Computer algebra independent integration tests

Summer 2022 edition

0-Independent-test-suites/11-Welz-Problems

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [116]. This is test number [11].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.0.1 (February 17, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.0.1.debian on Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem via sagemath 9.6.
5. Fricas 1.3.7 (June 30, 2021) based on based on ecl 21.2.1 on Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem via sagemath 9.6.
6. Giac/Xcas 1.9.0-7 (April 2022) on on Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem. Direct testing using C++ API.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem via sagemath 9.6.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Mathics 4.0 via sagemath 9.6.

Maxima, Fricas, Mathics are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems. Mathics was called using its own interface in Sagemath as in this example

```
from sage.interfaces.mathics import mathics
res = mathics('Integrate[Sin[x]/(3 + Cos[x])^2,x]')
```

Sympy was called directly from Python. Giac was also called directly via its C++ interface.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	97.41 (113)	2.59 (3)
Mathematica	87.07 (101)	12.93 (15)
Fricas	78.45 (91)	21.55 (25)
Maple	68.10 (79)	31.90 (37)
Mupad	31.90 (37)	68.10 (79)
Giac	29.31 (34)	70.69 (82)
Sympy	25.00 (29)	75.00 (87)
Mathics	24.14 (28)	75.86 (88)
Maxima	17.24 (20)	82.76 (96)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

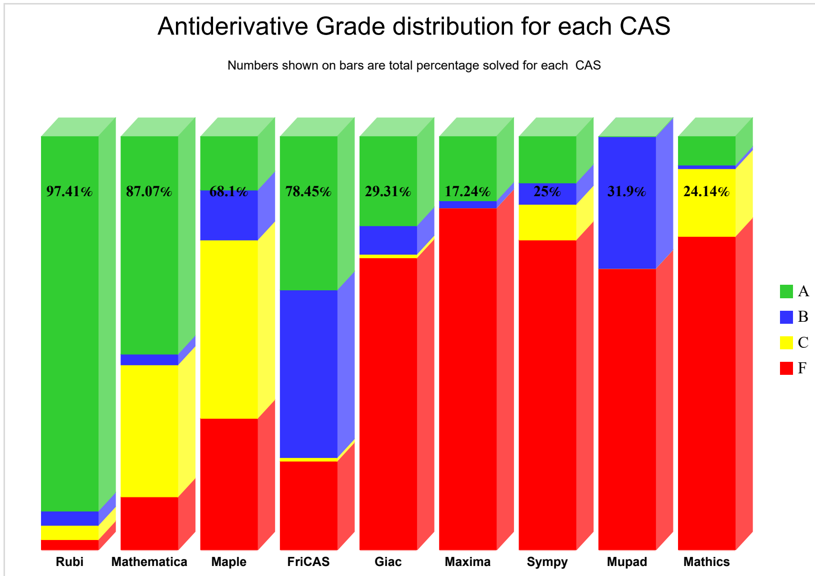
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

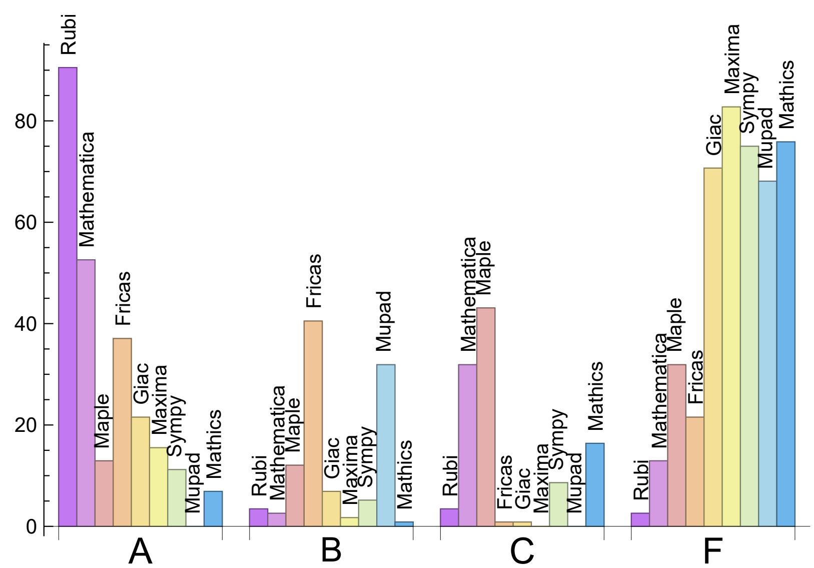
System	% A grade	% B grade	% C grade	% F grade
Rubi	90.52	3.45	3.45	2.59
Mathematica	52.59	2.59	31.90	12.93
Fricas	37.07	40.52	0.86	21.55
Giac	21.55	6.90	0.86	70.69
Maxima	15.52	1.72	0.00	82.76
Maple	12.93	12.07	43.10	31.90
Sympy	11.21	5.17	8.62	75.00
Mathics	6.90	0.86	16.38	75.86
Mupad	N/A	31.90	0.00	68.10

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	3	100.00 %	0.00 %	0.00 %
Mathematica	15	100.00 %	0.00 %	0.00 %
Maple	37	100.00 %	0.00 %	0.00 %
Fricas	25	40.00 %	28.00 %	32.00 %
Giac	82	100.00 %	0.00 %	0.00 %
Maxima	96	98.96 %	0.00 %	1.04 %
Sympy	87	90.80 %	6.90 %	2.30 %
Mupad	79	98.73 %	1.27 %	0.00 %
Mathics	88	0.00 %	31.82 %	68.18 %

Table 1.4: Failure statistics for each CAS

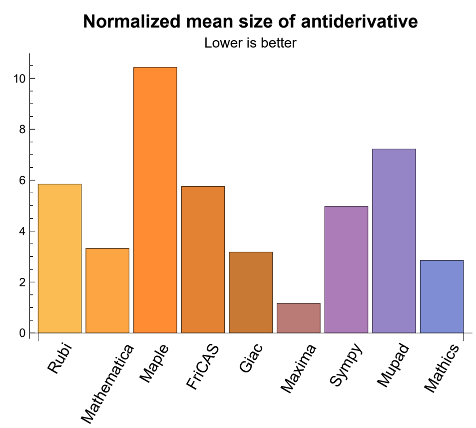
1.3 Performance

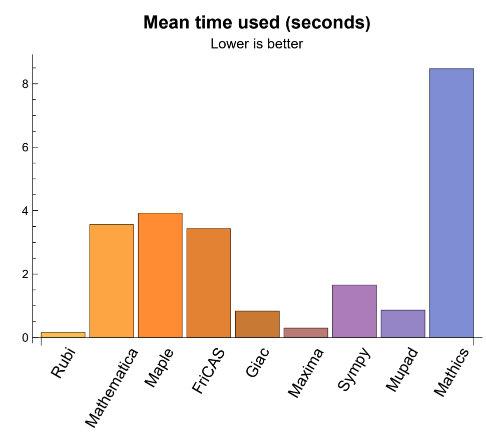
The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.16	162.17	5.85	88.00	1.00
Mathematica	3.56	680.89	3.32	86.00	1.07
Maple	3.92	1793.58	10.42	317.00	2.34
Maxima	0.30	81.80	1.16	62.00	1.05
Fricas	3.43	808.51	5.75	191.00	1.91
Sympy	1.65	202.10	4.96	37.00	0.82
Giac	0.83	770.82	3.18	84.50	1.41
Mupad	0.86	151.70	7.22	76.00	1.09
Mathics	8.47	97.57	2.85	29.50	0.78

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

Mathics {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{82\}$

Mathematica $\{53, 69, 70, 71, 72, 77, 78, 79, 80, 115\}$

Mathics $\{14, 15, 17, 19, 20, 22, 26, 27, 28, 30, 31, 33, 34, 35, 36, 56, 57, 106, 107\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi`, `Mathematica` and `Mathics`. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

For Giac, the call `taille(anti_derivative,RAND_MAX);` is used to find leaf size.

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

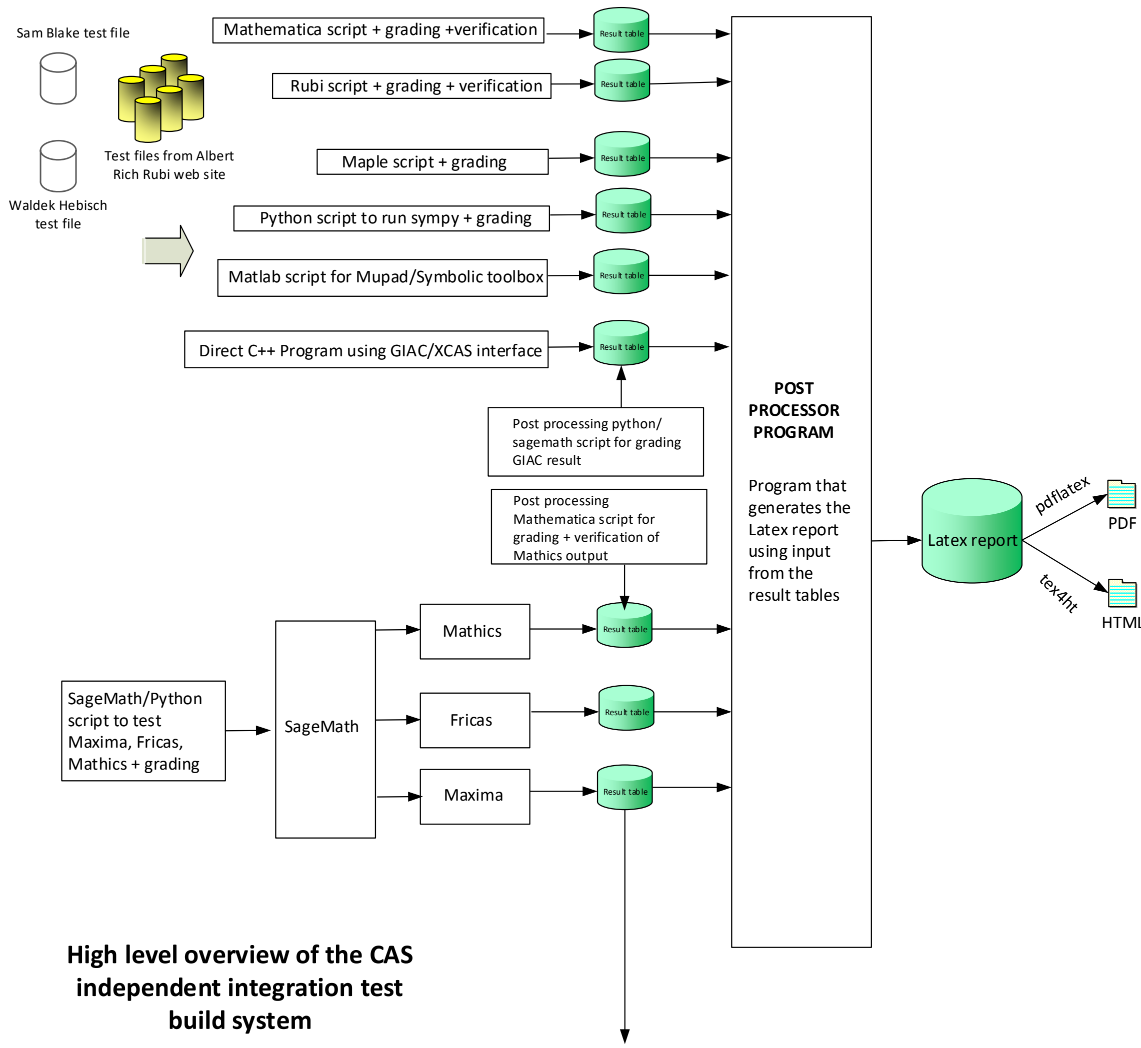
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi

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Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

B grade: { 10, 100, 101, 102 }

C grade: { 2, 52, 82, 83 }

F grade: { 43, 44, 45 }

2.1.2 Mathematica

A grade: { 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 41, 42, 43, 52, 55, 56, 57, 62, 66, 67, 84, 85, 90, 91, 92, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 113, 116 }

B grade: { 12, 13, 54 }

C grade: { 2, 24, 40, 47, 48, 49, 50, 51, 53, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 94, 96, 106, 114, 115 }

F grade: { 38, 44, 45, 46, 58, 59, 60, 61, 93, 95, 108, 109, 110, 111, 112 }

2.1.3 Maple

A grade: { 1, 7, 8, 16, 21, 22, 23, 32, 47, 48, 49, 62, 103, 104, 105 }

B grade: { 3, 4, 5, 6, 9, 10, 11, 17, 24, 31, 50, 51, 65, 68 }

C grade: { 2, 15, 28, 33, 34, 35, 36, 37, 38, 39, 40, 52, 55, 56, 57, 58, 59, 63, 64, 66, 67, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 99, 100, 101, 102, 106, 107, 110, 116 }

F grade: { 12, 13, 14, 18, 19, 20, 25, 26, 27, 29, 30, 41, 42, 43, 44, 45, 46, 53, 54, 60, 61, 69, 70, 71, 72, 80, 94, 95, 96, 98, 108, 109, 111, 112, 113, 114, 115 }

2.1.4 Maxima

A grade: { 1, 6, 16, 21, 22, 23, 29, 32, 33, 34, 35, 36, 56, 57, 62, 96, 99, 107 }

B grade: { 2, 106 }

C grade: { }

F grade: { 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 24, 25, 26, 27, 28, 30, 31, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 5, 6, 8, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 30, 31, 32, 33, 34, 36, 40, 52, 56, 57, 62, 63, 64, 65, 66, 67, 81, 83, 99, 106, 107, 113, 116 }

B grade: { 4, 7, 9, 10, 12, 13, 14, 24, 35, 37, 39, 41, 42, 43, 45, 47, 48, 49, 50, 51, 55, 59, 68, 73, 74, 75, 76, 77, 78, 79, 80, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 100, 101, 102, 110 }

C grade: { 82 }

F grade: { 29, 38, 44, 46, 53, 54, 58, 60, 61, 69, 70, 71, 72, 84, 92, 95, 103, 104, 105, 108, 109, 111, 112, 114, 115 }

2.1.6 Sympy

A grade: { 1, 2, 14, 15, 20, 21, 22, 23, 25, 26, 62, 63, 64 }

B grade: { 8, 16, 17, 19, 30, 31 }

C grade: { 27, 28, 33, 34, 35, 36, 56, 57, 106, 107 }

F grade: { 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 18, 24, 29, 32, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

2.1.7 Giac

A grade: { 1, 6, 8, 11, 16, 19, 20, 21, 22, 23, 25, 26, 30, 33, 34, 36, 41, 49, 57, 62, 63, 64, 96, 99, 107 }

B grade: { 2, 3, 4, 5, 7, 24, 47, 48 }

C grade: { 50 }

F grade: { 9, 10, 12, 13, 14, 15, 17, 18, 27, 28, 29, 31, 32, 35, 37, 38, 39, 40, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 5, 8, 16, 19, 20, 21, 22, 23, 25, 26, 30, 33, 34, 35, 36, 41, 47, 48, 49, 57, 62, 63, 64, 73, 74, 75, 81, 82, 84, 85, 96, 99, 106, 107 }

C grade: { }

F grade: { 4, 6, 7, 9, 10, 11, 12, 13, 14, 15, 17, 18, 24, 27, 28, 29, 31, 32, 37, 38, 39, 40, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 76, 77, 78, 79, 80, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

2.1.9 Mathics

A grade: $\{ 1, 8, 21, 23, 25, 62, 63, 64 \}$

B grade: $\{ 16 \}$

C grade: $\{ 14, 15, 17, 19, 20, 22, 26, 27, 28, 30, 31, 33, 34, 35, 36, 56, 57, 106, 107 \}$

F grade: $\{ 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 18, 24, 29, 32, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 \}$

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	15	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	1.00	0.87	0.87
time (sec)	N/A	0.001	0.007	0.088	0.245	0.300	0.030	0.006	0.027	1.589

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	C	C	C	B	A	A	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	52	37	42	41	1	42	93	43	0
N.S.	1	3.47	2.47	2.80	2.73	0.07	2.80	6.20	2.87	0.00
time (sec)	N/A	0.040	0.019	0.102	0.243	0.299	30.498	0.013	0.515	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	A	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	60	370	0	100	0	201	204	0
N.S.	1	1.00	0.73	4.51	0.00	1.22	0.00	2.45	2.49	0.00
time (sec)	N/A	0.049	0.190	0.165	0.000	0.310	0.000	0.021	0.569	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	71	172	0	80	0	108	-1	0
N.S.	1	1.00	1.65	4.00	0.00	1.86	0.00	2.51	-0.02	0.00
time (sec)	N/A	0.008	0.079	0.185	0.000	0.318	0.000	0.015	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	A	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	69	115	0	105	0	165	82	0
N.S.	1	1.00	0.93	1.55	0.00	1.42	0.00	2.23	1.11	0.00
time (sec)	N/A	0.038	0.074	0.015	0.000	0.319	0.000	0.028	1.707	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	66	150	53	89	0	114	-1	0
N.S.	1	1.00	1.03	2.34	0.83	1.39	0.00	1.78	-0.02	0.00
time (sec)	N/A	0.016	0.183	0.125	0.330	0.601	0.000	0.006	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	55	45	0	83	0	122	-1	0
N.S.	1	1.00	1.15	0.94	0.00	1.73	0.00	2.54	-0.02	0.00
time (sec)	N/A	0.008	0.106	0.138	0.000	0.307	0.000	0.003	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	26	21	0	18	53	53	21	32
N.S.	1	1.00	0.87	0.70	0.00	0.60	1.77	1.77	0.70	1.07
time (sec)	N/A	0.050	0.176	0.082	0.000	0.307	0.309	0.003	0.381	2.092

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	220	365	195	902	0	424	0	0	-1	0
N.S.	1	1.66	0.89	4.10	0.00	1.93	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.347	6.172	0.121	0.000	0.321	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	B	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	220	541	195	1637	0	424	0	0	-1	0
N.S.	1	2.46	0.89	7.44	0.00	1.93	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.439	6.357	0.251	0.000	0.334	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	126	278	0	161	0	213	-1	0
N.S.	1	1.00	0.91	2.01	0.00	1.17	0.00	1.54	-0.01	0.00
time (sec)	N/A	0.050	3.730	0.123	0.000	0.318	0.000	0.007	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	272	0	0	394	0	0	-1	0
N.S.	1	1.00	2.18	0.00	0.00	3.15	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.110	2.184	0.033	0.000	2.062	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	205	0	0	369	0	0	-1	0
N.S.	1	1.00	2.53	0.00	0.00	4.56	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.097	0.858	0.035	0.000	3.048	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	A	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	0	0	60	15	0	-1	21
N.S.	1	1.00	1.00	0.00	0.00	1.94	0.48	0.00	-0.03	0.68
time (sec)	N/A	0.033	0.130	0.020	0.000	0.422	0.965	0.000	0.000	2.564

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	A	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	22	0	29	15	0	-1	21
N.S.	1	1.00	1.00	0.67	0.00	0.88	0.45	0.00	-0.03	0.64
time (sec)	N/A	0.034	0.142	0.102	0.000	0.438	0.439	0.000	0.000	1.994

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	28	56	26	15	38
N.S.	1	1.00	1.00	0.84	0.79	1.47	2.95	1.37	0.79	2.00
time (sec)	N/A	0.174	0.017	0.160	0.245	0.307	1.162	0.011	0.400	2.842

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	A	B	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	120	0	32	2147	0	-1	738
N.S.	1	1.00	0.83	2.31	0.00	0.62	41.29	0.00	-0.02	14.19
time (sec)	N/A	0.016	0.123	0.032	0.000	0.318	1.735	0.000	0.000	94.031

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	50	0	0	33	0	0	-1	0
N.S.	1	1.00	0.89	0.00	0.00	0.59	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.016	0.138	0.020	0.000	0.316	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	B	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	0	15	311	16	15	269
N.S.	1	1.00	1.00	0.00	0.00	0.88	18.29	0.94	0.88	15.82
time (sec)	N/A	0.033	0.019	0.023	0.000	0.299	1.255	0.004	0.297	7.678

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	18	36	18	18	52
N.S.	1	1.00	1.00	0.00	0.00	0.90	1.80	0.90	0.90	2.60
time (sec)	N/A	0.035	0.020	0.040	0.000	0.305	0.765	0.003	0.296	2.557

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	39	43	40	52	36	45	58	57
N.S.	1	1.00	0.93	1.02	0.95	1.24	0.86	1.07	1.38	1.36
time (sec)	N/A	0.023	0.044	0.021	0.243	0.304	0.081	0.002	0.413	1.938

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	19	20	19	24	40
N.S.	1	1.00	1.00	0.95	0.91	0.86	0.91	0.86	1.09	1.82
time (sec)	N/A	0.016	0.026	0.015	0.249	0.294	0.063	0.001	0.385	1.745

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	49	50	51	58	51	70	47	70
N.S.	1	1.00	0.79	0.81	0.82	0.94	0.82	1.13	0.76	1.13
time (sec)	N/A	0.059	0.067	0.059	0.248	0.311	0.100	0.002	0.406	2.100

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	239	455	0	358	0	467	-1	0
N.S.	1	1.00	2.78	5.29	0.00	4.16	0.00	5.43	-0.01	0.00
time (sec)	N/A	0.055	0.311	0.736	0.000	0.325	0.000	0.017	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	0	0	15	15	19	15	15
N.S.	1	1.00	1.00	0.00	0.00	0.79	0.79	1.00	0.79	0.79
time (sec)	N/A	0.040	0.027	0.023	0.000	0.306	0.095	0.001	0.423	1.761

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	22	27	25	22	33
N.S.	1	1.00	1.00	0.00	0.00	0.85	1.04	0.96	0.85	1.27
time (sec)	N/A	0.060	0.038	180.000	0.000	0.596	0.514	0.002	0.506	2.126

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	56	0	0	198	46	0	-1	27
N.S.	1	1.00	0.89	0.00	0.00	3.14	0.73	0.00	-0.02	0.43
time (sec)	N/A	0.132	0.060	0.023	0.000	0.321	1.076	0.000	0.000	2.802

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	25	0	216	51	0	-1	33
N.S.	1	1.00	1.00	0.30	0.00	2.63	0.62	0.00	-0.01	0.40
time (sec)	N/A	0.045	0.009	0.020	0.000	0.470	1.789	0.000	0.000	3.425

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	A	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	606	606	412	0	518	0	0	0	-1	0
N.S.	1	1.00	0.68	0.00	0.85	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	2.887	0.183	0.092	0.258	0.303	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	B	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	0	15	311	16	15	269
N.S.	1	1.00	1.00	0.00	0.00	0.88	18.29	0.94	0.88	15.82
time (sec)	N/A	0.032	0.023	0.025	0.000	0.596	1.460	0.002	0.314	7.538

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	A	B	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	46	120	0	32	2147	0	-1	738
N.S.	1	1.00	0.88	2.31	0.00	0.62	41.29	0.00	-0.02	14.19
time (sec)	N/A	0.016	0.139	0.026	0.000	0.311	1.685	0.000	0.000	75.661

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	33	44	48	48	0	0	-1	0
N.S.	1	1.00	0.97	1.29	1.41	1.41	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.028	0.110	0.036	0.291	0.309	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	77	65	62	64	36	81	86	17
N.S.	1	1.00	1.33	1.12	1.07	1.10	0.62	1.40	1.48	0.29
time (sec)	N/A	0.024	0.042	1.092	0.323	0.306	0.422	0.001	0.543	2.030

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	77	48	62	64	37	82	76	17
N.S.	1	1.00	1.33	0.83	1.07	1.10	0.64	1.41	1.31	0.29
time (sec)	N/A	0.022	0.033	1.089	0.332	0.303	0.447	0.001	0.457	2.037

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	B	C	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	86	12	78	82	29	0	10	16
N.S.	1	1.00	1.76	0.24	1.59	1.67	0.59	0.00	0.20	0.33
time (sec)	N/A	0.003	0.026	1.203	0.337	0.478	0.406	0.000	0.334	2.025

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	79	65	62	64	32	77	80	16
N.S.	1	1.00	1.44	1.18	1.13	1.16	0.58	1.40	1.45	0.29
time (sec)	N/A	0.022	0.036	1.930	0.327	0.306	0.422	0.002	0.514	2.021

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	148	1163	0	301	0	0	-1	0
N.S.	1	1.00	1.53	11.99	0.00	3.10	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.026	0.732	5.241	0.000	1.940	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	F	C	F	F(-2)	F	F	F	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	0	1790	0	0	0	0	-1	0
N.S.	1	1.00	0.00	12.34	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.066	3.596	12.289	0.000	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	110	176	162	1069	0	277	0	0	-1	0
N.S.	1	1.60	1.47	9.72	0.00	2.52	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.018	0.184	5.380	0.000	1.111	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	131	85	433	0	120	0	0	-1	0
N.S.	1	1.62	1.05	5.35	0.00	1.48	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.050	0.010	0.895	0.000	0.456	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	117	130	0	0	415	0	97	37	0
N.S.	1	1.77	1.97	0.00	0.00	6.29	0.00	1.47	0.56	0.00
time (sec)	N/A	0.039	0.605	0.043	0.000	1.004	0.000	0.006	0.394	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	79	145	145	0	0	665	0	0	-1	0
N.S.	1	1.84	1.84	0.00	0.00	8.42	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.066	0.705	0.005	0.000	0.850	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	F	A	F	F	B	F	F	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	118	0	189	0	0	1496	0	0	-1	0
N.S.	1	0.00	1.60	0.00	0.00	12.68	0.00	0.00	-0.01	0.00
time (sec)	N/A	21.044	2.188	0.009	0.000	15.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	111	0	0	0	0	0	0	0	-1	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.380	41.062	0.027	0.000	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	F	F	F	F	B	F	F	F	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	176	0	0	0	0	932	0	0	-1	0
N.S.	1	0.00	0.00	0.00	0.00	5.30	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.296	10.423	0.023	0.000	54.991	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	F	F	F	F(-2)	F	F	F	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	493	570	0	0	0	0	0	0	-1	0
N.S.	1	1.16	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.286	10.240	0.086	0.000	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	178	415	0	957	0	1277	343	0
N.S.	1	1.00	0.44	1.02	0.00	2.35	0.00	3.14	0.84	0.00
time (sec)	N/A	0.423	4.101	0.888	0.000	0.326	0.000	0.273	0.474	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	648	648	253	550	0	1563	0	1534	567	0
N.S.	1	1.00	0.39	0.85	0.00	2.41	0.00	2.37	0.88	0.00
time (sec)	N/A	0.785	6.854	0.907	0.000	0.361	0.000	0.409	0.561	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	F	B	F(-2)	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1058	1058	1100	820	0	2763	0	2044	1017	0
N.S.	1	1.00	1.04	0.78	0.00	2.61	0.00	1.93	0.96	0.00
time (sec)	N/A	1.605	16.110	1.181	0.000	39.824	0.000	0.727	0.972	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	B	F	B	F	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	733	21028	0	1873	0	18547	-1	0
N.S.	1	1.00	1.94	55.63	0.00	4.96	0.00	49.07	-0.00	0.00
time (sec)	N/A	0.510	2.583	1.516	0.000	0.377	0.000	26.732	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	638	638	56025	86793	0	2775	0	0	-1	0
N.S.	1	1.00	87.81	136.04	0.00	4.35	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.873	20.245	4.553	0.000	0.490	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	C	A	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	204	99	787	0	546	0	0	-1	0
N.S.	1	3.09	1.50	11.92	0.00	8.27	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.755	1.246	0.171	0.000	0.351	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	145	0	0	0	0	0	-1	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.063	10.151	0.039	0.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	F	F	F(-2)	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	1072	0	0	0	0	0	-1	0
N.S.	1	1.00	5.41	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.056	8.611	0.043	0.000	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	168	1592	0	171	0	0	-1	0
N.S.	1	1.00	1.73	16.41	0.00	1.76	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.009	0.197	8.731	0.000	1.258	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	99	15	105	96	32	0	-1	19
N.S.	1	1.00	1.36	0.21	1.44	1.32	0.44	0.00	-0.01	0.26
time (sec)	N/A	0.008	0.138	1.000	0.330	0.307	0.510	0.000	0.000	2.117

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	88	49	71	73	37	87	83	21
N.S.	1	1.00	1.31	0.73	1.06	1.09	0.55	1.30	1.24	0.31
time (sec)	N/A	0.024	0.042	1.661	0.321	0.319	0.499	0.001	0.366	2.072

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	F	C	F	F(-2)	F	F	F	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	482	482	0	2990	0	0	0	0	-1	0
N.S.	1	1.00	0.00	6.20	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.205	32.468	12.523	0.000	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	F	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	280	408	0	925	0	3085	0	0	-1	0
N.S.	1	1.46	0.00	3.30	0.00	11.02	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.156	5.955	7.064	0.000	5.458	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	F	F	F	F(-2)	F	F	F	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	0	0	0	0	0	0	-1	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.118	33.695	0.007	0.000	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	F	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	0	0	0	0	0	0	-1	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.101	10.102	0.083	0.000	5.720	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	23	23	22	25	23	23
N.S.	1	1.00	1.00	0.96	0.92	0.92	0.88	1.00	0.92	0.92
time (sec)	N/A	0.014	0.008	0.013	0.242	0.303	0.053	0.002	0.074	1.707

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	86	62	0	43	73	56	53	38
N.S.	1	1.00	1.46	1.05	0.00	0.73	1.24	0.95	0.90	0.64
time (sec)	N/A	0.035	0.013	0.026	0.000	0.310	0.083	0.003	0.335	2.122

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	99	70	0	66	100	80	76	60
N.S.	1	1.00	1.27	0.90	0.00	0.85	1.28	1.03	0.97	0.77
time (sec)	N/A	0.046	0.013	0.029	0.000	0.300	0.093	0.003	0.090	2.406

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	57	113	0	56	0	0	-1	0
N.S.	1	1.00	1.16	2.31	0.00	1.14	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.005	0.125	0.417	0.000	0.340	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	365	0	61	0	0	-1	0
N.S.	1	1.00	0.83	6.89	0.00	1.15	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.009	0.154	0.283	0.000	0.345	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	81	1512	0	359	0	0	-1	0
N.S.	1	1.00	1.08	20.16	0.00	4.79	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.060	0.563	0.107	0.000	0.363	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	B	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	92	625	0	2667	0	0	-1	0
N.S.	1	1.00	0.54	3.65	0.00	15.60	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.050	0.234	0.111	0.000	1.748	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F(-1)	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	157	0	0	0	0	0	-1	0
N.S.	1	1.00	1.96	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.026	10.174	0.053	0.000	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F(-1)	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	162	0	0	0	0	0	-1	0
N.S.	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.028	10.171	0.031	0.000	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F(-1)	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	144	0	0	0	0	0	-1	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.027	10.155	0.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F(-1)	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	152	0	0	0	0	0	-1	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.023	10.129	0.035	0.000	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	28	164	0	1191	0	0	653	0
N.S.	1	1.00	0.22	1.29	0.00	9.38	0.00	0.00	5.14	0.00
time (sec)	N/A	0.013	10.029	5.863	0.000	0.452	0.000	0.000	0.449	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	B	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	54	240	0	1666	0	0	331	0
N.S.	1	1.00	0.34	1.53	0.00	10.61	0.00	0.00	2.11	0.00
time (sec)	N/A	0.021	10.047	0.232	0.000	0.525	0.000	0.000	15.034	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	B	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	48	421	0	547	0	0	533	0
N.S.	1	1.00	0.65	5.69	0.00	7.39	0.00	0.00	7.20	0.00
time (sec)	N/A	0.092	10.024	2.232	0.000	0.418	0.000	0.000	0.211	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	32	383	0	497	0	0	-1	0
N.S.	1	1.00	0.31	3.72	0.00	4.83	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.180	10.044	0.392	0.000	0.487	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	126	651	0	1792	0	0	-1	0
N.S.	1	1.00	1.56	8.04	0.00	22.12	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.007	2.860	3.617	0.000	1.280	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	126	239	0	345	0	0	-1	0
N.S.	1	1.00	1.56	2.95	0.00	4.26	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.007	2.690	1.178	0.000	0.957	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	118	938	0	1943	0	0	-1	0
N.S.	1	1.00	1.04	8.30	0.00	17.19	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.009	2.635	11.552	0.000	0.803	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	124	0	0	1685	0	0	-1	0
N.S.	1	1.00	1.14	0.00	0.00	15.46	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.008	2.735	0.048	0.000	0.779	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	159	206	0	85	0	0	217	0
N.S.	1	1.00	1.83	2.37	0.00	0.98	0.00	0.00	2.49	0.00
time (sec)	N/A	0.551	18.286	0.101	0.000	0.343	0.000	0.000	0.169	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	C	C	C	F	C	F	F	B	F(-2)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1	529	127	317	0	70	0	0	207	0
N.S.	1	529.00	127.00	317.00	0.00	70.00	0.00	0.00	207.00	0.00
time (sec)	N/A	1.072	17.903	0.096	0.000	0.329	0.000	0.000	0.478	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	C	C	C	F	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	180	133	536	0	63	0	0	-1	0
N.S.	1	3.91	2.89	11.65	0.00	1.37	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.965	20.530	0.105	0.000	0.324	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	F(-2)	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	258	0	0	0	0	67	0
N.S.	1	1.00	1.06	8.06	0.00	0.00	0.00	0.00	2.09	0.00
time (sec)	N/A	0.061	1.011	1.315	0.000	0.000	0.000	0.000	1.686	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	31	240	0	44	0	0	204	0
N.S.	1	1.00	1.35	10.43	0.00	1.91	0.00	0.00	8.87	0.00
time (sec)	N/A	0.037	0.640	0.273	0.000	0.329	0.000	0.000	0.219	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	47	353	0	7739	0	0	-1	0
N.S.	1	1.00	0.22	1.62	0.00	35.50	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.031	10.046	37.470	0.000	3.864	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	50	350	0	8237	0	0	-1	0
N.S.	1	1.00	0.24	1.67	0.00	39.22	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.023	10.061	37.867	0.000	4.329	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	65	349	0	7910	0	0	-1	0
N.S.	1	1.00	0.29	1.57	0.00	35.63	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.020	10.049	37.967	0.000	3.401	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	68	350	0	8105	0	0	-1	0
N.S.	1	1.00	0.32	1.64	0.00	37.87	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.021	10.047	37.459	0.000	3.410	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	77	327	0	323	0	0	-1	0
N.S.	1	1.00	1.18	5.03	0.00	4.97	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.083	8.075	0.422	0.000	0.405	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	77	311	0	112	0	0	-1	0
N.S.	1	1.00	1.22	4.94	0.00	1.78	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.081	8.047	0.434	0.000	0.398	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	F(-2)	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	92	544	0	0	0	0	-1	0
N.S.	1	1.00	1.74	10.26	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.036	0.574	2.296	0.000	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	F	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	0	1421	0	267	0	0	-1	0
N.S.	1	1.00	0.00	13.16	0.00	2.47	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.057	6.839	4.135	0.000	1.043	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	189	0	0	1252	0	0	-1	0
N.S.	1	1.00	1.93	0.00	0.00	12.78	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.016	0.868	0.045	0.000	118.524	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	F	F	F	F(-1)	F	F	F	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	0	0	0	0	0	0	-1	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.121	10.132	0.089	0.000	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	A	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	182	0	110	387	0	180	157	0
N.S.	1	1.00	1.90	0.00	1.15	4.03	0.00	1.88	1.64	0.00
time (sec)	N/A	0.049	0.332	0.046	0.330	0.339	0.000	0.004	0.590	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	114	907	0	253	0	0	-1	0
N.S.	1	1.00	1.30	10.31	0.00	2.88	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.008	0.220	1.590	0.000	1.564	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	283	0	0	373	0	0	-1	0
N.S.	1	1.00	1.21	0.00	0.00	1.60	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.069	0.744	0.045	0.000	1.368	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	104	657	86	90	0	145	100	0
N.S.	1	1.00	1.27	8.01	1.05	1.10	0.00	1.77	1.22	0.00
time (sec)	N/A	0.037	0.086	2.311	0.330	0.319	0.000	0.003	0.548	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	B	A	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	135	383	145	720	0	318	0	0	-1	0
N.S.	1	2.84	1.07	5.33	0.00	2.36	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.173	0.910	5.210	0.000	5.359	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	B	A	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	135	383	145	737	0	318	0	0	-1	0
N.S.	1	2.84	1.07	5.46	0.00	2.36	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.189	0.888	5.145	0.000	6.298	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	B	A	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	119	357	139	714	0	268	0	0	-1	0
N.S.	1	3.00	1.17	6.00	0.00	2.25	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.181	0.865	5.451	0.000	5.486	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.79	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.034	10.112	0.211	0.000	0.304	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.79	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.051	10.059	0.207	0.000	0.323	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	43	34	0	0	0	0	-1	0
N.S.	1	1.00	1.10	0.87	0.00	0.00	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.017	7.053	0.127	0.000	0.309	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	B	A	C	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	101	12	105	94	31	0	10	16
N.S.	1	1.00	1.51	0.18	1.57	1.40	0.46	0.00	0.15	0.24
time (sec)	N/A	0.007	0.084	1.026	0.356	0.335	0.515	0.000	0.342	2.144

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	90	66	73	75	41	91	91	23
N.S.	1	1.00	1.29	0.94	1.04	1.07	0.59	1.30	1.30	0.33
time (sec)	N/A	0.025	0.048	1.707	0.325	0.318	0.508	0.002	0.403	2.137

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	0	0	0	0	0	0	-1	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.286	14.245	0.004	0.000	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	0	0	0	0	0	0	-1	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.105	20.249	0.003	0.000	1.369	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	F	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	199	250	0	457	0	1827	0	0	-1	0
N.S.	1	1.26	0.00	2.30	0.00	9.18	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.171	12.055	24.621	0.000	1.341	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	0	0	0	0	0	0	-1	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.090	31.902	0.003	0.000	1.912	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	0	0	0	0	0	0	-1	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.097	20.129	0.003	0.000	1.886	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	204	0	0	191	0	0	-1	0
N.S.	1	1.00	1.55	0.00	0.00	1.45	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	0.318	0.043	0.000	0.327	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	26	0	0	0	0	0	-1	0
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.080	10.012	0.038	0.000	1.612	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	383	648	138	0	0	0	0	0	-1	0
N.S.	1	1.69	0.36	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.592	10.154	0.042	0.000	4.750	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	283	681	0	341	0	0	-1	0
N.S.	1	1.00	1.04	2.50	0.00	1.25	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.097	2.016	4.348	0.000	1.554	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [83] had the largest ratio of [51]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	10	0.100
2	C	5	2	3.47	37	0.054
3	A	9	6	1.00	15	0.400
4	A	3	3	1.00	19	0.158
5	A	8	7	1.00	17	0.412
6	A	6	6	1.00	17	0.353
7	A	3	3	1.00	17	0.176
8	A	4	2	1.00	23	0.087
9	A	18	12	1.66	27	0.444
10	B	25	13	2.46	39	0.333
11	A	7	3	1.00	45	0.067
12	A	7	4	1.00	32	0.125
13	A	5	3	1.00	32	0.094
14	A	2	2	1.00	27	0.074
15	A	2	2	1.00	29	0.069
16	A	2	1	1.00	30	0.033
17	A	3	2	1.00	13	0.154
18	A	3	2	1.00	15	0.133
19	A	2	2	1.00	23	0.087
20	A	2	2	1.00	25	0.080
21	A	3	2	1.00	11	0.182
22	A	2	2	1.00	18	0.111
23	A	6	6	1.00	20	0.300
24	A	6	5	1.00	31	0.161
25	A	2	2	1.00	29	0.069

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	2	1.00	35	0.057
27	A	5	5	1.00	32	0.156
28	A	6	6	1.00	21	0.286
29	A	359	30	1.00	14	2.143
30	A	2	2	1.00	23	0.087
31	A	3	2	1.00	13	0.154
32	A	2	2	1.00	33	0.061
33	A	5	5	1.00	15	0.333
34	A	5	5	1.00	15	0.333
35	A	1	1	1.00	11	0.091
36	A	5	5	1.00	15	0.333
37	A	1	1	1.00	17	0.059
38	A	3	3	1.00	18	0.167
39	A	2	2	1.60	16	0.125
40	A	5	5	1.62	17	0.294
41	A	5	5	1.77	13	0.385
42	A	5	5	1.84	16	0.312
43	F	0	0	N/A	0	N/A
44	F	0	0	N/A	0	N/A
45	F	0	0	N/A	0	N/A
46	A	19	14	1.16	32	0.438
47	A	19	9	1.00	20	0.450
48	A	29	9	1.00	20	0.450
49	A	49	9	1.00	20	0.450
50	A	14	6	1.00	23	0.261
51	A	24	6	1.00	23	0.261
52	C	9	8	3.09	48	0.167
53	A	7	7	1.00	24	0.292
54	A	7	7	1.00	24	0.292
55	A	1	1	1.00	18	0.056
56	A	2	2	1.00	13	0.154
57	A	6	6	1.00	15	0.400
58	A	25	15	1.00	17	0.882
59	A	19	12	1.46	22	0.546
60	A	12	11	1.00	17	0.647

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	9	8	1.00	21	0.381
62	A	1	1	1.00	38	0.026
63	A	1	1	1.00	33	0.030
64	A	2	2	1.00	38	0.053
65	A	1	1	1.00	19	0.053
66	A	4	4	1.00	19	0.210
67	A	4	4	1.00	24	0.167
68	A	1	1	1.00	24	0.042
69	A	7	7	1.00	24	0.292
70	A	7	7	1.00	24	0.292
71	A	3	3	1.00	26	0.115
72	A	3	3	1.00	22	0.136
73	A	1	1	1.00	22	0.045
74	A	1	1	1.00	23	0.043
75	A	8	8	1.00	18	0.444
76	A	8	7	1.00	23	0.304
77	A	1	1	1.00	21	0.048
78	A	1	1	1.00	19	0.053
79	A	1	1	1.00	19	0.053
80	A	1	1	1.00	19	0.053
81	A	4	4	1.00	34	0.118
82	C	5	5	529.00	40	0.125
83	C	7	7	3.91	51	0.137
84	A	2	2	1.00	29	0.069
85	A	2	2	1.00	18	0.111
86	A	1	1	1.00	25	0.040
87	A	1	1	1.00	25	0.040
88	A	1	1	1.00	25	0.040
89	A	1	1	1.00	25	0.040
90	A	2	2	1.00	40	0.050
91	A	2	2	1.00	40	0.050
92	A	1	1	1.00	18	0.056
93	A	3	3	1.00	15	0.200
94	A	1	1	1.00	21	0.048
95	A	8	7	1.00	23	0.304

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	5	5	1.00	24	0.208
97	A	1	1	1.00	19	0.053
98	A	8	8	1.00	20	0.400
99	A	5	5	1.00	22	0.227
100	B	16	12	2.84	25	0.480
101	B	17	13	2.84	24	0.542
102	B	16	12	3.00	23	0.522
103	A	5	4	1.00	20	0.200
104	A	5	4	1.00	25	0.160
105	A	3	3	1.00	19	0.158
106	A	2	2	1.00	11	0.182
107	A	6	6	1.00	15	0.400
108	A	13	12	1.00	19	0.632
109	A	13	12	1.00	22	0.546
110	A	14	12	1.26	27	0.444
111	A	5	5	1.00	17	0.294
112	A	6	6	1.00	27	0.222
113	A	3	3	1.00	19	0.158
114	A	10	10	1.00	20	0.500
115	A	17	6	1.69	24	0.250
116	A	14	8	1.00	19	0.421

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{1}{\sqrt{1-ax}}\,dx$

Optimal. Leaf size=15

$$-\frac{2\sqrt{1-ax}}{a}$$

```
[Out] -2*(-a*x+1)^(1/2)/a
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{32\}$

$$-\frac{2\sqrt{1-ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - a*x],x]
```

```
[Out] (-2*Sqrt[1 - a*x])/a
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\sqrt{1-ax}}{a}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$-\frac{2\sqrt{1-ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - a*x],x]
```

```
[Out] (-2*Sqrt[1 - a*x])/a
```

Mathics [A]
time = 1.59, size = 13, normalized size = 0.87

$$\frac{-2\sqrt{1-ax}}{a}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/Sqrt[1 - a*x],x]')
```

```
[Out] -2 Sqrt[1 - a x] / a
```

Maple [A]
time = 0.09, size = 14, normalized size = 0.93

method	result	size
gosper	$-\frac{2\sqrt{-ax+1}}{a}$	14
derivativedivides	$-\frac{2\sqrt{-ax+1}}{a}$	14
default	$-\frac{2\sqrt{-ax+1}}{a}$	14
trager	$-\frac{2\sqrt{-ax+1}}{a}$	14
risch	$\frac{2ax-2}{a\sqrt{-ax+1}}$	19
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-ax+1}}{\sqrt{\pi}\,a}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-a*x+1)^(1/2)/a
```

Maxima [A]
time = 0.24, size = 13, normalized size = 0.87

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-a*x + 1)/a
```

Fricas [A]
time = 0.30, size = 13, normalized size = 0.87

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-a*x + 1)/a
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.80

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)**(1/2),x)
```

```
[Out] -2*sqrt(-a*x + 1)/a
```

Giac [A]
time = 0.01, size = 15, normalized size = 1.00

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/2),x)
```

```
[Out] -2*sqrt(-a*x + 1)/a
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.87

$$-\frac{2\,\sqrt{1-a\,x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - a*x)^(1/2),x)
```

```
[Out] -(2*(1 - a*x)^(1/2))/a
```

3.2 $\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx$

<u>**Optimal**</u>. Leaf size=15

$$-\frac{2\sqrt{1-ax}}{a}$$

```
[Out] -2*(-a*x+1)^(1/2)/a
```

<u>**Rubi**</u> [C] Result contains higher order function than in optimal. Order 3 vs. order 2 in optimal.
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 3.47, number of steps used = 5, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {12, 2332}

$$\frac{\sqrt{ax-1}\,\log(ax-1)}{\pi a} - \frac{2\sqrt{ax-1}\,\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x])/(2*Pi*Sqrt[-1 + a*x]),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Log[-Sqrt[-1 + a*x]])/(a*Pi) + (Sqrt[-1 + a*x]*Log[-1 + a*x])/(a*Pi)
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 2332</u>

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

<u>Rubi steps</u>

$$\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx = \frac{\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{\sqrt{-1+ax}}\,dx}{2\pi}$$

$$= \frac{\text{Subst}\left(\int \left(-2\log(-x)+\log\left(x^2\right)\right)\,dx, x, \sqrt{-1+ax}\right)}{a\pi}$$

$$= \frac{\text{Subst}\left(\int \log\left(x^2\right)\,dx, x, \sqrt{-1+ax}\right)}{a\pi} - \frac{2\text{Subst}\left(\int \log(-x)\ldots\right.}{a\ldots}$$

$$= -\frac{2\sqrt{-1+ax}\,\log\left(-\sqrt{-1+ax}\right)}{a\pi} + \frac{\sqrt{-1+ax}\,\log(-1+\ldots}{a\pi}$$

Mathematica [C] Result contains higher order function than in optimal. Order 3 vs. order 2 in optimal.
time = 0.02, size = 37, normalized size = 2.47

$$\frac{\sqrt{-1+ax}\left(-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)\right)}{a\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x])/(2*Pi*Sqrt[-1 + a*x]),x
]
```

```
[Out] (Sqrt[-1 + a*x]*(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x]))/(a*Pi)
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Log[a*x - 1] - 2*Log[-Sqrt[a*x - 1]])/(2*Pi*Sqrt[a*x -
1]),x]')
```

```
[Out] cought exception:
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.10, size = 42, normalized size = 2.80

method	result	size
gosper	$\frac{\sqrt{ax-1}\left(\ln(ax-1)-2\ln\left(-\sqrt{ax-1}\right)\right)}{a\pi}$	34
derivativedivides	$\frac{-2\ln\left(-\sqrt{ax-1}\right)\sqrt{ax-1}+\sqrt{ax-1}\ \ln(ax-1)}{\pi a}$	42
default	$\frac{-2\ln\left(-\sqrt{ax-1}\right)\sqrt{ax-1}+\sqrt{ax-1}\ \ln(ax-1)}{\pi a}$	42
meijerg	$\frac{i\sqrt{-\mathrm{signum}\left(ax-1\right)}\left(-2\sqrt{\pi}+2\sqrt{\pi}\ \sqrt{-ax+1}\right)}{\sqrt{\pi}\ \sqrt{\mathrm{signum}\left(ax-1\right)}\ a}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*(ln(a*x-1)-2*ln(-(a*x-1)^(1/2)))/Pi/(a*x-1)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/Pi/a*(-2*ln(-(a*x-1)^(1/2))*(a*x-1)^(1/2)+(a*x-1)^(1/2)*ln(a*x-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(13) = 26$.
time = 0.24, size = 41, normalized size = 2.73

$$\frac{\sqrt{ax-1}\,\log\left(ax-1\right)-2\,\sqrt{ax-1}\,\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algori
thm="maxima")
```

```
[Out] (sqrt(a*x - 1)*log(a*x - 1) - 2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)))/(pi*a)
```

Fricas [A]
time = 0.30, size = 1, normalized size = 0.07

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algori
thm="fricas")
```

```
[Out] 0
```

Sympy [A]
time = 30.50, size = 42, normalized size = 2.80

$$\frac{\begin{cases}\frac{-2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)+\sqrt{ax-1}\log\left(ax-1\right)}{a} & \text{for } a\neq 0\\ \pi x & \text{otherwise}\end{cases}}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(ln(a*x-1)-2*ln(-(a*x-1)**(1/2)))/pi/(a*x-1)**(1/2),x)
```

```
[Out] Piecewise(((-2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)) + sqrt(a*x - 1)*log(a*x -
1))/a, Ne(a, 0)), (pi*x, True))/pi
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(13) = 26$.
time = 0.01, size = 93, normalized size = 6.20

$$\frac{-2\left(-\sqrt{ax-1}\,\ln\left(ax-1\right)+\frac{2\sqrt{ax-1}\,\sqrt{ax-1}}{\sqrt{ax-1}}\right)+4\left(-\sqrt{ax-1}\,\ln\left(-\sqrt{ax-1}\right)+\sqrt{ax-1}\right)}{2\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x)
```

```
[Out] (sqrt(a*x - 1)*log(a*x - 1) - 2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)))/(pi*a)
```

Mupad [**B**]
time = 0.51, size = 43, normalized size = 2.87

$$-\frac{2\,\ln\left(-\sqrt{a\,x-1}\right)\,\sqrt{a\,x-1}-\ln\left(a\,x-1\right)\,\sqrt{a\,x-1}}{\Pi\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(a*x - 1)/2 - log(-(a*x - 1)^(1/2)))/(Pi*(a*x - 1)^(1/2)),x)
```

```
[Out] -(2*log(-(a*x - 1)^(1/2))*(a*x - 1)^(1/2) - log(a*x - 1)*(a*x - 1)^(1/2))/(
Pi*a)
```

3.3 $\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2} \, dx$

Optimal. Leaf size=82

$$\frac{4x}{3(1-3x^2)} - \frac{2\sqrt{1+x^2}}{3(1-3x^2)} - \frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\,\sqrt{1+x^2}\right)}{3\sqrt{3}}$$

```
[Out] 4/3*x/(-3*x^2+1)-1/9*arctanh(x*3^(1/2))*3^(1/2)+1/9*arctanh(1/2*3^(1/2)*(x^
2+1)^(1/2))*3^(1/2)-2/3*(x^2+1)^(1/2)/(-3*x^2+1)
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6874, 205, 213, 455, 43, 65}

$$\frac{4x}{3(1-3x^2)} - \frac{2\sqrt{x^2+1}}{3(1-3x^2)} + \frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\,\sqrt{x^2+1}\right)}{3\sqrt{3}} - \frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + Sqrt[1 + x^2])^(-2),x]
```

```
[Out] (4*x)/(3*(1 - 3*x^2)) - (2*Sqrt[1 + x^2])/(3*(1 - 3*x^2)) - ArcTanh[Sqrt[3]
*x]/(3*Sqrt[3]) + ArcTanh[(Sqrt[3]*Sqrt[1 + x^2])/2]/(3*Sqrt[3])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
```

```
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx &= \int \left(\frac{8}{3\left(-1+3x^2\right)^2}-\frac{4x\sqrt{1+x^2}}{\left(-1+3x^2\right)^2}+\frac{5}{3\left(-1+3x^2\right)}\right)dx \\
&= \frac{5}{3}\int \frac{1}{-1+3x^2}\,dx+\frac{8}{3}\int \frac{1}{\left(-1+3x^2\right)^2}\,dx-4\int \frac{x\sqrt{1+x^2}}{\left(-1+3x^2\right)^2}\,dx \\
&= \frac{4x}{3\left(1-3x^2\right)}-\frac{5\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}-\frac{4}{3}\int \frac{1}{-1+3x^2}\,dx-2\text{Subst}\left(\int \frac{\sqrt{1+x}}{(-1+3x)^2}\,d\right. \\
&= \frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{1+x}\,(-1+3x)}\,d\right. \\
&= \frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-4+3x^2}\,dx,x,\sqrt{1}\right. \\
&= \frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\,\sqrt{1+x^2}\right)}{3\sqrt{3}}
\end{aligned}
$$

Mathematica [A]

time = 0.19, size = 60, normalized size = 0.73

$$\frac{6\left(-2x+\sqrt{1+x^2}\right)-2\sqrt{3}\,(-1+3x^2)\tanh^{-1}\left(\frac{x-\sqrt{1+x^2}}{\sqrt{3}}\right)}{-9+27x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + Sqrt[1 + x^2])^(-2),x]
```

```
[Out] (6*(-2*x + Sqrt[1 + x^2]) - 2*Sqrt[3]*(-1 + 3*x^2)*ArcTanh[(x - Sqrt[1 + x^
2])/Sqrt[3]])/(-9 + 27*x^2)
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x^2 + 1] + 2*x)^2,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(60) = 120.
time = 0.16, size = 370, normalized size = 4.51

method	result
trager	$-\frac{4x}{3(3x^2-1)}+\frac{2\sqrt{x^2+1}}{3(3x^2-1)}+\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{2\operatorname{RootOf}\left(\_Z^2-3\right)+3\sqrt{x^2+1}}{\operatorname{RootOf}\left(\_Z^2-3\right)x+1}\right)}{9}$

default	$-\frac{x}{2(3x^2-1)}-\frac{\operatorname{arctanh}\left(x\sqrt{3}\right)\sqrt{3}}{9}-\frac{5x}{18\left(x^2-\frac{1}{3}\right)}-\sqrt{3}\left(-\frac{\left(\left(x-\frac{\sqrt{3}}{3}\right)^2+\frac{2\sqrt{3}\left(x-\frac{\sqrt{3}}{3}\right)}{3}+\frac{4}{3}\right)^{\frac{3}{2}}}{12\left(x-\frac{\sqrt{3}}{3}\right)}+\frac{\sqrt{3}\left(\sqrt{\ldots}\right.}{\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x+(x^2+1)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x/(3*x^2-1)-1/9*arctanh(x*3^(1/2))*3^(1/2)-5/18*x/(x^2-1/3)-3^(1/2)*(-
1/12/(x-1/3*3^(1/2))*((x-1/3*3^(1/2))^2+2/3*3^(1/2)*(x-1/3*3^(1/2))+4/3)^(3
/2)+1/36*3^(1/2)*(1/3*(9*(x-1/3*3^(1/2))^2+6*3^(1/2)*(x-1/3*3^(1/2))+12)^(1
/2)+1/3*3^(1/2)*arcsinh(x)-2/3*3^(1/2)*arctanh(3/4*(8/3+2/3*3^(1/2)*(x-1/3*
3^(1/2)))*3^(1/2)/(9*(x-1/3*3^(1/2))^2+6*3^(1/2)*(x-1/3*3^(1/2))+12)^(1/2))
)+1/12*x*((x-1/3*3^(1/2))^2+2/3*3^(1/2)*(x-1/3*3^(1/2))+4/3)^(1/2)+1/12*arc
sinh(x))+3^(1/2)*(-1/12/(x+1/3*3^(1/2))*((x+1/3*3^(1/2))^2-2/3*3^(1/2)*(x+1
/3*3^(1/2))+4/3)^(3/2)-1/36*3^(1/2)*(1/3*(9*(x+1/3*3^(1/2))^2-6*3^(1/2)*(x+
1/3*3^(1/2))+12)^(1/2)-1/3*3^(1/2)*arcsinh(x)-2/3*3^(1/2)*arctanh(3/4*(8/3-
2/3*3^(1/2)*(x+1/3*3^(1/2)))*3^(1/2)/(9*(x+1/3*3^(1/2))^2-6*3^(1/2)*(x+1/3*
3^(1/2))+12)^(1/2)))+1/12*x*((x+1/3*3^(1/2))^2-2/3*3^(1/2)*(x+1/3*3^(1/2))+
4/3)^(1/2)+1/12*arcsinh(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((2*x + sqrt(x^2 + 1))^(-2), x)
```

Fricas [A]
time = 0.31, size = 100, normalized size = 1.22

$$\frac{\sqrt{3}\,(3\,x^2-1)\log\left(\frac{3\,x^2-2\,\sqrt{3}\,x+1}{3\,x^2-1}\right)+\sqrt{3}\,(3\,x^2-1)\log\left(\frac{3\,x^2+4\,\sqrt{3}\,\sqrt{x^2+1}+7}{3\,x^2-1}\right)-24\,x+12\,\sqrt{x^2+1}}{18\,(3\,x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/18*(sqrt(3)*(3*x^2 - 1)*log((3*x^2 - 2*sqrt(3)*x + 1)/(3*x^2 - 1)) + sqrt
(3)*(3*x^2 - 1)*log((3*x^2 + 4*sqrt(3)*sqrt(x^2 + 1) + 7)/(3*x^2 - 1)) - 24
*x + 12*sqrt(x^2 + 1))/(3*x^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(2x+\sqrt{x^2+1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x**2+1)**(1/2))**2,x)
```

```
[Out] Integral((2*x + sqrt(x**2 + 1))**(-2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(60) = 120.
time = 0.02, size = 201, normalized size = 2.45

$$-\frac{4x}{3\,(3x^2-1)}+\frac{\ln\left|\frac{6x-2\sqrt{3}}{6x+2\sqrt{3}}\right|}{2\cdot 3\sqrt{3}}+2\left(\frac{2\left(\sqrt{x^2+1}-x+\frac{1}{\sqrt{x^2+1}-x}\right)}{3\left(3\left(\sqrt{x^2+1}-x+\frac{1}{\sqrt{x^2+1}-x}\right)^2-16\right)}-\frac{\ln\left(\frac{\left|6\left(\sqrt{x^2+1}-x+\frac{1}{\sqrt{x^2+1}-x}\right)-8\sqrt{3}\right|}{6\left(\sqrt{x^2+1}-x+\frac{1}{\sqrt{x^2+1}-x}\right)+8\sqrt{3}}\right)}{12\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x)
```

```
[Out] -1/18*sqrt(3)*log(-1/2*abs(-6*x - 8*sqrt(3) + 6*sqrt(x^2 + 1) - 6/(x - sqrt
(x^2 + 1)))/(3*x - 4*sqrt(3) - 3*sqrt(x^2 + 1) + 3/(x - sqrt(x^2 + 1)))) +
1/18*sqrt(3)*log(abs((3*x - sqrt(3))/(3*x + sqrt(3)))) - 4/3*(x - sqrt(x^2
+ 1) + 1/(x - sqrt(x^2 + 1)))/(3*(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 + 1))
)^2 - 16) - 4/3*x/(3*x^2 - 1)
```

Mupad [B]
time = 0.57, size = 204, normalized size = 2.49

$$\frac{\sqrt{3}\,\left(\ln\left(x-\frac{\sqrt{3}}{3}\right)-\ln\left(x+\sqrt{3}+2\,\sqrt{x^2+1}\right)\right)}{18}-\frac{4\,x}{9\,\left(x^2-\frac{1}{3}\right)}+\frac{\sqrt{3}\,\left(\ln\left(x+\frac{\sqrt{3}}{3}\right)-\ln\left(x-\sqrt{3}-2\,\sqrt{x^2+1}\right)\right)}{18}-\frac{\sqrt{3}\,\left(6\,\ln\left(x-\frac{\sqrt{3}}{3}\right)-6\,\ln\left(x+\sqrt{3}+2\,\sqrt{x^2+1}\right)\right)}{54}-\frac{\sqrt{3}\,\left(6\,\ln\left(x+\frac{\sqrt{3}}{3}\right)-6\,\ln\left(x-\sqrt{3}-2\,\sqrt{x^2+1}\right)\right)}{54}+\frac{\sqrt{3}\,\sqrt{x^2+1}}{9\,\left(x-\frac{\sqrt{3}}{3}\right)}-\frac{\sqrt{3}\,\sqrt{x^2+1}}{9\,\left(x+\frac{\sqrt{3}}{3}\right)}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + (x^2 + 1)^(1/2))^2,x)
```

```
[Out] (3^(1/2)*(log(x - 3^(1/2)/3) - log(x + 3^(1/2) + 2*(x^2 + 1)^(1/2))))/18 +
(3^(1/2)*atan(3^(1/2)*x*1i)*1i)/9 - (4*x)/(9*(x^2 - 1/3)) + (3^(1/2)*(log(x
 + 3^(1/2)/3) - log(x - 3^(1/2) - 2*(x^2 + 1)^(1/2))))/18 - (3^(1/2)*(6*log
(x - 3^(1/2)/3) - 6*log(x + 3^(1/2) + 2*(x^2 + 1)^(1/2))))/54 - (3^(1/2)*(6
*log(x + 3^(1/2)/3) - 6*log(x - 3^(1/2) - 2*(x^2 + 1)^(1/2))))/54 + (3^(1/2
)*(x^2 + 1)^(1/2))/(9*(x - 3^(1/2)/3)) - (3^(1/2)*(x^2 + 1)^(1/2))/(9*(x +
3^(1/2)/3))
```

3.4 $\int \frac{1}{\sqrt{-1+x^2}\,(-4+3x^2)^2}\,dx$

Optimal. Leaf size=43

$$\frac{3x\sqrt{-1+x^2}}{8(4-3x^2)}+\frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{-1+x^2}}\right)$$

```
[Out] 5/16*arctanh(1/2*x/(x^2-1)^(1/2))+3/8*x*(x^2-1)^(1/2)/(-3*x^2+4)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {390, 385, 213}

$$\frac{3\sqrt{x^2-1}\,x}{8(4-3x^2)}+\frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
```

```
[Out] (3*x*Sqrt[-1 + x^2])/(8*(4 - 3*x^2)) + (5*ArcTanh[x/(2*Sqrt[-1 + x^2])])/16
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x^2}\,(-4+3x^2)^2}\,dx = \frac{3x\sqrt{-1+x^2}}{8(4-3x^2)} - \frac{5}{8}\int \frac{1}{\sqrt{-1+x^2}\,(-4+3x^2)}\,dx$$
$$= \frac{3x\sqrt{-1+x^2}}{8(4-3x^2)} - \frac{5}{8}\text{Subst}\left(\int \frac{1}{-4+x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)$$
$$= \frac{3x\sqrt{-1+x^2}}{8(4-3x^2)} + \frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{-1+x^2}}\right)$$

Mathematica [**A**]
time = 0.08, size = 71, normalized size = 1.65

$$-\frac{3x\sqrt{-1+x^2}}{8(-4+3x^2)} + \frac{5}{32}\log\left(2-x^2+x\sqrt{-1+x^2}\right) - \frac{5}{32}\log\left(2-3x^2+3x\sqrt{-1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
```

```
[Out] (-3*x*Sqrt[-1 + x^2])/(8*(-4 + 3*x^2)) + (5*Log[2 - x^2 + x*Sqrt[-1 + x^2]]
)/32 - (5*Log[2 - 3*x^2 + 3*x*Sqrt[-1 + x^2]])/32
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x^2 - 1]*(3*x^2 - 4)^2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(33) = 66$.
time = 0.18, size = 172, normalized size = 4.00

method	result
trager	$-\frac{3x\sqrt{x^2-1}}{8(3x^2-4)} - \frac{5\ln\left(-\frac{4\sqrt{x^2-1}\,x-5x^2+4}{3x^2-4}\right)}{32}$

risch	$-\frac{3x\sqrt{x^2-1}}{8(3x^2-4)}+\frac{5\,\operatorname{arctanh}\left(\frac{3\left(\frac{2}{3}+\frac{4\sqrt{3}\left(x-\frac{2\sqrt{3}}{3}\right)}{3}\right)\sqrt{3}}{2\sqrt{9\left(x-\frac{2\sqrt{3}}{3}\right)^2+12\sqrt{3}\left(x-\frac{2\sqrt{3}}{3}\right)+3}}\right)}{32}-\frac{5\,\operatorname{arctanh}\left(\frac{\cdots}{2\sqrt{9\left(x+\cdots\right.}}\right.}{\cdots}$
default	$-\frac{5\,\operatorname{arctanh}\left(\frac{3\left(\frac{2}{3}-\frac{4\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)}{3}\right)\sqrt{3}}{2\sqrt{9\left(x+\frac{2\sqrt{3}}{3}\right)^2-12\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)+3}}\right)}{32}-\frac{\sqrt{\left(x-\frac{2\sqrt{3}}{3}\right)^2+\frac{4\sqrt{3}\left(x-\frac{2\sqrt{3}}{3}\right)}{3}\cdots}}{16\left(x-\frac{2\sqrt{3}}{3}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-4)^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -5/32*arctanh(3/2*(2/3-4/3*3^(1/2)*(x+2/3*3^(1/2)))*3^(1/2)/(9*(x+2/3*3^(1/
2))^2-12*3^(1/2)*(x+2/3*3^(1/2))+3)^(1/2))-1/16/(x-2/3*3^(1/2))*((x-2/3*3^(
1/2))^2+4/3*3^(1/2)*(x-2/3*3^(1/2))+1/3)^(1/2)+5/32*arctanh(3/2*(2/3+4/3*3^
(1/2)*(x-2/3*3^(1/2)))*3^(1/2)/(9*(x-2/3*3^(1/2))^2+12*3^(1/2)*(x-2/3*3^(1/
2))+3)^(1/2))-1/16/(x+2/3*3^(1/2))*((x+2/3*3^(1/2))^2-4/3*3^(1/2)*(x+2/3*3^
(1/2))+1/3)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 4)^2*sqrt(x^2 - 1)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(33) = 66$.
time = 0.32, size = 80, normalized size = 1.86

$$-\frac{12\,x^2+5\,(3\,x^2-4)\log\left(3\,x^2-3\,\sqrt{x^2-1}\,x-2\right)-5\,(3\,x^2-4)\log\left(x^2-\sqrt{x^2-1}\,x-2\right)+12\,\sqrt{x^2-1}\,x-16}{32\,(3\,x^2-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*(12*x^2 + 5*(3*x^2 - 4)*log(3*x^2 - 3*sqrt(x^2 - 1)*x - 2) - 5*(3*x^2
 - 4)*log(x^2 - sqrt(x^2 - 1)*x - 2) + 12*sqrt(x^2 - 1)*x - 16)/(3*x^2 - 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\,(3x^2-4)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-4)**2/(x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(3*x**2 - 4)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(33) = 66.
time = 0.02, size = 108, normalized size = 2.51

$$-8\left(-\frac{5}{256}\ln\left|\left(\sqrt{x^2-1}-x\right)^2-3\right|+\frac{5}{256}\ln\left|3\left(\sqrt{x^2-1}-x\right)^2-1\right|+\frac{-5\left(\sqrt{x^2-1}-x\right)^2+3}{32\left(3\left(\sqrt{x^2-1}-x\right)^4-10\left(\sqrt{x^2-1}-x\right)^2+3\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x)
```

```
[Out] 1/4*(5*(x - sqrt(x^2 - 1))^2 - 3)/(3*(x - sqrt(x^2 - 1))^4 - 10*(x - sqrt(x
^2 - 1))^2 + 3) - 5/32*log(abs(3*(x - sqrt(x^2 - 1))^2 - 1)) + 5/32*log(abs
((x - sqrt(x^2 - 1))^2 - 3))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2-1}\,(3\,x^2-4)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(3*x^2 - 4)^2),x)
```

```
[Out] int(1/((x^2 - 1)^(1/2)*(3*x^2 - 4)^2), x)
```

3.5 $\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx$

Optimal. Leaf size=74

$$\frac{8}{9(1-3x)}-\frac{4\sqrt{x}\,\sqrt{1+x}}{3(1-3x)}-\frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right)+\frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{1+x}}\right)+\frac{5}{9}\log(1-3x)$$

```
[Out] 8/9/(1-3*x)-8/9*arcsinh(x^(1/2))+10/9*arctanh(2*x^(1/2)/(1+x)^(1/2))+5/9*ln
(1-3*x)-4/3*x^(1/2)*(1+x)^(1/2)/(1-3*x)
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.412$, Rules used = {6874, 99, 163, 56, 221, 95, 213}

$$-\frac{4\sqrt{x}\,\sqrt{x+1}}{3(1-3x)}+\frac{8}{9(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right)+\frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
```

```
[Out] 8/(9*(1 - 3*x)) - (4*Sqrt[x]*Sqrt[1 + x])/(3*(1 - 3*x)) - (8*ArcSinh[Sqrt[x
]])/9 + (10*ArcTanh[(2*Sqrt[x])/Sqrt[1 + x]])/9 + (5*Log[1 - 3*x])/9
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
```

```
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx &= \int \left(\frac{8}{3(-1+3x)^2}-\frac{4\sqrt{x}\,\sqrt{1+x}}{(-1+3x)^2}+\frac{5}{3(-1+3x)}\right)dx\\
&= \frac{8}{9(1-3x)}+\frac{5}{9}\log(1-3x)-4\int \frac{\sqrt{x}\,\sqrt{1+x}}{(-1+3x)^2}\,dx\\
&= \frac{8}{9(1-3x)}-\frac{4\sqrt{x}\,\sqrt{1+x}}{3(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{4}{3}\int \frac{\frac{1}{2}+x}{\sqrt{x}\,\sqrt{1+x}\,(-1+3x)}\,dx\\
&= \frac{8}{9(1-3x)}-\frac{4\sqrt{x}\,\sqrt{1+x}}{3(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{4}{9}\int \frac{1}{\sqrt{x}\,\sqrt{1+x}}\,dx-\frac{10}{9}\int \frac{}{\sqrt{}}\\
&= \frac{8}{9(1-3x)}-\frac{4\sqrt{x}\,\sqrt{1+x}}{3(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{8}{9}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{x}\right.\\
&= \frac{8}{9(1-3x)}-\frac{4\sqrt{x}\,\sqrt{1+x}}{3(1-3x)}-\frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right)+\frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{1+x}}\right)+\frac{5}{9}\text{lo}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 69, normalized size = 0.93

$$\frac{2\left(-4+6\sqrt{x}\,\sqrt{1+x}+(-1+3x)\tanh^{-1}\left(\sqrt{\frac{x}{1+x}}\right)+5(-1+3x)\log\left(1-x+\sqrt{x}\,\sqrt{1+x}\right)\right)}{-9+27x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
```

```
[Out] (2*(-4 + 6*Sqrt[x]*Sqrt[1 + x] + (-1 + 3*x)*ArcTanh[Sqrt[x/(1 + x)]] + 5*(-
1 + 3*x)*Log[1 - x + Sqrt[x]*Sqrt[1 + x]]))/(-9 + 27*x)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(2*Sqrt[x] + Sqrt[x + 1])^2,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(54) = 108$.
time = 0.02, size = 115, normalized size = 1.55

method	result
default	$-\frac{8}{9(3x-1)}+\frac{5\ln(3x-1)}{9}-\frac{\sqrt{x}\,\sqrt{1+x}\,\left(12\ln\left(\frac{1}{2}+x+\sqrt{x(1+x)}\right)x-15\,\mathrm{arctanh}\left(\frac{5x+1}{4\sqrt{x(1+x)}}\right)x-4\ln\left(\frac{1}{2}+\right.\right.}{9\sqrt{x(1+x)}\,(3x-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^(1/2)+(1+x)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -8/9/(3*x-1)+5/9*ln(3*x-1)-1/9*x^(1/2)*(1+x)^(1/2)*(12*ln(1/2+x+(x*(1+x))^(
1/2))*x-15*arctanh(1/4*(5*x+1)/(x*(1+x))^(1/2))*x-4*ln(1/2+x+(x*(1+x))^(1/2
))+5*arctanh(1/4*(5*x+1)/(x*(1+x))^(1/2))-12*(x*(1+x))^(1/2))/(x*(1+x))^(1/
2)/(3*x-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
[Out] integrate((sqrt(x + 1) + 2*sqrt(x))^(-2), x)
```

Fricas [A]
time = 0.32, size = 105, normalized size = 1.42

$$-\frac{5\,(3\,x-1)\log\left(3\,\sqrt{x+1}\,\sqrt{x}-3\,x-1\right)-4\,(3\,x-1)\log\left(2\,\sqrt{x+1}\,\sqrt{x}-2\,x-1\right)-5\,(3\,x-1)\log\left(\sqrt{x+1}\,\sqrt{x}-x+1\right)-5\,(3\,x-1)\log(3\,x-1)-12\,\sqrt{x+1}\,\sqrt{x}-12\,x+12}{9\,(3\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
[Out] -1/9*(5*(3*x - 1)*log(3*sqrt(x + 1)*sqrt(x) - 3*x - 1) - 4*(3*x - 1)*log(2*
sqrt(x + 1)*sqrt(x) - 2*x - 1) - 5*(3*x - 1)*log(sqrt(x + 1)*sqrt(x) - x +
1) - 5*(3*x - 1)*log(3*x - 1) - 12*sqrt(x + 1)*sqrt(x) - 12*x + 12)/(3*x -
1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(2\sqrt{x}+\sqrt{x+1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**(1/2)+(1+x)**(1/2))**2,x)
[Out] Integral((2*sqrt(x) + sqrt(x + 1))**(-2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(54) = 108.
time = 0.03, size = 165, normalized size = 2.23

$$2\left(\frac{-5x-1}{6\,(3x-1)}+\frac{5}{18}\ln|3x-1|+2\left(\frac{2}{9}\ln\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{5}{36}\ln\left|\left(\sqrt{x+1}-\sqrt{x}\right)^2-3\right|-\frac{5}{36}\ln\left|3\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right|-\frac{10\left(\sqrt{x+1}-\sqrt{x}\right)^2-6}{9\left(3\left(\sqrt{x+1}-\sqrt{x}\right)^4-10\left(\sqrt{x+1}-\sqrt{x}\right)^2+3\right)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x)
[Out] -8/9*(5*(sqrt(x + 1) - sqrt(x))^2 - 3)/(3*(sqrt(x + 1) - sqrt(x))^4 - 10*(s
qrt(x + 1) - sqrt(x))^2 + 3) - 1/3*(5*x + 1)/(3*x - 1) + 8/9*log(sqrt(x + 1
) - sqrt(x)) - 5/9*log(abs(3*(sqrt(x + 1) - sqrt(x))^2 - 1)) + 5/9*log(abs(
(sqrt(x + 1) - sqrt(x))^2 - 3)) + 5/9*log(abs(3*x - 1))
```

Mupad [B]
time = 1.71, size = 82, normalized size = 1.11

$$\frac{10\,\mathrm{atanh}\left(\frac{2662400\,\sqrt{x}}{81\left(\frac{665600\,x}{81\left(\sqrt{x+1}-1\right)^2}+\frac{665600}{81}\right)\left(\sqrt{x+1}-1\right)}\right)}{9}+\frac{5\,\ln\left(x-\frac{1}{3}\right)}{9}-\frac{16\,\mathrm{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)}{9}-\frac{8}{27\,\left(x-\frac{1}{3}\right)}+\frac{4\,\sqrt{x}\,\sqrt{x+1}}{3\,(3\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(1/2) + 2*x^(1/2))^2,x)
```

```
[Out] (10*atanh((2662400*x^(1/2))/(81*((665600*x)/(81*((x + 1)^(1/2) - 1)^2) + 66
5600/81)*((x + 1)^(1/2) - 1))))/9 + (5*log(x - 1/3))/9 - (16*atanh(x^(1/2)/
((x + 1)^(1/2) - 1)))/9 - 8/(27*(x - 1/3)) + (4*x^(1/2)*(x + 1)^(1/2))/(3*(
3*x - 1))
```

3.6 $\int \frac{\sqrt{-1+x^2}}{(-i+x)^2} \, dx$

Optimal. Leaf size=64

$$\frac{\sqrt{-1+x^2}}{i-x} - \frac{i\tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{-1+x^2}}\right)}{\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

```
[Out] arctanh(x/(x^2-1)^(1/2))-1/2*I*arctan(1/2*(1-I*x)*2^(1/2)/(x^2-1)^(1/2))*2^
(1/2)+(x^2-1)^(1/2)/(I-x)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {747, 858, 223, 212, 739, 210}

$$\frac{\sqrt{x^2-1}}{-x+i} - \frac{i\tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^2]/(-I + x)^2,x]
```

```
[Out] Sqrt[-1 + x^2]/(I - x) - (I*ArcTan[(1 - I*x)/(Sqrt[2]*Sqrt[-1 + x^2])])/Sqr
t[2] + ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx &= \frac{\sqrt{-1+x^2}}{i-x} + \int \frac{x}{(-i+x)\sqrt{-1+x^2}}\,dx \\
&= \frac{\sqrt{-1+x^2}}{i-x} + i\int \frac{1}{(-i+x)\sqrt{-1+x^2}}\,dx + \int \frac{1}{\sqrt{-1+x^2}}\,dx \\
&= \frac{\sqrt{-1+x^2}}{i-x} - i\text{Subst}\left(\int \frac{1}{-2-x^2}\,dx, x, \frac{-1+ix}{\sqrt{-1+x^2}}\right) + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right) \\
&= \frac{\sqrt{-1+x^2}}{i-x} - \frac{i\tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{-1+x^2}}\right)}{\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.18, size = 66, normalized size = 1.03

$$\frac{\sqrt{-1+x^2}}{i-x} + \tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right) - \sqrt{2}\tanh^{-1}\left(\frac{1+ix-i\sqrt{-1+x^2}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^2]/(-I + x)^2,x]
```

```
[Out] Sqrt[-1 + x^2]/(I - x) + ArcTanh[x/Sqrt[-1 + x^2]] - Sqrt[2]*ArcTanh[(1 + I
*x - I*Sqrt[-1 + x^2])/Sqrt[2]]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x^2 - 1]/(x - I)^2,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(52) = 104.
time = 0.12, size = 150, normalized size = 2.34

method	result
risch	$-\frac{\sqrt{x^2-1}}{x-i}+\ln\left(x+\sqrt{x^2-1}\right)+\frac{i\sqrt{2}\,\arctan\left(\frac{(-4+2i(x-i))\sqrt{2}}{4\sqrt{(x-i)^2+2i(x-i)-2}}\right)}{2}$
default	$\frac{\left((x-i)^2+2i(x-i)-2\right)^{\frac{3}{2}}}{2x-2i}-\frac{i\left(\sqrt{(x-i)^2+2i(x-i)-2}+i\ln\left(x+\sqrt{(x-i)^2+2i(x-i)-2}\right)-\sqrt{2}\,\mathrm{arctan}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(1/2)/(x-I)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(x-I)*((x-I)^2+2*I*(x-I)-2)^(3/2)-1/2*I*(((x-I)^2+2*I*(x-I)-2)^(1/2)+I*
ln(x+((x-I)^2+2*I*(x-I)-2)^(1/2))-2^(1/2)*arctan(1/4*(-4+2*I*(x-I))*2^(1/2)
/((x-I)^2+2*I*(x-I)-2)^(1/2)))-1/2*x*((x-I)^2+2*I*(x-I)-2)^(1/2)+1/2*ln(x+(
(x-I)^2+2*I*(x-I)-2)^(1/2))
```

Maxima [A]
time = 0.33, size = 53, normalized size = 0.83

$$\frac{1}{2}i\sqrt{2}\,\arcsin\left(\frac{i\,x}{|x-i|}-\frac{1}{|x-i|}\right)-\frac{\sqrt{x^2-1}}{x-i}+\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*I*sqrt(2)*arcsin(I*x/abs(x - I) - 1/abs(x - I)) - sqrt(x^2 - 1)/(x - I)
 + log(2*x + 2*sqrt(x^2 - 1))
```

Fricas [A]
time = 0.60, size = 89, normalized size = 1.39

$$-\frac{\sqrt{2}\,(x-i)\log\left(-x+i\sqrt{2}+\sqrt{x^2-1}+i\right)-\sqrt{2}\,(x-i)\log\left(-x-i\sqrt{2}+\sqrt{x^2-1}+i\right)+2\,(x-i)\log\left(-x+\sqrt{x^2-1}\right)+2\,x+2\sqrt{x^2-1}-2i}{2\,(x-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(x - I)*log(-x + I*sqrt(2) + sqrt(x^2 - 1) + I) - sqrt(2)*(x
- I)*log(-x - I*sqrt(2) + sqrt(x^2 - 1) + I) + 2*(x - I)*log(-x + sqrt(x^2
- 1)) + 2*x + 2*sqrt(x^2 - 1) - 2*I)/(x - I)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{(x-1)\,(x+1)}}{(x-i)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(1/2)/(-I+x)**2,x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1))/(x - I)**2, x)
```

Giac [A]
time = 0.01, size = 114, normalized size = 1.78

$$2\left(-\frac{\ln\left|\sqrt{x^2-1}-x\right|}{2}+\frac{2I\arctan\left(\frac{I+\sqrt{x^2-1}-x}{\sqrt{-I^2+1}}\right)}{2\sqrt{-I^2+1}}+\frac{\left(\sqrt{x^2-1}-x\right)I+1}{-\left(\sqrt{x^2-1}-x\right)^2-2\left(\sqrt{x^2-1}-x\right)I-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x)
```

```
[Out] I*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 - 1) - I)) - 2*(-I*x + I*sqrt(x
^2 - 1) + 1)/((x - sqrt(x^2 - 1))^2 - 2*I*x + 2*I*sqrt(x^2 - 1) + 1) - log(
abs(-x + sqrt(x^2 - 1)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{x^2-1}}{(x-\mathrm{i})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^(1/2)/(x - 1i)^2,x)
```

```
[Out] int((x^2 - 1)^(1/2)/(x - 1i)^2, x)
```

3.7 $\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)^2}\,dx$

Optimal. Leaf size=48

$$-\frac{x\sqrt{-1+x^2}}{4(1+x^2)}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right)}{4\sqrt{2}}$$

```
[Out] 3/8*arctanh(x*2^(1/2)/(x^2-1)^(1/2))*2^(1/2)-1/4*x*(x^2-1)^(1/2)/(x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {390, 385, 212}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2-1}}{4(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*(1 + x^2)^2),x]
```

```
[Out] -1/4*(x*Sqrt[-1 + x^2])/(1 + x^2) + (3*ArcTanh[(Sqrt[2]*x)/Sqrt[-1 + x^2]])
/(4*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)^2}\,dx &= -\frac{x\sqrt{-1+x^2}}{4(1+x^2)}+\frac{3}{4}\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)}\,dx\\ &= -\frac{x\sqrt{-1+x^2}}{4(1+x^2)}+\frac{3}{4}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right)\\ &= -\frac{x\sqrt{-1+x^2}}{4(1+x^2)}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 55, normalized size = 1.15

$$\frac{1}{8}\left(-\frac{2x\sqrt{-1+x^2}}{1+x^2}+3\sqrt{2}\,\tanh^{-1}\left(\frac{1+x^2-x\sqrt{-1+x^2}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(1 + x^2)^2),x]
[Out] ((-2*x*Sqrt[-1 + x^2])/(1 + x^2) + 3*Sqrt[2]*ArcTanh[(1 + x^2 - x*Sqrt[-1 +
 x^2])/Sqrt[2]])/8
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x^2 - 1]*(x^2 + 1)^2),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.14, size = 45, normalized size = 0.94

method	result	size
risch	$\frac{3\,\text{arctanh}\left(\frac{x\sqrt{2}}{\sqrt{x^2-1}}\right)\sqrt{2}}{8}-\frac{x\sqrt{x^2-1}}{4(x^2+1)}$	37

default	$-\frac{x}{8\sqrt{x^2-1}\,\left(\frac{x^2}{x^2-1}-\frac{1}{2}\right)}+\frac{3\,\mathrm{arctanh}\left(\frac{x\sqrt{2}}{\sqrt{x^2-1}}\right)\sqrt{2}}{8}$	45
trager	$-\frac{x\sqrt{x^2-1}}{4(x^2+1)}+\frac{3\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^2-2\right)x^2+4\sqrt{x^2-1}\,x-\mathrm{RootOf}\left(\_Z^2-2\right)}{x^2+1}\right)}{16}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*x/(x^2-1)^(1/2)/(x^2/(x^2-1)-1/2)+3/8*arctanh(x*2^(1/2)/(x^2-1)^(1/2))
*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 1)^2*sqrt(x^2 - 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(36) = 72$.
time = 0.31, size = 83, normalized size = 1.73

$$\frac{3\sqrt{2}\,(x^2+1)\log\left(\frac{9\,x^2+2\,\sqrt{2}\,(3\,x^2-1)+2\,\sqrt{x^2-1}\,\left(3\,\sqrt{2}\,x+4\,x\right)-3}{x^2+1}\right)-4\,x^2-4\,\sqrt{x^2-1}\,x-4}{16\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*(x^2 + 1)*log((9*x^2 + 2*sqrt(2)*(3*x^2 - 1) + 2*sqrt(x^2 -
 1)*(3*sqrt(2)*x + 4*x) - 3)/(x^2 + 1)) - 4*x^2 - 4*sqrt(x^2 - 1)*x - 4)/(x
^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{(x-1)\,(x+1)}\,\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2/(x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(x**2 + 1)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(36) = 72.
time = 0.00, size = 122, normalized size = 2.54

$$-8\left(\frac{3\left(\sqrt{x^2-1}-x\right)^2+1}{16\left(\left(\sqrt{x^2-1}-x\right)^4+6\left(\sqrt{x^2-1}-x\right)^2+1\right)}+\frac{3\ln\left(\frac{2\left(\sqrt{x^2-1}-x\right)^2+6-4\sqrt{2}}{2\left(\sqrt{x^2-1}-x\right)^2+6+4\sqrt{2}}\right)}{64\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x)
```

```
[Out] -3/16*sqrt(2)*log(((x - sqrt(x^2 - 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 -
1))^2 + 2*sqrt(2) + 3)) - 1/2*(3*(x - sqrt(x^2 - 1))^2 + 1)/((x - sqrt(x^2
- 1))^4 + 6*(x - sqrt(x^2 - 1))^2 + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2-1}\,\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(x^2 + 1)^2),x)
```

```
[Out] int(1/((x^2 - 1)^(1/2)*(x^2 + 1)^2), x)
```

3.8 $\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2\sqrt{-1+x}}\,dx$

Optimal. Leaf size=30

$$2\sqrt{-1+x}+\frac{4}{3}(-1+x)^{3/2}-\frac{4x^{3/2}}{3}$$

```
[Out] 4/3*(-1+x)^(3/2)-4/3*x^(3/2)+2*(-1+x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {6821, 45}

$$-\frac{4x^{3/2}}{3}+\frac{4}{3}(x-1)^{3/2}+2\sqrt{x-1}$$

Antiderivative was successfully verified.

```
[In] Int[1/((Sqrt[-1 + x] + Sqrt[x])^2*Sqrt[-1 + x]),x]
```

```
[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 - (4*x^(3/2))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2\sqrt{-1+x}}\,dx = \int \left(-\frac{1}{\sqrt{-1+x}}-2\sqrt{x}+\frac{2x}{\sqrt{-1+x}}\right)dx$$
$$= -2\sqrt{-1+x}-\frac{4x^{3/2}}{3}+2\int \frac{x}{\sqrt{-1+x}}\,dx$$
$$= -2\sqrt{-1+x}-\frac{4x^{3/2}}{3}+2\int \left(\frac{1}{\sqrt{-1+x}}+\sqrt{-1+x}\right)dx$$
$$= 2\sqrt{-1+x}+\frac{4}{3}(-1+x)^{3/2}-\frac{4x^{3/2}}{3}$$

Mathematica [A]
time = 0.18, size = 26, normalized size = 0.87

$$-\frac{4x^{3/2}}{3}+\frac{2}{3}\sqrt{-1+x}\,(1+2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((Sqrt[-1 + x] + Sqrt[x])^2*Sqrt[-1 + x]),x]
[Out] (-4*x^(3/2))/3 + (2*Sqrt[-1 + x]*(1 + 2*x))/3
```

Mathics [A]
time = 2.09, size = 32, normalized size = 1.07

$$\frac{2\left(-2\sqrt{x}-\sqrt{-1+x}\right)}{-3+6\sqrt{x}\sqrt{-1+x}+6x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(Sqrt[x - 1]*(Sqrt[x - 1] + Sqrt[x])^2),x]')
[Out] 2 (-2 Sqrt[x] - Sqrt[-1 + x]) / (3 (-1 + 2 Sqrt[x] Sqrt[-1 + x] + 2 x))
```

Maple [A]
time = 0.08, size = 21, normalized size = 0.70

method	result	size
default	$\frac{4(-1+x)^{\frac{3}{2}}}{3}-\frac{4x^{\frac{3}{2}}}{3}+2\sqrt{-1+x}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x,method=_RETURNVERBOSE)
[Out] 4/3*(-1+x)^(3/2)-4/3*x^(3/2)+2*(-1+x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="maxima")
[Out] integrate(1/(sqrt(x - 1)*(sqrt(x - 1) + sqrt(x))^2), x)
```

Fricas [A]
time = 0.31, size = 18, normalized size = 0.60

$$\frac{2}{3}\,(2\,x+1)\sqrt{x-1}-\frac{4}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="fricas")
[Out] 2/3*(2*x + 1)*sqrt(x - 1) - 4/3*x^(3/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(26) = 52$
time = 0.31, size = 53, normalized size = 1.77

$$-\frac{4\sqrt{x}}{6\sqrt{x}\,\sqrt{x-1}+6x-3}-\frac{2\sqrt{x-1}}{6\sqrt{x}\,\sqrt{x-1}+6x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**(1/2)/((-1+x)**(1/2)+x**(1/2))**2,x)
[Out] -4*sqrt(x)/(6*sqrt(x)*sqrt(x - 1) + 6*x - 3) - 2*sqrt(x - 1)/(6*sqrt(x)*sqr
t(x - 1) + 6*x - 3)
```

Giac [A]
time = 0.00, size = 53, normalized size = 1.77

$$2\left(\frac{2}{3}\sqrt{x-1}\,(x-1)+\sqrt{x-1}+2\left(-\frac{1}{3}-\frac{1}{3}\sqrt{x-1}\,\sqrt{x-1}\right)\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x)
[Out] 4/3*(x - 1)^(3/2) - 4/3*x^(3/2) + 2*sqrt(x - 1)
```

Mupad [B]
time = 0.38, size = 21, normalized size = 0.70

$$\frac{4\,x\,\sqrt{x-1}}{3}+\frac{2\,\sqrt{x-1}}{3}-\frac{4\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x - 1)^(1/2) + x^(1/2))^2*(x - 1)^(1/2)),x)
[Out] (4*x*(x - 1)^(1/2))/3 + (2*(x - 1)^(1/2))/3 - (4*x^(3/2))/3
```

3.9 $\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$

Optimal. Leaf size=220

$$\frac{2-4x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)}+\frac{1}{25}\sqrt{-110+50\sqrt{5}}\,\tan^{-1}\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}\,\sqrt{x}\right)-\frac{1}{50}\sqrt{-110+50\sqrt{5}}\,\tan^{-1}\Bigg($$

```
[Out] 1/5*(2-4*x)/(x^(1/2)+(x^2-1)^(1/2))-1/50*arctan((x^2-1)^(1/2)*(-2+2*5^(1/2)
)^(1/2)/(2-x*(-5^(1/2)+1)))*(-110+50*5^(1/2))^(1/2)+1/25*arctan(1/2*x^(1/2)
*(2+2*5^(1/2))^(1/2))*(-110+50*5^(1/2))^(1/2)-1/25*arctanh(1/2*x^(1/2)*(-2+
2*5^(1/2))^(1/2))*(110+50*5^(1/2))^(1/2)-1/50*arctanh((x^2-1)^(1/2)*(2+2*5^
(1/2))^(1/2)/(2-x-x*5^(1/2)))*(110+50*5^(1/2))^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 365, normalized size of antiderivative = 1.66, number of steps used = 18, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6874, 750, 840, 1180, 213, 209, 1032, 1048, 739, 212, 210, 999}

$$-\frac{2\sqrt{x^2-1}\,(1-2x)}{5\left(-x^2+x+1\right)}+\frac{2\sqrt{x}\,(1-2x)}{5\left(-x^2+x+1\right)}-\frac{2}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\,\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\,\sqrt{x^2-1}}\right)+\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\,\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\,\sqrt{x^2-1}}\right)-\frac{2}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\,\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{x^2-1}}\right)+\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\,\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{x^2-1}}\right)+\frac{1}{5}\sqrt{\frac{2}{5}\left(5\sqrt{5}-11\right)}\,\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,\sqrt{x}\right)-\frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)}\,\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

```
[Out] (2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - (2*(1 - 2*x)*Sqrt[-1 + x^2])/(5*(
1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))/5]*ArcTan[Sqrt[2/(-1 + Sqrt[5])
]*Sqrt[x]])/5 + Sqrt[2/(5*(-1 + Sqrt[5]))]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sq
rt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])] - (2*Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTan[
(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(
2*(11 + 5*Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]])/5 + Sqrt[2/(
5*(1 + Sqrt[5]))]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt
[-1 + x^2])] - (2*Sqrt[(2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sq
rt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 750

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 999

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[1/((b - q + 2*c*x)
*Sqrt[d + f*x^2]), x], x] - Dist[2*(c/q), Int[1/((b + q + 2*c*x)*Sqrt[d + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ
[b^2 - 4*a*c]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx = \int \left(-\frac{2\sqrt{x}}{(-1-x+x^2)^2} + \frac{2x}{\sqrt{-1+x^2}\,(-1-x+x^2)^2} + \frac{}{\sqrt{-1+}}\right.$$

$$= -\left(2\int \frac{\sqrt{x}}{(-1-x+x^2)^2}\,dx\right) + 2\int \frac{x}{\sqrt{-1+x^2}\,(-1-x+x^2)^2}$$

$$= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{2(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} - \frac{2}{5}\int \frac{-\frac{1}{2}-x}{\sqrt{x}\,(-1-x+x}$$

$$= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{2(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} - \frac{4}{5}\mathrm{Subst}\left(\int \frac{-\frac{1}{2}-x}{-1-x^2-}\right.$$

$$= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{2(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\ \mathrm{tan}$$

$$= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{2(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{1}{5}\sqrt{\frac{2}{5}\left(-11+5\sqrt{5}\right)}$$

Mathematica [A]
time = 6.17, size = 195, normalized size = 0.89

$$\frac{1}{25}\left(-\frac{10(-1+2x)\left(-\sqrt{x}+\sqrt{-1+x^2}\right)}{-1-x+x^2} + \sqrt{-110+50\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{x}\right) - \sqrt{-110+50\sqrt{5}}\tan^{-1}\left(\frac{\sqrt{-2+\sqrt{5}}\sqrt{-1+x^2}}{1+x}\right) - \sqrt{110+50\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\sqrt{x}\right) + \sqrt{110+50\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{5}}\sqrt{-1+x^2}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

```
[Out] ((-10*(-1 + 2*x)*(-Sqrt[x] + Sqrt[-1 + x^2]))/(-1 - x + x^2) + Sqrt[-110 +
50*Sqrt[5]]*ArcTan[Sqrt[(1 + Sqrt[5])/2]*Sqrt[x]] - Sqrt[-110 + 50*Sqrt[5]]
*ArcTan[(Sqrt[-2 + Sqrt[5]]*Sqrt[-1 + x^2])/(1 + x)] - Sqrt[110 + 50*Sqrt[5
]]*ArcTanh[Sqrt[(-1 + Sqrt[5])/2]*Sqrt[x]] + Sqrt[110 + 50*Sqrt[5]]*ArcTanh
[(Sqrt[2 + Sqrt[5]]*Sqrt[-1 + x^2])/(1 + x)])/25
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x^2 - 1]*(Sqrt[x^2 - 1] + Sqrt[x])^2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(158) = 316$.
time = 0.12, size = 902, normalized size = 4.10

method	result	size
default	Expression too large to display	902

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)
+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)+2/5/(1/2-1/2*5^(1/2))/(-2+2*
5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*
5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+
2-2*5^(1/2))^(1/2))*5^(1/2)-6/5/(1/2-1/2*5^(1/2))/(-2+2*5^(1/2))^(1/2)*arct
an(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(
x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))
+1/5*5^(1/2)/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(
-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)-1/5/(1/2+1/2*5^(1/2)
)/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2
)+1/2+1/2*5^(1/2))^(1/2)+6/5/(1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(
2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2
*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))+2/5/(
1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/
2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*
(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))*5^(1/2)-1/5*5^(1/2)/(1/2+1/2*5^(1/2
))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/
2)+1/2+1/2*5^(1/2))^(1/2)-6/25*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(
1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)
-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))-6/25*5^(1/2)/
(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/
(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)
-1/2)+2-2*5^(1/2))^(1/2))+2/5*x^(1/2)/(x+1/2*5^(1/2)-1/2)+4/5/(-2+2*5^(1/2)
)^(1/2)*arctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))-8/25/(-2+2*5^(1/2))^(1/2)*ar
ctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))*5^(1/2)+2/5*x^(1/2)/(x-1/2*5^(1/2)-1/2
)-4/5/(2+2*5^(1/2))^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))-8/25/(2+2*
5^(1/2))^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))*5^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(153) = 306$.
time = 0.32, size = 424, normalized size = 1.93

$$\frac{4\sqrt{5}\,(x^2-x-1)\sqrt{10\sqrt{5}-22}\,\arctan\left(\frac{1}{2}\sqrt{2x^2-\sqrt{x^2-1}\left(2x+\sqrt{5}-1\right)+\sqrt{5}\,x-x}\,\sqrt{10\sqrt{5}-22}\left(\sqrt{5}+2\right)+\frac{1}{4}\left(\sqrt{5}\,(2x+1)-2\sqrt{x^2-1}\left(\sqrt{5}+2\right)+4x+3\right)\sqrt{10\sqrt{5}-22}\right)-4\sqrt{5}\,(x^2-x-1)\sqrt{10\sqrt{5}-22}\,\arctan\left(\frac{1}{4}\left(\sqrt{2}\sqrt{2x+\sqrt{5}-1}\left(\sqrt{5}+2\right)-2\sqrt{x}\left(\sqrt{5}+2\right)\right)\sqrt{10\sqrt{5}-22}\right)-\sqrt{5}\,(x^2-x-1)\sqrt{10\sqrt{5}+22}\,\log\left(\sqrt{10\sqrt{5}+22}\left(\sqrt{5}-3\right)-4x+2\sqrt{5}+4\sqrt{x^2-1}+2\right)+\sqrt{5}\,(x^2-x-1)\sqrt{10\sqrt{5}+22}\,\log\left(\sqrt{10\sqrt{5}+22}\left(\sqrt{5}-3\right)+4\sqrt{x}\right)+\sqrt{5}\,(x^2-x-1)\sqrt{10\sqrt{5}+22}\,\log\left(-\sqrt{10\sqrt{5}+22}\left(\sqrt{5}-3\right)-4x+2\sqrt{5}+4\sqrt{x^2-1}+2\right)-\sqrt{5}\,(x^2-x-1)\sqrt{10\sqrt{5}+22}\,\log\left(-\sqrt{10\sqrt{5}+22}\left(\sqrt{5}-3\right)+4\sqrt{x}\right)-40x^2-20\sqrt{x^2-1}\,(2x-1)+20\,(2x-1)\sqrt{x}+40x+40}{50\,(x^2-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/50*(4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22)*arctan(1/2*sqrt(2*x^2 -
 sqrt(x^2 - 1)*(2*x + sqrt(5) - 1) + sqrt(5)*x - x)*sqrt(10*sqrt(5) - 22)*(
sqrt(5) + 2) + 1/4*(sqrt(5)*(2*x + 1) - 2*sqrt(x^2 - 1)*(sqrt(5) + 2) + 4*x
 + 3)*sqrt(10*sqrt(5) - 22)) - 4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22
)*arctan(1/4*(sqrt(2)*sqrt(2*x + sqrt(5) - 1)*(sqrt(5) + 2) - 2*sqrt(x)*(sq
rt(5) + 2))*sqrt(10*sqrt(5) - 22)) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
+ 22)*log(sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^
2 - 1) + 2) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(
5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
 + 22)*log(-sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(
x^2 - 1) + 2) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sq
rt(5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) - 40*x^2 - 20*sqrt(x^2 - 1)*(2*x - 1
) + 20*(2*x - 1)*sqrt(x) + 40*x + 40)/(x^2 - x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)\,(x+1)}\,\left(\sqrt{x}+\sqrt{x^2-1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(x**(1/2)+(x**2-1)**(1/2))**2,x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(sqrt(x) + sqrt(x**2 - 1))**2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{x^2-1}\,\left(\sqrt{x^2-1}+\sqrt{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*((x^2 - 1)^(1/2) + x^(1/2))^2),x)
```

```
[Out] int(1/((x^2 - 1)^(1/2)*((x^2 - 1)^(1/2) + x^(1/2))^2), x)
```

3.10 $\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{(1+x-x^2)^2\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=220

$$\frac{2-4x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)}+\frac{1}{25}\sqrt{-110+50\sqrt{5}}\,\tan^{-1}\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}\,\sqrt{x}\right)-\frac{1}{50}\sqrt{-110+50\sqrt{5}}\,\tan^{-1}\left(\right.$$

```
[Out] 1/5*(2-4*x)/(x^(1/2)+(x^2-1)^(1/2))-1/50*arctan((x^2-1)^(1/2)*(-2+2*5^(1/2)
)^(1/2)/(2-x*(-5^(1/2)+1)))*(-110+50*5^(1/2))^(1/2)+1/25*arctan(1/2*x^(1/2)
*(2+2*5^(1/2))^(1/2))*(-110+50*5^(1/2))^(1/2)-1/25*arctanh(1/2*x^(1/2)*(-2+
2*5^(1/2))^(1/2))*(110+50*5^(1/2))^(1/2)-1/50*arctanh((x^2-1)^(1/2)*(2+2*5^
(1/2))^(1/2)/(2-x-x*5^(1/2)))*(110+50*5^(1/2))^(1/2)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 541 vs. 2(220) = 440. time = 0.44, antiderivative size = 541, normalized size of antiderivative = 2.46, number of steps used = 25, number of rules used = 13, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6874, 750, 840, 1180, 213, 209, 989, 1048, 739, 212, 210, 1032, 1079}

$$-\frac{\sqrt{x^2-1}(1-2x)}{5(-x^2+x+1)}+\frac{2\sqrt{x}(1-2x)}{5(-x^2+x+1)}-\frac{(3-x)\sqrt{x^2-1}}{5(-x^2+x+1)}+\frac{(x+2)\sqrt{x^2-1}}{5(-x^2+x+1)}+\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)-\frac{1}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)-\frac{1}{5}\sqrt{\frac{1}{10}\left(5\sqrt{5}-11\right)}\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)+\frac{1}{5}\sqrt{\frac{1}{10}\left(11+5\sqrt{5}\right)}\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)-\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)-\frac{1}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)+\frac{1}{5}\sqrt{\frac{2}{5}\left(5\sqrt{5}-11\right)}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x}\right)-\frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 + x^2]),x]
```

```
[Out] (2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - ((1 - 2*x)*Sqrt[-1 + x^2])/(5*(1
+ x - x^2)) - ((3 - x)*Sqrt[-1 + x^2])/(5*(1 + x - x^2)) + ((2 + x)*Sqrt[-1
 + x^2])/(5*(1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))/5]*ArcTan[Sqrt[2/(
-1 + Sqrt[5])]*Sqrt[x]])/5 - (Sqrt[(-11 + 5*Sqrt[5])/10]*ArcTan[(2 - (1 - S
qrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(-2 + 5*Sqrt
[5])/5]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2]
)])/5 + (Sqrt[(2 + 5*Sqrt[5])/5]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 +
 Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(2*(11 + 5*Sqrt[5]))/5]*ArcTanh[Sqrt
[2/(1 + Sqrt[5])]*Sqrt[x]])/5 - (Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 +
 Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(2 + 5*Sqrt
[5])/5]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2]
)])/5 + (Sqrt[(11 + 5*Sqrt[5])/10]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1
 + Sqrt[5])]*Sqrt[-1 + x^2])])/5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 750

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 989

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*f
```

```
))*x)*(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1079

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2
)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((A*c - a*C)*(
(-b)*(c*d + a*f)) + (A*b)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(2*a*f)) + C*(b^2*d - 2*a*(c*d - a*f)))*x), x] + Dist[1/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) +
(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) -
c*d*(p + 2)) - (2*f*((A*c - a*C)*((-b)*(c*d + a*f)) + (A*b)*(2*c^2*d + b^2*
```

```
f - c*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*
C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f)
 - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d
, f, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*
d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{(1+x-x^2)^2\sqrt{-1+x^2}}\,dx = \int \left(-\frac{2\sqrt{x}}{(-1-x+x^2)^2} - \frac{1}{\sqrt{-1+x^2}\,(-1-x+x^2)^2} + \frac{x}{\sqrt{-1+x^2}\,(-1-\cdots}\right.$$

$$= -\left(2\int \frac{\sqrt{x}}{(-1-x+x^2)^2}\,dx\right) - \int \frac{1}{\sqrt{-1+x^2}\,(-1-x+x^2)^2}\,dx + \int \frac{\cdots}{\sqrt{-\cdots}}$$

$$= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} - \frac{(3-x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{(2+x)\sqrt{-\cdots}}{5(1+x-\cdots}$$

$$= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} - \frac{(3-x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{(2+x)\sqrt{-\cdots}}{5(1+x-\cdots}$$

$$= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} - \frac{(3-x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{(2+x)\sqrt{-\cdots}}{5(1+x-\cdots}$$

$$= \frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)} - \frac{(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)} - \frac{(3-x)\sqrt{-1+x^2}}{5(1+x-x^2)} + \frac{(2+x)\sqrt{-\cdots}}{5(1+x-\cdots}$$

Mathematica [A]

time = 6.36, size = 195, normalized size = 0.89

$$\frac{1}{25}\left(-\frac{10(-1+2x)\left(-\sqrt{x}+\sqrt{-1+x^2}\right)}{-1-x+x^2}+\sqrt{-110+50\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{x}\right)-\sqrt{-110+50\sqrt{5}}\tan^{-1}\left(\frac{\sqrt{-2+\sqrt{5}}\sqrt{-1+x^2}}{1+x}\right)-\sqrt{110+50\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\sqrt{x}\right)+\sqrt{110+50\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{5}}\sqrt{-1+x^2}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 + x^2]),x]
```

```
[Out] ((-10*(-1 + 2*x)*(-Sqrt[x] + Sqrt[-1 + x^2]))/(-1 - x + x^2) + Sqrt[-110 +
50*Sqrt[5]]*ArcTan[Sqrt[(1 + Sqrt[5])/2]*Sqrt[x]] - Sqrt[-110 + 50*Sqrt[5]]
*ArcTan[(Sqrt[-2 + Sqrt[5]]*Sqrt[-1 + x^2])/(1 + x)] - Sqrt[110 + 50*Sqrt[5
]]*ArcTanh[Sqrt[(-1 + Sqrt[5])/2]*Sqrt[x]] + Sqrt[110 + 50*Sqrt[5]]*ArcTanh
[(Sqrt[2 + Sqrt[5]]*Sqrt[-1 + x^2])/(1 + x)])/25
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 +
x^2]),x]')
```

```
[Out] Timed out
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1636 vs. $2(158) = 316$.
time = 0.25, size = 1637, normalized size = 7.44

method	result	size
default	Expression too large to display	1637

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/25*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^
(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1
/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))+(2/5-2/5*5^(1/2))*(-1/4/(1/2-1/2*5^(1/2
))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1
/2)+1/2-1/2*5^(1/2))^(1/2)-1/4*(-5^(1/2)+1)/(1/2-1/2*5^(1/2))/(-2+2*5^(1/2)
)^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2)
)^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(
1/2))^(1/2)))+(2/5+2/5*5^(1/2))*(-1/4/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)
*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1
/2)+1/4*(5^(1/2)+1)/(1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1
```

```
/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-
1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)))+2/25*5^(1/2)/
(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/
(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)
-1/2)+2-2*5^(1/2))^(1/2))+2/5*x^(1/2)/(x-1/2*5^(1/2)-1/2)-8/25*(5/2+5^(1/2)
)/(2+2*5^(1/2))^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))+2/5*x^(1/2)/(x
+1/2*5^(1/2)-1/2)-8/25*(-5/2+5^(1/2))/(-2+2*5^(1/2))^(1/2)*arctan(2*x^(1/2)
/(-2+2*5^(1/2))^(1/2))-4/25*5^(1/2)*(1/4*(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2
)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)+1/4*(5^(1/2)+1)*ln(x+((x-1/2*5^
(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2))-(1/2+1
/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(
1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/
2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)))-1/5/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/
2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))
^(3/2)+1/10*(-5^(1/2)+1)/(1/2-1/2*5^(1/2))*(1/2*(4*(x+1/2*5^(1/2)-1/2)^2+4*
(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)+1/2*(-5^(1/2)+1)*ln(x+(
(x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/
2))+2*(1/2-1/2*5^(1/2))/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+
1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5
^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)))+2/5/(1/2-1/2*5^(1/2))*(1
/2*x*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2
))^(1/2)+1/8*(2-2*5^(1/2)-(-5^(1/2)+1)^2)*ln(x+((x+1/2*5^(1/2)-1/2)^2+(-5^(
1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)))-1/5/(1/2+1/2*5^(1/2))/
(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+
1/2+1/2*5^(1/2))^(3/2)+1/10*(5^(1/2)+1)/(1/2+1/2*5^(1/2))*(1/2*(4*(x-1/2*5^
(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)+1/2*(5^(1
/2)+1)*ln(x+((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*
5^(1/2))^(1/2))-2*(1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2
)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/
2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)))+2/5/(1/2+1/2*5^
(1/2))*(1/2*x*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/
2*5^(1/2))^(1/2)+1/8*(2+2*5^(1/2)-(5^(1/2)+1)^2)*ln(x+((x-1/2*5^(1/2)-1/2)^
2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)))+4/25*5^(1/2)*(1/
4*(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^
(1/2)+1/4*(-5^(1/2)+1)*ln(x+((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1
/2)-1/2)+1/2-1/2*5^(1/2))^(1/2))+(1/2-1/2*5^(1/2))/(-2+2*5^(1/2))^(1/2)*arc
tan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*
(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm
="maxima")
```

```
[Out] -2/5*(x^(5/2) - 3*x^(3/2))/(x^2 - x - 1) + integrate(1/5*(x^(3/2) + sqrt(x)
)/(x^2 - x - 1), x) + integrate((x^2 + x - 1)/((x^4 - 2*x^3 - x^2 + 2*x + 1
)*sqrt(x + 1)*sqrt(x - 1)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(153) = 306$.
time = 0.33, size = 424, normalized size = 1.93

$$\frac{4\sqrt{5}(x^2-x-1)\sqrt{10\sqrt{5}-22}\arctan\left(\frac{1}{2}\sqrt{2x^2-\sqrt{x^2-1}\left(2x+\sqrt{5}-1\right)+\sqrt{5}x-x}\sqrt{10\sqrt{5}-22}\left(\sqrt{5}+2\right)+\frac{1}{4}\left(\sqrt{5}(2x+1)-2\sqrt{x^2-1}\left(\sqrt{5}+2\right)+4x+3\right)\sqrt{10\sqrt{5}-22}\right)-4\sqrt{5}(x^2-x-1)\sqrt{10\sqrt{5}-22}\arctan\left(\frac{1}{4}\left(\sqrt{2}\sqrt{2x+\sqrt{5}-1}\left(\sqrt{5}+2\right)-2\sqrt{x}\left(\sqrt{5}+2\right)\right)\sqrt{10\sqrt{5}-22}\right)-\sqrt{5}(x^2-x-1)\sqrt{10\sqrt{5}+22}\log\left(\sqrt{10\sqrt{5}+22}\left(\sqrt{5}-3\right)-4x+2\sqrt{5}+4\sqrt{x^2-1}+2\right)+\sqrt{5}(x^2-x-1)\sqrt{10\sqrt{5}+22}\log\left(\sqrt{10\sqrt{5}+22}\left(\sqrt{5}-3\right)+4\sqrt{x}\right)+\sqrt{5}(x^2-x-1)\sqrt{10\sqrt{5}+22}\log\left(-\sqrt{10\sqrt{5}+22}\left(\sqrt{5}-3\right)-4x+2\sqrt{5}+4\sqrt{x^2-1}+2\right)-\sqrt{5}(x^2-x-1)\sqrt{10\sqrt{5}+22}\log\left(-\sqrt{10\sqrt{5}+22}\left(\sqrt{5}-3\right)+4\sqrt{x}\right)-40x^2-20\sqrt{x^2-1}(2x-1)+20(2x-1)\sqrt{x}+40x+40}{50(x^2-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/50*(4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22)*arctan(1/2*sqrt(2*x^2 -
 sqrt(x^2 - 1)*(2*x + sqrt(5) - 1) + sqrt(5)*x - x)*sqrt(10*sqrt(5) - 22)*(
sqrt(5) + 2) + 1/4*(sqrt(5)*(2*x + 1) - 2*sqrt(x^2 - 1)*(sqrt(5) + 2) + 4*x
 + 3)*sqrt(10*sqrt(5) - 22)) - 4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22
)*arctan(1/4*(sqrt(2)*sqrt(2*x + sqrt(5) - 1)*(sqrt(5) + 2) - 2*sqrt(x)*(sq
rt(5) + 2))*sqrt(10*sqrt(5) - 22)) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
+ 22)*log(sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^
2 - 1) + 2) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(
5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
 + 22)*log(-sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(
x^2 - 1) + 2) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sq
rt(5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) - 40*x^2 - 20*sqrt(x^2 - 1)*(2*x - 1
) + 20*(2*x - 1)*sqrt(x) + 40*x + 40)/(x^2 - x - 1)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/2)-(x**2-1)**(1/2))**2/(-x**2+x+1)**2/(x**2-1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\sqrt{x^2-1}-\sqrt{x}\right)^2}{\sqrt{x^2-1}\,\left(-x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)^(1/2) - x^(1/2))^2/((x^2 - 1)^(1/2)*(x - x^2 + 1)^2),x)
```

```
[Out] int(((x^2 - 1)^(1/2) - x^(1/2))^2/((x^2 - 1)^(1/2)*(x - x^2 + 1)^2), x)
```

3.11 $\int \left(\frac{1}{\sqrt{2}\,(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}\,(1+x)^2\sqrt{i+x^2}}\right)dx$

Optimal. Leaf size=138

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}\,(1+x)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}\,(1+x)}+\frac{\tanh^{-1}\left(\frac{i+x}{\sqrt{1-i}\,\sqrt{-i+x^2}}\right)}{(1-i)^{3/2}\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{i-x}{\sqrt{1+i}\,\sqrt{i+x^2}}\right)}{(1+i)^{3/2}\sqrt{2}}$$

```
[Out] 1/2*arctanh((I+x)/(1-I)^(1/2)/(-I+x^2)^(1/2))/(1-I)^(3/2)*2^(1/2)-1/2*arcta
nh((I-x)/(1+I)^(1/2)/(I+x^2)^(1/2))/(1+I)^(3/2)*2^(1/2)-(1/4+1/4*I)*(-I+x^2
)^(1/2)/(1+x)*2^(1/2)+(-1/4+1/4*I)*(I+x^2)^(1/2)/(1+x)*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {745, 739, 212}

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x^2-i}}{\sqrt{2}\,(x+1)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{x^2+i}}{\sqrt{2}\,(x+1)}+\frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{1-i}\,\sqrt{x^2-i}}\right)}{(1-i)^{3/2}\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{-x+i}{\sqrt{1+i}\,\sqrt{x^2+i}}\right)}{(1+i)^{3/2}\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2]*(1 + x)^2*Sqrt[-I + x^2]) + 1/(Sqrt[2]*(1 + x)^2*Sqrt[I + x^
2]),x]
```

```
[Out] ((-1/2 - I/2)*Sqrt[-I + x^2])/(Sqrt[2]*(1 + x)) - ((1/2 - I/2)*Sqrt[I + x^2
])/(Sqrt[2]*(1 + x)) + ArcTanh[(I + x)/(Sqrt[1 - I]*Sqrt[-I + x^2])]/((1 -
I)^(3/2)*Sqrt[2]) - ArcTanh[(I - x)/(Sqrt[1 + I]*Sqrt[I + x^2])]/((1 + I)^(
3/2)*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
```

```
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rubi steps

$$\int \left(\frac{1}{\sqrt{2}\,(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}\,(1+x)^2\sqrt{i+x^2}}\right)dx=\frac{\int \frac{1}{(1+x)^2\sqrt{-i+x^2}}\,dx}{\sqrt{2}}+\frac{\int \frac{1}{(1+x)^2\sqrt{i+x^2}}\,dx}{\sqrt{2}}$$

$$=-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}\,(1+x)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}\,(1+x)}+\ldots$$

$$=-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}\,(1+x)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}\,(1+x)}+\ldots$$

$$=-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}\,(1+x)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}\,(1+x)}+\ldots$$

Mathematica [A]
time = 3.73, size = 126, normalized size = 0.91

$$-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(i\sqrt{-i+x^2}+\sqrt{i+x^2}+\frac{2(1+x)\tan^{-1}\left(\sqrt{-\frac{1}{2}-\frac{i}{2}}\left(1+x-\sqrt{-i+x^2}\right)\right)}{\sqrt{1-i}}+(1+i)^{3/2}(1+x)\tan^{-1}\left(\sqrt{-\frac{1}{2}+\frac{i}{2}}\left(1+x-\sqrt{i+x^2}\right)\right)\right)}{\sqrt{2}\,(1+x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2]*(1 + x)^2*Sqrt[-I + x^2]) + 1/(Sqrt[2]*(1 + x)^2*Sqrt[
I + x^2]),x]
```

```
[Out] ((-1/2 + I/2)*(I*Sqrt[-I + x^2] + Sqrt[I + x^2] + (2*(1 + x)*ArcTan[Sqrt[-1
/2 - I/2]*(1 + x - Sqrt[-I + x^2])])/Sqrt[1 - I] + (1 + I)^(3/2)*(1 + x)*Ar
cTan[Sqrt[-1/2 + I/2]*(1 + x - Sqrt[I + x^2])]))/(Sqrt[2]*(1 + x))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: Invalid comparison of non-real I

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[2]*(1 + x)^2*Sqrt[-I + x^2]) + 1/(Sqrt[2]*(1 + x
)^2*Sqrt[I + x^2]),x]')
```

```
[Out] cought exception: Invalid comparison of non-real I
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(101) = 202$.
time = 0.12, size = 278, normalized size = 2.01

method	result
default	$-\frac{\sqrt{2}\,\sqrt{(1+x)^2-2x-1-i}}{4(1+x)}-\frac{i\sqrt{2}\,\sqrt{(1+x)^2-2x-1-i}}{4(1+x)}-\frac{\sqrt{2}\,\ln\left(\frac{-2i-2x+2\sqrt{1-i}\,\sqrt{(1}}{1+}\right.}{4\sqrt{1-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/(1+x)^2*2^(1/2)/(x^2-I)^(1/2)+1/2/(1+x)^2*2^(1/2)/(x^2+I)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/4*2^(1/2)/(1+x)*((1+x)^2-2*x-1-I)^(1/2)-1/4*I*2^(1/2)/(1+x)*((1+x)^2-2*x
-1-I)^(1/2)-1/4*2^(1/2)/(1-I)^(1/2)*ln((-2*I-2*x+2*(1-I)^(1/2)*((1+x)^2-2*x
-1-I)^(1/2))/(1+x))-1/4*I*2^(1/2)/(1-I)^(1/2)*ln((-2*I-2*x+2*(1-I)^(1/2)*((
1+x)^2-2*x-1-I)^(1/2))/(1+x))-1/4*2^(1/2)/(1+x)*((1+x)^2-2*x-1+I)^(1/2)+1/4
*I*2^(1/2)/(1+x)*((1+x)^2-2*x-1+I)^(1/2)-1/4*2^(1/2)/(1+I)^(1/2)*ln((2*I-2*
x+2*(1+I)^(1/2)*((1+x)^2-2*x-1+I)^(1/2))/(1+x))+1/4*I*2^(1/2)/(1+I)^(1/2)*l
n((2*I-2*x+2*(1+I)^(1/2)*((1+x)^2-2*x-1+I)^(1/2))/(1+x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1
/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  1which is not of the expected type LIST
```

Fricas [A]
time = 0.32, size = 161, normalized size = 1.17

$$\frac{\sqrt{-\frac{1}{2}i+\frac{1}{2}}\,(-(i-1)\,x-i+1)\log\left(\sqrt{2}\,\sqrt{-\frac{1}{2}i+\frac{1}{2}}-x+\sqrt{x^2-i}-1\right)+\sqrt{-\frac{1}{2}i+\frac{1}{2}}\,((i-1)\,x+i-1)\log\left(-\sqrt{2}\,\sqrt{-\frac{1}{2}i+\frac{1}{2}}-x+\sqrt{x^2-i}-1\right)+\sqrt{-\frac{1}{2}i-\frac{1}{2}}\,(-(i+1)\,x-i-1)\log\left(i\sqrt{2}\,\sqrt{-\frac{1}{2}i-\frac{1}{2}}-x+\sqrt{x^2+i}-1\right)+\sqrt{-\frac{1}{2}i-\frac{1}{2}}\,((i+1)\,x+i+1)\log\left(-i\sqrt{2}\,\sqrt{-\frac{1}{2}i-\frac{1}{2}}-x+\sqrt{x^2+i}-1\right)+\sqrt{2}\,(-(i+1)\,x-i-1)-\sqrt{2}\,\sqrt{x^2+i}-i\sqrt{2}\,\sqrt{x^2-i}}{(2i+2)\,x+2i+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1
/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-1/2*I + 1/2)*(-(I - 1)*x - I + 1)*log(sqrt(2)*sqrt(-1/2*I + 1/2) - x
 + sqrt(x^2 - I) - 1) + sqrt(-1/2*I + 1/2)*((I - 1)*x + I - 1)*log(-sqrt(2)
*sqrt(-1/2*I + 1/2) - x + sqrt(x^2 - I) - 1) + sqrt(-1/2*I - 1/2)*(-(I + 1)
*x - I - 1)*log(I*sqrt(2)*sqrt(-1/2*I - 1/2) - x + sqrt(x^2 + I) - 1) + sqr
t(-1/2*I - 1/2)*((I + 1)*x + I + 1)*log(-I*sqrt(2)*sqrt(-1/2*I - 1/2) - x +
 sqrt(x^2 + I) - 1) + sqrt(2)*(-(I + 1)*x - I - 1) - sqrt(2)*sqrt(x^2 + I)
- I*sqrt(2)*sqrt(x^2 - I))/((2*I + 2)*x + 2*I + 2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)**2*2**(1/2)/(-I+x**2)**(1/2)+1/2/(1+x)**2*2**(1/2)/(I+x
**2)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A]
time = 0.01, size = 213, normalized size = 1.54

$$-\frac{4}{2}\sqrt{2}\left(\frac{\sqrt{-I+x^2}-x-I}{(2I-2)\left(\left(\sqrt{-I+x^2}-x\right)^2-2\left(\sqrt{-I+x^2}-x\right)+I\right)}+\frac{\arctan\left(\frac{\sqrt{-I+x^2}-x-1}{\sqrt{I-1}}\right)}{2(I-1)\sqrt{I-1}}\right)-\frac{4}{2}\sqrt{2}\left(\frac{-\sqrt{I+x^2}+x-I}{(2I+2)\left(\left(\sqrt{I+x^2}-x\right)^2-2\left(\sqrt{I+x^2}-x\right)-I\right)}-\frac{\arctan\left(\frac{\sqrt{I+x^2}-x-1}{\sqrt{-I-1}}\right)}{2(I+1)\sqrt{-I-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1
/2),x)
```

```
[Out] 1/2*sqrt(2)*(-(I - 1)*arctan(-(x - sqrt(x^2 + I) + 1)/sqrt(-I - 1))/sqrt(-I
 - 1) + ((I - 1)*x - (I - 1)*sqrt(x^2 + I) + I + 1)/((x - sqrt(x^2 + I))^2
+ 2*x - 2*sqrt(x^2 + I) - I)) + 1/2*sqrt(2)*((I + 1)*arctan(-(x - sqrt(x^2
- I) + 1)/sqrt(I - 1))/sqrt(I - 1) + (-(I + 1)*x + (I + 1)*sqrt(x^2 - I) -
I + 1)/((x - sqrt(x^2 - I))^2 + 2*x - 2*sqrt(x^2 - I) + I))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2}}{2\sqrt{x^2-\mathrm{i}}\,(x+1)^2}+\frac{\sqrt{2}}{2\sqrt{x^2+1\mathrm{i}}\,(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^(1/2)/(2*(x^2 - 1i)^(1/2)*(x + 1)^2) + 2^(1/2)/(2*(x^2 + 1i)^(1/2)*(x
 + 1)^2),x)
```

```
[Out] int(2^(1/2)/(2*(x^2 - 1i)^(1/2)*(x + 1)^2) + 2^(1/2)/(2*(x^2 + 1i)^(1/2)*(x
 + 1)^2), x)
```

3.12 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=125

$$-\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\,\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\,\sqrt{1+ix^2}}\right)$$

```
[Out] -1/4*(1-I)^(3/2)*arctanh((1+I*x)/(1-I)^(1/2)/(1-I*x^2)^(1/2))-1/4*(1+I)^(3/
2)*arctanh((1-I*x)/(1+I)^(1/2)/(1+I*x^2)^(1/2))-1/2*(1-I*x^2)^(1/2)/(1+x)-1
/2*(1+I*x^2)^(1/2)/(1+x)
```

Rubi [A]
time = 0.11, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2158, 745, 739, 212}

$$-\frac{\sqrt{1-ix^2}}{2(x+1)}-\frac{\sqrt{1+ix^2}}{2(x+1)}-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\,\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\,\sqrt{1+ix^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

```
[Out] -1/2*Sqrt[1 - I*x^2]/(1 + x) - Sqrt[1 + I*x^2]/(2*(1 + x)) - ((1 - I)^(3/2)
*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/4 - ((1 + I)^(3/2)*ArcTa
nh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2158

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx &= \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+x)^2\sqrt{1-ix^2}}\,dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1+x)^2\sqrt{1+ix^2}}\,dx\\
&= -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}-\frac{1}{2}i\int \frac{1}{(1+x)\sqrt{1-ix^2}}\,dx+\frac{1}{2}i\int \frac{1}{(1+x)\sqrt{1+ix^2}}\\
&= -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}+\frac{1}{2}i\text{Subst}\left(\int \frac{1}{(1-i)-x^2}\,dx,x,\frac{1+ix}{\sqrt{1-ix^2}}\right)-\frac{1}{2}i\text{Su}\\
&= -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\,\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 272 vs. $2(125) = 250$.
time = 2.18, size = 272, normalized size = 2.18

$$\frac{1}{2}\left(\frac{-1-2x^4-\sqrt{1+x^4}-x^2\left(1+2\sqrt{1+x^4}\right)}{(1+x)\left(x^2+\sqrt{1+x^4}\right)^{3/2}}+\frac{\tan^{-1}\left(\sqrt{1+\sqrt{2}}\,\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{-1+\sqrt{2}}}-\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)-\frac{\tanh^{-1}\left(\sqrt{-1+\sqrt{2}}\,\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{1+\sqrt{2}}}+\sqrt{-1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

```
[Out] ((-1 - 2*x^4 - Sqrt[1 + x^4] - x^2*(1 + 2*Sqrt[1 + x^4]))/((1 + x)*(x^2 + S
qrt[1 + x^4])^(3/2)) + ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]]/
Sqrt[-1 + Sqrt[2]] - Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(-1 + Sqrt[2])]*x*Sqr
t[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])] - ArcTanh[Sqrt[-1 + Sqrt
[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[1 + Sqrt[2]] + Sqrt[-1 + Sqrt[2]]*ArcT
anh[(Sqrt[2*(1 + Sqrt[2])]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 +
 x^4])])/2
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[Sqrt[x^4 + 1] + x^2]/((x + 1)^2*Sqrt[x^4 + 1]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{2}+\sqrt{x^{4}+1}}}{\left(1+x\right)^{2} \sqrt{x^{4}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(81) = 162.
time = 2.06, size = 394, normalized size = 3.15

$$\frac{4\,(x+1)\sqrt{\sqrt{2}+1}\,\arctan\left(\frac{2\left(x^{3}+x^{2}-\sqrt{2}\,(x^{3}+1)+\sqrt{x^{4}+1}\left(\sqrt{2}\,x-x-1\right)-x+1\right)\sqrt{x^{2}+\sqrt{x^{4}+1}}\sqrt{\sqrt{2}+1}+\left(2x^{2}-\sqrt{2}\,(x^{2}+1)+2\sqrt{x^{4}+1}\left(\sqrt{2}-1\right)+2\right)\sqrt{2\sqrt{2}+2}\sqrt{\sqrt{2}+1}}{2\,(x^{2}-2x+1)}\right)+(x+1)\sqrt{\sqrt{2}-1}\,\log\left(-\frac{\left(2x^{3}-\sqrt{2}\,(x^{3}-x^{2}-x-1)+\sqrt{x^{4}+1}\left(\sqrt{2}\,(x-1)-2x\right)-2\right)\sqrt{x^{2}+\sqrt{x^{4}+1}}+\left(\sqrt{2}\,(x^{2}+1)+2\sqrt{x^{4}+1}\right)\sqrt{\sqrt{2}-1}}{x^{2}+2x+1}\right)-(x+1)\sqrt{\sqrt{2}-1}\,\log\left(-\frac{\left(2x^{3}-\sqrt{2}\,(x^{3}-x^{2}-x-1)+\sqrt{x^{4}+1}\left(\sqrt{2}\,(x-1)-2x\right)-2\right)\sqrt{x^{2}+\sqrt{x^{4}+1}}-\left(\sqrt{2}\,(x^{2}+1)+2\sqrt{x^{4}+1}\right)\sqrt{\sqrt{2}-1}}{x^{2}+2x+1}\right)+4\sqrt{x^{2}+\sqrt{x^{4}+1}}\left(x^{2}-\sqrt{x^{4}+1}-1\right)}{8\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/8*(4*(x + 1)*sqrt(sqrt(2) + 1)*arctan(1/2*(2*(x^3 + x^2 - sqrt(2)*(x^3 +
1) + sqrt(x^4 + 1)*(sqrt(2)*x - x - 1) - x + 1)*sqrt(x^2 + sqrt(x^4 + 1))*s
qrt(sqrt(2) + 1) + (2*x^2 - sqrt(2)*(x^2 + 1) + 2*sqrt(x^4 + 1)*(sqrt(2) -
1) + 2)*sqrt(2*sqrt(2) + 2)*sqrt(sqrt(2) + 1))/(x^2 - 2*x + 1)) + (x + 1)*s
qrt(sqrt(2) - 1)*log(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)
*(sqrt(2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) + (sqrt(2)*(x^2 + 1
) + 2*sqrt(x^4 + 1))*sqrt(sqrt(2) - 1))/(x^2 + 2*x + 1)) - (x + 1)*sqrt(sqr
t(2) - 1)*log(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(
```

```
2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - (sqrt(2)*(x^2 + 1) + 2*s
qrt(x^4 + 1))*sqrt(sqrt(2) - 1))/(x^2 + 2*x + 1)) + 4*sqrt(x^2 + sqrt(x^4 +
 1))*(x^2 - sqrt(x^4 + 1) - 1))/(x + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{(x+1)^2 \sqrt{x^4 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)**2/(x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)**2*sqrt(x**4 + 1)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{x^4 + 1} + x^2}}{\sqrt{x^4 + 1}\, (x+1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)^2),x)
```

```
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)^2), x)
```

3.13 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=81

$$-\frac{1}{2}\sqrt{1-i}\,\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\,\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\,\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\,\sqrt{1+ix^2}}\right)$$

```
[Out] -1/2*arctanh((1+I*x)/(1-I)^(1/2)/(1-I*x^2)^(1/2))*(1-I)^(1/2)-1/2*arctanh((
1-I*x)/(1+I)^(1/2)/(1+I*x^2)^(1/2))*(1+I)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2158, 739, 212}

$$-\frac{1}{2}\sqrt{1-i}\,\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\,\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\,\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\,\sqrt{1+ix^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] -1/2*(Sqrt[1 - I]*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])]) - (Sqrt
[1 + I]*ArcTanh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2158

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+x)\sqrt{1-ix^2}}\,dx + \left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1+x)\sqrt{1+ix^2}}\,dx$$

$$= \left(-\frac{1}{2}-\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{(1+i)-x^2}\,dx, x, \frac{1-ix}{\sqrt{1+ix^2}}\right) + \left(-\frac{1}{2}+\frac{i}{2}\right)\text{Subst}\left(\int \frac{}{(}\right.$$

$$= -\frac{1}{2}\sqrt{1-i}\,\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\,\sqrt{1-ix^2}}\right) - \frac{1}{2}\sqrt{1+i}\,\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\,\sqrt{1+ix^2}}\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 205 vs. 2(81) = 162.
time = 0.86, size = 205, normalized size = 2.53

$$\frac{\sqrt{-1+\sqrt{2}}\left(\tan^{-1}\left(\sqrt{1+\sqrt{2}}\,\sqrt{x^2+\sqrt{1+x^4}}\right)-\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)-\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\sqrt{-1+\sqrt{2}}\,\sqrt{x^2+\sqrt{1+x^4}}\right)+\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] (Sqrt[-1 + Sqrt[2]]*(ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]] -
ArcTan[(Sqrt[2*(-1 + Sqrt[2])]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt
[1 + x^4])]) - Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x^2 + Sqrt
[1 + x^4]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[(Sqrt[2*(1 + Sqrt[2])]*x*Sqrt[x^2 +
 Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])])/Sqrt[2]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[Sqrt[x^4 + 1] + x^2]/((x + 1)*Sqrt[x^4 + 1]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(1+x)\,\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(49) = 98$.
time = 3.05, size = 369, normalized size = 4.56

$$\frac{1}{2}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{\left(2x^2-\sqrt{2}\left(x^3-x^2+x+1\right)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2\right)-2x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{2\sqrt{2}-2}+\left(x^2+\sqrt{2}\sqrt{x^4+1}+1\right)\sqrt{2\sqrt{2}+2}\sqrt{2\sqrt{2}-2}}{2\left(x^2-2x+1\right)}\right)-\frac{1}{8}\sqrt{2\sqrt{2}+2}\log\left(-\frac{\left(2x^3-\sqrt{2}\left(x^3-x^2-x-1\right)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2x\right)-2\right)\sqrt{x^2+\sqrt{x^4+1}}+\left(x^2-\sqrt{2}\left(x^2+1\right)+\sqrt{x^4+1}\left(\sqrt{2}-2\right)+1\right)\sqrt{2\sqrt{2}+2}}{x^2+2x+1}\right)+\frac{1}{8}\sqrt{2\sqrt{2}+2}\log\left(-\frac{\left(2x^3-\sqrt{2}\left(x^3-x^2-x-1\right)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2x\right)-2\right)\sqrt{x^2+\sqrt{x^4+1}}-\left(x^2-\sqrt{2}\left(x^2+1\right)+\sqrt{x^4+1}\left(\sqrt{2}-2\right)+1\right)\sqrt{2\sqrt{2}+2}}{x^2+2x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*sqrt(2*sqrt(2) - 2)*arctan(1/2*((2*x^2 - sqrt(2)*(x^3 - x^2 + x + 1) +
sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2) - 2*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(2
*sqrt(2) - 2) + (x^2 + sqrt(2)*sqrt(x^4 + 1) + 1)*sqrt(2*sqrt(2) + 2)*sqrt(
2*sqrt(2) - 2))/(x^2 - 2*x + 1)) - 1/8*sqrt(2*sqrt(2) + 2)*log(-((2*x^3 - s
qrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2*x) - 2)*sqr
t(x^2 + sqrt(x^4 + 1)) + (x^2 - sqrt(2)*(x^2 + 1) + sqrt(x^4 + 1)*(sqrt(2)
- 2) + 1)*sqrt(2*sqrt(2) + 2))/(x^2 + 2*x + 1)) + 1/8*sqrt(2*sqrt(2) + 2)*l
og(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1)
- 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - (x^2 - sqrt(2)*(x^2 + 1) + sqrt(x^4
 + 1)*(sqrt(2) - 2) + 1)*sqrt(2*sqrt(2) + 2))/(x^2 + 2*x + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)\sqrt{x^4+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)/(x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)*sqrt(x**4 + 1)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)),x)
```

```
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)), x)
```

3.14 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(x*2^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2157, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2157

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

Mathematica [A]
time = 0.13, size = 31, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.56, size = 21, normalized size = 0.68

$$\frac{\text{meijerg}\left[\left\{\{1,1\},\left\{\frac{1}{2}\right\}\right\},\left\{\left\{\frac{1}{4},\frac{3}{4}\right\},\{0\}\right\},x^4\right]}{4\sqrt{\text{Pi}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[Sqrt[x^4 + 1] + x^2]/Sqrt[x^4 + 1],x]')
[Out] meijerg[{{1, 1}, {1 / 2}}, {{1 / 4, 3 / 4}, {0}}, x ^ 4] / (4 Sqrt[Pi])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(24) = 48.
time = 0.42, size = 60, normalized size = 1.94

$$\frac{1}{4}\sqrt{2}\log\left(4x^4+4\sqrt{x^4+1}\,x^2+2\left(\sqrt{2}\,x^3+\sqrt{2}\,\sqrt{x^4+1}\,x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt
(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1)
```

Sympy [A]
time = 0.97, size = 15, normalized size = 0.48

$$\frac{G_{3,3}^{2,2}\left(\begin{matrix} 1, 1 & \frac{1}{2} \\ \frac{1}{4}, \frac{3}{4} & 0 \end{matrix} \middle| {x^{4}} \right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

```
[Out] meijerg(((1, 1), (1/2,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4 + 1)^(1/2),x)
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4 + 1)^(1/2), x)
```

3.15 $\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=33

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(x*2^(1/2)/(-x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2157, 209}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2157

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

Mathematica [**A**]
time = 0.14, size = 33, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Mathics [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.99, size = 21, normalized size = 0.64

$$\frac{\text{meijerg}\left[\left\{\left\{\frac{1}{2},1\right\},\{1\}\right\},\left\{\left\{\frac{1}{4},\frac{3}{4}\right\},\{0\}\right\},x^4\right]}{4\sqrt{\text{Pi}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[Sqrt[x^4 + 1] - x^2]/Sqrt[x^4 + 1],x]')
[Out] meijerg[{{1 / 2, 1}, {1}}, {{1 / 4, 3 / 4}, {0}}, x ^ 4] / (4 Sqrt[Pi])
```

Maple [**C**] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 0.10, size = 22, normalized size = 0.67

method	result	size
meijerg	$-\frac{\sqrt{2}\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4},\frac{5}{4}\right],\left[\frac{3}{2},\frac{3}{2}\right],-\frac{1}{x^4}\right)}{4x^2}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*2^(1/2)/x^2*hypergeom([1/2,3/4,5/4],[3/2,3/2],-1/x^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

Fricas [A]
time = 0.44, size = 29, normalized size = 0.88

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+\sqrt{x^4+1}}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + sqrt(x^4 + 1))/x)
```

Sympy [A]
time = 0.44, size = 15, normalized size = 0.45

$$\frac{G_{3,3}^{2,2}\left(\begin{matrix}\frac{1}{2},1 & 1\\ \frac{1}{4},\frac{3}{4} & 0\end{matrix}\middle|\,x^4\right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
[Out] meijerg(((1/2, 1), (1,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\sqrt{x^4+1}-x^2}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) - x^2)^(1/2)/(x^4 + 1)^(1/2),x)
[Out] int(((x^4 + 1)^(1/2) - x^2)^(1/2)/(x^4 + 1)^(1/2), x)
```

3.16 $\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{2}{\sqrt{-1+x}}-\frac{2}{\sqrt{1+x}}$$

```
[Out] -2/(-1+x)^(1/2)-2/(1+x)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {6820}

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx &= \int \left(\frac{1}{(-1+x)^{3/2}}+\frac{1}{(1+x)^{3/2}}\right)dx\\ &= -\frac{2}{\sqrt{-1+x}}-\frac{2}{\sqrt{1+x}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 19, normalized size = 1.00

$$-\frac{2}{\sqrt{-1+x}}-\frac{2}{\sqrt{1+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)),x
]
```

```
[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.
time = 2.84, size = 38, normalized size = 2.00

$$\frac{2\left(-x\sqrt{-1+x} - x\sqrt{1+x} + \sqrt{1+x} - \sqrt{-1+x}\right)}{-1+x^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[((x - 1)^(3/2) + (x + 1)^(3/2))/((x + 1)^(3/2)*(x - 1)^(
3/2)),x]')
```

```
[Out] 2 (-x Sqrt[-1 + x] - x Sqrt[1 + x] + Sqrt[1 + x] - Sqrt[-1 + x]) / (-1 + x
^ 2)
```

Maple [**A**]
time = 0.16, size = 16, normalized size = 0.84

method	result	size
default	$-\frac{2}{\sqrt{-1+x}} - \frac{2}{\sqrt{1+x}}$	16
meijerg	$\frac{2\sqrt{\pi} - \frac{2\sqrt{\pi}}{\sqrt{1+x}}}{\sqrt{\pi}} - \frac{2(-\mathrm{signum}(-1+x))^{\frac{3}{2}}\left(\sqrt{\pi} - \frac{\sqrt{\pi}}{\sqrt{1-x}}\right)}{\sqrt{\pi}\ \mathrm{signum}(-1+x)^{\frac{3}{2}}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -2/(-1+x)^(1/2)-2/(1+x)^(1/2)
```

Maxima [**A**]
time = 0.24, size = 15, normalized size = 0.79

$$-\frac{2}{\sqrt{x+1}} - \frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"maxima")
```

```
[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

Fricas [**A**]
time = 0.31, size = 28, normalized size = 1.47

$$-\frac{2\left((x+1)\sqrt{x-1} + \sqrt{x+1}\,(x-1)\right)}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"fricas")
```

```
[Out] -2*((x + 1)*sqrt(x - 1) + sqrt(x + 1)*(x - 1))/(x^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(17) = 34$
time = 1.16, size = 56, normalized size = 2.95

$$-\frac{2x\sqrt{x-1}}{x^2-1}-\frac{2x\sqrt{x+1}}{x^2-1}-\frac{2\sqrt{x-1}}{x^2-1}+\frac{2\sqrt{x+1}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)**(3/2)+(1+x)**(3/2))/(-1+x)**(3/2)/(1+x)**(3/2),x)
```

```
[Out] -2*x*sqrt(x - 1)/(x**2 - 1) - 2*x*sqrt(x + 1)/(x**2 - 1) - 2*sqrt(x - 1)/(x
**2 - 1) + 2*sqrt(x + 1)/(x**2 - 1)
```

Giac [A]
time = 0.01, size = 26, normalized size = 1.37

$$-\frac{2}{\sqrt{x-1}}-\frac{2\sqrt{x+1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x)
```

```
[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

Mupad [B]
time = 0.40, size = 15, normalized size = 0.79

$$-\frac{2}{\sqrt{x-1}}-\frac{2}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)^(3/2) + (x + 1)^(3/2))/((x - 1)^(3/2)*(x + 1)^(3/2)),x)
```

```
[Out] - 2/(x - 1)^(1/2) - 2/(x + 1)^(1/2)
```

3.17 $\int \left(x+\sqrt{a+x^2}\right)^b dx$

Optimal. Leaf size=52

$$-\frac{a\left(x+\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)}+\frac{\left(x+\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}$$

```
[Out] -1/2*a*(x+(x^2+a)^(1/2))^(-1+b)/(1-b)+1/2*(x+(x^2+a)^(1/2))^(1+b)/(1+b)
```

Rubi [A]
time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2142, 14}

$$\frac{\left(\sqrt{a+x^2}+x\right)^{b+1}}{2(b+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{b-1}}{2(1-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^b,x]
```

```
[Out] -1/2*(a*(x + Sqrt[a + x^2])^(-1 + b))/(1 - b) + (x + Sqrt[a + x^2])^(1 + b)
/(2*(1 + b))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(x+\sqrt{a+x^2}\right)^b dx = \frac{1}{2}\text{Subst}\left(\int x^{-2+b}\left(a+x^2\right) dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(ax^{-2+b}+x^b\right) dx, x, x+\sqrt{a+x^2}\right)$$
$$= -\frac{a\left(x+\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)} + \frac{\left(x+\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}$$

Mathematica [A]
time = 0.12, size = 43, normalized size = 0.83

$$\frac{\left(x+\sqrt{a+x^2}\right)^{-1+b}\left(ab+(-1+b)x\left(x+\sqrt{a+x^2}\right)\right)}{-1+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^b,x]
```

```
[Out] ((x + Sqrt[a + x^2])^(-1 + b)*(a*b + (-1 + b)*x*(x + Sqrt[a + x^2])))/(-1 +
 b^2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 94.03, size = 738, normalized size = 14.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x + Sqrt[x^2 + a])^b,x]')
```

```
[Out] Piecewise[{{(-a ^ 2 b + a ^ 2 b Cosh[ArcSinh[x / Sqrt[a]] (1 + b)] - a ^ (3
 / 2) x Cosh[b ArcSinh[x / Sqrt[a]]] + a ^ (3 / 2) b x Sqrt[1 + a / x ^ 2]
Sinh[b ArcSinh[x / Sqrt[a]]] - a x ^ 2 Sqrt[1 + a / x ^ 2] Sinh[ArcSinh[x /
 Sqrt[a]] (1 + b)] + a x ^ 2 Cosh[ArcSinh[x / Sqrt[a]] (1 + b)] - a b x ^ 2
 - a b x ^ 2 Sqrt[1 + a / x ^ 2] Sinh[ArcSinh[x / Sqrt[a]] (1 + b)] + 2 a b
 x ^ 2 Cosh[ArcSinh[x / Sqrt[a]] (1 + b)] - Sqrt[a] x ^ 3 Cosh[b ArcSinh[x
/ Sqrt[a]]] + Sqrt[a] b x ^ 3 Sqrt[1 + a / x ^ 2] Sinh[b ArcSinh[x / Sqrt[a
]]] - x ^ 4 Sqrt[1 + a / x ^ 2] Sinh[ArcSinh[x / Sqrt[a]] (1 + b)] + x ^ 4
Cosh[ArcSinh[x / Sqrt[a]] (1 + b)] - b x ^ 4 Sqrt[1 + a / x ^ 2] Sinh[ArcSi
nh[x / Sqrt[a]] (1 + b)] + b x ^ 4 Cosh[ArcSinh[x / Sqrt[a]] (1 + b)]) a ^
(-1 / 2 + b / 2) / (-a + a b ^ 2 - x ^ 2 + b ^ 2 x ^ 2), Abs[x ^ 2 / a] > 1
}}, 2 a ^ 3 b Cosh[b ArcSinh[x / Sqrt[a]] + ArcSinh[x / Sqrt[a]]] Gamma[1 -
 b / 2] a ^ (b / 2) / (-2 a ^ (5 / 2) Gamma[1 - b / 2] + 2 a ^ (5 / 2) b ^
```

```
2 Gamma[1 - b / 2]) - a ^ 3 b ^ 2 Gamma[-b / 2] a ^ (b / 2) Sqrt[1 + x ^ 2
/ a] Sinh[b ArcSinh[x / Sqrt[a]]] / (-2 a ^ (5 / 2) Gamma[1 - b / 2] + 2 a
^ (5 / 2) b ^ 2 Gamma[1 - b / 2]) - 2 a ^ (5 / 2) x Gamma[1 - b / 2] a ^ (b
 / 2) Sqrt[1 + x ^ 2 / a] Sinh[b ArcSinh[x / Sqrt[a]] + ArcSinh[x / Sqrt[a]
]] / (-2 a ^ (5 / 2) Gamma[1 - b / 2] + 2 a ^ (5 / 2) b ^ 2 Gamma[1 - b / 2
]) - 2 a ^ (5 / 2) b x Gamma[1 - b / 2] a ^ (b / 2) Sqrt[1 + x ^ 2 / a] Sin
h[b ArcSinh[x / Sqrt[a]] + ArcSinh[x / Sqrt[a]]] / (-2 a ^ (5 / 2) Gamma[1
- b / 2] + 2 a ^ (5 / 2) b ^ 2 Gamma[1 - b / 2]) + a ^ (5 / 2) b x Cosh[b A
rcSinh[x / Sqrt[a]]] Gamma[-b / 2] a ^ (b / 2) / (-2 a ^ (5 / 2) Gamma[1 -
b / 2] + 2 a ^ (5 / 2) b ^ 2 Gamma[1 - b / 2]) + 2 a ^ 2 x ^ 2 Cosh[b ArcSi
nh[x / Sqrt[a]] + ArcSinh[x / Sqrt[a]]] Gamma[1 - b / 2] a ^ (b / 2) / (-2
a ^ (5 / 2) Gamma[1 - b / 2] + 2 a ^ (5 / 2) b ^ 2 Gamma[1 - b / 2]) + 2 a
^ 2 b x ^ 2 Cosh[b ArcSinh[x / Sqrt[a]] + ArcSinh[x / Sqrt[a]]] Gamma[1 - b
 / 2] a ^ (b / 2) / (-2 a ^ (5 / 2) Gamma[1 - b / 2] + 2 a ^ (5 / 2) b ^ 2
Gamma[1 - b / 2])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(44) = 88$.
time = 0.03, size = 120, normalized size = 2.31

method	result	siz
meijerg	$\frac{a^{\frac{b}{2}+\frac{1}{2}} b \left(\frac{8\sqrt{\pi}\, x^{1+b} a^{-\frac{b}{2}-\frac{1}{2}} \left(\frac{a b}{x^{2}}+b-1\right) \left(\sqrt{1+\frac{a}{x^{2}}}+1\right)^{-1+b}}{(1+b) b (-2+2 b)}+\frac{4\sqrt{\pi}\, x^{1+b} a^{-\frac{b}{2}-\frac{1}{2}} \sqrt{1+\frac{a}{x^{2}}}\, \left(\sqrt{1+\frac{a}{x^{2}}}+1\right)^{-1+b}}{(1+b) b}\right)}{4\sqrt{\pi}}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^b,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^(1/2*b+1/2)/Pi^(1/2)*b*(8*Pi^(1/2)/(1+b)/b*x^(1+b)*a^(-1/2*b-1/2)*(a*
b/x^2+b-1)/(-2+2*b)*((1+1/x^2*a)^(1/2)+1)^(-1+b)+4*Pi^(1/2)/(1+b)/b*x^(1+b)
*a^(-1/2*b-1/2)*(1+1/x^2*a)^(1/2)*((1+1/x^2*a)^(1/2)+1)^(-1+b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^b, x)
```

Fricas [A]
time = 0.32, size = 32, normalized size = 0.62

$$\frac{\left(\sqrt{x^2+a}\, b-x\right)\left(x+\sqrt{x^2+a}\right)^{b}}{b^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b,x, algorithm="fricas")
```

```
[Out] (sqrt(x^2 + a)*b - x)*(x + sqrt(x^2 + a))^b/(b^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2147 vs. $2(37) = 74$
time = 1.73, size = 2147, normalized size = 41.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**b,x)
```

```
[Out] Piecewise((-a**(9/2)*a**(b/2)*b**2*x*sqrt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a
)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2)
 + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) +
a**(9/2)*a**(b/2)*b*x*cosh(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2
*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1
- b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) - a**(7/2)*a**(b/2)*b**2*x**3*sqrt
(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(1
- b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) -
2*a**(7/2)*x**2*gamma(1 - b/2)) + a**(7/2)*a**(b/2)*b*x**3*cosh(b*asinh(x/s
qrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 -
 b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2
)) + 2*a**5*a**(b/2)*b*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1
- b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(
7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) - 2*a**5*a*
*(b/2)*b*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(
1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 -
b/2)) - 2*a**4*a**(b/2)*b*x**2*sqrt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)) + a
sinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2
)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*ga
mma(1 - b/2)) + 4*a**4*a**(b/2)*b*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sq
rt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1
 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b
/2)) - 2*a**4*a**(b/2)*b*x**2*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2
) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**
(7/2)*x**2*gamma(1 - b/2)) - 2*a**4*a**(b/2)*x**2*sqrt(a/x**2 + 1)*sinh(b*a
sinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1
 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) -
 2*a**(7/2)*x**2*gamma(1 - b/2)) + 2*a**4*a**(b/2)*x**2*cosh(b*asinh(x/sqrt
(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2
*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)
*x**2*gamma(1 - b/2)) - 2*a**3*a**(b/2)*b*x**4*sqrt(a/x**2 + 1)*sinh(b*asin
h(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 -
```

```
b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*
a**(7/2)*x**2*gamma(1 - b/2)) + 2*a**3*a**(b/2)*b*x**4*cosh(b*asinh(x/sqrt(
a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*
a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*
x**2*gamma(1 - b/2)) - 2*a**3*a**(b/2)*x**4*sqrt(a/x**2 + 1)*sinh(b*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2
) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**
(7/2)*x**2*gamma(1 - b/2)) + 2*a**3*a**(b/2)*x**4*cosh(b*asinh(x/sqrt(a)) +
 asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9
/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*
gamma(1 - b/2)), Abs(x**2/a) > 1), (-2*a**(5/2)*a**(b/2)*b*x*sqrt(1 + x**2/
a)*sinh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(5/2)*b
**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 - b/2)) + a**(5/2)*a**(b/2)*b*x*cos
h(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5
/2)*gamma(1 - b/2)) - 2*a**(5/2)*a**(b/2)*x*sqrt(1 + x**2/a)*sinh(b*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2
) - 2*a**(5/2)*gamma(1 - b/2)) - a**3*a**(b/2)*b**2*sqrt(1 + x**2/a)*sinh(b
*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)
*gamma(1 - b/2)) + 2*a**3*a**(b/2)*b*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt
(a)))*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 -
 b/2)) + 2*a**2*a**(b/2)*b*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))
*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 - b/2)
) + 2*a**2*a**(b/2)*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(
1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 - b/2)), True
))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b,x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(x+\sqrt{x^2+a}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^b,x)
```

```
[Out] int((x + (a + x^2)^(1/2))^b, x)
```

3.18 $\int \left(x-\sqrt{a+x^2}\right)^b \, dx$

Optimal. Leaf size=56

$$-\frac{a\left(x-\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)}+\frac{\left(x-\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}$$

```
[Out] -1/2*a*(x-(x^2+a)^(1/2))^(-1+b)/(1-b)+1/2*(x-(x^2+a)^(1/2))^(1+b)/(1+b)
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2142, 14}

$$\frac{\left(x-\sqrt{a+x^2}\right)^{b+1}}{2(b+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{b-1}}{2(1-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^b,x]
```

```
[Out] -1/2*(a*(x - Sqrt[a + x^2])^(-1 + b))/(1 - b) + (x - Sqrt[a + x^2])^(1 + b)
/(2*(1 + b))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(x-\sqrt{a+x^2}\right)^b\,dx = \frac{1}{2}\text{Subst}\left(\int x^{-2+b}\left(a+x^2\right)\,dx, x, x-\sqrt{a+x^2}\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(ax^{-2+b}+x^b\right)\,dx, x, x-\sqrt{a+x^2}\right)$$
$$= -\frac{a\left(x-\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)} + \frac{\left(x-\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}$$

Mathematica [**A**]
time = 0.14, size = 50, normalized size = 0.89

$$\frac{1}{2}\left(x-\sqrt{a+x^2}\right)^{-1+b}\left(\frac{a}{-1+b}+\frac{\left(x-\sqrt{a+x^2}\right)^2}{1+b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^b,x]
```

```
[Out] ((x - Sqrt[a + x^2])^(-1 + b)*(a/(-1 + b) + (x - Sqrt[a + x^2])^2/(1 + b)))
/2
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x - Sqrt[x^2 + a])^b,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(x-\sqrt{x^2+a}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^b,x)
```

```
[Out] int((x-(x^2+a)^(1/2))^b,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^b, x)
```

Fricas [**A**]
time = 0.32, size = 33, normalized size = 0.59

$$-\frac{\left(\sqrt{x^2+a}\,b+x\right)\left(x-\sqrt{x^2+a}\right)^b}{b^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="fricas")
[Out] -(sqrt(x^2 + a)*b + x)*(x - sqrt(x^2 + a))^b/(b^2 - 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x-\sqrt{a+x^2}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**b,x)
[Out] Integral((x - sqrt(a + x**2))**b, x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b,x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int\left(x-\sqrt{x^2+a}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^b,x)
[Out] int((x - (a + x^2)^(1/2))^b, x)
```

3.19 $\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\left(x+\sqrt{a+x^2}\right)^b}{b}$$

```
[Out] (x+(x^2+a)^(1/2))^b/b
```

Rubi [A]
time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2147, 30}

$$\frac{\left(\sqrt{a+x^2}+x\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

```
[Out] (x + Sqrt[a + x^2])^b/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx &= \text{Subst}\left(\int x^{-1+b}\,dx, x, x+\sqrt{a+x^2}\right)\\ &= \frac{\left(x+\sqrt{a+x^2}\right)^b}{b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.00

$$\frac{\left(x+\sqrt{a+x^2}\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

```
[Out] (x + Sqrt[a + x^2])^b/b
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 7.68, size = 269, normalized size = 15.82

$$\text{Piecewise}\left[\left\{\left\{\frac{xa^{\frac{1}{2}+\frac{b}{2}}\sqrt{\frac{a+x^2}{x^2}}\,\text{Sinh}\left[\text{ArcSinh}\left[\frac{x}{\sqrt{a}}\right](-1+b)\right]+x^3a^{-\frac{1}{2}+\frac{b}{2}}\sqrt{\frac{a+x^2}{x^2}}\,\text{Sinh}\left[\text{ArcSinh}\left[\frac{x}{\sqrt{a}}\right](-1+b)\right]+(a+x^2)\left(\frac{x\text{Cosh}\left[\text{ArcSinh}\left[\frac{x}{\sqrt{a}}\right](-1+b)\right]}{\sqrt{a}}+\text{Cosh}\left[b\text{ArcSinh}\left[\frac{x}{\sqrt{a}}\right]\right]\right)a^{\frac{b}{2}}}{b(a+x^2)},\text{Abs}\left[\frac{x^2}{a}\right]>1\right\}\right\},-\frac{2\text{Cosh}\left[b\text{ArcSinh}\left[\frac{x}{\sqrt{a}}\right]\right]\text{Gamma}\left[1-\frac{b}{2}\right]a^{\frac{b}{2}}}{b^2\text{Gamma}\left[-\frac{b}{2}\right]}+\frac{a^{\frac{b}{2}}\text{Sinh}\left[b\text{ArcSinh}\left[\frac{x}{\sqrt{a}}\right]-\text{ArcSinh}\left[\frac{x}{\sqrt{a}}\right]\right]}{b\sqrt{1+\frac{x^2}{a}}}+\frac{x\text{Cosh}\left[b\text{ArcSinh}\left[\frac{x}{\sqrt{a}}\right]-\text{ArcSinh}\left[\frac{x}{\sqrt{a}}\right]\right]a^{\frac{b}{2}}}{\sqrt{a}\,b}+\frac{x^2a^{\frac{b}{2}}\text{Sinh}\left[b\text{ArcSinh}\left[\frac{x}{\sqrt{a}}\right]-\text{ArcSinh}\left[\frac{x}{\sqrt{a}}\right]\right]}{ab\sqrt{1+\frac{x^2}{a}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x + Sqrt[x^2 + a])^b/Sqrt[x^2 + a],x]')
```

```
[Out] Piecewise[{{(x a ^ (1 / 2 + b / 2) Sqrt[(a + x ^ 2) / x ^ 2] Sinh[ArcSinh[x
 / Sqrt[a]] (-1 + b)] + x ^ 3 a ^ (-1 / 2 + b / 2) Sqrt[(a + x ^ 2) / x ^ 2
] Sinh[ArcSinh[x / Sqrt[a]] (-1 + b)] + (a + x ^ 2) (x Cosh[ArcSinh[x / Sqr
t[a]] (-1 + b)] / Sqrt[a] + Cosh[b ArcSinh[x / Sqrt[a]]]) a ^ (b / 2)) / (b
 (a + x ^ 2)), Abs[x ^ 2 / a] > 1}}, -2 Cosh[b ArcSinh[x / Sqrt[a]]] Gamma[
1 - b / 2] a ^ (b / 2) / (b ^ 2 Gamma[-b / 2]) + a ^ (b / 2) Sinh[b ArcSinh
[x / Sqrt[a]] - ArcSinh[x / Sqrt[a]]] / (b Sqrt[1 + x ^ 2 / a]) + x Cosh[b
ArcSinh[x / Sqrt[a]] - ArcSinh[x / Sqrt[a]]] a ^ (b / 2) / (Sqrt[a] b) + x
^ 2 a ^ (b / 2) Sinh[b ArcSinh[x / Sqrt[a]] - ArcSinh[x / Sqrt[a]]] / (a b
Sqrt[1 + x ^ 2 / a])]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

```
[Out] int((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

Fricas [A]
time = 0.30, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] (x + sqrt(x^2 + a))^b/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(12) = 24$
time = 1.25, size = 311, normalized size = 18.29

$$\begin{cases} \frac{\sqrt{a}\,a^{\frac{b}{2}}\sinh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{bx\sqrt{\frac{a}{x^2}+1}} - \frac{2a^{\frac{b}{2}}\cosh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{b}{2}\right)}{b^2\Gamma\left(-\frac{b}{2}\right)} + \frac{a^{\frac{b}{2}}x\cosh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}\,b} + \frac{a^{\frac{b}{2}}x\sinh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}\,b\sqrt{\frac{a}{x^2}+1}} & \text{for}\ \left|\frac{x^2}{a}\right| > 1 \\ \frac{a^{\frac{b}{2}}\sinh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{b\sqrt{1+\frac{x^2}{a}}} - \frac{2a^{\frac{b}{2}}\cosh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{b}{2}\right)}{b^2\Gamma\left(-\frac{b}{2}\right)} + \frac{a^{\frac{b}{2}}x^2\sinh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{ab\sqrt{1+\frac{x^2}{a}}} + \frac{a^{\frac{b}{2}}x\cosh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}\,b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a)*a**(b/2)*sinh(b*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(b*
x*sqrt(a/x**2 + 1)) - 2*a**(b/2)*cosh(b*asinh(x/sqrt(a)))*gamma(1 - b/2)/(b
**2*gamma(-b/2)) + a**(b/2)*x*cosh(b*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(
sqrt(a)*b) + a**(b/2)*x*sinh(b*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(sqrt(a
)*b*sqrt(a/x**2 + 1)), Abs(x**2/a) > 1), (a**(b/2)*sinh(b*asinh(x/sqrt(a))
- asinh(x/sqrt(a)))/(b*sqrt(1 + x**2/a)) - 2*a**(b/2)*cosh(b*asinh(x/sqrt(a
)))*gamma(1 - b/2)/(b**2*gamma(-b/2)) + a**(b/2)*x**2*sinh(b*asinh(x/sqrt(a
)) - asinh(x/sqrt(a)))/(a*b*sqrt(1 + x**2/a)) + a**(b/2)*x*cosh(b*asinh(x/s
qrt(a)) - asinh(x/sqrt(a)))/(sqrt(a)*b), True))
```

Giac [A]
time = 0.00, size = 16, normalized size = 0.94

$$\frac{\left(x+\sqrt{a+x^2}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

```
[Out] (x + sqrt(x^2 + a))^b/b
```

Mupad [B]
time = 0.30, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^b/(a + x^2)^(1/2),x)
```

```
[Out] (x + (a + x^2)^(1/2))^b/b
```

3.20 $\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

[Out] -(x-(x^2+a)^(1/2))^b/b

Rubi [A]

time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2147, 30}

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

```
[Out] -((x - Sqrt[a + x^2])^b/b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx &= -\text{Subst}\left(\int x^{-1+b}\,dx, x, x-\sqrt{a+x^2}\right)\\ &= -\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 1.00

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

```
[Out] -((x - Sqrt[a + x^2])^b/b)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.56, size = 52, normalized size = 2.60

$$\text{Piecewise}\left[\left\{\left\{-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b},b!=0\right\}\right\},\text{Piecewise}\left[\left\{\left\{\text{ArcSinh}\left[x\sqrt{\frac{1}{a}}\right],a>0\right\},\left\{\text{ArcCosh}\left[x\sqrt{-\frac{1}{a}}\right],a<0\right\}\right\}\right]\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x - Sqrt[x^2 + a])^b/Sqrt[x^2 + a],x]')
```

```
[Out] Piecewise[{{-(x - Sqrt[a + x ^ 2]) ^ b / b, b != 0}}, Piecewise[{{ArcSinh[x
 Sqrt[1 / a]], a > 0}, {ArcCosh[x Sqrt[-1 / a]], a < 0}}]]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

```
[Out] int((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

Fricas [A]
time = 0.31, size = 18, normalized size = 0.90

$$-\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -(x - sqrt(x^2 + a))^b/b
```

Sympy [A]
time = 0.76, size = 36, normalized size = 1.80

$$\begin{cases} -\frac{\left(x-\sqrt{a+x^2}\right)^b}{b} & \text{for } b \neq 0 \\ \begin{cases} \operatorname{asinh}\left(x\sqrt{\frac{1}{a}}\right) & \text{for } a > 0 \\ \operatorname{acosh}\left(x\sqrt{-\frac{1}{a}}\right) & \text{for } a < 0 \end{cases} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-(x - sqrt(a + x**2))**b/b, Ne(b, 0)), (Piecewise((asinh(x*sqrt(
1/a)), a > 0), (acosh(x*sqrt(-1/a)), a < 0)), True))
```

Giac [A]
time = 0.00, size = 18, normalized size = 0.90

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

```
[Out] -(x - sqrt(x^2 + a))^b/b
```

Mupad [B]
time = 0.30, size = 18, normalized size = 0.90

$$-\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^b/(a + x^2)^(1/2),x)
```

```
[Out] -(x - (a + x^2)^(1/2))^b/b
```

3.21 $\int \frac{1}{(a+be^{px})^2} dx$

Optimal. Leaf size=42

$$\frac{1}{a\left(a+be^{px}\right)p}+\frac{x}{a^2}-\frac{\log\left(a+be^{px}\right)}{a^2p}$$

```
[Out] 1/a/(a+b*exp(p*x))/p+x/a^2-ln(a+b*exp(p*x))/a^2/p
```

Rubi [A]

time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2320, 46}

$$-\frac{\log\left(a+be^{px}\right)}{a^2p}+\frac{x}{a^2}+\frac{1}{ap\left(a+be^{px}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*E^(p*x))^(-2),x]
```

```
[Out] 1/(a*(a + b*E^(p*x))*p) + x/a^2 - Log[a + b*E^(p*x)]/(a^2*p)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+be^{px}\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx,x,e^{px}\right)}{p}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx,x,e^{px}\right)}{p}\\ &= \frac{1}{a\left(a+be^{px}\right)p}+\frac{x}{a^2}-\frac{\log\left(a+be^{px}\right)}{a^2p}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 39, normalized size = 0.93

$$\frac{\frac{a}{a+be^{px}}+\log\left(e^{px}\right)-\log\left(a+be^{px}\right)}{a^2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*E^(p*x))^(-2),x]
```

```
[Out] (a/(a + b*E^(p*x)) + Log[E^(p*x)] - Log[a + b*E^(p*x)])/(a^2*p)
```

Mathics [A]
time = 1.94, size = 57, normalized size = 1.36

$$\frac{a+px\left(a+bE^{px}\right)-\mathrm{Log}\left[\frac{a+bE^{px}}{b}\right]\left(a+bE^{px}\right)}{a^2p\left(a+bE^{px}\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a + b*E^(p*x))^2,x]')
```

```
[Out] (a + p x (a + b E ^ (p x)) - Log[(a + b E ^ (p x)) / b] (a + b E ^ (p x)))
/ (a ^ 2 p (a + b E ^ (p x)))
```

Maple [A]
time = 0.02, size = 43, normalized size = 1.02

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(a+b\,\mathrm{e}^{px}\right)}{a^2}+\frac{1}{a\left(a+b\,\mathrm{e}^{px}\right)}+\frac{\ln\left(\mathrm{e}^{px}\right)}{a^2}}{p}$	43
default	$\frac{-\frac{\ln\left(a+b\,\mathrm{e}^{px}\right)}{a^2}+\frac{1}{a\left(a+b\,\mathrm{e}^{px}\right)}+\frac{\ln\left(\mathrm{e}^{px}\right)}{a^2}}{p}$	43
risch	$\frac{x}{a^2}+\frac{1}{a(a+b\,\mathrm{e}^{px})p}-\frac{\ln\left(\mathrm{e}^{px}+\frac{a}{b}\right)}{a^2p}$	43
norman	$\frac{\frac{x}{a}+\frac{bx\,\mathrm{e}^{px}}{a^2}-\frac{b\,\mathrm{e}^{px}}{a^2p}}{a+b\,\mathrm{e}^{px}}-\frac{\ln(a+b\,\mathrm{e}^{px})}{a^2p}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*exp(p*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/p*(-1/a^2*ln(a+b*exp(p*x))+1/a/(a+b*exp(p*x))+1/a^2*ln(exp(p*x)))
```

Maxima [A]
time = 0.24, size = 40, normalized size = 0.95

$$\frac{x}{a^2}+\frac{1}{(abe^{(px)}+a^2)p}-\frac{\log\left(be^{(px)}+a\right)}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))^2,x, algorithm="maxima")
```

```
[Out] x/a^2 + 1/((a*b*e^(p*x) + a^2)*p) - log(b*e^(p*x) + a)/(a^2*p)
```

Fricas [A]
time = 0.30, size = 52, normalized size = 1.24

$$\frac{bpxe^{(px)} + apx - \left(be^{(px)} + a\right)\log\left(be^{(px)} + a\right) + a}{a^2bpe^{(px)} + a^3p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))^2,x, algorithm="fricas")
```

```
[Out] (b*p*x*e^(p*x) + a*p*x - (b*e^(p*x) + a)*log(b*e^(p*x) + a) + a)/(a^2*b*p*e
^(p*x) + a^3*p)
```

Sympy [A]
time = 0.08, size = 36, normalized size = 0.86

$$\frac{1}{a^2p + abpe^{px}} + \frac{x}{a^2} - \frac{\log\left(\frac{a}{b} + e^{px}\right)}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))**2,x)
```

```
[Out] 1/(a**2*p + a*b*p*exp(p*x)) + x/a**2 - log(a/b + exp(p*x))/(a**2*p)
```

Giac [A]
time = 0.00, size = 45, normalized size = 1.07

$$\frac{\frac{px}{a^2} - \frac{b\ln|e^{px}b+a|}{ba^2} + \frac{a}{a^2(e^{px}b+a)}}{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))^2,x)
```

```
[Out] (p*x/a^2 - log(abs(b*e^(p*x) + a))/a^2 + 1/((b*e^(p*x) + a)*a))/p
```

Mupad [B]
time = 0.41, size = 58, normalized size = 1.38

$$\frac{\frac{x}{a} + \frac{b\,x\,e^{p\,x}}{a^2} - \frac{b\,e^{p\,x}}{a^2\,p}}{a + b\,e^{p\,x}} - \frac{\ln\left(a + b\,e^{p\,x}\right)}{a^2\,p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*exp(p*x))^2,x)
```

```
[Out] (x/a + (b*x*exp(p*x))/a^2 - (b*exp(p*x))/(a^2*p))/(a + b*exp(p*x)) - log(a
+ b*exp(p*x))/(a^2*p)
```

3.22 $\int \frac{1}{(be^{-px}+ae^{px})^2}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2a\left(b+ae^{2px}\right)p}$$

```
[Out] -1/2/a/(b+a*exp(2*p*x))/p
```

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2320, 267}

$$-\frac{1}{2ap\left(ae^{2px}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b/E^(p*x) + a*E^(p*x))^(-2),x]
```

```
[Out] -1/2*1/(a*(b + a*E^(2*p*x))*p)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(be^{-px}+ae^{px}\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(b+ax^2)^2}\,dx,x,e^{px}\right)}{p}\\ &= -\frac{1}{2a\left(b+ae^{2px}\right)p}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 1.00

$$-\frac{1}{2a\left(b+ae^{2px}\right)p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/E^(p*x) + a*E^(p*x))^(-2),x]
```

```
[Out] -1/2*1/(a*(b + a*E^(2*p*x))*p)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.74, size = 40, normalized size = 1.82

$$\text{ConditionalExpression}\left[2abp+2b^2pE^{-2px},\left\{2abp+2b^2pE^{-2px}!{=}0\right\}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(a*E^(p*x) + b*E^(-p*x))^2,x]')
```

```
[Out] ConditionalExpression[2 a b p + 2 b ^ 2 p E ^ (-2 p x), {2 a b p + 2 b ^ 2
p E ^ (-2 p x) != 0}]
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.95

method	result	size
risch	$-\frac{1}{2a(b+a\,\mathrm{e}^{2px})p}$	20
derivativedivides	$-\frac{1}{2a(b+a\,\mathrm{e}^{2px})p}$	21
default	$-\frac{1}{2a(b+a\,\mathrm{e}^{2px})p}$	21
norman	$-\frac{1}{2a(b+a\,\mathrm{e}^{2px})p}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/exp(p*x)+a*exp(p*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/p/a/(a*exp(p*x)^2+b)
```

Maxima [A]
time = 0.25, size = 20, normalized size = 0.91

$$\frac{1}{2\left(b^2e^{(-2\,px)}+ab\right)p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2/((b^2*e^(-2*p*x) + a*b)*p)
```

Fricas [A]
time = 0.29, size = 19, normalized size = 0.86

$$-\frac{1}{2\left(a^2pe^{(2\,px)} + abp\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="fricas")
```

```
[Out] -1/2/(a^2*p*e^(2*p*x) + a*b*p)
```

Sympy [A]
time = 0.06, size = 20, normalized size = 0.91

$$\frac{1}{2abp + 2b^2pe^{-2px}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))**2,x)
```

```
[Out] 1/(2*a*b*p + 2*b**2*p*exp(-2*p*x))
```

Giac [A]
time = 0.00, size = 19, normalized size = 0.86

$$-\frac{1}{2a\left(\left(e^{px}\right)^2 a + b\right)p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x)
```

```
[Out] -1/2/((a*e^(2*p*x) + b)*a*p)
```

Mupad [B]
time = 0.39, size = 24, normalized size = 1.09

$$\frac{e^{2\,p\,x}}{2\,b\,p\,\left(b + a\,e^{2\,p\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*exp(p*x) + b*exp(-p*x))^2,x)
```

```
[Out] exp(2*p*x)/(2*b*p*(b + a*exp(2*p*x)))
```

3.23 $\int \frac{x}{(be^{-px}+ae^{px})^2}\,dx$

Optimal. Leaf size=62

$$\frac{x}{2abp}-\frac{x}{2a\left(b+ae^{2px}\right)p}-\frac{\log\left(b+ae^{2px}\right)}{4abp^2}$$

```
[Out] 1/2*x/a/b/p-1/2*x/a/(b+a*exp(2*p*x))/p-1/4*ln(b+a*exp(2*p*x))/a/b/p^2
```

Rubi [A]
time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2321, 2222, 2320, 36, 29, 31}

$$-\frac{\log\left(ae^{2px}+b\right)}{4abp^2}+\frac{x}{2abp}-\frac{x}{2ap\left(ae^{2px}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b/E^(p*x) + a*E^(p*x))^2,x]
```

```
[Out] x/(2*a*b*p) - x/(2*a*(b + a*E^(2*p*x))*p) - Log[b + a*E^(2*p*x)]/(4*a*b*p^2
)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2321

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Int[u*F^(n
*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ
[n, 0] && LinearQ[{v, w}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(be^{-px}+ae^{px})^2}\,dx &= \int \frac{e^{2px}x}{(b+ae^{2px})^2}\,dx\\
&= -\frac{x}{2a(b+ae^{2px})p}+\frac{\int \frac{1}{b+ae^{2px}}\,dx}{2ap}\\
&= -\frac{x}{2a(b+ae^{2px})p}+\frac{\operatorname{Subst}\left(\int \frac{1}{x(b+ax)}\,dx,x,e^{2px}\right)}{4ap^2}\\
&= -\frac{x}{2a(b+ae^{2px})p}-\frac{\operatorname{Subst}\left(\int \frac{1}{b+ax}\,dx,x,e^{2px}\right)}{4bp^2}+\frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,e^{2px}\right)}{4abp^2}\\
&= \frac{x}{2abp}-\frac{x}{2a(b+ae^{2px})p}-\frac{\log(b+ae^{2px})}{4abp^2}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 0.79

$$\frac{\frac{2e^{2px}px}{b+ae^{2px}}-\frac{\log(b+ae^{2px})}{a}}{4bp^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b/E^(p*x) + a*E^(p*x))^2,x]
```

```
[Out] ((2*E^(2*p*x)*p*x)/(b + a*E^(2*p*x)) - Log[b + a*E^(2*p*x)]/a)/(4*b*p^2)
```

Mathics [A]
time = 2.10, size = 70, normalized size = 1.13

$$\frac{2px\left(aE^{2px}+b\right)-2bpx-\operatorname{Log}\left[\frac{aE^{2px}+b}{a}\right]\left(aE^{2px}+b\right)}{4abp^2\left(aE^{2px}+b\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x/(a*E^(p*x) + b*E^(-p*x))^2,x]')
```

```
[Out] (2 p x (a E ^ (2 p x) + b) - 2 b p x - Log[(a E ^ (2 p x) + b) / a] (a E ^
(2 p x) + b)) / (4 a b p ^ 2 (a E ^ (2 p x) + b))
```

Maple [A]
time = 0.06, size = 50, normalized size = 0.81

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(b+a\,{\mathrm e}^{2 p x}\right)}{4 b a}+\frac{p x\,{\mathrm e}^{2 p x}}{2 b\left(b+a\,{\mathrm e}^{2 p x}\right)}}{p^{2}}$	50
default	$\frac{-\frac{\ln\left(b+a\,{\mathrm e}^{2 p x}\right)}{4 b a}+\frac{p x\,{\mathrm e}^{2 p x}}{2 b\left(b+a\,{\mathrm e}^{2 p x}\right)}}{p^{2}}$	50
norman	$\frac{x\,{\mathrm e}^{2 p x}}{2 b p\left(b+a\,{\mathrm e}^{2 p x}\right)}-\frac{\ln\left(b+a\,{\mathrm e}^{2 p x}\right)}{4 a b\,p^{2}}$	51
risch	$\frac{x}{2 a b p}-\frac{x}{2 a\left(b+a\,{\mathrm e}^{2 p x}\right) p}-\frac{\ln\left({\mathrm e}^{2 p x}+\frac{b}{a}\right)}{4 a b\,p^{2}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b/exp(p*x)+a*exp(p*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/p^2*(-1/4/b/a*ln(a*exp(p*x)^2+b)+1/2*p*x*exp(p*x)^2/b/(a*exp(p*x)^2+b))
```

Maxima [A]
time = 0.25, size = 51, normalized size = 0.82

$$\frac{xe^{(2\,px)}}{2\,(abpe^{(2\,px)}+b^2p)}-\frac{\log\left(\frac{ae^{(2\,px)}+b}{a}\right)}{4\,abp^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x*e^(2*p*x)/(a*b*p*e^(2*p*x) + b^2*p) - 1/4*log((a*e^(2*p*x) + b)/a)/(a
*b*p^2)
```

Fricas [A]
time = 0.31, size = 58, normalized size = 0.94

$$\frac{2\,apxe^{(2\,px)}-\left(ae^{(2\,px)}+b\right)\log\left(ae^{(2\,px)}+b\right)}{4\,(a^2bp^2e^{(2\,px)}+ab^2p^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*p*x*e^(2*p*x) - (a*e^(2*p*x) + b)*log(a*e^(2*p*x) + b))/(a^2*b*p^2
*e^(2*p*x) + a*b^2*p^2)
```

Sympy [A]
time = 0.10, size = 51, normalized size = 0.82

$$\frac{x}{2abp + 2b^2pe^{-2px}} - \frac{x}{2abp} - \frac{\log\left(\frac{a}{b} + e^{-2px}\right)}{4abp^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))**2,x)
```

```
[Out] x/(2*a*b*p + 2*b**2*p*exp(-2*p*x)) - x/(2*a*b*p) - log(a/b + exp(-2*p*x))/(
4*a*b*p**2)
```

Giac [A]
time = 0.00, size = 70, normalized size = 1.13

$$\frac{2apxe^{2px} - ae^{2px}\ln\left(-ae^{2px} - b\right) - b\ln\left(-ae^{2px} - b\right)}{\left(4a^2bpe^{2px} + 4ab^2p\right)p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x)
```

```
[Out] 1/4*(2*a*p*x*e^(2*p*x) - a*e^(2*p*x)*log(-a*e^(2*p*x) - b) - b*log(-a*e^(2*
p*x) - b))/((a^2*b*p*e^(2*p*x) + a*b^2*p)*p)
```

Mupad [B]
time = 0.41, size = 47, normalized size = 0.76

$$\frac{x\,e^{2\,p\,x}}{2\,b\,p\,\left(b + a\,e^{2\,p\,x}\right)} - \frac{\ln\left(b + a\,e^{2\,p\,x}\right)}{4\,a\,b\,p^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*exp(p*x) + b*exp(-p*x))^2,x)
```

```
[Out] (x*exp(2*p*x))/(2*b*p*(b + a*exp(2*p*x))) - log(b + a*exp(2*p*x))/(4*a*b*p^
2)
```

3.24 $\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\,(1+x+x^2)^2}\,dx$

Optimal. Leaf size=86

$$\frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,(1+x)}{\sqrt{1-x+x^2}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\,(1-x)}{\sqrt{1-x+x^2}}\right)}{\sqrt{6}}$$

```
[Out] arctan((1+x)*2^(1/2)/(x^2-x+1)^(1/2))*2^(1/2)-1/6*arctanh(1/3*(1-x)*6^(1/2)
/(x^2-x+1)^(1/2))*6^(1/2)+(1+x)*(x^2-x+1)^(1/2)/(x^2+x+1)
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1074, 1049, 1043, 212, 210}

$$\frac{\sqrt{x^2-x+1}\,(x+1)}{x^2+x+1}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,(x+1)}{\sqrt{x^2-x+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\,(1-x)}{\sqrt{x^2-x+1}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x + 3*x^2)/(Sqrt[1 - x + x^2]*(1 + x + x^2)^2),x]
```

```
[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) + Sqrt[2]*ArcTan[(Sqrt[2]*(1 + x)
)/Sqrt[1 - x + x^2]] - ArcTanh[(Sqrt[2/3]*(1 - x))/Sqrt[1 - x + x^2]]/Sqrt[
6]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\,(1+x+x^2)^2}\,dx &= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + \frac{1}{12}\int \frac{18-6x}{\sqrt{1-x+x^2}\,(1+x+x^2)}\,dx \\ &= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + \frac{1}{48}\int \frac{24+24x}{\sqrt{1-x+x^2}\,(1+x+x^2)}\,dx - \frac{1}{48}\int \\ &= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + 24\text{Subst}\left(\int \frac{1}{1728-2x^2}\,dx, x, \frac{-24+24x}{\sqrt{1-x+x^2}}\right. \\ &= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,(1+x)}{\sqrt{1-x+x^2}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{1-}}\right.}{\sqrt{6}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.31, size = 239, normalized size = 2.78

$$\frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} - \text{RootSum}\left[3+6\#1+\#1^2-2\#1^3+\#1^4\&, \frac{19\log\left(-x+\sqrt{1-x+x^2}-\#1\right)+6\log\left(-x+\sqrt{1-x+x^2}-\#1\right)\#1}{3+\#1-3\#1^2+2\#1^3}\&\right] - \frac{1}{2}\text{RootSum}\left[3+6\#1+\#1^2-2\#1^3+\#1^4\&, \frac{-36\log\left(-x+\sqrt{1-x+x^2}-\#1\right)-6\log\left(-x+\sqrt{1-x+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{1-x+x^2}-\#1\right)\#1^2}{3+\#1-3\#1^2+2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + 3*x^2)/(Sqrt[1 - x + x^2]*(1 + x + x^2)^2),x]
```

```
[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) - RootSum[3 + 6*#1 + #1^2 - 2*#1^
3 + #1^4 & , (19*Log[-x + Sqrt[1 - x + x^2] - #1] + 6*Log[-x + Sqrt[1 - x +
 x^2] - #1]*#1)/(3 + #1 - 3*#1^2 + 2*#1^3) & ] - RootSum[3 + 6*#1 + #1^2 -
2*#1^3 + #1^4 & , (-36*Log[-x + Sqrt[1 - x + x^2] - #1] - 6*Log[-x + Sqrt[1
 - x + x^2] - #1]*#1 + Log[-x + Sqrt[1 - x + x^2] - #1]*#1^2)/(3 + #1 - 3*#
1^2 + 2*#1^3) & ]/2
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x + 3*x^2)/((1 + x + x^2)^2*Sqrt[1 - x + x^2]),x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(71) = 142.
time = 0.74, size = 455, normalized size = 5.29

method	result
risch	$\frac{(1+x)\sqrt{x^2-x+1}}{x^2+x+1}+\frac{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\,\Bigg(6\sqrt{2}\,\arctan\Bigg(\frac{2\sqrt{2}\,(1+x)}{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\,(1-x)}\Bigg)-\sqrt{6}\,\text{arctanh}\Bigg(\frac{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\,\sqrt{6}}{4}}{6\sqrt{\frac{\frac{(1+x)^2}{(1-x)^2}+3}{\left(\frac{1+x}{1-x}+1\right)^2}}\,\left(\frac{1+x}{1-x}+1\right)}$
default	$\frac{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\,\Bigg(3\sqrt{2}\,\arctan\Bigg(\frac{2\sqrt{2}\,(1+x)}{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\,(1-x)}\Bigg)-\sqrt{6}\,\text{arctanh}\Bigg(\frac{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\,\sqrt{6}}{4}\Bigg)\Bigg)}{2\sqrt{\frac{\frac{(1+x)^2}{(1-x)^2}+3}{\left(\frac{1+x}{1-x}+1\right)^2}}\,\left(\frac{1+x}{1-x}+1\right)}-\frac{9\sqrt{2}\,\sqrt{\frac{(1+x)^2}{(1-x)^2}}}{}$
trager	$\frac{(1+x)\sqrt{x^2-x+1}}{x^2+x+1}-\frac{24\ln\Bigg(\frac{1728x\,\text{RootOf}\left(576\_Z^4+528\_Z^2+169\right)^5-744\,\text{RootOf}\left(576\_Z^4+528\_Z^2+169\right)^3x+1344\,\text{RootOf}\Big(576}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((1+x)^2/(1-x)^2+3)^(1/2)*(3*2^(1/2)*arctan(2*2^(1/2)/((1+x)^2/(1-x)^2+
3)^(1/2)*(1+x)/(1-x))-6^(1/2)*arctanh(1/4*((1+x)^2/(1-x)^2+3)^(1/2)*6^(1/2)
))/((((1+x)^2/(1-x)^2+3)/((1+x)/(1-x)+1)^2)^(1/2)/((1+x)/(1-x)+1)-1/6*(9*2^(
1/2)*((1+x)^2/(1-x)^2+3)^(1/2)*arctan(2*2^(1/2)/((1+x)^2/(1-x)^2+3)^(1/2)*(
1+x)/(1-x))*(1+x)^2/(1-x)^2-6*6^(1/2)*((1+x)^2/(1-x)^2+3)^(1/2)*arctanh(1/4
*((1+x)^2/(1-x)^2+3)^(1/2)*6^(1/2))*(1+x)^2/(1-x)^2+3*2^(1/2)*arctan(2*2^(1
/2)/((1+x)^2/(1-x)^2+3)^(1/2)*(1+x)/(1-x))*((1+x)^2/(1-x)^2+3)^(1/2)-2*6^(1
/2)*arctanh(1/4*((1+x)^2/(1-x)^2+3)^(1/2)*6^(1/2))*((1+x)^2/(1-x)^2+3)^(1/2
)-12*(1+x)^3/(1-x)^3-36*(1+x)/(1-x))/((((1+x)^2/(1-x)^2+3)/((1+x)/(1-x)+1)^2
)^(1/2)/((1+x)/(1-x)+1)/(3*(1+x)^2/(1-x)^2+1)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 - x + 1)/((x^2 + x + 1)^2*sqrt(x^2 - x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(69) = 138$.
time = 0.33, size = 358, normalized size = 4.16

$$-\frac{8\sqrt{6}\sqrt{3}(x^2+x+1)\arctan\left(\frac{2}{3}\sqrt{6}\sqrt{3}(x-1)+\frac{2}{3}\sqrt{2x^2-\sqrt{x^2-x+1}\left(2x-\sqrt{6}+1\right)-\sqrt{6}(x+1)+4}\left(\sqrt{6}\sqrt{3}+3\sqrt{3}\right)-\frac{2}{3}\sqrt{x^2-x+1}\left(\sqrt{6}\sqrt{3}+3\sqrt{3}\right)+\sqrt{3}(2x-1)\right)+8\sqrt{6}\sqrt{3}(x^2+x+1)\arctan\left(\frac{2}{3}\sqrt{6}\sqrt{3}(x-1)+\frac{2}{3}\sqrt{2x^2-\sqrt{x^2-x+1}\left(2x+\sqrt{6}+1\right)+\sqrt{6}(x+1)+4}\left(\sqrt{6}\sqrt{3}-3\sqrt{3}\right)-\frac{2}{3}\sqrt{x^2-x+1}\left(\sqrt{6}\sqrt{3}-3\sqrt{3}\right)-\sqrt{3}(2x-1)\right)-\sqrt{6}(x^2+x+1)\log\left(12168x^2-6084\sqrt{x^2-x+1}\left(2x+\sqrt{6}+1\right)+6084\sqrt{6}(x+1)+24336\right)+\sqrt{6}(x^2+x+1)\log\left(12168x^2-6084\sqrt{x^2-x+1}\left(2x-\sqrt{6}+1\right)-6084\sqrt{6}(x+1)+24336\right)-12x^2-12\sqrt{x^2-x+1}(x+1)-12x-12}{12(x^2+x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(8*sqrt(6)*sqrt(3)*(x^2 + x + 1)*arctan(2/3*sqrt(6)*sqrt(3)*(x - 1) +
 2/3*sqrt(2*x^2 - sqrt(x^2 - x + 1)*(2*x - sqrt(6) + 1) - sqrt(6)*(x + 1) +
 4)*(sqrt(6)*sqrt(3) + 3*sqrt(3)) - 2/3*sqrt(x^2 - x + 1)*(sqrt(6)*sqrt(3)
+ 3*sqrt(3)) + sqrt(3)*(2*x - 1)) + 8*sqrt(6)*sqrt(3)*(x^2 + x + 1)*arctan(
2/3*sqrt(6)*sqrt(3)*(x - 1) + 2/3*sqrt(2*x^2 - sqrt(x^2 - x + 1)*(2*x + sqr
t(6) + 1) + sqrt(6)*(x + 1) + 4)*(sqrt(6)*sqrt(3) - 3*sqrt(3)) - 2/3*sqrt(x
^2 - x + 1)*(sqrt(6)*sqrt(3) - 3*sqrt(3)) - sqrt(3)*(2*x - 1)) - sqrt(6)*(x
^2 + x + 1)*log(12168*x^2 - 6084*sqrt(x^2 - x + 1)*(2*x + sqrt(6) + 1) + 60
84*sqrt(6)*(x + 1) + 24336) + sqrt(6)*(x^2 + x + 1)*log(12168*x^2 - 6084*sq
rt(x^2 - x + 1)*(2*x - sqrt(6) + 1) - 6084*sqrt(6)*(x + 1) + 24336) - 12*x^
2 - 12*sqrt(x^2 - x + 1)*(x + 1) - 12*x - 12)/(x^2 + x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2 - x + 1}{\sqrt{x^2 - x + 1}\, \left(x^2 + x + 1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+1)/(x**2+x+1)**2/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral((3*x**2 - x + 1)/(sqrt(x**2 - x + 1)*(x**2 + x + 1)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(69) = 138$.
time = 0.02, size = 467, normalized size = 5.43

$$2\left(\frac{-\left(\sqrt{x^2-x+1}-x\right)^3+4\left(\sqrt{x^2-x+1}-x\right)^2+10\left(\sqrt{x^2-x+1}-x\right)+5}{2\left(\left(\sqrt{x^2-x+1}-x\right)^4-2\left(\sqrt{x^2-x+1}-x\right)^3+\left(\sqrt{x^2-x+1}-x\right)^2+6\left(\sqrt{x^2-x+1}-x\right)+3\right)}+\frac{\frac{1}{12}\sqrt{6}\ln\left(\left(12\left(\sqrt{x^2-x+1}-x\right)-6\sqrt{6}-6\right)\left(12\left(\sqrt{x^2-x+1}-x\right)-6\sqrt{6}-6\right)+\left(-2\sqrt{3}\sqrt{6}-6\sqrt{3}\right)\left(-2\sqrt{3}\sqrt{6}-6\sqrt{3}\right)\right)-\frac{2}{6}\sqrt{6}\sqrt{3}\arctan\left(\frac{6\sqrt{6}+6-12\left(\sqrt{x^2-x+1}-x\right)}{-6\sqrt{2}-6\sqrt{3}}\right)-\frac{1}{12}\sqrt{6}\ln\left(\left(12\left(\sqrt{x^2-x+1}-x\right)+6\sqrt{6}-6\right)\left(12\left(\sqrt{x^2-x+1}-x\right)+6\sqrt{6}-6\right)+\left(2\sqrt{3}\sqrt{6}-6\sqrt{3}\right)\left(2\sqrt{3}\sqrt{6}-6\sqrt{3}\right)\right)+\frac{2}{6}\sqrt{6}\sqrt{3}\arctan\left(\frac{-6\sqrt{6}+6-12\left(\sqrt{x^2-x+1}-x\right)}{6\sqrt{2}-6\sqrt{3}}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x)
```

```
[Out] -1/3*sqrt(6)*sqrt(3)*arctan(-(2*x + sqrt(6) - 2*sqrt(x^2 - x + 1) + 1)/(sqr
t(3) + sqrt(2))) + 1/3*sqrt(6)*sqrt(3)*arctan(-(2*x - sqrt(6) - 2*sqrt(x^2
- x + 1) + 1)/(sqrt(3) - sqrt(2))) + 1/12*sqrt(6)*log(4*(sqrt(6)*sqrt(3) +
3*sqrt(3))^2 + 36*(2*x + sqrt(6) - 2*sqrt(x^2 - x + 1) + 1)^2) - 1/12*sqrt(
```

```
6)*log(4*(sqrt(6)*sqrt(3) - 3*sqrt(3))^2 + 36*(2*x - sqrt(6) - 2*sqrt(x^2 -
 x + 1) + 1)^2) + ((x - sqrt(x^2 - x + 1))^3 + 4*(x - sqrt(x^2 - x + 1))^2
- 10*x + 10*sqrt(x^2 - x + 1) + 5)/((x - sqrt(x^2 - x + 1))^4 + 2*(x - sqrt
(x^2 - x + 1))^3 + (x - sqrt(x^2 - x + 1))^2 - 6*x + 6*sqrt(x^2 - x + 1) +
3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x^2-x+1}{\sqrt{x^2-x+1}\,{\left(x^2+x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - x + 1)/((x^2 - x + 1)^(1/2)*(x + x^2 + 1)^2),x)
[Out] int((3*x^2 - x + 1)/((x^2 - x + 1)^(1/2)*(x + x^2 + 1)^2), x)
```

3.25 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}} \, dx$

Optimal. Leaf size=19

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

```
[Out] 2*(x+(a^2+x^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2147, 30}

$$2\sqrt{\sqrt{a^2+x^2}+x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}} \, dx &= \text{Subst}\left(\int \frac{1}{\sqrt{x}} \, dx, x, x+\sqrt{a^2+x^2}\right) \\
&= 2\sqrt{x+\sqrt{a^2+x^2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 19, normalized size = 1.00

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]]
```

Mathics [A]
time = 1.76, size = 15, normalized size = 0.79

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]')
[Out] 2 Sqrt[x + Sqrt[a ^ 2 + x ^ 2]]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x)
[Out] int((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2), x)
```

Fricas [A]
time = 0.31, size = 15, normalized size = 0.79

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x + sqrt(a^2 + x^2))
```

Sympy [A]
time = 0.10, size = 15, normalized size = 0.79

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/(a**2+x**2)**(1/2),x)
[Out] 2*sqrt(x + sqrt(a**2 + x**2))
```

Giac [A]
time = 0.00, size = 19, normalized size = 1.00

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x)
[Out] 2*sqrt(x + sqrt(a^2 + x^2))
```

Mupad [B]
time = 0.42, size = 15, normalized size = 0.79

$$2\,\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a^2 + x^2)^(1/2))^(1/2)/(a^2 + x^2)^(1/2),x)
[Out] 2*(x + (a^2 + x^2)^(1/2))^(1/2)
```

3.26 $\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx$

Optimal. Leaf size=26

$$\frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b}$$

```
[Out] 2*(b*x+(b^2*x^2+a)^(1/2))^(1/2)/b
```

Rubi [A]
time = 0.06, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2147, 30}

$$\frac{2\sqrt{\sqrt{a+b^2x^2}+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2],x]
```

```
[Out] (2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, bx+\sqrt{a+b^2x^2}\right)}{b}\\ &= \frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 26, normalized size = 1.00

$$\frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2],x]
```

```
[Out] (2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.13, size = 33, normalized size = 1.27

$$\text{Piecewise}\left[\left\{\left\{\frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b}, b!{=}0\right\}\right\}, \frac{x}{a^{\frac{1}{4}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2],x]')
```

```
[Out] Piecewise[{{2 Sqrt[b x + Sqrt[a + b ^ 2 x ^ 2]] / b, b != 0}}, x / a ^ (1 /
 4)]
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+\sqrt{x^2b^2+a}}}{\sqrt{x^2b^2+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x)
```

```
[Out] int((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(b*x + sqrt(b^2*x^2 + a))/sqrt(b^2*x^2 + a), x)
```

Fricas [A]
time = 0.60, size = 22, normalized size = 0.85

$$\frac{2\sqrt{bx+\sqrt{b^2x^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2*sqrt(b*x + sqrt(b^2*x^2 + a))/b
```

Sympy [A]
time = 0.51, size = 27, normalized size = 1.04

$$\begin{cases} \frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b} & \text{for } b\neq 0 \\ \frac{x}{\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b**2*x**2+a)**(1/2))**(1/2)/(b**2*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(b*x + sqrt(a + b**2*x**2))/b, Ne(b, 0)), (x/a**(1/4), Tru
e))
```

Giac [A]
time = 0.00, size = 25, normalized size = 0.96

$$\frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x)
```

```
[Out] 2*sqrt(b*x + sqrt(b^2*x^2 + a))/b
```

Mupad [B]
time = 0.51, size = 22, normalized size = 0.85

$$\frac{2\sqrt{\sqrt{b^2\,x^2+a}+b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b^2*x^2)^(1/2) + b*x)^(1/2)/(a + b^2*x^2)^(1/2),x)
```

```
[Out] (2*((a + b^2*x^2)^(1/2) + b*x)^(1/2))/b
```

3.27 $\int \frac{1}{x\sqrt{a^2+x^2}\,\sqrt{x+\sqrt{a^2+x^2}}}\,dx$

Optimal. Leaf size=63

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -2*arctan((x+(a^2+x^2)^(1/2))^(1/2)/a^(1/2))/a^(3/2)-2*arctanh((x+(a^2+x^2)
^(1/2))^(1/2)/a^(1/2))/a^(3/2)
```

Rubi [A]
time = 0.13, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2145, 335, 218, 212, 209}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + x^2]*Sqrt[x + Sqrt[a^2 + x^2]]),x]
```

```
[Out] (-2*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]])/a^(3/2) - (2*ArcTanh[Sqrt[x
+ Sqrt[a^2 + x^2]]/Sqrt[a]])/a^(3/2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2145

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + p + 1)*e^(p + 1)*f^(2*
m)))*(i/c)^m, Subst[Int[x^(n - 2*m - p - 2)*((-a)*f^2 + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx &= 2\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(-a^2+x^2)}\,dx, x, x+\sqrt{a^2+x^2}\right)\\
&= 4\text{Subst}\left(\int \frac{1}{-a^2+x^4}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)\\
&= -\frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)}{a} - \frac{2\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \right.}{a}\\
&= -\frac{2\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 56, normalized size = 0.89

$$-\frac{2\left(\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)+\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + x^2]*Sqrt[x + Sqrt[a^2 + x^2]]),x]
```

```
[Out] (-2*(ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] + ArcTanh[Sqrt[x + Sqrt[a^2
+ x^2]]/Sqrt[a]]))/a^(3/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.80, size = 27, normalized size = 0.43

$$-\frac{\sqrt{2}\,\text{hyper}\left[\left\{\frac{3}{4},\frac{3}{4},\frac{5}{4}\right\},\left\{\frac{3}{2},\frac{7}{4}\right\},\frac{a^2\text{exp\_polar}[I\text{Pi}]}{x^2}\right]}{3x^{\frac{3}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*Sqrt[a^2 + x^2]*Sqrt[x + Sqrt[a^2 + x^2]]),x]')
```

```
[Out] -Sqrt[2] hyper[{3 / 4, 3 / 4, 5 / 4}, {3 / 2, 7 / 4}, a ^ 2 exp_polar[I Pi]
 / x ^ 2] / (3 x ^ (3 / 2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a^2+x^2}\,\sqrt{x+\sqrt{a^2+x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x), x)
```

Fricas [A] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(47) = 94.
time = 0.32, size = 198, normalized size = 3.14

$$\left[-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)-\sqrt{a}\log\left(\frac{a^2+\sqrt{a^2+x^2}\,a-\left((a-x)\sqrt{a}+\sqrt{a^2+x^2}\,\sqrt{a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)}{a^2}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\,\sqrt{x+\sqrt{a^2+x^2}}}{a}\right)-\sqrt{-a}\log\left(-\frac{a^2-\sqrt{a^2+x^2}\,a-\left(\sqrt{-a}\,(a+x)-\sqrt{a^2+x^2}\,\sqrt{-a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)}{a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-(2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) - sqrt(a)*log((a^2 +
 sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x + s
qrt(a^2 + x^2)))/x))/a^2, (2*sqrt(-a)*arctan(sqrt(-a)*sqrt(x + sqrt(a^2 + x
^2))/a) - sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a - (sqrt(-a)*(a + x) - sqrt
(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x))/a^2]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.08, size = 46, normalized size = 0.73

$$-\frac{\Gamma^2\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_3F_2\left(\begin{matrix}\frac{3}{4},\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4}\end{matrix}\middle|\frac{a^2e^{i\pi}}{x^2}\right)}{\pi x^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+x**2)**(1/2)/(x+(a**2+x**2)**(1/2))**(1/2),x)
```

```
[Out] -gamma(3/4)**2*gamma(5/4)*hyper((3/4, 3/4, 5/4), (3/2, 7/4), a**2*exp_polar
(I*pi)/x**2)/(pi*x**(3/2)*gamma(7/4))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\sqrt{x+\sqrt{a^2+x^2}}\,\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + (a^2 + x^2)^(1/2))^(1/2)*(a^2 + x^2)^(1/2)),x)
```

```
[Out] int(1/(x*(x + (a^2 + x^2)^(1/2))^(1/2)*(a^2 + x^2)^(1/2)), x)
```

3.28 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$

Optimal. Leaf size=82

$$2\sqrt{x+\sqrt{a^2+x^2}}-2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)-2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)$$

```
[Out] -2*arctan((x+(a^2+x^2)^(1/2))^(1/2)/a^(1/2))*a^(1/2)-2*arctanh((x+(a^2+x^2)
^(1/2))^(1/2)/a^(1/2))*a^(1/2)+2*(x+(a^2+x^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2144, 470, 335, 218, 212, 209}

$$2\sqrt{\sqrt{a^2+x^2}+x}-2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)-2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2144

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx &= \text{Subst}\left(\int \frac{a^2+x^2}{\sqrt{x}\,(-a^2+x^2)}\,dx, x, x+\sqrt{a^2+x^2}\right)\\
&= 2\sqrt{x+\sqrt{a^2+x^2}} + (2a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(-a^2+x^2)}\,dx, x, x+\sqrt{a^2+x^2}\right)\\
&= 2\sqrt{x+\sqrt{a^2+x^2}} + (4a^2)\,\text{Subst}\left(\int \frac{1}{-a^2+x^4}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)\\
&= 2\sqrt{x+\sqrt{a^2+x^2}} - (2a)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right) - (2a)\text{Sub}\\
&= 2\sqrt{x+\sqrt{a^2+x^2}} - 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{x+}}{}\right.
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 82, normalized size = 1.00

$$2\sqrt{x+\sqrt{a^2+x^2}} - 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.42, size = 33, normalized size = 0.40

$$-\frac{\sqrt{2}\sqrt{x}\,\text{Gamma}\left[-\frac{1}{4}\right]\text{hyper}\left[\left\{-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\right\},\left\{\frac{1}{2},\frac{3}{4}\right\},\frac{a^2\text{exp\_polar}[I\text{Pi}]}{x^2}\right]}{2\text{Gamma}\left[\frac{3}{4}\right]}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[Sqrt[a^2 + x^2] + x]/x,x]')
```

```
[Out] -Sqrt[2] Sqrt[x] Gamma[-1 / 4] hyper[{-1 / 4, -1 / 4, 1 / 4}, {1 / 2, 3 / 4
}, a ^ 2 exp_polar[I Pi] / x ^ 2] / (2 Gamma[3 / 4])
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 0.02, size = 25, normalized size = 0.30

method	result	size
meijerg	$2\sqrt{2}\,\sqrt{x}\,\text{ hypergeom}\left(\left[-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\right],\left[\frac{1}{2},\frac{3}{4}\right],-\frac{a^2}{x^2}\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(a^2+x^2)^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*2^(1/2)*x^(1/2)*hypergeom([-1/4,-1/4,1/4],[1/2,3/4],-a^2/x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

Fricas [A]
time = 0.47, size = 216, normalized size = 2.63

$$\left[-2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)+\sqrt{a}\log\left(\frac{a^2+\sqrt{a^2+x^2}\,a-\left((a-x)\sqrt{a}+\sqrt{a^2+x^2}\,\sqrt{a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)+2\sqrt{x+\sqrt{a^2+x^2}},2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\,\sqrt{x+\sqrt{a^2+x^2}}}{a}\right)+\sqrt{-a}\log\left(-\frac{a^2-\sqrt{a^2+x^2}\,a+\left(\sqrt{-a}\,(a+x)-\sqrt{a^2+x^2}\,\sqrt{-a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)+2\sqrt{x+\sqrt{a^2+x^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) + sqrt(a)*log((a^2 +
sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x + sq
rt(a^2 + x^2)))/x) + 2*sqrt(x + sqrt(a^2 + x^2)), 2*sqrt(-a)*arctan(sqrt(-a
)*sqrt(x + sqrt(a^2 + x^2))/a) + sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a + (
sqrt(-a)*(a + x) - sqrt(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x)
+ 2*sqrt(x + sqrt(a^2 + x^2))]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.79, size = 51, normalized size = 0.62

$$\frac{\sqrt{x}\,\Gamma^2\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_3F_2\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{3}{4}\end{matrix}\middle|\frac{a^2e^{i\pi}}{x^2}\right)}{8\pi\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)
```

```
[Out] sqrt(x)*gamma(-1/4)**2*gamma(1/4)*hyper((-1/4, -1/4, 1/4), (1/2, 3/4), a**2
*exp_polar(I*pi)/x**2)/(8*pi*gamma(3/4))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a^2 + x^2)^(1/2))^(1/2)/x,x)
```

```
[Out] int((x + (a^2 + x^2)^(1/2))^(1/2)/x, x)
```

3.29 $\int x^3 \log^3(2+x) \log(3+x)\,dx$

Optimal. Leaf size=606

$$-\frac{302177x}{1152}+\frac{8029x^2}{2304}-\frac{763x^3}{3456}+\frac{3x^4}{256}+\frac{377}{64}(2+x)^2-\frac{71}{216}(2+x)^3+\frac{3}{256}(2+x)^4+\frac{2069}{144}\log(2+x)-\frac{187}{64}x^2\log(2+x)$$

```
[Out] -302177/1152*x+377/64*(2+x)^2-71/216*(2+x)^3+3/256*(2+x)^4-5609/96*polylog(
2,-2-x)-563/8*polylog(3,-2-x)-195/2*polylog(4,-2-x)+3891/128*ln(3+x)+2069/1
44*ln(2+x)-25*x*ln(2+x)*ln(3+x)+13/4*x^2*ln(2+x)*ln(3+x)-7/12*x^3*ln(2+x)*l
n(3+x)+3/32*x^4*ln(2+x)*ln(3+x)+6*x*ln(2+x)^2*ln(3+x)-3/2*x^2*ln(2+x)^2*ln(
3+x)+1/2*x^3*ln(2+x)^2*ln(3+x)-3/16*x^4*ln(2+x)^2*ln(3+x)+1/4*x^4*ln(2+x)^3
*ln(3+x)+3/256*x^4+8029/2304*x^2-763/3456*x^3-43/12*ln(2+x)^2-4083/32*ln(2+
x)*ln(3+x)+963/16*ln(2+x)^2*ln(3+x)-81/4*ln(2+x)^3*ln(3+x)+563/8*ln(2+x)*po
lylog(2,-2-x)-195/4*ln(2+x)^2*polylog(2,-2-x)+195/2*ln(2+x)*polylog(3,-2-x)
-187/64*x^2*ln(2+x)+83/288*x^3*ln(2+x)-3/128*x^4*ln(2+x)+6733/32*(2+x)*ln(2
+x)-377/32*(2+x)^2*ln(2+x)+71/72*(2+x)^3*ln(2+x)-3/64*(2+x)^4*ln(2+x)-17/48
*x^3*ln(2+x)^2+3/64*x^4*ln(2+x)^2-1251/16*(2+x)*ln(2+x)^2+273/32*(2+x)^2*ln
(2+x)^2-3/4*(2+x)^3*ln(2+x)^2+3/64*(2+x)^4*ln(2+x)^2+65/4*(2+x)*ln(2+x)^3-3
3/8*(2+x)^2*ln(2+x)^3+3/4*(2+x)^3*ln(2+x)^3-1/16*(2+x)^4*ln(2+x)^3-115/48*x
^2*ln(3+x)+37/144*x^3*ln(3+x)-3/128*x^4*ln(3+x)+415/12*(3+x)*ln(3+x)
```

Rubi [A]
time = 2.89, antiderivative size = 606, normalized size of antiderivative = 1.00, number of steps used = 359, number of rules used = 30, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 2.143$, Rules used = {2489, 2463, 2436, 2333, 2332, 2448, 2437, 2342, 2341, 2443, 2481, 2421, 2430, 6724, 2458, 2388, 2339, 30, 2367, 2356, 45, 2372, 2338, 6874, 2479, 2440, 2438, 2441, 2442, 2445}

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[2 + x]^3*Log[3 + x],x]
```

```
[Out] (-302177*x)/1152 + (8029*x^2)/2304 - (763*x^3)/3456 + (3*x^4)/256 + (377*(2
 + x)^2)/64 - (71*(2 + x)^3)/216 + (3*(2 + x)^4)/256 + (2069*Log[2 + x])/14
4 - (187*x^2*Log[2 + x])/64 + (83*x^3*Log[2 + x])/288 - (3*x^4*Log[2 + x])/
128 + (6733*(2 + x)*Log[2 + x])/32 - (377*(2 + x)^2*Log[2 + x])/32 + (71*(2
 + x)^3*Log[2 + x])/72 - (3*(2 + x)^4*Log[2 + x])/64 - (43*Log[2 + x]^2)/12
 - (17*x^3*Log[2 + x]^2)/48 + (3*x^4*Log[2 + x]^2)/64 - (1251*(2 + x)*Log[2
 + x]^2)/16 + (273*(2 + x)^2*Log[2 + x]^2)/32 - (3*(2 + x)^3*Log[2 + x]^2)/
4 + (3*(2 + x)^4*Log[2 + x]^2)/64 + (65*(2 + x)*Log[2 + x]^3)/4 - (33*(2 +
x)^2*Log[2 + x]^3)/8 + (3*(2 + x)^3*Log[2 + x]^3)/4 - ((2 + x)^4*Log[2 + x]
^3)/16 + (3891*Log[3 + x])/128 - (115*x^2*Log[3 + x])/48 + (37*x^3*Log[3 +
x])/144 - (3*x^4*Log[3 + x])/128 + (415*(3 + x)*Log[3 + x])/12 - (4083*Log[
```

```
2 + x]*Log[3 + x])/32 - 25*x*Log[2 + x]*Log[3 + x] + (13*x^2*Log[2 + x]*Log
[3 + x])/4 - (7*x^3*Log[2 + x]*Log[3 + x])/12 + (3*x^4*Log[2 + x]*Log[3 + x
])/32 + (963*Log[2 + x]^2*Log[3 + x])/16 + 6*x*Log[2 + x]^2*Log[3 + x] - (3
*x^2*Log[2 + x]^2*Log[3 + x])/2 + (x^3*Log[2 + x]^2*Log[3 + x])/2 - (3*x^4*
Log[2 + x]^2*Log[3 + x])/16 - (81*Log[2 + x]^3*Log[3 + x])/4 + (x^4*Log[2 +
 x]^3*Log[3 + x])/4 - (5609*PolyLog[2, -2 - x])/96 + (563*Log[2 + x]*PolyLo
g[2, -2 - x])/8 - (195*Log[2 + x]^2*PolyLog[2, -2 - x])/4 - (563*PolyLog[3,
 -2 - x])/8 + (195*Log[2 + x]*PolyLog[3, -2 - x])/2 - (195*PolyLog[4, -2 -
x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
```

```
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2479

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
```

```
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[x*((a +
 b*Log[c*(d + e*x)^n])^p/(i + j*x)), x], x] - Dist[b*e*n*p, Int[x*(a + b*Lo
g[c*(d + e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2489

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Dist[g*j*(m/(r + 1)), Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
+ j*x)), x], x] - Dist[b*e*n*(p/(r + 1)), Int[x^(r + 1)*(a + b*Log[c*(d + e
*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^3 \log^3(2+x)\log(3+x)\,dx = \frac{1}{4}x^4\log^3(2+x)\log(3+x) - \frac{1}{4}\int \frac{x^4\log^3(2+x)}{3+x}\,dx - \frac{3}{4}\int \frac{x^4\log^2(2+x)}{2+}$$
$$= \frac{1}{4}x^4\log^3(2+x)\log(3+x) - \frac{1}{4}\int \Big(-27\log^3(2+x) + 9x\log^3(2+x) - 3x$$
$$= \frac{1}{4}x^4\log^3(2+x)\log(3+x) - \frac{1}{4}\int x^3\log^3(2+x)\,dx + \frac{3}{4}\int x^2\log^3(2+x)\,c$$
$$= 6x\log^2(2+x)\log(3+x) - \frac{3}{2}x^2\log^2(2+x)\log(3+x) + \frac{1}{2}x^3\log^2(2+x)\,\mathrm{l}$$
$$= \frac{27}{4}(2+x)\log^3(2+x) + 6x\log^2(2+x)\log(3+x) - \frac{3}{2}x^2\log^2(2+x)\log(3$$
$$= -\frac{81}{4}(2+x)\log^2(2+x) + \frac{27}{4}(2+x)\log^3(2+x) + 6x\log^2(2+x)\log(3+$$
$$= -\frac{81x}{2} + \frac{81}{2}(2+x)\log(2+x) - \frac{17}{48}x^3\log^2(2+x) + \frac{3}{64}x^4\log^2(2+x) - \frac{8}{4}$$
$$= -\frac{81x}{2} + \frac{81}{2}(2+x)\log(2+x) - \frac{17}{48}x^3\log^2(2+x) + \frac{3}{64}x^4\log^2(2+x) - \frac{7}{1}$$
$$= -\frac{765x}{8} + \frac{27}{32}(2+x)^2 - \frac{1}{6}(2+x)^3 + \frac{3}{512}(2+x)^4 + \frac{765}{8}(2+x)\log(2+x)$$
$$= -\frac{857x}{8} + \frac{79}{32}(2+x)^2 - \frac{71}{216}(2+x)^3 + \frac{3}{256}(2+x)^4 - \frac{75}{64}x^2\log(2+x) +$$
$$= -\frac{16463x}{96} + \frac{185}{64}(2+x)^2 - \frac{71}{216}(2+x)^3 + \frac{3}{256}(2+x)^4 - \frac{75}{64}x^2\log(2+x)$$
$$= -\frac{213473x}{1152} + \frac{6013x^2}{2304} - \frac{763x^3}{3456} + \frac{3x^4}{256} + \frac{185}{64}(2+x)^2 - \frac{71}{216}(2+x)^3 + \frac{3}{256}$$

Mathematica [A]
time = 0.18, size = 412, normalized size = 0.68

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[2 + x]^3*Log[3 + x],x]
```

```
[Out] (-195984 - 558290*x + 17705*x^2 - 1050*x^3 + 54*x^4 + 910528*Log[2 + x] + 4
00008*x*Log[2 + x] - 22836*x^2*Log[2 + x] + 2072*x^3*Log[2 + x] - 162*x^4*L
og[2 + x] - 302016*Log[2 + x]^2 - 118800*x*Log[2 + x]^2 + 11880*x^2*Log[2 +
 x]^2 - 1680*x^3*Log[2 + x]^2 + 216*x^4*Log[2 + x]^2 + 48384*Log[2 + x]^3 +
 15552*x*Log[2 + x]^3 - 2592*x^2*Log[2 + x]^3 + 576*x^3*Log[2 + x]^3 - 144*
```

```
x^4*Log[2 + x]^3 + 309078*Log[3 + x] + 79680*x*Log[3 + x] - 5520*x^2*Log[3
+ x] + 592*x^3*Log[3 + x] - 54*x^4*Log[3 + x] - 293976*Log[2 + x]*Log[3 + x
] - 57600*x*Log[2 + x]*Log[3 + x] + 7488*x^2*Log[2 + x]*Log[3 + x] - 1344*x
^3*Log[2 + x]*Log[3 + x] + 216*x^4*Log[2 + x]*Log[3 + x] + 138672*Log[2 + x
]^2*Log[3 + x] + 13824*x*Log[2 + x]^2*Log[3 + x] - 3456*x^2*Log[2 + x]^2*Lo
g[3 + x] + 1152*x^3*Log[2 + x]^2*Log[3 + x] - 432*x^4*Log[2 + x]^2*Log[3 +
x] - 46656*Log[2 + x]^3*Log[3 + x] + 576*x^4*Log[2 + x]^3*Log[3 + x] - 24*(
5609 - 6756*Log[2 + x] + 4680*Log[2 + x]^2)*PolyLog[2, -2 - x] + 288*(-563
+ 780*Log[2 + x])*PolyLog[3, -2 - x] - 224640*PolyLog[4, -2 - x])/2304
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3*Log[2 + x]^3*Log[3 + x],x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int x^{3} \ln \left(2+x\right)^{3} \ln \left(3+x\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(2+x)^3*ln(3+x),x)
```

```
[Out] int(x^3*ln(2+x)^3*ln(3+x),x)
```

Maxima [**A**]
time = 0.26, size = 518, normalized size = 0.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="maxima")
```

```
[Out] 3/128*x^4 + 1/16*(4*x^4*log(x + 3) - x^4 + 4*x^3 - 18*x^2 + 108*x - 324*log
(x + 3))*log(x + 2)^3 - 65/4*log(x + 3)*log(x + 2)^3 + 195/4*log(x + 3)*log
(x + 2)^2*log(-x - 2) - 175/384*x^3 + 1/96*(9*x^4 - 70*x^3 + 495*x^2 - 6*(3
*x^4 - 8*x^3 + 24*x^2 - 96*x)*log(x + 3) + 4680*log(x + 3)*log(-x - 2) - 49
50*x + 4680*dilog(x + 3) + 5778*log(x + 3) + 6048*log(x + 2))*log(x + 2)^2
+ 195/4*dilog(x + 3)*log(x + 2)^2 - 195/4*dilog(-x - 2)*log(x + 2)^2 + 563/
```

```
16*log(x + 3)*log(x + 2)^2 + 21*log(x + 2)^3 + 17705/2304*x^2 + 1/8*(780*lo
g(x + 2)^2 - 563*log(x + 2))*dilog(-x - 2) - 1/1152*(27*x^4 - 296*x^3 - 187
20*log(x + 2)^3 + 2760*x^2 + 40536*log(x + 2)^2 - 39840*x - 67308*log(x + 2
))*log(x + 3) - 1/1152*(81*x^4 - 1036*x^3 + 56160*log(x + 3)*log(x + 2)^2 +
 112320*log(x + 3)*log(x + 2)*log(-x - 2) + 11418*x^2 - 12*(9*x^4 - 56*x^3
+ 312*x^2 + 4680*log(x + 2)^2 - 2400*x - 6756*log(x + 2))*log(x + 3) + 1123
20*dilog(x + 3)*log(x + 2) + 112320*dilog(-x - 2)*log(x + 2) - 81072*log(x
+ 3)*log(x + 2) + 72576*log(x + 2)^2 - 200004*x - 81072*dilog(-x - 2) + 146
988*log(x + 3) + 302016*log(x + 2) - 112320*polylog(3, -x - 2))*log(x + 2)
+ 563/8*dilog(-x - 2)*log(x + 2) - 5609/96*log(x + 3)*log(x + 2) + 1573/12*
log(x + 2)^2 - 279145/1152*x - 5609/96*dilog(-x - 2) + 17171/128*log(x + 3)
 + 14227/36*log(x + 2) - 195/2*polylog(4, -x - 2) - 563/8*polylog(3, -x - 2
)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="fricas")
```

```
[Out] integral(x^3*log(x + 3)*log(x + 2)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\left(\frac{x^4\log(x+2)^3}{4}-\frac{3x^4\log(x+2)^2}{16}+\frac{3x^4\log(x+2)}{32}-\frac{3x^4}{128}+\frac{x^3\log(x+2)^2}{2}-\frac{7x^3\log(x+2)}{12}+\frac{37x^3}{144}-\frac{3x^2\log(x+2)^2}{2}+\frac{13x^2\log(x+2)}{4}-\frac{115x^2}{48}+6x\log(x+2)^2-25x\log(x+2)+\frac{415x}{12}-4\log(x+2)^3+25\log(x+2)^2-\frac{415\log(x+2)}{6}+\frac{10955281}{240000}\right)\log(x+3)-\frac{\int\frac{24900000x}{x+3}\,dx+\int\left(-\frac{1725000x^2}{x+3}\right)dx+\int\frac{185000x^3}{x+3}\,dx+\int\left(-\frac{16875x^4}{x+3}\right)dx+\int\left(-\frac{49800000\log(x+2)}{x+3}\right)dx+\int\frac{18000000\log(x+2)^2}{x+3}\,dx+\int\left(-\frac{2880000\log(x+2)^3}{x+3}\right)dx+\int\left(-\frac{18000000x\log(x+2)}{x+3}\right)dx+\int\frac{4320000x\log(x+2)^2}{x+3}\,dx+\int\frac{2340000x^2\log(x+2)}{x+3}\,dx+\int\left(-\frac{1080000x^2\log(x+2)^2}{x+3}\right)dx+\int\left(-\frac{420000x^3\log(x+2)}{x+3}\right)dx+\int\frac{360000x^3\log(x+2)^2}{x+3}\,dx+\int\frac{67500x^4\log(x+2)}{x+3}\,dx+\int\left(-\frac{135000x^4\log(x+2)^2}{x+3}\right)dx+\int\frac{180000x^4\log(x+2)^3}{x+3}\,dx+\int\frac{32865843}{x+3}\,dx}{720000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(2+x)**3*ln(3+x),x)
```

```
[Out] (x**4*log(x + 2)**3/4 - 3*x**4*log(x + 2)**2/16 + 3*x**4*log(x + 2)/32 - 3*
x**4/128 + x**3*log(x + 2)**2/2 - 7*x**3*log(x + 2)/12 + 37*x**3/144 - 3*x*
*2*log(x + 2)**2/2 + 13*x**2*log(x + 2)/4 - 115*x**2/48 + 6*x*log(x + 2)**2
 - 25*x*log(x + 2) + 415*x/12 - 4*log(x + 2)**3 + 25*log(x + 2)**2 - 415*lo
g(x + 2)/6 + 10955281/240000)*log(x + 3) - (Integral(24900000*x/(x + 3), x)
 + Integral(-1725000*x**2/(x + 3), x) + Integral(185000*x**3/(x + 3), x) +
Integral(-16875*x**4/(x + 3), x) + Integral(-49800000*log(x + 2)/(x + 3), x
) + Integral(18000000*log(x + 2)**2/(x + 3), x) + Integral(-2880000*log(x +
 2)**3/(x + 3), x) + Integral(-18000000*x*log(x + 2)/(x + 3), x) + Integral
(4320000*x*log(x + 2)**2/(x + 3), x) + Integral(2340000*x**2*log(x + 2)/(x
+ 3), x) + Integral(-1080000*x**2*log(x + 2)**2/(x + 3), x) + Integral(-420
000*x**3*log(x + 2)/(x + 3), x) + Integral(360000*x**3*log(x + 2)**2/(x + 3
), x) + Integral(67500*x**4*log(x + 2)/(x + 3), x) + Integral(-135000*x**4*
```

```
log(x + 2)**2/(x + 3), x) + Integral(180000*x**4*log(x + 2)**3/(x + 3), x)
+ Integral(32865843/(x + 3), x))/720000
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \ln(x+2)^3 \ln(x+3)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(x + 2)^3*log(x + 3),x)
```

```
[Out] int(x^3*log(x + 2)^3*log(x + 3), x)
```

3.30 $\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\left(x+\sqrt{b+x^2}\right)^a}{a}$$

[Out] (x+(x^2+b)^(1/2))^a/a

Rubi [A]
time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2147, 30}

$$\frac{\left(\sqrt{b+x^2}+x\right)^a}{a}$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2],x]

[Out] (x + Sqrt[b + x^2])^a/a

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx &= \text{Subst}\left(\int x^{-1+a}\,dx, x, x+\sqrt{b+x^2}\right)\\ &= \frac{\left(x+\sqrt{b+x^2}\right)^a}{a}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.00

$$\frac{\left(x+\sqrt{b+x^2}\right)^a}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2],x]
```

```
[Out] (x + Sqrt[b + x^2])^a/a
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 7.54, size = 269, normalized size = 15.82

$$\text{Piecewise}\left[\left\{\left\{\frac{xb^{\frac{1}{2}+\frac{a}{2}}\sqrt{\frac{b+x^2}{x^2}}\,\text{Sinh}\left[\text{ArcSinh}\left[\frac{x}{\sqrt{b}}\right](-1+a)\right]+x^3b^{-\frac{1}{2}+\frac{a}{2}}\sqrt{\frac{b+x^2}{x^2}}\,\text{Sinh}\left[\text{ArcSinh}\left[\frac{x}{\sqrt{b}}\right](-1+a)\right]+(b+x^2)\left(\frac{x\text{Cosh}\left[\text{ArcSinh}\left[\frac{x}{\sqrt{b}}\right](-1+a)\right]}{\sqrt{b}}+\text{Cosh}\left[a\text{ArcSinh}\left[\frac{x}{\sqrt{b}}\right]\right]\right)b^{\frac{a}{2}}}{a\,(b+x^2)},\text{Abs}\left[\frac{x^2}{b}\right]>1\right\}\right\},\frac{-2\text{Cosh}\left[a\text{ArcSinh}\left[\frac{x}{\sqrt{b}}\right]\right]\text{Gamma}\left[1-\frac{a}{2}\right]b^{\frac{a}{2}}}{a^2\text{Gamma}\left[-\frac{a}{2}\right]}+\frac{b^{\frac{a}{2}}\text{Sinh}\left[a\text{ArcSinh}\left[\frac{x}{\sqrt{b}}\right]-\text{ArcSinh}\left[\frac{x}{\sqrt{b}}\right]\right]}{a\sqrt{1+\frac{x^2}{b}}}+\frac{x\text{Cosh}\left[a\text{ArcSinh}\left[\frac{x}{\sqrt{b}}\right]-\text{ArcSinh}\left[\frac{x}{\sqrt{b}}\right]\right]b^{\frac{a}{2}}}{a\sqrt{b}}+\frac{x^2b^{\frac{a}{2}}\text{Sinh}\left[a\text{ArcSinh}\left[\frac{x}{\sqrt{b}}\right]-\text{ArcSinh}\left[\frac{x}{\sqrt{b}}\right]\right]}{ab\sqrt{1+\frac{x^2}{b}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Sqrt[x^2 + b] + x)^a/Sqrt[x^2 + b],x]')
```

```
[Out] Piecewise[{{(x b ^ (1 / 2 + a / 2) Sqrt[(b + x ^ 2) / x ^ 2] Sinh[ArcSinh[x
 / Sqrt[b]] (-1 + a)] + x ^ 3 b ^ (-1 / 2 + a / 2) Sqrt[(b + x ^ 2) / x ^ 2
] Sinh[ArcSinh[x / Sqrt[b]] (-1 + a)] + (b + x ^ 2) (x Cosh[ArcSinh[x / Sqr
t[b]] (-1 + a)] / Sqrt[b] + Cosh[a ArcSinh[x / Sqrt[b]]]) b ^ (a / 2)) / (a
 (b + x ^ 2)), Abs[x ^ 2 / b] > 1}}, -2 Cosh[a ArcSinh[x / Sqrt[b]]] Gamma[
1 - a / 2] b ^ (a / 2) / (a ^ 2 Gamma[-a / 2]) + b ^ (a / 2) Sinh[a ArcSinh
[x / Sqrt[b]] - ArcSinh[x / Sqrt[b]]] / (a Sqrt[1 + x ^ 2 / b]) + x Cosh[a
ArcSinh[x / Sqrt[b]] - ArcSinh[x / Sqrt[b]]] b ^ (a / 2) / (a Sqrt[b]) + x
^ 2 b ^ (a / 2) Sinh[a ArcSinh[x / Sqrt[b]] - ArcSinh[x / Sqrt[b]]] / (a b
Sqrt[1 + x ^ 2 / b])]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+b}\right)^a}{\sqrt{x^2+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x)
```

```
[Out] int((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b), x)
```

Fricas [A]
time = 0.60, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="fricas")
```

```
[Out] (x + sqrt(x^2 + b))^a/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(12) = 24$
time = 1.46, size = 311, normalized size = 18.29

$$\begin{cases} \frac{\sqrt{b}\, b^{\frac{a}{2}} \sinh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a x \sqrt{\frac{b}{x^2}+1}} + \frac{b^{\frac{a}{2}} x \cosh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}} + \frac{b^{\frac{a}{2}} x \sinh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}\sqrt{\frac{b}{x^2}+1}} - \frac{2 b^{\frac{a}{2}} \cosh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)\Gamma\left(1-\frac{a}{2}\right)}{a^2\Gamma\left(-\frac{a}{2}\right)} & \text{for}\ \left|\frac{x^2}{b}\right| > 1 \\ \frac{b^{\frac{a}{2}} \sinh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{1+\frac{x^2}{b}}} + \frac{b^{\frac{a}{2}} x^2 \sinh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{ab\sqrt{1+\frac{x^2}{b}}} + \frac{b^{\frac{a}{2}} x \cosh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}} - \frac{2 b^{\frac{a}{2}} \cosh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)\Gamma\left(1-\frac{a}{2}\right)}{a^2\Gamma\left(-\frac{a}{2}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+b)**(1/2))**a/(x**2+b)**(1/2),x)
```

```
[Out] Piecewise((sqrt(b)*b**(a/2)*sinh(a*asinh(x/sqrt(b)) - asinh(x/sqrt(b)))/(a*
x*sqrt(b/x**2 + 1)) + b**(a/2)*x*cosh(a*asinh(x/sqrt(b)) - asinh(x/sqrt(b))
)/(a*sqrt(b)) + b**(a/2)*x*sinh(a*asinh(x/sqrt(b)) - asinh(x/sqrt(b)))/(a*s
qrt(b)*sqrt(b/x**2 + 1)) - 2*b**(a/2)*cosh(a*asinh(x/sqrt(b)))*gamma(1 - a/
2)/(a**2*gamma(-a/2)), Abs(x**2/b) > 1), (b**(a/2)*sinh(a*asinh(x/sqrt(b))
- asinh(x/sqrt(b)))/(a*sqrt(1 + x**2/b)) + b**(a/2)*x**2*sinh(a*asinh(x/sqr
t(b)) - asinh(x/sqrt(b)))/(a*b*sqrt(1 + x**2/b)) + b**(a/2)*x*cosh(a*asinh(
x/sqrt(b)) - asinh(x/sqrt(b)))/(a*sqrt(b)) - 2*b**(a/2)*cosh(a*asinh(x/sqrt
(b)))*gamma(1 - a/2)/(a**2*gamma(-a/2)), True))
```

Giac [A]
time = 0.00, size = 16, normalized size = 0.94

$$\frac{\left(x+\sqrt{b+x^2}\right)^a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x)
```

```
[Out] (x + sqrt(x^2 + b))^a/a
```

Mupad [B]
time = 0.31, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (b + x^2)^(1/2))^a/(b + x^2)^(1/2),x)
```

```
[Out] (x + (b + x^2)^(1/2))^a/a
```

3.31 $\int \left(x+\sqrt{b+x^2}\right)^a \, dx$

Optimal. Leaf size=52

$$-\frac{b\left(x+\sqrt{b+x^2}\right)^{-1+a}}{2(1-a)}+\frac{\left(x+\sqrt{b+x^2}\right)^{1+a}}{2(1+a)}$$

```
[Out] -1/2*b*(x+(x^2+b)^(1/2))^(-1+a)/(1-a)+1/2*(x+(x^2+b)^(1/2))^(1+a)/(1+a)
```

Rubi [A]
time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2142, 14}

$$\frac{\left(\sqrt{b+x^2}+x\right)^{a+1}}{2(a+1)}-\frac{b\left(\sqrt{b+x^2}+x\right)^{a-1}}{2(1-a)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[b + x^2])^a,x]
```

```
[Out] -1/2*(b*(x + Sqrt[b + x^2])^(-1 + a))/(1 - a) + (x + Sqrt[b + x^2])^(1 + a)
/(2*(1 + a))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(x+\sqrt{b+x^2}\right)^a dx &= \frac{1}{2}\text{Subst}\left(\int x^{-2+a}\left(b+x^2\right)dx, x, x+\sqrt{b+x^2}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(bx^{-2+a}+x^a\right)dx, x, x+\sqrt{b+x^2}\right)\\ &= -\frac{b\left(x+\sqrt{b+x^2}\right)^{-1+a}}{2(1-a)}+\frac{\left(x+\sqrt{b+x^2}\right)^{1+a}}{2(1+a)}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 46, normalized size = 0.88

$$\frac{1}{2}\left(x+\sqrt{b+x^2}\right)^{-1+a}\left(\frac{b}{-1+a}+\frac{\left(x+\sqrt{b+x^2}\right)^2}{1+a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[b + x^2])^a,x]
```

```
[Out] ((x + Sqrt[b + x^2])^(-1 + a)*(b/(-1 + a) + (x + Sqrt[b + x^2])^2/(1 + a)))
/2
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 75.66, size = 738, normalized size = 14.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Sqrt[x^2 + b] + x)^a,x]')
```

```
[Out] Piecewise[{{(-a b ^ 2 + a b ^ 2 Cosh[ArcSinh[x / Sqrt[b]] (1 + a)] + a b ^
(3 / 2) x Sqrt[1 + b / x ^ 2] Sinh[a ArcSinh[x / Sqrt[b]]] - b x ^ 2 Sqrt[1
 + b / x ^ 2] Sinh[ArcSinh[x / Sqrt[b]] (1 + a)] + b x ^ 2 Cosh[ArcSinh[x /
 Sqrt[b]] (1 + a)] - a b x ^ 2 - a b x ^ 2 Sqrt[1 + b / x ^ 2] Sinh[ArcSinh
[x / Sqrt[b]] (1 + a)] + 2 a b x ^ 2 Cosh[ArcSinh[x / Sqrt[b]] (1 + a)] + a
 Sqrt[b] x ^ 3 Sqrt[1 + b / x ^ 2] Sinh[a ArcSinh[x / Sqrt[b]]] - x ^ 4 Sqr
t[1 + b / x ^ 2] Sinh[ArcSinh[x / Sqrt[b]] (1 + a)] + x ^ 4 Cosh[ArcSinh[x
/ Sqrt[b]] (1 + a)] - a x ^ 4 Sqrt[1 + b / x ^ 2] Sinh[ArcSinh[x / Sqrt[b]]
 (1 + a)] + a x ^ 4 Cosh[ArcSinh[x / Sqrt[b]] (1 + a)] - b ^ (3 / 2) x Cosh
[a ArcSinh[x / Sqrt[b]]] - Sqrt[b] x ^ 3 Cosh[a ArcSinh[x / Sqrt[b]]]) b ^
(-1 / 2 + a / 2) / (-b + a ^ 2 b - x ^ 2 + a ^ 2 x ^ 2), Abs[x ^ 2 / b] > 1
}}, 2 a b ^ 3 Cosh[a ArcSinh[x / Sqrt[b]] + ArcSinh[x / Sqrt[b]]] Gamma[1 -
 a / 2] b ^ (a / 2) / (-2 b ^ (5 / 2) Gamma[1 - a / 2] + 2 a ^ 2 b ^ (5 / 2
```

```
) Gamma[1 - a / 2]) - a ^ 2 b ^ 3 Gamma[-a / 2] b ^ (a / 2) Sqrt[1 + x ^ 2
/ b] Sinh[a ArcSinh[x / Sqrt[b]]] / (-2 b ^ (5 / 2) Gamma[1 - a / 2] + 2 a
^ 2 b ^ (5 / 2) Gamma[1 - a / 2]) + 2 a b ^ 2 x ^ 2 Cosh[a ArcSinh[x / Sqrt
[b]] + ArcSinh[x / Sqrt[b]]] Gamma[1 - a / 2] b ^ (a / 2) / (-2 b ^ (5 / 2)
 Gamma[1 - a / 2] + 2 a ^ 2 b ^ (5 / 2) Gamma[1 - a / 2]) - 2 b ^ (5 / 2) x
 Gamma[1 - a / 2] b ^ (a / 2) Sqrt[1 + x ^ 2 / b] Sinh[a ArcSinh[x / Sqrt[b
]] + ArcSinh[x / Sqrt[b]]] / (-2 b ^ (5 / 2) Gamma[1 - a / 2] + 2 a ^ 2 b ^
 (5 / 2) Gamma[1 - a / 2]) - 2 a b ^ (5 / 2) x Gamma[1 - a / 2] b ^ (a / 2)
 Sqrt[1 + x ^ 2 / b] Sinh[a ArcSinh[x / Sqrt[b]] + ArcSinh[x / Sqrt[b]]] /
(-2 b ^ (5 / 2) Gamma[1 - a / 2] + 2 a ^ 2 b ^ (5 / 2) Gamma[1 - a / 2]) +
a b ^ (5 / 2) x Cosh[a ArcSinh[x / Sqrt[b]]] Gamma[-a / 2] b ^ (a / 2) / (-
2 b ^ (5 / 2) Gamma[1 - a / 2] + 2 a ^ 2 b ^ (5 / 2) Gamma[1 - a / 2]) + 2
b ^ 2 x ^ 2 Cosh[a ArcSinh[x / Sqrt[b]] + ArcSinh[x / Sqrt[b]]] Gamma[1 - a
 / 2] b ^ (a / 2) / (-2 b ^ (5 / 2) Gamma[1 - a / 2] + 2 a ^ 2 b ^ (5 / 2)
Gamma[1 - a / 2])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(44) = 88$.
time = 0.03, size = 120, normalized size = 2.31

method	result	si
meijerg	$\frac{b^{\frac{a}{2}+\frac{1}{2}} a \left(\frac{8\sqrt{\pi}\, x^{1+a} b^{-\frac{a}{2}-\frac{1}{2}} \left(\frac{ab}{x^{2}}+a-1\right) \left(\sqrt{1+\frac{b}{x^{2}}}+1\right)^{a-1}}{(1+a)a(2a-2)}+\frac{4\sqrt{\pi}\, x^{1+a} b^{-\frac{a}{2}-\frac{1}{2}} \sqrt{1+\frac{b}{x^{2}}}\, \left(\sqrt{1+\frac{b}{x^{2}}}+1\right)^{a-1}}{(1+a)a}\right)}{4\sqrt{\pi}}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+b)^(1/2))^a,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^(1/2*a+1/2)/Pi^(1/2)*a*(8*Pi^(1/2)/(1+a)/a*x^(1+a)*b^(-1/2*a-1/2)*(a*
b/x^2+a-1)/(2*a-2)*((1+1/x^2*b)^(1/2)+1)^(a-1)+4*Pi^(1/2)/(1+a)/a*x^(1+a)*b
^(-1/2*a-1/2)*(1+1/x^2*b)^(1/2)*((1+1/x^2*b)^(1/2)+1)^(a-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + b))^a, x)
```

Fricas [A]
time = 0.31, size = 32, normalized size = 0.62

$$\frac{\left(\sqrt{x^{2}+b}\,a-x\right)\left(x+\sqrt{x^{2}+b}\right)^{a}}{a^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="fricas")
```

```
[Out] (sqrt(x^2 + b)*a - x)*(x + sqrt(x^2 + b))^a/(a^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2147 vs. $2(37) = 74$
time = 1.69, size = 2147, normalized size = 41.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+b)**(1/2))**a,x)
```

```
[Out] Piecewise((-a**2*b**(9/2)*b**(a/2)*x*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b
)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamm
a(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) -
a**2*b**(7/2)*b**(a/2)*x**3*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)))*gamma
(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2
) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) + a*b**(9/2
)*b**(a/2)*x*cosh(a*asinh(x/sqrt(b)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1
- a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) -
2*b**(7/2)*x**2*gamma(1 - a/2)) + a*b**(7/2)*b**(a/2)*x**3*cosh(a*asinh(x/s
qrt(b)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2
*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2
)) + 2*a*b**5*b**(a/2)*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1
- a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2
) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) - 2*a*b**5*
b**(a/2)*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x
**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 -
a/2)) - 2*a*b**4*b**(a/2)*x**2*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)) + a
sinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b*
*(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*ga
mma(1 - a/2)) + 4*a*b**4*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sq
rt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x*
*2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a
/2)) - 2*a*b**4*b**(a/2)*x**2*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2
) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**
(7/2)*x**2*gamma(1 - a/2)) - 2*a*b**3*b**(a/2)*x**4*sqrt(b/x**2 + 1)*sinh(a
*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma
```

```
(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2)
 - 2*b**(7/2)*x**2*gamma(1 - a/2)) + 2*a*b**3*b**(a/2)*x**4*cosh(a*asinh(x/
sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2)
 + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(
7/2)*x**2*gamma(1 - a/2)) - 2*b**4*b**(a/2)*x**2*sqrt(b/x**2 + 1)*sinh(a*as
inh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1
- a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) -
2*b**(7/2)*x**2*gamma(1 - a/2)) + 2*b**4*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(
b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*
a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*
x**2*gamma(1 - a/2)) - 2*b**3*b**(a/2)*x**4*sqrt(b/x**2 + 1)*sinh(a*asinh(x
/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2
) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**
(7/2)*x**2*gamma(1 - a/2)) + 2*b**3*b**(a/2)*x**4*cosh(a*asinh(x/sqrt(b)) +
 asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*
b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*
gamma(1 - a/2)), Abs(x**2/b) > 1), (-a**2*b**3*b**(a/2)*sqrt(1 + x**2/b)*si
nh(a*asinh(x/sqrt(b)))*gamma(-a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(
5/2)*gamma(1 - a/2)) - 2*a*b**(5/2)*b**(a/2)*x*sqrt(1 + x**2/b)*sinh(a*asin
h(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 -
a/2) - 2*b**(5/2)*gamma(1 - a/2)) + a*b**(5/2)*b**(a/2)*x*cosh(a*asinh(x/sq
rt(b)))*gamma(-a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 -
a/2)) + 2*a*b**3*b**(a/2)*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma
(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)) + 2*
a*b**2*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 -
a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)) - 2*b**(5
/2)*b**(a/2)*x*sqrt(1 + x**2/b)*sinh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))
*gamma(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)
) + 2*b**2*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(
1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)), True
))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(x+\sqrt{x^2+b}\right)}^a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (b + x^2)^(1/2))^a,x)
```

```
[Out] int((x + (b + x^2)^(1/2))^a, x)
```

3.32 $\int (6+3x^a+2x^{2a})^{\frac{1}{a}} (x^a+x^{2a}+x^{3a}) \, dx$

Optimal. Leaf size=34

$$\frac{x^{1+a} (6+3x^a+2x^{2a})^{1+\frac{1}{a}}}{6(1+a)}$$

```
[Out] 1/6*x^(1+a)*(6+3*x^a+2*x^(2*a))^(1+1/a)/(1+a)
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1608, 1761}

$$\frac{x^{a+1} (2x^{2a}+3x^a+6)^{\frac{1}{a}+1}}{6(a+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(6 + 3*x^a + 2*x^(2*a))^a^(-1)*(x^a + x^(2*a) + x^(3*a)),x]
```

```
[Out] (x^(1 + a)*(6 + 3*x^a + 2*x^(2*a))^(1 + a^(-1)))/(6*(1 + a))
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1761

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.)*
((d_) + (e_.)*(x_)^(n_.) + (f_.)*(x_)^(n2_.)), x_Symbol] :> Simp[d*(g*x)^(m
 + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*g*(m + 1))), x] /; FreeQ[{a, b, c
, d, e, f, g, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[a*e*(m + 1) - b*d*(m + n*
(p + 1) + 1), 0] && EqQ[a*f*(m + 1) - c*d*(m + 2*n*(p + 1) + 1), 0] && NeQ[
m, -1]
```

Rubi steps

$$\begin{aligned}
\int (6+3x^a+2x^{2a})^{\frac{1}{a}} (x^a+x^{2a}+x^{3a}) \, dx &= \int x^a (1+x^a+x^{2a}) (6+3x^a+2x^{2a})^{\frac{1}{a}} \, dx \\
&= \frac{x^{1+a} (6+3x^a+2x^{2a})^{1+\frac{1}{a}}}{6(1+a)}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 33, normalized size = 0.97

$$\frac{x^{1+a}\left(6+3x^a+2x^{2a}\right)^{1+\frac{1}{a}}}{6+6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(6 + 3*x^a + 2*x^(2*a))^a^(-1)*(x^a + x^(2*a) + x^(3*a)),x]
```

```
[Out] (x^(1 + a)*(6 + 3*x^a + 2*x^(2*a))^(1 + a^(-1)))/(6 + 6*a)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^(3*a) + x^(2*a) + x^a)*(2*x^(2*a) + 3*x^a + 6)^(1/a),
x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.04, size = 44, normalized size = 1.29

method	result	size
risch	$\frac{x\,x^{a}\left(6+3x^{a}+2x^{2a}\right)\left(6+3x^{a}+2x^{2a}\right)^{\frac{1}{a}}}{6+6a}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/6*x*x^a*(6+3*x^a+2*(x^a)^2)/(1+a)*(6+3*x^a+2*(x^a)^2)^(1/a)
```

Maxima [A]
time = 0.29, size = 48, normalized size = 1.41

$$\frac{(2\,xx^{3\,a}+3\,xx^{2\,a}+6\,xx^{a})(2\,x^{2\,a}+3\,x^{a}+6)^{\left(\frac{1}{a}\right)}}{6\,(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="max
ima")
```

```
[Out] 1/6*(2*x*x^(3*a) + 3*x*x^(2*a) + 6*x*x^a)*(2*x^(2*a) + 3*x^a + 6)^(1/a)/(a
+ 1)
```

Fricas [A]
time = 0.31, size = 48, normalized size = 1.41

$$\frac{(2\,xx^{3\,a}+3\,xx^{2\,a}+6\,xx^{a})(2\,x^{2\,a}+3\,x^{a}+6)^{\left(\frac{1}{a}\right)}}{6\,(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="fri
cas")
```

```
[Out] 1/6*(2*x*x^(3*a) + 3*x*x^(2*a) + 6*x*x^a)*(2*x^(2*a) + 3*x^a + 6)^(1/a)/(a
+ 1)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x**a+2*x**(2*a))**(1/a)*(x**a+x**(2*a)+x**(3*a)),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \left(x^{a}+x^{2\,a}+x^{3\,a}\right)\,\left(3\,x^{a}+2\,x^{2\,a}+6\right)^{1/a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^a + x^(2*a) + x^(3*a))*(3*x^a + 2*x^(2*a) + 6)^(1/a),x)
```

```
[Out] int((x^a + x^(2*a) + x^(3*a))*(3*x^a + 2*x^(2*a) + 6)^(1/a), x)
```

3.33 $\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx$

Optimal. Leaf size=58

$$\frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)$$

```
[Out] -1/2*ln(x)+3/4*ln(1-(-x^2+1)^(1/3))+1/2*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(
1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 57, 632, 210, 31}

$$\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)+\frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3*Log[1 -
 (1 - x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^2\right)\\ &= -\frac{\log(x)}{2} - \frac{3}{4}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^2}\right) + \frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right)\\ &= -\frac{\log(x)}{2} + \frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right) - \frac{3}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^2}\right)\\ &= \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right) - \frac{\log(x)}{2} + \frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 77, normalized size = 1.33

$$\frac{1}{4}\left(2\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right) + 2\log\left(-1+\sqrt[3]{1-x^2}\right) - \log\left(1+\sqrt[3]{1-x^2}+\left(1-x^2\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^2)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] + 2*Log[-1 + (1 - x^2)^(
1/3)] - Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)])/4
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.03, size = 17, normalized size = 0.29

$$3 - \frac{\text{hyper}\left[\left\{\frac{1}{3},\frac{1}{3}\right\},\left\{\frac{4}{3}\right\},\frac{1}{x^2}\right]}{2x^{\frac{2}{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*(1 - x^2)^(1/3)),x]')
```

```
[Out] 3 -1 ^ (2 / 3) hyper[{1 / 3, 1 / 3}, {4 / 3}, 1 / x ^ 2] / (2 x ^ (2 / 3))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 1.09, size = 65, normalized size = 1.12

method	result
meijerg	$\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+2\ln(x)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}+\frac{2\pi\sqrt{3}\,x^{2}\,\text{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^{2}\right)}{9\Gamma\left(\frac{2}{3}\right)}\right)}{4\pi}$
trager	$\frac{\ln\left(-\frac{4\,\text{RootOf}\left(\_Z^{2}+\_Z+1\right)^{2}x^{2}+15\,\text{RootOf}\left(\_Z^{2}+\_Z+1\right)\left(-x^{2}+1\right)^{\frac{2}{3}}+17\,\text{RootOf}\left(\_Z^{2}+\_Z+1\right)x^{2}+24\left(-x^{2}+1\right)^{\frac{2}{3}}+9\,\text{RootOf}\left(\_Z^{2}\right.}{x^{2}}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/Pi*3^(1/2)*GAMMA(2/3)*(2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x)+I*Pi)*Pi*
3^(1/2)/GAMMA(2/3)+2/9*Pi*3^(1/2)/GAMMA(2/3)*x^2*hypergeom([1,1,4/3],[2,2],
x^2))
```

Maxima [A]
time = 0.32, size = 62, normalized size = 1.07

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^{2}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^{2}+1\right)^{\frac{2}{3}}+\left(-x^{2}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^{2}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

Fricas [A]
time = 0.31, size = 64, normalized size = 1.10

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^{2}+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{4}\log\left(\left(-x^{2}+1\right)^{\frac{2}{3}}+\left(-x^{2}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^{2}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/4*log((-
x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.42, size = 36, normalized size = 0.62

$$-\frac{e^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{1}{x^2}\right)}{2x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+1)**(1/3),x)
```

```
[Out] -exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**(-2))/(2*x**(2/3)*gam
ma(4/3))
```

Giac [A]
time = 0.00, size = 81, normalized size = 1.40

$$\frac{3}{2}\left(-\frac{\ln\left(\left((-x^2+1)^{\frac{1}{3}}\right)^2+(-x^2+1)^{\frac{1}{3}}+1\right)}{6}+\frac{1}{3}\sqrt{3}\arctan\left(\frac{2\left((-x^2+1)^{\frac{1}{3}}+\frac{1}{2}\right)}{\sqrt{3}}\right)+\frac{\ln\left|(-x^2+1)^{\frac{1}{3}}-1\right|}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3),x)
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log(abs((-x^2 + 1)^(1/3) - 1))
```

Mupad [B]
time = 0.54, size = 86, normalized size = 1.48

$$\frac{\ln\left(\frac{9\,(1-x^2)^{1/3}}{4}-\frac{9}{4}\right)}{2}+\ln\left(\frac{9\,(1-x^2)^{1/3}}{4}-9\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)^2\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-\ln\left(\frac{9\,(1-x^2)^{1/3}}{4}-9\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)^2\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^2)^(1/3)),x)
```

```
[Out] log((9*(1 - x^2)^(1/3))/4 - 9/4)/2 + log((9*(1 - x^2)^(1/3))/4 - 9*((3^(1/2
)*1i)/4 - 1/4)^2)*((3^(1/2)*1i)/4 - 1/4) - log((9*(1 - x^2)^(1/3))/4 - 9*((
3^(1/2)*1i)/4 + 1/4)^2)*((3^(1/2)*1i)/4 + 1/4)
```

3.34 $\int \frac{1}{x(1-x^2)^{2/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)$$

```
[Out] -1/2*ln(x)+3/4*ln(1-(-x^2+1)^(1/3))-1/2*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(
1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 59, 632, 210, 31}

$$\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^2)^(2/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]) - Log[x]/2 + (3*Log[
1 - (1 - x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(1-x^2\right)^{2/3}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^2\right)\\
&= -\frac{\log(x)}{2}-\frac{3}{4}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^2}\right)-\frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-\right.\\
&= -\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)+\frac{3}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^2}\right)\\
&= -\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 77, normalized size = 1.33

$$\frac{1}{4}\left(-2\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)+2\log\left(-1+\sqrt[3]{1-x^2}\right)-\log\left(1+\sqrt[3]{1-x^2}+\left(1-x^2\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^2)^(2/3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] + 2*Log[-1 + (1 - x^2)^
(1/3)] - Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)])/4
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.04, size = 17, normalized size = 0.29

$$3-\frac{\text{hyper}\left[\left\{\frac{2}{3},\frac{2}{3}\right\},\left\{\frac{5}{3}\right\},\frac{1}{x^2}\right]}{4x^{\frac{4}{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*(1 - x^2)^(2/3)),x]')
```

```
[Out] 3 -1 ^ (1 / 3) hyper[{2 / 3, 2 / 3}, {5 / 3}, 1 / x ^ 2] / (4 x ^ (4 / 3))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 1.09, size = 48, normalized size = 0.83

method	result
meijerg	$\frac{\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+2\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)+\frac{2\Gamma\left(\frac{2}{3}\right)x^2\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],x^2\right)}{3}}{2\Gamma\left(\frac{2}{3}\right)}$
trager	$\frac{\ln\left(\frac{-4\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^2+15\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(-x^2+1\right)^{\frac{2}{3}}+9\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2-9\left(-x^2+1\right)^{\frac{2}{3}}+9\,\mathrm{RootOf}\left(\_Z^2+\_\right.}{x^2}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+1)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/GAMMA(2/3)*((1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x)+I*Pi)*GAMMA(2/3)+2/3*GAMM
A(2/3)*x^2*hypergeom([1,1,5/3],[2,2],x^2))
```

Maxima [A]
time = 0.33, size = 62, normalized size = 1.07

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

Fricas [A]
time = 0.30, size = 64, normalized size = 1.10

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/4*log((
-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.45, size = 37, normalized size = 0.64

$$-\frac{e^{-\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{1}{x^2}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+1)**(2/3),x)
```

```
[Out] -exp(-2*I*pi/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), x**(-2))/(2*x**(4/3)*g
amma(5/3))
```

Giac [A]
time = 0.00, size = 82, normalized size = 1.41

$$\frac{3}{2}\left(-\frac{\ln\left(\left((-x^2+1)^{\frac{1}{3}}\right)^2+(-x^2+1)^{\frac{1}{3}}+1\right)}{6}-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2\left((-x^2+1)^{\frac{1}{3}}+\frac{1}{2}\right)}{\sqrt{3}}\right)+\frac{\ln\left|(-x^2+1)^{\frac{1}{3}}-1\right|}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(2/3),x)
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log(abs((-x^2 + 1)^(1/3) - 1))
```

Mupad [B]
time = 0.46, size = 76, normalized size = 1.31

$$\frac{\ln\left(\frac{9\,(1-x^2)^{1/3}}{4}-\frac{9}{4}\right)}{2}+\ln\left(\frac{9\,(1-x^2)^{1/3}}{2}+\frac{9}{4}-\frac{\sqrt{3}\,9\mathrm{i}}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-\ln\left(\frac{9\,(1-x^2)^{1/3}}{2}+\frac{9}{4}+\frac{\sqrt{3}\,9\mathrm{i}}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^2)^(2/3)),x)
```

```
[Out] log((9*(1 - x^2)^(1/3))/4 - 9/4)/2 + log((9*(1 - x^2)^(1/3))/2 - (3^(1/2)*9
i)/4 + 9/4)*((3^(1/2)*1i)/4 - 1/4) - log((3^(1/2)*9i)/4 + (9*(1 - x^2)^(1/3
))/2 + 9/4)*((3^(1/2)*1i)/4 + 1/4)
```

3.35 $\int \frac{1}{\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=49

$$-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

```
[Out] 1/2*ln(x+(-x^3+1)^(1/3))-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(
1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{245\}$

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(-1/3),x]
```

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + Log[x + (1 - x^3)^
(1/3)]/2
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx=-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

Mathematica [A]
time = 0.03, size = 86, normalized size = 1.76

$$\frac{\tan^{-1}\left(\frac{-1+\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right)+\frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(-1/3),x]
```

```
[Out] ArcTan[(-1 + (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 + x^2/(1 - x^3
)^(2/3) - x/(1 - x^3)^(1/3)]/6 + Log[1 + x/(1 - x^3)^(1/3)]/3
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.03, size = 16, normalized size = 0.33

$$x\text{hyper}\left[\left\{\frac{1}{3},\frac{1}{3}\right\},\left\{\frac{4}{3}\right\},x^3\text{exp\_polar}\left[2I\text{Pi}\right]\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(1 - x^3)^(1/3),x]')
```

```
[Out] x hyper[{1 / 3, 1 / 3}, {4 / 3}, x ^ 3 exp_polar[2 I Pi]]
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 1.20, size = 12, normalized size = 0.24

method	result
meijerg	$x\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^3\right)$
trager	$\frac{\ln\left(-2\,\text{RootOf}\left(\_Z^2+\_Z+1\right)^2 x^3+3\,\text{RootOf}\left(\_Z^2+\_Z+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x-5\,\text{RootOf}\left(\_Z^2+\_Z+1\right)x^3+3x\left(-x^3+1\right)^{\frac{2}{3}}+3x^2\left(-\right.\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] x*hypergeom([1/3,1/3],[4/3],x^3)
```

Maxima [A]
time = 0.34, size = 78, normalized size = 1.59

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(-x^3+1\right)^{\frac{1}{3}}}{x}-1\right)\right)+\frac{1}{3}\log\left(\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+1\right)-\frac{1}{6}\log\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) + 1/3*log((-x^3
 + 1)^(1/3)/x + 1) - 1/6*log(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(40) = 80$.
time = 0.48, size = 82, normalized size = 1.67

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\,x - 2\sqrt{3}\,(-x^3+1)^{\frac{1}{3}}}{3\,x}\right) + \frac{1}{3}\log\left(\frac{x + (-x^3+1)^{\frac{1}{3}}}{x}\right) - \frac{1}{6}\log\left(\frac{x^2 - (-x^3+1)^{\frac{1}{3}}x + (-x^3+1)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/3*
log((x + (-x^3 + 1)^(1/3))/x) - 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 +
 1)^(2/3))/x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.41, size = 29, normalized size = 0.59

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(1/3),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/3
))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.33, size = 10, normalized size = 0.20

$$x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x^3)^(1/3),x)
```

```
[Out] x*hypergeom([1/3, 1/3], 4/3, x^3)
```

3.36 $\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=55

$$\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)$$

```
[Out] -1/2*ln(x)+1/2*ln(1-(-x^3+1)^(1/3))+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(
1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {272, 57, 632, 210, 31}

$$\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x
^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^3\right)\\
&= -\frac{\log(x)}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)\\
&= -\frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right)\\
&= \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 79, normalized size = 1.44

$$\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log\left(-1+\sqrt[3]{1-x^3}\right) - \frac{1}{6}\log\left(1+\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3)^(1/3)),x]
```

```
[Out] ArcTan[1/Sqrt[3] + (2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[-1 + (1 - x^3
)^(1/3)]/3 - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.02, size = 16, normalized size = 0.29

$$-\frac{\text{hyper}\left[\left\{\frac{1}{3},\frac{1}{3}\right\},\left\{\frac{4}{3}\right\},\frac{1}{x^3}\right]}{x}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*(1 - x^3)^(1/3)),x]')
```

```
[Out] -1 ^ (2 / 3) hyper[{1 / 3, 1 / 3}, {4 / 3}, 1 / x ^ 3] / x
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 1.93, size = 65, normalized size = 1.18

method	result
meijerg	$\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}+\frac{2\pi\sqrt{3}\,x^{3}\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^{3}\right)}{9\Gamma\left(\frac{2}{3}\right)}\right)}{6\pi}$
trager	$\ln\left(\frac{-211\,\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)^{2}x^{3}-3126\,\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)x^{3}+5502\left(-x^{3}+1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)-11543x^{3}-14247\left(-x^{3}+1 \right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/Pi*3^(1/2)*GAMMA(2/3)*(2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*Pi*
3^(1/2)/GAMMA(2/3)+2/9*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1,1,4/3],[2,2],
x^3))
```

Maxima [A]
time = 0.33, size = 62, normalized size = 1.13

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{6}\log\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 +
1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

Fricas [A]
time = 0.31, size = 64, normalized size = 1.16

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{6}\log\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) - 1/6*log((-
x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.42, size = 32, normalized size = 0.58

$$-\frac{e^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{1}{x^3}\right)}{3x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+1)**(1/3),x)
```

```
[Out] -exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**(-3))/(3*x*gamma(4/3)
)
```

Giac [A]
time = 0.00, size = 77, normalized size = 1.40

$$-\frac{\ln\left(\left((-x^3+1)^{\frac{1}{3}}\right)^2+(-x^3+1)^{\frac{1}{3}}+1\right)}{6}+\frac{1}{3}\sqrt{3}\arctan\left(\frac{2\left((-x^3+1)^{\frac{1}{3}}+\frac{1}{2}\right)}{\sqrt{3}}\right)+\frac{\ln\left|(-x^3+1)^{\frac{1}{3}}-1\right|}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3),x)
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 +
1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

Mupad [B]
time = 0.51, size = 80, normalized size = 1.45

$$\frac{\ln\left((1-x^3)^{1/3}-1\right)}{3}+\ln\left((1-x^3)^{1/3}-9\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left((1-x^3)^{1/3}-9\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^3)^(1/3)),x)
```

```
[Out] log((1 - x^3)^(1/3) - 1)/3 + log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 - 1/6)
^2)*((3^(1/2)*1i)/6 - 1/6) - log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 + 1/6)
^2)*((3^(1/2)*1i)/6 + 1/6)
```

3.37 $\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}}+\frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] -1/8*ln((1-x)*(1+x)^2)*2^(2/3)+3/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-
1/4*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [A]
time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2174}

$$\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(1
/3) - Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + (3*Log[-1 + x + 2^(2/3)*(1 - x^3
)^(1/3)])/(4*2^(1/3))
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}}+\frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1\right.}}{4\sqrt[3]{2}}$$

Mathematica [A]
time = 0.73, size = 148, normalized size = 1.53

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)+2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x+2\sqrt[3]{1-x^3}\right)-\log\left(2^{2/3}-2\,2^{2/3}x+2^{2/3}x^2-2(-1+x)\sqrt[3]{2-2x^3}+4\left(1-x^3\right)^{2/3}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1 - x^3
)^(1/3))] + 2*Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)] - Log[2^(2/3) -
 2*2^(2/3)*x + 2^(2/3)*x^2 - 2*(-1 + x)*(2 - 2*x^3)^(1/3) + 4*(1 - x^3)^(2/
3)])/(4*2^(1/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 + x)*(1 - x^3)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.24, size = 1163, normalized size = 11.99

method	result	size
trager	Expression too large to display	1163

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(-(12*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_
Z^3-4)^2*x+10*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z
^3-4)^3*x+8*RootOf(_Z^3-4)^2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*Ro
otOf(_Z^3-4)+4*_Z^2)-8*RootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^
2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-13*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x+8*Root
Of(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^
2)+42*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2+13*RootOf(_Z^
3-4)^2*(-x^3+1)^(1/3)+35*RootOf(_Z^3-4)*x^2+36*RootOf(RootOf(_Z^3-4)^2+2*_Z
*RootOf(_Z^3-4)+4*_Z^2)*x+30*RootOf(_Z^3-4)*x+52*(-x^3+1)^(2/3)+42*RootOf(R
ootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+35*RootOf(_Z^3-4))/(1+x)^2)*Roo
tOf(_Z^3-4)-1/2*ln(-(12*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)
^2*RootOf(_Z^3-4)^2*x+10*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2
```

```
)*RootOf(_Z^3-4)^3*x+8*RootOf(_Z^3-4)^2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4
)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)-8*RootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(Root
Of(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-13*RootOf(_Z^3-4)^2*(-x^3+1)^(1/
3)*x+8*RootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z
^3-4)+4*_Z^2)+42*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2+13
*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+35*RootOf(_Z^3-4)*x^2+36*RootOf(RootOf(_Z^
3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+30*RootOf(_Z^3-4)*x+52*(-x^3+1)^(2/3)+
42*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+35*RootOf(_Z^3-4))/(
1+x)^2)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+1/2*RootOf(Root
Of(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*ln(-(12*RootOf(RootOf(_Z^3-4)^2+2*
_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-4*RootOf(RootOf(_Z^3-4)^2+2*
_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x-8*RootOf(_Z^3-4)^2*(-x^3+1)^(2
/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+8*RootOf(_Z^3-4)*(-
x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-9*RootOf
(_Z^3-4)^2*(-x^3+1)^(1/3)*x-8*RootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(RootOf(_
Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)-42*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf
(_Z^3-4)+4*_Z^2)*x^2+9*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+14*RootOf(_Z^3-4)*x^
2-12*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+4*RootOf(_Z^3-4)
*x+36*(-x^3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)
+14*RootOf(_Z^3-4))/(1+x)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(71) = 142$.
time = 1.94, size = 301, normalized size = 3.10

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}(13x^6+2x^5+19x^4-4x^3+19x^2+2x+13)-4\sqrt{2}\,(5x^5-5x^4+6x^3-6x^2+5x-5)(-x^3+1)^{\frac{1}{3}}+16\cdot 2^{\frac{1}{6}}(x^4+2x^3+2x^2+2x+1)(-x^3+1)^{\frac{2}{3}}\right)}{6\,(3x^6-18x^5-3x^4-28x^3-3x^2-18x+3)}\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(\frac{4\cdot 2^{\frac{2}{3}}(-x^3+1)^{\frac{2}{3}}(x^2+1)+2^{\frac{1}{3}}(5x^4+6x^2+5)-2\,(3x^3-x^2+x-3)(-x^3+1)^{\frac{1}{3}}}{x^4+4x^3+6x^2+4x+1}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}(x^2+2x+1)-2\cdot 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}(x-1)-4\,(-x^3+1)^{\frac{2}{3}}}{x^2+2x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(13*x^6 + 2*x^5 +
19*x^4 - 4*x^3 + 19*x^2 + 2*x + 13) - 4*sqrt(2)*(5*x^5 - 5*x^4 + 6*x^3 - 6*
x^2 + 5*x - 5)*(-x^3 + 1)^(1/3) + 16*2^(1/6)*(x^4 + 2*x^3 + 2*x^2 + 2*x + 1
)*(-x^3 + 1)^(2/3))/(3*x^6 - 18*x^5 - 3*x^4 - 28*x^3 - 3*x^2 - 18*x + 3)) -
 1/24*2^(2/3)*log((4*2^(2/3)*(-x^3 + 1)^(2/3)*(x^2 + 1) + 2^(1/3)*(5*x^4 +
6*x^2 + 5) - 2*(3*x^3 - x^2 + x - 3)*(-x^3 + 1)^(1/3))/(x^4 + 4*x^3 + 6*x^2
```

```
 + 4*x + 1)) + 1/12*2^(2/3)*log((2^(2/3)*(x^2 + 2*x + 1) - 2*2^(1/3)*(-x^3
+ 1)^(1/3)*(x - 1) - 4*(-x^3 + 1)^(2/3))/(x^2 + 2*x + 1))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-\left(x-1\right)\left(x^{2}+x+1\right)}\,\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(1-x^{3}\right)}^{1/3}\,\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(1/3)*(x + 1)),x)
```

```
[Out] int(1/((1 - x^3)^(1/3)*(x + 1)), x)
```

3.38 $\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}}+\frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)-\frac{3\log\left(\right.}{}$$

[Out] 1/8*ln((1-x)*(1+x)^2)*2^(2/3)+1/2*ln(x+(-x^3+1)^(1/3))-3/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/4*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)

Rubi [A]
time = 0.07, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2177, 245, 2174}

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3
)) - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[(1 - x)*(1 +
 x)^2]/(4*2^(1/3)) + Log[x + (1 - x^3)^(1/3)]/2 - (3*Log[-1 + x + 2^(2/3)*(
1 - x^3)^(1/3)])/(4*2^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
```

```
a*d^3, 0]
```

Rule 2177

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rubi steps

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{\sqrt[3]{1-x^3}}\,dx - \int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$$
$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}} + \frac{1}{2}$$

Mathematica [F]
time = 3.60, size = 0, normalized size = 0.00

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/((1 + x)*(1 - x^3)^(1/3)),x]
[Out] Integrate[x/((1 + x)*(1 - x^3)^(1/3)), x]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/((1 + x)*(1 - x^3)^(1/3)),x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 12.29, size = 1790, normalized size = 12.34

method	result	size
trager	Expression too large to display	1790

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(-(10*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^
3+4)^3*x+12*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z
^3+4)^2*x-8*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3
+4)^2*(-x^3+1)^(2/3)-13*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x-8*(-x^3+1)^(1/3)*
RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*x+13*(-x
^3+1)^(1/3)*RootOf(_Z^3+4)^2+8*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*
RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)-35*x^2*RootOf(_Z^3+4)-42*RootOf(RootO
f(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2-30*x*RootOf(_Z^3+4)-36*RootOf(R
ootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x+52*(-x^3+1)^(2/3)-35*RootOf(_
Z^3+4)-42*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(1+x)^2)*Roo
tOf(_Z^3+4)-1/2*ln(-(10*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)
*RootOf(_Z^3+4)^3*x+12*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^
2*RootOf(_Z^3+4)^2*x-8*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*
RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)-13*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x-8*(-x^
3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+
4)*x+13*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2+8*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3
+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)-35*x^2*RootOf(_Z^3+4)-42*R
ootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2-30*x*RootOf(_Z^3+4)-
36*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x+52*(-x^3+1)^(2/3)-
35*RootOf(_Z^3+4)-42*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(
1+x)^2)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)+1/2*RootOf(Root
Of(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*ln((4*RootOf(RootOf(_Z^3+4)^2+2*_Z
*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x-12*RootOf(RootOf(_Z^3+4)^2+2*_Z*
RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x-8*RootOf(RootOf(_Z^3+4)^2+2*_Z*
RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)+9*(-x^3+1)^(1/3)*Roo
tOf(_Z^3+4)^2*x-8*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4
)+4*_Z^2)*RootOf(_Z^3+4)*x-9*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2+8*(-x^3+1)^(1/
3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)+14*x^
2*RootOf(_Z^3+4)-42*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2
+4*x*RootOf(_Z^3+4)-12*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*
x-36*(-x^3+1)^(2/3)+14*RootOf(_Z^3+4)-42*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootO
f(_Z^3+4)+4*_Z^2))/(1+x)^2)-1/6*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+4)^2+2*
_Z*RootOf(_Z^3+4)+4*_Z^2)*ln(-RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4
*_Z^2)^2*RootOf(_Z^3+4)^4*x^3+2*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+4)^2+2*
_Z*RootOf(_Z^3+4)+4*_Z^2)*x^3-2*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+4)^2+2*
_Z*RootOf(_Z^3+4)+4*_Z^2)+12*x*(-x^3+1)^(2/3)-12*x^2*(-x^3+1)^(1/3)+8*x^3-4
)+1/6*ln(RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+
```

```
4)^4*x^3+6*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+
4)^2*(-x^3+1)^(2/3)*x-6*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootO
f(_Z^3+4)+4*_Z^2)*(-x^3+1)^(1/3)*x^2+4*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+
4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^3-2*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+
4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)+4*x^3-4)*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z
^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)-1/3*ln(RootOf(RootOf(_Z^3+4)^2+2*_Z*Roo
tOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^4*x^3+6*RootOf(RootOf(_Z^3+4)^2+2*_Z*R
ootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)*x-6*RootOf(_Z^3+4)^2*
RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*(-x^3+1)^(1/3)*x^2+4*Ro
otOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^3-2*Ro
otOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)+4*x^3-4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(1-x^3\right)^{1/3}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(1/3)*(x + 1)),x)
```

```
[Out] int(x/((1 - x^3)^(1/3)*(x + 1)), x)
```

3.39 $\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx$

Optimal. Leaf size=110

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-x)}{\sqrt{3}\sqrt[3]{2-3x+x^2}}\right)}{2\sqrt[3]{2}}-\frac{\log(2-x)}{4\sqrt[3]{2}}-\frac{\log(x)}{2\sqrt[3]{2}}+\frac{3\log\left(2-x-2^{2/3}\sqrt[3]{2-3x+x^2}\right)}{4\sqrt[3]{2}}$$

```
[Out] -1/8*ln(2-x)*2^(2/3)-1/4*ln(x)*2^(2/3)+3/8*ln(2-x-2^(2/3)*(x^2-3*x+2)^(1/3)
)*2^(2/3)+1/4*arctan(-1/3*3^(1/2)-1/3*2^(1/3)*(2-x)/(x^2-3*x+2)^(1/3)*3^(1/
2))*3^(1/2)*2^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 176, normalized size of antiderivative = 1.60, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {769, 124}

$$\frac{3\sqrt[3]{x-2}\sqrt[3]{x-1}\log\left(-\frac{(x-2)^{2/3}}{\sqrt[3]{2}}-\sqrt[3]{2}\sqrt[3]{x-1}\right)}{4\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}-\frac{\sqrt[3]{x-2}\sqrt[3]{x-1}\log(x)}{2\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}-\frac{\sqrt{3}\sqrt[3]{x-2}\sqrt[3]{x-1}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{2}(x-2)^{2/3}}{\sqrt{3}\sqrt[3]{x-1}}\right)}{2\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*(-2 + x)^(1/3)*(-1 + x)^(1/3)*ArcTan[1/Sqrt[3] - (2^(1/3)*(-2
 + x)^(2/3))/(Sqrt[3]*(-1 + x)^(1/3))])/(2^(1/3)*(2 - 3*x + x^2)^(1/3)) + (
3*(-2 + x)^(1/3)*(-1 + x)^(1/3)*Log[-((-2 + x)^(2/3)/2^(1/3)) - 2^(1/3)*(-1
 + x)^(1/3)])/(4*2^(1/3)*(2 - 3*x + x^2)^(1/3)) - ((-2 + x)^(1/3)*(-1 + x)^
(1/3)*Log[x])/(2*2^(1/3)*(2 - 3*x + x^2)^(1/3))
```

Rule 124

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)*((e_.) + (f_.)*(x_))
^(1/3)), x_Symbol] :> With[{q = Rt[b*((b*e - a*f)/(b*c - a*d)^2), 3]}, Simp
[-Log[a + b*x]/(2*q*(b*c - a*d)), x] + (-Simp[Sqrt[3]*(ArcTan[1/Sqrt[3] + 2
*q*((c + d*x)^(2/3)/(Sqrt[3]*(e + f*x)^(1/3)))]/(2*q*(b*c - a*d))), x] + Si
mp[3*(Log[q*(c + d*x)^(2/3) - (e + f*x)^(1/3)]/(4*q*(b*c - a*d))), x])] /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - b*c*f - a*d*f, 0]
```

Rule 769

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(b + q + 2*c*x)^(1/3)*((b - q
+ 2*c*x)^(1/3)/(a + b*x + c*x^2)^(1/3)), Int[1/((d + e*x)*(b + q + 2*c*x)^(
1/3)*(b - q + 2*c*x)^(1/3)), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^
2 - 4*a*c, 0] && EqQ[c^2*d^2 - b*c*d*e - 2*b^2*e^2 + 9*a*c*e^2, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx = \frac{\left(\sqrt[3]{-4+2x}\,\sqrt[3]{-2+2x}\right)\int \frac{1}{x\sqrt[3]{-4+2x}\,\sqrt[3]{-2+2x}}\,dx}{\sqrt[3]{2-3x+x^2}}$$
$$= -\frac{\sqrt{3}\,\sqrt[3]{-2+x}\,\sqrt[3]{-1+x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{2}\,(-2+x)^{2/3}}{\sqrt{3}\,\sqrt[3]{-1+x}}\right)}{2\sqrt[3]{2}\,\sqrt[3]{2-3x+x^2}} + \frac{3\sqrt[3]{-2+x}\,\sqrt[3]{-1+}}{4}$$

Mathematica [A]
time = 0.18, size = 162, normalized size = 1.47

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{2-3x+x^2}}{2\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{2-3x+x^2}}\right)+2\log\left(-2\sqrt[3]{2}+\sqrt[3]{2}x+2\sqrt[3]{2-3x+x^2}\right)-\log\left(4\,2^{2/3}-4\,2^{2/3}x+2^{2/3}x^2-2\sqrt[3]{2}(-2+x)\sqrt[3]{2-3x+x^2}+4(2-3x+x^2)^{2/3}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(Sqrt[3]*(2 - 3*x + x^2)^(1/3))/(2*2^(1/3) - 2^(1/3)*x +
(2 - 3*x + x^2)^(1/3))] + 2*Log[-2*2^(1/3) + 2^(1/3)*x + 2*(2 - 3*x + x^2)^
(1/3)] - Log[4*2^(2/3) - 4*2^(2/3)*x + 2^(2/3)*x^2 - 2*2^(1/3)*(-2 + x)*(2
- 3*x + x^2)^(1/3) + 4*(2 - 3*x + x^2)^(2/3)])/(4*2^(1/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in ___instancecheck___

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*(x^2 - 3*x + 2)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in __instancecheck__
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.38, size = 1069, normalized size = 9.72

method	result	size
trager	Expression too large to display	1069

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2-3*x+2)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*RootOf(_Z^3-4)*ln(-(112*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_
Z^2)^2*RootOf(_Z^3-4)^2*x^2+68*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+
```

```
4*_Z^2)*RootOf(_Z^3-4)^3*x^2-504*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4
)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-216*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3
-4)+4*_Z^2)*RootOf(_Z^3-4)^2*(x^2-3*x+2)^(2/3)-306*RootOf(RootOf(_Z^3-4)^2+
2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x+504*RootOf(RootOf(_Z^3-4)^2+
2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2+258*RootOf(RootOf(_Z^3-4)^2+
2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*(x^2-3*x+2)^(1/3)*x+306*RootOf(R
ootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3-108*RootOf(_Z^
3-4)^2*(x^2-3*x+2)^(1/3)*x-516*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+
4*_Z^2)*RootOf(_Z^3-4)*(x^2-3*x+2)^(1/3)+196*RootOf(RootOf(_Z^3-4)^2+2*_Z*R
ootOf(_Z^3-4)+4*_Z^2)*x^2+216*RootOf(_Z^3-4)^2*(x^2-3*x+2)^(1/3)+119*RootOf
(_Z^3-4)*x^2-1680*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-948
*(x^2-3*x+2)^(2/3)-1020*RootOf(_Z^3-4)*x+1680*RootOf(RootOf(_Z^3-4)^2+2*_Z*
RootOf(_Z^3-4)+4*_Z^2)+1020*RootOf(_Z^3-4))/x^2)+1/2*RootOf(RootOf(_Z^3-4)^
2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*ln((68*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z
^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x^2+28*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf
(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x^2-306*RootOf(RootOf(_Z^3-4)^2+2*_Z*Root
Of(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-108*RootOf(RootOf(_Z^3-4)^2+2*_Z*Ro
otOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2*(x^2-3*x+2)^(2/3)-126*RootOf(RootOf(_
Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x+306*RootOf(RootOf(_
Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2-237*RootOf(RootOf(_
Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*(x^2-3*x+2)^(1/3)*x+126
*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3-54*Ro
otOf(_Z^3-4)^2*(x^2-3*x+2)^(1/3)*x+474*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(
_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*(x^2-3*x+2)^(1/3)+17*RootOf(RootOf(_Z^3-4)^2
+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2+108*RootOf(_Z^3-4)^2*(x^2-3*x+2)^(1/3)+7*R
ootOf(_Z^3-4)*x^2+408*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x
+258*(x^2-3*x+2)^(2/3)+168*RootOf(_Z^3-4)*x-408*RootOf(RootOf(_Z^3-4)^2+2*_
Z*RootOf(_Z^3-4)+4*_Z^2)-168*RootOf(_Z^3-4))/x^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(81) = 162.
time = 1.11, size = 277, normalized size = 2.52

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}(x^6+36\,x^5-612\,x^4+2880\,x^3-5760\,x^2+5184\,x-1728)+12\sqrt{2}\,(x^5-38\,x^4+252\,x^3-648\,x^2+720\,x-288)(x^2-3\,x+2)^{\frac{1}{3}}+48\cdot 2^{\frac{1}{6}}(x^4-6\,x^3+6\,x^2)(x^2-3\,x+2)^{\frac{2}{3}}\right)}{6\,(x^6-108\,x^5+972\,x^4-3456\,x^3+6048\,x^2-5184\,x+1728)}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}x^2+6\cdot 2^{\frac{1}{3}}(x^2-3\,x+2)^{\frac{1}{3}}(x-2)+12\,(x^2-3\,x+2)^{\frac{2}{3}}}{x^2}\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(\frac{12\cdot 2^{\frac{2}{3}}(x^2-3\,x+2)^{\frac{2}{3}}(x^2-6\,x+6)+2^{\frac{1}{3}}(x^4-36\,x^3+180\,x^2-288\,x+144)-6\,(x^3-14\,x^2+36\,x-24)(x^2-3\,x+2)^{\frac{1}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(x^6 + 36*x^5 - 6
12*x^4 + 2880*x^3 - 5760*x^2 + 5184*x - 1728) + 12*sqrt(2)*(x^5 - 38*x^4 +
252*x^3 - 648*x^2 + 720*x - 288)*(x^2 - 3*x + 2)^(1/3) + 48*2^(1/6)*(x^4 -
6*x^3 + 6*x^2)*(x^2 - 3*x + 2)^(2/3))/(x^6 - 108*x^5 + 972*x^4 - 3456*x^3 +
 6048*x^2 - 5184*x + 1728)) + 1/12*2^(2/3)*log((2^(2/3)*x^2 + 6*2^(1/3)*(x^
2 - 3*x + 2)^(1/3)*(x - 2) + 12*(x^2 - 3*x + 2)^(2/3))/x^2) - 1/24*2^(2/3)*
log((12*2^(2/3)*(x^2 - 3*x + 2)^(2/3)*(x^2 - 6*x + 6) + 2^(1/3)*(x^4 - 36*x
^3 + 180*x^2 - 288*x + 144) - 6*(x^3 - 14*x^2 + 36*x - 24)*(x^2 - 3*x + 2)^
(1/3))/x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{(x-2)(x-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2-3*x+2)**(1/3),x)
```

```
[Out] Integral(1/(x*((x - 2)*(x - 1))**(1/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(x^2-3\,x+2\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 - 3*x + 2)^(1/3)),x)
```

```
[Out] int(1/(x*(x^2 - 3*x + 2)^(1/3)), x)
```

3.40 $\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx$

Optimal. Leaf size=81

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2(-1+x)}{\sqrt{3}\sqrt[3]{-5+7x-3x^2+x^3}}\right)+\frac{1}{4}\log(1-x)-\frac{3}{4}\log\left(1-x+\sqrt[3]{-5+7x-3x^2+x^3}\right)$$

```
[Out] 1/4*ln(1-x)-3/4*ln(1-x+(x^3-3*x^2+7*x-5)^(1/3))+1/2*arctan(1/3*3^(1/2)+2/3*
(-1+x)/(x^3-3*x^2+7*x-5)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.62, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2092, 2036, 335, 281, 245}

$$\frac{\sqrt{3}\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1}\tan^{-1}\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{(x-1)^2+4}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{(x-1)^3+4(x-1)}}-\frac{3\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1}\log\left((x-1)^{2/3}-\sqrt[3]{(x-1)^2+4}\right)}{4\sqrt[3]{(x-1)^3+4(x-1)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 7*x - 3*x^2 + x^3)^(-1/3),x]
```

```
[Out] (Sqrt[3]*(4 + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*ArcTan[(1 + (2*(-1 + x)^(2/3
))/(4 + (-1 + x)^2)^(1/3))/Sqrt[3]])/(2*(4*(-1 + x) + (-1 + x)^3)^(1/3)) -
(3*(4 + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*Log[-(4 + (-1 + x)^2)^(1/3) + (-1
+ x)^(2/3)])/(4*(4*(-1 + x) + (-1 + x)^3)^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt[3]{4x+x^3}}\,dx, x, -1+x\right)$$

$$= \frac{\left(\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\sqrt[3]{4+x^2}}\,dx, x, -1+x\right)}{\sqrt[3]{4(-1+x)+(-1+x)^3}}$$

$$= \frac{\left(3\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{4+x^6}}\,dx, x, \sqrt[3]{-1+x}\right)}{\sqrt[3]{4(-1+x)+(-1+x)^3}}$$

$$= \frac{\left(3\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{4+x^3}}\,dx, x, (-1+x)^{2/3}\right)}{2\sqrt[3]{4(-1+x)+(-1+x)^3}}$$

$$= \frac{\sqrt{3}\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\tan^{-1}\left(\frac{1+\frac{2(-1+x)^{2/3}}{\sqrt[3]{4+(-1+x)^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{-4(1-x)+(-1+x)^3}} - \frac{3\sqrt[3]{4+ \cdots}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 0.01, size = 85, normalized size = 1.05

$$\frac{3\sqrt[3]{(2-i)+ix}\sqrt[3]{i(-1+x)}\,((-1+2i)+x)F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{1}{4}i((-1+2i)+x),-\frac{1}{2}i((-1+2i)+x)\right)}{4\sqrt[3]{-5+7x-3x^2+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 7*x - 3*x^2 + x^3)^(-1/3),x]
```

```
[Out] (3*((2 - I) + I*x)^(1/3)*(I*(-1 + x))^(1/3)*((-1 + 2*I) + x)*AppellF1[2/3,
1/3, 1/3, 5/3, (-1/4*I)*((-1 + 2*I) + x), (-1/2*I)*((-1 + 2*I) + x)])/(4*(-
5 + 7*x - 3*x^2 + x^3)^(1/3))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^3 - 3*x^2 + 7*x - 5)^(1/3),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.90, size = 433, normalized size = 5.35

method	result
trager	$-\frac{\ln\left(92\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2 x^2+624\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3-3x^2+7x-5\right)^{\frac{2}{3}}-675\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3-3x^2+7x-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-3*x^2+7*x-5)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(92*RootOf(_Z^2-_Z+1)^2*x^2+624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^
(2/3)-675*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)*x-184*RootOf(_Z^2-_Z+1)
^2*x-41*RootOf(_Z^2-_Z+1)*x^2+51*(x^3-3*x^2+7*x-5)^(2/3)+675*RootOf(_Z^2-_Z
+1)*(x^3-3*x^2+7*x-5)^(1/3)+624*(x^3-3*x^2+7*x-5)^(1/3)*x+82*RootOf(_Z^2-_Z
+1)*x-583*x^2-624*(x^3-3*x^2+7*x-5)^(1/3)-713*RootOf(_Z^2-_Z+1)+1166*x-1643
)+1/2*RootOf(_Z^2-_Z+1)*ln(212*RootOf(_Z^2-_Z+1)^2*x^2-624*RootOf(_Z^2-_Z+1
)*(x^3-3*x^2+7*x-5)^(2/3)-51*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)*x-42
4*RootOf(_Z^2-_Z+1)^2*x+463*RootOf(_Z^2-_Z+1)*x^2+675*(x^3-3*x^2+7*x-5)^(2/
3)+51*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)-624*(x^3-3*x^2+7*x-5)^(1/3)
*x-926*RootOf(_Z^2-_Z+1)*x+161*x^2+624*(x^3-3*x^2+7*x-5)^(1/3)+1643*RootOf(
_Z^2-_Z+1)-322*x+713)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x)
```

Fricas [A]
time = 0.46, size = 120, normalized size = 1.48

$$-\frac{1}{2}\,\sqrt{3}\,\arctan\left(\frac{22791076\,\sqrt{3}\,(x^3-3\,x^2+7\,x-5)^{\frac{1}{3}}(x-1)+\sqrt{3}\,(20389537\,x^2-40779074\,x+53222437)+17987998\,\sqrt{3}\,(x^3-3\,x^2+7\,x-5)^{\frac{2}{3}}}{7204617\,x^2-14409234\,x-20666867}\right)-\frac{1}{4}\,\log\left(3\,(x^3-3\,x^2+7\,x-5)^{\frac{1}{3}}(x-1)-3\,(x^3-3\,x^2+7\,x-5)^{\frac{2}{3}}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan((22791076*sqrt(3)*(x^3 - 3*x^2 + 7*x - 5)^(1/3)*(x - 1)
 + sqrt(3)*(20389537*x^2 - 40779074*x + 53222437) + 17987998*sqrt(3)*(x^3 -
 3*x^2 + 7*x - 5)^(2/3))/(7204617*x^2 - 14409234*x - 20666867)) - 1/4*log(3
*(x^3 - 3*x^2 + 7*x - 5)^(1/3)*(x - 1) - 3*(x^3 - 3*x^2 + 7*x - 5)^(2/3) +
4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x^3-3x^2+7x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-3*x**2+7*x-5)**(1/3),x)
```

```
[Out] Integral((x**3 - 3*x**2 + 7*x - 5)**(-1/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(x^3-3\,x^2+7\,x-5\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7*x - 3*x^2 + x^3 - 5)^(1/3),x)
```

```
[Out] int(1/(7*x - 3*x^2 + x^3 - 5)^(1/3), x)
```

3.41 $\int \frac{1}{\sqrt[3]{x(-q+x^2)}}\,dx$

Optimal. Leaf size=66

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2x}{\sqrt{3}\sqrt[3]{x(-q+x^2)}}\right)+\frac{\log(x)}{4}-\frac{3}{4}\log\left(-x+\sqrt[3]{x(-q+x^2)}\right)$$

```
[Out] 1/4*ln(x)-3/4*ln(-x+(x*(x^2-q))^(1/3))+1/2*arctan(1/3*3^(1/2)+2/3*x/(x*(x^2
-q))^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.77, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2004, 2036, 335, 281, 245}

$$\frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{x^2-q}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-q}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{x^3-qx}}-\frac{3\sqrt[3]{x}\sqrt[3]{x^2-q}\log\left(x^{2/3}-\sqrt[3]{x^2-q}\right)}{4\sqrt[3]{x^3-qx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-q + x^2))^(-1/3),x]
```

```
[Out] (Sqrt[3]*x^(1/3)*(-q + x^2)^(1/3)*ArcTan[(1 + (2*x^(2/3))/(-q + x^2)^(1/3))
/Sqrt[3]])/(2*(-(q*x) + x^3)^(1/3)) - (3*x^(1/3)*(-q + x^2)^(1/3)*Log[x^(2/
3) - (-q + x^2)^(1/3)])/(4*(-(q*x) + x^3)^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{x(-q+x^2)}}\,dx &= \int \frac{1}{\sqrt[3]{-qx+x^3}}\,dx \\
&= \frac{\left(\sqrt[3]{x}\,\sqrt[3]{-q+x^2}\right)\int \frac{1}{\sqrt[3]{x}\,\sqrt[3]{-q+x^2}}\,dx}{\sqrt[3]{-qx+x^3}} \\
&= \frac{\left(3\sqrt[3]{x}\,\sqrt[3]{-q+x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{-q+x^6}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-qx+x^3}} \\
&= \frac{\left(3\sqrt[3]{x}\,\sqrt[3]{-q+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-q+x^3}}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{-qx+x^3}} \\
&= \frac{\sqrt{3}\,\sqrt[3]{x}\,\sqrt[3]{-q+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-q+x^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{-qx+x^3}} - \frac{3\sqrt[3]{x}\,\sqrt[3]{-q+x^2}\,\log\left(x^{2/3}-\sqrt[3]{-\right.}{4\sqrt[3]{-qx+x^3}}
\end{aligned}$$

Mathematica [A]
time = 0.61, size = 130, normalized size = 1.97

$$\frac{\sqrt[3]{x}\,\sqrt[3]{-q+x^2}\left(2\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}+2\sqrt[3]{-q+x^2}}\right)-2\log\left(-x^{2/3}+\sqrt[3]{-q+x^2}\right)+\log\left(x^{4/3}+x^{2/3}\sqrt[3]{-q+x^2}+(-q+x^2)^{2/3}\right)\right)}{4\sqrt[3]{-qx+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-q + x^2))^(-1/3),x]
```

```
[Out] (x^(1/3)*(-q + x^2)^(1/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*x^(2/3))/(x^(2/3) + 2*
(-q + x^2)^(1/3))] - 2*Log[-x^(2/3) + (-q + x^2)^(1/3)] + Log[x^(4/3) + x^(
2/3)*(-q + x^2)^(1/3) + (-q + x^2)^(2/3)]))/(4*(-(q*x) + x^3)^(1/3))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*(x^2 - q))^(1/3),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x \left(x^{2}-q \right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2-q))^(1/3),x)
```

```
[Out] int(1/(x*(x^2-q))^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x^2-q))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x^2 - q)*x)^(-1/3), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(52) = 104.
time = 1.00, size = 415, normalized size = 6.29

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{4\sqrt{3}(q^{12}-15q^{10}+90q^{8}-351q^{6}+810q^{4}-1215q^{2}+729)(x^{3}-qx)^{\frac{1}{3}}x-2\sqrt{3}(q^{12}+6q^{11}-15q^{10}-54q^{9}+90q^{8}+270q^{7}-351q^{6}-810q^{5}+810q^{4}+1458q^{3}-1215q^{2}-1458q+729)(x^{3}-qx)^{\frac{2}{3}}-\sqrt{3}(q^{13}+10q^{12}-15q^{11}-282q^{10}+90q^{9}+2178q^{8}-351q^{7}-6534q^{6}+810q^{5}+7614q^{4}-1215q^{3}-(q^{12}-6q^{11}-15q^{10}+54q^{9}+90q^{8}-270q^{7}-351q^{6}+810q^{5}+810q^{4}-1458q^{3}-1215q^{2}+1458q+729)x^{2}-2430q^{2}+729q)}{q^{13}+18q^{12}+81q^{11}-162q^{10}-1350q^{9}+810q^{8}+6561q^{7}-2430q^{6}-12150q^{5}+4374q^{4}+6561q^{3}-9(q^{12}+2q^{11}-15q^{10}-18q^{9}+90q^{8}+90q^{7}-351q^{6}-270q^{5}+810q^{4}+486q^{3}-1215q^{2}-486q+729)x^{2}-4374q^{2}+729q}\right)-\frac{1}{4}\log\left(-3(x^{3}-qx)^{\frac{1}{3}}x+q+3(x^{3}-qx)^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x^2-q))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan((4*sqrt(3)*(q^12 - 15*q^10 + 90*q^8 - 351*q^6 + 810*q^4
- 1215*q^2 + 729)*(x^3 - q*x)^(1/3)*x - 2*sqrt(3)*(q^12 + 6*q^11 - 15*q^10
- 54*q^9 + 90*q^8 + 270*q^7 - 351*q^6 - 810*q^5 + 810*q^4 + 1458*q^3 - 1215
*q^2 - 1458*q + 729)*(x^3 - q*x)^(2/3) - sqrt(3)*(q^13 + 10*q^12 - 15*q^11
- 282*q^10 + 90*q^9 + 2178*q^8 - 351*q^7 - 6534*q^6 + 810*q^5 + 7614*q^4 -
1215*q^3 - (q^12 - 6*q^11 - 15*q^10 + 54*q^9 + 90*q^8 - 270*q^7 - 351*q^6 +
 810*q^5 + 810*q^4 - 1458*q^3 - 1215*q^2 + 1458*q + 729)*x^2 - 2430*q^2 + 7
29*q))/(q^13 + 18*q^12 + 81*q^11 - 162*q^10 - 1350*q^9 + 810*q^8 + 6561*q^7
 - 2430*q^6 - 12150*q^5 + 4374*q^4 + 6561*q^3 - 9*(q^12 + 2*q^11 - 15*q^10
- 18*q^9 + 90*q^8 + 90*q^7 - 351*q^6 - 270*q^5 + 810*q^4 + 486*q^3 - 1215*q
^2 - 486*q + 729)*x^2 - 4374*q^2 + 729*q)) - 1/4*log(-3*(x^3 - q*x)^(1/3)*x
 + q + 3*(x^3 - q*x)^(2/3))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x**2-q))**(1/3),x)
```

```
[Out] Integral((x*(-q + x**2))**(-1/3), x)
```

Giac [A]
time = 0.01, size = 97, normalized size = 1.47

$$-\frac{3q\left(-\frac{\ln\left(\left(\left(-q\left(\frac{1}{x}\right)^2+1\right)^{\frac{1}{3}}\right)^2+\left(-q\left(\frac{1}{x}\right)^2+1\right)^{\frac{1}{3}}+1\right)}{6}+\frac{1}{3}\sqrt{3}\arctan\left(\frac{2\left(\left(-q\left(\frac{1}{x}\right)^2+1\right)^{\frac{1}{3}}+\frac{1}{2}\right)}{\sqrt{3}}\right)+\frac{\ln\left|\left(-q\left(\frac{1}{x}\right)^2+1\right)^{\frac{1}{3}}-1\right|}{3}\right)}{2q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x^2-q))^(1/3),x)
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-q/x^2 + 1)^(1/3) + 1)) + 1/4*log((-q/x
^2 + 1)^(2/3) + (-q/x^2 + 1)^(1/3) + 1) - 1/2*log(abs((-q/x^2 + 1)^(1/3) -
1))
```

Mupad [B]
time = 0.39, size = 37, normalized size = 0.56

$$\frac{3\,x\,{\left(1-\frac{x^2}{q}\right)}^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ \frac{x^2}{q}\right)}{2\,{\left(x^3-q\,x\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x*(q - x^2))^(1/3),x)
[Out] (3*x*(1 - x^2/q)^(1/3)*hypergeom([1/3, 1/3], 4/3, x^2/q))/(2*(x^3 - q*x)^(1
/3))
```

3.42 $\int \frac{1}{\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\,dx$

Optimal. Leaf size=79

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2(-1+x)}{\sqrt{3}\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\right)+\frac{1}{4}\log(1-x)-\frac{3}{4}\log\left(1-x+\sqrt[3]{(-1+x)\left(q-2x+\right.}\right.$$

```
[Out] 1/4*ln(1-x)-3/4*ln(1-x+((-1+x)*(x^2+q-2*x))^(1/3))+1/2*arctan(1/3*3^(1/2)+2
/3*(-1+x)/((-1+x)*(x^2+q-2*x))^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 145, normalized size of antiderivative = 1.84, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2092, 2036, 335, 281, 245}

$$\frac{\sqrt{3}\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1}\tan^{-1}\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{q+(x-1)^2-1}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{(x-1)^3-(1-q)(x-1)}}-\frac{3\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1}\log\left((x-1)^{2/3}-\sqrt[3]{q+(x-1)^2-1}\right)}{4\sqrt[3]{(x-1)^3-(1-q)(x-1)}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)*(q - 2*x + x^2))^(-1/3),x]
```

```
[Out] (Sqrt[3]*(-1 + q + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*ArcTan[(1 + (2*(-1 + x)
^(2/3))/(-1 + q + (-1 + x)^2)^(1/3))/Sqrt[3]])/(2*(-((1 - q)*(-1 + x)) + (-
1 + x)^3)^(1/3)) - (3*(-1 + q + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*Log[-(-1 +
 q + (-1 + x)^2)^(1/3) + (-1 + x)^(2/3)])/(4*(-((1 - q)*(-1 + x)) + (-1 + x
)^3)^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt[3]{-(1-q)x+x^3}}\,dx, x, -1+x\right)\\
&= \frac{\left(\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,\sqrt[3]{-1+q+x^2}}\,dx, x\right.}{\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}}\\
&= \frac{\left(3\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{-1+q+x^6}}\,dx, x, \sqrt[3]{\cdots}\right.}{\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}}\\
&= \frac{\left(3\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+q+x^3}}\,dx, x, (-\cdots\right.}{2\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}}\\
&= \frac{\sqrt{3}\,\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\,\tan^{-1}\left(\frac{1+\frac{2(-1+x)^{2/3}}{\sqrt[3]{q-(2-x)x}}}{\sqrt{3}}\right)}{2\sqrt[3]{(1-q)(1-x)+(-1+x)^3}}-\frac{3\cdots}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.71, size = 145, normalized size = 1.84

$$\frac{\sqrt[3]{-1+x}\,\sqrt[3]{q+(-2+x)x}\left(2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}(-1+x)^{2/3}}{(-1+x)^{2/3}+2\sqrt[3]{q+(-2+x)x}}\right)-2\log\left(-(-1+x)^{2/3}+\sqrt[3]{q+(-2+x)x}\right)+\log\left((-1+x)^{4/3}+(-1+x)^{2/3}\sqrt[3]{q+(-2+x)x}+(q+(-2+x)x)^{2/3}\right)\right)}{4\sqrt[3]{(-1+x)(q+(-2+x)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)*(q - 2*x + x^2))^(-1/3),x]
```

```
[Out] ((-1 + x)^(1/3)*(q + (-2 + x)*x)^(1/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*(-1 + x)^
(2/3))/((-1 + x)^(2/3) + 2*(q + (-2 + x)*x)^(1/3))] - 2*Log[-(-1 + x)^(2/3)
 + (q + (-2 + x)*x)^(1/3)] + Log[(-1 + x)^(4/3) + (-1 + x)^(2/3)*(q + (-2 +
 x)*x)^(1/3) + (q + (-2 + x)*x)^(2/3)]))/(4*((-1 + x)*(q + (-2 + x)*x))^(1/
3))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((x - 1)*(x^2 - 2*x + q))^(1/3),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(-1+x\right)\left(x^{2}+q-2x\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)*(x^2+q-2*x))^(1/3),x)
```

```
[Out] int(1/((-1+x)*(x^2+q-2*x))^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. 2(65) = 130.
time = 0.85, size = 665, normalized size = 8.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan((2*sqrt(3)*(q^12 - 18*q^11 + 117*q^10 - 346*q^9 + 414*q^
8 - 18*q^7 + 69*q^6 - 774*q^5 - 234*q^4 + 1058*q^3 + 621*q^2 + 378*q - 539)
*(x^3 + (q + 2)*x - 3*x^2 - q)^(2/3) + 4*sqrt(3)*(q^12 - 12*q^11 + 51*q^10
- 70*q^9 - 90*q^8 + 288*q^7 - 57*q^6 + 54*q^5 - 810*q^4 + 320*q^3 + 291*q^2
 - (q^12 - 12*q^11 + 51*q^10 - 70*q^9 - 90*q^8 + 288*q^7 - 57*q^6 + 54*q^5
- 810*q^4 + 320*q^3 + 291*q^2 + 714*q + 49)*x + 714*q + 49)*(x^3 + (q + 2)*
x - 3*x^2 - q)^(1/3) - sqrt(3)*(q^13 - 22*q^12 + 177*q^11 - 514*q^10 - 434*
q^9 + 5346*q^8 - 8247*q^7 - 4542*q^6 + 19638*q^5 - 8050*q^4 - 10343*q^3 + (
q^12 - 6*q^11 - 15*q^10 + 206*q^9 - 594*q^8 + 594*q^7 - 183*q^6 + 882*q^5 -
 1386*q^4 - 418*q^3 - 39*q^2 + 1050*q + 637)*x^2 + 6186*q^2 - 2*(q^12 - 6*q
^11 - 15*q^10 + 206*q^9 - 594*q^8 + 594*q^7 - 183*q^6 + 882*q^5 - 1386*q^4
- 418*q^3 - 39*q^2 + 1050*q + 637)*x + 1501*q + 32))/(q^13 - 22*q^12 + 249*
q^11 - 1546*q^10 + 4702*q^9 - 4230*q^8 - 10623*q^7 + 25338*q^6 - 3546*q^5 -
 31306*q^4 + 18817*q^3 + 9*(q^12 - 14*q^11 + 73*q^10 - 162*q^9 + 78*q^8 + 1
86*q^7 - 15*q^6 - 222*q^5 - 618*q^4 + 566*q^3 + 401*q^2 + 602*q - 147)*x^2
+ 9714*q^2 - 18*(q^12 - 14*q^11 + 73*q^10 - 162*q^9 + 78*q^8 + 186*q^7 - 15
*q^6 - 222*q^5 - 618*q^4 + 566*q^3 + 401*q^2 + 602*q - 147)*x - 995*q + 8))
 - 1/4*log(3*(x^3 + (q + 2)*x - 3*x^2 - q)^(1/3)*(x - 1) + q - 3*(x^3 + (q
+ 2)*x - 3*x^2 - q)^(2/3) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(x-1)\,(q+x^2-2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x**2+q-2*x))**(1/3),x)
```

```
[Out] Integral(((x - 1)*(q + x**2 - 2*x))**(-1/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\left(x-1\right)\,\left(x^2-2\,x+q\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)*(q - 2*x + x^2))^(1/3),x)
[Out] int(1/((x - 1)*(q - 2*x + x^2))^(1/3), x)
```

3.43 $\int \frac{1}{x\sqrt[3]{(-1+x)(q-2qx+x^2)}}\,dx$

Optimal. Leaf size=118

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{q}(-1+x)}{\sqrt{3}\sqrt[3]{(-1+x)(q-2qx+x^2)}}\right)}{2\sqrt[3]{q}}+\frac{\log(1-x)}{4\sqrt[3]{q}}+\frac{\log(x)}{2\sqrt[3]{q}}-\frac{3\log\left(-\sqrt[3]{q}(-1+x)+\sqrt[3]{(-1}\right.}{4\sqrt[3]{q}}$$

```
[Out] 1/4*ln(1-x)/q^(1/3)+1/2*ln(x)/q^(1/3)-3/4*ln(-q^(1/3)*(-1+x)+((-1+x)*(-2*q*
x+x^2+q))^(1/3))/q^(1/3)+1/2*arctan(1/3*3^(1/2)+2/3*q^(1/3)*(-1+x)/((-1+x)*
(-2*q*x+x^2+q))^(1/3)*3^(1/2))*3^(1/2)/q^(1/3)
```

Rubi [F]
time = 21.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt[3]{(-1+x)(q-2qx+x^2)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]
```

```
[Out] ((-1 - 2*q - (1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[
-((-1 + q)^3*q)])^(2/3))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[-((-1 +
 q)^3*q)])^(1/3) + 3*x)^(1/3)*(-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*(1 - q)^2)/(
1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (1 + 6*q -
15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (3*(1 - 5*q + 4*q^2 +
 (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*((-1 - 2*q
)/3 + x))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3) +
9*((-1 - 2*q)/3 + x)^2)^(1/3)*Defer[Subst][Defer[Int][1/(((1 + 2*q)/3 + x)*
(-1/3*(1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)
^3*q])^(2/3))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3
) + x)^(1/3)*((-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*(1 - q)^2)/(1 + 6*q - 15*q^2
 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (1 + 6*q - 15*q^2 + 8*q^3 +
 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))/9 + ((1 - 5*q + 4*q^2 + (1 + 6*q - 15*
q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*x)/(3*(1 + 6*q - 15*q^2 +
 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3)) + x^2)^(1/3)), x], x, (-1 - 2*
q)/3 + x])/(3*(-q + 3*q*x + (-1 - 2*q)*x^2 + x^3)^(1/3))
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{3}(1+2q)+x\right)\sqrt[3]{-\frac{2}{27}(1-q)^2(1+8q)-\frac{1}{3}(1-4q)(1-\cdots}}\right.$$

$$\left(\sqrt[3]{-1-2q-\frac{1-5q+4q^2+\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-\cdots}\right.}{\sqrt[3]{1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)\cdots}}}}\right.$$

$$= \frac{\quad}{\quad}$$

Mathematica [A]
time = 2.19, size = 189, normalized size = 1.60

$$\frac{\sqrt[3]{-1+x}\sqrt[3]{q-2qx+x^2}\left(2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{q}(-1+x)^{2/3}}{\sqrt[3]{q}(-1+x)^{2/3}+2\sqrt[3]{q-2qx+x^2}}\right)-2\log\left(-\sqrt[3]{q}(-1+x)^{2/3}+\sqrt[3]{q-2qx+x^2}\right)+\log\left(q^{2/3}(-1+x)^{4/3}+\sqrt[3]{q}(-1+x)^{2/3}\sqrt[3]{q-2qx+x^2}+\left(q-2qx+x^2\right)^{2/3}\right)\right)}{4\sqrt[3]{q}\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]
```

```
[Out] ((-1 + x)^(1/3)*(q - 2*q*x + x^2)^(1/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*q^(1/3)*
(-1 + x)^(2/3))/(q^(1/3)*(-1 + x)^(2/3) + 2*(q - 2*q*x + x^2)^(1/3))] - 2*L
og[-(q^(1/3)*(-1 + x)^(2/3)) + (q - 2*q*x + x^2)^(1/3)] + Log[q^(2/3)*(-1 +
 x)^(4/3) + q^(1/3)*(-1 + x)^(2/3)*(q - 2*q*x + x^2)^(1/3) + (q - 2*q*x + x
^2)^(2/3)]))/(4*q^(1/3)*((-1 + x)*(q - 2*q*x + x^2))^(1/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*((x - 1)*(x^2 - 2*q*x + q))^(1/3)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left((-1+x)\left(-2qx+x^2+q\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x)
```

```
[Out] int(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(100) = 200$.
time = 15.00, size = 1496, normalized size = 12.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="fricas")
```

```
[Out] [1/12*(sqrt(3)*q*sqrt((-q)^(1/3)/q)*log(-((q^3 - 30*q^2 - 51*q - 1)*x^6 + 5
4*(q^3 + 6*q^2 + 2*q)*x^5 - 27*(17*q^3 + 26*q^2 + 2*q)*x^4 + 486*q^3*x + 54
0*(2*q^3 + q^2)*x^3 - 81*q^3 - 135*(8*q^3 + q^2)*x^2 + 9*((2*q^2 - q - 1)*x
^4 - 6*(q^2 - q)*x^3 + 3*(q^2 - q)*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^
(2/3)*(-q)^(1/3) + 9*((q^2 + 7*q + 1)*x^5 - (19*q^2 + 25*q + 1)*x^4 + 9*(7*
q^2 + 3*q)*x^3 + 45*q^2*x - 9*(9*q^2 + q)*x^2 - 9*q^2)*(-(2*q + 1)*x^2 + x^
3 + 3*q*x - q)^(1/3)*(-q)^(2/3) + sqrt(3)*(3*((4*q^2 + 13*q + 1)*x^4 - 6*(7
*q^2 + 5*q)*x^3 - 72*q^2*x + 3*(31*q^2 + 5*q)*x^2 + 18*q^2)*(-(2*q + 1)*x^2
 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^3 - 5*q^2 - 5*q)*x^5 + 5*(q^3
+ 7*q^2 + q)*x^4 - 45*q^3*x - 45*(q^3 + q^2)*x^3 + 9*q^3 + 15*(5*q^3 + q^2)
*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) + ((q^3 + 24*q^2 + 3*q - 1)*
x^6 - 54*(q^3 + 2*q^2)*x^5 + 81*(3*q^3 + 2*q^2)*x^4 - 162*q^3*x - 108*(4*q^
3 + q^2)*x^3 + 27*q^3 + 27*(14*q^3 + q^2)*x^2)*(-q)^(1/3))*sqrt((-q)^(1/3)/
q))/x^6) - 2*(-q)^(2/3)*log(((-q)^(2/3)*(q - 1)*x^2 + 3*(-(2*q + 1)*x^2 + x
^3 + 3*q*x - q)^(1/3)*(q*x - q)*(-q)^(1/3) + 3*(-(2*q + 1)*x^2 + x^3 + 3*q*
x - q)^(2/3)*q)/x^2) + (-q)^(2/3)*log((3*((2*q + 1)*x^2 - 6*q*x + 3*q)*(-(2
*q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^2 + 2*q)*x^3 + 9*q^
2*x - (7*q^2 + 2*q)*x^2 - 3*q^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) -
 ((q^2 + 7*q + 1)*x^4 - 18*(q^2 + q)*x^3 - 36*q^2*x + 9*(5*q^2 + q)*x^2 + 9
*q^2)*(-q)^(1/3))/x^4))/q, 1/12*(2*sqrt(3)*q*sqrt(-(-q)^(1/3)/q)*arctan(1/3
*sqrt(3)*(6*((2*q^2 - q - 1)*x^4 - 6*(q^2 - q)*x^3 + 3*(q^2 - q)*x^2)*(-(2*
q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) - 6*((q^3 + 7*q^2 + q)*x^5 -
```

```
(19*q^3 + 25*q^2 + q)*x^4 + 45*q^3*x + 9*(7*q^3 + 3*q^2)*x^3 - 9*q^3 - 9*(
9*q^3 + q^2)*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) - ((q^3 - 12*q^2
 - 15*q - 1)*x^6 + 18*(q^3 + 6*q^2 + 2*q)*x^5 - 9*(17*q^3 + 26*q^2 + 2*q)*x
^4 + 162*q^3*x + 180*(2*q^3 + q^2)*x^3 - 27*q^3 - 45*(8*q^3 + q^2)*x^2)*(-q
)^(1/3))*sqrt(-(-q)^(1/3)/q)/((q^3 + 24*q^2 + 3*q - 1)*x^6 - 54*(q^3 + 2*q^
2)*x^5 + 81*(3*q^3 + 2*q^2)*x^4 - 162*q^3*x - 108*(4*q^3 + q^2)*x^3 + 27*q^
3 + 27*(14*q^3 + q^2)*x^2)) - 2*(-q)^(2/3)*log(((-q)^(2/3)*(q - 1)*x^2 + 3*
(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3)*(q*x - q)*(-q)^(1/3) + 3*(-(2*q +
1)*x^2 + x^3 + 3*q*x - q)^(2/3)*q)/x^2) + (-q)^(2/3)*log((3*((2*q + 1)*x^2
- 6*q*x + 3*q)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^
2 + 2*q)*x^3 + 9*q^2*x - (7*q^2 + 2*q)*x^2 - 3*q^2)*(-(2*q + 1)*x^2 + x^3 +
 3*q*x - q)^(1/3) - ((q^2 + 7*q + 1)*x^4 - 18*(q^2 + q)*x^3 - 36*q^2*x + 9*
(5*q^2 + q)*x^2 + 9*q^2)*(-q)^(1/3))/x^4))/q]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{(x-1)\left(-2qx+q+x^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x**2+q))**(1/3),x)
[Out] Integral(1/(x*((x - 1)*(-2*q*x + q + x**2))**(1/3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(\left(x-1\right)\,\left(x^2-2\,q\,x+q\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((x - 1)*(q - 2*q*x + x^2))^(1/3)),x)
[Out] int(1/(x*((x - 1)*(q - 2*q*x + x^2))^(1/3)), x)
```

3.44 $\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx$

Optimal. Leaf size=111

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{k}\,x}{\sqrt[3]{(1-x)x(1-kx)}}}{\sqrt{3}}\right)}{\sqrt[3]{k}}+\frac{\log(x)}{2\sqrt[3]{k}}+\frac{\log(1-(1+k)x)}{2\sqrt[3]{k}}-\frac{3\log\left(-\sqrt[3]{k}\,x+\sqrt[3]{(1-x)x(1-kx)}\right)}{2\sqrt[3]{k}}$$

```
[Out] 1/2*ln(x)/k^(1/3)+1/2*ln(1-(1+k)*x)/k^(1/3)-3/2*ln(-k^(1/3)*x+((1-x)*x*(-k*
x+1))^(1/3))/k^(1/3)+arctan(1/3*(1+2*k^(1/3)*x/((1-x)*x*(-k*x+1))^(1/3))*3^
(1/2))*3^(1/2)/k^(1/3)
```

Rubi [F]
time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)),x]
```

```
[Out] (3*(1 - x)^(1/3)*x*(1 - k*x)^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, x, k*x])/(2
*((1 - x)*x*(1 - k*x))^(1/3)) + ((1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defe
r[Int][1/((1 - x)^(1/3)*x^(1/3)*(1 + (-1 - k)*x)*(1 - k*x)^(1/3)), x])/((1
- x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\begin{aligned}
\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx &= \frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{2-(1+k)x}{\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\,(1-(1+k)x)}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}\\
&= \frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{1}{\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}+\frac{\left(\sqrt[3]{1-\right.}{}\\
&= \frac{3\sqrt[3]{1-x}\,x\sqrt[3]{1-kx}\,F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};x,kx\right)}{2\sqrt[3]{(1-x)x(1-kx)}}+\frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-\right.}{}
\end{aligned}$$

Mathematica [F]
time = 41.06, size = 0, normalized size = 0.00

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)),x]
[Out] Integrate[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)), x]
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(2 - (k + 1)*x)/((1 - (k + 1)*x)*(x*(1 - x)*(1 - k*x))^(
1/3)),x]')
[Out] Timed out
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{2-(1+k)\,x}{((1-x)\,x\,(-kx+1))^{\frac{1}{3}}\,(1-(1+k)\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x)
[Out] int((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="ma
xima")
[Out] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx+x-2}{\sqrt[3]{x(x-1)(kx-1)}\,(kx+x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(1-(1+k)*x),x)
```

```
[Out] Integral((k*x + x - 2)/((x*(x - 1)*(k*x - 1))**(1/3)*(k*x + x - 1)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(k+1)-2}{(x\,(k+1)-1)\,(x\,(k\,x-1)\,(x-1))^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(k + 1) - 2)/((x*(k + 1) - 1)*(x*(k*x - 1)*(x - 1))^(1/3)),x)
```

```
[Out] int((x*(k + 1) - 2)/((x*(k + 1) - 1)*(x*(k*x - 1)*(x - 1))^(1/3)), x)
```

3.45 $\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$

Optimal. Leaf size=176

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\,(1-kx)}{\sqrt[3]{1-k}\,\sqrt[3]{(1-x)x(1-kx)}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt[3]{1-k}}+\frac{\log(1-(2-k)x)}{2^{2/3}\sqrt[3]{1-k}}+\frac{\log(1-kx)}{2\,2^{2/3}\sqrt[3]{1-k}}-\frac{3\log\left(-1+kx+2^{2}\right.}{2}$$

```
[Out] 1/2*ln(1-(2-k)*x)*2^(1/3)/(1-k)^(1/3)+1/4*ln(-k*x+1)*2^(1/3)/(1-k)^(1/3)-3/
4*ln(-1+k*x+2^(2/3)*(1-k)^(1/3)*((1-x)*x*(-k*x+1))^(1/3))*2^(1/3)/(1-k)^(1/
3)-1/2*arctan(1/3*(1+2^(1/3)*(-k*x+1)/(1-k)^(1/3)/((1-x)*x*(-k*x+1))^(1/3))
*3^(1/2))*3^(1/2)*2^(1/3)/(1-k)^(1/3)
```

Rubi [F]
time = 0.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)),x]
```

```
[Out] ((1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Int][(1 - k*x)^(1/3)/((1 - x)^
(2/3)*x^(2/3)*(1 + (-2 + k)*x)), x])/((1 - x)*x*(1 - k*x))^(2/3)
```

Rubi steps

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx=\frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{\sqrt[3]{1-kx}}{(1-x)^{2/3}x^{2/3}(1+(-2+k)x)}\,dx}{((1-x)x(1-kx))^{2/3}}$$

Mathematica [F]
time = 10.42, size = 0, normalized size = 0.00

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)),x]
```

```
[Out] Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)), x]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - k*x)/((1 + (k - 2)*x)*(x*(1 - x)*(1 - k*x))^(2/3)),
x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{-kx+1}{(1+(-2+k)\,x)\,((1-x)\,x\,(-kx+1))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x)
```

```
[Out] int((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="maxi
ma")
```

```
[Out] -integrate((k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 932 vs. 2(132) = 264.
time = 54.99, size = 932, normalized size = 5.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="fric
as")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/3*(24*sqrt(3)*2^(1/3)*((k^5 - 3*k^4 - 4*k^3 +
22*k^2 - 24*k + 8)*x^4 - 2*(k^4 - 10*k^3 + 27*k^2 - 26*k + 8)*x^3 - 6*(k^3
```

```
- 4*k^2 + 4*k - 1)*x^2 - 2*(k^2 - 1)*x + k - 1)*(k*x^3 - (k + 1)*x^2 + x)^(
2/3)/(k - 1)^(1/3) - 6*sqrt(3)*2^(2/3)*((k^6 + 27*k^5 - 40*k^4 - 20*k^3 + 4
8*k^2 - 16*k)*x^5 - (33*k^5 + 55*k^4 - 220*k^3 + 132*k^2 + 16*k - 16)*x^4 +
 2*(55*k^4 - 55*k^3 - 66*k^2 + 82*k - 16)*x^3 - 2*(55*k^3 - 99*k^2 + 38*k +
 6)*x^2 + (33*k^2 - 61*k + 28)*x - k + 1)*(k*x^3 - (k + 1)*x^2 + x)^(1/3)/(
k - 1)^(2/3) + sqrt(3)*((k^6 - 48*k^5 - 192*k^4 + 416*k^3 - 48*k^2 - 192*k
+ 64)*x^6 + 6*(7*k^5 + 104*k^4 - 80*k^3 - 176*k^2 + 176*k - 32)*x^5 - 3*(13
9*k^4 + 256*k^3 - 768*k^2 + 352*k + 16)*x^4 + 4*(203*k^3 - 192*k^2 - 120*k
+ 104)*x^3 - 3*(139*k^2 - 208*k + 64)*x^2 + 6*(7*k - 8)*x + 1))/((k^6 + 96*
k^5 - 48*k^4 - 160*k^3 + 240*k^2 - 192*k + 64)*x^6 - 6*(17*k^5 + 64*k^4 - 1
12*k^3 + 80*k^2 - 80*k + 32)*x^5 + 3*(149*k^4 + 32*k^3 - 96*k^2 - 160*k + 8
0)*x^4 - 4*(157*k^3 - 24*k^2 - 168*k + 40)*x^3 + 3*(149*k^2 - 128*k - 16)*x
^2 - 6*(17*k - 16)*x + 1))/(k - 1)^(1/3) - 1/12*2^(1/3)*log((12*2^(2/3)*(k*
x^3 - (k + 1)*x^2 + x)^(2/3)*((k^3 + k^2 - 4*k + 2)*x^2 - 2*(2*k^2 - 3*k +
1)*x + k - 1)/(k - 1)^(2/3) + 6*((k^3 + 8*k^2 - 8*k)*x^3 - (11*k^2 - 8)*x^2
 + (11*k - 8)*x - 1)*(k*x^3 - (k + 1)*x^2 + x)^(1/3) + 2^(1/3)*((k^4 + 28*k
^3 - 12*k^2 - 32*k + 16)*x^4 - 4*(8*k^3 + 15*k^2 - 30*k + 8)*x^3 + 6*(13*k^
2 - 10*k - 2)*x^2 - 4*(8*k - 7)*x + 1)/(k - 1)^(1/3))/((k^4 - 8*k^3 + 24*k^
2 - 32*k + 16)*x^4 + 4*(k^3 - 6*k^2 + 12*k - 8)*x^3 + 6*(k^2 - 4*k + 4)*x^2
 + 4*(k - 2)*x + 1))/(k - 1)^(1/3) + 1/6*2^(1/3)*log((6*2^(1/3)*(k*x^3 - (k
 + 1)*x^2 + x)^(1/3)*(k*x - 1)/(k - 1)^(1/3) - 2^(2/3)*((k^2 - 4*k + 4)*x^2
 + 2*(k - 2)*x + 1)/(k - 1)^(2/3) - 12*(k*x^3 - (k + 1)*x^2 + x)^(2/3))/((k
^2 - 4*k + 4)*x^2 + 2*(k - 2)*x + 1))/(k - 1)^(1/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{kx}{kx\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}-2x\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}+\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}}\,dx-\int\left(-\frac{1}{kx\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}-2x\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}+\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))**(2/3),x)
```

```
[Out] -Integral(k*x/(k*x*(k*x**3 - k*x**2 - x**2 + x)**(2/3) - 2*x*(k*x**3 - k*x*
*2 - x**2 + x)**(2/3) + (k*x**3 - k*x**2 - x**2 + x)**(2/3)), x) - Integral
(-1/(k*x*(k*x**3 - k*x**2 - x**2 + x)**(2/3) - 2*x*(k*x**3 - k*x**2 - x**2
+ x)**(2/3) + (k*x**3 - k*x**2 - x**2 + x)**(2/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{k\,x-1}{\left(x\,\left(k-2\right)+1\right)\,{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(k*x - 1)/((x*(k - 2) + 1)*(x*(k*x - 1)*(x - 1))^(2/3)),x)
[Out] -int((k*x - 1)/((x*(k - 2) + 1)*(x*(k*x - 1)*(x - 1))^(2/3)), x)
```

3.46 $\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=493

$$\frac{(a+b)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{(a+b)\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}-\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{(a-c)\tan^{-1}\left(\cdots\right)}{\sqrt[3]{2}\,\cdots}$$

```
[Out] 1/24*(a+b)*ln((1-x)*(1+x)^2)*2^(2/3)-1/12*(a-c)*ln(x^3+1)*2^(2/3)-1/12*(b+c
)*ln(x^3+1)*2^(2/3)+1/12*(a+b)*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*
(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*(a+b)*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*
2^(2/3)+1/4*(b+c)*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/4*(a-c)*ln(-2^(1/3)*
x-(-x^3+1)^(1/3))*2^(2/3)+1/2*c*ln(x+(-x^3+1)^(1/3))-1/8*(a+b)*ln(-1+x+2^(2
/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*(a+b)*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1
)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*(a+b)*arctan(1/3*(1+2^(1/3)*(1-x)/(-
x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/3*c*arctan(1/3*(1-2*x/(-x^3+1)^(1/
3))*3^(1/2))*3^(1/2)-1/6*(a-c)*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^
(1/2))*2^(2/3)*3^(1/2)+1/6*(b+c)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1
/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 570, normalized size of antiderivative = 1.16, number of steps used = 19, number of rules used = 14, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {2183, 384, 502, 2174, 206, 31, 648, 631, 210, 642, 455, 57, 494, 245}

$$\frac{(a+b)\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{(a+b)\log\left(\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{(a+b)\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{(a+b)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{(a+b)\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}+\frac{(a+b)\log\left((1-x)(x+1)^2\right)}{12\sqrt[3]{2}}-\frac{a\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{a\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\sqrt[3]{2}}-\frac{a\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}-\frac{(b+c)\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{(b+c)\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{(b+c)\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{c\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{c\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\sqrt[3]{2}}+\frac{1}{2}c\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] ((a + b)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/3
)*Sqrt[3]) + ((a + b)*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3
]])/(2*2^(1/3)*Sqrt[3]) - (c*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/S
qrt[3] - (a*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*S
qrt[3]) + (c*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*
Sqrt[3]) + ((b + c)*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/3)
*Sqrt[3]) + ((a + b)*Log[(1 - x)*(1 + x)^2])/(12*2^(1/3)) - (a*Log[1 + x^3]
)/(6*2^(1/3)) + (c*Log[1 + x^3])/(6*2^(1/3)) - ((b + c)*Log[1 + x^3])/(6*2^
(1/3)) + ((a + b)*Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1
 - x))/(1 - x^3)^(1/3)])/(6*2^(1/3)) - ((a + b)*Log[1 + (2^(1/3)*(1 - x))/(
1 - x^3)^(1/3)])/(3*2^(1/3)) + ((b + c)*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*
2^(1/3)) + (a*Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)])/(2*2^(1/3)) - (c*Log[-(2
```

```
^(1/3)*x) - (1 - x^3)^(1/3)])/(2*2^(1/3)) + (c*Log[x + (1 - x^3)^(1/3)])/2
- ((a + b)*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))]/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
```

```
a*d^3, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \left(\frac{c}{\sqrt[3]{1-x^3}} + \frac{a-c+(b+c)x}{(1-x+x^2)\sqrt[3]{1-x^3}}\right)dx$$

$$= c\int \frac{1}{\sqrt[3]{1-x^3}}\,dx + \int \frac{a-c+(b+c)x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$$

$$= -\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}c\log\left(x+\sqrt[3]{1-x^3}\right) + \int \left(\frac{b-\frac{i(2a+b-}{\sqrt{3}}}{\left(-1-i\sqrt{3}+2x\right.}\right.$$

$$= -\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}c\log\left(x+\sqrt[3]{1-x^3}\right) + \frac{1}{3}\left(3b-i\sqrt{3}(2a+b-\right.$$

$$= -\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\left(2a+b-i\sqrt{3}b-c-i\sqrt{3}c\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}(\ldots}{\sqrt[3]{\ldots}}}{2\ldots}\right.}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)}$$

Mathematica [F]
time = 10.24, size = 0, normalized size = 0.00

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)), x]
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{c\,x^{2}+b x +a}{\left(x^{2}-x +1\right) \left(-x^{3}+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)
```

```
[Out] int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + cx^2}{\sqrt[3]{-\left(x - 1\right)\left(x^2 + x + 1\right)}\,\left(x^2 - x + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(x**2-x+1)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral((a + b*x + c*x**2)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1
)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,x^2 + b\,x + a}{{\left(1 - x^3\right)}^{1/3}\,\left(x^2 - x + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((1 - x^3)^(1/3)*(x^2 - x + 1)),x)
```

```
[Out] int((a + b*x + c*x^2)/((1 - x^3)^(1/3)*(x^2 - x + 1)), x)
```

3.47 $\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx$

Optimal. Leaf size=407

$$-\frac{19255}{395136(3-2x)^{9/2}}-\frac{462025}{30118144(3-2x)^{7/2}}-\frac{38491}{8605184(3-2x)^{5/2}}-\frac{141045}{120472576(3-2x)^{3/2}}-\frac{38225}{240945152\sqrt{3-}}$$

[Out] -19255/395136/(3-2*x)^(9/2)-462025/30118144/(3-2*x)^(7/2)-38491/8605184/(3-2*x)^(5/2)-141045/120472576/(3-2*x)^(3/2)+1/28*x/(3-2*x)^(9/2)/(2*x^2+x+1)^4+1/1176*(23+73*x)/(3-2*x)^(9/2)/(2*x^2+x+1)^3+1/32928*(1387+3049*x)/(3-2*x)^(9/2)/(2*x^2+x+1)^2+5/153664*(3049+4377*x)/(3-2*x)^(9/2)/(2*x^2+x+1)-38225/240945152/(3-2*x)^(1/2)+5/13492928512*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(-298093007954+81630132224*14^(1/2))^(1/2)-5/13492928512*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(-298093007954+81630132224*14^(1/2))^(1/2)+5/6746464256*arctan((-2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(298093007954+81630132224*14^(1/2))^(1/2)-5/6746464256*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(298093007954+81630132224*14^(1/2))^(1/2)

Rubi [A]
time = 0.42, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {754, 836, 842, 840, 1183, 648, 632, 210, 642}

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}+\frac{5(4377x+3049)}{153664(3-2x)^{9/2}\left(2x^2+x+1\right)}+\frac{3049x+1387}{32928(3-2x)^{9/2}\left(2x^2+x+1\right)^2}+\frac{73x+23}{1176(3-2x)^{9/2}\left(2x^2+x+1\right)^3}-\frac{38225}{240945152\sqrt{3-2x}}-\frac{141045}{120472576(3-2x)^{3/2}}-\frac{38491}{8605184(3-2x)^{5/2}}-\frac{462025}{30118144(3-2x)^{7/2}}-\frac{19255}{395136(3-2x)^{9/2}}+\frac{5\sqrt{\frac{1}{2}\left(40815066112\sqrt{14}-149046503977\right)}\log\left(-2x-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3\right)}{6746464256}-\frac{5\sqrt{\frac{1}{2}\left(40815066112\sqrt{14}-149046503977\right)}\log\left(-2x+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3\right)}{6746464256}+\frac{5\sqrt{\frac{1}{2}\left(149046503977+40815066112\sqrt{14}\right)}\tan^{-1}\left(\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{\sqrt{2\sqrt{14}-7}}\right)}{3373232128}-\frac{5\sqrt{\frac{1}{2}\left(149046503977+40815066112\sqrt{14}\right)}\tan^{-1}\left(\frac{2\sqrt{3-2x}+\sqrt{7+2\sqrt{14}}}{\sqrt{2\sqrt{14}-7}}\right)}{3373232128}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - 2*x)^(11/2)*(1 + x + 2*x^2)^5),x]

[Out] -19255/(395136*(3 - 2*x)^(9/2)) - 462025/(30118144*(3 - 2*x)^(7/2)) - 38491/(8605184*(3 - 2*x)^(5/2)) - 141045/(120472576*(3 - 2*x)^(3/2)) - 38225/(240945152*Sqrt[3 - 2*x]) + x/(28*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^4) + (23 + 73*x)/(1176*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^3) + (1387 + 3049*x)/(32928*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^2) + (5*(3049 + 4377*x))/(153664*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)) + (5*Sqrt[(149046503977 + 40815066112*Sqrt[14])/2]*ArcTan[(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/3373232128 - (5*Sqrt[(149046503977 + 40815066112*Sqrt[14])/2]*ArcTan[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/3373232128 + (5*Sqrt[(-149046503977 + 40815066112*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/6746464256 - (5*Sqrt[(-149046503977 + 40815066112*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/6746464256

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
```

```
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 842

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)
^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx = \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{1}{784}\int \frac{700-644x}{(3-2x)^{11/2}\left(1+x+2x^2\right)^4}\,dx$$

$$= \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{\int \frac{32}{(3-2}}{}$$

$$= \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{}{32928}$$

$$= \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{}{32928}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} + \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+7}{1176(3-2x)^{9/2}(}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} + \frac{x}{28(3-2x)^{9/2}(1+x+2}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} + \frac{}{28}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{}{12}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{}{12}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{}{12}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{}{12}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{}{12}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{}{12}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{}{12}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.10, size = 178, normalized size = 0.44

$$\frac{-\frac{14(40289347-429812744x+135202154x^2-1073855156x^3+1627773523x^4-1470758860x^5+2888625656x^6-3106712560x^7+2343370048x^8-2443779648x^9+1873554048x^{10}-677249280x^{11}+88070400x^{12})}{(3-2x)^{9/2}(1+x+2x^2)^4}-45\sqrt{149046503977+12577271771i\sqrt{7}}\tan^{-1}\left(\frac{1}{2}\sqrt{-1-\frac{i}{\sqrt{7}}}\sqrt{3-2x}\right)-45\sqrt{149046503977-12577271771i\sqrt{7}}\tan^{-1}\left(\frac{1}{2}\sqrt{-1+\frac{i}{\sqrt{7}}}\sqrt{3-2x}\right)}{30359089152}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] ((-14*(40289347 - 429812744*x + 135202154*x^2 - 1073855156*x^3 + 1627773523
*x^4 - 1470758860*x^5 + 2888625656*x^6 - 3106712560*x^7 + 2343370048*x^8 -
2443779648*x^9 + 1873554048*x^10 - 677249280*x^11 + 88070400*x^12))/((3 - 2
*x)^(9/2)*(1 + x + 2*x^2)^4) - 45*Sqrt[149046503977 + (12577271771*I)*Sqrt[
7]]*ArcTan[(Sqrt[-1 - I/Sqrt[7]]*Sqrt[3 - 2*x])/2] - 45*Sqrt[149046503977 -
 (12577271771*I)*Sqrt[7]]*ArcTan[(Sqrt[-1 + I/Sqrt[7]]*Sqrt[3 - 2*x])/2])/3
0359089152
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 + x + 2*x^2)^5*(3 - 2*x)^(11/2)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.89, size = 415, normalized size = 1.02

method	result
derivativedivides	$\frac{1}{151263(3-2x)^{\frac{9}{2}}}+\frac{5}{235298(3-2x)^{\frac{7}{2}}}+\frac{19}{470596(3-2x)^{\frac{5}{2}}}+\frac{185}{2823576(3-2x)^{\frac{3}{2}}}+\frac{505}{3294172\sqrt{3-2x}}+\frac{56765162\cdots}{\cdots}$
default	$\frac{1}{151263(3-2x)^{\frac{9}{2}}}+\frac{5}{235298(3-2x)^{\frac{7}{2}}}+\frac{19}{470596(3-2x)^{\frac{5}{2}}}+\frac{185}{2823576(3-2x)^{\frac{3}{2}}}+\frac{505}{3294172\sqrt{3-2x}}+\frac{56765162\cdots}{\cdots}$
trager	Expression too large to display
risch	$-\frac{88070400x^{12}-677249280x^{11}+1873554048x^{10}-2443779648x^{9}+2343370048x^{8}-3106712560x^{7}+2888625656x^{6}-1470\cdots}{2168506368(2x^2+x+1)^4\sqrt{3-2x}\,(2\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/151263/(3-2*x)^(9/2)+5/235298/(3-2*x)^(7/2)+19/470596/(3-2*x)^(5/2)+185/2
823576/(3-2*x)^(3/2)+505/3294172/(3-2*x)^(1/2)+1/6588344*(567651623/32*(3-2
*x)^(1/2)-6194606411/192*(3-2*x)^(3/2)+9801432515/384*(3-2*x)^(5/2)-8763772
549/768*(3-2*x)^(7/2)+149630663/48*(3-2*x)^(9/2)-200063633/384*(3-2*x)^(11/
2)+18969965/384*(3-2*x)^(13/2)-526135/256*(3-2*x)^(15/2))/((3-2*x)^2-7+14*x
)^4+5/13492928512*(-146319*(7+2*14^(1/2))^(1/2)*14^(1/2)+569986*(7+2*14^(1/
2))^(1/2))*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))+5/33732321
28*(-115739*14^(1/2)+1/2*(-146319*(7+2*14^(1/2))^(1/2)*14^(1/2)+569986*(7+2
*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-
2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))+5/13492928512*(1463
19*(7+2*14^(1/2))^(1/2)*14^(1/2)-569986*(7+2*14^(1/2))^(1/2))*ln(3-2*x+14^(
1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))+5/3373232128*(-115739*14^(1/2)-1/2
*(146319*(7+2*14^(1/2))^(1/2)*14^(1/2)-569986*(7+2*14^(1/2))^(1/2))*(7+2*14
^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))
^(1/2))/(-7+2*14^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^5*(-2*x + 3)^(11/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(298) = 596$.
time = 0.33, size = 957, normalized size = 2.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/852282865707923134247251378176*(2263908918780*22241759018113166^(1/4)*sqr
t(79716926)*sqrt(14)*sqrt(7)*(512*x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10
+ 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3
 - 1242*x^2 - 162*x - 243)*sqrt(21292357711*sqrt(14) + 81630132224)*arctan(
1/10052187156951869469526908685753437228729401815040*22241759018113166^(3/4
)*sqrt(12577271771)*sqrt(79716926)*sqrt(-2089731384934400*22241759018113166
```

```
^(1/4)*sqrt(79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt(14) + 8163013222
4)*(7645*sqrt(14) - 115739) - 419041899350251499556867911188480 0*x + 209520
94967512574977843395559424 00*sqrt(14) + 628562849025377249335301866782720 0)
*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(21292357711*sqrt(14) + 816
30132224) - 1/195818453485129580290665890 2*22241759018113166^(3/4)*sqrt(797
16926)*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(-2*x + 3)*sqrt(21292
357711*sqrt(14) + 81630132224) - 2/7*sqrt(14)*sqrt(7) - sqrt(7)) + 22639089
18780*22241759018113166^(1/4)*sqrt(79716926)*sqrt(14)*sqrt(7)*(512*x^13 - 2
816*x^12 + 5632*x^11 - 5888*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^
6 + 4994*x^5 - 1707*x^4 + 936*x^3 - 1242*x^2 - 162*x - 243)*sqrt(2129235771
1*sqrt(14) + 81630132224)*arctan(1/24628619072593968384668700756050455442*2
2241759018113166^(3/4)*sqrt(12577271771)*sqrt(22241759018113166^(1/4)*sqrt(
79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt(14) + 81630132224)*(7645*sqr
t(14) - 115739) - 2005242886101391892*x + 1002621443050695946*sqrt(14) + 30
07864329152087838)*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(21292357
711*sqrt(14) + 81630132224) - 1/1958184534851295802906658902*22241759018113
166^(3/4)*sqrt(79716926)*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(-2
*x + 3)*sqrt(21292357711*sqrt(14) + 81630132224) + 2/7*sqrt(14)*sqrt(7) + s
qrt(7)) + 315*22241759018113166^(1/4)*sqrt(79716926)*(41794627698688*x^13 -
 229870452342784*x^12 + 459740904685568*x^11 - 480638218534912*x^10 + 55900
3145469952*x^9 - 734018148958208*x^8 + 498923368153088*x^7 - 34611176062976
0*x^6 + 407660880326656*x^5 - 139342635706368*x^4 + 76405803761664*x^3 - 10
1384624222208*x^2 - 21292357711*sqrt(14)*(512*x^13 - 2816*x^12 + 5632*x^11
 - 5888*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x
^4 + 936*x^3 - 1242*x^2 - 162*x - 243) - 13224081420288*x - 19836122130432)
*sqrt(21292357711*sqrt(14) + 81630132224)*log(2089731384934400/12577271771*
22241759018113166^(1/4)*sqrt(79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt
(14) + 81630132224)*(7645*sqrt(14) - 115739) - 333173924345386159308800*x +
 166586962172693079654400*sqrt(14) + 499760886518079238963200) - 315*222417
59018113166^(1/4)*sqrt(79716926)*(41794627698688*x^13 - 229870452342784*x^1
2 + 459740904685568*x^11 - 480638218534912*x^10 + 559003145469952*x^9 - 734
018148958208*x^8 + 498923368153088*x^7 - 346111760629760*x^6 + 407660880326
656*x^5 - 139342635706368*x^4 + 76405803761664*x^3 - 101384624222208*x^2 -
21292357711*sqrt(14)*(512*x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10 + 6848*x
^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3 - 1242*
x^2 - 162*x - 243) - 13224081420288*x - 19836122130432)*sqrt(21292357711*sq
rt(14) + 81630132224)*log(-2089731384934400/12577271771*22241759018113166^(
1/4)*sqrt(79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt(14) + 81630132224)
*(7645*sqrt(14) - 115739) - 333173924345386159308800*x + 166586962172693079
654400*sqrt(14) + 499760886518079238963200) + 393027605675872810832*(880704
00*x^12 - 677249280*x^11 + 1873554048*x^10 - 2443779648*x^9 + 2343370048*x^
8 - 3106712560*x^7 + 2888625656*x^6 - 1470758860*x^5 + 1627773523*x^4 - 107
3855156*x^3 + 135202154*x^2 - 429812744*x + 40289347)*sqrt(-2*x + 3))/(512*
x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 -
 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3 - 1242*x^2 - 162*x - 243)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(11/2)/(2*x**2+x+1)**5,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 795 vs. 2(298) = 596.
time = 0.27, size = 1277, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x)
```

```
[Out] -5/1511207993344*sqrt(7)*(22935*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-2*sqr
t(14) + 8) + 7645*14^(3/4)*sqrt(7)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) + 8) + 5
3515*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 160545*14^(3/4)*sqrt(2*
sqrt(14) + 8)*(sqrt(14) - 4) + 925912*14^(1/4)*sqrt(7)*sqrt(-2*sqrt(14) + 8
) + 6481384*14^(1/4)*sqrt(2*sqrt(14) + 8))*arctan(1/28*14^(3/4)*(14^(1/4)*s
qrt(1/2)*sqrt(sqrt(14) + 4) + 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2))
- 5/1511207993344*sqrt(7)*(22935*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-2*sq
rt(14) + 8) + 7645*14^(3/4)*sqrt(7)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) + 8) +
53515*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 160545*14^(3/4)*sqrt(2
*sqrt(14) + 8)*(sqrt(14) - 4) + 925912*14^(1/4)*sqrt(7)*sqrt(-2*sqrt(14) +
8) + 6481384*14^(1/4)*sqrt(2*sqrt(14) + 8))*arctan(-1/28*14^(3/4)*(14^(1/4)
*sqrt(1/2)*sqrt(sqrt(14) + 4) - 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2)
) - 5/3022415986688*sqrt(7)*(7645*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sq
rt(14) + 4) + 22935*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) -
160545*14^(3/4)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) + 8) - 53515*14^(3/4)*(sqrt
(14) - 4)*sqrt(-2*sqrt(14) + 8) + 925912*14^(1/4)*sqrt(7)*sqrt(2*sqrt(14) +
 8) - 6481384*14^(1/4)*sqrt(-2*sqrt(14) + 8))*log(14^(1/4)*sqrt(1/2)*sqrt(-
2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 5/3022415986688*sqrt(7)
*(7645*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 22935*14^(3/4
)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) - 160545*14^(3/4)*(sqrt(14) +
 4)*sqrt(-2*sqrt(14) + 8) - 53515*14^(3/4)*(sqrt(14) - 4)*sqrt(-2*sqrt(14)
+ 8) + 925912*14^(1/4)*sqrt(7)*sqrt(2*sqrt(14) + 8) - 6481384*14^(1/4)*sqrt
(-2*sqrt(14) + 8))*log(-14^(1/4)*sqrt(1/2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4
) - 2*x + sqrt(14) + 3) + 1/5059848192*(1578405*(2*x - 3)^7*sqrt(-2*x + 3)
+ 37939930*(2*x - 3)^6*sqrt(-2*x + 3) + 400127266*(2*x - 3)^5*sqrt(-2*x + 3
) + 2394090608*(2*x - 3)^4*sqrt(-2*x + 3) + 8763772549*(2*x - 3)^3*sqrt(-2*
x + 3) + 19602865030*(2*x - 3)^2*sqrt(-2*x + 3) - 24778425644*(-2*x + 3)^(3
```

```
/2) + 13623638952*sqrt(-2*x + 3))/((2*x - 3)^2 + 14*x - 7)^4 + 1/59295096*(
9090*(2*x - 3)^4 - 3885*(2*x - 3)^3 + 2394*(2*x - 3)^2 - 2520*x + 4172)/((2
*x - 3)^4*sqrt(-2*x + 3))
```

Mupad [B]
time = 0.47, size = 343, normalized size = 0.84

$$-\frac{\frac{272\,x}{441}-\frac{164\,(2\,x-3)^2}{441}+\frac{1966\,(2\,x-3)^3}{3087}-\frac{9091\,(2\,x-3)^4}{3087}-\frac{32070727\,(2\,x-3)^5}{5531904}-\frac{41014777\,(2\,x-3)^6}{11063808}-\frac{141921511\,(2\,x-3)^7}{154893312}+\frac{23262655\,(2\,x-3)^8}{309786624}+\frac{1571659\,(2\,x-3)^9}{15059072}+\frac{468427\,(2\,x-3)^{10}}{17210368}+\frac{394105\,(2\,x-3)^{11}}{120472576}+\frac{38225\,(2\,x-3)^{12}}{240945152}-\frac{520}{441}}{38416\,(3-2\,x)^{9/2}-76832\,(3-2\,x)^{11/2}+68600\,(3-2\,x)^{13/2}-35672\,(3-2\,x)^{15/2}+11809\,(3-2\,x)^{17/2}-2548\,(3-2\,x)^{19/2}+350\,(3-2\,x)^{21/2}-28\,(3-2\,x)^{23/2}+(3-2\,x)^{25/2}}+\frac{\mathrm{atan}\left(\frac{\sqrt{3-2\,x}\,\sqrt{-149046503977+\sqrt{7}\,12577271771\mathrm{i}}\,1572158971375\mathrm{i}}{391663056253676053933850624\,\left(-\frac{230036728532618625}{27975932589548289566703616}+\frac{\sqrt{7}\,181960107187971125\mathrm{i}}{19583152812683802696925312}\right)}-\frac{1572158971375\,\sqrt{7}\,\sqrt{3-2\,x}\,\sqrt{-149046503977+\sqrt{7}\,12577271771\mathrm{i}}}{391663056253676053933850624\,\left(-\frac{230036728532618625}{27975932589548289566703616}+\frac{\sqrt{7}\,181960107187971125\mathrm{i}}{19583152812683802696925312}\right)}\right)\,\sqrt{-149046503977+\sqrt{7}\,12577271771\mathrm{i}}\,5\mathrm{i}}{3373232128}-\frac{\mathrm{atan}\left(\frac{\sqrt{3-2\,x}\,\sqrt{-149046503977-\sqrt{7}\,12577271771\mathrm{i}}\,1572158971375\mathrm{i}}{391663056253676053933850624\,\left(\frac{230036728532618625}{27975932589548289566703616}+\frac{\sqrt{7}\,181960107187971125\mathrm{i}}{19583152812683802696925312}\right)}+\frac{1572158971375\,\sqrt{7}\,\sqrt{3-2\,x}\,\sqrt{-149046503977-\sqrt{7}\,12577271771\mathrm{i}}}{391663056253676053933850624\,\left(\frac{230036728532618625}{27975932589548289566703616}+\frac{\sqrt{7}\,181960107187971125\mathrm{i}}{19583152812683802696925312}\right)}\right)\,\sqrt{-149046503977-\sqrt{7}\,12577271771\mathrm{i}}\,5\mathrm{i}}{3373232128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - 2*x)^(11/2)*(x + 2*x^2 + 1)^5),x)
```

```
[Out] (atan(((3 - 2*x)^(1/2)*(7^(1/2)*12577271771i - 149046503977)^(1/2)*15721589
71375i)/(391663056253676053933850624*((7^(1/2)*181960107187971125i)/1958315
2812683802696925312 - 230036728532618625/27975932589548289566703616)) - (1
572158971375*7^(1/2)*(3 - 2*x)^(1/2)*(7^(1/2)*12577271771i - 149046503977)^
(1/2))/(391663056253676053933850624*((7^(1/2)*181960107187971125i)/19583152
812683802696925312 - 230036728532618625/27975932589548289566703616)))*(7^(
1/2)*12577271771i - 149046503977)^(1/2)*5i)/3373232128 - ((272*x)/441 - (16
4*(2*x - 3)^2)/441 + (1966*(2*x - 3)^3)/3087 - (9091*(2*x - 3)^4)/3087 - (3
2070727*(2*x - 3)^5)/5531904 - (41014777*(2*x - 3)^6)/11063808 - (141921511
*(2*x - 3)^7)/154893312 + (23262655*(2*x - 3)^8)/309786624 + (1571659*(2*x
- 3)^9)/15059072 + (468427*(2*x - 3)^10)/17210368 + (394105*(2*x - 3)^11)/1
20472576 + (38225*(2*x - 3)^12)/240945152 - 520/441)/(38416*(3 - 2*x)^(9/2)
 - 76832*(3 - 2*x)^(11/2) + 68600*(3 - 2*x)^(13/2) - 35672*(3 - 2*x)^(15/2)
 + 11809*(3 - 2*x)^(17/2) - 2548*(3 - 2*x)^(19/2) + 350*(3 - 2*x)^(21/2) -
28*(3 - 2*x)^(23/2) + (3 - 2*x)^(25/2)) - (atan(((3 - 2*x)^(1/2)*(- 7^(1/2)
*12577271771i - 149046503977)^(1/2)*1572158971375i)/(391663056253676053933
850624*((7^(1/2)*181960107187971125i)/19583152812683802696925312 + 23003672
8532618625/27975932589548289566703616)) + (1572158971375*7^(1/2)*(3 - 2*x)^
(1/2)*(- 7^(1/2)*12577271771i - 149046503977)^(1/2))/(391663056253676053933
850624*((7^(1/2)*181960107187971125i)/19583152812683802696925312 + 2300367
28532618625/27975932589548289566703616)))*(- 7^(1/2)*12577271771i - 1490465
03977)^(1/2)*5i)/3373232128
```

3.48 $\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx$

Optimal. Leaf size=648

$$\frac{4718120139975}{351733660450816(3-2x)^{19/2}}-\frac{815900548375}{629418129227776(3-2x)^{17/2}}-\frac{3029508823715}{1555033025150976(3-2x)^{15/2}}-\frac{135}{1347695288}$$

```
[Out] 4718120139975/351733660450816/(3-2*x)^(19/2)-815900548375/629418129227776/(
3-2*x)^(17/2)-3029508823715/1555033025150976/(3-2*x)^(15/2)-13515743021825/
13476952884641792/(3-2*x)^(13/2)-5846828446875/14513641568075776/(3-2*x)^(1
1/2)-37283626871975/261245548225363968/(3-2*x)^(9/2)-132355162272575/284467
3747342852096/(3-2*x)^(7/2)-11557581705725/812763927812243456/(3-2*x)^(5/2)
-46601678385075/11378694989371408384/(3-2*x)^(3/2)+1/63*x/(3-2*x)^(19/2)/(2
*x^2+x+1)^9+1/7056*(53+173*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^8+1/691488*(8477+2
1409*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^7+5/6453888*(21409+47471*x)/(3-2*x)^(19/
2)/(2*x^2+x+1)^6+41/90354432*(47471+92875*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^5+4
1/5059848192*(3436375+5677637*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^4+451/101196963
84*(811091+998691*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^3+451/283351498752*(2896203
9+14627273*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^2+11275/3966920982528*(14627273-35
058731*x)/(3-2*x)^(19/2)/(2*x^2+x+1)-24229218097975/22757389978742816768/(3
-2*x)^(1/2)+11275/127441383880959773 9008*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7
+2*14^(1/2))^(1/2))*(9756589235-2148932869*14^(1/2))*(-14+4*14^(1/2))^(1/2)
-11275/1274413838809597739008*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2)
)^(1/2))*(9756589235-2148932869*14^(1/2))*(-14+4*14^(1/2))^(1/2)+11275/6372
069194047988695 04*arctan((-2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(
1/2))^(1/2))*(9756589235+2148932869*14^(1/2))*(14+4*14^(1/2))^(1/2)-11275/6
372069194047988695 04*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14
^(1/2))^(1/2))*(9756589235+2148932869*14^(1/2))*(14+4*14^(1/2))^(1/2)
```

Rubi [A]
time = 0.78, antiderivative size = 648, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {754, 836, 842, 840, 1183, 648, 632, 210, 642}

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] 4718120139975/(351733660450816*(3 - 2*x)^(19/2)) - 815900548375/(6294181292
27776*(3 - 2*x)^(17/2)) - 3029508823715/(1555033025150976*(3 - 2*x)^(15/2))
 - 13515743021825/(13476952884641792*(3 - 2*x)^(13/2)) - 5846828446875/(145
```

```
13641568075776*(3 - 2*x)^(11/2)) - 37283626871975/(261245548225363968*(3 -
2*x)^(9/2)) - 132355162272575/(2844673747342852096*(3 - 2*x)^(7/2)) - 11557
581705725/(812763927812243456*(3 - 2*x)^(5/2)) - 46601678385075/(1137869498
9371408384*(3 - 2*x)^(3/2)) - 24229218097975/(22757389978742816768*Sqrt[3 -
 2*x]) + x/(63*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) + (53 + 173*x)/(7056*(3
- 2*x)^(19/2)*(1 + x + 2*x^2)^8) + (8477 + 21409*x)/(691488*(3 - 2*x)^(19/2
)*(1 + x + 2*x^2)^7) + (5*(21409 + 47471*x))/(6453888*(3 - 2*x)^(19/2)*(1 +
 x + 2*x^2)^6) + (41*(47471 + 92875*x))/(90354432*(3 - 2*x)^(19/2)*(1 + x +
 2*x^2)^5) + (41*(3436375 + 5677637*x))/(5059848192*(3 - 2*x)^(19/2)*(1 + x
 + 2*x^2)^4) + (451*(811091 + 998691*x))/(10119696384*(3 - 2*x)^(19/2)*(1 +
 x + 2*x^2)^3) + (451*(28962039 + 14627273*x))/(283351498752*(3 - 2*x)^(19/
2)*(1 + x + 2*x^2)^2) + (11275*(14627273 - 35058731*x))/(3966920982528*(3 -
 2*x)^(19/2)*(1 + x + 2*x^2)) + (11275*Sqrt[(7 + 2*Sqrt[14])/2]*(9756589235
 + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqr
t[-7 + 2*Sqrt[14]]])/318603459702399434752 - (11275*Sqrt[(7 + 2*Sqrt[14])/2
]*(9756589235 + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[
3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/318603459702399434752 + (11275*(975658923
5 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[
7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404798869504 - (11275*(97565
89235 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + S
qrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404798869504
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 842

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)
^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
```

```
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{\int \frac{1680-1484x}{(3-2x)^{21/2}\left(1+x+2x^2\right)^9}\,dx}{1764}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{\int \cdots}{(3\cdots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{\cdots}{691\cdots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{\cdots}{691\cdots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{\cdots}{691\cdots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{\cdots}{691\cdots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{\cdots}{691\cdots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{\cdots}{691\cdots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{\cdots}{691\cdots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} + \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{\cdots}{7056(\cdots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} + \frac{\cdots}{63(3\cdots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{\cdots}{1555\cdots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{\cdots}{1555\cdots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{\cdots}{1555\cdots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{\cdots}{1555\cdots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{\cdots}{1555\cdots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{\cdots}{1555\cdots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{\cdots}{1555\cdots}$$

$$4718120139975 \qquad 815900548375$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.85, size = 253, normalized size = 0.39

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] ((14*(-4884417100172357749737 + 20570245201454032279728 9*x + 111926768697602999806116*x^2 + 1362587089603925431664856*x^3 - 8099903620950442100549 58*x^4 + 36733030582778222253869 26*x^5 - 8685973988079840377705700*x^6 + 107181317259168931515550 68*x^7 - 2724660425107668955204 3953*x^8 + 41613884937255303086792337*x^9 - 5979110268149411757214917 6*x^10 + 10203157363431783454797 6132*x^11 - 1333125413772463861158902 40*x^12 + 17264969229461496927416889 6*x^13 - 2294081329841665219771663 36*x^14 + 25881925681516324984544793 6*x^15 - 2826446645399948270310067 20*x^16 + 30401059101096681115595520 0*x^17 - 287279159180291305208156160*x^18 + 253788172995391086570485760*x^19 - 216634228326470609547509760*x^20 + 162290307223249502039654400*x^21 - 106701725825102321939251200*x^22 + 65360120291258796757811200*x^23 - 33969890064381284111155200*x^24 + 12365045055896811105484800*x^25 - 2621948941596237063782400*x^26 + 240031204937714427494400*x^27))/((3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) - 426093525*Sqrt[229300295369923682239 3 + (30540258843957888971*I)*Sqrt[7]]*ArcTan[(Sqrt[-1 - I/Sqrt[7]]*Sqrt[3 - 2*x])/2] - 426093525*Sqrt[2293002953699236822393 - (30540258843957888971*I)*Sqrt[7]]*ArcTan[(Sqrt[-1 + I/Sqrt[7]]*Sqrt[3 - 2*x])/2])/12040343345613377038712832
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 + x + 2*x^2)^10*(3 - 2*x)^(21/2)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.91, size = 550, normalized size = 0.85

method	result
derivativedivides	$\frac{1}{5367029731(3-2x)^{\frac{19}{2}}}+\frac{5}{4802079233(3-2x)^{\frac{17}{2}}}+\frac{73}{23727920916(3-2x)^{\frac{15}{2}}}+\frac{165}{25705247659(3-2x)^{\frac{13}{2}}}+\frac{\cdots}{221460595\cdots}$

default	$\frac{1}{5367029731(3-2x)^{\frac{19}{2}}}+\frac{5}{4802079233(3-2x)^{\frac{17}{2}}}+\frac{73}{23727920916(3-2x)^{\frac{15}{2}}}+\frac{165}{25705247659(3-2x)^{\frac{13}{2}}}+\frac{23}{2214605952}$
trager	Expression too large to display
risch	$-\frac{2400312049377144274944 00x^{27}-26219489415962370637824 00x^{26}+12365045055896811105484800x^{25}-339698900643}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5367029731/(3-2*x)^(19/2)+5/4802079233/(3-2*x)^(17/2)+73/23727920916/(3-2
*x)^(15/2)+165/25705247659/(3-2*x)^(13/2)+2365/221460595216/(3-2*x)^(11/2)+
30349/1993145356944/(3-2*x)^(9/2)+854095/43406276662336/(3-2*x)^(7/2)+75933
/3100448333024/(3-2*x)^(5/2)+8519225/260437659974016/(3-2*x)^(3/2)+891605/1
2401793332096/(3-2*x)^(1/2)+1/86812553324672*(-165574989211387894481/65536*
(3-2*x)^(23/2)+45406001689183688581/131072*(3-2*x)^(25/2)-43462358811134257
841/1179648*(3-2*x)^(27/2)+192384852501874197/65536*(3-2*x)^(29/2)-13528410
99712333/8192*(3-2*x)^(31/2)+4606702222670185/786432*(3-2*x)^(33/2)-2586532
0405815/262144*(3-2*x)^(35/2)+544765170330150812273/1024*(3-2*x)^(1/2)-3476
987783905860258979/1536*(3-2*x)^(3/2)+9364999706478908741137/2048*(3-2*x)^(
5/2)-23851905772903279054347/4096*(3-2*x)^(7/2)+192983613795383541041317/36
864*(3-2*x)^(9/2)-57758421475348449750643/16384*(3-2*x)^(11/2)+603330358695
84695411551/32768*(3-2*x)^(13/2)-149770885083493978040723/196608*(3-2*x)^(1
5/2)+66256899944582155696811/262144*(3-2*x)^(17/2)-17729978841543630405471/
262144*(3-2*x)^(19/2)+2869878271121283060373/196608*(3-2*x)^(21/2))/((3-2*x
)^2-7+14*x)^9+11275/1274413838809597739008*(18352320711*(7+2*14^(1/2))^(1/2
)*14^(1/2)-69111417106*(7+2*14^(1/2))^(1/2))*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2
)*(7+2*14^(1/2))^(1/2))+11275/318603459702399434752*(-9756589235*14^(1/2)-1
/2*(18352320711*(7+2*14^(1/2))^(1/2)*14^(1/2)-69111417106*(7+2*14^(1/2))^(1
/2))*(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7
+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))-11275/1274413838809597739008*(18
352320711*(7+2*14^(1/2))^(1/2)*14^(1/2)-69111417106*(7+2*14^(1/2))^(1/2))*l
n(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))-11275/3186034597023994
34752*(9756589235*14^(1/2)+1/2*(18352320711*(7+2*14^(1/2))^(1/2)*14^(1/2)-6
9111417106*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2
)*arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^10*(-2*x + 3)^(21/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1563 vs. $2(491) = 982$.
time = 0.36, size = 1563, normalized size = 2.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="fricas")
```

```
[Out] 1/1094755373086200603246995644663447631605361478665641987670016*(4732002380
085251586622550100*478793617507582534294314731468 6^(1/4)*sqrt(1169607525756
986)*sqrt(14)*sqrt(7)*(524288*x^28 - 5505024*x^27 + 24772608*x^26 - 6468403
2*x^25 + 119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 386777088*x^21
+ 449261568*x^20 - 515594240*x^19 + 540503040*x^18 - 496581120*x^17 + 46771
2000*x^16 - 411828480*x^15 + 303534720*x^14 - 248434368*x^13 + 186495624*x^
12 - 105219828*x^11 + 83621482*x^10 - 49793667*x^9 + 19105065*x^8 - 2003648
4*x^7 + 5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 + 826686*x^2 +
 137781*x + 59049)*sqrt(32757185052846240319 9*sqrt(14) + 122642238092815735
1936)*arctan(1/3656217085193197024885534011338703535441745724187062686602 49
4537948900883272531121 9252*478793617507582534294314731468 6^(3/4)*sqrt(27763
87167632535361)*sqrt(12865682783326846)*sqrt(1169607525756986)*sqrt(4787936
17507582534294314731468 6^(1/4)*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(3
275718505284624031 99*sqrt(14) + 1226422380928157351936)*(2148932869*sqrt(14
) - 9756589235) - 714402331649189922096968266312 02812*x + 28280279689505005
187146*sqrt(22335021272086100802556094) + 1071603497473784883145452399468 04
218)*(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))*sqrt(3275718505284
62403199*sqrt(14) + 1226422380928157351936) - 1/10235736708061576766691001 4
425822844132744790009674 2*478793617507582534294314731468 6^(3/4)*sqrt(116960
7525756986)*(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))*sqrt(-2*x +
 3)*sqrt(327571850528462403199*sqrt(14) + 1226422380928157351936) + 2/7*sqr
t(14)*sqrt(7) + sqrt(7)) + 4732002380085251586622550100*478793617507582534 2
943147314686^(1/4)*sqrt(1169607525756986)*sqrt(14)*sqrt(7)*(524288*x^28 - 5
505024*x^27 + 24772608*x^26 - 64684032*x^25 + 119734272*x^24 - 194052096*x^
23 + 295206912*x^22 - 386777088*x^21 + 449261568*x^20 - 515594240*x^19 + 54
0503040*x^18 - 496581120*x^17 + 467712000*x^16 - 411828480*x^15 + 303534720
*x^14 - 248434368*x^13 + 186495624*x^12 - 105219828*x^11 + 83621482*x^10 -
49793667*x^9 + 19105065*x^8 - 20036484*x^7 + 5497632*x^6 - 2235114*x^5 + 32
76126*x^4 + 734832*x^3 + 826686*x^2 + 137781*x + 59049)*sqrt(32757185052846
2403199*sqrt(14) + 1226422380928157351936)*arctan(1/392966702348163030765 55
330542603297083388480635973027797585697454399143598928370335464344780800*47
879361750758253429431473146 86^(3/4)*sqrt(2776387167632535361)*sqrt(11696075
```

25756986)*sqrt(-148621074404098425452288907673600000*478793617507582534294314
47314686^(1/4)*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(32757185052846240
3199*sqrt(14) + 122642238092815735193 6)*(2148932869*sqrt(14) - 9756589235)
- 106175242086495654810909306149554239903819258556180943535846981632 0000*x
+ 420304555190263689316852795001664341102416628348354560000*sqrt(2233502127
2086100802556094) + 159262863129743482216363959224331359855728887834271415 3
037704724480000)*(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))*sqrt(3
27571850528462403199*sqrt(14) + 122642238092815735193 6) - 1/102357367080615
767666910014425822844132744790009674 2*4787936175075825342943147314686^(3/4)
sqrt(1169607525756986)(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))
*sqrt(-2*x + 3)*sqrt(327571850528462403199*sqrt(14) + 122642238092815735193
6) - 2/7*sqrt(14)*sqrt(7) - sqrt(7)) + 271150425*47879361750758253429431473
14686^(1/4)*sqrt(1169607525756986)*(642998537252061761731821568*x^28 - 6751
484641146648498184126464*x^27 + 30381680885159918241828569088*x^26 - 793299
445334731198536634859 52*x^25 + 146844790944939604835504750592*x^24 - 237989
833600419359560990457856*x^23 + 362048363881489025715123781632*x^22 - 47435
2077153419437787597242368*x^21 + 550984441886077267281495195648*x^20 - 6323
363154136437844718544486 40*x^19 + 662885025215707070319757885440*x^18 - 609
018199514371017360613048320*x^17 + 573612464628670331388690432000*x^16 - 50
5075664975624031448627937280*x^15 + 372261773996761581935835217920*x^14 - 3
046854691069420251327737364 48*x^13 + 228722407218762404519491928064*x^12 -
129043951976611196927641387008*x^11 + 102555257051181053298083889152*x^10 -
 61068067637283818105902989312*x^9 + 23430879305087206538965155840*x^8 - 24
573192412708929931548033024*x^7 + 6742418926906827559038615552*x^6 - 274119
3833525857491515080704*x^5 + 4017914249140640432768679936*x^4 + 90121441102
2199723237834752*x^3 + 1013866212399974688642564096*x^2 - 32757185052846240
3199*sqrt(14)*(524288*x^28 - 5505024*x^27 + 24772608*x^26 - 64684032*x^25 +
 119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 386777088*x^21 + 449261
568*x^20 - 515594240*x^19 + 540503040*x^18 - 496581120*x^17 + 467712000*x^1
6 - 411828480*x^15 + 303534720*x^14 - 248434368*x^13 + 186495624*x^12 - 105
219828*x^11 + 83621482*x^10 - 49793667*x^9 + 19105065*x^8 - 20036484*x^7 +
5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 + 826686*x^2 + 137781*
x + 59049) + 1689777020666624481070940 16*x + 7241901517142676347446886 4)*sq
rt(327571850528462403199*sqrt(14) + 122642238092815735193 6)*log(14862107440
4098425452288907673600000/27763871676325353 61*478793617507582534294314731468
6^(1/4)*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(327571850528462403199*sq
rt(14) + 122642238092815735193 6)*(2148932869*sqrt(14) - 9756589235) - 38242
231964006946013272086827269818478925709312 0000*x + 151385426388014656165701
481356328960000*sqrt(2233502127208610080255609 4) + 573633479460104190199081
302409047277183885639680000) - 271150425*4787936175075825342943147314686^(1
/4)*sqrt(1169607525756986)*(642998537252061761731821568*x^28 - 675148464114
6648498184126464*x^27 + 30381680885159918241828569088*x^26 - 79329944533473
119853663485952*x^25 + 146844790944939604835504750592*x^24 - 23798983360041
9359560990457856*x^23 + 362048363881489025715123781632*x^22 - 4743520771534
19437787597242368*x^21 + 550984441886077267281495195648*x^20 - 632336315413

643784471854448640*x^19 + 66288502521570707031975788544О*x^18 - 60901819951
4371017360613048320*x^17 + 573612464628670331388690432000*x^16 - 5050756649
756240314486279372 80*x^15 + 372261773996761581935835217920*x^14 - 304685469
10694202513277373644 8*x^13 + 228722407218762404519491928064*x^12 - 12904395
1976611196927641387008*x^11 + 102555257051181053298083889152*x^10 - 6106806
7637283818105902989312*x^9 + 234308793050872065389651558 40*x^8 - 2457319241
2708929931548033024*x^7 + 674241892690682755903861555 2*x^6 - 27411938335258
57491515080704*x^5 + 401791424914064043276867993 6*x^4 + 9012144110221997232
37834752*x^3 + 1013866212399974688642564096*x^2 - 3275718505284624031 99*sqr
t(14)*(524288*x^28 - 5505024*x^27 + 24772608*x^26 - 64684032*x^25 + 1197342
72*x^24 - 194052096*x^23 + 295206912*x^22 - 386777088*x^21 + 449261568*x^20
 - 515594240*x^19 + 540503040*x^18 - 496581120*x^17 + 467712000*x^16 - 4118
28480*x^15 + 303534720*x^14 - 248434368*x^13 + 186495624*x^12 - 105219828*x
^11 + 83621482*x^10 - 49793667*x^9 + 19105065*x^8 - 20036484*x^7 + 5497632*
x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 + 826686*x^2 + 137781*x + 5904
9) + 1689777020666624481070940 16*x + 724190151714267634744688 64)*sqrt(32757
1850528462403199*sqrt(14) + 1226422380928157351936)*log(-148621074404098425
45228890767360000/2776387167632535361*4787936175075825342943147314686^(1/4)
*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(327571850528462403199*sqrt(14)
+ 1226422380928157351936)*(2148932869*sqrt(14) - 9756589235) - 382422319640
06946013272086827269818478925709312000 0*x + 151385426388014656165701481356 3
28960000*sqrt(223350212720861008025560 94) + 5736334794601041901990813024090
47277183885639680000) + 127293506366582931573641618361052283 2*(240031204937
714427494400*x^27 - 2621948941596237063782400*x^26 + 123650450558968111054 8
4800*x^25 - 33969890064381284111155200*x^24 + 65360120291258796757811200*x^
23 - 106701725825102321939251200*x^22 + 162290307223249502039654400*x^21 -
216634228326470609547509760*x^20 + 253788172995391086570485760*x^19 - 28727
9159180291305208156160*x^18 + 304010591010966811155955200*x^17 - 2826446645
39994827031006720*x^16 + 258819256815163249845447936*x^15 - 229408132984166
521977166336*x^14 + 172649692294614969274168896*x^13 - 1333125413772463861 1
5890240*x^12 + 102031573634317834547976132*x^11 - 5979110268149411757214917
6*x^10 + 41613884937255303086792337*x^9 - 27246604251076689552043953*x^8 +
10718131725916893151555068*x^7 - 8685973988079840377705700*x^6 + 3673303058
277822225386926*x^5 - 8099903620950442100549 58*x^4 + 1362587089603925431664
856*x^3 + 1119267686976029998061 16*x^2 + 205702452014540322797289*x - 48844
17100172357749737)*sqrt(-2*x + 3))/(524288*x^28 - 5505024*x^27 + 24772608*x
^26 - 64684032*x^25 + 119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 38
6777088*x^21 + 449261568*x^20 - 515594240*x^19 + 540503040*x^18 - 496581120
*x^17 + 467712000*x^16 - 411828480*x^15 + 303534720*x^14 - 248434368*x^13 +
 186495624*x^12 - 105219828*x^11 + 83621482*x^10 - 49793667*x^9 + 19105065*
x^8 - 20036484*x^7 + 5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 +
 826686*x^2 + 137781*x + 59049)

Sympy [**F(-1)**]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(21/2)/(2*x**2+x+1)**10,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1000 vs. 2(491) = 982.
time = 0.41, size = 1534, normalized size = 2.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x)
```

```
[Out] -11275/142734349946674946768896*sqrt(7)*(6446798607*14^(3/4)*sqrt(7)*(sqrt(
14) + 4)*sqrt(-2*sqrt(14) + 8) + 2148932869*14^(3/4)*sqrt(7)*(sqrt(14) - 4)
*sqrt(-2*sqrt(14) + 8) - 15042530083*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14
) + 4) - 45127590249*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 7805271
3880*14^(1/4)*sqrt(7)*sqrt(-2*sqrt(14) + 8) - 546368997160*14^(1/4)*sqrt(2*
sqrt(14) + 8))*arctan(1/28*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4)
+ 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2)) - 11275/14273434994667494676
8896*sqrt(7)*(6446798607*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) +
 8) + 2148932869*14^(3/4)*sqrt(7)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) + 8) - 15
042530083*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) - 45127590249*14^(3/
4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 78052713880*14^(1/4)*sqrt(7)*sqrt(
-2*sqrt(14) + 8) - 546368997160*14^(1/4)*sqrt(2*sqrt(14) + 8))*arctan(-1/28
*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4) - 2*sqrt(-2*x + 3))/sqrt(-
1/8*sqrt(14) + 1/2)) - 11275/285468699893349893537792*sqrt(7)*(2148932869*1
4^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 6446798607*14^(3/4)*s
qrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 45127590249*14^(3/4)*(sqrt(14)
 + 4)*sqrt(-2*sqrt(14) + 8) + 15042530083*14^(3/4)*(sqrt(14) - 4)*sqrt(-2*s
qrt(14) + 8) + 78052713880*14^(1/4)*sqrt(7)*sqrt(2*sqrt(14) + 8) + 54636899
7160*14^(1/4)*sqrt(-2*sqrt(14) + 8))*log(14^(1/4)*sqrt(1/2)*sqrt(-2*x + 3)*
sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 11275/285468699893349893537792*s
qrt(7)*(2148932869*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 6
446798607*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 4512759024
9*14^(3/4)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) + 8) + 15042530083*14^(3/4)*(sqr
t(14) - 4)*sqrt(-2*sqrt(14) + 8) + 78052713880*14^(1/4)*sqrt(7)*sqrt(2*sqrt
(14) + 8) + 546368997160*14^(1/4)*sqrt(-2*sqrt(14) + 8))*log(-14^(1/4)*sqrt
(1/2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 1/204816509
808685350912*(232787883652335*(2*x - 3)^17*sqrt(-2*x + 3) + 138201066680105
55*(2*x - 3)^16*sqrt(-2*x + 3) + 389618236717151904*(2*x - 3)^15*sqrt(-2*x
```

```
+ 3) + 6925854690067471092*(2*x - 3)^14*sqrt(-2*x + 3) + 869247176222685156
82*(2*x - 3)^13*sqrt(-2*x + 3) + 817308030405306394458*(2*x - 3)^12*sqrt(-2
*x + 3) + 5960699611609964201316*(2*x - 3)^11*sqrt(-2*x + 3) + 344385392534
55396724476*(2*x - 3)^10*sqrt(-2*x + 3) + 159569809573892673649239*(2*x - 3
)^9*sqrt(-2*x + 3) + 596312099501239401271299*(2*x - 3)^8*sqrt(-2*x + 3) +
1797250621001927736488676*(2*x - 3)^7*sqrt(-2*x + 3) + 43439785826100980696
31672*(2*x - 3)^6*sqrt(-2*x + 3) + 8317212692450176764092592*(2*x - 3)^5*sq
rt(-2*x + 3) + 12350951282904546626644288*(2*x - 3)^4*sqrt(-2*x + 3) + 1373
8697725192288735303872*(2*x - 3)^3*sqrt(-2*x + 3) + 10788479661863702869789
824*(2*x - 3)^2*sqrt(-2*x + 3) - 5340653236079401357791744*(-2*x + 3)^(3/2)
 + 1255138952440667471476992*sqrt(-2*x + 3))/((2*x - 3)^2 + 14*x - 7)^9 + 1
/3280733202692679552*(235862511885*(2*x - 3)^9 - 107316677325*(2*x - 3)^8 +
 80348352084*(2*x - 3)^7 - 64554208290*(2*x - 3)^6 + 49954696792*(2*x - 3)^
5 - 35035280280*(2*x - 3)^4 + 21058773120*(2*x - 3)^3 - 10093321056*(2*x -
3)^2 + 6831901440*x - 10859127552)/((2*x - 3)^9*sqrt(-2*x + 3))
```

Mupad [B]
time = 0.56, size = 567, normalized size = 0.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - 2*x)^(21/2)*(x + 2*x^2 + 1)^10),x)
```

```
[Out] ((184192*(2*x - 3)^2)/47481 - (18944*x)/2261 - (15552*(2*x - 3)^3)/4199 + (
5666272*(2*x - 3)^4)/1440257 - (63490768*(2*x - 3)^5)/12962313 + (533495672
*(2*x - 3)^6)/70572593 - (1111521492*(2*x - 3)^7)/70572593 + (78007323158*(
2*x - 3)^8)/1482024453 - (250239440467*(2*x - 3)^9)/494008151 + (1118693654
785651073*(2*x - 3)^10)/453254454575104 + (1624300450152249301*(2*x - 3)^11
)/97125954551808 + (35048653520674948897*(2*x - 3)^12)/906508909150208 + (9
5527511967437577915*(2*x - 3)^13)/1813017818300416 + (564066299973141561054
7*(2*x - 3)^14)/114220122552926208 + (1737142288764447500149*(2*x - 3)^15)/
50764498912411648 + (12971210667229097601055*(2*x - 3)^16)/7107029847737630
72 + (32723441206946795665235*(2*x - 3)^17)/4264217908642578432 + (10264579
7034777710681325*(2*x - 3)^18)/39799367147330732032 + (14609317874302006653
15*(2*x - 3)^19)/2094703534070038528 + (687618468821894139745*(2*x - 3)^20)
/4528256169239642112 + (39968995676603847725*(2*x - 3)^21)/1509418723079880
704 + (5940132943613849875*(2*x - 3)^22)/1625527855624486912 + (57179785036
20010375*(2*x - 3)^23)/14629750700620382208 + (178056995818325525*(2*x - 3)
^24)/5689347494685704192 + (179665281323275*(2*x - 3)^25)/10159549097653043
2 + (1433237383402275*(2*x - 3)^26)/22757389978742816768 + (24229218097975*
(2*x - 3)^27)/22757389978742816768 + 37120/2261)/(20661046784*(3 - 2*x)^(19
/2) - 92974710528*(3 - 2*x)^(21/2) + 199231522560*(3 - 2*x)^(23/2) - 270069
397248*(3 - 2*x)^(25/2) + 259475340096*(3 - 2*x)^(27/2) - 187609683744*(3 -
 2*x)^(29/2) + 105782451264*(3 - 2*x)^(31/2) - 47554666992*(3 - 2*x)^(33/2)
 + 17278167438*(3 - 2*x)^(35/2) - 5111496103*(3 - 2*x)^(37/2) + 1234154817*
```

```
(3 - 2*x)^(39/2) - 242625852*(3 - 2*x)^(41/2) + 38550456*(3 - 2*x)^(43/2) -
 4883634*(3 - 2*x)^(45/2) + 482454*(3 - 2*x)^(47/2) - 35868*(3 - 2*x)^(49/2
) + 1890*(3 - 2*x)^(51/2) - 63*(3 - 2*x)^(53/2) + (3 - 2*x)^(55/2)) - (atan
(((- 7^(1/2)*30540258843957888971i - 229300295369923682239З)^(1/2)*(3 - 2*x
)^(1/2)*437746180358291443303165206406251i)/(3300086980477615835608700826192
63806430093600589158123831296*((7^(1/2)*427090967094607473872427449424977717
8671875i)/165004349023880791780435041309631903215046800294579061915648 + 80
336582919506134555067610693840117548437‍5/2357204986055439882577643447280455
76021495428992255802736‍64)) + (437746180358291443303165206406‍25*7^(1/2)*(-
7^(1/2)*30540258843957888971i - 2293002953699236822393)^(1/2)*(3 - 2*x)^(1/
2))/(3300086980477615835608700826192638064300936005891581238312‍96*((7^(1/2)
*4270909670946074738724274494249771786718‍75i)/165004349023880791780435041‍30
963190321504680029457906191564‍8 + 80336582919506134555067610693840117548437
5/23572049860554398825776434472804557602149542899225580273664))*(- 7^(1/2)
*30540258843957888971i - 2293002953699236822393)^(1/2)*11275i)/318603459702
399434752 + (atan(((7^(1/2)*30540258843957888971i - 2293002953699236822393)
^(1/2)*(3 - 2*x)^(1/2)*437746180358291443303165206406‍25i)/(3300086980477615
835608700826192638064300936005891581238312‍96*((7^(1/2)*427090967094607473‍87
242744942497717867187‍5i)/1650043490238807917804350413096319032150468002945‍7
9061915648 - 803365829195061345550676106938401175484375/2357204986055439882
577643447280455760214954289922558027366‍4)) - (437746180358291443303165206‍40
625*7^(1/2)*(7^(1/2)*30540258843957888971i - 2293002953699236822393)^(1/2)*
(3 - 2*x)^(1/2))/(3300086980477615835608700826192638064300936005891581238‍31
296*((7^(1/2)*4270909670946074738724274494249771786718‍75i)/165004349023880‍7
917804350413096319032150468002945790619156‍48 - 8033658291950613455506761069
38401175484375/23572049860554398825776434472804557602149542899225580273664)
))*(7^(1/2)*30540258843957888971i - 2293002953699236822393)^(1/2)*11275i)/3
18603459702399434752
```

3.49 $\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx$

Optimal. Leaf size=1058

result too large to display

```
[Out] 115/3248261265098830736532127368829731369648128*ln(3-2*x+14^(1/2)-(3-2*x)^(
1/2)*(7+2*14^(1/2))^(1/2))*(30297118912219360725028693061-80611109111432760
53983022787*14^(1/2))*(-14+4*14^(1/2))^(1/2)-115/32482612650988307365321273
68829731369648128*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(30
297118912219360725028693061-8061110911143276053983022787*14^(1/2))*(-14+4*1
4^(1/2))^(1/2)+115/1624130632549415368266063684414865684824064*arctan((-2*(
3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(3029711891221936
0725028693061+8061110911143276053983022787*14^(1/2))*(14+4*14^(1/2))^(1/2)-
115/1624130632549415368266063684414865684824064*arctan((2*(3-2*x)^(1/2)+(7+
2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(30297118912219360725028693061+80
61110911143276053983022787*14^(1/2))*(14+4*14^(1/2))^(1/2)-115/125891696652
967303050166272*(88411609113007981044643-5712269536245152162963*x)/(3-2*x)^
(39/2)/(2*x^2+x+1)^2+115/195831528126838026966925312*(285613476812257608148
15+965934812839019490346107*x)/(3-2*x)^(39/2)/(2*x^2+x+1)+1/133*x/(3-2*x)^(
39/2)/(2*x^2+x+1)^19+1/33516*(113+373*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^18+1/79
76808*(40657+107329*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^17+5/595601664*(751303+18
31285*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^16+1/25015269888*(184959785+429411497*x
)/(3-2*x)^(39/2)/(2*x^2+x+1)^15+1/4902992898048*(41652915209+92630823167*x)
/(3-2*x)^(39/2)/(2*x^2+x+1)^14+1/297448235814912*(2871555518177+61001563555
17*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^13+1/7138757659557888*(77559130805859+1562
74047129113*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^12+5/1099368679571914752*(2656658
801194921+5020880176134289*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^11+1/3420258114223
734784*(45187921585208601+78752911037377255*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^1
0+1/430952522392190582784*(6063974149878048635+9477172618423641847*x)/(3-2*
x)^(39/2)/(2*x^2+x+1)^9+1/48266682507925345271808*(691833601144925854831+91
9498192874055581221*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^8+23/15767116285922279455
45728*(919498192874055581221+908287136092467468517*x)/(3-2*x)^(39/2)/(2*x^2
+x+1)^7+115/10187982830903626725064704*(908287136092467468517+2982818849445
22225747*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^6+23/20375965661807253450129408*(259
9313568802265110081-10426142448623187379187*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^5
-23/20018492580021161284337664*(10426142448623187379187+2751372346319426238
3705*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^4-115/7643424439644433994743808*(265132
24428169016478843+30673415406553789342019*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^3-9
27027754781476746208047620505/58004665448193406009502274443388060172288/(3-
2*x)^(1/2)-143401467550777247627940437025/739855426635119974610998398512602
80832/(3-2*x)^(9/2)-4611053278117143010907562317585/72505831810241757511877
84305423507521536/(3-2*x)^(7/2)-405965372440630510720926890227/207159519457
8335928910795515835287863296/(3-2*x)^(5/2)-4986681479187781853417316522775/
87006998172290109014253411665082090258432/(3-2*x)^(3/2)+1734413681498043786
```

```
61935869705/896508488907352010051592447177261056/(3-2*x)^(19/2)-22724090823
469905152713519545/1604278348571050965355481221264572416/(3-2*x)^(17/2)-101
19027441277961867857327524 5/39635112141167141497017771348889436 16/(3-2*x)^(
15/2)-4605031904169582830874393371 35/343504305223448559640820685023708446 72
/(3-2*x)^(13/2)-2211619588790911794826342607495/406920484649315986036049119
181931544576/(3-2*x)^(11/2)-1305695962836335553428578542 5/10692401435725356
2723941220352/(3-2*x)^(39/2)-394819434329140174032199641 5/20288146313940419
5937734623232/(3-2*x)^(37/2)-304688229262620222736480811/537361713180043545
997243056128/(3-2*x)^(35/2)+212431584675656745565386292 5/16888510985658511 4
4562763890688/(3-2*x)^(33/2)+476575150745141187960959295 35/6663285243432539
9703658138959872/(3-2*x)^(31/2)+3491161999397471406217275198 5/1246679174577
7010267136038902169 6/(3-2*x)^(29/2)+149066309808794760843017404825/16249818
2065645168309566300173107 2/(3-2*x)^(27/2)+158486139641690665437343801 71/601
845118761648771516912222863360/(3-2*x)^(25/2)+111551682229707742323768911 45
/168516633253261656024735422401740 8/(3-2*x)^(23/2)+140118184980910202724749
56375/101109979951956993614841253441044 48/(3-2*x)^(21/2)
```

Rubi [A]

time = 1.60, antiderivative size = 1058, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {754, 836, 842, 840, 1183, 648, 632, 210, 642}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x)^(41/2)*(1 + x + 2*x^2)^20),x]
```

```
[Out] -13056959628363355534285785425/(106924014357253562723941220352*(3 - 2*x)^(3
9/2)) - 39481943432914017403219964 15/(20288146313940419593773462 3232*(3 - 2
*x)^(37/2)) - 304688229262620222736480811/(537361713180043545997243056128*(
3 - 2*x)^(35/2)) + 2124315846756567455653862925/(1688851098565851144562763 8
90688*(3 - 2*x)^(33/2)) + 47657515074514118796095929535/(666328524343253997
03658138959872*(3 - 2*x)^(31/2)) + 349116199939747140621727519 85/(124667917
4577701026713603890216 96*(3 - 2*x)^(29/2)) + 1490663098087947608430174048 25
/(162498182065645168309566300173107 2*(3 - 2*x)^(27/2)) + 158486139641690665
43734380171/(601845118761648771516912222863360*(3 - 2*x)^(25/2)) + 11155168
222970774232376891145/(1685166332532616560247354224017408*(3 - 2*x)^(23/2))
 + 140118184980910202724749563 75/(1011099799519569936148412534410444 8*(3 -
2*x)^(21/2)) + 17344136814980437866193586970 5/(89650848890735201005159244 71
77261056*(3 - 2*x)^(19/2)) - 22724090823469905152713519545/(160427834857105
0965355481221264572416*(3 - 2*x)^(17/2)) - 101190274412779618678573275245/(
396351121411671414970177713488894361 6*(3 - 2*x)^(15/2)) - 46050319041695828
3087439337135/(343504305223448559640820685023708446 72*(3 - 2*x)^(13/2)) - 2
2116195887909117948263426074 95/(40692048464931598603604911918193154457 6*(3
 - 2*x)^(11/2)) - 143401467550777247627940437025/(7398554266351199746109983 9
851260280832*(3 - 2*x)^(9/2)) - 461105327811714301090756231758 5/(7250583181
0241757511877843054235075215 36*(3 - 2*x)^(7/2)) - 405965372440630510720926 8
```

```
90227/(2071595194578335928910795515835287863296*(3 - 2*x)^(5/2)) - 49866814
79187781853417316522775/(8700699817229010901425341166508209025843 2*(3 - 2*x
)^(3/2)) - 927027754781476746208047620505/(5800466544819340600950227444338 8
060172288*Sqrt[3 - 2*x]) + x/(133*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^19) + (1
13 + 373*x)/(33516*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^18) + (40657 + 107329*x
)/(7976808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^17) + (5*(751303 + 1831285*x))/
(595601664*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^16) + (184959785 + 429411497*x)
/(25015269888*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^15) + (41652915209 + 9263082
3167*x)/(4902992898048*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^14) + (287155551817
7 + 6100156355517*x)/(297448235814912*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^13)
+ (77559130805859 + 156274047129113*x)/(7138757659557888*(3 - 2*x)^(39/2)*(
1 + x + 2*x^2)^12) + (5*(2656658801194921 + 5020880176134289*x))/(109936867
9571914752*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^11) + (45187921585208601 + 7875
2911037377255*x)/(3420258114223734784*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^10)
+ (6063974149878048635 + 9477172618423641847*x)/(430952522392190582784*(3 -
 2*x)^(39/2)*(1 + x + 2*x^2)^9) + (691833601144925854831 + 9194981928740555
81221*x)/(48266682507925345271808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^8) + (23
*(919498192874055581221 + 9082871360924674 68517*x))/(157671162859222794554 5
728*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^7) + (115*(9082871360924674 68517 + 298
281884944522225747*x))/(10187982830903626725064704*(3 - 2*x)^(39/2)*(1 + x
+ 2*x^2)^6) + (23*(2599313568802265110081 - 10426142448623187379187*x))/(20
375965661807253450129408*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^5) - (23*(1042614
2448623187379187 + 27513723463194262383705*x))/(200184925800211612843376 64*
(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^4) - (115*(265132244281690164788 43 + 30673
415406553789342019*x))/(7643424439644443399474 3808*(3 - 2*x)^(39/2)*(1 + x
+ 2*x^2)^3) - (115*(884116091130079810446 43 - 571226953624515216296 3*x))/(1
258916966529673030501662 72*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^2) + (115*(2856
134768122576081481 5 + 96593481283901949034610 7*x))/(195831528126838026966 92
5312*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)) + (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302
971189122193607250286930 61 + 806111091114327605398302278 7*Sqrt[14])*ArcTan[
(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/8120653162
747076841330318422074328424120 32 - (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302971189
1221936072502869306 1 + 806111091114327605398302278 7*Sqrt[14])*ArcTan[(Sqrt[
7 + 2*Sqrt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/8120653162747076
841330318422074328424120 32 + (115*(302971189122193607250286930 61 - 80611109
1114327605398302278 7*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] -
 Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/1624130632549415368266063684414
865684824064 - (115*(302971189122193607250286930 61 - 806111091114327605398 3
022787*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sq
rt[14]]*Sqrt[3 - 2*x] - 2*x])/16241306325494153682660636844148656848 24064
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 842

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)
^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx = \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{\int \frac{3640-3164x}{(3-2x)^{41/2}(1+x+2x^2)^{19}}\,dx}{3724}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.11, size = 1100, normalized size = 1.04

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x)^(41/2)*(1 + x + 2*x^2)^20),x]
```

```
[Out] x/(133*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^19) + ((44296 + 146216*x)/(3528*(3
 - 2*x)^(39/2)*(1 + x + 2*x^2)^18) + ((223125616 + 589021552*x)/(3332*(3 - 2
*x)^(39/2)*(1 + x + 2*x^2)^17) + ((865861681440 + 2110519336800*x)/(3136*(3
 - 2*x)^(39/2)*(1 + x + 2*x^2)^16) + ((2984274342235200 + 6928434268875840*
x)/(2940*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^15) + ((9408813737133390720 + 209
24013532366815360*x)/(2744*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^14) + ((2724306
5619141593598720 + 57873497074462503141120*x)/(2548*(3 - 2*x)^(39/2)*(1 + x
```

```
 + 2*x^2)^13) + ((7211037735478027891383552O + 1452953429486831061640166640*
x)/(2352*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^12) + ((1729014581089328963351798
01600 + 326770416680301421681066214400*x)/(2156*(3 - 2*x)^(39/2)*(1 + x + 2
*x^2)^11) + ((370557652515461812186329087129600 + 6458029672318863068265404
24448000*x)/(1960*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^10) + ((6961755986759734
38759010577554944000 + 1088028437838790621809440473088716800*x)/(1764*(3 -
2*x)^(39/2)*(1 + x + 2*x^2)^9) + ((1111965063471244015489248163496668569600
 + 1477884081820868038735185945420330393600*x)/(1568*(3 - 2*x)^(39/2)*(1 +
x + 2*x^2)^8) + ((1427636023038958525418189623276039160217600 + 14102294542
80293592108580217248432347955200*x)/(1372*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^
7) + ((1283308803395067168818807997696073436639232000 + 4214391612869991217
70135584246204836237312000*x)/(1176*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^6) + (
(359909043739097249991695788946258930146664448000 - 14436361213243981948316
93460992758930913796096000*x)/(980*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^5) + ((
-1152021624816869759475691381872221626869209284608000 - 3040089329780519199
031170166260953381570260254720000*x)/(784*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^
4) + ((-2255746282697145245681128263365627409125133109002240000 - 260969551
1325529255410382651665073470845732989009920000*x)/(588*(3 - 2*x)^(39/2)*(1
 + x + 2*x^2)^3) + ((-179025112076931306921152249904224040100017283046080512
0000 + 115668033214143596894295804604678509924267822733393920000*x)/(392*(3
 - 2*x)^(39/2)*(1 + x + 2*x^2)^2) + ((7287086092491046604340635690094746125
2288728322038169600000 + 24644670900872826929692130734587768100251906626103
43034880000*x)/(196*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)) + (-53055056666589708
7493026465460148012491929957574880460800000/(3 - 2*x)^(39/2) + (-1708089006
242241264480481073293611769771298388785813753364480000/(37*(3 - 2*x)^(37/2)
) + (-696740950089909200017539783692427216704271188038402697920512000/(3 -
2*x)^(35/2) + (757366667762147355602446006474261151597409525795681824661504
000000/(3 - 2*x)^(33/2) + (616772664905423340350737254793402194192083509401
0816556282758758400000/(31*(3 - 2*x)^(31/2)) + (980445504127015992472138196
645778610361943940861637274650890661068800000/(29*(3 - 2*x)^(29/2)) + (4496
423323436580179825935667807239175646629240803415910250222313472000000/(3 -
2*x)^(27/2) + (487904184130260773926886832047572655461484781443782543411352
841560457216000/(3 - 2*x)^(25/2) + (429268867215238023064148871550918822599
02542088067698170622802545418240000000/(3 - 2*x)^(23/2) + (2893692593980364
723231826294558630623656919099359688069727689450554368000000000/(3 - 2*x)^(
21/2) + (118767476492930264374166633243140666046068763101817907661320807641
190359040000000/(3 - 2*x)^(19/2) + (-23130641371662285970537372414163682847
22516912423159767489332810437803253760000000/(3 - 2*x)^(17/2) + (-992239519
653790860422623948957964852355985846800936213338418761762097950023680000000
/(3 - 2*x)^(15/2) + (-10941518315154632243157241587901809625083601209973176
6901467841654602614755123200000000/(3 - 2*x)^(13/2) + (-8073268485314233063
840337934095431560069216535225849300748018943930634745621913600000000/(3 -
2*x)^(11/2) + (-44337987226211231305207361494572283981715203938096393248399
6666511839997547213824000000000/(3 - 2*x)^(9/2) + (-18330190892216697744173
706790143700087358561576136178754174544727578117325359791923200000000/(3 -
```

```
2*x)^(7/2) + (-5535412100027359570488442147160282454990867464017235233247806605576616684137250589491200000000/(3 - 2*x)^(5/2) + (-11332385663391839740343974428370683887566771471384841151672642393999283182139266339840000000000/
(3 - 2*x)^(3/2) + (-13272202629081314874038396353552342714266551897543529306435677723641008864036246751334400000000/Sqrt[3 - 2*x] + ((Sqrt[(7 - I*Sqr
t[7])/2]*(-18581083680713840823653754894973279799973172656560941029009948813097412409650745451868160000000000 - (38534140062781031467679876224014966993363355559218658375420168852658974828331156900925440000000000*I)*Sqrt[7])*Arc
Tanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 - I*Sqrt[7]]])/(-14 + (2*I)*Sqrt[7]) +
(Sqrt[(7 + I*Sqrt[7])/2]*(-18581083680713840823653754894973279799973172656560941029009948813097412409650745451868160000000000 + (38534140062781031467679876224014966993363355559218658375420168852658974828331156900925440000000000
*I)*Sqrt[7])*ArcTanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 + I*Sqrt[7]]])/(-14 - (
2*I)*Sqrt[7]))/7)/42)/70)/98)/126)/154)/182)/210)/238)/266)/294)/322)/350)/
378)/406)/434)/462)/490)/518)/546)/196)/392)/588)/784)/980)/1176)/1372)/156
8)/1764)/1960)/2156)/2352)/2548)/2744)/2940)/3136)/3332)/3528)/3724
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 + x + 2*x^2)^20*(3 - 2*x)^(41/2)),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 7262 deep
```

Maple [**A**]
time = 1.18, size = 820, normalized size = 0.78

method	result	size
trager	Expression too large to display	732
risch	Expression too large to display	761
derivativedivides	Expression too large to display	820
default	Expression too large to display	820

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x,method=_RETURNVERBOSE)
[Out] 1/3014567765899607808257563033 6*(807597736492641378942268937217995835353849
465/1048576*(3-2*x)^(1/2)+180866897114899220649017210287078795487454118 1/33
4114095890432*(3-2*x)^(57/2)-1196897725308288065129289211139553093326521 9/2
5701084299264*(3-2*x)^(59/2)+3395565446412935417599589886148144605498 73/982
6885173248*(3-2*x)^(61/2)-64243396719140374998473027009027485263697/2948065
5519744*(3-2*x)^(63/2)+129886852748727110357425618672922324659/113387136614
```

4*(3-2*x)^(65/2)-503502693505289734438057515605193725/103079215104*(3-2*x)^
(67/2)+133883313322119397348791732981953297/824633720832*(3-2*x)^(69/2)-325
485074800348342966738850178379/824633720832*(3-2*x)^(71/2)+360433340020130
123942335063779145/5772436045824*(3-2*x)^(73/2)-928342237074576734557978321
305/1924145348608*(3-2*x)^(75/2)-447963293570690822971544737256709035462203
92558695/9070970929152*(3-2*x)^(43/2)+2860722331769322369839567258415059386
3016075796143/29480655519744*(3-2*x)^(45/2)-5059022664167725408892162874688
680417923742003781/29480655519744*(3-2*x)^(47/2)+73012476452577571533836489
036461787385135079265/2680059592704*(3-2*x)^(49/2)-193924292090153482145402
6903132433081580221023737/501171143835648*(3-2*x)^(51/2)+490738543064879423
955077165987434152441563270473/1002342287671296*(3-2*x)^(53/2)-550118352883
61289002011693179378316699033102675/1002342287671296*(3-2*x)^(55/2)-1006304
725834560333245233940167063186576585913370455/10720238370816*(3-2*x)^(39/2)
+13805722741822612586258592099428566280191230197271405/39307540692992*(3-2*
x)^(37/2)-17650942358963262675871173166229809316744939271143/51904512*(3-2*
x)^(11/2)+1186323846453826237212517196312193819452761764018822545/391539956
1216*(3-2*x)^(21/2)+2672239984790337844292019294315182385216573077301785/11
7922622078976*(3-2*x)^(41/2)-2239754632120948695306207437479573729995706356
5/3145728*(3-2*x)^(3/2)+404531566689883337048499233527781983599187634017/12
582912*(3-2*x)^(5/2)-1188598027552254830082683218064697188605612952419/1258
2912*(3-2*x)^(7/2)+3831583379166294091823572953989993625772471445345/188743
68*(3-2*x)^(9/2)+9977850126168010187169130424774568330973123412551261/21592
276992*(3-2*x)^(13/2)-1255696718499588580979726331572072320357969297077745/
2399141888*(3-2*x)^(15/2)-7559301164046828565701951901920324412946321609455
23631/3915399561216*(3-2*x)^(23/2)+8535085022072145119870938660211240879080
41634697244059/7830799122432*(3-2*x)^(25/2)-6886173809894005543994516442461
871486007042005189775/125627793408*(3-2*x)^(27/2)+1363299879672453951418482
53765147208279814148352958009/5527622909952*(3-2*x)^(29/2)-5506609142081759
0167865401986871791412011888132876913/5527622909952*(3-2*x)^(31/2)+27374875
28928439357869138774910126923363791747141675/755914244096*(3-2*x)^(33/2)-11
664572170215876884203668230743495214488310113371105/9826885173248*(3-2*x)^(
35/2)+12646629333382722716904430763732665179119615389552413/25098715136*(3-
2*x)^(17/2)-2593673203685044441695042001860835122939346700333136537/6199382
638592*(3-2*x)^(19/2))/((3-2*x)^2-7+14*x)^19-115/32482612650988307365321273
68829731369648128*(62541562556792464940960784209*(7+2*14^(1/2))^(1/2)*14^(1
/2)-234044028404883307655877091262*(7+2*14^(1/2))^(1/2))*ln(3-2*x+14^(1/2)-
(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))+115/32482612650988307365321273688297313
69648128*(62541562556792464940960784209*(7+2*14^(1/2))^(1/2)*14^(1/2)-23404
4028404883307655877091262*(7+2*14^(1/2))^(1/2))*ln(3-2*x+14^(1/2)+(3-2*x)^(
1/2)*(7+2*14^(1/2))^(1/2))-115/812065316274707684133031842207432842412032*(
30297118912219360725028693061*14^(1/2)+1/2*(62541562556792464940960784209*(
7+2*14^(1/2))^(1/2)*14^(1/2)-234044028404883307655877091262*(7+2*14^(1/2))^
(1/2))*(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)-
(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))+683151246370725/30145677658996
078082575630336/(3-2*x)^(1/2)+355/5266289575642392066/(3-2*x)^(33/2)+52865/

```
27703874858530886747 2/(3-2*x)^(31/2)+14333/32395660116830472406/(3-2*x)^(29
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^20*(-2*x + 3)^(41/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2763 vs. $2(821) = 1642$.
time = 39.82, size = 2763, normalized size = 2.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="fricas")
```

```
[Out] 1/39216486643313459145226570078538364602163784899079333870594297242905074108177191061306767655456730731382076063194779070544791852259532242520616485257216
0*(616525316537858546962128448983043227187951381815778781478549978900*5795904991921858556653045419515717067178458593845454142080244780765852057823
3279417434470132 6^(1/4)*sqrt(12868460882463048970358421717432178503450051394)*sqrt(14)*sqrt(7)*(549755813888*x^58 - 11269994184704*x^57 + 107064944754688*x^56 - 630638638006272*x^55 + 2618521301286912*x^54 - 8342252417974272*x^53 + 21849572376576000*x^52 - 49684091485814784*x^51 + 101394501297242112*x^50 - 188583312363618304*x^49 + 323261995581177856*x^48 - 517079841212727296*x^47 + 778117896260812800*x^46 - 1105641165387988992*x^45 + 14912870282
```

33404416*x^44 - 1919929663119949824*x^43 + 2363050939901804544*x^42 - 27862
74020645928960*x^41 + 3161145685194047488*x^40 - 3453753931369283584*x^39 +
 3634098467102523392*x^38 - 3697893960325791744*x^37 + 3640651752731836416*
x^36 - 3461798212247617536*x^35 + 3194540251789393920*x^34 - 28615445794952
97024*x^33 + 2477632938217930752*x^32 - 2088430257127768064*x^31 + 17127610
05459316736*x^30 - 1355447485390974976*x^29 + 1048940886155151360*x^28 - 79
0511024135089152*x^27 + 571750925528393856*x^26 - 408374103192240192*x^25 +
 282845069599813728*x^24 - 186113897194906128*x^23 + 123982890381352520*x^2
2 - 78116367732251996*x^21 + 46488580159296898*x^20 - 29591055660829971*x^1
9 + 16200795673453545*x^18 - 8941894120163277*x^17 + 5578893209169441*x^16
- 2296849711499532*x^15 + 1448289882400788*x^14 - 756896247319212*x^13 + 18
2213447974992*x^12 - 240797810407770*x^11 + 25549234281774*x^10 - 265002817
27302*x^9 + 25520701332582*x^8 + 9965507230260*x^7 + 10389354811164*x^6 + 3
755740313808*x^5 + 1820618017974*x^4 + 463742325333*x^3 + 139858796529*x^2
+ 19758444939*x + 3486784401)*sqrt(378148402880167888800346812933915372766 2
345024772741260943*sqrt(14) + 14149022371848728385570789036684124101210161 6
40127797919744)*arctan(1/3488555476273159707600878934940824497561724963674 9
13242575009589894914045286581081812447079130476773106112671051669997871458 0
82291658322630168235582320931564879831926785152574881809490600509573163099 2
227838434460546889854820576222503959439208139217 00*579590499192185855665304
541951571706717845859384545414208024478076585205782332794174344701326^(3/4)
*sqrt(1634857335323112850812492677092639503349451327418417311)*sqrt(6434230
441231524485179210858716089251725025697 0)*sqrt(128684608824630489703584217 1
7432178503450051394)*sqrt(579590499192185855665304541951571706717845859384 5
454142080244780765852057823327941743447013 26^(1/4)*sqrt(128684608824630489 7
0358421717432178503450051394)*sqrt(-2*x + 3)*sqrt(378148402880167888800346 8
1293391537276623450247727412609 43*sqrt(14) + 14149022371848728385570789036 6
8412410121016164012779791974 4)*(806111091114327605398302278 7*sqrt(14) - 302
971189122193607250286930 61) - 2103809766801325355695634432872368239054787 19
2594512041684573248748652161620808567413707456508928153 40*x + 9637320505996
2179442545630821934006082946806299988282066139 0*sqrt(166789371965963959581 0
9874281758628913067976481215647672103870657600799128903328172 6) + 315571465
02019880335434516493085523585821807888917680625268598731229782424312128511 2
056118476339223010)*(3029711891221936072502869306 1*sqrt(14)*sqrt(7) - 11285
55527560058647557623190 18*sqrt(7))*sqrt(378148402880167888800346812933915 37
27662345024772741260943*sqrt(14) + 1414902237184872838557078903668412410121
01616401277979197 44) - 1/331641722680775415760424069447358035430711841280 57
80544574064399284894720547513183329763987573259243427226688367795480452172 1
5840067297151273069035 10*579590499192185855665304541951571706717845859384 54
5414208024478076585205782332794174344701326^(3/4)*sqrt(12868460882463048970
358421717432178503450051394)*(302971189122193607250286930 61*sqrt(14)*sqrt(7
) - 112855552756005864755762319018*sqrt(7))*sqrt(-2*x + 3)*sqrt(37814840288
0167888800346812933915372766234502477274126094 3*sqrt(14) + 1414902237184872
8385570789036684124101210161640127779791974 4) + 2/7*sqrt(14)*sqrt(7) + sqrt(
7)) + 616525316537858546962128448983043227187951381815778781478549978900*57

9590499192185855665304541951571706717845859384545414208024478076585205782332794174344701326^(1/4)*sqrt(12868460882463048970358421717432178503450051394)*sqrt(14)*sqrt(7)*(549755813888*x^58 - 11269994184704*x^57 + 107064944754688*x^56 - 630638638006272*x^55 + 2618521301286912*x^54 - 8342252417974272*x^53 + 21849572376576000*x^52 - 49684091485814784*x^51 + 101394501297242112*x^50 - 188583312363618304*x^49 + 323261995581177856*x^48 - 517079841212727296*x^47 + 778117896260812800*x^46 - 1105641165387988992*x^45 + 1491287028233404416*x^44 - 1919929663119949824*x^43 + 2363050939901804544*x^42 - 2786274020645928960*x^41 + 3161145685194047488*x^40 - 3453753931369283584*x^39 + 3634098467102523392*x^38 - 3697893960325791744*x^37 + 3640651752731836416*x^36 - 3461798212247617536*x^35 + 3194540251789393920*x^34 - 2861544579495297024*x^33 + 2477632938217930752*x^32 - 2088430257127768064*x^31 + 1712761005459316736*x^30 - 1355447485390974976*x^29 + 1048940886155151360*x^28 - 790511024135089152*x^27 + 571750925528393856*x^26 - 408374103192240192*x^25 + 282845069599813728*x^24 - 186113897194906128*x^23 + 123982890381352520*x^22 - 78116367732251996*x^21 + 46488580159296898*x^20 - 29591055660829971*x^19 + 16200795673453545*x^18 - 8941894120163277*x^17 + 5578893209169441*x^16 - 2296849711499532*x^15 + 1448289882400788*x^14 - 756896247319212*x^13 + 182213447974992*x^12 - 240797810407770*x^11 + 25549234281774*x^10 - 26500281727302*x^9 + 25520701332582*x^8 + 9965507230260*x^7 + 10389354811164*x^6 + 3755740313808*x^5 + 1820618017974*x^4 + 463742325333*x^3 + 139858796529*x^2 + 19758444939*x + 3486784401)*sqrt(378148402880167888800346812933915372766234502477274126094 3*sqrt(14) + 14149022371848728385570789036684124101210161640127797919744)*arctan(1/88221268136991557850830347742157188347641479830426221595502224219175844282483036150488446475189965496243820538044661002745019234209862134823250046493206397567627980015263931446769927917308008043494406341475991998227625530289790494302092900288913988891810201600*5795904991921858556653045419515717067178458593845454142080244780765852057823327941743447 01326^(3/4)*sqrt(16348573353231128508124926770926395033494513274184173 11)*sqrt(12868460882463048970358421717432178503450051394)*sqrt(-411483036686051024414565090581703228290144092715017759351638763701587148 80*5795904991921858556653045419515717067178458593845454142080244780765852057823327941743447 01326^(1/4)*sqrt(12868460882463048970358421717432178503450051394)*sqrt(-2*x + 3)*sqrt(3781484028801678888003468129339153727662345024772741260943*sqrt(14) + 14149022371848728385570789036684124101210161640127797919744)*(806111091114327605398302278 7*sqrt(14) - 30297118912219360725028693061) - 865682031453182211872836099757277909777893967442114745364436442239548415161956222805222560979051258711715996977028730413019050179358393215741553349147629366849149087955025 9200*x + 396559390732407356996974648323070402280103349710579549130971764522752446561212531052976487731364574616035901168078584342496340344 83200*sqrt(16678937196596395958109874281758628913067976481215647672103870657600799128903328172 6) + 12985230471797733178092541496359168646668409511631721180466546633593226227429343342078338414685768880675739954655430956195285752690375898236123300237214440502737236319325388800)*(30297118912219360725028 693061*sqrt(14)*sqrt(7) - 112855552756005864755762319018*sqrt(7))*sqrt(37814

```
84028801678888003468129339153727662345024772741260943*sqrt(14) + 1414902237
```

2444760326144*x^29 + 14841488064956066674396728991150443932171972367710399727450886674237452451840*x^28 - 11184958165680426483017504655762227434727279734038187934149692650461581017088*x^27 + 80897166364264609042152707250524108677377533249635571542371192388023106928 64*x^26 - 57780943221506676835246040556560917881760320348406671868899247662607871508 48*x^25 + 40019812175348750941930164544740009926063843881646165362756970090082934456 32*x^24 - 26333296951226810998155099683374518555527981864824487684034876000717577912 32*x^23 + 17542366897322253251089903529813657607145515074994145464959180389051321548 80*x^22 - 11052702346511956082854508312652613727705452254323918656667725718651918090 24*x^21 + 65776796070937473101898342886098749096522587138982913916680305570607215411 2*x^20 - 41868450855170421700463282020153827146482312561502405682299542551828784742 4*x^19 + 22922542042544429419114764617634169952032350182849637446627875857522229248 0*x^18 - 12651905995289280780622087615549707531950410298498160382922225196622204108 8*x^17 + 78935884826693368065254828931908211891519394127734424705049022825815343104*x^16 - 32498177952781175771570102356850846229225588393916533119446726744429559808*x^15 + 20491885947010913333756876708782289162249577789257433177529423102866358272*x^14 - 1070934193648787869612347546010978156666623212627159296329711543032532172 8*x^13 + 2578142151849856182075133918988458447112007949561957497307465546935042048*x^12 - 340705360655172629909903757378974739082831055743924947119182311837401088 0*x^11 + 3614966874364248039306939047787007807694904874158409849416228071339458 56*x^10 - 374953079019890060800626107146254604879378550577160501972659130689650688*x^9 + 361092974099972372832558605269197722032357635208049111095191908288299008*x^8 + 141002184747768997009392595202128779106364989840476870319621019208253440*x^7 + 146999213652233656885845331514573869620245040110547854481309077147222016*x^6 + 53140053722923555611929392435834198275650668430851979724323340159025152*x^5 + 25759965066905016287490162796639138798257488660077636512705521601478656*x^4 + 65615005359097682996437207123514787504995489983216621305948026720747 52*x^3 + 19788652409886602808449316434075888295157369994938346108143055677685 76*x^2 - 37814840288016788880034681293391537276623450247727412609 43*sqrt(14)*(549755813888*x^58 - 11269994184704*x^57 + 107064944754688*x^56 - 630638638006272*x^55 + 2618521301286912*x^54 - 8342252417974272*x^53 + 21849572376576000*x^52 - 49684091485814784*x^51 + 101394501297242112*x^50 - 188583312363618304*x^49 + 323261995581177856*x^48 - 517079841212727296*x^47 + 778117896260812800*x^46 - 1105641165387988992*x^45 + 1491287028233404416*x^44 - 1919929663119949824*x^43 + 2363050939901804544*x^42 - 2786274020645928960*x^41 + 3161145685194047488*x^40 - 3453753931369283584*x^39 + 3634098467102523392*x^38 - 3697893960325791744*x^37 + 3640651752731836416*x^36 - 3461798212247617536*x^35 + 3194540251789393920*x^34 - 2861544579495297024*x^33 + 2477632938217930752*x^32 - 2088430257127768064*x^31 + 1712761005459316736*x^30 - 1355447485390974976*x^29 + 1048940886155151360*x^28 - 790511024135089152*x^27 + 571750925528393856*x^26 - 408374103192240192*x^25 + 282845069599813728*x^24 - 186113897194906128*x^23 + 123982890381352520*x^22 - 78116367732251996*x^21 + 46488580159296898*x^20 - 29591055660829971*x^19 + 16200795673453545*x^18 - 8941894120163277*x^17 + 5

578893209169441*x^16 - 2296849711499532*x^15 + 1448289882400788*x^14 - 7568
96247319212*x^13 + 182213447974992*x^12 - 240797810407770*x^11 + 2554923428
1774*x^10 - 26500281727302*x^9 + 25520701332582*x^8 + 9965507230260*x^7 + 1
0389354811164*x^6 + 3755740313808*x^5 + 1820618017974*x^4 + 463742325333*x^
3 + 139858796529*x^2 + 19758444939*x + 3486784401) + 2795626794748522834434
66797268108117189203842033755028120580564975616*x + 49334590495562167666494
140694372020680447736829486181433043629113344)*sqrt(37814840288016788880034
68129339153727662345024772741260943*sqrt(14) + 1414902237184872838557078903
6684124101210161640127797919744)*log(41148303668605102441456509058170322829
014409271501775935163876370158714880/16348573353231128508124926770926395033
49451327418417311*579590499192185855665304541951571706717845859384545414208
024478076585205782332794174344701326^(1/4)*sqrt(128684608824630489703584217
17432178503450051394)*sqrt(-2*x + 3)*sqrt(378148402880167888800346812933915
3727662345024772741260943*sqrt(14) + 14149022371848728385570789036684124101
210161640127797919744)*(8061110911143276053983022787*sqrt(14) - 30297118912
219360725028693061) - 52951533613915553191922904161192663574869868505075814
89439885963489257332413262896909021356322314724080758142729925427200*x + 24
256513529606021838214197524700823604475704121457581896019753481444860834611
200*sqrt(166789371965963959581098742817586289130679764812156476721038706576
007991289033281726) + 79427300420873329787884356241788995362304802757613722
34159828945233885998619894345363532034483472086121137214094888140800) - 131
989413465*57959049919218585566530454195157170671784585938454541420802447807
6585205782332794174344701326^(1/4)*sqrt(12868460882463048970358421717432178
503450051394)*(777850730975521785282731740230062813402918889820449450570205
6024604672*x^58 - 159459399849981965982960006747162876747598372413192137366
892148504395776*x^57 + 1514864298574828676838120064098047329102184537925325
304985475410791759872*x^56 - 8922920197702954279424536475114108048248233314
852830759853471017224634368*x^55 + 3704951647307096233413231449449553559163
9403546454145111565499223693590528*x^54 - 118034716093527123457170227067542
059725196738523651032986322545956112826368*x^53 + 3091500883715018126702794
56678545863084687431424928239641662378993516544000*x^52 - 70298132195777230
6733839830751157952666084707511427579277943471480080695296*x^51 + 143463306
7237123554683051124392269116634712360343909848074251531317913059328*x^50 -
266826950559017228004904436710909000228611047955821555812116193496004139417
6*x^49 + 457384120744655026269982119717501065035316336043943275791621249027
8077988864*x^48 - 731617424135086661987001679983408942583827681464044898144
8669826605566132224*x^47 + 110096075221301083037203271509647145491034311516
20256934238701471906607923200*x^46 - 15643741584311556183830093683288060491
254305358060645835780583727821171458048*x^45 + 2110025352532224532336938735
5308423247443333718020271970799243119655863189504*x^44 - 271651277558601625
19562776582318065307577993759440407412731079664252858925056*x^43 + 33434860
614488797445569848022836846490704329307177012620295046744354886516736*x^42
- 3942305345222015457641741476722802069850250206709428727323239502827040538
6240*x^41 + 447271210204836554576971636845736326692914998781626008688914810
89937948803072*x^40 - 48867241641804491090588875438285611681021058445732456

6353751739474948086824 96*x^39 + 514189405125347735489480063031695833076028 7
0096615653180441644588670698651648*x^38 - 5232158437337392140510238565610 58
5273747288618769796862678103801339296979353 6*x^37 + 5151166309751303829841 8
01730228004104255520220166332695790975840304570459750 4*x^36 - 4898106035191
7473116168661974190599957194981022285273133557262743720935030784*x^35 + 451
9962149033940434515349853481794660915616369562519347955038731657856155648 0*
x^34 - 404880582733214195930596097362336238083826588763975168633651775570 98
594041856*x^33 + 350560838720748004870675678552490641788358688917227892916 4
021930462584556748 8*x^32 - 295492464301465825833445981662612329746797874010
510009322612207469969182556 16*x^31 + 242338937838739945110707883369250065 80
4077743121445205880335495460844040355 84*x^30 - 191782567946630073729270559 4
5418126742331837591606363287110323072444760326144*x^29 + 148414880649560666
7439672899115044393217197236771039972745088667423745245 1840*x^28 - 11184958
165680426483017504655762227434727279734038187934149692650461581017088*x^27
+ 808971663642646090421527072505241086773775332496355715423711923880231069 2
864*x^26 - 577809432215066768352460405565609178817603203484066718688992476 6
260787150848*x^25 + 400198121753487509419301645447400099260638438816461653 6
275697009008293445632*x^24 - 263332969512268109981550996833745185555279818 6
4824487684034876000717577912 32*x^23 + 175423668973222532510899035298136576 0
7145515074994145464959180389051321548 80*x^22 - 110527023465119560828545083 1
265261372770545225432391865666772571865191809024*x^21 + 657767960709374731 0
1898342886098749096522587138982913916680305570607215411 2*x^20 - 41868450855
1704217004632820201538271464823125615024056822995425518287847424*x^19 + 229
2254204254442941911476461763416995203235018284963744662787585752222924 80*x^
18 - 126519059952892807806220876155497075319504102984981603829222251966222 0
41088*x^17 + 789358848266933680652548289319082118915193941277344247050490 22
825815343104*x^16 - 324981779527811757715701023568508462292255883939165331 1
9446726744429559808*x^15 + 204918859470109133337568767087822891622495777892
57433177529423102866358272*x^14 - 107093419364878786961234754601097815666 66
232126271592963297115430325321728*x^13 + 257814215184985618207513391898845 8
44711200794956195749730746554693504204 8*x^12 - 340705360655172629909903757 3
7897473908283105574392494711918231183740108 80*x^11 + 361496687436424803930 6
939047787007807694904874158409849416228071339458 56*x^10 - 374953079019890 06
0800626107146254604879378550577160501972659130689650688*x^9 + 3610929740999
7237283255860526919772203235763520804911109519190828829900 8*x^8 + 141002184
7477689970093925952021287791063649898404768703196210192082534 40*x^7 + 14699
921365223365688584533151457386962024504011054785448130907714722201 6*x^6 + 5
3140053722923555611929392435834198275650668430851979724323340159025152*x^5
+ 25759965066905016287490162796639138798257488660077636512705521601478656*x
^4 + 6561500535909768299643720712351478750499548998321662130594802672074752
*x^3 + 197886524098866028084493164340758882951573699949383461081430556776 85
76*x^2 - 3781484028801678888003468129339153727662345024772741260943*sqrt(14
)*(549755813888*x^58 - 11269994184704*x^57 + 107064944754688*x^56 - 6306386
38006272*x^55 + 2618521301286912*x^54 - 8342252417974272*x^53 + 21849572376
576000*x^52 - 49684091485814784*x^51 + 101394501297242112*x^50 - 1885833123

63618304*x^49 + 323261995581177856*x^48 - 517079841212727296*x^47 + 7781178
96260812800*x^46 - 1105641165387988992*x^45 + 1491287028233404416*x^44 - 19
19929663119949824*x^43 + 2363050939901804544*x^42 - 2786274020645928960*x^4
1 + 3161145685194047488*x^40 - 3453753931369283584*x^39 + 36340984671025233
92*x^38 - 3697893960325791744*x^37 + 3640651752731836416*x^36 - 34617982122
47617536*x^35 + 3194540251789393920*x^34 - 2861544579495297024*x^33 + 24776
32938217930752*x^32 - 2088430257127768064*x^31 + 1712761005459316736*x^30 -
 1355447485390974976*x^29 + 1048940886155151360*x^28 - 790511024135089152*x
^27 + 571750925528393856*x^26 - 408374103192240192*x^25 + 2828450695998137 2
8*x^24 - 186113897194906128*x^23 + 123982890381352520*x^22 - 78116367732251
996*x^21 + 46488580159296898*x^20 - 29591055660829971*x^19 + 16200795673453
545*x^18 - 8941894120163277*x^17 + 5578893209169441*x^16 - 2296849711499532
*x^15 + 1448289882400788*x^14 - 756896247319212*x^13 + 182213447974992*x^12
 - 240797810407770*x^11 + 25549234281774*x^10 - 26500281727302*x^9 + 255207
01332582*x^8 + 9965507230260*x^7 + 10389354811164*x^6 + 3755740313808*x^5 +
 1820618017974*x^4 + 463742325333*x^3 + 139858796529*x^2 + 19758444939*x +
3486784401) + 279562679474852283443466797268108117189203842033755028120580 5
64975616*x + 493345904955621676664941406943720206804477368294861814330436 29
113344)*sqrt(378148402880167888800346812933915372766234502477274126094 3*sqr
t(14) + 14149022371848728385570789036684124101210161640127797919744)*log(-4
114830366860510244145650905817032282901440927150177593516387637015871488 0/1
634857335323112850812492677092639503349451327418417311*5795904991921858556 6
5304541951571706717845859384545414208024478076585205782332794174344701326^(
1/4)*sqrt(12868460882463048970358421717432178503450051394)*sqrt(-2*x + 3)*s
qrt(3781484028801678888003468129339153727662345024772741260943*sqrt(14) + 1
4149022371848728385570789036684124101210161640127797919744)*(80611109111432
76053983022787*sqrt(14) - 302971189122193607250286930 61) - 5295153361391555
319192290416119266357486986850507581489439885963489257332413262896909021356
322314724080758142729925427200*x + 24256513529606021838214197524700823604 47
570412145758189601975348144486083461120 0*sqrt(1667893719659639595810987428 1
7586289130679764812156476721038706576007991289033281726) + 7942730042087332
978788435624178899536230480275761372234159828945233885998619894345363532034
48347208612113721409488814080 0) + 325965782049849620329125967464809624391 09
746225179791317800502510255796338156401518821079958557305776*(5285259508814
16658752513929485454513733769472507904 00*x^57 - 10989677950662733151628560 9
342129905944018374791004160 0*x^56 + 10607209489316853390896228799650834948 4
4457992021082112 0*x^55 - 635711675502347539940141044000742233465808803157 19
352320*x^54 + 2687511020850507521524837838166725999310311212834829107 20*x^5
3 - 870946973219521114804962921504691759517713269107195904000*x^52 + 231375
80219324483124253216493360849810295060724976084582 40*x^51 - 531660404716026
7290459856323292969345744886768161070776320*x^50 + 10935442488009047264366 4
4839127560436875431043788307431424 0*x^49 - 204765576911600011474715598862 37
05646599840563445635219456 0*x^48 + 35302794239198802111604239039735944127 46
2536376667298856960*x^47 - 567147089880685206131013139748919822977787771083
53803878400*x^46 + 856402416640309357300397975158829414085522674588022535 61

856*x^45 - 12206325070017431655342522094916509561349432305907127654809 6*x^4
4 + 16501806799621223134371667301124433392748840364433110309273 6*x^43 - 212
7625797424699058202268238216644653085591759434574043545 60*x^42 + 2622073258
5283145852092858573622401829922651309656318882611 2*x^41 - 30944053790611241
11186204458928150796845040115639697413242 88*x^40 + 3510873064125786600001 08
0192194053518260654731309727078154 24*x^39 - 383554582100586246362167645670 8
92818138191443491318786949120*x^38 + 403492607520849908998883514652547403 91
576326886092710137036 8*x^37 - 410091833382540310980618746942733242840005307
5285885468016 64*x^36 + 403232407441991792232348027512081003879684846626157 3
08542976*x^35 - 38299557981652752964191530266540999587508486258926597505024
0*x^34 + 352587259766861713156680120052199648639816399610100338851840*x^33
- 315079971582181801347294250924732868231627903206246048727040*x^32 + 27231
6634459399870536836933035003973818695505518285221314560*x^31 - 228671395190
67109702086956450087572679758981616542114327756 8*x^30 + 1868861116889859290
98566117844019918629526116042561389293568*x^29 - 147575029055999994839406 28
764884369390118188761027353386188 8*x^28 + 113537974641311616719165089124033
8469388884352161872510003 20*x^27 - 851964156232333961701971885129750263083 9
387449450605004697 6*x^26 + 614907175198867437939779042891506812095480715428
12762022208*x^25 - 4349992956862403378514767029243146544060998598702281930 9
056*x^24 + 3001530719918349241842611523291770226136474186651754731838 4*x^23
 - 197145306642523678936947946324421753937272206601878137222 24*x^22 + 12908
687419060491715559483506875260114803121732707547895900*x^21 - 8152620728427
620176711248504306621849196751343566681977176*x^20 + 482656622988964999865 1
0829185742816673107671860732691740 97*x^19 - 2980031288821257171626437270731
35846361369025874804487563 1*x^18 + 167438179771788833624008261913648191314 1
447194739865411447*x^17 - 893893211516133869906083243128705875958804128593 5
29339933*x^16 + 539470558336347193822687371553759571054898242285358894340*x
^15 - 242275403875001443743419975934494764357192021279244664252*x^14 + 1307
86287070310326986845647168054788265093887227255620788*x^13 - 73538381632205
950970872198730312615396368885742113789428*x^12 + 203326305537313866021172 9
32490188746689500078791161545 90*x^11 - 185841889627321318186553873625864802
126238511202776650 58*x^10 + 457852904347974424322212486408517702165206452 34
34159250*x^9 - 158997639731645917754275134081471967883696538641872875 8*x^8
+ 213688451814064520882203297270884420940114772593324864 4*x^7 + 52743183825
24294066481060984967338478430238303379087 72*x^6 + 591293371646480980468080 8
5686210395228519470244720623 2*x^5 + 153671770129689537528196360895808154174
885919188027188*x^4 + 772867990754595680781483763124945886247480770883376 25
*x^3 + 132031550647631419600701555288103131051996950069692 41*x^2 + 41100428
98499321701713055782797445718557813264221007*x + 142488114863139797187698 61
885292400390994452676362 7)*sqrt(-2*x + 3))/(549755813888*x^58 - 11269994184
704*x^57 + 107064944754688*x^56 - 630638638006272*x^55 + 2618521301286912*x
^54 - 8342252417974272*x^53 + 21849572376576000*x^52 - 49684091485814784*x^
51 + 101394501297242112*x^50 - 188583312363618304*x^49 + 323261995581177856
*x^48 - 517079841212727296*x^47 + 778117896260812800*x^46 - 110564116538798
8992*x^45 + 1491287028233404416*x^44 - 1919929663119949824*x^43 + 236305093

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9901804544*x^42 - 2786274020645928960*x^41 + 3161145685194047488*x^40 - 345
3753931369283584*x^39 + 3634098467102523392*x^38 - 3697893960325791744*x^37
 + 3640651752731836416*x^36 - 3461798212247617536*x^35 + 319454025178939392
0*x^34 - 2861544579495297024*x^33 + 2477632938217930752*x^32 - 208843025712
7768064*x^31 + 1712761005459316736*x^30 - 1355447485390974976*x^29 + 104894
0886155151360*x^28 - 790511024135089152*x^27 + 571750925528393856*x^26 - 40
8374103192240192*x^25 + 282845069599813728*x^24 - 186113897194906128*x^23 +
 123982890381352520*x^22 - 78116367732251996*x^21 + 46488580159296898*x^20
- 29591055660829971*x^19 + 16200795673453545*x^18 - 8941894120163277*x^17 +
 5578893209169441*x^16 - 2296849711499532*x^15 + 1448289882400788*x^14 - 75
6896247319212*x^13 + 182213447974992*x^12 - 240797810407770*x^11 + 25549234
281774*x^10 - 26500281727302*x^9 + 25520701332582*x^8 + 9965507230260*x^7 +
 10389354811164*x^6 + 3755740313808*x^5 + 1820618017974*x^4 + 463742325333*
x^3 + 139858796529*x^2 + 19758444939*x + 3486784401)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(41/2)/(2*x**2+x+1)**20,x)
```

```
[Out] Exception raised: SystemError
```

Giac [A]
time = 0.73, size = 2044, normalized size = 1.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x)
```

```
[Out] -115/363805261691069042491598265308929913400590336*sqrt(7)*(241833327334298
28161949068361*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) + 8) + 8061
1109111432760539830227 87*14^(3/4)*sqrt(7)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) +
 8) - 5642777637800293237788115950 9*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14)
 + 4) - 16928332913400879713364347852 7*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(
14) - 4) + 24237695129775488580022954448 8*14^(1/4)*sqrt(7)*sqrt(-2*sqrt(14)
 + 8) - 16966386590842842006016068114 16*14^(1/4)*sqrt(2*sqrt(14) + 8))*arct
an(1/28*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4) + 2*sqrt(-2*x + 3))
/sqrt(-1/8*sqrt(14) + 1/2)) - 115/36380526169106904249159826530892991340059
0336*sqrt(7)*(24183332733429828161949068361*14^(3/4)*sqrt(7)*(sqrt(14) + 4)
*sqrt(-2*sqrt(14) + 8) + 80611109111432760539830227 87*14^(3/4)*sqrt(7)*(sqr
t(14) - 4)*sqrt(-2*sqrt(14) + 8) - 5642777637800293237788115950 9*14^(3/4)*s
qrt(2*sqrt(14) + 8)*(sqrt(14) + 4) - 16928332913400879713364347852 7*14^(3/4
)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 24237695129775488580022954448 8*14^(
```

1/4)*sqrt(7)*sqrt(-2*sqrt(14) + 8) - 169663865908428420060160681141 6*14^(1/
4)*sqrt(2*sqrt(14) + 8))*arctan(-1/28*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqr
t(14) + 4) - 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2)) - 115/72761052338
2138084983196530617859826801180672*sqrt(7)*(806111091114327605398302278 7*14
^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 241833327334298281619 4
9068361*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 169283329134
008797133643478527*14^(3/4)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) + 8) + 56427776
3780029323778811595 09*14^(3/4)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) + 8) + 24237
69512977548858002295 44488*14^(1/4)*sqrt(7)*sqrt(2*sqrt(14) + 8) + 169663865
908428420060160681141 6*14^(1/4)*sqrt(-2*sqrt(14) + 8))*log(14^(1/4)*sqrt(1/
2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 115/7276105233
82138084983196530617859826801180672*sqrt(7)*(806111091114327605398302278 7*1
4^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 241833327334298281619
49068361*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 16928332913
4008797133643478527*14^(3/4)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) + 8) + 5642777
63780029323778811595 09*14^(3/4)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) + 8) + 2423
7695129775488580022954 4488*14^(1/4)*sqrt(7)*sqrt(2*sqrt(14) + 8) + 16966386
5908428420060160681141 6*14^(1/4)*sqrt(-2*sqrt(14) + 8))*log(-14^(1/4)*sqrt(
1/2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 1/2411259743
1479447071556104988390860001680293888*(385912796294138623132486146144809805
*(2*x - 3)^37*sqrt(-2*x + 3) + 4994416662656937088431754278268478521 5*(2*x
- 3)^36*sqrt(-2*x + 3) + 31571043251901908187904170157686721002 51*(2*x - 3)
^35*sqrt(-2*x + 3) + 12986266353974282972701016844877225753779 3*(2*x - 3)^3
4*sqrt(-2*x + 3) + 390705603293305902738518568283243321795620 0*(2*x - 3)^33
*sqrt(-2*x + 3) + 9162634230824006291365946903167694132884768 8*(2*x - 3)^32
*sqrt(-2*x + 3) + 17430518397837166544585701688089337301746270 04*(2*x - 3)^
31*sqrt(-2*x + 3) + 27638544507622729125093621837291437830917462708*(2*x -
3)^30*sqrt(-2*x + 3) + 372498510070445411629537388290851713705080145718*(2*
x - 3)^29*sqrt(-2*x + 3) + 432995351693068734233747201427266636396965158731
4*(2*x - 3)^28*sqrt(-2*x + 3) + 438994445601123086236053311571438967258284 1
5934650*(2*x - 3)^27*sqrt(-2*x + 3) + 391609357365773780316151578457972453 6
48367489837454*(2*x - 3)^26*sqrt(-2*x + 3) + 309503170175884957504062693739
936319820203275388425 2*(2*x - 3)^25*sqrt(-2*x + 3) + 217907196222246813794 1
65678259100933686683346767977 80*(2*x - 3)^24*sqrt(-2*x + 3) + 1372614029241
98725794062163116053277099106968046586092*(2*x - 3)^23*sqrt(-2*x + 3) + 776
17111830556525453848713885531739126913521685009518 76*(2*x - 3)^22*sqrt(-2*x
+ 3) + 395009552637699460788078433865593460380216799543316640 5*(2*x - 3)^21
*sqrt(-2*x + 3) + 181258038168328615978327668733398821189240151833380076 55*
(2*x - 3)^20*sqrt(-2*x + 3) + 750834145086940501444266399776850855400388047
54309758915*(2*x - 3)^19*sqrt(-2*x + 3) + 280932652073348343517776090631271
895235611343284275820345*(2*x - 3)^18*sqrt(-2*x + 3) + 949449516366891514 86
664177930959753647849048998795446258 0*(2*x - 3)^17*sqrt(-2*x + 3) + 2896666
953760570249650513456393600983703549509654469117900*(2*x - 3)^16*sqrt(-2*x
+ 3) + 7968283692957988567650795129108295704483768260379820818752*(2*x - 3)
^15*sqrt(-2*x + 3) + 1972749457881227765860600971283186162692222652326643 57

34336*(2*x - 3)^14*sqrt(-2*x + 3) + 438441033794236958424800303207601166664
91172278035172870400*(2*x - 3)^13*sqrt(-2*x + 3) + 871807724494537191124097
15850861698835279004734515297162496*(2*x - 3)^12*sqrt(-2*x + 3) + 154427451
620079851403012035013949923367197814895239131529728*(2*x - 3)^11*sqrt(-2*x
+ 3) + 242351725944359254347670713000225450988365795247877220072960*(2*x -
3)^10*sqrt(-2*x + 3) + 3346460914322591740452610992480923909021262686637826
08549888*(2*x - 3)^9*sqrt(-2*x + 3) + 4030345196682619867089916868903813178
41470126237337802123264*(2*x - 3)^8*sqrt(-2*x + 3) + 4186467946454733297148
9609516908707261586337434634780753920*(2*x - 3)^7*sqrt(-2*x + 3) + 3696217
15112196031007775193340564258755874521674193323966464*(2*x - 3)^6*sqrt(-2*x
+ 3) + 272008032423513780299697431707644217391623176190273099661312*(2*x -
3)^5*sqrt(-2*x + 3) + 1623771097205550225359730217062113881706506204116787
44248320*(2*x - 3)^4*sqrt(-2*x + 3) + 7555666674888429176622089229716660337
6040200755275694800896*(2*x - 3)^3*sqrt(-2*x + 3) + 25715217479147156311480
451271603595696519278112265697558528*(2*x - 3)^2*sqrt(-2*x + 3) - 569505889
8488457914056616763522088045930624578769252515840*(-2*x + 3)^(3/2) + 616047
393270423249767303997369406352855404127230297374720*sqrt(-2*x + 3))/((2*x -
3)^2 + 14*x - 7)^19 + 1/43768013439874312895399492130064309616640*(9918560
55479912729664933375*(2*x - 3)^19 - 465215115289202563341931875*(2*x - 3)^1
8 + 376870004361848629670138100*(2*x - 3)^17 - 347816399209073565143694750*
(2*x - 3)^16 + 333480450533749292133360000*(2*x - 3)^15 - 31977824826109400
5065228000*(2*x - 3)^14 + 300292311231869293365336000*(2*x - 3)^13 - 272225
522279980529558298000*(2*x - 3)^12 + 235508819476507302437712000*(2*x - 3)^
11 - 192403914635036320216640640*(2*x - 3)^10 + 146870291549367152461094400
*(2*x - 3)^9 - 103544963718981484751251200*(2*x - 3)^8 + 665207702174834449
75816704*(2*x - 3)^7 - 38308222816032989365145600*(2*x - 3)^6 + 19364536310
461049463275520*(2*x - 3)^5 - 8351885944887834417868800*(2*x - 3)^4 + 29503
96963171184804659200*(2*x - 3)^3 - 800398003403553957642240*(2*x - 3)^2 + 2
96499732880545408614400*x - 458814330239510651535360)/((2*x - 3)^19*sqrt(-2
*x + 3))

Mupad [B]
time = 0.97, size = 1017, normalized size = 0.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - 2*x)^(41/2)*(x + 2*x^2 + 1)^20),x)
```

```
[Out] ((64356352*(2*x - 3)^2)/38073 - (5767168*x)/1443 - (7517962240*(2*x - 3)^3)
/5444439 + (1357449428992*(2*x - 3)^4)/1181443263 - (34130408095744*(2*x -
3)^5)/34261854627 + (1965832636456960*(2*x - 3)^6)/2158496841501 - (9552588
571922432*(2*x - 3)^7)/10792484207505 + (69571472879183872*(2*x - 3)^8)/755
47389452535 - (5204838729946112*(2*x - 3)^9)/5036492630169 + (3250820527817
55904*(2*x - 3)^10)/257635969158645 - (461538785202937088*(2*x - 3)^11)/272
428464995505 + (17726678744562203264*(2*x - 3)^12)/6992330601551295 - (1432
```

```
471149647610304*(2*x - 3)^13)/332968123883395 + (2043463601243388704*(2*x -
 3)^14)/241114848329355 - (96972768477343976816*(2*x - 3)^15)/4840844262612
435 + (10833870670122545927656*(2*x - 3)^16)/181389282075536535 - (44340157
049832305729324*(2*x - 3)^17)/181389282075536535 + (69150977813218626180728
2*(2*x - 3)^18)/423241658176251915 - (135773583315370822397034407*(2*x - 3)^
19)/423241658176251915 + (50949594385895993964075303946506726149 81*(2*x - 3
)^20)/203594616979243053623625646080 + (4747534027372414822574988626088463 2
526403*(2*x - 3)^21)/203594616979243053623625646080 + (547362406727667345 86
817623075460075234 1499*(2*x - 3)^22)/518240843219891409223774371840 + (1363
21739916884674180325053144349616764 7559*(2*x - 3)^23)/438511482724523500112
424468480 + (40035704814224807138997531075224002020138 8159*(2*x - 3)^24)/59
856817391897457765345939947520 + (1678035321867106187517785121743165245 0855
3291*(2*x - 3)^25)/149642043479743644413364849 86880 + (5110877106069831531 9
124863093548144195799415067*(2*x - 3)^26)/3351981773946257634859372637061 12
0 + (3939870831872067350820038893812216643460900 53*(2*x - 3)^27)/2280259710
16752220058460723609 60 + (194509919512254900809288150922829785396777195281*
(2*x - 3)^28)/1168896208350489841899678663180 2880 + (3990494121741585984967
8112809547525787872838871677*(2*x - 3)^29)/28871736346257099094922062980553
11360 + (2982029082982520685654165296540313515739996589545 19*(2*x - 3)^30)/
29783475388770481171603812337833738240 + (1727837071783712649879020657943 55
72552986029824411*(2*x - 3)^31)/27075886717064073792367102125303398 40 + (13
6589909140623157483229616961110867609087469195457*(2*x - 3)^32)/37906241403
88970330931394297542475776 0 + (12124448510282132213121066777925721516746772
830847*(2*x - 3)^33)/66893367183334770545848134662514278 40 + (5268103225464
003924284598756770514565895682824129*(2*x - 3)^34)/645866993494266750097844
0588104826880 + (61717610092862026266313005902016039510287732711413*(2*x -
3)^35)/1873014281133373575283747770550399795 20 + (236279120368028123256783 3
3491117787914105632657738 7*(2*x - 3)^36)/1972268029364372858760322438733412
433920 + (10069182899664488193697417735778758301092236673 48001*(2*x - 3)^37
)/2563948438173684716388419170353436164096 0 + (834315251412234134041251370 6
840068954518337251868859*(2*x - 3)^38)/7179055626886317205887573676989621 25
946880 + (669016452611293431036170511813057767424944839195492 3*(2*x - 3)^39
)/215371668806589516176627210309688637784064 0 + (305581065207833944849384 01
5433140408874881230574613*(2*x - 3)^40)/407459913958412598171997424910221 74
7159040 + (731867339371195846981841457176808134814103613309*(2*x - 3)^41)/4
4775814720704681117801914825299093094 40 + (98156536112115492322904146290693
53244130713267641*(2*x - 3)^42)/305594935468809448628998068682666310369280
+ (111998015172594816786872871418593904041451326 17*(2*x - 3)^43)/1971580228
83102870083224560440429877657 6 + (136564747272427838170630719416707180545 54
74221*(2*x - 3)^44)/151422305976050793637586261153308213248 0 + (40305011659
0493478621865418191675419450056550 1*(2*x - 3)^45)/31462190241690553789142 92
315074292875264 0 + (142800962844555649098866729552205491584243363 1*(2*x - 3
)^46)/880941326767335506096001848220802005073 92 + (160089053926633694221849
8464088424576826036 21*(2*x - 3)^47)/880941326767335506096001848220802005073
92 + (21001998140967208924158271678544758006824603 89*(2*x - 3)^48)/11716519
```

646005562231076824581336666667483136 + (731521029491460764762993572365861799703833*(2*x - 3)^49)/47435302210548834943630868750350877196288 + (14527825011424680881745287944067060548347 7*(2*x - 3)^50)/1269591912105865876432473251847626419077 12 + (305417624689119903340176820462205459591 7*(2*x - 3)^51)/423197304035288625477491083949208806359 04 + (4322624121559696023583903787645234779 3*(2*x - 3)^52)/1139377357018084760900937533709408324812 8 + (16757214169421265746492710756597657 5*(2*x - 3)^53)/10357975972891679644553977579176439316 48 + (935756145095208333386444273642906999*(2*x - 3)^54)/174013996344580218028506823330164180516864 + (325001551972552320039960952878829 9*(2*x - 3)^55)/248591423349400311469295461900234543595 52 + (359910711199433658030176367535945*(2*x - 3)^56)/174013996344580218028506823330164180516864 + (927027754781476746208047620505*(2*x - 3)^57)/580046654481934060095022744433880601722 88 + 79953920/10101)/(5976303958948914397184*(3 - 2*x)^(39/2) - 56774887610014686773248*(3 - 2*x)^(41/2) + 263597692475068188590080*(3 - 2*x)^(43/2) - 796876101097706139353088*(3 - 2*x)^(45/2) + 1763207861643670399942656*(3 - 2*x)^(47/2) - 3043249843014358669590528*(3 - 2*x)^(49/2) + 4264137522753475514499072*(3 - 2*x)^(51/2) - 4984324075408572529754112*(3 - 2*x)^(53/2) + 4956568063057422401458176*(3 - 2*x)^(55/2) - 4255315771373708518529024*(3 - 2*x)^(57/2) + 3189779613484873345291264*(3 - 2*x)^(59/2) - 2106235539086912777861632*(3 - 2*x)^(61/2) + 1233708448609783150169088*(3 - 2*x)^(63/2) - 644615788666077029453568*(3 - 2*x)^(65/2) + 301787157080763250721664*(3 - 2*x)^(67/2) - 127037834354660188150464*(3 - 2*x)^(69/2) + 48214067552985728953272*(3 - 2*x)^(71/2) - 16530947936007918636468*(3 - 2*x)^(73/2) + 5127550624086495626518*(3 - 2*x)^(75/2) - 1440010379792375040419*(3 - 2*x)^(77/2) + 366253616006178259037*(3 - 2*x)^(79/2) - 84341571102081217533*(3 - 2*x)^(81/2) + 17570724326889842913*(3 - 2*x)^(83/2) - 3306899061710229804*(3 - 2*x)^(85/2) + 561126236614140036*(3 - 2*x)^(87/2) - 85611621840452988*(3 - 2*x)^(89/2) + 11703514272799272*(3 - 2*x)^(91/2) - 1427192816292922*(3 - 2*x)^(93/2) + 154386157043846*(3 - 2*x)^(95/2) - 14711313018374*(3 - 2*x)^(97/2) + 1223975378934*(3 - 2*x)^(99/2) - 87916389372*(3 - 2*x)^(101/2) + 5372380188*(3 - 2*x)^(103/2) - 273870408*(3 - 2*x)^(105/2) + 11333994*(3 - 2*x)^(107/2) - 365883*(3 - 2*x)^(109/2) + 8645*(3 - 2*x)^(111/2) - 133*(3 - 2*x)^(113/2) + (3 - 2*x)^(115/2)) - (atan(((3 - 2*x)^(1/2)*(- 7^(1/2)*8174286676615564254062463385463197516747256637092086555i - 2647038820161175221602427690537407609363641517340918882660 1)^(1/2)*124320682492976962848972490013663405232829839379374271393356 25i)/(546445444973747744833043391094451536038531013369836763902595689946460970498681963431302609552363376731445542355720493177241 6*((7^(1/2)*37665585007379907233596472018658739840629614558598888628455806290315259752942013758759812 5i)/273222722486873872416521695547225768019265506684918381951297844973230485249340981715651304776181688365722771177860246588620 8 + 777520974123765253499790235238946338576340593433713638916736753944556393731049211251114562 5/390318174981248389166459565067465382884665009549883402787568349961757836070487116736644721108830983379603958825514637983744)) + (124320682492976962848972490013663405232829839379374271393356 25*7^(1/2)*(3 - 2*x)^(1/2)*(- 7^(1/2)*817428667661556425406246338546319751 67

47256637092086555i - 264703882016117522160242769053740760936364151734091888
26601)^(1/2))/(54644544497374774483304339109445153603853101336983676390259 5
68994646097049868196343130260955236337673144554235572049317772416*((7^(1/2)*
37665585007379907233596472018658739840629614558598888628455806290315259752 9
420137587598125i)/2732227224868738724165216955472257680192655066849183819 51
2978449732304852493409817156513047761816883657227711778602465886208 + 77752
09741237652534997902352389463385763405934337136389167367539445563937310492 1
1251145625/390318174981248389166459565067465382884665009549883402787568349 9
61757836070487116736644721108830983379603958825514637983744)))*(- 7^(1/2)*8
174286676615564254062463385463197516747256637092086555i - 26470388201611752
2160242769053740760936364151734091888266 01)^(1/2)*115i)/8120653162747076841
3303184220743284241 2032 + (atan(((3 - 2*x)^(1/2)*(7^(1/2)*81742866766155642
5406246338546319751674725663709208 6555i - 264703882016117522160242769053740
7609363641517340918882660 1)^(1/2)*124320682492976962848972490013663405232 82
98393793742713933562 5i)/(546445444973747744833043391094451536038531013369 83
676390259568994646097049868196343130260955236337673144554235572049317772416*
((7^(1/2)*376655850073799072335964720186587398406296145585988886284558062 90
31525975294201375875981 25i)/27322272248687387241652169554722576801926550668
491838195129784497323048524934098171565130477618168836572277117786024658862
08 - 777520974123765253499790235238946338576340593433713638916736753944556 3
937310492112511456 25/3903181749812483891664595650674653828846650095498834 02
787568349961757836070487116736644721108830983379603958825514637983744)) - (
1243206824929769628489724900136634052328298393793742713933562 5*7^(1/2)*(3 -
 2*x)^(1/2)*(7^(1/2)*817428667661556425406246338546319751674725663709208655
5i - 26470388201611752216024276905374076093636415173409188826601)^(1/2))/(5
4644544497374774483304339109445153603853101336983676390259568994646097049 86
8196343130260955236337673144554235572049317772416*((7^(1/2)*3766558500737990
723359647201865873984062961455859888862845580629031525975294201375875981 25i
)/2732227224868738724165216955472257680192655066849183819512978449732304852
4934098171565130477618168836572277117786024658862 08 - 777520974123765253499
79023523894633857634059343371363891673675394455639373104921125114562 5/39031
8174981248389166459565067465382884665009549883402787568349961757836070487 11
673664472110883098337960395882551463798374 4)))*(7^(1/2)*817428667661556425 4
06246338546319751674725663709208655 5i - 264703882016117522160242769053740 76
0936364151734091888266 01)^(1/2)*115i)/812065316274707684133031842207432842 4
12032

3.50 $\int \frac{1}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)^5}\,dx$

Optimal. Leaf size=378

$$-\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}}-\frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}}-\frac{30316369-15043110x}{6860000000\left(3-2x+x^2\right)^{5/2}}-\frac{63043297-2}{41160000000\left(3\right.}$$

[Out] 1/123480000*(-3450497+2004270*x)/(x^2-2*x+3)^(9/2)+1/411600000*(-4878869+25
78034*x)/(x^2-2*x+3)^(7/2)+1/6860000000*(-30316369+15043110*x)/(x^2-2*x+3)^
(5/2)+1/41160000000*(-63043297+29625922*x)/(x^2-2*x+3)^(3/2)+1/280*(-1+10*x
)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)^4+1/1050*(28+67*x)/(x^2-2*x+3)^(9/2)/(2*x^2
+x+1)^3+1/117600*(5485+8878*x)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)^2+3/343000*(88
22+8233*x)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)-31/411600000000*(7434109-3088870*x
)/(x^2-2*x+3)^(1/2)-1/9604000000000*arctanh(1/7*(308108167+x*(932587773-620
347970*2^(1/2))-312239803*2^(1/2))*35^(1/2)/(-151363871237318045+1103204757
41093888*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(-1059547098661226315O+772243330
1876572160*2^(1/2))^(1/2)+1/9604000000000*arctan(1/7*(308108167+312239803*2
^(1/2)+x*(932587773+620347970*2^(1/2)))*35^(1/2)/(151363871237318045+110320
475741093888*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(1059547098661226315O+772243
3301876572160*2^(1/2))^(1/2)

Rubi [A]
time = 0.51, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {988, 1074, 1049, 1043, 212, 210}

$$-\frac{63043297-29625922x}{41160000000\left(x^2-2x+3\right)^{3/2}}-\frac{31(7434109-3088870x)}{411600000000\sqrt{x^2-2x+3}}+\frac{3(8233x+8822)}{343000\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)}+\frac{8878x+5485}{117600\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^2}-\frac{30316369-15043110x}{6860000000\left(x^2-2x+3\right)^{5/2}}+\frac{67x+28}{1050\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^3}-\frac{4878869-2578034x}{411600000\left(x^2-2x+3\right)^{7/2}}-\frac{1-10x}{280\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)^4}-\frac{3450497-2004270x}{123480000\left(x^2-2x+3\right)^{9/2}}+\frac{\sqrt{\frac{1}{70}\left(151363871237318045+110320475741093888\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{5}{7\left(151363871237318045+110320475741093888\sqrt{2}\right)}}\left(\left(932587773+620347970\sqrt{2}\right)x+312239803\sqrt{2}+308108167\right)}{\sqrt{x^2-2x+3}}\right)}{137200000000}-\frac{\sqrt{\frac{1}{70}\left(110320475741093888\sqrt{2}-151363871237318045\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{5}{7\left(110320475741093888\sqrt{2}-151363871237318045\right)}}\left(\left(932587773-620347970\sqrt{2}\right)x-312239803\sqrt{2}+308108167\right)}{\sqrt{x^2-2x+3}}\right)}{137200000000}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - 2*x + x^2)^(11/2)*(1 + x + 2*x^2)^5),x]

[Out] -1/123480000*(3450497 - 2004270*x)/(3 - 2*x + x^2)^(9/2) - (4878869 - 25780
34*x)/(411600000*(3 - 2*x + x^2)^(7/2)) - (30316369 - 15043110*x)/(68600000
00*(3 - 2*x + x^2)^(5/2)) - (63043297 - 29625922*x)/(41160000000*(3 - 2*x +
 x^2)^(3/2)) - (31*(7434109 - 3088870*x))/(411600000000*Sqrt[3 - 2*x + x^2]
) - (1 - 10*x)/(280*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^4) + (28 + 67*x)/
(1050*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^3) + (5485 + 8878*x)/(117600*(3
 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^2) + (3*(8822 + 8233*x))/(343000*(3 - 2
x + x^2)^(9/2)(1 + x + 2*x^2)) + (Sqrt[(151363871237318045 + 110320475741

```
093888*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(151363871237318045 + 110320475741093
888*Sqrt[2]))]*(308108167 + 312239803*Sqrt[2] + (932587773 + 620347970*Sqrt
[2])*x))/Sqrt[3 - 2*x + x^2]])/137200000000 - (Sqrt[(-151363871237318045 +
110320475741093888*Sqrt[2])/70]*ArcTanh[(Sqrt[5/(7*(-151363871237318045 + 1
10320475741093888*Sqrt[2]))]*(308108167 - 312239803*Sqrt[2] + (932587773 -
620347970*Sqrt[2])*x))/Sqrt[3 - 2*x + x^2]])/137200000000
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)^5}\,dx = -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} - \frac{\int \frac{-1235+1335x-800x^2}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)^4}\,d}{1400}$$

$$= -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}(1}$$

$$= -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}(1}$$

$$= -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}(1}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x\right.}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{}{28}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{}{68}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{}{68}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{}{68}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{}{68}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{}{68}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{}{68}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order

3 in optimal.
time = 2.58, size = 733, normalized size = 1.94

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x + x^2)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] ((-53205422447 + 261702502714*x - 266966654968*x^2 + 1002897791524*x^3 - 14
09335257371*x^4 + 2503427226914*x^5 - 3359813871472*x^6 + 4591320676952*x^7
 - 5134334619701*x^8 + 5380603084494*x^9 - 4915797913008*x^10 + 39996561325
32*x^11 - 2679143870481*x^12 + 1459208021718*x^13 - 606785954952*x^14 + 188
603773872*x^15 - 38639385552*x^16 + 4596238560*x^17)/((3 - 2*x + x^2)^(9/2)
*(1 + x + 2*x^2)^4) - 49392*RootSum[14 + 7*#1 - 5*#1^2 - #1^3 + #1^4 & , (-
6014*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 10727*Log[-x + Sqrt[3 - 2*x + x^2
] - #1]*#1 + 3229*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1^2)/(7 - 10*#1 - 3*#
1^2 + 4*#1^3) & ] - 56448*RootSum[14 + 7*#1 - 5*#1^2 - #1^3 + #1^4 & , (737
81*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 60407*Log[-x + Sqrt[3 - 2*x + x^2]
- #1]*#1 + 13104*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1^2)/(7 - 10*#1 - 3*#1
^2 + 4*#1^3) & ] - 504*RootSum[14 + 7*#1 - 5*#1^2 - #1^3 + #1^4 & , (275935
046*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 208696097*Log[-x + Sqrt[3 - 2*x +
x^2] - #1]*#1 + 50007219*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1^2)/(7 - 10*#
1 - 3*#1^2 + 4*#1^3) & ] + 1440*RootSum[14 + 7*#1 - 5*#1^2 - #1^3 + #1^4 &
, (3276009822*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 2447831621*Log[-x + Sqrt
[3 - 2*x + x^2] - #1]*#1 + 590084719*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1^
2)/(7 - 10*#1 - 3*#1^2 + 4*#1^3) & ] - 18*RootSum[14 + 7*#1 - 5*#1^2 - #1^3
 + #1^4 & , (254137663854*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 189631531133
*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1 + 45801521671*Log[-x + Sqrt[3 - 2*x
+ x^2] - #1]*#1^2)/(7 - 10*#1 - 3*#1^2 + 4*#1^3) & ])/1234800000000
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 + x + 2*x^2)^5*(3 - 2*x + x^2)^(11/2)),x]')
```

```
[Out] Timed out
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 21027 vs. 2(298) = 596.
time = 1.52, size = 21028, normalized size = 55.63

method	result

risch	$\frac{4596238560x^{17}-38639385552x^{16}+188603773872x^{15}-606785954952x^{14}+1459208021718x^{13}-2679143870481x^{12}+3999656132532x}{}$
trager	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^5*(x^2 - 2*x + 3)^(11/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1873 vs. 2(298) = 596.
time = 0.38, size = 1873, normalized size = 4.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/71086524447232167580280752950240000000000*(2646020608687651230198155914607
4412800*x^18 - 211681648695012098415852473168595302400*x^17 + 1018717934344
745723626290027123864892800*x^16 - 3214915039555496244690759436248041155200
*x^15 + 7688343631118056605744516779381246569200*x^14 - 1398091139115337718
7559506313807067863200*x^13 + 20977982138251784909414754860497120398000*x^1
```

2 - 25712705264922250829450580100197810638400*x^11 + 2875728272779347952619
7333249442997761200*x^10 - 27283780001330543747380735174495978898400*x^9 +
25562212842803140665733059982554512415600*x^8 - 180458605512497813899514233
37622749529600*x^7 + 15206349685551845663545027271759639106000*x^6 - 726663
4096608462190931685680490685615200*x^5 - 3602042876982878244*33780221308347
3608^(1/4)*sqrt(205487899)*sqrt(35)*sqrt(2)*(16*x^18 - 128*x^17 + 616*x^16
- 1944*x^15 + 4649*x^14 - 8454*x^13 + 12685*x^12 - 15548*x^11 + 17389*x^10
- 16498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*x^4 - 396*
x^3 + 1647*x^2 + 162*x + 243)*sqrt(151363871237318045*sqrt(2) + 22064095148
2187776)*arctan(1/964393622349963919677467835514205441102895152270484353118
304*sqrt(205487899)*(12071210867722009415131100925112940*sqrt(4167294734812
9)*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) + sqrt(205487899)*(5*337802213083473608
^(3/4)*sqrt(41672947348129)*sqrt(35)*(534678000*sqrt(2) - 573381349) + 2876
830586*337802213083473608^(1/4)*sqrt(41672947348129)*sqrt(35)*(201502465*sq
rt(2) + 108532744))*sqrt(151363871237318045*sqrt(2) + 220640951482187776) +
2414242173544401883026220185022588*sqrt(41672947348129)*sqrt(7)*(125*sqrt(
2) + 172))*sqrt(164483605088694913184970968*x^2 + sqrt(205487899)*(33780221
3083473608^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(89801606*sqrt(2) - 4
2834985) - 337802213083473608^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(89801606*x -
132636591) - 42834985*x + 222438197))*sqrt(151363871237318045*sqrt(2) + 22
0640951482187776) - 41120901272173728296242742*sqrt(x^2 - 2*x + 3)*(4*x + 1
) - 123362703816521184888728226*x + 205604506360868641481213710*sqrt(2) + 2
87846308905216098073699194) + 5/476*sqrt(7)*sqrt(2)*(sqrt(2)*(10*x - 19) +
9*x - 29) + 1/1149179274607135296320480808070751888*sqrt(205487899)*(5*3378
02213083473608^(3/4)*sqrt(35)*(sqrt(2)*(534678000*x + 38703349) - 573381349
*x - 495974651) + 2876830586*337802213083473608^(1/4)*sqrt(35)*(sqrt(2)*(20
1502465*x - 310035209) + 108532744*x - 511537674) - (5*337802213083473608^(
3/4)*sqrt(35)*(534678000*sqrt(2) - 573381349) + 2876830586*3378022130834736
08^(1/4)*sqrt(35)*(201502465*sqrt(2) + 108532744))*sqrt(x^2 - 2*x + 3))*sqr
t(151363871237318045*sqrt(2) + 220640951482187776) - 1/476*sqrt(x^2 - 2*x +
3)*(5*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) + sqrt(7)*(125*sqrt(2) + 172)) + 1/
476*sqrt(7)*(25*sqrt(2)*(5*x - 1) + 172*x - 82)) - 3602042876982878244*3378
02213083473608^(1/4)*sqrt(205487899)*sqrt(35)*sqrt(2)*(16*x^18 - 128*x^17 +
616*x^16 - 1944*x^15 + 4649*x^14 - 8454*x^13 + 12685*x^12 - 15548*x^11 + 1
7389*x^10 - 16498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*
x^4 - 396*x^3 + 1647*x^2 + 162*x + 243)*sqrt(151363871237318045*sqrt(2) + 2
20640951482187776)*arctan(-1/9643936223499639196774678355142054411028951522
70484353118304*sqrt(205487899)*(12071210867722009415131100925112940*sqrt(41
672947348129)*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) - sqrt(205487899)*(5*3378022
13083473608^(3/4)*sqrt(41672947348129)*sqrt(35)*(534678000*sqrt(2) - 573381
349) + 2876830586*337802213083473608^(1/4)*sqrt(41672947348129)*sqrt(35)*(2
01502465*sqrt(2) + 108532744))*sqrt(151363871237318045*sqrt(2) + 2206409514
82187776) + 2414242173544401883026220185022588*sqrt(41672947348129)*sqrt(7)
*(125*sqrt(2) + 172))*sqrt(164483605088694913184970968*x^2 - sqrt(205487899
)*(337802213083473608^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(89801606*

```
sqrt(2) - 42834985) - 337802213083473608^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(8
9801606*x - 132636591) - 42834985*x + 222438197))*sqrt(151363871237318045*s
qrt(2) + 220640951482187776) - 411209012721737282962427 42*sqrt(x^2 - 2*x +
3)*(4*x + 1) - 1233627038165211848887 28226*x + 2056045063608686414812137 10*
sqrt(2) + 2878463089052160980736991 94) - 5/476*sqrt(7)*sqrt(2)*(sqrt(2)*(10
*x - 19) + 9*x - 29) + 1/114917927460713529632048080807 0751888*sqrt(2054878
99)*(5*337802213083473608^(3/4)*sqrt(35)*(sqrt(2)*(534678000*x + 38703349)
- 573381349*x - 495974651) + 2876830586*337802213083473608^(1/4)*sqrt(35)*(
sqrt(2)*(201502465*x - 310035209) + 108532744*x - 511537674) - (5*337802213
083473608^(3/4)*sqrt(35)*(534678000*sqrt(2) - 573381349) + 2876830586*33780
2213083473608^(1/4)*sqrt(35)*(201502465*sqrt(2) + 108532744))*sqrt(x^2 - 2*
x + 3))*sqrt(151363871237318045*sqrt(2) + 220640951482187776) + 1/476*sqrt(
x^2 - 2*x + 3)*(5*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) + sqrt(7)*(125*sqrt(2) +
 172)) - 1/476*sqrt(7)*(25*sqrt(2)*(5*x - 1) + 172*x - 82)) + 9*33780221308
3473608^(1/4)*sqrt(205487899)*sqrt(35)*sqrt(7)*(3530255223715004416*x^18 -
28242041789720035328*x^17 + 135914826113027670016*x^16 - 428926009681373036
544*x^15 + 1025759783440690970624*x^14 - 1865298603830415458304*x^13 + 2798
830469551551938560*x^12 - 3430525513645055541248*x^11 + 3836725505323763236
864*x^10 - 3640134417553133928448*x^9 + 3410447187060176453632*x^8 - 240763
4062573633011712*x^7 + 2028793548878716600320*x^6 - 969496340812733087744*x
^5 + 972364673182001528832*x^4 - 87373816786946359296*x^3 + 363395647091163
267072*x^2 - 151363871237318045*sqrt(2)*(16*x^18 - 128*x^17 + 616*x^16 - 19
44*x^15 + 4649*x^14 - 8454*x^13 + 12685*x^12 - 15548*x^11 + 17389*x^10 - 16
498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*x^4 - 396*x^3
+ 1647*x^2 + 162*x + 243) + 35743834140114419712*x + 53615751210171629568)*
sqrt(151363871237318045*sqrt(2) + 220640951482187776)*log(19083512352618334
93759852130293986099 2*x^2 + 2369114176935798061127434 24/2041974420058321*sq
rt(205487899)*(337802213083473608^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3
)*(89801606*sqrt(2) - 42834985) - 337802213083473608^(1/4)*sqrt(35)*sqrt(7)
*(sqrt(2)*(89801606*x - 132636591) - 42834985*x + 222438197))*sqrt(15136387
1237318045*sqrt(2) + 220640951482187776) - 477087808815458373439963032573 49
65248*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 143126342644637512031988909772048957 4
4*x + 238543904407729186719981516286748262 40*sqrt(2) + 333961466170820861 40
797412280144756736) - 9*337802213083473608^(1/4)*sqrt(205487899)*sqrt(35)*s
qrt(7)*(3530255223715004416*x^18 - 28242041789720035328*x^17 + 135914826113
027670016*x^16 - 428926009681373036544*x^15 + 1025759783440690970624*x^14 -
 1865298603830415458304*x^13 + 2798830469551551938560*x^12 - 34305255136450
55541248*x^11 + 3836725505323763236864*x^10 - 3640134417553133928448*x^9 +
3410447187060176453632*x^8 - 2407634062573633011712*x^7 + 20287935488787166
00320*x^6 - 969496340812733087744*x^5 + 972364673182001528832*x^4 - 8737381
6786946359296*x^3 + 363395647091163267072*x^2 - 151363871237318045*sqrt(2)*
(16*x^18 - 128*x^17 + 616*x^16 - 1944*x^15 + 4649*x^14 - 8454*x^13 + 12685*
x^12 - 15548*x^11 + 17389*x^10 - 16498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x
^6 - 4394*x^5 + 4407*x^4 - 396*x^3 + 1647*x^2 + 162*x + 243) + 357438341401
14419712*x + 53615751210171629568)*sqrt(151363871237318045*sqrt(2) + 220640
```

```
951482187776)*log(1908351235261833493759852130293986099 2*x^2 - 236911417693
579806112743424/2041974420058321*sqrt(205487899)*(337802213083473608^(1/4)*
sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(89801606*sqrt(2) - 42834985) - 337802
213083473608^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(89801606*x - 132636591) - 428
34985*x + 222438197))*sqrt(151363871237318045*sqrt(2) + 220640951482187776)
 - 4770878088154583734399630325734965248*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 14
3126342644637512031988909772048957 44*x + 2385439044077291867199815162867482
6240*sqrt(2) + 3339614661708208614079741228014475 6736) + 728813301405404935
7177045697296871075600*x^4 - 6548901006501936794740435888653417168 00*x^3 +
2723747464067850985085226744599034867600*x^2 + 5756926178104321961473983880
*(4596238560*x^17 - 38639385552*x^16 + 188603773872*x^15 - 606785954952*x^1
4 + 1459208021718*x^13 - 2679143870481*x^12 + 3999656132532*x^11 - 49157979
13008*x^10 + 5380603084494*x^9 - 5134334619701*x^8 + 4591320676952*x^7 - 33
59813871472*x^6 + 2503427226914*x^5 - 1409335257371*x^4 + 1002897791524*x^3
 - 266966654968*x^2 + 261702502714*x - 53205422447)*sqrt(x^2 - 2*x + 3) + 2
6790958662962468705756328635400342960 0*x + 4018643799444370305863449295310 0
5144400)/(16*x^18 - 128*x^17 + 616*x^16 - 1944*x^15 + 4649*x^14 - 8454*x^13
 + 12685*x^12 - 15548*x^11 + 17389*x^10 - 16498*x^9 + 15457*x^8 - 10912*x^7
 + 9195*x^6 - 4394*x^5 + 4407*x^4 - 396*x^3 + 1647*x^2 + 162*x + 243)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-2x+3\right)^{\frac{11}{2}}\left(2x^2+x+1\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x+3)**(11/2)/(2*x**2+x+1)**5,x)
```

```
[Out] Integral(1/((x**2 - 2*x + 3)**(11/2)*(2*x**2 + x + 1)**5), x)
```

Giac [**C**] Result contains complex when optimal does not.
time = 26.73, size = 18547, normalized size = 49.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x)
```

```
[Out] 1/19208000000000*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*l
og(3136*(2474301535988301451359142266380914651280177790712513272161012361 81
29348555930033078502447011486458402660428462270 0*sqrt(7)*sqrt(2)*sqrt(77224
33301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2)
- 151363871237318045)^2 + 1443342562659842513292832988722200213246770377915
63274209392387772421199909591859624597607567004340682185832696575 0*sqrt(7)*
(110320475741093888*sqrt(2) - 151363871237318045)^3 + 2886685125319685026 58
566597744440042649354075583126548418784775544842399819183719249195215134008
```

```
6813643716653931500*sqrt(2)*(110320475741093888*sqrt(2) - 15136387123731804
5)^3 + 20619179466569178761326185553174288760668148255937610601341769681774
4571299416942320853725095720486688836903852250*sqrt(7722433301876572160*sqr
t(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 1513638712373180
45)^3 - 1049138541122969625735220807296230414367334042656225923210840932892
59251415027575686933144355006438004151420024881229481000*sqrt(7)*sqrt(2)*(1
10320475741093888*sqrt(2) - 151363871237318045)^2 - 10491385411229696257352
208072962304143673340426562259232108409328925925141502757568693314435500643
800415142002488122948100*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 1059547
0986612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 - 209827
708224593925147044161459246082873466808531245184642168186578518502830055151
37386628871001287600830284004976245896200*sqrt(2)*sqrt(7722433301876572160*
sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 1513638712373
18045)^2 - 1223994964643464563357757608512268816761889716432263577079314421
70802459984198838301422001747507511004843323362361434394500*(11032047574109
3888*sqrt(2) - 151363871237318045)^3 + 724503996256958016683144110303659048
527226579099834730629600402015843465282345034154512602754539838193840458016
13291562600472200700*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 105
95470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045) + 6339
409967248382662472854538412359683674181009662984901543522122388718802293934
79427155097805557896986440201529880194683449362574125*sqrt(7)*(110320475741
093888*sqrt(2) - 151363871237318045)^2 + 1267881993449676532082187318351088
361508312490869111205095341459358991548431951565218821053012281909331172952
868319415989224917443750*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237
318045)^2 + 126788199344967653538125603300215696332050217937699740680224518
030900924464663471430273419380295298646483255439984720301061537907975*sqrt(
7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqr
t(2) - 151363871237318045)^2 - 21264528220686985082784156444749824400286141
322404339508073021441899306648522000622634130083780322911432744853704088500
185306368048860002880*sqrt(7)*sqrt(2)*(110320475741093888*sqrt(2) - 1513638
71237318045) - 354408803678116418845746257791902502469699160932438355128709
627960208195159816911628734559796548815662417323599879097891861163466576081
8080*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(1103
20475741093888*sqrt(2) - 151363871237318045) - 7088176073562328374916566029
889536476564991204751185360922127716049447858985212646112610216597117728735
334198568887900615291459333783931760*sqrt(2)*sqrt(7722433301876572160*sqrt(
2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045
) - 62021540643670373420405091577929394267973114014403593823713156536262791
860899401705831007948594881042345033377486353135919026969012248710900*(1103
20475741093888*sqrt(2) - 151363871237318045)^2 + 32899118880973852710380417
819638428242533122398715783525424459521797423442868355883980260844124987688
79366524177397299680463206375263895127106986700*sqrt(7)*sqrt(2)*sqrt(772243
3301876572160*sqrt(2) - 10595470986612263150) + 575734580417042425329673296
871504559468760385065959616403976214712902997702584706455877479060569410162
15068838775424495834136105150442722719839966690*sqrt(7)*(110320475741093888
```

*sqrt(2) - 151363871237318045) + 115146916083408484993484259748605110296131
995582491774181190935500623259371276266538520099917051160383062730419039816
825148360727220332176154717624880*sqrt(2)*(110320475741093888*sqrt(2) - 151
363871237318045) + 191911526805680809288479094590285874434409383434343159 74
646074253344559590752575102628626876766807523046716011108365196910710221166
349350116972946360*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
*(110320475741093888*sqrt(2) - 151363871237318045) - 421908346813441135720 4
411829911575030935087599894152681923125521189411158761429808471398986722740
737297060377427265233396895882194727754358575990795926553 20*sqrt(7)*sqrt(2)
- 210954173406720568670297857045559135287403242478401319837374548657411401
676856640700014506909797905726420309651469993318236639490549988690120558780
166576108*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
- 421908346813441137138076397703623174719578634373819761152604209721240862 0
850995875571065161946256176959320357812676471048958085730443394260704501501
50098280*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) -
920589952140153149540860681042500826098422635875714185943723383762178092 67
922249720111767473806366273424910555828722901509465001692379638157996477568
878185171912773*sqrt(7) - 814539767127070039237583513123589170024107600674 9
872956875216413467916040501467073084588856054007249498459026692980650865954
421583924771666575361939322675559157565040950966 50*sqrt(2) - 92058995214015
311115855531990633917126372266976678439922113048655699796659517564449092929
141850974541546613899009112709389970185858812421123596605881876967985540965
*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) + 115328821454793
719911555111506838054918092483693711591974348115163386041674344938146595482
926045968304684846611487161656745020914533707787852282815214568718566601574
0023320320)^2 + 3136*(341243148066015550413679540409950260092031930837593 90
918637565727519131211355910825687206864529631615130 00*sqrt(7)*sqrt(2)*sqrt(
7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqr
t(2) - 151363871237318045)^2 + 199058503038509071074646398572470985053685 29
298859644702538580007719493206624281314984204004308951775492500*sqrt(7)*(11
0320475741093888*sqrt(2) - 151363871237318045)^3 + 398117006077018142149292
797144941970107370585977192894050771600154389864132485626299684080086179035
50985000*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 2843
692900550129586780662836749585500766932756979949243219797143959927600946325
902140600572044135967927500*sqrt(7722433301876572160*sqrt(2) - 105954709866
12263150)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 76858147264
002002271476515530447061472149684267202070486664776785256332938718528406617
898269719011792142 50*sqrt(7)*sqrt(2)*(110320475741093888*sqrt(2) - 15136387
1237318045)^2 + 768581472640020022714765155304470614721496842672020704866 64
776785256332938718528406617898269719011792142 5*sqrt(7)*sqrt(772243330187657
2160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 15136387
1237318045)^2 + 153716294528004004542953031060894122944299368534404140973 32
955357051266587743705681323579653943802358428 50*sqrt(2)*sqrt(77224333018765
72160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 1513638
71237318045)^2 + 896678384746690026500559347855215717175079649784024155677 7

5572916132388428504949807720881314672180424166625*(110320475741093888*sqrt(2
) - 151363871237318045)^3 + 97651082935133698531842609272064332805391755699
9006303431427669091931152434985476815319185951875160339805775571594750014000
0*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*
(110320475741093888*sqrt(2) - 151363871237318045) + 854446975682419864428577
1515706665814717081317737989161160454160393991529564500073813463684649514870
0981051126045418060425000*sqrt(7)*(110320475741093888*sqrt(2) - 1513638712
37318045)^2 + 170889395136483972828841572303130724558728369619768073207870428
06828084572689405859707072167406377935915009081120811676230000*sqrt(2)*(1
10320475741093888*sqrt(2) - 151363871237318045)^2 + 170889395136483973283832
4363911514584436344234997017533305796691850727246420616016306591378254759237
0164618233826987163070000*sqrt(7722433301876572160*sqrt(2) - 1059547098661
2263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 - 161566291032
1642985918250525155953853052845908562683937530778095691283476808165943145292
528342610931516231547450242131454340*sqrt(7)*sqrt(2)*(110320475741093888*s
qrt(2) - 151363871237318045) - 26927715172027383040078920360978657724993896
1671326854562560659627007661474391262636548122809722557868949842520038360635
590*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(1103
20475741093888*sqrt(2) - 151363871237318045) - 53855430344054766094797487819
862458739792842911493382621493024166773355244719162391167973693128993175050
520555382721992288 0*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986
612263150)*(110320475741093888*sqrt(2) - 151363871237318045) - 471235015510
4792023047027215704364836868338350696062783094015045632392519024289804526353
3063196497819345341126334532687062 00*(110320475741093888*sqrt(2) - 1513638
71237318045)^2 + 419385925470581756378654520411097941724961521827588380808369
9006997908745917921159286375728564093032299349286334300105037477765458575
2870920*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 1059547098661226
3150) + 733925369573518077568688728124769337212428798249901052299318450769533
84941467606270893463005503513404583010936669049988937100711791879589870 0*
sqrt(7)*(110320475741093888*sqrt(2) - 151363871237318045) + 146785073914703
6154160866626898201690081411926824917492012375174282776424232301401576206419
636158097776887638820246148744285006222232316501400*sqrt(2)*(1103204757410
93888*sqrt(2) - 151363871237318045) + 24464178985783936032882554361814959777
8089793408164936529659929761653877816458460985919888559249587651218708898851
31424329224393833979209210 0*sqrt(7722433301876572160*sqrt(2) - 1059547098
6612263150)*(110320475741093888*sqrt(2) - 151363871237318045) - 12920777533
126940193458934301499103263313648173982634148974314004813143066882043781215
921471858282975500842590978635984214415094963315999115 0*sqrt(7)*sqrt(2) + 7
587629160082400034098191013754109936929303905031237281898685620725368747731
98417845474059096092571231597614031305995383233253822344649520984786771597 2
44159013334688*x - 64603887665634701028843734757843820568795044140595103353
0044590248134676986715874118750732031211145579039572078252198592335248430 54
466661575*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
- 129207775331269402042300203703100565068677528624357666238053675568548792 6
22685475800752550765348440250164638699943637093422466523499441710582*sqrt(2

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)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) - 36018223914820
167914680328062277541844579926085292035680717885777055540822317688033834995
13928216083726730307366377332828337773366071083598072772386051930660*sqrt(7
) - 2566532515528593027457553740848822251225795581355385468029664178193344З
840848267805576609281074561095839963944023796921540484009785711469208886126
0268773750000*sqrt(2) - 758762916008240003409819101375410993692930390503123
728189868562072536874773198417845474059096092571231597614031305995383233253
82234464952098478677159724415901333468 8*sqrt(x^2 - 2*x + 3) - 3601822391482
016742539674834659882392115916103273640493101259172279652681774568854688273
07095828468411291786703019925453890905695664077533016012994490484508 0*sqrt(
7722433301876572160*sqrt(2) - 10595470986612263150) + 189691081878191176772
044460297494206938487074845844976108591897448936648822154611593038166610328
900989958412425584079869742930256624900190943273280656848430181850997)^2) -
 1/19208000000000*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*
log(3136*(2474301532945128077011663304566412660865899546342905585182468026З
46391921433693240822436073126340210807888656257 00*sqrt(7)*sqrt(2)*sqrt(7722
433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2)
 - 151363871237318045)^2 + 144334256088465804492347026099707405217177473536
669492468977301536872862083632105714642104265703178963793504948325 0*sqrt(7)
*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 288668512176931608 98
469405219941481043435494707333898493795460307374572416726421142928420853140
635792758700989665 00*sqrt(2)*(110320475741093888*sqrt(2) - 1513638712373180
45)^3 + 206191794412094006417638608713867721738824962195242132098539002195 5
3266011947443673520300609386168423399072135475 0*sqrt(7722433301876572160*sq
rt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318
045)^3 + 104913854900605463598088261225277871792243668954806319376337814136
029745746425269205752115419545432491274148221669945001000*sqrt(7)*sqrt(2)*(
110320475741093888*sqrt(2) - 151363871237318045)^2 + 104913854900605463598 0
882612252778717922436689548063193763378141360297457464252692057521154195454
324912741482216699450010 0*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 105954
70986612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 + 20982
770980121092719617652245055574358448733790961263875267562827205949149285053
84115042308390908649825482964433398900020 0*sqrt(2)*sqrt(7722433301876572160
*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237
318045)^2 + 122399497384039707531102971429490850424284280447274039272394116
49203470337082948074004413465613633790648650625861493583450 0*(1103204757410
93888*sqrt(2) - 151363871237318045)^3 + 724503995560791802483290151469992 35
208892617070047340278059551131392900835267393877060925502058563859855540246
51103233600369822970 0*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10
595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045) + 633
940996115692828822413237832995359418919269073856001343620769961624939096901
71398854437909880792765536002590786500681195813034787 5*sqrt(7)*(11032047574
1093888*sqrt(2) - 151363871237318045)^2 + 126788199223138565723244288684180
270600256132071997655920959161553276561399672542358602343795866698184031403
96280066451559348179862 50*sqrt(2)*(110320475741093888*sqrt(2) - 15136387123
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7318045)^2 + 12678819922313856605315115974353068086847790601418601070307910
10656639727573349458714545999870257969603562137129793592899786359662225*sqrt
(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sq
rt(2) - 151363871237318045)^2 + 21264527927126900961164377132668373311820322
94957143683565399829370447570678343578271402933641376843079911781112462090 2
43215454699798098396880*sqrt(7)*sqrt(2)*(110320475741093888*sqrt(2) - 151363
871237318045) + 35440879878544835015208327526337165880148748188307087674038
00505111750511407655000601839046131539324151071812411163027198287919028630 7
82880*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110
320475741093888*sqrt(2) - 151363871237318045) + 70881759757089670010433063 6
43035195836852113286085056911458168396557616061231611605173972209025973300 9
734142635479400694373491954926261 3360*sqrt(2)*sqrt(7722433301876572160*sqrt
(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 15136387123731804
5) + 62021539787453461399014070555129747821363280759028253853850789286729 67
32220280790025668866781314189126878913733316908824565448416159745349 00*(110
320475741093888*sqrt(2) - 151363871237318045)^2 + 3289911884763015221101541
1144055054747532701564810653878870517591440238490285735651535789555031009 92
85663679223721931723923009649817922361631420 9700*sqrt(7)*sqrt(2)*sqrt(77224
33301876572160*sqrt(2) - 10595470986612263150) + 575734579833527666590785 67
72641306665068371869928396900097820597735224485555484099581070415601388699 1
64524304214143405050306036039486243186707825479 0*sqrt(7)*(11032047574109388
8*sqrt(2) - 151363871237318045) + 11514691596670553324570673589674695312373
27517642866478505398893008403683149329750414543813391166094304121869089012 5
3978934468277888412836527277030080*sqrt(2)*(110320475741093888*sqrt(2) - 15
1363871237318045) + 19191152661117588970884988724230066021393869653903156 39
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6107328538508811760*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150
)*(110320475741093888*sqrt(2) - 151363871237318045) + 4219083267175436419 24
35882068986125477387841164893037938190180929920275078233100150837480351787 0
628151016355380370573185797674929411754956761480005114821080*sqrt(7)*sqrt(2
) + 21095416335877182177225666471166971125635837748319235420777168987856867
08945338404996962729098703004315518036561804858541608396971651230611582280 4
6039881708*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
 + 42190832671754364334199401583165465162529632769025946098011971581201199 1
65969696288572490469969280790154393567528142462119826677491671662473572082 1
529103720*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
- 920589964087560320916761581042015283905433780645215921762515407245365322 3
47939602047491610965392490295654216539646083128661631912861645825999759027 8
2992478976911083*sqrt(7) + 81453972796152704669483783306214586458459063445 2
97456508431975768331851585419547930175736322768552907736082746349927771232 2
108218563975398844449113459715493032206173237469850*sqrt(2) - 9205899640875
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26780610256882687440520985933897382743011055168688173583838809015223806948 3
5*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) - 11532881607348
6870377478683213467139945589116276762194723539787193562938947106189316000 85

322372043380298052663139565536675674564328460228800898840894611032414487006
75305025280)^2 + 3136*(3412431480660155504136795404099502600920319308375939
091863756572751913121135591082568720686452963161513000*sqrt(7)*sqrt(2)*sqrt
(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sq
rt(2) - 151363871237318045)^2 + 1990585030385090710746463985724709850536852
9298859644702538580007719493206624281314984204004308951775492500*sqrt(7)*(1
10320475741093888*sqrt(2) - 151363871237318045)^3 + 39811700607701814214929
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550985000*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 284
369290055012958678066283674958550076693275697994924321979714395992760094632
5902140600572044135967927500*sqrt(7722433301876572160*sqrt(2) - 10595470986
612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 7685814726
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789826971901179214250*sqrt(7)*sqrt(2)*(110320475741093888*sqrt(2) - 1513638
71237318045)^2 + 7685814726400200227147651553044706147214968426720207048666
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72160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 1513638
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871237318045)^2 + 896678384746690026500559347855215717175079649784024155677
7557291613238842850494980772088131467218042416625*(110320475741093888*sqrt(
2) - 151363871237318045)^3 + 9765108293513369853184260927206433280539175569
990063034314276690919311524349854768153191859518751603398057755715947500140
00*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
*(110320475741093888*sqrt(2) - 151363871237318045) + 8544469756824198644285
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237318045)^2 + 170889395136483972828841572303130724558728369619768073207870
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110320475741093888*sqrt(2) - 151363871237318045)^2 + 1708893951364839732838
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237016461823382698716307000*sqrt(7722433301876572160*sqrt(2) - 105954709866
12263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 - 16156629103
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92528342610931516231547450242131454340*sqrt(7)*sqrt(2)*(110320475741093888*
sqrt(2) - 151363871237318045) - 2692771517202738304007892036097865772499389
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320475741093888*sqrt(2) - 151363871237318045) - 538554303440547660947974878
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505205553827219922880*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 1059547098
6612263150)*(110320475741093888*sqrt(2) - 151363871237318045) - 47123501551
047920230470272157043648368683383506960627830940150456323925190242898045263
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871237318045)^2 + 4193859254705817563786545204110979417249615218275883808080
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52870920*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 105954709866122
63150) + 733925369573518077568688728124769337212428798249901052299318450769
533849414676062708934630055035134045830109366690499889371007117918795898700
sqrt(7)(110320475741093888*sqrt(2) - 151363871237318045) + 14678507391470
361541608666268982016900814119268249174920123751742827764242323014015762064
19636158097776887638820246148744285006222232316501400*sqrt(2)*(110320475741
093888*sqrt(2) - 151363871237318045) + 244641789857839360328825543618149597
778089793408164936529659929761653877816458460985919888559249587651218708898
851314243292243938339792092100*sqrt(7722433301876572160*sqrt(2) - 105954709
86612263150)*(110320475741093888*sqrt(2) - 151363871237318045) - 1292077753
312694019345893430149910326331364817398263414897431400481314306688204378121
59214718582829755008425909786359842144150949633159991150*sqrt(7)*sqrt(2) -
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198417845474059096092571231597614031305995383233253822344649520984786771597
244159013334688*x - 6460388766563470102884373475784382056879504414059510335
300445902481346769867158741187507320312111455790395720782521985923352484305
4466661575*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
 - 129207775331269402042300203703100565068677528624357666238053675568548792
622685475800752550765348440250164638699943637093422466523499441710582*sqrt(
2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) - 3601822391482
016791468032806227754184457992608529203568071788577705554082231768803383499
513928216083726730307366377332828337773366071083598072772386051930660*sqrt(
7) - 2566532515528593027457553740848822251225795581355385468029664178193344
384084826780557660928107456109583996394402379692154048400978571146920888612
60268773750000*sqrt(2) + 75876291600824000340981910137541099369293039050312
372818986856207253687477319841784547405909609257123159761403130599538323325
3822344649520984786771597244159013334688*sqrt(x^2 - 2*x + 3) - 360182239148
201674253967483465988239211591610327364049310125917227965268177456885468827
3070958284684112917867030199254538909056956640775330160129944904845080*sqrt
(7722433301876572160*sqrt(2) - 10595470986612263150) - 18969037612592882493
286509039021128990797812040571688798634238358733178856444459732969886293771
7384625840394590068917821873696654547424569549120105141773649324816347)^2)
+ 41672947348129/28000000000*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 105
95470986612263150)*arctan(-758762916008240003409819101375410993692930390503
123728189868562072536874773198417845474059096092571231597614031305995383233
253822344649520984786771597244159013334688*(4*x - 4*sqrt(x^2 - 2*x + 3) - I
*sqrt(20*sqrt(2) - 25) + 1)/(2404569034002236590242120764218575948765964811
188059545345978971635505204657923525607623616276623751473316663*(sqrt(7) +
2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 10595470986612263150))^7 - 1
427399375677509694877851438498272672608617745110511460150803990330057842381
327560366437338163352898015665578569812646*(sqrt(7) + 2*sqrt(2) + sqrt(7722
433301876572160*sqrt(2) - 10595470986612263150))^6 + 1035005708938511449879
694100445735935284668310570259154695166093989456713504357842860594681901380

```
0669603387143611635118473099638 6*(sqrt(7) + 2*sqrt(2) + sqrt(77224333018765
72160*sqrt(2) - 10595470986612263150))^5 - 50629829096873773899573990740257
783361032907165973215049765624930757608699456368874374542973258344666404687
384663029077837715816904194414*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572
160*sqrt(2) - 10595470986612263150))^4 + 10966372960324617518376520492953 90
353641982600736488153626251774799452192758375873916572220994961819156070680
9147945162088690245763393164652919228 47*(sqrt(7) + 2*sqrt(2) + sqrt(7722433
301876572160*sqrt(2) - 10595470986612263150))^3 - 421908346813441134201546 3
100849442848230569990476005516661368939297975317080268800700618350130102201
917477482441627065781217996707276477846427694382052541 46*(sqrt(7) + 2*sqrt(
2) + sqrt(7722433301876572160*sqrt(2) - 10595470986612263150))^2 - 36823598
085606129381210044942331473355861294289510039804332924533313682318158002472
147635428150206679235686050868347855430430947554077103367216545381319104549
5483200*sqrt(7) - 7364719617121225876242008988466294671172258857902007960 86
658490666273646363160049442952708563004133584713721017366957108608618951081
542067344330907626382090990966400*sqrt(2) - 368235980856061293812100449423 3
147335586129428951003980433292453331368231815800247214763542815020667923568
605086834785543043094755407710336721654538131910454954832 00*sqrt(7722433301
876572160*sqrt(2) - 10595470986612263150) + 14283521052038872882790385369 32
322893004029548936486004185262156116702645830414445699158147702914990438794
86110165753955023507391431740254479476659166189278113070264611396886))/(110
320475741093888*sqrt(2) - 151363871237318045) - 41672947348129/28000000000*
sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*arctan(-75
876291600824000340981910137541099369293039050312372818986856207253687477319
841784547405909609257123159761403130599538323325382234464952098478677159724
4159013334688*(4*x - 4*sqrt(x^2 - 2*x + 3) - I*sqrt(20*sqrt(2) - 25) + 1)/(
240456903104482806317945899374772853339737565242264877082844317429192606553
3229582917819313047949670367238733*(sqrt(7) + 2*sqrt(2) + sqrt(772243330187
6572160*sqrt(2) - 10595470986612263150))^7 + 1427399386402795423103241649 32
350845975841726469124244049439202906162919382891522728914442747680860532345
7798934284966*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 105
95470986612263150))^6 + 1035005707943988286737045590931491036645748300422 70
418676217772397611737399109743666577656693380270736279743910695330805730930
506*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 1059547098661
2263150))^5 + 50629828397921192657622134430905517819210041766723456843213 90
261731100045776430693261660418696870154376935391839699513483958286559484145
4*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 105954709866122
63150))^4 + 1096637294921005068525485165081893719521877793381198945537497 26
838300179564207424160536321027108218235029918594318831383181052429437744237
8547095677*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 105954
70986612263150))^3 + 4219083267175436404054639687522254737619823303456517 59
435786313555664940163096802031617835780482986046113584519114737539776217075
850964725785432795347393026*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160
*sqrt(2) - 10595470986612263150))^2 - 368235985635024162362460774967963334 0
042438977933035786955504105077991958445671969042859007463098310599443798853
```

```
874814743122235338322599695801057146658755284611383 80*sqrt(7) - 73647197127
004832472492154993592666800848779558660715739110082101559839168913439380857
180149261966211988875977077496294862444706766451993916021142933175105692227
6760*sqrt(2) - 368235985635024162362460774967963334004243897793303578695550
410507799195844567196904285900746309831059944379885387481474312223533832259
96958010571466587552846113 8380*sqrt(7722433301876572160*sqrt(2) - 105954709
86612263150) - 142835208193613592915330036722260451533458578946048957790307
351258262277239751604641916384506889981140070748629723613145062430716033621
311266813074832334518174840024828894486))/(110320475741093888*sqrt(2) - 151
363871237318045) + 1/205800000000*(108121281*(x - sqrt(x^2 - 2*x + 3))^15 +
 135317265*(x - sqrt(x^2 - 2*x + 3))^14 - 2309618731*(x - sqrt(x^2 - 2*x +
3))^13 - 4089866767*(x - sqrt(x^2 - 2*x + 3))^12 + 23951599406*(x - sqrt(x^
2 - 2*x + 3))^11 + 45641347654*(x - sqrt(x^2 - 2*x + 3))^10 - 149568395690*
(x - sqrt(x^2 - 2*x + 3))^9 - 288215430978*(x - sqrt(x^2 - 2*x + 3))^8 + 66
0704292769*(x - sqrt(x^2 - 2*x + 3))^7 + 1062639157153*(x - sqrt(x^2 - 2*x
+ 3))^6 - 2094971437979*(x - sqrt(x^2 - 2*x + 3))^5 - 2301192104575*(x - sq
rt(x^2 - 2*x + 3))^4 + 4977175786352*(x - sqrt(x^2 - 2*x + 3))^3 + 13029940
04424*(x - sqrt(x^2 - 2*x + 3))^2 - 6052879270032*x + 6052879270032*sqrt(x^
2 - 2*x + 3) + 2841437414928)/((x - sqrt(x^2 - 2*x + 3))^4 + (x - sqrt(x^2
- 2*x + 3))^3 - 5*(x - sqrt(x^2 - 2*x + 3))^2 - 7*x + 7*sqrt(x^2 - 2*x + 3)
 + 14)^4 + 1/3150000000*(3*((((((((29420*x - 332589)*x + 1860912)*x - 67437
44)*x + 17167416)*x - 31960026)*x + 43362368)*x - 42014736)*x + 26516604)*x
 - 27199867)/(x^2 - 2*x + 3)^(9/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(2\,x^2+x+1\right)^5\,\left(x^2-2\,x+3\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 2*x^2 + 1)^5*(x^2 - 2*x + 3)^(11/2)),x)
```

```
[Out] int(1/((x + 2*x^2 + 1)^5*(x^2 - 2*x + 3)^(11/2)), x)
```

3.51 $\int \frac{1}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx$

Optimal. Leaf size=638

$$\frac{37358055634422583-14024622879097678x}{1840124479200000000\left(3-2x+x^2\right)^{19/2}}+\frac{476849951294984711-125181871472148210x}{10427372048800000000\left(3-2x+x^2\right)^{17/2}}+\frac{78517583754}{156410580}$$

```
[Out] 1/1840124479200000000*(37358055634422583-14024622879097678*x)/(x^2-2*x+3)^(
19/2)+1/10427372048800000000*(476849951294984711-125181871472148210*x)/(x^
2-2*x+3)^(17/2)+1/156410580732000000000*(7851758375483333511+194216499620
4584234*x)/(x^2-2*x+3)^(15/2)-11/4066675099032000000000*(7502325106308201
089-7813986379726516886*x)/(x^2-2*x+3)^(13/2)-3/114701092536800000000000*(
69053268515296359011-44840736195018286006*x)/(x^2-2*x+3)^(11/2)+1/938463484
392000000000000*(-838519439380295335657+466189390555853643870*x)/(x^2-2*x+
3)^(9/2)+1/31282116146400000000000000*(-11176466647292384601 89+568839749685
437871554*x)/(x^2-2*x+3)^(7/2)+1/5213686024400000000000000*(-655140551156
5449301689+3127298559983309301910*x)/(x^2-2*x+3)^(5/2)+1/104273720488000000
0000000000*(-417903978239845985081 9+188699344558965240269 4*x)/(x^2-2*x+3)^(
3/2)+1/630*(-1+10*x)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1)^9+1/88200*(887+2218*x)/
(x^2-2*x+3)^(19/2)/(2*x^2+x+1)^8+1/1080450*(14453+29371*x)/(x^2-2*x+3)^(19/
2)/(2*x^2+x+1)^7+1/605052000*(8837931+17459234*x)/(x^2-2*x+3)^(19/2)/(2*x^2
+x+1)^6+1/26471025000*(447940041+813432205*x)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1
)^5+1/29647548000000*(592729157441+911061463974*x)/(x^2-2*x+3)^(19/2)/(2*x^
2+x+1)^4+1/12353145000000*(277010166219+310705340015*x)/(x^2-2*x+3)^(19/2)/
(2*x^2+x+1)^3+1/276710448000000*(5488221294349+1384103301166*x)/(x^2-2*x+3)
^(19/2)/(2*x^2+x+1)^2+1/2421216420000000*(-37857197792117-146548895467025*x
)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1)+1/10427372048800000000000000000*(-12105495
874518671061833+5117656435043679338190*x)/(x^2-2*x+3)^(1/2)-1/2259801992000
000000000000000*arctanh(1/7*(272944589523248381749+x*(656826642296538601431
-464885615909893491590*2^(1/2))-191941026386645109841*2^(1/2))*35^(1/2)/(-8
1042225921274689605478944797800854846405+573059225230017071260263638786665 0
0308992*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(-567295581448922827238352613584 6
059839248350+401141457661011949882184547150665502162944 0*2^(1/2))^(1/2)+1/2
259801992000000000000000000*arctan(1/7*(272944589523248381749+1919410263866
45109841*2^(1/2)+x*(656826642296538601431+464885615909893491590*2^(1/2)))*3
5^(1/2)/(81042225921274689605478944797800854846405+57305922523001707126026 3
6387866650030899 2*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(5672955814489228272383
52613584605983924835 0+4011414576610119498821845471506655021629440*2^(1/2))^
(1/2)
```

Rubi [A]
time = 0.87, antiderivative size = 638, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {988, 1074, 1049, 1043, 212, 210}

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] (37358055634422583 - 14024622879097678*x)/(18401244792000000*(3 - 2*x + x
^2)^(19/2)) + (476849951294984711 - 125181871472148210*x)/(1042737204880000
00000*(3 - 2*x + x^2)^(17/2)) + (78517583754833335 11 + 194216499620458 4234*
x)/(1564105807320000000000*(3 - 2*x + x^2)^(15/2)) - (11*(750232510630820 1
089 - 7813986379726516886*x))/(40666750990320000000000*(3 - 2*x + x^2)^(13
/2)) - (3*(69053268515296359011 - 44840736195018286006*x))/(114701092536800
0000000000*(3 - 2*x + x^2)^(11/2)) - (838519439380295335657 - 4661893905558
53643870*x)/(938463484392000000000000*(3 - 2*x + x^2)^(9/2)) - (1117646664
729238460189 - 568839749685437871554*x)/(3128211614640000000000000*(3 - 2*
x + x^2)^(7/2)) - (6551405511565449301689 - 3127298559983309301910*x)/(5213
686024400000000000000*(3 - 2*x + x^2)^(5/2)) - (41790397823984598508 19 -
18869934455896524026 94*x)/(104273720488000000000000000*(3 - 2*x + x^2)^(3/
2)) - (121054958745186710618 33 - 51176564350436793381 90*x)/(104273720488000
0000000000000*Sqrt[3 - 2*x + x^2]) - (1 - 10*x)/(630*(3 - 2*x + x^2)^(19/2
)*(1 + x + 2*x^2)^9) + (887 + 2218*x)/(88200*(3 - 2*x + x^2)^(19/2)*(1 + x
+ 2*x^2)^8) + (14453 + 29371*x)/(1080450*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*
x^2)^7) + (8837931 + 17459234*x)/(605052000*(3 - 2*x + x^2)^(19/2)*(1 + x +
 2*x^2)^6) + (447940041 + 813432205*x)/(26471025000*(3 - 2*x + x^2)^(19/2)*
(1 + x + 2*x^2)^5) + (592729157441 + 911061463974*x)/(29647548000000*(3 - 2
*x + x^2)^(19/2)*(1 + x + 2*x^2)^4) + (277010166219 + 310705340015*x)/(1235
3145000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^3) + (5488221294349 + 138
4103301166*x)/(276710448000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^2) -
(3785719779211 7 + 146548895467025*x)/(2421216420000000*(3 - 2*x + x^2)^(19/
2)*(1 + x + 2*x^2)) + (Sqrt[(810422259212746896054789447978008548 46405 + 57
30592252300170712602636387866650030 8992*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(810
42225921274689605478944797800854846405 + 573059225230017071260263638786665 0
0308992*Sqrt[2]))]*(27294458952324838174 9 + 19194102638664510984 1*Sqrt[2] +
 (656826642296538601431 + 464885615909893491590*Sqrt[2])*x))/Sqrt[3 - 2*x +
 x^2]])/32282885600000000000000000 - (Sqrt[(-81042225921274689605478944797 8
00854846405 + 57305922523001707126026363878666500308992*Sqrt[2])/70]*ArcTan
h[(Sqrt[5/(7*(-81042225921274689605478944797800854846405 + 573059225230017 0
7126026363878666500308992*Sqrt[2]))]*(272944589523248381749 - 191941026386 6
45109841*Sqrt[2] + (656826642296538601431 - 464885615909893491590*Sqrt[2])*
x))/Sqrt[3 - 2*x + x^2]])/32282885600000000000000000
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
```

```
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} - \frac{\int \frac{-2960+3060x-1800x^2}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^9}\,d}{3150}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}(}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}(}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}(}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}(}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}(}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}(}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}(}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}(}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000\left(3-2x+x^2\right)^{19/2}} - \frac{1-}{630\left(3-2x+x^2\right)^{1}}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000\left(3-2x+x^2\right)^{19/2}} + \frac{4768499512949847}{10427372048800}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000\left(3-2x+x^2\right)^{19/2}} + \frac{4768499512949847}{10427372048800}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000\left(3-2x+x^2\right)^{19/2}} + \frac{4768499512949847}{10427372048800}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000\left(3-2x+x^2\right)^{19/2}} + \frac{4768499512949847}{10427372048800}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000\left(3-2x+x^2\right)^{19/2}} + \frac{4768499512949847}{10427372048800}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000\left(3-2x+x^2\right)^{19/2}} + \frac{4768499512949847}{10427372048800}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000\left(3-2x+x^2\right)^{19/2}} + \frac{4768499512949847}{10427372048800}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 20.24, size = 56025, normalized size = 87.81

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] Result too large to show
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 + x + 2*x^2)^10*(3 - 2*x + x^2)^(21/2)),x]')
```

```
[Out] Timed out
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 86792 vs. $2(518) = 1036$.
time = 4.55, size = 86793, normalized size = 136.04

method	result
risch	$\frac{33722490019334222378242713 60x^{37} - 535022053996400313947961477 12x^{36} + 469149394082989701729494575872x^{35} - 28474992209\ldots}{\ldots}$
trager	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^10*(x^2 - 2*x + 3)^(21/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2775 vs. 2(518) = 1036.
time = 0.49, size = 2775, normalized size = 4.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="fricas")
```

```
[Out] 1/14344664239923376765642165360362728636455220318608571101268133326181128987
720320000000000000000000*(3604596077627223662808371797497205511119066017285
358396135728761934386631817942748278579200*x^38 - 5587123920322196677352976
28612066854223455232679227055140103795809982992793178112598317977600*x^37 +
 48121357636323435898491763496587693573439531330759233458841197897182406153
47695356895190323200*x^36 - 28710607758300836474268681367065241896063360827
677699962522107958880738952242991399003888332800*x^35 + 1315091821471322213
079849345669386715662902248081338714385016894218223678278587868742508613888
00*x^34 - 48615412586213217241775435956082154303807288316753404802358233229
5256693402493082467454965081600*x^33 + 150125118590038017914558770715112989
4284646412544039883821859865409233818038611634065993336166400*x^32 - 396012
076807241950819334539073291504431090617269457971847700920669696894988016137
7086262197766400*x^31 + 909142000002142860704234021157216421398787616081889
7418150894645435329015717575267031369642428000*x^30 - 184247649298721582706
989900442437618212098383039369354048250008452972140026730578162474910272628
00*x^29 + 33413073756673638925333625169011170445598811516221975115590200411
293389434416479555356860509015600*x^28 - 5481653256044929545971751700338269
9673242410936114304344629842103656622934490247108012261346586400*x^27 + 822
459830940635186677366276046635475885728402385815973257367014937498803836507
49401133206999014400*x^26 - 11372284806763969440259273586264909409387404544
3618754295078471234595964240139128161766283626302000*x^25 + 146086574413322
248286514192550522624098477614094095488624493581512454991258074867544318895
241990800*x^24 - 1750270940810010216829737529974120232517363052261271442728
11232619626419165679723993392477178363200*x^23 + 19688729160578415943345565
4443374481739030277196290989156609388218395099469530751149958413044135200*x
^22 - 208068683375682167383215047521697995267539026087882795784482813901791
360434798005710722616487282000*x^21 + 2081714449184784825196181653920157303
```

47012009814583465001141378703189206795143605224483243158516400*x^20 - 19622
755618454040835316742234157685550832000179582185155831117699557408106901596
9836642878534431200*x^19 + 176534941677723459681422280024952573032106299529
482816321219585323399086976471958310981405494523200*x^18 - 1491362557380113
805569548293989292587370076152040747303305658872207307833829238226195713407
37358000*x^17 + 12189081448358772438901196169673375625310538365442623433615
0913799569962877883235263704480534144400*x^16 - 919831860532221296355370692
78588580392985745730700928388526309371776740142438834607398588992195200*x^1
5 + 69317814132471559316390137037592557060398996838342232414889371690271398
098098738643314402130954400*x^14 - 4574307084113250024797073972709329687876
5897323708593659902862883667237249390700654758574610918000*x^13 + 329969655
216763949298031215090491433294517890491697894556446151291991903089176733485
18481311574800*x^12 - 17770083757788737971933739892049927033484890029804651
938270182161740937851280707834822272274354400*x^11 + 1354422526745145970196
036923825637435189936268397849855148372985225665526414709333739259622802880
0*x^10 - 481375973272848865172866855106995818624092546697867179982576756873
2599092879797201593187475517200*x^9 + 5091181133639025216832620106123280320
347641869015804163342220634415255665812683873707564839486000*x^8 - 46421311
850305640075834899457188406077339946202653776901797199608409580382714283736
3184426478400*x^7 + 1771233883264782126042267141811413849986971398265032235
916172889879027134542752439323372429279200*x^6 + 23911503454316320991841103
2521665649750496447867853609069487786445410804754998849116452338787600*x^5
+ 79817891129994413353362937273464455099835468*1264938752804265123815574105
117799608149057272418^(1/4)*sqrt(1590558865810545927822094)*sqrt(35)*sqrt(2
)*(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x^34 - 6905376
*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 261706983*x^29 + 4
74602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329345*x^25 + 207502
6563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895*x^21 + 295688552
9*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^17 + 1731347859*x
^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 + 468691803*x^12 -
252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*x^8 - 6593724*x^7
 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 1023516*x^2 + 1
37781*x + 59049)*sqrt(81042225921274689605478944797800854846405*sqrt(2) + 1
14611845046003414252052727757333000617984)*arctan(1/54206850781156887023310
518673090274966005685838243268724684064391985051350175945649154733957770247
43167351056637371274953501437271981836435236061968*sqrt(7952794329052729639
11047)*(9939513250523192816422116593216797292815016511001378462170679301990
*sqrt(11005224487862873621128239642490888848098)*sqrt(288886807671054271567
2947094311)*sqrt(7)*(10*sqrt(2) + 9) + sqrt(1590558865810545927822094)*(5*1
264938752804265123815574105117799608149057272418^(3/4)*sqrt(288886807671054
2715672947094311)*sqrt(35)*(340613697110906370000*sqrt(2) - 483753219647003
202703) + 5566956030336910747377329*126493875280426512381557410511779960814
9057272418^(1/4)*sqrt(2888868076710542715672947094311)*sqrt(35)*(4373478266
4604992355*sqrt(2) - 66269826580994560232))*sqrt(81042225921274689605478944
797800854846405*sqrt(2) + 114611845046003414252052727757333000617984) + 147

```
4618125404445687156966131141385579103594786769370421723255973728695229351822
724790786*sqrt(28888680767105427156729470943111)*sqrt(7)*(125*sqrt(2) + 172)
)*sqrt(5191798731734901573730421875012971256390643826285581511813805064*x^2
 + sqrt(1590558865810545927822094)*(126493875280426512381557410511779960814
9057272418^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(43268355662383849682
*sqrt(2) - 62135959399493560795) - 1264938752804265123815574105117799608149
057272418^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(43268355662383849682*x - 1054043
15061877410477) - 62135959399493560795*x + 148672670724261260159))*sqrt(810
42225921274689605478944797800854846405*sqrt(2) + 11461184504600341425205272
7757333000617984) - 1297949682933725393432605468753242814097660956571395377
953451266*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 389384904880117618029781640625972
8442292982869714186133860353798*x + 874869761179272589826814757400740628067
45190*sqrt(11005224487862873621128239642490888848098) + 9085647780536077754
028238281272699698683626695999767645674158862) + 5/35309486994022006419332*
sqrt(11005224487862873621128239642490888848098)*sqrt(7)*(sqrt(2)*(10*x - 19
) + 9*x - 29) + 1/701918227692516147086715878423299535653311118502220320740
26984349485892917146977000414136*sqrt(1590558865810545927822094)*(5*1264938
752804265123815574105117799608149057272418^(3/4)*sqrt(35)*(sqrt(2)*(3406136
97110906370000*x + 143139522536096832703) - 483753219647003202703*x - 19747
4174574809537297) + 5566956030336910747377329*12649387528042651238155741051
17799608149057272418^(1/4)*sqrt(35)*(sqrt(2)*(43734782664604992355*x + 2253
5043916389567877) - 66269826580994560232*x - 21199738748215424478) - (5*126
4938752804265123815574105117799608149057272418^(3/4)*sqrt(35)*(340613697110
906370000*sqrt(2) - 483753219647003202703) + 5566956030336910747377329*1264
938752804265123815574105117799608149057272418^(1/4)*sqrt(35)*(4373478266460
4992355*sqrt(2) - 66269826580994560232))*sqrt(x^2 - 2*x + 3))*sqrt(81042225
921274689605478944797800854846405*sqrt(2) + 1146118450460034142520527277573
33000617984) - 1/35309486994022006419332*sqrt(x^2 - 2*x + 3)*(5*sqrt(110052
24487862873621128239642490888848098)*sqrt(7)*(10*sqrt(2) + 9) + 74179594525
256316007*sqrt(7)*(125*sqrt(2) + 172)) + 1/476*sqrt(7)*(25*sqrt(2)*(5*x - 1
) + 172*x - 82)) + 79817891129994413353362937273464455099835468*12649387528
04265123815574105117799608149057272418^(1/4)*sqrt(1590558865810545927822094
)*sqrt(35)*sqrt(2)*(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 18679
68*x^34 - 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 2
61706983*x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329
345*x^25 + 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895
*x^21 + 2956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^
17 + 1731347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 +
468691803*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*
x^8 - 6593724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3
+ 1023516*x^2 + 137781*x + 59049)*sqrt(810422259212746896054789447978008548
46405*sqrt(2) + 114611845046003414252052727757333000617984)*arctan(-1/54206
850781156887023310518673090274966005685838243268724684064391985051350175945
64915473395777024743167351056637371274953501437271981836435236061968*sqrt(7
95279432905272963911047)*(9939513250523192816422116593216797292815016511001
```

378462170679301990*sqrt(1100522448786287362112823964249088848098)*sqrt(288
8868076710542715672947094311)*sqrt(7)*(10*sqrt(2) + 9) - sqrt(1590558865810
545927822094)*(5*12649387528042651238155741051177996081490572724l8^(3/4)*sq
rt(2888868076710542715672947094311)*sqrt(35)*(340613697110906370000*sqrt(2)
 - 483753219647003202703) + 556695603033691074737732 9*12649387528042651238l
5574105117799608149057272418^(1/4)*sqrt(2888868076710542715672947094311)*sq
rt(35)*(43734782664604992355*sqrt(2) - 66269826580994560232))*sqrt(81042225
92127468960547894479780085484 6405*sqrt(2) + 114611845046003414252052727757 3
33000617984) + 147461812540444568715696613114138557910359478676937042172325
5973728695229351827247907 86*sqrt(2888868076710542715672947094311)*sqrt(7)*(
125*sqrt(2) + 172))*sqrt(519179873173490157373042187501297125639064382628 55
81511813805064*x^2 - sqrt(1590558865810545927822094)*(1264938752804265123 81
5574105117799608149057272418^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(43
268355662383849682*sqrt(2) - 62135959399493560795) - 1264938752804265123815
574105117799608149057272418^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(43268355662383
849682*x - 105404315061877410477) - 62135959399493560795*x + 14867267072426
1260159))*sqrt(81042225921274689605478944797800854846405*sqrt(2) + 11461184
50460034142520527277573330006l7984) - 12979496829337253934326054687532428l4
09766095657139537795345126 6*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 389384904880117
6180297816406259728442292982869714186133860353798*x + 874869761179272589826
8147574007406280674519 0*sqrt(1100522448786287362112823964249088848098) + 9
0856477805360777540282382812726996986836266959997676456741588 62) - 5/353094
8699402200641933 2*sqrt(1100522448786287362112823964249088848098)*sqrt(7)*(
sqrt(2)*(10*x - 19) + 9*x - 29) + 1/701918227692516147086715878423299535653
31111850222032074026984349485892917146977000414136*sqrt(1590558865810545927
822094)*(5*126493875280426512381557410511779960814905727 2418^(3/4)*sqrt(35)
(sqrt(2)(340613697110906370000*x + 143139522536096832703) - 4837532196470
03202703*x - 197474174574809537297) + 556695603033691074737732 9*12649387528
04265123815574105117799608149057272418^(1/4)*sqrt(35)*(sqrt(2)*(43734782664
604992355*x + 22535043916389567877) - 66269826580994560232*x - 211997387482
15424478) - (5*126493875280426512381557410511779960814905727 2418^(3/4)*sqrt
(35)*(340613697110906370000*sqrt(2) - 483753219647003202703) + 556695603033
6910747377329*126493875280426512381557410511779960814905727 2418^(1/4)*sqrt(
35)*(43734782664604992355*sqrt(2) - 66269826580994560232))*sqrt(x^2 - 2*x +
 3))*sqrt(81042225921274689605478944797800854846405*sqrt(2) + 1146118450460
034142520527277573330006l7984) + 1/3530948699402200641933 2*sqrt(x^2 - 2*x +
 3)*(5*sqrt(1100522448786287362112823964249088848098)*sqrt(7)*(10*sqrt(2)
+ 9) + 741795945252563160 07*sqrt(7)*(125*sqrt(2) + 172)) - 1/476*sqrt(7)*(2
5*sqrt(2)*(5*x - 1) + 172*x - 82)) + 24453*12649387528042651238155741051177
99608149057272418^(1/4)*sqrt(1590558865810545927822094)*sqrt(35)*sqrt(7)*(5
8681264663553748097050996611754496316407808*x^38 - 909559602285083095504290
4474821946929043210 24*x^37 + 783394883258442537095630804766922525824044236 8
*x^36 - 4673962730452056035930111880126245631601881907 2*x^35 + 214091258966
892905713578429763409810498374336512*x^34 - 791437884096390872694182856990 0
21126475411881984*x^33 + 24439719810238523891830041696355042012098314895 36*

x^32 - 64469052811005676353501977392881165807935406735 36*x^31 + 14800438431
924516080565532176343356954693567774720*x^30 - 2999472018305304975160392093
8291975718069728182272*x^29 + 543950385039779684976755634437931462765495913
02144*x^28 - 89238943444544755685020562033988611037555993870336*x^27 + 1338
929022148270110928895284729232813282189440450 56*x^26 - 18513587658740219001
153199763371603483513268994 0480*x^25 + 2378226229048970415617021873730733 94
318812215508992*x^24 - 2849365368510540397648886779947929676842861781 31968*
x^23 + 32052399266923194172438848102331961245478278 7125248*x^22 - 338726814
722685956738928688519407226164440916295680*x^21 + 3388941060685178348864870
6925063457316146494675 3536*x^20 - 31944997194601654648963735003466931034597
1696140288*x^19 + 28739124750351132248997344249680842295832191 2250368*x^18
- 2427873721611128047920745808150073353142680105 77920*x^17 + 19843297253643
776777198157655776836216999227529 6256*x^16 - 14974464736529201535956289132 4
224536622995129499648*x^15 + 112846402465271023024054467724570114025686780 8
70656*x^14 - 74467740316666419201365857719494322341993062072320*x^13 + 5371
763229976795816995048942365255053104303518 5152*x^12 - 289289275588053521479
653590670201003027060028866 56*x^11 + 2204941276264425177330910467954280935 6
433843290112*x^10 - 783659258401410153171286014794040365856569740 4928*x^9 +
 82882226224310888136345304586172739794948744806 40*x^8 - 755718873364113816
635702120278992782166815932416*x^7 + 288349213189327895035480258909753471 72
93712375808*x^6 + 389268931244541502203228657135011133961927655424*x^5 + 77
026621102026789199647241685504778793625478758 4*x^4 + 151897599700059336983 3
5902525680408704502779084 8*x^3 + 117307057194105230541603999703274443460516
511744*x^2 - 810422259212746896054789447978008548464 05*sqrt(2)*(512*x^38 -
7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x^34 - 6905376*x^33 + 213239
04*x^32 - 56249904*x^31 + 129135330*x^30 - 261706983*x^29 + 474602241*x^28
- 778618854*x^27 + 1168229184*x^26 - 1615329345*x^25 + 2075026563*x^24 - 24
86100252*x^23 + 2796604422*x^22 - 2955425895*x^21 + 2956885529*x^20 - 27872
33482*x^19 + 2507517852*x^18 - 2118344505*x^17 + 1731347859*x^16 - 13065372
72*x^15 + 984596334*x^14 - 649738605*x^13 + 468691803*x^12 - 252407834*x^11
 + 192383368*x^10 - 68375067*x^9 + 72315585*x^8 - 6593724*x^7 + 25158762*x^
6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 1023516*x^2 + 137781*x + 5904
9) + 157913346222833964190620768831330981581464535 04*x + 6767714838121455 60
816946152134275635349133 7216)*sqrt(810422259212746896054789447978008548464 0
5*sqrt(2) + 114611845046003414252052727757333000617984)*log(514926300974084
616887160873794732709351351010668234952341442045423193866055445590 8352*x^2
+ 1651730760452563214106992734972755121667597949724571520 2048/1665374957748
9013357854121082231147111*sqrt(15905588658105459278220 94)*(1264938752804265
1238155741051177996081490572724 18^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3
)*(432683556623838496 82*sqrt(2) - 621359593994935607 95) - 12649387528042651
23815557410511779960814905727241 8^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(432683556
62383849682*x - 10540431506187741047 7) - 6213595939949356079 5*x + 148672670
724261260159))*sqrt(810422259212746896054789447978008548464 05*sqrt(2) + 114
611845046003414252052727757333000617984) - 1287315752435211542217902184486 8
31773378377526670587380853605113557984665138613977088*sqrt(x^2 - 2*x + 3)*(

4*x + 1) - 3861947257305634626653706553460495320135132580011762142560815340673953995415841931264*x + 86770206865778458070361238647050247531051756333085943213766737920*sqrt(11005224487862873621128239642490888848098) + 9011210267046480795525315291407822413648642686694111665975235794905892655970297839616) - 24453*1264938752804265123815574105117799608149057272418^(1/4)*sqrt(15905558865810545927822094)*sqrt(35)*sqrt(7)*(58681264663553748097050996611754496316407808*x^38 - 909559602285083095504290447482194692904321024*x^37 + 7833948832584425370956308047669225258240442368*x^36 - 46739627304520560359301118801262456316018819072*x^35 + 214091258966892905713578429763409810498374336512*x^34 - 791437884096390872694182856990021126475411881984*x^33 + 2443971981023852389183004169635504201209831489536*x^32 - 6446905281100567635350197739288116580793540673536*x^31 + 14800438431924516080565532176343356954693567774720*x^30 - 29994720183053049751603920938291975718069728182272*x^29 + 54395038503977968497675563443793146276549591302144*x^28 - 89238943444544755685020562033988611037555993870336*x^27 + 133892902214827011092889528472923281328218944045056*x^26 - 185135876587402190011531997633716034835132689940480*x^25 + 237822622904897041561702187373073394318812215508992*x^24 - 284936536851054039764888677994792967684286178131968*x^23 + 320523992669231941724388481023319612454782787125248*x^22 - 338726814722685956738928688519407226164440916295680*x^21 + 338894106068517834886487069250634573161464946753536*x^20 - 319449971946016546489637350034669310345971696140288*x^19 + 287391247503511322489973442496808422958321912250368*x^18 - 242787372161112804792074580815007335314268010577920*x^17 + 198432972536437767771981576557768362169992275296256*x^16 - 149744647365292015359562891324224536622995129499648*x^15 + 112846402465271023024054467724570114025686780870656*x^14 - 74467740316666419201365857719494322341993062072320*x^13 + 53717632299767958169950489423652550531043035185152*x^12 - 28928927558805352147965359067020100302706002886656*x^11 + 22049412762644251773309104679542809356433843290112*x^10 - 7836592584014101531712860147940403658565697404928*x^9 + 8288222622431088813634530458617273979494874480640*x^8 - 755718873364113816635702120278992782166815932416*x^7 + 2883492131893278950354802589097534717293712375808*x^6 + 389268931244541502203228657135011133961927655424*x^5 + 770266211020267891996472416855047787936254787584*x^4 + 151897599700059336983359025256804087045027790848*x^3 + 117307057194105230541603999703274443460516511744*x^2 - 81042225921274689605478944797800854846405*sqrt(2)*(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x^34 - 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 261706983*x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329345*x^25 + 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895*x^21 + 2956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^17 + 1731347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 + 468691803*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*x^8 - 6593724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 1023516*x^2 + 137781*x + 59049) + 15791334622283396419062076883133098158146453504*x + 6767714838121455608169461521342756353491337216)*sqrt(81042225921274689605478944797800854846405*sqrt(2) + 1146118450460034142

52052727757333000617984)*log(5149263009740846168871608737947327093513510106
682349523414420454231938660554455908352*x^2 - 16517307604525632141069927349
727551216675979497245715202048/16653749577489013357854121082231147111*sqrt(
1590558865810545927822094)*(12649387528042651238155741051177996081490572724
18^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(43268355662383849682*sqrt(2)
 - 62135959399493560795) - 126493875280426512381557410511779960814905727241
8^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(43268355662383849682*x - 105404315061877
410477) - 62135959399493560795*x + 148672670724261260159))*sqrt(81042225921
274689605478944797800854846405*sqrt(2) + 1146118450460034142520527277573330
00617984) - 128731575243521154221790218448683177337837752667058738085360511
3557984665138613977088*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 38619472573056346266
53706553460495320135132580011762142560815340673953995415841931264*x + 86770
206865778458070361238647050247531051756333085943213766737920*sqrt(110052244
87862873621128239642490888848098) + 901121026704648079552531529140782241364
8642686694111665975235794905892655970297839616) + 4731490670644819987632177
09555105306943512932580756046793648401639888862209988063963205432771600*x^4
 + 933056734920520960789163462383318633282684143000124192505535084262195413
27449647498113404575200*x^3 + 720578468552481534074799505602958944515340229
24762066351912610467773507163772995097552926305600*x^2 + 106889973888659738
28268515625026705527863090230587961936087245720*(33722490019334222378242713
60*x^37 - 53502205399640031394796147712*x^36 + 4691493940829897017294945758
72*x^35 - 2847499220912667753383035299072*x^34 + 13254252261100740556512388
253568*x^33 - 49770080058525077628064229832576*x^32 + 156010734937008739388
220889457760*x^31 - 417516398850754397130111919794336*x^30 + 97153817191336
5251873706873353652*x^29 - 1993653213575521837888601204380228*x^28 + 365555
3471852957606257345414140031*x^27 - 6054769996581738503753686155104785*x^26
 + 9155494158513869230271529746307221*x^25 - 127401066776850481786936051030
09787*x^24 + 16442770202470076313197215936814318*x^23 - 1977256973428874472
0189854470201506*x^22 + 22286437617621909921609206629636086*x^21 - 23584986
647560742443188031208946882*x^20 + 23579397211179175240196614296051673*x^19
 - 22218747553941794885903840542461607*x^18 + 19912295454080246583636391613
811979*x^17 - 16801760806053390242995145349148613*x^16 + 136134079650064752
88139078599341572*x^15 - 10279305650733178669223634020962076*x^14 + 7606288
378303449524327938977040824*x^13 - 5069838234992751929471190426115248*x^12
+ 3507425970596197680016078213030977*x^11 - 1974814483061344405275851094534
735*x^10 + 1357002388430055881833293557852283*x^9 - 56696901075916946161595
1049236597*x^8 + 458426000073846882432457044306894*x^7 - 947045576652534893
32536549937026*x^6 + 135183920426913231415208872303230*x^5 - 10230953189017
74638403186272874*x^4 + 29398041153524973343917601742151*x^3 + 193395719557
0062708781629134823*x^2 + 3397462350398947848063583843461*x - 8003871087155
5316861345369643)*sqrt(x^2 - 2*x + 3) + 97000947689757129586992241138859857
91552656932179508931988236024507972118200210878516740079600*x + 41571834724
181626965853817630939939106654243995055038279949582962177023363715189479357
45748400)/(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x^34 -
 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 261706983*

```
x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329345*x^25
+ 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895*x^21 + 2
956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^17 + 1731
347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 + 468691803
*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*x^8 - 659
3724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 1023516
*x^2 + 137781*x + 59049)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x+3)**(21/2)/(2*x**2+x+1)**10,x)
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x)
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(2\,x^2+x+1\right)^{10}\,\left(x^2-2\,x+3\right)^{21/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 2*x^2 + 1)^10*(x^2 - 2*x + 3)^(21/2)),x)
[Out] int(1/((x + 2*x^2 + 1)^10*(x^2 - 2*x + 3)^(21/2)), x)
```

3.52 $\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\left(1+x^2\right)}}\,dx$

Optimal. Leaf size=66

$$-\sqrt{2}\sqrt{a+\sqrt{1+a^2}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,(-a+x)}{\sqrt{(-a+x)\left(1+x^2\right)}}\right)$$

```
[Out] -arctan((-a+x)*2^(1/2)*(-a+(a^2+1)^(1/2))^(1/2)/((-a+x)*(x^2+1))^(1/2))*2^(
1/2)*(a+(a^2+1)^(1/2))^(1/2)
```

Rubi [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.76, antiderivative size = 204, normalized size of antiderivative = 3.09, number of steps used = 9, number of rules used = 8, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$,
Rules used = {6851, 6874, 733, 430, 946, 174, 552, 551}

$$\frac{4\sqrt{a^2+1}\sqrt{x^2+1}\sqrt{\frac{a-x}{a+i}}\,\Pi\left(\frac{2}{1-i\left(a-\sqrt{a^2+1}\right)};\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\left(1-i\left(a-\sqrt{a^2+1}\right)\right)\sqrt{-\left(\left(x^2+1\right)(a-x)\right)}}+\frac{2i\sqrt{x^2+1}\sqrt{\frac{a-x}{a+i}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{-\left(\left(x^2+1\right)(a-x)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a - Sqrt[1 + a^2] + x)/((-a + Sqrt[1 + a^2] + x)*Sqrt[(-a + x)*(1 + x
^2)]),x]
```

```
[Out] ((2*I)*Sqrt[(a - x)/(I + a)]*Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[1 - I*x]/S
qrt[2]], 2/(1 - I*a)])/Sqrt[-((a - x)*(1 + x^2))] + (4*Sqrt[1 + a^2]*Sqrt[(
a - x)/(I + a)]*Sqrt[1 + x^2]*EllipticPi[2/(1 - I*(a - Sqrt[1 + a^2])), Arc
Sin[Sqrt[1 - I*x]/Sqrt[2]], 2/(1 - I*a)])/((1 - I*(a - Sqrt[1 + a^2]))*Sqrt
[-((a - x)*(1 + x^2))])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\left(1+x^2\right)}}\,dx = \frac{\left(\sqrt{-a+x}\,\sqrt{1+x^2}\right)\int \frac{-a-\sqrt{1+a^2}+x}{\sqrt{-a+x}\left(-a+\sqrt{1+a^2}+x\right)\sqrt{}}}{\sqrt{(-a+x)\left(1+x^2\right)}}$$

$$= \frac{\left(\sqrt{-a+x}\,\sqrt{1+x^2}\right)\int \left(\frac{1}{\sqrt{-a+x}\,\sqrt{1+x^2}} - \frac{}{\sqrt{-a}}\right.}{\sqrt{(-a+x)\left(1+x^2\right.}}$$

$$= \frac{\left(\sqrt{-a+x}\,\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{-a+x}\,\sqrt{1+x^2}}\,dx}{\sqrt{(-a+x)\left(1+x^2\right)}} - \frac{\left(2\sqrt{}\right.}{}$$

$$= -\frac{\left(2\sqrt{1+a^2}\,\sqrt{-a+x}\,\sqrt{1+x^2}\right)\int \frac{}{\sqrt{1-ix}\,\sqrt{1+ix}}}{\sqrt{(-a+x)\left(1+x^2\right)}}$$

$$= \frac{2i\sqrt{\frac{a-x}{i+a}}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{-(a-x)\left(1+x^2\right)}} + $$

$$= \frac{2i\sqrt{\frac{a-x}{i+a}}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{-(a-x)\left(1+x^2\right)}} + $$

$$= \frac{2i\sqrt{\frac{a-x}{i+a}}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{-(a-x)\left(1+x^2\right)}} + \frac{4}{}$$

<u>Mathematica</u> [A]
time = 1.25, size = 99, normalized size = 1.50

$$-\frac{\sqrt{2}\,\sqrt{-a+x}\,\sqrt{1+x^2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{-a+\sqrt{1+a^2}}\,\sqrt{-a+x}}{\sqrt{1+x^2}}\right)}{\sqrt{-a+\sqrt{1+a^2}}\,\sqrt{(-a+x)\left(1+x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a - Sqrt[1 + a^2] + x)/((-a + Sqrt[1 + a^2] + x)*Sqrt[(-a + x)*
(1 + x^2)]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[-a + x]*Sqrt[1 + x^2]*ArcTan[(Sqrt[2]*Sqrt[-a + Sqrt[1 + a^
2]]*Sqrt[-a + x])/Sqrt[1 + x^2]])/(Sqrt[-a + Sqrt[1 + a^2]]*Sqrt[(-a + x)*(
1 + x^2)]))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x - a - Sqrt[a^2 + 1])/((x - a + Sqrt[a^2 + 1])*Sqrt[(x
 - a)*(x^2 + 1)]),x]')
```

```
[Out] Timed out
```

Maple [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.17, size = 787, normalized size = 11.92

method	result
default	$\frac{2i\sqrt{-i\left(x+i\right)}\,\sqrt{\frac{-a+x}{-i-a}}\,\sqrt{i\left(x-i\right)}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{-i\left(x+i\right)}}{2},\sqrt{2}\,\sqrt{-\frac{i}{-i-a}}\right)}{\sqrt{-a\,x^{2}+x^{3}-a+x}}-\frac{2\sqrt{a^{2}+1}\,\sqrt{-}}{}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),x,meth
od=_RETURNVERBOSE)
```

```
[Out] 2*I*(-I*(x+I))^(1/2)*((-a+x)/(-I-a))^(1/2)*(I*(x-I))^(1/2)/(-a*x^2+x^3-a+x)
^(1/2)*EllipticF(1/2*2^(1/2)*(-I*(x+I))^(1/2),2^(1/2)*(-I/(-I-a))^(1/2))-2*
(a^2+1)^(1/2)*(-(a-x)*(x^2+1)*(a^2+1))^(1/2)*(2*a*x-x^2+1)/(-a+x+(a^2+1)^(1
/2))/((-(a-x)*(x^2+1))^(1/2)*a^2+(-(a-x)*(x^2+1)*(a^2+1))^(1/2)*a-(-(a-x)*(
x^2+1)*(a^2+1))^(1/2)*x+(-(a-x)*(x^2+1))^(1/2))*(-I*(a^2+1)^(1/2)*(1-I*x)^(
1/2)*(-1/(-I-a)*a+1/(-I-a)*x)^(1/2)*(1+I*x)^(1/2)/(-a^3*x^2+a^2*x^3-a^3+a^2
*x-a*x^2+x^3-a+x)^(1/2)/(-I-a-(a^2+1)^(1/2))*EllipticPi(1/2*2^(1/2)*(-I*(x+
I))^(1/2),-2*I/(-I-a-(a^2+1)^(1/2)),2^(1/2)*(-I/(-I-a))^(1/2))+I*(a^2+1)^(1
/2)*(1-I*x)^(1/2)*(-1/(-I-a)*a+1/(-I-a)*x)^(1/2)*(1+I*x)^(1/2)/(-a^3*x^2+a^
2*x^3-a^3+a^2*x-a*x^2+x^3-a+x)^(1/2)/(-I-a+(a^2+1)^(1/2))*EllipticPi(1/2*2^
(1/2)*(-I*(x+I))^(1/2),-2*I/(-I-a+(a^2+1)^(1/2)),2^(1/2)*(-I/(-I-a))^(1/2))
+I*(1-I*x)^(1/2)*(-1/(-I-a)*a+1/(-I-a)*x)^(1/2)*(1+I*x)^(1/2)/(-a*x^2+x^3-a
```

```
+x)^(1/2)/(-I-a-(a^2+1)^(1/2))*EllipticPi(1/2*2^(1/2)*(-I*(x+I))^(1/2),-2*I
/(-I-a-(a^2+1)^(1/2)),2^(1/2)*(-I/(-I-a))^(1/2))+I*(1-I*x)^(1/2)*(-1/(-I-a)
*a+1/(-I-a)*x)^(1/2)*(1+I*x)^(1/2)/(-a*x^2+x^3-a+x)^(1/2)/(-I-a+(a^2+1)^(1/
2))*EllipticPi(1/2*2^(1/2)*(-I*(x+I))^(1/2),-2*I/(-I-a+(a^2+1)^(1/2)),2^(1/
2)*(-I/(-I-a))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x, algorithm="maxima")
```

```
[Out] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(a
^2 + 1))), x)
```

Fricas [A]
time = 0.35, size = 546, normalized size = 8.27

$$\left[\frac{1}{4}\sqrt{-2a-2\sqrt{a^2+1}}\log\left(-\frac{8ax^7+x^8+4(2a^2+15)x^6-8(4a^3+15a)x^5+2(8a^4+80a^2+67)x^4+64a^4-8(20a^3+37a)x^3+4(16a^4+74a^2+15)x^2+48a^2-4\left(ax^6+2(2a^2+3)x^5-(4a^3-a)x^4-8a^3-(4a^3+29a)x^2+20x^3+2(10a^2+3)x-(4ax^5+x^6-(4a^2-15)x^4-16ax^3+(4a^2+15)x^2+8a^2-20ax+1)\sqrt{a^2+1}-5a\right)\sqrt{-ax^2+x^3-a+x}\sqrt{-2a-2\sqrt{a^2+1}}-8(24a^3+13a)x+16(ax^6-x^7+15ax^4-7x^5-(12a^2+7)x^3+4a^3+(4a^3+15a)x^2-(12a^2+1)x+a)\sqrt{a^2+1}+1}{8ax^7-x^8-4(6a^2-1)x^6+8(4a^3-3a)x^5-2(8a^4-24a^2+3)x^4-8(4a^3-3a)x^3-4(6a^2-1)x^2-8ax-1}\right), -\frac{1}{2}\sqrt{2a+2\sqrt{a^2+1}}\arctan\left(-\frac{\sqrt{-ax^2+x^3-a+x}\left(2a^2-2ax-x^2-2\sqrt{a^2+1}(a-x)-1\right)\sqrt{2a+2\sqrt{a^2+1}}}{4(ax^2-x^3+a-x)}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(-2*a - 2*sqrt(a^2 + 1))*log(-(8*a*x^7 + x^8 + 4*(2*a^2 + 15)*x^6
- 8*(4*a^3 + 15*a)*x^5 + 2*(8*a^4 + 80*a^2 + 67)*x^4 + 64*a^4 - 8*(20*a^3 +
 37*a)*x^3 + 4*(16*a^4 + 74*a^2 + 15)*x^2 + 48*a^2 - 4*(a*x^6 + 2*(2*a^2 +
3)*x^5 - (4*a^3 - a)*x^4 - 8*a^3 - (4*a^3 + 29*a)*x^2 + 20*x^3 + 2*(10*a^2
+ 3)*x - (4*a*x^5 + x^6 - (4*a^2 - 15)*x^4 - 16*a*x^3 + (4*a^2 + 15)*x^2 +
8*a^2 - 20*a*x + 1)*sqrt(a^2 + 1) - 5*a)*sqrt(-a*x^2 + x^3 - a + x)*sqrt(-2
*a - 2*sqrt(a^2 + 1)) - 8*(24*a^3 + 13*a)*x + 16*(a*x^6 - x^7 + 15*a*x^4 -
7*x^5 - (12*a^2 + 7)*x^3 + 4*a^3 + (4*a^3 + 15*a)*x^2 - (12*a^2 + 1)*x + a)
*sqrt(a^2 + 1) + 1)/(8*a*x^7 - x^8 - 4*(6*a^2 - 1)*x^6 + 8*(4*a^3 - 3*a)*x^
5 - 2*(8*a^4 - 24*a^2 + 3)*x^4 - 8*(4*a^3 - 3*a)*x^3 - 4*(6*a^2 - 1)*x^2 -
8*a*x - 1)), -1/2*sqrt(2*a + 2*sqrt(a^2 + 1))*arctan(-1/4*sqrt(-a*x^2 + x^3
 - a + x)*(2*a^2 - 2*a*x - x^2 - 2*sqrt(a^2 + 1)*(a - x) - 1)*sqrt(2*a + 2*
sqrt(a^2 + 1))/(a*x^2 - x^3 + a - x))]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a**2+1)**(1/2))/(-a+x+(a**2+1)**(1/2))/((-a+x)*(x**2+1))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{a-x+\sqrt{a^2+1}}{\sqrt{-\left(x^2+1\right)\,\left(a-x\right)}\,\left(x-a+\sqrt{a^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a - x + (a^2 + 1)^(1/2))/((-(x^2 + 1)*(a - x))^(1/2)*(x - a + (a^2 +
1)^(1/2))),x)
```

```
[Out] int(-(a - x + (a^2 + 1)^(1/2))/((-(x^2 + 1)*(a - x))^(1/2)*(x - a + (a^2 +
1)^(1/2))), x)
```

3.53 $\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

Optimal. Leaf size=198

$$\frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}}+\frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{a\tanh^{-1}(x)}{6\ 2^{2/3}}+\frac{a\tanh^{-1}}{}$$

```
[Out] -1/12*a*arctanh(x)*2^(1/3)+1/4*a*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1
/3)-1/8*b*ln(x^2+3)*2^(1/3)+3/8*b*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3)+1/12*a
*arctan(3^(1/2)/x)*2^(1/3)*3^(1/2)+1/12*a*arctan((1-2^(1/3)*(-x^2+1)^(1/3))
*3^(1/2)/x)*2^(1/3)*3^(1/2)+1/4*b*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*
3^(1/2)*2^(1/3)
```

Rubi [A]
time = 0.06, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1024, 402, 455, 57, 631, 210, 31}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}+\frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{a\tanh^{-1}(x)}{6\ 2^{2/3}}-\frac{b\log\left(x^2+3\right)}{4\ 2^{2/3}}+\frac{3b\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (a*ArcTan[Sqrt[3]/x])/(2*2^(2/3)*Sqrt[3]) + (Sqrt[3]*b*ArcTan[(1 + (2 - 2*x
^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) + (a*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2
)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) - (a*ArcTanh[x])/(6*2^(2/3)) + (a*ArcTanh
[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(2*2^(2/3)) - (b*Log[3 + x^2])/(4*2^(2/3
)) + (3*b*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = a\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx + b\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}(x)}{6\,2^{2/3}} + \frac{a\tanh^{-1}}{}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}(x)}{6\,2^{2/3}} + \frac{a\tanh^{-1}}{}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}(x)}{6\,2^{2/3}} + \frac{a\tanh^{-1}}{}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\,2^{2/3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1-x^2}\right.}{x}\right.}{2\,2^{2/3}\sqrt{3}}$$

<u>Mathematica</u> [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.15, size = 145, normalized size = 0.73

$$\frac{1}{6}bx^2F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right) - \frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\,(3+x^2)\left(-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/3, 1, 2, x^2, -1/3*x^2])/6 - (9*a*x*AppellF1[1/2, 1/3,
 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)*(-9*AppellF1[1/2, 1/3,
1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2] -
 AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

<u>Mathics</u> [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)/((3 + x^2)*(1 - x^2)^(1/3)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{(-x^2+1)^{\frac{1}{3}}(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)
[Out] int((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
[Out] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+1)**(1/3)/(x**2+3),x)
[Out] Integral((a + b*x)/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x}{{\left(1-x^2\right)}^{1/3}\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] int((a + b*x)/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.54 $\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=198

$$-\frac{a\tan^{-1}(x)}{6\ 2^{2/3}}+\frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{a\tanh^{-1}\left(\right.}{}$$

```
[Out] -1/12*a*arctan(x)*2^(1/3)+1/4*a*arctan(x/(1+2^(1/3)*(x^2+1)^(1/3)))*2^(1/3)
+1/8*b*ln(-x^2+3)*2^(1/3)-3/8*b*ln(2^(2/3)-(x^2+1)^(1/3))*2^(1/3)-1/12*a*ar
ctanh(3^(1/2)/x)*2^(1/3)*3^(1/2)-1/12*a*arctanh((1-2^(1/3)*(x^2+1)^(1/3))*3
^(1/2)/x)*2^(1/3)*3^(1/2)-1/4*b*arctan(1/3*(1+2^(1/3)*(x^2+1)^(1/3))*3^(1/2
))*3^(1/2)*2^(1/3)
```

Rubi [A]
time = 0.06, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1024, 401, 455, 57, 631, 210, 31}

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{a\tan^{-1}(x)}{6\ 2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{b\log\left(3-x^2\right)}{4\ 2^{2/3}}-\frac{3b\log\left(2^{2/3}-\sqrt[3]{x^2+1}\right)}{4\ 2^{2/3}}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -1/6*(a*ArcTan[x])/2^(2/3) + (a*ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))])/(2
*2^(2/3)) - (Sqrt[3]*b*ArcTan[(1 + 2^(1/3)*(1 + x^2)^(1/3))/Sqrt[3]])/(2*2^
(2/3)) - (a*ArcTanh[Sqrt[3]/x])/(2*2^(2/3)*Sqrt[3]) - (a*ArcTanh[(Sqrt[3]*(
1 - 2^(1/3)*(1 + x^2)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) + (b*Log[3 - x^2])/(4
*2^(2/3)) - (3*b*Log[2^(2/3) - (1 + x^2)^(1/3)])/(4*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTanh[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (-Simp[q*(ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] + Simp[q*(ArcTan[q*x]/(6*2^(2/3)*a^(1/3)
*d)), x] + Simp[q*(ArcTanh[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = a\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx + b\int \frac{x}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$$

$$= -\frac{a\tan^{-1}(x)}{6\ 2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}\left(\cdots\right)}{\cdots}$$

$$= -\frac{a\tan^{-1}(x)}{6\ 2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}\left(\cdots\right)}{\cdots}$$

$$= -\frac{a\tan^{-1}(x)}{6\ 2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}\left(\cdots\right)}{\cdots}$$

$$= -\frac{a\tan^{-1}(x)}{6\ 2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}} - \cdots$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1072 vs. $2(198) = 396$.
time = 8.61, size = 1072, normalized size = 5.41

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] ((2*(Sqrt[3]*a^4 - 12*a^3*b + 18*Sqrt[3]*a^2*b^2 - 36*a*b^3 + 9*Sqrt[3]*b^4
)*ArcTan[(3*b + a*x + 6*2^(1/3)*b*(1 + x^2)^(1/3) - Sqrt[3]*(a + b*x + 2*2^
(1/3)*a*(1 + x^2)^(1/3)))/(3*b*(Sqrt[3] - x) + a*(-3 + Sqrt[3]*x))])/(Sqrt[
3]*a^3 - 9*a^2*b + 9*Sqrt[3]*a*b^2 - 9*b^3) - (2*(Sqrt[3]*a^4 + 12*a^3*b +
18*Sqrt[3]*a^2*b^2 + 36*a*b^3 + 9*Sqrt[3]*b^4)*ArcTan[(3*b + a*x + 6*2^(1/3
)*b*(1 + x^2)^(1/3) + Sqrt[3]*(a + b*x + 2*2^(1/3)*a*(1 + x^2)^(1/3)))/(3*b
*(Sqrt[3] + x) + a*(3 + Sqrt[3]*x))])/(Sqrt[3]*a^3 + 9*a^2*b + 9*Sqrt[3]*a*
b^2 + 9*b^3) + (2*(a^4 - 4*Sqrt[3]*a^3*b + 18*a^2*b^2 - 12*Sqrt[3]*a*b^3 +
9*b^4)*Log[3*b + a*x - 3*2^(1/3)*b*(1 + x^2)^(1/3) - Sqrt[3]*(a + b*x - 2^(
1/3)*a*(1 + x^2)^(1/3))])/(Sqrt[3]*a^3 - 9*a^2*b + 9*Sqrt[3]*a*b^2 - 9*b^3)
 - (2*(a^4 + 4*Sqrt[3]*a^3*b + 18*a^2*b^2 + 12*Sqrt[3]*a*b^3 + 9*b^4)*Log[-
3*b - a*x + 3*2^(1/3)*b*(1 + x^2)^(1/3) - Sqrt[3]*(a + b*x - 2^(1/3)*a*(1 +
 x^2)^(1/3))])/(Sqrt[3]*a^3 + 9*a^2*b + 9*Sqrt[3]*a*b^2 + 9*b^3) - ((a^4 -
4*Sqrt[3]*a^3*b + 18*a^2*b^2 - 12*Sqrt[3]*a*b^3 + 9*b^4)*Log[3*a^2 + 9*b^2
```

```
+ 12*a*b*x + a^2*x^2 + 3*b^2*x^2 + 3*2^(1/3)*(a^2 + 3*b^2 + 2*a*b*x)*(1 + x
^2)^(1/3) + 3*2^(2/3)*(a^2 + 3*b^2)*(1 + x^2)^(2/3) + Sqrt[3]*(-6*a*b - 2*a
^2*x - 6*b^2*x - 2*a*b*x^2 - 2^(1/3)*(6*a*b + a^2*x + 3*b^2*x)*(1 + x^2)^(1
/3) - 6*2^(2/3)*a*b*(1 + x^2)^(2/3))])/(Sqrt[3]*a^3 - 9*a^2*b + 9*Sqrt[3]*a
*b^2 - 9*b^3) + ((a^4 + 4*Sqrt[3]*a^3*b + 18*a^2*b^2 + 12*Sqrt[3]*a*b^3 + 9
*b^4)*Log[3*a^2 + 9*b^2 + 12*a*b*x + a^2*x^2 + 3*b^2*x^2 + 3*2^(1/3)*(a^2 +
 3*b^2 + 2*a*b*x)*(1 + x^2)^(1/3) + 3*2^(2/3)*(a^2 + 3*b^2)*(1 + x^2)^(2/3)
 + Sqrt[3]*(6*a*b + 2*a^2*x + 6*b^2*x + 2*a*b*x^2 + 2^(1/3)*(6*a*b + a^2*x
+ 3*b^2*x)*(1 + x^2)^(1/3) + 6*2^(2/3)*a*b*(1 + x^2)^(2/3))])/(Sqrt[3]*a^3
+ 9*a^2*b + 9*Sqrt[3]*a*b^2 + 9*b^3))/(12*2^(2/3))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)/((3 - x^2)*(1 + x^2)^(1/3)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(-x^{2}+3\right)\left(x^{2}+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)
[Out] int((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")
[Out] -integrate((b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{x^2\sqrt[3]{x^2+1} - 3\sqrt[3]{x^2+1}}\, dx - \int \frac{bx}{x^2\sqrt[3]{x^2+1} - 3\sqrt[3]{x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+3)/(x**2+1)**(1/3),x)
```

```
[Out] -Integral(a/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x) - Integral(b
*x/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,x}{{\left(x^2+1\right)}^{1/3}\,\left(x^2-3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x)/((x^2 + 1)^(1/3)*(x^2 - 3)),x)
```

```
[Out] int(-(a + b*x)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.55 $\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx$

Optimal. Leaf size=97

$$-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{4-6x+3x^2}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2\ 2^{2/3}}+\frac{\log\left(6-3x-3\sqrt[3]{2}\sqrt[3]{4-6x+3x^2}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/4*ln(x)*2^(1/3)+1/4*ln(6-3*x-3*2^(1/3)*(3*x^2-6*x+4)^(1/3))*2^(1/3)+1/6*
arctan(-1/3*3^(1/2)-1/3*2^(2/3)*(2-x)/(3*x^2-6*x+4)^(1/3)*3^(1/2))*2^(1/3)*
3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{764\}$

$$\frac{\log\left(-3\sqrt[3]{2}\sqrt[3]{3x^2-6x+4}-3x+6\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{3x^2-6x+4}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2^(2/3)*(2 - x))/(Sqrt[3]*(4 - 6*x + 3*x^2)^(1/3))]/(
2^(2/3)*Sqrt[3])) - Log[x]/(2*2^(2/3)) + Log[6 - 3*x - 3*2^(1/3)*(4 - 6*x +
 3*x^2)^(1/3)]/(2*2^(2/3))
```

Rule 764

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[3*c*e^2*(2*c*d - b*e), 3]}, Simp[(-Sqrt[3])*c*e*(ArcT
an[1/Sqrt[3] + 2*((c*d - b*e - c*e*x)/(Sqrt[3]*q*(a + b*x + c*x^2)^(1/3)))]
/q^2), x] + (-Simp[3*c*e*(Log[d + e*x]/(2*q^2)), x] + Simp[3*c*e*(Log[c*d -
 b*e - c*e*x - q*(a + b*x + c*x^2)^(1/3)]/(2*q^2)), x])] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[2*c*d - b*e, 0] && EqQ[c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*
c*e^2, 0] && PosQ[c*e^2*(2*c*d - b*e)]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{4-6x+3x^2}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2\ 2^{2/3}}+\frac{\log\left(6-3x-3\sqrt[3]{2}\sqrt[3]{4-6x\cdot\right.}}{2\ 2^{2/3}}$$

Mathematica [**A**]
time = 0.20, size = 168, normalized size = 1.73

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{2\,2^{2/3}-2^{2/3}x+\sqrt[3]{4-6x+3x^2}}{\sqrt{3}\sqrt[3]{4-6x+3x^2}}\right)-2\log\left(-22^{2/3}+2^{2/3}x+2\sqrt[3]{4-6x+3x^2}\right)+\log\left(-4\sqrt[3]{2}+4\sqrt[3]{2}x-\sqrt[3]{2}x^2+2^{2/3}(-2+x)\sqrt[3]{4-6x+3x^2}-2\left(4-6x+3x^2\right)^{2/3}\right)}{6\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(2*2^(2/3) - 2^(2/3)*x + (4 - 6*x + 3*x^2)^(1/3))/(S
qrt[3]*(4 - 6*x + 3*x^2)^(1/3))] - 2*Log[-2*2^(2/3) + 2^(2/3)*x + 2*(4 - 6*
x + 3*x^2)^(1/3)] + Log[-4*2^(1/3) + 4*2^(1/3)*x - 2^(1/3)*x^2 + 2^(2/3)*(-
2 + x)*(4 - 6*x + 3*x^2)^(1/3) - 2*(4 - 6*x + 3*x^2)^(2/3)])/2^(2/3)
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*(3*x^2 - 6*x + 4)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 8.73, size = 1592, normalized size = 16.41

method	result	size
trager	Expression too large to display	1592

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3*x^2-6*x+4)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*RootOf(_Z^3-2)*ln((96*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^
2)^2*RootOf(_Z^3-2)^2*x-12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z
^2)*RootOf(_Z^3-2)^3*x-96*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)-30*(3*x^2-6*
x+4)^(2/3)*x+24*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^3-240
*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2+480*RootOf(RootOf(
_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+96*RootOf(RootOf(_Z^3-2)^2+2*_Z*Roo
tOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(2/3)-24*RootOf(_Z^3-2)^
2*(3*x^2-6*x+4)^(1/3)*x^2+96*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)*x-72*Root
Of(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x+4
)^(1/3)-48*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^
3-2)^2*x^2+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^
3-2)^3*x^2-3*RootOf(_Z^3-2)*x^3+30*RootOf(_Z^3-2)*x^2-60*RootOf(_Z^3-2)*x-4
8*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(3*x
```

```
^2-6*x+4)^(2/3)*x-18*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*Ro
otOf(_Z^3-2)*(3*x^2-6*x+4)^(1/3)*x^2+72*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf
(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x+4)^(1/3)*x-320*RootOf(RootOf(_Z^
3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+8*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z
^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3+40*RootOf(_Z^3-2)+16*RootOf(RootOf(_Z^3-2)^2
+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-2*RootOf(RootOf(_Z^3-2)
^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3+60*(3*x^2-6*x+4)^(2/3)-
64*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2)/
x^3)+1/3*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*ln((12*RootOf(
RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x-24*RootOf
(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x+48*RootOf(
_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)+9*(3*x^2-6*x+4)^(2/3)*x-RootOf(RootOf(_Z^3-2)
^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^3+24*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_
Z^3-2)+4*_Z^2)*x^2-48*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x
-48*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(3
*x^2-6*x+4)^(2/3)+12*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)*x^2-48*RootOf(_Z^
3-2)^2*(3*x^2-6*x+4)^(1/3)*x+60*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)
+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x+4)^(1/3)-6*RootOf(RootOf(_Z^3-2)^2+2*_Z*
RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^2+12*RootOf(RootOf(_Z^3-2)^2+2*
_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^2+2*RootOf(_Z^3-2)*x^3-48*Root
Of(_Z^3-2)*x^2+96*RootOf(_Z^3-2)*x+24*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_
Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(2/3)*x+15*RootOf(RootOf(_Z^3
-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x+4)^(1/3)*x^2-60
*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-
6*x+4)^(1/3)*x+32*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+16*Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3-64*RootO
f(_Z^3-2)+2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z
^3-2)^2*x^3-4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z
^3-2)^3*x^3-18*(3*x^2-6*x+4)^(2/3)-8*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z
^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2)/x^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(74) = 148.
time = 1.26, size = 171, normalized size = 1.76

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}x^3+2\cdot 4^{\frac{2}{3}}(3\,x^2-6\,x+4)^{\frac{2}{3}}(x-2)+4\,(3\,x^2-6\,x+4)^{\frac{1}{3}}(x^2-4\,x+4)\right)}{6\,(x^3-12\,x^2+24\,x-16)}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{1}{3}}(x-2)+2\,(3\,x^2-6\,x+4)^{\frac{1}{3}}}{x}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}(3\,x^2-6\,x+4)^{\frac{2}{3}}+4^{\frac{1}{3}}(x^2-4\,x+4)-2\,(3\,x^2-6\,x+4)^{\frac{1}{3}}(x-2)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3)*x^3 + 2*4^(2/3)*(3
*x^2 - 6*x + 4)^(2/3)*(x - 2) + 4*(3*x^2 - 6*x + 4)^(1/3)*(x^2 - 4*x + 4))/
(x^3 - 12*x^2 + 24*x - 16)) + 1/12*4^(2/3)*log((4^(1/3)*(x - 2) + 2*(3*x^2
- 6*x + 4)^(1/3))/x) - 1/24*4^(2/3)*log((4^(2/3)*(3*x^2 - 6*x + 4)^(2/3) +
4^(1/3)*(x^2 - 4*x + 4) - 2*(3*x^2 - 6*x + 4)^(1/3)*(x - 2))/x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{3x^2-6x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x**2-6*x+4)**(1/3),x)
```

```
[Out] Integral(1/(x*(3*x**2 - 6*x + 4)**(1/3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(3\,x^2-6\,x+4\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x^2 - 6*x + 4)^(1/3)),x)
```

```
[Out] int(1/(x*(3*x^2 - 6*x + 4)^(1/3)), x)
```

3.56 $\int x\sqrt[3]{1-x^3}\,dx$

Optimal. Leaf size=73

$$\frac{1}{3}x^2\sqrt[3]{1-x^3}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(-x-\sqrt[3]{1-x^3}\right)$$

```
[Out] 1/3*x^2*(-x^3+1)^(1/3)-1/6*ln(-x-(-x^3+1)^(1/3))-1/9*arctan(1/3*(1-2*x/(-x^
3+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {285, 337}

$$-\frac{1}{6}\log\left(-\sqrt[3]{1-x^3}-x\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\sqrt[3]{1-x^3}\,x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - x^3)^(1/3),x]
```

```
[Out] (x^2*(1 - x^3)^(1/3))/3 - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(3*Sq
rt[3]) - Log[-x - (1 - x^3)^(1/3)]/6
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int x\sqrt[3]{1-x^3}\,dx = \frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{3}\int \frac{x}{(1-x^3)^{2/3}}\,dx$$
$$= \frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{3}\mathrm{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= \frac{1}{3}x^2\sqrt[3]{1-x^3} - \frac{1}{9}\mathrm{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{9}\mathrm{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \right.$$
$$= \frac{1}{3}x^2\sqrt[3]{1-x^3} - \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{18}\mathrm{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) +$$
$$= \frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{18}\log\left(1+\frac{x^2}{(1-x^3)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\mathrm{Su}$$
$$= \frac{1}{3}x^2\sqrt[3]{1-x^3} - \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{18}\log\left(1+\frac{x^2}{(1-x^3)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{9}\mathrm{lo}$$

Mathematica [A]
time = 0.14, size = 99, normalized size = 1.36

$$\frac{1}{18}\left(6x^2\sqrt[3]{1-x^3} - 2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2\sqrt[3]{1-x^3}}\right) - 2\log\left(x+\sqrt[3]{1-x^3}\right) + \log\left(x^2-x\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - x^3)^(1/3),x]
[Out] (6*x^2*(1 - x^3)^(1/3) - 2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/3
))] - 2*Log[x + (1 - x^3)^(1/3)] + Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^
(2/3)])/18
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.12, size = 19, normalized size = 0.26

$$\frac{x^2\mathrm{hyper}\left[\left\{-\frac{1}{3},\frac{2}{3}\right\},\left\{\frac{5}{3}\right\},x^3\mathrm{exp\_polar}\left[2I\mathrm{Pi}\right]\right]}{2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x*(1 - x^3)^(1/3),x]')
[Out] x ^ 2 hyper[{-1 / 3, 2 / 3}, {5 / 3}, x ^ 3 exp_polar[2 I Pi]] / 2
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 1.00, size = 15, normalized size = 0.21

method	result
meijerg	$\frac{x^2 \operatorname{hypergeom}\left(\left[-\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{2}$
risch	$-\frac{x^2(x^3-1)}{3(-x^3+1)^{\frac{2}{3}}}+\frac{(x^3-1)^{\frac{2}{3}}(-\operatorname{signum}(x^3-1))^{\frac{2}{3}}x^2 \operatorname{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{6\operatorname{signum}(x^3-1)^{\frac{2}{3}}(-x^3+1)^{\frac{2}{3}}}$
trager	$\frac{x^2(-x^3+1)^{\frac{1}{3}}}{3}-\frac{\ln\left(-2\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)^2x^3+3\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)(-x^3+1)^{\frac{2}{3}}x-\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)x^3+3x^2(-\right.}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*hypergeom([-1/3,2/3],[5/3],x^3)
```

Maxima [A]
time = 0.33, size = 105, normalized size = 1.44

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2(-x^3+1)^{\frac{1}{3}}}{x}-1\right)\right)-\frac{(-x^3+1)^{\frac{1}{3}}}{3x\left(\frac{x^3-1}{x^3}-1\right)}-\frac{1}{9}\log\left(\frac{(-x^3+1)^{\frac{1}{3}}}{x}+1\right)+\frac{1}{18}\log\left(-\frac{(-x^3+1)^{\frac{1}{3}}}{x}+\frac{(-x^3+1)^{\frac{2}{3}}}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) - 1/3*(-x^3 + 1
)^(1/3)/(x*((x^3 - 1)/x^3 - 1)) - 1/9*log((-x^3 + 1)^(1/3)/x + 1) + 1/18*lo
g(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)
```

Fricas [A]
time = 0.31, size = 96, normalized size = 1.32

$$\frac{1}{3}(-x^3+1)^{\frac{1}{3}}x^2-\frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3x}\right)-\frac{1}{9}\log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right)+\frac{1}{18}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*(-x^3 + 1)^(1/3)*x^2 - 1/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(
-x^3 + 1)^(1/3))/x) - 1/9*log((x + (-x^3 + 1)^(1/3))/x) + 1/18*log((x^2 - (
-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.51, size = 32, normalized size = 0.44

$$\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)**(1/3),x)
```

```
[Out] x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(5/3))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(1-x^3\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1 - x^3)^(1/3),x)
```

```
[Out] int(x*(1 - x^3)^(1/3), x)
```

3.57 $\int \frac{\sqrt[3]{1-x^3}}{x}\,dx$

Optimal. Leaf size=67

$$\sqrt[3]{1-x^3}-\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)$$

```
[Out] (-x^3+1)^(1/3)-1/2*ln(x)+1/2*ln(1-(-x^3+1)^(1/3))-1/3*arctan(1/3*(1+2*(-x^3
+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.400$, Rules used = {272, 52, 59, 632, 210, 31}

$$\sqrt[3]{1-x^3}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(1/3)/x,x]
```

```
[Out] (1 - x^3)^(1/3) - ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/
2 + Log[1 - (1 - x^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{1-x^3}}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{1-x}}{x}\,dx, x, x^3\right) \\
&= \sqrt[3]{1-x^3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^3\right) \\
&= \sqrt[3]{1-x^3} - \frac{\log(x)}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, \right. \\
&= \sqrt[3]{1-x^3} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right) \\
&= \sqrt[3]{1-x^3} - \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 88, normalized size = 1.31

$$
\sqrt[3]{1-x^3} - \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log\left(-1+\sqrt[3]{1-x^3}\right) - \frac{1}{6}\log\left(1+\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(1/3)/x,x]
```

```
[Out] (1 - x^3)^(1/3) - ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[-1
+ (1 - x^3)^(1/3)]/3 - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.07, size = 21, normalized size = 0.31

latex error

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x^3)^(1/3)/x,x]')
```

```
[Out] -1 ^ (1 / 3) x Gamma[-1 / 3] hyper[{-1 / 3, -1 / 3}, {2 / 3}, 1 / x ^ 3] /
(3 Gamma[2 / 3])
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 1.66, size = 49, normalized size = 0.73

method	result
meijerg	$-\frac{-3\left(3+\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)+\Gamma\left(\frac{2}{3}\right)x^3\,\mathrm{hypergeom}\left(\left[\frac{2}{3},1,1\right],[2,2],x^3\right)}{9\Gamma\left(\frac{2}{3}\right)}$
trager	$(-x^3+1)^{\frac{1}{3}}+\frac{\ln\left(-\frac{211\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3-2704\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3+5502\left(-x^3+1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)+862}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/GAMMA(2/3)*(-3*(3+1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*GAMMA(2/3)+GA
MMA(2/3)*x^3*hypergeom([2/3,1,1],[2,2],x^3))
```

Maxima [A]
time = 0.32, size = 71, normalized size = 1.06

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\left(-x^3+1\right)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + (-x^3 + 1)^(1/3
) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(
1/3) - 1)
```

Fricas [A]
time = 0.32, size = 73, normalized size = 1.09

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\left(-x^3+1\right)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + (-x^3 + 1
)^(1/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3
+ 1)^(1/3) - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.50, size = 37, normalized size = 0.55

$$-\frac{xe^{\frac{i\pi}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{1}{x^3}\right)}{3\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/x,x)
```

```
[Out] -x*exp(I*pi/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), x**(-3))/(3*gamma(2/
3))
```

Giac [A]
time = 0.00, size = 87, normalized size = 1.30

$$\left(-x^3+1\right)^{\frac{1}{3}}+\frac{\ln\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|}{3}-\frac{\ln\left(\left(\left(-x^3+1\right)^{\frac{1}{3}}\right)^2+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)}{6}-\frac{\arctan\left(\frac{2\left(-x^3+1\right)^{\frac{1}{3}}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/x,x)
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + (-x^3 + 1)^(1/3
) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 +
1)^(1/3) - 1))
```

Mupad [B]
time = 0.37, size = 83, normalized size = 1.24

$$\frac{\ln\left(\left(1-x^3\right)^{1/3}-1\right)}{3}+\ln\left(3\left(1-x^3\right)^{1/3}+\frac{3}{2}-\frac{\sqrt{3}\,3\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(3\left(1-x^3\right)^{1/3}+\frac{3}{2}+\frac{\sqrt{3}\,3\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\left(1-x^3\right)^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/x,x)
```

```
[Out] log((1 - x^3)^(1/3) - 1)/3 + log(3*(1 - x^3)^(1/3) - (3^(1/2)*3i)/2 + 3/2)*
((3^(1/2)*1i)/6 - 1/6) - log((3^(1/2)*3i)/2 + 3*(1 - x^3)^(1/3) + 3/2)*((3^
(1/2)*1i)/6 + 1/6) + (1 - x^3)^(1/3)
```

3.58 $\int \frac{\sqrt[3]{1-x^3}}{1+x} \, dx$

Optimal. Leaf size=482

$$\sqrt[3]{1-x^3}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1-\cdots}{\cdots}\right)}{\sqrt{3}}$$

```
[Out] (-x^3+1)^(1/3)-1/3*2^(1/3)*ln(x^3+1)+1/6*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*
2^(1/3)-1/6*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3
))*2^(1/3)+1/3*2^(1/3)*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))-1/12*ln(2*2^(1/3)
+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/2*ln(2^(1/3
)-(-x^3+1)^(1/3))*2^(1/3)-1/2*ln(-x-(-x^3+1)^(1/3))+1/2*ln(-2^(1/3)*x-(-x^3
+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))
*2^(1/3)*3^(1/2)-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/3
*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/3*2^(
1/3)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {2181, 420, 493, 298, 31, 648, 631, 210, 642, 495, 337, 503, 455, 52, 59}

$$\sqrt[3]{1-x^3}-\frac{1}{3}\sqrt[3]{2}\log\left(x^3+1\right)+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2^{2/3}}-\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2^{2/3}}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(1/3)/(1 + x),x]
```

```
[Out] (1 - x^3)^(1/3) + (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]])/Sqrt[3] + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]
]/(2^(2/3)*Sqrt[3]) - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] +
 (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - (2
^(1/3)*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - (2^(1/3)*Lo
g[1 + x^3])/3 + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[1
+ (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/
(3*2^(2/3)) + (2^(1/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 - Log[
2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/
(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/2^(2/3) - Log[-x - (1 - x^3)^(
1/3)]/2 + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/2^(2/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\int \frac{\sqrt[3]{1-x^3}}{1+x} dx = \int \frac{\sqrt[3]{1-x^3}}{1+x} dx$$

Mathematica [**F**]
time = 32.47, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{1-x^3}}{1+x} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^3)^(1/3)/(1 + x),x]
```

```
[Out] Integrate[(1 - x^3)^(1/3)/(1 + x), x]
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x^3)^(1/3)/(1 + x),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 12.52, size = 2990, normalized size = 6.20

method	result	size
risch	Expression too large to display	2990

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(1+x),x,method=_RETURNVERBOSE)
```

```
[Out] -(x^3-1)/(-x^3+1)^(2/3)+(1/2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2
)*ln(-(2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x
^3+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x^3
+2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x^2+4*R
ootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x^2+2*Roo
tOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x+4*RootOf(Ro
otOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x+5*(x^6-2*x^3+1)
^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)+8*R
ootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)*x^2+10*RootOf(RootOf(_Z^3-2)^2+_Z*RootO
f(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*(x^6-2*x^3+1)^(1/3)*x^2+7*RootOf(_Z^3-2)*x^4
+14*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^4+8*RootOf(_Z^3-2)^2*
(x^6-2*x^3+1)^(1/3)*x+10*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*Ro
otOf(_Z^3-2)*(x^6-2*x^3+1)^(1/3)*x+13*RootOf(_Z^3-2)*x^3+26*RootOf(RootOf(_
Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^3+8*RootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)
+10*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*(x^6-2*x
^3+1)^(1/3)+20*RootOf(_Z^3-2)*x^2+40*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3
-2)+_Z^2)*x^2+13*RootOf(_Z^3-2)*x+26*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3
-2)+_Z^2)*x+8*(x^6-2*x^3+1)^(2/3)+7*RootOf(_Z^3-2)+14*RootOf(RootOf(_Z^3-2)
^2+_Z*RootOf(_Z^3-2)+_Z^2))/(x^2+x+1)/(1+x)^2)-1/2*ln(-(2*RootOf(RootOf(_Z^
3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x^3+4*RootOf(RootOf(_Z^3-2)
^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x^3+2*RootOf(RootOf(_Z^3-2)^2
+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x^2+4*RootOf(RootOf(_Z^3-2)^2+_Z*
RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x^2+2*RootOf(RootOf(_Z^3-2)^2+_Z*Ro
otOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_
Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x-5*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*Ro
otOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)-2*RootOf(_Z^3-2)^2*(x^6-2*x^3
+1)^(1/3)*x^2-10*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^
3-2)*(x^6-2*x^3+1)^(1/3)*x^2-7*RootOf(_Z^3-2)*x^4-14*RootOf(RootOf(_Z^3-2)^
2+_Z*RootOf(_Z^3-2)+_Z^2)*x^4-2*RootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)*x-10*R
ootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*(x^6-2*x^3+1)
^(1/3)*x-9*RootOf(_Z^3-2)*x^3-18*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+
_Z^2)*x^3-2*RootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)-10*RootOf(RootOf(_Z^3-2)^2
+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*(x^6-2*x^3+1)^(1/3)-16*RootOf(_Z^3-
2)*x^2-32*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^2-9*RootOf(_Z^3
-2)*x-18*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x-2*(x^6-2*x^3+1)^
(2/3)-7*RootOf(_Z^3-2)-14*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2))/
(x^2+x+1)/(1+x)^2)*RootOf(_Z^3-2)-1/2*ln(-(2*RootOf(RootOf(_Z^3-2)^2+_Z*Roo
tOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x^3+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(
_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x^3+2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z
^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x^2+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2
```

```
)+_Z^2)^2*RootOf(_Z^3-2)^2*x^2+2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+
_Z^2)*RootOf(_Z^3-2)^3*x+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^
2*RootOf(_Z^3-2)^2*x-5*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_
Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)-2*RootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)*x^2
-10*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*(x^6-2*x
^3+1)^(1/3)*x^2-7*RootOf(_Z^3-2)*x^4-14*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_
Z^3-2)+_Z^2)*x^4-2*RootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)*x-10*RootOf(RootOf(
_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*(x^6-2*x^3+1)^(1/3)*x-9*Ro
otOf(_Z^3-2)*x^3-18*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^3-2*R
ootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)-10*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z
^3-2)+_Z^2)*RootOf(_Z^3-2)*(x^6-2*x^3+1)^(1/3)-16*RootOf(_Z^3-2)*x^2-32*Roo
tOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^2-9*RootOf(_Z^3-2)*x-18*Root
Of(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x-2*(x^6-2*x^3+1)^(2/3)-7*RootO
f(_Z^3-2)-14*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2))/(x^2+x+1)/(1+
x)^2)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)-1/3*ln((RootOf(RootOf
(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^6-RootOf(RootOf(_Z^
3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^3+8*RootOf(RootOf(_Z^3-
2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^6-6*(x^6-2*x^3+1)^(2/3)*Roo
tOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^2-10*RootOf
(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^3+16*x^6-12*(x
^6-2*x^3+1)^(1/3)*x^4+2*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(
_Z^3-2)+_Z^2)-24*x^3+12*(x^6-2*x^3+1)^(1/3)*x+8)/(-1+x)/(x^2+x+1))+1/6*ln((
RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^6-Root
Of(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^3+2*RootOf
(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^6-6*RootOf(_Z^
3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*(x^6-2*x^3+1)^(1/3)*
x^4+6*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootO
f(_Z^3-2)+_Z^2)*x^2-8*x^6+6*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*Roo
tOf(_Z^3-2)+_Z^2)*(x^6-2*x^3+1)^(1/3)*x+12*(x^6-2*x^3+1)^(2/3)*x^2-2*RootOf
(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)+16*x^3-8)/(-1+x)
/(x^2+x+1))*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2
)+1/3*ln((RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^
4*x^6-RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^
3+2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^6-6*
RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*(x^6-2*x^3
+1)^(1/3)*x^4+6*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^
2+_Z*RootOf(_Z^3-2)+_Z^2)*x^2-8*x^6+6*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2
)^2+_Z*RootOf(_Z^3-2)+_Z^2)*(x^6-2*x^3+1)^(1/3)*x+12*(x^6-2*x^3+1)^(2/3)*x^
2-2*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)+16*x^3
-8)/(-1+x)/(x^2+x+1)))/(-x^3+1)^(2/3)*((x^3-1)^2)^(1/3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 1), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(1+x),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x + 1), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(1-x^3)^{1/3}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/(x + 1),x)
```

```
[Out] int((1 - x^3)^(1/3)/(x + 1), x)
```

3.59 $\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2} \, dx$

Optimal. Leaf size=280

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}(-1+x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log}{}$$

```
[Out] -1/4*ln(-3*(-1+x)*(x^2-x+1))*2^(1/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)
+3/4*ln(-2^(1/3)*(-1+x)+(-x^3+1)^(1/3))*2^(1/3)+1/2*ln(x+(-x^3+1)^(1/3))-1/
4*ln(2^(1/3)*x+(-x^3+1)^(1/3))*2^(1/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)-1/6*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^
(1/2))*3^(1/2)-1/6*2^(1/3)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3
^(1/2)+1/2*arctan(1/3*(1+2*2^(1/3)*(-1+x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*
2^(1/3)
```

Rubi [A]

time = 0.16, antiderivative size = 408, normalized size of antiderivative = 1.46, number of steps used = 19, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2183, 420, 493, 298, 31, 648, 631, 210, 642, 495, 337, 503}

$$\frac{\log(x^3+1)}{3\,2^{2/3}}+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\,2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\,2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{6\,2^{2/3}}+\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2^{2/3}}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(1/3)/(1 - x + x^2),x]
```

```
[Out] (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]
) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - (2^(1/3)*ArcTan[(
1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + Log[1 + x^3]/(3*2^(2
/3)) + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[1 + (2^(2/3
)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(2/3
)) + (2^(1/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 - Log[2*2^(1/3)
 + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(2/3
)) + Log[-x - (1 - x^3)^(1/3)]/2 - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/2^(2
/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx &= \int \left(\frac{2i\sqrt[3]{1-x^3}}{\sqrt{3}\left(1+i\sqrt{3}-2x\right)}+\frac{2i\sqrt[3]{1-x^3}}{\sqrt{3}\left(-1+i\sqrt{3}+2x\right)}\right)dx\\ &= \frac{(2i)\int \frac{\sqrt[3]{1-x^3}}{1+i\sqrt{3}-2x}\,dx}{\sqrt{3}}+\frac{(2i)\int \frac{\sqrt[3]{1-x^3}}{-1+i\sqrt{3}+2x}\,dx}{\sqrt{3}}\end{aligned}$$

Mathematica [F]
time = 5.96, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^3)^(1/3)/(1 - x + x^2),x]
[Out] Integrate[(1 - x^3)^(1/3)/(1 - x + x^2), x]
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x^3)^(1/3)/(1 - x + x^2),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 7.06, size = 925, normalized size = 3.30

method	result	size
trager	Expression too large to display	925

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(x^2-x+1),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(-RootOf(_Z^6+108)^6*x^3+18*RootOf(_Z^6+108)^3*(-x^3+1)^(2/3)*x-18*Ro
otOf(_Z^6+108)^3*x^3+108*x*(-x^3+1)^(2/3)+216*x^2*(-x^3+1)^(1/3)+12*RootOf(
_Z^6+108)^3)-1/6*RootOf(_Z^6+108)*ln(-(RootOf(_Z^6+108)^5*x^4-2*RootOf(_Z^6
+108)^4*(-x^3+1)^(1/3)*x^3+2*RootOf(_Z^6+108)^5*x^3+6*RootOf(_Z^6+108)^4*(-
x^3+1)^(1/3)*x^2-x^2*RootOf(_Z^6+108)^5-2*RootOf(_Z^6+108)^4*(-x^3+1)^(1/3)
*x-2*RootOf(_Z^6+108)^5*x-6*RootOf(_Z^6+108)^2*x^4+36*RootOf(_Z^6+108)*(-x^
3+1)^(1/3)*x^3+RootOf(_Z^6+108)^5-12*RootOf(_Z^6+108)^2*x^3+144*(-x^3+1)^(2
/3)*x^2-108*RootOf(_Z^6+108)*(-x^3+1)^(1/3)*x^2+6*x^2*RootOf(_Z^6+108)^2-14
4*x*(-x^3+1)^(2/3)+36*RootOf(_Z^6+108)*(-x^3+1)^(1/3)*x+12*RootOf(_Z^6+108)
^2*x-6*RootOf(_Z^6+108)^2)/(x^2-x+1)^2)+1/72*ln(-(-3*RootOf(_Z^6+108)^4*(-x
^3+1)^(1/3)*x^3+RootOf(_Z^6+108)^4*(-x^3+1)^(1/3)*x^2+RootOf(_Z^6+108)^4*(-
x^3+1)^(1/3)*x-15*RootOf(_Z^6+108)^2*x^4+6*RootOf(_Z^6+108)^2*x^3+72*(-x^3+
1)^(2/3)*x^2+3*x^2*RootOf(_Z^6+108)^2-36*x*(-x^3+1)^(2/3)+6*RootOf(_Z^6+108
)^2*x-3*RootOf(_Z^6+108)^2)/(x^2-x+1)^2)*RootOf(_Z^6+108)^4+1/12*ln(-(-3*Ro
otOf(_Z^6+108)^4*(-x^3+1)^(1/3)*x^3+RootOf(_Z^6+108)^4*(-x^3+1)^(1/3)*x^2+R
ootOf(_Z^6+108)^4*(-x^3+1)^(1/3)*x-15*RootOf(_Z^6+108)^2*x^4+6*RootOf(_Z^6+
108)^2*x^3+72*(-x^3+1)^(2/3)*x^2+3*x^2*RootOf(_Z^6+108)^2-36*x*(-x^3+1)^(2/
3)+6*RootOf(_Z^6+108)^2*x-3*RootOf(_Z^6+108)^2)/(x^2-x+1)^2)*RootOf(_Z^6+10
8)-1/36*ln(RootOf(_Z^6+108)^6*x^3-36*RootOf(_Z^6+108)^3*(-x^3+1)^(2/3)*x-36
*(-x^3+1)^(1/3)*RootOf(_Z^6+108)^3*x^2+216*x*(-x^3+1)^(2/3)-216*x^2*(-x^3+1
)^(1/3)+12*RootOf(_Z^6+108)^3-324*x^3+216)*RootOf(_Z^6+108)^3-1/6*ln(RootOf
(_Z^6+108)^6*x^3-36*RootOf(_Z^6+108)^3*(-x^3+1)^(2/3)*x-36*(-x^3+1)^(1/3)*R
```

```
ootOf(_Z^6+108)^3*x^2+216*x*(-x^3+1)^(2/3)-216*x^2*(-x^3+1)^(1/3)+12*RootOf
(_Z^6+108)^3-324*x^3+216)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3085 vs. 2(218) = 436.
time = 5.46, size = 3085, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*2^(1/3)*arctan(1/3*(26795748*sqrt(3)*2^(2/3)*(586745*x^11 - 70
6109*x^10 - 191742*x^9 - 43779*x^8 + 396304*x^7 + 323715*x^6 - 462255*x^5 +
 73568*x^4 + 24102*x^3 + 2372*x^2 - 2008*x)*(-x^3 + 1)^(1/3) + 26795748*sqr
t(3)*2^(1/3)*(340975*x^10 + 46080*x^9 - 970873*x^8 + 685704*x^7 - 289743*x^
6 + 397966*x^5 - 203166*x^4 - 21912*x^3 + 29756*x^2 - 4016*x)*(-x^3 + 1)^(2
/3) + 7*sqrt(273426)*2^(1/6)*(6*sqrt(3)*2^(2/3)*(338078915*x^10 - 459916473
*x^9 - 111133574*x^8 + 235674676*x^7 + 297312537*x^6 - 494815414*x^5 + 2448
15194*x^4 - 34383000*x^3 - 8933924*x^2 + 2566224*x)*(-x^3 + 1)^(2/3) + sqrt
(3)*2^(1/3)*(2332175065*x^12 - 3283524318*x^11 + 1882024851*x^10 - 39193009
70*x^9 + 2796090405*x^8 + 610770276*x^7 + 98233512*x^6 + 140867400*x^5 - 11
45424564*x^4 + 430987096*x^3 + 108889824*x^2 - 54987072*x + 4032064) - 6*sq
rt(3)*(493920245*x^11 - 452201839*x^10 - 276972599*x^9 - 661557480*x^8 + 13
75964914*x^7 - 191435014*x^6 - 333786162*x^5 - 47180632*x^4 + 107411572*x^3
 - 13096840*x^2 - 2566224*x)*(-x^3 + 1)^(1/3)) - 3*sqrt(3)*(2247079524645*x
^12 - 5276442179264*x^11 + 3816306322874*x^10 - 3280399521884*x^9 + 6278089
258290*x^8 - 6181108351032*x^7 + 2698150339136*x^6 + 1210170331680*x^5 - 25
58541243960*x^4 + 1136906331664*x^3 - 42652634816*x^2 - 54080708992*x + 515
2977792))/(18230538112975*x^12 - 14115716188440*x^11 - 20854883745366*x^10
+ 1856205891292*x^9 + 11854156958820*x^8 + 23868971173080*x^7 - 27900743059
560*x^6 + 8785124358048*x^5 - 2880050871456*x^4 + 1047429829408*x^3 + 24296
4112512*x^2 - 141331907328*x + 8096384512)) + 1/18*sqrt(3)*2^(1/3)*arctan(-
1/3*(13397874*sqrt(3)*2^(2/3)*(18803*x^11 - 25367*x^10 - 203754*x^9 + 40802
1*x^8 - 139829*x^7 + 7128*x^6 - 233871*x^5 + 225275*x^4 - 47094*x^3 - 10225
*x^2 + 2921*x)*(-x^3 + 1)^(1/3) + 26795748*sqrt(3)*2^(1/3)*(10589*x^10 - 73
935*x^9 + 63883*x^8 + 142959*x^7 - 173613*x^6 - 31588*x^5 + 79410*x^4 - 437
```

```
7*x^3 - 13328*x^2 + 2921*x)*(-x^3 + 1)^(2/3) - 7*sqrt(273426)*(6*sqrt(3)*2^
(2/3)*(309683372*x^10 - 328552599*x^9 - 24698630*x^8 - 422031122*x^7 + 7021
64163*x^6 - 95703451*x^5 - 206316094*x^4 + 60985482*x^3 + 11167816*x^2 - 37
33038*x)*(-x^3 + 1)^(2/3) + sqrt(3)*2^(1/3)*(2345654785*x^12 - 2502234618*x
^11 - 252041853*x^10 - 4416416426*x^9 + 6899968311*x^8 - 1680852528*x^7 + 1
576960038*x^6 - 2990585436*x^5 + 642930363*x^4 + 528479914*x^3 - 117963261*
x^2 - 38399466*x + 8532241) - 6*sqrt(3)*(491687266*x^11 - 516958230*x^10 -
69305552*x^9 - 808934094*x^8 + 1418391515*x^7 - 385704187*x^6 - 112721241*x
^5 - 69510422*x^4 + 47121139*x^3 + 11465929*x^2 - 4799203*x)*(-x^3 + 1)^(1/
3))*sqrt((6*2^(2/3)*(4*x^10 - 27*x^9 + 32*x^8 + 6*x^7 + 12*x^6 - 65*x^5 + 4
8*x^4 - 6*x^3 - 4*x^2 + x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(35*x^12 - 66*x^11 -
201*x^10 + 338*x^9 + 90*x^8 - 90*x^7 - 249*x^6 - 18*x^5 + 306*x^4 - 166*x^3
 + 15*x^2 + 6*x - 1) - 6*(x^11 + 29*x^10 - 93*x^9 + 66*x^8 - 19*x^7 + 87*x^
6 - 99*x^5 + 10*x^4 + 27*x^3 - 11*x^2 + x)*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11
 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*x^5 + 90*x^4 - 50*x^
3 + 21*x^2 - 6*x + 1)) - 3*sqrt(3)*(2995162579*x^12 + 315959718008*x^11 - 8
49682072424*x^10 + 177300060912*x^9 - 508006765899*x^8 + 3583876884636*x^7
 - 3031033916540*x^6 - 1410763301208*x^5 + 2375077456341*x^4 - 546587071308*
x^3 - 175036021936*x^2 + 63861157012*x - 3114267965))/(367648430113*x^12 -
1408582980384*x^11 - 1269375810828*x^10 + 5714713216048*x^9 - 1087485936795
*x^8 - 126379999188*x^7 - 10319650860540*x^6 + 10854292018608*x^5 - 1383220
291365*x^4 - 1828745373668*x^3 + 426327416076*x^2 + 93479232396*x - 2492267
5961)) - 1/18*sqrt(3)*2^(1/3)*arctan(1/3*(13397874*sqrt(3)*2^(2/3)*(17344*x
^11 - 120304*x^10 + 110610*x^9 + 203214*x^8 - 213415*x^7 - 96387*x^6 + 3010
2*x^5 + 157561*x^4 - 101868*x^3 + 15151*x^2 + 913*x)*(-x^3 + 1)^(1/3) - 267
95748*sqrt(3)*2^(1/3)*(1277*x^10 + 57510*x^9 - 189677*x^8 + 108972*x^7 + 10
2426*x^6 - 47461*x^5 - 82155*x^4 + 56409*x^3 - 7301*x^2 - 913*x)*(-x^3 + 1)
^(2/3) + 7*sqrt(273426)*(6*sqrt(3)*2^(2/3)*(8733539*x^10 - 122586360*x^9 +
269810944*x^8 - 28009538*x^7 - 316185126*x^6 + 161786897*x^5 + 95479640*x^4
 - 80193978*x^3 + 11163982*x^2 + 1166814*x)*(-x^3 + 1)^(2/3) - sqrt(3)*2^(1
/3)*(1971824*x^12 - 78264612*x^11 + 705529692*x^10 - 1556393152*x^9 + 93384
9120*x^8 + 135726408*x^7 - 213906684*x^6 + 446158968*x^5 - 582881445*x^4 +
182390318*x^3 + 31120185*x^2 - 12999294*x - 833569) + 6*sqrt(3)*(12965988*x
^11 - 175265260*x^10 + 270273662*x^9 + 299814882*x^8 - 663644613*x^7 + 7755
3085*x^6 + 286893603*x^5 - 82332150*x^4 - 33723265*x^3 + 10863861*x^2 + 333
245*x)*(-x^3 + 1)^(1/3))*sqrt((6*2^(2/3)*(143*x^10 - 177*x^9 - 2*x^8 - 54*x
^7 + 141*x^6 - 31*x^5 - 18*x^4 - 6*x^3 + 7*x^2 - x)*(-x^3 + 1)^(2/3) + 2^(1
/3)*(1081*x^12 - 1338*x^11 - 15*x^10 - 1130*x^9 + 1962*x^8 - 234*x^7 + 33*x
^6 - 630*x^5 + 234*x^4 + 58*x^3 - 15*x^2 - 6*x + 1) - 6*(227*x^11 - 281*x^1
0 - 3*x^9 - 162*x^8 + 319*x^7 - 51*x^6 - 21*x^5 - 58*x^4 + 33*x^3 - x^2 - x
)*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 +
141*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) - 3*sqrt(3)*(67113
679084*x^12 - 61534090748*x^11 - 1006807736260*x^10 + 1996201310444*x^9 + 1
93806523788*x^8 - 2673973669800*x^7 + 775957356356*x^6 + 2110159119756*x^5
 - 1821028473882*x^4 + 377014646048*x^3 + 67410900094*x^2 - 19835743048*x -
```

```
1369553867))/(168032067092*x^12 - 2318893136652*x^11 + 4401905935020*x^10 +
 1550444734940*x^9 - 6210007783092*x^8 - 1634341806144*x^7 + 6341768478444*
x^6 - 948091553244*x^5 - 2281774840272*x^4 + 1036207535072*x^3 - 5948022808
2*x^2 - 20085678624*x - 761048497)) + 1/3*sqrt(3)*arctan((4*sqrt(3)*(-x^3 +
 1)^(1/3)*x^2 + 2*sqrt(3)*(-x^3 + 1)^(2/3)*x - sqrt(3)*(x^3 - 1))/(9*x^3 -
1)) + 1/48*2^(1/3)*log(7717175424*(6*2^(2/3)*(143*x^10 - 177*x^9 - 2*x^8 -
54*x^7 + 141*x^6 - 31*x^5 - 18*x^4 - 6*x^3 + 7*x^2 - x)*(-x^3 + 1)^(2/3) +
2^(1/3)*(1081*x^12 - 1338*x^11 - 15*x^10 - 1130*x^9 + 1962*x^8 - 234*x^7 +
33*x^6 - 630*x^5 + 234*x^4 + 58*x^3 - 15*x^2 - 6*x + 1) - 6*(227*x^11 - 281
*x^10 - 3*x^9 - 162*x^8 + 319*x^7 - 51*x^6 - 21*x^5 - 58*x^4 + 33*x^3 - x^2
 - x)*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^
7 + 141*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) + 1/48*2^(1/3)
*log(1929293856*(6*2^(2/3)*(143*x^10 - 177*x^9 - 2*x^8 - 54*x^7 + 141*x^6 -
 31*x^5 - 18*x^4 - 6*x^3 + 7*x^2 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(1081*x^12
 - 1338*x^11 - 15*x^10 - 1130*x^9 + 1962*x^8 - 234*x^7 + 33*x^6 - 630*x^5 +
 234*x^4 + 58*x^3 - 15*x^2 - 6*x + 1) - 6*(227*x^11 - 281*x^10 - 3*x^9 - 16
2*x^8 + 319*x^7 - 51*x^6 - 21*x^5 - 58*x^4 + 33*x^3 - x^2 - x)*(-x^3 + 1)^(
1/3))/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*
x^5 + 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) - 1/48*2^(1/3)*log(7717175424*(6
*2^(2/3)*(4*x^10 - 27*x^9 + 32*x^8 + 6*x^7 + 12*x^6 - 65*x^5 + 48*x^4 - 6*x
^3 - 4*x^2 + x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(35*x^12 - 66*x^11 - 201*x^10 +
338*x^9 + 90*x^8 - 90*x^7 - 249*x^6 - 18*x^5 + 306*x^4 - 166*x^3 + 15*x^2 +
 6*x - 1) - 6*(x^11 + 29*x^10 - 93*x^9 + 66*x^8 - 19*x^7 + 87*x^6 - 99*x^5
+ 10*x^4 + 27*x^3 - 11*x^2 + x)*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11 + 21*x^10
- 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2
- 6*x + 1)) - 1/48*2^(1/3)*log(1929293856*(6*2^(2/3)*(4*x^10 - 27*x^9 + 32*
x^8 + 6*x^7 + 12*x^6 - 65*x^5 + 48*x^4 - 6*x^3 - 4*x^2 + x)*(-x^3 + 1)^(2/3
) - 2^(1/3)*(35*x^12 - 66*x^11 - 201*x^10 + 338*x^9 + 90*x^8 - 90*x^7 - 249
*x^6 - 18*x^5 + 306*x^4 - 166*x^3 + 15*x^2 + 6*x - 1) - 6*(x^11 + 29*x^10 -
 93*x^9 + 66*x^8 - 19*x^7 + 87*x^6 - 99*x^5 + 10*x^4 + 27*x^3 - 11*x^2 + x)
*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 1
41*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) + 1/6*log(3*(-x^3 +
 1)^(1/3)*x^2 + 3*(-x^3 + 1)^(2/3)*x + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(x**2-x+1),x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x**2 - x + 1), x)
```

Giac [**F**] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(1-x^3)^{1/3}}{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/(x^2 - x + 1),x)
```

```
[Out] int((1 - x^3)^(1/3)/(x^2 - x + 1), x)
```

3.60 $\int \frac{\sqrt[3]{1-x^3}}{2+x} \, dx$

Optimal. Leaf size=232

$$\sqrt[3]{1-x^3}+\frac{1}{2}xF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-\frac{x^3}{8}\right)-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\sqrt[6]{3}\tan^{-1}\left(\frac{1-\frac{3^{2/3}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)-\sqrt[6]{3}\tan^{-1}$$

```
[Out] (-x^3+1)^(1/3)+1/2*x*AppellF1(1/3,-1/3,1,4/3,x^3,-1/8*x^3)-3^(1/6)*arctan(2
/9*(-x^3+1)^(1/3)*3^(5/6)+1/3*3^(1/2))+3^(1/6)*arctan(1/3*(1-3^(2/3)*x/(-x^
3+1)^(1/3))*3^(1/2))-1/3*ln(x^3+8)*3^(2/3)+1/2*3^(2/3)*ln(3^(2/3)-(-x^3+1)^
(1/3))-ln(-x-(-x^3+1)^(1/3))+1/2*3^(2/3)*ln(-1/2*3^(2/3)*x-(-x^3+1)^(1/3))-
2/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {2181, 440, 495, 337, 503, 455, 52, 59, 631, 210, 31}

$$\frac{1}{2}xF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-\frac{x^3}{8}\right)+\sqrt[3]{1-x^3}-\frac{\log\left(x^3+8\right)}{\sqrt[3]{3}}+\frac{1}{2}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{1-x^3}\right)-\log\left(-\sqrt[3]{1-x^3}-x\right)+\frac{1}{2}3^{2/3}\log\left(-\sqrt[3]{1-x^3}-\frac{1}{2}3^{2/3}x\right)-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\sqrt[6]{3}\tan^{-1}\left(\frac{1-\frac{3^{2/3}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)-\sqrt[6]{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}}{3\sqrt[6]{3}}+\frac{1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(1/3)/(2 + x),x]
```

```
[Out] (1 - x^3)^(1/3) + (x*AppellF1[1/3, -1/3, 1, 4/3, x^3, -1/8*x^3])/2 - (2*Arc
Tan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + 3^(1/6)*ArcTan[(1 - (3^
(2/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - 3^(1/6)*ArcTan[1/Sqrt[3] + (2*(1 - x^3
)^(1/3))/(3*3^(1/6))] - Log[8 + x^3]/3^(1/3) + (3^(2/3)*Log[3^(2/3) - (1 -
x^3)^(1/3)])/2 - Log[-x - (1 - x^3)^(1/3)] + (3^(2/3)*Log[-1/2*(3^(2/3)*x)
- (1 - x^3)^(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx=\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx$$

Mathematica [F]
time = 33.69, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^3)^(1/3)/(2 + x),x]
```

```
[Out] Integrate[(1 - x^3)^(1/3)/(2 + x), x]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x^3)^(1/3)/(2 + x),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(-x^3+1)^{\frac{1}{3}}}{2+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(2+x),x)
```

```
[Out] int((-x^3+1)^(1/3)/(2+x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(2+x),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x + 2), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(2+x),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(1-x^3\right)}^{1/3}}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/(x + 2),x)
[Out] int((1 - x^3)^(1/3)/(x + 2), x)
```

3.61 $\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=168

$$-\frac{x^2F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};x^3,-\frac{x^3}{2}\right)}{2\sqrt[3]{2}}+\frac{2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3^{5/6}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}+2\sqrt[3]{2+x^3}}{3^{5/6}}\right)}{3^{5/6}}+\frac{\log\left(1-x^3\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}\right.}{2}$$

```
[Out] -1/4*x^2*AppellF1(2/3,1,1/3,5/3,x^3,-1/2*x^3)*2^(2/3)+1/3*arctan(1/3*(3^(1/
3)+2*(x^3+2)^(1/3))*3^(1/6))*3^(1/6)+2/3*arctan(1/3*(1+2*3^(1/3)*x/(x^3+2)^
(1/3))*3^(1/2))*3^(1/6)+1/18*ln(-x^3+1)*3^(2/3)+1/6*ln(3^(1/3)-(x^3+2)^(1/3
))*3^(2/3)-1/3*ln(3^(1/3)*x-(x^3+2)^(1/3))*3^(2/3)
```

Rubi [A]
time = 0.10, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2183, 384, 524, 455, 57, 631, 210, 31}

$$-\frac{x^2F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};x^3,-\frac{x^3}{2}\right)}{2\sqrt[3]{2}}+\frac{\log\left(1-x^3\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}-\sqrt[3]{x^3+2}\right)}{2\sqrt[3]{3}}-\frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{x^3+2}\right)}{\sqrt[3]{3}}+\frac{2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+2}+\sqrt[3]{3}}{3^{5/6}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)),x]
```

```
[Out] -1/2*(x^2*AppellF1[2/3, 1, 1/3, 5/3, x^3, -1/2*x^3])/2^(1/3) + (2*ArcTan[(1
 + (2*3^(1/3)*x)/(2 + x^3)^(1/3))/Sqrt[3]])/3^(5/6) + ArcTan[(3^(1/3) + 2*(
2 + x^3)^(1/3))/3^(5/6)]/3^(5/6) + Log[1 - x^3]/(6*3^(1/3)) + Log[3^(1/3) -
 (2 + x^3)^(1/3)]/(2*3^(1/3)) - Log[3^(1/3)*x - (2 + x^3)^(1/3)]/3^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx = \int \left(\frac{1-i\sqrt{3}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{2+x^3}} + \frac{1+i\sqrt{3}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{2+x^3}}\right) dx$$
$$= \left(1-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{2+x^3}}\,dx + \left(1+i\sqrt{3}\right)\int \frac{}{\left(1+i\sqrt{3}\right.}$$

Mathematica [**F**]
time = 10.10, size = 0, normalized size = 0.00

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)),x]
[Out] Integrate[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)), x]
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{2+x}{(x^2+x+1)\,(x^3+2)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x)
[Out] int((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="maxima")
[Out] integrate((x + 2)/((x^3 + 2)^(1/3)*(x^2 + x + 1)), x)
```

Fricas [**F**]
time = 5.72, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="fricas")
[Out] integral((x^3 + 2)^(2/3)*(x + 2)/(x^5 + x^4 + x^3 + 2*x^2 + 2*x + 2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\sqrt[3]{x^3+2}\,(x^2+x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+x+1)/(x**3+2)**(1/3),x)
[Out] Integral((x + 2)/((x**3 + 2)**(1/3)*(x**2 + x + 1)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{{\left(x^3+2\right)}^{1/3}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((x^3 + 2)^(1/3)*(x + x^2 + 1)),x)
[Out] int((x + 2)/((x^3 + 2)^(1/3)*(x + x^2 + 1)), x)
```

3.62 $\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx$

Optimal. Leaf size=25

$$\frac{1}{8}\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

```
[Out] 1/8*ln(320*x^4+80*x^3-12*x^2+24*x+9)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1601}

$$\frac{1}{8}\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{1}{8}\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{8}\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^
4),x]
```

```
[Out] Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8
```

Mathics [A]
time = 1.71, size = 23, normalized size = 0.92

$$\frac{\text{Log}\left[9+24x-12x^2+80x^3+320x^4\right]}{8}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3
 + 320*x^4),x]')
```

```
[Out] Log[9 + 24 x - 12 x ^ 2 + 80 x ^ 3 + 320 x ^ 4] / 8
```

Maple [A]
time = 0.01, size = 24, normalized size = 0.96

method	result	size
default	$\frac{\ln(320x^4+80x^3-12x^2+24x+9)}{8}$	24
norman	$\frac{\ln(320x^4+80x^3-12x^2+24x+9)}{8}$	24
risch	$\frac{\ln(320x^4+80x^3-12x^2+24x+9)}{8}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/8*ln(320*x^4+80*x^3-12*x^2+24*x+9)
```

Maxima [A]
time = 0.24, size = 23, normalized size = 0.92

$$\frac{1}{8}\,\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorith
m="maxima")
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

Fricas [A]
time = 0.30, size = 23, normalized size = 0.92

$$\frac{1}{8}\,\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorith
m="fricas")
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

Sympy [A]
time = 0.05, size = 22, normalized size = 0.88

$$\frac{\log\left(320x^4+80x^3-12x^2+24x+9\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x**3+30*x**2-3*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
```

```
[Out] log(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9)/8
```

Giac [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{\ln\left(320x^4+80x^3-12x^2+24x+9\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x)
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

Mupad [B]
time = 0.07, size = 23, normalized size = 0.92

$$\frac{\ln\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((30*x^2 - 3*x + 160*x^3 + 3)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9),x)
```

```
[Out] log(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9)/8
```

3.63 $\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4} dx$

Optimal. Leaf size=59

$$-\frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}+\frac{\tan^{-1}\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right)}{2\sqrt{11}}$$

```
[Out] -1/22*arctan(1/55*(7-40*x)*11^(1/2))*11^(1/2)+1/22*arctan(1/66*(800*x^3-40*
x^2+30*x+57)*11^(1/2))*11^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {2115}

$$\frac{\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}}-\frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] -1/2*ArcTan[(7 - 40*x)/(5*Sqrt[11])]/Sqrt[11] + ArcTan[(57 + 30*x - 40*x^2
+ 800*x^3)/(6*Sqrt[11])]/(2*Sqrt[11])
```

Rule 2115

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 +
 (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-C)*(2*e*(B*d - 4
*A*e) + C*(d^2 - 4*c*e)), 2]}, Simp[2*(C^2/q)*ArcTan[(C*d - B*e + 2*C*e*x)/
q], x] - Simp[2*(C^2/q)*ArcTan[C*((4*B*c*C - 3*B^2*d - 4*A*C*d + 12*A*B*e +
 4*C*(2*c*C - B*d + 2*A*e)*x + 4*C*(2*C*d - B*e)*x^2 + 8*C^2*e*x^3)/(q*(B^2
 - 4*A*C))]], x]] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B^2*d + 2*C*
(b*C + A*d) - 2*B*(c*C + 2*A*e), 0] && EqQ[2*B^2*c*C - 8*a*C^3 - B^3*d - 4*
A*B*C*d + 4*A*(B^2 + 2*A*C)*e, 0] && NegQ[C*(2*e*(B*d - 4*A*e) + C*(d^2 - 4
*c*e))]
```

Rubi steps

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4} \, dx = -\frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}+\frac{\tan^{-1}\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right)}{2\sqrt{11}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 86, normalized size = 1.46

$$\frac{1}{8}\text{RootSum}\left[9+24\#1-12\#1^2+80\#1^3+320\#1^4\&,\frac{3\log(x-\#1)+12\log(x-\#1)\#1+20\log(x-\#1)\#1^2}{3-3\#1+30\#1^2+160\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (3*Log[x - #1] + 12*Lo
g[x - #1]*#1 + 20*Log[x - #1]*#1^2)/(3 - 3*#1 + 30*#1^2 + 160*#1^3) & ]/8
```

Mathics [A]
time = 2.12, size = 38, normalized size = 0.64

$$\frac{\sqrt{11}\left(\text{ArcTan}\left[\frac{\sqrt{11}\,(57+30x-40x^2+800x^3)}{66}\right]+\text{ArcTan}\left[\frac{\sqrt{11}\,(-7+40x)}{55}\right]\right)}{22}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^
4),x]')
```

```
[Out] Sqrt[11] (ArcTan[Sqrt[11] (57 + 30 x - 40 x ^ 2 + 800 x ^ 3) / 66] + ArcTan
[Sqrt[11] (-7 + 40 x) / 55]) / 22
```

Maple [C] Result contains complex when optimal does not.
time = 0.03, size = 62, normalized size = 1.05

method	result	size
risch	$\frac{\sqrt{11}\,\arctan\left(\frac{(40x-7)\sqrt{11}}{55}\right)}{22}+\frac{\sqrt{11}\,\arctan\left(-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}+\frac{400\sqrt{11}\,x^3}{33}\right)}{22}$	52
default	$\frac{i\sqrt{11}\,\ln\left(80x^2+\left(10i\sqrt{11}+10\right)x+3i\sqrt{11}-9\right)}{44}-\frac{i\sqrt{11}\,\ln\left(80x^2+\left(-10i\sqrt{11}+10\right)x-3i\sqrt{11}-9\right)}{44}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x,method=_RETURNVERBOSE)
```

```
[Out] 1/44*I*11^(1/2)*ln(80*x^2+(10*I*11^(1/2)+10)*x+3*I*11^(1/2)-9)-1/44*I*11^(1
/2)*ln(80*x^2+(-10*I*11^(1/2)+10)*x-3*I*11^(1/2)-9)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="maxi
ma")
```

```
[Out] integrate((20*x^2 + 12*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x)
```

Fricas [A]
time = 0.31, size = 43, normalized size = 0.73

$$\frac{1}{22}\sqrt{11}\arctan\left(\frac{1}{66}\sqrt{11}\left(800\,x^3-40\,x^2+30\,x+57\right)\right)+\frac{1}{22}\sqrt{11}\arctan\left(\frac{1}{55}\sqrt{11}\left(40\,x-7\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="fric
as")
```

```
[Out] 1/22*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) + 1/22*s
qrt(11)*arctan(1/55*sqrt(11)*(40*x - 7))
```

Sympy [A]
time = 0.08, size = 73, normalized size = 1.24

$$\frac{\sqrt{11}\cdot\left(2\,\mathrm{atan}\left(\frac{8\sqrt{11}\,x}{11}-\frac{7\sqrt{11}}{55}\right)+2\,\mathrm{atan}\left(\frac{400\sqrt{11}\,x^3}{33}-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}\right)\right)}{44}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x**2+12*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
```

```
[Out] sqrt(11)*(2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) + 2*atan(400*sqrt(11)*x**
3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22))/44
```

Giac [A]
time = 0.00, size = 56, normalized size = 0.95

$$-\frac{2}{44}\sqrt{11}\left(-\arctan\left(-\frac{-960x+168}{120\sqrt{11}}\right)-\arctan\left(\frac{76800x^3-3840x^2+2880x+5472}{576\sqrt{11}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x)
```

```
[Out] 1/22*sqrt(11)*(arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) + arcta
n(1/55*sqrt(11)*(40*x - 7)))
```

Mupad [B]
time = 0.34, size = 53, normalized size = 0.90

$$\frac{\sqrt{11}\,\mathrm{atan}\left(\frac{8\sqrt{11}\,x}{11}-\frac{7\sqrt{11}}{55}\right)}{22}+\frac{\sqrt{11}\,\mathrm{atan}\left(\frac{400\sqrt{11}\,x^3}{33}-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}\right)}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x + 20*x^2 + 3)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9),x)
```

```
[Out] (11^(1/2)*atan((8*11^(1/2)*x)/11 - (7*11^(1/2))/55))/22 + (11^(1/2)*atan((5
*11^(1/2)*x)/11 + (19*11^(1/2))/22 - (20*11^(1/2)*x^2)/33 + (400*11^(1/2)*x
^3)/33))/22
```

3.64 $\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx$

Optimal. Leaf size=78

$$2\sqrt{11}\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)-2\sqrt{11}\tan^{-1}\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right)+2\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

```
[Out] 2*ln(320*x^4+80*x^3-12*x^2+24*x+9)+2*arctan(1/55*(7-40*x)*11^(1/2))*11^(1/2
)-2*arctan(1/66*(800*x^3-40*x^2+30*x+57)*11^(1/2))*11^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2125, 2115}

$$-2\sqrt{11}\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)+2\log\left(320x^4+80x^3-12x^2+24x+9\right)+2\sqrt{11}\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-84 - 576*x - 400*x^2 + 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^
4),x]
```

```
[Out] 2*Sqrt[11]*ArcTan[(7 - 40*x)/(5*Sqrt[11])] - 2*Sqrt[11]*ArcTan[(57 + 30*x -
 40*x^2 + 800*x^3)/(6*Sqrt[11])] + 2*Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x
^4]
```

Rule 2115

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 +
 (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-C)*(2*e*(B*d - 4
*A*e) + C*(d^2 - 4*c*e)), 2]}, Simp[2*(C^2/q)*ArcTan[(C*d - B*e + 2*C*e*x)/
q], x] - Simp[2*(C^2/q)*ArcTan[C*((4*B*c*C - 3*B^2*d - 4*A*C*d + 12*A*B*e +
 4*C*(2*c*C - B*d + 2*A*e)*x + 4*C*(2*C*d - B*e)*x^2 + 8*C^2*e*x^3)/(q*(B^2
 - 4*A*C)))], x]] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B^2*d + 2*C*
(b*C + A*d) - 2*B*(c*C + 2*A*e), 0] && EqQ[2*B^2*c*C - 8*a*C^3 - B^3*d - 4*
A*B*C*d + 4*A*(B^2 + 2*A*C)*e, 0] && NegQ[C*(2*e*(B*d - 4*A*e) + C*(d^2 - 4
*c*e))]
```

Rule 2125

```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[Coeff[Pm, x, m]*(Log[Qn]/(n*Coeff[Qn, x, n])), x] + Dist[1/(n*Coeff[Qn,
x, n]), Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x]
/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

Rubi steps

$$\int -\frac{84+576x+400x^2-2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = 2\log\left(9+24x-12x^2+80x^3+320x^4\right) - \frac{\int \frac{168960+675840x+1126400x}{9+24x-12x^2+80x^3+320x}}{1280}$$

$$= 2\sqrt{11}\,\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right) - 2\sqrt{11}\,\tan^{-1}\left(\frac{57+30x-40x^2+80}{6\sqrt{11}}\right.$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 99, normalized size = 1.27

$$\frac{1}{2}\text{RootSum}\left[9+24\#1-12\#1^2+80\#1^3+320\#1^4\&,\frac{-21\log(x-\#1)-144\log(x-\#1)\#1-100\log(x-\#1)\#1^2+640\log(x-\#1)\#1^3}{3-3\#1+30\#1^2+160\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(-84 - 576*x - 400*x^2 + 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 +
320*x^4),x]
```

```
[Out] RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (-21*Log[x - #1] - 144
*Log[x - #1]*#1 - 100*Log[x - #1]*#1^2 + 640*Log[x - #1]*#1^3)/(3 - 3*#1 +
30*#1^2 + 160*#1^3) & ]/2
```

Mathics [**A**]
time = 2.41, size = 60, normalized size = 0.77

$$-2\sqrt{11}\left(\text{ArcTan}\left[\frac{\sqrt{11}\,(57+30x-40x^2+800x^3)}{66}\right]+\text{ArcTan}\left[\frac{\sqrt{11}\,(-7+40x)}{55}\right]\right)+2\text{Log}\left[\frac{9}{320}+\frac{3x}{40}-\frac{3x^2}{80}+\frac{x^3}{4}+x^4\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[-(84 + 576*x + 400*x^2 - 2560*x^3)/(9 + 24*x - 12*x^2 +
80*x^3 + 320*x^4),x]')
```

```
[Out] -2 Sqrt[11] (ArcTan[Sqrt[11] (57 + 30 x - 40 x ^ 2 + 800 x ^ 3) / 66] + Arc
Tan[Sqrt[11] (-7 + 40 x) / 55]) + 2 Log[9 / 320 + 3 x / 40 - 3 x ^ 2 / 80 +
 x ^ 3 / 4 + x ^ 4]
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.03, size = 70, normalized size = 0.90

method	result
default	$4\left(\frac{i\sqrt{11}}{4}+\frac{1}{2}\right)\ln\left(80x^2+\left(-10i\sqrt{11}+10\right)x-3i\sqrt{11}-9\right)+4\left(\frac{1}{2}-\frac{i\sqrt{11}}{4}\right)\ln\left(80x^2+(10\right.$
risch	$2\ln\left(6400x^4+1600x^3-240x^2+480x+180\right)-2\sqrt{11}\,\arctan\left(-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x,method=_RE
TURNVERBOSE)
```

```
[Out] 4*(1/4*I*11^(1/2)+1/2)*ln(80*x^2+(-10*I*11^(1/2)+10)*x-3*I*11^(1/2)-9)+4*(1
/2-1/4*I*11^(1/2))*ln(80*x^2+(10*I*11^(1/2)+10)*x+3*I*11^(1/2)-9)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, alg
orithm="maxima")
```

```
[Out] 4*integrate((640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 - 12*x^2 + 2
4*x + 9), x)
```

Fricas [A]
time = 0.30, size = 66, normalized size = 0.85

$$-2\sqrt{11}\arctan\left(\frac{1}{66}\sqrt{11}\left(800x^3-40x^2+30x+57\right)\right)-2\sqrt{11}\arctan\left(\frac{1}{55}\sqrt{11}\left(40x-7\right)\right)+2\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, alg
orithm="fricas")
```

```
[Out] -2*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - 2*sqrt(1
1)*arctan(1/55*sqrt(11)*(40*x - 7)) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 24*
x + 9)
```

Sympy [A]
time = 0.09, size = 100, normalized size = 1.28

$$\sqrt{11}\left(-2\operatorname{atan}\left(\frac{8\sqrt{11}x}{11}-\frac{7\sqrt{11}}{55}\right)-2\operatorname{atan}\left(\frac{400\sqrt{11}x^3}{33}-\frac{20\sqrt{11}x^2}{33}+\frac{5\sqrt{11}x}{11}+\frac{19\sqrt{11}}{22}\right)\right)+2\log\left(x^4+\frac{x^3}{4}-\frac{3x^2}{80}+\frac{3x}{40}+\frac{9}{320}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x**3-400*x**2-576*x-84)/(320*x**4+80*x**3-12*x**2+24*x+9),x
)
```

```
[Out] sqrt(11)*(-2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) - 2*atan(400*sqrt(11)*x*
*3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22)) + 2*log(x*
*4 + x**3/4 - 3*x**2/80 + 3*x/40 + 9/320)
```

Giac [A]
time = 0.00, size = 80, normalized size = 1.03

$$-\frac{1280}{1280}\cdot 2\sqrt{11}\left(\arctan\left(-\frac{-960x+168}{120\sqrt{11}}\right)+\arctan\left(\frac{76800x^3-3840x^2+2880x+5472}{576\sqrt{11}}\right)\right)+2\ln\left(320x^4+80x^3-12x^2+24x+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x)
```

```
[Out] -2*sqrt(11)*(arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) + arctan(
1/55*sqrt(11)*(40*x - 7))) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

Mupad [B]
time = 0.09, size = 76, normalized size = 0.97

$$2\ln\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)-2\sqrt{11}\,\mathrm{atan}\left(\frac{8\sqrt{11}\,x}{11}-\frac{7\sqrt{11}}{55}\right)-2\sqrt{11}\,\mathrm{atan}\left(\frac{400\sqrt{11}\,x^3}{33}-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(576*x + 400*x^2 - 2560*x^3 + 84)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 +
9),x)
```

```
[Out] 2*log(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9) - 2*11^(1/2)*atan((8*11^(1/2)*x
)/11 - (7*11^(1/2))/55) - 2*11^(1/2)*atan((5*11^(1/2)*x)/11 + (19*11^(1/2))
/22 - (20*11^(1/2)*x^2)/33 + (400*11^(1/2)*x^3)/33)
```

3.65 $\int \frac{\sqrt{1-x^4}}{1+x^4} \, dx$

Optimal. Leaf size=49

$$\frac{1}{2}\tan^{-1}\left(\frac{x(1+x^2)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x(1-x^2)}{\sqrt{1-x^4}}\right)$$

```
[Out] 1/2*arctan(x*(x^2+1)/(-x^4+1)^(1/2))+1/2*arctanh(x*(-x^2+1)/(-x^4+1)^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{414\}$

$$\frac{1}{2}\tan^{-1}\left(\frac{x(x^2+1)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x(1-x^2)}{\sqrt{1-x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^4]/(1 + x^4),x]
```

```
[Out] ArcTan[(x*(1 + x^2))/Sqrt[1 - x^4]]/2 + ArcTanh[(x*(1 - x^2))/Sqrt[1 - x^4]
]/2
```

Rule 414

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[(-a)*b, 4]}, Simp[(a/(2*c*q))*ArcTan[q*x*((a + q^2*x^2)/(a*Sqrt[a + b*x
^4]))], x] + Simp[(a/(2*c*q))*ArcTanh[q*x*((a - q^2*x^2)/(a*Sqrt[a + b*x^4]
))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && NegQ[a*b]
```

Rubi steps

$$\int \frac{\sqrt{1-x^4}}{1+x^4} \, dx = \frac{1}{2}\tan^{-1}\left(\frac{x(1+x^2)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x(1-x^2)}{\sqrt{1-x^4}}\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.13, size = 57, normalized size = 1.16

$$\left(\frac{1}{4}-\frac{i}{4}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{1-x^4}}\right)-\left(\frac{1}{4}+\frac{i}{4}\right)\tan^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{1-x^4}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^4]/(1 + x^4),x]
```

```
[Out] (1/4 - I/4)*ArcTan[((1 + I)*x)/Sqrt[1 - x^4]] - (1/4 + I/4)*ArcTan[((1/2 +
I/2)*Sqrt[1 - x^4])/x]
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[1 - x^4]/(1 + x^4),x]')
```

```
[Out] Timed out
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(41) = 82$.
time = 0.42, size = 113, normalized size = 2.31

method	result
default	$\frac{\left(-\frac{\arctan\left(1+\frac{\sqrt{-x^4+1}}{x}\right)\sqrt{2}}{4}-\frac{\arctan\left(-1+\frac{\sqrt{-x^4+1}}{x}\right)\sqrt{2}}{4}-\frac{\sqrt{2}\,\ln\left(\frac{1+\frac{-x^4+1}{2x^2}-\frac{\sqrt{-x^4+1}}{x}}{1+\frac{-x^4+1}{2x^2}+\frac{\sqrt{-x^4+1}}{x}}\right)}{8}\right)\sqrt{2}}{2}$
elliptic	$\frac{\left(-\frac{\arctan\left(1+\frac{\sqrt{-x^4+1}}{x}\right)\sqrt{2}}{4}-\frac{\arctan\left(-1+\frac{\sqrt{-x^4+1}}{x}\right)\sqrt{2}}{4}-\frac{\sqrt{2}\,\ln\left(\frac{1+\frac{-x^4+1}{2x^2}-\frac{\sqrt{-x^4+1}}{x}}{1+\frac{-x^4+1}{2x^2}+\frac{\sqrt{-x^4+1}}{x}}\right)}{8}\right)\sqrt{2}}{2}$
trager	$\frac{\ln\left(-\frac{4\,\mathrm{RootOf}\left(8\_Z^2-4\_Z+1\right)x+\sqrt{-x^4+1}}{4x^2\,\mathrm{RootOf}\left(8\_Z^2-4\_Z+1\right)-x^2-1}\right)}{2}-\ln\left(-\frac{4\,\mathrm{RootOf}\left(8\_Z^2-4\_Z+1\right)x+\sqrt{-x^4+1}}{4x^2\,\mathrm{RootOf}\left(8\_Z^2-4\_Z+1\right)-x^2-1}\right)\mathrm{RootOf}\,($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^(1/2)/(x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-1/4*arctan(1+(-x^4+1)^(1/2)/x)*2^(1/2)-1/4*arctan(-1+(-x^4+1)^(1/2)/x
)*2^(1/2)-1/8*2^(1/2)*ln((1+1/2*(-x^4+1)/x^2-(-x^4+1)^(1/2)/x)/(1+1/2*(-x^4
+1)/x^2+(-x^4+1)^(1/2)/x)))*2^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)
```

Fricas [A]
time = 0.34, size = 56, normalized size = 1.14

$$-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^4+1}\,x}{x^2-1}\right)+\frac{1}{4}\log\left(-\frac{x^4-2\,x^2-2\,\sqrt{-x^4+1}\,x-1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(sqrt(-x^4 + 1)*x/(x^2 - 1)) + 1/4*log(-(x^4 - 2*x^2 - 2*sqrt(-x
^4 + 1)*x - 1)/(x^4 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**(1/2)/(x**4+1),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/(x**4 + 1), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^4+1),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{1-x^4}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^4)^(1/2)/(x^4 + 1),x)
```

```
[Out] int((1 - x^4)^(1/2)/(x^4 + 1), x)
```

3.66 $\int \frac{\sqrt{1+x^4}}{1-x^4} \, dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctan(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)+1/4*arctanh(x*2^(1/2)/(x^4+1)^(
1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {413, 218, 212, 209}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^4]/(1 - x^4),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) + ArcTanh[(Sqrt[2]*x)/Sqrt[1
+ x^4]]/(2*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 413

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[a/c,
 Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^4}}{1-x^4}\,dx &= \text{Subst}\left(\int \frac{1}{1-4x^4}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 44, normalized size = 0.83

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^4]/(1 - x^4),x]
[Out] (ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]] + ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]])/(2
*Sqrt[2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[1 + x^4]/(1 - x^4),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.28, size = 365, normalized size = 6.89

method	result
elliptic	$\frac{\left(-\frac{\ln\left(-1+\frac{\sqrt{x^4+1}\,\sqrt{2}}{2x}\right)}{4}+\frac{\ln\left(1+\frac{\sqrt{x^4+1}\,\sqrt{2}}{2x}\right)}{4}-\frac{\arctan\left(\frac{\sqrt{x^4+1}\,\sqrt{2}}{2x}\right)}{2}\right)\sqrt{2}}{2}$
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2-2\right)x-\sqrt{x^4+1}}{(1+x)(-1+x)}\right)}{4}+\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)x-\sqrt{x^4+1}}{x^2+1}\right)}{4}$
default	$-\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\right.}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(1/2)/(-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2
)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+1/2*I/(1/2*2^(1/2)+1/2*I*2^(1/
2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticF(x*(1/2*2^(1/2)+
1/2*I*2^(1/2)),I)-(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*
EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/4))-1/2*I/(1/2*2^(1/2)+1/2*I*
2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*(EllipticF(x*(1/2*2^
(1/2)+1/2*I*2^(1/2)),I)-EllipticE(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I))-1/2*I/(
1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*El
lipticE(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^
2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x^4 + 1)/(x^4 - 1), x)
```

Fricas [A]
time = 0.35, size = 61, normalized size = 1.15

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)+\frac{1}{8}\sqrt{2}\log\left(\frac{x^4+2\sqrt{2}\,\sqrt{x^4+1}\,x+2\,x^2+1}{x^4-2\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1)) + 1/8*sqrt(2)*log((x^4 + 2*sqrt
(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2 + 1))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(1/2)/(-x**4+1),x)
```

```
[Out] -Integral(sqrt(x**4 + 1)/(x**4 - 1), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(-x^4+1),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 + 1)^(1/2)/(x^4 - 1),x)
```

```
[Out] -int((x^4 + 1)^(1/2)/(x^4 - 1), x)
```

3.67 $\int \frac{\sqrt{1+px^2+x^4}}{1-x^4} \, dx$

Optimal. Leaf size=75

$$\frac{1}{4}\sqrt{2-p}\tan^{-1}\left(\frac{\sqrt{2-p}\,x}{\sqrt{1+px^2+x^4}}\right)+\frac{1}{4}\sqrt{2+p}\tanh^{-1}\left(\frac{\sqrt{2+p}\,x}{\sqrt{1+px^2+x^4}}\right)$$

```
[Out] 1/4*arctan(x*(2-p)^(1/2)/(x^4+p*x^2+1)^(1/2))*(2-p)^(1/2)+1/4*arctanh(x*(2+
p)^(1/2)/(x^4+p*x^2+1)^(1/2))*(2+p)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2096, 1107, 211, 214}

$$\frac{1}{4}\sqrt{2-p}\tan^{-1}\left(\frac{\sqrt{2-p}\,x}{\sqrt{px^2+x^4+1}}\right)+\frac{1}{4}\sqrt{p+2}\tanh^{-1}\left(\frac{\sqrt{p+2}\,x}{\sqrt{px^2+x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + p*x^2 + x^4]/(1 - x^4),x]
```

```
[Out] (Sqrt[2 - p]*ArcTan[(Sqrt[2 - p]*x)/Sqrt[1 + p*x^2 + x^4]])/4 + (Sqrt[2 + p
]*ArcTanh[(Sqrt[2 + p]*x)/Sqrt[1 + p*x^2 + x^4]])/4
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 2096

```
Int[Sqrt[v_]/((d_) + (e_.)*(x_)^4), x_Symbol] :> With[{a = Coeff[v, x, 0],
b = Coeff[v, x, 2], c = Coeff[v, x, 4]}, Dist[a/d, Subst[Int[1/(1 - 2*b*x^2
```

```
 + (b^2 - 4*a*c)*x^4), x], x, x/Sqrt[v]], x] /; EqQ[c*d + a*e, 0] && PosQ[a
*c]] /; FreeQ[{d, e}, x] && PolyQ[v, x^2, 2]
```

Rubi steps

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4}\,dx = \text{Subst}\left(\int \frac{1}{1-2px^2+(-4+p^2)\,x^4}\,dx, x, \frac{x}{\sqrt{1+px^2+x^4}}\right)$$
$$= \frac{1}{4}\left(-4+p^2\right)\text{Subst}\left(\int \frac{1}{-2-p+(-4+p^2)\,x^2}\,dx, x, \frac{x}{\sqrt{1+px^2+x^4}}\right) - \frac{1}{4}\,(-4+$$
$$= \frac{1}{4}\sqrt{2-p}\,\tan^{-1}\left(\frac{\sqrt{2-p}\,x}{\sqrt{1+px^2+x^4}}\right) + \frac{1}{4}\sqrt{2+p}\,\tanh^{-1}\left(\frac{\sqrt{2+p}\,x}{\sqrt{1+px^2+x^4}}\right)$$

Mathematica [A]
time = 0.56, size = 81, normalized size = 1.08

$$\frac{1}{4}\left(-\sqrt{-2-p}\,\tan^{-1}\left(\frac{\sqrt{-2-p}\,x}{\sqrt{1+px^2+x^4}}\right) - \sqrt{2-p}\,\tan^{-1}\left(\frac{\sqrt{1+px^2+x^4}}{\sqrt{2-p}\,x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + p*x^2 + x^4]/(1 - x^4),x]
```

```
[Out] (-(Sqrt[-2 - p]*ArcTan[(Sqrt[-2 - p]*x)/Sqrt[1 + p*x^2 + x^4]]) - Sqrt[2 -
p]*ArcTan[Sqrt[1 + p*x^2 + x^4]/(Sqrt[2 - p]*x)])/4
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[1 + p*x^2 + x^4]/(1 - x^4),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.11, size = 1512, normalized size = 20.16

method	result	size

elliptic	$\frac{\left(\frac{4\left(\frac{1}{4}-\frac{p}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{x^4+p\,x^2+1}\,\sqrt{2}}{x\sqrt{2p-4}}\right)}{\sqrt{2p-4}}+\frac{4\left(\frac{1}{4}+\frac{p}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{x^4+p\,x^2+1}\,\sqrt{2}}{x\sqrt{4+2p}}\right)}{\sqrt{4+2p}}\right)\sqrt{2}}{2}$	89
default	Expression too large to display	1512

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+p*x^2+1)^(1/2)/(-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(1+p)/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^
(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2)/(x^4+p*x^2+1)^(1/2)*Elliptic
F(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)^(1/2)))^(1/2
))+2/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2)*
(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2)/(x^4+p*x^2+1)^(1/2)/(p+(p^2-4)^(1/
2))*(EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)
^(1/2)))^(1/2))-EllipticE(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-
1/2*(p^2-4)^(1/2)))^(1/2)))-1/4*(2+p)*(-1/2/(2+p)^(1/2)*arctanh(1/2*(p*x^2+
2*x^2+p+2)/(2+p)^(1/2)/(x^4+p*x^2+1)^(1/2))-1/(-1/2*p+1/2*(p^2-4)^(1/2))^(1
/2)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*
x^2)^(1/2)/(x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*
x,1/(-1/2*p+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2
*(p^2-4)^(1/2))^(1/2)))+1/2*(-1-p)/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2*p+
1/2*(p^2-4)^(1/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2)/(x^4
+p*x^2+1)^(1/2)*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-
1/2*(p^2-4)^(1/2)))^(1/2))+1/4*(2+p)*(-1/2/(2+p)^(1/2)*arctanh(1/2*(p*x^2+2
*x^2+p+2)/(2+p)^(1/2)/(x^4+p*x^2+1)^(1/2))+1/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/
2)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x
^2)^(1/2)/(x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*x
,1/(-1/2*p+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2*
(p^2-4)^(1/2))^(1/2)))+1/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*
(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1
)^(1/2)*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2
-4)^(1/2)))^(1/2))*p-1/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(p
^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1)^
(1/2)*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4
)^(1/2)))^(1/2))-2/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(p^2-4
)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1)^(1/2
)/(p+(p^2-4)^(1/2))*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/
2*p-1/2*(p^2-4)^(1/2)))^(1/2))+2/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-
1/2*x^2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4
+p*x^2+1)^(1/2)/(p+(p^2-4)^(1/2))*EllipticE(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1
/2),(-1-p*(-1/2*p-1/2*(p^2-4)^(1/2)))^(1/2))+1/(-1/2*p+1/2*(p^2-4)^(1/2))^(
1/2)*(1+1/2*p*x^2-1/2*x^2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)
```

```
^(1/2))^(1/2)/(x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/
2)*x,-1/(-1/2*p+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p
+1/2*(p^2-4)^(1/2))^(1/2))-1/2*p/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*
x^2-1/2*x^2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/
(x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*x,-1/(-1/2*
p+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2*(p^2-4)^(
1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x^4 + p*x^2 + 1)/(x^4 - 1), x)
```

Fricas [A]
time = 0.36, size = 359, normalized size = 4.79

$$\left[\frac{1}{8}\sqrt{p-2}\log\left(\frac{x^4+2(p-1)x^2-2\sqrt{x^4+px^2+1}\sqrt{p-2}\,x+1}{x^4+2x^2+1}\right)+\frac{1}{8}\sqrt{p+2}\log\left(\frac{x^4+2(p+1)x^2+2\sqrt{x^4+px^2+1}\sqrt{p+2}\,x+1}{x^4-2x^2+1}\right),\frac{1}{4}\sqrt{-p+2}\arctan\left(\frac{\sqrt{-p+2}\,x}{\sqrt{x^4+px^2+1}}\right)+\frac{1}{8}\sqrt{p+2}\log\left(\frac{x^4+2(p+1)x^2+2\sqrt{x^4+px^2+1}\sqrt{p+2}\,x+1}{x^4-2x^2+1}\right),-\frac{1}{4}\sqrt{-p-2}\arctan\left(\frac{\sqrt{x^4+px^2+1}\sqrt{-p-2}}{(p+2)x}\right)+\frac{1}{8}\sqrt{p-2}\log\left(\frac{x^4+2(p-1)x^2-2\sqrt{x^4+px^2+1}\sqrt{p-2}\,x+1}{x^4+2x^2+1}\right),\frac{1}{4}\sqrt{-p+2}\arctan\left(\frac{\sqrt{-p+2}\,x}{\sqrt{x^4+px^2+1}}\right)-\frac{1}{4}\sqrt{-p-2}\arctan\left(\frac{\sqrt{x^4+px^2+1}\sqrt{-p-2}}{(p+2)x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(p - 2)*log((x^4 + 2*(p - 1)*x^2 - 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p
- 2)*x + 1)/(x^4 + 2*x^2 + 1)) + 1/8*sqrt(p + 2)*log((x^4 + 2*(p + 1)*x^2 +
 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p + 2)*x + 1)/(x^4 - 2*x^2 + 1)), 1/4*sqrt(-p
 + 2)*arctan(sqrt(-p + 2)*x/sqrt(x^4 + p*x^2 + 1)) + 1/8*sqrt(p + 2)*log((x
^4 + 2*(p + 1)*x^2 + 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p + 2)*x + 1)/(x^4 - 2*x^
2 + 1)), -1/4*sqrt(-p - 2)*arctan(sqrt(x^4 + p*x^2 + 1)*sqrt(-p - 2)/((p +
2)*x)) + 1/8*sqrt(p - 2)*log((x^4 + 2*(p - 1)*x^2 - 2*sqrt(x^4 + p*x^2 + 1)
*sqrt(p - 2)*x + 1)/(x^4 + 2*x^2 + 1)), 1/4*sqrt(-p + 2)*arctan(sqrt(-p + 2
)*x/sqrt(x^4 + p*x^2 + 1)) - 1/4*sqrt(-p - 2)*arctan(sqrt(x^4 + p*x^2 + 1)*
sqrt(-p - 2)/((p + 2)*x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{px^2 + x^4 + 1}}{x^4 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+p*x**2+1)**(1/2)/(-x**4+1),x)
```

```
[Out] -Integral(sqrt(p*x**2 + x**4 + 1)/(x**4 - 1), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{x^4+p\,x^2+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(p*x^2 + x^4 + 1)^(1/2)/(x^4 - 1),x)
```

```
[Out] -int((p*x^2 + x^4 + 1)^(1/2)/(x^4 - 1), x)
```

3.68 $\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx$

Optimal. Leaf size=171

$$-\frac{\sqrt{p+\sqrt{4+p^2}}\tan^{-1}\left(\frac{\sqrt{p+\sqrt{4+p^2}}\,x\left(p-\sqrt{4+p^2}-2x^2\right)}{2\sqrt{2}\sqrt{1+px^2-x^4}}\right)}{2\sqrt{2}}+\frac{\sqrt{-p+\sqrt{4+p^2}}\tanh^{-1}\left(\frac{\sqrt{-p+\sqrt{\ldots}}}{2\sqrt{\ldots}}\right.}{2\sqrt{2}}$$

```
[Out] 1/4*arctanh(1/4*x*(p-2*x^2+(p^2+4)^(1/2))*(-p+(p^2+4)^(1/2))^(1/2)*2^(1/2)/
(-x^4+p*x^2+1)^(1/2))*(-p+(p^2+4)^(1/2))^(1/2)*2^(1/2)-1/4*arctan(1/4*x*(p-
2*x^2-(p^2+4)^(1/2))*(p+(p^2+4)^(1/2))^(1/2)*2^(1/2)/(-x^4+p*x^2+1)^(1/2))*
(p+(p^2+4)^(1/2))^(1/2)*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {2097}

$$\frac{\sqrt{\sqrt{p^2+4}-p}\tanh^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}-p}\,x\left(\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}-\frac{\sqrt{\sqrt{p^2+4}+p}\tan^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}+p}\,x\left(-\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + p*x^2 - x^4]/(1 + x^4),x]
```

```
[Out] -1/2*(Sqrt[p + Sqrt[4 + p^2]]*ArcTan[(Sqrt[p + Sqrt[4 + p^2]]*x*(p - Sqrt[4
 + p^2] - 2*x^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/Sqrt[2] + (Sqrt[-p +
Sqrt[4 + p^2]]*ArcTanh[(Sqrt[-p + Sqrt[4 + p^2]]*x*(p + Sqrt[4 + p^2] - 2*x
^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/(2*Sqrt[2])
```

Rule 2097

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^4), x_Symbo
l] :> With[{q = Sqrt[b^2 - 4*a*c]}, Simp[(-a)*(Sqrt[b + q]/(2*Sqrt[2]*Rt[(-
a)*c, 2]*d))*ArcTan[Sqrt[b + q]*x*((b - q + 2*c*x^2)/(2*Sqrt[2]*Rt[(-a)*c,
2]*Sqrt[a + b*x^2 + c*x^4]))], x] + Simp[a*(Sqrt[-b + q]/(2*Sqrt[2]*Rt[(-a)
*c, 2]*d))*ArcTanh[Sqrt[-b + q]*x*((b + q + 2*c*x^2)/(2*Sqrt[2]*Rt[(-a)*c,
2]*Sqrt[a + b*x^2 + c*x^4]))], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d
+ a*e, 0] && NegQ[a*c]
```

Rubi steps

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx = -\frac{\sqrt{p+\sqrt{4+p^2}}\,\tan^{-1}\left(\frac{\sqrt{p+\sqrt{4+p^2}}\,x\left(p-\sqrt{4+p^2}-2x^2\right)}{2\sqrt{2}\,\sqrt{1+px^2-x^4}}\right)}{2\sqrt{2}} + \frac{\sqrt{-p+\sqrt{4}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.23, size = 92, normalized size = 0.54

$$\frac{1}{4}i\left(\sqrt{-2i-p}\,\tan^{-1}\left(\frac{\sqrt{-2i-p}\,x}{\sqrt{1+px^2-x^4}}\right)-\sqrt{2i-p}\,\tan^{-1}\left(\frac{\sqrt{2i-p}\,x}{\sqrt{1+px^2-x^4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + p*x^2 - x^4]/(1 + x^4),x]
```

```
[Out] (I/4)*(Sqrt[-2*I - p]*ArcTan[(Sqrt[-2*I - p]*x)/Sqrt[1 + p*x^2 - x^4]] - Sq
rt[2*I - p]*ArcTan[(Sqrt[2*I - p]*x)/Sqrt[1 + p*x^2 - x^4]])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[1 + p*x^2 - x^4]/(1 + x^4),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(131) = 262.
time = 0.11, size = 625, normalized size = 3.65

method	result
default	$\left(-\frac{\sqrt{p+\sqrt{p^2+4}}\,\sqrt{p^2+4}\,\ln\left(\frac{-x^4+p\,x^2+1}{x^2}-\frac{\sqrt{-x^4+p\,x^2+1}\,\sqrt{2}\,\sqrt{p+\sqrt{p^2+4}}}{x}+\sqrt{p^2+4}\right)}{16}-\right.$

elliptic	$\left(-\frac{\sqrt{p+\sqrt{p^2+4}}\,\sqrt{p^2+4}\,\ln\left(\frac{-x^4+p\,x^2+1}{x^2}-\frac{\sqrt{-x^4+p\,x^2+1}\,\sqrt{2}\,\sqrt{p+\sqrt{p^2+4}}}{x}+\sqrt{p^2+4}\right)}{16}-\ldots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+p*x^2+1)^(1/2)/(x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-1/16*(p+(p^2+4)^(1/2))^(1/2)*(p^2+4)^(1/2)*ln((-x^4+p*x^2+1)/x^2-(-x^
4+p*x^2+1)^(1/2)*2^(1/2)/x*(p+(p^2+4)^(1/2))^(1/2)+(p^2+4)^(1/2))-1/8*(p^2+
4)^(1/2)*(p+(p^2+4)^(1/2))/(-p+(p^2+4)^(1/2))^(1/2)*arctan(1/2*(2*(-x^4+p*x
^2+1)^(1/2)*2^(1/2)/x-2*(p+(p^2+4)^(1/2))^(1/2))/(-p+(p^2+4)^(1/2))^(1/2))+
1/16*(p+(p^2+4)^(1/2))^(1/2)*p*ln((-x^4+p*x^2+1)/x^2-(-x^4+p*x^2+1)^(1/2)*2
^(1/2)/x*(p+(p^2+4)^(1/2))^(1/2)+(p^2+4)^(1/2))+1/8*p*(p+(p^2+4)^(1/2))/(-p
+(p^2+4)^(1/2))^(1/2)*arctan(1/2*(2*(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x-2*(p+(p^
2+4)^(1/2))^(1/2))/(-p+(p^2+4)^(1/2))^(1/2))+1/16*(p+(p^2+4)^(1/2))^(1/2)*(
p^2+4)^(1/2)*ln((-x^4+p*x^2+1)/x^2+(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x*(p+(p^2+4
)^(1/2))^(1/2)+(p^2+4)^(1/2))-1/8*(p^2+4)^(1/2)*(p+(p^2+4)^(1/2))/(-p+(p^2+
4)^(1/2))^(1/2)*arctan(1/2*(2*(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x+2*(p+(p^2+4)^(
1/2))^(1/2))/(-p+(p^2+4)^(1/2))^(1/2))-1/16*(p+(p^2+4)^(1/2))^(1/2)*p*ln((-
x^4+p*x^2+1)/x^2+(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x*(p+(p^2+4)^(1/2))^(1/2)+(p^
2+4)^(1/2))+1/8*p*(p+(p^2+4)^(1/2))/(-p+(p^2+4)^(1/2))^(1/2)*arctan(1/2*(2*
(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x+2*(p+(p^2+4)^(1/2))^(1/2))/(-p+(p^2+4)^(1/2)
)^(1/2)))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2667 vs. $2(135) = 270$.
time = 1.75, size = 2667, normalized size = 15.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(8*sqrt(2)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*arctan(1/4
*(2*(p^3 + 4*p)*x^12 - 2*(p^4 - 2*p^2 - 24)*x^10 - 20*(p^3 + 4*p)*x^8 + 2*(
3*p^4 + 4*p^2 - 32)*x^6 + 10*(p^3 + 4*p)*x^4 + 4*(p^2 + 4)*x^2 - 2*((p^2 +
4)*x^12 - (p^3 + 4*p)*x^10 - (p^3 + 4*p)*x^6 - (p^2 + 4)*x^4 + (p*x^12 - (p
^2 - 6)*x^10 - 10*p*x^8 + (3*p^2 - 8)*x^6 + 5*p*x^4 + 2*x^2)*sqrt(p^2 + 4))
*sqrt(p^2 + 4) + 2*((p^2 + 4)*x^12 - (p^3 + 4*p)*x^10 - (p^3 + 4*p)*x^6 - (
p^2 + 4)*x^4)*sqrt(p^2 + 4) + sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(2*(sqrt(2)*(
x^9 - p*x^7 - x^5)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + sqrt(2)*(x^11 - 2
*p*x^9 + (p^2 - 2)*x^7 + 2*p*x^5 + x^3)*sqrt(-x^4 + p*x^2 + 1))*(p^2 + 4)^(
3/4) - (sqrt(2)*(p*x^9 + 8*x^7 - 6*p*x^5 + 2*p^2*x^3 + p*x)*sqrt(-x^4 + p*x
^2 + 1)*sqrt(p^2 + 4) + sqrt(2)*((p^2 + 4)*x^9 + 4*(p^2 + 4)*x^5 - 2*(p^3 +
 4*p)*x^3 - (p^2 + 4)*x)*sqrt(-x^4 + p*x^2 + 1))*(p^2 + 4)^(1/4)) - (2*((p^
3 + 4*p)*x^8 + 4*(p^2 + 4)*x^6 - (p^3 + 4*p)*x^4)*sqrt(-x^4 + p*x^2 + 1)*sq
rt(p^2 + 4) + 2*((p^4 + 6*p^2 + 8)*x^8 + 4*(p^3 + 4*p)*x^6 - (p^4 - 4*p^2 -
 32)*x^4 - 4*(p^3 + 4*p)*x^2 - 2*p^2 - 8)*sqrt(-x^4 + p*x^2 + 1) - 2*((p*x^
10 - (p^2 - 4)*x^8 - 6*p*x^6 + (p^2 - 4)*x^4 + p*x^2)*sqrt(-x^4 + p*x^2 + 1
)*sqrt(p^2 + 4) + ((p^2 + 4)*x^10 - (p^3 + 4*p)*x^8 - 2*(p^2 + 4)*x^6 + (p^
3 + 4*p)*x^4 + (p^2 + 4)*x^2)*sqrt(-x^4 + p*x^2 + 1))*sqrt(p^2 + 4) - sqrt(
p^2 + sqrt(p^2 + 4)*p + 4)*((sqrt(2)*(x^11 - p*x^9 - p*x^5 - x^3)*sqrt(p^2
+ 4) + sqrt(2)*(2*x^13 - 5*p*x^11 + (3*p^2 - 8)*x^9 + 10*p*x^7 - (p^2 - 6)*
x^5 - p*x^3))*(p^2 + 4)^(3/4) - (sqrt(2)*(p*x^11 - (p^2 - 6)*x^9 - 10*p*x^7
 + (3*p^2 - 8)*x^5 + 5*p*x^3 + 2*x)*sqrt(p^2 + 4) + sqrt(2)*((p^2 + 4)*x^11
 - (p^3 + 4*p)*x^9 - (p^3 + 4*p)*x^5 - (p^2 + 4)*x^3))*(p^2 + 4)^(1/4)))*sq
rt(-((p^2 + 4)*x^4 - (p^2 + 4)^(3/2)*x^2 - sqrt(2)*sqrt(-x^4 + p*x^2 + 1)*s
qrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*x - (p^3 + 4*p)*x^2 - p^2 -
4)/((p^2 + 4)*x^4 + p^2 + 4)))/((p^2 + 4)*x^12 - 3*(p^3 + 4*p)*x^10 + (2*p^
4 + p^2 - 28)*x^8 + 10*(p^3 + 4*p)*x^6 - (2*p^4 + p^2 - 28)*x^4 - 3*(p^3 +
4*p)*x^2 - p^2 - 4)) + 8*sqrt(2)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^
(3/4)*arctan(-1/4*(2*(p^3 + 4*p)*x^12 - 2*(p^4 - 2*p^2 - 24)*x^10 - 20*(p^3
 + 4*p)*x^8 + 2*(3*p^4 + 4*p^2 - 32)*x^6 + 10*(p^3 + 4*p)*x^4 + 4*(p^2 + 4)
*x^2 - 2*((p^2 + 4)*x^12 - (p^3 + 4*p)*x^10 - (p^3 + 4*p)*x^6 - (p^2 + 4)*x
^4 + (p*x^12 - (p^2 - 6)*x^10 - 10*p*x^8 + (3*p^2 - 8)*x^6 + 5*p*x^4 + 2*x^
2)*sqrt(p^2 + 4))*sqrt(p^2 + 4) + 2*((p^2 + 4)*x^12 - (p^3 + 4*p)*x^10 - (p
^3 + 4*p)*x^6 - (p^2 + 4)*x^4)*sqrt(p^2 + 4) - sqrt(p^2 + sqrt(p^2 + 4)*p +
 4)*(2*(sqrt(2)*(x^9 - p*x^7 - x^5)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) +
sqrt(2)*(x^11 - 2*p*x^9 + (p^2 - 2)*x^7 + 2*p*x^5 + x^3)*sqrt(-x^4 + p*x^2
+ 1))*(p^2 + 4)^(3/4) - (sqrt(2)*(p*x^9 + 8*x^7 - 6*p*x^5 + 2*p^2*x^3 + p*x
)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + sqrt(2)*((p^2 + 4)*x^9 + 4*(p^2 +
4)*x^5 - 2*(p^3 + 4*p)*x^3 - (p^2 + 4)*x)*sqrt(-x^4 + p*x^2 + 1))*(p^2 + 4)
^(1/4)) - (2*((p^3 + 4*p)*x^8 + 4*(p^2 + 4)*x^6 - (p^3 + 4*p)*x^4)*sqrt(-x^
4 + p*x^2 + 1)*sqrt(p^2 + 4) + 2*((p^4 + 6*p^2 + 8)*x^8 + 4*(p^3 + 4*p)*x^6
 - (p^4 - 4*p^2 - 32)*x^4 - 4*(p^3 + 4*p)*x^2 - 2*p^2 - 8)*sqrt(-x^4 + p*x^
2 + 1) - 2*((p*x^10 - (p^2 - 4)*x^8 - 6*p*x^6 + (p^2 - 4)*x^4 + p*x^2)*sqrt
(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + ((p^2 + 4)*x^10 - (p^3 + 4*p)*x^8 - 2*(p
^2 + 4)*x^6 + (p^3 + 4*p)*x^4 + (p^2 + 4)*x^2)*sqrt(-x^4 + p*x^2 + 1))*sqrt
```

```
(p^2 + 4) + sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*((sqrt(2)*(x^11 - p*x^9 - p*x^5
 - x^3)*sqrt(p^2 + 4) + sqrt(2)*(2*x^13 - 5*p*x^11 + (3*p^2 - 8)*x^9 + 10*p
*x^7 - (p^2 - 6)*x^5 - p*x^3))*(p^2 + 4)^(3/4) - (sqrt(2)*(p*x^11 - (p^2 -
6)*x^9 - 10*p*x^7 + (3*p^2 - 8)*x^5 + 5*p*x^3 + 2*x)*sqrt(p^2 + 4) + sqrt(2
)*((p^2 + 4)*x^11 - (p^3 + 4*p)*x^9 - (p^3 + 4*p)*x^5 - (p^2 + 4)*x^3))*(p^
2 + 4)^(1/4)))*sqrt(-((p^2 + 4)*x^4 - (p^2 + 4)^(3/2)*x^2 + sqrt(2)*sqrt(-x
^4 + p*x^2 + 1)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*x - (p^3 +
4*p)*x^2 - p^2 - 4)/((p^2 + 4)*x^4 + p^2 + 4)))/((p^2 + 4)*x^12 - 3*(p^3 +
4*p)*x^10 + (2*p^4 + p^2 - 28)*x^8 + 10*(p^3 + 4*p)*x^6 - (2*p^4 + p^2 - 28
)*x^4 - 3*(p^3 + 4*p)*x^2 - p^2 - 4)) - (sqrt(2)*sqrt(p^2 + 4)*p - sqrt(2)*
(p^2 + 4))*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(1/4)*log(-((p^2 + 4)*
x^4 - (p^2 + 4)^(3/2)*x^2 + sqrt(2)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + sqrt(
p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*x - (p^3 + 4*p)*x^2 - p^2 - 4)/((p^2 + 4)*x
^4 + p^2 + 4)) + (sqrt(2)*sqrt(p^2 + 4)*p - sqrt(2)*(p^2 + 4))*sqrt(p^2 + s
qrt(p^2 + 4)*p + 4)*(p^2 + 4)^(1/4)*log(-((p^2 + 4)*x^4 - (p^2 + 4)^(3/2)*x
^2 - sqrt(2)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 +
4)^(3/4)*x - (p^3 + 4*p)*x^2 - p^2 - 4)/((p^2 + 4)*x^4 + p^2 + 4)))/(p^2 +
4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^2 - x^4 + 1}}{x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+p*x**2+1)**(1/2)/(x**4+1),x)
[Out] Integral(sqrt(p*x**2 - x**4 + 1)/(x**4 + 1), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-x^4 + p\,x^2 + 1}}{x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^2 - x^4 + 1)^(1/2)/(x^4 + 1),x)
[Out] int((p*x^2 - x^4 + 1)^(1/2)/(x^4 + 1), x)
```

3.69 $\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx$

Optimal. Leaf size=80

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}-b\tan^{-1}\left(\sqrt[4]{-1+x^2}\right)+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}+b\tanh^{-1}\left(\sqrt[4]{-1+x^2}\right)$$

```
[Out] -b*arctan((x^2-1)^(1/4))+b*arctanh((x^2-1)^(1/4))+1/4*a*arctan(1/2*x/(x^2-1
)^(1/4)*2^(1/2))*2^(1/2)+1/4*a*arctanh(1/2*x/(x^2-1)^(1/4)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1024, 407, 455, 65, 304, 209, 212}

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}-b\tan^{-1}\left(\sqrt[4]{x^2-1}\right)+b\tanh^{-1}\left(\sqrt[4]{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] (a*ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTan[(-1 + x^2)^
(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTanh[
(-1 + x^2)^(1/4)]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = a\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx + b\int \frac{x}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx$$
$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{1}{2}b\text{Subst}\left(\int \frac{}{(2}\right.$$
$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + (2b)\text{Subst}\left(\int \frac{}{1}\right.$$
$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + b\text{Subst}\left(\int \frac{1}{1-}\right.$$
$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} - b\tan^{-1}\left(\sqrt[4]{-1+x^2}\right) + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+}}\right.}{2\sqrt{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.17, size = 157, normalized size = 1.96

$$\frac{x\left(bx\sqrt[4]{1-x^2}\,(-2+x^2)\,F_1\left(1;\frac{1}{4},1;2;x^2,\frac{x^2}{2}\right) - \frac{24aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}{6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)+x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};x^2,\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};x^2,\frac{x^2}{2}\right)\right)}\right)}{4\,(-2+x^2)\sqrt[4]{-1+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] (x*(b*x*(1 - x^2)^(1/4)*(-2 + x^2)*AppellF1[1, 1/4, 1, 2, x^2, x^2/2] - (24
*a*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2])/(6*AppellF1[1/2, 1/4, 1, 3/2, x^
2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, x^2, x^2/2] + AppellF1[3/2, 5
/4, 1, 5/2, x^2, x^2/2]))))/(4*(-2 + x^2)*(-1 + x^2)^(1/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{bx + a}{(-x^2 + 2)(x^2 - 1)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)
[Out] int((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="maxima")
[Out] -integrate((b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{x^2 \sqrt[4]{x^2 - 1} - 2\sqrt[4]{x^2 - 1}}\, dx - \int \frac{bx}{x^2 \sqrt[4]{x^2 - 1} - 2\sqrt[4]{x^2 - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+2)/(x**2-1)**(1/4),x)
[Out] -Integral(a/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x) - Integral(b
*x/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,x}{{\left(x^2-1\right)}^{1/4}\,\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x)/((x^2 - 1)^(1/4)*(x^2 - 2)),x)
```

```
[Out] int(-(a + b*x)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

3.70 $\int \frac{a+bx}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx$

Optimal. Leaf size=88

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\,\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}}+b\tan^{-1}\left(\sqrt[4]{-1-x^2}\right)+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\,\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}}-b\tanh^{-1}\left(\sqrt[4]{-1-x^2}\right)$$

```
[Out] b*arctan((-x^2-1)^(1/4))-b*arctanh((-x^2-1)^(1/4))+1/4*a*arctan(1/2*x/(-x^2
-1)^(1/4)*2^(1/2))*2^(1/2)+1/4*a*arctanh(1/2*x/(-x^2-1)^(1/4)*2^(1/2))*2^(1
/2)
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1024, 407, 455, 65, 304, 209, 212}

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\,\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\,\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+b\tan^{-1}\left(\sqrt[4]{-x^2-1}\right)-b\tanh^{-1}\left(\sqrt[4]{-x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((-1 - x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (a*ArcTan[x/(Sqrt[2]*(-1 - x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTan[(-1 - x^2)^
(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1 - x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTanh[
(-1 - x^2)^(1/4)]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx = a\int \frac{1}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx + b\int \frac{x}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{1}{2}b\text{Subst}\left(\int \frac{}{\sqrt[4]{-}}\right.$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - (2b)\text{Subst}\left(\int \frac{3}{1-}\right.$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - b\text{Subst}\left(\int \frac{1}{1-x}\right.$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + b\tan^{-1}\left(\sqrt[4]{-1-x^2}\right) + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x}}\right.}{2\sqrt{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.17, size = 162, normalized size = 1.84

$$\frac{x\left(bx\sqrt[4]{1+x^2}\,F_1\left(1;\frac{1}{4},1;2;-x^2,-\frac{x^2}{2}\right) - \frac{24aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)}{(2+x^2)\left(-6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)+x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)\right)\right)}\right)}{4\sqrt[4]{-1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((-1 - x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (x*(b*x*(1 + x^2)^(1/4)*AppellF1[1, 1/4, 1, 2, -x^2, -1/2*x^2] - (24*a*Appe
llF1[1/2, 1/4, 1, 3/2, -x^2, -1/2*x^2])/((2 + x^2)*(-6*AppellF1[1/2, 1/4, 1
, 3/2, -x^2, -1/2*x^2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -x^2, -1/2*x^2]
+ AppellF1[3/2, 5/4, 1, 5/2, -x^2, -1/2*x^2])))))/(4*(-1 - x^2)^(1/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)/((2 + x^2)*(-1 - x^2)^(1/4)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(-x^{2}-1\right)^{\frac{1}{4}} \left(x^{2}+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)
[Out] int((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="maxima")
[Out] integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{\sqrt[4]{-x^{2}-1}\,\left(x^{2}+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2-1)**(1/4)/(x**2+2),x)
[Out] Integral((a + b*x)/((-x**2 - 1)**(1/4)*(x**2 + 2)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x}{{\left(-x^2-1\right)}^{1/4}\,\left(x^2+2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/((- x^2 - 1)^(1/4)*(x^2 + 2)),x)
```

```
[Out] int((a + b*x)/((- x^2 - 1)^(1/4)*(x^2 + 2)), x)
```

3.71 $\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx$

Optimal. Leaf size=149

$$\frac{b\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}+\frac{1}{2}a\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)+\frac{b\tanh^{-1}\left(\frac{1+\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}+\frac{1}{2}a\tanh^{-1}\left(\frac{1+\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)$$

```
[Out] 1/2*a*arctan((1-(-x^2+1)^(1/2))/x/(-x^2+1)^(1/4))+1/2*a*arctanh((1+(-x^2+1)
^(1/2))/x/(-x^2+1)^(1/4))+1/2*b*arctan(1/2*(1-(-x^2+1)^(1/2))/(-x^2+1)^(1/4
)*2^(1/2))*2^(1/2)+1/2*b*arctanh(1/2*(1+(-x^2+1)^(1/2))/(-x^2+1)^(1/4)*2^(1
/2))*2^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1024, 406, 450}

$$\frac{1}{2}a\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)+\frac{1}{2}a\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{x\sqrt[4]{1-x^2}}\right)+\frac{b\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((1 - x^2)^(1/4)*(2 - x^2)),x]
```

```
[Out] (b*ArcTan[(1 - Sqrt[1 - x^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a*ArcT
an[(1 - Sqrt[1 - x^2])/(x*(1 - x^2)^(1/4))])/2 + (b*ArcTanh[(1 + Sqrt[1 - x
^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a*ArcTanh[(1 + Sqrt[1 - x^2])/(
x*(1 - x^2)^(1/4))])/2
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rule 450

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
Simp[(-(Sqrt[2]*Rt[a, 4]*d)^(-1))*ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sq
rt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x] - Simp[(1/(Sqrt[2]*Rt[a, 4]*d))*ArcT
anh[(Rt[a, 4]^2 + Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx = a\int \frac{1}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx + b\int \frac{x}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx$$
$$= \frac{b\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{1}{2}a\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right) + \frac{b\tanh^{-1}\left(\frac{1+\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.16, size = 144, normalized size = 0.97

$$\frac{1}{4}bx^2F_1\left(1;\frac{1}{4},1;2;x^2,\frac{x^2}{2}\right) - \frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}{\sqrt[4]{1-x^2}\,(-2+x^2)\left(6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)+x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};x^2,\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};x^2,\frac{x^2}{2}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((1 - x^2)^(1/4)*(2 - x^2)),x]
[Out] (b*x^2*AppellF1[1, 1/4, 1, 2, x^2, x^2/2])/4 - (6*a*x*AppellF1[1/2, 1/4, 1,
 3/2, x^2, x^2/2])/((1 - x^2)^(1/4)*(-2 + x^2)*(6*AppellF1[1/2, 1/4, 1, 3/2
, x^2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, x^2, x^2/2] + AppellF1[3/
2, 5/4, 1, 5/2, x^2, x^2/2])))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)/((2 - x^2)*(1 - x^2)^(1/4)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{(-x^2+1)^{\frac{1}{4}}\,(-x^2+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)
```

```
[Out] int((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{x^2\sqrt[4]{1-x^2}-2\sqrt[4]{1-x^2}}\,dx-\int \frac{bx}{x^2\sqrt[4]{1-x^2}-2\sqrt[4]{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+1)**(1/4)/(-x**2+2),x)
```

```
[Out] -Integral(a/(x**2*(1 - x**2)**(1/4) - 2*(1 - x**2)**(1/4)), x) - Integral(b
*x/(x**2*(1 - x**2)**(1/4) - 2*(1 - x**2)**(1/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,x}{{\left(1-x^2\right)}^{1/4}\,\left(x^2-2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x)/((1 - x^2)^(1/4)*(x^2 - 2)),x)
[Out] int(-(a + b*x)/((1 - x^2)^(1/4)*(x^2 - 2)), x)
```

3.72 $\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx$

Optimal. Leaf size=135

$$-\frac{b\tan^{-1}\left(\frac{1-\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right)}{\sqrt{2}}-\frac{1}{2}a\tan^{-1}\left(\frac{1+\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right)-\frac{1}{2}a\tanh^{-1}\left(\frac{1-\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right)-\frac{b\tanh^{-1}\left(\frac{1+\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*a*arctan((1+(x^2+1)^(1/2))/x/(x^2+1)^(1/4))-1/2*a*arctanh((1-(x^2+1)^(
1/2))/x/(x^2+1)^(1/4))-1/2*b*arctan(1/2*(1-(x^2+1)^(1/2))/(x^2+1)^(1/4)*2^(
1/2))*2^(1/2)-1/2*b*arctanh(1/2*(1+(x^2+1)^(1/2))/(x^2+1)^(1/4)*2^(1/2))*2^
(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1024, 406, 450}

$$-\frac{1}{2}a\tan^{-1}\left(\frac{\sqrt{x^2+1}+1}{x\sqrt[4]{x^2+1}}\right)-\frac{1}{2}a\tanh^{-1}\left(\frac{1-\sqrt{x^2+1}}{x\sqrt[4]{x^2+1}}\right)-\frac{b\tan^{-1}\left(\frac{1-\sqrt{x^2+1}}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{x^2+1}+1}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((1 + x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] -((b*ArcTan[(1 - Sqrt[1 + x^2])/(Sqrt[2]*(1 + x^2)^(1/4))])/Sqrt[2]) - (a*A
rcTan[(1 + Sqrt[1 + x^2])/(x*(1 + x^2)^(1/4))])/2 - (a*ArcTanh[(1 - Sqrt[1
+ x^2])/(x*(1 + x^2)^(1/4))])/2 - (b*ArcTanh[(1 + Sqrt[1 + x^2])/(Sqrt[2]*(
1 + x^2)^(1/4))])/Sqrt[2]
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rule 450

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
Simp[(-(Sqrt[2]*Rt[a, 4]*d)^(-1))*ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sq
rt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x] - Simp[(1/(Sqrt[2]*Rt[a, 4]*d))*ArcT
anh[(Rt[a, 4]^2 + Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx = a\int \frac{1}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx + b\int \frac{x}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx$$

$$= -\frac{b\tan^{-1}\left(\frac{1-\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right)}{\sqrt{2}} - \frac{1}{2}a\tan^{-1}\left(\frac{1+\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right) - \frac{1}{2}a\tanh^{-1}\left(\frac{1-\sqrt{1+\dots}}{x\sqrt[4]{1+\dots}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.13, size = 152, normalized size = 1.13

$$\frac{1}{4}bx^2F_1\left(1;\frac{1}{4},1;2;-x^2,-\frac{x^2}{2}\right) - \frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)}{\sqrt[4]{1+x^2}\,(2+x^2)\left(-6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)+x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((1 + x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/4, 1, 2, -x^2, -1/2*x^2])/4 - (6*a*x*AppellF1[1/2, 1/4
, 1, 3/2, -x^2, -1/2*x^2])/((1 + x^2)^(1/4)*(2 + x^2)*(-6*AppellF1[1/2, 1/4
, 1, 3/2, -x^2, -1/2*x^2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -x^2, -1/2*x^
2] + AppellF1[3/2, 5/4, 1, 5/2, -x^2, -1/2*x^2])))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)/((2 + x^2)*(1 + x^2)^(1/4)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{(x^2+1)^{\frac{1}{4}}\,(x^2+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x)
```

```
[Out] int((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx}{\sqrt[4]{x^2 + 1}\,(x^2 + 2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x**2+1)**(1/4)/(x**2+2),x)
```

```
[Out] Integral((a + b*x)/((x**2 + 1)**(1/4)*(x**2 + 2)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x}{{\left(x^2+1\right)}^{1/4}\,\left(x^2+2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/((x^2 + 1)^(1/4)*(x^2 + 2)),x)
[Out] int((a + b*x)/((x^2 + 1)^(1/4)*(x^2 + 2)), x)
```

3.73 $\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx$

Optimal. Leaf size=127

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{1+\sqrt[3]{2}\,x}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

```
[Out] -1/6*arctanh((1+2^(1/3)*x)/(-x^3+1)^(1/2))*2^(1/3)+1/18*arctanh((-x^3+1)^(1
/2))*2^(1/3)-1/18*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*2^(1/3)*3^(1
/2)+1/18*arctan(1/3*(-x^3+1)^(1/2)*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {497}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}\,x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] -1/3*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(2^(2/3)*Sqrt[3]) + Ar
cTan[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)/S
qrt[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))
```

Rule 497

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{1+\sqrt[3]{2}\,x}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tan}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.03, size = 28, normalized size = 0.22

$$\frac{1}{8}x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,\frac{x^3}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/4])/8
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/((4 - x^3)*Sqrt[1 - x^3]),x]')
```

```
[Out] Timed out
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.86, size = 164, normalized size = 1.29

method	result
default	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^3-4\right)}\frac{\_\alpha^2\sqrt{2}\,\sqrt{i\left(-i\sqrt{3}+2x+1\right)}\,\sqrt{\frac{-1+x}{i\sqrt{3}-3}}\,\sqrt{-\frac{i\left(i\sqrt{3}+2x+1\right)}{2}}\,\left(-2\_\alpha^2+\_\alpha+1+i\sqrt{\cdots}\right.}{\cdots}\right.}{36}$

elliptic	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^3-4\right)}\frac{\_\alpha^2\sqrt{2}\,\sqrt{i\left(-i\sqrt{3}+2x+1\right)}\,\sqrt{\frac{-1+x}{i\sqrt{3}-3}}\,\sqrt{-\frac{i\left(i\sqrt{3}+2x+1\right)}{2}}\,\left(-2\_\alpha^2+\_\alpha+1+i\right.}{}\right.}{36}$
trager	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+4)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*I*2^(1/2)*sum(_alpha^2*(1/2*I*(-I*3^(1/2)+2*x+1))^(1/2)*((-1+x)/(I*3^(
1/2)-3))^(1/2)*(-1/2*I*(I*3^(1/2)+2*x+1))^(1/2)/(-x^3+1)^(1/2)*(-2*_alpha^2
+_alpha+1+I*3^(1/2)*(1-_alpha))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2),1/2*_alpha-1/3*I*_alpha^2*3^(1/2)-1/2+1/6*I*_alpha*3^(1
/2)+1/6*I*3^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^
3-4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1191 vs. 2(92) = 184.
time = 0.45, size = 1191, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/31104*432^(5/6)*sqrt(3)*log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2
/3)*(x^8 - 5*x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 -
 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3
888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) - 1/31104*432
```

```
^(5/6)*sqrt(3)*log(36*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*
x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*
x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(
x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^(5/6)*sqrt(3
)*log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2)
 - (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432
^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) -
 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^(5/6)*sqrt(3)*log(36*(36
*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6
- 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3
)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 -
 12*x^6 + 48*x^3 - 64)) - 1/1944*432^(5/6)*arctan(1/216*sqrt(-x^3 + 1)*(72*
432^(1/6)*x^2 + 432^(5/6)*x + 72*sqrt(3))/(2*x^3 - 1)) + 1/3888*432^(5/6)*a
rctan(-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4
) + 18*432^(1/6)*(x^5 + 8*x^2)) + (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sq
rt(3)*2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) - sqrt(-x^3 + 1)*(432^(5/
6)*(2*x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2)))*sqr
t((36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) + (2592
*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*s
qrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(
x^9 - 12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4)) + 1/3888*432^(5/6)*arctan(
-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4) + 18
*432^(1/6)*(x^5 + 8*x^2)) - (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sqrt(3)*
2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) + sqrt(-x^3 + 1)*(432^(5/6)*(2*
x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2)))*sqrt((36*
x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6 -
 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)
*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 -
12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^3\sqrt{1-x^3} - 4\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+4)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x**3*sqrt(1 - x**3) - 4*sqrt(1 - x**3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.45, size = 653, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((1 - x^3)^(1/2)*(x^3 - 4)),x)
```

```
[Out] - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1
/2)*1i)/2 + 3/2)/(2^(2/3) - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(1 - x^3)^(1/2)*(2^
(2/3) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1
i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x
- 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(2/
3)*((3^(1/2)*1i)/2 + 1/2) + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 + 1
/2)*(1 - x^3)^(1/2)*(2^(2/3)*((3^(1/2)*1i)/2 + 1/2) + 1)*(((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(
-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i
)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(2/3)*((3^(1/2)*1i)/2 - 1/2) -
1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 - 1/2)*(1 - x^3)^(1/2)*(2^(2/3)
*((3^(1/2)*1i)/2 - 1/2) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.74 $\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx$

Optimal. Leaf size=157

$$-\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2}\sqrt[3]{d}\,x}{\sqrt{-1+dx^3}}\right)}{3\ 2^{2/3}d^{2/3}}-\frac{\tan^{-1}\left(\sqrt{-1+dx^3}\right)}{9\ 2^{2/3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}\,x\right)}{\sqrt{-1+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}\,d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{-1+dx^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}\,d^{2/3}}$$

```
[Out] -1/6*arctan((1+2^(1/3)*d^(1/3)*x)/(d*x^3-1)^(1/2))*2^(1/3)/d^(2/3)-1/18*arc
tan((d*x^3-1)^(1/2))*2^(1/3)/d^(2/3)-1/18*arctanh((1-2^(1/3)*d^(1/3)*x)*3^(
1/2)/(d*x^3-1)^(1/2))*2^(1/3)/d^(2/3)*3^(1/2)-1/18*arctanh(1/3*(d*x^3-1)^(1
/2)*3^(1/2))*2^(1/3)/d^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {498}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{d}\,x+1}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}d^{2/3}}-\frac{\tan^{-1}\left(\sqrt{dx^3-1}\right)}{9\ 2^{2/3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}\,x\right)}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}\sqrt{3}\,d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{dx^3-1}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}\,d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]
```

```
[Out] -1/3*ArcTan[(1 + 2^(1/3)*d^(1/3)*x)/Sqrt[-1 + d*x^3]]/(2^(2/3)*d^(2/3)) - A
rcTan[Sqrt[-1 + d*x^3]]/(9*2^(2/3)*d^(2/3)) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)
*d^(1/3)*x))/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3)) - ArcTanh[Sqrt[-
1 + d*x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3))
```

Rule 498

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(-q)*(ArcTan[Sqrt[c + d*x^3]/Rt[-c, 2]]/(9*2^(2/3
)*b*Rt[-c, 2])), x] + (-Simp[q*(ArcTan[Rt[-c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c
+ d*x^3])]/(3*2^(2/3)*b*Rt[-c, 2])), x] - Simp[q*(ArcTanh[Sqrt[c + d*x^3]/(
Sqrt[3]*Rt[-c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[-c, 2])), x] - Simp[q*(ArcTanh[
Sqrt[3]*Rt[-c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b
*Rt[-c, 2])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*
b*c - a*d, 0] && NegQ[c]
```

Rubi steps

$$\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx = -\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2}\sqrt[3]{d}x}{\sqrt{-1+dx^3}}\right)}{3\,2^{2/3}d^{2/3}} - \frac{\tan^{-1}\left(\sqrt{-1+dx^3}\right)}{9\,2^{2/3}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{\right.}{\sqrt{-1+dx}}\right)}{3\,2^{2/3}\sqrt{3}\,d^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.05, size = 54, normalized size = 0.34

$$\frac{x^2\sqrt{1-dx^3}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};dx^3,\frac{dx^3}{4}\right)}{8\sqrt{-1+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 - d*x^3]*AppellF1[2/3, 1/2, 1, 5/3, d*x^3, (d*x^3)/4])/(8*Sqrt[
-1 + d*x^3])
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.23, size = 240, normalized size = 1.53

method	result

default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-4\right)}\frac{\sqrt{-\frac{i\left(2x+\frac{1}{d^{\frac{1}{3}}}+\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}{2}}\sqrt{\frac{x-\frac{1}{d^{\frac{1}{3}}}}{-\frac{3}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}}}\sqrt{2}\sqrt{i\left(2x+\frac{1}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}\,(-2 \cdots}{\cdots}\right.}{\cdots}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-4\right)}\frac{\sqrt{-\frac{i\left(2x+\frac{1}{d^{\frac{1}{3}}}+\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}{2}}\sqrt{\frac{x-\frac{1}{d^{\frac{1}{3}}}}{-\frac{3}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}}}\sqrt{2}\sqrt{i\left(2x+\frac{1}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}\,(-2 \cdots}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*I*2^(1/2)*sum(1/_alpha/d^(4/3)*(-1/2*I*(2*x+1/d^(1/3)+I*3^(1/2)/d^(1/3
))*d^(1/3))^(1/2)*((x-1/d^(1/3))/(-3/d^(1/3)-I*3^(1/2)/d^(1/3)))^(1/2)*(1/2
*I*(2*x+1/d^(1/3)-I*3^(1/2)/d^(1/3))*d^(1/3))^(1/2)/(d*x^3-1)^(1/2)*(-2*_al
pha^2*d+I*3^(1/2)*_alpha*d^(2/3)-I*3^(1/2)*d^(1/3)+_alpha*d^(2/3)+d^(1/3))*
EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/d^(1/3)+1/2*I*3^(1/2)/d^(1/3))*3^(1/2)*d^
(1/3))^(1/2),1/3*I*3^(1/2)*d^(2/3)*_alpha^2-1/6*I*3^(1/2)*d^(1/3)*_alpha-1/
6*I*3^(1/2)+1/2*d^(1/3)*_alpha-1/2,(-I*3^(1/2)/d^(1/3)/(-3/2/d^(1/3)-1/2*I*
3^(1/2)/d^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-4))
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1666 vs. $2(110) = 220$.
time = 0.53, size = 1666, normalized size = 10.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*(1/432)^(1/6)*(d^(-4))^(1/6)*arctan(1/3*(3*(sqrt(3)*sqrt(1/3)*
d^2*sqrt(d^(-4))*x + 2*sqrt(3)*(1/432)^(1/6)*d*(d^(-4))^(1/6)*x^2 - 24*sqrt
(3)*(1/432)^(5/6)*(d^4*x^3 - 4*d^3)*(d^(-4))^(5/6))*sqrt(d*x^3 - 1) + (2*sq
rt(3)*(1/2)^(1/3)*(d^2*x^3 - d)*(d^(-4))^(1/3) + sqrt(3)*(d*x^4 - x) + 3*(s
qrt(3)*sqrt(1/3)*d^2*sqrt(d^(-4))*x + 2*sqrt(3)*(1/432)^(1/6)*d*(d^(-4))^(1
/6)*x^2 + 24*sqrt(3)*(1/432)^(5/6)*(d^4*x^3 + 2*d^3)*(d^(-4))^(5/6))*sqrt(d
*x^3 - 1))*sqrt((d^3*x^9 - 60*d^2*x^6 - 24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4
 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^
2)*(d^(-4))^(1/3) + 12*(648*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3
)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*
d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^
6 + 48*d*x^3 - 64)))/(d*x^4 - x)) - 1/9*sqrt(3)*(1/432)^(1/6)*(d^(-4))^(1/6
)*arctan(1/3*(3*(sqrt(3)*sqrt(1/3)*d^2*sqrt(d^(-4))*x + 2*sqrt(3)*(1/432)^(
1/6)*d*(d^(-4))^(1/6)*x^2 - 24*sqrt(3)*(1/432)^(5/6)*(d^4*x^3 - 4*d^3)*(d^(
-4))^(5/6))*sqrt(d*x^3 - 1) - (2*sqrt(3)*(1/2)^(1/3)*(d^2*x^3 - d)*(d^(-4))
^(1/3) + sqrt(3)*(d*x^4 - x) - 3*(sqrt(3)*sqrt(1/3)*d^2*sqrt(d^(-4))*x + 2*
sqrt(3)*(1/432)^(1/6)*d*(d^(-4))^(1/6)*x^2 + 24*sqrt(3)*(1/432)^(5/6)*(d^4*
x^3 + 2*d^3)*(d^(-4))^(5/6))*sqrt(d*x^3 - 1))*sqrt((d^3*x^9 - 60*d^2*x^6 -
24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1
/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) - 12*(648*(1/432)^(5/6
)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^
(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*
x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)))/(d*x^4 - x)) + 1/18
*(1/432)^(1/6)*(d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 + 48*(1/
2)^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4
*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) + 6*(1296*(1/432)^(5/6)*d^5*(d
^(-4))^(5/6)*x^5 + sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4))
 + 2*(1/432)^(1/6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^
3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) - 1/18*(1/432)^(1/6)*(
d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 + 48*(1/2)^(2/3)*(d^5*x^
7 + d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5
 - 8*d^2*x^2)*(d^(-4))^(1/3) - 6*(1296*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5
 + sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4)) + 2*(1/432)^(1/
6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^
3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) - 1/36*(1/432)^(1/6)*(d^(-4))^(1/6)*lo
g((d^3*x^9 - 60*d^2*x^6 - 24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d
^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1
```

```
/3) + 12*(648*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 1
6*d^3*x^3 - 8*d^2)*sqrt(d^(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*
x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 -
 64)) + 1/36*(1/432)^(1/6)*(d^(-4))^(1/6)*log((d^3*x^9 - 60*d^2*x^6 - 24*(1
/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(
d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) - 12*(648*(1/432)^(5/6)*d^5
*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^(-4))
 - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 -
 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{dx^3\sqrt{dx^3-1}-4\sqrt{dx^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+4)/(d*x**3-1)**(1/2),x)
[Out] -Integral(x/(d*x**3*sqrt(d*x**3 - 1) - 4*sqrt(d*x**3 - 1)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x)
[Out] Could not integrate
```

Mupad [B]
time = 15.03, size = 331, normalized size = 2.11

$$\frac{\sqrt{3}\,314928^{1/3}\ln\left(\frac{\left(54\sqrt{d\,x^3-1}+54\sqrt{3}-54\,2^{1/3}\sqrt{3}\,d^{1/3}\,x\right)\left(\sqrt{d\,x^3-1}-\sqrt{3}+2^{1/3}\sqrt{3}\,d^{1/3}\,x\right)^3}{\left(2^{2/3}-d^{1/3}\,x\right)^6}\right)}{2916\,d^{2/3}}+\frac{\sqrt{3}\,314928^{1/3}\ln\left(\frac{\left(2\sqrt{d\,x^3-1}+2\sqrt{3}+2^{1/3}\sqrt{3}\,d^{1/3}\,x+2^{1/3}\,d^{1/3}\,x\,3\mathrm{i}\right)^3\left(108\sqrt{3}-108\sqrt{d\,x^3-1}+54\,2^{1/3}\sqrt{3}\,d^{1/3}\,x+2^{1/3}\,d^{1/3}\,x\,162\mathrm{i}\right)}{\left(2^{2/3}+2\,d^{1/3}\,x-2^{2/3}\sqrt{3}\,1\mathrm{i}\right)^6}\right)\sqrt{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}{2916\,d^{2/3}}+\frac{\sqrt{3}\,314928^{1/3}\ln\left(\frac{\left(2\sqrt{d\,x^3-1}-2\sqrt{3}-2^{1/3}\sqrt{3}\,d^{1/3}\,x+2^{1/3}\,d^{1/3}\,x\,3\mathrm{i}\right)^3\left(108\sqrt{d\,x^3-1}+108\sqrt{3}+54\,2^{1/3}\sqrt{3}\,d^{1/3}\,x-2^{1/3}\,d^{1/3}\,x\,162\mathrm{i}\right)}{\left(2^{2/3}+2\,d^{1/3}\,x+2^{2/3}\sqrt{3}\,1\mathrm{i}\right)^6}\right)\sqrt{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\,1\mathrm{i}}{2916\,d^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((d*x^3 - 1)^(1/2)*(d*x^3 - 4)),x)
[Out] (3^(1/2)*314928^(1/3)*log(((54*(d*x^3 - 1)^(1/2) + 54*3^(1/2) - 54*2^(1/3)*
3^(1/2)*d^(1/3)*x)*((d*x^3 - 1)^(1/2) - 3^(1/2) + 2^(1/3)*3^(1/2)*d^(1/3)*x
)^3)/(2^(2/3) - d^(1/3)*x)^6))/(2916*d^(2/3)) + (3^(1/2)*314928^(1/3)*log((
(2*(d*x^3 - 1)^(1/2) + 2*3^(1/2) + 2^(1/3)*d^(1/3)*x*3i + 2^(1/3)*3^(1/2)*d
^(1/3)*x)^3*(108*3^(1/2) - 108*(d*x^3 - 1)^(1/2) + 2^(1/3)*d^(1/3)*x*162i +
 54*2^(1/3)*3^(1/2)*d^(1/3)*x))/(2^(2/3) - 2^(2/3)*3^(1/2)*1i + 2*d^(1/3)*x
)^6)*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(2916*d^(2/3)) + (3^(1/2)*314928^(1/3)*l
og(((2*(d*x^3 - 1)^(1/2) - 2*3^(1/2) + 2^(1/3)*d^(1/3)*x*3i - 2^(1/3)*3^(1/
2)*d^(1/3)*x)^3*(108*(d*x^3 - 1)^(1/2) + 108*3^(1/2) - 2^(1/3)*d^(1/3)*x*16
2i + 54*2^(1/3)*3^(1/2)*d^(1/3)*x))/(2^(2/3)*3^(1/2)*1i + 2^(2/3) + 2*d^(1/
3)*x)^6)*((3^(1/2)*1i)/2 + 1/2)^(1/2)*1i)/(2916*d^(2/3))
```

3.75 $\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)}\,dx$

Optimal. Leaf size=74

$$\frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)+\frac{1}{18}\tan^{-1}\left(\frac{1}{3}\sqrt{-1+x^3}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\,(1-x)}{\sqrt{-1+x^3}}\right)}{6\sqrt{3}}$$

```
[Out] 1/18*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))+1/18*arctan(1/3*(x^3-1)^(1/2))-1/18*
arctanh((1-x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {499, 455, 65, 210, 2163, 209, 2170, 212}

$$\frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)+\frac{1}{18}\tan^{-1}\left(\frac{\sqrt{x^3-1}}{3}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\,(1-x)}{\sqrt{x^3-1}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x^3]*(8 + x^3)),x]
```

```
[Out] ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])]/18 + ArcTan[Sqrt[-1 + x^3]/3]/18 - Arc
Tanh[(Sqrt[3]*(1 - x))/Sqrt[-1 + x^3]]/(6*Sqrt[3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)}\,dx = -\left(\frac{1}{12}\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx\right) - \frac{1}{12}\int \frac{-2-2x+x^2}{(4-2x+x^2)\sqrt{-1+x^3}}\,dx - \frac{1}{4}\int$$

$$= -\left(\frac{1}{12}\text{Subst}\left(\int \frac{1}{(-8-x)\sqrt{-1+x}}\,dx, x, x^3\right)\right) + \frac{1}{6}\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x,\right.$$

$$= \frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\,(1-x)}{\sqrt{-1+x^3}}\right)}{6\sqrt{3}} - \frac{1}{6}\text{Subst}\left(\int \frac{1}{-9-x^2}\,dx\right.$$

$$= \frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) + \frac{1}{18}\tan^{-1}\left(\frac{1}{3}\sqrt{-1+x^3}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\,(1-x)}{\sqrt{-1+x^3}}\right.}{6\sqrt{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.02, size = 48, normalized size = 0.65

$$\frac{x^2\sqrt{1-x^3}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,-\frac{x^3}{8}\right)}{16\sqrt{-1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(8 + x^3)),x]
```

```
[Out] (x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, -1/8*x^3])/(16*Sqrt[-1 +
 x^3])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/((x^3 + 8)*Sqrt[x^3 - 1]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 2.23, size = 421, normalized size = 5.69

method	result

default	$-\frac{\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{i\sqrt{3}}{6}+\frac{1}{2},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{9\sqrt{x^3-1}}$
trager	Expression too large to display
elliptic	$\frac{\sqrt{-\frac{1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}+\frac{x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}+\frac{1}{3-i\sqrt{3}}-\frac{i\sqrt{3}}{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\sqrt{\frac{x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}+\frac{1}{i\sqrt{3}+3}+\frac{i\sqrt{3}}{i\sqrt{3}+3}}\ \mathrm{E}}{6\sqrt{x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3+8)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/
6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+1/9*I*(1/2
-1/2*I*3^(1/2))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((
x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2
+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I
*3^(1/2)))^(1/2),1/6*I*(1+I*3^(1/2))*3^(1/2)+1/3*I*3^(1/2),((3/2+1/2*I*3^(1
/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/9*I*(1/2+1/2*I*3^(1/2))*(-3/2-1/2*I*3^(1
/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*
3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(
1/2)*3^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/6*I*(1-I*3^(1
/2))*3^(1/2)-2/3*I*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 8)*sqrt(x^3 - 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(51) = 102.
time = 0.42, size = 547, normalized size = 7.39

$$\frac{1}{216}\sqrt{3}\log\left(\frac{4\left(x^6+48x^5+186x^4-56x^3+6\sqrt{3}\left(x^4+12x^3+12x^2-16x\right)\sqrt{x^3-1}-120x^2-96x+64\right)}{x^6-6x^5+24x^4-56x^3+96x^2-96x+64}\right)-\frac{1}{216}\sqrt{3}\log\left(\frac{4\left(x^6+48x^5+186x^4-56x^3-6\sqrt{3}\left(x^4+12x^3+12x^2-16x\right)\sqrt{x^3-1}-120x^2-96x+64\right)}{x^6-6x^5+24x^4-56x^3+96x^2-96x+64}\right)+\frac{1}{54}\arctan\left(\frac{\left(x^3-12x^2-6x-10\right)\sqrt{x^3-1}}{6\left(x^4-x^3-x+1\right)}\right)-\frac{1}{54}\arctan\left(-\frac{\sqrt{x^3-1}\left(x^2-8x+10\right)+\left(3\sqrt{3}\left(x^3+x^2-2x\right)-\sqrt{x^3-1}\left(x^2+10x-8\right)\right)\sqrt{\frac{x^6+48x^5+186x^4-56x^3+6\sqrt{3}\left(x^4+12x^3+12x^2-16x\right)\sqrt{x^3-1}-120x^2-96x+64}{x^6-6x^5+24x^4-56x^3+96x^2-96x+64}}}{3\left(x^3-3x^2+2\right)}\right)-\frac{1}{54}\arctan\left(-\frac{\sqrt{x^3-1}\left(x^2-8x+10\right)-\left(3\sqrt{3}\left(x^3+x^2-2x\right)+\sqrt{x^3-1}\left(x^2+10x-8\right)\right)\sqrt{\frac{x^6+48x^5+186x^4-56x^3-6\sqrt{3}\left(x^4+12x^3+12x^2-16x\right)\sqrt{x^3-1}-120x^2-96x+64}{x^6-6x^5+24x^4-56x^3+96x^2-96x+64}}}{3\left(x^3-3x^2+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/216*sqrt(3)*log(4*(x^6 + 48*x^5 + 186*x^4 - 56*x^3 + 6*sqrt(3)*(x^4 + 12*
x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) - 120*x^2 - 96*x + 64)/(x^6 - 6*x^5 + 24
*x^4 - 56*x^3 + 96*x^2 - 96*x + 64)) - 1/216*sqrt(3)*log(4*(x^6 + 48*x^5 +
186*x^4 - 56*x^3 - 6*sqrt(3)*(x^4 + 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) -
 120*x^2 - 96*x + 64)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64))
 + 1/54*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x +
 1)) - 1/54*arctan(-1/3*(sqrt(x^3 - 1)*(x^2 - 8*x + 10) + (3*sqrt(3)*(x^3 +
 x^2 - 2*x) - sqrt(x^3 - 1)*(x^2 + 10*x - 8))*sqrt((x^6 + 48*x^5 + 186*x^4
- 56*x^3 + 6*sqrt(3)*(x^4 + 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) - 120*x^2
 - 96*x + 64)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64)))/(x^3 -
 3*x^2 + 2)) - 1/54*arctan(-1/3*(sqrt(x^3 - 1)*(x^2 - 8*x + 10) - (3*sqrt(3
)*(x^3 + x^2 - 2*x) + sqrt(x^3 - 1)*(x^2 + 10*x - 8))*sqrt((x^6 + 48*x^5 +
186*x^4 - 56*x^3 - 6*sqrt(3)*(x^4 + 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) -
 120*x^2 - 96*x + 64)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64))
)/(x^3 - 3*x^2 + 2))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x-1)(x^2+x+1)}\,(x+2)(x^2-2x+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3+8)/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)*(x**2 - 2*x + 4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.21, size = 533, normalized size = 7.20

$$\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\,\Big|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{9\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{3};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\,\Big|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)2\mathrm{i}}{9\left(-1+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{3};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\,\Big|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)2\mathrm{i}}{9\left(1+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 - 1)^(1/2)*(x^3 + 8)),x)
```

```
[Out] (((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2)
)^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x -
 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
 - 3/2)))/(9*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (3^(1/2)*((3^(1/2)
*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((
x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)
*1i)/2 + 3/2))^(1/2)*ellipticPi(-(3^(1/2)*((3^(1/2)*1i)/2 + 3/2)*1i)/3, asi
n((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/
2)*1i)/2 - 3/2))*2i)/(9*(3^(1/2)*1i - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)
^(1/2)) - (3^(1/2)*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*((3^(1
/2)*1i)/2 + 3/2)*1i)/3, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3
^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*2i)/(9*(3^(1/2)*1i + 1)*(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.76 $\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx$

Optimal. Leaf size=103

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{d}\,x\right)}{\sqrt{1+dx^3}}\right)}{6\sqrt{3}\,d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(1+\sqrt[3]{d}\,x\right)^2}{3\sqrt{1+dx^3}}\right)}{18d^{2/3}}-\frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{1+dx^3}\right)}{18d^{2/3}}$$

```
[Out] 1/18*arctanh(1/3*(1+d^(1/3)*x)^2/(d*x^3+1)^(1/2))/d^(2/3)-1/18*arctanh(1/3*
(d*x^3+1)^(1/2))/d^(2/3)-1/18*arctan((1+d^(1/3)*x)*3^(1/2)/(d*x^3+1)^(1/2))
/d^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{d}\,x+1\right)}{\sqrt{dx^3+1}}\right)}{6\sqrt{3}\,d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{d}\,x+1\right)^2}{3\sqrt{dx^3+1}}\right)}{18d^{2/3}}-\frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]
```

```
[Out] -1/6*ArcTan[(Sqrt[3]*(1 + d^(1/3)*x))/Sqrt[1 + d*x^3]]/(Sqrt[3]*d^(2/3)) +
ArcTanh[(1 + d^(1/3)*x)^2/(3*Sqrt[1 + d*x^3])]/(18*d^(2/3)) - ArcTanh[Sqrt[
1 + d*x^3]/3]/(18*d^(2/3))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx = -\frac{\int \frac{2d^{2/3}-2dx-d^{4/3}x^2}{\left(4+2\sqrt[3]{d}\,x+d^{2/3}x^2\right)\sqrt{1+dx^3}}\,dx}{12d} + \frac{\int \frac{1+\sqrt[3]{d}\,x}{\left(2-\sqrt[3]{d}\,x\right)\sqrt{1+dx^3}}\,dx}{12\sqrt[3]{d}} - \frac{1}{4}\sqrt[3]{d}\int$$

$$= \frac{\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{\left(1+\sqrt[3]{d}\,x\right)^2}{\sqrt{1+dx^3}}\right)}{6d^{2/3}} - \frac{1}{12}\sqrt[3]{d}\,\text{Subst}\left(\int \frac{1}{(8-dx)\sqrt{1+dx}}\,dx\right.$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{d}\,x\right)}{\sqrt{1+dx^3}}\right)}{6\sqrt{3}\,d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(1+\sqrt[3]{d}\,x\right)^2}{3\sqrt{1+dx^3}}\right)}{18d^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x,\right.}{6d^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{d}\,x\right)}{\sqrt{1+dx^3}}\right)}{6\sqrt{3}\,d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(1+\sqrt[3]{d}\,x\right)^2}{3\sqrt{1+dx^3}}\right)}{18d^{2/3}} - \frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{1+dx^3}\right.}{18d^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.04, size = 32, normalized size = 0.31

$$\frac{1}{16}x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-dx^3,\frac{dx^3}{8}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, -(d*x^3), (d*x^3)/8])/16
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.39, size = 383, normalized size = 3.72

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8\right)}\frac{\left(-d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}+\left(-d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(\cdots\right.}{\cdots}}\cdots}{\cdots}\right.}{\cdots}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8\right)}\frac{\left(-d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}+\left(-d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(\cdots\right.}{\cdots}}\cdots}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*I/d^3*2^(1/2)*sum(1/_alpha*(-d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-d^2)^(1/3)+(-d^2)^(1/3)))/(-d^2)^(1/3))^(1/2)*(d*(x-1/d*(-d^2)^(1/3))/(-
3*(-d^2)^(1/3)+I*3^(1/2)*(-d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)
*(-d^2)^(1/3)+(-d^2)^(1/3)))/(-d^2)^(1/3))^(1/2)/(d*x^3+1)^(1/2)*(I*(-d^2)^
(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2)^(2/3)+2*_alpha^2*d^2-(-d^2)^(1/3)*_
alpha*d-(-d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-d^2)^(1/3))*3^(1/2)*d/(-d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2)^
(1/3)*_alpha^2*3^(1/2)*d-I*(-d^2)^(2/3)*_alpha*3^(1/2)-3*(-d^2)^(2/3)*_alph
```

```
a+I*3^(1/2)*d-3*d),(I*3^(1/2)/d*(-d^2)^(1/3)/(-3/2/d*(-d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(73) = 146$.
time = 0.49, size = 497, normalized size = 4.83

$$\frac{2\sqrt{3}\,(d^2)^{\frac{1}{6}} d \arctan\left(-\frac{\left(9\sqrt{3}\,d^3x^5-\sqrt{3}\,(d^2x^6-40\,dx^3-32)(d^2)^{\frac{2}{3}}+3\sqrt{3}\,(5\,d^2x^4+8\,dx)(d^2)^{\frac{1}{3}}\right)\sqrt{dx^3+1}\,(d^2)^{\frac{1}{6}}}{9\,(d^4x^7-7\,d^3x^4-8\,d^2x)}\right)+2\,(d^2)^{\frac{2}{3}}\log\left(\frac{d^4x^9+318\,d^3x^6+1200\,d^2x^3+18\,(5\,d^2x^7+64\,dx^4+32\,x)(d^2)^{\frac{2}{3}}+6\left(7\,d^3x^6+152\,d^2x^3+(d^2x^7+80\,dx^4+160\,x)(d^2)^{\frac{2}{3}}+6\,(5\,d^2x^5+32\,dx^2)(d^2)^{\frac{1}{3}}+64\,d\right)\sqrt{dx^3+1}+18\,(d^3x^8+38\,d^2x^5+64\,dx^2)(d^2)^{\frac{1}{3}}+640\,d}{d^3x^9-24\,d^2x^6+192\,dx^3-512}\right)-(d^2)^{\frac{2}{3}}\log\left(\frac{d^4x^9-276\,d^3x^6-1608\,d^2x^3-18\,(d^2x^7-52\,dx^4-80\,x)(d^2)^{\frac{2}{3}}-6\left(4\,d^3x^6+164\,d^2x^3+(d^2x^7-28\,dx^4-272\,x)(d^2)^{\frac{2}{3}}-24\,(d^2x^5+dx^2)(d^2)^{\frac{1}{3}}+160\,d\right)\sqrt{dx^3+1}+18\,(d^3x^8+20\,d^2x^5-8\,dx^2)(d^2)^{\frac{1}{3}}-1088\,d}{d^3x^9-24\,d^2x^6+192\,dx^3-512}\right)}{108\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="fricas")
[Out] 1/108*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(-1/9*(9*sqrt(3)*d^3*x^5 - sqrt(3)*(d^
2*x^6 - 40*d*x^3 - 32)*(d^2)^(2/3) + 3*sqrt(3)*(5*d^2*x^4 + 8*d*x)*(d^2)^(1
/3))*sqrt(d*x^3 + 1)*(d^2)^(1/6)/(d^4*x^7 - 7*d^3*x^4 - 8*d^2*x)) + 2*(d^2)
^(2/3)*log((d^4*x^9 + 318*d^3*x^6 + 1200*d^2*x^3 + 18*(5*d^2*x^7 + 64*d*x^4
 + 32*x)*(d^2)^(2/3) + 6*(7*d^3*x^6 + 152*d^2*x^3 + (d^2*x^7 + 80*d*x^4 + 1
60*x)*(d^2)^(2/3) + 6*(5*d^2*x^5 + 32*d*x^2)*(d^2)^(1/3) + 64*d)*sqrt(d*x^3
 + 1) + 18*(d^3*x^8 + 38*d^2*x^5 + 64*d*x^2)*(d^2)^(1/3) + 640*d)/(d^3*x^9
- 24*d^2*x^6 + 192*d*x^3 - 512)) - (d^2)^(2/3)*log((d^4*x^9 - 276*d^3*x^6 -
 1608*d^2*x^3 - 18*(d^2*x^7 - 52*d*x^4 - 80*x)*(d^2)^(2/3) - 6*(4*d^3*x^6 +
 164*d^2*x^3 + (d^2*x^7 - 28*d*x^4 - 272*x)*(d^2)^(2/3) - 24*(d^2*x^5 + d*x
^2)*(d^2)^(1/3) + 160*d)*sqrt(d*x^3 + 1) + 18*(d^3*x^8 + 20*d^2*x^5 - 8*d*x
^2)*(d^2)^(1/3) - 1088*d)/(d^3*x^9 - 24*d^2*x^6 + 192*d*x^3 - 512)))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{dx^3\sqrt{dx^3+1}-8\sqrt{dx^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+8)/(d*x**3+1)**(1/2),x)
[Out] -Integral(x/(d*x**3*sqrt(d*x**3 + 1) - 8*sqrt(d*x**3 + 1)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x}{\sqrt{d\,x^3+1}\,\left(d\,x^3-8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((d*x^3 + 1)^(1/2)*(d*x^3 - 8)),x)
```

```
[Out] -int(x/((d*x^3 + 1)^(1/2)*(d*x^3 - 8)), x)
```

3.77 $\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx$

Optimal. Leaf size=81

$$\frac{1}{4}\tan^{-1}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}\,x}\right)}{4\sqrt{3}}$$

```
[Out] 1/4*arctan((1-(-3*x^2+1)^(1/3))/x)+1/12*arctanh(1/3*x*3^(1/2))*3^(1/2)-1/12
*arctanh(1/9*(1-(-3*x^2+1)^(1/3))^2/x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {404}

$$\frac{1}{4}\tan^{-1}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)-\frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}\,x}\right)}{4\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - 3*x^2)^(1/3)*(3 - x^2)),x]
```

```
[Out] ArcTan[(1 - (1 - 3*x^2)^(1/3))/x]/4 + ArcTanh[x/Sqrt[3]]/(4*Sqrt[3]) - ArcT
anh[(1 - (1 - 3*x^2)^(1/3))^2/(3*Sqrt[3]*x)]/(4*Sqrt[3])
```

Rule 404

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx=\frac{1}{4}\tan^{-1}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}\,x}\right)}{4\sqrt{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 2.86, size = 126, normalized size = 1.56

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};3x^2,\frac{x^2}{3}\right)}{\sqrt[3]{1-3x^2}\,(-3+x^2)\left(9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};3x^2,\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};3x^2,\frac{x^2}{3}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};3x^2,\frac{x^2}{3}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - 3*x^2)^(1/3)*(3 - x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3])/((1 - 3*x^2)^(1/3)*(-3 + x^
2)*(9*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3,
2, 5/2, 3*x^2, x^2/3] + 3*AppellF1[3/2, 4/3, 1, 5/2, 3*x^2, x^2/3])))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((3 - x^2)*(1 - 3*x^2)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.62, size = 651, normalized size = 8.04

method	result
trager	$-48\ln\left(-\frac{18432\,\mathrm{RootOf}\left(2304\_Z^4+48\_Z^2+1\right)^5\left(-3x^2+1\right)^{\frac{1}{3}}x-36864\,\mathrm{RootOf}\left(2304\_Z^4+48\_Z^2+1\right)^5x+768\,\mathrm{RootOf}\left(2304\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+1)^(1/3)/(-x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -48*ln(-(18432*RootOf(2304*_Z^4+48*_Z^2+1)^5*(-3*x^2+1)^(1/3)*x-36864*RootO
f(2304*_Z^4+48*_Z^2+1)^5*x+768*RootOf(2304*_Z^4+48*_Z^2+1)^3*(-3*x^2+1)^(1/
3)*x-2304*RootOf(2304*_Z^4+48*_Z^2+1)^3*x-48*RootOf(2304*_Z^4+48*_Z^2+1)^2*
x^2-96*RootOf(2304*_Z^4+48*_Z^2+1)^2*(-3*x^2+1)^(1/3)-48*RootOf(2304*_Z^4+4
8*_Z^2+1)^2-32*RootOf(2304*_Z^4+48*_Z^2+1)*x-2*(-3*x^2+1)^(2/3)-x^2-1)/(x^2
-3))*RootOf(2304*_Z^4+48*_Z^2+1)^3+RootOf(2304*_Z^4+48*_Z^2+1)*ln(-(9216*Ro
otOf(2304*_Z^4+48*_Z^2+1)^5*(-3*x^2+1)^(1/3)*x-18432*RootOf(2304*_Z^4+48*_Z
^2+1)^5*x+576*RootOf(2304*_Z^4+48*_Z^2+1)^3*(-3*x^2+1)^(1/3)*x-768*RootOf(2
304*_Z^4+48*_Z^2+1)^3*x+24*RootOf(2304*_Z^4+48*_Z^2+1)^2*x^2+48*RootOf(2304
*_Z^4+48*_Z^2+1)^2*(-3*x^2+1)^(1/3)+8*RootOf(2304*_Z^4+48*_Z^2+1)*(-3*x^2+1
```

```
)^(1/3)*x+24*RootOf(2304*_Z^4+48*_Z^2+1)^2-(-3*x^2+1)^(2/3)+(-3*x^2+1)^(1/3
))/(x^2-3))-ln(-(18432*RootOf(2304*_Z^4+48*_Z^2+1)^5*(-3*x^2+1)^(1/3)*x-368
64*RootOf(2304*_Z^4+48*_Z^2+1)^5*x+768*RootOf(2304*_Z^4+48*_Z^2+1)^3*(-3*x^
2+1)^(1/3)*x-2304*RootOf(2304*_Z^4+48*_Z^2+1)^3*x-48*RootOf(2304*_Z^4+48*_Z
^2+1)^2*x^2-96*RootOf(2304*_Z^4+48*_Z^2+1)^2*(-3*x^2+1)^(1/3)-48*RootOf(230
4*_Z^4+48*_Z^2+1)^2-32*RootOf(2304*_Z^4+48*_Z^2+1)*x-2*(-3*x^2+1)^(2/3)-x^2
-1)/(x^2-3))*RootOf(2304*_Z^4+48*_Z^2+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1792 vs. 2(59) = 118.
time = 1.28, size = 1792, normalized size = 22.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(6)*sqrt(3)*sqrt(2)*arctan(1/9*(36*sqrt(6)*sqrt(3)*sqrt(2)*(3*x^11
 - 1117*x^9 + 3918*x^7 - 1866*x^5 + 255*x^3 - 9*x) + sqrt(3)*(sqrt(6)*sqrt(
3)*sqrt(2)*(x^12 + 2184*x^10 - 211215*x^8 + 94152*x^6 - 13581*x^4 + 432*x^2
 + 27) + 12*(sqrt(6)*sqrt(3)*sqrt(2)*(x^10 - 107*x^8 - 7262*x^6 + 2322*x^4
- 243*x^2 + 9) - 48*sqrt(3)*(5*x^9 - 245*x^7 + 183*x^5 - 15*x^3))*(-3*x^2 +
 1)^(2/3) - 12*sqrt(3)*(29*x^11 + 293*x^9 - 2670*x^7 + 4986*x^5 - 1215*x^3
+ 81*x) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(49*x^10 - 5043*x^8 + 3658*x^6 + 378*x
^4 - 171*x^2 + 9) - 2*sqrt(3)*(x^11 + 917*x^9 - 40566*x^7 + 15786*x^5 - 204
3*x^3 + 81*x))*(-3*x^2 + 1)^(1/3))*sqrt((x^6 - 93*x^4 + 4*sqrt(6)*sqrt(2)*(
x^5 + 13*x^3) - 117*x^2 - 2*(4*sqrt(6)*sqrt(2)*x^3 - 3*x^4 - 18*x^2 + 9)*(-
3*x^2 + 1)^(2/3) + (6*x^4 - sqrt(6)*sqrt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2
 - 18)*(-3*x^2 + 1)^(1/3) + 9)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 12*(2*sqrt(6)
*sqrt(3)*sqrt(2)*(35*x^9 - 4860*x^7 + 2106*x^5 - 396*x^3 + 27*x) - 3*sqrt(3
)*(x^10 + 589*x^8 + 3946*x^6 - 774*x^4 - 27*x^2 + 9))*(-3*x^2 + 1)^(2/3) -
3*sqrt(3)*(x^12 + 3150*x^10 + 77991*x^8 + 4260*x^6 - 14337*x^4 + 2862*x^2 -
 135) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(x^11 - 1591*x^9 + 42426*x^7 - 15102*x^5
 + 1269*x^3 - 27*x) - 6*sqrt(3)*(27*x^10 + 2307*x^8 + 4574*x^6 - 2538*x^4 +
 279*x^2 - 9))*(-3*x^2 + 1)^(1/3))/(x^12 - 4986*x^10 + 327519*x^8 - 159660*
x^6 + 25839*x^4 - 2106*x^2 + 81)) + 1/72*sqrt(6)*sqrt(3)*sqrt(2)*arctan(1/9
*(36*sqrt(6)*sqrt(3)*sqrt(2)*(3*x^11 - 1117*x^9 + 3918*x^7 - 1866*x^5 + 255
```

```
*x^3 - 9*x) + sqrt(3)*(sqrt(6)*sqrt(3)*sqrt(2)*(x^12 + 2184*x^10 - 211215*x
^8 + 94152*x^6 - 13581*x^4 + 432*x^2 + 27) + 12*(sqrt(6)*sqrt(3)*sqrt(2)*(x
^10 - 107*x^8 - 7262*x^6 + 2322*x^4 - 243*x^2 + 9) + 48*sqrt(3)*(5*x^9 - 24
5*x^7 + 183*x^5 - 15*x^3))*(-3*x^2 + 1)^(2/3) + 12*sqrt(3)*(29*x^11 + 293*x
^9 - 2670*x^7 + 4986*x^5 - 1215*x^3 + 81*x) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(4
9*x^10 - 5043*x^8 + 3658*x^6 + 378*x^4 - 171*x^2 + 9) + 2*sqrt(3)*(x^11 + 9
17*x^9 - 40566*x^7 + 15786*x^5 - 2043*x^3 + 81*x))*(-3*x^2 + 1)^(1/3))*sqrt
((x^6 - 93*x^4 - 4*sqrt(6)*sqrt(2)*(x^5 + 13*x^3) - 117*x^2 + 2*(4*sqrt(6)*
sqrt(2)*x^3 + 3*x^4 + 18*x^2 - 9)*(-3*x^2 + 1)^(2/3) + (6*x^4 + sqrt(6)*sqr
t(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2 - 18)*(-3*x^2 + 1)^(1/3) + 9)/(x^6 - 9
*x^4 + 27*x^2 - 27)) + 12*(2*sqrt(6)*sqrt(3)*sqrt(2)*(35*x^9 - 4860*x^7 + 2
106*x^5 - 396*x^3 + 27*x) + 3*sqrt(3)*(x^10 + 589*x^8 + 3946*x^6 - 774*x^4
- 27*x^2 + 9))*(-3*x^2 + 1)^(2/3) + 3*sqrt(3)*(x^12 + 3150*x^10 + 77991*x^8
 + 4260*x^6 - 14337*x^4 + 2862*x^2 - 135) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(x^1
1 - 1591*x^9 + 42426*x^7 - 15102*x^5 + 1269*x^3 - 27*x) + 6*sqrt(3)*(27*x^1
0 + 2307*x^8 + 4574*x^6 - 2538*x^4 + 279*x^2 - 9))*(-3*x^2 + 1)^(1/3))/(x^1
2 - 4986*x^10 + 327519*x^8 - 159660*x^6 + 25839*x^4 - 2106*x^2 + 81)) - 1/2
88*sqrt(6)*sqrt(2)*log(12*(x^6 - 93*x^4 + 4*sqrt(6)*sqrt(2)*(x^5 + 13*x^3)
- 117*x^2 - 2*(4*sqrt(6)*sqrt(2)*x^3 - 3*x^4 - 18*x^2 + 9)*(-3*x^2 + 1)^(2/
3) + (6*x^4 - sqrt(6)*sqrt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2 - 18)*(-3*x^2
 + 1)^(1/3) + 9)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/288*sqrt(6)*sqrt(2)*log(1
2*(x^6 - 93*x^4 - 4*sqrt(6)*sqrt(2)*(x^5 + 13*x^3) - 117*x^2 + 2*(4*sqrt(6)
*sqrt(2)*x^3 + 3*x^4 + 18*x^2 - 9)*(-3*x^2 + 1)^(2/3) + (6*x^4 + sqrt(6)*sq
rt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2 - 18)*(-3*x^2 + 1)^(1/3) + 9)/(x^6 -
9*x^4 + 27*x^2 - 27)) + 1/72*sqrt(3)*log(-(x^12 + 2598*x^10 + 55143*x^8 + 1
14228*x^6 - 22113*x^4 - 7290*x^2 + 8*(3*x^10 + 576*x^8 + 5598*x^6 + 5832*x^
4 - 729*x^2 - sqrt(3)*(41*x^9 + 1368*x^7 + 4482*x^5 + 864*x^3 - 243*x))*(-3
*x^2 + 1)^(2/3) - 4*sqrt(3)*(25*x^11 + 2359*x^9 + 15426*x^7 + 6966*x^5 - 43
47*x^3 + 243*x) - 4*(84*x^10 + 4536*x^8 + 20880*x^6 + 5832*x^4 - 2916*x^2 -
 sqrt(3)*(x^11 + 521*x^9 + 7362*x^7 + 10746*x^5 - 1971*x^3 - 243*x))*(-3*x^
2 + 1)^(1/3) + 729)/(x^12 - 18*x^10 + 135*x^8 - 540*x^6 + 1215*x^4 - 1458*x
^2 + 729))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^2\sqrt[3]{1-3x^2}-3\sqrt[3]{1-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+1)**(1/3)/(-x**2+3),x)
```

```
[Out] -Integral(1/(x**2*(1 - 3*x**2)**(1/3) - 3*(1 - 3*x**2)**(1/3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{(x^2-3)\,(1-3\,x^2)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^2 - 3)*(1 - 3*x^2)^(1/3)),x)
```

```
[Out] -int(1/((x^2 - 3)*(1 - 3*x^2)^(1/3)), x)
```

3.78 $\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx$

Optimal. Leaf size=81

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+3x^2}\right)^2}{3\sqrt{3}\,x}\right)}{4\sqrt{3}}-\frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{1+3x^2}}{x}\right)$$

```
[Out] -1/4*arctanh((1-(3*x^2+1)^(1/3))/x)+1/12*arctan(1/3*x*3^(1/2))*3^(1/2)+1/12
*arctan(1/9*(1-(3*x^2+1)^(1/3))^2/x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {403}

$$\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{3x^2+1}\right)^2}{3\sqrt{3}\,x}\right)}{4\sqrt{3}}-\frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{3x^2+1}}{x}\right)+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

```
[Out] ArcTan[x/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 - (1 + 3*x^2)^(1/3))^2/(3*Sqrt[3]
*x)]/(4*Sqrt[3]) - ArcTanh[(1 - (1 + 3*x^2)^(1/3))/x]/4
```

Rule 403

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx=\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+3x^2}\right)^2}{3\sqrt{3}\,x}\right)}{4\sqrt{3}}-\frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{1+3x^2}}{x}\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 2.69, size = 126, normalized size = 1.56

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-3x^2,-\frac{x^2}{3}\right)}{(3+x^2)\sqrt[3]{1+3x^2}\left(-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-3x^2,-\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-3x^2,-\frac{x^2}{3}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-3x^2,-\frac{x^2}{3}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -1/3*x^2])/((3 + x^2)*(1 + 3*x^2)^
(1/3)*(-9*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/
2, 1/3, 2, 5/2, -3*x^2, -1/3*x^2] + 3*AppellF1[3/2, 4/3, 1, 5/2, -3*x^2, -1
/3*x^2])))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.18, size = 239, normalized size = 2.95

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+3\right)\left(3x^2+1\right)^{\frac{1}{3}}x-\mathrm{RootOf}\left(\_Z^2+3\right)x^2+4\left(3x^2+1\right)^{\frac{2}{3}}+2\,\mathrm{RootOf}\left(\_Z^2+3\right)\left(3x^2+1\right)^{\frac{1}{3}}-2\left(3x^2+\right.}{x^2+3}\right.}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+3)/(3*x^2+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*RootOf(_Z^2+3)*ln(-(2*RootOf(_Z^2+3)*(3*x^2+1)^(1/3)*x-RootOf(_Z^2+3)
*x^2+4*(3*x^2+1)^(2/3)+2*RootOf(_Z^2+3)*(3*x^2+1)^(1/3)-2*(3*x^2+1)^(1/3)*x
-4*RootOf(_Z^2+3)*x-x^2-2*(3*x^2+1)^(1/3)+RootOf(_Z^2+3)-4*x+1)/(x^2+3))+1/
8*ln(-(2*(3*x^2+1)^(2/3)+2*(3*x^2+1)^(1/3)*x+x^2+2*(3*x^2+1)^(1/3)+4*x-1)/(
x^2+3))-1/24*ln(-(2*(3*x^2+1)^(2/3)+2*(3*x^2+1)^(1/3)*x+x^2+2*(3*x^2+1)^(1/
3)+4*x-1)/(x^2+3))*RootOf(_Z^2+3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(59) = 118$.
time = 0.96, size = 345, normalized size = 4.26

$$\frac{1}{36}\sqrt{3}\arctan\left(\frac{4\sqrt{3}(3x^4-10x^3-36x^2+18x+9)(3x^2+1)^{\frac{2}{3}}-4\sqrt{3}(x^5+15x^4-26x^3-54x^2+9x-9)(3x^2+1)^{\frac{1}{3}}+\sqrt{3}(x^6-2x^5-105x^4-28x^3+63x^2+126x+9)}{x^6+126x^5-225x^4-828x^3-81x^2-162x+81}\right)-\frac{1}{36}\sqrt{3}\arctan\left(\frac{2\left(2\sqrt{3}(23x^3+9x)(3x^2+1)^{\frac{2}{3}}+\sqrt{3}(x^5-80x^3-9x)(3x^2+1)^{\frac{1}{3}}+\sqrt{3}(11x^5+10x^3-9x)\right)}{x^6-657x^4-189x^2-27}\right)+\frac{1}{24}\log\left(\frac{x^6+108x^5+549x^4+360x^3+99x^2+6(3x^4+32x^3+42x^2+3)(3x^2+1)^{\frac{2}{3}}+6(x^5+27x^4+70x^3+18x^2+9x+3)(3x^2+1)^{\frac{1}{3}}+108x-9}{x^6+9x^4+27x^2+27}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(3)*arctan((4*sqrt(3)*(3*x^4 - 10*x^3 - 36*x^2 + 18*x + 9)*(3*x^2
+ 1)^(2/3) - 4*sqrt(3)*(x^5 + 15*x^4 - 26*x^3 - 54*x^2 + 9*x - 9)*(3*x^2 +
1)^(1/3) + sqrt(3)*(x^6 - 2*x^5 - 105*x^4 - 28*x^3 + 63*x^2 + 126*x + 9))/(
x^6 + 126*x^5 - 225*x^4 - 828*x^3 - 81*x^2 - 162*x + 81)) - 1/36*sqrt(3)*ar
ctan(2*(2*sqrt(3)*(23*x^3 + 9*x)*(3*x^2 + 1)^(2/3) + sqrt(3)*(x^5 - 80*x^3
- 9*x)*(3*x^2 + 1)^(1/3) + sqrt(3)*(11*x^5 + 10*x^3 - 9*x))/(x^6 - 657*x^4
- 189*x^2 - 27)) + 1/24*log((x^6 + 108*x^5 + 549*x^4 + 360*x^3 + 99*x^2 + 6
*(3*x^4 + 32*x^3 + 42*x^2 + 3)*(3*x^2 + 1)^(2/3) + 6*(x^5 + 27*x^4 + 70*x^3
 + 18*x^2 + 9*x + 3)*(3*x^2 + 1)^(1/3) + 108*x - 9)/(x^6 + 9*x^4 + 27*x^2 +
 27))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2+3)\sqrt[3]{3x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+3)/(3*x**2+1)**(1/3),x)
```

```
[Out] Integral(1/((x**2 + 3)*(3*x**2 + 1)**(1/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^2+3\right)\,{\left(3\,x^2+1\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 3)*(3*x^2 + 1)^(1/3)),x)
[Out] int(1/((x^2 + 3)*(3*x^2 + 1)^(1/3)), x)
```

3.79 $\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

Optimal. Leaf size=113

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}+\frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\,\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/12*arctanh(x)*2^(1/3)+1/4*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)+
1/12*arctan(3^(1/2)/x)*2^(1/3)*3^(1/2)+1/12*arctan((1-2^(1/3)*(-x^2+1)^(1/3
))*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {402}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\,\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x
^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1
+ 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}+\frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\ldots}\right)}{2\ 2^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 2.64, size = 118, normalized size = 1.04

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\,(3+x^2)\left(-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)
*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3,
2, 5/2, x^2, -1/3*x^2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((3 + x^2)*(1 - x^2)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 11.55, size = 938, normalized size = 8.30

method	result	size
trager	Expression too large to display	938

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*RootOf(_Z^6+108)*ln(-(-72*RootOf(_Z^6+108)^4*x^5+225*RootOf(_Z^6+108)^
4*x^4+72*RootOf(_Z^6+108)^4*x^3+1296*RootOf(_Z^6+108)*x^5-4050*RootOf(_Z^6+
108)*x^4-1296*RootOf(_Z^6+108)*x^3+3402*RootOf(_Z^6+108)*x^2-189*x^2*RootOf
(_Z^6+108)^4-1296*(-x^2+1)^(2/3)*x^4+9072*(-x^2+1)^(2/3)*x^3-3888*(-x^2+1)^
(2/3)*x^2-3888*(-x^2+1)^(2/3)*x+6*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^5-108
*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^4+RootOf(_Z^6+108)^4*x^6-18*RootOf(_Z^
6+108)*x^6+144*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^3+108*RootOf(_Z^6+108)^5
*(-x^2+1)^(1/3)*x^2-36*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^5-54*RootOf(_Z^6
+108)^5*(-x^2+1)^(1/3)*x+648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^4-864*Root
Of(_Z^6+108)^2*(-x^2+1)^(1/3)*x^3-648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^2
+324*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x-486*RootOf(_Z^6+108)+27*RootOf(_Z^
6+108)^4)/(x^2+3)^3)+1/432*ln((RootOf(_Z^6+108)^4*x^6-72*RootOf(_Z^6+108)^4
```

```
*x^5+225*RootOf(_Z^6+108)^4*x^4-36*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^5+72
*RootOf(_Z^6+108)^4*x^3+648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^4-189*x^2*R
ootOf(_Z^6+108)^4-864*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^3+648*(-x^2+1)^(2
/3)*x^4-648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^2-4536*(-x^2+1)^(2/3)*x^3+2
7*RootOf(_Z^6+108)^4+324*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x+1944*(-x^2+1)^
(2/3)*x^2+1944*(-x^2+1)^(2/3)*x)/(x^2+3)^3)*RootOf(_Z^6+108)^4+1/72*ln((Roo
tOf(_Z^6+108)^4*x^6-72*RootOf(_Z^6+108)^4*x^5+225*RootOf(_Z^6+108)^4*x^4-36
*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^5+72*RootOf(_Z^6+108)^4*x^3+648*RootOf
(_Z^6+108)^2*(-x^2+1)^(1/3)*x^4-189*x^2*RootOf(_Z^6+108)^4-864*RootOf(_Z^6+
108)^2*(-x^2+1)^(1/3)*x^3+648*(-x^2+1)^(2/3)*x^4-648*RootOf(_Z^6+108)^2*(-x
^2+1)^(1/3)*x^2-4536*(-x^2+1)^(2/3)*x^3+27*RootOf(_Z^6+108)^4+324*RootOf(_Z
^6+108)^2*(-x^2+1)^(1/3)*x+1944*(-x^2+1)^(2/3)*x^2+1944*(-x^2+1)^(2/3)*x)/(
x^2+3)^3)*RootOf(_Z^6+108)
```

Maximā [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1943 vs. 2(81) = 162.
time = 0.80, size = 1943, normalized size = 17.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/20736*432^(5/6)*sqrt(3)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 +
27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x)
+ 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 -
 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/20736*432
^(5/6)*sqrt(3)*log(2592*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) + 144*432
^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) + 216*2^(1/3)
*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 - 18*x^2 - 9*x
)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3
)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(
3)*(x^5 - x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^
2))*(-x^2 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)
^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3)*log(2592*(
6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 - x^3
) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1
```

```
)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6
+ 9*x^4 + 27*x^2 + 27)) - 1/1296*432^(5/6)*arctan(1/36*(432^(5/6)*(x^5 - 18
*x^3 + 9*x)*(-x^2 + 1)^(1/3) + sqrt(3)*2^(1/3)*(432^(5/6)*(x^4 + 9*x^2)*(-x
^2 + 1)^(2/3) - 288*sqrt(3)*(2*x^4 - 3*x^2)*(-x^2 + 1)^(1/3) + 6*432^(1/6)*
(x^6 + 141*x^4 - 153*x^2 + 27)) - 648*432^(1/6)*(3*x^3 - x)*(-x^2 + 1)^(2/3
) - 72*sqrt(3)*(7*x^5 + 6*x^3 - 9*x))/(x^6 - 225*x^4 + 243*x^2 - 27)) - 1/2
592*432^(5/6)*arctan(-1/18*(sqrt(2)*(18*sqrt(3)*2^(2/3)*(29*x^11 + 879*x^9
- 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) - 2*(-x^2 + 1)^(2/3)*(432^(5/6)
*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) - 216*sqrt(3)*2^(1/3)*(31*x^9 - 297*
x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)*(sqrt(3)*(x^11 + 1167*x^9 - 1
3158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) - 8*sqrt(3)*(13*x^10 - 6*x^8 - 140
4*x^6 + 1350*x^4 - 81*x^2)) - 3*432^(1/6)*(x^12 + 7620*x^10 - 92115*x^8 + 1
69776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sqrt((6*2^(2/3)*(x^6 + 225*x^4 -
 189*x^2 + 27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*
x^3 - 3*x) + 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4
 + 24*x^3 - 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) -
216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^6 + 2808*x^4 - 243*x^2) - 3*43
2^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^3 + 27*x))*(-x^2 + 1)^(2/3) -
18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 42660*x^6 + 58239*x^4 - 14094*x^2
 + 729) + 144*sqrt(3)*(11*x^11 - 807*x^9 + 4518*x^7 - 5238*x^5 + 3807*x^3 -
 243*x) - (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 - 1215*x^9 + 11754*x^7 - 21006*
x^5 + 5589*x^3 - 243*x) - 432*sqrt(3)*2^(1/3)*(13*x^10 - 120*x^8 + 1242*x^6
 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 110727*x^8 - 301860*x^6 + 18783
9*x^4 - 21870*x^2 + 729)) - 1/2592*432^(5/6)*arctan(1/18*(sqrt(2)*(18*sqrt(
3)*2^(2/3)*(29*x^11 + 879*x^9 - 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) +
 2*(-x^2 + 1)^(2/3)*(432^(5/6)*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) + 216*
sqrt(3)*2^(1/3)*(31*x^9 - 297*x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)
*(sqrt(3)*(x^11 + 1167*x^9 - 13158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) + 8*
sqrt(3)*(13*x^10 - 6*x^8 - 1404*x^6 + 1350*x^4 - 81*x^2)) + 3*432^(1/6)*(x^
12 + 7620*x^10 - 92115*x^8 + 169776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sq
rt((6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 -
 x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2
 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(
x^6 + 9*x^4 + 27*x^2 + 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^
6 + 2808*x^4 - 243*x^2) + 3*432^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^
3 + 27*x))*(-x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 426
60*x^6 + 58239*x^4 - 14094*x^2 + 729) - 144*sqrt(3)*(11*x^11 - 807*x^9 + 45
18*x^7 - 5238*x^5 + 3807*x^3 - 243*x) + (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 -
 1215*x^9 + 11754*x^7 - 21006*x^5 + 5589*x^3 - 243*x) + 432*sqrt(3)*2^(1/3)
*(13*x^10 - 120*x^8 + 1242*x^6 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 1
10727*x^8 - 301860*x^6 + 187839*x^4 - 21870*x^2 + 729))
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
[Out] Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(1-x^2)^{1/3}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/3)*(x^2 + 3)),x)
[Out] int(1/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.80 $\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

<u>**Optimal**</u>. Leaf size=109

$$-\frac{\tan^{-1}(x)}{6\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\ \sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\ \sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

```
[Out] -1/12*arctan(x)*2^(1/3)+1/4*arctan(x/(1+2^(1/3)*(x^2+1)^(1/3)))*2^(1/3)-1/1
2*arctanh(3^(1/2)/x)*2^(1/3)*3^(1/2)-1/12*arctanh((1-2^(1/3)*(x^2+1)^(1/3))
*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

<u>**Rubi**</u> [A]
time = 0.01, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {401}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\ \sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\ \sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tan^{-1}(x)}{6\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -1/6*ArcTan[x]/2^(2/3) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/(2*2^(2/3)
) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*
(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])
```

<u>Rule 401</u>

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTanh[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (-Simp[q*(ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] + Simp[q*(ArcTan[q*x]/(6*2^(2/3)*a^(1/3)
*d)), x] + Simp[q*(ArcTanh[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

<u>Rubi steps</u>

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx=-\frac{\tan^{-1}(x)}{6\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\ \sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\right.}{}\right)}{2\ 2^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 2.74, size = 124, normalized size = 1.14

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)}{(-3+x^2)\sqrt[3]{1+x^2}\left(9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,\frac{x^2}{3}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)*
(9*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5
/2, -x^2, x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((3 - x^2)*(1 + x^2)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(-x^2+3)(x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

```
[Out] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1685 vs. $2(77) = 154$.

time = 0.78, size = 1685, normalized size = 15.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2592*432^(5/6)*sqrt(3)*arctan(-1/54*(2592*x^11 - 393984*x^9 - 699840*x^7
- 373248*x^5 - 69984*x^3 - sqrt(6)*(18*sqrt(3)*2^(2/3)*(19*x^11 + 111*x^9 +
 6030*x^7 + 7182*x^5 + 2511*x^3 + 243*x) + 3*432^(1/6)*sqrt(3)*(x^12 + 924*
x^10 - 33363*x^8 - 60912*x^6 - 36693*x^4 - 8748*x^2 - 729) + (432^(5/6)*sqr
t(3)*(x^10 - 78*x^8 - 720*x^6 - 594*x^4 - 81*x^2) + 432*sqrt(3)*2^(1/3)*(13
*x^9 - 177*x^7 - 153*x^5 - 27*x^3))*(x^2 + 1)^(2/3) + 36*(96*x^10 - 4032*x^
8 - 2592*x^6 + sqrt(3)*(x^11 + 369*x^9 - 3654*x^7 - 5454*x^5 - 2187*x^3 - 2
43*x))*(x^2 + 1)^(1/3))*sqrt((2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) + (x^
2 + 1)^(2/3)*(432^(5/6)*(x^3 + x) + 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 -
18*x^2 + sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) - 8*432^(1/6)*(x^5 + 18*x^3 +
 9*x))/(x^6 - 9*x^4 + 27*x^2 - 27)) + 216*(sqrt(3)*2^(2/3)*(x^10 + 276*x^8
+ 1206*x^6 + 756*x^4 + 81*x^2) + 432^(1/6)*sqrt(3)*(31*x^9 - 1620*x^7 - 207
0*x^5 - 756*x^3 - 81*x))*(x^2 + 1)^(2/3) + 18*sqrt(3)*(x^12 + 1422*x^10 + 2
1447*x^8 + 27108*x^6 + 16767*x^4 + 6318*x^2 + 729) + (432^(5/6)*sqrt(3)*(x^
11 - 681*x^9 + 4338*x^7 + 6102*x^5 + 2349*x^3 + 243*x) + 3888*sqrt(3)*2^(1/
3)*(x^10 + 44*x^8 + 94*x^6 + 60*x^4 + 9*x^2))*(x^2 + 1)^(1/3))/(x^12 - 2178
*x^10 + 46791*x^8 + 83268*x^6 + 47871*x^4 + 10206*x^2 + 729)) + 1/2592*432^
(5/6)*sqrt(3)*arctan(-1/54*(2592*x^11 - 393984*x^9 - 699840*x^7 - 373248*x^
5 - 69984*x^3 + sqrt(6)*(18*sqrt(3)*2^(2/3)*(19*x^11 + 111*x^9 + 6030*x^7 +
 7182*x^5 + 2511*x^3 + 243*x) - 3*432^(1/6)*sqrt(3)*(x^12 + 924*x^10 - 3336
3*x^8 - 60912*x^6 - 36693*x^4 - 8748*x^2 - 729) - (432^(5/6)*sqrt(3)*(x^10
- 78*x^8 - 720*x^6 - 594*x^4 - 81*x^2) - 432*sqrt(3)*2^(1/3)*(13*x^9 - 177*
x^7 - 153*x^5 - 27*x^3))*(x^2 + 1)^(2/3) - 36*(96*x^10 - 4032*x^8 - 2592*x^
6 - sqrt(3)*(x^11 + 369*x^9 - 3654*x^7 - 5454*x^5 - 2187*x^3 - 243*x))*(x^2
 + 1)^(1/3))*sqrt((2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) - (x^2 + 1)^(2/3
)*(432^(5/6)*(x^3 + x) - 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^2 - sq
rt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) + 8*432^(1/6)*(x^5 + 18*x^3 + 9*x))/(x^6
 - 9*x^4 + 27*x^2 - 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 276*x^8 + 1206*x^6
+ 756*x^4 + 81*x^2) - 432^(1/6)*sqrt(3)*(31*x^9 - 1620*x^7 - 2070*x^5 - 756
*x^3 - 81*x))*(x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 + 1422*x^10 + 21447*x^8 +
27108*x^6 + 16767*x^4 + 6318*x^2 + 729) + (432^(5/6)*sqrt(3)*(x^11 - 681*x^
9 + 4338*x^7 + 6102*x^5 + 2349*x^3 + 243*x) - 3888*sqrt(3)*2^(1/3)*(x^10 +
44*x^8 + 94*x^6 + 60*x^4 + 9*x^2))*(x^2 + 1)^(1/3))/(x^12 - 2178*x^10 + 467
91*x^8 + 83268*x^6 + 47871*x^4 + 10206*x^2 + 729)) + 1/5184*432^(5/6)*log(-
(432^(5/6)*(x^6 + 69*x^4 + 63*x^2 + 27) + 864*(9*x^3 + sqrt(3)*(x^4 + 9*x^2
) + 9*x)*(x^2 + 1)^(2/3) + 432*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) + 432*(x^2 +
1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 + 9*x) + 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 -
```

```
 9*x^4 + 27*x^2 - 27)) - 1/5184*432^(5/6)*log((432^(5/6)*(x^6 + 69*x^4 + 63
*x^2 + 27) - 864*(9*x^3 - sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3) - 43
2*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) - 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x
^3 + 9*x) - 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 - 9*x^4 + 27*x^2 - 27)) - 1/10
368*432^(5/6)*log(31104*(2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) + (x^2 + 1
)^(2/3)*(432^(5/6)*(x^3 + x) + 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^
2 + sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) - 8*432^(1/6)*(x^5 + 18*x^3 + 9*x)
)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/10368*432^(5/6)*log(31104*(2*2^(2/3)*(x^
6 - 57*x^4 - 117*x^2 - 27) - (x^2 + 1)^(2/3)*(432^(5/6)*(x^3 + x) - 24*2^(1
/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^2 - sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/
3) + 8*432^(1/6)*(x^5 + 18*x^3 + 9*x))/(x^6 - 9*x^4 + 27*x^2 - 27))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+3)/(x**2+1)**(1/3),x)
[Out] -Integral(1/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{{\left(x^2+1\right)}^{1/3}\,\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x)
[Out] -int(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.81 $\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)x^2+x^3}}\,dx$

Optimal. Leaf size=87

$$-\frac{2\sqrt{x}\sqrt{a^2-(1+a^2)x+x^2}\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{a^2-(1+a^2)x+x^2}}\right)}{(1-a)\sqrt{a^2x-(1+a^2)x^2+x^3}}$$

```
[Out] -2*arctan((1-a)*x^(1/2)/(a^2-(a^2+1)*x+x^2)^(1/2))*x^(1/2)*(a^2-(a^2+1)*x+x
^2)^(1/2)/(1-a)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2081, 6865, 1712, 211}

$$-\frac{2\sqrt{x}\sqrt{-(a^2+1)x+a^2+x^2}\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{-(a^2+1)x+a^2+x^2}}\right)}{(1-a)\sqrt{-(a^2+1)x^2+a^2x+x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x)/((-a + x)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a^2 - (1 + a^2)*x + x^2]*ArcTan[((1 - a)*Sqrt[x])/Sqrt[a^2
 - (1 + a^2)*x + x^2]])/((1 - a)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{a^2-(1+a^2)x+x^2}\right)\int\frac{a+x}{\sqrt{x}(-a+x)\sqrt{a^2-(1+a^2)x+}}}{\sqrt{a^2x-(1+a^2)x^2+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{a^2-(1+a^2)x+x^2}\right)\text{Subst}\left(\int\frac{a+x^2}{(-a+x^2)\sqrt{a^2+(-1-}}\right.}{\sqrt{a^2x-(1+a^2)x^2+x^3}}$$

$$= \frac{\left(2a\sqrt{x}\sqrt{a^2-(1+a^2)x+x^2}\right)\text{Subst}\left(\int\frac{1}{-a-(-2a^2-a(-1-a^2))x^2}\right.}{\sqrt{a^2x-(1+a^2)x^2+x^3}}$$

$$= -\frac{2\sqrt{x}\sqrt{a^2-(1+a^2)x+x^2}\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{a^2-(1+a^2)x+x^2}}\right)}{(1-a)\sqrt{a^2x-(1+a^2)x^2+x^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 18.29, size = 159, normalized size = 1.83

$$-\frac{2i(a^2-x)^{3/2}\sqrt{\frac{-1+x}{-a^2+x}}\sqrt{\frac{x}{-a^2+x}}\left((1+a)F\left(i\sinh^{-1}\left(\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right)|1-\frac{1}{a^2}\right)-2\Pi\left(\frac{-1+a}{a};i\sinh^{-1}\left(\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right)|1-\frac{1}{a^2}\right)\right)}{(-1+a)\sqrt{-a^2}\sqrt{(-1+x)x(-a^2+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x)/((-a + x)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3]),x]
```

```
[Out] ((-2*I)*(a^2 - x)^(3/2)*Sqrt[(-1 + x)/(-a^2 + x)]*Sqrt[x/(-a^2 + x)]*((1 +
a)*EllipticF[I*ArcSinh[Sqrt[-a^2]/Sqrt[a^2 - x]], 1 - a^(-2)] - 2*EllipticP
i[(-1 + a)/a, I*ArcSinh[Sqrt[-a^2]/Sqrt[a^2 - x]], 1 - a^(-2)]))/((-1 + a)*
Sqrt[-a^2]*Sqrt[(-1 + x)*x*(-a^2 + x)])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x + a)/((x - a)*Sqrt[x^3 - x^2*(a^2 + 1) + a^2*x]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.10, size = 206, normalized size = 2.37

method	result
default	$-\frac{2a^2\sqrt{-\frac{-a^2+x}{a^2}}\sqrt{\frac{-1+x}{a^2-1}}\sqrt{\frac{x}{a^2}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{-a^2+x}{a^2}},\sqrt{\frac{a^2}{a^2-1}}\right)}{\sqrt{-a^2x^2+a^2x+x^3-x^2}}-\frac{4a^3\sqrt{-\frac{-a^2+x}{a^2}}\sqrt{\frac{-1+x}{a^2-1}}\sqrt{\frac{x}{a^2}}\,\mathrm{Ellipti}}{\sqrt{-a^2x^2+a^2x+}}$
elliptic	$-\frac{2a^2\sqrt{-\frac{-a^2+x}{a^2}}\sqrt{\frac{-1+x}{a^2-1}}\sqrt{\frac{x}{a^2}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{-a^2+x}{a^2}},\sqrt{\frac{a^2}{a^2-1}}\right)}{\sqrt{-a^2x^2+a^2x+x^3-x^2}}-\frac{4a^3\sqrt{-\frac{-a^2+x}{a^2}}\sqrt{\frac{-1+x}{a^2-1}}\sqrt{\frac{x}{a^2}}\,\mathrm{Ellipti}}{\sqrt{-a^2x^2+a^2x+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^2*(-(-a^2+x)/a^2)^(1/2)*((-1+x)/(a^2-1))^(1/2)*(x/a^2)^(1/2)/(-a^2*x^2
+a^2*x+x^3-x^2)^(1/2)*EllipticF((-(-a^2+x)/a^2)^(1/2),(a^2/(a^2-1))^(1/2))-
4*a^3*(-(-a^2+x)/a^2)^(1/2)*((-1+x)/(a^2-1))^(1/2)*(x/a^2)^(1/2)/(-a^2*x^2+
a^2*x+x^3-x^2)^(1/2)/(a^2-a)*EllipticPi((-(-a^2+x)/a^2)^(1/2),a^2/(a^2-a),(
a^2/(a^2-1))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)), x)
```

Fricas [A]
time = 0.34, size = 85, normalized size = 0.98

$$\frac{\arctan\left(\frac{\sqrt{a^2x-(a^2+1)x^2+x^3}\,(a^2-2\,(a^2-a+1)x+x^2)}{2\,((a-1)x^3-(a^3-a^2+a-1)x^2+(a^3-a^2)x)}\right)}{a-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(1/2*sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a^2 - 2*(a^2 - a + 1)*x + x^2
)/((a - 1)*x^3 - (a^3 - a^2 + a - 1)*x^2 + (a^3 - a^2)*x))/(a - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+x}{\sqrt{x\left(-a^{2}+x\right)\left(x-1\right)}\,\left(-a+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a**2*x-(a**2+1)*x**2+x**3)**(1/2),x)
```

```
[Out] Integral((a + x)/(sqrt(x*(-a**2 + x)*(x - 1))*(-a + x)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.17, size = 217, normalized size = 2.49

$$\frac{4\,a\,\left(a^2-1\right)\,\sqrt{\frac{x}{a^2}}\,\sqrt{\frac{x-1}{a^2-1}}\,\sqrt{-\frac{x-a^2}{a^2-1}}\,\Pi\left(-\frac{a^2-1}{a-a^2};\mathrm{asin}\left(\sqrt{-\frac{x-a^2}{a^2-1}}\right)\middle|\frac{a^2-1}{a^2}\right)}{\left(a-a^2\right)\,\sqrt{a^2\,x-x^2\,\left(a^2+1\right)+x^3}}-\frac{2\,\left(a^2-1\right)\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-a^2}{a^2-1}}\right)\middle|\frac{a^2-1}{a^2}\right)\,\sqrt{\frac{x}{a^2}}\,\sqrt{\frac{x-1}{a^2-1}}\,\sqrt{-\frac{x-a^2}{a^2-1}}}{\sqrt{a^2\,x-x^2\,\left(a^2+1\right)+x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + x)/((a - x)*(a^2*x - x^2*(a^2 + 1) + x^3)^(1/2)),x)
```

```
[Out] (4*a*(a^2 - 1)*(x/a^2)^(1/2)*((x - 1)/(a^2 - 1))^(1/2)*(-(x - a^2)/(a^2 - 1
))^(1/2)*ellipticPi(-(a^2 - 1)/(a - a^2), asin((-(x - a^2)/(a^2 - 1))^(1/2)
), (a^2 - 1)/a^2))/((a - a^2)*(a^2*x - x^2*(a^2 + 1) + x^3)^(1/2)) - (2*(a^
2 - 1)*ellipticF(asin((-(x - a^2)/(a^2 - 1))^(1/2)), (a^2 - 1)/a^2)*(x/a^2)
^(1/2)*((x - 1)/(a^2 - 1))^(1/2)*(-(x - a^2)/(a^2 - 1))^(1/2))/(a^2*x - x^2
*(a^2 + 1) + x^3)^(1/2)
```

3.82 $\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)x^2+x^3}}\,dx$

Optimal. Leaf size=1

$$0$$

```
[Out] 0
```

Rubi [C] Result contains higher order function than in optimal. Order 4 vs. order 1 in optimal.
time = 1.07, antiderivative size = 529, normalized size of antiderivative = 529.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2081, 6865, 1722, 1117, 1720}

$$\frac{2(1-a)\sqrt{x}\sqrt{-(-a^2+2a+1)x+(2-a)a+x^2}\tan^{-1}\left(\frac{\sqrt{-a^2+2a-1}\sqrt{x}}{\sqrt{-(-a^2+2a+1)x+(2-a)a+x^2}}\right)}{a\sqrt{-a^2+2a-1}\sqrt{-(-a^2+2a+1)x^2+(2-a)ax+x^3}}+\frac{((2-a)a)^{3/4}\sqrt{x}\left(\frac{x}{\sqrt{(2-a)a}}+1\right)\sqrt{\frac{-(-a^2+2a+1)x+(2-a)a+x^2}{(2-a)a\left(\frac{x}{\sqrt{(2-a)a}}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{(2-a)a}}\right)|\frac{1}{4}\left(\frac{-a^2+2a+1}{\sqrt{(2-a)a}}+2\right)\right)}{a\sqrt{-(-a^2+2a+1)x^2+(2-a)ax+x^3}}+\frac{(2-a)\left(1-\sqrt{(2-a)a}\right)\sqrt{x}\left(\frac{x}{\sqrt{(2-a)a}}+1\right)\sqrt{\frac{-(-a^2+2a+1)x+(2-a)a+x^2}{(2-a)a\left(\frac{x}{\sqrt{(2-a)a}}+1\right)^2}}\Pi\left(\frac{\left(\sqrt{2-a}+\sqrt{a}\right)^2}{4\sqrt{(2-a)a}};2\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{(2-a)a}}\right)|\frac{1}{4}\left(\frac{-a^2+2a+1}{\sqrt{(2-a)a}}+2\right)\right)}{((2-a)a)^{3/4}\sqrt{-(-a^2+2a+1)x^2+(2-a)ax+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-2 + a + x)/((-a + x)*Sqrt[(2 - a)*a*x + (-1 - 2*a + a^2)*x^2 + x^3]),
x]
```

```
[Out] (2*(1 - a)*Sqrt[x]*Sqrt[(2 - a)*a - (1 + 2*a - a^2)*x + x^2]*ArcTan[(Sqrt[-
1 + 2*a - a^2]*Sqrt[x])/Sqrt[(2 - a)*a - (1 + 2*a - a^2)*x + x^2]])/(a*Sqrt
[-1 + 2*a - a^2]*Sqrt[(2 - a)*a*x - (1 + 2*a - a^2)*x^2 + x^3]) + (((2 - a)
*a)^(3/4)*Sqrt[x]*(1 + x/Sqrt[(2 - a)*a])*Sqrt[((2 - a)*a - (1 + 2*a - a^2)
*x + x^2)/((2 - a)*a*(1 + x/Sqrt[(2 - a)*a])^2)]*EllipticF[2*ArcTan[Sqrt[x]
/((2 - a)*a)^(1/4)], (2 + (1 + 2*a - a^2)/Sqrt[(2 - a)*a])/4])/(a*Sqrt[(2 -
 a)*a*x - (1 + 2*a - a^2)*x^2 + x^3]) + ((2 - a)*(1 - Sqrt[(2 - a)*a])*Sqrt
[x]*(1 + x/Sqrt[(2 - a)*a])*Sqrt[((2 - a)*a - (1 + 2*a - a^2)*x + x^2)/((2
- a)*a*(1 + x/Sqrt[(2 - a)*a])^2)]*EllipticPi[(Sqrt[2 - a] + Sqrt[a])^2/(4*
Sqrt[(2 - a)*a]), 2*ArcTan[Sqrt[x]/((2 - a)*a)^(1/4)], (2 + (1 + 2*a - a^2)
/Sqrt[(2 - a)*a])/4])/(((2 - a)*a)^(3/4)*Sqrt[(2 - a)*a*x - (1 + 2*a - a^2)
*x^2 + x^3])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
```

```
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\,\sqrt{(2-a)a+(-1-2a+a^2)x+x^2}\right)\int \frac{}{\sqrt{\;}}}{\sqrt{(2-a)ax+(-1-}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt{(2-a)a+(-1-2a+a^2)x+x^2}\right)\,\mathrm{Sub}}{\sqrt{(2-a)ax}}$$

$$= \frac{\left(2\sqrt{(2-a)a}\,\sqrt{x}\,\sqrt{(2-a)a+(-1-2a+a^2)x}\right.}{a\sqrt{(2-a}}$$

$$= \frac{2(1-a)\sqrt{x}\,\sqrt{(2-a)a-(1+2a-a^2)x+x^2}\;\mathrm{t}}{a\sqrt{-1+2a-a^2}\,\sqrt{(2-a)ax}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 1 in optimal.
time = 17.90, size = 127, normalized size = 127.00

$$\frac{2\sqrt{1+\frac{1}{-1+x}}\sqrt{1+\frac{(-1+a)^2}{-1+x}}(-1+x)^{3/2}\left(F\left(\sin^{-1}\left(\frac{\sqrt{-(-1+a)^2}}{\sqrt{-1+x}}\right)|\frac{1}{(-1+a)^2}\right)-2\Pi\left(\frac{1}{1-a};\sin^{-1}\left(\frac{\sqrt{-(-1+a)^2}}{\sqrt{-1+x}}\right)|\frac{1}{(-1+a)^2}\right)\right)}{\sqrt{-(-1+a)^2}\sqrt{(-1+x)x(-2a+a^2+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + a + x)/((-a + x)*Sqrt[(2 - a)*a*x + (-1 - 2*a + a^2)*x^2 +
x^3]),x]
```

```
[Out] (2*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (-1 + a)^2/(-1 + x)]*(-1 + x)^(3/2)*(El
lipticF[ArcSin[Sqrt[-(-1 + a)^2]/Sqrt[-1 + x]], (-1 + a)^(-2)] - 2*Elliptic
Pi[(1 - a)^(-1), ArcSin[Sqrt[-(-1 + a)^2]/Sqrt[-1 + x]], (-1 + a)^(-2)]))/(
Sqrt[-(-1 + a)^2]*Sqrt[(-1 + x)*x*(-2*a + a^2 + x)])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x + a - 2)/((x - a)*Sqrt[x^3 + x^2*(a^2 - 2*a - 1) + a*
x*(2 - a)]),x]')
```

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 1.
time = 0.10, size = 317, normalized size = 317.00

method	result
default	$\frac{2(a^2-2a)\sqrt{\frac{a^2-2a+x}{a^2-2a}}\sqrt{\frac{-1+x}{-a^2+2a-1}}\sqrt{\frac{x}{-a^2+2a}}\,\text{EllipticF}\left(\sqrt{\frac{a^2-2a+x}{a^2-2a}},\sqrt{\frac{-a^2+2a}{-a^2+2a-1}}\right)}{\sqrt{a^2x^2-a^2x-2a\,x^2+x^3+2ax-x^2}}-\frac{2(-2a+2)(a^2-2a)\sqrt{\frac{a^2-}{a^2}}}{}$
elliptic	$\frac{2(a^2-2a)\sqrt{\frac{a^2-2a+x}{a^2-2a}}\sqrt{\frac{-1+x}{-a^2+2a-1}}\sqrt{\frac{x}{-a^2+2a}}\,\text{EllipticF}\left(\sqrt{\frac{a^2-2a+x}{a^2-2a}},\sqrt{\frac{-a^2+2a}{-a^2+2a-1}}\right)}{\sqrt{a^2x^2-a^2x-2a\,x^2+x^3+2ax-x^2}}+\frac{2(2a-2)(a^2-2a)\sqrt{\frac{a^2-2a}{a^2-}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 2*(a^2-2*a)*((a^2-2*a+x)/(a^2-2*a))^(1/2)*((-1+x)/(-a^2+2*a-1))^(1/2)*(x/(-
a^2+2*a))^(1/2)/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^(1/2)*EllipticF(((a^2
-2*a+x)/(a^2-2*a))^(1/2),((-a^2+2*a)/(-a^2+2*a-1))^(1/2))-2*(-2*a+2)*(a^2-2
*a)*((a^2-2*a+x)/(a^2-2*a))^(1/2)*((-1+x)/(-a^2+2*a-1))^(1/2)*(x/(-a^2+2*a)
)^(1/2)/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^(1/2)/(-a^2+a)*EllipticPi(((a
^2-2*a+x)/(a^2-2*a))^(1/2),(-a^2+2*a)/(-a^2+a),((-a^2+2*a)/(-a^2+2*a-1))^(1
/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorith
m="maxima")
```

```
[Out] -integrate((a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a -
 x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.33, size = 70, normalized size = 70.00

$$\frac{\log\left(-\frac{a^2-2\,(a^2-a)x-x^2+2\,\sqrt{(a^2-2\,a-1)x^2+x^3-(a^2-2\,a)x}\,a}{a^2-2\,ax+x^2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorith
m="fricas")
```

```
[Out] log(-(a^2 - 2*(a^2 - a)*x - x^2 + 2*sqrt((a^2 - 2*a - 1)*x^2 + x^3 - (a^2 -
 2*a)*x)*a)/(a^2 - 2*a*x + x^2))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + x - 2}{\sqrt{x\left(x - 1\right)\left(a^{2} - 2a + x\right)}\,\left(-a + x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a**2-2*a-1)*x**2+x**3)**(1/2),x)
```

```
[Out] Integral((a + x - 2)/(sqrt(x*(x - 1)*(a**2 - 2*a + x))*(-a + x)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.48, size = 207, normalized size = 207.00

$$\frac{2\,\sqrt{\frac{x}{2\,a-a^2}}\,\sqrt{-\frac{x-1}{a^2-2\,a+1}}\,{\left(a-1\right)}^2\,\sqrt{\frac{a^2-2\,a+x}{a^2-2\,a+1}}\,\left(a\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{a^2-2\,a+x}{a^2-2\,a+1}}\right)\middle|-\frac{a^2-2\,a+1}{2\,a-a^2}\right)-2\,\Pi\left(-\frac{a^2-2\,a+1}{a-a^2};\mathrm{asin}\left(\sqrt{\frac{a^2-2\,a+x}{a^2-2\,a+1}}\right)\middle|-\frac{a^2-2\,a+1}{2\,a-a^2}\right)\right)}{a\,\sqrt{x^3+\left(a^2-2\,a-1\right)\,x^2+\left(2\,a-a^2\right)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + x - 2)/((a - x)*(x^3 - x^2*(2*a - a^2 + 1) - a*x*(a - 2))^(1/2)),
x)
```

```
[Out] (2*(x/(2*a - a^2))^(1/2)*(-(x - 1)/(a^2 - 2*a + 1))^(1/2)*(a - 1)^2*((x - 2
*a + a^2)/(a^2 - 2*a + 1))^(1/2)*(a*ellipticF(asin(((x - 2*a + a^2)/(a^2 -
2*a + 1))^(1/2)), -(a^2 - 2*a + 1)/(2*a - a^2)) - 2*ellipticPi(-(a^2 - 2*a
+ 1)/(a - a^2), asin(((x - 2*a + a^2)/(a^2 - 2*a + 1))^(1/2)), -(a^2 - 2*a
+ 1)/(2*a - a^2))))/(a*(x*(2*a - a^2) - x^2*(2*a - a^2 + 1) + x^3)^(1/2))
```

3.83 $\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)x^2+(-1+2a)x^3}}\,dx$

Optimal. Leaf size=46

$$\log\left(\frac{-a^2+2ax+x^2-2\left(x+\sqrt{(1-x)x(a^2+x-2ax)}\right)}{(a-x)^2}\right)$$

```
[Out] ln((-a^2+2*a*x+x^2-2*x-2*((1-x)*x*(a^2-2*a*x+x))^(1/2))/(a-x)^2)
```

Rubi [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.96, antiderivative size = 180, normalized size of antiderivative = 3.91, number of steps used = 7, number of rules used = 7, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.137$, Rules used = {2081, 6865, 1724, 1118, 430, 1234, 551}

$$\frac{4(1-a)\sqrt{1-x}\sqrt{x}\sqrt{\frac{(1-2a)x}{a^2}+1}\,\Pi\left(\frac{1}{a};\sin^{-1}\left(\sqrt{x}\right)|-\frac{1-2a}{a^2}\right)}{\sqrt{(-a^2-2a+1)x^2+a^2x-((1-2a)x^3)}}-\frac{2(1-2a)\sqrt{1-x}\sqrt{x}\sqrt{\frac{(1-2a)x}{a^2}+1}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|-\frac{1-2a}{a^2}\right)}{\sqrt{(-a^2-2a+1)x^2+a^2x-((1-2a)x^3)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2 + (-1 +
 2*a)*x^3]),x]
```

```
[Out] (-2*(1 - 2*a)*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 + ((1 - 2*a)*x)/a^2]*EllipticF[Arc
Sin[Sqrt[x]], -((1 - 2*a)/a^2)])/Sqrt[a^2*x + (1 - 2*a - a^2)*x^2 - (1 - 2*
a)*x^3] + (4*(1 - a)*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 + ((1 - 2*a)*x)/a^2]*Ellipt
icPi[a^(-1), ArcSin[Sqrt[x]], -((1 - 2*a)/a^2)])/Sqrt[a^2*x + (1 - 2*a - a^
2)*x^2 - (1 - 2*a)*x^3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1118

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2
*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 1234

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(
Sqrt[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/((d + e*x^2)*Sq
rt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1724

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> Dist[B/e, Int[1/Sqrt[a + b*x^2 + c*x^4], x],
 x] + Dist[(e*A - d*B)/e, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x],
x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)\,x^2+(-1+2a)x^3}}\,dx = \frac{\left(\sqrt{x}\,\sqrt{a^2-(-1+2a+a^2)\,x+(-1+2a)x^2}\right.}{\sqrt{a^2x-(-1+}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt{a^2-(-1+2a+a^2)\,x+(-1+2a)x^2}\right.}{\sqrt{a^2x-}}$$

$$= -\frac{\left(4(1-a)a\sqrt{x}\,\sqrt{a^2-(-1+2a+a^2)\,x+(-\right.}{\sqrt{}}$$

$$= -\frac{\left(4(1-a)a\sqrt{1-x}\,\sqrt{x}\,\sqrt{1+\frac{(1-2a)x}{a^2}}\,\sqrt{\right.}{}$$

$$= -\frac{2(1-2a)\sqrt{1-x}\,\sqrt{x}\,\sqrt{1+\frac{(1-2a)x}{a^2}}\,F\,(\mathrm{si}}{\sqrt{a^2x+(1-2a-a^2)\,x^2-(1-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 20.53, size = 133, normalized size = 2.89

$$\frac{2i(-1+x)^{3/2}\sqrt{\frac{x}{-1+x}}\sqrt{-\frac{a^2+x-2ax}{(-1+2a)(-1+x)}}\left(-F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{-1+x}}\right)|-\frac{(-1+a)^2}{-1+2a}\right)+2a\Pi\left(1-a;i\sinh^{-1}\left(\frac{1}{\sqrt{-1+x}}\right)|-\frac{(-1+a)^2}{-1+2a}\right)\right)}{\sqrt{-((-1+x)x\,(a^2+x-2ax))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2 +
 (-1 + 2*a)*x^3]),x]
```

```
[Out] ((2*I)*(-1 + x)^(3/2)*Sqrt[x/(-1 + x)]*Sqrt[-((a^2 + x - 2*a*x)/((-1 + 2*a)
*(-1 + x)))]*(-EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], -((-1 + a)^2/(-1 + 2*a)
)] + 2*a*EllipticPi[1 - a, I*ArcSinh[1/Sqrt[-1 + x]], -((-1 + a)^2/(-1 + 2*
a))]))/Sqrt[-((-1 + x)*x*(a^2 + x - 2*a*x))]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x*(2*a - 1) - a)/((x - a)*Sqrt[x^3*(2*a - 1) - x^2*(a^2
 + 2*a - 1) + a^2*x]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.10, size = 536, normalized size = 11.65

method	result
elliptic	$-\frac{2a^2\sqrt{-\frac{\left(x-\frac{a^2}{-1+2a}\right)(-1+2a)}{a^2}}\sqrt{\frac{-1+x}{\frac{a^2}{-1+2a}-1}}\sqrt{\frac{x(-1+2a)}{a^2}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{\left(x-\frac{a^2}{-1+2a}\right)(-1+2a)}{a^2}},\sqrt{\frac{a^2}{(-1+2a)\left(\frac{a^2}{-1+2a}\cdots\right.}}\right.}{\sqrt{-a^2x^2+2a\,x^3+a^2x-2a\,x^2-x^3+x^2}}$
default	$-\frac{4a^3\sqrt{-\frac{\left(x-\frac{a^2}{-1+2a}\right)(-1+2a)}{a^2}}\sqrt{\frac{-1+x}{\frac{a^2}{-1+2a}-1}}\sqrt{\frac{x(-1+2a)}{a^2}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{\left(x-\frac{a^2}{-1+2a}\right)(-1+2a)}{a^2}},\sqrt{\frac{a^2}{(-1+2a)\left(\frac{a^2}{-1+2a}\cdots\right.}}\right.}{(-1+2a)\sqrt{-a^2x^2+2a\,x^3+a^2x-2a\,x^2-x^3+x^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -4*a^3/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((-1+x)/(a^2/(-1+2*a
)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)
^(1/2)*EllipticF((-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),(a^2/(-1+2*a)/(a^2/
(-1+2*a)-1))^(1/2))+2*a^2/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*(
(-1+x)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2
*x-2*a*x^2-x^3+x^2)^(1/2)*EllipticF((-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),
(a^2/(-1+2*a)/(a^2/(-1+2*a)-1))^(1/2))-4*a^3*(a-1)/(-1+2*a)*(-(x-a^2/(-1+2*
a))/a^2*(-1+2*a))^(1/2)*((-1+x)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1
/2)/(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)^(1/2)/(a^2/(-1+2*a)-a)*Ellipti
cPi((-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),a^2/(-1+2*a)/(a^2/(-1+2*a)-a),(a
^2/(-1+2*a)/(a^2/(-1+2*a)-1))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="maxima")
```

```
[Out] -integrate(((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*
x^2)*(a - x)), x)
```

Fricas [A]
time = 0.32, size = 63, normalized size = 1.37

$$\log\left(-\frac{a^2 - 2\,(a-1)x - x^2 + 2\,\sqrt{(2\,a-1)x^3 + a^2x - (a^2 + 2\,a - 1)x^2}}{a^2 - 2\,ax + x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="fricas")
```

```
[Out] log(-(a^2 - 2*(a - 1)*x - x^2 + 2*sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a -
 1)*x^2))/(a^2 - 2*a*x + x^2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2ax - a - x}{\sqrt{x\,(x-1)\,(-a^2 + 2ax - x)}\,\,(-a + x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a**2*x-(a**2+2*a-1)*x**2+(-1+2*a)*x**3)**
(1/2),x)
```

```
[Out] Integral((2*a*x - a - x)/(sqrt(x*(x - 1)*(-a**2 + 2*a*x - x))*(-a + x)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x)
```

```
[Out] Could not integrate
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - x*(2*a - 1))/((a - x)*(x^3*(2*a - 1) - x^2*(2*a + a^2 - 1) + a^2*x
)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

3.84 $\int \frac{1-\sqrt[3]{2}\, x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\, dx$

Optimal. Leaf size=32

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\, x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}$$

```
[Out] 2/3*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2162, 209}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}\, x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1-\sqrt[3]{2}\,x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = 2\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1+\sqrt[3]{2}\,x}{\sqrt{1+x^3}}\right)$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 1.01, size = 34, normalized size = 1.06

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{1+x^3}}{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
[Out] (-2*ArcTan[Sqrt[1 + x^3]/(Sqrt[3]*(1 + 2^(1/3)*x))])/Sqrt[3]
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]')
[Out] Timed out
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.32, size = 258, normalized size = 8.06

method	result
trager	$-\frac{2^{\frac{1}{3}}\,\text{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)\ln\left(\frac{12\sqrt{x^3+1}\,x+3\,2^{\frac{2}{3}}\,\text{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)x^2-\text{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)x^3+6\sqrt{x^3+1}\,2^{\frac{2}{3}}+6\,2^{\frac{1}{3}}\,\text{RootO}}{\left(2^{\frac{1}{3}}x+2\right)^3}\right.}{6}$
default	$-\frac{2\,2^{\frac{1}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{6\left(\frac{3}{2}\right.}{}$

elliptic	$-\frac{2\,2^{\frac{1}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{6(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*2^(1/3)*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I
*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),
((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+6*(3/2-1/2*I*3^(1/2))*((
1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))
)^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2
^(2/3)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))
/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt[3]{2}\,x}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\,dx-\int\left(-\frac{1}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2**(1/3)*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(2**(1/3)*x/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x) - Int
egral(-1/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Unable to divide, perhaps due to rounding error1,[1] / [1,0,0]:[1,0,0,-2],[1] Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] Unable to divide, perhaps due to rounding error {1,[1] } /  { {[1,0,
0]:[1,0,0,-2] },[1] } Error: Bad Argument Value
```

Mupad [B]
time = 1.69, size = 67, normalized size = 2.09

$$\frac{\sqrt{3}\ \ln\left(\frac{\left(\sqrt{3}\ 1\mathrm{i}+\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\ x\,1\mathrm{i}\right)\left(\sqrt{3}\ 1\mathrm{i}-\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\ x\,1\mathrm{i}\right)^3}{\left(x+2^{2/3}\right)^6}\right)\,1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2^(1/3)*x - 1)/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] (3^(1/2)*log(((3^(1/2)*1i + (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)*(3^(1/2
)*1i - (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)^3)/(x + 2^(2/3))^6)*1i)/3
```

3.85 $\int \frac{1+x}{(-2+x)\sqrt{1+x^3}} \, dx$

Optimal. Leaf size=23

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)$$

```
[Out] -2/3*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2163, 212}

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(-2+x)\sqrt{1+x^3}} \, dx &= -\left(2\text{Subst}\left(\int \frac{1}{9-x^2} \, dx, x, \frac{(1+x)^2}{\sqrt{1+x^3}}\right)\right)\\ &= -\frac{2}{3}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)\end{aligned}$$

Mathematica [A]
time = 0.64, size = 31, normalized size = 1.35

$$-\frac{2}{3}\tanh^{-1}\left(\frac{\frac{1}{3}+\frac{2x}{3}+\frac{x^2}{3}}{\sqrt{1+x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(1/3 + (2*x)/3 + x^2/3)/Sqrt[1 + x^3]])/3
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.27, size = 240, normalized size = 10.43

method	result
trager	$-\frac{\ln\left(\frac{x^3+6\sqrt{x^3+1}\,x+12x^2+6\sqrt{x^3+1}-6x+10}{(-2+x)^3}\right)}{3}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right.}{}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-2+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
```

```
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2*I*3^(1/2))/(
-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.
time = 0.33, size = 44, normalized size = 1.91

$$\frac{1}{3}\log\left(\frac{x^3+12\,x^2-6\,\sqrt{x^3+1}\,(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log((x^3 + 12*x^2 - 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 +
12*x - 8))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+1}{\sqrt{(x+1)\,(x^2-x+1)}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x - 2)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.22, size = 204, normalized size = 8.87

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^3 + 1)^(1/2)*(x - 2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
+ 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin(((x
+ 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/
2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.86 $\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=218

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{2\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{1+x^3}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\cdots\right.}{3\sqrt{2}\,\cdots}$$

```
[Out] -1/12*arctan(1/2*3^(1/4)*(1+x)*(1+3^(1/2))*2^(1/2)/(x^3+1)^(1/2))*(2-3^(1/2
))*3^(1/4)*2^(1/2)-1/18*arctan(1/6*(1-3^(1/2))*(x^3+1)^(1/2)*3^(1/4)*2^(1/2
))*(2-3^(1/2))*3^(1/4)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*(1+x)*(1-3^(1/2))*2
^(1/2)/(x^3+1)^(1/2))*(2-3^(1/2))*3^(3/4)*2^(1/2)-1/18*arctanh(1/2*3^(1/4)*
(1-2*x+3^(1/2))*2^(1/2)/(x^3+1)^(1/2))*(2-3^(1/2))*3^(3/4)*2^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{500\}$

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{x^3+1}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(-2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] -1/2*((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1
+ x^3])])/(Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[1 +
 x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTanh[(3^
(1/4)*(1 + Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 + x^3])])/(3*Sqrt[2]*3^(1/4)) -
((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^
3])])/(6*Sqrt[2]*3^(1/4))
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx = -\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{2\sqrt{2}\,3^{3/4}} - \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\ \cdots\right.}{\cdots}\right)}{3\sqrt{2}\,3^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.05, size = 47, normalized size = 0.22

$$\frac{x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-x^3,-\frac{x^3}{10+6\sqrt{3}}\right)}{20+12\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, -x^3, -(x^3/(10 + 6*Sqrt[3]))])/(20 + 12*Sq
rt[3])
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/((10 + 6*Sqrt[3] + x^3)*Sqrt[1 + x^3]),x]')
```

```
[Out] Timed out
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 37.47, size = 353, normalized size = 1.62

method	result

default	$-\frac{\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^2+\left(-1-\sqrt{3}\right)\_Z+2\sqrt{3}+4\right)}\frac{\left(-\sqrt{3}\,\_\alpha+\_\alpha-2\right)\left(3-i\sqrt{3}\right)\sqrt{\frac{1+x}{3-i\sqrt{3}}}\sqrt{\frac{-i\sqrt{3}+2x-1}{-3-i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}}{i\sqrt{}}}}{}\right.}{18}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(-\frac{\left(-1-\sqrt{3}\right)^2}{3}+\frac{2\left(-1-\sqrt{3}\right)^2\sqrt{3}}{9}-\frac{2}{3}-\frac{\sqrt{3}}{9}-\frac{2\sqrt{3}}{}\right.}{}$
trager	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*2^(1/2)*sum((-3^(1/2)*_alpha+_alpha-2)/(-1+2*_alpha-3^(1/2))*(3-I*3^(
1/2))*((1+x)/(3-I*3^(1/2)))^(1/2)*((-I*3^(1/2)+2*x-1)/(-3-I*3^(1/2)))^(1/2)
*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-3))^(1/2)/(x^3+1)^(1/2)*(-1+2*_alpha-3^(1/2)
*_alpha)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/2*I*_alpha+1/3*I*_
alpha*3^(1/2)+1/2*3^(1/2)*_alpha-_alpha-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(-1-3^(1/2))*_Z+2*3^(1
/2)+4))+1/9*(-1-3^(1/2))/(2+3^(1/2))*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*
3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+
1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi
(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7739 vs. 2(148) = 296.

time = 3.86, size = 7739, normalized size = 35.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/432*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(56*sqrt(3)
 + 97)*sqrt(-56*sqrt(3) + 97)*(-672*sqrt(3) + 1164)^(3/4)*arctan(-1/1296*(6
*sqrt(x^3 + 1)*((459*x^16 - 13425*x^15 - 33201*x^14 + 950652*x^13 - 997302*
x^12 - 14760972*x^11 + 47069892*x^10 - 49762248*x^9 - 8212536*x^8 + 8437780
8*x^7 - 88427328*x^6 + 25613856*x^5 + 27458496*x^4 - 36433344*x^3 + 1260979
2*x^2 + sqrt(3)*(265*x^16 - 7751*x^15 - 19167*x^14 + 548864*x^13 - 575818*x
^12 - 8522268*x^11 + 27175852*x^10 - 28730312*x^9 - 4741560*x^8 + 48715600*
x^7 - 51053600*x^6 + 14788128*x^5 + 15853184*x^4 - 21034816*x^3 + 7280256*x
^2 - 2488832*x - 1889792) + (3691*x^16 - 6128*x^15 - 537864*x^14 + 1586477*
x^13 + 16210952*x^12 - 77181756*x^11 + 84218362*x^10 + 71018320*x^9 - 25445
5812*x^8 + 196076008*x^7 + 120105208*x^6 - 256326864*x^5 + 134645168*x^4 +
78464672*x^3 - 78514944*x^2 + sqrt(3)*(2131*x^16 - 3538*x^15 - 310536*x^14
+ 915953*x^13 + 9359398*x^12 - 44560908*x^11 + 48623494*x^10 + 41002448*x^9
 - 146910132*x^8 + 113204536*x^7 + 69342776*x^6 - 147990384*x^5 + 77737424*
x^4 + 45301600*x^3 - 45330624*x^2 + 12242560*x + 7598336) + 21204736*x + 13
160704)*sqrt(-672*sqrt(3) + 1164) - 4310784*x - 3273216)*(-672*sqrt(3) + 11
64)^(3/4) + 3*(984*x^15 - 30612*x^14 + 164676*x^13 - 205368*x^12 - 289200*x
^11 + 183720*x^10 + 886752*x^9 - 71568*x^8 - 1960992*x^7 + 1849440*x^6 + 15
58464*x^5 - 2478912*x^4 + 66432*x^3 + 750336*x^2 + 4*sqrt(3)*(142*x^15 - 44
19*x^14 + 23781*x^13 - 29608*x^12 - 41940*x^11 + 26454*x^10 + 128152*x^9 -
10692*x^8 - 283320*x^7 + 267064*x^6 + 224784*x^5 - 357936*x^4 + 9632*x^3 +
108288*x^2 - 96000*x - 33920) + (4945*x^15 - 88617*x^14 + 738528*x^13 - 186
0046*x^12 - 784596*x^11 + 7668708*x^10 - 6570680*x^9 - 6903864*x^8 + 154441
44*x^7 - 4312832*x^6 - 9559200*x^5 + 9359808*x^4 - 155968*x^3 - 3016704*x^2
 + sqrt(3)*(2855*x^15 - 51163*x^14 + 426388*x^13 - 1073898*x^12 - 452980*x^
11 + 4427548*x^10 - 3793592*x^9 - 3985944*x^8 + 8916720*x^7 - 2490016*x^6 -
 5519008*x^5 + 5403904*x^4 - 90048*x^3 - 1741696*x^2 + 1543936*x + 545536)
+ 2674176*x + 944896)*sqrt(-672*sqrt(3) + 1164) - 665088*x - 235008)*(-672*
sqrt(3) + 1164)^(1/4))*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) +
 24)*sqrt(-56*sqrt(3) + 97) + 36*(144*x^17 - 5976*x^16 + 5544*x^15 + 299664
*x^14 - 1062360*x^13 + 116712*x^12 + 3600000*x^11 - 4761216*x^10 - 1046592*
x^9 + 8676864*x^8 - 6592896*x^7 - 2641536*x^6 + 7016832*x^5 - 3699072*x^4 -
 1861632*x^3 + 1640448*x^2 + 12*sqrt(3)*(7*x^17 - 286*x^16 + 238*x^15 + 142
55*x^14 - 50390*x^13 + 5942*x^12 + 171808*x^11 - 226888*x^10 - 48920*x^9 +
415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x^5 - 177344*x^4 - 89152*x^3
+ 78784*x^2 - 39040*x - 18176) - (1164*x^17 - 6276*x^16 - 26052*x^15 + 3328
44*x^14 - 1632156*x^13 + 4149132*x^12 - 5805024*x^11 + 318696*x^10 + 126210
```

72*x^9 - 19878720*x^8 + 9619008*x^7 + 13361088*x^6 - 20168256*x^5 + 1093612
8*x^4 + 6434304*x^3 - 6426240*x^2 + 24*sqrt(3)*(28*x^17 - 151*x^16 - 626*x^
15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11 + 7661*x^10 + 303610
*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080*x^4 + 154
784*x^3 - 154592*x^2 + 78464*x + 36544) + (2340*x^17 - 96354*x^16 + 84798*x
^15 + 4817124*x^14 - 17052930*x^13 + 1941678*x^12 + 57963744*x^11 - 7660368
0*x^10 - 16678512*x^9 + 139922496*x^8 - 106227360*x^7 - 42453216*x^6 + 1132
69536*x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x^2 + sqrt(3)*(1351*x^17
 - 55630*x^16 + 48958*x^15 + 2781167*x^14 - 9845510*x^13 + 1121030*x^12 + 3
3465376*x^11 - 44227144*x^10 - 9629336*x^9 + 80784280*x^8 - 61330384*x^7 -
24510368*x^6 + 65396192*x^5 - 34464704*x^4 - 17335360*x^3 + 15297472*x^2 -
7571584*x - 3526400) - 13114368*x - 6107904)*sqrt(-672*sqrt(3) + 1164) + 32
61696*x + 1519104)*sqrt(-672*sqrt(3) + 1164) + 12*(97*x^17 - 523*x^16 - 217
1*x^15 + 27737*x^14 - 136013*x^13 + 345761*x^12 - 483752*x^11 + 26558*x^10
+ 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x^6 - 1680688*x^5 + 9113
44*x^4 + 536192*x^3 - 535520*x^2 + 2*sqrt(3)*(28*x^17 - 151*x^16 - 626*x^15
 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11 + 7661*x^10 + 303610*x
^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080*x^4 + 15478
4*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 126592)*sqrt(-672*sqrt(3
) + 1164) - 811008*x - 377856)*sqrt(-56*sqrt(3) + 97) - (sqrt(x^3 + 1)*((45
9*x^16 - 1557*x^15 - 26415*x^14 - 1449954*x^13 + 4677912*x^12 + 12651948*x^
11 - 55684800*x^10 + 62834256*x^9 + 8526168*x^8 - 105313392*x^7 + 99605088*
x^6 - 18897984*x^5 - 42499296*x^4 + 37357632*x^3 - 8256960*x^2 + sqrt(3)*(2
65*x^16 - 899*x^15 - 15249*x^14 - 837130*x^13 + 2700776*x^12 + 7304604*x^11
 - 32149640*x^10 + 36277360*x^9 + 4922568*x^8 - 60802736*x^7 + 57507040*x^6
 - 10910784*x^5 - 24536992*x^4 + 21568448*x^3 - 4767168*x^2 + 1207168*x + 1
383424) + (3691*x^16 + 17731*x^15 - 951114*x^14 + 450359*x^13 + 4370159*x^1
2 + 30318522*x^11 - 78096668*x^10 + 9429316*x^9 + 146877876*x^8 - 197107784
*x^7 - 30834152*x^6 + 185125776*x^5 - 132260896*x^4 - 45545344*x^3 + 695175
36*x^2 + sqrt(3)*(2131*x^16 + 10237*x^15 - 549126*x^14 + 260015*x^13 + 2523
113*x^12 + 17504406*x^11 - 45089132*x^10 + 5444020*x^9 + 84799980*x^8 - 113
800232*x^7 - 17802104*x^6 + 106882416*x^5 - 76360864*x^4 - 26295616*x^3 + 4
0135968*x^2 - 7907648*x - 5562368) - 13696448*x - 9634304)*sqrt(-672*sqrt(3
) + 1164) + 2090880*x + 2396160)*(-672*sqrt(3) + 1164)^(3/4) + 3*(984*x^15
- 14712*x^14 - 53940*x^13 + 411732*x^12 - 280248*x^11 - 324624*x^10 + 18081
6*x^9 - 518544*x^8 + 974304*x^7 - 887136*x^6 - 1404096*x^5 + 1843584*x^4 +
135936*x^3 - 696192*x^2 + 4*sqrt(3)*(142*x^15 - 2124*x^14 - 7773*x^13 + 594
47*x^12 - 40626*x^11 - 46860*x^10 + 26308*x^9 - 75276*x^8 + 140472*x^7 - 12
7784*x^6 - 202896*x^5 + 266016*x^4 + 19712*x^3 - 100512*x^2 + 62400*x + 248
32) + (4945*x^15 - 37473*x^14 - 490698*x^13 + 2249468*x^12 + 474132*x^11 -
8423784*x^10 + 5853520*x^9 + 8451720*x^8 - 15320016*x^7 + 768064*x^6 + 1040
5056*x^5 - 6627744*x^4 - 700480*x^3 + 2799552*x^2 + sqrt(3)*(2855*x^15 - 21
635*x^14 - 283306*x^13 + 1298732*x^12 + 273748*x^11 - 4863472*x^10 + 337953
6*x^9 + 4879608*x^8 - 8845008*x^7 + 443456*x^6 + 6007360*x^5 - 3826528*x^4
- 404416*x^3 + 1616320*x^2 - 1003648*x - 399360) - 1738368*x - 691712)*sqrt

(-672*sqrt(3) + 1164) + 432384*x + 172032)*(-672*sqrt(3) + 1164)^(1/4))*sqr
t(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*sqrt(-56*sqrt(3) + 97
) - 6*(4680*x^16 - 60552*x^15 + 89856*x^14 + 278280*x^13 + 64440*x^12 - 128
5200*x^11 - 255600*x^10 + 3098880*x^9 - 1770336*x^8 - 3614400*x^7 + 3895488
*x^6 + 1199232*x^5 - 2905344*x^4 + 681984*x^3 + 649728*x^2 + 108*sqrt(3)*(2
5*x^16 - 324*x^15 + 489*x^14 + 1482*x^13 + 316*x^12 - 6984*x^11 - 1312*x^10
 + 16624*x^9 - 9792*x^8 - 19328*x^7 + 20976*x^6 + 6240*x^5 - 15552*x^4 + 37
12*x^3 + 3456*x^2 - 4096*x - 1280) + (1164*x^17 + 1248*x^16 - 246120*x^15 +
 518172*x^14 + 2607528*x^13 - 8301144*x^12 + 7017600*x^11 + 6258120*x^10 -
21360336*x^9 + 16998960*x^8 + 966336*x^7 - 18216672*x^6 + 15860544*x^5 - 47
20704*x^4 - 6023424*x^3 + 5362176*x^2 + 48*sqrt(3)*(14*x^17 + 15*x^16 - 296
0*x^15 + 6232*x^14 + 31362*x^13 - 99844*x^12 + 84404*x^11 + 75267*x^10 - 25
6916*x^9 + 204458*x^8 + 11616*x^7 - 219104*x^6 + 190768*x^5 - 56784*x^4 - 7
2448*x^3 + 64496*x^2 - 24480*x - 13376) + (2340*x^17 - 35850*x^16 - 106410*
x^15 - 2064744*x^14 + 11945946*x^13 - 1710042*x^12 - 46293732*x^11 + 591615
24*x^10 + 18480192*x^9 - 122366520*x^8 + 81203856*x^7 + 45222000*x^6 - 1005
98112*x^5 + 42207168*x^4 + 29609472*x^3 - 22458240*x^2 + sqrt(3)*(1351*x^17
 - 20698*x^16 - 61436*x^15 - 1192081*x^14 + 6896998*x^13 - 987292*x^12 - 26
727704*x^11 + 34156928*x^10 + 10669552*x^9 - 70648352*x^8 + 46883072*x^7 +
26108944*x^6 - 58080352*x^5 + 24368320*x^4 + 17095040*x^3 - 12966272*x^2 +
4724480*x + 2581504) + 8183040*x + 4471296)*sqrt(-672*sqrt(3) + 1164) - 203
5200*x - 1112064)*sqrt(-672*sqrt(3) + 1164) + 24*(627*x^16 - 14286*x^15 + 3
9762*x^14 + 50142*x^13 - 216816*x^12 + 112284*x^11 + 325707*x^10 - 586326*x
^9 - 3294*x^8 + 631752*x^7 - 539220*x^6 - 184392*x^5 + 483816*x^4 - 115296*
x^3 - 108576*x^2 + 2*sqrt(3)*(181*x^16 - 4124*x^15 + 11478*x^14 + 14474*x^1
3 - 62584*x^12 + 32412*x^11 + 94021*x^10 - 169244*x^9 - 954*x^8 + 182368*x^
7 - 155648*x^6 - 53232*x^5 + 139664*x^4 - 33280*x^3 - 31344*x^2 + 37024*x +
 11584) + 128256*x + 40128)*sqrt(-672*sqrt(3) + 1164) - 764928*x - 239616)*
sqrt(-56*sqrt(3) + 97))*sqrt((36*x^8 + 72*x^7 + 1656*x^6 + 720*x^5 + 1440*x
^4 + 2016*x^3 + (60*x^6 + 324*x^5 + 576*x^4 + 696*x^3 + 432*x^2 + 36*sqrt(3
)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + (123*x^6 + 2016*x^5 + 221
4*x^4 + 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1192*x
^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) + 144*x + 96)*sqrt(x^3
 + 1)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3
) + 1164)^(1/4) - 288*x^2 + 144*sqrt(3)*(x^7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^
3 + 6*x^2 + 4*x - 8) + 72*(26*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3 + 42*
x^2 + sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 + 12*x +
 4) + 20*x + 8)*sqrt(-672*sqrt(3) + 1164) - 576*x + 2304)/(x^8 - 4*x^7 + 16
*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)))/(x^17 + 13*x^16 - 5
22*x^15 + 1742*x^14 + 3008*x^13 - 16884*x^12 + 11656*x^11 + 23944*x^10 - 42
336*x^9 + 9136*x^8 + 36256*x^7 - 27360*x^6 - 256*x^5 + 13376*x^4 - 5760*x^3
 - 1664*x^2 + 256*x)) - 1/432*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) +
1164) + 24)*(56*sqrt(3) + 97)*sqrt(-56*sqrt(3) + 97)*(-672*sqrt(3) + 1164)^
(3/4)*arctan(-1/1296*(6*sqrt(x^3 + 1)*((459*x^16 - 13425*x^15 - 33201*x^14
+ 950652*x^13 - 997302*x^12 - 14760972*x^11 + 47069892*x^10 - 49762248*x^9

- 8212536*x^8 + 84377808*x^7 - 88427328*x^6 + 25613856*x^5 + 27458496*x^4 -
36433344*x^3 + 12609792*x^2 + sqrt(3)*(265*x^16 - 7751*x^15 - 19167*x^14 +
548864*x^13 - 575818*x^12 - 8522268*x^11 + 27175852*x^10 - 28730312*x^9 -
4741560*x^8 + 48715600*x^7 - 51053600*x^6 + 14788128*x^5 + 15853184*x^4 - 2
1034816*x^3 + 7280256*x^2 - 2488832*x - 1889792) + (3691*x^16 - 6128*x^15 -
537864*x^14 + 1586477*x^13 + 16210952*x^12 - 77181756*x^11 + 84218362*x^10
+ 71018320*x^9 - 254455812*x^8 + 196076008*x^7 + 120105208*x^6 - 256326864
*x^5 + 134645168*x^4 + 78464672*x^3 - 78514944*x^2 + sqrt(3)*(2131*x^16 - 3
538*x^15 - 310536*x^14 + 915953*x^13 + 9359398*x^12 - 44560908*x^11 + 48623
494*x^10 + 41002448*x^9 - 146910132*x^8 + 113204536*x^7 + 69342776*x^6 - 14
7990384*x^5 + 77737424*x^4 + 45301600*x^3 - 45330624*x^2 + 12242560*x + 759
8336) + 21204736*x + 13160704)*sqrt(-672*sqrt(3) + 1164) - 4310784*x - 3273
216)*(-672*sqrt(3) + 1164)^(3/4) + 3*(984*x^15 - 30612*x^14 + 164676*x^13 -
205368*x^12 - 289200*x^11 + 183720*x^10 + 886752*x^9 - 71568*x^8 - 1960992
*x^7 + 1849440*x^6 + 1558464*x^5 - 2478912*x^4 + 66432*x^3 + 750336*x^2 + 4
sqrt(3)(142*x^15 - 4419*x^14 + 23781*x^13 - 29608*x^12 - 41940*x^11 + 264
54*x^10 + 128152*x^9 - 10692*x^8 - 283320*x^7 + 267064*x^6 + 224784*x^5 - 3
57936*x^4 + 9632*x^3 + 108288*x^2 - 96000*x - 33920) + (4945*x^15 - 88617*x
^14 + 738528*x^13 - 1860046*x^12 - 784596*x^11 + 7668708*x^10 - 6570680*x^9
- 6903864*x^8 + 15444144*x^7 - 4312832*x^6 - 9559200*x^5 + 9359808*x^4 - 1
55968*x^3 - 3016704*x^2 + sqrt(3)*(2855*x^15 - 51163*x^14 + 426388*x^13 - 1
073898*x^12 - 452980*x^11 + 4427548*x^10 - 3793592*x^9 - 3985944*x^8 + 8916
720*x^7 - 2490016*x^6 - 5519008*x^5 + 5403904*x^4 - 90048*x^3 - 1741696*x^2
+ 1543936*x + 545536) + 2674176*x + 944896)*sqrt(-672*sqrt(3) + 1164) - 66
5088*x - 235008)*(-672*sqrt(3) + 1164)^(1/4))*sqrt(-2*(7*sqrt(3) + 12)*sqrt
(-672*sqrt(3) + 1164) + 24)*sqrt(-56*sqrt(3) + 97) - 36*(144*x^17 - 5976*x^
16 + 5544*x^15 + 299664*x^14 - 1062360*x^13 + 116712*x^12 + 3600000*x^11 -
4761216*x^10 - 1046592*x^9 + 8676864*x^8 - 6592896*x^7 - 2641536*x^6 + 7016
832*x^5 - 3699072*x^4 - 1861632*x^3 + 1640448*x^2 + 12*sqrt(3)*(7*x^17 - 28
6*x^16 + 238*x^15 + 14255*x^14 - 50390*x^13 + 5942*x^12 + 171808*x^11 - 226
888*x^10 - 48920*x^9 + 415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x^5 -
177344*x^4 - 89152*x^3 + 78784*x^2 - 39040*x - 18176) - (1164*x^17 - 6276*x
^16 - 26052*x^15 + 332844*x^14 - 1632156*x^13 + 4149132*x^12 - 5805024*x^11
+ 318696*x^10 + 12621072*x^9 - 19878720*x^8 + 9619008*x^7 + 13361088*x^6 -
20168256*x^5 + 10936128*x^4 + 6434304*x^3 - 6426240*x^2 + 24*sqrt(3)*(28*x
^17 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^
11 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176
*x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + (2340*x^17
- 96354*x^16 + 84798*x^15 + 4817124*x^14 - 17052930*x^13 + 1941678*x^12 +
57963744*x^11 - 76603680*x^10 - 16678512*x^9 + 139922496*x^8 - 106227360*x^
7 - 42453216*x^6 + 113269536*x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x
^2 + sqrt(3)*(1351*x^17 - 55630*x^16 + 48958*x^15 + 2781167*x^14 - 9845510*
x^13 + 1121030*x^12 + 33465376*x^11 - 44227144*x^10 - 9629336*x^9 + 8078428
0*x^8 - 61330384*x^7 - 24510368*x^6 + 65396192*x^5 - 34464704*x^4 - 1733536
0*x^3 + 15297472*x^2 - 7571584*x - 3526400) - 13114368*x - 6107904)*sqrt(-6

72*sqrt(3) + 1164) + 3261696*x + 1519104)*sqrt(-672*sqrt(3) + 1164) + 12*(9
7*x^17 - 523*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 345761*x^12 - 48
3752*x^11 + 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x
^6 - 1680688*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 + 2*sqrt(3)*(28*x^1
7 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11
 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x
^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 1
26592)*sqrt(-672*sqrt(3) + 1164) - 811008*x - 377856)*sqrt(-56*sqrt(3) + 97
) - (sqrt(x^3 + 1)*((459*x^16 - 1557*x^15 - 26415*x^14 - 1449954*x^13 + 467
7912*x^12 + 12651948*x^11 - 55684800*x^10 + 62834256*x^9 + 8526168*x^8 - 10
5313392*x^7 + 99605088*x^6 - 18897984*x^5 - 42499296*x^4 + 37357632*x^3 - 8
256960*x^2 + sqrt(3)*(265*x^16 - 899*x^15 - 15249*x^14 - 837130*x^13 + 2700
776*x^12 + 7304604*x^11 - 32149640*x^10 + 36277360*x^9 + 4922568*x^8 - 6080
2736*x^7 + 57507040*x^6 - 10910784*x^5 - 24536992*x^4 + 21568448*x^3 - 4767
168*x^2 + 1207168*x + 1383424) + (3691*x^16 + 17731*x^15 - 951114*x^14 + 45
0359*x^13 + 4370159*x^12 + 30318522*x^11 - 78096668*x^10 + 9429316*x^9 + 14
6877876*x^8 - 197107784*x^7 - 30834152*x^6 + 185125776*x^5 - 132260896*x^4
 - 45545344*x^3 + 69517536*x^2 + sqrt(3)*(2131*x^16 + 10237*x^15 - 549126*x^
14 + 260015*x^13 + 2523113*x^12 + 17504406*x^11 - 45089132*x^10 + 5444020*x
^9 + 84799980*x^8 - 113800232*x^7 - 17802104*x^6 + 106882416*x^5 - 76360864
*x^4 - 26295616*x^3 + 40135968*x^2 - 7907648*x - 5562368) - 13696448*x - 96
34304)*sqrt(-672*sqrt(3) + 1164) + 2090880*x + 2396160)*(-672*sqrt(3) + 116
4)^(3/4) + 3*(984*x^15 - 14712*x^14 - 53940*x^13 + 411732*x^12 - 280248*x^1
1 - 324624*x^10 + 180816*x^9 - 518544*x^8 + 974304*x^7 - 887136*x^6 - 14040
96*x^5 + 1843584*x^4 + 135936*x^3 - 696192*x^2 + 4*sqrt(3)*(142*x^15 - 2124
*x^14 - 7773*x^13 + 59447*x^12 - 40626*x^11 - 46860*x^10 + 26308*x^9 - 7527
6*x^8 + 140472*x^7 - 127784*x^6 - 202896*x^5 + 266016*x^4 + 19712*x^3 - 100
512*x^2 + 62400*x + 24832) + (4945*x^15 - 37473*x^14 - 490698*x^13 + 224946
8*x^12 + 474132*x^11 - 8423784*x^10 + 5853520*x^9 + 8451720*x^8 - 15320016*
x^7 + 768064*x^6 + 10405056*x^5 - 6627744*x^4 - 700480*x^3 + 2799552*x^2 +
sqrt(3)*(2855*x^15 - 21635*x^14 - 283306*x^13 + 1298732*x^12 + 273748*x^11
 - 4863472*x^10 + 3379536*x^9 + 4879608*x^8 - 8845008*x^7 + 443456*x^6 + 600
7360*x^5 - 3826528*x^4 - 404416*x^3 + 1616320*x^2 - 1003648*x - 399360) - 1
738368*x - 691712)*sqrt(-672*sqrt(3) + 1164) + 432384*x + 172032)*(-672*sqr
t(3) + 1164)^(1/4))*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24
)*sqrt(-56*sqrt(3) + 97) + 6*(4680*x^16 - 60552*x^15 + 89856*x^14 + 278280*
x^13 + 64440*x^12 - 1285200*x^11 - 255600*x^10 + 3098880*x^9 - 1770336*x^8
 - 3614400*x^7 + 3895488*x^6 + 1199232*x^5 - 2905344*x^4 + 681984*x^3 + 6497
28*x^2 + 108*sqrt(3)*(25*x^16 - 324*x^15 + 489*x^14 + 1482*x^13 + 316*x^12
 - 6984*x^11 - 1312*x^10 + 16624*x^9 - 9792*x^8 - 19328*x^7 + 20976*x^6 + 62
40*x^5 - 15552*x^4 + 3712*x^3 + 3456*x^2 - 4096*x - 1280) + (1164*x^17 + 12
48*x^16 - 246120*x^15 + 518172*x^14 + 2607528*x^13 - 8301144*x^12 + 7017600
*x^11 + 6258120*x^10 - 21360336*x^9 + 16998960*x^8 + 966336*x^7 - 18216672*
x^6 + 15860544*x^5 - 4720704*x^4 - 6023424*x^3 + 5362176*x^2 + 48*sqrt(3)*(
14*x^17 + 15*x^16 - 2960*x^15 + 6232*x^14 + 31362*x^13 - 99844*x^12 + 84404

*x^11 + 75267*x^10 - 256916*x^9 + 204458*x^8 + 11616*x^7 - 219104*x^6 + 190
768*x^5 - 56784*x^4 - 72448*x^3 + 64496*x^2 - 24480*x - 13376) + (2340*x^17
 - 35850*x^16 - 106410*x^15 - 2064744*x^14 + 11945946*x^13 - 1710042*x^12 -
 46293732*x^11 + 59161524*x^10 + 18480192*x^9 - 122366520*x^8 + 81203856*x^
7 + 45222000*x^6 - 100598112*x^5 + 42207168*x^4 + 29609472*x^3 - 22458240*x
^2 + sqrt(3)*(1351*x^17 - 20698*x^16 - 61436*x^15 - 1192081*x^14 + 6896998*
x^13 - 987292*x^12 - 26727704*x^11 + 34156928*x^10 + 10669552*x^9 - 7064835
2*x^8 + 46883072*x^7 + 26108944*x^6 - 58080352*x^5 + 24368320*x^4 + 1709504
0*x^3 - 12966272*x^2 + 4724480*x + 2581504) + 8183040*x + 4471296)*sqrt(-67
2*sqrt(3) + 1164) - 2035200*x - 1112064)*sqrt(-672*sqrt(3) + 1164) + 24*(62
7*x^16 - 14286*x^15 + 39762*x^14 + 50142*x^13 - 216816*x^12 + 112284*x^11 +
 325707*x^10 - 586326*x^9 - 3294*x^8 + 631752*x^7 - 539220*x^6 - 184392*x^5
 + 483816*x^4 - 115296*x^3 - 108576*x^2 + 2*sqrt(3)*(181*x^16 - 4124*x^15 +
 11478*x^14 + 14474*x^13 - 62584*x^12 + 32412*x^11 + 94021*x^10 - 169244*x^
9 - 954*x^8 + 182368*x^7 - 155648*x^6 - 53232*x^5 + 139664*x^4 - 33280*x^3
- 31344*x^2 + 37024*x + 11584) + 128256*x + 40128)*sqrt(-672*sqrt(3) + 1164
) - 764928*x - 239616)*sqrt(-56*sqrt(3) + 97))*sqrt((36*x^8 + 72*x^7 + 1656
*x^6 + 720*x^5 + 1440*x^4 + 2016*x^3 - (60*x^6 + 324*x^5 + 576*x^4 + 696*x^
3 + 432*x^2 + 36*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + (1
23*x^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 + 1164*
x^5 + 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164)
 + 144*x + 96)*sqrt(x^3 + 1)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1
164) + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 + 144*sqrt(3)*(x^7 + 4*x^6
 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) + 72*(26*x^7 + 38*x^6 + 42*x^5
+ 46*x^4 + 46*x^3 + 42*x^2 + sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 2
6*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(-672*sqrt(3) + 1164) - 576*x +
2304)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16
)))/(x^17 + 13*x^16 - 522*x^15 + 1742*x^14 + 3008*x^13 - 16884*x^12 + 11656
*x^11 + 23944*x^10 - 42336*x^9 + 9136*x^8 + 36256*x^7 - 27360*x^6 - 256*x^5
 + 13376*x^4 - 5760*x^3 - 1664*x^2 + 256*x)) + 1/5184*((7*sqrt(3) + 12)*sqr
t(-672*sqrt(3) + 1164) + 12)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1
164) + 24)*(-672*sqrt(3) + 1164)^(1/4)*log(1/36*(36*x^8 + 72*x^7 + 1656*x^6
 + 720*x^5 + 1440*x^4 + 2016*x^3 + (60*x^6 + 324*x^5 + 576*x^4 + 696*x^3 +
432*x^2 + 36*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + (123*x
^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 + 1164*x^5
+ 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) + 1
44*x + 96)*sqrt(x^3 + 1)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164)
 + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 + 144*sqrt(3)*(x^7 + 4*x^6 + 6
*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) + 72*(26*x^7 + 38*x^6 + 42*x^5 + 46
*x^4 + 46*x^3 + 42*x^2 + sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 26*x^
3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(-672*sqrt(3) + 1164) - 576*x + 2304
)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)) -
 1/5184*((7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 12)*sqrt(-2*(7*sqrt(3
) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3) + 1164)^(1/4)*log(1/3
6*(36*x^8 + 72*x^7 + 1656*x^6 + 720*x^5 + 1440*x^4 + 2016*x^3 - (60*x^6 + 3

```
24*x^5 + 576*x^4 + 696*x^3 + 432*x^2 + 36*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 1
0*x^3 + 8*x^2 + 4*x) + (123*x^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2
+ sqrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*
sqrt(-672*sqrt(3) + 1164) + 144*x + 96)*sqrt(x^3 + 1)*sqrt(-2*(7*sqrt(3) +
12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 +
 144*sqrt(3)*(x^7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) + 72*(
26*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3 + 42*x^2 + sqrt(3)*(15*x^7 + 22*
x^6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(-672*s
qrt(3) + 1164) - 576*x + 2304)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32
*x^3 + 64*x^2 + 32*x + 16)) + 1/36*sqrt(14*sqrt(3) - 24)*arctan(1/12*(3*x^2
 + sqrt(3)*(x^2 - 10*x - 8) - 18*x - 12)*sqrt(14*sqrt(3) - 24)/sqrt(x^3 + 1
))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x+1)(x^2-x+1)}\left(x^3+10+6\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x**3+6*3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x**3 + 10 + 6*sqrt(3))), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{x^3+1}\,\left(x^3+6\,\sqrt{3}+10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 + 1)^(1/2)*(6*3^(1/2) + x^3 + 10)),x)
```

```
[Out] int(x/((x^3 + 1)^(1/2)*(6*3^(1/2) + x^3 + 10)), x)
```

3.87 $\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=210

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}-2x\right)}{\sqrt{2}\sqrt{1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}}{}\right.}{2\sqrt{2}\,3^{3/}}$$

```
[Out] -1/18*arctan(1/2*3^(1/4)*(1-2*x-3^(1/2))*2^(1/2)/(x^3+1)^(1/2))*(2+3^(1/2))
*3^(3/4)*2^(1/2)-1/36*arctan(1/2*3^(1/4)*(1+x)*(1+3^(1/2))*2^(1/2)/(x^3+1)^
(1/2))*(2+3^(1/2))*3^(3/4)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*(1+x)*(1-3^(1/2
))*2^(1/2)/(x^3+1)^(1/2))*(2+3^(1/2))*3^(1/4)*2^(1/2)+1/18*arctanh(1/6*(1+3
^(1/2))*(x^3+1)^(1/2)*3^(1/4)*2^(1/2))*(2+3^(1/2))*3^(1/4)*2^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {500}

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(-2x-\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x^3+1}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x^3]*(10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] -1/3*((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 +
x^3])])/(Sqrt[2]*3^(1/4)) - ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1
 + x))/(Sqrt[2]*Sqrt[1 + x^3])])/(6*Sqrt[2]*3^(1/4)) + ((2 + Sqrt[3])*ArcTa
nh[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3])])/(2*Sqrt[2]*3^(
3/4)) + ((2 + Sqrt[3])*ArcTanh[((1 + Sqrt[3])*Sqrt[1 + x^3])/(Sqrt[2]*3^(3/
4))])/(3*Sqrt[2]*3^(3/4))
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}dx = -\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}-2x\right)}{\sqrt{2}\sqrt{1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\right.}{\sqrt{2}\sqrt{}}\right)}{6\sqrt{2}\sqrt[4]{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.06, size = 50, normalized size = 0.24

$$-\frac{x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-x^3,\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right)}{4\left(-5+3\sqrt{3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + x^3]*(10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] -1/4*(x^2*AppellF1[2/3, 1/2, 1, 5/3, -x^3, ((5 + 3*Sqrt[3])*x^3)/4])/(-5 +
3*Sqrt[3])
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/((10 - 6*Sqrt[3] + x^3)*Sqrt[1 + x^3]),x]')
```

```
[Out] Timed out
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 37.87, size = 350, normalized size = 1.67

method	result

default	$\frac{\left(\sqrt{3}-1\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)}{3}\right.}{9\left(-2+\sqrt{3}\right)\sqrt{x^{3}+1}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(-\frac{2\left(\sqrt{3}-1\right)^{2}\sqrt{3}}{9}-\frac{\left(\sqrt{3}-1\right)^{2}}{3}+\frac{2\left(\sqrt{3}-1\right)\sqrt{3}}{9}+\right.}{}$
trager	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*(3^(1/2)-1)/(-2+3^(1/2))*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2))
)^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^
(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((1+x)/
(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-3^(1/2)*_alpha-_alp
ha+2)/(1-2*_alpha-3^(1/2))*(3-I*3^(1/2))*((1+x)/(3-I*3^(1/2)))^(1/2)*((-I*3
^(1/2)+2*x-1)/(-3-I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-3))^(1/2)
/(x^3+1)^(1/2)*(2*_alpha-1+3^(1/2)*_alpha)*EllipticPi(((1+x)/(3/2-1/2*I*3^(
1/2)))^(1/2),1/3*I*_alpha*3^(1/2)+1/2*I*_alpha-1/2*3^(1/2)*_alpha-_alpha-1/
6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=R
ootOf(_Z^2+(3^(1/2)-1)*_Z-2*3^(1/2)+4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8237 vs. $2(146) = 292$.

time = 4.33, size = 8237, normalized size = 39.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/108*sqrt(3)*sqrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(67
2*sqrt(3) + 1164)^(1/4)*(56*sqrt(3) + 97)*(56*sqrt(3) - 97)*arctan(1/324*(2
16*sqrt(3)*(97*x^17 - 523*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 345
761*x^12 - 483752*x^11 + 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^
7 + 1113424*x^6 - 1680688*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*sq
rt(3)*(28*x^17 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12
- 139652*x^11 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x
^6 - 485176*x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) +
 271808*x + 126592)*(56*sqrt(3) + 97) - 36*sqrt(3)*(sqrt(3)*(2340*x^17 - 96
354*x^16 + 84798*x^15 + 4817124*x^14 - 17052930*x^13 + 1941678*x^12 + 57963
744*x^11 - 76603680*x^10 - 16678512*x^9 + 139922496*x^8 - 106227360*x^7 - 4
2453216*x^6 + 113269536*x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x^2 -
sqrt(3)*(1351*x^17 - 55630*x^16 + 48958*x^15 + 2781167*x^14 - 9845510*x^13
+ 1121030*x^12 + 33465376*x^11 - 44227144*x^10 - 9629336*x^9 + 80784280*x^8
 - 61330384*x^7 - 24510368*x^6 + 65396192*x^5 - 34464704*x^4 - 17335360*x^3
 + 15297472*x^2 - 7571584*x - 3526400) - 13114368*x - 6107904)*(56*sqrt(3)
+ 97) + 6*(97*x^17 - 523*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 3457
61*x^12 - 483752*x^11 + 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7
 + 1113424*x^6 - 1680688*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*sqr
t(3)*(28*x^17 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 -
 139652*x^11 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^
6 - 485176*x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) +
271808*x + 126592)*sqrt(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) + 3*sqrt(sq
rt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*((2*sqrt(3)*(3691*x^16 -
6128*x^15 - 537864*x^14 + 1586477*x^13 + 16210952*x^12 - 77181756*x^11 + 84
218362*x^10 + 71018320*x^9 - 254455812*x^8 + 196076008*x^7 + 120105208*x^6
- 256326864*x^5 + 134645168*x^4 + 78464672*x^3 - 78514944*x^2 - sqrt(3)*(21
31*x^16 - 3538*x^15 - 310536*x^14 + 915953*x^13 + 9359398*x^12 - 44560908*x
^11 + 48623494*x^10 + 41002448*x^9 - 146910132*x^8 + 113204536*x^7 + 693427
76*x^6 - 147990384*x^5 + 77737424*x^4 + 45301600*x^3 - 45330624*x^2 + 12242
560*x + 7598336) + 21204736*x + 13160704)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) +
 (459*x^16 - 13425*x^15 - 33201*x^14 + 950652*x^13 - 997302*x^12 - 14760972
*x^11 + 47069892*x^10 - 49762248*x^9 - 8212536*x^8 + 84377808*x^7 - 8842732
8*x^6 + 25613856*x^5 + 27458496*x^4 - 36433344*x^3 + 12609792*x^2 - sqrt(3)
*(265*x^16 - 7751*x^15 - 19167*x^14 + 548864*x^13 - 575818*x^12 - 8522268*x
^11 + 27175852*x^10 - 28730312*x^9 - 4741560*x^8 + 48715600*x^7 - 51053600*
x^6 + 14788128*x^5 + 15853184*x^4 - 21034816*x^3 + 7280256*x^2 - 2488832*x
```

- 1889792) - 4310784*x - 3273216)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672
sqrt(3) + 1164)^(3/4) + 6(sqrt(3)*(4945*x^15 - 88617*x^14 + 738528*x^13 -
1860046*x^12 - 784596*x^11 + 7668708*x^10 - 6570680*x^9 - 6903864*x^8 + 15
444144*x^7 - 4312832*x^6 - 9559200*x^5 + 9359808*x^4 - 155968*x^3 - 3016704
x^2 - sqrt(3)(2855*x^15 - 51163*x^14 + 426388*x^13 - 1073898*x^12 - 45298
0*x^11 + 4427548*x^10 - 3793592*x^9 - 3985944*x^8 + 8916720*x^7 - 2490016*x
^6 - 5519008*x^5 + 5403904*x^4 - 90048*x^3 - 1741696*x^2 + 1543936*x + 5455
36) + 2674176*x + 944896)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + 2*(246*x^15 - 7
653*x^14 + 41169*x^13 - 51342*x^12 - 72300*x^11 + 45930*x^10 + 221688*x^9 -
17892*x^8 - 490248*x^7 + 462360*x^6 + 389616*x^5 - 619728*x^4 + 16608*x^3
+ 187584*x^2 - sqrt(3)*(142*x^15 - 4419*x^14 + 23781*x^13 - 29608*x^12 - 41
940*x^11 + 26454*x^10 + 128152*x^9 - 10692*x^8 - 283320*x^7 + 267064*x^6 +
224784*x^5 - 357936*x^4 + 9632*x^3 + 108288*x^2 - 96000*x - 33920) - 166272
*x - 58752)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)
) + 108*(12*x^17 - 498*x^16 + 462*x^15 + 24972*x^14 - 88530*x^13 + 9726*x^1
2 + 300000*x^11 - 396768*x^10 - 87216*x^9 + 723072*x^8 - 549408*x^7 - 22012
8*x^6 + 584736*x^5 - 308256*x^4 - 155136*x^3 + 136704*x^2 - sqrt(3)*(7*x^17
- 286*x^16 + 238*x^15 + 14255*x^14 - 50390*x^13 + 5942*x^12 + 171808*x^11
- 226888*x^10 - 48920*x^9 + 415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x
^5 - 177344*x^4 - 89152*x^3 + 78784*x^2 - 39040*x - 18176) - 67584*x - 3148
8)*sqrt(56*sqrt(3) + 97) + (144*sqrt(3)*(627*x^16 - 14286*x^15 + 39762*x^14
+ 50142*x^13 - 216816*x^12 + 112284*x^11 + 325707*x^10 - 586326*x^9 - 3294
*x^8 + 631752*x^7 - 539220*x^6 - 184392*x^5 + 483816*x^4 - 115296*x^3 - 108
576*x^2 - 2*sqrt(3)*(181*x^16 - 4124*x^15 + 11478*x^14 + 14474*x^13 - 62584
*x^12 + 32412*x^11 + 94021*x^10 - 169244*x^9 - 954*x^8 + 182368*x^7 - 15564
8*x^6 - 53232*x^5 + 139664*x^4 - 33280*x^3 - 31344*x^2 + 37024*x + 11584) +
128256*x + 40128)*(56*sqrt(3) + 97) + 12*sqrt(3)*(sqrt(3)*(2340*x^17 - 358
50*x^16 - 106410*x^15 - 2064744*x^14 + 11945946*x^13 - 1710042*x^12 - 46293
732*x^11 + 59161524*x^10 + 18480192*x^9 - 122366520*x^8 + 81203856*x^7 + 45
222000*x^6 - 100598112*x^5 + 42207168*x^4 + 29609472*x^3 - 22458240*x^2 - s
qrt(3)*(1351*x^17 - 20698*x^16 - 61436*x^15 - 1192081*x^14 + 6896998*x^13 -
987292*x^12 - 26727704*x^11 + 34156928*x^10 + 10669552*x^9 - 70648352*x^8
+ 46883072*x^7 + 26108944*x^6 - 58080352*x^5 + 24368320*x^4 + 17095040*x^3
- 12966272*x^2 + 4724480*x + 2581504) + 8183040*x + 4471296)*(56*sqrt(3) +
97) + 6*(97*x^17 + 104*x^16 - 20510*x^15 + 43181*x^14 + 217294*x^13 - 69176
2*x^12 + 584800*x^11 + 521510*x^10 - 1780028*x^9 + 1416580*x^8 + 80528*x^7
- 1518056*x^6 + 1321712*x^5 - 393392*x^4 - 501952*x^3 + 446848*x^2 - 4*sqrt
(3)*(14*x^17 + 15*x^16 - 2960*x^15 + 6232*x^14 + 31362*x^13 - 99844*x^12 +
84404*x^11 + 75267*x^10 - 256916*x^9 + 204458*x^8 + 11616*x^7 - 219104*x^6
+ 190768*x^5 - 56784*x^4 - 72448*x^3 + 64496*x^2 - 24480*x - 13376) - 16960
0*x - 92672)*sqrt(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) - sqrt(sqrt(3)*sq
rt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*((2*sqrt(3)*(3691*x^16 + 17731*x^
15 - 951114*x^14 + 450359*x^13 + 4370159*x^12 + 30318522*x^11 - 78096668*x^
10 + 9429316*x^9 + 146877876*x^8 - 197107784*x^7 - 30834152*x^6 + 185125776
*x^5 - 132260896*x^4 - 45545344*x^3 + 69517536*x^2 - sqrt(3)*(2131*x^16 + 1

0237*x^15 - 549126*x^14 + 260015*x^13 + 2523113*x^12 + 17504406*x^11 - 4508
9132*x^10 + 5444020*x^9 + 84799980*x^8 - 113800232*x^7 - 17802104*x^6 + 106
882416*x^5 - 76360864*x^4 - 26295616*x^3 + 40135968*x^2 - 7907648*x - 55623
68) - 13696448*x - 9634304)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + (459*x^16 - 1
557*x^15 - 26415*x^14 - 1449954*x^13 + 4677912*x^12 + 12651948*x^11 - 55684
800*x^10 + 62834256*x^9 + 8526168*x^8 - 105313392*x^7 + 99605088*x^6 - 1889
7984*x^5 - 42499296*x^4 + 37357632*x^3 - 8256960*x^2 - sqrt(3)*(265*x^16 -
899*x^15 - 15249*x^14 - 837130*x^13 + 2700776*x^12 + 7304604*x^11 - 3214964
0*x^10 + 36277360*x^9 + 4922568*x^8 - 60802736*x^7 + 57507040*x^6 - 1091078
4*x^5 - 24536992*x^4 + 21568448*x^3 - 4767168*x^2 + 1207168*x + 1383424) +
2090880*x + 2396160)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 11
64)^(3/4) + 6*(sqrt(3)*(4945*x^15 - 37473*x^14 - 490698*x^13 + 2249468*x^12
 + 474132*x^11 - 8423784*x^10 + 5853520*x^9 + 8451720*x^8 - 15320016*x^7 +
768064*x^6 + 10405056*x^5 - 6627744*x^4 - 700480*x^3 + 2799552*x^2 - sqrt(3
)*(2855*x^15 - 21635*x^14 - 283306*x^13 + 1298732*x^12 + 273748*x^11 - 4863
472*x^10 + 3379536*x^9 + 4879608*x^8 - 8845008*x^7 + 443456*x^6 + 6007360*x
^5 - 3826528*x^4 - 404416*x^3 + 1616320*x^2 - 1003648*x - 399360) - 1738368
*x - 691712)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + 2*(246*x^15 - 3678*x^14 - 13
485*x^13 + 102933*x^12 - 70062*x^11 - 81156*x^10 + 45204*x^9 - 129636*x^8 +
 243576*x^7 - 221784*x^6 - 351024*x^5 + 460896*x^4 + 33984*x^3 - 174048*x^2
 - sqrt(3)*(142*x^15 - 2124*x^14 - 7773*x^13 + 59447*x^12 - 40626*x^11 - 46
860*x^10 + 26308*x^9 - 75276*x^8 + 140472*x^7 - 127784*x^6 - 202896*x^5 + 2
66016*x^4 + 19712*x^3 - 100512*x^2 + 62400*x + 24832) + 108096*x + 43008)*s
qrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) + 108*(130*
x^16 - 1682*x^15 + 2496*x^14 + 7730*x^13 + 1790*x^12 - 35700*x^11 - 7100*x^
10 + 86080*x^9 - 49176*x^8 - 100400*x^7 + 108208*x^6 + 33312*x^5 - 80704*x^
4 + 18944*x^3 + 18048*x^2 - 3*sqrt(3)*(25*x^16 - 324*x^15 + 489*x^14 + 1482
*x^13 + 316*x^12 - 6984*x^11 - 1312*x^10 + 16624*x^9 - 9792*x^8 - 19328*x^7
 + 20976*x^6 + 6240*x^5 - 15552*x^4 + 3712*x^3 + 3456*x^2 - 4096*x - 1280)
- 21248*x - 6656)*sqrt(56*sqrt(3) + 97))*sqrt((9*x^8 + 18*x^7 + 414*x^6 + 1
80*x^5 + 360*x^4 + 504*x^3 - 72*x^2 + 36*sqrt(3)*(26*x^7 + 38*x^6 + 42*x^5
+ 46*x^4 + 46*x^3 + 42*x^2 - sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 2
6*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(56*sqrt(3) + 97) + (sqrt(3)*(12
3*x^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 + 1164*x
^5 + 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(x^3 + 1)*sqrt(56*sqrt
(3) + 97) + 6*(5*x^6 + 27*x^5 + 48*x^4 + 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 +
 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + 12*x + 8)*sqrt(x^3 + 1))*sqrt(sqr
t(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(672*sqrt(3) + 1164)^(1/4)
 - 36*sqrt(3)*(x^7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) - 144
*x + 576)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x
+ 16)))/(x^17 + 13*x^16 - 522*x^15 + 1742*x^14 + 3008*x^13 - 16884*x^12 + 1
1656*x^11 + 23944*x^10 - 42336*x^9 + 9136*x^8 + 36256*x^7 - 27360*x^6 - 256
*x^5 + 13376*x^4 - 5760*x^3 - 1664*x^2 + 256*x)) - 1/108*sqrt(3)*sqrt(sqrt(
3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(672*sqrt(3) + 1164)^(1/4)*(
56*sqrt(3) + 97)*(56*sqrt(3) - 97)*arctan(-1/324*(216*sqrt(3)*(97*x^17 - 52

3*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 345761*x^12 - 483752*x^11 +
26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x^6 - 168068
8*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*sqrt(3)*(28*x^17 - 151*x^1
6 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11 + 7661*x^1
0 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080
*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 126592)*(56*
sqrt(3) + 97) - 36*sqrt(3)*(sqrt(3)*(2340*x^17 - 96354*x^16 + 84798*x^15 +
4817124*x^14 - 17052930*x^13 + 1941678*x^12 + 57963744*x^11 - 76603680*x^10
- 16678512*x^9 + 139922496*x^8 - 106227360*x^7 - 42453216*x^6 + 113269536*
x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x^2 - sqrt(3)*(1351*x^17 - 556
30*x^16 + 48958*x^15 + 2781167*x^14 - 9845510*x^13 + 1121030*x^12 + 3346537
6*x^11 - 44227144*x^10 - 9629336*x^9 + 80784280*x^8 - 61330384*x^7 - 245103
68*x^6 + 65396192*x^5 - 34464704*x^4 - 17335360*x^3 + 15297472*x^2 - 757158
4*x - 3526400) - 13114368*x - 6107904)*(56*sqrt(3) + 97) + 6*(97*x^17 - 523
*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 345761*x^12 - 483752*x^11 +
26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x^6 - 1680688
*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*sqrt(3)*(28*x^17 - 151*x^16
- 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11 + 7661*x^10
+ 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080*
x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 126592)*sqrt(
56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) - 3*sqrt(sqrt(3)*sqrt(56*sqrt(3) +
97)*(7*sqrt(3) - 12) + 6)*((2*sqrt(3)*(3691*x^16 - 6128*x^15 - 537864*x^14
+ 1586477*x^13 + 16210952*x^12 - 77181756*x^11 + 84218362*x^10 + 71018320*x
^9 - 254455812*x^8 + 196076008*x^7 + 120105208*x^6 - 256326864*x^5 + 134645
168*x^4 + 78464672*x^3 - 78514944*x^2 - sqrt(3)*(2131*x^16 - 3538*x^15 - 31
0536*x^14 + 915953*x^13 + 9359398*x^12 - 44560908*x^11 + 48623494*x^10 + 41
002448*x^9 - 146910132*x^8 + 113204536*x^7 + 69342776*x^6 - 147990384*x^5 +
77737424*x^4 + 45301600*x^3 - 45330624*x^2 + 12242560*x + 7598336) + 21204
736*x + 13160704)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + (459*x^16 - 13425*x^15
- 33201*x^14 + 950652*x^13 - 997302*x^12 - 14760972*x^11 + 47069892*x^10 -
49762248*x^9 - 8212536*x^8 + 84377808*x^7 - 88427328*x^6 + 25613856*x^5 + 2
7458496*x^4 - 36433344*x^3 + 12609792*x^2 - sqrt(3)*(265*x^16 - 7751*x^15 -
19167*x^14 + 548864*x^13 - 575818*x^12 - 8522268*x^11 + 27175852*x^10 - 28
730312*x^9 - 4741560*x^8 + 48715600*x^7 - 51053600*x^6 + 14788128*x^5 + 158
53184*x^4 - 21034816*x^3 + 7280256*x^2 - 2488832*x - 1889792) - 4310784*x -
3273216)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) +
6*(sqrt(3)*(4945*x^15 - 88617*x^14 + 738528*x^13 - 1860046*x^12 - 784596*x
^11 + 7668708*x^10 - 6570680*x^9 - 6903864*x^8 + 15444144*x^7 - 4312832*x^6
- 9559200*x^5 + 9359808*x^4 - 155968*x^3 - 3016704*x^2 - sqrt(3)*(2855*x^1
5 - 51163*x^14 + 426388*x^13 - 1073898*x^12 - 452980*x^11 + 4427548*x^10 -
3793592*x^9 - 3985944*x^8 + 8916720*x^7 - 2490016*x^6 - 5519008*x^5 + 54039
04*x^4 - 90048*x^3 - 1741696*x^2 + 1543936*x + 545536) + 2674176*x + 944896
)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + 2*(246*x^15 - 7653*x^14 + 41169*x^13 -
51342*x^12 - 72300*x^11 + 45930*x^10 + 221688*x^9 - 17892*x^8 - 490248*x^7
+ 462360*x^6 + 389616*x^5 - 619728*x^4 + 16608*x^3 + 187584*x^2 - sqrt(3)*(

142*x^15 - 4419*x^14 + 23781*x^13 - 29608*x^12 - 41940*x^11 + 26454*x^10 +
128152*x^9 - 10692*x^8 - 283320*x^7 + 267064*x^6 + 224784*x^5 - 357936*x^4
+ 9632*x^3 + 108288*x^2 - 96000*x - 33920) - 166272*x - 58752)*sqrt(x^3 + 1
)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) + 108*(12*x^17 - 498*x
^16 + 462*x^15 + 24972*x^14 - 88530*x^13 + 9726*x^12 + 300000*x^11 - 396768
*x^10 - 87216*x^9 + 723072*x^8 - 549408*x^7 - 220128*x^6 + 584736*x^5 - 308
256*x^4 - 155136*x^3 + 136704*x^2 - sqrt(3)*(7*x^17 - 286*x^16 + 238*x^15 +
14255*x^14 - 50390*x^13 + 5942*x^12 + 171808*x^11 - 226888*x^10 - 48920*x^
9 + 415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x^5 - 177344*x^4 - 89152*
x^3 + 78784*x^2 - 39040*x - 18176) - 67584*x - 31488)*sqrt(56*sqrt(3) + 97)
+ (144*sqrt(3)*(627*x^16 - 14286*x^15 + 39762*x^14 + 50142*x^13 - 216816*x
^12 + 112284*x^11 + 325707*x^10 - 586326*x^9 - 3294*x^8 + 631752*x^7 - 5392
20*x^6 - 184392*x^5 + 483816*x^4 - 115296*x^3 - 108576*x^2 - 2*sqrt(3)*(181
*x^16 - 4124*x^15 + 11478*x^14 + 14474*x^13 - 62584*x^12 + 32412*x^11 + 940
21*x^10 - 169244*x^9 - 954*x^8 + 182368*x^7 - 155648*x^6 - 53232*x^5 + 1396
64*x^4 - 33280*x^3 - 31344*x^2 + 37024*x + 11584) + 128256*x + 40128)*(56*s
qrt(3) + 97) + 12*sqrt(3)*(sqrt(3)*(2340*x^17 - 35850*x^16 - 106410*x^15 -
2064744*x^14 + 11945946*x^13 - 1710042*x^12 - 46293732*x^11 + 59161524*x^10
+ 18480192*x^9 - 122366520*x^8 + 81203856*x^7 + 45222000*x^6 - 100598112*x
^5 + 42207168*x^4 + 29609472*x^3 - 22458240*x^2 - sqrt(3)*(1351*x^17 - 2069
8*x^16 - 61436*x^15 - 1192081*x^14 + 6896998*x^13 - 987292*x^12 - 26727704*
x^11 + 34156928*x^10 + 10669552*x^9 - 70648352*x^8 + 46883072*x^7 + 2610894
4*x^6 - 58080352*x^5 + 24368320*x^4 + 17095040*x^3 - 12966272*x^2 + 4724480
*x + 2581504) + 8183040*x + 4471296)*(56*sqrt(3) + 97) + 6*(97*x^17 + 104*x
^16 - 20510*x^15 + 43181*x^14 + 217294*x^13 - 691762*x^12 + 584800*x^11 + 5
21510*x^10 - 1780028*x^9 + 1416580*x^8 + 80528*x^7 - 1518056*x^6 + 1321712*
x^5 - 393392*x^4 - 501952*x^3 + 446848*x^2 - 4*sqrt(3)*(14*x^17 + 15*x^16 -
2960*x^15 + 6232*x^14 + 31362*x^13 - 99844*x^12 + 84404*x^11 + 75267*x^10
- 256916*x^9 + 204458*x^8 + 11616*x^7 - 219104*x^6 + 190768*x^5 - 56784*x^4
- 72448*x^3 + 64496*x^2 - 24480*x - 13376) - 169600*x - 92672)*sqrt(56*sqr
t(3) + 97))*sqrt(56*sqrt(3) + 97) + sqrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*s
qrt(3) - 12) + 6)*((2*sqrt(3)*(3691*x^16 + 17731*x^15 - 951114*x^14 + 45035
9*x^13 + 4370159*x^12 + 30318522*x^11 - 78096668*x^10 + 9429316*x^9 + 14687
7876*x^8 - 197107784*x^7 - 30834152*x^6 + 185125776*x^5 - 132260896*x^4 - 4
5545344*x^3 + 69517536*x^2 - sqrt(3)*(2131*x^16 + 10237*x^15 - 549126*x^14
+ 260015*x^13 + 2523113*x^12 + 17504406*x^11 - 45089132*x^10 + 5444020*x^9
+ 84799980*x^8 - 113800232*x^7 - 17802104*x^6 + 106882416*x^5 - 76360864*x^
4 - 26295616*x^3 + 40135968*x^2 - 7907648*x - 5562368) - 13696448*x - 96343
04)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + (459*x^16 - 1557*x^15 - 26415*x^14 -
1449954*x^13 + 4677912*x^12 + 12651948*x^11 - 55684800*x^10 + 62834256*x^9
+ 8526168*x^8 - 105313392*x^7 + 99605088*x^6 - 18897984*x^5 - 42499296*x^4
+ 37357632*x^3 - 8256960*x^2 - sqrt(3)*(265*x^16 - 899*x^15 - 15249*x^14 -
837130*x^13 + 2700776*x^12 + 7304604*x^11 - 32149640*x^10 + 36277360*x^9 +
4922568*x^8 - 60802736*x^7 + 57507040*x^6 - 10910784*x^5 - 24536992*x^4 + 2
1568448*x^3 - 4767168*x^2 + 1207168*x + 1383424) + 2090880*x + 2396160)*sqr

t(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) + 6*(sqrt(3)*(
4945*x^15 - 37473*x^14 - 490698*x^13 + 2249468*x^12 + 474132*x^11 - 8423784
*x^10 + 5853520*x^9 + 8451720*x^8 - 15320016*x^7 + 768064*x^6 + 10405056*x^
5 - 6627744*x^4 - 700480*x^3 + 2799552*x^2 - sqrt(3)*(2855*x^15 - 21635*x^1
4 - 283306*x^13 + 1298732*x^12 + 273748*x^11 - 4863472*x^10 + 3379536*x^9 +
4879608*x^8 - 8845008*x^7 + 443456*x^6 + 6007360*x^5 - 3826528*x^4 - 40441
6*x^3 + 1616320*x^2 - 1003648*x - 399360) - 1738368*x - 691712)*sqrt(x^3 +
1)*(56*sqrt(3) + 97) + 2*(246*x^15 - 3678*x^14 - 13485*x^13 + 102933*x^12 -
70062*x^11 - 81156*x^10 + 45204*x^9 - 129636*x^8 + 243576*x^7 - 221784*x^6
- 351024*x^5 + 460896*x^4 + 33984*x^3 - 174048*x^2 - sqrt(3)*(142*x^15 - 2
124*x^14 - 7773*x^13 + 59447*x^12 - 40626*x^11 - 46860*x^10 + 26308*x^9 - 7
5276*x^8 + 140472*x^7 - 127784*x^6 - 202896*x^5 + 266016*x^4 + 19712*x^3 -
100512*x^2 + 62400*x + 24832) + 108096*x + 43008)*sqrt(x^3 + 1)*sqrt(56*sqr
t(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) + 108*(130*x^16 - 1682*x^15 + 2496*
x^14 + 7730*x^13 + 1790*x^12 - 35700*x^11 - 7100*x^10 + 86080*x^9 - 49176*x
^8 - 100400*x^7 + 108208*x^6 + 33312*x^5 - 80704*x^4 + 18944*x^3 + 18048*x^
2 - 3*sqrt(3)*(25*x^16 - 324*x^15 + 489*x^14 + 1482*x^13 + 316*x^12 - 6984*
x^11 - 1312*x^10 + 16624*x^9 - 9792*x^8 - 19328*x^7 + 20976*x^6 + 6240*x^5
- 15552*x^4 + 3712*x^3 + 3456*x^2 - 4096*x - 1280) - 21248*x - 6656)*sqrt(5
6*sqrt(3) + 97))*sqrt((9*x^8 + 18*x^7 + 414*x^6 + 180*x^5 + 360*x^4 + 504*x
^3 - 72*x^2 + 36*sqrt(3)*(26*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3 + 42*x
^2 - sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 + 12*x +
4) + 20*x + 8)*sqrt(56*sqrt(3) + 97) - (sqrt(3)*(123*x^6 + 2016*x^5 + 2214*
x^4 + 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1192*x^3
+ 228*x^2 - 112) - 192)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97) + 6*(5*x^6 + 2
7*x^5 + 48*x^4 + 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3
+ 8*x^2 + 4*x) + 12*x + 8)*sqrt(x^3 + 1))*sqrt(sqrt(3)*sqrt(56*sqrt(3) + 9
7)*(7*sqrt(3) - 12) + 6)*(672*sqrt(3) + 1164)^(1/4) - 36*sqrt(3)*(x^7 + 4*x
^6 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) - 144*x + 576)/(x^8 - 4*x^7 +
16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)))/(x^17 + 13*x^16
- 522*x^15 + 1742*x^14 + 3008*x^13 - 16884*x^12 + 11656*x^11 + 23944*x^10 -
42336*x^9 + 9136*x^8 + 36256*x^7 - 27360*x^6 - 256*x^5 + 13376*x^4 - 5760*
x^3 - 1664*x^2 + 256*x)) - 1/1296*sqrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqr
t(3) - 12) + 6)*(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) - 6)*(672*s
qrt(3) + 1164)^(1/4)*log(1/9*(9*x^8 + 18*x^7 + 414*x^6 + 180*x^5 + 360*x^4
+ 504*x^3 - 72*x^2 + 36*sqrt(3)*(26*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3
+ 42*x^2 - sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 +
12*x + 4) + 20*x + 8)*sqrt(56*sqrt(3) + 97) + (sqrt(3)*(123*x^6 + 2016*x^5
+ 2214*x^4 + 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1
192*x^3 + 228*x^2 - 112) - 192)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97) + 6*(5*
x^6 + 27*x^5 + 48*x^4 + 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 +
10*x^3 + 8*x^2 + 4*x) + 12*x + 8)*sqrt(x^3 + 1))*sqrt(sqrt(3)*sqrt(56*sqrt
(3) + 97)*(7*sqrt(3) - 12) + 6)*(672*sqrt(3) + 1164)^(1/4) - 36*sqrt(3)*(x^
7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) - 144*x + 576)/(x^8 -
4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)) + 1/1296*s

```
qrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(sqrt(3)*sqrt(56*sq
rt(3) + 97)*(7*sqrt(3) - 12) - 6)*(672*sqrt(3) + 1164)^(1/4)*log(1/9*(9*x^8
 + 18*x^7 + 414*x^6 + 180*x^5 + 360*x^4 + 504*x^3 - 72*x^2 + 36*sqrt(3)*(26
*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3 + 42*x^2 - sqrt(3)*(15*x^7 + 22*x^
6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(56*sqrt(
3) + 97) - (sqrt(3)*(123*x^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2 - s
qrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*sqr
t(x^3 + 1)*sqrt(56*sqrt(3) + 97) + 6*(5*x^6 + 27*x^5 + 48*x^4 + 58*x^3 + 36
*x^2 - 3*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + 12*x + 8)*
sqrt(x^3 + 1))*sqrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(67
2*sqrt(3) + 1164)^(1/4) - 36*sqrt(3)*(x^7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^3 +
 6*x^2 + 4*x - 8) - 144*x + 576)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 +
32*x^3 + 64*x^2 + 32*x + 16)) + 1/72*sqrt(14*sqrt(3) + 24)*log((x^8 - 16*x^
7 + 112*x^6 - 16*x^5 + 112*x^4 + 224*x^3 + 64*x^2 + 2*(5*x^6 - 54*x^5 + 96*
x^4 - 56*x^3 - 36*x^2 - 3*sqrt(3)*(x^6 - 10*x^5 + 20*x^4 - 8*x^3 - 4*x^2 +
8*x) + 24*x - 16)*sqrt(x^3 + 1)*sqrt(14*sqrt(3) + 24) + 16*sqrt(3)*(x^7 - 2
*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x + 4) + 128*x + 112)/(x^8 + 8*x^7
 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 - 64*x + 16))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x+1)\,(x^2-x+1)}\,\left(x^3-6\sqrt{3}+10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x**3-6*3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x**3 - 6*sqrt(3) + 10)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{x^3+1}\,\left(x^3-6\,\sqrt{3}+10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 + 1)^(1/2)*(x^3 - 6*3^(1/2) + 10)),x)
[Out] int(x/((x^3 + 1)^(1/2)*(x^3 - 6*3^(1/2) + 10)), x)
```

3.88 $\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=222

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}+2x\right)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\,(}{\sqrt{2}}\right.}{2\sqrt{2}\,3^{3/4}}$$

```
[Out] 1/36*arctan(1/2*3^(1/4)*(1-x)*(1-3^(1/2))*2^(1/2)/(x^3-1)^(1/2))*(2-3^(1/2)
)*3^(3/4)*2^(1/2)+1/18*arctan(1/2*3^(1/4)*(1+2*x+3^(1/2))*2^(1/2)/(x^3-1)^(
1/2))*(2-3^(1/2))*3^(3/4)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*(1-x)*(1+3^(1/2)
)*2^(1/2)/(x^3-1)^(1/2))*(2-3^(1/2))*3^(1/4)*2^(1/2)-1/18*arctanh(1/6*(1-3^
(1/2))*(x^3-1)^(1/2)*3^(1/4)*2^(1/2))*(2-3^(1/2))*3^(1/4)*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{501\}$

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{x^3-1}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x^3]*(-10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^
3])]])/(6*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3] + 2
*x))/(Sqrt[2]*Sqrt[-1 + x^3])]])/(3*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan
h[(3^(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])]])/(2*Sqrt[2]*3^(
3/4)) - ((2 - Sqrt[3])*ArcTanh[((1 - Sqrt[3])*Sqrt[-1 + x^3])/(Sqrt[2]*3^(3
/4))]])/(3*Sqrt[2]*3^(3/4))
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx = \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}}{\sqrt{2}}\right.}{3\sqrt{2}\sqrt[4]{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.05, size = 65, normalized size = 0.29

$$-\frac{x^2\sqrt{1-x^3}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,\frac{x^3}{10+6\sqrt{3}}\right)}{\left(20+12\sqrt{3}\right)\sqrt{-1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(-10 - 6*Sqrt[3] + x^3)),x]
[Out] -((x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/(10 + 6*Sqrt[3])])
/((20 + 12*Sqrt[3])*Sqrt[-1 + x^3]))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/((-10 - 6*Sqrt[3] + x^3)*Sqrt[-1 + x^3]),x]')
[Out] Timed out
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 37.97, size = 349, normalized size = 1.57

method	result

default	$\frac{\left(-1-\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{}}{2}\right.}{}\right.}{9\left(2+\sqrt{3}\right)\sqrt{x^3-1}}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(-\frac{2\left(1+\sqrt{3}\right)^2\sqrt{3}}{9}+\frac{\left(1+\sqrt{3}\right)^2}{3}-\frac{2\left(1+\sqrt{3}\right)\sqrt{3}}{9}+\right.}{}$
trager	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*(-1-3^(1/2))/(2+3^(1/2))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/
2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*
3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((-1+x
)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-3^(1/2)*_alpha+_alp
ha+2)/(-1-2*_alpha-3^(1/2))*(-3-I*3^(1/2))*((-1+x)/(-3-I*3^(1/2)))^(1/2)*((
-I*3^(1/2)+2*x+1)/(3-I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x+1)/(I*3^(1/2)+3))^(1
/2)/(x^3-1)^(1/2)*(1+2*_alpha-3^(1/2)*_alpha)*EllipticPi(((-1+x)/(-3/2-1/2*
I*3^(1/2)))^(1/2),1/3*I*_alpha*3^(1/2)-1/2*I*_alpha-1/2*3^(1/2)*_alpha+_alp
ha+1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)),_alph
a=RootOf(_Z^2+(1+3^(1/2))*_Z+2*3^(1/2)+4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7910 vs. $2(146) = 292$.

time = 3.40, size = 7910, normalized size = 35.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/432*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(56*sqrt(3) +
 97)*sqrt(-56*sqrt(3) + 97)*(-672*sqrt(3) + 1164)^(3/4)*arctan(1/1296*(6*sq
rt(x^3 - 1)*((459*x^16 + 13425*x^15 - 33201*x^14 - 950652*x^13 - 997302*x^1
2 + 14760972*x^11 + 47069892*x^10 + 49762248*x^9 - 8212536*x^8 - 84377808*x
^7 - 88427328*x^6 - 25613856*x^5 + 27458496*x^4 + 36433344*x^3 + 12609792*x
^2 + sqrt(3)*(265*x^16 + 7751*x^15 - 19167*x^14 - 548864*x^13 - 575818*x^12
 + 8522268*x^11 + 27175852*x^10 + 28730312*x^9 - 4741560*x^8 - 48715600*x^7
 - 51053600*x^6 - 14788128*x^5 + 15853184*x^4 + 21034816*x^3 + 7280256*x^2
+ 2488832*x - 1889792) - (3691*x^16 + 6128*x^15 - 537864*x^14 - 1586477*x^1
3 + 16210952*x^12 + 77181756*x^11 + 84218362*x^10 - 71018320*x^9 - 25445581
2*x^8 - 196076008*x^7 + 120105208*x^6 + 256326864*x^5 + 134645168*x^4 - 784
64672*x^3 - 78514944*x^2 + sqrt(3)*(2131*x^16 + 3538*x^15 - 310536*x^14 - 9
15953*x^13 + 9359398*x^12 + 44560908*x^11 + 48623494*x^10 - 41002448*x^9 -
146910132*x^8 - 113204536*x^7 + 69342776*x^6 + 147990384*x^5 + 77737424*x^4
 - 45301600*x^3 - 45330624*x^2 - 12242560*x + 7598336) - 21204736*x + 13160
704)*sqrt(-672*sqrt(3) + 1164) + 4310784*x - 3273216)*(-672*sqrt(3) + 1164)
^(3/4) + 3*(984*x^15 + 30612*x^14 + 164676*x^13 + 205368*x^12 - 289200*x^11
 - 183720*x^10 + 886752*x^9 + 71568*x^8 - 1960992*x^7 - 1849440*x^6 + 15584
64*x^5 + 2478912*x^4 + 66432*x^3 - 750336*x^2 + 4*sqrt(3)*(142*x^15 + 4419*
x^14 + 23781*x^13 + 29608*x^12 - 41940*x^11 - 26454*x^10 + 128152*x^9 + 106
92*x^8 - 283320*x^7 - 267064*x^6 + 224784*x^5 + 357936*x^4 + 9632*x^3 - 108
288*x^2 - 96000*x + 33920) - (4945*x^15 + 88617*x^14 + 738528*x^13 + 186004
6*x^12 - 784596*x^11 - 7668708*x^10 - 6570680*x^9 + 6903864*x^8 + 15444144*
x^7 + 4312832*x^6 - 9559200*x^5 - 9359808*x^4 - 155968*x^3 + 3016704*x^2 +
sqrt(3)*(2855*x^15 + 51163*x^14 + 426388*x^13 + 1073898*x^12 - 452980*x^11
- 4427548*x^10 - 3793592*x^9 + 3985944*x^8 + 8916720*x^7 + 2490016*x^6 - 55
19008*x^5 - 5403904*x^4 - 90048*x^3 + 1741696*x^2 + 1543936*x - 545536) + 2
674176*x - 944896)*sqrt(-672*sqrt(3) + 1164) - 665088*x + 235008)*(-672*sqr
t(3) + 1164)^(1/4))*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)
*sqrt(-56*sqrt(3) + 97) + 36*(144*x^17 + 5976*x^16 + 5544*x^15 - 299664*x^1
4 - 1062360*x^13 - 116712*x^12 + 3600000*x^11 + 4761216*x^10 - 1046592*x^9
- 8676864*x^8 - 6592896*x^7 + 2641536*x^6 + 7016832*x^5 + 3699072*x^4 - 186
1632*x^3 - 1640448*x^2 + 12*sqrt(3)*(7*x^17 + 286*x^16 + 238*x^15 - 14255*x
^14 - 50390*x^13 - 5942*x^12 + 171808*x^11 + 226888*x^10 - 48920*x^9 - 4153
84*x^8 - 315088*x^7 + 125600*x^6 + 336608*x^5 + 177344*x^4 - 89152*x^3 - 78
784*x^2 - 39040*x + 18176) + (1164*x^17 + 6276*x^16 - 26052*x^15 - 332844*x
^14 - 1632156*x^13 - 4149132*x^12 - 5805024*x^11 - 318696*x^10 + 12621072*x
```

^9 + 19878720*x^8 + 9619008*x^7 - 13361088*x^6 - 20168256*x^5 - 10936128*x^
4 + 6434304*x^3 + 6426240*x^2 + 24*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 -
 8006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9
 + 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*
x^3 + 154592*x^2 + 78464*x - 36544) - (2340*x^17 + 96354*x^16 + 84798*x^15
- 4817124*x^14 - 17052930*x^13 - 1941678*x^12 + 57963744*x^11 + 76603680*x^
10 - 16678512*x^9 - 139922496*x^8 - 106227360*x^7 + 42453216*x^6 + 11326953
6*x^5 + 59694624*x^4 - 30025728*x^3 - 26496000*x^2 + sqrt(3)*(1351*x^17 + 5
5630*x^16 + 48958*x^15 - 2781167*x^14 - 9845510*x^13 - 1121030*x^12 + 33465
376*x^11 + 44227144*x^10 - 9629336*x^9 - 80784280*x^8 - 61330384*x^7 + 2451
0368*x^6 + 65396192*x^5 + 34464704*x^4 - 17335360*x^3 - 15297472*x^2 - 7571
584*x + 3526400) - 13114368*x + 6107904)*sqrt(-672*sqrt(3) + 1164) + 326169
6*x - 1519104)*sqrt(-672*sqrt(3) + 1164) - 12*(97*x^17 + 523*x^16 - 2171*x^
15 - 27737*x^14 - 136013*x^13 - 345761*x^12 - 483752*x^11 - 26558*x^10 + 10
51756*x^9 + 1656560*x^8 + 801584*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x
^4 + 536192*x^3 + 535520*x^2 + 2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8
006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 +
 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^
3 + 154592*x^2 + 78464*x - 36544) + 271808*x - 126592)*sqrt(-672*sqrt(3) +
1164) - 811008*x + 377856)*sqrt(-56*sqrt(3) + 97) - (sqrt(x^3 - 1)*((459*x^
16 + 1557*x^15 - 26415*x^14 + 1449954*x^13 + 4677912*x^12 - 12651948*x^11 -
 55684800*x^10 - 62834256*x^9 + 8526168*x^8 + 105313392*x^7 + 99605088*x^6
+ 18897984*x^5 - 42499296*x^4 - 37357632*x^3 - 8256960*x^2 + sqrt(3)*(265*x
^16 + 899*x^15 - 15249*x^14 + 837130*x^13 + 2700776*x^12 - 7304604*x^11 - 3
2149640*x^10 - 36277360*x^9 + 4922568*x^8 + 60802736*x^7 + 57507040*x^6 + 1
0910784*x^5 - 24536992*x^4 - 21568448*x^3 - 4767168*x^2 - 1207168*x + 13834
24) - (3691*x^16 - 17731*x^15 - 951114*x^14 - 450359*x^13 + 4370159*x^12 -
30318522*x^11 - 78096668*x^10 - 9429316*x^9 + 146877876*x^8 + 197107784*x^7
 - 30834152*x^6 - 185125776*x^5 - 132260896*x^4 + 45545344*x^3 + 69517536*x
^2 + sqrt(3)*(2131*x^16 - 10237*x^15 - 549126*x^14 - 260015*x^13 + 2523113*
x^12 - 17504406*x^11 - 45089132*x^10 - 5444020*x^9 + 84799980*x^8 + 1138002
32*x^7 - 17802104*x^6 - 106882416*x^5 - 76360864*x^4 + 26295616*x^3 + 40135
968*x^2 + 7907648*x - 5562368) + 13696448*x - 9634304)*sqrt(-672*sqrt(3) +
1164) - 2090880*x + 2396160)*(-672*sqrt(3) + 1164)^(3/4) + 3*(984*x^15 + 14
712*x^14 - 53940*x^13 - 411732*x^12 - 280248*x^11 + 324624*x^10 + 180816*x^
9 + 518544*x^8 + 974304*x^7 + 887136*x^6 - 1404096*x^5 - 1843584*x^4 + 1359
36*x^3 + 696192*x^2 + 4*sqrt(3)*(142*x^15 + 2124*x^14 - 7773*x^13 - 59447*x
^12 - 40626*x^11 + 46860*x^10 + 26308*x^9 + 75276*x^8 + 140472*x^7 + 127784
*x^6 - 202896*x^5 - 266016*x^4 + 19712*x^3 + 100512*x^2 + 62400*x - 24832)
- (4945*x^15 + 37473*x^14 - 490698*x^13 - 2249468*x^12 + 474132*x^11 + 8423
784*x^10 + 5853520*x^9 - 8451720*x^8 - 15320016*x^7 - 768064*x^6 + 10405056
*x^5 + 6627744*x^4 - 700480*x^3 - 2799552*x^2 + sqrt(3)*(2855*x^15 + 21635*
x^14 - 283306*x^13 - 1298732*x^12 + 273748*x^11 + 4863472*x^10 + 3379536*x^
9 - 4879608*x^8 - 8845008*x^7 - 443456*x^6 + 6007360*x^5 + 3826528*x^4 - 40
4416*x^3 - 1616320*x^2 - 1003648*x + 399360) - 1738368*x + 691712)*sqrt(-67

2*sqrt(3) + 1164) + 432384*x - 172032)*(-672*sqrt(3) + 1164)^(1/4))*sqrt(2*
(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*sqrt(-56*sqrt(3) + 97) + 6
*(4680*x^16 + 60552*x^15 + 89856*x^14 - 278280*x^13 + 64440*x^12 + 1285200*
x^11 - 255600*x^10 - 3098880*x^9 - 1770336*x^8 + 3614400*x^7 + 3895488*x^6
- 1199232*x^5 - 2905344*x^4 - 681984*x^3 + 649728*x^2 + 108*sqrt(3)*(25*x^1
6 + 324*x^15 + 489*x^14 - 1482*x^13 + 316*x^12 + 6984*x^11 - 1312*x^10 - 16
624*x^9 - 9792*x^8 + 19328*x^7 + 20976*x^6 - 6240*x^5 - 15552*x^4 - 3712*x^
3 + 3456*x^2 + 4096*x - 1280) + (1164*x^17 - 1248*x^16 - 246120*x^15 - 5181
72*x^14 + 2607528*x^13 + 8301144*x^12 + 7017600*x^11 - 6258120*x^10 - 21360
336*x^9 - 16998960*x^8 + 966336*x^7 + 18216672*x^6 + 15860544*x^5 + 4720704
*x^4 - 6023424*x^3 - 5362176*x^2 + 48*sqrt(3)*(14*x^17 - 15*x^16 - 2960*x^1
5 - 6232*x^14 + 31362*x^13 + 99844*x^12 + 84404*x^11 - 75267*x^10 - 256916*
x^9 - 204458*x^8 + 11616*x^7 + 219104*x^6 + 190768*x^5 + 56784*x^4 - 72448*
x^3 - 64496*x^2 - 24480*x + 13376) - (2340*x^17 + 35850*x^16 - 106410*x^15
+ 2064744*x^14 + 11945946*x^13 + 1710042*x^12 - 46293732*x^11 - 59161524*x^
10 + 18480192*x^9 + 122366520*x^8 + 81203856*x^7 - 45222000*x^6 - 100598112
*x^5 - 42207168*x^4 + 29609472*x^3 + 22458240*x^2 + sqrt(3)*(1351*x^17 + 20
698*x^16 - 61436*x^15 + 1192081*x^14 + 6896998*x^13 + 987292*x^12 - 2672770
4*x^11 - 34156928*x^10 + 10669552*x^9 + 70648352*x^8 + 46883072*x^7 - 26108
944*x^6 - 58080352*x^5 - 24368320*x^4 + 17095040*x^3 + 12966272*x^2 + 47244
80*x - 2581504) + 8183040*x - 4471296)*sqrt(-672*sqrt(3) + 1164) - 2035200*
x + 1112064)*sqrt(-672*sqrt(3) + 1164) - 24*(627*x^16 + 14286*x^15 + 39762*
x^14 - 50142*x^13 - 216816*x^12 - 112284*x^11 + 325707*x^10 + 586326*x^9 -
3294*x^8 - 631752*x^7 - 539220*x^6 + 184392*x^5 + 483816*x^4 + 115296*x^3 -
108576*x^2 + 2*sqrt(3)*(181*x^16 + 4124*x^15 + 11478*x^14 - 14474*x^13 - 6
2584*x^12 - 32412*x^11 + 94021*x^10 + 169244*x^9 - 954*x^8 - 182368*x^7 - 1
55648*x^6 + 53232*x^5 + 139664*x^4 + 33280*x^3 - 31344*x^2 - 37024*x + 1158
4) - 128256*x + 40128)*sqrt(-672*sqrt(3) + 1164) + 764928*x - 239616)*sqrt(
-56*sqrt(3) + 97))*sqrt((36*x^8 - 72*x^7 + 1656*x^6 - 720*x^5 + 1440*x^4 -
2016*x^3 + (60*x^6 - 324*x^5 + 576*x^4 - 696*x^3 + 432*x^2 + 36*sqrt(3)*(x^
6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - (123*x^6 - 2016*x^5 + 2214*x^4
- 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3 +
228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) - 144*x + 96)*sqrt(x^3 - 1)
sqrt(2(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3) + 11
64)^(1/4) - 288*x^2 - 144*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*
x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^2 +
sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4) +
20*x - 8)*sqrt(-672*sqrt(3) + 1164) + 576*x + 2304)/(x^8 + 4*x^7 + 16*x^6 +
16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)))/(x^17 - 13*x^16 - 522*x^1
5 - 1742*x^14 + 3008*x^13 + 16884*x^12 + 11656*x^11 - 23944*x^10 - 42336*x^
9 - 9136*x^8 + 36256*x^7 + 27360*x^6 - 256*x^5 - 13376*x^4 - 5760*x^3 + 166
4*x^2 + 256*x)) + 1/432*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) +
24)*(56*sqrt(3) + 97)*sqrt(-56*sqrt(3) + 97)*(-672*sqrt(3) + 1164)^(3/4)*a
rctan(1/1296*(6*sqrt(x^3 - 1)*((459*x^16 + 13425*x^15 - 33201*x^14 - 950652
*x^13 - 997302*x^12 + 14760972*x^11 + 47069892*x^10 + 49762248*x^9 - 821253

6*x^8 - 84377808*x^7 - 88427328*x^6 - 25613856*x^5 + 27458496*x^4 + 3643334
4*x^3 + 12609792*x^2 + sqrt(3)*(265*x^16 + 7751*x^15 - 19167*x^14 - 548864*
x^13 - 575818*x^12 + 8522268*x^11 + 27175852*x^10 + 28730312*x^9 - 4741560*
x^8 - 48715600*x^7 - 51053600*x^6 - 14788128*x^5 + 15853184*x^4 + 21034816*
x^3 + 7280256*x^2 + 2488832*x - 1889792) - (3691*x^16 + 6128*x^15 - 537864*
x^14 - 1586477*x^13 + 16210952*x^12 + 77181756*x^11 + 84218362*x^10 - 71018
320*x^9 - 254455812*x^8 - 196076008*x^7 + 120105208*x^6 + 256326864*x^5 + 1
34645168*x^4 - 78464672*x^3 - 78514944*x^2 + sqrt(3)*(2131*x^16 + 3538*x^15
 - 310536*x^14 - 915953*x^13 + 9359398*x^12 + 44560908*x^11 + 48623494*x^10
 - 41002448*x^9 - 146910132*x^8 - 113204536*x^7 + 69342776*x^6 + 147990384*
x^5 + 77737424*x^4 - 45301600*x^3 - 45330624*x^2 - 12242560*x + 7598336) -
21204736*x + 13160704)*sqrt(-672*sqrt(3) + 1164) + 4310784*x - 3273216)*(-6
72*sqrt(3) + 1164)^(3/4) + 3*(984*x^15 + 30612*x^14 + 164676*x^13 + 205368*
x^12 - 289200*x^11 - 183720*x^10 + 886752*x^9 + 71568*x^8 - 1960992*x^7 - 1
849440*x^6 + 1558464*x^5 + 2478912*x^4 + 66432*x^3 - 750336*x^2 + 4*sqrt(3)
*(142*x^15 + 4419*x^14 + 23781*x^13 + 29608*x^12 - 41940*x^11 - 26454*x^10
+ 128152*x^9 + 10692*x^8 - 283320*x^7 - 267064*x^6 + 224784*x^5 + 357936*x^
4 + 9632*x^3 - 108288*x^2 - 96000*x + 33920) - (4945*x^15 + 88617*x^14 + 73
8528*x^13 + 1860046*x^12 - 784596*x^11 - 7668708*x^10 - 6570680*x^9 + 69038
64*x^8 + 15444144*x^7 + 4312832*x^6 - 9559200*x^5 - 9359808*x^4 - 155968*x^
3 + 3016704*x^2 + sqrt(3)*(2855*x^15 + 51163*x^14 + 426388*x^13 + 1073898*x
^12 - 452980*x^11 - 4427548*x^10 - 3793592*x^9 + 3985944*x^8 + 8916720*x^7
+ 2490016*x^6 - 5519008*x^5 - 5403904*x^4 - 90048*x^3 + 1741696*x^2 + 15439
36*x - 545536) + 2674176*x - 944896)*sqrt(-672*sqrt(3) + 1164) - 665088*x +
 235008)*(-672*sqrt(3) + 1164)^(1/4))*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqr
t(3) + 1164) + 24)*sqrt(-56*sqrt(3) + 97) - 36*(144*x^17 + 5976*x^16 + 5544
*x^15 - 299664*x^14 - 1062360*x^13 - 116712*x^12 + 3600000*x^11 + 4761216*x
^10 - 1046592*x^9 - 8676864*x^8 - 6592896*x^7 + 2641536*x^6 + 7016832*x^5 +
 3699072*x^4 - 1861632*x^3 - 1640448*x^2 + 12*sqrt(3)*(7*x^17 + 286*x^16 +
238*x^15 - 14255*x^14 - 50390*x^13 - 5942*x^12 + 171808*x^11 + 226888*x^10
- 48920*x^9 - 415384*x^8 - 315088*x^7 + 125600*x^6 + 336608*x^5 + 177344*x^
4 - 89152*x^3 - 78784*x^2 - 39040*x + 18176) + (1164*x^17 + 6276*x^16 - 260
52*x^15 - 332844*x^14 - 1632156*x^13 - 4149132*x^12 - 5805024*x^11 - 318696
*x^10 + 12621072*x^9 + 19878720*x^8 + 9619008*x^7 - 13361088*x^6 - 20168256
*x^5 - 10936128*x^4 + 6434304*x^3 + 6426240*x^2 + 24*sqrt(3)*(28*x^17 + 151
*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661
*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 26
3080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 36544) - (2340*x^17 + 96354*
x^16 + 84798*x^15 - 4817124*x^14 - 17052930*x^13 - 1941678*x^12 + 57963744*
x^11 + 76603680*x^10 - 16678512*x^9 - 139922496*x^8 - 106227360*x^7 + 42453
216*x^6 + 113269536*x^5 + 59694624*x^4 - 30025728*x^3 - 26496000*x^2 + sqrt
(3)*(1351*x^17 + 55630*x^16 + 48958*x^15 - 2781167*x^14 - 9845510*x^13 - 11
21030*x^12 + 33465376*x^11 + 44227144*x^10 - 9629336*x^9 - 80784280*x^8 - 6
1330384*x^7 + 24510368*x^6 + 65396192*x^5 + 34464704*x^4 - 17335360*x^3 - 1
5297472*x^2 - 7571584*x + 3526400) - 13114368*x + 6107904)*sqrt(-672*sqrt(3

) + 1164) + 3261696*x - 1519104)*sqrt(-672*sqrt(3) + 1164) - 12*(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^13 - 345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 801584*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^2 + 2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 36544) + 271808*x - 126592)*sqrt(-672*sqrt(3) + 1164) - 811008*x + 377856)*sqrt(-56*sqrt(3) + 97) - (sqrt(x^3 - 1)*((459*x^16 + 1557*x^15 - 26415*x^14 + 1449954*x^13 + 4677912*x^12 - 12651948*x^11 - 55684800*x^10 - 62834256*x^9 + 8526168*x^8 + 105313392*x^7 + 99605088*x^6 + 18897984*x^5 - 42499296*x^4 - 37357632*x^3 - 8256960*x^2 + sqrt(3)*(265*x^16 + 899*x^15 - 15249*x^14 + 837130*x^13 + 2700776*x^12 - 7304604*x^11 - 32149640*x^10 - 36277360*x^9 + 4922568*x^8 + 60802736*x^7 + 57507040*x^6 + 10910784*x^5 - 24536992*x^4 - 21568448*x^3 - 4767168*x^2 - 1207168*x + 1383424) - (3691*x^16 - 17731*x^15 - 951114*x^14 - 450359*x^13 + 4370159*x^12 - 30318522*x^11 - 78096668*x^10 - 9429316*x^9 + 146877876*x^8 + 197107784*x^7 - 30834152*x^6 - 185125776*x^5 - 132260896*x^4 + 45545344*x^3 + 69517536*x^2 + sqrt(3)*(2131*x^16 - 10237*x^15 - 549126*x^14 - 260015*x^13 + 2523113*x^12 - 17504406*x^11 - 45089132*x^10 - 5444020*x^9 + 84799980*x^8 + 113800232*x^7 - 17802104*x^6 - 106882416*x^5 - 76360864*x^4 + 26295616*x^3 + 40135968*x^2 + 7907648*x - 5562368) + 13696448*x - 9634304)*sqrt(-672*sqrt(3) + 1164) - 2090880*x + 2396160)*(-672*sqrt(3) + 1164)^(3/4) + 3*(984*x^15 + 14712*x^14 - 53940*x^13 - 411732*x^12 - 280248*x^11 + 324624*x^10 + 180816*x^9 + 518544*x^8 + 974304*x^7 + 887136*x^6 - 1404096*x^5 - 1843584*x^4 + 135936*x^3 + 696192*x^2 + 4*sqrt(3)*(142*x^15 + 2124*x^14 - 7773*x^13 - 59447*x^12 - 40626*x^11 + 46860*x^10 + 26308*x^9 + 75276*x^8 + 140472*x^7 + 127784*x^6 - 202896*x^5 - 266016*x^4 + 19712*x^3 + 100512*x^2 + 62400*x - 24832) - (4945*x^15 + 37473*x^14 - 490698*x^13 - 2249468*x^12 + 474132*x^11 + 8423784*x^10 + 5853520*x^9 - 8451720*x^8 - 15320016*x^7 - 768064*x^6 + 10405056*x^5 + 6627744*x^4 - 700480*x^3 - 2799552*x^2 + sqrt(3)*(2855*x^15 + 21635*x^14 - 283306*x^13 - 1298732*x^12 + 273748*x^11 + 4863472*x^10 + 3379536*x^9 - 4879608*x^8 - 8845008*x^7 - 443456*x^6 + 6007360*x^5 + 3826528*x^4 - 404416*x^3 - 1616320*x^2 - 1003648*x + 399360) - 1738368*x + 691712)*sqrt(-672*sqrt(3) + 1164) + 432384*x - 172032)*(-672*sqrt(3) + 1164)^(1/4))*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*sqrt(-56*sqrt(3) + 97) - 6*(4680*x^16 + 60552*x^15 + 89856*x^14 - 278280*x^13 + 64440*x^12 + 1285200*x^11 - 255600*x^10 - 3098880*x^9 - 1770336*x^8 + 3614400*x^7 + 3895488*x^6 - 1199232*x^5 - 2905344*x^4 - 681984*x^3 + 649728*x^2 + 108*sqrt(3)*(25*x^16 + 324*x^15 + 489*x^14 - 1482*x^13 + 316*x^12 + 6984*x^11 - 1312*x^10 - 16624*x^9 - 9792*x^8 + 19328*x^7 + 20976*x^6 - 6240*x^5 - 15552*x^4 - 3712*x^3 + 3456*x^2 + 4096*x - 1280) + (1164*x^17 - 1248*x^16 - 246120*x^15 - 518172*x^14 + 2607528*x^13 + 8301144*x^12 + 7017600*x^11 - 6258120*x^10 - 21360336*x^9 - 16998960*x^8 + 966336*x^7 + 18216672*x^6 + 15860544*x^5 + 4720704*x^4 - 6023424*x^3 - 5362176*x^2 + 48*sqrt(3)*(14*x^17 - 15*x^16 - 2960*x^15 - 6232*x^14 + 31362*x^13 + 99844*x^12 + 84404*x^11 - 75

267*x^10 - 256916*x^9 - 204458*x^8 + 11616*x^7 + 219104*x^6 + 190768*x^5 + 56784*x^4 - 72448*x^3 - 64496*x^2 - 24480*x + 13376) - (2340*x^17 + 35850*x^16 - 106410*x^15 + 2064744*x^14 + 11945946*x^13 + 1710042*x^12 - 46293732*x^11 - 59161524*x^10 + 18480192*x^9 + 122366520*x^8 + 81203856*x^7 - 45222000*x^6 - 100598112*x^5 - 42207168*x^4 + 29609472*x^3 + 22458240*x^2 + sqrt(3)*(1351*x^17 + 20698*x^16 - 61436*x^15 + 1192081*x^14 + 6896998*x^13 + 987292*x^12 - 26727704*x^11 - 34156928*x^10 + 10669552*x^9 + 70648352*x^8 + 46883072*x^7 - 26108944*x^6 - 58080352*x^5 - 24368320*x^4 + 17095040*x^3 + 12966272*x^2 + 4724480*x - 2581504) + 8183040*x - 4471296)*sqrt(-672*sqrt(3) + 1164) - 2035200*x + 1112064)*sqrt(-672*sqrt(3) + 1164) - 24*(627*x^16 + 14286*x^15 + 39762*x^14 - 50142*x^13 - 216816*x^12 - 112284*x^11 + 325707*x^10 + 586326*x^9 - 3294*x^8 - 631752*x^7 - 539220*x^6 + 184392*x^5 + 483816*x^4 + 115296*x^3 - 108576*x^2 + 2*sqrt(3)*(181*x^16 + 4124*x^15 + 11478*x^14 - 14474*x^13 - 62584*x^12 - 32412*x^11 + 94021*x^10 + 169244*x^9 - 954*x^8 - 182368*x^7 - 155648*x^6 + 53232*x^5 + 139664*x^4 + 33280*x^3 - 31344*x^2 - 37024*x + 11584) - 128256*x + 40128)*sqrt(-672*sqrt(3) + 1164) + 764928*x - 239616)*sqrt(-56*sqrt(3) + 97))*sqrt((36*x^8 - 72*x^7 + 1656*x^6 - 720*x^5 + 1440*x^4 - 2016*x^3 - (60*x^6 - 324*x^5 + 576*x^4 - 696*x^3 + 432*x^2 + 36*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - (123*x^6 - 2016*x^5 + 2214*x^4 - 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) - 144*x + 96)*sqrt(x^3 - 1)*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 - 144*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^2 + sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4) + 20*x - 8)*sqrt(-672*sqrt(3) + 1164) + 576*x + 2304)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)))/(x^17 - 13*x^16 - 522*x^15 - 1742*x^14 + 3008*x^13 + 16884*x^12 + 11656*x^11 - 23944*x^10 - 42336*x^9 - 9136*x^8 + 36256*x^7 + 27360*x^6 - 256*x^5 - 13376*x^4 - 5760*x^3 + 1664*x^2 + 256*x)) + 1/5184*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*((7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) - 12)*(-672*sqrt(3) + 1164)^(1/4)*log(1/36*(36*x^8 - 72*x^7 + 1656*x^6 - 720*x^5 + 1440*x^4 - 2016*x^3 + (60*x^6 - 324*x^5 + 576*x^4 - 696*x^3 + 432*x^2 + 36*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - (123*x^6 - 2016*x^5 + 2214*x^4 - 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) - 144*x + 96)*sqrt(x^3 - 1)*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 - 144*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^2 + sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4) + 20*x - 8)*sqrt(-672*sqrt(3) + 1164) + 576*x + 2304)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 1/5184*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*((7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) - 12)*(-672*sqrt(3) + 1164)^(1/4)*log(1/36*(36*x^8 - 72*x^7 + 1656*x^6 - 720*x^5 + 1440*x^4 - 2016*x^3 - (60*x^6 - 324*x^5 + 576*x

```
^4 - 696*x^3 + 432*x^2 + 36*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2
- 4*x) - (123*x^6 - 2016*x^5 + 2214*x^4 - 2064*x^3 + 396*x^2 + sqrt(3)*(71*
x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt
(3) + 1164) - 144*x + 96)*sqrt(x^3 - 1)*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*s
qrt(3) + 1164) + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 - 144*sqrt(3)*(x
^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6
 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^2 + sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 -
27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4) + 20*x - 8)*sqrt(-672*sqrt(3) + 1164)
+ 576*x + 2304)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 -
 32*x + 16)) + 1/72*sqrt(14*sqrt(3) - 24)*log((x^8 + 16*x^7 + 112*x^6 + 16*
x^5 + 112*x^4 - 224*x^3 + 64*x^2 - 2*(5*x^6 + 54*x^5 + 96*x^4 + 56*x^3 - 36
*x^2 + 3*sqrt(3)*(x^6 + 10*x^5 + 20*x^4 + 8*x^3 - 4*x^2 - 8*x) - 24*x - 16)
*sqrt(x^3 - 1)*sqrt(14*sqrt(3) - 24) + 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*
x^4 + 2*x^3 - 6*x^2 + 4*x - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^
5 - 56*x^4 - 32*x^3 + 64*x^2 + 64*x + 16))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x-1)\,(x^2+x+1)}\,\left(x^3-6\sqrt{3}-10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x**3-6*3**(1/2))/(x**3-1)**(1/2),x)
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x**3 - 6*sqrt(3) - 10)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x}{\sqrt{x^3-1}\,\left(-x^3+6\,\sqrt{3}+10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((x^3 - 1)^(1/2)*(6*3^(1/2) - x^3 + 10)),x)
[Out] int(-x/((x^3 - 1)^(1/2)*(6*3^(1/2) - x^3 + 10)), x)
```

3.89 $\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=214

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{2\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-1+x^3}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\right.}{6\sqrt{2}}$$

```
[Out] -1/12*arctan(1/2*3^(1/4)*(1-x)*(1-3^(1/2))*2^(1/2)/(x^3-1)^(1/2))*(2+3^(1/2
))*3^(1/4)*2^(1/2)+1/18*arctan(1/6*(1+3^(1/2))*(x^3-1)^(1/2)*3^(1/4)*2^(1/2
))*(2+3^(1/2))*3^(1/4)*2^(1/2)+1/18*arctanh(1/2*3^(1/4)*(1+2*x-3^(1/2))*2^(
1/2)/(x^3-1)^(1/2))*(2+3^(1/2))*3^(3/4)*2^(1/2)+1/36*arctanh(1/2*3^(1/4)*(1
-x)*(1+3^(1/2))*2^(1/2)/(x^3-1)^(1/2))*(2+3^(1/2))*3^(3/4)*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {501}

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x^3-1}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(2x-\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] -1/2*((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1
 + x^3])])/(Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-1
 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTanh[(
3^(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(6*Sqrt[2]*3^(1/4
)) + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3] + 2*x))/(Sqrt[2]*Sqrt[-1
+ x^3])])/(3*Sqrt[2]*3^(1/4))
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{2\sqrt{2}\,3^{3/4}} + \frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\left(\right.\right.}{3\sqrt{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.05, size = 68, normalized size = 0.32

$$\frac{x^2\sqrt{1-x^3}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,-\frac{x^3}{-10+6\sqrt{3}}\right)}{4\left(-5+3\sqrt{3}\right)\sqrt{-1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] (x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, -(x^3/(-10 + 6*Sqrt[3]))
])/(4*(-5 + 3*Sqrt[3])*Sqrt[-1 + x^3])
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/((-10 + 6*Sqrt[3] + x^3)*Sqrt[-1 + x^3]),x]')
```

```
[Out] Timed out
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 37.46, size = 350, normalized size = 1.64

method	result

default	$\frac{\left(\sqrt{3}-1\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)}{3}\right.}{9\left(-2+\sqrt{3}\right)\sqrt{x^{3}-1}}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\frac{2\left(1-\sqrt{3}\right)^{2}\sqrt{3}}{9}+\frac{\left(1-\sqrt{3}\right)^{2}}{3}+\frac{2\sqrt{3}\left(1-\sqrt{3}\right)}{9}+\right.}{}$
trager	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*(3^(1/2)-1)/(-2+3^(1/2))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/
2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*
3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((-1+x
)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-3^(1/2)*_alpha-_alph
a-2)/(-3^(1/2)+2*_alpha+1)*(-3-I*3^(1/2))*((-1+x)/(-3-I*3^(1/2)))^(1/2)*((-
I*3^(1/2)+2*x+1)/(3-I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x+1)/(I*3^(1/2)+3))^(1/
2)/(x^3-1)^(1/2)*(1+2*_alpha+3^(1/2)*_alpha)*EllipticPi(((-1+x)/(-3/2-1/2*I
*3^(1/2)))^(1/2),1/3*I*_alpha*3^(1/2)+1/2*I*_alpha+1/2*3^(1/2)*_alpha+_alph
a+1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)),_alpha
=RootOf(_Z^2+(1-3^(1/2))*_Z-2*3^(1/2)+4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8105 vs. $2(148) = 296$.

time = 3.41, size = 8105, normalized size = 37.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/216*sqrt(3)*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*
(672*sqrt(3) + 1164)^(1/4)*(56*sqrt(3) + 97)*(56*sqrt(3) - 97)*arctan(-1/64
8*(432*sqrt(3)*(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^13 -
 345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 80158
4*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^2 -
2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x
^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 3214
12*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 3654
4) + 271808*x - 126592)*(56*sqrt(3) + 97) + 72*sqrt(3)*(sqrt(3)*(2340*x^17
+ 96354*x^16 + 84798*x^15 - 4817124*x^14 - 17052930*x^13 - 1941678*x^12 + 5
7963744*x^11 + 76603680*x^10 - 16678512*x^9 - 139922496*x^8 - 106227360*x^7
 + 42453216*x^6 + 113269536*x^5 + 59694624*x^4 - 30025728*x^3 - 26496000*x^
2 - sqrt(3)*(1351*x^17 + 55630*x^16 + 48958*x^15 - 2781167*x^14 - 9845510*x
^13 - 1121030*x^12 + 33465376*x^11 + 44227144*x^10 - 9629336*x^9 - 80784280
*x^8 - 61330384*x^7 + 24510368*x^6 + 65396192*x^5 + 34464704*x^4 - 17335360
*x^3 - 15297472*x^2 - 7571584*x + 3526400) - 13114368*x + 6107904)*(56*sqrt
(3) + 97) - 6*(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^13 -
345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 801584
*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^2 - 2
*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x^
12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 32141
2*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 36544
) + 271808*x - 126592)*sqrt(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) - sqrt(
1/2)*(288*sqrt(3)*(627*x^16 + 14286*x^15 + 39762*x^14 - 50142*x^13 - 216816
*x^12 - 112284*x^11 + 325707*x^10 + 586326*x^9 - 3294*x^8 - 631752*x^7 - 53
9220*x^6 + 184392*x^5 + 483816*x^4 + 115296*x^3 - 108576*x^2 - 2*sqrt(3)*(1
81*x^16 + 4124*x^15 + 11478*x^14 - 14474*x^13 - 62584*x^12 - 32412*x^11 + 9
4021*x^10 + 169244*x^9 - 954*x^8 - 182368*x^7 - 155648*x^6 + 53232*x^5 + 13
9664*x^4 + 33280*x^3 - 31344*x^2 - 37024*x + 11584) - 128256*x + 40128)*(56
*sqrt(3) + 97) + 24*sqrt(3)*(sqrt(3)*(2340*x^17 + 35850*x^16 - 106410*x^15
+ 2064744*x^14 + 11945946*x^13 + 1710042*x^12 - 46293732*x^11 - 59161524*x^
10 + 18480192*x^9 + 122366520*x^8 + 81203856*x^7 - 45222000*x^6 - 100598112
*x^5 - 42207168*x^4 + 29609472*x^3 + 22458240*x^2 - sqrt(3)*(1351*x^17 + 20
698*x^16 - 61436*x^15 + 1192081*x^14 + 6896998*x^13 + 987292*x^12 - 2672770
4*x^11 - 34156928*x^10 + 10669552*x^9 + 70648352*x^8 + 46883072*x^7 - 26108
944*x^6 - 58080352*x^5 - 24368320*x^4 + 17095040*x^3 + 12966272*x^2 + 47244
80*x - 2581504) + 8183040*x - 4471296)*(56*sqrt(3) + 97) - 6*(97*x^17 - 104
```

*x^16 - 20510*x^15 - 43181*x^14 + 217294*x^13 + 691762*x^12 + 584800*x^11 -
521510*x^10 - 1780028*x^9 - 1416580*x^8 + 80528*x^7 + 1518056*x^6 + 132171
2*x^5 + 393392*x^4 - 501952*x^3 - 446848*x^2 - 4*sqrt(3)*(14*x^17 - 15*x^16
- 2960*x^15 - 6232*x^14 + 31362*x^13 + 99844*x^12 + 84404*x^11 - 75267*x^1
0 - 256916*x^9 - 204458*x^8 + 11616*x^7 + 219104*x^6 + 190768*x^5 + 56784*x
^4 - 72448*x^3 - 64496*x^2 - 24480*x + 13376) - 169600*x + 92672)*sqrt(56*s
qrt(3) + 97))*sqrt(56*sqrt(3) + 97) - sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)
*(7*sqrt(3) - 12) + 24)*((2*sqrt(3)*(3691*x^16 - 17731*x^15 - 951114*x^14 -
450359*x^13 + 4370159*x^12 - 30318522*x^11 - 78096668*x^10 - 9429316*x^9 +
146877876*x^8 + 197107784*x^7 - 30834152*x^6 - 185125776*x^5 - 132260896*x
^4 + 45545344*x^3 + 69517536*x^2 - sqrt(3)*(2131*x^16 - 10237*x^15 - 549126
*x^14 - 260015*x^13 + 2523113*x^12 - 17504406*x^11 - 45089132*x^10 - 544402
0*x^9 + 84799980*x^8 + 113800232*x^7 - 17802104*x^6 - 106882416*x^5 - 76360
864*x^4 + 26295616*x^3 + 40135968*x^2 + 7907648*x - 5562368) + 13696448*x -
9634304)*sqrt(x^3 - 1)*(56*sqrt(3) + 97) - (459*x^16 + 1557*x^15 - 26415*x
^14 + 1449954*x^13 + 4677912*x^12 - 12651948*x^11 - 55684800*x^10 - 6283425
6*x^9 + 8526168*x^8 + 105313392*x^7 + 99605088*x^6 + 18897984*x^5 - 4249929
6*x^4 - 37357632*x^3 - 8256960*x^2 - sqrt(3)*(265*x^16 + 899*x^15 - 15249*x
^14 + 837130*x^13 + 2700776*x^12 - 7304604*x^11 - 32149640*x^10 - 36277360*
x^9 + 4922568*x^8 + 60802736*x^7 + 57507040*x^6 + 10910784*x^5 - 24536992*x
^4 - 21568448*x^3 - 4767168*x^2 - 1207168*x + 1383424) - 2090880*x + 239616
0)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) + 6*(sqr
t(3)*(4945*x^15 + 37473*x^14 - 490698*x^13 - 2249468*x^12 + 474132*x^11 + 8
423784*x^10 + 5853520*x^9 - 8451720*x^8 - 15320016*x^7 - 768064*x^6 + 10405
056*x^5 + 6627744*x^4 - 700480*x^3 - 2799552*x^2 - sqrt(3)*(2855*x^15 + 216
35*x^14 - 283306*x^13 - 1298732*x^12 + 273748*x^11 + 4863472*x^10 + 3379536
*x^9 - 4879608*x^8 - 8845008*x^7 - 443456*x^6 + 6007360*x^5 + 3826528*x^4 -
404416*x^3 - 1616320*x^2 - 1003648*x + 399360) - 1738368*x + 691712)*sqrt(
x^3 - 1)*(56*sqrt(3) + 97) - 2*(246*x^15 + 3678*x^14 - 13485*x^13 - 102933*
x^12 - 70062*x^11 + 81156*x^10 + 45204*x^9 + 129636*x^8 + 243576*x^7 + 2217
84*x^6 - 351024*x^5 - 460896*x^4 + 33984*x^3 + 174048*x^2 - sqrt(3)*(142*x^
15 + 2124*x^14 - 7773*x^13 - 59447*x^12 - 40626*x^11 + 46860*x^10 + 26308*x
^9 + 75276*x^8 + 140472*x^7 + 127784*x^6 - 202896*x^5 - 266016*x^4 + 19712*
x^3 + 100512*x^2 + 62400*x - 24832) + 108096*x - 43008)*sqrt(x^3 - 1)*sqrt(
56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) - 216*(130*x^16 + 1682*x^15 +
2496*x^14 - 7730*x^13 + 1790*x^12 + 35700*x^11 - 7100*x^10 - 86080*x^9 - 4
9176*x^8 + 100400*x^7 + 108208*x^6 - 33312*x^5 - 80704*x^4 - 18944*x^3 + 18
048*x^2 - 3*sqrt(3)*(25*x^16 + 324*x^15 + 489*x^14 - 1482*x^13 + 316*x^12 +
6984*x^11 - 1312*x^10 - 16624*x^9 - 9792*x^8 + 19328*x^7 + 20976*x^6 - 624
0*x^5 - 15552*x^4 - 3712*x^3 + 3456*x^2 + 4096*x - 1280) + 21248*x - 6656)*
sqrt(56*sqrt(3) + 97))*sqrt((18*x^8 - 36*x^7 + 828*x^6 - 360*x^5 + 720*x^4
- 1008*x^3 - 144*x^2 + 72*sqrt(3)*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x
^3 - 42*x^2 - sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2
+ 12*x - 4) + 20*x - 8)*sqrt(56*sqrt(3) + 97) + (sqrt(3)*(123*x^6 - 2016*x^
5 + 2214*x^4 - 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 -

1192*x^3 + 228*x^2 - 112) - 192)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97) - 6*(
5*x^6 - 27*x^5 + 48*x^4 - 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 - 5*x^5 + 10*x^4
- 10*x^3 + 8*x^2 - 4*x) - 12*x + 8)*sqrt(x^3 - 1))*sqrt(-4*sqrt(3)*sqrt(56
sqrt(3) + 97)(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4) + 72*sqrt(
3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 288*x + 1152)/
(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 3
*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*((2*sqrt(3)*(
3691*x^16 + 6128*x^15 - 537864*x^14 - 1586477*x^13 + 16210952*x^12 + 771817
56*x^11 + 84218362*x^10 - 71018320*x^9 - 254455812*x^8 - 196076008*x^7 + 12
0105208*x^6 + 256326864*x^5 + 134645168*x^4 - 78464672*x^3 - 78514944*x^2 -
sqrt(3)*(2131*x^16 + 3538*x^15 - 310536*x^14 - 915953*x^13 + 9359398*x^12
+ 44560908*x^11 + 48623494*x^10 - 41002448*x^9 - 146910132*x^8 - 113204536*
x^7 + 69342776*x^6 + 147990384*x^5 + 77737424*x^4 - 45301600*x^3 - 45330624
*x^2 - 12242560*x + 7598336) - 21204736*x + 13160704)*sqrt(x^3 - 1)*(56*sqr
t(3) + 97) - (459*x^16 + 13425*x^15 - 33201*x^14 - 950652*x^13 - 997302*x^1
2 + 14760972*x^11 + 47069892*x^10 + 49762248*x^9 - 8212536*x^8 - 84377808*x
^7 - 88427328*x^6 - 25613856*x^5 + 27458496*x^4 + 36433344*x^3 + 12609792*x
^2 - sqrt(3)*(265*x^16 + 7751*x^15 - 19167*x^14 - 548864*x^13 - 575818*x^12
+ 8522268*x^11 + 27175852*x^10 + 28730312*x^9 - 4741560*x^8 - 48715600*x^7
- 51053600*x^6 - 14788128*x^5 + 15853184*x^4 + 21034816*x^3 + 7280256*x^2
+ 2488832*x - 1889792) + 4310784*x - 3273216)*sqrt(x^3 - 1)*sqrt(56*sqrt(3)
+ 97))*(672*sqrt(3) + 1164)^(3/4) + 6*(sqrt(3)*(4945*x^15 + 88617*x^14 + 7
38528*x^13 + 1860046*x^12 - 784596*x^11 - 7668708*x^10 - 6570680*x^9 + 6903
864*x^8 + 15444144*x^7 + 4312832*x^6 - 9559200*x^5 - 9359808*x^4 - 155968*x
^3 + 3016704*x^2 - sqrt(3)*(2855*x^15 + 51163*x^14 + 426388*x^13 + 1073898*
x^12 - 452980*x^11 - 4427548*x^10 - 3793592*x^9 + 3985944*x^8 + 8916720*x^7
+ 2490016*x^6 - 5519008*x^5 - 5403904*x^4 - 90048*x^3 + 1741696*x^2 + 1543
936*x - 545536) + 2674176*x - 944896)*sqrt(x^3 - 1)*(56*sqrt(3) + 97) - 2*(
246*x^15 + 7653*x^14 + 41169*x^13 + 51342*x^12 - 72300*x^11 - 45930*x^10 +
221688*x^9 + 17892*x^8 - 490248*x^7 - 462360*x^6 + 389616*x^5 + 619728*x^4
+ 16608*x^3 - 187584*x^2 - sqrt(3)*(142*x^15 + 4419*x^14 + 23781*x^13 + 296
08*x^12 - 41940*x^11 - 26454*x^10 + 128152*x^9 + 10692*x^8 - 283320*x^7 - 2
67064*x^6 + 224784*x^5 + 357936*x^4 + 9632*x^3 - 108288*x^2 - 96000*x + 339
20) - 166272*x + 58752)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) +
1164)^(1/4)) - 216*(12*x^17 + 498*x^16 + 462*x^15 - 24972*x^14 - 88530*x^1
3 - 9726*x^12 + 300000*x^11 + 396768*x^10 - 87216*x^9 - 723072*x^8 - 549408
*x^7 + 220128*x^6 + 584736*x^5 + 308256*x^4 - 155136*x^3 - 136704*x^2 - sqr
t(3)*(7*x^17 + 286*x^16 + 238*x^15 - 14255*x^14 - 50390*x^13 - 5942*x^12 +
171808*x^11 + 226888*x^10 - 48920*x^9 - 415384*x^8 - 315088*x^7 + 125600*x^
6 + 336608*x^5 + 177344*x^4 - 89152*x^3 - 78784*x^2 - 39040*x + 18176) - 67
584*x + 31488)*sqrt(56*sqrt(3) + 97))/(x^17 - 13*x^16 - 522*x^15 - 1742*x^1
4 + 3008*x^13 + 16884*x^12 + 11656*x^11 - 23944*x^10 - 42336*x^9 - 9136*x^8
+ 36256*x^7 + 27360*x^6 - 256*x^5 - 13376*x^4 - 5760*x^3 + 1664*x^2 + 256*
x)) + 1/216*sqrt(3)*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12)
+ 24)*(672*sqrt(3) + 1164)^(1/4)*(56*sqrt(3) + 97)*(56*sqrt(3) - 97)*arctan

(1/648*(432*sqrt(3)*(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x
^13 - 345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 +
801584*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x
^2 - 2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99
812*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 -
 321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x -
 36544) + 271808*x - 126592)*(56*sqrt(3) + 97) + 72*sqrt(3)*(sqrt(3)*(2340*
x^17 + 96354*x^16 + 84798*x^15 - 4817124*x^14 - 17052930*x^13 - 1941678*x^1
2 + 57963744*x^11 + 76603680*x^10 - 16678512*x^9 - 139922496*x^8 - 10622736
0*x^7 + 42453216*x^6 + 113269536*x^5 + 59694624*x^4 - 30025728*x^3 - 264960
00*x^2 - sqrt(3)*(1351*x^17 + 55630*x^16 + 48958*x^15 - 2781167*x^14 - 9845
510*x^13 - 1121030*x^12 + 33465376*x^11 + 44227144*x^10 - 9629336*x^9 - 807
84280*x^8 - 61330384*x^7 + 24510368*x^6 + 65396192*x^5 + 34464704*x^4 - 173
35360*x^3 - 15297472*x^2 - 7571584*x + 3526400) - 13114368*x + 6107904)*(56
sqrt(3) + 97) - 6(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^
13 - 345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 8
01584*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^
2 - 2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 998
12*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 -
321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x -
36544) + 271808*x - 126592)*sqrt(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) -
sqrt(1/2)*(288*sqrt(3)*(627*x^16 + 14286*x^15 + 39762*x^14 - 50142*x^13 - 2
16816*x^12 - 112284*x^11 + 325707*x^10 + 586326*x^9 - 3294*x^8 - 631752*x^7
 - 539220*x^6 + 184392*x^5 + 483816*x^4 + 115296*x^3 - 108576*x^2 - 2*sqrt(
3)*(181*x^16 + 4124*x^15 + 11478*x^14 - 14474*x^13 - 62584*x^12 - 32412*x^1
1 + 94021*x^10 + 169244*x^9 - 954*x^8 - 182368*x^7 - 155648*x^6 + 53232*x^5
 + 139664*x^4 + 33280*x^3 - 31344*x^2 - 37024*x + 11584) - 128256*x + 40128
)*(56*sqrt(3) + 97) + 24*sqrt(3)*(sqrt(3)*(2340*x^17 + 35850*x^16 - 106410*
x^15 + 2064744*x^14 + 11945946*x^13 + 1710042*x^12 - 46293732*x^11 - 591615
24*x^10 + 18480192*x^9 + 122366520*x^8 + 81203856*x^7 - 45222000*x^6 - 1005
98112*x^5 - 42207168*x^4 + 29609472*x^3 + 22458240*x^2 - sqrt(3)*(1351*x^17
 + 20698*x^16 - 61436*x^15 + 1192081*x^14 + 6896998*x^13 + 987292*x^12 - 26
727704*x^11 - 34156928*x^10 + 10669552*x^9 + 70648352*x^8 + 46883072*x^7 -
26108944*x^6 - 58080352*x^5 - 24368320*x^4 + 17095040*x^3 + 12966272*x^2 +
4724480*x - 2581504) + 8183040*x - 4471296)*(56*sqrt(3) + 97) - 6*(97*x^17
- 104*x^16 - 20510*x^15 - 43181*x^14 + 217294*x^13 + 691762*x^12 + 584800*x
^11 - 521510*x^10 - 1780028*x^9 - 1416580*x^8 + 80528*x^7 + 1518056*x^6 + 1
321712*x^5 + 393392*x^4 - 501952*x^3 - 446848*x^2 - 4*sqrt(3)*(14*x^17 - 15
*x^16 - 2960*x^15 - 6232*x^14 + 31362*x^13 + 99844*x^12 + 84404*x^11 - 7526
7*x^10 - 256916*x^9 - 204458*x^8 + 11616*x^7 + 219104*x^6 + 190768*x^5 + 56
784*x^4 - 72448*x^3 - 64496*x^2 - 24480*x + 13376) - 169600*x + 92672)*sqrt
(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) + sqrt(-4*sqrt(3)*sqrt(56*sqrt(3)
+ 97)*(7*sqrt(3) - 12) + 24)*((2*sqrt(3)*(3691*x^16 - 17731*x^15 - 951114*x
^14 - 450359*x^13 + 4370159*x^12 - 30318522*x^11 - 78096668*x^10 - 9429316*
x^9 + 146877876*x^8 + 197107784*x^7 - 30834152*x^6 - 185125776*x^5 - 132260

896*x^4 + 45545344*x^3 + 69517536*x^2 - sqrt(3)*(2131*x^16 - 10237*x^15 - 5
49126*x^14 - 260015*x^13 + 2523113*x^12 - 17504406*x^11 - 45089132*x^10 - 5
444020*x^9 + 84799980*x^8 + 113800232*x^7 - 17802104*x^6 - 106882416*x^5 -
76360864*x^4 + 26295616*x^3 + 40135968*x^2 + 7907648*x - 5562368) + 1369644
8*x - 9634304)*sqrt(x^3 - 1)*(56*sqrt(3) + 97) - (459*x^16 + 1557*x^15 - 26
415*x^14 + 1449954*x^13 + 4677912*x^12 - 12651948*x^11 - 55684800*x^10 - 62
834256*x^9 + 8526168*x^8 + 105313392*x^7 + 99605088*x^6 + 18897984*x^5 - 42
499296*x^4 - 37357632*x^3 - 8256960*x^2 - sqrt(3)*(265*x^16 + 899*x^15 - 15
249*x^14 + 837130*x^13 + 2700776*x^12 - 7304604*x^11 - 32149640*x^10 - 3627
7360*x^9 + 4922568*x^8 + 60802736*x^7 + 57507040*x^6 + 10910784*x^5 - 24536
992*x^4 - 21568448*x^3 - 4767168*x^2 - 1207168*x + 1383424) - 2090880*x + 2
396160)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) + 6
(sqrt(3)(4945*x^15 + 37473*x^14 - 490698*x^13 - 2249468*x^12 + 474132*x^1
1 + 8423784*x^10 + 5853520*x^9 - 8451720*x^8 - 15320016*x^7 - 768064*x^6 +
10405056*x^5 + 6627744*x^4 - 700480*x^3 - 2799552*x^2 - sqrt(3)*(2855*x^15
+ 21635*x^14 - 283306*x^13 - 1298732*x^12 + 273748*x^11 + 4863472*x^10 + 33
79536*x^9 - 4879608*x^8 - 8845008*x^7 - 443456*x^6 + 6007360*x^5 + 3826528*
x^4 - 404416*x^3 - 1616320*x^2 - 1003648*x + 399360) - 1738368*x + 691712)*
sqrt(x^3 - 1)*(56*sqrt(3) + 97) - 2*(246*x^15 + 3678*x^14 - 13485*x^13 - 10
2933*x^12 - 70062*x^11 + 81156*x^10 + 45204*x^9 + 129636*x^8 + 243576*x^7 +
221784*x^6 - 351024*x^5 - 460896*x^4 + 33984*x^3 + 174048*x^2 - sqrt(3)*(1
42*x^15 + 2124*x^14 - 7773*x^13 - 59447*x^12 - 40626*x^11 + 46860*x^10 + 26
308*x^9 + 75276*x^8 + 140472*x^7 + 127784*x^6 - 202896*x^5 - 266016*x^4 + 1
9712*x^3 + 100512*x^2 + 62400*x - 24832) + 108096*x - 43008)*sqrt(x^3 - 1)*
sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) - 216*(130*x^16 + 1682*x
^15 + 2496*x^14 - 7730*x^13 + 1790*x^12 + 35700*x^11 - 7100*x^10 - 86080*x^
9 - 49176*x^8 + 100400*x^7 + 108208*x^6 - 33312*x^5 - 80704*x^4 - 18944*x^3
+ 18048*x^2 - 3*sqrt(3)*(25*x^16 + 324*x^15 + 489*x^14 - 1482*x^13 + 316*x
^12 + 6984*x^11 - 1312*x^10 - 16624*x^9 - 9792*x^8 + 19328*x^7 + 20976*x^6
- 6240*x^5 - 15552*x^4 - 3712*x^3 + 3456*x^2 + 4096*x - 1280) + 21248*x - 6
656)*sqrt(56*sqrt(3) + 97))*sqrt((18*x^8 - 36*x^7 + 828*x^6 - 360*x^5 + 720
*x^4 - 1008*x^3 - 144*x^2 + 72*sqrt(3)*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 +
46*x^3 - 42*x^2 - sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24
*x^2 + 12*x - 4) + 20*x - 8)*sqrt(56*sqrt(3) + 97) - (sqrt(3)*(123*x^6 - 20
16*x^5 + 2214*x^4 - 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 - 1164*x^5 + 1278*
x^4 - 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97)
- 6*(5*x^6 - 27*x^5 + 48*x^4 - 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 - 5*x^5 + 1
0*x^4 - 10*x^3 + 8*x^2 - 4*x) - 12*x + 8)*sqrt(x^3 - 1))*sqrt(-4*sqrt(3)*sq
rt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4) + 72*
sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 288*x + 1
152)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)
) + 3*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*((2*sqrt
(3)*(3691*x^16 + 6128*x^15 - 537864*x^14 - 1586477*x^13 + 16210952*x^12 + 7
7181756*x^11 + 84218362*x^10 - 71018320*x^9 - 254455812*x^8 - 196076008*x^7
+ 120105208*x^6 + 256326864*x^5 + 134645168*x^4 - 78464672*x^3 - 78514944*

x^2 - sqrt(3)*(2131*x^16 + 3538*x^15 - 310536*x^14 - 915953*x^13 + 9359398*
x^12 + 44560908*x^11 + 48623494*x^10 - 41002448*x^9 - 146910132*x^8 - 11320
4536*x^7 + 69342776*x^6 + 147990384*x^5 + 77737424*x^4 - 45301600*x^3 - 453
30624*x^2 - 12242560*x + 7598336) - 21204736*x + 13160704)*sqrt(x^3 - 1)*(5
6*sqrt(3) + 97) - (459*x^16 + 13425*x^15 - 33201*x^14 - 950652*x^13 - 99730
2*x^12 + 14760972*x^11 + 47069892*x^10 + 49762248*x^9 - 8212536*x^8 - 84377
808*x^7 - 88427328*x^6 - 25613856*x^5 + 27458496*x^4 + 36433344*x^3 + 12609
792*x^2 - sqrt(3)*(265*x^16 + 7751*x^15 - 19167*x^14 - 548864*x^13 - 575818
*x^12 + 8522268*x^11 + 27175852*x^10 + 28730312*x^9 - 4741560*x^8 - 4871560
0*x^7 - 51053600*x^6 - 14788128*x^5 + 15853184*x^4 + 21034816*x^3 + 7280256
*x^2 + 2488832*x - 1889792) + 4310784*x - 3273216)*sqrt(x^3 - 1)*sqrt(56*sq
rt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) + 6*(sqrt(3)*(4945*x^15 + 88617*x^1
4 + 738528*x^13 + 1860046*x^12 - 784596*x^11 - 7668708*x^10 - 6570680*x^9 +
 6903864*x^8 + 15444144*x^7 + 4312832*x^6 - 9559200*x^5 - 9359808*x^4 - 155
968*x^3 + 3016704*x^2 - sqrt(3)*(2855*x^15 + 51163*x^14 + 426388*x^13 + 107
3898*x^12 - 452980*x^11 - 4427548*x^10 - 3793592*x^9 + 3985944*x^8 + 891672
0*x^7 + 2490016*x^6 - 5519008*x^5 - 5403904*x^4 - 90048*x^3 + 1741696*x^2 +
 1543936*x - 545536) + 2674176*x - 944896)*sqrt(x^3 - 1)*(56*sqrt(3) + 97)
 - 2*(246*x^15 + 7653*x^14 + 41169*x^13 + 51342*x^12 - 72300*x^11 - 45930*x^
10 + 221688*x^9 + 17892*x^8 - 490248*x^7 - 462360*x^6 + 389616*x^5 + 619728
*x^4 + 16608*x^3 - 187584*x^2 - sqrt(3)*(142*x^15 + 4419*x^14 + 23781*x^13
 + 29608*x^12 - 41940*x^11 - 26454*x^10 + 128152*x^9 + 10692*x^8 - 283320*x^
7 - 267064*x^6 + 224784*x^5 + 357936*x^4 + 9632*x^3 - 108288*x^2 - 96000*x
 + 33920) - 166272*x + 58752)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt
(3) + 1164)^(1/4)) - 216*(12*x^17 + 498*x^16 + 462*x^15 - 24972*x^14 - 8853
0*x^13 - 9726*x^12 + 300000*x^11 + 396768*x^10 - 87216*x^9 - 723072*x^8 - 5
49408*x^7 + 220128*x^6 + 584736*x^5 + 308256*x^4 - 155136*x^3 - 136704*x^2
 - sqrt(3)*(7*x^17 + 286*x^16 + 238*x^15 - 14255*x^14 - 50390*x^13 - 5942*x^
12 + 171808*x^11 + 226888*x^10 - 48920*x^9 - 415384*x^8 - 315088*x^7 + 1256
00*x^6 + 336608*x^5 + 177344*x^4 - 89152*x^3 - 78784*x^2 - 39040*x + 18176)
 - 67584*x + 31488)*sqrt(56*sqrt(3) + 97))/(x^17 - 13*x^16 - 522*x^15 - 174
2*x^14 + 3008*x^13 + 16884*x^12 + 11656*x^11 - 23944*x^10 - 42336*x^9 - 913
6*x^8 + 36256*x^7 + 27360*x^6 - 256*x^5 - 13376*x^4 - 5760*x^3 + 1664*x^2 +
 256*x)) + 1/2592*(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*sqrt
(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 116
4)^(1/4)*log(1/18*(18*x^8 - 36*x^7 + 828*x^6 - 360*x^5 + 720*x^4 - 1008*x^3
 - 144*x^2 + 72*sqrt(3)*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^
2 - sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4
) + 20*x - 8)*sqrt(56*sqrt(3) + 97) + (sqrt(3)*(123*x^6 - 2016*x^5 + 2214*x
^4 - 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3
 + 228*x^2 - 112) - 192)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97) - 6*(5*x^6 - 27
*x^5 + 48*x^4 - 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3
 + 8*x^2 - 4*x) - 12*x + 8)*sqrt(x^3 - 1))*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) +
 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4) + 72*sqrt(3)*(x^7 -
4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 288*x + 1152)/(x^8 + 4*x

```
^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 1/2592*(sqr
t(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*sqrt(-4*sqrt(3)*sqrt(56*sq
rt(3) + 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4)*log(1/18*(18*
x^8 - 36*x^7 + 828*x^6 - 360*x^5 + 720*x^4 - 1008*x^3 - 144*x^2 + 72*sqrt(3
)*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^2 - sqrt(3)*(15*x^7 -
22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4) + 20*x - 8)*sqrt(56*
sqrt(3) + 97) - (sqrt(3)*(123*x^6 - 2016*x^5 + 2214*x^4 - 2064*x^3 + 396*x^
2 - sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3 + 228*x^2 - 112) - 192
)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97) - 6*(5*x^6 - 27*x^5 + 48*x^4 - 58*x^3
 + 36*x^2 - 3*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - 12*x
+ 8)*sqrt(x^3 - 1))*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12)
+ 24)*(672*sqrt(3) + 1164)^(1/4) + 72*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4
- 4*x^3 - 6*x^2 + 4*x + 8) + 288*x + 1152)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 +
 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 1/36*sqrt(14*sqrt(3) + 24)*arctan
(-1/12*(3*x^2 - sqrt(3)*(x^2 + 10*x - 8) + 18*x - 12)*sqrt(14*sqrt(3) + 24)
/sqrt(x^3 - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x-1)(x^2+x+1)}\left(x^3-10+6\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x**3+6*3**(1/2))/(x**3-1)**(1/2),x)
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x**3 - 10 + 6*sqrt(3))), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{x^3-1}\,\left(x^3+6\,\sqrt{3}-10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 - 1)^(1/2)*(6*3^(1/2) + x^3 - 10)),x)
[Out] int(x/((x^3 - 1)^(1/2)*(6*3^(1/2) + x^3 - 10)), x)
```

3.90 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}\,x^2+x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{1}{3}\sqrt{-3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\,\sqrt{-4+4\sqrt{3}\,x^2+x^4}}\right)$$

```
[Out] 1/3*arctanh((1+x-3^(1/2))^2/(-9+6*3^(1/2))^(1/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/
2))*(-3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1754, 213}

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\,\tanh^{-1}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\,\sqrt{x^4+4\sqrt{3}\,x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3])]
*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1754

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Dist[(-A^2)*((B*d + A*e)/e), Subst[Int[1/(6*A^3*B
*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /;
 FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^
4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && E
qQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}\,x^2+x^4}}\,dx = -\left(\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{3\left(1-\sqrt{3}\right)^4+6\left(1-\sqrt{3}\right.}\right.\right.$$

$$= \frac{1}{3}\sqrt{-3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\,\sqrt{-4+4\sqrt{3}\,\right.}}\right.$$

Mathematica [A]
time = 8.08, size = 77, normalized size = 1.18

$$\frac{1}{3}\sqrt{-3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\sqrt{9+6\sqrt{3}}\,\sqrt{-4+4\sqrt{3}\,x^2+x^4}}{2+\left(-2-2\sqrt{3}\right)x+\left(2+\sqrt{3}\right)x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^
4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(Sqrt[9 + 6*Sqrt[3]]*Sqrt[-4 + 4*Sqrt[3]*x^2
+ x^4])/(2 + (-2 - 2*Sqrt[3])*x + (2 + Sqrt[3])*x^2)])/3
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]
*x^2 + x^4]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.42, size = 327, normalized size = 5.03

method	result

default	$\frac{\sqrt{1-\left(-1+\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{2}\right)x^2}\ \mathrm{EllipticF}\left(x\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right),i\sqrt{1+4\sqrt{3}\,\left(1+\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right)\sqrt{-4+x^4+4\sqrt{3}\,x^2}} -$
elliptic	$\frac{\sqrt{1-\left(-1+\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{2}\right)x^2}\ \mathrm{EllipticF}\left(x\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right),i\sqrt{1+4\sqrt{3}\,\left(1+\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right)\sqrt{-4+x^4+4\sqrt{3}\,x^2}} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/(1/2*I*3^(1/2)-1/2*I)*(1-(-1+1/2*3^(1/2))*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x
^2)^(1/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2*I*3^(1/2)-1/2*I),I*
(1+4*3^(1/2)*(1+1/2*3^(1/2)))^(1/2))-2*3^(1/2)*(-1/2/((-1-3^(1/2))^4+4*(-1-
3^(1/2))^2*3^(1/2)-4)^(1/2)*arctanh(1/2*(4*(-1-3^(1/2))^2*3^(1/2)-8+4*3^(1/
2)*x^2+2*x^2*(-1-3^(1/2))^2)/((-1-3^(1/2))^4+4*(-1-3^(1/2))^2*3^(1/2)-4)^(1
/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/2))-1/(-1+1/2*3^(1/2))^(1/2)/(-1-3^(1/2))*(1-
(-1+1/2*3^(1/2))*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4+4*3^(1/2)
*x^2)^(1/2)*EllipticPi((-1+1/2*3^(1/2))^(1/2)*x,1/(-1+1/2*3^(1/2))/(-1-3^(1
/2))^2,(1+1/2*3^(1/2))^(1/2)/(-1+1/2*3^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(47) = 94$.

time = 0.41, size = 323, normalized size = 4.97

$$\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(-\frac{37x^{12}-204x^{11}+804x^{10}-2408x^{9}+3708x^{8}-5472x^{7}+6432x^{6}+10944x^{5}+14832x^{4}+19264x^{3}+12864x^{2}+\left(54x^{10}-300x^{9}+1026x^{8}-2232x^{7}+3024x^{6}-3024x^{5}-1008x^{4}-2016x^{3}-2592x^{2}+\sqrt{3}(31x^{10}-176x^{9}+576x^{8}-1320x^{7}+1848x^{6}-1008x^{5}+1344x^{4}+1632x^{3}+1008x^{2}+832x+256)-1152x-480\right)\sqrt{x^{4}+4\sqrt{3}x^{2}-4}\sqrt{2\sqrt{3}-3}+3\sqrt{3}(7x^{12}-40x^{11}+160x^{10}-400x^{9}+924x^{8}-960x^{7}-1920x^{5}-3696x^{4}-3200x^{3}-2560x^{2}-1280x-448)+6528x+2368}{x^{12}+12x^{11}+48x^{10}+40x^{9}-180x^{8}-288x^{7}+384x^{6}+576x^{5}-720x^{4}-320x^{3}+768x^{2}-384x+64}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log(-(37*x^12 - 204*x^11 + 804*x^10 - 2408*x^9 + 3
708*x^8 - 5472*x^7 + 6432*x^6 + 10944*x^5 + 14832*x^4 + 19264*x^3 + 12864*x
^2 + (54*x^10 - 300*x^9 + 1026*x^8 - 2232*x^7 + 3024*x^6 - 3024*x^5 - 1008*
x^4 - 2016*x^3 - 2592*x^2 + sqrt(3)*(31*x^10 - 176*x^9 + 576*x^8 - 1320*x^7
 + 1848*x^6 - 1008*x^5 + 1344*x^4 + 1632*x^3 + 1008*x^2 + 832*x + 256) - 11
52*x - 480)*sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*sqrt(2*sqrt(3) - 3) + 3*sqrt(3)*(
7*x^12 - 40*x^11 + 160*x^10 - 400*x^9 + 924*x^8 - 960*x^7 - 1920*x^5 - 3696
*x^4 - 3200*x^3 - 2560*x^2 - 1280*x - 448) + 6528*x + 2368)/(x^12 + 12*x^11
 + 48*x^10 + 40*x^9 - 180*x^8 - 288*x^7 + 384*x^6 + 576*x^5 - 720*x^4 - 320
*x^3 + 768*x^2 - 384*x + 64))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\left(x+1+\sqrt{3}\right)\sqrt{x^4+4\sqrt{3}\,x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))*sqrt(x**4 + 4*sqrt(3)*x**2 -
4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\,\sqrt{x^4+4\,\sqrt{3}\,x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),x
)
```

```
[Out] int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),
x)
```

3.91 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\,dx$

Optimal. Leaf size=63

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\right)$$

```
[Out] -1/3*arctan((1+x+3^(1/2))^2/(9+6*3^(1/2))^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2
))*(3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1754, 209}

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}\,x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] -1/3*(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3]
)]*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4])])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1754

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Dist[(-A^2)*((B*d + A*e)/e), Subst[Int[1/(6*A^3*B
*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /;
 FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^
4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && E
qQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\,dx = -\left(\left(4\left(2+\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{6\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)^3+3\right.\right.$$

$$= -\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-4-4\sqrt{3}\,x^2+\right.$$

Mathematica [A]
time = 8.05, size = 77, normalized size = 1.22

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{-9+6\sqrt{3}}\sqrt{-4-4\sqrt{3}\,x^2+x^4}}{-2+\left(2-2\sqrt{3}\right)x+\left(-2+\sqrt{3}\right)x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^
4]),x]
```

```
[Out] -1/3*(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(Sqrt[-9 + 6*Sqrt[3]]*Sqrt[-4 - 4*Sqrt[3]*
x^2 + x^4])/(-2 + (2 - 2*Sqrt[3])*x + (-2 + Sqrt[3])*x^2)])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]
*x^2 + x^4]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.43, size = 311, normalized size = 4.94

method	result

default	$\frac{\sqrt{1-\left(-1-\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{3}}{2}\right)x^2}\,\text{EllipticF}\left(x\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right),i\sqrt{1-4\sqrt{3}\,\left(1-\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{-4+x^4-4\sqrt{3}\,x^2}}$
elliptic	$\frac{\sqrt{1-\left(-1-\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{3}}{2}\right)x^2}\,\text{EllipticF}\left(x\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right),i\sqrt{1-4\sqrt{3}\,\left(1-\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{-4+x^4-4\sqrt{3}\,x^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/(1/2*I+1/2*I*3^(1/2))*(1-(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*x
^2)^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2*I+1/2*I*3^(1/2)),I*
(1-4*3^(1/2)*(1-1/2*3^(1/2)))^(1/2))+2*3^(1/2)*(-1/2/((3^(1/2)-1)^4-4*(3^(1
/2)-1)^2*3^(1/2)-4)^(1/2)*arctanh(1/2*(-4*(3^(1/2)-1)^2*3^(1/2)-8-4*3^(1/2)
*x^2+2*x^2*(3^(1/2)-1)^2)/((3^(1/2)-1)^4-4*(3^(1/2)-1)^2*3^(1/2)-4)^(1/2)/(
-4+x^4-4*3^(1/2)*x^2)^(1/2))-1/(-1-1/2*3^(1/2))^(1/2)/(3^(1/2)-1)*(1-(-1-1/
2*3^(1/2))*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^
(1/2)*EllipticPi((-1-1/2*3^(1/2))^(1/2)*x,1/(-1-1/2*3^(1/2))/(3^(1/2)-1)^2,
(1-1/2*3^(1/2))^(1/2)/(-1-1/2*3^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(45) = 90$.

time = 0.40, size = 112, normalized size = 1.78

$$\frac{1}{6}\sqrt{2\sqrt{3}+3}\,\arctan\left(-\frac{\left(9x^4-30x^3+18x^2-2\sqrt{3}\,(2x^4-10x^3+3x^2-10x+2)+24\right)\sqrt{x^4-4\sqrt{3}\,x^2-4}\,\sqrt{2\sqrt{3}+3}}{11x^6-42x^5+66x^4-176x^3-132x^2-168x-88}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*sqrt(2*sqrt(3) + 3)*arctan(-(9*x^4 - 30*x^3 + 18*x^2 - 2*sqrt(3)*(2*x^4
 - 10*x^3 + 3*x^2 - 10*x + 2) + 24)*sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*sqrt(2*sq
rt(3) + 3)/(11*x^6 - 42*x^5 + 66*x^4 - 176*x^3 - 132*x^2 - 168*x - 88))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1+\sqrt{3}}{\left(x-\sqrt{3}+1\right)\sqrt{x^4-4\sqrt{3}\,x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)*sqrt(x**4 - 4*sqrt(3)*x**2 -
4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}\,x^2-4}\,\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),x
)
```

```
[Out] int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),
x)
```

3.92 $\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}} dx$

Optimal. Leaf size=53

$$\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)+\log(1+x)-\frac{3}{2}\log\left(2+x-\sqrt[3]{2+x^3}\right)$$

```
[Out] ln(1+x)-3/2*ln(2+x-(x^3+2)^(1/3))+arctan(1/3*(1+2*(2+x)/(x^3+2)^(1/3))*3^(1
/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2176}

$$-\frac{3}{2}\log\left(-\sqrt[3]{x^3+2}+x+2\right)+\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]] + Log[1 + x] - (3
*Log[2 + x - (2 + x^3)^(1/3)])/2
```

Rule 2176

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[Sqrt[3]*f*(ArcTan[(1 + 2*Rt[b, 3]*((2*c + d*x)/(d*(a + b*
x^3)^(1/3))))/Sqrt[3]]/(Rt[b, 3]*d)), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx=\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)+\log(1+x)-\frac{3}{2}\log\left(2+x-\sqrt[3]{2+x^3}\right)$$

Mathematica [A]
time = 0.57, size = 92, normalized size = 1.74

$$-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{2+x^3}}{4+2x+\sqrt[3]{2+x^3}}\right)-\log\left(-2-x+\sqrt[3]{2+x^3}\right)+\frac{1}{2}\log\left(4+4x+x^2+(2+x)\sqrt[3]{2+x^3}+\left(2+x^3\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*(2 + x^3)^(1/3))/(4 + 2*x + (2 + x^3)^(1/3))]) -
Log[-2 - x + (2 + x^3)^(1/3)] + Log[4 + 4*x + x^2 + (2 + x)*(2 + x^3)^(1/3)
 + (2 + x^3)^(2/3)]/2
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x - 1)/((x + 1)*(x^3 + 2)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.30, size = 544, normalized size = 10.26

method	result
trager	$-\ln\left(-\frac{787\,\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)^2 x^3+9008\,\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)\left(x^3+2\right)^{\frac{2}{3}} x-9678\,\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)\left(x^3+2\right)^{\frac{1}{3}} x}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(1+x)/(x^3+2)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(-(787*RootOf(_Z^2-2*_Z+4)^2*x^3+9008*RootOf(_Z^2-2*_Z+4)*(x^3+2)^(2/3)*
x-9678*RootOf(_Z^2-2*_Z+4)*(x^3+2)^(1/3)*x^2-1574*RootOf(_Z^2-2*_Z+4)^2*x^2
-904*RootOf(_Z^2-2*_Z+4)*x^3+18016*RootOf(_Z^2-2*_Z+4)*(x^3+2)^(2/3)+1340*x
*(x^3+2)^(2/3)-38712*RootOf(_Z^2-2*_Z+4)*(x^3+2)^(1/3)*x+18016*(x^3+2)^(1/3
)*x^2-3148*RootOf(_Z^2-2*_Z+4)^2*x+23844*RootOf(_Z^2-2*_Z+4)*x^2-16208*x^3+
2680*(x^3+2)^(2/3)-38712*RootOf(_Z^2-2*_Z+4)*(x^3+2)^(1/3)+72064*(x^3+2)^(1
/3)*x+47688*RootOf(_Z^2-2*_Z+4)*x-81040*x^2+72064*(x^3+2)^(1/3)+22036*RootO
f(_Z^2-2*_Z+4)-162080*x-113456)/(1+x)^2)+1/2*RootOf(_Z^2-2*_Z+4)*ln((1013*R
ootOf(_Z^2-2*_Z+4)^2*x^3+4504*RootOf(_Z^2-2*_Z+4)*(x^3+2)^(2/3)*x+335*RootO
f(_Z^2-2*_Z+4)*(x^3+2)^(1/3)*x^2-2026*RootOf(_Z^2-2*_Z+4)^2*x^2-6865*RootOf
(_Z^2-2*_Z+4)*x^3+9008*RootOf(_Z^2-2*_Z+4)*(x^3+2)^(2/3)-9678*x*(x^3+2)^(2/
3)+1340*RootOf(_Z^2-2*_Z+4)*(x^3+2)^(1/3)*x+9008*(x^3+2)^(1/3)*x^2-4052*Roo
tOf(_Z^2-2*_Z+4)^2*x-14634*RootOf(_Z^2-2*_Z+4)*x^2+4722*x^3-19356*(x^3+2)^(
2/3)+1340*RootOf(_Z^2-2*_Z+4)*(x^3+2)^(1/3)+36032*(x^3+2)^(1/3)*x-29268*Roo
```

```
tOf(_Z^2-2*_Z+4)*x+12592*x^2+36032*(x^3+2)^(1/3)-28364*RootOf(_Z^2-2*_Z+4)+
25184*x+22036)/(1+x)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="maxima")
[Out] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{(x+1)\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x**3+2)**(1/3),x)
[Out] Integral((x - 1)/((x + 1)*(x**3 + 2)**(1/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x)
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x-1}{\left(x^3+2\right)^{1/3}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((x^3 + 2)^(1/3)*(x + 1)),x)
```

```
[Out] int((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

3.93 $\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=108

$$\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)-\frac{1}{2}\log(1+x)+\frac{3}{4}\log\left(2+x-\sqrt[3]{2+x^3}\right)-\frac{1}{4}\log\left(-\right.$$

```
[Out] -1/2*ln(1+x)+3/4*ln(2+x-(x^3+2)^(1/3))-1/4*ln(-x+(x^3+2)^(1/3))+1/6*arctan(
1/3*(1+2*x/(x^3+2)^(1/3))*3^(1/2))*3^(1/2)-1/2*arctan(1/3*(1+2*(2+x)/(x^3+2
)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2175, 245, 2176}

$$\frac{3}{4}\log\left(-\sqrt[3]{x^3+2}+x+2\right)-\frac{1}{4}\log\left(\sqrt[3]{x^3+2}-x\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - (Sqrt[3]*ArcTan[(
1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]])/2 - Log[1 + x]/2 + (3*Log[2 + x
- (2 + x^3)^(1/3)])/4 - Log[-x + (2 + x^3)^(1/3)]/4
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2175

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Dist[
1/(2*c), Int[1/(a + b*x^3)^(1/3), x], x] + Dist[1/(2*c), Int[(c - d*x)/((c
+ d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[2*b*c^3
- a*d^3, 0]
```

Rule 2176

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[Sqrt[3]*f*(ArcTan[(1 + 2*Rt[b, 3]*((2*c + d*x)/(d*(a + b*
```

```
x^3)^(1/3))))/Sqrt[3]]/(Rt[b, 3]*d)), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt[3]{2+x^3}}\,dx + \frac{1}{2}\int \frac{1-x}{(1+x)\sqrt[3]{2+x^3}}\,dx$$
$$= \frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right) - \frac{1}{2}\log(1+x) + \frac{3}{4}\log\left(2\right.$$

Mathematica [F]
time = 6.84, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 + x)*(2 + x^3)^(1/3)),x]
[Out] Integrate[1/((1 + x)*(2 + x^3)^(1/3)), x]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((x + 1)*(x^3 + 2)^(1/3)),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.14, size = 1421, normalized size = 13.16

method	result	size
trager	Expression too large to display	1421

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x^3+2)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*ln((-4550781346817636-6728375859478224*x+4993190285176576*RootOf(_Z^2+_
Z+1)^2*x^3+9094739448000192*RootOf(_Z^2+_Z+1)^2*x^2+5884831407529536*RootOf
(_Z^2+_Z+1)^2*x-625895428788672*x^5-68457312523761*x^6-234710785795752*x^4-
469421571591504*x^2+2151515536461060*x^3+8816461926585488*RootOf(_Z^2+_Z+1)
*x^3+1055101552116528*RootOf(_Z^2+_Z+1)*x^2-21283128527537520*RootOf(_Z^2+_
Z+1)*x+16295099853018372*x*(x^3+2)^(2/3)-15559137585059152*RootOf(_Z^2+_Z+1
)+4346750471470680*RootOf(_Z^2+_Z+1)^2*x^5+153868976350327*RootOf(_Z^2+_Z+1
)*x^6+4547369724000096*RootOf(_Z^2+_Z+1)^2*x^4-868588603920114*RootOf(_Z^2+
_Z+1)*x^5+527550776058264*RootOf(_Z^2+_Z+1)*x^4-9125490357912936*(x^3+2)^(1
/3)-928201890361806*(x^3+2)^(2/3)*x^4-540325086981687*(x^3+2)^(1/3)*x^5-195
9537324097146*(x^3+2)^(2/3)*x^3+36303984101745*(x^3+2)^(2/3)*RootOf(_Z^2+_Z
+1)^2*x^4+288449229728205*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^5-13069434276
62820*(x^3+2)^(2/3)*RootOf(_Z^2+_Z+1)^2*x^3-601904942144643*RootOf(_Z^2+_Z+
1)*(x^3+2)^(2/3)*x^4+1286927332633530*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^4
-580773949503594*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)*x^5-3775614346581480*(x^3+
2)^(2/3)*RootOf(_Z^2+_Z+1)^2*x^2-3235955176589922*RootOf(_Z^2+_Z+1)*(x^3+2)
^(2/3)*x^3+1420057746354240*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^3-330458778
6698814*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)*x^4-2613886855325640*(x^3+2)^(2/3)*
RootOf(_Z^2+_Z+1)^2*x-1442113972435308*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^2-
310637632014990*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^2-4286079775383252*(x^3
+2)^(1/3)*RootOf(_Z^2+_Z+1)*x^3+7759251414704196*RootOf(_Z^2+_Z+1)*(x^3+2)^
(2/3)*x-798782482324260*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x+257120106922963
2*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)*x^2+7575270505902288*(x^3+2)^(1/3)*RootOf
(_Z^2+_Z+1)*x-480288966205944*(x^3+2)^(1/3)*x^4+5569211342170836*(x^3+2)^(2
/3)*x^2+1200722415514860*(x^3+2)^(1/3)*x^3+7115580883942020*RootOf(_Z^2+_Z+
1)*(x^3+2)^(2/3)+3372637147591320*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)+101070872
50606332*(x^3+2)^(2/3)+1326316169500028*RootOf(_Z^2+_Z+1)^2*x^6-46828174205
07954*(x^3+2)^(1/3)*x^2-14648813469281292*(x^3+2)^(1/3)*x)/(1+x)^6)+1/6*Roo
tOf(_Z^2+_Z+1)*ln((-15559137585059152-28889172364235904*x-1460422667173568*
RootOf(_Z^2+_Z+1)^2*x^3-2660055572351856*RootOf(_Z^2+_Z+1)^2*x^2-1721212429
168848*RootOf(_Z^2+_Z+1)^2*x-6486689165117784*x^5-1560371964117680*x^6-5349
846734117760*x^4-10699693468235520*x^2+2362848974235344*x^3-430209741588908
4*RootOf(_Z^2+_Z+1)*x^3-12224216591943552*RootOf(_Z^2+_Z+1)*x^2-14334419696
176608*RootOf(_Z^2+_Z+1)*x+5921961582988536*x*(x^3+2)^(2/3)-455078134681763
6*RootOf(_Z^2+_Z+1)-1271350089726990*RootOf(_Z^2+_Z+1)^2*x^5-17826982529917
68*RootOf(_Z^2+_Z+1)*x^6-1330027786175928*RootOf(_Z^2+_Z+1)^2*x^4-624399598
9986342*RootOf(_Z^2+_Z+1)*x^5-6112108295971776*RootOf(_Z^2+_Z+1)*x^4+912549
0357912936*(x^3+2)^(1/3)-289992964115418*(x^3+2)^(2/3)*x^4-240144483102972*
(x^3+2)^(1/3)*x^5-30525575170044*(x^3+2)^(2/3)*x^3+36303984101745*(x^3+2)^(
2/3)*RootOf(_Z^2+_Z+1)^2*x^4-1068918799812864*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+
```

```
1)^2*x^5-1306943427662820*(x^3+2)^(2/3)*RootOf(_Z^2+_Z+1)^2*x^3+67451291034
8133*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^4-4769022337626624*(x^3+2)^(1/3)*Roo
tOf(_Z^2+_Z+1)^2*x^4-1109367662334771*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)*x^5-3
775614346581480*(x^3+2)^(2/3)*RootOf(_Z^2+_Z+1)^2*x^2+622068321264282*RootO
f(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^3-5262369476001792*(x^3+2)^(1/3)*RootOf(_Z^2+_
Z+1)^2*x^3-7593321158119494*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)*x^4-26138868553
25640*(x^3+2)^(2/3)*RootOf(_Z^2+_Z+1)^2*x-6109114720727652*RootOf(_Z^2+_Z+1
)*(x^3+2)^(2/3)*x^2+1151143322875392*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^2-
10749171666899904*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)*x^3-12987025125355476*Roo
tOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x+2960082830251008*(x^3+2)^(1/3)*RootOf(_Z^2+_
Z+1)^2*x+8405161812612978*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)*x^2+2518416680543
4588*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)*x-3001806038787150*(x^3+2)^(1/3)*x^4+3
235710968024664*(x^3+2)^(2/3)*x^2-5043034145162412*(x^3+2)^(1/3)*x^3-711558
0883942020*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)+12498127505504256*(x^3+2)^(1/3)*
RootOf(_Z^2+_Z+1)+2991506366664312*(x^3+2)^(2/3)-387924770967979*RootOf(_Z^
2+_Z+1)^2*x^6+5523323111368356*(x^3+2)^(1/3)*x^2+16810113817208040*(x^3+2)^
(1/3)*x)/(1+x)^6)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(84) = 168$.
time = 1.04, size = 267, normalized size = 2.47

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{13910019318573948542\sqrt{3}(7114781247x^4 + 13663058416x^3 - 46178206896x^2 - 126842559344x - 77084338088)(x^3+2)^{\frac{2}{3}} - 27820038637147897084\sqrt{3}(1625757424x^5 + 16302821713x^4 + 26102613730x^3 - 26431113242x^2 - 80188343316x - 42779182428)(x^3+2)^{\frac{1}{3}} + \sqrt{3}(9329257083355943566313230188 5x^6 + 382151535711085278859235047618x^5 + 673924074224408772959625384792x^4 + 889426563183087468015580290048x^3 + 888876515195959220955879945824x^2 + 351260598258508240019971964880x - 4767400099559721105781688 4304)}{3(78905434814564721745708464883x^6 + 33774670583645822286334793450x^5 + 15598952776058587894336070976x^4 - 89543052531510010868478796 4824x^3 + 361667862240477028869533375352x^2 + 254180230101163251064597209 0336x + 1554815286823334092314485968880)}\right) + \frac{1}{12}\log\left(\frac{22x^6 + 6x^5 - 48x^4 + 44x^3 + 24x^2 + 3(7x^4 - 2x^3 - 32x^2 - 20x + 4)(x^3+2)^{\frac{2}{3}} + 3(7x^5 - 16x^3 + 34x^2 + 76x + 32)(x^3+2)^{\frac{1}{3}} - 192x - 140}{x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*(13910019318573948542*sqrt(3)*(7114781247*x^4 + 1366
3058416*x^3 - 46178206896*x^2 - 126842559344*x - 77084338088)*(x^3 + 2)^(2/
3) - 27820038637147897084*sqrt(3)*(1625757424*x^5 + 16302821713*x^4 + 26102
613730*x^3 - 26431113242*x^2 - 80188343316*x - 42779182428)*(x^3 + 2)^(1/3)
 + sqrt(3)*(93292570833559435663132301885*x^6 + 382151535711085278859235047
618*x^5 + 673924074224408772959625384792*x^4 + 8894265631830874680155802900
48*x^3 + 888876515195959220955879945824*x^2 + 35126059825850824001997196488
0*x - 47674000995597211057816884304))/(78905434814564721745708464883*x^6 +
337746705836458222863347934450*x^5 + 15598952776058587894336070976*x^4 - 89
```

```
54305253151001086847879648 24*x^3 + 3616678622404770288695333 75352*x^2 + 254
18023010116325106459720903 36*x + 155481528682333409231448596 8880)) + 1/12*l
og((22*x^6 + 6*x^5 - 48*x^4 + 44*x^3 + 24*x^2 + 3*(7*x^4 - 2*x^3 - 32*x^2 -
 20*x + 4)*(x^3 + 2)^(2/3) + 3*(7*x^5 - 16*x^3 + 34*x^2 + 76*x + 32)*(x^3 +
 2)^(1/3) - 192*x - 140)/(x^6 + 6*x^5 + 15*x^4 + 20*x^3 + 15*x^2 + 6*x + 1)
)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+1)\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**3+2)**(1/3),x)
```

```
[Out] Integral(1/((x + 1)*(x**3 + 2)**(1/3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(x^3+2\right)}^{1/3}\,\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 + 2)^(1/3)*(x + 1)),x)
```

```
[Out] int(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

3.94 $\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=98

$$\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a+b}}+\frac{\log(1-x^3)}{6\sqrt[3]{a+b}}-\frac{\log\left(\sqrt[3]{a+b}\,x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

```
[Out] 1/6*ln(-x^3+1)/(a+b)^(1/3)-1/2*ln((a+b)^(1/3)*x-(b*x^3+a)^(1/3))/(a+b)^(1/3
)+1/3*arctan(1/3*(1+2*(a+b)^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/(a+b)^(1/3)*3
^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{384\}$

$$\frac{\log(1-x^3)}{6\sqrt[3]{a+b}}-\frac{\log\left(x\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*(a + b)^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a + b
)^(1/3)) + Log[1 - x^3]/(6*(a + b)^(1/3)) - Log[(a + b)^(1/3)*x - (a + b*x^
3)^(1/3)]/(2*(a + b)^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx &= \text{Subst}\left(\int \frac{1}{1-(a+b)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) \\ &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{a+b}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{2+\sqrt[3]{a+}}{1+\sqrt[3]{a+b}\,x+(} \right. \\ &= -\frac{\log\left(1-\frac{\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{a+b}\,x+(a+b)^{2/3}x^2}\,dx, x, \frac{}{\sqrt[3]{a}} \right. \\ &= -\frac{\log\left(1-\frac{\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}} + \frac{\log\left(1+\frac{(a+b)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{a+b}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x} \right.}{} \\ &= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} - \frac{\log\left(1-\frac{\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}} + \frac{\log\left(1+\frac{(a+b)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{a}}{\sqrt[3]{a+}} \right.}{6\sqrt[3]{a+b}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.87, size = 189, normalized size = 1.93

$$\frac{-2\sqrt{-6+6i\sqrt{3}}\tan^{-1}\left(\frac{3\sqrt[3]{a+b}\,x}{\sqrt{3}\sqrt[3]{a+b}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{a+bx^3}}\right)+\left(1+i\sqrt{3}\right)\left(2\log\left(2\sqrt[3]{a+b}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{a+bx^3}\right)-\log\left(\left(-\sqrt[3]{a+b}\,x+\sqrt[3]{a+bx^3}\right)\left(2i\sqrt[3]{a+b}\,x+\left(i+\sqrt{3}\right)\sqrt[3]{a+bx^3}\right)\right)\right)}{12\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] (-2*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[(3*(a + b)^(1/3)*x)/(Sqrt[3]*(a + b)^(1
/3)*x - (3*I + Sqrt[3])*(a + b*x^3)^(1/3))] + (1 + I*Sqrt[3])*(2*Log[2*(a +
 b)^(1/3)*x + (1 + I*Sqrt[3])*(a + b*x^3)^(1/3)] - Log[(-((a + b)^(1/3)*x)
+ (a + b*x^3)^(1/3))*((2*I)*(a + b)^(1/3)*x + (I + Sqrt[3])*(a + b*x^3)^(1/
3))]))/(12*(a + b)^(1/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x^3)*(a + b*x^3)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^{3}+1\right) \left(b \,x^{3}+a \right)^{\frac{1}{3}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

```
[Out] int(1/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^3 + a)^(1/3)*(x^3 - 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(78) = 156$.
time = 118.52, size = 1252, normalized size = 12.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(a + b)*sqrt((-a - b)^(1/3)/(a + b))*log(-((a^3 - 27*a^2
*b - 108*a*b^2 - 81*b^3)*x^9 - 3*(10*a^3 + 54*a^2*b + 45*a*b^2)*x^6 - 3*(17
*a^3 + 18*a^2*b)*x^3 - a^3 + 9*((2*a^2 + 3*a*b)*x^7 - (a^2 + 3*a*b)*x^4 - a
^2*x)*(b*x^3 + a)^(2/3)*(-a - b)^(1/3) + 9*((a^2 + 9*a*b + 9*b^2)*x^8 + (7*
a^2 + 9*a*b)*x^5 + a^2*x^2)*(b*x^3 + a)^(1/3)*(-a - b)^(2/3) + 3*sqrt(1/3)*
(3*((4*a^2 + 21*a*b + 18*b^2)*x^7 + (13*a^2 + 15*a*b)*x^4 + a^2*x)*(b*x^3 +
 a)^(2/3)*(-a - b)^(2/3) + 3*((a^3 - 2*a^2*b - 12*a*b^2 - 9*b^3)*x^8 - 5*(a
^3 + 4*a^2*b + 3*a*b^2)*x^5 - 5*(a^3 + a^2*b)*x^2)*(b*x^3 + a)^(1/3) + ((a^
3 + 27*a^2*b + 54*a*b^2 + 27*b^3)*x^9 + 3*(8*a^3 + 18*a^2*b + 9*a*b^2)*x^6
+ 3*a^3*x^3 - a^3)*(-a - b)^(1/3))*sqrt((-a - b)^(1/3)/(a + b)))/(x^9 - 3*x
^6 + 3*x^3 - 1)) - 2*(-a - b)^(2/3)*log(-(3*(b*x^3 + a)^(1/3)*(a + b)*(-a -
 b)^(1/3)*x^2 + 3*(b*x^3 + a)^(2/3)*(a + b)*x + (a*x^3 - a)*(-a - b)^(2/3))
/(x^3 - 1)) + (-a - b)^(2/3)*log((3*((2*a + 3*b)*x^4 + a*x)*(b*x^3 + a)^(2/
3)*(-a - b)^(2/3) + 3*((a^2 + 4*a*b + 3*b^2)*x^5 + 2*(a^2 + a*b)*x^2)*(b*x^
3 + a)^(1/3) - ((a^2 + 9*a*b + 9*b^2)*x^6 + (7*a^2 + 9*a*b)*x^3 + a^2)*(-a
- b)^(1/3))/(x^6 - 2*x^3 + 1)))/(a + b), 1/18*(6*sqrt(1/3)*(a + b)*sqrt(-(-
```

```
a - b)^(1/3)/(a + b))*arctan(sqrt(1/3)*(6*((2*a^2 + 3*a*b)*x^7 - (a^2 + 3*a
*b)*x^4 - a^2*x)*(b*x^3 + a)^(2/3)*(-a - b)^(2/3) - 6*((a^3 + 10*a^2*b + 18
*a*b^2 + 9*b^3)*x^8 + (7*a^3 + 16*a^2*b + 9*a*b^2)*x^5 + (a^3 + a^2*b)*x^2)
*(b*x^3 + a)^(1/3) - ((a^3 - 9*a^2*b - 36*a*b^2 - 27*b^3)*x^9 - 3*(4*a^3 +
18*a^2*b + 15*a*b^2)*x^6 - 3*(5*a^3 + 6*a^2*b)*x^3 - a^3)*(-a - b)^(1/3))*s
qrt(-(-a - b)^(1/3)/(a + b))/((a^3 + 27*a^2*b + 54*a*b^2 + 27*b^3)*x^9 + 3*
(8*a^3 + 18*a^2*b + 9*a*b^2)*x^6 + 3*a^3*x^3 - a^3)) - 2*(-a - b)^(2/3)*log
(-(3*(b*x^3 + a)^(1/3)*(a + b)*(-a - b)^(1/3)*x^2 + 3*(b*x^3 + a)^(2/3)*(a
+ b)*x + (a*x^3 - a)*(-a - b)^(2/3))/(x^3 - 1)) + (-a - b)^(2/3)*log((3*((2
*a + 3*b)*x^4 + a*x)*(b*x^3 + a)^(2/3)*(-a - b)^(2/3) + 3*((a^2 + 4*a*b + 3
*b^2)*x^5 + 2*(a^2 + a*b)*x^2)*(b*x^3 + a)^(1/3) - ((a^2 + 9*a*b + 9*b^2)*x
^6 + (7*a^2 + 9*a*b)*x^3 + a^2)*(-a - b)^(1/3))/(x^6 - 2*x^3 + 1)))/(a + b)
]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^3\sqrt[3]{a+bx^3}-\sqrt[3]{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)/(b*x**3+a)**(1/3),x)
[Out] -Integral(1/(x**3*(a + b*x**3)**(1/3) - (a + b*x**3)**(1/3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(x^3-1\right)\,{\left(b\,x^3+a\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^3 - 1)*(a + b*x^3)^(1/3)),x)
[Out] -int(1/((x^3 - 1)*(a + b*x^3)^(1/3)), x)
```

3.95 $\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=154

$$\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a+b}}+\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a+b}}+\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}-\frac{\log\left(\sqrt[3]{a+b}\,x-\sqrt[3]{a+bx\right.}}{2\sqrt[3]{a+b}}$$

```
[Out] 1/2*ln((a+b)^(1/3)-(b*x^3+a)^(1/3))/(a+b)^(1/3)-1/2*ln((a+b)^(1/3)*x-(b*x^3
+a)^(1/3))/(a+b)^(1/3)+1/3*arctan(1/3*(1+2*(a+b)^(1/3)*x/(b*x^3+a)^(1/3))*3
^(1/2))/(a+b)^(1/3)*3^(1/2)+1/3*arctan(1/3*(1+2*(b*x^3+a)^(1/3)/(a+b)^(1/3)
)*3^(1/2))/(a+b)^(1/3)*3^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2183, 384, 455, 57, 631, 210, 31}

$$\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}-\frac{\log\left(x\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a+b}}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*(a + b)^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a + b
)^(1/3)) + ArcTan[(1 + (2*(a + b*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]]/(Sqrt[
3]*(a + b)^(1/3)) + Log[(a + b)^(1/3) - (a + b*x^3)^(1/3)]/(2*(a + b)^(1/3)
) - Log[(a + b)^(1/3)*x - (a + b*x^3)^(1/3)]/(2*(a + b)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx = \int \left(\frac{1-\frac{i}{\sqrt{3}}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{a+bx^3}}+\frac{1+\frac{i}{\sqrt{3}}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{a+bx^3}}\right)dx$$

$$= \frac{1}{3}\left(3-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{a+bx^3}}\,dx+\frac{1}{3}\left(3+i\sqrt{3}\right)\int \frac{}{\left(1+\right.}$$

Mathematica [F]
time = 10.13, size = 0, normalized size = 0.00

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)),x]
```

```
[Out] Integrate[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)), x]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1+x}{(x^2+x+1)\,(b\,x^3+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x)
```

```
[Out] int((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/((b*x^3 + a)^(1/3)*(x^2 + x + 1)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt[3]{a+bx^3}\,(x^2+x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+x+1)/(b*x**3+a)**(1/3),x)
```

```
[Out] Integral((x + 1)/((a + b*x**3)**(1/3)*(x**2 + x + 1)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+1}{{\left(b\,x^3+a\right)}^{1/3}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((a + b*x^3)^(1/3)*(x + x^2 + 1)),x)
```

```
[Out] int((x + 1)/((a + b*x^3)^(1/3)*(x + x^2 + 1)), x)
```

3.96 $\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=96

$$-\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}+\frac{\log\left(1-x^3\right)}{6\sqrt[3]{a+b}}-\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

```
[Out] 1/6*ln(-x^3+1)/(a+b)^(1/3)-1/2*ln((a+b)^(1/3)-(b*x^3+a)^(1/3))/(a+b)^(1/3)-
1/3*arctan(1/3*(1+2*(b*x^3+a)^(1/3)/(a+b)^(1/3))*3^(1/2))/(a+b)^(1/3)*3^(1/
2)
```

Rubi [A]
time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {455, 57, 631, 210, 31}

$$\frac{\log\left(1-x^3\right)}{6\sqrt[3]{a+b}}-\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[(1 + (2*(a + b*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a + b
)^(1/3))) + Log[1 - x^3]/(6*(a + b)^(1/3)) - Log[(a + b)^(1/3) - (a + b*x^3
)^(1/3)]/(2*(a + b)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt[3]{a+bx}}\,dx, x, x^3\right)$$

$$= \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+b)^{2/3}+\sqrt[3]{a+b}\,x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right) + \frac{\text{Su}}{}$$

$$= \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+}}{\sqrt[3]{a-}}\right.}{\sqrt[3]{a+b}}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.33, size = 182, normalized size = 1.90

$$\frac{2\sqrt{-6+6i\sqrt{3}}\tan^{-1}\left(\frac{1+\frac{\left(-1-i\sqrt{3}\right)\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right) - i\left(-i+\sqrt{3}\right)\left(\log\left(\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)\left(2\sqrt[3]{a+b}+\sqrt[3]{a+bx^3}-i\sqrt{3}\sqrt[3]{a+bx^3}\right)\right) - 2\log\left(2\sqrt[3]{a+b}+\left(1+i\sqrt{3}\right)\sqrt[3]{a+bx^3}\right)\right)}{12\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] (2*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[(1 + ((-1 - I*Sqrt[3])*(a + b*x^3)^(1/3)
)/(a + b)^(1/3))/Sqrt[3]] - I*(-I + Sqrt[3])*(Log[((a + b)^(1/3) - (a + b*x
^3)^(1/3))*(2*(a + b)^(1/3) + (a + b*x^3)^(1/3) - I*Sqrt[3]*(a + b*x^3)^(1/
3))] - 2*Log[2*(a + b)^(1/3) + (1 + I*Sqrt[3])*(a + b*x^3)^(1/3)]))/(12*(a
+ b)^(1/3))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2/((1 - x^3)*(a + b*x^3)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(-x^{3}+1\right) \left(b\,x^{3}+a \right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

```
[Out] int(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

Maxima [**A**]
time = 0.33, size = 110, normalized size = 1.15

$$-\frac{\frac{2\,\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,\left(bx^{3}+a\right)^{\frac{1}{3}}+\left(a+b\right)^{\frac{1}{3}}\right)}{3\,\left(a+b\right)^{\frac{1}{3}}}\right)}{\left(a+b\right)^{\frac{1}{3}}}-\frac{b\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(a+b\right)^{\frac{1}{3}}+\left(a+b\right)^{\frac{2}{3}}\right)}{\left(a+b\right)^{\frac{1}{3}}}+\frac{2\,b\log\left(\left(bx^{3}+a\right)^{\frac{1}{3}}-\left(a+b\right)^{\frac{1}{3}}\right)}{\left(a+b\right)^{\frac{1}{3}}}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (a + b)^(1/3))/
(a + b)^(1/3))/(a + b)^(1/3) - b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*
(a + b)^(1/3) + (a + b)^(2/3))/(a + b)^(1/3) + 2*b*log((b*x^3 + a)^(1/3) -
(a + b)^(1/3))/(a + b)^(1/3))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(75) = 150$.
time = 0.34, size = 387, normalized size = 4.03

$$\left[\frac{3\sqrt{\frac{1}{3}}(a+b)\sqrt{\frac{(-a-b)^{\frac{1}{3}}}{a+b}}\log\left(\frac{2bx^3+3\sqrt{\frac{1}{3}}\left((bx^3+a)^{\frac{1}{3}}(a+b)-(a+b)(-a-b)^{\frac{1}{3}}-2(bx^3+a)^{\frac{2}{3}}(-a-b)^{\frac{2}{3}}\right)\sqrt{\frac{(-a-b)^{\frac{1}{3}}}{a+b}}+3a-3(bx^3+a)^{\frac{1}{3}}(-a-b)^{\frac{2}{3}}+b}{x^3-1}\right)+(-a-b)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}(-a-b)^{\frac{1}{3}}+(-a-b)^{\frac{2}{3}}\right)-2(-a-b)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{1}{3}}+(-a-b)^{\frac{1}{3}}\right)}{6(a+b)}, -\frac{6\sqrt{\frac{1}{3}}(a+b)\sqrt{-\frac{(-a-b)^{\frac{1}{3}}}{a+b}}\arctan\left(\sqrt{\frac{1}{3}}\left(2(bx^3+a)^{\frac{1}{3}}-(-a-b)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a-b)^{\frac{1}{3}}}{a+b}}\right)-(-a-b)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}(-a-b)^{\frac{1}{3}}+(-a-b)^{\frac{2}{3}}\right)+2(-a-b)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{1}{3}}+(-a-b)^{\frac{1}{3}}\right)}{6(a+b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(a + b)*sqrt((-a - b)^(1/3)/(a + b))*log((2*b*x^3 + 3*sqr
t(1/3)*((b*x^3 + a)^(1/3)*(a + b) - (a + b)*(-a - b)^(1/3) - 2*(b*x^3 + a)^
(2/3)*(-a - b)^(2/3))*sqrt((-a - b)^(1/3)/(a + b)) + 3*a - 3*(b*x^3 + a)^(1
/3)*(-a - b)^(2/3) + b)/(x^3 - 1)) + (-a - b)^(2/3)*log((b*x^3 + a)^(2/3) -
 (b*x^3 + a)^(1/3)*(-a - b)^(1/3) + (-a - b)^(2/3)) - 2*(-a - b)^(2/3)*log(
(b*x^3 + a)^(1/3) + (-a - b)^(1/3)))/(a + b), -1/6*(6*sqrt(1/3)*(a + b)*sqr
t(-(-a - b)^(1/3)/(a + b))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) - (-a - b)
^(1/3))*sqrt(-(-a - b)^(1/3)/(a + b))) - (-a - b)^(2/3)*log((b*x^3 + a)^(2/
3) - (b*x^3 + a)^(1/3)*(-a - b)^(1/3) + (-a - b)^(2/3)) + 2*(-a - b)^(2/3)*
log((b*x^3 + a)^(1/3) + (-a - b)^(1/3)))/(a + b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^3\sqrt[3]{a+bx^3} - \sqrt[3]{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)/(b*x**3+a)**(1/3),x)
```

```
[Out] -Integral(x**2/(x**3*(a + b*x**3)**(1/3) - (a + b*x**3)**(1/3)), x)
```

Giac [A]
time = 0.00, size = 180, normalized size = 1.88

$$\frac{\left((a+b)^{\frac{1}{3}}\right)^2\ln\left(\left((a+bx^3)^{\frac{1}{3}}\right)^2+(a+b)^{\frac{1}{3}}(a+bx^3)^{\frac{1}{3}}+(a+b)^{\frac{1}{3}}(a+b)^{\frac{1}{3}}\right)}{6a+6b}+\frac{\left((a+b)^{\frac{1}{3}}\right)^2\arctan\left(\frac{2\left((a+bx^3)^{\frac{1}{3}}+\frac{(a+b)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,(a+b)^{\frac{1}{3}}}\right)}{-\sqrt{3}\,a-\sqrt{3}\,b}+\frac{(a+b)^{\frac{1}{3}}(a+b)^{\frac{1}{3}}\ln\left|(a+bx^3)^{\frac{1}{3}}-(a+b)^{\frac{1}{3}}\right|}{3(-b-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

```
[Out] -(a + b)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (a + b)^(1/3))/(a
+ b)^(1/3))/(sqrt(3)*a + sqrt(3)*b) + 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 +
a)^(1/3)*(a + b)^(1/3) + (a + b)^(2/3))/(a + b)^(1/3) - 1/3*log(abs((b*x^3
+ a)^(1/3) - (a + b)^(1/3)))/(a + b)^(1/3)
```

Mupad [B]
time = 0.59, size = 157, normalized size = 1.64

$$\frac{\ln\left((b\,x^3+a)^{1/3}-\frac{9\,a+9\,b}{9\,(-a-b)^{2/3}}\right)}{3\,(-a-b)^{1/3}}+\frac{\ln\left((b\,x^3+a)^{1/3}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,(9\,a+9\,b)}{36\,(-a-b)^{2/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-a-b)^{1/3}}-\frac{\ln\left((b\,x^3+a)^{1/3}-\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\,(9\,a+9\,b)}{36\,(-a-b)^{2/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-a-b)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^3 - 1)*(a + b*x^3)^(1/3)),x)
```

```
[Out] log((a + b*x^3)^(1/3) - (9*a + 9*b)/(9*(- a - b)^(2/3)))/(3*(- a - b)^(1/3)
) + (log((a + b*x^3)^(1/3) - ((3^(1/2)*1i - 1)^2*(9*a + 9*b))/(36*(- a - b)
^(2/3)))*(3^(1/2)*1i - 1))/(6*(- a - b)^(1/3)) - (log((a + b*x^3)^(1/3) - (
(3^(1/2)*1i + 1)^2*(9*a + 9*b))/(36*(- a - b)^(2/3)))*(3^(1/2)*1i + 1))/(6*
(- a - b)^(1/3))
```

3.97 $\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=88

$$-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}\,x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/12*ln(x^3+1)*2^(2/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/6*arcta
n(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {384}

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) -
Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}\,x}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x,\right.$$

$$= \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subs}}{}$$

$$= -\frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.}{\sqrt[3]{2}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}} - \frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right.}{3\sqrt[3]{2}}$$

Mathematica [**A**]
time = 0.22, size = 114, normalized size = 1.30

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-2\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\,(1-x^3)^{2/3}\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*Log[2
*x + 2^(2/3)*(1 - x^3)^(1/3)] + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^
(1/3)*(1 - x^3)^(2/3)])/2^(1/3)
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 + x^3)*(1 - x^3)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.59, size = 907, normalized size = 10.31

method	result	size
trager	Expression too large to display	907

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*RootOf(_Z^3-4)*ln(-(3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z
^2)*RootOf(_Z^3-4)^3*x^3+54*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*
_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(_Z^3-4)^2*RootOf(Root
Of(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x-5*RootOf(_Z^3-4)^2*(-x^3+1)^(1/
3)*x^2-6*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2
)*RootOf(_Z^3-4)*x^2-RootOf(_Z^3-4)*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*Roo
tOf(_Z^3-4)+36*_Z^2)*x^3-2*x*(-x^3+1)^(2/3)+RootOf(_Z^3-4)+18*RootOf(RootOf
(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(1+x)/(x^2-x+1))-1/6*ln((-3*RootOf
(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+36*Root
Of(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*
(-x^3+1)^(2/3)*RootOf(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*x+RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2+30*(-x^3+1)^(1/3)*RootOf(Ro
otOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-3*RootOf(_Z^
3-4)*x^3+36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+10*x*(
-x^3+1)^(2/3)+RootOf(_Z^3-4)-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2))/(1+x)/(x^2-x+1))*RootOf(_Z^3-4)-ln((-3*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+36*RootOf(RootOf(_Z^3-4)^2
+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*Root
Of(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x+RootOf(
_Z^3-4)^2*(-x^3+1)^(1/3)*x^2+30*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-3*RootOf(_Z^3-4)*x^3+36*RootOf(
RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+10*x*(-x^3+1)^(2/3)+RootO
f(_Z^3-4)-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(1+x)/(x
^2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(67) = 134$.
time = 1.56, size = 253, normalized size = 2.88

$$-\frac{1}{18}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-\sqrt{6}\,2^{\frac{1}{3}}(71x^9-111x^6+33x^3-1)+12\sqrt{6}\,(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}\right)}{6\,(109x^9-105x^6+3x^3+1)}\right)+\frac{1}{18}\cdot 2^{\frac{2}{3}}\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}x^2+2^{\frac{2}{3}}(x^3+1)+6\,(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\right)-\frac{1}{36}\cdot 2^{\frac{2}{3}}\log\left(\frac{3\cdot 2^{\frac{2}{3}}(5x^4-x)(-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(19x^6-16x^3+1)-12\,(2x^5-x^2)(-x^3+1)^{\frac{1}{3}}}{x^6+2x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7 + 4*x^4
- x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1) + 1
2*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 + 3*
x^3 + 1)) + 1/18*2^(2/3)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2/3)*(x^3
 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/36*2^(2/3)*log((3*2^(2/3)*(5*x
^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)
*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{{\left(1-x^3\right)}^{1/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int(1/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.98 $\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=233

$$\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}+\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-$$

```
[Out] 1/24*ln((1-x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(
1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2
^(2/3)-1/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3*(1-2*2^(1
/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/3*(1+2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [A]
time = 0.07, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {502, 2174, 206, 31, 648, 631, 210, 642}

$$\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}+\frac{\log\left((1-x)(x+1)^2\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[
3]) + Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1
- x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (2^
(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x^3
)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Mathematica [**A**]
time = 0.74, size = 283, normalized size = 1.21

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)-2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x+2\sqrt[3]{1-x^3}\right)+2\log\left(2^{2/3}-2\,2^{2/3}x+2^{2/3}x^2+(-1+x)\sqrt[3]{2-2x^3}+(1-x^3)^{2/3}\right)+\log\left(2^{2/3}-2\,2^{2/3}x+2^{2/3}x^2-2(-1+x)\sqrt[3]{2-2x^3}+4(1-x^3)^{2/3}\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1 - x^
3)^(1/3))] - 4*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2*2^(
1/3)*x + (1 - x^3)^(1/3))] - 4*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/3)]
- 2*Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)] + 2*Log[2^(2/3) - 2*2^(2/
3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)] + Log[2^
(2/3) - 2*2^(2/3)*x + 2^(2/3)*x^2 - 2*(-1 + x)*(2 - 2*x^3)^(1/3) + 4*(1 - x
^3)^(2/3)])/(12*2^(1/3))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/((1 + x^3)*(1 - x^3)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x}{(-x^3+1)^{\frac{1}{3}}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(171) = 342$.
time = 1.37, size = 373, normalized size = 1.60

$$-\frac{1}{36}\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{2^{\frac{1}{6}}\left(24\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}(x^{14}-2x^{11}-6x^{8}-2x^{5}+x^{2})(-x^{3}+1)^{\frac{2}{3}}+12\sqrt{6}\,(-1)^{\frac{1}{3}}(x^{16}-33x^{13}+110x^{10}-110x^{7}+33x^{4}-x)(-x^{3}+1)^{\frac{1}{3}}+\sqrt{6}\,2^{\frac{1}{3}}(x^{18}+42x^{15}-417x^{12}+812x^{9}-417x^{6}+42x^{3}+1)\right)}{6(x^{18}-102x^{15}+447x^{12}-628x^{9}+447x^{6}-102x^{3}+1)}\right)-\frac{1}{72}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{12\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^{8}-4x^{5}+x^{2})(-x^{3}+1)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}(x^{12}-32x^{9}+78x^{6}-32x^{3}+1)-6(x^{10}-11x^{7}+11x^{4}-x)(-x^{3}+1)^{\frac{1}{3}}}{x^{12}+4x^{9}+6x^{6}+4x^{3}+1}\right)+\frac{1}{36}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{12(-x^{3}+1)^{\frac{2}{3}}x^{2}-6\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}(x^{4}-x)(-x^{3}+1)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^{6}+2x^{3}+1)}{x^{6}+2x^{3}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(24*sqrt(6)*2^(2/3)*(-1
)^(2/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) + 12*sqrt(6)
*(-1)^(1/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(
1/3) + sqrt(6)*2^(1/3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*
x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) -
 1/72*2^(2/3)*(-1)^(1/3)*log(-(12*2^(2/3)*(-1)^(1/3)*(x^8 - 4*x^5 + x^2)*(-
x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) -
 6*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4
*x^3 + 1)) + 1/36*2^(2/3)*(-1)^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^2 - 6*2^(1
/3)*(-1)^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3)*(x^6 + 2*x^3
 + 1))/(x^6 + 2*x^3 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(1-x^3\right)^{1/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int(x/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.99 $\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=82

$$\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/12*ln(x^3+1)*2^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1
/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {455, 57, 631, 210, 31}

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x
^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)$$

$$= -\frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-\cdots}\right.}{\sqrt[3]{2}}$$

$$= \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

Mathematica [A]
time = 0.09, size = 104, normalized size = 1.27

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+2\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\,(1-x^3)^{2/3}\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 2*Log[-2 + 2^(2/
3)*(1 - x^3)^(1/3)] - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(
2/3)])/(6*2^(1/3))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2/((1 + x^3)*(1 - x^3)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.31, size = 657, normalized size = 8.01

method	result	size
trager	Expression too large to display	657

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*ln(-(12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z
^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootO
f(_Z^3-4)^2*x^3+12*RootOf(_Z^3-4)*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootO
f(_Z^3-4)+36*_Z^2)*x^3-21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)-42*(-x^3+1)^(2/3)
-28*RootOf(_Z^3-4)+42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))
/(1+x)/(x^2-x+1))*RootOf(_Z^3-4)-ln(-(12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootO
f(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*Roo
tOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*RootOf(_Z^3-4)*x^3-18*RootOf
(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-21*RootOf(_Z^3-4)^2*(-x^
3+1)^(1/3)-42*(-x^3+1)^(2/3)-28*RootOf(_Z^3-4)+42*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2))/(1+x)/(x^2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*
RootOf(_Z^3-4)+36*_Z^2)+RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2
)*ln((15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4
)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z
^3-4)^2*x^3-5*RootOf(_Z^3-4)*x^3-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3
-4)+36*_Z^2)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)+35*Ro
otOf(_Z^3-4)+42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(1+x)
/(x^2-x+1))
```

Maxima [**A**]
time = 0.33, size = 86, normalized size = 1.05

$$\frac{1}{6}\,\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Fricas [A]
time = 0.32, size = 90, normalized size = 1.10

$$\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 +
 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 +
 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2}}{\sqrt[3]{-\left(x-1\right)\left(x^{2}+x+1\right)}\,\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [A]
time = 0.00, size = 145, normalized size = 1.77

$$-\frac{1}{24}\left(3\cdot\left(2^{\frac{1}{3}}\right)^{2}-\left(2^{\frac{1}{3}}\right)^{2}\right)\ln\left(\left(\left(-x^{3}+1\right)^{\frac{1}{3}}\right)^{2}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\cdot 2^{\frac{1}{3}}\right)+\frac{1}{6}\cdot\left(2^{\frac{1}{3}}\right)^{2}\sqrt{3}\arctan\left(\frac{2\left(\left(-x^{3}+1\right)^{\frac{1}{3}}+\frac{2^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\cdot 2^{\frac{1}{3}}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\cdot 2^{\frac{1}{3}}\ln\left|\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

Mupad [B]
time = 0.55, size = 100, normalized size = 1.22

$$\frac{2^{2/3}\ln\left(\left(1-x^{3}\right)^{1/3}-2^{1/3}\right)}{6}+\frac{2^{2/3}\ln\left(\left(1-x^{3}\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{2/3}\ln\left(\left(1-x^{3}\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)

[Out] (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2^(2/3)*log((1 - x^3)^(1/3) -
 (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x
^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

3.100 $\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=135

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

```
[Out] 1/4*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/
3)-1/2*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/2*arctan(1/3*(1-2*2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 383 vs. $2(135) = 270$.
time = 0.17, antiderivative size = 383, normalized size of antiderivative = 2.84, number of steps used = 16, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2183, 384, 502, 2174, 206, 31, 648, 631, 210, 642, 455, 57}

$$-\frac{\log(x^3+1)}{3\sqrt[3]{2}}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}+\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log((1-x)(x+1)^2)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]
) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) +
 ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[(1 -
 x)*(1 + x)^2]/(6*2^(1/3)) - Log[1 + x^3]/(3*2^(1/3)) + Log[1 + (2^(2/3)*(1
 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) -
 (2^(2/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 + Log[2^(1/3) - (1
- x^3)^(1/3)]/(2*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))
 - Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \left(\frac{1-i\sqrt{3}}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}+\frac{1+i\sqrt{3}}{\left(-1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\right)dx$$

$$= \left(1-i\sqrt{3}\right)\int \frac{1}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\,dx+\left(1+i\sqrt{3}\right)\int \frac{\cdots}{\left(-1+i\sqrt{\cdots}\right.}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1-i\sqrt{3}+2x\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)}+\frac{\left(3+i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1\cdots\right.}{\sqrt[3]{1\cdots}}}{2\cdots}\right.}{2\sqrt[3]{2}\left(i-\sqrt{3}\right)\cdots}$$

Mathematica [A]
time = 0.91, size = 145, normalized size = 1.07

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)+\log\left(2^{2/3}-2\,2^{2/3}x+2^{2/3}x^2+(-1+x)\sqrt[3]{2-2x^3}+\left(1-x^3\right)^{2/3}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2*2^(1/3)*x + (1
 - x^3)^(1/3))] - 2*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/3)] + Log[2^(2/
3) - 2*2^(2/3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^(2/
3)])/(2*2^(1/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.21, size = 720, normalized size = 5.33

method	result	size
trager	Expression too large to display	720

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*ln((RootOf(_Z^3+4)^2*Ro
otOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x+RootOf(RootOf(_Z^3+4)^2
+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*(-x^3+1)^(2/3)-2*(-x^3+1)^(1/3)
*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x+2*(-x^3+1)^(1/3)*Roo
tOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)-x^2+x-1)/(x^2-x+1))-1/2*ln
(-(RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x+R
ootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1
)^(2/3)-2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2
)*RootOf(_Z^3+4)*x-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x+2*(-x^3+1)^(1/3)*RootO
f(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)+(-x^3+1)^(1/3
)*RootOf(_Z^3+4)^2+x^2*RootOf(_Z^3+4)-3*x*RootOf(_Z^3+4)-2*(-x^3+1)^(2/3)+R
ootOf(_Z^3+4))/(x^2-x+1))*RootOf(_Z^3+4)-ln(-(RootOf(RootOf(_Z^3+4)^2+2*_Z*
```

```
RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x+RootOf(RootOf(_Z^3+4)^2+2*_Z*Root
Of(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)-2*(-x^3+1)^(1/3)*RootOf(
RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*x-(-x^3+1)^(1/3
)*RootOf(_Z^3+4)^2*x+2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_
Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)+(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2+x^2*RootOf(_Z
^3+4)-3*x*RootOf(_Z^3+4)-2*(-x^3+1)^(2/3)+RootOf(_Z^3+4))/(x^2-x+1))*RootOf
(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(101) = 202$.
time = 5.36, size = 318, normalized size = 2.36

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(4\cdot 2^{\frac{1}{6}}(-1)^{\frac{2}{3}}(x^4-4x^3+5x^2-4x+1)(-x^3+1)^{\frac{2}{3}}-4\sqrt{2}\,(-1)^{\frac{1}{3}}(x^5-x^4-3x^3+3x^2+x-1)(-x^3+1)^{\frac{1}{3}}+2^{\frac{5}{6}}(x^6-7x^5+10x^4-7x^3+10x^2-7x+1)\right)}{6(3x^6-9x^5+6x^4-x^3+6x^2-9x+3)}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(-x^3+1)^{\frac{2}{3}}(x^2-3x+1)+2^{\frac{1}{3}}(-1)^{\frac{2}{3}}(x^4-3x^2+1)+4(-x^3+1)^{\frac{1}{3}}(x^2-x)}{x^4-2x^3+3x^2-2x+1}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{2\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}(x-1)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^2-x+1)-2(-x^3+1)^{\frac{2}{3}}}{x^2-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(4*2^(1/6)*(-1)^(
2/3)*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) - 4*sqrt(2)*(-1)^(1/3
)*(x^5 - x^4 - 3*x^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 2^(5/6)*(x^6 - 7*x
^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x
^2 - 9*x + 3)) - 1/12*2^(2/3)*(-1)^(1/3)*log(-(2^(2/3)*(-1)^(1/3)*(-x^3 + 1
)^(2/3)*(x^2 - 3*x + 1) + 2^(1/3)*(-1)^(2/3)*(x^4 - 3*x^2 + 1) + 4*(-x^3 +
1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/6*2^(2/3)*(-1)^(1/
3)*log(-(2*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) + 2^(2/3)*(-1)^(1/3)
*(x^2 - x + 1) - 2*(-x^3 + 1)^(2/3))/(x^2 - x + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+1}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2-x+1)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral((x + 1)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+1}{{\left(1-x^3\right)}^{1/3}\,\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((1 - x^3)^(1/3)*(x^2 - x + 1)),x)
```

```
[Out] int((x + 1)/((1 - x^3)^(1/3)*(x^2 - x + 1)), x)
```

3.101 $\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=135

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

```
[Out] 1/4*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/
3)-1/2*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/2*arctan(1/3*(1-2*2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 383 vs. 2(135) = 270.
time = 0.19, antiderivative size = 383, normalized size of antiderivative = 2.84, number of steps used = 17, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {1600, 2183, 384, 502, 2174, 206, 31, 648, 631, 210, 642, 455, 57}

$$-\frac{\log(x^3+1)}{3\sqrt[3]{2}}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}+\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left((1-x)(x+1)^2\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]
) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) +
 ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[(1 -
 x)*(1 + x)^2]/(6*2^(1/3)) - Log[1 + x^3]/(3*2^(1/3)) + Log[1 + (2^(2/3)*(1
 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) -
 (2^(2/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 + Log[2^(1/3) - (1
- x^3)^(1/3)]/(2*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))
 - Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$$

$$= \int \left(\frac{1-i\sqrt{3}}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}} + \frac{1+i\sqrt{3}}{\left(-1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}} \right) dx$$

$$= \left(1-i\sqrt{3}\right)\int \frac{1}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\,dx + \left(1+i\sqrt{3}\right)\int \frac{1}{\left(-1+i\sqrt{3}+\cdots\right.}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1-i\sqrt{3}+2x\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)} + \frac{\left(3+i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1+i\sqrt{3}\cdots\right.}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right.}{2\sqrt[3]{2}\left(i-\sqrt{3}\right)}$$

Mathematica [**A**]
time = 0.89, size = 145, normalized size = 1.07

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right) - 2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right) + \log\left(2^{2/3}-2\,2^{2/3}x+2^{2/3}x^2+(-1+x)\sqrt[3]{2-2x^3}+(1-x^3)^{2/3}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2*2^(1/3)*x + (1
 - x^3)^(1/3))] - 2*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/3)] + Log[2^(2/
3) - 2*2^(2/3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^(2/
3)])/(2*2^(1/3))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^2/((1 + x^3)*(1 - x^3)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.14, size = 737, normalized size = 5.46

method	result	size
trager	Expression too large to display	737

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln((2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^
3+4)^2*x+RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)
^2*(-x^3+1)^(2/3)-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x+(-x^3+1)^(1/3)*RootOf(_
Z^3+4)^2-2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2+2*RootOf
(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x-2*RootOf(RootOf(_Z^3+4)^2+2
*_Z*RootOf(_Z^3+4)+4*_Z^2))/(x^2-x+1))*RootOf(_Z^3+4)-ln((2*RootOf(RootOf(_
Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x+RootOf(RootOf(_Z^
3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)-(-x^3+1)
^(1/3)*RootOf(_Z^3+4)^2*x+(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2-2*RootOf(RootOf(_
Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*Roo
tOf(_Z^3+4)+4*_Z^2)*x-2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)
)/(x^2-x+1))*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)+1/2*RootOf
(_Z^3+4)*ln(-(2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootO
f(_Z^3+4)^2*x+RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z
^3+4)^2*(-x^3+1)^(2/3)+2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf
(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*x-2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+
2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*R
ootOf(_Z^3+4)+4*_Z^2)*x^2-6*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_
Z^2)*x-2*(-x^3+1)^(2/3)+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^
2))/(x^2-x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)^2/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(101) = 202$.
time = 6.30, size = 318, normalized size = 2.36

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(4\cdot 2^{\frac{1}{6}}\,(-1)^{\frac{2}{3}}\,(x^4-4\,x^3+5\,x^2-4\,x+1)(-x^3+1)^{\frac{2}{3}}-4\,\sqrt{2}\,(-1)^{\frac{1}{3}}\,(x^5-x^4-3\,x^3+3\,x^2+x-1)(-x^3+1)^{\frac{1}{3}}+2^{\frac{5}{6}}(x^6-7\,x^5+10\,x^4-7\,x^3+10\,x^2-7\,x+1)\right)}{6\,(3\,x^6-9\,x^5+6\,x^4-x^3+6\,x^2-9\,x+3)}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\log\left(-\frac{2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\,(-x^3+1)^{\frac{2}{3}}(x^2-3\,x+1)+2^{\frac{1}{3}}\,(-1)^{\frac{2}{3}}\,(x^4-3\,x^2+1)+4\,(-x^3+1)^{\frac{1}{3}}(x^2-x)}{x^4-2\,x^3+3\,x^2-2\,x+1}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\log\left(-\frac{2\cdot 2^{\frac{1}{3}}\,(-1)^{\frac{2}{3}}\,(-x^3+1)^{\frac{1}{3}}(x-1)+2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\,(x^2-x+1)-2\,(-x^3+1)^{\frac{2}{3}}}{x^2-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(4*2^(1/6)*(-1)^(
2/3)*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) - 4*sqrt(2)*(-1)^(1/3
)*(x^5 - x^4 - 3*x^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 2^(5/6)*(x^6 - 7*x
^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x
^2 - 9*x + 3)) - 1/12*2^(2/3)*(-1)^(1/3)*log(-(2^(2/3)*(-1)^(1/3)*(-x^3 + 1
)^(2/3)*(x^2 - 3*x + 1) + 2^(1/3)*(-1)^(2/3)*(x^4 - 3*x^2 + 1) + 4*(-x^3 +
1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/6*2^(2/3)*(-1)^(1/
3)*log(-(2*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) + 2^(2/3)*(-1)^(1/3)
*(x^2 - x + 1) - 2*(-x^3 + 1)^(2/3))/(x^2 - x + 1))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral((x + 1)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x+1)^2}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int((x + 1)^2/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.102 $\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx$

Optimal. Leaf size=119

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(1+\frac{2^{2/3}(1+x)^2}{(1+x^3)^{2/3}}-\frac{\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}\right)}{2\sqrt[3]{2}}+\frac{\log\left(1+\frac{\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}\right)}{\sqrt[3]{2}}$$

```
[Out] -1/4*ln(1+2^(2/3)*(1+x)^2/(x^3+1)^(2/3)-2^(1/3)*(1+x)/(x^3+1)^(1/3))*2^(2/3
)+1/2*ln(1+2^(1/3)*(1+x)/(x^3+1)^(1/3))*2^(2/3)-1/2*arctan(1/3*(1-2*2^(1/3)
*(1+x)/(x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 357 vs. 2(119) = 238.
time = 0.18, antiderivative size = 357, normalized size of antiderivative = 3.00, number of steps used = 16, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$,
Rules used = {2183, 384, 502, 2174, 206, 31, 648, 631, 210, 642, 455, 57}

$$\frac{\log(1-x^3)}{3\sqrt[3]{2}}-\frac{\log\left(\frac{2^{2/3}(x+1)^2}{(x^3+1)^{2/3}}-\frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}+1\right)}{3\sqrt[3]{2}}+\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}+1\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{x^3+1}\right)}{2\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}x-\sqrt[3]{x^3+1}\right)}{2\sqrt[3]{2}}+\frac{\log\left(-2^{2/3}\sqrt[3]{x^3+1}+x+1\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left((1-x)^2(x+1)\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*2^(1/3)*x)/(1 + x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - (2^
(2/3)*ArcTan[(1 - (2*2^(1/3)*(1 + x))/(1 + x^3)^(1/3))/Sqrt[3]])/Sqrt[3] -
ArcTan[(1 + (2^(1/3)*(1 + x))/(1 + x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) -
 ArcTan[(1 + 2^(2/3)*(1 + x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[(1 -
 x)^2*(1 + x)]/(6*2^(1/3)) + Log[1 - x^3]/(3*2^(1/3)) - Log[1 + (2^(2/3)*(1
 + x)^2)/(1 + x^3)^(2/3) - (2^(1/3)*(1 + x))/(1 + x^3)^(1/3)]/(3*2^(1/3)) +
 (2^(2/3)*Log[1 + (2^(1/3)*(1 + x))/(1 + x^3)^(1/3)])/3 - Log[2^(1/3) - (1
+ x^3)^(1/3)]/(2*2^(1/3)) - Log[2^(1/3)*x - (1 + x^3)^(1/3)]/(2*2^(1/3)) +
Log[1 + x - 2^(2/3)*(1 + x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx &= \int \left(\frac{-1-i\sqrt{3}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1+x^3}}+\frac{-1+i\sqrt{3}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{1+x^3}}\right)dx \\
&= \left(-1-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1+x^3}}\,dx+\left(-1+i\sqrt{3}\right)\int \frac{}{\left(1+i\sqrt{\right.}} \\
&= \frac{\left(3-i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1-i\sqrt{3}-2x\right)}{\sqrt[3]{1+x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)}-\frac{\left(3+i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1+i\right.}{\sqrt[3]{1-}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i-\sqrt{3}\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.86, size = 139, normalized size = 1.17

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1+x^3}}{-2\sqrt[3]{2}-2\sqrt[3]{2}x+\sqrt[3]{1+x^3}}\right)+2\log\left(\sqrt[3]{2}+\sqrt[3]{2}x+\sqrt[3]{1+x^3}\right)-\log\left(2^{2/3}+2\,2^{2/3}x+2^{2/3}x^2-\sqrt[3]{2}(1+x)\sqrt[3]{1+x^3}+\left(1+x^3\right)^{2/3}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 + x^3)^(1/3))/(-2*2^(1/3) - 2*2^(1/3)*x + (1
+ x^3)^(1/3))] + 2*Log[2^(1/3) + 2^(1/3)*x + (1 + x^3)^(1/3)] - Log[2^(2/3)
 + 2*2^(2/3)*x + 2^(2/3)*x^2 - 2^(1/3)*(1 + x)*(1 + x^3)^(1/3) + (1 + x^3)^
(2/3)])/(2*2^(1/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.45, size = 714, normalized size = 6.00

method	result	size
trager	Expression too large to display	714

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(-(2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z
^3-4)^2*x+(x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)
*RootOf(_Z^3-4)^2-(x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x-2*RootOf(RootOf(_Z^3-4)^
2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-(x^3+1)^(1/3)*RootOf(_Z^3-4)^2-2*RootOf(R
ootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-2*RootOf(RootOf(_Z^3-4)^2+2*_
Z*RootOf(_Z^3-4)+4*_Z^2))/(x^2+x+1))*RootOf(_Z^3-4)-ln(-(2*RootOf(RootOf(_Z
^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x+(x^3+1)^(2/3)*Root
Of(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2-(x^3+1)^(1
/3)*RootOf(_Z^3-4)^2*x-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2
)*x^2-(x^3+1)^(1/3)*RootOf(_Z^3-4)^2-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(
_Z^3-4)+4*_Z^2)*x-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(x
```

```
^2+x+1))*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+1/2*RootOf(_Z^
3-4)*ln((2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^
3-4)^2*x+(x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*
RootOf(_Z^3-4)^2+2*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-
4)+4*_Z^2)*(x^3+1)^(1/3)*x+2*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+2*_Z*Ro
otOf(_Z^3-4)+4*_Z^2)*(x^3+1)^(1/3)+2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z
^3-4)+4*_Z^2)*x^2+6*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+2
*(x^3+1)^(2/3)+2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(x^2+
x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate((x - 1)/((x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(93) = 186$.
time = 5.49, size = 268, normalized size = 2.25

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}(x^6+7x^5+10x^4+7x^3+10x^2+7x+1)-4\sqrt{2}\,(x^5+x^4-3x^3-3x^2+x+1)(x^3+1)^{\frac{1}{3}}+4\cdot 2^{\frac{1}{6}}(x^4+4x^3+5x^2+4x+1)(x^3+1)^{\frac{2}{3}}\right)}{6\,(3x^6+9x^5+6x^4+x^3+6x^2+9x+3)}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}(x^3+1)^{\frac{2}{3}}(x^2+3x+1)-2^{\frac{1}{3}}(x^4-3x^2+1)-4\,(x^3+1)^{\frac{1}{3}}(x^2+x)}{x^4+2x^3+3x^2+2x+1}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}(x^2+x+1)+2\cdot 2^{\frac{1}{3}}(x^3+1)^{\frac{1}{3}}(x+1)+2\,(x^3+1)^{\frac{2}{3}}}{x^2+x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(x^6 + 7*x^5 + 10*x
^4 + 7*x^3 + 10*x^2 + 7*x + 1) - 4*sqrt(2)*(x^5 + x^4 - 3*x^3 - 3*x^2 + x +
 1)*(x^3 + 1)^(1/3) + 4*2^(1/6)*(x^4 + 4*x^3 + 5*x^2 + 4*x + 1)*(x^3 + 1)^(
2/3))/(3*x^6 + 9*x^5 + 6*x^4 + x^3 + 6*x^2 + 9*x + 3)) - 1/12*2^(2/3)*log((
2^(2/3)*(x^3 + 1)^(2/3)*(x^2 + 3*x + 1) - 2^(1/3)*(x^4 - 3*x^2 + 1) - 4*(x^
3 + 1)^(1/3)*(x^2 + x))/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)) + 1/6*2^(2/3)*log(
(2^(2/3)*(x^2 + x + 1) + 2*2^(1/3)*(x^3 + 1)^(1/3)*(x + 1) + 2*(x^3 + 1)^(2
/3))/(x^2 + x + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x}{x^2\sqrt[3]{x^3+1}+x\sqrt[3]{x^3+1}+\sqrt[3]{x^3+1}}\,dx-\int\left(-\frac{1}{x^2\sqrt[3]{x^3+1}+x\sqrt[3]{x^3+1}+\sqrt[3]{x^3+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x**2+x+1)/(x**3+1)**(1/3),x)
```

```
[Out] -Integral(x/(x**2*(x**3 + 1)**(1/3) + x*(x**3 + 1)**(1/3) + (x**3 + 1)**(1/
3)), x) - Integral(-1/(x**2*(x**3 + 1)**(1/3) + x*(x**3 + 1)**(1/3) + (x**3
 + 1)**(1/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x-1}{{\left(x^3+1\right)}^{1/3}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((x^3 + 1)^(1/3)*(x + x^2 + 1)),x)
```

```
[Out] -int((x - 1)/((x^3 + 1)^(1/3)*(x + x^2 + 1)), x)
```

3.103 $\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2} \, dx$

Optimal. Leaf size=43

$$\frac{1}{\sqrt[3]{1-x^3}}+\frac{x}{\sqrt[3]{1-x^3}}-x^2\,{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};x^3\right)$$

```
[Out] 1/(-x^3+1)^(1/3)+x/(-x^3+1)^(1/3)-x^2*hypergeom([2/3, 4/3],[5/3],x^3)
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2183, 197, 371, 267}

$$x^2\left(-{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};x^3\right)\right)+\frac{x}{\sqrt[3]{1-x^3}}+\frac{1}{\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3)/(1 + x + x^2)^2,x]
```

```
[Out] (1 - x^3)^(-1/3) + x/(1 - x^3)^(1/3) - x^2*Hypergeometric2F1[2/3, 4/3, 5/3,
 x^3]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
```

```
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2}\,dx = \int \left(-\frac{4(1-x^3)^{2/3}}{3\left(-1+i\sqrt{3}-2x\right)^2}+\frac{4i(1-x^3)^{2/3}}{3\sqrt{3}\left(-1+i\sqrt{3}-2x\right)}-\frac{4(1-x^3)^{2/3}}{3\left(1+i\sqrt{3}+2x\right)^2}+\frac{}{3}\right.$$

$$= -\left(\frac{4}{3}\int \frac{(1-x^3)^{2/3}}{\left(-1+i\sqrt{3}-2x\right)^2}\,dx\right)-\frac{4}{3}\int \frac{(1-x^3)^{2/3}}{\left(1+i\sqrt{3}+2x\right)^2}\,dx+\frac{(4i)\int \frac{(1-x^3)^{2/3}}{-1+i\sqrt{3}-2x}}{3\sqrt{3}}$$

Mathematica [**A**]
time = 10.11, size = 43, normalized size = 1.00

$$\frac{(1+2x)\,(1-x^3)^{2/3}}{1+x+x^2}+x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(2/3)/(1 + x + x^2)^2,x]
[Out] ((1 + 2*x)*(1 - x^3)^(2/3))/(1 + x + x^2) + x^2*Hypergeometric2F1[1/3, 2/3,
 5/3, x^3]
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x^3)^(2/3)/(1 + x + x^2)^2,x]')
[Out] Timed out
```

Maple [**A**]
time = 0.21, size = 34, normalized size = 0.79

method	result	size
risch	$-\frac{(-1+x)(1+2x)}{(-x^3+1)^{\frac{1}{3}}}+x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(x^2+x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-1+x)*(1+2*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 + x + 1)^2, x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}}{\left(x^{2}+x+1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(x**2+x+1)**2,x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x**2 + x + 1)**2, x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(1-x^3\right)^{2/3}}{\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(x + x^2 + 1)^2,x)
[Out] int((1 - x^3)^(2/3)/(x + x^2 + 1)^2, x)
```

3.104 $\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=43

$$\frac{1}{\sqrt[3]{1-x^3}}+\frac{x}{\sqrt[3]{1-x^3}}-x^2\,{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};x^3\right)$$

```
[Out] 1/(-x^3+1)^(1/3)+x/(-x^3+1)^(1/3)-x^2*hypergeom([2/3, 4/3],[5/3],x^3)
```

Rubi [A]
time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.160$, Rules used = {2183, 197, 371, 267}

$$x^2\left(-\,{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};x^3\right)\right)+\frac{x}{\sqrt[3]{1-x^3}}+\frac{1}{\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/((1 + x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] (1 - x^3)^(-1/3) + x/(1 - x^3)^(1/3) - x^2*Hypergeometric2F1[2/3, 4/3, 5/3,
 x^3]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
```

```
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \left(\frac{-1-i\sqrt{3}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}+\frac{-1+i\sqrt{3}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\right)dx$$
$$= \left(-1-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\,dx+\left(-1+i\sqrt{3}\right)\int \frac{}{\left(1+i\sqrt{3}\right.}$$

Mathematica [A]
time = 10.06, size = 43, normalized size = 1.00

$$\frac{(1+2x)\left(1-x^3\right)^{2/3}}{1+x+x^2}+x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/((1 + x + x^2)*(1 - x^3)^(1/3)),x]
[Out] ((1 + 2*x)*(1 - x^3)^(2/3))/(1 + x + x^2) + x^2*Hypergeometric2F1[1/3, 2/3,
 5/3, x^3]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)/((1 + x + x^2)*(1 - x^3)^(1/3)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.21, size = 34, normalized size = 0.79

method	result	size
risch	$-\frac{(-1+x)(1+2x)}{(-x^3+1)^{\frac{1}{3}}}+x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -(-1+x)*(1+2*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate((x - 1)/((-x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^2\sqrt[3]{1-x^3}+x\sqrt[3]{1-x^3}+\sqrt[3]{1-x^3}}\,dx-\int \left(-\frac{1}{x^2\sqrt[3]{1-x^3}+x\sqrt[3]{1-x^3}+\sqrt[3]{1-x^3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x**2+x+1)/(-x**3+1)**(1/3),x)
```

```
[Out] -Integral(x/(x**2*(1 - x**3)**(1/3) + x*(1 - x**3)**(1/3) + (1 - x**3)**(1/
3)), x) - Integral(-1/(x**2*(1 - x**3)**(1/3) + x*(1 - x**3)**(1/3) + (1 -
x**3)**(1/3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x-1}{\left(1-x^3\right)^{1/3}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((1 - x^3)^(1/3)*(x + x^2 + 1)),x)
[Out] -int((x - 1)/((1 - x^3)^(1/3)*(x + x^2 + 1)), x)
```

3.105 $\int \frac{(1-x)^2}{(1-x^3)^{4/3}} dx$

Optimal. Leaf size=39

$$\frac{1+(1-2x)x}{\sqrt[3]{1-x^3}}+x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$$

```
[Out] (1+(1-2*x)*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3, 2/3],[5/3],x^3)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1868, 12, 371}

$$x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(1-2x)x+1}{\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^2/(1 - x^3)^(4/3),x]
```

```
[Out] (1 + (1 - 2*x)*x)/(1 - x^3)^(1/3) + x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^
3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(1-x)^2}{(1-x^3)^{4/3}}\,dx = \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}} - \int -\frac{2x}{\sqrt[3]{1-x^3}}\,dx$$
$$= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}} + 2\int \frac{x}{\sqrt[3]{1-x^3}}\,dx$$
$$= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}} + x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$$

Mathematica [A]
time = 7.05, size = 43, normalized size = 1.10

$$\frac{1}{\sqrt[3]{1-x^3}} + \frac{x}{\sqrt[3]{1-x^3}} - x^2\,{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^2/(1 - x^3)^(4/3),x]
```

```
[Out] (1 - x^3)^(-1/3) + x/(1 - x^3)^(1/3) - x^2*Hypergeometric2F1[2/3, 4/3, 5/3,
 x^3]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^2/(1 - x^3)^(4/3),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.13, size = 34, normalized size = 0.87

method	result	size
risch	$-\frac{(-1+x)(1+2x)}{(-x^3+1)^{\frac{1}{3}}} + x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)$	34
meijerg	$\frac{x}{(-x^3+1)^{\frac{1}{3}}} - x^2\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{4}{3}\right],\left[\frac{5}{3}\right],x^3\right) + \frac{x^3\,\mathrm{hypergeom}\left(\left[1,\frac{4}{3}\right],[2],x^3\right)}{3}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^2/(-x^3+1)^(4/3),x,method=_RETURNVERBOSE)
```

[Out] -(-1+x)*(1+2*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="maxima")

[Out] x/(-x^3 + 1)^(1/3) - integrate((x^2 - 2*x)/((x^3 - 1)*(x^2 + x + 1)^(1/3)*(
-x + 1)^(1/3)), x)

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="fricas")

[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)^2}{(-(x-1)(x^2+x+1))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)**2/(-x**3+1)**(4/3),x)

[Out] Integral((x - 1)**2/(-(x - 1)*(x**2 + x + 1))**(4/3), x)

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^2/(-x^3+1)^(4/3),x)

[Out] Could not integrate

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(x-1)^2}{(1-x^3)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^2/(1 - x^3)^(4/3),x)
[Out] int((x - 1)^2/(1 - x^3)^(4/3), x)
```

3.106 $\int (1-x^3)^{2/3}\,dx$

Optimal. Leaf size=67

$$\frac{1}{3}x\left(1-x^3\right)^{2/3}-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(x+\sqrt[3]{1-x^3}\right)$$

```
[Out] 1/3*x*(-x^3+1)^(2/3)+1/3*ln(x+(-x^3+1)^(1/3))-2/9*arctan(1/3*(1-2*x/(-x^3+1
)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {201, 245}

$$\frac{1}{3}\left(1-x^3\right)^{2/3}x+\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3),x]
```

```
[Out] (x*(1 - x^3)^(2/3))/3 - (2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*
Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/3
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \left(1-x^3\right)^{2/3} dx = \frac{1}{3}x\left(1-x^3\right)^{2/3} + \frac{2}{3}\int \frac{1}{\sqrt[3]{1-x^3}}\,dx$$
$$= \frac{1}{3}x\left(1-x^3\right)^{2/3} - \frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{3}\log\left(x+\sqrt[3]{1-x^3}\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 0.08, size = 101, normalized size = 1.51

$$\frac{3(-1+x)\left(1-x^3\right)^{2/3} F_1\left(\frac{5}{3};-\frac{2}{3},-\frac{2}{3};\frac{8}{3};-\frac{-1+x}{1-(-1)^{2/3}},-\frac{-1+x}{1+\sqrt[3]{-1}}\right)}{5\left(1+\frac{-1+x}{1+\sqrt[3]{-1}}\right)^{2/3}\left(1+\frac{-1+x}{1-(-1)^{2/3}}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(2/3),x]
```

```
[Out] (3*(-1 + x)*(1 - x^3)^(2/3)*AppellF1[5/3, -2/3, -2/3, 8/3, -((-1 + x)/(1 -
(-1)^(2/3))), -((-1 + x)/(1 + (-1)^(1/3)))])/(5*(1 + (-1 + x)/(1 + (-1)^(1/
3)))^(2/3)*(1 + (-1 + x)/(1 - (-1)^(2/3)))^(2/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.14, size = 16, normalized size = 0.24

$$x\text{hyper}\left[\left\{-\frac{2}{3},\frac{1}{3}\right\},\left\{\frac{4}{3}\right\},x^3\text{exp\_polar}\left[2I\text{Pi}\right]\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x^3)^(2/3),x]')
```

```
[Out] x hyper[{-2 / 3, 1 / 3}, {4 / 3}, x ^ 3 exp_polar[2 I Pi]]
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 1.03, size = 12, normalized size = 0.18

method	result
meijerg	$x\,\text{hypergeom}\left(\left[-\frac{2}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^3\right)$
risch	$-\frac{x\left(x^3-1\right)}{3\left(-x^3+1\right)^{\frac{1}{3}}}+\frac{2x\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^3\right)}{3}$

trager	$\frac{x(-x^3+1)^{\frac{2}{3}}}{3}+\frac{2\,\mathrm{RootOf}(\_Z^2+\_Z+1)\ln\left(\mathrm{RootOf}(\_Z^2+\_Z+1)^2x^3+3\,\mathrm{RootOf}(\_Z^2+\_Z+1)(-x^3+1)^{\frac{2}{3}}x-3\,\mathrm{RootOf}(\_}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] x*hypergeom([-2/3,1/3],[4/3],x^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(52)=104$.
time = 0.36, size = 105, normalized size = 1.57

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2(-x^3+1)^{\frac{1}{3}}}{x}-1\right)\right)-\frac{(-x^3+1)^{\frac{2}{3}}}{3x^2\left(\frac{x^3-1}{x^3}-1\right)}+\frac{2}{9}\log\left(\frac{(-x^3+1)^{\frac{1}{3}}}{x}+1\right)-\frac{1}{9}\log\left(-\frac{(-x^3+1)^{\frac{1}{3}}}{x}+\frac{(-x^3+1)^{\frac{2}{3}}}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) - 1/3*(-x^3 + 1
)^(2/3)/(x^2*((x^3 - 1)/x^3 - 1)) + 2/9*log((-x^3 + 1)^(1/3)/x + 1) - 1/9*l
og(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)
```

Fricas [A]
time = 0.34, size = 94, normalized size = 1.40

$$\frac{1}{3}(-x^3+1)^{\frac{2}{3}}x-\frac{2}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\,x-2\sqrt{3}\,(-x^3+1)^{\frac{1}{3}}}{3x}\right)+\frac{2}{9}\log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right)-\frac{1}{9}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/3*(-x^3 + 1)^(2/3)*x - 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x
^3 + 1)^(1/3))/x) + 2/9*log((x + (-x^3 + 1)^(1/3))/x) - 1/9*log((x^2 - (-x^
3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.52, size = 31, normalized size = 0.46

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3),x)
```

```
[Out] x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/
3))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.34, size = 10, normalized size = 0.15

$$x\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{1}{3};\ \frac{4}{3};\ x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3),x)
```

```
[Out] x*hypergeom([-2/3, 1/3], 4/3, x^3)
```

3.107 $\int \frac{(1-x^3)^{2/3}}{x}\,dx$

Optimal. Leaf size=70

$$\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)$$

```
[Out] 1/2*(-x^3+1)^(2/3)-1/2*ln(x)+1/2*ln(1-(-x^3+1)^(1/3))+1/3*arctan(1/3*(1+2*(
-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {272, 52, 57, 632, 210, 31}

$$\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3)/x,x]
```

```
[Out] (1 - x^3)^(2/3)/2 + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x
]/2 + Log[1 - (1 - x^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x^3)^{2/3}}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(1-x)^{2/3}}{x}\,dx, x, x^3\right)\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^3\right)\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{\log(x)}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\right.\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-}\right.\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 90, normalized size = 1.29

$$\frac{1}{6}\left(3\left(1-x^3\right)^{2/3} + 2\sqrt{3}\,\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 2\log\left(-1+\sqrt[3]{1-x^3}\right) - \log\left(1+\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(2/3)/x,x]
```

```
[Out] (3*(1 - x^3)^(2/3) + 2*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] + 2*
Log[-1 + (1 - x^3)^(1/3)] - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)])/6
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.14, size = 23, normalized size = 0.33

latex error

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x^3)^(2/3)/x,x]')
```

```
[Out] -1 ^ (2 / 3) x ^ 2 Gamma[-2 / 3] hyper[{-2 / 3, -2 / 3}, {1 / 3}, 1 / x ^ 3
] / (3 Gamma[1 / 3])
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 1.71, size = 66, normalized size = 0.94

method	result
meijerg	$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{\left(\frac{3}{2}-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)}+\frac{2\pi\sqrt{3}\,x^{3}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},1,1\right],[2,2],x^{3}\right)}{3\Gamma\left(\frac{2}{3}\right)}\right)}{9\pi}$
trager	$\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{2}+\frac{\ln\left(\frac{-211\,\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)^{2}x^{3}-3126\,\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)x^{3}+5502\left(-x^{3}+1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)-11543}{}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/Pi*3^(1/2)*GAMMA(2/3)*(-(3/2-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*Pi
*3^(1/2)/GAMMA(2/3)+2/3*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1/3,1,1],[2,2]
,x^3))
```

Maxima [A]
time = 0.33, size = 73, normalized size = 1.04

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2}\left(-x^{3}+1\right)^{\frac{2}{3}}-\frac{1}{6}\log\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2*(-x^3 + 1)^(
2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1
)^(1/3) - 1)
```

Fricas [A]
time = 0.32, size = 75, normalized size = 1.07

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 1/2*(-x^3
+ 1)^(2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x
^3 + 1)^(1/3) - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.51, size = 41, normalized size = 0.59

$$-\frac{x^2e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{1}{x^3}\right)}{3\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/x,x)
```

```
[Out] -x**2*exp(2*I*pi/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), x**(-3))/(3*gam
ma(1/3))
```

Giac [A]
time = 0.00, size = 91, normalized size = 1.30

$$\frac{\ln\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|}{3}-\frac{\ln\left(\left(\left(-x^3+1\right)^{\frac{1}{3}}\right)^2+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)}{6}+\frac{\arctan\left(\frac{2\left(-x^3+1\right)^{\frac{1}{3}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\left(\left(-x^3+1\right)^{\frac{1}{3}}\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/x,x)
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2*(-x^3 + 1)^(
2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3
 + 1)^(1/3) - 1))
```

Mupad [B]
time = 0.40, size = 91, normalized size = 1.30

$$\frac{\ln\left(\left(1-x^3\right)^{1/3}-1\right)}{3}+\ln\left(\left(1-x^3\right)^{1/3}-9\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(\left(1-x^3\right)^{1/3}-9\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{\left(1-x^3\right)^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/x,x)
```

```
[Out] log((1 - x^3)^(1/3) - 1)/3 + log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 - 1/6)
^2)*((3^(1/2)*1i)/6 - 1/6) - log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 + 1/6)
^2)*((3^(1/2)*1i)/6 + 1/6) + (1 - x^3)^(2/3)/2
```

3.108 $\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx$

Optimal. Leaf size=384

$$\frac{(1-x^3)^{2/3}}{2b}-\frac{(a^3+b^3)\,x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-\frac{b^3x^3}{a^3}\right)}{2a^2b^2}+\frac{a^2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^3}-\frac{(a^3+b^3)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a^3+}}{a\sqrt[3]{1-}}}{\sqrt{3}}\right.}{\sqrt{3}\,b^3}$$

```
[Out] 1/2*(-x^3+1)^(2/3)/b-1/2*(a^3+b^3)*x^2*AppellF1(2/3,1/3,1,5/3,x^3,-b^3*x^3/
a^3)/a^2/b^2+1/2*a*x^2*hypergeom([1/3, 2/3],[5/3],x^3)/b^2-1/3*(a^3+b^3)^(2
/3)*ln(b^3*x^3+a^3)/b^3+1/2*(a^3+b^3)^(2/3)*ln(-(a^3+b^3)^(1/3)*x/a-(-x^3+1
)^(1/3))/b^3-1/2*a^2*ln(x+(-x^3+1)^(1/3))/b^3+1/2*(a^3+b^3)^(2/3)*ln((a^3+b
^3)^(1/3)-b*(-x^3+1)^(1/3))/b^3+1/3*a^2*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3
^(1/2))/b^3*3^(1/2)-1/3*(a^3+b^3)^(2/3)*arctan(1/3*(1-2*(a^3+b^3)^(1/3)*x/a
/(-x^3+1)^(1/3))*3^(1/2))/b^3*3^(1/2)+1/3*(a^3+b^3)^(2/3)*arctan(1/3*(1+2*b
*(-x^3+1)^(1/3)/(a^3+b^3)^(1/3))*3^(1/2))/b^3*3^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {2178, 2177, 245, 2181, 384, 524, 455, 57, 631, 210, 31, 371}

$$-\frac{(a^3+b^3)^{2/3}\log(a^3+b^3x^3)}{3b^3}+\frac{(a^3+b^3)^{2/3}\log\left(-\frac{x\sqrt[3]{a^3+b^3}}{a}-\sqrt[3]{1-x^3}\right)}{2b^3}+\frac{(a^3+b^3)^{2/3}\log\left(\sqrt[3]{a^3+b^3}-b\sqrt[3]{1-x^3}\right)}{2b^3}-\frac{(a^3+b^3)^{2/3}\tan^{-1}\left(\frac{1-\frac{2x\sqrt[3]{a^3+b^3}}{a\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^3}+\frac{(a^3+b^3)^{2/3}\tan^{-1}\left(\frac{\frac{2b\sqrt[3]{1-x^3}}{\sqrt[3]{a^3+b^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^3}-\frac{a^2\log\left(\sqrt[3]{1-x^3}+x\right)}{2b^3}+\frac{a^2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^3}-\frac{x^2(a^3+b^3)F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-\frac{b^3x^3}{a^3}\right)}{2a^2b^2}+\frac{ax^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)}{2b^2}+\frac{(1-x^3)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3)/(a + b*x),x]
```

```
[Out] (1 - x^3)^(2/3)/(2*b) - ((a^3 + b^3)*x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -(
(b^3*x^3)/a^3)])/(2*a^2*b^2) + (a^2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt
[3]])/(Sqrt[3]*b^3) - ((a^3 + b^3)^(2/3)*ArcTan[(1 - (2*(a^3 + b^3)^(1/3)*x
)/(a*(1 - x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*b^3) + ((a^3 + b^3)^(2/3)*ArcTan[
(1 + (2*b*(1 - x^3)^(1/3))/(a^3 + b^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^3) + (a*
x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/(2*b^2) - ((a^3 + b^3)^(2/3)*Log
[a^3 + b^3*x^3])/(3*b^3) + ((a^3 + b^3)^(2/3)*Log[-(((a^3 + b^3)^(1/3)*x)/a
) - (1 - x^3)^(1/3)])/(2*b^3) - (a^2*Log[x + (1 - x^3)^(1/3)])/(2*b^3) + ((
a^3 + b^3)^(2/3)*Log[(a^3 + b^3)^(1/3) - b*(1 - x^3)^(1/3)])/(2*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2177

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rule 2178

```
Int[((a_) + (b_.)*(x_)^3)^(2/3)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(a +
 b*x^3)^(2/3)/(2*d), x] + (Dist[1/d^2, Int[(a*d^2 + b*c^2*x)/((c + d*x)*(a
+ b*x^3)^(1/3)), x], x] - Dist[b*(c/d^2), Int[x/(a + b*x^3)^(1/3), x], x])
/; FreeQ[{a, b, c, d}, x]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx=\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx$$

Mathematica [F]
time = 14.24, size = 0, normalized size = 0.00

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^3)^(2/3)/(a + b*x),x]
[Out] Integrate[(1 - x^3)^(2/3)/(a + b*x), x]
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x^3)^(2/3)/(a + b*x),x]')
[Out] Timed out
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-x^3+1)^{\frac{2}{3}}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(b*x+a),x)
[Out] int((-x^3+1)^(2/3)/(b*x+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)/(b*x + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(b*x+a),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(a + b*x), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(1-x^3\right)}^{2/3}}{a+b\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(a + b*x),x)
```

```
[Out] int((1 - x^3)^(2/3)/(a + b*x), x)
```

3.109 $\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx$

Optimal. Leaf size=234

$$-\frac{(1-x^3)^{2/3}}{3(1+x^3)}+\frac{x(1-x^3)^{2/3}}{3(1+x^3)}+\frac{2x^2(1-x^3)^{2/3}}{3(1+x^3)}-\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2^{2/3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}}+$$

```
[Out] -1/3*(-x^3+1)^(2/3)/(x^3+1)+1/3*x*(-x^3+1)^(2/3)/(x^3+1)+2/3*x^2*(-x^3+1)^(
2/3)/(x^3+1)+1/3*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/6*ln(2^(1/3)-(-x^3+1
)^(1/3))*2^(2/3)+1/6*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/9*arctan(1/3*(
1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/9*arctan(1/3*(1+2^
(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2183, 386, 384, 480, 21, 371, 455, 43, 57, 631, 210, 31}

$$\frac{1}{3}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(1-x^3)^{2/3}x}{3(x^3+1)}-\frac{(1-x^3)^{2/3}}{3(x^3+1)}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{3\sqrt[3]{2}}-\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2(1-x^3)^{2/3}x^2}{3(x^3+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3)/(1 - x + x^2)^2,x]
```

```
[Out] -1/3*(1 - x^3)^(2/3)/(1 + x^3) + (x*(1 - x^3)^(2/3))/(3*(1 + x^3)) + (2*x^2
*(1 - x^3)^(2/3))/(3*(1 + x^3)) - (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x
^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) - (2^(2/3)*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(
1/3))/Sqrt[3]])/(3*Sqrt[3]) + (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/3
 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(3*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3
)^(1/3)]/(3*2^(1/3))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx = \int \left(-\frac{4\,(1-x^3)^{2/3}}{3\left(1+i\sqrt{3}-2x\right)^2}+\frac{4i\,(1-x^3)^{2/3}}{3\sqrt{3}\left(1+i\sqrt{3}-2x\right)}-\frac{4\,(1-x^3)^{2/3}}{3\left(-1+i\sqrt{3}+2x\right)^2}+\frac{\quad}{3\sqrt{\quad}}\right.$$

$$= -\left(\frac{4}{3}\int \frac{(1-x^3)^{2/3}}{\left(1+i\sqrt{3}-2x\right)^2}\,dx\right)-\frac{4}{3}\int \frac{(1-x^3)^{2/3}}{\left(-1+i\sqrt{3}+2x\right)^2}\,dx+\frac{(4i)\int \frac{(1-x^3)^{2/3}}{1+i\sqrt{3}-2x}}{3\sqrt{3}}$$

Mathematica [F]

time = 20.25, size = 0, normalized size = 0.00

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^3)^(2/3)/(1 - x + x^2)^2,x]
```

```
[Out] Integrate[(1 - x^3)^(2/3)/(1 - x + x^2)^2, x]
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x^3)^(2/3)/(1 - x + x^2)^2,x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-x^3+1)^{\frac{2}{3}}}{(x^2-x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(x^2-x+1)^2,x)
```

```
[Out] int((-x^3+1)^(2/3)/(x^2-x+1)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 - x + 1)^2, x)
```

Fricas [**F**]
time = 1.37, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}}{\left(x^{2}-x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(x**2-x+1)**2,x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x**2 - x + 1)**2, x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-x^{3}\right)^{2/3}}{\left(x^{2}-x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(x^2 - x + 1)^2,x)
```

```
[Out] int((1 - x^3)^(2/3)/(x^2 - x + 1)^2, x)
```

3.110 $\int \frac{(1-2x)(1-x^3)^{2/3}}{(1-x+x^2)^2} \, dx$

Optimal. Leaf size=199

$$\frac{(1-x^3)^{2/3}}{1-x+x^2}-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2^{2/3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log\left(\sqrt[3]{2}\right.}{}$$

```
[Out] (-x^3+1)^(2/3)/(x^2-x+1)+1/2*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(-2^(
1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+ln(x+(-x^3+1)^(1/3))-2/3*arctan(1/3*(1-2*x/(
-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/3*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3
))*3^(1/2))*2^(2/3)*3^(1/2)+1/3*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/
2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 250, normalized size of antiderivative = 1.26, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$,
Rules used = {2183, 386, 384, 455, 43, 57, 631, 210, 31, 478, 544, 245}

$$\frac{(1-x^3)^{2/3}x}{x^3+1}+\frac{(1-x^3)^{2/3}}{x^3+1}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}-\frac{2}{3}2^{2/3}\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}+\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2,x]
```

```
[Out] (1 - x^3)^(2/3)/(1 + x^3) + (x*(1 - x^3)^(2/3))/(1 + x^3) - (2*ArcTan[(1 -
(2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*
x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + (2^(2/3)*ArcTan[(1 + 2^(2/3)*(1 - x
^3)^(1/3))/Sqrt[3]])/Sqrt[3] + Log[2^(1/3) - (1 - x^3)^(1/3)]/2^(1/3) + Log
[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(3*2^(1/3)) - (2*2^(2/3)*Log[-(2^(1/3)*x)
- (1 - x^3)^(1/3)])/3 + Log[x + (1 - x^3)^(1/3)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx = \int \left(\frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2} - \frac{2x\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\right)dx$$

$$= -\left(2\int \frac{x\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx\right) + \int \frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$$

$$= -\left(2\int \left(-\frac{2\left(1+i\sqrt{3}\right)\left(1-x^3\right)^{2/3}}{3\left(1+i\sqrt{3}-2x\right)^2} + \frac{2i\left(1-x^3\right)^{2/3}}{3\sqrt{3}\left(1+i\sqrt{3}-2x\right)} - \frac{2\left(1-i\sqrt{3}\right.}{3\left(-1+i\right.}\right.\right.$$

$$= -\left(\frac{4}{3}\int \frac{\left(1-x^3\right)^{2/3}}{\left(1+i\sqrt{3}-2x\right)^2}\,dx\right) - \frac{4}{3}\int \frac{\left(1-x^3\right)^{2/3}}{\left(-1+i\sqrt{3}+2x\right)^2}\,dx + \frac{1}{3}\left(4\left(1-i\right.\right.$$

Mathematica [F]
time = 12.06, size = 0, normalized size = 0.00

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2,x]
```

```
[Out] Integrate[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2, x]
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - 2*x)*(1 - x^3)^(2/3)/(1 - x + x^2)^2,x]')
```

```
[Out] Timed out
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 24.62, size = 457, normalized size = 2.30

method	result
trager	$\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^2-x+1}+\frac{\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^6+432\right)^4 x^2+\operatorname{RootOf}\left(\_Z^6+432\right)^4 x-\operatorname{RootOf}\left(\_Z^6+432\right)^4+12\operatorname{RootOf}\left(\_Z^6+432\right)^2\left(-x^3+1\right)^{\frac{1}{3}} x+72}{x^2-x+1}\right.}{72}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-x^3+1)^(2/3)/(x^2-x+1)+1/72*ln(-(RootOf(_Z^6+432)^4*x^2+RootOf(_Z^6+432)^
4*x-RootOf(_Z^6+432)^4+12*RootOf(_Z^6+432)^2*(-x^3+1)^(1/3)*x+72*(-x^3+1)^(
2/3))/(x^2-x+1))*RootOf(_Z^6+432)^4+1/6*ln(-(RootOf(_Z^6+432)^4*x^2+RootOf(
_Z^6+432)^4*x-RootOf(_Z^6+432)^4+12*RootOf(_Z^6+432)^2*(-x^3+1)^(1/3)*x+72*
(-x^3+1)^(2/3))/(x^2-x+1))*RootOf(_Z^6+432)+1/3*RootOf(_Z^6+432)*ln(-(RootO
f(_Z^6+432)^5*(-x^3+1)^(1/3)*x-RootOf(_Z^6+432)^4*x^2-RootOf(_Z^6+432)^4*x+
RootOf(_Z^6+432)^4-12*RootOf(_Z^6+432)^2*(-x^3+1)^(1/3)*x+36*RootOf(_Z^6+43
2)*x^2+36*RootOf(_Z^6+432)*x+144*(-x^3+1)^(2/3)-36*RootOf(_Z^6+432))/(x^2-x
+1))-1/12*ln((-x^3+1)^(2/3)-x*(-x^3+1)^(1/3)+x^2)*RootOf(_Z^6+432)^3-ln((-x
^3+1)^(2/3)-x*(-x^3+1)^(1/3)+x^2)+1/18*RootOf(_Z^6+432)^3*ln(-RootOf(_Z^6+4
```

32)^6*x^3+24*RootOf(_Z^6+432)^3+1728*x*(-x^3+1)^(2/3)-1728*x^2*(-x^3+1)^(1/3)+1296*x^3-864)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] -integrate((-x^3 + 1)^(2/3)*(2*x - 1)/(x^2 - x + 1)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1827 vs. $2(163) = 326$.
time = 1.34, size = 1827, normalized size = 9.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] -1/72*(8*4^(1/3)*sqrt(3)*(x^2 - x + 1)*arctan(-1/6*(3822*4^(2/3)*sqrt(3)*(5
0*x^4 - 74*x^3 - 207*x^2 + 143*x + 19)*(-x^3 + 1)^(2/3) + 7644*4^(1/3)*sqrt
(3)*(19*x^5 - 150*x^4 + 43*x^3 + 112*x^2 + 57*x - 50)*(-x^3 + 1)^(1/3) - 7*
sqrt(39)*(6*4^(1/3)*sqrt(3)*(1150*x^4 - 3974*x^3 - 1911*x^2 + 1522*x + 3898
)*(-x^3 + 1)^(2/3) - 4^(2/3)*sqrt(3)*(1778*x^6 - 6366*x^5 - 8412*x^4 + 1725
4*x^3 + 15117*x^2 - 4227*x - 16105) + 12*sqrt(3)*(437*x^5 - 1539*x^4 - 333*
x^3 - 2074*x^2 + 372*x + 3261)*(-x^3 + 1)^(1/3))*sqrt((6*4^(1/3)*(5*x^4 + 4
*x^3 - 3*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 + 15*x^5 - 12*x^
4 - 25*x^3 - 12*x^2 + 15*x + 1) - 12*(4*x^5 + 3*x^4 - 2*x^3 - 5*x^2 + 1)*(-
x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 6*sqrt(3
)*(29494*x^6 - 17582*x^5 + 153824*x^4 - 266248*x^3 - 129950*x^2 + 238106*x
- 29747))/(138718*x^6 - 463746*x^5 - 296508*x^4 - 115072*x^3 + 1093704*x^2
- 70446*x - 256859)) + 8*4^(1/3)*sqrt(3)*(x^2 - x + 1)*arctan(1/6*(3822*4^(
2/3)*sqrt(3)*(19*x^4 - 181*x^3 + 36*x^2 + 169*x - 31)*(-x^3 + 1)^(2/3) - 76
44*4^(1/3)*sqrt(3)*(31*x^5 + 57*x^4 - 131*x^3 - 119*x^2 + 93*x + 19)*(-x^3
+ 1)^(1/3) + 7*sqrt(39)*(6*4^(1/3)*sqrt(3)*(3385*x^4 + 3574*x^3 - 1911*x^2
- 2948*x + 124)*(-x^3 + 1)^(2/3) + 4^(2/3)*sqrt(3)*(13027*x^6 + 16539*x^5 -
 8961*x^4 - 32644*x^3 - 2361*x^2 + 17139*x - 239) - 12*sqrt(3)*(2748*x^5 +
3450*x^4 - 4126*x^3 - 2385*x^2 + 1539*x - 76)*(-x^3 + 1)^(1/3))*sqrt((6*4^(
1/3)*(x^4 - 4*x^3 - 3*x^2 + 4*x + 5)*(-x^3 + 1)^(2/3) + 4^(2/3)*(x^6 + 15*x
^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 19) + 12*(x^5 - 5*x^3 - 2*x^2 + 3*x
+ 4)*(-x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 6
*sqrt(3)*(53953*x^6 - 12994*x^5 - 396521*x^4 + 169424*x^3 + 300029*x^2 - 62
294*x - 41597))/(52723*x^6 + 682854*x^5 - 325173*x^4 - 1353400*x^3 + 193623
*x^2 + 640446*x - 16073)) + 16*4^(1/3)*sqrt(3)*(x^2 - x + 1)*arctan(1/6*(76
```

```
44*4^(2/3)*sqrt(3)*(5*x^4 - 107*x^3 - 243*x^2 + 26*x + 157)*(-x^3 + 1)^(2/3
) - 7644*4^(1/3)*sqrt(3)*(307*x^5 + 300*x^4 - 140*x^3 - 221*x^2 - 186*x - 9
8)*(-x^3 + 1)^(1/3) + 7*sqrt(39)*4^(1/3)*(6*4^(1/3)*sqrt(3)*(3109*x^4 + 400
*x^3 - 3822*x^2 + 1426*x + 3622)*(-x^3 + 1)^(2/3) + 4^(2/3)*sqrt(3)*(15505*
x^6 + 11493*x^5 - 22383*x^4 - 22720*x^3 - 5454*x^2 + 13032*x + 10888) - 12*
sqrt(3)*(2111*x^5 + 3450*x^4 - 941*x^3 - 1111*x^2 - 372*x - 2624)*(-x^3 + 1
)^(1/3)) + 6*sqrt(3)*(307479*x^6 + 239258*x^5 - 543668*x^4 - 607716*x^3 + 1
9112*x^2 + 232000*x + 343788))/(933353*x^6 + 1472754*x^5 + 285042*x^4 - 100
8596*x^3 - 1598208*x^2 - 560184*x + 468980)) + 48*sqrt(3)*(x^2 - x + 1)*arc
tan((4*sqrt(3)*(-x^3 + 1)^(1/3)*x^2 + 2*sqrt(3)*(-x^3 + 1)^(2/3)*x - sqrt(3
)*(x^3 - 1))/(9*x^3 - 1)) - 3*4^(1/3)*(x^2 - x + 1)*log(39626496*(6*4^(1/3)
*(5*x^4 + 4*x^3 - 3*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 + 15*
x^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 1) - 12*(4*x^5 + 3*x^4 - 2*x^3 - 5*
x^2 + 1)*(-x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1))
 - 3*4^(1/3)*(x^2 - x + 1)*log(9906624*(6*4^(1/3)*(5*x^4 + 4*x^3 - 3*x^2 -
4*x + 1)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 + 15*x^5 - 12*x^4 - 25*x^3 - 12
*x^2 + 15*x + 1) - 12*(4*x^5 + 3*x^4 - 2*x^3 - 5*x^2 + 1)*(-x^3 + 1)^(1/3))
/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 3*4^(1/3)*(x^2 - x + 1)
*log(39626496*(6*4^(1/3)*(x^4 - 4*x^3 - 3*x^2 + 4*x + 5)*(-x^3 + 1)^(2/3) +
 4^(2/3)*(x^6 + 15*x^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 19) + 12*(x^5 -
5*x^3 - 2*x^2 + 3*x + 4)*(-x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6
*x^2 - 3*x + 1)) + 3*4^(1/3)*(x^2 - x + 1)*log(9906624*(6*4^(1/3)*(x^4 - 4*
x^3 - 3*x^2 + 4*x + 5)*(-x^3 + 1)^(2/3) + 4^(2/3)*(x^6 + 15*x^5 - 12*x^4 -
25*x^3 - 12*x^2 + 15*x + 19) + 12*(x^5 - 5*x^3 - 2*x^2 + 3*x + 4)*(-x^3 + 1
)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) - 24*(x^2 - x + 1
)*log(3*(-x^3 + 1)^(1/3)*x^2 + 3*(-x^3 + 1)^(2/3)*x + 1) - 72*(-x^3 + 1)^(2
/3))/(x^2 - x + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{(1-x^3)^{\frac{2}{3}}}{x^4-2x^3+3x^2-2x+1}\right)dx-\int\frac{2x\,(1-x^3)^{\frac{2}{3}}}{x^4-2x^3+3x^2-2x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x**3+1)**(2/3)/(x**2-x+1)**2,x)
```

```
[Out] -Integral(-(1 - x**3)**(2/3)/(x**4 - 2*x**3 + 3*x**2 - 2*x + 1), x) - Integ
ral(2*x*(1 - x**3)**(2/3)/(x**4 - 2*x**3 + 3*x**2 - 2*x + 1), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{(2x-1)\left(1-x^3\right)^{2/3}}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*x - 1)*(1 - x^3)^(2/3))/(x^2 - x + 1)^2,x)
```

```
[Out] -int(((2*x - 1)*(1 - x^3)^(2/3))/(x^2 - x + 1)^2, x)
```

3.111 $\int \frac{(1-x^3)^{2/3}}{1+x}\,dx$

Optimal. Leaf size=177

$$\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log((1-x)(1}{2\sqrt[3]{2}}$$

```
[Out] 1/2*(-x^3+1)^(2/3)+1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/4*ln((1-x)*(1+
x)^2)*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+3/4*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*
2^(2/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/2*arctan(1
/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [A]
time = 0.09, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2178, 2177, 245, 2174, 371}

$$\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log\left((1-x)(x+1)^2\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3)/(1 + x),x]
```

```
[Out] (1 - x^3)^(2/3)/2 - (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]])/2^(1/3) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] +
(x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[(1 - x)*(1 + x)^2]/(2*2
^(1/3)) - Log[x + (1 - x^3)^(1/3)]/2 + (3*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1
/3)])/(2*2^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2177

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rule 2178

```
Int[((a_) + (b_.)*(x_)^3)^(2/3)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(a +
 b*x^3)^(2/3)/(2*d), x] + (Dist[1/d^2, Int[(a*d^2 + b*c^2*x)/((c + d*x)*(a
+ b*x^3)^(1/3)), x], x] - Dist[b*(c/d^2), Int[x/(a + b*x^3)^(1/3), x], x])
/; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{1+x}\,dx = \int \frac{(1-x^3)^{2/3}}{1+x}\,dx$$

Mathematica [F]
time = 31.90, size = 0, normalized size = 0.00

$$\int \frac{(1-x^3)^{2/3}}{1+x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^3)^(2/3)/(1 + x),x]
```

```
[Out] Integrate[(1 - x^3)^(2/3)/(1 + x), x]
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x^3)^(2/3)/(1 + x),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{1+x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(1+x),x)
```

```
[Out] int((-x^3+1)^(2/3)/(1+x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x + 1), x)
```

Fricas [F]
time = 1.91, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)/(x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}}{x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(1+x),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x + 1), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(1+x),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-x^3\right)^{2/3}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(x + 1),x)
```

```
[Out] int((1 - x^3)^(2/3)/(x + 1), x)
```

3.112 $\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx$

Optimal. Leaf size=177

$$\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log((1-x)(1}{2\sqrt[3]{2}}$$

```
[Out] 1/2*(-x^3+1)^(2/3)+1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/4*ln((1-x)*(1+
x)^2)*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+3/4*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*
2^(2/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/2*arctan(1
/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [A]
time = 0.10, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1600, 2178, 2177, 245, 2174, 371}

$$\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log\left((1-x)(x+1)^2\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] (1 - x^3)^(2/3)/2 - (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]])/2^(1/3) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] +
(x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[(1 - x)*(1 + x)^2]/(2*2
^(1/3)) - Log[x + (1 - x^3)^(1/3)]/2 + (3*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1
/3)])/(2*2^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2177

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rule 2178

```
Int[((a_) + (b_.)*(x_)^3)^(2/3)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(a +
 b*x^3)^(2/3)/(2*d), x] + (Dist[1/d^2, Int[(a*d^2 + b*c^2*x)/((c + d*x)*(a
+ b*x^3)^(1/3)), x], x] - Dist[b*(c/d^2), Int[x/(a + b*x^3)^(1/3), x], x])
/; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int \frac{(1-x+x^2)\,(1-x^3)^{2/3}}{1+x^3}\,dx = \int \frac{(1-x^3)^{2/3}}{1+x}\,dx$$

Mathematica [F]
time = 20.13, size = 0, normalized size = 0.00

$$\int \frac{(1-x+x^2)\,(1-x^3)^{2/3}}{1+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] Integrate[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3), x]
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x + x^2)*(1 - x^3)^(2/3)/(1 + x^3),x]')
[Out] Timed out
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2-x+1)\,(-x^3+1)^{\frac{2}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
[Out] int((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)*(x^2 - x + 1)/(x^3 + 1), x)
```

Fricas [**F**]
time = 1.89, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x + 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(x-1)\,(x^2+x+1))^{\frac{2}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+1)*(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x + 1), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-x^3\right)^{2/3}\,\left(x^2-x+1\right)}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - x^3)^(2/3)*(x^2 - x + 1))/(x^3 + 1),x)
```

```
[Out] int(((1 - x^3)^(2/3)*(x^2 - x + 1))/(x^3 + 1), x)
```

3.113 $\int \frac{(1-x^3)^{2/3}}{1+x^3} dx$

Optimal. Leaf size=132

$$\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(1+x^3)}{3\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}-\frac{1}{2}\log\left(x+\right.$$

```
[Out] -1/6*ln(x^3+1)*2^(2/3)+1/2*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(x+(
-x^3+1)^(1/3))+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/3*a
rctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {399, 245, 384}

$$-\frac{\log(x^3+1)}{3\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{\sqrt[3]{2}}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3)/(1 + x^3),x]
```

```
[Out] ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - (2^(2/3)*ArcTan[(1 -
(2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - Log[1 + x^3]/(3*2^(1/3))
 + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/2^(1/3) - Log[x + (1 - x^3)^(1/3)]/2
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{1+x^3}\,dx = xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)$$

Mathematica [A]
time = 0.32, size = 204, normalized size = 1.55

$$\frac{1}{6}\left(2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)-2\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-2\log\left(x+\sqrt[3]{1-x^3}\right)+2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+\log\left(x^2-x\sqrt[3]{1-x^3}+(1-x^3)^{2/3}\right)-2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(2/3)/(1 + x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/3))] - 2*2^(2/3)*Sqrt[3]*
ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*Log[x + (1 - x^3)^(1/
3)] + 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + Log[x^2 - x*(1 - x^3)^
(1/3) + (1 - x^3)^(2/3)] - 2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) -
 2^(1/3)*(1 - x^3)^(2/3)])/6
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x^3)^(2/3)/(1 + x^3),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(-x^3+1)^{\frac{2}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int((-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x^3 + 1), x)
```

Fricas [A]
time = 0.33, size = 191, normalized size = 1.45

$$-\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\,x-4^{\frac{1}{3}}\sqrt{3}\,(-x^3+1)^{\frac{1}{3}}}{3\,x}\right)+\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\,x-2\sqrt{3}\,(-x^3+1)^{\frac{1}{3}}}{3\,x}\right)+\frac{1}{3}\cdot 4^{\frac{1}{3}}\log\left(\frac{4^{\frac{2}{3}}x+2\,(-x^3+1)^{\frac{1}{3}}}{x}\right)-\frac{1}{6}\cdot 4^{\frac{1}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}x^2-4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}x+2\,(-x^3+1)^{\frac{2}{3}}}{x^2}\right)-\frac{1}{3}\log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right)+\frac{1}{6}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*4^(1/3)*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(-x^3 + 1)^(1
/3))/x) + 1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/
x) + 1/3*4^(1/3)*log((4^(2/3)*x + 2*(-x^3 + 1)^(1/3))/x) - 1/6*4^(1/3)*log(
(2*4^(1/3)*x^2 - 4^(2/3)*(-x^3 + 1)^(1/3)*x + 2*(-x^3 + 1)^(2/3))/x^2) - 1/
3*log((x + (-x^3 + 1)^(1/3))/x) + 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3
 + 1)^(2/3))/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/((x + 1)*(x**2 - x + 1)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x^3)^{2/3}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(x^3 + 1),x)
```

```
[Out] int((1 - x^3)^(2/3)/(x^3 + 1), x)
```

3.114 $\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx$

Optimal. Leaf size=250

$$\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left((1-x)(1+x)^2\right)}{6\sqrt[3]{2}}+\frac{\log\left(\right.}{}$$

```
[Out] -1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/12*ln((1-x)*(1+x)^2)*2^(2/3)+1/6
*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-
1/3*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/4*ln(-1+x+2^(2/3)*(-x^3+1)
^(1/3))*2^(2/3)+1/3*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*
2^(2/3)*3^(1/2)+1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^
(1/2)*2^(2/3)
```

Rubi [A]

time = 0.08, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {495, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}+\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left((1-x)(x+1)^2\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]
) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 + Log[(1 - x)*(1 + x)^2]/
(6*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x
))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - (2^(2/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x
^3)^(1/3)])/3 - Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{x\left(1-x^3\right)^{2/3}}{1+x^3}\,dx=\frac{1}{2}x^2F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 10.01, size = 26, normalized size = 0.10

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 - x^3)^(2/3))/(1 + x^3),x]
[Out] (x^2*AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3])/2
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x*(1 - x^3)^(2/3)/(1 + x^3),x]')
[Out] Timed out
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x\left(-x^3+1\right)^{\frac{2}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(x*(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)
```

Fricas [F]
time = 1.61, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}}{\left(x+1\right)\left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x*(-(x - 1)*(x**2 + x + 1))**(2/3)/((x + 1)*(x**2 - x + 1)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(1-x^3\right)^{2/3}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - x^3)^(2/3))/(x^3 + 1),x)
[Out] int((x*(1 - x^3)^(2/3))/(x^3 + 1), x)
```

3.115 $\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$

Optimal. Leaf size=383

$$-\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/12*ln((1-x)*(1+x)^2)*2^(2/3)-1/6*
ln(x^3+1)*2^(2/3)-1/6*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x
^3+1)^(1/3))*2^(2/3)+1/3*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/2*ln(
-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+1/4*ln(-1+x+2^(
2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/3*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/
3))*3^(1/2))*2^(2/3)*3^(1/2)-1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)*2^(2/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3
^(1/2)-1/3*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/
2)
```

Rubi [A]
time = 0.59, antiderivative size = 648, normalized size of antiderivative = 1.69, number of steps used = 17, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$,
Rules used = {6857, 2178, 2177, 245, 2174, 371}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] -((2^(2/3)*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
) + (2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) + ((1 - (-1
)^(1/3))*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) + ((1 + (
-1)^(2/3))*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) - ((1 -
 (-1)^(1/3))*ArcTan[(1 - (2^(1/3)*((-1)^(1/3) + x))/(1 - x^3)^(1/3))/Sqrt[3
]])/(2^(1/3)*Sqrt[3]) - ((1 + (-1)^(2/3))*ArcTan[(1 + ((-1)^(2/3)*2^(1/3)*(
1 + (-1)^(1/3)*x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]) + (x^2*Hype
rgeometric2F1[1/3, 2/3, 5/3, x^3])/3 + ((1 - (-1)^(1/3))*x^2*Hypergeometric
2F1[1/3, 2/3, 5/3, x^3])/6 + ((1 + (-1)^(2/3))*x^2*Hypergeometric2F1[1/3, 2
/3, 5/3, x^3])/6 - Log[-((1 - x)*(1 + x)^2)]/(3*2^(1/3)) - ((1 + (-1)^(2/3)
)*Log[-((-1)^(2/3)*((-1)^(2/3) + x)^2*(1 + (-1)^(1/3)*x))])/(6*2^(1/3)) - (
(1 - (-1)^(1/3))*Log[(-1)^(2/3)*((-1)^(1/3) + x)*(1 + (-1)^(2/3)*x)^2])/(6*
2^(1/3)) - Log[x + (1 - x^3)^(1/3)]/3 - ((1 - (-1)^(1/3))*Log[x + (1 - x^3)
^(1/3)])/6 - ((1 + (-1)^(2/3))*Log[x + (1 - x^3)^(1/3)])/6 + ((1 - (-1)^(1/
```

```
3))*Log[1 - (-1)^(2/3)*x - (-2)^(2/3)*(1 - x^3)^(1/3)])/(2*2^(1/3)) + Log[1
 - x - 2^(2/3)*(1 - x^3)^(1/3)]/2^(1/3) + ((1 + (-1)^(2/3))*Log[1 + (-1)^(1
/3)*x + (-1)^(1/3)*2^(2/3)*(1 - x^3)^(1/3)])/(2*2^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2177

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rule 2178

```
Int[((a_) + (b_.)*(x_)^3)^(2/3)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(a +
 b*x^3)^(2/3)/(2*d), x] + (Dist[1/d^2, Int[(a*d^2 + b*c^2*x)/((c + d*x)*(a
+ b*x^3)^(1/3)), x], x] - Dist[b*(c/d^2), Int[x/(a + b*x^3)^(1/3), x], x])
/; FreeQ[{a, b, c, d}, x]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(1-x)(1-x^3)^{2/3}}{1+x^3}\,dx = \int \left(-\frac{2(1-x^3)^{2/3}}{3(-1-x)} + \frac{(-1-(-1)^{2/3})(1-x^3)^{2/3}}{3(-1+\sqrt[3]{-1}\,x)} + \frac{(-1+\sqrt[3]{-1})(1-x^3)^{2/3}}{3(-1-(-1)^{2/3}x)}\right)$$
$$= -\left(\frac{2}{3}\int \frac{(1-x^3)^{2/3}}{-1-x}\,dx\right) + \frac{1}{3}(-1+\sqrt[3]{-1})\int \frac{(1-x^3)^{2/3}}{-1-(-1)^{2/3}x}\,dx + \frac{1}{3}(-1-(-$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 10.15, size = 138, normalized size = 0.36

$$-\frac{1}{2}x^2F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right) - \frac{4x(1-x^3)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)}{(1+x^3)\left(-4F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)+x^3\left(3F_1\left(\frac{4}{3};-\frac{2}{3},2;\frac{7}{3};x^3,-x^3\right)+2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - x)*(1 - x^3)^(2/3))/(1 + x^3),x]
[Out] -1/2*(x^2*AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3]) - (4*x*(1 - x^3)^(2/3)*Ap
pellF1[1/3, -2/3, 1, 4/3, x^3, -x^3])/((1 + x^3)*(-4*AppellF1[1/3, -2/3, 1,
 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -2/3, 2, 7/3, x^3, -x^3] + 2*Appell
F1[4/3, 1/3, 1, 7/3, x^3, -x^3])))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)*(1 - x^3)^(2/3)/(1 + x^3),x]')
[Out] Timed out
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(1-x)(-x^3+1)^{\frac{2}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*(-x^3+1)^(2/3)/(x^3+1),x)
[Out] int((1-x)*(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] -integrate((-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

Fricas [F]
time = 4.75, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
[Out] integral(-(-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\left(1-x^{3}\right)^{\frac{2}{3}}}{x^{3}+1}\right)dx-\int\frac{x\left(1-x^{3}\right)^{\frac{2}{3}}}{x^{3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x**3+1)**(2/3)/(x**3+1),x)
[Out] -Integral(-(1 - x**3)**(2/3)/(x**3 + 1), x) - Integral(x*(1 - x**3)**(2/3)/
(x**3 + 1), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int\frac{\left(1-x^{3}\right)^{2/3}\,\left(x-1\right)}{x^{3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((1 - x^3)^(2/3)*(x - 1))/(x^3 + 1),x)
[Out] -int(((1 - x^3)^(2/3)*(x - 1))/(x^3 + 1), x)
```

3.116 $\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx$

Optimal. Leaf size=272

$$\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-}{\sqrt[3]{1-}}\right.}{3\ 2^{2/3}}$$

```
[Out] 1/6*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*2^(1/3)-1/6*ln(1+2^(2/3)*(1-x)^2/(-x^
3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*ln(1+2^(1/3)*(
1-x)/(-x^3+1)^(1/3))-1/12*ln(2*2^(1/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)
/(-x^3+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)
^(1/3))*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^
(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {420, 493, 298, 31, 648, 631, 210, 642}

$$\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(1/3)/(1 + x^3),x]
```

```
[Out] (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]
) + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[1 + (2^(2/3)*(
1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(2/3))
+ (2^(1/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 - Log[2*2^(1/3) +
(1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx = xF_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; x^3, -x^3\right)$$

Mathematica [A]
time = 2.02, size = 283, normalized size = 1.04

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)+4\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)-2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x+2\sqrt[3]{1-x^3}\right)+2\log\left(2^{2/3}-2\,2^{2/3}x+2^{2/3}x^2+(-1+x)\sqrt[3]{2-2x^3}+(1-x^3)^{2/3}\right)+\log\left(2^{2/3}-2\,2^{2/3}x+2^{2/3}x^2-2(-1+x)\sqrt[3]{2-2x^3}+4(1-x^3)^{2/3}\right)}{6\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(1/3)/(1 + x^3),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1
- x^3)^(1/3))] + 4*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2
*2^(1/3)*x + (1 - x^3)^(1/3))] - 4*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/
3)] - 2*Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)] + 2*Log[2^(2/3) - 2*2
^(2/3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)] + Lo
g[2^(2/3) - 2*2^(2/3)*x + 2^(2/3)*x^2 - 2*(-1 + x)*(2 - 2*x^3)^(1/3) + 4*(1
 - x^3)^(2/3)])/2^(2/3)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x^3)^(1/3)/(1 + x^3),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.35, size = 681, normalized size = 2.50

method	result	size
trager	Expression too large to display	681

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*RootOf(_Z^3-2)*ln(-(18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z
^2)^2*RootOf(_Z^3-2)^2*x^3+12*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9
*_Z^2)*RootOf(_Z^3-2)^3*x^3-3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9
*_Z^2)*x^6-2*RootOf(_Z^3-2)*x^6+18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3
-2)+9*_Z^2)*RootOf(_Z^3-2)*(-x^3+1)^(2/3)*x^2+6*(-x^3+1)^(1/3)*x^4+18*RootO
f(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^3+12*RootOf(_Z^3-2)*x^3-6*
x*(-x^3+1)^(1/3)-3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)-2*Ro
otOf(_Z^3-2))/(1+x)^2/(x^2-x+1)^2)+1/2*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(
_Z^3-2)+9*_Z^2)*ln((-18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)
^2*RootOf(_Z^3-2)^3*x^3-12*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z
```

```
^2)*RootOf(_Z^3-2)^4*x^3-3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z
^2)*RootOf(_Z^3-2)*x^6-2*RootOf(_Z^3-2)^2*x^6+18*RootOf(RootOf(_Z^3-2)^2+3*
_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^2*(-x^3+1)^(2/3)*x^2-18*RootOf(Roo
tOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*(-x^3+1)^(1/3)*x^4+6*RootOf(RootO
f(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)*x^3+4*RootOf(_Z^3-2)
^2*x^3+12*(-x^3+1)^(2/3)*x^2+18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)
+9*_Z^2)*(-x^3+1)^(1/3)*x-3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_
Z^2)*RootOf(_Z^3-2)-2*RootOf(_Z^3-2)^2)/(1+x)^2/(x^2-x+1)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(1/3)/(x^3 + 1), x)
```

Fricas [A]
time = 1.55, size = 341, normalized size = 1.25

$$\frac{1}{18}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(-\frac{6\sqrt{3}\,2^{\frac{2}{3}}(x^{16}-33x^{13}+110x^{10}-110x^{7}+33x^{4}-x)(-x^{3}+1)^{\frac{1}{3}}-24\sqrt{3}\,2^{\frac{1}{3}}(x^{14}-2x^{11}-6x^{8}-2x^{5}+x^{2})(-x^{3}+1)^{\frac{2}{3}}-\sqrt{3}(x^{18}+42x^{15}-417x^{12}+812x^{9}-417x^{6}+42x^{3}+1)}{3(x^{18}-102x^{15}+447x^{12}-628x^{9}+447x^{6}-102x^{3}+1)}\right)+\frac{1}{18}\cdot 2^{\frac{1}{3}}\log\left(-\frac{12(-x^{3}+1)^{\frac{2}{3}}x^{2}+2^{\frac{2}{3}}(x^{6}+2x^{3}+1)-6\cdot 2^{\frac{1}{3}}(x^{4}-x)(-x^{3}+1)^{\frac{1}{3}}}{x^{6}+2x^{3}+1}\right)-\frac{1}{36}\cdot 2^{\frac{1}{3}}\log\left(\frac{12\cdot 2^{\frac{2}{3}}(x^{8}-4x^{5}+x^{2})(-x^{3}+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(x^{12}-32x^{9}+78x^{6}-32x^{3}+1)+6(x^{10}-11x^{7}+11x^{4}-x)(-x^{3}+1)^{\frac{1}{3}}}{x^{12}+4x^{9}+6x^{6}+4x^{3}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
[Out] 1/18*sqrt(3)*2^(1/3)*arctan(-1/3*(6*sqrt(3)*2^(2/3)*(x^16 - 33*x^13 + 110*x
^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3) - 24*sqrt(3)*2^(1/3)*(x^14 - 2
*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) - sqrt(3)*(x^18 + 42*x^15 - 4
17*x^12 + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 62
8*x^9 + 447*x^6 - 102*x^3 + 1)) + 1/18*2^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^
2 + 2^(2/3)*(x^6 + 2*x^3 + 1) - 6*2^(1/3)*(x^4 - x)*(-x^3 + 1)^(1/3))/(x^6
+ 2*x^3 + 1)) - 1/36*2^(1/3)*log((12*2^(2/3)*(x^8 - 4*x^5 + x^2)*(-x^3 + 1)
^(2/3) + 2^(1/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) + 6*(x^10 - 11*x^7 +
 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/((x + 1)*(x**2 - x + 1)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-x^3\right)^{1/3}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/(x^3 + 1),x)
```

```
[Out] int((1 - x^3)^(1/3)/(x^3 + 1), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
```

```
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count i
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
```

```
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues."
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal
```

```
#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.


if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;


if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A","";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A","";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
```

```
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                      convert(ExpnType_result,string)," vs. order ",
                      convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
```

```
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,''^'') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,''+'') or type(expn,''*'') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
```

```
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
```

```
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
```

```
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,''+'') or type(expn,''*'')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;
```

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
```

```
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
```

```
    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:    #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation ="none"
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```