Computer algebra independent integration tests

Summer 2022 edition

0-Independent-test-suites/2-Bondarenko-Problems

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [35]. This is test number [2].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.0.1 (February 17, 2022) on windows 10.

2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.

3. Maple 2022.1 (June 1, 2022) on windows 10.

4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.0.1.debian on Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem via sagemath 9.6.

5. Fricas 1.3.7 (June 30, 2021) based on based on ecl 21.2.1 on Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem via sagemath 9.6.

6. Giac/Xcas 1.9.0-7 (April 2022) on on Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem. Direct testing using C++ API.

7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem via sagemath 9.6.

8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

9. Mathics 4.0 via sagemath 9.6.

Maxima, Fricas, Mathics are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems. Mathics was called using its own interface in Sagemath as in this example

```
from sage.interfaces.mathics import mathics
res = mathics('Integrate[Sin[x]/(3 + Cos[x])^2,x]')
```

Sympy was called directly from Python. Giac was also called directly via its C++ interface.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (35)	0.00 (0)
Rubi	94.29 (33)	5.71 (2)
Maple	80.00 (28)	20.00 (7)
Fricas	68.57 (24)	31.43 (11)
Giac	48.57 (17)	51.43 (18)
Maxima	42.86 (15)	57.14 (20)
Mupad	25.71 (9)	74.29 (26)
Sympy	20.00 (7)	80.00 (28)
Mathics	17.14 (6)	82.86 (29)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	91.43	2.86	0.00	5.71
Mathematica	65.71	11.43	22.86	0.00
Maple	48.57	17.14	14.29	20.00
Fricas	37.14	31.43	0.00	31.43
Maxima	37.14	2.86	2.86	57.14
Giac	28.57	17.14	2.86	51.43
Sympy	11.43	8.57	0.00	80.00
Mathics	8.57	2.86	5.71	82.86
Mupad	N/A	25.71	0.00	74.29

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.
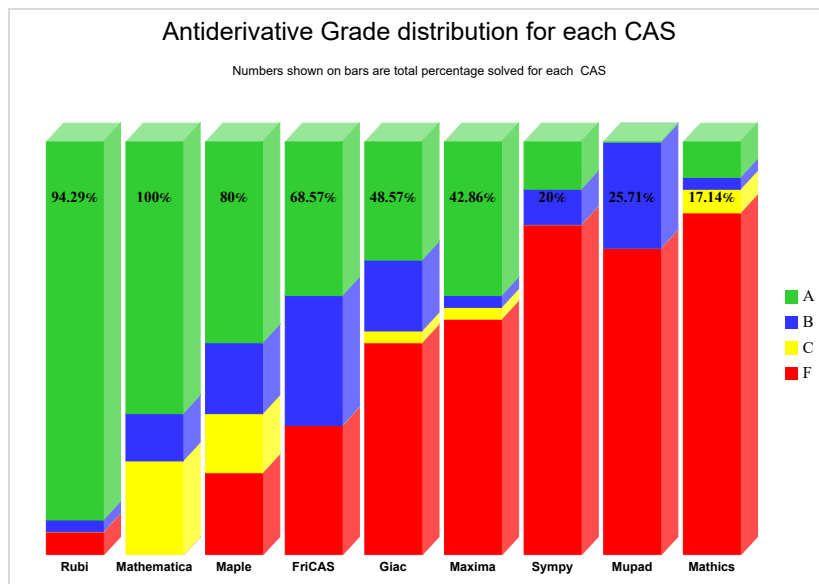
The figure below compares the CAS systems for each grade level.
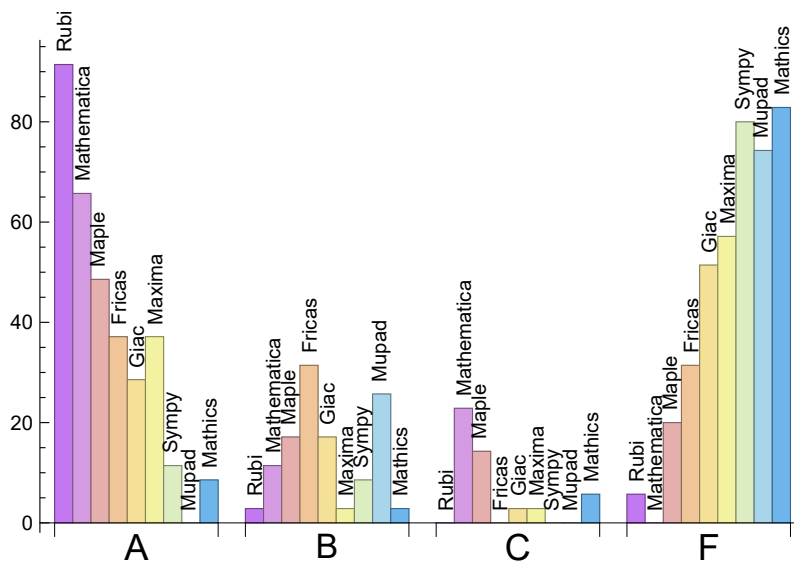
The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	2	100.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	7	100.00 %	0.00 %	0.00 %
Fricas	11	81.82 %	0.00 %	18.18 %
Giac	18	100.00 %	0.00 %	0.00 %
Maxima	20	95.00 %	5.00 %	0.00 %
Sympy	28	92.86 %	7.14 %	0.00 %
Mupad	26	100.00 %	0.00 %	0.00 %
Mathics	29	0.00 %	20.69 %	79.31 %

Table 1.4: Failure statistics for each CAS

1.3   Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.20	183.58	1.23	83.00	1.00
Mathematica	1.81	213.74	1.53	102.00	1.00
Maple	0.10	136.64	1.21	79.00	0.86
Maxima	0.35	94.33	1.20	51.00	1.23
Fricas	0.85	571.00	2.88	77.00	1.52
Sympy	2.79	168.29	3.95	61.00	2.44
Giac	0.02	148.12	2.16	92.00	1.38
Mupad	0.59	160.00	1.47	49.00	0.94
Mathics	5.59	43.83	1.24	35.00	1.30

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.
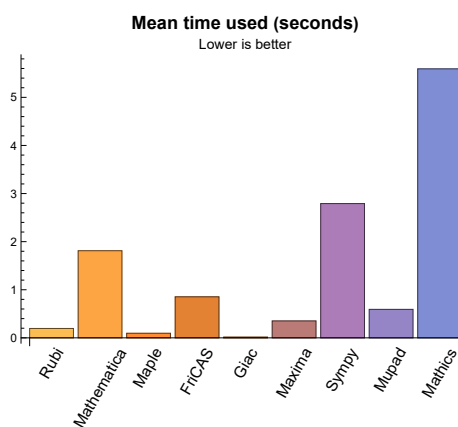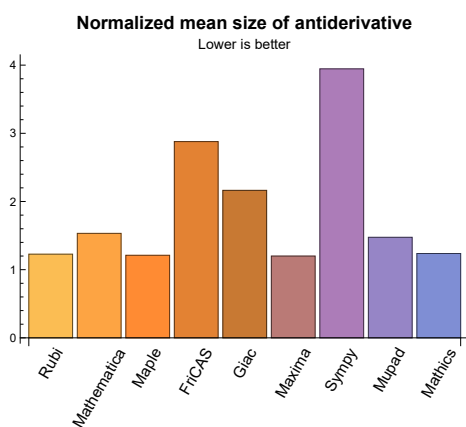
1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

Mathics {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{7, 21, 26, 31, 35\}$

Mathics $\{19\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi`, `Mathematica` and `Mathics`. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```python
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```python
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
        leafCount =1
```

For Giac, the call `taille(anti_derivative,RAND_MAX);` is used to find leaf size.

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10   Design of the test system

The following diagram gives a high level view of the current test build system.

High level overview of the CAS independent integration test build system
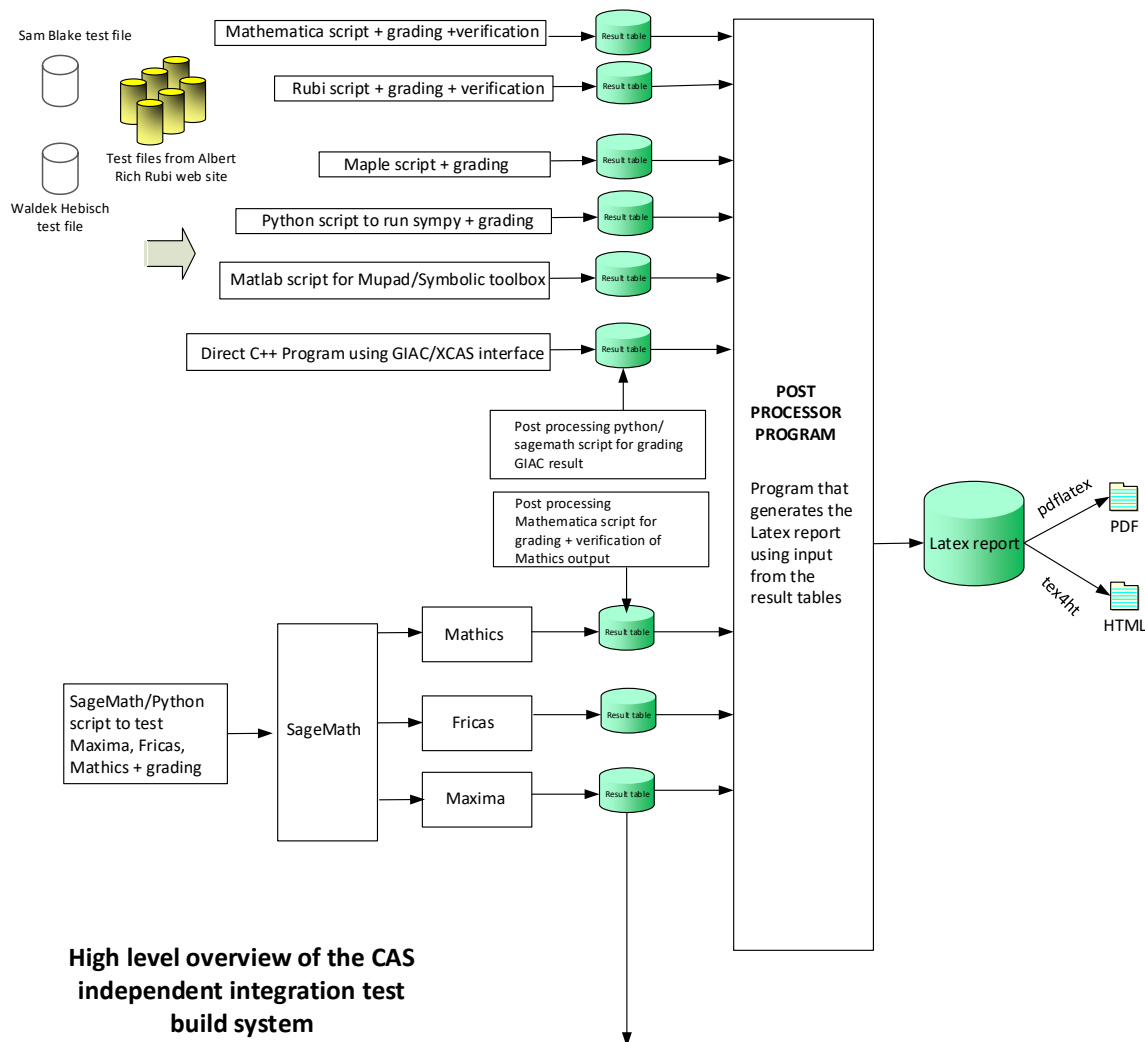
One record (line) per one integral result. The line is CSV comma separated. This is description of each record
1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.
    *The following fields present only in **Rubi** Table file*
15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi

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Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35 }

B grade: { 21 }

C grade: { }

F grade: { 7, 8 }

2.1.2 Mathematica

A grade: { 2, 3, 5, 7, 9, 10, 11, 12, 15, 16, 17, 18, 22, 23, 24, 27, 28, 29, 30, 31, 32, 34, 35 }

B grade: { 8, 19, 20, 33 }

C grade: { 1, 4, 6, 13, 14, 21, 25, 26 }

F grade: { }

2.1.3 Maple

A grade: { 1, 3, 4, 5, 10, 12, 20, 21, 22, 23, 25, 27, 30, 32, 33, 34, 35 }

B grade: { 2, 11, 17, 19, 24, 26 }

C grade: { 6, 7, 8, 13, 14 }

F grade: { 9, 15, 16, 18, 28, 29, 31 }

2.1.4 Maxima

A grade: { 1, 3, 5, 7, 8, 10, 11, 12, 19, 20, 22, 33, 34 }

B grade: { 23 }

C grade: { 4 }

F grade: { 2, 6, 9, 13, 14, 15, 16, 17, 18, 21, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 9, 12, 15, 16, 17, 18, 19, 22 }

B grade: { 6, 10, 11, 13, 14, 20, 21, 23, 24, 26, 33 }

C grade: { }

F grade: { 7, 8, 25, 27, 28, 29, 30, 31, 32, 34, 35 }

2.1.6 Sympy

A grade: { $3, 6, 10, 19$ }

B grade: { $1, 5, 22$ }

C grade: { }

F grade: { $2, 4, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35$ }

2.1.7 Giac

A grade: { $1, 3, 5, 6, 10, 11, 12, 21, 22, 23$ }

B grade: { $2, 17, 19, 20, 24, 26$ }

C grade: { 4 }

F grade: { $7, 8, 9, 13, 14, 15, 16, 18, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35$ }

2.1.8 Mupad

A grade: { }

B grade: { $1, 2, 3, 5, 6, 21, 22, 23, 26$ }

C grade: { }

F grade: { $4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35$ }

2.1.9 Mathics

A grade: { $1, 3, 22$ }

B grade: { 5 }

C grade: { $6, 19$ }

F grade: { $2, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35$ }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	77	21	20	25	61	24	16	32
N.S.	1	1.00	3.50	0.95	0.91	1.14	2.77	1.09	0.73	1.45
time (sec)	N/A	0.011	0.045	0.090	0.406	0.355	4.142	0.003	0.141	6.104

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	A	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	41	50	0	44	0	208	49	0
N.S.	1	1.00	1.28	1.56	0.00	1.38	0.00	6.50	1.53	0.00
time (sec)	N/A	0.022	0.089	0.010	0.000	0.346	0.000	0.026	0.622	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	16	16	23	20	14	21	14	14
N.S.	1	1.00	0.64	0.64	0.92	0.80	0.56	0.84	0.56	0.56
time (sec)	N/A	0.012	0.008	0.017	0.279	0.373	0.186	0.000	0.163	1.848

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	C	A	F	C	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	68	42	112	46	0	80	-1	0
N.S.	1	1.00	1.17	0.72	1.93	0.79	0.00	1.38	-0.02	0.00
time (sec)	N/A	0.061	0.012	0.059	0.375	0.445	0.000	0.002	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	26	42	56	67	838	39	88	86
N.S.	1	1.00	0.52	0.84	1.12	1.34	16.76	0.78	1.76	1.72
time (sec)	N/A	0.020	0.025	0.121	0.273	0.343	3.527	0.002	0.269	15.130

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	30	36	0	1058	26	344	313	37
N.S.	1	1.00	0.09	0.11	0.00	3.17	0.08	1.03	0.94	0.11
time (sec)	N/A	0.248	0.004	0.018	0.000	0.390	1.248	0.007	0.262	3.092

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	F	A	C	A	F(-2)	F	F	F	F(-2)
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	291	0	430	172	366	0	0	0	-1	0
N.S.	1	0.00	1.48	0.59	1.26	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.031	0.737	0.148	0.377	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	F	B	C	A	F(-2)	F	F	F	F(-2)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	308	0	654	199	378	0	0	0	-1	0
N.S.	1	0.00	2.12	0.65	1.23	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.028	0.442	0.011	0.375	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	105	0	0	66	0	0	-1	0
N.S.	1	1.00	1.25	0.00	0.00	0.79	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.039	0.120	0.010	0.000	0.336	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	34	51	101	112	71	-1	0
N.S.	1	1.00	1.00	0.83	1.24	2.46	2.73	1.73	-0.02	0.00
time (sec)	N/A	0.134	0.054	0.056	0.343	0.348	8.044	0.006	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	69	175	63	112	0	92	-1	0
N.S.	1	1.00	0.95	2.40	0.86	1.53	0.00	1.26	-0.01	0.00
time (sec)	N/A	0.084	0.059	0.096	0.363	0.349	0.000	0.021	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	52	60	77	110	0	106	-1	0
N.S.	1	1.00	0.71	0.82	1.05	1.51	0.00	1.45	-0.01	0.00
time (sec)	N/A	0.197	0.047	0.018	0.370	0.417	0.000	0.005	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	217	109	0	5235	0	0	-1	0
N.S.	1	1.00	0.59	0.30	0.00	14.34	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.605	0.155	0.059	0.000	5.354	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	212	105	0	4535	0	0	-1	0
N.S.	1	1.00	0.63	0.31	0.00	13.46	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.399	0.143	0.059	0.000	3.374	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	74	0	0	56	0	0	-1	0
N.S.	1	1.00	0.96	0.00	0.00	0.73	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.047	10.041	0.007	0.000	0.621	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	112	0	0	73	0	0	-1	0
N.S.	1	1.00	0.95	0.00	0.00	0.62	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.127	10.055	0.012	0.000	0.825	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	77	298	0	81	0	262	-1	0
N.S.	1	1.00	0.93	3.59	0.00	0.98	0.00	3.16	-0.01	0.00
time (sec)	N/A	0.070	0.147	0.018	0.000	1.862	0.000	0.020	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	89	0	0	122	0	0	-1	0
N.S.	1	1.00	0.93	0.00	0.00	1.27	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.056	0.356	0.014	0.000	1.897	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	A	A	A	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	65	49	36	34	61	91	-1	61
N.S.	1	1.00	2.60	1.96	1.44	1.36	2.44	3.64	-0.04	2.44
time (sec)	N/A	0.050	0.106	0.036	0.337	0.344	2.023	0.015	0.000	4.681

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	126	33	35	55	0	93	-1	0
N.S.	1	1.00	5.04	1.32	1.40	2.20	0.00	3.72	-0.04	0.00
time (sec)	N/A	0.034	0.082	0.069	0.386	0.360	0.000	0.015	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	B	C	A	F(-1)	B	F	A	B	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	108	786	478	95	0	219	0	128	307	0
N.S.	1	7.28	4.43	0.88	0.00	2.03	0.00	1.19	2.84	0.00
time (sec)	N/A	0.790	6.233	0.129	0.000	0.390	0.000	0.004	1.247	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	56	27	40	46	66	40	26	33
N.S.	1	1.00	1.93	0.93	1.38	1.59	2.28	1.38	0.90	1.14
time (sec)	N/A	0.013	0.023	0.086	0.273	0.347	0.370	0.002	0.278	2.700

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	20	43	68	0	26	19	0
N.S.	1	1.00	1.00	0.77	1.65	2.62	0.00	1.00	0.73	0.00
time (sec)	N/A	0.009	0.013	0.056	0.360	0.335	0.000	0.006	0.162	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	110	147	121	364	0	859	0	489	-1	0
N.S.	1	1.34	1.10	3.31	0.00	7.81	0.00	4.45	-0.01	0.00
time (sec)	N/A	0.390	0.078	0.097	0.000	0.364	0.000	0.091	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	54	75	0	0	0	0	-1	0
N.S.	1	1.00	1.35	1.88	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.052	32.138	0.151	0.000	0.372	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	185	349	910	392	0	452	0	404	608	0
N.S.	1	1.89	4.92	2.12	0.00	2.44	0.00	2.18	3.29	0.00
time (sec)	N/A	0.697	0.343	0.101	0.000	0.359	0.000	0.031	2.183	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	102	83	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.81	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.096	0.014	0.023	0.000	0.329	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	122	0	0	0	0	0	-1	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.127	0.060	0.099	0.000	0.322	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-1)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	347	0	0	0	0	0	-1	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.354	0.153	0.075	0.000	0.337	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	981	981	868	722	0	0	0	0	-1	0
N.S.	1	1.00	0.88	0.74	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.859	0.323	0.051	0.000	0.341	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	555	555	1076	0	0	0	0	0	-1	0
N.S.	1	1.00	1.94	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.438	0.890	0.007	0.000	0.342	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	303	252	0	0	0	0	-1	0
N.S.	1	1.00	0.97	0.81	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.234	0.056	0.014	0.000	0.342	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	262	113	84	220	0	0	-1	0
N.S.	1	1.00	3.28	1.41	1.05	2.75	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.051	0.162	0.253	0.376	0.349	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	44	71	31	0	0	0	-1	0
N.S.	1	1.00	0.77	1.25	0.54	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.047	0.049	0.493	0.386	0.337	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	F	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	131	171	0	0	0	0	-1	0
N.S.	1	1.00	1.08	1.41	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.065	0.138	0.331	0.000	0.344	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [16] had the largest ratio of [36]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	12	0.083
2	A	4	3	1.00	19	0.158
3	A	2	2	1.00	6	0.333
4	A	5	5	1.00	10	0.500
5	A	3	2	1.00	7	0.286
6	A	22	9	1.00	8	1.125
7	F	0	0	N/A	0	N/A
8	F	0	0	N/A	0	N/A
9	A	4	3	1.00	19	0.158
10	A	6	3	1.00	25	0.120
11	A	5	2	1.00	19	0.105
12	A	6	3	1.00	21	0.143
13	A	20	8	1.00	28	0.286
14	A	22	9	1.00	21	0.429
15	A	2	1	1.00	27	0.037
16	A	3	2	1.00	36	0.056
17	A	7	5	1.00	17	0.294
18	A	7	5	1.00	17	0.294
19	A	6	6	1.00	25	0.240
20	A	7	7	1.00	14	0.500
21	B	45	7	7.28	9	0.778
22	A	3	3	1.00	8	0.375
23	A	2	2	1.00	10	0.200
24	A	11	7	1.34	16	0.438
25	A	5	5	1.00	11	0.454
						Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	31	12	1.89	16	0.750
27	A	12	10	1.00	16	0.625
28	A	13	12	1.00	12	1.000
29	A	28	15	1.00	13	1.154
30	A	44	10	1.00	18	0.556
31	A	35	16	1.00	18	0.889
32	A	21	7	1.00	14	0.500
33	A	7	4	1.00	5	0.800
34	A	5	5	1.00	8	0.625
35	A	10	7	1.00	14	0.500

Chapter 3

Listing of integrals

Local contents

3.1 $\int \dfrac{1}{\sqrt{2}+\cos(z)+\sin(z)}\,dz$

Optimal. Leaf size=22

$$-\frac{1-\sqrt{2}\,\sin(z)}{\cos(z)-\sin(z)}$$

[Out] (-1+sin(z)*2^(1/2))/(cos(z)-sin(z))

Rubi [A]

time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\dfrac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3193}

$$-\frac{1-\sqrt{2}\,\sin(z)}{\cos(z)-\sin(z)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[2] + Cos[z] + Sin[z])^(-1),z]

[Out] -((1 - Sqrt[2]*Sin[z])/(Cos[z] - Sin[z]))

Rule 3193

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Simp[-(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\int \frac{1}{\sqrt{2}+\cos(z)+\sin(z)}\,dz = -\frac{1-\sqrt{2}\,\sin(z)}{\cos(z)-\sin(z)}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.05, size = 77, normalized size = 3.50

$$\frac{-\left(\left((1+3i)+\sqrt{2}\right)\cos\left(\frac{z}{2}\right)\right)+\left((1+i)-i\sqrt{2}\right)\sin\left(\frac{z}{2}\right)}{\left((1+i)+\sqrt{2}\right)\cos\left(\frac{z}{2}\right)+i\left((-1-i)+\sqrt{2}\right)\sin\left(\frac{z}{2}\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[2] + Cos[z] + Sin[z])^(-1),z]

```
[Out] (-(((1 + 3*I) + Sqrt[2])*Cos[z/2]) + ((1 + I) - I*Sqrt[2])*Sin[z/2])/(((1 +
 I) + Sqrt[2])*Cos[z/2] + I*((-1 - I) + Sqrt[2])*Sin[z/2])
```

Mathics [A]

time $= 6.10$, size $= 32$, normalized size $= 1.45$

$$\frac{-198 + 140\sqrt{2}}{99 - 70\sqrt{2} - 239\text{Tan}\left[\frac{z}{2}\right] + 169\sqrt{2}\,\text{Tan}\left[\frac{z}{2}\right]}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(Sqrt[2] + Sin[z] + Cos[z]),z]')
```

```
[Out] (-198 + 140 Sqrt[2]) / (99 - 70 Sqrt[2] - 239 Tan[z / 2] + 169 Sqrt[2] Tan[
z / 2])
```

Maple [A]

time $= 0.09$, size $= 21$, normalized size $= 0.95$

method	result	size
default	$-\dfrac{2}{\left(\sqrt{2}-1\right)\left(\tan\left(\frac{z}{2}\right)+\sqrt{2}+1\right)}$	21
norman	$\dfrac{\left(-2-2\sqrt{2}\right)\tan\left(\frac{z}{2}\right)+2}{\tan^2\left(\frac{z}{2}\right)+2\tan\left(\frac{z}{2}\right)-1}$	32
risch	$-\dfrac{2}{\sqrt{2}+2\,e^{iz}+i\sqrt{2}} + \dfrac{2i}{\sqrt{2}+2\,e^{iz}+i\sqrt{2}}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(z)+sin(z)+2^(1/2)),z,method=_RETURNVERBOSE)
```

```
[Out] -2/(2^(1/2)-1)/(tan(1/2*z)+2^(1/2)+1)
```

Maxima [A]

time $= 0.41$, size $= 20$, normalized size $= 0.91$

$$-\frac{2}{\dfrac{\left(\sqrt{2}-1\right)\sin(z)}{\cos(z)+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(z)+sin(z)+2^(1/2)),z, algorithm="maxima")
```

```
[Out] -2/((sqrt(2) - 1)*sin(z)/(cos(z) + 1) + 1)
```

Fricas [A]

time $= 0.36$, size $= 25$, normalized size $= 1.14$

$$\frac{\sqrt{2}\,\cos\left(z\right) + \sqrt{2}\,\sin\left(z\right) - 2}{2\left(\cos\left(z\right) - \sin\left(z\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(z)+sin(z)+2^(1/2)),z, algorithm="fricas")
[Out] 1/2*(sqrt(2)*cos(z) + sqrt(2)*sin(z) - 2)/(cos(z) - sin(z))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(15) = 30$
time $= 4.14$, size $= 61$, normalized size $= 2.77$

$$-\frac{198}{-239\tan\left(\frac{z}{2}\right) + 169\sqrt{2}\,\tan\left(\frac{z}{2}\right) - 70\sqrt{2} + 99} + \frac{140\sqrt{2}}{-239\tan\left(\frac{z}{2}\right) + 169\sqrt{2}\,\tan\left(\frac{z}{2}\right) - 70\sqrt{2} + 99}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(z)+sin(z)+2**(1/2)),z)
[Out] -198/(-239*tan(z/2) + 169*sqrt(2)*tan(z/2) - 70*sqrt(2) + 99) + 140*sqrt(2)
/(-239*tan(z/2) + 169*sqrt(2)*tan(z/2) - 70*sqrt(2) + 99)
```

Giac [A]
time $= 0.00$, size $= 24$, normalized size $= 1.09$

$$\frac{2\left(-\sqrt{2} - 1\right)}{\tan\left(\frac{z}{2}\right) + \sqrt{2} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(z)+sin(z)+2^(1/2)),z)
[Out] -2*(sqrt(2) + 1)/(sqrt(2) + tan(1/2*z) + 1)
```

Mupad [B]
time $= 0.14$, size $= 16$, normalized size $= 0.73$

$$-\frac{2}{\tan\left(\frac{z}{2}\right)\left(\sqrt{2} - 1\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(z) + sin(z) + 2^(1/2)),z)
[Out] -2/(tan(z/2)*(2^(1/2) - 1) + 1)
```

3.2 $\quad \int \dfrac{1}{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}\,dx$

Optimal. Leaf size=32

$$-\frac{1}{2x}+\frac{\sqrt{1-x^2}}{2x}+\frac{1}{2}\sin^{-1}(x)$$

[Out] -1/2/x+1/2*arcsin(x)+1/2*(-x^2+1)^(1/2)/x

Rubi [A]

time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6822, 283, 222}

$$\frac{\sqrt{1-x^2}}{2x}-\frac{1}{2x}+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^(-2),x]

[Out] -1/2*1/x + Sqrt[1 - x^2]/(2*x) + ArcSin[x]/2

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 283

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]

Rule 6822

Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]

Rubi steps

$$\int \frac{1}{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}\,dx = \frac{1}{4}\int\left(\frac{2}{x^2}-\frac{2\sqrt{1-x^2}}{x^2}\right)dx$$

$$= -\frac{1}{2x}-\frac{1}{2}\int\frac{\sqrt{1-x^2}}{x^2}\,dx$$

$$= -\frac{1}{2x}+\frac{\sqrt{1-x^2}}{2x}+\frac{1}{2}\int\frac{1}{\sqrt{1-x^2}}\,dx$$

$$= -\frac{1}{2x}+\frac{\sqrt{1-x^2}}{2x}+\frac{1}{2}\sin^{-1}(x)$$

Mathematica [A]
time = 0.09, size = 41, normalized size = 1.28

$$\frac{-1+\sqrt{1-x^2}+2x\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^(-2),x]
[Out] (-1 + Sqrt[1 - x^2] + 2*x*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]])/(2*x)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[1 + x] + Sqrt[1 - x])^2,x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(24) = 48$.
time = 0.01, size = 50, normalized size = 1.56

method	result	size
default	$-\dfrac{1}{2x}-\dfrac{\left(-\arcsin(x)x-\sqrt{-x^2+1}\right)\sqrt{1+x}\,\sqrt{1-x}}{2x\sqrt{-x^2+1}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1-x)^(1/2)+(1+x)^(1/2))^2,x,method=_RETURNVERBOSE)
[Out] -1/2/x-1/2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(1+x)^(1/2)*(1-x)^(1/2)/x/(-x^2+1)
^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Failed to integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
[Out] integrate((sqrt(x + 1) + sqrt(-x + 1))^(-2), x)
```

Fricas [A]
time = 0.35, size = 44, normalized size = 1.38

$$-\frac{2\,x\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}\,-1}{x}\right)-\sqrt{x+1}\,\sqrt{-x+1}\,+1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
[Out] -1/2*(2*x*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - sqrt(x + 1)*sqrt(-x +
1) + 1)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sqrt{1-x}\,+\sqrt{x+1}\,\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)**(1/2)+(1+x)**(1/2))**2,x)
[Out] Integral((sqrt(1 - x) + sqrt(x + 1))**(-2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(24) = 48.
time = 0.03, size = 208, normalized size = 6.50

$$-\frac{\pi}{2}-\frac{2\left(\frac{2\sqrt{-x+1}}{-2\sqrt{x+1}\,+2\sqrt{2}}-\frac{-2\sqrt{x+1}\,+2\sqrt{2}}{2\sqrt{-x+1}}\right)}{-\left(\frac{2\sqrt{-x+1}}{-2\sqrt{x+1}\,+2\sqrt{2}}-\frac{-2\sqrt{x+1}\,+2\sqrt{2}}{2\sqrt{-x+1}}\right)^2+4}-\arctan\left(\frac{\sqrt{-x+1}\,\left(\left(-\frac{-2\sqrt{x+1}\,+2\sqrt{2}}{2\sqrt{-x+1}}\right)^2-1\right)}{-2\sqrt{x+1}\,+2\sqrt{2}}\right)-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)^(1/2)+(1+x)^(1/2))^2,x)
```

```
[Out] -1/2*pi - 2*((sqrt(2) - sqrt(x + 1))/sqrt(-x + 1) - sqrt(-x + 1)/(sqrt(2) -
 sqrt(x + 1)))/(((sqrt(2) - sqrt(x + 1))/sqrt(-x + 1) - sqrt(-x + 1)/(sqrt(
2) - sqrt(x + 1)))^2 - 4) - 1/2/x - arctan(-1/2*sqrt(-x + 1)*((sqrt(2) - sq
rt(x + 1))^2/(x - 1) + 1)/(sqrt(2) - sqrt(x + 1)))
```

Mupad [B]
time $= 0.62$, size $= 49$, normalized size $= 1.53$

$$\frac{\left(\frac{x}{2}+\frac{1}{2}\right)\sqrt{1-x}}{x\sqrt{x+1}} - \frac{1}{2\,x} - 2\operatorname{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] ((x/2 + 1/2)*(1 - x)^(1/2))/(x*(x + 1)^(1/2)) - 1/(2*x) - 2*atan(((1 - x)^(
1/2) - 1)/((x + 1)^(1/2) - 1))
```

3.3 $\int \frac{1}{(1+\cos(x))^2}\,dx$

Optimal. Leaf size=25

$$\frac{\sin(x)}{3(1+\cos(x))^2} + \frac{\sin(x)}{3(1+\cos(x))}$$

[Out] 1/3*sin(x)/(1+cos(x))^2+1/3*sin(x)/(1+cos(x))

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2729, 2727}

$$\frac{\sin(x)}{3(\cos(x)+1)} + \frac{\sin(x)}{3(\cos(x)+1)^2}$$

Antiderivative was successfully verified.

[In] Int[(1 + Cos[x])^(-2),x]

[Out] Sin[x]/(3*(1 + Cos[x])^2) + Sin[x]/(3*(1 + Cos[x]))

Rule 2727

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2729

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c + d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rubi steps

$$\int \frac{1}{(1+\cos(x))^2}\,dx = \frac{\sin(x)}{3(1+\cos(x))^2} + \frac{1}{3}\int \frac{1}{1+\cos(x)}\,dx$$
$$= \frac{\sin(x)}{3(1+\cos(x))^2} + \frac{\sin(x)}{3(1+\cos(x))}$$

Mathematica [A]

time = 0.01, size = 16, normalized size = 0.64

$$\frac{(2+\cos(x))\sin(x)}{3(1+\cos(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])^(-2),x]
```

```
[Out] ((2 + Cos[x])*Sin[x])/(3*(1 + Cos[x])^2)
```

Mathics [A]
time = 1.85, size = 14, normalized size = 0.56

$$\frac{\left(3 + \text{Tan}\left[\frac{x}{2}\right]^2\right) \text{Tan}\left[\frac{x}{2}\right]}{6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(1 + Cos[x])^2,x]')
```

```
[Out] (3 + Tan[x / 2] ^ 2) Tan[x / 2] / 6
```

Maple [A]
time = 0.02, size = 16, normalized size = 0.64

method	result	size
default	$\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{6} + \frac{\tan\left(\frac{x}{2}\right)}{2}$	16
norman	$\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{6} + \frac{\tan\left(\frac{x}{2}\right)}{2}$	16
risch	$\frac{2i\left(1 + 3\,e^{ix}\right)}{3\left(1 + e^{ix}\right)^3}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*tan(1/2*x)^3+1/2*tan(1/2*x)
```

Maxima [A]
time = 0.28, size = 23, normalized size = 0.92

$$\frac{\sin\left(x\right)}{2\left(\cos\left(x\right) + 1\right)} + \frac{\sin\left(x\right)^3}{6\left(\cos\left(x\right) + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*sin(x)/(cos(x) + 1) + 1/6*sin(x)^3/(cos(x) + 1)^3
```

Fricas [A]
time = 0.37, size = 20, normalized size = 0.80

$$\frac{\left(\cos\left(x\right) + 2\right)\sin\left(x\right)}{3\left(\cos\left(x\right)^2 + 2\cos\left(x\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x))^2,x, algorithm="fricas")
[Out] 1/3*(cos(x) + 2)*sin(x)/(cos(x)^2 + 2*cos(x) + 1)
```

Sympy [A]
time = 0.19, size = 14, normalized size = 0.56

$$\frac{\tan^3\left(\frac{x}{2}\right)}{6} + \frac{\tan\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x))**2,x)
[Out] tan(x/2)**3/6 + tan(x/2)/2
```

Giac [A]
time = 0.00, size = 21, normalized size = 0.84

$$\frac{2}{4}\left(\frac{1}{3}\tan^3\left(\frac{x}{2}\right) + \tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x))^2,x)
[Out] 1/6*tan(1/2*x)^3 + 1/2*tan(1/2*x)
```

Mupad [B]
time = 0.16, size = 14, normalized size = 0.56

$$\frac{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2 + 3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) + 1)^2,x)
[Out] (tan(x/2)*(tan(x/2)^2 + 3))/6
```

3.4 $\int \dfrac{\sin(x)}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=58

$$\sqrt{2\pi}\,\cos(1)S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{1+x}\right) - \sqrt{2\pi}\,C\left(\sqrt{\frac{2}{\pi}}\,\sqrt{1+x}\right)\sin(1)$$

[Out] cos(1)*FresnelS(2^(1/2)/Pi^(1/2)*(1+x)^(1/2))*2^(1/2)*Pi^(1/2)-FresnelC(2^(
1/2)/Pi^(1/2)*(1+x)^(1/2))*sin(1)*2^(1/2)*Pi^(1/2)

Rubi [A]

time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\dfrac{\text{number of rules}}{\text{integrand size}} = 0.500$,

Rules used = {3387, 3386, 3432, 3385, 3433}

$$\sqrt{2\pi}\,\cos(1)S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{x+1}\right) - \sqrt{2\pi}\,\sin(1)C\left(\sqrt{\frac{2}{\pi}}\,\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/Sqrt[1 + x],x]

[Out] Sqrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]] - Sqrt[2*Pi]*FresnelC[Sq
rt[2/Pi]*Sqrt[1 + x]]*Sin[1]

Rule 3385

Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3386

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]

Rule 3387

Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin(x)}{\sqrt{1+x}}\,dx = \cos(1)\int \frac{\sin(1+x)}{\sqrt{1+x}}\,dx - \sin(1)\int \frac{\cos(1+x)}{\sqrt{1+x}}\,dx$$

$$= (2\cos(1))\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{1+x}\,\right) - (2\sin(1))\text{Subst}\left(\int \cos\left(x^2\right)\,dx, x, \sqrt{1+x}\,\right)$$

$$= \sqrt{2\pi}\,\cos(1)S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{1+x}\,\right) - \sqrt{2\pi}\,C\left(\sqrt{\frac{2}{\pi}}\,\sqrt{1+x}\,\right)\sin(1)$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.01, size = 68, normalized size = 1.17

$$-\frac{e^{-i}\left(\sqrt{-i(1+x)}\,\Gamma\left(\frac{1}{2}, -i(1+x)\right) + e^{2i}\sqrt{i(1+x)}\,\Gamma\left(\frac{1}{2}, i(1+x)\right)\right)}{2\sqrt{1+x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/Sqrt[1 + x],x]

[Out] -1/2*(Sqrt[(-I)*(1 + x)]*Gamma[1/2, (-I)*(1 + x)] + E^(2*I)*Sqrt[I*(1 + x)]
Gamma[1/2, I(1 + x)])/(E^I*Sqrt[1 + x])

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sin[x]/Sqrt[1 + x],x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject

Maple [A]
time = 0.06, size = 42, normalized size = 0.72

method	result	size
derivativedivides	$\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(1\right)\mathrm{S}\left(\dfrac{\sqrt{2}\,\sqrt{1+x}}{\sqrt{\pi}}\right)-\sin\left(1\right)\mathrm{FresnelC}\left(\dfrac{\sqrt{2}\,\sqrt{1+x}}{\sqrt{\pi}}\right)\right)$	42
default	$\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(1\right)\mathrm{S}\left(\dfrac{\sqrt{2}\,\sqrt{1+x}}{\sqrt{\pi}}\right)-\sin\left(1\right)\mathrm{FresnelC}\left(\dfrac{\sqrt{2}\,\sqrt{1+x}}{\sqrt{\pi}}\right)\right)$	42

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/(1+x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2^(1/2)*Pi^(1/2)*(cos(1)*FresnelS(2^(1/2)/Pi^(1/2)*(1+x)^(1/2))-sin(1)*Fres
nelC(2^(1/2)/Pi^(1/2)*(1+x)^(1/2)))

Maxima [C] Result contains complex when optimal does not.
time = 0.37, size = 112, normalized size = 1.93

$$\tfrac{1}{8}\sqrt{\pi}\left(\left((i+1)\,\sqrt{2}\cos(1)+(i-1)\,\sqrt{2}\sin(1)\right)\mathrm{erf}\left(\left(\tfrac{1}{2}i+\tfrac{1}{2}\right)\sqrt{2}\,\sqrt{x+1}\right)+\left((i-1)\,\sqrt{2}\cos(1)+(i+1)\,\sqrt{2}\sin(1)\right)\mathrm{erf}\left(\left(\tfrac{1}{2}i-\tfrac{1}{2}\right)\sqrt{2}\,\sqrt{x+1}\right)+\left(-(i-1)\,\sqrt{2}\cos(1)-(i+1)\,\sqrt{2}\sin(1)\right)\mathrm{erf}\left(\sqrt{-i}\,\sqrt{x+1}\right)+\left((i+1)\,\sqrt{2}\cos(1)+(i-1)\,\sqrt{2}\sin(1)\right)\mathrm{erf}\left((-1)^{\frac{1}{4}}\sqrt{x+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(1+x)^(1/2),x, algorithm="maxima")

[Out] 1/8*sqrt(pi)*(((I + 1)*sqrt(2)*cos(1) + (I - 1)*sqrt(2)*sin(1))*erf((1/2*I
+ 1/2)*sqrt(2)*sqrt(x + 1)) + ((I - 1)*sqrt(2)*cos(1) + (I + 1)*sqrt(2)*sin
(1))*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x + 1)) + (-(I - 1)*sqrt(2)*cos(1) - (I
+ 1)*sqrt(2)*sin(1))*erf(sqrt(-I)*sqrt(x + 1)) + ((I + 1)*sqrt(2)*cos(1) +
(I - 1)*sqrt(2)*sin(1))*erf((-1)^(1/4)*sqrt(x + 1)))

Fricas [A]
time = 0.45, size = 46, normalized size = 0.79

$$\sqrt{2}\,\sqrt{\pi}\,\cos\left(1\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{\sqrt{\pi}}\right)-\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{\sqrt{\pi}}\right)\sin\left(1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(1+x)^(1/2),x, algorithm="fricas")

[Out] sqrt(2)*sqrt(pi)*cos(1)*fresnel_sin(sqrt(2)*sqrt(x + 1)/sqrt(pi)) - sqrt(2)
*sqrt(pi)*fresnel_cos(sqrt(2)*sqrt(x + 1)/sqrt(pi))*sin(1)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(x\right)}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+x)**(1/2),x)
```

```
[Out] Integral(sin(x)/sqrt(x + 1), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.00, size = 80, normalized size = 1.38

$$
2\left(\frac{2\sqrt{\pi}\,e^{-i}\mathrm{erf}\left(\frac{i\sqrt{2}\,\sqrt{x+1}}{1-i}\right)}{(-1+i)\,\sqrt{2}\cdot 22i} + \frac{2i\sqrt{\pi}\,e^{i}\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{x+1}}{1-i}\right)}{(-1-i)\,\sqrt{2}\cdot 2\cdot 2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+x)^(1/2),x)
```

```
[Out] -(1/4*I + 1/4)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x + 1))*e^I
 + (1/4*I - 1/4)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x + 1))*e^
(-I)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$
\int \frac{\sin(x)}{\sqrt{x+1}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(x + 1)^(1/2),x)
```

```
[Out] int(sin(x)/(x + 1)^(1/2), x)
```

3.5 $\quad \int \dfrac{1}{(\cos(x)+\sin(x))^6}\, dx$

Optimal. Leaf size=50

$$-\frac{\cos(x)-\sin(x)}{10(\cos(x)+\sin(x))^5}-\frac{\cos(x)-\sin(x)}{15(\cos(x)+\sin(x))^3}+\frac{2\sin(x)}{15(\cos(x)+\sin(x))}$$

[Out] 1/10*(-cos(x)+sin(x))/(cos(x)+sin(x))^5+1/15*(-cos(x)+sin(x))/(cos(x)+sin(x))^3+2/15*sin(x)/(cos(x)+sin(x))

Rubi [A]

time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\dfrac{\text{number of rules}}{\text{integrand size}}=0.286$,

Rules used = {3155, 3154}

$$-\frac{\cos(x)-\sin(x)}{15(\sin(x)+\cos(x))^3}-\frac{\cos(x)-\sin(x)}{10(\sin(x)+\cos(x))^5}+\frac{2\sin(x)}{15(\sin(x)+\cos(x))}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x] + Sin[x])^(-6),x]

[Out] -1/10*(Cos[x] - Sin[x])/(Cos[x] + Sin[x])^5 - (Cos[x] - Sin[x])/(15*(Cos[x] + Sin[x])^3) + (2*Sin[x])/(15*(Cos[x] + Sin[x]))

Rule 3154

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 3155

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]

Rubi steps

$$\int \frac{1}{(\cos(x)+\sin(x))^6}\, dx = -\frac{\cos(x)-\sin(x)}{10(\cos(x)+\sin(x))^5}+\frac{2}{5}\int \frac{1}{(\cos(x)+\sin(x))^4}\, dx$$
$$= -\frac{\cos(x)-\sin(x)}{10(\cos(x)+\sin(x))^5}-\frac{\cos(x)-\sin(x)}{15(\cos(x)+\sin(x))^3}+\frac{2}{15}\int \frac{1}{(\cos(x)+\sin(x))^2}\, dx$$
$$= -\frac{\cos(x)-\sin(x)}{10(\cos(x)+\sin(x))^5}-\frac{\cos(x)-\sin(x)}{15(\cos(x)+\sin(x))^3}+\frac{2\sin(x)}{15(\cos(x)+\sin(x))}$$

Mathematica [A]
time $= 0.02$, size $= 26$, normalized size $= 0.52$

$$-\frac{5\cos(3x) - 10\sin(x) + \sin(5x)}{30(\cos(x) + \sin(x))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sin[x])^(-6),x]
```

```
[Out] -1/30*(5*Cos[3*x] - 10*Sin[x] + Sin[5*x])/(Cos[x] + Sin[x])^5
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 110 vs. $2(50) = 100$.
time $= 15.13$, size $= 86$, normalized size $= 1.72$

$$\frac{2\left(-15 - 100\mathrm{Tan}\left[\frac{x}{2}\right]^2 - 100\mathrm{Tan}\left[\frac{x}{2}\right]^6 - 60\mathrm{Tan}\left[\frac{x}{2}\right] - 20\mathrm{Tan}\left[\frac{x}{2}\right]^5 - 15\mathrm{Tan}\left[\frac{x}{2}\right]^8 + 20\mathrm{Tan}\left[\frac{x}{2}\right]^3 + 60\mathrm{Tan}\left[\frac{x}{2}\right]^7 + 118\mathrm{Tan}\left[\frac{x}{2}\right]^4\right)\mathrm{Tan}\left[\frac{x}{2}\right]}{15\left(-1 + \mathrm{Tan}\left[\frac{x}{2}\right]^2 - 2\mathrm{Tan}\left[\frac{x}{2}\right]\right)^5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(Cos[x] + Sin[x])^6,x]')
```

```
[Out] 2 (-15 - 100 Tan[x / 2] ^ 2 - 100 Tan[x / 2] ^ 6 - 60 Tan[x / 2] - 20 Tan[x
 / 2] ^ 5 - 15 Tan[x / 2] ^ 8 + 20 Tan[x / 2] ^ 3 + 60 Tan[x / 2] ^ 7 + 118
 Tan[x / 2] ^ 4) Tan[x / 2] / (15 (-1 + Tan[x / 2] ^ 2 - 2 Tan[x / 2]) ^ 5)
```

Maple [A]
time $= 0.12$, size $= 42$, normalized size $= 0.84$

method	result	size
risch	$\dfrac{-\frac{2}{15} + \frac{4e^{4ix}}{3} + \frac{2ie^{2ix}}{3}}{\left(e^{2ix}+i\right)^5}$	30
default	$\dfrac{2}{(\tan(x)+1)^2} - \dfrac{8}{3(\tan(x)+1)^3} - \dfrac{1}{\tan(x)+1} + \dfrac{2}{(\tan(x)+1)^4} - \dfrac{4}{5(\tan(x)+1)^5}$	42
norman	$\dfrac{-8\left(\tan^2\left(\frac{x}{2}\right)\right) - 2\tan\left(\frac{x}{2}\right) - 2\left(\tan^9\left(\frac{x}{2}\right)\right) + 8\left(\tan^8\left(\frac{x}{2}\right)\right) - \frac{40\left(\tan^3\left(\frac{x}{2}\right)\right)}{3} - \frac{40\left(\tan^7\left(\frac{x}{2}\right)\right)}{3} - \frac{8\left(\tan^6\left(\frac{x}{2}\right)\right)}{3} + \frac{8\left(\tan^4\left(\frac{x}{2}\right)\right)}{3} + \frac{236\left(\tan^5\left(\frac{x}{2}\right)\right)}{15}}{\left(\tan^2\left(\frac{x}{2}\right) - 2\tan\left(\frac{x}{2}\right) - 1\right)^5}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)+sin(x))^6,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(tan(x)+1)^2-8/3/(tan(x)+1)^3-1/(tan(x)+1)+2/(tan(x)+1)^4-4/5/(tan(x)+1)^
5
```

Maxima [A]
time $= 0.27$, size $= 56$, normalized size $= 1.12$

$$-\frac{15\tan(x)^4 + 30\tan(x)^3 + 40\tan(x)^2 + 20\tan(x) + 7}{15\left(\tan(x)^5 + 5\tan(x)^4 + 10\tan(x)^3 + 10\tan(x)^2 + 5\tan(x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x))^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*tan(x)^4 + 30*tan(x)^3 + 40*tan(x)^2 + 20*tan(x) + 7)/(tan(x)^5 +
 5*tan(x)^4 + 10*tan(x)^3 + 10*tan(x)^2 + 5*tan(x) + 1)
```

Fricas [A]
time = 0.34, size = 67, normalized size = 1.34

$$-\frac{8\cos\left(x\right)^5 - 20\cos\left(x\right)^3 - \left(8\cos\left(x\right)^4 + 4\cos\left(x\right)^2 - 7\right)\sin\left(x\right) + 5\cos\left(x\right)}{30\left(4\cos\left(x\right)^5 + \left(4\cos\left(x\right)^4 - 8\cos\left(x\right)^2 - 1\right)\sin\left(x\right) - 5\cos\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x))^6,x, algorithm="fricas")
```

```
[Out] -1/30*(8*cos(x)^5 - 20*cos(x)^3 - (8*cos(x)^4 + 4*cos(x)^2 - 7)*sin(x) + 5*
cos(x))/(4*cos(x)^5 + (4*cos(x)^4 - 8*cos(x)^2 - 1)*sin(x) - 5*cos(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 838 vs.
$2(51) = 102$
time = 3.53, size = 838, normalized size = 16.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x))**6,x)
```

```
[Out] -30*tan(x/2)**9/(15*tan(x/2)**10 - 150*tan(x/2)**9 + 525*tan(x/2)**8 - 600*
tan(x/2)**7 - 450*tan(x/2)**6 + 1020*tan(x/2)**5 + 450*tan(x/2)**4 - 600*ta
n(x/2)**3 - 525*tan(x/2)**2 - 150*tan(x/2) - 15) + 120*tan(x/2)**8/(15*tan(
x/2)**10 - 150*tan(x/2)**9 + 525*tan(x/2)**8 - 600*tan(x/2)**7 - 450*tan(x/
2)**6 + 1020*tan(x/2)**5 + 450*tan(x/2)**4 - 600*tan(x/2)**3 - 525*tan(x/2)
**2 - 150*tan(x/2) - 15) - 200*tan(x/2)**7/(15*tan(x/2)**10 - 150*tan(x/2)*
*9 + 525*tan(x/2)**8 - 600*tan(x/2)**7 - 450*tan(x/2)**6 + 1020*tan(x/2)**5
 + 450*tan(x/2)**4 - 600*tan(x/2)**3 - 525*tan(x/2)**2 - 150*tan(x/2) - 15)
 - 40*tan(x/2)**6/(15*tan(x/2)**10 - 150*tan(x/2)**9 + 525*tan(x/2)**8 - 60
0*tan(x/2)**7 - 450*tan(x/2)**6 + 1020*tan(x/2)**5 + 450*tan(x/2)**4 - 600*
tan(x/2)**3 - 525*tan(x/2)**2 - 150*tan(x/2) - 15) + 236*tan(x/2)**5/(15*ta
n(x/2)**10 - 150*tan(x/2)**9 + 525*tan(x/2)**8 - 600*tan(x/2)**7 - 450*tan(
x/2)**6 + 1020*tan(x/2)**5 + 450*tan(x/2)**4 - 600*tan(x/2)**3 - 525*tan(x/
2)**2 - 150*tan(x/2) - 15) + 40*tan(x/2)**4/(15*tan(x/2)**10 - 150*tan(x/2)
**9 + 525*tan(x/2)**8 - 600*tan(x/2)**7 - 450*tan(x/2)**6 + 1020*tan(x/2)**
5 + 450*tan(x/2)**4 - 600*tan(x/2)**3 - 525*tan(x/2)**2 - 150*tan(x/2) - 15
) - 200*tan(x/2)**3/(15*tan(x/2)**10 - 150*tan(x/2)**9 + 525*tan(x/2)**8 -
600*tan(x/2)**7 - 450*tan(x/2)**6 + 1020*tan(x/2)**5 + 450*tan(x/2)**4 - 60
0*tan(x/2)**3 - 525*tan(x/2)**2 - 150*tan(x/2) - 15) - 120*tan(x/2)**2/(15*
```

```
tan(x/2)**10 - 150*tan(x/2)**9 + 525*tan(x/2)**8 - 600*tan(x/2)**7 - 450*ta
n(x/2)**6 + 1020*tan(x/2)**5 + 450*tan(x/2)**4 - 600*tan(x/2)**3 - 525*tan(
x/2)**2 - 150*tan(x/2) - 15) - 30*tan(x/2)/(15*tan(x/2)**10 - 150*tan(x/2)*
*9 + 525*tan(x/2)**8 - 600*tan(x/2)**7 - 450*tan(x/2)**6 + 1020*tan(x/2)**5
 + 450*tan(x/2)**4 - 600*tan(x/2)**3 - 525*tan(x/2)**2 - 150*tan(x/2) - 15)
```

Giac [A]
time = 0.00, size = 39, normalized size = 0.78

$$\frac{2\left(-15\tan^4 x - 30\tan^3 x - 40\tan^2 x - 20\tan x - 7\right)}{30\left(\tan x + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)+sin(x))^6,x)

[Out] -1/15*(15*tan(x)^4 + 30*tan(x)^3 + 40*tan(x)^2 + 20*tan(x) + 7)/(tan(x) + 1
)^5

Mupad [B]
time = 0.27, size = 88, normalized size = 1.76

$$\frac{2\tan\left(\frac{x}{2}\right)\left(15\tan\left(\frac{x}{2}\right)^8 - 60\tan\left(\frac{x}{2}\right)^7 + 100\tan\left(\frac{x}{2}\right)^6 + 20\tan\left(\frac{x}{2}\right)^5 - 118\tan\left(\frac{x}{2}\right)^4 - 20\tan\left(\frac{x}{2}\right)^3 + 100\tan\left(\frac{x}{2}\right)^2 + 60\tan\left(\frac{x}{2}\right) + 15\right)}{15\left(-\tan\left(\frac{x}{2}\right)^2 + 2\tan\left(\frac{x}{2}\right) + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x) + sin(x))^6,x)

[Out] (2*tan(x/2)*(60*tan(x/2) + 100*tan(x/2)^2 - 20*tan(x/2)^3 - 118*tan(x/2)^4
+ 20*tan(x/2)^5 + 100*tan(x/2)^6 - 60*tan(x/2)^7 + 15*tan(x/2)^8 + 15))/(15
*(2*tan(x/2) - tan(x/2)^2 + 1)^5)

3.6 $\quad \int \log\left(\frac{1}{x^4} + x^4\right)\,dx$

Optimal. Leaf size=334

$$-4x - \sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}} - 2x}{\sqrt{2+\sqrt{2}}}\right) - \sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}} - 2x}{\sqrt{2-\sqrt{2}}}\right) + \sqrt{2+\sqrt{2}}\,\tan^{-1}$$

```
[Out] -4*x+x*ln(1/x^4+x^4)-arctan((-2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-
2^(1/2))^(1/2)+arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2)
)^(1/2)-1/2*ln(1+x^2-x*(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/2*ln(1+x^2+x*
(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)-arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2^(
1/2))^(1/2))*(2+2^(1/2))^(1/2)+arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(
1/2))*(2+2^(1/2))^(1/2)-1/2*ln(1+x^2-x*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)
+1/2*ln(1+x^2+x*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.125$,

Rules used = {2603, 12, 396, 219, 1183, 648, 632, 210, 642}

$$x\log\left(x^4+\frac{1}{x^4}\right) - \frac{1}{2}\sqrt{2-\sqrt{2}}\,\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right) + \frac{1}{2}\sqrt{2-\sqrt{2}}\,\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right) - \frac{1}{2}\sqrt{2+\sqrt{2}}\,\log\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right) + \frac{1}{2}\sqrt{2+\sqrt{2}}\,\log\left(x^2+\sqrt{2+\sqrt{2}}\,x+1\right) - 4x - \sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right) - \sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right) + \sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right) + \sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x^(-4) + x^4],x]
```

```
[Out] -4*x - Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]
] - Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]] +
 Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]] + Sq
rt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]] - (Sqrt
[2 - Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/2 + (Sqrt[2 - Sqrt[2]]*Lo
g[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/2 - (Sqrt[2 + Sqrt[2]]*Log[1 - Sqrt[2 + S
qrt[2]]*x + x^2])/2 + (Sqrt[2 + Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2]
)/2 + x*Log[x^(-4) + x^4]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```


Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b,
 4]], s = Denominator[Rt[a/b, 4]]}, Dist[r/(2*Sqrt[2]*a), Int[(Sqrt[2]*r -
s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Dist[r/(2*Sq
rt[2]*a), Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n
/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
```

```
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \log\left(\frac{1}{x^4} + x^4\right) dx = x\log\left(\frac{1}{x^4} + x^4\right) - \int \frac{4(-1+x^8)}{1+x^8} dx$$

$$= x\log\left(\frac{1}{x^4} + x^4\right) - 4\int \frac{-1+x^8}{1+x^8} dx$$

$$= -4x + x\log\left(\frac{1}{x^4} + x^4\right) + 8\int \frac{1}{1+x^8} dx$$

$$= -4x + x\log\left(\frac{1}{x^4} + x^4\right) + \left(2\sqrt{2}\right)\int \frac{\sqrt{2} - x^2}{1 - \sqrt{2}\,x^2 + x^4} dx + \left(2\sqrt{2}\right)\int \frac{\sqrt{2} + }{1 + \sqrt{2}\,x^2}$$

$$= -4x + x\log\left(\frac{1}{x^4} + x^4\right) + \sqrt{2 - \sqrt{2}}\int \frac{\sqrt{2\left(2+\sqrt{2}\right)} - \left(1+\sqrt{2}\right)x}{1 - \sqrt{2+\sqrt{2}}\,x + x^2} dx + \sqrt{}$$

$$= -4x + x\log\left(\frac{1}{x^4} + x^4\right) - \frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{-\sqrt{2-\sqrt{2}} + 2x}{1 - \sqrt{2-\sqrt{2}}\,x + x^2} dx + \frac{1}{2}\sqrt{2-\sqrt{2}}$$

$$= -4x - \frac{1}{2}\sqrt{2-\sqrt{2}}\,\log\left(1 - \sqrt{2-\sqrt{2}}\,x + x^2\right) + \frac{1}{2}\sqrt{2-\sqrt{2}}\,\log\left(1 + \sqrt{2-}\right.$$

$$= -4x - \sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}} - 2x}{\sqrt{2+\sqrt{2}}}\right) - \sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.00, size = 30, normalized size = 0.09

$$-4x + 8x\,{}_2F_1\left(\frac{1}{8}, 1; \frac{9}{8}; -x^8\right) + x\log\left(\frac{1}{x^4} + x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x^(-4) + x^4],x]
```

```
[Out] -4*x + 8*x*Hypergeometric2F1[1/8, 1, 9/8, -x^8] + x*Log[x^(-4) + x^4]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 3.09, size = 37, normalized size = 0.11

$$-4x + x\text{Log}\left[\frac{1+x^8}{x^4}\right] - \text{RootSum}\left[1 + \#1^8\&, \text{Log}\left[x - \#1\right]\#1\&\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Log[x^4 + 1/x^4],x]')
[Out] -4 x + x Log[(1 + x ^ 8) / x ^ 4] - RootSum[1 + #1 ^ 8&, Log[x - #1] #1&]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 36, normalized size = 0.11

method	result	size
risch	$x \ln\left(\frac{1}{x^4} + x^4\right) - 4x + \left(\sum\limits_{\_R=\text{RootOf}(\_Z^8+1)} \frac{\ln\left(-\_R+x\right)}{\_R^7}\right)$	34
default	$x \ln\left(\frac{x^8+1}{x^4}\right) - 4x + \left(\sum\limits_{\_R=\text{RootOf}(\_Z^8+1)} \frac{\ln\left(-\_R+x\right)}{\_R^7}\right)$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1/x^4+x^4),x,method=_RETURNVERBOSE)
[Out] x*ln((x^8+1)/x^4)-4*x+sum(1/_R^7*ln(-_R+x),_R=RootOf(_Z^8+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

<center>Failed to integrate</center>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/x^4+x^4),x, algorithm="maxima")
[Out] x*log(x^8 + 1) - 4*x*log(x) - 4*x + 8*integrate(1/(x^8 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1058 vs.
2(252) = 504.
time = 0.39, size = 1058, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/x^4+x^4),x, algorithm="fricas")
[Out] -1/2*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sq
rt(2)*x - sqrt(2)*sqrt(4*x^2 + 2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 2*sqrt(2)*x*
sqrt(-sqrt(2) + 2) + 4) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqr
t(2) + 2) + sqrt(-sqrt(2) + 2))) - 1/2*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)
*sqrt(-sqrt(2) + 2))*arctan(-(2*sqrt(2)*x - sqrt(2)*sqrt(4*x^2 - 2*sqrt(2)*
x*sqrt(sqrt(2) + 2) + 2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 4) - sqrt(sqrt(2) +
2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) + 1/2*(s
```

```
qrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((2*sqrt(2)*x
- sqrt(2)*sqrt(4*x^2 + 2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 2*sqrt(2)*x*sqrt(-sq
rt(2) + 2) + 4) + sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2
) - sqrt(-sqrt(2) + 2))) + 1/2*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-s
qrt(2) + 2))*arctan((2*sqrt(2)*x - sqrt(2)*sqrt(4*x^2 - 2*sqrt(2)*x*sqrt(sq
rt(2) + 2) - 2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 4) - sqrt(sqrt(2) + 2) - sqrt
(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) + 1/8*(sqrt(2)*sq
rt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(4*x^2 + 2*sqrt(2)*x*sqrt(
sqrt(2) + 2) + 2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 4) + 1/8*(sqrt(2)*sqrt(sqrt
(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(4*x^2 + 2*sqrt(2)*x*sqrt(sqrt(2)
 + 2) - 2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 4) - 1/8*(sqrt(2)*sqrt(sqrt(2) + 2
) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(4*x^2 - 2*sqrt(2)*x*sqrt(sqrt(2) + 2) +
 2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 4) - 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) + sqr
t(2)*sqrt(-sqrt(2) + 2))*log(4*x^2 - 2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 2*sqrt
(2)*x*sqrt(-sqrt(2) + 2) + 4) + x*log((x^8 + 1)/x^4) - sqrt(sqrt(2) + 2)*ar
ctan(-(2*x - 2*sqrt(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + sqrt(-sqrt(2) + 2))/s
qrt(sqrt(2) + 2)) - sqrt(sqrt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 - x*sqrt(-s
qrt(2) + 2) + 1) - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) - sqrt(-sqrt(2) +
 2)*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2
))/sqrt(-sqrt(2) + 2)) - sqrt(-sqrt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 - x*s
qrt(sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/4*sqrt(s
qrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1/4*sqrt(sqrt(2) + 2)*log(
x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/4*sqrt(-sqrt(2) + 2)*log(x^2 + x*sqrt(-s
qrt(2) + 2) + 1) - 1/4*sqrt(-sqrt(2) + 2)*log(x^2 - x*sqrt(-sqrt(2) + 2) +
1) - 4*x
```

Sympy [A]
time = 1.25, size = 26, normalized size = 0.08

$$ x \log \left(x^4 + \frac{1}{x^4} \right) - 4x - \text{RootSum} \left(t^8 + 1, (t \mapsto t \log (-t + x)) \right) $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1/x**4+x**4),x)
```

```
[Out] x*log(x**4 + x**(-4)) - 4*x - RootSum(_t**8 + 1, Lambda(_t, _t*log(-_t + x)
))
```

Giac [A]
time = 0.01, size = 344, normalized size = 1.03

$$ -\tfrac{1}{2}\sqrt{\sqrt{2}+2}\,\ln\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right)+\sqrt{-\sqrt{2}+2}\,\arctan\left(\frac{x-\frac{\sqrt{2+\sqrt{2}}}{2}}{\frac{\sqrt{2-\sqrt{2}}}{2}}\right)-\tfrac{1}{2}\sqrt{-\sqrt{2}+2}\,\ln\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)+\sqrt{\sqrt{2}+2}\,\arctan\left(\frac{x-\frac{\sqrt{2-\sqrt{2}}}{2}}{\frac{\sqrt{2+\sqrt{2}}}{2}}\right)+\tfrac{1}{2}\sqrt{-\sqrt{2}+2}\,\ln\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)+\sqrt{\sqrt{2}+2}\,\arctan\left(\frac{x+\frac{\sqrt{2-\sqrt{2}}}{2}}{\frac{\sqrt{2+\sqrt{2}}}{2}}\right)+\tfrac{1}{2}\sqrt{\sqrt{2}+2}\,\ln\left(x^2+\sqrt{2+\sqrt{2}}\,x+1\right)+\sqrt{-\sqrt{2}+2}\,\arctan\left(\frac{x+\frac{\sqrt{2+\sqrt{2}}}{2}}{\frac{\sqrt{2-\sqrt{2}}}{2}}\right)-4x+x\ln\left(\frac{1}{x^4}+x^4\right) $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/x^4+x^4),x)
```

```
[Out] x*log(x^4 + 1/x^4) + sqrt(sqrt(2) + 2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sq
rt(sqrt(2) + 2)) + sqrt(sqrt(2) + 2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt
(sqrt(2) + 2)) + sqrt(-sqrt(2) + 2)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-
sqrt(2) + 2)) + sqrt(-sqrt(2) + 2)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-s
qrt(2) + 2)) + 1/2*sqrt(sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1
/2*sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/2*sqrt(-sqrt(2)
 + 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/2*sqrt(-sqrt(2) + 2)*log(x^2
- x*sqrt(-sqrt(2) + 2) + 1) - 4*x
```

Mupad [B]

time = 0.26, size = 313, normalized size = 0.94

$$x\ln\left(\frac{1}{x^4}+x^4\right)-4x+\operatorname{atan}\left(\frac{x\sqrt{\sqrt{2}-2}\,2097152i}{2097152\sqrt{2-\sqrt{2}}\,\sqrt{-\sqrt{2}-2}+2097152\sqrt{2}}-\frac{x\sqrt{2-\sqrt{2}}\,2097152i}{2097152\sqrt{2-\sqrt{2}}\,\sqrt{-\sqrt{2}-2}+2097152\sqrt{2}}\right)\left(\sqrt{-\sqrt{2}-2}\,1i-\sqrt{2-\sqrt{2}}\,1i\right)-\operatorname{atan}\left(\frac{x\sqrt{\sqrt{2}-2}\,2097152i}{2097152\sqrt{2}+2097152\sqrt{\sqrt{2}-2}\,\sqrt{\sqrt{2}+2}}+\frac{x\sqrt{\sqrt{2}+2}\,2097152i}{2097152\sqrt{2}+2097152\sqrt{\sqrt{2}-2}\,\sqrt{\sqrt{2}+2}}\right)\left(\sqrt{\sqrt{2}-2}\,1i+\sqrt{\sqrt{2}+2}\,1i\right)+\operatorname{atan}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{2}+2}}{2}+x\sqrt{\sqrt{2}+2}\left(\frac{1}{2}+\frac{1}{2}i\right)\right)\left(\frac{\sqrt{2}}{2}1i-\frac{1}{2}-\frac{1}{2}i\right)\sqrt{\sqrt{2}+2}\,2i-\operatorname{atan}\left(x\sqrt{\sqrt{2}+2}\left(\frac{1}{2}-\frac{1}{2}i\right)+\frac{\sqrt{2}\,x\sqrt{\sqrt{2}+2}\,1i}{2}\right)\left(\frac{\sqrt{2}}{2}-\frac{1}{2}-\frac{1}{2}i\right)\sqrt{\sqrt{2}+2}\,2i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1/x^4 + x^4),x)
```

```
[Out] x*log(1/x^4 + x^4) - 4*x + atan((x*(- 2^(1/2) - 2)^(1/2)*2097152i)/(2097152
*(2 - 2^(1/2))^(1/2)*(- 2^(1/2) - 2)^(1/2) + 2097152*2^(1/2)) - (x*(2 - 2^(
1/2))^(1/2)*2097152i)/(2097152*(2 - 2^(1/2))^(1/2)*(- 2^(1/2) - 2)^(1/2) +
2097152*2^(1/2)))*((- 2^(1/2) - 2)^(1/2)*1i - (2 - 2^(1/2))^(1/2)*1i) - ata
n((x*(2^(1/2) - 2)^(1/2)*2097152i)/(2097152*2^(1/2) + 2097152*(2^(1/2) - 2)
^(1/2)*(2^(1/2) + 2)^(1/2)) + (x*(2^(1/2) + 2)^(1/2)*2097152i)/(2097152*2^(
1/2) + 2097152*(2^(1/2) - 2)^(1/2)*(2^(1/2) + 2)^(1/2)))*((2^(1/2) - 2)^(1/
2)*1i + (2^(1/2) + 2)^(1/2)*1i) + atan(x*(2^(1/2) + 2)^(1/2)*(1/2 + 1i/2) -
 (2^(1/2)*x*(2^(1/2) + 2)^(1/2))/2)*((2^(1/2)*1i)/2 - (1/2 + 1i/2))*(2^(1/2
) + 2)^(1/2)*2i - atan(x*(2^(1/2) + 2)^(1/2)*(1/2 - 1i/2) + (2^(1/2)*x*(2^(
1/2) + 2)^(1/2)*1i)/2)*(2^(1/2)/2 - (1/2 - 1i/2))*(2^(1/2) + 2)^(1/2)*2i
```

3.7 $\displaystyle\int \frac{\log(1+x)}{x\sqrt{1+\sqrt{1+x}}}\,dx$

Optimal. Leaf size=291

$$-8\tanh^{-1}\left(\sqrt{1+\sqrt{1+x}}\right) - \frac{2\log(1+x)}{\sqrt{1+\sqrt{1+x}}} - \sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1+\sqrt{1+x}}}{\sqrt{2}}\right)\log(1+x) + 2\sqrt{2}\,\tanh^{-1}$$

```
[Out] -8*arctanh((1+(1+x)^(1/2))^(1/2))-arctanh(1/2*(1+(1+x)^(1/2))^(1/2)*2^(1/2)
)*ln(1+x)*2^(1/2)+2*arctanh(1/2*2^(1/2))*ln(1-(1+(1+x)^(1/2))^(1/2))*2^(1/2
)-2*arctanh(1/2*2^(1/2))*ln(1+(1+(1+x)^(1/2))^(1/2))*2^(1/2)+polylog(2,-2^(
1/2)*(1-(1+(1+x)^(1/2))^(1/2))/(2-2^(1/2)))*2^(1/2)-polylog(2,2^(1/2)*(1-(1
+(1+x)^(1/2))^(1/2))/(2+2^(1/2)))*2^(1/2)-polylog(2,-2^(1/2)*(1+(1+(1+x)^(1
/2))^(1/2))/(2-2^(1/2)))*2^(1/2)+polylog(2,2^(1/2)*(1+(1+(1+x)^(1/2))^(1/2)
)/(2+2^(1/2)))*2^(1/2)-2*ln(1+x)/(1+(1+x)^(1/2))^(1/2)
```

Rubi [F]

time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(1+x)}{x\sqrt{1+\sqrt{1+x}}}\,dx$$

Verification is not applicable to the result.

```
[In]  Int[Log[1 + x]/(x*Sqrt[1 + Sqrt[1 + x]]),x]
```

```
[Out] Defer[Int][Log[1 + x]/(x*Sqrt[1 + Sqrt[1 + x]]), x]
```

Rubi steps

$$\int \frac{\log(1+x)}{x\sqrt{1+\sqrt{1+x}}}\,dx = \int \frac{\log(1+x)}{x\sqrt{1+\sqrt{1+x}}}\,dx$$

Mathematica [A]

time = 0.74, size = 430, normalized size = 1.48

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[1 + x]/(x*Sqrt[1 + Sqrt[1 + x]]),x]
```

```
[Out] -8*ArcTanh[1/Sqrt[1 + Sqrt[1 + x]]] - (2*Log[1 + x])/Sqrt[1 + Sqrt[1 + x]]
+ (Log[1 + x]*Log[1 - Sqrt[2]/Sqrt[1 + Sqrt[1 + x]]])/Sqrt[2] - (Log[1 + x]
*Log[1 + Sqrt[2]/Sqrt[1 + Sqrt[1 + x]]])/Sqrt[2] - Sqrt[2]*(Log[1 + Sqrt[2]
]*Log[1 + 1/Sqrt[1 + Sqrt[1 + x]]] + Log[-((2 + Sqrt[2])*(-1 + 1/Sqrt[1 + S
qrt[1 + x]]))]*Log[1 - Sqrt[2]/Sqrt[1 + Sqrt[1 + x]]] + 2*PolyLog[2, Sqrt[2
]/Sqrt[1 + Sqrt[1 + x]]] - PolyLog[2, -((-2 + Sqrt[2])*(1 + 1/Sqrt[1 + Sqrt
[1 + x]]))] + PolyLog[2, (1 + Sqrt[2])*(-1 + Sqrt[2]/Sqrt[1 + Sqrt[1 + x]])
]) + Sqrt[2]*(Log[1 + Sqrt[2]]*Log[1 - 1/Sqrt[1 + Sqrt[1 + x]]] + Log[(2 +
Sqrt[2])*(1 + 1/Sqrt[1 + Sqrt[1 + x]])]*Log[1 + Sqrt[2]/Sqrt[1 + Sqrt[1 + x
]]] + 2*PolyLog[2, -(Sqrt[2]/Sqrt[1 + Sqrt[1 + x]])] - PolyLog[2, (-2 + Sqr
t[2])*(-1 + 1/Sqrt[1 + Sqrt[1 + x]])] + PolyLog[2, -((1 + Sqrt[2])*(1 + Sqr
t[2]/Sqrt[1 + Sqrt[1 + x]]))])]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Log[1 + x]/(x*Sqrt[1 + Sqrt[1 + x]]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 172, normalized size = 0.59

method	result
derivativedivides	$8\left(\displaystyle\sum_{\_\alpha=\mathrm{RootOf}(\_Z^2-2)}\dfrac{\frac{\ln\left(\sqrt{1+\sqrt{1+x}}-\_\alpha\right)\ln(1+x)}{2}-\mathrm{dilog}\left(\frac{1+\sqrt{1+\sqrt{1+x}}}{1+\_\alpha}\right)-\ln\left(\sqrt{1+\cdots}\right)}{\cdots}\right.$
default	$8\left(\displaystyle\sum_{\_\alpha=\mathrm{RootOf}(\_Z^2-2)}\dfrac{\frac{\ln\left(\sqrt{1+\sqrt{1+x}}-\_\alpha\right)\ln(1+x)}{2}-\mathrm{dilog}\left(\frac{1+\sqrt{1+\sqrt{1+x}}}{1+\_\alpha}\right)-\ln\left(\sqrt{1+\cdots}\right)}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+x)/x/(1+(1+x)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

[Out] 8*Sum(1/8*(1/2*ln((1+(1+x)^(1/2))^(1/2)-_alpha)*ln(1+x)-dilog((1+(1+(1+x)^(1/2))^(1/2))/(1+_alpha))-ln((1+(1+x)^(1/2))^(1/2)-_alpha)*ln((1+(1+(1+x)^(1/2))^(1/2))/(1+_alpha))-dilog(((1+(1+x)^(1/2))^(1/2)-1)/(-1+_alpha))-ln((1+(1+x)^(1/2))^(1/2)-_alpha)*ln(((1+(1+x)^(1/2))^(1/2)-1)/(-1+_alpha)))*_alpha,_alpha=RootOf(_Z^2-2))-2*ln(1+x)/(1+(1+x)^(1/2))^(1/2)-8*arctanh((1+(1+x)^(1/2))^(1/2))

Maxima [A]
time = 0.38, size = 366, normalized size = 1.26

$$\frac{1}{8}\left(\sqrt{2}\ln\left(-\frac{\sqrt{2}-\sqrt{\sqrt{x+1}+1}}{\sqrt{2}+\sqrt{\sqrt{x+1}+1}}\right)-\frac{4}{\sqrt{\sqrt{x+1}+1}}\right)\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(1+x)/x/(1+(1+x)^(1/2))^(1/2),x, algorithm="maxima")

[Out] 1/2*(sqrt(2)*log(-(sqrt(2) - sqrt(sqrt(x + 1) + 1))/(sqrt(2) + sqrt(sqrt(x + 1) + 1))) - 4/sqrt(sqrt(x + 1) + 1)*log(x + 1) + sqrt(2)*(log(sqrt(2) + sqrt(sqrt(x + 1) + 1))*log(-(sqrt(2) + sqrt(sqrt(x + 1) + 1))/(sqrt(2) + 1) + 1) + dilog((sqrt(2) + sqrt(sqrt(x + 1) + 1))/(sqrt(2) + 1))) - sqrt(2)*(log(-sqrt(2) + sqrt(sqrt(x + 1) + 1))*log(-(sqrt(2) - sqrt(sqrt(x + 1) + 1))/(sqrt(2) + 1) + 1) + dilog((sqrt(2) - sqrt(sqrt(x + 1) + 1))/(sqrt(2) + 1))) + sqrt(2)*(log(sqrt(2) + sqrt(sqrt(x + 1) + 1))*log(-(sqrt(2) + sqrt(sqrt(x + 1) + 1))/(sqrt(2) - 1) + 1) + dilog((sqrt(2) + sqrt(sqrt(x + 1) + 1))/(sqrt(2) - 1))) - sqrt(2)*(log(-sqrt(2) + sqrt(sqrt(x + 1) + 1))*log(-(sqrt(2) - sqrt(sqrt(x + 1) + 1))/(sqrt(2) - 1) + 1) + dilog((sqrt(2) - sqrt(sqrt(x + 1) + 1))/(sqrt(2) - 1))) - 4*log(sqrt(sqrt(x + 1) + 1) + 1) + 4*log(sqrt(sqrt(x + 1) + 1) - 1)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(1+x)/x/(1+(1+x)^(1/2))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log{\left(x + 1 \right)}}{x \sqrt{\sqrt{x + 1} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+x)/x/(1+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(log(x + 1)/(x*sqrt(sqrt(x + 1) + 1)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

<div align="center">Could not integrate</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x)/x/(1+(1+x)^(1/2))^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln(x+1)}{x\sqrt{\sqrt{x+1}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + 1)/(x*((x + 1)^(1/2) + 1)^(1/2)),x)
```

```
[Out] int(log(x + 1)/(x*((x + 1)^(1/2) + 1)^(1/2)), x)
```

3.8 $\int \dfrac{\sqrt{1+\sqrt{1+x}}\ \log(1+x)}{x}\,dx$

Optimal. Leaf size=308

$$-16\sqrt{1+\sqrt{1+x}}+16\tanh^{-1}\left(\sqrt{1+\sqrt{1+x}}\right)+4\sqrt{1+\sqrt{1+x}}\ \log(1+x)-2\sqrt{2}\ \tanh^{-1}\left(\frac{\sqrt{1+\sqrt{1}}}{\sqrt{2}}\right)$$

```
[Out] 16*arctanh((1+(1+x)^(1/2))^(1/2))-2*arctanh(1/2*(1+(1+x)^(1/2))^(1/2)*2^(1/
2))*ln(1+x)*2^(1/2)+4*arctanh(1/2*2^(1/2))*ln(1-(1+(1+x)^(1/2))^(1/2))*2^(1
/2)-4*arctanh(1/2*2^(1/2))*ln(1+(1+(1+x)^(1/2))^(1/2)*2^(1/2)+2*polylog(2,
-2^(1/2)*(1-(1+(1+x)^(1/2))^(1/2))/(2-2^(1/2)))*2^(1/2)-2*polylog(2,2^(1/2)
*(1-(1+(1+x)^(1/2))^(1/2))/(2+2^(1/2)))*2^(1/2)-2*polylog(2,-2^(1/2)*(1+(1+
(1+x)^(1/2))^(1/2))/(2-2^(1/2)))*2^(1/2)+2*polylog(2,2^(1/2)*(1+(1+(1+x)^(1
/2))^(1/2))/(2+2^(1/2)))*2^(1/2)-16*(1+(1+x)^(1/2))^(1/2)+4*ln(1+x)*(1+(1+x
)^(1/2))^(1/2)
```

Rubi [F]

time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\dfrac{\text{number of rules}}{\text{integrand size}} = 0.000$,
Rules used = {}

$$\int \frac{\sqrt{1+\sqrt{1+x}}\ \log(1+x)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[1 + Sqrt[1 + x]]*Log[1 + x])/x,x]
```

```
[Out] Defer[Int][(Sqrt[1 + Sqrt[1 + x]]*Log[1 + x])/x, x]
```

Rubi steps

$$\int \frac{\sqrt{1+\sqrt{1+x}}\ \log(1+x)}{x}\,dx = \int \frac{\sqrt{1+\sqrt{1+x}}\ \log(1+x)}{x}\,dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 654 vs. 2(308) = 616.

time = 0.44, size = 654, normalized size = 2.12

Antiderivative was successfully verified.

`[In] Integrate[(Sqrt[1 + Sqrt[1 + x]]*Log[1 + x])/x,x]`

`[Out] -16*Sqrt[1 + Sqrt[1 + x]] + 4*Sqrt[1 + Sqrt[1 + x]]*Log[1 + x] + Sqrt[2]*Lo`
`g[1 + x]*Log[Sqrt[2] - Sqrt[1 + Sqrt[1 + x]]] - 8*Log[-1 + Sqrt[1 + Sqrt[1`
`+ x]]] - 2*Sqrt[2]*Log[Sqrt[2] - Sqrt[1 + Sqrt[1 + x]]]*Log[-1 + Sqrt[1 + S`
`qrt[1 + x]]] + 8*Log[1 + Sqrt[1 + Sqrt[1 + x]]] - 2*Sqrt[2]*Log[Sqrt[2] - S`
`qrt[1 + Sqrt[1 + x]]]*Log[1 + Sqrt[1 + Sqrt[1 + x]]] - Sqrt[2]*Log[1 + x]*L`
`og[Sqrt[2] + Sqrt[1 + Sqrt[1 + x]]] + 2*Sqrt[2]*Log[-1 + Sqrt[1 + Sqrt[1 +`
`x]]]*Log[Sqrt[2] + Sqrt[1 + Sqrt[1 + x]]] + 2*Sqrt[2]*Log[1 + Sqrt[1 + Sqrt`
`[1 + x]]]*Log[Sqrt[2] + Sqrt[1 + Sqrt[1 + x]]] - 2*Sqrt[2]*Log[-1 + Sqrt[1`
`+ Sqrt[1 + x]]]*Log[(-1 + Sqrt[2])*(Sqrt[2] + Sqrt[1 + Sqrt[1 + x]])] - 2*S`
`qrt[2]*Log[1 + Sqrt[1 + Sqrt[1 + x]]]*Log[2 + Sqrt[2] + Sqrt[1 + Sqrt[1 + x`
`]] + Sqrt[2]*Sqrt[1 + Sqrt[1 + x]]] + 2*Sqrt[2]*Log[-1 + Sqrt[1 + Sqrt[1 +`
`x]]]*Log[1 - (1 + Sqrt[2])*(-1 + Sqrt[1 + Sqrt[1 + x]])] + 2*Sqrt[2]*Log[1`
`+ Sqrt[1 + Sqrt[1 + x]]]*Log[1 - (-1 + Sqrt[2])*(1 + Sqrt[1 + Sqrt[1 + x]])`
`] - 2*Sqrt[2]*PolyLog[2, -((-1 + Sqrt[2])*(-1 + Sqrt[1 + Sqrt[1 + x]]))] +`
`2*Sqrt[2]*PolyLog[2, (1 + Sqrt[2])*(-1 + Sqrt[1 + Sqrt[1 + x]])] + 2*Sqrt[2`
`]*PolyLog[2, (-1 + Sqrt[2])*(1 + Sqrt[1 + Sqrt[1 + x]])] - 2*Sqrt[2]*PolyLo`
`g[2, -((1 + Sqrt[2])*(1 + Sqrt[1 + Sqrt[1 + x]]))]`

Mathics **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

   cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Log[1 + x]/x*Sqrt[1 + Sqrt[1 + x]],x]')`

`[Out] cought exception: maximum recursion depth exceeded while calling a Python o`
`bject`

Maple **[C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.01, size = 199, normalized size = 0.65

method	result
derivativedivides	$4\ln(1+x)\sqrt{1+\sqrt{1+x}} - 16\sqrt{1+\sqrt{1+x}} - 8\ln\left(\sqrt{1+\sqrt{1+x}}-1\right) + 8\ln\left(1\right.$

default	$4\ln\left(1+x\right)\sqrt{1+\sqrt{1+x}}-16\sqrt{1+\sqrt{1+x}}-8\ln\left(\sqrt{1+\sqrt{1+x}}-1\right)+8\ln\left($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+x)*(1+(1+x)^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 4*ln(1+x)*(1+(1+x)^(1/2))^(1/2)-16*(1+(1+x)^(1/2))^(1/2)-8*ln((1+(1+x)^(1/2
))^(1/2)-1)+8*ln(1+(1+(1+x)^(1/2))^(1/2))+8*Sum(1/4*(1/2*ln((1+(1+x)^(1/2))
^(1/2)-_alpha)*ln(1+x)-dilog((1+(1+(1+x)^(1/2))^(1/2))/(1+_alpha))-ln((1+(1
+x)^(1/2))^(1/2)-_alpha)*ln((1+(1+(1+x)^(1/2))^(1/2))/(1+_alpha))-dilog(((1
+(1+x)^(1/2))^(1/2)-1)/(-1+_alpha))-ln((1+(1+x)^(1/2))^(1/2)-_alpha)*ln(((1
+(1+x)^(1/2))^(1/2)-1)/(-1+_alpha)))*_alpha,_alpha=RootOf(_Z^2-2))
```

Maxima [A]
time = 0.38, size = 378, normalized size = 1.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x)*(1+(1+x)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] (sqrt(2)*log(-(sqrt(2) - sqrt(sqrt(x + 1) + 1))/(sqrt(2) + sqrt(sqrt(x + 1)
 + 1))) + 4*sqrt(sqrt(x + 1) + 1))*log(x + 1) + 2*sqrt(2)*(log(sqrt(2) + sq
rt(sqrt(x + 1) + 1))*log(-(sqrt(2) + sqrt(sqrt(x + 1) + 1))/(sqrt(2) + 1) +
 1) + dilog((sqrt(2) + sqrt(sqrt(x + 1) + 1))/(sqrt(2) + 1))) - 2*sqrt(2)*(
log(-sqrt(2) + sqrt(sqrt(x + 1) + 1))*log(-(sqrt(2) - sqrt(sqrt(x + 1) + 1)
)/(sqrt(2) + 1) + 1) + dilog((sqrt(2) - sqrt(sqrt(x + 1) + 1))/(sqrt(2) + 1
))) + 2*sqrt(2)*(log(sqrt(2) + sqrt(sqrt(x + 1) + 1))*log(-(sqrt(2) + sqrt(
sqrt(x + 1) + 1))/(sqrt(2) - 1) + 1) + dilog((sqrt(2) + sqrt(sqrt(x + 1) +
1))/(sqrt(2) - 1))) - 2*sqrt(2)*(log(-sqrt(2) + sqrt(sqrt(x + 1) + 1))*log(
-(sqrt(2) - sqrt(sqrt(x + 1) + 1))/(sqrt(2) - 1) + 1) + dilog((sqrt(2) - sq
rt(sqrt(x + 1) + 1))/(sqrt(2) - 1))) - 16*sqrt(sqrt(x + 1) + 1) + 8*log(sqr
t(sqrt(x + 1) + 1) + 1) - 8*log(sqrt(sqrt(x + 1) + 1) - 1)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x)*(1+(1+x)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:    integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x+1}+1}\,\log{(x+1)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+x)*(1+(1+x)**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(sqrt(x + 1) + 1)*log(x + 1)/x, x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x)*(1+(1+x)^(1/2))^(1/2)/x,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln{(x+1)}\,\sqrt{\sqrt{x+1}+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x + 1)*((x + 1)^(1/2) + 1)^(1/2))/x,x)
```

```
[Out] int((log(x + 1)*((x + 1)^(1/2) + 1)^(1/2))/x, x)
```

3.9 $\quad \int \dfrac{1}{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=84

$$-\frac{1}{2\left(x+\sqrt{1+x^2}\right)}+\frac{1}{\sqrt{x+\sqrt{1+x^2}}}+\sqrt{x+\sqrt{1+x^2}}+\frac{1}{2}\log\left(x+\sqrt{1+x^2}\right)-2\log\left(1+\sqrt{x+\sqrt{1+x^2}}\right)$$

[Out] 1/2*ln(x+(x^2+1)^(1/2))-2*ln(1+(x+(x^2+1)^(1/2))^(1/2))-1/2/(x+(x^2+1)^(1/2))+1/(x+(x^2+1)^(1/2))^(1/2)+(x+(x^2+1)^(1/2))^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$,

Rules used = {2142, 1835, 1634}

$$\sqrt{\sqrt{x^2+1}+x}+\frac{1}{\sqrt{\sqrt{x^2+1}+x}}-\frac{1}{2\left(\sqrt{x^2+1}+x\right)}+\frac{1}{2}\log\left(\sqrt{x^2+1}+x\right)-2\log\left(\sqrt{\sqrt{x^2+1}+x}+1\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[x + Sqrt[1 + x^2]])^(-1),x]

[Out] -1/2*1/(x + Sqrt[1 + x^2]) + 1/Sqrt[x + Sqrt[1 + x^2]] + Sqrt[x + Sqrt[1 + x^2]] + Log[x + Sqrt[1 + x^2]]/2 - 2*Log[1 + Sqrt[x + Sqrt[1 + x^2]]]

Rule 1634

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]

Rule 1835

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]

Rule 2142

Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\int \frac{1}{1+\sqrt{x+\sqrt{1+x^2}}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{\left(1+\sqrt{x}\right)x^2}\,dx, x, x+\sqrt{1+x^2}\right)$$

$$= \text{Subst}\left(\int \frac{1+x^4}{x^3(1+x)}\,dx, x, \sqrt{x+\sqrt{1+x^2}}\right)$$

$$= \text{Subst}\left(\int \left(1+\frac{1}{x^3}-\frac{1}{x^2}+\frac{1}{x}-\frac{2}{1+x}\right)\,dx, x, \sqrt{x+\sqrt{1+x^2}}\right)$$

$$= -\frac{1}{2\left(x+\sqrt{1+x^2}\right)}+\frac{1}{\sqrt{x+\sqrt{1+x^2}}}+\sqrt{x+\sqrt{1+x^2}}+\frac{1}{2}\log\left(x+\sqrt{1}\right.$$

Mathematica [A]
time = 0.12, size = 105, normalized size = 1.25

$$\frac{1}{2}\left(\frac{-1+5x+2(1+x)\sqrt{x+\sqrt{1+x^2}}+\sqrt{1+x^2}\left(5+2\sqrt{x+\sqrt{1+x^2}}\right)}{x+\sqrt{1+x^2}}+\log\left(x+\sqrt{1+x^2}\right)-4\log\left(1+\sqrt{x+\sqrt{1+x^2}}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sqrt[x + Sqrt[1 + x^2]])^(-1),x]

[Out] ((-1 + 5*x + 2*(1 + x)*Sqrt[x + Sqrt[1 + x^2]] + Sqrt[1 + x^2]*(5 + 2*Sqrt[
x + Sqrt[1 + x^2]]))/(x + Sqrt[1 + x^2]) + Log[x + Sqrt[1 + x^2]] - 4*Log[1
 + Sqrt[x + Sqrt[1 + x^2]]])/2

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(1 + Sqrt[x + Sqrt[1 + x^2]]),x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{1+\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+(x+(x^2+1)^(1/2))^(1/2)),x)

[Out] int(1/(1+(x+(x^2+1)^(1/2))^(1/2)),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

<div align="center">Failed to integrate</div>

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x + sqrt(x^2 + 1)) + 1), x)

Fricas [A]
time = 0.34, size = 66, normalized size = 0.79

$$-\sqrt{x + \sqrt{x^2 + 1}}\left(x - \sqrt{x^2 + 1} - 1\right) + \frac{1}{2}x - \frac{1}{2}\sqrt{x^2 + 1} - 2\log\left(\sqrt{x + \sqrt{x^2 + 1}} + 1\right) + \log\left(\sqrt{x + \sqrt{x^2 + 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="fricas")

[Out] -sqrt(x + sqrt(x^2 + 1))*(x - sqrt(x^2 + 1) - 1) + 1/2*x - 1/2*sqrt(x^2 + 1
) - 2*log(sqrt(x + sqrt(x^2 + 1)) + 1) + log(sqrt(x + sqrt(x^2 + 1)))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x + \sqrt{x^2 + 1}} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(x+(x**2+1)**(1/2))**(1/2)),x)

[Out] Integral(1/(sqrt(x + sqrt(x**2 + 1)) + 1), x)

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

<div align="center">Could not integrate</div>

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(x+(x^2+1)^(1/2))^(1/2)),x)

[Out] Could not integrate

Mupad [F]
time $= 0.00$, size $= -1$, normalized size $= -0.01$

$$\int \frac{1}{\sqrt{x + \sqrt{x^2 + 1}} + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x + (x^2 + 1)^(1/2))^(1/2) + 1),x)

[Out] int(1/((x + (x^2 + 1)^(1/2))^(1/2) + 1), x)

3.10 $\int \dfrac{\sqrt{1+x}}{x+\sqrt{1+\sqrt{1+x}}}\, dx$

Optimal. Leaf size=41

$$2\sqrt{1+x} + \dfrac{8\tanh^{-1}\left(\dfrac{1+2\sqrt{1+\sqrt{1+x}}}{\sqrt{5}}\right)}{\sqrt{5}}$$

[Out] 8/5*arctanh(1/5*(1+2*(1+(1+x)^(1/2))^(1/2))*5^(1/2))*5^(1/2)+2*(1+x)^(1/2)

Rubi [A]

time = 0.13, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 25, $\dfrac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {814, 632, 212}

$$2\sqrt{x+1} + \dfrac{8\tanh^{-1}\left(\dfrac{2\sqrt{\sqrt{x+1}+1}+1}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x]/(x + Sqrt[1 + Sqrt[1 + x]]),x]

[Out] 2*Sqrt[1 + x] + (8*ArcTanh[(1 + 2*Sqrt[1 + Sqrt[1 + x]])/Sqrt[5]])/Sqrt[5]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]

Rule 814

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\int \frac{\sqrt{1+x}}{x+\sqrt{1+\sqrt{1+x}}}\,dx = 2\mathrm{Subst}\left(\int \frac{x^2}{-1+x^2+\sqrt{1+x}}\,dx, x, \sqrt{1+x}\right)$$

$$= 4\mathrm{Subst}\left(\int \frac{(-1+x)(1+x)^2}{-1+x+x^2}\,dx, x, \sqrt{1+\sqrt{1+x}}\right)$$

$$= 4\mathrm{Subst}\left(\int \left(x-\frac{1}{-1+x+x^2}\right)\,dx, x, \sqrt{1+\sqrt{1+x}}\right)$$

$$= 2\sqrt{1+x} - 4\mathrm{Subst}\left(\int \frac{1}{-1+x+x^2}\,dx, x, \sqrt{1+\sqrt{1+x}}\right)$$

$$= 2\sqrt{1+x} + 8\mathrm{Subst}\left(\int \frac{1}{5-x^2}\,dx, x, 1+2\sqrt{1+\sqrt{1+x}}\right)$$

$$= 2\sqrt{1+x} + \frac{8\tanh^{-1}\left(\frac{1+2\sqrt{1+\sqrt{1+x}}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Mathematica [A]
time = 0.05, size = 41, normalized size = 1.00

$$2\sqrt{1+x} + \frac{8\tanh^{-1}\left(\frac{1+2\sqrt{1+\sqrt{1+x}}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + x]/(x + Sqrt[1 + Sqrt[1 + x]]),x]

[Out] 2*Sqrt[1 + x] + (8*ArcTanh[(1 + 2*Sqrt[1 + Sqrt[1 + x]])/Sqrt[5]])/Sqrt[5]

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sqrt[1 + x]/(x + Sqrt[1 + Sqrt[1 + x]]),x]')

[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'gen
s'

Maple [A]

time $= 0.06$, size $= 34$, normalized size $= 0.83$

method	result	size
derivativedivides	$2\sqrt{1+x} + 2 + \dfrac{8\,\mathrm{arctanh}\left(\dfrac{\left(1+2\sqrt{1+\sqrt{1+x}}\right)\sqrt{5}}{5}\right)\sqrt{5}}{5}$	34
default	$2\sqrt{1+x} + 2 + \dfrac{8\,\mathrm{arctanh}\left(\dfrac{\left(1+2\sqrt{1+\sqrt{1+x}}\right)\sqrt{5}}{5}\right)\sqrt{5}}{5}$	34

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+x)^(1/2)/(x+(1+(1+x)^(1/2))^(1/2)),x,method=_RETURNVERBOSE)`

[Out] `2*(1+x)^(1/2)+2+8/5*arctanh(1/5*(1+2*(1+(1+x)^(1/2))^(1/2))*5^(1/2))*5^(1/2`
`)`

Maxima [A]

time $= 0.34$, size $= 51$, normalized size $= 1.24$

$$-\frac{4}{5}\sqrt{5}\,\log\left(-\frac{\sqrt{5}-2\sqrt{\sqrt{x+1}+1}-1}{\sqrt{5}+2\sqrt{\sqrt{x+1}+1}+1}\right) + 2\sqrt{x+1} + 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)^(1/2)/(x+(1+(1+x)^(1/2))^(1/2)),x, algorithm="maxima")`

[Out] `-4/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(sqrt(x + 1) + 1) - 1)/(sqrt(5) + 2*sqrt`
`(sqrt(x + 1) + 1) + 1)) + 2*sqrt(x + 1) + 2`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(32) = 64$.

time $= 0.35$, size $= 101$, normalized size $= 2.46$

$$\frac{4}{5}\sqrt{5}\,\log\left(\frac{2x^2-\sqrt{5}\,(3x+1)-\left(\sqrt{5}\,(x+2)-5x\right)\sqrt{x+1}+\left(\sqrt{5}\,(x+2)+\left(\sqrt{5}\,(2x-1)-5\right)\sqrt{x+1}-5x\right)\sqrt{\sqrt{x+1}+1}+3x+3}{x^2-x-1}\right)+2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)^(1/2)/(x+(1+(1+x)^(1/2))^(1/2)),x, algorithm="fricas")`

[Out] `4/5*sqrt(5)*log((2*x^2 - sqrt(5)*(3*x + 1) - (sqrt(5)*(x + 2) - 5*x)*sqrt(x`
`+ 1) + (sqrt(5)*(x + 2) + (sqrt(5)*(2*x - 1) - 5)*sqrt(x + 1) - 5*x)*sqrt(`
`sqrt(x + 1) + 1) + 3*x + 3)/(x^2 - x - 1)) + 2*sqrt(x + 1)`

Sympy [A]

time = 8.04, size = 112, normalized size = 2.73

$$2\sqrt{x+1}-16\left(\left(\begin{cases}-\dfrac{\sqrt{5}\,\operatorname{acoth}\left(\dfrac{2\sqrt{5}\left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)}{5}\right)}{10} & \text{for }\left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)^2>\frac{5}{4}\\[2em] -\dfrac{\sqrt{5}\,\operatorname{atanh}\left(\dfrac{2\sqrt{5}\left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)}{5}\right)}{10} & \text{for }\left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)^2<\frac{5}{4}\end{cases}\right)\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)**(1/2)/(x+(1+(1+x)**(1/2))**(1/2)),x)

[Out] 2*sqrt(x + 1) - 16*Piecewise((-sqrt(5)*acoth(2*sqrt(5)*(sqrt(sqrt(x + 1) + 1) + 1/2)/5)/10, (sqrt(sqrt(x + 1) + 1) + 1/2)**2 > 5/4), (-sqrt(5)*atanh(2 *sqrt(5)*(sqrt(sqrt(x + 1) + 1) + 1/2)/5)/10, (sqrt(sqrt(x + 1) + 1) + 1/2) **2 < 5/4)) + 2

Giac [A]

time = 0.01, size = 71, normalized size = 1.73

$$4\left(\frac{\sqrt{\sqrt{x+1}+1}}{2}-\frac{\ln\left(\dfrac{2\sqrt{\sqrt{x+1}+1}+1-\sqrt{5}}{2\sqrt{\sqrt{x+1}+1}+1+\sqrt{5}}\right)}{\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^(1/2)/(x+(1+(1+x)^(1/2))^(1/2)),x)

[Out] -4/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(sqrt(x + 1) + 1) - 1)/(sqrt(5) + 2*sqrt (sqrt(x + 1) + 1) + 1)) + 2*sqrt(x + 1) + 2

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{x+1}}{x+\sqrt{\sqrt{x+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 1)^(1/2)/(x + ((x + 1)^(1/2) + 1)^(1/2)),x)

[Out] int((x + 1)^(1/2)/(x + ((x + 1)^(1/2) + 1)^(1/2)), x)

3.11 $\int \dfrac{1}{x - \sqrt{1 + \sqrt{1 + x}}} \, dx$

Optimal. Leaf size=73

$$\frac{2}{5}\left(5 + \sqrt{5}\right)\log\left(1 - \sqrt{5} - 2\sqrt{1 + \sqrt{1 + x}}\right) + \frac{2}{5}\left(5 - \sqrt{5}\right)\log\left(1 + \sqrt{5} - 2\sqrt{1 + \sqrt{1 + x}}\right)$$

[Out] 2/5*ln(1+5^(1/2)-2*(1+(1+x)^(1/2))^(1/2))*(5-5^(1/2))+2/5*ln(1-5^(1/2)-2*(1+(1+x)^(1/2))^(1/2))*(5+5^(1/2))

Rubi [A]

time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\dfrac{\text{number of rules}}{\text{integrand size}} = 0.105$,

Rules used = {646, 31}

$$\frac{2}{5}\left(5 + \sqrt{5}\right)\log\left(-2\sqrt{\sqrt{x+1} + 1} - \sqrt{5} + 1\right) + \frac{2}{5}\left(5 - \sqrt{5}\right)\log\left(-2\sqrt{\sqrt{x+1} + 1} + \sqrt{5} + 1\right)$$

Antiderivative was successfully verified.

[In] Int[(x - Sqrt[1 + Sqrt[1 + x]])^(-1),x]

[Out] (2*(5 + Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[1 + Sqrt[1 + x]]])/5 + (2*(5 - Sqrt[5])*Log[1 + Sqrt[5] - 2*Sqrt[1 + Sqrt[1 + x]]])/5

Rule 31

Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 646

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\int \frac{1}{x - \sqrt{1 + \sqrt{1 + x}}}\, dx = 2\mathrm{Subst}\left(\int \frac{x}{-1 + x^2 - \sqrt{1 + x}}\, dx, x, \sqrt{1 + x}\right)$$

$$= 4\mathrm{Subst}\left(\int \frac{-1 + x}{-1 - x + x^2}\, dx, x, \sqrt{1 + \sqrt{1 + x}}\right)$$

$$= \frac{1}{5}\left(2\left(5 - \sqrt{5}\right)\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{2} - \frac{\sqrt{5}}{2} + x}\, dx, x, \sqrt{1 + \sqrt{1 + x}}\right) + \frac{1}{5}\left(2\left(5\right.\right.$$

$$= \frac{2}{5}\left(5 + \sqrt{5}\right)\log\left(1 - \sqrt{5} - 2\sqrt{1 + \sqrt{1 + x}}\right) + \frac{2}{5}\left(5 - \sqrt{5}\right)\log\left(1 + \sqrt{5}\right.$$

Mathematica [A]
time = 0.06, size = 69, normalized size = 0.95

$$-\frac{2}{5}\left(-5 + \sqrt{5}\right)\log\left(1 + \sqrt{5} - 2\sqrt{1 + \sqrt{1 + x}}\right) + \frac{2}{5}\left(5 + \sqrt{5}\right)\log\left(-1 + \sqrt{5} + 2\sqrt{1 + \sqrt{1 + x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 + Sqrt[1 + x]])^(-1),x]
```

```
[Out] (-2*(-5 + Sqrt[5])*Log[1 + Sqrt[5] - 2*Sqrt[1 + Sqrt[1 + x]]])/5 + (2*(5 +
Sqrt[5])*Log[-1 + Sqrt[5] + 2*Sqrt[1 + Sqrt[1 + x]]])/5
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x - Sqrt[1 + Sqrt[1 + x]]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(53) = 106.
time = 0.10, size = 175, normalized size = 2.40

method	result

derivativedivides	$2\ln\left(\sqrt{1+x} - \sqrt{1+\sqrt{1+x}}\right) + \dfrac{4\sqrt{5}\ \operatorname{arctanh}\left(\dfrac{\left(2\sqrt{1+\sqrt{1+x}}-1\right)\sqrt{5}}{5}\right)}{5}$
default	$\dfrac{\sqrt{5}\ \operatorname{arctanh}\left(\dfrac{\left(1+2\sqrt{1+x}\right)\sqrt{5}}{5}\right)}{5} + \dfrac{\sqrt{5}\ \operatorname{arctanh}\left(\dfrac{\left(2\sqrt{1+x}-1\right)\sqrt{5}}{5}\right)}{5} - \ln\left(\sqrt{1+x}+\sqrt{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(1+(1+x)^(1/2))^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*5^(1/2)*arctanh(1/5*(1+2*(1+x)^(1/2))*5^(1/2))+1/5*5^(1/2)*arctanh(1/5*
(2*(1+x)^(1/2)-1)*5^(1/2))-ln((1+x)^(1/2)+(1+(1+x)^(1/2))^(1/2))+1/2*ln(x-(
1+x)^(1/2))+1/5*5^(1/2)*arctanh(1/5*(2*x-1)*5^(1/2))+1/2*ln(x^2-x-1)+2/5*5^
(1/2)*arctanh(1/5*(2*(1+(1+x)^(1/2))^(1/2)-1)*5^(1/2))+ln((1+x)^(1/2)-(1+(1
+x)^(1/2))^(1/2))-1/2*ln(x+(1+x)^(1/2))+2/5*arctanh(1/5*(1+2*(1+(1+x)^(1/2)
)^(1/2))*5^(1/2))*5^(1/2)
```

Maxima [A]
time = 0.36, size = 63, normalized size = 0.86

$$-\frac{2}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{\sqrt{x+1}+1}+1}{\sqrt{5}+2\sqrt{\sqrt{x+1}+1}-1}\right) + 2\log\left(\sqrt{x+1} - \sqrt{\sqrt{x+1}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+(1+x)^(1/2))^(1/2)),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(sqrt(x + 1) + 1) + 1)/(sqrt(5) + 2*sqrt
(sqrt(x + 1) + 1) - 1)) + 2*log(sqrt(x + 1) - sqrt(sqrt(x + 1) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(51) = 102.
time = 0.35, size = 112, normalized size = 1.53

$$\frac{2}{5}\sqrt{5}\log\left(\frac{2x^2 + \sqrt{5}\,(3x+1) + \left(\sqrt{5}\,(x+2)+5x\right)\sqrt{x+1} + \left(\sqrt{5}\,(x+2)+\left(\sqrt{5}\,(2x-1)+5\right)\sqrt{x+1}+5x\right)\sqrt{\sqrt{x+1}+1} + 3x+3}{x^2-x-1}\right) + 2\log\left(\sqrt{x+1} - \sqrt{\sqrt{x+1}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+(1+x)^(1/2))^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*log((2*x^2 + sqrt(5)*(3*x + 1) + (sqrt(5)*(x + 2) + 5*x)*sqrt(x
 + 1) + (sqrt(5)*(x + 2) + (sqrt(5)*(2*x - 1) + 5)*sqrt(x + 1) + 5*x)*sqrt(
```

```
sqrt(x + 1) + 1) + 3*x + 3)/(x^2 - x - 1)) + 2*log(sqrt(x + 1) - sqrt(sqrt(
x + 1) + 1))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x - \sqrt{\sqrt{x+1}\ +1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(1+(1+x)**(1/2))**(1/2)),x)

[Out] Integral(1/(x - sqrt(sqrt(x + 1) + 1)), x)

Giac [A]

time = 0.02, size = 92, normalized size = 1.26

$$4\left(\frac{\ln\left|\sqrt{x+1}\ +1 - \sqrt{\sqrt{x+1}\ +1}\ -1\right|}{2} - \frac{\ln\left(\frac{\left|2\sqrt{\sqrt{x+1}\ +1}\ -1-\sqrt{5}\right|}{\left|2\sqrt{\sqrt{x+1}\ +1}\ -1+\sqrt{5}\right|}\right)}{2\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(1+(1+x)^(1/2))^(1/2)),x)

[Out] -2/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(sqrt(x + 1) + 1) - 1)/abs(sqrt(5) +
2*sqrt(sqrt(x + 1) + 1) - 1)) + 2*log(abs(sqrt(x + 1) - sqrt(sqrt(x + 1) +
1)))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x - \sqrt{\sqrt{x+1}\ +1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x - ((x + 1)^(1/2) + 1)^(1/2)),x)

[Out] int(1/(x - ((x + 1)^(1/2) + 1)^(1/2)), x)

3.12 $\int \dfrac{x}{x+\sqrt{1-\sqrt{1+x}}}\, dx$

Optimal. Leaf size=73

$$2\sqrt{1+x}-4\sqrt{1-\sqrt{1+x}}+\left(1-\sqrt{1+x}\right)^2+\dfrac{8\tanh^{-1}\left(\dfrac{1+2\sqrt{1-\sqrt{1+x}}}{\sqrt{5}}\right)}{\sqrt{5}}$$

[Out] 8/5*arctanh(1/5*(1+2*(1-(1+x)^(1/2))^(1/2))*5^(1/2))*5^(1/2)+(1-(1+x)^(1/2))^2+2*(1+x)^(1/2)-4*(1-(1+x)^(1/2))^(1/2)

Rubi [A]

time = 0.20, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 21, $\dfrac{\text{number of rules}}{\text{integrand size}}=0.143$,

Rules used = $\{1642, 632, 212\}$

$$\left(1-\sqrt{x+1}\right)^2-4\sqrt{1-\sqrt{x+1}}+2\sqrt{x+1}+\dfrac{8\tanh^{-1}\left(\dfrac{2\sqrt{1-\sqrt{x+1}}+1}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[x/(x + Sqrt[1 - Sqrt[1 + x]]),x]

[Out] 2*Sqrt[1 + x] - 4*Sqrt[1 - Sqrt[1 + x]] + (1 - Sqrt[1 + x])^2 + (8*ArcTanh[(1 + 2*Sqrt[1 - Sqrt[1 + x]])/Sqrt[5]])/Sqrt[5]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1642

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\int \frac{x}{x + \sqrt{1 - \sqrt{1+x}}}\, dx = 2\text{Subst}\left(\int \frac{x\left(-1+x^2\right)}{-1+\sqrt{1-x}+x^2}\, dx, x, \sqrt{1+x}\right)$$

$$= 4\text{Subst}\left(\int \frac{x^2(1+x)\left(-2+x^2\right)}{-1+x+x^2}\, dx, x, \sqrt{1-\sqrt{1+x}}\right)$$

$$= 4\text{Subst}\left(\int \left(-1-x+x^3-\frac{1}{-1+x+x^2}\right) dx, x, \sqrt{1-\sqrt{1+x}}\right)$$

$$= 2\sqrt{1+x} - 4\sqrt{1-\sqrt{1+x}} + \left(1-\sqrt{1+x}\right)^2 - 4\text{Subst}\left(\int \frac{1}{-1+x+x^2}\, dx \right.$$

$$= 2\sqrt{1+x} - 4\sqrt{1-\sqrt{1+x}} + \left(1-\sqrt{1+x}\right)^2 + 8\text{Subst}\left(\int \frac{1}{5-x^2}\, dx, x, 1 \right.$$

$$= 2\sqrt{1+x} - 4\sqrt{1-\sqrt{1+x}} + \left(1-\sqrt{1+x}\right)^2 + \frac{8\tanh^{-1}\left(\frac{1+2\sqrt{1-\sqrt{1+x}}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Mathematica [A]
time = 0.05, size = 52, normalized size = 0.71

$$x - 4\sqrt{1-\sqrt{1+x}} + \frac{8\tanh^{-1}\left(\frac{1+2\sqrt{1-\sqrt{1+x}}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(x + Sqrt[1 - Sqrt[1 + x]]),x]

[Out] x - 4*Sqrt[1 - Sqrt[1 + x]] + (8*ArcTanh[(1 + 2*Sqrt[1 - Sqrt[1 + x]])/Sqrt[5]])/Sqrt[5]

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x/(x + Sqrt[1 - Sqrt[1 + x]]),x]')

[Out] cought exception: maximum recursion depth exceeded in comparison

Maple [A]

time $= 0.02$, size $= 60$, normalized size $= 0.82$

method	result
derivativedivides	$\left(1 - \sqrt{1+x}\ \right)^2 - 2 + 2\sqrt{1+x}\ - 4\sqrt{1 - \sqrt{1+x}\ } + \dfrac{8\arctanh\left(\dfrac{\left(1+2\sqrt{1 - \sqrt{1+x}\ }\right)\sqrt{\cdots}}{5}\right)}{5}$
default	$\left(1 - \sqrt{1+x}\ \right)^2 - 2 + 2\sqrt{1+x}\ - 4\sqrt{1 - \sqrt{1+x}\ } + \dfrac{8\arctanh\left(\dfrac{\left(1+2\sqrt{1 - \sqrt{1+x}\ }\right)\sqrt{\cdots}}{5}\right)}{5}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x+(1-(1+x)^(1/2))^(1/2)),x,method=_RETURNVERBOSE)

[Out] (1-(1+x)^(1/2))^2-2+2*(1+x)^(1/2)-4*(1-(1+x)^(1/2))^(1/2)+8/5*arctanh(1/5*(
1+2*(1-(1+x)^(1/2))^(1/2))*5^(1/2))*5^(1/2)

Maxima [A]

time $= 0.37$, size $= 77$, normalized size $= 1.05$

$$\left(\sqrt{x+1}\ - 1\right)^2 - \frac{4}{5}\sqrt{5}\log\left(-\frac{\sqrt{5} - 2\sqrt{-\sqrt{x+1}\ + 1}\ - 1}{\sqrt{5} + 2\sqrt{-\sqrt{x+1}\ + 1}\ + 1}\right) + 2\sqrt{x+1}\ - 4\sqrt{-\sqrt{x+1}\ + 1}\ - 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x+(1-(1+x)^(1/2))^(1/2)),x, algorithm="maxima")

[Out] (sqrt(x + 1) - 1)^2 - 4/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(-sqrt(x + 1) + 1)
- 1)/(sqrt(5) + 2*sqrt(-sqrt(x + 1) + 1) + 1)) + 2*sqrt(x + 1) - 4*sqrt(-sq
rt(x + 1) + 1) - 2

Fricas [A]

time $= 0.42$, size $= 110$, normalized size $= 1.51$

$$\frac{4}{5}\sqrt{5}\log\left(\frac{2x^2 - \sqrt{5}(3x+1) + \left(\sqrt{5}(x+2) - 5x\right)\sqrt{x+1} + \left(\sqrt{5}(x+2) - \left(\sqrt{5}(2x-1) - 5\right)\sqrt{x+1}\ - 5x\right)\sqrt{-\sqrt{x+1}\ + 1}\ + 3x+3}{x^2 - x - 1}\right) + x - 4\sqrt{-\sqrt{x+1}\ + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x+(1-(1+x)^(1/2))^(1/2)),x, algorithm="fricas")

[Out] 4/5*sqrt(5)*log((2*x^2 - sqrt(5)*(3*x + 1) + (sqrt(5)*(x + 2) - 5*x)*sqrt(x
+ 1) + (sqrt(5)*(x + 2) - (sqrt(5)*(2*x - 1) - 5)*sqrt(x + 1) - 5*x)*sqrt(
-sqrt(x + 1) + 1) + 3*x + 3)/(x^2 - x - 1)) + x - 4*sqrt(-sqrt(x + 1) + 1)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x + \sqrt{1 - \sqrt{x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x+(1-(1+x)**(1/2))**(1/2)),x)

[Out] Integral(x/(x + sqrt(1 - sqrt(x + 1))), x)

Giac [A]
time = 0.00, size = 106, normalized size = 1.45

$$4\left(\frac{\left(-\sqrt{x+1}+1\right)^2}{4} - \frac{-\sqrt{x+1}+1}{2} - \sqrt{-\sqrt{x+1}+1} - \frac{\ln\left(\frac{\left|2\sqrt{-\sqrt{x+1}+1}+1-\sqrt{5}\right|}{2\sqrt{-\sqrt{x+1}+1}+1+\sqrt{5}}\right)}{\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x+(1-(1+x)^(1/2))^(1/2)),x)

[Out] (sqrt(x + 1) - 1)^2 - 4/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(-sqrt(x + 1) +
1) + 1)/(sqrt(5) + 2*sqrt(-sqrt(x + 1) + 1) + 1)) + 2*sqrt(x + 1) - 4*sqrt(
-sqrt(x + 1) + 1) - 2

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{x + \sqrt{1 - \sqrt{x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x + (1 - (x + 1)^(1/2))^(1/2)),x)

[Out] int(x/(x + (1 - (x + 1)^(1/2))^(1/2)), x)

3.13 $\int \dfrac{\sqrt{x+\sqrt{1+x}}}{\sqrt{1+x}\,(1+x^2)}\,dx$

Optimal. Leaf size=365

$$-\frac{i\tan^{-1}\left(\dfrac{2+\sqrt{1-i}-\left(1-2\sqrt{1-i}\right)\sqrt{1+x}}{2\sqrt{i+\sqrt{1-i}}\,\sqrt{x+\sqrt{1+x}}}\right)}{2\sqrt{\dfrac{1-i}{i+\sqrt{1-i}}}}+\frac{i\tan^{-1}\left(\dfrac{2+\sqrt{1+i}-\left(1-2\sqrt{1+i}\right)\sqrt{1+x}}{2\sqrt{-i+\sqrt{1+i}}\,\sqrt{x+\sqrt{1+x}}}\right)}{2\sqrt{-\dfrac{1+i}{i-\sqrt{1+i}}}}+\frac{i\tanh^{-1}\left(\right.}{}$$

```
[Out] 1/2*I*arctanh(1/2*(2-(1-I)^(1/2)-(1+2*(1-I)^(1/2))*(1+x)^(1/2))/(-I+(1-I)^(
1/2))^(1/2)/(x+(1+x)^(1/2))^(1/2))/((-1+I)/(I-(1-I)^(1/2)))^(1/2)-1/2*I*arc
tan(1/2*(2+(1-I)^(1/2)-(1-2*(1-I)^(1/2))*(1+x)^(1/2))/(I+(1-I)^(1/2))^(1/2)
/(x+(1+x)^(1/2))^(1/2))/((1-I)/(I+(1-I)^(1/2)))^(1/2)+1/2*I*arctan(1/2*(2+(
1+I)^(1/2)-(1-2*(1+I)^(1/2))*(1+x)^(1/2))/(-I+(1+I)^(1/2))^(1/2)/(x+(1+x)^(
1/2))^(1/2))/((-1-I)/(I-(1+I)^(1/2)))^(1/2)-1/2*I*arctanh(1/2*(2-(1+I)^(1/2
)-(1+2*(1+I)^(1/2))*(1+x)^(1/2))/(I+(1+I)^(1/2))^(1/2)/(x+(1+x)^(1/2))^(1/2
))/((1+I)/(I+(1+I)^(1/2)))^(1/2)
```

Rubi [A]

time = 0.61, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 28, $\dfrac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6873, 6860, 1004, 635, 212, 1047, 738, 210}

$$-\frac{i\tan^{-1}\left(\dfrac{-\left(\left(1-2\sqrt{1-i}\right)\sqrt{x+1}\right)+\sqrt{1-i}+2}{2\sqrt{i+\sqrt{1-i}}\,\sqrt{x+\sqrt{x+1}}}\right)}{2\sqrt{\dfrac{1-i}{i+\sqrt{1-i}}}}+\frac{i\tan^{-1}\left(\dfrac{-\left(\left(1-2\sqrt{1+i}\right)\sqrt{x+1}\right)+\sqrt{1+i}+2}{2\sqrt{\sqrt{1+i}-i}\,\sqrt{x+\sqrt{x+1}}}\right)}{2\sqrt{-\dfrac{1+i}{i-\sqrt{1+i}}}}+\frac{i\tanh^{-1}\left(\dfrac{-\left(\left(1+2\sqrt{1-i}\right)\sqrt{x+1}\right)-\sqrt{1-i}+2}{2\sqrt{\sqrt{1-i}-i}\,\sqrt{x+\sqrt{x+1}}}\right)}{2\sqrt{-\dfrac{1-i}{i-\sqrt{1-i}}}}-\frac{i\tanh^{-1}\left(\dfrac{-\left(\left(1+2\sqrt{1+i}\right)\sqrt{x+1}\right)-\sqrt{1+i}+2}{2\sqrt{i+\sqrt{1+i}}\,\sqrt{x+\sqrt{x+1}}}\right)}{2\sqrt{\dfrac{1+i}{i+\sqrt{1+i}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[1 + x]]/(Sqrt[1 + x]*(1 + x^2)),x]
```

```
[Out] ((-1/2*I)*ArcTan[(2 + Sqrt[1 - I] - (1 - 2*Sqrt[1 - I])*Sqrt[1 + x])/(2*Sqr
t[I + Sqrt[1 - I]]*Sqrt[x + Sqrt[1 + x]])])/Sqrt[(1 - I)/(I + Sqrt[1 - I])]
 + ((I/2)*ArcTan[(2 + Sqrt[1 + I] - (1 - 2*Sqrt[1 + I])*Sqrt[1 + x])/(2*Sqr
t[-I + Sqrt[1 + I]]*Sqrt[x + Sqrt[1 + x]])])/Sqrt[(-1 - I)/(I - Sqrt[1 + I]
)] + ((I/2)*ArcTanh[(2 - Sqrt[1 - I] - (1 + 2*Sqrt[1 - I])*Sqrt[1 + x])/(2*
Sqrt[-I + Sqrt[1 - I]]*Sqrt[x + Sqrt[1 + x]])])/Sqrt[(-1 + I)/(I - Sqrt[1 -
 I])] - ((I/2)*ArcTanh[(2 - Sqrt[1 + I] - (1 + 2*Sqrt[1 + I])*Sqrt[1 + x])/
(2*Sqrt[I + Sqrt[1 + I]]*Sqrt[x + Sqrt[1 + x]])])/Sqrt[(1 + I)/(I + Sqrt[1
+ I])]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

& (LtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

Rule 635

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 738

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1004

Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1047

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]

Rule 6860

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rule 6873

Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]

Rubi steps

$$\int \frac{\sqrt{x+\sqrt{1+x}}}{\sqrt{1+x}\,(1+x^2)}\,dx = 2\text{Subst}\left(\int \frac{\sqrt{-1+x+x^2}}{1+\left(-1+x^2\right)^2}\,dx, x, \sqrt{1+x}\right)$$

$$= 2\text{Subst}\left(\int \frac{\sqrt{-1+x+x^2}}{2-2x^2+x^4}\,dx, x, \sqrt{1+x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{i\sqrt{-1+x+x^2}}{(2+2i)-2x^2}+\frac{i\sqrt{-1+x+x^2}}{(-2+2i)+2x^2}\right)dx, x, \sqrt{1+x}\right)$$

$$= 2i\text{Subst}\left(\int \frac{\sqrt{-1+x+x^2}}{(2+2i)-2x^2}\,dx, x, \sqrt{1+x}\right) + 2i\text{Subst}\left(\int \frac{\sqrt{-1+x+x^2}}{(-2+2i)+2x^2}\,dx\right.$$

$$= i\text{Subst}\left(\int \frac{2i+2x}{\left((2+2i)-2x^2\right)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right) - i\text{Subst}\left(\int \frac{}{\sqrt{-1}}\right.$$

$$= -\left(\frac{1}{2}\left(i\left(-2-(1-i)^{3/2}\right)\right)\right)\text{Subst}\left(\int \frac{1}{\left(-2\sqrt{1-i}+2x\right)\sqrt{-1+x+x^2}}\,dx, x,\right.$$

$$= \left(i\left(-2-(1-i)^{3/2}\right)\right)\text{Subst}\left(\int \frac{1}{-16i+16\sqrt{1-i}-x^2}\,dx, x, \frac{-4+2\sqrt{1-i}-}{\sqrt{x}}\right.$$

$$= -\frac{i\sqrt{i+\sqrt{1-i}}\,\tan^{-1}\left(\frac{2+\sqrt{1-i}-\left(1-2\sqrt{1-i}\right)\sqrt{1+x}}{2\sqrt{i+\sqrt{1-i}}\,\sqrt{x+\sqrt{1+x}}}\right)}{2\sqrt{1-i}} + \frac{1}{4}(1+i)^{3/2}\sqrt{-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.16, size = 217, normalized size = 0.59

$$-\tfrac{1}{2}\text{RootSum}\left[1-8\#1+40\#1^2-48\#1^3+20\#1^4+8\#1^5-4\#1^6+\#1^8\&, \frac{-\log\left(-\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}-\#1\right)+5\log\left(-\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}-\#1\right)\#1^2-5\log\left(-\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}-\#1\right)\#1^4+2\log\left(-\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}-\#1\right)\#1^5}{-1+10\#1-18\#1^2+10\#1^3+5\#1^4-3\#1^5+\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[1 + x]]/(Sqrt[1 + x]*(1 + x^2)),x]

[Out] -1/2*RootSum[1 - 8*#1 + 40*#1^2 - 48*#1^3 + 20*#1^4 + 8*#1^5 - 4*#1^6 + #1^
8 & , (-Log[-Sqrt[1 + x] + Sqrt[x + Sqrt[1 + x]] - #1] + 5*Log[-Sqrt[1 + x]
 + Sqrt[x + Sqrt[1 + x]] - #1]*#1^2 - 5*Log[-Sqrt[1 + x] + Sqrt[x + Sqrt[1
+ x]] - #1]*#1^4 + 2*Log[-Sqrt[1 + x] + Sqrt[x + Sqrt[1 + x]] - #1]*#1^5)/(
-1 + 10*#1 - 18*#1^2 + 10*#1^3 + 5*#1^4 - 3*#1^5 + #1^7) & ]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sqrt[Sqrt[1 + x] + x]/((1 + x^2)*Sqrt[1 + x]),x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 109, normalized size = 0.30

method	result
derivativedivides	$-\dfrac{\left(\sum\limits_{\_R=\mathrm{RootOf}\left(\_Z^8-4\_Z^6+8\_Z^5+20\_Z^4-48\_Z^3+40\_Z^2-8\_Z+1\right)}\dfrac{\left(2\_R^5-5\_R^4+5\_R^2-1\right)\ln\left(\sqrt{x+\sqrt{1+}}\right)}{\_R^7-3\_R^5+5\_R^4+10\_R^3-18\_}\right)}{2}$
default	$-\dfrac{\left(\sum\limits_{\_R=\mathrm{RootOf}\left(\_Z^8-4\_Z^6+8\_Z^5+20\_Z^4-48\_Z^3+40\_Z^2-8\_Z+1\right)}\dfrac{\left(2\_R^5-5\_R^4+5\_R^2-1\right)\ln\left(\sqrt{x+\sqrt{1+}}\right)}{\_R^7-3\_R^5+5\_R^4+10\_R^3-18\_}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(1+x)^(1/2))^(1/2)/(x^2+1)/(1+x)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2*sum((2*_R^5-5*_R^4+5*_R^2-1)/(_R^7-3*_R^5+5*_R^4+10*_R^3-18*_R^2+10*_R
-1)*ln((x+(1+x)^(1/2))^(1/2)-(1+x)^(1/2)-_R),_R=RootOf(_Z^8-4*_Z^6+8*_Z^5+2
0*_Z^4-48*_Z^3+40*_Z^2-8*_Z+1))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(1+x)^(1/2))^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x + sqrt(x + 1))/((x^2 + 1)*sqrt(x + 1)), x)

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than
twice the leaf count of optimal. 5235 vs. 2(201) = 402.
time = 5.35, size = 5235, normalized size = 14.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="fricas")

[Out] -1/4*sqrt(sqrt(1/8*I + 1/8) + sqrt(-1/8*I + 1/8) - 2*sqrt(-3/64*(4*sqrt(1/8
*I + 1/8) - I + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt
(1/8*I + 1/8) - I + 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1
/8) + 1/8*I + 1/8) - 1/2)*log(1/4*(2*(((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*
sqrt(-1/8*I + 1/8) + I + 1) - 3*(3*x - 1)*sqrt(x + 1) - 7*x + 9)*sqrt(x + s
qrt(x + 1))*(4*sqrt(1/8*I + 1/8) - I + 1)^2 + 2*(((3*x - 1)*sqrt(x + 1) + 4
*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 4*((3*x - 1)*sqrt(x + 1) + 4*x -
 3)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 2*(7*x + 1)*sqrt(x + 1) + 12*x - 14)*s
qrt(x + sqrt(x + 1))*(4*sqrt(1/8*I + 1/8) - I + 1) + 16*((((3*x - 1)*sqrt(x
 + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 3*(3*x - 1)*sqrt(x + 1) -
 7*x + 9)*sqrt(x + sqrt(x + 1))*(4*sqrt(1/8*I + 1/8) - I + 1) - ((3*(3*x -
1)*sqrt(x + 1) + 7*x - 9)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 2*(11*x - 7)*sqr
t(x + 1) - 16*x + 22)*sqrt(x + sqrt(x + 1)))*sqrt(-3/64*(4*sqrt(1/8*I + 1/8
) - I + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt(1/8*I +
 1/8) - I + 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1/8) + 1/
8*I + 1/8) - 2*((3*(3*x - 1)*sqrt(x + 1) + 7*x - 9)*(4*sqrt(-1/8*I + 1/8) +
 I + 1)^2 - 2*((7*x + 1)*sqrt(x + 1) + 6*x - 7)*(4*sqrt(-1/8*I + 1/8) + I +
 1) - 4*sqrt(x + 1)*(x - 7) - 12*x + 44)*sqrt(x + sqrt(x + 1)) + ((3*x^2 -
(x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1) +
2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(1/8*I + 1/8) - I + 1)^2 + (3*x
^2 + 2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2
- 68*x^2 - ((x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8)
 + I + 1)^2 - 2*x^2 - 4*(x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-
1/8*I + 1/8) + I + 1) + 4*(7*x + 6)*sqrt(x + 1) + 24*x + 30)*(4*sqrt(1/8*I
+ 1/8) - I + 1) + 2*(x^2 - 2*(7*x + 6)*sqrt(x + 1) - 12*x - 15)*(4*sqrt(-1/
8*I + 1/8) + I + 1) - 8*(14*x^2 - (3*x^2 - (x^2 + 2*(3*x + 4)*sqrt(x + 1) +
 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 2*(9*x + 7)*sqrt(x + 1) + 24*x +
 15)*(4*sqrt(1/8*I + 1/8) - I + 1) - (3*x^2 + 2*(9*x + 7)*sqrt(x + 1) + 24*
x + 15)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 4*(11*x + 8)*sqrt(x + 1) + 72*x +
30)*sqrt(-3/64*(4*sqrt(1/8*I + 1/8) - I + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8)
 + I + 1)^2 - 1/32*(4*sqrt(1/8*I + 1/8) - I + 1)*(4*sqrt(-1/8*I + 1/8) + I
- 3) + 1/2*sqrt(-1/8*I + 1/8) + 1/8*I + 1/8) - 8*(11*x + 3)*sqrt(x + 1) - 6
4*x - 20)*sqrt(sqrt(1/8*I + 1/8) + sqrt(-1/8*I + 1/8) - 2*sqrt(-3/64*(4*sqr
t(1/8*I + 1/8) - I + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4
*sqrt(1/8*I + 1/8) - I + 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*
I + 1/8) + 1/8*I + 1/8) - 1/2))/(x^2 + 1)) + 1/4*sqrt(sqrt(1/8*I + 1/8) + s
qrt(-1/8*I + 1/8) - 2*sqrt(-3/64*(4*sqrt(1/8*I + 1/8) - I + 1)^2 - 3/64*(4*
sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt(1/8*I + 1/8) - I + 1)*(4*sqrt(
-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1/8) + 1/8*I + 1/8) - 1/2)*log(1
/4*(2*(((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 3
*(3*x - 1)*sqrt(x + 1) - 7*x + 9)*sqrt(x + sqrt(x + 1))*(4*sqrt(1/8*I + 1/8
```

) - I + 1)^2 + 2*(((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) +
I + 1)^2 - 4*((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I +
1) + 2*(7*x + 1)*sqrt(x + 1) + 12*x - 14)*sqrt(x + sqrt(x + 1))*(4*sqrt(1/
8*I + 1/8) - I + 1) + 16*((((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I
 + 1/8) + I + 1) - 3*(3*x - 1)*sqrt(x + 1) - 7*x + 9)*sqrt(x + sqrt(x + 1))
*(4*sqrt(1/8*I + 1/8) - I + 1) - ((3*(3*x - 1)*sqrt(x + 1) + 7*x - 9)*(4*sq
rt(-1/8*I + 1/8) + I + 1) - 2*(11*x - 7)*sqrt(x + 1) - 16*x + 22)*sqrt(x +
sqrt(x + 1)))*sqrt(-3/64*(4*sqrt(1/8*I + 1/8) - I + 1)^2 - 3/64*(4*sqrt(-1/
8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt(1/8*I + 1/8) - I + 1)*(4*sqrt(-1/8*I +
 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1/8) + 1/8*I + 1/8) - 2*((3*(3*x - 1)*sq
rt(x + 1) + 7*x - 9)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 2*((7*x + 1)*sqrt(x
 + 1) + 6*x - 7)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 4*sqrt(x + 1)*(x - 7) - 1
2*x + 44)*sqrt(x + sqrt(x + 1)) - ((3*x^2 - (x^2 + 2*(3*x + 4)*sqrt(x + 1)
+ 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 2*(9*x + 7)*sqrt(x + 1) + 24*x
+ 15)*(4*sqrt(1/8*I + 1/8) - I + 1)^2 + (3*x^2 + 2*(9*x + 7)*sqrt(x + 1) +
24*x + 15)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 68*x^2 - ((x^2 + 2*(3*x + 4)*
sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 2*x^2 - 4*(x^2 +
2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 4*(7*x
+ 6)*sqrt(x + 1) + 24*x + 30)*(4*sqrt(1/8*I + 1/8) - I + 1) + 2*(x^2 - 2*(7
*x + 6)*sqrt(x + 1) - 12*x - 15)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 8*(14*x^2
 - (3*x^2 - (x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8)
 + I + 1) + 2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(1/8*I + 1/8) - I +
 1) - (3*x^2 + 2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(-1/8*I + 1/8) +
 I + 1) + 4*(11*x + 8)*sqrt(x + 1) + 72*x + 30)*sqrt(-3/64*(4*sqrt(1/8*I +
1/8) - I + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt(1/8*
I + 1/8) - I + 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1/8) +
 1/8*I + 1/8) - 8*(11*x + 3)*sqrt(x + 1) - 64*x - 20)*sqrt(sqrt(1/8*I + 1/8
) + sqrt(-1/8*I + 1/8) - 2*sqrt(-3/64*(4*sqrt(1/8*I + 1/8) - I + 1)^2 - 3/6
4*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt(1/8*I + 1/8) - I + 1)*(4*
sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1/8) + 1/8*I + 1/8) - 1/2))
/(x^2 + 1)) - 1/4*sqrt(sqrt(1/8*I + 1/8) + sqrt(-1/8*I + 1/8) + 2*sqrt(-3/6
4*(4*sqrt(1/8*I + 1/8) - I + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 -
 1/32*(4*sqrt(1/8*I + 1/8) - I + 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sq
rt(-1/8*I + 1/8) + 1/8*I + 1/8) - 1/2)*log(1/4*(2*(((3*x - 1)*sqrt(x + 1) +
 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 3*(3*x - 1)*sqrt(x + 1) - 7*x +
9)*sqrt(x + sqrt(x + 1))*(4*sqrt(1/8*I + 1/8) - I + 1)^2 + 2*(((3*x - 1)*sq
rt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 4*((3*x - 1)*sqrt(x
 + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 2*(7*x + 1)*sqrt(x + 1) +
 12*x - 14)*sqrt(x + sqrt(x + 1))*(4*sqrt(1/8*I + 1/8) - I + 1) - 16*((((3*
x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 3*(3*x - 1)*
sqrt(x + 1) - 7*x + 9)*sqrt(x + sqrt(x + 1))*(4*sqrt(1/8*I + 1/8) - I + 1)
- ((3*(3*x - 1)*sqrt(x + 1) + 7*x - 9)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 2*(
11*x - 7)*sqrt(x + 1) - 16*x + 22)*sqrt(x + sqrt(x + 1)))*sqrt(-3/64*(4*sqr
t(1/8*I + 1/8) - I + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4
*sqrt(1/8*I + 1/8) - I + 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*

```
I + 1/8) + 1/8*I + 1/8) - 2*((3*(3*x - 1)*sqrt(x + 1) + 7*x - 9)*(4*sqrt(-1
/8*I + 1/8) + I + 1)^2 - 2*((7*x + 1)*sqrt(x + 1) + 6*x - 7)*(4*sqrt(-1/8*I
 + 1/8) + I + 1) - 4*sqrt(x + 1)*(x - 7) - 12*x + 44)*sqrt(x + sqrt(x + 1))
 + ((3*x^2 - (x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8
) + I + 1) + 2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(1/8*I + 1/8) - I
+ 1)^2 + (3*x^2 + 2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(-1/8*I + 1/8
) + I + 1)^2 - 68*x^2 - ((x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(
-1/8*I + 1/8) + I + 1)^2 - 2*x^2 - 4*(x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x +
 5)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 4*(7*x + 6)*sqrt(x + 1) + 24*x + 30)*(
4*sqrt(1/8*I + 1/8) - I + 1) + 2*(x^2 - 2*(7*x + 6)*sqrt(x + 1) - 12*x - 15
)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 8*(14*x^2 - (3*x^2 - (x^2 + 2*(3*x + 4)*
sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 2*(9*x + 7)*sqrt(x
+ 1) + 24*x + 15)*(4*sqrt(1/8*I + 1/8) - I + 1) - (3*x^2 + 2*(9*x + 7)*sqrt
(x + 1) + 24*x + 15)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 4*(11*x + 8)*sqrt(x +
 1) + 72*x + 30)*sqrt(-3/64*(4*sqrt(1/8*I + 1/8) - I + 1)^2 - 3/64*(4*sqrt(
-1/8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt(1/8*I + 1/8) - I + 1)*(4*sqrt(-1/8*
I + 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1/8) + 1/8*I + 1/8) - 8*(11*x + 3)*sq
rt(x + 1) - 64*x - 20)*sqrt(sqrt(1/8*I + 1/8) + sqrt(-1/8*I + 1/8) + 2*sqrt
(-3/64*(4*sqrt(1/8*I + 1/8) - I + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1
)^2 - 1/32*(4*sqrt(1/8*I + 1/8) - I + 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1
/2*sqrt(-1/8*I + 1/8) + 1/8*I + 1/8) - 1/2))/(x^2 + 1)) + 1/4*sqrt(sqrt(1/8
*I + 1/8) + sqrt(-1/8*I + 1/8) + 2*sqrt(-3/64*(4*sqrt(1/8*I + 1/8) - I + 1)
^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt(1/8*I + 1/8) - I
+ 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1/8) + 1/8*I + 1/8)
 - 1/2)*log(1/4*(2*(((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8)
 + I + 1) - 3*(3*x - 1)*sqrt(x + 1) - 7*x + 9)*sqrt(x + sqrt(x + 1))*(4*sqr
t(1/8*I + 1/8) - I + 1)^2 + 2*(((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1
/8*I + 1/8) + I + 1)^2 - 4*((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I
 + 1/8) + I + 1) + 2*(7*x + 1)*sqrt(x + 1) + 12*x - 14)*sqrt(x + sqrt(x + 1
))*(4*sqrt(1/8*I + 1/8) - I + 1) - 16*((((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(
4*sqrt(-1/8*I + 1/8) + I + 1) - 3*(3*x - 1)*sqrt(x + 1) - 7*x + 9)*sqrt(x +
 sqrt(x + 1))*(4*sqrt(1/8*I + 1/8) - I + 1) - ((3*(3*x - 1)*sqrt(x + 1) + 7
*x - 9)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 2*(11*x - 7)*sqrt(x + 1) - 16*x +
22)*sqrt(x + sqrt(x + 1)))*sqrt(-3/64*(4*sqrt(1/8*I + 1/8) - I + 1)^2 - 3/6
4*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt(1/8*I + 1/8) - I + 1)*(4*
sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1/8) + 1/8*I + 1/8) - 2*((3
*(3*x - 1)*sqrt(x + 1) + 7*x - 9)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 2*((7*
x + 1)*sqrt(x + 1) + 6*x - 7)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 4*sqrt(x + 1
)*(x - 7) - 12*x + 44)*sqrt(x + sqrt(x + 1)) - ((3*x^2 - (x^2 + 2*(3*x + 4)
*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 2*(9*x + 7)*sqrt(x
 + 1) + 24*x + 15)*(4*sqrt(1/8*I + 1/8) - I + 1)^2 + (3*x^2 + 2*(9*x + 7)*s
qrt(x + 1) + 24*x + 15)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 68*x^2 - ((x^2 +
 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 2*x^
2 - 4*(x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I +
 1) + 4*(7*x + 6)*sqrt(x + 1) + 24*x + 30)*(4*sqrt(1/8*I + 1/8) - I + 1) +
```

```
2*(x^2 - 2*(7*x + 6)*sqrt(x + 1) - 12*x - 15)*(4*sqrt(-1/8*I + 1/8) + I + 1
) + 8*(14*x^2 - (3*x^2 - (x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(
-1/8*I + 1/8) + I + 1) + 2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(1/8*I
 + 1/8) - I + 1) - (3*x^2 + 2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(-1
/8*I + 1/8) + I + 1) + 4*(11*x + 8)*sqrt(x + 1) + 72*x + 30)*sqrt(-3/64*(4*
sqrt(1/8*I + 1/8) - I + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32
*(4*sqrt(1/8*I + 1/8) - I + 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1
/8*I + 1/8) + 1/8*I + 1/8) - 8*(11*x + 3)*sqrt(x + 1) - 64*x - 20)*sqrt(sqr
t(1/8*I + 1/8) + sqrt(-1/8*I + 1/8) + 2*sqrt(-3/64*(4*sqrt(1/8*I + 1/8) - I
 + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt(1/8*I + 1/8)
 - I + 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1/8) + 1/8*I +
 1/8) - 1/2))/(x^2 + 1)) + 1/2*sqrt(-1/2*sqrt(1/8*I + 1/8) + 1/8*I - 1/8)*l
og(-((((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 3*
(3*x - 1)*sqrt(x + 1) - 7*x + 9)*sqrt(x + sqrt(x + 1))*(4*sqrt(1/8*I + 1/8)
 - I + 1)^2 + (((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I
+ 1)^2 - 4*((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1)
 + 2*(7*x + 1)*sqrt(x + 1) + 12*x - 14)*sqrt(x + sqrt(x + 1))*(4*sqrt(1/8*I
 + 1/8) - I + 1) + (((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8)
 + I + 1)^3 - 4*((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I
 + 1)^2 + 4*((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1
) + 2*(3*x - 1)*sqrt(x + 1) - 2*x + 14)*sqrt(x + sqrt(x + 1)) + ((x^2 + 2*(
3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1)^3 - (3*x^2 -
 (x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1) +
 2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(1/8*I + 1/8) - I + 1)^2 - 4*(
x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 -
 18*x^2 + ((x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8)
+ I + 1)^2 - 2*x^2 - 4*(x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1
/8*I + 1/8) + I + 1) + 4*(7*x + 6)*sqrt(x + 1) + 24*x + 30)*(4*sqrt(1/8*I +
 1/8) - I + 1) + 4*(x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I
 + 1/8) + I + 1) - 4*(7*x + 1)*sqrt(x + 1) + 16*x - 10)*sqrt(-1/2*sqrt(1/8*
I + 1/8) + 1/8*I - 1/8))/(x^2 + 1)) - 1/2*sqrt(-1/2*sqrt(1/8*I + 1/8) + 1/8
*I - 1/8)*log(-(((((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) +
I + 1) - 3*(3*x - 1)*sqrt(x + 1) - 7*x + 9)*sqrt(x + sqrt(x + 1))*(4*sqrt(1
/8*I + 1/8) - I + 1)^2 + (((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I
+ 1/8) + I + 1)^2 - 4*((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/
8) + I + 1) + 2*(7*x + 1)*sqrt(x + 1) + 12*x - 14)*sqrt(x + sqrt(x + 1))*(4
*sqrt(1/8*I + 1/8) - I + 1) + (((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1
/8*I + 1/8) + I + 1)^3 - 4*((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I
 + 1/8) + I + 1)^2 + 4*((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1
/8) + I + 1) + 2*(3*x - 1)*sqrt(x + 1) - 2*x + 14)*sqrt(x + sqrt(x + 1)) -
((x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1)^3
 - (3*x^2 - (x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8)
 + I + 1) + 2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(1/8*I + 1/8) - I +
 1)^2 - 4*(x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) +
 I + 1)^2 - 18*x^2 + ((x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/
```

```
8*I + 1/8) + I + 1)^2 - 2*x^2 - 4*(x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)
*(4*sqrt(-1/8*I + 1/8) + I + 1) + 4*(7*x + 6)*sqrt(x + 1) + 24*x + 30)*(4*s
qrt(1/8*I + 1/8) - I + 1) + 4*(x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*
sqrt(-1/8*I + 1/8) + I + 1) - 4*(7*x + 1)*sqrt(x + 1) + 16*x - 10)*sqrt(-1/
2*sqrt(1/8*I + 1/8) + 1/8*I - 1/8))/(x^2 + 1)) + 1/2*sqrt(-1/2*sqrt(-1/8*I
+ 1/8) - 1/8*I - 1/8)*log(((((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*
I + 1/8) + I + 1)^3 - ((3*x - 1)*sqrt(x + 1) + 9*x - 3)*(4*sqrt(-1/8*I + 1/
8) + I + 1)^2 - 2*((x + 3)*sqrt(x + 1) - 2*x - 1)*(4*sqrt(-1/8*I + 1/8) + I
 + 1) - 10*(3*x - 1)*sqrt(x + 1) - 30*x + 10)*sqrt(x + sqrt(x + 1)) + ((x^2
 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1)^3 - (x
^2 + 6*(x + 3)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 + 10
*x^2 + 2*(3*x^2 - 2*sqrt(x + 1)*(x - 2) + 4*x - 5)*(4*sqrt(-1/8*I + 1/8) +
I + 1) - 20*(x + 3)*sqrt(x + 1) - 80*x - 30)*sqrt(-1/2*sqrt(-1/8*I + 1/8) -
 1/8*I - 1/8))/(x^2 + 1)) - 1/2*sqrt(-1/2*sqrt(-1/8*I + 1/8) - 1/8*I - 1/8)
*log((((((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1)^3 -
 ((3*x - 1)*sqrt(x + 1) + 9*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 2*((x
 + 3)*sqrt(x + 1) - 2*x - 1)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 10*(3*x - 1)*
sqrt(x + 1) - 30*x + 10)*sqrt(x + sqrt(x + 1)) - ((x^2 + 2*(3*x + 4)*sqrt(x
 + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1)^3 - (x^2 + 6*(x + 3)*sqrt(x
 + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 + 10*x^2 + 2*(3*x^2 - 2*s
qrt(x + 1)*(x - 2) + 4*x - 5)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 20*(x + 3)*s
qrt(x + 1) - 80*x - 30)*sqrt(-1/2*sqrt(-1/8*I + 1/8) - 1/8*I - 1/8))/(x^2 +
 1))
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x + \sqrt{x + 1}}}{\sqrt{x + 1}\, (x^2 + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)**(1/2))**(1/2)/(x**2+1)/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x + sqrt(x + 1))/(sqrt(x + 1)*(x**2 + 1)), x)
```

Giac [**F**] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(x^2+1)/(1+x)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time $= 0.00$, size $= $ -1, normalized size $= $ -0.00

$$\int \frac{\sqrt{x + \sqrt{x+1}}}{(x^2 + 1)\ \sqrt{x+1}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x + 1)^(1/2))^(1/2)/((x^2 + 1)*(x + 1)^(1/2)),x)
[Out] int((x + (x + 1)^(1/2))^(1/2)/((x^2 + 1)*(x + 1)^(1/2)), x)
```

3.14 $\quad \int \dfrac{\sqrt{x + \sqrt{1+x}}}{1+x^2}\, dx$

Optimal. Leaf size=337

$$\frac{1}{2}i\sqrt{i+\sqrt{1-i}}\ \tan^{-1}\left(\frac{2+\sqrt{1-i}-\left(1-2\sqrt{1-i}\right)\sqrt{1+x}}{2\sqrt{i+\sqrt{1-i}}\ \sqrt{x+\sqrt{1+x}}}\right)-\frac{1}{2}i\sqrt{-i+\sqrt{1+i}}\ \tan^{-1}\left(\frac{2+\sqrt{1-}}{2\sqrt{-}}\right.$$

```
[Out] 1/2*I*arctanh(1/2*(2-(1-I)^(1/2)-(1+2*(1-I)^(1/2))*(1+x)^(1/2))/(-I+(1-I)^(
1/2))^(1/2)/(x+(1+x)^(1/2))^(1/2))*(-I+(1-I)^(1/2))^(1/2)+1/2*I*arctan(1/2*
(2+(1-I)^(1/2)-(1-2*(1-I)^(1/2))*(1+x)^(1/2))/(I+(1-I)^(1/2))^(1/2)/(x+(1+x
)^(1/2))^(1/2))*(I+(1-I)^(1/2))^(1/2)-1/2*I*arctan(1/2*(2+(1+I)^(1/2)-(1-2*
(1+I)^(1/2))*(1+x)^(1/2))/(-I+(1+I)^(1/2))^(1/2)/(x+(1+x)^(1/2))^(1/2))*(-I
+(1+I)^(1/2))^(1/2)-1/2*I*arctanh(1/2*(2-(1+I)^(1/2)-(1+2*(1+I)^(1/2))*(1+x
)^(1/2))/(I+(1+I)^(1/2))^(1/2)/(x+(1+x)^(1/2))^(1/2))*(I+(1+I)^(1/2))^(1/2)
```

Rubi [A]

time $= 0.40$, antiderivative size $= 337$, normalized size of antiderivative $= 1.00$, number of steps used $= 22$, number of rules used $= 9$, integrand size $= 21$, $\dfrac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used $= \{6873, 6860, 1035, 1092, 635, 212, 1047, 738, 210\}$

$$\tfrac{1}{2}i\sqrt{i+\sqrt{1-i}}\,\tan^{-1}\!\left(\frac{-\left(\left(1-2\sqrt{1-i}\right)\sqrt{x+1}\right)+\sqrt{1-i}+2}{2\sqrt{i+\sqrt{1-i}}\,\sqrt{x+\sqrt{x+1}}}\right)-\tfrac{1}{2}i\sqrt{\sqrt{1+i}-i}\,\tan^{-1}\!\left(\frac{-\left(\left(1-2\sqrt{1+i}\right)\sqrt{x+1}\right)+\sqrt{1+i}+2}{2\sqrt{\sqrt{1+i}-i}\,\sqrt{x+\sqrt{x+1}}}\right)+\tfrac{1}{2}i\sqrt{\sqrt{1-i}-i}\,\tanh^{-1}\!\left(\frac{-\left(\left(1+2\sqrt{1-i}\right)\sqrt{x+1}\right)-\sqrt{1-i}+2}{2\sqrt{\sqrt{1-i}-i}\,\sqrt{x+\sqrt{x+1}}}\right)-\tfrac{1}{2}i\sqrt{i+\sqrt{1+i}}\,\tanh^{-1}\!\left(\frac{-\left(\left(1+2\sqrt{1+i}\right)\sqrt{x+1}\right)-\sqrt{1+i}+2}{2\sqrt{i+\sqrt{1+i}}\,\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[1 + x]]/(1 + x^2),x]
```

```
[Out] (I/2)*Sqrt[I + Sqrt[1 - I]]*ArcTan[(2 + Sqrt[1 - I] - (1 - 2*Sqrt[1 - I])*S
qrt[1 + x])/(2*Sqrt[I + Sqrt[1 - I]]*Sqrt[x + Sqrt[1 + x]])] - (I/2)*Sqrt[-
I + Sqrt[1 + I]]*ArcTan[(2 + Sqrt[1 + I] - (1 - 2*Sqrt[1 + I])*Sqrt[1 + x])
/(2*Sqrt[-I + Sqrt[1 + I]]*Sqrt[x + Sqrt[1 + x]])] + (I/2)*Sqrt[-I + Sqrt[1
 - I]]*ArcTanh[(2 - Sqrt[1 - I] - (1 + 2*Sqrt[1 - I])*Sqrt[1 + x])/(2*Sqrt[
-I + Sqrt[1 - I]]*Sqrt[x + Sqrt[1 + x]])] - (I/2)*Sqrt[I + Sqrt[1 + I]]*Arc
Tanh[(2 - Sqrt[1 + I] - (1 + 2*Sqrt[1 + I])*Sqrt[1 + x])/(2*Sqrt[I + Sqrt[1
 + I]]*Sqrt[x + Sqrt[1 + x]])]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

Q[a, 0] || LtQ[b, 0])

Rule 635

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 738

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1035

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d + f*x^2)^(q + 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*((-b)*f) + c*(-2*g*f)*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1047

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]

Rule 1092

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]

Rule 6860

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\int \frac{\sqrt{x + \sqrt{1+x}}}{1+x^2} \, dx = 2\text{Subst}\left(\int \frac{x\sqrt{-1+x+x^2}}{1+(-1+x^2)^2} \, dx, x, \sqrt{1+x}\right)$$

$$= 2\text{Subst}\left(\int \frac{x\sqrt{-1+x+x^2}}{2-2x^2+x^4} \, dx, x, \sqrt{1+x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{ix\sqrt{-1+x+x^2}}{(2+2i)-2x^2} + \frac{ix\sqrt{-1+x+x^2}}{(-2+2i)+2x^2}\right) \, dx, x, \sqrt{1+x}\right)$$

$$= 2i\text{Subst}\left(\int \frac{x\sqrt{-1+x+x^2}}{(2+2i)-2x^2} \, dx, x, \sqrt{1+x}\right) + 2i\text{Subst}\left(\int \frac{x\sqrt{-1+x+x^2}}{(-2+2i)+2x^2} \, d\right.$$

$$= i\text{Subst}\left(\int \frac{(1+i)+2ix+x^2}{((2+2i)-2x^2)\sqrt{-1+x+x^2}} \, dx, x, \sqrt{1+x}\right) - i\text{Subst}\left(\int \frac{}{\sqrt{-1+}}\right.$$

$$= -\left(\frac{1}{2}i\text{Subst}\left(\int \frac{(-4-4i)-4ix}{((2+2i)-2x^2)\sqrt{-1+x+x^2}} \, dx, x, \sqrt{1+x}\right)\right) - \frac{1}{2}i\text{Subst}\left(\right.$$

$$= -\left(\left(-1-i\sqrt{1-i}\right)\text{Subst}\left(\int \frac{1}{\left(-2\sqrt{1-i}+2x\right)\sqrt{-1+x+x^2}} \, dx, x, \sqrt{1+}\right.\right.$$

$$= -\left(\left(2\left(1-i\sqrt{1-i}\right)\right)\text{Subst}\left(\int \frac{1}{-16i-16\sqrt{1-i}-x^2} \, dx, x, \frac{-4-2\sqrt{1-i}}{}\right.\right.$$

$$= \frac{1}{2}i\sqrt{i+\sqrt{1-i}} \tan^{-1}\left(\frac{2+\sqrt{1-i}-\left(1-2\sqrt{1-i}\right)\sqrt{1+x}}{2\sqrt{i+\sqrt{1-i}}\sqrt{x+\sqrt{1+x}}}\right) - \frac{1}{2}i\sqrt{-i+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.14, size = 212, normalized size = 0.63

$$\frac{1}{2}\text{RootSum}\left[1-8\#1+40\#1^2-48\#1^3+20\#1^4+8\#1^5-4\#1^6+\#1^8\&, \frac{\log\left(-\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}-\#1\right)+2\log\left(-\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}-\#1\right)\#1-2\log\left(-\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}-\#1\right)\#1^5+\log\left(-\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}-\#1\right)\#1^6}{-1+10\#1-18\#1^2+10\#1^3+5\#1^4-3\#1^5+\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[1 + x]]/(1 + x^2),x]
```

```
[Out] RootSum[1 - 8*#1 + 40*#1^2 - 48*#1^3 + 20*#1^4 + 8*#1^5 - 4*#1^6 + #1^8 & ,
  (Log[-Sqrt[1 + x] + Sqrt[x + Sqrt[1 + x]] - #1] + 2*Log[-Sqrt[1 + x] + Sqr
t[x + Sqrt[1 + x]] - #1]*#1 - 2*Log[-Sqrt[1 + x] + Sqrt[x + Sqrt[1 + x]] -
#1]*#1^5 + Log[-Sqrt[1 + x] + Sqrt[x + Sqrt[1 + x]] - #1]*#1^6)/(-1 + 10*#1
  - 18*#1^2 + 10*#1^3 + 5*#1^4 - 3*#1^5 + #1^7) & ]/2
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x + Sqrt[1 + x]]/(1 + x^2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 105, normalized size = 0.31

method	result
derivativedivides	$$\frac{\left(\displaystyle\sum_{\_R=\text{RootOf}\left(\_Z^8-4\_Z^6+8\_Z^5+20\_Z^4-48\_Z^3+40\_Z^2-8\_Z+1\right)}\frac{\left(\_R^6-2\_R^5+2\_R+1\right)\ln\left(\sqrt{x+\sqrt{1+x}}\cdots\right)}{\_R^7-3\_R^5+5\_R^4+10\_R^3-18\_R^2\cdots}\right)}{2}$$
default	$$\frac{\left(\displaystyle\sum_{\_R=\text{RootOf}\left(\_Z^8-4\_Z^6+8\_Z^5+20\_Z^4-48\_Z^3+40\_Z^2-8\_Z+1\right)}\frac{\left(\_R^6-2\_R^5+2\_R+1\right)\ln\left(\sqrt{x+\sqrt{1+x}}\cdots\right)}{\_R^7-3\_R^5+5\_R^4+10\_R^3-18\_R^2\cdots}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(1+x)^(1/2))^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum((_R^6-2*_R^5+2*_R+1)/(_R^7-3*_R^5+5*_R^4+10*_R^3-18*_R^2+10*_R-1)*l
n((x+(1+x)^(1/2))^(1/2)-(1+x)^(1/2)-_R),_R=RootOf(_Z^8-4*_Z^6+8*_Z^5+20*_Z^
4-48*_Z^3+40*_Z^2-8*_Z+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(x^2+1),x, algorithm="maxima")
```

[Out] `integrate(sqrt(x + sqrt(x + 1))/(x^2 + 1), x)`

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4535 vs. 2(185) = 370.
time = 3.37, size = 4535, normalized size = 13.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+(1+x)^(1/2))^(1/2)/(x^2+1),x, algorithm="fricas")`

[Out] `-1/4*sqrt(sqrt(1/4*I + 1/4) + sqrt(-1/4*I + 1/4) - 2*sqrt(-3/16*(2*sqrt(1/4`
`*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I + 1/4) + I)*(2*sqrt(-1/4*I + 1/4) - I)`
` - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2))*log(-1/4*(2*(((2*x + 1)*sqrt(x + 1)`
` - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I) + 4*(2*x + 1)*sqrt(x + 1) - x - 8)*sqr`
`t(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I)^2 + 2*(((2*x + 1)*sqrt(x + 1)`
` - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I)^2 + (3*x - 16)*sqrt(x + 1) + 4*x - 3)`
`*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I) + 8*((((2*x + 1)*sqrt(x +`
` 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I) + 4*(2*x + 1)*sqrt(x + 1) - x - 8)`
`*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I) + ((4*(2*x + 1)*sqrt(x +`
` 1) - x - 8)*(2*sqrt(1/4*I + 1/4) + I) - (3*x - 16)*sqrt(x + 1) - 4*x + 3)*s`
`qrt(x + sqrt(x + 1)))*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(`
`1/4*I + 1/4) + I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) -`
` I)^2) + 2*((4*(2*x + 1)*sqrt(x + 1) - x - 8)*(2*sqrt(1/4*I + 1/4) + I)^2 +`
` ((3*x - 16)*sqrt(x + 1) + 4*x - 3)*(2*sqrt(1/4*I + 1/4) + I) + 12*(2*x + 1`
`)*sqrt(x + 1) + 32*x + 46)*sqrt(x + sqrt(x + 1)) + ((3*x^2 + 8*sqrt(x + 1)*`
`(x - 2) - 16*x + 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + (3*x^2 - 2*(4*x^2 - sqrt(`
`x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*sqrt(x + 1)*(x - 2)`
` - 16*x + 5)*(2*sqrt(-1/4*I + 1/4) - I)^2 + 44*x^2 - 2*(6*x^2 + (16*x + 3)*`
`sqrt(x + 1) + 3*x + 10)*(2*sqrt(1/4*I + 1/4) + I) - 2*((4*x^2 - sqrt(x + 1)`
`*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + 6*x^2 + (16*x + 3)*sqrt(x`
` + 1) + 3*x + 10)*(2*sqrt(-1/4*I + 1/4) - I) + 4*(12*x^2 + (3*x^2 + 8*sqrt(`
`x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(1/4*I + 1/4) + I) + (3*x^2 - 2*(4*x^2 -`
`sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*sqrt(x + 1)*(x`
` - 2) - 16*x + 5)*(2*sqrt(-1/4*I + 1/4) - I) + 2*(16*x + 3)*sqrt(x + 1) + 6`
`*x + 20)*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I + 1/4)`
`+ I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2) + 24*s`
`qrt(x + 1)*(x - 2) + 92*x - 20)*sqrt(sqrt(1/4*I + 1/4) + sqrt(-1/4*I + 1/4)`
` - 2*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I + 1/4) + I)`
`*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2)))/(x^2 + 1`
`)) + 1/4*sqrt(sqrt(1/4*I + 1/4) + sqrt(-1/4*I + 1/4) - 2*sqrt(-3/16*(2*sqrt`
`(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I + 1/4) + I)*(2*sqrt(-1/4*I + 1/4)`
` - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2))*log(-1/4*(2*(((2*x + 1)*sqrt(x +`
` 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I) + 4*(2*x + 1)*sqrt(x + 1) - x - 8)`
`*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I)^2 + 2*(((2*x + 1)*sqrt(x`
`+ 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I)^2 + (3*x - 16)*sqrt(x + 1) + 4*x`

- 3)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I) + 8*((((2*x + 1)*sqrt
(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I) + 4*(2*x + 1)*sqrt(x + 1) - x
- 8)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I) + ((4*(2*x + 1)*sqrt(
x + 1) - x - 8)*(2*sqrt(1/4*I + 1/4) + I) - (3*x - 16)*sqrt(x + 1) - 4*x +
3)*sqrt(x + sqrt(x + 1)))*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*s
qrt(1/4*I + 1/4) + I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/
4) - I)^2) + 2*((4*(2*x + 1)*sqrt(x + 1) - x - 8)*(2*sqrt(1/4*I + 1/4) + I)
^2 + ((3*x - 16)*sqrt(x + 1) + 4*x - 3)*(2*sqrt(1/4*I + 1/4) + I) + 12*(2*x
 + 1)*sqrt(x + 1) + 32*x + 46)*sqrt(x + sqrt(x + 1)) - ((3*x^2 + 8*sqrt(x +
 1)*(x - 2) - 16*x + 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + (3*x^2 - 2*(4*x^2 - s
qrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*sqrt(x + 1)*(x
- 2) - 16*x + 5)*(2*sqrt(-1/4*I + 1/4) - I)^2 + 44*x^2 - 2*(6*x^2 + (16*x +
 3)*sqrt(x + 1) + 3*x + 10)*(2*sqrt(1/4*I + 1/4) + I) - 2*((4*x^2 - sqrt(x
+ 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + 6*x^2 + (16*x + 3)*sq
rt(x + 1) + 3*x + 10)*(2*sqrt(-1/4*I + 1/4) - I) + 4*(12*x^2 + (3*x^2 + 8*s
qrt(x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(1/4*I + 1/4) + I) + (3*x^2 - 2*(4*x^
2 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*sqrt(x + 1
)*(x - 2) - 16*x + 5)*(2*sqrt(-1/4*I + 1/4) - I) + 2*(16*x + 3)*sqrt(x + 1)
 + 6*x + 20)*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I + 1
/4) + I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2) +
24*sqrt(x + 1)*(x - 2) + 92*x - 20)*sqrt(sqrt(1/4*I + 1/4) + sqrt(-1/4*I +
1/4) - 2*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I + 1/4)
+ I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2)))/(x^2
 + 1)) - 1/4*sqrt(sqrt(1/4*I + 1/4) + sqrt(-1/4*I + 1/4) + 2*sqrt(-3/16*(2*
sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I + 1/4) + I)*(2*sqrt(-1/4*I + 1
/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2))*log(-1/4*(2*(((2*x + 1)*sqrt
(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I) + 4*(2*x + 1)*sqrt(x + 1) - x
- 8)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I)^2 + 2*(((2*x + 1)*sqr
t(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I)^2 + (3*x - 16)*sqrt(x + 1) +
4*x - 3)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I) - 8*((((2*x + 1)*
sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I) + 4*(2*x + 1)*sqrt(x + 1)
- x - 8)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I) + ((4*(2*x + 1)*s
qrt(x + 1) - x - 8)*(2*sqrt(1/4*I + 1/4) + I) - (3*x - 16)*sqrt(x + 1) - 4*
x + 3)*sqrt(x + sqrt(x + 1)))*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*
(2*sqrt(1/4*I + 1/4) + I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I
+ 1/4) - I)^2) + 2*((4*(2*x + 1)*sqrt(x + 1) - x - 8)*(2*sqrt(1/4*I + 1/4)
+ I)^2 + ((3*x - 16)*sqrt(x + 1) + 4*x - 3)*(2*sqrt(1/4*I + 1/4) + I) + 12*
(2*x + 1)*sqrt(x + 1) + 32*x + 46)*sqrt(x + sqrt(x + 1)) + ((3*x^2 + 8*sqrt
(x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + (3*x^2 - 2*(4*x^2
 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*sqrt(x + 1)
*(x - 2) - 16*x + 5)*(2*sqrt(-1/4*I + 1/4) - I)^2 + 44*x^2 - 2*(6*x^2 + (16
*x + 3)*sqrt(x + 1) + 3*x + 10)*(2*sqrt(1/4*I + 1/4) + I) - 2*((4*x^2 - sqr
t(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + 6*x^2 + (16*x + 3
)*sqrt(x + 1) + 3*x + 10)*(2*sqrt(-1/4*I + 1/4) - I) - 4*(12*x^2 + (3*x^2 +
 8*sqrt(x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(1/4*I + 1/4) + I) + (3*x^2 - 2*(

```
4*x^2 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*sqrt(x
 + 1)*(x - 2) - 16*x + 5)*(2*sqrt(-1/4*I + 1/4) - I) + 2*(16*x + 3)*sqrt(x
+ 1) + 6*x + 20)*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I
 + 1/4) + I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2
) + 24*sqrt(x + 1)*(x - 2) + 92*x - 20)*sqrt(sqrt(1/4*I + 1/4) + sqrt(-1/4*
I + 1/4) + 2*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I + 1
/4) + I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2)))/
(x^2 + 1)) + 1/4*sqrt(sqrt(1/4*I + 1/4) + sqrt(-1/4*I + 1/4) + 2*sqrt(-3/16
*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I + 1/4) + I)*(2*sqrt(-1/4*I
 + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2))*log(-1/4*(2*(((2*x + 1)*
sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I) + 4*(2*x + 1)*sqrt(x + 1)
- x - 8)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I)^2 + 2*(((2*x + 1)
*sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I)^2 + (3*x - 16)*sqrt(x + 1
) + 4*x - 3)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I) - 8*((((2*x +
 1)*sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I) + 4*(2*x + 1)*sqrt(x +
 1) - x - 8)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I) + ((4*(2*x +
1)*sqrt(x + 1) - x - 8)*(2*sqrt(1/4*I + 1/4) + I) - (3*x - 16)*sqrt(x + 1)
- 4*x + 3)*sqrt(x + sqrt(x + 1)))*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 -
1/8*(2*sqrt(1/4*I + 1/4) + I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/
4*I + 1/4) - I)^2) + 2*((4*(2*x + 1)*sqrt(x + 1) - x - 8)*(2*sqrt(1/4*I + 1
/4) + I)^2 + ((3*x - 16)*sqrt(x + 1) + 4*x - 3)*(2*sqrt(1/4*I + 1/4) + I) +
 12*(2*x + 1)*sqrt(x + 1) + 32*x + 46)*sqrt(x + sqrt(x + 1)) - ((3*x^2 + 8*
sqrt(x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + (3*x^2 - 2*(4
*x^2 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*sqrt(x
+ 1)*(x - 2) - 16*x + 5)*(2*sqrt(-1/4*I + 1/4) - I)^2 + 44*x^2 - 2*(6*x^2 +
 (16*x + 3)*sqrt(x + 1) + 3*x + 10)*(2*sqrt(1/4*I + 1/4) + I) - 2*((4*x^2 -
 sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + 6*x^2 + (16*x
 + 3)*sqrt(x + 1) + 3*x + 10)*(2*sqrt(-1/4*I + 1/4) - I) - 4*(12*x^2 + (3*x
^2 + 8*sqrt(x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(1/4*I + 1/4) + I) + (3*x^2 -
 2*(4*x^2 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*sq
rt(x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(-1/4*I + 1/4) - I) + 2*(16*x + 3)*sqr
t(x + 1) + 6*x + 20)*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1
/4*I + 1/4) + I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) -
I)^2) + 24*sqrt(x + 1)*(x - 2) + 92*x - 20)*sqrt(sqrt(1/4*I + 1/4) + sqrt(-
1/4*I + 1/4) + 2*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I
 + 1/4) + I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2
)))/(x^2 + 1)) + 1/2*sqrt(-1/2*sqrt(-1/4*I + 1/4) + 1/4*I)*log(((((2*x + 1)
*sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I) + 4*(2*x + 1)*sqrt(x + 1)
 - x - 8)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I)^2 + (((2*x + 1)*
sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I)^2 + (3*x - 16)*sqrt(x + 1)
 + 4*x - 3)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I) + (((2*x + 1)*
sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I)^3 - 6*(2*x + 1)*sqrt(x + 1
) - 16*x - 23)*sqrt(x + sqrt(x + 1)) + (2*(4*x^2 - sqrt(x + 1)*(x - 2) + 2*
x - 5)*(2*sqrt(1/4*I + 1/4) + I)^3 - (3*x^2 - 2*(4*x^2 - sqrt(x + 1)*(x - 2
) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*sqrt(x + 1)*(x - 2) - 16*x + 5)*
```

```
(2*sqrt(-1/4*I + 1/4) - I)^2 + 22*x^2 + 2*((4*x^2 - sqrt(x + 1)*(x - 2) + 2
*x - 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + 6*x^2 + (16*x + 3)*sqrt(x + 1) + 3*x
+ 10)*(2*sqrt(-1/4*I + 1/4) - I) + 12*sqrt(x + 1)*(x - 2) + 46*x - 10)*sqrt
(-1/2*sqrt(-1/4*I + 1/4) + 1/4*I))/(x^2 + 1)) - 1/2*sqrt(-1/2*sqrt(-1/4*I +
 1/4) + 1/4*I)*log(((((2*x + 1)*sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4)
 + I) + 4*(2*x + 1)*sqrt(x + 1) - x - 8)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4
*I + 1/4) - I)^2 + (((2*x + 1)*sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4)
+ I)^2 + (3*x - 16)*sqrt(x + 1) + 4*x - 3)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1
/4*I + 1/4) - I) + (((2*x + 1)*sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4)
+ I)^3 - 6*(2*x + 1)*sqrt(x + 1) - 16*x - 23)*sqrt(x + sqrt(x + 1)) - (2*(4
*x^2 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I)^3 - (3*x^2
- 2*(4*x^2 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*s
qrt(x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(-1/4*I + 1/4) - I)^2 + 22*x^2 + 2*((
4*x^2 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + 6*x^2
+ (16*x + 3)*sqrt(x + 1) + 3*x + 10)*(2*sqrt(-1/4*I + 1/4) - I) + 12*sqrt(x
 + 1)*(x - 2) + 46*x - 10)*sqrt(-1/2*sqrt(-1/4*I + 1/4) + 1/4*I))/(x^2 + 1)
) + 1/2*sqrt(-1/2*sqrt(1/4*I + 1/4) - 1/4*I)*log(-(((((2*x + 1)*sqrt(x + 1)
 - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I)^3 - (4*(2*x + 1)*sqrt(x + 1) - x - 8)*
(2*sqrt(1/4*I + 1/4) + I)^2 - ((3*x - 16)*sqrt(x + 1) + 4*x - 3)*(2*sqrt(1/
4*I + 1/4) + I) + 10*(2*x + 1)*sqrt(x + 1) - 20*x + 15)*sqrt(x + sqrt(x + 1
)) + (2*(4*x^2 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I)^3
 + (3*x^2 + 8*sqrt(x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(1/4*I + 1/4) + I)^2 +
 10*x^2 - 2*(6*x^2 + (16*x + 3)*sqrt(x + 1) + 3*x + 10)*(2*sqrt(1/4*I + 1/4
) + I) - 20*sqrt(x + 1)*(x - 2) - 30*x - 30)*sqrt(-1/2*sqrt(1/4*I + 1/4) -
1/4*I))/(x^2 + 1)) - 1/2*sqrt(-1/2*sqrt(1/4*I + 1/4) - 1/4*I)*log(-(((((2*x
+ 1)*sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I)^3 - (4*(2*x + 1)*sqrt
(x + 1) - x - 8)*(2*sqrt(1/4*I + 1/4) + I)^2 - ((3*x - 16)*sqrt(x + 1) + 4*
x - 3)*(2*sqrt(1/4*I + 1/4) + I) + 10*(2*x + 1)*sqrt(x + 1) - 20*x + 15)*sq
rt(x + sqrt(x + 1)) - (2*(4*x^2 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/
4*I + 1/4) + I)^3 + (3*x^2 + 8*sqrt(x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(1/4*
I + 1/4) + I)^2 + 10*x^2 - 2*(6*x^2 + (16*x + 3)*sqrt(x + 1) + 3*x + 10)*(2
*sqrt(1/4*I + 1/4) + I) - 20*sqrt(x + 1)*(x - 2) - 30*x - 30)*sqrt(-1/2*sqr
t(1/4*I + 1/4) - 1/4*I))/(x^2 + 1))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x + \sqrt{x + 1}}}{x^2 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+(1+x)**(1/2))**(1/2)/(x**2+1),x)

[Out] Integral(sqrt(x + sqrt(x + 1))/(x**2 + 1), x)
```

Giac **[F]** N/A

time $= 0.00$, size $= 0$, normalized size $= 0.00$

<div align="center">Could not integrate</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(x^2+1),x)
```

```
[Out] Could not integrate
```

Mupad **[F]**

time $= 0.00$, size $= -1$, normalized size $= -0.00$

$$\int \frac{\sqrt{x + \sqrt{x + 1}}}{x^2 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x + 1)^(1/2))^(1/2)/(x^2 + 1),x)
```

```
[Out] int((x + (x + 1)^(1/2))^(1/2)/(x^2 + 1), x)
```

3.15 $\int \sqrt{1 + \sqrt{x} + \sqrt{1 + 2\sqrt{x} + 2x}}\ dx$

Optimal. Leaf size=77

$$\frac{2\sqrt{1 + \sqrt{x} + \sqrt{1 + 2\sqrt{x} + 2x}}\ \left(2 + \sqrt{x} + 6x^{3/2} - \left(2 - \sqrt{x}\right)\sqrt{1 + 2\sqrt{x} + 2x}\right)}{15\sqrt{x}}$$

[Out] 2/15*(2+6*x^(3/2)+x^(1/2)-(2-x^(1/2))*(1+2*x+2*x^(1/2))^(1/2))*(1+x^(1/2)+(1+2*x+2*x^(1/2))^(1/2))^(1/2)/x^(1/2)

Rubi [A]

time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$,

Rules used = {2139}

$$\frac{2\sqrt{\sqrt{x} + \sqrt{2x + 2\sqrt{x} + 1} + 1}\ \left(6x^{3/2} + \sqrt{x} - \left(2 - \sqrt{x}\right)\sqrt{2x + 2\sqrt{x} + 1} + 2\right)}{15\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + Sqrt[x] + Sqrt[1 + 2*Sqrt[x] + 2*x]],x]

[Out] (2*Sqrt[1 + Sqrt[x] + Sqrt[1 + 2*Sqrt[x] + 2*x]]*(2 + Sqrt[x] + 6*x^(3/2) - (2 - Sqrt[x])*Sqrt[1 + 2*Sqrt[x] + 2*x]))/(15*Sqrt[x])

Rule 2139

Int[((g_.) + (h_.)*(x_))*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]], x_Symbol] :> Simp[2*((f*(5*b*c*g^2 - 2*b^2*g*h - 3*a*c*g*h + 2*a*b*h^2) + c*f*(10*c*g^2 - b*g*h + a*h^2)*x + 9*c^2*f*g*h*x^2 + 3*c^2*f*h^2*x^3 - (e*g - d*h)*(5*c*g - 2*b*h + c*h*x)*Sqrt[a + b*x + c*x^2])/(15*c^2*f*(g + h*x)))*Sqrt[d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && EqQ[(e*g - d*h)^2 - f^2*(c*g^2 - b*g*h + a*h^2), 0] && EqQ[2*e^2*g - 2*d*e*h - f^2*(2*c*g - b*h), 0]

Rule steps

$$\int \sqrt{1 + \sqrt{x} + \sqrt{1 + 2\sqrt{x} + 2x}}\ dx = 2\text{Subst}\left(\int x\sqrt{1 + x + \sqrt{1 + 2x + 2x^2}}\ dx, x, \sqrt{x}\right)$$

$$= \frac{2\sqrt{1 + \sqrt{x} + \sqrt{1 + 2\sqrt{x} + 2x}}\ \left(2 + \sqrt{x} + 6x^{3/2} - \left(2 - \sqrt{x}\right)\right.}{15\sqrt{x}}$$

Mathematica [A]
time = 10.04, size = 74, normalized size = 0.96

$$\frac{2\sqrt{1 + \sqrt{x} + \sqrt{1 + 2\sqrt{x} + 2x}}\ \left(2 + \sqrt{x} + 6x^{3/2} + \left(-2 + \sqrt{x}\right)\sqrt{1 + 2\sqrt{x} + 2x}\right)}{15\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[x] + Sqrt[1 + 2*Sqrt[x] + 2*x]],x]
```

```
[Out] (2*Sqrt[1 + Sqrt[x] + Sqrt[1 + 2*Sqrt[x] + 2*x]]*(2 + Sqrt[x] + 6*x^(3/2) +
  (-2 + Sqrt[x])*Sqrt[1 + 2*Sqrt[x] + 2*x]))/(15*Sqrt[x])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[1 + Sqrt[x] + Sqrt[1 + 2*Sqrt[x] + 2*x]],x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{1 + \sqrt{x} + \sqrt{1 + 2x + 2\sqrt{x}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/2)+(1+2*x+2*x^(1/2))^(1/2))^(1/2),x)
```

```
[Out] int((1+x^(1/2)+(1+2*x+2*x^(1/2))^(1/2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2)+(1+2*x+2*x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(2*x + 2*sqrt(x) + 1) + sqrt(x) + 1), x)
```

Fricas [A]

time = 0.62, size = 56, normalized size = 0.73

$$
\frac{2\left(6\,x^2 + \sqrt{2\,x + 2\,\sqrt{x}\ + 1}\ (x - 2\,\sqrt{x}\,) + x + 2\,\sqrt{x}\,\right)\sqrt{\sqrt{2\,x + 2\,\sqrt{x}\ + 1}\ + \sqrt{x}\ + 1}}{15\,x}
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x^(1/2)+(1+2*x+2*x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")

[Out] 2/15*(6*x^2 + sqrt(2*x + 2*sqrt(x) + 1)*(x - 2*sqrt(x)) + x + 2*sqrt(x))*sq
rt(sqrt(2*x + 2*sqrt(x) + 1) + sqrt(x) + 1)/x

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$
\int \sqrt{\sqrt{x}\ + \sqrt{2\sqrt{x}\ + 2x + 1}\ + 1}\ dx
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x**(1/2)+(1+2*x+2*x**(1/2))**(1/2))**(1/2),x)

[Out] Integral(sqrt(sqrt(x) + sqrt(2*sqrt(x) + 2*x + 1) + 1), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

$$
\text{Could not integrate}
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x^(1/2)+(1+2*x+2*x^(1/2))^(1/2))^(1/2),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$
\int \sqrt{\sqrt{2\,x + 2\,\sqrt{x}\ + 1}\ + \sqrt{x}\ + 1}\ dx
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2*x + 2*x^(1/2) + 1)^(1/2) + x^(1/2) + 1)^(1/2),x)

[Out] int(((2*x + 2*x^(1/2) + 1)^(1/2) + x^(1/2) + 1)^(1/2), x)

3.16 $\quad \int \sqrt{\sqrt{2} + \sqrt{x} + \sqrt{2 + 2\sqrt{2}\,\sqrt{x} + 2x}}\ dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{2}\,\sqrt{\sqrt{2} + \sqrt{x} + \sqrt{2}\,\sqrt{1 + \sqrt{2}\,\sqrt{x} + x}}\ \left(4 + \sqrt{2}\,\sqrt{x} + 3\sqrt{2}\,x^{3/2} - \sqrt{2}\,\left(2\sqrt{2} - \sqrt{x}\right)\sqrt{1 + \sqrt{\cdots}}\right)}{15\sqrt{x}}$$

[Out] 2/15*2^(1/2)*(4+3*x^(3/2)*2^(1/2)+2^(1/2)*x^(1/2)-2^(1/2)*(2*2^(1/2)-x^(1/2)))*(1+x+2^(1/2)*x^(1/2))^(1/2))*(2^(1/2)+x^(1/2)+2^(1/2)*(1+x+2^(1/2)*x^(1/2))^(1/2))^(1/2)/x^(1/2)

Rubi [A]

time = 0.13, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2140, 2139}

$$\frac{2\sqrt{2}\,\sqrt{\sqrt{x} + \sqrt{2}\,\sqrt{x + \sqrt{2}\,\sqrt{x} + 1} + \sqrt{2}}\ \left(3\sqrt{2}\,x^{3/2} + \sqrt{2}\,\sqrt{x} - \sqrt{2}\,\left(2\sqrt{2} - \sqrt{x}\right)\sqrt{x + \sqrt{2}\,\sqrt{x} + 1} + 4\right)}{15\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sqrt[2] + Sqrt[x] + Sqrt[2 + 2*Sqrt[2]*Sqrt[x] + 2*x]],x]

[Out] (2*Sqrt[2]*Sqrt[Sqrt[2] + Sqrt[x] + Sqrt[2]*Sqrt[1 + Sqrt[2]*Sqrt[x] + x]]*
(4 + Sqrt[2]*Sqrt[x] + 3*Sqrt[2]*x^(3/2) - Sqrt[2]*(2*Sqrt[2] - Sqrt[x])*Sq
rt[1 + Sqrt[2]*Sqrt[x] + x]))/(15*Sqrt[x])

Rule 2139

Int[((g_.) + (h_.)*(x_))*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)
(x_) + (c_.)(x_)^2]], x_Symbol] :> Simp[2*((f*(5*b*c*g^2 - 2*b^2*g*h - 3*
a*c*g*h + 2*a*b*h^2) + c*f*(10*c*g^2 - b*g*h + a*h^2)*x + 9*c^2*f*g*h*x^2 +
 3*c^2*f*h^2*x^3 - (e*g - d*h)*(5*c*g - 2*b*h + c*h*x)*Sqrt[a + b*x + c*x^2
])/(15*c^2*f*(g + h*x)))*Sqrt[d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && EqQ[(e*g - d*h)^2 - f^2*(c*g^2 - b*g*h +
 a*h^2), 0] && EqQ[2*e^2*g - 2*d*e*h - f^2*(2*c*g - b*h), 0]

Rule 2140

Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*((g_.) + (h_.)*(x_))^(m_.
), x_Symbol] :> Int[(g + h*x)^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandT
oSum[v, x]])^n, x] /; FreeQ[{f, g, h, j, k, m, n}, x] && LinearQ[u, x] && Q
uadraticQ[v, x] && !(LinearMatchQ[u, x] && QuadraticMatchQ[v, x] && (EqQ[j
, 0] || EqQ[f, 1])) && EqQ[(Coefficient[u, x, 1]*g - h*(Coefficient[u, x, 0

```
] + f*j))^2 - f^2*k^2*(Coefficient[v, x, 2]*g^2 - Coefficient[v, x, 1]*g*h
+ Coefficient[v, x, 0]*h^2), 0]
```

Rubi steps

$$\int \sqrt{\sqrt{2} + \sqrt{x} + \sqrt{2 + 2\sqrt{2}\sqrt{x} + 2x}}\ dx = 2\text{Subst}\left(\int x\sqrt{x + \sqrt{2}\left(1 + \sqrt{1 + \sqrt{2}\,x + x^2}\right)}\ dx, x, \sqrt{}\right)$$

$$= 2\text{Subst}\left(\int x\sqrt{\sqrt{2} + x + \sqrt{2}\sqrt{1 + \sqrt{2}\,x + x^2}}\ dx, x, \sqrt{}\right)$$

$$= \frac{2\sqrt{2}\sqrt{\sqrt{2} + \sqrt{x} + \sqrt{2}\sqrt{1 + \sqrt{2}\sqrt{x} + x}}\left(4 + \sqrt{2}\sqrt{}\right)}{15}$$

Mathematica [A]
time = 10.05, size = 112, normalized size = 0.95

$$\frac{2\sqrt{2}\left(4 + \sqrt{2}\sqrt{x} + 3\sqrt{2}\,x^{3/2} + \sqrt{2}\left(-2\sqrt{2} + \sqrt{x}\right)\sqrt{1 + \sqrt{2}\sqrt{x} + x}\right)\sqrt{\sqrt{x} + \sqrt{2}\left(1 + \sqrt{1 + \sqrt{2}\sqrt{x} + x}\right)}}{15\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sqrt[2] + Sqrt[x] + Sqrt[2 + 2*Sqrt[2]*Sqrt[x] + 2*x]],x]
```

```
[Out] (2*Sqrt[2]*(4 + Sqrt[2]*Sqrt[x] + 3*Sqrt[2]*x^(3/2) + Sqrt[2]*(-2*Sqrt[2] +
 Sqrt[x])*Sqrt[1 + Sqrt[2]*Sqrt[x] + x])*Sqrt[Sqrt[x] + Sqrt[2]*(1 + Sqrt[1
 + Sqrt[2]*Sqrt[x] + x])])/(15*Sqrt[x])
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[Sqrt[2] + Sqrt[x] + Sqrt[2 + Sqrt[8]*Sqrt[x] + 2*x]
],x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{2} + \sqrt{x} + \sqrt{2 + 2x + 2\sqrt{2}\sqrt{x}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((2^(1/2)+x^(1/2)+(2+2*x+2*2^(1/2)*x^(1/2))^(1/2))^(1/2),x)`

`[Out] int((2^(1/2)+x^(1/2)+(2+2*x+2*2^(1/2)*x^(1/2))^(1/2))^(1/2),x)`

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Failed to integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((2^(1/2)+x^(1/2)+(2+2*x+2*2^(1/2)*x^(1/2))^(1/2))^(1/2),x, algori`
`thm="maxima")`

`[Out] integrate(sqrt(sqrt(2) + sqrt(2*sqrt(2)*sqrt(x) + 2*x + 2) + sqrt(x)), x)`

Fricas [A]
time = 0.83, size = 73, normalized size = 0.62

$$\frac{2\left(6x^2 + \left(\sqrt{2}\,x - 4\sqrt{x}\right)\sqrt{2\sqrt{2}\,\sqrt{x} + 2x + 2} + 4\sqrt{2}\,\sqrt{x} + 2x\right)\sqrt{\sqrt{2} + \sqrt{2\sqrt{2}\,\sqrt{x} + 2x + 2} + \sqrt{x}}}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((2^(1/2)+x^(1/2)+(2+2*x+2*2^(1/2)*x^(1/2))^(1/2))^(1/2),x, algori`
`thm="fricas")`

`[Out] 2/15*(6*x^2 + (sqrt(2)*x - 4*sqrt(x))*sqrt(2*sqrt(2)*sqrt(x) + 2*x + 2) + 4`
`*sqrt(2)*sqrt(x) + 2*x)*sqrt(sqrt(2) + sqrt(2*sqrt(2)*sqrt(x) + 2*x + 2) +`
`sqrt(x))/x`

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x} + \sqrt{2\sqrt{2}\,\sqrt{x} + 2x + 2} + \sqrt{2}}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((2**(1/2)+x**(1/2)+(2+2*x+2*2**(1/2)*x**(1/2))**(1/2))**(1/2),x)`

`[Out] Integral(sqrt(sqrt(x) + sqrt(2*sqrt(2)*sqrt(x) + 2*x + 2) + sqrt(2)), x)`

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

$$\text{Could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(1/2)+x^(1/2)+(2+2*x+2*2^(1/2)*x^(1/2))^(1/2))^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sqrt{2\,x + 2\,\sqrt{2}\,\sqrt{x} + 2} + \sqrt{2} + \sqrt{x}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 2*2^(1/2)*x^(1/2) + 2)^(1/2) + 2^(1/2) + x^(1/2))^(1/2),x)
```

```
[Out] int(((2*x + 2*2^(1/2)*x^(1/2) + 2)^(1/2) + 2^(1/2) + x^(1/2))^(1/2), x)
```

3.17 $\quad \int \dfrac{\sqrt{x + \sqrt{1+x}}}{x^2}\, dx$

Optimal. Leaf size=83

$$-\frac{\sqrt{x + \sqrt{1+x}}}{x} - \frac{1}{4}\tan^{-1}\left(\frac{3 + \sqrt{1+x}}{2\sqrt{x + \sqrt{1+x}}}\right) + \frac{3}{4}\tanh^{-1}\left(\frac{1 - 3\sqrt{1+x}}{2\sqrt{x + \sqrt{1+x}}}\right)$$

```
[Out] -1/4*arctan(1/2*(3+(1+x)^(1/2))/(x+(1+x)^(1/2))^(1/2))+3/4*arctanh(1/2*(1-3
*(1+x)^(1/2))/(x+(1+x)^(1/2))^(1/2))-(x+(1+x)^(1/2))^(1/2)/x
```

Rubi [A]

time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 17, $\dfrac{\text{number of rules}}{\text{integrand size}} = 0.294$,

Rules used = {1028, 1047, 738, 212, 210}

$$-\frac{\sqrt{x + \sqrt{x+1}}}{x} - \frac{1}{4}\tan^{-1}\left(\frac{\sqrt{x+1} + 3}{2\sqrt{x + \sqrt{x+1}}}\right) + \frac{3}{4}\tanh^{-1}\left(\frac{1 - 3\sqrt{x+1}}{2\sqrt{x + \sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[1 + x]]/x^2,x]
```

```
[Out] -(Sqrt[x + Sqrt[1 + x]]/x) - ArcTan[(3 + Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 +
x]])]/4 + (3*ArcTanh[(1 - 3*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1028

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a*h - g*c*x)*(a + c*x^2)^(p + 1)*((d +
 e*x + f*x^2)^q/(2*a*c*(p + 1))), x] + Dist[2/(4*a*c*(p + 1)), Int[(a + c*x
^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[g*c*d*(2*p + 3) - a*(h*e*q) + (g
*c*e*(2*p + q + 3) - a*(2*h*f*q))*x + g*c*f*(2*p + 2*q + 3)*x^2, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] &&
 GtQ[q, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\int \frac{\sqrt{x + \sqrt{1+x}}}{x^2}\, dx = 2\mathrm{Subst}\left(\int \frac{x\sqrt{-1+x+x^2}}{(-1+x^2)^2}\, dx, x, \sqrt{1+x}\right)$$

$$= -\frac{\sqrt{x + \sqrt{1+x}}}{x} + \mathrm{Subst}\left(\int \frac{\frac{1}{2}+x}{(-1+x^2)\sqrt{-1+x+x^2}}\, dx, x, \sqrt{1+x}\right)$$

$$= -\frac{\sqrt{x + \sqrt{1+x}}}{x} + \frac{1}{4}\mathrm{Subst}\left(\int \frac{1}{(1+x)\sqrt{-1+x+x^2}}\, dx, x, \sqrt{1+x}\right) + \frac{3}{4}\mathrm{Subs}$$

$$= -\frac{\sqrt{x + \sqrt{1+x}}}{x} - \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{-4-x^2}\, dx, x, \frac{-3-\sqrt{1+x}}{\sqrt{x + \sqrt{1+x}}}\right) - \frac{3}{2}\mathrm{Subst}\left(\int\right.$$

$$= -\frac{\sqrt{x + \sqrt{1+x}}}{x} - \frac{1}{4}\tan^{-1}\left(\frac{3+\sqrt{1+x}}{2\sqrt{x + \sqrt{1+x}}}\right) + \frac{3}{4}\tanh^{-1}\left(\frac{1-3\sqrt{1+x}}{2\sqrt{x + \sqrt{1+x}}}\right)$$

Mathematica [A]

time = 0.15, size = 77, normalized size = 0.93

$$-\frac{\sqrt{x + \sqrt{1+x}}}{x} - \frac{1}{2}\tan^{-1}\left(1 + \sqrt{1+x} - \sqrt{x + \sqrt{1+x}}\right) - \frac{3}{2}\tanh^{-1}\left(1 - \sqrt{1+x} + \sqrt{x + \sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[1 + x]]/x^2,x]
```

```
[Out] -(Sqrt[x + Sqrt[1 + x]]/x) - ArcTan[1 + Sqrt[1 + x] - Sqrt[x + Sqrt[1 + x]]
]/2 - (3*ArcTanh[1 - Sqrt[1 + x] + Sqrt[x + Sqrt[1 + x]]])/2
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x + Sqrt[1 + x]]/x^2,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(59) = 118$.
time = 0.02, size = 298, normalized size = 3.59

method	result
derivativedivides	$-\dfrac{\left(\left(-1+\sqrt{1+x}\right)^2-2+3\sqrt{1+x}\right)^{\frac{3}{2}}}{2\left(-1+\sqrt{1+x}\right)} + \dfrac{3\sqrt{\left(-1+\sqrt{1+x}\right)^2-2+3\sqrt{1+x}}}{4} + \dfrac{\ln\left(\frac{1}{2}+\sqrt{}\right.}{}$
default	$-\dfrac{\left(\left(-1+\sqrt{1+x}\right)^2-2+3\sqrt{1+x}\right)^{\frac{3}{2}}}{2\left(-1+\sqrt{1+x}\right)} + \dfrac{3\sqrt{\left(-1+\sqrt{1+x}\right)^2-2+3\sqrt{1+x}}}{4} + \dfrac{\ln\left(\frac{1}{2}+\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(1+x)^(1/2))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-1+(1+x)^(1/2))*((-1+(1+x)^(1/2))^2-2+3*(1+x)^(1/2))^(3/2)+3/4*((-1+(
1+x)^(1/2))^2-2+3*(1+x)^(1/2))^(1/2)+1/2*ln(1/2+(1+x)^(1/2)+((-1+(1+x)^(1/2
))^2-2+3*(1+x)^(1/2))^(1/2))-3/4*arctanh(1/2*(-1+3*(1+x)^(1/2))/((-1+(1+x)^
(1/2))^2-2+3*(1+x)^(1/2))^(1/2))+1/4*(1+2*(1+x)^(1/2))*((-1+(1+x)^(1/2))^2-
2+3*(1+x)^(1/2))^(1/2)-1/2/(1+(1+x)^(1/2))*((1+(1+x)^(1/2))^2-(1+x)^(1/2)-2
)^(3/2)-1/4*((1+(1+x)^(1/2))^2-(1+x)^(1/2)-2)^(1/2)-1/2*ln(1/2+(1+x)^(1/2)+
((1+(1+x)^(1/2))^2-(1+x)^(1/2)-2)^(1/2))+1/4*arctan(1/2*(-3-(1+x)^(1/2))/((
1+(1+x)^(1/2))^2-(1+x)^(1/2)-2)^(1/2))+1/4*(1+2*(1+x)^(1/2))*((1+(1+x)^(1/2
))^2-(1+x)^(1/2)-2)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(1+x)^(1/2))^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(x + sqrt(x + 1))/x^2, x)

Fricas [A]

time = 1.86, size = 81, normalized size = 0.98

$$\frac{x \arctan\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}-3\right)}{x-8}\right) + 3x \log\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}+1\right)-3x-2\sqrt{x+1}-2}{x}\right) - 4\sqrt{x+\sqrt{x+1}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(1+x)^(1/2))^(1/2)/x^2,x, algorithm="fricas")

[Out] 1/4*(x*arctan(2*sqrt(x + sqrt(x + 1))*(sqrt(x + 1) - 3)/(x - 8)) + 3*x*log(
(2*sqrt(x + sqrt(x + 1))*(sqrt(x + 1) + 1) - 3*x - 2*sqrt(x + 1) - 2)/x) -
4*sqrt(x + sqrt(x + 1)))/x

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(1+x)**(1/2))**(1/2)/x**2,x)

[Out] Integral(sqrt(x + sqrt(x + 1))/x**2, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(59) = 118.

time = 0.02, size = 262, normalized size = 3.16

$$4\left(\frac{3}{16}\ln\left|\sqrt{x+1+\sqrt{x+1}-1}-\sqrt{x+1}\right| - \frac{3}{16}\ln\left|\sqrt{x+1+\sqrt{x+1}-1}-\sqrt{x+1}+2\right| + \frac{\arctan\left(\sqrt{x+1+\sqrt{x+1}-1}-\sqrt{x+1}-1\right)}{8} - \frac{2\left(\sqrt{x+1+\sqrt{x+1}-1}-\sqrt{x+1}\right)^3 - 3\left(\sqrt{x+1+\sqrt{x+1}-1}-\sqrt{x+1}\right)^2 - \sqrt{x+1+\sqrt{x+1}-1}+\sqrt{x+1}+1}{4\left(\left(\sqrt{x+1+\sqrt{x+1}-1}-\sqrt{x+1}\right)^4 - 2\left(\sqrt{x+1+\sqrt{x+1}-1}-\sqrt{x+1}\right)^2 + 4\left(\sqrt{x+1+\sqrt{x+1}-1}-\sqrt{x+1}\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(1+x)^(1/2))^(1/2)/x^2,x)

```
[Out] -(2*(sqrt(x + sqrt(x + 1)) - sqrt(x + 1))^3 - 3*(sqrt(x + sqrt(x + 1)) - sq
rt(x + 1))^2 - sqrt(x + sqrt(x + 1)) + sqrt(x + 1) + 1)/((sqrt(x + sqrt(x +
 1)) - sqrt(x + 1))^4 - 2*(sqrt(x + sqrt(x + 1)) - sqrt(x + 1))^2 + 4*sqrt(
x + sqrt(x + 1)) - 4*sqrt(x + 1)) + 1/2*arctan(sqrt(x + sqrt(x + 1)) - sqrt
(x + 1) - 1) - 3/4*log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x + 1) + 2)) + 3/4*
log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x + 1)))
```

Mupad **[F]**

time $= 0.00$, size $= $ -1, normalized size $= $ -0.01

$$\int \frac{\sqrt{x + \sqrt{x + 1}}}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x + 1)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int((x + (x + 1)^(1/2))^(1/2)/x^2, x)
```

3.18 $\int \sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\ dx$

Optimal. Leaf size=96

$$\sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\,x+\frac{1}{4}\tan^{-1}\left(\frac{3+\sqrt{1+\frac{1}{x}}}{2\sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}}\right)-\frac{3}{4}\tanh^{-1}\left(\frac{1-3\sqrt{1+\frac{1}{x}}}{2\sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}}\right)$$

[Out] 1/4*arctan(1/2*(3+(1+1/x)^(1/2))/(1/x+(1+1/x)^(1/2))^(1/2))-3/4*arctanh(1/2
(1-3(1+1/x)^(1/2))/(1/x+(1+1/x)^(1/2))^(1/2))+x*(1/x+(1+1/x)^(1/2))^(1/2)

Rubi [A]

time = 0.06, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1028, 1047, 738, 212, 210}

$$\sqrt{\sqrt{\frac{1}{x}+1}+\frac{1}{x}}\,x+\frac{1}{4}\tan^{-1}\left(\frac{\sqrt{\frac{1}{x}+1}+3}{2\sqrt{\sqrt{\frac{1}{x}+1}+\frac{1}{x}}}\right)-\frac{3}{4}\tanh^{-1}\left(\frac{1-3\sqrt{\frac{1}{x}+1}}{2\sqrt{\sqrt{\frac{1}{x}+1}+\frac{1}{x}}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sqrt[1 + x^(-1)] + x^(-1)],x]

[Out] Sqrt[Sqrt[1 + x^(-1)] + x^(-1)]*x + ArcTan[(3 + Sqrt[1 + x^(-1)])/(2*Sqrt[S
qrt[1 + x^(-1)] + x^(-1)])]/4 - (3*ArcTanh[(1 - 3*Sqrt[1 + x^(-1)])/(2*Sqrt
[Sqrt[1 + x^(-1)] + x^(-1)])])/4

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

Rule 738

```
Int[1/((((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1028

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a*h - g*c*x)*(a + c*x^2)^(p + 1)*((d +
 e*x + f*x^2)^q/(2*a*c*(p + 1))), x] + Dist[2/(4*a*c*(p + 1)), Int[(a + c*x
^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[g*c*d*(2*p + 3) - a*(h*e*q) + (g
*c*e*(2*p + q + 3) - a*(2*h*f*q))*x + g*c*f*(2*p + 2*q + 3)*x^2, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] &&
GtQ[q, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\int \sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\;dx = -\left(2\mathrm{Subst}\left(\int \frac{x\sqrt{-1+x+x^2}}{(-1+x^2)^2}\;dx, x, \sqrt{1+\frac{1}{x}}\right)\right)$$

$$= \sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\;x - \mathrm{Subst}\left(\int \frac{\frac{1}{2}+x}{(-1+x^2)\sqrt{-1+x+x^2}}\;dx, x, \sqrt{1+\frac{1}{x}}\right)$$

$$= \sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\;x - \frac{1}{4}\mathrm{Subst}\left(\int \frac{1}{(1+x)\sqrt{-1+x+x^2}}\;dx, x, \sqrt{1+\frac{1}{x}}\right) - \frac{3}{4}\mathrm{Su}$$

$$= \sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\;x + \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{-4-x^2}\;dx, x, \frac{-3-\sqrt{1+\frac{1}{x}}}{\sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}}\right) + \frac{3}{2}\mathrm{Subst}$$

$$= \sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\;x + \frac{1}{4}\tan^{-1}\left(\frac{3+\sqrt{1+\frac{1}{x}}}{2\sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}}\right) - \frac{3}{4}\tanh^{-1}\left(\frac{1-3\sqrt{1+\frac{1}{x}}}{2\sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}}\right)$$

Mathematica [A]
time = 0.36, size = 89, normalized size = 0.93

$$\frac{1}{2}\left(2\sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\;x + \tan^{-1}\left(1+\sqrt{1+\frac{1}{x}}-\sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\right) + 3\tanh^{-1}\left(1-\sqrt{1+\frac{1}{x}}+\sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sqrt[1 + x^(-1)] + x^(-1)],x]
```

```
[Out] (2*Sqrt[Sqrt[1 + x^(-1)] + x^(-1)]*x + ArcTan[1 + Sqrt[1 + x^(-1)] - Sqrt[S
qrt[1 + x^(-1)] + x^(-1)]] + 3*ArcTanh[1 - Sqrt[1 + x^(-1)] + Sqrt[Sqrt[1 +
 x^(-1)] + x^(-1)]])/2
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[1/x + Sqrt[1 + 1/x]],x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple **[F]**

time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{1}{x} + \sqrt{1 + \frac{1}{x}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x+(1+1/x)^(1/2))^(1/2),x)
```

```
[Out] int((1/x+(1+1/x)^(1/2))^(1/2),x)
```

Maxima **[F]**

time = 0.00, size = 0, normalized size = 0.00

$$\text{Failed to integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+(1+1/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(1/x + 1) + 1/x), x)
```

Fricas **[A]**

time = 1.90, size = 122, normalized size = 1.27

$$x\sqrt{\frac{x\sqrt{\frac{x+1}{x}}+1}{x}} + \frac{1}{4}\arctan\left(\frac{2\left(x\sqrt{\frac{x+1}{x}}-3x\right)\sqrt{\frac{x\sqrt{\frac{x+1}{x}}+1}{x}}}{8x-1}\right) + \frac{3}{4}\log\left(2\left(x\sqrt{\frac{x+1}{x}}+x\right)\sqrt{\frac{x\sqrt{\frac{x+1}{x}}+1}{x}}+2x\sqrt{\frac{x+1}{x}}+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+(1+1/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] x*sqrt((x*sqrt((x + 1)/x) + 1)/x) + 1/4*arctan(2*(x*sqrt((x + 1)/x) - 3*x)*
sqrt((x*sqrt((x + 1)/x) + 1)/x)/(8*x - 1)) + 3/4*log(2*(x*sqrt((x + 1)/x) +
 x)*sqrt((x*sqrt((x + 1)/x) + 1)/x) + 2*x*sqrt((x + 1)/x) + 2*x + 3)
```

Sympy **[F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{1 + \frac{1}{x}} + \frac{1}{x}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+(1+1/x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(1 + 1/x) + 1/x), x)
```

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

$$\text{Could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+(1+1/x)^(1/2))^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sqrt{\frac{1}{x}+1} + \frac{1}{x}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/x + 1)^(1/2) + 1/x)^(1/2),x)
```

```
[Out] int(((1/x + 1)^(1/2) + 1/x)^(1/2), x)
```

3.19 $\quad \int \frac{\sqrt{1+e^{-x}}}{-e^{-x}+e^x}\,dx$

Optimal. Leaf size=25

$$-\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1+e^{-x}}}{\sqrt{2}}\right)$$

`[Out] -arctanh(1/2*(1+exp(-x))^(1/2)*2^(1/2))*2^(1/2)`

Rubi [A]

time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2320, 1460, 1483, 641, 65, 212}

$$-\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{e^{-x}+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

`[In] Int[Sqrt[1 + E^(-x)]/(-E^(-x) + E^x),x]`

`[Out] -(Sqrt[2]*ArcTanh[Sqrt[1 + E^(-x)]/Sqrt[2]])`

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 1460

```
Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_S
ymbol] :> Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x] /; FreeQ[{a,
```

c, d, e, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]

Rule 1483

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]

Rule 2320

Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$
\int \frac{\sqrt{1 + e^{-x}}}{-e^{-x} + e^x} \, dx = \text{Subst}\left(\int \frac{\sqrt{1 + \frac{1}{x}}}{-1 + x^2} \, dx, x, e^x \right)
$$

$$
= \text{Subst}\left(\int \frac{\sqrt{1 + \frac{1}{x}}}{\left(1 - \frac{1}{x^2}\right) x^2} \, dx, x, e^x \right)
$$

$$
= -\text{Subst}\left(\int \frac{\sqrt{1 + x}}{1 - x^2} \, dx, x, e^{-x} \right)
$$

$$
= -\text{Subst}\left(\int \frac{1}{(1 - x)\sqrt{1 + x}} \, dx, x, e^{-x} \right)
$$

$$
= -\left(2\text{Subst}\left(\int \frac{1}{2 - x^2} \, dx, x, \sqrt{1 + e^{-x}} \right) \right)
$$

$$
= -\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{1 + e^{-x}}}{\sqrt{2}} \right)
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 65 vs. 2(25) =
50.

time $= 0.11$, size $= 65$, normalized size $= 2.60$

$$-\frac{\sqrt{2}\,e^{x/2}\sqrt{1+e^{-x}}\,\tanh^{-1}\left(\frac{1-e^x+e^{x/2}\sqrt{1+e^x}}{\sqrt{2}}\right)}{\sqrt{1+e^x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + E^(-x)]/(-E^(-x) + E^x),x]
```

```
[Out] -((Sqrt[2]*E^(x/2)*Sqrt[1 + E^(-x)]*ArcTanh[(1 - E^x + E^(x/2)*Sqrt[1 + E^x
]))/Sqrt[2]])/Sqrt[1 + E^x])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time $= 4.68$, size $= 61$, normalized size $= 2.44$

$$\text{Piecewise}\left[\left\{\left\{-\sqrt{2}\,\text{ArcCoth}\left[\frac{\sqrt{2}\,\sqrt{1+E^{-x}}}{2}\right], E^{-x}>1\right\}, \left\{-\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{1+E^{-x}}}{2}\right], E^{-x}<1\right\}, \{0, \text{True}\}\right\}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[1 + Exp[-x]]/(Exp[x] - Exp[-x]),x]')
```

```
[Out] Piecewise[{{-Sqrt[2] ArcCoth[Sqrt[2] Sqrt[1 + E ^ (-x)] / 2], E ^ (-x) > 1}
, {-Sqrt[2] ArcTanh[Sqrt[2] Sqrt[1 + E ^ (-x)] / 2], E ^ (-x) < 1}, {0, Tru
e}}]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(19) = 38$.

time $= 0.04$, size $= 49$, normalized size $= 1.96$

method	result	size
default	$-\dfrac{\sqrt{(1+e^x)\,e^{-x}}\,e^x\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{(1+3\,e^x)\,\sqrt{2}}{4\sqrt{e^x+e^{2x}}}\right)}{2\sqrt{(1+e^x)\,e^x}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+exp(-x))^(1/2)/(-exp(-x)+exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((1+exp(x))/exp(x))^(1/2)*exp(x)/((1+exp(x))*exp(x))^(1/2)*2^(1/2)*arc
tanh(1/4*(1+3*exp(x))*2^(1/2)/(exp(x)^2+exp(x))^(1/2))
```

Maxima [A]
time $= 0.34$, size $= 36$, normalized size $= 1.44$

$$\frac{1}{2}\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}-\sqrt{e^{(-x)}+1}}{\sqrt{2}+\sqrt{e^{(-x)}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+exp(-x))^(1/2)/(-exp(-x)+exp(x)),x, algorithm="maxima")

[Out] 1/2*sqrt(2)*log(-(sqrt(2) - sqrt(e^(-x) + 1))/(sqrt(2) + sqrt(e^(-x) + 1)))

Fricas [A]
time = 0.34, size = 34, normalized size = 1.36

$$\frac{1}{2}\sqrt{2}\,\log\left(\frac{2\sqrt{2}\,\sqrt{e^x+1}\,e^{\left(\frac{1}{2}x\right)}-3\,e^x-1}{e^x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+exp(-x))^(1/2)/(-exp(-x)+exp(x)),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*log((2*sqrt(2)*sqrt(e^x + 1)*e^(1/2*x) - 3*e^x - 1)/(e^x - 1))

Sympy [A]
time = 2.02, size = 61, normalized size = 2.44

$$2\left(\begin{cases} -\dfrac{\sqrt{2}\,\operatorname{acoth}\left(\dfrac{\sqrt{2}\,\sqrt{1+e^{-x}}}{2}\right)}{2} & \text{for } e^{-x}>1 \\[3ex] -\dfrac{\sqrt{2}\,\operatorname{atanh}\left(\dfrac{\sqrt{2}\,\sqrt{1+e^{-x}}}{2}\right)}{2} & \text{for } e^{-x}<1 \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+exp(-x))**(1/2)/(-exp(-x)+exp(x)),x)

[Out] 2*Piecewise((-sqrt(2)*acoth(sqrt(2)*sqrt(1 + exp(-x))/2)/2, exp(-x) > 1), (
-sqrt(2)*atanh(sqrt(2)*sqrt(1 + exp(-x))/2)/2, exp(-x) < 1))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(19) = 38.
time = 0.02, size = 91, normalized size = 3.64

$$-\frac{\ln\left(\dfrac{\sqrt{2}-1}{\sqrt{2}+1}\right)}{\sqrt{2}}+\frac{\ln\left(\dfrac{\left|2\left(-e^x+\sqrt{(e^x)^2+e^x}\right)-2\sqrt{2}+2\right|}{\left|2\left(-e^x+\sqrt{(e^x)^2+e^x}\right)+2\sqrt{2}+2\right|}\right)}{\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+exp(-x))^(1/2)/(-exp(-x)+exp(x)),x)

[Out] -1/2*sqrt(2)*log((sqrt(2) - 1)/(sqrt(2) + 1)) + 1/2*sqrt(2)*log(abs(-2*sqrt
(2) + 2*sqrt(e^(2*x) + e^x) - 2*e^x + 2)/abs(2*sqrt(2) + 2*sqrt(e^(2*x) + e
^x) - 2*e^x + 2))

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{\sqrt{e^{-x}+1}}{e^{-x}-e^{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(exp(-x) + 1)^(1/2)/(exp(-x) - exp(x)),x)

[Out] -int((exp(-x) + 1)^(1/2)/(exp(-x) - exp(x)), x)

3.20 $\int \sqrt{1 + e^{-x}}\, \text{csch}(x)\, dx$

Optimal. Leaf size=25

$$-2\sqrt{2}\, \tanh^{-1}\left(\frac{\sqrt{1 + e^{-x}}}{\sqrt{2}}\right)$$

[Out] -2*arctanh(1/2*(1+exp(-x))^(1/2)*2^(1/2))*2^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2320, 12, 1460, 1483, 641, 65, 212}

$$-2\sqrt{2}\, \tanh^{-1}\left(\frac{\sqrt{e^{-x} + 1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + E^(-x)]*Csch[x],x]

[Out] -2*Sqrt[2]*ArcTanh[Sqrt[1 + E^(-x)]/Sqrt[2]]

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

Rule 641

Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege

rQ[m + p]))

Rule 1460

Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_S
ymbol] :> Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x] /; FreeQ[{a,
c, d, e, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]

Rule 1483

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]

Rule 2320

Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\int \sqrt{1+e^{-x}}\,\mathrm{csch}(x)\,dx = \mathrm{Subst}\left(\int \frac{2\sqrt{1+\frac{1}{x}}}{-1+x^2}\,dx,\,x,\,e^x\right)$$

$$= 2\mathrm{Subst}\left(\int \frac{\sqrt{1+\frac{1}{x}}}{-1+x^2}\,dx,\,x,\,e^x\right)$$

$$= 2\mathrm{Subst}\left(\int \frac{\sqrt{1+\frac{1}{x}}}{\left(1-\frac{1}{x^2}\right)x^2}\,dx,\,x,\,e^x\right)$$

$$= -\left(2\mathrm{Subst}\left(\int \frac{\sqrt{1+x}}{1-x^2}\,dx,\,x,\,e^{-x}\right)\right)$$

$$= -\left(2\mathrm{Subst}\left(\int \frac{1}{(1-x)\sqrt{1+x}}\,dx,\,x,\,e^{-x}\right)\right)$$

$$= -\left(4\mathrm{Subst}\left(\int \frac{1}{2-x^2}\,dx,\,x,\,\sqrt{1+e^{-x}}\right)\right)$$

$$= -2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1+e^{-x}}}{\sqrt{2}}\right)$$

Mathematica **[B]** Leaf count is larger than twice the leaf count of optimal. 126 vs. $2(25) = 50$.

time = 0.08, size = 126, normalized size = 5.04

$$\frac{\sqrt{2}\,e^{x/2}\sqrt{1+e^{-x}}\left(\log\left(1-e^{-x/2}\right)+\log\left(1+e^{-x/2}\right)-\log\left(e^{-x/2}\left(-1+e^{x/2}+\sqrt{2}\,\sqrt{1+e^x}\right)\right)-\log\left(e^{-x/2}\left(1+e^{x/2}+\sqrt{2}\,\sqrt{1+e^x}\right)\right)\right)}{\sqrt{1+e^x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + E^(-x)]*Csch[x],x]
```

```
[Out] (Sqrt[2]*E^(x/2)*Sqrt[1 + E^(-x)]*(Log[1 - E^(-1/2*x)] + Log[1 + E^(-1/2*x)
] - Log[(-1 + E^(x/2) + Sqrt[2]*Sqrt[1 + E^x])/E^(x/2)] - Log[(1 + E^(x/2)
+ Sqrt[2]*Sqrt[1 + E^x])/E^(x/2)]))/Sqrt[1 + E^x]
```

Mathics **[F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

   cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Sqrt[1 + Exp[-x]]/Sinh[x],x]')`

`[Out] cought exception: maximum recursion depth exceeded while calling a Python object`

Maple [A]
time = 0.07, size = 33, normalized size = 1.32

method	result	size
default	$-2\sqrt{2}\ \sqrt{\frac{1}{\tanh\left(\frac{x}{2}\right)+1}}\ \sqrt{\tanh\left(\frac{x}{2}\right)+1}\ \operatorname{arctanh}\left(\sqrt{\tanh\left(\frac{x}{2}\right)+1}\right)$	33

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((1+exp(-x))^(1/2)/sinh(x),x,method=_RETURNVERBOSE)`

`[Out] -2*2^(1/2)*(1/(tanh(1/2*x)+1))^(1/2)*(tanh(1/2*x)+1)^(1/2)*arctanh((tanh(1/2*x)+1)^(1/2))`

Maxima [A]
time = 0.39, size = 35, normalized size = 1.40

$$\sqrt{2}\,\log\left(-\frac{\sqrt{2}-\sqrt{e^{(-x)}+1}}{\sqrt{2}+\sqrt{e^{(-x)}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((1+exp(-x))^(1/2)/sinh(x),x, algorithm="maxima")`

`[Out] sqrt(2)*log(-(sqrt(2) - sqrt(e^(-x) + 1))/(sqrt(2) + sqrt(e^(-x) + 1)))`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(19) = 38.
time = 0.36, size = 55, normalized size = 2.20

$$\sqrt{2}\,\log\left(\frac{2\left(\sqrt{2}\,\cosh\left(x\right)+\sqrt{2}\,\sinh\left(x\right)\right)\sqrt{\frac{\cosh\left(x\right)+\sinh\left(x\right)+1}{\cosh\left(x\right)+\sinh\left(x\right)}}-3\cosh\left(x\right)-3\sinh\left(x\right)-1}{\cosh\left(x\right)+\sinh\left(x\right)-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((1+exp(-x))^(1/2)/sinh(x),x, algorithm="fricas")`

`[Out] sqrt(2)*log((2*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt((cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x))) - 3*cosh(x) - 3*sinh(x) - 1)/(cosh(x) + sinh(x) - 1))`

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1 + e^{-x}}}{\sinh{(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(-x))**(1/2)/sinh(x),x)
```

```
[Out] Integral(sqrt(1 + exp(-x))/sinh(x), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(19) = 38.
time = 0.02, size = 93, normalized size = 3.72

$$2 \left(-\frac{\ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)}{\sqrt{2}} + \frac{\ln\left(\left|\frac{2\left(-e^x + \sqrt{(e^x)^2 + e^x}\right) - 2\sqrt{2} + 2}{2\left(-e^x + \sqrt{(e^x)^2 + e^x}\right) + 2\sqrt{2} + 2}\right|\right)}{\sqrt{2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(-x))^(1/2)/sinh(x),x)
```

```
[Out] -sqrt(2)*log((sqrt(2) - 1)/(sqrt(2) + 1)) + sqrt(2)*log(abs(-2*sqrt(2) + 2*
sqrt(e^(2*x) + e^x) - 2*e^x + 2)/abs(2*sqrt(2) + 2*sqrt(e^(2*x) + e^x) - 2*
e^x + 2))
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e^{-x} + 1}}{\sinh{(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(-x) + 1)^(1/2)/sinh(x),x)
```

```
[Out] int((exp(-x) + 1)^(1/2)/sinh(x), x)
```

3.21 $\int \frac{1}{(\cos(x)+\cos(3x))^5}\, dx$

Optimal. Leaf size=108

$$-\frac{523}{256}\tanh^{-1}(\sin(x))+\frac{1483\tanh^{-1}\left(\sqrt{2}\,\sin(x)\right)}{512\sqrt{2}}+\frac{\sin(x)}{32\left(1-2\sin^2(x)\right)^4}-\frac{17\sin(x)}{192\left(1-2\sin^2(x)\right)^3}+\frac{203\sin(x}{768\left(1-2\sin(x}$$

```
[Out] -523/256*arctanh(sin(x))+1/32*sin(x)/(1-2*sin(x)^2)^4-17/192*sin(x)/(1-2*si
n(x)^2)^3+203/768*sin(x)/(1-2*sin(x)^2)^2-437/512*sin(x)/(1-2*sin(x)^2)+148
3/1024*arctanh(sin(x)*2^(1/2))*2^(1/2)-43/256*sec(x)*tan(x)-1/128*sec(x)^3*
tan(x)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 786 vs. 2(108) = 216. time = 0.79, antiderivative size = 786, normalized size of antiderivative = 7.28, number of steps used = 45, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {12, 2098, 213, 652, 628, 632, 212}

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + Cos[3*x])^(-5),x]
```

```
[Out] (-523*ArcTanh[Sin[x]])/256 - (1483*Log[2 + Sqrt[2] + Cos[x] + Sqrt[2]*Cos[x
] - Sin[x] - Sqrt[2]*Sin[x]])/(2048*Sqrt[2]) - (1483*Log[2 - Sqrt[2] + Cos[
x] - Sqrt[2]*Cos[x] + Sin[x] - Sqrt[2]*Sin[x]])/(2048*Sqrt[2]) + (1483*Log[
2 - Sqrt[2] + Cos[x] - Sqrt[2]*Cos[x] - Sin[x] + Sqrt[2]*Sin[x]])/(2048*Sqr
t[2]) + (1483*Log[2 + Sqrt[2] + Cos[x] + Sqrt[2]*Cos[x] + Sin[x] + Sqrt[2]*
Sin[x]])/(2048*Sqrt[2]) - 1/(128*(1 - Tan[x/2])^4) + 1/(64*(1 - Tan[x/2])^3
) - 47/(256*(1 - Tan[x/2])^2) + 45/(256*(1 - Tan[x/2])) + 1/(128*(1 + Tan[x
/2])^4) - 1/(64*(1 + Tan[x/2])^3) + 47/(256*(1 + Tan[x/2])^2) - 45/(256*(1
+ Tan[x/2])) - (7 - 17*Tan[x/2])/(4*(1 - 2*Tan[x/2] - Tan[x/2]^2)^4) + (119
*(1 + Tan[x/2]))/(48*(1 - 2*Tan[x/2] - Tan[x/2]^2)^3) - (11*(1 + 3*Tan[x/2]
))/(12*(1 - 2*Tan[x/2] - Tan[x/2]^2)^3) - (1 - 43*Tan[x/2])/(32*(1 - 2*Tan[
x/2] - Tan[x/2]^2)^2) - (65*(1 + Tan[x/2]))/(384*(1 - 2*Tan[x/2] - Tan[x/2]
^2)^2) + (451*(1 + Tan[x/2]))/(512*(1 - 2*Tan[x/2] - Tan[x/2]^2)) - (89 + 1
5*Tan[x/2])/(64*(1 - 2*Tan[x/2] - Tan[x/2]^2)) + (7 + 17*Tan[x/2])/(4*(1 +
2*Tan[x/2] - Tan[x/2]^2)^4) + (11*(1 - 3*Tan[x/2]))/(12*(1 + 2*Tan[x/2] - T
an[x/2]^2)^3) - (119*(1 - Tan[x/2]))/(48*(1 + 2*Tan[x/2] - Tan[x/2]^2)^3) +
 (65*(1 - Tan[x/2]))/(384*(1 + 2*Tan[x/2] - Tan[x/2]^2)^2) + (1 + 43*Tan[x/
2])/(32*(1 + 2*Tan[x/2] - Tan[x/2]^2)^2) + (89 - 15*Tan[x/2])/(64*(1 + 2*Ta
n[x/2] - Tan[x/2]^2)) - (451*(1 - Tan[x/2]))/(512*(1 + 2*Tan[x/2] - Tan[x/2
]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2098

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(\cos(x)+\cos(3x))^5}\,dx = 2\mathrm{Subst}\left(\int \frac{(1+x^2)^{14}}{32\left(1-7x^2+7x^4-x^6\right)^5}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$

$$= \frac{1}{16}\mathrm{Subst}\left(\int \frac{(1+x^2)^{14}}{\left(1-7x^2+7x^4-x^6\right)^5}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$

$$= \frac{1}{16}\mathrm{Subst}\left(\int \left(\frac{1}{2(-1+x)^5}+\frac{3}{4(-1+x)^4}+\frac{47}{8(-1+x)^3}+\frac{45}{16(-1+x)^2}-\frac{}{2(1}\right)\right.$$

$$= -\frac{1}{128\left(1-\tan\left(\frac{x}{2}\right)\right)^4}+\frac{1}{64\left(1-\tan\left(\frac{x}{2}\right)\right)^3}-\frac{47}{256\left(1-\tan\left(\frac{x}{2}\right)\right)^2}+\frac{45}{256\left(1-\mathrm{ta}\right.}$$

$$= -\frac{523}{256}\tanh^{-1}(\sin(x))-\frac{1}{128\left(1-\tan\left(\frac{x}{2}\right)\right)^4}+\frac{1}{64\left(1-\tan\left(\frac{x}{2}\right)\right)^3}-\frac{47}{256\left(1-\mathrm{ta}\right.}$$

$$= -\frac{523}{256}\tanh^{-1}(\sin(x))-\frac{9\log\left(2+\sqrt{2}+\cos(x)+\sqrt{2}\,\cos(x)-\sin(x)-\sqrt{2}\right)}{16\sqrt{2}}$$

$$= -\frac{523}{256}\tanh^{-1}(\sin(x))-\frac{129\log\left(2+\sqrt{2}+\cos(x)+\sqrt{2}\,\cos(x)-\sin(x)-\sqrt{}\right)}{256\sqrt{2}}$$

$$= -\frac{523}{256}\tanh^{-1}(\sin(x))-\frac{387\log\left(2+\sqrt{2}+\cos(x)+\sqrt{2}\,\cos(x)-\sin(x)-\sqrt{}\right)}{512\sqrt{2}}$$

$$= -\frac{523}{256}\tanh^{-1}(\sin(x))-\frac{111\log\left(2+\sqrt{2}+\cos(x)+\sqrt{2}\,\cos(x)-\sin(x)-\sqrt{}\right)}{256\sqrt{2}}$$

$$= -\frac{523}{256}\tanh^{-1}(\sin(x))-\frac{1483\log\left(2+\sqrt{2}+\cos(x)+\sqrt{2}\,\cos(x)-\sin(x)-\right)}{2048\sqrt{2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.23, size = 478, normalized size = 4.43

$$\text{[illegible long expression]}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[x] + Cos[3*x])^(-5),x]
```

```
[Out] (((-1483*I)/1024)*ArcTan[(Cos[x/2] - Sin[x/2] - Sqrt[2]*Sin[x/2])/(-Cos[x/2
] + Sqrt[2]*Cos[x/2] - Sin[x/2])])/Sqrt[2] + ((1483/2048 + (1483*I)/2048)*(
(-1 - I) + Sqrt[2])*ArcTan[(Cos[x/2] + Sin[x/2] - Sqrt[2]*Sin[x/2])/(Cos[x/
2] + Sqrt[2]*Cos[x/2] - Sin[x/2])])/((-1 + I) + Sqrt[2]) + (523*Log[Cos[x/2
```

```
] - Sin[x/2]])/256 - (523*Log[Cos[x/2] + Sin[x/2]])/256 + (1483*Log[Sqrt[2]
 + 2*Sin[x]])/(1024*Sqrt[2]) - (1483*Log[2 - Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x
]])/(2048*Sqrt[2]) + ((1483/4096 - (1483*I)/4096)*((-1 - I) + Sqrt[2])*Log[
2 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]])/((-1 + I) + Sqrt[2]) - 1/(512*(Cos[x/
2] - Sin[x/2])^4) - 43/(512*(Cos[x/2] - Sin[x/2])^2) + 1/(512*(Cos[x/2] + S
in[x/2])^4) + 43/(512*(Cos[x/2] + Sin[x/2])^2) - 17/(768*(Cos[x] - Sin[x])^
3) - 437/(1024*(Cos[x] - Sin[x])) + Sin[x]/(128*(Cos[x] - Sin[x])^4) + (83*
Sin[x])/(512*(Cos[x] - Sin[x])^2) + Sin[x]/(128*(Cos[x] + Sin[x])^4) + 17/(
768*(Cos[x] + Sin[x])^3) + (83*Sin[x])/(512*(Cos[x] + Sin[x])^2) + 437/(102
4*(Cos[x] + Sin[x]))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(Cos[x] + Cos[3*x])^5,x]')

[Out] Timed out

Maple [A]
time = 0.13, size = 95, normalized size = 0.88

method	result
default	$-\dfrac{4\left(-\frac{437\left(\sin^7(x)\right)}{256}+\frac{3527\left(\sin^5(x)\right)}{1536}-\frac{3257\left(\sin^3(x)\right)}{3072}+\frac{331\sin(x)}{2048}\right)}{\left(2\left(\sin^2(x)\right)-1\right)^4}+\dfrac{1483\,\text{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{1024}-\dfrac{1}{512(\sin(x)-1)^2}+\dfrac{4}{512(\sin}$
risch	$\dfrac{i\left(1827\,e^{23ix}+3733\,e^{21ix}+6115\,e^{19ix}+9109\,e^{17ix}+5746\,e^{15ix}+2382\,e^{13ix}-2382\,e^{11ix}-5746\,e^{9ix}-9109\,e^{7ix}-6115\,e^{5ix}-3733\,e^{3ix}-182\right.}{1536(e^{6ix}+e^{4ix}+e^{2ix}+1)^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)+cos(3*x))^5,x,method=_RETURNVERBOSE)

[Out] -4*(-437/256*sin(x)^7+3527/1536*sin(x)^5-3257/3072*sin(x)^3+331/2048*sin(x)
)/(2*sin(x)^2-1)^4+1483/1024*arctanh(sin(x)*2^(1/2))*2^(1/2)-1/512/(sin(x)-
1)^2+43/512/(sin(x)-1)+523/512*ln(sin(x)-1)+1/512/(sin(x)+1)^2+43/512/(sin(
x)+1)-523/512*ln(sin(x)+1)

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(cos(x)+cos(3*x))^5,x, algorithm="maxima")`

[Out] Timed out

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(88) = 176$.
time $= 0.39$, size $= 219$, normalized size $= 2.03$

$$\frac{4449\left(16\sqrt{2}\cos(x)^{12}-32\sqrt{2}\cos(x)^{10}+24\sqrt{2}\cos(x)^{8}-8\sqrt{2}\cos(x)^{6}+\sqrt{2}\cos(x)^{4}\right)\log\left(-\frac{2\cos(x)^{2}-2\sqrt{2}\sin(x)-3}{2\cos(x)^{2}-1}\right)-6276\left(16\cos(x)^{12}-32\cos(x)^{10}+24\cos(x)^{8}-8\cos(x)^{6}+\cos(x)^{4}\right)\log(\sin(x)+1)+6276\left(16\cos(x)^{12}-32\cos(x)^{10}+24\cos(x)^{8}-8\cos(x)^{6}+\cos(x)^{4}\right)\log(-\sin(x)+1)-4\left(14616\cos(x)^{10}-25420\cos(x)^{8}+15570\cos(x)^{6}-3677\cos(x)^{4}+162\cos(x)^{2}+12\right)\sin(x)}{6144\left(16\cos(x)^{12}-32\cos(x)^{10}+24\cos(x)^{8}-8\cos(x)^{6}+\cos(x)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(cos(x)+cos(3*x))^5,x, algorithm="fricas")`

[Out] `1/6144*(4449*(16*sqrt(2)*cos(x)^12 - 32*sqrt(2)*cos(x)^10 + 24*sqrt(2)*cos(x)^8 - 8*sqrt(2)*cos(x)^6 + sqrt(2)*cos(x)^4)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 6276*(16*cos(x)^12 - 32*cos(x)^10 + 24*cos(x)^8 - 8*cos(x)^6 + cos(x)^4)*log(sin(x) + 1) + 6276*(16*cos(x)^12 - 32*cos(x)^10 + 24*cos(x)^8 - 8*cos(x)^6 + cos(x)^4)*log(-sin(x) + 1) - 4*(14616*cos(x)^10 - 25420*cos(x)^8 + 15570*cos(x)^6 - 3677*cos(x)^4 + 162*cos(x)^2 + 12)*sin(x))/(16*cos(x)^12 - 32*cos(x)^10 + 24*cos(x)^8 - 8*cos(x)^6 + cos(x)^4)`

Sympy [F]
time $= 0.00$, size $= 0$, normalized size $= 0.00$

$$\int \frac{1}{\left(\cos\left(x\right)+\cos\left(3x\right)\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(cos(x)+cos(3*x))**5,x)`

[Out] `Integral((cos(x) + cos(3*x))**(-5), x)`

Giac [A]
time $= 0.00$, size $= 128$, normalized size $= 1.19$

$$2\left(-\frac{-43\sin^{3}x+45\sin x}{512\left(\sin^{2}x-1\right)^{2}}+\frac{10488\sin^{7}x-14108\sin^{5}x+6514\sin^{3}x-993\sin x}{3072\left(2\sin^{2}x-1\right)^{4}}+\frac{523}{1024}\ln\left(-\sin x+1\right)-\frac{523}{1024}\ln\left(\sin x+1\right)-\frac{1483\ln\left(\left|\frac{4\sin x-2\sqrt{2}}{4\sin x+2\sqrt{2}}\right|\right)}{2\cdot1024\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(cos(x)+cos(3*x))^5,x)`

[Out] `-1483/2048*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) + 1/256*(43*sin(x)^3 - 45*sin(x))/(sin(x)^2 - 1)^2 + 1/1536*(10488*sin(x)^7 - 14108*sin(x)^5 + 6514*sin(x)^3 - 993*sin(x))/(2*sin(x)^2 - 1)^4 - 523/512*log(sin(x) + 1) + 523/512*log(-sin(x) + 1)`

Mupad [B]

time = 1.25, size = 307, normalized size = 2.84

$$\text{(small illegible equation)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(3*x) + cos(x))^5,x)
```

[Out] -(11492*sin(3*x) + 18218*sin(5*x) + 12230*sin(7*x) + 7466*sin(9*x) + 3654*s
in(11*x) + 276144*atanh(sin(x/2)/cos(x/2)) + 4764*sin(x) + 502080*cos(2*x)*
atanh(sin(x/2)/cos(x/2)) + 389112*cos(4*x)*atanh(sin(x/2)/cos(x/2)) + 25104
0*cos(6*x)*atanh(sin(x/2)/cos(x/2)) + 125520*cos(8*x)*atanh(sin(x/2)/cos(x/
2)) + 50208*cos(10*x)*atanh(sin(x/2)/cos(x/2)) + 12552*cos(12*x)*atanh(sin(
x/2)/cos(x/2)) - 97878*2^(1/2)*atanh(2^(1/2)*sin(x)) - 177960*2^(1/2)*atanh
(2^(1/2)*sin(x))*cos(2*x) - 137919*2^(1/2)*atanh(2^(1/2)*sin(x))*cos(4*x) -
 88980*2^(1/2)*atanh(2^(1/2)*sin(x))*cos(6*x) - 44490*2^(1/2)*atanh(2^(1/2)
*sin(x))*cos(8*x) - 17796*2^(1/2)*atanh(2^(1/2)*sin(x))*cos(10*x) - 4449*2^
(1/2)*atanh(2^(1/2)*sin(x))*cos(12*x))/(122880*cos(2*x) + 95232*cos(4*x) +
61440*cos(6*x) + 30720*cos(8*x) + 12288*cos(10*x) + 3072*cos(12*x) + 67584)
```

# 3.22 $\int \frac{1}{(1+\cos(x)+\sin(x))^2}\,dx$

**Optimal**. Leaf size=29

$$-\log\left(1+\tan\left(\frac{x}{2}\right)\right) - \frac{\cos(x)-\sin(x)}{1+\cos(x)+\sin(x)}$$

[Out] -ln(1+tan(1/2*x))+(-cos(x)+sin(x))/(1+cos(x)+sin(x))

**Rubi [A]**

time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3208, 3203, 31}

$$-\log\left(\tan\left(\frac{x}{2}\right)+1\right) - \frac{\cos(x)-\sin(x)}{\sin(x)+\cos(x)+1}$$

Antiderivative was successfully verified.

[In] Int[(1 + Cos[x] + Sin[x])^(-2),x]

[Out] -Log[1 + Tan[x/2]] - (Cos[x] - Sin[x])/(1 + Cos[x] + Sin[x])

Rule 31

Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]


Rule 3203

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]


Rule 3208

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Simp[(((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2)))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]


Rubi steps

$$\int \frac{1}{(1+\cos(x)+\sin(x))^2}\,dx = -\frac{\cos(x)-\sin(x)}{1+\cos(x)+\sin(x)} - \int \frac{1}{1+\cos(x)+\sin(x)}\,dx$$

$$= -\frac{\cos(x)-\sin(x)}{1+\cos(x)+\sin(x)} - 2\mathrm{Subst}\left(\int \frac{1}{2+2x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$

$$= -\log\left(1+\tan\left(\frac{x}{2}\right)\right) - \frac{\cos(x)-\sin(x)}{1+\cos(x)+\sin(x)}$$

## Mathematica [A]
time = 0.02, size = 56, normalized size = 1.93

$$\log\left(\cos\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right) + \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)} + \frac{1}{2}\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x] + Sin[x])^(-2),x]
```

```
[Out] Log[Cos[x/2]] - Log[Cos[x/2] + Sin[x/2]] + Sin[x/2]/(Cos[x/2] + Sin[x/2]) +
 Tan[x/2]/2
```

Mathics [A]
time = 2.70, size = 33, normalized size = 1.14

$$\frac{-3+\mathrm{Tan}\left[\frac{x}{2}\right]^2 - 2\mathrm{Log}\left[1+\mathrm{Tan}\left[\frac{x}{2}\right]\right]\left(1+\mathrm{Tan}\left[\frac{x}{2}\right]\right)}{2+2\mathrm{Tan}\left[\frac{x}{2}\right]}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(Cos[x] + Sin[x] + 1)^2,x]')
```

```
[Out] (-3 + Tan[x / 2] ^ 2 - 2 Log[1 + Tan[x / 2]] (1 + Tan[x / 2])) / (2 (1 + Ta
n[x / 2]))
```

Maple [A]
time = 0.09, size = 27, normalized size = 0.93

method	result	size
default	$\frac{\tan\left(\frac{x}{2}\right)}{2} - \frac{1}{1+\tan\left(\frac{x}{2}\right)} - \ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	27
norman	$\frac{\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{3}{2}}{1+\tan\left(\frac{x}{2}\right)} - \ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	30
risch	$\frac{(-1+i)\left(e^{ix}+1+i\right)}{e^{2ix}+i+e^{ix}+ie^{ix}} + \ln\left(1+e^{ix}\right) - \ln\left(e^{ix}+i\right)$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)+sin(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*tan(1/2*x)-1/(1+tan(1/2*x))-ln(1+tan(1/2*x))
```

<u>Maxima [A]</u>

time = 0.27, size = 40, normalized size = 1.38

$$\frac{\sin\left(x\right)}{2\left(\cos\left(x\right)+1\right)} - \frac{1}{\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1} - \log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)+sin(x))^2,x, algorithm="maxima")
[Out] 1/2*sin(x)/(cos(x) + 1) - 1/(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/(cos(x)
+ 1) + 1)
```

<u>Fricas [A]</u>

time = 0.35, size = 46, normalized size = 1.59

$$\frac{\left(\cos\left(x\right)+\sin\left(x\right)+1\right)\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right) - \left(\cos\left(x\right)+\sin\left(x\right)+1\right)\log\left(\sin\left(x\right)+1\right) - 2\cos\left(x\right)+2\sin\left(x\right)}{2\left(\cos\left(x\right)+\sin\left(x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)+sin(x))^2,x, algorithm="fricas")
[Out] 1/2*((cos(x) + sin(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + sin(x) + 1)*lo
g(sin(x) + 1) - 2*cos(x) + 2*sin(x))/(cos(x) + sin(x) + 1)
```

<u>Sympy [B]</u> Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(22) = 44$

time = 0.37, size = 66, normalized size = 2.28

$$-\frac{2\log\left(\tan\left(\frac{x}{2}\right)+1\right)\tan\left(\frac{x}{2}\right)}{2\tan\left(\frac{x}{2}\right)+2} - \frac{2\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{2\tan\left(\frac{x}{2}\right)+2} + \frac{\tan^2\left(\frac{x}{2}\right)}{2\tan\left(\frac{x}{2}\right)+2} - \frac{3}{2\tan\left(\frac{x}{2}\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)+sin(x))**2,x)
[Out] -2*log(tan(x/2) + 1)*tan(x/2)/(2*tan(x/2) + 2) - 2*log(tan(x/2) + 1)/(2*tan
(x/2) + 2) + tan(x/2)**2/(2*tan(x/2) + 2) - 3/(2*tan(x/2) + 2)
```

<u>Giac [A]</u>

time = 0.00, size = 40, normalized size = 1.38

$$2\left(\frac{\tan\left(\frac{x}{2}\right)}{4} + \frac{\tan\left(\frac{x}{2}\right)}{2\left(\tan\left(\frac{x}{2}\right)+1\right)} - \frac{\ln\left|\tan\left(\frac{x}{2}\right)+1\right|}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)+sin(x))^2,x)
```

```
[Out] tan(1/2*x)/(tan(1/2*x) + 1) - log(abs(tan(1/2*x) + 1)) + 1/2*tan(1/2*x)
```

Mupad [B]
time $= 0.28$, size $= 26$, normalized size $= 0.90$

$$\frac{\tan\left(\frac{x}{2}\right)}{2} - \ln\left(\tan\left(\frac{x}{2}\right) + 1\right) - \frac{1}{\tan\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) + sin(x) + 1)^2,x)
```

```
[Out] tan(x/2)/2 - log(tan(x/2) + 1) - 1/(tan(x/2) + 1)
```

3.23 $\quad \int \sqrt{1 + \tanh(4x)}\; dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\dfrac{\sqrt{1 + \tanh(4x)}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

[Out] 1/4*arctanh(1/2*(1+tanh(4*x))^(1/2)*2^(1/2))*2^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\dfrac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3561, 212}

$$\frac{\tanh^{-1}\left(\dfrac{\sqrt{\tanh(4x) + 1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + Tanh[4*x]],x]

[Out] ArcTanh[Sqrt[1 + Tanh[4*x]]/Sqrt[2]]/(2*Sqrt[2])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

Rule 3561

Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \sqrt{1 + \tanh(4x)}\; dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{2 - x^2}\; dx, x, \sqrt{1 + \tanh(4x)}\right)$$

$$= \frac{\tanh^{-1}\left(\dfrac{\sqrt{1 + \tanh(4x)}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Mathematica [A]
time $= 0.01$, size $= 26$, normalized size $= 1.00$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{1+\tanh(4x)}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Tanh[4*x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[4*x]]/Sqrt[2]]/(2*Sqrt[2])
```

Mathics [F(-2)]
time $= 0.00$, size $= 0$, normalized size $= 0.00$

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[1 + Tanh[4*x]],x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time $= 0.06$, size $= 20$, normalized size $= 0.77$

method	result	size
derivativedivides	$\dfrac{\text{arctanh}\left(\dfrac{\sqrt{1+\tanh\left(4x\right)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}$	20
default	$\dfrac{\text{arctanh}\left(\dfrac{\sqrt{1+\tanh\left(4x\right)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tanh(4*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*arctanh(1/2*(1+tanh(4*x))^(1/2)*2^(1/2))*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(19) = 38$.
time $= 0.36$, size $= 43$, normalized size $= 1.65$

$$-\frac{1}{8}\sqrt{2}\,\log\left(-\frac{\sqrt{2}-\dfrac{\sqrt{2}}{\sqrt{e^{(-8x)}+1}}}{\sqrt{2}+\dfrac{\sqrt{2}}{\sqrt{e^{(-8x)}+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(4*x))^(1/2),x, algorithm="maxima")
[Out] -1/8*sqrt(2)*log(-(sqrt(2) - sqrt(2)/sqrt(e^(-8*x) + 1))/(sqrt(2) + sqrt(2)
/sqrt(e^(-8*x) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(19) = 38$.
time $= 0.34$, size $= 68$, normalized size $= 2.62$

$$\frac{1}{8}\sqrt{2}\log\left(-2\sqrt{2}\sqrt{\frac{\cosh\left(4\,x\right)}{\cosh\left(4\,x\right)-\sinh\left(4\,x\right)}}\left(\cosh\left(4\,x\right)+\sinh\left(4\,x\right)\right)-2\cosh\left(4\,x\right)^2-4\cosh\left(4\,x\right)\sinh\left(4\,x\right)-2\sinh\left(4\,x\right)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(4*x))^(1/2),x, algorithm="fricas")
[Out] 1/8*sqrt(2)*log(-2*sqrt(2)*sqrt(cosh(4*x)/(cosh(4*x) - sinh(4*x)))*(cosh(4*
x) + sinh(4*x)) - 2*cosh(4*x)^2 - 4*cosh(4*x)*sinh(4*x) - 2*sinh(4*x)^2 - 1
)
```

Sympy [F]
time $= 0.00$, size $= 0$, normalized size $= 0.00$

$$\int \sqrt{\tanh\left(4x\right)+1}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(4*x))**(1/2),x)
[Out] Integral(sqrt(tanh(4*x) + 1), x)
```

Giac [A]
time $= 0.01$, size $= 26$, normalized size $= 1.00$

$$-\frac{1}{4}\sqrt{2}\ln\left(\sqrt{(e^x)^8+1}-(e^x)^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(4*x))^(1/2),x)
[Out] -1/4*sqrt(2)*log(sqrt(e^(8*x) + 1) - e^(4*x))
```

Mupad [B]
time $= 0.16$, size $= 19$, normalized size $= 0.73$

$$\frac{\sqrt{2}\ \text{atanh}\left(\frac{\sqrt{2}\ \sqrt{\tanh\left(4\,x\right)+1}}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(4*x) + 1)^(1/2),x)
[Out] (2^(1/2)*atanh((2^(1/2)*(tanh(4*x) + 1)^(1/2))/2))/4
```

3.24 $\quad \int \dfrac{\tanh(x)}{\sqrt{e^x + e^{2x}}}\, dx$

Optimal. Leaf size=110

$$2e^{-x}\sqrt{e^x + e^{2x}} - \frac{\tan^{-1}\left(\frac{i-(1-2i)e^x}{2\sqrt{1+i}\,\sqrt{e^x+e^{2x}}}\right)}{\sqrt{1+i}} + \frac{\tan^{-1}\left(\frac{i+(1+2i)e^x}{2\sqrt{1-i}\,\sqrt{e^x+e^{2x}}}\right)}{\sqrt{1-i}}$$

[Out] arctan(1/2*(I+(1+2*I)*exp(x))/(1-I)^(1/2)/(exp(x)+exp(2*x))^(1/2))/(1-I)^(1
/2)-arctan(1/2*(I+(-1+2*I)*exp(x))/(1+I)^(1/2)/(exp(x)+exp(2*x))^(1/2))/(1+
I)^(1/2)+2*(exp(x)+exp(2*x))^(1/2)/exp(x)

Rubi [A]

time = 0.39, antiderivative size = 147, normalized size of antiderivative = 1.34, number of steps used = 11, number of rules used = 7, integrand size = 16, $\dfrac{\text{number of rules}}{\text{integrand size}} = 0.438$,

Rules used = {2320, 6856, 1600, 6857, 96, 95, 214}

$$\frac{2\left(e^x + 1\right)}{\sqrt{e^x + e^{2x}}} - \frac{(1-i)^{3/2}\sqrt{e^x}\,\sqrt{e^x+1}\,\tanh^{-1}\left(\frac{\sqrt{1-i}\,\sqrt{e^x}}{\sqrt{e^x+1}}\right)}{\sqrt{e^x + e^{2x}}} - \frac{(1+i)^{3/2}\sqrt{e^x}\,\sqrt{e^x+1}\,\tanh^{-1}\left(\frac{\sqrt{1+i}\,\sqrt{e^x}}{\sqrt{e^x+1}}\right)}{\sqrt{e^x + e^{2x}}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]/Sqrt[E^x + E^(2*x)],x]

[Out] (2*(1 + E^x))/Sqrt[E^x + E^(2*x)] - ((1 - I)^(3/2)*Sqrt[E^x]*Sqrt[1 + E^x]*
ArcTanh[(Sqrt[1 - I]*Sqrt[E^x])/Sqrt[1 + E^x]])/Sqrt[E^x + E^(2*x)] - ((1 +
 I)^(3/2)*Sqrt[E^x]*Sqrt[1 + E^x]*ArcTanh[(Sqrt[1 + I]*Sqrt[E^x])/Sqrt[1 +
E^x]])/Sqrt[E^x + E^(2*x)]

Rule 95

Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 96

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] || !SumSimplerQ[p, 1]) && NeQ[m, -1]

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6856

```
Int[(u_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(m_), x_Symbol] :> With[{v
= (a*x^r + b*x^s)^FracPart[m]/(x^(r*FracPart[m])*(a + b*x^(s - r))^FracPart
[m])}, Dist[v, Int[u*x^(m*r)*(a + b*x^(s - r))^m, x], x] /; NeQ[Simplify[v]
, 1]] /; FreeQ[{a, b, m, r, s}, x] &&  !IntegerQ[m] && PosQ[s - r]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\tanh(x)}{\sqrt{e^x + e^{2x}}}\, dx = \text{Subst}\left(\int \frac{-1+x^2}{x\left(1+x^2\right)\sqrt{x+x^2}}\, dx, x, e^x\right)$$

$$= \frac{\left(\sqrt{e^x}\,\sqrt{1+e^x}\right)\text{Subst}\left(\int \frac{-1+x^2}{x^{3/2}\sqrt{1+x}\,(1+x^2)}\, dx, x, e^x\right)}{\sqrt{e^x + e^{2x}}}$$

$$= \frac{\left(\sqrt{e^x}\,\sqrt{1+e^x}\right)\text{Subst}\left(\int \frac{(-1+x)\sqrt{1+x}}{x^{3/2}(1+x^2)}\, dx, x, e^x\right)}{\sqrt{e^x + e^{2x}}}$$

$$= \frac{\left(\sqrt{e^x}\,\sqrt{1+e^x}\right)\text{Subst}\left(\int \left(-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{1+x}}{(i-x)x^{3/2}} + \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{1+x}}{x^{3/2}(i+x)}\right)\, dx, x, e^x\right)}{\sqrt{e^x + e^{2x}}}$$

$$= -\frac{\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{e^x}\,\sqrt{1+e^x}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{(i-x)x^{3/2}}\, dx, x, e^x\right)}{\sqrt{e^x + e^{2x}}} + \frac{\left(\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{e^x}\,\sqrt{1+e^x}\right)\text{S}}{\sqrt{e^x}}$$

$$= \frac{2\left(1+e^x\right)}{\sqrt{e^x + e^{2x}}} - \frac{\left(\sqrt{e^x}\,\sqrt{1+e^x}\right)\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\,\sqrt{1+x}}\, dx, x, e^x\right)}{\sqrt{e^x + e^{2x}}} + \frac{\left(\sqrt{e^x}\,\sqrt{1+e}\right)}{}$$

$$= \frac{2\left(1+e^x\right)}{\sqrt{e^x + e^{2x}}} - \frac{\left(2\sqrt{e^x}\,\sqrt{1+e^x}\right)\text{Subst}\left(\int \frac{1}{i-(1+i)x^2}\, dx, x, \frac{\sqrt{e^x}}{\sqrt{1+e^x}}\right)}{\sqrt{e^x + e^{2x}}} + \frac{\left(2\sqrt{e^x}\,\sqrt{1+}\right)}{}$$

$$= \frac{2\left(1+e^x\right)}{\sqrt{e^x + e^{2x}}} - \frac{(1-i)^{3/2}\sqrt{e^x}\,\sqrt{1+e^x}\,\tanh^{-1}\left(\frac{\sqrt{1-i}\,\sqrt{e^x}}{\sqrt{1+e^x}}\right)}{\sqrt{e^x + e^{2x}}} - \frac{(1+i)^{3/2}\sqrt{e^x}\,\sqrt{1+}}{}$$

Mathematica [A]
time = 0.08, size = 121, normalized size = 1.10

$$\frac{2 + 2e^x - (1-i)^{3/2}e^{x/2}\sqrt{1+e^x}\,\tanh^{-1}\left(\frac{\sqrt{1-i}\,e^{x/2}}{\sqrt{1+e^x}}\right) - (1+i)^{3/2}e^{x/2}\sqrt{1+e^x}\,\tanh^{-1}\left(\frac{\sqrt{1+i}\,e^{x/2}}{\sqrt{1+e^x}}\right)}{\sqrt{e^x\left(1+e^x\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[E^x + E^(2*x)],x]
```

```
[Out] (2 + 2*E^x - (1 - I)^(3/2)*E^(x/2)*Sqrt[1 + E^x]*ArcTanh[(Sqrt[1 - I]*E^(x/
2))/Sqrt[1 + E^x]] - (1 + I)^(3/2)*E^(x/2)*Sqrt[1 + E^x]*ArcTanh[(Sqrt[1 +
I]*E^(x/2))/Sqrt[1 + E^x]])/Sqrt[E^x*(1 + E^x)]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Tanh[x]/Sqrt[Exp[2*x] + Exp[x]],x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $\overline{2(81) = 162}$.
time = 0.10, size = 364, normalized size = 3.31

method	result
default	$-\dfrac{\sqrt{2}\left(\sqrt{2}\ \sqrt{2+2\sqrt{2}}\ \sqrt{\tanh\left(\frac{x}{2}\right)+1}\ \ln\left(\tanh(\frac{x}{2})+1-\sqrt{\tanh\left(\frac{x}{2}\right)+1}\ \sqrt{2+2\sqrt{2}}+\sqrt{2}\right)\sqrt{-2-}\right.}{\ }$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(exp(x)+exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*2^(1/2)*(2^(1/2)*(2+2*2^(1/2))^(1/2)*(tanh(1/2*x)+1)^(1/2)*ln(tanh(1/2
*x)+1-(tanh(1/2*x)+1)^(1/2)*(2+2*2^(1/2))^(1/2)+2^(1/2))*(-2+2*2^(1/2))^(1/
2)-2^(1/2)*(2+2*2^(1/2))^(1/2)*(tanh(1/2*x)+1)^(1/2)*ln(tanh(1/2*x)+1+(tanh
(1/2*x)+1)^(1/2)*(2+2*2^(1/2))^(1/2)+2^(1/2))*(-2+2*2^(1/2))^(1/2)-(2+2*2^(
1/2))^(1/2)*(tanh(1/2*x)+1)^(1/2)*ln(tanh(1/2*x)+1-(tanh(1/2*x)+1)^(1/2)*(2
+2*2^(1/2))^(1/2)+2^(1/2))*(-2+2*2^(1/2))^(1/2)+(2+2*2^(1/2))^(1/2)*(tanh(1
/2*x)+1)^(1/2)*ln(tanh(1/2*x)+1+(tanh(1/2*x)+1)^(1/2)*(2+2*2^(1/2))^(1/2)+2
^(1/2))*(-2+2*2^(1/2))^(1/2)-4*(tanh(1/2*x)+1)^(1/2)*arctan((-2*(tanh(1/2*x
)+1)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+4*(tanh(1/2*x)+1)^(1/
2)*arctan((2*(tanh(1/2*x)+1)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2
))+8*(-2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)/(tanh(1/2*x)-1)/((tanh(1/2*
x)+1)/(tanh(1/2*x)-1)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

<div align="center">Failed to integrate</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(exp(x)+exp(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/sqrt(e^(2*x) + e^x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(67) = 134$.
time = 0.36, size = 859, normalized size = 7.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(exp(x)+exp(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(8^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*e^x*log(2*(8^(1/4)*(sqrt(2)
 - 1)*e^x - 8^(1/4))*sqrt(2*sqrt(2) + 4) - 2*(8^(1/4)*sqrt(2*sqrt(2) + 4)*(
sqrt(2) - 1) + 4*e^x)*sqrt(e^(2*x) + e^x) + 4*sqrt(2) + 8*e^(2*x) + 4*e^x +
 4) - 8^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*e^x*log(-2*(8^(1/4)*(sqrt(2
) - 1)*e^x - 8^(1/4))*sqrt(2*sqrt(2) + 4) + 2*(8^(1/4)*sqrt(2*sqrt(2) + 4)*
(sqrt(2) - 1) - 4*e^x)*sqrt(e^(2*x) + e^x) + 4*sqrt(2) + 8*e^(2*x) + 4*e^x
+ 4) + 4*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(1/7*(sqrt(2)*(5*sqrt(2)
 + 6) + 8*sqrt(2) + 4)*e^x + 1/112*(8*sqrt(2)*(5*sqrt(2) + 6) + (8^(3/4)*(5
*sqrt(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1))*sqrt(2*sqrt(2) + 4) + 64*sqrt(2)
 + 32)*sqrt(2*(8^(1/4)*(sqrt(2) - 1)*e^x - 8^(1/4))*sqrt(2*sqrt(2) + 4) - 2
*(8^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1) + 4*e^x)*sqrt(e^(2*x) + e^x) +
4*sqrt(2) + 8*e^(2*x) + 4*e^x + 4) + 1/56*((8^(3/4)*(5*sqrt(2) + 6) + 8*8^(
1/4)*(2*sqrt(2) + 1))*e^x + 8^(3/4)*(sqrt(2) + 4) - 8*8^(1/4)*(sqrt(2) - 3)
)*sqrt(2*sqrt(2) + 4) + 1/7*sqrt(2)*(sqrt(2) + 4) - 1/56*(8*sqrt(2)*(5*sqrt
(2) + 6) + (8^(3/4)*(5*sqrt(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1))*sqrt(2*sqr
t(2) + 4) + 64*sqrt(2) + 32)*sqrt(e^(2*x) + e^x) + 3/7*sqrt(2) + 5/7)*e^x +
 4*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(-1/7*(sqrt(2)*(5*sqrt(2) + 6)
 + 8*sqrt(2) + 4)*e^x - 1/112*(8*sqrt(2)*(5*sqrt(2) + 6) - (8^(3/4)*(5*sqrt
(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1))*sqrt(2*sqrt(2) + 4) + 64*sqrt(2) + 32
)*sqrt(-2*(8^(1/4)*(sqrt(2) - 1)*e^x - 8^(1/4))*sqrt(2*sqrt(2) + 4) + 2*(8^
(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1) - 4*e^x)*sqrt(e^(2*x) + e^x) + 4*sq
rt(2) + 8*e^(2*x) + 4*e^x + 4) + 1/56*((8^(3/4)*(5*sqrt(2) + 6) + 8*8^(1/4)
*(2*sqrt(2) + 1))*e^x + 8^(3/4)*(sqrt(2) + 4) - 8*8^(1/4)*(sqrt(2) - 3))*sq
rt(2*sqrt(2) + 4) - 1/7*sqrt(2)*(sqrt(2) + 4) + 1/56*(8*sqrt(2)*(5*sqrt(2)
+ 6) - (8^(3/4)*(5*sqrt(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1))*sqrt(2*sqrt(2)
 + 4) + 64*sqrt(2) + 32)*sqrt(e^(2*x) + e^x) - 3/7*sqrt(2) - 5/7)*e^x - 16*
sqrt(e^(2*x) + e^x) - 16*e^x)*e^(-x)
```

Sympy **[F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh{(x)}}{\sqrt{(e^x + 1)\, e^x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(exp(x)+exp(2*x))**(1/2),x)
```

```
[Out] Integral(tanh(x)/sqrt((exp(x) + 1)*exp(x)), x)
```

Giac **[B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(67) = 134$.

time = 0.09, size = 489, normalized size = 4.45

$$\left(\ldots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(exp(x)+exp(2*x))^(1/2),x)
```

```
[Out] -(1/4*I + 1/4)*sqrt(2*sqrt(2) + 2)*(I/(sqrt(2) + 1) + 1)*log(4*sqrt(2)*(sqr
t(e^(2*x) + e^x) - e^x) + 2*sqrt(2)*sqrt(2*sqrt(2) - 2) - 4*I*sqrt(2) - (2*
I + 2)*sqrt(2*sqrt(2) - 2) - 4*sqrt(e^(2*x) + e^x) + 4*e^x + 4*I) + (1/4*I
+ 1/4)*sqrt(2*sqrt(2) + 2)*(I/(sqrt(2) + 1) + 1)*log(4*sqrt(2)*(sqrt(e^(2*x
) + e^x) - e^x) - 2*sqrt(2)*sqrt(2*sqrt(2) - 2) - 4*I*sqrt(2) + (2*I + 2)*s
qrt(2*sqrt(2) - 2) - 4*sqrt(e^(2*x) + e^x) + 4*e^x + 4*I) - (1/4*I + 1/4)*s
qrt(2*sqrt(2) - 2)*(I/(sqrt(2) - 1) + 1)*log(-20*sqrt(2)*(-(I + 2)*sqrt(e^(
2*x) + e^x) + (I + 2)*e^x) + 10*sqrt(2)*sqrt(10*sqrt(2) - 14) + (40*I - 20)
*sqrt(2) - (2*I + 14)*sqrt(10*sqrt(2) - 14) - (28*I + 56)*sqrt(e^(2*x) + e^
x) + (28*I + 56)*e^x - 56*I + 28) + (1/4*I + 1/4)*sqrt(2*sqrt(2) - 2)*(I/(s
qrt(2) - 1) + 1)*log(-20*sqrt(2)*(-(I + 2)*sqrt(e^(2*x) + e^x) + (I + 2)*e^
x) - 10*sqrt(2)*sqrt(10*sqrt(2) - 14) + (40*I - 20)*sqrt(2) + (2*I + 14)*sq
rt(10*sqrt(2) - 14) - (28*I + 56)*sqrt(e^(2*x) + e^x) + (28*I + 56)*e^x - 5
6*I + 28) + 2/(sqrt(e^(2*x) + e^x) - e^x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)}{\sqrt{e^{2x} + e^x}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(exp(2*x) + exp(x))^(1/2),x)
```

```
[Out] int(tanh(x)/(exp(2*x) + exp(x))^(1/2), x)
```

3.25 $\int \sqrt{\operatorname{sech}(x)\sinh(2x)}\ dx$

Optimal. Leaf size=40

$$\frac{2i\sqrt{2}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\,\middle|\,2\right)\sqrt{\sinh(x)}}{\sqrt{i\sinh(x)}}$$

[Out] 2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))*2^(1/2)*sinh(x)^(1/2)/(I*sinh(x))^(1/2)

Rubi [A]

time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$,

Rules used = {4483, 4485, 4306, 4394, 2719}

$$\frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\,\middle|\,2\right)\sqrt{\sinh(2x)\operatorname{sech}(x)}}{\sqrt{i\sinh(x)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sech[x]*Sinh[2*x]],x]

[Out] ((2*I)*EllipticE[Pi/4 - (I/2)*x, 2]*Sqrt[Sech[x]*Sinh[2*x]])/Sqrt[I*Sinh[x]]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 4306

Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a + b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rule 4394

Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/((e*Cos[a + b*x])^p*Sin[a + b*x]^p), Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p]

Rule 4483

Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = ActivateTrig[v]}, Dist[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p]), Int[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] && !IntegerQ[p] && !InertTrigFreeQ[

v]

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \sqrt{\text{sech}(x)\sinh(2x)}\ dx = \frac{\sqrt{\text{sech}(x)\sinh(2x)}\int \sqrt{i\,\text{sech}(x)\sinh(2x)}\ dx}{\sqrt{i\,\text{sech}(x)\sinh(2x)}}$$

$$= \frac{\sqrt{\text{sech}(x)\sinh(2x)}\int \sqrt{\text{sech}(x)}\sqrt{i\sinh(2x)}\ dx}{\sqrt{\text{sech}(x)}\sqrt{i\sinh(2x)}}$$

$$= \frac{\left(\sqrt{\cosh(x)}\sqrt{\text{sech}(x)\sinh(2x)}\right)\int \frac{\sqrt{i\sinh(2x)}}{\sqrt{\cosh(x)}}\ dx}{\sqrt{i\sinh(2x)}}$$

$$= \frac{\sqrt{\text{sech}(x)\sinh(2x)}\int \sqrt{i\sinh(x)}\ dx}{\sqrt{i\sinh(x)}}$$

$$= \frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\Big|\,2\right)\sqrt{\text{sech}(x)\sinh(2x)}}{\sqrt{i\sinh(x)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 32.14, size = 54, normalized size = 1.35

$$-\frac{2}{3}\left(-3+\ {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\tanh^2\left(\frac{x}{2}\right)\right)\sqrt{\text{sech}^2\left(\frac{x}{2}\right)}\right)\sqrt{\text{sech}(x)\sinh(2x)}\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sech[x]*Sinh[2*x]],x]

[Out] (-2*(-3 + Hypergeometric2F1[1/2, 3/4, 7/4, Tanh[x/2]^2]*Sqrt[Sech[x/2]^2])*
Sqrt[Sech[x]*Sinh[2*x]]*Tanh[x/2])/3

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[Sinh[2*x]/Cosh[x]],x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.15, size = 75, normalized size = 1.88

method	result
default	$\dfrac{2\sqrt{-i\left(\sinh\left(x\right)+i\right)}\,\sqrt{-i\left(-\sinh\left(x\right)+i\right)}\,\sqrt{i\sinh\left(x\right)}\,\left(2\operatorname{EllipticE}\left(\sqrt{1-i\sinh\left(x\right)},\frac{\sqrt{2}}{2}\right)-\operatorname{Elliptic}\right.}{\cosh(x)\sqrt{\sinh\left(x\right)}}$
risch	$2\sqrt{\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)}+\dfrac{\left(-\dfrac{4\left(\mathrm{e}^{2x}-1\right)}{\sqrt{\mathrm{e}^{x}\left(\mathrm{e}^{2x}-1\right)}}+\dfrac{2\sqrt{1+\mathrm{e}^{x}}\,\sqrt{2-2\,\mathrm{e}^{x}}\,\sqrt{-\mathrm{e}^{x}}\,\left(-2\operatorname{EllipticE}\left(\sqrt{1+\mathrm{e}^{x}},\frac{\sqrt{2}}{2}\right)+\mathrm{E}\right.}{\sqrt{\mathrm{e}^{3x}-\mathrm{e}^{x}}}\right)}{\mathrm{e}^{2x}-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(2*x)/cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-I*(sinh(x)+I))^(1/2)*(-I*(-sinh(x)+I))^(1/2)*(I*sinh(x))^(1/2)*(2*Ellip
ticE((1-I*sinh(x))^(1/2),1/2*2^(1/2))-EllipticF((1-I*sinh(x))^(1/2),1/2*2^(
1/2)))/cosh(x)/sinh(x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Failed to integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(2*x)/cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sinh(2*x)/cosh(x)), x)
```

Fricas [F]
time = 0.37, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(2*x)/cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sinh(2*x)/cosh(x)), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{\sinh\left(2x\right)}{\cosh\left(x\right)}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sinh(2*x)/cosh(x))**(1/2),x)

[Out] Integral(sqrt(sinh(2*x)/cosh(x)), x)

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sinh(2*x)/cosh(x))^(1/2),x)

[Out] Could not integrate

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{\sinh\left(2\,x\right)}{\cosh\left(x\right)}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sinh(2*x)/cosh(x))^(1/2),x)

[Out] int((sinh(2*x)/cosh(x))^(1/2), x)

3.26 $\quad \int \log\left(x^2 + \sqrt{1 - x^2}\,\right)\,dx$

Optimal. Leaf size=185

$$-2x-\sin^{-1}(x)+\sqrt{\frac{1}{2}\left(1+\sqrt{5}\,\right)}\,\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)+\sqrt{\frac{1}{2}\left(1+\sqrt{5}\,\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\,\right)}\,x}{\sqrt{1-x^2}}\right)+$$

[Out] -2*x-arcsin(x)+x*ln(x^2+(-x^2+1)^(1/2))+1/2*arctanh(x*2^(1/2)/(5^(1/2)-1)^(
1/2))*(-2+2*5^(1/2))^(1/2)-1/2*arctanh(1/2*x*(-2+2*5^(1/2))^(1/2)/(-x^2+1)^
(1/2))*(-2+2*5^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)/(5^(1/2)+1)^(1/2))*(2+2*5^
(1/2))^(1/2)+1/2*arctan(1/2*x*(2+2*5^(1/2))^(1/2)/(-x^2+1)^(1/2))*(2+2*5^(1
/2))^(1/2)

Rubi [A]

time = 0.70, antiderivative size = 349, normalized size of antiderivative = 1.89, number of steps used = 31, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$,
Rules used = {2628, 6874, 1307, 222, 1706, 385, 213, 209, 1180, 1144, 1188, 399}

$$x\log\left(x^2+\sqrt{1-x^2}\right)+2\sqrt{\tfrac{1}{5}\left(2+\sqrt{5}\right)}\,\tan^{-1}\left(\tfrac{\sqrt{\tfrac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right)-\sqrt{\tfrac{1}{10}\left(1+\sqrt{5}\right)}\,\tan^{-1}\left(\tfrac{\sqrt{\tfrac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right)-\sqrt{\tfrac{1}{10}\left(\sqrt{5}-1\right)}\,\tanh^{-1}\left(\tfrac{\sqrt{\tfrac{1}{2}\left(\sqrt{5}-1\right)}\,x}{\sqrt{1-x^2}}\right)-2\sqrt{\tfrac{1}{5}\left(\sqrt{5}-2\right)}\,\tanh^{-1}\left(\tfrac{\sqrt{\tfrac{1}{2}\left(\sqrt{5}-1\right)}\,x}{\sqrt{1-x^2}}\right)-2x-\sin^{-1}(x)+2\sqrt{\tfrac{1}{5}\left(2+\sqrt{5}\right)}\,\tan^{-1}\left(\sqrt{\tfrac{2}{1+\sqrt{5}}}\,x\right)-\sqrt{\tfrac{1}{10}\left(1+\sqrt{5}\right)}\,\tan^{-1}\left(\sqrt{\tfrac{2}{1+\sqrt{5}}}\,x\right)+\sqrt{\tfrac{1}{10}\left(\sqrt{5}-1\right)}\,\tanh^{-1}\left(\sqrt{\tfrac{2}{\sqrt{5}-1}}\,x\right)+2\sqrt{\tfrac{1}{5}\left(\sqrt{5}-2\right)}\,\tanh^{-1}\left(\sqrt{\tfrac{2}{\sqrt{5}-1}}\,x\right)$$

Antiderivative was successfully verified.

[In] Int[Log[x^2 + Sqrt[1 - x^2]],x]

[Out] -2*x - ArcSin[x] - Sqrt[(1 + Sqrt[5])/10]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] +
 2*Sqrt[(2 + Sqrt[5])/5]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] - Sqrt[(1 + Sqrt[5
])/10]*ArcTan[(Sqrt[(1 + Sqrt[5])/2]*x)/Sqrt[1 - x^2]] + 2*Sqrt[(2 + Sqrt[5
])/5]*ArcTan[(Sqrt[(1 + Sqrt[5])/2]*x)/Sqrt[1 - x^2]] + 2*Sqrt[(-2 + Sqrt[5
])/5]*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x] + Sqrt[(-1 + Sqrt[5])/10]*ArcTanh[S
qrt[2/(-1 + Sqrt[5])]*x] - 2*Sqrt[(-2 + Sqrt[5])/5]*ArcTanh[(Sqrt[(-1 + Sqr
t[5])/2]*x)/Sqrt[1 - x^2]] - Sqrt[(-1 + Sqrt[5])/10]*ArcTanh[(Sqrt[(-1 + Sq
rt[5])/2]*x)/Sqrt[1 - x^2]] + x*Log[x^2 + Sqrt[1 - x^2]]

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[2*(c/r), Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1307

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[e*(f^2/c), Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[a
```

```
*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \log\left(x^2 + \sqrt{1-x^2}\right)\, dx = x\log\left(x^2 + \sqrt{1-x^2}\right) - \int \frac{x^2\left(2 - \frac{1}{\sqrt{1-x^2}}\right)}{x^2 + \sqrt{1-x^2}}\, dx$$

$$= x\log\left(x^2 + \sqrt{1-x^2}\right) - \int\left(\frac{2x^2}{x^2 + \sqrt{1-x^2}} - \frac{x^2}{1 - x^2 + x^2\sqrt{1-x^2}}\right) dx$$

$$= x\log\left(x^2 + \sqrt{1-x^2}\right) - 2\int \frac{x^2}{x^2 + \sqrt{1-x^2}}\, dx + \int \frac{x^2}{1 - x^2 + x^2\sqrt{1-x^2}}\, dx$$

$$= x\log\left(x^2 + \sqrt{1-x^2}\right) - 2\int\left(1 - \frac{x^2\sqrt{1-x^2}}{-1 + x^2 + x^4} + \frac{1 - x^2}{-1 + x^2 + x^4}\right) dx + \int\Big($$

$$= -2x + x\log\left(x^2 + \sqrt{1-x^2}\right) + 2\int \frac{x^2\sqrt{1-x^2}}{-1 + x^2 + x^4}\, dx - 2\int \frac{1 - x^2}{-1 + x^2 + x^4}\, dx$$

$$= -2x + \sin^{-1}(x) + x\log\left(x^2 + \sqrt{1-x^2}\right) - 2\int \frac{1}{\sqrt{1-x^2}}\, dx - 2\int \frac{1}{\sqrt{1-x^2}}$$

$$= -2x - \sin^{-1}(x) - \sqrt{\frac{1}{10}\left(1 + \sqrt{5}\right)}\, \tan^{-1}\left(\sqrt{\frac{2}{1 + \sqrt{5}}}\, x\right) + 2\sqrt{\frac{1}{5}\left(2 + \sqrt{5}\right)}$$

$$= -2x - \sin^{-1}(x) - \sqrt{\frac{1}{10}\left(1 + \sqrt{5}\right)}\, \tan^{-1}\left(\sqrt{\frac{2}{1 + \sqrt{5}}}\, x\right) + 2\sqrt{\frac{1}{5}\left(2 + \sqrt{5}\right)}$$

$$= -2x - \sin^{-1}(x) - \sqrt{\frac{1}{10}\left(1 + \sqrt{5}\right)}\, \tan^{-1}\left(\sqrt{\frac{2}{1 + \sqrt{5}}}\, x\right) + 2\sqrt{\frac{1}{5}\left(2 + \sqrt{5}\right)}$$

$$= -2x - \sin^{-1}(x) - \sqrt{\frac{1}{10}\left(1 + \sqrt{5}\right)}\, \tan^{-1}\left(\sqrt{\frac{2}{1 + \sqrt{5}}}\, x\right) + 2\sqrt{\frac{1}{5}\left(2 + \sqrt{5}\right)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.34, size = 910, normalized size = 4.92

Warning: Unable to verify antiderivative.

[In] Integrate[Log[x^2 + Sqrt[1 - x^2]],x]

[Out] (-8*Sqrt[5]*x - 4*Sqrt[5]*ArcSin[x] + 5*Sqrt[2*(-1 + Sqrt[5])]*ArcTan[Sqrt[
2/(1 + Sqrt[5])]*x] + Sqrt[10*(-1 + Sqrt[5])]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*

```
x] - (-5 + Sqrt[5])*Sqrt[2*(1 + Sqrt[5])]*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x]
 - 5*Sqrt[2 + Sqrt[5]]*Log[-Sqrt[(-1 + Sqrt[5])/2] + x] + 3*Sqrt[5*(2 + Sqr
t[5])]*Log[-Sqrt[(-1 + Sqrt[5])/2] + x] + 5*Sqrt[2 + Sqrt[5]]*Log[Sqrt[(-1
+ Sqrt[5])/2] + x] - 3*Sqrt[5*(2 + Sqrt[5])]*Log[Sqrt[(-1 + Sqrt[5])/2] + x
] - (5*I)*Sqrt[-2 + Sqrt[5]]*Log[(-I)*Sqrt[(1 + Sqrt[5])/2] + x] - (3*I)*Sq
rt[5*(-2 + Sqrt[5])]*Log[(-I)*Sqrt[(1 + Sqrt[5])/2] + x] + (5*I)*Sqrt[-2 +
Sqrt[5]]*Log[I*Sqrt[(1 + Sqrt[5])/2] + x] + (3*I)*Sqrt[5*(-2 + Sqrt[5])]*Lo
g[I*Sqrt[(1 + Sqrt[5])/2] + x] + 4*Sqrt[5]*x*Log[x^2 + Sqrt[1 - x^2]] + (5*
I)*Sqrt[-2 + Sqrt[5]]*Log[2 - I*Sqrt[2*(1 + Sqrt[5])]*x + Sqrt[2*(3 + Sqrt[
5])]*Sqrt[1 - x^2]] + (3*I)*Sqrt[5*(-2 + Sqrt[5])]*Log[2 - I*Sqrt[2*(1 + Sq
rt[5])]*x + Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 - x^2]] - (5*I)*Sqrt[-2 + Sqrt[5]]
*Log[2 + I*Sqrt[2*(1 + Sqrt[5])]*x + Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 - x^2]] -
 (3*I)*Sqrt[5*(-2 + Sqrt[5])]*Log[2 + I*Sqrt[2*(1 + Sqrt[5])]*x + Sqrt[2*(3
 + Sqrt[5])]*Sqrt[1 - x^2]] + 5*Sqrt[2 + Sqrt[5]]*Log[2 - Sqrt[2*(-1 + Sqrt
[5])]*x + Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)]] - 3*Sqrt[5*(2 + Sqrt[5])
]*Log[2 - Sqrt[2*(-1 + Sqrt[5])]*x + Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)
]] - 5*Sqrt[2 + Sqrt[5]]*Log[2 + Sqrt[2*(-1 + Sqrt[5])]*x + Sqrt[2]*Sqrt[(-
3 + Sqrt[5])*(-1 + x^2)]] + 3*Sqrt[5*(2 + Sqrt[5])]*Log[2 + Sqrt[2*(-1 + Sq
rt[5])]*x + Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)]])/(4*Sqrt[5])
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Log[x^2 + Sqrt[1 - x^2]],x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(138) = 276$.
time = 0.10, size = 392, normalized size = 2.12

method	result
default	$x\ln\left(x^2 + \sqrt{-x^2+1}\right) + \dfrac{\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}} + \dfrac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}} - \dfrac{\arctanh\left(\frac{2x}{\sqrt{-2+2}}\right)}{\sqrt{-2+2\sqrt{}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(x^2+(-x^2+1)^(1/2))+1/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/
2))+5^(1/2)/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))-1/(-2+2*5^(
1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))+5^(1/2)/(-2+2*5^(1/2))^(1/2)*
```

```
arctanh(2*x/(-2+2*5^(1/2))^(1/2))-2*x-1/2*5^(1/2)/(2+5^(1/2))^(1/2)*arctanh
(((-x^2+1)^(1/2)-1)/x/(2+5^(1/2))^(1/2))-1/2/(2+5^(1/2))^(1/2)*arctanh(((-x
^2+1)^(1/2)-1)/x/(2+5^(1/2))^(1/2))-1/2*5^(1/2)/(-2+5^(1/2))^(1/2)*arctan((
(-x^2+1)^(1/2)-1)/x/(-2+5^(1/2))^(1/2))+1/2/(-2+5^(1/2))^(1/2)*arctan(((-x^
2+1)^(1/2)-1)/x/(-2+5^(1/2))^(1/2))-3/2/(2+5^(1/2))^(1/2)*arctan(((-x^2+1)^
(1/2)-1)/x/(2+5^(1/2))^(1/2))-1/2*5^(1/2)/(2+5^(1/2))^(1/2)*arctan(((-x^2+1
)^(1/2)-1)/x/(2+5^(1/2))^(1/2))+3/2/(-2+5^(1/2))^(1/2)*arctanh(((-x^2+1)^(1
/2)-1)/x/(-2+5^(1/2))^(1/2))-1/2*5^(1/2)/(-2+5^(1/2))^(1/2)*arctanh(((-x^2+
1)^(1/2)-1)/x/(-2+5^(1/2))^(1/2))-arcsin(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

[Out] x*log(x^2 + sqrt(x + 1)*sqrt(-x + 1)) - x - integrate((x^4 - 2*x^2)/(x^4 -
x^2 + (x^2 - 1)*e^(1/2*log(x + 1) + 1/2*log(-x + 1))), x) + 1/2*log(x + 1)
- 1/2*log(-x + 1)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs.
2(138) = 276.
time = 0.36, size = 452, normalized size = 2.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

[Out] -sqrt(2)*sqrt(sqrt(5) + 1)*arctan(1/8*sqrt(4*x^2 + 2*sqrt(5) + 2)*(sqrt(5)*
sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 1) - 1/4*(sqrt(5)*sqrt(2)*x - sqrt(2)*x)*
sqrt(sqrt(5) + 1)) - sqrt(2)*sqrt(sqrt(5) + 1)*arctan(1/8*(sqrt(2)*(sqrt(-x
^2 + 1)*(sqrt(5)*sqrt(2) - sqrt(2)) + sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(
5) + 1)*sqrt((x^4 - 4*x^2 - sqrt(5)*(x^4 - 2*x^2) - 2*(sqrt(5)*x^2 - x^2 +
2)*sqrt(-x^2 + 1) + 4)/x^4) + 2*sqrt(-x^2 + 1)*(sqrt(5)*sqrt(2) - sqrt(2))*
sqrt(sqrt(5) + 1))/x) + x*log(x^2 + sqrt(-x^2 + 1)) + 1/4*sqrt(2)*sqrt(sqrt
(5) - 1)*log(2*x + sqrt(2)*sqrt(sqrt(5) - 1)) - 1/4*sqrt(2)*sqrt(sqrt(5) -
1)*log(2*x - sqrt(2)*sqrt(sqrt(5) - 1)) + 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*log
(-(2*x^2 + (sqrt(2)*sqrt(-x^2 + 1)*x - sqrt(2)*x)*sqrt(sqrt(5) - 1) + 2*sqr
t(-x^2 + 1) - 2)/x^2) - 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*log(-(2*x^2 - (sqrt(2
)*sqrt(-x^2 + 1)*x - sqrt(2)*x)*sqrt(sqrt(5) - 1) + 2*sqrt(-x^2 + 1) - 2)/x
^2) - 2*x + 2*arctan((sqrt(-x^2 + 1) - 1)/x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(x^2 + \sqrt{1 - x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(x**2+(-x**2+1)**(1/2)),x)

[Out] Integral(log(x**2 + sqrt(1 - x**2)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(138) = 276.

time = 0.03, size = 404, normalized size = 2.18

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x^2+(-x^2+1)^(1/2)),x)

[Out] x*log(x^2 + sqrt(-x^2 + 1)) - 1/2*pi*sgn(x) + 1/2*sqrt(2*sqrt(5) + 2)*arcta
n(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/2*sqrt(2*sqrt(5) + 2)*arctan(-(x/(sqrt(-x^
2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt(2*sqrt(5) + 2)) + 1/4*sqrt(2*sqr
t(5) - 2)*log(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/4*sqrt(2*sqrt(5) - 2)*l
og(abs(x - sqrt(1/2*sqrt(5) - 1/2))) - 1/4*sqrt(2*sqrt(5) - 2)*log(abs(sqrt
(2*sqrt(5) - 2) - x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x)) + 1/4*s
qrt(2*sqrt(5) - 2)*log(abs(-sqrt(2*sqrt(5) - 2) - x/(sqrt(-x^2 + 1) - 1) +
(sqrt(-x^2 + 1) - 1)/x)) - 2*x - arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2
- 1)/(sqrt(-x^2 + 1) - 1))

Mupad [B]

time = 2.18, size = 608, normalized size = 3.29

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(x^2 + (1 - x^2)^(1/2)),x)

[Out] x*log(x^2 + (1 - x^2)^(1/2)) - asin(x) - 2*x + (log(x - (2^(1/2)*(5^(1/2) -
 1)^(1/2))/2)*(5^(1/2)/2 - 5/2))/(2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2
- 1/2)^(3/2)) - (log(x + (2^(1/2)*(5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 - 5/2)
)/(2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2)) - (log(x - (2^(1/
2)*(- 5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 + 5/2))/(2*(- 5^(1/2)/2 - 1/2)^(1/2
) + 4*(- 5^(1/2)/2 - 1/2)^(3/2)) + (log(x + (2^(1/2)*(- 5^(1/2) - 1)^(1/2))

/2)*(5^(1/2)/2 + 5/2))/(2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)
^(3/2)) + (log((((x*(5^(1/2)/2 - 1/2)^(1/2) + 1)*1i)/(3/2 - 5^(1/2)/2)^(1/2
) + (1 - x^2)^(1/2)*1i)/(x + (5^(1/2)/2 - 1/2)^(1/2)))*((3*5^(1/2))/2 - 5/2
))/((2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(3/2 - 5^(1/2)/
2)^(1/2)) - (log((((x*(- 5^(1/2)/2 - 1/2)^(1/2) + 1)*1i)/(5^(1/2)/2 + 3/2)^
(1/2) + (1 - x^2)^(1/2)*1i)/(x + (- 5^(1/2)/2 - 1/2)^(1/2)))*((3*5^(1/2))/2
 + 5/2))/((2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(5^(1
/2)/2 + 3/2)^(1/2)) - (log((((x*(5^(1/2)/2 - 1/2)^(1/2) - 1)*1i)/(3/2 - 5^(
1/2)/2)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (5^(1/2)/2 - 1/2)^(1/2)))*((3*5^(1
/2))/2 - 5/2))/((2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(3/
2 - 5^(1/2)/2)^(1/2)) + (log((((x*(- 5^(1/2)/2 - 1/2)^(1/2) - 1)*1i)/(5^(1/
2)/2 + 3/2)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (- 5^(1/2)/2 - 1/2)^(1/2)))*((
3*5^(1/2))/2 + 5/2))/((2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^
(3/2))*(5^(1/2)/2 + 3/2)^(1/2))

3.27 $\int \frac{\log(1+e^x)}{1+e^{2x}}\,dx$

Optimal. Leaf size=102

$$-\frac{1}{2}\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(i-e^x)\right)\log\left(1+e^x\right)-\frac{1}{2}\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(i+e^x)\right)\log\left(1+e^x\right)-\mathrm{Li}_2\left(-e^x\right)-\frac{1}{2}\mathrm{Li}_2\left(\left(\frac{1}{2}\right.\right.$$

```
[Out] -1/2*ln((1/2-1/2*I)*(I-exp(x)))*ln(1+exp(x))-1/2*ln((-1/2-1/2*I)*(I+exp(x))
)*ln(1+exp(x))-polylog(2,-exp(x))-1/2*polylog(2,(1/2-1/2*I)*(1+exp(x)))-1/2
*polylog(2,(1/2+1/2*I)*(1+exp(x)))
```

Rubi [A]

time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$,

Rules used = {2320, 272, 36, 29, 31, 2463, 2438, 266, 2441, 2440}

$$-\mathrm{Li}_2\left(-e^x\right)-\frac{1}{2}\mathrm{Li}_2\left(\left(\frac{1}{2}-\frac{i}{2}\right)(1+e^x)\right)-\frac{1}{2}\mathrm{Li}_2\left(\left(\frac{1}{2}+\frac{i}{2}\right)(1+e^x)\right)-\frac{1}{2}\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(-e^x+i)\right)\log\left(e^x+1\right)-\frac{1}{2}\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(e^x+i)\right)\log\left(e^x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + E^x]/(1 + E^(2*x)),x]
```

```
[Out] -1/2*(Log[(1/2 - I/2)*(I - E^x)]*Log[1 + E^x]) - (Log[(-1/2 - I/2)*(I + E^x
)]*Log[1 + E^x])/2 - PolyLog[2, -E^x] - PolyLog[2, (1/2 - I/2)*(1 + E^x)]/2
 - PolyLog[2, (1/2 + I/2)*(1 + E^x)]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\int \frac{\log\left(1+e^x\right)}{1+e^{2x}}\,dx = \text{Subst}\left(\int \frac{\log(1+x)}{x\left(1+x^2\right)}\,dx, x, e^x\right)$$

$$= \text{Subst}\left(\int \left(\frac{\log(1+x)}{x} - \frac{x\log(1+x)}{1+x^2}\right)\,dx, x, e^x\right)$$

$$= \text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^x\right) - \text{Subst}\left(\int \frac{x\log(1+x)}{1+x^2}\,dx, x, e^x\right)$$

$$= -\text{Li}_2\left(-e^x\right) - \text{Subst}\left(\int \left(-\frac{\log(1+x)}{2(i-x)} + \frac{\log(1+x)}{2(i+x)}\right)\,dx, x, e^x\right)$$

$$= -\text{Li}_2\left(-e^x\right) + \frac{1}{2}\text{Subst}\left(\int \frac{\log(1+x)}{i-x}\,dx, x, e^x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1+x)}{i+x}\,dx, x, e^x\right)$$

$$= -\frac{1}{2}\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(i-e^x)\right)\log\left(1+e^x\right) - \frac{1}{2}\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(i+e^x)\right)\log\left(1+e^x\right) -$$

$$= -\frac{1}{2}\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(i-e^x)\right)\log\left(1+e^x\right) - \frac{1}{2}\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(i+e^x)\right)\log\left(1+e^x\right) -$$

$$= -\frac{1}{2}\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(i-e^x)\right)\log\left(1+e^x\right) - \frac{1}{2}\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(i+e^x)\right)\log\left(1+e^x\right) -$$

Mathematica [A]
time = 0.01, size = 102, normalized size = 1.00

$$-\frac{1}{2}\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(i-e^x)\right)\log\left(1+e^x\right) - \frac{1}{2}\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(i+e^x)\right)\log\left(1+e^x\right) - \text{Li}_2\left(-e^x\right) - \frac{1}{2}\text{Li}_2\left(\left(\frac{1}{2}-\frac{i}{2}\right)(1+e^x)\right) - \frac{1}{2}\text{Li}_2\left(\left(\frac{1}{2}+\frac{i}{2}\right)(1+e^x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + E^x]/(1 + E^(2*x)),x]
```

```
[Out] -1/2*(Log[(1/2 - I/2)*(I - E^x)]*Log[1 + E^x]) - (Log[(-1/2 - I/2)*(I + E^x
)]*Log[1 + E^x])/2 - PolyLog[2, -E^x] - PolyLog[2, (1/2 - I/2)*(1 + E^x)]/2
 - PolyLog[2, (1/2 + I/2)*(1 + E^x)]/2
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

<div align="center">Timed out</div>

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Log[1 + Exp[x]]/(1 + Exp[2*x]),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.02, size = 83, normalized size = 0.81

method	result
risch	$-\operatorname{dilog}\left(1+\mathrm{e}^x\right)-\dfrac{\ln(1+\mathrm{e}^x)\ln\left(\frac{1}{2}-\frac{\mathrm{e}^x}{2}+\frac{i(1+\mathrm{e}^x)}{2}\right)}{2}-\dfrac{\ln(1+\mathrm{e}^x)\ln\left(\frac{1}{2}-\frac{\mathrm{e}^x}{2}-\frac{i(1+\mathrm{e}^x)}{2}\right)}{2}-\dfrac{\operatorname{dilog}\left(\frac{1}{2}-\frac{\mathrm{e}^x}{2}+\frac{i(1+\mathrm{e}^x)}{2}\right)}{2}-\dfrac{\operatorname{dilog}\left(\frac{1}{2}-\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(1+exp(x))/(1+exp(2*x)),x,method=_RETURNVERBOSE)

[Out] -dilog(1+exp(x))-1/2*ln(1+exp(x))*ln(1/2-1/2*exp(x)+1/2*I*(1+exp(x)))-1/2*l
n(1+exp(x))*ln(1/2-1/2*exp(x)-1/2*I*(1+exp(x)))-1/2*dilog(1/2-1/2*exp(x)+1/
2*I*(1+exp(x)))-1/2*dilog(1/2-1/2*exp(x)-1/2*I*(1+exp(x)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Failed to integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(1+exp(x))/(1+exp(2*x)),x, algorithm="maxima")

[Out] integrate(log(e^x + 1)/(e^(2*x) + 1), x)

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(1+exp(x))/(1+exp(2*x)),x, algorithm="fricas")

[Out] integral(log(e^x + 1)/(e^(2*x) + 1), x)

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(1+exp(x))/(1+exp(2*x)),x)

[Out] Timed out

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

$$\text{Could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+exp(x))/(1+exp(2*x)),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(e^{x}+1\right)}{e^{2x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(exp(x) + 1)/(exp(2*x) + 1),x)
```

```
[Out] int(log(exp(x) + 1)/(exp(2*x) + 1), x)
```

3.28 $\int \cosh(x) \log^2 \left(1 + \cosh^2(x)\right) \, dx$

Optimal. Leaf size=159

$$-8\sqrt{2}\,\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)+4i\sqrt{2}\,\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)^2+8\sqrt{2}\,\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+i\sinh(x)}\right)+4\sqrt{2}$$

[Out] 8*sinh(x)-4*ln(2+sinh(x)^2)*sinh(x)+ln(2+sinh(x)^2)^2*sinh(x)-8*arctan(1/2*
sinh(x)*2^(1/2))*2^(1/2)+4*I*arctan(1/2*sinh(x)*2^(1/2))^2*2^(1/2)+4*arctan
(1/2*sinh(x)*2^(1/2))*ln(2+sinh(x)^2)*2^(1/2)+8*arctan(1/2*sinh(x)*2^(1/2))
*ln(2*2^(1/2)/(I*sinh(x)+2^(1/2)))*2^(1/2)+4*I*polylog(2,1-2*2^(1/2)/(I*sin
h(x)+2^(1/2)))*2^(1/2)

Rubi [A]

time = 0.13, antiderivative size = 159, normalized size of antiderivative = 1.00, number of
steps used = 13, number of rules used = 12, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000,
Rules used = {4443, 2500, 2526, 2498, 327, 209, 2520, 12, 5040, 4964, 2449, 2352}

$$4i\sqrt{2}\,\text{Li}_2\left(1-\frac{2\sqrt{2}}{i\sinh(x)+\sqrt{2}}\right)+8\sinh(x)+\sinh(x)\log^2\left(\sinh^2(x)+2\right)-4\sinh(x)\log\left(\sinh^2(x)+2\right)+4i\sqrt{2}\,\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)^2-8\sqrt{2}\,\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)+4\sqrt{2}\,\log\left(\sinh^2(x)+2\right)\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)+8\sqrt{2}\,\log\left(\frac{2\sqrt{2}}{\sqrt{2}+i\sinh(x)}\right)\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]*Log[1 + Cosh[x]^2]^2,x]

[Out] -8*Sqrt[2]*ArcTan[Sinh[x]/Sqrt[2]] + (4*I)*Sqrt[2]*ArcTan[Sinh[x]/Sqrt[2]]^
2 + 8*Sqrt[2]*ArcTan[Sinh[x]/Sqrt[2]]*Log[(2*Sqrt[2])/(Sqrt[2] + I*Sinh[x])
] + 4*Sqrt[2]*ArcTan[Sinh[x]/Sqrt[2]]*Log[2 + Sinh[x]^2] + (4*I)*Sqrt[2]*Po
lyLog[2, 1 - (2*Sqrt[2])/(Sqrt[2] + I*Sinh[x])] + 8*Sinh[x] - 4*Log[2 + Sin
h[x]^2]*Sinh[x] + Log[2 + Sinh[x]^2]^2*Sinh[x]

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])

Rule 327

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4443

```
Int[Cosh[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sinh[c*(a +
b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a +
```

```
b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \cosh(x) \log^2\left(1 + \cosh^2(x)\right) \, dx = \text{Subst}\left(\int \log^2\left(2 + x^2\right) \, dx, x, \sinh(x)\right)$$

$$= \log^2\left(2 + \sinh^2(x)\right) \sinh(x) - 4\text{Subst}\left(\int \frac{x^2 \log\left(2 + x^2\right)}{2 + x^2} \, dx, x, \sinh(x)\right)$$

$$= \log^2\left(2 + \sinh^2(x)\right) \sinh(x) - 4\text{Subst}\left(\int \left(\log\left(2 + x^2\right) - \frac{2\log\left(2 + x^2\right)}{2 + x^2}\right)\right)$$

$$= \log^2\left(2 + \sinh^2(x)\right) \sinh(x) - 4\text{Subst}\left(\int \log\left(2 + x^2\right) \, dx, x, \sinh(x)\right)$$

$$= 4\sqrt{2} \, \tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right) \log\left(2 + \sinh^2(x)\right) - 4\log\left(2 + \sinh^2(x)\right) \sinh(x)$$

$$= 4\sqrt{2} \, \tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right) \log\left(2 + \sinh^2(x)\right) + 8\sinh(x) - 4\log\left(2 + \sinh^2(x)\right)$$

$$= -8\sqrt{2} \, \tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right) + 4i\sqrt{2} \, \tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)^2 + 4\sqrt{2} \, \tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)$$

$$= -8\sqrt{2} \, \tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right) + 4i\sqrt{2} \, \tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)^2 + 8\sqrt{2} \, \tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)$$

$$= -8\sqrt{2} \, \tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right) + 4i\sqrt{2} \, \tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)^2 + 8\sqrt{2} \, \tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)$$

$$= -8\sqrt{2} \, \tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right) + 4i\sqrt{2} \, \tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)^2 + 8\sqrt{2} \, \tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)$$

Mathematica [A]
time = 0.06, size = 122, normalized size = 0.77

$$4\sqrt{2}\,\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)\left(-2 + i\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right) + 2\log\left(\frac{4i}{2i - \sqrt{2}\,\sinh(x)}\right) + \log\left(2 + \sinh^2(x)\right)\right) + 4i\sqrt{2}\,\text{Li}_2\left(\frac{2i + \sqrt{2}\,\sinh(x)}{-2i + \sqrt{2}\,\sinh(x)}\right) + \left(8 - 4\log\left(2 + \sinh^2(x)\right) + \log^2\left(2 + \sinh^2(x)\right)\right)\sinh(x)$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]*Log[1 + Cosh[x]^2]^2,x]

[Out] 4*Sqrt[2]*ArcTan[Sinh[x]/Sqrt[2]]*(-2 + I*ArcTan[Sinh[x]/Sqrt[2]] + 2*Log[(
4*I)/(2*I - Sqrt[2]*Sinh[x])] + Log[2 + Sinh[x]^2]) + (4*I)*Sqrt[2]*PolyLog
[2, (2*I + Sqrt[2]*Sinh[x])/(-2*I + Sqrt[2]*Sinh[x])] + (8 - 4*Log[2 + Sinh
[x]^2] + Log[2 + Sinh[x]^2]^2)*Sinh[x]

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

<div align="center">Timed out</div>

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Log[1 + Cosh[x]^2]^2*Cosh[x],x]')

[Out] Timed out

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \cosh{(x)} \ln{\left(1 + \cosh^2{(x)}\right)}^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)*ln(1+cosh(x)^2)^2,x)

[Out] int(cosh(x)*ln(1+cosh(x)^2)^2,x)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

<div align="center">Failed to integrate</div>

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)*log(1+cosh(x)^2)^2,x, algorithm="maxima")

[Out] 1/2*(e^(2*x) - 1)*e^(-x)*log(e^(4*x) + 6*e^(2*x) + 1)^2 - 2*(e^(-x) + integ
rate((e^(2*x) + 6)*e^x/(e^(4*x) + 6*e^(2*x) + 1), x))*log(2)^2 + 2*(e^x - i
ntegrate((6*e^(2*x) + 1)*e^x/(e^(4*x) + 6*e^(2*x) + 1), x))*log(2)^2 + 14*i
ntegrate(e^(3*x)/(e^(4*x) + 6*e^(2*x) + 1), x)*log(2)^2 + 14*integrate(e^x/
(e^(4*x) + 6*e^(2*x) + 1), x)*log(2)^2 + 4*integrate(x*e^(6*x)/(e^(5*x) + 6
*e^(3*x) + e^x), x)*log(2) + 28*integrate(x*e^(4*x)/(e^(5*x) + 6*e^(3*x) +
e^x), x)*log(2) + 28*integrate(x*e^(2*x)/(e^(5*x) + 6*e^(3*x) + e^x), x)*lo
g(2) - 2*integrate(e^(6*x)*log(e^(4*x) + 6*e^(2*x) + 1)/(e^(5*x) + 6*e^(3*x
) + e^x), x)*log(2) - 14*integrate(e^(4*x)*log(e^(4*x) + 6*e^(2*x) + 1)/(e^
(5*x) + 6*e^(3*x) + e^x), x)*log(2) - 14*integrate(e^(2*x)*log(e^(4*x) + 6*
e^(2*x) + 1)/(e^(5*x) + 6*e^(3*x) + e^x), x)*log(2) + 4*integrate(x/(e^(5*x
) + 6*e^(3*x) + e^x), x)*log(2) - 2*integrate(log(e^(4*x) + 6*e^(2*x) + 1)/

```
(e^(5*x) + 6*e^(3*x) + e^x), x)*log(2) + 2*integrate(x^2*e^(6*x)/(e^(5*x) +
 6*e^(3*x) + e^x), x) + 14*integrate(x^2*e^(4*x)/(e^(5*x) + 6*e^(3*x) + e^x
), x) + 14*integrate(x^2*e^(2*x)/(e^(5*x) + 6*e^(3*x) + e^x), x) - 2*integr
ate(x*e^(6*x)*log(e^(4*x) + 6*e^(2*x) + 1)/(e^(5*x) + 6*e^(3*x) + e^x), x)
 - 14*integrate(x*e^(4*x)*log(e^(4*x) + 6*e^(2*x) + 1)/(e^(5*x) + 6*e^(3*x)
 + e^x), x) - 14*integrate(x*e^(2*x)*log(e^(4*x) + 6*e^(2*x) + 1)/(e^(5*x) +
 6*e^(3*x) + e^x), x) + 2*integrate(x^2/(e^(5*x) + 6*e^(3*x) + e^x), x) - 2
*integrate(x*log(e^(4*x) + 6*e^(2*x) + 1)/(e^(5*x) + 6*e^(3*x) + e^x), x) -
 4*integrate(e^(6*x)*log(e^(4*x) + 6*e^(2*x) + 1)/(e^(5*x) + 6*e^(3*x) + e^
x), x) - 8*integrate(e^(4*x)*log(e^(4*x) + 6*e^(2*x) + 1)/(e^(5*x) + 6*e^(3
*x) + e^x), x) + 12*integrate(e^(2*x)*log(e^(4*x) + 6*e^(2*x) + 1)/(e^(5*x)
 + 6*e^(3*x) + e^x), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*log(1+cosh(x)^2)^2,x, algorithm="fricas")
[Out] integral(cosh(x)*log(cosh(x)^2 + 1)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\cosh^2\left(x\right)+1\right)^2 \cosh\left(x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*ln(1+cosh(x)**2)**2,x)
[Out] Integral(log(cosh(x)**2 + 1)**2*cosh(x), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

$$\text{Could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*log(1+cosh(x)^2)^2,x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(\cosh\left(x\right)^2+1\right)^2 \cosh\left(x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cosh(x)^2 + 1)^2*cosh(x),x)
[Out] int(log(cosh(x)^2 + 1)^2*cosh(x), x)
```

3.29 $\int \cosh(x) \log^2\left(\cosh^2(x) + \sinh(x)\right)\, dx$

Optimal. Leaf size=395

$$-4\sqrt{3}\,\tan^{-1}\left(\frac{1+2\sinh(x)}{\sqrt{3}}\right) - \frac{1}{2}\left(1 - i\sqrt{3}\,\right)\log^2\left(1 - i\sqrt{3}\, + 2\sinh(x)\right) - \left(1 + i\sqrt{3}\,\right)\log\left(\frac{i\left(1 - i\sqrt{3}\right)}{2\sqrt{}}\right.$$

```
[Out] -2*ln(1+sinh(x)+sinh(x)^2)+8*sinh(x)-4*ln(1+sinh(x)+sinh(x)^2)*sinh(x)+ln(1
+sinh(x)+sinh(x)^2)^2*sinh(x)+ln(1+sinh(x)+sinh(x)^2)*ln(1+2*sinh(x)-I*3^(1
/2))*(1-I*3^(1/2))-1/2*ln(1+2*sinh(x)-I*3^(1/2))^2*(1-I*3^(1/2))-ln(1+2*sin
h(x)-I*3^(1/2))*ln(-1/6*I*(1+2*sinh(x)+I*3^(1/2))*3^(1/2))*(1-I*3^(1/2))-po
lylog(2,1/6*(I+2*I*sinh(x)+3^(1/2))*3^(1/2))*(1-I*3^(1/2))+ln(1+sinh(x)+sin
h(x)^2)*ln(1+2*sinh(x)+I*3^(1/2))*(1+I*3^(1/2))-1/2*ln(1+2*sinh(x)+I*3^(1/2
))^2*(1+I*3^(1/2))-ln(1+2*sinh(x)+I*3^(1/2))*ln(1/6*I*(1+2*sinh(x)-I*3^(1/2
))*3^(1/2))*(1+I*3^(1/2))-polylog(2,1/6*(-I-2*I*sinh(x)+3^(1/2))*3^(1/2))*(
1+I*3^(1/2))-4*arctan(1/3*(1+2*sinh(x))*3^(1/2))*3^(1/2)
```

Rubi [A]

time $= 0.35$, antiderivative size $= 395$, normalized size of antiderivative $= 1.00$, number of steps used $= 28$, number of rules used $= 15$, integrand size $= 13$, $\frac{\text{number of rules}}{\text{integrand size}} = 1.154$, Rules used $= \{4443, 2603, 2608, 787, 648, 632, 210, 642, 2604, 2465, 2437, 2338, 2441, 2440, 2438\}$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Log[Cosh[x]^2 + Sinh[x]]^2,x]
```

```
[Out] -4*Sqrt[3]*ArcTan[(1 + 2*Sinh[x])/Sqrt[3]] - ((1 - I*Sqrt[3])*Log[1 - I*Sqr
t[3] + 2*Sinh[x]]^2)/2 - (1 + I*Sqrt[3])*Log[((I/2)*(1 - I*Sqrt[3] + 2*Sinh
[x]))/Sqrt[3]]*Log[1 + I*Sqrt[3] + 2*Sinh[x]] - ((1 + I*Sqrt[3])*Log[1 + I*
Sqrt[3] + 2*Sinh[x]]^2)/2 - (1 - I*Sqrt[3])*Log[1 - I*Sqrt[3] + 2*Sinh[x]]*
Log[((-1/2*I)*(1 + I*Sqrt[3] + 2*Sinh[x]))/Sqrt[3]] - 2*Log[1 + Sinh[x] + S
inh[x]^2] + (1 - I*Sqrt[3])*Log[1 - I*Sqrt[3] + 2*Sinh[x]]*Log[1 + Sinh[x]
+ Sinh[x]^2] + (1 + I*Sqrt[3])*Log[1 + I*Sqrt[3] + 2*Sinh[x]]*Log[1 + Sinh[
x] + Sinh[x]^2] - (1 + I*Sqrt[3])*PolyLog[2, -1/2*(I - Sqrt[3] + (2*I)*Sinh
[x])/Sqrt[3]] - (1 - I*Sqrt[3])*PolyLog[2, (I + Sqrt[3] + (2*I)*Sinh[x])/(2
*Sqrt[3])] + 8*Sinh[x] - 4*Log[1 + Sinh[x] + Sinh[x]^2]*Sinh[x] + Log[1 + S
inh[x] + Sinh[x]^2]^2*Sinh[x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[((c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 4443

```
Int[Cosh[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sinh[c*(a +
b*x)]/d], u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a +
b*x)]/d], u, x]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \cosh(x) \log^2\left(\cosh^2(x) + \sinh(x)\right) \, dx = \mathrm{Subst}\left(\int \log^2\left(1 + x + x^2\right) \, dx, x, \sinh(x)\right)$$

$$= \log^2\left(1 + \sinh(x) + \sinh^2(x)\right)\sinh(x) - 2\mathrm{Subst}\left(\int \frac{x(1 + 2x)\log}{1 + x}\right.$$

$$= \log^2\left(1 + \sinh(x) + \sinh^2(x)\right)\sinh(x) - 2\mathrm{Subst}\left(\int \left(2\log\left(1 + x\right.\right.\right.$$

$$= \log^2\left(1 + \sinh(x) + \sinh^2(x)\right)\sinh(x) + 2\mathrm{Subst}\left(\int \frac{(2 + x)\log\left(1\right.}{1 + x +}\right.$$

$$= -4\log\left(1 + \sinh(x) + \sinh^2(x)\right)\sinh(x) + \log^2\left(1 + \sinh(x) + \sinh\right.$$

$$= 8\sinh(x) - 4\log\left(1 + \sinh(x) + \sinh^2(x)\right)\sinh(x) + \log^2\left(1 + \sinh\right.$$

$$= \left(1 - i\sqrt{3}\right)\log\left(1 - i\sqrt{3} + 2\sinh(x)\right)\log\left(1 + \sinh(x) + \sinh^2(x\right.$$

$$= -2\log\left(1 + \sinh(x) + \sinh^2(x)\right) + \left(1 - i\sqrt{3}\right)\log\left(1 - i\sqrt{3} + 2\,\right.$$

$$= -4\sqrt{3}\,\tan^{-1}\left(\frac{1 + 2\sinh(x)}{\sqrt{3}}\right) - 2\log\left(1 + \sinh(x) + \sinh^2(x)\right) +$$

$$= -4\sqrt{3}\,\tan^{-1}\left(\frac{1 + 2\sinh(x)}{\sqrt{3}}\right) - \left(1 + i\sqrt{3}\right)\log\left(\frac{i\left(1 - i\sqrt{3} + \right.}{2\sqrt{3}}\right.$$

$$= -4\sqrt{3}\,\tan^{-1}\left(\frac{1 + 2\sinh(x)}{\sqrt{3}}\right) - \frac{1}{2}\left(1 - i\sqrt{3}\right)\log^2\left(1 - i\sqrt{3} + \right.$$

$$= -4\sqrt{3}\,\tan^{-1}\left(\frac{1 + 2\sinh(x)}{\sqrt{3}}\right) - \frac{1}{2}\left(1 - i\sqrt{3}\right)\log^2\left(1 - i\sqrt{3} + \right.$$

Mathematica [A]

time = 0.15, size = 347, normalized size = 0.88

$$-4\sqrt{3}\tan^{-1}\left(\tfrac{1+2\sinh(x)}{\sqrt{3}}\right)-2\log\left(1+\sinh(x)+\sinh^2(x)\right)+\left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2\sinh(x)\right)\log\left(1+\sinh(x)+\sinh^2(x)\right)+\left(1+i\sqrt{3}\right)\log\left(1+i\sqrt{3}+2\sinh(x)\right)\log\left(1+\sinh(x)+\sinh^2(x)\right)-\tfrac{1}{2}\left(-i+\sqrt{3}\right)\left(\log\left(1+i\sqrt{3}+2\sinh(x)\right)\left(2\log\left(\tfrac{i+\sqrt{3}+2i\sinh(x)}{2\sqrt{3}}\right)+\log\left(1+i\sqrt{3}+2\sinh(x)\right)\right)+2\text{Li}_2\left(\tfrac{-i+\sqrt{3}-2i\sinh(x)}{2\sqrt{3}}\right)\right)+\tfrac{1}{2}\left(i+\sqrt{3}\right)\left(\log\left(1-i\sqrt{3}+2\sinh(x)\right)\left(2\log\left(\tfrac{-i+\sqrt{3}-2i\sinh(x)}{2\sqrt{3}}\right)+\log\left(1-i\sqrt{3}+2\sinh(x)\right)\right)+2\text{Li}_2\left(\tfrac{i+\sqrt{3}+2i\sinh(x)}{2\sqrt{3}}\right)\right)+8\sinh(x)-4\log\left(1+\sinh(x)+\sinh^2(x)\right)\sinh(x)+\log^2\left(1+\sinh(x)+\sinh^2(x)\right)\sinh(x)$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]*Log[Cosh[x]^2 + Sinh[x]]^2,x]

[Out] -4*Sqrt[3]*ArcTan[(1 + 2*Sinh[x])/Sqrt[3]] - 2*Log[1 + Sinh[x] + Sinh[x]^2]
 + (1 - I*Sqrt[3])*Log[1 - I*Sqrt[3] + 2*Sinh[x]]*Log[1 + Sinh[x] + Sinh[x]
^2] + (1 + I*Sqrt[3])*Log[1 + I*Sqrt[3] + 2*Sinh[x]]*Log[1 + Sinh[x] + Sinh
[x]^2] - (I/2)*(-I + Sqrt[3])*(Log[1 + I*Sqrt[3] + 2*Sinh[x]]*(2*Log[(I + S
qrt[3] + (2*I)*Sinh[x])/(2*Sqrt[3])] + Log[1 + I*Sqrt[3] + 2*Sinh[x]]) + 2*
PolyLog[2, (-I + Sqrt[3] - (2*I)*Sinh[x])/(2*Sqrt[3])]) + (I/2)*(I + Sqrt[3
])*(Log[1 - I*Sqrt[3] + 2*Sinh[x]]*(2*Log[(-I + Sqrt[3] - (2*I)*Sinh[x])/(2
*Sqrt[3])] + Log[1 - I*Sqrt[3] + 2*Sinh[x]]) + 2*PolyLog[2, (I + Sqrt[3] +
(2*I)*Sinh[x])/(2*Sqrt[3])]) + 8*Sinh[x] - 4*Log[1 + Sinh[x] + Sinh[x]^2]*S
inh[x] + Log[1 + Sinh[x] + Sinh[x]^2]^2*Sinh[x]

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Log[Sinh[x] + Cosh[x]^2]^2*Cosh[x],x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \cosh\left(x\right)\ln\left(\cosh^2\left(x\right)+\sinh\left(x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)*ln(cosh(x)^2+sinh(x))^2,x)

[Out] int(cosh(x)*ln(cosh(x)^2+sinh(x))^2,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)*log(cosh(x)^2+sinh(x))^2,x, algorithm="maxima")

```
[Out] 1/2*(e^(2*x) - 1)*e^(-x)*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)^2
 + 2*(2*x - e^(-x) - integrate((2*e^(3*x) + 5*e^(2*x) + 6*e^x - 2)*e^x/(e^(
4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1), x))*log(2)^2 - 4*(x - integrate(
(e^(3*x) + 2*e^(2*x) + 2*e^x - 2)*e^x/(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*
e^x + 1), x))*log(2)^2 + 2*(e^x - integrate((2*e^(3*x) + 2*e^(2*x) - 2*e^x
 + 1)*e^x/(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1), x))*log(2)^2 + 4*in
tegrate(e^(4*x)/(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1), x)*log(2)^2
+ 6*integrate(e^(3*x)/(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1), x)*log
(2)^2 + 6*integrate(e^x/(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1), x)*l
og(2)^2 + 4*integrate(x*e^(6*x)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x
) + e^x), x)*log(2) + 8*integrate(x*e^(5*x)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x
) - 2*e^(2*x) + e^x), x)*log(2) + 12*integrate(x*e^(4*x)/(e^(5*x) + 2*e^(4*
x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x)*log(2) + 12*integrate(x*e^(2*x)/(e^(5
*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x)*log(2) - 8*integrate(x*e
^x/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x)*log(2) - 2*integ
rate(e^(6*x)*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*
e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x)*log(2) - 4*integrate(e^(5*x)*log
(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) + 2*e^(3
*x) - 2*e^(2*x) + e^x), x)*log(2) - 6*integrate(e^(4*x)*log(e^(4*x) + 2*e^(
3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x)
+ e^x), x)*log(2) - 6*integrate(e^(2*x)*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x)
 - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x)*log(2
) + 4*integrate(e^x*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*
x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x)*log(2) + 4*integrate(x/(e
^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x)*log(2) - 2*integrate(
log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) + 2*e
^(3*x) - 2*e^(2*x) + e^x), x)*log(2) + 2*integrate(x^2*e^(6*x)/(e^(5*x) + 2
*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x) + 4*integrate(x^2*e^(5*x)/(e^(5
*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x) + 6*integrate(x^2*e^(4*x
)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x) + 6*integrate(x^2
*e^(2*x)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x) - 4*integr
ate(x^2*e^x/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x) - 2*int
egrate(x*e^(6*x)*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x)
+ 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x) - 4*integrate(x*e^(5*x)*log(
e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*
x) - 2*e^(2*x) + e^x), x) - 6*integrate(x*e^(4*x)*log(e^(4*x) + 2*e^(3*x) +
 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x)
, x) - 6*integrate(x*e^(2*x)*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x +
1)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x) + 4*integrate(x*
e^x*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) +
 2*e^(3*x) - 2*e^(2*x) + e^x), x) + 2*integrate(x^2/(e^(5*x) + 2*e^(4*x) +
2*e^(3*x) - 2*e^(2*x) + e^x), x) - 2*integrate(x*log(e^(4*x) + 2*e^(3*x) +
2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x),
 x) - 4*integrate(e^(6*x)*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/
(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x) - 6*integrate(e^(5*
```

x)*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) +
2*e^(3*x) - 2*e^(2*x) + e^x), x) + 8*integrate(e^(3*x)*log(e^(4*x) + 2*e^(3
*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) +
 e^x), x) + 4*integrate(e^(2*x)*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x
 + 1)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x) - 2*integrate
(e^x*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x)
+ 2*e^(3*x) - 2*e^(2*x) + e^x), x)

Fricas [**F**]
time = 0.34, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)*log(cosh(x)^2+sinh(x))^2,x, algorithm="fricas")

[Out] integral(cosh(x)*log(cosh(x)^2 + sinh(x))^2, x)

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)*ln(cosh(x)**2+sinh(x))**2,x)

[Out] Timed out

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

$$\text{Could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)*log(cosh(x)^2+sinh(x))^2,x)

[Out] Could not integrate

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh\left(x\right) \ln\left(\cosh\left(x\right)^2 + \sinh\left(x\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)*log(sinh(x) + cosh(x)^2)^2,x)

[Out] int(cosh(x)*log(sinh(x) + cosh(x)^2)^2, x)

3.30 $\quad \displaystyle\int \frac{\log\left(x+\sqrt{1+x}\,\right)}{1+x^2}\,dx$

Optimal. Leaf size=981

$$\frac{1}{2}i\log\left(\sqrt{1-i}\,-\sqrt{1+x}\,\right)\log\left(x+\sqrt{1+x}\,\right)-\frac{1}{2}i\log\left(\sqrt{1+i}\,-\sqrt{1+x}\,\right)\log\left(x+\sqrt{1+x}\,\right)+\frac{1}{2}i\log\left(\sqrt{}\right.$$

```
[Out] 1/2*I*ln((1+I)^(1/2)+(1+x)^(1/2))*ln((1-5^(1/2)+2*(1+x)^(1/2))/(1-2*(1+I)^(
1/2)-5^(1/2)))+1/2*I*polylog(2,2*((1+I)^(1/2)-(1+x)^(1/2))/(1+2*(1+I)^(1/2)
+5^(1/2)))-1/2*I*polylog(2,-2*((1-I)^(1/2)+(1+x)^(1/2))/(1-2*(1-I)^(1/2)+5^
(1/2)))-1/2*I*polylog(2,2*((1-I)^(1/2)-(1+x)^(1/2))/(1+2*(1-I)^(1/2)-5^(1/2
)))+1/2*I*ln((1-I)^(1/2)-(1+x)^(1/2))*ln(x+(1+x)^(1/2))-1/2*I*ln((1+I)^(1/2
)-(1+x)^(1/2))*ln(x+(1+x)^(1/2))-1/2*I*ln(x+(1+x)^(1/2))*ln((1+I)^(1/2)+(1+
x)^(1/2))+1/2*I*polylog(2,-2*((1+I)^(1/2)+(1+x)^(1/2))/(1-2*(1+I)^(1/2)-5^(
1/2)))+1/2*I*polylog(2,2*((1+I)^(1/2)-(1+x)^(1/2))/(1+2*(1+I)^(1/2)-5^(1/2)
))-1/2*I*ln((1-I)^(1/2)+(1+x)^(1/2))*ln((1-5^(1/2)+2*(1+x)^(1/2))/(1-2*(1-I
)^(1/2)-5^(1/2)))+1/2*I*ln((1+I)^(1/2)+(1+x)^(1/2))*ln((1+5^(1/2)+2*(1+x)^(
1/2))/(1-2*(1+I)^(1/2)+5^(1/2)))-1/2*I*polylog(2,2*((1-I)^(1/2)-(1+x)^(1/2)
)/(1+2*(1-I)^(1/2)+5^(1/2)))-1/2*I*ln((1-I)^(1/2)-(1+x)^(1/2))*ln((1-5^(1/2
)+2*(1+x)^(1/2))/(1+2*(1-I)^(1/2)-5^(1/2)))+1/2*I*ln((1+I)^(1/2)-(1+x)^(1/2
))*ln((1+5^(1/2)+2*(1+x)^(1/2))/(1+2*(1+I)^(1/2)+5^(1/2)))+1/2*I*ln(x+(1+x)
^(1/2))*ln((1-I)^(1/2)+(1+x)^(1/2))+1/2*I*ln((1+I)^(1/2)-(1+x)^(1/2))*ln((1
-5^(1/2)+2*(1+x)^(1/2))/(1+2*(1+I)^(1/2)-5^(1/2)))-1/2*I*ln((1-I)^(1/2)+(1+
x)^(1/2))*ln((1+5^(1/2)+2*(1+x)^(1/2))/(1-2*(1-I)^(1/2)+5^(1/2)))-1/2*I*pol
ylog(2,-2*((1-I)^(1/2)+(1+x)^(1/2))/(1-2*(1-I)^(1/2)-5^(1/2)))+1/2*I*polylo
g(2,-2*((1+I)^(1/2)+(1+x)^(1/2))/(1-2*(1+I)^(1/2)+5^(1/2)))-1/2*I*ln((1-I)^
(1/2)-(1+x)^(1/2))*ln((1+5^(1/2)+2*(1+x)^(1/2))/(1+2*(1-I)^(1/2)+5^(1/2)))
```

Rubi [A]

time = 0.86, antiderivative size = 981, normalized size of antiderivative = 1.00, number of steps used = 44, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {2610, 1605, 209, 6873, 2608, 2604, 2465, 2441, 2440, 2438}

Antiderivative was successfully verified.

```
[In] Int[Log[x + Sqrt[1 + x]]/(1 + x^2),x]
```

```
[Out] (I/2)*Log[Sqrt[1 - I] - Sqrt[1 + x]]*Log[x + Sqrt[1 + x]] - (I/2)*Log[Sqrt[
1 + I] - Sqrt[1 + x]]*Log[x + Sqrt[1 + x]] + (I/2)*Log[Sqrt[1 - I] + Sqrt[1
 + x]]*Log[x + Sqrt[1 + x]] - (I/2)*Log[Sqrt[1 + I] + Sqrt[1 + x]]*Log[x +
Sqrt[1 + x]] - (I/2)*Log[Sqrt[1 - I] + Sqrt[1 + x]]*Log[(1 - Sqrt[5] + 2*Sq
```

rt[1 + x])/(1 - 2*Sqrt[1 - I] - Sqrt[5])] - (I/2)*Log[Sqrt[1 - I] - Sqrt[1
+ x]]*Log[(1 - Sqrt[5] + 2*Sqrt[1 + x])/(1 + 2*Sqrt[1 - I] - Sqrt[5])] + (I
/2)*Log[Sqrt[1 + I] + Sqrt[1 + x]]*Log[(1 - Sqrt[5] + 2*Sqrt[1 + x])/(1 - 2
*Sqrt[1 + I] - Sqrt[5])] + (I/2)*Log[Sqrt[1 + I] - Sqrt[1 + x]]*Log[(1 - Sq
rt[5] + 2*Sqrt[1 + x])/(1 + 2*Sqrt[1 + I] - Sqrt[5])] - (I/2)*Log[Sqrt[1 -
I] + Sqrt[1 + x]]*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 - 2*Sqrt[1 - I] + Sq
rt[5])] - (I/2)*Log[Sqrt[1 - I] - Sqrt[1 + x]]*Log[(1 + Sqrt[5] + 2*Sqrt[1
+ x])/(1 + 2*Sqrt[1 - I] + Sqrt[5])] + (I/2)*Log[Sqrt[1 + I] + Sqrt[1 + x]]
*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 - 2*Sqrt[1 + I] + Sqrt[5])] + (I/2)*L
og[Sqrt[1 + I] - Sqrt[1 + x]]*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 + 2*Sqrt
[1 + I] + Sqrt[5])] - (I/2)*PolyLog[2, (2*(Sqrt[1 - I] - Sqrt[1 + x]))/(1 +
 2*Sqrt[1 - I] - Sqrt[5])] - (I/2)*PolyLog[2, (2*(Sqrt[1 - I] - Sqrt[1 + x]
))/(1 + 2*Sqrt[1 - I] + Sqrt[5])] + (I/2)*PolyLog[2, (2*(Sqrt[1 + I] - Sqrt
[1 + x]))/(1 + 2*Sqrt[1 + I] - Sqrt[5])] + (I/2)*PolyLog[2, (2*(Sqrt[1 + I]
 - Sqrt[1 + x]))/(1 + 2*Sqrt[1 + I] + Sqrt[5])] - (I/2)*PolyLog[2, (-2*(Sqr
t[1 - I] + Sqrt[1 + x]))/(1 - 2*Sqrt[1 - I] - Sqrt[5])] - (I/2)*PolyLog[2,
(-2*(Sqrt[1 - I] + Sqrt[1 + x]))/(1 - 2*Sqrt[1 - I] + Sqrt[5])] + (I/2)*Pol
yLog[2, (-2*(Sqrt[1 + I] + Sqrt[1 + x]))/(1 - 2*Sqrt[1 + I] - Sqrt[5])] + (
I/2)*PolyLog[2, (-2*(Sqrt[1 + I] + Sqrt[1 + x]))/(1 - 2*Sqrt[1 + I] + Sqrt[
5])]

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])

Rule 1605

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]

Rule 2438

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 2440

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2610

```
Int[((a_.) + Log[u_]*(b_.))*(RFx_), x_Symbol] :> With[{lst = SubstForFracti
onalPowerOfLinear[RFx*(a + b*Log[u]), x]}, Dist[lst[[2]]*lst[[4]], Subst[In
t[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x] /;  !FalseQ[lst]] /; FreeQ[{a
, b}, x] && RationalFunctionQ[RFx, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\int \frac{\log\left(x + \sqrt{1+x}\right)}{1+x^2}\,dx = 2\mathrm{Subst}\left(\int \frac{x\log\left(-1+x+x^2\right)}{1+\left(-1+x^2\right)^2}\,dx, x, \sqrt{1+x}\right)$$

$$= 2\mathrm{Subst}\left(\int \frac{x\log\left(-1+x+x^2\right)}{2-2x^2+x^4}\,dx, x, \sqrt{1+x}\right)$$

$$= 2\mathrm{Subst}\left(\int \left(\frac{ix\log\left(-1+x+x^2\right)}{(2+2i)-2x^2} + \frac{ix\log\left(-1+x+x^2\right)}{(-2+2i)+2x^2}\right) dx, x, \sqrt{1+x}\right)$$

$$= 2i\,\mathrm{Subst}\left(\int \frac{x\log\left(-1+x+x^2\right)}{(2+2i)-2x^2}\,dx, x, \sqrt{1+x}\right) + 2i\,\mathrm{Subst}\left(\int \frac{x\log\left(-1+x\right)}{(-2+2i)+}\right.$$

$$= 2i\,\mathrm{Subst}\left(\int \left(-\frac{\log\left(-1+x+x^2\right)}{4\left(\sqrt{1-i}\,-x\right)} + \frac{\log\left(-1+x+x^2\right)}{4\left(\sqrt{1-i}\,+x\right)}\right) dx, x, \sqrt{1+x}\right) +$$

$$= -\left(\frac{1}{2}i\,\mathrm{Subst}\left(\int \frac{\log\left(-1+x+x^2\right)}{\sqrt{1-i}\,-x}\,dx, x, \sqrt{1+x}\right)\right) + \frac{1}{2}i\,\mathrm{Subst}\left(\int \frac{\log\left(-1-\right.}{\sqrt{1+}}\right.$$

$$= \frac{1}{2}i\log\left(\sqrt{1-i}\,-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \frac{1}{2}i\log\left(\sqrt{1+i}\,-\sqrt{1+x}\right)$$

$$= \frac{1}{2}i\log\left(\sqrt{1-i}\,-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \frac{1}{2}i\log\left(\sqrt{1+i}\,-\sqrt{1+x}\right)$$

$$= \frac{1}{2}i\log\left(\sqrt{1-i}\,-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \frac{1}{2}i\log\left(\sqrt{1+i}\,-\sqrt{1+x}\right)$$

$$= \frac{1}{2}i\log\left(\sqrt{1-i}\,-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \frac{1}{2}i\log\left(\sqrt{1+i}\,-\sqrt{1+x}\right)$$

$$= \frac{1}{2}i\log\left(\sqrt{1-i}\,-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \frac{1}{2}i\log\left(\sqrt{1+i}\,-\sqrt{1+x}\right)$$

$$= \frac{1}{2}i\log\left(\sqrt{1-i}\,-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \frac{1}{2}i\log\left(\sqrt{1+i}\,-\sqrt{1+x}\right)$$

<u>**Mathematica [A]**</u>
time = 0.32, size = 868, normalized size = 0.88

Antiderivative was successfully verified.

```
[In] Integrate[Log[x + Sqrt[1 + x]]/(1 + x^2),x]
```

```
[Out] (I/2)*((2*I)*ArcTan[x]*Log[1/2 - Sqrt[5]/2 + Sqrt[1 + x]] + Log[(2*(Sqrt[1
 - I] - Sqrt[1 + x]))/(1 + 2*Sqrt[1 - I] - Sqrt[5])]*Log[1/2 - Sqrt[5]/2 + S
qrt[1 + x]] - Log[(2*(Sqrt[1 + I] - Sqrt[1 + x]))/(1 + 2*Sqrt[1 + I] - Sqrt
[5])]*Log[1/2 - Sqrt[5]/2 + Sqrt[1 + x]] + Log[(2*(Sqrt[1 - I] + Sqrt[1 + x
]))/(-1 + 2*Sqrt[1 - I] + Sqrt[5])]*Log[1/2 - Sqrt[5]/2 + Sqrt[1 + x]] - Lo
g[(2*(Sqrt[1 + I] + Sqrt[1 + x]))/(-1 + 2*Sqrt[1 + I] + Sqrt[5])]*Log[1/2 -
 Sqrt[5]/2 + Sqrt[1 + x]] + (2*I)*ArcTan[x]*Log[(1 + Sqrt[5])/2 + Sqrt[1 +
x]] + Log[(2*(Sqrt[1 - I] - Sqrt[1 + x]))/(1 + 2*Sqrt[1 - I] + Sqrt[5])]*Lo
g[(1 + Sqrt[5])/2 + Sqrt[1 + x]] - Log[(2*(Sqrt[1 + I] - Sqrt[1 + x]))/(1 +
 2*Sqrt[1 + I] + Sqrt[5])]*Log[(1 + Sqrt[5])/2 + Sqrt[1 + x]] + Log[(2*(Sqr
t[1 - I] + Sqrt[1 + x]))/(-1 + 2*Sqrt[1 - I] - Sqrt[5])]*Log[(1 + Sqrt[5])/
2 + Sqrt[1 + x]] - Log[(2*(Sqrt[1 + I] + Sqrt[1 + x]))/(-1 + 2*Sqrt[1 + I]
- Sqrt[5])]*Log[(1 + Sqrt[5])/2 + Sqrt[1 + x]] - (2*I)*ArcTan[x]*Log[x + Sq
rt[1 + x]] + PolyLog[2, (-1 + Sqrt[5] - 2*Sqrt[1 + x])/(-1 + 2*Sqrt[1 - I]
+ Sqrt[5])] - PolyLog[2, (-1 + Sqrt[5] - 2*Sqrt[1 + x])/(-1 + 2*Sqrt[1 + I]
 + Sqrt[5])] + PolyLog[2, (1 - Sqrt[5] + 2*Sqrt[1 + x])/(1 + 2*Sqrt[1 - I]
- Sqrt[5])] - PolyLog[2, (1 - Sqrt[5] + 2*Sqrt[1 + x])/(1 + 2*Sqrt[1 + I] -
 Sqrt[5])] + PolyLog[2, (1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 - 2*Sqrt[1 - I] +
Sqrt[5])] + PolyLog[2, (1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 + 2*Sqrt[1 - I] + S
qrt[5])] - PolyLog[2, (1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 - 2*Sqrt[1 + I] + Sq
rt[5])] - PolyLog[2, (1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 + 2*Sqrt[1 + I] + Sqr
t[5])])
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

<div align="center">Timed out</div>

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Log[x + Sqrt[1 + x]]/(1 + x^2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.05, size = 722, normalized size = 0.74

method	result
derivativedivides	$i\left(\ln\left(\sqrt{1+x}-\sqrt{1-i}\right)\ln\left(x+\sqrt{1+x}\right)-\ln\left(\sqrt{1+x}-\sqrt{1-i}\right)\ln\left(\frac{1-\sqrt{5}+2\sqrt{1+x}}{1+2\sqrt{1-i}-\sqrt{5}}\right)-\ln\left(\sqrt{\ldots}\right.$

default	$i\left(\ln\left(\sqrt{1+x}-\sqrt{1-i}\right)\ln\left(x+\sqrt{1+x}\right)-\ln\left(\sqrt{1+x}-\sqrt{1-i}\right)\ln\left(\frac{1-\sqrt{5}+2\sqrt{1+x}}{1+2\sqrt{1-i}-\sqrt{5}}\right)-\ln\left(\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+(1+x)^(1/2))/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*(ln((1+x)^(1/2)-(1-I)^(1/2))*ln(x+(1+x)^(1/2))-ln((1+x)^(1/2)-(1-I)^(
1/2))*ln((1-5^(1/2)+2*(1+x)^(1/2))/(1+2*(1-I)^(1/2)-5^(1/2)))-ln((1+x)^(1/2
)-(1-I)^(1/2))*ln((1+5^(1/2)+2*(1+x)^(1/2))/(1+2*(1-I)^(1/2)+5^(1/2)))-dilo
g((1-5^(1/2)+2*(1+x)^(1/2))/(1+2*(1-I)^(1/2)-5^(1/2)))-dilog((1+5^(1/2)+2*(
1+x)^(1/2))/(1+2*(1-I)^(1/2)+5^(1/2))))+1/2*I*(ln((1-I)^(1/2)+(1+x)^(1/2))*
ln(x+(1+x)^(1/2))-ln((1-I)^(1/2)+(1+x)^(1/2))*ln((1-5^(1/2)+2*(1+x)^(1/2))/
(1-2*(1-I)^(1/2)-5^(1/2)))-ln((1-I)^(1/2)+(1+x)^(1/2))*ln((1+5^(1/2)+2*(1+x
)^(1/2))/(1-2*(1-I)^(1/2)+5^(1/2)))-dilog((1-5^(1/2)+2*(1+x)^(1/2))/(1-2*(1
-I)^(1/2)-5^(1/2)))-dilog((1+5^(1/2)+2*(1+x)^(1/2))/(1-2*(1-I)^(1/2)+5^(1/2
))))-1/2*I*(ln((1+x)^(1/2)-(1+I)^(1/2))*ln(x+(1+x)^(1/2))-ln((1+x)^(1/2)-(1
+I)^(1/2))*ln((1-5^(1/2)+2*(1+x)^(1/2))/(1+2*(1+I)^(1/2)-5^(1/2)))-ln((1+x)
^(1/2)-(1+I)^(1/2))*ln((1+5^(1/2)+2*(1+x)^(1/2))/(1+2*(1+I)^(1/2)+5^(1/2)))
-dilog((1-5^(1/2)+2*(1+x)^(1/2))/(1+2*(1+I)^(1/2)-5^(1/2)))-dilog((1+5^(1/2
)+2*(1+x)^(1/2))/(1+2*(1+I)^(1/2)+5^(1/2))))-1/2*I*(ln((1+I)^(1/2)+(1+x)^(1
/2))*ln(x+(1+x)^(1/2))-ln((1+I)^(1/2)+(1+x)^(1/2))*ln((1-5^(1/2)+2*(1+x)^(1
/2))/(1-2*(1+I)^(1/2)-5^(1/2)))-ln((1+I)^(1/2)+(1+x)^(1/2))*ln((1+5^(1/2)+2
*(1+x)^(1/2))/(1-2*(1+I)^(1/2)+5^(1/2)))-dilog((1-5^(1/2)+2*(1+x)^(1/2))/(1
-2*(1+I)^(1/2)-5^(1/2)))-dilog((1+5^(1/2)+2*(1+x)^(1/2))/(1-2*(1+I)^(1/2)+5
^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

<center>Failed to integrate</center>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(1+x)^(1/2))/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(log(x + sqrt(x + 1))/(x^2 + 1), x)
```

Fricas [F]
time = 0.34, size = 0, normalized size = 0.00

<center>could not integrate</center>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(1+x)^(1/2))/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(log(x + sqrt(x + 1))/(x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(x + \sqrt{x+1}\right)}{x^2 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x+(1+x)**(1/2))/(x**2+1),x)
```

```
[Out] Integral(log(x + sqrt(x + 1))/(x**2 + 1), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(1+x)^(1/2))/(x^2+1),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(x + \sqrt{x+1}\right)}{x^2 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + (x + 1)^(1/2))/(x^2 + 1),x)
```

```
[Out] int(log(x + (x + 1)^(1/2))/(x^2 + 1), x)
```

3.31 $\displaystyle\int \frac{\log^2\left(x+\sqrt{1+x}\right)}{(1+x)^2}\,dx$

Optimal. Leaf size=555

$$\log(1+x)+\frac{2\log\left(x+\sqrt{1+x}\right)}{\sqrt{1+x}}-6\log\left(\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right)-\frac{\log^2\left(x+\sqrt{1+x}\right)}{1+x}-\left(1+\sqrt{5}\right)\text{lo}$$

[Out] ln(1+x)-3*ln(1+x)*ln(x+(1+x)^(1/2))-ln(x+(1+x)^(1/2))^2/(1+x)+6*ln(1/2*5^(1
/2)-1/2)*ln(1-5^(1/2)+2*(1+x)^(1/2))+3*ln(1+x)*ln(1+2*(1+x)^(1/2)/(5^(1/2)+
1))+6*polylog(2,-2*(1+x)^(1/2)/(5^(1/2)+1))-6*polylog(2,1+2*(1+x)^(1/2)/(-5
^(1/2)+1))-ln(1+5^(1/2)+2*(1+x)^(1/2))*(-5^(1/2)+1)+ln(x+(1+x)^(1/2))*ln(1+
5^(1/2)+2*(1+x)^(1/2))*(3-5^(1/2))-ln(1/10*(-1+5^(1/2)-2*(1+x)^(1/2))*5^(1/
2))*ln(1+5^(1/2)+2*(1+x)^(1/2))*(3-5^(1/2))-1/2*ln(1+5^(1/2)+2*(1+x)^(1/2))
^2*(3-5^(1/2))-polylog(2,1/10*(1+5^(1/2)+2*(1+x)^(1/2))*5^(1/2))*(3-5^(1/2)
)-ln(1-5^(1/2)+2*(1+x)^(1/2))*(5^(1/2)+1)+ln(x+(1+x)^(1/2))*ln(1-5^(1/2)+2*
(1+x)^(1/2))*(3+5^(1/2))-1/2*ln(1-5^(1/2)+2*(1+x)^(1/2))^2*(3+5^(1/2))-ln(1
-5^(1/2)+2*(1+x)^(1/2))*ln(1/10*(1+5^(1/2)+2*(1+x)^(1/2))*5^(1/2))*(3+5^(1/
2))-polylog(2,1/10*(-1+5^(1/2)-2*(1+x)^(1/2))*5^(1/2))*(3+5^(1/2))+2*ln(x+(
1+x)^(1/2))/(1+x)^(1/2)

Rubi [A]

time = 0.44, antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 16, integrand size = 18, $\dfrac{\text{number of rules}}{\text{integrand size}} = 0.889$,
Rules used = {2605, 2608, 814, 646, 31, 2604, 2404, 2353, 2352, 2354, 2438, 2465, 2437, 2338, 2441, 2440}

Antiderivative was successfully verified.

[In] Int[Log[x + Sqrt[1 + x]]^2/(1 + x)^2,x]

[Out] Log[1 + x] + (2*Log[x + Sqrt[1 + x]])/Sqrt[1 + x] - 6*Log[Sqrt[1 + x]]*Log[
x + Sqrt[1 + x]] - Log[x + Sqrt[1 + x]]^2/(1 + x) - (1 + Sqrt[5])*Log[1 - S
qrt[5] + 2*Sqrt[1 + x]] + 6*Log[(-1 + Sqrt[5])/2]*Log[1 - Sqrt[5] + 2*Sqrt[
1 + x]] + (3 + Sqrt[5])*Log[x + Sqrt[1 + x]]*Log[1 - Sqrt[5] + 2*Sqrt[1 + x
]] - ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 + x]]^2)/2 - (1 - Sqrt[5])*L
og[1 + Sqrt[5] + 2*Sqrt[1 + x]] + (3 - Sqrt[5])*Log[x + Sqrt[1 + x]]*Log[1
+ Sqrt[5] + 2*Sqrt[1 + x]] - (3 - Sqrt[5])*Log[-1/2*(1 - Sqrt[5] + 2*Sqrt[1
 + x])/Sqrt[5]]*Log[1 + Sqrt[5] + 2*Sqrt[1 + x]] - ((3 - Sqrt[5])*Log[1 + S
qrt[5] + 2*Sqrt[1 + x]]^2)/2 - (3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 + x
]]*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])/(2*Sqrt[5])] + 6*Log[Sqrt[1 + x]]*Log[
1 + (2*Sqrt[1 + x])/(1 + Sqrt[5])] + 6*PolyLog[2, (-2*Sqrt[1 + x])/(1 + Sqr

t[5])] - (3 + Sqrt[5])*PolyLog[2, -1/2*(1 - Sqrt[5] + 2*Sqrt[1 + x])/Sqrt[5
]] - (3 - Sqrt[5])*PolyLog[2, (1 + Sqrt[5] + 2*Sqrt[1 + x])/(2*Sqrt[5])] -
6*PolyLog[2, 1 + (2*Sqrt[1 + x])/(1 - Sqrt[5])]

Rule 31

Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]

Rule 646

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 814

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 2338

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]

Rule 2352

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2353

Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
(a + b*Log[(-c)*(d/e)])*(Log[d + e*x]/e), x] + Dist[b, Int[Log[(-e)*(x/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[(-c)*(d/e), 0]

Rule 2354

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\log^2\left(x + \sqrt{1+x}\right)}{(1+x)^2}\, dx = 2\text{Subst}\left(\int \frac{\log^2\left(-1 + x + x^2\right)}{x^3}\, dx, x, \sqrt{1+x}\right)$$

$$= -\frac{\log^2\left(x + \sqrt{1+x}\right)}{1+x} + 2\text{Subst}\left(\int \frac{(1+2x)\log\left(-1 + x + x^2\right)}{x^2\left(-1 + x + x^2\right)}\, dx, x, \sqrt{1+x}\right.$$

$$= -\frac{\log^2\left(x + \sqrt{1+x}\right)}{1+x} + 2\text{Subst}\left(\int \left(-\frac{\log\left(-1 + x + x^2\right)}{x^2} - \frac{3\log\left(-1 + x + \right.}{x}\right.\right.$$

$$= -\frac{\log^2\left(x + \sqrt{1+x}\right)}{1+x} - 2\text{Subst}\left(\int \frac{\log\left(-1 + x + x^2\right)}{x^2}\, dx, x, \sqrt{1+x}\right) + 2\text{S}$$

$$= \frac{2\log\left(x + \sqrt{1+x}\right)}{\sqrt{1+x}} - 6\log\left(\sqrt{1+x}\right)\log\left(x + \sqrt{1+x}\right) - \frac{\log^2\left(x + \sqrt{1}\right.}{1+x}$$

$$= \frac{2\log\left(x + \sqrt{1+x}\right)}{\sqrt{1+x}} - 6\log\left(\sqrt{1+x}\right)\log\left(x + \sqrt{1+x}\right) - \frac{\log^2\left(x + \sqrt{1}\right.}{1+x}$$

$$= \log(1+x) + \frac{2\log\left(x + \sqrt{1+x}\right)}{\sqrt{1+x}} - 6\log\left(\sqrt{1+x}\right)\log\left(x + \sqrt{1+x}\right) - \frac{\text{lo}}{}$$

$$= \log(1+x) + \frac{2\log\left(x + \sqrt{1+x}\right)}{\sqrt{1+x}} - 6\log\left(\sqrt{1+x}\right)\log\left(x + \sqrt{1+x}\right) - \frac{\text{lo}}{}$$

$$= \log(1+x) + \frac{2\log\left(x + \sqrt{1+x}\right)}{\sqrt{1+x}} - 6\log\left(\sqrt{1+x}\right)\log\left(x + \sqrt{1+x}\right) - \frac{\text{lo}}{}$$

$$= \log(1+x) + \frac{2\log\left(x + \sqrt{1+x}\right)}{\sqrt{1+x}} - 6\log\left(\sqrt{1+x}\right)\log\left(x + \sqrt{1+x}\right) - \frac{\text{lo}}{}$$

$$= \log(1+x) + \frac{2\log\left(x + \sqrt{1+x}\right)}{\sqrt{1+x}} - 6\log\left(\sqrt{1+x}\right)\log\left(x + \sqrt{1+x}\right) - \frac{\text{lo}}{}$$

$$= \log(1+x) + \frac{2\log\left(x + \sqrt{1+x}\right)}{\sqrt{1+x}} - 6\log\left(\sqrt{1+x}\right)\log\left(x + \sqrt{1+x}\right) - \frac{\text{lo}}{}$$

Mathematica [A]
time = 0.89, size = 1076, normalized size = 1.94

Warning: Unable to verify antiderivative.

[In] Integrate[Log[x + Sqrt[1 + x]]^2/(1 + x)^2,x]

[Out] (2*Log[1 + x] - 6*Log[(1 + Sqrt[5])/2]*Log[1 + x] - 12*Log[(2*Sqrt[1 + x])/
(-1 + Sqrt[5])]*Log[1/2 - Sqrt[5]/2 + Sqrt[1 + x]] + 6*Log[1 + x]*Log[1/2 -
Sqrt[5]/2 + Sqrt[1 + x]] - 6*Log[-1 + Sqrt[5] - 2*Sqrt[1 + x]]*Log[1/2 - S
qrt[5]/2 + Sqrt[1 + x]] - 2*Sqrt[5]*Log[-1 + Sqrt[5] - 2*Sqrt[1 + x]]*Log[1
/2 - Sqrt[5]/2 + Sqrt[1 + x]] + 3*Log[1/2 - Sqrt[5]/2 + Sqrt[1 + x]]^2 + Sq
rt[5]*Log[1/2 - Sqrt[5]/2 + Sqrt[1 + x]]^2 + 6*Log[1 + x]*Log[(1 + Sqrt[5])
/2 + Sqrt[1 + x]] - 6*Log[-1 + Sqrt[5] - 2*Sqrt[1 + x]]*Log[(1 + Sqrt[5])/2
 + Sqrt[1 + x]] - 2*Sqrt[5]*Log[-1 + Sqrt[5] - 2*Sqrt[1 + x]]*Log[(1 + Sqrt
[5])/2 + Sqrt[1 + x]] + 3*Log[(1 + Sqrt[5])/2 + Sqrt[1 + x]]^2 - Sqrt[5]*Lo
g[(1 + Sqrt[5])/2 + Sqrt[1 + x]]^2 + (4*Log[x + Sqrt[1 + x]])/Sqrt[1 + x] -
 6*Log[1 + x]*Log[x + Sqrt[1 + x]] + 6*Log[-1 + Sqrt[5] - 2*Sqrt[1 + x]]*Lo
g[x + Sqrt[1 + x]] + 2*Sqrt[5]*Log[-1 + Sqrt[5] - 2*Sqrt[1 + x]]*Log[x + Sq
rt[1 + x]] - (2*Log[x + Sqrt[1 + x]]^2)/(1 + x) - 2*Log[1 - Sqrt[5] + 2*Sqr
t[1 + x]] - 2*Sqrt[5]*Log[1 - Sqrt[5] + 2*Sqrt[1 + x]] + 3*Log[5]*Log[1 - S
qrt[5] + 2*Sqrt[1 + x]] + Sqrt[5]*Log[5]*Log[1 - Sqrt[5] + 2*Sqrt[1 + x]] -
 2*Log[1 + Sqrt[5] + 2*Sqrt[1 + x]] + 2*Sqrt[5]*Log[1 + Sqrt[5] + 2*Sqrt[1
+ x]] - 6*Log[1/2 - Sqrt[5]/2 + Sqrt[1 + x]]*Log[1 + Sqrt[5] + 2*Sqrt[1 + x
]] + 2*Sqrt[5]*Log[1/2 - Sqrt[5]/2 + Sqrt[1 + x]]*Log[1 + Sqrt[5] + 2*Sqrt[
1 + x]] - 6*Log[(1 + Sqrt[5])/2 + Sqrt[1 + x]]*Log[1 + Sqrt[5] + 2*Sqrt[1 +
 x]] + 2*Sqrt[5]*Log[(1 + Sqrt[5])/2 + Sqrt[1 + x]]*Log[1 + Sqrt[5] + 2*Sqr
t[1 + x]] + 6*Log[x + Sqrt[1 + x]]*Log[1 + Sqrt[5] + 2*Sqrt[1 + x]] - 2*Sqr
t[5]*Log[x + Sqrt[1 + x]]*Log[1 + Sqrt[5] + 2*Sqrt[1 + x]] + 6*Log[1/2 - Sq
rt[5]/2 + Sqrt[1 + x]]*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])/(2*Sqrt[5])] - 2*S
qrt[5]*Log[1/2 - Sqrt[5]/2 + Sqrt[1 + x]]*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])
/(2*Sqrt[5])] + 12*PolyLog[2, (-2*Sqrt[1 + x])/(1 + Sqrt[5])] - 4*Sqrt[5]*P
olyLog[2, (-1 + Sqrt[5] - 2*Sqrt[1 + x])/(2*Sqrt[5])] - 12*PolyLog[2, (-1 +
 Sqrt[5] - 2*Sqrt[1 + x])/(-1 + Sqrt[5])])/2

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

    cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Log[x + Sqrt[1 + x]]^2/(1 + x)^2,x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject

Maple [**F**]

time $= 0.01$, size $= 0$, normalized size $= 0.00$

$$\int \frac{\ln\left(x + \sqrt{1+x}\,\right)^2}{(1+x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(x+(1+x)^(1/2))^2/(1+x)^2,x)

[Out] int(ln(x+(1+x)^(1/2))^2/(1+x)^2,x)

Maxima [**F**]

time $= 0.00$, size $= 0$, normalized size $= 0.00$

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x+(1+x)^(1/2))^2/(1+x)^2,x, algorithm="maxima")

[Out] -log(x + sqrt(x + 1))^2/(x + 1) + integrate((2*x + sqrt(x + 1) + 2)*log(x + sqrt(x + 1))/(x^3 + 2*x^2 + (x^2 + 2*x + 1)*sqrt(x + 1) + x), x)

Fricas [**F**]

time $= 0.34$, size $= 0$, normalized size $= 0.00$

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x+(1+x)^(1/2))^2/(1+x)^2,x, algorithm="fricas")

[Out] integral(log(x + sqrt(x + 1))^2/(x^2 + 2*x + 1), x)

Sympy [**F**]

time $= 0.00$, size $= 0$, normalized size $= 0.00$

$$\int \frac{\log\left(x + \sqrt{x+1}\,\right)^2}{(x+1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(x+(1+x)**(1/2))**2/(1+x)**2,x)

[Out] Integral(log(x + sqrt(x + 1))**2/(x + 1)**2, x)

Giac [**F**] N/A

time $= 0.00$, size $= 0$, normalized size $= 0.00$

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(1+x)^(1/2))^2/(1+x)^2,x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(x + \sqrt{x+1}\,\right)^2}{(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + (x + 1)^(1/2))^2/(x + 1)^2,x)
```

```
[Out] int(log(x + (x + 1)^(1/2))^2/(x + 1)^2, x)
```

3.32 $\displaystyle\int \frac{\log\left(x+\sqrt{1+x}\right)}{x}\, dx$

Optimal. Leaf size=313

$$\log\left(-1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right)+\log\left(1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right)-\log\left(-1+\sqrt{1+x}\right)\log$$

```
[Out] ln(-1+(1+x)^(1/2))*ln(x+(1+x)^(1/2))+ln(1+(1+x)^(1/2))*ln(x+(1+x)^(1/2))-ln
(-1+(1+x)^(1/2))*ln((1-5^(1/2)+2*(1+x)^(1/2))/(3-5^(1/2)))-ln(1+(1+x)^(1/2)
)*ln((-1+5^(1/2)-2*(1+x)^(1/2))/(5^(1/2)+1))-ln(1+(1+x)^(1/2))*ln((-1-5^(1/
2)-2*(1+x)^(1/2))/(-5^(1/2)+1))-ln(-1+(1+x)^(1/2))*ln((1+5^(1/2)+2*(1+x)^(1
/2))/(3+5^(1/2)))-polylog(2,2*(1-(1+x)^(1/2))/(3-5^(1/2)))-polylog(2,2*(1-(
1+x)^(1/2))/(3+5^(1/2)))-polylog(2,2*(1+(1+x)^(1/2))/(-5^(1/2)+1))-polylog(
2,2*(1+(1+x)^(1/2))/(5^(1/2)+1))
```

Rubi [A]

time = 0.23, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 14, $\dfrac{\text{number of rules}}{\text{integrand size}} = 0.500$,

Rules used = {2610, 2608, 2604, 2465, 2441, 2440, 2438}

$$-\mathrm{Li}_2\left(\tfrac{2\left(1-\sqrt{x+1}\right)}{3-\sqrt{5}}\right)-\mathrm{Li}_2\left(\tfrac{2\left(1-\sqrt{x+1}\right)}{3+\sqrt{5}}\right)-\mathrm{Li}_2\left(\tfrac{2\left(\sqrt{x+1}+1\right)}{1-\sqrt{5}}\right)-\mathrm{Li}_2\left(\tfrac{2\left(\sqrt{x+1}+1\right)}{1+\sqrt{5}}\right)+\log\left(\sqrt{x+1}-1\right)\log\left(x+\sqrt{x+1}\right)+\log\left(\sqrt{x+1}+1\right)\log\left(x+\sqrt{x+1}\right)-\log\left(\sqrt{x+1}-1\right)\log\left(\tfrac{2\sqrt{x+1}-\sqrt{5}+1}{3-\sqrt{5}}\right)-\log\left(\sqrt{x+1}+1\right)\log\left(-\tfrac{2\sqrt{x+1}-\sqrt{5}+1}{1+\sqrt{5}}\right)-\log\left(\sqrt{x+1}+1\right)\log\left(-\tfrac{2\sqrt{x+1}+\sqrt{5}+1}{1-\sqrt{5}}\right)-\log\left(\sqrt{x+1}-1\right)\log\left(\tfrac{2\sqrt{x+1}+\sqrt{5}+1}{3+\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x + Sqrt[1 + x]]/x,x]
```

```
[Out] Log[-1 + Sqrt[1 + x]]*Log[x + Sqrt[1 + x]] + Log[1 + Sqrt[1 + x]]*Log[x + S
qrt[1 + x]] - Log[-1 + Sqrt[1 + x]]*Log[(1 - Sqrt[5] + 2*Sqrt[1 + x])/(3 -
Sqrt[5])] - Log[1 + Sqrt[1 + x]]*Log[-((1 - Sqrt[5] + 2*Sqrt[1 + x])/(1 + S
qrt[5]))] - Log[1 + Sqrt[1 + x]]*Log[-((1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 - S
qrt[5]))] - Log[-1 + Sqrt[1 + x]]*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])/(3 + Sq
rt[5])] - PolyLog[2, (2*(1 - Sqrt[1 + x]))/(3 - Sqrt[5])] - PolyLog[2, (2*(
1 - Sqrt[1 + x]))/(3 + Sqrt[5])] - PolyLog[2, (2*(1 + Sqrt[1 + x]))/(1 - Sq
rt[5])] - PolyLog[2, (2*(1 + Sqrt[1 + x]))/(1 + Sqrt[5])]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

(e*f - d*g), 0]

Rule 2441

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]

Rule 2465

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]

Rule 2604

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

Rule 2608

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]

Rule 2610

Int[((a_.) + Log[u_]*(b_.))*(RFx_), x_Symbol] :> With[{lst = SubstForFracti
onalPowerOfLinear[RFx*(a + b*Log[u]), x]}, Dist[lst[[2]]*lst[[4]], Subst[In
t[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x] /; !FalseQ[lst]] /; FreeQ[{a
, b}, x] && RationalFunctionQ[RFx, x]

Rubi steps

$$\int \frac{\log\left(x+\sqrt{1+x}\right)}{x}\,dx = 2\text{Subst}\left(\int \frac{x\log\left(-1+x+x^2\right)}{-1+x^2}\,dx, x, \sqrt{1+x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{\log\left(-1+x+x^2\right)}{2(-1+x)} + \frac{\log\left(-1+x+x^2\right)}{2(1+x)}\right)dx, x, \sqrt{1+x}\right)$$

$$= \text{Subst}\left(\int \frac{\log\left(-1+x+x^2\right)}{-1+x}\,dx, x, \sqrt{1+x}\right) + \text{Subst}\left(\int \frac{\log\left(-1+x+x^2\right)}{1+x}\,d\right.$$

$$= \log\left(-1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) + \log\left(1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right.$$

$$= \log\left(-1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) + \log\left(1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right.$$

$$= \log\left(-1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) + \log\left(1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right.$$

$$= \log\left(-1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) + \log\left(1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right.$$

$$= \log\left(-1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) + \log\left(1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right.$$

$$= \log\left(-1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) + \log\left(1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right.$$

Mathematica [A]
time = 0.06, size = 303, normalized size = 0.97

$$\log\left(1-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) + \log\left(1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \log\left(\tfrac{1}{2}\left(3-\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2\sqrt{1+x}\right) - \log\left(\tfrac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2\sqrt{1+x}\right) - \log\left(\tfrac{1}{2}\left(3+\sqrt{5}\right)\right)\log\left(1+\sqrt{5}+2\sqrt{1+x}\right) - \log\left(1+\sqrt{1+x}\right)\log\left(-\tfrac{1+\sqrt{5}+2\sqrt{1+x}}{1-\sqrt{5}}\right) - \text{Li}_2\left(\tfrac{2\left(1+\sqrt{1+x}\right)}{1-\sqrt{5}}\right) + \text{Li}_2\left(\tfrac{1-\sqrt{5}+2\sqrt{1+x}}{3-\sqrt{5}}\right) + \text{Li}_2\left(\tfrac{1-\sqrt{5}+2\sqrt{1+x}}{1+\sqrt{5}}\right) + \text{Li}_2\left(\tfrac{1+\sqrt{5}+2\sqrt{1+x}}{3+\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x + Sqrt[1 + x]]/x,x]

[Out] Log[1 - Sqrt[1 + x]]*Log[x + Sqrt[1 + x]] + Log[1 + Sqrt[1 + x]]*Log[x + Sq
rt[1 + x]] - Log[(3 - Sqrt[5])/2]*Log[1 - Sqrt[5] + 2*Sqrt[1 + x]] - Log[(1
 + Sqrt[5])/2]*Log[1 - Sqrt[5] + 2*Sqrt[1 + x]] - Log[(3 + Sqrt[5])/2]*Log[
1 + Sqrt[5] + 2*Sqrt[1 + x]] - Log[1 + Sqrt[1 + x]]*Log[-((1 + Sqrt[5] + 2*
Sqrt[1 + x])/(1 - Sqrt[5]))] - PolyLog[2, (2*(1 + Sqrt[1 + x]))/(1 - Sqrt[5
])] + PolyLog[2, (1 - Sqrt[5] + 2*Sqrt[1 + x])/(3 - Sqrt[5])] + PolyLog[2,
```

```
-((1 - Sqrt[5] + 2*Sqrt[1 + x])/(1 + Sqrt[5]))] + PolyLog[2, (1 + Sqrt[5] +
 2*Sqrt[1 + x])/(3 + Sqrt[5])]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Log[x + Sqrt[1 + x]]/x,x]')
```

[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject

Maple [A]
time = 0.01, size = 252, normalized size = 0.81

method	result
derivativedivides	$\ln\left(-1+\sqrt{1+x}\right)\ln\left(x+\sqrt{1+x}\right)-\ln\left(-1+\sqrt{1+x}\right)\ln\left(\frac{-1+\sqrt{5}-2\sqrt{1+x}}{\sqrt{5}-3}\right)-\ln$
default	$\ln\left(-1+\sqrt{1+x}\right)\ln\left(x+\sqrt{1+x}\right)-\ln\left(-1+\sqrt{1+x}\right)\ln\left(\frac{-1+\sqrt{5}-2\sqrt{1+x}}{\sqrt{5}-3}\right)-\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+(1+x)^(1/2))/x,x,method=_RETURNVERBOSE)
```

[Out] ln(-1+(1+x)^(1/2))*ln(x+(1+x)^(1/2))-ln(-1+(1+x)^(1/2))*ln((-1+5^(1/2)-2*(1
+x)^(1/2))/(5^(1/2)-3))-ln(-1+(1+x)^(1/2))*ln((1+5^(1/2)+2*(1+x)^(1/2))/(3+
5^(1/2)))-dilog((-1+5^(1/2)-2*(1+x)^(1/2))/(5^(1/2)-3))-dilog((1+5^(1/2)+2*
(1+x)^(1/2))/(3+5^(1/2)))+ln(1+(1+x)^(1/2))*ln(x+(1+x)^(1/2))-ln(1+(1+x)^(1
/2))*ln((-1+5^(1/2)-2*(1+x)^(1/2))/(5^(1/2)+1))-ln(1+(1+x)^(1/2))*ln((1+5^(
1/2)+2*(1+x)^(1/2))/(5^(1/2)-1))-dilog((-1+5^(1/2)-2*(1+x)^(1/2))/(5^(1/2)+
1))-dilog((1+5^(1/2)+2*(1+x)^(1/2))/(5^(1/2)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(1+x)^(1/2))/x,x, algorithm="maxima")
```

[Out] integrate(log(x + sqrt(x + 1))/x, x)

Fricas [F]
time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(log(x+(1+x)^(1/2))/x,x, algorithm="fricas")`

`[Out] integral(log(x + sqrt(x + 1))/x, x)`

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(x + \sqrt{x+1}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(ln(x+(1+x)**(1/2))/x,x)`

`[Out] Integral(log(x + sqrt(x + 1))/x, x)`

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(log(x+(1+x)^(1/2))/x,x)`

`[Out] Could not integrate`

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(x + \sqrt{x+1}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(log(x + (x + 1)^(1/2))/x,x)`

`[Out] int(log(x + (x + 1)^(1/2))/x, x)`

3.33 $\quad \int \tan^{-1}(2\tan(x))\,dx$

Optimal. Leaf size=80

$$x\tan^{-1}(2\tan(x)) + \frac{1}{2}ix\log\left(1 - 3e^{2ix}\right) - \frac{1}{2}ix\log\left(1 - \frac{1}{3}e^{2ix}\right) - \frac{1}{4}\text{Li}_2\left(\frac{1}{3}e^{2ix}\right) + \frac{1}{4}\text{Li}_2\left(3e^{2ix}\right)$$

[Out] x*arctan(2*tan(x))+1/2*I*x*ln(1-3*exp(2*I*x))-1/2*I*x*ln(1-1/3*exp(2*I*x))-
1/4*polylog(2,1/3*exp(2*I*x))+1/4*polylog(2,3*exp(2*I*x))

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of
steps used = 7, number of rules used = 4, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$,
Rules used = {5275, 2221, 2317, 2438}

$$-\frac{1}{4}\text{Li}_2\left(\frac{1}{3}e^{2ix}\right) + \frac{1}{4}\text{Li}_2\left(3e^{2ix}\right) + \frac{1}{2}ix\log\left(1 - 3e^{2ix}\right) - \frac{1}{2}ix\log\left(1 - \frac{1}{3}e^{2ix}\right) + x\tan^{-1}(2\tan(x))$$

Antiderivative was successfully verified.

[In] Int[ArcTan[2*Tan[x]],x]

[Out] x*ArcTan[2*Tan[x]] + (I/2)*x*Log[1 - 3*E^((2*I)*x)] - (I/2)*x*Log[1 - E^((2
*I)*x)/3] - PolyLog[2, E^((2*I)*x)/3]/4 + PolyLog[2, 3*E^((2*I)*x)]/4

Rule 2221

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2317

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2438

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 5275

Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcT
an[c + d*Tan[a + b*x]], x] + (Dist[b*(1 - I*c - d), Int[x*(E^(2*I*a + 2*I*b

```
*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x] - Dist[b*(1
+ I*c + d), Int[x*(E^(2*I*a + 2*I*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2*I*
a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(2\tan(x))\,dx = x\tan^{-1}(2\tan(x)) - 3\int \frac{e^{2ix}x}{-1+3e^{2ix}}\,dx - \int \frac{e^{2ix}x}{3-e^{2ix}}\,dx$$

$$= x\tan^{-1}(2\tan(x)) + \frac{1}{2}ix\log\left(1-3e^{2ix}\right) - \frac{1}{2}ix\log\left(1-\frac{1}{3}e^{2ix}\right) - \frac{1}{2}i\int \log\left(1-3e^{2}\right.$$

$$= x\tan^{-1}(2\tan(x)) + \frac{1}{2}ix\log\left(1-3e^{2ix}\right) - \frac{1}{2}ix\log\left(1-\frac{1}{3}e^{2ix}\right) - \frac{1}{4}\text{Subst}\left(\int \frac{\log(1}{}\right.$$

$$= x\tan^{-1}(2\tan(x)) + \frac{1}{2}ix\log\left(1-3e^{2ix}\right) - \frac{1}{2}ix\log\left(1-\frac{1}{3}e^{2ix}\right) - \frac{1}{4}\text{Li}_2\left(\frac{1}{3}e^{2ix}\right) +$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 262 vs. $2(80) = 160$.
time $= 0.16$, size $= 262$, normalized size $= 3.28$

$$x\tan^{-1}(2\tan(x)) - \frac{1}{4}i\left(4ix\tan^{-1}\left(\frac{\cot(x)}{2}\right) + 2i\cos^{-1}\left(\frac{5}{3}\right)\tan^{-1}(2\tan(x)) + \left(\cos^{-1}\left(\frac{5}{3}\right) + 2\tan^{-1}\left(\frac{\cot(x)}{2}\right) + 2\tan^{-1}(2\tan(x))\right)\log\left(\frac{2i\sqrt{\frac{2}{3}}e^{-ix}}{\sqrt{-5+3\cos(2x)}}\right) + \left(\cos^{-1}\left(\frac{5}{3}\right) - 2\tan^{-1}\left(\frac{\cot(x)}{2}\right) - 2\tan^{-1}(2\tan(x))\right)\log\left(\frac{2i\sqrt{\frac{2}{3}}e^{ix}}{\sqrt{-5+3\cos(2x)}}\right) - \left(\cos^{-1}\left(\frac{5}{3}\right) - 2\tan^{-1}(2\tan(x))\right)\log\left(\frac{4i-4\tan(x)}{i+2\tan(x)}\right) - \left(\cos^{-1}\left(\frac{5}{3}\right) + 2\tan^{-1}(2\tan(x))\right)\log\left(\frac{4(i+\tan(x))}{3i+6\tan(x)}\right) + i\left(-\text{Li}_2\left(\frac{-3i+6\tan(x)}{i+2\tan(x)}\right) + \text{Li}_2\left(\frac{-i+2\tan(x)}{3i+6\tan(x)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[2*Tan[x]],x]
```

```
[Out] x*ArcTan[2*Tan[x]] - (I/4)*((4*I)*x*ArcTan[Cot[x]/2] + (2*I)*ArcCos[5/3]*Ar
cTan[2*Tan[x]] + (ArcCos[5/3] + 2*ArcTan[Cot[x]/2] + 2*ArcTan[2*Tan[x]])*Lo
g[((2*I)*Sqrt[2/3])/(E^(I*x)*Sqrt[-5 + 3*Cos[2*x]])] + (ArcCos[5/3] - 2*Arc
Tan[Cot[x]/2] - 2*ArcTan[2*Tan[x]])*Log[((2*I)*Sqrt[2/3]*E^(I*x))/Sqrt[-5 +
 3*Cos[2*x]]] - (ArcCos[5/3] - 2*ArcTan[2*Tan[x]])*Log[(4*I - 4*Tan[x])/(I
+ 2*Tan[x])] - (ArcCos[5/3] + 2*ArcTan[2*Tan[x]])*Log[(4*(I + Tan[x]))/(3*I
 + 6*Tan[x])] + I*(-PolyLog[2, (-3*I + 6*Tan[x])/(I + 2*Tan[x])] + PolyLog[
2, (-I + 2*Tan[x])/(3*I + 6*Tan[x])]))
```

Mathics [F(-2)]
time $= 0.00$, size $= 0$, normalized size $= 0.00$

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[ArcTan[2*Tan[x]],x]')
```

[Out] cought exception: maximum recursion depth exceeded

Maple [A]
time = 0.25, size = 113, normalized size = 1.41

method	result
derivativedivides	$\arctan\left(2\tan\left(x\right)\right)\arctan\left(\tan\left(x\right)\right) + \dfrac{i\arctan(\tan(x))\ln\left(1-\frac{3(1+i\tan(x))^2}{1+\tan^2(x)}\right)}{2} + \dfrac{\operatorname{polylog}\left(2,\frac{3(1+i\tan(x))^2}{1+\tan^2(x)}\right)}{4}$
default	$\arctan\left(2\tan\left(x\right)\right)\arctan\left(\tan\left(x\right)\right) + \dfrac{i\arctan(\tan(x))\ln\left(1-\frac{3(1+i\tan(x))^2}{1+\tan^2(x)}\right)}{2} + \dfrac{\operatorname{polylog}\left(2,\frac{3(1+i\tan(x))^2}{1+\tan^2(x)}\right)}{4}$
risch	$\dfrac{x\pi\operatorname{csgn}\left(\frac{i}{e^{2ix}+1}\right)\operatorname{csgn}\left(\frac{i\left(e^{2ix}-\frac13\right)}{e^{2ix}+1}\right)^2}{4} + \dfrac{x\pi\operatorname{csgn}(i(e^{2ix}-\frac13))\operatorname{csgn}\left(\frac{i\left(e^{2ix}-\frac13\right)}{e^{2ix}+1}\right)^2}{4} - \dfrac{x\pi\operatorname{csgn}(i(e^{2ix}-3))\operatorname{csgn}\left(\frac{i\left(e^{2ix}-\right.}{e^{2ix}+}\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(2*tan(x)),x,method=_RETURNVERBOSE)

[Out] arctan(2*tan(x))*arctan(tan(x))+1/2*I*arctan(tan(x))*ln(1-3*(1+I*tan(x))^2/
(1+tan(x)^2))+1/4*polylog(2,3*(1+I*tan(x))^2/(1+tan(x)^2))-1/2*I*arctan(tan
(x))*ln(1-1/3*(1+I*tan(x))^2/(1+tan(x)^2))-1/4*polylog(2,1/3*(1+I*tan(x))^2
/(1+tan(x)^2))

Maxima [A]
time = 0.38, size = 84, normalized size = 1.05

$x\arctan\left(2\tan\left(x\right)\right)-\frac18\log\left(4\tan\left(x\right)^2+4\right)\log\left(4\tan\left(x\right)^2+1\right)+\frac18\log\left(4\tan\left(x\right)^2+1\right)\log\left(\frac49\tan\left(x\right)^2+\frac49\right)-\frac14\operatorname{Li}_2\left(2i\tan\left(x\right)-1\right)+\frac14\operatorname{Li}_2\left(\frac23i\tan\left(x\right)+\frac13\right)+\frac14\operatorname{Li}_2\left(-\frac23i\tan\left(x\right)+\frac13\right)-\frac14\operatorname{Li}_2\left(-2i\tan\left(x\right)-1\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(2*tan(x)),x, algorithm="maxima")

[Out] x*arctan(2*tan(x)) - 1/8*log(4*tan(x)^2 + 4)*log(4*tan(x)^2 + 1) + 1/8*log(
4*tan(x)^2 + 1)*log(4/9*tan(x)^2 + 4/9) - 1/4*dilog(2*I*tan(x) - 1) + 1/4*d
ilog(2/3*I*tan(x) + 1/3) + 1/4*dilog(-2/3*I*tan(x) + 1/3) - 1/4*dilog(-2*I*
tan(x) - 1)

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(50) = 100.
time = 0.35, size = 220, normalized size = 2.75

$x\arctan\left(2\tan\left(x\right)\right)-\frac14ix\log\left(\frac{2\left(2\tan\left(x\right)^2+3i\tan\left(x\right)-1\right)}{\tan\left(x\right)^2+1}\right)+\frac14ix\log\left(\frac{2\left(2\tan\left(x\right)^2+i\tan\left(x\right)+1\right)}{3\left(\tan\left(x\right)^2+1\right)}\right)-\frac14ix\log\left(\frac{2\left(2\tan\left(x\right)^2-i\tan\left(x\right)+1\right)}{3\left(\tan\left(x\right)^2+1\right)}\right)+\frac14ix\log\left(\frac{2\left(2\tan\left(x\right)^2-3i\tan\left(x\right)-1\right)}{\tan\left(x\right)^2+1}\right)+\frac18\operatorname{Li}_2\left(-\frac{2\left(2\tan\left(x\right)^2+3i\tan\left(x\right)-1\right)}{\tan\left(x\right)^2+1}+1\right)-\frac18\operatorname{Li}_2\left(-\frac{2\left(2\tan\left(x\right)^2+i\tan\left(x\right)+1\right)}{3\left(\tan\left(x\right)^2+1\right)}+1\right)-\frac18\operatorname{Li}_2\left(-\frac{2\left(2\tan\left(x\right)^2-i\tan\left(x\right)+1\right)}{3\left(\tan\left(x\right)^2+1\right)}+1\right)+\frac18\operatorname{Li}_2\left(-\frac{2\left(2\tan\left(x\right)^2-3i\tan\left(x\right)-1\right)}{\tan\left(x\right)^2+1}+1\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(2*tan(x)),x, algorithm="fricas")

[Out] x*arctan(2*tan(x)) - 1/4*I*x*log(2*(2*tan(x)^2 + 3*I*tan(x) - 1)/(tan(x)^2
+ 1)) + 1/4*I*x*log(2/3*(2*tan(x)^2 + I*tan(x) + 1)/(tan(x)^2 + 1)) - 1/4*I

```
*x*log(2/3*(2*tan(x)^2 - I*tan(x) + 1)/(tan(x)^2 + 1)) + 1/4*I*x*log(2*(2*t
an(x)^2 - 3*I*tan(x) - 1)/(tan(x)^2 + 1)) + 1/8*dilog(-2*(2*tan(x)^2 + 3*I*
tan(x) - 1)/(tan(x)^2 + 1) + 1) - 1/8*dilog(-2/3*(2*tan(x)^2 + I*tan(x) + 1
)/(tan(x)^2 + 1) + 1) - 1/8*dilog(-2/3*(2*tan(x)^2 - I*tan(x) + 1)/(tan(x)^
2 + 1) + 1) + 1/8*dilog(-2*(2*tan(x)^2 - 3*I*tan(x) - 1)/(tan(x)^2 + 1) + 1
)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}\left(2\tan\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(2*tan(x)),x)
```

```
[Out] Integral(atan(2*tan(x)), x)
```

Giac [**F**] N/A

time = 0.00, size = 0, normalized size = 0.00

$$\text{Could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(2*tan(x)),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atan}\left(2\tan\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(2*tan(x)),x)
```

```
[Out] int(atan(2*tan(x)), x)
```

3.34 $\quad \int \dfrac{\tan^{-1}(x)\log(x)}{x}\,dx$

Optimal. Leaf size=57

$$\frac{1}{2}i\log(x)\mathrm{Li}_2(-ix) - \frac{1}{2}i\log(x)\mathrm{Li}_2(ix) - \frac{1}{2}i\mathrm{Li}_3(-ix) + \frac{1}{2}i\mathrm{Li}_3(ix)$$

[Out] 1/2*I*ln(x)*polylog(2,-I*x)-1/2*I*ln(x)*polylog(2,I*x)-1/2*I*polylog(3,-I*x)+1/2*I*polylog(3,I*x)

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\dfrac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4940, 2438, 5125, 2421, 6724}

$$-\frac{1}{2}i\mathrm{Li}_3(-ix) + \frac{1}{2}i\mathrm{Li}_3(ix) + \frac{1}{2}i\mathrm{Li}_2(-ix)\log(x) - \frac{1}{2}i\mathrm{Li}_2(ix)\log(x)$$

Antiderivative was successfully verified.

[In] Int[(ArcTan[x]*Log[x])/x,x]

[Out] (I/2)*Log[x]*PolyLog[2, (-I)*x] - (I/2)*Log[x]*PolyLog[2, I*x] - (I/2)*PolyLog[3, (-I)*x] + (I/2)*PolyLog[3, I*x]

Rule 2421

Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d*e, 1]

Rule 2438

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 4940

Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x] + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 + I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]

Rule 5125

Int[(ArcTan[(c_.)*(x_)^(n_.)]*Log[(d_.)*(x_)^(m_.)])/(x_), x_Symbol] :> Dist[I/2, Int[Log[d*x^m]*(Log[1 - I*c*x^n]/x), x], x] - Dist[I/2, Int[Log[d*x^

```
m]*(Log[1 + I*c*x^n]/x), x], x] /; FreeQ[{c, d, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tan^{-1}(x)\log(x)}{x}\,dx = \frac{1}{2}i\int \frac{\log(1-ix)\log(x)}{x}\,dx - \frac{1}{2}i\int \frac{\log(1+ix)\log(x)}{x}\,dx$$

$$= \frac{1}{2}i\log(x)\mathrm{Li}_2(-ix) - \frac{1}{2}i\log(x)\mathrm{Li}_2(ix) - \frac{1}{2}i\int \frac{\mathrm{Li}_2(-ix)}{x}\,dx + \frac{1}{2}i\int \frac{\mathrm{Li}_2(ix)}{x}\,dx$$

$$= \frac{1}{2}i\log(x)\mathrm{Li}_2(-ix) - \frac{1}{2}i\log(x)\mathrm{Li}_2(ix) - \frac{1}{2}i\mathrm{Li}_3(-ix) + \frac{1}{2}i\mathrm{Li}_3(ix)$$

Mathematica [A]
time = 0.05, size = 44, normalized size = 0.77

$$\frac{1}{2}i(\log(x)\mathrm{Li}_2(-ix) - \log(x)\mathrm{Li}_2(ix) - \mathrm{Li}_3(-ix) + \mathrm{Li}_3(ix))$$

Antiderivative was successfully verified.

```
[In] Integrate[(ArcTan[x]*Log[x])/x,x]
```

```
[Out] (I/2)*(Log[x]*PolyLog[2, (-I)*x] - Log[x]*PolyLog[2, I*x] - PolyLog[3, (-I)
*x] + PolyLog[3, I*x])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[ArcTan[x]*Log[x]/x,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [A]
time = 0.49, size = 71, normalized size = 1.25

| method | result |

risch	$\dfrac{i\ln(x)^2\ln(-i(x+i))}{4} - \dfrac{i\ln(x)^2\ln(-ix+1)}{4} - \dfrac{i\ln(x)\,\text{polylog}(2,ix)}{2} + \dfrac{i\,\text{polylog}(3,ix)}{2} + \dfrac{i\ln(x)\,\text{polylog}(2,-ix)}{2} - \dfrac{i\,\text{polylog}(3,-ix}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(x)*ln(x)/x,x,method=_RETURNVERBOSE)

[Out] 1/4*I*ln(x)^2*ln(-I*(x+I))-1/4*I*ln(x)^2*ln(1-I*x)-1/2*I*ln(x)*polylog(2,I*x)+1/2*I*polylog(3,I*x)+1/2*I*ln(x)*polylog(2,-I*x)-1/2*I*polylog(3,-I*x)

Maxima [A]
time = 0.39, size = 31, normalized size = 0.54

$$-\frac{1}{2}i\,\text{Li}_2\left(i\,x\right)\log\left(x\right) + \frac{1}{2}i\,\text{Li}_2\left(-i\,x\right)\log\left(x\right) + \frac{1}{2}i\,\text{Li}_3(i\,x) - \frac{1}{2}i\,\text{Li}_3(-i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(x)*log(x)/x,x, algorithm="maxima")

[Out] -1/2*I*dilog(I*x)*log(x) + 1/2*I*dilog(-I*x)*log(x) + 1/2*I*polylog(3, I*x) - 1/2*I*polylog(3, -I*x)

Fricas [F]
time = 0.34, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(x)*log(x)/x,x, algorithm="fricas")

[Out] integral(arctan(x)*log(x)/x, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(x\right)\text{atan}\left(x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(x)*ln(x)/x,x)

[Out] Integral(log(x)*atan(x)/x, x)

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

$$\text{Could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x)/x,x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atan}(x)\ln(x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(x)*log(x))/x,x)
```

```
[Out] int((atan(x)*log(x))/x, x)
```

```
   f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5000

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)), Int[(d + e*x^2)^(q - 1)*(a + b*Arc
Tan[c*x])^p, x], x] + Dist[b^2*d*p*((p - 1)/(2*q*(2*q + 1))), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[x*(d + e*x^2)^q*((a +
 b*ArcTan[c*x])^p/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5008

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \sqrt{1+x^2}\,\tan^{-1}(x)^2\,dx = -\sqrt{1+x^2}\,\tan^{-1}(x) + \frac{1}{2}x\sqrt{1+x^2}\,\tan^{-1}(x)^2 + \frac{1}{2}\int \frac{\tan^{-1}(x)^2}{\sqrt{1+x^2}}\,dx + \int \frac{1}{\sqrt{1+x}}$$

$$= \sinh^{-1}(x) - \sqrt{1+x^2}\,\tan^{-1}(x) + \frac{1}{2}x\sqrt{1+x^2}\,\tan^{-1}(x)^2 + \frac{1}{2}\mathrm{Subst}\left(\int x^2\sec(x)\right)$$

$$= \sinh^{-1}(x) - \sqrt{1+x^2}\,\tan^{-1}(x) + \frac{1}{2}x\sqrt{1+x^2}\,\tan^{-1}(x)^2 - i\tan^{-1}\left(e^{i\tan^{-1}(x)}\right)t$$

$$= \sinh^{-1}(x) - \sqrt{1+x^2}\,\tan^{-1}(x) + \frac{1}{2}x\sqrt{1+x^2}\,\tan^{-1}(x)^2 - i\tan^{-1}\left(e^{i\tan^{-1}(x)}\right)t$$

$$= \sinh^{-1}(x) - \sqrt{1+x^2}\,\tan^{-1}(x) + \frac{1}{2}x\sqrt{1+x^2}\,\tan^{-1}(x)^2 - i\tan^{-1}\left(e^{i\tan^{-1}(x)}\right)t$$

$$= \sinh^{-1}(x) - \sqrt{1+x^2}\,\tan^{-1}(x) + \frac{1}{2}x\sqrt{1+x^2}\,\tan^{-1}(x)^2 - i\tan^{-1}\left(e^{i\tan^{-1}(x)}\right)t$$

Mathematica [A]
time = 0.14, size = 131, normalized size = 1.08

$$-\sqrt{1+x^2}\,\tan^{-1}(x) + \frac{1}{2}x\sqrt{1+x^2}\,\tan^{-1}(x)^2 - i\tan^{-1}\left(e^{i\tan^{-1}(x)}\right)\tan^{-1}(x)^2 + \tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) + i\tan^{-1}(x)\mathrm{Li}_2\left(-ie^{i\tan^{-1}(x)}\right) - i\tan^{-1}(x)\mathrm{Li}_2\left(ie^{i\tan^{-1}(x)}\right) - \mathrm{Li}_3\left(-ie^{i\tan^{-1}(x)}\right) + \mathrm{Li}_3\left(ie^{i\tan^{-1}(x)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 + x^2]*ArcTan[x]^2,x]
```

```
[Out] -(Sqrt[1 + x^2]*ArcTan[x]) + (x*Sqrt[1 + x^2]*ArcTan[x]^2)/2 - I*ArcTan[E^(
I*ArcTan[x])]*ArcTan[x]^2 + ArcTanh[x/Sqrt[1 + x^2]] + I*ArcTan[x]*PolyLog[
2, (-I)*E^(I*ArcTan[x])] - I*ArcTan[x]*PolyLog[2, I*E^(I*ArcTan[x])] - Poly
Log[3, (-I)*E^(I*ArcTan[x])] + PolyLog[3, I*E^(I*ArcTan[x])]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[ArcTan[x]^2*Sqrt[1 + x^2],x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.33, size = 171, normalized size = 1.41

method	result

default	$\dfrac{\sqrt{x^2+1}\ \arctan(x)(x\arctan(x)-2)}{2} + \dfrac{\arctan(x)^2\ln\left(1-\dfrac{i(ix+1)}{\sqrt{x^2+1}}\right)}{2} - \dfrac{\arctan(x)^2\ln\left(1+\dfrac{i(ix+1)}{\sqrt{x^2+1}}\right)}{2} - i\arctan($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)^2*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x^2+1)^(1/2)*arctan(x)*(x*arctan(x)-2)+1/2*arctan(x)^2*ln(1-I*(1+I*x)/
(x^2+1)^(1/2))-1/2*arctan(x)^2*ln(1+I*(1+I*x)/(x^2+1)^(1/2))-I*arctan(x)*po
lylog(2,I*(1+I*x)/(x^2+1)^(1/2))+I*arctan(x)*polylog(2,-I*(1+I*x)/(x^2+1)^(
1/2))+polylog(3,I*(1+I*x)/(x^2+1)^(1/2))-polylog(3,-I*(1+I*x)/(x^2+1)^(1/2)
)-2*I*arctan((1+I*x)/(x^2+1)^(1/2))
```

Maxima [F]
time $= 0.00$, size $= 0$, normalized size $= 0.00$

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2*(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 1)*arctan(x)^2, x)
```

Fricas [F]
time $= 0.34$, size $= 0$, normalized size $= 0.00$

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2*(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 1)*arctan(x)^2, x)
```

Sympy [F]
time $= 0.00$, size $= 0$, normalized size $= 0.00$

$$\int \sqrt{x^2+1}\ \text{atan}^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)**2*(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)*atan(x)**2, x)
```

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

$$\text{Could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2*(x^2+1)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atan}(x)^2 \sqrt{x^2 + 1} \ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x)^2*(x^2 + 1)^(1/2),x)
```

```
[Out] int(atan(x)^2*(x^2 + 1)^(1/2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*   "F" if the result fails to integrate an expression that*)
(*       is integrable*)
(*   "C" if result involves higher level functions than necessary*)
(*   "B" if result is more than twice the size of the optimal*)
(*       antiderivative*)
(*   "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
```

```
                            ,(*ELSE*)
                                finalresult={"B","Both result and optimal contain complex but leaf count is
                            ]
                        ,(*ELSE*)
                            finalresult={"C","Result contains complex when optimal does not."}
                        ]
                    ,(*ELSE*)(*result does not contains complex*)
                        If[leafCountResult<=2*leafCountOptimal,
                            finalresult={"A","none"}
                        ,(*ELSE*)
                            finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
                        ]
                ]
            ,(*ELSE*) (*expnResult>expnOptimal*)
                If[FreeQ[result,Integrate] && FreeQ[result,Int],
                    finalresult={"C","Result contains higher order function than in optimal. Order "<>"
                    ,
                    finalresult={"F","Contains unresolved integral."}
                ]
            ];

        finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
```

```
        If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
          1,
        Max[ExpnType[expn[[1]]],2]],
      Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]],
    If[Head[expn]===Plus || Head[expn]===Times,
      Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
    If[ElementaryFunctionQ[Head[expn]],
      Max[3,ExpnType[expn[[1]]]],
    If[SpecialFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
    If[HypergeometricFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
    If[AppellFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
    If[Head[expn]===RootSum,
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    If[Head[expn]===Integrate || Head[expn]===Int,
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
    9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade


GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues."
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result   := ExpnType(result);
        ExpnType_optimal  := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#    "F" if the result fails to integrate an expression that
#        is integrable
#    "C" if result involves higher level functions than necessary
#    "B" if result is more than twice the size of the optimal
#        antiderivative
#    "A" if result can be considered optimal
```

```
#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.


if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;


if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A","";
            else
                return "B",cat("Both result and optimal contain complex but leaf count of
                            convert(leaf_count_result,string)," vs. $2 (",
                            convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
        # this assumes optimal do not as well. No check is needed here.
        if debug then
                print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A","";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
```

```
                    fi;
                    return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_coun
              fi;
          fi;
      else #ExpnType(result) > ExpnType(optimal)
          if debug then
              print("ExpnType(result) > ExpnType(optimal)");
          fi;
          return "C",cat("Result contains higher order function than in optimal. Order ",
                         convert(ExpnType_result,string)," vs. order ",
                         convert(ExpnType_optimal,string),".");
      fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
     1
  elif type(expn,'list') then
     apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
     if type(op(1,expn),'rational') then
        1
     else
```

```
          max(2,ExpnType(op(1,expn)))
      end if
  elif type(expn,''^'') then
      if type(op(2,expn),'integer') then
        ExpnType(op(1,expn))
      elif type(op(2,expn),'rational') then
        if type(op(1,expn),'rational') then
            1
        else
            max(2,ExpnType(op(1,expn)))
        end if
      else
            max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
      end if
  elif type(expn,''+'') or type(expn,''*'') then
      max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
      max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
      max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
      max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
      max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
      max(8,apply(max,map(ExpnType,[op(expn)]))) else
      9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
```

```
end proc:

HypergeometricFunctionQ := proc(func)
   member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
   member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
   if nops(u)=2 then
      op(2,u)
   else
      apply(op(0,u),op(2..nops(u),u))
   end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3   Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#          Port of original Maple grading function by
#          Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added 'RootSum'. See problem 177, Timofeev file
#          added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
```

```python
            return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
            fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
            gamma,loggamma,digamma,zeta,polylog,LambertW,
            elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'^')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
```

```python
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,"+") or type(expn,"*")
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9


#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4   SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;
```

```python
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """

    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
    'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
```

```
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
```

```python
    if debug:
        print (">>>>>>Enter expnType, expn=", expn)
        print (">>>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation ="none"
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```