &= \log(x) - \log(1+x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 9, normalized size = 1.00

$$\log(x) - \log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x)),x]
```

```
[Out] Log[x] - Log[1 + x]
```

Mathics [A]

time = 1.77, size = 9, normalized size = 1.00

$$\text{Log}[x] - \text{Log}[1 + x]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(-1)*(x+1)^(-1),x]')
```

```
[Out] Log[x] - Log[1 + x]
```

Maple [A]

time = 0.03, size = 10, normalized size = 1.11

method	result	size
default	$\ln(x) - \ln(1 + x)$	10
norman	$\ln(x) - \ln(1 + x)$	10
meijerg	$\ln(x) - \ln(1 + x)$	10
risch	$\ln(x) - \ln(1 + x)$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-ln(1+x)
```

Maxima [A]

time = 0.27, size = 9, normalized size = 1.00

$$-\log(x + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x),x, algorithm="maxima")
```

```
[Out] -log(x + 1) + log(x)
```

Fricas [A]

time = 0.33, size = 9, normalized size = 1.00

$$-\log(x + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1+x),x, algorithm="fricas")`

[Out] $-\log(x + 1) + \log(x)$

Sympy [A]

time = 0.04, size = 7, normalized size = 0.78

$$\log(x) - \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1+x),x)`

[Out] $\log(x) - \log(x + 1)$

Giac [A]

time = 0.00, size = 10, normalized size = 1.11

$$\ln|x| - \ln|x + 1|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1+x),x)`

[Out] $-\log(\text{abs}(x + 1)) + \log(\text{abs}(x))$

Mupad [B]

time = 0.20, size = 8, normalized size = 0.89

$$-\ln\left(\frac{1}{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(x + 1)),x)`

[Out] $-\log(1/x + 1)$

$$3.222 \quad \int \frac{1}{\sqrt{x}(-1+2x)} dx$$

Optimal. Leaf size=19

$$-\sqrt{2} \tanh^{-1}\left(\sqrt{2} \sqrt{x}\right)$$

[Out] -arctanh(2^(1/2)*x^(1/2))*2^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$,

Rules used = {65, 213}

$$-\sqrt{2} \tanh^{-1}\left(\sqrt{2} \sqrt{x}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*(-1 + 2*x)),x]

[Out] -(Sqrt[2]*ArcTanh[Sqrt[2]*Sqrt[x]])

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1)*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x}(-1+2x)} dx &= 2\text{Subst}\left(\int \frac{1}{-1+2x^2} dx, x, \sqrt{x}\right) \\ &= -\sqrt{2} \tanh^{-1}\left(\sqrt{2} \sqrt{x}\right) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 19, normalized size = 1.00

$$-\sqrt{2} \tanh^{-1}\left(\sqrt{2} \sqrt{x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*(-1 + 2*x)),x]

[Out] -(Sqrt[2]*ArcTanh[Sqrt[2]*Sqrt[x]])

Mathics [A]

time = 2.07, size = 28, normalized size = 1.47

$$\frac{\sqrt{2} \left(\operatorname{Log} \left[-\frac{\sqrt{2}}{2} + \sqrt{x} \right] - \operatorname{Log} \left[\frac{\sqrt{2}}{2} + \sqrt{x} \right] \right)}{2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^(-1/2)*(2*x-1)^(-1),x]')

[Out] Sqrt[2] (Log[-Sqrt[2] / 2 + Sqrt[x]] - Log[Sqrt[2] / 2 + Sqrt[x]]) / 2

Maple [A]

time = 0.07, size = 14, normalized size = 0.74

method	result	size
derivativedivides	$-\operatorname{arctanh}(\sqrt{2}\sqrt{x})\sqrt{2}$	14
default	$-\operatorname{arctanh}(\sqrt{2}\sqrt{x})\sqrt{2}$	14
meijerg	$-\operatorname{arctanh}(\sqrt{2}\sqrt{x})\sqrt{2}$	14
trager	$-\frac{\operatorname{RootOf}(-Z^2-2) \ln\left(\frac{2\operatorname{RootOf}(-Z^2-2)x+4\sqrt{x}+\operatorname{RootOf}(-Z^2-2)}{2x-1}\right)}{2}$	39

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(1/2)/(2*x-1),x,method=_RETURNVERBOSE)

[Out] -arctanh(2^(1/2)*x^(1/2))*2^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(13) = 26.

time = 0.35, size = 28, normalized size = 1.47

$$\frac{1}{2} \sqrt{2} \log \left(-\frac{\sqrt{2} - 2\sqrt{x}}{\sqrt{2} + 2\sqrt{x}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(-1+2*x),x, algorithm="maxima")

[Out] 1/2*sqrt(2)*log(-(sqrt(2) - 2*sqrt(x))/(sqrt(2) + 2*sqrt(x)))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(13) = 26.

time = 0.30, size = 28, normalized size = 1.47

$$\frac{1}{2} \sqrt{2} \log \left(-\frac{2 \sqrt{2} \sqrt{x} - 2x - 1}{2x - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(-1+2*x),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*log(-(2*sqrt(2)*sqrt(x) - 2*x - 1)/(2*x - 1))

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34

time = 0.12, size = 39, normalized size = 2.05

$$\frac{\sqrt{2} \log \left(\sqrt{x} - \frac{\sqrt{2}}{2} \right)}{2} - \frac{\sqrt{2} \log \left(\sqrt{x} + \frac{\sqrt{2}}{2} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(1/2)/(-1+2*x),x)

[Out] sqrt(2)*log(sqrt(x) - sqrt(2)/2)/2 - sqrt(2)*log(sqrt(x) + sqrt(2)/2)/2

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(13) = 26.

time = 0.00, size = 52, normalized size = 2.74

$$2 \left(\frac{1}{4} \sqrt{2} \ln \left| \sqrt{x} - \frac{\sqrt{2}}{2} \right| - \frac{1}{4} \sqrt{2} \ln \left(\sqrt{x} + \frac{\sqrt{2}}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(-1+2*x),x)

[Out] -1/2*sqrt(2)*log(1/2*sqrt(2) + sqrt(x)) + 1/2*sqrt(2)*log(abs(-1/2*sqrt(2) + sqrt(x)))

Mupad [B]

time = 0.08, size = 13, normalized size = 0.68

$$-\sqrt{2} \operatorname{atanh}(\sqrt{2x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/2)*(2*x - 1)),x)

[Out] -2^(1/2)*atanh((2*x)^(1/2))

3.223 $\int \sqrt{x} (1 + x^2) dx$

Optimal. Leaf size=19

$$\frac{2x^{3/2}}{3} + \frac{2x^{7/2}}{7}$$

[Out] $2/3*x^{(3/2)}+2/7*x^{(7/2)}$

Rubi [A]

time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{2x^{7/2}}{7} + \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]*(1 + x^2),x]

[Out] $(2*x^{(3/2)})/3 + (2*x^{(7/2)})/7$

Rule 14

Int[(u_)*((c_)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \sqrt{x} (1 + x^2) dx &= \int (\sqrt{x} + x^{5/2}) dx \\ &= \frac{2x^{3/2}}{3} + \frac{2x^{7/2}}{7} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 16, normalized size = 0.84

$$\frac{2}{21}x^{3/2}(7 + 3x^2)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]*(1 + x^2),x]

[Out] $(2*x^{(3/2)}*(7 + 3*x^2))/21$

Mathics [A]

time = 2.10, size = 12, normalized size = 0.63

$$\frac{2x^{\frac{3}{2}}(7+3x^2)}{21}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(x^2+1)*x^(1/2),x]')``[Out] 2 x ^ (3 / 2) (7 + 3 x ^ 2) / 21`**Maple [A]**

time = 0.01, size = 12, normalized size = 0.63

method	result	size
derivativdivides	$\frac{2x^{\frac{3}{2}}}{3} + \frac{2x^{\frac{7}{2}}}{7}$	12
default	$\frac{2x^{\frac{3}{2}}}{3} + \frac{2x^{\frac{7}{2}}}{7}$	12
gosper	$\frac{2x^{\frac{3}{2}}(3x^2+7)}{21}$	13
trager	$\frac{2x^{\frac{3}{2}}(3x^2+7)}{21}$	13
risch	$\frac{2x^{\frac{3}{2}}(3x^2+7)}{21}$	13

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^(1/2)*(x^2+1),x,method=_RETURNVERBOSE)``[Out] 2/3*x^(3/2)+2/7*x^(7/2)`**Maxima [A]**

time = 0.26, size = 11, normalized size = 0.58

$$\frac{2}{7}x^{\frac{7}{2}} + \frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^(1/2)*(x^2+1),x, algorithm="maxima")``[Out] 2/7*x^(7/2) + 2/3*x^(3/2)`**Fricas [A]**

time = 0.32, size = 14, normalized size = 0.74

$$\frac{2}{21}(3x^3 + 7x)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)*(x^2+1),x, algorithm="fricas")`

[Out] $2/21*(3*x^3 + 7*x)*\sqrt{x}$

Sympy [A]

time = 0.52, size = 15, normalized size = 0.79

$$\frac{2x^{\frac{7}{2}}}{7} + \frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1/2)*(x**2+1),x)`

[Out] $2*x**(7/2)/7 + 2*x**(3/2)/3$

Giac [A]

time = 0.00, size = 23, normalized size = 1.21

$$\frac{2}{7}\sqrt{x} x^3 + \frac{2}{3}\sqrt{x} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)*(x^2+1),x)`

[Out] $2/7*x^(7/2) + 2/3*x^(3/2)$

Mupad [B]

time = 0.02, size = 12, normalized size = 0.63

$$\frac{2x^{3/2}(3x^2 + 7)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*(x^2 + 1),x)`

[Out] $(2*x^(3/2)*(3*x^2 + 7))/21$

$$3.224 \quad \int \frac{\sqrt[3]{-a+x}}{x} dx$$

Optimal. Leaf size=88

$$3\sqrt[3]{-a+x} + \sqrt{3} \sqrt[3]{a} \tan^{-1} \left(\frac{\sqrt[3]{a} - 2\sqrt[3]{-a+x}}{\sqrt{3} \sqrt[3]{a}} \right) + \frac{1}{2} \sqrt[3]{a} \log(x) - \frac{3}{2} \sqrt[3]{a} \log(\sqrt[3]{a} + \sqrt[3]{-a+x})$$

[Out] $3*(-a+x)^{(1/3)}+1/2*a^{(1/3)}*\ln(x)-3/2*a^{(1/3)}*\ln(a^{(1/3)}+(-a+x)^{(1/3)})+a^{(1/3)}*\arctan(1/3*(a^{(1/3)}-2*(-a+x)^{(1/3)})/a^{(1/3)}*3^{(1/2)})*3^{(1/2)}$

Rubi [A]

time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {52, 60, 631, 210, 31}

$$3\sqrt[3]{x-a} + \frac{1}{2} \sqrt[3]{a} \log(x) - \frac{3}{2} \sqrt[3]{a} \log(\sqrt[3]{x-a} + \sqrt[3]{a}) + \sqrt{3} \sqrt[3]{a} \tan^{-1} \left(\frac{\sqrt[3]{a} - 2\sqrt[3]{x-a}}{\sqrt{3} \sqrt[3]{a}} \right)$$

Antiderivative was successfully verified.

[In] Int[(-a + x)^(1/3)/x,x]

[Out] $3*(-a+x)^{(1/3)} + \text{Sqrt}[3]*a^{(1/3)}*\text{ArcTan}[(a^{(1/3)} - 2*(-a+x)^{(1/3)})/(\text{Sqrt}[3]*a^{(1/3)})] + (a^{(1/3)}*\text{Log}[x])/2 - (3*a^{(1/3)}*\text{Log}[a^{(1/3)} + (-a+x)^{(1/3)})/2$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 52

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 60

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x

]]) /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 210

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 631

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt[3]{-a+x}}{x} dx &= 3\sqrt[3]{-a+x} - a \int \frac{1}{x(-a+x)^{2/3}} dx \\ &= 3\sqrt[3]{-a+x} + \frac{1}{2}\sqrt[3]{a} \log(x) - \frac{1}{2}(3\sqrt[3]{a}) \operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x} dx, x, \sqrt[3]{-a+x}\right) - \frac{1}{2}(3a^2) \\ &= 3\sqrt[3]{-a+x} + \frac{1}{2}\sqrt[3]{a} \log(x) - \frac{3}{2}\sqrt[3]{a} \log(\sqrt[3]{a} + \sqrt[3]{-a+x}) - (3\sqrt[3]{a}) \operatorname{Subst}\left(\int \frac{1}{-3-x} dx, x, \sqrt[3]{-a+x}\right) \\ &= 3\sqrt[3]{-a+x} + \sqrt{3} \sqrt[3]{a} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{-a+x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + \frac{1}{2}\sqrt[3]{a} \log(x) - \frac{3}{2}\sqrt[3]{a} \log(\sqrt[3]{a} + \sqrt[3]{-a+x}) \end{aligned}$$

Mathematica [A]

time = 0.06, size = 112, normalized size = 1.27

$$3\sqrt[3]{-a+x} + \sqrt{3} \sqrt[3]{a} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{-a+x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - \sqrt[3]{a} \log(\sqrt[3]{a} + \sqrt[3]{-a+x}) + \frac{1}{2}\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{-a+x} + (-a+x)^{2/3})$$

Antiderivative was successfully verified.

[In] Integrate[(-a + x)^(1/3)/x,x]

[Out] 3*(-a + x)^(1/3) + Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*(-a + x)^(1/3))/a^(1/3))/Sqrt[3]] - a^(1/3)*Log[a^(1/3) + (-a + x)^(1/3)] + (a^(1/3)*Log[a^(2/3) - a^(1/3)*(-a + x)^(1/3) + (-a + x)^(2/3)])/2

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 3.73, size = 87, normalized size = 0.99

$$-a^{\frac{1}{3}} \operatorname{Log} \left[1 - \frac{(-a+x)^{\frac{1}{3}} \exp_{\text{polar}}[I\text{Pi}]}{a^{\frac{1}{3}}} \right] - a^{\frac{1}{3}} \operatorname{Log} \left[1 - \frac{(-a+x)^{\frac{1}{3}} \exp_{\text{polar}}\left[\frac{5I}{3}\text{Pi}\right]}{a^{\frac{1}{3}}} \right] + 3(-a+x)^{\frac{1}{3}}$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[x^(-1)*(x-a)^(1/3),x]')`

[Out] $-1^{\frac{2}{3}} a^{\frac{1}{3}} \operatorname{Log}[1 - (-a+x)^{\frac{1}{3}} \exp_{\text{polar}}[I/3 \text{Pi}] / a^{\frac{1}{3}}] - a^{\frac{1}{3}} \operatorname{Log}[1 - (-a+x)^{\frac{1}{3}} \exp_{\text{polar}}[I \text{Pi}] / a^{\frac{1}{3}}] + -1^{\frac{1}{3}} a^{\frac{1}{3}} \operatorname{Log}[1 - (-a+x)^{\frac{1}{3}} \exp_{\text{polar}}[5I/3 \text{Pi}] / a^{\frac{1}{3}}] + 3(-a+x)^{\frac{1}{3}}$

Maple [A]

time = 0.04, size = 89, normalized size = 1.01

method	result
derivativedivides	$3(-a+x)^{\frac{1}{3}} - 3 \left(\frac{\ln\left(a^{\frac{1}{3}} + (-a+x)^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}} - \frac{\ln\left((-a+x)^{\frac{2}{3}} - a^{\frac{1}{3}}(-a+x)^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}} + \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3} \left(\frac{2(-a+x)}{a^{\frac{1}{3}}}\right)}{3}\right)}{3a^{\frac{2}{3}}}\right)$
default	$3(-a+x)^{\frac{1}{3}} - 3 \left(\frac{\ln\left(a^{\frac{1}{3}} + (-a+x)^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}} - \frac{\ln\left((-a+x)^{\frac{2}{3}} - a^{\frac{1}{3}}(-a+x)^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}} + \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3} \left(\frac{2(-a+x)}{a^{\frac{1}{3}}}\right)}{3}\right)}{3a^{\frac{2}{3}}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-a+x)^(1/3)/x,x,method=_RETURNVERBOSE)`

[Out] $3*(-a+x)^{\frac{1}{3}} - 3*(1/3/a^{\frac{2}{3}}*\ln(a^{\frac{1}{3}}+(-a+x)^{\frac{1}{3}})) - 1/6/a^{\frac{2}{3}}*\ln((-a+x)^{\frac{2}{3}} - a^{\frac{1}{3}}*(-a+x)^{\frac{1}{3}} + a^{\frac{2}{3}}) + 1/3/a^{\frac{2}{3}}*3^{\frac{1}{2}}*\arctan(1/3*3^{\frac{1}{2}}*(2/a^{\frac{1}{3}}*(-a+x)^{\frac{1}{3}} - 1)))*a$

Maxima [A]

time = 0.34, size = 86, normalized size = 0.98

$$-\sqrt{3} a^{\frac{1}{3}} \arctan\left(-\frac{\sqrt{3} \left(a^{\frac{1}{3}} - 2(-a+x)^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right) + \frac{1}{2} a^{\frac{1}{3}} \log\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}(-a+x)^{\frac{1}{3}} + (-a+x)^{\frac{2}{3}}\right) - a^{\frac{1}{3}} \log\left(a^{\frac{1}{3}} + (-a+x)^{\frac{1}{3}}\right) + 3(-a+x)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+x)^(1/3)/x,x, algorithm="maxima")

[Out] -sqrt(3)*a^(1/3)*arctan(-1/3*sqrt(3)*(a^(1/3) - 2*(-a + x)^(1/3))/a^(1/3)) + 1/2*a^(1/3)*log(a^(2/3) - a^(1/3)*(-a + x)^(1/3) + (-a + x)^(2/3)) - a^(1/3)*log(a^(1/3) + (-a + x)^(1/3)) + 3*(-a + x)^(1/3)

Fricas [A]

time = 0.33, size = 104, normalized size = 1.18

$$\sqrt{3} (-a)^{\frac{1}{3}} \arctan\left(-\frac{\sqrt{3}a - 2\sqrt{3}(-a)^{\frac{2}{3}}(-a+x)^{\frac{1}{3}}}{3a}\right) - \frac{1}{2} (-a)^{\frac{1}{3}} \log\left((-a)^{\frac{2}{3}} + (-a)^{\frac{1}{3}}(-a+x)^{\frac{1}{3}} + (-a+x)^{\frac{2}{3}}\right) + (-a)^{\frac{1}{3}} \log\left(-(-a)^{\frac{1}{3}} + (-a+x)^{\frac{1}{3}}\right) + 3(-a+x)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+x)^(1/3)/x,x, algorithm="fricas")

[Out] sqrt(3)*(-a)^(1/3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(-a)^(2/3)*(-a + x)^(1/3))/a) - 1/2*(-a)^(1/3)*log((-a)^(2/3) + (-a)^(1/3)*(-a + x)^(1/3) + (-a + x)^(2/3)) + (-a)^(1/3)*log(-(-a)^(1/3) + (-a + x)^(1/3)) + 3*(-a + x)^(1/3)

Sympy [C] Result contains complex when optimal does not.

time = 0.92, size = 153, normalized size = 1.74

$$\frac{4\sqrt[3]{a} e^{-\frac{i\pi}{3}} \log\left(1 - \frac{\sqrt[3]{-a+x} e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right) \Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)} - \frac{4\sqrt[3]{a} \log\left(1 - \frac{\sqrt[3]{-a+x} e^{i\pi}}{\sqrt[3]{a}}\right) \Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{4\sqrt[3]{a} e^{\frac{i\pi}{3}} \log\left(1 - \frac{\sqrt[3]{-a+x} e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right) \Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{4\sqrt[3]{-a+x} \Gamma\left(\frac{4}{3}\right)}{\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+x)**(1/3)/x,x)

[Out] 4*a**(1/3)*exp(-I*pi/3)*log(1 - (-a + x)**(1/3)*exp_polar(I*pi/3)/a**(1/3)) *gamma(4/3)/(3*gamma(7/3)) - 4*a**(1/3)*log(1 - (-a + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*a**(1/3)*exp(I*pi/3)*log(1 - (-a + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*(-a + x)**(1/3)*gamma(4/3)/gamma(7/3)

Giac [A]

time = 0.00, size = 139, normalized size = 1.58

$$-\frac{1}{2}(-a)^{\frac{1}{3}} \ln\left(\left((-a+x)^{\frac{1}{3}}\right)^2 + (-a)^{\frac{1}{3}}(-a+x)^{\frac{1}{3}} + (-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}\right) - \sqrt{3}(-a)^{\frac{1}{3}} \arctan\left(\frac{2\left(\left((-a+x)^{\frac{1}{3}} + \frac{(-a)^{\frac{1}{3}}}{2}\right)\right)}{\sqrt{3}(-a)^{\frac{1}{3}}}\right) + \frac{3a(-a)^{\frac{1}{3}} \ln\left|(-a+x)^{\frac{1}{3}} - (-a)^{\frac{1}{3}}\right|}{3a} + 3(-a+x)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+x)^(1/3)/x,x)

[Out] -sqrt(3)*(-a)^(1/3)*arctan(1/3*sqrt(3)*((-a)^(1/3) + 2*(-a + x)^(1/3))/(-a)^(1/3)) - 1/2*(-a)^(1/3)*log((-a)^(2/3) + (-a)^(1/3)*(-a + x)^(1/3) + (-a +

$x)^{2/3}) + (-a)^{1/3} \cdot \log(\text{abs}(-(-a)^{1/3} + (-a + x)^{1/3})) + 3 \cdot (-a + x)^{1/3}$

Mupad [B]

time = 0.18, size = 119, normalized size = 1.35

$$(-a)^{1/3} \ln(-9(-a)^{4/3} - 9a(x-a)^{1/3}) + 3(x-a)^{1/3} + \frac{(-a)^{1/3} \ln\left(\frac{9(-a)^{4/3}(-1+\sqrt{3}i)}{2} + 9a(x-a)^{1/3}\right)(-1+\sqrt{3}i)}{2} - \frac{(-a)^{1/3} \ln\left(\frac{9(-a)^{4/3}(1+\sqrt{3}i)}{2} - 9a(x-a)^{1/3}\right)(1+\sqrt{3}i)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x - a)^(1/3)/x,x)`

[Out] $(-a)^{1/3} \cdot \log(-9 \cdot (-a)^{4/3} - 9 \cdot a \cdot (x - a)^{1/3}) + 3 \cdot (x - a)^{1/3} + ((-a)^{1/3} \cdot \log((9 \cdot (-a)^{4/3} \cdot (3^{1/2} \cdot i - 1))/2 + 9 \cdot a \cdot (x - a)^{1/3}) \cdot (3^{1/2} \cdot i - 1))/2 - ((-a)^{1/3} \cdot \log((9 \cdot (-a)^{4/3} \cdot (3^{1/2} \cdot i + 1))/2 - 9 \cdot a \cdot (x - a)^{1/3}) \cdot (3^{1/2} \cdot i + 1))/2$

3.225 $\int x \sinh(x) dx$

Optimal. Leaf size=9

$$x \cosh(x) - \sinh(x)$$

[Out] x*cosh(x)-sinh(x)

Rubi [A]

time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3377, 2717}

$$x \cosh(x) - \sinh(x)$$

Antiderivative was successfully verified.

[In] Int[x*Sinh[x],x]

[Out] x*Cosh[x] - Sinh[x]

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 3377

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-c + d*x)^m*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rubi steps

$$\begin{aligned} \int x \sinh(x) dx &= x \cosh(x) - \int \cosh(x) dx \\ &= x \cosh(x) - \sinh(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 9, normalized size = 1.00

$$x \cosh(x) - \sinh(x)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sinh[x],x]

[Out] $x \cdot \text{Cosh}[x] - \text{Sinh}[x]$

Mathics [A]

time = 1.78, size = 9, normalized size = 1.00

$$x \text{Cosh}[x] - \text{Sinh}[x]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x*Sinh[x],x]')`

[Out] $x \text{Cosh}[x] - \text{Sinh}[x]$

Maple [A]

time = 0.02, size = 10, normalized size = 1.11

method	result	size
default	$x \cosh(x) - \sinh(x)$	10
meijerg	$x \cosh(x) - \sinh(x)$	10
risch	$(-\frac{1}{2} + \frac{x}{2}) e^x + (\frac{1}{2} + \frac{x}{2}) e^{-x}$	20

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*sinh(x),x,method=_RETURNVERBOSE)`

[Out] $x \cdot \cosh(x) - \sinh(x)$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(9) = 18$.

time = 0.26, size = 34, normalized size = 3.78

$$\frac{1}{2} x^2 \sinh(x) + \frac{1}{4} (x^2 + 2x + 2) e^{(-x)} - \frac{1}{4} (x^2 - 2x + 2) e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sinh(x),x, algorithm="maxima")`

[Out] $1/2 \cdot x^2 \cdot \sinh(x) + 1/4 \cdot (x^2 + 2 \cdot x + 2) \cdot e^{(-x)} - 1/4 \cdot (x^2 - 2 \cdot x + 2) \cdot e^x$

Fricas [A]

time = 0.30, size = 9, normalized size = 1.00

$$x \cosh(x) - \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sinh(x),x, algorithm="fricas")`

[Out] $x \cdot \cosh(x) - \sinh(x)$

Sympy [A]

time = 0.08, size = 7, normalized size = 0.78

$$x \cosh(x) - \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sinh(x),x)

[Out] x*cosh(x) - sinh(x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

time = 0.00, size = 17, normalized size = 1.89

$$\frac{(x-1)e^x + (x+1)e^{-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sinh(x),x)

[Out] 1/2*e^x*(x-1) + 1/2*(x+1)/e^x

Mupad [B]

time = 0.02, size = 9, normalized size = 1.00

$$x \cosh(x) - \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sinh(x),x)

[Out] x*cosh(x) - sinh(x)

3.226 $\int x \cosh(x) dx$

Optimal. Leaf size=9

$$-\cosh(x) + x \sinh(x)$$

[Out] `-cosh(x)+x*sinh(x)`

Rubi [A]

time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3377, 2718}

$$x \sinh(x) - \cosh(x)$$

Antiderivative was successfully verified.

[In] `Int[x*Cosh[x],x]`

[Out] `-Cosh[x] + x*Sinh[x]`

Rule 2718

`Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rule 3377

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rubi steps

$$\begin{aligned} \int x \cosh(x) dx &= x \sinh(x) - \int \sinh(x) dx \\ &= -\cosh(x) + x \sinh(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 9, normalized size = 1.00

$$-\cosh(x) + x \sinh(x)$$

Antiderivative was successfully verified.

[In] `Integrate[x*Cosh[x],x]`

[Out] $-\text{Cosh}[x] + x*\text{Sinh}[x]$

Mathics [A]

time = 1.77, size = 9, normalized size = 1.00

$$x\text{Sinh}[x] - \text{Cosh}[x]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x*Cosh[x],x]')`

[Out] $x \text{ Sinh}[x] - \text{Cosh}[x]$

Maple [A]

time = 0.02, size = 10, normalized size = 1.11

method	result	size
default	$-\cosh(x) + x \sinh(x)$	10
risch	$\left(-\frac{1}{2} + \frac{x}{2}\right) e^x + \left(-\frac{1}{2} - \frac{x}{2}\right) e^{-x}$	20
meijerg	$-2\sqrt{\pi} \left(-\frac{1}{2\sqrt{\pi}} + \frac{\cosh(x)}{2\sqrt{\pi}} - \frac{x \sinh(x)}{2\sqrt{\pi}}\right)$	27

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cosh(x),x,method=_RETURNVERBOSE)`

[Out] $-\cosh(x) + x*\sinh(x)$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(9) = 18.

time = 0.26, size = 34, normalized size = 3.78

$$\frac{1}{2}x^2 \cosh(x) - \frac{1}{4}(x^2 + 2x + 2)e^{(-x)} - \frac{1}{4}(x^2 - 2x + 2)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cosh(x),x, algorithm="maxima")`

[Out] $1/2*x^2*\cosh(x) - 1/4*(x^2 + 2*x + 2)*e^{(-x)} - 1/4*(x^2 - 2*x + 2)*e^x$

Fricas [A]

time = 0.29, size = 9, normalized size = 1.00

$$x \sinh(x) - \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cosh(x),x, algorithm="fricas")`

[Out] $x*\sinh(x) - \cosh(x)$

Sympy [A]

time = 0.08, size = 7, normalized size = 0.78

$$x \sinh(x) - \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cosh(x),x)`

[Out] $x*\sinh(x) - \cosh(x)$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.

time = 0.00, size = 18, normalized size = 2.00

$$\frac{(x-1)e^x + (-x-1)e^{-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cosh(x),x)`

[Out] $1/2*e^x*(x-1) - 1/2*(x+1)/e^x$

Mupad [B]

time = 0.03, size = 9, normalized size = 1.00

$$x \sinh(x) - \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cosh(x),x)`

[Out] $x*\sinh(x) - \cosh(x)$

3.227 $\int \tanh(2x) dx$

Optimal. Leaf size=9

$$\frac{1}{2} \log(\cosh(2x))$$

[Out] 1/2*ln(cosh(2*x))

Rubi [A]

time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3556}

$$\frac{1}{2} \log(\cosh(2x))$$

Antiderivative was successfully verified.

[In] Int[Tanh[2*x],x]

[Out] Log[Cosh[2*x]]/2

Rule 3556

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \tanh(2x) dx = \frac{1}{2} \log(\cosh(2x))$$

Mathematica [A]

time = 0.01, size = 9, normalized size = 1.00

$$\frac{1}{2} \log(\cosh(2x))$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[2*x],x]

[Out] Log[Cosh[2*x]]/2

Mathics [A]

time = 1.84, size = 7, normalized size = 0.78

$$\frac{\text{Log}[\text{Cosh}[2x]]}{2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sinh[2*x]/Cosh[2*x],x]')
```

```
[Out] Log[Cosh[2 x]] / 2
```

Maple [A]

time = 0.01, size = 8, normalized size = 0.89

method	result	size
derivativedivides	$\frac{\ln(\cosh(2x))}{2}$	8
default	$\frac{\ln(\cosh(2x))}{2}$	8
risch	$-x + \frac{\ln(e^{4x}+1)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(2*x)/cosh(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(cosh(2*x))
```

Maxima [A]

time = 0.27, size = 7, normalized size = 0.78

$$\frac{1}{2} \log(\cosh(2x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(2*x)/cosh(2*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(cosh(2*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(7) = 14.
time = 0.30, size = 26, normalized size = 2.89

$$-x + \frac{1}{2} \log\left(\frac{2 \cosh(2x)}{\cosh(2x) - \sinh(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(2*x)/cosh(2*x),x, algorithm="fricas")
```

```
[Out] -x + 1/2*log(2*cosh(2*x)/(cosh(2*x) - sinh(2*x)))
```

Sympy [A]

time = 0.07, size = 7, normalized size = 0.78

$$\frac{\log(\cosh(2x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(2*x)/cosh(2*x),x)`

[Out] `log(cosh(2*x))/2`

Giac [A]

time = 0.00, size = 15, normalized size = 1.67

$$-x + \frac{\ln\left((e^{2x})^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(2*x)/cosh(2*x),x)`

[Out] `-x + 1/2*log(e^(4*x) + 1)`

Mupad [B]

time = 0.19, size = 7, normalized size = 0.78

$$\frac{\ln(\cosh(2x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(2*x)/cosh(2*x),x)`

[Out] `log(cosh(2*x))/2`

$$3.228 \quad \int \frac{-1 + i \text{eps} \sinh(x)}{ia - x + i \text{eps} \cosh(x)} dx$$

Optimal. Leaf size=12

$$\log(a + ix + \text{eps} \cosh(x))$$

[Out] $\ln(a + I*x + \text{eps}*\cosh(x))$

Rubi [A]

time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {6816}

$$\log(a + \text{eps} \cosh(x) + ix)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-1 + I*\text{eps}*\text{Sinh}[x])/(I*a - x + I*\text{eps}*\text{Cosh}[x]), x]$

[Out] $\text{Log}[a + I*x + \text{eps}*\text{Cosh}[x]]$

Rule 6816

$\text{Int}[(u_)/(y_), x_Symbol] \text{ :> With}[\{q = \text{DerivativeDivides}[y, u, x]\}, \text{Simp}[q*\text{Log}[\text{RemoveContent}[y, x]], x] \text{ /; } \text{!FalseQ}[q]]$

Rubi steps

$$\int \frac{-1 + i \text{eps} \sinh(x)}{ia - x + i \text{eps} \cosh(x)} dx = \log(a + ix + \text{eps} \cosh(x))$$

Mathematica [A]

time = 0.07, size = 12, normalized size = 1.00

$$\log(a + ix + \text{eps} \cosh(x))$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(-1 + I*\text{eps}*\text{Sinh}[x])/(I*a - x + I*\text{eps}*\text{Cosh}[x]), x]$

[Out] $\text{Log}[a + I*x + \text{eps}*\text{Cosh}[x]]$

Mathics [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

time = 2.10, size = 29, normalized size = 2.42

$$-x + \text{Log} \left[\frac{\text{eps} (1 + E^{2x}) + 2 (a + Ix) E^x}{\text{eps}} \right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(I*eps*Sinh[x]-1)/(eps*I*Cosh[x]+I*a-x),x]')
```

```
[Out] -x + Log[(eps (1 + E ^ (2 x)) + 2 (a + I x) E ^ x) / eps]
```

Maple [A]

time = 0.03, size = 16, normalized size = 1.33

method	result	size
derivativedivides	$\ln(ia - x + i \text{eps} \cosh(x))$	16
default	$\ln(ia - x + i \text{eps} \cosh(x))$	16
risch	$-x + \ln\left(1 + \frac{2(ix+a)e^x}{\text{eps}} + e^{2x}\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(I*a-x+I*eps*cosh(x))
```

Maxima [A]

time = 0.25, size = 13, normalized size = 1.08

$$\log(i \text{eps} \cosh(x) + ia - x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x, algorithm="maxima")
```

```
[Out] log(I*eps*cosh(x) + I*a - x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

time = 0.31, size = 26, normalized size = 2.17

$$-x + \log\left(\frac{\text{epse}^{(2x)} + 2(a + ix)e^x + \text{eps}}{\text{eps}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x, algorithm="fricas")
```

```
[Out] -x + log((eps*e^(2*x) + 2*(a + I*x)*e^x + eps)/eps)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$

time = 0.17, size = 22, normalized size = 1.83

$$-x + \log\left(e^{2x} + 1 + \frac{(2a + 2ix)e^x}{\text{eps}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x)

[Out] -x + log(exp(2*x) + 1 + (2*a + 2*I*x)*exp(x)/eps)

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(10) = 20.

time = 0.02, size = 27, normalized size = 2.25

$$-x + \ln(2Iae^x + I \cdot \text{eps} \cdot (e^x)^2 + I \cdot \text{eps} - 2xe^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x)

[Out] -x + log(2*I*a*e^x + I*e^(2*x)*eps - 2*e^x*x + I*eps)

Mupad [B]

time = 0.32, size = 13, normalized size = 1.08

$$\ln(x - a \text{ li} - \text{eps} \cosh(x) \text{ li})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((eps*sinh(x)*1i - 1)/(a*1i - x + eps*cosh(x)*1i),x)

[Out] log(x - a*1i - eps*cosh(x)*1i)

3.229 $\int \cos^2(x) \sin(3 + 2x) dx$

Optimal. Leaf size=28

$$-\frac{1}{4} \cos(3 + 2x) - \frac{1}{16} \cos(3 + 4x) + \frac{1}{4} x \sin(3)$$

[Out] $-1/4*\cos(3+2*x)-1/16*\cos(3+4*x)+1/4*x*\sin(3)$

Rubi [A]

time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4670, 2718}

$$\frac{1}{4} x \sin(3) - \frac{1}{4} \cos(2x + 3) - \frac{1}{16} \cos(4x + 3)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[x]^2*\text{Sin}[3 + 2*x], x]$

[Out] $-1/4*\text{Cos}[3 + 2*x] - \text{Cos}[3 + 4*x]/16 + (x*\text{Sin}[3])/4$

Rule 2718

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[-\text{Cos}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 4670

$\text{Int}[\text{Cos}[w_]^{(q_.)}*\text{Sin}[v_]^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[\text{Sin}[v]^p*\text{Cos}[w]^q, x], x] /;$ IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))

Rubi steps

$$\begin{aligned} \int \cos^2(x) \sin(3 + 2x) dx &= \int \left(\frac{\sin(3)}{4} + \frac{1}{2} \sin(3 + 2x) + \frac{1}{4} \sin(3 + 4x) \right) dx \\ &= \frac{1}{4} x \sin(3) + \frac{1}{4} \int \sin(3 + 4x) dx + \frac{1}{2} \int \sin(3 + 2x) dx \\ &= -\frac{1}{4} \cos(3 + 2x) - \frac{1}{16} \cos(3 + 4x) + \frac{1}{4} x \sin(3) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 28, normalized size = 1.00

$$-\frac{1}{4} \cos(3 + 2x) - \frac{1}{16} \cos(3 + 4x) + \frac{1}{4} x \sin(3)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^2*Sin[3 + 2*x],x]

[Out] -1/4*Cos[3 + 2*x] - Cos[3 + 4*x]/16 + (x*Sin[3])/4

Mathics [A]

time = 2.78, size = 26, normalized size = 0.93

$$\frac{\sin[3] x}{4} - \frac{\cos[3 + 2x]}{4} - \frac{3\cos[3]}{16} - \frac{\cos[3 + 4x]}{16}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Sin[2*x+3]*Cos[x]^2,x]')

[Out] Sin[3] x / 4 - Cos[3 + 2 x] / 4 - 3 Cos[3] / 16 - Cos[3 + 4 x] / 16

Maple [A]

time = 0.05, size = 23, normalized size = 0.82

method	result
default	$-\frac{\cos(3+2x)}{4} - \frac{\cos(3+4x)}{16} + \frac{x \sin(3)}{4}$
risch	$-\frac{\cos(3+2x)}{4} - \frac{\cos(3+4x)}{16} + \frac{x \sin(3)}{4}$
norman	$\frac{x(\tan^3(\frac{x}{2}) - \frac{(\tan^4(\frac{x}{2}))}{3} + \frac{(\tan^2(\frac{3}{2}+x))}{3}) + x \tan(\frac{x}{2})(\tan^2(\frac{3}{2}+x)) + \frac{\tan(\frac{x}{2})\tan(\frac{3}{2}+x)}{3} - \frac{(\tan^3(\frac{x}{2}))\tan(\frac{3}{2}+x)}{3} + \frac{(\tan^4(\frac{x}{2}))(\tan^2(\frac{3}{2}+x))}{3}}{(1+\tan^2(\frac{x}{2}))^2(1+\tan^2(\frac{3}{2}+x))}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2*sin(3+2*x),x,method=_RETURNVERBOSE)

[Out] -1/4*cos(3+2*x)-1/16*cos(3+4*x)+1/4*x*sin(3)

Maxima [A]

time = 0.27, size = 22, normalized size = 0.79

$$\frac{1}{4} x \sin(3) - \frac{1}{16} \cos(4x + 3) - \frac{1}{4} \cos(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2*sin(3+2*x),x, algorithm="maxima")

[Out] 1/4*x*sin(3) - 1/16*cos(4*x + 3) - 1/4*cos(2*x + 3)

Fricas [A]

time = 0.32, size = 32, normalized size = 1.14

$$-\frac{1}{2} \cos(3) \cos(x)^4 + \frac{1}{4} x \sin(3) + \frac{1}{4} (2 \cos(x)^3 \sin(3) + \cos(x) \sin(3)) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2*sin(3+2*x),x, algorithm="fricas")

[Out] -1/2*cos(3)*cos(x)^4 + 1/4*x*sin(3) + 1/4*(2*cos(x)^3*sin(3) + cos(x)*sin(3))*sin(x)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(22) = 44

time = 0.35, size = 75, normalized size = 2.68

$$-\frac{x \sin^2(x) \sin(2x+3)}{4} - \frac{x \sin(x) \cos(x) \cos(2x+3)}{2} + \frac{x \sin(2x+3) \cos^2(x)}{4} - \frac{\sin(x) \sin(2x+3) \cos(x)}{4} - \frac{\cos^2(x) \cos(2x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**2*sin(3+2*x),x)

[Out] -x*sin(x)**2*sin(2*x + 3)/4 - x*sin(x)*cos(x)*cos(2*x + 3)/2 + x*sin(2*x + 3)*cos(x)**2/4 - sin(x)*sin(2*x + 3)*cos(x)/4 - cos(x)**2*cos(2*x + 3)/2

Giac [A]

time = 0.00, size = 27, normalized size = 0.96

$$-\frac{\cos(2x+3)}{4} - \frac{\cos(4x+3)}{16} + \frac{1}{4} \sin(3)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2*sin(3+2*x),x)

[Out] 1/4*x*sin(3) - 1/16*cos(4*x + 3) - 1/4*cos(2*x + 3)

Mupad [B]

time = 0.33, size = 22, normalized size = 0.79

$$\frac{x \sin(3)}{4} - \frac{\cos(4x+3)}{16} - \frac{\cos(2x+3)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x + 3)*cos(x)^2,x)

[Out] (x*sin(3))/4 - cos(4*x + 3)/16 - cos(2*x + 3)/4

3.230 $\int x \tan^{-1}(x) dx$

Optimal. Leaf size=21

$$-\frac{x}{2} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} x^2 \tan^{-1}(x)$$

[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4946, 327, 209}

$$\frac{1}{2} x^2 \tan^{-1}(x) - \frac{x}{2} + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x*ArcTan[x],x]

[Out] -1/2*x + ArcTan[x]/2 + (x^2*ArcTan[x])/2

Rule 209

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 327

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n-1)*(c*x)^(m-n+1)*((a+b*x^n)^(p+1)/(b*(m+n*p+1))), x] - Dist[a*c^n*((m-n+1)/(b*(m+n*p+1))), Int[(c*x)^(m-n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 4946

Int[((a_) + ArcTan[(c_)*(x_)^(n_)])*(b_)^(p_)*(x_)^(m_), x_Symbol] := Simp[x^(m+1)*((a+b*ArcTan[c*x^n])^p/(m+1)), x] - Dist[b*c^n*(p/(m+1)), Int[x^(m+n)*((a+b*ArcTan[c*x^n])^(p-1)/(1+c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] && IntegerQ[m])) && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int x \tan^{-1}(x) dx &= \frac{1}{2}x^2 \tan^{-1}(x) - \frac{1}{2} \int \frac{x^2}{1+x^2} dx \\
 &= -\frac{x}{2} + \frac{1}{2}x^2 \tan^{-1}(x) + \frac{1}{2} \int \frac{1}{1+x^2} dx \\
 &= -\frac{x}{2} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2}x^2 \tan^{-1}(x)
 \end{aligned}$$

Mathematica [A]

time = 0.00, size = 21, normalized size = 1.00

$$-\frac{x}{2} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2}x^2 \tan^{-1}(x)$$

Antiderivative was successfully verified.

`[In] Integrate[x*ArcTan[x], x]``[Out] -1/2*x + ArcTan[x]/2 + (x^2*ArcTan[x])/2`**Mathics [A]**

time = 1.75, size = 15, normalized size = 0.71

$$-\frac{x}{2} + \frac{x^2 \text{ArcTan}[x]}{2} + \frac{\text{ArcTan}[x]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x*ArcTan[x], x]')``[Out] -x / 2 + x ^ 2 ArcTan[x] / 2 + ArcTan[x] / 2`**Maple [A]**

time = 0.00, size = 16, normalized size = 0.76

method	result	size
default	$-\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{x^2 \arctan(x)}{2}$	16
meijerg	$-\frac{x}{2} + \frac{(3x^2+3) \arctan(x)}{6}$	16
risch	$-\frac{ix^2 \ln(ix+1)}{4} + \frac{ix^2 \ln(-ix+1)}{4} - \frac{x}{2} + \frac{\arctan(x)}{2}$	35

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*arctan(x), x, method=_RETURNVERBOSE)``[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)`

Maxima [A]

time = 0.34, size = 15, normalized size = 0.71

$$\frac{1}{2} x^2 \arctan(x) - \frac{1}{2} x + \frac{1}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*arctan(x),x, algorithm="maxima")``[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)`**Fricas [A]**

time = 0.31, size = 13, normalized size = 0.62

$$\frac{1}{2} (x^2 + 1) \arctan(x) - \frac{1}{2} x$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*arctan(x),x, algorithm="fricas")``[Out] 1/2*(x^2 + 1)*arctan(x) - 1/2*x`**Sympy [A]**

time = 0.09, size = 15, normalized size = 0.71

$$\frac{x^2 \operatorname{atan}(x)}{2} - \frac{x}{2} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*atan(x),x)``[Out] x**2*atan(x)/2 - x/2 + atan(x)/2`**Giac [A]**

time = 0.00, size = 19, normalized size = 0.90

$$-\frac{x}{2} + \frac{\arctan x}{2} + \frac{1}{2} x^2 \arctan x$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*arctan(x),x)``[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)`**Mupad [B]**

time = 0.02, size = 14, normalized size = 0.67

$$\operatorname{atan}(x) \left(\frac{x^2}{2} + \frac{1}{2} \right) - \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*atan(x),x)``[Out] atan(x)*(x^2/2 + 1/2) - x/2`

3.231 $\int x \cot^{-1}(x) dx$

Optimal. Leaf size=21

$$\frac{x}{2} + \frac{1}{2}x^2 \cot^{-1}(x) - \frac{1}{2} \tan^{-1}(x)$$

[Out] 1/2*x+1/2*x^2*arccot(x)-1/2*arctan(x)

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4947, 327, 209}

$$\frac{1}{2}x^2 \cot^{-1}(x) + \frac{x}{2} - \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x*ArcCot[x],x]

[Out] x/2 + (x^2*ArcCot[x])/2 - ArcTan[x]/2

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 327

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n-1)*(c*x)^(m-n+1)*((a+b*x^n)^(p+1)/(b*(m+n*p+1))), x] - Dist[a*c^n*((m-n+1)/(b*(m+n*p+1))), Int[(c*x)^(m-n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 4947

Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Simp[x^(m+1)*((a+b*ArcCot[c*x^n])^p/(m+1)), x] + Dist[b*c*n*(p/(m+1)), Int[x^(m+n)*((a+b*ArcCot[c*x^n])^(p-1)/(1+c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] && IntegerQ[m])) && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int x \cot^{-1}(x) dx &= \frac{1}{2}x^2 \cot^{-1}(x) + \frac{1}{2} \int \frac{x^2}{1+x^2} dx \\
&= \frac{x}{2} + \frac{1}{2}x^2 \cot^{-1}(x) - \frac{1}{2} \int \frac{1}{1+x^2} dx \\
&= \frac{x}{2} + \frac{1}{2}x^2 \cot^{-1}(x) - \frac{1}{2} \tan^{-1}(x)
\end{aligned}$$

Mathematica [A]

time = 0.00, size = 21, normalized size = 1.00

$$\frac{x}{2} + \frac{1}{2}x^2 \cot^{-1}(x) - \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

`[In] Integrate[x*ArcCot[x], x]``[Out] x/2 + (x^2*ArcCot[x])/2 - ArcTan[x]/2`**Mathics [A]**

time = 1.74, size = 15, normalized size = 0.71

$$\frac{x}{2} + \frac{x^2 \text{ArcCot}[x]}{2} + \frac{\text{ArcCot}[x]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x*ArcCot[x], x]')``[Out] x / 2 + x ^ 2 ArcCot[x] / 2 + ArcCot[x] / 2`**Maple [A]**

time = 0.03, size = 16, normalized size = 0.76

method	result	size
default	$\frac{x}{2} + \frac{x^2 \operatorname{arccot}(x)}{2} - \frac{\arctan(x)}{2}$	16
risch	$\frac{ix^2 \ln(ix+1)}{4} - \frac{ix^2 \ln(-ix+1)}{4} + \frac{\pi x^2}{4} + \frac{x}{2} - \frac{\arctan(x)}{2}$	41

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*arccot(x), x, method=_RETURNVERBOSE)``[Out] 1/2*x+1/2*x^2*arccot(x)-1/2*arctan(x)`

Maxima [A]

time = 0.36, size = 15, normalized size = 0.71

$$\frac{1}{2}x^2 \operatorname{arccot}(x) + \frac{1}{2}x - \frac{1}{2} \operatorname{arctan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*arccot(x),x, algorithm="maxima")``[Out] 1/2*x^2*arccot(x) + 1/2*x - 1/2*arctan(x)`**Fricas [A]**

time = 0.30, size = 13, normalized size = 0.62

$$\frac{1}{2}(x^2 + 1) \operatorname{arccot}(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*arccot(x),x, algorithm="fricas")``[Out] 1/2*(x^2 + 1)*arccot(x) + 1/2*x`**Sympy [A]**

time = 0.10, size = 15, normalized size = 0.71

$$\frac{x^2 \operatorname{acot}(x)}{2} + \frac{x}{2} + \frac{\operatorname{acot}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*acot(x),x)``[Out] x**2*acot(x)/2 + x/2 + acot(x)/2`**Giac [A]**

time = 0.00, size = 21, normalized size = 1.00

$$\frac{x}{2} + \frac{\operatorname{arctan}\left(\frac{1}{x}\right)}{2} + \frac{1}{2}(-x)^2 \operatorname{arctan}\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*arccot(x),x)``[Out] 1/2*x^2*arctan(1/x) + 1/2*x + 1/2*arctan(1/x)`**Mupad [B]**

time = 0.05, size = 15, normalized size = 0.71

$$\frac{x}{2} - \frac{\operatorname{atan}(x)}{2} + \frac{x^2 \operatorname{acot}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*acot(x),x)``[Out] x/2 - atan(x)/2 + (x^2*acot(x))/2`

3.232 $\int x \log(a + x^2) dx$

Optimal. Leaf size=23

$$-\frac{x^2}{2} + \frac{1}{2}(a + x^2) \log(a + x^2)$$

[Out] -1/2*x^2+1/2*(x^2+a)*ln(x^2+a)

Rubi [A]

time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2504, 2436, 2332}

$$\frac{1}{2}(a + x^2) \log(a + x^2) - \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[x*Log[a + x^2],x]

[Out] -1/2*x^2 + ((a + x^2)*Log[a + x^2])/2

Rule 2332

Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]

Rule 2436

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.), x_Symbol] :> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

Rule 2504

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

Rubi steps

$$\begin{aligned}
\int x \log(a+x^2) dx &= \frac{1}{2} \text{Subst} \left(\int \log(a+x) dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \log(x) dx, x, a+x^2 \right) \\
&= -\frac{x^2}{2} + \frac{1}{2} (a+x^2) \log(a+x^2)
\end{aligned}$$

Mathematica [A]

time = 0.00, size = 22, normalized size = 0.96

$$\frac{1}{2} (-x^2 + (a+x^2) \log(a+x^2))$$

Antiderivative was successfully verified.

`[In] Integrate[x*Log[a + x^2],x]``[Out] (-x^2 + (a + x^2)*Log[a + x^2])/2`**Mathics [A]**

time = 1.83, size = 26, normalized size = 1.13

$$\frac{a \text{Log}[a+x^2]}{2} - \frac{x^2}{2} + \frac{x^2 \text{Log}[a+x^2]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x*Log[x^2+a],x]')``[Out] a Log[a + x ^ 2] / 2 - x ^ 2 / 2 + x ^ 2 Log[a + x ^ 2] / 2`**Maple [A]**

time = 0.01, size = 23, normalized size = 1.00

method	result	size
derivativdivides	$\frac{(x^2+a) \ln(x^2+a)}{2} - \frac{x^2}{2} - \frac{a}{2}$	23
default	$\frac{(x^2+a) \ln(x^2+a)}{2} - \frac{x^2}{2} - \frac{a}{2}$	23
norman	$-\frac{x^2}{2} + \frac{\ln(x^2+a)a}{2} + \frac{\ln(x^2+a)x^2}{2}$	27
risch	$-\frac{x^2}{2} + \frac{\ln(x^2+a)a}{2} + \frac{\ln(x^2+a)x^2}{2}$	27

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*ln(x^2+a),x,method=_RETURNVERBOSE)`

[Out] $1/2*(x^2+a)*\ln(x^2+a)-1/2*x^2-1/2*a$

Maxima [A]

time = 0.26, size = 22, normalized size = 0.96

$$-\frac{1}{2}x^2 + \frac{1}{2}(x^2 + a)\log(x^2 + a) - \frac{1}{2}a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(x^2+a),x, algorithm="maxima")`

[Out] $-1/2*x^2 + 1/2*(x^2 + a)*\log(x^2 + a) - 1/2*a$

Fricas [A]

time = 0.30, size = 19, normalized size = 0.83

$$-\frac{1}{2}x^2 + \frac{1}{2}(x^2 + a)\log(x^2 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(x^2+a),x, algorithm="fricas")`

[Out] $-1/2*x^2 + 1/2*(x^2 + a)*\log(x^2 + a)$

Sympy [A]

time = 0.07, size = 26, normalized size = 1.13

$$\frac{a \log(a + x^2)}{2} + \frac{x^2 \log(a + x^2)}{2} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*ln(x**2+a),x)`

[Out] $a*\log(a + x**2)/2 + x**2*\log(a + x**2)/2 - x**2/2$

Giac [A]

time = 0.00, size = 22, normalized size = 0.96

$$\frac{-a - x^2 + (a + x^2)\ln(a + x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(x^2+a),x)`

[Out] $-1/2*x^2 + 1/2*(x^2 + a)*\log(x^2 + a) - 1/2*a$

Mupad [B]

time = 0.14, size = 41, normalized size = 1.78

$$\frac{a \ln(x + \sqrt{-a})}{2} + \frac{x^2 \ln(x^2 + a)}{2} + \frac{a \ln(x - \sqrt{-a})}{2} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(a + x^2),x)
```

```
[Out] (a*log(x + (-a)^(1/2)))/2 + (x^2*log(a + x^2))/2 + (a*log(x - (-a)^(1/2)))/  
2 - x^2/2
```

3.233 $\int \cos(x) \sin(a + x) dx$

Optimal. Leaf size=18

$$-\frac{1}{4} \cos(a + 2x) + \frac{1}{2} x \sin(a)$$

[Out] -1/4*cos(a+2*x)+1/2*x*sin(a)

Rubi [A]

time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4670, 2718}

$$\frac{1}{2} x \sin(a) - \frac{1}{4} \cos(a + 2x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[a + x],x]

[Out] -1/4*Cos[a + 2*x] + (x*Sin[a])/2

Rule 2718

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4670

Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] := Int[ExpandTrigReduce[Sin[v]^p * Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))

Rubi steps

$$\begin{aligned} \int \cos(x) \sin(a + x) dx &= \int \left(\frac{\sin(a)}{2} + \frac{1}{2} \sin(a + 2x) \right) dx \\ &= \frac{1}{2} x \sin(a) + \frac{1}{2} \int \sin(a + 2x) dx \\ &= -\frac{1}{4} \cos(a + 2x) + \frac{1}{2} x \sin(a) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 18, normalized size = 1.00

$$\frac{1}{4} (-\cos(a + 2x) + 2x \sin(a))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sin[a + x],x]

[Out] (-Cos[a + 2*x] + 2*x*Sin[a])/4

Mathics [A]

time = 2.03, size = 14, normalized size = 0.78

$$\frac{x \sin[a]}{2} + \frac{\sin[x] \sin[a + x]}{2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Sin[x+a]*Cos[x],x]')

[Out] x Sin[a] / 2 + Sin[x] Sin[a + x] / 2

Maple [A]

time = 0.07, size = 15, normalized size = 0.83

method	result	size
default	$-\frac{\cos(a+2x)}{4} + \frac{x \sin(a)}{2}$	15
risch	$-\frac{\cos(a+2x)}{4} + \frac{x \sin(a)}{2}$	15
norman	$\frac{x \tan\left(\frac{a+x}{2}\right) + x \tan\left(\frac{x}{2}\right) \left(\tan^2\left(\frac{a+x}{2}\right) + 2 \tan\left(\frac{x}{2}\right) \tan\left(\frac{a+x}{2}\right) - x \tan\left(\frac{x}{2}\right) - x \left(\tan^2\left(\frac{x}{2}\right) \tan\left(\frac{a+x}{2}\right)\right)}{\left(1 + \tan^2\left(\frac{x}{2}\right)\right) \left(1 + \tan^2\left(\frac{a+x}{2}\right)\right)}$	91

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(a+x),x,method=_RETURNVERBOSE)

[Out] -1/4*cos(a+2*x)+1/2*x*sin(a)

Maxima [A]

time = 0.26, size = 14, normalized size = 0.78

$$\frac{1}{2} x \sin(a) - \frac{1}{4} \cos(a + 2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(a+x),x, algorithm="maxima")

[Out] 1/2*x*sin(a) - 1/4*cos(a + 2*x)

Fricas [A]

time = 0.32, size = 28, normalized size = 1.56

$$-\frac{1}{2} \cos(a+x)^2 \cos(a) - \frac{1}{2} \cos(a+x) \sin(a+x) \sin(a) + \frac{1}{2} x \sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(a+x),x, algorithm="fricas")

[Out] $-1/2*\cos(a + x)^2*\cos(a) - 1/2*\cos(a + x)*\sin(a + x)*\sin(a) + 1/2*x*\sin(a)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$

time = 0.14, size = 32, normalized size = 1.78

$$-\frac{x \sin(x) \cos(a+x)}{2} + \frac{x \sin(a+x) \cos(x)}{2} + \frac{\sin(x) \sin(a+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(a+x),x)

[Out] $-x*\sin(x)*\cos(a + x)/2 + x*\sin(a + x)*\cos(x)/2 + \sin(x)*\sin(a + x)/2$

Giac [A]

time = 0.00, size = 17, normalized size = 0.94

$$-\frac{\cos(a+2x)}{4} + \frac{1}{2} \sin a \cdot x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(a+x),x)

[Out] $1/2*x*\sin(a) - 1/4*\cos(a + 2*x)$

Mupad [B]

time = 0.03, size = 14, normalized size = 0.78

$$\frac{x \sin(a)}{2} - \frac{\cos(a+2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a + x)*cos(x),x)

[Out] $(x*\sin(a))/2 - \cos(a + 2*x)/4$

3.234 $\int \cos(a + x) \sin(x) dx$

Optimal. Leaf size=18

$$-\frac{1}{4} \cos(a + 2x) - \frac{1}{2} x \sin(a)$$

[Out] -1/4*cos(a+2*x)-1/2*x*sin(a)

Rubi [A]

time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4670, 2718}

$$-\frac{1}{2} x \sin(a) - \frac{1}{4} \cos(a + 2x)$$

Antiderivative was successfully verified.

[In] Int[Cos[a + x]*Sin[x],x]

[Out] -1/4*Cos[a + 2*x] - (x*Sin[a])/2

Rule 2718

Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4670

Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] := Int[ExpandTrigReduce[Sin[v]^p * Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))

Rubi steps

$$\begin{aligned} \int \cos(a + x) \sin(x) dx &= \int \left(-\frac{\sin(a)}{2} + \frac{1}{2} \sin(a + 2x) \right) dx \\ &= -\frac{1}{2} x \sin(a) + \frac{1}{2} \int \sin(a + 2x) dx \\ &= -\frac{1}{4} \cos(a + 2x) - \frac{1}{2} x \sin(a) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 18, normalized size = 1.00

$$\frac{1}{4}(-\cos(a + 2x) - 2x \sin(a))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + x]*Sin[x],x]

[Out] (-Cos[a + 2*x] - 2*x*Sin[a])/4

Mathics [A]

time = 2.02, size = 14, normalized size = 0.78

$$-\frac{x \sin[a]}{2} - \frac{\cos[x] \cos[a+x]}{2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Cos[x+a]*Sin[x],x]')

[Out] -x Sin[a] / 2 - Cos[x] Cos[a + x] / 2

Maple [A]

time = 0.05, size = 15, normalized size = 0.83

method	result	size
default	$-\frac{\cos(a+2x)}{4} - \frac{x \sin(a)}{2}$	15
risch	$-\frac{\cos(a+2x)}{4} - \frac{x \sin(a)}{2}$	15
meijerg	$\frac{\cos(a) \sqrt{\pi} \left(\frac{1}{\sqrt{\pi}} - \frac{\cos(2x)}{\sqrt{\pi}} \right)}{4} - \frac{\sin(a) \sqrt{\pi} \left(\frac{2x}{\sqrt{\pi}} - \frac{\sin(2x)}{\sqrt{\pi}} \right)}{4}$	45
norman	$\frac{x \tan\left(\frac{x}{2}\right) + x \left(\tan^2\left(\frac{x}{2}\right)\right) \tan\left(\frac{a}{2} + \frac{x}{2}\right) + 2 \tan\left(\frac{x}{2}\right) \tan\left(\frac{a}{2} + \frac{x}{2}\right) - x \tan\left(\frac{a}{2} + \frac{x}{2}\right) - x \tan\left(\frac{x}{2}\right) \left(\tan^2\left(\frac{a}{2} + \frac{x}{2}\right)\right)}{\left(1 + \tan^2\left(\frac{a}{2} + \frac{x}{2}\right)\right) \left(1 + \tan^2\left(\frac{x}{2}\right)\right)}$	91

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+x)*sin(x),x,method=_RETURNVERBOSE)

[Out] -1/4*cos(a+2*x)-1/2*x*sin(a)

Maxima [A]

time = 0.27, size = 14, normalized size = 0.78

$$-\frac{1}{2} x \sin(a) - \frac{1}{4} \cos(a + 2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+x)*sin(x),x, algorithm="maxima")

[Out] -1/2*x*sin(a) - 1/4*cos(a + 2*x)

Fricas [A]

time = 0.31, size = 28, normalized size = 1.56

$$-\frac{1}{2} \cos(a+x)^2 \cos(a) - \frac{1}{2} \cos(a+x) \sin(a+x) \sin(a) - \frac{1}{2} x \sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+x)*sin(x),x, algorithm="fricas")`

[Out] $-1/2*\cos(a + x)^2*\cos(a) - 1/2*\cos(a + x)*\sin(a + x)*\sin(a) - 1/2*x*\sin(a)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$

time = 0.14, size = 32, normalized size = 1.78

$$\frac{x \sin(x) \cos(a+x)}{2} - \frac{x \sin(a+x) \cos(x)}{2} + \frac{\sin(x) \sin(a+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+x)*sin(x),x)`

[Out] $x*\sin(x)*\cos(a + x)/2 - x*\sin(a + x)*\cos(x)/2 + \sin(x)*\sin(a + x)/2$

Giac [A]

time = 0.00, size = 18, normalized size = 1.00

$$-\frac{\cos(a+2x)}{4} - \frac{1}{2} \sin a \cdot x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+x)*sin(x),x)`

[Out] $-1/2*x*\sin(a) - 1/4*\cos(a + 2*x)$

Mupad [B]

time = 0.02, size = 14, normalized size = 0.78

$$-\frac{\cos(a+2x)}{4} - \frac{x \sin(a)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + x)*sin(x),x)`

[Out] $-\cos(a + 2*x)/4 - (x*\sin(a))/2$

3.235 $\int \sqrt{1 + \sin(x)} dx$

Optimal. Leaf size=12

$$-\frac{2 \cos(x)}{\sqrt{1 + \sin(x)}}$$

[Out] $-2*\cos(x)/(1+\sin(x))^{(1/2)}$

Rubi [A]

time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2725}

$$-\frac{2 \cos(x)}{\sqrt{\sin(x) + 1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + Sin[x]],x]

[Out] $(-2*\cos[x])/Sqrt[1 + Sin[x]]$

Rule 2725

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[-2*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \sqrt{1 + \sin(x)} dx = -\frac{2 \cos(x)}{\sqrt{1 + \sin(x)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(12) = 24.

time = 0.01, size = 40, normalized size = 3.33

$$\frac{2 \left(-\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right) \right) \sqrt{1 + \sin(x)}}{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + Sin[x]],x]

[Out] $(2*(-\cos[x/2] + \sin[x/2])*Sqrt[1 + Sin[x]])/(Cos[x/2] + Sin[x/2])$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(1+Sin[x])^(1/2),x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [A]**

time = 0.06, size = 17, normalized size = 1.42

method	result	size
default	$\frac{2(\sin(x)-1)\sqrt{\sin(x)+1}}{\cos(x)}$	17
risch	$-\frac{i\sqrt{2}\sqrt{2+2\sin(x)}(e^{ix}-i)(e^{ix}+i)}{e^{2ix}+2ie^{ix}-1}$	48

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((sin(x)+1)^(1/2),x,method=_RETURNVERBOSE)``[Out] 2*(sin(x)-1)*(sin(x)+1)^(1/2)/cos(x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((1+sin(x))^(1/2),x, algorithm="maxima")``[Out] integrate(sqrt(sin(x) + 1), x)`**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(10) = 20.

time = 0.30, size = 24, normalized size = 2.00

$$\frac{2(\cos(x) - \sin(x) + 1)\sqrt{\sin(x) + 1}}{\cos(x) + \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((1+sin(x))^(1/2),x, algorithm="fricas")`

[Out] $-2*(\cos(x) - \sin(x) + 1)*\sqrt{\sin(x) + 1}/(\cos(x) + \sin(x) + 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+sin(x))**(1/2),x)`

[Out] `Integral(sqrt(sin(x) + 1), x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.

time = 0.00, size = 29, normalized size = 2.42

$$2\sqrt{2} \operatorname{sign}\left(\cos\left(-\frac{\pi}{4} + \frac{x}{2}\right)\right) \sin\left(\frac{-\pi + 2x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+sin(x))^(1/2),x)`

[Out] `2*sqrt(2)*sgn(cos(-1/4*pi + 1/2*x))*sin(-1/4*pi + 1/2*x)`

Mupad [B]

time = 0.14, size = 16, normalized size = 1.33

$$\frac{2 (\sin(x) - 1) \sqrt{\sin(x) + 1}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((sin(x) + 1)^(1/2),x)`

[Out] `(2*(sin(x) - 1)*(sin(x) + 1)^(1/2))/cos(x)`

3.236 $\int \sqrt{1 - \sin(x)} dx$

Optimal. Leaf size=14

$$\frac{2 \cos(x)}{\sqrt{1 - \sin(x)}}$$

[Out] 2*cos(x)/(1-sin(x))^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2725}

$$\frac{2 \cos(x)}{\sqrt{1 - \sin(x)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - Sin[x]],x]

[Out] (2*Cos[x])/Sqrt[1 - Sin[x]]

Rule 2725

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \sqrt{1 - \sin(x)} dx = \frac{2 \cos(x)}{\sqrt{1 - \sin(x)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(14) = 28.

time = 0.01, size = 42, normalized size = 3.00

$$\frac{2 \left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right) \right) \sqrt{1 - \sin(x)}}{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - Sin[x]],x]

[Out] (2*(Cos[x/2] + Sin[x/2])*Sqrt[1 - Sin[x]])/(Cos[x/2] - Sin[x/2])

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(1-Sin[x])^(1/2),x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.09, size = 23, normalized size = 1.64

method	result	size
default	$-\frac{2(\sin(x)-1)(\sin(x)+1)}{\cos(x)\sqrt{1-\sin(x)}}$	23
risch	$\frac{i\sqrt{2-2\sin(x)}\sqrt{-i(2ie^{2ix}-e^{3ix}+e^{ix})}\sqrt{2}^{(e^{ix}-i)(e^{ix}+i)}}{(2ie^{ix}-e^{2ix}+1)\sqrt{i(e^{3ix}-2ie^{2ix}-e^{ix})}}$	102

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((1-sin(x))^(1/2),x,method=_RETURNVERBOSE)``[Out] -2*(sin(x)-1)*(sin(x)+1)/cos(x)/(1-sin(x))^(1/2)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((1-sin(x))^(1/2),x, algorithm="maxima")``[Out] integrate(sqrt(-sin(x) + 1), x)`**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

time = 0.31, size = 26, normalized size = 1.86

$$\frac{2(\cos(x) + \sin(x) + 1)\sqrt{-\sin(x) + 1}}{\cos(x) - \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((1-sin(x))^(1/2),x, algorithm="fricas")``[Out] 2*(cos(x) + sin(x) + 1)*sqrt(-sin(x) + 1)/(cos(x) - sin(x) + 1)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1 - \sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sin(x))**(1/2),x)

[Out] Integral(sqrt(1 - sin(x)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(12) = 24.

time = 0.00, size = 46, normalized size = 3.29

$$\sqrt{2} \left(2\text{sign} \left(\sin \left(\frac{x}{2} - \frac{\pi}{4} \right) \right) - 2\text{sign} \left(\sin \left(-\frac{\pi}{4} + \frac{x}{2} \right) \right) \cos \left(\frac{-\pi + 2x}{4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sin(x))^(1/2),x)

[Out] -2*sqrt(2)*(cos(-1/4*pi + 1/2*x)*sgn(sin(-1/4*pi + 1/2*x)) - sgn(sin(-1/4*pi + 1/2*x)))

Mupad [B]

time = 0.14, size = 18, normalized size = 1.29

$$\frac{2 \sqrt{1 - \sin(x)} (\sin(x) + 1)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - sin(x))^(1/2),x)

[Out] (2*(1 - sin(x))^(1/2)*(sin(x) + 1))/cos(x)

3.237 $\int \sqrt{1 + \cos(x)} dx$

Optimal. Leaf size=12

$$\frac{2 \sin(x)}{\sqrt{1 + \cos(x)}}$$

[Out] 2*sin(x)/(1+cos(x))^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2725}

$$\frac{2 \sin(x)}{\sqrt{\cos(x) + 1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + Cos[x]],x]

[Out] (2*Sin[x])/Sqrt[1 + Cos[x]]

Rule 2725

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[-2*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \sqrt{1 + \cos(x)} dx = \frac{2 \sin(x)}{\sqrt{1 + \cos(x)}}$$

Mathematica [A]

time = 0.01, size = 16, normalized size = 1.33

$$2\sqrt{1 + \cos(x)} \tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + Cos[x]],x]

[Out] 2*Sqrt[1 + Cos[x]]*Tan[x/2]

Mathics [A]

time = 2.37, size = 12, normalized size = 1.00

$$2\sqrt{1 + \cos[x]} \tan\left[\frac{x}{2}\right]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(1+Cos[x])^(1/2),x]')`[Out] `2 Sqrt[1 + Cos[x]] Tan[x / 2]`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

time = 0.04, size = 22, normalized size = 1.83

method	result	size
default	$\frac{4 \cos(\frac{x}{2}) \sin(\frac{x}{2}) \sqrt{2}}{\sqrt{2 \cos(x) + 2}}$	22
risch	$-\frac{i\sqrt{2} \sqrt{(1 + e^{ix})^2 e^{-ix} (e^{ix} - 1)}}{1 + e^{ix}}$	40

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+cos(x))^(1/2),x,method=_RETURNVERBOSE)`[Out] `2*cos(1/2*x)*sin(1/2*x)*2^(1/2)/(cos(1/2*x)^2)^(1/2)`**Maxima [A]**

time = 0.39, size = 9, normalized size = 0.75

$$2\sqrt{2} \sin\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+cos(x))^(1/2),x, algorithm="maxima")`[Out] `2*sqrt(2)*sin(1/2*x)`**Fricas [A]**

time = 0.31, size = 10, normalized size = 0.83

$$\frac{2 \sin(x)}{\sqrt{\cos(x) + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+cos(x))^(1/2),x, algorithm="fricas")`

[Out] $2*\sin(x)/\sqrt{\cos(x) + 1}$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(12) = 24$

time = 0.37, size = 36, normalized size = 3.00

$$2\sqrt{1 - \frac{\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) + 1} + \frac{1}{\tan^2\left(\frac{x}{2}\right) + 1}} \tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+cos(x))**(1/2), x)`

[Out] $2*\sqrt{1 - \tan(x/2)**2/(\tan(x/2)**2 + 1) + 1/(\tan(x/2)**2 + 1)}*\tan(x/2)$

Giac [A]

time = 0.00, size = 18, normalized size = 1.50

$$2\sqrt{2} \operatorname{sign}\left(\cos\left(\frac{x}{2}\right)\right) \sin\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+cos(x))^(1/2), x)`

[Out] $2*\sqrt{2}*sgn(\cos(1/2*x))*\sin(1/2*x)$

Mupad [B]

time = 0.15, size = 10, normalized size = 0.83

$$\frac{2 \sin(x)}{\sqrt{\cos(x) + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(x) + 1)^(1/2), x)`

[Out] $(2*\sin(x))/(\cos(x) + 1)^(1/2)$

3.238 $\int \sqrt{1 - \cos(x)} dx$

Optimal. Leaf size=14

$$-\frac{2 \sin(x)}{\sqrt{1 - \cos(x)}}$$

[Out] $-2*\sin(x)/(1-\cos(x))^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2725}

$$-\frac{2 \sin(x)}{\sqrt{1 - \cos(x)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - Cos[x]],x]

[Out] $(-2*\sin[x])/Sqrt[1 - Cos[x]]$

Rule 2725

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \sqrt{1 - \cos(x)} dx = -\frac{2 \sin(x)}{\sqrt{1 - \cos(x)}}$$

Mathematica [A]

time = 0.01, size = 18, normalized size = 1.29

$$-2\sqrt{1 - \cos(x)} \cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - Cos[x]],x]

[Out] $-2*Sqrt[1 - Cos[x]]*Cot[x/2]$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1-Cos[x])^(1/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python object
```

Maple [A]

time = 0.05, size = 22, normalized size = 1.57

method	result	size
default	$-\frac{4 \sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right) \sqrt{2}}{\sqrt{2 - 2 \cos(x)}}$	22
risch	$-\frac{i \sqrt{2} \sqrt{-(e^{ix} - 1)^2 e^{-ix}} (1 + e^{ix})}{e^{ix} - 1}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*sin(1/2*x)*cos(1/2*x)*2^(1/2)/(sin(1/2*x)^2)^(1/2)
```

Maxima [A]

time = 0.35, size = 20, normalized size = 1.43

$$-\frac{2\sqrt{2}}{\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(2)/sqrt(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A]

time = 0.30, size = 18, normalized size = 1.29

$$-\frac{2(\cos(x)+1)\sqrt{-\cos(x)+1}}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-cos(x))^(1/2),x, algorithm="fricas")

[Out] -2*(cos(x) + 1)*sqrt(-cos(x) + 1)/sin(x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1 - \cos(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-cos(x))**(1/2),x)

[Out] Integral(sqrt(1 - cos(x)), x)

Giac [A]

time = 0.00, size = 29, normalized size = 2.07

$$\sqrt{2} \left(2\operatorname{sign}\left(\sin\left(\frac{x}{2}\right)\right) - 2\operatorname{sign}\left(\sin\left(\frac{x}{2}\right)\right)\cos\left(\frac{x}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-cos(x))^(1/2),x)

[Out] -2*sqrt(2)*(cos(1/2*x)*sgn(sin(1/2*x)) - sgn(sin(1/2*x)))

Mupad [B]

time = 0.03, size = 12, normalized size = 0.86

$$-\frac{2 \sin(x)}{\sqrt{1 - \cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - cos(x))^(1/2),x)

[Out] -(2*sin(x))/(1 - cos(x))^(1/2)

$$3.239 \quad \int \frac{1}{-\sqrt{-1+x} + \sqrt{x}} dx$$

Optimal. Leaf size=21

$$\frac{2}{3}(-1+x)^{3/2} + \frac{2x^{3/2}}{3}$$

[Out] 2/3*(-1+x)^(3/2)+2/3*x^(3/2)

Rubi [A]

time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2131, 30, 32}

$$\frac{2x^{3/2}}{3} + \frac{2}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(-Sqrt[-1 + x] + Sqrt[x])^(-1), x]

[Out] (2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 32

Int[((a_) + (b_)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2131

Int[(u_)/((d_)*(x_)^(n_) + (c_)*Sqrt[(a_) + (b_)*(x_)^(p_)]), x_Symbol] := Dist[-b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{-\sqrt{-1+x} + \sqrt{x}} dx &= \int \sqrt{-1+x} dx + \int \sqrt{x} dx \\ &= \frac{2}{3}(-1+x)^{3/2} + \frac{2x^{3/2}}{3} \end{aligned}$$

Mathematica [A]

time = 0.06, size = 17, normalized size = 0.81

$$\frac{2}{3} \left((-1+x)^{3/2} + x^{3/2} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(-Sqrt[-1 + x] + Sqrt[x])^(-1), x]``[Out] (2*((-1 + x)^(3/2) + x^(3/2)))/3`**Mathics [A]**

time = 2.26, size = 30, normalized size = 1.43

$$\frac{2(-1 - \sqrt{x} \sqrt{-1+x} + 2x)}{3\sqrt{x} - 3\sqrt{-1+x}}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^(1/2)-(x-1)^(1/2)), x]')``[Out] 2(-1 - Sqrt[x] Sqrt[-1 + x] + 2 x) / (3 (Sqrt[x] - Sqrt[-1 + x]))`**Maple [A]**

time = 0.02, size = 14, normalized size = 0.67

method	result	size
default	$\frac{2(-1+x)^{\frac{3}{2}}}{3} + \frac{2x^{\frac{3}{2}}}{3}$	14

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(-(-1+x)^(1/2)+x^(1/2)), x, method=_RETURNVERBOSE)``[Out] 2/3*(-1+x)^(3/2)+2/3*x^(3/2)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(-(-1+x)^(1/2)+x^(1/2)), x, algorithm="maxima")``[Out] -integrate(1/(sqrt(x - 1) - sqrt(x)), x)`**Fricas [A]**

time = 0.30, size = 13, normalized size = 0.62

$$\frac{2}{3} (x-1)^{\frac{3}{2}} + \frac{2}{3} x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="fricas")

[Out] 2/3*(x - 1)^(3/2) + 2/3*x^(3/2)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(17) = 34

time = 0.22, size = 63, normalized size = 3.00

$$\frac{2\sqrt{x}\sqrt{x-1}}{-3\sqrt{x}+3\sqrt{x-1}} - \frac{4x}{-3\sqrt{x}+3\sqrt{x-1}} + \frac{2}{-3\sqrt{x}+3\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)**(1/2)+x**(1/2)),x)

[Out] 2*sqrt(x)*sqrt(x - 1)/(-3*sqrt(x) + 3*sqrt(x - 1)) - 4*x/(-3*sqrt(x) + 3*sqrt(x - 1)) + 2/(-3*sqrt(x) + 3*sqrt(x - 1))

Giac [A]

time = 0.00, size = 43, normalized size = 2.05

$$2\left(\frac{1}{3}\sqrt{x-1}(x-1)\right) + 2\left(\frac{1}{6}\sqrt{x-1}\sqrt{x-1} + \frac{1}{6}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)^(1/2)+x^(1/2)),x)

[Out] 2/3*(x - 1)^(3/2) + 2/3*x^(3/2)

Mupad [B]

time = 0.19, size = 21, normalized size = 1.00

$$\frac{2x\sqrt{x-1}}{3} - \frac{2\sqrt{x-1}}{3} + \frac{2x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/((x - 1)^(1/2) - x^(1/2)),x)

[Out] (2*x*(x - 1)^(1/2))/3 - (2*(x - 1)^(1/2))/3 + (2*x^(3/2))/3

$$3.240 \quad \int \frac{1}{1 - \sqrt{1+x}} dx$$

Optimal. Leaf size=24

$$-2\sqrt{1+x} - 2\log\left(1 - \sqrt{1+x}\right)$$

[Out] -2*ln(1-(1+x)^(1/2))-2*(1+x)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {253, 196, 45}

$$-2\sqrt{x+1} - 2\log\left(1 - \sqrt{x+1}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[1 + x])^(-1), x]

[Out] -2*Sqrt[1 + x] - 2*Log[1 - Sqrt[1 + x]]

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 196

Int[((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(1/n - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]

Rule 253

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{1 - \sqrt{1+x}} dx &= \text{Subst} \left(\int \frac{1}{1 - \sqrt{x}} dx, x, 1+x \right) \\
&= 2\text{Subst} \left(\int \frac{x}{1-x} dx, x, \sqrt{1+x} \right) \\
&= 2\text{Subst} \left(\int \left(-1 + \frac{1}{1-x} \right) dx, x, \sqrt{1+x} \right) \\
&= -2\sqrt{1+x} - 2\log(1 - \sqrt{1+x})
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 22, normalized size = 0.92

$$-2\sqrt{1+x} - 2\log(-1 + \sqrt{1+x})$$

Antiderivative was successfully verified.

`[In] Integrate[(1 - Sqrt[1 + x])^(-1), x]``[Out] -2*Sqrt[1 + x] - 2*Log[-1 + Sqrt[1 + x]]`**Mathics [A]**

time = 1.77, size = 18, normalized size = 0.75

$$-2\text{Log}[-1 + \sqrt{1+x}] - 2\sqrt{1+x}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(1-(x+1)^(1/2)), x]')``[Out] -2 Log[-1 + Sqrt[1 + x]] - 2 Sqrt[1 + x]`**Maple [A]**

time = 0.04, size = 31, normalized size = 1.29

method	result	size
derivativedivides	$-2\sqrt{1+x} - 2\ln(-1 + \sqrt{1+x})$	19
trager	$-2\sqrt{1+x} - \ln(2\sqrt{1+x} - 2 - x)$	24
default	$-\ln(x) - 2\sqrt{1+x} - \ln(-1 + \sqrt{1+x}) + \ln(1 + \sqrt{1+x})$	31

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(1-(1+x)^(1/2)), x, method=_RETURNVERBOSE)`

[Out] $-\ln(x) - 2*(1+x)^{(1/2)} - \ln(-1+(1+x)^{(1/2)}) + \ln(1+(1+x)^{(1/2)})$

Maxima [A]

time = 0.27, size = 18, normalized size = 0.75

$$-2\sqrt{x+1} - 2\log(\sqrt{x+1} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-(1+x)^(1/2)),x, algorithm="maxima")`

[Out] $-2*\text{sqrt}(x + 1) - 2*\log(\text{sqrt}(x + 1) - 1)$

Fricas [A]

time = 0.30, size = 18, normalized size = 0.75

$$-2\sqrt{x+1} - 2\log(\sqrt{x+1} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-(1+x)^(1/2)),x, algorithm="fricas")`

[Out] $-2*\text{sqrt}(x + 1) - 2*\log(\text{sqrt}(x + 1) - 1)$

Sympy [A]

time = 0.06, size = 20, normalized size = 0.83

$$-2\sqrt{x+1} - 2\log(\sqrt{x+1} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-(1+x)**(1/2)),x)`

[Out] $-2*\text{sqrt}(x + 1) - 2*\log(\text{sqrt}(x + 1) - 1)$

Giac [A]

time = 0.00, size = 22, normalized size = 0.92

$$-2\left(\sqrt{x+1} + \ln\left|\sqrt{x+1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-(1+x)^(1/2)),x)`

[Out] $-2*\text{sqrt}(x + 1) - 2*\log(\text{abs}(\text{sqrt}(x + 1) - 1))$

Mupad [B]

time = 0.11, size = 18, normalized size = 0.75

$$-2\ln(\sqrt{x+1} - 1) - 2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/((x + 1)^(1/2) - 1),x)`

[Out] $-2*\log((x + 1)^{(1/2)} - 1) - 2*(x + 1)^{(1/2)}$

$$3.241 \quad \int \frac{x}{\sqrt{36 + x^4}} dx$$

Optimal. Leaf size=12

$$\frac{1}{2} \sinh^{-1} \left(\frac{x^2}{6} \right)$$

[Out] 1/2*arcsinh(1/6*x^2)

Rubi [A]

time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {281, 221}

$$\frac{1}{2} \sinh^{-1} \left(\frac{x^2}{6} \right)$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[36 + x^4],x]

[Out] ArcSinh[x^2/6]/2

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 281

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{36 + x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{36 + x^2}} dx, x, x^2 \right) \\ &= \frac{1}{2} \sinh^{-1} \left(\frac{x^2}{6} \right) \end{aligned}$$

Mathematica [A]

time = 0.06, size = 18, normalized size = 1.50

$$\frac{1}{2} \tanh^{-1} \left(\frac{x^2}{\sqrt{36 + x^4}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[36 + x^4],x]

[Out] ArcTanh[x^2/Sqrt[36 + x^4]]/2

Mathics [A]

time = 2.03, size = 8, normalized size = 0.67

$$\frac{\text{ArcSinh}\left[\frac{x^2}{6}\right]}{2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x/(x^4+36)^(1/2),x]')

[Out] ArcSinh[x ^ 2 / 6] / 2

Maple [A]

time = 0.08, size = 9, normalized size = 0.75

method	result	size
default	$\frac{\text{arcsinh}\left(\frac{x^2}{6}\right)}{2}$	9
meijerg	$\frac{\text{arcsinh}\left(\frac{x^2}{6}\right)}{2}$	9
elliptic	$\frac{\text{arcsinh}\left(\frac{x^2}{6}\right)}{2}$	9
trager	$-\frac{\ln\left(x^2 - \sqrt{x^4 + 36}\right)}{2}$	17

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^4+36)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*arcsinh(1/6*x^2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(8) = 16.

time = 0.26, size = 33, normalized size = 2.75

$$\frac{1}{4} \log\left(\frac{\sqrt{x^4 + 36}}{x^2} + 1\right) - \frac{1}{4} \log\left(\frac{\sqrt{x^4 + 36}}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^4+36)^(1/2),x, algorithm="maxima")

[Out] 1/4*log(sqrt(x^4 + 36)/x^2 + 1) - 1/4*log(sqrt(x^4 + 36)/x^2 - 1)

Fricas [A]

time = 0.29, size = 16, normalized size = 1.33

$$-\frac{1}{2} \log \left(-x^2 + \sqrt{x^4 + 36} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x/(x^4+36)^(1/2),x, algorithm="fricas")``[Out] -1/2*log(-x^2 + sqrt(x^4 + 36))`**Sympy [A]**

time = 0.42, size = 7, normalized size = 0.58

$$\frac{\operatorname{asinh} \left(\frac{x^2}{6} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x/(x**4+36)**(1/2),x)``[Out] asinh(x**2/6)/2`**Giac [A]**

time = 0.00, size = 19, normalized size = 1.58

$$-\frac{\ln \left(\sqrt{x^4 + 36} - x^2 \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x/(x^4+36)^(1/2),x)``[Out] -1/2*log(-x^2 + sqrt(x^4 + 36))`**Mupad [B]**

time = 0.04, size = 8, normalized size = 0.67

$$\frac{\operatorname{asinh} \left(\frac{x^2}{6} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x/(x^4 + 36)^(1/2),x)``[Out] asinh(x^2/6)/2`

$$3.242 \quad \int \frac{1}{\sqrt[3]{x} + \sqrt{x}} dx$$

Optimal. Leaf size=32

$$6\sqrt[6]{x} - 3\sqrt[3]{x} + 2\sqrt{x} - 6 \log(1 + \sqrt[6]{x})$$

[Out] 6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1607, 272, 45}

$$2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6 \log(\sqrt[6]{x} + 1)$$

Antiderivative was successfully verified.

[In] Int[(x^(1/3) + Sqrt[x])^(-1), x]

[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1607

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{x} + \sqrt{x}} dx &= \int \frac{1}{(1 + \sqrt[6]{x}) \sqrt[3]{x}} dx \\
&= 6 \text{Subst} \left(\int \frac{x^3}{1+x} dx, x, \sqrt[6]{x} \right) \\
&= 6 \text{Subst} \left(\int \left(1 + \frac{1}{-1-x} - x + x^2 \right) dx, x, \sqrt[6]{x} \right) \\
&= 6\sqrt[6]{x} - 3\sqrt[3]{x} + 2\sqrt{x} - 6 \log(1 + \sqrt[6]{x})
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 33, normalized size = 1.03

$$(6 - 3\sqrt[6]{x} + 2\sqrt[3]{x}) \sqrt[6]{x} - 6 \log(1 + \sqrt[6]{x})$$

Antiderivative was successfully verified.

`[In] Integrate[(x^(1/3) + Sqrt[x])^(-1), x]``[Out] (6 - 3*x^(1/6) + 2*x^(1/3))*x^(1/6) - 6*Log[1 + x^(1/6)]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x^(1/3)+x^(1/2)), x]')``[Out] cought exception: maximum recursion depth exceeded in comparison`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(24) = 48.

time = 0.03, size = 92, normalized size = 2.88

method	result
derivativdivides	$6x^{\frac{1}{6}} - 3x^{\frac{1}{3}} - 6 \ln(1 + x^{\frac{1}{6}}) + 2\sqrt{x}$
meijerg	$\frac{x^{\frac{1}{6}}(4x^{\frac{1}{3}} - 6x^{\frac{1}{6}} + 12)}{2} - 6 \ln(1 + x^{\frac{1}{6}})$
default	$2 \ln(-1 + x^{\frac{1}{6}}) - \ln(x^{\frac{1}{3}} + x^{\frac{1}{6}} + 1) - 2 \ln(1 + x^{\frac{1}{6}}) + \ln(x^{\frac{1}{3}} - x^{\frac{1}{6}} + 1) + 2\sqrt{x} + \ln($

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)`

[Out] $2*\ln(-1+x^{1/6})-\ln(x^{1/3}+x^{1/6}+1)-2*\ln(1+x^{1/6})+\ln(x^{1/3}-x^{1/6}+1)+2*x^{1/2}+\ln(x^{1/2}-1)-\ln(1+x^{1/2})+6*x^{1/6}-\ln(-1+x)-2*\ln(-1+x^{1/3})+\ln(x^{2/3}+x^{1/3}+1)-3*x^{1/3}$

Maxima [A]

time = 0.27, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="maxima")`

[Out] $2*\sqrt{x} - 3*x^{1/3} + 6*x^{1/6} - 6*\log(x^{1/6} + 1)$

Fricas [A]

time = 0.30, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="fricas")`

[Out] $2*\sqrt{x} - 3*x^{1/3} + 6*x^{1/6} - 6*\log(x^{1/6} + 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x} + \sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**(1/3)+x**(1/2)),x)`

[Out] `Integral(1/(x**(1/3) + sqrt(x)), x)`

Giac [A]

time = 0.00, size = 39, normalized size = 1.22

$$6\left(\frac{1}{3}x^{\frac{1}{6}}x^{\frac{1}{3}} - \frac{x^{\frac{1}{3}}}{2} + x^{\frac{1}{6}} - \ln\left(x^{\frac{1}{6}} + 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^(1/3)+x^(1/2)),x)`

[Out] $2*\sqrt{x} - 3*x^{(1/3)} + 6*x^{(1/6)} - 6*\log(x^{(1/6)} + 1)$

Mupad [B]

time = 0.03, size = 24, normalized size = 0.75

$$2\sqrt{x} - 6 \ln(x^{1/6} + 1) - 3x^{1/3} + 6x^{1/6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^{(1/2)} + x^{(1/3)}), x)$

[Out] $2*x^{(1/2)} - 6*\log(x^{(1/6)} + 1) - 3*x^{(1/3)} + 6*x^{(1/6)}$

3.243 $\int \log(2 + 3x^2) dx$

Optimal. Leaf size=33

$$-2x + 2\sqrt{\frac{2}{3}} \tan^{-1}\left(\sqrt{\frac{3}{2}}x\right) + x \log(2 + 3x^2)$$

[Out] $-2*x+x*\ln(3*x^2+2)+2/3*\arctan(1/2*x*6^{(1/2)})*6^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2498, 327, 209}

$$x \log(3x^2 + 2) - 2x + 2\sqrt{\frac{2}{3}} \tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Log}[2 + 3*x^2], x]$

[Out] $-2*x + 2*\text{Sqrt}[2/3]*\text{ArcTan}[\text{Sqrt}[3/2]*x] + x*\text{Log}[2 + 3*x^2]$

Rule 209

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))*\text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 327

$\text{Int}[(c \cdot x)^m * (a + (b \cdot x)^n)^p, x_Symbol] \rightarrow \text{Simp}[c^{(n-1)} * (c \cdot x)^{m-n+1} * (a + b \cdot x^n)^{p+1} / (b * (m + n * p + 1))], x] - \text{Dist}[a * c^n * ((m - n + 1) / (b * (m + n * p + 1))), \text{Int}[(c \cdot x)^{m-n} * (a + b \cdot x^n)^p, x], x] /;$ FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n * p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2498

$\text{Int}[\text{Log}[c \cdot (d + (e \cdot x)^n)^p], x_Symbol] \rightarrow \text{Simp}[x * \text{Log}[c * (d + e \cdot x^n)^p], x] - \text{Dist}[e * n * p, \text{Int}[x^n / (d + e \cdot x^n), x], x] /;$ FreeQ[{c, d, e, n, p}, x]

Rubi steps

$$\begin{aligned}
\int \log(2 + 3x^2) dx &= x \log(2 + 3x^2) - 6 \int \frac{x^2}{2 + 3x^2} dx \\
&= -2x + x \log(2 + 3x^2) + 4 \int \frac{1}{2 + 3x^2} dx \\
&= -2x + 2\sqrt{\frac{2}{3}} \tan^{-1}\left(\sqrt{\frac{3}{2}} x\right) + x \log(2 + 3x^2)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 33, normalized size = 1.00

$$-2x + 2\sqrt{\frac{2}{3}} \tan^{-1}\left(\sqrt{\frac{3}{2}} x\right) + x \log(2 + 3x^2)$$

Antiderivative was successfully verified.

`[In] Integrate[Log[2 + 3*x^2], x]``[Out] -2*x + 2*sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]`**Mathics [A]**

time = 1.97, size = 26, normalized size = 0.79

$$-2x + x \operatorname{Log}[2 + 3x^2] + \frac{2\sqrt{6} \operatorname{ArcTan}\left[\frac{\sqrt{6}x}{2}\right]}{3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Log[2+3*x^2], x]')``[Out] -2 x + x Log[2 + 3 x ^ 2] + 2 Sqrt[6] ArcTan[Sqrt[6] x / 2] / 3`**Maple [A]**

time = 0.02, size = 27, normalized size = 0.82

method	result	size
default	$-2x + x \ln(3x^2 + 2) + \frac{2 \arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{3}$	27
risch	$-2x + x \ln(3x^2 + 2) + \frac{2 \arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{3}$	27

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(3*x^2+2),x,method=_RETURNVERBOSE)`

[Out] `-2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)`

Maxima [A]

time = 0.42, size = 26, normalized size = 0.79

$$x \log(3x^2 + 2) + \frac{2}{3} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6} x\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(3*x^2+2),x, algorithm="maxima")`

[Out] `x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x`

Fricas [A]

time = 0.31, size = 32, normalized size = 0.97

$$\frac{2}{3} \sqrt{3} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{3} \sqrt{2} x\right) + x \log(3x^2 + 2) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(3*x^2+2),x, algorithm="fricas")`

[Out] `2/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x) + x*log(3*x^2 + 2) - 2*x`

Sympy [A]

time = 0.07, size = 31, normalized size = 0.94

$$x \log(3x^2 + 2) - 2x + \frac{2\sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6} x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(3*x**2+2),x)`

[Out] `x*log(3*x**2 + 2) - 2*x + 2*sqrt(6)*atan(sqrt(6)*x/2)/3`

Giac [A]

time = 0.00, size = 41, normalized size = 1.24

$$x \ln(3x^2 + 2) - 6 \left(\frac{x}{3} - \frac{4 \arctan\left(\frac{3x}{\sqrt{6}}\right)}{2 \cdot 3\sqrt{6}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(3*x^2+2),x)`

[Out] `x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x`

Mupad [B]

time = 0.06, size = 26, normalized size = 0.79

$$\frac{2\sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{3} - 2x + x \ln(3x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(3*x^2 + 2),x)`

[Out] `(2*6^(1/2)*atan((6^(1/2)*x)/2))/3 - 2*x + x*log(3*x^2 + 2)`

3.244 $\int \cot(x) dx$

Optimal. Leaf size=3

$$\log(\sin(x))$$

[Out] $\ln(\sin(x))$

Rubi [A]

time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556}

$$\log(\sin(x))$$

Antiderivative was successfully verified.

[In] `Int[Cot[x],x]`

[Out] `Log[Sin[x]]`

Rule 3556

`Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\int \cot(x) dx = \log(\sin(x))$$

Mathematica [A]

time = 0.00, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Antiderivative was successfully verified.

[In] `Integrate[Cot[x],x]`

[Out] `Log[Sin[x]]`

Mathics [A]

time = 1.62, size = 3, normalized size = 1.00

$$\text{Log}[\text{Sin}[x]]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[Cot[x],x]')`

[Out] `Log[Sin[x]]`

Maple [A]

time = 0.00, size = 4, normalized size = 1.33

method	result	size
lookup	$\ln(\sin(x))$	4
default	$\ln(\sin(x))$	4
derivativedivides	$-\frac{\ln(\cot^2(x)+1)}{2}$	10
norman	$-\frac{\ln(1+\tan^2(x))}{2} + \ln(\tan(x))$	14
risch	$-ix + \ln(e^{2ix} - 1)$	14

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(x),x,method=_RETURNVERBOSE)`

[Out] `ln(sin(x))`

Maxima [A]

time = 0.27, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x),x, algorithm="maxima")`

[Out] `log(sin(x))`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.

time = 0.32, size = 11, normalized size = 3.67

$$\frac{1}{2} \log\left(-\frac{1}{2} \cos(2x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x),x, algorithm="fricas")`

[Out] `1/2*log(-1/2*cos(2*x) + 1/2)`

Sympy [A]

time = 0.03, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x),x)`

[Out] `log(sin(x))`

Giac [A]

time = 0.00, size = 4, normalized size = 1.33

$$\ln |\sin x|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x),x)`

[Out] `log(abs(sin(x)))`

Mupad [B]

time = 0.02, size = 3, normalized size = 1.00

$$\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(x),x)`

[Out] `log(sin(x))`

3.245 $\int \cot^4(x) dx$

Optimal. Leaf size=12

$$x + \cot(x) - \frac{\cot^3(x)}{3}$$

[Out] x+cot(x)-1/3*cot(x)^3

Rubi [A]

time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 8}

$$x - \frac{1}{3} \cot^3(x) + \cot(x)$$

Antiderivative was successfully verified.

[In] Int[Cot[x]^4,x]

[Out] x + Cot[x] - Cot[x]^3/3

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3554

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[b*((b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int \cot^4(x) dx &= -\frac{1}{3} \cot^3(x) - \int \cot^2(x) dx \\ &= \cot(x) - \frac{\cot^3(x)}{3} + \int 1 dx \\ &= x + \cot(x) - \frac{\cot^3(x)}{3} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 18, normalized size = 1.50

$$x + \frac{4 \cot(x)}{3} - \frac{1}{3} \cot(x) \csc^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]^4,x]

[Out] $x + (4*\text{Cot}[x])/3 - (\text{Cot}[x]*\text{Csc}[x]^2)/3$

Mathics [A]

time = 1.88, size = 12, normalized size = 1.00

$$x + \frac{1}{\text{Tan}[x]} - \frac{1}{3\text{Tan}[x]^3}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Cot[x]^4,x]')

[Out] $x + 1 / \text{Tan}[x] - 1 / (3 \text{Tan}[x] ^ 3)$

Maple [A]

time = 0.00, size = 16, normalized size = 1.33

method	result	size
derivativedivides	$-\frac{(\cot^3(x))}{3} + \cot(x) - \frac{\pi}{2} + \text{arccot}(\cot(x))$	16
default	$-\frac{(\cot^3(x))}{3} + \cot(x) - \frac{\pi}{2} + \text{arccot}(\cot(x))$	16
norman	$\frac{-\frac{1}{3} + \tan^2(x) + x(\tan^3(x))}{\tan(x)^3}$	18
risch	$x + \frac{4i(3e^{4ix} - 3e^{2ix} + 2)}{3(e^{2ix} - 1)^3}$	31

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)^4,x,method=_RETURNVERBOSE)

[Out] $-1/3*\cot(x)^3 + \cot(x) - 1/2*Pi + \text{arccot}(\cot(x))$

Maxima [A]

time = 0.36, size = 16, normalized size = 1.33

$$x + \frac{3 \tan(x)^2 - 1}{3 \tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^4,x, algorithm="maxima")

[Out] $x + 1/3*(3*\tan(x)^2 - 1)/\tan(x)^3$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(10) = 20$.

time = 0.32, size = 48, normalized size = 4.00

$$\frac{4 \cos(2x)^2 + 3(x \cos(2x) - x) \sin(2x) + 2 \cos(2x) - 2}{3(\cos(2x) - 1) \sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^4,x, algorithm="fricas")

[Out] 1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)/((cos(2*x) - 1)*sin(2*x))

Sympy [A]

time = 0.03, size = 19, normalized size = 1.58

$$x + \frac{\cos(x)}{\sin(x)} - \frac{\cos^3(x)}{3 \sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)**4,x)

[Out] x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(10) = 20$.

time = 0.00, size = 52, normalized size = 4.33

$$2 \left(\frac{\frac{256}{3} \tan^3\left(\frac{x}{2}\right) - 1280 \tan\left(\frac{x}{2}\right)}{4096} + \frac{15 \tan^2\left(\frac{x}{2}\right) - 1}{48 \tan^3\left(\frac{x}{2}\right)} + \frac{x}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^4,x)

[Out] 1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 - 5/8*tan(1/2*x)

Mupad [B]

time = 0.03, size = 10, normalized size = 0.83

$$-\frac{\cot(x)^3}{3} + \cot(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)^4,x)

[Out] x + cot(x) - cot(x)^3/3

3.246 $\int \tanh(x) dx$

Optimal. Leaf size=3

$$\log(\cosh(x))$$

[Out] $\ln(\cosh(x))$

Rubi [A]

time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556}

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Tanh}[x], x]$

[Out] $\text{Log}[\text{Cosh}[x]]$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d * x], x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rubi steps

$$\int \tanh(x) dx = \log(\cosh(x))$$

Mathematica [A]

time = 0.00, size = 3, normalized size = 1.00

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Tanh}[x], x]$

[Out] $\text{Log}[\text{Cosh}[x]]$

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 9 vs. 2(3) = 6. time = 1.96, size = 9, normalized size = 3.00

$$x - \text{Log}[1 + \text{Tanh}[x]]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[Tanh[x],x]')`

[Out] `x - Log[1 + Tanh[x]]`

Maple [A]

time = 0.01, size = 4, normalized size = 1.33

method	result	size
lookup	$\ln(\cosh(x))$	4
default	$\ln(\cosh(x))$	4
risch	$-x + \ln(1 + e^{2x})$	12
derivativedivides	$-\frac{\ln(\tanh(x)-1)}{2} - \frac{\ln(\tanh(x)+1)}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x),x,method=_RETURNVERBOSE)`

[Out] `ln(cosh(x))`

Maxima [A]

time = 0.28, size = 3, normalized size = 1.00

$$\log(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x),x, algorithm="maxima")`

[Out] `log(cosh(x))`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(3) = 6$.
time = 0.31, size = 18, normalized size = 6.00

$$-x + \log\left(\frac{2 \cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x),x, algorithm="fricas")`

[Out] `-x + log(2*cosh(x)/(cosh(x) - sinh(x)))`

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$
time = 0.06, size = 7, normalized size = 2.33

$$x - \log(\tanh(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x),x)`

[Out] `x - log(tanh(x) + 1)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(3) = 6$.
time = 0.00, size = 10, normalized size = 3.33

$$-x + \ln(e^{2x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x),x)`

[Out] `-x + log(e^(2*x) + 1)`

Mupad [B]

time = 0.01, size = 3, normalized size = 1.00

$$\ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x),x)`

[Out] `log(cosh(x))`

3.247 $\int \coth(x) dx$

Optimal. Leaf size=3

$$\log(\sinh(x))$$

[Out] $\ln(\sinh(x))$

Rubi [A]

time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556}

$$\log(\sinh(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Coth}[x], x]$

[Out] $\text{Log}[\text{Sinh}[x]]$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d * x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\int \coth(x) dx = \log(\sinh(x))$$

Mathematica [A]

time = 0.00, size = 3, normalized size = 1.00

$$\log(\sinh(x))$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Coth}[x], x]$

[Out] $\text{Log}[\text{Sinh}[x]]$

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 12 vs. $2(3) = 6$. time = 2.05, size = 12, normalized size = 4.00

$$x + \text{Log}[\text{Tanh}[x]] - \text{Log}[1 + \text{Tanh}[x]]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Coth[x],x]')
```

```
[Out] x + Log[Tanh[x]] - Log[1 + Tanh[x]]
```

Maple [A]

time = 0.01, size = 4, normalized size = 1.33

method	result	size
lookup	$\ln(\sinh(x))$	4
default	$\ln(\sinh(x))$	4
risch	$-x + \ln(e^{2x} - 1)$	12
derivativedivides	$-\frac{\ln(\coth(x)-1)}{2} - \frac{\ln(\coth(x)+1)}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(sinh(x))
```

Maxima [A]

time = 0.27, size = 3, normalized size = 1.00

$$\log(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x, algorithm="maxima")
```

```
[Out] log(sinh(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(3) = 6$.
time = 0.32, size = 18, normalized size = 6.00

$$-x + \log\left(\frac{2 \sinh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x, algorithm="fricas")
```

```
[Out] -x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(3) = 6$
time = 0.15, size = 12, normalized size = 4.00

$$x - \log(\tanh(x) + 1) + \log(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x),x)`

[Out] `x - log(tanh(x) + 1) + log(tanh(x))`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(3) = 6$.
time = 0.00, size = 11, normalized size = 3.67

$$-x + \ln |e^{2x} - 1|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x),x)`

[Out] `-x + log(abs(e^(2*x) - 1))`

Mupad [B]

time = 0.14, size = 3, normalized size = 1.00

$$\ln(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x),x)`

[Out] `log(sinh(x))`

3.248 $\int b^x dx$

Optimal. Leaf size=8

$$\frac{b^x}{\log(b)}$$

[Out] $b^x/\ln(b)$

Rubi [A]

time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2225}

$$\frac{b^x}{\log(b)}$$

Antiderivative was successfully verified.

[In] Int[b^x,x]

[Out] b^x/Log[b]

Rule 2225

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\int b^x dx = \frac{b^x}{\log(b)}$$

Mathematica [A]

time = 0.00, size = 8, normalized size = 1.00

$$\frac{b^x}{\log(b)}$$

Antiderivative was successfully verified.

[In] Integrate[b^x,x]

[Out] b^x/Log[b]

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.69, size = 16, normalized size = 2.00

$$\text{Piecewise} \left[\left\{ \left\{ \frac{b^x}{\text{Log}[b]}, \text{Log}[b] \neq 0 \right\} \right\}, x \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[b^x,x]')`

[Out] `Piecewise[{{b ^ x / Log[b], Log[b] != 0}}, x]`

Maple [A]

time = 0.01, size = 9, normalized size = 1.12

method	result	size
gospers	$\frac{b^x}{\ln(b)}$	9
derivativedivides	$\frac{b^x}{\ln(b)}$	9
default	$\frac{b^x}{\ln(b)}$	9
risch	$\frac{b^x}{\ln(b)}$	9
norman	$\frac{e^{x \ln(b)}}{\ln(b)}$	11
meijerg	$-\frac{1-e^{x \ln(b)}}{\ln(b)}$	16

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(b^x,x,method=_RETURNVERBOSE)`

[Out] `b^x/ln(b)`

Maxima [A]

time = 0.27, size = 8, normalized size = 1.00

$$\frac{b^x}{\log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(b^x,x, algorithm="maxima")`

[Out] `b^x/log(b)`

Fricas [A]

time = 0.31, size = 8, normalized size = 1.00

$$\frac{b^x}{\log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(b^x,x, algorithm="fricas")`

[Out] $b^x/\log(b)$

Sympy [A]

time = 0.04, size = 8, normalized size = 1.00

$$\begin{cases} \frac{b^x}{\log(b)} & \text{for } \log(b) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(b**x,x)`

[Out] `Piecewise((b**x/log(b), Ne(log(b), 0)), (x, True))`

Giac [A]

time = 0.00, size = 7, normalized size = 0.88

$$\frac{b^x}{\ln b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(b^x,x)`

[Out] $b^x/\log(b)$

Mupad [B]

time = 0.16, size = 8, normalized size = 1.00

$$\frac{b^x}{\ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(b^x,x)`

[Out] $b^x/\log(b)$

$$3.249 \quad \int \sqrt{2 + \frac{1}{x^4} + x^4} dx$$

Optimal. Leaf size=49

$$-\frac{x\sqrt{2 + \frac{1}{x^4} + x^4}}{1 + x^4} + \frac{x^5\sqrt{2 + \frac{1}{x^4} + x^4}}{3(1 + x^4)}$$

[Out] $-x*(2+1/x^4+x^4)^{(1/2)}/(x^4+1)+1/3*x^5*(2+1/x^4+x^4)^{(1/2)}/(x^4+1)$

Rubi [A]

time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1365, 1369, 14}

$$\frac{x^5\sqrt{x^4 + \frac{1}{x^4} + 2}}{3(x^4 + 1)} - \frac{x\sqrt{x^4 + \frac{1}{x^4} + 2}}{x^4 + 1}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 + x^(-4) + x^4], x]

[Out] $-((x*\text{Sqrt}[2 + x^{(-4)} + x^4])/(1 + x^4)) + (x^5*\text{Sqrt}[2 + x^{(-4)} + x^4])/(3*(1 + x^4))$

Rule 14

Int[(u_)*((c_)*(x_)^(m_)), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1365

Int[((a_) + (c_)*(x_)^(n_) + (b_)*(x_)^(mn))^(p_), x_Symbol] := Dist[x^(n*FracPart[p])*((a + b/x^n + c*x^n)^FracPart[p]/(b + a*x^n + c*x^(2*n))^FracPart[p]), Int[(b + a*x^n + c*x^(2*n))^p/x^(n*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[mn, -n] && !IntegerQ[p] && PosQ[n]

Rule 1369

Int[((d_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
 \int \sqrt{2 + \frac{1}{x^4} + x^4} dx &= \frac{\left(x^2 \sqrt{2 + \frac{1}{x^4} + x^4}\right) \int \frac{\sqrt{1 + 2x^4 + x^8}}{x^2} dx}{\sqrt{1 + 2x^4 + x^8}} \\
 &= \frac{\left(x^2 \sqrt{2 + \frac{1}{x^4} + x^4}\right) \int \frac{1+x^4}{x^2} dx}{1 + x^4} \\
 &= \frac{\left(x^2 \sqrt{2 + \frac{1}{x^4} + x^4}\right) \int \left(\frac{1}{x^2} + x^2\right) dx}{1 + x^4} \\
 &= -\frac{x \sqrt{2 + \frac{1}{x^4} + x^4}}{1 + x^4} + \frac{x^5 \sqrt{2 + \frac{1}{x^4} + x^4}}{3(1 + x^4)}
 \end{aligned}$$

Mathematica [A]

time = 0.01, size = 29, normalized size = 0.59

$$\frac{x(-3 + x^4) \sqrt{2 + \frac{1}{x^4} + x^4}}{3(1 + x^4)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[2 + x^(-4) + x^4], x]

[Out] (x*(-3 + x^4)*Sqrt[2 + x^(-4) + x^4])/(3*(1 + x^4))

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(x^4+x^(-4)+2)^(1/2), x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.01, size = 32, normalized size = 0.65

method	result	size
gospers	$\frac{x(x^4-3)\sqrt{\frac{x^8+2x^4+1}{x^4}}}{3x^4+3}$	32
default	$\frac{x(x^4-3)\sqrt{\frac{x^8+2x^4+1}{x^4}}}{3x^4+3}$	32
risch	$\frac{\sqrt{\frac{(x^4+1)^2}{x^4}}}{3x^4+3} x^5 - \frac{\sqrt{\frac{(x^4+1)^2}{x^4}}}{x^4+1} x$	50

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2+1/x^4+x^4)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `1/3*x*(x^4-3)*((x^8+2*x^4+1)/x^4)^(1/2)/(x^4+1)`

Maxima [A]

time = 0.36, size = 10, normalized size = 0.20

$$\frac{x^4 - 3}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+1/x^4+x^4)^(1/2),x, algorithm="maxima")`

[Out] `1/3*(x^4 - 3)/x`

Fricas [A]

time = 0.31, size = 10, normalized size = 0.20

$$\frac{x^4 - 3}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+1/x^4+x^4)^(1/2),x, algorithm="fricas")`

[Out] `1/3*(x^4 - 3)/x`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4 + 2 + \frac{1}{x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+1/x**4+x**4)**(1/2),x)`

[Out] Integral(sqrt(x**4 + 2 + x**(-4)), x)

Giac [A]

time = 0.00, size = 11, normalized size = 0.22

$$\frac{1}{3}x^3 - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+1/x^4+x^4)^(1/2),x)

[Out] 1/3*x^3 - 1/x

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{1}{x^4} + x^4 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/x^4 + x^4 + 2)^(1/2),x)

[Out] int((1/x^4 + x^4 + 2)^(1/2), x)

3.250 $\int \frac{1+2x}{2+3x} dx$

Optimal. Leaf size=16

$$\frac{2x}{3} - \frac{1}{9} \log(2 + 3x)$$

[Out] 2/3*x-1/9*ln(2+3*x)

Rubi [A]

time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{2x}{3} - \frac{1}{9} \log(3x + 2)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)/(2 + 3*x), x]

[Out] (2*x)/3 - Log[2 + 3*x]/9

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{1+2x}{2+3x} dx &= \int \left(\frac{2}{3} - \frac{1}{3(2+3x)} \right) dx \\ &= \frac{2x}{3} - \frac{1}{9} \log(2+3x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 17, normalized size = 1.06

$$\frac{1}{9}(4 + 6x - \log(2 + 3x))$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x)/(2 + 3*x), x]

[Out] (4 + 6*x - Log[2 + 3*x])/9

Mathics [A]

time = 1.73, size = 12, normalized size = 0.75

$$\frac{2x}{3} - \frac{\text{Log}[2 + 3x]}{9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(2*x+1)/(3*x+2),x]')
```

```
[Out] 2 x / 3 - Log[2 + 3 x] / 9
```

Maple [A]

time = 0.03, size = 13, normalized size = 0.81

method	result	size
default	$\frac{2x}{3} - \frac{\ln(2+3x)}{9}$	13
norman	$\frac{2x}{3} - \frac{\ln(2+3x)}{9}$	13
meijerg	$-\frac{\ln(1+\frac{3x}{2})}{9} + \frac{2x}{3}$	13
risch	$\frac{2x}{3} - \frac{\ln(2+3x)}{9}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(2+3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x-1/9*ln(2+3*x)
```

Maxima [A]

time = 0.36, size = 12, normalized size = 0.75

$$\frac{2}{3}x - \frac{1}{9}\log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(2+3*x),x, algorithm="maxima")
```

```
[Out] 2/3*x - 1/9*log(3*x + 2)
```

Fricas [A]

time = 0.32, size = 12, normalized size = 0.75

$$\frac{2}{3}x - \frac{1}{9}\log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(2+3*x),x, algorithm="fricas")
```

[Out] $\frac{2}{3}x - \frac{1}{9}\log(3x + 2)$

Sympy [A]

time = 0.04, size = 12, normalized size = 0.75

$$\frac{2x}{3} - \frac{\log(3x + 2)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)/(2+3*x),x)`

[Out] $\frac{2x}{3} - \log(3x + 2)/9$

Giac [A]

time = 0.00, size = 17, normalized size = 1.06

$$\frac{2}{3}x - \frac{\ln|3x + 2|}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)/(2+3*x),x)`

[Out] $\frac{2}{3}x - \frac{1}{9}\log(\text{abs}(3x + 2))$

Mupad [B]

time = 0.14, size = 10, normalized size = 0.62

$$\frac{2x}{3} - \frac{\ln\left(x + \frac{2}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x + 1)/(3*x + 2),x)`

[Out] $\frac{(2x)}{3} - \log(x + 2/3)/9$

3.251 $\int x \log \left(x + \sqrt{1 + x^2} \right) dx$

Optimal. Leaf size=40

$$-\frac{1}{4}x\sqrt{1+x^2} + \frac{1}{4}\sinh^{-1}(x) + \frac{1}{2}x^2 \log \left(x + \sqrt{1+x^2} \right)$$

[Out] 1/4*arcsinh(x)+1/2*x^2*ln(x+(x^2+1)^(1/2))-1/4*x*(x^2+1)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2616, 327, 221}

$$-\frac{1}{4}\sqrt{x^2+1}x + \frac{1}{2}x^2 \log \left(\sqrt{x^2+1} + x \right) + \frac{1}{4}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x*Log[x + Sqrt[1 + x^2]],x]

[Out] -1/4*(x*Sqrt[1 + x^2]) + ArcSinh[x]/4 + (x^2*Log[x + Sqrt[1 + x^2]])/2

Rule 221

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 327

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n-1)*(c*x)^(m-n+1)*((a + b*x^n)^(p+1)/(b*(m+n*p+1))), x] - Dist[a*c^n*((m-n+1)/(b*(m+n*p+1))), Int[(c*x)^(m-n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2616

Int[Log[(d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (c_)*(x_)^2]]*((g_)*(x_))^(m_), x_Symbol] := Simp[(g*x)^(m+1)*(Log[d + e*x + f*Sqrt[a + c*x^2]]/(g*(m+1))), x] - Dist[a*c*(f^2/(g*(m+1))), Int[(g*x)^(m+1)/(d*e*(a + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned}
\int x \log(x + \sqrt{1+x^2}) dx &= \frac{1}{2}x^2 \log(x + \sqrt{1+x^2}) - \frac{1}{2} \int \frac{x^2}{\sqrt{1+x^2}} dx \\
&= -\frac{1}{4}x\sqrt{1+x^2} + \frac{1}{2}x^2 \log(x + \sqrt{1+x^2}) + \frac{1}{4} \int \frac{1}{\sqrt{1+x^2}} dx \\
&= -\frac{1}{4}x\sqrt{1+x^2} + \frac{1}{4} \sinh^{-1}(x) + \frac{1}{2}x^2 \log(x + \sqrt{1+x^2})
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 36, normalized size = 0.90

$$\frac{1}{4} \left(-x\sqrt{1+x^2} + \sinh^{-1}(x) + 2x^2 \log(x + \sqrt{1+x^2}) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[x*Log[x + Sqrt[1 + x^2]],x]``[Out] (-(x*Sqrt[1 + x^2]) + ArcSinh[x] + 2*x^2*Log[x + Sqrt[1 + x^2]])/4`**Mathics [F(-1)]**

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[x*Log[x+(x^2+1)^(1/2)],x]')``[Out] Timed out`**Maple [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int x \ln(x + \sqrt{x^2 + 1}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*ln(x+(x^2+1)^(1/2)),x)``[Out] int(x*ln(x+(x^2+1)^(1/2)),x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*log(x+(x^2+1)^(1/2)),x, algorithm="maxima")

[Out] 1/2*x^2*log(x + sqrt(x^2 + 1)) - 1/4*x^2 - integrate(1/2*x^2/(x^3 + (x^2 + 1)^(3/2) + x), x) + 1/4*log(x^2 + 1)

Fricas [A]

time = 0.32, size = 30, normalized size = 0.75

$$\frac{1}{4} (2x^2 + 1) \log(x + \sqrt{x^2 + 1}) - \frac{1}{4} \sqrt{x^2 + 1} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*log(x+(x^2+1)^(1/2)),x, algorithm="fricas")

[Out] 1/4*(2*x^2 + 1)*log(x + sqrt(x^2 + 1)) - 1/4*sqrt(x^2 + 1)*x

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x \log(x + \sqrt{x^2 + 1}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*ln(x+(x**2+1)**(1/2)),x)

[Out] Integral(x*log(x + sqrt(x**2 + 1)), x)

Giac [A]

time = 0.00, size = 51, normalized size = 1.28

$$-\frac{2}{8}x\sqrt{x^2 + 1} - \frac{\ln(\sqrt{x^2 + 1} - x)}{4} + \frac{1}{2}x^2 \ln(x + \sqrt{x^2 + 1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*log(x+(x^2+1)^(1/2)),x)

[Out] 1/2*x^2*log(x + sqrt(x^2 + 1)) - 1/4*sqrt(x^2 + 1)*x - 1/4*log(-x + sqrt(x^2 + 1))

Mupad [B]

time = 0.04, size = 32, normalized size = 0.80

$$x \ln(x + \sqrt{x^2 + 1}) \left(\frac{x}{2} + \frac{1}{4x} \right) - \frac{x \sqrt{x^2 + 1}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*log(x + (x^2 + 1)^(1/2)),x)

[Out] x*log(x + (x^2 + 1)^(1/2))*(x/2 + 1/(4*x)) - (x*(x^2 + 1)^(1/2))/4

3.252 $\int x (1 + e^x \sin(x))^2 dx$

Optimal. Leaf size=128

$$-\frac{3e^{2x}}{32} + \frac{1}{8}e^{2x}x + \frac{x^2}{2} + e^x \cos(x) - e^x x \cos(x) - \frac{1}{32}e^{2x} \cos(2x) + e^x x \sin(x) + \frac{1}{16}e^{2x} \cos(x) \sin(x) - \frac{1}{4}e^{2x} x \cos(x) \sin(x)$$

[Out] -3/32*exp(2*x)+1/8*exp(2*x)*x+1/2*x^2+exp(x)*cos(x)-exp(x)*x*cos(x)-1/32*exp(2*x)*cos(2*x)+exp(x)*x*sin(x)+1/16*exp(2*x)*cos(x)*sin(x)-1/4*exp(2*x)*x*cos(x)*sin(x)-1/16*exp(2*x)*sin(x)^2+1/4*exp(2*x)*x*sin(x)^2+1/32*exp(2*x)*sin(2*x)

Rubi [A]

time = 0.12, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$,

Rules used = {6874, 4517, 4553, 4518, 4519, 2225, 4557, 12}

$$\frac{x^2}{2} + \frac{1}{8}e^{2x}x - \frac{3e^{2x}}{32} + \frac{1}{4}e^{2x}x \sin^2(x) - \frac{1}{16}e^{2x} \sin^2(x) + e^x x \sin(x) + \frac{1}{32}e^{2x} \sin(2x) - e^x x \cos(x) + e^x \cos(x) - \frac{1}{32}e^{2x} \cos(2x) - \frac{1}{4}e^{2x} x \sin(x) \cos(x) + \frac{1}{16}e^{2x} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[x*(1 + E^x*Sin[x])^2,x]

[Out] (-3*E^(2*x))/32 + (E^(2*x)*x)/8 + x^2/2 + E^x*Cos[x] - E^x*x*Cos[x] - (E^(2*x)*Cos[2*x])/32 + E^x*x*Sin[x] + (E^(2*x)*Cos[x]*Sin[x])/16 - (E^(2*x)*x*Cos[x]*Sin[x])/4 - (E^(2*x)*Sin[x]^2)/16 + (E^(2*x)*x*Sin[x]^2)/4 + (E^(2*x)*Sin[2*x])/32

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2225

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 4517

Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] := Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]

Rule 4518

Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] := Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x]

] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]

Rule 4519

Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] := Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]

Rule 4553

Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*(x_)]^(n_.), x_Symbol] := Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]

Rule 4557

Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] := Int[ExpandTrigReduce[F^(c*(a + b*x)), Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 6874

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int x(1 + e^x \sin(x))^2 dx &= \int (x + 2e^x x \sin(x) + e^{2x} x \sin^2(x)) dx \\
&= \frac{x^2}{2} + 2 \int e^x x \sin(x) dx + \int e^{2x} x \sin^2(x) dx \\
&= \frac{1}{8} e^{2x} x + \frac{x^2}{2} - e^x x \cos(x) + e^x x \sin(x) - \frac{1}{4} e^{2x} x \cos(x) \sin(x) + \frac{1}{4} e^{2x} x \sin^2(x) - 2 \int e^{2x} \cos(x) \sin(x) dx \\
&= \frac{1}{8} e^{2x} x + \frac{x^2}{2} - e^x x \cos(x) + e^x x \sin(x) - \frac{1}{4} e^{2x} x \cos(x) \sin(x) + \frac{1}{4} e^{2x} x \sin^2(x) - \frac{1}{8} e^{2x} \cos(2x) \\
&= -\frac{e^{2x}}{16} + \frac{1}{8} e^{2x} x + \frac{x^2}{2} + e^x \cos(x) - e^x x \cos(x) + e^x x \sin(x) + \frac{1}{16} e^{2x} \cos(x) \sin(x) - \frac{1}{8} e^{2x} \cos(2x) \\
&= -\frac{3e^{2x}}{32} + \frac{1}{8} e^{2x} x + \frac{x^2}{2} + e^x \cos(x) - e^x x \cos(x) + e^x x \sin(x) + \frac{1}{16} e^{2x} \cos(x) \sin(x) - \frac{1}{8} e^{2x} \cos(2x) \\
&= -\frac{3e^{2x}}{32} + \frac{1}{8} e^{2x} x + \frac{x^2}{2} + e^x \cos(x) - e^x x \cos(x) - \frac{1}{32} e^{2x} \cos(2x) + e^x x \sin(x) + \frac{1}{16} e^{2x} \cos(x) \sin(x)
\end{aligned}$$

Mathematica [A]

time = 0.12, size = 67, normalized size = 0.52

$$\frac{1}{8} (4x^2 + e^{2x}(-1 + 2x) - 8e^x(-1 + x) \cos(x) - e^{2x} x \cos(2x) + 8e^x x \sin(x) - e^{2x}(-1 + 2x) \cos(x) \sin(x))$$

Antiderivative was successfully verified.

`[In] Integrate[x*(1 + E^x*Sin[x])^2,x]`

```
[Out] (4*x^2 + E^(2*x)*(-1 + 2*x) - 8*E^x*(-1 + x)*Cos[x] - E^(2*x)*x*Cos[2*x] +
8*E^x*x*Sin[x] - E^(2*x)*(-1 + 2*x)*Cos[x]*Sin[x])/8
```

Mathics [A]

time = 3.08, size = 72, normalized size = 0.56

$$-\sqrt{2} x \cos\left[\frac{\pi}{4} + x\right] E^x - \frac{\sqrt{2} x E^{2x} \sin\left[\frac{\pi}{4} + 2x\right]}{8} + \frac{x E^{2x}}{4} + \frac{x^2}{2} - \frac{E^{2x}}{8} + \frac{E^{2x} \sin[2x]}{16} + \cos[x] E^x$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x*(E^x*Sin[x]+1)^2,x]')`

```
[Out] -Sqrt[2] x Cos[Pi / 4 + x] E ^ x - Sqrt[2] x E ^ (2 x) Sin[Pi / 4 + 2 x] /
8 + x E ^ (2 x) / 4 + x ^ 2 / 2 - E ^ (2 x) / 8 + E ^ (2 x) Sin[2 x] / 16 +
Cos[x] E ^ x
```

Maple [A]

time = 0.04, size = 63, normalized size = 0.49

method	result
default	$\frac{x^2}{2} + 2\left(-\frac{x}{2} + \frac{1}{2}\right)e^x \cos(x) + e^x x \sin(x) + \frac{e^{2x}x}{4} - \frac{e^{2x}}{8} - \frac{x e^{2x} \cos(2x)}{8} + \frac{(-\frac{x}{4} + \frac{1}{8})e^{2x} \sin(2x)}{2}$
risch	$\frac{x^2}{2} + \left(-\frac{1}{8} + \frac{x}{4}\right)e^{2x} + \left(-\frac{1}{64} + \frac{i}{64}\right)(-1 + i + 4x)e^{(2+2i)x} + \left(-\frac{1}{4} - \frac{i}{4}\right)(-1 + i + 2x)e^{(1+i)x} + \left(-\frac{1}{4}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(1+exp(x)*sin(x))^2,x,method=_RETURNVERBOSE)`

[Out] `1/2*x^2+2*(-1/2*x+1/2)*exp(x)*cos(x)+exp(x)*x*sin(x)+1/4*exp(x)^2*x-1/8*exp(x)^2-1/8*x*exp(2*x)*cos(2*x)+1/2*(-1/4*x+1/8)*exp(2*x)*sin(2*x)`

Maxima [A]

time = 0.28, size = 58, normalized size = 0.45

$$-\frac{1}{8}x \cos(2x) e^{(2x)} - (x-1) \cos(x) e^x - \frac{1}{16}(2x-1)e^{(2x)} \sin(2x) + x e^x \sin(x) + \frac{1}{2}x^2 + \frac{1}{8}(2x-1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+exp(x)*sin(x))^2,x, algorithm="maxima")`

[Out] `-1/8*x*cos(2*x)*e^(2*x) - (x - 1)*cos(x)*e^x - 1/16*(2*x - 1)*e^(2*x)*sin(2*x) + x*e^x*sin(x) + 1/2*x^2 + 1/8*(2*x - 1)*e^(2*x)`

Fricas [A]

time = 0.31, size = 55, normalized size = 0.43

$$-(x-1) \cos(x) e^x + \frac{1}{2}x^2 - \frac{1}{8}(2x \cos(x)^2 - 3x + 1)e^{(2x)} - \frac{1}{8}((2x-1) \cos(x) e^{(2x)} - 8x e^x) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+exp(x)*sin(x))^2,x, algorithm="fricas")`

[Out] `-(x - 1)*cos(x)*e^x + 1/2*x^2 - 1/8*(2*x*cos(x)^2 - 3*x + 1)*e^(2*x) - 1/8*((2*x - 1)*cos(x)*e^(2*x) - 8*x*e^x)*sin(x)`

Sympy [A]

time = 0.72, size = 109, normalized size = 0.85

$$\frac{x^2}{2} + \frac{3x e^{2x} \sin^2(x)}{8} - \frac{x e^{2x} \sin(x) \cos(x)}{4} + \frac{x e^{2x} \cos^2(x)}{8} + x e^x \sin(x) - x e^x \cos(x) - \frac{e^{2x} \sin^2(x)}{8} + \frac{e^{2x} \sin(x) \cos(x)}{8} - \frac{e^{2x} \cos^2(x)}{8} + e^x \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+exp(x)*sin(x))**2,x)`

[Out] `x**2/2 + 3*x*exp(2*x)*sin(x)**2/8 - x*exp(2*x)*sin(x)*cos(x)/4 + x*exp(2*x)*cos(x)**2/8 + x*exp(x)*sin(x) - x*exp(x)*cos(x) - exp(2*x)*sin(x)**2/8 + exp(2*x)*sin(x)*cos(x)/8 - exp(2*x)*cos(x)**2/8 + exp(x)*cos(x)`

Giac [A]

time = 0.00, size = 71, normalized size = 0.55

$$\frac{1}{2}x^2 + e^x \left(-\frac{1}{2}(2x-2)\cos x + \frac{2}{2}x\sin x \right) + \frac{1}{8}(2x-1)e^{2x} + e^{2x} \left(-\frac{2}{16}x\cos(2x) + \frac{1}{16}(-2x+1)\sin(2x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(1+exp(x)*sin(x))^2,x)**[Out]** -1/16*(2*x*cos(2*x) + (2*x - 1)*sin(2*x))*e^(2*x) - ((x - 1)*cos(x) - x*sin(x))*e^x + 1/8*e^(2*x)*(2*x - 1) + 1/2*x^2**Mupad [B]**

time = 0.28, size = 69, normalized size = 0.54

$$\frac{3xe^{2x}}{8} - \frac{e^{2x}}{8} + e^x \cos(x) + \frac{x^2}{2} - \frac{xe^{2x}\cos(x)^2}{4} + \frac{e^{2x}\cos(x)\sin(x)}{8} - xe^x \cos(x) + xe^x \sin(x) - \frac{xe^{2x}\cos(x)\sin(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(exp(x)*sin(x) + 1)^2,x)**[Out]** (3*x*exp(2*x))/8 - exp(2*x)/8 + exp(x)*cos(x) + x^2/2 - (x*exp(2*x)*cos(x)^2)/4 + (exp(2*x)*cos(x)*sin(x))/8 - x*exp(x)*cos(x) + x*exp(x)*sin(x) - (x*exp(2*x)*cos(x)*sin(x))/4

3.253 $\int e^x x \cos(x) dx$

Optimal. Leaf size=30

$$\frac{1}{2}e^x x \cos(x) - \frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x)$$

[Out] 1/2*exp(x)*x*cos(x)-1/2*exp(x)*sin(x)+1/2*exp(x)*x*sin(x)

Rubi [A]

time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4518, 4554, 4517}

$$-\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

Antiderivative was successfully verified.

[In] Int[E^x*x*Cos[x],x]

[Out] (E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :=
  Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :=
  Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] := Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int e^x x \cos(x) dx &= \frac{1}{2} e^x x \cos(x) + \frac{1}{2} e^x x \sin(x) - \int \left(\frac{1}{2} e^x \cos(x) + \frac{1}{2} e^x \sin(x) \right) dx \\
&= \frac{1}{2} e^x x \cos(x) + \frac{1}{2} e^x x \sin(x) - \frac{1}{2} \int e^x \cos(x) dx - \frac{1}{2} \int e^x \sin(x) dx \\
&= \frac{1}{2} e^x x \cos(x) - \frac{1}{2} e^x \sin(x) + \frac{1}{2} e^x x \sin(x)
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 18, normalized size = 0.60

$$\frac{1}{2} e^x (x \cos(x) + (-1 + x) \sin(x))$$

Antiderivative was successfully verified.

`[In] Integrate[E^x*x*Cos[x], x]``[Out] (E^x*(x*Cos[x] + (-1 + x)*Sin[x]))/2`**Mathics [A]**

time = 1.89, size = 21, normalized size = 0.70

$$\frac{\left(\sqrt{2} x \sin \left[\frac{\text{Pi}}{4} + x \right] - \sin [x] \right) E^x}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x*E^x*Cos[x], x]')``[Out] (Sqrt[2] x Sin[Pi / 4 + x] - Sin[x]) E ^ x / 2`**Maple [A]**

time = 0.03, size = 20, normalized size = 0.67

method	result	size
default	$\frac{e^x x \cos(x)}{2} - \left(-\frac{x}{2} + \frac{1}{2}\right) e^x \sin(x)$	20
risch	$\left(\frac{1}{8} - \frac{i}{8}\right) (-1 + i + 2x) e^{(1+i)x} + \left(\frac{1}{8} + \frac{i}{8}\right) (-1 - i + 2x) e^{(1-i)x}$	36
norman	$\frac{e^x x \tan\left(\frac{x}{2}\right) + \frac{e^x x}{2} - e^x \tan\left(\frac{x}{2}\right) - \frac{e^x x \left(\tan^2\left(\frac{x}{2}\right)\right)}{2}}{1 + \tan^2\left(\frac{x}{2}\right)}$	45

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(exp(x)*x*cos(x), x, method=_RETURNVERBOSE)`

[Out] $1/2*\exp(x)*x*\cos(x)-(-1/2*x+1/2)*\exp(x)*\sin(x)$

Maxima [A]

time = 0.26, size = 17, normalized size = 0.57

$$\frac{1}{2}x \cos(x) e^x + \frac{1}{2}(x-1)e^x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x*cos(x),x, algorithm="maxima")`

[Out] $1/2*x*\cos(x)*e^x + 1/2*(x-1)*e^x*\sin(x)$

Fricas [A]

time = 0.33, size = 17, normalized size = 0.57

$$\frac{1}{2}x \cos(x) e^x + \frac{1}{2}(x-1)e^x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x*cos(x),x, algorithm="fricas")`

[Out] $1/2*x*\cos(x)*e^x + 1/2*(x-1)*e^x*\sin(x)$

Sympy [A]

time = 0.18, size = 27, normalized size = 0.90

$$\frac{x e^x \sin(x)}{2} + \frac{x e^x \cos(x)}{2} - \frac{e^x \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x*cos(x),x)`

[Out] $x*\exp(x)*\sin(x)/2 + x*\exp(x)*\cos(x)/2 - \exp(x)*\sin(x)/2$

Giac [A]

time = 0.00, size = 18, normalized size = 0.60

$$e^x \left(\frac{1}{2}x \cos x + \frac{1}{2}(x-1) \sin x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x*cos(x),x)`

[Out] $1/2*(x*\cos(x) + (x-1)*\sin(x))*e^x$

Mupad [B]

time = 0.17, size = 17, normalized size = 0.57

$$\frac{e^x (x \cos(x) - \sin(x) + x \sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x)*cos(x),x)
```

```
[Out] (exp(x)*(x*cos(x) - sin(x) + x*sin(x)))/2
```

$$3.254 \quad \int \frac{1}{(-3+x)^4} dx$$

Optimal. Leaf size=11

$$\frac{1}{3(3-x)^3}$$

[Out] 1/3/(3-x)^3

Rubi [A]

time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {32}

$$\frac{1}{3(3-x)^3}$$

Antiderivative was successfully verified.

[In] Int[(-3 + x)^(-4), x]

[Out] 1/(3*(3 - x)^3)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{1}{(-3+x)^4} dx = \frac{1}{3(3-x)^3}$$

Mathematica [A]

time = 0.00, size = 9, normalized size = 0.82

$$-\frac{1}{3(-3+x)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + x)^(-4), x]

[Out] -1/3*1/(-3 + x)^3

Mathics [A]

time = 1.77, size = 19, normalized size = 1.73

$$-\frac{1}{-81 + 81x - 27x^2 + 3x^3}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(x-3)^4,x]')`

[Out] $-1 / (-81 + 81 x - 27 x^2 + 3 x^3)$

Maple [A]

time = 0.04, size = 8, normalized size = 0.73

method	result	size
gospers	$-\frac{1}{3(-3+x)^3}$	8
default	$-\frac{1}{3(-3+x)^3}$	8
norman	$-\frac{1}{3(-3+x)^3}$	8
risch	$-\frac{1}{3(-3+x)^3}$	8
meijerg	$\frac{x(\frac{1}{9}x^2-x+3)}{243(1-\frac{x}{3})^3}$	21

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-3+x)^4,x,method=_RETURNVERBOSE)`

[Out] $-1/3/(-3+x)^3$

Maxima [A]

time = 0.27, size = 7, normalized size = 0.64

$$-\frac{1}{3(x-3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3+x)^4,x, algorithm="maxima")`

[Out] $-1/3/(x-3)^3$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$.

time = 0.31, size = 17, normalized size = 1.55

$$-\frac{1}{3(x^3 - 9x^2 + 27x - 27)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3+x)^4,x, algorithm="fricas")`

[Out] $-1/3/(x^3 - 9*x^2 + 27*x - 27)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$
time = 0.05, size = 17, normalized size = 1.55

$$-\frac{1}{3x^3 - 27x^2 + 81x - 81}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3+x)**4,x)`

[Out] `-1/(3*x**3 - 27*x**2 + 81*x - 81)`

Giac [A]

time = 0.00, size = 9, normalized size = 0.82

$$-\frac{1}{3(x-3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3+x)^4,x)`

[Out] `-1/3/(x - 3)^3`

Mupad [B]

time = 0.07, size = 7, normalized size = 0.64

$$-\frac{1}{3(x-3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x - 3)^4,x)`

[Out] `-1/(3*(x - 3)^3)`

3.255 $\int \frac{x}{-1+x^3} dx$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log(1+x+x^2)$$

[Out] 1/3*ln(1-x)-1/6*ln(x^2+x+1)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {298, 31, 648, 632, 210, 642}

$$-\frac{1}{6}\log(x^2+x+1) + \frac{1}{3}\log(1-x) + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x/(-1 + x^3), x]

[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x + x^2]/6

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(n_+1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 298

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := Dist[-(3*Rt[a, 3]*Rt[b, 3])^(n_+1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{x}{-1+x^3} dx &= \frac{1}{3} \int \frac{1}{-1+x} dx - \frac{1}{3} \int \frac{-1+x}{1+x+x^2} dx \\ &= \frac{1}{3} \log(1-x) - \frac{1}{6} \int \frac{1+2x}{1+x+x^2} dx + \frac{1}{2} \int \frac{1}{1+x+x^2} dx \\ &= \frac{1}{3} \log(1-x) - \frac{1}{6} \log(1+x+x^2) - \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1+2x \right) \\ &= \frac{\tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{1}{3} \log(1-x) - \frac{1}{6} \log(1+x+x^2) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 40, normalized size = 1.00

$$\frac{\tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{1}{3} \log(1-x) - \frac{1}{6} \log(1+x+x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x/(-1 + x^3), x]

[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x + x^2]/6

Mathics [A]

time = 1.88, size = 32, normalized size = 0.80

$$-\frac{\text{Log}[1+x+x^2]}{6} + \frac{\sqrt{3} \text{ArcTan} \left[\frac{\sqrt{3}(1+2x)}{3} \right]}{3} + \frac{\text{Log}[-1+x]}{3}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x/(x^3-1),x]')`

[Out] $-\text{Log}[1 + x + x^2] / 6 + \text{Sqrt}[3] \text{ArcTan}[\text{Sqrt}[3] (1 + 2x) / 3] / 3 + \text{Log}[-1 + x] / 3$

Maple [A]

time = 0.04, size = 33, normalized size = 0.82

method	result	size
default	$\frac{\ln(-1+x)}{3} - \frac{\ln(x^2+x+1)}{6} + \frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	33
risch	$-\frac{\ln(4x^2+4x+4)}{6} + \frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3} + \frac{\ln(-1+x)}{3}$	37
meijerg	$\frac{x^2 \left(\ln\left(1 - (x^3)^{\frac{1}{3}}\right) - \frac{\ln\left(1 + (x^3)^{\frac{1}{3}} + (x^3)^{\frac{2}{3}}\right)}{2} + \sqrt{3} \arctan\left(\frac{\sqrt{3} (x^3)^{\frac{1}{3}}}{2 + (x^3)^{\frac{1}{3}}}\right) \right)}{3(x^3)^{\frac{2}{3}}}$	63

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(x^3-1),x,method=_RETURNVERBOSE)`

[Out] $1/3*\ln(-1+x)-1/6*\ln(x^2+x+1)+1/3*\arctan(1/3*(1+2*x)*3^{(1/2)})*3^{(1/2)}$

Maxima [A]

time = 0.34, size = 32, normalized size = 0.80

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) - \frac{1}{6} \log(x^2 + x + 1) + \frac{1}{3} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^3-1),x, algorithm="maxima")`

[Out] $1/3*\text{sqrt}(3)*\arctan(1/3*\text{sqrt}(3)*(2*x + 1)) - 1/6*\log(x^2 + x + 1) + 1/3*\log(x - 1)$

Fricas [A]

time = 0.31, size = 32, normalized size = 0.80

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) - \frac{1}{6} \log(x^2 + x + 1) + \frac{1}{3} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^3-1),x, algorithm="fricas")`

[Out] $\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\log(x^2+x+1) + \frac{1}{3}\log(x-1)$

Sympy [A]

time = 0.06, size = 41, normalized size = 1.02

$$\frac{\log(x-1)}{3} - \frac{\log(x^2+x+1)}{6} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x**3-1),x)`

[Out] $\log(x-1)/3 - \log(x^2+x+1)/6 + \sqrt{3}\operatorname{atan}(2\sqrt{3}x/3 + \sqrt{3}/3)/3$

Giac [A]

time = 0.00, size = 43, normalized size = 1.08

$$-\frac{\ln(x^2+x+1)}{6} + \frac{1}{3}\sqrt{3} \arctan\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + \frac{\ln|x-1|}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^3-1),x)`

[Out] $\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\log(x^2+x+1) + \frac{1}{3}\log(\operatorname{abs}(x-1))$

Mupad [B]

time = 0.11, size = 46, normalized size = 1.15

$$\frac{\ln(x-1)}{3} - \ln\left(x + \frac{1}{2} - \frac{\sqrt{3} \operatorname{li}}{2}\right) \left(\frac{1}{6} + \frac{\sqrt{3} \operatorname{li}}{6}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2}\right) \left(-\frac{1}{6} + \frac{\sqrt{3} \operatorname{li}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(x^3-1),x)`

[Out] $\log(x-1)/3 - \log(x - (3^{1/2}*1i)/2 + 1/2)*((3^{1/2}*1i)/6 + 1/6) + \log(x + (3^{1/2}*1i)/2 + 1/2)*((3^{1/2}*1i)/6 - 1/6)$

3.256

$$\int \frac{x}{-1+x^4} dx$$

Optimal. Leaf size=8

$$-\frac{1}{2} \tanh^{-1}(x^2)$$

[Out] -1/2*arctanh(x^2)

Rubi [A]

time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {281, 213}

$$-\frac{1}{2} \tanh^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Int[x/(-1 + x^4), x]

[Out] -1/2*ArcTanh[x^2]

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 281

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x}{-1+x^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, x^2 \right) \\ &= -\frac{1}{2} \tanh^{-1}(x^2) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(8) = 16. time = 0.00, size = 23, normalized size = 2.88

$$\frac{1}{4} \log(1-x^2) - \frac{1}{4} \log(1+x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x/(-1 + x^4),x]

[Out] Log[1 - x^2]/4 - Log[1 + x^2]/4

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(8) = 16.
time = 1.71, size = 17, normalized size = 2.12

$$-\frac{\text{Log}[1 + x^2]}{4} + \frac{\text{Log}[-1 + x^2]}{4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x/(x^4-1),x]')

[Out] -Log[1 + x ^ 2] / 4 + Log[-1 + x ^ 2] / 4

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(6) = 12.
time = 0.03, size = 22, normalized size = 2.75

method	result	size
meijerg	$-\frac{\text{arctanh}(x^2)}{2}$	7
risch	$-\frac{\ln(x^2+1)}{4} + \frac{\ln(x^2-1)}{4}$	18
default	$\frac{\ln(-1+x)}{4} + \frac{\ln(1+x)}{4} - \frac{\ln(x^2+1)}{4}$	22
norman	$\frac{\ln(-1+x)}{4} + \frac{\ln(1+x)}{4} - \frac{\ln(x^2+1)}{4}$	22

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^4-1),x,method=_RETURNVERBOSE)

[Out] 1/4*ln(-1+x)+1/4*ln(1+x)-1/4*ln(x^2+1)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.
time = 0.25, size = 17, normalized size = 2.12

$$-\frac{1}{4} \log(x^2 + 1) + \frac{1}{4} \log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^4-1),x, algorithm="maxima")

[Out] -1/4*log(x^2 + 1) + 1/4*log(x^2 - 1)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.
time = 0.30, size = 17, normalized size = 2.12

$$-\frac{1}{4} \log(x^2 + 1) + \frac{1}{4} \log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^4-1),x, algorithm="fricas")

[Out] -1/4*log(x^2 + 1) + 1/4*log(x^2 - 1)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$
time = 0.05, size = 15, normalized size = 1.88

$$\frac{\log(x^2 - 1)}{4} - \frac{\log(x^2 + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x**4-1),x)

[Out] log(x**2 - 1)/4 - log(x**2 + 1)/4

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(6) = 12$.
time = 0.00, size = 24, normalized size = 3.00

$$\frac{\frac{\ln|x^2-1|}{2} - \frac{\ln(x^2+1)}{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^4-1),x)

[Out] -1/4*log(x^2 + 1) + 1/4*log(abs(x^2 - 1))

Mupad [B]

time = 0.07, size = 6, normalized size = 0.75

$$-\frac{\operatorname{atanh}(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^4 - 1),x)

[Out] -atanh(x^2)/2

$$3.257 \quad \int \frac{(1+x^3) \log(x)}{2+x^4} dx$$

Optimal. Leaf size=227

$$\frac{1}{8} (2 + i\sqrt[4]{-2}) \log(x) \log\left(1 - \frac{(1+i)x}{2^{3/4}}\right) + \frac{1}{16} (4 + (1-i)2^{3/4}) \log(x) \log\left(1 + \frac{(1+i)x}{2^{3/4}}\right) + \frac{1}{8} (2 + \sqrt[4]{-2}) \log(x) \log\left(1 - \frac{(1-i)x}{2^{3/4}}\right) + \frac{1}{16} (4 + (1+i)2^{3/4}) \log(x) \log\left(1 + \frac{(1-i)x}{2^{3/4}}\right)$$

```
[Out] 1/8*(2+I*(-2)^(1/4))*ln(x)*ln(1-(1/2+1/2*I)*x*2^(1/4))+1/16*(4+(1-I)*2^(3/4))
)*ln(x)*ln(1+(1/2+1/2*I)*x*2^(1/4))+1/8*(2+(-2)^(1/4))*ln(x)*ln(1-1/2*(-1)
)^(3/4)*x*2^(3/4))+1/8*(2-(-2)^(1/4))*ln(x)*ln(1+1/2*(-1)^(3/4)*x*2^(3/4))+1
/16*(4+(1-I)*2^(3/4))*polylog(2,(-1/2-1/2*I)*x*2^(1/4))+1/8*(2+I*(-2)^(1/4)
)*polylog(2,(1/2+1/2*I)*x*2^(1/4))+1/8*(2-(-2)^(1/4))*polylog(2,-1/2*(-1)^(
3/4)*x*2^(3/4))+1/8*(2+(-2)^(1/4))*polylog(2,1/2*(-1)^(3/4)*x*2^(3/4))
```

Rubi [A]

time = 0.12, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2404, 2354, 2438}

$$\frac{1}{16} (4 + (1-i)2^{3/4}) \operatorname{Li}_2\left(\frac{1+i\sqrt[4]{-2}x}{2^{3/4}}\right) + \frac{1}{8} (2 + i\sqrt[4]{-2}) \operatorname{Li}_2\left(\frac{1+i\sqrt[4]{-2}x}{2^{3/4}}\right) + \frac{1}{8} (2 - \sqrt[4]{-2}) \operatorname{Li}_2\left(\frac{-1-i\sqrt[4]{-2}x}{\sqrt{2}}\right) + \frac{1}{8} (2 + \sqrt[4]{-2}) \operatorname{Li}_2\left(\frac{-1-i\sqrt[4]{-2}x}{\sqrt{2}}\right) + \frac{1}{8} (2 + i\sqrt[4]{-2}) \log(x) \log\left(1 - \frac{1+i\sqrt[4]{-2}x}{2^{3/4}}\right) + \frac{1}{16} (4 + (1-i)2^{3/4}) \log(x) \log\left(1 + \frac{1+i\sqrt[4]{-2}x}{2^{3/4}}\right) + \frac{1}{8} (2 + \sqrt[4]{-2}) \log(x) \log\left(1 - \frac{-1-i\sqrt[4]{-2}x}{\sqrt{2}}\right) + \frac{1}{8} (2 - \sqrt[4]{-2}) \log(x) \log\left(\frac{-1-i\sqrt[4]{-2}x}{\sqrt{2}} + 1\right)$$

Antiderivative was successfully verified.

[In] Int[((1 + x^3)*Log[x])/(2 + x^4),x]

```
[Out] ((2 + I*(-2)^(1/4))*Log[x]*Log[1 - ((1 + I)*x)/2^(3/4)]/8 + ((4 + (1 - I)*
2^(3/4))*Log[x]*Log[1 + ((1 + I)*x)/2^(3/4)]/16 + ((2 + (-2)^(1/4))*Log[x]
*Log[1 - ((-1)^(3/4)*x)/2^(1/4)]/8 + ((2 - (-2)^(1/4))*Log[x]*Log[1 + ((-1)
)^(3/4)*x)/2^(1/4)]/8 + ((4 + (1 - I)*2^(3/4))*PolyLog[2, ((-1 - I)*x)/2^(
3/4)]/16 + ((2 + I*(-2)^(1/4))*PolyLog[2, ((1 + I)*x)/2^(3/4)]/8 + ((2 -
(-2)^(1/4))*PolyLog[2, -(((-1)^(3/4)*x)/2^(1/4)]])/8 + ((2 + (-2)^(1/4))*Po
lyLog[2, ((-1)^(3/4)*x)/2^(1/4)]/8
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int \frac{(1+x^3)\log(x)}{2+x^4} dx &= \int \left(\frac{(-2+\sqrt[4]{-2})\log(x)}{8(\sqrt[4]{-2}-x)} + \frac{(-2i+\sqrt[4]{-2})\log(x)}{8(\sqrt[4]{-2}-ix)} + \frac{(2i+\sqrt[4]{-2})\log(x)}{8(\sqrt[4]{-2}+ix)} + \frac{(2+\sqrt[4]{-2})\log(x)}{8(\sqrt[4]{-2}+x)} \right) dx \\ &= \frac{1}{8}(-2+\sqrt[4]{-2}) \int \frac{\log(x)}{\sqrt[4]{-2}-x} dx + \frac{1}{8}(-2i+\sqrt[4]{-2}) \int \frac{\log(x)}{\sqrt[4]{-2}-ix} dx + \frac{1}{8}(2i+\sqrt[4]{-2}) \int \frac{\log(x)}{\sqrt[4]{-2}+ix} dx + \frac{1}{8}(2+\sqrt[4]{-2}) \int \frac{\log(x)}{\sqrt[4]{-2}+x} dx \\ &= \frac{1}{8}(2+i\sqrt[4]{-2})\log(x)\log\left(1-\frac{(1+i)x}{2^{3/4}}\right) + \frac{1}{8}(2-i\sqrt[4]{-2})\log(x)\log\left(1+\frac{(1+i)x}{2^{3/4}}\right) \\ &+ \frac{1}{8}(2+i\sqrt[4]{-2})\log(x)\log\left(1-\frac{(1-i)x}{2^{3/4}}\right) + \frac{1}{8}(2-i\sqrt[4]{-2})\log(x)\log\left(1+\frac{(1-i)x}{2^{3/4}}\right) \end{aligned}$$

Mathematica [A]

time = 0.18, size = 194, normalized size = 0.85

$$\frac{1}{8} \left((2+i\sqrt[4]{-2})\log(x)\log\left(1-\frac{(1+i)x}{2^{3/4}}\right) + (2+\frac{1-i}{\sqrt[4]{-2}})\log(x)\log\left(1+\frac{(1-i)x}{2^{3/4}}\right) - (-2+\sqrt[4]{-2})\log(x)\log\left(1-\frac{(1-i)x}{2^{3/4}}\right) + (2+\sqrt[4]{-2})\log(x)\log\left(1+\frac{(1-i)x}{2^{3/4}}\right) + (2+\frac{1-i}{\sqrt[4]{-2}})\text{Li}_2\left(-\frac{(1+i)x}{2^{3/4}}\right) + (2+\sqrt[4]{-2})\text{Li}_2\left(-\frac{(1-i)x}{2^{3/4}}\right) - (-2+\sqrt[4]{-2})\text{Li}_2\left(\frac{(1-i)x}{2^{3/4}}\right) + (2+i\sqrt[4]{-2})\text{Li}_2\left(\frac{(1+i)x}{2^{3/4}}\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)*Log[x])/(2 + x^4), x]
```

```
[Out] ((2 + I*(-2)^(1/4))*Log[x]*Log[1 - (-1/2)^(1/4)*x] + (2 + (1 - I)/2^(1/4))*Log[x]*Log[1 + (-1/2)^(1/4)*x] - (-2 + (-2)^(1/4))*Log[x]*Log[1 - ((1 - I)*x)/2^(3/4)] + (2 + (-2)^(1/4))*Log[x]*Log[1 + ((1 - I)*x)/2^(3/4)] + (2 + (1 - I)/2^(1/4))*PolyLog[2, ((-1 - I)*x)/2^(3/4)] + (2 + (-2)^(1/4))*PolyLog[2, ((-1 + I)*x)/2^(3/4)] - (-2 + (-2)^(1/4))*PolyLog[2, ((1 - I)*x)/2^(3/4)] + (2 + I*(-2)^(1/4))*PolyLog[2, ((1 + I)*x)/2^(3/4)]/8
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Log[x]*(x^3+1)/(x^4+2), x]')
```

```
[Out] Timed out
```


Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(171) = 342$.
time = 0.04, size = 394, normalized size = 1.74

method	result
default	$\frac{\left(\left(\frac{\frac{3}{2} + i\frac{3}{2}}{2}\right)^3 + 1\right) \left(\ln(x) \ln\left(\frac{\frac{3}{2} + i\frac{3}{2} - x}{\frac{3}{2} + i\frac{3}{2}}\right) + \operatorname{dilog}\left(\frac{\frac{3}{2} + i\frac{3}{2} - x}{\frac{3}{2} + i\frac{3}{2}}\right)\right)}{4\left(\frac{\frac{3}{2} + i\frac{3}{2}}{2}\right)^3} + \frac{\left(\left(\frac{i\frac{3}{2} - \frac{3}{2}}{2}\right)^3 + 1\right) \left(\ln(x) \ln\left(\frac{\frac{3}{2} - i\frac{3}{2} - x}{\frac{3}{2} - i\frac{3}{2}}\right) + \operatorname{dilog}\left(\frac{\frac{3}{2} - i\frac{3}{2} - x}{\frac{3}{2} - i\frac{3}{2}}\right)\right)}{4\left(\frac{\frac{3}{2} - i\frac{3}{2}}{2}\right)^3}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3+1)*ln(x)/(x^4+2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{4} * \left(\frac{(1/2 * 2^{3/4} + 1/2 * I * 2^{3/4})^3 + 1}{(1/2 * 2^{3/4} + 1/2 * I * 2^{3/4})^3} * (\ln(x) * \ln\left(\frac{1/2 * 2^{3/4} + 1/2 * I * 2^{3/4} - x}{1/2 * 2^{3/4} + 1/2 * I * 2^{3/4}}\right) + \operatorname{dilog}\left(\frac{1/2 * 2^{3/4} + 1/2 * I * 2^{3/4} - x}{1/2 * 2^{3/4} + 1/2 * I * 2^{3/4}}\right)) + \frac{(1/2 * I * 2^{3/4} - 1/2 * 2^{3/4})^3 + 1}{(1/2 * I * 2^{3/4} - 1/2 * 2^{3/4})^3} * (\ln(x) * \ln\left(\frac{1/2 * I * 2^{3/4} - 1/2 * 2^{3/4} - x}{1/2 * I * 2^{3/4} - 1/2 * 2^{3/4}}\right) + \operatorname{dilog}\left(\frac{1/2 * I * 2^{3/4} - 1/2 * 2^{3/4} - x}{1/2 * I * 2^{3/4} - 1/2 * 2^{3/4}}\right)) + \frac{(-1/2 * 2^{3/4} - 1/2 * I * 2^{3/4})^3 + 1}{(-1/2 * 2^{3/4} - 1/2 * I * 2^{3/4})^3} * (\ln(x) * \ln\left(\frac{-1/2 * 2^{3/4} - 1/2 * I * 2^{3/4} - x}{-1/2 * 2^{3/4} - 1/2 * I * 2^{3/4}}\right) + \operatorname{dilog}\left(\frac{-1/2 * 2^{3/4} - 1/2 * I * 2^{3/4} - x}{-1/2 * 2^{3/4} - 1/2 * I * 2^{3/4}}\right)) + \frac{(-1/2 * I * 2^{3/4} + 1/2 * 2^{3/4})^3 + 1}{(-1/2 * I * 2^{3/4} + 1/2 * 2^{3/4})^3} * (\ln(x) * \ln\left(\frac{-1/2 * I * 2^{3/4} + 1/2 * 2^{3/4} - x}{-1/2 * I * 2^{3/4} + 1/2 * 2^{3/4}}\right) + \operatorname{dilog}\left(\frac{-1/2 * I * 2^{3/4} + 1/2 * 2^{3/4} - x}{-1/2 * I * 2^{3/4} + 1/2 * 2^{3/4}}\right)) \right)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+1)*log(x)/(x^4+2),x, algorithm="maxima")`

[Out] `integrate((x^3 + 1)*log(x)/(x^4 + 2), x)`

Fricas [F]

time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+1)*log(x)/(x^4+2),x, algorithm="fricas")`

[Out] `integral((x^3 + 1)*log(x)/(x^4 + 2), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)(x^2-x+1)\log(x)}{x^4+2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+1)*ln(x)/(x**4+2),x)**[Out]** Integral((x + 1)*(x**2 - x + 1)*log(x)/(x**4 + 2), x)**Giac [F]** N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)*log(x)/(x^4+2),x)**[Out]** Could not integrate**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln(x)(x^3+1)}{x^4+2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((log(x)*(x^3 + 1))/(x^4 + 2),x)**[Out]** int((log(x)*(x^3 + 1))/(x^4 + 2), x)

3.258 $\int (\log(x) + \log(1 + x) + \log(2 + x)) dx$

Optimal. Leaf size=24

$$-3x + x \log(x) + (1 + x) \log(1 + x) + (2 + x) \log(2 + x)$$

[Out] $-3*x+x*\ln(x)+(1+x)*\ln(1+x)+(2+x)*\ln(2+x)$

Rubi [A]

time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2332, 2436}

$$-3x + x \log(x) + (x + 1) \log(x + 1) + (x + 2) \log(x + 2)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Log}[x] + \text{Log}[1 + x] + \text{Log}[2 + x], x]$

[Out] $-3*x + x*\text{Log}[x] + (1 + x)*\text{Log}[1 + x] + (2 + x)*\text{Log}[2 + x]$

Rule 2332

$\text{Int}[\text{Log}[(c_.)*(x_)^{(n_.)}], x_Symbol] \rightarrow \text{Simp}[x*\text{Log}[c*x^n], x] - \text{Simp}[n*x, x] /; \text{FreeQ}\{c, n, x\}$

Rule 2436

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_))^{(n_.)}]* (b_.)^{(p_.)}], x_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x]$

Rubi steps

$$\begin{aligned} \int (\log(x) + \log(1 + x) + \log(2 + x)) dx &= \int \log(x) dx + \int \log(1 + x) dx + \int \log(2 + x) dx \\ &= -x + x \log(x) + \text{Subst}\left(\int \log(x) dx, x, 1 + x\right) + \text{Subst}\left(\int \log(x) dx, x, 2 + x\right) \\ &= -3x + x \log(x) + (1 + x) \log(1 + x) + (2 + x) \log(2 + x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 30, normalized size = 1.25

$$-3x + x \log(x) + \log(1 + x) + x \log(1 + x) + 2 \log(2 + x) + x \log(2 + x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x] + Log[1 + x] + Log[2 + x], x]
```

```
[Out] -3*x + x*Log[x] + Log[1 + x] + x*Log[1 + x] + 2*Log[2 + x] + x*Log[2 + x]
```

Mathics [A]

time = 2.48, size = 30, normalized size = 1.25

$$-3x + x\log(x) + x\log(1+x) + x\log(2+x) + \log(1+x) + 2\log(2+x)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Log[x]+Log[x+1]+Log[x+2], x]')
```

```
[Out] -3 x + x Log[x] + x Log[1 + x] + x Log[2 + x] + Log[1 + x] + 2 Log[2 + x]
```

Maple [A]

time = 0.01, size = 26, normalized size = 1.08

method	result	size
default	$-3x + x \ln(x) + (1+x) \ln(1+x) - 3 + (2+x) \ln(2+x)$	26
risch	$-3x + x \ln(x) + \ln(1+x)x + \ln(1+x) + \ln(2+x)x + 2 \ln(2+x)$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)+ln(1+x)+ln(2+x), x, method=_RETURNVERBOSE)
```

```
[Out] -3*x+x*ln(x)+(1+x)*ln(1+x)-3+(2+x)*ln(2+x)
```

Maxima [A]

time = 0.26, size = 25, normalized size = 1.04

$$(x+2)\log(x+2) + (x+1)\log(x+1) + x\log(x) - 3x - 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)+log(1+x)+log(2+x), x, algorithm="maxima")
```

```
[Out] (x + 2)*log(x + 2) + (x + 1)*log(x + 1) + x*log(x) - 3*x - 3
```

Fricas [A]

time = 0.30, size = 24, normalized size = 1.00

$$(x+2)\log(x+2) + (x+1)\log(x+1) + x\log(x) - 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)+log(1+x)+log(2+x), x, algorithm="fricas")
```

[Out] $(x + 2) \cdot \log(x + 2) + (x + 1) \cdot \log(x + 1) + x \cdot \log(x) - 3 \cdot x$

Sympy [A]

time = 0.68, size = 37, normalized size = 1.54

$$x \log(x) - 3x + \left(x + \frac{1}{2}\right) \log(x + 1) + (x + 1) \log(x + 2) + \frac{\log(x + 1)}{2} + \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(x)+ln(1+x)+ln(2+x),x)`

[Out] $x \cdot \log(x) - 3 \cdot x + (x + 1/2) \cdot \log(x + 1) + (x + 1) \cdot \log(x + 2) + \log(x + 1)/2 + \log(x + 2)$

Giac [A]

time = 0.00, size = 29, normalized size = 1.21

$$x \ln x - x - 1 - x + (1 + x) \ln(1 + x) - 2 - x + (2 + x) \ln(2 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x)+log(1+x)+log(2+x),x)`

[Out] $(x + 2) \cdot \log(x + 2) + (x + 1) \cdot \log(x + 1) + x \cdot \log(x) - 3 \cdot x - 3$

Mupad [B]

time = 0.44, size = 51, normalized size = 2.12

$$\ln(x + 1) - 3x + 2 \ln(x + 2) + x \ln(x + 1) + x \ln(x) + \frac{\ln(x + 2) (x^3 + 3x^2 + 2x)}{(x + 1)(x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(x + 1) + log(x + 2) + log(x),x)`

[Out] $\log(x + 1) - 3 \cdot x + 2 \cdot \log(x + 2) + x \cdot \log(x + 1) + x \cdot \log(x) + (\log(x + 2) \cdot (2 \cdot x + 3 \cdot x^2 + x^3)) / ((x + 1) \cdot (x + 2))$

3.259 $\int \frac{1}{5+x^3} dx$

Optimal. Leaf size=78

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{5}-2x}{\sqrt{3}\sqrt[3]{5}}\right)}{\sqrt{3}5^{2/3}} + \frac{\log(\sqrt[3]{5}+x)}{35^{2/3}} - \frac{\log(5^{2/3}-\sqrt[3]{5}x+x^2)}{65^{2/3}}$$

[Out] 1/15*ln(5^(1/3)+x)*5^(1/3)-1/30*ln(5^(2/3)-5^(1/3)*x+x^2)*5^(1/3)-1/15*arctan(1/15*(5^(1/3)-2*x)*5^(2/3)*3^(1/2))*5^(1/3)*3^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {206, 31, 648, 631, 210, 642}

$$-\frac{\log(x^2-\sqrt[3]{5}x+5^{2/3})}{65^{2/3}} + \frac{\log(x+\sqrt[3]{5})}{35^{2/3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{5}-2x}{\sqrt{3}\sqrt[3]{5}}\right)}{\sqrt{3}5^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(5 + x^3)^(-1), x]

[Out] -(ArcTan[(5^(1/3) - 2*x)/(Sqrt[3]*5^(1/3))]/(Sqrt[3]*5^(2/3))) + Log[5^(1/3) + x]/(3*5^(2/3)) - Log[5^(2/3) - 5^(1/3)*x + x^2]/(6*5^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(n-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{5+x^3} dx &= \frac{\int \frac{1}{\sqrt[3]{5}+x} dx}{3 \cdot 5^{2/3}} + \frac{\int \frac{2\sqrt[3]{5}-x}{5^{2/3}-\sqrt[3]{5}x+x^2} dx}{3 \cdot 5^{2/3}} \\
 &= \frac{\log(\sqrt[3]{5}+x)}{3 \cdot 5^{2/3}} - \frac{\int \frac{-\sqrt[3]{5}+2x}{5^{2/3}-\sqrt[3]{5}x+x^2} dx}{6 \cdot 5^{2/3}} + \frac{\int \frac{1}{5^{2/3}-\sqrt[3]{5}x+x^2} dx}{2\sqrt[3]{5}} \\
 &= \frac{\log(\sqrt[3]{5}+x)}{3 \cdot 5^{2/3}} - \frac{\log(5^{2/3}-\sqrt[3]{5}x+x^2)}{6 \cdot 5^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1-\frac{2x}{\sqrt[3]{5}}\right)}{5^{2/3}} \\
 &= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{5}-2x}{\sqrt{3}\sqrt[3]{5}}\right)}{\sqrt{3} \cdot 5^{2/3}} + \frac{\log(\sqrt[3]{5}+x)}{3 \cdot 5^{2/3}} - \frac{\log(5^{2/3}-\sqrt[3]{5}x+x^2)}{6 \cdot 5^{2/3}}
 \end{aligned}$$

Mathematica [A]

time = 0.02, size = 71, normalized size = 0.91

$$\frac{2\sqrt{3} \tan^{-1}\left(\frac{-5+2 \cdot 5^{2/3}x}{5\sqrt{3}}\right) + 2 \log(5+5^{2/3}x) - \log(5-5^{2/3}x+\sqrt[3]{5}x^2)}{6 \cdot 5^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x^3)^(-1), x]

[Out] $(2\sqrt{3}\text{ArcTan}[-5 + 2\cdot 5^{2/3}x]/(5\sqrt{3})) + 2\text{Log}[5 + 5^{2/3}x] - \text{Log}[5 - 5^{2/3}x + 5^{1/3}x^2]/(6\cdot 5^{2/3})$

Mathics [A]

time = 2.13, size = 49, normalized size = 0.63

$$\frac{5^{\frac{1}{3}} \left(-\text{Log} \left[5^{\frac{2}{3}} - 5^{\frac{1}{3}}x + x^2 \right] + 2\sqrt{3} \text{ArcTan} \left[\frac{\sqrt{3}(-5+2\cdot 5^{\frac{2}{3}}x)}{15} \right] + 2\text{Log} \left[5^{\frac{1}{3}} + x \right] \right)}{30}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(x^3+5),x]')`

[Out] $5^{1/3}(-\text{Log}[5^{2/3} - 5^{1/3}x + x^2] + 2\sqrt{3}\text{ArcTan}[\sqrt{3}(-5 + 2\cdot 5^{2/3}x)/15] + 2\text{Log}[5^{1/3} + x])/30$

Maple [A]

time = 0.03, size = 54, normalized size = 0.69

method	result	size
risch	$\frac{\sum_{R=\text{RootOf}(_Z^3+5)} \frac{\ln(-R+x)}{-R^2}}{3}$	22
default	$\frac{\ln(5^{\frac{1}{3}}+x)5^{\frac{1}{3}}}{15} - \frac{\ln(5^{\frac{2}{3}}-5^{\frac{1}{3}}x+x^2)5^{\frac{1}{3}}}{30} + \frac{5^{\frac{1}{3}}\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(\frac{2\cdot 5^{\frac{2}{3}}x-1}{3}\right)}{3}\right)}{15}$	54
meijerg	$\frac{5^{\frac{1}{3}} \left(\frac{x \ln\left(1 + \frac{5^{\frac{2}{3}}(x^3)^{\frac{1}{3}}}{5}\right)}{(x^3)^{\frac{1}{3}}} - \frac{x \ln\left(1 - \frac{5^{\frac{2}{3}}(x^3)^{\frac{1}{3}}}{5} + \frac{5^{\frac{1}{3}}(x^3)^{\frac{2}{3}}}{5}\right)}{2(x^3)^{\frac{1}{3}}} + \frac{x\sqrt{3} \arctan\left(\frac{\sqrt{3}5^{\frac{2}{3}}(x^3)^{\frac{1}{3}}}{10-5^{\frac{2}{3}}(x^3)^{\frac{1}{3}}}\right)}{(x^3)^{\frac{1}{3}}} \right)}{15}$	96

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3+5),x,method=_RETURNVERBOSE)`

[Out] $1/15\cdot\ln(5^{1/3}+x)\cdot 5^{1/3}-1/30\cdot\ln(5^{2/3}-5^{1/3}\cdot x+x^2)\cdot 5^{1/3}+1/15\cdot 5^{1/3}\cdot 3^{1/2}\cdot\arctan(1/3\cdot 3^{1/2}\cdot(2/5\cdot 5^{2/3}\cdot x-1))$

Maxima [A]

time = 0.34, size = 57, normalized size = 0.73

$$\frac{1}{15} \cdot 5^{\frac{1}{3}} \sqrt{3} \arctan \left(\frac{1}{15} \cdot 5^{\frac{2}{3}} \sqrt{3} \left(2x - 5^{\frac{1}{3}} \right) \right) - \frac{1}{30} \cdot 5^{\frac{1}{3}} \log \left(x^2 - 5^{\frac{1}{3}}x + 5^{\frac{2}{3}} \right) + \frac{1}{15} \cdot 5^{\frac{1}{3}} \log \left(x + 5^{\frac{1}{3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3+5),x, algorithm="maxima")

[Out] $\frac{1}{15} \cdot 5^{1/3} \cdot \sqrt{3} \cdot \arctan\left(\frac{1}{15} \cdot 5^{2/3} \cdot \sqrt{3} \cdot (2x - 5^{1/3})\right) - \frac{1}{30} \cdot 5^{1/3} \cdot \log(x^2 - 5^{1/3} \cdot x + 5^{2/3}) + \frac{1}{15} \cdot 5^{1/3} \cdot \log(x + 5^{1/3})$

Fricas [A]

time = 0.31, size = 69, normalized size = 0.88

$$\frac{1}{15} \cdot 25^{1/6} \sqrt{3} \arctan\left(\frac{1}{75} \cdot 25^{1/6} (2 \cdot 25^{2/3} \sqrt{3} x - 5 \cdot 25^{1/3} \sqrt{3})\right) - \frac{1}{150} \cdot 25^{2/3} \log(5x^2 - 25^{2/3}x + 5 \cdot 25^{1/3}) + \frac{1}{75} \cdot 25^{2/3} \log(5x + 25^{2/3})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3+5),x, algorithm="fricas")

[Out] $\frac{1}{15} \cdot 25^{1/6} \cdot \sqrt{3} \cdot \arctan\left(\frac{1}{75} \cdot 25^{1/6} \cdot (2 \cdot 25^{2/3} \cdot \sqrt{3} \cdot x - 5 \cdot 25^{1/3})\right) - \frac{1}{150} \cdot 25^{2/3} \cdot \log(5x^2 - 25^{2/3} \cdot x + 5 \cdot 25^{1/3}) + \frac{1}{75} \cdot 25^{2/3} \cdot \log(5x + 25^{2/3})$

Sympy [A]

time = 0.15, size = 73, normalized size = 0.94

$$\frac{\sqrt[3]{5} \log(x + \sqrt[3]{5})}{15} - \frac{\sqrt[3]{5} \log(x^2 - \sqrt[3]{5} x + 5^{2/3})}{30} + \frac{\sqrt{3} \cdot \sqrt[3]{5} \operatorname{atan}\left(\frac{2\sqrt{3} \cdot 5^{2/3} x - \sqrt{3}}{15}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**3+5),x)

[Out] $5^{1/3} \cdot \log(x + 5^{1/3}) / 15 - 5^{1/3} \cdot \log(x^2 - 5^{1/3} \cdot x + 5^{2/3}) / 30 + \sqrt{3} \cdot 5^{1/3} \cdot \operatorname{atan}(2 \cdot \sqrt{3} \cdot 5^{2/3} \cdot x / 15 - \sqrt{3} / 3) / 15$

Giac [A]

time = 0.00, size = 93, normalized size = 1.19

$$-\frac{1}{30} \cdot 5^{1/3} \ln(x^2 - 5^{1/3}x + 5^{1/3} \cdot 5^{1/3}) - \frac{1}{15} \cdot 5^{1/3} \sqrt{3} \arctan\left(-\frac{x - \frac{5^{1/3}}{2}}{\frac{1}{2} \sqrt{3} \cdot 5^{1/3}}\right) + \frac{1}{15} \cdot 5^{1/3} \ln|x + 5^{1/3}|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3+5),x)

[Out] $-\frac{1}{15} \cdot 5^{1/3} \cdot \sqrt{3} \cdot \arctan\left(-\frac{1}{15} \cdot 5^{2/3} \cdot \sqrt{3} \cdot (2x - 5^{1/3})\right) - \frac{1}{30} \cdot 5^{1/3} \cdot \log(x^2 - 5^{1/3} \cdot x + 5^{2/3}) + \frac{1}{15} \cdot 5^{1/3} \cdot \log(\operatorname{abs}(x + 5^{1/3}))$

Mupad [B]

time = 0.27, size = 70, normalized size = 0.90

$$\frac{5^{1/3} \ln(x + 5^{1/3})}{15} + \frac{5^{1/3} \ln\left(x + \frac{5^{1/3}(-1 + \sqrt{3} \operatorname{li})}{2}\right) (-1 + \sqrt{3} \operatorname{li})}{30} - \frac{5^{1/3} \ln\left(x - \frac{5^{1/3}(1 + \sqrt{3} \operatorname{li})}{2}\right) (1 + \sqrt{3} \operatorname{li})}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 + 5),x)
```

```
[Out] (5^(1/3)*log(x + 5^(1/3)))/15 + (5^(1/3)*log(x + (5^(1/3))*(3^(1/2)*1i - 1))  
/2*(3^(1/2)*1i - 1))/30 - (5^(1/3)*log(x - (5^(1/3))*(3^(1/2)*1i + 1))/2)*(  
3^(1/2)*1i + 1))/30
```

$$3.260 \quad \int \frac{1}{\sqrt{1+x^2}} dx$$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

[Out] arcsinh(x)

Rubi [A]

time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {221}

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 + x^2], x]

[Out] ArcSinh[x]

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}} dx = \sinh^{-1}(x)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 12 vs. 2(2) = 4. time = 0.00, size = 12, normalized size = 6.00

$$\tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 + x^2], x]

[Out] ArcTanh[x/Sqrt[1 + x^2]]

Mathics [A]

time = 1.59, size = 2, normalized size = 1.00

$$\text{ArcSinh}[x]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/Sqrt[1+x^2],x]')
```

```
[Out] ArcSinh[x]
```

Maple [A]

time = 0.03, size = 3, normalized size = 1.50

method	result	size
default	$\operatorname{arcsinh}(x)$	3
meijerg	$\operatorname{arcsinh}(x)$	3
trager	$\ln(x + \sqrt{x^2 + 1})$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsinh(x)
```

Maxima [A]

time = 0.39, size = 2, normalized size = 1.00

$$\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(2) = 4$.
time = 0.31, size = 14, normalized size = 7.00

$$-\log(-x + \sqrt{x^2 + 1})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 1))
```

Sympy [A]

time = 0.06, size = 2, normalized size = 1.00

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+1)**(1/2),x)

[Out] asinh(x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(2) = 4$.
time = 0.00, size = 14, normalized size = 7.00

$$-\ln\left(\sqrt{x^2+1}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)^(1/2),x)

[Out] -log(-x + sqrt(x^2 + 1))

Mupad [B]

time = 0.00, size = 2, normalized size = 1.00

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2 + 1)^(1/2),x)

[Out] asinh(x)

3.261 $\int \sqrt{3 + x^2} dx$

Optimal. Leaf size=27

$$\frac{1}{2}x\sqrt{3+x^2} + \frac{3}{2}\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

[Out] 3/2*arcsinh(1/3*x*3^(1/2))+1/2*x*(x^2+3)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {201, 221}

$$\frac{1}{2}\sqrt{x^2+3}x + \frac{3}{2}\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 + x^2],x]

[Out] (x*Sqrt[3 + x^2])/2 + (3*ArcSinh[x/Sqrt[3]])/2

Rule 201

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x*((a + b*x^n)^p/(n*p + 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned}\int \sqrt{3+x^2} dx &= \frac{1}{2}x\sqrt{3+x^2} + \frac{3}{2} \int \frac{1}{\sqrt{3+x^2}} dx \\ &= \frac{1}{2}x\sqrt{3+x^2} + \frac{3}{2} \sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A]

time = 0.02, size = 31, normalized size = 1.15

$$\frac{1}{2}x\sqrt{3+x^2} + \frac{3}{2} \tanh^{-1}\left(\frac{x}{\sqrt{3+x^2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 + x^2],x]

[Out] (x*Sqrt[3 + x^2])/2 + (3*ArcTanh[x/Sqrt[3 + x^2]])/2

Mathics [A]

time = 1.82, size = 20, normalized size = 0.74

$$\frac{x\sqrt{3+x^2}}{2} + \frac{3\text{ArcSinh}\left[\frac{\sqrt{3}x}{3}\right]}{2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Sqrt[x^2+3],x]')

[Out] x Sqrt[3 + x ^ 2] / 2 + 3 ArcSinh[Sqrt[3] x / 3] / 2

Maple [A]

time = 0.05, size = 21, normalized size = 0.78

method	result	size
default	$\frac{3 \operatorname{arcsinh}\left(\frac{x\sqrt{3}}{3}\right)}{2} + \frac{x\sqrt{x^2+3}}{2}$	21
risch	$\frac{3 \operatorname{arcsinh}\left(\frac{x\sqrt{3}}{3}\right)}{2} + \frac{x\sqrt{x^2+3}}{2}$	21
trager	$\frac{x\sqrt{x^2+3}}{2} - \frac{3 \ln\left(x - \sqrt{x^2+3}\right)}{2}$	26
meijerg	$3 \frac{\left(-\frac{{}_2F_1\left(\frac{1}{2}, \frac{x\sqrt{3}}{3}\right)}{3} - 2\sqrt{\pi} \operatorname{arcsinh}\left(\frac{x\sqrt{3}}{3}\right)\right)}{4\sqrt{\pi}}$	37

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+3)^(1/2),x,method=_RETURNVERBOSE)

[Out] 3/2*arcsinh(1/3*x*3^(1/2))+1/2*x*(x^2+3)^(1/2)

Maxima [A]

time = 0.35, size = 20, normalized size = 0.74

$$\frac{1}{2} \sqrt{x^2+3} x + \frac{3}{2} \operatorname{arsinh}\left(\frac{1}{3} \sqrt{3} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3)^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(x^2 + 3)*x + 3/2*arcsinh(1/3*sqrt(3)*x)

Fricas [A]

time = 0.31, size = 25, normalized size = 0.93

$$\frac{1}{2} \sqrt{x^2 + 3} x - \frac{3}{2} \log(-x + \sqrt{x^2 + 3})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3)^(1/2),x, algorithm="fricas")

[Out] 1/2*sqrt(x^2 + 3)*x - 3/2*log(-x + sqrt(x^2 + 3))

Sympy [A]

time = 0.08, size = 24, normalized size = 0.89

$$\frac{x \sqrt{x^2 + 3}}{2} + \frac{3 \operatorname{asinh}\left(\frac{\sqrt{3} x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+3)**(1/2),x)

[Out] x*sqrt(x**2 + 3)/2 + 3*asinh(sqrt(3)*x/3)/2

Giac [A]

time = 0.00, size = 33, normalized size = 1.22

$$\frac{2}{4} x \sqrt{x^2 + 3} - \frac{3}{2} \ln(\sqrt{x^2 + 3} - x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3)^(1/2),x)

[Out] 1/2*sqrt(x^2 + 3)*x - 3/2*log(-x + sqrt(x^2 + 3))

Mupad [B]

time = 0.05, size = 20, normalized size = 0.74

$$\frac{3 \operatorname{asinh}\left(\frac{\sqrt{3} x}{3}\right)}{2} + \frac{x \sqrt{x^2 + 3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 3)^(1/2),x)

[Out] (3*asinh((3^(1/2)*x)/3))/2 + (x*(x^2 + 3)^(1/2))/2

3.262 $\int \frac{x}{(1+x)^2} dx$

Optimal. Leaf size=10

$$\frac{1}{1+x} + \log(1+x)$$

[Out] 1/(1+x)+ln(1+x)

Rubi [A]

time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {45}

$$\frac{1}{x+1} + \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[x/(1 + x)^2,x]

[Out] (1 + x)^(-1) + Log[1 + x]

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int [ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{x}{(1+x)^2} dx &= \int \left(-\frac{1}{(1+x)^2} + \frac{1}{1+x} \right) dx \\ &= \frac{1}{1+x} + \log(1+x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{1+x} + \log(1+x)$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 + x)^2,x]

[Out] $(1 + x)^{-1} + \text{Log}[1 + x]$

Mathics [A]

time = 1.64, size = 16, normalized size = 1.60

$$\frac{1 + \text{Log}1 + x}{1 + x}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x/(x+1)^2,x]')`

[Out] $(1 + \text{Log}1 + x) / (1 + x)$

Maple [A]

time = 0.03, size = 11, normalized size = 1.10

method	result	size
default	$\frac{1}{1+x} + \ln(1 + x)$	11
norman	$\frac{1}{1+x} + \ln(1 + x)$	11
risch	$\frac{1}{1+x} + \ln(1 + x)$	11
meijerg	$-\frac{x}{1+x} + \ln(1 + x)$	14

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(1+x)^2,x,method=_RETURNVERBOSE)`

[Out] $1/(1+x)+\ln(1+x)$

Maxima [A]

time = 0.25, size = 10, normalized size = 1.00

$$\frac{1}{x + 1} + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(1+x)^2,x, algorithm="maxima")`

[Out] $1/(x + 1) + \log(x + 1)$

Fricas [A]

time = 0.30, size = 16, normalized size = 1.60

$$\frac{(x + 1) \log(x + 1) + 1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)^2,x, algorithm="fricas")

[Out] ((x + 1)*log(x + 1) + 1)/(x + 1)

Sympy [A]

time = 0.04, size = 8, normalized size = 0.80

$$\log(x + 1) + \frac{1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)**2,x)

[Out] log(x + 1) + 1/(x + 1)

Giac [A]

time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{x + 1} + \ln|x + 1|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)^2,x)

[Out] 1/(x + 1) + log(abs(x + 1))

Mupad [B]

time = 0.03, size = 10, normalized size = 1.00

$$\ln(x + 1) + \frac{1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x + 1)^2,x)

[Out] log(x + 1) + 1/(x + 1)

3.263 $\int \sin^{-1}(x) dx$

Optimal. Leaf size=16

$$\sqrt{1-x^2} + x \sin^{-1}(x)$$

[Out] x*arcsin(x)+(-x^2+1)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {4715, 267}

$$\sqrt{1-x^2} + x \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[ArcSin[x], x]

[Out] Sqrt[1 - x^2] + x*ArcSin[x]

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 4715

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*ArcSin[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \sin^{-1}(x) dx &= x \sin^{-1}(x) - \int \frac{x}{\sqrt{1-x^2}} dx \\ &= \sqrt{1-x^2} + x \sin^{-1}(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 16, normalized size = 1.00

$$\sqrt{1-x^2} + x \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[x],x]

[Out] Sqrt[1 - x^2] + x*ArcSin[x]

Mathics [A]

time = 1.72, size = 14, normalized size = 0.88

$$x \operatorname{ArcSin}[x] + \sqrt{1 - x^2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[ArcSin[x],x]')

[Out] x ArcSin[x] + Sqrt[1 - x ^ 2]

Maple [A]

time = 0.00, size = 15, normalized size = 0.94

method	result	size
lookup	$\arcsin(x) x + \sqrt{-x^2 + 1}$	15
default	$\arcsin(x) x + \sqrt{-x^2 + 1}$	15

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(x),x,method=_RETURNVERBOSE)

[Out] arcsin(x)*x+(-x^2+1)^(1/2)

Maxima [A]

time = 0.34, size = 14, normalized size = 0.88

$$x \arcsin(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(x),x, algorithm="maxima")

[Out] x*arcsin(x) + sqrt(-x^2 + 1)

Fricas [A]

time = 0.32, size = 14, normalized size = 0.88

$$x \arcsin(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(x),x, algorithm="fricas")

[Out] x*arcsin(x) + sqrt(-x^2 + 1)

Sympy [A]

time = 0.06, size = 12, normalized size = 0.75

$$x \operatorname{asin}(x) + \sqrt{1 - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(asin(x), x)``[Out] x*asin(x) + sqrt(1 - x**2)`**Giac [A]**

time = 0.00, size = 15, normalized size = 0.94

$$x \arcsin x + \sqrt{1 - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(arcsin(x), x)``[Out] x*arcsin(x) + sqrt(-x^2 + 1)`**Mupad [B]**

time = 0.22, size = 14, normalized size = 0.88

$$x \operatorname{asin}(x) + \sqrt{1 - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(asin(x), x)``[Out] x*asin(x) + (1 - x^2)^(1/2)`

3.264 $\int x^2 \sin^{-1}(x) dx$

Optimal. Leaf size=40

$$\frac{\sqrt{1-x^2}}{3} - \frac{1}{9}(1-x^2)^{3/2} + \frac{1}{3}x^3 \sin^{-1}(x)$$

[Out] $-1/9*(-x^2+1)^{(3/2)}+1/3*x^3*\arcsin(x)+1/3*(-x^2+1)^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4723, 272, 45}

$$\frac{1}{3}x^3 \sin^{-1}(x) - \frac{1}{9}(1-x^2)^{3/2} + \frac{\sqrt{1-x^2}}{3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*\text{ArcSin}[x], x]$

[Out] $\text{Sqrt}[1 - x^2]/3 - (1 - x^2)^{(3/2)}/9 + (x^3*\text{ArcSin}[x])/3$

Rule 45

$\text{Int}[(a_.) + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

$\text{Int}[(x_.)^{(m_.)*((a_.) + (b_.)*(x_.)^{(n_.))^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n) - 1}*(a + b*x)^p, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4723

$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.))^{(n_.)*((d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m + 1)*((a + b*\text{ArcSin}[c*x])^n/(d*(m + 1))), x] - \text{Dist}[b*c*(n/(d*(m + 1))), \text{Int}[(d*x)^{(m + 1)*((a + b*\text{ArcSin}[c*x])^{(n - 1)}/\text{Sqrt}[1 - c^2*x^2]), x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int x^2 \sin^{-1}(x) dx &= \frac{1}{3}x^3 \sin^{-1}(x) - \frac{1}{3} \int \frac{x^3}{\sqrt{1-x^2}} dx \\
&= \frac{1}{3}x^3 \sin^{-1}(x) - \frac{1}{6} \text{Subst} \left(\int \frac{x}{\sqrt{1-x}} dx, x, x^2 \right) \\
&= \frac{1}{3}x^3 \sin^{-1}(x) - \frac{1}{6} \text{Subst} \left(\int \left(\frac{1}{\sqrt{1-x}} - \sqrt{1-x} \right) dx, x, x^2 \right) \\
&= \frac{\sqrt{1-x^2}}{3} - \frac{1}{9} (1-x^2)^{3/2} + \frac{1}{3}x^3 \sin^{-1}(x)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 29, normalized size = 0.72

$$\frac{1}{9} \left(\sqrt{1-x^2} (2+x^2) + 3x^3 \sin^{-1}(x) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[x^2*ArcSin[x],x]``[Out] (Sqrt[1 - x^2]*(2 + x^2) + 3*x^3*ArcSin[x])/9`**Mathics** [A]

time = 1.91, size = 33, normalized size = 0.82

$$\frac{x^2 \sqrt{1-x^2}}{9} + \frac{x^3 \text{ArcSin}[x]}{3} + \frac{2\sqrt{1-x^2}}{9}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^2*ArcSin[x],x]')``[Out] x ^ 2 Sqrt[1 - x ^ 2] / 9 + x ^ 3 ArcSin[x] / 3 + 2 Sqrt[1 - x ^ 2] / 9`**Maple** [A]

time = 0.00, size = 34, normalized size = 0.85

method	result	size
default	$\frac{x^3 \arcsin(x)}{3} + \frac{x^2 \sqrt{-x^2 + 1}}{9} + \frac{2\sqrt{-x^2 + 1}}{9}$	34

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2*arcsin(x),x,method=_RETURNVERBOSE)``[Out] 1/3*x^3*arcsin(x)+1/9*x^2*(-x^2+1)^(1/2)+2/9*(-x^2+1)^(1/2)`

Maxima [A]

time = 0.34, size = 33, normalized size = 0.82

$$\frac{1}{3} x^3 \arcsin(x) + \frac{1}{9} \sqrt{-x^2 + 1} x^2 + \frac{2}{9} \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2*arcsin(x),x, algorithm="maxima")``[Out] 1/3*x^3*arcsin(x) + 1/9*sqrt(-x^2 + 1)*x^2 + 2/9*sqrt(-x^2 + 1)`**Fricas [A]**

time = 0.31, size = 24, normalized size = 0.60

$$\frac{1}{3} x^3 \arcsin(x) + \frac{1}{9} (x^2 + 2) \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2*arcsin(x),x, algorithm="fricas")``[Out] 1/3*x^3*arcsin(x) + 1/9*(x^2 + 2)*sqrt(-x^2 + 1)`**Sympy [A]**

time = 0.11, size = 32, normalized size = 0.80

$$\frac{x^3 \arcsin(x)}{3} + \frac{x^2 \sqrt{1-x^2}}{9} + \frac{2\sqrt{1-x^2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x**2*asin(x),x)``[Out] x**3*asin(x)/3 + x**2*sqrt(1 - x**2)/9 + 2*sqrt(1 - x**2)/9`**Giac [A]**

time = 0.00, size = 59, normalized size = 1.48

$$\frac{1}{12} x \arcsin x - \frac{1}{9} \sqrt{-x^2 + 1} (-x^2 + 1) + \frac{\sqrt{-x^2 + 1}}{3} + \frac{1}{4} x \arcsin x - \frac{1}{3} x (-x^2 + 1) \arcsin x$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2*arcsin(x),x)``[Out] 1/3*(x^2 - 1)*x*arcsin(x) + 1/3*x*arcsin(x) - 1/9*(-x^2 + 1)^(3/2) + 1/3*sqrt(-x^2 + 1)`**Mupad [B]**

time = 0.03, size = 24, normalized size = 0.60

$$\frac{x^3 \arcsin(x)}{3} + \frac{\sqrt{1-x^2} (x^2 + 2)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asin(x),x)
```

```
[Out] (x^3*asin(x))/3 + ((1 - x^2)^(1/2)*(x^2 + 2))/9
```

$$3.265 \quad \int \frac{\sec^2(x)}{1 + \sec^2(x) - 3 \tan(x)} dx$$

Optimal. Leaf size=21

$$-\log(\cos(x) - \sin(x)) + \log(2 \cos(x) - \sin(x))$$

[Out] `-ln(cos(x)-sin(x))+ln(2*cos(x)-sin(x))`

Rubi [A]

time = 0.08, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {630, 31}

$$\log(2 \cos(x) - \sin(x)) - \log(\cos(x) - \sin(x))$$

Antiderivative was successfully verified.

[In] `Int[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]`

[Out] `-Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]`

Rule 31

`Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rule 630

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]`

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)}{1 + \sec^2(x) - 3 \tan(x)} dx &= \text{Subst} \left(\int \frac{1}{2 - 3x + x^2} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \frac{1}{-2 + x} dx, x, \tan(x) \right) - \text{Subst} \left(\int \frac{1}{-1 + x} dx, x, \tan(x) \right) \\ &= -\log(1 - \tan(x)) + \log(2 - \tan(x)) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 29, normalized size = 1.38

$$2 \left(-\frac{1}{2} \log(\cos(x) - \sin(x)) + \frac{1}{2} \log(2 \cos(x) - \sin(x)) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]
```

```
[Out] 2*(-1/2*Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]/2)
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sec[x]^2/(1+Sec[x]^2-3*Tan[x]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]

time = 0.10, size = 14, normalized size = 0.67

method	result	size
default	$\ln(\tan(x) - 2) - \ln(\tan(x) - 1)$	14
risch	$-\ln(e^{2ix} - i) + \ln(e^{2ix} + \frac{3}{5} - \frac{4i}{5})$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(1+sec(x)^2-3*tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(tan(x)-2)-ln(tan(x)-1)
```

Maxima [A]

time = 0.26, size = 13, normalized size = 0.62

$$-\log(\tan(x) - 1) + \log(\tan(x) - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="maxima")
```

```
[Out] -log(tan(x) - 1) + log(tan(x) - 2)
```

Fricas [A]

time = 0.32, size = 29, normalized size = 1.38

$$\frac{1}{2} \log\left(\frac{3}{4} \cos(x)^2 - \cos(x) \sin(x) + \frac{1}{4}\right) - \frac{1}{2} \log(-2 \cos(x) \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="fricas")
```

[Out] $1/2*\log(3/4*\cos(x)^2 - \cos(x)*\sin(x) + 1/4) - 1/2*\log(-2*\cos(x)*\sin(x) + 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{-3 \tan(x) + \sec^2(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)**2/(1+sec(x)**2-3*tan(x)),x)`

[Out] `Integral(sec(x)**2/(-3*tan(x) + sec(x)**2 + 1), x)`

Giac [A]

time = 0.00, size = 14, normalized size = 0.67

$$\ln |\tan x - 2| - \ln |\tan x - 1|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x)`

[Out] `-log(abs(tan(x) - 1)) + log(abs(tan(x) - 2))`

Mupad [B]

time = 0.81, size = 9, normalized size = 0.43

$$-2 \operatorname{atanh}(2 \tan(x) - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cos(x)^2*(1/cos(x)^2 - 3*tan(x) + 1)),x)`

[Out] `-2*atanh(2*tan(x) - 3)`

3.266 $\int \cos^2(x) dx$

Optimal. Leaf size=14

$$\frac{x}{2} + \frac{1}{2} \cos(x) \sin(x)$$

[Out] 1/2*x+1/2*cos(x)*sin(x)

Rubi [A]

time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2715, 8}

$$\frac{x}{2} + \frac{1}{2} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2,x]

[Out] x/2 + (Cos[x]*Sin[x])/2

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*SIn[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIn[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos^2(x) dx &= \frac{1}{2} \cos(x) \sin(x) + \frac{\int 1 dx}{2} \\ &= \frac{x}{2} + \frac{1}{2} \cos(x) \sin(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2} + \frac{1}{4} \sin(2x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^2,x]

[Out] x/2 + Sin[2*x]/4

Mathics [A]

time = 1.67, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin[2x]}{4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/Sec[x]^2,x]')

[Out] x / 2 + Sin[2 x] / 4

Maple [A]

time = 0.02, size = 11, normalized size = 0.79

method	result	size
default	$\frac{x}{2} + \frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2} + \frac{\sin(2x)}{4}$	11
norman	$\frac{x(\tan^2(\frac{x}{2})) + \frac{x}{2} - (\tan^3(\frac{x}{2})) + \frac{x(\tan^4(\frac{x}{2}))}{2} + \tan(\frac{x}{2})}{(1+\tan^2(\frac{x}{2}))^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(x)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*x+1/2*cos(x)*sin(x)

Maxima [A]

time = 0.26, size = 10, normalized size = 0.71

$$\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(x)^2,x, algorithm="maxima")

[Out] 1/2*x + 1/4*sin(2*x)

Fricas [A]

time = 0.32, size = 10, normalized size = 0.71

$$\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(x)^2,x, algorithm="fricas")

[Out] 1/2*cos(x)*sin(x) + 1/2*x

Sympy [A]

time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(x) \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(x)**2,x)

[Out] x/2 + sin(x)*cos(x)/2

Giac [A]

time = 0.00, size = 19, normalized size = 1.36

$$2 \left(\frac{\tan x}{4(\tan^2 x + 1)} + \frac{x}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(x)^2,x)

[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)

Mupad [B]

time = 0.00, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2,x)

[Out] x/2 + sin(2*x)/4

$$3.267 \quad \int \frac{-2-3x+5x^2}{(-2+x)x^2} dx$$

Optimal. Leaf size=18

$$-\frac{1}{x} + 3 \log(2-x) + 2 \log(x)$$

[Out] -1/x+3*ln(2-x)+2*ln(x)

Rubi [A]

time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {907}

$$-\frac{1}{x} + 3 \log(2-x) + 2 \log(x)$$

Antiderivative was successfully verified.

[In] Int[(-2 - 3*x + 5*x^2)/((-2 + x)*x^2), x]

[Out] -x^(-1) + 3*Log[2 - x] + 2*Log[x]

Rule 907

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{-2-3x+5x^2}{(-2+x)x^2} dx &= \int \left(\frac{3}{-2+x} + \frac{1}{x^2} + \frac{2}{x} \right) dx \\ &= -\frac{1}{x} + 3 \log(2-x) + 2 \log(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 18, normalized size = 1.00

$$-\frac{1}{x} + 3 \log(2-x) + 2 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-2 - 3*x + 5*x^2)/((-2 + x)*x^2),x]

[Out] -x^(-1) + 3*Log[2 - x] + 2*Log[x]

Mathics [A]

time = 1.73, size = 16, normalized size = 0.89

$$-\frac{1}{x} + 2\text{Log}[x] + 3\text{Log}[-2 + x]$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(5*x^2-3*x-2)/(x^2*(x-2)),x]')

[Out] -1 / x + 2 Log[x] + 3 Log[-2 + x]

Maple [A]

time = 0.04, size = 17, normalized size = 0.94

method	result	size
default	$-\frac{1}{x} + 2 \ln(x) + 3 \ln(-2 + x)$	17
norman	$-\frac{1}{x} + 2 \ln(x) + 3 \ln(-2 + x)$	17
risch	$-\frac{1}{x} + 2 \ln(x) + 3 \ln(-2 + x)$	17
meijerg	$-\frac{1}{x} + 2 \ln(x) - 2 \ln(2) + 2i\pi + 3 \ln\left(1 - \frac{x}{2}\right)$	27

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2-3*x-2)/(-2+x)/x^2,x,method=_RETURNVERBOSE)

[Out] -1/x+2*ln(x)+3*ln(-2+x)

Maxima [A]

time = 0.27, size = 16, normalized size = 0.89

$$-\frac{1}{x} + 3 \log(x - 2) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="maxima")

[Out] -1/x + 3*log(x - 2) + 2*log(x)

Fricas [A]

time = 0.32, size = 18, normalized size = 1.00

$$\frac{3x \log(x - 2) + 2x \log(x) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="fricas")

[Out] (3*x*log(x - 2) + 2*x*log(x) - 1)/x

Sympy [A]

time = 0.06, size = 14, normalized size = 0.78

$$2 \log(x) + 3 \log(x - 2) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2-3*x-2)/(-2+x)/x**2,x)

[Out] 2*log(x) + 3*log(x - 2) - 1/x

Giac [A]

time = 0.00, size = 16, normalized size = 0.89

$$3 \ln|x - 2| + 2 \ln|x| - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2-3*x-2)/(-2+x)/x^2,x)

[Out] -1/x + 3*log(abs(x - 2)) + 2*log(abs(x))

Mupad [B]

time = 0.15, size = 16, normalized size = 0.89

$$3 \ln(x - 2) + 2 \ln(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3*x - 5*x^2 + 2)/(x^2*(x - 2)),x)

[Out] 3*log(x - 2) + 2*log(x) - 1/x

$$3.268 \quad \int \frac{1}{\sqrt{9 + 4x^2}} dx$$

Optimal. Leaf size=10

$$\frac{1}{2} \sinh^{-1} \left(\frac{2x}{3} \right)$$

[Out] 1/2*arcsinh(2/3*x)

Rubi [A]

time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {221}

$$\frac{1}{2} \sinh^{-1} \left(\frac{2x}{3} \right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[9 + 4*x^2],x]

[Out] ArcSinh[(2*x)/3]/2

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\int \frac{1}{\sqrt{9 + 4x^2}} dx = \frac{1}{2} \sinh^{-1} \left(\frac{2x}{3} \right)$$

Mathematica [A]

time = 0.01, size = 20, normalized size = 2.00

$$-\frac{1}{2} \log \left(-2x + \sqrt{9 + 4x^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[9 + 4*x^2],x]

[Out] -1/2*Log[-2*x + Sqrt[9 + 4*x^2]]

Mathics [A]

time = 1.65, size = 6, normalized size = 0.60

$$\frac{\text{ArcSinh}\left[\frac{2x}{3}\right]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(4*x^2+9)^(1/2),x]')``[Out] ArcSinh[2 x / 3] / 2`**Maple [A]**

time = 0.05, size = 7, normalized size = 0.70

method	result	size
default	$\frac{\text{arcsinh}\left(\frac{2x}{3}\right)}{2}$	7
meijerg	$\frac{\text{arcsinh}\left(\frac{2x}{3}\right)}{2}$	7
trager	$\frac{\ln\left(2x + \sqrt{4x^2 + 9}\right)}{2}$	17

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)``[Out] 1/2*arcsinh(2/3*x)`**Maxima [A]**

time = 0.35, size = 6, normalized size = 0.60

$$\frac{1}{2} \text{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="maxima")``[Out] 1/2*arcsinh(2/3*x)`**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.
time = 0.32, size = 16, normalized size = 1.60

$$-\frac{1}{2} \log\left(-2x + \sqrt{4x^2 + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="fricas")`

[Out] $-1/2*\log(-2*x + \text{sqrt}(4*x^2 + 9))$

Sympy [A]

time = 0.07, size = 7, normalized size = 0.70

$$\frac{\text{asinh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(4*x**2+9)**(1/2),x)`

[Out] $\text{asinh}(2*x/3)/2$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.
time = 0.00, size = 21, normalized size = 2.10

$$-\frac{\ln\left(\sqrt{4x^2 + 9} - 2x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(4*x^2+9)^(1/2),x)`

[Out] $-1/2*\log(-2*x + \text{sqrt}(4*x^2 + 9))$

Mupad [B]

time = 0.04, size = 6, normalized size = 0.60

$$\frac{\text{asinh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(4*x^2 + 9)^(1/2),x)`

[Out] $\text{asinh}((2*x)/3)/2$

$$3.269 \quad \int \frac{1}{\sqrt{4+x^2}} dx$$

Optimal. Leaf size=6

$$\sinh^{-1}\left(\frac{x}{2}\right)$$

[Out] arcsinh(1/2*x)

Rubi [A]

time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {221}

$$\sinh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[4 + x^2], x]

[Out] ArcSinh[x/2]

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\int \frac{1}{\sqrt{4+x^2}} dx = \sinh^{-1}\left(\frac{x}{2}\right)$$

Mathematica [A]

time = 0.01, size = 12, normalized size = 2.00

$$\tanh^{-1}\left(\frac{x}{\sqrt{4+x^2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[4 + x^2], x]

[Out] ArcTanh[x/Sqrt[4 + x^2]]

Mathics [A]

time = 1.62, size = 4, normalized size = 0.67

$$\text{ArcSinh}\left[\frac{x}{2}\right]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(x^2+4)^(-1/2),x]')`

[Out] `ArcSinh[x / 2]`

Maple [A]

time = 0.05, size = 5, normalized size = 0.83

method	result	size
default	$\operatorname{arcsinh}\left(\frac{x}{2}\right)$	5
meijerg	$\operatorname{arcsinh}\left(\frac{x}{2}\right)$	5
trager	$\ln\left(x + \sqrt{x^2 + 4}\right)$	11

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2+4)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `arcsinh(1/2*x)`

Maxima [A]

time = 0.37, size = 4, normalized size = 0.67

$$\operatorname{arsinh}\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2+4)^(1/2),x, algorithm="maxima")`

[Out] `arcsinh(1/2*x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(4) = 8$.
time = 0.32, size = 14, normalized size = 2.33

$$-\log\left(-x + \sqrt{x^2 + 4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2+4)^(1/2),x, algorithm="fricas")`

[Out] `-log(-x + sqrt(x^2 + 4))`

Sympy [A]

time = 0.07, size = 3, normalized size = 0.50

$$\operatorname{asinh}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+4)**(1/2),x)

[Out] asinh(x/2)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(4) = 8.
time = 0.00, size = 14, normalized size = 2.33

$$-\ln\left(\sqrt{x^2+4}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+4)^(1/2),x)

[Out] -log(-x + sqrt(x^2 + 4))

Mupad [B]

time = 0.03, size = 4, normalized size = 0.67

$$\operatorname{asinh}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2 + 4)^(1/2),x)

[Out] asinh(x/2)

$$3.270 \quad \int \frac{1}{10-12x+9x^2} dx$$

Optimal. Leaf size=21

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{6}}\right)}{3\sqrt{6}}$$

[Out] -1/18*arctan(1/6*(2-3*x)*6^(1/2))*6^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {632, 210}

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{6}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(10 - 12*x + 9*x^2)^(-1), x]

[Out] -1/3*ArcTan[(2 - 3*x)/Sqrt[6]]/Sqrt[6]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{10-12x+9x^2} dx &= -\left(2\text{Subst}\left(\int \frac{1}{-216-x^2} dx, x, -12+18x\right)\right) \\ &= -\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{6}}\right)}{3\sqrt{6}} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 21, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{-2+3x}{\sqrt{6}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

`[In] Integrate[(10 - 12*x + 9*x^2)^(-1),x]``[Out] ArcTan[(-2 + 3*x)/Sqrt[6]]/(3*Sqrt[6])`**Mathics [A]**

time = 1.72, size = 16, normalized size = 0.76

$$\frac{\sqrt{6} \operatorname{ArcTan}\left[\frac{\sqrt{6}(-2+3x)}{6}\right]}{18}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(9*x^2-12*x+10),x]')``[Out] Sqrt[6] ArcTan[Sqrt[6] (-2 + 3 x) / 6] / 18`**Maple [A]**

time = 0.16, size = 17, normalized size = 0.81

method	result	size
default	$\frac{\sqrt{6} \arctan\left(\frac{(18x-12)\sqrt{6}}{36}\right)}{18}$	17
risch	$\frac{\sqrt{6} \arctan\left(\frac{(-2+3x)\sqrt{6}}{6}\right)}{18}$	17

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(9*x^2-12*x+10),x,method=_RETURNVERBOSE)``[Out] 1/18*6^(1/2)*arctan(1/36*(18*x-12)*6^(1/2))`**Maxima [A]**

time = 0.34, size = 16, normalized size = 0.76

$$\frac{1}{18} \sqrt{6} \arctan\left(\frac{1}{6} \sqrt{6} (3x - 2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(9*x^2-12*x+10),x, algorithm="maxima")

[Out] 1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))

Fricas [A]

time = 0.31, size = 16, normalized size = 0.76

$$\frac{1}{18} \sqrt{6} \arctan \left(\frac{1}{6} \sqrt{6} (3x - 2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(9*x^2-12*x+10),x, algorithm="fricas")

[Out] 1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))

Sympy [A]

time = 0.06, size = 22, normalized size = 1.05

$$\frac{\sqrt{6} \operatorname{atan} \left(\frac{\sqrt{6} x}{2} - \frac{\sqrt{6}}{3} \right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(9*x**2-12*x+10),x)

[Out] sqrt(6)*atan(sqrt(6)*x/2 - sqrt(6)/3)/18

Giac [A]

time = 0.00, size = 23, normalized size = 1.10

$$\frac{2 \arctan \left(\frac{3x-2}{\sqrt{6}} \right)}{6\sqrt{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(9*x^2-12*x+10),x)

[Out] 1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))

Mupad [B]

time = 0.14, size = 16, normalized size = 0.76

$$\frac{\sqrt{6} \operatorname{atan} \left(\frac{\sqrt{6} (3x-2)}{6} \right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(9*x^2 - 12*x + 10),x)

[Out] (6^(1/2)*atan((6^(1/2)*(3*x - 2))/6))/18

$$3.271 \quad \int \frac{1}{x^4 - 2x^5 + 2x^6 - 2x^7 + x^8} dx$$

Optimal. Leaf size=53

$$\frac{1}{2(1-x)} - \frac{1}{3x^3} - \frac{1}{x^2} - \frac{2}{x} - \frac{5}{2} \log(1-x) + 2 \log(x) + \frac{1}{4} \log(1+x^2)$$

[Out] 1/2/(1-x)-1/3/x^3-1/x^2-2/x-5/2*ln(1-x)+2*ln(x)+1/4*ln(x^2+1)

Rubi [A]

time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2083, 266}

$$-\frac{1}{3x^3} - \frac{1}{x^2} + \frac{1}{4} \log(x^2 + 1) + \frac{1}{2(1-x)} - \frac{2}{x} - \frac{5}{2} \log(1-x) + 2 \log(x)$$

Antiderivative was successfully verified.

[In] Int[(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8)^(-1), x]

[Out] 1/(2*(1 - x)) - 1/(3*x^3) - x^(-2) - 2/x - (5*Log[1 - x])/2 + 2*Log[x] + Log[1 + x^2]/4

Rule 266

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 2083

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^4 - 2x^5 + 2x^6 - 2x^7 + x^8} dx &= \int \left(\frac{1}{2(-1+x)^2} - \frac{5}{2(-1+x)} + \frac{1}{x^4} + \frac{2}{x^3} + \frac{2}{x^2} + \frac{2}{x} + \frac{x}{2(1+x^2)} \right) dx \\ &= \frac{1}{2(1-x)} - \frac{1}{3x^3} - \frac{1}{x^2} - \frac{2}{x} - \frac{5}{2} \log(1-x) + 2 \log(x) + \frac{1}{2} \int \frac{x}{1+x^2} dx \\ &= \frac{1}{2(1-x)} - \frac{1}{3x^3} - \frac{1}{x^2} - \frac{2}{x} - \frac{5}{2} \log(1-x) + 2 \log(x) + \frac{1}{4} \log(1+x^2) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 51, normalized size = 0.96

$$-\frac{1}{2(-1+x)} - \frac{1}{3x^3} - \frac{1}{x^2} - \frac{2}{x} - \frac{5}{2} \log(1-x) + 2 \log(x) + \frac{1}{4} \log(1+x^2)$$

Antiderivative was successfully verified.

[In] Integrate[(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8)^(-1),x]

[Out] -1/2*1/(-1 + x) - 1/(3*x^3) - x^(-2) - 2/x - (5*Log[1 - x])/2 + 2*Log[x] + Log[1 + x^2]/4

Mathics [A]

time = 2.02, size = 50, normalized size = 0.94

$$\frac{4 + 8x + 12x^2 - 30x^3 + 3x^3(-1 + x)(\text{Log}[1 + x^2] - 10\text{Log}[-1 + x] + 8\text{Log}[x])}{12x^3(-1 + x)}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x]')

[Out] (4 + 8 x + 12 x ^ 2 - 30 x ^ 3 + 3 x ^ 3 (-1 + x) (Log[1 + x ^ 2] - 10 Log[-1 + x] + 8 Log[x])) / (12 x ^ 3 (-1 + x))

Maple [A]

time = 0.02, size = 42, normalized size = 0.79

method	result	size
default	$-\frac{1}{3x^3} - \frac{1}{x^2} - \frac{2}{x} + 2 \ln(x) - \frac{1}{2(-1+x)} - \frac{5 \ln(-1+x)}{2} + \frac{\ln(x^2+1)}{4}$	42
norman	$\frac{\frac{1}{3}+x^2-\frac{5}{2}x^3+\frac{2}{3}x}{x^3(-1+x)} + 2 \ln(x) - \frac{5 \ln(-1+x)}{2} + \frac{\ln(x^2+1)}{4}$	42
risch	$\frac{\frac{1}{3}+x^2-\frac{5}{2}x^3+\frac{2}{3}x}{x^3(-1+x)} + 2 \ln(x) - \frac{5 \ln(-1+x)}{2} + \frac{\ln(x^2+1)}{4}$	42

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x,method=_RETURNVERBOSE)

[Out] -1/3/x^3-1/x^2-2/x+2*ln(x)-1/2/(-1+x)-5/2*ln(-1+x)+1/4*ln(x^2+1)

Maxima [A]

time = 0.34, size = 47, normalized size = 0.89

$$-\frac{15x^3 - 6x^2 - 4x - 2}{6(x^4 - x^3)} + \frac{1}{4} \log(x^2 + 1) - \frac{5}{2} \log(x - 1) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x, algorithm="maxima")

[Out] -1/6*(15*x^3 - 6*x^2 - 4*x - 2)/(x^4 - x^3) + 1/4*log(x^2 + 1) - 5/2*log(x - 1) + 2*log(x)

Fricas [A]

time = 0.31, size = 73, normalized size = 1.38

$$\frac{30x^3 - 12x^2 - 3(x^4 - x^3)\log(x^2 + 1) + 30(x^4 - x^3)\log(x - 1) - 24(x^4 - x^3)\log(x) - 8x - 4}{12(x^4 - x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x, algorithm="fricas")`

```
[Out] -1/12*(30*x^3 - 12*x^2 - 3*(x^4 - x^3)*log(x^2 + 1) + 30*(x^4 - x^3)*log(x - 1) - 24*(x^4 - x^3)*log(x) - 8*x - 4)/(x^4 - x^3)
```

Sympy [A]

time = 0.09, size = 46, normalized size = 0.87

$$2\log(x) - \frac{5\log(x-1)}{2} + \frac{\log(x^2+1)}{4} + \frac{-15x^3+6x^2+4x+2}{6x^4-6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(x**8-2*x**7+2*x**6-2*x**5+x**4),x)`

```
[Out] 2*log(x) - 5*log(x - 1)/2 + log(x**2 + 1)/4 + (-15*x**3 + 6*x**2 + 4*x + 2)/(6*x**4 - 6*x**3)
```

Giac [A]

time = 0.00, size = 52, normalized size = 0.98

$$\frac{\ln(x^2+1)}{4} - \frac{5}{2}\ln|x-1| + 2\ln|x| + \frac{\frac{1}{6}(-15x^3+6x^2+4x+2)}{x^3(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x)`

```
[Out] -1/6*(15*x^3 - 6*x^2 - 4*x - 2)/((x - 1)*x^3) + 1/4*log(x^2 + 1) - 5/2*log(abs(x - 1)) + 2*log(abs(x))
```

Mupad [B]

time = 0.06, size = 45, normalized size = 0.85

$$\frac{\ln(x^2+1)}{4} - \frac{5\ln(x-1)}{2} + 2\ln(x) - \frac{-\frac{5x^3}{2} + x^2 + \frac{2x}{3} + \frac{1}{3}}{x^3 - x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8),x)`

```
[Out] log(x^2 + 1)/4 - (5*log(x - 1))/2 + 2*log(x) - ((2*x)/3 + x^2 - (5*x^3)/2 + 1/3)/(x^3 - x^4)
```

$$3.272 \quad \int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)} dx$$

Optimal. Leaf size=49

$$ax + \frac{1}{12}(27a + 9b + 3c + d) \log(3 - x) - \frac{1}{3}d \log(x) - \frac{1}{4}(a - b + c - d) \log(1 + x)$$

[Out] a*x+1/12*(27*a+9*b+3*c+d)*ln(3-x)-1/3*d*ln(x)-1/4*(a-b+c-d)*ln(1+x)

Rubi [A]

time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {1626}

$$\frac{1}{12} \log(3 - x)(27a + 9b + 3c + d) - \frac{1}{4} \log(x + 1)(a - b + c - d) + ax - \frac{1}{3}d \log(x)$$

Antiderivative was successfully verified.

[In] Int[(d + c*x + b*x^2 + a*x^3)/((-3 + x)*x*(1 + x)),x]

[Out] a*x + ((27*a + 9*b + 3*c + d)*Log[3 - x])/12 - (d*Log[x])/3 - ((a - b + c - d)*Log[1 + x])/4

Rule 1626

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[m, n]

Rubi steps

$$\begin{aligned} \int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)} dx &= \int \left(a + \frac{27a+9b+3c+d}{12(-3+x)} - \frac{d}{3x} + \frac{-a+b-c+d}{4(1+x)} \right) dx \\ &= ax + \frac{1}{12}(27a + 9b + 3c + d) \log(3 - x) - \frac{1}{3}d \log(x) - \frac{1}{4}(a - b + c - d) \log(1 + x) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 49, normalized size = 1.00

$$ax + \frac{1}{12}(27a + 9b + 3c + d) \log(3 - x) - \frac{1}{3}d \log(x) + \frac{1}{4}(-a + b - c + d) \log(1 + x)$$

Antiderivative was successfully verified.

[In] Integrate[(d + c*x + b*x^2 + a*x^3)/((-3 + x)*x*(1 + x)),x]

[Out] a*x + ((27*a + 9*b + 3*c + d)*Log[3 - x])/12 - (d*Log[x])/3 + ((-a + b - c + d)*Log[1 + x])/4

Mathics [A]

time = 65.47, size = 41, normalized size = 0.84

$$ax - \frac{d \operatorname{Log}[x]}{3} - \frac{\operatorname{Log}[1+x](a-b+c-d)}{4} + \frac{\operatorname{Log}[-3+x](27a+9b+3c+d)}{12}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(a*x^3+b*x^2+c*x+d)/((x+1)*x*(x-3)),x]')

[Out] a x - d Log[x] / 3 - Log[1 + x] (a - b + c - d) / 4 + Log[-3 + x] (27 a + 9 b + 3 c + d) / 12

Maple [A]

time = 0.04, size = 46, normalized size = 0.94

method	result
default	$ax + \left(\frac{9a}{4} + \frac{3b}{4} + \frac{c}{4} + \frac{d}{12}\right) \ln(-3+x) - \frac{d \ln(x)}{3} + \left(-\frac{a}{4} + \frac{b}{4} - \frac{c}{4} + \frac{d}{4}\right) \ln(1+x)$
norman	$ax + \left(\frac{9a}{4} + \frac{3b}{4} + \frac{c}{4} + \frac{d}{12}\right) \ln(-3+x) - \frac{d \ln(x)}{3} + \left(-\frac{a}{4} + \frac{b}{4} - \frac{c}{4} + \frac{d}{4}\right) \ln(1+x)$
risch	$ax + \frac{9 \ln(3-x)a}{4} + \frac{3 \ln(3-x)b}{4} + \frac{\ln(3-x)c}{4} + \frac{\ln(3-x)d}{12} - \frac{d \ln(x)}{3} - \frac{\ln(-1-x)a}{4} + \frac{\ln(-1-x)b}{4} - \frac{\ln(-1-x)c}{4} + \frac{\ln(-1-x)d}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x,method=_RETURNVERBOSE)

[Out] a*x+(9/4*a+3/4*b+1/4*c+1/12*d)*ln(-3+x)-1/3*d*ln(x)+(-1/4*a+1/4*b-1/4*c+1/4*d)*ln(1+x)

Maxima [A]

time = 0.26, size = 41, normalized size = 0.84

$$ax - \frac{1}{4}(a-b+c-d) \log(x+1) + \frac{1}{12}(27a+9b+3c+d) \log(x-3) - \frac{1}{3}d \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x, algorithm="maxima")

[Out] a*x - 1/4*(a - b + c - d)*log(x + 1) + 1/12*(27*a + 9*b + 3*c + d)*log(x - 3) - 1/3*d*log(x)

Fricas [A]

time = 0.33, size = 41, normalized size = 0.84

$$ax - \frac{1}{4}(a-b+c-d) \log(x+1) + \frac{1}{12}(27a+9b+3c+d) \log(x-3) - \frac{1}{3}d \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x, algorithm="fricas")
```

```
[Out] a*x - 1/4*(a - b + c - d)*log(x + 1) + 1/12*(27*a + 9*b + 3*c + d)*log(x - 3) - 1/3*d*log(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(49) = 98

time = 55.71, size = 762, normalized size = 15.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b*x**2+c*x+d)/(-3+x)/x/(1+x),x)
```

```
[Out] a*x - d*log(x)/3 - (a - b + c - d)*log(x + (-1512*a**2*d + 1134*a**2*(a - b + c - d) - 864*a*b*d + 648*a*b*(a - b + c - d) - 432*a*c*d + 324*a*c*(a - b + c - d) - 144*a*d**2 + 81*a*(a - b + c - d)**2 - 216*b**2*d + 162*b**2*(a - b + c - d) - 288*b*d**2 + 108*b*d*(a - b + c - d) + 81*b*(a - b + c - d)**2 - 72*c**2*d + 54*c**2*(a - b + c - d) + 144*c*d**2 - 72*c*d*(a - b + c - d) - 27*c*(a - b + c - d)**2 - 136*d**3 - 54*d**2*(a - b + c - d) + 117*d*(a - b + c - d)**2)/(1215*a**3 - 567*a**2*b + 1593*a**2*c - 2691*a**2*d - 567*a*b**2 + 378*a*b*c - 1638*a*b*d + 405*a*c**2 - 702*a*c*d - 351*a*d**2 - 81*b**3 - 27*b**2*c - 207*b**2*d + 81*b*c**2 - 270*b*c*d - 27*b*d**2 + 27*c**3 - 27*c**2*d - 99*c*d**2 + 35*d**3))/4 + (27*a + 9*b + 3*c + d)*log(x + (-1512*a**2*d - 378*a**2*(27*a + 9*b + 3*c + d) - 864*a*b*d - 216*a*b*(27*a + 9*b + 3*c + d) - 432*a*c*d - 108*a*c*(27*a + 9*b + 3*c + d) - 144*a*d**2 + 9*a*(27*a + 9*b + 3*c + d)**2 - 216*b**2*d - 54*b**2*(27*a + 9*b + 3*c + d) - 288*b*d**2 - 36*b*d*(27*a + 9*b + 3*c + d) + 9*b*(27*a + 9*b + 3*c + d)**2 - 72*c**2*d - 18*c**2*(27*a + 9*b + 3*c + d) + 144*c*d**2 + 24*c*d*(27*a + 9*b + 3*c + d) - 3*c*(27*a + 9*b + 3*c + d)**2 - 136*d**3 + 18*d**2*(27*a + 9*b + 3*c + d) + 13*d*(27*a + 9*b + 3*c + d)**2)/(1215*a**3 - 567*a**2*b + 1593*a**2*c - 2691*a**2*d - 567*a*b**2 + 378*a*b*c - 1638*a*b*d + 405*a*c**2 - 702*a*c*d - 351*a*d**2 - 81*b**3 - 27*b**2*c - 207*b**2*d + 81*b*c**2 - 270*b*c*d - 27*b*d**2 + 27*c**3 - 27*c**2*d - 99*c*d**2 + 35*d**3))/12
```

Giac [A]

time = 0.00, size = 46, normalized size = 0.94

$$-\frac{1}{3}d \ln|x| + \frac{1}{12}(27a + 9b + 3c + d) \ln|x - 3| + \frac{1}{4}(-a + b - c + d) \ln|x + 1| + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x)
```

[Out] $a*x - 1/4*(a - b + c - d)*\log(\text{abs}(x + 1)) + 1/12*(27*a + 9*b + 3*c + d)*\log(\text{abs}(x - 3)) - 1/3*d*\log(\text{abs}(x))$

Mupad [B]

time = 0.21, size = 46, normalized size = 0.94

$$\ln(x - 3) \left(\frac{9a}{4} + \frac{3b}{4} + \frac{c}{4} + \frac{d}{12} \right) - \ln(x + 1) \left(\frac{a}{4} - \frac{b}{4} + \frac{c}{4} - \frac{d}{4} \right) + ax - \frac{d \ln(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d + c*x + a*x^3 + b*x^2)/(x*(x + 1)*(x - 3)), x)$

[Out] $\log(x - 3)*((9*a)/4 + (3*b)/4 + c/4 + d/12) - \log(x + 1)*(a/4 - b/4 + c/4 - d/4) + a*x - (d*\log(x))/3$

$$3.273 \quad \int \frac{1}{(2 - \log(1 + x^2))^5} dx$$

Optimal. Leaf size=15

$$\text{Int} \left(\frac{1}{(2 - \log(1 + x^2))^5}, x \right)$$

[Out] Unintegrable(1/(2-ln(x^2+1))^5,x)

Rubi [A]

time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(2 - \log(1 + x^2))^5} dx$$

Verification is not applicable to the result.

[In] Int[(2 - Log[1 + x^2])^(-5), x]

[Out] Defer[Int] [(2 - Log[1 + x^2])^(-5), x]

Rubi steps

$$\int \frac{1}{(2 - \log(1 + x^2))^5} dx = \int \frac{1}{(2 - \log(1 + x^2))^5} dx$$

Mathematica [A]

time = 2.64, size = 0, normalized size = 0.00

$$\int \frac{1}{(2 - \log(1 + x^2))^5} dx$$

Verification is not applicable to the result.

[In] Integrate[(2 - Log[1 + x^2])^(-5), x]

[Out] Integrate[(2 - Log[1 + x^2])^(-5), x]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(2-Log[x^2+1])^5,x]')

[Out] caught exception: maximum recursion depth exceeded

Maple [A]

time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(2 - \ln(x^2 + 1))^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2-ln(x^2+1))^5,x)

[Out] int(1/(2-ln(x^2+1))^5,x)

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2-log(x^2+1))^5,x, algorithm="maxima")

[Out] 1/384*(32*x^8 + 56*x^6 + 120*x^4 + (x^8 - 10*x^4 - 24*x^2 - 15)*log(x^2 + 1))^3 - 2*(2*x^8 - x^6 - 33*x^4 - 75*x^2 - 45)*log(x^2 + 1)^2 + 216*x^2 + 4*(3*x^8 - 2*x^6 - 38*x^4 - 78*x^2 - 45)*log(x^2 + 1) + 120)/(x^7*log(x^2 + 1)^4 - 8*x^7*log(x^2 + 1)^3 + 24*x^7*log(x^2 + 1)^2 - 32*x^7*log(x^2 + 1) + 16*x^7) - integrate(1/384*(x^8 + 30*x^4 + 120*x^2 + 105)/(x^8*log(x^2 + 1) - 2*x^8), x)

Fricas [A]

time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2-log(x^2+1))^5,x, algorithm="fricas")

[Out] integral(-1/(log(x^2 + 1)^5 - 10*log(x^2 + 1)^4 + 40*log(x^2 + 1)^3 - 80*log(x^2 + 1)^2 + 80*log(x^2 + 1) - 32), x)

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{120x^2}{x^8 \log(x^2+1)-2x^8} dx + \int \frac{30x^4}{x^8 \log(x^2+1)-2x^8} dx + \int \frac{x^6}{x^8 \log(x^2+1)-2x^8} dx + \int \frac{105}{x^8 \log(x^2+1)-2x^8} dx + \frac{2x^6}{3} + \frac{7x^4}{6} + \frac{5x^2}{2} + \frac{9x^2}{2} + \left(\frac{x^6}{38} - \frac{5x^4}{24} - \frac{x^2}{16}\right) \log(x^2+1)^3 + \left(-\frac{x^6}{12} + \frac{x^4}{24} + \frac{11x^2}{8} + \frac{23x^2}{8} + \frac{15}{8}\right) \log(x^2+1)^2 + \left(\frac{x^6}{4} - \frac{x^4}{6} - \frac{19x^2}{6} - \frac{13x^2}{2} - \frac{15}{4}\right) \log(x^2+1) + \frac{5}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2-ln(x**2+1))**5,x)

[Out] -(Integral(120*x**2/(x**8*log(x**2 + 1) - 2*x**8), x) + Integral(30*x**4/(x**8*log(x**2 + 1) - 2*x**8), x) + Integral(x**8/(x**8*log(x**2 + 1) - 2*x**8), x) + Integral(105/(x**8*log(x**2 + 1) - 2*x**8), x))/384 + (2*x**8/3 + 7*x**6/6 + 5*x**4/2 + 9*x**2/2 + (x**8/48 - 5*x**4/24 - x**2/2 - 5/16)*log(x**2 + 1)**3 + (-x**8/12 + x**6/24 + 11*x**4/8 + 25*x**2/8 + 15/8)*log(x**2 + 1)**2 + (x**8/4 - x**6/6 - 19*x**4/6 - 13*x**2/2 - 15/4)*log(x**2 + 1) + 5/2)/(8*x**7*log(x**2 + 1)**4 - 64*x**7*log(x**2 + 1)**3 + 192*x**7*log(x**2 + 1)**2 - 256*x**7*log(x**2 + 1) + 128*x**7)

Giac [A] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2-log(x^2+1))^5,x)

[Out] Could not integrate

Mupad [A]

time = 0.00, size = -1, normalized size = -0.07

$$\int -\frac{1}{(\ln(x^2 + 1) - 2)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(log(x^2 + 1) - 2)^5,x)

[Out] int(-1/(log(x^2 + 1) - 2)^5, x)

$$3.274 \quad \int \left(\frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2 + \log(x)}{(x + \log^2(x))^2} + \frac{1 + \frac{1}{x} + \frac{2\log(x)}{x}}{x + \log^2(x)} \right) dx$$

Optimal. Leaf size=28

$$e^{x^2} \log(x) - \frac{\log(x)}{x + \log^2(x)} + \log(x + \log^2(x))$$

[Out] exp(x^2)*ln(x)-ln(x)/(x+ln(x)^2)+ln(x+ln(x)^2)

Rubi [A]

time = 0.13, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2241, 2240, 2634, 12, 2627, 6874, 2618}

$$e^{x^2} \log(x) - \frac{\log(x)}{x + \log^2(x)} + \log(x + \log^2(x))$$

Antiderivative was successfully verified.

[In] Int[E^x^2/x + 2*E^x^2*x*Log[x] + (-2 + Log[x])/(x + Log[x]^2)^2 + (1 + x^(-1) + (2*Log[x])/x)/(x + Log[x]^2),x]

[Out] E^x^2*Log[x] - Log[x]/(x + Log[x]^2) + Log[x + Log[x]^2]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2240

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n * Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]

Rule 2241

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]

Rule 2618

Int[Log[(c_.)*(x_))^(n_.)]^(r_.)/((x_)*(Log[(c_.)*(x_))^(n_.)]^(q_.)*(b_.) + (a_.)*(x_))^(m_.), x_Symbol] := Simp[Log[a*x^m + b*Log[c*x^n]^q]/(b*n*q), x]

```
] - Dist[a*(m/(b*n*q)), Int[x^(m - 1)/(a*x^m + b*Log[c*x^n]^q), x], x] /; FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, q - 1]
```

Rule 2627

```
Int[(Log[(c_.)*(x_)^(n_.)]*(e_.) + (d_.))/(Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.) + (a_.)*(x_)^2, x_Symbol] := Simp[(-e)*(Log[c*x^n]/(a*(a*x + b*Log[c*x^n]^q))), x] + Dist[(d + e*n)/a, Int[1/(x*(a*x + b*Log[c*x^n]^q)), x], x] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[d + e*n*q, 0]
```

Rule 2634

```
Int[Log[u]*(v_), x_Symbol] := With[{w = IntHide[v, x]}, Dist[Log[u], w, x] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x]] /; InverseFunctionFreeQ[u, x]
```

Rule 6874

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned} \int \left(\frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2 + \log(x)}{(x + \log^2(x))^2} + \frac{1 + \frac{1}{x} + \frac{2\log(x)}{x}}{x + \log^2(x)} \right) dx &= 2 \int e^{x^2} x \log(x) dx + \int \frac{e^{x^2}}{x} dx + \int \frac{-2}{(x + \log^2(x))^2} dx \\ &= \frac{\text{Ei}(x^2)}{2} + e^{x^2} \log(x) - \frac{\log(x)}{x + \log^2(x)} - 2 \int \frac{1}{x + \log^2(x)} dx \\ &= \frac{\text{Ei}(x^2)}{2} + e^{x^2} \log(x) - \frac{\log(x)}{x + \log^2(x)} + 2 \int \frac{1}{x + \log^2(x)} dx \\ &= e^{x^2} \log(x) - \frac{\log(x)}{x + \log^2(x)} + \log(x + \log^2(x)) \end{aligned}$$

Mathematica [A]

time = 33.45, size = 28, normalized size = 1.00

$$e^{x^2} \log(x) - \frac{\log(x)}{x + \log^2(x)} + \log(x + \log^2(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2/x + 2*E^x^2*x*Log[x] + (-2 + Log[x])/(x + Log[x]^2)^2 + (1 + x^(-1) + (2*Log[x])/x)/(x + Log[x]^2), x]
```


[Out] $E^{x^2} \text{Log}[x] - \text{Log}[x]/(x + \text{Log}[x]^2) + \text{Log}[x + \text{Log}[x]^2]$

Mathics [A]

time = 1.97, size = 37, normalized size = 1.32

$$\frac{-\text{Log}[x] + (x + \text{Log}[x]^2) \left(\text{Log}[x + \text{Log}[x]^2] + \text{Log}[x] E^{x^2} \right)}{x + \text{Log}[x]^2}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[2*x*E^(x^2)*Log[x]+E^(x^2)/x+(Log[x]-2)/(Log[x]^2+x)^2+(2/x)*Log[x]+(1/x)+1)/(Log[x]^2+x),x']`

[Out] $(-\text{Log}[x] + (x + \text{Log}[x]^2) (\text{Log}[x + \text{Log}[x]^2] + \text{Log}[x] E^{x^2})) / (x + \text{Log}[x]^2)$

Maple [A]

time = 0.19, size = 28, normalized size = 1.00

method	result	size
default	$e^{x^2} \ln(x) - \frac{\ln(x)}{x+\ln(x)^2} + \ln(x + \ln(x)^2)$	28
risch	$e^{x^2} \ln(x) - \frac{\ln(x)}{x+\ln(x)^2} + \ln(x + \ln(x)^2)$	28

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x^2)/x+2*exp(x^2)*x*ln(x)+(-2+ln(x))/(x+ln(x)^2)^2+(1+1/x+2*ln(x)/x)/(x+ln(x)^2),x,method=_RETURNVERBOSE)`

[Out] $\exp(x^2) \ln(x) - \ln(x)/(x + \ln(x)^2) + \ln(x + \ln(x)^2)$

Maxima [A]

time = 0.29, size = 27, normalized size = 0.96

$$e^{(x^2)} \log(x) - \frac{\log(x)}{\log(x)^2 + x} + \log(\log(x)^2 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2)/x+2*exp(x^2)*x*log(x)+(-2+log(x))/(x+log(x)^2)^2+(1+1/x+2*log(x)/x)/(x+log(x)^2),x, algorithm="maxima")`

[Out] $e^{(x^2)} \log(x) - \log(x)/(\log(x)^2 + x) + \log(\log(x)^2 + x)$

Fricas [A]

time = 0.32, size = 44, normalized size = 1.57

$$\frac{e^{(x^2)} \log(x)^3 + (\log(x)^2 + x) \log(\log(x)^2 + x) + (xe^{(x^2)} - 1) \log(x)}{\log(x)^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^2)/x+2*exp(x^2)*x*log(x)+(-2+log(x))/(x+log(x)^2)^2+(1+1/x+2*log(x)/x)/(x+log(x)^2),x, algorithm="fricas")

[Out] (e^(x^2)*log(x)^3 + (log(x)^2 + x)*log(log(x)^2 + x) + (x*e^(x^2) - 1)*log(x))/(log(x)^2 + x)

Sympy [A]

time = 0.17, size = 26, normalized size = 0.93

$$e^{x^2} \log(x) + \log(x + \log(x)^2) - \frac{\log(x)}{x + \log(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x**2)/x+2*exp(x**2)*x*ln(x)+(-2+ln(x))/(x+ln(x)**2)**2+(1+1/x+2*ln(x)/x)/(x+ln(x)**2),x)

[Out] exp(x**2)*log(x) + log(x + log(x)**2) - log(x)/(x + log(x)**2)

Giac [A]

time = 0.01, size = 71, normalized size = 2.54

$$\frac{\text{Ei}(x^2)}{2} + 2 \left(\frac{1}{2} e^{x^2} \ln x - \frac{\text{Ei}(x^2)}{4} \right) + \frac{\ln x}{-\ln^2 x - x} + \frac{\ln x}{-\ln^2 x - x} + \frac{\ln x}{-\ln^2 x - x} + \ln(x + \ln^2 x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^2)/x+2*exp(x^2)*x*log(x)+(-2+log(x))/(x+log(x)^2)^2+(1+1/x+2*log(x)/x)/(x+log(x)^2),x)

[Out] e^(x^2)*log(x) - 3*log(x)/(log(x)^2 + x) + log(log(x)^2 + x)

Mupad [B]

time = 0.34, size = 27, normalized size = 0.96

$$\ln(\ln(x)^2 + x) + e^{x^2} \ln(x) - \frac{\ln(x)}{\ln(x)^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((log(x) - 2)/(x + log(x)^2)^2 + ((2*log(x))/x + 1/x + 1)/(x + log(x)^2) + exp(x^2)/x + 2*x*exp(x^2)*log(x),x)

[Out] log(x + log(x)^2) + exp(x^2)*log(x) - log(x)/(x + log(x)^2)

3.275 $\int e^{\frac{x}{2}+xz} x^4 \sin^4(\pi z) dz$

Optimal. Leaf size=199

$$\frac{24e^{\frac{x}{2}+xz}\pi^4x^3}{64\pi^4+20\pi^2x^2+x^4} - \frac{24e^{\frac{x}{2}+xz}\pi^3x^4\cos(\pi z)\sin(\pi z)}{64\pi^4+20\pi^2x^2+x^4} + \frac{12e^{\frac{x}{2}+xz}\pi^2x^5\sin^2(\pi z)}{64\pi^4+20\pi^2x^2+x^4} - \frac{4e^{\frac{x}{2}+xz}\pi x^4\cos(\pi z)\sin^3(\pi z)}{16\pi^2+x^2} + e^{\frac{x}{2}+xz}\pi^4x^4\sin^4(\pi z)$$

[Out] 24*exp(1/2*x+x*z)*Pi^4*x^3/(64*Pi^4+20*Pi^2*x^2+x^4)-24*exp(1/2*x+x*z)*Pi^3*x^4*cos(Pi*z)*sin(Pi*z)/(64*Pi^4+20*Pi^2*x^2+x^4)+12*exp(1/2*x+x*z)*Pi^2*x^5*sin(Pi*z)^2/(64*Pi^4+20*Pi^2*x^2+x^4)-4*exp(1/2*x+x*z)*Pi*x^4*cos(Pi*z)*sin(Pi*z)^3/(16*Pi^2+x^2)+exp(1/2*x+x*z)*x^5*sin(Pi*z)^4/(16*Pi^2+x^2)

Rubi [A]

time = 0.07, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {12, 4519, 2225}

$$\frac{x^5 e^{xz+\frac{x}{2}} \sin^4(\pi z)}{x^2+16\pi^2} - \frac{4\pi x^4 e^{xz+\frac{x}{2}} \sin^3(\pi z) \cos(\pi z)}{x^2+16\pi^2} - \frac{24\pi^3 x^4 e^{xz+\frac{x}{2}} \sin(\pi z) \cos(\pi z)}{x^4+20\pi^2x^2+64\pi^4} + \frac{12\pi^2 x^5 e^{xz+\frac{x}{2}} \sin^2(\pi z)}{x^4+20\pi^2x^2+64\pi^4} + \frac{24\pi^4 x^3 e^{xz+\frac{x}{2}}}{x^4+20\pi^2x^2+64\pi^4}$$

Antiderivative was successfully verified.

[In] Int[E^(x/2 + x*z)*x^4*Sin[Pi*z]^4,z]

[Out] (24*E^(x/2 + x*z)*Pi^4*x^3)/(64*Pi^4 + 20*Pi^2*x^2 + x^4) - (24*E^(x/2 + x*z)*Pi^3*x^4*Cos[Pi*z]*Sin[Pi*z])/(64*Pi^4 + 20*Pi^2*x^2 + x^4) + (12*E^(x/2 + x*z)*Pi^2*x^5*Sin[Pi*z]^2)/(64*Pi^4 + 20*Pi^2*x^2 + x^4) - (4*E^(x/2 + x*z)*Pi*x^4*Cos[Pi*z]*Sin[Pi*z]^3)/(16*Pi^2 + x^2) + (E^(x/2 + x*z)*x^5*Sin[Pi*z]^4)/(16*Pi^2 + x^2)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2225

Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 4519

Int[(F_)^((c_)*((a_) + (b_)*(x_)))*Sin[(d_) + (e_)*(x_)]^(n_), x_Symbol] := Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]

Rubi steps

$$\begin{aligned}
\int e^{\frac{x}{2}+xz} x^4 \sin^4(\pi z) dz &= x^4 \int e^{\frac{x}{2}+xz} \sin^4(\pi z) dz \\
&= -\frac{4e^{\frac{x}{2}+xz} \pi x^4 \cos(\pi z) \sin^3(\pi z)}{16\pi^2 + x^2} + \frac{e^{\frac{x}{2}+xz} x^5 \sin^4(\pi z)}{16\pi^2 + x^2} + \frac{(12\pi^2 x^4) \int e^{\frac{x}{2}+xz} \sin^2(\pi z) dz}{16\pi^2 + x^2} \\
&= -\frac{24e^{\frac{x}{2}+xz} \pi^3 x^4 \cos(\pi z) \sin(\pi z)}{64\pi^4 + 20\pi^2 x^2 + x^4} + \frac{12e^{\frac{x}{2}+xz} \pi^2 x^5 \sin^2(\pi z)}{64\pi^4 + 20\pi^2 x^2 + x^4} - \frac{4e^{\frac{x}{2}+xz} \pi x^4 \cos(\pi z) \sin^3(\pi z)}{16\pi^2 + x^2} \\
&= \frac{24e^{\frac{x}{2}+xz} \pi^4 x^3}{64\pi^4 + 20\pi^2 x^2 + x^4} - \frac{24e^{\frac{x}{2}+xz} \pi^3 x^4 \cos(\pi z) \sin(\pi z)}{64\pi^4 + 20\pi^2 x^2 + x^4} + \frac{12e^{\frac{x}{2}+xz} \pi^2 x^5 \sin^2(\pi z)}{64\pi^4 + 20\pi^2 x^2 + x^4} - \frac{4e^{\frac{x}{2}+xz} \pi x^4 \cos(\pi z) \sin^3(\pi z)}{16\pi^2 + x^2}
\end{aligned}$$

Mathematica [A]

time = 0.15, size = 136, normalized size = 0.68

$$\frac{e^{x(\frac{1}{2}+z)} x^4 (192\pi^4 + 60\pi^2 x^2 + 3x^4 - 4x^2(16\pi^2 + x^2) \cos(2\pi z) + x^2(4\pi^2 + x^2) \cos(4\pi z) - 128\pi^3 x \sin(2\pi z) - 8\pi x^3 \sin(2\pi z) + 16\pi^3 x \sin(4\pi z) + 4\pi x^3 \sin(4\pi z))}{8(64\pi^4 x + 20\pi^2 x^3 + x^5)}$$

Antiderivative was successfully verified.

[In] Integrate[E^(x/2 + x*z)*x^4*Sin[Pi*z]^4,z]

[Out] (E^(x*(1/2 + z))*x^4*(192*Pi^4 + 60*Pi^2*x^2 + 3*x^4 - 4*x^2*(16*Pi^2 + x^2)*Cos[2*Pi*z] + x^2*(4*Pi^2 + x^2)*Cos[4*Pi*z] - 128*Pi^3*x*Sin[2*Pi*z] - 8*Pi*x^3*Sin[2*Pi*z] + 16*Pi^3*x*Sin[4*Pi*z] + 4*Pi*x^3*Sin[4*Pi*z]))/(8*(64*Pi^4*x + 20*Pi^2*x^3 + x^5))

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 164.25, size = 422, normalized size = 2.12



Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Exp[x*z+x/2]*Sin[Pi*z]^4*x^4,z]')

[Out] Piecewise[{{x^4 (I / 16 - Pi z / 16 + I Pi z Sin[4 Pi z] / 16 + Pi z Cos[2 Pi z]^2 / 8 - I Cos[2 Pi z] / 6 - Sin[4 Pi z] / 64 + I Cos[2 Pi z]^2 / 16 + Sin[2 Pi z] / 12) E^(-4 I Pi z) / Pi, x == 0 || x == -4 I Pi}, {x^4 (-5 I + I 6 Pi z Sin[2 Pi z] + 6 Pi z Cos[2 Pi z] - 2 Sin[2 Pi z] - Sin[4 Pi z] + I 5 Cos[2 Pi z] + I Cos[2 Pi z]^2) E^(-2 I Pi z) / (24 Pi), x == -2 I Pi}, {x^4 (5 I - 6 I Pi z Sin[2 Pi z] + 6 Pi z Cos[2 Pi z] - 5 I Cos[2 Pi z] - 2 Sin[2 Pi z] - I Cos[2 Pi z]^2 - Sin[4 Pi z]) E^(I 2 Pi z) / (24 Pi), x == I 2 Pi}, {x^4 (-I / 16 - I Pi z Sin[4 Pi z] / 16 - Pi z Cos[2 Pi z]^2 / 8 - I Cos[2 Pi z]^2 / 16 - Sin[4 Pi z]

/ 64 + I Cos[2 Pi z] / 6 + Sin[2 Pi z] / 12) E ^ (I 4 Pi z) / Pi, x == I 4 Pi}, {x ^ 3 (48 Pi ^ 4 + x (2 x ^ 3 Sin[Pi z] ^ 4 - 2 Pi x ^ 2 Sin[2 Pi z] + Pi x ^ 2 Sin[4 Pi z] + 32 Pi ^ 2 x Sin[Pi z] ^ 4 - 32 Pi ^ 3 Sin[2 Pi z] + 4 Pi ^ 3 Sin[4 Pi z]) + 3 Pi ^ 2 x ^ 2 (1 - Cos[4 Pi z])) E ^ (x (1 + 2 z) / 2) / (2 (64 Pi ^ 4 + x ^ 4 + 20 Pi ^ 2 x ^ 2)), True}}]

Maple [A]

time = 0.18, size = 128, normalized size = 0.64

method	result
default	$x^4 \left(\frac{-3e^{\frac{1}{2}x+xz}}{x} - \frac{x e^{\frac{1}{2}x+xz} \cos(4\pi z)}{16\pi^2+x^2} - \frac{4\pi e^{\frac{1}{2}x+xz} \sin(4\pi z)}{16\pi^2+x^2} + \frac{4x e^{\frac{1}{2}x+xz} \cos(2\pi z)}{4\pi^2+x^2} + \frac{8\pi e^{\frac{1}{2}x+xz} \sin(2\pi z)}{4\pi^2+x^2} \right) / 8$
risch	$\frac{3x^3 e^{\frac{x(1+2z)}{2}}}{8} + \frac{x^5 e^{\frac{1}{2}x+xz} \cos(4\pi z)}{128\pi^2+8x^2} + \frac{x^4 e^{\frac{1}{2}x+xz} \pi \sin(4\pi z)}{32\pi^2+2x^2} - \frac{x^5 e^{\frac{1}{2}x+xz} \cos(2\pi z)}{2(4\pi^2+x^2)} - \frac{x^4 e^{\frac{1}{2}x+xz} \pi \sin(2\pi z)}{4\pi^2+x^2}$
norman	$\frac{24 e^{\frac{1}{2}x+xz} \pi^4 x^3}{64\pi^4+20\pi^2 x^2+x^4} - \frac{48\pi^3 x^4 e^{\frac{1}{2}x+xz} \tan\left(\frac{\pi z}{2}\right)}{64\pi^4+20\pi^2 x^2+x^4} + \frac{48\pi^3 x^4 e^{\frac{1}{2}x+xz} \left(\tan^7\left(\frac{\pi z}{2}\right)\right)}{64\pi^4+20\pi^2 x^2+x^4} + \frac{16(9\pi^4+10\pi^2 x^2+x^4) x^3 e^{\frac{1}{2}x+xz} \left(\tan^4\left(\frac{\pi z}{2}\right)\right)}{64\pi^4+20\pi^2 x^2+x^4} + \frac{24 e^{\frac{1}{2}x+xz} \pi^4 x^3}{64\pi^4+20\pi^2 x^2+x^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*exp(1/2*x+x*z)*sin(Pi*z)^4,z,method=_RETURNVERBOSE)

[Out] -1/8*x^4*(-3*exp(1/2*x+x*z)/x-x/(16*Pi^2+x^2)*exp(1/2*x+x*z)*cos(4*Pi*z)-4*Pi/(16*Pi^2+x^2)*exp(1/2*x+x*z)*sin(4*Pi*z)+4*x/(4*Pi^2+x^2)*exp(1/2*x+x*z)*cos(2*Pi*z)+8*Pi/(4*Pi^2+x^2)*exp(1/2*x+x*z)*sin(2*Pi*z))

Maxima [A]

time = 0.28, size = 160, normalized size = 0.80

$$\frac{((4\pi^2 x^2 + x^4) \cos(4\pi z) e^{(xz + \frac{1}{2}x)} - 4(16\pi^2 x^2 + x^4) \cos(2\pi z) e^{(xz + \frac{1}{2}x)} + 4(4\pi^3 x + \pi x^3) e^{(xz + \frac{1}{2}x)} \sin(4\pi z) - 8(16\pi^3 x + \pi x^3) e^{(xz + \frac{1}{2}x)} \sin(2\pi z) + 3(64\pi^4 + 20\pi^2 x^2 + x^4) e^{(xz + \frac{1}{2}x)}) x^4}{8(64\pi^4 x + 20\pi^2 x^3 + x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*exp(1/2*x+x*z)*sin(pi*z)^4,z, algorithm="maxima")

[Out] 1/8*((4*pi^2*x^2 + x^4)*cos(4*pi*z)*e^(x*z + 1/2*x) - 4*(16*pi^2*x^2 + x^4)*cos(2*pi*z)*e^(x*z + 1/2*x) + 4*(4*pi^3*x + pi*x^3)*e^(x*z + 1/2*x)*sin(4*pi*z) - 8*(16*pi^3*x + pi*x^3)*e^(x*z + 1/2*x)*sin(2*pi*z) + 3*(64*pi^4 + 20*pi^2*x^2 + x^4)*e^(x*z + 1/2*x))*x^4/(64*pi^4*x + 20*pi^2*x^3 + x^5)

Fricas [A]

time = 0.32, size = 145, normalized size = 0.73

$$\frac{4((4\pi^3 x^4 + \pi x^6) \cos(\pi z)^3 - (10\pi^3 x^4 + \pi x^6) \cos(\pi z) e^{(xz + \frac{1}{2}x)} \sin(\pi z) + (24\pi^4 x^3 + 16\pi^2 x^5 + x^7 + (4\pi^2 x^5 + x^7) \cos(\pi z)^4 - 2(10\pi^2 x^5 + x^7) \cos(\pi z)^2) e^{(xz + \frac{1}{2}x)})}{64\pi^4 + 20\pi^2 x^2 + x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*exp(1/2*x+x*z)*sin(pi*z)^4,z, algorithm="fricas")

```
[Out] (4*((4*pi^3*x^4 + pi*x^6)*cos(pi*z)^3 - (10*pi^3*x^4 + pi*x^6)*cos(pi*z))*e
^(x*z + 1/2*x)*sin(pi*z) + (24*pi^4*x^3 + 16*pi^2*x^5 + x^7 + (4*pi^2*x^5 +
x^7)*cos(pi*z)^4 - 2*(10*pi^2*x^5 + x^7)*cos(pi*z)^2)*e^(x*z + 1/2*x))/(64
*pi^4 + 20*pi^2*x^2 + x^4)
```

Sympy [A]

time = 135.78, size = 1277, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*exp(1/2*x+x*z)*sin(pi*z)**4,z)
```

```
[Out] x**4*Piecewise((3*z*sin(pi*z)**4/8 + 3*z*sin(pi*z)**2*cos(pi*z)**2/4 + 3*z*
cos(pi*z)**4/8 - 5*sin(pi*z)**3*cos(pi*z)/(8*pi) - 3*sin(pi*z)*cos(pi*z)**3
/(8*pi), Eq(x, 0)), (z*exp(-4*I*pi*z)*sin(pi*z)**4/16 - I*z*exp(-4*I*pi*z)*
sin(pi*z)**3*cos(pi*z)/4 - 3*z*exp(-4*I*pi*z)*sin(pi*z)**2*cos(pi*z)**2/8 +
I*z*exp(-4*I*pi*z)*sin(pi*z)*cos(pi*z)**3/4 + z*exp(-4*I*pi*z)*cos(pi*z)**
4/16 + 7*I*exp(-4*I*pi*z)*sin(pi*z)**4/(24*pi) + 11*exp(-4*I*pi*z)*sin(pi*z)
**3*cos(pi*z)/(48*pi) + 5*exp(-4*I*pi*z)*sin(pi*z)*cos(pi*z)**3/(48*pi) -
I*exp(-4*I*pi*z)*cos(pi*z)**4/(24*pi), Eq(x, -4*I*pi)), (-z*exp(-2*I*pi*z)*
sin(pi*z)**4/4 + I*z*exp(-2*I*pi*z)*sin(pi*z)**3*cos(pi*z)/2 + I*z*exp(-2*I
*pi*z)*sin(pi*z)*cos(pi*z)**3/2 + z*exp(-2*I*pi*z)*cos(pi*z)**4/4 - 3*I*exp
(-2*I*pi*z)*sin(pi*z)**4/(8*pi) - I*exp(-2*I*pi*z)*sin(pi*z)**2*cos(pi*z)**
2/(2*pi) - exp(-2*I*pi*z)*sin(pi*z)*cos(pi*z)**3/(3*pi) + I*exp(-2*I*pi*z)*
cos(pi*z)**4/(24*pi), Eq(x, -2*I*pi)), (-z*exp(2*I*pi*z)*sin(pi*z)**4/4 - I
*z*exp(2*I*pi*z)*sin(pi*z)**3*cos(pi*z)/2 - I*z*exp(2*I*pi*z)*sin(pi*z)*cos
(pi*z)**3/2 + z*exp(2*I*pi*z)*cos(pi*z)**4/4 + 3*I*exp(2*I*pi*z)*sin(pi*z)*
**4/(8*pi) + I*exp(2*I*pi*z)*sin(pi*z)**2*cos(pi*z)**2/(2*pi) - exp(2*I*pi*z)
*sin(pi*z)*cos(pi*z)**3/(3*pi) - I*exp(2*I*pi*z)*cos(pi*z)**4/(24*pi), Eq(
x, 2*I*pi)), (z*exp(4*I*pi*z)*sin(pi*z)**4/16 + I*z*exp(4*I*pi*z)*sin(pi*z)
**3*cos(pi*z)/4 - 3*z*exp(4*I*pi*z)*sin(pi*z)**2*cos(pi*z)**2/8 - I*z*exp(4
*I*pi*z)*sin(pi*z)*cos(pi*z)**3/4 + z*exp(4*I*pi*z)*cos(pi*z)**4/16 - 7*I*exp
(4*I*pi*z)*sin(pi*z)**4/(24*pi) + 11*exp(4*I*pi*z)*sin(pi*z)**3*cos(pi*z)
/(48*pi) + 5*exp(4*I*pi*z)*sin(pi*z)*cos(pi*z)**3/(48*pi) + I*exp(4*I*pi*z)
*cos(pi*z)**4/(24*pi), Eq(x, 4*I*pi)), (x**4*exp(x/2)*exp(x*z)*sin(pi*z)**4
/(x**5 + 20*pi**2*x**3 + 64*pi**4*x) - 4*pi*x**3*exp(x/2)*exp(x*z)*sin(pi*z)
**3*cos(pi*z)/(x**5 + 20*pi**2*x**3 + 64*pi**4*x) + 16*pi**2*x**2*exp(x/2)
*exp(x*z)*sin(pi*z)**4/(x**5 + 20*pi**2*x**3 + 64*pi**4*x) + 12*pi**2*x**2*
exp(x/2)*exp(x*z)*sin(pi*z)**2*cos(pi*z)**2/(x**5 + 20*pi**2*x**3 + 64*pi**
4*x) - 40*pi**3*x*exp(x/2)*exp(x*z)*sin(pi*z)**3*cos(pi*z)/(x**5 + 20*pi**2
*x**3 + 64*pi**4*x) - 24*pi**3*x*exp(x/2)*exp(x*z)*sin(pi*z)*cos(pi*z)**3/(
x**5 + 20*pi**2*x**3 + 64*pi**4*x) + 24*pi**4*exp(x/2)*exp(x*z)*sin(pi*z)**
4/(x**5 + 20*pi**2*x**3 + 64*pi**4*x) + 48*pi**4*exp(x/2)*exp(x*z)*sin(pi*z)
**2*cos(pi*z)**2/(x**5 + 20*pi**2*x**3 + 64*pi**4*x) + 24*pi**4*exp(x/2)*exp
(x*z)*cos(pi*z)**4/(x**5 + 20*pi**2*x**3 + 64*pi**4*x), True))
```

Giac [A]

time = 0.00, size = 127, normalized size = 0.64

$$x^4 \left(\frac{3e^{\frac{2xz+x}{2}}}{8x} + e^{\frac{2xz+x}{2}} \left(-\frac{2x \cos(2\pi z)}{(2x)^2 + (4\pi)^2} - \frac{4\pi \sin(2\pi z)}{(2x)^2 + (4\pi)^2} \right) + e^{\frac{2xz+x}{2}} \left(\frac{8x \cos(4\pi z)}{(8x)^2 + (32\pi)^2} + \frac{32\pi \sin(4\pi z)}{(8x)^2 + (32\pi)^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*exp(1/2*x+x*z)*sin(pi*z)^4,z)

[Out] 1/8*(e^(x*z + 1/2*x)*(x*cos(4*pi*z)/(16*pi^2 + x^2) + 4*pi*sin(4*pi*z)/(16*pi^2 + x^2)) - 4*e^(x*z + 1/2*x)*(x*cos(2*pi*z)/(4*pi^2 + x^2) + 2*pi*sin(2*pi*z)/(4*pi^2 + x^2)) + 3*e^(x*z + 1/2*x)/x)*x^4

Mupad [B]

time = 1.11, size = 140, normalized size = 0.70

$$\frac{x^3 e^{\frac{x}{2}+xz} \left(24\Pi^4 - \frac{x^4 \cos(2\Pi z)}{2} + \frac{x^4 \cos(4\Pi z)}{8} + \frac{3x^4}{8} + \frac{15\Pi^2 x^2}{2} - \Pi x^3 \sin(2\Pi z) - 16\Pi^3 x \sin(2\Pi z) + \frac{\Pi x^3 \sin(4\Pi z)}{2} + 2\Pi^3 x \sin(4\Pi z) - 8\Pi^2 x^2 \cos(2\Pi z) + \frac{\Pi^2 x^2 \cos(4\Pi z)}{2} \right)}{64\Pi^4 + 20\Pi^2 x^2 + x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*exp(x/2 + x*z)*sin(Pi*z)^4,z)

[Out] (x^3*exp(x/2 + x*z)*(24*Pi^4 - (x^4*cos(2*Pi*z))/2 + (x^4*cos(4*Pi*z))/8 + (3*x^4)/8 + (15*Pi^2*x^2)/2 - Pi*x^3*sin(2*Pi*z) - 16*Pi^3*x*sin(2*Pi*z) + (Pi*x^3*sin(4*Pi*z))/2 + 2*Pi^3*x*sin(4*Pi*z) - 8*Pi^2*x^2*cos(2*Pi*z) + (Pi^2*x^2*cos(4*Pi*z))/2))/(64*Pi^4 + x^4 + 20*Pi^2*x^2)

3.276 $\int \operatorname{erf}(x) dx$

Optimal. Leaf size=18

$$\frac{e^{-x^2}}{\sqrt{\pi}} + x\operatorname{erf}(x)$$

[Out] $x*\operatorname{erf}(x)+1/\exp(x^2)/\operatorname{Pi}^{(1/2)}$

Rubi [A]

time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6484}

$$x\operatorname{erf}(x) + \frac{e^{-x^2}}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

[In] `Int[Erf[x],x]`

[Out] `1/(E^x^2*Sqrt[Pi]) + x*Erf[x]`

Rule 6484

`Int[Erf[(a_.) + (b_.)*(x_)], x_Symbol] := Simp[(a + b*x)*(Erf[a + b*x]/b), x] + Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]`

Rubi steps

$$\int \operatorname{erf}(x) dx = \frac{e^{-x^2}}{\sqrt{\pi}} + x\operatorname{erf}(x)$$

Mathematica [A]

time = 0.01, size = 18, normalized size = 1.00

$$\frac{e^{-x^2}}{\sqrt{\pi}} + x\operatorname{erf}(x)$$

Antiderivative was successfully verified.

[In] `Integrate[Erf[x],x]`

[Out] `1/(E^x^2*Sqrt[Pi]) + x*Erf[x]`

Mathics [A]

time = 1.79, size = 16, normalized size = 0.89

$$x \operatorname{Erf}[x] + \frac{E^{-x^2}}{\sqrt{\text{Pi}}}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Erf[x],x]')``[Out] x Erf[x] + E ^ (-x ^ 2) / Sqrt[Pi]`**Maple [A]**

time = 0.01, size = 16, normalized size = 0.89

method	result	size
default	$x \operatorname{erf}(x) + \frac{e^{-x^2}}{\sqrt{\pi}}$	16
meijerg	$\frac{-2+2e^{-x^2}+2x\sqrt{\pi}\operatorname{erf}(x)}{2\sqrt{\pi}}$	24

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(erf(x),x,method=_RETURNVERBOSE)``[Out] x*erf(x)+1/Pi^(1/2)*exp(-x^2)`**Maxima [A]**

time = 0.27, size = 15, normalized size = 0.83

$$x \operatorname{erf}(x) + \frac{e^{(-x^2)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(erf(x),x, algorithm="maxima")``[Out] x*erf(x) + e^(-x^2)/sqrt(pi)`**Fricas [A]**

time = 0.32, size = 20, normalized size = 1.11

$$\frac{\pi x \operatorname{erf}(x) + \sqrt{\pi} e^{(-x^2)}}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(erf(x),x, algorithm="fricas")``[Out] (pi*x*erf(x) + sqrt(pi)*e^(-x^2))/pi`

Sympy [A]

time = 0.15, size = 15, normalized size = 0.83

$$x \operatorname{erf}(x) + \frac{e^{-x^2}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(erf(x),x)**[Out]** x*erf(x) + exp(-x**2)/sqrt(pi)**Giac [A]**

time = 0.00, size = 21, normalized size = 1.17

$$x \operatorname{erf}(x) + \frac{2\sqrt{\pi} e^{-x^2}}{\pi \cdot 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(erf(x),x)**[Out]** x*erf(x) + 1/(sqrt(pi)*e^(x^2))**Mupad [B]**

time = 0.15, size = 15, normalized size = 0.83

$$\frac{e^{-x^2}}{\sqrt{\pi}} + x \operatorname{erf}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(erf(x),x)**[Out]** exp(-x^2)/pi^(1/2) + x*erf(x)

3.277 $\int \operatorname{erf}(a + x) dx$

Optimal. Leaf size=24

$$\frac{e^{-(a+x)^2}}{\sqrt{\pi}} + (a+x)\operatorname{erf}(a+x)$$

[Out] (a+x)*erf(a+x)+1/exp((a+x)^2)/Pi^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6484}

$$(a+x)\operatorname{erf}(a+x) + \frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

[In] Int[Erf[a + x], x]

[Out] 1/(E^(a + x)^2*Sqrt[Pi]) + (a + x)*Erf[a + x]

Rule 6484

Int[Erf[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Erf[a + b*x]/b), x] + Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]

Rubi steps

$$\int \operatorname{erf}(a + x) dx = \frac{e^{-(a+x)^2}}{\sqrt{\pi}} + (a+x)\operatorname{erf}(a+x)$$

Mathematica [A]

time = 0.02, size = 24, normalized size = 1.00

$$\frac{e^{-(a+x)^2}}{\sqrt{\pi}} + (a+x)\operatorname{erf}(a+x)$$

Antiderivative was successfully verified.

[In] Integrate[Erf[a + x], x]

[Out] 1/(E^(a + x)^2*Sqrt[Pi]) + (a + x)*Erf[a + x]

Mathics [A]

time = 2.07, size = 34, normalized size = 1.42

$$a\text{Erf}[a + x] + x\text{Erf}[a + x] + \frac{E^{-a^2 - 2ax - x^2}}{\sqrt{\text{Pi}}}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Erf[x+a],x]')``[Out] a Erf[a + x] + x Erf[a + x] + E ^ (-a ^ 2 - 2 a x - x ^ 2) / Sqrt[Pi]`**Maple [A]**

time = 0.02, size = 22, normalized size = 0.92

method	result	size
derivativedivides	$(a + x) \text{erf}(a + x) + \frac{e^{-(a+x)^2}}{\sqrt{\pi}}$	22
default	$(a + x) \text{erf}(a + x) + \frac{e^{-(a+x)^2}}{\sqrt{\pi}}$	22

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(erf(a+x),x,method=_RETURNVERBOSE)``[Out] (a+x)*erf(a+x)+1/Pi^(1/2)*exp(-(a+x)^2)`**Maxima [A]**

time = 0.33, size = 21, normalized size = 0.88

$$(a + x) \text{erf}(a + x) + \frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(erf(a+x),x, algorithm="maxima")``[Out] (a + x)*erf(a + x) + e^(-(a + x)^2)/sqrt(pi)`**Fricas [A]**

time = 0.30, size = 37, normalized size = 1.54

$$\frac{(\pi a + \pi x) \text{erf}(a + x) + \sqrt{\pi} e^{(-a^2 - 2ax - x^2)}}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(erf(a+x),x, algorithm="fricas")`

[Out] $((\pi*a + \pi*x)*\text{erf}(a + x) + \sqrt{\pi})e^{-(a^2 - 2*a*x - x^2)}/\pi$

Sympy [A]

time = 0.24, size = 36, normalized size = 1.50

$$a \operatorname{erf}(a + x) + x \operatorname{erf}(a + x) + \frac{e^{-a^2} e^{-x^2} e^{-2ax}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(erf(a+x),x)`

[Out] $a*\text{erf}(a + x) + x*\text{erf}(a + x) + \exp(-a**2)*\exp(-x**2)*\exp(-2*a*x)/\sqrt{\pi}$

Giac [A]

time = 0.00, size = 50, normalized size = 2.08

$$x \operatorname{erf}(a + x) - \frac{2\sqrt{\pi} \left(-\frac{e^{-(a^2+2ax+x^2)}}{2} - \frac{1}{2}a\sqrt{\pi} \operatorname{erf}\left(x + \frac{2}{2}a\right) \right)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(erf(a+x),x)`

[Out] $x*\text{erf}(a + x) + (\sqrt{\pi})a*\text{erf}(a + x) + e^{-(a^2 - 2*a*x - x^2)}/\sqrt{\pi}$

Mupad [B]

time = 0.07, size = 21, normalized size = 0.88

$$\operatorname{erf}(a + x) (a + x) + \frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(erf(a + x),x)`

[Out] $\text{erf}(a + x)*(a + x) + \exp(-(a + x)^2)/\pi^{(1/2)}$

$$3.278 \quad \int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2 \sqrt{1+2x^2+4x^3+x^4}} dx$$

Optimal. Leaf size=94

$$\frac{(1+2x)\sqrt{1+2x^2+4x^3+x^4}}{2(-1+2x^2)} - \tanh^{-1}\left(\frac{x(2+x)(7-x+27x^2+33x^3)}{(2+37x^2+31x^3)\sqrt{1+2x^2+4x^3+x^4}}\right)$$

[Out] $-\operatorname{arctanh}(x(2+x)(33x^3+27x^2-x+7)/(31x^3+37x^2+2)/(x^4+4x^3+2x^2+1)^{1/2})+1/2(1+2x)(x^4+4x^3+2x^2+1)^{1/2}/(2x^2-1)$

Rubi [F]

time = 1.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2 \sqrt{1+2x^2+4x^3+x^4}} dx$$

Verification is not applicable to the result.

[In] $\operatorname{Int}[(-8-8x-x^2-3x^3+7x^4+4x^5+2x^6)/((-1+2x^2)^2 \operatorname{Sqrt}[1+2x^2+4x^3+x^4]), x]$

[Out] $(9*\operatorname{Defer}[\operatorname{Int}[1/\operatorname{Sqrt}[1+2x^2+4x^3+x^4], x])/4 - (13*\operatorname{Defer}[\operatorname{Int}[1/((\operatorname{Sqrt}[2]-2x)^2 \operatorname{Sqrt}[1+2x^2+4x^3+x^4]), x])/4 + \operatorname{Defer}[\operatorname{Int}[x/\operatorname{Sqrt}[1+2x^2+4x^3+x^4], x] + \operatorname{Defer}[\operatorname{Int}[x^2/\operatorname{Sqrt}[1+2x^2+4x^3+x^4], x]/2 - (13*\operatorname{Defer}[\operatorname{Int}[1/((\operatorname{Sqrt}[2]+2x)^2 \operatorname{Sqrt}[1+2x^2+4x^3+x^4]), x])/4 - (13*\operatorname{Defer}[\operatorname{Int}[1/((1-\operatorname{Sqrt}[2]*x)*\operatorname{Sqrt}[1+2x^2+4x^3+x^4]), x])/8 - ((15+\operatorname{Sqrt}[2])*\operatorname{Defer}[\operatorname{Int}[1/((1-\operatorname{Sqrt}[2]*x)*\operatorname{Sqrt}[1+2x^2+4x^3+x^4]), x])/8 - (13*\operatorname{Defer}[\operatorname{Int}[1/((1+\operatorname{Sqrt}[2]*x)*\operatorname{Sqrt}[1+2x^2+4x^3+x^4]), x])/8 - ((15-\operatorname{Sqrt}[2])*\operatorname{Defer}[\operatorname{Int}[1/((1+\operatorname{Sqrt}[2]*x)*\operatorname{Sqrt}[1+2x^2+4x^3+x^4]), x])/8 - (17*\operatorname{Defer}[\operatorname{Int}[x/((-1+2x^2)^2 \operatorname{Sqrt}[1+2x^2+4x^3+x^4]), x])/2$

Rubi steps

$$\begin{aligned}
\int \frac{-8 - 8x - x^2 - 3x^3 + 7x^4 + 4x^5 + 2x^6}{(-1 + 2x^2)^2 \sqrt{1 + 2x^2 + 4x^3 + x^4}} dx &= \int \left(\frac{9}{4\sqrt{1 + 2x^2 + 4x^3 + x^4}} + \frac{x}{\sqrt{1 + 2x^2 + 4x^3 + x^4}} + \frac{1}{2\sqrt{1 + 2x^2 + 4x^3 + x^4}} \right) dx \\
&= \frac{1}{4} \int \frac{15 + 2x}{(-1 + 2x^2) \sqrt{1 + 2x^2 + 4x^3 + x^4}} dx + \frac{1}{2} \int \frac{1}{\sqrt{1 + 2x^2 + 4x^3 + x^4}} dx \\
&= \frac{1}{4} \int \left(-\frac{15 + \sqrt{2}}{2(1 - \sqrt{2}x) \sqrt{1 + 2x^2 + 4x^3 + x^4}} - \frac{1}{2(1 + \sqrt{2}x) \sqrt{1 + 2x^2 + 4x^3 + x^4}} \right) dx \\
&= \frac{1}{2} \int \frac{x^2}{\sqrt{1 + 2x^2 + 4x^3 + x^4}} dx + \frac{9}{4} \int \frac{1}{\sqrt{1 + 2x^2 + 4x^3 + x^4}} dx \\
&= \frac{1}{2} \int \frac{x^2}{\sqrt{1 + 2x^2 + 4x^3 + x^4}} dx + \frac{9}{4} \int \frac{1}{\sqrt{1 + 2x^2 + 4x^3 + x^4}} dx \\
&= \frac{1}{2} \int \frac{x^2}{\sqrt{1 + 2x^2 + 4x^3 + x^4}} dx + \frac{9}{4} \int \frac{1}{\sqrt{1 + 2x^2 + 4x^3 + x^4}} dx \\
&= \frac{1}{2} \int \frac{x^2}{\sqrt{1 + 2x^2 + 4x^3 + x^4}} dx + \frac{9}{4} \int \frac{1}{\sqrt{1 + 2x^2 + 4x^3 + x^4}} dx \\
&= \frac{1}{2} \int \frac{x^2}{\sqrt{1 + 2x^2 + 4x^3 + x^4}} dx - \frac{13}{8} \int \frac{1}{(1 - \sqrt{2}x) \sqrt{1 + 2x^2 + 4x^3 + x^4}} dx
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 16.34, size = 5137, normalized size = 54.65

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[(-8 - 8*x - x^2 - 3*x^3 + 7*x^4 + 4*x^5 + 2*x^6)/((-1 + 2*x^2)^2*Sqrt[1 + 2*x^2 + 4*x^3 + x^4]),x]

[Out] Result too large to show

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(2*x^6+4*x^5+7*x^4-3*x^3-x*x-8*x-8)/((2*x^2-1)^2*Sqrt[x^4+4*x^3+2*x^2+1]),x]')
```

[Out] Timed out

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.34, size = 1197351, normalized size = 12737.78

method	result
trager	$\frac{(1+2x)\sqrt{x^4+4x^3+2x^2+1}}{4x^2-2} + \frac{\ln\left(-\frac{1025x^{10}+1023\sqrt{x^4+4x^3+2x^2+1}x^8-6138x^9+4104\sqrt{x^4+4x^3+2x^2+1}x^7}{x^8-6138x^9+4104\sqrt{x^4+4x^3+2x^2+1}x^7}\right)}{2(2x^2-1)}$
risch	Expression too large to display
elliptic	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] result too large to display

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^6 + 4*x^5 + 7*x^4 - 3*x^3 - x^2 - 8*x - 8)/(sqrt(x^4 + 4*x^3 + 2*x^2 + 1)*(2*x^2 - 1)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(88) = 176.

time = 0.37, size = 179, normalized size = 1.90

$$\frac{(2x^2-1)\log\left(\frac{1025x^{10}+6138x^9+12307x^8+10188x^7+4503x^6+3134x^5+1589x^4+140x^3+176x^2-(1023x^8+4104x^7+5084x^6+2182x^5+805x^4+624x^3+10x^2+28x)\sqrt{x^4+4x^3+2x^2+1}+2}{32x^{10}-80x^9+80x^8-40x^4+10x^2-1}\right)+\sqrt{x^4+4x^3+2x^2+1}(2x+1)}{2(2x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+1)^(1/2),x, algorithm="fricas")
```


[Out] $\frac{1}{2} \left((2x^2 - 1) \log((1025x^{10} + 6138x^9 + 12307x^8 + 10188x^7 + 4503x^6 + 3134x^5 + 1589x^4 + 140x^3 + 176x^2 - (1023x^8 + 4104x^7 + 5084x^6 + 2182x^5 + 805x^4 + 624x^3 + 10x^2 + 28x) \sqrt{x^4 + 4x^3 + 2x^2 + 1}) + 2) / (32x^{10} - 80x^8 + 80x^6 - 40x^4 + 10x^2 - 1) + \sqrt{x^4 + 4x^3 + 2x^2 + 1} (2x + 1) / (2x^2 - 1) \right)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^6 + 4x^5 + 7x^4 - 3x^3 - x^2 - 8x - 8}{\sqrt{(x+1)(x^3+3x^2-x+1)}(2x^2-1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**6+4*x**5+7*x**4-3*x**3-x**2-8*x-8)/(2*x**2-1)**2/(x**4+4*x**3+2*x**2+1)**(1/2),x)`

[Out] `Integral((2*x**6 + 4*x**5 + 7*x**4 - 3*x**3 - x**2 - 8*x - 8)/(sqrt((x + 1)*(x**3 + 3*x**2 - x + 1))*(2*x**2 - 1)**2), x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+1)^(1/2),x)`

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{-2x^6 - 4x^5 - 7x^4 + 3x^3 + x^2 + 8x + 8}{(2x^2 - 1)^2 \sqrt{x^4 + 4x^3 + 2x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(8*x + x^2 + 3*x^3 - 7*x^4 - 4*x^5 - 2*x^6 + 8)/((2*x^2 - 1)^2*(2*x^2 + 4*x^3 + x^4 + 1)^(1/2)),x)`

[Out] `int(-(8*x + x^2 + 3*x^3 - 7*x^4 - 4*x^5 - 2*x^6 + 8)/((2*x^2 - 1)^2*(2*x^2 + 4*x^3 + x^4 + 1)^(1/2)), x)`

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + 2*y)*Sqrt[1 - 5*y - 5*y^2])/(y*(1 + y)*(2 + y)*Sqrt[1 - y - y^2]),y]
```

```
[Out] ((-1 - 2/Sqrt[5])*(1 + Sqrt[5] + 2*y)^2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(5 + 5*Sqrt[5] + 10*y)]*(20*(-4*Sqrt[(-5 + 3*Sqrt[5] - 10*y)/(1 + Sqrt[5] + 2*y)]*Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*y)] + Sqrt[5]*Sqrt[(-5 + 3*Sqrt[5] - 10*y)/(1 + Sqrt[5] + 2*y)]*Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*y)]) + 5*Sqrt[-((-5 + Sqrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))]*Sqrt[-((-3 + Sqrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))] - 2*Sqrt[5]*Sqrt[-((-5 + Sqrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))] *Sqrt[-((-3 + Sqrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))])*EllipticF[ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y))]/Sqrt[15]], 15/16] + Sqrt[(-5 + 3*Sqrt[5] - 10*y)/(1 + Sqrt[5] + 2*y)]*Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*y)]*(9*Sqrt[5]*EllipticPi[5/8 - Sqrt[5]/8, ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y))]/Sqrt[15]], 15/16] + (-20 + 9*Sqrt[5])*EllipticPi[(-3*(-5 + Sqrt[5]))/8, ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y))]/Sqrt[15]], 15/16] + 2*Sqrt[5]*EllipticPi[(3*(5 + Sqrt[5]))/8, ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y))]/Sqrt[15]], 15/16)))/(16*Sqrt[1 - 5*y - 5*y^2]*Sqrt[1 - y - y^2])
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1+2*y)*Sqrt[1-5*y-5*y^2]/(y*(1+y)*(2+y)*Sqrt[1-y-y^2]),y]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.34, size = 354, normalized size = 2.49

method	result
default	$300\sqrt{-5y^2 - 5y + 1} \sqrt{-y^2 - y + 1} \sqrt{-\frac{10y+5+3\sqrt{5}}{-10y-5+3\sqrt{5}}} \left(-10y-5+3\sqrt{5}\right)^2 \sqrt{\frac{-2y+\sqrt{5}-1}{-10y-5+3\sqrt{5}}} \sqrt{5} \sqrt{\frac{2}{-1}}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y,method=_RETURNVERBOSE)
```

```
[Out] 300*(-5*y^2-5*y+1)^(1/2)*(-y^2-y+1)^(1/2)*(-(10*y+5+3*5^(1/2))/(-10*y-5+3*5^(1/2)))^(1/2)*(-10*y-5+3*5^(1/2))^2*((-2*y+5^(1/2)-1)/(-10*y-5+3*5^(1/2)))^(1/2)*5^(1/2)*((2*y+1+5^(1/2))/(-10*y-5+3*5^(1/2)))^(1/2)*(3*EllipticPi(2*(-(10*y+5+3*5^(1/2))/(-10*y-5+3*5^(1/2)))^(1/2),-1/4*(5+5^(1/2))/(5^(1/2)-5),1/4)-EllipticPi(2*(-(10*y+5+3*5^(1/2))/(-10*y-5+3*5^(1/2)))^(1/2),-1/4*(3*5^(1/2)-5)/(5+3*5^(1/2)),1/4)-2*EllipticPi(2*(-(10*y+5+3*5^(1/2))/(-10*y-5+3*5^(1/2)))^(1/2),-1/4*(5+3*5^(1/2))/(3*5^(1/2)-5),1/4))/(5*y^4+10*y^3-y^2-6*y+1)^(1/2)/((10*y+5+3*5^(1/2))*(-10*y-5+3*5^(1/2))*(-2*y+5^(1/2)-1)*(2*y+1+5^(1/2)))^(1/2)/(5+5^(1/2))/(5^(1/2)-5)/(3*5^(1/2)-5)/(5+3*5^(1/2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y, algorithm="maxima")
```

```
[Out] integrate(sqrt(-5*y^2 - 5*y + 1)*(2*y + 1)/(sqrt(-y^2 - y + 1)*(y + 2)*(y + 1)*y), y)
```

Fricas [A]

time = 0.37, size = 223, normalized size = 1.57

$$\frac{9}{8} \log\left(\frac{235y^4 + 935y^3 - 3(35y^2 + 104y + 77)\sqrt{-y^2 - y + 1}\sqrt{-5y^2 - 5y + 1} + 1086y^2 + 131y - 281}{y^4 + 8y^3 + 24y^2 + 32y + 16}\right) + \frac{1}{4} \log\left(\frac{35y^4 + 125y^3 + (15y^2 + 38y + 24)\sqrt{-y^2 - y + 1}\sqrt{-5y^2 - 5y + 1} + 131y^2 + 16y - 26}{y^4 + 4y^3 + 6y^2 + 4y + 1}\right) - \frac{1}{8} \log\left(\frac{35y^4 + 15y^3 + (15y^2 - 8y + 1)\sqrt{-y^2 - y + 1}\sqrt{-5y^2 - 5y + 1} - 34y^2 + 11y - 1}{y^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y, algorithm="fricas")
```

```
[Out] 9/8*log(-(235*y^4 + 935*y^3 - 3*(35*y^2 + 104*y + 77)*sqrt(-y^2 - y + 1)*sqrt(-5*y^2 - 5*y + 1) + 1086*y^2 + 131*y - 281)/(y^4 + 8*y^3 + 24*y^2 + 32*y + 16)) + 1/4*log((35*y^4 + 125*y^3 + (15*y^2 + 38*y + 24)*sqrt(-y^2 - y + 1)*sqrt(-5*y^2 - 5*y + 1) + 131*y^2 + 16*y - 26)/(y^4 + 4*y^3 + 6*y^2 + 4*y + 1)) + 1/8*log((35*y^4 + 15*y^3 + (15*y^2 - 8*y + 1)*sqrt(-y^2 - y + 1)*sqrt(-5*y^2 - 5*y + 1) - 34*y^2 + 11*y - 1)/y^4)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2y + 1) \sqrt{-5y^2 - 5y + 1}}{y(y + 1)(y + 2) \sqrt{-y^2 - y + 1}} dy$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*y)*(-5*y**2-5*y+1)**(1/2)/y/(1+y)/(2+y)/(-y**2-y+1)**(1/2),y)

[Out] Integral((2*y + 1)*sqrt(-5*y**2 - 5*y + 1)/(y*(y + 1)*(y + 2)*sqrt(-y**2 - y + 1)), y)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2y + 1) \sqrt{-5y^2 - 5y + 1}}{y(y + 1)(y + 2) \sqrt{-y^2 - y + 1}} dy$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2*y + 1)*(1 - 5*y^2 - 5*y)^(1/2))/(y*(y + 1)*(y + 2)*(1 - y^2 - y)^(1/2)),y)

[Out] int(((2*y + 1)*(1 - 5*y^2 - 5*y)^(1/2))/(y*(y + 1)*(y + 2)*(1 - y^2 - y)^(1/2)), y)

3.280

$$\int \frac{x \left(-\sqrt{-4+x^2} + x^2 \sqrt{-4+x^2} - 4\sqrt{-1+x^2} + x^2 \sqrt{-1+x^2} \right)}{(4-5x^2+x^4) \left(1 + \sqrt{-4+x^2} + \sqrt{-1+x^2} \right)} dx$$

Optimal. Leaf size=21

$$\log \left(1 + \sqrt{-4+x^2} + \sqrt{-1+x^2} \right)$$

[Out] $\ln(1+(x^2-4)^{(1/2)}+(x^2-1)^{(1/2}))$ **Rubi [A]**

time = 0.17, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 85, $\frac{\text{number of rules}}{\text{integrand size}} = 0.012$, Rules used = {6816}

$$\log \left(\sqrt{x^2-4} + \sqrt{x^2-1} + 1 \right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*(-\text{Sqrt}[-4+x^2]+x^2*\text{Sqrt}[-4+x^2]-4*\text{Sqrt}[-1+x^2]+x^2*\text{Sqrt}[-1+x^2]))/((4-5*x^2+x^4)*(1+\text{Sqrt}[-4+x^2]+\text{Sqrt}[-1+x^2])),x]$

[Out] $\text{Log}[1+\text{Sqrt}[-4+x^2]+\text{Sqrt}[-1+x^2]]$

Rule 6816

$\text{Int}[(u_)/(y_), x_Symbol] :> \text{With}[\{q = \text{DerivativeDivides}[y, u, x]\}, \text{Simp}[q*\text{L}\text{og}[\text{RemoveContent}[y, x]], x] /; \text{!FalseQ}[q]]$

Rubi steps

$$\int \frac{x \left(-\sqrt{-4+x^2} + x^2 \sqrt{-4+x^2} - 4\sqrt{-1+x^2} + x^2 \sqrt{-1+x^2} \right)}{(4-5x^2+x^4) \left(1 + \sqrt{-4+x^2} + \sqrt{-1+x^2} \right)} dx = \log \left(1 + \sqrt{-4+x^2} + \sqrt{-1+x^2} \right)$$

Mathematica [A]

time = 0.57, size = 31, normalized size = 1.48

$$2 \tanh^{-1} \left(1 - \frac{2}{3} \sqrt{-4+x^2} + \frac{2}{3} \sqrt{-1+x^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x*(-Sqrt[-4 + x^2] + x^2*Sqrt[-4 + x^2] - 4*Sqrt[-1 + x^2] + x^2*Sqrt[-1 + x^2]))/((4 - 5*x^2 + x^4)*(1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2])), x]

[Out] 2*ArcTanh[1 - (2*Sqrt[-4 + x^2])/3 + (2*Sqrt[-1 + x^2])/3]

Mathics [A]

time = 261.50, size = 17, normalized size = 0.81

$$\text{Log}\left[1 + \sqrt{-4 + x^2} + \sqrt{-1 + x^2}\right]$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x*(Sqrt[x^2-1]*x^2-4*Sqrt[x^2-1]+Sqrt[x^2-4]*x^2-Sqrt[x^2-4])/((1+Sqrt[x^2-4]+Sqrt[x^2-1])*(x^4-5*x^2+4)), x]')

[Out] Log[1 + Sqrt[-4 + x ^ 2] + Sqrt[-1 + x ^ 2]]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1087 vs. 2(17) = 34.

time = 0.20, size = 1088, normalized size = 51.81

method	result
elliptic	$\sqrt{(x^2 - 4)(x^2 - 1)} \left(\frac{\ln(x^2 - 5)}{4} + \frac{\ln\left(-\frac{5}{2} + x^2 + \sqrt{x^4 - 5x^2 + 4}\right)}{4} + \frac{\operatorname{arctanh}\left(\frac{5x^2 - 17}{4\sqrt{(x^2 - 5)^2 + 5x^2 - 21}}\right)}{4} + \dots \right)$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(1/2))/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)), x, method=_RETURNVERBOSE)

[Out] -1/4/(2+5^(1/2))/(-2+5^(1/2))*((-2+x)^2+4*x-8)^(1/2)-1/2/(2+5^(1/2))/(-2+5^(1/2))*ln(x+((-2+x)^2+4*x-8)^(1/2))-1/4/(2+5^(1/2))/(-2+5^(1/2))*((2+x)^2-4*x-8)^(1/2)+1/2/(2+5^(1/2))/(-2+5^(1/2))*ln(x+((2+x)^2-4*x-8)^(1/2))+1/2/(5^(1/2)+1)/(5^(1/2)-1)*((1+x)^2-2*x-2)^(1/2)-1/2/(5^(1/2)+1)/(5^(1/2)-1)*ln(x+((1+x)^2-2*x-2)^(1/2))+1/2/(5^(1/2)+1)/(5^(1/2)-1)*((-1+x)^2-2+2*x)^(1/2)+1/2/(5^(1/2)+1)/(5^(1/2)-1)*ln(x+((-1+x)^2-2+2*x)^(1/2))+1/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/5^(1/2)*ln(x+((x-5^(1/2))^2+2*5^(1/2)*(x-5^(1/2))+1)^(1/2))-1/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/5^(1/2)*ln(x+((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+1)^(1/2))-1/2/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/5^(1/2)*ln(x+((x-5^(1/2))^2+2*5^(1/2)*(x-5

[Out] $-1/4*\log(4*x^4 - (4*x^2 - 1)*\sqrt{x^2 - 1}*\sqrt{x^2 - 4} - 21*x^2 + 23) - 1/4*\log(x^2 - \sqrt{x^2 - 1}*(x + 2) + 2*x - 1) + 1/4*\log(x^2 - \sqrt{x^2 - 4}*(x + 1) + x - 4) - 1/4*\log(x^2 - \sqrt{x^2 - 4}*(x - 1) - x - 4) + 1/4*\log(x^2 - \sqrt{x^2 - 1}*(x - 2) - 2*x - 1) + 1/4*\log(x^2 - 5) + 1/4*\log(-x^2 + \sqrt{x^2 - 1}*\sqrt{x^2 - 4} + 7)$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-(x**2-4)**(1/2)+x**2*(x**2-4)**(1/2)-4*(x**2-1)**(1/2)+x**2*(x**2-1)**(1/2))/(x**4-5*x**2+4)/(1+(x**2-4)**(1/2)+(x**2-1)**(1/2)),x)`

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(17) = 34.

time = 0.01, size = 98, normalized size = 4.67

$$\frac{\ln|\sqrt{x^2-4}-1|}{2} + 2 \left(-\frac{\ln(\sqrt{x^2-1}-\sqrt{x^2-4})}{4} - \frac{\ln|\sqrt{x^2-1}-\sqrt{x^2-4}-1|}{4} + \frac{\ln|\sqrt{x^2-1}-\sqrt{x^2-4}+3|}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(1/2))/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x)`

[Out] $-1/2*\log(\sqrt{x^2 - 1} - \sqrt{x^2 - 4}) + 1/2*\log(\text{abs}(\sqrt{x^2 - 1} - \sqrt{x^2 - 4} + 3)) - 1/2*\log(\text{abs}(\sqrt{x^2 - 1} - \sqrt{x^2 - 4} - 1)) + 1/2*\log(\text{abs}(\sqrt{x^2 - 4} - 1))$

Mupad [B]

time = 2.37, size = 172, normalized size = 8.19

$$\frac{\ln(x-\sqrt{5})}{4} - \operatorname{atanh}\left(\frac{\sqrt{3}-\sqrt{x^2-1}}{\sqrt{x^2-4}}\right) + \frac{\operatorname{atanh}\left(\frac{\sqrt{x^2-1}}{2}\right)}{2} + \frac{\ln(x+\sqrt{5})}{4} - \frac{7 \operatorname{atanh}\left(\frac{4(\sqrt{3}-\sqrt{x^2-1})}{\sqrt{x^2-4}\left(\frac{(\sqrt{3}-\sqrt{x^2-1})^2}{x^2-4}+1\right)}\right)}{4} + \frac{5 \operatorname{atanh}\left(\frac{12150(\sqrt{3}-\sqrt{x^2-1})}{\sqrt{x^2-4}\left(\frac{6075(\sqrt{3}-\sqrt{x^2-1})^2}{x^2-4}+\frac{6075}{2}\right)}\right)}{4} - \frac{\operatorname{atanh}\left(\frac{\sqrt{x^2-4}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x*(4*(x^2-1)^(1/2)+(x^2-4)^(1/2)-x^2*(x^2-1)^(1/2)-x^2*(x^2-4)^(1/2)))/((x^4-5*x^2+4)*((x^2-1)^(1/2)+(x^2-4)^(1/2)+1)),x)`

[Out] $\log(x - 5^{(1/2)})/4 - \operatorname{atanh}((3^{(1/2)} - (x^2 - 1)^{(1/2)})/(x^2 - 4)^{(1/2)}) + \operatorname{atanh}((x^2 - 1)^{(1/2)}/2)/2 + \log(x + 5^{(1/2)})/4 - (7*\operatorname{atanh}((4*(3^{(1/2)} - (x^2 - 1)^{(1/2)})/(x^2 - 4)^{(1/2)*((3^{(1/2)} - (x^2 - 1)^{(1/2)})^2/(x^2 - 4) + 1))))/4 + (5*\operatorname{atanh}((12150*(3^{(1/2)} - (x^2 - 1)^{(1/2)})/(x^2 - 4)^{(1/2)*((6075*(3^{(1/2)} - (x^2 - 1)^{(1/2)})^2/(2*(x^2 - 4)) + 6075/2)))))/4 - \operatorname{atanh}((x^2 - 4)^{(1/2)}/2)$

$$3.281 \quad \int \left(\sqrt{9 - 4\sqrt{2}} x - \sqrt{2} \sqrt{1 + 4x + 2x^2 + x^4} \right) dx$$

Optimal. Leaf size=4030

result too large to display

[Out] $\frac{1}{2}x^2(-1+2^{1/2})-2^{1/2}(-1/3(x^4+2x^2+4x+1)^{1/2}+1/3(1+x)(x^4+2x^2+4x+1)^{1/2}+4I(-13+3\sqrt{33})^{1/3}(x^4+2x^2+4x+1)^{1/2}/(4\sqrt{2/3})(-I+3^{1/2})-2I(-13+3\sqrt{33})^{1/3}+6Ix(-13+3\sqrt{33})^{1/3}+2^{1/3}(3^{1/2}+I)(-13+3\sqrt{33})^{2/3})-8\sqrt{2/3}\text{EllipticE}((26-6\sqrt{33})(1/2)+6x(-13+3\sqrt{33})^{1/2}+(-13-13I\sqrt{3}^{1/2}+9I\sqrt{11}^{1/2}+3\sqrt{33}^{1/2})(-26+6\sqrt{33})^{1/3}+4I(3^{1/2}+I)(-26+6\sqrt{33})^{2/3})^{1/2}/((39+13I\sqrt{3}^{1/2}-9I\sqrt{11}^{1/2}-9\sqrt{33}^{1/2}+4(3-I\sqrt{3}^{1/2})(-26+6\sqrt{33})^{1/3})/(39-13I\sqrt{3}^{1/2}+9I\sqrt{11}^{1/2}-9\sqrt{33}^{1/2}+4(3+I\sqrt{3}^{1/2})(-26+6\sqrt{33})^{1/3}))^{1/2}/(26-6\sqrt{33})^{1/2}+6x(-13+3\sqrt{33})^{1/2}+(-13+13I\sqrt{3}^{1/2}-9I\sqrt{11}^{1/2}+3\sqrt{33}^{1/2})(-26+6\sqrt{33})^{1/3}+(-4-4I\sqrt{3}^{1/2})(-26+6\sqrt{33})^{2/3})^{1/2}, ((84+28I\sqrt{3}^{1/2}-12I\sqrt{11}^{1/2}-12\sqrt{33}^{1/2}+(3-I\sqrt{3}^{1/2}-3I\sqrt{11}^{1/2}+3\sqrt{33}^{1/2})(-26+6\sqrt{33})^{1/3})/(84-28I\sqrt{3}^{1/2}+12I\sqrt{11}^{1/2}-12\sqrt{33}^{1/2}+(3+I\sqrt{3}^{1/2}+3I\sqrt{11}^{1/2}+3\sqrt{33}^{1/2})(-26+6\sqrt{33})^{1/3}))^{1/2})\sqrt{2}x^2+4x+1)^{1/2}3^{1/2}/(-13+3\sqrt{33})^{1/2}+4(-26+6\sqrt{33})^{1/3})^{1/2}(I(-19899+x(59697-10335\sqrt{33})^{1/2}+3445\sqrt{33}^{1/2}+(-26+6\sqrt{33})^{2/3})(-2574+466\sqrt{33})^{1/2}+(-26+6\sqrt{33})^{1/3})(-19899+3445\sqrt{33})^{1/2}))/(-39-13I\sqrt{3}^{1/2}+9I\sqrt{11}^{1/2}+9\sqrt{33}^{1/2}+4I(3I\sqrt{3}^{1/2}+3^{1/2})(-26+6\sqrt{33})^{1/3})/(26-6\sqrt{33})^{1/2}+6x(-13+3\sqrt{33})^{1/2}+(-13+13I\sqrt{3}^{1/2}-9I\sqrt{11}^{1/2}+3\sqrt{33}^{1/2})(-26+6\sqrt{33})^{1/3}+(-4-4I\sqrt{3}^{1/2})(-26+6\sqrt{33})^{2/3}))^{1/2}/(4\sqrt{2/3}-(-13+3\sqrt{33})^{1/3}+3x(-13+3\sqrt{33})^{1/3}-2^{1/3}(-13+3\sqrt{33})^{2/3})/(26-6\sqrt{33})^{1/2}+6x(-13+3\sqrt{33})^{1/2}+(-13+13I\sqrt{3}^{1/2}-9I\sqrt{11}^{1/2}+3\sqrt{33}^{1/2})(-26+6\sqrt{33})^{1/3}+(-4-4I\sqrt{3}^{1/2})(-26+6\sqrt{33})^{2/3}))^{1/2}/(I(1+x)/(26-6\sqrt{33})^{1/2}+6x(-13+3\sqrt{33})^{1/2}+(-13+13I\sqrt{3}^{1/2}-9I\sqrt{11}^{1/2}+3\sqrt{33}^{1/2})(-26+6\sqrt{33})^{1/3}+(-4-4I\sqrt{3}^{1/2})(-26+6\sqrt{33})^{2/3}))/((104-24\sqrt{33})^{1/2}+(-13-13I\sqrt{3}^{1/2}+9I\sqrt{11}^{1/2}+3\sqrt{33}^{1/2})(-26+6\sqrt{33})^{1/3}+4I(3^{1/2}+I)(-26+6\sqrt{33})^{2/3}))^{1/2}/(26-6\sqrt{33})^{1/2}+6x(-13+3\sqrt{33})^{1/2}+(-13-13I\sqrt{3}^{1/2}+9I\sqrt{11}^{1/2}+3\sqrt{33}^{1/2})(-26+6\sqrt{33})^{1/3}+4I(3^{1/2}+I)(-26+6\sqrt{33})^{2/3})^{1/2}+1/6\text{EllipticPi}(1/6(13-3\sqrt{33})^{1/2}-2^{1/3})(-13+3\sqrt{33})^{4/3}+4(-26+6\sqrt{33})^{2/3}+x(-39+9\sqrt{33})^{1/2}))^{1/2}2^{5/6}3^{1/2}/(-13+3\sqrt{33})^{1/2})^{2/3}/(1+x)^{1/2}/((-39+13I\sqrt{3}^{1/2}-9I\sqrt{11}^{1/2}+9\sqrt{33}^{1/2}-4I(-3I\sqrt{3}^{1/2}+3^{1/2})(-26+6\sqrt{33})^{1/3})/(104-24\sqrt{33})^{1/2}+(-13+13I\sqrt{3}^{1/2}-9I\sqrt{11}^{1/2}+3\sqrt{33}^{1/2})(-26+6\sqrt{33})^{1/3}+(-4-4I\sqrt{3}^{1/2})(-26+6\sqrt{33})^{2/3}))^{1/2}, 2^{1/3}(4\sqrt{2/3})(-3I\sqrt{3}^{1/2}+3I\sqrt{3}^{1/2})(-13+3\sqrt{33})^{1/2})^{2/3})/(4\sqrt{2/3})(-I+3^{1/2})-8I(-13+3\sqrt{33})^{1/3}+2^{1/3}(3^{1/2}+I)(-13+3\sqrt{33})^{2/3}), ((84-28I\sqrt{3}^{1/2}+12I\sqrt{11}^{1/2}-12\sqrt{33}^{1/2}+(3+I\sqrt{3}^{1/2}+3I\sqrt{11}^{1/2}+3\sqrt{33}^{1/2})(-26+6\sqrt{33})^{1/3})/(84+28I\sqrt{3}^{1/2}-12I\sqrt{11}^{1/2}-12\sqrt{33}^{1/2}+(3-I\sqrt{3}^{1/2}-3I\sqrt{11}^{1/2}+3\sqrt{33}^{1/2})(-26+6\sqrt{33})^{1/3}))^{1/2})\sqrt{2}x^2+4x+1)^{1/2}+2(-13+3\sqrt{33})^{1/3}-2^{1/3}(-13+3\sqrt{33})^{2/3})$

$$\begin{aligned}
& 3^{(1/2)})^{(2/3)} * (4 * 2^{(2/3)} * (3^{(1/2)} + I) - 4 * I * (-13 + 3 * 33^{(1/2)}))^{(1/3)} + 2^{(1/3)} * (-I + 3^{(1/2)}) * (-13 + 3 * 33^{(1/2)})^{(2/3)} * (4 * 2^{(2/3)} * (-I + 3^{(1/2)}) + 4 * I * (-13 + 3 * 33^{(1/2)}))^{(1/3)} + 2^{(1/3)} * (3^{(1/2)} + I) * (-13 + 3 * 33^{(1/2)})^{(2/3)} * (1 + x)^{(1/2)} * (x^4 + 2 * x^2 + 4 * x + 1)^{(1/2)} * ((-39 + 13 * I * 3^{(1/2)} - 9 * I * 11^{(1/2)} + 9 * 33^{(1/2)} - 4 * I * (-3 * I + 3^{(1/2)})) * (-26 + 6 * 33^{(1/2)})^{(1/3)} / (104 - 24 * 33^{(1/2)} + (-13 + 13 * I * 3^{(1/2)} - 9 * I * 11^{(1/2)} + 3 * 33^{(1/2)}) * (-26 + 6 * 33^{(1/2)})^{(1/3)} + (-4 - 4 * I * 3^{(1/2)}) * (-26 + 6 * 33^{(1/2)})^{(2/3)})^{(1/2)} * ((104 - 24 * 33^{(1/2)} + 2 * (1 + 14 * I * 3^{(1/2)} - 6 * I * 11^{(1/2)} + 33^{(1/2)}) * (-26 + 6 * 33^{(1/2)})^{(1/3)} + (-7 - I * 3^{(1/2)} - 3 * I * 11^{(1/2)} + 33^{(1/2)}) * (-26 + 6 * 33^{(1/2)})^{(2/3)} + 2 * x * (-52 + 12 * 33^{(1/2)} + 2^{(1/3)} * (-13 + 3 * 33^{(1/2)})^{(4/3)} - 4 * (-26 + 6 * 33^{(1/2)})^{(2/3)})) / (1 + x) / (-39 + 13 * I * 3^{(1/2)} - 9 * I * 11^{(1/2)} + 9 * 33^{(1/2)} - 4 * I * (-3 * I + 3^{(1/2)}) * (-26 + 6 * 33^{(1/2)})^{(1/3)})^{(1/2)} * ((104 - 24 * 33^{(1/2)} + 2 * (1 - 14 * I * 3^{(1/2)} + 6 * I * 11^{(1/2)} + 33^{(1/2)}) * (-26 + 6 * 33^{(1/2)})^{(1/3)} + (-7 + I * 3^{(1/2)} + 3 * I * 11^{(1/2)} + 33^{(1/2)}) * (-26 + 6 * 33^{(1/2)})^{(2/3)} + 2 * x * (-52 + 12 * 33^{(1/2)} + 2^{(1/3)} * (-13 + 3 * 33^{(1/2)})^{(4/3)} - 4 * (-26 + 6 * 33^{(1/2)})^{(2/3)})) / (1 + x) / (-39 - 13 * I * 3^{(1/2)} + 9 * I * 11^{(1/2)} + 9 * 33^{(1/2)} + 4 * I * (3 * I + 3^{(1/2)}) * (-26 + 6 * 33^{(1/2)})^{(1/3)})^{(1/2)} * 2^{(5/6)} * 3^{(1/2)} / (4 * 2^{(2/3)} * (3^{(1/2)} + I) + 2 * I * (-13 + 3 * 33^{(1/2)})^{(1/3)} - 6 * I * x * (-13 + 3 * 33^{(1/2)})^{(1/3)} + 2^{(1/3)} * (-I + 3^{(1/2)}) * (-13 + 3 * 33^{(1/2)})^{(2/3)}) / (4 * 2^{(2/3)} * (-I + 3^{(1/2)}) - 2 * I * (-13 + 3 * 33^{(1/2)})^{(1/3)} + 6 * I * x * (-13 + 3 * 33^{(1/2)})^{(1/3)} + 2^{(1/3)} * (3^{(1/2)} + I) * (-13 + 3 * 33^{(1/2)})^{(2/3)}) / (13 - 3 * 33^{(1/2)} - 2^{(1/3)} * (-13 + 3 * 33^{(1/2)})^{(4/3)} + 4 * (-26 + 6 * 33^{(1/2)})^{(2/3)} + x * (-39 + 9 * 33^{(1/2)}))^{(1/2)} + 1 / 18 * I * EllipticF(1/6 * I * (-52 + 12 * 33^{(1/2)} + 2^{(1/3)} * (-13 + 3 * 33^{(1/2)})^{(4/3)} - 4 * (-26 + 6 * 33^{(1/2)})^{(2/3)})^{(1/2)} * (26 - 6 * 33^{(1/2)} + 6 * x * (-13 + 3 * 33^{(1/2)}) + (-13 - 13 * I * 3^{(1/2)} + 9 * I * 11^{(1/2)} + 3 * 33^{(1/2)}) * (-26 + 6 * 33^{(1/2)})^{(1/3)} + 4 * I * (3^{(1/2)} + I) * (-26 + 6 * 33^{(1/2)})^{(2/3)})^{(1/2)} * 2^{(5/6)} * 3^{(1/2)} / (-13 + 3 * 33^{(1/2)})^{(2/3)} / (1 + x)^{(1/2)} / (39 + 13 * I * 3^{(1/2)} - 9 * I * 11^{(1/2)} - 9 * 33^{(1/2)} + 4 * (3 - I * 3^{(1/2)}) * (-26 + 6 * 33^{(1/2)})^{(1/3)})^{(1/2)}, ((84 * I - 28 * 3^{(1/2)} + 12 * 11^{(1/2)} - 12 * I * 33^{(1/2)} + (3 * I + 3^{(1/2)} + 3 * 11^{(1/2)} + 3 * I * 33^{(1/2)}) * (-26 + 6 * 33^{(1/2)})^{(1/3)}) / (-56 * 3^{(1/2)} + 24 * 11^{(1/2)} + 2 * (-26 + 6 * 33^{(1/2)})^{(1/3)} * (3^{(1/2)} + 3 * 11^{(1/2)}))^{(1/2)} * (2^{(1/3)} * (13 - 13 * I * 3^{(1/2)} + 9 * I * 11^{(1/2)} - 3 * 33^{(1/2)}) + 4 * 2^{(2/3)} * (1 + I * 3^{(1/2)}) * (-13 + 3 * 33^{(1/2)})^{(1/3)} + 20 * (-13 + 3 * 33^{(1/2)})^{(2/3)}) * (4 * 2^{(2/3)} * (3^{(1/2)} + I) + 8 * I * (-13 + 3 * 33^{(1/2)})^{(1/3)} + 2^{(1/3)} * (-I + 3^{(1/2)}) * (-13 + 3 * 33^{(1/2)})^{(2/3)}) * (x^4 + 2 * x^2 + 4 * x + 1)^{(1/2)} * (1 / (-13 + 3 * 33^{(1/2)} + 4 * (-26 + 6 * 33^{(1/2)})^{(1/3)}) * (-52 + 12 * 33^{(1/2)} + 2^{(1/3)} * (-13 + 3 * 33^{(1/2)})^{(4/3)} - 4 * (-26 + 6 * 33^{(1/2)})^{(2/3)}))^{(1/2)} * ((-8 * I * (-13 + 3 * 33^{(1/2)}) + (-43 * I - 13 * 3^{(1/2)} + 9 * 11^{(1/2)} + 5 * I * 33^{(1/2)}) * (-26 + 6 * 33^{(1/2)})^{(1/3)} + (2 * I + 4 * 3^{(1/2)} - 2 * I * 33^{(1/2)}) * (-26 + 6 * 33^{(1/2)})^{(2/3)} + x * (8 * I * (-13 + 3 * 33^{(1/2)}) + (13 * I - 13 * 3^{(1/2)} + 9 * 11^{(1/2)} - 3 * I * 33^{(1/2)}) * (-26 + 6 * 33^{(1/2)})^{(1/3)} + 4 * (3^{(1/2)} + I) * (-26 + 6 * 33^{(1/2)})^{(2/3)})) / (1 + x)^{(1/2)} * 2^{(1/3)} * 3^{(1/4)} / (-13 + 3 * 33^{(1/2)})^{(1/3)} / (4 * 2^{(2/3)} * (-I + 3^{(1/2)}) - 2 * I * (-13 + 3 * 33^{(1/2)})^{(1/3)} + 6 * I * x * (-13 + 3 * 33^{(1/2)})^{(1/3)} + 2^{(1/3)} * (3^{(1/2)} + I) * (-13 + 3 * 33^{(1/2)})^{(2/3)}) / (1 + x)^{(1/2)} / (39 + 13 * I * 3^{(1/2)} - 9 * I * 11^{(1/2)} - 9 * 33^{(1/2)} + 4 * (3 - I * 3^{(1/2)}) * (-26 + 6 * 33^{(1/2)})^{(1/3)})^{(1/2)} / (26 - 6 * 33^{(1/2)} + 6 * x * (-13 + 3 * 33^{(1/2)}) + (-13 - 13 * I * 3^{(1/2)} + 9 * I * 11^{(1/2)} + 3 * 33^{(1/2)}) * (-26 + 6 * 33^{(1/2)})^{(1/3)} + 4 * I * (3^{(1/2)} + I) * (-26 + 6 * 33^{(1/2)})^{(2/3)})^{(1/2)} / ((-104 + 24 * 33^{(1/2)} - (5 - 3 * I * 3^{(1/2)} + 3 * I * 11^{(1/2)} + 33^{(1/2)}) * (-26 + 6 * 33^{(1/2)})^{(2/3)} + (-26 + 6 * 33^{(1/2)})^{(1/3)} * (-41 + 15 * I * 3^{(1/2)} - 3 * I * 11^{(1/2)} + 7 * 33^{(1/2)})) + x * (104 - 24 * 33^{(1/2)} + (-13 - 13 * I * 3^{(1/2)} + 9 * I * 11^{(1/2)} + 3 * 33^{(1/2)})) * (-26 + 6 * 33^{(1/2)})^{(1/3)} + 4 * I * (3^{(1/2)} + I) * (-26 + 6 * 33^{(1/2)})^{(2/3)}) / (1 + x) / (-
\end{aligned}$$

39-13*I*3^(1/2)+9*I*11^(1/2)+9*33^(1/2)+4*I*(3*I+3^(1/2))*(-26+6*33^(1/2))^(1/3))^(1/2))

Rubi [F]

time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(\sqrt{9 - 4\sqrt{2}} x - \sqrt{2} \sqrt{1 + 4x + 2x^2 + x^4} \right) dx$$

Verification is not applicable to the result.

[In] Int[Sqrt[9 - 4*Sqrt[2]]*x - Sqrt[2]*Sqrt[1 + 4*x + 2*x^2 + x^4], x]

[Out] (Sqrt[9 - 4*Sqrt[2]]*x^2)/2 - Sqrt[2]*Defer[Int][Sqrt[1 + 4*x + 2*x^2 + x^4], x]

Rubi steps

$$\int \left(\sqrt{9 - 4\sqrt{2}} x - \sqrt{2} \sqrt{1 + 4x + 2x^2 + x^4} \right) dx = \frac{1}{2} \sqrt{9 - 4\sqrt{2}} x^2 - \sqrt{2} \int \sqrt{1 + 4x + 2x^2 + x^4} dx$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

time = 8.02, size = 3168, normalized size = 0.79

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[9 - 4*Sqrt[2]]*x - Sqrt[2]*Sqrt[1 + 4*x + 2*x^2 + x^4], x]

[Out] (Sqrt[9 - 4*Sqrt[2]]*x^2)/2 - (Sqrt[2]*x*Sqrt[1 + 4*x + 2*x^2 + x^4])/3 - (2*Sqrt[2]*((6*(x - Root[1 + 3*#1 - #1^2 + #1^3 &, 1, 0])^2*(-EllipticF[ArcSin[Sqrt[-((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 &, 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 &, 3, 0])]/((x - Root[1 + 3*#1 - #1^2 + #1^3 &, 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 &, 3, 0])]))], ((Root[1 + 3*#1 - #1^2 + #1^3 &, 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 &, 2, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 &, 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 &, 2, 0])*(Root[1 + 3*#1 - #1^2 + #1^3 &, 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 &, 3, 0]))]*Root[1 + 3*#1 - #1^2 + #1^3 &, 1, 0]) + EllipticPi[(1 + Root[1 + 3*#1 - #1^2 + #1^3 &, 3, 0])/(-Root[1 + 3*#1 - #1^2 + #1^3 &, 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 &, 3, 0]), ArcSin[Sqrt[-((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 &, 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 &, 3, 0])]/((x - Root[1 + 3*#1

$$\begin{aligned}
& - \#1^2 + \#1^3 \& , 1, 0])*(1 + \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0]))], \\
& ((\text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 1, 0] - \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 2 \\
& , 0])*(1 + \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0]))/((1 + \text{Root}[1 + 3*\#1 - \#1 \\
& ^2 + \#1^3 \& , 2, 0])*(\text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 1, 0] - \text{Root}[1 + 3*\#1 \\
& - \#1^2 + \#1^3 \& , 3, 0]))*(1 + \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 1, 0]))*Sq \\
& \text{rt}[(x - \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 2, 0])/((x - \text{Root}[1 + 3*\#1 - \#1^2 + \\
& \#1^3 \& , 1, 0])*(1 + \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 2, 0]))]*(-1 - \text{Root}[1 \\
& + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0])*Sqrt[(x - \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , \\
& 3, 0])/((x - \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 1, 0])*(1 + \text{Root}[1 + 3*\#1 - \#1 \\
& ^2 + \#1^3 \& , 3, 0]))]*Sqrt[-(((1 + x)*(\text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 1, \\
& 0] - \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0]))/((x - \text{Root}[1 + 3*\#1 - \#1^2 + \# \\
& 1^3 \& , 1, 0])*(1 + \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0])))]/(Sqrt[1 + 4* \\
& x + 2*x^2 + x^4]*(\text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 1, 0] - \text{Root}[1 + 3*\#1 - \# \\
& 1^2 + \#1^3 \& , 3, 0])) + (2*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(1 + x)*(-\text{Root}[1 + 3*\#1 \\
& - \#1^2 + \#1^3 \& , 1, 0] + \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0]))]/((x - \text{Ro \\
& o}t[1 + 3*\#1 - \#1^2 + \#1^3 \& , 1, 0])*(1 + \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 3, \\
& 0])))], ((\text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 1, 0] - \text{Root}[1 + 3*\#1 - \#1^2 + \# \\
& 1^3 \& , 2, 0])*(-1 - \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0]))/((-1 - \text{Root}[1 \\
& + 3*\#1 - \#1^2 + \#1^3 \& , 2, 0])*(\text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 1, 0] - \text{Ro \\
& o}t[1 + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0]))*(x - \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , \\
& 1, 0])^2*Sqrt[(x - \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 2, 0])/((x - \text{Root}[1 + 3 \\
& *\#1 - \#1^2 + \#1^3 \& , 1, 0])*(1 + \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 2, 0]))]* \\
& (-1 - \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0])*Sqrt[(x - \text{Root}[1 + 3*\#1 - \#1^2 \\
& + \#1^3 \& , 3, 0])/((x - \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 1, 0])*(1 + \text{Root}[1 \\
& + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0]))]*Sqrt[((1 + x)*(-\text{Root}[1 + 3*\#1 - \#1^2 + \# \\
& 1^3 \& , 1, 0] + \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0]))/((x - \text{Root}[1 + 3*\#1 \\
& - \#1^2 + \#1^3 \& , 1, 0])*(1 + \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0])))]/(S \\
& \text{qrt}[1 + 4*x + 2*x^2 + x^4]*(-\text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 1, 0] + \text{Root}[1 \\
& + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0])) + ((1 + x)*(x - \text{Root}[1 + 3*\#1 - \#1^2 + \#1 \\
& ^3 \& , 2, 0])*(x - \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0]) + (x - \text{Root}[1 + 3 \\
& *\#1 - \#1^2 + \#1^3 \& , 1, 0])^2*(1 + \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 1, 0])* \\
& \text{Sqrt}[(x - \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 2, 0])/((x - \text{Root}[1 + 3*\#1 - \#1^2 \\
& + \#1^3 \& , 1, 0])*(1 + \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 2, 0]))]*Sqrt[(x - \\
& \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0])/((x - \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& \\
& , 1, 0])*(1 + \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0]))]*Sqrt[-(((1 + x)*(\text{Ro \\
& o}t[1 + 3*\#1 - \#1^2 + \#1^3 \& , 1, 0] - \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0]) \\
&)/((x - \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 1, 0])*(1 + \text{Root}[1 + 3*\#1 - \#1^2 + \\
& \#1^3 \& , 3, 0])))]*(1 + \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0])*((\text{EllipticE} \\
& \text{ArcSin}[\text{Sqrt}[-(((1 + x)*(\text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 1, 0] - \text{Root}[1 + 3* \\
& \#1 - \#1^2 + \#1^3 \& , 3, 0]))/((x - \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 1, 0])*(\\
& 1 + \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0])))]], ((\text{Root}[1 + 3*\#1 - \#1^2 + \#1 \\
& ^3 \& , 1, 0] - \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 2, 0])*(1 + \text{Root}[1 + 3*\#1 - \\
& \#1^2 + \#1^3 \& , 3, 0]))/((1 + \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 2, 0])*(\text{Root}[\\
& 1 + 3*\#1 - \#1^2 + \#1^3 \& , 1, 0] - \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 3, 0]))] \\
& *(1 + \text{Root}[1 + 3*\#1 - \#1^2 + \#1^3 \& , 2, 0]))/(1 + \text{Root}[1 + 3*\#1 - \#1^2 + \#
\end{aligned}$$

```

1^3 & , 1, 0]) - (EllipticPi[(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])/(-
Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 3,
0]), ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1
+ 3*#1 - #1^2 + #1^3 & , 3, 0]))/(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1,
0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))], ((Root[1 + 3*#1 - #1^2
+ #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*
#1 - #1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(
Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3,
0])))*(1 - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #
1^3 & , 2, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/(-Root[1 + 3*#1 - #
1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]) + (EllipticF[
ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*
#1 - #1^2 + #1^3 & , 3, 0]))/(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(
1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))], ((Root[1 + 3*#1 - #1^2 + #1
^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*#1 -
#1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(Root[
1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]
*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 1
, 0]*(-Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3
& , 3, 0]) - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 -
#1^2 + #1^3 & , 1, 0])*(-Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1 + 3
*#1 - #1^2 + #1^3 & , 3, 0])))/Sqrt[1 + 4*x + 2*x^2 + x^4]))/3

```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[-4*Sqrt[2] + 9]*x - Sqrt[x^4 + 2*x^2 + 4*x + 1]*Sqr
t[2],x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]

time = 0.87, size = 4640, normalized size = 1.15

method	result	size
default	Expression too large to display	4640
elliptic	Expression too large to display	4646

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x,method=_RETURNVERBO
SE)
```

[Out] $\frac{1}{2}x^2(-1+2\sqrt{2})-2^{1/2}*(1/3*x*(x^4+2*x^2+4*x+1)^{1/2}+4/3*(-4/3-1/6$
 $* (26+6*33^{1/2})^{1/3}+4/3/(26+6*33^{1/2})^{1/3}+1/2*I*3^{1/2}*(-1/3*(26+6*$
 $33^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3}))*((1/2*(26+6*33^{1/2})^{1/3}-4/($
 $26+6*33^{1/2})^{1/3}-1/2*I*3^{1/2}*(-1/3*(26+6*33^{1/2})^{1/3}-8/3/(26+6*33$
 $^{1/2})^{1/3}))*(1+x)/(1/6*(26+6*33^{1/2})^{1/3}-4/3/(26+6*33^{1/2})^{1/3}+$
 $4/3-1/2*I*3^{1/2}*(-1/3*(26+6*33^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3}))/$
 $(x+1/3*(26+6*33^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3}-1/3)^{1/2}*(x+1/3*(2$
 $6+6*33^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3}-1/3)^2*((-1/3*(26+6*33^{1/2})$
 $^{1/3}+8/3/(26+6*33^{1/2})^{1/3}+4/3)*(x-1/6*(26+6*33^{1/2})^{1/3}+4/3/(26+$
 $6*33^{1/2})^{1/3}-1/3-1/2*I*3^{1/2}*(-1/3*(26+6*33^{1/2})^{1/3}-8/3/(26+6*3$
 $3^{1/2})^{1/3}))/ (1/6*(26+6*33^{1/2})^{1/3}-4/3/(26+6*33^{1/2})^{1/3}+4/3+1$
 $/2*I*3^{1/2}*(-1/3*(26+6*33^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3}))/ (x+1/3$
 $*(26+6*33^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3}-1/3)^{1/2}*((-1/3*(26+6*3$
 $3^{1/2})^{1/3}+8/3/(26+6*33^{1/2})^{1/3}+4/3)*(x-1/6*(26+6*33^{1/2})^{1/3}+$
 $4/3/(26+6*33^{1/2})^{1/3}-1/3+1/2*I*3^{1/2}*(-1/3*(26+6*33^{1/2})^{1/3}-8/3$
 $/(26+6*33^{1/2})^{1/3}))/ (1/6*(26+6*33^{1/2})^{1/3}-4/3/(26+6*33^{1/2})^{1/3}-1/$
 $3)+4/3-1/2*I*3^{1/2}*(-1/3*(26+6*33^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3})$
 $)/ (x+1/3*(26+6*33^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3}-1/3)^{1/2} / (1/2*($
 $26+6*33^{1/2})^{1/3}-4/(26+6*33^{1/2})^{1/3}-1/2*I*3^{1/2}*(-1/3*(26+6*33^{1/2}$
 $^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3}))/ (-1/3*(26+6*33^{1/2})^{1/3}+8/3/(26$
 $+6*33^{1/2})^{1/3}+4/3) / ((1+x)*(x+1/3*(26+6*33^{1/2})^{1/3}-8/3/(26+6*33^{1/2}$
 $^{1/2})^{1/3}-1/3)*(x-1/6*(26+6*33^{1/2})^{1/3}+4/3/(26+6*33^{1/2})^{1/3}-1/3-$
 $1/2*I*3^{1/2}*(-1/3*(26+6*33^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3}))* (x-1/$
 $6*(26+6*33^{1/2})^{1/3}+4/3/(26+6*33^{1/2})^{1/3}-1/3+1/2*I*3^{1/2}*(-1/3*($
 $26+6*33^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3})))^{1/2} * \text{EllipticF}(((1/2*(26$
 $+6*33^{1/2})^{1/3}-4/(26+6*33^{1/2})^{1/3}-1/2*I*3^{1/2}*(-1/3*(26+6*33^{1/2}$
 $^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3}))* (1+x)/(1/6*(26+6*33^{1/2})^{1/3}-4/3/$
 $(26+6*33^{1/2})^{1/3}+4/3-1/2*I*3^{1/2}*(-1/3*(26+6*33^{1/2})^{1/3}-8/3/(26$
 $+6*33^{1/2})^{1/3}))/ (x+1/3*(26+6*33^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3}$
 $-1/3)^{1/2}, ((-1/2*(26+6*33^{1/2})^{1/3}+4/(26+6*33^{1/2})^{1/3}-1/2*I*3^{1/2}$
 $(1/2)*(-1/3*(26+6*33^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3}))* (-4/3-1/6*(26+$
 $6*33^{1/2})^{1/3}+4/3/(26+6*33^{1/2})^{1/3}+1/2*I*3^{1/2}*(-1/3*(26+6*33^{1/2}$
 $^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3}))/ (-4/3-1/6*(26+6*33^{1/2})^{1/3}+4/3/$
 $(26+6*33^{1/2})^{1/3}-1/2*I*3^{1/2}*(-1/3*(26+6*33^{1/2})^{1/3}-8/3/(26+6*3$
 $3^{1/2})^{1/3}))/ (-1/2*(26+6*33^{1/2})^{1/3}+4/(26+6*33^{1/2})^{1/3}+1/2*I*$
 $3^{1/2}*(-1/3*(26+6*33^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3})))^{1/2} + 4*($
 $-4/3-1/6*(26+6*33^{1/2})^{1/3}+4/3/(26+6*33^{1/2})^{1/3}+1/2*I*3^{1/2}*(-1/$
 $3*(26+6*33^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3}))* ((1/2*(26+6*33^{1/2})^{1/2}$
 $(1/3)-4/(26+6*33^{1/2})^{1/3}-1/2*I*3^{1/2}*(-1/3*(26+6*33^{1/2})^{1/3}-8/3/$
 $(26+6*33^{1/2})^{1/3}))* (1+x)/(1/6*(26+6*33^{1/2})^{1/3}-4/3/(26+6*33^{1/2}$
 $)^{1/3}+4/3-1/2*I*3^{1/2}*(-1/3*(26+6*33^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3}$
 $))/ (x+1/3*(26+6*33^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3}-1/3)^{1/2}*($
 $x+1/3*(26+6*33^{1/2})^{1/3}-8/3/(26+6*33^{1/2})^{1/3}-1/3)^2*((-1/3*(26+6*3$
 $3^{1/2})^{1/3}+8/3/(26+6*33^{1/2})^{1/3}+4/3)*(x-1/6*(26+6*33^{1/2})^{1/3}+$
 $4/3/(26+6*33^{1/2})^{1/3}-1/3-1/2*I*3^{1/2}*(-1/3*(26+6*33^{1/2})^{1/3}-8/3$

$$\begin{aligned}
& /((26+6*33^{(1/2)})^{(1/3)}) / ((1/6*(26+6*33^{(1/2)})^{(1/3)} - 4/3/(26+6*33^{(1/2)})^{(1/3)} \\
& + 4/3 + 1/2*I*3^{(1/2)} * (-1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)}) \\
&) / (x + 1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)} - 1/3)^{(1/2)} * ((-1/3 \\
& * (26+6*33^{(1/2)})^{(1/3)} + 8/3/(26+6*33^{(1/2)})^{(1/3)} + 4/3) * (x - 1/6*(26+6*33^{(1/2)}) \\
&)^{(1/3)} + 4/3/(26+6*33^{(1/2)})^{(1/3)} - 1/3 + 1/2*I*3^{(1/2)} * (-1/3*(26+6*33^{(1/2)})^{(1/3)} \\
& - 8/3/(26+6*33^{(1/2)})^{(1/3)}) / ((1/6*(26+6*33^{(1/2)})^{(1/3)} - 4/3/(26+6*33^{(1/2)})^{(1/3)} \\
&)^{(1/3)} + 4/3 - 1/2*I*3^{(1/2)} * (-1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)} \\
&)^{(1/3)}) / (x + 1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)} - 1/3)^{(1/2)} \\
&) / (1/2*(26+6*33^{(1/2)})^{(1/3)} - 4/(26+6*33^{(1/2)})^{(1/3)} - 1/2*I*3^{(1/2)} * (-1/3*(26+6*33^{(1/2)})^{(1/3)} \\
& - 8/3/(26+6*33^{(1/2)})^{(1/3)}) / (-1/3*(26+6*33^{(1/2)})^{(1/3)} + 8/3/(26+6*33^{(1/2)})^{(1/3)} + 4/3) / ((1+x) * (x + 1/3*(26+6*33^{(1/2)})^{(1/3)} \\
& - 8/3/(26+6*33^{(1/2)})^{(1/3)} - 1/3) * (x - 1/6*(26+6*33^{(1/2)})^{(1/3)} + 4/3/(26+6*33^{(1/2)})^{(1/3)} \\
& - 1/3 - 1/2*I*3^{(1/2)} * (-1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)} \\
&)) * (x - 1/6*(26+6*33^{(1/2)})^{(1/3)} + 4/3/(26+6*33^{(1/2)})^{(1/3)} - 1/3 + 1/2*I*3^{(1/2)} \\
& * (-1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)}))^{(1/2)} * ((-1/3*(26+6*33^{(1/2)})^{(1/3)} \\
& + 8/3/(26+6*33^{(1/2)})^{(1/3)} + 1/3) * \text{EllipticF}(((1/2*(26+6*33^{(1/2)})^{(1/3)} - 4/(26+6*33^{(1/2)})^{(1/3)} \\
& - 1/2*I*3^{(1/2)} * (-1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)})) * (1+x) / (1/6*(26+6*33^{(1/2)})^{(1/3)} \\
& - 4/3/(26+6*33^{(1/2)})^{(1/3)} + 4/3 - 1/2*I*3^{(1/2)} * (-1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)} \\
&)^{(1/3)}) / (x + 1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)} - 1/3)^{(1/2)}, ((-1/2*(26+6*33^{(1/2)})^{(1/3)} \\
& + 4/(26+6*33^{(1/2)})^{(1/3)} - 1/2*I*3^{(1/2)} * (-1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)})) * (-4/3 - 1/6*(26+6*33^{(1/2)})^{(1/3)} \\
& + 4/3/(26+6*33^{(1/2)})^{(1/3)} + 1/2*I*3^{(1/2)} * (-1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)})) / (-4/3 - 1/6*(26+6*33^{(1/2)})^{(1/3)} \\
& + 4/3/(26+6*33^{(1/2)})^{(1/3)} - 1/2*I*3^{(1/2)} * (-1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)}))^{(1/2)} \\
& + (-4/3 + 1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)}) * \text{EllipticPi}(((1/2*(26+6*33^{(1/2)})^{(1/3)} - 4/(26+6*33^{(1/2)})^{(1/3)} \\
& - 1/2*I*3^{(1/2)} * (-1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)})) * (1+x) / (1/6*(26+6*33^{(1/2)})^{(1/3)} \\
& - 4/3/(26+6*33^{(1/2)})^{(1/3)} + 4/3 - 1/2*I*3^{(1/2)} * (-1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)} \\
&)^{(1/3)}) / (x + 1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)} - 1/3)^{(1/2)}, (1/6*(26+6*33^{(1/2)})^{(1/3)} \\
& - 4/3/(26+6*33^{(1/2)})^{(1/3)} + 4/3 - 1/2*I*3^{(1/2)} * (-1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)} \\
&)^{(1/3)}) / (1/2*(26+6*33^{(1/2)})^{(1/3)} - 4/(26+6*33^{(1/2)})^{(1/3)} - 1/2*I*3^{(1/2)} * (-1/3*(26+6*33^{(1/2)})^{(1/3)} \\
& - 8/3/(26+6*33^{(1/2)})^{(1/3)})) / ((-1/2*(26+6*33^{(1/2)})^{(1/3)} + 4/(26+6*33^{(1/2)})^{(1/3)} - 1/2*I*3^{(1/2)} * (-1/3*(26+6*33^{(1/2)})^{(1/3)} \\
& - 8/3/(26+6*33^{(1/2)})^{(1/3)})) * (-4/3 - 1/6*(26+6*33^{(1/2)})^{(1/3)} + 4/3/(26+6*33^{(1/2)})^{(1/3)} + 1/2*I*3^{(1/2)} * \\
& (-1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)})) / (-4/3 - 1/6*(26+6*33^{(1/2)})^{(1/3)} + 4/3/(26+6*33^{(1/2)})^{(1/3)} \\
& - 1/2*I*3^{(1/2)} * (-1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)}))^{(1/2)} + 2/3 * ((1+x) * (x - 1/6*(26+6*33^{(1/2)})^{(1/3)} \\
& + 4/3/(26+6*33^{(1/2)})^{(1/3)} - 1/3 - 1/2*I*3^{(1/2)} * (-1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)})) * (x - 1/6*(26+6*33^{(1/2)})^{(1/3)} \\
& + 4/3/(26+6*33^{(1/2)})^{(1/3)} - 1/3 + 1/2*I*3^{(1/2)} * (-1/3*(26+6*33^{(1/2)})^{(1/3)} - 8/3/(26+6*33^{(1/2)})^{(1/3)}))^{(1/2)}
\end{aligned}$$

$$\begin{aligned} & \left(\frac{(-1/3*(26+6*33^{(1/2)})^{(1/3)}-8/3/(26+6*33^{(1/2)})^{(1/3)})}{(-4/3-1/6*(26+6*33^{(1/2)})^{(1/3)}+4/3/(26+6*33^{(1/2)})^{(1/3)}-1/2*I*3^{(1/2)}*(-1/3*(26+6*33^{(1/2)})^{(1/3)}-8/3/(26+6*33^{(1/2)})^{(1/3)})} \right) / \left(\frac{(-1/2*(26+6*33^{(1/2)})^{(1/3)}+4/(26+6*33^{(1/2)})^{(1/3)}+1/2*I*3^{(1/2)}*(-1/3*(26+6*33^{(1/2)})^{(1/3)}-8/3/(26+6*33^{(1/2)})^{(1/3)})}{(-1/3*(26+6*33^{(1/2)})^{(1/3)}+8/3/(26+6*33^{(1/2)})^{(1/3)}+4/3)} \right) \\ & \left(\frac{((1+x)*(x+1/3*(26+6*33^{(1/2)})^{(1/3)}-8/3/(26+6*33^{(1/2)})^{(1/3)}-1/3*(x-1/6*(26+6*33^{(1/2)})^{(1/3)}+4/3/(26+6*33^{(1/2)})^{(1/3)}-1/3-1/2*I*3^{(1/2)}*(-1/3*(26+6*33^{(1/2)})^{(1/3)}-8/3/(26+6*33^{(1/2)})^{(1/3)}))*(x-1/6*(26+6*33^{(1/2)})^{(1/3)}+4/3/(26+6*33^{(1/2)})^{(1/3)}-1/3+1/2*I*3^{(1/2)}*(-1/3*(26+6*33^{(1/2)})^{(1/3)}-8/3/(26+6*33^{(1/2)})^{(1/3)}))}{(1+x)*(x+1/3*(26+6*33^{(1/2)})^{(1/3)}-8/3/(26+6*33^{(1/2)})^{(1/3)}-1/3*(x-1/6*(26+6*33^{(1/2)})^{(1/3)}+4/3/(26+6*33^{(1/2)})^{(1/3)}-1/3-1/2*I*3^{(1/2)}*(-1/3*(26+6*33^{(1/2)})^{(1/3)}-8/3/(26+6*33^{(1/2)})^{(1/3)}))} \right) \end{aligned}$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x, algorithm="maxima")

[Out] 1/2*x^2*(2*sqrt(2) - 1) - sqrt(2)*integrate(sqrt(x^3 - x^2 + 3*x + 1)*sqrt(x + 1), x)

Fricas [F]

time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x, algorithm="fricas")

[Out] integral(2*sqrt(2)*x - sqrt(2)*sqrt(x^4 + 2*x^2 + 4*x + 1) - x, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x \left(-1 + 2\sqrt{2} \right) - \sqrt{2} \sqrt{x^4 + 2x^2 + 4x + 1} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-2**(1/2)*(x**4+2*x**2+4*x+1)**(1/2)+x*(-1+2*2**(1/2)),x)

[Out] Integral(x*(-1 + 2*sqrt(2)) - sqrt(2)*sqrt(x**4 + 2*x**2 + 4*x + 1), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x)`

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x \left(2\sqrt{2} - 1 \right) - \sqrt{2} \sqrt{x^4 + 2x^2 + 4x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(2*2^(1/2) - 1) - 2^(1/2)*(4*x + 2*x^2 + x^4 + 1)^(1/2),x)`

[Out] `int(x*(2*2^(1/2) - 1) - 2^(1/2)*(4*x + 2*x^2 + x^4 + 1)^(1/2), x)`

$$3.282 \quad \int \frac{e^{-\frac{x}{y}} \left(\pi^2 (-3mc^8 + 4mc^9 + 24mc^6x - 48mc^7x - 144mc^5x^2 - 24mc^2x^3 + 176mc^3x^3 + 3x^4 + 12mc^*x^4) + 12mc^3\pi^2(3mc - 12mc^2 - 8x) * x^2 * \text{Log}[x/mc^2] \right)}{384x} dx$$

Optimal. Leaf size=330

$$\frac{e^{-\frac{x}{y}}(3-4mc)mc^8\pi^2}{384x} + \frac{3}{8}e^{-\frac{x}{y}}mc^5\pi^2y + \frac{1}{48}e^{-\frac{x}{y}}(3-22mc)mc^2\pi^2xy - \frac{1}{128}e^{-\frac{x}{y}}(1+4mc)\pi^2x^2y + \frac{1}{48}e^{-\frac{x}{y}}(3-22mc)mc^2x^3 + \frac{3}{16}e^{-\frac{x}{y}}mc^3x^3 + \frac{1}{4}e^{-\frac{x}{y}}x^4 + \frac{1}{12}e^{-\frac{x}{y}}mc^*x^4 + \frac{1}{32}e^{-\frac{x}{y}}mc^3\pi^2(3mc - 12mc^2 - 8x) * x^2 * \text{Log}[x/mc^2]$$

[Out] $\frac{1}{384}*(3-4*mc)*mc^8*\pi^2/\exp(x/y)/x + \frac{3}{8}*mc^5*\pi^2*y/\exp(x/y) + \frac{1}{48}*(3-22*mc)*mc^2*\pi^2*x*y/\exp(x/y) - \frac{1}{128}*(1+4*mc)*\pi^2*x^2*y/\exp(x/y) + \frac{1}{48}*(3-22*mc)*mc^2*\pi^2*y^2/\exp(x/y) + \frac{1}{4}*mc^3*\pi^2*y^2/\exp(x/y) - \frac{1}{64}*(1+4*mc)*\pi^2*x*y^2/\exp(x/y) - \frac{1}{64}*(1+4*mc)*\pi^2*y^3/\exp(x/y) + \frac{1}{16}*(1-2*mc)*mc^6*\pi^2*Ei(-x/y) + \frac{1}{384}*(3-4*mc)*mc^8*\pi^2*Ei(-x/y)/y + \frac{1}{32}*(3-22*mc)*mc^2*(3*mc - 12*mc^2 - 8*x)*y*Ei(-x/y) - \frac{1}{32}*(3-22*mc)*mc^3*(3*(1-4*mc)*mc - 8*x)*y*\ln(x/mc^2)/\exp(x/y) + \frac{1}{4}*mc^3*\pi^2*y^2*\ln(x/mc^2)/\exp(x/y)$

Rubi [A]

time = 0.57, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 107, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {12, 6874, 2230, 2225, 2208, 2209, 2207, 2634}

$$\frac{\pi^2(3-4mc)mc^8Ei\left(-\frac{x}{y}\right)}{384y} + \frac{3}{8}e^{-\frac{x}{y}}mc^5\pi^2 + \frac{1}{48}e^{-\frac{x}{y}}(3-22mc)mc^2\pi^2xy + \frac{1}{128}e^{-\frac{x}{y}}(1+4mc)\pi^2x^2y + \frac{1}{48}e^{-\frac{x}{y}}(3-22mc)mc^2x^3 + \frac{3}{16}e^{-\frac{x}{y}}mc^3x^3 + \frac{1}{4}e^{-\frac{x}{y}}x^4 + \frac{1}{12}e^{-\frac{x}{y}}mc^*x^4 + \frac{1}{32}e^{-\frac{x}{y}}mc^3\pi^2(3mc - 12mc^2 - 8x) * x^2 * \text{Log}[x/mc^2]$$

Antiderivative was successfully verified.

[In] Int[(Pi^2*(-3*mc^8 + 4*mc^9 + 24*mc^6*x - 48*mc^7*x - 144*mc^5*x^2 - 24*mc^2*x^3 + 176*mc^3*x^3 + 3*x^4 + 12*mc*x^4) + 12*mc^3*Pi^2*(3*mc - 12*mc^2 - 8*x)*x^2*Log[x/mc^2])/(384*E^(x/y)*x^2), x]

[Out] $\left(\frac{(3-4*mc)*mc^8*\pi^2}{384*E^{(x/y)}*x} + \frac{3*mc^5*\pi^2*y}{8*E^{(x/y)}} + \left(\frac{(3-22*mc)*mc^2*\pi^2*x*y}{48*E^{(x/y)}} - \frac{(1+4*mc)*\pi^2*x^2*y}{128*E^{(x/y)}}\right) + \left(\frac{(3-22*mc)*mc^2*\pi^2*y^2}{48*E^{(x/y)}} + \frac{mc^3*\pi^2*y^2}{4*E^{(x/y)}} - \frac{(1+4*mc)*\pi^2*x*y^2}{64*E^{(x/y)}} - \frac{(1+4*mc)*\pi^2*y^3}{64*E^{(x/y)}}\right) + \frac{(1-2*mc)*mc^6*\pi^2*\text{ExpIntegralEi}[-(x/y)]}{16} + \frac{(3-4*mc)*mc^8*\pi^2*\text{ExpIntegralEi}[-(x/y)]}{384*y} + \frac{mc^3*\pi^2*(3*mc - 12*mc^2 - 8*x)*y*\text{ExpIntegralEi}[-(x/y)]}{32} - \frac{mc^3*\pi^2*(3*(1-4*mc)*mc - 8*x)*y*\text{Log}[x/mc^2]}{32*E^{(x/y)}} + \frac{mc^3*\pi^2*y^2*\text{Log}[x/mc^2]}{4*E^{(x/y)}}\right)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2207

Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),

```
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !TrueQ[$UseGamma]
```

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2230

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] := Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F,
c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] && !TrueQ[$UseGamma]
```

Rule 2634

```
Int[Log[u]*(v_), x_Symbol] := With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
- Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 6874

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{e^{-\frac{x}{y}} (\pi^2 (-3mc^8 + 4mc^9 + 24mc^6x - 48mc^7x - 144mc^5x^2 - 24mc^2x^3 + 176mc^3x^3 + 3x^4 + 12mcx^4) + 12mc^3\pi^2)}{384x^2} dx$$

Mathematica [A]

time = 0.09, size = 181, normalized size = 0.55

$$\frac{1}{384} \pi^2 \left(-\frac{mc^8 (-3mc^5 + 4mc^6 - 24mc^3y + 48mc^4y - 36mc^2y^2 + 144mc^2y^2 + 96y^3) \operatorname{Ei}\left(-\frac{x}{y}\right)}{y} + \frac{e^{-\frac{x}{y}} (3mc^8 - 4mc^9 + 144mc^3xy + 24mc^2xy(x+y) - 16mc^3xy(11x+5y) - 3xy(x^2+2xy+2y^2) - 12mc^2xy(x^2+2xy+2y^2) + 12mc^3xy(-3mc+12mc^2+8(x+y)) \log\left(\frac{mx}{my}\right))}{x} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Pi^2*(-3*mc^8 + 4*mc^9 + 24*mc^6*x - 48*mc^7*x - 144*mc^5*x^2 - 24*mc^2*x^3 + 176*mc^3*x^3 + 3*x^4 + 12*mc*x^4) + 12*mc^3*Pi^2*(3*mc - 12*mc^2 - 8*x)*x^2*Log[x/mc^2])/(384*E^(x/y)*x^2), x]
```

```
[Out] (Pi^2*(-((mc^3*(-3*mc^5 + 4*mc^6 - 24*mc^3*y + 48*mc^4*y - 36*mc*y^2 + 144*mc^2*y^2 + 96*y^3)*ExpIntegralEi[-(x/y)]))/y) + (3*mc^8 - 4*mc^9 + 144*mc^5*x*y + 24*mc^2*x*y*(x+y) - 16*mc^3*x*y*(11*x+5*y) - 3*x*y*(x^2+2*x*y+2*y^2) - 12*mc*x*y*(x^2+2*x*y+2*y^2) + 12*mc^3*x*y*(-3*mc+12*mc^2+8*(x+y))*Log[x/mc^2])/(E^(x/y)*x))/384
```

Mathics [A]

time = 14.51, size = 212, normalized size = 0.64

$$\frac{\pi^2 (mc^8 \operatorname{ExpIntegralEi}\left[\frac{2}{y}\right] (3-4mc) E^{\frac{x}{y}} + 24mc^6 \operatorname{ExpIntegralEi}\left[-\frac{x}{y}\right] (1-2mc) E^{\frac{x}{y}} + xy (-12mc(x^2+2xy+2y^2) + 24mc^2(x+y) - 176mc^3(x+y) + 96mc^2(y - y \operatorname{ExpIntegralEi}\left[-\frac{x}{y}\right] E^{\frac{x}{y}} + \operatorname{Log}\left[\frac{mx}{my}\right] (x+y)) + 36mc^4 (-\operatorname{Log}\left[\frac{mx}{my}\right] + \operatorname{ExpIntegralEi}\left[-\frac{x}{y}\right] E^{\frac{x}{y}}) - 144mc^2 (-\operatorname{Log}\left[\frac{mx}{my}\right] + \operatorname{ExpIntegralEi}\left[-\frac{x}{y}\right] E^{\frac{x}{y}}) + 144mc^2 (-3x^2 - 6xy - 6y^2)) E^{-\frac{x}{y}})}{384}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(12*Log[x/mc^2]*x^2*Pi^2*mc^3*(-8*x-12*mc^2+3*mc) + Pi^2
*(12*x^4*mc+3*x^4+176*x^3*mc^3-24*x^3*mc^2 - 144*x^2*mc^5-48*x*mc^7+24*x*mc
^6+4*mc^9-3*mc^8))/(384*E^(x/y)*x^2),x]')
```

```
[Out] Pi ^ 2 (mc ^ 8 ExpIntegralE[2, x / y] (3 - 4 mc) E ^ (x / y) + 24 mc ^ 6 x
ExpIntegralEi[-x / y] (1 - 2 mc) E ^ (x / y) + x y (-12 mc (x ^ 2 + 2 x y +
2 y ^ 2) + 24 mc ^ 2 (x + y) - 176 mc ^ 3 (x + y) + 96 mc ^ 3 (y - y ExpIn
tegralEi[-x / y] E ^ (x / y) + Log[x / mc ^ 2] (x + y)) + 36 mc ^ 4 (-Log[x
/ mc ^ 2] + ExpIntegralEi[-x / y] E ^ (x / y)) - 144 mc ^ 5 (-Log[x / mc ^
2] + ExpIntegralEi[-x / y] E ^ (x / y)) + 144 mc ^ 5 - 3 x ^ 2 - 6 x y - 6
y ^ 2)) E ^ (-x / y) / (384 x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 1356, normalized size = 4.11

method	result	size
risch	Expression too large to display	1356

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/384*(Pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*mc^3*x^
3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*Pi^2*(-12*mc^2+3*mc-8*x)*x^2*ln(x/mc
^2))/exp(x/y)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*y*Pi^2*ln(mc)*mc^3*x*exp(-x/y)+1/8*I*y^2*Pi^3*mc^3*csgn(I*mc^2)^3*exp(-
-x/y)-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I*mc^2)^3+3/64*I*y*Pi^3*exp(-x/y)*m
c^4*csgn(I/mc^2*x)^3+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I*mc^2)^3-3/16*I*y*P
i^3*exp(-x/y)*mc^5*csgn(I/mc^2*x)^3-1/8*I*y^2*Pi^3*mc^3*csgn(I/mc^2*x)^3*ex
p(-x/y)-1/8*I*y*Pi^3*mc^3*csgn(I*x)*csgn(I/mc^2)*csgn(I/mc^2*x)*x*exp(-x/y)
+1/8*I*y*Pi^3*mc^3*csgn(I/mc^2)*csgn(I/mc^2*x)^2*x*exp(-x/y)-1/8*I*y^2*Pi^3
*mc^3*csgn(I*x)*csgn(I/mc^2)*csgn(I/mc^2*x)*exp(-x/y)+1/8*I*y*Pi^3*mc^3*csg
n(I*x)*csgn(I/mc^2*x)^2*x*exp(-x/y)+1/8*I*y*Pi^3*mc^3*csgn(I*mc)^2*csgn(I*m
c^2)*x*exp(-x/y)-1/4*I*y*Pi^3*mc^3*csgn(I*mc)*csgn(I*mc^2)^2*x*exp(-x/y)+3/
64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I*x)*csgn(I/mc^2)*csgn(I/mc^2*x)-3/16*I*y*P
i^3*exp(-x/y)*mc^5*csgn(I*x)*csgn(I/mc^2)*csgn(I/mc^2*x)-1/16*y^3*Pi^2*mc*e
xp(-x/y)+1/16*y^2*Pi^2*mc^2*exp(-x/y)-1/128*y*Pi^2*x^2*exp(-x/y)-1/64*y^2*P
i^2*x*exp(-x/y)+3/8*y*Pi^2*exp(-x/y)*mc^5-1/128/y*Pi^2*mc^8*Ei(1,x/y)+1/96/
y*Pi^2*mc^9*Ei(1,x/y)+1/128*Pi^2*mc^8/x*exp(-x/y)-1/96*Pi^2*mc^9/x*exp(-x/y)
)-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I*mc)^2*csgn(I*mc^2)+1/8*I*y^2*Pi^3*mc^
3*csgn(I*x)*csgn(I/mc^2*x)^2*exp(-x/y)+1/8*I*y^2*Pi^3*mc^3*csgn(I/mc^2)*csg
n(I/mc^2*x)^2*exp(-x/y)-1/8*I*y*Pi^3*mc^3*csgn(I/mc^2*x)^3*x*exp(-x/y)+1/8*
I*y*Pi^3*mc^3*csgn(I*mc^2)^3*x*exp(-x/y)+1/8*I*y^2*Pi^3*mc^3*csgn(I*mc)^2*c
sgn(I*mc^2)*exp(-x/y)-1/4*I*y^2*Pi^3*mc^3*csgn(I*mc)*csgn(I*mc^2)^2*exp(-x/
y)+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I/mc^2)*csgn(I/mc^2*x)^2+3/16*I*y*Pi^3
*exp(-x/y)*mc^5*csgn(I*mc)^2*csgn(I*mc^2)-3/8*I*y*Pi^3*exp(-x/y)*mc^5*csgn(
```

$$I*mc)*csgn(I*mc^2)^2-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I*x)*csgn(I/mc^2*x)^2-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I/mc^2)*csgn(I/mc^2*x)^2+3/32*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I*mc)*csgn(I*mc^2)^2+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I*x)*csgn(I/mc^2*x)^2+1/384*(144*Pi^2*mc^5*y-36*Pi^2*mc^4*y+96*Pi^2*mc^3*x*y+96*Pi^2*mc^3*y^2)*exp(-x/y)*ln(x)+1/4*Pi^2*mc^3*y^2*Ei(1,x/y)-3/32*Pi^2*mc^4*y*Ei(1,x/y)+3/8*Pi^2*mc^5*y*Ei(1,x/y)-5/24*mc^3*Pi^2*y^2*exp(-x/y)+1/8*Pi^2*mc^7*Ei(1,x/y)-1/16*Pi^2*mc^6*Ei(1,x/y)-1/64*y^3*Pi^2*exp(-x/y)-11/24*y*Pi^2*mc^3*x*exp(-x/y)-1/32*y*Pi^2*mc*x^2*exp(-x/y)-1/16*y^2*Pi^2*mc*x*exp(-x/y)+1/16*y*Pi^2*mc^2*x*exp(-x/y)-1/2*y^2*Pi^2*ln(mc)*mc^3*exp(-x/y)-3/4*y*Pi^2*exp(-x/y)*ln(mc)*mc^5+3/16*y*Pi^2*exp(-x/y)*ln(mc)*mc^4$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/384*(pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*mc^3*x^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*pi^2*(-12*mc^2+3*mc-8*x)*x^2*log(x/mc^2))/exp(x/y)/x^2,x, algorithm="maxima")

[Out] $-1/96*\pi^2*mc^9*\gamma(-1, x/y)/y - 1/8*\pi^2*mc^7*Ei(-x/y) + 1/128*\pi^2*mc^8*\gamma(-1, x/y)/y + 3/8*\pi^2*mc^5*y*e^{(-x/y)}*\log(x/mc^2) + 1/16*\pi^2*mc^6*Ei(-x/y) - 3/8*\pi^2*mc^5*y*Ei(-x/y) + 3/8*\pi^2*mc^5*y*e^{(-x/y)} - 3/32*\pi^2*mc^4*y*e^{(-x/y)}*\log(x/mc^2) + 3/32*\pi^2*mc^4*y*Ei(-x/y) - 11/24*\pi^2*(x*y + y^2)*mc^3*e^{(-x/y)} + 1/4*\pi^2*((x*y + y^2)*e^{(-x/y)}*\log(x) + \text{integrate}((2*x^2*\log(mc) - x*y - y^2)*e^{(-x/y)}/x, x))*mc^3 + 1/16*\pi^2*(x*y + y^2)*mc^2*e^{(-x/y)} - 1/32*\pi^2*(x^2*y + 2*x*y^2 + 2*y^3)*mc*e^{(-x/y)} - 1/128*\pi^2*(x^2*y + 2*x*y^2 + 2*y^3)*e^{(-x/y)}$

Fricas [A]

time = 0.31, size = 269, normalized size = 0.82

$\frac{12(8^2 mc^2 xy^4 + (8^2 mc^2 x^2 + 3^2(4 mc^2 - mc^2) y^2) x^2) \log(\frac{x}{mc^2}) - (96^2 mc^2 xy^3 + 36^2(4 mc^2 - mc^2) y^2 + 24^2(2 mc^2 - mc^2) xy + x^2(4 mc^2 - 3 mc^2) x) Ei(-\frac{x}{y}) - (6\pi^2(4 mc + 1) xy^4 + x^2(4 mc^2 - 3 mc^2) y + 2(3\pi^2(4 mc + 1) x^2 + 4\pi^2(10 mc^2 - 3 mc^2) y^2 - (144^2 mc^2 x - 3\pi^2(4 mc + 1) x^2 - 8\pi^2(22 mc^2 - 3 mc^2) y^2) x^2)}{384 xy}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/384*(pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*mc^3*x^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*pi^2*(-12*mc^2+3*mc-8*x)*x^2*log(x/mc^2))/exp(x/y)/x^2,x, algorithm="fricas")

[Out] $1/384*(12*(8*\pi^2*mc^3*x*y^3 + (8*\pi^2*mc^3*x^2 + 3*\pi^2*(4*mc^5 - mc^4)*x)*y^2)*e^{(-x/y)}*\log(x/mc^2) - (96*\pi^2*mc^3*x*y^3 + 36*\pi^2*(4*mc^5 - mc^4)*x*y^2 + 24*\pi^2*(2*mc^7 - mc^6)*x*y + \pi^2*(4*mc^9 - 3*mc^8)*x)*Ei(-x/y) - (6*\pi^2*(4*mc + 1)*x*y^4 + \pi^2*(4*mc^9 - 3*mc^8)*y + 2*(3*\pi^2*(4*mc + 1)*x^2 + 4*\pi^2*(10*mc^3 - 3*mc^2)*x)*y^3 - (144*\pi^2*mc^5*x - 3*\pi^2*(4*mc + 1)*x^3 - 8*\pi^2*(22*mc^3 - 3*mc^2)*x^2)*y^2)*e^{(-x/y)}/(x*y)$

Sympy [A]

time = 9.48, size = 330, normalized size = 1.00

$$\frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right)}{96x} + \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right)}{128x} - \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right)}{8} + \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right)}{16} - \frac{3\pi^2 \operatorname{erf}\left(\frac{x}{y}\right)}{8} - \frac{3\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{8} - \frac{3\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{32} - \frac{11\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{24} - \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{4} - \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{16} - \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{32} - \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/384*(pi**2*(4*mc**9-3*mc**8-48*mc**7*x+24*mc**6*x-144*mc**5*x**2+176*mc**3*x**3-24*mc**2*x**3+12*mc*x**4+3*x**4))+12*mc**3*pi**2*(-12*mc**2+3*mc-8*x)*x**2*ln(x/mc**2))/exp(x/y)/x**2,x)
```

```
[Out] -pi**2*mc**9*expint(2, x/y)/(96*x) + pi**2*mc**8*expint(2, x/y)/(128*x) - pi**2*mc**7*Ei(-x/y)/8 + pi**2*mc**6*Ei(-x/y)/16 + 3*pi**2*mc**5*y*exp(-x/y)/8 - 3*pi**2*mc**5*(y*Ei(-x/y) - y*exp(-x/y)*log(x/mc**2))/8 + 3*pi**2*mc**4*(y*Ei(-x/y) - y*exp(-x/y)*log(x/mc**2))/32 + 11*pi**2*mc**3*(-x*y*exp(-x/y) - y**2*exp(-x/y))/24 - pi**2*mc**3*(y**2*Ei(-x/y) - y**2*exp(-x/y) + (-x*y*exp(-x/y) - y**2*exp(-x/y))*log(x/mc**2))/4 - pi**2*mc**2*(-x*y*exp(-x/y) - y**2*exp(-x/y))/16 + pi**2*mc*(-x**2*y*exp(-x/y) - 2*x*y**2*exp(-x/y) - 2*y**3*exp(-x/y))/32 + pi**2*(-x**2*y*exp(-x/y) - 2*x*y**2*exp(-x/y) - 2*y**3*exp(-x/y))/128
```

Giac [A]

time = 0.02, size = 465, normalized size = 1.41

$$\frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right)}{96x} + \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right)}{128x} - \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right)}{8} + \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right)}{16} - \frac{3\pi^2 \operatorname{erf}\left(\frac{x}{y}\right)}{8} - \frac{3\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{8} - \frac{3\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{32} - \frac{11\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{24} - \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{4} - \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{16} - \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{32} - \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/384*(pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*mc^3*x^3-24*mc^2*x^3+12*mc*x^4+3*x^4))+12*mc^3*pi^2*(-12*mc^2+3*mc-8*x)*x^2*log(x/mc^2))/exp(x/y)/x^2,x)
```

```
[Out] -1/384*(4*pi^2*mc^9*x*Ei(-x/y) - 3*pi^2*mc^8*x*Ei(-x/y) + 48*pi^2*mc^7*x*y*Ei(-x/y) + 4*pi^2*mc^9*y/e^(x/y) - 24*pi^2*mc^6*x*y*Ei(-x/y) + 144*pi^2*mc^5*x*y^2*Ei(-x/y) - 3*pi^2*mc^8*y/e^(x/y) - 36*pi^2*mc^4*x*y^2*Ei(-x/y) + 96*pi^2*mc^3*x*y^3*Ei(-x/y) - 144*pi^2*mc^5*x*y^2*log(x/mc^2)/e^(x/y) - 144*pi^2*mc^5*x*y^2/e^(x/y) + 36*pi^2*mc^4*x*y^2*log(x/mc^2)/e^(x/y) - 96*pi^2*mc^3*x^2*y^2*log(x/mc^2)/e^(x/y) - 96*pi^2*mc^3*x*y^3*log(x/mc^2)/e^(x/y) + 176*pi^2*mc^3*x^2*y^2/e^(x/y) + 80*pi^2*mc^3*x*y^3/e^(x/y) - 24*pi^2*mc^2*x^2*y^2/e^(x/y) + 12*pi^2*mc*x^3*y^2/e^(x/y) - 24*pi^2*mc^2*x*y^3/e^(x/y) + 24*pi^2*mc*x^2*y^3/e^(x/y) + 24*pi^2*mc*x*y^4/e^(x/y) + 3*pi^2*x^3*y^2/e^(x/y) + 6*pi^2*x^2*y^3/e^(x/y) + 6*pi^2*x*y^4/e^(x/y))/(x*y)
```

Mupad [B]

time = 0.77, size = 265, normalized size = 0.80

$$\frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right)}{96x} + \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right)}{128x} - \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right)}{8} + \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right)}{16} - \frac{3\pi^2 \operatorname{erf}\left(\frac{x}{y}\right)}{8} - \frac{3\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{8} - \frac{3\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{32} - \frac{11\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{24} - \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{4} - \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{16} - \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{32} - \frac{\pi^2 \operatorname{erf}\left(\frac{x}{y}\right) \log\left(\frac{x}{y}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(-x/y)*((Pi^2*(176*mc^3*x^3 - 24*mc^2*x^3 - 144*mc^5*x^2 + 12*mc*x^4 + 24*mc^6*x - 48*mc^7*x - 3*mc^8 + 4*mc^9 + 3*x^4))/384 - (Pi^2*mc^3*x^2*log(x/mc^2)*(8*x - 3*mc + 12*mc^2))/32))/x^2,x)
```

```
[Out] ei(-x/y)*(((Pi^2*mc^8)/128 - (Pi^2*mc^9)/96)/y + (Pi^2*mc^6)/16 - (Pi^2*mc^7)/8 + y*((3*Pi^2*mc^4)/32 - (3*Pi^2*mc^5)/8) - (Pi^2*mc^3*y^2)/4 - (2*Pi^2*x^2*y*exp(-x/y)*(12*mc*y^2 - 12*mc^2*y + 40*mc^3*y - 72*mc^5 + 3*y^2) + 2*Pi^2*x^3*y*exp(-x/y)*(3*y + 12*mc*y - 12*mc^2 + 88*mc^3) + Pi^2*mc^8*x*exp(-x/y)*(4*mc - 3) + 3*Pi^2*x^4*y*exp(-x/y)*(4*mc + 1) - 96*Pi^2*mc^3*x^3*y*log(x/mc^2)*exp(-x/y) - 12*Pi^2*mc^3*x^2*y*log(x/mc^2)*exp(-x/y)*(8*y - 3*mc + 12*mc^2))/(384*x^2)
```

3.283 $\int \sec(x) \sin(2x) dx$

Optimal. Leaf size=4

$$-2 \cos(x)$$

[Out] -2*cos(x)

Rubi [A]

time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4372, 2718}

$$-2 \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]*Sin[2*x],x]

[Out] -2*Cos[x]

Rule 2718

Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4372

Int[(cos[(a_.) + (b_.)*(x_.)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_.)]^(p_.), x_Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \sec(x) \sin(2x) dx &= 2 \int \sin(x) dx \\ &= -2 \cos(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 4, normalized size = 1.00

$$-2 \cos(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]*Sin[2*x],x]

[Out] -2*Cos[x]

Mathics [A]

time = 1.90, size = 4, normalized size = 1.00

$$-2\cos(x)$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Sin[2*x]/Cos[x],x]')

[Out] -2 Cos[x]

Maple [A]

time = 0.03, size = 5, normalized size = 1.25

method	result	size
derivativedivides	$-2 \cos(x)$	5
default	$-2 \cos(x)$	5
risch	$-2 \cos(x)$	5

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)/cos(x),x,method=_RETURNVERBOSE)

[Out] -2*cos(x)

Maxima [A]

time = 0.27, size = 4, normalized size = 1.00

$$-2 \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)/cos(x),x, algorithm="maxima")

[Out] -2*cos(x)

Fricas [A]

time = 0.33, size = 4, normalized size = 1.00

$$-2 \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)/cos(x),x, algorithm="fricas")

[Out] -2*cos(x)

Sympy [A]

time = 0.38, size = 5, normalized size = 1.25

$$-2 \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)/cos(x),x)

[Out] -2*cos(x)

Giac [A]

time = 0.00, size = 5, normalized size = 1.25

$$-2 \cos x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)/cos(x),x)

[Out] -2*cos(x)

Mupad [B]

time = 0.02, size = 4, normalized size = 1.00

$$-2 \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)/cos(x),x)

[Out] -2*cos(x)

$$3.284 \quad \int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}} dx$$

Optimal. Leaf size=71

$$\frac{1}{2} \left((1 + \sqrt{2}) \log(1 + x + \sqrt{2}x + \sqrt{2}x^2 - x^7) - (-1 + \sqrt{2}) \log(-1 + (-1 + \sqrt{2})x + \sqrt{2}x^2 + x^7) \right)$$

[Out] $-1/2*\ln(-1+x^7+x*(2^{(1/2)}-1)+x^2*2^{(1/2)})*(2^{(1/2)}-1)+1/2*\ln(1+x-x^7+x*2^{(1/2)}+x^2*2^{(1/2)})*(1+2^{(1/2)})$

Rubi [F]

time = 0.47, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{3 + 3x - 4x^2 - 4x^3 - 7x^6 + 4x^7 + 10x^8 + 7x^{13}}{1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}} dx$$

Verification is not applicable to the result.

[In] $\text{Int}[(3 + 3*x - 4*x^2 - 4*x^3 - 7*x^6 + 4*x^7 + 10*x^8 + 7*x^{13})/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^{14}), x]$

[Out] $\text{Log}[1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^{14}]/2 + 2*\text{Defer}[\text{Int}[(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^{14})^{-1}, x] + 4*\text{Defer}[\text{Int}[x/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^{14}), x] + 2*\text{Defer}[\text{Int}[x^2/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^{14}), x] + 12*\text{Defer}[\text{Int}[x^7/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^{14}), x] + 10*\text{Defer}[\text{Int}[x^8/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^{14}), x]$

Rubi steps

$$\begin{aligned} \int \frac{3 + 3x - 4x^2 - 4x^3 - 7x^6 + 4x^7 + 10x^8 + 7x^{13}}{1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}} dx &= \frac{1}{2} \log(1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}) - \\ &= \frac{1}{2} \log(1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}) - \\ &= \frac{1}{2} \log(1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}) - \\ &= \frac{1}{2} \log(1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}) - \\ &= \frac{1}{2} \log(1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}) - \end{aligned}$$

Mathematica [A]

time = 0.02, size = 71, normalized size = 1.00

$$\frac{1}{2} \left((1 + \sqrt{2}) \log(1 + x + \sqrt{2}x + \sqrt{2}x^2 - x^7) - (-1 + \sqrt{2}) \log(-1 + (-1 + \sqrt{2})x + \sqrt{2}x^2 + x^7) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 3*x - 4*x^2 - 4*x^3 - 7*x^6 + 4*x^7 + 10*x^8 + 7*x^13)/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14), x]

[Out] ((1 + Sqrt[2])*Log[1 + x + Sqrt[2]*x + Sqrt[2]*x^2 - x^7] - (-1 + Sqrt[2])*Log[-1 + (-1 + Sqrt[2])*x + Sqrt[2]*x^2 + x^7])/2

Mathics [A]

time = 2.38, size = 60, normalized size = 0.85

$$\frac{(1 + \sqrt{2}) \operatorname{Log} \left[-1 + (-1 - \sqrt{2})x - \sqrt{2}x^2 + x^7 \right]}{2} + \frac{(1 - \sqrt{2}) \operatorname{Log} \left[-1 + (-1 + \sqrt{2})x + \sqrt{2}x^2 + x^7 \right]}{2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2*x^4-4*x^3-x^2+2*x+1), x]')

[Out] (1 + Sqrt[2]) Log[-1 + (-1 - Sqrt[2]) x - Sqrt[2] x ^ 2 + x ^ 7] / 2 + (1 - Sqrt[2]) Log[-1 + (-1 + Sqrt[2]) x + Sqrt[2] x ^ 2 + x ^ 7] / 2

Maple [A]

time = 0.04, size = 61, normalized size = 0.86

method	result
default	$\left(\frac{1}{2} + \frac{\sqrt{2}}{2}\right) \ln(x^7 - x^2\sqrt{2} + (-1 - \sqrt{2})x - 1) + \left(-\frac{\sqrt{2}}{2} + \frac{1}{2}\right) \ln(-1 + x^7 + x(\sqrt{2} - 1) - x^2\sqrt{2})$
risch	$\frac{\ln(x^7 - x^2\sqrt{2} + (-1 - \sqrt{2})x - 1)}{2} + \frac{\ln(x^7 - x^2\sqrt{2} + (-1 - \sqrt{2})x - 1)\sqrt{2}}{2} - \frac{\ln(-1 + x^7 + x(\sqrt{2} - 1) + x^2\sqrt{2})}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2*x^4-4*x^3-x^2+2*x+1), x, method=_RETURNVERBOSE)

[Out] (1/2+1/2*2^(1/2))*ln(x^7-x^2*2^(1/2)+(-1-2^(1/2))*x-1)+(-1/2*2^(1/2)+1/2)*ln(-1+x^7+x*(2^(1/2)-1)+x^2*2^(1/2))

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2*x^4-4*x^3-x^2+2*x+1),x, algorithm="maxima")

[Out] integrate((7*x^13 + 10*x^8 + 4*x^7 - 7*x^6 - 4*x^3 - 4*x^2 + 3*x + 3)/(x^14 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(56) = 112.

time = 0.32, size = 137, normalized size = 1.93

$$\frac{1}{2}\sqrt{2}\log\left(\frac{x^{14}-2x^8-2x^7+2x^4+4x^3+3x^2-2\sqrt{2}(x^9+x^8-x^3-2x^2-x)+2x+1}{x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1}\right)+\frac{1}{2}\log(x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2*x^4-4*x^3-x^2+2*x+1),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*log((x^14 - 2*x^8 - 2*x^7 + 2*x^4 + 4*x^3 + 3*x^2 - 2*sqrt(2)*(x^9 + x^8 - x^3 - 2*x^2 - x) + 2*x + 1)/(x^14 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1)) + 1/2*log(x^14 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1)

Sympy [A]

time = 0.12, size = 76, normalized size = 1.07

$$\left(\frac{1}{2} + \frac{\sqrt{2}}{2}\right)\log\left(x^7 - \sqrt{2}x^2 - 2x\left(\frac{1}{2} + \frac{\sqrt{2}}{2}\right) - 1\right) + \left(\frac{1}{2} - \frac{\sqrt{2}}{2}\right)\log\left(x^7 + \sqrt{2}x^2 - 2x\left(\frac{1}{2} - \frac{\sqrt{2}}{2}\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((7*x**13+10*x**8+4*x**7-7*x**6-4*x**3-4*x**2+3*x+3)/(x**14-2*x**8-2*x**7-2*x**4-4*x**3-x**2+2*x+1),x)

[Out] (1/2 + sqrt(2)/2)*log(x**7 - sqrt(2)*x**2 - 2*x*(1/2 + sqrt(2)/2) - 1) + (1/2 - sqrt(2)/2)*log(x**7 + sqrt(2)*x**2 - 2*x*(1/2 - sqrt(2)/2) - 1)

Giac [A]

time = 0.01, size = 113, normalized size = 1.59

$$\frac{-7\sqrt{2}\ln|x^7+x^2\sqrt{2}+x\sqrt{2}-x-1|+7\sqrt{2}\ln|x^7-x^2\sqrt{2}-x\sqrt{2}-x-1|}{14}+\frac{\ln|x^{14}-2x^8-2x^7-2x^4-4x^3-x^2+2x+1|}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2*x^4-4*x^3-x^2+2*x+1),x)

[Out] $-1/2*\sqrt{2}*\log(\text{abs}(x^7 + \sqrt{2}*x^2 + \sqrt{2}*x - x - 1)) + 1/2*\sqrt{2}*\log(\text{abs}(x^7 - \sqrt{2}*x^2 - \sqrt{2}*x - x - 1)) + 1/2*\log(\text{abs}(x^{14} - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1))$

Mupad [B]

time = 0.28, size = 103, normalized size = 1.45

$$\frac{\ln(\sqrt{2}x - x + \sqrt{2}x^2 + x^7 - 1)}{2} + \frac{\ln(x^7 - \sqrt{2}x - \sqrt{2}x^2 - x - 1)}{2} - \frac{\sqrt{2} \ln(\sqrt{2}x - x + \sqrt{2}x^2 + x^7 - 1)}{2} + \frac{\sqrt{2} \ln(x^7 - \sqrt{2}x - \sqrt{2}x^2 - x - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(-(3*x - 4*x^2 - 4*x^3 - 7*x^6 + 4*x^7 + 10*x^8 + 7*x^{13} + 3)/(x^2 - 2*x + 4*x^3 + 2*x^4 + 2*x^7 + 2*x^8 - x^{14} - 1), x)$

[Out] $\log(2^{(1/2)}*x - x + 2^{(1/2)}*x^2 + x^7 - 1)/2 + \log(x^7 - 2^{(1/2)}*x - 2^{(1/2)}*x^2 - x - 1)/2 - (2^{(1/2)}*\log(2^{(1/2)}*x - x + 2^{(1/2)}*x^2 + x^7 - 1))/2 + (2^{(1/2)}*\log(x^7 - 2^{(1/2)}*x - 2^{(1/2)}*x^2 - x - 1))/2$

Chapter 4

Appendix

Local contents

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4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*      Small rewrite of logic in main function to make it*)
(*      match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A","none"}

```

```

        ,(*ELSE*)
        finalresult={"B","Both result and optimal contain complex but leaf count is
    ]
    ,(*ELSE*)
        finalresult={"C","Result contains complex when optimal does not."}
    ]
    ,(*ELSE*)(*result does not contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
            finalresult={"A","none"}
        ,(*ELSE*)
            finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"
        ]
    ]
    ,(*ELSE*)(*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>"}
        ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]==Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]==Rational,

```

```

    If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
      1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]

```

```

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp,Log,
    Sin,Cos,Tan,Cot,Sec,Csc,
    ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
    Sinh,Cosh,Tanh,Coth,Sech,Csch,
    ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
  },func]

```

```

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  },func]

```

```

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems
#Nasser 4/07/2022 add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
        return "B","result has leaf size over 500,000. Avoiding possible recursion issues."
      fi;

      leaf_count_optimal := leafcount(optimal);
      ExpnType_result := ExpnType(result);
      ExpnType_optimal := ExpnType(optimal);

      if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal
```



```

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

```

```

if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;

```

```

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A","";
            else
                return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (" ,
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                                end if
            else #result contains complex but optimal is not
                if debug then
                    print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
            # this assumes optimal do not as well. No check is needed here.
            if debug then
                print("result do not contain complex, this assumes optimal do not as well");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
            fi;
        fi;
    fi;
fi;

```

```

        fi;
        return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                        convert(leaf_count_result,string)," $ vs. $2(",
                        convert(leaf_count_optimal,string),")=",convert(2*leaf_count

    fi;
    fi;
else #ExpnType(result) > ExpnType(optimal)
    if debug then
        print("ExpnType(result) > ExpnType(optimal)");
    fi;
    return "C",cat("Result contains higher order function than in optimal. Order ",
                  convert(ExpnType_result,string)," vs. order ",
                  convert(ExpnType_optimal,string),".");
fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
    return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
    if type(expn,'atomic') then
        1
    elif type(expn,'list') then
        apply(max,map(ExpnType,expn))
    elif type(expn,'sqrt') then
        if type(op(1,expn),'rational') then
            1
        else

```

```

    max(2,ExpnType(op(1,expn)))
  end if
elif type(expn,'^') then
  if type(op(2,expn),'integer') then
    ExpnType(op(1,expn))
  elif type(op(2,expn),'rational') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
  end if
elif type(expn,'+') or type(expn,'*') then
  max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif ElementaryFunctionQ(op(0,expn)) then
  max(3,ExpnType(op(1,expn)))
elif SpecialFunctionQ(op(0,expn)) then
  max(4,apply(max,map(ExpnType,[op(expn)])))
elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
end if
end proc:

```

```

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

```

```

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

```

```

end proc:

HypergeometricFunctionQ := proc(func)
  member(func, [Hypergeometric1F1, hypergeom, HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func, [AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u), op(2..nops(u), u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

4.2.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#      Port of original Maple grading function by
#      Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added 'RootSum'. See problem 177, Timofeev file
#      added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr, Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:

```

```

    return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
                    asinh,acosh,atanh,acoth,asech,acsch
                    ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
                    gamma,loggamma,digamma,zeta,polylog,LambertW,
                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
                    ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')

```

```

    return expnType(expn.args[0]) #ExpnType(op(1,expn))
elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
    else:
        return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
else:
    return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+') or type(expn,'*')
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

#main function

```
def grade_antiderivative(result,optimal):
```

```
    #print ("Enter grade_antiderivative for sagemath")
```

```
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)
```

```
leaf_count_result = leaf_count(result)
```

```
leaf_count_optimal = leaf_count(optimal)
```

```
    #print("leaf_count_result=",leaf_count_result)
```

```
    #print("leaf_count_optimal=",leaf_count_optimal)
```

```
expnType_result = expnType(result)
```

```
expnType_optimal = expnType(optimal)
```

```

if str(result).find("Integral") != -1:
    grade = "F"
    grade_annotation = ""
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Both result and optimal contain complex but leaf count of result is large"
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation = "Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = ""
            else:
                grade = "B"
                grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result) + " vs " + str(leaf_count_optimal)
    else:
        grade = "C"
        grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType_result) + " vs " + str(ExpnType_optimal)

    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```

4.2.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
# Albert Rich to use with Sagemath. This is used to
# grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
# 'arctan2', 'floor', 'abs', 'log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
# issue later when reading the file.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

```

```

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False

```



```

if debug: print ("type(func)=", type(func))

m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
    'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
    'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
    'polylog','lambert_w','elliptic_f','elliptic_e',
    'elliptic_pi','exp_integral_e','log_integral']

if debug:
    print ("m=",m)
    if m:
        print ("func ", func ," is special_function")
    else:
        print ("func ", func ," is NOT special_function")

return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric'] #[appellf1] can't find this in sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-try:
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False

def expnType(expn):

```

```

if debug:
    print (">>>>>Enter expnType, expn=", expn)
    print (">>>>>is_atom(expn)=", is_atom(expn))

if is_atom(expn):
    return 1
elif type(expn)==list: #instance(expn,list):
    return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
elif is_sqrt(expn):
    if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],Rational):
        return 1
    else:
        return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
elif expn.operator() == operator.pow: #instance(expn,Pow)
    if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
        return expnType(expn.operands()[0]) #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational: #instance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #instance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #instance(expn,Add) or instan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

```
#main function
```

```

def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = "none"
                else:
                    grade = "B"
                    grade_annotation = "Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation = "Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = "none"
            else:
                grade = "B"
                grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation = "Result contains higher order function than in optimal. Order "+str(expnType_resu

    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation

```