

Computer algebra independent integration tests

Summer 2022 edition

0-Independent-test-suites/10-Timofeev-Problems

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June 6, 2022

Compiled on June 6, 2022 at 12:15am

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [705]. This is test number [10].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.0.1 (February 17, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.0.1.debian on Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem via sagemath 9.6.
5. Fricas 1.3.7 (June 30, 2021) based on based on ecl 21.2.1 on Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem via sagemath 9.6.
6. Giac/Xcas 1.9.0-7 (April 2022) on on Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem. Direct testing using C++ API.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem via sagemath 9.6.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Mathics 4.0 via sagemath 9.6.

Maxima, Fricas, Mathics are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems. Mathics was called using its own interface in Sagemath as in this example

```
from sage.interfaces.mathics import mathics
res = mathics('Integrate[Sin[x]/(3 + Cos[x])^2,x]')
```

Sympy was called directly from Python. Giac was also called directly via its C++ interface.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (705)	0.00 (0)
Mathematica	100.00 (705)	0.00 (0)
Maple	92.91 (655)	7.09 (50)
Fricas	92.62 (653)	7.38 (52)
Giac	83.69 (590)	16.31 (115)
Maxima	80.00 (564)	20.00 (141)
Mupad	76.88 (542)	23.12 (163)
Sympy	61.84 (436)	38.16 (269)
Mathics	60.43 (426)	39.57 (279)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

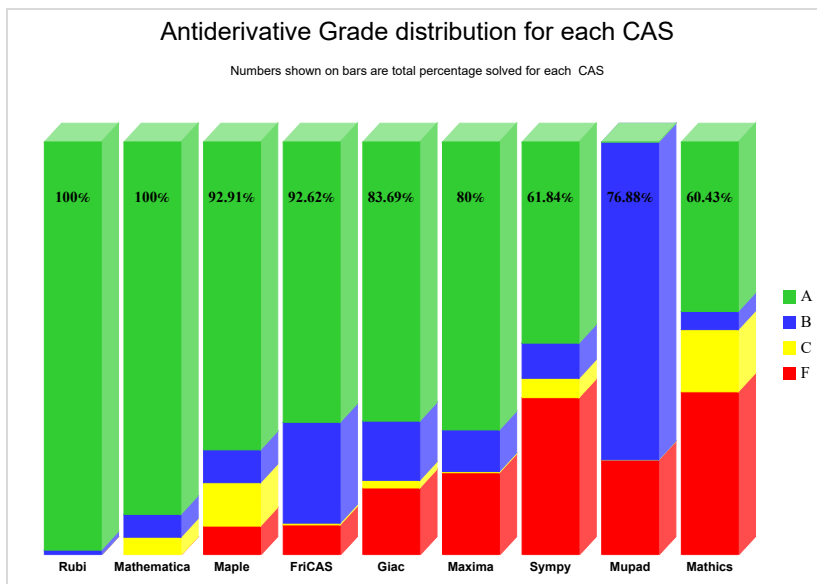
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

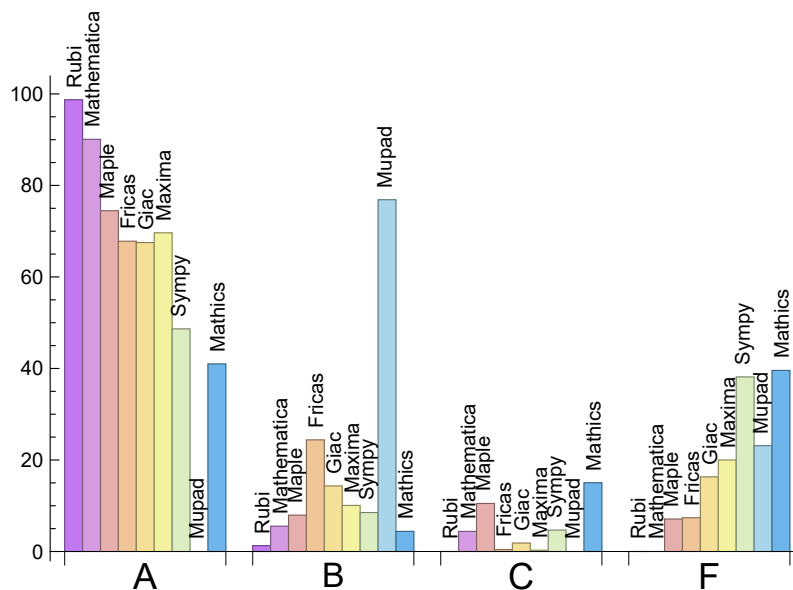
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.72	1.28	0.00	0.00
Mathematica	90.07	5.53	4.40	0.00
Maple	74.47	7.94	10.50	7.09
Maxima	69.65	10.07	0.28	20.00
Fricas	67.80	24.40	0.43	7.38
Giac	67.52	14.33	1.84	16.31
Sympy	48.65	8.51	4.68	38.16
Mathics	40.99	4.40	15.04	39.57
Mupad	N/A	76.88	0.00	23.12

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	50	100.00 %	0.00 %	0.00 %
Fricas	52	65.38 %	25.00 %	9.62 %
Giac	115	100.00 %	0.00 %	0.00 %
Maxima	141	87.23 %	2.84 %	9.93 %
Sympy	269	75.46 %	14.50 %	10.04 %
Mupad	163	100.00 %	0.00 %	0.00 %
Mathics	279	0.36 %	26.88 %	72.76 %

Table 1.4: Failure statistics for each CAS

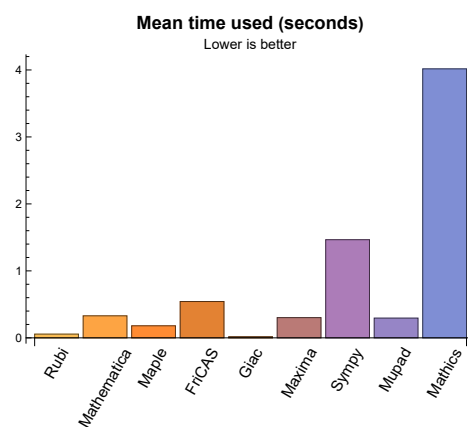
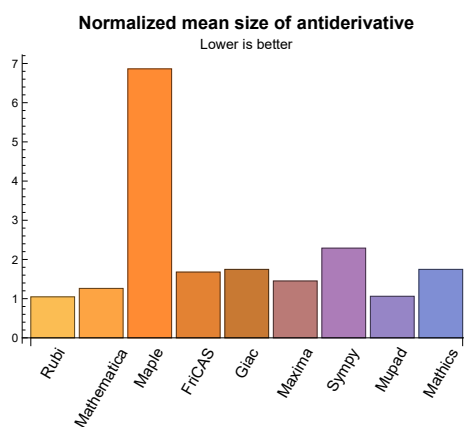
1.3 Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.06	51.38	1.05	38.00	1.00
Mathematica	0.33	53.67	1.26	37.00	1.00
Maple	0.18	490.19	6.86	35.00	0.94
Maxima	0.30	57.60	1.45	33.00	0.91
Fricas	0.54	75.26	1.68	40.00	1.11
Sympy	1.47	95.58	2.29	36.00	1.05
Giac	0.02	77.71	1.75	44.00	1.16
Mupad	0.29	42.33	1.06	28.00	0.85
Mathics	4.02	71.60	1.75	33.00	0.97

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.



1.4 list of integrals that has no closed form antiderivative

{

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

Mathics {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {222, 417}

Mathematica {222, 417, 426, 438, 452}

Mathics {3, 6, 11, 12, 13, 18, 48, 50, 51, 52, 53, 67, 68, 79, 89, 126, 133, 145, 154, 217, 240, 250, 251, 292, 294, 295, 296, 297, 298, 301, 302, 303, 304, 305, 316, 322, 323, 376, 378, 381, 461, 462, 463, 470, 477, 494, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 517, 518, 519, 520, 527, 531, 536, 537, 544, 545, 583, 585, 586, 587, 603, 608, 609, 616, 618, 621}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for **Rubi**, **Mathematica** and **Mathics**. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima `integrate` was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

For Giac, the call `taille(anti_derivative,RAND_MAX)`; is used to find leaf size.

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

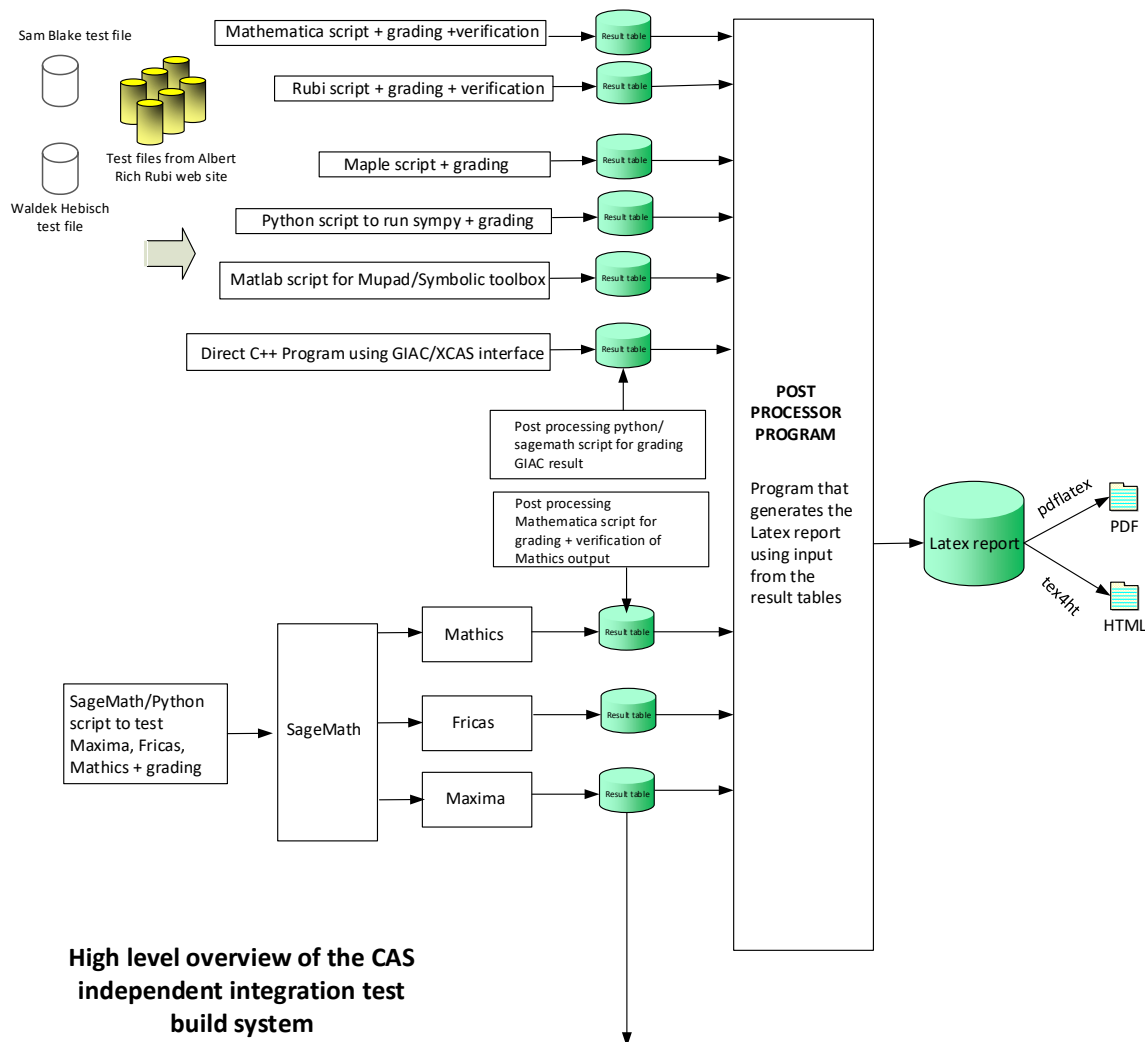
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand,the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

The following fields present only in Rubi Table file

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,...}" which is list of the rules used by Rubi

Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705 }

B grade: { 226, 228, 232, 335, 377, 413, 416, 447, 695 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 57, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 239, 240, 241, 242, 243, 244, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 313, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 385, 386, 387, 388, 390, 391, 392, 393, 394, 395, 396, 398, 400, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 418, 419, 420, 421, 422, 423, 425, 427, 428, 429, 430, 431, 432, 433, 435, 436, 437, 441, 442, 447, 449, 450, 453, 455, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 556, 558, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 704 }

B grade: { 3, 4, 41, 52, 53, 56, 63, 76, 99, 236, 237, 249, 311, 322, 323, 338, 357, 361, 438, 439, 444, 445, 446, 451, 452, 456, 488, 553, 554, 555, 557, 559, 574, 579, 592, 622, 623, 689, 705 }

C grade: { 37, 58, 113, 193, 198, 222, 238, 245, 246, 247, 248, 312, 328, 343, 384, 389, 397, 399, 401, 416, 417, 424, 426, 434, 440, 443, 448, 454, 677, 678, 703 }

F grade: { }

2.1.3 Maple

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 129, 130, 131, 132, 134, 135, 140, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 218, 220, 223, 224, 225, 227, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 243, 244, 245, 250, 251, 252, 254, 255, 256, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 307, 309, 310, 312, 318, 322, 323, 326, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 356, 357, 358, 360, 361, 362, 364, 365, 366, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 387, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 401, 412, 419, 420, 421, 422, 424, 425, 426, 428, 430, 440, 441, 448, 451, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 475, 476, 477, 478, 479, 480, 481, 483, 484, 485, 486, 488, 489, 490, 491, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 534, 535, 536, 537, 538, 539, 540, 541, 542, 544, 545, 546, 547, 558, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 579, 580, 581, 583, 584, 585, 590, 591, 593, 596, 597, 598, 599, 600, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 633, 634, 635, 636, 637, 638, 639, 640, 645, 646, 647, 648, 649, 650, 651, 652, 654, 655, 659, 660, 663, 664, 665, 666, 668, 669, 670, 671, 673, 675, 676, 677, 678, 679, 680, 681, 682, 689, 695, 696, 697, 698, 700, 701, 702 }

B grade: { 1, 13, 35, 128, 141, 213, 217, 219, 240, 241, 242, 246, 247, 248, 249, 253, 257, 260, 311, 335, 355, 359, 363, 367, 374, 383, 400, 423, 429, 431, 432, 433, 437, 438, 439, 447, 456, 474, 482, 548, 578, 586, 587, 588, 595, 601, 602, 603, 642, 644, 657, 661, 662, 672, 674, 683 }

C grade: { 81, 136, 137, 138, 139, 142, 143, 144, 226, 228, 232, 301, 302, 303, 304, 305, 306, 308, 313, 314, 315, 316, 317, 319, 320, 321, 324, 325, 327, 392, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 413, 416, 417, 434, 435, 436, 457, 487, 492, 493, 543, 556, 561, 563, 589, 592, 594, 641, 643, 653, 656, 658, 667, 684, 685, 686, 687, 688, 690, 691, 692, 693, 694, 704 }

F grade: { 67, 126, 133, 145, 193, 198, 221, 222, 328, 329, 352, 414, 415, 418, 427, 442, 443, 444, 445, 446, 449, 450, 452, 453, 454, 455, 500, 506, 511, 516, 521, 529, 532, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 582, 632, 699, 703, 705 }

2.1.4 Maxima

A grade: { 2, 3, 6, 7, 9, 10, 11, 12, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 229, 235, 236, 237, 243, 250, 251, 252, 253, 254, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 282, 284, 285, 286, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 310, 312, 313, 316, 326, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 370, 371, 372, 375, 376, 377, 378, 379, 380, 381, 382, 392, 396, 397, 398, 400, 401, 412, 414, 415, 418, 419, 420, 422, 424, 428, 439, 444, 445, 448, 450, 453, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 475, 476, 477, 478, 479, 480, 481, 483, 484, 485, 486, 490, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 532, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 585, 590, 591, 598, 600, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 638, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 658, 659, 660, 661, 662, 663, 664, 666, 667, 669, 670, 671, 673, 676, 677, 678, 679, 680, 681, 682, 685, 686, 687, 690, 692, 694, 697, 698, 700, 702, 704, 705 }

B grade: { 1, 4, 5, 8, 13, 19, 31, 41, 62, 76, 81, 82, 86, 128, 159, 240, 241, 257, 311, 318, 322, 323, 367, 369, 373, 374, 384, 385, 386, 387, 388, 389, 393, 394, 399, 423, 425, 429, 430, 431, 433, 437, 449, 451, 456, 474, 482, 488, 489, 491, 492, 493, 577, 578, 579, 580, 581, 584, 586, 588, 589, 593, 594, 596, 597, 599, 601, 643, 691, 695, 696 }

C grade: { 421, 426 }

F grade: { 69, 126, 133, 145, 149, 193, 194, 195, 196, 197, 198, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 238, 239, 242, 244, 245, 246, 247, 248, 249, 255, 256, 258, 259, 260, 279, 281, 283, 287, 288, 289, 290, 291, 306, 307, 308, 309, 314, 315, 317, 319, 320, 321, 324, 325, 327, 328, 329, 352, 383, 390, 391, 395, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 413, 416, 417, 427, 432, 434, 435, 436, 438, 440, 441, 442, 443, 446, 447, 452, 454, 455, 457, 473, 487, 494, 495, 500, 506, 511, 516, 521, 529, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 582, 583, 587, 592, 595, 603, 621, 637, 650, 657, 665, 668, 672, 674, 675, 683, 684, 688, 689, 693, 699, 701, 703 }

2.1.5 FriCAS

A grade: { 1, 2, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 58, 59, 60, 61, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 134, 140, 141, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 178, 179, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 194, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 225, 227, 230, 231, 232, 233, 234, 238, 241, 243, 250, 251, 252, 258, 260, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 282, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 307, 308, 309, 310, 312, 313, 316, 318, 320, 325, 326, 327, 330, 331, 332, 333, 334, 336, 339, 341, 344, 345, 346, 347, 348, 349, 350, 351, 356, 359, 362, 364, 366, 368, 370, 373, 374, 375, 376, 378, 380, 381, 385, 386, 387, 395, 396, 398, 409, 412, 414, 415, 418, 419, 420, 422, 425, 428, 436, 437, 439, 440, 441, 448, 450, 454, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 492, 493, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 532, 534, 535, 536, 537, 538, 539, 540, 541, 542, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 576, 583, 585, 590, 598, 599, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 658, 659, 660, 663, 664, 666, 667, 668, 669, 670, 671, 673, 676, 677, 679, 680, 681, 682, 684, 685, 686, 687, 688, 690, 691, 692, 693, 694, 695, 696, 697, 699, 700, 705 }

B grade: { 3, 4, 5, 19, 30, 31, 36, 37, 41, 53, 54, 55, 56, 57, 62, 63, 76, 99, 135, 149, 159, 161, 165, 174, 180, 187, 195, 196, 197, 202, 221, 222, 223, 224, 226, 228, 229, 235, 236, 237, 239, 240, 242, 244, 245, 246, 247, 249, 253, 254, 255, 256, 257, 259, 268, 269, 276, 281, 283, 284, 303, 311, 314, 315, 317, 319, 321, 322, 323, 324, 328, 335, 337, 338, 340, 342, 343, 353, 354, 355, 357, 358, 360, 361, 363, 365, 367, 369, 371, 372, 377, 379, 382, 383, 384, 388, 389, 390, 391, 392, 393, 394, 397, 399, 400, 402, 403, 404, 405, 406, 407, 408, 410, 411, 413, 416, 423, 424, 427, 429, 430, 431, 432, 433, 434, 435, 438, 443, 447, 451, 452, 453, 456, 457, 490, 491, 505, 510, 572, 573, 574, 575, 577, 578, 579, 580, 581, 582, 584, 586, 587, 588, 589, 591, 592, 593, 594, 595, 596, 597, 600, 601, 602, 603, 625, 635, 661, 662, 701, 702, 703, 704 }

C grade: { 177, 401, 421 }

F grade: { 126, 133, 136, 137, 138, 139, 142, 143, 144, 145, 193, 198, 248, 329, 352, 417, 426, 442, 444, 445, 446, 449, 455, 500, 506, 511, 516, 521, 529, 533, 543, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 616, 657, 665, 672, 674, 675, 678, 683, 689, 698 }

2.1.6 Sympy

A grade: { 3, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 60, 63, 65, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 120, 121, 124, 128, 129, 131, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 212, 240, 241, 250, 251, 292, 293, 300, 322, 323, 330, 331, 332, 333, 334, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 355, 356, 357, 358, 359, 361, 364, 365, 366, 367, 368, 371, 372, 377, 385, 387, 441, 458, 459, 460, 463, 464, 465, 466, 470, 471, 472, 476, 479, 480, 483, 484, 485, 486, 494, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 530, 531, 535, 537, 538, 539, 541, 542, 543, 544, 545, 546, 564, 565, 566, 567, 568, 569, 570, 571, 577, 578, 582, 583, 584, 585, 586, 587, 596, 597, 598, 599, 603, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 628, 629, 630, 631, 632, 634, 635, 637, 638, 641, 642, 644, 646, 647, 648, 651, 652, 653, 654, 656, 659, 660, 661, 662, 663, 664, 666, 667, 668, 669, 671, 676, 677, 679, 680, 681, 699, 700 }

B grade: { 1, 4, 8, 19, 21, 30, 31, 36, 41, 62, 149, 159, 194, 195, 196, 197, 211, 214, 252, 253, 254, 299, 312, 335, 340, 353, 354, 369, 370, 376, 378, 379, 382, 383, 388, 389, 396, 467, 468, 475, 487, 488, 489, 493, 547, 548, 572, 573, 576, 580, 588, 589, 590, 591, 602, 604, 655, 670, 697, 704 }

C grade: { 2, 114, 118, 119, 122, 123, 125, 126, 127, 130, 132, 133, 134, 145, 175, 207, 215, 294, 295, 296, 297, 298, 301, 302, 303, 304, 305, 313, 316, 461, 462, 477, 536 }

F grade: { 54, 55, 56, 57, 58, 59, 61, 64, 66, 69, 193, 198, 213, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 242, 243, 244, 245, 246, 247, 248, 249, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 306, 307, 308, 309, 310, 311, 314, 315, 317, 318, 319, 320, 321, 324, 325, 326, 327, 328, 329, 352, 360, 362, 363, 373, 374, 375, 380, 381, 384, 386, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 469, 473, 474, 478, 481, 482, 490, 491, 492, 495, 500, 506, 511, 516, 521, 528, 529, 532, 533, 534, 540, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 574, 575, 579, 581, 592, 593, 594, 595, 600, 601, 605, 606, 607, 622, 623, 624, 625, 626, 627, 633, 636, 639, 640, 643, 645, 649, 650, 657, 658, 665, 672, 673, 674, 675, 678, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 698, 701, 702, 703, 705 }

2.1.7 Giac

A grade: { 2, 4, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 56, 57, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 77, 78, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 227, 229, 230, 231, 235, 236, 237, 241, 250, 251, 252, 253, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 278, 282, 284, 285, 286, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309, 310, 312, 313, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 344, 345, 346, 347, 348, 349, 350, 351, 354, 355, 356, 358, 359, 360, 361, 362, 363, 364, 366, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 381, 382, 385, 386, 387, 390, 391, 392, 393, 396, 397, 398, 400, 401, 419, 420, 422, 423, 424, 425, 428, 429, 433, 440, 441, 442, 443, 444, 445, 450, 451, 452, 454, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 475, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 492, 493, 496, 497, 498, 499, 501, 502, 507, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 534, 535, 536, 539, 541, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 574, 583, 584, 585, 587, 589, 593, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 609, 610, 611, 612, 613, 614, 617, 618, 619, 621, 622, 624, 625, 627, 628, 629, 630, 631, 633, 634, 637, 638, 639, 640, 641, 642, 643, 644, 646, 647, 650, 651, 652, 653, 654, 655, 656, 659, 660, 661, 662, 663, 664, 667, 668, 669, 670, 671, 676, 677, 686, 687, 688, 691, 697, 700, 701, 705 }

B grade: { 1, 3, 5, 13, 19, 22, 52, 55, 58, 73, 76, 128, 197, 220, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 249, 254, 255, 256, 257, 258, 259, 265, 274, 279, 280, 281, 283, 287, 308, 311, 322, 323, 338, 342, 343, 353, 357, 365, 367, 374, 383, 384, 388, 389, 394, 395, 399, 431, 438, 447, 449, 455, 456, 474, 476, 487, 488, 489, 532, 538, 542, 570, 571, 572, 573, 575, 576, 577, 578, 579, 580, 581, 586, 588, 590, 591, 592, 595, 596, 597, 620, 635, 645, 649, 658, 666, 685, 690, 696, 702, 704 }

C grade: { 79, 421, 437, 457, 494, 503, 504, 505, 508, 509, 510, 537, 703 }

F grade: { 86, 126, 133, 145, 154, 193, 198, 221, 222, 223, 224, 225, 226, 228, 232, 233, 234, 291, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 327, 328, 329, 352, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 426, 427, 430, 432, 434, 435, 436, 439, 446, 448, 453, 473, 490, 491, 495, 500, 506, 511, 516, 521, 529, 533, 540, 543, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 582, 594, 608, 615, 616, 623, 626, 632, 636, 648, 657, 665, 672, 673, 674, 675, 678, 679, 680, 681, 682, 683, 684, 689, 692, 693, 694, 695, 698, 699 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 224, 225, 229, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 266, 268, 269, 271, 272, 273, 274, 282, 284, 285, 286, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 306, 307, 308, 309, 311, 312, 313, 318, 322, 323, 326, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 397, 398, 399, 400, 401, 408, 409, 410, 412, 414, 415, 419, 422, 425, 430, 431, 437, 439, 440, 441, 442, 443, 444, 451, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 593, 594, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 642, 643, 644, 647, 648, 669, 670, 671, 673, 676, 677, 678, 679, 680, 681, 682, 696, 697, 700, 704 }

C grade: { }

F grade: { 69, 86, 126, 133, 145, 193, 198, 221, 222, 226, 227, 228, 230, 231, 232, 243, 244, 245, 246, 247, 248, 249, 257, 264, 265, 267, 270, 275, 276, 277, 278, 279, 280, 281, 283, 287, 288, 290, 291, 304, 305, 310, 314, 315, 316, 317, 319, 320, 321, 324, 325, 327, 328, 329, 394, 395, 396, 402, 403, 404, 405, 406, 407, 411, 413, 416, 417, 418, 420, 421, 423, 424, 426, 427, 428, 429, 432, 433, 434, 435, 436, 438, 445, 446, 447, 448, 449, 450, 452, 453, 454, 455, 456, 457, 474, 490, 491, 492, 493, 500, 506, 511, 532, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 592, 595, 616, 641, 645, 646, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 672, 674, 675, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 698, 699, 701, 702, 703, 705 }

2.1.9 Mathics

A grade: { 7, 8, 9, 10, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 53, 60, 62, 63, 65, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 82, 84, 85, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 120, 121, 124, 128, 129, 131, 140, 141, 146, 147, 148, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 241, 251, 252, 254, 293, 299, 300, 312, 322, 323, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 354, 355, 356, 357, 358, 361, 364, 365, 366, 368, 369, 370, 371, 372, 377, 379, 385, 387, 396, 441, 458, 459, 460, 464, 465, 466, 467, 468, 471, 472, 475, 476, 479, 480, 483, 484, 485, 486, 496, 497, 498, 499, 522, 524, 535, 538, 539, 541, 542, 543, 546, 547, 548, 564, 565, 566, 567, 568, 569, 570, 571, 575, 576, 577, 578, 584, 589, 591, 596, 597, 598, 599, 602, 604, 610, 611, 612, 613, 614, 615, 617, 620, 628, 629, 630, 631, 632, 634, 635, 637, 638, 641, 642, 644, 646, 647, 648, 653, 654, 656, 659, 660, 664, 667, 668, 669, 670, 671, 676, 677, 679, 680, 681, 697, 699, 700 }

B grade: { 1, 4, 5, 19, 81, 99, 149, 195, 196, 197, 214, 253, 340, 353, 367, 382, 383, 388, 389, 487, 488, 489, 493, 572, 573, 574, 579, 588, 590, 655, 704 }

C grade: { 2, 3, 6, 11, 12, 13, 14, 15, 16, 17, 18, 21, 48, 50, 51, 52, 68, 79, 89, 114, 118, 119, 122, 123, 125, 126, 127, 130, 132, 133, 134, 135, 136, 137, 138, 139, 142, 143, 144, 145, 154, 175, 177, 207, 215, 217, 240, 250, 292, 294, 295, 296, 297, 298, 301, 302, 303, 304, 305, 316, 376, 378, 381, 461, 462, 463, 470, 477, 494, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 517, 518, 519, 520, 523, 525, 526, 527, 530, 531, 536, 537, 544, 545, 580, 583, 585, 586, 587, 603, 608, 609, 616, 618, 621 }

F grade: { 31, 54, 55, 56, 57, 58, 59, 61, 64, 66, 69, 83, 86, 174, 193, 198, 213, 216, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 242, 243, 244, 245, 246, 247, 248, 249, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 306, 307, 308, 309, 310, 311, 313, 314, 315, 317, 318, 319, 320, 321, 324, 325, 326, 327, 328, 329, 352, 359, 360, 362, 363, 373, 374, 375, 380, 384, 386, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 469, 473, 474, 478, 481, 482, 490, 491, 492, 495, 500, 501, 506, 511, 516, 521, 528, 529, 532, 533, 534, 540, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 581, 582, 592, 593, 594, 595, 600, 601, 605, 606, 607, 619, 622, 623, 624, 625, 626, 627, 633, 636, 639, 640, 643, 645, 649, 650, 651, 652, 657, 658, 661, 662, 663, 665, 666, 672, 673, 674, 675, 678, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 698, 701, 702, 703, 705 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	31	31	25	20	35	14	28
N.S.	1	1.00	1.00	2.21	2.21	1.79	1.43	2.50	1.00	2.00
time (sec)	N/A	0.005	0.003	0.046	0.264	0.338	0.072	0.006	0.058	1.989

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	26	13	14	29
N.S.	1	1.00	1.00	1.07	1.00	1.00	1.86	0.93	1.00	2.07
time (sec)	N/A	0.003	0.003	0.049	0.352	0.342	0.069	0.006	0.040	2.057

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	37	18	17	26	27	46	11	31
N.S.	1	1.00	2.85	1.38	1.31	2.00	2.08	3.54	0.85	2.38
time (sec)	N/A	0.004	0.007	0.022	0.271	0.337	0.055	0.010	0.233	2.219

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	15	19	23	22	13	7	19
N.S.	1	1.00	2.09	1.36	1.73	2.09	2.00	1.18	0.64	1.73
time (sec)	N/A	0.002	0.007	0.031	0.266	0.343	0.054	0.008	0.070	1.972

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	21	27	29	22	56	24	23
N.S.	1	1.00	1.00	1.40	1.80	1.93	1.47	3.73	1.60	1.53
time (sec)	N/A	0.002	0.005	0.130	0.283	0.357	0.103	0.008	0.207	2.551

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	4	4	3	3	12	8
N.S.	1	1.00	1.00	1.50	2.00	2.00	1.50	1.50	6.00	4.00
time (sec)	N/A	0.004	0.002	0.011	0.274	0.348	0.030	0.005	0.255	1.622

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	6	6	5	4	6	6
N.S.	1	1.00	1.00	1.25	1.50	1.50	1.25	1.00	1.50	1.50
time (sec)	N/A	0.004	0.002	0.000	0.270	0.336	0.031	0.004	0.208	1.763

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	27	4	7	6	4	4
N.S.	1	1.00	1.00	0.83	4.50	0.67	1.17	1.00	0.67	0.67
time (sec)	N/A	0.015	0.011	0.026	0.263	0.330	0.394	0.008	0.180	2.005

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	9	9	3	9	4	4
N.S.	1	1.00	0.67	0.56	1.00	1.00	0.33	1.00	0.44	0.44
time (sec)	N/A	0.006	0.004	0.016	0.274	0.327	0.092	0.004	0.189	1.752

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	10	10	7	11	6	8
N.S.	1	1.00	0.67	0.75	0.83	0.83	0.58	0.92	0.50	0.67
time (sec)	N/A	0.007	0.008	0.008	0.266	0.325	0.170	0.002	0.002	1.800

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	13	12	15	12	13	27
N.S.	1	1.00	1.00	1.08	1.08	1.00	1.25	1.00	1.08	2.25
time (sec)	N/A	0.015	0.014	0.033	0.273	0.377	0.176	0.001	0.198	1.897

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	15	31	16	15	53
N.S.	1	1.00	1.00	1.07	1.00	1.00	2.07	1.07	1.00	3.53
time (sec)	N/A	0.017	0.008	0.057	0.363	0.387	0.266	0.003	0.047	1.967

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	33	33	26	44	41	15	73
N.S.	1	1.00	1.00	2.20	2.20	1.73	2.93	2.73	1.00	4.87
time (sec)	N/A	0.018	0.007	0.069	0.269	0.362	0.276	0.002	0.184	2.011

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	21	32	18	48	36
N.S.	1	1.00	1.00	1.06	1.00	1.24	1.88	1.06	2.82	2.12
time (sec)	N/A	0.024	0.007	0.091	0.264	0.356	1.371	0.001	0.606	3.925

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	20	23	34	21	48	38
N.S.	1	1.00	1.00	1.05	1.05	1.21	1.79	1.11	2.53	2.00
time (sec)	N/A	0.025	0.008	0.138	0.263	0.345	1.401	0.001	0.491	4.050

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	22	22	19	18	18	34	19	58	37
N.S.	1	1.22	1.22	1.06	1.00	1.00	1.89	1.06	3.22	2.06
time (sec)	N/A	0.026	0.011	0.087	0.259	0.359	1.363	0.002	0.406	4.025

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	22	22	19	19	19	32	20	58	37
N.S.	1	1.22	1.22	1.06	1.06	1.06	1.78	1.11	3.22	2.06
time (sec)	N/A	0.027	0.009	0.166	0.265	0.357	1.403	0.001	0.379	4.073

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	19	14	13	31	61	57	26	37
N.S.	1	1.00	0.46	0.34	0.32	0.76	1.49	1.39	0.63	0.90
time (sec)	N/A	0.011	0.018	0.037	0.358	0.373	0.235	0.001	0.232	2.681

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	15	15	15	19	15	17
N.S.	1	1.00	1.00	1.00	2.50	2.50	2.50	3.17	2.50	2.83
time (sec)	N/A	0.011	0.002	0.006	0.265	0.332	0.054	0.000	0.004	1.843

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	3	4	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	1.00	1.33	1.00	1.00
time (sec)	N/A	0.008	0.004	0.003	0.265	0.356	0.043	0.000	0.176	1.712

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	15	3	3	22
N.S.	1	1.00	1.00	1.33	1.00	1.00	5.00	1.00	1.00	7.33
time (sec)	N/A	0.012	0.009	0.004	0.344	0.323	0.067	0.000	0.336	1.918

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	7	10	7	7	7	24	7	7
N.S.	1	1.00	0.78	1.11	0.78	0.78	0.78	2.67	0.78	0.78
time (sec)	N/A	0.012	0.007	0.006	0.275	0.345	0.048	0.000	0.206	1.722

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	8	9	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.78	0.89	1.00	1.00
time (sec)	N/A	0.011	0.013	0.007	0.256	0.319	0.050	0.001	0.209	1.796

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	22	24	22	27	24	21
N.S.	1	1.00	1.00	0.88	0.88	0.96	0.88	1.08	0.96	0.84
time (sec)	N/A	0.012	0.016	0.027	0.258	0.333	0.171	0.001	0.043	1.830

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	21	20	22	27	43	22	20
N.S.	1	1.00	1.00	0.70	0.67	0.73	0.90	1.43	0.73	0.67
time (sec)	N/A	0.060	0.020	0.007	0.269	0.368	2.001	0.002	0.034	3.815

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	8	16	8	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.80	1.60	0.80	1.00
time (sec)	N/A	0.003	0.002	0.054	0.271	0.324	0.034	0.000	0.063	1.748

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	24	36	40	42	51	47	35
N.S.	1	1.00	1.00	0.51	0.77	0.85	0.89	1.09	1.00	0.74
time (sec)	N/A	0.015	0.020	0.054	0.353	0.324	0.055	0.002	0.132	1.998

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	23	23	27	46	21	21
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.90	1.53	0.70	0.70
time (sec)	N/A	0.005	0.009	0.060	0.354	0.325	0.056	0.000	0.037	1.853

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	24	22	26	13	13
N.S.	1	1.00	1.00	0.67	0.62	1.14	1.05	1.24	0.62	0.62
time (sec)	N/A	0.005	0.012	0.055	0.277	0.338	0.143	0.001	0.167	1.933

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	24	26	13	11	21
N.S.	1	1.00	1.00	0.80	0.73	1.60	1.73	0.87	0.73	1.40
time (sec)	N/A	0.005	0.007	0.063	0.278	0.350	0.142	0.001	0.060	2.047

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	186	89	138	16	118	0
N.S.	1	1.00	1.00	0.95	8.86	4.24	6.57	0.76	5.62	0.00
time (sec)	N/A	0.022	0.056	0.067	0.361	0.426	0.665	0.009	1.334	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	13	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.93	0.71	0.71
time (sec)	N/A	0.003	0.002	0.000	0.284	0.346	0.028	0.000	0.028	1.841

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	12	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.86	0.71	0.71
time (sec)	N/A	0.004	0.002	0.016	0.259	0.336	0.027	0.000	0.026	1.718

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	10	12	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	1.25	1.50	0.75
time (sec)	N/A	0.007	0.001	0.023	0.259	0.325	0.029	0.000	0.030	1.701

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	32	14	22	15	16	11	14
N.S.	1	1.00	1.00	2.91	1.27	2.00	1.36	1.45	1.00	1.27
time (sec)	N/A	0.009	0.005	0.027	0.271	0.336	0.042	0.001	0.072	1.774

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	6	15	9	18	12	7	6	8
N.S.	1	1.00	0.86	2.14	1.29	2.57	1.71	1.00	0.86	1.14
time (sec)	N/A	0.014	0.006	0.021	0.262	0.327	0.032	0.001	0.201	1.726

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	28	18	12	23	19	36	10	12
N.S.	1	1.00	2.00	1.29	0.86	1.64	1.36	2.57	0.71	0.86
time (sec)	N/A	0.004	0.010	0.013	0.370	0.324	0.032	0.009	0.170	1.813

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	26	19	12	20	17	20	18	16
N.S.	1	1.00	1.62	1.19	0.75	1.25	1.06	1.25	1.12	1.00
time (sec)	N/A	0.006	0.010	0.027	0.370	0.351	0.065	0.004	0.270	1.917

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	12	19	10	11	12	12
N.S.	1	1.00	1.00	1.10	1.20	1.90	1.00	1.10	1.20	1.20
time (sec)	N/A	0.017	0.012	0.033	0.374	0.345	0.154	0.003	0.267	1.870

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	15	14	25	10	19	14	14
N.S.	1	1.00	0.86	1.07	1.00	1.79	0.71	1.36	1.00	1.00
time (sec)	N/A	0.039	0.007	0.040	0.369	0.337	0.646	0.004	0.275	3.460

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	19	28	24	29	15	12	18
N.S.	1	1.00	2.27	1.73	2.55	2.18	2.64	1.36	1.09	1.64
time (sec)	N/A	0.013	0.027	0.027	0.353	0.369	0.242	0.003	0.235	2.258

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	21	17	23	14	8	17	10	12
N.S.	1	1.00	1.31	1.06	1.44	0.88	0.50	1.06	0.62	0.75
time (sec)	N/A	0.015	0.016	0.024	0.355	0.327	0.210	0.004	0.166	1.839

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	33	29	18	22	19	32	18	21
N.S.	1	1.00	1.32	1.16	0.72	0.88	0.76	1.28	0.72	0.84
time (sec)	N/A	0.015	0.018	0.016	0.269	0.339	0.055	0.001	0.216	1.789

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	13	12	18	8	13
N.S.	1	1.00	1.00	0.67	0.62	0.62	0.57	0.86	0.38	0.62
time (sec)	N/A	0.003	0.002	0.080	0.272	0.334	0.048	0.000	0.257	1.748

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	12	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.86	0.71	0.71
time (sec)	N/A	0.010	0.004	0.013	0.431	0.334	0.052	0.000	0.063	1.796

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	47	29	35	45	42	54	34	35
N.S.	1	1.00	0.92	0.57	0.69	0.88	0.82	1.06	0.67	0.69
time (sec)	N/A	0.011	0.017	0.100	0.355	0.369	0.059	0.000	0.122	1.927

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	47	25	25	29	34	25	25
N.S.	1	1.00	1.00	1.42	0.76	0.76	0.88	1.03	0.76	0.76
time (sec)	N/A	0.008	0.013	0.094	0.265	0.343	0.060	0.001	0.203	1.835

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	18	68	73	26	19	92
N.S.	1	1.00	1.00	0.66	0.62	2.34	2.52	0.90	0.66	3.17
time (sec)	N/A	0.009	0.015	0.054	0.356	0.328	0.437	0.001	0.205	2.132

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	20	23	22	19	37	38	20	19
N.S.	1	1.00	0.74	0.85	0.81	0.70	1.37	1.41	0.74	0.70
time (sec)	N/A	0.007	0.011	0.048	0.345	0.325	0.160	0.000	0.042	2.001

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	25	29	12	18	33
N.S.	1	1.00	1.00	0.86	0.82	1.14	1.32	0.55	0.82	1.50
time (sec)	N/A	0.006	0.082	0.053	0.351	0.328	0.455	0.003	0.094	2.352

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	41	12	26	22	23	24	35
N.S.	1	1.00	1.00	1.86	0.55	1.18	1.00	1.05	1.09	1.59
time (sec)	N/A	0.008	0.014	0.055	0.353	0.341	0.458	0.001	0.258	2.361

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	53	37	34	25	22	47	21	35
N.S.	1	1.00	2.30	1.61	1.48	1.09	0.96	2.04	0.91	1.52
time (sec)	N/A	0.009	0.031	0.052	0.265	0.337	0.472	0.001	0.445	2.340

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	49	35	12	40	7	43	26	11
N.S.	1	1.00	2.33	1.67	0.57	1.90	0.33	2.05	1.24	0.52
time (sec)	N/A	0.008	0.023	0.053	0.263	0.324	0.442	0.001	0.094	2.076

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	21	7	8	30	0	9	6	0
N.S.	1	1.00	1.75	0.58	0.67	2.50	0.00	0.75	0.50	0.00
time (sec)	N/A	0.004	0.051	0.099	0.344	0.360	0.000	0.001	0.182	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	33	15	16	38	0	41	26	0
N.S.	1	1.00	1.74	0.79	0.84	2.00	0.00	2.16	1.37	0.00
time (sec)	N/A	0.009	0.054	0.226	0.355	0.331	0.000	0.002	0.294	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	38	7	6	16	0	6	6	0
N.S.	1	1.00	4.75	0.88	0.75	2.00	0.00	0.75	0.75	0.00
time (sec)	N/A	0.002	0.024	0.075	0.348	0.334	0.000	0.001	0.156	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	36	22	21	43	0	26	40	0
N.S.	1	1.00	1.33	0.81	0.78	1.59	0.00	0.96	1.48	0.00
time (sec)	N/A	0.006	0.081	0.096	0.356	0.318	0.000	0.004	0.315	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	A	A	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	25	33	39	0	82	28	0
N.S.	1	1.00	1.06	0.78	1.03	1.22	0.00	2.56	0.88	0.00
time (sec)	N/A	0.007	0.063	0.106	0.364	0.325	0.000	0.004	0.338	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	23	21	22	17	17	0	32	19	0
N.S.	1	1.10	1.00	1.05	0.81	0.81	0.00	1.52	0.90	0.00
time (sec)	N/A	0.004	0.085	0.090	0.355	0.317	0.000	0.005	0.221	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	54	23	33	39	22	44	22	22
N.S.	1	1.00	1.93	0.82	1.18	1.39	0.79	1.57	0.79	0.79
time (sec)	N/A	0.084	0.042	0.069	0.273	0.351	0.316	0.004	0.278	2.025

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	48	17	14	13	31	0	57	26	0
N.S.	1	1.30	0.46	0.38	0.35	0.84	0.00	1.54	0.70	0.00
time (sec)	N/A	0.013	0.033	0.039	0.353	0.343	0.000	0.001	0.236	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	21	11	47	32	27	16	10	21
N.S.	1	1.00	1.50	0.79	3.36	2.29	1.93	1.14	0.71	1.50
time (sec)	N/A	0.022	0.015	0.077	0.268	0.338	0.700	0.003	0.215	2.594

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	4	15	45	15	20	3	15
N.S.	1	1.00	2.09	0.36	1.36	4.09	1.36	1.82	0.27	1.36
time (sec)	N/A	0.019	0.006	0.020	0.265	0.336	0.071	0.002	0.352	1.795

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-2)	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	19	19	15	14	14	0	79	14	0
N.S.	1	1.06	1.06	0.83	0.78	0.78	0.00	4.39	0.78	0.00
time (sec)	N/A	0.028	0.011	0.037	0.278	0.341	0.000	0.002	0.315	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	15	17	24	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.83	0.94	1.33	0.78	0.78
time (sec)	N/A	0.029	0.014	0.065	0.263	0.333	1.036	0.002	0.304	2.749

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	0	16	37	0
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.00	0.94	2.18	0.00
time (sec)	N/A	0.009	0.020	0.041	0.348	0.342	0.000	0.000	0.293	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	18	16	7	24	16	8
N.S.	1	1.00	1.00	0.00	0.90	0.80	0.35	1.20	0.80	0.40
time (sec)	N/A	0.015	0.023	0.007	0.270	0.323	0.356	0.003	0.341	1.880

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	15	11	12	21
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.25	0.92	1.00	1.75
time (sec)	N/A	0.015	0.004	0.138	0.262	0.350	0.938	0.001	0.232	2.506

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	0	38	0	24	-1	0
N.S.	1	1.00	1.00	0.90	0.00	0.90	0.00	0.57	-0.02	0.00
time (sec)	N/A	0.018	0.028	0.071	0.000	0.317	0.000	0.005	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	8	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	1.00	0.75	0.75
time (sec)	N/A	0.014	0.005	0.054	0.349	0.313	0.511	0.001	0.349	2.002

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	17	22	22	26	17	17
N.S.	1	1.00	1.00	0.82	0.61	0.79	0.79	0.93	0.61	0.61
time (sec)	N/A	0.006	0.002	0.000	0.279	0.319	0.053	0.001	0.034	1.725

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	11	15	18	9	11
N.S.	1	1.00	1.00	0.82	0.76	0.65	0.88	1.06	0.53	0.65
time (sec)	N/A	0.004	0.002	0.008	0.269	0.334	0.052	0.000	0.166	1.733

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	38	38	53	28	28	26	88	40	28
N.S.	1	1.06	1.06	1.47	0.78	0.78	0.72	2.44	1.11	0.78
time (sec)	N/A	0.016	0.005	0.030	0.266	0.330	0.062	0.002	0.345	1.878

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	23	17	15	18	17	19	21	15
N.S.	1	1.00	1.21	0.89	0.79	0.95	0.89	1.00	1.11	0.79
time (sec)	N/A	0.005	0.002	0.011	0.263	0.386	0.031	0.000	0.002	1.812

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	26	18	25	31	28	26	30
N.S.	1	1.00	0.88	0.76	0.53	0.74	0.91	0.82	0.76	0.88
time (sec)	N/A	0.020	0.010	0.021	0.262	0.324	0.030	0.001	0.022	1.919

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	71	26	42	69	46	100	33	31
N.S.	1	1.00	2.73	1.00	1.62	2.65	1.77	3.85	1.27	1.19
time (sec)	N/A	0.007	0.006	0.076	0.261	0.346	0.073	0.001	0.291	2.421

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	14	18	11	17	17	17	11	16
N.S.	1	1.00	0.61	0.78	0.48	0.74	0.74	0.74	0.48	0.70
time (sec)	N/A	0.005	0.009	0.025	0.277	0.321	0.183	0.000	0.023	1.885

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	19	21	26	22	19	20
N.S.	1	1.00	0.81	0.81	0.70	0.78	0.96	0.81	0.70	0.74
time (sec)	N/A	0.006	0.024	0.024	0.270	0.310	0.108	0.001	0.036	1.922

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	32	24	20	107	383	20	83
N.S.	1	1.00	0.65	1.03	0.77	0.65	3.45	12.35	0.65	2.68
time (sec)	N/A	0.008	0.015	0.033	0.262	0.373	0.292	0.004	0.026	2.469

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	10	13	15	15	13	13
N.S.	1	1.00	1.00	0.82	0.59	0.76	0.88	0.88	0.76	0.76
time (sec)	N/A	0.002	0.003	0.016	0.268	0.315	0.126	0.000	0.195	1.862

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	67	94	22	15	14	35	31
N.S.	1	1.00	1.00	5.58	7.83	1.83	1.25	1.17	2.92	2.58
time (sec)	N/A	0.012	0.012	0.119	0.353	0.316	20.090	0.007	0.428	4.453

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	20	107	21	19	27	13	17
N.S.	1	1.00	1.00	1.33	7.13	1.40	1.27	1.80	0.87	1.13
time (sec)	N/A	0.009	0.013	0.006	0.356	0.318	0.074	0.006	0.023	1.903

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	33	39	22	41	20	0
N.S.	1	1.00	1.00	0.95	1.50	1.77	1.00	1.86	0.91	0.00
time (sec)	N/A	0.010	0.003	0.001	0.353	0.353	0.958	0.003	0.021	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	23	23	22	25	22	23
N.S.	1	1.00	1.00	0.96	0.92	0.92	0.88	1.00	0.88	0.92
time (sec)	N/A	0.020	0.005	0.001	0.354	0.313	0.077	0.002	0.049	2.057

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	24	19	19	21	19	19
N.S.	1	1.00	1.00	0.87	1.04	0.83	0.83	0.91	0.83	0.83
time (sec)	N/A	0.030	0.010	0.079	0.372	0.316	0.131	0.001	0.220	2.067

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	57	49	45	78	30	63	0	-1	0
N.S.	1	1.50	1.29	1.18	2.05	0.79	1.66	0.00	-0.03	0.00
time (sec)	N/A	0.020	0.036	0.008	0.356	0.317	4.760	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	25	21	20
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	1.00	0.84	0.80
time (sec)	N/A	0.005	0.002	0.040	0.264	0.310	0.027	0.000	0.043	1.706

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	30	29	29	34	40	29	28
N.S.	1	1.00	1.00	0.77	0.74	0.74	0.87	1.03	0.74	0.72
time (sec)	N/A	0.009	0.001	0.084	0.271	0.321	0.030	0.000	0.032	1.874

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	29	23	27	75	20	23	112
N.S.	1	1.00	1.00	1.26	1.00	1.17	3.26	0.87	1.00	4.87
time (sec)	N/A	0.005	0.032	0.077	0.270	0.324	19.412	0.003	0.725	22.194

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	27	23	22	22	20	28	20	22
N.S.	1	1.00	0.90	0.77	0.73	0.73	0.67	0.93	0.67	0.73
time (sec)	N/A	0.007	0.005	0.052	0.269	0.318	0.036	0.000	0.035	1.870

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	35	27	26	32	34	43	32	26
N.S.	1	1.00	0.85	0.66	0.63	0.78	0.83	1.05	0.78	0.63
time (sec)	N/A	0.008	0.009	0.061	0.358	0.307	0.049	0.001	0.041	1.999

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	15	14	20	8	13
N.S.	1	1.00	1.00	0.76	0.71	0.71	0.67	0.95	0.38	0.62
time (sec)	N/A	0.004	0.002	0.081	0.267	0.321	0.049	0.000	0.089	1.870

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	29	28	28	36	33	30	28
N.S.	1	1.00	0.97	0.88	0.85	0.85	1.09	1.00	0.91	0.85
time (sec)	N/A	0.012	0.007	0.173	0.357	0.325	0.057	0.001	0.044	2.049

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	23	21	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.92	0.84	0.84
time (sec)	N/A	0.010	0.003	0.079	0.362	0.312	0.054	0.001	0.040	1.886

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	39	38	38	46	51	40	36
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.98	1.09	0.85	0.77
time (sec)	N/A	0.039	0.012	0.122	0.356	0.315	0.058	0.001	0.197	2.081

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	17	17	17	23	17	17
N.S.	1	1.00	1.00	0.72	0.68	0.68	0.68	0.92	0.68	0.68
time (sec)	N/A	0.022	0.004	0.015	0.269	0.317	0.065	0.001	0.227	1.883

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	30	25	25	26	34	25	25
N.S.	1	1.00	0.88	0.91	0.76	0.76	0.79	1.03	0.76	0.76
time (sec)	N/A	0.034	0.010	0.020	0.271	0.320	0.134	0.001	0.088	2.048

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	61	37	26	25	25	29	36	29	25
N.S.	1	1.65	1.00	0.70	0.68	0.68	0.78	0.97	0.78	0.68
time (sec)	N/A	0.022	0.005	0.019	0.314	0.334	0.091	0.001	0.061	2.032

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	69	26	43	51	60	73	25	49
N.S.	1	1.00	2.23	0.84	1.39	1.65	1.94	2.35	0.81	1.58
time (sec)	N/A	0.006	0.013	0.020	0.351	0.335	0.320	0.003	0.083	2.537

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	26	25	25	26	35	15	25
N.S.	1	1.00	1.00	0.63	0.61	0.61	0.63	0.85	0.37	0.61
time (sec)	N/A	0.014	0.005	0.082	0.269	0.354	0.086	0.001	0.246	2.041

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	16	20	22	29	17	22	22	29
N.S.	1	1.00	0.64	0.80	0.88	1.16	0.68	0.88	0.88	1.16
time (sec)	N/A	0.007	0.007	0.052	0.263	0.328	0.045	0.000	0.168	1.950

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	32	39	31	38	32	36
N.S.	1	1.00	1.00	0.92	0.89	1.08	0.86	1.06	0.89	1.00
time (sec)	N/A	0.011	0.010	0.046	0.264	0.319	0.041	0.000	0.029	2.039

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	37	28	30	45	31	35	30	39
N.S.	1	1.00	0.90	0.68	0.73	1.10	0.76	0.85	0.73	0.95
time (sec)	N/A	0.024	0.014	0.015	0.260	0.312	0.072	0.001	0.064	2.077

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	29	45	26	33	29	38
N.S.	1	1.00	1.00	1.04	1.07	1.67	0.96	1.22	1.07	1.41
time (sec)	N/A	0.022	0.012	0.056	0.258	0.311	0.069	0.001	0.191	1.998

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	47	32	29	45	32	40	31	37
N.S.	1	1.00	1.21	0.82	0.74	1.15	0.82	1.03	0.79	0.95
time (sec)	N/A	0.023	0.011	0.071	0.266	0.323	0.077	0.001	0.047	2.034

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	35	40	65	37	47	40	44
N.S.	1	1.00	0.87	0.76	0.87	1.41	0.80	1.02	0.87	0.96
time (sec)	N/A	0.015	0.010	0.017	0.261	0.318	0.080	0.000	0.183	2.189

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	23	23	22	27	29	23
N.S.	1	1.00	1.00	0.83	0.79	0.79	0.76	0.93	1.00	0.79
time (sec)	N/A	0.018	0.006	0.016	0.354	0.321	0.063	0.001	0.049	1.890

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	21	21	22	29	27	21
N.S.	1	1.00	1.00	0.81	0.78	0.78	0.81	1.07	1.00	0.78
time (sec)	N/A	0.015	0.005	0.066	0.349	0.341	0.093	0.001	0.049	1.932

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	32	26	25	25	29	38	17	25
N.S.	1	1.00	1.33	1.08	1.04	1.04	1.21	1.58	0.71	1.04
time (sec)	N/A	0.006	0.008	0.018	0.352	0.336	0.076	0.001	0.059	1.954

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	32	44	39	49	49	40
N.S.	1	1.00	1.00	0.80	0.78	1.07	0.95	1.20	1.20	0.98
time (sec)	N/A	0.051	0.016	0.020	0.354	0.323	0.081	0.001	0.252	1.983

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	37	36	57	37	47	38	42
N.S.	1	1.00	0.87	0.80	0.78	1.24	0.80	1.02	0.83	0.91
time (sec)	N/A	0.130	0.014	0.062	0.347	0.329	0.110	0.001	0.190	2.056

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	37	34	36	59	36	39	42	45
N.S.	1	1.00	0.79	0.72	0.77	1.26	0.77	0.83	0.89	0.96
time (sec)	N/A	0.026	0.018	0.063	0.342	0.322	0.085	0.001	0.043	2.046

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	73	54	53	53	70	67	47	53
N.S.	1	1.00	1.09	0.81	0.79	0.79	1.04	1.00	0.70	0.79
time (sec)	N/A	0.024	0.037	0.007	0.348	0.320	0.098	0.001	0.094	2.258

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	67	46	54	58	306	82	73	71
N.S.	1	1.00	1.40	0.96	1.12	1.21	6.38	1.71	1.52	1.48
time (sec)	N/A	0.029	0.012	0.076	0.351	0.315	0.322	0.001	0.167	4.944

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	46	41	43	66	46	51	44	53
N.S.	1	1.00	0.79	0.71	0.74	1.14	0.79	0.88	0.76	0.91
time (sec)	N/A	0.036	0.018	0.109	0.357	0.311	0.095	0.001	0.203	2.165

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	47	36	35	35	44	52	35	35
N.S.	1	1.00	0.92	0.71	0.69	0.69	0.86	1.02	0.69	0.69
time (sec)	N/A	0.172	0.016	0.083	0.376	0.338	0.209	0.001	0.242	2.162

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	34	33	33	32	42	33	33
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.78	1.02	0.80	0.80
time (sec)	N/A	0.184	0.006	0.073	0.266	0.326	0.092	0.001	0.056	2.015

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	52	51	49	45	73	59	64	54
N.S.	1	1.00	0.93	0.91	0.88	0.80	1.30	1.05	1.14	0.96
time (sec)	N/A	0.020	0.008	0.064	0.348	0.361	0.066	0.001	0.440	2.503

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	50	51	49	43	71	54	68	53
N.S.	1	1.00	0.89	0.91	0.88	0.77	1.27	0.96	1.21	0.95
time (sec)	N/A	0.018	0.004	0.046	0.348	0.325	0.061	0.001	0.118	2.672

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	12	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	1.00	0.83	0.83
time (sec)	N/A	0.002	0.002	0.046	0.263	0.307	0.048	0.000	0.030	1.826

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	34	23	18	19	29	18	17
N.S.	1	1.00	1.00	1.55	1.05	0.82	0.86	1.32	0.82	0.77
time (sec)	N/A	0.005	0.003	0.046	0.268	0.314	0.106	0.001	0.249	1.901

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	60	57	53	83	69	88	62
N.S.	1	1.00	0.95	0.95	0.90	0.84	1.32	1.10	1.40	0.98
time (sec)	N/A	0.022	0.009	0.053	0.373	0.334	0.083	0.001	0.249	2.916

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	68	60	57	62	80	72	86	67
N.S.	1	1.00	1.05	0.92	0.88	0.95	1.23	1.11	1.32	1.03
time (sec)	N/A	0.022	0.010	0.052	0.348	0.319	0.094	0.002	0.252	2.753

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	43	31	33	29	47	29	31
N.S.	1	1.00	1.00	1.30	0.94	1.00	0.88	1.42	0.88	0.94
time (sec)	N/A	0.012	0.004	0.049	0.258	0.361	0.134	0.001	0.073	2.346

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	74	66	66	68	90	81	99	76
N.S.	1	1.00	1.01	0.90	0.90	0.93	1.23	1.11	1.36	1.04
time (sec)	N/A	0.027	0.008	0.054	0.346	0.324	0.110	0.002	0.104	3.094

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	0	92	0	-1	30
N.S.	1	1.00	0.98	0.00	0.00	0.00	2.00	0.00	-0.02	0.65
time (sec)	N/A	0.007	0.089	0.023	0.000	0.330	1.253	0.000	0.000	3.987

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	38	33	32	26	37	40	18	34
N.S.	1	1.00	1.41	1.22	1.19	0.96	1.37	1.48	0.67	1.26
time (sec)	N/A	0.005	0.003	0.052	0.373	0.341	0.074	0.001	0.070	2.296

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	30	29	26	24	39	13	26
N.S.	1	1.00	1.00	2.00	1.93	1.73	1.60	2.60	0.87	1.73
time (sec)	N/A	0.004	0.003	0.126	0.274	0.331	0.077	0.001	0.048	1.978

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	41	25	20	19	30	20	19
N.S.	1	1.00	1.00	1.71	1.04	0.83	0.79	1.25	0.83	0.79
time (sec)	N/A	0.006	0.004	0.056	0.271	0.318	0.159	0.001	0.275	1.940

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	46	41	40	36	44	48	31	43
N.S.	1	1.00	1.31	1.17	1.14	1.03	1.26	1.37	0.89	1.23
time (sec)	N/A	0.009	0.005	0.062	0.351	0.323	0.099	0.001	0.223	2.304

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	50	43	37	41	34	44	22	39
N.S.	1	1.00	1.92	1.65	1.42	1.58	1.31	1.69	0.85	1.50
time (sec)	N/A	0.007	0.005	0.052	0.265	0.317	0.110	0.001	0.200	2.163

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	48	41	40	45	48	50	31	48
N.S.	1	1.00	1.30	1.11	1.08	1.22	1.30	1.35	0.84	1.30
time (sec)	N/A	0.008	0.005	0.053	0.352	0.321	0.111	0.001	0.080	2.242

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	0	0	0	95	0	-1	30
N.S.	1	1.00	0.98	0.00	0.00	0.00	2.11	0.00	-0.02	0.67
time (sec)	N/A	0.006	0.143	0.026	0.000	0.325	0.471	0.000	0.000	3.018

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	29	17	13	28
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.93	1.13	0.87	1.87
time (sec)	N/A	0.004	0.003	0.055	0.365	0.304	0.071	0.001	0.185	2.003

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	79	85	98	199	19	166	33	28
N.S.	1	1.00	0.72	0.78	0.90	1.83	0.17	1.52	0.30	0.26
time (sec)	N/A	0.039	0.018	0.045	0.360	0.340	0.069	0.002	0.100	2.049

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	F(-1)	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	204	101	180	0	39	236	174	49
N.S.	1	1.00	1.01	0.50	0.90	0.00	0.19	1.17	0.87	0.24
time (sec)	N/A	0.242	0.094	0.053	0.353	0.000	0.070	0.003	0.588	2.288

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	F(-1)	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	204	97	160	0	41	235	182	51
N.S.	1	1.00	1.01	0.48	0.80	0.00	0.20	1.17	0.91	0.25
time (sec)	N/A	0.181	0.031	0.049	0.358	0.000	0.067	0.009	0.591	2.458

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	F(-1)	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	204	101	160	0	41	235	202	51
N.S.	1	1.00	1.01	0.50	0.80	0.00	0.20	1.17	1.00	0.25
time (sec)	N/A	0.221	0.039	0.049	0.359	0.000	0.076	0.010	0.797	2.435

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	F(-1)	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	204	97	180	0	39	224	202	51
N.S.	1	1.00	1.01	0.48	0.90	0.00	0.19	1.11	1.00	0.25
time (sec)	N/A	0.218	0.023	0.052	0.349	0.000	0.072	0.011	0.410	2.355

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	12	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	1.00	0.83	0.83
time (sec)	N/A	0.002	0.002	0.049	0.261	0.318	0.054	0.001	0.184	2.469

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	49	23	18	19	29	18	17
N.S.	1	1.00	1.00	2.23	1.05	0.82	0.86	1.32	0.82	0.77
time (sec)	N/A	0.006	0.003	0.141	0.307	0.323	0.134	0.003	0.268	1.925

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	F(-1)	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	172	109	192	0	48	241	210	61
N.S.	1	1.00	0.82	0.52	0.92	0.00	0.23	1.15	1.00	0.29
time (sec)	N/A	0.226	0.091	0.052	0.344	0.000	0.091	0.010	0.394	2.557

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	F(-1)	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	174	105	173	0	51	245	210	62
N.S.	1	1.00	0.82	0.50	0.82	0.00	0.24	1.16	1.00	0.29
time (sec)	N/A	0.224	0.088	0.056	0.348	0.000	0.103	0.011	0.782	2.548

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	F(-1)	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	175	109	172	0	51	245	214	62
N.S.	1	1.00	0.83	0.52	0.82	0.00	0.24	1.16	1.01	0.29
time (sec)	N/A	0.195	0.070	0.051	0.352	0.000	0.103	0.010	0.692	2.766

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	0	92	0	-1	30
N.S.	1	1.00	0.98	0.00	0.00	0.00	2.00	0.00	-0.02	0.65
time (sec)	N/A	0.007	0.164	0.026	0.000	0.314	8.466	0.000	0.000	11.343

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	21	28	27	9	8	42	9	9
N.S.	1	1.00	0.60	0.80	0.77	0.26	0.23	1.20	0.26	0.26
time (sec)	N/A	0.285	0.005	0.056	0.368	0.307	0.062	0.002	0.034	1.737

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	52	54	71	63	53	45	71
N.S.	1	1.00	0.85	0.87	0.90	1.18	1.05	0.88	0.75	1.18
time (sec)	N/A	0.015	0.018	0.180	0.349	0.429	0.081	0.001	0.079	4.076

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	30	28	38	52	36	32	27	53
N.S.	1	1.00	0.70	0.65	0.88	1.21	0.84	0.74	0.63	1.23
time (sec)	N/A	0.008	0.009	0.051	0.344	0.306	0.069	0.001	0.182	2.044

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	88	103	0	448	323	95	159	377
N.S.	1	1.00	0.98	1.14	0.00	4.98	3.59	1.06	1.77	4.19
time (sec)	N/A	0.047	0.054	0.151	0.000	0.436	0.565	0.003	0.287	9.110

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	33	50	31	41	41	44
N.S.	1	1.00	1.00	0.92	0.87	1.32	0.82	1.08	1.08	1.16
time (sec)	N/A	0.017	0.010	0.086	0.353	0.306	0.069	0.001	0.202	2.749

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	53	42	58	53	54	60	49
N.S.	1	1.00	0.86	0.93	0.74	1.02	0.93	0.95	1.05	0.86
time (sec)	N/A	0.016	0.015	0.058	0.418	0.410	0.078	0.000	0.082	2.114

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	29	31	40	32	34	29	44
N.S.	1	1.00	0.92	0.81	0.86	1.11	0.89	0.94	0.81	1.22
time (sec)	N/A	0.014	0.011	0.049	0.353	0.306	0.070	0.001	0.180	1.960

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	78	41	40	40	46	51	52	40
N.S.	1	1.00	1.59	0.84	0.82	0.82	0.94	1.04	1.06	0.82
time (sec)	N/A	0.024	0.010	0.052	0.390	0.455	0.071	0.001	0.086	2.075

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	20	13	14	14	13	14	0	14	27
N.S.	1	1.54	1.00	1.08	1.08	1.00	1.08	0.00	1.08	2.08
time (sec)	N/A	0.028	0.017	0.072	0.254	0.311	0.825	0.000	0.322	2.671

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	38	35	34	34	42	45	34	32
N.S.	1	1.00	0.93	0.85	0.83	0.83	1.02	1.10	0.83	0.78
time (sec)	N/A	0.025	0.008	0.016	0.344	0.317	0.061	0.001	0.202	2.035

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	25	24	24	24	29	22	22
N.S.	1	1.00	1.00	0.78	0.75	0.75	0.75	0.91	0.69	0.69
time (sec)	N/A	0.014	0.004	0.016	0.336	0.472	0.060	0.001	0.086	1.911

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	17	17	15	25	16	17
N.S.	1	1.00	1.00	0.72	0.68	0.68	0.60	1.00	0.64	0.68
time (sec)	N/A	0.010	0.003	0.016	0.258	0.311	0.053	0.000	0.584	1.866

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	27	32	46	29	27	22	44
N.S.	1	1.00	0.67	0.75	0.89	1.28	0.81	0.75	0.61	1.22
time (sec)	N/A	0.014	0.009	0.057	0.267	0.306	0.057	0.001	0.039	1.997

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	24	30	72	72	70	26	29	72
N.S.	1	1.00	0.53	0.67	1.60	1.60	1.56	0.58	0.64	1.60
time (sec)	N/A	0.008	0.004	0.057	0.262	0.353	0.080	0.001	0.099	2.284

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	41	46	48	67	42	41	43	67
N.S.	1	1.00	0.75	0.84	0.87	1.22	0.76	0.75	0.78	1.22
time (sec)	N/A	0.025	0.008	0.051	0.269	0.444	0.069	0.001	0.048	3.256

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	38	59	41	45	31	49
N.S.	1	1.00	1.06	0.97	1.06	1.64	1.14	1.25	0.86	1.36
time (sec)	N/A	0.011	0.017	0.133	0.255	0.308	0.069	0.001	0.039	2.100

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	28	31	48	29	39	34	34
N.S.	1	1.00	0.89	0.80	0.89	1.37	0.83	1.11	0.97	0.97
time (sec)	N/A	0.010	0.011	0.076	0.254	0.309	0.066	0.001	0.220	1.972

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	46	42	50	85	51	51	45	68
N.S.	1	1.00	0.75	0.69	0.82	1.39	0.84	0.84	0.74	1.11
time (sec)	N/A	0.008	0.016	0.073	0.257	0.341	0.081	0.001	0.123	2.196

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	36	44	44	62	42	40	39	62
N.S.	1	1.00	0.71	0.86	0.86	1.22	0.82	0.78	0.76	1.22
time (sec)	N/A	0.010	0.012	0.125	0.361	0.355	0.079	0.001	0.203	2.140

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	49	60	105	58	52	55	83
N.S.	1	1.00	1.00	0.91	1.11	1.94	1.07	0.96	1.02	1.54
time (sec)	N/A	0.017	0.015	0.052	0.261	0.314	0.112	0.001	0.051	2.272

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	42	53	32	40	53	49	58	33	52
N.S.	1	1.17	1.47	0.89	1.11	1.47	1.36	1.61	0.92	1.44
time (sec)	N/A	0.009	0.022	0.084	0.385	0.362	0.056	0.001	0.252	1.997

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	32	41	59	39	36	40	61
N.S.	1	1.00	0.69	0.55	0.71	1.02	0.67	0.62	0.69	1.05
time (sec)	N/A	0.010	0.013	0.050	0.343	0.308	0.073	0.001	0.074	3.030

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	37	36	45	42	46	36	45
N.S.	1	1.00	1.00	0.86	0.84	1.05	0.98	1.07	0.84	1.05
time (sec)	N/A	0.013	0.022	0.186	0.356	0.370	0.071	0.001	0.191	2.065

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	62	37	47	68	58	63	34	55
N.S.	1	1.00	1.44	0.86	1.09	1.58	1.35	1.47	0.79	1.28
time (sec)	N/A	0.016	0.025	0.110	0.348	0.355	0.076	0.003	0.215	2.251

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	28	30	38	24	43	32	38
N.S.	1	1.00	0.68	0.76	0.81	1.03	0.65	1.16	0.86	1.03
time (sec)	N/A	0.009	0.008	0.049	0.343	0.309	0.079	0.003	0.037	1.883

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	39	26	29	28	38	26	34	28	38
N.S.	1	1.22	0.81	0.91	0.88	1.19	0.81	1.06	0.88	1.19
time (sec)	N/A	0.017	0.010	0.045	0.274	0.429	0.068	0.001	0.189	1.938

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	19	20	14	11	23
N.S.	1	1.00	1.00	0.92	0.85	1.46	1.54	1.08	0.85	1.77
time (sec)	N/A	0.002	0.003	0.046	0.259	0.380	0.132	0.001	0.199	1.875

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	52	57	81	51	63	52	70
N.S.	1	1.00	0.85	0.96	1.06	1.50	0.94	1.17	0.96	1.30
time (sec)	N/A	0.021	0.019	0.056	0.266	0.317	0.232	0.005	0.106	2.272

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	134	124	147	338	66	215	76	0
N.S.	1	1.00	0.85	0.79	0.94	2.15	0.42	1.37	0.48	0.00
time (sec)	N/A	0.064	0.067	0.053	0.350	0.328	0.212	0.007	0.114	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	75	53	60	78	78	65	53	93
N.S.	1	1.00	1.17	0.83	0.94	1.22	1.22	1.02	0.83	1.45
time (sec)	N/A	0.019	0.032	0.056	0.347	0.312	0.230	0.005	0.222	6.021

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	34	34	45	27	39	29	45
N.S.	1	1.00	0.85	0.87	0.87	1.15	0.69	1.00	0.74	1.15
time (sec)	N/A	0.016	0.009	0.051	0.343	0.308	0.082	0.015	0.050	3.432

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	C	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	293	354	335	1751	37	481	285	65
N.S.	1	1.00	0.92	1.11	1.05	5.49	0.12	1.51	0.89	0.20
time (sec)	N/A	0.379	0.190	0.104	0.354	3.219	0.144	0.154	1.604	2.248

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	65	39	56	73	58	73	41	68
N.S.	1	1.00	1.10	0.66	0.95	1.24	0.98	1.24	0.69	1.15
time (sec)	N/A	0.015	0.045	0.062	0.348	0.306	0.082	0.001	0.243	2.254

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	29	31	40	32	34	29	44
N.S.	1	1.00	0.92	0.81	0.86	1.11	0.89	0.94	0.81	1.22
time (sec)	N/A	0.016	0.004	0.045	0.342	0.313	0.070	0.001	0.002	2.052

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	31	38	65	36	40	33	51
N.S.	1	1.00	0.84	0.82	1.00	1.71	0.95	1.05	0.87	1.34
time (sec)	N/A	0.038	0.016	0.031	0.258	0.313	0.069	0.001	0.065	2.139

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	61	50	73	60	68	63	60
N.S.	1	1.00	1.02	0.95	0.78	1.14	0.94	1.06	0.98	0.94
time (sec)	N/A	0.046	0.022	0.062	0.395	0.317	0.114	0.001	0.105	2.228

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	54	59	97	68	70	73	73
N.S.	1	1.00	1.00	0.86	0.94	1.54	1.08	1.11	1.16	1.16
time (sec)	N/A	0.068	0.025	0.108	0.344	0.314	0.101	0.001	0.254	2.253

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	36	36	55	34	43	25	42
N.S.	1	1.00	1.07	0.84	0.84	1.28	0.79	1.00	0.58	0.98
time (sec)	N/A	0.013	0.012	0.061	0.260	0.307	0.072	0.000	0.054	4.275

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	20	24	43	43	39	22	18	43
N.S.	1	1.00	0.74	0.89	1.59	1.59	1.44	0.81	0.67	1.59
time (sec)	N/A	0.012	0.006	0.056	0.263	0.308	0.089	0.000	0.112	2.083

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	39	39	55	36	47	34	52
N.S.	1	1.00	0.96	0.85	0.85	1.20	0.78	1.02	0.74	1.13
time (sec)	N/A	0.019	0.010	0.051	0.263	0.358	0.064	0.001	0.187	2.045

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	50	39	35	37	56	34	43	33	47
N.S.	1	1.14	0.89	0.80	0.84	1.27	0.77	0.98	0.75	1.07
time (sec)	N/A	0.011	0.015	0.068	0.256	0.307	0.071	0.000	0.048	2.102

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	49	60	105	58	52	55	83
N.S.	1	1.00	1.00	0.91	1.11	1.94	1.07	0.96	1.02	1.54
time (sec)	N/A	0.013	0.002	0.000	0.276	0.307	0.084	0.001	0.002	2.456

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	32	39	31	38	32	36
N.S.	1	1.00	1.00	0.92	0.89	1.08	0.86	1.06	0.89	1.00
time (sec)	N/A	0.011	0.009	0.043	0.254	0.307	0.041	0.000	0.002	1.851

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	39	38	38	39	47	37	37
N.S.	1	1.00	0.93	0.89	0.86	0.86	0.89	1.07	0.84	0.84
time (sec)	N/A	0.025	0.009	0.030	0.260	0.307	0.030	0.000	0.049	1.844

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	91	95	91	91	100	114	88	87
N.S.	1	1.00	0.95	0.99	0.95	0.95	1.04	1.19	0.92	0.91
time (sec)	N/A	0.069	0.019	0.098	0.258	0.302	0.038	0.001	0.190	2.384

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	167	237	171	171	189	208	164	154
N.S.	1	1.00	1.00	1.42	1.02	1.02	1.13	1.25	0.98	0.92
time (sec)	N/A	0.123	0.021	0.082	0.277	0.321	0.047	0.002	0.216	2.802

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	263	363	273	273	313	343	263	247
N.S.	1	1.00	1.00	1.38	1.04	1.04	1.19	1.30	1.00	0.94
time (sec)	N/A	0.219	0.040	0.085	0.262	0.299	0.058	0.001	0.258	7.522

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	267	0	0	0	0	0	-1	0
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.058	0.434	0.061	0.000	0.315	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	66	63	0	203	246	66	155	100
N.S.	1	1.00	1.02	0.97	0.00	3.12	3.78	1.02	2.38	1.54
time (sec)	N/A	0.028	0.028	0.135	0.000	0.321	0.425	0.002	0.271	6.205

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	88	103	0	447	323	95	159	377
N.S.	1	1.00	0.99	1.16	0.00	5.02	3.63	1.07	1.79	4.24
time (sec)	N/A	0.030	0.053	0.102	0.000	0.324	0.560	0.002	0.283	8.918

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	127	155	0	1104	622	212	360	681
N.S.	1	1.00	0.98	1.19	0.00	8.49	4.78	1.63	2.77	5.24
time (sec)	N/A	0.047	0.090	0.109	0.000	0.332	1.033	0.003	0.505	31.650

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	168	206	0	1950	1027	387	640	1215
N.S.	1	1.00	0.97	1.19	0.00	11.27	5.94	2.24	3.70	7.02
time (sec)	N/A	0.072	0.140	0.115	0.000	0.352	1.715	0.004	0.706	33.003

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	264	0	0	0	0	0	-1	0
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.051	0.517	0.059	0.000	0.317	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	31	41	52	46	57	36	35
N.S.	1	1.00	0.90	0.63	0.84	1.06	0.94	1.16	0.73	0.71
time (sec)	N/A	0.012	0.015	0.080	0.357	0.364	0.056	0.000	0.171	2.650

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	70	56	67	97	76	72	53	85
N.S.	1	1.00	1.15	0.92	1.10	1.59	1.25	1.18	0.87	1.39
time (sec)	N/A	0.014	0.032	0.069	0.342	0.335	0.088	0.003	0.209	2.273

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	28	30	45	31	36	25	38
N.S.	1	1.00	0.92	0.78	0.83	1.25	0.86	1.00	0.69	1.06
time (sec)	N/A	0.005	0.012	0.069	0.251	0.322	0.060	0.001	0.051	2.001

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	70	90	165	88	65	65	128
N.S.	1	1.00	1.00	0.80	1.03	1.90	1.01	0.75	0.75	1.47
time (sec)	N/A	0.015	0.023	0.066	0.260	0.322	0.102	0.000	0.094	2.453

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	62	69	116	80	85	77	79
N.S.	1	1.00	0.86	0.77	0.85	1.43	0.99	1.05	0.95	0.98
time (sec)	N/A	0.038	0.023	0.188	0.357	0.312	0.115	0.001	0.109	2.367

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	87	70	90	165	90	74	90	130
N.S.	1	1.00	0.84	0.67	0.87	1.59	0.87	0.71	0.87	1.25
time (sec)	N/A	0.047	0.021	0.068	0.258	0.300	0.103	0.001	0.213	2.702

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	99	80	94	173	90	69	85	132
N.S.	1	1.00	0.97	0.78	0.92	1.70	0.88	0.68	0.83	1.29
time (sec)	N/A	0.023	0.037	0.076	0.260	0.300	0.116	0.002	0.213	3.531

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	39	39	37	53	37	34
N.S.	1	1.00	1.00	0.88	0.98	0.98	0.92	1.32	0.92	0.85
time (sec)	N/A	0.015	0.004	0.057	0.257	0.336	0.124	0.001	0.204	2.053

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	62	56	83	113	102	71	79	130
N.S.	1	1.00	0.75	0.67	1.00	1.36	1.23	0.86	0.95	1.57
time (sec)	N/A	0.025	0.015	0.071	0.356	0.320	0.366	0.002	0.146	12.580

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	43	32	31	37	42	65	31	31
N.S.	1	1.00	0.88	0.65	0.63	0.76	0.86	1.33	0.63	0.63
time (sec)	N/A	0.034	0.016	0.066	0.256	0.381	0.317	0.001	0.190	2.651

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	32	31	31	48	120	31	31
N.S.	1	1.00	1.00	0.58	0.56	0.56	0.87	2.18	0.56	0.56
time (sec)	N/A	0.148	0.111	0.068	0.256	0.301	0.874	0.005	0.066	4.348

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	31	18	18	19	21	18	18
N.S.	1	1.00	1.00	1.41	0.82	0.82	0.86	0.95	0.82	0.82
time (sec)	N/A	0.006	0.010	0.039	0.263	0.303	0.061	0.000	0.134	7.996

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	18	13	12	11	22	23	11	19
N.S.	1	1.00	1.20	0.87	0.80	0.73	1.47	1.53	0.73	1.27
time (sec)	N/A	0.004	0.010	0.012	0.359	0.297	0.463	0.001	0.030	7.060

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	21	20	19	20	25	19	19
N.S.	1	1.00	0.96	0.84	0.80	0.76	0.80	1.00	0.76	0.76
time (sec)	N/A	0.011	0.013	0.048	0.262	0.300	0.069	0.001	0.219	4.883

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	111	25	25	0	31	25	0
N.S.	1	1.00	0.94	3.36	0.76	0.76	0.00	0.94	0.76	0.00
time (sec)	N/A	0.013	0.311	0.028	0.258	0.312	0.000	0.002	0.245	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	20	19	19	134	46	16	64
N.S.	1	1.00	0.83	0.69	0.66	0.66	4.62	1.59	0.55	2.21
time (sec)	N/A	0.006	0.018	0.054	0.261	0.317	0.665	0.002	0.543	3.152

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	36	73	55	63	3966	71	43	1410
N.S.	1	1.00	0.69	1.40	1.06	1.21	76.27	1.37	0.83	27.12
time (sec)	N/A	0.006	0.037	0.069	0.273	0.317	1.577	0.001	0.048	28.731

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	70	157	111	125	0	139	96	0
N.S.	1	1.00	0.59	1.33	0.94	1.06	0.00	1.18	0.81	0.00
time (sec)	N/A	0.018	0.062	0.098	0.319	0.299	0.000	0.002	0.210	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F(-1)	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	90	158	105	86	0	130	120	95
N.S.	1	1.00	0.87	1.52	1.01	0.83	0.00	1.25	1.15	0.91
time (sec)	N/A	0.026	0.150	0.313	0.343	0.306	0.000	0.002	0.212	155.098

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	66	39	43	32	0	31	43	0
N.S.	1	1.00	1.74	1.03	1.13	0.84	0.00	0.82	1.13	0.00
time (sec)	N/A	0.009	0.065	0.080	0.337	0.303	0.000	0.005	0.194	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	92	95	99	196	130	73	0	113	140	0
N.S.	1	1.03	1.08	2.13	1.41	0.79	0.00	1.23	1.52	0.00
time (sec)	N/A	0.036	0.164	0.051	0.354	0.381	0.000	0.015	0.345	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	47	64	0	65	0	101	95	0
N.S.	1	1.00	0.87	1.19	0.00	1.20	0.00	1.87	1.76	0.00
time (sec)	N/A	0.053	0.068	0.092	0.000	0.384	0.000	0.009	0.609	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	304	319	195	0	0	577	0	0	-1	0
N.S.	1	1.05	0.64	0.00	0.00	1.90	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.492	15.863	0.043	0.000	0.376	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	B	F(-2)	F	F	F(-2)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	292	522	455	0	0	865	0	0	-1	0
N.S.	1	1.79	1.56	0.00	0.00	2.96	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.179	29.551	0.033	0.000	0.370	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	29	25	22	0	47	0	0	25	0
N.S.	1	1.16	1.00	0.88	0.00	1.88	0.00	0.00	1.00	0.00
time (sec)	N/A	0.010	0.021	0.023	0.000	0.301	0.000	0.000	0.236	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	30	25	22	0	69	0	0	25	0
N.S.	1	1.20	1.00	0.88	0.00	2.76	0.00	0.00	1.00	0.00
time (sec)	N/A	0.016	0.027	0.030	0.000	0.302	0.000	0.000	0.260	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	63	30	27	0	77	0	0	30	0
N.S.	1	1.19	0.57	0.51	0.00	1.45	0.00	0.00	0.57	0.00
time (sec)	N/A	0.014	0.062	0.025	0.000	0.301	0.000	0.000	0.227	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	B	A	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	188	110	370	0	128	0	0	-1	0
N.S.	1	2.81	1.64	5.52	0.00	1.91	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.078	0.068	0.254	0.000	0.312	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	122	133	96	118	0	142	0	89	-1	0
N.S.	1	1.09	0.79	0.97	0.00	1.16	0.00	0.73	-0.01	0.00
time (sec)	N/A	0.210	0.069	0.125	0.000	0.332	0.000	0.034	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	B	A	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	150	404	229	1247	0	280	0	0	-1	0
N.S.	1	2.69	1.53	8.31	0.00	1.87	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.207	0.330	2.970	0.000	0.337	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	39	36	51	64	0	48	27	0
N.S.	1	1.00	0.91	0.84	1.19	1.49	0.00	1.12	0.63	0.00
time (sec)	N/A	0.004	0.148	0.079	0.252	0.313	0.000	0.003	0.050	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	45	0	62	0	41	-1	0
N.S.	1	1.00	0.88	1.07	0.00	1.48	0.00	0.98	-0.02	0.00
time (sec)	N/A	0.027	0.018	0.053	0.000	0.310	0.000	0.003	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	58	144	0	138	0	97	-1	0
N.S.	1	1.00	0.42	1.04	0.00	0.99	0.00	0.70	-0.01	0.00
time (sec)	N/A	0.076	0.043	0.058	0.000	0.317	0.000	0.008	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	B	A	C	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	75	188	110	670	0	128	0	0	-1	0
N.S.	1	2.51	1.47	8.93	0.00	1.71	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.076	0.068	0.245	0.000	0.316	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	23	20	0	22	0	0	24	0
N.S.	1	1.00	0.79	0.69	0.00	0.76	0.00	0.00	0.83	0.00
time (sec)	N/A	0.021	0.020	0.015	0.000	0.306	0.000	0.000	0.052	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	32	29	0	44	0	0	37	0
N.S.	1	1.00	0.35	0.32	0.00	0.48	0.00	0.00	0.40	0.00
time (sec)	N/A	0.055	0.038	0.021	0.000	0.309	0.000	0.000	0.080	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	32	15	16	33	0	21	16	0
N.S.	1	1.00	1.68	0.79	0.84	1.74	0.00	1.11	0.84	0.00
time (sec)	N/A	0.010	0.061	0.135	0.357	0.318	0.000	0.002	0.197	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	23	5	8	29	0	4	4	0
N.S.	1	1.00	2.88	0.62	1.00	3.62	0.00	0.50	0.50	0.00
time (sec)	N/A	0.004	0.051	0.108	0.338	0.401	0.000	0.001	0.184	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	33	12	11	40	0	14	11	0
N.S.	1	1.00	2.75	1.00	0.92	3.33	0.00	1.17	0.92	0.00
time (sec)	N/A	0.003	0.059	0.115	0.352	0.303	0.000	0.002	0.222	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	F	A	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	42	29	0	23	0	70	79	0
N.S.	1	1.00	1.35	0.94	0.00	0.74	0.00	2.26	2.55	0.00
time (sec)	N/A	0.005	0.081	0.090	0.000	0.326	0.000	0.004	0.516	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	40	22	0	54	0	72	61	0
N.S.	1	1.00	1.29	0.71	0.00	1.74	0.00	2.32	1.97	0.00
time (sec)	N/A	0.007	0.081	0.089	0.000	0.322	0.000	0.003	0.485	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	100	112	48	61	54	78	63
N.S.	1	1.00	1.00	4.17	4.67	2.00	2.54	2.25	3.25	2.62
time (sec)	N/A	0.012	0.032	0.125	0.353	0.319	2.314	0.001	0.789	5.620

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	100	101	32	24	27	83	20
N.S.	1	1.00	1.00	4.00	4.04	1.28	0.96	1.08	3.32	0.80
time (sec)	N/A	0.012	0.022	0.132	0.354	0.314	2.293	0.001	0.760	3.965

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	57	70	0	72	0	91	107	0
N.S.	1	1.00	1.33	1.63	0.00	1.67	0.00	2.12	2.49	0.00
time (sec)	N/A	0.013	0.097	0.148	0.000	0.307	0.000	0.006	0.114	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	49	54	74	0	136	-1	0
N.S.	1	1.00	1.00	0.79	0.87	1.19	0.00	2.19	-0.02	0.00
time (sec)	N/A	0.023	0.141	0.194	0.358	0.362	0.000	0.010	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	90	69	0	151	0	201	-1	0
N.S.	1	1.00	1.10	0.84	0.00	1.84	0.00	2.45	-0.01	0.00
time (sec)	N/A	0.075	0.223	0.359	0.000	0.321	0.000	0.011	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	F	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	102	53	0	307	0	292	-1	0
N.S.	1	1.00	1.62	0.84	0.00	4.87	0.00	4.63	-0.02	0.00
time (sec)	N/A	0.035	0.125	0.486	0.000	0.337	0.000	0.025	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	B	F	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	103	128	0	303	0	259	-1	0
N.S.	1	1.00	1.84	2.29	0.00	5.41	0.00	4.62	-0.02	0.00
time (sec)	N/A	0.027	0.116	0.446	0.000	0.330	0.000	0.013	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	B	F	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	123	158	0	311	0	545	-1	0
N.S.	1	1.00	1.76	2.26	0.00	4.44	0.00	7.79	-0.01	0.00
time (sec)	N/A	0.041	0.168	0.632	0.000	0.321	0.000	0.017	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	B	F	F(-1)	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	80	89	124	192	0	0	0	1186	-1	0
N.S.	1	1.11	1.55	2.40	0.00	0.00	0.00	14.82	-0.01	0.00
time (sec)	N/A	0.078	0.830	0.214	0.000	0.000	0.000	0.098	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	F	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	85	94	0	81	0	293	-1	0
N.S.	1	1.00	2.24	2.47	0.00	2.13	0.00	7.71	-0.03	0.00
time (sec)	N/A	0.015	0.362	0.179	0.000	0.300	0.000	6.001	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	47	49	48	42	153	45	35	117
N.S.	1	1.00	0.72	0.75	0.74	0.65	2.35	0.69	0.54	1.80
time (sec)	N/A	0.011	0.041	0.083	0.347	0.297	4.495	0.003	0.028	6.500

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	28	38	37	31	41	59	25	28
N.S.	1	1.00	0.57	0.78	0.76	0.63	0.84	1.20	0.51	0.57
time (sec)	N/A	0.007	0.031	0.059	0.352	0.311	1.457	0.003	0.027	3.159

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	28	38	37	44	139	41	187	38
N.S.	1	1.00	0.57	0.78	0.76	0.90	2.84	0.84	3.82	0.78
time (sec)	N/A	0.005	0.037	0.052	0.251	0.306	3.578	0.004	0.048	6.846

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	328	10	62	53	13	154	42
N.S.	1	1.00	1.00	27.33	0.83	5.17	4.42	1.08	12.83	3.50
time (sec)	N/A	0.029	0.013	0.034	0.256	0.305	1.207	0.001	0.270	3.338

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	22	11	10	37	29	31	24	10
N.S.	1	1.00	1.83	0.92	0.83	3.08	2.42	2.58	2.00	0.83
time (sec)	N/A	0.008	0.102	0.055	0.333	0.301	5.504	0.003	0.195	7.553

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	44	23	0	67	0	83	77	0
N.S.	1	1.00	1.63	0.85	0.00	2.48	0.00	3.07	2.85	0.00
time (sec)	N/A	0.007	0.074	0.104	0.000	0.307	0.000	0.003	0.173	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	55	46	0	83	0	122	117	0
N.S.	1	1.00	1.15	0.96	0.00	1.73	0.00	2.54	2.44	0.00
time (sec)	N/A	0.008	0.090	0.112	0.000	0.301	0.000	0.005	0.093	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	50	100	107	77	0	89	-1	0
N.S.	1	1.00	1.22	2.44	2.61	1.88	0.00	2.17	-0.02	0.00
time (sec)	N/A	0.014	0.121	0.163	0.354	0.295	0.000	0.008	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	70	0	50	0	158	115	0
N.S.	1	1.00	1.00	1.49	0.00	1.06	0.00	3.36	2.45	0.00
time (sec)	N/A	0.013	0.143	0.127	0.000	0.301	0.000	0.007	0.133	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	108	82	0	160	0	185	159	0
N.S.	1	1.00	1.23	0.93	0.00	1.82	0.00	2.10	1.81	0.00
time (sec)	N/A	0.147	0.208	0.442	0.000	0.330	0.000	0.020	0.382	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	104	656	0	170	0	237	216	0
N.S.	1	1.00	0.76	4.82	0.00	1.25	0.00	1.74	1.59	0.00
time (sec)	N/A	0.946	0.332	0.030	0.000	0.312	0.000	0.016	0.615	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	66	64	63	51	0	54	56	0
N.S.	1	1.00	1.03	1.00	0.98	0.80	0.00	0.84	0.88	0.00
time (sec)	N/A	0.013	0.085	0.069	0.426	0.318	0.000	0.005	0.100	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	74	83	81	60	0	67	75	0
N.S.	1	1.00	0.83	0.93	0.91	0.67	0.00	0.75	0.84	0.00
time (sec)	N/A	0.019	0.089	0.069	0.367	0.309	0.000	0.006	0.325	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	82	104	101	68	0	78	96	0
N.S.	1	1.00	0.73	0.92	0.89	0.60	0.00	0.69	0.85	0.00
time (sec)	N/A	0.026	0.099	0.069	0.346	0.309	0.000	0.006	0.137	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	78	42	54	62	0	88	-1	0
N.S.	1	1.00	1.59	0.86	1.10	1.27	0.00	1.80	-0.02	0.00
time (sec)	N/A	0.018	0.115	0.153	0.345	0.322	0.000	0.008	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	78	74	66	64	0	192	-1	0
N.S.	1	1.00	0.99	0.94	0.84	0.81	0.00	2.43	-0.01	0.00
time (sec)	N/A	0.027	0.201	0.111	0.350	0.310	0.000	0.005	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	20	10	11	18	0	19	12	0
N.S.	1	1.00	1.67	0.83	0.92	1.50	0.00	1.58	1.00	0.00
time (sec)	N/A	0.006	0.045	0.128	0.430	0.303	0.000	0.003	0.240	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	47	47	48	39	0	51	-1	0
N.S.	1	1.00	0.89	0.89	0.91	0.74	0.00	0.96	-0.02	0.00
time (sec)	N/A	0.015	0.065	0.119	0.355	0.322	0.000	0.005	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	22	34	0	27	13	0
N.S.	1	1.00	1.00	0.84	1.16	1.79	0.00	1.42	0.68	0.00
time (sec)	N/A	0.001	0.079	0.105	0.257	0.324	0.000	0.007	0.032	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	27	22	28	0	29	15	0
N.S.	1	1.00	1.00	1.59	1.29	1.65	0.00	1.71	0.88	0.00
time (sec)	N/A	0.002	0.077	0.107	0.254	0.300	0.000	0.003	0.024	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	47	61	47	64	0	58	-1	0
N.S.	1	1.00	0.84	1.09	0.84	1.14	0.00	1.04	-0.02	0.00
time (sec)	N/A	0.015	0.099	0.120	0.354	0.310	0.000	0.004	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	52	49	56	44	0	57	61	0
N.S.	1	1.00	0.80	0.75	0.86	0.68	0.00	0.88	0.94	0.00
time (sec)	N/A	0.015	0.060	0.118	0.411	0.321	0.000	0.005	0.131	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	52	43	56	44	0	59	43	0
N.S.	1	1.00	0.95	0.78	1.02	0.80	0.00	1.07	0.78	0.00
time (sec)	N/A	0.009	0.099	0.111	0.345	0.313	0.000	0.005	0.213	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	62	58	77	54	0	73	56	0
N.S.	1	1.00	0.84	0.78	1.04	0.73	0.00	0.99	0.76	0.00
time (sec)	N/A	0.012	0.166	0.121	0.346	0.309	0.000	0.005	0.075	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	31	37	52	0	77	31	0
N.S.	1	1.00	0.87	0.82	0.97	1.37	0.00	2.03	0.82	0.00
time (sec)	N/A	0.008	0.063	0.124	0.345	0.308	0.000	0.004	0.028	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	42	44	50	63	0	98	-1	0
N.S.	1	1.00	0.74	0.77	0.88	1.11	0.00	1.72	-0.02	0.00
time (sec)	N/A	0.014	0.075	0.122	0.343	0.304	0.000	0.006	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	45	56	58	94	0	109	-1	0
N.S.	1	1.00	0.73	0.90	0.94	1.52	0.00	1.76	-0.02	0.00
time (sec)	N/A	0.015	0.121	0.122	0.413	0.301	0.000	0.005	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	52	69	71	107	0	147	-1	0
N.S.	1	1.00	0.66	0.87	0.90	1.35	0.00	1.86	-0.01	0.00
time (sec)	N/A	0.022	0.129	0.125	0.346	0.303	0.000	0.006	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	22	25	30	0	41	-1	0
N.S.	1	1.00	0.82	1.00	1.14	1.36	0.00	1.86	-0.05	0.00
time (sec)	N/A	0.006	0.071	0.129	0.342	0.295	0.000	0.005	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	85	69	0	110	0	179	-1	0
N.S.	1	1.00	0.99	0.80	0.00	1.28	0.00	2.08	-0.01	0.00
time (sec)	N/A	0.175	0.167	0.241	0.000	0.313	0.000	0.016	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	71	91	61	92	0	148	-1	0
N.S.	1	1.00	1.15	1.47	0.98	1.48	0.00	2.39	-0.02	0.00
time (sec)	N/A	0.030	0.175	0.194	0.347	0.304	0.000	0.010	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	84	123	0	174	0	273	-1	0
N.S.	1	1.00	1.11	1.62	0.00	2.29	0.00	3.59	-0.01	0.00
time (sec)	N/A	0.041	0.418	0.561	0.000	0.310	0.000	0.008	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	26	31	28	22	0	33	19	0
N.S.	1	1.00	0.72	0.86	0.78	0.61	0.00	0.92	0.53	0.00
time (sec)	N/A	0.073	0.068	0.110	0.342	0.299	0.000	0.004	0.269	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	95	69	0	147	0	186	-1	0
N.S.	1	1.00	1.09	0.79	0.00	1.69	0.00	2.14	-0.01	0.00
time (sec)	N/A	0.132	0.342	1.540	0.000	0.319	0.000	0.007	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	39	53	76	98	0	63	69	0
N.S.	1	1.00	0.67	0.91	1.31	1.69	0.00	1.09	1.19	0.00
time (sec)	N/A	0.006	0.213	0.131	0.259	0.297	0.000	0.006	0.229	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	33	40	59	73	0	53	29	0
N.S.	1	1.00	0.70	0.85	1.26	1.55	0.00	1.13	0.62	0.00
time (sec)	N/A	0.005	0.140	0.073	0.271	0.295	0.000	0.008	0.055	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	33	40	59	51	0	55	29	0
N.S.	1	1.00	0.70	0.85	1.26	1.09	0.00	1.17	0.62	0.00
time (sec)	N/A	0.005	0.146	0.122	0.288	0.301	0.000	0.009	0.204	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	43	40	0	39	0	66	-1	0
N.S.	1	1.00	1.48	1.38	0.00	1.34	0.00	2.28	-0.03	0.00
time (sec)	N/A	0.024	0.223	0.053	0.000	0.299	0.000	0.003	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	59	54	52	0	63	0	80	-1	0
N.S.	1	1.31	1.20	1.16	0.00	1.40	0.00	1.78	-0.02	0.00
time (sec)	N/A	0.020	0.075	0.063	0.000	0.302	0.000	0.003	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	64	59	0	54	0	76	71	0
N.S.	1	1.00	0.81	0.75	0.00	0.68	0.00	0.96	0.90	0.00
time (sec)	N/A	0.084	0.126	0.039	0.000	0.299	0.000	0.005	0.071	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	72	80	0	99	0	126	-1	0
N.S.	1	1.00	0.90	1.00	0.00	1.24	0.00	1.58	-0.01	0.00
time (sec)	N/A	0.216	0.147	0.235	0.000	0.320	0.000	0.020	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	159	140	0	155	0	0	-1	0
N.S.	1	1.00	1.01	0.89	0.00	0.98	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.340	5.222	0.012	0.000	0.313	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	31	30	38	28	131	44	29	120
N.S.	1	1.00	0.76	0.73	0.93	0.68	3.20	1.07	0.71	2.93
time (sec)	N/A	0.007	0.028	0.106	0.342	0.299	1.952	0.001	0.044	3.928

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	17	10	12	11	11
N.S.	1	1.00	1.00	0.92	0.85	1.31	0.77	0.92	0.85	0.85
time (sec)	N/A	0.002	0.017	0.022	0.270	0.287	0.269	0.002	0.460	1.780

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	85	109	76	80	541	100	90	81
N.S.	1	1.00	1.20	1.54	1.07	1.13	7.62	1.41	1.27	1.14
time (sec)	N/A	0.019	0.074	0.420	0.344	0.319	1.269	0.003	0.211	6.509

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	23	29	28	29	178	242	23	95
N.S.	1	1.00	0.58	0.72	0.70	0.72	4.45	6.05	0.58	2.38
time (sec)	N/A	0.005	0.016	0.071	0.255	0.296	0.844	0.002	0.201	2.928

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	53	52	52	51	77	36	25
N.S.	1	1.00	1.00	1.10	1.08	1.08	1.06	1.60	0.75	0.52
time (sec)	N/A	0.011	0.029	0.073	0.341	0.309	1.137	0.010	0.867	2.802

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	36	46	45	25	3303	108	45	1506
N.S.	1	1.00	0.52	0.67	0.65	0.36	47.87	1.57	0.65	21.83
time (sec)	N/A	0.013	0.017	0.062	0.262	0.310	1.122	0.002	0.300	13.254

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	120	125	157	202	44	225	77	23
N.S.	1	1.00	0.62	0.65	0.81	1.05	0.23	1.17	0.40	0.12
time (sec)	N/A	0.070	0.211	0.079	0.345	0.317	3.181	0.009	1.278	4.821

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	26	20	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	2.00	1.54	0.69	0.69
time (sec)	N/A	0.002	0.009	0.069	0.254	0.297	0.145	0.000	0.291	1.768

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	12	18	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.92	1.38	0.69	0.69
time (sec)	N/A	0.002	0.012	0.063	0.257	0.288	0.219	0.001	0.319	1.709

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	82	74	64	66	42	124	76	23
N.S.	1	1.00	1.39	1.25	1.08	1.12	0.71	2.10	1.29	0.39
time (sec)	N/A	0.025	0.050	6.229	0.350	0.300	0.481	0.001	0.455	2.125

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	83	76	66	79	34	94	92	19
N.S.	1	1.00	1.19	1.09	0.94	1.13	0.49	1.34	1.31	0.27
time (sec)	N/A	0.026	0.076	6.227	0.342	0.300	0.791	0.002	0.367	2.382

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	B	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	20	83	146	41	104	18	29
N.S.	1	1.00	1.00	0.29	1.22	2.15	0.60	1.53	0.26	0.43
time (sec)	N/A	0.013	0.129	2.437	0.347	2.261	0.556	0.010	0.423	2.209

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	82	19	130	105	41	144	-1	22
N.S.	1	1.00	0.88	0.20	1.40	1.13	0.44	1.55	-0.01	0.24
time (sec)	N/A	0.018	0.215	2.111	0.341	0.307	0.967	0.013	0.000	2.648

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	82	20	129	105	39	143	-1	23
N.S.	1	1.00	0.88	0.22	1.39	1.13	0.42	1.54	-0.01	0.25
time (sec)	N/A	0.018	0.210	2.077	0.358	0.336	1.028	0.009	0.000	2.761

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	93	129	137	15	0	99	0	97	29	0
N.S.	1	1.39	1.47	0.16	0.00	1.06	0.00	1.04	0.31	0.00
time (sec)	N/A	0.046	0.443	2.046	0.000	0.428	0.000	0.004	0.370	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	126	114	69	108	0	87	0	103	27	0
N.S.	1	0.90	0.55	0.86	0.00	0.69	0.00	0.82	0.21	0.00
time (sec)	N/A	0.066	0.124	0.063	0.000	43.777	0.000	0.011	0.286	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	41	60	0	49	0	92	35	0
N.S.	1	1.00	1.21	1.76	0.00	1.44	0.00	2.71	1.03	0.00
time (sec)	N/A	0.019	0.159	0.146	0.000	0.306	0.000	0.015	0.515	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	47	0	47	0	62	43	0
N.S.	1	1.00	0.95	0.81	0.00	0.81	0.00	1.07	0.74	0.00
time (sec)	N/A	0.023	0.070	0.226	0.000	0.301	0.000	0.009	0.292	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	57	60	52	81	0	124	-1	0
N.S.	1	1.00	0.80	0.85	0.73	1.14	0.00	1.75	-0.01	0.00
time (sec)	N/A	0.031	0.085	0.123	0.342	0.306	0.000	0.010	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	52	46	52	59	0	79	49	0
N.S.	1	1.00	2.48	2.19	2.48	2.81	0.00	3.76	2.33	0.00
time (sec)	N/A	0.018	0.082	0.104	0.261	0.311	0.000	0.009	0.815	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	75	14	13	13	36	30	21	13
N.S.	1	1.00	4.41	0.82	0.76	0.76	2.12	1.76	1.24	0.76
time (sec)	N/A	0.006	10.048	0.087	0.268	0.300	0.236	0.002	0.279	2.104

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	C	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	116	34	35	221	73	34	0
N.S.	1	1.00	0.87	2.52	0.74	0.76	4.80	1.59	0.74	0.00
time (sec)	N/A	0.122	0.035	0.095	0.261	0.305	5.894	0.005	0.373	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	110	904	0	232	0	0	-1	0
N.S.	1	1.00	1.41	11.59	0.00	2.97	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.010	0.197	1.701	0.000	1.576	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	76	150	0	388	0	0	-1	0
N.S.	1	1.00	0.54	1.06	0.00	2.75	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.039	0.162	0.746	0.000	2.482	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	90	29	94	86	71	0	-1	43
N.S.	1	1.00	1.43	0.46	1.49	1.37	1.13	0.00	-0.02	0.68
time (sec)	N/A	0.007	0.141	1.012	0.343	0.302	1.080	0.000	0.000	3.140

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	227	0	242	0	0	-1	0
N.S.	1	1.00	1.00	3.07	0.00	3.27	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.018	0.221	1.687	0.000	3.645	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	59	26	23	73	40	0	0	23	0
N.S.	1	1.23	0.54	0.48	1.52	0.83	0.00	0.00	0.48	0.00
time (sec)	N/A	0.008	0.745	0.097	0.287	0.300	0.000	0.000	0.262	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	180	2515	0	458	0	0	-1	0
N.S.	1	1.00	2.00	27.94	0.00	5.09	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.026	0.342	8.535	0.000	5.499	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	112	0	18	0	0	-1	0
N.S.	1	1.00	1.00	4.87	0.00	0.78	0.00	0.00	-0.04	0.00
time (sec)	N/A	0.020	0.004	0.129	0.000	0.516	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	112	0	42	0	0	-1	0
N.S.	1	1.00	1.00	4.87	0.00	1.83	0.00	0.00	-0.04	0.00
time (sec)	N/A	0.021	0.003	0.147	0.000	0.332	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	23	35	18	57	49	14	59	17	13
N.S.	1	1.44	2.19	1.12	3.56	3.06	0.88	3.69	1.06	0.81
time (sec)	N/A	0.015	0.177	0.109	0.339	0.302	4.350	0.005	0.152	6.174

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	23	37	18	57	47	14	59	17	13
N.S.	1	1.44	2.31	1.12	3.56	2.94	0.88	3.69	1.06	0.81
time (sec)	N/A	0.014	0.057	0.106	0.336	0.307	2.578	0.005	0.234	4.251

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	184	0	45	0	0	-1	0
N.S.	1	1.00	1.00	7.08	0.00	1.73	0.00	0.00	-0.04	0.00
time (sec)	N/A	0.026	0.149	0.266	0.000	0.591	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	188	0	13	0	0	-1	0
N.S.	1	1.00	1.00	12.53	0.00	0.87	0.00	0.00	-0.07	0.00
time (sec)	N/A	0.023	0.118	0.219	0.000	0.327	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	22	14	0	0	14	0
N.S.	1	1.00	1.00	0.94	1.38	0.88	0.00	0.00	0.88	0.00
time (sec)	N/A	0.011	0.249	0.043	0.294	0.300	0.000	0.000	0.058	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	65	247419	0	252	0	0	-1	0
N.S.	1	1.00	0.88	3343.50	0.00	3.41	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.120	0.605	0.174	0.000	0.459	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	96	0	0	62	0	0	-1	0
N.S.	1	1.00	4.36	0.00	0.00	2.82	0.00	0.00	-0.05	0.00
time (sec)	N/A	0.039	0.207	0.025	0.000	0.664	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F(-2)	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	0	0	0	0	0	-1	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.04	0.00
time (sec)	N/A	0.038	0.105	0.024	0.000	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	12	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.86	0.71	0.71
time (sec)	N/A	0.004	0.002	0.013	0.248	0.312	0.028	0.000	0.185	1.604

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	15	11	9	10	8	11	9	9
N.S.	1	1.00	1.36	1.00	0.82	0.91	0.73	1.00	0.82	0.82
time (sec)	N/A	0.004	0.002	0.064	0.296	0.310	0.028	0.000	0.035	1.573

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	18	16	19	24	21	16	16
N.S.	1	1.00	0.92	0.75	0.67	0.79	1.00	0.88	0.67	0.67
time (sec)	N/A	0.006	0.002	0.069	0.253	0.308	0.028	0.000	0.032	1.716

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	24	24	25	36	31	22	32
N.S.	1	1.00	0.88	0.71	0.71	0.74	1.06	0.91	0.65	0.94
time (sec)	N/A	0.011	0.002	0.078	0.252	0.314	0.028	0.000	0.038	1.831

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	30	30	31	48	39	28	44
N.S.	1	1.00	0.86	0.68	0.68	0.70	1.09	0.89	0.64	1.00
time (sec)	N/A	0.014	0.002	0.089	0.258	0.319	0.029	0.000	0.027	1.876

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	B	A	B	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	64	21	39	23	37	99	21	20	14
N.S.	1	3.20	1.05	1.95	1.15	1.85	4.95	1.05	1.00	0.70
time (sec)	N/A	0.012	0.014	0.091	0.261	0.311	0.164	0.002	0.273	2.667

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	23	23	22	39	31	22	35
N.S.	1	1.00	1.00	0.74	0.74	0.71	1.26	1.00	0.71	1.13
time (sec)	N/A	0.007	0.013	0.268	0.252	0.311	0.105	0.028	0.254	2.266

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	18	20	37	32	25	27	23
N.S.	1	1.00	1.29	0.86	0.95	1.76	1.52	1.19	1.29	1.10
time (sec)	N/A	0.006	0.004	0.079	0.253	0.306	0.028	0.002	0.196	1.791

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	95	32	54	93	60	133	44	44
N.S.	1	1.00	2.64	0.89	1.50	2.58	1.67	3.69	1.22	1.22
time (sec)	N/A	0.012	0.006	0.092	0.249	0.325	0.081	0.003	0.249	2.367

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	57	37	33	40	66	45	51	40
N.S.	1	1.00	1.39	0.90	0.80	0.98	1.61	1.10	1.24	0.98
time (sec)	N/A	0.010	0.005	0.082	0.265	0.321	0.029	0.001	0.213	2.008

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	40	51	70	388	64	35	166
N.S.	1	1.00	1.00	1.00	1.28	1.75	9.70	1.60	0.88	4.15
time (sec)	N/A	0.008	0.009	0.333	0.252	0.334	0.550	0.010	0.621	9.731

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	30	21	18	18	31	20	18	18
N.S.	1	1.00	1.36	0.95	0.82	0.82	1.41	0.91	0.82	0.82
time (sec)	N/A	0.008	0.004	0.010	0.343	0.314	0.035	0.001	0.035	1.754

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	16	26	22	40	19	43	26	14
N.S.	1	1.00	0.80	1.30	1.10	2.00	0.95	2.15	1.30	0.70
time (sec)	N/A	0.008	0.003	0.027	0.261	0.313	0.053	0.002	0.270	1.717

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	40	38	30	70	20	92	24	26
N.S.	1	1.00	1.25	1.19	0.94	2.19	0.62	2.88	0.75	0.81
time (sec)	N/A	0.008	0.013	0.046	0.357	0.329	0.083	0.021	0.088	2.007

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	46	42	24	37	56	43	38	64
N.S.	1	1.00	0.82	0.75	0.43	0.66	1.00	0.77	0.68	1.14
time (sec)	N/A	0.039	0.018	0.081	0.263	0.323	0.030	0.001	0.043	1.959

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	55	38	25	25	27	33	25	26
N.S.	1	1.00	1.67	1.15	0.76	0.76	0.82	1.00	0.76	0.79
time (sec)	N/A	0.018	0.025	0.032	0.256	0.329	0.032	0.001	0.196	1.761

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	40	38	44	52	36	36
N.S.	1	1.00	1.00	0.80	0.87	0.83	0.96	1.13	0.78	0.78
time (sec)	N/A	0.017	0.008	0.035	0.262	0.336	0.046	0.001	0.036	1.863

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	53	56	37	55	44	31	27	39
N.S.	1	1.00	1.29	1.37	0.90	1.34	1.07	0.76	0.66	0.95
time (sec)	N/A	0.020	0.019	0.075	0.261	0.324	0.034	0.003	0.122	2.045

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	14	19	10	19	14	13	18	10
N.S.	1	1.00	0.58	0.79	0.42	0.79	0.58	0.54	0.75	0.42
time (sec)	N/A	0.017	0.005	0.026	0.254	0.313	0.032	0.001	0.044	1.647

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	22	36	16	31	31	21	32	22
N.S.	1	1.00	0.48	0.78	0.35	0.67	0.67	0.46	0.70	0.48
time (sec)	N/A	0.033	0.008	0.083	0.258	0.315	0.033	0.001	0.045	1.794

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	30	52	24	43	46	32	44	34
N.S.	1	1.00	0.44	0.76	0.35	0.63	0.68	0.47	0.65	0.50
time (sec)	N/A	0.050	0.011	0.079	0.268	0.311	0.034	0.001	0.028	1.947

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	38	68	30	55	61	39	56	46
N.S.	1	1.00	0.42	0.76	0.33	0.61	0.68	0.43	0.62	0.51
time (sec)	N/A	0.070	0.016	0.083	0.266	0.339	0.035	0.001	0.036	2.085

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	58	0	0	0	0	0	52	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.024	0.038	0.023	0.000	0.334	0.000	0.000	0.749	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	45	25	41	71	54	149	24	52
N.S.	1	1.00	1.41	0.78	1.28	2.22	1.69	4.66	0.75	1.62
time (sec)	N/A	0.014	0.043	0.058	0.252	0.330	0.626	0.033	0.284	5.013

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	11	6	14	17	7	6	6
N.S.	1	1.00	1.00	1.38	0.75	1.75	2.12	0.88	0.75	0.75
time (sec)	N/A	0.013	0.002	0.036	0.250	0.311	0.031	0.001	0.028	1.644

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	32	14	22	15	16	11	14
N.S.	1	1.00	1.00	2.91	1.27	2.00	1.36	1.45	1.00	1.27
time (sec)	N/A	0.009	0.005	0.069	0.264	0.400	0.047	0.001	0.317	1.635

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	8	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	1.00	0.75	0.75
time (sec)	N/A	0.008	0.004	0.025	0.249	0.314	0.031	0.001	0.323	1.550

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	71	36	38	68	41	47	24	31
N.S.	1	1.00	2.73	1.38	1.46	2.62	1.58	1.81	0.92	1.19
time (sec)	N/A	0.019	0.013	0.046	0.249	0.328	0.072	0.002	0.323	2.107

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	14	30	15	16	13	14
N.S.	1	1.00	1.00	1.29	0.82	1.76	0.88	0.94	0.76	0.82
time (sec)	N/A	0.016	0.006	0.037	0.249	0.311	0.048	0.001	0.068	1.641

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	24	49	19	20	39	30	22	0
N.S.	1	1.00	0.77	1.58	0.61	0.65	1.26	0.97	0.71	0.00
time (sec)	N/A	0.017	0.034	0.109	0.269	0.319	16.076	0.002	1.118	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	22	26	13	27	0	29	32	0
N.S.	1	1.00	1.05	1.24	0.62	1.29	0.00	1.38	1.52	0.00
time (sec)	N/A	0.015	0.025	0.282	0.253	0.338	0.000	0.003	1.152	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	95	52	54	93	56	51	57	44
N.S.	1	1.00	2.50	1.37	1.42	2.45	1.47	1.34	1.50	1.16
time (sec)	N/A	0.033	0.014	0.053	0.251	0.324	0.077	0.001	0.280	2.669

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	74	76	74	82	0	109	75	0
N.S.	1	1.00	0.97	1.00	0.97	1.08	0.00	1.43	0.99	0.00
time (sec)	N/A	0.026	0.050	0.384	0.337	0.333	0.000	0.069	6.376	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	F(-2)	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	88	84	127	210	115	178	0	166	133	0
N.S.	1	0.95	1.44	2.39	1.31	2.02	0.00	1.89	1.51	0.00
time (sec)	N/A	0.365	1.355	0.335	0.350	0.465	0.000	0.006	0.381	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	74	66	54	54	148	100	94	65
N.S.	1	1.00	1.06	0.94	0.77	0.77	2.11	1.43	1.34	0.93
time (sec)	N/A	0.096	0.050	0.148	0.259	0.321	0.268	0.003	0.385	3.054

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	35	36	71	39	101	75	34
N.S.	1	1.00	0.88	1.06	1.09	2.15	1.18	3.06	2.27	1.03
time (sec)	N/A	0.036	0.022	0.040	0.352	0.330	0.244	0.011	0.518	2.180

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	20	21	14	26	24	23	26	14
N.S.	1	1.00	1.25	1.31	0.88	1.62	1.50	1.44	1.62	0.88
time (sec)	N/A	0.026	0.013	0.115	0.345	0.332	10.301	0.001	0.312	11.941

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	22	21	27	24	32	10	21
N.S.	1	1.00	1.00	1.83	1.75	2.25	2.00	2.67	0.83	1.75
time (sec)	N/A	0.015	0.008	0.084	0.271	0.316	0.825	0.001	0.269	2.746

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	18	20	15	13	18
N.S.	1	1.00	1.00	0.82	0.76	1.06	1.18	0.88	0.76	1.06
time (sec)	N/A	0.005	0.007	0.069	0.252	0.326	0.139	0.001	0.193	2.047

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	31	54	43	75	47	33	46
N.S.	1	1.00	1.00	1.19	2.08	1.65	2.88	1.81	1.27	1.77
time (sec)	N/A	0.019	0.035	0.109	0.252	0.355	10.242	0.002	0.234	12.669

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	29	30	31	139	35	36	48
N.S.	1	1.00	1.00	0.76	0.79	0.82	3.66	0.92	0.95	1.26
time (sec)	N/A	0.019	0.014	0.094	0.280	0.339	2.287	0.001	0.301	4.980

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	35	33	43	22	23	21	20
N.S.	1	1.00	1.00	1.75	1.65	2.15	1.10	1.15	1.05	1.00
time (sec)	N/A	0.030	0.009	0.085	0.276	0.328	13.290	0.002	0.067	15.817

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	19	19	25	14	16	35	12
N.S.	1	1.00	1.00	1.58	1.58	2.08	1.17	1.33	2.92	1.00
time (sec)	N/A	0.014	0.008	0.076	0.262	0.341	3.321	0.002	0.298	5.138

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	65	39	35	208	64	0	89	47	0
N.S.	1	1.02	0.61	0.55	3.25	1.00	0.00	1.39	0.73	0.00
time (sec)	N/A	0.050	0.026	0.152	0.369	0.327	0.000	0.007	0.342	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	A	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	17	17	27	92	17	0	26	25	0
N.S.	1	1.55	1.55	2.45	8.36	1.55	0.00	2.36	2.27	0.00
time (sec)	N/A	0.021	0.009	0.090	0.262	0.334	0.000	0.001	0.616	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	12	9	11	0	9	5	0
N.S.	1	1.00	1.00	1.71	1.29	1.57	0.00	1.29	0.71	0.00
time (sec)	N/A	0.024	0.026	0.102	0.394	0.324	0.000	0.001	0.266	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	83	33	43	27	38	107	97	23	39
N.S.	1	1.57	0.62	0.81	0.51	0.72	2.02	1.83	0.43	0.74
time (sec)	N/A	0.068	0.043	0.112	0.352	0.358	4.902	0.002	0.214	9.953

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	B	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	94	22	20	39	66	39	49	17	30
N.S.	1	2.85	0.67	0.61	1.18	2.00	1.18	1.48	0.52	0.91
time (sec)	N/A	0.045	0.031	0.070	0.335	0.326	0.316	0.005	0.081	2.415

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	9	8	7	21	219	20	16	169
N.S.	1	1.00	0.33	0.30	0.26	0.78	8.11	0.74	0.59	6.26
time (sec)	N/A	0.014	0.017	0.060	0.341	0.323	5.091	0.001	0.283	9.799

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	41	31	28	46	102	36	38	50
N.S.	1	1.00	1.46	1.11	1.00	1.64	3.64	1.29	1.36	1.79
time (sec)	N/A	0.026	0.029	0.113	0.337	0.319	0.241	0.004	0.320	3.025

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	58	39	38	61	0	89	120	0
N.S.	1	1.00	0.87	0.58	0.57	0.91	0.00	1.33	1.79	0.00
time (sec)	N/A	0.030	0.082	0.122	0.335	0.324	0.000	0.001	0.434	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	54	29	28	58	0	74	77	69
N.S.	1	1.00	0.98	0.53	0.51	1.05	0.00	1.35	1.40	1.25
time (sec)	N/A	0.023	0.062	0.078	0.337	0.326	0.000	0.001	0.386	12.501

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	70	45	44	77	252	48	48	91
N.S.	1	1.00	1.67	1.07	1.05	1.83	6.00	1.14	1.14	2.17
time (sec)	N/A	0.057	0.144	0.052	0.341	0.332	0.263	0.005	0.284	4.660

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	20	0	19	76	26	7	69
N.S.	1	1.00	1.00	2.22	0.00	2.11	8.44	2.89	0.78	7.67
time (sec)	N/A	0.014	0.006	0.081	0.000	0.316	2.171	0.003	0.362	4.786

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	B	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	174	13	129	33	0	60	12	0
N.S.	1	1.00	11.60	0.87	8.60	2.20	0.00	4.00	0.80	0.00
time (sec)	N/A	0.009	0.239	0.057	0.344	0.329	0.000	0.007	0.126	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	28	21	128	26	22	27	9	22
N.S.	1	1.00	1.65	1.24	7.53	1.53	1.29	1.59	0.53	1.29
time (sec)	N/A	0.026	0.011	0.074	0.351	0.324	0.560	0.004	0.242	2.411

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	81	19	0	26	15	0
N.S.	1	1.00	1.00	0.86	3.86	0.90	0.00	1.24	0.71	0.00
time (sec)	N/A	0.025	0.012	0.106	0.366	0.336	0.000	0.002	0.116	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	34	129	19	17	26	17	17
N.S.	1	1.00	1.00	1.62	6.14	0.90	0.81	1.24	0.81	0.81
time (sec)	N/A	0.016	0.006	0.083	0.337	0.318	0.569	0.002	0.260	2.382

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	28	171	50	294	59	27	83
N.S.	1	1.00	1.00	1.08	6.58	1.92	11.31	2.27	1.04	3.19
time (sec)	N/A	0.016	0.016	0.098	0.361	0.347	3.493	0.003	0.466	8.430

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	218	28	171	50	294	59	27	83
N.S.	1	1.00	8.38	1.08	6.58	1.92	11.31	2.27	1.04	3.19
time (sec)	N/A	0.022	0.208	0.104	0.352	0.366	7.803	0.004	0.288	13.085

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	25	22	0	34	0	23	21	0
N.S.	1	1.00	1.56	1.38	0.00	2.12	0.00	1.44	1.31	0.00
time (sec)	N/A	0.006	0.010	0.104	0.000	0.380	0.000	0.003	0.226	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	27	31	0	35	0	39	23	0
N.S.	1	1.00	1.59	1.82	0.00	2.06	0.00	2.29	1.35	0.00
time (sec)	N/A	0.008	0.010	0.098	0.000	0.307	0.000	0.002	0.223	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	47	9	41	55	0	42	13	0
N.S.	1	1.00	1.74	0.33	1.52	2.04	0.00	1.56	0.48	0.00
time (sec)	N/A	0.008	0.012	0.046	0.394	0.325	0.000	0.003	0.053	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	33	17	101	58	0	18	28	0
N.S.	1	1.00	1.10	0.57	3.37	1.93	0.00	0.60	0.93	0.00
time (sec)	N/A	0.009	0.012	0.076	0.404	0.322	0.000	0.005	0.245	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	61	52	433	107	0	123	-1	0
N.S.	1	1.00	1.15	0.98	8.17	2.02	0.00	2.32	-0.02	0.00
time (sec)	N/A	0.018	0.096	0.091	0.398	0.316	0.000	0.009	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	76	47	0	71	0	161	-1	0
N.S.	1	1.00	1.04	0.64	0.00	0.97	0.00	2.21	-0.01	0.00
time (sec)	N/A	0.021	0.108	0.180	0.000	0.610	0.000	0.012	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	36	31	43	40	230	67	-1	27
N.S.	1	1.00	0.65	0.56	0.78	0.73	4.18	1.22	-0.02	0.49
time (sec)	N/A	0.091	0.051	0.691	0.262	0.309	43.340	0.011	0.000	35.791

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	A	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	24	49	80	180	0	120	65	0
N.S.	1	1.00	0.24	0.50	0.82	1.84	0.00	1.22	0.66	0.00
time (sec)	N/A	0.041	0.010	0.052	0.360	0.312	0.000	0.002	0.127	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	69	69	53	52	54	0	83	67	0
N.S.	1	1.21	1.21	0.93	0.91	0.95	0.00	1.46	1.18	0.00
time (sec)	N/A	0.039	0.034	0.059	0.346	0.309	0.000	0.037	0.687	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	B	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	73	130	3213	541	0	234	63	0
N.S.	1	1.00	0.84	1.49	36.93	6.22	0.00	2.69	0.72	0.00
time (sec)	N/A	0.065	0.065	0.132	0.582	0.568	0.000	0.008	0.443	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	38	219	30	82	0	44	105	0
N.S.	1	1.00	0.95	5.48	0.75	2.05	0.00	1.10	2.62	0.00
time (sec)	N/A	0.087	0.909	0.472	0.258	0.322	0.000	0.006	1.421	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	A	C	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	133	62	94	117	603	0	155	228	0
N.S.	1	1.58	0.74	1.12	1.39	7.18	0.00	1.85	2.71	0.00
time (sec)	N/A	0.224	0.184	0.050	0.356	1.246	0.000	0.009	1.271	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	266	0	137	0	0	-1	0
N.S.	1	1.00	1.00	8.58	0.00	4.42	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.010	0.029	0.147	0.000	0.325	0.000	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	29	98	0	137	0	0	-1	0
N.S.	1	1.00	0.94	3.16	0.00	4.42	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.009	0.022	0.106	0.000	0.336	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	171	0	151	0	0	-1	0
N.S.	1	1.00	0.91	3.80	0.00	3.36	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.017	0.027	0.103	0.000	0.332	0.000	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	43	443	0	76	0	0	-1	0
N.S.	1	1.00	0.91	9.43	0.00	1.62	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.057	0.044	0.175	0.000	0.319	0.000	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	510	0	181	0	0	-1	0
N.S.	1	1.00	0.82	8.36	0.00	2.97	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.048	0.061	0.143	0.000	0.329	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	1108	0	205	0	0	-1	0
N.S.	1	1.00	0.92	18.16	0.00	3.36	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.049	0.051	0.162	0.000	0.334	0.000	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	508	0	39	0	0	18	0
N.S.	1	1.00	1.00	31.75	0.00	2.44	0.00	0.00	1.12	0.00
time (sec)	N/A	0.014	0.023	0.158	0.000	0.475	0.000	0.000	0.562	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	286	0	32	0	0	20	0
N.S.	1	1.00	0.65	9.23	0.00	1.03	0.00	0.00	0.65	0.00
time (sec)	N/A	0.025	0.021	0.122	0.000	0.316	0.000	0.000	0.385	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	121	0	43	0	0	29	0
N.S.	1	1.00	0.83	4.17	0.00	1.48	0.00	0.00	1.00	0.00
time (sec)	N/A	0.026	0.022	0.092	0.000	0.320	0.000	0.000	0.430	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F(-1)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	95	111	761	0	136	0	0	-1	0
N.S.	1	1.40	1.63	11.19	0.00	2.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.552	1.050	0.362	0.000	0.329	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	29	19	16	6	15	0	0	15	0
N.S.	1	1.53	1.00	0.84	0.32	0.79	0.00	0.00	0.79	0.00
time (sec)	N/A	0.072	0.008	0.201	0.343	0.314	0.000	0.000	0.514	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	B	A	C	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	92	204	66	318	0	1006	0	0	-1	0
N.S.	1	2.22	0.72	3.46	0.00	10.93	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.146	0.053	0.255	0.000	29.396	0.000	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	63	0	13	29	0	0	32	0
N.S.	1	1.00	1.34	0.00	0.28	0.62	0.00	0.00	0.68	0.00
time (sec)	N/A	0.046	0.097	0.378	0.426	0.312	0.000	0.000	3.937	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	35	0	77	101	0	0	110	0
N.S.	1	1.00	0.50	0.00	1.10	1.44	0.00	0.00	1.57	0.00
time (sec)	N/A	0.118	0.036	0.276	0.360	0.316	0.000	0.000	3.552	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	B	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	108	234	105	247	0	611	0	0	-1	0
N.S.	1	2.17	0.97	2.29	0.00	5.66	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.981	0.181	0.523	0.000	4.708	0.000	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	364	665	434	22968	0	0	0	0	-1	0
N.S.	1	1.83	1.19	63.10	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	2.889	7.969	9.418	0.000	0.000	0.000	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	125	141	58	0	60	56	0	0	-1	0
N.S.	1	1.13	0.46	0.00	0.48	0.45	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.632	0.225	1.064	0.384	0.324	0.000	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-2)	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	67	61	56	55	108	0	73	43	0
N.S.	1	0.92	0.84	0.77	0.75	1.48	0.00	1.00	0.59	0.00
time (sec)	N/A	0.029	0.082	0.044	0.323	0.350	0.000	0.007	0.133	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	48	103	53	88	0	56	-1	0
N.S.	1	1.00	0.70	1.49	0.77	1.28	0.00	0.81	-0.01	0.00
time (sec)	N/A	0.033	0.047	0.157	0.323	0.333	0.000	0.011	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	C	C	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	61	82	36	123	0	55	-1	0
N.S.	1	1.00	1.05	1.41	0.62	2.12	0.00	0.95	-0.02	0.00
time (sec)	N/A	0.031	0.048	0.175	0.343	0.333	0.000	0.006	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-2)	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	37	44	43	51	0	50	28	0
N.S.	1	1.00	0.67	0.80	0.78	0.93	0.00	0.91	0.51	0.00
time (sec)	N/A	0.033	0.059	0.086	0.245	0.351	0.000	0.013	0.658	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	58	716	100	0	49	-1	0
N.S.	1	1.00	1.00	1.49	18.36	2.56	0.00	1.26	-0.03	0.00
time (sec)	N/A	0.051	0.063	0.177	0.392	0.335	0.000	0.016	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	63	53	69	131	0	54	-1	0
N.S.	1	1.00	1.31	1.10	1.44	2.73	0.00	1.12	-0.02	0.00
time (sec)	N/A	0.051	0.199	0.124	0.330	0.332	0.000	0.018	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	F(-2)	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	28	46	192	1	0	54	28	0
N.S.	1	1.00	0.57	0.94	3.92	0.02	0.00	1.10	0.57	0.00
time (sec)	N/A	0.071	0.058	0.082	0.277	0.332	0.000	0.005	0.633	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	C	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	111	119	295	131	115	0	0	0	-1	0
N.S.	1	1.07	2.66	1.18	1.04	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.368	1.104	0.304	0.352	0.000	0.000	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F(-1)	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	156	0	0	195	0	0	-1	0
N.S.	1	1.00	1.39	0.00	0.00	1.74	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.277	0.224	180.000	0.000	0.378	0.000	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	38	25	26	0	169	-1	0
N.S.	1	1.00	0.88	1.15	0.76	0.79	0.00	5.12	-0.03	0.00
time (sec)	N/A	0.037	0.027	0.144	0.244	0.353	0.000	0.013	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	62	488	77	0	36	-1	0
N.S.	1	1.00	0.97	1.88	14.79	2.33	0.00	1.09	-0.03	0.00
time (sec)	N/A	0.014	0.013	0.147	0.394	0.332	0.000	0.015	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	55	790	103	0	0	29	0
N.S.	1	1.00	0.89	1.00	14.36	1.87	0.00	0.00	0.53	0.00
time (sec)	N/A	0.020	0.067	0.089	0.378	0.324	0.000	0.000	0.419	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	39	90	39	0	88	12	0
N.S.	1	1.00	1.00	2.44	5.62	2.44	0.00	5.50	0.75	0.00
time (sec)	N/A	0.009	0.027	0.151	0.337	0.320	0.000	0.058	0.347	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	100	0	118	0	0	-1	0
N.S.	1	1.00	1.00	2.04	0.00	2.41	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.036	0.062	0.162	0.000	0.342	0.000	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	87	91	62	180	1359	163	0	79	-1	0
N.S.	1	1.05	0.71	2.07	15.62	1.87	0.00	0.91	-0.01	0.00
time (sec)	N/A	0.125	0.135	0.312	0.442	0.338	0.000	0.022	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	115	754	0	130	0	0	-1	0
N.S.	1	1.00	1.69	11.09	0.00	1.91	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.030	0.139	0.458	0.000	0.329	0.000	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	79	473	0	115	0	0	-1	0
N.S.	1	1.00	1.98	11.82	0.00	2.88	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.019	0.080	0.442	0.000	0.324	0.000	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F(-1)	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	131	975	0	97	0	0	-1	0
N.S.	1	1.00	1.39	10.37	0.00	1.03	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.107	0.350	1.355	0.000	0.374	0.000	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	A	F	C	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	64	63	33	0	193	20	0
N.S.	1	1.00	0.92	1.64	1.62	0.85	0.00	4.95	0.51	0.00
time (sec)	N/A	0.094	0.055	0.535	0.258	0.323	0.000	0.015	0.759	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	255	1611	0	257	0	161	-1	0
N.S.	1	1.00	3.49	22.07	0.00	3.52	0.00	2.21	-0.01	0.00
time (sec)	N/A	0.708	3.765	1.528	0.000	0.386	0.000	0.046	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	A	A	F(-1)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	116	117	47	84	0	0	113	0
N.S.	1	1.00	2.04	2.05	0.82	1.47	0.00	0.00	1.98	0.00
time (sec)	N/A	0.526	0.628	0.667	0.331	0.327	0.000	0.000	1.473	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	F	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	49	61	0	50	0	84	90	0
N.S.	1	1.00	0.74	0.92	0.00	0.76	0.00	1.27	1.36	0.00
time (sec)	N/A	0.044	0.048	0.046	0.000	0.325	0.000	0.007	1.569	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	71	41	0	76	46	57	172	40
N.S.	1	1.00	1.31	0.76	0.00	1.41	0.85	1.06	3.19	0.74
time (sec)	N/A	0.036	0.263	0.047	0.000	0.327	3.469	0.008	0.511	5.418

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F(-1)	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	105	0	0	0	0	274	250	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	2.06	1.88	0.00
time (sec)	N/A	0.102	0.091	0.034	0.000	0.000	0.000	0.022	1.447	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	42	0	0	117	0	104	101	0
N.S.	1	1.00	0.61	0.00	0.00	1.70	0.00	1.51	1.46	0.00
time (sec)	N/A	0.056	0.060	0.030	0.000	0.673	0.000	0.011	0.717	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	F	A	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	256	0	71	0	0	84	46	0
N.S.	1	1.00	4.92	0.00	1.37	0.00	0.00	1.62	0.88	0.00
time (sec)	N/A	0.051	0.191	0.038	0.342	0.000	0.000	0.095	0.414	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	F	A	F(-1)	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	245	0	74	0	0	87	-1	0
N.S.	1	1.00	4.54	0.00	1.37	0.00	0.00	1.61	-0.02	0.00
time (sec)	N/A	0.049	0.180	0.039	0.343	0.000	0.000	0.092	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	F	F(-1)	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	133	174	370	0	0	0	0	0	-1	0
N.S.	1	1.31	2.78	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	3.186	54.123	180.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	B	A	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	100	208	169	559	0	271	0	230	-1	0
N.S.	1	2.08	1.69	5.59	0.00	2.71	0.00	2.30	-0.01	0.00
time (sec)	N/A	0.649	3.064	0.643	0.000	0.655	0.000	0.052	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	47	127	136	185	0	0	-1	0
N.S.	1	1.00	0.42	1.13	1.21	1.65	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.103	0.033	0.307	0.326	0.336	0.000	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	B	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	95	162	154	0	145	0	0	189	-1	0
N.S.	1	1.71	1.62	0.00	1.53	0.00	0.00	1.99	-0.01	0.00
time (sec)	N/A	0.169	0.089	0.115	0.342	0.000	0.000	0.016	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	30	0	37	46	0	67	-1	0
N.S.	1	1.00	0.61	0.00	0.76	0.94	0.00	1.37	-0.02	0.00
time (sec)	N/A	0.062	0.325	0.832	0.241	0.370	0.000	0.005	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	B	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	42	26	86	35	0	40	43	0
N.S.	1	1.00	2.10	1.30	4.30	1.75	0.00	2.00	2.15	0.00
time (sec)	N/A	0.133	0.139	0.054	0.291	0.320	0.000	0.031	0.621	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	F	F	B	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	35	2512	0	0	93	0	56	-1	0
N.S.	1	1.30	93.04	0.00	0.00	3.44	0.00	2.07	-0.04	0.00
time (sec)	N/A	0.576	45.134	0.421	0.000	1.027	0.000	0.047	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	126	89	0	100	461	0	0	-1	0
N.S.	1	1.25	0.88	0.00	0.99	4.56	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.856	0.264	0.697	0.333	43.317	0.000	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	140	0	0	26	0	128	-1	0
N.S.	1	1.00	5.60	0.00	0.00	1.04	0.00	5.12	-0.04	0.00
time (sec)	N/A	0.038	4.368	0.128	0.000	0.321	0.000	0.008	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F(-1)	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	102	154	153	0	0	0	0	244	-1	0
N.S.	1	1.51	1.50	0.00	0.00	0.00	0.00	2.39	-0.01	0.00
time (sec)	N/A	0.122	0.076	0.309	0.000	0.000	0.000	0.009	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	18	45	88	259	50	0	111	-1	0
N.S.	1	1.06	2.65	5.18	15.24	2.94	0.00	6.53	-0.06	0.00
time (sec)	N/A	0.012	0.050	0.280	0.352	0.358	0.000	0.030	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F(-2)	B	F	C	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	52	242	0	115	0	171	-1	0
N.S.	1	1.00	1.62	7.56	0.00	3.59	0.00	5.34	-0.03	0.00
time (sec)	N/A	0.025	0.046	0.478	0.000	0.337	0.000	0.046	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	27	30	24	38	24	23
N.S.	1	1.00	1.00	0.77	0.87	0.97	0.77	1.23	0.77	0.74
time (sec)	N/A	0.010	0.003	0.081	0.247	0.300	0.061	0.001	0.308	1.722

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	29	30	32	32	43	30	28
N.S.	1	1.00	1.00	0.74	0.77	0.82	0.82	1.10	0.77	0.72
time (sec)	N/A	0.009	0.009	0.081	0.343	0.297	0.067	0.001	0.035	1.743

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	60	46	73	41	48	44	61
N.S.	1	1.00	0.69	1.03	0.79	1.26	0.71	0.83	0.76	1.05
time (sec)	N/A	0.022	0.013	0.088	0.246	0.303	0.078	0.001	0.082	1.929

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	46	37	33	65	984	55	34	613
N.S.	1	1.00	0.88	0.71	0.63	1.25	18.92	1.06	0.65	11.79
time (sec)	N/A	0.015	0.040	0.112	0.327	0.309	2.086	0.001	0.471	6.711

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	49	46	42	47	165	76	46	138
N.S.	1	1.00	0.80	0.75	0.69	0.77	2.70	1.25	0.75	2.26
time (sec)	N/A	0.023	0.036	0.136	0.322	0.615	3.927	0.001	0.467	5.717

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	16	14	15	15	16	24	22
N.S.	1	1.00	0.94	1.00	0.88	0.94	0.94	1.00	1.50	1.38
time (sec)	N/A	0.002	0.002	0.006	0.255	0.308	0.029	0.000	0.456	1.661

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	36	33	35	49	32	42	35	45
N.S.	1	1.00	0.78	0.72	0.76	1.07	0.70	0.91	0.76	0.98
time (sec)	N/A	0.015	0.009	0.082	0.239	0.314	0.060	0.001	0.055	1.860

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	23	58	36	36	32	24	36	36
N.S.	1	1.00	0.58	1.45	0.90	0.90	0.80	0.60	0.90	0.90
time (sec)	N/A	0.022	0.006	0.074	0.245	0.302	0.067	0.001	0.066	1.817

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	17	19	21	41	34	20	21
N.S.	1	1.00	0.93	0.63	0.70	0.78	1.52	1.26	0.74	0.78
time (sec)	N/A	0.007	0.014	0.079	0.241	0.313	3.964	0.001	0.378	4.778

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	22	28	33	82	46	21	21
N.S.	1	1.00	0.66	0.58	0.74	0.87	2.16	1.21	0.55	0.55
time (sec)	N/A	0.010	0.018	0.076	0.259	0.309	1.077	0.001	0.454	3.226

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	24	26	25	34	61	33	99	19
N.S.	1	1.00	0.73	0.79	0.76	1.03	1.85	1.00	3.00	0.58
time (sec)	N/A	0.003	0.030	0.066	0.242	0.306	0.967	0.004	0.045	3.108

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	26	36	51	61	0	41	22	0
N.S.	1	1.00	0.60	0.84	1.19	1.42	0.00	0.95	0.51	0.00
time (sec)	N/A	0.004	0.148	0.125	0.240	0.308	0.000	0.005	0.283	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	28	36	35	40	153	81	24	127
N.S.	1	1.00	0.60	0.77	0.74	0.85	3.26	1.72	0.51	2.70
time (sec)	N/A	0.006	0.033	0.079	0.327	0.307	1.718	0.003	0.449	3.846

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	19	17	16	15	24	34	17	15
N.S.	1	1.00	0.68	0.61	0.57	0.54	0.86	1.21	0.61	0.54
time (sec)	N/A	0.003	0.012	0.056	0.244	0.306	1.150	0.001	0.272	2.591

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	49	47	53	74	49	66	51	65
N.S.	1	1.00	0.94	0.90	1.02	1.42	0.94	1.27	0.98	1.25
time (sec)	N/A	0.019	0.018	0.082	0.243	0.310	0.084	0.001	0.052	2.037

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	0	21	0	0	15	0
N.S.	1	1.00	1.00	0.88	0.00	0.84	0.00	0.00	0.60	0.00
time (sec)	N/A	0.033	6.362	0.107	0.000	0.462	0.000	0.000	0.450	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	35	121	79	69	0	134	-1	0
N.S.	1	1.00	0.70	2.42	1.58	1.38	0.00	2.68	-0.02	0.00
time (sec)	N/A	0.009	0.192	0.139	0.322	0.308	0.000	0.020	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	30	22	21	41	49	33	54	21
N.S.	1	1.00	1.25	0.92	0.88	1.71	2.04	1.38	2.25	0.88
time (sec)	N/A	0.004	0.043	0.104	0.327	0.309	0.215	0.003	0.303	2.005

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	46	58	45	45	32	113	33	33
N.S.	1	1.00	1.15	1.45	1.12	1.12	0.80	2.82	0.82	0.82
time (sec)	N/A	0.005	0.054	0.111	0.340	0.312	0.507	0.006	0.041	2.125

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	28	38	37	44	291	43	179	187
N.S.	1	1.00	0.57	0.78	0.76	0.90	5.94	0.88	3.65	3.82
time (sec)	N/A	0.005	0.165	0.076	0.241	0.306	3.786	0.007	0.276	6.944

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	31	40	59	47	0	48	29	0
N.S.	1	1.00	0.66	0.85	1.26	1.00	0.00	1.02	0.62	0.00
time (sec)	N/A	0.004	0.141	0.155	0.254	0.319	0.000	0.007	0.289	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	32	32	34	40	32	33
N.S.	1	1.00	1.00	0.92	0.89	0.89	0.94	1.11	0.89	0.92
time (sec)	N/A	0.007	0.001	0.123	0.237	0.303	0.029	0.001	0.033	1.664

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	30	29	29	34	40	29	28
N.S.	1	1.00	1.00	0.77	0.74	0.74	0.87	1.03	0.74	0.72
time (sec)	N/A	0.009	0.001	0.104	0.255	0.295	0.031	0.001	0.029	1.650

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	33	54	59	73	0	54	29	0
N.S.	1	1.00	0.70	1.15	1.26	1.55	0.00	1.15	0.62	0.00
time (sec)	N/A	0.006	0.203	0.176	0.252	0.313	0.000	0.009	0.365	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	33	86	76	73	0	55	29	0
N.S.	1	1.00	0.73	1.91	1.69	1.62	0.00	1.22	0.64	0.00
time (sec)	N/A	0.013	0.351	0.145	0.241	0.313	0.000	0.011	0.187	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	67	70	55	57	112	72	69	66
N.S.	1	1.00	0.81	0.84	0.66	0.69	1.35	0.87	0.83	0.80
time (sec)	N/A	0.056	0.038	0.136	0.242	0.325	0.518	0.001	0.396	2.690

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	57	49	52	92	60	59	59
N.S.	1	1.00	0.70	0.78	0.67	0.71	1.26	0.82	0.81	0.81
time (sec)	N/A	0.050	0.065	0.124	0.254	0.321	0.343	0.001	0.335	2.418

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	70	96	66	72	192	85	88	122
N.S.	1	1.00	0.67	0.91	0.63	0.69	1.83	0.81	0.84	1.16
time (sec)	N/A	0.064	0.056	0.168	0.246	0.318	0.757	0.001	0.398	3.482

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	39	32	35	36	53	45	40	34
N.S.	1	1.00	0.89	0.73	0.80	0.82	1.20	1.02	0.91	0.77
time (sec)	N/A	0.025	0.083	0.102	0.241	0.324	0.235	0.001	0.075	2.079

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F(-2)	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	60	0	45	507	89	56	67
N.S.	1	1.00	1.00	1.82	0.00	1.36	15.36	2.70	1.70	2.03
time (sec)	N/A	0.030	0.022	0.150	0.000	0.342	0.815	0.010	0.492	8.544

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	104	30	619	47	551	391	35	171
N.S.	1	1.00	3.47	1.00	20.63	1.57	18.37	13.03	1.17	5.70
time (sec)	N/A	0.028	0.082	0.200	0.355	0.323	0.729	0.095	0.496	9.220

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	132	15	128	61	16	61
N.S.	1	1.00	1.00	0.81	8.25	0.94	8.00	3.81	1.00	3.81
time (sec)	N/A	0.011	0.012	0.095	0.252	0.314	0.463	0.005	0.352	4.119

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	57	66	113	0	0	-1	0
N.S.	1	1.00	0.92	0.92	1.06	1.82	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.042	0.013	0.082	0.332	0.340	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	59	210	138	0	0	-1	0
N.S.	1	1.00	0.92	1.00	3.56	2.34	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.037	0.011	0.274	0.340	0.357	0.000	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	44	78	13	0	10	-1	0
N.S.	1	1.00	1.17	3.67	6.50	1.08	0.00	0.83	-0.08	0.00
time (sec)	N/A	0.069	0.154	0.869	0.337	0.331	0.000	0.006	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	B	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	29	69	19	66	13	-1	86
N.S.	1	1.00	0.95	1.45	3.45	0.95	3.30	0.65	-0.05	4.30
time (sec)	N/A	0.022	0.148	0.250	0.243	0.319	0.792	0.004	0.000	3.499

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	22	42	319	22	51
N.S.	1	1.00	1.00	1.05	0.00	1.00	1.91	14.50	1.00	2.32
time (sec)	N/A	0.016	0.018	0.033	0.000	0.316	0.320	0.005	0.327	1.950

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	41	46	42	0	52	0	0	34	0
N.S.	1	1.21	1.35	1.24	0.00	1.53	0.00	0.00	1.00	0.00
time (sec)	N/A	0.134	0.042	0.042	0.000	0.312	0.000	0.000	0.493	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	11	7	6	4	4
N.S.	1	1.00	1.00	0.89	0.78	1.22	0.78	0.67	0.44	0.44
time (sec)	N/A	0.002	0.006	0.013	0.258	0.316	0.049	0.000	0.051	1.608

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	16	21	17	28	8	8
N.S.	1	1.00	1.00	0.86	0.73	0.95	0.77	1.27	0.36	0.36
time (sec)	N/A	0.011	0.008	0.019	0.239	0.302	0.058	0.001	0.056	1.710

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	24	23	24	24	30	23	11
N.S.	1	1.00	0.97	0.77	0.74	0.77	0.77	0.97	0.74	0.35
time (sec)	N/A	0.012	0.022	0.027	0.246	0.593	0.098	0.001	0.307	1.784

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	31	28	31	31	42	28	16
N.S.	1	1.00	1.03	0.86	0.78	0.86	0.86	1.17	0.78	0.44
time (sec)	N/A	0.016	0.036	0.026	0.240	0.306	0.079	0.001	0.344	1.806

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	52	45	0	0	0	0	0	-1	0
N.S.	1	1.08	0.94	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.030	0.051	0.007	0.000	0.315	0.000	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	34	44	41	39	53	49	41	0
N.S.	1	1.00	0.79	1.02	0.95	0.91	1.23	1.14	0.95	0.00
time (sec)	N/A	0.018	0.056	0.034	0.239	0.307	0.135	0.011	0.411	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	27	26	29	23	26	91
N.S.	1	1.00	1.00	1.04	1.00	0.96	1.07	0.85	0.96	3.37
time (sec)	N/A	0.008	0.010	0.025	0.244	0.304	0.075	0.001	0.362	2.267

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	55	51	62	250	677	68	277
N.S.	1	1.00	1.00	1.04	0.96	1.17	4.72	12.77	1.28	5.23
time (sec)	N/A	0.052	0.041	0.039	0.244	0.318	0.524	0.015	0.384	2.697

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	65	84	77	130	665	1035	81	636
N.S.	1	1.00	0.82	1.06	0.97	1.65	8.42	13.10	1.03	8.05
time (sec)	N/A	0.071	0.070	0.059	0.252	0.305	2.719	0.017	0.390	5.302

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	80	109	99	205	1350	1361	106	1211
N.S.	1	1.00	0.82	1.11	1.01	2.09	13.78	13.89	1.08	12.36
time (sec)	N/A	0.083	0.073	0.043	0.247	0.319	21.807	0.021	0.419	23.855

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	80	73	0	0	0	0	0	-1	0
N.S.	1	1.11	1.01	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.059	0.029	0.031	0.000	0.322	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	28	29	24	27	93
N.S.	1	1.00	1.00	1.04	1.00	1.00	1.04	0.86	0.96	3.32
time (sec)	N/A	0.007	0.006	0.019	0.242	0.316	0.073	0.000	0.310	2.346

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	55	51	64	248	677	69	275
N.S.	1	1.00	1.00	1.04	0.96	1.21	4.68	12.77	1.30	5.19
time (sec)	N/A	0.049	0.039	0.030	0.255	0.334	0.525	0.011	0.365	2.750

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	66	84	77	131	663	1035	81	633
N.S.	1	1.00	0.84	1.06	0.97	1.66	8.39	13.10	1.03	8.01
time (sec)	N/A	0.059	0.064	0.045	0.245	0.307	2.714	0.015	0.364	5.343

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	80	109	99	207	1348	1361	106	1209
N.S.	1	1.00	0.82	1.11	1.01	2.11	13.76	13.89	1.08	12.34
time (sec)	N/A	0.071	0.066	0.033	0.246	0.315	21.989	0.019	0.350	23.885

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	82	75	0	0	0	0	0	-1	0
N.S.	1	1.11	1.01	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.041	0.028	0.035	0.000	0.336	0.000	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	19	15	13	15	29
N.S.	1	1.00	1.00	1.07	1.00	1.27	1.00	0.87	1.00	1.93
time (sec)	N/A	0.004	0.006	0.018	0.243	0.321	0.046	0.000	0.319	1.896

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	35	32	31	29	44	30	26	46
N.S.	1	1.00	1.06	0.97	0.94	0.88	1.33	0.91	0.79	1.39
time (sec)	N/A	0.009	0.015	0.019	0.243	0.308	0.062	0.001	0.318	2.227

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	45	41	46	39	70	41	34	56
N.S.	1	1.00	0.90	0.82	0.92	0.78	1.40	0.82	0.68	1.12
time (sec)	N/A	0.011	0.019	0.022	0.243	0.313	0.075	0.001	0.326	2.589

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	50	61	47	87	50	42	64
N.S.	1	1.00	0.82	0.77	0.94	0.72	1.34	0.77	0.65	0.98
time (sec)	N/A	0.013	0.020	0.025	0.239	0.302	0.084	0.002	0.330	2.842

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	0	0	0	55	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.38	0.00
time (sec)	N/A	0.014	0.036	0.012	0.000	0.306	0.000	0.000	0.315	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	21	19	14	16	28
N.S.	1	1.00	1.00	1.06	1.00	1.31	1.19	0.88	1.00	1.75
time (sec)	N/A	0.003	0.004	0.017	0.237	0.329	0.048	0.000	0.297	1.901

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	35	32	31	29	44	31	27	44
N.S.	1	1.00	1.06	0.97	0.94	0.88	1.33	0.94	0.82	1.33
time (sec)	N/A	0.009	0.016	0.019	0.245	0.308	0.064	0.001	0.303	2.228

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	45	41	46	39	70	43	35	54
N.S.	1	1.00	0.90	0.82	0.92	0.78	1.40	0.86	0.70	1.08
time (sec)	N/A	0.011	0.019	0.023	0.246	0.310	0.078	0.001	0.316	2.617

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	50	61	47	87	52	43	62
N.S.	1	1.00	0.82	0.77	0.94	0.72	1.34	0.80	0.66	0.95
time (sec)	N/A	0.013	0.020	0.041	0.262	0.307	0.090	0.002	0.311	2.862

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0	57	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.30	0.00
time (sec)	N/A	0.013	0.036	0.015	0.000	0.313	0.000	0.000	0.316	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	38	29	23	22	15	26	22	25
N.S.	1	1.00	1.58	1.21	0.96	0.92	0.62	1.08	0.92	1.04
time (sec)	N/A	0.011	0.018	0.016	0.241	0.308	0.065	0.001	0.331	1.808

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	100	123	97	95	100	311	22	142	104	28
N.S.	1	1.23	0.97	0.95	1.00	3.11	0.22	1.42	1.04	0.28
time (sec)	N/A	0.062	0.076	0.033	0.331	0.327	0.096	0.002	1.512	1.911

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	15	14	11	11	8	10	11	12
N.S.	1	1.00	1.25	1.17	0.92	0.92	0.67	0.83	0.92	1.00
time (sec)	N/A	0.014	0.012	0.014	0.247	0.315	0.041	0.001	0.049	1.670

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	38	37	39	22	50	39	30
N.S.	1	1.00	0.96	0.81	0.79	0.83	0.47	1.06	0.83	0.64
time (sec)	N/A	0.036	0.039	0.025	0.339	0.307	0.076	0.001	0.325	1.910

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	29	28	28	48	31	28	51
N.S.	1	1.00	0.95	0.74	0.72	0.72	1.23	0.79	0.72	1.31
time (sec)	N/A	0.038	0.044	0.046	0.330	0.380	0.108	0.001	0.088	2.240

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	32	37	94	27	29	144
N.S.	1	1.00	1.00	1.10	1.07	1.23	3.13	0.90	0.97	4.80
time (sec)	N/A	0.024	0.058	0.026	0.249	0.316	0.486	0.003	0.355	2.280

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	57	56	56	0	79	33	0
N.S.	1	1.00	1.00	1.06	1.04	1.04	0.00	1.46	0.61	0.00
time (sec)	N/A	0.014	0.029	0.049	0.345	0.307	0.000	0.003	0.347	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0	75	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.021	0.034	0.012	0.000	0.324	0.000	0.000	0.404	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	7	18	31	18	14	37
N.S.	1	1.00	1.00	0.83	0.39	1.00	1.72	1.00	0.78	2.06
time (sec)	N/A	0.017	0.026	0.024	0.243	0.309	0.365	0.001	0.406	1.942

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	20	20	31	19	16	37
N.S.	1	1.00	1.00	0.85	1.00	1.00	1.55	0.95	0.80	1.85
time (sec)	N/A	0.018	0.028	0.023	0.248	0.521	0.380	0.002	0.409	1.964

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	0	39	46	0	76	-1	0
N.S.	1	1.00	1.05	0.00	0.98	1.15	0.00	1.90	-0.02	0.00
time (sec)	N/A	0.063	0.112	0.011	0.329	0.305	0.000	0.013	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	57	54	0	0	0	0	0	-1	0
N.S.	1	1.54	1.46	0.00	0.00	0.00	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.032	0.032	0.006	0.000	0.312	0.000	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	44	37	49	30	0	93	30	0
N.S.	1	1.00	0.60	0.51	0.67	0.41	0.00	1.27	0.41	0.00
time (sec)	N/A	0.030	0.036	0.020	0.239	0.304	0.000	0.001	0.106	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	23	41	19	19	20	32	21	22
N.S.	1	1.00	0.52	0.93	0.43	0.43	0.45	0.73	0.48	0.50
time (sec)	N/A	0.021	0.014	0.019	0.240	0.309	0.045	0.000	0.028	1.640

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	26	31	7	23	32	42	22	31
N.S.	1	1.00	0.67	0.79	0.18	0.59	0.82	1.08	0.56	0.79
time (sec)	N/A	0.021	0.021	0.044	0.267	0.489	0.688	0.001	0.272	2.412

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	28	27	27	37	1076	27	41
N.S.	1	1.00	0.66	0.64	0.61	0.61	0.84	24.45	0.61	0.93
time (sec)	N/A	0.013	0.015	0.021	0.246	0.308	0.060	0.003	0.059	1.714

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	18	9	8	18	9	10
N.S.	1	1.00	1.00	0.83	1.50	0.75	0.67	1.50	0.75	0.83
time (sec)	N/A	0.029	0.019	0.021	0.247	0.306	0.046	0.001	0.049	1.572

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	26	25	33	24	39	26	40
N.S.	1	1.00	0.97	0.81	0.78	1.03	0.75	1.22	0.81	1.25
time (sec)	N/A	0.033	0.038	0.027	0.336	0.306	0.063	0.001	0.327	1.815

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	20	21	12	0	0	12	0
N.S.	1	1.00	1.00	1.33	1.40	0.80	0.00	0.00	0.80	0.00
time (sec)	N/A	0.035	0.166	0.092	0.277	0.307	0.000	0.000	0.448	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	19	21	26	24	19	20
N.S.	1	1.00	0.81	0.81	0.70	0.78	0.96	0.89	0.70	0.74
time (sec)	N/A	0.006	0.021	0.036	0.241	0.339	0.196	0.001	0.028	1.790

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	22	39	21	29	56	19	18
N.S.	1	1.00	0.74	0.63	1.11	0.60	0.83	1.60	0.54	0.51
time (sec)	N/A	0.070	0.043	0.061	0.247	0.305	0.271	0.002	0.104	1.844

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	F(-2)	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	37	37	31	0	70	0	33	30
N.S.	1	1.00	0.65	0.65	0.54	0.00	1.23	0.00	0.58	0.53
time (sec)	N/A	0.018	0.047	0.053	0.329	0.000	0.570	0.000	0.037	2.645

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	39	45	45	37	265	45	37	178
N.S.	1	1.00	0.72	0.83	0.83	0.69	4.91	0.83	0.69	3.30
time (sec)	N/A	0.015	0.024	0.067	0.245	0.315	0.445	0.001	0.052	3.471

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	64	68	73	65	638	71	47	342
N.S.	1	1.00	0.78	0.83	0.89	0.79	7.78	0.87	0.57	4.17
time (sec)	N/A	0.024	0.122	0.095	0.249	0.316	1.394	0.001	0.058	6.477

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	36	38	27	42	76	49	39	28
N.S.	1	1.00	0.46	0.48	0.34	0.53	0.96	0.62	0.49	0.35
time (sec)	N/A	0.028	0.030	0.093	0.249	0.315	0.536	0.002	0.296	2.450

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	21	28	27	40	70	30	18	46
N.S.	1	1.00	0.58	0.78	0.75	1.11	1.94	0.83	0.50	1.28
time (sec)	N/A	0.027	0.025	0.044	0.243	0.315	0.738	0.001	0.412	2.779

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	21	63	27	50	99	34	18	21
N.S.	1	1.00	0.58	1.75	0.75	1.39	2.75	0.94	0.50	0.58
time (sec)	N/A	0.027	0.028	0.061	0.248	0.312	0.732	0.001	0.394	2.970

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	85	97	0	0	0	0	0	-1	0
N.S.	1	1.47	1.67	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.048	0.173	0.006	0.000	0.308	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	90	0	0	0	0	0	-1	0
N.S.	1	1.00	2.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.012	0.142	0.026	0.000	0.314	0.000	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	77	66	0	0	0	0	0	-1	0
N.S.	1	1.51	1.29	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.023	0.036	0.062	0.000	0.313	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.04	0.00
time (sec)	N/A	0.021	0.008	0.014	0.000	0.325	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	84	0	0	0	0	0	-1	0
N.S.	1	1.00	3.23	0.00	0.00	0.00	0.00	0.00	-0.04	0.00
time (sec)	N/A	0.020	0.054	0.022	0.000	0.312	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	61	0	0	0	0	0	-1	0
N.S.	1	1.00	2.03	0.00	0.00	0.00	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.021	0.415	0.018	0.000	0.314	0.000	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	61	0	0	0	0	0	-1	0
N.S.	1	1.00	2.03	0.00	0.00	0.00	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.022	0.442	0.036	0.000	0.314	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	11	21	22	12	0	10	8	0
N.S.	1	1.00	0.73	1.40	1.47	0.80	0.00	0.67	0.53	0.00
time (sec)	N/A	0.019	0.152	0.066	0.292	0.304	0.000	0.005	0.387	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	45	100	0	0	0	0	0	-1	0
N.S.	1	1.10	2.44	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.070	0.151	0.049	0.000	0.304	0.000	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	8	22	11	0	8	7	0
N.S.	1	1.00	0.83	0.67	1.83	0.92	0.00	0.67	0.58	0.00
time (sec)	N/A	0.017	0.128	0.051	0.298	0.311	0.000	0.003	0.366	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	44	87	0	0	0	0	0	-1	0
N.S.	1	1.05	2.07	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.068	0.153	0.053	0.000	0.326	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	48	72	0	0	0	0	0	-1	0
N.S.	1	1.04	1.57	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.078	0.495	0.062	0.000	0.309	0.000	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	23	21	22	24	0	22	24	0
N.S.	1	1.00	1.64	1.50	1.57	1.71	0.00	1.57	1.71	0.00
time (sec)	N/A	0.014	0.050	0.098	0.296	0.310	0.000	0.005	0.374	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	73	0	0	0	0	0	-1	0
N.S.	1	1.09	1.70	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.076	0.132	0.051	0.000	0.312	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	23	21	22	24	0	21	20	0
N.S.	1	1.00	1.77	1.62	1.69	1.85	0.00	1.62	1.54	0.00
time (sec)	N/A	0.014	0.043	0.083	0.298	0.310	0.000	0.005	0.424	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	18	20	17	17	27	18	17	21
N.S.	1	1.00	0.60	0.67	0.57	0.57	0.90	0.60	0.57	0.70
time (sec)	N/A	0.022	0.019	0.020	0.245	0.315	0.190	0.000	0.066	1.856

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	25	27	26	26	48	59	21	44
N.S.	1	1.00	0.50	0.54	0.52	0.52	0.96	1.18	0.42	0.88
time (sec)	N/A	0.064	0.028	0.044	0.245	0.301	0.364	0.001	0.327	2.140

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	38	36	33	37	80	74	39	40
N.S.	1	1.00	0.51	0.48	0.44	0.49	1.07	0.99	0.52	0.53
time (sec)	N/A	0.087	0.025	0.048	0.249	0.308	0.465	0.000	0.107	2.257

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	187	253	72	78	77	72	202	185	83	90
N.S.	1	1.35	0.39	0.42	0.41	0.39	1.08	0.99	0.44	0.48
time (sec)	N/A	0.294	0.100	0.114	0.255	0.339	1.413	0.002	0.313	4.022

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	40	40	41	41	85	89	51	52
N.S.	1	1.00	0.46	0.46	0.47	0.47	0.98	1.02	0.59	0.60
time (sec)	N/A	0.099	0.054	0.046	0.271	0.313	0.369	0.001	0.353	2.496

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	76	78	77	72	202	177	83	90
N.S.	1	1.00	0.41	0.42	0.42	0.39	1.09	0.96	0.45	0.49
time (sec)	N/A	0.211	0.141	0.079	0.256	0.313	1.403	0.001	0.527	4.011

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	10	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	5.00	1.00	1.00
time (sec)	N/A	0.002	0.003	0.011	0.241	0.299	0.059	0.000	0.016	1.489

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	9	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	4.50	1.00	1.00
time (sec)	N/A	0.002	0.002	0.013	0.238	0.298	0.059	0.000	0.018	1.495

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	18	7	10	3	9
N.S.	1	1.00	1.00	1.33	1.00	6.00	2.33	3.33	1.00	3.00
time (sec)	N/A	0.002	0.003	0.009	0.248	0.310	0.061	0.001	0.020	1.766

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	18	12	11	3	12
N.S.	1	1.00	1.00	1.33	1.00	6.00	4.00	3.67	1.00	4.00
time (sec)	N/A	0.002	0.003	0.009	0.238	0.322	0.157	0.001	0.027	1.893

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	9	4	3	8	0	5	5	7
N.S.	1	1.00	3.00	1.33	1.00	2.67	0.00	1.67	1.67	2.33
time (sec)	N/A	0.002	0.003	0.013	0.243	0.310	0.000	0.000	0.022	1.648

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	7	6	5	17	0	21	5	5
N.S.	1	1.00	1.40	1.20	1.00	3.40	0.00	4.20	1.00	1.00
time (sec)	N/A	0.002	0.004	0.010	0.242	0.302	0.000	0.000	0.011	1.629

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	16	10	24	31	10	10
N.S.	1	1.00	1.00	0.79	1.14	0.71	1.71	2.21	0.71	0.71
time (sec)	N/A	0.004	0.002	0.018	0.251	0.332	0.081	0.001	0.026	1.831

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	23	18	35	42	29	47	15	18
N.S.	1	1.00	1.21	0.95	1.84	2.21	1.53	2.47	0.79	0.95
time (sec)	N/A	0.007	0.002	0.084	0.238	0.311	0.229	0.001	0.026	2.023

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	26	38	68	10	30	12	12
N.S.	1	1.00	1.29	1.86	2.71	4.86	0.71	2.14	0.86	0.86
time (sec)	N/A	0.006	0.003	0.016	0.251	0.313	0.089	0.001	0.066	1.752

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	36	11	45	211	0	47	16	24
N.S.	1	1.00	2.25	0.69	2.81	13.19	0.00	2.94	1.00	1.50
time (sec)	N/A	0.007	0.004	0.084	0.238	0.315	0.000	0.002	0.290	2.058

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	21	61	461	422	63	22	112
N.S.	1	1.00	1.15	0.81	2.35	17.73	16.23	2.42	0.85	4.31
time (sec)	N/A	0.011	0.004	0.098	0.339	0.306	1.382	0.004	0.078	9.596

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	35	257	0	50	35	0
N.S.	1	1.00	1.00	0.94	1.94	14.28	0.00	2.78	1.94	0.00
time (sec)	N/A	0.014	0.006	0.026	0.337	0.321	0.000	0.003	0.357	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	0	0	359	41	0	120	0
N.S.	1	1.00	0.87	0.00	0.00	11.58	1.32	0.00	3.87	0.00
time (sec)	N/A	0.019	0.035	0.071	0.000	0.313	45.537	0.000	0.165	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	42	41	36	0	175	126	36	43	205
N.S.	1	1.02	1.00	0.88	0.00	4.27	3.07	0.88	1.05	5.00
time (sec)	N/A	0.031	0.023	0.042	0.000	0.320	1.977	0.001	0.163	3.653

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	16	16	49	58	14	18	14	15
N.S.	1	1.00	0.64	0.64	1.96	2.32	0.56	0.72	0.56	0.60
time (sec)	N/A	0.012	0.009	0.023	0.255	0.298	0.185	0.001	0.288	1.827

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	55	41	42	146	42	35	150
N.S.	1	1.00	0.74	1.41	1.05	1.08	3.74	1.08	0.90	3.85
time (sec)	N/A	0.029	0.046	0.031	0.241	0.313	0.288	0.001	0.132	4.013

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	98	76	288	1149	92	109	1298
N.S.	1	1.00	1.00	3.16	2.45	9.29	37.06	2.97	3.52	41.87
time (sec)	N/A	0.022	0.044	0.092	0.334	0.318	16.944	0.002	0.652	21.041

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	74	0	388	874	54	106	1175
N.S.	1	1.00	1.00	2.11	0.00	11.09	24.97	1.54	3.03	33.57
time (sec)	N/A	0.023	0.031	0.070	0.000	0.325	16.062	0.001	0.381	20.189

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	55	69	113	908	59	63	359
N.S.	1	1.00	0.96	2.20	2.76	4.52	36.32	2.36	2.52	14.36
time (sec)	N/A	0.011	0.074	0.056	0.345	0.314	2.767	0.002	0.400	30.108

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	37	78	70	74	102	32	22	50
N.S.	1	1.00	1.12	2.36	2.12	2.24	3.09	0.97	0.67	1.52
time (sec)	N/A	0.091	0.092	0.132	0.349	0.313	0.570	0.003	0.370	3.865

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	42	44	114	66	40	66
N.S.	1	1.00	1.00	0.77	1.40	1.47	3.80	2.20	1.33	2.20
time (sec)	N/A	0.021	0.013	0.080	0.242	0.309	1.353	0.001	0.403	4.546

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	42	111	138	88	40	40
N.S.	1	1.00	1.00	0.77	1.40	3.70	4.60	2.93	1.33	1.33
time (sec)	N/A	0.020	0.012	0.133	0.240	0.307	1.289	0.001	0.416	4.666

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	C	F	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	69	102	160	987	0	376	0	160	-1	0
N.S.	1	1.48	2.32	14.30	0.00	5.45	0.00	2.32	-0.01	0.00
time (sec)	N/A	0.619	2.105	0.315	0.000	0.330	0.000	0.028	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	26	30	125	474	0	67	57	0
N.S.	1	1.00	0.70	0.81	3.38	12.81	0.00	1.81	1.54	0.00
time (sec)	N/A	0.030	0.043	0.035	0.263	0.327	0.000	0.022	0.139	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	B	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	21	28	177	161	0	0	47	0
N.S.	1	1.00	0.72	0.97	6.10	5.55	0.00	0.00	1.62	0.00
time (sec)	N/A	0.065	0.035	0.108	0.375	0.318	0.000	0.000	0.486	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	63	0	482	0	75	-1	0
N.S.	1	1.00	1.00	4.20	0.00	32.13	0.00	5.00	-0.07	0.00
time (sec)	N/A	0.011	0.009	0.170	0.000	0.392	0.000	0.007	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	28	49	93	17	52	21	19
N.S.	1	1.00	1.00	1.75	3.06	5.81	1.06	3.25	1.31	1.19
time (sec)	N/A	0.011	0.014	0.017	0.379	0.318	0.079	0.003	0.312	1.923

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	28	53	95	22	54	27	24
N.S.	1	1.00	1.00	1.75	3.31	5.94	1.38	3.38	1.69	1.50
time (sec)	N/A	0.011	0.014	0.028	0.280	0.315	0.331	0.002	0.293	2.621

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	23	16	13	20	26	20	16	9
N.S.	1	1.00	1.15	0.80	0.65	1.00	1.30	1.00	0.80	0.45
time (sec)	N/A	0.041	0.053	0.194	0.250	0.302	0.218	0.001	0.061	2.056

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	20	16	35	20	12	13	17	14
N.S.	1	1.00	1.33	1.07	2.33	1.33	0.80	0.87	1.13	0.93
time (sec)	N/A	0.084	0.047	0.053	0.246	0.311	0.175	0.001	0.286	1.798

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	20	22	75	0	28	24	0
N.S.	1	1.00	1.00	1.00	1.10	3.75	0.00	1.40	1.20	0.00
time (sec)	N/A	0.012	0.008	0.152	0.266	0.304	0.000	0.003	0.331	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	22	75	102	0	14	19	0
N.S.	1	1.00	1.00	1.69	5.77	7.85	0.00	1.08	1.46	0.00
time (sec)	N/A	0.010	0.007	0.100	0.239	0.320	0.000	0.001	0.311	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	22	6	16	12	7	6	7
N.S.	1	1.00	1.00	2.44	0.67	1.78	1.33	0.78	0.67	0.78
time (sec)	N/A	0.010	0.002	0.060	0.252	0.307	0.239	0.000	0.317	1.806

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	19	18	26	0	25	32	14	14	43
N.S.	1	1.46	1.38	2.00	0.00	1.92	2.46	1.08	1.08	3.31
time (sec)	N/A	0.019	0.018	0.094	0.000	0.497	0.265	0.001	0.109	2.213

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	10	1	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	10.00	1.00	1.00	1.00
time (sec)	N/A	0.009	0.000	0.050	0.240	0.293	0.199	0.000	0.288	1.705

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	36	24	16	26	0	16	16	0
N.S.	1	1.00	1.64	1.09	0.73	1.18	0.00	0.73	0.73	0.00
time (sec)	N/A	0.015	0.019	0.033	0.253	0.301	0.000	0.001	0.059	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	18	18	11	21	0	10	11	0
N.S.	1	1.00	1.38	1.38	0.85	1.62	0.00	0.77	0.85	0.00
time (sec)	N/A	0.019	0.018	0.035	0.245	0.299	0.000	0.001	0.303	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	20	18	11	22	0	10	11	0
N.S.	1	1.00	1.33	1.20	0.73	1.47	0.00	0.67	0.73	0.00
time (sec)	N/A	0.022	0.019	0.049	0.247	0.297	0.000	0.002	0.044	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	19	19	26	25	56	0	32	36
N.S.	1	1.00	0.73	0.73	1.00	0.96	2.15	0.00	1.23	1.38
time (sec)	N/A	0.007	0.006	0.015	0.241	0.342	0.219	0.000	0.401	2.055

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	30	41	42	45	155	85	43	63
N.S.	1	1.00	0.71	0.98	1.00	1.07	3.69	2.02	1.02	1.50
time (sec)	N/A	0.015	0.008	0.019	0.248	0.321	0.362	0.004	0.354	2.932

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	21	23	22	17	34	49	17	17
N.S.	1	1.00	0.62	0.68	0.65	0.50	1.00	1.44	0.50	0.50
time (sec)	N/A	0.012	0.004	0.058	0.237	0.422	1.087	0.004	0.037	2.536

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	27	25	25	22	27	21	22
N.S.	1	1.00	1.00	0.96	0.89	0.89	0.79	0.96	0.75	0.79
time (sec)	N/A	0.007	0.003	0.012	0.240	0.301	0.061	0.001	0.332	1.687

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	81	69	69	69	71	81	71	66
N.S.	1	1.00	1.21	1.03	1.03	1.03	1.06	1.21	1.06	0.99
time (sec)	N/A	0.021	0.012	0.086	0.246	0.300	0.082	0.001	0.381	1.982

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	36	23	26	48	20	20
N.S.	1	1.00	1.00	1.04	1.57	1.00	1.13	2.09	0.87	0.87
time (sec)	N/A	0.009	0.003	0.022	0.252	0.300	0.062	0.001	0.294	1.654

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	53	66	48	51	77	51	43
N.S.	1	1.00	1.00	0.88	1.10	0.80	0.85	1.28	0.85	0.72
time (sec)	N/A	0.054	0.006	0.014	0.241	0.309	0.089	0.002	0.374	1.897

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	37	8	34	32	0	29	33
N.S.	1	1.00	1.00	0.86	0.19	0.79	0.74	0.00	0.67	0.77
time (sec)	N/A	0.035	0.017	0.014	0.261	0.300	0.390	0.000	0.279	1.966

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	F	A	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	30	32	25	0	177	0	-1	284
N.S.	1	1.00	1.03	1.10	0.86	0.00	6.10	0.00	-0.03	9.79
time (sec)	N/A	0.012	0.003	0.079	0.245	0.647	3.793	0.000	0.000	6.418

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	30	38	34	24	37	35	41
N.S.	1	1.00	0.93	1.03	1.31	1.17	0.83	1.28	1.21	1.41
time (sec)	N/A	0.008	0.009	0.081	0.259	0.317	0.123	0.001	0.392	1.921

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	15	11	22	21
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.25	0.92	1.83	1.75
time (sec)	N/A	0.011	0.003	0.018	0.243	0.322	0.453	0.001	0.333	1.931

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	22	36	17	19	0
N.S.	1	1.00	1.00	1.05	1.00	1.16	1.89	0.89	1.00	0.00
time (sec)	N/A	0.017	0.006	0.024	0.243	0.325	0.634	0.001	0.450	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	28	11	13
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	2.55	1.00	1.18
time (sec)	N/A	0.014	0.009	0.015	0.245	0.304	0.065	0.000	0.294	1.642

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	0	27	71	19	22	104
N.S.	1	1.00	1.00	1.04	0.00	1.17	3.09	0.83	0.96	4.52
time (sec)	N/A	0.020	0.007	0.023	0.000	0.644	5.988	0.001	0.427	7.552

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	46	15	7	18	0	18	14	0
N.S.	1	1.00	2.88	0.94	0.44	1.12	0.00	1.12	0.88	0.00
time (sec)	N/A	0.023	0.013	0.015	0.245	0.302	0.000	0.001	0.430	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	50	17	20	20	0	0	16	0
N.S.	1	1.00	2.78	0.94	1.11	1.11	0.00	0.00	0.89	0.00
time (sec)	N/A	0.025	0.014	0.015	0.239	0.308	0.000	0.000	0.392	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	7	25	0	9	16	0
N.S.	1	1.00	1.00	0.94	0.39	1.39	0.00	0.50	0.89	0.00
time (sec)	N/A	0.025	0.014	0.014	0.326	0.312	0.000	0.002	0.612	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	37	13	44	0	43	27	0
N.S.	1	1.00	1.00	1.68	0.59	2.00	0.00	1.95	1.23	0.00
time (sec)	N/A	0.053	0.009	0.013	0.241	0.316	0.000	0.001	0.581	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	39	37	27	0	0	22	0
N.S.	1	1.00	1.00	1.62	1.54	1.12	0.00	0.00	0.92	0.00
time (sec)	N/A	0.060	0.010	0.015	0.249	0.308	0.000	0.000	0.608	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	43	13	27	0	20	25	0
N.S.	1	1.00	1.00	1.87	0.57	1.17	0.00	0.87	1.09	0.00
time (sec)	N/A	0.059	0.009	0.014	0.323	0.312	0.000	0.002	0.524	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	10	10	8	8
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.91	0.91	0.73	0.73
time (sec)	N/A	0.005	0.005	0.013	0.236	0.309	0.077	0.000	0.322	1.626

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	15	20	24	21	15	15
N.S.	1	1.00	1.00	1.05	0.75	1.00	1.20	1.05	0.75	0.75
time (sec)	N/A	0.011	0.006	0.020	0.238	0.318	0.112	0.001	0.377	1.698

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	30	22	29	36	31	29	22
N.S.	1	1.00	1.00	1.03	0.76	1.00	1.24	1.07	1.00	0.76
time (sec)	N/A	0.012	0.006	0.020	0.250	0.338	0.143	0.001	0.361	1.793

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	39	29	38	48	40	38	29
N.S.	1	1.00	1.00	1.03	0.76	1.00	1.26	1.05	1.00	0.76
time (sec)	N/A	0.014	0.006	0.024	0.241	0.366	0.174	0.001	0.371	1.884

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	29	14	24	0	24	24
N.S.	1	1.00	1.00	0.00	1.21	0.58	1.00	0.00	1.00	1.00
time (sec)	N/A	0.022	0.019	0.005	0.263	0.462	0.972	0.000	0.451	2.553

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	0	5	4	0
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.00	1.25	1.00	0.00
time (sec)	N/A	0.012	0.007	0.115	0.241	0.315	0.000	0.001	0.396	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	10	9	12	7	7	9	9
N.S.	1	1.00	1.00	1.43	1.29	1.71	1.00	1.00	1.29	1.29
time (sec)	N/A	0.028	0.005	0.041	0.254	0.315	0.892	0.001	0.387	2.285

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	46	10	46	8	8
N.S.	1	1.00	1.00	1.09	1.00	4.18	0.91	4.18	0.73	0.73
time (sec)	N/A	0.007	0.008	0.079	0.245	0.304	0.231	0.001	0.365	1.791

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	0	0	16	0
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.00	0.00	1.78	0.00
time (sec)	N/A	0.009	0.003	0.086	0.245	0.305	0.000	0.000	0.439	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	22	20	25	22	22
N.S.	1	1.00	1.00	0.88	0.00	0.85	0.77	0.96	0.85	0.85
time (sec)	N/A	0.003	0.013	0.007	0.000	0.307	5.128	0.002	0.078	5.374

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	27	26	25	26	26	34	25	26
N.S.	1	1.00	0.77	0.74	0.71	0.74	0.74	0.97	0.71	0.74
time (sec)	N/A	0.009	0.008	0.026	0.264	0.313	0.077	0.001	0.065	1.718

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-2)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	24	24	20	26	0	20	24	0
N.S.	1	1.00	0.75	0.75	0.62	0.81	0.00	0.62	0.75	0.00
time (sec)	N/A	0.032	0.016	0.020	0.243	0.304	0.000	0.004	0.380	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	42	43	42	42	0	65	31	0
N.S.	1	1.00	0.81	0.83	0.81	0.81	0.00	1.25	0.60	0.00
time (sec)	N/A	0.025	0.017	0.017	0.242	0.312	0.000	0.002	0.548	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	197	25	24	42	28	-1	24
N.S.	1	1.00	1.00	6.57	0.83	0.80	1.40	0.93	-0.03	0.80
time (sec)	N/A	0.022	0.006	0.184	0.245	0.328	0.922	0.001	0.000	2.562

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	55	25	39	46	28	148	26
N.S.	1	1.00	0.97	1.83	0.83	1.30	1.53	0.93	4.93	0.87
time (sec)	N/A	0.037	0.042	0.299	0.253	0.325	13.140	0.003	1.803	13.675

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	B	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	32	164	56	32	0	30	39	0
N.S.	1	1.00	1.14	5.86	2.00	1.14	0.00	1.07	1.39	0.00
time (sec)	N/A	0.029	0.051	0.147	0.241	0.318	0.000	0.006	0.576	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	56	106	86	53	88	56	164	38
N.S.	1	1.00	0.93	1.77	1.43	0.88	1.47	0.93	2.73	0.63
time (sec)	N/A	0.090	0.086	0.276	0.327	0.316	3.235	0.038	0.787	4.344

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	52	52	51	44	0	145	-1	0
N.S.	1	1.00	0.80	0.80	0.78	0.68	0.00	2.23	-0.02	0.00
time (sec)	N/A	0.068	0.021	0.050	0.330	0.323	0.000	0.015	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	42	37	47	36	54	81	-1	47
N.S.	1	1.00	0.69	0.61	0.77	0.59	0.89	1.33	-0.02	0.77
time (sec)	N/A	0.057	0.012	0.099	0.334	0.317	0.173	0.005	0.000	1.918

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	37	42	44	36	44	47	41	41
N.S.	1	1.00	0.70	0.79	0.83	0.68	0.83	0.89	0.77	0.77
time (sec)	N/A	0.071	0.009	0.054	0.338	0.312	0.183	0.004	0.344	1.872

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	48	64	53	53	0	44	52
N.S.	1	1.00	0.92	0.79	1.05	0.87	0.87	0.00	0.72	0.85
time (sec)	N/A	0.077	0.010	0.079	0.325	0.327	0.292	0.000	0.105	1.963

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	42	56	95	35	0	152	-1	0
N.S.	1	1.00	0.67	0.89	1.51	0.56	0.00	2.41	-0.02	0.00
time (sec)	N/A	0.040	0.023	0.048	0.390	0.325	0.000	0.014	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	92	174	0	77	0	169	-1	0
N.S.	1	1.00	0.62	1.18	0.00	0.52	0.00	1.14	-0.01	0.00
time (sec)	N/A	0.087	0.039	0.151	0.000	0.341	0.000	0.013	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	31	30	26	39	36	-1	0
N.S.	1	1.00	0.88	0.91	0.88	0.76	1.15	1.06	-0.03	0.00
time (sec)	N/A	0.020	0.006	0.094	0.322	0.319	1.103	0.007	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	33	30	26	39	34	-1	0
N.S.	1	1.00	0.88	0.97	0.88	0.76	1.15	1.00	-0.03	0.00
time (sec)	N/A	0.018	0.010	0.109	0.334	0.319	1.144	0.008	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	26	134	22	27	37	36	-1	38
N.S.	1	1.00	0.87	4.47	0.73	0.90	1.23	1.20	-0.03	1.27
time (sec)	N/A	0.018	0.015	0.197	0.329	0.312	0.207	0.005	0.000	1.912

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	42	54	50	39	53	73	-1	47
N.S.	1	1.00	0.71	0.92	0.85	0.66	0.90	1.24	-0.02	0.80
time (sec)	N/A	0.032	0.017	0.133	0.332	0.319	0.370	0.012	0.000	2.084

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	37	27	37	63	46	-1	59
N.S.	1	1.00	0.95	1.00	0.73	1.00	1.70	1.24	-0.03	1.59
time (sec)	N/A	0.022	0.010	0.108	0.333	0.312	0.534	0.007	0.000	2.333

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	286	49	47	88	77	-1	80
N.S.	1	1.00	0.77	4.69	0.80	0.77	1.44	1.26	-0.02	1.31
time (sec)	N/A	0.046	0.030	0.312	0.341	0.319	9.127	0.010	0.000	8.093

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	119	230	0	0	0	0	-1	0
N.S.	1	1.00	1.25	2.42	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.097	0.159	0.288	0.000	0.317	0.000	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	36	201	35	44	0	189	-1	0
N.S.	1	1.00	0.88	4.90	0.85	1.07	0.00	4.61	-0.02	0.00
time (sec)	N/A	0.036	0.022	0.722	0.321	0.323	0.000	0.016	0.000	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	32	32	26	26	37	-1	26
N.S.	1	1.00	0.82	0.94	0.94	0.76	0.76	1.09	-0.03	0.76
time (sec)	N/A	0.037	0.007	0.100	0.328	0.638	0.127	0.006	0.000	1.766

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	43	54	52	39	53	77	-1	47
N.S.	1	1.00	0.70	0.89	0.85	0.64	0.87	1.26	-0.02	0.77
time (sec)	N/A	0.065	0.013	0.136	0.326	0.322	0.272	0.006	0.000	2.054

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	46	25	44	20	32	-1	0
N.S.	1	1.00	1.00	2.42	1.32	2.32	1.05	1.68	-0.05	0.00
time (sec)	N/A	0.023	0.007	0.095	0.325	0.332	4.648	0.004	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	32	47	25	44	20	32	-1	0
N.S.	1	1.00	1.88	2.76	1.47	2.59	1.18	1.88	-0.06	0.00
time (sec)	N/A	0.024	0.027	0.099	0.329	0.323	5.059	0.008	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	45	63	48	61	78	61	-1	0
N.S.	1	1.00	0.73	1.02	0.77	0.98	1.26	0.98	-0.02	0.00
time (sec)	N/A	0.025	0.046	0.094	0.330	0.322	15.330	0.009	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	40	61	45	57	37	45	-1	45
N.S.	1	1.00	1.11	1.69	1.25	1.58	1.03	1.25	-0.03	1.25
time (sec)	N/A	0.043	0.033	0.419	0.328	0.325	8.572	0.006	0.000	9.974

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	112	97	0	0	0	0	-1	0
N.S.	1	1.00	1.81	1.56	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.071	0.123	0.426	0.000	0.340	0.000	0.000	0.000	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	38	43	42	36	49	134	-1	0
N.S.	1	1.00	0.70	0.80	0.78	0.67	0.91	2.48	-0.02	0.00
time (sec)	N/A	0.057	0.023	0.112	0.330	0.323	7.022	0.007	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	158	52	41	78	74	-1	71
N.S.	1	1.00	0.76	2.39	0.79	0.62	1.18	1.12	-0.02	1.08
time (sec)	N/A	0.046	0.028	0.188	0.332	0.318	0.395	0.014	0.000	2.293

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	69	0	49	66	80	-1	57
N.S.	1	1.00	0.82	0.95	0.00	0.67	0.90	1.10	-0.01	0.78
time (sec)	N/A	0.094	0.014	0.118	0.000	0.323	0.261	0.011	0.000	2.053

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	21	27	26	19	31	28	21	21
N.S.	1	1.00	0.66	0.84	0.81	0.59	0.97	0.88	0.66	0.66
time (sec)	N/A	0.017	0.011	0.091	0.336	0.312	0.245	0.001	0.078	1.901

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	36	37	39	38	88	41	26	41
N.S.	1	1.00	0.82	0.84	0.89	0.86	2.00	0.93	0.59	0.93
time (sec)	N/A	0.018	0.013	0.093	0.332	0.328	0.360	0.003	0.319	2.380

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	24	19	19	21	19	19
N.S.	1	1.00	1.00	0.87	1.04	0.83	0.83	0.91	0.83	0.83
time (sec)	N/A	0.030	0.010	0.069	0.336	0.317	0.136	0.002	0.309	1.742

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	57	128	0	0	0	0	-1	0
N.S.	1	1.00	0.85	1.91	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.057	0.016	0.080	0.000	0.315	0.000	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	29	40	26	0	0	23	0
N.S.	1	1.00	0.82	0.85	1.18	0.76	0.00	0.00	0.68	0.00
time (sec)	N/A	0.030	0.015	0.108	0.340	0.314	0.000	0.000	0.065	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	64	139	0	0	0	0	-1	0
N.S.	1	1.00	0.81	1.76	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.068	0.041	0.089	0.000	0.326	0.000	0.000	0.000	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	70	149	0	0	0	0	-1	0
N.S.	1	1.00	0.79	1.67	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.139	0.125	0.086	0.000	0.314	0.000	0.000	0.000	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	21	26	19	23	22	22
N.S.	1	1.00	1.00	1.05	0.95	1.18	0.86	1.05	1.00	1.00
time (sec)	N/A	0.021	0.005	0.050	0.321	0.315	0.147	0.003	0.066	1.769

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	59	30	31	26	34	39	30	32
N.S.	1	1.00	1.90	0.97	1.00	0.84	1.10	1.26	0.97	1.03
time (sec)	N/A	0.014	0.005	0.050	0.325	0.320	0.297	0.003	0.322	1.840

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	81	79	71	0	0	0	53	0
N.S.	1	1.00	1.29	1.25	1.13	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.054	0.007	0.096	0.357	0.312	0.000	0.000	0.498	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	27	29	22	0	24	24
N.S.	1	1.00	1.00	0.89	0.96	1.04	0.79	0.00	0.86	0.86
time (sec)	N/A	0.035	0.005	0.115	0.338	0.317	0.201	0.000	0.089	1.803

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	38	34	36	38	32	0	31	33
N.S.	1	1.00	0.97	0.87	0.92	0.97	0.82	0.00	0.79	0.85
time (sec)	N/A	0.042	0.011	0.066	0.327	0.317	0.213	0.000	0.074	1.807

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	56	57	71	54	61	0	51	60
N.S.	1	1.00	0.93	0.95	1.18	0.90	1.02	0.00	0.85	1.00
time (sec)	N/A	0.048	0.017	0.084	0.336	0.317	0.298	0.000	0.120	1.991

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	79	82	47	78	94	51	0	0	56	0
N.S.	1	1.04	0.59	0.99	1.19	0.65	0.00	0.00	0.71	0.00
time (sec)	N/A	0.081	0.027	0.137	0.332	0.310	0.000	0.000	0.360	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	107	116	116	708	0	0	0	0	-1	0
N.S.	1	1.08	1.08	6.62	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.096	0.118	0.464	0.000	0.332	0.000	0.000	0.000	0.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	106	133	86	305	0	51	0	0	-1	0
N.S.	1	1.25	0.81	2.88	0.00	0.48	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.121	0.201	0.408	0.000	0.321	0.000	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	48	329	27	23	0	85	-1	0
N.S.	1	1.00	1.17	8.02	0.66	0.56	0.00	2.07	-0.02	0.00
time (sec)	N/A	0.032	0.028	0.484	0.328	0.322	0.000	0.026	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	67	67	128	48	75	0	72	-1	0
N.S.	1	1.03	1.03	1.97	0.74	1.15	0.00	1.11	-0.02	0.00
time (sec)	N/A	0.019	0.072	0.417	0.299	0.323	0.000	0.017	0.000	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	53	61	121	46	68	0	66	-1	0
N.S.	1	1.04	1.20	2.37	0.90	1.33	0.00	1.29	-0.02	0.00
time (sec)	N/A	0.039	0.064	0.512	0.292	0.330	0.000	0.014	0.000	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	84	72	197	0	69	0	77	-1	0
N.S.	1	1.02	0.88	2.40	0.00	0.84	0.00	0.94	-0.01	0.00
time (sec)	N/A	0.057	0.112	0.650	0.000	0.343	0.000	0.016	0.000	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	175	232	383	240	0	0	0	0	-1	0
N.S.	1	1.33	2.19	1.37	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.193	1.126	0.841	0.000	0.315	0.000	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	35	178	17	16	0	47	-1	0
N.S.	1	1.00	1.52	7.74	0.74	0.70	0.00	2.04	-0.04	0.00
time (sec)	N/A	0.030	0.020	0.377	0.340	0.312	0.000	0.016	0.000	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	B	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	91	79	702	123	81	0	117	-1	0
N.S.	1	1.30	1.13	10.03	1.76	1.16	0.00	1.67	-0.01	0.00
time (sec)	N/A	0.053	0.086	0.633	0.546	0.327	0.000	0.028	0.000	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	101	76	330	58	37	0	0	-1	0
N.S.	1	1.36	1.03	4.46	0.78	0.50	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.111	0.036	0.532	0.340	0.317	0.000	0.000	0.000	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	133	172	84	386	0	59	0	0	-1	0
N.S.	1	1.29	0.63	2.90	0.00	0.44	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.126	0.167	0.583	0.000	0.315	0.000	0.000	0.000	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	110	146	92	536	93	57	0	0	-1	0
N.S.	1	1.33	0.84	4.87	0.85	0.52	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.116	0.047	0.540	0.552	0.312	0.000	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	B	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	55	125	99	86	103	51	0	0	-1	0
N.S.	1	2.27	1.80	1.56	1.87	0.93	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.422	0.092	0.034	0.326	0.332	0.000	0.000	0.000	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	71	66	89	58	0	154	36	0
N.S.	1	1.00	1.78	1.65	2.22	1.45	0.00	3.85	0.90	0.00
time (sec)	N/A	0.028	0.057	0.051	0.326	0.317	0.000	0.028	0.364	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	32	31	50	153	40	31	65
N.S.	1	1.00	0.90	0.82	0.79	1.28	3.92	1.03	0.79	1.67
time (sec)	N/A	0.020	0.020	0.105	0.335	0.309	0.277	0.001	0.359	2.946

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	105	102	118	0	0	0	-1	0
N.S.	1	1.00	0.86	0.84	0.97	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.059	0.017	0.112	0.362	0.324	0.000	0.000	0.000	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	14	22	0	-1	26
N.S.	1	1.00	1.00	0.00	0.00	0.50	0.79	0.00	-0.04	0.93
time (sec)	N/A	0.021	0.011	0.065	0.000	0.313	0.470	0.000	0.000	2.096

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	25	25	26	31	25	25
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.84	1.00	0.81	0.81
time (sec)	N/A	0.023	0.019	0.073	0.248	0.349	0.702	0.003	0.571	2.228

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	61	70	0	90	0	92	-1	0
N.S.	1	1.00	1.07	1.23	0.00	1.58	0.00	1.61	-0.02	0.00
time (sec)	N/A	0.020	0.066	0.071	0.000	0.345	0.000	0.015	0.000	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	140	72	76	116	125	0	566	-1	0
N.S.	1	1.71	0.88	0.93	1.41	1.52	0.00	6.90	-0.01	0.00
time (sec)	N/A	0.048	0.051	0.079	0.760	0.317	0.000	0.101	0.000	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	B	F(-1)	C	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	66	0	0	519	0	269	-1	0
N.S.	1	1.00	1.35	0.00	0.00	10.59	0.00	5.49	-0.02	0.00
time (sec)	N/A	0.087	0.143	0.041	0.000	0.338	0.000	0.061	0.000	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	40	854	54	423	214	90	103	102
N.S.	1	1.00	1.11	23.72	1.50	11.75	5.94	2.50	2.86	2.83
time (sec)	N/A	0.076	0.121	0.520	0.335	0.332	88.205	0.026	0.536	66.424

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	F	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	64	0	16	26	0	27	-1	0
N.S.	1	1.00	2.29	0.00	0.57	0.93	0.00	0.96	-0.04	0.00
time (sec)	N/A	0.047	0.532	0.026	0.338	0.327	0.000	0.007	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [446] had the largest ratio of [61]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	14	0.071
2	A	1	1	1.00	13	0.077
3	A	1	1	1.00	5	0.200
4	A	2	2	1.00	10	0.200
5	A	1	1	1.00	12	0.083
6	A	2	2	1.00	5	0.400
7	A	2	2	1.00	5	0.400
8	A	2	1	1.00	7	0.143
9	A	1	1	1.00	6	0.167
10	A	1	1	1.00	8	0.125
11	A	2	2	1.00	12	0.167
12	A	2	2	1.00	17	0.118
13	A	2	2	1.00	18	0.111
14	A	3	2	1.00	19	0.105
15	A	3	2	1.00	20	0.100
16	A	3	2	1.22	19	0.105
17	A	3	2	1.22	20	0.100
18	A	2	2	1.00	10	0.200
19	A	2	2	1.00	13	0.154
20	A	2	2	1.00	8	0.250
21	A	2	1	1.00	12	0.083
22	A	2	2	1.00	12	0.167
23	A	2	2	1.00	14	0.143
24	A	3	2	1.00	16	0.125
25	A	4	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	1	1.00	13	0.077
27	A	3	2	1.00	15	0.133
28	A	3	3	1.00	15	0.200
29	A	1	1	1.00	9	0.111
30	A	1	1	1.00	9	0.111
31	A	4	3	1.00	10	0.300
32	A	2	2	1.00	4	0.500
33	A	2	2	1.00	4	0.500
34	A	2	2	1.00	7	0.286
35	A	2	1	1.00	7	0.143
36	A	3	2	1.00	9	0.222
37	A	2	2	1.00	8	0.250
38	A	2	2	1.00	8	0.250
39	A	4	2	1.00	9	0.222
40	A	4	4	1.00	9	0.444
41	A	2	2	1.00	9	0.222
42	A	2	2	1.00	11	0.182
43	A	3	2	1.00	19	0.105
44	A	3	2	1.00	10	0.200
45	A	3	3	1.00	16	0.188
46	A	3	2	1.00	14	0.143
47	A	4	3	1.00	11	0.273
48	A	2	2	1.00	13	0.154
49	A	3	2	1.00	13	0.154
50	A	3	3	1.00	15	0.200
51	A	3	3	1.00	17	0.176
52	A	3	3	1.00	17	0.176
53	A	3	3	1.00	15	0.200
54	A	2	2	1.00	12	0.167
55	A	2	2	1.00	14	0.143
56	A	2	2	1.00	11	0.182
57	A	3	3	1.00	18	0.167
58	A	2	2	1.00	16	0.125
59	A	1	1	1.10	18	0.056
60	A	7	5	1.00	17	0.294

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	2	2	1.30	10	0.200
62	A	3	1	1.00	11	0.091
63	A	2	1	1.00	17	0.059
64	A	3	2	1.06	19	0.105
65	A	3	2	1.00	19	0.105
66	A	3	3	1.00	11	0.273
67	A	3	3	1.00	17	0.176
68	A	1	1	1.00	12	0.083
69	A	1	1	1.00	24	0.042
70	A	1	1	1.00	16	0.062
71	A	2	2	1.00	6	0.333
72	A	1	1	1.00	6	0.167
73	A	5	4	1.06	12	0.333
74	A	2	1	1.00	4	0.250
75	A	4	3	1.00	9	0.333
76	A	3	2	1.00	4	0.500
77	A	1	1	1.00	8	0.125
78	A	1	1	1.00	10	0.100
79	A	1	1	1.00	6	0.167
80	A	1	1	1.00	3	0.333
81	A	3	4	1.00	8	0.500
82	A	3	3	1.00	6	0.500
83	A	4	4	1.00	6	0.667
84	A	3	3	1.00	4	0.750
85	A	4	4	1.00	13	0.308
86	A	6	6	1.50	12	0.500
87	A	2	1	1.00	11	0.091
88	A	2	1	1.00	16	0.062
89	A	1	1	1.00	15	0.067
90	A	2	1	1.00	11	0.091
91	A	3	2	1.00	13	0.154
92	A	3	2	1.00	12	0.167
93	A	4	4	1.00	16	0.250
94	A	5	5	1.00	14	0.357
95	A	7	6	1.00	29	0.207

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	2	1.00	19	0.105
97	A	2	1	1.00	39	0.026
98	A	12	5	1.65	21	0.238
99	A	3	2	1.00	20	0.100
100	A	2	1	1.00	21	0.048
101	A	2	1	1.00	11	0.091
102	A	2	1	1.00	9	0.111
103	A	2	1	1.00	25	0.040
104	A	2	1	1.00	24	0.042
105	A	3	2	1.00	19	0.105
106	A	2	1	1.00	19	0.053
107	A	5	4	1.00	19	0.210
108	A	5	4	1.00	16	0.250
109	A	3	3	1.00	14	0.214
110	A	6	5	1.00	33	0.152
111	A	5	4	1.00	21	0.190
112	A	5	4	1.00	21	0.190
113	A	9	5	1.00	10	0.500
114	A	10	9	1.00	17	0.529
115	A	6	5	1.00	26	0.192
116	A	6	2	1.00	29	0.069
117	A	3	2	1.00	30	0.067
118	A	6	6	1.00	9	0.667
119	A	6	6	1.00	11	0.546
120	A	1	1	1.00	13	0.077
121	A	4	4	1.00	13	0.308
122	A	7	7	1.00	13	0.538
123	A	7	7	1.00	13	0.538
124	A	3	2	1.00	13	0.154
125	A	8	7	1.00	13	0.538
126	A	1	1	1.00	15	0.067
127	A	3	3	1.00	11	0.273
128	A	2	2	1.00	13	0.154
129	A	4	4	1.00	15	0.267
130	A	4	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	3	3	1.00	15	0.200
132	A	4	4	1.00	15	0.267
133	A	1	1	1.00	17	0.059
134	A	2	2	1.00	11	0.182
135	A	9	6	1.00	13	0.462
136	A	6	6	1.00	9	0.667
137	A	6	6	1.00	11	0.546
138	A	6	6	1.00	13	0.462
139	A	6	6	1.00	13	0.462
140	A	1	1	1.00	13	0.077
141	A	4	4	1.00	13	0.308
142	A	7	7	1.00	13	0.538
143	A	7	7	1.00	13	0.538
144	A	7	7	1.00	13	0.538
145	A	1	1	1.00	15	0.067
146	A	22	8	1.00	13	0.615
147	A	4	3	1.00	10	0.300
148	A	4	4	1.00	16	0.250
149	A	3	3	1.00	19	0.158
150	A	5	5	1.00	26	0.192
151	A	7	7	1.00	7	1.000
152	A	4	4	1.00	18	0.222
153	A	7	7	1.00	9	0.778
154	A	5	4	1.54	18	0.222
155	A	5	5	1.00	18	0.278
156	A	5	4	1.00	16	0.250
157	A	4	3	1.00	16	0.188
158	A	2	1	1.00	16	0.062
159	A	2	1	1.00	9	0.111
160	A	3	2	1.00	15	0.133
161	A	3	2	1.00	11	0.182
162	A	2	1	1.00	11	0.091
163	A	5	3	1.00	10	0.300
164	A	4	3	1.00	10	0.300
165	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	4	3	1.17	11	0.273
167	A	5	3	1.00	11	0.273
168	A	3	3	1.00	18	0.167
169	A	3	3	1.00	18	0.167
170	A	4	4	1.00	11	0.364
171	A	3	3	1.22	21	0.143
172	A	1	1	1.00	13	0.077
173	A	3	2	1.00	13	0.154
174	A	12	8	1.00	13	0.615
175	A	5	4	1.00	13	0.308
176	A	3	2	1.00	13	0.154
177	A	8	8	1.00	13	0.615
178	A	5	4	1.00	16	0.250
179	A	4	4	1.00	18	0.222
180	A	2	1	1.00	44	0.023
181	A	7	6	1.00	16	0.375
182	A	7	6	1.00	22	0.273
183	A	2	1	1.00	15	0.067
184	A	3	2	1.00	13	0.154
185	A	3	2	1.00	13	0.154
186	A	2	1	1.14	11	0.091
187	A	2	1	1.00	11	0.091
188	A	2	1	1.00	9	0.111
189	A	2	1	1.00	17	0.059
190	A	2	1	1.00	19	0.053
191	A	2	1	1.00	19	0.053
192	A	2	1	1.00	19	0.053
193	A	2	2	1.00	19	0.105
194	A	4	4	1.00	19	0.210
195	A	3	3	1.00	19	0.158
196	A	4	4	1.00	19	0.210
197	A	5	4	1.00	19	0.210
198	A	2	2	1.00	21	0.095
199	A	3	2	1.00	14	0.143
200	A	4	4	1.00	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	4	3	1.00	14	0.214
202	A	7	3	1.00	10	0.300
203	A	7	6	1.00	16	0.375
204	A	8	5	1.00	14	0.357
205	A	7	5	1.00	20	0.250
206	A	3	2	1.00	14	0.143
207	A	6	4	1.00	13	0.308
208	A	9	4	1.00	24	0.167
209	A	5	3	1.00	33	0.091
210	A	4	3	1.00	11	0.273
211	A	2	1	1.00	13	0.077
212	A	4	3	1.00	21	0.143
213	A	5	4	1.00	19	0.210
214	A	3	2	1.00	17	0.118
215	A	5	4	1.00	11	0.364
216	A	9	4	1.00	13	0.308
217	A	8	5	1.00	11	0.454
218	A	3	3	1.00	15	0.200
219	A	4	4	1.03	19	0.210
220	A	6	6	1.00	27	0.222
221	A	33	16	1.05	52	0.308
222	A	46	21	1.79	56	0.375
223	A	2	2	1.16	15	0.133
224	A	2	2	1.20	15	0.133
225	A	3	3	1.19	15	0.200
226	B	3	3	2.81	13	0.231
227	A	9	9	1.09	19	0.474
228	B	6	6	2.69	17	0.353
229	A	2	2	1.00	12	0.167
230	A	4	4	1.00	17	0.235
231	A	7	6	1.00	17	0.353
232	B	3	3	2.51	17	0.176
233	A	3	3	1.00	17	0.176
234	A	5	4	1.00	17	0.235
235	A	2	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	2	2	1.00	14	0.143
237	A	2	2	1.00	14	0.143
238	A	2	2	1.00	19	0.105
239	A	2	2	1.00	19	0.105
240	A	3	3	1.00	22	0.136
241	A	3	3	1.00	22	0.136
242	A	5	4	1.00	17	0.235
243	A	5	5	1.00	21	0.238
244	A	9	7	1.00	20	0.350
245	A	5	5	1.00	24	0.208
246	A	5	4	1.00	21	0.190
247	A	5	4	1.00	30	0.133
248	A	5	4	1.11	32	0.125
249	A	2	2	1.00	30	0.067
250	A	4	3	1.00	15	0.200
251	A	3	2	1.00	13	0.154
252	A	3	2	1.00	11	0.182
253	A	3	2	1.00	20	0.100
254	A	2	2	1.00	18	0.111
255	A	4	4	1.00	17	0.235
256	A	3	3	1.00	17	0.176
257	A	5	5	1.00	20	0.250
258	A	4	4	1.00	24	0.167
259	A	15	9	1.00	33	0.273
260	A	32	14	1.00	44	0.318
261	A	4	4	1.00	16	0.250
262	A	5	5	1.00	18	0.278
263	A	6	5	1.00	18	0.278
264	A	5	5	1.00	19	0.263
265	A	4	4	1.00	24	0.167
266	A	2	2	1.00	10	0.200
267	A	4	4	1.00	14	0.286
268	A	1	1	1.00	10	0.100
269	A	1	1	1.00	12	0.083
270	A	4	4	1.00	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	5	5	1.00	14	0.357
272	A	4	3	1.00	10	0.300
273	A	5	3	1.00	10	0.300
274	A	3	3	1.00	14	0.214
275	A	4	4	1.00	14	0.286
276	A	4	4	1.00	14	0.286
277	A	5	5	1.00	14	0.357
278	A	2	2	1.00	16	0.125
279	A	10	7	1.00	22	0.318
280	A	6	6	1.00	18	0.333
281	A	6	6	1.00	28	0.214
282	A	4	4	1.00	34	0.118
283	A	14	10	1.00	24	0.417
284	A	3	2	1.00	12	0.167
285	A	2	2	1.00	14	0.143
286	A	2	2	1.00	14	0.143
287	A	5	4	1.00	16	0.250
288	A	3	2	1.31	14	0.143
289	A	7	6	1.00	23	0.261
290	A	26	8	1.00	29	0.276
291	A	36	8	1.00	31	0.258
292	A	4	3	1.00	11	0.273
293	A	1	1	1.00	15	0.067
294	A	6	6	1.00	13	0.462
295	A	2	1	1.00	13	0.077
296	A	6	6	1.00	17	0.353
297	A	3	2	1.00	15	0.133
298	A	13	9	1.00	17	0.529
299	A	1	1	1.00	13	0.077
300	A	1	1	1.00	13	0.077
301	A	5	5	1.00	15	0.333
302	A	6	6	1.00	13	0.462
303	A	5	5	1.00	15	0.333
304	A	6	5	1.00	15	0.333
305	A	6	6	1.00	15	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	6	6	1.39	13	0.462
307	A	8	6	0.90	13	0.462
308	A	3	3	1.00	22	0.136
309	A	5	5	1.00	16	0.312
310	A	5	5	1.00	18	0.278
311	A	3	3	1.00	23	0.130
312	A	1	1	1.00	23	0.043
313	A	9	4	1.00	39	0.103
314	A	1	1	1.00	17	0.059
315	A	10	7	1.00	17	0.412
316	A	2	2	1.00	15	0.133
317	A	5	5	1.00	17	0.294
318	A	3	2	1.23	17	0.118
319	A	3	3	1.00	32	0.094
320	A	2	2	1.00	24	0.083
321	A	2	2	1.00	24	0.083
322	A	6	6	1.44	18	0.333
323	A	6	6	1.44	18	0.333
324	A	2	2	1.00	27	0.074
325	A	2	2	1.00	27	0.074
326	A	1	1	1.00	21	0.048
327	A	1	1	1.00	44	0.023
328	A	2	2	1.00	27	0.074
329	A	2	2	1.00	31	0.065
330	A	2	2	1.00	4	0.500
331	A	2	1	1.00	4	0.250
332	A	3	2	1.00	4	0.500
333	A	4	2	1.00	4	0.500
334	A	5	2	1.00	4	0.500
335	B	3	2	3.20	14	0.143
336	A	2	1	1.00	14	0.071
337	A	2	1	1.00	4	0.250
338	A	4	2	1.00	4	0.500
339	A	2	1	1.00	4	0.250
340	A	2	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	4	2	1.00	4	0.500
342	A	3	2	1.00	4	0.500
343	A	3	2	1.00	14	0.143
344	A	6	3	1.00	9	0.333
345	A	3	2	1.00	9	0.222
346	A	4	3	1.00	7	0.429
347	A	3	2	1.00	9	0.222
348	A	3	3	1.00	9	0.333
349	A	5	3	1.00	9	0.333
350	A	7	3	1.00	9	0.333
351	A	9	3	1.00	9	0.333
352	A	1	1	1.00	13	0.077
353	A	3	2	1.00	23	0.087
354	A	2	2	1.00	9	0.222
355	A	2	1	1.00	7	0.143
356	A	2	2	1.00	7	0.286
357	A	3	3	1.00	9	0.333
358	A	3	2	1.00	9	0.222
359	A	3	2	1.00	11	0.182
360	A	3	2	1.00	11	0.182
361	A	4	3	1.00	9	0.333
362	A	3	3	1.00	29	0.103
363	A	8	5	0.95	22	0.227
364	A	15	7	1.00	17	0.412
365	A	4	3	1.00	17	0.176
366	A	4	2	1.00	9	0.222
367	A	4	3	1.00	7	0.429
368	A	1	1	1.00	7	0.143
369	A	4	4	1.00	9	0.444
370	A	6	2	1.00	9	0.222
371	A	4	3	1.00	9	0.333
372	A	3	1	1.00	9	0.111
373	A	7	6	1.02	11	0.546
374	A	5	5	1.55	9	0.556
375	A	5	4	1.00	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	3	3	1.57	14	0.214
377	B	3	3	2.85	12	0.250
378	A	2	1	1.00	16	0.062
379	A	6	4	1.00	10	0.400
380	A	4	3	1.00	9	0.333
381	A	3	2	1.00	9	0.222
382	A	4	4	1.00	21	0.190
383	A	2	1	1.00	9	0.111
384	A	2	2	1.00	7	0.286
385	A	4	3	1.00	9	0.333
386	A	4	3	1.00	9	0.333
387	A	5	5	1.00	7	0.714
388	A	4	2	1.00	7	0.286
389	A	4	2	1.00	9	0.222
390	A	1	1	1.00	10	0.100
391	A	1	1	1.00	12	0.083
392	A	2	2	1.00	10	0.200
393	A	2	2	1.00	12	0.167
394	A	3	3	1.00	12	0.250
395	A	3	2	1.00	14	0.143
396	A	4	2	1.00	32	0.062
397	A	11	8	1.00	6	1.333
398	A	9	9	1.21	8	1.125
399	A	6	5	1.00	12	0.417
400	A	4	2	1.00	32	0.062
401	A	19	13	1.58	13	1.000
402	A	1	1	1.00	11	0.091
403	A	1	1	1.00	11	0.091
404	A	2	2	1.00	11	0.182
405	A	6	5	1.00	16	0.312
406	A	4	3	1.00	13	0.231
407	A	4	3	1.00	13	0.231
408	A	1	1	1.00	13	0.077
409	A	2	2	1.00	13	0.154
410	A	3	3	1.00	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	6	4	1.40	35	0.114
412	A	5	4	1.53	11	0.364
413	B	13	9	2.22	11	0.818
414	A	5	3	1.00	13	0.231
415	A	4	2	1.00	13	0.154
416	B	27	11	2.17	27	0.407
417	A	66	21	1.83	41	0.512
418	A	13	4	1.13	28	0.143
419	A	5	3	0.92	15	0.200
420	A	5	3	1.00	18	0.167
421	A	5	4	1.00	20	0.200
422	A	4	3	1.00	20	0.150
423	A	3	3	1.00	19	0.158
424	A	4	3	1.00	22	0.136
425	A	4	3	1.00	23	0.130
426	A	18	13	1.07	33	0.394
427	A	27	6	1.00	39	0.154
428	A	5	3	1.00	17	0.176
429	A	3	3	1.00	11	0.273
430	A	5	4	1.00	11	0.364
431	A	1	1	1.00	11	0.091
432	A	6	6	1.00	13	0.462
433	A	11	9	1.05	28	0.321
434	A	7	6	1.00	12	0.500
435	A	4	4	1.00	12	0.333
436	A	10	10	1.00	22	0.454
437	A	5	4	1.00	23	0.174
438	A	16	12	1.00	31	0.387
439	A	10	7	1.00	48	0.146
440	A	7	5	1.00	15	0.333
441	A	6	5	1.00	15	0.333
442	A	6	6	1.00	19	0.316
443	A	7	7	1.00	15	0.467
444	A	6	6	1.00	19	0.316
445	A	6	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	29	16	1.31	61	0.262
447	B	22	11	2.08	29	0.379
448	A	7	7	1.00	20	0.350
449	A	14	10	1.71	15	0.667
450	A	4	3	1.00	19	0.158
451	A	2	2	1.00	33	0.061
452	A	15	9	1.30	31	0.290
453	A	14	10	1.25	52	0.192
454	A	4	3	1.00	15	0.200
455	A	14	10	1.51	15	0.667
456	A	3	3	1.06	11	0.273
457	A	6	5	1.00	11	0.454
458	A	3	2	1.00	11	0.182
459	A	4	2	1.00	11	0.182
460	A	3	2	1.00	11	0.182
461	A	5	4	1.00	13	0.308
462	A	6	4	1.00	13	0.308
463	A	1	1	1.00	7	0.143
464	A	3	2	1.00	11	0.182
465	A	4	3	1.00	19	0.158
466	A	3	2	1.00	13	0.154
467	A	3	2	1.00	13	0.154
468	A	2	2	1.00	11	0.182
469	A	2	2	1.00	12	0.167
470	A	3	2	1.00	13	0.154
471	A	2	1	1.00	13	0.077
472	A	3	2	1.00	11	0.182
473	A	3	3	1.00	23	0.130
474	A	2	2	1.00	19	0.105
475	A	2	2	1.00	15	0.133
476	A	3	2	1.00	15	0.133
477	A	3	2	1.00	11	0.182
478	A	2	2	1.00	14	0.143
479	A	2	1	1.00	12	0.083
480	A	2	1	1.00	16	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	2	2	1.00	20	0.100
482	A	2	2	1.00	25	0.080
483	A	9	4	1.00	8	0.500
484	A	8	4	1.00	8	0.500
485	A	13	4	1.00	8	0.500
486	A	4	4	1.00	10	0.400
487	A	6	5	1.00	10	0.500
488	A	5	4	1.00	8	0.500
489	A	3	3	1.00	8	0.375
490	A	8	8	1.00	8	1.000
491	A	7	7	1.00	6	1.167
492	A	2	2	1.00	18	0.111
493	A	3	3	1.00	15	0.200
494	A	2	2	1.00	11	0.182
495	A	9	4	1.21	22	0.182
496	A	3	1	1.00	11	0.091
497	A	4	3	1.00	13	0.231
498	A	3	2	1.00	13	0.154
499	A	4	3	1.00	13	0.231
500	A	4	4	1.08	13	0.308
501	A	4	3	1.00	15	0.200
502	A	3	1	1.00	11	0.091
503	A	6	2	1.00	13	0.154
504	A	7	2	1.00	13	0.154
505	A	8	2	1.00	13	0.154
506	A	2	2	1.11	13	0.154
507	A	3	1	1.00	13	0.077
508	A	6	2	1.00	15	0.133
509	A	7	2	1.00	15	0.133
510	A	8	2	1.00	15	0.133
511	A	2	2	1.11	15	0.133
512	A	2	1	1.00	7	0.143
513	A	3	2	1.00	9	0.222
514	A	3	2	1.00	9	0.222
515	A	3	2	1.00	9	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	2	2	1.00	9	0.222
517	A	2	1	1.00	9	0.111
518	A	3	2	1.00	11	0.182
519	A	3	2	1.00	11	0.182
520	A	3	2	1.00	11	0.182
521	A	2	2	1.00	11	0.182
522	A	4	4	1.00	11	0.364
523	A	7	7	1.23	15	0.467
524	A	3	2	1.00	13	0.154
525	A	5	5	1.00	24	0.208
526	A	6	5	1.00	29	0.172
527	A	2	2	1.00	21	0.095
528	A	6	6	1.00	15	0.400
529	A	2	2	1.00	15	0.133
530	A	3	3	1.00	17	0.176
531	A	3	3	1.00	19	0.158
532	A	3	3	1.00	39	0.077
533	A	4	4	1.54	17	0.235
534	A	3	2	1.00	21	0.095
535	A	4	2	1.00	11	0.182
536	A	3	2	1.00	11	0.182
537	A	3	2	1.00	9	0.222
538	A	5	3	1.00	12	0.250
539	A	6	6	1.00	13	0.462
540	A	1	1	1.00	25	0.040
541	A	1	1	1.00	10	0.100
542	A	6	4	1.00	21	0.190
543	A	2	2	1.00	16	0.125
544	A	2	2	1.00	10	0.200
545	A	2	2	1.00	10	0.200
546	A	3	3	1.00	16	0.188
547	A	4	3	1.00	14	0.214
548	A	4	3	1.00	22	0.136
549	A	5	3	1.47	10	0.300
550	A	1	1	1.00	10	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	2	2	1.51	10	0.200
552	A	2	2	1.00	10	0.200
553	A	2	2	1.00	12	0.167
554	A	2	2	1.00	10	0.200
555	A	2	2	1.00	12	0.167
556	A	1	1	1.00	18	0.056
557	A	7	6	1.10	16	0.375
558	A	1	1	1.00	14	0.071
559	A	7	6	1.05	16	0.375
560	A	7	6	1.04	18	0.333
561	A	1	1	1.00	16	0.062
562	A	7	6	1.09	14	0.429
563	A	1	1	1.00	16	0.062
564	A	4	3	1.00	7	0.429
565	A	11	5	1.00	9	0.556
566	A	11	5	1.00	11	0.454
567	A	31	8	1.35	15	0.533
568	A	11	5	1.00	13	0.385
569	A	24	6	1.00	17	0.353
570	A	1	1	1.00	2	0.500
571	A	1	1	1.00	2	0.500
572	A	1	1	1.00	2	0.500
573	A	1	1	1.00	2	0.500
574	A	1	1	1.00	2	0.500
575	A	1	1	1.00	2	0.500
576	A	2	2	1.00	4	0.500
577	A	2	1	1.00	4	0.250
578	A	3	2	1.00	4	0.500
579	A	2	2	1.00	4	0.500
580	A	3	2	1.00	4	0.500
581	A	4	3	1.00	7	0.429
582	A	3	2	1.00	11	0.182
583	A	2	2	1.02	8	0.250
584	A	2	2	1.00	6	0.333
585	A	2	2	1.00	8	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	2	2	1.00	14	0.143
587	A	2	2	1.00	15	0.133
588	A	3	3	1.00	10	0.300
589	A	5	3	1.00	23	0.130
590	A	5	2	1.00	11	0.182
591	A	5	2	1.00	15	0.133
592	A	8	4	1.48	31	0.129
593	A	3	3	1.00	15	0.200
594	A	5	3	1.00	21	0.143
595	A	2	2	1.00	11	0.182
596	A	3	3	1.00	6	0.500
597	A	3	3	1.00	6	0.500
598	A	13	7	1.00	16	0.438
599	A	8	4	1.00	13	0.308
600	A	3	3	1.00	10	0.300
601	A	3	3	1.00	10	0.300
602	A	2	2	1.00	13	0.154
603	A	3	3	1.46	13	0.231
604	A	2	2	1.00	11	0.182
605	A	4	3	1.00	12	0.250
606	A	3	2	1.00	14	0.143
607	A	3	2	1.00	18	0.111
608	A	1	1	1.00	6	0.167
609	A	2	2	1.00	8	0.250
610	A	2	2	1.00	10	0.200
611	A	2	1	1.00	8	0.125
612	A	4	4	1.00	10	0.400
613	A	6	2	1.00	14	0.143
614	A	13	8	1.00	16	0.500
615	A	5	3	1.00	8	0.375
616	A	2	2	1.00	10	0.200
617	A	2	2	1.00	10	0.200
618	A	2	2	1.00	8	0.250
619	A	2	2	1.00	12	0.167
620	A	2	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	2	2	1.00	14	0.143
622	A	3	2	1.00	16	0.125
623	A	3	2	1.00	18	0.111
624	A	3	2	1.00	18	0.111
625	A	4	3	1.00	20	0.150
626	A	4	3	1.00	22	0.136
627	A	4	3	1.00	22	0.136
628	A	1	1	1.00	7	0.143
629	A	3	2	1.00	9	0.222
630	A	4	2	1.00	9	0.222
631	A	5	2	1.00	9	0.222
632	A	3	2	1.00	9	0.222
633	A	3	3	1.00	8	0.375
634	A	3	1	1.00	8	0.125
635	A	2	2	1.00	6	0.333
636	A	2	3	1.00	6	0.500
637	A	2	2	1.00	14	0.143
638	A	3	2	1.00	8	0.250
639	A	8	6	1.00	20	0.300
640	A	6	6	1.00	14	0.429
641	A	4	4	1.00	8	0.500
642	A	4	3	1.00	8	0.375
643	A	4	5	1.00	14	0.357
644	A	6	7	1.00	12	0.583
645	A	5	5	1.00	8	0.625
646	A	5	5	1.00	8	0.625
647	A	10	7	1.00	8	0.875
648	A	13	8	1.00	8	1.000
649	A	5	5	1.00	8	0.625
650	A	10	5	1.00	8	0.625
651	A	3	3	1.00	14	0.214
652	A	3	3	1.00	14	0.214
653	A	2	1	1.00	15	0.067
654	A	6	5	1.00	14	0.357
655	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	4	5	1.00	17	0.294
657	A	10	7	1.00	17	0.412
658	A	4	3	1.00	17	0.176
659	A	3	3	1.00	17	0.176
660	A	5	3	1.00	17	0.176
661	A	2	2	1.00	15	0.133
662	A	2	2	1.00	15	0.133
663	A	4	4	1.00	14	0.286
664	A	3	5	1.00	17	0.294
665	A	8	6	1.00	17	0.353
666	A	4	4	1.00	17	0.235
667	A	5	4	1.00	17	0.235
668	A	6	4	1.00	19	0.210
669	A	3	3	1.00	11	0.273
670	A	4	3	1.00	11	0.273
671	A	4	4	1.00	13	0.308
672	A	8	8	1.00	13	0.615
673	A	2	2	1.00	13	0.154
674	A	8	8	1.00	13	0.615
675	A	17	11	1.00	13	0.846
676	A	8	8	1.00	11	0.727
677	A	3	2	1.00	11	0.182
678	A	12	6	1.00	13	0.462
679	A	7	7	1.00	13	0.538
680	A	8	7	1.00	8	0.875
681	A	11	8	1.00	13	0.615
682	A	4	4	1.04	15	0.267
683	A	9	7	1.08	15	0.467
684	A	11	8	1.25	15	0.533
685	A	4	4	1.00	15	0.267
686	A	4	6	1.03	12	0.500
687	A	4	5	1.04	15	0.333
688	A	5	6	1.02	15	0.400
689	A	16	11	1.33	15	0.733
690	A	2	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
691	A	5	8	1.30	15	0.533
692	A	6	4	1.36	17	0.235
693	A	11	9	1.29	17	0.529
694	A	8	5	1.33	17	0.294
695	B	8	7	2.27	16	0.438
696	A	4	4	1.00	16	0.250
697	A	5	4	1.00	8	0.500
698	A	5	5	1.00	13	0.385
699	A	2	2	1.00	24	0.083
700	A	2	3	1.00	21	0.143
701	A	5	5	1.00	12	0.417
702	A	7	6	1.71	10	0.600
703	A	5	6	1.00	8	0.750
704	A	6	6	1.00	10	0.600
705	A	5	6	1.00	7	0.857

Chapter 3

Listing of integrals

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3.14	$\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)} dx$	235
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3.27	$\int \frac{-5+2x}{-2+3x^2} dx$	275
3.28	$\int \frac{-5+2x}{2+3x^2} dx$	278
3.29	$\int \sin\left(\frac{x}{4}\right) \sin(x) dx$	282
3.30	$\int \cos(3x) \cos(4x) dx$	285
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3.32	$\int \sin^2(x) dx$	292
3.33	$\int \cos^2(x) dx$	295
3.34	$\int \cos^3(x) \sin(x) dx$	298
3.35	$\int \cot^3(x) \csc(x) dx$	301
3.36	$\int \csc^2(x) \sec^2(x) dx$	304
3.37	$\int \cot^2\left(\frac{3x}{4}\right) dx$	307
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3.39	$\int (-\cot(x) + \tan(x))^2 dx$	313
3.40	$\int (-\sec(x) + \tan(x))^2 dx$	316
3.41	$\int \frac{\sin(x)}{1+\sin(x)} dx$	320
3.42	$\int \frac{\cos(x)}{1-\cos(x)} dx$	323
3.43	$\int e^{-x/2} (-1 + e^{x/2})^3 dx$	326
3.44	$\int \frac{1}{5-6x+x^2} dx$	329
3.45	$\int \frac{x^2}{13-6x^3+x^6} dx$	332
3.46	$\int \frac{2+x}{-1-4x+x^2} dx$	336
3.47	$\int \frac{1}{1+\sqrt[3]{1+x}} dx$	339
3.48	$\int \frac{1}{\sqrt{x} (b+ax)} dx$	343
3.49	$\int x^3 \sqrt{1+x^2} dx$	347
3.50	$\int \frac{x}{\sqrt{a^4-x^4}} dx$	350
3.51	$\int \frac{1}{x\sqrt{-a^2+x^2}} dx$	354
3.52	$\int \frac{1}{x\sqrt{a^2-x^2}} dx$	358
3.53	$\int \frac{1}{x\sqrt{a^2+x^2}} dx$	362
3.54	$\int \frac{1}{\sqrt{2+x-x^2}} dx$	366
3.55	$\int \frac{1}{\sqrt{5-4x+3x^2}} dx$	369
3.56	$\int \frac{1}{\sqrt{x-x^2}} dx$	372
3.57	$\int \frac{1+2x}{\sqrt{2+x-x^2}} dx$	375
3.58	$\int \frac{1}{x\sqrt{2+x-x^2}} dx$	379
3.59	$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}} dx$	383

3.60	$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)} dx$	386
3.61	$\int \frac{1}{2+3\cos^2(x)} dx$	390
3.62	$\int \csc(2x)(1-\tan(x)) dx$	394
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3.65	$\int \frac{\sin(2x)}{\sqrt[3]{a^2 - 4\sin^2(x)}} dx$	403
3.66	$\int \frac{1}{\sqrt{-1+a^{2x}}} dx$	406
3.67	$\int \frac{e^{x/2}}{\sqrt{-1+e^x}} dx$	410
3.68	$\int \frac{\tan^{-1}(x)^n}{1+x^2} dx$	413
3.69	$\int \frac{\sin^{-1}(\frac{x}{a})^{3/2}}{\sqrt{a^2-x^2}} dx$	416
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3.71	$\int x \log^2(x) dx$	422
3.72	$\int \frac{\log(x)}{x^5} dx$	425
3.73	$\int x^2 \log\left(\frac{-1+x}{x}\right) dx$	428
3.74	$\int \cos^5(x) dx$	432
3.75	$\int \cos^4(x) \sin^2(x) dx$	435
3.76	$\int \csc^5(x) dx$	439
3.77	$\int e^{-x} \sin(x) dx$	443
3.78	$\int e^{2x} \sin(3x) dx$	446
3.79	$\int a^x \cos(x) dx$	449
3.80	$\int \cos(\log(x)) dx$	452
3.81	$\int \log(\cos(x)) \sec^2(x) dx$	455
3.82	$\int x \tan^2(x) dx$	459
3.83	$\int \frac{\sin^{-1}(x)}{x^2} dx$	462
3.84	$\int \sin^{-1}(x)^2 dx$	466
3.85	$\int \frac{x^2 \tan^{-1}(x)}{1+x^2} dx$	469
3.86	$\int \cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) dx$	472
3.87	$\int (2x+3x^2)^3 dx$	476
3.88	$\int (-1+x)(-1+2x+3x^2)^2 dx$	479
3.89	$\int x^{-1+k} (a+bx^k)^n dx$	482
3.90	$\int \frac{x^3}{1+2x} dx$	485
3.91	$\int \frac{x^5}{2+3x^2} dx$	488
3.92	$\int \frac{1}{2-7x+3x^2} dx$	492
3.93	$\int \frac{-1+3x}{1-x+x^2} dx$	495
3.94	$\int \frac{x^2}{5+2x+x^2} dx$	499
3.95	$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2} dx$	503
3.96	$\int \frac{-1+x+x^2}{-6x+x^2+x^3} dx$	507
3.97	$\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3} dx$	510

3.98	$\int \frac{2-x+x^2}{4-5x^2+x^4} dx$	513
3.99	$\int \frac{-5+2x^2}{6-5x^2+x^4} dx$	517
3.100	$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)} dx$	521
3.101	$\int \frac{1+x^2}{(-1+x)^3} dx$	524
3.102	$\int \frac{x^5}{(3+x)^2} dx$	527
3.103	$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4} dx$	530
3.104	$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2} dx$	533
3.105	$\int \frac{2+x^2+x^3}{x(-1+x^2)^2} dx$	536
3.106	$\int \frac{1}{x^3-x^4-x^5+x^6} dx$	539
3.107	$\int \frac{1+x^4}{-1+x-x^2+x^3} dx$	542
3.108	$\int \frac{1}{x(1+x)(1+x^2)} dx$	546
3.109	$\int \frac{x^2}{-2+x^2+x^4} dx$	550
3.110	$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4} dx$	554
3.111	$\int \frac{x}{(1+x)(1+2x)^2(1+x^2)} dx$	558
3.112	$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)} dx$	562
3.113	$\int \frac{1}{1+x^2+x^4} dx$	566
3.114	$\int \frac{3+2x^3}{-9x+x^5} dx$	570
3.115	$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)} dx$	575
3.116	$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)} dx$	579
3.117	$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)} dx$	583
3.118	$\int \frac{1}{a^3+x^3} dx$	586
3.119	$\int \frac{x}{a^3+x^3} dx$	590
3.120	$\int \frac{x^2}{a^3+x^3} dx$	594
3.121	$\int \frac{1}{x(a^3+x^3)} dx$	597
3.122	$\int \frac{1}{x^2(a^3+x^3)} dx$	601
3.123	$\int \frac{1}{x^3(a^3+x^3)} dx$	606
3.124	$\int \frac{1}{x^4(a^3+x^3)} dx$	611
3.125	$\int \frac{1}{x^5(a^3+x^3)} dx$	614
3.126	$\int \frac{x^{-m}}{a^3+x^3} dx$	619
3.127	$\int \frac{1}{a^4-x^4} dx$	622
3.128	$\int \frac{x}{a^4-x^4} dx$	625
3.129	$\int \frac{1}{x(a^4-x^4)} dx$	628
3.130	$\int \frac{1}{x^2(a^4-x^4)} dx$	632
3.131	$\int \frac{1}{x^3(a^4-x^4)} dx$	636
3.132	$\int \frac{1}{x^4(a^4-x^4)} dx$	640
3.133	$\int \frac{x^{-m}}{a^4-x^4} dx$	644
3.134	$\int \frac{x}{a^4+x^4} dx$	647
3.135	$\int \frac{x^2}{a^4+x^4} dx$	650

3.136	$\int \frac{1}{a^5+x^5} dx$	655
3.137	$\int \frac{x}{a^5+x^5} dx$	660
3.138	$\int \frac{x^2}{a^5+x^5} dx$	665
3.139	$\int \frac{x^3}{a^5+x^5} dx$	670
3.140	$\int \frac{x^4}{a^5+x^5} dx$	675
3.141	$\int \frac{1}{x(a^5+x^5)} dx$	678
3.142	$\int \frac{1}{x^2(a^5+x^5)} dx$	682
3.143	$\int \frac{1}{x^3(a^5+x^5)} dx$	687
3.144	$\int \frac{1}{x^4(a^5+x^5)} dx$	692
3.145	$\int \frac{x^{-m}}{a^5+x^5} dx$	697
3.146	$\int \frac{1+x^4}{1+x^6} dx$	700
3.147	$\int \frac{1}{(5+3x+x^2)^3} dx$	705
3.148	$\int \frac{1+x^2+x^4}{(1+x^2)^4} dx$	709
3.149	$\int \frac{B+Ax}{(c+2bx+ax^2)^2} dx$	713
3.150	$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2} dx$	718
3.151	$\int \frac{1}{(-1+x^3)^2} dx$	722
3.152	$\int \frac{4+3x^4}{x^2(1+x^2)^3} dx$	727
3.153	$\int \frac{x}{1+x^6} dx$	731
3.154	$\int \frac{-1+x^{-1+n}}{-nx+x^n} dx$	735
3.155	$\int \frac{x^3}{1-2x^2+3x^4} dx$	739
3.156	$\int \frac{x^5}{-4+x^2+3x^4} dx$	743
3.157	$\int \frac{x^2}{9-10x^3+x^6} dx$	747
3.158	$\int \frac{1-4x^2+x^3}{(-2+x)^4} dx$	751
3.159	$\int \frac{x^3}{(-1+x)^{12}} dx$	754
3.160	$\int \frac{-3x+x^4}{(1+2x)^5} dx$	757
3.161	$\int \frac{1}{(-1+x)^2(1+x)^3} dx$	760
3.162	$\int \frac{1}{(5-6x)^2x^2} dx$	763
3.163	$\int \frac{1}{(-3-2x+x^2)^3} dx$	766
3.164	$\int \frac{1}{(13-4x+x^2)^3} dx$	770
3.165	$\int \frac{1}{(2+x)^3(3+x)^4} dx$	774
3.166	$\int \frac{x^6}{(-2+x^2)^2} dx$	777
3.167	$\int \frac{x^8}{(4+x^2)^4} dx$	781
3.168	$\int \frac{-4+7x}{(5+2x+3x^2)^2} dx$	785
3.169	$\int \frac{5-4x}{(-2-4x+3x^2)^2} dx$	789
3.170	$\int \frac{x^5}{(1+x^4)^3} dx$	793
3.171	$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2} dx$	797

3.172	$\int \frac{x^3}{(a^4+x^4)^3} dx$	801
3.173	$\int \frac{1}{x(a^4+x^4)^3} dx$	804
3.174	$\int \frac{1}{x^2(a^4+x^4)^3} dx$	808
3.175	$\int \frac{1}{x^3(a^4+x^4)^3} dx$	814
3.176	$\int \frac{x^{14}}{(3+2x^5)^3} dx$	818
3.177	$\int \frac{x^6}{(3+2x^5)^3} dx$	821
3.178	$\int \frac{9}{5x^2(3-2x^2)^3} dx$	829
3.179	$\int \frac{4+3x^4}{x^2(1+x^2)^3} dx$	833
3.180	$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5} dx$	837
3.181	$\int \frac{1+x^2}{x(1+x^3)^2} dx$	840
3.182	$\int \frac{-2-3x+x^2}{(1+x)^2(1+x+x^2)^2} dx$	845
3.183	$\int \frac{1}{(1-4x)^3(2-3x)} dx$	850
3.184	$\int \frac{x^3}{(2-5x^2)^7} dx$	853
3.185	$\int \frac{x^7}{(2-5x^2)^3} dx$	856
3.186	$\int \frac{1}{(-2+x)^3(1+x)^2} dx$	859
3.187	$\int \frac{1}{(2+x)^3(3+x)^4} dx$	862
3.188	$\int \frac{x^5}{(3+x)^2} dx$	865
3.189	$\int (b_1 + c_1x)(a + 2bx + cx^2) dx$	868
3.190	$\int (b_1 + c_1x)(a + 2bx + cx^2)^2 dx$	871
3.191	$\int (b_1 + c_1x)(a + 2bx + cx^2)^3 dx$	874
3.192	$\int (b_1 + c_1x)(a + 2bx + cx^2)^4 dx$	878
3.193	$\int (b_1 + c_1x)(a + 2bx + cx^2)^n dx$	882
3.194	$\int \frac{b_1+c_1x}{a+2bx+cx^2} dx$	885
3.195	$\int \frac{b_1+c_1x}{(a+2bx+cx^2)^2} dx$	889
3.196	$\int \frac{b_1+c_1x}{(a+2bx+cx^2)^3} dx$	894
3.197	$\int \frac{b_1+c_1x}{(a+2bx+cx^2)^4} dx$	900
3.198	$\int (b_1 + c_1x)(a + 2bx + cx^2)^{-n} dx$	907
3.199	$\int \frac{x}{3+6x+2x^2} dx$	910
3.200	$\int \frac{-3+2x}{(3+6x+2x^2)^3} dx$	913
3.201	$\int \frac{-1+x}{(4+5x+x^2)^2} dx$	917
3.202	$\int \frac{1}{(2+3x+x^2)^5} dx$	921
3.203	$\int \frac{1}{x^3(7-6x+2x^2)^2} dx$	925
3.204	$\int \frac{x^9}{(2+3x+x^2)^5} dx$	930
3.205	$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5} dx$	935
3.206	$\int \frac{(a-bx^2)^3}{x^7} dx$	940

3.207	$\int \frac{x^{13}}{(a^4+x^4)^5} dx$	943
3.208	$\int (2\sqrt{x} - x)^2 x^{3/2} (1+x^2) dx$	947
3.209	$\int (-3x^{3/5} + x^{3/2})^2 \left(-\frac{x^{2/3}}{3} + 4x^{3/2}\right) dx$	951
3.210	$\int \frac{1}{1+\sqrt{1+x}} dx$	955
3.211	$\int \frac{x}{1+\sqrt{1+x}} dx$	959
3.212	$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}} dx$	962
3.213	$\int \frac{1}{-\sqrt{1+x} + (1+x)^{2/3}} dx$	965
3.214	$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}} dx$	969
3.215	$\int \frac{1}{x^3(1+x)^{3/2}} dx$	972
3.216	$\int \frac{1}{(1-x)^{7/2}x^5} dx$	980
3.217	$\int \frac{1}{(-1+x)^{2/3}x^5} dx$	985
3.218	$\int \sqrt{\frac{1-x}{1+x}} dx$	990
3.219	$\int x \sqrt{\frac{-a+x}{b-x}} dx$	994
3.220	$\int \frac{\sqrt{-5+x} \sqrt{3+x}}{(-1+x)(-25+x^2)} dx$	998
3.221	$\int \frac{x^2 \sqrt{1+x} \sqrt[4]{1-x^2}}{\sqrt{1-x} (\sqrt{1-x} - \sqrt{1+x})} dx$	1002
3.222	$\int \frac{\sqrt{1-x} x(1+x)^{2/3}}{-(1-x)^{5/6} \sqrt[3]{1+x} + (1-x)^{2/3} \sqrt{1+x}} dx$	1010
3.223	$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}} dx$	1019
3.224	$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}} dx$	1022
3.225	$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}} dx$	1025
3.226	$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}} dx$	1029
3.227	$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}} dx$	1033
3.228	$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2} dx$	1038
3.229	$\int \frac{1}{(-3-2x+x^2)^{5/2}} dx$	1044
3.230	$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}} dx$	1047
3.231	$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}} dx$	1051
3.232	$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}} dx$	1056
3.233	$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}} dx$	1060
3.234	$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}} dx$	1064
3.235	$\int \frac{1}{\sqrt{4+3x-2x^2}} dx$	1068

3.236	$\int \frac{1}{\sqrt{-3+4x-x^2}} dx$	1071
3.237	$\int \frac{1}{\sqrt{-2-5x-3x^2}} dx$	1074
3.238	$\int \frac{1}{\sqrt{1-x^2}(4+x^2)} dx$	1077
3.239	$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}} dx$	1081
3.240	$\int \frac{x}{(3-x^2)\sqrt{5-x^2}} dx$	1085
3.241	$\int \frac{x}{\sqrt{3-x^2}(5-x^2)} dx$	1089
3.242	$\int \frac{1}{\sqrt{2+x^2}(-1+x^4)} dx$	1093
3.243	$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}} dx$	1097
3.244	$\int \frac{1}{\sqrt{5+2x+x^2}(-8+x^3)} dx$	1101
3.245	$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}} dx$	1106
3.246	$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}} dx$	1111
3.247	$\int \frac{1+2x}{\sqrt{-1+6x+x^2}(4+4x+3x^2)} dx$	1115
3.248	$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}} dx$	1120
3.249	$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}} dx$	1125
3.250	$\int x^4\sqrt{5-x^2} dx$	1129
3.251	$\int \frac{1}{x^6\sqrt{2+x^2}} dx$	1133
3.252	$\int \frac{1}{(3+2x^2)^{7/2}} dx$	1137
3.253	$\int \frac{x}{1+x^2+a\sqrt{1+x^2}} dx$	1141
3.254	$\int \frac{1-x+x^2}{(1+x^2)^{3/2}} dx$	1145
3.255	$\int \frac{\sqrt{1+x^2}}{2+x^2} dx$	1148
3.256	$\int \frac{1}{\sqrt{1+x^2}(2+x^2)^2} dx$	1152
3.257	$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}} dx$	1156
3.258	$\int \frac{5+x^2}{\sqrt{1-x^2}(1+x^2)^2} dx$	1160
3.259	$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}} dx$	1164
3.260	$\int \frac{x^2(2-\sqrt{1+x^2})}{\sqrt{1+x^2}(1-x^3+(1+x^2)^{3/2})} dx$	1169
3.261	$\int x\sqrt{2rx-x^2} dx$	1176
3.262	$\int x^2\sqrt{2rx-x^2} dx$	1180
3.263	$\int x^3\sqrt{2rx-x^2} dx$	1184
3.264	$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}} dx$	1188
3.265	$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}} dx$	1192

3.266	$\int \frac{1}{\sqrt{1+x+x^2}} dx$	1196
3.267	$\int \frac{1}{x^3 \sqrt{1+x+x^2}} dx$	1199
3.268	$\int \frac{1}{(1+x+x^2)^{3/2}} dx$	1203
3.269	$\int \frac{x}{(1+x+x^2)^{3/2}} dx$	1206
3.270	$\int \frac{x^3}{(1+x+x^2)^{3/2}} dx$	1209
3.271	$\int x^2 \sqrt{1+x+x^2} dx$	1213
3.272	$\int (1+x+x^2)^{3/2} dx$	1217
3.273	$\int (1+x+x^2)^{5/2} dx$	1221
3.274	$\int \frac{1}{x^2 \sqrt{1+x+x^2}} dx$	1225
3.275	$\int \frac{1}{x^3 \sqrt{1+x+x^2}} dx$	1229
3.276	$\int \frac{1}{x^2(1+x+x^2)^{3/2}} dx$	1233
3.277	$\int \frac{1}{x^3(1+x+x^2)^{3/2}} dx$	1237
3.278	$\int \frac{1}{(1+x)\sqrt{1+x+x^2}} dx$	1241
3.279	$\int \frac{1}{\sqrt{4+2x+x^2}(-x+x^3)} dx$	1244
3.280	$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2} dx$	1249
3.281	$\int \frac{3+2x}{(3+2x+x^2)^2 \sqrt{4+2x+x^2}} dx$	1254
3.282	$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}(-3+x+2x^2)} dx$	1259
3.283	$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}} dx$	1263
3.284	$\int \frac{1}{(4+2x+x^2)^{7/2}} dx$	1268
3.285	$\int \frac{1}{(1+8x+3x^2)^{5/2}} dx$	1272
3.286	$\int \frac{1}{(5+4x-3x^2)^{5/2}} dx$	1275
3.287	$\int \frac{1}{1+\sqrt{2+2x+x^2}} dx$	1278
3.288	$\int \frac{1}{x+\sqrt{1+x+x^2}} dx$	1282
3.289	$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}} dx$	1286
3.290	$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}} dx$	1290
3.291	$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}} dx$	1295
3.292	$\int \frac{1}{\sqrt{-1+x} x^3} dx$	1300
3.293	$\int \frac{1}{(1-\frac{3}{x})^{4/3} x^2} dx$	1304
3.294	$\int \frac{(-1+3x)^{4/3}}{x^2} dx$	1307
3.295	$\int (4-3x)^{4/3} x^2 dx$	1312
3.296	$\int \frac{(1-2\sqrt[3]{x})^{3/4}}{x} dx$	1315

3.297	$\int \frac{x}{(3-2\sqrt{x})^{3/4}} dx$	1319
3.298	$\int \frac{(-1+2\sqrt{x})^{5/4}}{x^2} dx$	1326
3.299	$\int x^6 \sqrt[3]{1+x^7} dx$	1332
3.300	$\int \frac{x^6}{(1+x^7)^{5/3}} dx$	1335
3.301	$\int \frac{1}{x(-27+2x^7)^{2/3}} dx$	1338
3.302	$\int \frac{(1+x^7)^{2/3}}{x^8} dx$	1342
3.303	$\int \frac{\sqrt[4]{3+4x^4}}{x^2} dx$	1347
3.304	$\int x^2 (3+4x^4)^{5/4} dx$	1351
3.305	$\int x^6 \sqrt[4]{3+4x^4} dx$	1356
3.306	$\int \sqrt[3]{x(1-x^2)} dx$	1361
3.307	$\int \sqrt{(1+\sqrt[3]{x})x} dx$	1366
3.308	$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}} dx$	1371
3.309	$\int x^9 \sqrt{1+x^5+x^{10}} dx$	1375
3.310	$\int \frac{1}{x^5 \sqrt{4+2x^2+x^4}} dx$	1379
3.311	$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}} dx$	1383
3.312	$\int (-3x+2x^3)(-3x^2+x^4)^{3/5} dx$	1387
3.313	$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}} dx$	1390
3.314	$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}} dx$	1394
3.315	$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}} dx$	1398
3.316	$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}} dx$	1403
3.317	$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2} dx$	1407
3.318	$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}} dx$	1411
3.319	$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}} dx$	1414
3.320	$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}} dx$	1420
3.321	$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}} dx$	1424
3.322	$\int \frac{1+x^2}{x\sqrt{1+x^4}} dx$	1428
3.323	$\int \frac{-1+x^2}{x\sqrt{1+x^4}} dx$	1432
3.324	$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}} dx$	1436
3.325	$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}} dx$	1440
3.326	$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}} dx$	1443

3.327	$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}} dx$	1446
3.328	$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}} dx$	1450
3.329	$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}} dx$	1453
3.330	$\int \cos^2(x) dx$	1456
3.331	$\int \cos^3(x) dx$	1459
3.332	$\int \sin^4(x) dx$	1462
3.333	$\int \cos^6(x) dx$	1465
3.334	$\int \sin^8(x) dx$	1468
3.335	$\int \cos^4\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$	1471
3.336	$\int -\sin^3\left(\frac{\pi}{12} - 3x\right) dx$	1474
3.337	$\int \csc^6(x) dx$	1477
3.338	$\int \csc^7(x) dx$	1480
3.339	$\int \sec^{12}(x) dx$	1484
3.340	$\int \sec^3\left(\frac{\pi}{4} + 3x\right) dx$	1487
3.341	$\int \tan^6(x) dx$	1491
3.342	$\int \cot^5(x) dx$	1494
3.343	$\int \cot^4\left(\frac{\pi}{4} + \frac{x}{3}\right) dx$	1497
3.344	$\int \cos^6(x) \sin^4(x) dx$	1500
3.345	$\int \cos^6(x) \sin^7(x) dx$	1504
3.346	$\int \sin^{10}(x) \tan(x) dx$	1507
3.347	$\int \csc^6(x) \sec^6(x) dx$	1511
3.348	$\int \cos^2(x) \sin^2(x) dx$	1514
3.349	$\int \cos^4(x) \sin^4(x) dx$	1517
3.350	$\int \cos^6(x) \sin^6(x) dx$	1521
3.351	$\int \cos^8(x) \sin^8(x) dx$	1525
3.352	$\int \cos^{2m}(x) \sin^{2m}(x) dx$	1529
3.353	$\int \csc^3\left(\frac{\pi}{4} + 2x\right) \sec\left(\frac{\pi}{4} + 2x\right) dx$	1532
3.354	$\int \sec^2(x) \tan^2(x) dx$	1536
3.355	$\int \cot^3(x) \csc(x) dx$	1539
3.356	$\int \sec^3(x) \tan(x) dx$	1542
3.357	$\int \cot^2(x) \csc^3(x) dx$	1545
3.358	$\int \cot^3(x) \csc^4(x) dx$	1549
3.359	$\int \sec^{\frac{13}{2}}(x) \sin^5(x) dx$	1552
3.360	$\int \sec^4(x) \tan^{\frac{3}{2}}(x) dx$	1555
3.361	$\int \cot^4(x) \csc^3(x) dx$	1558
3.362	$\int \sec^3\left(\frac{\pi}{4} + \frac{x}{2}\right) \tan^2\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$	1562
3.363	$\int (1 + \cot^3(x)) (a \sec^2(x) - \sin(2x))^2 dx$	1566
3.364	$\int (4 - 3 \cos(x)) \left(1 - \frac{\sin(x)}{2}\right)^4 dx$	1570
3.365	$\int \left(\frac{1}{2} - 3 \cot(x)\right) (3 - 2 \cot(x))^3 dx$	1574
3.366	$\int \cos(5x) \sec^5(x) dx$	1578
3.367	$\int \cos(4x) \sec(x) dx$	1581

3.368	$\int \cos(x) \cos(4x) dx$	1585
3.369	$\int \cos(4x) \sec^5(x) dx$	1588
3.370	$\int \cos^4(x) \cos(4x) dx$	1592
3.371	$\int \cos(5x) \csc^5(x) dx$	1595
3.372	$\int \csc^4(x) \sin(4x) dx$	1599
3.373	$\int \frac{\cot(x)}{2+\sin(2x)} dx$	1602
3.374	$\int \cos(x) \cot(x) \sec(3x) dx$	1606
3.375	$\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)} dx$	1610
3.376	$\int \frac{1}{4+\sqrt{3} \cos(x)+\sin(x)} dx$	1613
3.377	$\int \frac{1}{3+4 \cos(x)+4 \sin(x)} dx$	1617
3.378	$\int \frac{1}{4-3 \cos^2(x)+5 \sin^2(x)} dx$	1621
3.379	$\int \frac{1}{4+4 \cot(x)+\tan(x)} dx$	1624
3.380	$\int \frac{1}{(2 \sec(x)+\sin(x))^2} dx$	1628
3.381	$\int \frac{1}{(\cos(x)+2 \sec(x))^2} dx$	1632
3.382	$\int \frac{5-\tan(x)-6 \tan^2(x)}{(1+3 \tan(x))^3} dx$	1636
3.383	$\int \cos^2(x) \sec(3x) dx$	1640
3.384	$\int \sec(2x) \sin(x) dx$	1643
3.385	$\int \sec(2x) \sin^2(x) dx$	1647
3.386	$\int \sec(3x) \sin^3(x) dx$	1651
3.387	$\int \cos(x) \csc(3x) dx$	1655
3.388	$\int \csc(4x) \sin(x) dx$	1659
3.389	$\int \csc(4x) \sin^3(x) dx$	1663
3.390	$\int \sqrt{1+\sin(2x)} dx$	1667
3.391	$\int \sqrt{1-\sin(2x)} dx$	1670
3.392	$\int \frac{1}{\sqrt{1+\cos(2x)}} dx$	1673
3.393	$\int \frac{1}{\sqrt{1-\cos(2x)}} dx$	1676
3.394	$\int \frac{1}{(1-\cos(3x))^{3/2}} dx$	1680
3.395	$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2} dx$	1684
3.396	$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}} dx$	1688
3.397	$\int \sqrt{\tan(x)} dx$	1692
3.398	$\int \frac{1}{\sqrt[3]{\tan(5x)}} dx$	1697
3.399	$\int \frac{1}{(4+3 \tan(2x))^{3/2}} dx$	1702
3.400	$\int \frac{\sec^2(x)\left(-\sqrt{4-3 \tan(x)}+3 \tan(x)\right)}{(4-3 \tan(x))^{3/2}} dx$	1709
3.401	$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2} dx$	1713
3.402	$\int \frac{\sin(x)}{\sqrt{\sin(2x)}} dx$	1720

3.403	$\int \frac{\cos(x)}{\sqrt{\sin(2x)}} dx$	1723
3.404	$\int \sin(x) \sqrt{\sin(2x)} dx$	1726
3.405	$\int (\cos(x) - \sin(x)) \sqrt{\sin(2x)} dx$	1729
3.406	$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)} dx$	1733
3.407	$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)} dx$	1737
3.408	$\int \csc^5(x) \sin^{\frac{3}{2}}(2x) dx$	1742
3.409	$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}} dx$	1745
3.410	$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)} dx$	1748
3.411	$\int \frac{\cos^3(x)(\cos(2x) - 3 \tan(x))}{(\sin^2(x) - \sin(2x)) \sin^{\frac{5}{2}}(2x)} dx$	1752
3.412	$\int \sqrt{\sec^4(x) \tan(x)} dx$	1757
3.413	$\int \sqrt{\sin^4(x) \tan(x)} dx$	1761
3.414	$\int \sqrt[3]{\sec^{12}(x) \tan^2(x)} dx$	1767
3.415	$\int \frac{1}{\sqrt[4]{\cos^{11}(x) \sin^{13}(x)}} dx$	1771
3.416	$\int \frac{\cos(2x) - \sqrt{\sin(2x)}}{\sqrt{\cos^3(x) \sin(x)}} dx$	1775
3.417	$\int \frac{\sqrt{\cos(x) \sin^3(x)} - 2 \sin(2x)}{-\sqrt{\cos^3(x) \sin(x)} + \sqrt{\tan(x)}} dx$	1782
3.418	$\int \frac{-3 \tan(x) + \sqrt[3]{\sec^6(x) \tan(x)}}{(\cos^5(x) \sin(x))^{2/3}} dx$	1793
3.419	$\int (1 + 2 \cos^2(x))^{5/2} \sin(x) dx$	1797
3.420	$\int \cos(x) (5 \cos^2(x) + \sin^2(x))^{5/2} dx$	1801
3.421	$\int \cos(x) (-\cos^2(x) - 5 \sin^2(x))^{3/2} dx$	1805
3.422	$\int \frac{\sin(x)}{(5 \cos^2(x) - 2 \sin^2(x))^{7/2}} dx$	1809
3.423	$\int \frac{\cos(x) \cos(2x)}{(2 - 5 \sin^2(x))^{3/2}} dx$	1813
3.424	$\int \frac{\sin(5x)}{(5 \cos^2(x) + 9 \sin^2(x))^{5/2}} dx$	1817
3.425	$\int \frac{\cos(x) \cos(2x) \sin(3x)}{(-5 + 4 \sin^2(x))^{5/2}} dx$	1821
3.426	$\int \frac{\csc^2(x) (-2 \cos^3(x) (-1 + \sin(x)) + \cos(2x) \sin(x))}{\sqrt{-5 + \sin^2(x)}} dx$	1825
3.427	$\int \frac{\cos(3x)}{-\sqrt{-1 + 8 \cos^2(x)} + \sqrt{3 \cos^2(x) - \sin^2(x)}} dx$	1831
3.428	$\int (2 - 3 \sin^2(x))^{3/5} \sin(4x) dx$	1836
3.429	$\int \cos(x) \sqrt{\cos(2x)} dx$	1840
3.430	$\int \cos^{\frac{3}{2}}(2x) \sin(x) dx$	1844
3.431	$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)} dx$	1848
3.432	$\int \cos^{\frac{3}{2}}(2x) \sec^3(x) dx$	1851

- 3.433 $\int \frac{\sin^2(x)(3\sin^3(x) - \cos(x)\sin(4x))}{\cos^{\frac{7}{2}}(2x)} dx \dots\dots\dots 1855$
- 3.434 $\int (4 - 5\sec^2(x))^{3/2} dx \dots\dots\dots 1861$
- 3.435 $\int \frac{1}{(4-5\sec^2(x))^{3/2}} dx \dots\dots\dots 1866$
- 3.436 $\int \frac{-2\cot^2(x) + \sin(x)}{(1+5\tan^2(x))^{3/2}} dx \dots\dots\dots 1870$
- 3.437 $\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}} dx \dots\dots\dots 1876$
- 3.438 $\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))(5-4\sec^2(x))^{3/2}} dx \dots\dots\dots 1880$
- 3.439 $\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{(4\sec^2(x)+5\tan^2(x))^{3/2}} dx \dots\dots\dots 1888$
- 3.440 $\int \tan(x)(1+5\tan^2(x))^{5/2} dx \dots\dots\dots 1893$
- 3.441 $\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}} dx \dots\dots\dots 1897$
- 3.442 $\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}} dx \dots\dots\dots 1902$
- 3.443 $\int \tan(x)(1-7\tan^2(x))^{2/3} dx \dots\dots\dots 1907$
- 3.444 $\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}} dx \dots\dots\dots 1912$
- 3.445 $\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}} dx \dots\dots\dots 1916$
- 3.446 $\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)} dx \dots\dots\dots 1920$
- 3.447 $\int \frac{\sec^2(x)(-\cos(2x)+2\tan^2(x))}{(\tan(x)\tan(2x))^{3/2}} dx \dots\dots\dots 1927$
- 3.448 $\int \frac{\tan(x)}{(a^3-b^3\cos^n(x))^{4/3}} dx \dots\dots\dots 1934$
- 3.449 $\int (1+2\cos^9(x))^{5/6}\tan(x) dx \dots\dots\dots 1939$
- 3.450 $\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}} dx \dots\dots\dots 1944$
- 3.451 $\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}} dx \dots\dots\dots 1948$
- 3.452 $\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}} dx \dots\dots\dots 1952$
- 3.453 $\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)} dx \dots\dots\dots 1959$
- 3.454 $\int \cos^3(x)\cos^{\frac{2}{3}}(2x)\sin(x) dx \dots\dots\dots 1965$
- 3.455 $\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)} dx \dots\dots\dots 1968$
- 3.456 $\int \sqrt{\tan(x)\tan(2x)} dx \dots\dots\dots 1973$
- 3.457 $\int \sqrt{\cot(2x)\tan(x)} dx \dots\dots\dots 1977$
- 3.458 $\int \frac{1}{x^5(5+x^2)} dx \dots\dots\dots 1981$
- 3.459 $\int \frac{1}{x^6(5+x^2)} dx \dots\dots\dots 1984$
- 3.460 $\int \frac{1}{x(-4+x^2)^4} dx \dots\dots\dots 1988$

3.461	$\int \frac{1}{x(-2+x^2)^{5/2}} dx$	1992
3.462	$\int \frac{(-10+x^2)^{5/2}}{x} dx$	1997
3.463	$\int x^{1+2n} dx$	2002
3.464	$\int \frac{x^7}{(-5+x^2)^3} dx$	2005
3.465	$\int \frac{-4x^3+3x^5}{(-1+x^2)^5} dx$	2008
3.466	$\int x^3(1+x^2)^{9/14} dx$	2012
3.467	$\int \frac{x^5}{(-4+x^2)^{13/6}} dx$	2015
3.468	$\int \frac{1}{(1+2x^2)^{5/2}} dx$	2019
3.469	$\int \frac{1}{(-1-2x+x^2)^{5/2}} dx$	2022
3.470	$\int \frac{1}{x^4(-8+x^2)^{3/2}} dx$	2025
3.471	$\int \frac{(5+x^2)^2}{x^{13/3}} dx$	2029
3.472	$\int \frac{1}{x^7(1+x^2)^3} dx$	2032
3.473	$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}} dx$	2036
3.474	$\int \frac{x^4}{\left(\sqrt{10-x^2}\right)^{9/2}} dx$	2040
3.475	$\int \frac{x^2}{(3-x^2)^{3/2}} dx$	2045
3.476	$\int \frac{(25-x^2)^{3/2}}{x^4} dx$	2049
3.477	$\int \frac{1}{(1-2x^2)^{7/2}} dx$	2053
3.478	$\int \frac{1}{(-7+6x-x^2)^{5/2}} dx$	2057
3.479	$\int (1-2x-2x^2)^3 dx$	2060
3.480	$\int (-1+5x)(-1-x+x^2)^2 dx$	2063
3.481	$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}} dx$	2066
3.482	$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}} dx$	2069
3.483	$\int x^2 \cos^5(x) dx$	2073
3.484	$\int x^3 \sin^3(x) dx$	2077
3.485	$\int x^2 \sin^6(x) dx$	2081
3.486	$\int x^2 \cos(x) \sin^2(x) dx$	2085
3.487	$\int x \cos^2(x) \cot^2(x) dx$	2089
3.488	$\int x \sec(x) \tan^3(x) dx$	2093
3.489	$\int x \sec^2(x) \tan(x) dx$	2098
3.490	$\int x \sin^2(x) \tan(x) dx$	2102
3.491	$\int x \tan^3(x) dx$	2106
3.492	$\int \frac{2x+\sin(2x)}{(\cos(x)+x \sin(x))^2} dx$	2110
3.493	$\int \frac{x^2}{(x \cos(x)-\sin(x))^2} dx$	2113
3.494	$\int a^{mx} b^{nx} dx$	2116
3.495	$\int a^{-x} b^{-x} (a^x - b^x)^2 dx$	2120
3.496	$\int (-e^{-x} + e^x) dx$	2124

3.497	$\int (-e^{-x} + e^x)^2 dx$	2127
3.498	$\int (-e^{-x} + e^x)^3 dx$	2131
3.499	$\int (-e^{-x} + e^x)^4 dx$	2134
3.500	$\int (-e^{-x} + e^x)^n dx$	2138
3.501	$\int (a^{-4x} - a^{2x})^3 dx$	2142
3.502	$\int (a^{kx} + a^{lx}) dx$	2146
3.503	$\int (a^{kx} + a^{lx})^2 dx$	2149
3.504	$\int (a^{kx} + a^{lx})^3 dx$	2153
3.505	$\int (a^{kx} + a^{lx})^4 dx$	2158
3.506	$\int (a^{kx} + a^{lx})^n dx$	2164
3.507	$\int (a^{kx} - a^{lx}) dx$	2167
3.508	$\int (a^{kx} - a^{lx})^2 dx$	2170
3.509	$\int (a^{kx} - a^{lx})^3 dx$	2174
3.510	$\int (a^{kx} - a^{lx})^4 dx$	2179
3.511	$\int (a^{kx} - a^{lx})^n dx$	2185
3.512	$\int (1 + a^{mx}) dx$	2188
3.513	$\int (1 + a^{mx})^2 dx$	2191
3.514	$\int (1 + a^{mx})^3 dx$	2194
3.515	$\int (1 + a^{mx})^4 dx$	2198
3.516	$\int (1 + a^{mx})^n dx$	2202
3.517	$\int (1 - a^{mx}) dx$	2205
3.518	$\int (1 - a^{mx})^2 dx$	2208
3.519	$\int (1 - a^{mx})^3 dx$	2211
3.520	$\int (1 - a^{mx})^4 dx$	2215
3.521	$\int (1 - a^{mx})^n dx$	2219
3.522	$\int \frac{1}{b+ae^{nx}} dx$	2222
3.523	$\int \frac{e^x}{b+ae^{3x}} dx$	2226
3.524	$\int \frac{-1+e^x}{1+e^x} dx$	2231
3.525	$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}} dx$	2234
3.526	$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}} dx$	2238
3.527	$\int e^{nx} (a + be^{nx})^{r/s} dx$	2242
3.528	$\int \sqrt[4]{1 - 2e^{x/3}} dx$	2246
3.529	$\int (a + be^{nx})^{r/s} dx$	2250
3.530	$\int \frac{e^x}{\sqrt{a^2 + e^{2x}}} dx$	2253
3.531	$\int \frac{e^x}{\sqrt{-a^2 + e^{2x}}} dx$	2257
3.532	$\int \frac{e^{3x/4}}{(-2+e^{3x/4})\sqrt{-2 + e^{3x/4} + e^{3x/2}}} dx$	2261
3.533	$\int e^{-2x} (-3 + e^{7x})^{2/3} dx$	2265
3.534	$\int \frac{e^{2x}}{(3-e^{x/2})^{3/4}} dx$	2269
3.535	$\int e^{-x/2} x^3 dx$	2273

3.536	$\int \frac{e^{-x/2}}{x^3} dx$	2276
3.537	$\int a^{3x} x^2 dx$	2279
3.538	$\int e^{x^2} x (1 + x^2) dx$	2283
3.539	$\int \frac{x}{(e^{-x} + e^x)^2} dx$	2287
3.540	$\int \frac{e^x (1 - x - x^2)}{\sqrt{1 - x^2}} dx$	2291
3.541	$\int e^{-3x} \cos(2x) dx$	2294
3.542	$\int \frac{\cos(\frac{x}{2}) + \sin(\frac{x}{2})}{\sqrt[3]{e^x}} dx$	2297
3.543	$\int \frac{\cos(\frac{3x}{2})}{\sqrt[4]{3^{3x}}} dx$	2301
3.544	$\int e^{mx} \cos^2(x) dx$	2304
3.545	$\int e^{mx} \sin^3(x) dx$	2308
3.546	$\int \frac{\cos^3(\frac{x}{3})}{\sqrt{e^x}} dx$	2312
3.547	$\int e^{2x} \cos^2(x) \sin^2(x) dx$	2316
3.548	$\int e^{3x} \cos^2(\frac{3x}{2}) \sin^2(\frac{3x}{2}) dx$	2320
3.549	$\int e^{mx} \tan^2(x) dx$	2324
3.550	$\int e^{mx} \csc^2(x) dx$	2328
3.551	$\int e^{mx} \sec^3(x) dx$	2332
3.552	$\int \frac{e^x}{1 + \cos(x)} dx$	2337
3.553	$\int \frac{e^x}{1 - \cos(x)} dx$	2340
3.554	$\int \frac{e^x}{1 + \sin(x)} dx$	2343
3.555	$\int \frac{e^x}{1 - \sin(x)} dx$	2346
3.556	$\int \frac{e^x (1 - \sin(x))}{1 - \cos(x)} dx$	2349
3.557	$\int \frac{e^x (1 + \sin(x))}{1 - \cos(x)} dx$	2352
3.558	$\int \frac{e^x (1 + \sin(x))}{1 + \cos(x)} dx$	2356
3.559	$\int \frac{e^x (1 - \sin(x))}{1 + \cos(x)} dx$	2359
3.560	$\int \frac{e^x (1 - \cos(x))}{1 - \sin(x)} dx$	2363
3.561	$\int \frac{e^x (1 + \cos(x))}{1 - \sin(x)} dx$	2367
3.562	$\int \frac{e^x (1 + \cos(x))}{1 + \sin(x)} dx$	2370
3.563	$\int \frac{e^x (1 - \cos(x))}{1 + \sin(x)} dx$	2374
3.564	$\int e^x x \cos(x) dx$	2377
3.565	$\int e^x x^2 \sin(x) dx$	2381
3.566	$\int e^{-3x} x^2 \sin(x) dx$	2385
3.567	$\int e^{x/2} x^2 \cos^3(x) dx$	2389
3.568	$\int e^{2x} x^2 \sin(4x) dx$	2394
3.569	$\int e^{x/2} x^2 \cos(x) \sin^2(x) dx$	2398
3.570	$\int \cosh(x) dx$	2403
3.571	$\int \sinh(x) dx$	2406
3.572	$\int \tanh(x) dx$	2409
3.573	$\int \coth(x) dx$	2412

3.574	$\int \operatorname{sech}(x) dx$	2415
3.575	$\int \operatorname{csch}(x) dx$	2418
3.576	$\int \cosh^2(x) dx$	2421
3.577	$\int \sinh^5(x) dx$	2424
3.578	$\int \tanh^4(x) dx$	2427
3.579	$\int \operatorname{csch}^3(x) dx$	2430
3.580	$\int \operatorname{sech}^5(x) dx$	2433
3.581	$\int \sinh^4(x) \tanh(x) dx$	2437
3.582	$\int \operatorname{sech}^{\frac{23}{4}}(x) \sinh^5(x) dx$	2441
3.583	$\int \frac{1}{a+b \cosh(x)} dx$	2445
3.584	$\int \frac{1}{(1+\cosh(x))^2} dx$	2449
3.585	$\int \frac{1}{a+b \tanh(x)} dx$	2452
3.586	$\int \frac{1}{a^2+b^2 \cosh^2(x)} dx$	2456
3.587	$\int \frac{1}{a^2-b^2 \cosh^2(x)} dx$	2461
3.588	$\int \frac{1}{1-\sinh^4(x)} dx$	2466
3.589	$\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)} dx$	2471
3.590	$\int \cosh(x) \cosh(2x) \cosh(3x) dx$	2475
3.591	$\int \cosh\left(\frac{3x}{2}\right) \sinh(x) \sinh\left(\frac{5x}{2}\right) dx$	2478
3.592	$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)} (\sinh^2(x)+\sinh(2x))} dx$	2482
3.593	$\int \frac{\sinh(x)}{(-9+4 \cosh^2(x))^{5/2}} dx$	2488
3.594	$\int \frac{\sinh^2(x) \sinh(2x)}{(1-\sinh^2(x))^{3/2}} dx$	2492
3.595	$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}} dx$	2496
3.596	$\int x \tanh^2(x) dx$	2500
3.597	$\int x \coth^2(x) dx$	2503
3.598	$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)} dx$	2506
3.599	$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)} dx$	2510
3.600	$\int e^{2x} \operatorname{csch}^4(x) dx$	2514
3.601	$\int e^{-2x} \operatorname{sech}^4(x) dx$	2518
3.602	$\int \frac{e^x}{\cosh(x)-\sinh(x)} dx$	2522
3.603	$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)} dx$	2525
3.604	$\int \frac{e^x}{\cosh(x)+\sinh(x)} dx$	2528
3.605	$\int \frac{e^x}{1-\cosh(x)} dx$	2531
3.606	$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)} dx$	2535
3.607	$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)} dx$	2538
3.608	$\int x^m \log(x) dx$	2541
3.609	$\int x^m \log^2(x) dx$	2544
3.610	$\int \frac{\log^2(x)}{x^{5/2}} dx$	2548
3.611	$\int (a+bx) \log(x) dx$	2551

3.612	$\int (a + bx)^3 \log(x) dx$	2554
3.613	$\int (-1 - 8 \log^2(x) + 3 \log^3(x)) dx$	2558
3.614	$\int (1 + x^4) (1 - 2 \log(x) + \log^3(x)) dx$	2561
3.615	$\int \frac{1}{x^3 \log^4(x)} dx$	2565
3.616	$\int \frac{\log(x)}{a+bx} dx$	2569
3.617	$\int \frac{\log(x)}{(a+bx)^2} dx$	2573
3.618	$\int \frac{\log^n(x)}{x} dx$	2576
3.619	$\int \frac{(a+b \log(x))^n}{x} dx$	2579
3.620	$\int \frac{1}{x(a+b \log(x))} dx$	2582
3.621	$\int \frac{(a+b \log(x))^{-n}}{x} dx$	2585
3.622	$\int \frac{1}{x \sqrt{a^2 + \log^2(x)}} dx$	2588
3.623	$\int \frac{1}{x \sqrt{-a^2 + \log^2(x)}} dx$	2592
3.624	$\int \frac{1}{x \sqrt{a^2 - \log^2(x)}} dx$	2596
3.625	$\int \frac{1}{x \log(x) \sqrt{a^2 + \log^2(x)}} dx$	2600
3.626	$\int \frac{1}{x \log(x) \sqrt{a^2 - \log^2(x)}} dx$	2604
3.627	$\int \frac{1}{x \log(x) \sqrt{-a^2 + \log^2(x)}} dx$	2608
3.628	$\int \frac{\log(\log(x))}{x} dx$	2612
3.629	$\int \frac{\log^2(\log(x))}{x} dx$	2615
3.630	$\int \frac{\log^3(\log(x))}{x} dx$	2618
3.631	$\int \frac{\log^4(\log(x))}{x} dx$	2621
3.632	$\int \frac{\log^n(\log(x))}{x} dx$	2624
3.633	$\int \frac{\cot(x)}{\log(\sin(x))} dx$	2627
3.634	$\int (\cos(x) + \sec(x)) \tan(x) dx$	2630
3.635	$\int \log(\cosh(x)) \sinh(x) dx$	2633
3.636	$\int \log(\cosh(x)) \tanh(x) dx$	2636
3.637	$\int \log(x - \sqrt{1 + x^2}) dx$	2639
3.638	$\int \frac{\log(-1+x)}{x^3} dx$	2642
3.639	$\int (-e^{-x} + e^x) \log(1 + e^{2x}) dx$	2645
3.640	$\int e^{3x/2} \log(-1 + e^x) dx$	2649
3.641	$\int \cos^3(x) \log(\sin(x)) dx$	2653
3.642	$\int \log(\tan(x)) \sec^4(x) dx$	2657
3.643	$\int \frac{\log(\cos(\frac{x}{2}))}{1+\cos(x)} dx$	2661
3.644	$\int \frac{\cos(x) \log(\sin(x))}{(1+\cos(x))^2} dx$	2665
3.645	$\int \frac{\cos^{-1}(x)^2}{x^5} dx$	2670

3.646	$\int x^2 \sin^{-1}(x)^2 dx$	2674
3.647	$\int x^3 \tan^{-1}(x)^2 dx$	2678
3.648	$\int \frac{\tan^{-1}(x)^2}{x^5} dx$	2682
3.649	$\int x^3 \csc^{-1}(x)^2 dx$	2686
3.650	$\int \frac{\sec^{-1}(x)^4}{x^5} dx$	2690
3.651	$\int \sqrt{1-x^2} \sin^{-1}(x) dx$	2694
3.652	$\int \sqrt{1-x^2} \cos^{-1}(x) dx$	2697
3.653	$\int x\sqrt{1-x^2} \cos^{-1}(x) dx$	2700
3.654	$\int (1-x^2)^{3/2} \sin^{-1}(x) dx$	2703
3.655	$\int x(1-x^2)^{3/2} \sin^{-1}(x) dx$	2707
3.656	$\int x^3(1-x^2)^{3/2} \cos^{-1}(x) dx$	2710
3.657	$\int \frac{(1-x^2)^{3/2} \cos^{-1}(x)}{x} dx$	2714
3.658	$\int \frac{(1-x^2)^{3/2} \sin^{-1}(x)}{x^6} dx$	2718
3.659	$\int \frac{x^2 \sin^{-1}(x)}{\sqrt{1-x^2}} dx$	2722
3.660	$\int \frac{x^4 \sin^{-1}(x)}{\sqrt{1-x^2}} dx$	2725
3.661	$\int \frac{x \sin^{-1}(x)}{(1-x^2)^{3/2}} dx$	2729
3.662	$\int \frac{x \cos^{-1}(x)}{(1-x^2)^{3/2}} dx$	2732
3.663	$\int \frac{\sin^{-1}(x)}{(1-x^2)^{5/2}} dx$	2735
3.664	$\int \frac{x^3 \sin^{-1}(x)}{(1-x^2)^{3/2}} dx$	2739
3.665	$\int \frac{\sin^{-1}(x)}{x(1-x^2)^{3/2}} dx$	2743
3.666	$\int \frac{\cos^{-1}(x)}{x^4 \sqrt{1-x^2}} dx$	2747
3.667	$\int x\sqrt{1-x^2} \cos^{-1}(x)^2 dx$	2751
3.668	$\int \frac{x^2 \sin^{-1}(x)^3}{\sqrt{1-x^2}} dx$	2755
3.669	$\int \frac{x \tan^{-1}(x)}{(1+x^2)^2} dx$	2759
3.670	$\int \frac{x \tan^{-1}(x)}{(1+x^2)^3} dx$	2763
3.671	$\int \frac{x^2 \tan^{-1}(x)}{1+x^2} dx$	2767
3.672	$\int \frac{x^3 \tan^{-1}(x)}{1+x^2} dx$	2770
3.673	$\int \frac{x^2 \tan^{-1}(x)}{(1+x^2)^2} dx$	2774
3.674	$\int \frac{x^3 \tan^{-1}(x)}{(1+x^2)^2} dx$	2777
3.675	$\int \frac{x^5 \tan^{-1}(x)}{(1+x^2)^2} dx$	2782
3.676	$\int \frac{(1+x^2) \tan^{-1}(x)}{x^2} dx$	2787
3.677	$\int \frac{(1+x^2) \tan^{-1}(x)}{x^5} dx$	2791
3.678	$\int \frac{(1+x^2)^{3/2} \tan^{-1}(x)}{x^5} dx$	2794
3.679	$\int \frac{\tan^{-1}(x)}{x^2(1+x^2)} dx$	2798

3.680	$\int \frac{\tan^{-1}(x)^2}{x^3} dx$	2802
3.681	$\int \frac{(1+x^2) \tan^{-1}(x)^2}{x^5} dx$	2806
3.682	$\int \frac{x^3 \tan^{-1}(x)^2}{(1+x^2)^3} dx$	2811
3.683	$\int \frac{\sqrt{-1+x^2} \sec^{-1}(x)}{x^2} dx$	2815
3.684	$\int \frac{(-1+x^2)^{5/2} \csc^{-1}(x)}{x^3} dx$	2820
3.685	$\int \frac{\sqrt{-1+x^2} \sec^{-1}(x)}{x^4} dx$	2825
3.686	$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}} dx$	2829
3.687	$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}} dx$	2833
3.688	$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}} dx$	2837
3.689	$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}} dx$	2841
3.690	$\int \frac{\sec^{-1}(x)}{x^2 \sqrt{-1+x^2}} dx$	2847
3.691	$\int \frac{\csc^{-1}(x)}{x^2 (-1+x^2)^{5/2}} dx$	2851
3.692	$\int \frac{\csc^{-1}(x)^4}{x^2 \sqrt{-1+x^2}} dx$	2856
3.693	$\int \frac{(-1+x^2)^{3/2} \sec^{-1}(x)^2}{x^5} dx$	2860
3.694	$\int \frac{\sqrt{-1+x^2} \sec^{-1}(x)^3}{x^4} dx$	2865
3.695	$\int \sin^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right) dx$	2870
3.696	$\int \tan^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right) dx$	2875
3.697	$\int \frac{\tan^{-1}(x)}{(1+x)^3} dx$	2879
3.698	$\int \frac{-\tan^{-1}(a-x)}{a+x} dx$	2883
3.699	$\int \frac{\sin^{-1}(\sqrt{1-x^2})}{\sqrt{1-x^2}} dx$	2887
3.700	$\int \frac{x \tan^{-1}(\sqrt{1+x^2})}{\sqrt{1+x^2}} dx$	2890
3.701	$\int \frac{\sin^{-1}(x)}{(1-x)^{5/2}} dx$	2893
3.702	$\int (-1+x)^{5/2} \csc^{-1}(x) dx$	2897
3.703	$\int \sin^{-1}(\sinh(x)) \operatorname{sech}^4(x) dx$	2902
3.704	$\int \cot^{-1}(\cosh(x)) \coth(x) \operatorname{csch}^3(x) dx$	2907
3.705	$\int e^x \sin^{-1}(\tanh(x)) dx$	2912

3.1 $\int \frac{1}{a^2 - b^2 x^2} dx$

Optimal. Leaf size=14

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{ab}$$

[Out] arctanh(b*x/a)/a/b

Rubi [A]

time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {214}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Antiderivative was successfully verified.

[In] Int[(a^2 - b^2*x^2)^(-1), x]

[Out] ArcTanh[(b*x)/a]/(a*b)

Rule 214

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\int \frac{1}{a^2 - b^2 x^2} dx = \frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Mathematica [A]

time = 0.00, size = 14, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 - b^2*x^2)^(-1), x]

[Out] ArcTanh[(b*x)/a]/(a*b)

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 30 vs. 2(14) = 28. time = 1.99, size = 28, normalized size = 2.00

$$\frac{\text{Log}\left[\frac{a}{b} + x\right] - \text{Log}\left[-\frac{a}{b} + x\right]}{2ab}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(a^2 - b^2*x^2),x]')`

[Out] $(\text{Log}[a / b + x] - \text{Log}[-a / b + x]) / (2 a b)$

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.

time = 0.05, size = 31, normalized size = 2.21

method	result	size
default	$-\frac{\ln(-bx+a)}{2ab} + \frac{\ln(bx+a)}{2ab}$	31
norman	$-\frac{\ln(-bx+a)}{2ab} + \frac{\ln(bx+a)}{2ab}$	31
risch	$-\frac{\ln(-bx+a)}{2ab} + \frac{\ln(bx+a)}{2ab}$	31

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)`

[Out] $-1/2/a/b*\ln(-b*x+a)+1/2*\ln(b*x+a)/a/b$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

time = 0.26, size = 31, normalized size = 2.21

$$\frac{\log(bx+a)}{2ab} - \frac{\log(bx-a)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b^2*x^2+a^2),x, algorithm="maxima")`

[Out] $1/2*\log(b*x + a)/(a*b) - 1/2*\log(b*x - a)/(a*b)$

Fricas [A]

time = 0.34, size = 25, normalized size = 1.79

$$\frac{\log(bx+a) - \log(bx-a)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b^2*x^2+a^2),x, algorithm="fricas")`

[Out] $1/2*(\log(b*x + a) - \log(b*x - a))/(a*b)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$

time = 0.07, size = 20, normalized size = 1.43

$$-\frac{\frac{\log(-\frac{a}{b}+x)}{2} - \frac{\log(\frac{a}{b}+x)}{2}}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b**2*x**2+a**2),x)`

[Out] $-(\log(-a/b + x)/2 - \log(a/b + x)/2)/(a*b)$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.
time = 0.01, size = 35, normalized size = 2.50

$$-\frac{b \ln |xb - a|}{2b^2a} + \frac{b \ln |xb + a|}{2b^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b^2*x^2+a^2),x)`

[Out] $1/2*\log(\text{abs}(b*x + a))/(a*b) - 1/2*\log(\text{abs}(b*x - a))/(a*b)$

Mupad [B]

time = 0.06, size = 14, normalized size = 1.00

$$\frac{\text{atanh}\left(\frac{bx}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^2 - b^2*x^2),x)`

[Out] $\text{atanh}((b*x)/a)/(a*b)$

3.2 $\int \frac{1}{a^2+b^2x^2} dx$

Optimal. Leaf size=14

$$\frac{\tan^{-1}\left(\frac{bx}{a}\right)}{ab}$$

[Out] arctan(b*x/a)/a/b

Rubi [A]

time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {211}

$$\frac{\tan^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + b^2*x^2)^(-1), x]

[Out] ArcTan[(b*x)/a]/(a*b)

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\int \frac{1}{a^2 + b^2x^2} dx = \frac{\tan^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Mathematica [A]

time = 0.00, size = 14, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + b^2*x^2)^(-1), x]

[Out] ArcTan[(b*x)/a]/(a*b)

Mathics [C] Result contains complex when optimal does not.

time = 2.06, size = 29, normalized size = 2.07

$$\frac{\frac{i}{2} \left(\text{Log} \left[\frac{Ia}{b} + x \right] - \text{Log} \left[\frac{-Ia}{b} + x \right] \right)}{ab}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(a^2 + b^2*x^2),x]')`

[Out] $I / 2 (\text{Log}[I a / b + x] - \text{Log}[-I a / b + x]) / (a b)$

Maple [A]

time = 0.05, size = 15, normalized size = 1.07

method	result	size
default	$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$	15
risch	$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$	15

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b^2*x^2+a^2),x,method=_RETURNVERBOSE)`

[Out] $\arctan(b*x/a)/a/b$

Maxima [A]

time = 0.35, size = 14, normalized size = 1.00

$$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b^2*x^2+a^2),x, algorithm="maxima")`

[Out] $\arctan(b*x/a)/(a*b)$

Fricas [A]

time = 0.34, size = 14, normalized size = 1.00

$$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b^2*x^2+a^2),x, algorithm="fricas")`

[Out] $\arctan(b*x/a)/(a*b)$

Sympy [C] Result contains complex when optimal does not.

time = 0.07, size = 26, normalized size = 1.86

$$\frac{-\frac{i \log\left(-\frac{ia}{b}+x\right)}{2} + \frac{i \log\left(\frac{ia}{b}+x\right)}{2}}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b**2*x**2+a**2),x)`

[Out] $(-I*\log(-I*a/b + x)/2 + I*\log(I*a/b + x)/2)/(a*b)$

Giac [A]

time = 0.01, size = 13, normalized size = 0.93

$$\frac{2 \arctan\left(\frac{bx}{a}\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b^2*x^2+a^2),x)`

[Out] $\arctan(b*x/a)/(a*b)$

Mupad [B]

time = 0.04, size = 14, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{bx}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^2 + b^2*x^2),x)`

[Out] $\operatorname{atan}((b*x)/a)/(a*b)$

3.3 $\int \sec(2ax) dx$

Optimal. Leaf size=13

$$\frac{\tanh^{-1}(\sin(2ax))}{2a}$$

[Out] 1/2*arctanh(sin(2*a*x))/a

Rubi [A]

time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3855}

$$\frac{\tanh^{-1}(\sin(2ax))}{2a}$$

Antiderivative was successfully verified.

[In] Int[Sec[2*a*x],x]

[Out] ArcTanh[Sin[2*a*x]]/(2*a)

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \sec(2ax) dx = \frac{\tanh^{-1}(\sin(2ax))}{2a}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(13) = 26.

time = 0.01, size = 37, normalized size = 2.85

$$-\frac{\log(\cos(ax) - \sin(ax))}{2a} + \frac{\log(\cos(ax) + \sin(ax))}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2*a*x],x]

[Out] -1/2*Log[Cos[a*x] - Sin[a*x]]/a + Log[Cos[a*x] + Sin[a*x]]/(2*a)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.22, size = 31, normalized size = 2.38

$$\text{Piecewise} \left[\left\{ \left\{ \frac{\text{Log}[1 + \text{Sin}[2ax]] - \text{Log}[-1 + \text{Sin}[2ax]]}{4a}, a \neq 0 \right\} \right\}, x \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[Sec[2*a*x],x]')`

[Out] `Piecewise[{{(Log[1 + Sin[2 a x]] - Log[-1 + Sin[2 a x]]) / (4 a), a != 0}}, x]`

Maple [A]

time = 0.02, size = 18, normalized size = 1.38

method	result	size
derivativedivides	$\frac{\ln(\sec(2ax) + \tan(2ax))}{2a}$	18
default	$\frac{\ln(\sec(2ax) + \tan(2ax))}{2a}$	18
norman	$-\frac{\ln(\tan(ax)-1)}{2a} + \frac{\ln(\tan(ax)+1)}{2a}$	26
risch	$-\frac{\ln(e^{2iax}-i)}{2a} + \frac{\ln(e^{2iax}+i)}{2a}$	32

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(2*a*x),x,method=_RETURNVERBOSE)`

[Out] `1/2/a*ln(sec(2*a*x)+tan(2*a*x))`

Maxima [A]

time = 0.27, size = 17, normalized size = 1.31

$$\frac{\log(\sec(2ax) + \tan(2ax))}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2*a*x),x, algorithm="maxima")`

[Out] `1/2*log(sec(2*a*x) + tan(2*a*x))/a`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(11) = 22.

time = 0.34, size = 26, normalized size = 2.00

$$\frac{\log(\sin(2ax) + 1) - \log(-\sin(2ax) + 1)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*a*x),x, algorithm="fricas")

[Out] 1/4*(log(sin(2*a*x) + 1) - log(-sin(2*a*x) + 1))/a

Sympy [A]

time = 0.05, size = 27, normalized size = 2.08

$$\begin{cases} \frac{-\frac{\log(\sin(2ax)-1)}{2} + \frac{\log(\sin(2ax)+1)}{2}}{2a} & \text{for } a \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*a*x),x)

[Out] Piecewise((((-log(sin(2*a*x) - 1)/2 + log(sin(2*a*x) + 1)/2)/(2*a), Ne(a, 0)), (x, True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(11) = 22.

time = 0.01, size = 46, normalized size = 3.54

$$\frac{2 \left(-\frac{\ln \left| \sin(2ax) + \frac{1}{\sin(2ax)} - 2 \right|}{8} + \frac{\ln \left| \sin(2ax) + \frac{1}{\sin(2ax)} + 2 \right|}{8} \right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*a*x),x)

[Out] 1/8*(log(abs(1/sin(2*a*x) + sin(2*a*x) + 2)) - log(abs(1/sin(2*a*x) + sin(2*a*x) - 2)))/a

Mupad [B]

time = 0.23, size = 11, normalized size = 0.85

$$\frac{\operatorname{atanh}(\sin(2ax))}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(2*a*x),x)

[Out] atanh(sin(2*a*x))/(2*a)

3.4 $\int \frac{1}{4} \csc\left(\frac{x}{3}\right) dx$

Optimal. Leaf size=11

$$-\frac{3}{4} \tanh^{-1}\left(\cos\left(\frac{x}{3}\right)\right)$$

[Out] -3/4*arctanh(cos(1/3*x))

Rubi [A]

time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {12, 3855}

$$-\frac{3}{4} \tanh^{-1}\left(\cos\left(\frac{x}{3}\right)\right)$$

Antiderivative was successfully verified.

[In] Int[Csc[x/3]/4,x]

[Out] (-3*ArcTanh[Cos[x/3]])/4

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{4} \csc\left(\frac{x}{3}\right) dx &= \frac{1}{4} \int \csc\left(\frac{x}{3}\right) dx \\ &= -\frac{3}{4} \tanh^{-1}\left(\cos\left(\frac{x}{3}\right)\right) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

time = 0.01, size = 23, normalized size = 2.09

$$\frac{1}{4} \left(-3 \log\left(\cos\left(\frac{x}{6}\right)\right) + 3 \log\left(\sin\left(\frac{x}{6}\right)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x/3]/4,x]

[Out] (-3*Log[Cos[x/6]] + 3*Log[Sin[x/6]])/4

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.
time = 1.97, size = 19, normalized size = 1.73

$$\frac{-3\text{Log}\left[1 + \text{Cos}\left[\frac{x}{3}\right]\right]}{8} + \frac{3\text{Log}\left[-1 + \text{Cos}\left[\frac{x}{3}\right]\right]}{8}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(4*Sin[x/3]),x]')

[Out] -3 Log[1 + Cos[x / 3]] / 8 + 3 Log[-1 + Cos[x / 3]] / 8

Maple [A]

time = 0.03, size = 15, normalized size = 1.36

method	result	size
norman	$\frac{3 \ln(\tan(\frac{x}{6}))}{4}$	8
derivativedivides	$\frac{3 \ln(\csc(\frac{x}{3}) - \cot(\frac{x}{3}))}{4}$	15
default	$\frac{3 \ln(\csc(\frac{x}{3}) - \cot(\frac{x}{3}))}{4}$	15
risch	$-\frac{3 \ln(e^{\frac{ix}{3}} + 1)}{4} + \frac{3 \ln(e^{\frac{ix}{3}} - 1)}{4}$	22

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/4/sin(1/3*x),x,method=_RETURNVERBOSE)

[Out] 3/4*ln(csc(1/3*x)-cot(1/3*x))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

time = 0.27, size = 19, normalized size = 1.73

$$-\frac{3}{8} \log\left(\cos\left(\frac{1}{3}x\right) + 1\right) + \frac{3}{8} \log\left(\cos\left(\frac{1}{3}x\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/4/sin(1/3*x),x, algorithm="maxima")

[Out] -3/8*log(cos(1/3*x) + 1) + 3/8*log(cos(1/3*x) - 1)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(7) = 14.
time = 0.34, size = 23, normalized size = 2.09

$$-\frac{3}{8} \log\left(\frac{1}{2} \cos\left(\frac{1}{3}x\right) + \frac{1}{2}\right) + \frac{3}{8} \log\left(-\frac{1}{2} \cos\left(\frac{1}{3}x\right) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/4/sin(1/3*x),x, algorithm="fricas")

[Out] $-3/8*\log(1/2*\cos(1/3*x) + 1/2) + 3/8*\log(-1/2*\cos(1/3*x) + 1/2)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$

time = 0.05, size = 22, normalized size = 2.00

$$\frac{3 \log\left(\cos\left(\frac{x}{3}\right) - 1\right)}{8} - \frac{3 \log\left(\cos\left(\frac{x}{3}\right) + 1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/4/sin(1/3*x),x)

[Out] $3*\log(\cos(x/3) - 1)/8 - 3*\log(\cos(x/3) + 1)/8$

Giac [A]

time = 0.01, size = 13, normalized size = 1.18

$$\frac{3}{4} \ln \left| \tan\left(\frac{x}{3 \cdot 2}\right) \right|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/4/sin(1/3*x),x)

[Out] $3/4*\log(\text{abs}(\tan(1/6*x)))$

Mupad [B]

time = 0.07, size = 7, normalized size = 0.64

$$\frac{3 \ln\left(\tan\left(\frac{x}{6}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*sin(x/3)),x)

[Out] $(3*\log(\tan(x/6)))/4$

3.5 $\int -\sec\left(\frac{\pi}{4} + 2x\right) dx$

Optimal. Leaf size=15

$$-\frac{1}{2} \tanh^{-1}\left(\sin\left(\frac{\pi}{4} + 2x\right)\right)$$

[Out] -1/2*arctanh(sin(1/4*Pi+2*x))

Rubi [A]

time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3855}

$$-\frac{1}{2} \tanh^{-1}\left(\sin\left(2x + \frac{\pi}{4}\right)\right)$$

Antiderivative was successfully verified.

[In] Int[-Sec[Pi/4 + 2*x],x]

[Out] -1/2*ArcTanh[Sin[Pi/4 + 2*x]]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int -\sec\left(\frac{\pi}{4} + 2x\right) dx = -\frac{1}{2} \tanh^{-1}\left(\sin\left(\frac{\pi}{4} + 2x\right)\right)$$

Mathematica [A]

time = 0.01, size = 15, normalized size = 1.00

$$-\frac{1}{2} \tanh^{-1}\left(\sin\left(\frac{\pi}{4} + 2x\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[-Sec[Pi/4 + 2*x],x]

[Out] -1/2*ArcTanh[Sin[Pi/4 + 2*x]]

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$. time = 2.55, size = 23, normalized size = 1.53

$$-\frac{\text{Log}\left[1 + \text{Tan}\left[\frac{\text{Pi}}{8} + x\right]\right]}{2} + \frac{\text{Log}\left[-1 + \text{Tan}\left[\frac{\text{Pi}}{8} + x\right]\right]}{2}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/Cos[3*Pi/4 - 2*x],x]')`

[Out] `-Log[1 + Tan[Pi / 8 + x]] / 2 + Log[-1 + Tan[Pi / 8 + x]] / 2`

Maple [A]

time = 0.13, size = 21, normalized size = 1.40

method	result	size
derivativdivides	$-\frac{\ln(\sec(\frac{\pi}{4}+2x)+\tan(\frac{\pi}{4}+2x))}{2}$	21
default	$-\frac{\ln(\sec(\frac{\pi}{4}+2x)+\tan(\frac{\pi}{4}+2x))}{2}$	21
norman	$\frac{\ln(\tan(\frac{\pi}{8}+x)-1)}{2} - \frac{\ln(\tan(\frac{\pi}{8}+x)+1)}{2}$	24
risch	$\frac{\ln\left(e^{\frac{i(\pi+8x)}{4}}-i\right)}{2} - \frac{\ln\left(e^{\frac{i(\pi+8x)}{4}}+i\right)}{2}$	32

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/cos(1/4*Pi+2*x),x,method=_RETURNVERBOSE)`

[Out] `-1/2*ln(sec(1/4*Pi+2*x)+tan(1/4*Pi+2*x))`

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

time = 0.28, size = 27, normalized size = 1.80

$$-\frac{1}{4} \log\left(\sin\left(\frac{1}{4}\pi + 2x\right) + 1\right) + \frac{1}{4} \log\left(\sin\left(\frac{1}{4}\pi + 2x\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/cos(1/4*pi+2*x),x, algorithm="maxima")`

[Out] `-1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(sin(1/4*pi + 2*x) - 1)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(11) = 22.

time = 0.36, size = 29, normalized size = 1.93

$$-\frac{1}{4} \log\left(\sin\left(\frac{1}{4}\pi + 2x\right) + 1\right) + \frac{1}{4} \log\left(-\sin\left(\frac{1}{4}\pi + 2x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/cos(1/4*pi+2*x),x, algorithm="fricas")`

[Out] `-1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(-sin(1/4*pi + 2*x) + 1)`

Sympy [A]

time = 0.10, size = 22, normalized size = 1.47

$$\frac{\log\left(\tan\left(x + \frac{\pi}{8}\right) - 1\right)}{2} - \frac{\log\left(\tan\left(x + \frac{\pi}{8}\right) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/cos(1/4*pi+2*x),x)**[Out]** log(tan(x + pi/8) - 1)/2 - log(tan(x + pi/8) + 1)/2**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(11) = 22.

time = 0.01, size = 56, normalized size = 3.73

$$\frac{\ln\left|\sin\left(\frac{\pi+8x}{4}\right) + \frac{1}{\sin\left(\frac{\pi+8x}{4}\right)} - 2\right|}{8} - \frac{\ln\left|\sin\left(\frac{\pi+8x}{4}\right) + \frac{1}{\sin\left(\frac{\pi+8x}{4}\right)} + 2\right|}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/cos(1/4*pi+2*x),x)**[Out]** -1/8*log(abs(1/sin(1/4*pi + 2*x) + sin(1/4*pi + 2*x) + 2)) + 1/8*log(abs(1/sin(1/4*pi + 2*x) + sin(1/4*pi + 2*x) - 2))**Mupad [B]**

time = 0.21, size = 24, normalized size = 1.60

$$\frac{\ln\left(\frac{\sin\left(\frac{\pi}{4}+2x\right)+1}{\cos\left(\frac{\pi}{4}+2x\right)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/cos(Pi/4 + 2*x),x)**[Out]** -log((sin(Pi/4 + 2*x) + 1)/cos(Pi/4 + 2*x))/2

3.6 $\int \sec(x) \tan(x) dx$

Optimal. Leaf size=2

$\sec(x)$

[Out] $\sec(x)$

Rubi [A]

time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2686, 8}

$\sec(x)$

Antiderivative was successfully verified.

[In] Int[Sec[x]*Tan[x],x]

[Out] Sec[x]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2686

Int[((a_.)*sec[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^(n - 1)/2], x], x, Sec[e + f*x], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rubi steps

$$\int \sec(x) \tan(x) dx = \text{Subst}\left(\int 1 dx, x, \sec(x)\right) = \sec(x)$$

Mathematica [A]

time = 0.00, size = 2, normalized size = 1.00

$\sec(x)$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]*Tan[x],x]

[Out] Sec[x]

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.62, size = 8, normalized size = 4.00

ConditionalExpression[Cos[x], {Cos[x] != 0}]

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sec[x]*Tan[x],x]')

[Out] ConditionalExpression[Cos[x], {Cos[x] != 0}]

Maple [A]

time = 0.01, size = 3, normalized size = 1.50

method	result	size
derivativedivides	$\sec(x)$	3
default	$\sec(x)$	3
risch	$\frac{2e^{ix}}{e^{2ix}+1}$	17

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)*tan(x),x,method=_RETURNVERBOSE)

[Out] sec(x)

Maxima [A]

time = 0.27, size = 4, normalized size = 2.00

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x),x, algorithm="maxima")

[Out] 1/cos(x)

Fricas [A]

time = 0.35, size = 4, normalized size = 2.00

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x),x, algorithm="fricas")

[Out] $1/\cos(x)$

Sympy [A]

time = 0.03, size = 3, normalized size = 1.50

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)*tan(x),x)`

[Out] $1/\cos(x)$

Giac [A]

time = 0.00, size = 3, normalized size = 1.50

$$\frac{1}{\cos x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)*tan(x),x)`

[Out] $1/\cos(x)$

Mupad [B]

time = 0.25, size = 12, normalized size = 6.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(x)/cos(x),x)`

[Out] $-2/(\tan(x/2)^2 - 1)$

3.7 $\int \cot(x) \csc(x) dx$

Optimal. Leaf size=4

$$- \csc(x)$$

[Out] -csc(x)

Rubi [A]

time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2686, 8}

$$- \csc(x)$$

Antiderivative was successfully verified.

[In] Int[Cot[x]*Csc[x],x]

[Out] -Csc[x]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2686

Int[((a_.)*sec[(e_.) + (f_.)*(x_.)]^(m_.))*((b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rubi steps

$$\begin{aligned} \int \cot(x) \csc(x) dx &= -\text{Subst}\left(\int 1 dx, x, \csc(x)\right) \\ &= -\csc(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 4, normalized size = 1.00

$$- \csc(x)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]*Csc[x],x]

[Out] $-\text{Csc}[x]$

Mathics [A]

time = 1.76, size = 6, normalized size = 1.50

$$-\frac{1}{\text{Sin}[x]}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[Csc[x]*Cot[x],x]')`

[Out] $-1 / \text{Sin}[x]$

Maple [A]

time = 0.00, size = 5, normalized size = 1.25

method	result	size
derivativedivides	$-\text{csc}(x)$	5
default	$-\text{csc}(x)$	5
risch	$-\frac{2ie^{ix}}{e^{2ix}-1}$	18

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(x)*csc(x),x,method=_RETURNVERBOSE)`

[Out] $-\text{csc}(x)$

Maxima [A]

time = 0.27, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)*csc(x),x, algorithm="maxima")`

[Out] $-1/\sin(x)$

Fricas [A]

time = 0.34, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)*csc(x),x, algorithm="fricas")`

[Out] $-1/\sin(x)$

Sympy [A]

time = 0.03, size = 5, normalized size = 1.25

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)*csc(x),x)`

[Out] $-1/\sin(x)$

Giac [A]

time = 0.00, size = 4, normalized size = 1.00

$$-\frac{1}{\sin x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)*csc(x),x)`

[Out] $-1/\sin(x)$

Mupad [B]

time = 0.21, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(x)/sin(x),x)`

[Out] $-1/\sin(x)$

3.8 $\int \csc(2x) \tan(x) dx$

Optimal. Leaf size=6

$$\frac{\tan(x)}{2}$$

[Out] 1/2*tan(x)

Rubi [A]

time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {8}

$$\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Csc[2*x]*Tan[x],x]

[Out] Tan[x]/2

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \csc(2x) \tan(x) dx &= \text{Subst} \left(\int \frac{1}{2} dx, x, \tan(x) \right) \\ &= \frac{\tan(x)}{2} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 6, normalized size = 1.00

$$\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[2*x]*Tan[x],x]

[Out] Tan[x]/2

Mathics [A]

time = 2.01, size = 4, normalized size = 0.67

$$\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[Tan[x]/Sin[2*x],x]')`

[Out] `Tan[x] / 2`

Maple [A]

time = 0.03, size = 5, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\tan(x)}{2}$	5
default	$\frac{\tan(x)}{2}$	5
norman	$\frac{\tan(x)}{2}$	5
risch	$\frac{i}{e^{2ix}+1}$	13

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(x)/sin(2*x),x,method=_RETURNVERBOSE)`

[Out] `1/2*tan(x)`

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(4) = 8$.

time = 0.26, size = 27, normalized size = 4.50

$$\frac{\sin(2x)}{\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)/sin(2*x),x, algorithm="maxima")`

[Out] `sin(2*x)/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)`

Fricas [A]

time = 0.33, size = 4, normalized size = 0.67

$$\frac{1}{2} \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)/sin(2*x),x, algorithm="fricas")

[Out] 1/2*tan(x)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6
time = 0.39, size = 7, normalized size = 1.17

$$\frac{\sin(x)}{2 \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)/sin(2*x),x)

[Out] sin(x)/(2*cos(x))

Giac [A]

time = 0.01, size = 6, normalized size = 1.00

$$\frac{2}{4} \tan x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)/sin(2*x),x)

[Out] 1/2*tan(x)

Mupad [B]

time = 0.18, size = 4, normalized size = 0.67

$$\frac{\tan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)/sin(2*x),x)

[Out] tan(x)/2

3.9 $\int \frac{1}{1+\cos(x)} dx$

Optimal. Leaf size=9

$$\frac{\sin(x)}{1 + \cos(x)}$$

[Out] sin(x)/(1+cos(x))

Rubi [A]

time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2727}

$$\frac{\sin(x)}{\cos(x) + 1}$$

Antiderivative was successfully verified.

[In] Int[(1 + Cos[x])^(-1), x]

[Out] Sin[x]/(1 + Cos[x])

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \frac{1}{1 + \cos(x)} dx = \frac{\sin(x)}{1 + \cos(x)}$$

Mathematica [A]

time = 0.00, size = 6, normalized size = 0.67

$$\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Cos[x])^(-1), x]

[Out] Tan[x/2]

Mathics [A]

time = 1.75, size = 4, normalized size = 0.44

$$\text{Tan} \left[\frac{x}{2} \right]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(1 + Cos[x]),x]')`[Out] `Tan[x / 2]`**Maple [A]**

time = 0.02, size = 5, normalized size = 0.56

method	result	size
default	$\tan\left(\frac{x}{2}\right)$	5
norman	$\tan\left(\frac{x}{2}\right)$	5
risch	$\frac{2i}{1+e^{ix}}$	13

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1+cos(x)),x,method=_RETURNVERBOSE)`[Out] `tan(1/2*x)`**Maxima [A]**

time = 0.27, size = 9, normalized size = 1.00

$$\frac{\sin(x)}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+cos(x)),x, algorithm="maxima")`[Out] `sin(x)/(cos(x) + 1)`**Fricas [A]**

time = 0.33, size = 9, normalized size = 1.00

$$\frac{\sin(x)}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+cos(x)),x, algorithm="fricas")`[Out] `sin(x)/(cos(x) + 1)`

Sympy [A]

time = 0.09, size = 3, normalized size = 0.33

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+cos(x)),x)**[Out]** tan(x/2)**Giac [A]**

time = 0.00, size = 9, normalized size = 1.00

$$\frac{2}{2} \tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+cos(x)),x)**[Out]** tan(1/2*x)**Mupad [B]**

time = 0.19, size = 4, normalized size = 0.44

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x) + 1),x)**[Out]** tan(x/2)

3.10 $\int \frac{1}{1-\cos(x)} dx$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

[Out] `-sin(x)/(1-cos(x))`

Rubi [A]

time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2727}

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

[In] `Int[(1 - Cos[x])^(-1), x]`

[Out] `-(Sin[x]/(1 - Cos[x]))`

Rule 2727

`Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

Rubi steps

$$\int \frac{1}{1-\cos(x)} dx = -\frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A]

time = 0.01, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] `Integrate[(1 - Cos[x])^(-1), x]`

[Out] `-Cot[x/2]`

Mathics [A]

time = 1.80, size = 8, normalized size = 0.67

$$-\frac{1}{\tan\left[\frac{x}{2}\right]}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(1 - Cos[x]),x]')``[Out] -1 / Tan[x / 2]`**Maple [A]**

time = 0.01, size = 9, normalized size = 0.75

method	result	size
default	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
norman	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
risch	$-\frac{2i}{e^{ix}-1}$	13

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(1-cos(x)),x,method=_RETURNVERBOSE)``[Out] -1/tan(1/2*x)`**Maxima [A]**

time = 0.27, size = 10, normalized size = 0.83

$$-\frac{\cos(x) + 1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(1-cos(x)),x, algorithm="maxima")``[Out] -(cos(x) + 1)/sin(x)`**Fricas [A]**

time = 0.32, size = 10, normalized size = 0.83

$$-\frac{\cos(x) + 1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(1-cos(x)),x, algorithm="fricas")`

[Out] $-(\cos(x) + 1)/\sin(x)$

Sympy [A]

time = 0.17, size = 7, normalized size = 0.58

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-cos(x)),x)`

[Out] $-1/\tan(x/2)$

Giac [A]

time = 0.00, size = 11, normalized size = 0.92

$$-\frac{2}{2 \tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-cos(x)),x)`

[Out] $-1/\tan(1/2*x)$

Mupad [B]

time = 0.00, size = 6, normalized size = 0.50

$$-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(cos(x) - 1),x)`

[Out] $-\cot(x/2)$

3.11 $\int \frac{\sin(x)}{a-b \cos(x)} dx$

Optimal. Leaf size=12

$$\frac{\log(a - b \cos(x))}{b}$$

[Out] ln(a-b*cos(x))/b

Rubi [A]

time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2747, 31}

$$\frac{\log(a - b \cos(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(a - b*Cos[x]),x]

[Out] Log[a - b*Cos[x]]/b

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2747

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b² - x²)^{((p - 1)/2)}, x], x, b*SIN[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a² - b², 0]

Rubi steps

$$\begin{aligned} \int \frac{\sin(x)}{a - b \cos(x)} dx &= \frac{\text{Subst} \left(\int \frac{1}{a+x} dx, x, -b \cos(x) \right)}{b} \\ &= \frac{\log(a - b \cos(x))}{b} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 12, normalized size = 1.00

$$\frac{\log(a - b \cos(x))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/(a - b*Cos[x]),x]

[Out] Log[a - b*Cos[x]]/b

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.90, size = 27, normalized size = 2.25

$$\text{Piecewise} \left[\left\{ \left\{ \frac{\text{Log} \left[-\frac{a}{b} + \text{Cos}[x] \right]}{b}, b \neq 0 \right\} \right\}, -\frac{\text{Cos}[x]}{a} \right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sin[x]/(a - b*Cos[x]),x]')

[Out] Piecewise[{{Log[-a / b + Cos[x]] / b, b != 0}}, -Cos[x] / a]

Maple [A]

time = 0.03, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$\frac{\ln(a-b \cos(x))}{b}$	13
default	$\frac{\ln(a-b \cos(x))}{b}$	13
risch	$-\frac{ix}{b} + \frac{\ln(e^{2ix} - \frac{2a}{b}e^{ix} + 1)}{b}$	32
norman	$\frac{\ln(a(\tan^2(\frac{x}{2})) + b(\tan^2(\frac{x}{2})) + a - b)}{b} - \frac{\ln(1 + \tan^2(\frac{x}{2}))}{b}$	42

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/(a-b*cos(x)),x,method=_RETURNVERBOSE)

[Out] ln(a-b*cos(x))/b

Maxima [A]

time = 0.27, size = 13, normalized size = 1.08

$$\frac{\log(b \cos(x) - a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a-b*cos(x)),x, algorithm="maxima")

[Out] log(b*cos(x) - a)/b

Fricas [A]

time = 0.38, size = 12, normalized size = 1.00

$$\frac{\log(-b \cos(x) + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a-b*cos(x)),x, algorithm="fricas")

[Out] log(-b*cos(x) + a)/b

Sympy [A]

time = 0.18, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\log(-\frac{a}{b} + \cos(x))}{b} & \text{for } b \neq 0 \\ -\frac{\cos(x)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a-b*cos(x)),x)

[Out] Piecewise((log(-a/b + cos(x))/b, Ne(b, 0)), (-cos(x)/a, True))

Giac [A]

time = 0.00, size = 12, normalized size = 1.00

$$\frac{\ln|\cos x \cdot b - a|}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a-b*cos(x)),x)

[Out] log(abs(b*cos(x) - a))/b

Mupad [B]

time = 0.20, size = 13, normalized size = 1.08

$$\frac{\ln(b \cos(x) - a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/(a - b*cos(x)),x)

[Out] log(b*cos(x) - a)/b

$$3.12 \quad \int \frac{\cos(x)}{a^2 + b^2 \sin^2(x)} dx$$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{b \sin(x)}{a}\right)}{ab}$$

[Out] arctan(b*sin(x)/a)/a/b

Rubi [A]

time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3269, 211}

$$\frac{\tan^{-1}\left(\frac{b \sin(x)}{a}\right)}{ab}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/(a^2 + b^2*Sin[x]^2),x]

[Out] ArcTan[(b*Sin[x])/a]/(a*b)

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3269

Int[cos[(e_) + (f_)*(x_)]^(m_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{\cos(x)}{a^2 + b^2 \sin^2(x)} dx &= \text{Subst} \left(\int \frac{1}{a^2 + b^2 x^2} dx, x, \sin(x) \right) \\ &= \frac{\tan^{-1}\left(\frac{b \sin(x)}{a}\right)}{ab} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 15, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{b \sin(x)}{a}\right)}{ab}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/(a^2 + b^2*Sin[x]^2),x]

[Out] ArcTan[(b*Sin[x])/a]/(a*b)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.97, size = 53, normalized size = 3.53

$$\text{Piecewise} \left[\left\{ \left\{ \text{DirectedInfinity} \left[\frac{1}{\text{Sin}[x]} \right], a==0 \&\& b==0 \right\}, \left\{ -\frac{1}{b^2 \text{Sin}[x]}, a==0 \right\}, \left\{ \frac{\text{Sin}[x]}{a^2}, b==0 \right\} \right\}, \frac{\text{ArcTan} \left[\frac{b \text{Sin}[x]}{a} \right]}{ab} \right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Cos[x]/(a^2 + b^2*Sin[x]^2),x]')

[Out] Piecewise[{{DirectedInfinity[1 / Sin[x]], a == 0 && b == 0}, {-1 / (b ^ 2 S in[x]), a == 0}, {Sin[x] / a ^ 2, b == 0}}, ArcTan[b Sin[x] / a] / (a b)]

Maple [A]

time = 0.06, size = 16, normalized size = 1.07

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{b \sin(x)}{a}\right)}{ab}$	16
default	$\frac{\arctan\left(\frac{b \sin(x)}{a}\right)}{ab}$	16
risch	$-\frac{i \ln\left(e^{2ix} + \frac{2a}{b}e^{ix} - 1\right)}{2ba} + \frac{i \ln\left(e^{2ix} - \frac{2a}{b}e^{ix} - 1\right)}{2ba}$	58

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(a^2+b^2*sin(x)^2),x,method=_RETURNVERBOSE)

[Out] arctan(b*sin(x)/a)/a/b

Maxima [A]

time = 0.36, size = 15, normalized size = 1.00

$$\frac{\arctan\left(\frac{b \sin(x)}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(a^2+b^2*sin(x)^2),x, algorithm="maxima")

[Out] arctan(b*sin(x)/a)/(a*b)

Fricas [A]

time = 0.39, size = 15, normalized size = 1.00

$$\frac{\arctan\left(\frac{b \sin(x)}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(a^2+b^2*sin(x)^2),x, algorithm="fricas")

[Out] arctan(b*sin(x)/a)/(a*b)

Sympy [A]

time = 0.27, size = 31, normalized size = 2.07

$$\begin{cases} \frac{\infty}{\sin(x)} & \text{for } a = 0 \wedge b = 0 \\ -\frac{1}{b^2 \sin(x)} & \text{for } a = 0 \\ \frac{\sin(x)}{a^2} & \text{for } b = 0 \\ \frac{\operatorname{atan}\left(\frac{b \sin(x)}{a}\right)}{ab} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(a**2+b**2*sin(x)**2),x)

[Out] Piecewise((zoo/sin(x), Eq(a, 0) & Eq(b, 0)), (-1/(b**2*sin(x)), Eq(a, 0)), (sin(x)/a**2, Eq(b, 0)), (atan(b*sin(x)/a)/(a*b), True))

Giac [A]

time = 0.00, size = 16, normalized size = 1.07

$$\frac{\frac{1}{2} \cdot 2 \arctan\left(\frac{b \sin x}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(a^2+b^2*sin(x)^2),x)

[Out] arctan(b*sin(x)/a)/(a*b)

Mupad [B]

time = 0.05, size = 15, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{b \sin(x)}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(b^2*sin(x)^2 + a^2),x)

[Out] atan((b*sin(x))/a)/(a*b)

3.13

$$\int \frac{\cos(x)}{a^2 - b^2 \sin^2(x)} dx$$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\frac{b \sin(x)}{a}\right)}{ab}$$

[Out] arctanh(b*sin(x)/a)/a/b

Rubi [A]

time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3269, 214}

$$\frac{\tanh^{-1}\left(\frac{b \sin(x)}{a}\right)}{ab}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/(a^2 - b^2*Sin[x]^2), x]

[Out] ArcTanh[(b*Sin[x])/a]/(a*b)

Rule 214

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3269

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{\cos(x)}{a^2 - b^2 \sin^2(x)} dx &= \text{Subst} \left(\int \frac{1}{a^2 - b^2 x^2} dx, x, \sin(x) \right) \\ &= \frac{\tanh^{-1}\left(\frac{b \sin(x)}{a}\right)}{ab} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 15, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{b \sin(x)}{a}\right)}{ab}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/(a^2 - b^2*Sin[x]^2),x]

[Out] ArcTanh[(b*Sin[x])/a]/(a*b)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.01, size = 73, normalized size = 4.87

Piecewise $\left[\left\{ \left\{ \text{DirectedInfinity} \left[\frac{1}{\text{Sin}[x]} \right], a==0 \&\& b==0 \right\}, \left\{ \frac{1}{b^2 \text{Sin}[x]}, a==0 \right\}, \left\{ \frac{\text{Sin}[x]}{a^2}, b==0 \right\} \right\}, -\frac{\text{Log} \left[-\frac{a}{b} + \text{Sin}[x] \right]}{2ab} + \frac{\text{Log} \left[\frac{a}{b} + \text{Sin}[x] \right]}{2ab} \right]$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Cos[x]/(a^2 - b^2*Sin[x]^2),x]')

[Out] Piecewise[{{DirectedInfinity[1 / Sin[x]], a == 0 && b == 0}, {1 / (b ^ 2 Sin[x]), a == 0}, {Sin[x] / a ^ 2, b == 0}}, -Log[-a / b + Sin[x]] / (2 a b) + Log[a / b + Sin[x]] / (2 a b)]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(15) = 30.

time = 0.07, size = 33, normalized size = 2.20

method	result	size
derivativedivides	$\frac{\ln(a+b \sin(x))}{2ab} - \frac{\ln(-b \sin(x)+a)}{2ab}$	33
default	$\frac{\ln(a+b \sin(x))}{2ab} - \frac{\ln(-b \sin(x)+a)}{2ab}$	33
norman	$-\frac{\ln(a(\tan^2(\frac{x}{2}))-2b \tan(\frac{x}{2}))+a}{2ab} + \frac{\ln(a(\tan^2(\frac{x}{2}))+2b \tan(\frac{x}{2}))+a}{2ab}$	54
risch	$-\frac{\ln(e^{2ix} - \frac{2ia}{b}e^{ix} - 1)}{2ab} + \frac{\ln(e^{2ix} + \frac{2ia}{b}e^{ix} - 1)}{2ab}$	58

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(a^2-b^2*sin(x)^2),x,method=_RETURNVERBOSE)

[Out] 1/2/a/b*ln(a+b*sin(x))-1/2/a/b*ln(-b*sin(x)+a)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(15) = 30.

time = 0.27, size = 33, normalized size = 2.20

$$\frac{\log(b \sin(x) + a)}{2ab} - \frac{\log(b \sin(x) - a)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(a^2-b^2*sin(x)^2),x, algorithm="maxima")

[Out] $1/2*\log(b*\sin(x) + a)/(a*b) - 1/2*\log(b*\sin(x) - a)/(a*b)$

Fricas [A]

time = 0.36, size = 26, normalized size = 1.73

$$\frac{\log(b \sin(x) + a) - \log(-b \sin(x) + a)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/(a^2-b^2*sin(x)^2),x, algorithm="fricas")`

[Out] $1/2*(\log(b*\sin(x) + a) - \log(-b*\sin(x) + a))/(a*b)$

Sympy [A]

time = 0.28, size = 44, normalized size = 2.93

$$\begin{cases} \frac{\infty}{\sin(x)} & \text{for } a = 0 \wedge b = 0 \\ \frac{1}{b^2 \sin(x)} & \text{for } a = 0 \\ \frac{\sin(x)}{a^2} & \text{for } b = 0 \\ -\frac{\log(-\frac{a}{b} + \sin(x))}{2ab} + \frac{\log(\frac{a}{b} + \sin(x))}{2ab} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/(a**2-b**2*sin(x)**2),x)`

[Out] `Piecewise((zoo/sin(x), Eq(a, 0) & Eq(b, 0)), (1/(b**2*sin(x)), Eq(a, 0)), (sin(x)/a**2, Eq(b, 0)), (-log(-a/b + sin(x))/(2*a*b) + log(a/b + sin(x))/(2*a*b), True))`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.

time = 0.00, size = 41, normalized size = 2.73

$$-\frac{\frac{1}{2}b \ln |\sin x \cdot b - a|}{ab^2} + \frac{\frac{1}{2}b \ln |\sin x \cdot b + a|}{ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/(a^2-b^2*sin(x)^2),x)`

[Out] $1/2*\log(\text{abs}(b*\sin(x) + a))/(a*b) - 1/2*\log(\text{abs}(b*\sin(x) - a))/(a*b)$

Mupad [B]

time = 0.18, size = 15, normalized size = 1.00

$$\frac{\operatorname{atanh}\left(\frac{b \sin(x)}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-cos(x)/(b^2*sin(x)^2 - a^2),x)`

[Out] $\operatorname{atanh}((b*\sin(x))/a)/(a*b)$

$$3.14 \quad \int \frac{\sin(2x)}{a^2 + b^2 \sin^2(x)} dx$$

Optimal. Leaf size=17

$$\frac{\log(a^2 + b^2 \sin^2(x))}{b^2}$$

[Out] $\ln(a^2 + b^2 \sin(x)^2) / b^2$

Rubi [A]

time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {12, 266}

$$\frac{\log(a^2 + b^2 \sin^2(x))}{b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sin}[2*x]/(a^2 + b^2*\text{Sin}[x]^2), x]$

[Out] $\text{Log}[a^2 + b^2*\text{Sin}[x]^2]/b^2$

Rule 12

$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

Rule 266

$\text{Int}[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned} \int \frac{\sin(2x)}{a^2 + b^2 \sin^2(x)} dx &= \text{Subst} \left(\int \frac{2x}{a^2 + b^2 x^2} dx, x, \sin(x) \right) \\ &= 2 \text{Subst} \left(\int \frac{x}{a^2 + b^2 x^2} dx, x, \sin(x) \right) \\ &= \frac{\log(a^2 + b^2 \sin^2(x))}{b^2} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 17, normalized size = 1.00

$$\frac{\log(a^2 + b^2 \sin^2(x))}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/(a^2 + b^2*Sin[x]^2),x]
```

```
[Out] Log[a^2 + b^2*Sin[x]^2]/b^2
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 3.93, size = 36, normalized size = 2.12

$$\text{Piecewise} \left[\left\{ \left\{ -\frac{\text{Cos}[x]^2}{a^2}, b^2 == 0 \right\}, \left\{ \frac{\text{Log}[a^2 + b^2 \text{Sin}[x]^2]}{b^2}, \text{True} \right\} \right\} \right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sin[2*x]/(a^2 + b^2*Sin[x]^2),x]')
```

```
[Out] Piecewise[{{-Cos[x]^2/a^2, b^2 == 0}, {Log[a^2 + b^2 Sin[x]^2]/b^2, True}}]
```

Maple [A]

time = 0.09, size = 18, normalized size = 1.06

method	result	size
derivativdivides	$\frac{\ln(a^2 + b^2(\sin^2(x)))}{b^2}$	18
default	$\frac{\ln(a^2 + b^2(\sin^2(x)))}{b^2}$	18
risch	$-\frac{2ix}{b^2} + \frac{\ln\left(e^{4ix} - \frac{2(2a^2 + b^2)e^{2ix}}{b^2} + 1\right)}{b^2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(a^2+b^2*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a^2+b^2*sin(x)^2)/b^2
```

Maxima [A]

time = 0.26, size = 17, normalized size = 1.00

$$\frac{\log(b^2 \sin(x)^2 + a^2)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2+b^2*sin(x)^2),x, algorithm="maxima")
```

```
[Out] log(b^2*sin(x)^2 + a^2)/b^2
```


Fricas [A]

time = 0.36, size = 21, normalized size = 1.24

$$\frac{\log(-b^2 \cos(x)^2 + a^2 + b^2)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(2*x)/(a^2+b^2*sin(x)^2),x, algorithm="fricas")``[Out] log(-b^2*cos(x)^2 + a^2 + b^2)/b^2`**Sympy [A]**

time = 1.37, size = 32, normalized size = 1.88

$$2 \left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ \frac{\log(a^2 + b^2 \sin^2(x))}{2b^2} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(2*x)/(a**2+b**2*sin(x)**2),x)``[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (log(a**2 + b**2*sin(x)**2)/(2*b**2), True))`**Giac [A]**

time = 0.00, size = 18, normalized size = 1.06

$$\frac{\ln(\sin^2 x \cdot b^2 + a^2)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(2*x)/(a^2+b^2*sin(x)^2),x)``[Out] log(b^2*sin(x)^2 + a^2)/b^2`**Mupad [B]**

time = 0.61, size = 48, normalized size = 2.82

$$\frac{\operatorname{atan}\left(\frac{b^2 \sin(x)^2}{a^2 \cos(x)^2 + a^2 \sin(x)^2 + b^2 \sin(x)^2}\right) 2i}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sin(2*x)/(b^2*sin(x)^2 + a^2),x)``[Out] (atan((b^2*sin(x)^2)/(a^2*cos(x)^2 + a^2*sin(x)^2 + b^2*sin(x)^2))*2i)/b^2`

3.15

$$\int \frac{\sin(2x)}{a^2 - b^2 \sin^2(x)} dx$$

Optimal. Leaf size=19

$$-\frac{\log(a^2 - b^2 \sin^2(x))}{b^2}$$

[Out] $-\ln(a^2 - b^2 \sin(x)^2)/b^2$

Rubi [A]

time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {12, 266}

$$-\frac{\log(a^2 - b^2 \sin^2(x))}{b^2}$$

Antiderivative was successfully verified.

[In] `Int[Sin[2*x]/(a^2 - b^2*Sin[x]^2),x]`

[Out] `-(Log[a^2 - b^2*Sin[x]^2]/b^2)`

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 266

`Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]`

Rubi steps

$$\begin{aligned} \int \frac{\sin(2x)}{a^2 - b^2 \sin^2(x)} dx &= \text{Subst} \left(\int \frac{2x}{a^2 - b^2 x^2} dx, x, \sin(x) \right) \\ &= 2 \text{Subst} \left(\int \frac{x}{a^2 - b^2 x^2} dx, x, \sin(x) \right) \\ &= -\frac{\log(a^2 - b^2 \sin^2(x))}{b^2} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 19, normalized size = 1.00

$$-\frac{\log(a^2 - b^2 \sin^2(x))}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[2*x]/(a^2 - b^2*Sin[x]^2),x]

[Out] -(Log[a^2 - b^2*Sin[x]^2]/b^2)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 4.05, size = 38, normalized size = 2.00

$$\text{Piecewise} \left[\left[\left\{ \left\{ -\frac{\text{Cos}[x]^2}{a^2}, b^2 == 0 \right\}, \left\{ -\frac{\text{Log}[a^2 - b^2 \text{Sin}[x]^2]}{b^2}, \text{True} \right\} \right\} \right] \right]$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Sin[2*x]/(a^2 - b^2*Sin[x]^2),x]')

[Out] Piecewise[{{-Cos[x]^2/a^2, b^2 == 0}, {-Log[a^2 - b^2 Sin[x]^2]/b^2, True}}]

Maple [A]

time = 0.14, size = 20, normalized size = 1.05

method	result	size
derivativdivides	$-\frac{\ln(a^2 - b^2 \sin^2(x))}{b^2}$	20
default	$-\frac{\ln(a^2 - b^2 \sin^2(x))}{b^2}$	20
risch	$\frac{2ix}{b^2} - \frac{\ln\left(e^{4ix} + \frac{2(2a^2 - b^2)e^{2ix}}{b^2} + 1\right)}{b^2}$	43

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)/(a^2-b^2*sin(x)^2),x,method=_RETURNVERBOSE)

[Out] -ln(a^2-b^2*sin(x)^2)/b^2

Maxima [A]

time = 0.26, size = 20, normalized size = 1.05

$$-\frac{\log(b^2 \sin(x)^2 - a^2)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)/(a^2-b^2*sin(x)^2),x, algorithm="maxima")

[Out] -log(b^2*sin(x)^2 - a^2)/b^2

Fricas [A]

time = 0.35, size = 23, normalized size = 1.21

$$-\frac{\log(b^2 \cos(x)^2 + a^2 - b^2)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(2*x)/(a^2-b^2*sin(x)^2),x, algorithm="fricas")``[Out] -log(b^2*cos(x)^2 + a^2 - b^2)/b^2`**Sympy [A]**

time = 1.40, size = 34, normalized size = 1.79

$$2 \left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ -\frac{\log(a^2 - b^2 \sin^2(x))}{2b^2} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(2*x)/(a**2-b**2*sin(x)**2),x)``[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (-log(a**2 - b**2*sin(x)**2)/(2*b**2), True))`**Giac [A]**

time = 0.00, size = 21, normalized size = 1.11

$$-\frac{\ln|\sin^2 x \cdot b^2 - a^2|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(2*x)/(a^2-b^2*sin(x)^2),x)``[Out] -log(abs(b^2*sin(x)^2 - a^2))/b^2`**Mupad [B]**

time = 0.49, size = 48, normalized size = 2.53

$$\frac{\operatorname{atan}\left(\frac{b^2 \sin(x)^2}{a^2 \cos(x)^2 + a^2 \sin(x)^2 - b^2 \sin(x)^2}\right) 2i}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(-sin(2*x)/(b^2*sin(x)^2 - a^2),x)``[Out] (atan((b^2*sin(x)^2)/(a^2*cos(x)^2 + a^2*sin(x)^2 - b^2*sin(x)^2)) * 2i)/b^2`

$$3.16 \quad \int \frac{\sin(2x)}{a^2 + b^2 \cos^2(x)} dx$$

Optimal. Leaf size=18

$$-\frac{\log(a^2 + b^2 \cos^2(x))}{b^2}$$

[Out] $-\ln(a^2 + b^2 \cos(x)^2)/b^2$

Rubi [A]

time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {12, 266}

$$-\frac{\log(a^2 - b^2 \sin^2(x) + b^2)}{b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sin}[2*x]/(a^2 + b^2*\text{Cos}[x]^2), x]$

[Out] $-(\text{Log}[a^2 + b^2 - b^2*\text{Sin}[x]^2]/b^2)$

Rule 12

$\text{Int}[(a_)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

Rule 266

$\text{Int}[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned} \int \frac{\sin(2x)}{a^2 + b^2 \cos^2(x)} dx &= \text{Subst} \left(\int \frac{2x}{a^2 + b^2 - b^2 x^2} dx, x, \sin(x) \right) \\ &= 2 \text{Subst} \left(\int \frac{x}{a^2 + b^2 - b^2 x^2} dx, x, \sin(x) \right) \\ &= -\frac{\log(a^2 + b^2 - b^2 \sin^2(x))}{b^2} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 22, normalized size = 1.22

$$-\frac{\log(a^2 + b^2 - b^2 \sin^2(x))}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[2*x]/(a^2 + b^2*Cos[x]^2),x]

[Out] -(Log[a^2 + b^2 - b^2*Sin[x]^2]/b^2)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 4.03, size = 37, normalized size = 2.06

$$\text{Piecewise} \left[\left\{ \left\{ -\frac{\text{Cos}[x]^2}{a^2}, b^2 == 0 \right\}, \left\{ -\frac{\text{Log}[a^2 + b^2 \text{Cos}[x]^2]}{b^2}, \text{True} \right\} \right\} \right]$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Sin[2*x]/(a^2 + b^2*Cos[x]^2),x]')

[Out] Piecewise[{{-Cos[x]^2/a^2, b^2 == 0}, {-Log[a^2 + b^2 Cos[x]^2]/b^2, True}}]

Maple [A]

time = 0.09, size = 19, normalized size = 1.06

method	result	size
derivativedivides	$-\frac{\ln(a^2 + b^2(\cos^2(x)))}{b^2}$	19
default	$-\frac{\ln(a^2 + b^2(\cos^2(x)))}{b^2}$	19
risch	$\frac{2ix}{b^2} - \frac{\ln\left(e^{4ix} + \frac{2(2a^2 + b^2)e^{2ix}}{b^2} + 1\right)}{b^2}$	41

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)/(a^2+b^2*cos(x)^2),x,method=_RETURNVERBOSE)

[Out] -ln(a^2+b^2*cos(x)^2)/b^2

Maxima [A]

time = 0.26, size = 18, normalized size = 1.00

$$-\frac{\log(b^2 \cos(x)^2 + a^2)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)/(a^2+b^2*cos(x)^2),x, algorithm="maxima")

[Out] -log(b^2*cos(x)^2 + a^2)/b^2

Fricas [A]

time = 0.36, size = 18, normalized size = 1.00

$$-\frac{\log(b^2 \cos(x)^2 + a^2)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(2*x)/(a^2+b^2*cos(x)^2),x, algorithm="fricas")``[Out] -log(b^2*cos(x)^2 + a^2)/b^2`**Sympy [A]**

time = 1.36, size = 34, normalized size = 1.89

$$2 \left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ -\frac{\log(a^2 + b^2 \cos^2(x))}{2b^2} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(2*x)/(a**2+b**2*cos(x)**2),x)``[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (-log(a**2 + b**2*cos(x)**2)/(2*b**2), True))`**Giac [A]**

time = 0.00, size = 19, normalized size = 1.06

$$-\frac{\ln(\cos^2 x \cdot b^2 + a^2)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(2*x)/(a^2+b^2*cos(x)^2),x)``[Out] -log(b^2*cos(x)^2 + a^2)/b^2`**Mupad [B]**

time = 0.41, size = 58, normalized size = 3.22

$$\frac{2 \operatorname{atanh}\left(\frac{b^2}{2a^2 + b^2 \cos(x)^2 + b^2} - \frac{b^2 \cos(x)^2}{2a^2 + b^2 \cos(x)^2 + b^2}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sin(2*x)/(b^2*cos(x)^2 + a^2),x)``[Out] (2*atanh(b^2/(b^2*cos(x)^2 + 2*a^2 + b^2) - (b^2*cos(x)^2)/(b^2*cos(x)^2 + 2*a^2 + b^2)))/b^2`

$$3.17 \quad \int \frac{\sin(2x)}{a^2 - b^2 \cos^2(x)} dx$$

Optimal. Leaf size=18

$$\frac{\log(a^2 - b^2 \cos^2(x))}{b^2}$$

[Out] $\ln(a^2 - b^2 \cos(x)^2) / b^2$

Rubi [A]

time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {12, 266}

$$\frac{\log(a^2 + b^2 \sin^2(x) - b^2)}{b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sin}[2*x]/(a^2 - b^2*\text{Cos}[x]^2), x]$

[Out] $\text{Log}[a^2 - b^2 + b^2*\text{Sin}[x]^2]/b^2$

Rule 12

$\text{Int}[(a_*)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]$

Rule 266

$\text{Int}[(x_)^(m_)/((a_) + (b_*)*(x_)^(n_)), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned} \int \frac{\sin(2x)}{a^2 - b^2 \cos^2(x)} dx &= \text{Subst} \left(\int \frac{2x}{a^2 - b^2 + b^2 x^2} dx, x, \sin(x) \right) \\ &= 2 \text{Subst} \left(\int \frac{x}{a^2 - b^2 + b^2 x^2} dx, x, \sin(x) \right) \\ &= \frac{\log(a^2 - b^2 + b^2 \sin^2(x))}{b^2} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 22, normalized size = 1.22

$$\frac{\log(a^2 - b^2 + b^2 \sin^2(x))}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/(a^2 - b^2*Cos[x]^2),x]
```

```
[Out] Log[a^2 - b^2 + b^2*Sin[x]^2]/b^2
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 4.07, size = 37, normalized size = 2.06

$$\text{Piecewise} \left[\left\{ \left\{ -\frac{\text{Cos}[x]^2}{a^2}, b^2 == 0 \right\}, \left\{ \frac{\text{Log}[a^2 - b^2 \text{Cos}[x]^2]}{b^2}, \text{True} \right\} \right\} \right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sin[2*x]/(a^2 - b^2*Cos[x]^2),x]')
```

```
[Out] Piecewise[{{-Cos[x]^2/a^2, b^2 == 0}, {Log[a^2 - b^2 Cos[x]^2]/b^2, True}}
```

Maple [A]

time = 0.17, size = 19, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\ln(a^2 - b^2 \cos^2(x))}{b^2}$	19
default	$\frac{\ln(a^2 - b^2 \cos^2(x))}{b^2}$	19
risch	$-\frac{2ix}{b^2} + \frac{\ln\left(e^{4ix} - \frac{2(2a^2 - b^2)e^{2ix}}{b^2} + 1\right)}{b^2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(a^2-b^2*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a^2-b^2*cos(x)^2)/b^2
```

Maxima [A]

time = 0.26, size = 19, normalized size = 1.06

$$\frac{\log(b^2 \cos(x)^2 - a^2)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-b^2*cos(x)^2),x, algorithm="maxima")
```

```
[Out] log(b^2*cos(x)^2 - a^2)/b^2
```

Fricas [A]

time = 0.36, size = 19, normalized size = 1.06

$$\frac{\log(b^2 \cos(x)^2 - a^2)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(2*x)/(a^2-b^2*cos(x)^2),x, algorithm="fricas")``[Out] log(b^2*cos(x)^2 - a^2)/b^2`**Sympy [A]**

time = 1.40, size = 32, normalized size = 1.78

$$2 \left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ \frac{\log(a^2 - b^2 \cos^2(x))}{2b^2} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(2*x)/(a**2-b**2*cos(x)**2),x)``[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (log(a**2 - b**2*cos(x)**2)/(2*b**2), True))`**Giac [A]**

time = 0.00, size = 20, normalized size = 1.11

$$\frac{\ln|\cos^2 x \cdot b^2 - a^2|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(2*x)/(a^2-b^2*cos(x)^2),x)``[Out] log(abs(b^2*cos(x)^2 - a^2))/b^2`**Mupad [B]**

time = 0.38, size = 58, normalized size = 3.22

$$-\frac{2 \operatorname{atanh}\left(\frac{b^2}{-2a^2 + b^2 \cos(x)^2 + b^2} - \frac{b^2 \cos(x)^2}{-2a^2 + b^2 \cos(x)^2 + b^2}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(-sin(2*x)/(b^2*cos(x)^2 - a^2),x)``[Out] -(2*atanh(b^2/(b^2*cos(x)^2 - 2*a^2 + b^2) - (b^2*cos(x)^2)/(b^2*cos(x)^2 - 2*a^2 + b^2)))/b^2`

3.18

$$\int \frac{1}{4 - \cos^2(x)} dx$$

Optimal. Leaf size=41

$$\frac{x}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{3+2\sqrt{3}+\sin^2(x)}\right)}{2\sqrt{3}}$$

[Out] 1/6*x*3^(1/2)+1/6*arctan(cos(x)*sin(x)/(3+sin(x)^2+2*3^(1/2)))*3^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3260, 209}

$$\frac{x}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+2\sqrt{3}+3}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(4 - Cos[x]^2)^(-1), x]

[Out] x/(2*Sqrt[3]) + ArcTan[(Cos[x]*Sin[x])/(3 + 2*Sqrt[3] + Sin[x]^2)]/(2*Sqrt[3])

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3260

Int[((a_) + (b_.)*sin[e_.] + (f_.)*(x_)^2)^(-1), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{4 - \cos^2(x)} dx &= -\text{Subst}\left(\int \frac{1}{4 + 3x^2} dx, x, \cot(x)\right) \\ &= \frac{x}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{3+2\sqrt{3}+\sin^2(x)}\right)}{2\sqrt{3}} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 19, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{2\tan(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

`[In] Integrate[(4 - Cos[x]^2)^(-1), x]``[Out] ArcTan[(2*Tan[x])/Sqrt[3]]/(2*Sqrt[3])`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.68, size = 37, normalized size = 0.90

$$\frac{\sqrt{3} \left(2\text{PiFloor} \left[-\frac{1}{2} + \frac{x}{2\text{Pi}} \right] + \text{ArcTan} \left[\frac{\sqrt{3} \text{Tan} \left[\frac{x}{2} \right]}{3} \right] + \text{ArcTan} \left[\sqrt{3} \text{Tan} \left[\frac{x}{2} \right] \right] \right)}{6}$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(4 - Cos[x]^2), x]')``[Out] Sqrt[3] (2 Pi Floor[-1 / 2 + x / (2 Pi)] + ArcTan[Sqrt[3] Tan[x / 2] / 3] + ArcTan[Sqrt[3] Tan[x / 2]]) / 6`**Maple [A]**

time = 0.04, size = 14, normalized size = 0.34

method	result	size
default	$\frac{\sqrt{3} \arctan\left(\frac{2\tan(x)\sqrt{3}}{3}\right)}{6}$	14
risch	$\frac{i\sqrt{3} \ln\left(e^{2ix} - 4\sqrt{3} - 7\right)}{12} - \frac{i\sqrt{3} \ln\left(e^{2ix} + 4\sqrt{3} - 7\right)}{12}$	40

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(4-cos(x)^2), x, method=_RETURNVERBOSE)``[Out] 1/6*3^(1/2)*arctan(2/3*tan(x)*3^(1/2))`**Maxima [A]**

time = 0.36, size = 13, normalized size = 0.32

$$\frac{1}{6} \sqrt{3} \arctan\left(\frac{2}{3} \sqrt{3} \tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-cos(x)^2),x, algorithm="maxima")

[Out] 1/6*sqrt(3)*arctan(2/3*sqrt(3)*tan(x))

Fricas [A]

time = 0.37, size = 31, normalized size = 0.76

$$-\frac{1}{12} \sqrt{3} \arctan \left(\frac{7 \sqrt{3} \cos(x)^2 - 4 \sqrt{3}}{12 \cos(x) \sin(x)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-cos(x)^2),x, algorithm="fricas")

[Out] -1/12*sqrt(3)*arctan(1/12*(7*sqrt(3)*cos(x)^2 - 4*sqrt(3))/(cos(x)*sin(x)))

Sympy [A]

time = 0.23, size = 61, normalized size = 1.49

$$\frac{\sqrt{3} \left(\operatorname{atan} \left(\frac{\sqrt{3} \tan \left(\frac{x}{2} \right)}{3} \right) + \pi \left\lfloor \frac{\frac{x}{2} - \frac{\pi}{2}}{\pi} \right\rfloor \right)}{6} + \frac{\sqrt{3} \left(\operatorname{atan} \left(\sqrt{3} \tan \left(\frac{x}{2} \right) \right) + \pi \left\lfloor \frac{\frac{x}{2} - \frac{\pi}{2}}{\pi} \right\rfloor \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-cos(x)**2),x)

[Out] sqrt(3)*(atan(sqrt(3)*tan(x/2)/3) + pi*floor((x/2 - pi/2)/pi))/6 + sqrt(3)*(atan(sqrt(3)*tan(x/2)) + pi*floor((x/2 - pi/2)/pi))/6

Giac [A]

time = 0.00, size = 57, normalized size = 1.39

$$\frac{2 \left(\arctan \left(\frac{-\sqrt{3} \sin(2x) + 2 \sin(2x)}{\sqrt{3} \cos(2x) + \sqrt{3} - 2 \cos(2x) + 2} \right) + x \right)}{4\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-cos(x)^2),x)

[Out] 1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) - 2*sin(2*x))/(sqrt(3)*cos(2*x) + sqrt(3) - 2*cos(2*x) + 2)))

Mupad [B]

time = 0.23, size = 26, normalized size = 0.63

$$\frac{\sqrt{3} (x - \operatorname{atan}(\tan(x)))}{6} + \frac{\sqrt{3} \operatorname{atan} \left(\frac{2\sqrt{3} \tan(x)}{3} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(cos(x)^2 - 4),x)`

[Out] $(3^{1/2}*(x - \operatorname{atan}(\tan(x))))/6 + (3^{1/2}*\operatorname{atan}((2*3^{1/2}*\tan(x))/3))/6$

$$3.19 \quad \int \frac{e^x}{-1+e^{2x}} dx$$

Optimal. Leaf size=6

$$-\tanh^{-1}(e^x)$$

[Out] -arctanh(exp(x))

Rubi [A]

time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2281, 213}

$$-\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Int[E^x/(-1 + E^(2*x)),x]

[Out] -ArcTanh[E^x]

Rule 213

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2281

Int[((a_) + (b_)*(F_)^(e_)*((c_) + (d_)*(x_)))^(p_)*(G_)^(h_)*((f_) + (g_)*(x_)), x_Symbol] := With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denominator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{e^x}{-1+e^{2x}} dx &= \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, e^x \right) \\ &= -\tanh^{-1}(e^x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 6, normalized size = 1.00

$$-\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(-1 + E^(2*x)),x]

[Out] -ArcTanh[E^x]

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 21 vs. $2(6) = 12$.
time = 1.84, size = 17, normalized size = 2.83

$$-\frac{\text{Log}[1 + E^x]}{2} + \frac{\text{Log}[-1 + E^x]}{2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[E^x/(-1 + E^(2*x)),x]')

[Out] -Log[1 + E ^ x] / 2 + Log[-1 + E ^ x] / 2

Maple [A]

time = 0.01, size = 6, normalized size = 1.00

method	result	size
default	$-\text{arctanh}(e^x)$	6
norman	$\frac{\ln(-1+e^x)}{2} - \frac{\ln(1+e^x)}{2}$	16
risch	$\frac{\ln(-1+e^x)}{2} - \frac{\ln(1+e^x)}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(exp(2*x)-1),x,method=_RETURNVERBOSE)

[Out] -arctanh(exp(x))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

time = 0.26, size = 15, normalized size = 2.50

$$-\frac{1}{2} \log(e^x + 1) + \frac{1}{2} \log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="maxima")

[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.
time = 0.33, size = 15, normalized size = 2.50

$$-\frac{1}{2} \log(e^x + 1) + \frac{1}{2} \log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(-1+exp(2*x)),x, algorithm="fricas")`

[Out] $-1/2*\log(e^x + 1) + 1/2*\log(e^x - 1)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$

time = 0.05, size = 15, normalized size = 2.50

$$\frac{\log(e^x - 1)}{2} - \frac{\log(e^x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(-1+exp(2*x)),x)`

[Out] $\log(\exp(x) - 1)/2 - \log(\exp(x) + 1)/2$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(5) = 10$.
time = 0.00, size = 19, normalized size = 3.17

$$\frac{\ln|e^x - 1|}{2} - \frac{\ln(e^x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(-1+exp(2*x)),x)`

[Out] $-1/2*\log(e^x + 1) + 1/2*\log(\text{abs}(e^x - 1))$

Mupad [B]

time = 0.00, size = 15, normalized size = 2.50

$$\frac{\ln(e^x - 1)}{2} - \frac{\ln(e^x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)/(exp(2*x) - 1),x)`

[Out] $\log(\exp(x) - 1)/2 - \log(\exp(x) + 1)/2$

3.20

$$\int \frac{1}{x \log(x)} dx$$

Optimal. Leaf size=3

$$\log(\log(x))$$

[Out] ln(ln(x))

Rubi [A]

time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2339, 29}

$$\log(\log(x))$$

Antiderivative was successfully verified.

[In] Int[1/(x*Log[x]),x]

[Out] Log[Log[x]]

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 2339

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{x \log(x)} dx &= \text{Subst} \left(\int \frac{1}{x} dx, x, \log(x) \right) \\ &= \log(\log(x)) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 3, normalized size = 1.00

$$\log(\log(x))$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Log[x]),x]

[Out] $\text{Log}[\text{Log}[x]]$

Mathics [A]

time = 1.71, size = 3, normalized size = 1.00

$\text{Log}[\text{Log}[x]]$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(x*Log[x]),x]')`

[Out] $\text{Log}[\text{Log}[x]]$

Maple [A]

time = 0.00, size = 4, normalized size = 1.33

method	result	size
derivativedivides	$\ln(\ln(x))$	4
default	$\ln(\ln(x))$	4
norman	$\ln(\ln(x))$	4
risch	$\ln(\ln(x))$	4

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/ln(x),x,method=_RETURNVERBOSE)`

[Out] $\ln(\ln(x))$

Maxima [A]

time = 0.26, size = 3, normalized size = 1.00

$\log(\log(x))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/log(x),x, algorithm="maxima")`

[Out] $\log(\log(x))$

Fricas [A]

time = 0.36, size = 3, normalized size = 1.00

$\log(\log(x))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/log(x),x, algorithm="fricas")`

[Out] $\log(\log(x))$

Sympy [A]

time = 0.04, size = 3, normalized size = 1.00

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/ln(x),x)

[Out] log(log(x))

Giac [A]

time = 0.00, size = 4, normalized size = 1.33

$$\ln|\ln x|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(x),x)

[Out] log(abs(log(x)))

Mupad [B]

time = 0.18, size = 3, normalized size = 1.00

$$\ln(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*log(x)),x)

[Out] log(log(x))

$$3.21 \quad \int \frac{1}{x(1+\log^2(x))} dx$$

Optimal. Leaf size=3

$$\tan^{-1}(\log(x))$$

[Out] arctan(ln(x))

Rubi [A]

time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$,

Rules used = {209}

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 + Log[x]^2)),x]

[Out] ArcTan[Log[x]]

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\int \frac{1}{x(1+\log^2(x))} dx = \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \log(x) \right) \\ = \tan^{-1}(\log(x))$$

Mathematica [A]

time = 0.01, size = 3, normalized size = 1.00

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 + Log[x]^2)),x]

[Out] ArcTan[Log[x]]

Mathics [C] Result contains complex when optimal does not.

time = 1.92, size = 22, normalized size = 7.33

$$\text{RootSum}[1 + 4\#1^2 \&, \text{Log}[\text{Log}[x] + 2\#1] \#1 \&]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x*(1 + Log[x]^2)),x]')
```

```
[Out] RootSum[1 + 4 #1 ^ 2&, Log[Log[x] + 2 #1] #1&]
```

Maple [A]

time = 0.00, size = 4, normalized size = 1.33

method	result	size
derivativdivides	$\arctan(\ln(x))$	4
default	$\arctan(\ln(x))$	4
risch	$\frac{i \ln(\ln(x)+i)}{2} - \frac{i \ln(\ln(x)-i)}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+ln(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(ln(x))
```

Maxima [A]

time = 0.34, size = 3, normalized size = 1.00

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(log(x))
```

Fricas [A]

time = 0.32, size = 3, normalized size = 1.00

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
```

```
[Out] arctan(log(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$
time = 0.07, size = 15, normalized size = 5.00

$$\text{RootSum}(4z^2 + 1, (i \mapsto i \log(2i + \log(x))))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+ln(x)**2),x)
```

```
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

Giac [A]

time = 0.00, size = 3, normalized size = 1.00

$$\arctan(\ln x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x)
```

```
[Out] arctan(log(x))
```

Mupad [B]

time = 0.34, size = 3, normalized size = 1.00

$$\operatorname{atan}(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x)^2 + 1)),x)
```

```
[Out] atan(log(x))
```

3.22

$$\int \frac{1}{x(1-\log(x))} dx$$

Optimal. Leaf size=9

$$-\log(1 - \log(x))$$

[Out] -ln(1-ln(x))

Rubi [A]

time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2339, 29}

$$-\log(1 - \log(x))$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 - Log[x])),x]

[Out] -Log[1 - Log[x]]

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 2339

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{x(1-\log(x))} dx &= -\text{Subst} \left(\int \frac{1}{x} dx, x, 1 - \log(x) \right) \\ &= -\log(1 - \log(x)) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 7, normalized size = 0.78

$$-\log(-1 + \log(x))$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 - Log[x])),x]

[Out] $-\text{Log}[-1 + \text{Log}[x]]$

Mathics [A]

time = 1.72, size = 7, normalized size = 0.78

$-\text{Log}[-1 + \text{Log}[x]]$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(x*(1 - Log[x])),x]')`

[Out] $-\text{Log}[-1 + \text{Log}[x]]$

Maple [A]

time = 0.01, size = 10, normalized size = 1.11

method	result	size
norman	$-\ln(-1 + \ln(x))$	8
risch	$-\ln(-1 + \ln(x))$	8
derivativedivides	$-\ln(1 - \ln(x))$	10
default	$-\ln(1 - \ln(x))$	10

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(1-ln(x)),x,method=_RETURNVERBOSE)`

[Out] $-\ln(1-\ln(x))$

Maxima [A]

time = 0.28, size = 7, normalized size = 0.78

$-\log(\log(x) - 1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1-log(x)),x, algorithm="maxima")`

[Out] $-\log(\log(x) - 1)$

Fricas [A]

time = 0.34, size = 7, normalized size = 0.78

$-\log(\log(x) - 1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1-log(x)),x, algorithm="fricas")`

[Out] $-\log(\log(x) - 1)$

Sympy [A]

time = 0.05, size = 7, normalized size = 0.78

$$-\log(\log(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(1-ln(x)),x)**[Out]** -log(log(x) - 1)**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.

time = 0.00, size = 24, normalized size = 2.67

$$-\frac{\ln\left(\left(\ln|x| - 1\right)^2 + \left(\frac{1}{2}\pi(1 - \operatorname{sign}(x))\right)^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(1-log(x)),x)**[Out]** -1/2*log(1/4*pi^2*(sgn(x) - 1)^2 + (log(abs(x)) - 1)^2)**Mupad [B]**

time = 0.21, size = 7, normalized size = 0.78

$$-\ln(\ln(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(x*(log(x) - 1)),x)**[Out]** -log(log(x) - 1)

$$3.23 \quad \int \frac{1}{x(1+\log(\frac{x}{a}))} dx$$

Optimal. Leaf size=9

$$\log\left(1 + \log\left(\frac{x}{a}\right)\right)$$

[Out] ln(1+ln(x/a))

Rubi [A]

time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2339, 29}

$$\log\left(\log\left(\frac{x}{a}\right) + 1\right)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 + Log[x/a])),x]

[Out] Log[1 + Log[x/a]]

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 2339

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{x(1+\log(\frac{x}{a}))} dx &= \text{Subst}\left(\int \frac{1}{x} dx, x, 1 + \log\left(\frac{x}{a}\right)\right) \\ &= \log\left(1 + \log\left(\frac{x}{a}\right)\right) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 9, normalized size = 1.00

$$\log\left(1 + \log\left(\frac{x}{a}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 + Log[x/a])),x]

[Out] Log[1 + Log[x/a]]

Mathics [A]

time = 1.80, size = 9, normalized size = 1.00

$$\text{Log} \left[1 + \text{Log} \left[\frac{x}{a} \right] \right]$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x*(1 + Log[x/a])),x]')

[Out] Log[1 + Log[x / a]]

Maple [A]

time = 0.01, size = 10, normalized size = 1.11

method	result	size
derivativdivides	$\ln \left(1 + \ln \left(\frac{x}{a} \right) \right)$	10
default	$\ln \left(1 + \ln \left(\frac{x}{a} \right) \right)$	10
norman	$\ln \left(1 + \ln \left(\frac{x}{a} \right) \right)$	10
risch	$\ln \left(1 + \ln \left(\frac{x}{a} \right) \right)$	10

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(1+ln(x/a)),x,method=_RETURNVERBOSE)

[Out] ln(1+ln(x/a))

Maxima [A]

time = 0.26, size = 9, normalized size = 1.00

$$\log \left(\log \left(\frac{x}{a} \right) + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(1+log(x/a)),x, algorithm="maxima")

[Out] log(log(x/a) + 1)

Fricas [A]

time = 0.32, size = 9, normalized size = 1.00

$$\log \left(\log \left(\frac{x}{a} \right) + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(1+log(x/a)),x, algorithm="fricas")

[Out] log(log(x/a) + 1)

Sympy [A]

time = 0.05, size = 7, normalized size = 0.78

$$\log\left(\log\left(\frac{x}{a}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(1+ln(x/a)),x)

[Out] log(log(x/a) + 1)

Giac [A]

time = 0.00, size = 8, normalized size = 0.89

$$\ln\left(\ln\left(\frac{x}{a}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(1+log(x/a)),x)

[Out] log(log(x/a) + 1)

Mupad [B]

time = 0.21, size = 9, normalized size = 1.00

$$\ln\left(\ln\left(\frac{x}{a}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(log(x/a) + 1)),x)

[Out] log(log(x/a) + 1)

$$3.24 \quad \int \frac{(1 - \sqrt{x} + x)^2}{x^2} dx$$

Optimal. Leaf size=25

$$-\frac{1}{x} + \frac{4}{\sqrt{x}} - 4\sqrt{x} + x + 3\log(x)$$

[Out] -1/x+x+3*ln(x)+4/x^(1/2)-4*x^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1371, 712}

$$x - 4\sqrt{x} + \frac{4}{\sqrt{x}} - \frac{1}{x} + 3\log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[x] + x)^2/x^2,x]

[Out] -x^(-1) + 4/Sqrt[x] - 4*Sqrt[x] + x + 3*Log[x]

Rule 712

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rule 1371

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{(1 - \sqrt{x} + x)^2}{x^2} dx &= 2\text{Subst} \left(\int \frac{(1 - x + x^2)^2}{x^3} dx, x, \sqrt{x} \right) \\ &= 2\text{Subst} \left(\int \left(-2 + \frac{1}{x^3} - \frac{2}{x^2} + \frac{3}{x} + x \right) dx, x, \sqrt{x} \right) \\ &= -\frac{1}{x} + \frac{4}{\sqrt{x}} - 4\sqrt{x} + x + 3\log(x) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 25, normalized size = 1.00

$$-\frac{1}{x} + \frac{4}{\sqrt{x}} - 4\sqrt{x} + x + 3 \log(x)$$

Antiderivative was successfully verified.

`[In] Integrate[(1 - Sqrt[x] + x)^2/x^2,x]``[Out] -x^(-1) + 4/Sqrt[x] - 4*Sqrt[x] + x + 3*Log[x]`**Mathics [A]**

time = 1.83, size = 21, normalized size = 0.84

$$-\frac{1}{x} + \frac{4}{\sqrt{x}} - 4\sqrt{x} + x + 3\text{Log}[x]$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(1 - Sqrt[x] + x)^2/x^2,x]')``[Out] -1 / x + 4 / Sqrt[x] - 4 Sqrt[x] + x + 3 Log[x]`**Maple [A]**

time = 0.03, size = 22, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{1}{x} + x + 3 \ln(x) + \frac{4}{\sqrt{x}} - 4\sqrt{x}$	22
default	$-\frac{1}{x} + x + 3 \ln(x) + \frac{4}{\sqrt{x}} - 4\sqrt{x}$	22
trager	$\frac{(1+x)(-1+x)}{x} - \frac{4(-1+x)}{\sqrt{x}} - 3 \ln\left(\frac{1}{x}\right)$	26

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((1+x-x^(1/2))^2/x^2,x,method=_RETURNVERBOSE)``[Out] -1/x+x+3*ln(x)+4/x^(1/2)-4*x^(1/2)`**Maxima [A]**

time = 0.26, size = 22, normalized size = 0.88

$$x - 4\sqrt{x} + \frac{4\sqrt{x} - 1}{x} + 3 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((1+x-x^(1/2))^2/x^2,x, algorithm="maxima")`

[Out] $x - 4\sqrt{x} + (4\sqrt{x} - 1)/x + 3\log(x)$

Fricas [A]

time = 0.33, size = 24, normalized size = 0.96

$$\frac{x^2 + 6x \log(\sqrt{x}) - 4(x-1)\sqrt{x} - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x-x^(1/2))^2/x^2,x, algorithm="fricas")`

[Out] $(x^2 + 6x\log(\sqrt{x}) - 4(x-1)\sqrt{x} - 1)/x$

Sympy [A]

time = 0.17, size = 22, normalized size = 0.88

$$-4\sqrt{x} + x + 3\log(x) - \frac{1}{x} + \frac{4}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x-x**(1/2))**2/x**2,x)`

[Out] $-4\sqrt{x} + x + 3\log(x) - 1/x + 4/\sqrt{x}$

Giac [A]

time = 0.00, size = 27, normalized size = 1.08

$$x - 4\sqrt{x} + \frac{4\sqrt{x} - 1}{x} + 3 \ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x-x^(1/2))^2/x^2,x)`

[Out] $x - 4\sqrt{x} + (4\sqrt{x} - 1)/x + 3\log(\text{abs}(x))$

Mupad [B]

time = 0.04, size = 24, normalized size = 0.96

$$x + 6 \ln(\sqrt{x}) + \frac{4\sqrt{x} - 1}{x} - 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x - x^(1/2) + 1)^2/x^2,x)`

[Out] $x + 6\log(x^{1/2}) + (4x^{1/2} - 1)/x - 4x^{1/2}$

$$3.25 \quad \int \frac{(2-x^{2/3})(\sqrt{x}+x)}{x^{3/2}} dx$$

Optimal. Leaf size=30

$$4\sqrt{x} - \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7} + 2\log(x)$$

[Out] $-3/2*x^{(2/3)}-6/7*x^{(7/6)}+2*\ln(x)+4*x^{(1/2)}$

Rubi [A]

time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1598, 1834}

$$-\frac{6x^{7/6}}{7} - \frac{3x^{2/3}}{2} + 4\sqrt{x} + 2\log(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2 - x^{(2/3)})*(\text{Sqrt}[x] + x)/x^{(3/2)}, x]$

[Out] $4*\text{Sqrt}[x] - (3*x^{(2/3)})/2 - (6*x^{(7/6)})/7 + 2*\text{Log}[x]$

Rule 1598

$\text{Int}[(u_*)*(x_*)^{(m_*)}*((a_*)*(x_*)^{(p_*)} + (b_*)*(x_*)^{(q_*)})^{(n_*)}, x_Symbol]$
 $:\> \text{Int}[u*x^{(m+n*p)}*(a+b*x^{(q-p)})^n, x] /;$ $\text{FreeQ}\{a, b, m, p, q\}, x]$
 $\&\& \text{IntegerQ}[n] \ \&\& \text{PosQ}[q-p]$

Rule 1834

$\text{Int}[(Pq_*)*((c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol] :\>$
 $\text{Int}[\text{ExpandIntegrand}[(c*x)^m*Pq*(a+b*x^n)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, m,$
 $n\}, x] \ \&\& \text{PolyQ}[Pq, x] \ \&\& (\text{IGtQ}[p, 0] \ || \ \text{EqQ}[n, 1])$

Rubi steps

$$\begin{aligned} \int \frac{(2-x^{2/3})(\sqrt{x}+x)}{x^{3/2}} dx &= \int \frac{(1+\sqrt{x})(2-x^{2/3})}{x} dx \\ &= -\left(6\text{Subst}\left(\int \frac{(1+x^3)(-2+x^4)}{x} dx, x, \sqrt[6]{x}\right)\right) \\ &= -\left(6\text{Subst}\left(\int \left(-\frac{2}{x} - 2x^2 + x^3 + x^6\right) dx, x, \sqrt[6]{x}\right)\right) \\ &= 4\sqrt{x} - \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7} + 2\log(x) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 30, normalized size = 1.00

$$4\sqrt{x} - \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7} + 2\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[((2 - x^(2/3))*(Sqrt[x] + x))/x^(3/2), x]

[Out] 4*Sqrt[x] - (3*x^(2/3))/2 - (6*x^(7/6))/7 + 2*Log[x]

Mathics [A]

time = 3.82, size = 20, normalized size = 0.67

$$4\sqrt{x} - \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7} + 2\text{Log}[x]$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(2 - x^(2/3))*(x + Sqrt[x])/x^(3/2), x]')

[Out] 4 Sqrt[x] - 3 x ^ (2 / 3) / 2 - 6 x ^ (7 / 6) / 7 + 2 Log[x]

Maple [A]

time = 0.01, size = 21, normalized size = 0.70

method	result	size
derivativedivides	$-\frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7} + 2\ln(x) + 4\sqrt{x}$	21
default	$-\frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7} + 2\ln(x) + 4\sqrt{x}$	21

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2-x^(2/3))*(x+x^(1/2))/x^(3/2), x, method=_RETURNVERBOSE)

[Out] -3/2*x^(2/3)-6/7*x^(7/6)+2*ln(x)+4*x^(1/2)

Maxima [A]

time = 0.27, size = 20, normalized size = 0.67

$$-\frac{6}{7}x^{7/6} - \frac{3}{2}x^{2/3} + 4\sqrt{x} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-x^(2/3))*(x+x^(1/2))/x^(3/2), x, algorithm="maxima")

[Out] -6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 2*log(x)

Fricas [A]

time = 0.37, size = 22, normalized size = 0.73

$$-\frac{6}{7}x^{\frac{7}{6}} - \frac{3}{2}x^{\frac{2}{3}} + 4\sqrt{x} + 12\log\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x, algorithm="fricas")

[Out] -6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 12*log(x^(1/6))

Sympy [A]

time = 2.00, size = 27, normalized size = 0.90

$$-\frac{6x^{\frac{7}{6}}}{7} - \frac{3x^{\frac{2}{3}}}{2} + 4\sqrt{x} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-x**(2/3))*(x+x**(1/2))/x**(3/2),x)

[Out] -6*x**(7/6)/7 - 3*x**(2/3)/2 + 4*sqrt(x) + 2*log(x)

Giac [A]

time = 0.00, size = 43, normalized size = 1.43

$$-\frac{6}{7}x^{\frac{1}{6}}x - \frac{3}{2}\left(x^{\frac{1}{3}}\right)^2 + 4x^{\frac{1}{6}}x^{\frac{1}{3}} + \frac{6}{3}\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x)

[Out] -6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 2*log(abs(x))

Mupad [B]

time = 0.03, size = 22, normalized size = 0.73

$$12\ln\left(x^{1/6}\right) + 4\sqrt{x} - \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-((x^(2/3) - 2)*(x + x^(1/2)))/x^(3/2),x)

[Out] 12*log(x^(1/6)) + 4*x^(1/2) - (3*x^(2/3))/2 - (6*x^(7/6))/7

3.26

$$\int \frac{-1+2x}{3+2x} dx$$

Optimal. Leaf size=10

$$x - 2 \log(3 + 2x)$$

[Out] x-2*ln(3+2*x)

Rubi [A]

time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$,

Rules used = {45}

$$x - 2 \log(2x + 3)$$

Antiderivative was successfully verified.

[In] Int[(-1 + 2*x)/(3 + 2*x), x]

[Out] x - 2*Log[3 + 2*x]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{-1+2x}{3+2x} dx &= \int \left(1 - \frac{4}{3+2x}\right) dx \\ &= x - 2 \log(3 + 2x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 10, normalized size = 1.00

$$x - 2 \log(3 + 2x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 2*x)/(3 + 2*x), x]

[Out] x - 2*Log[3 + 2*x]

Mathics [A]

time = 1.75, size = 10, normalized size = 1.00

$$x - 2\text{Log}[3 + 2x]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(2*x - 1)/(2*x + 3),x]')`[Out] `x - 2 Log[3 + 2 x]`**Maple [A]**

time = 0.05, size = 11, normalized size = 1.10

method	result	size
default	$x - 2 \ln(3 + 2x)$	11
norman	$x - 2 \ln(3 + 2x)$	11
meijerg	$-2 \ln\left(1 + \frac{2x}{3}\right) + x$	11
risch	$x - 2 \ln(3 + 2x)$	11

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x-1)/(3+2*x),x,method=_RETURNVERBOSE)`[Out] `x-2*ln(3+2*x)`**Maxima [A]**

time = 0.27, size = 10, normalized size = 1.00

$$x - 2 \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+2*x)/(3+2*x),x, algorithm="maxima")`[Out] `x - 2*log(2*x + 3)`**Fricas [A]**

time = 0.32, size = 10, normalized size = 1.00

$$x - 2 \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+2*x)/(3+2*x),x, algorithm="fricas")`[Out] `x - 2*log(2*x + 3)`

Sympy [A]

time = 0.03, size = 8, normalized size = 0.80

$$x - 2 \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+2*x)/(3+2*x),x)

[Out] x - 2*log(2*x + 3)

Giac [A]

time = 0.00, size = 16, normalized size = 1.60

$$\frac{2}{2}x - 2 \ln |2x + 3|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+2*x)/(3+2*x),x)

[Out] x - 2*log(abs(2*x + 3))

Mupad [B]

time = 0.06, size = 8, normalized size = 0.80

$$x - 2 \ln \left(x + \frac{3}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x - 1)/(2*x + 3),x)

[Out] x - 2*log(x + 3/2)

3.27 $\int \frac{-5+2x}{-2+3x^2} dx$

Optimal. Leaf size=47

$$\frac{1}{12} (4 - 5\sqrt{6}) \log(\sqrt{6} - 3x) + \frac{1}{12} (4 + 5\sqrt{6}) \log(\sqrt{6} + 3x)$$

[Out] 1/12*ln(-3*x+6^(1/2))*(4-5*6^(1/2))+1/12*ln(3*x+6^(1/2))*(4+5*6^(1/2))

Rubi [A]

time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {647, 31}

$$\frac{1}{12} (4 - 5\sqrt{6}) \log(\sqrt{6} - 3x) + \frac{1}{12} (4 + 5\sqrt{6}) \log(3x + \sqrt{6})$$

Antiderivative was successfully verified.

[In] Int[(-5 + 2*x)/(-2 + 3*x^2), x]

[Out] ((4 - 5*Sqrt[6])*Log[Sqrt[6] - 3*x])/12 + ((4 + 5*Sqrt[6])*Log[Sqrt[6] + 3*x])/12

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 647

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[(-a)*c]

Rubi steps

$$\begin{aligned} \int \frac{-5+2x}{-2+3x^2} dx &= \frac{1}{4} (4 - 5\sqrt{6}) \int \frac{1}{-\sqrt{6} + 3x} dx + \frac{1}{4} (4 + 5\sqrt{6}) \int \frac{1}{\sqrt{6} + 3x} dx \\ &= \frac{1}{12} (4 - 5\sqrt{6}) \log(\sqrt{6} - 3x) + \frac{1}{12} (4 + 5\sqrt{6}) \log(\sqrt{6} + 3x) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 47, normalized size = 1.00

$$\frac{1}{12} (4 - 5\sqrt{6}) \log(\sqrt{6} - 3x) + \frac{1}{12} (4 + 5\sqrt{6}) \log(\sqrt{6} + 3x)$$

Antiderivative was successfully verified.

[In] Integrate[(-5 + 2*x)/(-2 + 3*x^2),x]

[Out] ((4 - 5*sqrt[6])*Log[Sqrt[6] - 3*x])/12 + ((4 + 5*sqrt[6])*Log[Sqrt[6] + 3*x])/12

Mathics [A]

time = 2.00, size = 35, normalized size = 0.74

$$\frac{(4 - 5\sqrt{6}) \operatorname{Log}\left[-\frac{\sqrt{6}}{3} + x\right]}{12} + \frac{(4 + 5\sqrt{6}) \operatorname{Log}\left[\frac{\sqrt{6}}{3} + x\right]}{12}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(2*x - 5)/(3*x^2 - 2),x]')

[Out] (4 - 5 Sqrt[6]) Log[-Sqrt[6] / 3 + x] / 12 + (4 + 5 Sqrt[6]) Log[Sqrt[6] / 3 + x] / 12

Maple [A]

time = 0.05, size = 24, normalized size = 0.51

method	result	size
default	$\frac{\ln(3x^2-2)}{3} + \frac{5\sqrt{6} \operatorname{arctanh}\left(\frac{x\sqrt{6}}{2}\right)}{6}$	24
meijerg	$\frac{5\sqrt{6} \operatorname{arctanh}\left(\frac{x\sqrt{2}\sqrt{3}}{2}\right)}{6} + \frac{\ln\left(1-\frac{3x^2}{2}\right)}{3}$	27
risch	$\frac{\ln(3x+\sqrt{6})}{3} + \frac{5\ln(3x+\sqrt{6})\sqrt{6}}{12} + \frac{\ln(3x-\sqrt{6})}{3} - \frac{5\ln(3x-\sqrt{6})\sqrt{6}}{12}$	52

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x-5)/(3*x^2-2),x,method=_RETURNVERBOSE)

[Out] 1/3*ln(3*x^2-2)+5/6*6^(1/2)*arctanh(1/2*x*6^(1/2))

Maxima [A]

time = 0.35, size = 36, normalized size = 0.77

$$-\frac{5}{12}\sqrt{6}\log\left(\frac{3x-\sqrt{6}}{3x+\sqrt{6}}\right) + \frac{1}{3}\log(3x^2-2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+2*x)/(3*x^2-2),x, algorithm="maxima")

[Out] $-5/12*\sqrt{6}*\log((3*x - \sqrt{6})/(3*x + \sqrt{6})) + 1/3*\log(3*x^2 - 2)$

Fricas [A]

time = 0.32, size = 40, normalized size = 0.85

$$\frac{5}{12} \sqrt{6} \log\left(\frac{3x^2 + 2\sqrt{6}x + 2}{3x^2 - 2}\right) + \frac{1}{3} \log(3x^2 - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-5+2*x)/(3*x^2-2),x, algorithm="fricas")`

[Out] $5/12*\sqrt{6}*\log((3*x^2 + 2*\sqrt{6}*x + 2)/(3*x^2 - 2)) + 1/3*\log(3*x^2 - 2)$

Sympy [A]

time = 0.06, size = 42, normalized size = 0.89

$$\left(\frac{1}{3} - \frac{5\sqrt{6}}{12}\right) \log\left(x - \frac{\sqrt{6}}{3}\right) + \left(\frac{1}{3} + \frac{5\sqrt{6}}{12}\right) \log\left(x + \frac{\sqrt{6}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-5+2*x)/(3*x**2-2),x)`

[Out] $(1/3 - 5*\sqrt{6}/12)*\log(x - \sqrt{6}/3) + (1/3 + 5*\sqrt{6}/12)*\log(x + \sqrt{6}/3)$

Giac [A]

time = 0.00, size = 51, normalized size = 1.09

$$\frac{1}{12} (-5\sqrt{6} + 4) \ln\left|x - \frac{\sqrt{6}}{3}\right| + \frac{1}{12} (5\sqrt{6} + 4) \ln\left|x + \frac{\sqrt{6}}{3}\right|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-5+2*x)/(3*x^2-2),x)`

[Out] $1/12*(5*\sqrt{6} + 4)*\log(\text{abs}(x + 1/3*\sqrt{6})) - 1/12*(5*\sqrt{6} - 4)*\log(\text{abs}(x - 1/3*\sqrt{6}))$

Mupad [B]

time = 0.13, size = 47, normalized size = 1.00

$$\frac{\ln\left(x - \frac{\sqrt{6}}{3}\right)}{3} + \frac{\ln\left(x + \frac{\sqrt{6}}{3}\right)}{3} - \frac{5\sqrt{6} \ln\left(x - \frac{\sqrt{6}}{3}\right)}{12} + \frac{5\sqrt{6} \ln\left(x + \frac{\sqrt{6}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x - 5)/(3*x^2 - 2),x)`

[Out] $\log(x - 6^{(1/2)}/3)/3 + \log(x + 6^{(1/2)}/3)/3 - (5*6^{(1/2)}*\log(x - 6^{(1/2)}/3))/12 + (5*6^{(1/2)}*\log(x + 6^{(1/2)}/3))/12$

3.28 $\int \frac{-5+2x}{2+3x^2} dx$

Optimal. Leaf size=30

$$-\frac{5 \tan^{-1}\left(\sqrt{\frac{3}{2}} x\right)}{\sqrt{6}} + \frac{1}{3} \log(2 + 3x^2)$$

[Out] 1/3*ln(3*x^2+2)-5/6*arctan(1/2*x*6^(1/2))*6^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {649, 209, 266}

$$\frac{1}{3} \log(3x^2 + 2) - \frac{5 \tan^{-1}\left(\sqrt{\frac{3}{2}} x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(-5 + 2*x)/(2 + 3*x^2), x]

[Out] (-5*ArcTan[Sqrt[3/2]*x])/Sqrt[6] + Log[2 + 3*x^2]/3

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 649

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[(-a)*c]

Rubi steps

$$\int \frac{-5 + 2x}{2 + 3x^2} dx = 2 \int \frac{x}{2 + 3x^2} dx - 5 \int \frac{1}{2 + 3x^2} dx$$

$$= -\frac{5 \tan^{-1} \left(\sqrt{\frac{3}{2}} x \right)}{\sqrt{6}} + \frac{1}{3} \log(2 + 3x^2)$$

Mathematica [A]

time = 0.01, size = 30, normalized size = 1.00

$$-\frac{5 \tan^{-1} \left(\sqrt{\frac{3}{2}} x \right)}{\sqrt{6}} + \frac{1}{3} \log(2 + 3x^2)$$

Antiderivative was successfully verified.

`[In] Integrate[(-5 + 2*x)/(2 + 3*x^2), x]``[Out] (-5*ArcTan[Sqrt[3/2]*x])/Sqrt[6] + Log[2 + 3*x^2]/3`**Mathics [A]**

time = 1.85, size = 21, normalized size = 0.70

$$\frac{-5\sqrt{6} \operatorname{ArcTan} \left[\frac{\sqrt{6} x}{2} \right]}{6} + \frac{\operatorname{Log} \left[\frac{2}{3} + x^2 \right]}{3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(2*x - 5)/(3*x^2 + 2), x]')``[Out] -5 Sqrt[6] ArcTan[Sqrt[6] x / 2] / 6 + Log[2 / 3 + x ^ 2] / 3`**Maple [A]**

time = 0.06, size = 24, normalized size = 0.80

method	result	size
default	$\frac{\ln(3x^2+2)}{3} - \frac{5 \arctan \left(\frac{x\sqrt{6}}{2} \right) \sqrt{6}}{6}$	24
risch	$\frac{\ln(9x^2+6)}{3} - \frac{5 \arctan \left(\frac{x\sqrt{6}}{2} \right) \sqrt{6}}{6}$	24

meijerg	$-\frac{5\sqrt{6} \arctan\left(\frac{x\sqrt{2}\sqrt{3}}{2}\right)}{6} + \frac{\ln\left(1+\frac{3x^2}{2}\right)}{3}$	27
---------	---	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x-5)/(3*x^2+2),x,method=_RETURNVERBOSE)`

[Out] $1/3*\ln(3*x^2+2)-5/6*\arctan(1/2*x*6^{(1/2)})*6^{(1/2)}$

Maxima [A]

time = 0.35, size = 23, normalized size = 0.77

$$-\frac{5}{6}\sqrt{6} \arctan\left(\frac{1}{2}\sqrt{6}x\right) + \frac{1}{3}\log(3x^2+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-5+2*x)/(3*x^2+2),x, algorithm="maxima")`

[Out] $-5/6*\sqrt{6}*\arctan(1/2*\sqrt{6}*x) + 1/3*\log(3*x^2 + 2)$

Fricas [A]

time = 0.33, size = 23, normalized size = 0.77

$$-\frac{5}{6}\sqrt{6} \arctan\left(\frac{1}{2}\sqrt{6}x\right) + \frac{1}{3}\log(3x^2+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-5+2*x)/(3*x^2+2),x, algorithm="fricas")`

[Out] $-5/6*\sqrt{6}*\arctan(1/2*\sqrt{6}*x) + 1/3*\log(3*x^2 + 2)$

Sympy [A]

time = 0.06, size = 27, normalized size = 0.90

$$\frac{\log\left(x^2 + \frac{2}{3}\right)}{3} - \frac{5\sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-5+2*x)/(3*x**2+2),x)`

[Out] $\log(x**2 + 2/3)/3 - 5*\sqrt{6}*\operatorname{atan}(\sqrt{6}*x/2)/6$

Giac [A]

time = 0.00, size = 46, normalized size = 1.53

$$\frac{\ln\left(x^2 + \frac{\sqrt{6}\sqrt{6}}{3\cdot 3}\right)}{3} - \frac{5}{6}\sqrt{6} \arctan\left(\frac{x}{\frac{\sqrt{6}}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+2*x)/(3*x^2+2),x)

[Out] -5/6*sqrt(6)*arctan(1/2*sqrt(6)*x) + 1/3*log(x^2 + 2/3)

Mupad [B]

time = 0.04, size = 21, normalized size = 0.70

$$\frac{\ln\left(x^2 + \frac{2}{3}\right)}{3} - \frac{5\sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x - 5)/(3*x^2 + 2),x)

[Out] log(x^2 + 2/3)/3 - (5*6^(1/2)*atan((6^(1/2)*x)/2))/6

3.29 $\int \sin\left(\frac{x}{4}\right) \sin(x) dx$

Optimal. Leaf size=21

$$\frac{2}{3} \sin\left(\frac{3x}{4}\right) - \frac{2}{5} \sin\left(\frac{5x}{4}\right)$$

[Out] 2/3*sin(3/4*x)-2/5*sin(5/4*x)

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4367}

$$\frac{2}{3} \sin\left(\frac{3x}{4}\right) - \frac{2}{5} \sin\left(\frac{5x}{4}\right)$$

Antiderivative was successfully verified.

[In] Int[Sin[x/4]*Sin[x],x]

[Out] (2*Sin[(3*x)/4])/3 - (2*Sin[(5*x)/4])/5

Rule 4367

Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\int \sin\left(\frac{x}{4}\right) \sin(x) dx = \frac{2}{3} \sin\left(\frac{3x}{4}\right) - \frac{2}{5} \sin\left(\frac{5x}{4}\right)$$

Mathematica [A]

time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{3} \sin\left(\frac{3x}{4}\right) - \frac{2}{5} \sin\left(\frac{5x}{4}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x/4]*Sin[x],x]

[Out] (2*Sin[(3*x)/4])/3 - (2*Sin[(5*x)/4])/5

Mathics [A]

time = 1.93, size = 13, normalized size = 0.62

$$\frac{-2\sin\left[\frac{5x}{4}\right]}{5} + \frac{2\sin\left[\frac{3x}{4}\right]}{3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sin[x/4]*Sin[x],x]')
```

```
[Out] -2 Sin[5 x / 4] / 5 + 2 Sin[3 x / 4] / 3
```

Maple [A]

time = 0.06, size = 14, normalized size = 0.67

method	result	size
default	$\frac{2\sin\left(\frac{3x}{4}\right)}{3} - \frac{2\sin\left(\frac{5x}{4}\right)}{5}$	14
risch	$\frac{2\sin\left(\frac{3x}{4}\right)}{3} - \frac{2\sin\left(\frac{5x}{4}\right)}{5}$	14
norman	$-\frac{8\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{8}\right)\right)}{15} + \frac{32\left(\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{8}\right)}{15} + \frac{8\tan\left(\frac{x}{2}\right)}{15} - \frac{32\tan\left(\frac{x}{8}\right)}{15}$ $\frac{\phantom{-\frac{8\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{8}\right)\right)}{15} + \frac{32\left(\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{8}\right)}{15} + \frac{8\tan\left(\frac{x}{2}\right)}{15} - \frac{32\tan\left(\frac{x}{8}\right)}{15}}{\left(1+\tan^2\left(\frac{x}{8}\right)\right)\left(1+\tan^2\left(\frac{x}{2}\right)\right)}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/4*x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*sin(3/4*x)-2/5*sin(5/4*x)
```

Maxima [A]

time = 0.28, size = 13, normalized size = 0.62

$$-\frac{2}{5}\sin\left(\frac{5}{4}x\right) + \frac{2}{3}\sin\left(\frac{3}{4}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -2/5*sin(5/4*x) + 2/3*sin(3/4*x)
```

Fricas [A]

time = 0.34, size = 24, normalized size = 1.14

$$-\frac{16}{15}\left(6\cos\left(\frac{1}{4}x\right)^4 - 7\cos\left(\frac{1}{4}x\right)^2 + 1\right)\sin\left(\frac{1}{4}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4*x)*sin(x),x, algorithm="fricas")
```

[Out] $-16/15*(6*\cos(1/4*x)^4 - 7*\cos(1/4*x)^2 + 1)*\sin(1/4*x)$

Sympy [A]

time = 0.14, size = 22, normalized size = 1.05

$$-\frac{16 \sin\left(\frac{x}{4}\right) \cos(x)}{15} + \frac{4 \sin(x) \cos\left(\frac{x}{4}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(1/4*x)*sin(x),x)`

[Out] $-16*\sin(x/4)*\cos(x)/15 + 4*\sin(x)*\cos(x/4)/15$

Giac [A]

time = 0.00, size = 26, normalized size = 1.24

$$4 \left(-\frac{8}{5} \sin^5\left(\frac{x}{4}\right) + \frac{4}{3} \sin^3\left(\frac{x}{4}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(1/4*x)*sin(x),x)`

[Out] $-32/5*\sin(1/4*x)^5 + 16/3*\sin(1/4*x)^3$

Mupad [B]

time = 0.17, size = 13, normalized size = 0.62

$$\frac{2 \sin\left(\frac{3x}{4}\right)}{3} - \frac{2 \sin\left(\frac{5x}{4}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x/4)*sin(x),x)`

[Out] $(2*\sin((3*x)/4))/3 - (2*\sin((5*x)/4))/5$

3.30 $\int \cos(3x) \cos(4x) dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2} + \frac{1}{14} \sin(7x)$$

[Out] 1/2*sin(x)+1/14*sin(7*x)

Rubi [A]

time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4368}

$$\frac{\sin(x)}{2} + \frac{1}{14} \sin(7x)$$

Antiderivative was successfully verified.

[In] Int[Cos[3*x]*Cos[4*x],x]

[Out] Sin[x]/2 + Sin[7*x]/14

Rule 4368

Int[cos[(a_.) + (b_.)*(x_.)]*cos[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\int \cos(3x) \cos(4x) dx = \frac{\sin(x)}{2} + \frac{1}{14} \sin(7x)$$

Mathematica [A]

time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sin(x)}{2} + \frac{1}{14} \sin(7x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[3*x]*Cos[4*x],x]

[Out] Sin[x]/2 + Sin[7*x]/14

Mathics [A]

time = 2.05, size = 21, normalized size = 1.40

$$\frac{-3\cos[4x]\sin[3x]}{7} + \frac{4\cos[3x]\sin[4x]}{7}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Cos[3*x]*Cos[4*x],x]')
```

```
[Out] -3 Cos[4 x] Sin[3 x] / 7 + 4 Cos[3 x] Sin[4 x] / 7
```

Maple [A]

time = 0.06, size = 12, normalized size = 0.80

method	result	size
default	$\frac{\sin(x)}{2} + \frac{\sin(7x)}{14}$	12
risch	$\frac{\sin(x)}{2} + \frac{\sin(7x)}{14}$	12
norman	$\frac{-\frac{8 \tan(2x) \left(\tan^2\left(\frac{3x}{2}\right)\right)}{7} + \frac{6 \left(\tan^2(2x)\right) \tan\left(\frac{3x}{2}\right)}{7} + \frac{8 \tan(2x)}{7} - \frac{6 \tan\left(\frac{3x}{2}\right)}{7}}{\left(1 + \tan^2\left(\frac{3x}{2}\right)\right) \left(1 + \tan^2(2x)\right)}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*cos(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(x)+1/14*sin(7*x)
```

Maxima [A]

time = 0.28, size = 11, normalized size = 0.73

$$\frac{1}{14} \sin(7x) + \frac{1}{2} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="maxima")
```

```
[Out] 1/14*sin(7*x) + 1/2*sin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(11) = 22.

time = 0.35, size = 24, normalized size = 1.60

$$\frac{1}{7} (32 \cos(x)^6 - 40 \cos(x)^4 + 12 \cos(x)^2 + 3) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="fricas")
```

[Out] $1/7*(32*\cos(x)^6 - 40*\cos(x)^4 + 12*\cos(x)^2 + 3)*\sin(x)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$

time = 0.14, size = 26, normalized size = 1.73

$$-\frac{3 \sin(3x) \cos(4x)}{7} + \frac{4 \sin(4x) \cos(3x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(3*x)*cos(4*x),x)`

[Out] $-3*\sin(3*x)*\cos(4*x)/7 + 4*\sin(4*x)*\cos(3*x)/7$

Giac [A]

time = 0.00, size = 13, normalized size = 0.87

$$\frac{\sin x}{2} + \frac{\sin(7x)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(3*x)*cos(4*x),x)`

[Out] $1/14*\sin(7*x) + 1/2*\sin(x)$

Mupad [B]

time = 0.06, size = 11, normalized size = 0.73

$$\frac{\sin(7x)}{14} + \frac{\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(3*x)*cos(4*x),x)`

[Out] $\sin(7*x)/14 + \sin(x)/2$

3.31 $\int -\tan(a-x)\tan(x)dx$

Optimal. Leaf size=21

$$-x + \cot(a) \log(\cos(a-x)) - \cot(a) \log(\cos(x))$$

[Out] `-x-cot(a)*ln(cos(x))+cot(a)*ln(cos(a-x))`

Rubi [A]

time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4708, 4706, 3556}

$$\cot(a) \log(\cos(a-x)) - \cot(a) \log(\cos(x)) - x$$

Antiderivative was successfully verified.

[In] `Int[-(Tan[a-x]*Tan[x]),x]`

[Out] `-x + Cot[a]*Log[Cos[a-x]] - Cot[a]*Log[Cos[x]]`

Rule 3556

`Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 4706

`Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] := Dist[-Csc[(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]`

Rule 4708

`Int[Tan[(a_.) + (b_.)*(x_)]*Tan[(c_) + (d_.)*(x_)], x_Symbol] := Simp[(-b)*(x/d), x] + Dist[(b/d)*Cos[(b*c - a*d)/d], Int[Sec[a + b*x]*Sec[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]`

Rubi steps

$$\begin{aligned} \int -\tan(a-x)\tan(x)dx &= -x + \cos(a) \int \sec(a-x)\sec(x)dx \\ &= -x + \cot(a) \int \tan(a-x)dx + \cot(a) \int \tan(x)dx \\ &= -x + \cot(a) \log(\cos(a-x)) - \cot(a) \log(\cos(x)) \end{aligned}$$

Mathematica [A]

time = 0.06, size = 21, normalized size = 1.00

$$-x + \cot(a) \log(\cos(a - x)) - \cot(a) \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[-(Tan[a - x]*Tan[x]),x]

[Out] -x + Cot[a]*Log[Cos[a - x]] - Cot[a]*Log[Cos[x]]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Tan[x]*Tan[x - a],x]')

[Out] cought exception:

Maple [A]

time = 0.07, size = 20, normalized size = 0.95

method	result	size
derivativedivides	$-\arctan(\tan(x)) + \frac{\ln(1+\tan(x)\tan(a))}{\tan(a)}$	20
default	$-\arctan(\tan(x)) + \frac{\ln(1+\tan(x)\tan(a))}{\tan(a)}$	20
risch	$-x + \frac{i \ln(e^{2ia} + e^{2ix})e^{2ia}}{e^{2ia} - 1} + \frac{i \ln(e^{2ia} + e^{2ix})}{e^{2ia} - 1} - \frac{i \ln(e^{2ix} + 1)e^{2ia}}{e^{2ia} - 1} - \frac{i \ln(e^{2ix} + 1)}{e^{2ia} - 1}$	103

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-tan(x)*tan(a-x),x,method=_RETURNVERBOSE)

[Out] -arctan(tan(x))+1/tan(a)*ln(1+tan(x)*tan(a))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(21) = 42.

time = 0.36, size = 186, normalized size = 8.86

$$\frac{(\cos(2a)^2 + \sin(2a)^2 - 2\cos(2a) + 1)x + (\cos(2a)^2 + \sin(2a)^2 - 1)\arctan(\sin(2a) + \sin(2x), \cos(2a) + \cos(2x)) - (\cos(2a)^2 + \sin(2a)^2 - 1)\arctan(\sin(2x), \cos(2x) + 1) - \log(\cos(2a)^2 + 2\cos(2a)\cos(2x) + \cos(2x)^2 + \sin(2a)^2 + 2\sin(2a)\sin(2x) + \sin(2x)^2)\sin(2a) + \log(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1)\sin(2a)}{\cos(2a)^2 + \sin(2a)^2 - 2\cos(2a) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-tan(x)*tan(a-x),x, algorithm="maxima")

[Out] -((cos(2*a)^2 + sin(2*a)^2 - 2*cos(2*a) + 1)*x + (cos(2*a)^2 + sin(2*a)^2 - 1)*arctan2(sin(2*a) + sin(2*x), cos(2*a) + cos(2*x)) - (cos(2*a)^2 + sin(2

$*a)^2 - 1) \cdot \arctan 2(\sin(2*x), \cos(2*x) + 1) - \log(\cos(2*a)^2 + 2*\cos(2*a)*\cos(2*x) + \cos(2*x)^2 + \sin(2*a)^2 + 2*\sin(2*a)*\sin(2*x) + \sin(2*x)^2) * \sin(2*a) + \log(\cos(2*x)^2 + \sin(2*x)^2 + 2*\cos(2*x) + 1) * \sin(2*a) / (\cos(2*a)^2 + \sin(2*a)^2 - 2*\cos(2*a) + 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(21) = 42.

time = 0.43, size = 89, normalized size = 4.24

$$\frac{(\cos(2a) + 1) \log\left(-\frac{(\cos(2a)-1)\tan(x)^2 - 2\sin(2a)\tan(x) - \cos(2a)-1}{(\cos(2a)+1)\tan(x)^2 + \cos(2a)+1}\right) - (\cos(2a) + 1) \log\left(\frac{1}{\tan(x)^2 + 1}\right) - 2x \sin(2a)}{2 \sin(2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-tan(x)*tan(a-x),x, algorithm="fricas")

[Out] $1/2*((\cos(2*a) + 1)*\log(-((\cos(2*a) - 1)*\tan(x)^2 - 2*\sin(2*a)*\tan(x) - \cos(2*a) - 1)/((\cos(2*a) + 1)*\tan(x)^2 + \cos(2*a) + 1))) - (\cos(2*a) + 1)*\log(1/(\tan(x)^2 + 1)) - 2*x*\sin(2*a))/\sin(2*a)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(19) = 38

time = 0.66, size = 138, normalized size = 6.57

$$-\left(\begin{cases} \frac{2x \tan(a)}{2 \tan^2(a)+2} - \frac{2 \log\left(\frac{\tan(x)+\frac{1}{\tan(a)}}{\tan^2(a)+2}\right)}{2 \tan^2(a)+2} + \frac{\log(\tan^2(x)+1)}{2 \tan^2(a)+2} & \text{for } a \neq 0 \\ \frac{\log(\tan^2(x)+1)}{2} & \text{otherwise} \end{cases}\right) \tan(a) + \begin{cases} -\frac{2x \tan(a)}{2 \tan^3(a)+2 \tan(a)} + \frac{2 \log\left(\frac{\tan(x)+\frac{1}{\tan(a)}}{\tan^3(a)+2 \tan(a)}\right)}{2 \tan^3(a)+2 \tan(a)} + \frac{\log(\tan^2(x)+1) \tan^2(a)}{2 \tan^3(a)+2 \tan(a)} & \text{for } a \neq 0 \\ -x + \tan(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-tan(x)*tan(a-x),x)

[Out] $-\text{Piecewise}((2*x*\tan(a)/(2*\tan(a)**2 + 2) - 2*\log(\tan(x) + 1/\tan(a))/(2*\tan(a)**2 + 2) + \log(\tan(x)**2 + 1)/(2*\tan(a)**2 + 2), \text{Ne}(a, 0)), (\log(\tan(x)**2 + 1)/2, \text{True}))*\tan(a) + \text{Piecewise}((-2*x*\tan(a)/(2*\tan(a)**3 + 2*\tan(a)) + 2*\log(\tan(x) + 1/\tan(a))/(2*\tan(a)**3 + 2*\tan(a)) + \log(\tan(x)**2 + 1)*\tan(a)**2/(2*\tan(a)**3 + 2*\tan(a)), \text{Ne}(a, 0)), (-x + \tan(x), \text{True}))$

Giac [A]

time = 0.01, size = 16, normalized size = 0.76

$$\frac{\ln |\tan x \cdot \tan a + 1|}{\tan a} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-tan(x)*tan(a-x),x)

[Out] $-x + \log(\text{abs}(\tan(a)*\tan(x) + 1))/\tan(a)$

Mupad [B]

time = 1.33, size = 118, normalized size = 5.62

$$-x - \frac{\sin(2a) \ln(\sin(2a+x)^2 \sin(2a)^2 \sin(x)^2 \sin(4a) - \sin(2x) + \sin(4a+2x))}{2} - \frac{\sin(2a) \ln(\sin(2a)(2\sin(a)^2 - 1) - \sin(2a)^2 \sin(x)^2 - 1) - \sin(2a) \sin(2x) \sin(2a)}{2} \frac{1}{\sin(a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-tan(a - x)*tan(x),x)`

[Out] `- x - ((sin(2*a)*log(sin(4*a) - sin(2*x) + sin(4*a + 2*x) - sin(x)^2*2i + sin(2*a + x)^2*2i + sin(2*a)^2*2i))/2 - (sin(2*a)*log(sin(2*a)*(2*sin(a)^2 - 1) - sin(2*a)^2*1i + sin(2*a)*(2*sin(x)^2 - 1) - sin(2*a)*sin(2*x)*1i))/2)/sin(a)^2`

3.32 $\int \sin^2(x) dx$

Optimal. Leaf size=14

$$\frac{x}{2} - \frac{1}{2} \cos(x) \sin(x)$$

[Out] 1/2*x-1/2*cos(x)*sin(x)

Rubi [A]

time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2715, 8}

$$\frac{x}{2} - \frac{1}{2} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^2,x]

[Out] x/2 - (Cos[x]*Sin[x])/2

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sine[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sine[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \sin^2(x) dx &= -\frac{1}{2} \cos(x) \sin(x) + \frac{\int 1 dx}{2} \\ &= \frac{x}{2} - \frac{1}{2} \cos(x) \sin(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2} - \frac{1}{4} \sin(2x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^2,x]

[Out] x/2 - Sin[2*x]/4

Mathics [A]

time = 1.84, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin(2x)}{4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Sin[x]^2,x]')

[Out] x / 2 - Sin[2 x] / 4

Maple [A]

time = 0.00, size = 11, normalized size = 0.79

method	result	size
default	$\frac{x}{2} - \frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2} - \frac{\sin(2x)}{4}$	11
meijerg	$\frac{\sqrt{\pi} \left(\frac{2x}{\sqrt{\pi}} - \frac{\sin(2x)}{\sqrt{\pi}} \right)}{4}$	22
norman	$\frac{\tan^3\left(\frac{x}{2}\right) + x\left(\tan^2\left(\frac{x}{2}\right) + \frac{x}{2} + \frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2} - \tan\left(\frac{x}{2}\right)\right)}{\left(1 + \tan^2\left(\frac{x}{2}\right)\right)^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*x-1/2*cos(x)*sin(x)

Maxima [A]

time = 0.28, size = 10, normalized size = 0.71

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2,x, algorithm="maxima")

[Out] 1/2*x - 1/4*sin(2*x)

Fricas [A]

time = 0.35, size = 10, normalized size = 0.71

$$-\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2,x, algorithm="fricas")

[Out] -1/2*cos(x)*sin(x) + 1/2*x

Sympy [A]

time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**2,x)

[Out] x/2 - sin(x)*cos(x)/2

Giac [A]

time = 0.00, size = 13, normalized size = 0.93

$$\frac{x}{2} - \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2,x)

[Out] 1/2*x - 1/4*sin(2*x)

Mupad [B]

time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^2,x)

[Out] x/2 - sin(2*x)/4

3.33 $\int \cos^2(x) dx$

Optimal. Leaf size=14

$$\frac{x}{2} + \frac{1}{2} \cos(x) \sin(x)$$

[Out] 1/2*x+1/2*cos(x)*sin(x)

Rubi [A]

time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2715, 8}

$$\frac{x}{2} + \frac{1}{2} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2,x]

[Out] x/2 + (Cos[x]*Sin[x])/2

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((b*SIN[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos^2(x) dx &= \frac{1}{2} \cos(x) \sin(x) + \frac{\int 1 dx}{2} \\ &= \frac{x}{2} + \frac{1}{2} \cos(x) \sin(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2} + \frac{1}{4} \sin(2x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^2,x]

[Out] x/2 + Sin[2*x]/4

Mathics [A]

time = 1.72, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin[2x]}{4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Cos[x]^2,x]')

[Out] x / 2 + Sin[2 x] / 4

Maple [A]

time = 0.02, size = 11, normalized size = 0.79

method	result	size
default	$\frac{x}{2} + \frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2} + \frac{\sin(2x)}{4}$	11
norman	$\frac{x(\tan^2(\frac{x}{2}) + \frac{x}{2} - (\tan^3(\frac{x}{2})) + \frac{x(\tan^4(\frac{x}{2}))}{2}) + \tan(\frac{x}{2})}{(1 + \tan^2(\frac{x}{2}))^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*x+1/2*cos(x)*sin(x)

Maxima [A]

time = 0.26, size = 10, normalized size = 0.71

$$\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2,x, algorithm="maxima")

[Out] 1/2*x + 1/4*sin(2*x)

Fricas [A]

time = 0.34, size = 10, normalized size = 0.71

$$\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2,x, algorithm="fricas")

[Out] 1/2*cos(x)*sin(x) + 1/2*x

Sympy [A]

time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(x) \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**2,x)

[Out] x/2 + sin(x)*cos(x)/2

Giac [A]

time = 0.00, size = 12, normalized size = 0.86

$$\frac{x}{2} + \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2,x)

[Out] 1/2*x + 1/4*sin(2*x)

Mupad [B]

time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2,x)

[Out] x/2 + sin(2*x)/4

3.34 $\int \cos^3(x) \sin(x) dx$

Optimal. Leaf size=8

$$-\frac{1}{4} \cos^4(x)$$

[Out] -1/4*cos(x)^4

Rubi [A]

time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2645, 30}

$$-\frac{1}{4} \cos^4(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^3*Sin[x],x]

[Out] -1/4*Cos[x]^4

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2645

Int[(cos[(e_) + (f_)*(x_)]*(a_))^(m_)*sin[(e_) + (f_)*(x_)]^(n_), x_Symbol] := Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])

Rubi steps

$$\begin{aligned} \int \cos^3(x) \sin(x) dx &= -\text{Subst} \left(\int x^3 dx, x, \cos(x) \right) \\ &= -\frac{1}{4} \cos^4(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 8, normalized size = 1.00

$$-\frac{1}{4} \cos^4(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Sin[x],x]
```

```
[Out] -1/4*Cos[x]^4
```

Mathics [A]

time = 1.70, size = 6, normalized size = 0.75

$$-\frac{\text{Cos}[x]^4}{4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sin[x]*Cos[x]^3,x]')
```

```
[Out] -Cos[x]^4 / 4
```

Maple [A]

time = 0.02, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{\cos^4(x)}{4}$	7
default	$-\frac{\cos^4(x)}{4}$	7
risch	$-\frac{\cos(4x)}{32} - \frac{\cos(2x)}{8}$	14
norman	$\frac{2(\tan^2(\frac{x}{2})) + 2(\tan^6(\frac{x}{2}))}{(1 + \tan^2(\frac{x}{2}))^4}$	29
meijerg	$\frac{\sqrt{\pi} \left(\frac{1}{\sqrt{\pi}} - \frac{\cos(2x)}{\sqrt{\pi}} \right)}{8} + \frac{\sqrt{\pi} \left(\frac{1}{\sqrt{\pi}} - \frac{\cos(4x)}{\sqrt{\pi}} \right)}{32}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*cos(x)^4
```

Maxima [A]

time = 0.26, size = 6, normalized size = 0.75

$$-\frac{1}{4} \cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x),x, algorithm="maxima")
```

```
[Out] -1/4*cos(x)^4
```

Fricas [A]

time = 0.33, size = 6, normalized size = 0.75

$$-\frac{1}{4} \cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)^3*sin(x),x, algorithm="fricas")``[Out] -1/4*cos(x)^4`**Sympy [A]**

time = 0.03, size = 7, normalized size = 0.88

$$-\frac{\cos^4(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)**3*sin(x),x)``[Out] -cos(x)**4/4`**Giac [A]**

time = 0.00, size = 10, normalized size = 1.25

$$-\frac{\cos^4 x}{2 \cdot 2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)^3*sin(x),x)``[Out] -1/4*cos(x)^4`**Mupad [B]**

time = 0.03, size = 12, normalized size = 1.50

$$-\frac{\sin(x)^2 (\sin(x)^2 - 2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x)^3*sin(x),x)``[Out] -(sin(x)^2*(sin(x)^2 - 2))/4`

3.35 $\int \cot^3(x) \csc(x) dx$

Optimal. Leaf size=11

$$\csc(x) - \frac{\csc^3(x)}{3}$$

[Out] $-1/3/\sin(x)^3+1/\sin(x)$

Rubi [A]

time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2686}

$$\csc(x) - \frac{\csc^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[Cot[x]^3*Csc[x],x]

[Out] Csc[x] - Csc[x]^3/3

Rule 2686

Int[((a_.)*sec[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rubi steps

$$\begin{aligned} \int \cot^3(x) \csc(x) dx &= -\text{Subst} \left(\int (-1 + x^2) dx, x, \csc(x) \right) \\ &= \csc(x) - \frac{\csc^3(x)}{3} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 11, normalized size = 1.00

$$\csc(x) - \frac{\csc^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]^3*Csc[x],x]

[Out] Csc[x] - Csc[x]^3/3

Mathics [A]

time = 1.77, size = 14, normalized size = 1.27

$$3 - \frac{1}{\text{Sin}[x]^2}$$

$$\frac{3 \text{Sin}[x]}{3 \text{Sin}[x]}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Cos[x]^3/Sin[x]^4,x]')

[Out] (3 - 1 / Sin[x] ^ 2) / (3 Sin[x])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

time = 0.03, size = 32, normalized size = 2.91

method	result	size
default	$-\frac{\cos^4(x)}{3 \sin(x)^3} + \frac{\cos^4(x)}{3 \sin(x)} + \frac{(2+\cos^2(x)) \sin(x)}{3}$	32
norman	$-\frac{1}{24} + \frac{3(\tan^2(\frac{x}{2}))}{8} + \frac{3(\tan^4(\frac{x}{2}))}{8} - \frac{(\tan^6(\frac{x}{2}))}{24}$ $\frac{\tan(\frac{x}{2})}{\tan(\frac{x}{2})^3}$	34
risch	$\frac{2i(3e^{5ix} - 2e^{3ix} + 3e^{ix})}{3(e^{2ix} - 1)^3}$	35

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^3/sin(x)^4,x,method=_RETURNVERBOSE)

[Out] -1/3/sin(x)^3*cos(x)^4+1/3/sin(x)*cos(x)^4+1/3*(2+cos(x)^2)*sin(x)

Maxima [A]

time = 0.27, size = 14, normalized size = 1.27

$$\frac{3 \sin(x)^2 - 1}{3 \sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3/sin(x)^4,x, algorithm="maxima")

[Out] 1/3*(3*sin(x)^2 - 1)/sin(x)^3

Fricas [A]

time = 0.34, size = 22, normalized size = 2.00

$$\frac{3 \cos(x)^2 - 2}{3 (\cos(x)^2 - 1) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^3/sin(x)^4,x, algorithm="fricas")`

[Out] `1/3*(3*cos(x)^2 - 2)/((cos(x)^2 - 1)*sin(x))`

Sympy [A]

time = 0.04, size = 15, normalized size = 1.36

$$-\frac{1 - 3 \sin^2(x)}{3 \sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)**3/sin(x)**4,x)`

[Out] `-(1 - 3*sin(x)**2)/(3*sin(x)**3)`

Giac [A]

time = 0.00, size = 16, normalized size = 1.45

$$\frac{3 \sin^2 x - 1}{3 \sin^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^3/sin(x)^4,x)`

[Out] `1/3*(3*sin(x)^2 - 1)/sin(x)^3`

Mupad [B]

time = 0.07, size = 11, normalized size = 1.00

$$\frac{\sin(x)^2 - \frac{1}{3}}{\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)^3/sin(x)^4,x)`

[Out] `(sin(x)^2 - 1/3)/sin(x)^3`

3.36 $\int \csc^2(x) \sec^2(x) dx$

Optimal. Leaf size=7

$$-\cot(x) + \tan(x)$$

[Out] -cot(x)+tan(x)

Rubi [A]

time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2700, 14}

$$\tan(x) - \cot(x)$$

Antiderivative was successfully verified.

[In] Int[Csc[x]^2*Sec[x]^2,x]

[Out] -Cot[x] + Tan[x]

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 2700

Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Dist[1/f, Subst[Int[(1 + x^2)^(m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]

Rubi steps

$$\begin{aligned} \int \csc^2(x) \sec^2(x) dx &= \text{Subst} \left(\int \frac{1+x^2}{x^2} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \left(1 + \frac{1}{x^2} \right) dx, x, \tan(x) \right) \\ &= -\cot(x) + \tan(x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 6, normalized size = 0.86

$$-2 \cot(2x)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^2*Sec[x]^2,x]

[Out] -2*Cot[2*x]

Mathics [A]

time = 1.73, size = 8, normalized size = 1.14

$$\frac{-2}{\tan[2x]}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(Sin[x]^2*Cos[x]^2),x]')

[Out] -2 / Tan[2 x]

Maple [A]

time = 0.02, size = 15, normalized size = 2.14

method	result	size
default	$\frac{1}{\sin(x)\cos(x)} - 2\cot(x)$	15
risch	$-\frac{4i}{(e^{2ix}+1)(e^{2ix}-1)}$	22
norman	$\frac{\frac{1}{2}-3(\tan^2(\frac{x}{2}))+\frac{(\tan^4(\frac{x}{2}))}{2}}{(\tan^2(\frac{x}{2})-1)\tan(\frac{x}{2})}$	36

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(x)^2/sin(x)^2,x,method=_RETURNVERBOSE)

[Out] 1/sin(x)/cos(x)-2*cot(x)

Maxima [A]

time = 0.26, size = 9, normalized size = 1.29

$$-\frac{1}{\tan(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="maxima")

[Out] -1/tan(x) + tan(x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(7) = 14$.

time = 0.33, size = 18, normalized size = 2.57

$$\frac{2\cos(x)^2 - 1}{\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="fricas")

[Out] $-(2*\cos(x)^2 - 1)/(\cos(x)*\sin(x))$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(5) = 10$

time = 0.03, size = 12, normalized size = 1.71

$$-\frac{2 \cos(2x)}{\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(x)**2/sin(x)**2,x)

[Out] $-2*\cos(2*x)/\sin(2*x)$

Giac [A]

time = 0.00, size = 7, normalized size = 1.00

$$\tan x - \frac{1}{\tan x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(x)^2/sin(x)^2,x)

[Out] $-1/\tan(x) + \tan(x)$

Mupad [B]

time = 0.20, size = 6, normalized size = 0.86

$$-2 \cot(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2*sin(x)^2),x)

[Out] $-2*\cot(2*x)$

3.37 $\int \cot^2\left(\frac{3x}{4}\right) dx$

Optimal. Leaf size=14

$$-x - \frac{4}{3} \cot\left(\frac{3x}{4}\right)$$

[Out] $-x - \frac{4}{3} \cot(3/4 * x)$

Rubi [A]

time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3554, 8}

$$-x - \frac{4}{3} \cot\left(\frac{3x}{4}\right)$$

Antiderivative was successfully verified.

[In] Int[Cot[(3*x)/4]^2,x]

[Out] $-x - (4 * \text{Cot}[(3*x)/4])/3$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3554

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[b*((b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int \cot^2\left(\frac{3x}{4}\right) dx &= -\frac{4}{3} \cot\left(\frac{3x}{4}\right) - \int 1 dx \\ &= -x - \frac{4}{3} \cot\left(\frac{3x}{4}\right) \end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.01, size = 28, normalized size = 2.00

$$-\frac{4}{3} \cot\left(\frac{3x}{4}\right) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; -\tan^2\left(\frac{3x}{4}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[(3*x)/4]^2,x]

[Out] $(-4*\text{Cot}[(3*x)/4]*\text{Hypergeometric2F1}[-1/2, 1, 1/2, -\text{Tan}[(3*x)/4]^2])/3$

Mathics [A]

time = 1.81, size = 12, normalized size = 0.86

$$-x - \frac{4}{3 \text{Tan}\left[\frac{3x}{4}\right]}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Cot[3*x/4]^2,x]')

[Out] $-x - 4 / (3 \text{Tan}[3 x / 4])$

Maple [A]

time = 0.01, size = 18, normalized size = 1.29

method	result	size
norman	$\frac{-\frac{4}{3} - x \tan\left(\frac{3x}{4}\right)}{\tan\left(\frac{3x}{4}\right)}$	17
risch	$-x - \frac{8i}{3\left(e^{\frac{3ix}{2}} - 1\right)}$	17
derivativdivides	$-\frac{4 \cot\left(\frac{3x}{4}\right)}{3} + \frac{2\pi}{3} - \frac{4 \operatorname{arccot}\left(\cot\left(\frac{3x}{4}\right)\right)}{3}$	18
default	$-\frac{4 \cot\left(\frac{3x}{4}\right)}{3} + \frac{2\pi}{3} - \frac{4 \operatorname{arccot}\left(\cot\left(\frac{3x}{4}\right)\right)}{3}$	18

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(3/4*x)^2,x,method=_RETURNVERBOSE)`

[Out] $-4/3*\text{cot}(3/4*x)+2/3*\text{Pi}-4/3*\text{arccot}(\text{cot}(3/4*x))$

Maxima [A]

time = 0.37, size = 12, normalized size = 0.86

$$-x - \frac{4}{3 \tan\left(\frac{3}{4}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(3/4*x)^2,x, algorithm="maxima")`

[Out] $-x - 4/3/\text{tan}(3/4*x)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

time = 0.32, size = 23, normalized size = 1.64

$$-\frac{3x \sin\left(\frac{3}{2}x\right) + 4 \cos\left(\frac{3}{2}x\right) + 4}{3 \sin\left(\frac{3}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(3/4*x)^2,x, algorithm="fricas")`

[Out] `-1/3*(3*x*sin(3/2*x) + 4*cos(3/2*x) + 4)/sin(3/2*x)`

Sympy [A]

time = 0.03, size = 19, normalized size = 1.36

$$-x - \frac{4 \cos\left(\frac{3x}{4}\right)}{3 \sin\left(\frac{3x}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(3/4*x)**2,x)`

[Out] `-x - 4*cos(3*x/4)/(3*sin(3*x/4))`

Giac [A]

time = 0.01, size = 36, normalized size = 2.57

$$\frac{8}{3} \left(\frac{\tan\left(\frac{3x}{4 \cdot 2}\right)}{4} - \frac{1}{4 \tan\left(\frac{3x}{4 \cdot 2}\right)} - \frac{3x}{4 \cdot 2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(3/4*x)^2,x)`

[Out] `-x - 2/3/tan(3/8*x) + 2/3*tan(3/8*x)`

Mupad [B]

time = 0.17, size = 10, normalized size = 0.71

$$-x - \frac{4 \cot\left(\frac{3x}{4}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot((3*x)/4)^2,x)`

[Out] `- x - (4*cot((3*x)/4))/3`

3.38 $\int (1 + \tan(2x))^2 dx$

Optimal. Leaf size=16

$$-\log(\cos(2x)) + \frac{1}{2} \tan(2x)$$

[Out] $-\ln(\cos(2*x))+1/2*\tan(2*x)$

Rubi [A]

time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3558, 3556}

$$\frac{1}{2} \tan(2x) - \log(\cos(2x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + \text{Tan}[2*x])^2, x]$

[Out] $-\text{Log}[\text{Cos}[2*x]] + \text{Tan}[2*x]/2$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] \text{ ; FreeQ}\{c, d\}, x]$

Rule 3558

$\text{Int}[((a_.) + (b_.)*\tan[(c_.) + (d_.)*(x_.)])^2, x_Symbol] \rightarrow \text{Simp}[(a^2 - b^2)*x, x] + (\text{Dist}[2*a*b, \text{Int}[\text{Tan}[c + d*x], x], x] + \text{Simp}[b^2*(\text{Tan}[c + d*x]/d), x]) \text{ ; FreeQ}\{a, b, c, d\}, x]$

Rubi steps

$$\begin{aligned} \int (1 + \tan(2x))^2 dx &= \frac{1}{2} \tan(2x) + 2 \int \tan(2x) dx \\ &= -\log(\cos(2x)) + \frac{1}{2} \tan(2x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 26, normalized size = 1.62

$$x - \frac{1}{2} \tan^{-1}(\tan(2x)) - \log(\cos(2x)) + \frac{1}{2} \tan(2x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Tan[2*x])^2,x]

[Out] x - ArcTan[Tan[2*x]]/2 - Log[Cos[2*x]] + Tan[2*x]/2

Mathics [A]

time = 1.92, size = 16, normalized size = 1.00

$$\frac{\text{Log}\left[\frac{1}{\text{Cos}[2x]^2}\right]}{2} + \frac{\text{Tan}[2x]}{2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(1 + Tan[2*x])^2,x]')

[Out] Log[1 / Cos[2 x] ^ 2] / 2 + Tan[2 x] / 2

Maple [A]

time = 0.03, size = 19, normalized size = 1.19

method	result	size
derivativedivides	$\frac{\tan(2x)}{2} + \frac{\ln(1+\tan^2(2x))}{2}$	19
default	$\frac{\tan(2x)}{2} + \frac{\ln(1+\tan^2(2x))}{2}$	19
norman	$\frac{\tan(2x)}{2} + \frac{\ln(1+\tan^2(2x))}{2}$	19
risch	$2ix + \frac{i}{e^{4ix}+1} - \ln(e^{4ix} + 1)$	28

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+tan(2*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/2*tan(2*x)+1/2*ln(1+tan(2*x)^2)

Maxima [A]

time = 0.37, size = 12, normalized size = 0.75

$$\log(\sec(2x)) + \frac{1}{2} \tan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+tan(2*x))^2,x, algorithm="maxima")

[Out] log(sec(2*x)) + 1/2*tan(2*x)

Fricas [A]

time = 0.35, size = 20, normalized size = 1.25

$$-\frac{1}{2} \log\left(\frac{1}{\tan(2x)^2 + 1}\right) + \frac{1}{2} \tan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+tan(2*x))^2,x, algorithm="fricas")

[Out] -1/2*log(1/(tan(2*x)^2 + 1)) + 1/2*tan(2*x)

Sympy [A]

time = 0.07, size = 17, normalized size = 1.06

$$\frac{\log(\tan^2(2x) + 1)}{2} + \frac{\tan(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+tan(2*x))**2,x)

[Out] log(tan(2*x)**2 + 1)/2 + tan(2*x)/2

Giac [A]

time = 0.00, size = 20, normalized size = 1.25

$$\frac{\tan(2x)}{2} + \frac{\ln(\tan^2(2x) + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+tan(2*x))^2,x)

[Out] 1/2*log(tan(2*x)^2 + 1) + 1/2*tan(2*x)

Mupad [B]

time = 0.27, size = 18, normalized size = 1.12

$$\frac{\tan(2x)}{2} + \frac{\ln(\tan(2x)^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(2*x) + 1)^2,x)

[Out] tan(2*x)/2 + log(tan(2*x)^2 + 1)/2

3.39 $\int (-\cot(x) + \tan(x))^2 dx$

Optimal. Leaf size=10

$$-4x - \cot(x) + \tan(x)$$

[Out] $-4*x - \cot(x) + \tan(x)$

Rubi [A]

time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {472, 209}

$$-4x + \tan(x) - \cot(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-\text{Cot}[x] + \text{Tan}[x])^2, x]$

[Out] $-4*x - \text{Cot}[x] + \text{Tan}[x]$

Rule 209

$\text{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))* \text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 472

$\text{Int}[(e_.)*(x_)^{(m_)}*((a_ + (b_.)*(x_)^{(n_)})^{(p_)} / ((c_ + (d_.)*(x_)^{(n_)})), x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)), x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{IGtQ}[2*(m + 1), 0] \ || \ !\text{RationalQ}[m])$

Rubi steps

$$\begin{aligned} \int (-\cot(x) + \tan(x))^2 dx &= \text{Subst} \left(\int \frac{(1-x^2)^2}{x^2(1+x^2)} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \left(1 + \frac{1}{x^2} - \frac{4}{1+x^2} \right) dx, x, \tan(x) \right) \\ &= -\cot(x) + \tan(x) - 4 \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \tan(x) \right) \\ &= -4x - \cot(x) + \tan(x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 10, normalized size = 1.00

$$-4x - \cot(x) + \tan(x)$$

Antiderivative was successfully verified.

`[In] Integrate[(-Cot[x] + Tan[x])^2,x]``[Out] -4*x - Cot[x] + Tan[x]`**Mathics [A]**

time = 1.87, size = 12, normalized size = 1.20

$$-4x + \text{Tan}[x] - \frac{1}{\text{Tan}[x]}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(Tan[x] - Cot[x])^2,x]')``[Out] -4 x + Tan[x] - 1 / Tan[x]`**Maple [A]**

time = 0.03, size = 11, normalized size = 1.10

method	result	size
default	$-4x - \cot(x) + \tan(x)$	11
norman	$\frac{-1 + \tan^2(x) - 4x \tan(x)}{\tan(x)}$	17
risch	$-4x - \frac{4i}{(e^{2ix} + 1)(e^{2ix} - 1)}$	26

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((-cot(x)+tan(x))^2,x,method=_RETURNVERBOSE)``[Out] -4*x-cot(x)+tan(x)`**Maxima [A]**

time = 0.37, size = 12, normalized size = 1.20

$$-4x - \frac{1}{\tan(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((-cot(x)+tan(x))^2,x, algorithm="maxima")``[Out] -4*x - 1/tan(x) + tan(x)`

Fricas [A]

time = 0.35, size = 19, normalized size = 1.90

$$-\frac{4x \tan(x) - \tan(x)^2 + 1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cot(x)+tan(x))^2,x, algorithm="fricas")

[Out] -(4*x*tan(x) - tan(x)^2 + 1)/tan(x)

Sympy [A]

time = 0.15, size = 10, normalized size = 1.00

$$-4x + \tan(x) - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cot(x)+tan(x))**2,x)

[Out] -4*x + tan(x) - 1/tan(x)

Giac [A]

time = 0.00, size = 11, normalized size = 1.10

$$\tan x - \frac{1}{\tan x} - 4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cot(x)+tan(x))^2,x)

[Out] -4*x - 1/tan(x) + tan(x)

Mupad [B]

time = 0.27, size = 12, normalized size = 1.20

$$\tan(x) - 4x - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(x) - tan(x))^2,x)

[Out] tan(x) - 4*x - 1/tan(x)

3.40 $\int (-\sec(x) + \tan(x))^2 dx$

Optimal. Leaf size=14

$$-x - \frac{2 \cos(x)}{1 + \sin(x)}$$

[Out] $-x - 2 \cos(x) / (1 + \sin(x))$

Rubi [A]

time = 0.04, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4476, 2749, 2759, 8}

$$-x - \frac{2 \cos(x)}{\sin(x) + 1}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-\text{Sec}[x] + \text{Tan}[x])^2, x]$

[Out] $-x - (2 \cos[x]) / (1 + \sin[x])$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2749

$\text{Int}[(\cos[(e_.) + (f_.)*(x_.)]*(g_.))^{(p_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}, x_Symbol] \rightarrow \text{Dist}[(a/g)^{(2*m)}, \text{Int}[(g*\cos[e + f*x])^{(2*m + p)} / (a - b*\sin[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, e, f, g\}, x \} \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{LtQ}[p, -1] \&\& \text{GeQ}[2*m + p, 0]$

Rule 2759

$\text{Int}[(\cos[(e_.) + (f_.)*(x_.)]*(g_.))^{(p_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[2*g*(g*\cos[e + f*x])^{(p - 1)}*((a + b*\sin[e + f*x])^{(m + 1)} / (b*f*(2*m + p + 1))), x] + \text{Dist}[g^2*((p - 1) / (b^2*(2*m + p + 1))), \text{Int}[(g*\cos[e + f*x])^{(p - 2)}*(a + b*\sin[e + f*x])^{(m + 2)}, x], x] /; \text{FreeQ}\{a, b, e, f, g\}, x \} \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LeQ}[m, -2] \&\& \text{GtQ}[p, 1] \&\& \text{NeQ}[2*m + p + 1, 0] \&\& \text{!IntegerQ}[m + p + 1, 0] \&\& \text{IntegerQ}[2*m, 2*p]$

Rule 4476

$\text{Int}[(u_.)*((b_.)*\sec[(c_.) + (d_.)*(x_.)]^{(n_.)} + (a_.)*\tan[(c_.) + (d_.)*(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ActivateTrig}[u]*\text{Sec}[c + d*x]^{(n*p)}*(b + a*\sin[c + d*x]^n)^p, x] /; \text{FreeQ}\{a, b, c, d\}, x \} \&\& \text{IntegerQ}[n, p]$

Rubi steps

$$\begin{aligned}
\int (-\sec(x) + \tan(x))^2 dx &= \int \sec^2(x)(-1 + \sin(x))^2 dx \\
&= \int \frac{\cos^2(x)}{(-1 - \sin(x))^2} dx \\
&= -\frac{2 \cos(x)}{1 + \sin(x)} - \int 1 dx \\
&= -x - \frac{2 \cos(x)}{1 + \sin(x)}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 12, normalized size = 0.86

$$-x - 2 \sec(x) + 2 \tan(x)$$

Antiderivative was successfully verified.

`[In] Integrate[(-Sec[x] + Tan[x])^2, x]``[Out] -x - 2*Sec[x] + 2*Tan[x]`**Mathics [A]**

time = 3.46, size = 14, normalized size = 1.00

$$-x - \frac{2}{\text{Cos}[x]} + 2 \text{Tan}[x]$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(Tan[x] - Sec[x])^2, x]')``[Out] -x - 2 / Cos[x] + 2 Tan[x]`**Maple [A]**

time = 0.04, size = 15, normalized size = 1.07

method	result	size
default	$2 \tan(x) - \frac{2}{\cos(x)} - x$	15
risch	$-x - \frac{4}{e^{ix} + i}$	17

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((-sec(x)+tan(x))^2, x, method=_RETURNVERBOSE)`

[Out] $2*\tan(x)-2/\cos(x)-x$

Maxima [A]

time = 0.37, size = 14, normalized size = 1.00

$$-x - \frac{2}{\cos(x)} + 2 \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-sec(x)+tan(x))^2,x, algorithm="maxima")`

[Out] $-x - 2/\cos(x) + 2*\tan(x)$

Fricas [A]

time = 0.34, size = 25, normalized size = 1.79

$$-\frac{(x+2)\cos(x) + (x-2)\sin(x) + x + 2}{\cos(x) + \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-sec(x)+tan(x))^2,x, algorithm="fricas")`

[Out] $-((x+2)*\cos(x) + (x-2)*\sin(x) + x + 2)/(\cos(x) + \sin(x) + 1)$

Sympy [A]

time = 0.65, size = 10, normalized size = 0.71

$$-x + 2 \tan(x) - 2 \sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-sec(x)+tan(x))**2,x)`

[Out] $-x + 2*\tan(x) - 2*\sec(x)$

Giac [A]

time = 0.00, size = 19, normalized size = 1.36

$$2 \left(-\frac{2}{\tan\left(\frac{x}{2}\right) + 1} - \frac{x}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-sec(x)+tan(x))^2,x)`

[Out] $-x - 4/(\tan(1/2*x) + 1)$

Mupad [B]

time = 0.27, size = 14, normalized size = 1.00

$$-x - \frac{4}{\tan\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(x) - 1/cos(x))^2,x)

[Out] - x - 4/(tan(x/2) + 1)

$$3.41 \quad \int \frac{\sin(x)}{1+\sin(x)} dx$$

Optimal. Leaf size=11

$$x + \frac{\cos(x)}{1 + \sin(x)}$$

[Out] x+cos(x)/(1+sin(x))

Rubi [A]

time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2814, 2727}

$$x + \frac{\cos(x)}{\sin(x) + 1}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(1 + Sin[x]),x]

[Out] x + Cos[x]/(1 + Sin[x])

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2814

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sin(x)}{1 + \sin(x)} dx &= x - \int \frac{1}{1 + \sin(x)} dx \\ &= x + \frac{\cos(x)}{1 + \sin(x)} \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

time = 0.03, size = 25, normalized size = 2.27

$$x - \frac{2 \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/(1 + Sin[x]),x]

[Out] x - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])

Mathics [A]

time = 2.26, size = 18, normalized size = 1.64

$$\frac{2 + x + x \operatorname{Tan} \left[\frac{x}{2} \right]}{1 + \operatorname{Tan} \left[\frac{x}{2} \right]}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Sin[x]/(1 + Sin[x]),x]')

[Out] (2 + x + x Tan[x / 2]) / (1 + Tan[x / 2])

Maple [A]

time = 0.03, size = 19, normalized size = 1.73

method	result	size
risch	$x + \frac{2}{e^{ix} + i}$	15
default	$2 \arctan \left(\tan \left(\frac{x}{2} \right) \right) + \frac{2}{1 + \tan \left(\frac{x}{2} \right)}$	19
norman	$\frac{x + x \tan \left(\frac{x}{2} \right) + x \left(\tan^2 \left(\frac{x}{2} \right) \right) + x \left(\tan^3 \left(\frac{x}{2} \right) \right) + 2 \left(\tan^2 \left(\frac{x}{2} \right) \right) + 2}{\left(1 + \tan^2 \left(\frac{x}{2} \right) \right) \left(1 + \tan \left(\frac{x}{2} \right) \right)}$	53

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/(sin(x)+1),x,method=_RETURNVERBOSE)

[Out] 2*arctan(tan(1/2*x))+2/(1+tan(1/2*x))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(11) = 22.

time = 0.35, size = 28, normalized size = 2.55

$$\frac{2}{\frac{\sin(x)}{\cos(x)+1} + 1} + 2 \arctan \left(\frac{\sin(x)}{\cos(x) + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(1+sin(x)),x, algorithm="maxima")

[Out] 2/(sin(x)/(cos(x) + 1) + 1) + 2*arctan(sin(x)/(cos(x) + 1))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(11) = 22.

time = 0.37, size = 24, normalized size = 2.18

$$\frac{(x + 1) \cos(x) + (x - 1) \sin(x) + x + 1}{\cos(x) + \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/(1+sin(x)),x, algorithm="fricas")`

[Out] `((x + 1)*cos(x) + (x - 1)*sin(x) + x + 1)/(cos(x) + sin(x) + 1)`

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(8) = 16$

time = 0.24, size = 29, normalized size = 2.64

$$\frac{x \tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right) + 1} + \frac{x}{\tan\left(\frac{x}{2}\right) + 1} + \frac{2}{\tan\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/(1+sin(x)),x)`

[Out] `x*tan(x/2)/(tan(x/2) + 1) + x/(tan(x/2) + 1) + 2/(tan(x/2) + 1)`

Giac [A]

time = 0.00, size = 15, normalized size = 1.36

$$2 \left(\frac{1}{\tan\left(\frac{x}{2}\right) + 1} + \frac{x}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/(1+sin(x)),x)`

[Out] `x + 2/(tan(1/2*x) + 1)`

Mupad [B]

time = 0.24, size = 12, normalized size = 1.09

$$x + \frac{2}{\tan\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/(sin(x) + 1),x)`

[Out] `x + 2/(tan(x/2) + 1)`

$$3.42 \quad \int \frac{\cos(x)}{1-\cos(x)} dx$$

Optimal. Leaf size=16

$$-x - \frac{\sin(x)}{1 - \cos(x)}$$

[Out] -x-sin(x)/(1-cos(x))

Rubi [A]

time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2814, 2727}

$$-x - \frac{\sin(x)}{1 - \cos(x)}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/(1 - Cos[x]),x]

[Out] -x - Sin[x]/(1 - Cos[x])

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2814

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos(x)}{1-\cos(x)} dx &= -x + \int \frac{1}{1-\cos(x)} dx \\ &= -x - \frac{\sin(x)}{1-\cos(x)} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 21, normalized size = 1.31

$$\frac{2x \sin^2\left(\frac{x}{2}\right) + \sin(x)}{-1 + \cos(x)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/(1 - Cos[x]),x]

[Out] (2*x*Sin[x/2]^2 + Sin[x])/(-1 + Cos[x])

Mathics [A]

time = 1.84, size = 12, normalized size = 0.75

$$-x - \frac{1}{\operatorname{Tan}\left[\frac{x}{2}\right]}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Cos[x]/(1 - Cos[x]),x]')

[Out] -x - 1 / Tan[x / 2]

Maple [A]

time = 0.02, size = 17, normalized size = 1.06

method	result	size
default	$-2 \arctan\left(\tan\left(\frac{x}{2}\right)\right) - \frac{1}{\tan\left(\frac{x}{2}\right)}$	17
risch	$-x - \frac{2i}{e^{ix} - 1}$	17
norman	$\frac{-1 - (\tan^2(\frac{x}{2})) - x \tan(\frac{x}{2}) - x (\tan^3(\frac{x}{2}))}{(1 + \tan^2(\frac{x}{2})) \tan(\frac{x}{2})}$	44

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(1-cos(x)),x,method=_RETURNVERBOSE)

[Out] -2*arctan(tan(1/2*x))-1/tan(1/2*x)

Maxima [A]

time = 0.35, size = 23, normalized size = 1.44

$$-\frac{\cos(x) + 1}{\sin(x)} - 2 \arctan\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(1-cos(x)),x, algorithm="maxima")

[Out] -(cos(x) + 1)/sin(x) - 2*arctan(sin(x)/(cos(x) + 1))

Fricas [A]

time = 0.33, size = 14, normalized size = 0.88

$$-\frac{x \sin(x) + \cos(x) + 1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/(1-cos(x)),x, algorithm="fricas")`

[Out] `-(x*sin(x) + cos(x) + 1)/sin(x)`

Sympy [A]

time = 0.21, size = 8, normalized size = 0.50

$$-x - \frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/(1-cos(x)),x)`

[Out] `-x - 1/tan(x/2)`

Giac [A]

time = 0.00, size = 17, normalized size = 1.06

$$2 \left(-\frac{1}{2 \tan\left(\frac{x}{2}\right)} - \frac{x}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/(1-cos(x)),x)`

[Out] `-x - 1/tan(1/2*x)`

Mupad [B]

time = 0.17, size = 10, normalized size = 0.62

$$-x - \cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-cos(x)/(cos(x) - 1),x)`

[Out] `- x - cot(x/2)`

$$3.43 \quad \int e^{-x/2} (-1 + e^{x/2})^3 dx$$

Optimal. Leaf size=25

$$2e^{-x/2} - 6e^{x/2} + e^x + 3x$$

[Out] 2/exp(1/2*x)-6*exp(1/2*x)+exp(x)+3*x

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2280, 45}

$$3x + 2e^{-x/2} - 6e^{x/2} + e^x$$

Antiderivative was successfully verified.

[In] Int[(-1 + E^(x/2))^3/E^(x/2), x]

[Out] 2/E^(x/2) - 6*E^(x/2) + E^x + 3*x

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2280

Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log[F]))]}, Dist[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])), Subst[Int[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/Denominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

Rubi steps

$$\begin{aligned} \int e^{-x/2} (-1 + e^{x/2})^3 dx &= 2\text{Subst} \left(\int \frac{(-1 + x)^3}{x^2} dx, x, e^{x/2} \right) \\ &= 2\text{Subst} \left(\int \left(-3 - \frac{1}{x^2} + \frac{3}{x} + x \right) dx, x, e^{x/2} \right) \\ &= 2e^{-x/2} - 6e^{x/2} + e^x + 3x \end{aligned}$$

Mathematica [A]

time = 0.02, size = 33, normalized size = 1.32

$$e^{-x/2} (2 - 6e^x + e^{3x/2}) + 6 \log(e^{x/2})$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + E^(x/2))^3/E^(x/2), x]

[Out] (2 - 6*E^x + E^((3*x)/2))/E^(x/2) + 6*Log[E^(x/2)]

Mathics [A]

time = 1.79, size = 21, normalized size = 0.84

$$3x + E^x - 6E^{\frac{x}{2}} + 2E^{-\frac{x}{2}}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(E^(x/2) - 1)^3*E^(-x/2), x]')

[Out] 3 x + E ^ x - 6 E ^ (x / 2) + 2 E ^ (-x / 2)

Maple [A]

time = 0.02, size = 29, normalized size = 1.16

method	result	size
risch	$e^x + 3x - 6e^{\frac{x}{2}} + 2e^{-\frac{x}{2}}$	19
derivativedivides	$e^x - 6e^{\frac{x}{2}} + 2e^{-\frac{x}{2}} + 6 \ln(e^{\frac{x}{2}})$	29
default	$e^x - 6e^{\frac{x}{2}} + 2e^{-\frac{x}{2}} + 6 \ln(e^{\frac{x}{2}})$	29
norman	$(2 + e^{\frac{3x}{2}} - 6e^x + 3xe^{\frac{x}{2}})e^{-\frac{x}{2}}$	31

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+exp(1/2*x))^3/exp(1/2*x), x, method=_RETURNVERBOSE)

[Out] exp(1/2*x)^2-6*exp(1/2*x)+2/exp(1/2*x)+6*ln(exp(1/2*x))

Maxima [A]

time = 0.27, size = 18, normalized size = 0.72

$$3x - 6e^{(\frac{1}{2}x)} + 2e^{(-\frac{1}{2}x)} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+exp(1/2*x))^3/exp(1/2*x), x, algorithm="maxima")

[Out] 3*x - 6*e^(1/2*x) + 2*e^(-1/2*x) + e^x

Fricas [A]

time = 0.34, size = 22, normalized size = 0.88

$$\left(3xe^{\left(\frac{1}{2}x\right)} + e^{\left(\frac{3}{2}x\right)} - 6e^x + 2\right)e^{\left(-\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+exp(1/2*x))^3/exp(1/2*x),x, algorithm="fricas")

[Out] (3*x*e^(1/2*x) + e^(3/2*x) - 6*e^x + 2)*e^(-1/2*x)

Sympy [A]

time = 0.05, size = 19, normalized size = 0.76

$$3x - 6e^{\frac{x}{2}} + e^x + 2e^{-\frac{x}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+exp(1/2*x))**3/exp(1/2*x),x)

[Out] 3*x - 6*exp(x/2) + exp(x) + 2*exp(-x/2)

Giac [A]

time = 0.00, size = 32, normalized size = 1.28

$$2 \left(\frac{\left(e^{\frac{x}{2}}\right)^2}{2} - 3e^{\frac{x}{2}} + \frac{1}{e^{\frac{x}{2}}} + \frac{3}{2}x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+exp(1/2*x))^3/exp(1/2*x),x)

[Out] -6*e^(1/2*x) + e^x + 3*x + 2/e^(1/2*x)

Mupad [B]

time = 0.22, size = 18, normalized size = 0.72

$$3x + 2e^{-\frac{x}{2}} - 6e^{x/2} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-x/2)*(exp(x/2) - 1)^3,x)

[Out] 3*x + 2*exp(-x/2) - 6*exp(x/2) + exp(x)

3.44 $\int \frac{1}{5-6x+x^2} dx$

Optimal. Leaf size=21

$$-\frac{1}{4} \log(1-x) + \frac{1}{4} \log(5-x)$$

[Out] -1/4*ln(1-x)+1/4*ln(5-x)

Rubi [A]

time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {630, 31}

$$\frac{1}{4} \log(5-x) - \frac{1}{4} \log(1-x)$$

Antiderivative was successfully verified.

[In] Int[(5 - 6*x + x^2)^(-1), x]

[Out] -1/4*Log[1 - x] + Log[5 - x]/4

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 630

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{1}{5-6x+x^2} dx &= \frac{1}{4} \int \frac{1}{-5+x} dx - \frac{1}{4} \int \frac{1}{-1+x} dx \\ &= -\frac{1}{4} \log(1-x) + \frac{1}{4} \log(5-x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 21, normalized size = 1.00

$$-\frac{1}{4} \log(1-x) + \frac{1}{4} \log(5-x)$$

Antiderivative was successfully verified.

[In] Integrate[(5 - 6*x + x^2)^(-1),x]

[Out] -1/4*Log[1 - x] + Log[5 - x]/4

Mathics [A]

time = 1.75, size = 13, normalized size = 0.62

$$-\frac{\text{Log}[-1+x]}{4} + \frac{\text{Log}[-5+x]}{4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(5 - 6*x + x^2),x]')

[Out] -Log[-1 + x] / 4 + Log[-5 + x] / 4

Maple [A]

time = 0.08, size = 14, normalized size = 0.67

method	result	size
default	$\frac{\ln(x-5)}{4} - \frac{\ln(-1+x)}{4}$	14
norman	$\frac{\ln(x-5)}{4} - \frac{\ln(-1+x)}{4}$	14
risch	$\frac{\ln(x-5)}{4} - \frac{\ln(-1+x)}{4}$	14

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2-6*x+5),x,method=_RETURNVERBOSE)

[Out] 1/4*ln(x-5)-1/4*ln(-1+x)

Maxima [A]

time = 0.27, size = 13, normalized size = 0.62

$$-\frac{1}{4} \log(x-1) + \frac{1}{4} \log(x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-6*x+5),x, algorithm="maxima")

[Out] -1/4*log(x - 1) + 1/4*log(x - 5)

Fricas [A]

time = 0.33, size = 13, normalized size = 0.62

$$-\frac{1}{4} \log(x-1) + \frac{1}{4} \log(x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2-6*x+5),x, algorithm="fricas")`

[Out] $-1/4*\log(x - 1) + 1/4*\log(x - 5)$

Sympy [A]

time = 0.05, size = 12, normalized size = 0.57

$$\frac{\log(x - 5)}{4} - \frac{\log(x - 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2-6*x+5),x)`

[Out] $\log(x - 5)/4 - \log(x - 1)/4$

Giac [A]

time = 0.00, size = 18, normalized size = 0.86

$$\frac{\ln|x - 5|}{4} - \frac{\ln|x - 1|}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2-6*x+5),x)`

[Out] $-1/4*\log(\text{abs}(x - 1)) + 1/4*\log(\text{abs}(x - 5))$

Mupad [B]

time = 0.26, size = 8, normalized size = 0.38

$$-\frac{\operatorname{atanh}\left(\frac{x}{2} - \frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2 - 6*x + 5),x)`

[Out] $-\operatorname{atanh}(x/2 - 3/2)/2$

3.45

$$\int \frac{x^2}{13-6x^3+x^6} dx$$

Optimal. Leaf size=14

$$\frac{1}{6} \tan^{-1} \left(\frac{1}{2} (-3 + x^3) \right)$$

[Out] 1/6*arctan(1/2*x^3-3/2)

Rubi [A]

time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1366, 632, 210}

$$\frac{1}{6} \tan^{-1} \left(\frac{1}{2} (x^3 - 3) \right)$$

Antiderivative was successfully verified.

[In] Int[x^2/(13 - 6*x^3 + x^6),x]

[Out] ArcTan[(-3 + x^3)/2]/6

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1366

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{13 - 6x^3 + x^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{13 - 6x + x^2} dx, x, x^3 \right) \\ &= - \left(\frac{2}{3} \text{Subst} \left(\int \frac{1}{-16 - x^2} dx, x, 2(-3 + x^3) \right) \right) \\ &= \frac{1}{6} \tan^{-1} \left(\frac{1}{2} (-3 + x^3) \right) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 14, normalized size = 1.00

$$\frac{1}{6} \tan^{-1} \left(\frac{1}{2} (-3 + x^3) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[x^2/(13 - 6*x^3 + x^6),x]``[Out] ArcTan[(-3 + x^3)/2]/6`**Mathics [A]**

time = 1.80, size = 10, normalized size = 0.71

$$\frac{\text{ArcTan} \left[-\frac{3}{2} + \frac{x^3}{2} \right]}{6}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^2/(13 - 6*x^3 + x^6),x]')``[Out] ArcTan[-3 / 2 + x ^ 3 / 2] / 6`**Maple [A]**

time = 0.01, size = 11, normalized size = 0.79

method	result	size
default	$\frac{\arctan\left(\frac{x^3}{2} - \frac{3}{2}\right)}{6}$	11
risch	$\frac{\arctan\left(\frac{x^3}{2} - \frac{3}{2}\right)}{6}$	11

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2/(x^6-6*x^3+13),x,method=_RETURNVERBOSE)``[Out] 1/6*arctan(1/2*x^3-3/2)`

Maxima [A]

time = 0.43, size = 10, normalized size = 0.71

$$\frac{1}{6} \arctan\left(\frac{1}{2}x^3 - \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2/(x^6-6*x^3+13),x, algorithm="maxima")``[Out] 1/6*arctan(1/2*x^3 - 3/2)`**Fricas [A]**

time = 0.33, size = 10, normalized size = 0.71

$$\frac{1}{6} \arctan\left(\frac{1}{2}x^3 - \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2/(x^6-6*x^3+13),x, algorithm="fricas")``[Out] 1/6*arctan(1/2*x^3 - 3/2)`**Sympy [A]**

time = 0.05, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^3}{2} - \frac{3}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x**2/(x**6-6*x**3+13),x)``[Out] atan(x**3/2 - 3/2)/6`**Giac [A]**

time = 0.00, size = 12, normalized size = 0.86

$$\frac{\arctan\left(\frac{x^3-3}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2/(x^6-6*x^3+13),x)``[Out] 1/6*arctan(1/2*x^3 - 3/2)`

Mupad [B]

time = 0.06, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^3}{2} - \frac{3}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(x^6 - 6*x^3 + 13),x)`

[Out] `atan(x^3/2 - 3/2)/6`

3.46 $\int \frac{2+x}{-1-4x+x^2} dx$

Optimal. Leaf size=51

$$\frac{1}{10} (5 - 4\sqrt{5}) \log(2 - \sqrt{5} - x) + \frac{1}{10} (5 + 4\sqrt{5}) \log(2 + \sqrt{5} - x)$$

[Out] 1/10*ln(2-x-5^(1/2))*(5-4*5^(1/2))+1/10*ln(2-x+5^(1/2))*(5+4*5^(1/2))

Rubi [A]

time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {646, 31}

$$\frac{1}{10} (5 - 4\sqrt{5}) \log(-x - \sqrt{5} + 2) + \frac{1}{10} (5 + 4\sqrt{5}) \log(-x + \sqrt{5} + 2)$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/(-1 - 4*x + x^2), x]

[Out] ((5 - 4*Sqrt[5])*Log[2 - Sqrt[5] - x])/10 + ((5 + 4*Sqrt[5])*Log[2 + Sqrt[5] - x])/10

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 646

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{2+x}{-1-4x+x^2} dx &= - \left(\frac{1}{10} (-5 + 4\sqrt{5}) \int \frac{1}{-2 + \sqrt{5} + x} dx \right) + \frac{1}{10} (5 + 4\sqrt{5}) \int \frac{1}{-2 - \sqrt{5} + x} dx \\ &= \frac{1}{10} (5 - 4\sqrt{5}) \log(2 - \sqrt{5} - x) + \frac{1}{10} (5 + 4\sqrt{5}) \log(2 + \sqrt{5} - x) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 47, normalized size = 0.92

$$\frac{1}{10} (5 + 4\sqrt{5}) \log(2 + \sqrt{5} - x) + \frac{1}{10} (5 - 4\sqrt{5}) \log(-2 + \sqrt{5} + x)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x)/(-1 - 4*x + x^2),x]

[Out] ((5 + 4*Sqrt[5])*Log[2 + Sqrt[5] - x])/10 + ((5 - 4*Sqrt[5])*Log[-2 + Sqrt[5] + x])/10

Mathics [A]

time = 1.93, size = 35, normalized size = 0.69

$$\frac{(5 - 4\sqrt{5}) \operatorname{Log}[-2 + \sqrt{5} + x]}{10} + \frac{(5 + 4\sqrt{5}) \operatorname{Log}[-2 - \sqrt{5} + x]}{10}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(2 + x)/(-1 - 4*x + x^2),x]')

[Out] (5 - 4 Sqrt[5]) Log[-2 + Sqrt[5] + x] / 10 + (5 + 4 Sqrt[5]) Log[-2 - Sqrt[5] + x] / 10

Maple [A]

time = 0.10, size = 29, normalized size = 0.57

method	result	size
default	$\frac{\ln(x^2-4x-1)}{2} - \frac{4\sqrt{5} \operatorname{arctanh}\left(\frac{(2x-4)\sqrt{5}}{10}\right)}{5}$	29
risch	$\frac{\ln(x-2-\sqrt{5})}{2} + \frac{2\ln(x-2-\sqrt{5})\sqrt{5}}{5} + \frac{\ln(x+\sqrt{5}-2)}{2} - \frac{2\ln(x+\sqrt{5}-2)\sqrt{5}}{5}$	48

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+x)/(x^2-4*x-1),x,method=_RETURNVERBOSE)

[Out] 1/2*ln(x^2-4*x-1)-4/5*5^(1/2)*arctanh(1/10*(2*x-4)*5^(1/2))

Maxima [A]

time = 0.36, size = 35, normalized size = 0.69

$$\frac{2}{5} \sqrt{5} \log\left(\frac{x - \sqrt{5} - 2}{x + \sqrt{5} - 2}\right) + \frac{1}{2} \log(x^2 - 4x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(x^2-4*x-1),x, algorithm="maxima")

[Out] 2/5*sqrt(5)*log((x - sqrt(5) - 2)/(x + sqrt(5) - 2)) + 1/2*log(x^2 - 4*x - 1)

Fricas [A]

time = 0.37, size = 45, normalized size = 0.88

$$\frac{2}{5}\sqrt{5}\log\left(\frac{x^2 - 2\sqrt{5}(x-2) - 4x + 9}{x^2 - 4x - 1}\right) + \frac{1}{2}\log(x^2 - 4x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((2+x)/(x^2-4*x-1),x, algorithm="fricas")``[Out] 2/5*sqrt(5)*log((x^2 - 2*sqrt(5)*(x - 2) - 4*x + 9)/(x^2 - 4*x - 1)) + 1/2*log(x^2 - 4*x - 1)`**Sympy [A]**

time = 0.06, size = 42, normalized size = 0.82

$$\left(\frac{1}{2} - \frac{2\sqrt{5}}{5}\right)\log(x - 2 + \sqrt{5}) + \left(\frac{1}{2} + \frac{2\sqrt{5}}{5}\right)\log(x - \sqrt{5} - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((2+x)/(x**2-4*x-1),x)``[Out] (1/2 - 2*sqrt(5)/5)*log(x - 2 + sqrt(5)) + (1/2 + 2*sqrt(5)/5)*log(x - sqrt(5) - 2)`**Giac [A]**

time = 0.00, size = 54, normalized size = 1.06

$$\frac{\ln|x^2 - 4x - 1|}{2} + \frac{4\ln\left|\frac{2x-4-2\sqrt{5}}{2x-4+2\sqrt{5}}\right|}{2\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((2+x)/(x^2-4*x-1),x)``[Out] 2/5*sqrt(5)*log(abs((x - sqrt(5) - 2)/(x + sqrt(5) - 2))) + 1/2*log(abs(x^2 - 4*x - 1))`**Mupad [B]**

time = 0.12, size = 34, normalized size = 0.67

$$\ln(x - \sqrt{5} - 2) \left(\frac{2\sqrt{5}}{5} + \frac{1}{2}\right) - \ln(x + \sqrt{5} - 2) \left(\frac{2\sqrt{5}}{5} - \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(-(x + 2)/(4*x - x^2 + 1),x)``[Out] log(x - 5^(1/2) - 2)*((2*5^(1/2))/5 + 1/2) - log(x + 5^(1/2) - 2)*((2*5^(1/2))/5 - 1/2)`

$$3.47 \quad \int \frac{1}{1 + \sqrt[3]{1+x}} dx$$

Optimal. Leaf size=33

$$-3\sqrt[3]{1+x} + \frac{3}{2}(1+x)^{2/3} + 3 \log\left(1 + \sqrt[3]{1+x}\right)$$

[Out] $-3*(1+x)^{(1/3)}+3/2*(1+x)^{(2/3)}+3*\ln(1+(1+x)^{(1/3)})$

Rubi [A]

time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {253, 196, 45}

$$\frac{3}{2}(x+1)^{2/3} - 3\sqrt[3]{x+1} + 3 \log\left(\sqrt[3]{x+1} + 1\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + (1 + x)^{(1/3)})^{(-1)}, x]$

[Out] $-3*(1 + x)^{(1/3)} + (3*(1 + x)^{(2/3)})/2 + 3*\text{Log}[1 + (1 + x)^{(1/3)}]$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 196

$\text{Int}[(a_. + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(1/n)} - 1]*(a + b*x)^p, x], x, x^n], x] /;$ FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]

Rule 253

$\text{Int}[(a_. + (b_.)*(v_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/\text{Coefficient}[v, x, 1], \text{Subst}[\text{Int}[(a + b*x^n)^p, x], x, v], x] /;$ FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{1 + \sqrt[3]{1+x}} dx &= \text{Subst} \left(\int \frac{1}{1 + \sqrt[3]{x}} dx, x, 1+x \right) \\
&= 3 \text{Subst} \left(\int \frac{x^2}{1+x} dx, x, \sqrt[3]{1+x} \right) \\
&= 3 \text{Subst} \left(\int \left(-1 + x + \frac{1}{1+x} \right) dx, x, \sqrt[3]{1+x} \right) \\
&= -3\sqrt[3]{1+x} + \frac{3}{2}(1+x)^{2/3} + 3 \log \left(1 + \sqrt[3]{1+x} \right)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 33, normalized size = 1.00

$$\frac{3}{2} \sqrt[3]{1+x} \left(-2 + \sqrt[3]{1+x} \right) + 3 \log \left(1 + \sqrt[3]{1+x} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + (1 + x)^(1/3))^(−1), x]``[Out] (3*(1 + x)^(1/3)*(-2 + (1 + x)^(1/3)))/2 + 3*Log[1 + (1 + x)^(1/3)]`**Mathics [A]**

time = 1.83, size = 25, normalized size = 0.76

$$-3(1+x)^{\frac{1}{3}} + \frac{3(1+x)^{\frac{2}{3}}}{2} + 3 \text{Log} \left[1 + (1+x)^{\frac{1}{3}} \right]$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(1 + (1 + x)^(1/3)), x]')``[Out] -3 (1 + x) ^ (1 / 3) + 3 (1 + x) ^ (2 / 3) / 2 + 3 Log[1 + (1 + x) ^ (1 / 3)]`**Maple [A]**

time = 0.09, size = 47, normalized size = 1.42

method	result
derivativedivides	$-3(1+x)^{\frac{1}{3}} + \frac{3(1+x)^{\frac{2}{3}}}{2} + 3 \ln \left(1 + (1+x)^{\frac{1}{3}} \right)$
trager	$-3(1+x)^{\frac{1}{3}} + \frac{3(1+x)^{\frac{2}{3}}}{2} + \ln \left(-3(1+x)^{\frac{2}{3}} - 3(1+x)^{\frac{1}{3}} - x - 2 \right)$
default	$\ln(2+x) + \frac{3(1+x)^{\frac{2}{3}}}{2} + 2 \ln \left(1 + (1+x)^{\frac{1}{3}} \right) - \ln \left((1+x)^{\frac{2}{3}} - (1+x)^{\frac{1}{3}} + 1 \right) - 3(1+x)^{\frac{1}{3}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1+(1+x)^(1/3)),x,method=_RETURNVERBOSE)`

[Out] $\ln(2+x)+3/2*(1+x)^(2/3)+2*\ln(1+(1+x)^(1/3))- \ln((1+x)^(2/3)-(1+x)^(1/3)+1)-3*(1+x)^(1/3)$

Maxima [A]

time = 0.26, size = 25, normalized size = 0.76

$$\frac{3}{2}(x+1)^{\frac{2}{3}} - 3(x+1)^{\frac{1}{3}} + 3 \log\left((x+1)^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+(1+x)^(1/3)),x, algorithm="maxima")`

[Out] $3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*\log((x + 1)^(1/3) + 1)$

Fricas [A]

time = 0.34, size = 25, normalized size = 0.76

$$\frac{3}{2}(x+1)^{\frac{2}{3}} - 3(x+1)^{\frac{1}{3}} + 3 \log\left((x+1)^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+(1+x)^(1/3)),x, algorithm="fricas")`

[Out] $3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*\log((x + 1)^(1/3) + 1)$

Sympy [A]

time = 0.06, size = 29, normalized size = 0.88

$$\frac{3(x+1)^{\frac{2}{3}}}{2} - 3\sqrt[3]{x+1} + 3 \log\left(\sqrt[3]{x+1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+(1+x)**(1/3)),x)`

[Out] $3*(x + 1)**(2/3)/2 - 3*(x + 1)**(1/3) + 3*\log((x + 1)**(1/3) + 1)$

Giac [A]

time = 0.00, size = 34, normalized size = 1.03

$$3 \left(\frac{\left((x+1)^{\frac{1}{3}} \right)^2}{2} - (x+1)^{\frac{1}{3}} + \ln \left| (x+1)^{\frac{1}{3}} + 1 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+(1+x)^(1/3)),x)`

[Out] $3/2*(x + 1)^{2/3} - 3*(x + 1)^{1/3} + 3*\log(\text{abs}((x + 1)^{1/3} + 1))$

Mupad [B]

time = 0.20, size = 25, normalized size = 0.76

$$3 \ln \left((x + 1)^{1/3} + 1 \right) - 3(x + 1)^{1/3} + \frac{3(x + 1)^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((x + 1)^(1/3) + 1),x)`

[Out] $3*\log((x + 1)^{1/3} + 1) - 3*(x + 1)^{1/3} + (3*(x + 1)^{2/3})/2$

$$3.48 \quad \int \frac{1}{\sqrt{x}(b+ax)} dx$$

Optimal. Leaf size=29

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{a} \sqrt{x}}{\sqrt{b}} \right)}{\sqrt{a} \sqrt{b}}$$

[Out] 2*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(1/2)/b^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {65, 211}

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{a} \sqrt{x}}{\sqrt{b}} \right)}{\sqrt{a} \sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*(b + a*x)),x]

[Out] (2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x}(b+ax)} dx &= 2 \text{Subst} \left(\int \frac{1}{b+ax^2} dx, x, \sqrt{x} \right) \\ &= \frac{2 \tan^{-1} \left(\frac{\sqrt{a} \sqrt{x}}{\sqrt{b}} \right)}{\sqrt{a} \sqrt{b}} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 29, normalized size = 1.00

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{a} \sqrt{x}}{\sqrt{b}} \right)}{\sqrt{a} \sqrt{b}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(Sqrt[x]*(b + a*x)),x]``[Out] (2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.13, size = 92, normalized size = 3.17

$$\text{Piecewise} \left[\left\{ \left\{ \text{DirectedInfinity}[\sqrt{x}], a==0 \&\& b==0 \right\}, \left\{ \frac{2\sqrt{x}}{b}, a==0 \right\}, \left\{ \frac{-2}{a\sqrt{x}}, b==0 \right\} \right\}, -\frac{\text{Log} \left[\sqrt{x} + \sqrt{-\frac{b}{a}} \right]}{a\sqrt{-\frac{b}{a}}} + \frac{\text{Log} \left[\sqrt{x} - \sqrt{-\frac{b}{a}} \right]}{a\sqrt{-\frac{b}{a}}} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(Sqrt[x]*(a*x + b)),x]')``[Out] Piecewise[{{DirectedInfinity[Sqrt[x]], a == 0 && b == 0}, {2 Sqrt[x] / b, a == 0}, {-2 / (a Sqrt[x]), b == 0}}, -Log[Sqrt[x] + Sqrt[-b / a]] / (a Sqrt[-b / a]) + Log[Sqrt[x] - Sqrt[-b / a]] / (a Sqrt[-b / a])]`**Maple [A]**

time = 0.05, size = 19, normalized size = 0.66

method	result	size
derivativedivides	$\frac{2 \arctan \left(\frac{a\sqrt{x}}{\sqrt{ab}} \right)}{\sqrt{ab}}$	19
default	$\frac{2 \arctan \left(\frac{a\sqrt{x}}{\sqrt{ab}} \right)}{\sqrt{ab}}$	19

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(a*x+b)/x^(1/2),x,method=_RETURNVERBOSE)``[Out] 2/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2))`

Maxima [A]

time = 0.36, size = 18, normalized size = 0.62

$$\frac{2 \arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a*x+b)/x^(1/2),x, algorithm="maxima")``[Out] 2*arctan(a*sqrt(x)/sqrt(a*b))/sqrt(a*b)`**Fricas [A]**

time = 0.33, size = 68, normalized size = 2.34

$$\left[-\frac{\sqrt{-ab} \log\left(\frac{ax-b-2\sqrt{-ab}\sqrt{x}}{ax+b}\right)}{ab}, -\frac{2\sqrt{ab} \arctan\left(\frac{\sqrt{ab}}{a\sqrt{x}}\right)}{ab} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a*x+b)/x^(1/2),x, algorithm="fricas")``[Out] [-sqrt(-a*b)*log((a*x - b - 2*sqrt(-a*b)*sqrt(x))/(a*x + b))/(a*b), -2*sqrt(a*b)*arctan(sqrt(a*b)/(a*sqrt(x)))/(a*b)]`**Sympy [A]**

time = 0.44, size = 73, normalized size = 2.52

$$\begin{cases} \infty\sqrt{x} & \text{for } a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{b} & \text{for } a = 0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b = 0 \\ \frac{\log\left(\sqrt{x} - \sqrt{-\frac{b}{a}}\right)}{a\sqrt{-\frac{b}{a}}} - \frac{\log\left(\sqrt{x} + \sqrt{-\frac{b}{a}}\right)}{a\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a*x+b)/x**(1/2),x)``[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b, Eq(a, 0)), (-2/(a*sqrt(x)), Eq(b, 0)), (log(sqrt(x) - sqrt(-b/a))/(a*sqrt(-b/a)) - log(sqrt(x) + sqrt(-b/a))/(a*sqrt(-b/a)), True))`

Giac [A]

time = 0.00, size = 26, normalized size = 0.90

$$\frac{2 \arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a*x+b)/x^(1/2), x)``[Out] 2*arctan(a*sqrt(x)/sqrt(a*b))/sqrt(a*b)`**Mupad [B]**

time = 0.20, size = 19, normalized size = 0.66

$$\frac{2 \operatorname{atan}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(x^(1/2)*(b + a*x)), x)``[Out] (2*atan((a^(1/2)*x^(1/2))/b^(1/2)))/(a^(1/2)*b^(1/2))`

3.49 $\int x^3 \sqrt{1+x^2} dx$

Optimal. Leaf size=27

$$-\frac{1}{3}(1+x^2)^{3/2} + \frac{1}{5}(1+x^2)^{5/2}$$

[Out] -1/3*(x^2+1)^(3/2)+1/5*(x^2+1)^(5/2)

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x^3*Sqrt[1+x^2],x]

[Out] -1/3*(1+x^2)^(3/2) + (1+x^2)^(5/2)/5

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int x^3 \sqrt{1+x^2} dx &= \frac{1}{2} \text{Subst} \left(\int x \sqrt{1+x} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(-\sqrt{1+x} + (1+x)^{3/2} \right) dx, x, x^2 \right) \\ &= -\frac{1}{3} (1+x^2)^{3/2} + \frac{1}{5} (1+x^2)^{5/2} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 20, normalized size = 0.74

$$\frac{1}{15} (1 + x^2)^{3/2} (-2 + 3x^2)$$

Antiderivative was successfully verified.

`[In] Integrate[x^3*Sqrt[1 + x^2],x]``[Out] ((1 + x^2)^(3/2)*(-2 + 3*x^2))/15`**Mathics [A]**

time = 2.00, size = 19, normalized size = 0.70

$$\frac{(-2 + x^2 + 3x^4) \sqrt{1 + x^2}}{15}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^3*Sqrt[1 + x^2],x]')``[Out] (-2 + x ^ 2 + 3 x ^ 4) Sqrt[1 + x ^ 2] / 15`**Maple [A]**

time = 0.05, size = 23, normalized size = 0.85

method	result	size
gospers	$\frac{(x^2+1)^{\frac{3}{2}}(3x^2-2)}{15}$	17
risch	$\frac{(3x^4+x^2-2)\sqrt{x^2+1}}{15}$	20
trager	$\left(\frac{1}{5}x^4 + \frac{1}{15}x^2 - \frac{2}{15}\right)\sqrt{x^2+1}$	21
default	$\frac{x^2(x^2+1)^{\frac{3}{2}}}{5} - \frac{2(x^2+1)^{\frac{3}{2}}}{15}$	23
meijerg	$-\frac{8\sqrt{\pi}}{15} + \frac{4\sqrt{\pi}(x^2+1)^{\frac{3}{2}}(-3x^2+2)}{15}$ $\frac{4\sqrt{\pi}}{15}$	31

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^3*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)``[Out] 1/5*x^2*(x^2+1)^(3/2)-2/15*(x^2+1)^(3/2)`**Maxima [A]**

time = 0.35, size = 22, normalized size = 0.81

$$\frac{1}{5} (x^2 + 1)^{\frac{3}{2}} x^2 - \frac{2}{15} (x^2 + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(x^2+1)^(1/2),x, algorithm="maxima")

[Out] 1/5*(x^2 + 1)^(3/2)*x^2 - 2/15*(x^2 + 1)^(3/2)

Fricas [A]

time = 0.33, size = 19, normalized size = 0.70

$$\frac{1}{15} (3x^4 + x^2 - 2) \sqrt{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(x^2+1)^(1/2),x, algorithm="fricas")

[Out] 1/15*(3*x^4 + x^2 - 2)*sqrt(x^2 + 1)

Sympy [A]

time = 0.16, size = 37, normalized size = 1.37

$$\frac{x^4 \sqrt{x^2 + 1}}{5} + \frac{x^2 \sqrt{x^2 + 1}}{15} - \frac{2 \sqrt{x^2 + 1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(x**2+1)**(1/2),x)

[Out] x**4*sqrt(x**2 + 1)/5 + x**2*sqrt(x**2 + 1)/15 - 2*sqrt(x**2 + 1)/15

Giac [A]

time = 0.00, size = 38, normalized size = 1.41

$$\frac{1}{5} \sqrt{x^2 + 1} (x^2 + 1)^2 - \frac{1}{3} \sqrt{x^2 + 1} (x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(x^2+1)^(1/2),x)

[Out] 1/5*(x^2 + 1)^(5/2) - 1/3*(x^2 + 1)^(3/2)

Mupad [B]

time = 0.04, size = 20, normalized size = 0.74

$$\sqrt{x^2 + 1} \left(\frac{x^4}{5} + \frac{x^2}{15} - \frac{2}{15} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(x^2 + 1)^(1/2),x)

[Out] (x^2 + 1)^(1/2)*(x^2/15 + x^4/5 - 2/15)

3.50

$$\int \frac{x}{\sqrt{a^4 - x^4}} dx$$

Optimal. Leaf size=22

$$\frac{1}{2} \tan^{-1} \left(\frac{x^2}{\sqrt{a^4 - x^4}} \right)$$

[Out] 1/2*arctan(x^2/(a^4-x^4)^(1/2))

Rubi [A]

time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {281, 223, 209}

$$\frac{1}{2} \tan^{-1} \left(\frac{x^2}{\sqrt{a^4 - x^4}} \right)$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a^4 - x^4],x]

[Out] ArcTan[x^2/Sqrt[a^4 - x^4]]/2

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{a^4 - x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{a^4 - x^2}} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \frac{x^2}{\sqrt{a^4 - x^4}} \right) \\ &= \frac{1}{2} \tan^{-1} \left(\frac{x^2}{\sqrt{a^4 - x^4}} \right) \end{aligned}$$

Mathematica [A]

time = 0.08, size = 22, normalized size = 1.00

$$-\frac{1}{2} \tan^{-1} \left(\frac{\sqrt{a^4 - x^4}}{x^2} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[x/Sqrt[a^4 - x^4],x]``[Out] -1/2*ArcTan[Sqrt[a^4 - x^4]/x^2]`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.35, size = 33, normalized size = 1.50

$$\text{Piecewise} \left[\left\{ \left\{ \left(-\frac{I}{2} \right) \text{ArcCosh} \left[\frac{x^2}{a^2} \right], \text{Abs} \left[\frac{x^4}{a^4} \right] > 1 \right\} \right\}, \frac{\text{ArcSin} \left[\frac{x^2}{a^2} \right]}{2} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[x/Sqrt[a^4 - x^4],x]')``[Out] Piecewise[{{(-I / 2) ArcCosh[x ^ 2 / a ^ 2], Abs[x ^ 4 / a ^ 4] > 1}}, ArcSin[x ^ 2 / a ^ 2] / 2]`**Maple [A]**

time = 0.05, size = 19, normalized size = 0.86

method	result	size
default	$\frac{\arctan\left(\frac{x^2}{\sqrt{a^4 - x^4}}\right)}{2}$	19

elliptic	$\frac{\arctan\left(\frac{x^2}{\sqrt{a^4 - x^4}}\right)}{2}$	19
----------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a^4-x^4)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `1/2*arctan(x^2/(a^4-x^4)^(1/2))`

Maxima [A]

time = 0.35, size = 18, normalized size = 0.82

$$-\frac{1}{2} \arctan\left(\frac{\sqrt{a^4 - x^4}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a^4-x^4)^(1/2),x, algorithm="maxima")`

[Out] `-1/2*arctan(sqrt(a^4 - x^4)/x^2)`

Fricas [A]

time = 0.33, size = 25, normalized size = 1.14

$$-\arctan\left(-\frac{a^2 - \sqrt{a^4 - x^4}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a^4-x^4)^(1/2),x, algorithm="fricas")`

[Out] `-arctan(-(a^2 - sqrt(a^4 - x^4))/x^2)`

Sympy [A]

time = 0.46, size = 29, normalized size = 1.32

$$\begin{cases} -\frac{i \operatorname{acosh}\left(\frac{x^2}{a^2}\right)}{2} & \text{for } \left|\frac{x^4}{a^4}\right| > 1 \\ \frac{\operatorname{asin}\left(\frac{x^2}{a^2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a**4-x**4)**(1/2),x)`

[Out] `Piecewise((-I*acosh(x**2/a**2)/2, Abs(x**4/a**4) > 1), (asin(x**2/a**2)/2, True))`

Giac [A]

time = 0.00, size = 12, normalized size = 0.55

$$\frac{\arcsin\left(\frac{x^2}{a^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a^4-x^4)^(1/2),x)**[Out]** 1/2*arcsin(x^2/a^2)**Mupad [B]**

time = 0.09, size = 18, normalized size = 0.82

$$\frac{\operatorname{atan}\left(\frac{x^2}{\sqrt{a^4 - x^4}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a^4 - x^4)^(1/2),x)**[Out]** atan(x^2/(a^4 - x^4)^(1/2))/2

$$3.51 \quad \int \frac{1}{x \sqrt{-a^2 + x^2}} dx$$

Optimal. Leaf size=22

$$\frac{\tan^{-1}\left(\frac{\sqrt{-a^2 + x^2}}{a}\right)}{a}$$

[Out] arctan((-a^2+x^2)^(1/2)/a)/a

Rubi [A]

time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {272, 65, 209}

$$\frac{\tan^{-1}\left(\frac{\sqrt{x^2 - a^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/(x*sqrt[-a^2 + x^2]),x]

[Out] ArcTan[Sqrt[-a^2 + x^2]/a]/a

Rule 65

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{-a^2+x^2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{-a^2+x}} dx, x, x^2 \right) \\
&= \text{Subst} \left(\int \frac{1}{a^2+x^2} dx, x, \sqrt{-a^2+x^2} \right) \\
&= \frac{\tan^{-1} \left(\frac{\sqrt{-a^2+x^2}}{a} \right)}{a}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 22, normalized size = 1.00

$$\frac{\tan^{-1} \left(\frac{\sqrt{-a^2+x^2}}{a} \right)}{a}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x*Sqrt[-a^2 + x^2]),x]``[Out] ArcTan[Sqrt[-a^2 + x^2]/a]/a`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.36, size = 35, normalized size = 1.59

$$\text{Piecewise} \left[\left\{ \left\{ \frac{I \text{ArcCosh} \left[\frac{a}{x} \right]}{a}, \text{Abs} \left[\frac{a^2}{x^2} \right] > 1 \right\} \right\}, -\frac{\text{ArcSin} \left[\frac{a}{x} \right]}{a} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x*Sqrt[x^2 - a^2]),x]')``[Out] Piecewise[{{I ArcCosh[a / x] / a, Abs[a ^ 2 / x ^ 2] > 1}}, -ArcSin[a / x] / a]`**Maple [A]**

time = 0.06, size = 41, normalized size = 1.86

method	result	size
default	$-\frac{\ln \left(\frac{-2a^2+2\sqrt{-a^2} \sqrt{-a^2+x^2}}{x} \right)}{\sqrt{-a^2}}$	41

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(-a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/(-a^2)^{(1/2)} * \ln((-2*a^2+2*(-a^2)^{(1/2)}*(-a^2+x^2)^{(1/2)})/x)$

Maxima [A]

time = 0.35, size = 12, normalized size = 0.55

$$-\frac{\arcsin\left(\frac{a}{|x|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-a^2+x^2)^(1/2),x, algorithm="maxima")`

[Out] $-\arcsin(a/\text{abs}(x))/a$

Fricas [A]

time = 0.34, size = 26, normalized size = 1.18

$$\frac{2 \arctan\left(-\frac{x - \sqrt{-a^2 + x^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-a^2+x^2)^(1/2),x, algorithm="fricas")`

[Out] $2 * \arctan(-(x - \text{sqrt}(-a^2 + x^2))/a)/a$

Sympy [A]

time = 0.46, size = 22, normalized size = 1.00

$$\begin{cases} \frac{i \operatorname{acosh}\left(\frac{a}{x}\right)}{a} & \text{for } \left|\frac{a^2}{x^2}\right| > 1 \\ -\frac{\operatorname{asin}\left(\frac{a}{x}\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-a**2+x**2)**(1/2),x)`

[Out] `Piecewise((I*acosh(a/x)/a, Abs(a**2/x**2) > 1), (-asin(a/x)/a, True))`

Giac [A]

time = 0.00, size = 23, normalized size = 1.05

$$\frac{\frac{1}{2} \cdot 2 \arctan\left(\frac{\sqrt{-a^2 + x^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-a^2+x^2)^(1/2),x)`

[Out] `arctan(sqrt(-a^2 + x^2)/a)/a`

Mupad [B]

time = 0.26, size = 24, normalized size = 1.09

$$\frac{\operatorname{atan}\left(\frac{\sqrt{x^2 - a^2}}{\sqrt{a^2}}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(x^2 - a^2)^(1/2)),x)`

[Out] `atan((x^2 - a^2)^(1/2)/(a^2)^(1/2))/(a^2)^(1/2)`

$$3.52 \quad \int \frac{1}{x \sqrt{a^2 - x^2}} dx$$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2 - x^2}}{a}\right)}{a}$$

[Out] -arctanh((a^2-x^2)^(1/2)/a)/a

Rubi [A]

time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {272, 65, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2 - x^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/(x*sqrt[a^2 - x^2]),x]

[Out] -(ArcTanh[Sqrt[a^2 - x^2]/a]/a)

Rule 65

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{a^2 - x^2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{a^2 - x} x} dx, x, x^2 \right) \\ &= -\text{Subst} \left(\int \frac{1}{a^2 - x^2} dx, x, \sqrt{a^2 - x^2} \right) \\ &= -\frac{\tanh^{-1} \left(\frac{\sqrt{a^2 - x^2}}{a} \right)}{a} \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 53 vs. 2(23) = 46.

time = 0.03, size = 53, normalized size = 2.30

$$-\frac{\log(a + \sqrt{a^2 - x^2})}{2a} + \frac{\log(-a^2 + a\sqrt{a^2 - x^2})}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*sqrt[a^2 - x^2]),x]

[Out] -1/2*Log[a + Sqrt[a^2 - x^2]]/a + Log[-a^2 + a*Sqrt[a^2 - x^2]]/(2*a)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.34, size = 35, normalized size = 1.52

$$\text{Piecewise} \left[\left\{ \left\{ -\frac{\text{ArcCosh} \left[\frac{a}{x} \right]}{a}, \text{Abs} \left[\frac{a^2}{x^2} \right] > 1 \right\} \right\}, \frac{I \text{ArcSin} \left[\frac{a}{x} \right]}{a} \right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x*sqrt[a^2 - x^2]),x]')

[Out] Piecewise[{{-ArcCosh[a / x] / a, Abs[a ^ 2 / x ^ 2] > 1}}, I ArcSin[a / x] / a]

Maple [A]

time = 0.05, size = 37, normalized size = 1.61

method	result	size
default	$-\frac{\ln \left(\frac{2a^2 + 2\sqrt{a^2} \sqrt{a^2 - x^2}}{x} \right)}{\sqrt{a^2}}$	37

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/(a^2)^{(1/2)}*\ln((2*a^2+2*(a^2)^{(1/2)}*(a^2-x^2)^{(1/2)})/x)$

Maxima [A]

time = 0.27, size = 34, normalized size = 1.48

$$\frac{\log\left(\frac{2a^2}{|x|} + \frac{2\sqrt{a^2-x^2}a}{|x|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^2-x^2)^(1/2),x, algorithm="maxima")`

[Out] $-\log(2*a^2/\text{abs}(x) + 2*\text{sqrt}(a^2 - x^2)*a/\text{abs}(x))/a$

Fricas [A]

time = 0.34, size = 25, normalized size = 1.09

$$\frac{\log\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^2-x^2)^(1/2),x, algorithm="fricas")`

[Out] $\log(-(a - \text{sqrt}(a^2 - x^2))/x)/a$

Sympy [A]

time = 0.47, size = 22, normalized size = 0.96

$$\begin{cases} -\frac{\text{acosh}\left(\frac{a}{x}\right)}{a} & \text{for } \left|\frac{a^2}{x^2}\right| > 1 \\ \frac{i \text{asin}\left(\frac{a}{x}\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a**2-x**2)**(1/2),x)`

[Out] `Piecewise((-acosh(a/x)/a, Abs(a**2/x**2) > 1), (I*asin(a/x)/a, True))`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(21) = 42$.

time = 0.00, size = 47, normalized size = 2.04

$$\frac{\frac{1}{2} \ln \left| \sqrt{a^2 - x^2} - a \right|}{a} - \frac{\frac{1}{2} \ln \left| \sqrt{a^2 - x^2} + a \right|}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a^2-x^2)^(1/2),x)

[Out] $-1/2*\log(\text{abs}(a + \text{sqrt}(a^2 - x^2)))/a + 1/2*\log(\text{abs}(-a + \text{sqrt}(a^2 - x^2)))/a$

Mupad [B]

time = 0.44, size = 21, normalized size = 0.91

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{a^2 - x^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a^2 - x^2)^(1/2)),x)

[Out] $-\operatorname{atanh}((a^2 - x^2)^{1/2}/a)/a$

$$3.53 \quad \int \frac{1}{x \sqrt{a^2 + x^2}} dx$$

Optimal. Leaf size=21

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2 + x^2}}{a}\right)}{a}$$

[Out] -arctanh((a^2+x^2)^(1/2)/a)/a

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {272, 65, 213}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2 + x^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/(x*sqrt[a^2 + x^2]),x]

[Out] -(ArcTanh[Sqrt[a^2 + x^2]/a]/a)

Rule 65

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{a^2+x^2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{a^2+x}} dx, x, x^2 \right) \\ &= \text{Subst} \left(\int \frac{1}{-a^2+x^2} dx, x, \sqrt{a^2+x^2} \right) \\ &= -\frac{\tanh^{-1} \left(\frac{\sqrt{a^2+x^2}}{a} \right)}{a} \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 49 vs. 2(21) = 42.

time = 0.02, size = 49, normalized size = 2.33

$$-\frac{\log(a + \sqrt{a^2+x^2})}{2a} + \frac{\log(-a^2 + a\sqrt{a^2+x^2})}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a^2 + x^2]),x]

[Out] -1/2*Log[a + Sqrt[a^2 + x^2]]/a + Log[-a^2 + a*Sqrt[a^2 + x^2]]/(2*a)

Mathics [A]

time = 2.08, size = 11, normalized size = 0.52

$$-\frac{\text{ArcSinh} \left[\frac{a}{x} \right]}{a}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x*Sqrt[x^2 + a^2]),x]')

[Out] -ArcSinh[a / x] / a

Maple [A]

time = 0.05, size = 35, normalized size = 1.67

method	result	size
default	$-\frac{\ln \left(\frac{2a^2+2\sqrt{a^2} \sqrt{a^2+x^2}}{x} \right)}{\sqrt{a^2}}$	35

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] $-1/(a^2)^{(1/2)} * \ln((2*a^2 + 2*(a^2)^{(1/2)} * (a^2 + x^2)^{(1/2)})/x)$

Maxima [A]

time = 0.26, size = 12, normalized size = 0.57

$$-\frac{\operatorname{arsinh}\left(\frac{a}{|x|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^2+x^2)^(1/2),x, algorithm="maxima")`

[Out] `-arcsinh(a/abs(x))/a`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(19) = 38.

time = 0.32, size = 40, normalized size = 1.90

$$-\frac{\log\left(a - x + \sqrt{a^2 + x^2}\right) - \log\left(-a - x + \sqrt{a^2 + x^2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^2+x^2)^(1/2),x, algorithm="fricas")`

[Out] `-(log(a - x + sqrt(a^2 + x^2)) - log(-a - x + sqrt(a^2 + x^2)))/a`

Sympy [A]

time = 0.44, size = 7, normalized size = 0.33

$$-\frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a**2+x**2)**(1/2),x)`

[Out] `-asinh(a/x)/a`

Giac [A]

time = 0.00, size = 43, normalized size = 2.05

$$\frac{\frac{1}{2} \ln\left(\sqrt{a^2 + x^2} - a\right)}{a} - \frac{\frac{1}{2} \ln\left(\sqrt{a^2 + x^2} + a\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^2+x^2)^(1/2),x)`

[Out] `-1/2*log(a + sqrt(a^2 + x^2))/a + 1/2*log(-a + sqrt(a^2 + x^2))/a`

Mupad [B]

time = 0.09, size = 26, normalized size = 1.24

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a^2 + x^2}}{\sqrt{-a^2}}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a^2 + x^2)^(1/2)),x)`

[Out] `atan((a^2 + x^2)^(1/2)/(-a^2)^(1/2))/(-a^2)^(1/2)`

$$3.54 \quad \int \frac{1}{\sqrt{2+x-x^2}} dx$$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

[Out] arcsin(-1/3+2/3*x)

Rubi [A]

time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {633, 222}

$$-\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[2 + x - x^2], x]

[Out] -ArcSin[(1 - 2*x)/3]

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 633

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*(-4*(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{2+x-x^2}} dx &= - \left(\frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}} dx, x, 1-2x \right) \right) \\ &= -\sin^{-1}\left(\frac{1}{3}(1-2x)\right) \end{aligned}$$

Mathematica [A]

time = 0.05, size = 21, normalized size = 1.75

$$-2 \tan^{-1} \left(\frac{\sqrt{2+x-x^2}}{1+x} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[1/Sqrt[2 + x - x^2],x]``[Out] -2*ArcTan[Sqrt[2 + x - x^2]/(1 + x)]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/Sqrt[2 + x - x^2],x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.10, size = 7, normalized size = 0.58

method	result	size
default	$\arcsin\left(-\frac{1}{3} + \frac{2x}{3}\right)$	7
trager	$\text{RootOf}(_Z^2 + 1) \ln\left(-2 \text{RootOf}(_Z^2 + 1) x + 2\sqrt{-x^2 + x + 2} + \text{RootOf}(_Z^2 + 1)\right)$	37

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)``[Out] arcsin(-1/3+2/3*x)`**Maxima [A]**

time = 0.34, size = 8, normalized size = 0.67

$$-\arcsin\left(-\frac{2}{3}x + \frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(-x^2+x+2)^(1/2),x, algorithm="maxima")``[Out] -arcsin(-2/3*x + 1/3)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(6) = 12$.
time = 0.36, size = 30, normalized size = 2.50

$$-\arctan\left(\frac{\sqrt{-x^2+x+2}(2x-1)}{2(x^2-x-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-x^2+x+2)^(1/2),x, algorithm="fricas")`

[Out] `-\arctan(1/2*sqrt(-x^2 + x + 2)*(2*x - 1)/(x^2 - x - 2))`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2+x+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-x**2+x+2)**(1/2),x)`

[Out] `Integral(1/sqrt(-x**2 + x + 2), x)`

Giac [A]

time = 0.00, size = 9, normalized size = 0.75

$$\arcsin\left(\frac{2x-1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-x^2+x+2)^(1/2),x)`

[Out] `arcsin(2/3*x - 1/3)`

Mupad [B]

time = 0.18, size = 6, normalized size = 0.50

$$\operatorname{asin}\left(\frac{2x}{3} - \frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x - x^2 + 2)^(1/2),x)`

[Out] `asin((2*x)/3 - 1/3)`

$$3.55 \quad \int \frac{1}{\sqrt{5 - 4x + 3x^2}} dx$$

Optimal. Leaf size=19

$$-\frac{\sinh^{-1}\left(\frac{2-3x}{\sqrt{11}}\right)}{\sqrt{3}}$$

[Out] -1/3*arcsinh(1/11*(2-3*x)*11^(1/2))*3^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {633, 221}

$$-\frac{\sinh^{-1}\left(\frac{2-3x}{\sqrt{11}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[5 - 4*x + 3*x^2],x]

[Out] -(ArcSinh[(2 - 3*x)/Sqrt[11]]/Sqrt[3])

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 633

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\int \frac{1}{\sqrt{5 - 4x + 3x^2}} dx = \frac{\text{Subst} \left(\int \frac{1}{\sqrt{1 + \frac{x^2}{44}}} dx, x, -4 + 6x \right)}{2\sqrt{33}} = -\frac{\sinh^{-1}\left(\frac{2-3x}{\sqrt{11}}\right)}{\sqrt{3}}$$

Mathematica [A]

time = 0.05, size = 33, normalized size = 1.74

$$-\frac{\log\left(2 - 3x + \sqrt{3} \sqrt{5 - 4x + 3x^2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/Sqrt[5 - 4*x + 3*x^2],x]``[Out] -(Log[2 - 3*x + Sqrt[3]*Sqrt[5 - 4*x + 3*x^2]]/Sqrt[3])`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/Sqrt[5 - 4*x + 3*x^2],x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [A]**

time = 0.23, size = 15, normalized size = 0.79

method	result	size
default	$\frac{\sqrt{3} \operatorname{arcsinh}\left(\frac{{}^3\sqrt{11} \left(x - \frac{2}{3}\right)}{11}\right)}{3}$	15
trager	$\frac{\operatorname{RootOf}(_Z^2 - 3) \ln\left(3 \operatorname{RootOf}(_Z^2 - 3) x + 3 \sqrt{3x^2 - 4x + 5} - 2 \operatorname{RootOf}(_Z^2 - 3)\right)}{3}$	42

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(3*x^2-4*x+5)^(1/2),x,method=_RETURNVERBOSE)``[Out] 1/3*3^(1/2)*arcsinh(3/11*11^(1/2)*(x-2/3))`**Maxima [A]**

time = 0.36, size = 16, normalized size = 0.84

$$\frac{1}{3} \sqrt{3} \operatorname{arsinh}\left(\frac{1}{11} \sqrt{11} (3x - 2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(3*x^2-4*x+5)^(1/2),x, algorithm="maxima")`

[Out] $\frac{1}{3}\sqrt{3}\operatorname{arcsinh}\left(\frac{1}{11}\sqrt{11}\cdot(3x-2)\right)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(16) = 32$.

time = 0.33, size = 38, normalized size = 2.00

$$\frac{1}{6}\sqrt{3}\log\left(-2\sqrt{3}\sqrt{3x^2-4x+5}(3x-2)-18x^2+24x-19\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3*x^2-4*x+5)^(1/2),x, algorithm="fricas")`

[Out] $\frac{1}{6}\sqrt{3}\log(-2\sqrt{3}\sqrt{3x^2-4x+5}\cdot(3x-2)-18x^2+24x-19)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^2-4x+5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3*x**2-4*x+5)**(1/2),x)`

[Out] `Integral(1/sqrt(3*x**2 - 4*x + 5), x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(16) = 32$.

time = 0.00, size = 41, normalized size = 2.16

$$\frac{\ln\left(\sqrt{3}\left(\sqrt{3x^2-4x+5}-\sqrt{3}x\right)+2\right)}{\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3*x^2-4*x+5)^(1/2),x)`

[Out] $-\frac{1}{3}\sqrt{3}\log(-\sqrt{3}\cdot(\sqrt{3}x-\sqrt{3x^2-4x+5}))+2)$

Mupad [B]

time = 0.29, size = 26, normalized size = 1.37

$$\frac{\sqrt{3}\ln\left(\sqrt{3}\left(x-\frac{2}{3}\right)+\sqrt{3x^2-4x+5}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*x^2 - 4*x + 5)^(1/2),x)`

[Out] $(3^{1/2}\log(3^{1/2}\cdot(x-2/3)+\sqrt{3x^2-4x+5}))/3$

$$3.56 \quad \int \frac{1}{\sqrt{x-x^2}} dx$$

Optimal. Leaf size=8

$$-\sin^{-1}(1-2x)$$

[Out] arcsin(-1+2*x)

Rubi [A]

time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {633, 222}

$$-\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x - x^2], x]

[Out] -ArcSin[1 - 2*x]

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 633

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*(-4*(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x-x^2}} dx &= -\text{Subst} \left(\int \frac{1}{\sqrt{1-x^2}} dx, x, 1-2x \right) \\ &= -\sin^{-1}(1-2x) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 38 vs. 2(8) = 16. time = 0.02, size = 38, normalized size = 4.75

$$\frac{2\sqrt{-1+x} \sqrt{x} \tanh^{-1} \left(\frac{\sqrt{x}}{\sqrt{-1+x}} \right)}{\sqrt{-((-1+x)x)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x - x^2],x]

[Out] (2*sqrt[-1 + x]*sqrt[x]*ArcTanh[sqrt[x]/sqrt[-1 + x]])/sqrt[-((-1 + x)*x)]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/Sqrt[x - x^2],x]')

[Out] cought exception: maximum recursion depth exceeded in comparison

Maple [A]

time = 0.08, size = 7, normalized size = 0.88

method	result	size
default	$\arcsin(2x - 1)$	7
meijerg	$2 \arcsin(\sqrt{x})$	7
trager	$\text{RootOf}(_Z^2 + 1) \ln(-2 \text{RootOf}(_Z^2 + 1) x + 2\sqrt{-x^2 + x} + \text{RootOf}(_Z^2 + 1))$	36

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^2+x)^(1/2),x,method=_RETURNVERBOSE)

[Out] arcsin(2*x-1)

Maxima [A]

time = 0.35, size = 6, normalized size = 0.75

$$\arcsin(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+x)^(1/2),x, algorithm="maxima")

[Out] arcsin(2*x - 1)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.
time = 0.33, size = 16, normalized size = 2.00

$$-2 \arctan\left(\frac{\sqrt{-x^2 + x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+x)^(1/2),x, algorithm="fricas")

[Out] -2*arctan(sqrt(-x^2 + x)/x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2 + x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**2+x)**(1/2),x)

[Out] Integral(1/sqrt(-x**2 + x), x)

Giac [A]

time = 0.00, size = 6, normalized size = 0.75

$$\arcsin(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+x)^(1/2),x)

[Out] arcsin(2*x - 1)

Mupad [B]

time = 0.16, size = 6, normalized size = 0.75

$$\operatorname{asin}(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x - x^2)^(1/2),x)

[Out] asin(2*x - 1)

$$3.57 \quad \int \frac{1+2x}{\sqrt{2+x-x^2}} dx$$

Optimal. Leaf size=27

$$-2\sqrt{2+x-x^2} - 2\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

[Out] 2*arcsin(-1/3+2/3*x)-2*(-x^2+x+2)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {654, 633, 222}

$$-2\sqrt{-x^2+x+2} - 2\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)/Sqrt[2 + x - x^2],x]

[Out] -2*Sqrt[2 + x - x^2] - 2*ArcSin[(1 - 2*x)/3]

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 633

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 654

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{1+2x}{\sqrt{2+x-x^2}} dx &= -2\sqrt{2+x-x^2} + 2 \int \frac{1}{\sqrt{2+x-x^2}} dx \\
&= -2\sqrt{2+x-x^2} - \frac{2}{3} \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}} dx, x, 1-2x \right) \\
&= -2\sqrt{2+x-x^2} - 2 \sin^{-1} \left(\frac{1}{3}(1-2x) \right)
\end{aligned}$$

Mathematica [A]

time = 0.08, size = 36, normalized size = 1.33

$$-2\sqrt{2+x-x^2} - 4 \tan^{-1} \left(\frac{\sqrt{2+x-x^2}}{1+x} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + 2*x)/Sqrt[2 + x - x^2], x]``[Out] -2*Sqrt[2 + x - x^2] - 4*ArcTan[Sqrt[2 + x - x^2]/(1 + x)]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(1 + 2*x)/Sqrt[2 + x - x^2], x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.10, size = 22, normalized size = 0.81

method	result
default	$2 \arcsin \left(-\frac{1}{3} + \frac{2x}{3} \right) - 2\sqrt{-x^2 + x + 2}$
risch	$\frac{2x^2-2x-4}{\sqrt{-x^2+x+2}} + 2 \arcsin \left(-\frac{1}{3} + \frac{2x}{3} \right)$
trager	$-2\sqrt{-x^2+x+2} + 2 \text{RootOf}(_Z^2+1) \ln(-2 \text{RootOf}(_Z^2+1)x + 2\sqrt{-x^2+x+2}) + \text{RootOf}(_Z^2+1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+2*x)/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `2*arcsin(-1/3+2/3*x)-2*(-x^2+x+2)^(1/2)`

Maxima [A]

time = 0.36, size = 21, normalized size = 0.78

$$-2\sqrt{-x^2+x+2} - 2\arcsin\left(-\frac{2}{3}x + \frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)/(-x^2+x+2)^(1/2),x, algorithm="maxima")`

[Out] `-2*sqrt(-x^2 + x + 2) - 2*arcsin(-2/3*x + 1/3)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(21) = 42$.

time = 0.32, size = 43, normalized size = 1.59

$$-2\sqrt{-x^2+x+2} - 2\arctan\left(\frac{\sqrt{-x^2+x+2}(2x-1)}{2(x^2-x-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)/(-x^2+x+2)^(1/2),x, algorithm="fricas")`

[Out] `-2*sqrt(-x^2 + x + 2) - 2*arctan(1/2*sqrt(-x^2 + x + 2)*(2*x - 1)/(x^2 - x - 2))`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\sqrt{-(x-2)(x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)/(-x**2+x+2)**(1/2),x)`

[Out] `Integral((2*x + 1)/sqrt(-(x - 2)*(x + 1)), x)`

Giac [A]

time = 0.00, size = 26, normalized size = 0.96

$$-2\sqrt{-x^2+x+2} + 2\arcsin\left(\frac{2x-1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)/(-x^2+x+2)^(1/2),x)`

[Out] $-2\sqrt{-x^2 + x + 2} + 2\arcsin(2/3x - 1/3)$

Mupad [B]

time = 0.31, size = 40, normalized size = 1.48

$$\arcsin\left(\frac{2x}{3} - \frac{1}{3}\right) - 2\sqrt{-x^2 + x + 2} - \ln\left(x\text{1i} + \sqrt{-x^2 + x + 2} - \frac{1}{2}\text{i}\right)\text{1i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((2x + 1)/(x - x^2 + 2)^{(1/2)}, x)$

[Out] $\arcsin((2x)/3 - 1/3) - \log(x\text{1i} + (x - x^2 + 2)^{(1/2)} - \text{1i}/2)\text{1i} - 2*(x - x^2 + 2)^{(1/2)}$

$$3.58 \quad \int \frac{1}{x \sqrt{2+x-x^2}} dx$$

Optimal. Leaf size=32

$$-\frac{\tanh^{-1}\left(\frac{4+x}{2\sqrt{2}\sqrt{2+x-x^2}}\right)}{\sqrt{2}}$$

[Out] $-1/2*\operatorname{arctanh}(1/4*(4+x)*2^{(1/2)/(-x^2+x+2)^{(1/2)})*2^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {738, 212}

$$-\frac{\tanh^{-1}\left(\frac{x+4}{2\sqrt{2}\sqrt{-x^2+x+2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[1/(x*\operatorname{Sqrt}[2+x-x^2]),x]$

[Out] $-(\operatorname{ArcTanh}[(4+x)/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[2+x-x^2])]/\operatorname{Sqrt}[2])$

Rule 212

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 738

$\operatorname{Int}[1/(((d_+) + (e_+)*(x_+))*\operatorname{Sqrt}[(a_+) + (b_+)*(x_+) + (c_+)*(x_+)^2]), x_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/\operatorname{Sqrt}[a + b*x + c*x^2]], x] /; \operatorname{FreeQ}\{a, b, c, d, e\}, x \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \ \&\& \operatorname{NeQ}[2*c*d - b*e, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{x \sqrt{2+x-x^2}} dx &= - \left(2 \operatorname{Subst} \left(\int \frac{1}{8-x^2} dx, x, \frac{4+x}{\sqrt{2+x-x^2}} \right) \right) \\ &= - \frac{\tanh^{-1}\left(\frac{4+x}{2\sqrt{2}\sqrt{2+x-x^2}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.06, size = 34, normalized size = 1.06

$$i\sqrt{2} \tan^{-1} \left(\frac{x + i\sqrt{2+x-x^2}}{\sqrt{2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[2 + x - x^2]),x]

[Out] I*Sqrt[2]*ArcTan[(x + I*Sqrt[2 + x - x^2])/Sqrt[2]]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x*Sqrt[2 + x - x^2]),x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [A]

time = 0.11, size = 25, normalized size = 0.78

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{(4+x)\sqrt{2}}{4\sqrt{-x^2+x+2}}\right)\sqrt{2}}{2}$	25
trager	$\frac{\operatorname{RootOf}(-Z^2-2) \ln\left(\frac{-\operatorname{RootOf}(-Z^2-2)x+4\sqrt{-x^2+x+2}-4\operatorname{RootOf}(-Z^2-2)}{x}\right)}{2}$	44

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2*arctanh(1/4*(4+x)*2^(1/2)/(-x^2+x+2)^(1/2))*2^(1/2)

Maxima [A]

time = 0.36, size = 33, normalized size = 1.03

$$-\frac{1}{2}\sqrt{2} \log\left(\frac{2\sqrt{2}\sqrt{-x^2+x+2}}{|x|} + \frac{4}{|x|} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^2+x+2)^(1/2),x, algorithm="maxima")

[Out] -1/2*sqrt(2)*log(2*sqrt(2)*sqrt(-x^2 + x + 2)/abs(x) + 4/abs(x) + 1)

Fricas [A]

time = 0.32, size = 39, normalized size = 1.22

$$\frac{1}{4} \sqrt{2} \log \left(-\frac{4 \sqrt{2} \sqrt{-x^2 + x + 2} (x + 4) + 7x^2 - 16x - 32}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^2+x+2)^(1/2),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log(-4*sqrt(2)*sqrt(-x^2 + x + 2)*(x + 4) + 7*x^2 - 16*x - 32)/x^2)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{-(x-2)(x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x**2+x+2)**(1/2),x)

[Out] Integral(1/(x*sqrt(-(x - 2)*(x + 1))), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(24) = 48.

time = 0.00, size = 82, normalized size = 2.56

$$\frac{\ln \left(\frac{\left| -4\sqrt{2} - 6 + \frac{2(-2\sqrt{-x^2 + x + 2} + 3)}{-2x+1} \right|}{\left| 4\sqrt{2} - 6 + \frac{2(-2\sqrt{-x^2 + x + 2} + 3)}{-2x+1} \right|} \right)}{\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^2+x+2)^(1/2),x)

[Out] -1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*(2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1) - 6)/abs(4*sqrt(2) + 2*(2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1) - 6))

Mupad [B]

time = 0.34, size = 28, normalized size = 0.88

$$\frac{\sqrt{2} \ln \left(\frac{x+2 \sqrt{2} \sqrt{-x^2 + x + 2} + 4}{x} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(x - x^2 + 2)^(1/2)),x)`

[Out] $-(2^{1/2}*\log((x + 2*2^{1/2}*(x - x^2 + 2)^{1/2} + 4)/x))/2$

$$3.59 \quad \int \frac{1}{(-2+x)\sqrt{2+x-x^2}} dx$$

Optimal. Leaf size=21

$$\frac{2\sqrt{2+x-x^2}}{3(-2+x)}$$

[Out] 2/3*(-x^2+x+2)^(1/2)/(-2+x)

Rubi [A]

time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.10, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {664}

$$-\frac{2\sqrt{-x^2+x+2}}{3(2-x)}$$

Antiderivative was successfully verified.

[In] Int[1/((-2 + x)*Sqrt[2 + x - x^2]),x]

[Out] (-2*Sqrt[2 + x - x^2])/(3*(2 - x))

Rule 664

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]

Rubi steps

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}} dx = -\frac{2\sqrt{2+x-x^2}}{3(2-x)}$$

Mathematica [A]

time = 0.09, size = 21, normalized size = 1.00

$$\frac{2\sqrt{2+x-x^2}}{3(-2+x)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((-2 + x)*Sqrt[2 + x - x^2]),x]

[Out] $(2*\text{Sqrt}[2 + x - x^2])/(3*(-2 + x))$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[1/((x - 2)*Sqrt[2 + x - x^2]),x]')`

[Out] cought exception: maximum recursion depth exceeded

Maple [A]

time = 0.09, size = 22, normalized size = 1.05

method	result	size
gospers	$-\frac{2(1+x)}{3\sqrt{-x^2+x+2}}$	16
risch	$-\frac{2(1+x)}{3\sqrt{-x^2+x+2}}$	16
trager	$\frac{2\sqrt{-x^2+x+2}}{3(-2+x)}$	18
default	$\frac{2\sqrt{-(-2+x)^2+6-3x}}{3(-2+x)}$	22

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-2+x)/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $2/3/(-2+x)*(-(-2+x)^2+6-3*x)^(1/2)$

Maxima [A]

time = 0.35, size = 17, normalized size = 0.81

$$\frac{2\sqrt{-x^2+x+2}}{3(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-2+x)/(-x^2+x+2)^(1/2),x, algorithm="maxima")`

[Out] $2/3*\text{sqrt}(-x^2 + x + 2)/(x - 2)$

Fricas [A]

time = 0.32, size = 17, normalized size = 0.81

$$\frac{2\sqrt{-x^2+x+2}}{3(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-2+x)/(-x^2+x+2)^(1/2),x, algorithm="fricas")`

[Out] $2/3*\sqrt{-x^2 + x + 2}/(x - 2)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(x-2)(x+1)}(x-2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-2+x)/(-x**2+x+2)**(1/2),x)`

[Out] `Integral(1/(sqrt(-(x - 2)*(x + 1))*(x - 2)), x)`

Giac [A]

time = 0.00, size = 32, normalized size = 1.52

$$-\frac{4}{3 \left(1 + \frac{-2\sqrt{-x^2 + x + 2} + 3}{-2x+1} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-2+x)/(-x^2+x+2)^(1/2),x)`

[Out] $-4/3/((2*\sqrt{-x^2 + x + 2} - 3)/(2*x - 1) + 1)$

Mupad [B]

time = 0.22, size = 19, normalized size = 0.90

$$\frac{2\sqrt{-x^2 + x + 2}}{3(x - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((x - 2)*(x - x^2 + 2)^(1/2)),x)`

[Out] $(2*(x - x^2 + 2)^(1/2))/(3*(x - 2))$

$$3.60 \quad \int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)} dx$$

Optimal. Leaf size=28

$$-\tanh^{-1}(\cos(x)) - \frac{1}{1-\cos(x)} - \frac{3\sin(x)}{1-\cos(x)}$$

[Out] `-arctanh(cos(x))-1/(1-cos(x))-3*sin(x)/(1-cos(x))`

Rubi [A]

time = 0.08, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {4486, 2727, 2746, 46, 213}

$$-\frac{1}{1-\cos(x)} - \frac{3\sin(x)}{1-\cos(x)} - \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] `Int[(Csc[x]*(2 + 3*Sin[x]))/(1 - Cos[x]),x]`

[Out] `-ArcTanh[Cos[x]] - (1 - Cos[x])^(-1) - (3*Sin[x])/(1 - Cos[x])`

Rule 46

`Int[((a_) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])`

Rule 213

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1)]*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

Rule 2727

`Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

Rule 2746

`Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In`

tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 4486

Int[u_, x_Symbol] := With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /; !InertTrigFreeQ[u]

Rubi steps

$$\begin{aligned}
 \int \frac{\csc(x)(2 + 3 \sin(x))}{1 - \cos(x)} dx &= \int \left(-\frac{3}{-1 + \cos(x)} - \frac{2 \csc(x)}{-1 + \cos(x)} \right) dx \\
 &= - \left(2 \int \frac{\csc(x)}{-1 + \cos(x)} dx \right) - 3 \int \frac{1}{-1 + \cos(x)} dx \\
 &= -\frac{3 \sin(x)}{1 - \cos(x)} + 2 \text{Subst} \left(\int \frac{1}{(-1 - x)(-1 + x)^2} dx, x, \cos(x) \right) \\
 &= -\frac{3 \sin(x)}{1 - \cos(x)} + 2 \text{Subst} \left(\int \left(-\frac{1}{2(-1 + x)^2} + \frac{1}{2(-1 + x^2)} \right) dx, x, \cos(x) \right) \\
 &= -\frac{1}{1 - \cos(x)} - \frac{3 \sin(x)}{1 - \cos(x)} + \text{Subst} \left(\int \frac{1}{-1 + x^2} dx, x, \cos(x) \right) \\
 &= -\tanh^{-1}(\cos(x)) - \frac{1}{1 - \cos(x)} - \frac{3 \sin(x)}{1 - \cos(x)}
 \end{aligned}$$

Mathematica [A]

time = 0.04, size = 54, normalized size = 1.93

$$\frac{1}{2} \csc^2\left(\frac{x}{2}\right) \left(-1 - \log\left(\cos\left(\frac{x}{2}\right)\right) + \cos(x) \left(\log\left(\cos\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right)\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right) - 3 \sin(x)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x]*(2 + 3*Sin[x]))/(1 - Cos[x]), x]

[Out] (Csc[x/2]^2*(-1 - Log[Cos[x/2]] + Cos[x]*(Log[Cos[x/2]] - Log[Sin[x/2]]) + Log[Sin[x/2]] - 3*Sin[x])/2

Mathics [A]

time = 2.03, size = 22, normalized size = 0.79

$$\text{Log} \left[\text{Tan} \left[\frac{x}{2} \right] \right] - \frac{3}{\text{Tan} \left[\frac{x}{2} \right]} - \frac{1}{2 \text{Tan} \left[\frac{x}{2} \right]^2}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(2 + 3*Sin[x])/(Sin[x]*(1 - Cos[x])),x]')`

[Out] `Log[Tan[x / 2]] - 3 / Tan[x / 2] - 1 / (2 Tan[x / 2] ^ 2)`

Maple [A]

time = 0.07, size = 23, normalized size = 0.82

method	result	size
default	$-\frac{1}{2 \tan\left(\frac{x}{2}\right)^2} - \frac{3}{\tan\left(\frac{x}{2}\right)} + \ln\left(\tan\left(\frac{x}{2}\right)\right)$	23
risch	$\frac{\left(\frac{1}{5} - \frac{3i}{5}\right)(10e^{ix} - 9 + 3i)}{(e^{ix} - 1)^2} - \ln(1 + e^{ix}) + \ln(e^{ix} - 1)$	44
norman	$-\frac{1}{2} - \frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2} - 3\left(\tan^3\left(\frac{x}{2}\right)\right) - 3 \tan\left(\frac{x}{2}\right)}{(1 + \tan^2\left(\frac{x}{2}\right)) \tan\left(\frac{x}{2}\right)^2} + \ln\left(\tan\left(\frac{x}{2}\right)\right)$	48

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2+3*sin(x))/(1-cos(x))/sin(x),x,method=_RETURNVERBOSE)`

[Out] `-1/2/tan(1/2*x)^2-3/tan(1/2*x)+ln(tan(1/2*x))`

Maxima [A]

time = 0.27, size = 33, normalized size = 1.18

$$-\frac{(\cos(x) + 1)^2}{2 \sin(x)^2} - \frac{3(\cos(x) + 1)}{\sin(x)} + \log\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+3*sin(x))/(1-cos(x))/sin(x),x, algorithm="maxima")`

[Out] `-1/2*(cos(x) + 1)^2/sin(x)^2 - 3*(cos(x) + 1)/sin(x) + log(sin(x)/(cos(x) + 1))`

Fricas [A]

time = 0.35, size = 39, normalized size = 1.39

$$-\frac{(\cos(x) - 1) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) - (\cos(x) - 1) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) - 6 \sin(x) - 2}{2(\cos(x) - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+3*sin(x))/(1-cos(x))/sin(x),x, algorithm="fricas")`

[Out] `-1/2*((cos(x) - 1)*log(1/2*cos(x) + 1/2) - (cos(x) - 1)*log(-1/2*cos(x) + 1/2) - 6*sin(x) - 2)/(cos(x) - 1)`

Sympy [A]

time = 0.32, size = 22, normalized size = 0.79

$$\log\left(\tan\left(\frac{x}{2}\right)\right) - \frac{3}{\tan\left(\frac{x}{2}\right)} - \frac{1}{2 \tan^2\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*sin(x))/(1-cos(x))/sin(x),x)

[Out] log(tan(x/2)) - 3/tan(x/2) - 1/(2*tan(x/2)**2)

Giac [A]

time = 0.00, size = 44, normalized size = 1.57

$$2 \left(\frac{-3 \tan^2\left(\frac{x}{2}\right) - 6 \tan\left(\frac{x}{2}\right) - 1}{4 \tan^2\left(\frac{x}{2}\right)} + \frac{\ln\left|\tan\left(\frac{x}{2}\right)\right|}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*sin(x))/(1-cos(x))/sin(x),x)

[Out] -1/2*(3*tan(1/2*x)^2 + 6*tan(1/2*x) + 1)/tan(1/2*x)^2 + log(abs(tan(1/2*x)))

Mupad [B]

time = 0.28, size = 22, normalized size = 0.79

$$\ln\left(\tan\left(\frac{x}{2}\right)\right) - \frac{3 \tan\left(\frac{x}{2}\right) + \frac{1}{2}}{\tan\left(\frac{x}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3*sin(x) + 2)/(sin(x)*(cos(x) - 1)),x)

[Out] log(tan(x/2)) - (3*tan(x/2) + 1/2)/tan(x/2)^2

3.61 $\int \frac{1}{2+3\cos^2(x)} dx$

Optimal. Leaf size=37

$$\frac{x}{\sqrt{10}} - \frac{\tan^{-1}\left(\frac{3\cos(x)\sin(x)}{2+\sqrt{10}+3\cos^2(x)}\right)}{\sqrt{10}}$$

[Out] 1/10*x*10^(1/2)-1/10*arctan(3*cos(x)*sin(x)/(2+3*cos(x)^2+10^(1/2)))*10^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3260, 209}

$$\frac{x}{\sqrt{10}} - \frac{\tan^{-1}\left(\frac{\left(\sqrt{\frac{5}{2}}-1\right)\sin(x)\cos(x)}{\left(\sqrt{\frac{5}{2}}-1\right)\cos^2(x)+1}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*Cos[x]^2)^(-1), x]

[Out] x/Sqrt[10] - ArcTan[(-1 + Sqrt[5/2])*Cos[x]*Sin[x]]/(1 + (-1 + Sqrt[5/2])*Cos[x]^2)/Sqrt[10]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\int \frac{1}{2 + 3 \cos^2(x)} dx = -\text{Subst} \left(\int \frac{1}{2 + 5x^2} dx, x, \cot(x) \right)$$

$$= \frac{x}{\sqrt{10}} - \frac{\tan^{-1} \left(\frac{\left(-1 + \sqrt{\frac{5}{2}} \right) \cos(x) \sin(x)}{1 + \left(-1 + \sqrt{\frac{5}{2}} \right) \cos^2(x)} \right)}{\sqrt{10}}$$

Mathematica [A]

time = 0.03, size = 17, normalized size = 0.46

$$\frac{\tan^{-1} \left(\sqrt{\frac{2}{5}} \tan(x) \right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

`[In] Integrate[(2 + 3*Cos[x]^2)^(-1),x]``[Out] ArcTan[Sqrt[2/5]*Tan[x]]/Sqrt[10]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(2 + 3*Cos[x]^2),x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [A]**

time = 0.04, size = 14, normalized size = 0.38

method	result	size
default	$\frac{\sqrt{10} \arctan\left(\frac{\tan(x)\sqrt{10}}{5}\right)}{10}$	14
risch	$\frac{i\sqrt{10} \ln\left(e^{2ix} + \frac{2\sqrt{10}}{3} + \frac{7}{3}\right)}{20} - \frac{i\sqrt{10} \ln\left(e^{2ix} - \frac{2\sqrt{10}}{3} + \frac{7}{3}\right)}{20}$	40

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2+3*cos(x)^2),x,method=_RETURNVERBOSE)`

[Out] `1/10*10^(1/2)*arctan(1/5*tan(x)*10^(1/2))`

Maxima [A]

time = 0.35, size = 13, normalized size = 0.35

$$\frac{1}{10} \sqrt{10} \arctan \left(\frac{1}{5} \sqrt{10} \tan(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2+3*cos(x)^2),x, algorithm="maxima")`

[Out] `1/10*sqrt(10)*arctan(1/5*sqrt(10)*tan(x))`

Fricas [A]

time = 0.34, size = 31, normalized size = 0.84

$$-\frac{1}{20} \sqrt{10} \arctan \left(\frac{7 \sqrt{10} \cos(x)^2 - 2 \sqrt{10}}{20 \cos(x) \sin(x)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2+3*cos(x)^2),x, algorithm="fricas")`

[Out] `-1/20*sqrt(10)*arctan(1/20*(7*sqrt(10)*cos(x)^2 - 2*sqrt(10))/(cos(x)*sin(x)))`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{3 \cos^2(x) + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2+3*cos(x)**2),x)`

[Out] `Integral(1/(3*cos(x)**2 + 2), x)`

Giac [A]

time = 0.00, size = 57, normalized size = 1.54

$$\frac{2 \left(\arctan \left(\frac{-\sqrt{10} \sin(2x) + 2 \sin(2x)}{\sqrt{10} \cos(2x) + \sqrt{10} - 2 \cos(2x) + 2} \right) + x \right)}{2\sqrt{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2+3*cos(x)^2),x)`

[Out] `1/10*sqrt(10)*(x + arctan(-(sqrt(10)*sin(2*x) - 2*sin(2*x))/(sqrt(10)*cos(2*x) + sqrt(10) - 2*cos(2*x) + 2)))`

Mupad [B]

time = 0.24, size = 26, normalized size = 0.70

$$\frac{\sqrt{10} (x - \operatorname{atan}(\tan(x)))}{10} + \frac{\sqrt{10} \operatorname{atan}\left(\frac{\sqrt{10} \tan(x)}{5}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*cos(x)^2 + 2),x)`

[Out] `(10^(1/2)*(x - atan(tan(x))))/10 + (10^(1/2)*atan((10^(1/2)*tan(x))/5))/10`

3.62 $\int \csc(2x)(1 - \tan(x)) dx$

Optimal. Leaf size=14

$$\frac{1}{2} \log(\tan(x)) - \frac{\tan(x)}{2}$$

[Out] 1/2*ln(tan(x))-1/2*tan(x)

Rubi [A]

time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {12}

$$\frac{1}{2} \log(\tan(x)) - \frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Csc[2*x]*(1 - Tan[x]),x]

[Out] Log[Tan[x]]/2 - Tan[x]/2

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rubi steps

$$\begin{aligned} \int \csc(2x)(1 - \tan(x)) dx &= \text{Subst} \left(\int \frac{1}{2} \left(-1 + \frac{1}{x} \right) dx, x, \tan(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(-1 + \frac{1}{x} \right) dx, x, \tan(x) \right) \\ &= \frac{1}{2} \log(\tan(x)) - \frac{\tan(x)}{2} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 21, normalized size = 1.50

$$-\frac{1}{2} \log(\cos(x)) + \frac{1}{2} \log(\sin(x)) - \frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[2*x]*(1 - Tan[x]),x]

[Out] $-1/2*\text{Log}[\text{Cos}[x]] + \text{Log}[\text{Sin}[x]]/2 - \text{Tan}[x]/2$

Mathics [A]

time = 2.59, size = 21, normalized size = 1.50

$$-\frac{\text{Tan}[x]}{2} - \frac{\text{Log}[\text{Cos}[x]^2]}{4} + \frac{\text{Log}[-1 + \text{Cos}[x]^2]}{4}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(1 - Tan[x])/Sin[2*x], x]')`

[Out] $-\text{Tan}[x] / 2 - \text{Log}[\text{Cos}[x]^2] / 4 + \text{Log}[-1 + \text{Cos}[x]^2] / 4$

Maple [A]

time = 0.08, size = 11, normalized size = 0.79

method	result	size
default	$\frac{\ln(\tan(x))}{2} - \frac{\tan(x)}{2}$	11
norman	$\frac{\ln(\tan(x))}{2} - \frac{\tan(x)}{2}$	11
risch	$-\frac{i}{e^{2ix}+1} - \frac{\ln(e^{2ix}+1)}{2} + \frac{\ln(e^{2ix}-1)}{2}$	34

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1-tan(x))/sin(2*x),x,method=_RETURNVERBOSE)`

[Out] $1/2*\ln(\tan(x))-1/2*\tan(x)$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(10) = 20.

time = 0.27, size = 47, normalized size = 3.36

$$-\frac{\sin(2x)}{\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1} - \frac{1}{4} \log(\cos(2x) + 1) + \frac{1}{4} \log(\cos(2x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-tan(x))/sin(2*x),x, algorithm="maxima")`

[Out] $-\sin(2*x)/(\cos(2*x)^2 + \sin(2*x)^2 + 2*\cos(2*x) + 1) - 1/4*\log(\cos(2*x) + 1) + 1/4*\log(\cos(2*x) - 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(10) = 20.

time = 0.34, size = 32, normalized size = 2.29

$$\frac{1}{4} \log\left(\frac{\tan(x)^2}{\tan(x)^2 + 1}\right) - \frac{1}{4} \log\left(\frac{1}{\tan(x)^2 + 1}\right) - \frac{1}{2} \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-tan(x))/sin(2*x),x, algorithm="fricas")

[Out] $1/4*\log(\tan(x)^2/(\tan(x)^2 + 1)) - 1/4*\log(1/(\tan(x)^2 + 1)) - 1/2*\tan(x)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$

time = 0.70, size = 27, normalized size = 1.93

$$\frac{\log(\cos(2x) - 1)}{4} - \frac{\log(\cos(2x) + 1)}{4} - \frac{\sin(x)}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-tan(x))/sin(2*x),x)

[Out] $\log(\cos(2*x) - 1)/4 - \log(\cos(2*x) + 1)/4 - \sin(x)/(2*\cos(x))$

Giac [A]

time = 0.00, size = 16, normalized size = 1.14

$$2 \left(-\frac{\tan x}{4} + \frac{\ln|\tan x|}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-tan(x))/sin(2*x),x)

[Out] $1/2*\log(\text{abs}(\tan(x))) - 1/2*\tan(x)$

Mupad [B]

time = 0.21, size = 10, normalized size = 0.71

$$\frac{\ln(\tan(x))}{2} - \frac{\tan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(tan(x) - 1)/sin(2*x),x)

[Out] $\log(\tan(x))/2 - \tan(x)/2$

3.63

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)} dx$$

Optimal. Leaf size=11

$$\frac{1}{2} \tanh^{-1}(2 \cos(x) \sin(x))$$

[Out] 1/2*arctanh(2*cos(x)*sin(x))

Rubi [A]

time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {212}

$$\frac{1}{2} \tanh^{-1}(2 \sin(x) \cos(x))$$

Antiderivative was successfully verified.

[In] Int[(1 + Tan[x]^2)/(1 - Tan[x]^2), x]

[Out] ArcTanh[2*Cos[x]*Sin[x]]/2

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1+\tan^2(x)}{1-\tan^2(x)} dx &= \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \tan(x) \right) \\ &= \frac{1}{2} \tanh^{-1}(2 \cos(x) \sin(x)) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

time = 0.01, size = 23, normalized size = 2.09

$$-\frac{1}{2} \log(\cos(x) - \sin(x)) + \frac{1}{2} \log(\cos(x) + \sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Tan[x]^2)/(1 - Tan[x]^2), x]

[Out] $-1/2*\text{Log}[\text{Cos}[x] - \text{Sin}[x]] + \text{Log}[\text{Cos}[x] + \text{Sin}[x]]/2$

Mathics [A]

time = 1.80, size = 15, normalized size = 1.36

$$-\frac{\text{Log}[-1 + \text{Tan}[x]]}{2} + \frac{\text{Log}[1 + \text{Tan}[x]]}{2}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(1 + Tan[x]^2)/(1 - Tan[x]^2), x]')`

[Out] $-\text{Log}[-1 + \text{Tan}[x]] / 2 + \text{Log}[1 + \text{Tan}[x]] / 2$

Maple [A]

time = 0.02, size = 4, normalized size = 0.36

method	result	size
derivativedivides	$\text{arctanh}(\tan(x))$	4
default	$\text{arctanh}(\tan(x))$	4
norman	$-\frac{\ln(-1+\tan(x))}{2} + \frac{\ln(\tan(x)+1)}{2}$	16
risch	$\frac{\ln(e^{2ix}+i)}{2} - \frac{\ln(e^{2ix}-i)}{2}$	24

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+tan(x)^2)/(1-tan(x)^2), x, method=_RETURNVERBOSE)`

[Out] $\text{arctanh}(\tan(x))$

Maxima [A]

time = 0.26, size = 15, normalized size = 1.36

$$\frac{1}{2} \log(\tan(x) + 1) - \frac{1}{2} \log(\tan(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+tan(x)^2)/(1-tan(x)^2), x, algorithm="maxima")`

[Out] $1/2*\log(\tan(x) + 1) - 1/2*\log(\tan(x) - 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(9) = 18$.
time = 0.34, size = 45, normalized size = 4.09

$$\frac{1}{4} \log\left(\frac{\tan(x)^2 + 2 \tan(x) + 1}{\tan(x)^2 + 1}\right) - \frac{1}{4} \log\left(\frac{\tan(x)^2 - 2 \tan(x) + 1}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+tan(x)^2)/(1-tan(x)^2),x, algorithm="fricas")

[Out] 1/4*log((tan(x)^2 + 2*tan(x) + 1)/(tan(x)^2 + 1)) - 1/4*log((tan(x)^2 - 2*tan(x) + 1)/(tan(x)^2 + 1))

Sympy [A]

time = 0.07, size = 15, normalized size = 1.36

$$-\frac{\log(\tan(x) - 1)}{2} + \frac{\log(\tan(x) + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+tan(x)**2)/(1-tan(x)**2),x)

[Out] -log(tan(x) - 1)/2 + log(tan(x) + 1)/2

Giac [A]

time = 0.00, size = 20, normalized size = 1.82

$$-\frac{\ln|\tan x - 1|}{2} + \frac{\ln|\tan x + 1|}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+tan(x)^2)/(1-tan(x)^2),x)

[Out] 1/2*log(abs(tan(x) + 1)) - 1/2*log(abs(tan(x) - 1))

Mupad [B]

time = 0.35, size = 3, normalized size = 0.27

$$\operatorname{atanh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(tan(x)^2 + 1)/(tan(x)^2 - 1),x)

[Out] atanh(tan(x))

3.64 $\int (a^2 - 4 \cos^2(x))^{3/4} \sin(2x) dx$

Optimal. Leaf size=18

$$\frac{1}{7} (a^2 - 4 \cos^2(x))^{7/4}$$

[Out] 1/7*(a^2-4*cos(x)^2)^(7/4)

Rubi [A]

time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {12, 267}

$$\frac{1}{7} (a^2 + 4 \sin^2(x) - 4)^{7/4}$$

Antiderivative was successfully verified.

[In] Int[(a^2 - 4*Cos[x]^2)^(3/4)*Sin[2*x],x]

[Out] (-4 + a^2 + 4*Sin[x]^2)^(7/4)/7

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 267

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int (a^2 - 4 \cos^2(x))^{3/4} \sin(2x) dx &= \text{Subst} \left(\int 2x (-4 + a^2 + 4x^2)^{3/4} dx, x, \sin(x) \right) \\ &= 2 \text{Subst} \left(\int x (-4 + a^2 + 4x^2)^{3/4} dx, x, \sin(x) \right) \\ &= \frac{1}{7} (-4 + a^2 + 4 \sin^2(x))^{7/4} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 19, normalized size = 1.06

$$\frac{1}{7} (-4 + a^2 + 4 \sin^2(x))^{7/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - 4*Cos[x]^2)^(3/4)*Sin[2*x],x]
```

```
[Out] (-4 + a^2 + 4*Sin[x]^2)^(7/4)/7
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a^2 - 4*Cos[x]^2)^(3/4)*Sin[2*x],x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5005 deep
```

Maple [A]

time = 0.04, size = 15, normalized size = 0.83

method	result	size
derivativedivides	$\frac{(a^2 - 4(\cos^2(x)))^{7/4}}{7}$	15
default	$\frac{(a^2 - 4(\cos^2(x)))^{7/4}}{7}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*(a^2-4*cos(x)^2)^(7/4)
```

Maxima [A]

time = 0.28, size = 14, normalized size = 0.78

$$\frac{1}{7} (a^2 - 4 \cos(x)^2)^{7/4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="maxima")
```

```
[Out] 1/7*(a^2 - 4*cos(x)^2)^(7/4)
```

Fricas [A]

time = 0.34, size = 14, normalized size = 0.78

$$\frac{1}{7} (a^2 - 4 \cos(x)^2)^{7/4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="fricas")

[Out] 1/7*(a^2 - 4*cos(x)^2)^(7/4)

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2-4*cos(x)**2)**(3/4)*sin(2*x),x)

[Out] Exception raised: SystemError

Giac [A]

time = 0.00, size = 79, normalized size = 4.39

$$\frac{1}{7} \left((a^2 - 4 \cos^2 x)^{\frac{1}{4}} \right)^3 (a^2 - 4 \cos^2 x) - \frac{1}{3} \left((a^2 - 4 \cos^2 x)^{\frac{1}{4}} \right)^3 a^2 + \frac{1}{3} a^2 \left((a^2 - 4 \cos^2 x)^{\frac{1}{4}} \right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x)

[Out] 1/7*(a^2 - 4*cos(x)^2)^(7/4)

Mupad [B]

time = 0.31, size = 14, normalized size = 0.78

$$\frac{(a^2 - 4 \cos(x)^2)^{7/4}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)*(a^2 - 4*cos(x)^2)^(3/4),x)

[Out] (a^2 - 4*cos(x)^2)^(7/4)/7

$$3.65 \quad \int \frac{\sin(2x)}{\sqrt[3]{a^2 - 4\sin^2(x)}} dx$$

Optimal. Leaf size=18

$$-\frac{3}{8} (a^2 - 4\sin^2(x))^{2/3}$$

[Out] -3/8*(a^2-4*sin(x)^2)^(2/3)

Rubi [A]

time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {12, 267}

$$-\frac{3}{8} (a^2 - 4\sin^2(x))^{2/3}$$

Antiderivative was successfully verified.

[In] Int[Sin[2*x]/(a^2 - 4*Sin[x]^2)^(1/3),x]

[Out] (-3*(a^2 - 4*Sin[x]^2)^(2/3))/8

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 267

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sin(2x)}{\sqrt[3]{a^2 - 4\sin^2(x)}} dx &= \text{Subst} \left(\int \frac{2x}{\sqrt[3]{a^2 - 4x^2}} dx, x, \sin(x) \right) \\ &= 2\text{Subst} \left(\int \frac{x}{\sqrt[3]{a^2 - 4x^2}} dx, x, \sin(x) \right) \\ &= -\frac{3}{8} (a^2 - 4\sin^2(x))^{2/3} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 18, normalized size = 1.00

$$-\frac{3}{8} (a^2 - 4\sin^2(x))^{2/3}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[2*x]/(a^2 - 4*Sin[x]^2)^(1/3),x]

[Out] (-3*(a^2 - 4*Sin[x]^2)^(2/3))/8

Mathics [A]

time = 2.75, size = 14, normalized size = 0.78

$$\frac{-3(a^2 - 4\sin[x]^2)^{\frac{2}{3}}}{8}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Sin[2*x]/(a^2 - 4*Sin[x]^2)^(1/3),x]')

[Out] -3 (a ^ 2 - 4 Sin[x] ^ 2) ^ (2 / 3) / 8

Maple [A]

time = 0.06, size = 15, normalized size = 0.83

method	result	size
derivativedivides	$-\frac{3(a^2-4(\sin^2(x)))^{\frac{2}{3}}}{8}$	15
default	$-\frac{3(a^2-4(\sin^2(x)))^{\frac{2}{3}}}{8}$	15

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x,method=_RETURNVERBOSE)

[Out] -3/8*(a^2-4*sin(x)^2)^(2/3)

Maxima [A]

time = 0.26, size = 14, normalized size = 0.78

$$-\frac{3}{8} (a^2 - 4 \sin(x)^2)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x, algorithm="maxima")

[Out] -3/8*(a^2 - 4*sin(x)^2)^(2/3)

Fricas [A]

time = 0.33, size = 15, normalized size = 0.83

$$-\frac{3}{8} (a^2 + 4 \cos(x)^2 - 4)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x, algorithm="fricas")

[Out] $-3/8*(a^2 + 4*\cos(x)^2 - 4)^(2/3)$

Sympy [A]

time = 1.04, size = 17, normalized size = 0.94

$$\frac{3(a^2 - 4\sin^2(x))^{\frac{2}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)/(a**2-4*sin(x)**2)**(1/3),x)

[Out] $-3*(a**2 - 4*\sin(x)**2)**(2/3)/8$

Giac [A]

time = 0.00, size = 24, normalized size = 1.33

$$\frac{3\left((a^2 - 4\sin^2 x)^{\frac{1}{3}}\right)^2}{4 \cdot 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x)

[Out] $-3/8*(a^2 - 4*\sin(x)^2)^(2/3)$

Mupad [B]

time = 0.30, size = 14, normalized size = 0.78

$$\frac{3(a^2 - 4\sin(x)^2)^{2/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)/(a^2 - 4*sin(x)^2)^(1/3),x)

[Out] $-(3*(a^2 - 4*\sin(x)^2)^(2/3))/8$

$$3.66 \quad \int \frac{1}{\sqrt{-1 + a^{2x}}} dx$$

Optimal. Leaf size=17

$$\frac{\tan^{-1}\left(\sqrt{-1 + a^{2x}}\right)}{\log(a)}$$

[Out] arctan((-1+a^(2*x))^(1/2))/ln(a)

Rubi [A]

time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2320, 65, 209}

$$\frac{\tan^{-1}\left(\sqrt{a^{2x} - 1}\right)}{\log(a)}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-1 + a^(2*x)],x]

[Out] ArcTan[Sqrt[-1 + a^(2*x)]]/Log[a]

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1 + a^{2x}}} dx = \frac{\text{Subst} \left(\int \frac{1}{\sqrt{-1 + x}} dx, x, a^{2x} \right)}{2 \log(a)}$$

$$= \frac{\text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt{-1 + a^{2x}} \right)}{\log(a)}$$

$$= \frac{\tan^{-1} \left(\sqrt{-1 + a^{2x}} \right)}{\log(a)}$$

Mathematica [A]

time = 0.02, size = 17, normalized size = 1.00

$$\frac{\tan^{-1} \left(\sqrt{-1 + a^{2x}} \right)}{\log(a)}$$

Antiderivative was successfully verified.

`[In] Integrate[1/Sqrt[-1 + a^(2*x)], x]``[Out] ArcTan[Sqrt[-1 + a^(2*x)]]/Log[a]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/Sqrt[a^(2*x) - 1], x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [A]**

time = 0.04, size = 16, normalized size = 0.94

method	result	size
derivativedivides	$\frac{\arctan\left(\sqrt{-1 + a^{2x}}\right)}{\ln(a)}$	16
default	$\frac{\arctan\left(\sqrt{-1 + a^{2x}}\right)}{\ln(a)}$	16

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-1+a^(2*x))^(1/2),x,method=_RETURNVERBOSE)`

[Out] `arctan((-1+a^(2*x))^(1/2))/ln(a)`

Maxima [A]

time = 0.35, size = 15, normalized size = 0.88

$$\frac{\arctan\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1+a^(2*x))^(1/2),x, algorithm="maxima")`

[Out] `arctan(sqrt(a^(2*x) - 1))/log(a)`

Fricas [A]

time = 0.34, size = 15, normalized size = 0.88

$$\frac{\arctan\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1+a^(2*x))^(1/2),x, algorithm="fricas")`

[Out] `arctan(sqrt(a^(2*x) - 1))/log(a)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^{2x}-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1+a**(2*x))**(1/2),x)`

[Out] `Integral(1/sqrt(a**(2*x) - 1), x)`

Giac [A]

time = 0.00, size = 16, normalized size = 0.94

$$\frac{\arctan\left(\sqrt{(a^x)^2-1}\right)}{\ln a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+a^(2*x))^(1/2),x)

[Out] arctan(sqrt(a^(2*x) - 1))/log(a)

Mupad [B]

time = 0.29, size = 37, normalized size = 2.18

$$\frac{a^x \operatorname{asin}\left(\frac{1}{a^x}\right) \sqrt{1 - \frac{1}{a^{2x}}}}{\ln(a) \sqrt{a^{2x} - 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^(2*x) - 1)^(1/2),x)

[Out] -(a^x*asin(1/a^x)*(1 - 1/a^(2*x))^(1/2))/(log(a)*(a^(2*x) - 1)^(1/2))

$$3.67 \quad \int \frac{e^{x/2}}{\sqrt{-1 + e^x}} dx$$

Optimal. Leaf size=20

$$2 \tanh^{-1} \left(\frac{e^{x/2}}{\sqrt{-1 + e^x}} \right)$$

[Out] 2*arctanh(exp(1/2*x)/(-1+exp(x))^(1/2))

Rubi [A]

time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2281, 223, 212}

$$2 \tanh^{-1} \left(\frac{e^{x/2}}{\sqrt{e^x - 1}} \right)$$

Antiderivative was successfully verified.

[In] Int[E^(x/2)/Sqrt[-1 + E^x],x]

[Out] 2*ArcTanh[E^(x/2)/Sqrt[-1 + E^x]]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 223

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 2281

Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denominator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{e^{x/2}}{\sqrt{-1+e^x}} dx &= 2\text{Subst} \left(\int \frac{1}{\sqrt{-1+x^2}} dx, x, e^{x/2} \right) \\ &= 2\text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \frac{e^{x/2}}{\sqrt{-1+e^x}} \right) \\ &= 2 \tanh^{-1} \left(\frac{e^{x/2}}{\sqrt{-1+e^x}} \right) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 20, normalized size = 1.00

$$2 \tanh^{-1} \left(\frac{e^{x/2}}{\sqrt{-1+e^x}} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[E^(x/2)/Sqrt[-1 + E^x], x]``[Out] 2*ArcTanh[E^(x/2)/Sqrt[-1 + E^x]]`**Mathics [A]**

time = 1.88, size = 8, normalized size = 0.40

$$2\text{ArcCosh} \left[E^{\frac{x}{2}} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[E^(x/2)/Sqrt[E^x - 1], x]')``[Out] 2 ArcCosh[E ^ (x / 2)]`**Maple [F]**

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{e^{\frac{x}{2}}}{\sqrt{-1+e^x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(exp(1/2*x)/(-1+exp(x))^(1/2), x)``[Out] int(exp(1/2*x)/(-1+exp(x))^(1/2), x)`**Maxima [A]**

time = 0.27, size = 18, normalized size = 0.90

$$2 \log \left(2 \sqrt{e^x - 1} + 2 e^{(\frac{1}{2}x)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1/2*x)/(-1+exp(x))^(1/2),x, algorithm="maxima")

[Out] 2*log(2*sqrt(e^x - 1) + 2*e^(1/2*x))

Fricas [A]

time = 0.32, size = 16, normalized size = 0.80

$$-2 \log \left(\sqrt{e^x - 1} - e^{(\frac{1}{2}x)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1/2*x)/(-1+exp(x))^(1/2),x, algorithm="fricas")

[Out] -2*log(sqrt(e^x - 1) - e^(1/2*x))

Sympy [A]

time = 0.36, size = 7, normalized size = 0.35

$$2 \operatorname{acosh} \left(e^{\frac{x}{2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1/2*x)/(-1+exp(x))**(1/2),x)

[Out] 2*acosh(exp(x/2))

Giac [A]

time = 0.00, size = 24, normalized size = 1.20

$$-2 \ln \left(-\sqrt{\left(e^{\frac{x}{2}}\right)^2 - 1} + e^{\frac{x}{2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1/2*x)/(-1+exp(x))^(1/2),x)

[Out] -2*log(e^(1/2*x) - sqrt(e^x - 1))

Mupad [B]

time = 0.34, size = 16, normalized size = 0.80

$$\ln \left(e^x + \sqrt{e^x} \sqrt{e^x - 1} - \frac{1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x/2)/(exp(x) - 1)^(1/2),x)

[Out] log(exp(x) + exp(x)^(1/2)*(exp(x) - 1)^(1/2) - 1/2)

$$3.68 \quad \int \frac{\tan^{-1}(x)^n}{1+x^2} dx$$

Optimal. Leaf size=12

$$\frac{\tan^{-1}(x)^{1+n}}{1+n}$$

[Out] arctan(x)^(1+n)/(1+n)

Rubi [A]

time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5004}

$$\frac{\tan^{-1}(x)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[x]^n/(1 + x^2), x]

[Out] ArcTan[x]^(1 + n)/(1 + n)

Rule 5004

Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]

Rubi steps

$$\int \frac{\tan^{-1}(x)^n}{1+x^2} dx = \frac{\tan^{-1}(x)^{1+n}}{1+n}$$

Mathematica [A]

time = 0.00, size = 12, normalized size = 1.00

$$\frac{\tan^{-1}(x)^{1+n}}{1+n}$$

Antiderivative was successfully verified.

[In] Integrate[ArcTan[x]^n/(1 + x^2), x]

[Out] ArcTan[x]^(1 + n)/(1 + n)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.51, size = 21, normalized size = 1.75

$$\text{Piecewise} \left[\left\{ \left\{ \frac{\text{ArcTan}[x]^{1+n}}{1+n}, n \neq -1 \right\} \right\}, \text{Log}[\text{ArcTan}[x]] \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[ArcTan[x]^n/(1 + x^2),x]')`

[Out] `Piecewise[{{ArcTan[x]^(1+n)/(1+n), n != -1}}, Log[ArcTan[x]]]`

Maple [A]

time = 0.14, size = 13, normalized size = 1.08

method	result	size
derivativdivides	$\frac{\arctan(x)^{1+n}}{1+n}$	13
default	$\frac{\arctan(x)^{1+n}}{1+n}$	13
risch	$\frac{i(\ln(-ix+1)-\ln(ix+1))\left(\frac{i(\ln(-ix+1)-\ln(ix+1))}{2}\right)^n}{2+2n}$	48

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arctan(x)^n/(x^2+1),x,method=_RETURNVERBOSE)`

[Out] `arctan(x)^(1+n)/(1+n)`

Maxima [A]

time = 0.26, size = 12, normalized size = 1.00

$$\frac{\arctan(x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(x)^n/(x^2+1),x, algorithm="maxima")`

[Out] `arctan(x)^(n+1)/(n+1)`

Fricas [A]

time = 0.35, size = 12, normalized size = 1.00

$$\frac{\arctan(x)^n \arctan(x)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(x)^n/(x^2+1),x, algorithm="fricas")

[Out] arctan(x)^n*arctan(x)/(n + 1)

Sympy [A]

time = 0.94, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{atan}^{n+1}(x)}{n+1} & \text{for } n \neq -1 \\ \log(\operatorname{atan}(x)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(x)**n/(x**2+1),x)

[Out] Piecewise((atan(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(atan(x)), True))

Giac [A]

time = 0.00, size = 11, normalized size = 0.92

$$\frac{(\arctan x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(x)^n/(x^2+1),x)

[Out] arctan(x)^(n + 1)/(n + 1)

Mupad [B]

time = 0.23, size = 12, normalized size = 1.00

$$\frac{\operatorname{atan}(x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atan(x)^n/(x^2 + 1),x)

[Out] atan(x)^(n + 1)/(n + 1)

$$3.69 \quad \int \frac{\sin^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2 - x^2}} dx$$

Optimal. Leaf size=42

$$\frac{2a\sqrt{1 - \frac{x^2}{a^2}} \sin^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2 - x^2}}$$

[Out] $2/5*a*\arcsin(x/a)^{(5/2)*(1-x^2/a^2)^{(1/2)}/(a^2-x^2)^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {4737}

$$\frac{2a\sqrt{1 - \frac{x^2}{a^2}} \sin^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2 - x^2}}$$

Antiderivative was successfully verified.

[In] `Int[ArcSin[x/a]^(3/2)/Sqrt[a^2 - x^2],x]`

[Out] $(2*a*\text{Sqrt}[1 - x^2/a^2]*\text{ArcSin}[x/a]^{(5/2)})/(5*\text{Sqrt}[a^2 - x^2])$

Rule 4737

`Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

Rubi steps

$$\begin{aligned} \int \frac{\sin^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2 - x^2}} dx &= \frac{\sqrt{1 - \frac{x^2}{a^2}} \int \frac{\sin^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{1 - \frac{x^2}{a^2}}} dx}{\sqrt{a^2 - x^2}} \\ &= \frac{2a\sqrt{1 - \frac{x^2}{a^2}} \sin^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2 - x^2}} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 42, normalized size = 1.00

$$\frac{2a\sqrt{1-\frac{x^2}{a^2}}\sin^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[x/a]^(3/2)/Sqrt[a^2 - x^2],x]

[Out] (2*a*Sqrt[1 - x^2/a^2]*ArcSin[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[ArcSin[x/a]^(3/2)/Sqrt[a^2 - x^2],x]')

[Out] Timed out

Maple [A]

time = 0.07, size = 38, normalized size = 0.90

method	result	size
default	$\frac{2\arcsin\left(\frac{x}{a}\right)^{\frac{5}{2}}a\sqrt{\frac{a^2-x^2}{a^2}}}{5\sqrt{a^2-x^2}}$	38

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/5*arcsin(x/a)^(5/2)*a/(a^2-x^2)^(1/2)*((a^2-x^2)/a^2)^(1/2)

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.

Fricas [A]

time = 0.32, size = 38, normalized size = 0.90

$$\frac{2}{5} \sqrt{-\arctan\left(-\frac{x}{\sqrt{a^2-x^2}}\right)} \arctan\left(-\frac{x}{\sqrt{a^2-x^2}}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(-arctan(-x/sqrt(a^2 - x^2)))*arctan(-x/sqrt(a^2 - x^2))^2
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\sqrt{-(-a+x)(a+x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x/a)**(3/2)/(a**2-x**2)**(1/2),x)
```

```
[Out] Integral(asin(x/a)**(3/2)/sqrt(-(-a + x)*(a + x)), x)
```

Giac [A]

time = 0.00, size = 24, normalized size = 0.57

$$\frac{2|a| \sqrt{\arcsin\left(\frac{x}{a}\right)} \arcsin^2\left(\frac{x}{a}\right)}{a \cdot 5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x)
```

```
[Out] 2/5*abs(a)*arcsin(x/a)^(5/2)/a
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asin}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x/a)^(3/2)/(a^2 - x^2)^(1/2),x)
```

```
[Out] int(asin(x/a)^(3/2)/(a^2 - x^2)^(1/2), x)
```

$$3.70 \quad \int \frac{1}{\sqrt{1-x^2} \cos^{-1}(x)^3} dx$$

Optimal. Leaf size=8

$$\frac{1}{2 \cos^{-1}(x)^2}$$

[Out] 1/2/arccos(x)^2

Rubi [A]

time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4738}

$$\frac{1}{2 \cos^{-1}(x)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 - x^2]*ArcCos[x]^3),x]

[Out] 1/(2*ArcCos[x]^2)

Rule 4738

Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^ (n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(- (b*c*(n + 1))^-1)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2} \cos^{-1}(x)^3} dx = \frac{1}{2 \cos^{-1}(x)^2}$$

Mathematica [A]

time = 0.00, size = 8, normalized size = 1.00

$$\frac{1}{2 \cos^{-1}(x)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[1 - x^2]*ArcCos[x]^3),x]

[Out] 1/(2*ArcCos[x]^2)

Mathics [A]

time = 2.00, size = 6, normalized size = 0.75

$$\frac{1}{2\text{ArcCos}[x]^2}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(ArcCos[x]^3*Sqrt[1 - x^2]),x]')`[Out] `1 / (2 ArcCos[x] ^ 2)`**Maple [A]**

time = 0.05, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\frac{1}{2\arccos(x)^2}$	7
default	$\frac{1}{2\arccos(x)^2}$	7

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/arccos(x)^3/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)`[Out] `1/2/arccos(x)^2`**Maxima [A]**

time = 0.35, size = 6, normalized size = 0.75

$$\frac{1}{2\arccos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/arccos(x)^3/(-x^2+1)^(1/2),x, algorithm="maxima")`[Out] `1/2/arccos(x)^2`**Fricas [A]**

time = 0.31, size = 6, normalized size = 0.75

$$\frac{1}{2\arccos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/arccos(x)^3/(-x^2+1)^(1/2),x, algorithm="fricas")`[Out] `1/2/arccos(x)^2`

Sympy [A]

time = 0.51, size = 7, normalized size = 0.88

$$\frac{1}{2 \operatorname{acos}^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/acos(x)**3/(-x**2+1)**(1/2),x)

[Out] 1/(2*acos(x)**2)

Giac [A]

time = 0.00, size = 8, normalized size = 1.00

$$\frac{1}{2 \operatorname{arccos}^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccos(x)^3/(-x^2+1)^(1/2),x)

[Out] 1/2/arccos(x)^2

Mupad [B]

time = 0.35, size = 6, normalized size = 0.75

$$\frac{1}{2 \operatorname{acos}(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(acos(x)^3*(1 - x^2)^(1/2)),x)

[Out] 1/(2*acos(x)^2)

3.71 $\int x \log^2(x) dx$

Optimal. Leaf size=28

$$\frac{x^2}{4} - \frac{1}{2}x^2 \log(x) + \frac{1}{2}x^2 \log^2(x)$$

[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2

Rubi [A]

time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2342, 2341}

$$\frac{x^2}{4} + \frac{1}{2}x^2 \log^2(x) - \frac{1}{2}x^2 \log(x)$$

Antiderivative was successfully verified.

[In] Int[x*Log[x]^2,x]

[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2

Rule 2341

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 2342

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int x \log^2(x) dx &= \frac{1}{2}x^2 \log^2(x) - \int x \log(x) dx \\ &= \frac{x^2}{4} - \frac{1}{2}x^2 \log(x) + \frac{1}{2}x^2 \log^2(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 28, normalized size = 1.00

$$\frac{x^2}{4} - \frac{1}{2}x^2 \log(x) + \frac{1}{2}x^2 \log^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[x*Log[x]^2,x]

[Out] $x^2/4 - (x^2 \cdot \text{Log}[x])/2 + (x^2 \cdot \text{Log}[x]^2)/2$

Mathics [A]

time = 1.73, size = 17, normalized size = 0.61

$$\frac{x^2 (1 - 2\text{Log}[x] + 2\text{Log}[x]^2)}{4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x*Log[x]^2,x]')

[Out] $x^2 (1 - 2 \text{Log}[x] + 2 \text{Log}[x]^2) / 4$

Maple [A]

time = 0.00, size = 23, normalized size = 0.82

method	result	size
default	$\frac{x^2}{4} - \frac{x^2 \ln(x)}{2} + \frac{x^2 \ln(x)^2}{2}$	23
norman	$\frac{x^2}{4} - \frac{x^2 \ln(x)}{2} + \frac{x^2 \ln(x)^2}{2}$	23
risch	$\frac{x^2}{4} - \frac{x^2 \ln(x)}{2} + \frac{x^2 \ln(x)^2}{2}$	23

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*ln(x)^2,x,method=_RETURNVERBOSE)

[Out] $1/4*x^2-1/2*x^2*\ln(x)+1/2*x^2*\ln(x)^2$

Maxima [A]

time = 0.28, size = 17, normalized size = 0.61

$$\frac{1}{4} (2 \log(x)^2 - 2 \log(x) + 1) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*log(x)^2,x, algorithm="maxima")

[Out] $1/4*(2*\log(x)^2 - 2*\log(x) + 1)*x^2$

Fricas [A]

time = 0.32, size = 22, normalized size = 0.79

$$\frac{1}{2} x^2 \log(x)^2 - \frac{1}{2} x^2 \log(x) + \frac{1}{4} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*log(x)^2,x, algorithm="fricas")

[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2

Sympy [A]

time = 0.05, size = 22, normalized size = 0.79

$$\frac{x^2 \log(x)^2}{2} - \frac{x^2 \log(x)}{2} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*ln(x)**2,x)

[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4

Giac [A]

time = 0.00, size = 26, normalized size = 0.93

$$\frac{1}{2}x^2 \ln^2 x + \frac{x^2}{4} - \frac{1}{2}x^2 \ln x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*log(x)^2,x)

[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2

Mupad [B]

time = 0.03, size = 17, normalized size = 0.61

$$\frac{x^2 (2 \ln(x)^2 - 2 \ln(x) + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*log(x)^2,x)

[Out] (x^2*(2*log(x)^2 - 2*log(x) + 1))/4

$$3.72 \quad \int \frac{\log(x)}{x^5} dx$$

Optimal. Leaf size=17

$$-\frac{1}{16x^4} - \frac{\log(x)}{4x^4}$$

[Out] $-1/16/x^4 - 1/4*\ln(x)/x^4$

Rubi [A]

time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2341}

$$-\frac{1}{16x^4} - \frac{\log(x)}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[Log[x]/x^5,x]

[Out] $-1/16*1/x^4 - \text{Log}[x]/(4*x^4)$

Rule 2341

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_)^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{\log(x)}{x^5} dx = -\frac{1}{16x^4} - \frac{\log(x)}{4x^4}$$

Mathematica [A]

time = 0.00, size = 17, normalized size = 1.00

$$-\frac{1}{16x^4} - \frac{\log(x)}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[Log[x]/x^5,x]

[Out] $-1/16*1/x^4 - \text{Log}[x]/(4*x^4)$

Mathics [A]

time = 1.73, size = 11, normalized size = 0.65

$$\frac{-1 - 4\text{Log}[x]}{16x^4}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Log[x]/x^5,x]')``[Out] (-1 - 4 Log[x]) / (16 x ^ 4)`**Maple** [A]

time = 0.01, size = 14, normalized size = 0.82

method	result	size
norman	$-\frac{\frac{1}{16} - \frac{\ln(x)}{4}}{x^4}$	11
default	$-\frac{1}{16x^4} - \frac{\ln(x)}{4x^4}$	14
risch	$-\frac{1}{16x^4} - \frac{\ln(x)}{4x^4}$	14

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(ln(x)/x^5,x,method=_RETURNVERBOSE)``[Out] -1/16/x^4-1/4*ln(x)/x^4`**Maxima** [A]

time = 0.27, size = 13, normalized size = 0.76

$$-\frac{\log(x)}{4x^4} - \frac{1}{16x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(log(x)/x^5,x, algorithm="maxima")``[Out] -1/4*log(x)/x^4 - 1/16/x^4`**Fricas** [A]

time = 0.33, size = 11, normalized size = 0.65

$$-\frac{4 \log(x) + 1}{16x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(log(x)/x^5,x, algorithm="fricas")``[Out] -1/16*(4*log(x) + 1)/x^4`

Sympy [A]

time = 0.05, size = 15, normalized size = 0.88

$$-\frac{\log(x)}{4x^4} - \frac{1}{16x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(x)/x**5,x)

[Out] -log(x)/(4*x**4) - 1/(16*x**4)

Giac [A]

time = 0.00, size = 18, normalized size = 1.06

$$-\frac{\ln x}{4x^4} - \frac{1}{16x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)/x^5,x)

[Out] -1/4*log(x)/x^4 - 1/16/x^4

Mupad [B]

time = 0.17, size = 9, normalized size = 0.53

$$-\frac{\ln(x) + \frac{1}{4}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(x)/x^5,x)

[Out] -(log(x) + 1/4)/(4*x^4)

3.73 $\int x^2 \log\left(\frac{-1+x}{x}\right) dx$

Optimal. Leaf size=36

$$-\frac{x}{3} - \frac{x^2}{6} - \frac{1}{3} \log(-1+x) + \frac{1}{3} x^3 \log\left(\frac{-1+x}{x}\right)$$

[Out] -1/3*x-1/6*x^2-1/3*ln(-1+x)+1/3*x^3*ln((-1+x)/x)

Rubi [A]

time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2511, 2505, 269, 45}

$$\frac{1}{3} x^3 \log\left(1 - \frac{1}{x}\right) - \frac{x^2}{6} - \frac{x}{3} - \frac{1}{3} \log(1-x)$$

Antiderivative was successfully verified.

[In] Int[x^2*Log[(-1 + x)/x], x]

[Out] -1/3*x - x^2/6 + (x^3*Log[1 - x^(-1)])/3 - Log[1 - x]/3

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 269

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

Rule 2505

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^(m_.), x_Symbol] := Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m + 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

Rule 2511

Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbol] := Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, f, m, p, q}, x] && BinomialQ[v, x] && !BinomialMatchQ[v, x]

Rubi steps

$$\begin{aligned}
\int x^2 \log\left(\frac{-1+x}{x}\right) dx &= \int x^2 \log\left(1 - \frac{1}{x}\right) dx \\
&= \frac{1}{3}x^3 \log\left(1 - \frac{1}{x}\right) - \frac{1}{3} \int \frac{x}{1 - \frac{1}{x}} dx \\
&= \frac{1}{3}x^3 \log\left(1 - \frac{1}{x}\right) - \frac{1}{3} \int \frac{x^2}{-1+x} dx \\
&= \frac{1}{3}x^3 \log\left(1 - \frac{1}{x}\right) - \frac{1}{3} \int \left(1 + \frac{1}{-1+x} + x\right) dx \\
&= -\frac{x}{3} - \frac{x^2}{6} + \frac{1}{3}x^3 \log\left(1 - \frac{1}{x}\right) - \frac{1}{3} \log(1-x)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 38, normalized size = 1.06

$$-\frac{x}{3} - \frac{x^2}{6} - \frac{1}{3} \log(1-x) + \frac{1}{3}x^3 \log\left(\frac{-1+x}{x}\right)$$

Antiderivative was successfully verified.

`[In] Integrate[x^2*Log[(-1 + x)/x], x]``[Out] -1/3*x - x^2/6 - Log[1 - x]/3 + (x^3*Log[(-1 + x)/x])/3`**Mathics [A]**

time = 1.88, size = 28, normalized size = 0.78

$$-\frac{x}{3} - \frac{x^2}{6} + \frac{x^3 \text{Log}\left[\frac{-1+x}{x}\right]}{3} - \frac{\text{Log}[-1+x]}{3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^2*Log[(x - 1)/x], x]')``[Out] -x / 3 - x ^ 2 / 6 + x ^ 3 Log[(-1 + x) / x] / 3 - Log[-1 + x] / 3`**Maple [A]**

time = 0.03, size = 53, normalized size = 1.47

method	result	size
risch	$-\frac{x}{3} - \frac{x^2}{6} - \frac{\ln(-1+x)}{3} + \frac{x^3 \ln\left(\frac{-1+x}{x}\right)}{3}$	29

derivativedivides	$-\frac{x^2}{6} + \frac{\ln(-\frac{1}{x})}{3} - \frac{x}{3} + \frac{\ln(1-\frac{1}{x})(1-\frac{1}{x})\left((1-\frac{1}{x})^2 + \frac{3}{x}\right)x^3}{3}$	53
default	$-\frac{x^2}{6} + \frac{\ln(-\frac{1}{x})}{3} - \frac{x}{3} + \frac{\ln(1-\frac{1}{x})(1-\frac{1}{x})\left((1-\frac{1}{x})^2 + \frac{3}{x}\right)x^3}{3}$	53

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*ln((-1+x)/x),x,method=_RETURNVERBOSE)`

[Out] $-1/6*x^2+1/3*\ln(-1/x)-1/3*x+1/3*\ln(1-1/x)*(1-1/x)*((1-1/x)^2+3/x)*x^3$

Maxima [A]

time = 0.27, size = 28, normalized size = 0.78

$$\frac{1}{3}x^3 \log\left(\frac{x-1}{x}\right) - \frac{1}{6}x^2 - \frac{1}{3}x - \frac{1}{3} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*log((-1+x)/x),x, algorithm="maxima")`

[Out] $1/3*x^3*\log((x-1)/x) - 1/6*x^2 - 1/3*x - 1/3*\log(x-1)$

Fricas [A]

time = 0.33, size = 28, normalized size = 0.78

$$\frac{1}{3}x^3 \log\left(\frac{x-1}{x}\right) - \frac{1}{6}x^2 - \frac{1}{3}x - \frac{1}{3} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*log((-1+x)/x),x, algorithm="fricas")`

[Out] $1/3*x^3*\log((x-1)/x) - 1/6*x^2 - 1/3*x - 1/3*\log(x-1)$

Sympy [A]

time = 0.06, size = 26, normalized size = 0.72

$$\frac{x^3 \log\left(\frac{x-1}{x}\right)}{3} - \frac{x^2}{6} - \frac{x}{3} - \frac{\log(x-1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*ln((-1+x)/x),x)`

[Out] $x**3*\log((x-1)/x)/3 - x**2/6 - x/3 - \log(x-1)/3$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(28) = 56.

time = 0.00, size = 88, normalized size = 2.44

$$-\frac{\ln\left(\frac{|x-1|}{|x|}\right)}{3} + \frac{\ln\left|\frac{x-1}{x} - 1\right|}{3} + \frac{\frac{2(x-1)}{x} - 3}{6\left(\frac{x-1}{x} - 1\right)^2} - \frac{\ln\left(\left(-1 + \frac{1}{-\frac{x-1}{x} + 1}\right)\left(-\frac{x-1}{x} + 1\right)\right)}{3\left(\frac{x-1}{x} - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*log((-1+x)/x),x)`

[Out] $\frac{1}{6} * (2 * (x - 1) / x - 3) / ((x - 1) / x - 1)^2 - \frac{1}{3} * \log(((x - 1) / x - 1) * (1 / ((x - 1) / x - 1) + 1)) / ((x - 1) / x - 1)^3 - \frac{1}{3} * \log(\text{abs}(x - 1) / \text{abs}(x)) + \frac{1}{3} * \log(\text{abs}((x - 1) / x - 1))$

Mupad [B]

time = 0.35, size = 40, normalized size = 1.11

$$\frac{x^3 \ln\left(\frac{x-1}{x}\right)}{3} - \frac{\ln(x(x-1))}{6} - \frac{\ln\left(\frac{x-1}{x}\right)}{6} - \frac{x}{3} - \frac{x^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*log((x - 1)/x),x)`

[Out] $(x^3 * \log((x - 1) / x)) / 3 - \log(x * (x - 1)) / 6 - \log((x - 1) / x) / 6 - x / 3 - x^2 / 6$

3.74 $\int \cos^5(x) dx$

Optimal. Leaf size=19

$$\sin(x) - \frac{2 \sin^3(x)}{3} + \frac{\sin^5(x)}{5}$$

[Out] $\sin(x) - 2/3 * \sin(x)^3 + 1/5 * \sin(x)^5$

Rubi [A]

time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2713}

$$\frac{\sin^5(x)}{5} - \frac{2 \sin^3(x)}{3} + \sin(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^5,x]

[Out] Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned} \int \cos^5(x) dx &= -\text{Subst} \left(\int (1 - 2x^2 + x^4) dx, x, -\sin(x) \right) \\ &= \sin(x) - \frac{2 \sin^3(x)}{3} + \frac{\sin^5(x)}{5} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 23, normalized size = 1.21

$$\frac{5 \sin(x)}{8} + \frac{5}{48} \sin(3x) + \frac{1}{80} \sin(5x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^5,x]

[Out] (5*Sin[x])/8 + (5*Sin[3*x])/48 + Sin[5*x]/80

Mathics [A]

time = 1.81, size = 15, normalized size = 0.79

$$\sin [x] - \frac{2\sin [x]^3}{3} + \frac{\sin [x]^5}{5}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Cos[x]^5,x]')``[Out] Sin[x] - 2 Sin[x]^3 / 3 + Sin[x]^5 / 5`**Maple [A]**

time = 0.01, size = 17, normalized size = 0.89

method	result	size
default	$\frac{\left(\frac{8}{3} + \cos^4(x) + \frac{4(\cos^2(x))}{3}\right) \sin(x)}{5}$	17
risch	$\frac{5 \sin(x)}{8} + \frac{\sin(5x)}{80} + \frac{5 \sin(3x)}{48}$	18

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x)^5,x,method=_RETURNVERBOSE)``[Out] 1/5*(8/3+cos(x)^4+4/3*cos(x)^2)*sin(x)`**Maxima [A]**

time = 0.26, size = 15, normalized size = 0.79

$$\frac{1}{5} \sin (x)^5 - \frac{2}{3} \sin (x)^3 + \sin (x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)^5,x, algorithm="maxima")``[Out] 1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)`**Fricas [A]**

time = 0.39, size = 18, normalized size = 0.95

$$\frac{1}{15} (3 \cos (x)^4 + 4 \cos (x)^2 + 8) \sin (x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)^5,x, algorithm="fricas")``[Out] 1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)`

Sympy [A]

time = 0.03, size = 17, normalized size = 0.89

$$\frac{\sin^5(x)}{5} - \frac{2\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**5,x)**[Out]** sin(x)**5/5 - 2*sin(x)**3/3 + sin(x)**Giac [A]**

time = 0.00, size = 19, normalized size = 1.00

$$\frac{\sin^5 x}{5} - \frac{2}{3} \sin^3 x + \sin x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^5,x)**[Out]** 1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)**Mupad [B]**

time = 0.00, size = 21, normalized size = 1.11

$$\frac{\sin(x) \cos(x)^4}{5} + \frac{4 \sin(x) \cos(x)^2}{15} + \frac{8 \sin(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^5,x)**[Out]** (8*sin(x))/15 + (4*cos(x)^2*sin(x))/15 + (cos(x)^4*sin(x))/5

3.75 $\int \cos^4(x) \sin^2(x) dx$

Optimal. Leaf size=34

$$\frac{x}{16} + \frac{1}{16} \cos(x) \sin(x) + \frac{1}{24} \cos^3(x) \sin(x) - \frac{1}{6} \cos^5(x) \sin(x)$$

[Out] 1/16*x+1/16*cos(x)*sin(x)+1/24*cos(x)^3*sin(x)-1/6*cos(x)^5*sin(x)

Rubi [A]

time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2648, 2715, 8}

$$\frac{x}{16} - \frac{1}{6} \sin(x) \cos^5(x) + \frac{1}{24} \sin(x) \cos^3(x) + \frac{1}{16} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^4*Sin[x]^2,x]

[Out] x/16 + (Cos[x]*Sin[x])/16 + (Cos[x]^3*Sin[x])/24 - (Cos[x]^5*Sin[x])/6

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2648

Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^n]*((a_.)*sin[(e_.) + (f_.)*(x_)])^m, x_Symbol] := Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m - 1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^n, x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
\int \cos^4(x) \sin^2(x) dx &= -\frac{1}{6} \cos^5(x) \sin(x) + \frac{1}{6} \int \cos^4(x) dx \\
&= \frac{1}{24} \cos^3(x) \sin(x) - \frac{1}{6} \cos^5(x) \sin(x) + \frac{1}{8} \int \cos^2(x) dx \\
&= \frac{1}{16} \cos(x) \sin(x) + \frac{1}{24} \cos^3(x) \sin(x) - \frac{1}{6} \cos^5(x) \sin(x) + \frac{\int 1 dx}{16} \\
&= \frac{x}{16} + \frac{1}{16} \cos(x) \sin(x) + \frac{1}{24} \cos^3(x) \sin(x) - \frac{1}{6} \cos^5(x) \sin(x)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 30, normalized size = 0.88

$$\frac{x}{16} + \frac{1}{64} \sin(2x) - \frac{1}{64} \sin(4x) - \frac{1}{192} \sin(6x)$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[x]^4*Sin[x]^2,x]``[Out] x/16 + Sin[2*x]/64 - Sin[4*x]/64 - Sin[6*x]/192`**Mathics [A]**

time = 1.92, size = 30, normalized size = 0.88

$$\frac{x}{16} - \frac{(1 + \cos[2x])^2 \sin[2x]}{48} + \frac{\sin[4x]}{192} + \frac{\sin[2x]}{24}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sin[x]^2*Cos[x]^4,x]')``[Out] x / 16 - (1 + Cos[2 x]) ^ 2 Sin[2 x] / 48 + Sin[4 x] / 192 + Sin[2 x] / 24`**Maple [A]**

time = 0.02, size = 26, normalized size = 0.76

method	result
risch	$\frac{x}{16} - \frac{\sin(6x)}{192} - \frac{\sin(4x)}{64} + \frac{\sin(2x)}{64}$
default	$-\frac{(\cos^5(x) \sin(x))}{6} + \frac{(\cos^3(x) + \frac{3 \cos(x)}{2}) \sin(x)}{24} + \frac{x}{16}$
norman	$\frac{x}{16} + \frac{47(\tan^3(\frac{x}{2}))}{24} - \frac{13(\tan^5(\frac{x}{2}))}{4} + \frac{13(\tan^7(\frac{x}{2}))}{4} - \frac{47(\tan^9(\frac{x}{2}))}{24} + \frac{(\tan^{11}(\frac{x}{2}))}{8} + \frac{3x(\tan^2(\frac{x}{2}))}{8} + \frac{15x(\tan^4(\frac{x}{2}))}{16} + \frac{5x(\tan^6(\frac{x}{2}))}{4} + \frac{15x(\tan^8(\frac{x}{2}))}{16} - \frac{1}{(1 + \tan^2(\frac{x}{2}))^6}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)^4*sin(x)^2,x,method=_RETURNVERBOSE)`

[Out] `-1/6*cos(x)^5*sin(x)+1/24*(cos(x)^3+3/2*cos(x))*sin(x)+1/16*x`

Maxima [A]

time = 0.26, size = 18, normalized size = 0.53

$$\frac{1}{48} \sin(2x)^3 + \frac{1}{16} x - \frac{1}{64} \sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^4*sin(x)^2,x, algorithm="maxima")`

[Out] `1/48*sin(2*x)^3 + 1/16*x - 1/64*sin(4*x)`

Fricas [A]

time = 0.32, size = 25, normalized size = 0.74

$$-\frac{1}{48} (8 \cos(x)^5 - 2 \cos(x)^3 - 3 \cos(x)) \sin(x) + \frac{1}{16} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^4*sin(x)^2,x, algorithm="fricas")`

[Out] `-1/48*(8*cos(x)^5 - 2*cos(x)^3 - 3*cos(x))*sin(x) + 1/16*x`

Sympy [A]

time = 0.03, size = 31, normalized size = 0.91

$$\frac{x}{16} - \frac{\sin(x) \cos^5(x)}{6} + \frac{\sin(x) \cos^3(x)}{24} + \frac{\sin(x) \cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)**4*sin(x)**2,x)`

[Out] `x/16 - sin(x)*cos(x)**5/6 + sin(x)*cos(x)**3/24 + sin(x)*cos(x)/16`

Giac [A]

time = 0.00, size = 28, normalized size = 0.82

$$\frac{x}{16} + \frac{\sin(2x)}{64} - \frac{\sin(4x)}{64} - \frac{\sin(6x)}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^4*sin(x)^2,x)

[Out] 1/16*x - 1/192*sin(6*x) - 1/64*sin(4*x) + 1/64*sin(2*x)

Mupad [B]

time = 0.02, size = 26, normalized size = 0.76

$$\left(\frac{\cos(x)^3}{6} + \frac{\cos(x)}{8} \right) \sin(x)^3 - \frac{\cos(x) \sin(x)}{16} + \frac{x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^4*sin(x)^2,x)

[Out] x/16 - (cos(x)*sin(x))/16 + sin(x)^3*(cos(x)/8 + cos(x)^3/6)

3.76 $\int \csc^5(x) dx$

Optimal. Leaf size=26

$$-\frac{3}{8} \tanh^{-1}(\cos(x)) - \frac{3}{8} \cot(x) \csc(x) - \frac{1}{4} \cot(x) \csc^3(x)$$

[Out] $-3/8*\operatorname{arctanh}(\cos(x))-3/8*\cot(x)*\csc(x)-1/4*\cot(x)*\csc(x)^3$

Rubi [A]

time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3853, 3855}

$$-\frac{3}{8} \tanh^{-1}(\cos(x)) - \frac{1}{4} \cot(x) \csc^3(x) - \frac{3}{8} \cot(x) \csc(x)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csc}[x]^5, x]$

[Out] $(-3*\operatorname{ArcTanh}[\operatorname{Cos}[x]])/8 - (3*\operatorname{Cot}[x]*\operatorname{Csc}[x])/8 - (\operatorname{Cot}[x]*\operatorname{Csc}[x]^3)/4$

Rule 3853

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_)]*(b_.)^n), x_Symbol] \rightarrow \operatorname{Simp}[(-b)*\operatorname{Cos}[c + d*x]*((b*\operatorname{Csc}[c + d*x])^{n-1}/(d*(n-1))), x] + \operatorname{Dist}[b^2*((n-2)/(n-1)), \operatorname{Int}[(b*\operatorname{Csc}[c + d*x])^{n-2}, x], x] /;$ $\operatorname{FreeQ}\{b, c, d, x\} \ \&\& \ \operatorname{GtQ}[n, 1] \ \& \ \operatorname{IntegerQ}[2*n]$

Rule 3855

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_)], x_Symbol] \rightarrow \operatorname{Simp}[-\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /;$ $\operatorname{FreeQ}\{c, d, x\}$

Rubi steps

$$\begin{aligned} \int \csc^5(x) dx &= -\frac{1}{4} \cot(x) \csc^3(x) + \frac{3}{4} \int \csc^3(x) dx \\ &= -\frac{3}{8} \cot(x) \csc(x) - \frac{1}{4} \cot(x) \csc^3(x) + \frac{3}{8} \int \csc(x) dx \\ &= -\frac{3}{8} \tanh^{-1}(\cos(x)) - \frac{3}{8} \cot(x) \csc(x) - \frac{1}{4} \cot(x) \csc^3(x) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 71 vs. $2(26) = 52$.

time = 0.01, size = 71, normalized size = 2.73

$$-\frac{3}{32} \csc^2\left(\frac{x}{2}\right) - \frac{1}{64} \csc^4\left(\frac{x}{2}\right) - \frac{3}{8} \log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{3}{8} \log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{3}{32} \sec^2\left(\frac{x}{2}\right) + \frac{1}{64} \sec^4\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^5,x]

[Out] (-3*Csc[x/2]^2)/32 - Csc[x/2]^4/64 - (3*Log[Cos[x/2]])/8 + (3*Log[Sin[x/2]])/8 + (3*Sec[x/2]^2)/32 + Sec[x/2]^4/64

Mathics [A]

time = 2.42, size = 31, normalized size = 1.19

$$\frac{-3\cos[x]}{8\sin[x]^2} - \frac{\cos[x]}{4\sin[x]^4} - \frac{3\log[1+\cos[x]]}{16} + \frac{3\log[-1+\cos[x]]}{16}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/Sin[x]^5,x]')

[Out] -3 Cos[x] / (8 Sin[x]^2) - Cos[x] / (4 Sin[x]^4) - 3 Log[1 + Cos[x]] / 16 + 3 Log[-1 + Cos[x]] / 16

Maple [A]

time = 0.08, size = 26, normalized size = 1.00

method	result	size
default	$\left(-\frac{\csc^3(x)}{4} - \frac{3\csc(x)}{8}\right) \cot(x) + \frac{3\ln(\csc(x)-\cot(x))}{8}$	26
norman	$\frac{-\frac{1}{64} - \frac{(\tan^2(\frac{x}{2}))}{8} + \frac{(\tan^6(\frac{x}{2}))}{8} + \frac{(\tan^8(\frac{x}{2}))}{64}}{\tan(\frac{x}{2})^4} + \frac{3\ln(\tan(\frac{x}{2}))}{8}$	42
risch	$\frac{3e^{7ix} - 11e^{5ix} - 11e^{3ix} + 3e^{ix}}{4(e^{2ix} - 1)^4} - \frac{3\ln(1+e^{ix})}{8} + \frac{3\ln(e^{ix}-1)}{8}$	62

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sin(x)^5,x,method=_RETURNVERBOSE)

[Out] (-1/4*csc(x)^3-3/8*csc(x))*cot(x)+3/8*ln(csc(x)-cot(x))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

time = 0.26, size = 42, normalized size = 1.62

$$\frac{3\cos(x)^3 - 5\cos(x)}{8(\cos(x)^4 - 2\cos(x)^2 + 1)} - \frac{3}{16} \log(\cos(x) + 1) + \frac{3}{16} \log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(x)^5,x, algorithm="maxima")

[Out] $\frac{1}{8} \cdot (3 \cos(x)^3 - 5 \cos(x)) / (\cos(x)^4 - 2 \cos(x)^2 + 1) - \frac{3}{16} \log(\cos(x) + 1) + \frac{3}{16} \log(\cos(x) - 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(20) = 40.

time = 0.35, size = 69, normalized size = 2.65

$$\frac{6 \cos(x)^3 - 3(\cos(x)^4 - 2 \cos(x)^2 + 1) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 3(\cos(x)^4 - 2 \cos(x)^2 + 1) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) - 10 \cos(x)}{16(\cos(x)^4 - 2 \cos(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(x)^5,x, algorithm="fricas")

[Out] $\frac{1}{16} \cdot (6 \cos(x)^3 - 3(\cos(x)^4 - 2 \cos(x)^2 + 1) \log(1/2 \cos(x) + 1/2) + 3(\cos(x)^4 - 2 \cos(x)^2 + 1) \log(-1/2 \cos(x) + 1/2) - 10 \cos(x)) / (\cos(x)^4 - 2 \cos(x)^2 + 1)$

Sympy [A]

time = 0.07, size = 46, normalized size = 1.77

$$\frac{3 \cos^3(x) - 5 \cos(x)}{8 \cos^4(x) - 16 \cos^2(x) + 8} + \frac{3 \log(\cos(x) - 1)}{16} - \frac{3 \log(\cos(x) + 1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(x)**5,x)

[Out] $(3 \cos(x)^3 - 5 \cos(x)) / (8 \cos(x)^4 - 16 \cos(x)^2 + 8) + 3 \log(\cos(x) - 1) / 16 - 3 \log(\cos(x) + 1) / 16$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(20) = 40.

time = 0.00, size = 100, normalized size = 3.85

$$2 \left(\frac{32 \left(\frac{1 - \cos x}{1 + \cos x} \right)^2 + \frac{256(1 - \cos x)}{1 + \cos x}}{4096} + \frac{-18 \left(\frac{1 - \cos x}{1 + \cos x} \right)^2 - \frac{8(1 - \cos x)}{1 + \cos x} - 1}{128 \left(\frac{1 - \cos x}{1 + \cos x} \right)^2} + \frac{3}{32} \ln \left(\frac{1 - \cos x}{1 + \cos x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(x)^5,x)

[Out] $\frac{1}{64} \cdot (8(\cos(x) - 1) / (\cos(x) + 1) - 18(\cos(x) - 1)^2 / (\cos(x) + 1)^2 - 1) \cdot (\cos(x) + 1)^2 / (\cos(x) - 1)^2 - 1/8 \cdot (\cos(x) - 1) / (\cos(x) + 1) + 1/64 \cdot (\cos(x) - 1)^2 / (\cos(x) + 1)^2 + 3/16 \cdot \log(-(\cos(x) - 1) / (\cos(x) + 1)))$

Mupad [B]

time = 0.29, size = 33, normalized size = 1.27

$$-\frac{3 \operatorname{atanh}(\cos(x))}{8} - \frac{\frac{5 \cos(x)}{8} - \frac{3 \cos(x)^3}{8}}{\cos(x)^4 - 2 \cos(x)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^5,x)
```

```
[Out] - (3*atanh(cos(x)))/8 - ((5*cos(x))/8 - (3*cos(x)^3)/8)/(cos(x)^4 - 2*cos(x)^2 + 1)
```

3.77 $\int e^{-x} \sin(x) dx$

Optimal. Leaf size=23

$$-\frac{1}{2}e^{-x} \cos(x) - \frac{1}{2}e^{-x} \sin(x)$$

[Out] $-1/2*\cos(x)/\exp(x)-1/2*\sin(x)/\exp(x)$

Rubi [A]

time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4517}

$$-\frac{1}{2}e^{-x} \sin(x) - \frac{1}{2}e^{-x} \cos(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sin}[x]/E^x, x]$

[Out] $-1/2*\text{Cos}[x]/E^x - \text{Sin}[x]/(2*E^x)$

Rule 4517

$\text{Int}[(F_)^{((c_)*(a_)+(b_)*(x_))}*\text{Sin}[(d_)+(e_)*(x_)], x_Symbol] :>$
 $\text{Simp}[b*c*\text{Log}[F]*F^{(c*(a+b*x))*(\text{Sin}[d+e*x]/(e^2+b^2*c^2*\text{Log}[F]^2))}, x$
 $] - \text{Simp}[e*F^{(c*(a+b*x))*(\text{Cos}[d+e*x]/(e^2+b^2*c^2*\text{Log}[F]^2))}, x] /;$ F
 $\text{reeQ}\{F, a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[e^2+b^2*c^2*\text{Log}[F]^2, 0]$

Rubi steps

$$\int e^{-x} \sin(x) dx = -\frac{1}{2}e^{-x} \cos(x) - \frac{1}{2}e^{-x} \sin(x)$$

Mathematica [A]

time = 0.01, size = 14, normalized size = 0.61

$$-\frac{1}{2}e^{-x}(\cos(x) + \sin(x))$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Sin}[x]/E^x, x]$

[Out] $-1/2*(\text{Cos}[x] + \text{Sin}[x])/E^x$

Mathics [A]

time = 1.89, size = 16, normalized size = 0.70

$$-\frac{\sqrt{2} E^{-x} \operatorname{Sin}\left[\frac{\text{Pi}}{4} + x\right]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sin[x]/E^x,x]')``[Out] -Sqrt[2] E ^ (-x) Sin[Pi / 4 + x] / 2`**Maple [A]**

time = 0.02, size = 18, normalized size = 0.78

method	result	size
default	$-\frac{e^{-x} \cos(x)}{2} - \frac{e^{-x} \sin(x)}{2}$	18
norman	$\frac{\left(-\frac{1}{2} + \frac{\tan^2\left(\frac{x}{2}\right)}{2}\right) - \tan\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)} e^{-x}$	32
risch	$-\frac{e^{(-1+i)x}}{4} + \frac{ie^{(-1+i)x}}{4} - \frac{e^{(-1-i)x}}{4} - \frac{ie^{(-1-i)x}}{4}$	36

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sin(x)/exp(x),x,method=_RETURNVERBOSE)``[Out] -1/2*exp(-x)*cos(x)-1/2*exp(-x)*sin(x)`**Maxima [A]**

time = 0.28, size = 11, normalized size = 0.48

$$-\frac{1}{2} (\cos(x) + \sin(x)) e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(x)/exp(x),x, algorithm="maxima")``[Out] -1/2*(cos(x) + sin(x))*e^(-x)`**Fricas [A]**

time = 0.32, size = 17, normalized size = 0.74

$$-\frac{1}{2} \cos(x) e^{(-x)} - \frac{1}{2} e^{(-x)} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sin(x)/exp(x),x, algorithm="fricas")`

[Out] $-1/2*\cos(x)*e^{-x} - 1/2*e^{-x}*\sin(x)$

Sympy [A]

time = 0.18, size = 17, normalized size = 0.74

$$-\frac{e^{-x} \sin(x)}{2} - \frac{e^{-x} \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/exp(x),x)`

[Out] $-\exp(-x)*\sin(x)/2 - \exp(-x)*\cos(x)/2$

Giac [A]

time = 0.00, size = 17, normalized size = 0.74

$$e^{-x} \left(-\frac{\cos x}{2} - \frac{\sin x}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/exp(x),x)`

[Out] $-1/2*(\cos(x) + \sin(x))/e^x$

Mupad [B]

time = 0.02, size = 11, normalized size = 0.48

$$-\frac{e^{-x} (\cos(x) + \sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-x)*sin(x),x)`

[Out] $-(\exp(-x)*(\cos(x) + \sin(x)))/2$

3.78 $\int e^{2x} \sin(3x) dx$

Optimal. Leaf size=27

$$-\frac{3}{13}e^{2x} \cos(3x) + \frac{2}{13}e^{2x} \sin(3x)$$

[Out] -3/13*exp(2*x)*cos(3*x)+2/13*exp(2*x)*sin(3*x)

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4517}

$$\frac{2}{13}e^{2x} \sin(3x) - \frac{3}{13}e^{2x} \cos(3x)$$

Antiderivative was successfully verified.

[In] Int[E^(2*x)*Sin[3*x],x]

[Out] (-3*E^(2*x)*Cos[3*x])/13 + (2*E^(2*x)*Sin[3*x])/13

Rule 4517

Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]

Rubi steps

$$\int e^{2x} \sin(3x) dx = -\frac{3}{13}e^{2x} \cos(3x) + \frac{2}{13}e^{2x} \sin(3x)$$

Mathematica [A]

time = 0.02, size = 22, normalized size = 0.81

$$\frac{1}{13}e^{2x}(-3 \cos(3x) + 2 \sin(3x))$$

Antiderivative was successfully verified.

[In] Integrate[E^(2*x)*Sin[3*x],x]

[Out] (E^(2*x)*(-3*Cos[3*x] + 2*Sin[3*x]))/13

Mathics [A]

time = 1.92, size = 20, normalized size = 0.74

$$\frac{(-3\cos[3x] + 2\sin[3x]) E^{2x}}{13}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[E^(2*x)*Sin[3*x],x]')
```

```
[Out] (-3 Cos[3 x] + 2 Sin[3 x]) E ^ (2 x) / 13
```

Maple [A]

time = 0.02, size = 22, normalized size = 0.81

method	result	size
default	$-\frac{3e^{2x}\cos(3x)}{13} + \frac{2e^{2x}\sin(3x)}{13}$	22
risch	$-\frac{3e^{(2+3i)x}}{26} - \frac{ie^{(2+3i)x}}{13} - \frac{3e^{(2-3i)x}}{26} + \frac{ie^{(2-3i)x}}{13}$	36
norman	$\frac{4e^{2x}\tan\left(\frac{3x}{2}\right)}{13} + \frac{3e^{2x}\left(\tan^2\left(\frac{3x}{2}\right)\right)}{13} - \frac{3e^{2x}}{13}$ $\frac{\quad}{1+\tan^2\left(\frac{3x}{2}\right)}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)*sin(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] -3/13*exp(2*x)*cos(3*x)+2/13*exp(2*x)*sin(3*x)
```

Maxima [A]

time = 0.27, size = 19, normalized size = 0.70

$$-\frac{1}{13} (3 \cos(3x) - 2 \sin(3x)) e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] -1/13*(3*cos(3*x) - 2*sin(3*x))*e^(2*x)
```

Fricas [A]

time = 0.31, size = 21, normalized size = 0.78

$$-\frac{3}{13} \cos(3x) e^{(2x)} + \frac{2}{13} e^{(2x)} \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*sin(3*x),x, algorithm="fricas")
```

[Out] $-3/13*\cos(3*x)*e^{(2*x)} + 2/13*e^{(2*x)}*\sin(3*x)$

Sympy [A]

time = 0.11, size = 26, normalized size = 0.96

$$\frac{2e^{2x} \sin(3x)}{13} - \frac{3e^{2x} \cos(3x)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(2*x)*sin(3*x),x)`

[Out] $2*\exp(2*x)*\sin(3*x)/13 - 3*\exp(2*x)*\cos(3*x)/13$

Giac [A]

time = 0.00, size = 22, normalized size = 0.81

$$e^{2x} \left(-\frac{3}{13} \cos(3x) + \frac{2}{13} \sin(3x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(2*x)*sin(3*x),x)`

[Out] $-1/13*e^{(2*x)}*(3*\cos(3*x) - 2*\sin(3*x))$

Mupad [B]

time = 0.04, size = 19, normalized size = 0.70

$$-\frac{e^{2x} (3 \cos(3x) - 2 \sin(3x))}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(3*x)*exp(2*x),x)`

[Out] $-(\exp(2*x)*(3*\cos(3*x) - 2*\sin(3*x)))/13$

3.79 $\int a^x \cos(x) dx$

Optimal. Leaf size=31

$$\frac{a^x \cos(x) \log(a)}{1 + \log^2(a)} + \frac{a^x \sin(x)}{1 + \log^2(a)}$$

[Out] $a^x \cos(x) \ln(a) / (1 + \ln(a)^2) + a^x \sin(x) / (1 + \ln(a)^2)$

Rubi [A]

time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4518}

$$\frac{a^x \sin(x)}{\log^2(a) + 1} + \frac{a^x \log(a) \cos(x)}{\log^2(a) + 1}$$

Antiderivative was successfully verified.

[In] Int[a^x*Cos[x],x]

[Out] (a^x*Cos[x]*Log[a])/(1 + Log[a]^2) + (a^x*Sin[x])/(1 + Log[a]^2)

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_.)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :=
  Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int a^x \cos(x) dx = \frac{a^x \cos(x) \log(a)}{1 + \log^2(a)} + \frac{a^x \sin(x)}{1 + \log^2(a)}$$

Mathematica [A]

time = 0.01, size = 20, normalized size = 0.65

$$\frac{a^x (\cos(x) \log(a) + \sin(x))}{1 + \log^2(a)}$$

Antiderivative was successfully verified.

[In] Integrate[a^x*Cos[x],x]

[Out] (a^x*(Cos[x]*Log[a] + Sin[x]))/(1 + Log[a]^2)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.47, size = 83, normalized size = 2.68

$$\text{Piecewise}\left[\left\{\left\{\frac{(I x \sin [x]+x \cos [x]+\sin [x]) E^{-I x}}{2}, a==E^{-I}\right\},\left\{\frac{(-I x \sin [x]+x \cos [x]+\sin [x]) E^{I x}}{2}, a==E^I\right\}\right\}, \frac{\cos [x] \log [a] a^x}{1+\log [a]^2}+\frac{a^x \sin [x]}{1+\log [a]^2}\right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[a^x*cos[x],x]')`

[Out] `Piecewise[{{(I x Sin[x] + x Cos[x] + Sin[x]) E ^ (-I x) / 2, a == E ^ (-I)}, {(-I x Sin[x] + x Cos[x] + Sin[x]) E ^ (I x) / 2, a == E ^ I}}, Cos[x] Log[a] a ^ x / (1 + Log[a] ^ 2) + a ^ x Sin[x] / (1 + Log[a] ^ 2)]`

Maple [A]

time = 0.03, size = 32, normalized size = 1.03

method	result	size
risch	$\frac{a^x \cos(x) \ln(a)}{1+\ln(a)^2} + \frac{a^x \sin(x)}{1+\ln(a)^2}$	32
norman	$\frac{\frac{\ln(a)e^x \ln(a)}{1+\ln(a)^2} + \frac{2 e^x \ln(a) \tan\left(\frac{x}{2}\right)}{1+\ln(a)^2} - \frac{\ln(a)e^x \ln(a) \left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\ln(a)^2}}{1+\tan^2\left(\frac{x}{2}\right)}$	71

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a^x*cos(x),x,method=_RETURNVERBOSE)`

[Out] `a^x*cos(x)*ln(a)/(1+ln(a)^2)+a^x*sin(x)/(1+ln(a)^2)`

Maxima [A]

time = 0.26, size = 24, normalized size = 0.77

$$\frac{a^x \cos(x) \log(a) + a^x \sin(x)}{\log(a)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a^x*cos(x),x, algorithm="maxima")`

[Out] `(a^x*cos(x)*log(a) + a^x*sin(x))/(log(a)^2 + 1)`

Fricas [A]

time = 0.37, size = 20, normalized size = 0.65

$$\frac{(\cos(x) \log(a) + \sin(x)) a^x}{\log(a)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x*cos(x),x, algorithm="fricas")

[Out] (cos(x)*log(a) + sin(x))*a^x/(log(a)^2 + 1)

Sympy [A]

time = 0.29, size = 107, normalized size = 3.45

$$\begin{cases} \frac{ixe^{-ix} \sin(x)}{2} + \frac{xe^{-ix} \cos(x)}{2} + \frac{ie^{-ix} \cos(x)}{2} & \text{for } a = e^{-i} \\ -\frac{ixe^{ix} \sin(x)}{2} + \frac{xe^{ix} \cos(x)}{2} - \frac{ie^{ix} \cos(x)}{2} & \text{for } a = e^i \\ \frac{a^x \log(a) \cos(x)}{\log(a)^2 + 1} + \frac{a^x \sin(x)}{\log(a)^2 + 1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a**x*cos(x),x)

[Out] Piecewise((I*x*exp(-I*x)*sin(x)/2 + x*exp(-I*x)*cos(x)/2 + I*exp(-I*x)*cos(x)/2, Eq(a, exp(-I))), (-I*x*exp(I*x)*sin(x)/2 + x*exp(I*x)*cos(x)/2 - I*exp(I*x)*cos(x)/2, Eq(a, exp(I))), (a**x*log(a)*cos(x)/(log(a)**2 + 1) + a**x*sin(x)/(log(a)**2 + 1), True))

Giac [C] Result contains complex when optimal does not.

time = 0.00, size = 383, normalized size = 12.35

$$e^{i\pi/4} \left(\frac{4 - 2\ln|a| \cdot \cos\left(\frac{\operatorname{atan2}(-x+2i)}{2}\right) - 2(-2\operatorname{sign}(a) + 2x - 4)\sin\left(\frac{\operatorname{atan2}(-x+2i)}{2}\right)}{(4\ln|a|)^2 + (2\operatorname{sign}(a) - 2x + 4)^2} \right) + e^{i\pi/4} \left(\frac{4 - 2\ln|a| \cdot \cos\left(\frac{\operatorname{atan2}(-x-2i)}{2}\right) - 2(-2\operatorname{sign}(a) + 2x + 4)\sin\left(\frac{\operatorname{atan2}(-x-2i)}{2}\right)}{(4\ln|a|)^2 + (2\operatorname{sign}(a) - 2x - 4)^2} \right) + \frac{e^{i\pi/4} \left(-\frac{2e^{-\frac{1}{2}i\operatorname{atan2}(-x+2i)}}{4\ln|a| + 2\operatorname{sign}(a) - 2x - 4} + \frac{2e^{-\frac{1}{2}i\operatorname{atan2}(-x+2i)}}{4\ln|a| - 2\operatorname{sign}(a) + 2x + 4} \right)}{2} + \frac{e^{i\pi/4} \left(-\frac{2e^{-\frac{1}{2}i\operatorname{atan2}(-x-2i)}}{4\ln|a| + 2\operatorname{sign}(a) - 2x - 4} + \frac{2e^{-\frac{1}{2}i\operatorname{atan2}(-x-2i)}}{4\ln|a| - 2\operatorname{sign}(a) + 2x + 4} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^x*cos(x),x)

[Out] abs(a)^x*(2*cos(1/2*pi*x*sgn(a) - 1/2*pi*x + x)*log(abs(a))/((pi - pi*sgn(a) - 2)^2 + 4*log(abs(a))^2) - (pi - pi*sgn(a) - 2)*sin(1/2*pi*x*sgn(a) - 1/2*pi*x + x)/((pi - pi*sgn(a) - 2)^2 + 4*log(abs(a))^2)) + abs(a)^x*(2*cos(1/2*pi*x*sgn(a) - 1/2*pi*x - x)*log(abs(a))/((pi - pi*sgn(a) + 2)^2 + 4*log(abs(a))^2) - (pi - pi*sgn(a) + 2)*sin(1/2*pi*x*sgn(a) - 1/2*pi*x - x)/((pi - pi*sgn(a) + 2)^2 + 4*log(abs(a))^2)) + I*abs(a)^x*(I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sgn(a) + 4*log(abs(a)) + 4*I) - I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*pi*x - I*x)/(2*I*pi - 2*I*pi*sgn(a) + 4*log(abs(a)) - 4*I)) + I*abs(a)^x*(I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi*x - I*x)/(-2*I*pi + 2*I*pi*sgn(a) + 4*log(abs(a)) - 4*I) - I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*pi*x + I*x)/(2*I*pi - 2*I*pi*sgn(a) + 4*log(abs(a)) + 4*I))

Mupad [B]

time = 0.03, size = 20, normalized size = 0.65

$$\frac{a^x (\sin(x) + \ln(a) \cos(x))}{\ln(a)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a^x*cos(x),x)

[Out] (a^x*(sin(x) + log(a)*cos(x)))/(log(a)^2 + 1)

3.80 $\int \cos(\log(x)) dx$

Optimal. Leaf size=17

$$\frac{1}{2}x \cos(\log(x)) + \frac{1}{2}x \sin(\log(x))$$

[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))

Rubi [A]

time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4564}

$$\frac{1}{2}x \sin(\log(x)) + \frac{1}{2}x \cos(\log(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[Log[x]],x]

[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2

Rule 4564

Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] + Simp[b*d*n*x*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b^2*d^2*n^2 + 1, 0]

Rubi steps

$$\int \cos(\log(x)) dx = \frac{1}{2}x \cos(\log(x)) + \frac{1}{2}x \sin(\log(x))$$

Mathematica [A]

time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2}x \cos(\log(x)) + \frac{1}{2}x \sin(\log(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[Log[x]],x]

[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2

Mathics [A]

time = 1.86, size = 13, normalized size = 0.76

$$\frac{\sqrt{2} x \sin \left[\frac{\text{Pi}}{4} + \text{Log}[x] \right]}{2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Cos[Log[x]],x]')
```

```
[Out] Sqrt[2] x Sin[Pi / 4 + Log[x]] / 2
```

Maple [A]

time = 0.02, size = 14, normalized size = 0.82

method	result	size
lookup	$\frac{x \cos(\ln(x))}{2} + \frac{x \sin(\ln(x))}{2}$	14
default	$\frac{x \cos(\ln(x))}{2} + \frac{x \sin(\ln(x))}{2}$	14
risch	$\left(\frac{1}{4} - \frac{i}{4}\right) x x^i + \left(\frac{1}{4} + \frac{i}{4}\right) x x^{-i}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Maxima [A]

time = 0.27, size = 10, normalized size = 0.59

$$\frac{1}{2} x (\cos(\log(x)) + \sin(\log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x*(cos(log(x)) + sin(log(x)))
```

Fricas [A]

time = 0.32, size = 13, normalized size = 0.76

$$\frac{1}{2} x \cos(\log(x)) + \frac{1}{2} x \sin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A]

time = 0.13, size = 15, normalized size = 0.88

$$\frac{x \sin(\log(x))}{2} + \frac{x \cos(\log(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(ln(x)),x)``[Out] x*sin(log(x))/2 + x*cos(log(x))/2`**Giac [A]**

time = 0.00, size = 15, normalized size = 0.88

$$\frac{1}{2}x \sin(\ln x) + \frac{1}{2}x \cos(\ln x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(log(x)),x)``[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))`**Mupad [B]**

time = 0.20, size = 13, normalized size = 0.76

$$\frac{\sqrt{2} x \sin\left(\frac{\pi}{4} + \ln(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(log(x)),x)``[Out] (2^(1/2)*x*sin(pi/4 + log(x)))/2`

3.81 $\int \log(\cos(x)) \sec^2(x) dx$

Optimal. Leaf size=12

$$-x + \tan(x) + \log(\cos(x)) \tan(x)$$

[Out] $-x + \tan(x) + \ln(\cos(x)) * \tan(x)$

Rubi [A]

time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3852, 8, 2634, 3554}

$$-x + \tan(x) + \tan(x) \log(\cos(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Log}[\text{Cos}[x]] * \text{Sec}[x]^2, x]$

[Out] $-x + \text{Tan}[x] + \text{Log}[\text{Cos}[x]] * \text{Tan}[x]$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2634

$\text{Int}[\text{Log}[u_]*(v_), x_Symbol] \rightarrow \text{With}[\{w = \text{IntHide}[v, x]\}, \text{Dist}[\text{Log}[u], w, x] - \text{Int}[\text{SimplifyIntegrand}[w*(D[u, x]/u), x], x] /; \text{InverseFunctionFreeQ}[w, x]] /; \text{InverseFunctionFreeQ}[u, x]$

Rule 3554

$\text{Int}[\{(b_.)*\tan[(c_.) + (d_.)*(x_.)]\}^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[b*((b*\text{Tan}[c + d*x])^{(n - 1)/(d*(n - 1))}), x] - \text{Dist}[b^2, \text{Int}[(b*\text{Tan}[c + d*x])^{(n - 2)}, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \&\& \text{GtQ}[n, 1]$

Rule 3852

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[-d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /; \text{FreeQ}[\{c, d\}, x] \&\& \text{IGtQ}[n/2, 0]$

Rubi steps

$$\begin{aligned}
\int \log(\cos(x)) \sec^2(x) dx &= \log(\cos(x)) \tan(x) + \int \tan^2(x) dx \\
&= \tan(x) + \log(\cos(x)) \tan(x) - \int 1 dx \\
&= -x + \tan(x) + \log(\cos(x)) \tan(x)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 12, normalized size = 1.00

$$-x + \tan(x) + \log(\cos(x)) \tan(x)$$

Antiderivative was successfully verified.

`[In] Integrate[Log[Cos[x]]*Sec[x]^2,x]``[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]`**Mathics [B]** Leaf count is larger than twice the leaf count of optimal. 35 vs. $2(12) = 24$.
time = 4.45, size = 31, normalized size = 2.58

$$-\frac{x}{2} + \frac{x}{2\cos[x]} + \frac{x}{-1 + \tan\left[\frac{x}{2}\right]^2} + \tan[x] + \log[\cos[x]] \tan[x]$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sec[x]^2*Log[Cos[x]],x]')``[Out] -x / 2 + x / (2 Cos[x]) + x / (-1 + Tan[x / 2] ^ 2) + Tan[x] + Log[Cos[x]] Tan[x]`**Maple [C]** Result contains complex when optimal does not.
time = 0.12, size = 67, normalized size = 5.58

method	result
norman	$\frac{x - x(\tan^2(\frac{x}{2})) - 2 \tan(\frac{x}{2}) \ln\left(\frac{1 - (\tan^2(\frac{x}{2}))}{1 + \tan^2(\frac{x}{2})}\right) - 2 \tan(\frac{x}{2})}{\tan^2(\frac{x}{2}) - 1}$
default	$-4i \left(\frac{e^{2ix} \ln((e^{2ix} + 1)e^{-ix})}{e^{2ix} + 1} - \frac{1}{2} - \frac{\ln(e^{2ix} + 1)}{4} + \frac{\ln(2)}{2e^{2ix} + 2} \right)$
risch	$-\frac{2i \ln(e^{ix})}{e^{2ix} + 1} + \frac{\pi \operatorname{csgn}(ie^{-ix}) \operatorname{csgn}(i(e^{2ix} + 1)) \operatorname{csgn}(i \cos(x)) - \pi \operatorname{csgn}(ie^{-ix}) \operatorname{csgn}(i \cos(x))^2 - \pi \operatorname{csgn}(i(e^{2ix} + 1)) \operatorname{csgn}(i \cos(x))^2 + \pi \operatorname{csgn}(i \cos(x))}{e^{2ix} + 1}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(cos(x))*sec(x)^2,x,method=_RETURNVERBOSE)`

[Out] $-4 \cdot I \cdot \left(\frac{1}{2} \exp(2 \cdot I \cdot x) \cdot \ln \left(\frac{\exp(I \cdot x)^2 + 1}{\exp(I \cdot x)} \right) - \frac{1}{2} \right) / \left(\exp(2 \cdot I \cdot x) + 1 \right) - \frac{1}{4} \cdot \ln \left(\exp(2 \cdot I \cdot x) + 1 \right) + \frac{1}{2} \cdot \ln(2) / \left(\exp(I \cdot x)^2 + 1 \right)$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(12) = 24$.

time = 0.35, size = 94, normalized size = 7.83

$$-\frac{2 \log \left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2} - 1}{\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1} \right) \sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} - 1 \right) (\cos(x) + 1)} - \frac{2 \sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} - 1 \right) (\cos(x) + 1)} - 2 \arctan \left(\frac{\sin(x)}{\cos(x) + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(cos(x))*sec(x)^2,x, algorithm="maxima")`

[Out] $-2 \cdot \log \left(-\frac{\sin(x)^2}{(\cos(x) + 1)^2} - 1 \right) / \left(\frac{\sin(x)^2}{(\cos(x) + 1)^2} + 1 \right) \cdot \sin(x) / \left(\left(\frac{\sin(x)^2}{(\cos(x) + 1)^2} - 1 \right) \cdot (\cos(x) + 1) \right) - 2 \cdot \sin(x) / \left(\left(\frac{\sin(x)^2}{(\cos(x) + 1)^2} - 1 \right) \cdot (\cos(x) + 1) \right) - 2 \cdot \arctan(\sin(x) / (\cos(x) + 1))$

Fricas [A]

time = 0.32, size = 22, normalized size = 1.83

$$\frac{x \cos(x) - \log(\cos(x)) \sin(x) - \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(cos(x))*sec(x)^2,x, algorithm="fricas")`

[Out] $-(x \cdot \cos(x) - \log(\cos(x)) \cdot \sin(x) - \sin(x)) / \cos(x)$

Sympy [A]

time = 20.09, size = 15, normalized size = 1.25

$$-x + \log(\cos(x)) \tan(x) + \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(cos(x))*sec(x)**2,x)`

[Out] $-x + \log(\cos(x)) \cdot \tan(x) + \sin(x) / \cos(x)$

Giac [A]

time = 0.01, size = 14, normalized size = 1.17

$$\tan x - x + \frac{2}{2} \tan x \cdot \ln(\cos x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(cos(x))*sec(x)^2,x)`

[Out] `log(cos(x))*tan(x) - x + tan(x)`

Mupad [B]

time = 0.43, size = 35, normalized size = 2.92

$$\tan(x) - 2x + \ln(\cos(x)) \tan(x) + \ln(\cos(x)) \operatorname{li} - \ln(\cos(2x) + 1 + \sin(2x)) \operatorname{li} \operatorname{li}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(cos(x))/cos(x)^2,x)`

[Out] `log(cos(x))*1i - 2*x - log(cos(2*x) + sin(2*x)*1i + 1)*1i + tan(x) + log(cos(x))*tan(x)`

3.82 $\int x \tan^2(x) dx$

Optimal. Leaf size=15

$$-\frac{x^2}{2} + \log(\cos(x)) + x \tan(x)$$

[Out] $-1/2*x^2+\ln(\cos(x))+x*\tan(x)$

Rubi [A]

time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3801, 3556, 30}

$$-\frac{x^2}{2} + x \tan(x) + \log(\cos(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Tan}[x]^2,x]$

[Out] $-1/2*x^2 + \text{Log}[\text{Cos}[x]] + x*\text{Tan}[x]$

Rule 30

$\text{Int}[(x_)^{(m_.)}, x_Symbol] \text{ :> } \text{Simp}[x^{(m + 1)}/(m + 1), x] \text{ /; } \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

Rule 3556

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x_Symbol] \text{ :> } \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] \text{ /; } \text{FreeQ}[\{c, d\}, x]$

Rule 3801

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*((b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] \text{ :> } \text{Simp}[b*(c + d*x)^m*((b*\text{Tan}[e + f*x])^{(n - 1)}/(f*(n - 1))), x] + (-\text{Dist}[b*d*(m/(f*(n - 1))), \text{Int}[(c + d*x)^{(m - 1)}*(b*\text{Tan}[e + f*x])^{(n - 1)}, x], x] - \text{Dist}[b^2, \text{Int}[(c + d*x)^m*(b*\text{Tan}[e + f*x])^{(n - 2)}, x], x]) \text{ /; } \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{GtQ}[m, 0]$

Rubi steps

$$\begin{aligned} \int x \tan^2(x) dx &= x \tan(x) - \int x dx - \int \tan(x) dx \\ &= -\frac{x^2}{2} + \log(\cos(x)) + x \tan(x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 15, normalized size = 1.00

$$-\frac{x^2}{2} + \log(\cos(x)) + x \tan(x)$$

Antiderivative was successfully verified.

`[In] Integrate[x*Tan[x]^2,x]``[Out] -1/2*x^2 + Log[Cos[x]] + x*Tan[x]`**Mathics [A]**

time = 1.90, size = 17, normalized size = 1.13

$$x \text{Tan}[x] - \frac{x^2}{2} - \frac{\text{Log}\left[\frac{1}{\text{Cos}[x]^2}\right]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x*Tan[x]^2,x]')``[Out] x Tan[x] - x ^ 2 / 2 - Log[1 / Cos[x] ^ 2] / 2`**Maple [A]**

time = 0.01, size = 20, normalized size = 1.33

method	result	size
norman	$x \tan(x) - \frac{x^2}{2} - \frac{\ln(1+\tan^2(x))}{2}$	20
risch	$-\frac{x^2}{2} - 2ix + \frac{2ix}{e^{2ix}+1} + \ln(e^{2ix} + 1)$	32

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*tan(x)^2,x,method=_RETURNVERBOSE)``[Out] x*tan(x)-1/2*x^2-1/2*ln(1+tan(x)^2)`**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(13) = 26$.

time = 0.36, size = 107, normalized size = 7.13

$$\frac{-x^2 \cos(2x)^2 + x^2 \sin(2x)^2 + 2x^2 \cos(2x) + x^2 - (\cos(2x)^2 + \sin(2x)^2 + 2 \cos(2x) + 1) \log(\cos(2x)^2 + \sin(2x)^2 + 2 \cos(2x) + 1) - 4x \sin(2x)}{2(\cos(2x)^2 + \sin(2x)^2 + 2 \cos(2x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*tan(x)^2,x, algorithm="maxima")`

[Out] $-1/2*(x^2*\cos(2*x)^2 + x^2*\sin(2*x)^2 + 2*x^2*\cos(2*x) + x^2 - (\cos(2*x)^2 + \sin(2*x)^2 + 2*\cos(2*x) + 1)*\log(\cos(2*x)^2 + \sin(2*x)^2 + 2*\cos(2*x) + 1) - 4*x*\sin(2*x))/(\cos(2*x)^2 + \sin(2*x)^2 + 2*\cos(2*x) + 1)$

Fricas [A]

time = 0.32, size = 21, normalized size = 1.40

$$-\frac{1}{2}x^2 + x \tan(x) + \frac{1}{2} \log\left(\frac{1}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(x)^2,x, algorithm="fricas")`

[Out] $-1/2*x^2 + x*\tan(x) + 1/2*\log(1/(\tan(x)^2 + 1))$

Sympy [A]

time = 0.07, size = 19, normalized size = 1.27

$$-\frac{x^2}{2} + x \tan(x) - \frac{\log(\tan^2(x) + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(x)**2,x)`

[Out] $-x**2/2 + x*\tan(x) - \log(\tan(x)**2 + 1)/2$

Giac [A]

time = 0.01, size = 27, normalized size = 1.80

$$-\frac{1}{2}x^2 + x \tan x + \frac{1}{2} \ln\left(\frac{4}{\tan^2 x + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(x)^2,x)`

[Out] $-1/2*x^2 + x*\tan(x) + 1/2*\log(4/(\tan(x)^2 + 1))$

Mupad [B]

time = 0.02, size = 13, normalized size = 0.87

$$\ln(\cos(x)) + x \tan(x) - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*tan(x)^2,x)`

[Out] $\log(\cos(x)) + x*\tan(x) - x^2/2$

3.83 $\int \frac{\sin^{-1}(x)}{x^2} dx$

Optimal. Leaf size=22

$$-\frac{\sin^{-1}(x)}{x} - \tanh^{-1}\left(\sqrt{1-x^2}\right)$$

[Out] -arcsin(x)/x-arctanh((-x^2+1)^(1/2))

Rubi [A]

time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {4723, 272, 65, 212}

$$-\tanh^{-1}\left(\sqrt{1-x^2}\right) - \frac{\sin^{-1}(x)}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[x]/x^2,x]

[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4723

Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*

$x^2]), x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \frac{\sin^{-1}(x)}{x^2} dx &= -\frac{\sin^{-1}(x)}{x} + \int \frac{1}{x\sqrt{1-x^2}} dx \\ &= -\frac{\sin^{-1}(x)}{x} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1-x}x} dx, x, x^2 \right) \\ &= -\frac{\sin^{-1}(x)}{x} - \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \sqrt{1-x^2} \right) \\ &= -\frac{\sin^{-1}(x)}{x} - \tanh^{-1} \left(\sqrt{1-x^2} \right) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 22, normalized size = 1.00

$$-\frac{\sin^{-1}(x)}{x} - \tanh^{-1} \left(\sqrt{1-x^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[x]/x^2,x]

[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[ArcSin[x]/x^2,x]')

[Out] cought exception:

Maple [A]

time = 0.00, size = 21, normalized size = 0.95

method	result	size
default	$-\frac{\arcsin(x)}{x} - \operatorname{arctanh} \left(\frac{1}{\sqrt{-x^2 + 1}} \right)$	21

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsin(x)/x^2,x,method=_RETURNVERBOSE)`

[Out] `-arcsin(x)/x-arctanh(1/(-x^2+1)^(1/2))`

Maxima [A]

time = 0.35, size = 33, normalized size = 1.50

$$-\frac{\arcsin(x)}{x} - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(x)/x^2,x, algorithm="maxima")`

[Out] `-arcsin(x)/x - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))`

Fricas [A]

time = 0.35, size = 39, normalized size = 1.77

$$\frac{x \log\left(\sqrt{-x^2+1} + 1\right) - x \log\left(\sqrt{-x^2+1} - 1\right) + 2 \arcsin(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(x)/x^2,x, algorithm="fricas")`

[Out] `-1/2*(x*log(sqrt(-x^2 + 1) + 1) - x*log(sqrt(-x^2 + 1) - 1) + 2*arcsin(x))/x`

Sympy [A]

time = 0.96, size = 22, normalized size = 1.00

$$\begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|} > 1 \\ i \operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases} - \frac{\operatorname{asin}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asin(x)/x**2,x)`

[Out] `Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True)) - asin(x)/x`

Giac [A]

time = 0.00, size = 41, normalized size = 1.86

$$\frac{\ln\left(-\sqrt{-x^2+1} + 1\right)}{2} - \frac{\ln\left(\sqrt{-x^2+1} + 1\right)}{2} - \frac{\arcsin x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(x)/x^2,x)

[Out] -arcsin(x)/x - 1/2*log(sqrt(-x^2 + 1) + 1) + 1/2*log(-sqrt(-x^2 + 1) + 1)

Mupad [B]

time = 0.02, size = 20, normalized size = 0.91

$$-\operatorname{atanh}\left(\frac{1}{\sqrt{1-x^2}}\right) - \frac{\operatorname{asin}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(x)/x^2,x)

[Out] - atanh(1/(1 - x^2)^(1/2)) - asin(x)/x

3.84 $\int \sin^{-1}(x)^2 dx$

Optimal. Leaf size=25

$$-2x + 2\sqrt{1-x^2} \sin^{-1}(x) + x \sin^{-1}(x)^2$$

[Out] $-2*x+x*\arcsin(x)^2+2*\arcsin(x)*(-x^2+1)^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4715, 4767, 8}

$$2\sqrt{1-x^2} \sin^{-1}(x) - 2x + x \sin^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{ArcSin}[x]^2, x]$

[Out] $-2*x + 2*\text{Sqrt}[1 - x^2]*\text{ArcSin}[x] + x*\text{ArcSin}[x]^2$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 4715

$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{ArcSin}[c*x])^n, x] - \text{Dist}[b*c*n, \text{Int}[x*((a + b*\text{ArcSin}[c*x])^{(n-1)})/\text{Sqrt}[1 - c^2*x^2]), x], x] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \text{GtQ}[n, 0]$

Rule 4767

$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*(x_.)*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(2*e*(p+1))), x] + \text{Dist}[b*(n/(2*c*(p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p], \text{Int}[(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x \ \&\& \text{EqQ}[c^2*d + e, 0] \ \&\& \text{GtQ}[n, 0] \ \&\& \text{NeQ}[p, -1]$

Rubi steps

$$\begin{aligned} \int \sin^{-1}(x)^2 dx &= x \sin^{-1}(x)^2 - 2 \int \frac{x \sin^{-1}(x)}{\sqrt{1-x^2}} dx \\ &= 2\sqrt{1-x^2} \sin^{-1}(x) + x \sin^{-1}(x)^2 - 2 \int 1 dx \\ &= -2x + 2\sqrt{1-x^2} \sin^{-1}(x) + x \sin^{-1}(x)^2 \end{aligned}$$

Mathematica [A]

time = 0.00, size = 25, normalized size = 1.00

$$-2x + 2\sqrt{1-x^2} \sin^{-1}(x) + x \sin^{-1}(x)^2$$

Antiderivative was successfully verified.

`[In] Integrate[ArcSin[x]^2,x]``[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2`**Mathics [A]**

time = 2.06, size = 23, normalized size = 0.92

$$-2x + x \operatorname{ArcSin}[x]^2 + 2 \operatorname{ArcSin}[x] \sqrt{1-x^2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[ArcSin[x]^2,x]')``[Out] -2 x + x ArcSin[x] ^ 2 + 2 ArcSin[x] Sqrt[1 - x ^ 2]`**Maple [A]**

time = 0.00, size = 24, normalized size = 0.96

method	result	size
default	$-2x + x \arcsin(x)^2 + 2 \arcsin(x) \sqrt{-x^2 + 1}$	24

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(arcsin(x)^2,x,method=_RETURNVERBOSE)``[Out] -2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)`**Maxima [A]**

time = 0.35, size = 23, normalized size = 0.92

$$x \arcsin(x)^2 + 2 \sqrt{-x^2 + 1} \arcsin(x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(arcsin(x)^2,x, algorithm="maxima")``[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x`**Fricas [A]**

time = 0.31, size = 23, normalized size = 0.92

$$x \arcsin(x)^2 + 2 \sqrt{-x^2 + 1} \arcsin(x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(x)^2,x, algorithm="fricas")

[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x

Sympy [A]

time = 0.08, size = 22, normalized size = 0.88

$$x \operatorname{asin}^2(x) - 2x + 2\sqrt{1-x^2} \operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(x)**2,x)

[Out] x*asin(x)**2 - 2*x + 2*sqrt(1 - x**2)*asin(x)

Giac [A]

time = 0.00, size = 25, normalized size = 1.00

$$x \operatorname{arcsin}^2 x + 2\sqrt{-x^2 + 1} \operatorname{arcsin} x - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(x)^2,x)

[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x

Mupad [B]

time = 0.05, size = 22, normalized size = 0.88

$$2 \operatorname{asin}(x) \sqrt{1-x^2} + x (\operatorname{asin}(x)^2 - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(x)^2,x)

[Out] 2*asin(x)*(1 - x^2)^(1/2) + x*(asin(x)^2 - 2)

3.85

$$\int \frac{x^2 \tan^{-1}(x)}{1+x^2} dx$$

Optimal. Leaf size=23

$$x \tan^{-1}(x) - \frac{1}{2} \tan^{-1}(x)^2 - \frac{1}{2} \log(1+x^2)$$

[Out] x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)

Rubi [A]

time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5036, 4930, 266, 5004}

$$-\frac{1}{2} \log(x^2 + 1) - \frac{1}{2} \tan^{-1}(x)^2 + x \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x^2*ArcTan[x])/(1 + x^2),x]

[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2

Rule 266

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 4930

Int[((a_) + ArcTan[(c_)*(x_)^(n_)])*(b_)^(p_), x_Symbol] :> Simp[x*(a + b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p - 1))/(1 + c^2*x^(2*n))], x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] && (EqQ[n, 1] || EqQ[p, 1])

Rule 5004

Int[((a_) + ArcTan[(c_)*(x_)])*(b_)^(p_)/((d_) + (e_)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]

Rule 5036

Int[((a_) + ArcTan[(c_)*(x_)])*(b_)^(p_)*((f_)*(x_)^(m_))/((d_) + (e_)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \tan^{-1}(x)}{1+x^2} dx &= \int \tan^{-1}(x) dx - \int \frac{\tan^{-1}(x)}{1+x^2} dx \\
&= x \tan^{-1}(x) - \frac{1}{2} \tan^{-1}(x)^2 - \int \frac{x}{1+x^2} dx \\
&= x \tan^{-1}(x) - \frac{1}{2} \tan^{-1}(x)^2 - \frac{1}{2} \log(1+x^2)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 23, normalized size = 1.00

$$x \tan^{-1}(x) - \frac{1}{2} \tan^{-1}(x)^2 - \frac{1}{2} \log(1+x^2)$$

Antiderivative was successfully verified.

`[In] Integrate[(x^2*ArcTan[x])/(1+x^2),x]``[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1+x^2]/2`**Mathics [A]**

time = 2.07, size = 19, normalized size = 0.83

$$x \text{ArcTan}[x] - \frac{\text{Log}[1+x^2]}{2} - \frac{\text{ArcTan}[x]^2}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[ArcTan[x]*x^2/(1+x^2),x]')``[Out] x ArcTan[x] - Log[1+x^2]/2 - ArcTan[x]^2/2`**Maple [A]**

time = 0.08, size = 20, normalized size = 0.87

method	result	size
default	$x \arctan(x) - \frac{\arctan(x)^2}{2} - \frac{\ln(x^2+1)}{2}$	20
risch	$\frac{\ln(ix+1)^2}{8} + \frac{i(-x + \frac{i \ln(-ix+1)}{2}) \ln(ix+1)}{2} + \frac{\ln(-ix+1)^2}{8} + \frac{ix \ln(-ix+1)}{2} - \frac{\ln(x^2+1)}{2}$	67

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2*arctan(x)/(x^2+1),x,method=_RETURNVERBOSE)``[Out] x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)`

Maxima [A]

time = 0.37, size = 24, normalized size = 1.04

$$(x - \arctan(x)) \arctan(x) + \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="maxima")``[Out] (x - arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)`**Fricas [A]**

time = 0.32, size = 19, normalized size = 0.83

$$x \arctan(x) - \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="fricas")``[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)`**Sympy [A]**

time = 0.13, size = 19, normalized size = 0.83

$$x \operatorname{atan}(x) - \frac{\log(x^2 + 1)}{2} - \frac{\operatorname{atan}^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x**2*atan(x)/(x**2+1),x)``[Out] x*atan(x) - log(x**2 + 1)/2 - atan(x)**2/2`**Giac [A]**

time = 0.00, size = 21, normalized size = 0.91

$$\frac{2x \arctan x - \arctan^2 x - \ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2*arctan(x)/(x^2+1),x)``[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)`**Mupad [B]**

time = 0.22, size = 19, normalized size = 0.83

$$-\frac{\operatorname{atan}(x)^2}{2} + x \operatorname{atan}(x) - \frac{\ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((x^2*atan(x))/(x^2 + 1),x)``[Out] x*atan(x) - atan(x)^2/2 - log(x^2 + 1)/2`

3.86 $\int \cos^{-1} \left(\sqrt{\frac{x}{1+x}} \right) dx$

Optimal. Leaf size=38

$$(1+x) \left(\sqrt{\frac{1}{1+x}} \sqrt{\frac{x}{1+x}} + \cos^{-1} \left(\sqrt{\frac{x}{1+x}} \right) \right)$$

[Out] (1+x)*(arccos((x/(1+x))^(1/2))+(1/(1+x))^(1/2)*(x/(1+x))^(1/2))

Rubi [A]

time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.50, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4925, 12, 6851, 52, 65, 209}

$$\sqrt{\frac{x}{(x+1)^2}} (x+1) + x \cos^{-1} \left(\sqrt{\frac{x}{x+1}} \right) - \frac{\sqrt{\frac{x}{(x+1)^2}} (x+1) \tan^{-1}(\sqrt{x})}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[Sqrt[x/(1+x)]],x]

[Out] Sqrt[x/(1+x)^2]*(1+x) + x*ArcCos[Sqrt[x/(1+x)]] - (Sqrt[x/(1+x)^2]*(1+x)*ArcTan[Sqrt[x]])/Sqrt[x]

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 52

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m+1)*((c + d*x)^n/(b*(m+n+1))), x] + Dist[n*(b*c - a*d)/(b*(m+n+1)), Int[(a + b*x)^m*(c + d*x)^(n-1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n+2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m+1)-1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 209

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 4925

Int[ArcCos[u_], x_Symbol] := Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[x*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] && !FunctionOfExponentialQ[u, x]

Rule 6851

Int[(u_)*((a_)*(v_)^(m_)*(w_)^(n_))^(p_), x_Symbol] := Dist[a^IntPart[p]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x]

Rubi steps

$$\begin{aligned}
 \int \cos^{-1} \left(\sqrt{\frac{x}{1+x}} \right) dx &= x \cos^{-1} \left(\sqrt{\frac{x}{1+x}} \right) + \int \frac{1}{2} \sqrt{\frac{x}{(1+x)^2}} dx \\
 &= x \cos^{-1} \left(\sqrt{\frac{x}{1+x}} \right) + \frac{1}{2} \int \sqrt{\frac{x}{(1+x)^2}} dx \\
 &= x \cos^{-1} \left(\sqrt{\frac{x}{1+x}} \right) + \frac{\left(\sqrt{\frac{x}{(1+x)^2}} (1+x) \right) \int \frac{\sqrt{x}}{1+x} dx}{2\sqrt{x}} \\
 &= \sqrt{\frac{x}{(1+x)^2}} (1+x) + x \cos^{-1} \left(\sqrt{\frac{x}{1+x}} \right) - \frac{\left(\sqrt{\frac{x}{(1+x)^2}} (1+x) \right) \int \frac{1}{\sqrt{x} (1+x)}}{2\sqrt{x}} \\
 &= \sqrt{\frac{x}{(1+x)^2}} (1+x) + x \cos^{-1} \left(\sqrt{\frac{x}{1+x}} \right) - \frac{\left(\sqrt{\frac{x}{(1+x)^2}} (1+x) \right) \text{Subst} \left(\int \frac{1}{\sqrt{x} (1+x)} \right)}{\sqrt{x}} \\
 &= \sqrt{\frac{x}{(1+x)^2}} (1+x) + x \cos^{-1} \left(\sqrt{\frac{x}{1+x}} \right) - \frac{\sqrt{\frac{x}{(1+x)^2}} (1+x) \tan^{-1}(\sqrt{x})}{\sqrt{x}}
 \end{aligned}$$

Mathematica [A]

time = 0.04, size = 49, normalized size = 1.29

$$x \cos^{-1} \left(\sqrt{\frac{x}{1+x}} \right) + \frac{\sqrt{\frac{x}{(1+x)^2}} (1+x) (\sqrt{x} - \tan^{-1}(\sqrt{x}))}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[Sqrt[x/(1 + x)]],x]

[Out] x*ArcCos[Sqrt[x/(1 + x)]] + (Sqrt[x/(1 + x)^2]*(1 + x)*(Sqrt[x] - ArcTan[Sqrt[x]]))/Sqrt[x]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[ArcCos[Sqrt[x/(1 + x)]],x]')

[Out] cought exception:

Maple [A]

time = 0.01, size = 45, normalized size = 1.18

method	result	size
default	$x \arccos \left(\sqrt{\frac{x}{1+x}} \right) - \frac{\sqrt{x} \sqrt{\frac{1}{1+x}} (-\sqrt{x} + \arctan(\sqrt{x}))}{\sqrt{\frac{x}{1+x}}}$	45

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos((x/(1+x))^(1/2)),x,method=_RETURNVERBOSE)

[Out] x*arccos((x/(1+x))^(1/2))-1/(x/(1+x))^(1/2)*x^(1/2)*(1/(1+x))^(1/2)*(-x^(1/2)+arctan(x^(1/2)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(30) = 60.

time = 0.36, size = 78, normalized size = 2.05

$$-\frac{\arccos \left(\sqrt{\frac{x}{x+1}} \right)}{\frac{x}{x+1} - 1} - \frac{\sqrt{-\frac{x}{x+1} + 1}}{2 \left(\sqrt{\frac{x}{x+1}} + 1 \right)} - \frac{\sqrt{-\frac{x}{x+1} + 1}}{2 \left(\sqrt{\frac{x}{x+1}} - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos((x/(1+x))^(1/2)),x, algorithm="maxima")

[Out] $-\arccos(\sqrt{x/(x+1)})/(x/(x+1)-1) - 1/2*\sqrt{-x/(x+1)+1}/(\sqrt{x/(x+1)+1}) - 1/2*\sqrt{-x/(x+1)+1}/(\sqrt{x/(x+1)-1})$

Fricas [A]

time = 0.32, size = 30, normalized size = 0.79

$$(x+1)\arccos\left(\sqrt{\frac{x}{x+1}}\right) + \sqrt{x+1}\sqrt{\frac{x}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos((x/(1+x))^(1/2)),x, algorithm="fricas")

[Out] $(x+1)*\arccos(\sqrt{x/(x+1)}) + \sqrt{x+1}*\sqrt{x/(x+1)}$

Sympy [A]

time = 4.76, size = 63, normalized size = 1.66

$$x \operatorname{acos}\left(\sqrt{\frac{x}{x+1}}\right) - 2 \left(\left\{ -\frac{\sqrt{\frac{x}{x+1}}}{2\sqrt{-\frac{x}{x+1}+1}} + \frac{\operatorname{asin}\left(\sqrt{\frac{x}{x+1}}\right)}{2} \text{ for } \sqrt{\frac{x}{x+1}} > -1 \wedge \sqrt{\frac{x}{x+1}} < 1 \right\} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos((x/(1+x))^(1/2)),x)

[Out] $x*\operatorname{acos}(\sqrt{x/(x+1)}) - 2*\operatorname{Piecewise}((- \sqrt{x/(x+1)})/(2*\sqrt{-x/(x+1)+1}) + \operatorname{asin}(\sqrt{x/(x+1)})/2, (\sqrt{x/(x+1)} > -1) \& (\sqrt{x/(x+1)} < 1))$

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos((x/(1+x))^(1/2)),x)

[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \operatorname{acos}\left(\sqrt{\frac{x}{x+1}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acos((x/(x+1))^(1/2)),x)

[Out] int(acos((x/(x+1))^(1/2)), x)

3.87 $\int (2x + 3x^2)^3 dx$

Optimal. Leaf size=25

$$2x^4 + \frac{36x^5}{5} + 9x^6 + \frac{27x^7}{7}$$

[Out] $2x^4 + 36/5x^5 + 9x^6 + 27/7x^7$

Rubi [A]

time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {625}

$$\frac{27x^7}{7} + 9x^6 + \frac{36x^5}{5} + 2x^4$$

Antiderivative was successfully verified.

[In] Int[(2*x + 3*x^2)^3,x]

[Out] $2x^4 + (36x^5)/5 + 9x^6 + (27x^7)/7$

Rule 625

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && (EqQ[a, 0] || !PerfectSquareQ[b^2 - 4*a*c])

Rubi steps

$$\begin{aligned} \int (2x + 3x^2)^3 dx &= \int (8x^3 + 36x^4 + 54x^5 + 27x^6) dx \\ &= 2x^4 + \frac{36x^5}{5} + 9x^6 + \frac{27x^7}{7} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 25, normalized size = 1.00

$$2x^4 + \frac{36x^5}{5} + 9x^6 + \frac{27x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(2*x + 3*x^2)^3,x]

[Out] $2x^4 + (36x^5)/5 + 9x^6 + (27x^7)/7$

Mathics [A]

time = 1.71, size = 20, normalized size = 0.80

$$\frac{x^4 (70 + 252x + 315x^2 + 135x^3)}{35}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(3*x^2 + 2*x)^3,x]')
```

```
[Out] x ^ 4 (70 + 252 x + 315 x ^ 2 + 135 x ^ 3) / 35
```

Maple [A]

time = 0.04, size = 22, normalized size = 0.88

method	result	size
gosper	$\frac{x^4(135x^3+315x^2+252x+70)}{35}$	21
default	$2x^4 + \frac{36}{5}x^5 + 9x^6 + \frac{27}{7}x^7$	22
norman	$2x^4 + \frac{36}{5}x^5 + 9x^6 + \frac{27}{7}x^7$	22
risch	$2x^4 + \frac{36}{5}x^5 + 9x^6 + \frac{27}{7}x^7$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^4+36/5*x^5+9*x^6+27/7*x^7
```

Maxima [A]

time = 0.26, size = 21, normalized size = 0.84

$$\frac{27}{7}x^7 + 9x^6 + \frac{36}{5}x^5 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x)^3,x, algorithm="maxima")
```

```
[Out] 27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

Fricas [A]

time = 0.31, size = 21, normalized size = 0.84

$$\frac{27}{7}x^7 + 9x^6 + \frac{36}{5}x^5 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x)^3,x, algorithm="fricas")
```

[Out] $27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4$

Sympy [A]

time = 0.03, size = 22, normalized size = 0.88

$$\frac{27x^7}{7} + 9x^6 + \frac{36x^5}{5} + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2+2*x)**3,x)`

[Out] $27*x**7/7 + 9*x**6 + 36*x**5/5 + 2*x**4$

Giac [A]

time = 0.00, size = 25, normalized size = 1.00

$$\frac{27}{7}x^7 + 9x^6 + \frac{36}{5}x^5 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+2*x)^3,x)`

[Out] $27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4$

Mupad [B]

time = 0.04, size = 21, normalized size = 0.84

$$\frac{27 x^7}{7} + 9 x^6 + \frac{36 x^5}{5} + 2 x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x + 3*x^2)^3,x)`

[Out] $2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7$

3.88 $\int (-1 + x) (-1 + 2x + 3x^2)^2 dx$

Optimal. Leaf size=39

$$-x + \frac{5x^2}{2} - \frac{2x^3}{3} - \frac{7x^4}{2} + \frac{3x^5}{5} + \frac{3x^6}{2}$$

[Out] $-x+5/2*x^2-2/3*x^3-7/2*x^4+3/5*x^5+3/2*x^6$

Rubi [A]

time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$,

Rules used = {645}

$$\frac{3x^6}{2} + \frac{3x^5}{5} - \frac{7x^4}{2} - \frac{2x^3}{3} + \frac{5x^2}{2} - x$$

Antiderivative was successfully verified.

[In] Int[(-1 + x)*(-1 + 2*x + 3*x^2)^2,x]

[Out] $-x + (5*x^2)/2 - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2$

Rule 645

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || EqQ[a, 0])

Rubi steps

$$\begin{aligned} \int (-1 + x) (-1 + 2x + 3x^2)^2 dx &= \int (-1 + 5x - 2x^2 - 14x^3 + 3x^4 + 9x^5) dx \\ &= -x + \frac{5x^2}{2} - \frac{2x^3}{3} - \frac{7x^4}{2} + \frac{3x^5}{5} + \frac{3x^6}{2} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 39, normalized size = 1.00

$$-x + \frac{5x^2}{2} - \frac{2x^3}{3} - \frac{7x^4}{2} + \frac{3x^5}{5} + \frac{3x^6}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x)*(-1 + 2*x + 3*x^2)^2,x]

[Out] $-x + (5x^2)/2 - (2x^3)/3 - (7x^4)/2 + (3x^5)/5 + (3x^6)/2$

Mathics [A]

time = 1.87, size = 28, normalized size = 0.72

$$\frac{x(-30 + 75x - 20x^2 - 105x^3 + 18x^4 + 45x^5)}{30}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(3*x^2 + 2*x - 1)^2*(x - 1),x]')`

[Out] $x(-30 + 75x - 20x^2 - 105x^3 + 18x^4 + 45x^5) / 30$

Maple [A]

time = 0.08, size = 30, normalized size = 0.77

method	result	size
gospers	$\frac{x(45x^5+18x^4-105x^3-20x^2+75x-30)}{30}$	29
default	$-x + \frac{5}{2}x^2 - \frac{2}{3}x^3 - \frac{7}{2}x^4 + \frac{3}{5}x^5 + \frac{3}{2}x^6$	30
norman	$-x + \frac{5}{2}x^2 - \frac{2}{3}x^3 - \frac{7}{2}x^4 + \frac{3}{5}x^5 + \frac{3}{2}x^6$	30
risch	$-x + \frac{5}{2}x^2 - \frac{2}{3}x^3 - \frac{7}{2}x^4 + \frac{3}{5}x^5 + \frac{3}{2}x^6$	30

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-1+x)*(3*x^2+2*x-1)^2,x,method=_RETURNVERBOSE)`

[Out] $-x+5/2*x^2-2/3*x^3-7/2*x^4+3/5*x^5+3/2*x^6$

Maxima [A]

time = 0.27, size = 29, normalized size = 0.74

$$\frac{3}{2}x^6 + \frac{3}{5}x^5 - \frac{7}{2}x^4 - \frac{2}{3}x^3 + \frac{5}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+x)*(3*x^2+2*x-1)^2,x, algorithm="maxima")`

[Out] $3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x$

Fricas [A]

time = 0.32, size = 29, normalized size = 0.74

$$\frac{3}{2}x^6 + \frac{3}{5}x^5 - \frac{7}{2}x^4 - \frac{2}{3}x^3 + \frac{5}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)*(3*x^2+2*x-1)^2,x, algorithm="fricas")

[Out] $\frac{3}{2}x^6 + \frac{3}{5}x^5 - \frac{7}{2}x^4 - \frac{2}{3}x^3 + \frac{5}{2}x^2 - x$

Sympy [A]

time = 0.03, size = 34, normalized size = 0.87

$$\frac{3x^6}{2} + \frac{3x^5}{5} - \frac{7x^4}{2} - \frac{2x^3}{3} + \frac{5x^2}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)*(3*x**2+2*x-1)**2,x)

[Out] $3*x**6/2 + 3*x**5/5 - 7*x**4/2 - 2*x**3/3 + 5*x**2/2 - x$

Giac [A]

time = 0.00, size = 40, normalized size = 1.03

$$\frac{3}{2}x^6 + \frac{3}{5}x^5 - \frac{7}{2}x^4 - \frac{2}{3}x^3 + \frac{5}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)*(3*x^2+2*x-1)^2,x)

[Out] $\frac{3}{2}x^6 + \frac{3}{5}x^5 - \frac{7}{2}x^4 - \frac{2}{3}x^3 + \frac{5}{2}x^2 - x$

Mupad [B]

time = 0.03, size = 29, normalized size = 0.74

$$\frac{3x^6}{2} + \frac{3x^5}{5} - \frac{7x^4}{2} - \frac{2x^3}{3} + \frac{5x^2}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - 1)*(2*x + 3*x^2 - 1)^2,x)

[Out] $(5*x^2)/2 - x - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2$

$$3.89 \quad \int x^{-1+k} (a + bx^k)^n dx$$

Optimal. Leaf size=23

$$\frac{(a + bx^k)^{1+n}}{bk(1+n)}$$

[Out] (a+b*x^k)^(1+n)/b/k/(1+n)

Rubi [A]

time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {267}

$$\frac{(a + bx^k)^{n+1}}{bk(n+1)}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + k)*(a + b*x^k)^n,x]

[Out] (a + b*x^k)^(1 + n)/(b*k*(1 + n))

Rule 267

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int x^{-1+k} (a + bx^k)^n dx = \frac{(a + bx^k)^{1+n}}{bk(1+n)}$$

Mathematica [A]

time = 0.03, size = 23, normalized size = 1.00

$$\frac{(a + bx^k)^{1+n}}{bk(1+n)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + k)*(a + b*x^k)^n,x]

[Out] (a + b*x^k)^(1 + n)/(b*k*(1 + n))

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 22.19, size = 112, normalized size = 4.87

Piecewise $\left[\left\{ \left\{ \frac{\text{Log}[x]}{a}, b=0 \&\& k=0 \&\& n=-1 \right\}, \left\{ \frac{a^n x^k}{k}, b=0 \right\}, \{\text{Log}[x] (a+b)^n, k=0\}, \left\{ \frac{\text{Log}\left[\frac{a}{b} + x^k\right]}{bk}, n=-1 \right\}, \frac{a(a+bx^k)^n}{bk+bkn} + \frac{bx^k(a+bx^k)^n}{bk+bkn} \right]$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(a + b*x^k)^n*x^(k - 1),x]')`

[Out] `Piecewise[{{Log[x] / a, b == 0 && k == 0 && n == -1}, {a ^ n x ^ k / k, b = 0}, {Log[x] (a + b) ^ n, k == 0}, {Log[a / b + x ^ k] / (b k), n == -1}}, a (a + b x ^ k) ^ n / (b k + b k n) + b x ^ k (a + b x ^ k) ^ n / (b k + b k n)]`

Maple [A]

time = 0.08, size = 29, normalized size = 1.26

method	result	size
risch	$\frac{(a+bx^k)(a+bx^k)^n}{b(1+n)k}$	29

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-1+k)*(a+b*x^k)^n,x,method=_RETURNVERBOSE)`

[Out] `(a+b*x^k)/b/(1+n)/k*(a+b*x^k)^n`

Maxima [A]

time = 0.27, size = 23, normalized size = 1.00

$$\frac{(bx^k + a)^{n+1}}{bk(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+k)*(a+b*x^k)^n,x, algorithm="maxima")`

[Out] `(b*x^k + a)^(n + 1)/(b*k*(n + 1))`

Fricas [A]

time = 0.32, size = 27, normalized size = 1.17

$$\frac{(bx^k + a)(bx^k + a)^n}{bkn + bk}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+k)*(a+b*x^k)^n,x, algorithm="fricas")`

[Out] $(b*x^k + a)*(b*x^k + a)^n/(b*k*n + b*k)$

Sympy [A]

time = 19.41, size = 75, normalized size = 3.26

$$\begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge k = 0 \wedge n = -1 \\ \frac{a^n x^k}{k} & \text{for } b = 0 \\ (a + b)^n \log(x) & \text{for } k = 0 \\ \frac{\log(\frac{a}{b} + x^k)}{bk} & \text{for } n = -1 \\ \frac{a(a+bx^k)^n}{bkn+bk} + \frac{bx^k(a+bx^k)^n}{bkn+bk} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1+k)*(a+b*x**k)**n,x)`

[Out] `Piecewise((log(x)/a, Eq(b, 0) & Eq(k, 0) & Eq(n, -1)), (a**n*x**k/k, Eq(b, 0)), ((a + b)**n*log(x), Eq(k, 0)), (log(a/b + x**k)/(b*k), Eq(n, -1)), (a*(a + b*x**k)**n/(b*k*n + b*k) + b*x**k*(a + b*x**k)**n/(b*k*n + b*k), True))`

Giac [A]

time = 0.00, size = 20, normalized size = 0.87

$$\frac{(a + bx^k)^{n+1}}{bk(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+k)*(a+b*x^k)^n,x)`

[Out] $(b*x^k + a)^{(n + 1)}/(b*k*(n + 1))$

Mupad [B]

time = 0.73, size = 23, normalized size = 1.00

$$\frac{(a + bx^k)^{n+1}}{bk(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(k - 1)*(a + b*x^k)^n,x)`

[Out] $(a + b*x^k)^{(n + 1)}/(b*k*(n + 1))$

3.90 $\int \frac{x^3}{1+2x} dx$

Optimal. Leaf size=30

$$\frac{x}{8} - \frac{x^2}{8} + \frac{x^3}{6} - \frac{1}{16} \log(1+2x)$$

[Out] 1/8*x-1/8*x^2+1/6*x^3-1/16*ln(1+2*x)

Rubi [A]

time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{x^3}{6} - \frac{x^2}{8} + \frac{x}{8} - \frac{1}{16} \log(2x+1)$$

Antiderivative was successfully verified.

[In] Int[x^3/(1 + 2*x), x]

[Out] x/8 - x^2/8 + x^3/6 - Log[1 + 2*x]/16

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int [ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{x^3}{1+2x} dx &= \int \left(\frac{1}{8} - \frac{x}{4} + \frac{x^2}{2} - \frac{1}{8(1+2x)} \right) dx \\ &= \frac{x}{8} - \frac{x^2}{8} + \frac{x^3}{6} - \frac{1}{16} \log(1+2x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 27, normalized size = 0.90

$$\frac{1}{96} (11 + 12x - 12x^2 + 16x^3 - 6 \log(1 + 2x))$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(1 + 2*x), x]

[Out] $(11 + 12x - 12x^2 + 16x^3 - 6\text{Log}[1 + 2x])/96$

Mathics [A]

time = 1.87, size = 22, normalized size = 0.73

$$\frac{x}{8} - \frac{x^2}{8} + \frac{x^3}{6} - \frac{\text{Log}[1 + 2x]}{16}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x^3/(1 + 2*x),x]')`

[Out] $x / 8 - x^2 / 8 + x^3 / 6 - \text{Log}[1 + 2x] / 16$

Maple [A]

time = 0.05, size = 23, normalized size = 0.77

method	result	size
default	$\frac{x}{8} - \frac{x^2}{8} + \frac{x^3}{6} - \frac{\ln(1+2x)}{16}$	23
norman	$\frac{x}{8} - \frac{x^2}{8} + \frac{x^3}{6} - \frac{\ln(1+2x)}{16}$	23
meijerg	$\frac{x(16x^2-12x+12)}{96} - \frac{\ln(1+2x)}{16}$	23
risch	$\frac{x}{8} - \frac{x^2}{8} + \frac{x^3}{6} - \frac{\ln(1+2x)}{16}$	23

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(1+2*x),x,method=_RETURNVERBOSE)`

[Out] $1/8*x-1/8*x^2+1/6*x^3-1/16*\ln(1+2*x)$

Maxima [A]

time = 0.27, size = 22, normalized size = 0.73

$$\frac{1}{6}x^3 - \frac{1}{8}x^2 + \frac{1}{8}x - \frac{1}{16}\log(2x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(1+2*x),x, algorithm="maxima")`

[Out] $1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*\log(2*x + 1)$

Fricas [A]

time = 0.32, size = 22, normalized size = 0.73

$$\frac{1}{6}x^3 - \frac{1}{8}x^2 + \frac{1}{8}x - \frac{1}{16}\log(2x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(1+2*x),x, algorithm="fricas")

[Out] 1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(2*x + 1)

Sympy [A]

time = 0.04, size = 20, normalized size = 0.67

$$\frac{x^3}{6} - \frac{x^2}{8} + \frac{x}{8} - \frac{\log(2x + 1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(1+2*x),x)

[Out] x**3/6 - x**2/8 + x/8 - log(2*x + 1)/16

Giac [A]

time = 0.00, size = 28, normalized size = 0.93

$$\frac{\frac{4}{3}x^3 - x^2 + x}{8} - \frac{\ln|2x + 1|}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(1+2*x),x)

[Out] 1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(abs(2*x + 1))

Mupad [B]

time = 0.03, size = 20, normalized size = 0.67

$$\frac{x}{8} - \frac{\ln\left(x + \frac{1}{2}\right)}{16} - \frac{x^2}{8} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(2*x + 1),x)

[Out] x/8 - log(x + 1/2)/16 - x^2/8 + x^3/6

3.91 $\int \frac{x^6}{2+3x^2} dx$

Optimal. Leaf size=41

$$\frac{4x}{27} - \frac{2x^3}{27} + \frac{x^5}{15} - \frac{4}{27} \sqrt{\frac{2}{3}} \tan^{-1} \left(\sqrt{\frac{3}{2}} x \right)$$

[Out] $4/27*x-2/27*x^3+1/15*x^5-4/81*\arctan(1/2*x*6^{(1/2)})*6^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {308, 209}

$$\frac{x^5}{15} - \frac{2x^3}{27} + \frac{4x}{27} - \frac{4}{27} \sqrt{\frac{2}{3}} \tan^{-1} \left(\sqrt{\frac{3}{2}} x \right)$$

Antiderivative was successfully verified.

[In] Int[x^6/(2 + 3*x^2),x]

[Out] $(4*x)/27 - (2*x^3)/27 + x^5/15 - (4*\text{Sqrt}[2/3]*\text{ArcTan}[\text{Sqrt}[3/2]*x])/27$

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 308

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rubi steps

$$\begin{aligned} \int \frac{x^6}{2+3x^2} dx &= \int \left(\frac{4}{27} - \frac{2x^2}{9} + \frac{x^4}{3} - \frac{8}{27(2+3x^2)} \right) dx \\ &= \frac{4x}{27} - \frac{2x^3}{27} + \frac{x^5}{15} - \frac{8}{27} \int \frac{1}{2+3x^2} dx \\ &= \frac{4x}{27} - \frac{2x^3}{27} + \frac{x^5}{15} - \frac{4}{27} \sqrt{\frac{2}{3}} \tan^{-1} \left(\sqrt{\frac{3}{2}} x \right) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 35, normalized size = 0.85

$$\frac{1}{405} \left(60x - 30x^3 + 27x^5 - 20\sqrt{6} \tan^{-1} \left(\sqrt{\frac{3}{2}} x \right) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[x^6/(2 + 3*x^2),x]``[Out] (60*x - 30*x^3 + 27*x^5 - 20*Sqrt[6]*ArcTan[Sqrt[3/2]*x])/405`**Mathics [A]**

time = 2.00, size = 26, normalized size = 0.63

$$\frac{4x}{27} - \frac{2x^3}{27} + \frac{x^5}{15} - \frac{4\sqrt{6} \operatorname{ArcTan} \left[\frac{\sqrt{6} x}{2} \right]}{81}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^6/(2 + 3*x^2),x]')``[Out] 4 x / 27 - 2 x ^ 3 / 27 + x ^ 5 / 15 - 4 Sqrt[6] ArcTan[Sqrt[6] x / 2] / 81`**Maple [A]**

time = 0.06, size = 27, normalized size = 0.66

method	result	size
default	$\frac{4x}{27} - \frac{2x^3}{27} + \frac{x^5}{15} - \frac{4 \arctan \left(\frac{x\sqrt{6}}{2} \right) \sqrt{6}}{81}$	27
risch	$\frac{4x}{27} - \frac{2x^3}{27} + \frac{x^5}{15} - \frac{4 \arctan \left(\frac{x\sqrt{6}}{2} \right) \sqrt{6}}{81}$	27
meijerg	$\frac{2\sqrt{2} \sqrt{3} \left(\frac{x\sqrt{2} \sqrt{3} \left(\frac{189}{4}x^4 - \frac{105}{2}x^2 + 105 \right) - 2 \arctan \left(\frac{x\sqrt{2} \sqrt{3}}{2} \right) \right)}{81}$	43

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^6/(3*x^2+2),x,method=_RETURNVERBOSE)``[Out] 4/27*x-2/27*x^3+1/15*x^5-4/81*arctan(1/2*x*6^(1/2))*6^(1/2)`**Maxima [A]**

time = 0.36, size = 26, normalized size = 0.63

$$\frac{1}{15} x^5 - \frac{2}{27} x^3 - \frac{4}{81} \sqrt{6} \arctan \left(\frac{1}{2} \sqrt{6} x \right) + \frac{4}{27} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(3*x^2+2),x, algorithm="maxima")

[Out] 1/15*x^5 - 2/27*x^3 - 4/81*sqrt(6)*arctan(1/2*sqrt(6)*x) + 4/27*x

Fricas [A]

time = 0.31, size = 32, normalized size = 0.78

$$\frac{1}{15}x^5 - \frac{2}{27}x^3 - \frac{4}{81}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}x\right) + \frac{4}{27}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(3*x^2+2),x, algorithm="fricas")

[Out] 1/15*x^5 - 2/27*x^3 - 4/81*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x) + 4/27*x

Sympy [A]

time = 0.05, size = 34, normalized size = 0.83

$$\frac{x^5}{15} - \frac{2x^3}{27} + \frac{4x}{27} - \frac{4\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(3*x**2+2),x)

[Out] x**5/15 - 2*x**3/27 + 4*x/27 - 4*sqrt(6)*atan(sqrt(6)*x/2)/81

Giac [A]

time = 0.00, size = 43, normalized size = 1.05

$$\frac{\frac{81}{5}x^5 - 18x^3 + 36x}{243} - \frac{16\arctan\left(\frac{3x}{\sqrt{6}}\right)}{2 \cdot 27\sqrt{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(3*x^2+2),x)

[Out] 1/15*x^5 - 2/27*x^3 - 4/81*sqrt(6)*arctan(1/2*sqrt(6)*x) + 4/27*x

Mupad [B]

time = 0.04, size = 32, normalized size = 0.78

$$\frac{4x}{27} - \frac{2x^3}{27} + \frac{x^5}{15} - \frac{4\sqrt{2}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{3}x}{2}\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2 + 2),x)
```

```
[Out] (4*x)/27 - (2*x^3)/27 + x^5/15 - (4*2^(1/2)*3^(1/2)*atan((2^(1/2)*3^(1/2)*x)/2))/81
```

3.92 $\int \frac{1}{2-7x+3x^2} dx$

Optimal. Leaf size=21

$$-\frac{1}{5} \log(1-3x) + \frac{1}{5} \log(2-x)$$

[Out] -1/5*ln(1-3*x)+1/5*ln(2-x)

Rubi [A]

time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {630, 31}

$$\frac{1}{5} \log(2-x) - \frac{1}{5} \log(1-3x)$$

Antiderivative was successfully verified.

[In] Int[(2 - 7*x + 3*x^2)^(-1), x]

[Out] -1/5*Log[1 - 3*x] + Log[2 - x]/5

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 630

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{1}{2-7x+3x^2} dx &= \frac{3}{5} \int \frac{1}{-6+3x} dx - \frac{3}{5} \int \frac{1}{-1+3x} dx \\ &= -\frac{1}{5} \log(1-3x) + \frac{1}{5} \log(2-x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 21, normalized size = 1.00

$$-\frac{1}{5} \log(1-3x) + \frac{1}{5} \log(2-x)$$

Antiderivative was successfully verified.

[In] Integrate[(2 - 7*x + 3*x^2)^(-1),x]

[Out] -1/5*Log[1 - 3*x] + Log[2 - x]/5

Mathics [A]

time = 1.87, size = 13, normalized size = 0.62

$$-\frac{\operatorname{Log}\left[-\frac{1}{3} + x\right]}{5} + \frac{\operatorname{Log}[-2 + x]}{5}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(3*x^2 - 7*x + 2),x]')

[Out] -Log[-1 / 3 + x] / 5 + Log[-2 + x] / 5

Maple [A]

time = 0.08, size = 16, normalized size = 0.76

method	result	size
default	$\frac{\ln(-2+x)}{5} - \frac{\ln(3x-1)}{5}$	16
norman	$\frac{\ln(-2+x)}{5} - \frac{\ln(3x-1)}{5}$	16
risch	$\frac{\ln(-2+x)}{5} - \frac{\ln(3x-1)}{5}$	16

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x^2-7*x+2),x,method=_RETURNVERBOSE)

[Out] 1/5*ln(-2+x)-1/5*ln(3*x-1)

Maxima [A]

time = 0.27, size = 15, normalized size = 0.71

$$-\frac{1}{5} \log(3x - 1) + \frac{1}{5} \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2-7*x+2),x, algorithm="maxima")

[Out] -1/5*log(3*x - 1) + 1/5*log(x - 2)

Fricas [A]

time = 0.32, size = 15, normalized size = 0.71

$$-\frac{1}{5} \log(3x - 1) + \frac{1}{5} \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2-7*x+2),x, algorithm="fricas")

[Out] -1/5*log(3*x - 1) + 1/5*log(x - 2)

Sympy [A]

time = 0.05, size = 14, normalized size = 0.67

$$\frac{\log(x-2)}{5} - \frac{\log\left(x - \frac{1}{3}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x**2-7*x+2),x)

[Out] log(x - 2)/5 - log(x - 1/3)/5

Giac [A]

time = 0.00, size = 20, normalized size = 0.95

$$\frac{\ln|x-2|}{5} - \frac{\ln|3x-1|}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2-7*x+2),x)

[Out] -1/5*log(abs(3*x - 1)) + 1/5*log(abs(x - 2))

Mupad [B]

time = 0.09, size = 8, normalized size = 0.38

$$-\frac{2 \operatorname{atanh}\left(\frac{6x}{5} - \frac{7}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x^2 - 7*x + 2),x)

[Out] -(2*atanh((6*x)/5 - 7/5))/5

3.93 $\int \frac{-1+3x}{1-x+x^2} dx$

Optimal. Leaf size=33

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2} \log(1-x+x^2)$$

[Out] 3/2*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {648, 632, 210, 642}

$$\frac{3}{2} \log(x^2 - x + 1) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + 3*x)/(1 - x + x^2), x]

[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + (3*Log[1 - x + x^2])/2

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

`[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rubi steps

$$\begin{aligned} \int \frac{-1+3x}{1-x+x^2} dx &= \frac{1}{2} \int \frac{1}{1-x+x^2} dx + \frac{3}{2} \int \frac{-1+2x}{1-x+x^2} dx \\ &= \frac{3}{2} \log(1-x+x^2) - \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x \right) \\ &= \frac{\tan^{-1} \left(\frac{-1+2x}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{3}{2} \log(1-x+x^2) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 32, normalized size = 0.97

$$\frac{\tan^{-1} \left(\frac{-1+2x}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{3}{2} \log(1-x+x^2)$$

Antiderivative was successfully verified.

`[In] Integrate[(-1 + 3*x)/(1 - x + x^2), x]`

`[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + (3*Log[1 - x + x^2])/2`

Mathics [A]

time = 2.05, size = 28, normalized size = 0.85

$$\frac{\sqrt{3} \text{ArcTan} \left[\frac{\sqrt{3}(-1+2x)}{3} \right]}{3} + \frac{3 \text{Log}[1-x+x^2]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(3*x - 1)/(x^2 - x + 1), x]')`

`[Out] Sqrt[3] ArcTan[Sqrt[3] (-1 + 2 x) / 3] / 3 + 3 Log[1 - x + x ^ 2] / 2`

Maple [A]

time = 0.17, size = 29, normalized size = 0.88

method	result	size
--------	--------	------

default	$\frac{3 \ln(x^2 - x + 1)}{2} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	29
risch	$\frac{3 \ln(4x^2 - 4x + 4)}{2} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	31

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x-1)/(x^2-x+1),x,method=_RETURNVERBOSE)`

[Out] $3/2*\ln(x^2-x+1)+1/3*3^{(1/2)}*\arctan(1/3*(2*x-1)*3^{(1/2)})$

Maxima [A]

time = 0.36, size = 28, normalized size = 0.85

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{3}{2} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+3*x)/(x^2-x+1),x, algorithm="maxima")`

[Out] $1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) + 3/2*\log(x^2 - x + 1)$

Fricas [A]

time = 0.32, size = 28, normalized size = 0.85

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{3}{2} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+3*x)/(x^2-x+1),x, algorithm="fricas")`

[Out] $1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) + 3/2*\log(x^2 - x + 1)$

Sympy [A]

time = 0.06, size = 36, normalized size = 1.09

$$\frac{3 \log(x^2 - x + 1)}{2} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+3*x)/(x**2-x+1),x)`

[Out] $3*\log(x**2 - x + 1)/2 + \sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 - \sqrt{3}/3)/3$

Giac [A]

time = 0.00, size = 33, normalized size = 1.00

$$\frac{3}{2} \ln(x^2 - x + 1) + \frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((-1+3*x)/(x^2-x+1),x)``[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/2*log(x^2 - x + 1)`**Mupad [B]**

time = 0.04, size = 30, normalized size = 0.91

$$\frac{3 \ln(x^2 - x + 1)}{2} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x - \sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((3*x - 1)/(x^2 - x + 1),x)``[Out] (3*log(x^2 - x + 1))/2 + (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3`

3.94 $\int \frac{x^2}{5+2x+x^2} dx$

Optimal. Leaf size=25

$$x - \frac{3}{2} \tan^{-1} \left(\frac{1+x}{2} \right) - \log(5 + 2x + x^2)$$

[Out] x-3/2*arctan(1/2+1/2*x)-ln(x^2+2*x+5)

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {717, 648, 632, 210, 642}

$$-\log(x^2 + 2x + 5) + x - \frac{3}{2} \tan^{-1} \left(\frac{x+1}{2} \right)$$

Antiderivative was successfully verified.

[In] Int[x^2/(5 + 2*x + x^2),x]

[Out] x - (3*ArcTan[(1 + x)/2])/2 - Log[5 + 2*x + x^2]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] := Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
+ a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{5 + 2x + x^2} dx &= x + \int \frac{-5 - 2x}{5 + 2x + x^2} dx \\ &= x - 3 \int \frac{1}{5 + 2x + x^2} dx - \int \frac{2 + 2x}{5 + 2x + x^2} dx \\ &= x - \log(5 + 2x + x^2) + 6 \text{Subst} \left(\int \frac{1}{-16 - x^2} dx, x, 2 + 2x \right) \\ &= x - \frac{3}{2} \tan^{-1} \left(\frac{1 + x}{2} \right) - \log(5 + 2x + x^2) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 25, normalized size = 1.00

$$x - \frac{3}{2} \tan^{-1} \left(\frac{1 + x}{2} \right) - \log(5 + 2x + x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(5 + 2*x + x^2), x]
```

```
[Out] x - (3*ArcTan[(1 + x)/2])/2 - Log[5 + 2*x + x^2]
```

Mathics [A]

time = 1.89, size = 21, normalized size = 0.84

$$x - \frac{3 \text{ArcTan} \left[\frac{1}{2} + \frac{x}{2} \right]}{2} - \text{Log} [5 + 2x + x^2]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2/(5 + 2*x + x^2), x]')
```

```
[Out] x - 3 ArcTan[1 / 2 + x / 2] / 2 - Log[5 + 2 x + x ^ 2]
```

Maple [A]

time = 0.08, size = 22, normalized size = 0.88

method	result	size
default	$x - \frac{3 \arctan(\frac{1}{2} + \frac{x}{2})}{2} - \ln(x^2 + 2x + 5)$	22
risch	$x - \frac{3 \arctan(\frac{1}{2} + \frac{x}{2})}{2} - \ln(x^2 + 2x + 5)$	22

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(x^2+2*x+5),x,method=_RETURNVERBOSE)`

[Out] `x-3/2*arctan(1/2+1/2*x)-ln(x^2+2*x+5)`

Maxima [A]

time = 0.36, size = 21, normalized size = 0.84

$$x - \frac{3}{2} \arctan\left(\frac{1}{2}x + \frac{1}{2}\right) - \log(x^2 + 2x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(x^2+2*x+5),x, algorithm="maxima")`

[Out] `x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)`

Fricas [A]

time = 0.31, size = 21, normalized size = 0.84

$$x - \frac{3}{2} \arctan\left(\frac{1}{2}x + \frac{1}{2}\right) - \log(x^2 + 2x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(x^2+2*x+5),x, algorithm="fricas")`

[Out] `x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)`

Sympy [A]

time = 0.05, size = 22, normalized size = 0.88

$$x - \log(x^2 + 2x + 5) - \frac{3 \operatorname{atan}\left(\frac{x}{2} + \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(x**2+2*x+5),x)`

[Out] `x - log(x**2 + 2*x + 5) - 3*atan(x/2 + 1/2)/2`

Giac [A]

time = 0.00, size = 23, normalized size = 0.92

$$x - \ln(x^2 + 2x + 5) - \frac{3}{2} \arctan\left(\frac{x + 1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(x^2+2*x+5),x)`

[Out] `x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)`

Mupad [B]

time = 0.04, size = 21, normalized size = 0.84

$$x - \ln(x^2 + 2x + 5) - \frac{3 \operatorname{atan}\left(\frac{x}{2} + \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(2*x + x^2 + 5),x)`

[Out] `x - log(2*x + x^2 + 5) - (3*atan(x/2 + 1/2))/2`

3.95

$$\int \frac{4x^2 - 5x^3 + 6x^4}{1 - x + 2x^2} dx$$

Optimal. Leaf size=47

$$-\frac{x^2}{2} + x^3 - \frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{2\sqrt{7}} + \frac{1}{4} \log(1 - x + 2x^2)$$

[Out] $-1/2*x^2+x^3+1/4*\ln(2*x^2-x+1)-1/14*\arctan(1/7*(1-4*x)*7^(1/2))*7^(1/2)$

Rubi [A]

time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1608, 1642, 648, 632, 210, 642}

$$x^3 - \frac{x^2}{2} + \frac{1}{4} \log(2x^2 - x + 1) - \frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(4*x^2 - 5*x^3 + 6*x^4)/(1 - x + 2*x^2), x]$

[Out] $-1/2*x^2 + x^3 - \text{ArcTan}[(1 - 4*x)/\text{Sqrt}[7]]/(2*\text{Sqrt}[7]) + \text{Log}[1 - x + 2*x^2]/4$

Rule 210

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 632

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[(d_ + (e_)*(x_))/((a_ + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 648

$\text{Int}[(d_ + (e_)*(x_))/((a_ + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{In}$

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1608

```
Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x
_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{4x^2 - 5x^3 + 6x^4}{1 - x + 2x^2} dx &= \int \frac{x^2(4 - 5x + 6x^2)}{1 - x + 2x^2} dx \\
&= \int \left(-x + 3x^2 + \frac{x}{1 - x + 2x^2} \right) dx \\
&= -\frac{x^2}{2} + x^3 + \int \frac{x}{1 - x + 2x^2} dx \\
&= -\frac{x^2}{2} + x^3 + \frac{1}{4} \int \frac{1}{1 - x + 2x^2} dx + \frac{1}{4} \int \frac{-1 + 4x}{1 - x + 2x^2} dx \\
&= -\frac{x^2}{2} + x^3 + \frac{1}{4} \log(1 - x + 2x^2) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{-7 - x^2} dx, x, -1 + 4x \right) \\
&= -\frac{x^2}{2} + x^3 - \frac{\tan^{-1} \left(\frac{1-4x}{\sqrt{7}} \right)}{2\sqrt{7}} + \frac{1}{4} \log(1 - x + 2x^2)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 47, normalized size = 1.00

$$-\frac{x^2}{2} + x^3 + \frac{\tan^{-1} \left(\frac{-1+4x}{\sqrt{7}} \right)}{2\sqrt{7}} + \frac{1}{4} \log(1 - x + 2x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*x^2 - 5*x^3 + 6*x^4)/(1 - x + 2*x^2), x]
```

[Out] $-1/2*x^2 + x^3 + \text{ArcTan}[(-1 + 4*x)/\text{Sqrt}[7]]/(2*\text{Sqrt}[7]) + \text{Log}[1 - x + 2*x^2]/4$

Mathics [A]

time = 2.08, size = 36, normalized size = 0.77

$$-\frac{x^2}{2} + x^3 + \frac{\sqrt{7} \text{ArcTan}\left[\frac{\sqrt{7}(-1+4x)}{7}\right]}{14} + \frac{\text{Log}\left[\frac{1}{2} - \frac{x}{2} + x^2\right]}{4}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(6*x^4 - 5*x^3 + 4*x^2)/(2*x^2 - x + 1),x]')`

[Out] $-x^2/2 + x^3 + \text{Sqrt}[7] \text{ArcTan}[\text{Sqrt}[7](-1 + 4x)/7]/14 + \text{Log}[1 - x/2 + x^2]/4$

Maple [A]

time = 0.12, size = 39, normalized size = 0.83

method	result	size
default	$x^3 - \frac{x^2}{2} + \frac{\ln(2x^2-x+1)}{4} + \frac{\sqrt{7} \arctan\left(\frac{(-1+4x)\sqrt{7}}{7}\right)}{14}$	39
risch	$x^3 - \frac{x^2}{2} + \frac{\ln(16x^2-8x+8)}{4} + \frac{\sqrt{7} \arctan\left(\frac{(-1+4x)\sqrt{7}}{7}\right)}{14}$	39

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x,method=_RETURNVERBOSE)`

[Out] $x^3 - 1/2*x^2 + 1/4*\ln(2*x^2-x+1) + 1/14*7^{(1/2)}*\arctan(1/7*(-1+4*x)*7^{(1/2)})$

Maxima [A]

time = 0.36, size = 38, normalized size = 0.81

$$x^3 - \frac{1}{2}x^2 + \frac{1}{14}\sqrt{7} \arctan\left(\frac{1}{7}\sqrt{7}(4x-1)\right) + \frac{1}{4}\log(2x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x, algorithm="maxima")`

[Out] $x^3 - 1/2*x^2 + 1/14*\text{sqrt}(7)*\arctan(1/7*\text{sqrt}(7)*(4*x - 1)) + 1/4*\log(2*x^2 - x + 1)$

Fricas [A]

time = 0.32, size = 38, normalized size = 0.81

$$x^3 - \frac{1}{2}x^2 + \frac{1}{14}\sqrt{7} \arctan\left(\frac{1}{7}\sqrt{7}(4x-1)\right) + \frac{1}{4}\log(2x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x, algorithm="fricas")

[Out] $x^3 - 1/2*x^2 + 1/14*\sqrt{7}*\arctan(1/7*\sqrt{7}*(4*x - 1)) + 1/4*\log(2*x^2 - x + 1)$

Sympy [A]

time = 0.06, size = 46, normalized size = 0.98

$$x^3 - \frac{x^2}{2} + \frac{\log\left(x^2 - \frac{x}{2} + \frac{1}{2}\right)}{4} + \frac{\sqrt{7} \operatorname{atan}\left(\frac{4\sqrt{7}x}{7} - \frac{\sqrt{7}}{7}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((6*x**4-5*x**3+4*x**2)/(2*x**2-x+1),x)

[Out] $x^{**3} - x^{**2}/2 + \log(x^{**2} - x/2 + 1/2)/4 + \sqrt{7}*\operatorname{atan}(4*\sqrt{7}*x/7 - \sqrt{7}(7)/7)/14$

Giac [A]

time = 0.00, size = 51, normalized size = 1.09

$$\frac{\ln(2x^2 - x + 1)}{4} + \frac{\arctan\left(\frac{4x-1}{\sqrt{7}}\right)}{2\sqrt{7}} + \frac{8x^3 - 4x^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x)

[Out] $x^3 - 1/2*x^2 + 1/14*\sqrt{7}*\arctan(1/7*\sqrt{7}*(4*x - 1)) + 1/4*\log(2*x^2 - x + 1)$

Mupad [B]

time = 0.20, size = 40, normalized size = 0.85

$$\frac{\ln(2x^2 - x + 1)}{4} + \frac{\sqrt{7} \operatorname{atan}\left(\frac{4\sqrt{7}x}{7} - \frac{\sqrt{7}}{7}\right)}{14} - \frac{x^2}{2} + x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2 - 5*x^3 + 6*x^4)/(2*x^2 - x + 1),x)

[Out] $\log(2*x^2 - x + 1)/4 + (7^{(1/2)}*\operatorname{atan}((4*7^{(1/2)}*x)/7 - 7^{(1/2)}/7))/14 - x^2/2 + x^3$

3.96

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3} dx$$

Optimal. Leaf size=25

$$\frac{1}{2} \log(2-x) + \frac{\log(x)}{6} + \frac{1}{3} \log(3+x)$$

[Out] 1/2*ln(2-x)+1/6*ln(x)+1/3*ln(3+x)

Rubi [A]

time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1608, 1642}

$$\frac{1}{2} \log(2-x) + \frac{\log(x)}{6} + \frac{1}{3} \log(x+3)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x + x^2)/(-6*x + x^2 + x^3), x]

[Out] Log[2 - x]/2 + Log[x]/6 + Log[3 + x]/3

Rule 1608

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n_.], x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1642

Int[(Pq_.)*((d_.) + (e_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p_.], x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{-1+x+x^2}{-6x+x^2+x^3} dx &= \int \frac{-1+x+x^2}{x(-6+x+x^2)} dx \\ &= \int \left(\frac{1}{2(-2+x)} + \frac{1}{6x} + \frac{1}{3(3+x)} \right) dx \\ &= \frac{1}{2} \log(2-x) + \frac{\log(x)}{6} + \frac{1}{3} \log(3+x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 25, normalized size = 1.00

$$\frac{1}{2} \log(2 - x) + \frac{\log(x)}{6} + \frac{1}{3} \log(3 + x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x + x^2)/(-6*x + x^2 + x^3), x]

[Out] Log[2 - x]/2 + Log[x]/6 + Log[3 + x]/3

Mathics [A]

time = 1.88, size = 17, normalized size = 0.68

$$\frac{\text{Log}[x]}{6} + \frac{\text{Log}[3 + x]}{3} + \frac{\text{Log}[-2 + x]}{2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(x^2 + x - 1)/(x^3 + x^2 - 6*x), x]')

[Out] Log[x] / 6 + Log[3 + x] / 3 + Log[-2 + x] / 2

Maple [A]

time = 0.02, size = 18, normalized size = 0.72

method	result	size
default	$\frac{\ln(x)}{6} + \frac{\ln(-2+x)}{2} + \frac{\ln(3+x)}{3}$	18
norman	$\frac{\ln(x)}{6} + \frac{\ln(-2+x)}{2} + \frac{\ln(3+x)}{3}$	18
risch	$\frac{\ln(x)}{6} + \frac{\ln(-2+x)}{2} + \frac{\ln(3+x)}{3}$	18

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+x-1)/(x^3+x^2-6*x), x, method=_RETURNVERBOSE)

[Out] 1/6*ln(x)+1/2*ln(-2+x)+1/3*ln(3+x)

Maxima [A]

time = 0.27, size = 17, normalized size = 0.68

$$\frac{1}{3} \log(x + 3) + \frac{1}{2} \log(x - 2) + \frac{1}{6} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x-1)/(x^3+x^2-6*x), x, algorithm="maxima")

[Out] 1/3*log(x + 3) + 1/2*log(x - 2) + 1/6*log(x)

Fricas [A]

time = 0.32, size = 17, normalized size = 0.68

$$\frac{1}{3} \log(x + 3) + \frac{1}{2} \log(x - 2) + \frac{1}{6} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x-1)/(x^3+x^2-6*x),x, algorithm="fricas")

[Out] 1/3*log(x + 3) + 1/2*log(x - 2) + 1/6*log(x)

Sympy [A]

time = 0.07, size = 17, normalized size = 0.68

$$\frac{\log(x)}{6} + \frac{\log(x - 2)}{2} + \frac{\log(x + 3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+x-1)/(x**3+x**2-6*x),x)

[Out] log(x)/6 + log(x - 2)/2 + log(x + 3)/3

Giac [A]

time = 0.00, size = 23, normalized size = 0.92

$$\frac{\ln|x|}{6} + \frac{\ln|x - 2|}{2} + \frac{\ln|x + 3|}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x-1)/(x^3+x^2-6*x),x)

[Out] 1/3*log(abs(x + 3)) + 1/2*log(abs(x - 2)) + 1/6*log(abs(x))

Mupad [B]

time = 0.23, size = 17, normalized size = 0.68

$$\frac{\ln(x - 2)}{2} + \frac{\ln(x + 3)}{3} + \frac{\ln(x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^2 - 1)/(x^2 - 6*x + x^3),x)

[Out] log(x - 2)/2 + log(x + 3)/3 + log(x)/6

$$3.97 \quad \int \frac{11a^2 - 7ax + 5x^2}{-6a^3 + 11a^2x - 6ax^2 + x^3} dx$$

Optimal. Leaf size=33

$$\frac{9}{2} \log(a - x) - 17 \log(2a - x) + \frac{35}{2} \log(3a - x)$$

[Out] 9/2*ln(a-x)-17*ln(2*a-x)+35/2*ln(3*a-x)

Rubi [A]

time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {2099}

$$\frac{9}{2} \log(a - x) - 17 \log(2a - x) + \frac{35}{2} \log(3a - x)$$

Antiderivative was successfully verified.

[In] Int[(11*a^2 - 7*a*x + 5*x^2)/(-6*a^3 + 11*a^2*x - 6*a*x^2 + x^3),x]

[Out] (9*Log[a - x])/2 - 17*Log[2*a - x] + (35*Log[3*a - x])/2

Rule 2099

Int[(P_)^(p_)*(Q_)^(q_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned} \int \frac{11a^2 - 7ax + 5x^2}{-6a^3 + 11a^2x - 6ax^2 + x^3} dx &= \int \left(-\frac{9}{2(a-x)} + \frac{17}{2a-x} - \frac{35}{2(3a-x)} \right) dx \\ &= \frac{9}{2} \log(a-x) - 17 \log(2a-x) + \frac{35}{2} \log(3a-x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 29, normalized size = 0.88

$$\frac{35}{2} \log(-3a + x) - 17 \log(-2a + x) + \frac{9}{2} \log(-a + x)$$

Antiderivative was successfully verified.

[In] Integrate[(11*a^2 - 7*a*x + 5*x^2)/(-6*a^3 + 11*a^2*x - 6*a*x^2 + x^3),x]

[Out] $(35*\text{Log}[-3*a + x])/2 - 17*\text{Log}[-2*a + x] + (9*\text{Log}[-a + x])/2$

Mathics [A]

time = 2.05, size = 25, normalized size = 0.76

$$-17\text{Log}[-2a + x] + \frac{9\text{Log}[-a + x]}{2} + \frac{35\text{Log}[-3a + x]}{2}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(5*x^2 - 7*a*x + 11*a^2)/(x^3 - 6*a x^2 + 11*a^2*x - 6*a^3),x]')`

[Out] $-17 \text{Log}[-2 a + x] + 9 \text{Log}[-a + x] / 2 + 35 \text{Log}[-3 a + x] / 2$

Maple [A]

time = 0.02, size = 30, normalized size = 0.91

method	result	size
risch	$\frac{9 \ln(-a+x)}{2} - 17 \ln(x - 2a) + \frac{35 \ln(-3a+x)}{2}$	26
default	$\frac{9 \ln(a-x)}{2} - 17 \ln(-x + 2a) + \frac{35 \ln(3a-x)}{2}$	30
norman	$\frac{9 \ln(a-x)}{2} - 17 \ln(-x + 2a) + \frac{35 \ln(3a-x)}{2}$	30

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x,method=_RETURNVERBOSE)`

[Out] $9/2*\ln(a-x)-17*\ln(-x+2*a)+35/2*\ln(3*a-x)$

Maxima [A]

time = 0.27, size = 25, normalized size = 0.76

$$\frac{9}{2} \log(-a + x) - 17 \log(-2a + x) + \frac{35}{2} \log(-3a + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x, algorithm="maxima")`

[Out] $9/2*\log(-a + x) - 17*\log(-2*a + x) + 35/2*\log(-3*a + x)$

Fricas [A]

time = 0.32, size = 25, normalized size = 0.76

$$\frac{9}{2} \log(-a + x) - 17 \log(-2a + x) + \frac{35}{2} \log(-3a + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x, algorithm="fricas")

[Out] 9/2*log(-a + x) - 17*log(-2*a + x) + 35/2*log(-3*a + x)

Sympy [A]

time = 0.13, size = 26, normalized size = 0.79

$$\frac{35 \log(-3a + x)}{2} - 17 \log(-2a + x) + \frac{9 \log(-a + x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((11*a**2-7*a*x+5*x**2)/(-6*a**3+11*a**2*x-6*a*x**2+x**3),x)

[Out] 35*log(-3*a + x)/2 - 17*log(-2*a + x) + 9*log(-a + x)/2

Giac [A]

time = 0.00, size = 34, normalized size = 1.03

$$\frac{35}{2} \ln|x - 3a| - 17 \ln|x - 2a| + \frac{9}{2} \ln|x - a|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x)

[Out] 9/2*log(abs(-a + x)) - 17*log(abs(-2*a + x)) + 35/2*log(abs(-3*a + x))

Mupad [B]

time = 0.09, size = 25, normalized size = 0.76

$$\frac{9 \ln(x - a)}{2} - 17 \ln(x - 2a) + \frac{35 \ln(x - 3a)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(11*a^2 - 7*a*x + 5*x^2)/(6*a*x^2 - 11*a^2*x + 6*a^3 - x^3),x)

[Out] (9*log(x - a))/2 - 17*log(x - 2*a) + (35*log(x - 3*a))/2

$$3.98 \quad \int \frac{2-x+x^2}{4-5x^2+x^4} dx$$

Optimal. Leaf size=37

$$-\frac{1}{3} \log(1-x) + \frac{1}{3} \log(2-x) + \frac{2}{3} \log(1+x) - \frac{2}{3} \log(2+x)$$

[Out] -1/3*ln(1-x)+1/3*ln(2-x)+2/3*ln(1+x)-2/3*ln(2+x)

Rubi [A]

time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.65, number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1687, 1175, 630, 31, 1121}

$$\frac{1}{6} \log(1-x^2) - \frac{1}{6} \log(4-x^2) - \frac{1}{2} \log(1-x) + \frac{1}{2} \log(2-x) + \frac{1}{2} \log(x+1) - \frac{1}{2} \log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(2 - x + x^2)/(4 - 5*x^2 + x^4), x]

[Out] -1/2*Log[1 - x] + Log[2 - x]/2 + Log[1 + x]/2 - Log[2 + x]/2 + Log[1 - x^2]/6 - Log[4 - x^2]/6

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 630

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 1121

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1175

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[2*(d/e) - b/c, 0] || (!LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],

0]))

Rule 1687

```
Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Module[{q
= Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
&& !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned} \int \frac{2-x+x^2}{4-5x^2+x^4} dx &= -\int \frac{x}{4-5x^2+x^4} dx + \int \frac{2+x^2}{4-5x^2+x^4} dx \\ &= \frac{1}{2} \int \frac{1}{2-3x+x^2} dx + \frac{1}{2} \int \frac{1}{2+3x+x^2} dx - \frac{1}{2} \text{Subst} \left(\int \frac{1}{4-5x+x^2} dx, x, x^2 \right) \\ &= -\left(\frac{1}{6} \text{Subst} \left(\int \frac{1}{-4+x} dx, x, x^2 \right) \right) + \frac{1}{6} \text{Subst} \left(\int \frac{1}{-1+x} dx, x, x^2 \right) + \frac{1}{2} \int \frac{1}{-2+x} dx \\ &= -\frac{1}{2} \log(1-x) + \frac{1}{2} \log(2-x) + \frac{1}{2} \log(1+x) - \frac{1}{2} \log(2+x) + \frac{1}{6} \log(1-x^2) - \frac{1}{6} \log(1-x^2) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 37, normalized size = 1.00

$$-\frac{1}{3} \log(1-x) + \frac{1}{3} \log(2-x) + \frac{2}{3} \log(1+x) - \frac{2}{3} \log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + x^2)/(4 - 5*x^2 + x^4), x]
```

```
[Out] -1/3*Log[1 - x] + Log[2 - x]/3 + (2*Log[1 + x])/3 - (2*Log[2 + x])/3
```

Mathics [A]

time = 2.03, size = 25, normalized size = 0.68

$$\frac{-2\text{Log}[2+x]}{3} - \frac{\text{Log}[-1+x]}{3} + \frac{\text{Log}[-2+x]}{3} + \frac{2\text{Log}[1+x]}{3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(x^2 - x + 2)/(x^4 - 5*x^2 + 4), x]')
```

```
[Out] -2 Log[2 + x] / 3 - Log[-1 + x] / 3 + Log[-2 + x] / 3 + 2 Log[1 + x] / 3
```

Maple [A]

time = 0.02, size = 26, normalized size = 0.70

method	result	size
default	$\frac{\ln(-2+x)}{3} - \frac{\ln(-1+x)}{3} - \frac{2\ln(2+x)}{3} + \frac{2\ln(1+x)}{3}$	26
norman	$\frac{\ln(-2+x)}{3} - \frac{\ln(-1+x)}{3} - \frac{2\ln(2+x)}{3} + \frac{2\ln(1+x)}{3}$	26
risch	$\frac{\ln(-2+x)}{3} - \frac{\ln(-1+x)}{3} - \frac{2\ln(2+x)}{3} + \frac{2\ln(1+x)}{3}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+2)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(-2+x)-1/3*ln(-1+x)-2/3*ln(2+x)+2/3*ln(1+x)
```

Maxima [A]

time = 0.31, size = 25, normalized size = 0.68

$$-\frac{2}{3} \log(x+2) + \frac{2}{3} \log(x+1) - \frac{1}{3} \log(x-1) + \frac{1}{3} \log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] -2/3*log(x + 2) + 2/3*log(x + 1) - 1/3*log(x - 1) + 1/3*log(x - 2)
```

Fricas [A]

time = 0.33, size = 25, normalized size = 0.68

$$-\frac{2}{3} \log(x+2) + \frac{2}{3} \log(x+1) - \frac{1}{3} \log(x-1) + \frac{1}{3} \log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] -2/3*log(x + 2) + 2/3*log(x + 1) - 1/3*log(x - 1) + 1/3*log(x - 2)
```

Sympy [A]

time = 0.09, size = 29, normalized size = 0.78

$$\frac{\log(x-2)}{3} - \frac{\log(x-1)}{3} + \frac{2\log(x+1)}{3} - \frac{2\log(x+2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+2)/(x**4-5*x**2+4),x)
```

```
[Out] log(x - 2)/3 - log(x - 1)/3 + 2*log(x + 1)/3 - 2*log(x + 2)/3
```

Giac [A]

time = 0.00, size = 36, normalized size = 0.97

$$\frac{\ln|x-2|}{3} - \frac{\ln|x-1|}{3} + \frac{2}{3}\ln|x+1| - \frac{2}{3}\ln|x+2|$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((x^2-x+2)/(x^4-5*x^2+4),x)``[Out] -2/3*log(abs(x + 2)) + 2/3*log(abs(x + 1)) - 1/3*log(abs(x - 1)) + 1/3*log(abs(x - 2))`**Mupad [B]**

time = 0.06, size = 29, normalized size = 0.78

$$\frac{2 \operatorname{atanh}\left(\frac{64}{3(24x-16)} - \frac{5}{3}\right)}{3} + \frac{4 \operatorname{atanh}\left(\frac{128}{3(48x+32)} + \frac{5}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((x^2 - x + 2)/(x^4 - 5*x^2 + 4),x)``[Out] (2*atanh(64/(3*(24*x - 16)) - 5/3))/3 + (4*atanh(128/(3*(48*x + 32)) + 5/3))/3`

$$3.99 \quad \int \frac{-5+2x^2}{6-5x^2+x^4} dx$$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] $-1/2*\operatorname{arctanh}(1/2*x*2^{(1/2)})*2^{(1/2)}-1/3*\operatorname{arctanh}(1/3*x*3^{(1/2)})*3^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1180, 213}

$$-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(-5 + 2*x^2)/(6 - 5*x^2 + x^4), x]$

[Out] $-(\operatorname{ArcTanh}[x/\operatorname{Sqrt}[2]]/\operatorname{Sqrt}[2]) - \operatorname{ArcTanh}[x/\operatorname{Sqrt}[3]]/\operatorname{Sqrt}[3]$

Rule 213

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2])^{-1})*\operatorname{ArcTanh}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{LtQ}[a, 0] \ || \ \operatorname{GtQ}[b, 0])$

Rule 1180

$\operatorname{Int}[(d_ + (e_)*(x_)^2)/((a_ + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :$
 $> \operatorname{With}\{q = \operatorname{Rt}[b^2 - 4*a*c, 2]\}, \operatorname{Dist}[e/2 + (2*c*d - b*e)/(2*q), \operatorname{Int}[1/(b/2 - q/2 + c*x^2), x], x] + \operatorname{Dist}[e/2 - (2*c*d - b*e)/(2*q), \operatorname{Int}[1/(b/2 + q/2 + c*x^2), x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e\}, x \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \ \&\& \operatorname{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \operatorname{PosQ}[b^2 - 4*a*c]$

Rubi steps

$$\begin{aligned} \int \frac{-5+2x^2}{6-5x^2+x^4} dx &= \int \frac{1}{-3+x^2} dx + \int \frac{1}{-2+x^2} dx \\ &= -\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 69 vs. $2(31) = 62$.

time = 0.01, size = 69, normalized size = 2.23

$$\frac{1}{12} \left(3\sqrt{2} \log(\sqrt{2} - x) + 2\sqrt{3} \log(\sqrt{3} - x) - 3\sqrt{2} \log(\sqrt{2} + x) - 2\sqrt{3} \log(\sqrt{3} + x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-5 + 2*x^2)/(6 - 5*x^2 + x^4), x]

[Out] (3*Sqrt[2]*Log[Sqrt[2] - x] + 2*Sqrt[3]*Log[Sqrt[3] - x] - 3*Sqrt[2]*Log[Sqrt[2] + x] - 2*Sqrt[3]*Log[Sqrt[3] + x])/12

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 73 vs. $2(31) = 62$.

time = 2.54, size = 49, normalized size = 1.58

$$-\frac{\sqrt{2} \operatorname{Log}[\sqrt{2} + x]}{4} - \frac{\sqrt{3} \operatorname{Log}[\sqrt{3} + x]}{6} + \frac{\sqrt{3} \operatorname{Log}[-\sqrt{3} + x]}{6} + \frac{\sqrt{2} \operatorname{Log}[-\sqrt{2} + x]}{4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(2*x^2 - 5)/(x^4 - 5*x^2 + 6), x]')

[Out] -Sqrt[2] Log[Sqrt[2] + x] / 4 - Sqrt[3] Log[Sqrt[3] + x] / 6 + Sqrt[3] Log[-Sqrt[3] + x] / 6 + Sqrt[2] Log[-Sqrt[2] + x] / 4

Maple [A]

time = 0.02, size = 26, normalized size = 0.84

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2} - \frac{\operatorname{arctanh}\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}$	26
risch	$\frac{\sqrt{2} \ln(x - \sqrt{2})}{4} - \frac{\sqrt{2} \ln(x + \sqrt{2})}{4} + \frac{\sqrt{3} \ln(x - \sqrt{3})}{6} - \frac{\sqrt{3} \ln(x + \sqrt{3})}{6}$	50

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-5)/(x^4-5*x^2+6), x, method=_RETURNVERBOSE)

[Out] -1/2*arctanh(1/2*x*2^(1/2))*2^(1/2)-1/3*arctanh(1/3*x*3^(1/2))*3^(1/2)

Maxima [A]

time = 0.35, size = 43, normalized size = 1.39

$$\frac{1}{6} \sqrt{3} \log\left(\frac{x - \sqrt{3}}{x + \sqrt{3}}\right) + \frac{1}{4} \sqrt{2} \log\left(\frac{x - \sqrt{2}}{x + \sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-5)/(x^4-5*x^2+6),x, algorithm="maxima")

[Out] 1/6*sqrt(3)*log((x - sqrt(3))/(x + sqrt(3))) + 1/4*sqrt(2)*log((x - sqrt(2))/(x + sqrt(2)))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(25) = 50.

time = 0.34, size = 51, normalized size = 1.65

$$\frac{1}{4} \sqrt{2} \log \left(\frac{x^2 - 2\sqrt{2}x + 2}{x^2 - 2} \right) + \frac{1}{6} \sqrt{3} \log \left(\frac{x^2 - 2\sqrt{3}x + 3}{x^2 - 3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-5)/(x^4-5*x^2+6),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*x + 2)/(x^2 - 2)) + 1/6*sqrt(3)*log((x^2 - 2*sqrt(3)*x + 3)/(x^2 - 3))

Sympy [A]

time = 0.32, size = 60, normalized size = 1.94

$$\frac{\sqrt{2} \log(x - \sqrt{2})}{4} - \frac{\sqrt{2} \log(x + \sqrt{2})}{4} + \frac{\sqrt{3} \log(x - \sqrt{3})}{6} - \frac{\sqrt{3} \log(x + \sqrt{3})}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-5)/(x**4-5*x**2+6),x)

[Out] sqrt(2)*log(x - sqrt(2))/4 - sqrt(2)*log(x + sqrt(2))/4 + sqrt(3)*log(x - sqrt(3))/6 - sqrt(3)*log(x + sqrt(3))/6

Giac [A]

time = 0.00, size = 73, normalized size = 2.35

$$\frac{\ln \left| \frac{2x-2\sqrt{3}}{2x+2\sqrt{3}} \right|}{2\sqrt{3}} + \frac{\ln \left| \frac{2x-2\sqrt{2}}{2x+2\sqrt{2}} \right|}{2\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-5)/(x^4-5*x^2+6),x)

[Out] 1/6*sqrt(3)*log(abs((x - sqrt(3))/(x + sqrt(3)))) + 1/4*sqrt(2)*log(abs((x - sqrt(2))/(x + sqrt(2))))

Mupad [B]

time = 0.08, size = 25, normalized size = 0.81

$$-\frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2} x}{2}\right)}{2} - \frac{\sqrt{3} \operatorname{atanh}\left(\frac{\sqrt{3} x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - 5)/(x^4 - 5*x^2 + 6),x)`

[Out] `-(2^(1/2)*atanh((2^(1/2)*x)/2))/2 - (3^(1/2)*atanh((3^(1/2)*x)/3))/3`

$$3.100 \quad \int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)} dx$$

Optimal. Leaf size=41

$$-\frac{1}{6} \log(1-x) + \frac{1}{2} \log(2-x) - \frac{1}{2} \log(3-x) + \frac{1}{6} \log(4-x)$$

[Out] -1/6*ln(1-x)+1/2*ln(2-x)-1/2*ln(3-x)+1/6*ln(4-x)

Rubi [A]

time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {186}

$$-\frac{1}{6} \log(1-x) + \frac{1}{2} \log(2-x) - \frac{1}{2} \log(3-x) + \frac{1}{6} \log(4-x)$$

Antiderivative was successfully verified.

[In] Int[1/((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)),x]

[Out] -1/6*Log[1 - x] + Log[2 - x]/2 - Log[3 - x]/2 + Log[4 - x]/6

Rule 186

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && IntegersQ[p, q]

Rubi steps

$$\begin{aligned} \int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)} dx &= \int \left(\frac{1}{6(-4+x)} - \frac{1}{2(-3+x)} + \frac{1}{2(-2+x)} - \frac{1}{6(-1+x)} \right) dx \\ &= -\frac{1}{6} \log(1-x) + \frac{1}{2} \log(2-x) - \frac{1}{2} \log(3-x) + \frac{1}{6} \log(4-x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 41, normalized size = 1.00

$$-\frac{1}{6} \log(1-x) + \frac{1}{2} \log(2-x) - \frac{1}{2} \log(3-x) + \frac{1}{6} \log(4-x)$$

Antiderivative was successfully verified.

[In] Integrate[1/((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)),x]

[Out] $-1/6*\text{Log}[1 - x] + \text{Log}[2 - x]/2 - \text{Log}[3 - x]/2 + \text{Log}[4 - x]/6$

Mathics [A]

time = 2.04, size = 25, normalized size = 0.61

$$-\frac{\text{Log}[-3 + x]}{2} - \frac{\text{Log}[-1 + x]}{6} + \frac{\text{Log}[-4 + x]}{6} + \frac{\text{Log}[-2 + x]}{2}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/((x - 1)*(x - 2)*(x - 3)*(x - 4)),x]')`

[Out] $-\text{Log}[-3 + x] / 2 - \text{Log}[-1 + x] / 6 + \text{Log}[-4 + x] / 6 + \text{Log}[-2 + x] / 2$

Maple [A]

time = 0.08, size = 26, normalized size = 0.63

method	result	size
default	$-\frac{\ln(-3+x)}{2} + \frac{\ln(-2+x)}{2} - \frac{\ln(-1+x)}{6} + \frac{\ln(x-4)}{6}$	26
norman	$-\frac{\ln(-3+x)}{2} + \frac{\ln(-2+x)}{2} - \frac{\ln(-1+x)}{6} + \frac{\ln(x-4)}{6}$	26
risch	$-\frac{\ln(-3+x)}{2} + \frac{\ln(-2+x)}{2} - \frac{\ln(-1+x)}{6} + \frac{\ln(x-4)}{6}$	26

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x-4)/(-3+x)/(-2+x)/(-1+x),x,method=_RETURNVERBOSE)`

[Out] $-1/2*\ln(-3+x)+1/2*\ln(-2+x)-1/6*\ln(-1+x)+1/6*\ln(x-4)$

Maxima [A]

time = 0.27, size = 25, normalized size = 0.61

$$-\frac{1}{6} \log(x - 1) + \frac{1}{2} \log(x - 2) - \frac{1}{2} \log(x - 3) + \frac{1}{6} \log(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")`

[Out] $-1/6*\log(x - 1) + 1/2*\log(x - 2) - 1/2*\log(x - 3) + 1/6*\log(x - 4)$

Fricas [A]

time = 0.35, size = 25, normalized size = 0.61

$$-\frac{1}{6} \log(x - 1) + \frac{1}{2} \log(x - 2) - \frac{1}{2} \log(x - 3) + \frac{1}{6} \log(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")`

[Out] $-1/6*\log(x - 1) + 1/2*\log(x - 2) - 1/2*\log(x - 3) + 1/6*\log(x - 4)$

Sympy [A]

time = 0.09, size = 26, normalized size = 0.63

$$\frac{\log(x - 4)}{6} - \frac{\log(x - 3)}{2} + \frac{\log(x - 2)}{2} - \frac{\log(x - 1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x)`

[Out] $\log(x - 4)/6 - \log(x - 3)/2 + \log(x - 2)/2 - \log(x - 1)/6$

Giac [A]

time = 0.00, size = 35, normalized size = 0.85

$$\frac{\ln|x - 4|}{6} - \frac{\ln|x - 3|}{2} + \frac{\ln|x - 2|}{2} - \frac{\ln|x - 1|}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x)`

[Out] $-1/6*\log(\text{abs}(x - 1)) + 1/2*\log(\text{abs}(x - 2)) - 1/2*\log(\text{abs}(x - 3)) + 1/6*\log(\text{abs}(x - 4))$

Mupad [B]

time = 0.25, size = 15, normalized size = 0.37

$$\operatorname{atanh}(2x - 5) - \frac{\operatorname{atanh}\left(\frac{2x}{3} - \frac{5}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((x - 1)*(x - 2)*(x - 3)*(x - 4)),x)`

[Out] $\operatorname{atanh}(2*x - 5) - \operatorname{atanh}((2*x)/3 - 5/3)/3$

$$3.101 \quad \int \frac{1+x^2}{(-1+x)^3} dx$$

Optimal. Leaf size=25

$$-\frac{1}{(1-x)^2} + \frac{2}{1-x} + \log(1-x)$$

[Out] -1/(1-x)^2+2/(1-x)+ln(1-x)

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {711}

$$\frac{2}{1-x} - \frac{1}{(1-x)^2} + \log(1-x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(-1 + x)^3,x]

[Out] -(1 - x)^(-2) + 2/(1 - x) + Log[1 - x]

Rule 711

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{(-1+x)^3} dx &= \int \left(\frac{2}{(-1+x)^3} + \frac{2}{(-1+x)^2} + \frac{1}{-1+x} \right) dx \\ &= -\frac{1}{(1-x)^2} + \frac{2}{1-x} + \log(1-x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 16, normalized size = 0.64

$$\frac{1-2x}{(-1+x)^2} + \log(-1+x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(-1 + x)^3,x]

[Out] $(1 - 2x)/(-1 + x)^2 + \text{Log}[-1 + x]$

Mathics [A]

time = 1.95, size = 29, normalized size = 1.16

$$\frac{1 - 2x + \text{Log}[-1 + x](1 - 2x + x^2)}{1 - 2x + x^2}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(x^2 + 1)/(x - 1)^3,x]')`

[Out] $(1 - 2x + \text{Log}[-1 + x](1 - 2x + x^2)) / (1 - 2x + x^2)$

Maple [A]

time = 0.05, size = 20, normalized size = 0.80

method	result	size
norman	$\frac{1-2x}{(-1+x)^2} + \ln(-1+x)$	17
risch	$\frac{1-2x}{(-1+x)^2} + \ln(-1+x)$	17
default	$-\frac{1}{(-1+x)^2} + \ln(-1+x) - \frac{2}{-1+x}$	20
meijerg	$-\frac{x(2-x)}{2(1-x)^2} + \frac{x(-9x+6)}{6(1-x)^2} + \ln(1-x)$	38

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+1)/(-1+x)^3,x,method=_RETURNVERBOSE)`

[Out] $-1/(-1+x)^2 + \ln(-1+x) - 2/(-1+x)$

Maxima [A]

time = 0.26, size = 22, normalized size = 0.88

$$-\frac{2x - 1}{x^2 - 2x + 1} + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)/(-1+x)^3,x, algorithm="maxima")`

[Out] $-(2x - 1)/(x^2 - 2x + 1) + \log(x - 1)$

Fricas [A]

time = 0.33, size = 29, normalized size = 1.16

$$\frac{(x^2 - 2x + 1) \log(x - 1) - 2x + 1}{x^2 - 2x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(-1+x)^3,x, algorithm="fricas")

[Out] ((x^2 - 2*x + 1)*log(x - 1) - 2*x + 1)/(x^2 - 2*x + 1)

Sympy [A]

time = 0.04, size = 17, normalized size = 0.68

$$\frac{1 - 2x}{x^2 - 2x + 1} + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/(-1+x)**3,x)

[Out] (1 - 2*x)/(x**2 - 2*x + 1) + log(x - 1)

Giac [A]

time = 0.00, size = 22, normalized size = 0.88

$$\frac{\frac{1}{2}(-4x + 2)}{(x - 1)^2} + \ln|x - 1|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(-1+x)^3,x)

[Out] -(2*x - 1)/(x - 1)^2 + log(abs(x - 1))

Mupad [B]

time = 0.17, size = 22, normalized size = 0.88

$$\ln(x - 1) - \frac{2x - 1}{x^2 - 2x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)/(x - 1)^3,x)

[Out] log(x - 1) - (2*x - 1)/(x^2 - 2*x + 1)

3.102 $\int \frac{x^5}{(3+x)^2} dx$

Optimal. Leaf size=36

$$-108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405 \log(3+x)$$

[Out] $-108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*\ln(3+x)$

Rubi [A]

time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$,

Rules used = {45}

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + \frac{243}{x+3} + 405 \log(x+3)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^5/(3+x)^2, x]$

[Out] $-108*x + (27*x^2)/2 - 2*x^3 + x^4/4 + 243/(3+x) + 405*\text{Log}[3+x]$

Rule 45

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{x^5}{(3+x)^2} dx &= \int \left(-108 + 27x - 6x^2 + x^3 - \frac{243}{(3+x)^2} + \frac{405}{3+x} \right) dx \\ &= -108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405 \log(3+x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 36, normalized size = 1.00

$$\frac{1}{4} \left(-2079 - 432x + 54x^2 - 8x^3 + x^4 + \frac{972}{3+x} \right) + 405 \log(3+x)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(3 + x)^2,x]

[Out] $(-2079 - 432x + 54x^2 - 8x^3 + x^4 + 972/(3 + x))/4 + 405\text{Log}[3 + x]$

Mathics [A]

time = 2.04, size = 36, normalized size = 1.00

$$\frac{972 + (3 + x)(-432x + 54x^2 - 8x^3 + x^4 + 1620\text{Log}[3 + x])}{12 + 4x}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^5/(3 + x)^2,x]')

[Out] $(972 + (3 + x)(-432x + 54x^2 - 8x^3 + x^4 + 1620\text{Log}[3 + x])) / (4(3 + x))$

Maple [A]

time = 0.05, size = 33, normalized size = 0.92

method	result	size
default	$-108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405 \ln(3 + x)$	33
risch	$-108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405 \ln(3 + x)$	33
norman	$\frac{-\frac{135}{2}x^2 + \frac{15}{2}x^3 - \frac{5}{4}x^4 + \frac{1}{4}x^5 + 1215}{3+x} + 405 \ln(3 + x)$	36
meijerg	$-\frac{9x(-\frac{1}{27}x^4 + \frac{5}{27}x^3 - \frac{10}{9}x^2 + 10x + 60)}{4(1 + \frac{x}{3})} + 405 \ln(1 + \frac{x}{3})$	40

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(3+x)^2,x,method=_RETURNVERBOSE)

[Out] $-108x + 27/2x^2 - 2x^3 + 1/4x^4 + 243/(3+x) + 405\ln(3+x)$

Maxima [A]

time = 0.26, size = 32, normalized size = 0.89

$$\frac{1}{4}x^4 - 2x^3 + \frac{27}{2}x^2 - 108x + \frac{243}{x+3} + 405 \log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(3+x)^2,x, algorithm="maxima")

[Out] $1/4x^4 - 2x^3 + 27/2x^2 - 108x + 243/(x + 3) + 405\log(x + 3)$

Fricas [A]

time = 0.32, size = 39, normalized size = 1.08

$$\frac{x^5 - 5x^4 + 30x^3 - 270x^2 + 1620(x+3)\log(x+3) - 1296x + 972}{4(x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁵/(3+x)²,x, algorithm="fricas")

[Out] 1/4*(x⁵ - 5*x⁴ + 30*x³ - 270*x² + 1620*(x + 3)*log(x + 3) - 1296*x + 972)/(x + 3)

Sympy [A]

time = 0.04, size = 31, normalized size = 0.86

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + 405 \log(x + 3) + \frac{243}{x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(3+x)**2,x)

[Out] x**4/4 - 2*x**3 + 27*x**2/2 - 108*x + 405*log(x + 3) + 243/(x + 3)

Giac [A]

time = 0.00, size = 38, normalized size = 1.06

$$\frac{1}{4}x^4 - 2x^3 + \frac{27}{2}x^2 - 108x + \frac{243}{x + 3} + 405 \ln|x + 3|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁵/(3+x)²,x)

[Out] 1/4*x⁴ - 2*x³ + 27/2*x² - 108*x + 243/(x + 3) + 405*log(abs(x + 3))

Mupad [B]

time = 0.03, size = 32, normalized size = 0.89

$$405 \ln(x + 3) - 108x + \frac{243}{x + 3} + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x⁵/(x + 3)²,x)

[Out] 405*log(x + 3) - 108*x + 243/(x + 3) + (27*x²)/2 - 2*x³ + x⁴/4

$$3.103 \quad \int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4} dx$$

Optimal. Leaf size=41

$$-\frac{133}{8(3-x)^2} + \frac{407}{16(3-x)} + \frac{313}{64} \log(3-x) + \frac{7}{64} \log(1+x)$$

[Out] -133/8/(3-x)^2+407/16/(3-x)+313/64*ln(3-x)+7/64*ln(1+x)

Rubi [A]

time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2099}

$$\frac{407}{16(3-x)} - \frac{133}{8(3-x)^2} + \frac{313}{64} \log(3-x) + \frac{7}{64} \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-2 + 5*x^3)/(-27 + 18*x^2 - 8*x^3 + x^4), x]

[Out] -133/(8*(3 - x)^2) + 407/(16*(3 - x)) + (313*Log[3 - x])/64 + (7*Log[1 + x])/64

Rule 2099

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned} \int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4} dx &= \int \left(\frac{133}{4(-3+x)^3} + \frac{407}{16(-3+x)^2} + \frac{313}{64(-3+x)} + \frac{7}{64(1+x)} \right) dx \\ &= -\frac{133}{8(3-x)^2} + \frac{407}{16(3-x)} + \frac{313}{64} \log(3-x) + \frac{7}{64} \log(1+x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 37, normalized size = 0.90

$$-\frac{133}{8(-3+x)^2} - \frac{407}{16(-3+x)} + \frac{313}{64} \log(3-x) + \frac{7}{64} \log(1+x)$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + 5*x^3)/(-27 + 18*x^2 - 8*x^3 + x^4),x]

[Out] -133/(8*(-3 + x)^2) - 407/(16*(-3 + x)) + (313*Log[3 - x])/64 + (7*Log[1 + x])/64

Mathics [A]

time = 2.08, size = 39, normalized size = 0.95

$$\frac{3820 - 1628x + (9 - 6x + x^2)(7\text{Log}[1 + x] + 313\text{Log}[-3 + x])}{576 - 384x + 64x^2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(5*x^3 - 2)/(x^4 - 8*x^3 + 18*x^2 - 27),x]')

[Out] (3820 - 1628 x + (9 - 6 x + x ^ 2) (7 Log[1 + x] + 313 Log[-3 + x])) / (64 (9 - 6 x + x ^ 2))

Maple [A]

time = 0.02, size = 28, normalized size = 0.68

method	result	size
norman	$-\frac{407x + 955}{(-3+x)^2} + \frac{313 \ln(-3+x)}{64} + \frac{7 \ln(1+x)}{64}$	25
default	$-\frac{133}{8(-3+x)^2} - \frac{407}{16(-3+x)} + \frac{313 \ln(-3+x)}{64} + \frac{7 \ln(1+x)}{64}$	28
risch	$-\frac{407x + 955}{x^2 - 6x + 9} + \frac{313 \ln(-3+x)}{64} + \frac{7 \ln(1+x)}{64}$	30

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x,method=_RETURNVERBOSE)

[Out] -133/8/(-3+x)^2-407/16/(-3+x)+313/64*ln(-3+x)+7/64*ln(1+x)

Maxima [A]

time = 0.26, size = 30, normalized size = 0.73

$$-\frac{407x - 955}{16(x^2 - 6x + 9)} + \frac{7}{64} \log(x + 1) + \frac{313}{64} \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x, algorithm="maxima")

[Out] -1/16*(407*x - 955)/(x^2 - 6*x + 9) + 7/64*log(x + 1) + 313/64*log(x - 3)

Fricas [A]

time = 0.31, size = 45, normalized size = 1.10

$$\frac{7(x^2 - 6x + 9) \log(x + 1) + 313(x^2 - 6x + 9) \log(x - 3) - 1628x + 3820}{64(x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x, algorithm="fricas")

[Out] 1/64*(7*(x^2 - 6*x + 9)*log(x + 1) + 313*(x^2 - 6*x + 9)*log(x - 3) - 1628*x + 3820)/(x^2 - 6*x + 9)

Sympy [A]

time = 0.07, size = 31, normalized size = 0.76

$$\frac{955 - 407x}{16x^2 - 96x + 144} + \frac{313 \log(x - 3)}{64} + \frac{7 \log(x + 1)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**3-2)/(x**4-8*x**3+18*x**2-27),x)

[Out] (955 - 407*x)/(16*x**2 - 96*x + 144) + 313*log(x - 3)/64 + 7*log(x + 1)/64

Giac [A]

time = 0.00, size = 35, normalized size = 0.85

$$\frac{7}{64} \ln|x + 1| + \frac{313}{64} \ln|x - 3| + \frac{\frac{1}{128}(-3256x + 7640)}{(x - 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x)

[Out] -1/16*(407*x - 955)/(x - 3)^2 + 7/64*log(abs(x + 1)) + 313/64*log(abs(x - 3))

Mupad [B]

time = 0.06, size = 30, normalized size = 0.73

$$\frac{7 \ln(x + 1)}{64} + \frac{313 \ln(x - 3)}{64} - \frac{\frac{407x}{16} - \frac{955}{16}}{x^2 - 6x + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^3 - 2)/(18*x^2 - 8*x^3 + x^4 - 27),x)

[Out] (7*log(x + 1))/64 + (313*log(x - 3))/64 - ((407*x)/16 - 955/16)/(x^2 - 6*x + 9)

$$3.104 \quad \int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2} dx$$

Optimal. Leaf size=27

$$\frac{99}{3+x} + \frac{181}{4+x} + 264 \log(3+x) - 263 \log(4+x)$$

[Out] 99/(3+x)+181/(4+x)+264*ln(3+x)-263*ln(4+x)

Rubi [A]

time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1634}

$$\frac{99}{x+3} + \frac{181}{x+4} + 264 \log(x+3) - 263 \log(x+4)$$

Antiderivative was successfully verified.

[In] Int[(-9 + 3*x - 6*x^2 + x^3)/((3 + x)^2*(4 + x)^2), x]

[Out] 99/(3 + x) + 181/(4 + x) + 264*Log[3 + x] - 263*Log[4 + x]

Rule 1634

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
 := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2} dx &= \int \left(-\frac{99}{(3+x)^2} + \frac{264}{3+x} - \frac{181}{(4+x)^2} - \frac{263}{4+x} \right) dx \\ &= \frac{99}{3+x} + \frac{181}{4+x} + 264 \log(3+x) - 263 \log(4+x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 27, normalized size = 1.00

$$\frac{99}{3+x} + \frac{181}{4+x} + 264 \log(3+x) - 263 \log(4+x)$$

Antiderivative was successfully verified.

[In] Integrate[(-9 + 3*x - 6*x^2 + x^3)/((3 + x)^2*(4 + x)^2), x]

[Out] $99/(3 + x) + 181/(4 + x) + 264*\text{Log}[3 + x] - 263*\text{Log}[4 + x]$

Mathics [A]

time = 2.00, size = 38, normalized size = 1.41

$$\frac{939 + 280x + (12 + 7x + x^2)(-263\text{Log}[4 + x] + 264\text{Log}[3 + x])}{12 + 7x + x^2}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(x^3 - 6*x^2 + 3*x - 9)/((x + 3)^2*(x + 4)^2),x]')`

[Out] $(939 + 280 x + (12 + 7 x + x^2)(-263 \text{Log}[4 + x] + 264 \text{Log}[3 + x])) / (12 + 7 x + x^2)$

Maple [A]

time = 0.06, size = 28, normalized size = 1.04

method	result	size
default	$\frac{99}{3+x} + \frac{181}{4+x} + 264 \ln(3+x) - 263 \ln(4+x)$	28
norman	$\frac{280x+939}{(4+x)(3+x)} + 264 \ln(3+x) - 263 \ln(4+x)$	30
risch	$\frac{280x+939}{(4+x)(3+x)} + 264 \ln(3+x) - 263 \ln(4+x)$	30

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x,method=_RETURNVERBOSE)`

[Out] $99/(3+x)+181/(4+x)+264*\ln(3+x)-263*\ln(4+x)$

Maxima [A]

time = 0.26, size = 29, normalized size = 1.07

$$\frac{280x + 939}{x^2 + 7x + 12} - 263 \log(x + 4) + 264 \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x, algorithm="maxima")`

[Out] $(280*x + 939)/(x^2 + 7*x + 12) - 263*\log(x + 4) + 264*\log(x + 3)$

Fricas [A]

time = 0.31, size = 45, normalized size = 1.67

$$\frac{263(x^2 + 7x + 12) \log(x + 4) - 264(x^2 + 7x + 12) \log(x + 3) - 280x - 939}{x^2 + 7x + 12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x, algorithm="fricas")

[Out] $-(263*(x^2 + 7*x + 12)*\log(x + 4) - 264*(x^2 + 7*x + 12)*\log(x + 3) - 280*x - 939)/(x^2 + 7*x + 12)$

Sympy [A]

time = 0.07, size = 26, normalized size = 0.96

$$\frac{280x + 939}{x^2 + 7x + 12} + 264 \log(x + 3) - 263 \log(x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-6*x**2+3*x-9)/(3+x)**2/(4+x)**2,x)

[Out] $(280*x + 939)/(x**2 + 7*x + 12) + 264*\log(x + 3) - 263*\log(x + 4)$

Giac [A]

time = 0.00, size = 33, normalized size = 1.22

$$264 \ln|x + 3| - 263 \ln|x + 4| + \frac{\frac{1}{2}(560x + 1878)}{(x + 3)(x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x)

[Out] $(280*x + 939)/((x + 4)*(x + 3)) - 263*\log(\text{abs}(x + 4)) + 264*\log(\text{abs}(x + 3))$

Mupad [B]

time = 0.19, size = 29, normalized size = 1.07

$$264 \ln(x + 3) - 263 \ln(x + 4) + \frac{280x + 939}{x^2 + 7x + 12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x - 6*x^2 + x^3 - 9)/((x + 3)^2*(x + 4)^2),x)

[Out] $264*\log(x + 3) - 263*\log(x + 4) + (280*x + 939)/(7*x + x^2 + 12)$

3.105

$$\int \frac{2+x^2+x^3}{x(-1+x^2)^2} dx$$

Optimal. Leaf size=39

$$\frac{3+x}{2(1-x^2)} - \frac{3}{4} \log(1-x) + 2 \log(x) - \frac{5}{4} \log(1+x)$$

[Out] 1/2*(3+x)/(-x^2+1)-3/4*ln(1-x)+2*ln(x)-5/4*ln(1+x)

Rubi [A]

time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$,

Rules used = {1819, 815}

$$\frac{x+3}{2(1-x^2)} - \frac{3}{4} \log(1-x) + 2 \log(x) - \frac{5}{4} \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(2 + x^2 + x^3)/(x*(-1 + x^2)^2),x]

[Out] (3 + x)/(2*(1 - x^2)) - (3*Log[1 - x])/4 + 2*Log[x] - (5*Log[1 + x])/4

Rule 815

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 1819

Int[(Pq_)*((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)), x_Symbol] :> With[{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]

Rubi steps

$$\begin{aligned} \int \frac{2+x^2+x^3}{x(-1+x^2)^2} dx &= \frac{3+x}{2(1-x^2)} + \frac{1}{2} \int \frac{-4+x}{x(-1+x^2)} dx \\ &= \frac{3+x}{2(1-x^2)} + \frac{1}{2} \int \left(-\frac{3}{2(-1+x)} + \frac{4}{x} - \frac{5}{2(1+x)} \right) dx \\ &= \frac{3+x}{2(1-x^2)} - \frac{3}{4} \log(1-x) + 2 \log(x) - \frac{5}{4} \log(1+x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 47, normalized size = 1.21

$$\frac{1}{4} \left(-\frac{2}{-1+x} - \frac{4}{-1+x^2} + \log(1-x) + 8 \log(x) - \log(1+x) - 4 \log(1-x^2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x^2 + x^3)/(x*(-1 + x^2)^2), x]

[Out] (-2/(-1 + x) - 4/(-1 + x^2) + Log[1 - x] + 8*Log[x] - Log[1 + x] - 4*Log[1 - x^2])/4

Mathics [A]

time = 2.03, size = 37, normalized size = 0.95

$$\frac{-6 - 2x + (-1 + x^2) (-5 \operatorname{Log}[1 + x] - 3 \operatorname{Log}[-1 + x] + 8 \operatorname{Log}[x])}{-4 + 4x^2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(x^3 + x^2 + 2)/(x*(x^2 - 1)^2), x]')

[Out] (-6 - 2 x + (-1 + x ^ 2) (-5 Log[1 + x] - 3 Log[-1 + x] + 8 Log[x])) / (4 (-1 + x ^ 2))

Maple [A]

time = 0.07, size = 32, normalized size = 0.82

method	result	size
norman	$\frac{-\frac{x}{2} - \frac{3}{2}}{x^2 - 1} + 2 \ln(x) - \frac{3 \ln(-1+x)}{4} - \frac{5 \ln(1+x)}{4}$	31
risch	$\frac{-\frac{x}{2} - \frac{3}{2}}{x^2 - 1} + 2 \ln(x) - \frac{3 \ln(-1+x)}{4} - \frac{5 \ln(1+x)}{4}$	31
default	$2 \ln(x) - \frac{1}{-1+x} - \frac{3 \ln(-1+x)}{4} + \frac{1}{2+2x} - \frac{5 \ln(1+x)}{4}$	32
meijerg	$\frac{i \left(-\frac{ix}{-x^2+1} + i \operatorname{arctanh}(x) \right)}{2} + \frac{3x^2}{-2x^2+2} + 1 + 2 \ln(x) + i\pi - \ln(-x^2 + 1)$	71

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+2)/x/(x^2-1)^2,x,method=_RETURNVERBOSE)

[Out] 2*ln(x)-1/(-1+x)-3/4*ln(-1+x)+1/2/(1+x)-5/4*ln(1+x)

Maxima [A]

time = 0.27, size = 29, normalized size = 0.74

$$-\frac{x+3}{2(x^2-1)} - \frac{5}{4} \log(x+1) - \frac{3}{4} \log(x-1) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+2)/x/(x^2-1)^2,x, algorithm="maxima")

[Out] -1/2*(x + 3)/(x^2 - 1) - 5/4*log(x + 1) - 3/4*log(x - 1) + 2*log(x)

Fricas [A]

time = 0.32, size = 45, normalized size = 1.15

$$-\frac{5(x^2 - 1)\log(x + 1) + 3(x^2 - 1)\log(x - 1) - 8(x^2 - 1)\log(x) + 2x + 6}{4(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+2)/x/(x^2-1)^2,x, algorithm="fricas")

[Out] -1/4*(5*(x^2 - 1)*log(x + 1) + 3*(x^2 - 1)*log(x - 1) - 8*(x^2 - 1)*log(x) + 2*x + 6)/(x^2 - 1)

Sympy [A]

time = 0.08, size = 32, normalized size = 0.82

$$\frac{-x - 3}{2x^2 - 2} + 2\log(x) - \frac{3\log(x - 1)}{4} - \frac{5\log(x + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x**2+2)/x/(x**2-1)**2,x)

[Out] (-x - 3)/(2*x**2 - 2) + 2*log(x) - 3*log(x - 1)/4 - 5*log(x + 1)/4

Giac [A]

time = 0.00, size = 40, normalized size = 1.03

$$2\ln|x| - \frac{3}{4}\ln|x - 1| - \frac{5}{4}\ln|x + 1| + \frac{\frac{1}{2}(-x - 3)}{(x - 1)(x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+2)/x/(x^2-1)^2,x)

[Out] -1/2*(x + 3)/((x + 1)*(x - 1)) - 5/4*log(abs(x + 1)) - 3/4*log(abs(x - 1)) + 2*log(abs(x))

Mupad [B]

time = 0.05, size = 31, normalized size = 0.79

$$2\ln(x) - \frac{5\ln(x + 1)}{4} - \frac{3\ln(x - 1)}{4} - \frac{\frac{x}{2} + \frac{3}{2}}{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + x^3 + 2)/(x*(x^2 - 1)^2),x)

[Out] 2*log(x) - (5*log(x + 1))/4 - (3*log(x - 1))/4 - (x/2 + 3/2)/(x^2 - 1)

3.106

$$\int \frac{1}{x^3 - x^4 - x^5 + x^6} dx$$

Optimal. Leaf size=46

$$\frac{1}{2(1-x)} - \frac{1}{2x^2} - \frac{1}{x} - \frac{7}{4} \log(1-x) + 2 \log(x) - \frac{1}{4} \log(1+x)$$

[Out] 1/2/(1-x)-1/2/x^2-1/x-7/4*ln(1-x)+2*ln(x)-1/4*ln(1+x)

Rubi [A]

time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2083}

$$-\frac{1}{2x^2} + \frac{1}{2(1-x)} - \frac{1}{x} - \frac{7}{4} \log(1-x) + 2 \log(x) - \frac{1}{4} \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(x^3 - x^4 - x^5 + x^6)^(-1), x]

[Out] 1/(2*(1 - x)) - 1/(2*x^2) - x^(-1) - (7*Log[1 - x])/4 + 2*Log[x] - Log[1 + x]/4

Rule 2083

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 - x^4 - x^5 + x^6} dx &= \int \left(\frac{1}{2(-1+x)^2} - \frac{7}{4(-1+x)} + \frac{1}{x^3} + \frac{1}{x^2} + \frac{2}{x} - \frac{1}{4(1+x)} \right) dx \\ &= \frac{1}{2(1-x)} - \frac{1}{2x^2} - \frac{1}{x} - \frac{7}{4} \log(1-x) + 2 \log(x) - \frac{1}{4} \log(1+x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 40, normalized size = 0.87

$$\frac{1}{4} \left(-\frac{2}{-1+x} - \frac{2}{x^2} - \frac{4}{x} - 7 \log(1-x) + 8 \log(x) - \log(1+x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^3 - x^4 - x^5 + x^6)^(-1), x]

[Out] $(-2/(-1 + x) - 2/x^2 - 4/x - 7*\text{Log}[1 - x] + 8*\text{Log}[x] - \text{Log}[1 + x])/4$

Mathics [A]

time = 2.19, size = 44, normalized size = 0.96

$$\frac{2 + 2x - 6x^2 + x^2(-1 + x)(-7\text{Log}[-1 + x] - \text{Log}[1 + x] + 8\text{Log}[x])}{4x^2(-1 + x)}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(x^3 - x^4 - x^5 + x^6),x]')`

[Out] $(2 + 2x - 6x^2 + x^2(-1 + x)(-7\text{Log}[-1 + x] - \text{Log}[1 + x] + 8\text{Log}[x])) / (4x^2(-1 + x))$

Maple [A]

time = 0.02, size = 35, normalized size = 0.76

method	result	size
default	$-\frac{1}{2x^2} - \frac{1}{x} + 2\ln(x) - \frac{1}{2(-1+x)} - \frac{7\ln(-1+x)}{4} - \frac{\ln(1+x)}{4}$	35
norman	$\frac{\frac{1}{2} - \frac{3}{2}x^2 + \frac{1}{2}x}{x^2(-1+x)} + 2\ln(x) - \frac{7\ln(-1+x)}{4} - \frac{\ln(1+x)}{4}$	37
risch	$\frac{\frac{1}{2} - \frac{3}{2}x^2 + \frac{1}{2}x}{x^2(-1+x)} + 2\ln(x) - \frac{7\ln(-1+x)}{4} - \frac{\ln(1+x)}{4}$	37

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^6-x^5-x^4+x^3),x,method=_RETURNVERBOSE)`

[Out] $-1/2/x^2 - 1/x + 2*\ln(x) - 1/2/(-1+x) - 7/4*\ln(-1+x) - 1/4*\ln(1+x)$

Maxima [A]

time = 0.26, size = 40, normalized size = 0.87

$$-\frac{3x^2 - x - 1}{2(x^3 - x^2)} - \frac{1}{4}\log(x + 1) - \frac{7}{4}\log(x - 1) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^6-x^5-x^4+x^3),x, algorithm="maxima")`

[Out] $-1/2*(3*x^2 - x - 1)/(x^3 - x^2) - 1/4*\log(x + 1) - 7/4*\log(x - 1) + 2*\log(x)$

Fricas [A]

time = 0.32, size = 65, normalized size = 1.41

$$\frac{6x^2 + (x^3 - x^2)\log(x + 1) + 7(x^3 - x^2)\log(x - 1) - 8(x^3 - x^2)\log(x) - 2x - 2}{4(x^3 - x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-x^5-x^4+x^3),x, algorithm="fricas")

[Out] $-1/4*(6*x^2 + (x^3 - x^2)*\log(x + 1) + 7*(x^3 - x^2)*\log(x - 1) - 8*(x^3 - x^2)*\log(x) - 2*x - 2)/(x^3 - x^2)$

Sympy [A]

time = 0.08, size = 37, normalized size = 0.80

$$2 \log(x) - \frac{7 \log(x-1)}{4} - \frac{\log(x+1)}{4} + \frac{-3x^2 + x + 1}{2x^3 - 2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**6-x**5-x**4+x**3),x)

[Out] $2*\log(x) - 7*\log(x - 1)/4 - \log(x + 1)/4 + (-3*x**2 + x + 1)/(2*x**3 - 2*x**2)$

Giac [A]

time = 0.00, size = 47, normalized size = 1.02

$$-\frac{\ln|x+1|}{4} - \frac{7}{4} \ln|x-1| + 2 \ln|x| + \frac{\frac{1}{4}(-6x^2 + 2x + 2)}{x^2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-x^5-x^4+x^3),x)

[Out] $-1/2*(3*x^2 - x - 1)/((x - 1)*x^2) - 1/4*\log(\text{abs}(x + 1)) - 7/4*\log(\text{abs}(x - 1)) + 2*\log(\text{abs}(x))$

Mupad [B]

time = 0.18, size = 40, normalized size = 0.87

$$2 \ln(x) - \frac{\ln(x+1)}{4} - \frac{7 \ln(x-1)}{4} - \frac{-\frac{3x^2}{2} + \frac{x}{2} + \frac{1}{2}}{x^2 - x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3 - x^4 - x^5 + x^6),x)

[Out] $2*\log(x) - \log(x + 1)/4 - (7*\log(x - 1))/4 - (x/2 - (3*x^2)/2 + 1/2)/(x^2 - x^3)$

$$3.107 \quad \int \frac{1+x^4}{-1+x-x^2+x^3} dx$$

Optimal. Leaf size=29

$$x + \frac{x^2}{2} - \tan^{-1}(x) + \log(1-x) - \frac{1}{2} \log(1+x^2)$$

[Out] x+1/2*x^2-arctan(x)+ln(1-x)-1/2*ln(x^2+1)

Rubi [A]

time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2099, 649, 209, 266}

$$\frac{x^2}{2} - \frac{1}{2} \log(x^2 + 1) + x + \log(1 - x) - \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)/(-1 + x - x^2 + x^3), x]

[Out] x + x^2/2 - ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 649

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[(-a)*c]

Rule 2099

Int[(P_)^(p_.)*(Q_)^(q_.), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{-1+x-x^2+x^3} dx &= \int \left(1 + \frac{1}{-1+x} + x + \frac{-1-x}{1+x^2} \right) dx \\
&= x + \frac{x^2}{2} + \log(1-x) + \int \frac{-1-x}{1+x^2} dx \\
&= x + \frac{x^2}{2} + \log(1-x) - \int \frac{1}{1+x^2} dx - \int \frac{x}{1+x^2} dx \\
&= x + \frac{x^2}{2} - \tan^{-1}(x) + \log(1-x) - \frac{1}{2} \log(1+x^2)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 29, normalized size = 1.00

$$x + \frac{x^2}{2} - \tan^{-1}(x) + \log(1-x) - \frac{1}{2} \log(1+x^2)$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + x^4)/(-1 + x - x^2 + x^3), x]``[Out] x + x^2/2 - ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2`**Mathics [A]**

time = 1.89, size = 23, normalized size = 0.79

$$x + \frac{x^2}{2} + \text{Log}[-1 + x] - \text{ArcTan}[x] - \frac{\text{Log}[1 + x^2]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(x^4 + 1)/(x^3 - x^2 + x - 1), x]')``[Out] x + x^2 / 2 + Log[-1 + x] - ArcTan[x] - Log[1 + x^2] / 2`**Maple [A]**

time = 0.02, size = 24, normalized size = 0.83

method	result	size
default	$x + \frac{x^2}{2} + \ln(-1 + x) - \frac{\ln(x^2+1)}{2} - \arctan(x)$	24
risch	$x + \frac{x^2}{2} + \ln(-1 + x) - \frac{\ln(x^2+1)}{2} - \arctan(x)$	24

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((x^4+1)/(x^3-x^2+x-1), x, method=_RETURNVERBOSE)`

[Out] $x + \frac{1}{2}x^2 + \ln(-1+x) - \frac{1}{2}\ln(x^2+1) - \arctan(x)$

Maxima [A]

time = 0.35, size = 23, normalized size = 0.79

$$\frac{1}{2}x^2 + x - \arctan(x) - \frac{1}{2}\log(x^2 + 1) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+1)/(x^3-x^2+x-1),x, algorithm="maxima")`

[Out] $\frac{1}{2}x^2 + x - \arctan(x) - \frac{1}{2}\log(x^2 + 1) + \log(x - 1)$

Fricas [A]

time = 0.32, size = 23, normalized size = 0.79

$$\frac{1}{2}x^2 + x - \arctan(x) - \frac{1}{2}\log(x^2 + 1) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+1)/(x^3-x^2+x-1),x, algorithm="fricas")`

[Out] $\frac{1}{2}x^2 + x - \arctan(x) - \frac{1}{2}\log(x^2 + 1) + \log(x - 1)$

Sympy [A]

time = 0.06, size = 22, normalized size = 0.76

$$\frac{x^2}{2} + x + \log(x - 1) - \frac{\log(x^2 + 1)}{2} - \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**4+1)/(x**3-x**2+x-1),x)`

[Out] $x^{**2}/2 + x + \log(x - 1) - \log(x^{**2} + 1)/2 - \operatorname{atan}(x)$

Giac [A]

time = 0.00, size = 27, normalized size = 0.93

$$\frac{1}{2}x^2 + x + \ln|x - 1| - \frac{\ln(x^2 + 1)}{2} - \arctan x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+1)/(x^3-x^2+x-1),x)`

[Out] $\frac{1}{2}x^2 + x - \arctan(x) - \frac{1}{2}\log(x^2 + 1) + \log(\operatorname{abs}(x - 1))$

Mupad [B]

time = 0.05, size = 29, normalized size = 1.00

$$x + \ln(x - 1) + \frac{x^2}{2} + \ln(x - i) \left(-\frac{1}{2} + \frac{1}{2}i \right) + \ln(x + i) \left(-\frac{1}{2} - \frac{1}{2}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x - x^2 + x^3 - 1),x)
```

```
[Out] x + log(x - 1) - log(x - 1i)*(1/2 - 1i/2) - log(x + 1i)*(1/2 + 1i/2) + x^2/2
```

$$3.108 \quad \int \frac{1}{x(1+x)(1+x^2)} dx$$

Optimal. Leaf size=27

$$-\frac{1}{2} \tan^{-1}(x) + \log(x) - \frac{1}{2} \log(1+x) - \frac{1}{4} \log(1+x^2)$$

[Out] -1/2*arctan(x)+ln(x)-1/2*ln(1+x)-1/4*ln(x^2+1)

Rubi [A]

time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {908, 649, 209, 266}

$$-\frac{1}{4} \log(x^2 + 1) + \log(x) - \frac{1}{2} \log(x + 1) - \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1+x)*(1+x^2)),x]

[Out] -1/2*ArcTan[x] + Log[x] - Log[1+x]/2 - Log[1+x^2]/4

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 649

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[(-a)*c]

Rule 908

Int[((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)^n)*((a_) + (c_.)*(x_)^2)^p, x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(1+x)(1+x^2)} dx &= \int \left(\frac{1}{x} - \frac{1}{2(1+x)} + \frac{-1-x}{2(1+x^2)} \right) dx \\
&= \log(x) - \frac{1}{2} \log(1+x) + \frac{1}{2} \int \frac{-1-x}{1+x^2} dx \\
&= \log(x) - \frac{1}{2} \log(1+x) - \frac{1}{2} \int \frac{1}{1+x^2} dx - \frac{1}{2} \int \frac{x}{1+x^2} dx \\
&= -\frac{1}{2} \tan^{-1}(x) + \log(x) - \frac{1}{2} \log(1+x) - \frac{1}{4} \log(1+x^2)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 27, normalized size = 1.00

$$-\frac{1}{2} \tan^{-1}(x) + \log(x) - \frac{1}{2} \log(1+x) - \frac{1}{4} \log(1+x^2)$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x*(1+x)*(1+x^2)),x]``[Out] -1/2*ArcTan[x] + Log[x] - Log[1+x]/2 - Log[1+x^2]/4`**Mathics [A]**

time = 1.93, size = 21, normalized size = 0.78

$$\text{Log}[x] - \frac{\text{ArcTan}[x]}{2} - \frac{\text{Log}[1+x]}{2} - \frac{\text{Log}[1+x^2]}{4}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x*(1+x)*(1+x^2)),x]')``[Out] Log[x] - ArcTan[x] / 2 - Log[1+x] / 2 - Log[1+x^2] / 4`**Maple [A]**

time = 0.07, size = 22, normalized size = 0.81

method	result	size
default	$-\frac{\arctan(x)}{2} + \ln(x) - \frac{\ln(1+x)}{2} - \frac{\ln(x^2+1)}{4}$	22
risch	$-\frac{\arctan(x)}{2} + \ln(x) - \frac{\ln(1+x)}{2} - \frac{\ln(x^2+1)}{4}$	22

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/x/(1+x)/(x^2+1),x,method=_RETURNVERBOSE)`

[Out] $-1/2*\arctan(x)+\ln(x)-1/2*\ln(1+x)-1/4*\ln(x^2+1)$

Maxima [A]

time = 0.35, size = 21, normalized size = 0.78

$$-\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) - \frac{1}{2} \log(x + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1+x)/(x^2+1),x, algorithm="maxima")`

[Out] $-1/2*\arctan(x) - 1/4*\log(x^2 + 1) - 1/2*\log(x + 1) + \log(x)$

Fricas [A]

time = 0.34, size = 21, normalized size = 0.78

$$-\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) - \frac{1}{2} \log(x + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1+x)/(x^2+1),x, algorithm="fricas")`

[Out] $-1/2*\arctan(x) - 1/4*\log(x^2 + 1) - 1/2*\log(x + 1) + \log(x)$

Sympy [A]

time = 0.09, size = 22, normalized size = 0.81

$$\log(x) - \frac{\log(x + 1)}{2} - \frac{\log(x^2 + 1)}{4} - \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1+x)/(x**2+1),x)`

[Out] $\log(x) - \log(x + 1)/2 - \log(x**2 + 1)/4 - \operatorname{atan}(x)/2$

Giac [A]

time = 0.00, size = 29, normalized size = 1.07

$$\ln|x| - \frac{\ln|x + 1|}{2} - \frac{\ln(x^2 + 1)}{4} - \frac{\arctan x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1+x)/(x^2+1),x)`

[Out] $-1/2*\arctan(x) - 1/4*\log(x^2 + 1) - 1/2*\log(\operatorname{abs}(x + 1)) + \log(\operatorname{abs}(x))$

Mupad [B]

time = 0.05, size = 27, normalized size = 1.00

$$\ln(x) - \frac{\ln(x + 1)}{2} + \ln(x - i) \left(-\frac{1}{4} + \frac{1}{4}i \right) + \ln(x + i) \left(-\frac{1}{4} - \frac{1}{4}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 + 1)*(x + 1)),x)
```

```
[Out] log(x) - log(x - 1i)*(1/4 - 1i/4) - log(x + 1i)*(1/4 + 1i/4) - log(x + 1)/2
```

$$3.109 \quad \int \frac{x^2}{-2+x^2+x^4} dx$$

Optimal. Leaf size=24

$$\frac{1}{3}\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{3} \tanh^{-1}(x)$$

[Out] -1/3*arctanh(x)+1/3*arctan(1/2*x*2^(1/2))*2^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1144, 209, 213}

$$\frac{1}{3}\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{3} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x^2/(-2 + x^2 + x^4),x]

[Out] (Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - ArcTanh[x]/3

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1144

Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 - q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 2]

Rubi steps

$$\int \frac{x^2}{-2 + x^2 + x^4} dx = \frac{1}{3} \int \frac{1}{-1 + x^2} dx + \frac{2}{3} \int \frac{1}{2 + x^2} dx$$

$$= \frac{1}{3} \sqrt{2} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) - \frac{1}{3} \tanh^{-1}(x)$$

Mathematica [A]

time = 0.01, size = 32, normalized size = 1.33

$$\frac{1}{6} \left(2\sqrt{2} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) + \log(1 - x) - \log(1 + x) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[x^2/(-2 + x^2 + x^4),x]``[Out] (2*sqrt[2]*ArcTan[x/Sqrt[2]] + Log[1 - x] - Log[1 + x])/6`**Mathics [A]**

time = 1.95, size = 25, normalized size = 1.04

$$-\frac{\text{Log}[1 + x]}{6} + \frac{\text{Log}[-1 + x]}{6} + \frac{\sqrt{2} \text{ArcTan} \left[\frac{\sqrt{2} x}{2} \right]}{3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^2/(x^4 + x^2 - 2),x]')``[Out] -Log[1 + x] / 6 + Log[-1 + x] / 6 + Sqrt[2] ArcTan[Sqrt[2] x / 2] / 3`**Maple [A]**

time = 0.02, size = 26, normalized size = 1.08

method	result	size
default	$\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3} + \frac{\ln(-1+x)}{6} - \frac{\ln(1+x)}{6}$	26
risch	$\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3} + \frac{\ln(-1+x)}{6} - \frac{\ln(1+x)}{6}$	26

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2/(x^4+x^2-2),x,method=_RETURNVERBOSE)``[Out] 1/3*arctan(1/2*x*2^(1/2))*2^(1/2)+1/6*ln(-1+x)-1/6*ln(1+x)`

Maxima [A]

time = 0.35, size = 25, normalized size = 1.04

$$\frac{1}{3} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) - \frac{1}{6} \log(x+1) + \frac{1}{6} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2/(x^4+x^2-2),x, algorithm="maxima")``[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(x + 1) + 1/6*log(x - 1)`**Fricas [A]**

time = 0.34, size = 25, normalized size = 1.04

$$\frac{1}{3} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) - \frac{1}{6} \log(x+1) + \frac{1}{6} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2/(x^4+x^2-2),x, algorithm="fricas")``[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(x + 1) + 1/6*log(x - 1)`**Sympy [A]**

time = 0.08, size = 29, normalized size = 1.21

$$\frac{\log(x-1)}{6} - \frac{\log(x+1)}{6} + \frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x**2/(x**4+x**2-2),x)``[Out] log(x - 1)/6 - log(x + 1)/6 + sqrt(2)*atan(sqrt(2)*x/2)/3`**Giac [A]**

time = 0.00, size = 38, normalized size = 1.58

$$\frac{\ln|x-1|}{6} - \frac{\ln|x+1|}{6} + \frac{4 \arctan\left(\frac{x}{\sqrt{2}}\right)}{2 \cdot 3\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2/(x^4+x^2-2),x)``[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(abs(x + 1)) + 1/6*log(abs(x - 1))`

Mupad [B]

time = 0.06, size = 17, normalized size = 0.71

$$\frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} x}{2}\right)}{3} - \frac{\operatorname{atanh}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(x^2 + x^4 - 2),x)`

[Out] `(2^(1/2)*atan((2^(1/2)*x)/2))/3 - atanh(x)/3`

$$3.110 \quad \int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4} dx$$

Optimal. Leaf size=41

$$\frac{1}{1+x} + \frac{4}{3}\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{3}\log(1+x) + \frac{2}{3}\log(2+x^2)$$

[Out] 1/(1+x)-1/3*ln(1+x)+2/3*ln(x^2+2)+4/3*arctan(1/2*x*2^(1/2))*2^(1/2)

Rubi [A]

time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1608, 2100, 649, 209, 266}

$$\frac{2}{3}\log(x^2+2) + \frac{1}{x+1} - \frac{1}{3}\log(x+1) + \frac{4}{3}\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(6*x + 4*x^2 + x^3)/(2 + 4*x + 3*x^2 + 2*x^3 + x^4), x]

[Out] (1 + x)^(-1) + (4*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - Log[1 + x]/3 + (2*Log[2 + x^2])/3

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 649

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[(-a)*c]

Rule 1608

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q-p) + c*x^(r-p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rule 2100

`Int[(P_)^(p_)*(Qm_), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P, x] && ILtQ[p, 0]`

Rubi steps

$$\begin{aligned}
 \int \frac{6x + 4x^2 + x^3}{2 + 4x + 3x^2 + 2x^3 + x^4} dx &= \int \frac{x(6 + 4x + x^2)}{2 + 4x + 3x^2 + 2x^3 + x^4} dx \\
 &= \int \left(-\frac{1}{(1+x)^2} - \frac{1}{3(1+x)} + \frac{4(2+x)}{3(2+x^2)} \right) dx \\
 &= \frac{1}{1+x} - \frac{1}{3} \log(1+x) + \frac{4}{3} \int \frac{2+x}{2+x^2} dx \\
 &= \frac{1}{1+x} - \frac{1}{3} \log(1+x) + \frac{4}{3} \int \frac{x}{2+x^2} dx + \frac{8}{3} \int \frac{1}{2+x^2} dx \\
 &= \frac{1}{1+x} + \frac{4}{3} \sqrt{2} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) - \frac{1}{3} \log(1+x) + \frac{2}{3} \log(2+x^2)
 \end{aligned}$$

Mathematica [A]

time = 0.02, size = 41, normalized size = 1.00

$$\frac{1}{1+x} + \frac{4}{3} \sqrt{2} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) - \frac{1}{3} \log(1+x) + \frac{2}{3} \log(2+x^2)$$

Antiderivative was successfully verified.

[In] `Integrate[(6*x + 4*x^2 + x^3)/(2 + 4*x + 3*x^2 + 2*x^3 + x^4), x]`

[Out] `(1 + x)^(-1) + (4*sqrt[2]*ArcTan[x/sqrt[2]])/3 - Log[1 + x]/3 + (2*Log[2 + x^2])/3`

Mathics [A]

time = 1.98, size = 40, normalized size = 0.98

$$\frac{3 + (1+x) \left(-\log[1+x] + 2 \log[2+x^2] + 4\sqrt{2} \operatorname{ArcTan} \left[\frac{\sqrt{2}x}{2} \right] \right)}{3 + 3x}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(x^3 + 4*x^2 + 6*x)/(x^4 + 2*x^3 + 3*x^2 + 4*x + 2), x]')`

[Out] `(3 + (1 + x) (-Log[1 + x] + 2 Log[2 + x ^ 2] + 4 Sqrt[2] ArcTan[Sqrt[2] x / 2])) / (3 (1 + x))`

Maple [A]

time = 0.02, size = 33, normalized size = 0.80

method	result	size
default	$\frac{1}{1+x} - \frac{\ln(1+x)}{3} + \frac{2\ln(x^2+2)}{3} + \frac{4 \arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}$	33
risch	$\frac{1}{1+x} - \frac{\ln(1+x)}{3} + \frac{2\ln(x^2+2)}{3} + \frac{4 \arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}$	33

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x,method=_RETURNVERBOSE)

[Out] 1/(1+x)-1/3*ln(1+x)+2/3*ln(x^2+2)+4/3*arctan(1/2*x*2^(1/2))*2^(1/2)

Maxima [A]

time = 0.35, size = 32, normalized size = 0.78

$$\frac{4}{3} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + \frac{1}{x+1} + \frac{2}{3} \log(x^2+2) - \frac{1}{3} \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x, algorithm="maxima")

[Out] 4/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/(x + 1) + 2/3*log(x^2 + 2) - 1/3*log(x + 1)

Fricas [A]

time = 0.32, size = 44, normalized size = 1.07

$$\frac{4 \sqrt{2} (x+1) \arctan\left(\frac{1}{2} \sqrt{2} x\right) + 2 (x+1) \log(x^2+2) - (x+1) \log(x+1) + 3}{3 (x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x, algorithm="fricas")

[Out] 1/3*(4*sqrt(2)*(x + 1)*arctan(1/2*sqrt(2)*x) + 2*(x + 1)*log(x^2 + 2) - (x + 1)*log(x + 1) + 3)/(x + 1)

Sympy [A]

time = 0.08, size = 39, normalized size = 0.95

$$-\frac{\log(x+1)}{3} + \frac{2\log(x^2+2)}{3} + \frac{4\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{3} + \frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+4*x**2+6*x)/(x**4+2*x**3+3*x**2+4*x+2),x)

[Out] $-\log(x + 1)/3 + 2*\log(x^2 + 2)/3 + 4*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x/2)/3 + 1/(x + 1)$

Giac [A]

time = 0.00, size = 49, normalized size = 1.20

$$\frac{2}{3} \ln(x^2 + 2) + \frac{16 \arctan\left(\frac{x}{\sqrt{2}}\right)}{2 \cdot 3\sqrt{2}} - \frac{\ln|x + 1|}{3} + \frac{\frac{1}{3} \cdot 3}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x)

[Out] $4/3*\sqrt{2}*\arctan(1/2*\sqrt{2}*x) + 1/(x + 1) + 2/3*\log(x^2 + 2) - 1/3*\log(\operatorname{abs}(x + 1))$

Mupad [B]

time = 0.25, size = 49, normalized size = 1.20

$$\frac{1}{x + 1} - \frac{\ln(x + 1)}{3} - \ln\left(x - \sqrt{2} \operatorname{li}\right) \left(-\frac{2}{3} + \frac{\sqrt{2} \operatorname{2i}}{3}\right) + \ln\left(x + \sqrt{2} \operatorname{li}\right) \left(\frac{2}{3} + \frac{\sqrt{2} \operatorname{2i}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((6*x + 4*x^2 + x^3)/(4*x + 3*x^2 + 2*x^3 + x^4 + 2),x)

[Out] $1/(x + 1) - \log(x + 1)/3 - \log(x - 2^{(1/2)}*1i)*((2^{(1/2)}*2i)/3 - 2/3) + \log(x + 2^{(1/2)}*1i)*((2^{(1/2)}*2i)/3 + 2/3)$

$$3.111 \quad \int \frac{x}{(1+x)(1+2x)^2(1+x^2)} dx$$

Optimal. Leaf size=46

$$\frac{2}{5(1+2x)} + \frac{1}{50} \tan^{-1}(x) - \frac{1}{2} \log(1+x) + \frac{16}{25} \log(1+2x) - \frac{7}{100} \log(1+x^2)$$

[Out] 2/5/(1+2*x)+1/50*arctan(x)-1/2*ln(1+x)+16/25*ln(1+2*x)-7/100*ln(x^2+1)

Rubi [A]

time = 0.13, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6857, 649, 209, 266}

$$-\frac{7}{100} \log(x^2 + 1) + \frac{2}{5(2x + 1)} - \frac{1}{2} \log(x + 1) + \frac{16}{25} \log(2x + 1) + \frac{1}{50} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x/((1 + x)*(1 + 2*x)^2*(1 + x^2)),x]

[Out] 2/(5*(1 + 2*x)) + ArcTan[x]/50 - Log[1 + x]/2 + (16*Log[1 + 2*x])/25 - (7*Log[1 + x^2])/100

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 649

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[(-a)*c]

Rule 6857

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x}{(1+x)(1+2x)^2(1+x^2)} dx &= \int \left(-\frac{1}{2(1+x)} - \frac{4}{5(1+2x)^2} + \frac{32}{25(1+2x)} + \frac{1-7x}{50(1+x^2)} \right) dx \\
&= \frac{2}{5(1+2x)} - \frac{1}{2} \log(1+x) + \frac{16}{25} \log(1+2x) + \frac{1}{50} \int \frac{1-7x}{1+x^2} dx \\
&= \frac{2}{5(1+2x)} - \frac{1}{2} \log(1+x) + \frac{16}{25} \log(1+2x) + \frac{1}{50} \int \frac{1}{1+x^2} dx - \frac{7}{50} \int \frac{x}{1+x^2} dx \\
&= \frac{2}{5(1+2x)} + \frac{1}{50} \tan^{-1}(x) - \frac{1}{2} \log(1+x) + \frac{16}{25} \log(1+2x) - \frac{7}{100} \log(1+x^2)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 40, normalized size = 0.87

$$\frac{1}{100} \left(\frac{40}{1+2x} + 2 \tan^{-1}(x) - 50 \log(1+x) + 64 \log(1+2x) - 7 \log(1+x^2) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[x/((1+x)*(1+2*x)^2*(1+x^2)),x]``[Out] (40/(1+2*x) + 2*ArcTan[x] - 50*Log[1+x] + 64*Log[1+2*x] - 7*Log[1+x^2])/100`**Mathics [A]**

time = 2.06, size = 42, normalized size = 0.91

$$\frac{40 + (1+2x) \left(-50 \operatorname{Log}[1+x] - 7 \operatorname{Log}[1+x^2] + 2 \operatorname{ArcTan}[x] + 64 \operatorname{Log}\left[\frac{1}{2} + x\right] \right)}{100 + 200x}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x/((1+x)*(1+2*x)^2*(1+x^2)),x]')``[Out] (40 + (1 + 2 x) (-50 Log[1 + x] - 7 Log[1 + x ^ 2] + 2 ArcTan[x] + 64 Log[1 / 2 + x])) / (100 (1 + 2 x))`**Maple [A]**

time = 0.06, size = 37, normalized size = 0.80

method	result	size
risch	$\frac{1}{5x+\frac{5}{2}} + \frac{16 \ln(1+2x)}{25} - \frac{7 \ln(x^2+1)}{100} + \frac{\arctan(x)}{50} - \frac{\ln(1+x)}{2}$	35
default	$\frac{2}{5(1+2x)} + \frac{\arctan(x)}{50} - \frac{\ln(1+x)}{2} + \frac{16 \ln(1+2x)}{25} - \frac{7 \ln(x^2+1)}{100}$	37

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(1+x)/(1+2*x)^2/(x^2+1),x,method=_RETURNVERBOSE)`

[Out] $2/5/(1+2*x)+1/50*\arctan(x)-1/2*\ln(1+x)+16/25*\ln(1+2*x)-7/100*\ln(x^2+1)$

Maxima [A]

time = 0.35, size = 36, normalized size = 0.78

$$\frac{2}{5(2x+1)} + \frac{1}{50} \arctan(x) - \frac{7}{100} \log(x^2+1) + \frac{16}{25} \log(2x+1) - \frac{1}{2} \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(1+x)/(1+2*x)^2/(x^2+1),x, algorithm="maxima")`

[Out] $2/5/(2*x + 1) + 1/50*\arctan(x) - 7/100*\log(x^2 + 1) + 16/25*\log(2*x + 1) - 1/2*\log(x + 1)$

Fricas [A]

time = 0.33, size = 57, normalized size = 1.24

$$\frac{2(2x+1)\arctan(x) - 7(2x+1)\log(x^2+1) + 64(2x+1)\log(2x+1) - 50(2x+1)\log(x+1) + 40}{100(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(1+x)/(1+2*x)^2/(x^2+1),x, algorithm="fricas")`

[Out] $1/100*(2*(2*x + 1)*\arctan(x) - 7*(2*x + 1)*\log(x^2 + 1) + 64*(2*x + 1)*\log(2*x + 1) - 50*(2*x + 1)*\log(x + 1) + 40)/(2*x + 1)$

Sympy [A]

time = 0.11, size = 37, normalized size = 0.80

$$\frac{16 \log(x + \frac{1}{2})}{25} - \frac{\log(x + 1)}{2} - \frac{7 \log(x^2 + 1)}{100} + \frac{\operatorname{atan}(x)}{50} + \frac{2}{10x + 5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(1+x)/(1+2*x)**2/(x**2+1),x)`

[Out] $16*\log(x + 1/2)/25 - \log(x + 1)/2 - 7*\log(x**2 + 1)/100 + \operatorname{atan}(x)/50 + 2/(10*x + 5)$

Giac [A]

time = 0.00, size = 47, normalized size = 1.02

$$-\frac{\ln|x+1|}{2} - \frac{7}{100} \ln(x^2+1) + \frac{\arctan x}{50} + \frac{16}{25} \ln|2x+1| + \frac{\frac{1}{25} \cdot 10}{2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/(1+2*x)^2/(x^2+1),x)

[Out] 2/5/(2*x + 1) + 1/50*arctan(x) - 7/100*log(x^2 + 1) + 16/25*log(abs(2*x + 1)) - 1/2*log(abs(x + 1))

Mupad [B]

time = 0.19, size = 38, normalized size = 0.83

$$\frac{16 \ln\left(x + \frac{1}{2}\right)}{25} - \frac{\ln(x+1)}{2} + \frac{1}{5\left(x + \frac{1}{2}\right)} + \ln(x-i) \left(-\frac{7}{100} - \frac{1}{100}i\right) + \ln(x+1i) \left(-\frac{7}{100} + \frac{1}{100}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((2*x + 1)^2*(x^2 + 1)*(x + 1)),x)

[Out] (16*log(x + 1/2))/25 - log(x + 1)/2 - log(x - 1i)*(7/100 + 1i/100) - log(x + 1i)*(7/100 - 1i/100) + 1/(5*(x + 1/2))

$$3.112 \quad \int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)} dx$$

Optimal. Leaf size=47

$$-\frac{1}{2(1-x)^2} + \frac{5}{2(1-x)} - \tan^{-1}(x) - \frac{3}{2} \log(1-x) + \frac{3}{4} \log(1+x^2)$$

[Out] -1/2/(1-x)^2+5/2/(1-x)-arctan(x)-3/2*ln(1-x)+3/4*ln(x^2+1)

Rubi [A]

time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1643, 649, 209, 266}

$$\frac{3}{4} \log(x^2 + 1) + \frac{5}{2(1-x)} - \frac{1}{2(1-x)^2} - \frac{3}{2} \log(1-x) - \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-2 + x + 3*x^2)/((-1 + x)^3*(1 + x^2)),x]

[Out] -1/2*1/(1 - x)^2 + 5/(2*(1 - x)) - ArcTan[x] - (3*Log[1 - x])/2 + (3*Log[1 + x^2])/4

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 649

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[(-a)*c]

Rule 1643

Int[(Pq_)*((d_) + (e_.)*(x_)^(m_.))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)} dx &= \int \left(\frac{1}{(-1+x)^3} + \frac{5}{2(-1+x)^2} - \frac{3}{2(-1+x)} + \frac{-2+3x}{2(1+x^2)} \right) dx \\
&= -\frac{1}{2(1-x)^2} + \frac{5}{2(1-x)} - \frac{3}{2} \log(1-x) + \frac{1}{2} \int \frac{-2+3x}{1+x^2} dx \\
&= -\frac{1}{2(1-x)^2} + \frac{5}{2(1-x)} - \frac{3}{2} \log(1-x) + \frac{3}{2} \int \frac{x}{1+x^2} dx - \int \frac{1}{1+x^2} dx \\
&= -\frac{1}{2(1-x)^2} + \frac{5}{2(1-x)} - \tan^{-1}(x) - \frac{3}{2} \log(1-x) + \frac{3}{4} \log(1+x^2)
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 37, normalized size = 0.79

$$\frac{1}{4} \left(-\frac{2}{(-1+x)^2} - \frac{10}{-1+x} - 4 \tan^{-1}(x) - 6 \log(-1+x) + 3 \log(1+x^2) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(-2 + x + 3*x^2)/((-1 + x)^3*(1 + x^2)), x]``[Out] (-2/(-1 + x)^2 - 10/(-1 + x) - 4*ArcTan[x] - 6*Log[-1 + x] + 3*Log[1 + x^2])/4`**Mathics [A]**

time = 2.05, size = 45, normalized size = 0.96

$$\frac{8 - 10x + (1 - 2x + x^2)(-6 \operatorname{Log}[-1 + x] - 4 \operatorname{ArcTan}[x] + 3 \operatorname{Log}[1 + x^2])}{4 - 8x + 4x^2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(3*x^2 + x - 2)/((x - 1)^3*(x^2 + 1)), x]')``[Out] (8 - 10 x + (1 - 2 x + x ^ 2) (-6 Log[-1 + x] - 4 ArcTan[x] + 3 Log[1 + x ^ 2])) / (4 (1 - 2 x + x ^ 2))`**Maple [A]**

time = 0.06, size = 34, normalized size = 0.72

method	result	size
risch	$\frac{-\frac{5x}{2}+2}{(-1+x)^2} + \frac{3 \ln(x^2+1)}{4} - \arctan(x) - \frac{3 \ln(-1+x)}{2}$	31
default	$-\frac{1}{2(-1+x)^2} - \frac{5}{2(-1+x)} - \frac{3 \ln(-1+x)}{2} + \frac{3 \ln(x^2+1)}{4} - \arctan(x)$	34

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2+x-2)/(-1+x)^3/(x^2+1),x,method=_RETURNVERBOSE)`

[Out] $-1/2/(-1+x)^2-5/2/(-1+x)-3/2*\ln(-1+x)+3/4*\ln(x^2+1)-\arctan(x)$

Maxima [A]

time = 0.34, size = 36, normalized size = 0.77

$$-\frac{5x-4}{2(x^2-2x+1)} - \arctan(x) + \frac{3}{4} \log(x^2+1) - \frac{3}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+x-2)/(-1+x)^3/(x^2+1),x, algorithm="maxima")`

[Out] $-1/2*(5*x - 4)/(x^2 - 2*x + 1) - \arctan(x) + 3/4*\log(x^2 + 1) - 3/2*\log(x - 1)$

Fricas [A]

time = 0.32, size = 59, normalized size = 1.26

$$\frac{4(x^2-2x+1)\arctan(x) - 3(x^2-2x+1)\log(x^2+1) + 6(x^2-2x+1)\log(x-1) + 10x - 8}{4(x^2-2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+x-2)/(-1+x)^3/(x^2+1),x, algorithm="fricas")`

[Out] $-1/4*(4*(x^2 - 2*x + 1)*\arctan(x) - 3*(x^2 - 2*x + 1)*\log(x^2 + 1) + 6*(x^2 - 2*x + 1)*\log(x - 1) + 10*x - 8)/(x^2 - 2*x + 1)$

Sympy [A]

time = 0.09, size = 36, normalized size = 0.77

$$\frac{4-5x}{2x^2-4x+2} - \frac{3\log(x-1)}{2} + \frac{3\log(x^2+1)}{4} - \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2+x-2)/(-1+x)**3/(x**2+1),x)`

[Out] $(4 - 5*x)/(2*x**2 - 4*x + 2) - 3*\log(x - 1)/2 + 3*\log(x**2 + 1)/4 - \operatorname{atan}(x)$

Giac [A]

time = 0.00, size = 39, normalized size = 0.83

$$\frac{3}{4} \ln(x^2+1) - \arctan x - \frac{3}{2} \ln|x-1| + \frac{\frac{1}{4}(-10x+8)}{(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+x-2)/(-1+x)^3/(x^2+1),x)

[Out] -1/2*(5*x - 4)/(x - 1)^2 - arctan(x) + 3/4*log(x^2 + 1) - 3/2*log(abs(x - 1))

Mupad [B]

time = 0.04, size = 42, normalized size = 0.89

$$-\frac{3 \ln(x-1)}{2} - \frac{\frac{5x}{2} - 2}{x^2 - 2x + 1} + \ln(x-i) \left(\frac{3}{4} + \frac{1}{2}i\right) + \ln(x+1i) \left(\frac{3}{4} - \frac{1}{2}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 - 2)/((x^2 + 1)*(x - 1)^3),x)

[Out] log(x - 1i)*(3/4 + 1i/2) - (3*log(x - 1))/2 + log(x + 1i)*(3/4 - 1i/2) - ((5*x)/2 - 2)/(x^2 - 2*x + 1)

3.113 $\int \frac{1}{1+x^2+x^4} dx$

Optimal. Leaf size=67

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4} \log(1-x+x^2) + \frac{1}{4} \log(1+x+x^2)$$

[Out] $-1/4*\ln(x^2-x+1)+1/4*\ln(x^2+x+1)-1/6*\arctan(1/3*(1-2*x)*3^{(1/2)})*3^{(1/2)}+1/6*\arctan(1/3*(1+2*x)*3^{(1/2)})*3^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1108, 648, 632, 210, 642}

$$-\frac{1}{4} \log(x^2 - x + 1) + \frac{1}{4} \log(x^2 + x + 1) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] `Int[(1 + x^2 + x^4)^(-1), x]`

[Out] $-1/2*\text{ArcTan}[(1 - 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3] + \text{ArcTan}[(1 + 2*x)/\text{Sqrt}[3]]/(2*\text{Sqrt}[3]) - \text{Log}[1 - x + x^2]/4 + \text{Log}[1 + x + x^2]/4$

Rule 210

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 632

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 642

`Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
+ x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{1+x^2+x^4} dx &= \frac{1}{2} \int \frac{1-x}{1-x+x^2} dx + \frac{1}{2} \int \frac{1+x}{1+x+x^2} dx \\ &= \frac{1}{4} \int \frac{1}{1-x+x^2} dx - \frac{1}{4} \int \frac{-1+2x}{1-x+x^2} dx + \frac{1}{4} \int \frac{1}{1+x+x^2} dx + \frac{1}{4} \int \frac{1+2x}{1+x+x^2} dx \\ &= -\frac{1}{4} \log(1-x+x^2) + \frac{1}{4} \log(1+x+x^2) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x \right) - \frac{1}{2} \\ &= -\frac{\tan^{-1} \left(\frac{1-2x}{\sqrt{3}} \right)}{2\sqrt{3}} + \frac{\tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)}{2\sqrt{3}} - \frac{1}{4} \log(1-x+x^2) + \frac{1}{4} \log(1+x+x^2) \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.04, size = 73, normalized size = 1.09

$$\frac{i \left(\sqrt{1-i\sqrt{3}} \tan^{-1} \left(\frac{1}{2} (-i+\sqrt{3}) x \right) - \sqrt{1+i\sqrt{3}} \tan^{-1} \left(\frac{1}{2} (i+\sqrt{3}) x \right) \right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2 + x^4)^(-1), x]

[Out] (I*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x)/2] - Sqrt[1 + I*Sqrt[3]]*ArcTan[((I + Sqrt[3])*x)/2]))/Sqrt[6]

Mathics [A]

time = 2.26, size = 53, normalized size = 0.79

$$-\frac{\text{Log}[1-x+x^2]}{4} + \frac{\sqrt{3} \text{ArcTan} \left[\frac{\sqrt{3}(-1+2x)}{3} \right]}{6} + \frac{\sqrt{3} \text{ArcTan} \left[\frac{\sqrt{3}(1+2x)}{3} \right]}{6} + \frac{\text{Log}[1+x+x^2]}{4}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(x^4 + x^2 + 1),x]')`

[Out] $-\text{Log}[1 - x + x^2] / 4 + \text{Sqrt}[3] \text{ArcTan}[\text{Sqrt}[3] (-1 + 2x) / 3] / 6 + \text{Sqrt}[3] \text{ArcTan}[\text{Sqrt}[3] (1 + 2x) / 3] / 6 + \text{Log}[1 + x + x^2] / 4$

Maple [A]

time = 0.01, size = 54, normalized size = 0.81

method	result	size
default	$\frac{\ln(x^2+x+1)}{4} + \frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6} - \frac{\ln(x^2-x+1)}{4} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$	54
risch	$-\frac{\ln(4x^2-4x+4)}{4} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6} + \frac{\ln(4x^2+4x+4)}{4}$	60

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4+x^2+1),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{4} \ln(x^2+x+1) + \frac{1}{6} \arctan(1/3*(1+2*x)*3^{(1/2)}) * 3^{(1/2)} - \frac{1}{4} \ln(x^2-x+1) + \frac{1}{6} * 3^{(1/2)} * \arctan(1/3*(2*x-1)*3^{(1/2)})$

Maxima [A]

time = 0.35, size = 53, normalized size = 0.79

$$\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x-1)\right) + \frac{1}{4} \log(x^2+x+1) - \frac{1}{4} \log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4+x^2+1),x, algorithm="maxima")`

[Out] $\frac{1}{6} \sqrt{3} \arctan(1/3 \sqrt{3} (2x+1)) + \frac{1}{6} \sqrt{3} \arctan(1/3 \sqrt{3} (2x-1)) + \frac{1}{4} \log(x^2+x+1) - \frac{1}{4} \log(x^2-x+1)$

Fricas [A]

time = 0.32, size = 53, normalized size = 0.79

$$\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x-1)\right) + \frac{1}{4} \log(x^2+x+1) - \frac{1}{4} \log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4+x^2+1),x, algorithm="fricas")`

[Out] $\frac{1}{6} \sqrt{3} \arctan(1/3 \sqrt{3} (2x+1)) + \frac{1}{6} \sqrt{3} \arctan(1/3 \sqrt{3} (2x-1)) + \frac{1}{4} \log(x^2+x+1) - \frac{1}{4} \log(x^2-x+1)$

Sympy [A]

time = 0.10, size = 70, normalized size = 1.04

$$-\frac{\log(x^2 - x + 1)}{4} + \frac{\log(x^2 + x + 1)}{4} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x - \sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x + \sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**4+x**2+1),x)

[Out] -log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6

Giac [A]

time = 0.00, size = 67, normalized size = 1.00

$$-\frac{\ln(x^2 - x + 1)}{4} + \frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\ln(x^2 + x + 1)}{4} + \frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+x^2+1),x)

[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)

Mupad [B]

time = 0.09, size = 47, normalized size = 0.70

$$\operatorname{atanh}\left(\frac{2x}{-1 + \sqrt{3} \operatorname{li}}\right) \left(-\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{6}\right) + \operatorname{atanh}\left(\frac{2x}{1 + \sqrt{3} \operatorname{li}}\right) \left(\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2 + x^4 + 1),x)

[Out] atanh((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) + atanh((2*x)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/6 + 1/2)

$$3.114 \quad \int \frac{3+2x^3}{-9x+x^5} dx$$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{3} + \frac{1}{12} \log(9-x^4)$$

[Out] -1/3*ln(x)+1/12*ln(-x^4+9)+1/3*arctan(1/3*x*3^(1/2))*3^(1/2)-1/3*arctanh(1/3*x*3^(1/2))*3^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1607, 1845, 272, 36, 31, 29, 304, 209, 212}

$$\frac{1}{12} \log(9-x^4) - \frac{\log(x)}{3} + \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(3 + 2*x^3)/(-9*x + x^5), x]

[Out] ArcTan[x/Sqrt[3]]/Sqrt[3] - ArcTanh[x/Sqrt[3]]/Sqrt[3] - Log[x]/3 + Log[9 - x^4]/12

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 209

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-a/b,
2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a
/b, 0]
```

Rule 1607

```
Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1845

```
Int[((Pq_)*((c_)*(x_)^(m_)))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii])*x^(n/2)
)/(c^ii*(a + b*x^n))], {ii, 0, n/2 - 1}}, Int[v, x] /; SumQ[v] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{3 + 2x^3}{-9x + x^5} dx &= \int \frac{3 + 2x^3}{x(-9 + x^4)} dx \\
&= \int \left(\frac{3}{x(-9 + x^4)} + \frac{2x^2}{-9 + x^4} \right) dx \\
&= 2 \int \frac{x^2}{-9 + x^4} dx + 3 \int \frac{1}{x(-9 + x^4)} dx \\
&= \frac{3}{4} \text{Subst} \left(\int \frac{1}{(-9 + x)x} dx, x, x^4 \right) - \int \frac{1}{3 - x^2} dx + \int \frac{1}{3 + x^2} dx \\
&= \frac{\tan^{-1} \left(\frac{x}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{\tanh^{-1} \left(\frac{x}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{1}{12} \text{Subst} \left(\int \frac{1}{-9 + x} dx, x, x^4 \right) - \frac{1}{12} \text{Subst} \left(\int \frac{1}{x} dx, x, x^4 \right) \\
&= \frac{\tan^{-1} \left(\frac{x}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{\tanh^{-1} \left(\frac{x}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{\log(x)}{3} + \frac{1}{12} \log(9 - x^4)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 67, normalized size = 1.40

$$\frac{1}{12} \left(4\sqrt{3} \tan^{-1} \left(\frac{x}{\sqrt{3}} \right) - 4\log(x) + 2\sqrt{3} \log(3 - \sqrt{3}x) - 2\sqrt{3} \log(3 + \sqrt{3}x) + \log(9 - x^4) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(3 + 2*x^3)/(-9*x + x^5), x]`

```
[Out] (4*Sqrt[3]*ArcTan[x/Sqrt[3]] - 4*Log[x] + 2*Sqrt[3]*Log[3 - Sqrt[3]*x] - 2*
Sqrt[3]*Log[3 + Sqrt[3]*x] + Log[9 - x^4])/12
```

Mathics [C] Result contains complex when optimal does not.

time = 4.94, size = 71, normalized size = 1.48

$$-\frac{\text{Log}[x]}{3} + \frac{(1 - 2\sqrt{3}) \text{Log}[\sqrt{3} + x]}{12} + \frac{(1 - 2I\sqrt{3}) \text{Log}[-I\sqrt{3} + x]}{12} + \frac{(1 + 2I\sqrt{3}) \text{Log}[I\sqrt{3} + x]}{12} + \frac{(1 + 2\sqrt{3}) \text{Log}[-\sqrt{3} + x]}{12}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(2*x^3 + 3)/(x^5 - 9*x), x]')`

```
[Out] -Log[x] / 3 + (1 - 2 Sqrt[3]) Log[Sqrt[3] + x] / 12 + (1 - 2 I Sqrt[3]) Log
[-I Sqrt[3] + x] / 12 + (1 + 2 I Sqrt[3]) Log[I Sqrt[3] + x] / 12 + (1 + 2
Sqrt[3]) Log[-Sqrt[3] + x] / 12
```

Maple [A]

time = 0.08, size = 46, normalized size = 0.96

method	result
default	$\frac{\ln(x^2+3)}{12} + \frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3} - \frac{\ln(x)}{3} + \frac{\ln(x^2-3)}{12} - \frac{\operatorname{arctanh}\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}$
risch	$\frac{\ln(x-\sqrt{3})}{12} + \frac{\sqrt{3}\ln(x-\sqrt{3})}{6} + \frac{\ln(x+\sqrt{3})}{12} - \frac{\sqrt{3}\ln(x+\sqrt{3})}{6} + \frac{\ln(x^2+3)}{12} + \frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3} - \frac{\ln(x)}{3}$
meijerg	$-\frac{\ln(x)}{3} + \frac{\ln(3)}{6} - \frac{i\pi}{12} + \frac{\ln\left(1-\frac{x^4}{9}\right)}{12} + \frac{x^3\sqrt{3}\left(\ln\left(1-\frac{\sqrt{3}(x^4)^{\frac{1}{4}}}{3}\right) - \ln\left(1+\frac{\sqrt{3}(x^4)^{\frac{1}{4}}}{3}\right) + 2\arctan\left(\frac{\sqrt{3}(x^4)^{\frac{1}{4}}}{3}\right)\right)}{6(x^4)^{\frac{3}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^3+3)/(x^5-9*x),x,method=_RETURNVERBOSE)`

[Out] $1/12*\ln(x^2+3)+1/3*\arctan(1/3*x*3^{(1/2)})*3^{(1/2)}-1/3*\ln(x)+1/12*\ln(x^2-3)-1/3*\operatorname{arctanh}(1/3*x*3^{(1/2)})*3^{(1/2)}$

Maxima [A]

time = 0.35, size = 54, normalized size = 1.12

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) + \frac{1}{6}\sqrt{3}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right) + \frac{1}{12}\log(x^2+3) + \frac{1}{12}\log(x^2-3) - \frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^3+3)/(x^5-9*x),x, algorithm="maxima")`

[Out] $1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*x) + 1/6*\sqrt{3}*\log((x - \sqrt{3})/(x + \sqrt{3})) + 1/12*\log(x^2 + 3) + 1/12*\log(x^2 - 3) - 1/3*\log(x)$

Fricas [A]

time = 0.31, size = 58, normalized size = 1.21

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) + \frac{1}{6}\sqrt{3}\log\left(\frac{x^2-2\sqrt{3}x+3}{x^2-3}\right) + \frac{1}{12}\log(x^2+3) + \frac{1}{12}\log(x^2-3) - \frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^3+3)/(x^5-9*x),x, algorithm="fricas")`

[Out] $1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*x) + 1/6*\sqrt{3}*\log((x^2 - 2*\sqrt{3}*x + 3)/(x^2 - 3)) + 1/12*\log(x^2 + 3) + 1/12*\log(x^2 - 3) - 1/3*\log(x)$

Sympy [C] Result contains complex when optimal does not.

time = 0.32, size = 306, normalized size = 6.38

$$\frac{\log\left(\frac{1}{3}\sqrt{3}\right)\log\left(\frac{1}{3}\sqrt{3}\right) + \frac{1311}{1154}\sqrt{3}\log\left(\frac{1311}{1154}\sqrt{3}\right) + \frac{1300}{401}\sqrt{3}\log\left(\frac{1300}{401}\sqrt{3}\right) + \frac{4139}{401}\sqrt{3}\log\left(\frac{4139}{401}\sqrt{3}\right) + \frac{10000}{401}\sqrt{3}\log\left(\frac{10000}{401}\sqrt{3}\right)}{\left(\frac{1}{3}\sqrt{3}\right)\log\left(\frac{1}{3}\sqrt{3}\right) + \frac{1311}{1154}\sqrt{3}\log\left(\frac{1311}{1154}\sqrt{3}\right) + \frac{1300}{401}\sqrt{3}\log\left(\frac{1300}{401}\sqrt{3}\right) + \frac{4139}{401}\sqrt{3}\log\left(\frac{4139}{401}\sqrt{3}\right) + \frac{10000}{401}\sqrt{3}\log\left(\frac{10000}{401}\sqrt{3}\right)} + \frac{1}{12}\log(x^2+3) + \frac{1}{12}\log(x^2-3) - \frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3)/(x**5-9*x),x)

[Out] $-\log(x)/3 + (1/12 + \sqrt{3}*I/6)*\log(x + 17413/11544 - 943*\sqrt{3}*I/5772 + 1368*(1/12 + \sqrt{3}*I/6)**3/481 + 4158*(1/12 + \sqrt{3}*I/6)**2/481 - 108000*(1/12 + \sqrt{3}*I/6)**4/481) + (1/12 - \sqrt{3}*I/6)*\log(x + 17413/11544 - 108000*(1/12 - \sqrt{3}*I/6)**4/481 + 4158*(1/12 - \sqrt{3}*I/6)**2/481 + 1368*(1/12 - \sqrt{3}*I/6)**3/481 + 943*\sqrt{3}*I/5772) + (1/12 - \sqrt{3}/6)*\log(x - 108000*(1/12 - \sqrt{3}/6)**4/481 + 1368*(1/12 - \sqrt{3}/6)**3/481 + 943*\sqrt{3}/5772 + 4158*(1/12 - \sqrt{3}/6)**2/481 + 17413/11544) + (1/12 + \sqrt{3}/6)*\log(x - 108000*(1/12 + \sqrt{3}/6)**4/481 - 943*\sqrt{3}/5772 + 1368*(1/12 + \sqrt{3}/6)**3/481 + 4158*(1/12 + \sqrt{3}/6)**2/481 + 17413/11544)$

Giac [A]

time = 0.00, size = 82, normalized size = 1.71

$$-\frac{\ln|x|}{3} + \frac{\ln|x^2-3|}{12} + \frac{\ln\left|\frac{2x-2\sqrt{3}}{2x+2\sqrt{3}}\right|}{2\sqrt{3}} + \frac{\ln(x^2+3)}{12} + \frac{2\arctan\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3)/(x^5-9*x),x)

[Out] $1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*x) + 1/6*\sqrt{3}*\log(\text{abs}((x - \sqrt{3})/(x + \sqrt{3}))) + 1/12*\log(x^2 + 3) + 1/12*\log(\text{abs}(x^2 - 3)) - 1/3*\log(\text{abs}(x))$

Mupad [B]

time = 0.17, size = 73, normalized size = 1.52

$$\ln(x - \sqrt{3}) \left(\frac{\sqrt{3}}{6} + \frac{1}{12} \right) - \ln(x + \sqrt{3}) \left(\frac{\sqrt{3}}{6} - \frac{1}{12} \right) - \frac{\ln(x)}{3} - \ln(x - \sqrt{3} \text{ li}) \left(-\frac{1}{12} + \frac{\sqrt{3} \text{ li}}{6} \right) + \ln(x + \sqrt{3} \text{ li}) \left(\frac{1}{12} + \frac{\sqrt{3} \text{ li}}{6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2*x^3 + 3)/(9*x - x^5),x)

[Out] $\log(x - 3^{(1/2)})*(3^{(1/2)}/6 + 1/12) - \log(x + 3^{(1/2)})*(3^{(1/2)}/6 - 1/12) - \log(x)/3 - \log(x - 3^{(1/2)}*1i)*((3^{(1/2)}*1i)/6 - 1/12) + \log(x + 3^{(1/2)}*1i)*((3^{(1/2)}*1i)/6 + 1/12)$

$$3.115 \quad \int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)} dx$$

Optimal. Leaf size=58

$$-\frac{83}{4(4-x)^2} + \frac{41}{4(4-x)} - \frac{3}{16} \tan^{-1}\left(1 - \frac{x}{2}\right) - \frac{45}{16} \log(4-x) + \frac{45}{32} \log(8-4x+x^2)$$

[Out] -83/4/(4-x)^2+41/4/(4-x)+3/16*arctan(-1+1/2*x)-45/16*ln(4-x)+45/32*ln(x^2-4*x+8)

Rubi [A]

time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1642, 648, 631, 210, 642}

$$\frac{45}{32} \log(x^2 - 4x + 8) + \frac{41}{4(4-x)} - \frac{83}{4(4-x)^2} - \frac{45}{16} \log(4-x) - \frac{3}{16} \tan^{-1}\left(1 - \frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[(-20 + 8*x + 5*x^3)/((-4 + x)^3*(8 - 4*x + x^2)),x]

[Out] -83/(4*(4 - x)^2) + 41/(4*(4 - x)) - (3*ArcTan[1 - x/2])/16 - (45*Log[4 - x])/16 + (45*Log[8 - 4*x + x^2])/32

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 631

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \int \frac{-20 + 8x + 5x^3}{(-4 + x)^3(8 - 4x + x^2)} dx &= \int \left(\frac{83}{2(-4 + x)^3} + \frac{41}{4(-4 + x)^2} - \frac{45}{16(-4 + x)} + \frac{3(-28 + 15x)}{16(8 - 4x + x^2)} \right) dx \\ &= -\frac{83}{4(4 - x)^2} + \frac{41}{4(4 - x)} - \frac{45}{16} \log(4 - x) + \frac{3}{16} \int \frac{-28 + 15x}{8 - 4x + x^2} dx \\ &= -\frac{83}{4(4 - x)^2} + \frac{41}{4(4 - x)} - \frac{45}{16} \log(4 - x) + \frac{3}{8} \int \frac{1}{8 - 4x + x^2} dx + \frac{45}{32} \int \frac{-28 + 15x}{8 - 4x + x^2} dx \\ &= -\frac{83}{4(4 - x)^2} + \frac{41}{4(4 - x)} - \frac{45}{16} \log(4 - x) + \frac{45}{32} \log(8 - 4x + x^2) + \frac{3}{16} \text{Subst} \int \frac{1}{8 - 4x + x^2} dx \\ &= -\frac{83}{4(4 - x)^2} + \frac{41}{4(4 - x)} - \frac{3}{16} \tan^{-1} \left(1 - \frac{x}{2} \right) - \frac{45}{16} \log(4 - x) + \frac{45}{32} \log(8 - 4x + x^2) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 46, normalized size = 0.79

$$\frac{1}{32} \left(-\frac{664}{(-4 + x)^2} - \frac{328}{-4 + x} + 6 \tan^{-1} \left(\frac{1}{2}(-2 + x) \right) - 90 \log(-4 + x) + 45 \log(8 - 4x + x^2) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-20 + 8*x + 5*x^3)/((-4 + x)^3*(8 - 4*x + x^2)),x]
```

```
[Out] (-664/(-4 + x)^2 - 328/(-4 + x) + 6*ArcTan[(-2 + x)/2] - 90*Log[-4 + x] + 4
5*Log[8 - 4*x + x^2])/32
```

Mathics [A]

time = 2.17, size = 53, normalized size = 0.91

$$\frac{648 - 328x + 3(16 - 8x + x^2) \left(-30 \log(-4 + x) + 2 \text{ArcTan} \left[-1 + \frac{x}{2} \right] + 15 \log(8 - 4x + x^2) \right)}{512 - 256x + 32x^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(5*x^3 + 8*x - 20)/((x - 4)^3*(x^2 - 4*x + 8)),x]')
```

[Out] $(648 - 328x + 3(16 - 8x + x^2)(-30 \operatorname{Log}[-4 + x] + 2 \operatorname{ArcTan}[-1 + x/2]) + 15 \operatorname{Log}[8 - 4x + x^2])) / (32(16 - 8x + x^2))$

Maple [A]

time = 0.11, size = 41, normalized size = 0.71

method	result	size
risch	$\frac{-\frac{41x}{4} + \frac{81}{4}}{(x-4)^2} - \frac{45 \ln(x-4)}{16} + \frac{45 \ln(x^2-4x+8)}{32} + \frac{3 \arctan(-1+\frac{x}{2})}{16}$	38
default	$-\frac{83}{4(x-4)^2} - \frac{41}{4(x-4)} - \frac{45 \ln(x-4)}{16} + \frac{45 \ln(x^2-4x+8)}{32} + \frac{3 \arctan(-1+\frac{x}{2})}{16}$	41

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^3+8*x-20)/(x-4)^3/(x^2-4*x+8),x,method=_RETURNVERBOSE)`

[Out] $-83/4/(x-4)^2 - 41/4/(x-4) - 45/16 \ln(x-4) + 45/32 \ln(x^2-4x+8) + 3/16 \arctan(-1+1/2x)$

Maxima [A]

time = 0.36, size = 43, normalized size = 0.74

$$-\frac{41x - 81}{4(x^2 - 8x + 16)} + \frac{3}{16} \arctan\left(\frac{1}{2}x - 1\right) + \frac{45}{32} \log(x^2 - 4x + 8) - \frac{45}{16} \log(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x, algorithm="maxima")`

[Out] $-1/4*(41*x - 81)/(x^2 - 8*x + 16) + 3/16*\arctan(1/2*x - 1) + 45/32*\log(x^2 - 4*x + 8) - 45/16*\log(x - 4)$

Fricas [A]

time = 0.31, size = 66, normalized size = 1.14

$$\frac{6(x^2 - 8x + 16) \arctan\left(\frac{1}{2}x - 1\right) + 45(x^2 - 8x + 16) \log(x^2 - 4x + 8) - 90(x^2 - 8x + 16) \log(x - 4) - 328x + 648}{32(x^2 - 8x + 16)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x, algorithm="fricas")`

[Out] $1/32*(6*(x^2 - 8*x + 16)*\arctan(1/2*x - 1) + 45*(x^2 - 8*x + 16)*\log(x^2 - 4*x + 8) - 90*(x^2 - 8*x + 16)*\log(x - 4) - 328*x + 648)/(x^2 - 8*x + 16)$

Sympy [A]

time = 0.09, size = 46, normalized size = 0.79

$$\frac{81 - 41x}{4x^2 - 32x + 64} - \frac{45 \log(x - 4)}{16} + \frac{45 \log(x^2 - 4x + 8)}{32} + \frac{3 \operatorname{atan}\left(\frac{x}{2} - 1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**3+8*x-20)/(-4+x)**3/(x**2-4*x+8),x)

[Out] (81 - 41*x)/(4*x**2 - 32*x + 64) - 45*log(x - 4)/16 + 45*log(x**2 - 4*x + 8)/32 + 3*atan(x/2 - 1)/16

Giac [A]

time = 0.00, size = 51, normalized size = 0.88

$$\frac{45}{32} \ln(x^2 - 4x + 8) + \frac{3}{16} \arctan\left(\frac{x-2}{2}\right) - \frac{45}{16} \ln|x-4| + \frac{\frac{1}{32}(-328x + 648)}{(x-4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x)

[Out] -1/4*(41*x - 81)/(x - 4)^2 + 3/16*arctan(1/2*x - 1) + 45/32*log(x^2 - 4*x + 8) - 45/16*log(abs(x - 4))

Mupad [B]

time = 0.20, size = 44, normalized size = 0.76

$$-\frac{45 \ln(x-4)}{16} - \frac{\frac{41x}{4} - \frac{81}{4}}{x^2 - 8x + 16} + \ln(x-2-2i) \left(\frac{45}{32} - \frac{3}{32}i\right) + \ln(x-2+2i) \left(\frac{45}{32} + \frac{3}{32}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x + 5*x^3 - 20)/((x - 4)^3*(x^2 - 4*x + 8)),x)

[Out] log(x - (2 + 2i))*(45/32 - 3i/32) - (45*log(x - 4))/16 + log(x - (2 - 2i))*(45/32 + 3i/32) - ((41*x)/4 - 81/4)/(x^2 - 8*x + 16)

$$3.116 \quad \int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)} dx$$

Optimal. Leaf size=51

$$-\frac{1}{12} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{6} \tan^{-1}(x) - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

[Out] -1/12*arctan(1/2*x)+1/6*arctan(x)-1/4*arctan(1/2*x*2^(1/2))*2^(1/2)+1/6*arctan(1/3*x*3^(1/2))*3^(1/2)

Rubi [A]

time = 0.17, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {6857, 209}

$$-\frac{1}{12} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{6} \tan^{-1}(x) - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)), x]

[Out] -1/12*ArcTan[x/2] + ArcTan[x]/6 - ArcTan[x/Sqrt[2]]/(2*Sqrt[2]) + ArcTan[x/Sqrt[3]]/(2*Sqrt[3])

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 6857

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)} dx &= \int \left(\frac{1}{6(1+x^2)} - \frac{1}{2(2+x^2)} + \frac{1}{2(3+x^2)} - \frac{1}{6(4+x^2)} \right) dx \\ &= \frac{1}{6} \int \frac{1}{1+x^2} dx - \frac{1}{6} \int \frac{1}{4+x^2} dx - \frac{1}{2} \int \frac{1}{2+x^2} dx + \frac{1}{2} \int \frac{1}{3+x^2} dx \\ &= -\frac{1}{12} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{6} \tan^{-1}(x) - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 47, normalized size = 0.92

$$\frac{1}{12} \left(-\tan^{-1}\left(\frac{x}{2}\right) + 2 \tan^{-1}(x) - 3\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[1/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]``[Out] (-ArcTan[x/2] + 2*ArcTan[x] - 3*Sqrt[2]*ArcTan[x/Sqrt[2]] + 2*Sqrt[3]*ArcTan[x/Sqrt[3]])/12`**Mathics [A]**

time = 2.16, size = 35, normalized size = 0.69

$$-\frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2} x}{2}\right]}{4} - \frac{\operatorname{ArcTan}\left[\frac{x}{2}\right]}{12} + \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{\sqrt{3} x}{3}\right]}{6} + \frac{\operatorname{ArcTan}[x]}{6}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/((x^2 + 1)*(x^2 + 2)*(x^2 + 3)*(x^2 + 4)),x]')``[Out] -Sqrt[2] ArcTan[Sqrt[2] x / 2] / 4 - ArcTan[x / 2] / 12 + Sqrt[3] ArcTan[Sqrt[3] x / 3] / 6 + ArcTan[x] / 6`**Maple [A]**

time = 0.08, size = 36, normalized size = 0.71

method	result	size
default	$-\frac{\arctan\left(\frac{x}{2}\right)}{12} + \frac{\arctan(x)}{6} - \frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{4} + \frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{6}$	36

risch	$-\frac{\arctan\left(\frac{x}{2}\right)}{12} + \frac{\arctan(x)}{6} - \frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{4} + \frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{6}$	36
-------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x,method=_RETURNVERBOSE)`

[Out] $-1/12*\arctan(1/2*x)+1/6*\arctan(x)-1/4*\arctan(1/2*x*2^{(1/2)})*2^{(1/2)}+1/6*\arctan(1/3*x*3^{(1/2)})*3^{(1/2)}$

Maxima [A]

time = 0.38, size = 35, normalized size = 0.69

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) - \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{12}\arctan\left(\frac{1}{2}x\right) + \frac{1}{6}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="maxima")`

[Out] $1/6*\sqrt{3}*\arctan(1/3*\sqrt{3}*x) - 1/4*\sqrt{2}*\arctan(1/2*\sqrt{2}*x) - 1/12*\arctan(1/2*x) + 1/6*\arctan(x)$

Fricas [A]

time = 0.34, size = 35, normalized size = 0.69

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) - \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{12}\arctan\left(\frac{1}{2}x\right) + \frac{1}{6}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="fricas")`

[Out] $1/6*\sqrt{3}*\arctan(1/3*\sqrt{3}*x) - 1/4*\sqrt{2}*\arctan(1/2*\sqrt{2}*x) - 1/12*\arctan(1/2*x) + 1/6*\arctan(x)$

Sympy [A]

time = 0.21, size = 44, normalized size = 0.86

$$-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{12} + \frac{\operatorname{atan}(x)}{6} - \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2+1)/(x**2+2)/(x**2+3)/(x**2+4),x)`

[Out] $-\operatorname{atan}(x/2)/12 + \operatorname{atan}(x)/6 - \sqrt{2}*\operatorname{atan}(\sqrt{2}*x/2)/4 + \sqrt{3}*\operatorname{atan}(\sqrt{3}*x/3)/6$

Giac [A]

time = 0.00, size = 52, normalized size = 1.02

$$\frac{\arctan x}{6} - \frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}} + \frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\arctan\left(\frac{x}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x)``[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/12*arctan(1/2*x) + 1/6*arctan(x)`**Mupad [B]**

time = 0.24, size = 35, normalized size = 0.69

$$\frac{\operatorname{atan}(x)}{6} - \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{12} - \frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/((x^2 + 1)*(x^2 + 2)*(x^2 + 3)*(x^2 + 4)),x)``[Out] atan(x)/6 - atan(x/2)/12 - (2^(1/2)*atan((2^(1/2)*x)/2))/4 + (3^(1/2)*atan((3^(1/2)*x)/3))/6`

$$3.117 \quad \int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)} dx$$

Optimal. Leaf size=41

$$\frac{1}{12} \log(1+x^2) - \frac{1}{4} \log(2+x^2) + \frac{1}{4} \log(3+x^2) - \frac{1}{12} \log(4+x^2)$$

[Out] 1/12*ln(x^2+1)-1/4*ln(x^2+2)+1/4*ln(x^2+3)-1/12*ln(x^2+4)

Rubi [A]

time = 0.18, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {6826, 186}

$$\frac{1}{12} \log(x^2+1) - \frac{1}{4} \log(x^2+2) + \frac{1}{4} \log(x^2+3) - \frac{1}{12} \log(x^2+4)$$

Antiderivative was successfully verified.

[In] Int[x/((1+x^2)*(2+x^2)*(3+x^2)*(4+x^2)),x]

[Out] Log[1+x^2]/12 - Log[2+x^2]/4 + Log[3+x^2]/4 - Log[4+x^2]/12

Rule 186

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && IntegersQ[p, q]

Rule 6826

Int[(u_)*((c_.) + (d_.)*(v_))^(n_)*((e_.) + (f_.)*(w_))^(p_)*((a_.) + (b_.)*(y_))^(m_)*((g_.) + (h_.)*(z_))^(q_), x_Symbol] := With[{r = DerivativeDivides[y, u, x]}, Dist[r, Subst[Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x, y], x] /; !FalseQ[r]] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q}, x] && EqQ[v, y] && EqQ[w, y] && EqQ[z, y]

Rubi steps

$$\begin{aligned} \int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1+x)(2+x)(3+x)(4+x)} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{6(1+x)} - \frac{1}{2(2+x)} + \frac{1}{2(3+x)} - \frac{1}{6(4+x)} \right) dx \right) \\ &= \frac{1}{12} \log(1+x^2) - \frac{1}{4} \log(2+x^2) + \frac{1}{4} \log(3+x^2) - \frac{1}{12} \log(4+x^2) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 41, normalized size = 1.00

$$\frac{1}{12} \log(1 + x^2) - \frac{1}{4} \log(2 + x^2) + \frac{1}{4} \log(3 + x^2) - \frac{1}{12} \log(4 + x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)), x]

[Out] Log[1 + x^2]/12 - Log[2 + x^2]/4 + Log[3 + x^2]/4 - Log[4 + x^2]/12

Mathics [A]

time = 2.01, size = 33, normalized size = 0.80

$$-\frac{\text{Log}[2 + x^2]}{4} - \frac{\text{Log}[4 + x^2]}{12} + \frac{\text{Log}[1 + x^2]}{12} + \frac{\text{Log}[3 + x^2]}{4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x/((x^2 + 1)*(x^2 + 2)*(x^2 + 3)*(x^2 + 4)), x]')

[Out] -Log[2 + x ^ 2] / 4 - Log[4 + x ^ 2] / 12 + Log[1 + x ^ 2] / 12 + Log[3 + x ^ 2] / 4

Maple [A]

time = 0.07, size = 34, normalized size = 0.83

method	result	size
default	$\frac{\ln(x^2+1)}{12} - \frac{\ln(x^2+2)}{4} + \frac{\ln(x^2+3)}{4} - \frac{\ln(x^2+4)}{12}$	34
norman	$\frac{\ln(x^2+1)}{12} - \frac{\ln(x^2+2)}{4} + \frac{\ln(x^2+3)}{4} - \frac{\ln(x^2+4)}{12}$	34
risch	$\frac{\ln(x^2+1)}{12} - \frac{\ln(x^2+2)}{4} + \frac{\ln(x^2+3)}{4} - \frac{\ln(x^2+4)}{12}$	34

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4), x, method=_RETURNVERBOSE)

[Out] 1/12*ln(x^2+1)-1/4*ln(x^2+2)+1/4*ln(x^2+3)-1/12*ln(x^2+4)

Maxima [A]

time = 0.27, size = 33, normalized size = 0.80

$$-\frac{1}{12} \log(x^2 + 4) + \frac{1}{4} \log(x^2 + 3) - \frac{1}{4} \log(x^2 + 2) + \frac{1}{12} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4), x, algorithm="maxima")

[Out] $-1/12*\log(x^2 + 4) + 1/4*\log(x^2 + 3) - 1/4*\log(x^2 + 2) + 1/12*\log(x^2 + 1)$
)

Fricas [A]

time = 0.33, size = 33, normalized size = 0.80

$$-\frac{1}{12} \log(x^2 + 4) + \frac{1}{4} \log(x^2 + 3) - \frac{1}{4} \log(x^2 + 2) + \frac{1}{12} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="fricas")`

[Out] $-1/12*\log(x^2 + 4) + 1/4*\log(x^2 + 3) - 1/4*\log(x^2 + 2) + 1/12*\log(x^2 + 1)$
)

Sympy [A]

time = 0.09, size = 32, normalized size = 0.78

$$\frac{\log(x^2 + 1)}{12} - \frac{\log(x^2 + 2)}{4} + \frac{\log(x^2 + 3)}{4} - \frac{\log(x^2 + 4)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x**2+1)/(x**2+2)/(x**2+3)/(x**2+4),x)`

[Out] $\log(x^2 + 1)/12 - \log(x^2 + 2)/4 + \log(x^2 + 3)/4 - \log(x^2 + 4)/12$

Giac [A]

time = 0.00, size = 42, normalized size = 1.02

$$\frac{\frac{\ln(x^2+1)}{6} - \frac{\ln(x^2+2)}{2} + \frac{\ln(x^2+3)}{2} - \frac{\ln(x^2+4)}{6}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x)`

[Out] $-1/12*\log(x^2 + 4) + 1/4*\log(x^2 + 3) - 1/4*\log(x^2 + 2) + 1/12*\log(x^2 + 1)$
)

Mupad [B]

time = 0.06, size = 33, normalized size = 0.80

$$\frac{\operatorname{atanh}\left(\frac{3072}{5(1280x^2+3072)} - \frac{1}{5}\right)}{2} - \frac{\operatorname{atanh}\left(\frac{1024}{405\left(\frac{640x^2}{243} + \frac{1024}{243}\right)} - \frac{3}{5}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/((x^2 + 1)*(x^2 + 2)*(x^2 + 3)*(x^2 + 4)),x)`

[Out] $\operatorname{atanh}(3072/(5*(1280*x^2 + 3072)) - 1/5)/2 - \operatorname{atanh}(1024/(405*((640*x^2)/243 + 1024/243)) - 3/5)/6$

3.118 $\int \frac{1}{a^3+x^3} dx$

Optimal. Leaf size=56

$$-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^2} + \frac{\log(a+x)}{3a^2} - \frac{\log(a^2-ax+x^2)}{6a^2}$$

[Out] 1/3*ln(a+x)/a^2-1/6*ln(a^2-a*x+x^2)/a^2-1/3*arctan(1/3*(a-2*x)/a*3^(1/2))/a^2*3^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {206, 31, 648, 631, 210, 642}

$$-\frac{\log(a^2-ax+x^2)}{6a^2} + \frac{\log(a+x)}{3a^2} - \frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^2}$$

Antiderivative was successfully verified.

[In] Int[(a^3 + x^3)^(-1),x]

[Out] -(ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^2)) + Log[a + x]/(3*a^2) - Log[a^2 - a*x + x^2]/(6*a^2)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(n+1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{a^3 + x^3} dx &= \int \frac{1}{a+x} dx + \int \frac{2a-x}{a^2-ax+x^2} dx \\ &= \frac{\log(a+x)}{3a^2} - \frac{\int \frac{-a+2x}{a^2-ax+x^2} dx}{6a^2} + \frac{\int \frac{1}{a^2-ax+x^2} dx}{2a} \\ &= \frac{\log(a+x)}{3a^2} - \frac{\log(a^2-ax+x^2)}{6a^2} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2x}{a}\right)}{a^2} \\ &= -\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^2} + \frac{\log(a+x)}{3a^2} - \frac{\log(a^2-ax+x^2)}{6a^2} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 52, normalized size = 0.93

$$\frac{2\sqrt{3} \tan^{-1}\left(\frac{-a+2x}{\sqrt{3}a}\right) + 2\log(a+x) - \log(a^2 - ax + x^2)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + x^3)^(-1), x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] + 2*Log[a + x] - Log[a^2 - a*x + x^2])/(6*a^2)
```

Mathics [C] Result contains complex when optimal does not.

time = 2.50, size = 54, normalized size = 0.96

$$\frac{2\operatorname{Log}[a+x] + (-1 - I\sqrt{3}) \operatorname{Log}\left[-\frac{(1+I\sqrt{3})^a}{2} + x\right] + (-1 + I\sqrt{3}) \operatorname{Log}\left[-\frac{(1-I\sqrt{3})^a}{2} + x\right]}{6a^2}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(a^3 + x^3),x]')`

[Out] $(2 \operatorname{Log}[a+x] + (-1 - I \operatorname{Sqrt}[3]) \operatorname{Log}[-(1 + I \operatorname{Sqrt}[3]) a / 2 + x] + (-1 + I \operatorname{Sqrt}[3]) \operatorname{Log}[-(1 - I \operatorname{Sqrt}[3]) a / 2 + x]) / (6 a^2)$

Maple [A]

time = 0.06, size = 51, normalized size = 0.91

method	result	size
risch	$\frac{\ln(a+x)}{3a^2} + \frac{\left(\sum_{R=\operatorname{RootOf}(a^4-Z^2+a^2-Z+1)} -R \ln(-R a^3+x)\right)}{3}$	41
default	$\frac{-\frac{\ln(a^2-ax+x^2)}{2} + \sqrt{3} \arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)}{3a^2} + \frac{\ln(a+x)}{3a^2}$	51

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^3+x^3),x,method=_RETURNVERBOSE)`

[Out] $1/3/a^2*(-1/2*\ln(a^2-a*x+x^2)+3^{(1/2)*\arctan(1/3*(2*x-a)*3^{(1/2)/a})}+1/3/a^2*\ln(a+x)$

Maxima [A]

time = 0.35, size = 49, normalized size = 0.88

$$\frac{\sqrt{3} \arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^2} - \frac{\log(a^2 - ax + x^2)}{6a^2} + \frac{\log(a+x)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a^3+x^3),x, algorithm="maxima")`

[Out] $1/3*\operatorname{sqrt}(3)*\arctan(-1/3*\operatorname{sqrt}(3)*(a-2*x)/a)/a^2 - 1/6*\log(a^2 - a*x + x^2)/a^2 + 1/3*\log(a+x)/a^2$

Fricas [A]

time = 0.36, size = 45, normalized size = 0.80

$$\frac{2\sqrt{3} \arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right) - \log(a^2 - ax + x^2) + 2 \log(a+x)}{6a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^3+x^3),x, algorithm="fricas")

[Out] $\frac{1}{6}*(2*\sqrt{3}*\arctan(-1/3*\sqrt{3}*(a - 2*x)/a) - \log(a^2 - a*x + x^2) + 2*\log(a + x))/a^2$

Sympy [C] Result contains complex when optimal does not.

time = 0.07, size = 73, normalized size = 1.30

$$\frac{\frac{\log(a+x)}{3} + \left(-\frac{1}{6} - \frac{\sqrt{3}i}{6}\right) \log\left(3a\left(-\frac{1}{6} - \frac{\sqrt{3}i}{6}\right) + x\right) + \left(-\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) \log\left(3a\left(-\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) + x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a**3+x**3),x)

[Out] $(\log(a + x)/3 + (-1/6 - \sqrt{3}*I/6)*\log(3*a*(-1/6 - \sqrt{3}*I/6) + x) + (-1/6 + \sqrt{3}*I/6)*\log(3*a*(-1/6 + \sqrt{3}*I/6) + x))/a**2$

Giac [A]

time = 0.00, size = 59, normalized size = 1.05

$$\frac{\ln|x+a|}{3a^2} - \frac{\ln(x^2 - xa + a^2)}{6a^2} + \frac{\arctan\left(\frac{-a+2x}{a\sqrt{3}}\right)}{a\sqrt{3}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^3+x^3),x)

[Out] $\frac{1}{3}*\sqrt{3}*\arctan(-1/3*\sqrt{3}*(a - 2*x)/a)/a^2 - 1/6*\log(a^2 - a*x + x^2)/a^2 + 1/3*\log(\text{abs}(a + x))/a^2$

Mupad [B]

time = 0.44, size = 64, normalized size = 1.14

$$\frac{\ln(a+x)}{3a^2} + \frac{\ln\left(x + \frac{a(-1+\sqrt{3}i)}{2}\right) (-1 + \sqrt{3}i)}{6a^2} - \frac{\ln\left(x - \frac{a(1+\sqrt{3}i)}{2}\right) (1 + \sqrt{3}i)}{6a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^3 + x^3),x)

[Out] $\log(a + x)/(3*a^2) + (\log(x + (a*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^2) - (\log(x - (a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^2)$

3.119 $\int \frac{x}{a^3+x^3} dx$

Optimal. Leaf size=56

$$-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a} - \frac{\log(a+x)}{3a} + \frac{\log(a^2-ax+x^2)}{6a}$$

[Out] $-1/3*\ln(a+x)/a+1/6*\ln(a^2-a*x+x^2)/a-1/3*\arctan(1/3*(a-2*x)/a*3^{(1/2)})/a*3^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {298, 31, 648, 631, 210, 642}

$$\frac{\log(a^2-ax+x^2)}{6a} - \frac{\log(a+x)}{3a} - \frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x/(a^3 + x^3), x]$

[Out] $-(\text{ArcTan}[(a - 2*x)/(\text{Sqrt}[3]*a)]/(\text{Sqrt}[3]*a)) - \text{Log}[a + x]/(3*a) + \text{Log}[a^2 - a*x + x^2]/(6*a)$

Rule 31

$\text{Int}[(a_ + (b_)*(x_))^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

Rule 210

$\text{Int}[(a_ + (b_)*(x_)^2)^{(-1)}, x_Symbol] \rightarrow \text{Simp}[(-(\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{(-1)})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])]$

Rule 298

$\text{Int}[(x_)/((a_ + (b_)*(x_)^3), x_Symbol] \rightarrow \text{Dist}[-(3*\text{Rt}[a, 3]*\text{Rt}[b, 3])^{(-1)}, \text{Int}[1/(\text{Rt}[a, 3] + \text{Rt}[b, 3]*x), x], x] + \text{Dist}[1/(3*\text{Rt}[a, 3]*\text{Rt}[b, 3]), \text{Int}[(\text{Rt}[a, 3] + \text{Rt}[b, 3]*x)/(\text{Rt}[a, 3]^2 - \text{Rt}[a, 3]*\text{Rt}[b, 3]*x + \text{Rt}[b, 3]^2*x^2), x], x] /; \text{FreeQ}\{a, b\}, x]$

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{a^3 + x^3} dx &= -\int \frac{1}{a+x} dx + \int \frac{a+x}{a^2-ax+x^2} dx \\ &= -\frac{\log(a+x)}{3a} + \frac{1}{2} \int \frac{1}{a^2-ax+x^2} dx + \frac{\int \frac{-a+2x}{a^2-ax+x^2} dx}{6a} \\ &= -\frac{\log(a+x)}{3a} + \frac{\log(a^2-ax+x^2)}{6a} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2x}{a}\right)}{a} \\ &= -\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a} - \frac{\log(a+x)}{3a} + \frac{\log(a^2-ax+x^2)}{6a} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 50, normalized size = 0.89

$$\frac{2\sqrt{3} \tan^{-1}\left(\frac{-a+2x}{\sqrt{3}a}\right) - 2\log(a+x) + \log(a^2-ax+x^2)}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^3 + x^3), x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^2 - a*x + x^2])/(6*a)
```

Mathics [C] Result contains complex when optimal does not.

time = 2.67, size = 53, normalized size = 0.95

$$\frac{-2\operatorname{Log}[a+x] + \left(1 - I\sqrt{3}\right)\operatorname{Log}\left[-\frac{a}{2} - \frac{I}{2}\sqrt{3}a+x\right] + \left(1 + I\sqrt{3}\right)\operatorname{Log}\left[-\frac{\left(1 - I\sqrt{3}\right)a}{2} + x\right]}{6a}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x/(a^3 + x^3),x]')`

[Out] `(-2 Log[a + x] + (1 - I Sqrt[3]) Log[-a / 2 - I / 2 Sqrt[3] a + x] + (1 + I Sqrt[3]) Log[-(1 - I Sqrt[3]) a / 2 + x]) / (6 a)`

Maple [A]

time = 0.05, size = 51, normalized size = 0.91

method	result	size
risch	$-\frac{\ln(a+x)}{3a} + \frac{\left(\sum_{-R=\text{RootOf}(-Z^2a^2-aZ+1)} -R \ln(a^2 - R - a + x)\right)}{3}$	43
default	$\frac{\frac{\ln(a^2 - ax + x^2)}{2} + \sqrt{3} \arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)}{3a} - \frac{\ln(a+x)}{3a}$	51

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a^3+x^3),x,method=_RETURNVERBOSE)`

[Out] `1/3/a*(1/2*ln(a^2-a*x+x^2)+3^(1/2)*arctan(1/3*(2*x-a)*3^(1/2)/a))-1/3*ln(a+x)/a`

Maxima [A]

time = 0.35, size = 49, normalized size = 0.88

$$\frac{\sqrt{3} \arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a} + \frac{\log(a^2 - ax + x^2)}{6a} - \frac{\log(a+x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a^3+x^3),x, algorithm="maxima")`

[Out] `1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a + 1/6*log(a^2 - a*x + x^2)/a - 1/3*log(a + x)/a`

Fricas [A]

time = 0.33, size = 43, normalized size = 0.77

$$\frac{2\sqrt{3} \arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right) + \log(a^2 - ax + x^2) - 2\log(a+x)}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a^3+x^3),x, algorithm="fricas")

[Out] $\frac{1}{6}*(2*\sqrt{3}*\arctan(-1/3*\sqrt{3}*(a - 2*x)/a) + \log(a^2 - a*x + x^2) - 2*\log(a + x))/a$

Sympy [C] Result contains complex when optimal does not.

time = 0.06, size = 71, normalized size = 1.27

$$\frac{-\frac{\log(a+x)}{3} + \left(\frac{1}{6} - \frac{\sqrt{3}i}{6}\right) \log\left(9a\left(\frac{1}{6} - \frac{\sqrt{3}i}{6}\right)^2 + x\right) + \left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) \log\left(9a\left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)^2 + x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a**3+x**3),x)

[Out] $(-\log(a + x)/3 + (1/6 - \sqrt{3}*I/6)*\log(9*a*(1/6 - \sqrt{3}*I/6)**2 + x) + (1/6 + \sqrt{3}*I/6)*\log(9*a*(1/6 + \sqrt{3}*I/6)**2 + x))/a$

Giac [A]

time = 0.00, size = 54, normalized size = 0.96

$$-\frac{\ln|x+a|}{3a} + \frac{\ln(x^2 - xa + a^2)}{6a} + \frac{\arctan\left(\frac{-a+2x}{a\sqrt{3}}\right)}{\sqrt{3}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a^3+x^3),x)

[Out] $\frac{1}{3}*\sqrt{3}*\arctan(-1/3*\sqrt{3}*(a - 2*x)/a)/a + \frac{1}{6}*\log(a^2 - a*x + x^2)/a - \frac{1}{3}*\log(\text{abs}(a + x))/a$

Mupad [B]

time = 0.12, size = 68, normalized size = 1.21

$$\frac{\ln(a+x)}{3a} - \frac{\ln\left(x + \frac{a(-1+\sqrt{3}i)^2}{4}\right)(-1+\sqrt{3}i)}{6a} + \frac{\ln\left(x + \frac{a(1+\sqrt{3}i)^2}{4}\right)(1+\sqrt{3}i)}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a^3 + x^3),x)

[Out] $(\log(x + (a*(3^{1/2}*1i + 1)^2)/4)*(3^{1/2}*1i + 1))/(6*a) - (\log(x + (a*(3^{1/2}*1i - 1)^2)/4)*(3^{1/2}*1i - 1))/(6*a) - \log(a + x)/(3*a)$

$$3.120 \quad \int \frac{x^2}{a^3+x^3} dx$$

Optimal. Leaf size=12

$$\frac{1}{3} \log(a^3 + x^3)$$

[Out] 1/3*ln(a^3+x^3)

Rubi [A]

time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {266}

$$\frac{1}{3} \log(a^3 + x^3)$$

Antiderivative was successfully verified.

[In] Int[x^2/(a^3 + x^3),x]

[Out] Log[a^3 + x^3]/3

Rule 266

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\int \frac{x^2}{a^3 + x^3} dx = \frac{1}{3} \log(a^3 + x^3)$$

Mathematica [A]

time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{3} \log(a^3 + x^3)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a^3 + x^3),x]

[Out] Log[a^3 + x^3]/3

Mathics [A]

time = 1.83, size = 10, normalized size = 0.83

$$\frac{\text{Log}[a^3 + x^3]}{3}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x^2/(a^3 + x^3),x]')`

[Out] `Log[a ^ 3 + x ^ 3] / 3`

Maple [A]

time = 0.05, size = 11, normalized size = 0.92

method	result	size
derivativdivides	$\frac{\ln(a^3+x^3)}{3}$	11
default	$\frac{\ln(a^3+x^3)}{3}$	11
risch	$\frac{\ln(a^3+x^3)}{3}$	11
norman	$\frac{\ln(a+x)}{3} + \frac{\ln(a^2-ax+x^2)}{3}$	22

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a^3+x^3),x,method=_RETURNVERBOSE)`

[Out] `1/3*ln(a^3+x^3)`

Maxima [A]

time = 0.26, size = 10, normalized size = 0.83

$$\frac{1}{3} \log(a^3 + x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(a^3+x^3),x, algorithm="maxima")`

[Out] `1/3*log(a^3 + x^3)`

Fricas [A]

time = 0.31, size = 10, normalized size = 0.83

$$\frac{1}{3} \log(a^3 + x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(a^3+x^3),x, algorithm="fricas")`

[Out] `1/3*log(a^3 + x^3)`

Sympy [A]

time = 0.05, size = 8, normalized size = 0.67

$$\frac{\log(a^3 + x^3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(a**3+x**3),x)

[Out] log(a**3 + x**3)/3

Giac [A]

time = 0.00, size = 12, normalized size = 1.00

$$\frac{\ln |x^3 + a^3|}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a^3+x^3),x)

[Out] 1/3*log(abs(a^3 + x^3))

Mupad [B]

time = 0.03, size = 10, normalized size = 0.83

$$\frac{\ln (a^3 + x^3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a^3 + x^3),x)

[Out] log(a^3 + x^3)/3

3.121 $\int \frac{1}{x(a^3+x^3)} dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a^3} - \frac{\log(a^3 + x^3)}{3a^3}$$

[Out] $\ln(x)/a^3 - 1/3*\ln(a^3+x^3)/a^3$

Rubi [A]

time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {272, 36, 29, 31}

$$\frac{\log(x)}{a^3} - \frac{\log(a^3 + x^3)}{3a^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x*(a^3 + x^3)), x]$

[Out] $\text{Log}[x]/a^3 - \text{Log}[a^3 + x^3]/(3*a^3)$

Rule 29

$\text{Int}[(x_)^{(-1)}, x_Symbol] \text{ :> } \text{Simp}[\text{Log}[x], x]$

Rule 31

$\text{Int}[(a_) + (b_)*(x_)^{(-1)}, x_Symbol] \text{ :> } \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] \text{ /; } \text{FreeQ}[\{a, b\}, x]$

Rule 36

$\text{Int}[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] \text{ :> } \text{Dist}[b/(b*c - a*d), \text{Int}[1/(a + b*x), x], x] - \text{Dist}[d/(b*c - a*d), \text{Int}[1/(c + d*x), x], x] \text{ /; } \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 272

$\text{Int}[(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \text{ :> } \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] \text{ /; } \text{FreeQ}[\{a, b, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{1}{x(a^3 + x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x(a^3 + x)} dx, x, x^3 \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{x} dx, x, x^3 \right)}{3a^3} - \frac{\text{Subst} \left(\int \frac{1}{a^3+x} dx, x, x^3 \right)}{3a^3} \\ &= \frac{\log(x)}{a^3} - \frac{\log(a^3 + x^3)}{3a^3} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a^3} - \frac{\log(a^3 + x^3)}{3a^3}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x*(a^3 + x^3)),x]``[Out] Log[x]/a^3 - Log[a^3 + x^3]/(3*a^3)`**Mathics [A]**

time = 1.90, size = 17, normalized size = 0.77

$$\frac{\text{Log}[x] - \frac{\text{Log}[a^3+x^3]}{3}}{a^3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x*(a^3 + x^3)),x]')``[Out] (Log[x] - Log[a ^ 3 + x ^ 3] / 3) / a ^ 3`**Maple [A]**

time = 0.05, size = 34, normalized size = 1.55

method	result	size
risch	$\frac{\ln(x)}{a^3} - \frac{\ln(a^3+x^3)}{3a^3}$	21
default	$\frac{\ln(x)}{a^3} - \frac{\ln(a^2-ax+x^2)}{3a^3} - \frac{\ln(a+x)}{3a^3}$	34
norman	$\frac{\ln(x)}{a^3} - \frac{\ln(a^2-ax+x^2)}{3a^3} - \frac{\ln(a+x)}{3a^3}$	34

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/x/(a^3+x^3),x,method=_RETURNVERBOSE)`

[Out] $\ln(x)/a^3 - 1/3/a^3 \ln(a^2 - ax + x^2) - 1/3 \ln(a+x)/a^3$

Maxima [A]

time = 0.27, size = 23, normalized size = 1.05

$$-\frac{\log(a^3 + x^3)}{3a^3} + \frac{\log(x^3)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^3+x^3),x, algorithm="maxima")`

[Out] $-1/3 \log(a^3 + x^3)/a^3 + 1/3 \log(x^3)/a^3$

Fricas [A]

time = 0.31, size = 18, normalized size = 0.82

$$-\frac{\log(a^3 + x^3) - 3 \log(x)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^3+x^3),x, algorithm="fricas")`

[Out] $-1/3 * (\log(a^3 + x^3) - 3 * \log(x)) / a^3$

Sympy [A]

time = 0.11, size = 19, normalized size = 0.86

$$\frac{\log(x)}{a^3} - \frac{\log(a^3 + x^3)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a**3+x**3),x)`

[Out] $\log(x)/a^{**3} - \log(a^{**3} + x^{**3})/(3*a^{**3})$

Giac [A]

time = 0.00, size = 29, normalized size = 1.32

$$\frac{3 \ln|x|}{3a^3} - \frac{\ln|x^3 + a^3|}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^3+x^3),x)`

[Out] $-1/3 * \log(\text{abs}(a^3 + x^3)) / a^3 + \log(\text{abs}(x)) / a^3$

Mupad [B]

time = 0.25, size = 18, normalized size = 0.82

$$-\frac{\ln(a^3 + x^3) - 3 \ln(x)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^3 + x^3)),x)
```

```
[Out] -(log(a^3 + x^3) - 3*log(x))/(3*a^3)
```

$$3.122 \quad \int \frac{1}{x^2(a^3+x^3)} dx$$

Optimal. Leaf size=63

$$-\frac{1}{a^3x} + \frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^4} + \frac{\log(a+x)}{3a^4} - \frac{\log(a^2-ax+x^2)}{6a^4}$$

[Out] $-1/a^3/x + 1/3*\ln(a+x)/a^4 - 1/6*\ln(a^2-a*x+x^2)/a^4 + 1/3*\arctan(1/3*(a-2*x)/a*3^{(1/2)})/a^4*3^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {331, 298, 31, 648, 631, 210, 642}

$$\frac{\log(a+x)}{3a^4} + \frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^4} - \frac{1}{a^3x} - \frac{\log(a^2-ax+x^2)}{6a^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a^3 + x^3)),x]

[Out] $-(1/(a^3*x)) + \text{ArcTan}[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^4) + \text{Log}[a + x]/(3*a^4) - \text{Log}[a^2 - a*x + x^2]/(6*a^4)$

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

Rule 298

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := Dist[-(3*Rt[a, 3]*Rt[b, 3])^-1, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^2(a^3 + x^3)} dx &= -\frac{1}{a^3 x} - \frac{\int \frac{x}{a^3 + x^3} dx}{a^3} \\
 &= -\frac{1}{a^3 x} + \frac{\int \frac{1}{a+x} dx}{3a^4} - \frac{\int \frac{a+x}{a^2-ax+x^2} dx}{3a^4} \\
 &= -\frac{1}{a^3 x} + \frac{\log(a+x)}{3a^4} - \frac{\int \frac{-a+2x}{a^2-ax+x^2} dx}{6a^4} - \frac{\int \frac{1}{a^2-ax+x^2} dx}{2a^3} \\
 &= -\frac{1}{a^3 x} + \frac{\log(a+x)}{3a^4} - \frac{\log(a^2 - ax + x^2)}{6a^4} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2x}{a}\right)}{a^4} \\
 &= -\frac{1}{a^3 x} + \frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^4} + \frac{\log(a+x)}{3a^4} - \frac{\log(a^2 - ax + x^2)}{6a^4}
 \end{aligned}$$

Mathematica [A]

time = 0.01, size = 60, normalized size = 0.95

$$\frac{6a + 2\sqrt{3} x \tan^{-1}\left(\frac{-a+2x}{\sqrt{3}a}\right) - 2x \log(a+x) + x \log(a^2 - ax + x^2)}{6a^4x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a^3 + x^3)),x]

[Out] -1/6*(6*a + 2*sqrt[3]*x*ArcTan[(-a + 2*x)/(sqrt[3]*a)] - 2*x*Log[a + x] + x*Log[a^2 - a*x + x^2])/(a^4*x)

Mathics [C] Result contains complex when optimal does not.

time = 2.92, size = 62, normalized size = 0.98

$$\frac{-a + x \left(2 \operatorname{Log}[a+x] + (-1-I\sqrt{3}) \operatorname{Log}\left[-\frac{(1-I\sqrt{3})^a}{2} + x\right] + (-1+I\sqrt{3}) \operatorname{Log}\left[-\frac{a}{2} - \frac{I}{2}\sqrt{3}a+x\right] \right)}{a^4x}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x^2*(a^3 + x^3)),x]')

[Out] (-a + x (2 Log[a + x] + (-1 - I Sqrt[3]) Log[-(1 - I Sqrt[3]) a / 2 + x] + (-1 + I Sqrt[3]) Log[-a / 2 - I / 2 Sqrt[3] a + x]) / 6) / (a ^ 4 x)

Maple [A]

time = 0.05, size = 60, normalized size = 0.95

method	result	size
default	$-\frac{1}{a^3x} + \frac{-\frac{\ln(a^2-ax+x^2)}{2} - \sqrt{3} \arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right) + \frac{\ln(a+x)}{3a^4}}{3a^4}$	60
risch	$-\frac{1}{a^3x} + \frac{\left(\sum_{R=\text{RootOf}(a^8-Z^2+a^4-Z+1)} -R \ln\left((-4-R^3a^{12}+3)x-a^9-R^2\right)\right)}{3} + \frac{\ln(-a-x)}{3a^4}$	67

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a^3+x^3),x,method=_RETURNVERBOSE)

[Out] -1/a^3/x+1/3/a^4*(-1/2*ln(a^2-a*x+x^2)-3^(1/2)*arctan(1/3*(2*x-a)*3^(1/2)/a))+1/3*ln(a+x)/a^4

Maxima [A]

time = 0.37, size = 57, normalized size = 0.90

$$-\frac{\sqrt{3} \arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^4} - \frac{\log(a^2 - ax + x^2)}{6a^4} + \frac{\log(a+x)}{3a^4} - \frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a^3+x^3),x, algorithm="maxima")

[Out] $-1/3*\sqrt{3}*\arctan(-1/3*\sqrt{3}*(a - 2*x)/a)/a^4 - 1/6*\log(a^2 - a*x + x^2)/a^4 + 1/3*\log(a + x)/a^4 - 1/(a^3*x)$

Fricas [A]

time = 0.33, size = 53, normalized size = 0.84

$$\frac{2\sqrt{3}x\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right) + x\log(a^2 - ax + x^2) - 2x\log(a+x) + 6a}{6a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a^3+x^3),x, algorithm="fricas")

[Out] $-1/6*(2*\sqrt{3}*x*\arctan(-1/3*\sqrt{3}*(a - 2*x)/a) + x*\log(a^2 - a*x + x^2) - 2*x*\log(a + x) + 6*a)/(a^4*x)$

Sympy [C] Result contains complex when optimal does not.

time = 0.08, size = 83, normalized size = 1.32

$$-\frac{1}{a^3x} + \frac{\frac{\log(a+x)}{3} + \left(-\frac{1}{6} - \frac{\sqrt{3}i}{6}\right)\log\left(9a\left(-\frac{1}{6} - \frac{\sqrt{3}i}{6}\right)^2 + x\right) + \left(-\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)\log\left(9a\left(-\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)^2 + x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(a**3+x**3),x)

[Out] $-1/(a**3*x) + (\log(a + x)/3 + (-1/6 - \sqrt{3}i/6)*\log(9*a*(-1/6 - \sqrt{3}i/6)**2 + x) + (-1/6 + \sqrt{3}i/6)*\log(9*a*(-1/6 + \sqrt{3}i/6)**2 + x))/a**4$

Giac [A]

time = 0.00, size = 69, normalized size = 1.10

$$\frac{\ln|x+a|}{3a^4} - \frac{\ln(x^2 - xa + a^2)}{6a^4} - \frac{\arctan\left(\frac{-a+2x}{a\sqrt{3}}\right)}{a^3\sqrt{3}a} - \frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a^3+x^3),x)

[Out] $-1/3*\sqrt{3}*\arctan(-1/3*\sqrt{3}*(a - 2*x)/a)/a^4 - 1/6*\log(a^2 - a*x + x^2)/a^4 + 1/3*\log(\text{abs}(a + x))/a^4 - 1/(a^3*x)$

Mupad [B]

time = 0.25, size = 88, normalized size = 1.40

$$\frac{\ln(a+x)}{3a^4} - \frac{1}{a^3x} + \frac{\ln\left(\frac{(-1+\sqrt{3}i)^2 a^4}{4} + x a^3\right) (-1 + \sqrt{3}i)}{6a^4} - \frac{\ln\left(\frac{(1+\sqrt{3}i)^2 a^4}{4} + x a^3\right) (1 + \sqrt{3}i)}{6a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(a^3 + x^3)),x)`

[Out] `log(a + x)/(3*a^4) - 1/(a^3*x) + (log(a^3*x + (a^4*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(6*a^4) - (log(a^3*x + (a^4*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(6*a^4)`

3.123 $\int \frac{1}{x^3(a^3+x^3)} dx$

Optimal. Leaf size=65

$$-\frac{1}{2a^3x^2} + \frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5} - \frac{\log(a+x)}{3a^5} + \frac{\log(a^2-ax+x^2)}{6a^5}$$

[Out] $-1/2/a^3/x^2-1/3*\ln(a+x)/a^5+1/6*\ln(a^2-a*x+x^2)/a^5+1/3*\arctan(1/3*(a-2*x)/a*3^{(1/2)})/a^5*3^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {331, 206, 31, 648, 631, 210, 642}

$$-\frac{\log(a+x)}{3a^5} + \frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5} - \frac{1}{2a^3x^2} + \frac{\log(a^2-ax+x^2)}{6a^5}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^3*(a^3 + x^3)),x]`

[Out] $-1/2*1/(a^3*x^2) + \text{ArcTan}[(a - 2*x)/(\text{Sqrt}[3]*a)]/(\text{Sqrt}[3]*a^5) - \text{Log}[a + x]/(3*a^5) + \text{Log}[a^2 - a*x + x^2]/(6*a^5)$

Rule 31

`Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rule 206

`Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]`

Rule 210

`Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(n_+1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 331

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*c^n*(m + 1))], Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3(a^3 + x^3)} dx &= -\frac{1}{2a^3x^2} - \frac{\int \frac{1}{a^3+x^3} dx}{a^3} \\ &= -\frac{1}{2a^3x^2} - \frac{\int \frac{1}{a+x} dx}{3a^5} - \frac{\int \frac{2a-x}{a^2-ax+x^2} dx}{3a^5} \\ &= -\frac{1}{2a^3x^2} - \frac{\log(a+x)}{3a^5} + \frac{\int \frac{-a+2x}{a^2-ax+x^2} dx}{6a^5} - \frac{\int \frac{1}{a^2-ax+x^2} dx}{2a^4} \\ &= -\frac{1}{2a^3x^2} - \frac{\log(a+x)}{3a^5} + \frac{\log(a^2-ax+x^2)}{6a^5} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2x}{a}\right)}{a^5} \\ &= -\frac{1}{2a^3x^2} + \frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5} - \frac{\log(a+x)}{3a^5} + \frac{\log(a^2-ax+x^2)}{6a^5} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 68, normalized size = 1.05

$$-\frac{1}{2a^3x^2} - \frac{\tan^{-1}\left(\frac{-a+2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5} - \frac{\log(a+x)}{3a^5} + \frac{\log(a^2-ax+x^2)}{6a^5}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a^3 + x^3)),x]

[Out] -1/2*1/(a^3*x^2) - ArcTan[(-a + 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^5) - Log[a + x]/(3*a^5) + Log[a^2 - a*x + x^2]/(6*a^5)

Mathics [C] Result contains complex when optimal does not.

time = 2.75, size = 67, normalized size = 1.03

$$\frac{-3a^2 + x^2 \left(-2\text{Log}[a + x] + (1 - I\sqrt{3}) \text{Log}\left[-\frac{(1 - I\sqrt{3})a}{2} + x\right] + (1 + I\sqrt{3}) \text{Log}\left[-\frac{(1 + I\sqrt{3})a}{2} + x\right] \right)}{6a^5x^2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x^3*(a^3 + x^3)),x]')

[Out] (-3 a ^ 2 + x ^ 2 (-2 Log[a + x] + (1 - I Sqrt[3]) Log[-(1 - I Sqrt[3]) a / 2 + x] + (1 + I Sqrt[3]) Log[-(1 + I Sqrt[3]) a / 2 + x])) / (6 a ^ 5 x ^ 2)

Maple [A]

time = 0.05, size = 60, normalized size = 0.92

method	result	size
default	$-\frac{1}{2a^3x^2} + \frac{\frac{\ln(a^2-ax+x^2)}{2} - \sqrt{3} \arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)}{3a^5} - \frac{\ln(a+x)}{3a^5}$	60
risch	$-\frac{1}{2a^3x^2} + \frac{\left(\sum_{R=\text{RootOf}(a^{10}Z^2-a^5Z+1)} -R \ln\left(\left(-4-R^3a^{15}-3\right)x-a^6-R\right)\right)}{3} - \frac{\ln(a+x)}{3a^5}$	62

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a^3+x^3),x,method=_RETURNVERBOSE)

[Out] -1/2/a^3/x^2+1/3/a^5*(1/2*ln(a^2-a*x+x^2)-3^(1/2)*arctan(1/3*(2*x-a)*3^(1/2)/a))-1/3*ln(a+x)/a^5

Maxima [A]

time = 0.35, size = 57, normalized size = 0.88

$$-\frac{\sqrt{3} \arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^5} + \frac{\log(a^2-ax+x^2)}{6a^5} - \frac{\log(a+x)}{3a^5} - \frac{1}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a^3+x^3),x, algorithm="maxima")

[Out] $-1/3*\sqrt{3}*\arctan(-1/3*\sqrt{3}*(a - 2*x)/a)/a^5 + 1/6*\log(a^2 - a*x + x^2)/a^5 - 1/3*\log(a + x)/a^5 - 1/2/(a^3*x^2)$

Fricas [A]

time = 0.32, size = 62, normalized size = 0.95

$$\frac{2\sqrt{3}x^2\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right) - x^2\log(a^2 - ax + x^2) + 2x^2\log(a + x) + 3a^2}{6a^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a^3+x^3),x, algorithm="fricas")

[Out] $-1/6*(2*\sqrt{3}*x^2*\arctan(-1/3*\sqrt{3}*(a - 2*x)/a) - x^2*\log(a^2 - a*x + x^2) + 2*x^2*\log(a + x) + 3*a^2)/(a^5*x^2)$

Sympy [C] Result contains complex when optimal does not.

time = 0.09, size = 80, normalized size = 1.23

$$-\frac{1}{2a^3x^2} + \frac{-\frac{\log(a+x)}{3} + \left(\frac{1}{6} - \frac{\sqrt{3}i}{6}\right)\log\left(-3a\left(\frac{1}{6} - \frac{\sqrt{3}i}{6}\right) + x\right) + \left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)\log\left(-3a\left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) + x\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(a**3+x**3),x)

[Out] $-1/(2*a**3*x**2) + (-\log(a + x)/3 + (1/6 - \sqrt{3}*I/6)*\log(-3*a*(1/6 - \sqrt{3}*I/6) + x) + (1/6 + \sqrt{3}*I/6)*\log(-3*a*(1/6 + \sqrt{3}*I/6) + x))/a**5$

Giac [A]

time = 0.00, size = 72, normalized size = 1.11

$$-\frac{\ln|x+a|}{3a^5} + \frac{\ln(x^2 - xa + a^2)}{6a^5} - \frac{\arctan\left(\frac{-a+2x}{a\sqrt{3}}\right)}{a^4\sqrt{3}a} - \frac{1}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a^3+x^3),x)

[Out] $-1/3*\sqrt{3}*\arctan(-1/3*\sqrt{3}*(a - 2*x)/a)/a^5 + 1/6*\log(a^2 - a*x + x^2)/a^5 - 1/3*\log(\text{abs}(a + x))/a^5 - 1/2/(a^3*x^2)$

Mupad [B]

time = 0.25, size = 86, normalized size = 1.32

$$-\frac{\ln(a+x)}{3a^5} - \frac{1}{2a^3x^2} - \frac{\ln\left(\frac{3a^7(-1+\sqrt{3}i)}{2} + 3a^6x\right)(-1+\sqrt{3}i)}{6a^5} + \frac{\ln\left(\frac{3a^7(1+\sqrt{3}i)}{2} - 3a^6x\right)(1+\sqrt{3}i)}{6a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*(a^3 + x^3)),x)`

[Out] `(log((3*a^7*(3^(1/2)*1i + 1))/2 - 3*a^6*x)*(3^(1/2)*1i + 1))/(6*a^5) - 1/(2*a^3*x^2) - (log((3*a^7*(3^(1/2)*1i - 1))/2 + 3*a^6*x)*(3^(1/2)*1i - 1))/(6*a^5) - log(a + x)/(3*a^5)`

3.124 $\int \frac{1}{x^4(a^3+x^3)} dx$

Optimal. Leaf size=33

$$-\frac{1}{3a^3x^3} - \frac{\log(x)}{a^6} + \frac{\log(a^3+x^3)}{3a^6}$$

[Out] $-1/3/a^3/x^3 - \ln(x)/a^6 + 1/3*\ln(a^3+x^3)/a^6$

Rubi [A]

time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 46}

$$-\frac{\log(x)}{a^6} - \frac{1}{3a^3x^3} + \frac{\log(a^3+x^3)}{3a^6}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a^3 + x^3)),x]

[Out] $-1/3*1/(a^3*x^3) - \text{Log}[x]/a^6 + \text{Log}[a^3 + x^3]/(3*a^6)$

Rule 46

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^4(a^3+x^3)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2(a^3+x)} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{a^3x^2} - \frac{1}{a^6x} + \frac{1}{a^6(a^3+x)} \right) dx, x, x^3 \right) \\ &= -\frac{1}{3a^3x^3} - \frac{\log(x)}{a^6} + \frac{\log(a^3+x^3)}{3a^6} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 33, normalized size = 1.00

$$-\frac{1}{3a^3x^3} - \frac{\log(x)}{a^6} + \frac{\log(a^3 + x^3)}{3a^6}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x^4*(a^3 + x^3)),x]``[Out] -1/3*1/(a^3*x^3) - Log[x]/a^6 + Log[a^3 + x^3]/(3*a^6)`**Mathics [A]**

time = 2.35, size = 31, normalized size = 0.94

$$\frac{-a^3 + x^3 (\text{Log}[a^3 + x^3] - 3\text{Log}[x])}{3a^6x^3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^4*(a^3 + x^3)),x]')``[Out] (-a ^ 3 + x ^ 3 (Log[a ^ 3 + x ^ 3] - 3 Log[x])) / (3 a ^ 6 x ^ 3)`**Maple [A]**

time = 0.05, size = 43, normalized size = 1.30

method	result	size
risch	$-\frac{1}{3a^3x^3} - \frac{\ln(x)}{a^6} + \frac{\ln(-a^3-x^3)}{3a^6}$	34
default	$-\frac{1}{3a^3x^3} - \frac{\ln(x)}{a^6} + \frac{\ln(a^2-ax+x^2)}{3a^6} + \frac{\ln(a+x)}{3a^6}$	43
norman	$-\frac{1}{3a^3x^3} - \frac{\ln(x)}{a^6} + \frac{\ln(a^2-ax+x^2)}{3a^6} + \frac{\ln(a+x)}{3a^6}$	43

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/x^4/(a^3+x^3),x,method=_RETURNVERBOSE)``[Out] -1/3/a^3/x^3-ln(x)/a^6+1/3/a^6*ln(a^2-a*x+x^2)+1/3*ln(a+x)/a^6`**Maxima [A]**

time = 0.26, size = 31, normalized size = 0.94

$$\frac{\log(a^3 + x^3)}{3a^6} - \frac{\log(x^3)}{3a^6} - \frac{1}{3a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/x^4/(a^3+x^3),x, algorithm="maxima")``[Out] 1/3*log(a^3 + x^3)/a^6 - 1/3*log(x^3)/a^6 - 1/3/(a^3*x^3)`

Fricas [A]

time = 0.36, size = 33, normalized size = 1.00

$$\frac{x^3 \log(a^3 + x^3) - 3x^3 \log(x) - a^3}{3a^6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a^3+x^3),x, algorithm="fricas")**[Out]** 1/3*(x^3*log(a^3 + x^3) - 3*x^3*log(x) - a^3)/(a^6*x^3)**Sympy [A]**

time = 0.13, size = 29, normalized size = 0.88

$$-\frac{1}{3a^3 x^3} - \frac{\log(x)}{a^6} + \frac{\log(a^3 + x^3)}{3a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(a**3+x**3),x)**[Out]** -1/(3*a**3*x**3) - log(x)/a**6 + log(a**3 + x**3)/(3*a**6)**Giac [A]**

time = 0.00, size = 47, normalized size = 1.42

$$\frac{\ln|x^3 + a^3|}{3a^6} - \frac{3 \ln|x|}{3a^6} + \frac{x^3 - a^3}{3a^6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a^3+x^3),x)**[Out]** 1/3*log(abs(a^3 + x^3))/a^6 - log(abs(x))/a^6 - 1/3*(a^3 - x^3)/(a^6*x^3)**Mupad [B]**

time = 0.07, size = 29, normalized size = 0.88

$$\frac{\ln(a^3 + x^3)}{3a^6} - \frac{\ln(x)}{a^6} - \frac{1}{3a^3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(a^3 + x^3)),x)**[Out]** log(a^3 + x^3)/(3*a^6) - log(x)/a^6 - 1/(3*a^3*x^3)

3.125 $\int \frac{1}{x^5(a^3+x^3)} dx$

Optimal. Leaf size=73

$$-\frac{1}{4a^3x^4} + \frac{1}{a^6x} - \frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7} - \frac{\log(a+x)}{3a^7} + \frac{\log(a^2-ax+x^2)}{6a^7}$$

[Out] $-1/4/a^3/x^4+1/a^6/x-1/3*\ln(a+x)/a^7+1/6*\ln(a^2-a*x+x^2)/a^7-1/3*\arctan(1/3*(a-2*x)/a*3^(1/2))/a^7*3^(1/2)$

Rubi [A]

time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {331, 298, 31, 648, 631, 210, 642}

$$-\frac{\log(a+x)}{3a^7} - \frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7} + \frac{1}{a^6x} - \frac{1}{4a^3x^4} + \frac{\log(a^2-ax+x^2)}{6a^7}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^5*(a^3 + x^3)), x]$

[Out] $-1/4*1/(a^3*x^4) + 1/(a^6*x) - \text{ArcTan}[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^7) - \text{Log}[a + x]/(3*a^7) + \text{Log}[a^2 - a*x + x^2]/(6*a^7)$

Rule 31

$\text{Int}[(a_ + (b_)*(x_))^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] \text{ ; FreeQ}\{a, b\}, x]$

Rule 210

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] \text{ ; FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \& \ \& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])]$

Rule 298

$\text{Int}[(x_)/((a_ + (b_)*(x_)^3), x_Symbol] \rightarrow \text{Dist}[-(3*\text{Rt}[a, 3]*\text{Rt}[b, 3])^{-1}, \text{Int}[1/(\text{Rt}[a, 3] + \text{Rt}[b, 3]*x), x] + \text{Dist}[1/(3*\text{Rt}[a, 3]*\text{Rt}[b, 3]), \text{Int}[(\text{Rt}[a, 3] + \text{Rt}[b, 3]*x)/(\text{Rt}[a, 3]^2 - \text{Rt}[a, 3]*\text{Rt}[b, 3]*x + \text{Rt}[b, 3]^2*x^2), x], x] \text{ ; FreeQ}\{a, b\}, x]$

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*c^n*(m + 1))], Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^5(a^3+x^3)} dx &= -\frac{1}{4a^3x^4} - \frac{\int \frac{1}{x^2(a^3+x^3)} dx}{a^3} \\
 &= -\frac{1}{4a^3x^4} + \frac{1}{a^6x} + \frac{\int \frac{x}{a^3+x^3} dx}{a^6} \\
 &= -\frac{1}{4a^3x^4} + \frac{1}{a^6x} - \frac{\int \frac{1}{a+x} dx}{3a^7} + \frac{\int \frac{a+x}{a^2-ax+x^2} dx}{3a^7} \\
 &= -\frac{1}{4a^3x^4} + \frac{1}{a^6x} - \frac{\log(a+x)}{3a^7} + \frac{\int \frac{-a+2x}{a^2-ax+x^2} dx}{6a^7} + \frac{\int \frac{1}{a^2-ax+x^2} dx}{2a^6} \\
 &= -\frac{1}{4a^3x^4} + \frac{1}{a^6x} - \frac{\log(a+x)}{3a^7} + \frac{\log(a^2-ax+x^2)}{6a^7} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1-\frac{2x}{a}\right)}{a^7} \\
 &= -\frac{1}{4a^3x^4} + \frac{1}{a^6x} - \frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7} - \frac{\log(a+x)}{3a^7} + \frac{\log(a^2-ax+x^2)}{6a^7}
 \end{aligned}$$

Mathematica [A]

time = 0.01, size = 74, normalized size = 1.01

$$-\frac{1}{4a^3x^4} + \frac{1}{a^6x} + \frac{\tan^{-1}\left(\frac{-a+2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7} - \frac{\log(a+x)}{3a^7} + \frac{\log(a^2-ax+x^2)}{6a^7}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x^5*(a^3 + x^3)),x]`

```
[Out] -1/4*1/(a^3*x^4) + 1/(a^6*x) + ArcTan[(-a + 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^7)
- Log[a + x]/(3*a^7) + Log[a^2 - a*x + x^2]/(6*a^7)
```

Mathics [C] Result contains complex when optimal does not.

time = 3.09, size = 76, normalized size = 1.04

$$\frac{3a(-a^3 + 4x^3) + 2x^4 \left(-2\text{Log}[a+x] + (1 - I\sqrt{3}) \text{Log}\left[-\frac{a}{2} - \frac{I}{2}\sqrt{3}a+x\right] + (1 + I\sqrt{3}) \text{Log}\left[-\frac{(1-I\sqrt{3})a}{2} + x\right] \right)}{12a^7x^4}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^5*(a^3 + x^3)),x]')`

```
[Out] (3 a (-a ^ 3 + 4 x ^ 3) + 2 x ^ 4 (-2 Log[a + x] + (1 - I Sqrt[3]) Log[-a /
2 - I / 2 Sqrt[3] a + x] + (1 + I Sqrt[3]) Log[-(1 - I Sqrt[3]) a / 2 + x]
)) / (12 a ^ 7 x ^ 4)
```

Maple [A]

time = 0.05, size = 66, normalized size = 0.90

method	result	size
default	$-\frac{1}{4a^3x^4} + \frac{1}{a^6x} + \frac{\frac{\ln(a^2-ax+x^2)}{2} + \sqrt{3} \arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)}{3a^7} - \frac{\ln(a+x)}{3a^7}$	66
risch	$\frac{x^3}{a^6} - \frac{1}{4a^3x^4} - \frac{\ln(a+x)}{3a^7} + \frac{\left(\sum_{-R=\text{RootOf}(a^{14}-Z^2-a^7-Z+1)} -R \ln\left((-4-R^3a^{21}-3)x+a^{15}-R^2\right) \right)}{3}$	72

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/x^5/(a^3+x^3),x,method=_RETURNVERBOSE)`

```
[Out] -1/4/a^3/x^4+1/a^6/x+1/3/a^7*(1/2*ln(a^2-a*x+x^2)+3^(1/2)*arctan(1/3*(2*x-a)
)*3^(1/2)/a))-1/3*ln(a+x)/a^7
```

Maxima [A]

time = 0.35, size = 66, normalized size = 0.90

$$\frac{\sqrt{3} \arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^7} + \frac{\log(a^2 - ax + x^2)}{6a^7} - \frac{\log(a+x)}{3a^7} - \frac{a^3 - 4x^3}{4a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(a^3+x^3),x, algorithm="maxima")**[Out]** 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^7 + 1/6*log(a^2 - a*x + x^2)/a^7 - 1/3*log(a + x)/a^7 - 1/4*(a^3 - 4*x^3)/(a^6*x^4)**Fricas [A]**

time = 0.32, size = 68, normalized size = 0.93

$$\frac{4\sqrt{3}x^4 \arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right) + 2x^4 \log(a^2 - ax + x^2) - 4x^4 \log(a+x) - 3a^4 + 12ax^3}{12a^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(a^3+x^3),x, algorithm="fricas")**[Out]** 1/12*(4*sqrt(3)*x^4*arctan(-1/3*sqrt(3)*(a - 2*x)/a) + 2*x^4*log(a^2 - a*x + x^2) - 4*x^4*log(a + x) - 3*a^4 + 12*a*x^3)/(a^7*x^4)**Sympy [C]** Result contains complex when optimal does not.

time = 0.11, size = 90, normalized size = 1.23

$$\frac{-a^3 + 4x^3}{4a^6x^4} + \frac{-\frac{\log(a+x)}{3} + \left(\frac{1}{6} - \frac{\sqrt{3}i}{6}\right) \log\left(9a\left(\frac{1}{6} - \frac{\sqrt{3}i}{6}\right)^2 + x\right) + \left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) \log\left(9a\left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)^2 + x\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(a**3+x**3),x)**[Out]** (-a**3 + 4*x**3)/(4*a**6*x**4) + (-log(a + x)/3 + (1/6 - sqrt(3)*I/6)*log(9*a*(1/6 - sqrt(3)*I/6)**2 + x) + (1/6 + sqrt(3)*I/6)*log(9*a*(1/6 + sqrt(3)*I/6)**2 + x))/a**7**Giac [A]**

time = 0.00, size = 81, normalized size = 1.11

$$-\frac{\ln|x+a|}{3a^7} + \frac{\ln(x^2 - xa + a^2)}{6a^7} + \frac{\arctan\left(\frac{-a+2x}{a\sqrt{3}}\right)}{a^6\sqrt{3}a} + \frac{4x^3 - a^3}{4a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(a^3+x^3),x)

[Out] $\frac{1}{3}\sqrt{3}\arctan(-1/3\sqrt{3}(a-2x)/a)/a^7 + 1/6\log(a^2 - ax + x^2)/a^7 - 1/3\log(\text{abs}(a+x))/a^7 - 1/4(a^3 - 4x^3)/(a^6x^4)$

Mupad [B]

time = 0.10, size = 99, normalized size = 1.36

$$-\frac{\frac{1}{4a^3} - \frac{x^3}{a^6}}{x^4} - \frac{\ln(a+x)}{3a^7} - \frac{\ln\left(\frac{(-1+\sqrt{3}i)^2 a^7}{4} + x a^6\right) (-1+\sqrt{3}i)}{6a^7} + \frac{\ln\left(\frac{(1+\sqrt{3}i)^2 a^7}{4} + x a^6\right) (1+\sqrt{3}i)}{6a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(a^3 + x^3)),x)

[Out] $(\log(a^6x + (a^7(3^{(1/2)}i + 1)^2)/4)*(3^{(1/2)}i + 1))/(6a^7) - \log(a+x)/(3a^7) - (\log(a^6x + (a^7(3^{(1/2)}i - 1)^2)/4)*(3^{(1/2)}i - 1))/(6a^7) - (1/(4a^3) - x^3/a^6)/x^4$

$$3.126 \quad \int \frac{x^{-m}}{a^3+x^3} dx$$

Optimal. Leaf size=46

$$\frac{x^{1-m} {}_2F_1\left(1, \frac{1-m}{3}; \frac{4-m}{3}; -\frac{x^3}{a^3}\right)}{a^3(1-m)}$$

[Out] $x^{(1-m)}*\text{hypergeom}([1, 1/3-1/3*m], [4/3-1/3*m], -x^3/a^3)/a^3/(1-m)$

Rubi [A]

time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {371}

$$\frac{x^{1-m} {}_2F_1\left(1, \frac{1-m}{3}; \frac{4-m}{3}; -\frac{x^3}{a^3}\right)}{a^3(1-m)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^m*(a^3 + x^3)),x]

[Out] $(x^{(1-m)}*\text{Hypergeometric2F1}[1, (1-m)/3, (4-m)/3, -(x^3/a^3)])/(a^3*(1-m))$

Rule 371

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p * ((c*x)^(m+1)/(c*(m+1)))*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\int \frac{x^{-m}}{a^3+x^3} dx = \frac{x^{1-m} {}_2F_1\left(1, \frac{1-m}{3}; \frac{4-m}{3}; -\frac{x^3}{a^3}\right)}{a^3(1-m)}$$

Mathematica [A]

time = 0.09, size = 45, normalized size = 0.98

$$\frac{x^{1-m} {}_2F_1\left(1, \frac{1}{3} - \frac{m}{3}; \frac{4}{3} - \frac{m}{3}; -\frac{x^3}{a^3}\right)}{a^3(-1+m)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^m*(a^3 + x^3)),x]

[Out] -((x^(1 - m)*Hypergeometric2F1[1, 1/3 - m/3, 4/3 - m/3, -(x^3/a^3)])/(a^3*(-1 + m)))

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

time = 3.99, size = 30, normalized size = 0.65

$$\frac{\text{LerchPhi}\left[\frac{x^3 \exp_{\text{polar}}[i\text{Pi}]}{a^3}, 1, \frac{1}{3} - \frac{m}{3}\right] x^{1-m}}{3a^3}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x^m*(a^3 + x^3)),x]')

[Out] LerchPhi[x ^ 3 exp_polar[I Pi] / a ^ 3, 1, 1 / 3 - m / 3] x ^ (1 - m) / (3 a ^ 3)

Maple [F]

time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{-m}}{a^3 + x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^m)/(a^3+x^3),x)

[Out] int(1/(x^m)/(a^3+x^3),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^m)/(a^3+x^3),x, algorithm="maxima")

[Out] integrate(1/((a^3 + x^3)*x^m), x)

Fricas [F]

time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^m)/(a^3+x^3),x, algorithm="fricas")

[Out] integral(1/((a^3 + x^3)*x^m), x)

Sympy [C] Result contains complex when optimal does not.

time = 1.25, size = 92, normalized size = 2.00

$$-\frac{mxx^{-m}\Phi\left(\frac{x^3e^{i\pi}}{a^3}, 1, \frac{1}{3} - \frac{m}{3}\right)\Gamma\left(\frac{1}{3} - \frac{m}{3}\right)}{9a^3\Gamma\left(\frac{4}{3} - \frac{m}{3}\right)} + \frac{xx^{-m}\Phi\left(\frac{x^3e^{i\pi}}{a^3}, 1, \frac{1}{3} - \frac{m}{3}\right)\Gamma\left(\frac{1}{3} - \frac{m}{3}\right)}{9a^3\Gamma\left(\frac{4}{3} - \frac{m}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**m)/(a**3+x**3),x)

[Out] -m*x*lerchphi(x**3*exp_polar(I*pi)/a**3, 1, 1/3 - m/3)*gamma(1/3 - m/3)/(9*a**3*x**m*gamma(4/3 - m/3)) + x*lerchphi(x**3*exp_polar(I*pi)/a**3, 1, 1/3 - m/3)*gamma(1/3 - m/3)/(9*a**3*x**m*gamma(4/3 - m/3))

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^m)/(a^3+x^3),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^m (a^3 + x^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^m*(a^3 + x^3)),x)

[Out] int(1/(x^m*(a^3 + x^3)), x)

3.127 $\int \frac{1}{a^4 - x^4} dx$

Optimal. Leaf size=27

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^3} + \frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^3}$$

[Out] 1/2*arctan(x/a)/a^3+1/2*arctanh(x/a)/a^3

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {218, 212, 209}

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^3} + \frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^3}$$

Antiderivative was successfully verified.

[In] Int[(a^4 - x^4)^(-1),x]

[Out] ArcTan[x/a]/(2*a^3) + ArcTanh[x/a]/(2*a^3)

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 218

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{a^4 - x^4} dx &= \int \frac{1}{a^2 - x^2} dx + \int \frac{1}{a^2 + x^2} dx \\ &= \frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^3} + \frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^3} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 38, normalized size = 1.41

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^3} - \frac{\log(a-x)}{4a^3} + \frac{\log(a+x)}{4a^3}$$

Antiderivative was successfully verified.

`[In] Integrate[(a^4 - x^4)^(-1),x]``[Out] ArcTan[x/a]/(2*a^3) - Log[a - x]/(4*a^3) + Log[a + x]/(4*a^3)`**Mathics [C]** Result contains complex when optimal does not.

time = 2.30, size = 34, normalized size = 1.26

$$\frac{\text{Log}[a+x] - \text{Log}[-a+x] - I\text{Log}[-Ia+x] + I\text{Log}[Ia+x]}{4a^3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(a^4 - x^4),x]')``[Out] (Log[a + x] - Log[-a + x] - I Log[-I a + x] + I Log[I a + x]) / (4 a ^ 3)`**Maple [A]**

time = 0.05, size = 33, normalized size = 1.22

method	result	size
default	$\frac{\arctan\left(\frac{x}{a}\right)}{2a^3} + \frac{\ln(a+x)}{4a^3} - \frac{\ln(a-x)}{4a^3}$	33
risch	$\frac{\ln(a+x)}{4a^3} - \frac{\ln(-a+x)}{4a^3} + \frac{\left(\sum_{-R=\text{RootOf}(a^6-Z^2+1)} -R \ln(-R a^4+x)\right)}{4}$	47

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(a^4-x^4),x,method=_RETURNVERBOSE)``[Out] 1/2*arctan(x/a)/a^3+1/4*ln(a+x)/a^3-1/4/a^3*ln(a-x)`**Maxima [A]**

time = 0.37, size = 32, normalized size = 1.19

$$\frac{\arctan\left(\frac{x}{a}\right)}{2a^3} + \frac{\log(a+x)}{4a^3} - \frac{\log(-a+x)}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a^4-x^4),x, algorithm="maxima")`

[Out] $1/2*\arctan(x/a)/a^3 + 1/4*\log(a + x)/a^3 - 1/4*\log(-a + x)/a^3$

Fricas [A]

time = 0.34, size = 26, normalized size = 0.96

$$\frac{2 \arctan\left(\frac{x}{a}\right) + \log(a + x) - \log(-a + x)}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a^4-x^4),x, algorithm="fricas")`

[Out] $1/4*(2*\arctan(x/a) + \log(a + x) - \log(-a + x))/a^3$

Sympy [C] Result contains complex when optimal does not.

time = 0.07, size = 37, normalized size = 1.37

$$-\frac{\frac{\log(-a+x)}{4} - \frac{\log(a+x)}{4} + \frac{i \log(-ia+x)}{4} - \frac{i \log(ia+x)}{4}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a**4-x**4),x)`

[Out] $-(\log(-a + x)/4 - \log(a + x)/4 + I*\log(-I*a + x)/4 - I*\log(I*a + x)/4)/a**3$

Giac [A]

time = 0.00, size = 40, normalized size = 1.48

$$-\frac{\ln|x - a|}{4a^3} + \frac{\ln|x + a|}{4a^3} + \frac{\arctan\left(\frac{x}{a}\right)}{2a^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a^4-x^4),x)`

[Out] $1/2*\arctan(x/a)/a^3 + 1/4*\log(\text{abs}(a + x))/a^3 - 1/4*\log(\text{abs}(-a + x))/a^3$

Mupad [B]

time = 0.07, size = 18, normalized size = 0.67

$$\frac{\text{atan}\left(\frac{x}{a}\right) + \text{atanh}\left(\frac{x}{a}\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^4 - x^4),x)`

[Out] $(\text{atan}(x/a) + \text{atanh}(x/a))/(2*a^3)$

3.128 $\int \frac{x}{a^4 - x^4} dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

[Out] 1/2*arctanh(x^2/a^2)/a^2

Rubi [A]

time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {281, 212}

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a^4 - x^4),x]

[Out] ArcTanh[x^2/a^2]/(2*a^2)

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 281

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x}{a^4 - x^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{a^4 - x^2} dx, x, x^2 \right) \\ &= \frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 15, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a^4 - x^4),x]

[Out] ArcTanh[x^2/a^2]/(2*a^2)

Mathics [A]

time = 1.98, size = 26, normalized size = 1.73

$$\frac{\text{Log}[a^2 + x^2] - \text{Log}[-a^2 + x^2]}{4a^2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x/(a^4 - x^4),x]')

[Out] (Log[a^2 + x^2] - Log[-a^2 + x^2]) / (4 a^2)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.

time = 0.13, size = 30, normalized size = 2.00

method	result	size
default	$\frac{\ln(a^2+x^2)}{4a^2} - \frac{\ln(a^2-x^2)}{4a^2}$	30
risch	$-\frac{\ln(-a^2+x^2)}{4a^2} + \frac{\ln(a^2+x^2)}{4a^2}$	30
norman	$-\frac{\ln(a-x)}{4a^2} - \frac{\ln(a+x)}{4a^2} + \frac{\ln(a^2+x^2)}{4a^2}$	35

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a^4-x^4),x,method=_RETURNVERBOSE)

[Out] 1/4/a^2*ln(a^2+x^2)-1/4/a^2*ln(a^2-x^2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.

time = 0.27, size = 29, normalized size = 1.93

$$\frac{\log(a^2 + x^2)}{4a^2} - \frac{\log(-a^2 + x^2)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a^4-x^4),x, algorithm="maxima")

[Out] 1/4*log(a^2 + x^2)/a^2 - 1/4*log(-a^2 + x^2)/a^2

Fricas [A]

time = 0.33, size = 26, normalized size = 1.73

$$\frac{\log(a^2 + x^2) - \log(-a^2 + x^2)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a^4-x^4),x, algorithm="fricas")

[Out] 1/4*(log(a^2 + x^2) - log(-a^2 + x^2))/a^2

Sympy [A]

time = 0.08, size = 24, normalized size = 1.60

$$-\frac{\frac{\log(-a^2+x^2)}{4} - \frac{\log(a^2+x^2)}{4}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a**4-x**4),x)

[Out] -(log(-a**2 + x**2)/4 - log(a**2 + x**2)/4)/a**2

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(13) = 26.
time = 0.00, size = 39, normalized size = 2.60

$$\frac{-\frac{1}{2} \ln|x^2-a^2| + \frac{1}{2} \ln(x^2+a^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a^4-x^4),x)

[Out] 1/4*log(a^2 + x^2)/a^2 - 1/4*log(abs(-a^2 + x^2))/a^2

Mupad [B]

time = 0.05, size = 13, normalized size = 0.87

$$\frac{\operatorname{atanh}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a^4 - x^4),x)

[Out] atanh(x^2/a^2)/(2*a^2)

$$3.129 \quad \int \frac{1}{x(a^4 - x^4)} dx$$

Optimal. Leaf size=24

$$\frac{\log(x)}{a^4} - \frac{\log(a^4 - x^4)}{4a^4}$$

[Out] $\ln(x)/a^4 - 1/4 * \ln(a^4 - x^4)/a^4$

Rubi [A]

time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 36, 31, 29}

$$\frac{\log(x)}{a^4} - \frac{\log(a^4 - x^4)}{4a^4}$$

Antiderivative was successfully verified.

[In] `Int[1/(x*(a^4 - x^4)),x]`

[Out] `Log[x]/a^4 - Log[a^4 - x^4]/(4*a^4)`

Rule 29

`Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]`

Rule 31

`Int[((a_) + (b_)*(x_))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rule 36

`Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]`

Rule 272

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]`

Rubi steps

$$\begin{aligned} \int \frac{1}{x(a^4 - x^4)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{(a^4 - x)x} dx, x, x^4 \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{a^4 - x} dx, x, x^4 \right)}{4a^4} + \frac{\text{Subst} \left(\int \frac{1}{x} dx, x, x^4 \right)}{4a^4} \\ &= \frac{\log(x)}{a^4} - \frac{\log(a^4 - x^4)}{4a^4} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 24, normalized size = 1.00

$$\frac{\log(x)}{a^4} - \frac{\log(-a^4 + x^4)}{4a^4}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x*(a^4 - x^4)),x]``[Out] Log[x]/a^4 - Log[-a^4 + x^4]/(4*a^4)`**Mathics [A]**

time = 1.94, size = 19, normalized size = 0.79

$$\frac{\text{Log}[x] - \frac{\text{Log}[-a^4 + x^4]}{4}}{a^4}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x*(a^4 - x^4)),x]')``[Out] (Log[x] - Log[-a ^ 4 + x ^ 4] / 4) / a ^ 4`**Maple [A]**

time = 0.06, size = 41, normalized size = 1.71

method	result	size
risch	$\frac{\ln(x)}{a^4} - \frac{\ln(-a^4 + x^4)}{4a^4}$	23
default	$\frac{\ln(x)}{a^4} - \frac{\ln(a^2 + x^2)}{4a^4} - \frac{\ln(a+x)}{4a^4} - \frac{\ln(a-x)}{4a^4}$	41
norman	$\frac{\ln(x)}{a^4} - \frac{\ln(a^2 + x^2)}{4a^4} - \frac{\ln(a+x)}{4a^4} - \frac{\ln(a-x)}{4a^4}$	41

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/x/(a^4-x^4),x,method=_RETURNVERBOSE)`

[Out] $\ln(x)/a^4 - 1/4/a^4 * \ln(a^2 + x^2) - 1/4 * \ln(a+x)/a^4 - 1/4/a^4 * \ln(a-x)$

Maxima [A]

time = 0.27, size = 25, normalized size = 1.04

$$-\frac{\log(-a^4 + x^4)}{4a^4} + \frac{\log(x^4)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^4-x^4),x, algorithm="maxima")`

[Out] $-1/4 * \log(-a^4 + x^4)/a^4 + 1/4 * \log(x^4)/a^4$

Fricas [A]

time = 0.32, size = 20, normalized size = 0.83

$$-\frac{\log(-a^4 + x^4) - 4 \log(x)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^4-x^4),x, algorithm="fricas")`

[Out] $-1/4 * (\log(-a^4 + x^4) - 4 * \log(x))/a^4$

Sympy [A]

time = 0.16, size = 19, normalized size = 0.79

$$\frac{\log(x)}{a^4} - \frac{\log(-a^4 + x^4)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a**4-x**4),x)`

[Out] $\log(x)/a^{**4} - \log(-a^{**4} + x^{**4})/(4*a^{**4})$

Giac [A]

time = 0.00, size = 30, normalized size = 1.25

$$\frac{\ln(x^4)}{4a^4} - \frac{\ln|x^4 - a^4|}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^4-x^4),x)`

[Out] $1/4 * \log(x^4)/a^4 - 1/4 * \log(\text{abs}(-a^4 + x^4))/a^4$

Mupad [B]

time = 0.28, size = 20, normalized size = 0.83

$$-\frac{\ln(x^4 - a^4) - 4 \ln(x)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^4 - x^4)),x)
```

```
[Out] -(log(x^4 - a^4) - 4*log(x))/(4*a^4)
```

3.130 $\int \frac{1}{x^2(a^4-x^4)} dx$

Optimal. Leaf size=35

$$-\frac{1}{a^4x} - \frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^5} + \frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^5}$$

[Out] $-1/a^4/x - 1/2*\arctan(x/a)/a^5 + 1/2*\operatorname{arctanh}(x/a)/a^5$

Rubi [A]

time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {331, 304, 209, 212}

$$-\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^5} + \frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^5} - \frac{1}{a^4x}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^2*(a^4 - x^4)),x]`

[Out] `-(1/(a^4*x)) - ArcTan[x/a]/(2*a^5) + ArcTanh[x/a]/(2*a^5)`

Rule 209

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

Rule 212

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 304

`Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]`

Rule 331

`Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m+1)*((a + b*x^n)^(p+1)/(a*c*(m+1))), x] - Dist[b*((m+n*(p+1)+1)/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p]`

x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (a^4 - x^4)} dx &= -\frac{1}{a^4 x} + \frac{\int \frac{x^2}{a^4 - x^4} dx}{a^4} \\
&= -\frac{1}{a^4 x} + \frac{\int \frac{1}{a^2 - x^2} dx}{2a^4} - \frac{\int \frac{1}{a^2 + x^2} dx}{2a^4} \\
&= -\frac{1}{a^4 x} - \frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^5} + \frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^5}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 46, normalized size = 1.31

$$-\frac{1}{a^4 x} - \frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^5} - \frac{\log(a-x)}{4a^5} + \frac{\log(a+x)}{4a^5}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x^2*(a^4 - x^4)),x]``[Out] -(1/(a^4*x)) - ArcTan[x/a]/(2*a^5) - Log[a - x]/(4*a^5) + Log[a + x]/(4*a^5)`**Mathics [C]** Result contains complex when optimal does not.

time = 2.30, size = 43, normalized size = 1.23

$$-a + \frac{x \left(\text{Log}[a+x] - \text{Log}[-a+x] - I \text{Log}[Ia+x] + I \text{Log}[-Ia+x] \right)}{a^5 x}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^2*(a^4 - x^4)),x]')``[Out] (-a + x (Log[a + x] - Log[-a + x] - I Log[I a + x] + I Log[-I a + x])) / 4 / (a ^ 5 x)`**Maple [A]**

time = 0.06, size = 41, normalized size = 1.17

method	result	size
default	$-\frac{1}{a^4 x} - \frac{\arctan\left(\frac{x}{a}\right)}{2a^5} + \frac{\ln(a+x)}{4a^5} - \frac{\ln(a-x)}{4a^5}$	41

risch	$-\frac{1}{a^4x} + \frac{\ln(a+x)}{4a^5} - \frac{\ln(-a+x)}{4a^5} + \frac{\left(\sum_{-R=\text{RootOf}(a^{10}-Z^2+1)} -R \ln((5-R^4 a^{20}-4)x+a^{16}-R^3) \right)}{4}$	68
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(a^4-x^4),x,method=_RETURNVERBOSE)`

[Out] $-1/a^4/x - 1/2*\arctan(x/a)/a^5 + 1/4*\ln(a+x)/a^5 - 1/4/a^5*\ln(a-x)$

Maxima [A]

time = 0.35, size = 40, normalized size = 1.14

$$-\frac{\arctan\left(\frac{x}{a}\right)}{2a^5} + \frac{\log(a+x)}{4a^5} - \frac{\log(-a+x)}{4a^5} - \frac{1}{a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(a^4-x^4),x, algorithm="maxima")`

[Out] $-1/2*\arctan(x/a)/a^5 + 1/4*\log(a+x)/a^5 - 1/4*\log(-a+x)/a^5 - 1/(a^4*x)$

Fricas [A]

time = 0.32, size = 36, normalized size = 1.03

$$-\frac{2x \arctan\left(\frac{x}{a}\right) - x \log(a+x) + x \log(-a+x) + 4a}{4a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(a^4-x^4),x, algorithm="fricas")`

[Out] $-1/4*(2*x*\arctan(x/a) - x*\log(a+x) + x*\log(-a+x) + 4*a)/(a^5*x)$

Sympy [C] Result contains complex when optimal does not.

time = 0.10, size = 44, normalized size = 1.26

$$-\frac{1}{a^4x} - \frac{\frac{\log(-a+x)}{4} - \frac{\log(a+x)}{4} - \frac{i \log(-ia+x)}{4} + \frac{i \log(ia+x)}{4}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(a**4-x**4),x)`

[Out] $-1/(a**4*x) - (\log(-a+x)/4 - \log(a+x)/4 - I*\log(-I*a+x)/4 + I*\log(I*a+x)/4)/a**5$

Giac [A]

time = 0.00, size = 48, normalized size = 1.37

$$-\frac{\ln|x-a|}{4a^5} + \frac{\ln|x+a|}{4a^5} - \frac{\arctan\left(\frac{x}{a}\right)}{2a^4a} - \frac{1}{a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(a^4-x^4),x)`

[Out] $-1/2*\arctan(x/a)/a^5 + 1/4*\log(\text{abs}(a + x))/a^5 - 1/4*\log(\text{abs}(-a + x))/a^5 - 1/(a^4*x)$

Mupad [B]

time = 0.22, size = 31, normalized size = 0.89

$$\frac{\operatorname{atanh}\left(\frac{x}{a}\right)}{2a^5} - \frac{\operatorname{atan}\left(\frac{x}{a}\right)}{2a^5} - \frac{1}{a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(a^4 - x^4)),x)`

[Out] $\operatorname{atanh}(x/a)/(2*a^5) - \operatorname{atan}(x/a)/(2*a^5) - 1/(a^4*x)$

3.131 $\int \frac{1}{x^3(a^4-x^4)} dx$

Optimal. Leaf size=26

$$-\frac{1}{2a^4x^2} + \frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^6}$$

[Out] $-1/2/a^4/x^2+1/2*\operatorname{arctanh}(x^2/a^2)/a^6$

Rubi [A]

time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {281, 331, 212}

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^6} - \frac{1}{2a^4x^2}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^3*(a^4 - x^4)),x]`

[Out] $-1/2*1/(a^4*x^2) + \operatorname{ArcTanh}[x^2/a^2]/(2*a^6)$

Rule 212

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 281

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]`

Rule 331

`Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]`

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3(a^4 - x^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2(a^4 - x^2)} dx, x, x^2 \right) \\ &= -\frac{1}{2a^4x^2} + \frac{\text{Subst} \left(\int \frac{1}{a^4 - x^2} dx, x, x^2 \right)}{2a^4} \\ &= -\frac{1}{2a^4x^2} + \frac{\tanh^{-1} \left(\frac{x^2}{a^2} \right)}{2a^6} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 50, normalized size = 1.92

$$-\frac{1}{2a^4x^2} - \frac{\log(a-x)}{4a^6} - \frac{\log(a+x)}{4a^6} + \frac{\log(a^2+x^2)}{4a^6}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x^3*(a^4 - x^4)),x]``[Out] -1/2*1/(a^4*x^2) - Log[a - x]/(4*a^6) - Log[a + x]/(4*a^6) + Log[a^2 + x^2]/(4*a^6)`**Mathics [A]**

time = 2.16, size = 39, normalized size = 1.50

$$\frac{-2a^2 + x^2 (\text{Log}[a^2 + x^2] - \text{Log}[-a^2 + x^2])}{4a^6x^2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^3*(a^4 - x^4)),x]')``[Out] (-2 a ^ 2 + x ^ 2 (Log[a ^ 2 + x ^ 2] - Log[-a ^ 2 + x ^ 2])) / (4 a ^ 6 x ^ 2)`**Maple [A]**

time = 0.05, size = 43, normalized size = 1.65

method	result	size
risch	$-\frac{1}{2a^4x^2} + \frac{\ln(-a^2-x^2)}{4a^6} - \frac{\ln(a^2-x^2)}{4a^6}$	42
default	$\frac{\ln(a^2+x^2)}{4a^6} - \frac{1}{2a^4x^2} - \frac{\ln(a+x)}{4a^6} - \frac{\ln(a-x)}{4a^6}$	43
norman	$\frac{\ln(a^2+x^2)}{4a^6} - \frac{1}{2a^4x^2} - \frac{\ln(a+x)}{4a^6} - \frac{\ln(a-x)}{4a^6}$	43

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(a^4-x^4),x,method=_RETURNVERBOSE)`

[Out] $1/4/a^6*\ln(a^2+x^2)-1/2/a^4/x^2-1/4*\ln(a+x)/a^6-1/4/a^6*\ln(a-x)$

Maxima [A]

time = 0.26, size = 37, normalized size = 1.42

$$\frac{\log(a^2 + x^2)}{4a^6} - \frac{\log(-a^2 + x^2)}{4a^6} - \frac{1}{2a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(a^4-x^4),x, algorithm="maxima")`

[Out] $1/4*\log(a^2 + x^2)/a^6 - 1/4*\log(-a^2 + x^2)/a^6 - 1/2/(a^4*x^2)$

Fricas [A]

time = 0.32, size = 41, normalized size = 1.58

$$\frac{x^2 \log(a^2 + x^2) - x^2 \log(-a^2 + x^2) - 2a^2}{4a^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(a^4-x^4),x, algorithm="fricas")`

[Out] $1/4*(x^2*\log(a^2 + x^2) - x^2*\log(-a^2 + x^2) - 2*a^2)/(a^6*x^2)$

Sympy [A]

time = 0.11, size = 34, normalized size = 1.31

$$-\frac{1}{2a^4x^2} - \frac{\frac{\log(-a^2+x^2)}{4} - \frac{\log(a^2+x^2)}{4}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(a**4-x**4),x)`

[Out] $-1/(2*a**4*x**2) - (\log(-a**2 + x**2)/4 - \log(a**2 + x**2)/4)/a**6$

Giac [A]

time = 0.00, size = 44, normalized size = 1.69

$$-\frac{\ln|x^2 - a^2|}{4a^6} + \frac{\ln(x^2 + a^2)}{4a^6} - \frac{1}{2a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a^4-x^4),x)

[Out] $\frac{1}{4} \log(a^2 + x^2)/a^6 - \frac{1}{4} \log(\text{abs}(-a^2 + x^2))/a^6 - \frac{1}{2}/(a^4 x^2)$

Mupad [B]

time = 0.20, size = 22, normalized size = 0.85

$$\frac{\operatorname{atanh}\left(\frac{x^2}{a^2}\right)}{2 a^6} - \frac{1}{2 a^4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a^4 - x^4)),x)

[Out] $\operatorname{atanh}(x^2/a^2)/(2*a^6) - 1/(2*a^4*x^2)$

$$3.132 \quad \int \frac{1}{x^4(a^4-x^4)} dx$$

Optimal. Leaf size=37

$$-\frac{1}{3a^4x^3} + \frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^7} + \frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^7}$$

[Out] $-1/3/a^4/x^3+1/2*\arctan(x/a)/a^7+1/2*\operatorname{arctanh}(x/a)/a^7$

Rubi [A]

time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {331, 218, 212, 209}

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^7} + \frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^7} - \frac{1}{3a^4x^3}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^4*(a^4 - x^4)),x]`

[Out] $-1/3*1/(a^4*x^3) + \operatorname{ArcTan}[x/a]/(2*a^7) + \operatorname{ArcTanh}[x/a]/(2*a^7)$

Rule 209

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

Rule 212

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 218

`Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]`

Rule 331

`Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m+1)*((a + b*x^n)^(p+1)/(a*c*(m+1))), x] - Dist[b*((m+n*(p+1)+1)/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,`

x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (a^4 - x^4)} dx &= -\frac{1}{3a^4 x^3} + \frac{\int \frac{1}{a^4 - x^4} dx}{a^4} \\
&= -\frac{1}{3a^4 x^3} + \frac{\int \frac{1}{a^2 - x^2} dx}{2a^6} + \frac{\int \frac{1}{a^2 + x^2} dx}{2a^6} \\
&= -\frac{1}{3a^4 x^3} + \frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^7} + \frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^7}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 48, normalized size = 1.30

$$-\frac{1}{3a^4 x^3} + \frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^7} - \frac{\log(a-x)}{4a^7} + \frac{\log(a+x)}{4a^7}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x^4*(a^4 - x^4)),x]``[Out] -1/3*1/(a^4*x^3) + ArcTan[x/a]/(2*a^7) - Log[a - x]/(4*a^7) + Log[a + x]/(4*a^7)`**Mathics [C]** Result contains complex when optimal does not.

time = 2.24, size = 48, normalized size = 1.30

$$\frac{-4a^3 + 3x^3 (\text{Log}[a+x] - \text{Log}[-a+x] - I\text{Log}[-Ia+x] + I\text{Log}[Ia+x])}{12a^7 x^3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^4*(a^4 - x^4)),x]')``[Out] (-4 a ^ 3 + 3 x ^ 3 (Log[a + x] - Log[-a + x] - I Log[-I a + x] + I Log[I a + x])) / (12 a ^ 7 x ^ 3)`**Maple [A]**

time = 0.05, size = 41, normalized size = 1.11

method	result	size
default	$-\frac{1}{3a^4 x^3} + \frac{\arctan\left(\frac{x}{a}\right)}{2a^7} + \frac{\ln(a+x)}{4a^7} - \frac{\ln(a-x)}{4a^7}$	41

risch	$-\frac{1}{3a^4x^3} + \frac{\ln(-a-x)}{4a^7} + \frac{\left(\sum_{-R=\text{RootOf}(a^{14}-Z^2+1)} -R \ln\left(\left(-5-R^4 a^{28}+4\right)x-a^8-R\right)\right)}{4} - \frac{\ln(a-x)}{4a^7}$	71
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(a^4-x^4),x,method=_RETURNVERBOSE)`

[Out] $-1/3/a^4/x^3+1/2*\arctan(x/a)/a^7+1/4*\ln(a+x)/a^7-1/4/a^7*\ln(a-x)$

Maxima [A]

time = 0.35, size = 40, normalized size = 1.08

$$\frac{\arctan\left(\frac{x}{a}\right)}{2a^7} + \frac{\log(a+x)}{4a^7} - \frac{\log(-a+x)}{4a^7} - \frac{1}{3a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(a^4-x^4),x, algorithm="maxima")`

[Out] $1/2*\arctan(x/a)/a^7 + 1/4*\log(a + x)/a^7 - 1/4*\log(-a + x)/a^7 - 1/3/(a^4*x^3)$

Fricas [A]

time = 0.32, size = 45, normalized size = 1.22

$$\frac{6x^3 \arctan\left(\frac{x}{a}\right) + 3x^3 \log(a+x) - 3x^3 \log(-a+x) - 4a^3}{12a^7x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(a^4-x^4),x, algorithm="fricas")`

[Out] $1/12*(6*x^3*\arctan(x/a) + 3*x^3*\log(a + x) - 3*x^3*\log(-a + x) - 4*a^3)/(a^7*x^3)$

Sympy [C] Result contains complex when optimal does not.

time = 0.11, size = 48, normalized size = 1.30

$$-\frac{1}{3a^4x^3} - \frac{\frac{\log(-a+x)}{4} - \frac{\log(a+x)}{4} + \frac{i \log(-ia+x)}{4} - \frac{i \log(ia+x)}{4}}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(a**4-x**4),x)`

[Out] $-1/(3*a**4*x**3) - (\log(-a + x)/4 - \log(a + x)/4 + I*\log(-I*a + x)/4 - I*\log(I*a + x)/4)/a**7$

Giac [A]

time = 0.00, size = 50, normalized size = 1.35

$$-\frac{\ln|x-a|}{4a^7} + \frac{\ln|x+a|}{4a^7} + \frac{\arctan\left(\frac{x}{a}\right)}{2a^6a} - \frac{1}{3a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a^4-x^4),x)**[Out]** 1/2*arctan(x/a)/a^7 + 1/4*log(abs(a + x))/a^7 - 1/4*log(abs(-a + x))/a^7 - 1/3/(a^4*x^3)**Mupad [B]**

time = 0.08, size = 31, normalized size = 0.84

$$\frac{\operatorname{atan}\left(\frac{x}{a}\right)}{2a^7} + \frac{\operatorname{atanh}\left(\frac{x}{a}\right)}{2a^7} - \frac{1}{3a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(a^4 - x^4)),x)**[Out]** atan(x/a)/(2*a^7) + atanh(x/a)/(2*a^7) - 1/(3*a^4*x^3)

3.133 $\int \frac{x^{-m}}{a^4 - x^4} dx$

Optimal. Leaf size=45

$$\frac{x^{1-m} {}_2F_1\left(1, \frac{1-m}{4}; \frac{5-m}{4}, \frac{x^4}{a^4}\right)}{a^4(1-m)}$$

[Out] $x^{(1-m)} \cdot \text{hypergeom}\left([1, 1/4-1/4*m], [5/4-1/4*m], x^4/a^4\right)/a^4/(1-m)$

Rubi [A]

time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {371}

$$\frac{x^{1-m} {}_2F_1\left(1, \frac{1-m}{4}; \frac{5-m}{4}, \frac{x^4}{a^4}\right)}{a^4(1-m)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^m(a^4 - x^4)), x]$

[Out] $(x^{(1-m)} \cdot \text{Hypergeometric2F1}[1, (1-m)/4, (5-m)/4, x^4/a^4]) / (a^4 \cdot (1-m))$

Rule 371

$\text{Int}[\frac{(c \cdot x)^m \cdot (a + b \cdot x^n)^p}{a^4 - x^4}, x_Symbol] \rightarrow \text{Simp}[a^p \cdot \frac{(c \cdot x)^{m+1}}{c \cdot (m+1)} \cdot \text{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n + 1, (-b) \cdot (x^n/a)], x] /; \text{FreeQ}\{a, b, c, m, n, p\}, x \ \&\& \ !\text{IGtQ}[p, 0] \ \&\& \ (\text{ILtQ}[p, 0] \ || \ \text{GtQ}[a, 0])$

Rubi steps

$$\int \frac{x^{-m}}{a^4 - x^4} dx = \frac{x^{1-m} {}_2F_1\left(1, \frac{1-m}{4}; \frac{5-m}{4}, \frac{x^4}{a^4}\right)}{a^4(1-m)}$$

Mathematica [A]

time = 0.14, size = 44, normalized size = 0.98

$$\frac{x^{1-m} {}_2F_1\left(1, \frac{1}{4} - \frac{m}{4}; \frac{5}{4} - \frac{m}{4}, \frac{x^4}{a^4}\right)}{a^4(-1+m)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^m*(a^4 - x^4)),x]

[Out] -((x^(1 - m)*Hypergeometric2F1[1, 1/4 - m/4, 5/4 - m/4, x^4/a^4])/(a^4*(-1 + m)))

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

time = 3.02, size = 30, normalized size = 0.67

$$\frac{\text{LerchPhi}\left[\frac{x^4 \exp_{\text{polar}}[2i\text{Pi}]}{a^4}, 1, \frac{1}{4} - \frac{m}{4}\right] x^{1-m}}{4a^4}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x^m*(a^4 - x^4)),x]')

[Out] LerchPhi[x ^ 4 exp_polar[2 I Pi] / a ^ 4, 1, 1 / 4 - m / 4] x ^ (1 - m) / (4 a ^ 4)

Maple [F]

time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{-m}}{a^4 - x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^m)/(a^4-x^4),x)

[Out] int(1/(x^m)/(a^4-x^4),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^m)/(a^4-x^4),x, algorithm="maxima")

[Out] integrate(1/((a^4 - x^4)*x^m), x)

Fricas [F]

time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^m)/(a^4-x^4),x, algorithm="fricas")

[Out] `integral(1/((a^4 - x^4)*x^m), x)`

Sympy [C] Result contains complex when optimal does not.
time = 0.47, size = 95, normalized size = 2.11

$$-\frac{mxx^{-m}\Phi\left(\frac{x^4e^{2i\pi}}{a^4}, 1, \frac{1}{4} - \frac{m}{4}\right)\Gamma\left(\frac{1}{4} - \frac{m}{4}\right)}{16a^4\Gamma\left(\frac{5}{4} - \frac{m}{4}\right)} + \frac{xx^{-m}\Phi\left(\frac{x^4e^{2i\pi}}{a^4}, 1, \frac{1}{4} - \frac{m}{4}\right)\Gamma\left(\frac{1}{4} - \frac{m}{4}\right)}{16a^4\Gamma\left(\frac{5}{4} - \frac{m}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**m)/(a**4-x**4), x)`

[Out] `-m*x*lerchphi(x**4*exp_polar(2*I*pi)/a**4, 1, 1/4 - m/4)*gamma(1/4 - m/4)/(16*a**4*x**m*gamma(5/4 - m/4)) + x*lerchphi(x**4*exp_polar(2*I*pi)/a**4, 1, 1/4 - m/4)*gamma(1/4 - m/4)/(16*a**4*x**m*gamma(5/4 - m/4))`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^m)/(a^4-x^4), x)`

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^m (a^4 - x^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^m*(a^4 - x^4)), x)`

[Out] `int(1/(x^m*(a^4 - x^4)), x)`

3.134 $\int \frac{x}{a^4+x^4} dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

[Out] 1/2*arctan(x^2/a^2)/a^2

Rubi [A]

time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {281, 209}

$$\frac{\tan^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a^4 + x^4),x]

[Out] ArcTan[x^2/a^2]/(2*a^2)

Rule 209

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 281

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x}{a^4+x^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{a^4+x^2} dx, x, x^2 \right) \\ &= \frac{\tan^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 15, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a^4 + x^4),x]

[Out] ArcTan[x^2/a^2]/(2*a^2)

Mathics [C] Result contains complex when optimal does not.
time = 2.00, size = 28, normalized size = 1.87

$$\frac{\frac{1}{4} (\text{Log}[Ia^2 + x^2] - \text{Log}[-Ia^2 + x^2])}{a^2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x/(a^4 + x^4),x]')

[Out] I / 4 (Log[I a ^ 2 + x ^ 2] - Log[-I a ^ 2 + x ^ 2]) / a ^ 2

Maple [A]

time = 0.06, size = 14, normalized size = 0.93

method	result	size
default	$\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$	14
risch	$\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$	14

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a^4+x^4),x,method=_RETURNVERBOSE)

[Out] 1/2*arctan(x^2/a^2)/a^2

Maxima [A]

time = 0.36, size = 13, normalized size = 0.87

$$\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a^4+x^4),x, algorithm="maxima")

[Out] 1/2*arctan(x^2/a^2)/a^2

Fricas [A]

time = 0.30, size = 13, normalized size = 0.87

$$\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a^4+x^4),x, algorithm="fricas")`

[Out] $1/2*\arctan(x^2/a^2)/a^2$

Sympy [C] Result contains complex when optimal does not.
time = 0.07, size = 29, normalized size = 1.93

$$\frac{-\frac{i \log(-ia^2+x^2)}{4} + \frac{i \log(ia^2+x^2)}{4}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a**4+x**4),x)`

[Out] $(-I*\log(-I*a**2 + x**2)/4 + I*\log(I*a**2 + x**2)/4)/a**2$

Giac [A]

time = 0.00, size = 17, normalized size = 1.13

$$\frac{\frac{1}{2} \arctan\left(\frac{x^2}{a^2}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a^4+x^4),x)`

[Out] $1/2*\arctan(x^2/a^2)/a^2$

Mupad [B]

time = 0.19, size = 13, normalized size = 0.87

$$\frac{\operatorname{atan}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a^4 + x^4),x)`

[Out] $\operatorname{atan}(x^2/a^2)/(2*a^2)$

3.135 $\int \frac{x^2}{a^4+x^4} dx$

Optimal. Leaf size=109

$$\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a} + \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a} + \frac{\log\left(a^2 - \sqrt{2}ax + x^2\right)}{4\sqrt{2}a} - \frac{\log\left(a^2 + \sqrt{2}ax + x^2\right)}{4\sqrt{2}a}$$

[Out] $-1/4*\arctan(1-x*2^{(1/2)}/a)/a*2^{(1/2)}+1/4*\arctan(1+x*2^{(1/2)}/a)/a*2^{(1/2)}+1/8*\ln(a^2+x^2-a*x*2^{(1/2)})/a*2^{(1/2)}-1/8*\ln(a^2+x^2+a*x*2^{(1/2)})/a*2^{(1/2)}$

Rubi [A]

time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {303, 1176, 631, 210, 1179, 642}

$$\frac{\log\left(a^2 - \sqrt{2}ax + x^2\right)}{4\sqrt{2}a} - \frac{\log\left(a^2 + \sqrt{2}ax + x^2\right)}{4\sqrt{2}a} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a} + \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{a} + 1\right)}{2\sqrt{2}a}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2/(a^4 + x^4), x]$

[Out] $-1/2*\text{ArcTan}[1 - (\text{Sqrt}[2]*x)/a]/(\text{Sqrt}[2]*a) + \text{ArcTan}[1 + (\text{Sqrt}[2]*x)/a]/(2*\text{Sqrt}[2]*a) + \text{Log}[a^2 - \text{Sqrt}[2]*a*x + x^2]/(4*\text{Sqrt}[2]*a) - \text{Log}[a^2 + \text{Sqrt}[2]*a*x + x^2]/(4*\text{Sqrt}[2]*a)$

Rule 210

$\text{Int}[(a_.) + (b_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 303

$\text{Int}[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] \rightarrow \text{With}\{r = \text{Numerator}[\text{Rt}[a/b, 2]], s = \text{Denominator}[\text{Rt}[a/b, 2]]\}, \text{Dist}[1/(2*s), \text{Int}[(r + s*x^2)/(a + b*x^4), x], x] - \text{Dist}[1/(2*s), \text{Int}[(r - s*x^2)/(a + b*x^4), x], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ (\text{GtQ}[a/b, 0] \ || \ (\text{PosQ}[a/b] \ \&\& \ \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, a]] \ \&\& \ \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, b]]))$

Rule 631

$\text{Int}[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{With}\{q = 1 - 4*\text{Simplify}[a*(c/b^2)]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ \text{!RationalQ}[b^2 - 4*a*c]) /; \text{Free}$

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol] \ :> \ \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rubi steps

$$\begin{aligned} \int \frac{x^2}{a^4 + x^4} dx &= - \left(\frac{1}{2} \int \frac{a^2 - x^2}{a^4 + x^4} dx \right) + \frac{1}{2} \int \frac{a^2 + x^2}{a^4 + x^4} dx \\ &= \frac{1}{4} \int \frac{1}{a^2 - \sqrt{2} ax + x^2} dx + \frac{1}{4} \int \frac{1}{a^2 + \sqrt{2} ax + x^2} dx + \frac{\int \frac{\sqrt{2} a + 2x}{-a^2 - \sqrt{2} ax - x^2} dx}{4\sqrt{2} a} + \frac{\int \frac{\sqrt{2} a - 2x}{-a^2 + \sqrt{2} ax + x^2} dx}{4\sqrt{2} a} \\ &= \frac{\log(a^2 - \sqrt{2} ax + x^2)}{4\sqrt{2} a} - \frac{\log(a^2 + \sqrt{2} ax + x^2)}{4\sqrt{2} a} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1 - \frac{\sqrt{2} x}{a}\right)}{2\sqrt{2} a} \\ &= -\frac{\tan^{-1}\left(1 - \frac{\sqrt{2} x}{a}\right)}{2\sqrt{2} a} + \frac{\tan^{-1}\left(1 + \frac{\sqrt{2} x}{a}\right)}{2\sqrt{2} a} + \frac{\log(a^2 - \sqrt{2} ax + x^2)}{4\sqrt{2} a} - \frac{\log(a^2 + \sqrt{2} ax + x^2)}{4\sqrt{2} a} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 79, normalized size = 0.72

$$\frac{-2 \tan^{-1}\left(1 - \frac{\sqrt{2} x}{a}\right) + 2 \tan^{-1}\left(1 + \frac{\sqrt{2} x}{a}\right) + \log(a^2 - \sqrt{2} ax + x^2) - \log(a^2 + \sqrt{2} ax + x^2)}{4\sqrt{2} a}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a^4 + x^4), x]

[Out] $(-2*\text{ArcTan}[1 - (\text{Sqrt}[2]*x)/a] + 2*\text{ArcTan}[1 + (\text{Sqrt}[2]*x)/a] + \text{Log}[a^2 - \text{Sqrt}[2]*a*x + x^2] - \text{Log}[a^2 + \text{Sqrt}[2]*a*x + x^2])/(4*\text{Sqrt}[2]*a)$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.05, size = 28, normalized size = 0.26

$$\frac{\text{RootSum}\left[1 + 256\#1^4 \&, \text{Log}\left[64a\#1^3 + x\right] \#1\&\right]}{a}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^2/(a^4 + x^4), x]')

[Out] $\text{RootSum}[1 + 256 \#1^4 \&, \text{Log}[64 a \#1^3 + x] \#1\&] / a$

Maple [A]

time = 0.04, size = 85, normalized size = 0.78

method	result	size
risch	$\frac{\left(\sum_{R=\text{RootOf}(-Z^4+a^4)} \frac{\ln\left(-\frac{R+x}{-R}\right)}{-R}\right)}{4}$	24
default	$\frac{\sqrt{2} \left(\ln\left(\frac{x^2 - (a^4)^{\frac{1}{4}} x \sqrt{2} + \sqrt{a^4}}{x^2 + (a^4)^{\frac{1}{4}} x \sqrt{2} + \sqrt{a^4}}\right) + 2 \arctan\left(\frac{\sqrt{2} x}{(a^4)^{\frac{1}{4}} + 1}\right) + 2 \arctan\left(\frac{\sqrt{2} x}{(a^4)^{\frac{1}{4}} - 1}\right) \right)}{8(a^4)^{\frac{1}{4}}}$	85

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a^4+x^4), x, method=_RETURNVERBOSE)

[Out] $1/8/(a^4)^{(1/4)}*2^{(1/2)}*(\ln((x^2-(a^4)^{(1/4)}*x*2^{(1/2)}+(a^4)^{(1/2)})/(x^2+(a^4)^{(1/4)}*x*2^{(1/2)}+(a^4)^{(1/2)}))+2*\arctan(2^{(1/2)}/(a^4)^{(1/4)}*x+1)+2*\arctan(2^{(1/2)}/(a^4)^{(1/4)}*x-1))$

Maxima [A]

time = 0.36, size = 98, normalized size = 0.90

$$\frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}(\sqrt{2}a+2x)}{2a}\right)}{4a} + \frac{\sqrt{2} \arctan\left(-\frac{\sqrt{2}(\sqrt{2}a-2x)}{2a}\right)}{4a} - \frac{\sqrt{2} \log\left(\sqrt{2}ax + a^2 + x^2\right)}{8a} + \frac{\sqrt{2} \log\left(-\sqrt{2}ax + a^2 + x^2\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a^4+x^4), x, algorithm="maxima")

[Out] $\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{2}a+2x}{a}\right)\right)/a + \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{2}a-2x}{a}\right)\right)/a - \frac{1}{8}\sqrt{2}\log(\sqrt{2}ax+a^2+x^2)/a + \frac{1}{8}\sqrt{2}\log(-\sqrt{2}ax+a^2+x^2)/a$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(85) = 170.

time = 0.34, size = 199, normalized size = 1.83

$$-\frac{1}{2}\sqrt{2}\frac{1}{a^2}\arctan\left(-\sqrt{2}\frac{1}{a^2}x+\sqrt{2}\sqrt{\sqrt{2}a^2\frac{1}{a^2}x+a^2}\sqrt{\frac{1}{a^2}+x^2}\frac{1}{a^2}-1\right)-\frac{1}{2}\sqrt{2}\frac{1}{a^2}\arctan\left(-\sqrt{2}\frac{1}{a^2}x+\sqrt{2}\sqrt{-\sqrt{2}a^2\frac{1}{a^2}x+a^2}\sqrt{\frac{1}{a^2}+x^2}\frac{1}{a^2}+1\right)-\frac{1}{8}\sqrt{2}\frac{1}{a^2}\log\left(\sqrt{2}a^2\frac{1}{a^2}x+a^2\sqrt{\frac{1}{a^2}+x^2}\right)+\frac{1}{8}\sqrt{2}\frac{1}{a^2}\log\left(-\sqrt{2}a^2\frac{1}{a^2}x+a^2\sqrt{\frac{1}{a^2}+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(a^4+x^4),x, algorithm="fricas")`

[Out] $-\frac{1}{2}\sqrt{2}\left(\frac{a^{-4}}{a^2}\right)^{1/4}\arctan\left(-\sqrt{2}\left(\frac{a^{-4}}{a^2}\right)^{1/4}x+\sqrt{2}\sqrt{\sqrt{2}a^4\left(\frac{a^{-4}}{a^2}\right)^{3/4}x+a^4}\sqrt{\frac{a^{-4}}{a^2}+x^2}\left(\frac{a^{-4}}{a^2}\right)^{1/4}-1\right)-\frac{1}{2}\sqrt{2}\left(\frac{a^{-4}}{a^2}\right)^{1/4}\arctan\left(-\sqrt{2}\left(\frac{a^{-4}}{a^2}\right)^{1/4}x+\sqrt{2}\sqrt{-\sqrt{2}a^4\left(\frac{a^{-4}}{a^2}\right)^{3/4}x+a^4}\sqrt{\frac{a^{-4}}{a^2}+x^2}\left(\frac{a^{-4}}{a^2}\right)^{1/4}+1\right)-\frac{1}{8}\sqrt{2}\left(\frac{a^{-4}}{a^2}\right)^{1/4}\log\left(\sqrt{2}a^4\left(\frac{a^{-4}}{a^2}\right)^{3/4}x+a^4\sqrt{\frac{a^{-4}}{a^2}+x^2}\right)+\frac{1}{8}\sqrt{2}\left(\frac{a^{-4}}{a^2}\right)^{1/4}\log\left(-\sqrt{2}a^4\left(\frac{a^{-4}}{a^2}\right)^{3/4}x+a^4\sqrt{\frac{a^{-4}}{a^2}+x^2}\right)$

Sympy [A]

time = 0.07, size = 19, normalized size = 0.17

$$\frac{\text{RootSum}(256t^4 + 1, (t \mapsto t \log(64t^3a + x)))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(a**4+x**4),x)`

[Out] `RootSum(256*_t**4 + 1, Lambda(_t, _t*log(64*_t**3*a + x)))/a`

Giac [A]

time = 0.00, size = 166, normalized size = 1.52

$$\frac{|a|a^2\ln\left(x^2-\sqrt{2}|a|x+|a||a|\right)}{4\sqrt{2}a^4}+\frac{|a|a^2\arctan\left(\frac{x-\frac{1}{2}\sqrt{2}|a|}{\frac{1}{2}\sqrt{2}|a|}\right)}{2\sqrt{2}a^4}-\frac{|a|a^2\ln\left(x^2+\sqrt{2}|a|x+|a||a|\right)}{(4\sqrt{2})a^4}+\frac{|a|a^2\arctan\left(\frac{x+\frac{1}{2}\sqrt{2}|a|}{\frac{1}{2}\sqrt{2}|a|}\right)}{2\sqrt{2}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(a^4+x^4),x)`

[Out] $\frac{1}{4}\sqrt{2}\text{abs}(a)\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{2}\text{abs}(a)+2x}{\text{abs}(a)}\right)\right)/a^2 + \frac{1}{4}\sqrt{2}\text{abs}(a)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{2}\text{abs}(a)-2x}{\text{abs}(a)}\right)\right)/a^2 - \frac{1}{8}\sqrt{2}\text{abs}(a)\log(\sqrt{2}x\text{abs}(a)+x^2+\text{abs}(a)^2)/a^2 + \frac{1}{8}\sqrt{2}\text{abs}(a)\log(-\sqrt{2}x\text{abs}(a)+x^2+\text{abs}(a)^2)/a^2$

Mupad [B]

time = 0.10, size = 33, normalized size = 0.30

$$\frac{(-1)^{1/4} \operatorname{atan}\left(\frac{(-1)^{1/4} x}{a}\right) - (-1)^{1/4} \operatorname{atanh}\left(\frac{(-1)^{1/4} x}{a}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a^4 + x^4),x)`

[Out] `((-1)^(1/4)*atan((-1)^(1/4)*x/a) - (-1)^(1/4)*atanh((-1)^(1/4)*x/a))/(2*a)`

3.136 $\int \frac{1}{a^5+x^5} dx$

Optimal. Leaf size=201

$$\frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{5a^4} - \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})} \left((1+\sqrt{5})^{a-4x}\right)}{2a}\right)}{5a^4}$$

[Out] $1/5*\ln(a+x)/a^4-1/20*\ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(-5^(1/2)+1)/a^4-1/20$
 $*\ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(5^(1/2)+1)/a^4-1/10*\arctan(1/20*(-4*x+a*($
 $5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10-2*5^(1/2))^(1/2)/a^4-1/10*\arctan(($
 $-4*x+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)/a^4$

Rubi [A]

time = 0.24, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {207, 648, 632, 210, 642, 31}

$$\frac{\log(a+x)}{5a^4} - \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{5a^4} - \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})} \left((1+\sqrt{5})^{a-4x}\right)}{2a}\right)}{5a^4} - \frac{(1-\sqrt{5}) \log\left(a^2 - \frac{1}{2}(1-\sqrt{5})ax + x^2\right)}{20a^4} - \frac{(1+\sqrt{5}) \log\left(a^2 - \frac{1}{2}(1+\sqrt{5})ax + x^2\right)}{20a^4}$$

Antiderivative was successfully verified.

[In] Int[(a^5 + x^5)^(-1), x]

[Out] $-1/5*(\text{Sqrt}[(5 + \text{Sqrt}[5])/2]*\text{ArcTan}[\frac{((1 - \text{Sqrt}[5])*a - 4*x)/(\text{Sqrt}[2*(5 + \text{Sqrt}[5]))*a]}{a}])/a^4 - (\text{Sqrt}[(5 - \text{Sqrt}[5])/2]*\text{ArcTan}[\frac{(\text{Sqrt}[(5 + \text{Sqrt}[5])/10]*((1 + \text{Sqrt}[5])*a - 4*x))/(2*a)}{5*a^4} + \text{Log}[a + x]/(5*a^4) - ((1 - \text{Sqrt}[5])*\text{Log}[a^2 - ((1 - \text{Sqrt}[5])*a*x)/2 + x^2])/(20*a^4) - ((1 + \text{Sqrt}[5])*\text{Log}[a^2 - ((1 + \text{Sqrt}[5])*a*x)/2 + x^2])/(20*a^4)$

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 207

Int[((a_) + (b_)*(x_)^(n_))^(m_), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; (r/(a*n))*Int[1/(r + s*x), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 1)/2}], x, x] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && PosQ[a/b]

Rule 210

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{a^5 + x^5} dx &= \frac{2 \int \frac{a^{-\frac{1}{4}}(1-\sqrt{5})x}{a^{2-\frac{1}{2}}(1-\sqrt{5})^{ax+x^2}} dx}{5a^4} + \frac{2 \int \frac{a^{-\frac{1}{4}}(1+\sqrt{5})x}{a^{2-\frac{1}{2}}(1+\sqrt{5})^{ax+x^2}} dx}{5a^4} + \frac{\int \frac{1}{a+x} dx}{5a^4} \\
 &= \frac{\log(a+x)}{5a^4} - \frac{(1-\sqrt{5}) \int \frac{-\frac{1}{2}(1-\sqrt{5})^{a+2x}}{a^{2-\frac{1}{2}}(1-\sqrt{5})^{ax+x^2}} dx}{20a^4} - \frac{(1+\sqrt{5}) \int \frac{-\frac{1}{2}(1+\sqrt{5})^{a+2x}}{a^{2-\frac{1}{2}}(1+\sqrt{5})^{ax+x^2}} dx}{20a^4} + \dots \\
 &= \frac{\log(a+x)}{5a^4} - \frac{(1+\sqrt{5}) \log(2a^2 - ax - \sqrt{5}ax + 2x^2)}{20a^4} - \frac{(1-\sqrt{5}) \log(2a^2 - ax + \sqrt{5}ax + 2x^2)}{20a^4} + \dots \\
 &= -\frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{5a^4} - \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}}{\dots}\right)}{5a^4}
 \end{aligned}$$

Mathematica [A]

time = 0.09, size = 204, normalized size = 1.01

$$\frac{-2\sqrt{2(5+\sqrt{5})} \tan^{-1}\left(\frac{((1+\sqrt{5})a+x)}{\sqrt{2(5+\sqrt{5})}a}\right) - 2\sqrt{10-2\sqrt{5}} \tan^{-1}\left(\frac{((1+\sqrt{5})a+x)}{\sqrt{10-2\sqrt{5}}a}\right) - 4\log(a+x) + \log(a^2 + \frac{1}{2}(-1+\sqrt{5})ax + x^2) - \sqrt{5} \log(a^2 + \frac{1}{2}(-1+\sqrt{5})ax + x^2) + \log(a^2 - \frac{1}{2}(1+\sqrt{5})ax + x^2) + \sqrt{5} \log(a^2 - \frac{1}{2}(1+\sqrt{5})ax + x^2)}{20a^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a^5 + x^5)^(-1), x]

[Out]
$$\frac{-1/20*(-2*\text{Sqrt}[2*(5 + \text{Sqrt}[5])]*\text{ArcTan}[((-1 + \text{Sqrt}[5])*a + 4*x)/(\text{Sqrt}[2*(5 + \text{Sqrt}[5])]*a)] - 2*\text{Sqrt}[10 - 2*\text{Sqrt}[5]]*\text{ArcTan}[(-((1 + \text{Sqrt}[5])*a) + 4*x)/(\text{Sqrt}[10 - 2*\text{Sqrt}[5]]*a)] - 4*\text{Log}[a + x] + \text{Log}[a^2 + ((-1 + \text{Sqrt}[5])*a*x)/2 + x^2] - \text{Sqrt}[5]*\text{Log}[a^2 + ((-1 + \text{Sqrt}[5])*a*x)/2 + x^2] + \text{Log}[a^2 - ((1 + \text{Sqrt}[5])*a*x)/2 + x^2] + \text{Sqrt}[5]*\text{Log}[a^2 - ((1 + \text{Sqrt}[5])*a*x)/2 + x^2])}{a^4}$$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.29, size = 49, normalized size = 0.24

$$\frac{\text{RootSum}[1 + 5\#1 + 25\#1^2 + 125\#1^3 + 625\#1^4 \&, \text{Log}[5a\#1 + x] \#1 \&] + \frac{\text{Log}[a+x]}{5}}{a^4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(a^5 + x^5), x]')

[Out]
$$\frac{(\text{RootSum}[1 + 5 \#1 + 25 \#1^2 + 125 \#1^3 + 625 \#1^4 \&, \text{Log}[5 a \#1 + x] \#1 \&] + \text{Log}[a + x] / 5) / a^4}$$

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 0.05, size = 101, normalized size = 0.50

method	result	size
risch	$\frac{\left(\sum_{-R=\text{RootOf}(a^{16}-Z^4+a^{12}-Z^3+a^8-Z^2+a^4-Z+1)} -R \ln(-R a^5+x) \right)}{5} + \frac{\ln(a+x)}{5a^4}$	55
default	$\frac{\ln(a+x)}{5a^4} + \frac{\sum_{-R=\text{RootOf}(-Z^4-a-Z^3+_Z^2a^2-a^3-Z+a^4)} (-R^3+_2R^2a-3a^2-R+_4a^3) \ln(-R+x)}{4R^3-3R^2a+2a^2-R-a^3}$	101

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^5+x^5), x, method=_RETURNVERBOSE)

[Out]
$$\frac{1}{5} \ln(a+x) / a^4 + \frac{1}{5} / a^4 \sum((-R^3+2R^2a-3R+a^2+4a^3) / (4R^3-3R^2a+2R-a^3) * \ln(-R+x), R=\text{RootOf}(-Z^4-Z^3a+_Z^2a^2-_Za^3+a^4))$$

Maxima [A]

time = 0.35, size = 180, normalized size = 0.90

$$\frac{\sqrt{5}(\sqrt{5}+1)\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{5a^4\sqrt{2\sqrt{5}+10}} + \frac{\sqrt{5}(\sqrt{5}-1)\arctan\left(\frac{-a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{5a^4\sqrt{-2\sqrt{5}+10}} - \frac{(\sqrt{5}+3)\log(-ax(\sqrt{5}+1)+2a^2+2x^2)}{10a^4(\sqrt{5}+1)} - \frac{(\sqrt{5}-3)\log(ax(\sqrt{5}-1)+2a^2+2x^2)}{10a^4(\sqrt{5}-1)} + \frac{\log(a+x)}{5a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^5+x^5),x, algorithm="maxima")

[Out] 1/5*sqrt(5)*(sqrt(5) + 1)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a^4*sqrt(2*sqrt(5) + 10)) + 1/5*sqrt(5)*(sqrt(5) - 1)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/(a^4*sqrt(-2*sqrt(5) + 10)) - 1/10*(sqrt(5) + 3)*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^4*(sqrt(5) + 1)) - 1/10*(sqrt(5) - 3)*log(a*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a^4*(sqrt(5) - 1)) + 1/5*log(a + x)/a^4

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^5+x^5),x, algorithm="fricas")**[Out]** Timed out**Sympy [A]**

time = 0.07, size = 39, normalized size = 0.19

$$\frac{\frac{\log(a+x)}{5} + \text{RootSum}(625t^4 + 125t^3 + 25t^2 + 5t + 1, (t \mapsto t \log(5ta + x)))}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a**5+x**5),x)

[Out] (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda(_t, _t*log(5*_t*a + x))))/a**4

Giac [A]

time = 0.00, size = 236, normalized size = 1.17

$$\frac{\ln(x^2 + \frac{1}{2}(a\sqrt{5}-a)x + aa)}{4\sqrt{5}a^4} - \frac{8\sqrt{2(\sqrt{5}+5)}\arctan\left(\frac{-x+\frac{1}{2}(\sqrt{5}-1)a}{\frac{1}{2}\sqrt{2\sqrt{5}+10}a}\right)}{80a^4} + \frac{\ln|x+ a|}{5a^4} - \frac{\ln(x^2 + \frac{1}{2}(-a\sqrt{5}-a)x + aa)}{(4\sqrt{5})a^4} - \frac{8\sqrt{2(-\sqrt{5}+5)}\arctan\left(\frac{-x-\frac{1}{2}(\sqrt{5}+1)a}{\frac{1}{2}\sqrt{-2\sqrt{5}+10}a}\right)}{80a^4} - \frac{\ln|x^4 - x^3a + x^2a^2 - xa^3 + a^4|}{20a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^5+x^5),x)

[Out] $-1/10*\sqrt{2*\sqrt{5} + 10}*\arctan(-(a*(\sqrt{5} - 1) + 4*x)/(a*\sqrt{2*\sqrt{5} + 10} + 10))/a^4 - 1/10*\sqrt{-2*\sqrt{5} + 10}*\arctan((a*(\sqrt{5} + 1) - 4*x)/(a*\sqrt{-2*\sqrt{5} + 10}))/a^4 - 1/20*\sqrt{5}*\log(a^2 - 1/2*(\sqrt{5}*a + a)*x + x^2)/a^4 + 1/20*\sqrt{5}*\log(a^2 + 1/2*(\sqrt{5}*a - a)*x + x^2)/a^4 - 1/20*\log(\text{abs}(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^4 + 1/5*\log(\text{abs}(a + x))/a^4$

Mupad [B]

time = 0.59, size = 174, normalized size = 0.87

$$\frac{\ln(a+x)}{5a^4} - \frac{\ln\left(x - \frac{(\sqrt{5}-\sqrt{2\sqrt{5}-10+1)})}{4}\right)(\sqrt{5}-\sqrt{2\sqrt{5}-10+1})}{20a^4} - \frac{\ln\left(x - \frac{(\sqrt{-2\sqrt{5}-10}-\sqrt{5+1})}{4}\right)(\sqrt{-2\sqrt{5}-10}-\sqrt{5+1})}{20a^4} + \frac{\ln\left(x + \frac{(\sqrt{5}+\sqrt{-2\sqrt{5}-10-1})}{4}\right)(\sqrt{5}+\sqrt{-2\sqrt{5}-10-1})}{20a^4} - \frac{\ln\left(x - \frac{(\sqrt{5}+\sqrt{2\sqrt{5}-10+1})}{4}\right)(\sqrt{5}+\sqrt{2\sqrt{5}-10+1})}{20a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(a^5 + x^5), x)$

[Out] $\log(a + x)/(5*a^4) - (\log(x - (a*(5^{(1/2)} - (2*5^{(1/2)} - 10)^{(1/2)} + 1))/4) * (5^{(1/2)} - (2*5^{(1/2)} - 10)^{(1/2)} + 1))/(20*a^4) - (\log(x - (a*((- 2*5^{(1/2)} - 10)^{(1/2)} - 5^{(1/2)} + 1))/4) * ((- 2*5^{(1/2)} - 10)^{(1/2)} - 5^{(1/2)} + 1)) / (20*a^4) + (\log(x + (a*(5^{(1/2)} + (- 2*5^{(1/2)} - 10)^{(1/2)} - 1))/4) * (5^{(1/2)} + (- 2*5^{(1/2)} - 10)^{(1/2)} - 1)) / (20*a^4) - (\log(x - (a*(5^{(1/2)} + (2*5^{(1/2)} - 10)^{(1/2)} + 1))/4) * (5^{(1/2)} + (2*5^{(1/2)} - 10)^{(1/2)} + 1)) / (20*a^4)$

3.137 $\int \frac{x}{a^5+x^5} dx$

Optimal. Leaf size=201

$$\frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{5a^3} - \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}((1+\sqrt{5})^{a-4x})}{2a}\right)}{5a^3}$$

[Out] $-1/5*\ln(a+x)/a^3+1/20*\ln(a^2+x^2-1/2*a*x*(5^{(1/2)+1}))*(-5^{(1/2)+1})/a^3+1/20$
 $*\ln(a^2+x^2-1/2*a*x*(-5^{(1/2)+1}))*5^{(1/2)+1}/a^3+1/10*\arctan((-4*x+a*(-5^{(1/2)+1}))/a/(10+2*5^{(1/2)})^{(1/2)})*(10-2*5^{(1/2)})^{(1/2)}/a^3-1/10*\arctan(1/20*$
 $(-4*x+a*(5^{(1/2)+1}))*50+10*5^{(1/2)})^{(1/2)}/a*(10+2*5^{(1/2)})^{(1/2)}/a^3$

Rubi [A]

time = 0.18, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {299, 648, 632, 210, 642, 31}

$$\frac{\frac{\log(a+x)}{5a^3} + \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{5a^3} - \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}((1+\sqrt{5})^{a-4x})}{2a}\right)}{5a^3} + \frac{(1+\sqrt{5}) \log(a^2 - \frac{1}{2}(1-\sqrt{5})ax + x^2)}{20a^3} + \frac{(1-\sqrt{5}) \log(a^2 - \frac{1}{2}(1+\sqrt{5})ax + x^2)}{20a^3}}$$

Antiderivative was successfully verified.

[In] Int[x/(a^5 + x^5), x]

[Out] $(\text{Sqrt}[(5 - \text{Sqrt}[5])/2]*\text{ArcTan}[\frac{(1 - \text{Sqrt}[5])*a - 4*x}{(\text{Sqrt}[2*(5 + \text{Sqrt}[5])]*a)]})/(5*a^3) - (\text{Sqrt}[(5 + \text{Sqrt}[5])/2]*\text{ArcTan}[\frac{(\text{Sqrt}[(5 + \text{Sqrt}[5])/10]*((1 + \text{Sqrt}[5])*a - 4*x))/(2*a)]})/(5*a^3) - \text{Log}[a + x]/(5*a^3) + ((1 + \text{Sqrt}[5])* \text{Log}[a^2 - ((1 - \text{Sqrt}[5])*a*x)/2 + x^2])/(20*a^3) + ((1 - \text{Sqrt}[5])* \text{Log}[a^2 - ((1 + \text{Sqrt}[5])*a*x)/2 + x^2])/(20*a^3)$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^{(-1)) * ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])}

Rule 299

Int[(x_)^{(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r * Cos[(2*k}

$- 1)*m*(\text{Pi}/n)] - s*\text{Cos}[(2*k - 1)*(m + 1)*(\text{Pi}/n)]*x)/(r^2 - 2*r*s*\text{Cos}[(2*k - 1)*(\text{Pi}/n)]*x + s^2*x^2), x]; (-(-r)^(m + 1)/(a*n*s^m))*\text{Int}[1/(r + s*x), x] + \text{Dist}[2*(r^(m + 1)/(a*n*s^m)), \text{Sum}[u, \{k, 1, (n - 1)/2\}], x], x]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{IGtQ}[(n - 1)/2, 0] \&\& \text{IGtQ}[m, 0] \&\& \text{LtQ}[m, n - 1] \&\& \text{PosQ}[a/b]$

Rule 632

$\text{Int}[(a_. + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[(d_. + (e_.)*(x_))/((a_. + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 648

$\text{Int}[(d_. + (e_.)*(x_))/((a_. + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rubi steps

$$\begin{aligned} \int \frac{x}{a^5 + x^5} dx &= \frac{2 \int \frac{\frac{1}{4}(1-\sqrt{5})^{a-\frac{1}{4}}(-1-\sqrt{5})^x}{a^2-\frac{1}{2}(1-\sqrt{5})^{ax+x^2}} dx}{5a^3} + \frac{2 \int \frac{\frac{1}{4}(1+\sqrt{5})^{a-\frac{1}{4}}(-1+\sqrt{5})^x}{a^2-\frac{1}{2}(1+\sqrt{5})^{ax+x^2}} dx}{5a^3} - \frac{\int \frac{1}{a+x} dx}{5a^3} \\ &= -\frac{\log(a+x)}{5a^3} + \frac{(1-\sqrt{5}) \int \frac{-\frac{1}{2}(1+\sqrt{5})^{a+2x}}{a^2-\frac{1}{2}(1+\sqrt{5})^{ax+x^2}} dx}{20a^3} + \frac{(1+\sqrt{5}) \int \frac{-\frac{1}{2}(1-\sqrt{5})^{a+2x}}{a^2-\frac{1}{2}(1-\sqrt{5})^{ax+x^2}} dx}{20a^3} \\ &= -\frac{\log(a+x)}{5a^3} + \frac{(1-\sqrt{5}) \log(2a^2 - ax - \sqrt{5}ax + 2x^2)}{20a^3} + \frac{(1+\sqrt{5}) \log(2a^2 - ax + \sqrt{5}ax + 2x^2)}{20a^3} \\ &= \frac{\sqrt{\frac{2}{5(5+\sqrt{5})}} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}a}\right)}{a^3} - \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}}{a}\right)}{5a^3} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 204, normalized size = 1.01

$$\frac{-2\sqrt{10-2\sqrt{5}} \tan^{-1}\left(\frac{(-1+\sqrt{5})a+x}{2(5+\sqrt{5})a}\right) + 2\sqrt{2(5+\sqrt{5})} \tan^{-1}\left(\frac{-((1+\sqrt{5})a)+x}{\sqrt{10-2\sqrt{5}}a}\right) - 4\log(a+x) + \log(a^2 + \frac{1}{2}(-1+\sqrt{5})ax + x^2) + \sqrt{5}\log(a^2 + \frac{1}{2}(-1+\sqrt{5})ax + x^2) + \log(a^2 - \frac{1}{2}(1+\sqrt{5})ax + x^2) - \sqrt{5}\log(a^2 - \frac{1}{2}(1+\sqrt{5})ax + x^2)}{20a^3}$$

Antiderivative was successfully verified.

`[In] Integrate[x/(a^5 + x^5), x]`

```
[Out] (-2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 + Sqrt[5]])*a)] + 2*Sqrt[2*(5 + Sqrt[5]])*ArcTan[(-(1 + Sqrt[5])*a) + 4*x]/(Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2] - Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^3)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.46, size = 51, normalized size = 0.25

$$\frac{\text{RootSum}[1 - 125\#1^3 - 5\#1 + 25\#1^2 + 625\#1^4 \&, \text{Log}[-125a\#1^3 + x] \#1\&] - \frac{\text{Log}[a+x]}{5}}{a^3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x/(a^5 + x^5), x]')`

```
[Out] (RootSum[1 - 125 #1 ^ 3 - 5 #1 + 25 #1 ^ 2 + 625 #1 ^ 4 &, Log[-125 a #1 ^ 3 + x] #1&] - Log[a + x] / 5) / a ^ 3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 0.05, size = 97, normalized size = 0.48

method	result	size
risch	$\frac{\left(\sum_{R=\text{RootOf}(a^{12}Z^4 - a^9Z^3 + a^6Z^2 - a^3Z + 1)} -R \ln(-a^{10} - R^3 + x) \right)}{5} - \frac{\ln(a+x)}{5a^3}$	60
default	$-\frac{\ln(a+x)}{5a^3} + \frac{\sum_{R=\text{RootOf}(Z^4 - aZ^3 + Z^2a^2 - a^3Z + a^4)} \frac{(-R^3 - 2R^2a + 3a^2R + a^3) \ln(-R+x)}{4R^3 - 3R^2a + 2a^2R - a^3}}{5a^3}$	97

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x/(a^5+x^5), x, method=_RETURNVERBOSE)`

```
[Out] -1/5*ln(a+x)/a^3+1/5/a^3*sum((R^3-2*R^2*a+3*R*a^2+a^3)/(4*R^3-3*R^2*a+2*R*a^2-a^3)*ln(-R+x), R=RootOf(Z^4-Z^3*a+Z^2*a^2-Z*a^3+a^4))
```

Maxima [A]

time = 0.36, size = 160, normalized size = 0.80

$$-\frac{2\sqrt{5}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{5a^3\sqrt{2\sqrt{5}+10}} + \frac{2\sqrt{5}\arctan\left(\frac{-a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{5a^3\sqrt{-2\sqrt{5}+10}} - \frac{\log(a+x)}{5a^3} - \frac{\log(-ax(\sqrt{5}+1)+2a^2+2x^2)}{5a^3(\sqrt{5}+1)} + \frac{\log(ax(\sqrt{5}-1)+2a^2+2x^2)}{5a^3(\sqrt{5}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a^5+x^5),x, algorithm="maxima")

[Out] $-2/5*\sqrt{5}*\arctan((a*(\sqrt{5}-1)+4*x)/(a*\sqrt{2*\sqrt{5}+10}))/a^3*\sqrt{2*\sqrt{5}+10} + 2/5*\sqrt{5}*\arctan(-(a*(\sqrt{5}+1)-4*x)/(a*\sqrt{-2*\sqrt{5}+10}))/a^3*\sqrt{-2*\sqrt{5}+10} - 1/5*\log(a+x)/a^3 - 1/5*\log(-a*x*(\sqrt{5}+1)+2*a^2+2*x^2)/(a^3*(\sqrt{5}+1)) + 1/5*\log(a*x*(\sqrt{5}-1)+2*a^2+2*x^2)/(a^3*(\sqrt{5}-1))$

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a^5+x^5),x, algorithm="fricas")**[Out]** Timed out**Sympy [A]**

time = 0.07, size = 41, normalized size = 0.20

$$\frac{-\frac{\log(a+x)}{5} + \text{RootSum}(625t^4 - 125t^3 + 25t^2 - 5t + 1, (t \mapsto t \log(-125t^3a + x)))}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a**5+x**5),x)

[Out] $(-\log(a+x)/5 + \text{RootSum}(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t + 1, \text{Lambd} a(_t, _t*\log(-125*_t**3*a + x))))/a**3$

Giac [A]

time = 0.01, size = 235, normalized size = 1.17

$$\frac{8\sqrt{5}\ln(x^2 + \frac{1}{2}(a\sqrt{5}-a)x + aa)}{160a^3} + \frac{4\sqrt{2(-\sqrt{5}+5)}\arctan\left(\frac{-x+i(\sqrt{5}-1)a}{\frac{1}{2}\sqrt{2\sqrt{5}+10}a}\right)}{40a^3} - \frac{\ln|x+a|}{5a^3} - \frac{8\sqrt{5}\ln(x^2 + \frac{1}{2}(-a\sqrt{5}-a)x + aa)}{160a^3} - \frac{4\sqrt{2(\sqrt{5}+5)}\arctan\left(\frac{-x-i(\sqrt{5}+1)a}{\frac{1}{2}\sqrt{-2\sqrt{5}+10}a}\right)}{40a^3} + \frac{\ln|x^4 - x^3a + x^2a^2 - xa^3 + a^4|}{20a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a^5+x^5),x)

[Out] $\frac{1}{10}\sqrt{-2\sqrt{5} + 10}\arctan\left(\frac{-a(\sqrt{5} - 1) + 4x}{a\sqrt{2\sqrt{5} + 10}}\right) + \frac{1}{10}\sqrt{2\sqrt{5} + 10}\arctan\left(\frac{a(\sqrt{5} + 1) - 4x}{a\sqrt{-2\sqrt{5} + 10}}\right) + \frac{1}{20}\sqrt{5}\log(a^2 - \frac{1}{2}(\sqrt{5}a + a)x + x^2) + \frac{1}{20}\sqrt{5}\log(a^2 + \frac{1}{2}(\sqrt{5}a - a)x + x^2) + \frac{1}{20}\log(\text{abs}(a^4 - a^3x + a^2x^2 - ax^3 + x^4)) - \frac{1}{5}\log(\text{abs}(a + x))$

Mupad [B]

time = 0.59, size = 182, normalized size = 0.91

$$\frac{\ln\left(x - \frac{(\sqrt{5} - \sqrt{2\sqrt{5} - 10 + 1})^2}{64}\right)(\sqrt{5} - \sqrt{2\sqrt{5} - 10} + 1)}{20a^3} - \frac{\ln(a+x)}{5a^3} + \frac{\ln\left(x - \frac{(\sqrt{-2\sqrt{5} - 10} - \sqrt{5} + 1)^2}{64}\right)(\sqrt{-2\sqrt{5} - 10} - \sqrt{5} + 1)}{20a^3} - \frac{\ln\left(x + \frac{(\sqrt{5} + \sqrt{-2\sqrt{5} - 10} - 1)^2}{64}\right)(\sqrt{5} + \sqrt{-2\sqrt{5} - 10} - 1)}{20a^3} + \frac{\ln\left(x - \frac{(\sqrt{5} + \sqrt{2\sqrt{5} - 10} + 1)^2}{64}\right)(\sqrt{5} + \sqrt{2\sqrt{5} - 10} + 1)}{20a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x/(a^5 + x^5), x)$

[Out] $(\log(x - (a(5^{1/2} - (2*5^{1/2} - 10)^{1/2} + 1)^3)/64)*(5^{1/2} - (2*5^{1/2} - 10)^{1/2} + 1))/(20*a^3) - \log(a + x)/(5*a^3) + (\log(x - (a((- 2*5^{1/2} - 10)^{1/2} - 5^{1/2} + 1)^3)/64)*((- 2*5^{1/2} - 10)^{1/2} - 5^{1/2} + 1))/(20*a^3) - (\log(x + (a(5^{1/2} + (- 2*5^{1/2} - 10)^{1/2} - 1)^3)/64)*(5^{1/2} + (- 2*5^{1/2} - 10)^{1/2} - 1))/(20*a^3) + (\log(x - (a(5^{1/2} + (2*5^{1/2} - 10)^{1/2} + 1)^3)/64)*(5^{1/2} + (2*5^{1/2} - 10)^{1/2} + 1))/(20*a^3)$

$$3.138 \quad \int \frac{x^2}{a^5+x^5} dx$$

Optimal. Leaf size=201

$$\frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{5a^2} - \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})} \left((1+\sqrt{5})^{a-4x}\right)}{2a}\right)}{5a^2}$$

[Out] 1/5*ln(a+x)/a^2-1/20*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(-5^(1/2)+1)/a^2-1/20*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(5^(1/2)+1)/a^2+1/10*arctan((-4*x+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)/a^2-1/10*arctan(1/20*(-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10+2*5^(1/2))^(1/2)/a^2

Rubi [A]

time = 0.22, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {299, 648, 632, 210, 642, 31}

$$-\frac{(1+\sqrt{5}) \log\left(a^2 - \frac{1}{2}(1-\sqrt{5})ax + x^2\right)}{20a^2} - \frac{(1-\sqrt{5}) \log\left(a^2 - \frac{1}{2}(1+\sqrt{5})ax + x^2\right)}{20a^2} + \frac{\log(a+x)}{5a^2} + \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{5a^2} - \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})} \left((1+\sqrt{5})^{a-4x}\right)}{2a}\right)}{5a^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a^5 + x^5), x]

[Out] (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5 + Sqrt[5]])*a)])/(5*a^2) - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^2) + Log[a + x]/(5*a^2) - ((1 + Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^2) - ((1 - Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^2)

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

Rule 299

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*cos[(2*k

$- 1) * m * (\text{Pi}/n)] - s * \text{Cos}[(2 * k - 1) * (m + 1) * (\text{Pi}/n)] * x / (r^2 - 2 * r * s * \text{Cos}[(2 * k - 1) * (\text{Pi}/n)] * x + s^2 * x^2), x]; (-r)^{(m + 1)} / (a * n * s^m) * \text{Int}[1 / (r + s * x), x] + \text{Dist}[2 * (r^{(m + 1)} / (a * n * s^m)), \text{Sum}[u, \{k, 1, (n - 1) / 2\}], x], x] /;$ FreeQ[{a, b}, x] && IGtQ[(n - 1) / 2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a / b]

Rule 632

$\text{Int}[(a_.) + (b_.) * (x_.) + (c_.) * (x_.)^2]^{-1}, x_Symbol] := \text{Dist}[-2, \text{Subst}[\text{Int}[1 / \text{Simp}[b^2 - 4 * a * c - x^2, x], x], x, b + 2 * c * x], x] /;$ FreeQ[{a, b, c}, x] && NeQ[b^2 - 4 * a * c, 0]

Rule 642

$\text{Int}[(d_.) + (e_.) * (x_.)] / [(a_.) + (b_.) * (x_.) + (c_.) * (x_.)^2], x_Symbol] := \text{Simp}[d * (\text{Log}[\text{RemoveContent}[a + b * x + c * x^2, x]] / b), x] /;$ FreeQ[{a, b, c, d, e}, x] && EqQ[2 * c * d - b * e, 0]

Rule 648

$\text{Int}[(d_.) + (e_.) * (x_.)] / [(a_.) + (b_.) * (x_.) + (c_.) * (x_.)^2], x_Symbol] := \text{Dist}[(2 * c * d - b * e) / (2 * c), \text{Int}[1 / (a + b * x + c * x^2), x], x] + \text{Dist}[e / (2 * c), \text{Int}[(b + 2 * c * x) / (a + b * x + c * x^2), x], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[2 * c * d - b * e, 0] && NeQ[b^2 - 4 * a * c, 0] && !NiceSqrtQ[b^2 - 4 * a * c]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{a^5 + x^5} dx &= \frac{2 \int \frac{\frac{1}{4}(-1-\sqrt{5})^{a-\frac{1}{4}}(1+\sqrt{5})^x}{a^2-\frac{1}{2}(1-\sqrt{5})^{ax+x^2}} dx}{5a^2} + \frac{2 \int \frac{\frac{1}{4}(-1+\sqrt{5})^{a-\frac{1}{4}}(1-\sqrt{5})^x}{a^2-\frac{1}{2}(1+\sqrt{5})^{ax+x^2}} dx}{5a^2} + \frac{\int \frac{1}{a+x} dx}{5a^2} \\ &= \frac{\log(a+x)}{5a^2} - \frac{(1-\sqrt{5}) \int \frac{-\frac{1}{2}(1+\sqrt{5})^{a+2x}}{a^2-\frac{1}{2}(1+\sqrt{5})^{ax+x^2}} dx}{20a^2} - \frac{(1+\sqrt{5}) \int \frac{-\frac{1}{2}(1-\sqrt{5})^{a+2x}}{a^2-\frac{1}{2}(1-\sqrt{5})^{ax+x^2}} dx}{20a^2} \\ &= \frac{\log(a+x)}{5a^2} - \frac{(1-\sqrt{5}) \log(2a^2 - ax - \sqrt{5} ax + 2x^2)}{20a^2} - \frac{(1+\sqrt{5}) \log(2a^2 - ax + \sqrt{5} ax + 2x^2)}{20a^2} \\ &= \frac{\sqrt{\frac{2}{5(5+\sqrt{5})}} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right) - \sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}}{2a}\right)}{a^2} - \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}}{2a}\right)}{5a^2} \end{aligned}$$

Mathematica [A]

time = 0.04, size = 204, normalized size = 1.01

$$\frac{2\sqrt{10-2\sqrt{5}} \tan^{-1}\left(\frac{(-1+\sqrt{5})a+4x}{\sqrt{2(5+\sqrt{5})}a}\right) - 2\sqrt{2(5+\sqrt{5})} \tan^{-1}\left(\frac{(-1+\sqrt{5})a+4x}{\sqrt{10-2\sqrt{5}}a}\right) - 4\log(a+x) + \log(a^2 + \frac{1}{2}(-1+\sqrt{5})ax + x^2) + \sqrt{5}\log(a^2 + \frac{1}{2}(-1+\sqrt{5})ax + x^2) + \log(a^2 - \frac{1}{2}(1+\sqrt{5})ax + x^2) - \sqrt{5}\log(a^2 - \frac{1}{2}(1+\sqrt{5})ax + x^2)}{20a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a^5 + x^5),x]

[Out]
$$\frac{-1/20*(2*\text{Sqrt}[10 - 2*\text{Sqrt}[5]]*\text{ArcTan}[\frac{((-1 + \text{Sqrt}[5])*a + 4*x)}{(\text{Sqrt}[2*(5 + \text{Sqrt}[5]))*a}] - 2*\text{Sqrt}[2*(5 + \text{Sqrt}[5]))*\text{ArcTan}[\frac{-((1 + \text{Sqrt}[5])*a) + 4*x]}{(\text{Sqrt}[10 - 2*\text{Sqrt}[5]]*a)}] - 4*\text{Log}[a + x] + \text{Log}[a^2 + ((-1 + \text{Sqrt}[5])*a*x)/2 + x^2] + \text{Sqrt}[5]*\text{Log}[a^2 + ((-1 + \text{Sqrt}[5])*a*x)/2 + x^2] + \text{Log}[a^2 - ((1 + \text{Sqrt}[5])*a*x)/2 + x^2] - \text{Sqrt}[5]*\text{Log}[a^2 - ((1 + \text{Sqrt}[5])*a*x)/2 + x^2])}{a^2}$$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.44, size = 51, normalized size = 0.25

$$\frac{\text{RootSum}[1 + 5\#1 + 25\#1^2 + 125\#1^3 + 625\#1^4 \&, \text{Log}[25a\#1^2 + x] \#1\&] + \frac{\text{Log}[a+x]}{5}}{a^2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^2/(a^5 + x^5),x]')

[Out]
$$\frac{(\text{RootSum}[1 + 5\#1 + 25\#1^2 + 125\#1^3 + 625\#1^4 \&, \text{Log}[25a\#1^2 + x] \#1\&] + \text{Log}[a + x] / 5) / a^2}$$

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 0.05, size = 101, normalized size = 0.50

method	result	size
risch	$\frac{\ln(a+x)}{5a^2} + \frac{\left(\sum_{R=\text{RootOf}(a^8-Z^4+a^6-Z^3+a^4-Z^2+a^2-Z+1)} -R \ln(x - R^3 a^5 + 1) \right)}{5}$	58
default	$\frac{\ln(a+x)}{5a^2} + \frac{\sum_{R=\text{RootOf}(-Z^4 - aZ^3 + Z^2 a^2 - a^3 Z + a^4)} \frac{(-R^3 + 2R^2 a + 2a^2 - R - a^3) \ln(-R+x)}{4R^3 - 3R^2 a + 2a^2 - R - a^3}}{5a^2}$	101

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a^5+x^5),x,method=_RETURNVERBOSE)

[Out]
$$\frac{1}{5} \frac{\ln(a+x)}{a^2} + \frac{1}{5} \frac{\sum((-R^3+2R^2a+2R^2a^2-R^2a^3)/(4R^3-3R^2a+2R^2a^2-R^2a^3)*\ln(-R+x), R=\text{RootOf}(-Z^4-Z^3a+Z^2a^2-Za^3+a^4))}{a^2}$$

Maxima [A]

time = 0.36, size = 160, normalized size = 0.80

$$-\frac{2\sqrt{5}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{5a^2\sqrt{2\sqrt{5}+10}} + \frac{2\sqrt{5}\arctan\left(\frac{-a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{5a^2\sqrt{-2\sqrt{5}+10}} + \frac{\log(a+x)}{5a^2} + \frac{\log(-ax(\sqrt{5}+1)+2a^2+2x^2)}{5a^2(\sqrt{5}+1)} - \frac{\log(ax(\sqrt{5}-1)+2a^2+2x^2)}{5a^2(\sqrt{5}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a^5+x^5),x, algorithm="maxima")

[Out] -2/5*sqrt(5)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a^2*sqrt(2*sqrt(5) + 10)) + 2/5*sqrt(5)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/(a^2*sqrt(-2*sqrt(5) + 10)) + 1/5*log(a + x)/a^2 + 1/5*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^2*(sqrt(5) + 1)) - 1/5*log(a*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a^2*(sqrt(5) - 1))

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a^5+x^5),x, algorithm="fricas")**[Out]** Timed out**Sympy [A]**

time = 0.08, size = 41, normalized size = 0.20

$$\frac{\frac{\log(a+x)}{5} + \text{RootSum}(625t^4 + 125t^3 + 25t^2 + 5t + 1, (t \mapsto t \log(25t^2a + x)))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(a**5+x**5),x)

[Out] (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda(_t, _t*log(25*_t**2*a + x))))/a**2

Giac [A]

time = 0.01, size = 235, normalized size = 1.17

$$-\frac{8\sqrt{5}\ln\left(x^2 + \frac{1}{2}(a\sqrt{5}-a)x + aa\right)}{160a^2} + \frac{4\sqrt{2(-\sqrt{5}+5)}\arctan\left(\frac{-x+\frac{1}{2}(\sqrt{5}-1)a}{\frac{1}{2}\sqrt{2\sqrt{5}+10}a}\right)}{40a^2} + \frac{\ln|x+ a|}{5a^2} + \frac{8\sqrt{5}\ln\left(x^2 + \frac{1}{2}(-a\sqrt{5}-a)x + aa\right)}{160a^2} - \frac{4\sqrt{2(\sqrt{5}+5)}\arctan\left(\frac{-x+\frac{1}{2}(\sqrt{5}+1)a}{\frac{1}{2}\sqrt{-2\sqrt{5}+10}a}\right)}{40a^2} - \frac{\ln|x^4 - x^3a + x^2a^2 - xa^3 + a^4|}{20a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a^5+x^5),x)

[Out] $\frac{1}{10}\sqrt{-2\sqrt{5} + 10}\arctan\left(\frac{-a(\sqrt{5} - 1) + 4x}{a\sqrt{2\sqrt{5} + 10}}\right) - \frac{1}{10}\sqrt{2\sqrt{5} + 10}\arctan\left(\frac{a(\sqrt{5} + 1) - 4x}{a\sqrt{-2\sqrt{5} + 10}}\right) + \frac{1}{20}\sqrt{5}\log(a^2 - \frac{1}{2}(\sqrt{5}a + a)x + x^2) - \frac{1}{20}\sqrt{5}\log(a^2 + \frac{1}{2}(\sqrt{5}a - a)x + x^2) - \frac{1}{20}\log(\text{abs}(a^4 - a^3x + a^2x^2 - ax^3 + x^4)) + \frac{1}{5}\log(\text{abs}(a + x))$

Mupad [B]

time = 0.80, size = 202, normalized size = 1.00

$$\frac{\ln\left(\frac{x(\sqrt{5} + \sqrt{-2\sqrt{5} - 10})}{a}\right)}{5a^2} + \frac{\ln\left(\frac{x(\sqrt{5} + \sqrt{-2\sqrt{5} - 10})}{a}\right)}{20a^2} (\sqrt{5} + \sqrt{-2\sqrt{5} - 10} - 1) - \frac{\ln\left(\frac{x(\sqrt{5} + \sqrt{-2\sqrt{5} - 10})}{a}\right)}{20a^2} (\sqrt{5} + \sqrt{-2\sqrt{5} - 10} + 1) - \frac{\ln\left(\frac{x(\sqrt{5} - \sqrt{-2\sqrt{5} - 10})}{a}\right)}{20a^2} (\sqrt{5} - \sqrt{-2\sqrt{5} - 10} + 1) - \frac{\ln\left(\frac{x(\sqrt{5} - \sqrt{-2\sqrt{5} - 10})}{a}\right)}{20a^2} (\sqrt{-2\sqrt{5} - 10} - \sqrt{5} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2/(a^5 + x^5), x)$

[Out] $\log(a + x)/(5a^2) + (\log(a^5 + (a^4x(5^{1/2} + (-2*5^{1/2} - 10)^{1/2} - 1)^3)/64)*(5^{1/2} + (-2*5^{1/2} - 10)^{1/2} - 1))/(20a^2) - (\log(a^5 - (a^4x(5^{1/2} + (2*5^{1/2} - 10)^{1/2} + 1)^3)/64)*(5^{1/2} + (2*5^{1/2} - 10)^{1/2} + 1))/(20a^2) - (\log(a^5 - (a^4x(5^{1/2} - (2*5^{1/2} - 10)^{1/2} - 10)^{1/2} + 1)^3)/64)*(5^{1/2} - (2*5^{1/2} - 10)^{1/2} + 1))/(20a^2) - (\log(a^5 - (a^4x((-2*5^{1/2} - 10)^{1/2} - 5^{1/2} + 1)^3)/64)*((-2*5^{1/2} - 10)^{1/2} - 5^{1/2} + 1))/(20a^2)$

3.139 $\int \frac{x^3}{a^5+x^5} dx$

Optimal. Leaf size=201

$$\frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{5a} - \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}((1+\sqrt{5})^{a-4x})}{2a}\right)}{5a}$$

[Out] $-1/5*\ln(a+x)/a+1/20*\ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(-5^(1/2)+1)/a+1/20*\ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(5^(1/2)+1)/a-1/10*\arctan(1/20*(-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10-2*5^(1/2))^(1/2)/a-1/10*\arctan((-4*x+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)/a$

Rubi [A]

time = 0.22, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {299, 648, 632, 210, 642, 31}

$$\frac{(1-\sqrt{5}) \log\left(a^2 - \frac{1}{2}(1-\sqrt{5})ax + x^2\right)}{20a} + \frac{(1+\sqrt{5}) \log\left(a^2 - \frac{1}{2}(1+\sqrt{5})ax + x^2\right)}{20a} - \frac{\log(a+x)}{5a} - \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{5a} - \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}((1+\sqrt{5})^{a-4x})}{2a}\right)}{5a}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3/(a^5 + x^5), x]$

[Out] $-1/5*(\text{Sqrt}[(5 + \text{Sqrt}[5])/2]*\text{ArcTan}[\frac{(1 - \text{Sqrt}[5])*a - 4*x}{\text{Sqrt}[2*(5 + \text{Sqrt}[5])*a]})]/a - (\text{Sqrt}[(5 - \text{Sqrt}[5])/2]*\text{ArcTan}[\frac{(\text{Sqrt}[(5 + \text{Sqrt}[5])/10]*((1 + \text{Sqrt}[5])*a - 4*x))/(2*a)}])/(5*a) - \text{Log}[a + x]/(5*a) + ((1 - \text{Sqrt}[5])*\text{Log}[a^2 - ((1 - \text{Sqrt}[5])*a*x)/2 + x^2])/(20*a) + ((1 + \text{Sqrt}[5])*\text{Log}[a^2 - ((1 + \text{Sqrt}[5])*a*x)/2 + x^2])/(20*a)$

Rule 31

$\text{Int}[(a_ + (b_)*(x_))^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /;$ FreeQ[{a, b}, x]

Rule 210

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 299

$\text{Int}[(x_)^{(m_)} / ((a_ + (b_)*(x_)^n)), x_Symbol] \rightarrow \text{Module}\{r = \text{Numerator}[\text{Rt}[a/b, n]], s = \text{Denominator}[\text{Rt}[a/b, n]], k, u\}, \text{Simp}[u = \text{Int}[(r*\text{Cos}[(2*k$

$- 1)*m*(\text{Pi}/n)] - s*\text{Cos}[(2*k - 1)*(m + 1)*(\text{Pi}/n)]*x)/(r^2 - 2*r*s*\text{Cos}[(2*k - 1)*(\text{Pi}/n)]*x + s^2*x^2), x]; (-(-r)^(m + 1)/(a*n*s^m))*\text{Int}[1/(r + s*x), x] + \text{Dist}[2*(r^(m + 1)/(a*n*s^m)), \text{Sum}[u, \{k, 1, (n - 1)/2\}], x], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{IGtQ}[(n - 1)/2, 0] \&\& \text{IGtQ}[m, 0] \&\& \text{LtQ}[m, n - 1] \&\& \text{PosQ}[a/b]$

Rule 632

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[(d_.) + (e_.)*(x_)]/[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 648

$\text{Int}[(d_.) + (e_.)*(x_)]/[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rubi steps

$$\begin{aligned} \int \frac{x^3}{a^5 + x^5} dx &= \frac{2 \int \frac{\frac{1}{4}(1+\sqrt{5})^{a-\frac{1}{4}}(-1+\sqrt{5})^x}{a^{2-\frac{1}{2}}(1-\sqrt{5})^{ax+x^2}} dx}{5a} + \frac{2 \int \frac{\frac{1}{4}(1-\sqrt{5})^{a-\frac{1}{4}}(-1-\sqrt{5})^x}{a^{2-\frac{1}{2}}(1+\sqrt{5})^{ax+x^2}} dx}{5a} - \frac{\int \frac{1}{a+x} dx}{5a} \\ &= -\frac{\log(a+x)}{5a} + \frac{1}{20}(5-\sqrt{5}) \int \frac{1}{a^{2-\frac{1}{2}}(1+\sqrt{5})^{ax+x^2}} dx + \frac{1}{20}(5+\sqrt{5}) \int \frac{1}{a^{2-\frac{1}{2}}(1-\sqrt{5})^{ax+x^2}} dx \\ &= -\frac{\log(a+x)}{5a} + \frac{(1+\sqrt{5}) \log(2a^2 - ax - \sqrt{5}ax + 2x^2)}{20a} + \frac{(1-\sqrt{5}) \log(2a^2 - ax + \sqrt{5}ax + 2x^2)}{20a} \\ &= -\frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{5a} - \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}}{\sqrt{2(5-\sqrt{5})}^a}\right)}{5a} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 204, normalized size = 1.01

$$\frac{2\sqrt{2(5+\sqrt{5})} \tan^{-1}\left(\frac{(-1+\sqrt{5})a+4x}{\sqrt{2(5+\sqrt{5})}a}\right) + 2\sqrt{10-2\sqrt{5}} \tan^{-1}\left(\frac{-((1+\sqrt{5})a)+4x}{\sqrt{10-2\sqrt{5}}a}\right) - 4\log(a+x) + \log\left(a^2 + \frac{1}{2}(-1+\sqrt{5})ax + x^2\right) - \sqrt{5} \log\left(a^2 + \frac{1}{2}(-1+\sqrt{5})ax + x^2\right) + \log\left(a^2 - \frac{1}{2}(1+\sqrt{5})ax + x^2\right) + \sqrt{5} \log\left(a^2 - \frac{1}{2}(1+\sqrt{5})ax + x^2\right)}{20a}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a^5 + x^5), x]

[Out] (2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-1 + Sqrt[5])*a + 4*x]/(Sqrt[2*(5 + Sqrt[5])]*a) + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[-((1 + Sqrt[5])*a) + 4*x]/(Sqrt[10 - 2*Sqrt[5]]*a) - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] - Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2] + Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.36, size = 51, normalized size = 0.25

$$\frac{\text{RootSum}\left[1 - 125\#1^3 - 5\#1 + 25\#1^2 + 625\#1^4 \&, \text{Log}\left[625a\#1^4 + x\right] \#1\&\right] - \frac{\text{Log}[a+x]}{5}}{a}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^3/(a^5 + x^5), x]')

[Out] (RootSum[1 - 125 #1 ^ 3 - 5 #1 + 25 #1 ^ 2 + 625 #1 ^ 4 &, Log[625 a #1 ^ 4 + x] #1&] - Log[a + x] / 5) / a

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 0.05, size = 97, normalized size = 0.48

method	result	size
risch	$-\frac{\ln(a+x)}{5a} + \frac{\left(\sum_{R=\text{RootOf}(a^4 - Z^4 - a^3 - Z^3 + Z^2 a^2 - a - Z + 1)} -R \ln(-R^3 a^4 - R^2 a^3 + a^2 - R - a + x)\right)}{5}$	73
default	$-\frac{\ln(a+x)}{5a} + \frac{\sum_{R=\text{RootOf}(Z^4 - a - Z^3 + Z^2 a^2 - a^3 - Z + a^4)} \left(\frac{(-R^3 + 3R^2 a - 2a^2 - R + a^3) \ln(-R + x)}{4R^3 - 3R^2 a + 2a^2 - R - a^3}\right)}{5a}$	97

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a^5+x^5), x, method=_RETURNVERBOSE)

[Out] -1/5*ln(a+x)/a+1/5/a*sum((R^3+3*R^2*a-2*R*a^2+a^3)/(4*R^3-3*R^2*a+2*R*a^2-a^3)*ln(-R+x), R=RootOf(Z^4-Z^3*a+Z^2*a^2-Z*a^3+a^4))

Maxima [A]

time = 0.35, size = 180, normalized size = 0.90

$$\frac{\sqrt{5}(\sqrt{5}+1)\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{5a\sqrt{2\sqrt{5}+10}} + \frac{\sqrt{5}(\sqrt{5}-1)\arctan\left(\frac{-a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{5a\sqrt{-2\sqrt{5}+10}} + \frac{(\sqrt{5}+3)\log(-ax(\sqrt{5}+1)+2a^2+2x^2)}{10a(\sqrt{5}+1)} + \frac{(\sqrt{5}-3)\log(ax(\sqrt{5}-1)+2a^2+2x^2)}{10a(\sqrt{5}-1)} - \frac{\log(a+x)}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a^5+x^5),x, algorithm="maxima")

[Out] 1/5*sqrt(5)*(sqrt(5) + 1)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a*sqrt(2*sqrt(5) + 10)) + 1/5*sqrt(5)*(sqrt(5) - 1)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/(a*sqrt(-2*sqrt(5) + 10)) + 1/10*(sqrt(5) + 3)*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a*(sqrt(5) + 1)) + 1/10*(sqrt(5) - 3)*log(a*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a*(sqrt(5) - 1)) - 1/5*log(a + x)/a

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a^5+x^5),x, algorithm="fricas")**[Out]** Timed out**Sympy [A]**

time = 0.07, size = 39, normalized size = 0.19

$$\frac{-\frac{\log(a+x)}{5} + \text{RootSum}(625t^4 - 125t^3 + 25t^2 - 5t + 1, (t \mapsto t \log(625t^4 a + x)))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(a**5+x**5),x)

[Out] (-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t + 1, Lambda(a(_t, _t*log(625*_t**4*a + x)))))/a

Giac [A]

time = 0.01, size = 224, normalized size = 1.11

$$\frac{8\sqrt{5}\ln\left(x^2 + \frac{1}{2}(a\sqrt{5}-a)x + aa\right)}{160a} - \frac{8\sqrt{2(\sqrt{5}+5)}\arctan\left(\frac{-x+\frac{1}{2}(\sqrt{5}-1)a}{\frac{1}{2}\sqrt{2\sqrt{5}+10}a}\right)}{80a} - \frac{\ln|x+a|}{5a} + \frac{8\sqrt{5}\ln\left(x^2 + \frac{1}{2}(-a\sqrt{5}-a)x + aa\right)}{160a} - \frac{8\sqrt{2(-\sqrt{5}+5)}\arctan\left(\frac{-x-\frac{1}{2}(\sqrt{5}+1)a}{\frac{1}{2}\sqrt{-2\sqrt{5}+10}a}\right)}{80a} + \frac{\ln|x^4 - x^3a + x^2a^2 - xa^3 + a^4|}{20a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a^5+x^5),x)

[Out] $-1/10*\sqrt{2*\sqrt{5} + 10}*\arctan(-(a*(\sqrt{5} - 1) + 4*x)/(a*\sqrt{2*\sqrt{5} + 10} + 10))/a - 1/10*\sqrt{-2*\sqrt{5} + 10}*\arctan((a*(\sqrt{5} + 1) - 4*x)/(a*\sqrt{-2*\sqrt{5} + 10}))/a + 1/20*\sqrt{5}*\log(a^2 - 1/2*(\sqrt{5}*a + a)*x + x^2)/a - 1/20*\sqrt{5}*\log(a^2 + 1/2*(\sqrt{5}*a - a)*x + x^2)/a + 1/20*\log(\text{abs}(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a - 1/5*\log(\text{abs}(a + x))/a$

Mupad [B]

time = 0.41, size = 202, normalized size = 1.00

$$\frac{\ln\left(\frac{5a^{10} - \frac{1}{4}(\sqrt{5} + \sqrt{2\sqrt{5} - 10})^2}{20a}\right)(\sqrt{5} + \sqrt{2\sqrt{5} - 10} + 1) - \ln\left(\frac{5a^{10} + \frac{1}{4}(\sqrt{5} + \sqrt{2\sqrt{5} - 10})^2}{20a}\right)(\sqrt{5} + \sqrt{2\sqrt{5} - 10} - 1) - \frac{\ln(a+x)}{5a} + \ln\left(\frac{5a^{10} - \frac{1}{4}(\sqrt{5} - \sqrt{2\sqrt{5} - 10})^2}{20a}\right)(\sqrt{5} - \sqrt{2\sqrt{5} - 10} + 1) + \ln\left(\frac{5a^{10} - \frac{1}{4}(\sqrt{5} - \sqrt{2\sqrt{5} - 10})^2}{20a}\right)(\sqrt{5} - \sqrt{2\sqrt{5} - 10} - 1)}{20a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^3/(a^5 + x^5), x)$

[Out] $(\log(5*a^{10} - (5*a^9*x*(5^{(1/2)} + (2*5^{(1/2)} - 10)^{(1/2)} + 1))/4)*(5^{(1/2)} + (2*5^{(1/2)} - 10)^{(1/2)} + 1))/(20*a) - (\log(5*a^{10} + (5*a^9*x*(5^{(1/2)} + (-2*5^{(1/2)} - 10)^{(1/2)} - 1))/4)*(5^{(1/2)} + (-2*5^{(1/2)} - 10)^{(1/2)} - 1))/(20*a) - \log(a + x)/(5*a) + (\log(5*a^{10} - (5*a^9*x*(5^{(1/2)} - (2*5^{(1/2)} - 10)^{(1/2)} + 1))/4)*(5^{(1/2)} - (2*5^{(1/2)} - 10)^{(1/2)} + 1))/(20*a) + (\log(5*a^{10} - (5*a^9*x*((-2*5^{(1/2)} - 10)^{(1/2)} - 5^{(1/2)} + 1))/4)*((-2*5^{(1/2)} - 10)^{(1/2)} - 5^{(1/2)} + 1))/(20*a)$

$$3.140 \quad \int \frac{x^4}{a^5+x^5} dx$$

Optimal. Leaf size=12

$$\frac{1}{5} \log(a^5 + x^5)$$

[Out] 1/5*ln(a^5+x^5)

Rubi [A]

time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {266}

$$\frac{1}{5} \log(a^5 + x^5)$$

Antiderivative was successfully verified.

[In] Int[x^4/(a^5 + x^5),x]

[Out] Log[a^5 + x^5]/5

Rule 266

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\int \frac{x^4}{a^5+x^5} dx = \frac{1}{5} \log(a^5 + x^5)$$

Mathematica [A]

time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{5} \log(a^5 + x^5)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a^5 + x^5),x]

[Out] Log[a^5 + x^5]/5

Mathics [A]

time = 2.47, size = 10, normalized size = 0.83

$$\frac{\text{Log}[a^5 + x^5]}{5}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x^4/(a^5 + x^5),x]')`

[Out] `Log[a ^ 5 + x ^ 5] / 5`

Maple [A]

time = 0.05, size = 11, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\ln(a^5+x^5)}{5}$	11
default	$\frac{\ln(a^5+x^5)}{5}$	11
risch	$\frac{\ln(a^5+x^5)}{5}$	11
norman	$\frac{\ln(a+x)}{5} + \frac{\ln(a^4-a^3x+a^2x^2-ax^3+x^4)}{5}$	37

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(a^5+x^5),x,method=_RETURNVERBOSE)`

[Out] `1/5*ln(a^5+x^5)`

Maxima [A]

time = 0.26, size = 10, normalized size = 0.83

$$\frac{1}{5} \log(a^5 + x^5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(a^5+x^5),x, algorithm="maxima")`

[Out] `1/5*log(a^5 + x^5)`

Fricas [A]

time = 0.32, size = 10, normalized size = 0.83

$$\frac{1}{5} \log(a^5 + x^5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(a^5+x^5),x, algorithm="fricas")`

[Out] `1/5*log(a^5 + x^5)`

Sympy [A]

time = 0.05, size = 8, normalized size = 0.67

$$\frac{\log(a^5 + x^5)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(a**5+x**5),x)`

[Out] `log(a**5 + x**5)/5`

Giac [A]

time = 0.00, size = 12, normalized size = 1.00

$$\frac{\ln |x^5 + a^5|}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(a^5+x^5),x)`

[Out] `1/5*log(abs(a^5 + x^5))`

Mupad [B]

time = 0.18, size = 10, normalized size = 0.83

$$\frac{\ln (a^5 + x^5)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(a^5 + x^5),x)`

[Out] `log(a^5 + x^5)/5`

$$3.141 \quad \int \frac{1}{x(a^5+x^5)} dx$$

Optimal. Leaf size=22

$$\frac{\log(x)}{a^5} - \frac{\log(a^5+x^5)}{5a^5}$$

[Out] $\ln(x)/a^5 - 1/5 * \ln(a^5+x^5)/a^5$

Rubi [A]

time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {272, 36, 29, 31}

$$\frac{\log(x)}{a^5} - \frac{\log(a^5+x^5)}{5a^5}$$

Antiderivative was successfully verified.

[In] `Int[1/(x*(a^5 + x^5)),x]`

[Out] `Log[x]/a^5 - Log[a^5 + x^5]/(5*a^5)`

Rule 29

`Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]`

Rule 31

`Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rule 36

`Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]`

Rule 272

`Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]`

Rubi steps

$$\begin{aligned} \int \frac{1}{x(a^5 + x^5)} dx &= \frac{1}{5} \text{Subst} \left(\int \frac{1}{x(a^5 + x)} dx, x, x^5 \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{x} dx, x, x^5 \right)}{5a^5} - \frac{\text{Subst} \left(\int \frac{1}{a^5+x} dx, x, x^5 \right)}{5a^5} \\ &= \frac{\log(x)}{a^5} - \frac{\log(a^5 + x^5)}{5a^5} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a^5} - \frac{\log(a^5 + x^5)}{5a^5}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x*(a^5 + x^5)),x]``[Out] Log[x]/a^5 - Log[a^5 + x^5]/(5*a^5)`**Mathics [A]**

time = 1.93, size = 17, normalized size = 0.77

$$\frac{\text{Log}[x] - \frac{\text{Log}[a^5+x^5]}{5}}{a^5}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x*(a^5 + x^5)),x]')``[Out] (Log[x] - Log[a ^ 5 + x ^ 5] / 5) / a ^ 5`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.

time = 0.14, size = 49, normalized size = 2.23

method	result	size
risch	$\frac{\ln(x)}{a^5} - \frac{\ln(a^5+x^5)}{5a^5}$	21
default	$\frac{\ln(x)}{a^5} - \frac{\ln(a+x)}{5a^5} - \frac{\ln(a^4-a^3x+a^2x^2-ax^3+x^4)}{5a^5}$	49
norman	$\frac{\ln(x)}{a^5} - \frac{\ln(a+x)}{5a^5} - \frac{\ln(a^4-a^3x+a^2x^2-ax^3+x^4)}{5a^5}$	49

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/x/(a^5+x^5),x,method=_RETURNVERBOSE)`

[Out] $\ln(x)/a^5 - 1/5 * \ln(a+x)/a^5 - 1/5/a^5 * \ln(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4)$

Maxima [A]

time = 0.31, size = 23, normalized size = 1.05

$$-\frac{\log(a^5 + x^5)}{5a^5} + \frac{\log(x^5)}{5a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^5+x^5),x, algorithm="maxima")`

[Out] $-1/5 * \log(a^5 + x^5)/a^5 + 1/5 * \log(x^5)/a^5$

Fricas [A]

time = 0.32, size = 18, normalized size = 0.82

$$-\frac{\log(a^5 + x^5) - 5 \log(x)}{5a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^5+x^5),x, algorithm="fricas")`

[Out] $-1/5 * (\log(a^5 + x^5) - 5 * \log(x))/a^5$

Sympy [A]

time = 0.13, size = 19, normalized size = 0.86

$$\frac{\log(x)}{a^5} - \frac{\log(a^5 + x^5)}{5a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a**5+x**5),x)`

[Out] $\log(x)/a^{**5} - \log(a^{**5} + x^{**5})/(5*a^{**5})$

Giac [A]

time = 0.00, size = 29, normalized size = 1.32

$$\frac{5 \ln|x|}{5a^5} - \frac{\ln|x^5 + a^5|}{5a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^5+x^5),x)`

[Out] $-1/5 * \log(\text{abs}(a^5 + x^5))/a^5 + \log(\text{abs}(x))/a^5$

Mupad [B]

time = 0.27, size = 18, normalized size = 0.82

$$-\frac{\ln(a^5 + x^5) - 5 \ln(x)}{5a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^5 + x^5)),x)
```

```
[Out] -(log(a^5 + x^5) - 5*log(x))/(5*a^5)
```

3.142 $\int \frac{1}{x^2(a^5+x^5)} dx$

Optimal. Leaf size=209

$$-\frac{1}{a^5 x} + \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{5a^6} + \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})} \left(\frac{(1+\sqrt{5})^a}{2a}\right)}{5a^6}\right)}{5a^6}$$

[Out] $-1/a^5/x + 1/5*\ln(a+x)/a^6 - 1/20*\ln(a^2+x^2 - 1/2*a*x*(-5^(1/2)+1))*(-5^(1/2)+1)/a^6 - 1/20*\ln(a^2+x^2 - 1/2*a*x*(5^(1/2)+1))*(5^(1/2)+1)/a^6 + 1/10*\arctan(1/20*(-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10-2*5^(1/2))^(1/2)/a^6 + 1/10*\arctan((-4*x+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)/a^6$

Rubi [A]

time = 0.23, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$,

Rules used = {331, 299, 648, 632, 210, 642, 31}

$$\frac{\log(a+x)}{5a^6} + \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{5a^6} + \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})} \left(\frac{(1+\sqrt{5})^{a-4x}}{2a}\right)}{5a^6}\right)}{5a^6} - \frac{1}{a^5 x} - \frac{(1-\sqrt{5}) \log(a^2 - \frac{1}{2}(1-\sqrt{5})ax + x^2)}{20a^6} - \frac{(1+\sqrt{5}) \log(a^2 - \frac{1}{2}(1+\sqrt{5})ax + x^2)}{20a^6}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a^5 + x^5)),x]

[Out] $-(1/(a^5*x)) + (\text{Sqrt}[(5 + \text{Sqrt}[5])/2]*\text{ArcTan}[\frac{((1 - \text{Sqrt}[5])*a - 4*x)}{(\text{Sqrt}[2*(5 + \text{Sqrt}[5]])*a)}])/5*a^6 + (\text{Sqrt}[(5 - \text{Sqrt}[5])/2]*\text{ArcTan}[\frac{(\text{Sqrt}[(5 + \text{Sqrt}[5])/10]*((1 + \text{Sqrt}[5])*a - 4*x))}{(2*a)}])/5*a^6 + \text{Log}[a + x]/(5*a^6) - ((1 - \text{Sqrt}[5])*Log[a^2 - ((1 - \text{Sqrt}[5])*a*x)/2 + x^2])/(20*a^6) - ((1 + \text{Sqrt}[5])*Log[a^2 - ((1 + \text{Sqrt}[5])*a*x)/2 + x^2])/(20*a^6)$

Rule 31

Int[((a_) + (b_.)*(x_))^(−1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(−1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(−1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

Rule 299

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*cos[(2*k
- 1)*m*(Pi/n)] - s*cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*cos[(2*k -
1)*(Pi/n)]*x + s^2*x^2), x]; (-r)^(m + 1)/(a*n*s^m)*Int[1/(r + s*x), x]
+ Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 1)/2}], x], x] /; Free
Q[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a
/b]
```

Rule 331

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p +
1) + 1)/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 632

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int
[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (a^5 + x^5)} dx &= -\frac{1}{a^5 x} - \frac{\int \frac{x^3}{a^5 + x^5} dx}{a^5} \\
&= -\frac{1}{a^5 x} + \frac{\int \frac{1}{a+x} dx}{5a^6} - \frac{2 \int \frac{\frac{1}{4}(1+\sqrt{5})^{a-\frac{1}{4}}(-1+\sqrt{5})^x}{a^2-\frac{1}{2}(1-\sqrt{5})^{ax+x^2}} dx}{5a^6} - \frac{2 \int \frac{\frac{1}{4}(1-\sqrt{5})^{a-\frac{1}{4}}(-1-\sqrt{5})^x}{a^2-\frac{1}{2}(1+\sqrt{5})^{ax+x^2}} dx}{5a^6} \\
&= -\frac{1}{a^5 x} + \frac{\log(a+x)}{5a^6} - \frac{(1-\sqrt{5}) \int \frac{-\frac{1}{2}(1-\sqrt{5})^{a+2x}}{a^2-\frac{1}{2}(1-\sqrt{5})^{ax+x^2}} dx}{20a^6} - \frac{(1+\sqrt{5}) \int \frac{-\frac{1}{2}(1+\sqrt{5})^{a+2x}}{a^2-\frac{1}{2}(1+\sqrt{5})^{ax+x^2}} dx}{20a^6} \\
&= -\frac{1}{a^5 x} + \frac{\log(a+x)}{5a^6} - \frac{(1+\sqrt{5}) \log(2a^2 - ax - \sqrt{5} ax + 2x^2)}{20a^6} - \frac{(1-\sqrt{5}) \log(2a^2 - ax + \sqrt{5} ax + 2x^2)}{20a^6} \\
&= -\frac{1}{a^5 x} + \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right) + \sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{5a^6}
\end{aligned}$$

Mathematica [A]

time = 0.09, size = 172, normalized size = 0.82

$$\frac{\frac{20a}{x} + 2\sqrt{2(5+\sqrt{5})} \tan^{-1}\left(\frac{(-1+\sqrt{5})^{a+4x}}{\sqrt{2(5+\sqrt{5})}^a}\right) + 2\sqrt{10-2\sqrt{5}} \tan^{-1}\left(\frac{-((1+\sqrt{5})^a)^{4x}}{\sqrt{10-2\sqrt{5}}^a}\right) - 4\log(a+x) - (-1+\sqrt{5}) \log(a^2 + \frac{1}{2}(-1+\sqrt{5})ax + x^2) + (1+\sqrt{5}) \log(a^2 - \frac{1}{2}(1+\sqrt{5})ax + x^2)}{20a^6}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x^2*(a^5 + x^5)),x]`

```
[Out] -1/20*((20*a)/x + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a)] + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] - (-1 + Sqrt[5])*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + (1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/a^6
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.56, size = 61, normalized size = 0.29

$$-a + \frac{x(\text{Log}[a+x] + 5\text{RootSum}[1+5\#1+25\#1^2+125\#1^3+625\#1^4 \&, \text{Log}[625a\#1^4+x]\#1\&])}{a^6 x}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(x^2*(a^5 + x^5)),x]')`

[Out] $(-a + x (\text{Log}[a + x] + 5 \text{RootSum}[1 + 5 \#1 + 25 \#1^2 + 125 \#1^3 + 625 \#1^4], \text{Log}[625 a \#1^4 + x] \#1]) / 5) / (a^6 x)$

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 109, normalized size = 0.52

method	result	size
risch	$-\frac{1}{a^5 x} + \frac{\ln(a+x)}{5a^6} + \frac{\left(\sum_{R=\text{RootOf}(a^{24} Z^4 + a^{18} Z^3 + a^{12} Z^2 + a^6 Z + 1)} -R \ln\left(\left(6 - R^5 a^{30} - 5\right)x + a^{25} - R^4\right)\right)}{5}$	76
default	$-\frac{1}{a^5 x} + \frac{\ln(a+x)}{5a^6} + \frac{\sum_{R=\text{RootOf}(-Z^4 - a Z^3 + Z^2 a^2 - a^3 Z + a^4)} \frac{\left(-R^3 - 3R^2 a + 2a^2 R - a^3\right) \ln(-R+x)}{4R^3 - 3R^2 a + 2a^2 R - a^3}}{5a^6}$	109

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(a^5+x^5),x,method=_RETURNVERBOSE)`

[Out] $-1/a^5/x + 1/5 * \ln(a+x)/a^6 + 1/5/a^6 * \text{sum}\left(\frac{(-R^3 - 3R^2 a + 2R a^2 - a^3)}{(4R^3 - 3R^2 a + 2R a^2 - a^3)} * \ln(-R+x), R=\text{RootOf}(Z^4 - Z^3 a + Z^2 a^2 - Z a^3 + a^4)\right)$

Maxima [A]

time = 0.34, size = 192, normalized size = 0.92

$$\frac{2\sqrt{5}(\sqrt{5}+1) \arctan\left(\frac{a(\sqrt{5}-1)+x}{a\sqrt{2\sqrt{5}+10}}\right) + 2\sqrt{5}(\sqrt{5}-1) \arctan\left(\frac{a(\sqrt{5}+1)-x}{a\sqrt{-2\sqrt{5}+10}}\right) + \frac{(\sqrt{5}+3) \log(-ax(\sqrt{5}+1)+2a^2+2x^2)}{a(\sqrt{5}+1)} + \frac{(\sqrt{5}-3) \log(ax(\sqrt{5}-1)+2a^2+2x^2)}{a(\sqrt{5}-1)} - \frac{2 \log(a+x)}{a}}{10a^5} - \frac{1}{a^5 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(a^5+x^5),x, algorithm="maxima")`

[Out] $-1/10 * (2 * \sqrt{5} * (\sqrt{5} + 1) * \arctan((a * (\sqrt{5} - 1) + 4 * x) / (a * \sqrt{2 * \sqrt{5} + 10}))) / (a * \sqrt{2 * \sqrt{5} + 10}) + 2 * \sqrt{5} * (\sqrt{5} - 1) * \arctan(-(a * (\sqrt{5} + 1) - 4 * x) / (a * \sqrt{-2 * \sqrt{5} + 10}))) / (a * \sqrt{-2 * \sqrt{5} + 10}) + (\sqrt{5} + 3) * \log(-a * x * (\sqrt{5} + 1) + 2 * a^2 + 2 * x^2) / (a * (\sqrt{5} + 1)) + (\sqrt{5} - 3) * \log(a * x * (\sqrt{5} - 1) + 2 * a^2 + 2 * x^2) / (a * (\sqrt{5} - 1)) - 2 * \log(a + x) / a / a^5 - 1 / (a^5 * x)$

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a^5+x^5),x, algorithm="fricas")

[Out] Timed out

Sympy [A]

time = 0.09, size = 48, normalized size = 0.23

$$-\frac{1}{a^5 x} + \frac{\log\left(\frac{a+x}{5}\right) + \text{RootSum}\left(625t^4 + 125t^3 + 25t^2 + 5t + 1, (t \mapsto t \log(625t^4 a + x))\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(a**5+x**5),x)

[Out] -1/(a**5*x) + (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda(_t, _t*log(625*_t**4*a + x))))/a**6

Giac [A]

time = 0.01, size = 241, normalized size = 1.15

$$\frac{8\sqrt{5} \ln\left(x^2 + \frac{1}{2}(a\sqrt{5} - a)x + aa\right)}{160a^6} + \frac{8\sqrt{2(\sqrt{5}+5)} \arctan\left(\frac{x + \frac{1}{2}(\sqrt{5}-1)a}{\sqrt{2\sqrt{5}+10}a}\right)}{80a^6} + \frac{\ln|x+a|}{5a^6} - \frac{8\sqrt{5} \ln\left(x^2 + \frac{1}{2}(-a\sqrt{5} - a)x + aa\right)}{160a^6} + \frac{8\sqrt{2(-\sqrt{5}+5)} \arctan\left(\frac{x - \frac{1}{2}(\sqrt{5}+1)a}{\sqrt{-2\sqrt{5}+10}a}\right)}{80a^6} - \frac{\ln|x^2 - x^2a + x^2a^2 - xa^3 + a^4|}{20a^6} - \frac{1}{a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a^5+x^5),x)

[Out] 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/a^6 + 1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/a^6 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*x + x^2)/a^6 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^6 - 1/20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^6 + 1/5*log(abs(a + x))/a^6 - 1/(a^5*x)

Mupad [B]

time = 0.39, size = 210, normalized size = 1.00

$$\frac{\ln\left(\frac{a+x}{5a^6}\right) - \frac{1}{a^2 x} + \frac{\ln\left(5a^6 + \frac{5a^6(\sqrt{5} + \sqrt{-2\sqrt{5}-10})}{4}\right) (\sqrt{5} + \sqrt{-2\sqrt{5}-10})}{20a^6} - \frac{\ln\left(5a^6 - \frac{5a^6(\sqrt{5} + \sqrt{-2\sqrt{5}-10})}{4}\right) (\sqrt{5} + \sqrt{-2\sqrt{5}-10})}{20a^6} - \frac{\ln\left(5a^6 - \frac{5a^6(\sqrt{5} - \sqrt{-2\sqrt{5}-10})}{4}\right) (\sqrt{5} - \sqrt{-2\sqrt{5}-10})}{20a^6} - \frac{\ln\left(5a^6 + \frac{5a^6(\sqrt{5} - \sqrt{-2\sqrt{5}-10})}{4}\right) (\sqrt{-2\sqrt{5}-10} - \sqrt{5} + 1)}{20a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a^5 + x^5)),x)

[Out] log(a + x)/(5*a^6) - 1/(a^5*x) + (log(5*a^30 + (5*a^29*x*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/4)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/(20*a^6) - (log(5*a^30 - (5*a^29*x*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/4)*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^6) - (log(5*a^30 - (5*a^29*x*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/4)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^6) - (log(5*a^30 - (5*a^29*x*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/4))*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/(20*a^6)

3.143 $\int \frac{1}{x^3(a^5+x^5)} dx$

Optimal. Leaf size=211

$$\frac{1}{2a^5x^2} - \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{5a^7} + \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}^{((1+\sqrt{5})^{a-4x})}}{2a}\right)}{5a^7}$$

[Out] $-1/2/a^5/x^2-1/5*\ln(a+x)/a^7+1/20*\ln(a^2+x^2-1/2*a*x*(5^{(1/2)+1}))*(-5^{(1/2)+1})/a^7+1/20*\ln(a^2+x^2-1/2*a*x*(-5^{(1/2)+1}))*(-5^{(1/2)+1})/a^7-1/10*\arctan((-4*x+a*(-5^{(1/2)+1}))/a/(10+2*5^{(1/2)})^{(1/2)})*(10-2*5^{(1/2)})^{(1/2)}/a^7+1/10*\arctan(1/20*(-4*x+a*(5^{(1/2)+1}))*((50+10*5^{(1/2)})^{(1/2)}/a)*(10+2*5^{(1/2)})^{(1/2)}/a^7$

Rubi [A]

time = 0.22, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$,

Rules used = {331, 299, 648, 632, 210, 642, 31}

$$\frac{-\frac{\log(a+x)}{5a^7} - \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{5a^7} + \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}^{((1+\sqrt{5})^{a-4x})}}{2a}\right)}{5a^7}}{-\frac{1}{2a^5x^2} + \frac{(1+\sqrt{5}) \log(a^2 - \frac{1}{2}(1-\sqrt{5})ax + x^2)}{20a^7} + \frac{(1-\sqrt{5}) \log(a^2 - \frac{1}{2}(1+\sqrt{5})ax + x^2)}{20a^7}}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^3*(a^5 + x^5)),x]`

[Out] $-1/2*1/(a^5*x^2) - (\text{Sqrt}[(5 - \text{Sqrt}[5])/2]*\text{ArcTan}[\frac{((1 - \text{Sqrt}[5])*a - 4*x)}{(\text{Sqrt}[2*(5 + \text{Sqrt}[5]))*a}])]/(5*a^7) + (\text{Sqrt}[(5 + \text{Sqrt}[5])/2]*\text{ArcTan}[\frac{(\text{Sqrt}[(5 + \text{Sqrt}[5])/10]*((1 + \text{Sqrt}[5])*a - 4*x))/(2*a)}])]/(5*a^7) - \text{Log}[a + x]/(5*a^7) + ((1 + \text{Sqrt}[5])*\text{Log}[a^2 - ((1 - \text{Sqrt}[5])*a*x)/2 + x^2])/(20*a^7) + ((1 - \text{Sqrt}[5])*\text{Log}[a^2 - ((1 + \text{Sqrt}[5])*a*x)/2 + x^2])/(20*a^7)$

Rule 31

`Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rule 210

`Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & (LtQ[a, 0] || LtQ[b, 0])`

Rule 299

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
1)*(Pi/n)]*x + s^2*x^2), x]; (-r)^(m + 1)/(a*n*s^m)*Int[1/(r + s*x), x]
+ Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 1)/2}], x], x] /; Free
Q[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a
/b]
```

Rule 331

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p +
1) + 1)/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 632

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 (a^5 + x^5)} dx &= -\frac{1}{2a^5 x^2} - \frac{\int \frac{x^2}{a^5 + x^5} dx}{a^5} \\
&= -\frac{1}{2a^5 x^2} - \frac{\int \frac{1}{a+x} dx}{5a^7} - \frac{2 \int \frac{\frac{1}{4}(-1-\sqrt{5})^{a-\frac{1}{4}}(1+\sqrt{5})^x}{a^2-\frac{1}{2}(1-\sqrt{5})ax+x^2} dx}{5a^7} - \frac{2 \int \frac{\frac{1}{4}(-1+\sqrt{5})^{a-\frac{1}{4}}(1-\sqrt{5})^x}{a^2-\frac{1}{2}(1+\sqrt{5})ax+x^2} dx}{5a^7} \\
&= -\frac{1}{2a^5 x^2} - \frac{\log(a+x)}{5a^7} + \frac{(1-\sqrt{5}) \int \frac{-\frac{1}{2}(1+\sqrt{5})^{a+2x}}{a^2-\frac{1}{2}(1+\sqrt{5})ax+x^2} dx}{20a^7} + \frac{(1+\sqrt{5}) \int \frac{-\frac{1}{2}(1-\sqrt{5})^{a+2x}}{a^2-\frac{1}{2}(1-\sqrt{5})ax+x^2} dx}{20a^7} \\
&= -\frac{1}{2a^5 x^2} - \frac{\log(a+x)}{5a^7} + \frac{(1-\sqrt{5}) \log(2a^2 - ax - \sqrt{5} ax + 2x^2)}{20a^7} + \frac{(1+\sqrt{5}) \log(2a^2 - ax + \sqrt{5} ax + 2x^2)}{20a^7} \\
&= -\frac{1}{2a^5 x^2} - \frac{\sqrt{\frac{2}{5(5+\sqrt{5})}} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{a^7} + \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{(1+\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{a^7}
\end{aligned}$$

Mathematica [A]

time = 0.09, size = 174, normalized size = 0.82

$$\frac{\frac{10a^2}{2^2} - 2\sqrt{10-2\sqrt{5}} \tan^{-1}\left(\frac{(-1+\sqrt{5})^{a+4x}}{\sqrt{2(5+\sqrt{5})}^a}\right) + 2\sqrt{2(5+\sqrt{5})} \tan^{-1}\left(\frac{-(1+\sqrt{5})^a}{\sqrt{10-2\sqrt{5}}^a}\right) + 4\log(a+x) - (1+\sqrt{5}) \log(a^2 + \frac{1}{2}(-1+\sqrt{5})ax + x^2) + (-1+\sqrt{5}) \log(a^2 - \frac{1}{2}(1+\sqrt{5})ax + x^2)}{20a^7}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a^5 + x^5)),x]

[Out] $-1/20*((10*a^2)/x^2 - 2*\text{Sqrt}[10 - 2*\text{Sqrt}[5]]*\text{ArcTan}[((-1 + \text{Sqrt}[5])*a + 4*x)/(\text{Sqrt}[2*(5 + \text{Sqrt}[5])]*a)] + 2*\text{Sqrt}[2*(5 + \text{Sqrt}[5])]*\text{ArcTan}[(-(1 + \text{Sqrt}[5])*a) + 4*x)/(\text{Sqrt}[10 - 2*\text{Sqrt}[5]]*a)] + 4*\text{Log}[a + x] - (1 + \text{Sqrt}[5])*Log[a^2 + ((-1 + \text{Sqrt}[5])*a*x)/2 + x^2] + (-1 + \text{Sqrt}[5])*Log[a^2 - ((1 + \text{Sqrt}[5])*a*x)/2 + x^2])/a^7$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.55, size = 62, normalized size = 0.29

$$-\frac{\text{Log}[a+x]}{5a^7} + \frac{\text{RootSum}[1 - 125\#1^3 - 5\#1 + 25\#1^2 + 625\#1^4 \&, \text{Log}[25a\#1^2 + x] \#1 \&]}{a^7} - \frac{1}{2a^5 x^2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x^3*(a^5 + x^5)),x]')

[Out] $-\text{Log}[a + x] / (5 a^7) + \text{RootSum}[1 - 125 \#1^3 - 5 \#1 + 25 \#1^2 + 625 \#1^4 \&, \text{Log}[25 a \#1^2 + x] \#1\&] / a^7 - 1 / (2 a^5 x^2)$

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 105, normalized size = 0.50

method	result	size
risch	$-\frac{1}{2a^5x^2} - \frac{\ln(a+x)}{5a^7} + \frac{\left(\sum_{R=\text{RootOf}(a^{28}Z^4 - a^{21}Z^3 + a^{14}Z^2 - a^7Z + 1)} -R \ln\left(\frac{(-6R^5 a^{35} - 5)x + a^{15}R^2}{5}\right) \right)}{5}$	78
default	$-\frac{1}{2a^5x^2} - \frac{\ln(a+x)}{5a^7} + \frac{\sum_{R=\text{RootOf}(-Z^4 - aZ^3 + Z^2 a^2 - a^3Z + a^4)} \frac{\left(-R^3 - 2R^2 a - 2a^2 R + a^3 \right) \ln(-R+x)}{4R^3 - 3R^2 a + 2a^2 R - a^3}}{5a^7}$	105

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a^5+x^5),x,method=_RETURNVERBOSE)

[Out] $-1/2/a^5/x^2 - 1/5*\ln(a+x)/a^7 + 1/5/a^7*\text{sum}((R^3 - 2R^2a - 2R*a^2 + a^3)/(4R^3 - 3R^2a + 2R*a^2 - a^3)*\ln(-R+x), R=\text{RootOf}(Z^4 - Z^3*a + Z^2*a^2 - Z*a^3 + a^4))$

Maxima [A]

time = 0.35, size = 173, normalized size = 0.82

$$\frac{2\sqrt{5} \arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{a^2\sqrt{2\sqrt{5}+10}} - \frac{2\sqrt{5} \arctan\left(\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{a^2\sqrt{-2\sqrt{5}+10}} - \frac{\log(a+x)}{a^2} - \frac{\log(-ax(\sqrt{5}+1)+2a^2+2x^2)}{a^2(\sqrt{5}+1)} + \frac{\log(ax(\sqrt{5}-1)+2a^2+2x^2)}{a^2(\sqrt{5}-1)} - \frac{1}{2a^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a^5+x^5),x, algorithm="maxima")

[Out] $1/5*(2*\text{sqrt}(5)*\arctan((a*(\text{sqrt}(5) - 1) + 4*x)/(a*\text{sqrt}(2*\text{sqrt}(5) + 10)))/(a^2*\text{sqrt}(2*\text{sqrt}(5) + 10)) - 2*\text{sqrt}(5)*\arctan(-(a*(\text{sqrt}(5) + 1) - 4*x)/(a*\text{sqrt}(-2*\text{sqrt}(5) + 10)))/(a^2*\text{sqrt}(-2*\text{sqrt}(5) + 10)) - \log(a + x)/a^2 - \log(-a*x*(\text{sqrt}(5) + 1) + 2*a^2 + 2*x^2)/(a^2*(\text{sqrt}(5) + 1)) + \log(a*x*(\text{sqrt}(5) - 1) + 2*a^2 + 2*x^2)/(a^2*(\text{sqrt}(5) - 1)))/a^5 - 1/2/(a^5*x^2)$

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a^5+x^5),x, algorithm="fricas")

[Out] Timed out

Sympy [A]

time = 0.10, size = 51, normalized size = 0.24

$$-\frac{1}{2a^5x^2} + \frac{-\frac{\log(a+x)}{5} + \text{RootSum}(625t^4 - 125t^3 + 25t^2 - 5t + 1, (t \mapsto t \log(25t^2a + x)))}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(a**5+x**5),x)

[Out] -1/(2*a**5*x**2) + (-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t + 1, Lambda(_t, _t*log(25*_t**2*a + x))))/a**7

Giac [A]

time = 0.01, size = 245, normalized size = 1.16

$$\frac{8\sqrt{5} \ln(x^2 + \frac{1}{2}(a\sqrt{5} - a)x + aa)}{160a^7} - \frac{4\sqrt{2(-\sqrt{5} + 5)} \arctan\left(\frac{x + \frac{1}{2}(\sqrt{5} - 1)a}{\frac{1}{2}\sqrt{2\sqrt{5} + 10}a}\right)}{40a^7} - \frac{\ln|x+a|}{5a^7} - \frac{8\sqrt{5} \ln(x^2 + \frac{1}{2}(-a\sqrt{5} - a)x + aa)}{160a^7} + \frac{4\sqrt{2(\sqrt{5} + 5)} \arctan\left(\frac{x - \frac{1}{2}(\sqrt{5} + 1)a}{\frac{1}{2}\sqrt{-2\sqrt{5} + 10}a}\right)}{40a^7} + \frac{\ln|x^4 - x^3a + x^2a^2 - xa^3 + a^4|}{20a^7} - \frac{1}{2a^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a^5+x^5),x)

[Out] -1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/a^7 + 1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/a^7 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*x + x^2)/a^7 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^7 + 1/20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^7 - 1/5*log(abs(a + x))/a^7 - 1/2/(a^5*x^2)

Mupad [B]

time = 0.78, size = 210, normalized size = 1.00

$$\frac{\ln\left(\frac{a^{20} - \frac{a^{19}\sqrt{5+2\sqrt{5}-10+1}}{a}}{a^{20}}\right) \left(\sqrt{5} + \sqrt{2\sqrt{5}-10} + 1\right)}{20a^7} - \frac{1}{2a^5x^2} - \frac{\ln\left(\frac{a^{20} + \frac{a^{19}\sqrt{-2\sqrt{5}-10-1}}{a}}{a^{20}}\right) \left(\sqrt{5} + \sqrt{-2\sqrt{5}-10} - 1\right)}{20a^7} - \frac{\ln(a+x)}{5a^7} + \frac{\ln\left(\frac{a^{20} - \frac{a^{19}\sqrt{5-\sqrt{2\sqrt{5}-10+1}}}{a}}{a^{20}}\right) \left(\sqrt{5} - \sqrt{2\sqrt{5}-10} + 1\right)}{20a^7} + \frac{\ln\left(\frac{a^{20} - \frac{a^{19}\sqrt{-2\sqrt{5}-10-\sqrt{5}+1}}{a}}{a^{20}}\right) \left(\sqrt{-2\sqrt{5}-10} - \sqrt{5} + 1\right)}{20a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a^5 + x^5)),x)

[Out] (log(a^20 - (a^19*x*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1)/(20*a^7) - 1/(2*a^5*x^2) - (log(a^20 + (a^19*x*(5^(1/2) + (-2*5^(1/2) - 10)^(1/2) - 1)^3)/64)*(5^(1/2) + (-2*5^(1/2) - 10)^(1/2) - 1)/(20*a^7) - log(a + x)/(5*a^7) + (log(a^20 - (a^19*x*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1)/(20*a^7) + (log(a^20 - (a^19*x*((-2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1)^3)/64))*((-2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1)/(20*a^7)

3.144 $\int \frac{1}{x^4(a^5+x^5)} dx$

Optimal. Leaf size=211

$$\frac{1}{3a^5x^3} - \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{5a^8} + \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}((1+\sqrt{5})^{a-4x})}{2a}\right)}{5a^8}$$

[Out] $-1/3/a^5/x^3+1/5*\ln(a+x)/a^8-1/20*\ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(-5^(1/2)+1)/a^8-1/20*\ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(5^(1/2)+1)/a^8-1/10*\arctan((-4*x+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)/a^8+1/10*\arctan(1/20*(-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10+2*5^(1/2))^(1/2)/a^8$

Rubi [A]

time = 0.19, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$,

Rules used = {331, 299, 648, 632, 210, 642, 31}

$$\frac{\log(a+x)}{5a^8} - \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{5a^8} + \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}((1+\sqrt{5})^{a-4x})}{2a}\right)}{5a^8} - \frac{1}{3a^5x^3} - \frac{(1+\sqrt{5}) \log(a^2 - \frac{1}{2}(1-\sqrt{5})ax + x^2)}{20a^8} - \frac{(1-\sqrt{5}) \log(a^2 - \frac{1}{2}(1+\sqrt{5})ax + x^2)}{20a^8}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a^5 + x^5)),x]

[Out] $-1/3*1/(a^5*x^3) - (\text{Sqrt}[(5 - \text{Sqrt}[5])/2]*\text{ArcTan}[\frac{((1 - \text{Sqrt}[5])*a - 4*x)}{(\text{Sqrt}[2*(5 + \text{Sqrt}[5]))*a}])]/(5*a^8) + (\text{Sqrt}[(5 + \text{Sqrt}[5])/2]*\text{ArcTan}[\frac{(\text{Sqrt}[(5 + \text{Sqrt}[5])/10]*((1 + \text{Sqrt}[5])*a - 4*x))/(2*a)}])]/(5*a^8) + \text{Log}[a + x]/(5*a^8) - ((1 + \text{Sqrt}[5])*\text{Log}[a^2 - ((1 - \text{Sqrt}[5])*a*x)/2 + x^2])/(20*a^8) - ((1 - \text{Sqrt}[5])*\text{Log}[a^2 - ((1 + \text{Sqrt}[5])*a*x)/2 + x^2])/(20*a^8)$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] :> Simp[(-Rt[-a, 2]*Rt[-b, 2])⁽⁻¹⁾)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

Rule 299


```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*cos[(2*k
- 1)*m*(Pi/n)] - s*cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*cos[(2*k
- 1)*(Pi/n)]*x + s^2*x^2), x]; (-(-r)^(m + 1)/(a*n*s^m))*Int[1/(r + s*x), x]
+ Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 1)/2}], x], x] /; Free
Q[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a
/b]
```

Rule 331

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 632

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int
[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (a^5 + x^5)} dx &= -\frac{1}{3a^5 x^3} - \frac{\int \frac{x}{a^5 + x^5} dx}{a^5} \\
&= -\frac{1}{3a^5 x^3} + \frac{\int \frac{1}{a+x} dx}{5a^8} - \frac{2 \int \frac{\frac{1}{4}(1-\sqrt{5})^{a-\frac{1}{4}}(-1-\sqrt{5})^x}{a^2-\frac{1}{2}(1-\sqrt{5})^{ax+x^2}} dx}{5a^8} - \frac{2 \int \frac{\frac{1}{4}(1+\sqrt{5})^{a-\frac{1}{4}}(-1+\sqrt{5})^x}{a^2-\frac{1}{2}(1+\sqrt{5})^{ax+x^2}} dx}{5a^8} \\
&= -\frac{1}{3a^5 x^3} + \frac{\log(a+x)}{5a^8} - \frac{(1-\sqrt{5}) \int \frac{-\frac{1}{2}(1+\sqrt{5})^{a+2x}}{a^2-\frac{1}{2}(1+\sqrt{5})^{ax+x^2}} dx}{20a^8} - \frac{(1+\sqrt{5}) \int \frac{-\frac{1}{2}(1-\sqrt{5})^{a+2x}}{a^2-\frac{1}{2}(1-\sqrt{5})^{ax+x^2}} dx}{20a^8} \\
&= -\frac{1}{3a^5 x^3} + \frac{\log(a+x)}{5a^8} - \frac{(1-\sqrt{5}) \log(2a^2 - ax - \sqrt{5} ax + 2x^2)}{20a^8} - \frac{(1+\sqrt{5}) \log(2a^2 + ax + \sqrt{5} ax + 2x^2)}{20a^8} \\
&= -\frac{1}{3a^5 x^3} - \frac{\sqrt{\frac{2}{5(5+\sqrt{5})}} \tan^{-1}\left(\frac{(1-\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{a^8} + \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\frac{(1+\sqrt{5})^{a-4x}}{\sqrt{2(5+\sqrt{5})}^a}\right)}{a^8}
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 175, normalized size = 0.83

$$\frac{-\frac{20a^3}{x^3} + 6\sqrt{10-2\sqrt{5}} \tan^{-1}\left(\frac{(-1+\sqrt{5})^{a+4x}}{\sqrt{2(5+\sqrt{5})}^a}\right) - 6\sqrt{2(5+\sqrt{5})} \tan^{-1}\left(\frac{-(1+\sqrt{5})^{a+4x}}{\sqrt{10-2\sqrt{5}}^a}\right) + 12\log(a+x) - 3(1+\sqrt{5}) \log(a^2 + \frac{1}{2}(-1+\sqrt{5})ax + x^2) + 3(-1+\sqrt{5}) \log(a^2 - \frac{1}{2}(1+\sqrt{5})ax + x^2)}{60a^8}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x^4*(a^5 + x^5)),x]`

```
[Out] ((-20*a^3)/x^3 + 6*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 + Sqrt[5]])*a)] - 6*Sqrt[2*(5 + Sqrt[5]])*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] + 12*Log[a + x] - 3*(1 + Sqrt[5])*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + 3*(-1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(60*a^8)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.77, size = 62, normalized size = 0.29

$$\frac{\log[a+x]}{5a^8} + \frac{\text{RootSum}[1 + 5\#1 + 25\#1^2 + 125\#1^3 + 625\#1^4 \&, \text{Log}[125a\#1^3 + x] \#1 \&]}{a^8} - \frac{1}{3a^5 x^3}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/(x^4*(a^5 + x^5)),x]')`

[Out] $\text{Log}[a + x] / (5 a^8) + \text{RootSum}[1 + 5 \#1 + 25 \#1^2 + 125 \#1^3 + 625 \#1^4 \&, \text{Log}[125 a \#1^3 + x] \#1\&] / a^8 - 1 / (3 a^5 x^3)$

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 0.05, size = 109, normalized size = 0.52

method	result	size
risch	$-\frac{1}{3a^5x^3} + \frac{\ln(-a-x)}{5a^8} + \frac{\left(\sum_{-R=\text{RootOf}(a^{32}Z^4+a^{24}Z^3+a^{16}Z^2+a^8Z+1)} \frac{-R \ln\left(\left(-6R^5a^{40}+5\right)x-a^{25}R^3\right)}{5} \right)}{5}$	81
default	$-\frac{1}{3a^5x^3} + \frac{\ln(a+x)}{5a^8} + \frac{\sum_{-R=\text{RootOf}(-Z^4-aZ^3+Z^2a^2-a^3Z+a^4)} \frac{\left(-R^3+2R^2a-3a^2R-a^3\right) \ln(-R+x)}{4R^3-3R^2a+2a^2R-a^3}}{5a^8}$	109

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(a^5+x^5),x,method=_RETURNVERBOSE)`

[Out] $-1/3/a^5/x^3+1/5*\ln(a+x)/a^8+1/5/a^8*\text{sum}\left(\left(-R^3+2R^2a-3R^2a^2-a^3\right)/\left(4R^3-3R^2a+2R^2a^2-a^3\right)*\ln(-R+x), R=\text{RootOf}\left(-Z^4-Z^3a+Z^2a^2-Za^3+a^4\right)\right)$

Maxima [A]

time = 0.35, size = 172, normalized size = 0.82

$$\frac{2\sqrt{5} \arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{a^3\sqrt{2\sqrt{5}+10}} - \frac{2\sqrt{5} \arctan\left(\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{a^3\sqrt{-2\sqrt{5}+10}} + \frac{\log(a+x)}{a^3} + \frac{\log(-ax(\sqrt{5}+1)+2a^2+2x^2)}{a^3(\sqrt{5}+1)} - \frac{\log(ax(\sqrt{5}-1)+2a^2+2x^2)}{a^3(\sqrt{5}-1)} - \frac{1}{3a^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(a^5+x^5),x, algorithm="maxima")`

[Out] $1/5*(2*\text{sqrt}(5)*\arctan((a*(\text{sqrt}(5)-1)+4*x)/(a*\text{sqrt}(2*\text{sqrt}(5)+10)))/(a^3*\text{sqrt}(2*\text{sqrt}(5)+10)) - 2*\text{sqrt}(5)*\arctan(-(a*(\text{sqrt}(5)+1)-4*x)/(a*\text{sqrt}(-2*\text{sqrt}(5)+10)))/(a^3*\text{sqrt}(-2*\text{sqrt}(5)+10)) + \log(a+x)/a^3 + \log(-a*x*(\text{sqrt}(5)+1)+2*a^2+2*x^2)/(a^3*(\text{sqrt}(5)+1)) - \log(a*x*(\text{sqrt}(5)-1)+2*a^2+2*x^2)/(a^3*(\text{sqrt}(5)-1)))/a^5 - 1/3/(a^5*x^3)$

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a^5+x^5),x, algorithm="fricas")

[Out] Timed out

Sympy [A]

time = 0.10, size = 51, normalized size = 0.24

$$-\frac{1}{3a^5x^3} + \frac{\frac{\log(a+x)}{5} + \text{RootSum}(625t^4 + 125t^3 + 25t^2 + 5t + 1, (t \mapsto t \log(125t^3a + x)))}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(a**5+x**5),x)

[Out] -1/(3*a**5*x**3) + (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda(_t, _t*log(125*_t**3*a + x))))/a**8

Giac [A]

time = 0.01, size = 245, normalized size = 1.16

$$\frac{8\sqrt{5} \ln(x^2 + \frac{1}{2}(a\sqrt{5} - a)x + aa)}{160a^8} - \frac{4\sqrt{2(-\sqrt{5} + 5)} \arctan\left(\frac{x + \frac{1}{2}(\sqrt{5} - 1)a}{\frac{1}{2}\sqrt{2\sqrt{5} + 10}a}\right)}{40a^8} + \frac{\ln|x+ a|}{5a^8} + \frac{8\sqrt{5} \ln(x^2 + \frac{1}{2}(-a\sqrt{5} - a)x + aa)}{160a^8} + \frac{4\sqrt{2(\sqrt{5} + 5)} \arctan\left(\frac{x - \frac{1}{2}(\sqrt{5} + 1)a}{\frac{1}{2}\sqrt{-2\sqrt{5} + 10}a}\right)}{40a^8} - \frac{\ln|x^4 - x^3a + x^2a^2 - xa^3 + a^4|}{20a^8} - \frac{1}{3a^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a^5+x^5),x)

[Out] -1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/a^8 + 1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/a^8 + 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*x + x^2)/a^8 - 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^8 - 1/20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^8 + 1/5*log(abs(a + x))/a^8 - 1/3/(a^5*x^3)

Mupad [B]

time = 0.69, size = 214, normalized size = 1.01

$$\frac{\ln\left(a^5x - \frac{a^5(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1)}{a}\right)(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1)}{20a^8} - \frac{\ln\left(a^5x - \frac{a^5(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1)}{a}\right)(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1)}{20a^8} + \frac{\ln\left(\frac{(\sqrt{5}+\sqrt{2\sqrt{5}-10}-1)a^5}{a} + xa^5\right)(\sqrt{5}+\sqrt{2\sqrt{5}-10}-1)}{20a^8} - \frac{\ln\left(a^5x - \frac{a^5(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1)}{a}\right)(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1)}{20a^8} - \frac{1}{5a^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(a^5 + x^5)),x)

[Out] log(a + x)/(5*a^8) - 1/(3*a^5*x^3) - (log(a^15*x - (a^16*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1)/(20*a^8) - (log(a^15*x - (a^16*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1)^3)/64)*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1)/(20*a^8) + (log(a^15*x + (a^16*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1)^3)/64)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1)/(20*a^8) - (log(a^15*x - (a^16*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1)/(20*a^8)

3.145 $\int \frac{x^{-m}}{a^5+x^5} dx$

Optimal. Leaf size=46

$$\frac{x^{1-m} {}_2F_1\left(1, \frac{1-m}{5}; \frac{6-m}{5}; -\frac{x^5}{a^5}\right)}{a^5(1-m)}$$

[Out] $x^{(1-m)}*\text{hypergeom}([1, 1/5-1/5*m], [6/5-1/5*m], -x^5/a^5)/a^5/(1-m)$

Rubi [A]

time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {371}

$$\frac{x^{1-m} {}_2F_1\left(1, \frac{1-m}{5}; \frac{6-m}{5}; -\frac{x^5}{a^5}\right)}{a^5(1-m)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^m*(a^5 + x^5)),x]

[Out] $(x^{(1-m)}*\text{Hypergeometric2F1}[1, (1-m)/5, (6-m)/5, -(x^5/a^5)])/(a^5*(1-m))$

Rule 371

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p * ((c*x)^(m+1)/(c*(m+1)))*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\int \frac{x^{-m}}{a^5+x^5} dx = \frac{x^{1-m} {}_2F_1\left(1, \frac{1-m}{5}; \frac{6-m}{5}; -\frac{x^5}{a^5}\right)}{a^5(1-m)}$$

Mathematica [A]

time = 0.16, size = 45, normalized size = 0.98

$$\frac{x^{1-m} {}_2F_1\left(1, \frac{1}{5} - \frac{m}{5}; \frac{6}{5} - \frac{m}{5}; -\frac{x^5}{a^5}\right)}{a^5(-1+m)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^m*(a^5 + x^5)),x]

[Out] -((x^(1 - m)*Hypergeometric2F1[1, 1/5 - m/5, 6/5 - m/5, -(x^5/a^5)])/(a^5*(-1 + m)))

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

time = 11.34, size = 30, normalized size = 0.65

$$\frac{\text{LerchPhi}\left[\frac{x^5 \exp_{\text{polar}}[i\text{Pi}]}{a^5}, 1, \frac{1}{5} - \frac{m}{5}\right] x^{1-m}}{5a^5}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x^m*(a^5 + x^5)),x]')

[Out] LerchPhi[x ^ 5 exp_polar[I Pi] / a ^ 5, 1, 1 / 5 - m / 5] x ^ (1 - m) / (5 a ^ 5)

Maple [F]

time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{-m}}{a^5 + x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^m)/(a^5+x^5),x)

[Out] int(1/(x^m)/(a^5+x^5),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^m)/(a^5+x^5),x, algorithm="maxima")

[Out] integrate(1/((a^5 + x^5)*x^m), x)

Fricas [F]

time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^m)/(a^5+x^5),x, algorithm="fricas")

[Out] integral(1/((a^5 + x^5)*x^m), x)

Sympy [C] Result contains complex when optimal does not.

time = 8.47, size = 92, normalized size = 2.00

$$-\frac{mxx^{-m}\Phi\left(\frac{x^5e^{i\pi}}{a^5}, 1, \frac{1}{5} - \frac{m}{5}\right)\Gamma\left(\frac{1}{5} - \frac{m}{5}\right)}{25a^5\Gamma\left(\frac{6}{5} - \frac{m}{5}\right)} + \frac{xx^{-m}\Phi\left(\frac{x^5e^{i\pi}}{a^5}, 1, \frac{1}{5} - \frac{m}{5}\right)\Gamma\left(\frac{1}{5} - \frac{m}{5}\right)}{25a^5\Gamma\left(\frac{6}{5} - \frac{m}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**m)/(a**5+x**5),x)

[Out] -m*x*lerchphi(x**5*exp_polar(I*pi)/a**5, 1, 1/5 - m/5)*gamma(1/5 - m/5)/(25*a**5*x**m*gamma(6/5 - m/5)) + x*lerchphi(x**5*exp_polar(I*pi)/a**5, 1, 1/5 - m/5)*gamma(1/5 - m/5)/(25*a**5*x**m*gamma(6/5 - m/5))

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^m)/(a^5+x^5),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^m (a^5 + x^5)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^m*(a^5 + x^5)),x)

[Out] int(1/(x^m*(a^5 + x^5)), x)

3.146 $\int \frac{1+x^4}{1+x^6} dx$

Optimal. Leaf size=35

$$-\frac{1}{3} \tan^{-1}(\sqrt{3} - 2x) + \frac{2}{3} \tan^{-1}(x) + \frac{1}{3} \tan^{-1}(\sqrt{3} + 2x)$$

[Out] 2/3*arctan(x)+1/3*arctan(2*x-3^(1/2))+1/3*arctan(2*x+3^(1/2))

Rubi [A]

time = 0.29, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {1890, 215, 648, 632, 210, 642, 209, 301}

$$-\frac{1}{3} \tan^{-1}(\sqrt{3} - 2x) + \frac{2}{3} \tan^{-1}(x) + \frac{1}{3} \tan^{-1}(2x + \sqrt{3})$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)/(1 + x^6), x]

[Out] -1/3*ArcTan[Sqrt[3] - 2*x] + (2*ArcTan[x])/3 + ArcTan[Sqrt[3] + 2*x]/3

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 215

Int[((a_) + (b_.)*(x_)^(n_))(-1), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[{k, 1, (n - 2)/4}], x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a/b]

Rule 301

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k


```

- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]

```

Rule 632

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]

```

Rule 642

```

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]

```

Rule 648

```

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

```

Rule 1890

```

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}}], Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n

```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{1+x^6} dx &= \int \left(\frac{1}{1+x^6} + \frac{x^4}{1+x^6} \right) dx \\
&= \int \frac{1}{1+x^6} dx + \int \frac{x^4}{1+x^6} dx \\
&= \frac{1}{3} \int \frac{1 - \frac{\sqrt{3}x}{2}}{1 - \sqrt{3}x + x^2} dx + \frac{1}{3} \int \frac{-\frac{1}{2} + \frac{\sqrt{3}x}{2}}{1 - \sqrt{3}x + x^2} dx + \frac{1}{3} \int \frac{-\frac{1}{2} - \frac{\sqrt{3}x}{2}}{1 + \sqrt{3}x + x^2} dx + \frac{1}{3} \int \frac{1 + \frac{\sqrt{3}x}{2}}{1 + \sqrt{3}x + x^2} dx \\
&= \frac{2}{3} \tan^{-1}(x) + 2 \left(\frac{1}{12} \int \frac{1}{1 - \sqrt{3}x + x^2} dx \right) + 2 \left(\frac{1}{12} \int \frac{1}{1 + \sqrt{3}x + x^2} dx \right) \\
&= \frac{2}{3} \tan^{-1}(x) - 2 \left(\frac{1}{6} \text{Subst} \left(\int \frac{1}{-1 - x^2} dx, x, -\sqrt{3} + 2x \right) \right) - 2 \left(\frac{1}{6} \text{Subst} \left(\int \frac{1}{-1 - x^2} dx, x, \sqrt{3} + 2x \right) \right) \\
&= -\frac{1}{3} \tan^{-1}(\sqrt{3} - 2x) + \frac{2}{3} \tan^{-1}(x) + \frac{1}{3} \tan^{-1}(\sqrt{3} + 2x)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 21, normalized size = 0.60

$$\frac{2}{3} \tan^{-1}(x) - \frac{1}{3} \tan^{-1} \left(\frac{x}{-1 + x^2} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + x^4)/(1 + x^6), x]``[Out] (2*ArcTan[x])/3 - ArcTan[x/(-1 + x^2)]/3`**Mathics [A]**

time = 1.74, size = 9, normalized size = 0.26

$$\text{ArcTan}[x] + \frac{\text{ArcTan}[x^3]}{3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(x^4 + 1)/(x^6 + 1), x]')``[Out] ArcTan[x] + ArcTan[x ^ 3] / 3`**Maple [A]**

time = 0.06, size = 28, normalized size = 0.80

method	result
--------	--------

risch	$\arctan(x) + \frac{\arctan(x^3)}{3}$
default	$\frac{2\arctan(x)}{3} + \frac{\arctan(2x-\sqrt{3})}{3} + \frac{\arctan(2x+\sqrt{3})}{3}$
meijerg	$\frac{x^5\sqrt{3}\ln\left(1-\sqrt{3}(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{12(x^6)^{\frac{5}{6}}} + \frac{x^5\arctan\left(\frac{(x^6)^{\frac{1}{6}}}{2-\sqrt{3}(x^6)^{\frac{1}{6}}}\right)}{6(x^6)^{\frac{5}{6}}} + \frac{x^5\arctan\left((x^6)^{\frac{1}{6}}\right)}{3(x^6)^{\frac{5}{6}}} - \frac{x^5\sqrt{3}\ln\left(1+\sqrt{3}(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{12(x^6)^{\frac{5}{6}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4+1)/(x^6+1),x,method=_RETURNVERBOSE)`

[Out] `2/3*arctan(x)+1/3*arctan(2*x-3^(1/2))+1/3*arctan(2*x+3^(1/2))`

Maxima [A]

time = 0.37, size = 27, normalized size = 0.77

$$\frac{1}{3} \arctan(2x + \sqrt{3}) + \frac{1}{3} \arctan(2x - \sqrt{3}) + \frac{2}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+1)/(x^6+1),x, algorithm="maxima")`

[Out] `1/3*arctan(2*x + sqrt(3)) + 1/3*arctan(2*x - sqrt(3)) + 2/3*arctan(x)`

Fricas [A]

time = 0.31, size = 9, normalized size = 0.26

$$\frac{1}{3} \arctan(x^3) + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+1)/(x^6+1),x, algorithm="fricas")`

[Out] `1/3*arctan(x^3) + arctan(x)`

Sympy [A]

time = 0.06, size = 8, normalized size = 0.23

$$\operatorname{atan}(x) + \frac{\operatorname{atan}(x^3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**4+1)/(x**6+1),x)`

[Out] `atan(x) + atan(x**3)/3`

Giac [A]

time = 0.00, size = 42, normalized size = 1.20

$$\frac{\arctan\left(2\left(x - \frac{\sqrt{3}}{2}\right)\right)}{3} + \frac{2}{3}\arctan x + \frac{\arctan\left(2\left(x + \frac{\sqrt{3}}{2}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((x^4+1)/(x^6+1),x)``[Out] 1/3*arctan(2*x + sqrt(3)) + 1/3*arctan(2*x - sqrt(3)) + 2/3*arctan(x)`**Mupad [B]**

time = 0.03, size = 9, normalized size = 0.26

$$\frac{\operatorname{atan}(x^3)}{3} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((x^4 + 1)/(x^6 + 1),x)``[Out] atan(x^3)/3 + atan(x)`

$$3.147 \quad \int \frac{1}{(5+3x+x^2)^3} dx$$

Optimal. Leaf size=60

$$\frac{3+2x}{22(5+3x+x^2)^2} + \frac{3(3+2x)}{121(5+3x+x^2)} + \frac{12 \tan^{-1}\left(\frac{3+2x}{\sqrt{11}}\right)}{121\sqrt{11}}$$

[Out] 1/22*(3+2*x)/(x^2+3*x+5)^2+3/121*(3+2*x)/(x^2+3*x+5)+12/1331*arctan(1/11*(3+2*x)*11^(1/2))*11^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {628, 632, 210}

$$\frac{3(2x+3)}{121(x^2+3x+5)} + \frac{2x+3}{22(x^2+3x+5)^2} + \frac{12 \tan^{-1}\left(\frac{2x+3}{\sqrt{11}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

[In] Int[(5 + 3*x + x^2)^(-3), x]

[Out] (3 + 2*x)/(22*(5 + 3*x + x^2)^2) + (3*(3 + 2*x))/(121*(5 + 3*x + x^2)) + (12*ArcTan[(3 + 2*x)/Sqrt[11]])/(121*Sqrt[11])

Rule 210

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p + 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(5+3x+x^2)^3} dx &= \frac{3+2x}{22(5+3x+x^2)^2} + \frac{3}{11} \int \frac{1}{(5+3x+x^2)^2} dx \\
&= \frac{3+2x}{22(5+3x+x^2)^2} + \frac{3(3+2x)}{121(5+3x+x^2)} + \frac{6}{121} \int \frac{1}{5+3x+x^2} dx \\
&= \frac{3+2x}{22(5+3x+x^2)^2} + \frac{3(3+2x)}{121(5+3x+x^2)} - \frac{12}{121} \text{Subst} \left(\int \frac{1}{-11-x^2} dx, x, 3+2x \right) \\
&= \frac{3+2x}{22(5+3x+x^2)^2} + \frac{3(3+2x)}{121(5+3x+x^2)} + \frac{12 \tan^{-1} \left(\frac{3+2x}{\sqrt{11}} \right)}{121\sqrt{11}}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 51, normalized size = 0.85

$$\frac{\frac{11(3+2x)(41+18x+6x^2)}{(5+3x+x^2)^2} + 24\sqrt{11} \tan^{-1} \left(\frac{3+2x}{\sqrt{11}} \right)}{2662}$$

Antiderivative was successfully verified.

`[In] Integrate[(5 + 3*x + x^2)^(-3), x]`

```
[Out] ((11*(3 + 2*x)*(41 + 18*x + 6*x^2))/(5 + 3*x + x^2)^2 + 24*sqrt[11]*ArcTan[
(3 + 2*x)/sqrt[11]])/2662
```

Mathics [A]

time = 4.08, size = 71, normalized size = 1.18

$$\frac{1353 + 1496x + 594x^2 + 132x^3 + 24\sqrt{11} \text{ArcTan} \left[\frac{\sqrt{11}(3+2x)}{11} \right] (25 + 30x + 19x^2 + 6x^3 + x^4)}{66550 + 79860x + 50578x^2 + 15972x^3 + 2662x^4}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^2 + 3*x + 5)^3, x]')`

```
[Out] (1353 + 1496 x + 594 x ^ 2 + 132 x ^ 3 + 24 Sqrt[11] ArcTan[Sqrt[11] (3 + 2
x) / 11] (25 + 30 x + 19 x ^ 2 + 6 x ^ 3 + x ^ 4)) / (2662 (25 + 30 x + 19
x ^ 2 + 6 x ^ 3 + x ^ 4))
```

Maple [A]

time = 0.18, size = 52, normalized size = 0.87

method	result	size
risch	$\frac{\frac{6}{121}x^3 + \frac{27}{121}x^2 + \frac{68}{121}x + \frac{123}{242}}{(x^2+3x+5)^2} + \frac{12 \arctan\left(\frac{(3+2x)\sqrt{11}}{11}\right)\sqrt{11}}{1331}$	44
default	$\frac{3+2x}{22(x^2+3x+5)^2} + \frac{\frac{9}{121} + \frac{6x}{121}}{x^2+3x+5} + \frac{12 \arctan\left(\frac{(3+2x)\sqrt{11}}{11}\right)\sqrt{11}}{1331}$	52

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2+3*x+5)^3,x,method=_RETURNVERBOSE)`

[Out] $1/22*(3+2*x)/(x^2+3*x+5)^2+3/121*(3+2*x)/(x^2+3*x+5)+12/1331*\arctan(1/11*(3+2*x)*11^{(1/2)})*11^{(1/2)}$

Maxima [A]

time = 0.35, size = 54, normalized size = 0.90

$$\frac{12}{1331} \sqrt{11} \arctan\left(\frac{1}{11} \sqrt{11} (2x + 3)\right) + \frac{12x^3 + 54x^2 + 136x + 123}{242(x^4 + 6x^3 + 19x^2 + 30x + 25)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2+3*x+5)^3,x, algorithm="maxima")`

[Out] $12/1331*\sqrt{11}*\arctan(1/11*\sqrt{11}*(2*x + 3)) + 1/242*(12*x^3 + 54*x^2 + 136*x + 123)/(x^4 + 6*x^3 + 19*x^2 + 30*x + 25)$

Fricas [A]

time = 0.43, size = 71, normalized size = 1.18

$$\frac{132x^3 + 24\sqrt{11}(x^4 + 6x^3 + 19x^2 + 30x + 25)\arctan\left(\frac{1}{11}\sqrt{11}(2x + 3)\right) + 594x^2 + 1496x + 1353}{2662(x^4 + 6x^3 + 19x^2 + 30x + 25)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2+3*x+5)^3,x, algorithm="fricas")`

[Out] $1/2662*(132*x^3 + 24*\sqrt{11}*(x^4 + 6*x^3 + 19*x^2 + 30*x + 25)*\arctan(1/11*\sqrt{11}*(2*x + 3)) + 594*x^2 + 1496*x + 1353)/(x^4 + 6*x^3 + 19*x^2 + 30*x + 25)$

Sympy [A]

time = 0.08, size = 63, normalized size = 1.05

$$\frac{12x^3 + 54x^2 + 136x + 123}{242x^4 + 1452x^3 + 4598x^2 + 7260x + 6050} + \frac{12\sqrt{11} \operatorname{atan}\left(\frac{2\sqrt{11}x}{11} + \frac{3\sqrt{11}}{11}\right)}{1331}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+3*x+5)**3,x)

[Out] (12*x**3 + 54*x**2 + 136*x + 123)/(242*x**4 + 1452*x**3 + 4598*x**2 + 7260*x + 6050) + 12*sqrt(11)*atan(2*sqrt(11)*x/11 + 3*sqrt(11)/11)/1331

Giac [A]

time = 0.00, size = 53, normalized size = 0.88

$$\frac{12x^3 + 54x^2 + 136x + 123}{242(x^2 + 3x + 5)^2} + \frac{12 \arctan\left(\frac{2x+3}{\sqrt{11}}\right)}{121\sqrt{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+3*x+5)^3,x)

[Out] 12/1331*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 3)) + 1/242*(12*x^3 + 54*x^2 + 136*x + 123)/(x^2 + 3*x + 5)^2

Mupad [B]

time = 0.08, size = 45, normalized size = 0.75

$$6 \left(x + \frac{3}{2}\right) \left(\frac{1}{121(x^2 + 3x + 5)} + \frac{1}{66(x^2 + 3x + 5)^2}\right) + \frac{12\sqrt{11} \operatorname{atan}\left(\frac{2\sqrt{11}(x + \frac{3}{2})}{11}\right)}{1331}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x + x^2 + 5)^3,x)

[Out] 6*(x + 3/2)*(1/(121*(3*x + x^2 + 5)) + 1/(66*(3*x + x^2 + 5)^2)) + (12*11^(1/2)*atan((2*11^(1/2)*(x + 3/2))/11))/1331

$$3.148 \quad \int \frac{1+x^2+x^4}{(1+x^2)^4} dx$$

Optimal. Leaf size=43

$$\frac{x}{6(1+x^2)^3} - \frac{x}{24(1+x^2)^2} + \frac{7x}{16(1+x^2)} + \frac{7}{16} \tan^{-1}(x)$$

[Out] 1/6*x/(x^2+1)^3-1/24*x/(x^2+1)^2+7/16*x/(x^2+1)+7/16*arctan(x)

Rubi [A]

time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1171, 393, 205, 209}

$$\frac{7x}{16(x^2+1)} - \frac{x}{24(x^2+1)^2} + \frac{x}{6(x^2+1)^3} + \frac{7}{16} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2 + x^4)/(1 + x^2)^4,x]

[Out] x/(6*(1 + x^2)^3) - x/(24*(1 + x^2)^2) + (7*x)/(16*(1 + x^2)) + (7*ArcTan[x])/16

Rule 205

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 393

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(-b*c - a*d)*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2,
x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x],
0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
+ 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^2+x^4}{(1+x^2)^4} dx &= \frac{x}{6(1+x^2)^3} - \frac{1}{6} \int \frac{-5-6x^2}{(1+x^2)^3} dx \\ &= \frac{x}{6(1+x^2)^3} - \frac{x}{24(1+x^2)^2} + \frac{7}{8} \int \frac{1}{(1+x^2)^2} dx \\ &= \frac{x}{6(1+x^2)^3} - \frac{x}{24(1+x^2)^2} + \frac{7x}{16(1+x^2)} + \frac{7}{16} \int \frac{1}{1+x^2} dx \\ &= \frac{x}{6(1+x^2)^3} - \frac{x}{24(1+x^2)^2} + \frac{7x}{16(1+x^2)} + \frac{7}{16} \tan^{-1}(x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 30, normalized size = 0.70

$$\frac{1}{48} \left(\frac{x(27 + 40x^2 + 21x^4)}{(1+x^2)^3} + 21 \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^4)/(1 + x^2)^4, x]
```

```
[Out] ((x*(27 + 40*x^2 + 21*x^4))/(1 + x^2)^3 + 21*ArcTan[x])/48
```

Mathics [A]

time = 2.04, size = 53, normalized size = 1.23

$$\frac{x(27 + 40x^2 + 21x^4) + 21 \text{ArcTan}[x](1 + 3x^2 + 3x^4 + x^6)}{48 + 144x^2 + 144x^4 + 48x^6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(x^4 + x^2 + 1)/(x^2 + 1)^4, x]')
```

```
[Out] (x (27 + 40 x ^ 2 + 21 x ^ 4) + 21 ArcTan[x] (1 + 3 x ^ 2 + 3 x ^ 4 + x ^ 6
)) / (48 (1 + 3 x ^ 2 + 3 x ^ 4 + x ^ 6))
```

Maple [A]

time = 0.05, size = 28, normalized size = 0.65

method	result	size
default	$\frac{\frac{7}{16}x^5 + \frac{5}{6}x^3 + \frac{9}{16}x}{(x^2+1)^3} + \frac{7 \arctan(x)}{16}$	28
risch	$\frac{\frac{7}{16}x^5 + \frac{5}{6}x^3 + \frac{9}{16}x}{(x^2+1)^3} + \frac{7 \arctan(x)}{16}$	28
meijerg	$\frac{x(15x^4+40x^2+33)}{48(x^2+1)^3} + \frac{7 \arctan(x)}{16} - \frac{x(-15x^4+40x^2+15)}{240(x^2+1)^3} - \frac{x(-3x^4-8x^2+3)}{48(x^2+1)^3}$	72

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+x^2+1)/(x^2+1)^4,x,method=_RETURNVERBOSE)

[Out] (7/16*x^5+5/6*x^3+9/16*x)/(x^2+1)^3+7/16*arctan(x)

Maxima [A]

time = 0.34, size = 38, normalized size = 0.88

$$\frac{21x^5 + 40x^3 + 27x}{48(x^6 + 3x^4 + 3x^2 + 1)} + \frac{7}{16} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2+1)/(x^2+1)^4,x, algorithm="maxima")

[Out] 1/48*(21*x^5 + 40*x^3 + 27*x)/(x^6 + 3*x^4 + 3*x^2 + 1) + 7/16*arctan(x)

Fricas [A]

time = 0.31, size = 52, normalized size = 1.21

$$\frac{21x^5 + 40x^3 + 21(x^6 + 3x^4 + 3x^2 + 1) \arctan(x) + 27x}{48(x^6 + 3x^4 + 3x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2+1)/(x^2+1)^4,x, algorithm="fricas")

[Out] 1/48*(21*x^5 + 40*x^3 + 21*(x^6 + 3*x^4 + 3*x^2 + 1)*arctan(x) + 27*x)/(x^6 + 3*x^4 + 3*x^2 + 1)

Sympy [A]

time = 0.07, size = 36, normalized size = 0.84

$$\frac{21x^5 + 40x^3 + 27x}{48x^6 + 144x^4 + 144x^2 + 48} + \frac{7 \operatorname{atan}(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+x**2+1)/(x**2+1)**4,x)

[Out] (21*x**5 + 40*x**3 + 27*x)/(48*x**6 + 144*x**4 + 144*x**2 + 48) + 7*atan(x)/16

Giac [A]

time = 0.00, size = 32, normalized size = 0.74

$$\frac{21x^5 + 40x^3 + 27x}{48(x^2 + 1)^3} + \frac{7}{16} \arctan x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2+1)/(x^2+1)^4,x)

[Out] 1/48*(21*x^5 + 40*x^3 + 27*x)/(x^2 + 1)^3 + 7/16*arctan(x)

Mupad [B]

time = 0.18, size = 27, normalized size = 0.63

$$\frac{7 \operatorname{atan}(x)}{16} + \frac{\frac{7x^5}{16} + \frac{5x^3}{6} + \frac{9x}{16}}{(x^2 + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + x^4 + 1)/(x^2 + 1)^4,x)

[Out] (7*atan(x))/16 + ((9*x)/16 + (5*x^3)/6 + (7*x^5)/16)/(x^2 + 1)^3

$$3.149 \quad \int \frac{B+Ax}{(c+2bx+ax^2)^2} dx$$

Optimal. Leaf size=90

$$-\frac{bB - Ac - (Ab - aB)x}{2(b^2 - ac)(c + 2bx + ax^2)} - \frac{(Ab - aB) \tanh^{-1}\left(\frac{b+ax}{\sqrt{b^2 - ac}}\right)}{2(b^2 - ac)^{3/2}}$$

[Out] $1/2*(-b*B+A*c+(A*b-B*a)*x)/(-a*c+b^2)/(a*x^2+2*b*x+c)-1/2*(A*b-B*a)*\operatorname{arctanh}((a*x+b)/(-a*c+b^2)^{(1/2)})/(-a*c+b^2)^{(3/2)}$

Rubi [A]

time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {652, 632, 212}

$$-\frac{-x(Ab - aB) - Ac + bB}{2(b^2 - ac)(ax^2 + 2bx + c)} - \frac{(Ab - aB) \tanh^{-1}\left(\frac{ax+b}{\sqrt{b^2 - ac}}\right)}{2(b^2 - ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(B + A*x)/(c + 2*b*x + a*x^2)^2, x]$

[Out] $-1/2*(b*B - A*c - (A*b - a*B)*x)/((b^2 - a*c)*(c + 2*b*x + a*x^2)) - ((A*b - a*B)*\operatorname{ArcTanh}[(b + a*x)/\operatorname{Sqrt}[b^2 - a*c]])/(2*(b^2 - a*c)^{(3/2)})$

Rule 212

$\operatorname{Int}[(a_) + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{Gt} Q[a, 0] \ || \operatorname{Lt} Q[b, 0])$

Rule 632

$\operatorname{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0]$

Rule 652

$\operatorname{Int}[(d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \operatorname{Simp}[(b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c))*(a + b*x + c*x^2)^{(p + 1)}, x] - \operatorname{Dist}[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c))), \operatorname{Int}[(a + b*x + c*x^2)^{(p + 1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e\}, x \ \&\& \operatorname{NeQ}[2*c*d - b*e, 0] \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \ \&\& \operatorname{Lt} Q[p, -1] \ \&\& \operatorname{NeQ}[p, -3/2]$

Rubi steps

$$\begin{aligned} \int \frac{B + Ax}{(c + 2bx + ax^2)^2} dx &= -\frac{bB - Ac - (Ab - aB)x}{2(b^2 - ac)(c + 2bx + ax^2)} + \frac{(Ab - aB) \int \frac{1}{c + 2bx + ax^2} dx}{2(b^2 - ac)} \\ &= -\frac{bB - Ac - (Ab - aB)x}{2(b^2 - ac)(c + 2bx + ax^2)} - \frac{(Ab - aB) \text{Subst}\left(\int \frac{1}{4(b^2 - ac) - x^2} dx, x, 2b + 2ax\right)}{b^2 - ac} \\ &= -\frac{bB - Ac - (Ab - aB)x}{2(b^2 - ac)(c + 2bx + ax^2)} - \frac{(Ab - aB) \tanh^{-1}\left(\frac{b + ax}{\sqrt{b^2 - ac}}\right)}{2(b^2 - ac)^{3/2}} \end{aligned}$$

Mathematica [A]

time = 0.05, size = 88, normalized size = 0.98

$$\frac{-bB + Ac + Abx - aBx}{c + x(2b + ax)} + \frac{(Ab - aB) \tan^{-1}\left(\frac{b + ax}{\sqrt{-b^2 + ac}}\right)}{\sqrt{-b^2 + ac}}}{2(b^2 - ac)}$$

Antiderivative was successfully verified.

[In] Integrate[(B + A*x)/(c + 2*b*x + a*x^2)^2,x]

[Out] ((-(b*B) + A*c + A*b*x - a*B*x)/(c + x*(2*b + a*x)) + ((A*b - a*B)*ArcTan[(b + a*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(2*(b^2 - a*c))

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 399 vs. 2(90) = 180.
time = 9.11, size = 377, normalized size = 4.19

$$\frac{-\frac{bB}{c} + \frac{A}{a} - \frac{x(Ab - Ba)}{c} + \frac{\left(\frac{A^2 - Ba + a^2(-Ac - Ba)}{ac^2 + a^2(ac - b^2) - b^2c + 2bx(ac - b^2)} \left(\frac{A^2 - Ba + a^2(-Ac - Ba)}{ac - b^2} \sqrt{\frac{1}{(ac - b^2)^2}} + \frac{2a^2c(A - Ba)}{ac - b^2} \sqrt{\frac{1}{(ac - b^2)^2}} + \frac{2a(A - Ba) + (-Ac - Ba)}{ac - b^2} \sqrt{\frac{1}{(ac - b^2)^2}} \right) - \frac{A^2 - Ba + a^2(A - Ba)}{ac - b^2} \sqrt{\frac{1}{(ac - b^2)^2}} - \frac{2a^2c(A - Ba)}{ac - b^2} \sqrt{\frac{1}{(ac - b^2)^2}} + \frac{2a(A - Ba) + (-Ac - Ba)}{ac - b^2} \sqrt{\frac{1}{(ac - b^2)^2}} \right) \sqrt{\frac{1}{(ac - b^2)^2}}}{ac^2 + a^2(ac - b^2) - b^2c + 2bx(ac - b^2)}}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(A*x + B)/(a*x^2 + 2*b*x + c)^2,x]')

[Out] (-A c / 2 + B b / 2 - x (A b - B a) / 2 + (A b - B a) (a c ^ 2 + a x ^ 2 (a c - b ^ 2) - b ^ 2 c + 2 b x (a c - b ^ 2)) (Log[(A b ^ 2 - B a b + a ^ 2 c ^ 2 (-A b + B a) Sqrt[-1 / (a c - b ^ 2) ^ 3] + 2 a b ^ 2 c (A b - B a) Sqrt[-1 / (a c - b ^ 2) ^ 3] + a x (A b - B a) + b ^ 4 (-A b + B a) Sqrt[-1 / (a c - b ^ 2) ^ 3]) / (a (A b - B a))] - Log[(A b ^ 2 - B a b + a ^ 2 c ^ 2 (A b - B a) Sqrt[-1 / (a c - b ^ 2) ^ 3] - 2 a b ^ 2 c (A b - B a) Sqrt[-1 / (a c - b ^ 2) ^ 3] + a x (A b - B a) + b ^ 4 (A b - B a) Sqrt[-1 / (a c - b ^ 2) ^ 3]) / (a (A b - B a))] Sqrt[-1 / (a c - b ^ 2) ^ 3] / 4) / (a c ^ 2 + a x ^ 2 (a c - b ^ 2) - b ^ 2 c + 2 b x (a c - b ^ 2))

Maple [A]

time = 0.15, size = 103, normalized size = 1.14

method	result
default	$\frac{(-2Ab+2Ba)x+2bB-2Ac}{(4ac-4b^2)(ax^2+2bx+c)} + \frac{(-2Ab+2Ba) \arctan\left(\frac{2ax+2b}{2\sqrt{ac-b^2}}\right)}{(4ac-4b^2)\sqrt{ac-b^2}}$
risch	$\frac{-\frac{(Ab-Ba)x}{2(ac-b^2)} - \frac{Ac-bB}{2(ac-b^2)}}{ax^2+2bx+c} + \frac{\ln\left(\frac{(-a^2c+ab^2)x - (-ac+b^2)^{\frac{3}{2}} - abc + b^3}{4(-ac+b^2)^{\frac{3}{2}}}\right) Ab}{4(-ac+b^2)^{\frac{3}{2}}} - \frac{\ln\left(\frac{(-a^2c+ab^2)x - (-ac+b^2)^{\frac{3}{2}} - abc + b^3}{4(-ac+b^2)^{\frac{3}{2}}}\right) Ba}{4(-ac+b^2)^{\frac{3}{2}}} - \ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A*x+B)/(a*x^2+2*b*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((-2*A*b+2*B*a)*x+2*b*B-2*A*c)/(4*a*c-4*b^2)/(a*x^2+2*b*x+c)+(-2*A*b+2*B*a)/(4*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*a*x+2*b)/(a*c-b^2)^(1/2))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x+B)/(a*x^2+2*b*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see 'assume?' for more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(81) = 162.

time = 0.44, size = 448, normalized size = 4.98

$$\frac{2B^2 + 2Aac^2 - ((Ba^2 - Ab^2)x^2 + (Ba - Ab)c + 2(Bab - Ab^2)x)\sqrt{b^2 - ac} \log\left(\frac{a^2x^2 + 2abx + b^2 - ac}{a^2x^2 + 2abx + b^2}\right) - 2(Bab + Ab^2)c + 2(Bab^2 - Ab^3 - (Ba^2 - Ab^2)c)x}{4(b^2c - 2ab^2c + ac^2 + (ab^2 - 2a^2bc + a^2c^2)x^2 + 2(b^2 - 2ab^2c + a^2bc^2)x)} - \frac{Bb^2 + Aac^2 - ((Ba^2 - Ab^2)x^2 + (Ba - Ab)c + 2(Bab - Ab^2)x)\sqrt{-b^2 + ac} \arctan\left(\frac{\sqrt{-b^2 + ac}(ax+b)}{a^2x^2 + 2abx + b^2}\right) - (Bab + Ab^2)c + (Bab^2 - Ab^3 - (Ba^2 - Ab^2)c)x}{2(b^2c - 2ab^2c + ac^2 + (ab^2 - 2a^2bc + a^2c^2)x^2 + 2(b^2 - 2ab^2c + a^2bc^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x+B)/(a*x^2+2*b*x+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*b^3 + 2*A*a*c^2 - ((B*a^2 - A*a*b)*x^2 + (B*a - A*b)*c + 2*(B*a*b - A*b^2)*x)*sqrt(b^2 - a*c)*log((a^2*x^2 + 2*a*b*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(a*x + b))/(a*x^2 + 2*b*x + c)) - 2*(B*a*b + A*b^2)*c + 2*(B*a*b^2 - A*b^3 - (B*a^2 - A*a*b)*c)*x]/(b^4*c - 2*a*b^2*c^2 + a^2*c^3 + (a*b^4 - 2*a^2*b^2*c + a^3*c^2)*x^2 + 2*(b^5 - 2*a*b^3*c + a^2*b*c^2)*x), -1/2*(B*b^3 + A*a*c^2 - ((B*a^2 - A*a*b)*x^2 + (B*a - A*b)*c + 2*(B*a*b - A*b^2)*x
```

) $\sqrt{-b^2 + ac}$)*arctan(-sqrt(-b² + a*c)*(a*x + b)/(b² - a*c)) - (B*a*b + A*b²)*c + (B*a*b² - A*b³ - (B*a² - A*a*b)*c)*x)/(b⁴*c - 2*a*b²*c² + a²*c³ + (a*b⁴ - 2*a²*b²*c + a³*c²)*x² + 2*(b⁵ - 2*a*b³*c + a²*b*c²)*x]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(75) = 150

time = 0.56, size = 323, normalized size = 3.59

$$\frac{\sqrt{\frac{1}{(ac-b^2)^2}} (-Ab+Ba) \log\left(x + \frac{-Ab^2+Ba^2-x^2 \sqrt{\frac{1}{(ac-b^2)^2}} (-Ab+Ba)+2a^2 \sqrt{\frac{1}{(ac-b^2)^2}} (-Ab+Ba)-b^2 \sqrt{\frac{1}{(ac-b^2)^2}} (-Ab+Ba)}{-2abx}\right)}{4} + \frac{\sqrt{\frac{1}{(ac-b^2)^2}} (-Ab+Ba) \log\left(x + \frac{-Ab^2+Ba^2-x^2 \sqrt{\frac{1}{(ac-b^2)^2}} (-Ab+Ba)-2a^2 \sqrt{\frac{1}{(ac-b^2)^2}} (-Ab+Ba)+b^2 \sqrt{\frac{1}{(ac-b^2)^2}} (-Ab+Ba)}{2abx}\right)}{4} + \frac{-Ac+Bb+x(-Ab+Ba)}{2ac^3-2Bc+x^2(2a^2c-2ab^2)+x(4abc-4b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A*x+B)/(a*x**2+2*b*x+c)**2,x)

[Out] -sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a)*log(x + (-A*b**2 + B*a*b - a**2*c**2)*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a) + 2*a*b**2*c*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a) - b**4*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a))/(-A*a*b + B*a**2))/4 + sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a)*log(x + (-A*b**2 + B*a*b + a**2*c**2)*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a) - 2*a*b**2*c*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a) + b**4*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a))/(-A*a*b + B*a**2))/4 + (-A*c + B*b + x*(-A*b + B*a))/(2*a*c**2 - 2*b**2*c + x**2*(2*a**2*c - 2*a*b**2) + x*(4*a*b*c - 4*b**3))

Giac [A]

time = 0.00, size = 95, normalized size = 1.06

$$\frac{xAb - xBa + Ac - Bb}{(-2ac + 2b^2)(x^2a + 2xb + c)} + \frac{(Ab - Ba) \arctan\left(\frac{ax+b}{\sqrt{-b^2 + ac}}\right)}{2(-ac + b^2) \sqrt{-b^2 + ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A*x+B)/(a*x^2+2*b*x+c)^2,x)

[Out] -1/2*(B*a - A*b)*arctan((a*x + b)/sqrt(-b² + a*c))/((b² - a*c)*sqrt(-b² + a*c)) - 1/2*(B*a*x - A*b*x + B*b - A*c)/((a*x² + 2*b*x + c)*(b² - a*c))

Mupad [B]

time = 0.29, size = 159, normalized size = 1.77

$$\frac{\operatorname{atan}\left(\frac{2(ac-b^2) \left(\frac{(4b^3-4abc)(Ab-Ba)}{8(ac-b^2)^{5/2}} - \frac{ax(Ab-Ba)}{2(ac-b^2)^{3/2}}\right)}{Ab-Ba}\right) (Ab - Ba)}{2(ac - b^2)^{3/2}} - \frac{\frac{Ac-Bb}{2(ac-b^2)} + \frac{x(Ab-Ba)}{2(ac-b^2)}}{ax^2 + 2bx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B + A*x)/(c + 2*b*x + a*x^2)^2,x)`

[Out]
$$\left(\operatorname{atan}\left(\frac{2*(a*c - b^2)*((4*b^3 - 4*a*b*c)*(A*b - B*a))/(8*(a*c - b^2)^{(5/2)}) - (a*x*(A*b - B*a))/(2*(a*c - b^2)^{(3/2)})}{(A*b - B*a)*(A*b - B*a)/(2*(a*c - b^2)^{(3/2)})} \right) - \left(\frac{A*c - B*b}{2*(a*c - b^2)} + \frac{x*(A*b - B*a)}{2*(a*c - b^2)} \right) \right) / (c + 2*b*x + a*x^2)$$

$$3.150 \quad \int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2} dx$$

Optimal. Leaf size=38

$$\frac{1-x}{5-4x+x^2} - 2 \tan^{-1}(2-x) + \frac{5}{2} \log(5-4x+x^2)$$

[Out] (1-x)/(x^2-4*x+5)+2*arctan(-2+x)+5/2*ln(x^2-4*x+5)

Rubi [A]

time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1674, 648, 632, 210, 642}

$$\frac{1-x}{x^2-4x+5} + \frac{5}{2} \log(x^2-4x+5) - 2 \tan^{-1}(2-x)$$

Antiderivative was successfully verified.

[In] Int[(-41 + 55*x - 27*x^2 + 5*x^3)/(5 - 4*x + x^2)^2,x]

[Out] (1 - x)/(5 - 4*x + x^2) - 2*ArcTan[2 - x] + (5*Log[5 - 4*x + x^2])/2

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x])/b], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(
p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{-41 + 55x - 27x^2 + 5x^3}{(5 - 4x + x^2)^2} dx &= \frac{1 - x}{5 - 4x + x^2} + \frac{1}{4} \int \frac{-32 + 20x}{5 - 4x + x^2} dx \\ &= \frac{1 - x}{5 - 4x + x^2} + 2 \int \frac{1}{5 - 4x + x^2} dx + \frac{5}{2} \int \frac{-4 + 2x}{5 - 4x + x^2} dx \\ &= \frac{1 - x}{5 - 4x + x^2} + \frac{5}{2} \log(5 - 4x + x^2) - 4 \operatorname{Subst} \left(\int \frac{1}{-4 - x^2} dx, x, -4 + 2x \right) \\ &= \frac{1 - x}{5 - 4x + x^2} - 2 \tan^{-1}(2 - x) + \frac{5}{2} \log(5 - 4x + x^2) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 38, normalized size = 1.00

$$\frac{1 - x}{5 - 4x + x^2} - 2 \tan^{-1}(2 - x) + \frac{5}{2} \log(5 - 4x + x^2)$$

Antiderivative was successfully verified.

[In] Integrate[(-41 + 55*x - 27*x^2 + 5*x^3)/(5 - 4*x + x^2)^2,x]

[Out] (1 - x)/(5 - 4*x + x^2) - 2*ArcTan[2 - x] + (5*Log[5 - 4*x + x^2])/2

Mathics [A]

time = 2.75, size = 44, normalized size = 1.16

$$\frac{1 - x + \frac{(5 - 4x + x^2)(4 \operatorname{ArcTan}[-2 + x] + 5 \operatorname{Log}[5 - 4x + x^2])}{2}}{5 - 4x + x^2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(5*x^3 - 27*x^2 + 55*x - 41)/(x^2 - 4*x + 5)^2,x]')

[Out] $(1 - x + (5 - 4x + x^2) (4 \operatorname{ArcTan}[-2 + x] + 5 \operatorname{Log}[5 - 4x + x^2]) / 2) / (5 - 4x + x^2)$

Maple [A]

time = 0.09, size = 35, normalized size = 0.92

method	result	size
default	$\frac{1-x}{x^2-4x+5} + 2 \arctan(-2+x) + \frac{5 \ln(x^2-4x+5)}{2}$	35
risch	$\frac{1-x}{x^2-4x+5} + 2 \arctan(-2+x) + \frac{5 \ln(x^2-4x+5)}{2}$	35

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x,method=_RETURNVERBOSE)`

[Out] $(1-x)/(x^2-4*x+5)+2*\arctan(-2+x)+5/2*\ln(x^2-4*x+5)$

Maxima [A]

time = 0.35, size = 33, normalized size = 0.87

$$-\frac{x-1}{x^2-4x+5} + 2 \arctan(x-2) + \frac{5}{2} \log(x^2-4x+5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x, algorithm="maxima")`

[Out] $-(x-1)/(x^2-4*x+5) + 2*\arctan(x-2) + 5/2*\log(x^2-4*x+5)$

Fricas [A]

time = 0.31, size = 50, normalized size = 1.32

$$\frac{4(x^2-4x+5) \arctan(x-2) + 5(x^2-4x+5) \log(x^2-4x+5) - 2x+2}{2(x^2-4x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x, algorithm="fricas")`

[Out] $1/2*(4*(x^2-4*x+5)*\arctan(x-2) + 5*(x^2-4*x+5)*\log(x^2-4*x+5) - 2*x+2)/(x^2-4*x+5)$

Sympy [A]

time = 0.07, size = 31, normalized size = 0.82

$$\frac{1-x}{x^2-4x+5} + \frac{5 \log(x^2-4x+5)}{2} + 2 \operatorname{atan}(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**3-27*x**2+55*x-41)/(x**2-4*x+5)**2,x)

[Out] (1 - x)/(x**2 - 4*x + 5) + 5*log(x**2 - 4*x + 5)/2 + 2*atan(x - 2)

Giac [A]

time = 0.00, size = 41, normalized size = 1.08

$$\frac{5}{2} \ln(x^2 - 4x + 5) + 2 \arctan(x - 2) + \frac{\frac{1}{2}(-2x + 2)}{x^2 - 4x + 5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x)

[Out] -(x - 1)/(x^2 - 4*x + 5) + 2*arctan(x - 2) + 5/2*log(x^2 - 4*x + 5)

Mupad [B]

time = 0.20, size = 41, normalized size = 1.08

$$2 \operatorname{atan}(x - 2) + \frac{5 \ln(x^2 - 4x + 5)}{2} - \frac{x}{x^2 - 4x + 5} + \frac{1}{x^2 - 4x + 5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((55*x - 27*x^2 + 5*x^3 - 41)/(x^2 - 4*x + 5)^2,x)

[Out] 2*atan(x - 2) + (5*log(x^2 - 4*x + 5))/2 - x/(x^2 - 4*x + 5) + 1/(x^2 - 4*x + 5)

$$3.151 \quad \int \frac{1}{(-1+x^3)^2} dx$$

Optimal. Leaf size=57

$$\frac{x}{3(1-x^3)} + \frac{2 \tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2}{9} \log(1-x) + \frac{1}{9} \log(1+x+x^2)$$

[Out] 1/3*x/(-x^3+1)-2/9*ln(1-x)+1/9*ln(x^2+x+1)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {205, 206, 31, 648, 632, 210, 642}

$$\frac{x}{3(1-x^3)} + \frac{1}{9} \log(x^2+x+1) - \frac{2}{9} \log(1-x) + \frac{2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^3)^(-2), x]

[Out] x/(3*(1 - x^3)) + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (2*Log[1 - x])/9 + Log[1 + x + x^2]/9

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 205

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 206

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 210

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & (LtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(-1+x^3)^2} dx &= \frac{x}{3(1-x^3)} - \frac{2}{3} \int \frac{1}{-1+x^3} dx \\
 &= \frac{x}{3(1-x^3)} - \frac{2}{9} \int \frac{1}{-1+x} dx - \frac{2}{9} \int \frac{-2-x}{1+x+x^2} dx \\
 &= \frac{x}{3(1-x^3)} - \frac{2}{9} \log(1-x) + \frac{1}{9} \int \frac{1+2x}{1+x+x^2} dx + \frac{1}{3} \int \frac{1}{1+x+x^2} dx \\
 &= \frac{x}{3(1-x^3)} - \frac{2}{9} \log(1-x) + \frac{1}{9} \log(1+x+x^2) - \frac{2}{3} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1+2x \right) \\
 &= \frac{x}{3(1-x^3)} + \frac{2 \tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)}{3\sqrt{3}} - \frac{2}{9} \log(1-x) + \frac{1}{9} \log(1+x+x^2)
 \end{aligned}$$

Mathematica [A]

time = 0.01, size = 49, normalized size = 0.86

$$\frac{1}{9} \left(-\frac{3x}{-1+x^3} + 2\sqrt{3} \tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right) - 2 \log(1-x) + \log(1+x+x^2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^3)^(-2), x]

[Out] ((-3*x)/(-1 + x^3) + 2*sqrt[3]*ArcTan[(1 + 2*x)/sqrt[3]] - 2*Log[1 - x] + Log[1 + x + x^2])/9

Mathics [A]

time = 2.11, size = 49, normalized size = 0.86

$$\frac{-3x + (-1 + x^3) \left(\text{Log}[1 + x + x^2] - 2\text{Log}[-1 + x] + 2\sqrt{3} \text{ArcTan} \left[\frac{\sqrt{3}(1+2x)}{3} \right] \right)}{-9 + 9x^3}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x^3 - 1)^2, x]')

[Out] (-3 x + (-1 + x ^ 3) (Log[1 + x + x ^ 2] - 2 Log[-1 + x] + 2 Sqrt[3] ArcTan[Sqrt[3] (1 + 2 x) / 3])) / (9 (-1 + x ^ 3))

Maple [A]

time = 0.06, size = 53, normalized size = 0.93

method	result	size
risch	$-\frac{x}{3(x^3-1)} + \frac{\ln(4x^2+4x+4)}{9} + \frac{2 \arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9} - \frac{2 \ln(-1+x)}{9}$	47
default	$-\frac{1}{9(-1+x)} - \frac{2 \ln(-1+x)}{9} + \frac{-1+x}{9x^2+9x+9} + \frac{\ln(x^2+x+1)}{9} + \frac{2 \arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}$	53
meijerg	$-\frac{(-1)^{\frac{2}{3}} \left(\frac{3x(-1)^{\frac{1}{3}}}{-3x^3+3} - \frac{2x(-1)^{\frac{1}{3}} \left(\ln\left(1 - (x^3)^{\frac{1}{3}}\right) - \frac{\ln\left(1 + (x^3)^{\frac{1}{3}} + (x^3)^{\frac{2}{3}}\right)}{2} - \sqrt{3} \arctan\left(\frac{\sqrt{3} (x^3)^{\frac{1}{3}}}{2 + (x^3)^{\frac{1}{3}}}\right) \right)}{3(x^3)^{\frac{1}{3}}} \right)}{3}$	86

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3-1)^2, x, method=_RETURNVERBOSE)

[Out] -1/9/(-1+x)-2/9*ln(-1+x)+1/9*(-1+x)/(x^2+x+1)+1/9*ln(x^2+x+1)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)

Maxima [A]

time = 0.42, size = 42, normalized size = 0.74

$$\frac{2}{9} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x + 1) \right) - \frac{x}{3(x^3 - 1)} + \frac{1}{9} \log(x^2 + x + 1) - \frac{2}{9} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-1)^2,x, algorithm="maxima")

[Out] $\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}x/(x^3-1) + \frac{1}{9}\log(x^2+x+1) - \frac{2}{9}\log(x-1)$

Fricas [A]

time = 0.41, size = 58, normalized size = 1.02

$$\frac{2\sqrt{3}(x^3-1)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + (x^3-1)\log(x^2+x+1) - 2(x^3-1)\log(x-1) - 3x}{9(x^3-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-1)^2,x, algorithm="fricas")

[Out] $\frac{1}{9}(2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + (x^3-1)\log(x^2+x+1) - 2(x^3-1)\log(x-1) - 3x)/(x^3-1)$

Sympy [A]

time = 0.08, size = 53, normalized size = 0.93

$$-\frac{x}{3x^3-3} - \frac{2\log(x-1)}{9} + \frac{\log(x^2+x+1)}{9} + \frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**3-1)**2,x)

[Out] $-\frac{x}{(3x^3-3)} - \frac{2\log(x-1)}{9} + \frac{\log(x^2+x+1)}{9} + \frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$

Giac [A]

time = 0.00, size = 54, normalized size = 0.95

$$-\frac{2}{9}\ln|x-1| + \frac{\ln(x^2+x+1)}{9} + \frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{x}{3(x^3-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-1)^2,x)

[Out] $\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}x/(x^3-1) + \frac{1}{9}\log(x^2+x+1) - \frac{2}{9}\log(\operatorname{abs}(x-1))$

Mupad [B]

time = 0.08, size = 60, normalized size = 1.05

$$-\frac{2\ln(x-1)}{9} - \frac{x}{3(x^3-1)} - \ln\left(x + \frac{1}{2} - \frac{\sqrt{3}\operatorname{li}}{2}\right) \left(-\frac{1}{9} + \frac{\sqrt{3}\operatorname{li}}{9}\right) + \ln\left(2x+1 + \sqrt{3}\operatorname{li}\right) \left(\frac{1}{9} + \frac{\sqrt{3}\operatorname{li}}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 - 1)^2,x)
```

```
[Out] log(2*x + 3^(1/2)*1i + 1)*((3^(1/2)*1i)/9 + 1/9) - x/(3*(x^3 - 1)) - log(x  
- (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 - 1/9) - (2*log(x - 1))/9
```

$$3.152 \quad \int \frac{4+3x^4}{x^2(1+x^2)^3} dx$$

Optimal. Leaf size=36

$$-\frac{4}{x} - \frac{7x}{4(1+x^2)^2} - \frac{25x}{8(1+x^2)} - \frac{57}{8} \tan^{-1}(x)$$

[Out] $-4/x - 7/4*x/(x^2+1)^2 - 25/8*x/(x^2+1) - 57/8*\arctan(x)$

Rubi [A]

time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1274, 467, 464, 209}

$$-\frac{25x}{8(x^2+1)} - \frac{7x}{4(x^2+1)^2} - \frac{4}{x} - \frac{57}{8} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(4 + 3*x^4)/(x^2*(1 + x^2)^3), x]$

[Out] $-4/x - (7*x)/(4*(1 + x^2)^2) - (25*x)/(8*(1 + x^2)) - (57*\text{ArcTan}[x])/8$

Rule 209

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))*\text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 464

$\text{Int}[(e_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})), x_Symbol] \rightarrow \text{Simp}[c*(e*x)^{(m+1)}*((a + b*x^n)^{(p+1)}/(a*e^{(m+1)})), x] + \text{Dist}[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), \text{Int}[(e*x)^{(m+n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ (\text{IntegerQ}[n] \ || \ \text{GtQ}[e, 0]) \ \&\& \ ((\text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[m, -1]) \ || \ (\text{LtQ}[n, 0] \ \&\& \ \text{GtQ}[m+n, -1])) \ \&\& \ !\text{ILtQ}[p, -1]$

Rule 467

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^2)^{(p_)}*((c_ + (d_)*(x_)^2)), x_Symbol] \rightarrow \text{Simp}[(-a)^{(m/2-1)}*(b*c - a*d)*x*((a + b*x^2)^{(p+1)}/(2*b^{(m/2+1)}*(p+1))), x] + \text{Dist}[1/(2*b^{(m/2+1)}*(p+1)), \text{Int}[x^m*(a + b*x^2)^{(p+1)}*\text{ExpandToSum}[2*b*(p+1)*\text{Together}[(b^{(m/2)}*(c + d*x^2) - (-a)^{(m/2-1)}*(b*c - a*d)*x^{(-m+2)})/(a + b*x^2)] - ((-a)^{(m/2-1)}*(b*c - a*d))/x^m, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{ILtQ}[m/2$

, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1274

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*x*((d + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + c*x^4)^p - ((c*d^2 + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)], x], x] /; FreeQ[{a, c, d, e}, x] && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{4 + 3x^4}{x^2(1+x^2)^3} dx &= -\frac{7x}{4(1+x^2)^2} - \frac{1}{4} \int \frac{-16 + 9x^2}{x^2(1+x^2)^2} dx \\ &= -\frac{7x}{4(1+x^2)^2} - \frac{25x}{8(1+x^2)} + \frac{1}{8} \int \frac{32 - 25x^2}{x^2(1+x^2)} dx \\ &= -\frac{4}{x} - \frac{7x}{4(1+x^2)^2} - \frac{25x}{8(1+x^2)} - \frac{57}{8} \int \frac{1}{1+x^2} dx \\ &= -\frac{4}{x} - \frac{7x}{4(1+x^2)^2} - \frac{25x}{8(1+x^2)} - \frac{57}{8} \tan^{-1}(x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 33, normalized size = 0.92

$$-\frac{32 + 103x^2 + 57x^4}{8x(1+x^2)^2} - \frac{57}{8} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 3*x^4)/(x^2*(1 + x^2)^3), x]

[Out] -1/8*(32 + 103*x^2 + 57*x^4)/(x*(1 + x^2)^2) - (57*ArcTan[x])/8

Mathics [A]

time = 1.96, size = 44, normalized size = 1.22

$$\frac{-32 - 57x \operatorname{ArcTan}[x] (1 + 2x^2 + x^4) - 103x^2 - 57x^4}{8x(1 + 2x^2 + x^4)}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(3*x^4 + 4)/(x^2*(x^2 + 1)^3),x]')`

[Out] $(-32 - 57 x \operatorname{ArcTan}[x] (1 + 2 x^2 + x^4) - 103 x^2 - 57 x^4) / (8 x (1 + 2 x^2 + x^4))$

Maple [A]

time = 0.05, size = 29, normalized size = 0.81

method	result	size
default	$-\frac{4}{x} - \frac{\frac{25}{8}x^3 + \frac{39}{8}x}{(x^2+1)^2} - \frac{57 \arctan(x)}{8}$	29
risch	$\frac{-\frac{57}{8}x^4 - \frac{103}{8}x^2 - 4}{(x^2+1)^2 x} - \frac{57 \arctan(x)}{8}$	29
meijerg	$-\frac{15x^4 + 25x^2 + 8}{2x(x^2+1)^2} - \frac{57 \arctan(x)}{8} - \frac{x(-3x^2+3)}{8(x^2+1)^2}$	47

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^4+4)/x^2/(x^2+1)^3,x,method=_RETURNVERBOSE)`

[Out] $-4/x - (25/8*x^3 + 39/8*x)/(x^2+1)^2 - 57/8*\arctan(x)$

Maxima [A]

time = 0.35, size = 31, normalized size = 0.86

$$-\frac{57x^4 + 103x^2 + 32}{8(x^5 + 2x^3 + x)} - \frac{57}{8} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="maxima")`

[Out] $-1/8*(57*x^4 + 103*x^2 + 32)/(x^5 + 2*x^3 + x) - 57/8*\arctan(x)$

Fricas [A]

time = 0.31, size = 40, normalized size = 1.11

$$-\frac{57x^4 + 103x^2 + 57(x^5 + 2x^3 + x) \arctan(x) + 32}{8(x^5 + 2x^3 + x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="fricas")`

[Out] $-1/8*(57*x^4 + 103*x^2 + 57*(x^5 + 2*x^3 + x)*\arctan(x) + 32)/(x^5 + 2*x^3 + x)$

Sympy [A]

time = 0.07, size = 32, normalized size = 0.89

$$\frac{-57x^4 - 103x^2 - 32}{8x^5 + 16x^3 + 8x} - \frac{57 \operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**4+4)/x**2/(x**2+1)**3,x)

[Out] (-57*x**4 - 103*x**2 - 32)/(8*x**5 + 16*x**3 + 8*x) - 57*atan(x)/8

Giac [A]

time = 0.00, size = 34, normalized size = 0.94

$$-\frac{4}{x} + \frac{-25x^3 - 39x}{8(x^2 + 1)^2} - \frac{57}{8} \arctan x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x)

[Out] -1/8*(25*x^3 + 39*x)/(x^2 + 1)^2 - 4/x - 57/8*arctan(x)

Mupad [B]

time = 0.18, size = 29, normalized size = 0.81

$$-\frac{57 \operatorname{atan}(x)}{8} - \frac{\frac{57x^4}{8} + \frac{103x^2}{8} + 4}{x(x^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^4 + 4)/(x^2*(x^2 + 1)^3),x)

[Out] - (57*atan(x))/8 - ((103*x^2)/8 + (57*x^4)/8 + 4)/(x*(x^2 + 1)^2)

3.153 $\int \frac{x}{1+x^6} dx$

Optimal. Leaf size=49

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{6} \log(1+x^2) - \frac{1}{12} \log(1-x^2+x^4)$$

[Out] 1/6*ln(x^2+1)-1/12*ln(x^4-x^2+1)-1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {281, 206, 31, 648, 632, 210, 642}

$$\frac{1}{6} \log(x^2+1) - \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{12} \log(x^4-x^2+1)$$

Antiderivative was successfully verified.

[In] Int[x/(1 + x^6), x]

[Out] -1/2*ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] + Log[1 + x^2]/6 - Log[1 - x^2 + x^4]/12

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(n-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & (LtQ[a, 0] || LtQ[b, 0])

Rule 281

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x

$x^k, x] /; k \neq 1] /; \text{FreeQ}\{a, b, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$

Rule 632

$\text{Int}[(a_.) + (b_.)(x_) + (c_.)(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4ac - x^2, x], x], x, b + 2cx], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4ac, 0]$

Rule 642

$\text{Int}[(d_.) + (e_.)(x_)] / [(a_.) + (b_.)(x_) + (c_.)(x_)^2], x_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]/b), x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2cd - be, 0]$

Rule 648

$\text{Int}[(d_.) + (e_.)(x_)] / [(a_.) + (b_.)(x_) + (c_.)(x_)^2], x_Symbol] \rightarrow \text{Dist}[(2cd - be)/(2c), \text{Int}[1/(a + bx + cx^2), x], x] + \text{Dist}[e/(2c), \text{Int}[(b + 2cx)/(a + bx + cx^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2cd - be, 0] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4ac]$

Rubi steps

$$\begin{aligned} \int \frac{x}{1+x^6} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+x^3} dx, x, x^2 \right) \\ &= \frac{1}{6} \text{Subst} \left(\int \frac{1}{1+x} dx, x, x^2 \right) + \frac{1}{6} \text{Subst} \left(\int \frac{2-x}{1-x+x^2} dx, x, x^2 \right) \\ &= \frac{1}{6} \log(1+x^2) - \frac{1}{12} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^2 \right) + \frac{1}{4} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^2 \right) \\ &= \frac{1}{6} \log(1+x^2) - \frac{1}{12} \log(1-x^2+x^4) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^2 \right) \\ &= -\frac{\tan^{-1} \left(\frac{1-2x^2}{\sqrt{3}} \right)}{2\sqrt{3}} + \frac{1}{6} \log(1+x^2) - \frac{1}{12} \log(1-x^2+x^4) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 78, normalized size = 1.59

$$\frac{1}{12} \left(-2\sqrt{3} \tan^{-1}(\sqrt{3} - 2x) - 2\sqrt{3} \tan^{-1}(\sqrt{3} + 2x) + 2\log(1+x^2) - \log(1 - \sqrt{3}x + x^2) - \log(1 + \sqrt{3}x + x^2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 + x^6),x]

[Out] (-2*Sqrt[3]*ArcTan[Sqrt[3] - 2*x] - 2*Sqrt[3]*ArcTan[Sqrt[3] + 2*x] + 2*Log[1 + x^2] - Log[1 - Sqrt[3]*x + x^2] - Log[1 + Sqrt[3]*x + x^2])/12

Mathics [A]

time = 2.07, size = 40, normalized size = 0.82

$$-\frac{\text{Log}[1 - x^2 + x^4]}{12} + \frac{\sqrt{3} \text{ArcTan}\left[\frac{\sqrt{3}(-1+2x^2)}{3}\right]}{6} + \frac{\text{Log}[1 + x^2]}{6}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x/(x^6 + 1),x]')

[Out] -Log[1 - x^2 + x^4] / 12 + Sqrt[3] ArcTan[Sqrt[3] (-1 + 2 x^2) / 3] / 6 + Log[1 + x^2] / 6

Maple [A]

time = 0.05, size = 41, normalized size = 0.84

method	result	size
default	$\frac{\ln(x^2+1)}{6} - \frac{\ln(x^4-x^2+1)}{12} + \frac{\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	41
risch	$-\frac{\ln(4x^4-4x^2+4)}{12} + \frac{\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)\sqrt{3}}{6} + \frac{\ln(x^2+1)}{6}$	43
meijerg	$\frac{x^2 \ln\left(1+(x^6)^{\frac{1}{3}}\right)}{6(x^6)^{\frac{1}{3}}} - \frac{x^2 \ln\left(1-(x^6)^{\frac{1}{3}}+(x^6)^{\frac{2}{3}}\right)}{12(x^6)^{\frac{1}{3}}} + \frac{x^2 \sqrt{3} \arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{3}}}{2-(x^6)^{\frac{1}{3}}}\right)}{6(x^6)^{\frac{1}{3}}}$	80

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^6+1),x,method=_RETURNVERBOSE)

[Out] 1/6*ln(x^2+1)-1/12*ln(x^4-x^2+1)+1/6*arctan(1/3*(2*x^2-1)*3^(1/2))*3^(1/2)

Maxima [A]

time = 0.39, size = 40, normalized size = 0.82

$$\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 - 1)\right) - \frac{1}{12} \log(x^4 - x^2 + 1) + \frac{1}{6} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^6+1),x, algorithm="maxima")

[Out] $\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2 - 1)\right) - \frac{1}{12}\log(x^4 - x^2 + 1) + \frac{1}{6}\log(x^2 + 1)$

Fricas [A]

time = 0.45, size = 40, normalized size = 0.82

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2 - 1)\right) - \frac{1}{12}\log(x^4 - x^2 + 1) + \frac{1}{6}\log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^6+1),x, algorithm="fricas")`

[Out] $\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2 - 1)\right) - \frac{1}{12}\log(x^4 - x^2 + 1) + \frac{1}{6}\log(x^2 + 1)$

Sympy [A]

time = 0.07, size = 46, normalized size = 0.94

$$\frac{\log(x^2 + 1)}{6} - \frac{\log(x^4 - x^2 + 1)}{12} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} - \frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x**6+1),x)`

[Out] $\log(x^{**2} + 1)/6 - \log(x^{**4} - x^{**2} + 1)/12 + \sqrt{3}\operatorname{atan}(2*\sqrt{3}*x^{**2}/3 - \sqrt{3}/3)/6$

Giac [A]

time = 0.00, size = 51, normalized size = 1.04

$$\frac{-\frac{\ln(x^4 - x^2 + 1)}{6} + \frac{1}{3}\sqrt{3}\arctan\left(\frac{2(x^2 - \frac{1}{2})}{\sqrt{3}}\right) + \frac{\ln(x^2 + 1)}{3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^6+1),x)`

[Out] $\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2 - 1)\right) - \frac{1}{12}\log(x^4 - x^2 + 1) + \frac{1}{6}\log(x^2 + 1)$

Mupad [B]

time = 0.09, size = 52, normalized size = 1.06

$$\frac{\ln(x^2 + 1)}{6} - \ln\left(x^2 - \frac{\sqrt{3} \operatorname{li}}{2} - \frac{1}{2}\right) \left(\frac{1}{12} + \frac{\sqrt{3} \operatorname{li}}{12}\right) + \ln\left(x^2 + \frac{\sqrt{3} \operatorname{li}}{2} - \frac{1}{2}\right) \left(-\frac{1}{12} + \frac{\sqrt{3} \operatorname{li}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(x^6 + 1),x)`

[Out] $\log(x^2 + 1)/6 - \log(x^2 - (3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/12 + 1/12) + \log((3^{(1/2)}*1i)/2 + x^2 - 1/2)*((3^{(1/2)}*1i)/12 - 1/12)$

$$3.154 \quad \int \frac{-1+x^{-1+n}}{-nx+x^n} dx$$

Optimal. Leaf size=13

$$\frac{\log(-nx+x^n)}{n}$$

[Out] $\ln(-n*x+x^n)/n$

Rubi [A]

time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.54, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1607, 528, 457, 78}

$$\frac{\log(1-nx^{1-n})}{n} + \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-1 + x^{(-1 + n)})/(-n*x) + x^n, x]$

[Out] $\text{Log}[x] + \text{Log}[1 - n*x^{(1 - n)}]/n$

Rule 78

$\text{Int}[(a_. + (b_.)*(x_.))*((c_. + (d_.)*(x_.))^{(n_.)*((e_. + (f_.)*(x_.))^{(p_.)}, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& ((\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) \|\ \text{EqQ}[p, 1] \|\ (\text{IGtQ}[p, 0] \&\& (!\text{IntegerQ}[n] \|\ \text{LeQ}[9*p + 5*(n + 2), 0] \|\ \text{GeQ}[n + p + 1, 0] \|\ (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

Rule 457

$\text{Int}[(x_.)^{(m_.)*((a_. + (b_.)*(x_.)^{(n_.))^{(p_.)*((c_. + (d_.)*(x_.)^{(n_.))^{(q_.)}, x_Symbol] :> \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 528

$\text{Int}[(x_.)^{(m_.)*((c_. + (d_.)*(x_.)^{(mn_.))^{(q_.)*((a_. + (b_.)*(x_.)^{(n_.))^{(p_.)}, x_Symbol] :> \text{Int}[x^{(m - n*q)}*(a + b*x^n)^p*(d + c*x^n)^q, x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x\} \&\& \text{EqQ}[mn, -n] \&\& \text{IntegerQ}[q] \&\& (\text{PosQ}[n] \|\ !\text{IntegerQ}[p])$

Rule 1607

$\text{Int}[(u_.)*((a_.)*(x_.)^{(p_.)} + (b_.)*(x_.)^{(q_.))^{(n_.)}, x_Symbol] :> \text{Int}[u*x^{(n*p)}*(a + b*x^{(q - p)})^n, x] /; \text{FreeQ}\{a, b, p, q\}, x\} \&\& \text{IntegerQ}[n] \&\&$

PosQ[q - p]

Rubi steps

$$\begin{aligned}
 \int \frac{-1 + x^{-1+n}}{-nx + x^n} dx &= \int \frac{x^{-n}(-1 + x^{-1+n})}{1 - nx^{1-n}} dx \\
 &= \int \frac{1 - x^{1-n}}{x(1 - nx^{1-n})} dx \\
 &= \frac{\text{Subst}\left(\int \frac{1-x}{x(1-nx)} dx, x, x^{1-n}\right)}{1-n} \\
 &= \frac{\text{Subst}\left(\int \left(\frac{1}{x} + \frac{1-n}{-1+nx}\right) dx, x, x^{1-n}\right)}{1-n} \\
 &= \log(x) + \frac{\log(1 - nx^{1-n})}{n}
 \end{aligned}$$

Mathematica [A]

time = 0.02, size = 13, normalized size = 1.00

$$\frac{\log(-nx + x^n)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^(-1 + n))/(-n*x) + x^n], x]

[Out] Log[-(n*x) + x^n]/n

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.67, size = 27, normalized size = 2.08

$$\text{Piecewise}\left[\left[\left[\left[\frac{\text{Log}\left[-\frac{x^n}{n} + x\right]}{n}, n \neq 0\right]\right], -x + \text{Log}[x]\right]\right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(x^(n - 1) - 1)/(x^n - n*x), x]')

[Out] Piecewise[{{Log[-x^n / n + x] / n, n != 0}}, -x + Log[x]]

Maple [A]

time = 0.07, size = 14, normalized size = 1.08

method	result	size
risch	$\frac{\ln(-nx+x^n)}{n}$	14
norman	$\frac{\ln(nx-e^n \ln(x))}{n}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x^(-1+n))/(-n*x+x^n),x,method=_RETURNVERBOSE)
```

```
[Out] ln(-n*x+x^n)/n
```

Maxima [A]

time = 0.25, size = 14, normalized size = 1.08

$$\frac{\log(nx - x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(-1+n))/(-n*x+x^n),x, algorithm="maxima")
```

```
[Out] log(n*x - x^n)/n
```

Fricas [A]

time = 0.31, size = 13, normalized size = 1.00

$$\frac{\log(-nx + x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(-1+n))/(-n*x+x^n),x, algorithm="fricas")
```

```
[Out] log(-n*x + x^n)/n
```

Sympy [A]

time = 0.83, size = 14, normalized size = 1.08

$$\begin{cases} \frac{\log(-nx+x^n)}{n} & \text{for } n \neq 0 \\ -x + \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(-1+n))/(-n*x+x**n),x)
```

```
[Out] Piecewise((log(-n*x + x**n)/n, Ne(n, 0)), (-x + log(x), True))
```

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(-1+n))/(-n*x+x^n),x)
```

```
[Out] Could not integrate
```

Mupad [B]

time = 0.32, size = 14, normalized size = 1.08

$$\frac{\ln(n x - x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^(n - 1) - 1)/(n*x - x^n),x)
```

```
[Out] log(n*x - x^n)/n
```

$$3.155 \quad \int \frac{x^3}{1-2x^2+3x^4} dx$$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{1-3x^2}{\sqrt{2}}\right)}{6\sqrt{2}} + \frac{1}{12} \log(1-2x^2+3x^4)$$

[Out] 1/12*ln(3*x^4-2*x^2+1)-1/12*arctan(1/2*(-3*x^2+1)*2^(1/2))*2^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1128, 648, 632, 210, 642}

$$\frac{1}{12} \log(3x^4 - 2x^2 + 1) - \frac{\tan^{-1}\left(\frac{1-3x^2}{\sqrt{2}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(1 - 2*x^2 + 3*x^4),x]

[Out] -1/6*ArcTan[(1 - 3*x^2)/Sqrt[2]]/Sqrt[2] + Log[1 - 2*x^2 + 3*x^4]/12

Rule 210

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{1 - 2x^2 + 3x^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x}{1 - 2x + 3x^2} dx, x, x^2 \right) \\ &= \frac{1}{12} \text{Subst} \left(\int \frac{-2 + 6x}{1 - 2x + 3x^2} dx, x, x^2 \right) + \frac{1}{6} \text{Subst} \left(\int \frac{1}{1 - 2x + 3x^2} dx, x, x^2 \right) \\ &= \frac{1}{12} \log(1 - 2x^2 + 3x^4) - \frac{1}{3} \text{Subst} \left(\int \frac{1}{-8 - x^2} dx, x, 2(-1 + 3x^2) \right) \\ &= -\frac{\tan^{-1} \left(\frac{1-3x^2}{\sqrt{2}} \right)}{6\sqrt{2}} + \frac{1}{12} \log(1 - 2x^2 + 3x^4) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 38, normalized size = 0.93

$$\frac{1}{12} \left(\sqrt{2} \tan^{-1} \left(\frac{-1 + 3x^2}{\sqrt{2}} \right) + \log(1 - 2x^2 + 3x^4) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - 2*x^2 + 3*x^4),x]
```

```
[Out] (Sqrt[2]*ArcTan[(-1 + 3*x^2)/Sqrt[2]] + Log[1 - 2*x^2 + 3*x^4])/12
```

Mathics [A]

time = 2.03, size = 32, normalized size = 0.78

$$\frac{\sqrt{2} \text{ArcTan} \left[\frac{\sqrt{2}(-1+3x^2)}{2} \right]}{12} + \frac{\text{Log} \left[\frac{1}{3} - \frac{2x^2}{3} + x^4 \right]}{12}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3/(3*x^4 - 2*x^2 + 1),x]')
```


[Out] $\text{Sqrt}[2] \text{ArcTan}[\text{Sqrt}[2] (-1 + 3 x^2) / 2] / 12 + \text{Log}[1 / 3 - 2 x^2 / 3 + x^4] / 12$

Maple [A]

time = 0.02, size = 35, normalized size = 0.85

method	result	size
default	$\frac{\ln(3x^4-2x^2+1)}{12} + \frac{\sqrt{2} \arctan\left(\frac{(6x^2-2)\sqrt{2}}{4}\right)}{12}$	35
risch	$\frac{\ln(9x^4-6x^2+3)}{12} + \frac{\sqrt{2} \arctan\left(\frac{(3x^2-1)\sqrt{2}}{2}\right)}{12}$	35

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(3*x^4-2*x^2+1),x,method=_RETURNVERBOSE)`

[Out] $1/12*\ln(3*x^4-2*x^2+1)+1/12*2^{(1/2)}*\arctan(1/4*(6*x^2-2)*2^{(1/2)})$

Maxima [A]

time = 0.34, size = 34, normalized size = 0.83

$$\frac{1}{12} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} (3x^2 - 1)\right) + \frac{1}{12} \log(3x^4 - 2x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(3*x^4-2*x^2+1),x, algorithm="maxima")`

[Out] $1/12*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*(3*x^2 - 1)) + 1/12*\log(3*x^4 - 2*x^2 + 1)$

Fricas [A]

time = 0.32, size = 34, normalized size = 0.83

$$\frac{1}{12} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} (3x^2 - 1)\right) + \frac{1}{12} \log(3x^4 - 2x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(3*x^4-2*x^2+1),x, algorithm="fricas")`

[Out] $1/12*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*(3*x^2 - 1)) + 1/12*\log(3*x^4 - 2*x^2 + 1)$

Sympy [A]

time = 0.06, size = 42, normalized size = 1.02

$$\frac{\log\left(x^4 - \frac{2x^2}{3} + \frac{1}{3}\right)}{12} + \frac{\sqrt{2} \operatorname{atan}\left(\frac{3\sqrt{2}x^2}{2} - \frac{\sqrt{2}}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(3*x**4-2*x**2+1),x)

[Out] log(x**4 - 2*x**2/3 + 1/3)/12 + sqrt(2)*atan(3*sqrt(2)*x**2/2 - sqrt(2)/2)/12

Giac [A]

time = 0.00, size = 45, normalized size = 1.10

$$\frac{\frac{\ln(3x^4-2x^2+1)}{6} + \frac{\arctan\left(\frac{3x^2-1}{\sqrt{2}}\right)}{3\sqrt{2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(3*x^4-2*x^2+1),x)

[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*x^2 - 1)) + 1/12*log(3*x^4 - 2*x^2 + 1)

Mupad [B]

time = 0.20, size = 34, normalized size = 0.83

$$\frac{\ln\left(x^4 - \frac{2x^2}{3} + \frac{1}{3}\right)}{12} - \frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}x^2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(3*x^4 - 2*x^2 + 1),x)

[Out] log(x^4 - (2*x^2)/3 + 1/3)/12 - (2^(1/2)*atan(2^(1/2)/2 - (3*2^(1/2)*x^2)/2))/12

$$3.156 \quad \int \frac{x^5}{-4+x^2+3x^4} dx$$

Optimal. Leaf size=32

$$\frac{x^2}{6} + \frac{1}{14} \log(1-x^2) - \frac{8}{63} \log(4+3x^2)$$

[Out] 1/6*x^2+1/14*ln(-x^2+1)-8/63*ln(3*x^2+4)

Rubi [A]

time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1128, 717, 646, 31}

$$\frac{x^2}{6} + \frac{1}{14} \log(1-x^2) - \frac{8}{63} \log(3x^2+4)$$

Antiderivative was successfully verified.

[In] Int[x^5/(-4 + x^2 + 3*x^4),x]

[Out] x^2/6 + Log[1 - x^2]/14 - (8*Log[4 + 3*x^2])/63

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 646

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 717

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[e*((d + e*x)^(m-1)/(c*(m-1))), x] + Dist[1/c, Int[(d + e*x)^(m-2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]

Rule 1128

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m-1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free

Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{x^5}{-4 + x^2 + 3x^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{-4 + x + 3x^2} dx, x, x^2 \right) \\
 &= \frac{x^2}{6} + \frac{1}{6} \text{Subst} \left(\int \frac{4 - x}{-4 + x + 3x^2} dx, x, x^2 \right) \\
 &= \frac{x^2}{6} + \frac{3}{14} \text{Subst} \left(\int \frac{1}{-3 + 3x} dx, x, x^2 \right) - \frac{8}{21} \text{Subst} \left(\int \frac{1}{4 + 3x} dx, x, x^2 \right) \\
 &= \frac{x^2}{6} + \frac{1}{14} \log(1 - x^2) - \frac{8}{63} \log(4 + 3x^2)
 \end{aligned}$$

Mathematica [A]

time = 0.00, size = 32, normalized size = 1.00

$$\frac{x^2}{6} + \frac{1}{14} \log(1 - x^2) - \frac{8}{63} \log(4 + 3x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(-4 + x^2 + 3*x^4), x]

[Out] x^2/6 + Log[1 - x^2]/14 - (8*Log[4 + 3*x^2])/63

Mathics [A]

time = 1.91, size = 22, normalized size = 0.69

$$\frac{x^2}{6} - \frac{8 \text{Log} \left[\frac{4}{3} + x^2 \right]}{63} + \frac{\text{Log}[-1 + x^2]}{14}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^5/(3*x^4 + x^2 - 4), x]')

[Out] x ^ 2 / 6 - 8 Log[4 / 3 + x ^ 2] / 63 + Log[-1 + x ^ 2] / 14

Maple [A]

time = 0.02, size = 25, normalized size = 0.78

method	result	size
default	$\frac{x^2}{6} + \frac{\ln(x^2-1)}{14} - \frac{8 \ln(3x^2+4)}{63}$	25
risch	$\frac{x^2}{6} + \frac{\ln(x^2-1)}{14} - \frac{8 \ln(3x^2+4)}{63}$	25

norman	$\frac{x^2}{6} + \frac{\ln(-1+x)}{14} + \frac{\ln(1+x)}{14} - \frac{8 \ln(3x^2+4)}{63}$	29
--------	---	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(3*x^4+x^2-4),x,method=_RETURNVERBOSE)`

[Out] $1/6*x^2+1/14*\ln(x^2-1)-8/63*\ln(3*x^2+4)$

Maxima [A]

time = 0.34, size = 24, normalized size = 0.75

$$\frac{1}{6}x^2 - \frac{8}{63} \log(3x^2 + 4) + \frac{1}{14} \log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(3*x^4+x^2-4),x, algorithm="maxima")`

[Out] $1/6*x^2 - 8/63*\log(3*x^2 + 4) + 1/14*\log(x^2 - 1)$

Fricas [A]

time = 0.47, size = 24, normalized size = 0.75

$$\frac{1}{6}x^2 - \frac{8}{63} \log(3x^2 + 4) + \frac{1}{14} \log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(3*x^4+x^2-4),x, algorithm="fricas")`

[Out] $1/6*x^2 - 8/63*\log(3*x^2 + 4) + 1/14*\log(x^2 - 1)$

Sympy [A]

time = 0.06, size = 24, normalized size = 0.75

$$\frac{x^2}{6} + \frac{\log(x^2 - 1)}{14} - \frac{8 \log(x^2 + \frac{4}{3})}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(3*x**4+x**2-4),x)`

[Out] $x**2/6 + \log(x**2 - 1)/14 - 8*\log(x**2 + 4/3)/63$

Giac [A]

time = 0.00, size = 29, normalized size = 0.91

$$\frac{\ln|x^2 - 1|}{14} - \frac{8}{63} \ln(3x^2 + 4) + \frac{x^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(3*x^4+x^2-4),x)

[Out] 1/6*x^2 - 8/63*log(3*x^2 + 4) + 1/14*log(abs(x^2 - 1))

Mupad [B]

time = 0.09, size = 22, normalized size = 0.69

$$\frac{\ln(x^2 - 1)}{14} - \frac{8 \ln\left(x^2 + \frac{4}{3}\right)}{63} + \frac{x^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(x^2 + 3*x^4 - 4),x)

[Out] log(x^2 - 1)/14 - (8*log(x^2 + 4/3))/63 + x^2/6

3.157

$$\int \frac{x^2}{9-10x^3+x^6} dx$$

Optimal. Leaf size=25

$$-\frac{1}{24} \log(1-x^3) + \frac{1}{24} \log(9-x^3)$$

[Out] -1/24*ln(-x^3+1)+1/24*ln(-x^3+9)

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1366, 630, 31}

$$\frac{1}{24} \log(9-x^3) - \frac{1}{24} \log(1-x^3)$$

Antiderivative was successfully verified.

[In] Int[x^2/(9 - 10*x^3 + x^6),x]

[Out] -1/24*Log[1 - x^3] + Log[9 - x^3]/24

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 630

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 1366

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{9 - 10x^3 + x^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{9 - 10x + x^2} dx, x, x^3 \right) \\
&= \frac{1}{24} \text{Subst} \left(\int \frac{1}{-9 + x} dx, x, x^3 \right) - \frac{1}{24} \text{Subst} \left(\int \frac{1}{-1 + x} dx, x, x^3 \right) \\
&= -\frac{1}{24} \log(1 - x^3) + \frac{1}{24} \log(9 - x^3)
\end{aligned}$$

Mathematica [A]

time = 0.00, size = 25, normalized size = 1.00

$$-\frac{1}{24} \log(1 - x^3) + \frac{1}{24} \log(9 - x^3)$$

Antiderivative was successfully verified.

`[In] Integrate[x^2/(9 - 10*x^3 + x^6), x]``[Out] -1/24*Log[1 - x^3] + Log[9 - x^3]/24`**Mathics [A]**

time = 1.87, size = 17, normalized size = 0.68

$$-\frac{\text{Log}[-1 + x^3]}{24} + \frac{\text{Log}[-9 + x^3]}{24}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^2/(9 - 10*x^3 + x^6), x]')``[Out] -Log[-1 + x ^ 3] / 24 + Log[-9 + x ^ 3] / 24`**Maple [A]**

time = 0.02, size = 18, normalized size = 0.72

method	result	size
default	$-\frac{\ln(x^3-1)}{24} + \frac{\ln(x^3-9)}{24}$	18
risch	$-\frac{\ln(x^3-1)}{24} + \frac{\ln(x^3-9)}{24}$	18
norman	$-\frac{\ln(-1+x)}{24} + \frac{\ln(x^3-9)}{24} - \frac{\ln(x^2+x+1)}{24}$	25

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2/(x^6-10*x^3+9), x, method=_RETURNVERBOSE)``[Out] -1/24*ln(x^3-1)+1/24*ln(x^3-9)`

Maxima [A]

time = 0.26, size = 17, normalized size = 0.68

$$-\frac{1}{24} \log(x^3 - 1) + \frac{1}{24} \log(x^3 - 9)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2/(x^6-10*x^3+9),x, algorithm="maxima")``[Out] -1/24*log(x^3 - 1) + 1/24*log(x^3 - 9)`**Fricas [A]**

time = 0.31, size = 17, normalized size = 0.68

$$-\frac{1}{24} \log(x^3 - 1) + \frac{1}{24} \log(x^3 - 9)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2/(x^6-10*x^3+9),x, algorithm="fricas")``[Out] -1/24*log(x^3 - 1) + 1/24*log(x^3 - 9)`**Sympy [A]**

time = 0.05, size = 15, normalized size = 0.60

$$\frac{\log(x^3 - 9)}{24} - \frac{\log(x^3 - 1)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x**2/(x**6-10*x**3+9),x)``[Out] log(x**3 - 9)/24 - log(x**3 - 1)/24`**Giac [A]**

time = 0.00, size = 25, normalized size = 1.00

$$\frac{\frac{\ln|x^3-9|}{8} - \frac{\ln|x^3-1|}{8}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2/(x^6-10*x^3+9),x)``[Out] -1/24*log(abs(x^3 - 1)) + 1/24*log(abs(x^3 - 9))`**Mupad [B]**

time = 0.58, size = 16, normalized size = 0.64

$$\frac{\operatorname{atanh}\left(\frac{81}{320\left(\frac{5x^3}{4} - \frac{9}{8}\right)} - \frac{41}{40}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6 - 10*x^3 + 9),x)
```

```
[Out] atanh(81/(320*((5*x^3)/4 - 9/8)) - 41/40)/12
```

$$3.158 \quad \int \frac{1-4x^2+x^3}{(-2+x)^4} dx$$

Optimal. Leaf size=36

$$-\frac{7}{3(2-x)^3} + \frac{2}{(2-x)^2} + \frac{2}{2-x} + \log(2-x)$$

[Out] -7/3/(2-x)^3+2/(2-x)^2+2/(2-x)+ln(2-x)

Rubi [A]

time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1864}

$$\frac{2}{2-x} + \frac{2}{(2-x)^2} - \frac{7}{3(2-x)^3} + \log(2-x)$$

Antiderivative was successfully verified.

[In] Int[(1 - 4*x^2 + x^3)/(-2 + x)^4,x]

[Out] -7/(3*(2 - x)^3) + 2/(2 - x)^2 + 2/(2 - x) + Log[2 - x]

Rule 1864

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int \frac{1-4x^2+x^3}{(-2+x)^4} dx &= \int \left(-\frac{7}{(-2+x)^4} - \frac{4}{(-2+x)^3} + \frac{2}{(-2+x)^2} + \frac{1}{-2+x} \right) dx \\ &= -\frac{7}{3(2-x)^3} + \frac{2}{(2-x)^2} + \frac{2}{2-x} + \log(2-x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 24, normalized size = 0.67

$$\frac{-29 + 30x - 6x^2}{3(-2+x)^3} + \log(-2+x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 4*x^2 + x^3)/(-2 + x)^4,x]

[Out] $(-29 + 30x - 6x^2)/(3(-2 + x)^3) + \text{Log}[-2 + x]$

Mathics [A]

time = 2.00, size = 44, normalized size = 1.22

$$\frac{-\frac{29}{3} + 10x - 2x^2 + \text{Log}[-2 + x](-8 + 12x - 6x^2 + x^3)}{-8 + 12x - 6x^2 + x^3}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(x^3 - 4*x^2 + 1)/(x - 2)^4,x]')`

[Out] $(-29 / 3 + 10 x - 2 x ^ 2 + \text{Log}[-2 + x] (-8 + 12 x - 6 x ^ 2 + x ^ 3)) / (-8 + 12 x - 6 x ^ 2 + x ^ 3)$

Maple [A]

time = 0.06, size = 27, normalized size = 0.75

method	result	size
norman	$\frac{-2x^2+10x-\frac{29}{3}}{(-2+x)^3} + \ln(-2+x)$	22
risch	$\frac{-2x^2+10x-\frac{29}{3}}{(-2+x)^3} + \ln(-2+x)$	22
default	$\ln(-2+x) - \frac{2}{-2+x} + \frac{2}{(-2+x)^2} + \frac{7}{3(-2+x)^3}$	27
meijerg	$\frac{x(\frac{1}{4}x^2-\frac{3}{2}x+3)}{48(1-\frac{x}{2})^3} + \frac{x(\frac{11}{2}x^2-15x+12)}{24(1-\frac{x}{2})^3} + \ln(1-\frac{x}{2}) - \frac{x^3}{12(1-\frac{x}{2})^3}$	60

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3-4*x^2+1)/(-2+x)^4,x,method=_RETURNVERBOSE)`

[Out] $\ln(-2+x) - 2/(-2+x) + 2/(-2+x)^2 + 7/3/(-2+x)^3$

Maxima [A]

time = 0.27, size = 32, normalized size = 0.89

$$-\frac{6x^2 - 30x + 29}{3(x^3 - 6x^2 + 12x - 8)} + \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3-4*x^2+1)/(-2+x)^4,x, algorithm="maxima")`

[Out] $-1/3*(6*x^2 - 30*x + 29)/(x^3 - 6*x^2 + 12*x - 8) + \log(x - 2)$

Fricas [A]

time = 0.31, size = 46, normalized size = 1.28

$$-\frac{6x^2 - 3(x^3 - 6x^2 + 12x - 8)\log(x - 2) - 30x + 29}{3(x^3 - 6x^2 + 12x - 8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-4*x^2+1)/(-2+x)^4,x, algorithm="fricas")

[Out] -1/3*(6*x^2 - 3*(x^3 - 6*x^2 + 12*x - 8)*log(x - 2) - 30*x + 29)/(x^3 - 6*x^2 + 12*x - 8)

Sympy [A]

time = 0.06, size = 29, normalized size = 0.81

$$\frac{-6x^2 + 30x - 29}{3x^3 - 18x^2 + 36x - 24} + \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-4*x**2+1)/(-2+x)**4,x)

[Out] (-6*x**2 + 30*x - 29)/(3*x**3 - 18*x**2 + 36*x - 24) + log(x - 2)

Giac [A]

time = 0.00, size = 27, normalized size = 0.75

$$\frac{\frac{1}{6}(-12x^2 + 60x - 58)}{(x - 2)^3} + \ln|x - 2|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-4*x^2+1)/(-2+x)^4,x)

[Out] -1/3*(6*x^2 - 30*x + 29)/(x - 2)^3 + log(abs(x - 2))

Mupad [B]

time = 0.04, size = 22, normalized size = 0.61

$$\ln(x - 2) - \frac{2x^2 - 10x + \frac{29}{3}}{(x - 2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 - 4*x^2 + 1)/(x - 2)^4,x)

[Out] log(x - 2) - (2*x^2 - 10*x + 29/3)/(x - 2)^3

$$3.159 \quad \int \frac{x^3}{(-1+x)^{12}} dx$$

Optimal. Leaf size=45

$$\frac{1}{11(1-x)^{11}} - \frac{3}{10(1-x)^{10}} + \frac{1}{3(1-x)^9} - \frac{1}{8(1-x)^8}$$

[Out] 1/11/(1-x)^11-3/10/(1-x)^10+1/3/(1-x)^9-1/8/(1-x)^8

Rubi [A]

time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$-\frac{1}{8(1-x)^8} + \frac{1}{3(1-x)^9} - \frac{3}{10(1-x)^{10}} + \frac{1}{11(1-x)^{11}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(-1 + x)^12,x]

[Out] 1/(11*(1 - x)^11) - 3/(10*(1 - x)^10) + 1/(3*(1 - x)^9) - 1/(8*(1 - x)^8)

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(-1+x)^{12}} dx &= \int \left(\frac{1}{(-1+x)^{12}} + \frac{3}{(-1+x)^{11}} + \frac{3}{(-1+x)^{10}} + \frac{1}{(-1+x)^9} \right) dx \\ &= \frac{1}{11(1-x)^{11}} - \frac{3}{10(1-x)^{10}} + \frac{1}{3(1-x)^9} - \frac{1}{8(1-x)^8} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 24, normalized size = 0.53

$$\frac{1 - 11x + 55x^2 - 165x^3}{1320(-1 + x)^{11}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(-1 + x)^12,x]

[Out] (1 - 11*x + 55*x^2 - 165*x^3)/(1320*(-1 + x)^11)

Mathics [A]

time = 2.28, size = 72, normalized size = 1.60

$$\frac{1 - 11x + 55x^2 - 165x^3}{-1320 + 14520x - 72600x^2 + 217800x^3 - 435600x^4 + 609840x^5 - 609840x^6 + 435600x^7 - 217800x^8 + 72600x^9 - 14520x^{10} + 1320x^{11}}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^3/(x - 1)^12,x]')

[Out] (1 - 11 x + 55 x ^ 2 - 165 x ^ 3) / (1320 (-1 + 11 x - 55 x ^ 2 + 165 x ^ 3 - 330 x ^ 4 + 462 x ^ 5 - 462 x ^ 6 + 330 x ^ 7 - 165 x ^ 8 + 55 x ^ 9 - 11 x ^ 10 + x ^ 11))

Maple [A]

time = 0.06, size = 30, normalized size = 0.67

method	result	size
norman	$\frac{-\frac{1}{8}x^3 + \frac{1}{24}x^2 - \frac{1}{120}x + \frac{1}{1320}}{(-1+x)^{11}}$	22
risch	$\frac{-\frac{1}{8}x^3 + \frac{1}{24}x^2 - \frac{1}{120}x + \frac{1}{1320}}{(-1+x)^{11}}$	22
gospers	$-\frac{165x^3 - 55x^2 + 11x - 1}{1320(-1+x)^{11}}$	23
default	$-\frac{1}{3(-1+x)^9} - \frac{1}{11(-1+x)^{11}} - \frac{1}{8(-1+x)^8} - \frac{3}{10(-1+x)^{10}}$	30
meijerg	$\frac{x^4(-x^7 + 11x^6 - 55x^5 + 165x^4 - 330x^3 + 462x^2 - 462x + 330)}{1320(1-x)^{11}}$	48

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(-1+x)^12,x,method=_RETURNVERBOSE)

[Out] -1/3/(-1+x)^9-1/11/(-1+x)^11-1/8/(-1+x)^8-3/10/(-1+x)^10

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(29) = 58.

time = 0.26, size = 72, normalized size = 1.60

$$\frac{165x^3 - 55x^2 + 11x - 1}{1320(x^{11} - 11x^{10} + 55x^9 - 165x^8 + 330x^7 - 462x^6 + 462x^5 - 330x^4 + 165x^3 - 55x^2 + 11x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(-1+x)^12,x, algorithm="maxima")

[Out] -1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x^11 - 11*x^10 + 55*x^9 - 165*x^8 + 330*x^7 - 462*x^6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^2 + 11*x - 1)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(29) = 58$.

time = 0.35, size = 72, normalized size = 1.60

$$\frac{165x^3 - 55x^2 + 11x - 1}{1320(x^{11} - 11x^{10} + 55x^9 - 165x^8 + 330x^7 - 462x^6 + 462x^5 - 330x^4 + 165x^3 - 55x^2 + 11x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(-1+x)^12,x, algorithm="fricas")

[Out] -1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x^11 - 11*x^10 + 55*x^9 - 165*x^8 + 330*x^7 - 462*x^6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^2 + 11*x - 1)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$

time = 0.08, size = 70, normalized size = 1.56

$$\frac{-165x^3 + 55x^2 - 11x + 1}{1320x^{11} - 14520x^{10} + 72600x^9 - 217800x^8 + 435600x^7 - 609840x^6 + 609840x^5 - 435600x^4 + 217800x^3 - 72600x^2 + 14520x - 1320}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(-1+x)**12,x)

[Out] (-165*x**3 + 55*x**2 - 11*x + 1)/(1320*x**11 - 14520*x**10 + 72600*x**9 - 217800*x**8 + 435600*x**7 - 609840*x**6 + 609840*x**5 - 435600*x**4 + 217800*x**3 - 72600*x**2 + 14520*x - 1320)

Giac [A]

time = 0.00, size = 26, normalized size = 0.58

$$\frac{-165x^3 + 55x^2 - 11x + 1}{1320(x - 1)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(-1+x)^12,x)

[Out] -1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x - 1)^11

Mupad [B]

time = 0.10, size = 29, normalized size = 0.64

$$-\frac{1}{8(x-1)^8} - \frac{1}{3(x-1)^9} - \frac{3}{10(x-1)^{10}} - \frac{1}{11(x-1)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x - 1)^12,x)

[Out] - 1/(8*(x - 1)^8) - 1/(3*(x - 1)^9) - 3/(10*(x - 1)^10) - 1/(11*(x - 1)^11)

$$3.160 \quad \int \frac{-3x+x^4}{(1+2x)^5} dx$$

Optimal. Leaf size=55

$$-\frac{25}{128(1+2x)^4} + \frac{7}{24(1+2x)^3} - \frac{3}{32(1+2x)^2} + \frac{1}{8(1+2x)} + \frac{1}{32} \log(1+2x)$$

[Out] -25/128/(1+2*x)^4+7/24/(1+2*x)^3-3/32/(1+2*x)^2+1/8/(1+2*x)+1/32*ln(1+2*x)

Rubi [A]

time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1607, 1634}

$$\frac{1}{8(2x+1)} - \frac{3}{32(2x+1)^2} + \frac{7}{24(2x+1)^3} - \frac{25}{128(2x+1)^4} + \frac{1}{32} \log(2x+1)$$

Antiderivative was successfully verified.

[In] Int[(-3*x + x^4)/(1 + 2*x)^5, x]

[Out] -25/(128*(1 + 2*x)^4) + 7/(24*(1 + 2*x)^3) - 3/(32*(1 + 2*x)^2) + 1/(8*(1 + 2*x)) + Log[1 + 2*x]/32

Rule 1607

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1634

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{-3x+x^4}{(1+2x)^5} dx &= \int \frac{x(-3+x^3)}{(1+2x)^5} dx \\ &= \int \left(\frac{25}{16(1+2x)^5} - \frac{7}{4(1+2x)^4} + \frac{3}{8(1+2x)^3} - \frac{1}{4(1+2x)^2} + \frac{1}{16(1+2x)} \right) dx \\ &= -\frac{25}{128(1+2x)^4} + \frac{7}{24(1+2x)^3} - \frac{3}{32(1+2x)^2} + \frac{1}{8(1+2x)} + \frac{1}{32} \log(1+2x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 41, normalized size = 0.75

$$\frac{49 + 368x + 432x^2 + 384x^3 + 12(1 + 2x)^4 \log(1 + 2x)}{384(1 + 2x)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(-3*x + x^4)/(1 + 2*x)^5,x]

[Out] (49 + 368*x + 432*x^2 + 384*x^3 + 12*(1 + 2*x)^4*Log[1 + 2*x])/(384*(1 + 2*x)^4)

Mathics [A]

time = 3.26, size = 67, normalized size = 1.22

$$\frac{49 + 368x + 432x^2 + 384x^3 + 12\text{Log}[1 + 2x] (1 + 8x + 24x^2 + 32x^3 + 16x^4)}{384 + 3072x + 9216x^2 + 12288x^3 + 6144x^4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(x^4 - 3*x)/(1 + 2*x)^5,x]')

[Out] (49 + 368 x + 432 x ^ 2 + 384 x ^ 3 + 12 Log[1 + 2 x] (1 + 8 x + 24 x ^ 2 + 32 x ^ 3 + 16 x ^ 4)) / (384 (1 + 8 x + 24 x ^ 2 + 32 x ^ 3 + 16 x ^ 4))

Maple [A]

time = 0.05, size = 46, normalized size = 0.84

method	result	size
risch	$\frac{x^3 + \frac{9}{8}x^2 + \frac{23}{24}x + \frac{49}{384}}{(1+2x)^4} + \frac{\ln(1+2x)}{32}$	34
norman	$\frac{-\frac{37}{12}x^3 - \frac{31}{16}x^2 - \frac{1}{16}x - \frac{49}{24}x^4}{(1+2x)^4} + \frac{\ln(1+2x)}{32}$	37
default	$-\frac{25}{128(1+2x)^4} + \frac{7}{24(1+2x)^3} - \frac{3}{32(1+2x)^2} + \frac{1}{8+16x} + \frac{\ln(1+2x)}{32}$	46
meijerg	$-\frac{x(1000x^3+1040x^2+420x+60)}{960(1+2x)^4} + \frac{\ln(1+2x)}{32} - \frac{x^2(4x^2+8x+6)}{4(1+2x)^4}$	57

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-3*x)/(1+2*x)^5,x,method=_RETURNVERBOSE)

[Out] -25/128/(1+2*x)^4+7/24/(1+2*x)^3-3/32/(1+2*x)^2+1/8/(1+2*x)+1/32*ln(1+2*x)

Maxima [A]

time = 0.27, size = 48, normalized size = 0.87

$$\frac{384x^3 + 432x^2 + 368x + 49}{384(16x^4 + 32x^3 + 24x^2 + 8x + 1)} + \frac{1}{32} \log(2x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-3*x)/(1+2*x)^5,x, algorithm="maxima")

[Out] 1/384*(384*x^3 + 432*x^2 + 368*x + 49)/(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1) + 1/32*log(2*x + 1)

Fricas [A]

time = 0.44, size = 67, normalized size = 1.22

$$\frac{384x^3 + 432x^2 + 12(16x^4 + 32x^3 + 24x^2 + 8x + 1)\log(2x + 1) + 368x + 49}{384(16x^4 + 32x^3 + 24x^2 + 8x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-3*x)/(1+2*x)^5,x, algorithm="fricas")

[Out] 1/384*(384*x^3 + 432*x^2 + 12*(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1)*log(2*x + 1) + 368*x + 49)/(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1)

Sympy [A]

time = 0.07, size = 42, normalized size = 0.76

$$\frac{384x^3 + 432x^2 + 368x + 49}{6144x^4 + 12288x^3 + 9216x^2 + 3072x + 384} + \frac{\log(2x + 1)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4-3*x)/(1+2*x)**5,x)

[Out] (384*x**3 + 432*x**2 + 368*x + 49)/(6144*x**4 + 12288*x**3 + 9216*x**2 + 3072*x + 384) + log(2*x + 1)/32

Giac [A]

time = 0.00, size = 41, normalized size = 0.75

$$\frac{\frac{1}{48}(48x^3 + 54x^2 + 46x + \frac{49}{8})}{(2x + 1)^4} + \frac{\ln|2x + 1|}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-3*x)/(1+2*x)^5,x)

[Out] 1/384*(384*x^3 + 432*x^2 + 368*x + 49)/(2*x + 1)^4 + 1/32*log(abs(2*x + 1))

Mupad [B]

time = 0.05, size = 43, normalized size = 0.78

$$\frac{\ln\left(x + \frac{1}{2}\right)}{32} + \frac{\frac{x^3}{16} + \frac{9x^2}{128} + \frac{23x}{384} + \frac{49}{6144}}{x^4 + 2x^3 + \frac{3x^2}{2} + \frac{x}{2} + \frac{1}{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3*x - x^4)/(2*x + 1)^5,x)

[Out] log(x + 1/2)/32 + ((23*x)/384 + (9*x^2)/128 + x^3/16 + 49/6144)/(x/2 + (3*x^2)/2 + 2*x^3 + x^4 + 1/16)

$$3.161 \quad \int \frac{1}{(-1+x)^2(1+x)^3} dx$$

Optimal. Leaf size=36

$$\frac{1}{8(1-x)} - \frac{1}{8(1+x)^2} - \frac{1}{4(1+x)} + \frac{3}{8} \tanh^{-1}(x)$$

[Out] 1/8/(1-x)-1/8/(1+x)^2-1/4/(1+x)+3/8*arctanh(x)

Rubi [A]

time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {46, 213}

$$\frac{1}{8(1-x)} - \frac{1}{4(x+1)} - \frac{1}{8(x+1)^2} + \frac{3}{8} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[1/((-1 + x)^2*(1 + x)^3),x]

[Out] 1/(8*(1 - x)) - 1/(8*(1 + x)^2) - 1/(4*(1 + x)) + (3*ArcTanh[x])/8

Rule 46

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])]

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(-1+x)^2(1+x)^3} dx &= \int \left(\frac{1}{8(-1+x)^2} + \frac{1}{4(1+x)^3} + \frac{1}{4(1+x)^2} - \frac{3}{8(-1+x^2)} \right) dx \\ &= \frac{1}{8(1-x)} - \frac{1}{8(1+x)^2} - \frac{1}{4(1+x)} - \frac{3}{8} \int \frac{1}{-1+x^2} dx \\ &= \frac{1}{8(1-x)} - \frac{1}{8(1+x)^2} - \frac{1}{4(1+x)} + \frac{3}{8} \tanh^{-1}(x) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 38, normalized size = 1.06

$$\frac{1}{16} \left(\frac{4 - 6x - 6x^2}{(-1 + x)(1 + x)^2} - 3 \log(-1 + x) + 3 \log(1 + x) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[1/((-1 + x)^2*(1 + x)^3),x]``[Out] ((4 - 6*x - 6*x^2)/((-1 + x)*(1 + x)^2) - 3*Log[-1 + x] + 3*Log[1 + x])/16`**Mathics [A]**

time = 2.10, size = 49, normalized size = 1.36

$$\frac{4 - 6x - 6x^2 + 3(-1 - x + x^2 + x^3)(\text{Log}[1 + x] - \text{Log}[-1 + x])}{-16 - 16x + 16x^2 + 16x^3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/((x + 1)^3*(x - 1)^2),x]')``[Out] (4 - 6 x - 6 x ^ 2 + 3 (-1 - x + x ^ 2 + x ^ 3) (Log[1 + x] - Log[-1 + x])) / (16 (-1 - x + x ^ 2 + x ^ 3))`**Maple [A]**

time = 0.13, size = 35, normalized size = 0.97

method	result	size
default	$-\frac{1}{8(-1+x)} - \frac{3 \ln(-1+x)}{16} - \frac{1}{8(1+x)^2} - \frac{1}{4(1+x)} + \frac{3 \ln(1+x)}{16}$	35
norman	$\frac{-\frac{3}{8}x - \frac{3}{8}x^2 + \frac{1}{4}}{(-1+x)(1+x)^2} - \frac{3 \ln(-1+x)}{16} + \frac{3 \ln(1+x)}{16}$	35
risch	$\frac{-\frac{3}{8}x - \frac{3}{8}x^2 + \frac{1}{4}}{(-1+x)(1+x)^2} - \frac{3 \ln(-1+x)}{16} + \frac{3 \ln(1+x)}{16}$	35

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(-1+x)^2/(1+x)^3,x,method=_RETURNVERBOSE)``[Out] -1/8/(-1+x)-3/16*ln(-1+x)-1/8/(1+x)^2-1/4/(1+x)+3/16*ln(1+x)`**Maxima [A]**

time = 0.26, size = 38, normalized size = 1.06

$$-\frac{3x^2 + 3x - 2}{8(x^3 + x^2 - x - 1)} + \frac{3}{16} \log(x + 1) - \frac{3}{16} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+x)^2/(1+x)^3,x, algorithm="maxima")

[Out] $-1/8*(3*x^2 + 3*x - 2)/(x^3 + x^2 - x - 1) + 3/16*\log(x + 1) - 3/16*\log(x - 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(26) = 52.

time = 0.31, size = 59, normalized size = 1.64

$$\frac{6x^2 - 3(x^3 + x^2 - x - 1)\log(x + 1) + 3(x^3 + x^2 - x - 1)\log(x - 1) + 6x - 4}{16(x^3 + x^2 - x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+x)^2/(1+x)^3,x, algorithm="fricas")

[Out] $-1/16*(6*x^2 - 3*(x^3 + x^2 - x - 1)*\log(x + 1) + 3*(x^3 + x^2 - x - 1)*\log(x - 1) + 6*x - 4)/(x^3 + x^2 - x - 1)$

Sympy [A]

time = 0.07, size = 41, normalized size = 1.14

$$\frac{-3x^2 - 3x + 2}{8x^3 + 8x^2 - 8x - 8} - \frac{3\log(x - 1)}{16} + \frac{3\log(x + 1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+x)**2/(1+x)**3,x)

[Out] $(-3*x**2 - 3*x + 2)/(8*x**3 + 8*x**2 - 8*x - 8) - 3*\log(x - 1)/16 + 3*\log(x + 1)/16$

Giac [A]

time = 0.00, size = 45, normalized size = 1.25

$$-\frac{3}{16} \ln|x - 1| + \frac{3}{16} \ln|x + 1| + \frac{\frac{1}{32}(-12x^2 - 12x + 8)}{(x - 1)(x + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+x)^2/(1+x)^3,x)

[Out] $-1/8*(3*x^2 + 3*x - 2)/((x + 1)^2*(x - 1)) + 3/16*\log(\text{abs}(x + 1)) - 3/16*\log(\text{abs}(x - 1))$

Mupad [B]

time = 0.04, size = 31, normalized size = 0.86

$$\frac{3 \operatorname{atanh}(x)}{8} + \frac{\frac{3x^2}{8} + \frac{3x}{8} - \frac{1}{4}}{-x^3 - x^2 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x - 1)^2*(x + 1)^3),x)

[Out] $(3*\operatorname{atanh}(x))/8 + ((3*x)/8 + (3*x^2)/8 - 1/4)/(x - x^2 - x^3 + 1)$

$$3.162 \quad \int \frac{1}{(5-6x)^2 x^2} dx$$

Optimal. Leaf size=35

$$\frac{6}{25(5-6x)} - \frac{1}{25x} - \frac{12}{125} \log(5-6x) + \frac{12 \log(x)}{125}$$

[Out] 6/25/(5-6*x)-1/25/x-12/125*ln(5-6*x)+12/125*ln(x)

Rubi [A]

time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$\frac{6}{25(5-6x)} - \frac{1}{25x} - \frac{12}{125} \log(5-6x) + \frac{12 \log(x)}{125}$$

Antiderivative was successfully verified.

[In] Int[1/((5 - 6*x)^2*x^2), x]

[Out] 6/(25*(5 - 6*x)) - 1/(25*x) - (12*Log[5 - 6*x])/125 + (12*Log[x])/125

Rule 46

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(5-6x)^2 x^2} dx &= \int \left(\frac{1}{25x^2} + \frac{12}{125x} + \frac{36}{25(-5+6x)^2} - \frac{72}{125(-5+6x)} \right) dx \\ &= \frac{6}{25(5-6x)} - \frac{1}{25x} - \frac{12}{125} \log(5-6x) + \frac{12 \log(x)}{125} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 31, normalized size = 0.89

$$\frac{1}{125} \left(\frac{30}{5-6x} - \frac{5}{x} - 12 \log(5-6x) + 12 \log(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((5 - 6*x)^2*x^2),x]

[Out] (30/(5 - 6*x) - 5/x - 12*Log[5 - 6*x] + 12*Log[x])/125

Mathics [A]

time = 1.97, size = 34, normalized size = 0.97

$$\frac{25 - 60x + 12x(-5 + 6x)(\text{Log}[x] - \text{Log}[-\frac{5}{6} + x])}{125x(-5 + 6x)}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x^2*(5 - 6*x)^2),x]')

[Out] (25 - 60 x + 12 x (-5 + 6 x) (Log[x] - Log[-5 / 6 + x])) / (125 x (-5 + 6 x))

Maple [A]

time = 0.08, size = 28, normalized size = 0.80

method	result	size
default	$-\frac{1}{25x} + \frac{12 \ln(x)}{125} - \frac{6}{25(-5+6x)} - \frac{12 \ln(-5+6x)}{125}$	28
risch	$\frac{-\frac{12x}{25} + \frac{1}{5}}{x(-5+6x)} + \frac{12 \ln(x)}{125} - \frac{12 \ln(-5+6x)}{125}$	31
norman	$\frac{\frac{1}{5} - \frac{72x^2}{125}}{x(-5+6x)} + \frac{12 \ln(x)}{125} - \frac{12 \ln(-5+6x)}{125}$	32
meijerg	$-\frac{1}{25x} + \frac{6}{125} + \frac{12 \ln(x)}{125} + \frac{12 \ln(2)}{125} + \frac{12 \ln(3)}{125} - \frac{12 \ln(5)}{125} + \frac{12i\pi}{125} + \frac{108x}{625(3-\frac{18x}{5})} - \frac{12 \ln(1-\frac{6x}{5})}{125}$	46

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5-6*x)^2/x^2,x,method=_RETURNVERBOSE)

[Out] -1/25/x+12/125*ln(x)-6/25/(-5+6*x)-12/125*ln(-5+6*x)

Maxima [A]

time = 0.25, size = 31, normalized size = 0.89

$$-\frac{12x - 5}{25(6x^2 - 5x)} - \frac{12}{125} \log(6x - 5) + \frac{12}{125} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5-6*x)^2/x^2,x, algorithm="maxima")

[Out] -1/25*(12*x - 5)/(6*x^2 - 5*x) - 12/125*log(6*x - 5) + 12/125*log(x)

Fricas [A]

time = 0.31, size = 48, normalized size = 1.37

$$\frac{12(6x^2 - 5x) \log(6x - 5) - 12(6x^2 - 5x) \log(x) + 60x - 25}{125(6x^2 - 5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5-6*x)^2/x^2,x, algorithm="fricas")

[Out] $-1/125*(12*(6*x^2 - 5*x)*\log(6*x - 5) - 12*(6*x^2 - 5*x)*\log(x) + 60*x - 25)/(6*x^2 - 5*x)$

Sympy [A]

time = 0.07, size = 29, normalized size = 0.83

$$\frac{5 - 12x}{150x^2 - 125x} + \frac{12 \log(x)}{125} - \frac{12 \log\left(x - \frac{5}{6}\right)}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5-6*x)**2/x**2,x)

[Out] $(5 - 12*x)/(150*x**2 - 125*x) + 12*\log(x)/125 - 12*\log(x - 5/6)/125$

Giac [A]

time = 0.00, size = 39, normalized size = 1.11

$$\frac{12}{125} \ln|x| - \frac{12}{125} \ln|6x - 5| - \frac{12x - 5}{25(6x^2 - 5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5-6*x)^2/x^2,x)

[Out] $-1/25*(12*x - 5)/(6*x^2 - 5*x) - 12/125*\log(\text{abs}(6*x - 5)) + 12/125*\log(\text{abs}(x))$

Mupad [B]

time = 0.22, size = 34, normalized size = 0.97

$$\frac{1}{5x(6x - 5)} - \frac{12}{25(6x - 5)} - \frac{12 \ln\left(\frac{6x-5}{x}\right)}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(6*x - 5)^2),x)

[Out] $1/(5*x*(6*x - 5)) - 12/(25*(6*x - 5)) - (12*\log((6*x - 5)/x))/125$

$$3.163 \quad \int \frac{1}{(-3-2x+x^2)^3} dx$$

Optimal. Leaf size=61

$$\frac{1-x}{16(3+2x-x^2)^2} + \frac{3(1-x)}{128(3+2x-x^2)} + \frac{3}{512} \log(3-x) - \frac{3}{512} \log(1+x)$$

[Out] 1/16*(1-x)/(-x^2+2*x+3)^2+3/128*(1-x)/(-x^2+2*x+3)+3/512*ln(3-x)-3/512*ln(1+x)

Rubi [A]

time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {628, 630, 31}

$$\frac{3(1-x)}{128(-x^2+2x+3)} + \frac{1-x}{16(-x^2+2x+3)^2} + \frac{3}{512} \log(3-x) - \frac{3}{512} \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-3 - 2*x + x^2)^(-3), x]

[Out] (1 - x)/(16*(3 + 2*x - x^2)^2) + (3*(1 - x))/(128*(3 + 2*x - x^2)) + (3*Log[3 - x])/512 - (3*Log[1 + x])/512

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 628

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p + 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 630

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-3-2x+x^2)^3} dx &= \frac{1-x}{16(3+2x-x^2)^2} - \frac{3}{16} \int \frac{1}{(-3-2x+x^2)^2} dx \\
&= \frac{1-x}{16(3+2x-x^2)^2} + \frac{3(1-x)}{128(3+2x-x^2)} + \frac{3}{128} \int \frac{1}{-3-2x+x^2} dx \\
&= \frac{1-x}{16(3+2x-x^2)^2} + \frac{3(1-x)}{128(3+2x-x^2)} + \frac{3}{512} \int \frac{1}{-3+x} dx - \frac{3}{512} \int \frac{1}{1+x} dx \\
&= \frac{1-x}{16(3+2x-x^2)^2} + \frac{3(1-x)}{128(3+2x-x^2)} + \frac{3}{512} \log(3-x) - \frac{3}{512} \log(1+x)
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 46, normalized size = 0.75

$$\frac{1}{512} \left(\frac{4(17-11x-9x^2+3x^3)}{(-3-2x+x^2)^2} + 3\log(3-x) - 3\log(1+x) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(-3 - 2*x + x^2)^(-3), x]``[Out] ((4*(17 - 11*x - 9*x^2 + 3*x^3))/(-3 - 2*x + x^2)^2 + 3*Log[3 - x] - 3*Log[1 + x])/512`**Mathics [A]**

time = 2.20, size = 68, normalized size = 1.11

$$\frac{68 - 44x - 36x^2 + 12x^3 + 3(9 + 12x - 2x^2 - 4x^3 + x^4)(\text{Log}[-3 + x] - \text{Log}[1 + x])}{4608 + 6144x - 1024x^2 - 2048x^3 + 512x^4}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^2 - 2*x - 3)^3, x]')``[Out] (68 - 44 x - 36 x ^ 2 + 12 x ^ 3 + 3 (9 + 12 x - 2 x ^ 2 - 4 x ^ 3 + x ^ 4) (Log[-3 + x] - Log[1 + x])) / (512 (9 + 12 x - 2 x ^ 2 - 4 x ^ 3 + x ^ 4))`**Maple [A]**

time = 0.07, size = 42, normalized size = 0.69

method	result	size
norman	$\frac{\frac{3}{128}x^3 - \frac{9}{128}x^2 - \frac{11}{128}x + \frac{17}{128}}{(x^2-2x-3)^2} + \frac{3\ln(-3+x)}{512} - \frac{3\ln(1+x)}{512}$	40
risch	$\frac{\frac{3}{128}x^3 - \frac{9}{128}x^2 - \frac{11}{128}x + \frac{17}{128}}{(x^2-2x-3)^2} + \frac{3\ln(-3+x)}{512} - \frac{3\ln(1+x)}{512}$	40

default	$-\frac{1}{128(-3+x)^2} + \frac{3}{256(-3+x)} + \frac{3\ln(-3+x)}{512} + \frac{1}{128(1+x)^2} + \frac{3}{256(1+x)} - \frac{3\ln(1+x)}{512}$	42
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2-2*x-3)^3,x,method=_RETURNVERBOSE)`

[Out] $-1/128/(-3+x)^2+3/256/(-3+x)+3/512*\ln(-3+x)+1/128/(1+x)^2+3/256/(1+x)-3/512*\ln(1+x)$

Maxima [A]

time = 0.26, size = 50, normalized size = 0.82

$$\frac{3x^3 - 9x^2 - 11x + 17}{128(x^4 - 4x^3 - 2x^2 + 12x + 9)} - \frac{3}{512} \log(x + 1) + \frac{3}{512} \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2-2*x-3)^3,x, algorithm="maxima")`

[Out] $1/128*(3*x^3 - 9*x^2 - 11*x + 17)/(x^4 - 4*x^3 - 2*x^2 + 12*x + 9) - 3/512*\log(x + 1) + 3/512*\log(x - 3)$

Fricas [A]

time = 0.34, size = 85, normalized size = 1.39

$$\frac{12x^3 - 36x^2 - 3(x^4 - 4x^3 - 2x^2 + 12x + 9)\log(x + 1) + 3(x^4 - 4x^3 - 2x^2 + 12x + 9)\log(x - 3) - 44x + 68}{512(x^4 - 4x^3 - 2x^2 + 12x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2-2*x-3)^3,x, algorithm="fricas")`

[Out] $1/512*(12*x^3 - 36*x^2 - 3*(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)*\log(x + 1) + 3*(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)*\log(x - 3) - 44*x + 68)/(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)$

Sympy [A]

time = 0.08, size = 51, normalized size = 0.84

$$\frac{3x^3 - 9x^2 - 11x + 17}{128x^4 - 512x^3 - 256x^2 + 1536x + 1152} + \frac{3\log(x - 3)}{512} - \frac{3\log(x + 1)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2-2*x-3)**3,x)`

[Out] $(3*x**3 - 9*x**2 - 11*x + 17)/(128*x**4 - 512*x**3 - 256*x**2 + 1536*x + 1152) + 3*\log(x - 3)/512 - 3*\log(x + 1)/512$

Giac [A]

time = 0.00, size = 51, normalized size = 0.84

$$\frac{3}{512} \ln|x-3| - \frac{3}{512} \ln|x+1| - \frac{-3x^3 + 9x^2 + 11x - 17}{128(x^2 - 2x - 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-2*x-3)^3,x)**[Out]** 1/128*(3*x^3 - 9*x^2 - 11*x + 17)/(x^2 - 2*x - 3)^2 - 3/512*log(abs(x + 1)) + 3/512*log(abs(x - 3))**Mupad [B]**

time = 0.12, size = 45, normalized size = 0.74

$$-\frac{3 \ln\left(\frac{x+1}{x-3}\right)}{512} - 6 \left(\frac{1}{256(-x^2 + 2x + 3)} + \frac{1}{96(-x^2 + 2x + 3)^2} \right) (x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(2*x - x^2 + 3)^3,x)**[Out]** - (3*log((x + 1)/(x - 3)))/512 - 6*(1/(256*(2*x - x^2 + 3)) + 1/(96*(2*x - x^2 + 3)^2))*(x - 1)

$$3.164 \quad \int \frac{1}{(13-4x+x^2)^3} dx$$

Optimal. Leaf size=51

$$-\frac{2-x}{36(13-4x+x^2)^2} - \frac{2-x}{216(13-4x+x^2)} + \frac{1}{648} \tan^{-1}\left(\frac{1}{3}(-2+x)\right)$$

[Out] 1/36*(-2+x)/(x^2-4*x+13)^2+1/216*(-2+x)/(x^2-4*x+13)+1/648*arctan(-2/3+1/3*x)

Rubi [A]

time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {628, 632, 210}

$$-\frac{2-x}{216(x^2-4x+13)} - \frac{2-x}{36(x^2-4x+13)^2} + \frac{1}{648} \tan^{-1}\left(\frac{x-2}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[(13 - 4*x + x^2)^(-3), x]

[Out] -1/36*(2 - x)/(13 - 4*x + x^2)^2 - (2 - x)/(216*(13 - 4*x + x^2)) + ArcTan[(-2 + x)/3]/648

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p + 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(13 - 4x + x^2)^3} dx &= -\frac{2 - x}{36(13 - 4x + x^2)^2} + \frac{1}{12} \int \frac{1}{(13 - 4x + x^2)^2} dx \\
&= -\frac{2 - x}{36(13 - 4x + x^2)^2} - \frac{2 - x}{216(13 - 4x + x^2)} + \frac{1}{216} \int \frac{1}{13 - 4x + x^2} dx \\
&= -\frac{2 - x}{36(13 - 4x + x^2)^2} - \frac{2 - x}{216(13 - 4x + x^2)} - \frac{1}{108} \text{Subst} \left(\int \frac{1}{-36 - x^2} dx, x, -4 + \right. \\
&= -\frac{2 - x}{36(13 - 4x + x^2)^2} - \frac{2 - x}{216(13 - 4x + x^2)} + \frac{1}{648} \tan^{-1} \left(\frac{1}{3}(-2 + x) \right)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 36, normalized size = 0.71

$$\frac{1}{648} \left(\frac{3(-2 + x)(19 - 4x + x^2)}{(13 - 4x + x^2)^2} + \tan^{-1} \left(\frac{1}{3}(-2 + x) \right) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(13 - 4*x + x^2)^(-3),x]``[Out] ((3*(-2 + x)*(19 - 4*x + x^2))/(13 - 4*x + x^2)^2 + ArcTan[(-2 + x)/3])/648`**Mathics [A]**

time = 2.14, size = 62, normalized size = 1.22

$$\frac{-114 + 81x - 18x^2 + 3x^3 + \text{ArcTan} \left[-\frac{2}{3} + \frac{x}{3} \right] (169 - 104x + 42x^2 - 8x^3 + x^4)}{109512 - 67392x + 27216x^2 - 5184x^3 + 648x^4}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^2 - 4*x + 13)^3,x]')``[Out] (-114 + 81 x - 18 x ^ 2 + 3 x ^ 3 + ArcTan[-2 / 3 + x / 3] (169 - 104 x + 4 2 x ^ 2 - 8 x ^ 3 + x ^ 4)) / (648 (169 - 104 x + 42 x ^ 2 - 8 x ^ 3 + x ^ 4))`**Maple [A]**

time = 0.12, size = 44, normalized size = 0.86

method	result	size
risch	$\frac{\frac{1}{216}x^3 - \frac{1}{36}x^2 + \frac{1}{8}x - \frac{19}{108}}{(x^2 - 4x + 13)^2} + \frac{\arctan(-\frac{2}{3} + \frac{x}{3})}{648}$	36
default	$\frac{2x-4}{72(x^2-4x+13)^2} + \frac{2x-4}{432x^2-1728x+5616} + \frac{\arctan(-\frac{2}{3} + \frac{x}{3})}{648}$	44

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2-4*x+13)^3,x,method=_RETURNVERBOSE)`

[Out] $1/72*(2*x-4)/(x^2-4*x+13)^2+1/432/(x^2-4*x+13)*(2*x-4)+1/648*\arctan(-2/3+1/3*x)$

Maxima [A]

time = 0.36, size = 44, normalized size = 0.86

$$\frac{x^3 - 6x^2 + 27x - 38}{216(x^4 - 8x^3 + 42x^2 - 104x + 169)} + \frac{1}{648} \arctan\left(\frac{1}{3}x - \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2-4*x+13)^3,x, algorithm="maxima")`

[Out] $1/216*(x^3 - 6*x^2 + 27*x - 38)/(x^4 - 8*x^3 + 42*x^2 - 104*x + 169) + 1/648*\arctan(1/3*x - 2/3)$

Fricas [A]

time = 0.36, size = 62, normalized size = 1.22

$$\frac{3x^3 - 18x^2 + (x^4 - 8x^3 + 42x^2 - 104x + 169) \arctan\left(\frac{1}{3}x - \frac{2}{3}\right) + 81x - 114}{648(x^4 - 8x^3 + 42x^2 - 104x + 169)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2-4*x+13)^3,x, algorithm="fricas")`

[Out] $1/648*(3*x^3 - 18*x^2 + (x^4 - 8*x^3 + 42*x^2 - 104*x + 169)*\arctan(1/3*x - 2/3) + 81*x - 114)/(x^4 - 8*x^3 + 42*x^2 - 104*x + 169)$

Sympy [A]

time = 0.08, size = 42, normalized size = 0.82

$$\frac{x^3 - 6x^2 + 27x - 38}{216x^4 - 1728x^3 + 9072x^2 - 22464x + 36504} + \frac{\operatorname{atan}\left(\frac{x}{3} - \frac{2}{3}\right)}{648}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2-4*x+13)**3,x)`

[Out] $(x**3 - 6*x**2 + 27*x - 38)/(216*x**4 - 1728*x**3 + 9072*x**2 - 22464*x + 36504) + \operatorname{atan}(x/3 - 2/3)/648$

Giac [A]

time = 0.00, size = 40, normalized size = 0.78

$$\frac{x^3 - 6x^2 + 27x - 38}{216(x^2 - 4x + 13)^2} + \frac{\arctan\left(\frac{x-2}{3}\right)}{648}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-4*x+13)^3,x)

[Out] 1/216*(x^3 - 6*x^2 + 27*x - 38)/(x^2 - 4*x + 13)^2 + 1/648*arctan(1/3*x - 2/3)

Mupad [B]

time = 0.20, size = 39, normalized size = 0.76

$$\frac{\operatorname{atan}\left(\frac{x}{3} - \frac{2}{3}\right)}{648} + 6(x - 2) \left(\frac{1}{1296(x^2 - 4x + 13)} + \frac{1}{216(x^2 - 4x + 13)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2 - 4*x + 13)^3,x)

[Out] atan(x/3 - 2/3)/648 + 6*(x - 2)*(1/(1296*(x^2 - 4*x + 13)) + 1/(216*(x^2 - 4*x + 13)^2))

$$3.165 \quad \int \frac{1}{(2+x)^3(3+x)^4} dx$$

Optimal. Leaf size=54

$$-\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10 \log(2+x) - 10 \log(3+x)$$

[Out] $-1/2/(2+x)^2+4/(2+x)+1/3/(3+x)^3+3/2/(3+x)^2+6/(3+x)+10*\ln(2+x)-10*\ln(3+x)$

Rubi [A]

time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$\frac{4}{x+2} + \frac{6}{x+3} - \frac{1}{2(x+2)^2} + \frac{3}{2(x+3)^2} + \frac{1}{3(x+3)^3} + 10 \log(x+2) - 10 \log(x+3)$$

Antiderivative was successfully verified.

[In] Int[1/((2 + x)^3*(3 + x)^4), x]

[Out] $-1/2*1/(2 + x)^2 + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 + x) + 10*\text{Log}[2 + x] - 10*\text{Log}[3 + x]$

Rule 46

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\int \frac{1}{(2+x)^3(3+x)^4} dx = \int \left(\frac{1}{(2+x)^3} - \frac{4}{(2+x)^2} + \frac{10}{2+x} - \frac{1}{(3+x)^4} - \frac{3}{(3+x)^3} - \frac{6}{(3+x)^2} - \frac{10}{3+x} \right) dx$$

$$= -\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10 \log(2+x) - 10 \log(3+x)$$

Mathematica [A]

time = 0.01, size = 54, normalized size = 1.00

$$-\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10 \log(2+x) - 10 \log(3+x)$$

Antiderivative was successfully verified.

[In] Integrate[1/((2 + x)^3*(3 + x)^4),x]

[Out] $-1/2*1/(2 + x)^2 + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 + x) + 10*\text{Log}[2 + x] - 10*\text{Log}[3 + x]$

Mathics [A]

time = 2.27, size = 83, normalized size = 1.54

$$\frac{2627 + 4175x + 2450x^2 + 630x^3 + 60x^4 + 60(108 + 216x + 171x^2 + 67x^3 + 13x^4 + x^5)(\text{Log}[2 + x] - \text{Log}[3 + x])}{648 + 1296x + 1026x^2 + 402x^3 + 78x^4 + 6x^5}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/((x + 2)^3*(x + 3)^4),x]')

[Out] $(2627 + 4175x + 2450x^2 + 630x^3 + 60x^4 + 60(108 + 216x + 171x^2 + 67x^3 + 13x^4 + x^5)(\text{Log}[2 + x] - \text{Log}[3 + x])) / (6(108 + 216x + 171x^2 + 67x^3 + 13x^4 + x^5))$

Maple [A]

time = 0.05, size = 49, normalized size = 0.91

method	result	size
norman	$\frac{10x^4 + 105x^3 + \frac{1225}{3}x^2 + \frac{4175}{6}x + \frac{2627}{6}}{(2+x)^2(3+x)^3} + 10 \ln(2+x) - 10 \ln(3+x)$	45
risch	$\frac{10x^4 + 105x^3 + \frac{1225}{3}x^2 + \frac{4175}{6}x + \frac{2627}{6}}{(2+x)^2(3+x)^3} + 10 \ln(2+x) - 10 \ln(3+x)$	45
default	$-\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10 \ln(2+x) - 10 \ln(3+x)$	49

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2+x)^3/(3+x)^4,x,method=_RETURNVERBOSE)

[Out] $-1/2/(2+x)^2 + 4/(2+x) + 1/3/(3+x)^3 + 3/2/(3+x)^2 + 6/(3+x) + 10*\ln(2+x) - 10*\ln(3+x)$

Maxima [A]

time = 0.26, size = 60, normalized size = 1.11

$$\frac{60x^4 + 630x^3 + 2450x^2 + 4175x + 2627}{6(x^5 + 13x^4 + 67x^3 + 171x^2 + 216x + 108)} - 10 \log(x + 3) + 10 \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="maxima")

[Out] $1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108) - 10*\log(x + 3) + 10*\log(x + 2)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(48) = 96.

time = 0.31, size = 105, normalized size = 1.94

$$\frac{60x^4 + 630x^3 + 2450x^2 - 60(x^5 + 13x^4 + 67x^3 + 171x^2 + 216x + 108)\log(x + 3) + 60(x^5 + 13x^4 + 67x^3 + 171x^2 + 216x + 108)\log(x + 2) + 4175x + 2627}{6(x^5 + 13x^4 + 67x^3 + 171x^2 + 216x + 108)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="fricas")

[Out] $\frac{1}{6}(60x^4 + 630x^3 + 2450x^2 - 60(x^5 + 13x^4 + 67x^3 + 171x^2 + 216x + 108)\log(x + 3) + 60(x^5 + 13x^4 + 67x^3 + 171x^2 + 216x + 108)\log(x + 2) + 4175x + 2627)/(x^5 + 13x^4 + 67x^3 + 171x^2 + 216x + 108)$

Sympy [A]

time = 0.11, size = 58, normalized size = 1.07

$$\frac{60x^4 + 630x^3 + 2450x^2 + 4175x + 2627}{6x^5 + 78x^4 + 402x^3 + 1026x^2 + 1296x + 648} + 10 \log(x + 2) - 10 \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+x)**3/(3+x)**4,x)

[Out] $(60x^{**4} + 630x^{**3} + 2450x^{**2} + 4175x + 2627)/(6x^{**5} + 78x^{**4} + 402x^{**3} + 1026x^{**2} + 1296x + 648) + 10*\log(x + 2) - 10*\log(x + 3)$

Giac [A]

time = 0.00, size = 52, normalized size = 0.96

$$10 \ln|x + 2| - 10 \ln|x + 3| + \frac{\frac{1}{6}(60x^4 + 630x^3 + 2450x^2 + 4175x + 2627)}{(x + 2)^2(x + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+x)^3/(3+x)^4,x)

[Out] $\frac{1}{6}(60x^4 + 630x^3 + 2450x^2 + 4175x + 2627)/((x + 3)^3(x + 2)^2) - 10*\log(\text{abs}(x + 3)) + 10*\log(\text{abs}(x + 2))$

Mupad [B]

time = 0.05, size = 55, normalized size = 1.02

$$\frac{10x^4 + 105x^3 + \frac{1225x^2}{3} + \frac{4175x}{6} + \frac{2627}{6}}{x^5 + 13x^4 + 67x^3 + 171x^2 + 216x + 108} - 20 \operatorname{atanh}(2x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x + 2)^3*(x + 3)^4),x)

[Out] $((4175x)/6 + (1225x^2)/3 + 105x^3 + 10x^4 + 2627/6)/(216x + 171x^2 + 67x^3 + 13x^4 + x^5 + 108) - 20*\operatorname{atanh}(2x + 5)$

$$3.166 \quad \int \frac{x^6}{(-2+x^2)^2} dx$$

Optimal. Leaf size=36

$$4x + \frac{x^3}{3} - \frac{2x}{-2+x^2} - 5\sqrt{2} \tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

[Out] 4*x+1/3*x^3-2*x/(x^2-2)-5*arctanh(1/2*x*2^(1/2))*2^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {294, 308, 213}

$$\frac{5x^3}{6} + \frac{x^5}{2(2-x^2)} + 5x - 5\sqrt{2} \tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^6/(-2 + x^2)^2,x]

[Out] 5*x + (5*x^3)/6 + x^5/(2*(2 - x^2)) - 5*Sqrt[2]*ArcTanh[x/Sqrt[2]]

Rule 213

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 294

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n-1)*(c*x)^(m-n+1)*((a+b*x^n)^(p+1)/(b*n*(p+1))), x] - Dist[c^n*((m-n+1)/(b*n*(p+1))), Int[(c*x)^(m-n)*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !I LtQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 308

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a+b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n-1]

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(-2+x^2)^2} dx &= \frac{x^5}{2(2-x^2)} + \frac{5}{2} \int \frac{x^4}{-2+x^2} dx \\
&= \frac{x^5}{2(2-x^2)} + \frac{5}{2} \int \left(2+x^2 + \frac{4}{-2+x^2} \right) dx \\
&= 5x + \frac{5x^3}{6} + \frac{x^5}{2(2-x^2)} + 10 \int \frac{1}{-2+x^2} dx \\
&= 5x + \frac{5x^3}{6} + \frac{x^5}{2(2-x^2)} - 5\sqrt{2} \tanh^{-1} \left(\frac{x}{\sqrt{2}} \right)
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 53, normalized size = 1.47

$$4x + \frac{x^3}{3} - \frac{2x}{-2+x^2} + \frac{5 \log(\sqrt{2}-x)}{\sqrt{2}} - \frac{5 \log(\sqrt{2}+x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

`[In] Integrate[x^6/(-2 + x^2)^2,x]``[Out] 4*x + x^3/3 - (2*x)/(-2 + x^2) + (5*Log[Sqrt[2] - x])/Sqrt[2] - (5*Log[Sqrt[2] + x])/Sqrt[2]`**Mathics [A]**

time = 2.00, size = 52, normalized size = 1.44

$$\frac{-12x + (-2 + x^2) \left(24x + 2x^3 - 15\sqrt{2} \operatorname{Log}[\sqrt{2} + x] + 15\sqrt{2} \operatorname{Log}[-\sqrt{2} + x] \right)}{-12 + 6x^2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^6/(x^2 - 2)^2,x]')``[Out] (-12 x + (-2 + x ^ 2) (24 x + 2 x ^ 3 - 15 Sqrt[2] Log[Sqrt[2] + x] + 15 Sqrt[2] Log[-Sqrt[2] + x])) / (6 (-2 + x ^ 2))`**Maple [A]**

time = 0.08, size = 32, normalized size = 0.89

method	result	size
default	$4x + \frac{x^3}{3} - \frac{2x}{x^2-2} - 5 \operatorname{arctanh} \left(\frac{x\sqrt{2}}{2} \right) \sqrt{2}$	32

risch	$\frac{x^3}{3} + 4x - \frac{2x}{x^2-2} + \frac{5\sqrt{2} \ln(x-\sqrt{2})}{2} - \frac{5\sqrt{2} \ln(x+\sqrt{2})}{2}$	44
meijerg	$i\sqrt{2} \left(-\frac{ix\sqrt{2} \left(-\frac{7}{2}x^4 - 35x^2 + 105 \right)}{42 \left(-\frac{x^2}{2} + 1 \right)} + 5i \operatorname{arctanh} \left(\frac{x\sqrt{2}}{2} \right) \right)$	46

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(x^2-2)^2,x,method=_RETURNVERBOSE)`

[Out] `4*x+1/3*x^3-2*x/(x^2-2)-5*arctanh(1/2*x*2^(1/2))*2^(1/2)`

Maxima [A]

time = 0.39, size = 40, normalized size = 1.11

$$\frac{1}{3}x^3 + \frac{5}{2}\sqrt{2} \log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right) + 4x - \frac{2x}{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(x^2-2)^2,x, algorithm="maxima")`

[Out] `1/3*x^3 + 5/2*sqrt(2)*log((x - sqrt(2))/(x + sqrt(2))) + 4*x - 2*x/(x^2 - 2)`

Fricas [A]

time = 0.36, size = 53, normalized size = 1.47

$$\frac{2x^5 + 20x^3 + 15\sqrt{2}(x^2-2) \log\left(\frac{x^2-2\sqrt{2}x+2}{x^2-2}\right) - 60x}{6(x^2-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(x^2-2)^2,x, algorithm="fricas")`

[Out] `1/6*(2*x^5 + 20*x^3 + 15*sqrt(2)*(x^2 - 2)*log((x^2 - 2*sqrt(2)*x + 2)/(x^2 - 2)) - 60*x)/(x^2 - 2)`

Sympy [A]

time = 0.06, size = 49, normalized size = 1.36

$$\frac{x^3}{3} + 4x - \frac{2x}{x^2-2} + \frac{5\sqrt{2} \log(x-\sqrt{2})}{2} - \frac{5\sqrt{2} \log(x+\sqrt{2})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(x**2-2)**2,x)`

[Out] $x^3/3 + 4x - 2x/(x^2 - 2) + 5\sqrt{2}\log(x - \sqrt{2})/2 - 5\sqrt{2}\log(x + \sqrt{2})/2$

Giac [A]

time = 0.00, size = 58, normalized size = 1.61

$$\frac{1}{3}x^3 + 4x + \frac{2x}{-x^2 + 2} + \frac{10 \ln \left| \frac{2x-2\sqrt{2}}{2x+2\sqrt{2}} \right|}{2\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(x^2-2)^2,x)`

[Out] $1/3x^3 + 5/2\sqrt{2}\log(\text{abs}((x - \sqrt{2})/(x + \sqrt{2}))) + 4x - 2x/(x^2 - 2)$

Mupad [B]

time = 0.25, size = 33, normalized size = 0.92

$$4x - \frac{2x}{x^2 - 2} + \frac{x^3}{3} + \sqrt{2} \operatorname{atan} \left(\frac{\sqrt{2} x \operatorname{li}}{2} \right) 5i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(x^2 - 2)^2,x)`

[Out] $4x + 2^{(1/2)}\operatorname{atan}((2^{(1/2)}x \operatorname{li})/2)*5i - (2x)/(x^2 - 2) + x^3/3$

$$3.167 \quad \int \frac{x^8}{(4+x^2)^4} dx$$

Optimal. Leaf size=58

$$\frac{35x}{16} - \frac{x^7}{6(4+x^2)^3} - \frac{7x^5}{24(4+x^2)^2} - \frac{35x^3}{48(4+x^2)} - \frac{35}{8} \tan^{-1}\left(\frac{x}{2}\right)$$

[Out] 35/16*x-1/6*x^7/(x^2+4)^3-7/24*x^5/(x^2+4)^2-35/48*x^3/(x^2+4)-35/8*arctan(1/2*x)

Rubi [A]

time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {294, 327, 209}

$$-\frac{x^7}{6(x^2+4)^3} - \frac{7x^5}{24(x^2+4)^2} - \frac{35x^3}{48(x^2+4)} + \frac{35x}{16} - \frac{35}{8} \tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[x^8/(4 + x^2)^4,x]

[Out] (35*x)/16 - x^7/(6*(4 + x^2)^3) - (7*x^5)/(24*(4 + x^2)^2) - (35*x^3)/(48*(4 + x^2)) - (35*ArcTan[x/2])/8

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 294

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n-1)*(c*x)^(m-n+1)*((a+b*x^n)^(p+1)/(b*n*(p+1))), x] - Dist[c^n*((m-n+1)/(b*n*(p+1))), Int[(c*x)^(m-n)*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !ILtQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 327

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n-1)*(c*x)^(m-n+1)*((a+b*x^n)^(p+1)/(b*(m+n*p+1))), x] - Dist[a*c^n*((m-n+1)/(b*(m+n*p+1))), Int[(c*x)^(m-n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(4+x^2)^4} dx &= -\frac{x^7}{6(4+x^2)^3} + \frac{7}{6} \int \frac{x^6}{(4+x^2)^3} dx \\
&= -\frac{x^7}{6(4+x^2)^3} - \frac{7x^5}{24(4+x^2)^2} + \frac{35}{24} \int \frac{x^4}{(4+x^2)^2} dx \\
&= -\frac{x^7}{6(4+x^2)^3} - \frac{7x^5}{24(4+x^2)^2} - \frac{35x^3}{48(4+x^2)} + \frac{35}{16} \int \frac{x^2}{4+x^2} dx \\
&= \frac{35x}{16} - \frac{x^7}{6(4+x^2)^3} - \frac{7x^5}{24(4+x^2)^2} - \frac{35x^3}{48(4+x^2)} - \frac{35}{4} \int \frac{1}{4+x^2} dx \\
&= \frac{35x}{16} - \frac{x^7}{6(4+x^2)^3} - \frac{7x^5}{24(4+x^2)^2} - \frac{35x^3}{48(4+x^2)} - \frac{35}{8} \tan^{-1}\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 40, normalized size = 0.69

$$\frac{x(1680 + 1120x^2 + 231x^4 + 12x^6)}{12(4+x^2)^3} - \frac{35}{8} \tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

`[In] Integrate[x^8/(4 + x^2)^4, x]``[Out] (x*(1680 + 1120*x^2 + 231*x^4 + 12*x^6))/(12*(4 + x^2)^3) - (35*ArcTan[x/2])/8`**Mathics [A]**

time = 3.03, size = 61, normalized size = 1.05

$$\frac{\frac{x(912+544x^2+87x^4)}{12} + \frac{(64+48x^2+12x^4+x^6)(8x-35\text{ArcTan}[\frac{x}{2}])}{8}}{64+48x^2+12x^4+x^6}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^8/(x^2 + 4)^4, x]')``[Out] (x (912 + 544 x ^ 2 + 87 x ^ 4) / 12 + (64 + 48 x ^ 2 + 12 x ^ 4 + x ^ 6) (8 x - 35 ArcTan[x / 2]) / 8) / (64 + 48 x ^ 2 + 12 x ^ 4 + x ^ 6)`**Maple [A]**

time = 0.05, size = 32, normalized size = 0.55

method	result	size
risch	$x + \frac{29x^5 + \frac{136}{3}x^3 + 76x}{(x^2+4)^3} - \frac{35 \arctan(\frac{x}{2})}{8}$	31
default	$x - \frac{16(-\frac{29}{64}x^5 - \frac{17}{6}x^3 - \frac{19}{4}x)}{(x^2+4)^3} - \frac{35 \arctan(\frac{x}{2})}{8}$	32
meijerg	$\frac{x(\frac{9}{4}x^6 + \frac{693}{16}x^4 + 210x^2 + 315)}{144(1 + \frac{x^2}{4})^3} - \frac{35 \arctan(\frac{x}{2})}{8}$	37

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8/(x^2+4)^4,x,method=_RETURNVERBOSE)`

[Out] $x - 16(-\frac{29}{64}x^5 - \frac{17}{6}x^3 - \frac{19}{4}x)/(x^2+4)^3 - 35/8 \arctan(1/2*x)$

Maxima [A]

time = 0.34, size = 41, normalized size = 0.71

$$x + \frac{87x^5 + 544x^3 + 912x}{12(x^6 + 12x^4 + 48x^2 + 64)} - \frac{35}{8} \arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8/(x^2+4)^4,x, algorithm="maxima")`

[Out] $x + 1/12*(87*x^5 + 544*x^3 + 912*x)/(x^6 + 12*x^4 + 48*x^2 + 64) - 35/8*\arctan(1/2*x)$

Fricas [A]

time = 0.31, size = 59, normalized size = 1.02

$$\frac{24x^7 + 462x^5 + 2240x^3 - 105(x^6 + 12x^4 + 48x^2 + 64)\arctan\left(\frac{1}{2}x\right) + 3360x}{24(x^6 + 12x^4 + 48x^2 + 64)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8/(x^2+4)^4,x, algorithm="fricas")`

[Out] $1/24*(24*x^7 + 462*x^5 + 2240*x^3 - 105*(x^6 + 12*x^4 + 48*x^2 + 64)*\arctan(1/2*x) + 3360*x)/(x^6 + 12*x^4 + 48*x^2 + 64)$

Sympy [A]

time = 0.07, size = 39, normalized size = 0.67

$$x + \frac{87x^5 + 544x^3 + 912x}{12x^6 + 144x^4 + 576x^2 + 768} - \frac{35 \operatorname{atan}\left(\frac{x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8/(x**2+4)**4,x)`

[Out] $x + (87x^5 + 544x^3 + 912x)/(12x^6 + 144x^4 + 576x^2 + 768) - 35 \operatorname{atan}(x/2)/8$

Giac [A]

time = 0.00, size = 36, normalized size = 0.62

$$x + \frac{87x^5 + 544x^3 + 912x}{12(x^2 + 4)^3} - \frac{35}{8} \operatorname{arctan}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8/(x^2+4)^4,x)`

[Out] $x + 1/12*(87*x^5 + 544*x^3 + 912*x)/(x^2 + 4)^3 - 35/8*\operatorname{arctan}(1/2*x)$

Mupad [B]

time = 0.07, size = 40, normalized size = 0.69

$$x - \frac{35 \operatorname{atan}\left(\frac{x}{2}\right)}{8} + \frac{\frac{29x^5}{4} + \frac{136x^3}{3} + 76x}{x^6 + 12x^4 + 48x^2 + 64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8/(x^2 + 4)^4,x)`

[Out] $x - (35*\operatorname{atan}(x/2))/8 + (76*x + (136*x^3)/3 + (29*x^5)/4)/(48*x^2 + 12*x^4 + x^6 + 64)$

$$3.168 \quad \int \frac{-4+7x}{(5+2x+3x^2)^2} dx$$

Optimal. Leaf size=43

$$-\frac{39+19x}{28(5+2x+3x^2)} - \frac{19 \tan^{-1}\left(\frac{1+3x}{\sqrt{14}}\right)}{28\sqrt{14}}$$

[Out] 1/28*(-39-19*x)/(3*x^2+2*x+5)-19/392*arctan(1/14*(1+3*x)*14^(1/2))*14^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {652, 632, 210}

$$-\frac{19x+39}{28(3x^2+2x+5)} - \frac{19 \tan^{-1}\left(\frac{3x+1}{\sqrt{14}}\right)}{28\sqrt{14}}$$

Antiderivative was successfully verified.

[In] Int[(-4 + 7*x)/(5 + 2*x + 3*x^2)^2,x]

[Out] -1/28*(39 + 19*x)/(5 + 2*x + 3*x^2) - (19*ArcTan[(1 + 3*x)/Sqrt[14]])/(28*Sqrt[14])

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 652

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rubi steps

$$\begin{aligned}
\int \frac{-4 + 7x}{(5 + 2x + 3x^2)^2} dx &= -\frac{39 + 19x}{28(5 + 2x + 3x^2)} - \frac{19}{28} \int \frac{1}{5 + 2x + 3x^2} dx \\
&= -\frac{39 + 19x}{28(5 + 2x + 3x^2)} + \frac{19}{14} \text{Subst} \left(\int \frac{1}{-56 - x^2} dx, x, 2 + 6x \right) \\
&= -\frac{39 + 19x}{28(5 + 2x + 3x^2)} - \frac{19 \tan^{-1} \left(\frac{1+3x}{\sqrt{14}} \right)}{28\sqrt{14}}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 43, normalized size = 1.00

$$\frac{-39 - 19x}{28(5 + 2x + 3x^2)} - \frac{19 \tan^{-1} \left(\frac{1+3x}{\sqrt{14}} \right)}{28\sqrt{14}}$$

Antiderivative was successfully verified.

`[In] Integrate[(-4 + 7*x)/(5 + 2*x + 3*x^2)^2,x]``[Out] (-39 - 19*x)/(28*(5 + 2*x + 3*x^2)) - (19*ArcTan[(1 + 3*x)/Sqrt[14]])/(28*Sqrt[14])`**Mathics [A]**

time = 2.07, size = 45, normalized size = 1.05

$$\frac{-546 - 266x - 19\sqrt{14} \text{ArcTan} \left[\frac{\sqrt{14}(1+3x)}{14} \right] (5 + 2x + 3x^2)}{1960 + 784x + 1176x^2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(7*x - 4)/(3*x^2 + 2*x + 5)^2,x]')``[Out] (-546 - 266 x - 19 Sqrt[14] ArcTan[Sqrt[14] (1 + 3 x) / 14] (5 + 2 x + 3 x ^ 2)) / (392 (5 + 2 x + 3 x ^ 2))`**Maple [A]**

time = 0.19, size = 37, normalized size = 0.86

method	result	size
risch	$ \frac{-\frac{19x}{84} - \frac{13}{28}}{x^2 + \frac{2}{3}x + \frac{5}{3}} - \frac{19 \arctan \left(\frac{(1+3x)\sqrt{14}}{14} \right) \sqrt{14}}{392} $	34

default	$\frac{-38x-78}{168x^2+112x+280} - \frac{19\sqrt{14} \arctan\left(\frac{(6x+2)\sqrt{14}}{28}\right)}{392}$	37
---------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-4+7*x)/(3*x^2+2*x+5)^2,x,method=_RETURNVERBOSE)`

[Out] $1/56*(-38*x-78)/(3*x^2+2*x+5)-19/392*14^{(1/2)}*\arctan(1/28*(6*x+2)*14^{(1/2)})$

Maxima [A]

time = 0.36, size = 36, normalized size = 0.84

$$-\frac{19}{392}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(3x+1)\right) - \frac{19x+39}{28(3x^2+2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-4+7*x)/(3*x^2+2*x+5)^2,x, algorithm="maxima")`

[Out] $-19/392*\sqrt{14}*\arctan(1/14*\sqrt{14}*(3*x+1)) - 1/28*(19*x+39)/(3*x^2+2*x+5)$

Fricas [A]

time = 0.37, size = 45, normalized size = 1.05

$$-\frac{19\sqrt{14}(3x^2+2x+5)\arctan\left(\frac{1}{14}\sqrt{14}(3x+1)\right) + 266x + 546}{392(3x^2+2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-4+7*x)/(3*x^2+2*x+5)^2,x, algorithm="fricas")`

[Out] $-1/392*(19*\sqrt{14}*(3*x^2+2*x+5)*\arctan(1/14*\sqrt{14}*(3*x+1)) + 266*x + 546)/(3*x^2+2*x+5)$

Sympy [A]

time = 0.07, size = 42, normalized size = 0.98

$$\frac{-19x-39}{84x^2+56x+140} - \frac{19\sqrt{14}\operatorname{atan}\left(\frac{3\sqrt{14}x+\sqrt{14}}{14}\right)}{392}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-4+7*x)/(3*x**2+2*x+5)**2,x)`

[Out] $(-19*x-39)/(84*x**2+56*x+140) - 19*\sqrt{14}*\operatorname{atan}(3*\sqrt{14}*x/14 + \sqrt{14}/14)/392$

Giac [A]

time = 0.00, size = 46, normalized size = 1.07

$$\frac{-19x - 39}{28(3x^2 + 2x + 5)} - \frac{19 \arctan\left(\frac{3x+1}{\sqrt{14}}\right)}{2 \cdot 14\sqrt{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4+7*x)/(3*x^2+2*x+5)^2,x)**[Out]** -19/392*sqrt(14)*arctan(1/14*sqrt(14)*(3*x + 1)) - 1/28*(19*x + 39)/(3*x^2 + 2*x + 5)**Mupad [B]**

time = 0.19, size = 36, normalized size = 0.84

$$-\frac{\frac{19x}{84} + \frac{13}{28}}{x^2 + \frac{2x}{3} + \frac{5}{3}} - \frac{19\sqrt{14} \operatorname{atan}\left(\frac{3\sqrt{14}x}{14} + \frac{\sqrt{14}}{14}\right)}{392}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((7*x - 4)/(2*x + 3*x^2 + 5)^2,x)**[Out]** - ((19*x)/84 + 13/28)/((2*x)/3 + x^2 + 5/3) - (19*14^(1/2)*atan((3*14^(1/2)*x)/14 + 14^(1/2)/14))/392

$$3.169 \quad \int \frac{5-4x}{(-2-4x+3x^2)^2} dx$$

Optimal. Leaf size=43

$$-\frac{18-7x}{20(2+4x-3x^2)} - \frac{7 \tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

[Out] 1/20*(-18+7*x)/(-3*x^2+4*x+2)-7/200*arctanh(1/10*(2-3*x)*10^(1/2))*10^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {652, 632, 212}

$$-\frac{18-7x}{20(-3x^2+4x+2)} - \frac{7 \tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

Antiderivative was successfully verified.

[In] Int[(5 - 4*x)/(-2 - 4*x + 3*x^2)^2,x]

[Out] -1/20*(18 - 7*x)/(2 + 4*x - 3*x^2) - (7*ArcTanh[(2 - 3*x)/Sqrt[10]])/(20*Sqrt[10])

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 652

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rubi steps

$$\begin{aligned}
\int \frac{5-4x}{(-2-4x+3x^2)^2} dx &= -\frac{18-7x}{20(2+4x-3x^2)} - \frac{7}{20} \int \frac{1}{-2-4x+3x^2} dx \\
&= -\frac{18-7x}{20(2+4x-3x^2)} + \frac{7}{10} \text{Subst} \left(\int \frac{1}{40-x^2} dx, x, -4+6x \right) \\
&= -\frac{18-7x}{20(2+4x-3x^2)} - \frac{7 \tanh^{-1} \left(\frac{2-3x}{\sqrt{10}} \right)}{20\sqrt{10}}
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 62, normalized size = 1.44

$$\frac{18-7x}{20(-2-4x+3x^2)} - \frac{7 \log(2+\sqrt{10}-3x)}{40\sqrt{10}} + \frac{7 \log(-2+\sqrt{10}+3x)}{40\sqrt{10}}$$

Antiderivative was successfully verified.

`[In] Integrate[(5 - 4*x)/(-2 - 4*x + 3*x^2)^2, x]`

```
[Out] (18 - 7*x)/(20*(-2 - 4*x + 3*x^2)) - (7*Log[2 + Sqrt[10] - 3*x])/(40*Sqrt[10]) + (7*Log[-2 + Sqrt[10] + 3*x])/(40*Sqrt[10])
```

Mathics [A]

time = 2.25, size = 55, normalized size = 1.28

$$\frac{-360 + 140x + 7\sqrt{10} (2 + 4x - 3x^2) \left(\text{Log} \left[-\frac{2}{3} + \frac{\sqrt{10}}{3} + x \right] - \text{Log} \left[-\frac{2}{3} - \frac{\sqrt{10}}{3} + x \right] \right)}{800 + 1600x - 1200x^2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(5 - 4*x)/(3*x^2 - 4*x - 2)^2, x]')`

```
[Out] (-360 + 140 x + 7 Sqrt[10] (2 + 4 x - 3 x ^ 2) (Log[-2 / 3 + Sqrt[10] / 3 + x] - Log[-2 / 3 - Sqrt[10] / 3 + x])) / (400 (2 + 4 x - 3 x ^ 2))
```

Maple [A]

time = 0.11, size = 37, normalized size = 0.86

method	result	size
default	$-\frac{14x-36}{40(3x^2-4x-2)} + \frac{7\sqrt{10} \operatorname{arctanh}\left(\frac{(6x-4)\sqrt{10}}{20}\right)}{200}$	37

risch	$\frac{-\frac{7x}{60} + \frac{3}{10}}{x^2 - \frac{4}{3}x - \frac{2}{3}} + \frac{7\sqrt{10} \ln(3x-2+\sqrt{10})}{400} - \frac{7\sqrt{10} \ln(3x-2-\sqrt{10})}{400}$	48
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5-4*x)/(3*x^2-4*x-2)^2,x,method=_RETURNVERBOSE)`

[Out] $-1/40*(14*x-36)/(3*x^2-4*x-2)+7/200*10^{(1/2)}*\operatorname{arctanh}(1/20*(6*x-4)*10^{(1/2)})$

Maxima [A]

time = 0.35, size = 47, normalized size = 1.09

$$-\frac{7}{400}\sqrt{10}\log\left(\frac{3x-\sqrt{10}-2}{3x+\sqrt{10}-2}\right) - \frac{7x-18}{20(3x^2-4x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5-4*x)/(3*x^2-4*x-2)^2,x, algorithm="maxima")`

[Out] $-7/400*\operatorname{sqrt}(10)*\log((3*x - \operatorname{sqrt}(10) - 2)/(3*x + \operatorname{sqrt}(10) - 2)) - 1/20*(7*x - 18)/(3*x^2 - 4*x - 2)$

Fricas [A]

time = 0.35, size = 68, normalized size = 1.58

$$\frac{7\sqrt{10}(3x^2-4x-2)\log\left(\frac{9x^2+2\sqrt{10}(3x-2)-12x+14}{3x^2-4x-2}\right) - 140x + 360}{400(3x^2-4x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5-4*x)/(3*x^2-4*x-2)^2,x, algorithm="fricas")`

[Out] $1/400*(7*\operatorname{sqrt}(10)*(3*x^2 - 4*x - 2)*\log((9*x^2 + 2*\operatorname{sqrt}(10)*(3*x - 2) - 12*x + 14)/(3*x^2 - 4*x - 2)) - 140*x + 360)/(3*x^2 - 4*x - 2)$

Sympy [A]

time = 0.08, size = 58, normalized size = 1.35

$$-\frac{7x-18}{60x^2-80x-40} + \frac{7\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{400} - \frac{7\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{400}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5-4*x)/(3*x**2-4*x-2)**2,x)`

[Out] $-(7*x - 18)/(60*x**2 - 80*x - 40) + 7*\operatorname{sqrt}(10)*\log(x - 2/3 + \operatorname{sqrt}(10)/3)/400 - 7*\operatorname{sqrt}(10)*\log(x - \operatorname{sqrt}(10)/3 - 2/3)/400$

Giac [A]

time = 0.00, size = 63, normalized size = 1.47

$$\frac{-7x + 18}{20(3x^2 - 4x - 2)} - \frac{7 \ln \left| \frac{6x-4-2\sqrt{10}}{6x-4+2\sqrt{10}} \right|}{2 \cdot 20\sqrt{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5-4*x)/(3*x^2-4*x-2)^2,x)**[Out]** -7/400*sqrt(10)*log(abs((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2))) - 1/20*(7*x - 18)/(3*x^2 - 4*x - 2)**Mupad [B]**

time = 0.21, size = 34, normalized size = 0.79

$$\frac{7\sqrt{10} \operatorname{atanh}\left(\sqrt{10}\left(\frac{3x}{10} - \frac{1}{5}\right)\right)}{200} + \frac{\frac{7x}{60} - \frac{3}{10}}{-x^2 + \frac{4x}{3} + \frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(4*x - 5)/(4*x - 3*x^2 + 2)^2,x)**[Out]** (7*10^(1/2)*atanh(10^(1/2)*((3*x)/10 - 1/5)))/200 + ((7*x)/60 - 3/10)/((4*x)/3 - x^2 + 2/3)

$$3.170 \quad \int \frac{x^5}{(1+x^4)^3} dx$$

Optimal. Leaf size=37

$$-\frac{x^2}{8(1+x^4)^2} + \frac{x^2}{16(1+x^4)} + \frac{1}{16} \tan^{-1}(x^2)$$

[Out] $-1/8*x^2/(x^4+1)^2+1/16*x^2/(x^4+1)+1/16*\arctan(x^2)$

Rubi [A]

time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {281, 294, 205, 209}

$$\frac{1}{16} \tan^{-1}(x^2) + \frac{x^2}{16(x^4+1)} - \frac{x^2}{8(x^4+1)^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^5/(1+x^4)^3, x]$

[Out] $-1/8*x^2/(1+x^4)^2 + x^2/(16*(1+x^4)) + \text{ArcTan}[x^2]/16$

Rule 205

$\text{Int}[(a_+ + (b_+)(x_+)^{n_+})^{p_+}, x_Symbol] \rightarrow \text{Simp}[(-x) * ((a + b*x^n)^{(p+1)} / (a*n*(p+1))), x] + \text{Dist}[(n*(p+1) + 1) / (a*n*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)}, x], x] /;$ FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 209

$\text{Int}[(a_+ + (b_+)(x_+)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2])) * \text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 281

$\text{Int}[(x_+)^{m_+} * ((a_+ + (b_+)(x_+)^{n_+})^{p_+}), x_Symbol] \rightarrow \text{With}[\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{(m+1)/k - 1} * (a + b*x^{(n/k)})^p, x], x, x^k], x] /;$ k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 294

$\text{Int}[(c_+)(x_+)^{m_+} * ((a_+ + (b_+)(x_+)^{n_+})^{p_+}), x_Symbol] \rightarrow \text{Simp}[c^{(n-1)} * (c*x)^{(m-n+1)} * ((a + b*x^n)^{(p+1)} / (b*n*(p+1))), x] - \text{Dist}[c^n$

```

*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(1+x^4)^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(1+x^2)^3} dx, x, x^2 \right) \\
&= -\frac{x^2}{8(1+x^4)^2} + \frac{1}{8} \text{Subst} \left(\int \frac{1}{(1+x^2)^2} dx, x, x^2 \right) \\
&= -\frac{x^2}{8(1+x^4)^2} + \frac{x^2}{16(1+x^4)} + \frac{1}{16} \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, x^2 \right) \\
&= -\frac{x^2}{8(1+x^4)^2} + \frac{x^2}{16(1+x^4)} + \frac{1}{16} \tan^{-1}(x^2)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 25, normalized size = 0.68

$$\frac{1}{16} \left(\frac{x^2(-1+x^4)}{(1+x^4)^2} + \tan^{-1}(x^2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(1 + x^4)^3, x]

[Out] ((x^2*(-1 + x^4))/(1 + x^4)^2 + ArcTan[x^2])/16

Mathics [A]

time = 1.88, size = 38, normalized size = 1.03

$$\frac{-x^2 + x^6 + \text{ArcTan}[x^2] (1 + 2x^4 + x^8)}{16 + 32x^4 + 16x^8}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^5/(x^4 + 1)^3, x]')

[Out] (-x ^ 2 + x ^ 6 + ArcTan[x ^ 2] (1 + 2 x ^ 4 + x ^ 8)) / (16 (1 + 2 x ^ 4 + x ^ 8))

Maple [A]

time = 0.05, size = 28, normalized size = 0.76

method	result	size
meijerg	$-\frac{x^2(-3x^4+3)}{48(x^4+1)^2} + \frac{\arctan(x^2)}{16}$	27
risch	$\frac{\frac{1}{16}x^6 - \frac{1}{16}x^2}{(x^4+1)^2} + \frac{\arctan(x^2)}{16}$	27
default	$\frac{\frac{1}{8}x^6 - \frac{1}{8}x^2}{2(x^4+1)^2} + \frac{\arctan(x^2)}{16}$	28

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(x^4+1)^3,x,method=_RETURNVERBOSE)`

[Out] $1/2*(1/8*x^6-1/8*x^2)/(x^4+1)^2+1/16*\arctan(x^2)$

Maxima [A]

time = 0.34, size = 30, normalized size = 0.81

$$\frac{x^6 - x^2}{16(x^8 + 2x^4 + 1)} + \frac{1}{16} \arctan(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(x^4+1)^3,x, algorithm="maxima")`

[Out] $1/16*(x^6 - x^2)/(x^8 + 2*x^4 + 1) + 1/16*\arctan(x^2)$

Fricas [A]

time = 0.31, size = 38, normalized size = 1.03

$$\frac{x^6 - x^2 + (x^8 + 2x^4 + 1) \arctan(x^2)}{16(x^8 + 2x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(x^4+1)^3,x, algorithm="fricas")`

[Out] $1/16*(x^6 - x^2 + (x^8 + 2*x^4 + 1)*\arctan(x^2))/(x^8 + 2*x^4 + 1)$

Sympy [A]

time = 0.08, size = 24, normalized size = 0.65

$$\frac{x^6 - x^2}{16x^8 + 32x^4 + 16} + \frac{\operatorname{atan}(x^2)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(x**4+1)**3,x)`

[Out] $(x**6 - x**2)/(16*x**8 + 32*x**4 + 16) + \operatorname{atan}(x**2)/16$

Giac [A]

time = 0.00, size = 43, normalized size = 1.16

$$\frac{x^2 - \frac{1}{x^2}}{16 \left(\left(x^2 - \frac{1}{x^2} \right)^2 + 4 \right)} + \frac{\arctan \left(\frac{x^4 - 1}{2x^2} \right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^5/(x^4+1)^3,x)``[Out] 1/16*(x^2 - 1/x^2)/((x^2 - 1/x^2)^2 + 4) + 1/32*arctan(1/2*(x^4 - 1)/x^2)`**Mupad [B]**

time = 0.04, size = 32, normalized size = 0.86

$$\frac{\operatorname{atan}(x^2)}{16} - \frac{\frac{x^2}{16} - \frac{x^6}{16}}{x^8 + 2x^4 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^5/(x^4 + 1)^3,x)``[Out] atan(x^2)/16 - (x^2/16 - x^6/16)/(2*x^4 + x^8 + 1)`

$$3.171 \quad \int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2} dx$$

Optimal. Leaf size=32

$$\frac{1}{4(2+2x^2+x^4)} + \frac{1}{4} \log(2+2x^2+x^4)$$

[Out] 1/4/(x^4+2*x^2+2)+1/4*ln(x^4+2*x^2+2)

Rubi [A]

time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1261, 700, 642}

$$\frac{1}{4} \log(x^4 + 2x^2 + 2) - \frac{(x^2 + 1)^2}{4(x^4 + 2x^2 + 2)}$$

Antiderivative was successfully verified.

[In] Int[(x*(1 + x^2)^3)/(2 + 2*x^2 + x^4)^2,x]

[Out] -1/4*(1 + x^2)^2/(2 + 2*x^2 + x^4) + Log[2 + 2*x^2 + x^4]/4

Rule 642

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 700

Int[((d_) + (e_)*(x_)^m)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^p, x_Symbol] := Simp[d*(d + e*x)^(m-1)*((a + b*x + c*x^2)^(p+1)/(b*(p+1))), x] - Dist[d*e*((m-1)/(b*(p+1))), Int[(d + e*x)^(m-2)*(a + b*x + c*x^2)^(p+1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]

Rule 1261

Int[(x)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned} \int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(1+x)^3}{(2+2x+x^2)^2} dx, x, x^2 \right) \\ &= -\frac{(1+x^2)^2}{4(2+2x^2+x^4)} + \frac{1}{2} \text{Subst} \left(\int \frac{1+x}{2+2x+x^2} dx, x, x^2 \right) \\ &= -\frac{(1+x^2)^2}{4(2+2x^2+x^4)} + \frac{1}{4} \log(2+2x^2+x^4) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 26, normalized size = 0.81

$$\frac{1}{4} \left(\frac{1}{1+(1+x^2)^2} + \log(1+(1+x^2)^2) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(x*(1 + x^2)^3)/(2 + 2*x^2 + x^4)^2, x]``[Out] ((1 + (1 + x^2)^2)^(-1) + Log[1 + (1 + x^2)^2])/4`**Mathics [A]**

time = 1.94, size = 38, normalized size = 1.19

$$\frac{1 + \text{Log}[2 + 2x^2 + x^4] (2 + 2x^2 + x^4)}{8 + 8x^2 + 4x^4}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x*((x^2 + 1)^3/(x^4 + 2*x^2 + 2)^2), x]')``[Out] (1 + Log[2 + 2 x ^ 2 + x ^ 4] (2 + 2 x ^ 2 + x ^ 4)) / (4 (2 + 2 x ^ 2 + x ^ 4))`**Maple [A]**

time = 0.04, size = 29, normalized size = 0.91

method	result	size
default	$\frac{1}{4x^4+8x^2+8} + \frac{\ln(x^4+2x^2+2)}{4}$	29
norman	$\frac{1}{4x^4+8x^2+8} + \frac{\ln(x^4+2x^2+2)}{4}$	29
risch	$\frac{1}{4x^4+8x^2+8} + \frac{\ln(x^4+2x^2+2)}{4}$	29

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x,method=_RETURNVERBOSE)`

[Out] $1/4/(x^4+2x^2+2)+1/4*\ln(x^4+2x^2+2)$

Maxima [A]

time = 0.27, size = 28, normalized size = 0.88

$$\frac{1}{4(x^4 + 2x^2 + 2)} + \frac{1}{4} \log(x^4 + 2x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x, algorithm="maxima")`

[Out] $1/4/(x^4 + 2x^2 + 2) + 1/4*\log(x^4 + 2x^2 + 2)$

Fricas [A]

time = 0.43, size = 38, normalized size = 1.19

$$\frac{(x^4 + 2x^2 + 2) \log(x^4 + 2x^2 + 2) + 1}{4(x^4 + 2x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x, algorithm="fricas")`

[Out] $1/4*((x^4 + 2x^2 + 2)*\log(x^4 + 2x^2 + 2) + 1)/(x^4 + 2x^2 + 2)$

Sympy [A]

time = 0.07, size = 26, normalized size = 0.81

$$\frac{\log(x^4 + 2x^2 + 2)}{4} + \frac{1}{4x^4 + 8x^2 + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x**2+1)**3/(x**4+2*x**2+2)**2,x)`

[Out] $\log(x**4 + 2*x**2 + 2)/4 + 1/(4*x**4 + 8*x**2 + 8)$

Giac [A]

time = 0.00, size = 34, normalized size = 1.06

$$\frac{\frac{1}{4\left(\frac{x^4}{2}+x^2+1\right)} + \frac{\ln\left(\frac{x^4}{2}+x^2+1\right)}{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x)`

[Out] $1/4/(x^4 + 2x^2 + 2) + 1/4*\log(1/2*x^4 + x^2 + 1)$

Mupad [B]

time = 0.19, size = 28, normalized size = 0.88

$$\frac{\ln(x^4 + 2x^2 + 2)}{4} + \frac{1}{4(x^4 + 2x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(x^2 + 1)^3)/(2*x^2 + x^4 + 2)^2,x)

[Out] log(2*x^2 + x^4 + 2)/4 + 1/(4*(2*x^2 + x^4 + 2))

$$3.172 \quad \int \frac{x^3}{(a^4+x^4)^3} dx$$

Optimal. Leaf size=13

$$-\frac{1}{8(a^4+x^4)^2}$$

[Out] -1/8/(a^4+x^4)^2

Rubi [A]

time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$-\frac{1}{8(a^4+x^4)^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a^4 + x^4)^3,x]

[Out] -1/8*1/(a^4 + x^4)^2

Rule 267

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int \frac{x^3}{(a^4+x^4)^3} dx = -\frac{1}{8(a^4+x^4)^2}$$

Mathematica [A]

time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{8(a^4+x^4)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a^4 + x^4)^3,x]

[Out] -1/8*1/(a^4 + x^4)^2

Mathics [A]

time = 1.87, size = 23, normalized size = 1.77

$$-\frac{1}{8a^8 + 16a^4x^4 + 8x^8}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3/(a^4 + x^4)^3,x]')
```

```
[Out] -1 / (8 a ^ 8 + 16 a ^ 4 x ^ 4 + 8 x ^ 8)
```

Maple [A]

time = 0.05, size = 12, normalized size = 0.92

method	result	size
gosper	$-\frac{1}{8(a^4+x^4)^2}$	12
derivativedivides	$-\frac{1}{8(a^4+x^4)^2}$	12
default	$-\frac{1}{8(a^4+x^4)^2}$	12
norman	$-\frac{1}{8(a^4+x^4)^2}$	12
risch	$-\frac{1}{8(a^4+x^4)^2}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^4+x^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/(a^4+x^4)^2
```

Maxima [A]

time = 0.26, size = 11, normalized size = 0.85

$$-\frac{1}{8(a^4 + x^4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^4+x^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8/(a^4 + x^4)^2
```

Fricas [A]

time = 0.38, size = 19, normalized size = 1.46

$$-\frac{1}{8(a^8 + 2a^4x^4 + x^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a^4+x^4)^3,x, algorithm="fricas")

[Out] -1/8/(a^8 + 2*a^4*x^4 + x^8)

Sympy [A]

time = 0.13, size = 20, normalized size = 1.54

$$-\frac{1}{8a^8 + 16a^4x^4 + 8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(a**4+x**4)**3,x)

[Out] -1/(8*a**8 + 16*a**4*x**4 + 8*x**8)

Giac [A]

time = 0.00, size = 14, normalized size = 1.08

$$-\frac{1}{8(x^4 + a^4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a^4+x^4)^3,x)

[Out] -1/8/(a^4 + x^4)^2

Mupad [B]

time = 0.20, size = 11, normalized size = 0.85

$$-\frac{1}{8(a^4 + x^4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a^4 + x^4)^3,x)

[Out] -1/(8*(a^4 + x^4)^2)

$$3.173 \quad \int \frac{1}{x(a^4+x^4)^3} dx$$

Optimal. Leaf size=54

$$\frac{1}{8a^4(a^4+x^4)^2} + \frac{1}{4a^8(a^4+x^4)} + \frac{\log(x)}{a^{12}} - \frac{\log(a^4+x^4)}{4a^{12}}$$

[Out] 1/8/a^4/(a^4+x^4)^2+1/4/a^8/(a^4+x^4)+ln(x)/a^12-1/4*ln(a^4+x^4)/a^12

Rubi [A]

time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 46}

$$\frac{\log(x)}{a^{12}} + \frac{1}{8a^4(a^4+x^4)^2} - \frac{\log(a^4+x^4)}{4a^{12}} + \frac{1}{4a^8(a^4+x^4)}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a^4 + x^4)^3), x]

[Out] 1/(8*a^4*(a^4 + x^4)^2) + 1/(4*a^8*(a^4 + x^4)) + Log[x]/a^12 - Log[a^4 + x^4]/(4*a^12)

Rule 46

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{1}{x(a^4+x^4)^3} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x(a^4+x)^3} dx, x, x^4 \right) \\ &= \frac{1}{4} \text{Subst} \left(\int \left(\frac{1}{a^{12}x} - \frac{1}{a^4(a^4+x)^3} - \frac{1}{a^8(a^4+x)^2} - \frac{1}{a^{12}(a^4+x)} \right) dx, x, x^4 \right) \\ &= \frac{1}{8a^4(a^4+x^4)^2} + \frac{1}{4a^8(a^4+x^4)} + \frac{\log(x)}{a^{12}} - \frac{\log(a^4+x^4)}{4a^{12}} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 46, normalized size = 0.85

$$\frac{3a^8 + 2a^4x^4}{(a^4 + x^4)^2} + 8 \log(x) - 2 \log(a^4 + x^4)}{8a^{12}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a^4 + x^4)^3),x]

[Out] ((3*a^8 + 2*a^4*x^4)/(a^4 + x^4)^2 + 8*Log[x] - 2*Log[a^4 + x^4])/(8*a^12)

Mathics [A]

time = 2.27, size = 70, normalized size = 1.30

$$\frac{a^4(3a^4 + 2x^4) + 2(a^8 + 2a^4x^4 + x^8)(-\text{Log}[a^4 + x^4] + 4\text{Log}[x])}{8a^{12}(a^8 + 2a^4x^4 + x^8)}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x*(a^4 + x^4)^3),x]')

[Out] (a^4(3a^4 + 2x^4) + 2(a^8 + 2a^4x^4 + x^8)(-Log[a^4 + x^4] + 4Log[x])) / (8a^12(a^8 + 2a^4x^4 + x^8))

Maple [A]

time = 0.06, size = 52, normalized size = 0.96

method	result	size
norman	$\frac{\frac{3}{8a^4} + \frac{x^4}{4a^8}}{(a^4 + x^4)^2} + \frac{\ln(x)}{a^{12}} - \frac{\ln(a^4 + x^4)}{4a^{12}}$	45
risch	$\frac{\frac{3}{8a^4} + \frac{x^4}{4a^8}}{(a^4 + x^4)^2} + \frac{\ln(x)}{a^{12}} - \frac{\ln(a^4 + x^4)}{4a^{12}}$	45
default	$-\frac{a^4}{2(a^4 + x^4)} + \frac{\ln(a^4 + x^4)}{2} - \frac{a^8}{4(a^4 + x^4)^2} + \frac{\ln(x)}{a^{12}}$	52

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a^4+x^4)^3,x,method=_RETURNVERBOSE)

[Out] -1/2/a^12*(-1/2*a^4/(a^4+x^4)+1/2*ln(a^4+x^4)-1/4*a^8/(a^4+x^4)^2)+ln(x)/a^12

Maxima [A]

time = 0.27, size = 57, normalized size = 1.06

$$\frac{3a^4 + 2x^4}{8(a^{16} + 2a^{12}x^4 + a^8x^8)} - \frac{\log(a^4 + x^4)}{4a^{12}} + \frac{\log(x^4)}{4a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a^4+x^4)^3,x, algorithm="maxima")

[Out] $\frac{1}{8} \cdot \frac{3a^4 + 2x^4}{(a^{16} + 2a^{12}x^4 + a^8x^8)} - \frac{1}{4} \cdot \frac{\log(a^4 + x^4)}{a^{12}} + \frac{1}{4} \cdot \frac{\log(x^4)}{a^{12}}$

Fricas [A]

time = 0.32, size = 81, normalized size = 1.50

$$\frac{3a^8 + 2a^4x^4 - 2(a^8 + 2a^4x^4 + x^8)\log(a^4 + x^4) + 8(a^8 + 2a^4x^4 + x^8)\log(x)}{8(a^{20} + 2a^{16}x^4 + a^{12}x^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a^4+x^4)^3,x, algorithm="fricas")

[Out] $\frac{1}{8} \cdot \frac{3a^8 + 2a^4x^4 - 2(a^8 + 2a^4x^4 + x^8)\log(a^4 + x^4) + 8(a^8 + 2a^4x^4 + x^8)\log(x)}{(a^{20} + 2a^{16}x^4 + a^{12}x^8)}$

Sympy [A]

time = 0.23, size = 51, normalized size = 0.94

$$\frac{3a^4 + 2x^4}{8a^{16} + 16a^{12}x^4 + 8a^8x^8} + \frac{\log(x)}{a^{12}} - \frac{\log(a^4 + x^4)}{4a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a**4+x**4)**3,x)

[Out] $\frac{(3a^{**4} + 2x^{**4})}{(8a^{**16} + 16a^{**12}x^{**4} + 8a^{**8}x^{**8})} + \frac{\log(x)}{a^{**12}} - \frac{\log(a^{**4} + x^{**4})}{(4a^{**12})}$

Giac [A]

time = 0.00, size = 63, normalized size = 1.17

$$\frac{\ln(x^4)}{4a^{12}} - \frac{\ln(x^4 + a^4)}{4a^{12}} + \frac{3x^8 + 8x^4a^4 + 6a^8}{8a^{12}(x^4 + a^4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a^4+x^4)^3,x)

[Out] $-\frac{1}{4} \cdot \frac{\log(a^4 + x^4)}{a^{12}} + \frac{1}{4} \cdot \frac{\log(x^4)}{a^{12}} + \frac{1}{8} \cdot \frac{(6a^8 + 8a^4x^4 + 3x^8)}{(a^4 + x^4)^2 a^{12}}$

Mupad [B]

time = 0.11, size = 52, normalized size = 0.96

$$\frac{\ln(x)}{a^{12}} + \frac{\frac{3}{8a^4} + \frac{x^4}{4a^8}}{a^8 + 2a^4x^4 + x^8} - \frac{\ln(a^4 + x^4)}{4a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^4 + x^4)^3),x)
```

```
[Out] log(x)/a^12 + (3/(8*a^4) + x^4/(4*a^8))/(a^8 + x^8 + 2*a^4*x^4) - log(a^4 + x^4)/(4*a^12)
```

$$3.174 \quad \int \frac{1}{x^2(a^4+x^4)^3} dx$$

Optimal. Leaf size=157

$$-\frac{45}{32a^{12}x} + \frac{1}{8a^4x(a^4+x^4)^2} + \frac{9}{32a^8x(a^4+x^4)} + \frac{45 \tan^{-1}\left(1 - \frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}} - \frac{45 \tan^{-1}\left(1 + \frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}} - \frac{45 \log(a^2 - \sqrt{2}ax + x^2)}{128\sqrt{2}a^{13}}$$

[Out] $-45/32/a^{12}/x+1/8/a^4/x/(a^4+x^4)^2+9/32/a^8/x/(a^4+x^4)+45/128*\arctan(1-x*\sqrt{2}/a)/a^{13}*2^{(1/2)}-45/128*\arctan(1+x*\sqrt{2}/a)/a^{13}*2^{(1/2)}-45/256*\ln(a^2+x^2-a*x*\sqrt{2})/a^{13}*2^{(1/2)}+45/256*\ln(a^2+x^2+a*x*\sqrt{2})/a^{13}*2^{(1/2)}$

Rubi [A]

time = 0.06, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {296, 331, 303, 1176, 631, 210, 1179, 642}

$$\frac{45 \tan^{-1}\left(1 - \frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}} - \frac{45 \tan^{-1}\left(\frac{\sqrt{2}x}{a} + 1\right)}{64\sqrt{2}a^{13}} - \frac{45}{32a^{12}x} + \frac{1}{8a^4x(a^4+x^4)^2} - \frac{45 \log(a^2 - \sqrt{2}ax + x^2)}{128\sqrt{2}a^{13}} + \frac{45 \log(a^2 + \sqrt{2}ax + x^2)}{128\sqrt{2}a^{13}} + \frac{9}{32a^8x(a^4+x^4)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a^4 + x^4)^3), x]

[Out] $-45/(32*a^{12}*x) + 1/(8*a^4*x*(a^4 + x^4)^2) + 9/(32*a^8*x*(a^4 + x^4)) + (45*ArcTan[1 - (Sqrt[2]*x)/a])/(64*Sqrt[2]*a^{13}) - (45*ArcTan[1 + (Sqrt[2]*x)/a])/(64*Sqrt[2]*a^{13}) - (45*Log[a^2 - Sqrt[2]*a*x + x^2])/(128*Sqrt[2]*a^{13}) + (45*Log[a^2 + Sqrt[2]*a*x + x^2])/(128*Sqrt[2]*a^{13})$

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 296

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(-(c*x)^(m+1))*((a+b*x^n)^(p+1)/(a*c*n*(p+1))), x] + Dist[(m+n*(p+1)+1)/(a*n*(p+1)), Int[(c*x)^m*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 303

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4)

), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 331

Int[((c_.)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 631

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1176

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1179

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (a^4 + x^4)^3} dx &= \frac{1}{8a^4 x (a^4 + x^4)^2} + \frac{9 \int \frac{1}{x^2 (a^4 + x^4)^2} dx}{8a^4} \\
&= \frac{1}{8a^4 x (a^4 + x^4)^2} + \frac{9}{32a^8 x (a^4 + x^4)} + \frac{45 \int \frac{1}{x^2 (a^4 + x^4)} dx}{32a^8} \\
&= -\frac{45}{32a^{12}x} + \frac{1}{8a^4 x (a^4 + x^4)^2} + \frac{9}{32a^8 x (a^4 + x^4)} - \frac{45 \int \frac{x^2}{a^4 + x^4} dx}{32a^{12}} \\
&= -\frac{45}{32a^{12}x} + \frac{1}{8a^4 x (a^4 + x^4)^2} + \frac{9}{32a^8 x (a^4 + x^4)} + \frac{45 \int \frac{a^2 - x^2}{a^4 + x^4} dx}{64a^{12}} - \frac{45 \int \frac{a^2 + x^2}{a^4 + x^4} dx}{64a^{12}} \\
&= -\frac{45}{32a^{12}x} + \frac{1}{8a^4 x (a^4 + x^4)^2} + \frac{9}{32a^8 x (a^4 + x^4)} - \frac{45 \int \frac{\sqrt{2} a + 2x}{-a^2 - \sqrt{2} ax - x^2} dx}{128\sqrt{2} a^{13}} - \frac{45 \int \frac{\sqrt{2}}{-a^2 + \sqrt{2} ax + x^2} dx}{128\sqrt{2} a^{13}} \\
&= -\frac{45}{32a^{12}x} + \frac{1}{8a^4 x (a^4 + x^4)^2} + \frac{9}{32a^8 x (a^4 + x^4)} - \frac{45 \log(a^2 - \sqrt{2} ax + x^2)}{128\sqrt{2} a^{13}} + \frac{45 \log(a^2 + \sqrt{2} ax + x^2)}{128\sqrt{2} a^{13}} \\
&= -\frac{45}{32a^{12}x} + \frac{1}{8a^4 x (a^4 + x^4)^2} + \frac{9}{32a^8 x (a^4 + x^4)} + \frac{45 \tan^{-1}\left(1 - \frac{\sqrt{2}x}{a}\right)}{64\sqrt{2} a^{13}} - \frac{45 \tan^{-1}\left(1 + \frac{\sqrt{2}x}{a}\right)}{64\sqrt{2} a^{13}}
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 134, normalized size = 0.85

$$\frac{\frac{256a}{x} + \frac{32a^5 x^3}{(a^4 + x^4)^2} + \frac{104ax^3}{a^4 + x^4} - 90\sqrt{2} \tan^{-1}\left(1 - \frac{\sqrt{2}x}{a}\right) + 90\sqrt{2} \tan^{-1}\left(1 + \frac{\sqrt{2}x}{a}\right) + 45\sqrt{2} \log(a^2 - \sqrt{2}ax + x^2) - 45\sqrt{2} \log(a^2 + \sqrt{2}ax + x^2)}{256a^{13}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x^2*(a^4 + x^4)^3), x]`

```
[Out] -1/256*((256*a)/x + (32*a^5*x^3)/(a^4 + x^4)^2 + (104*a*x^3)/(a^4 + x^4) -
90*Sqrt[2]*ArcTan[1 - (Sqrt[2]*x)/a] + 90*Sqrt[2]*ArcTan[1 + (Sqrt[2]*x)/a]
+ 45*Sqrt[2]*Log[a^2 - Sqrt[2]*a*x + x^2] - 45*Sqrt[2]*Log[a^2 + Sqrt[2]*a
*x + x^2])/a^13
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x^2*(a^4 + x^4)^3), x]')`

[Out] Exception raised: AttributeError >> 'SympyExpression' object has no attribute 'expr'

Maple [A]

time = 0.05, size = 124, normalized size = 0.79

method	result	size
risch	$\frac{-\frac{45x^8}{32a^{12}} - \frac{81x^4}{32a^8} - \frac{1}{a^4}}{x(a^4+x^4)^2} + \frac{45 \left(\sum_{R=\text{RootOf}(a^{52}-Z^4+1)} -R \ln((5-R^4 a^{52}+4)x+R^3 a^{40}) \right)}{128}$	75
default	$\frac{\frac{17}{32}a^4x^3 + \frac{13}{32}x^7}{(a^4+x^4)^2} + \frac{45\sqrt{2} \left(\ln \left(\frac{x^2-(a^4)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^4}}{x^2+(a^4)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^4}} \right) + 2 \arctan \left(\frac{\sqrt{2}x}{(a^4)^{\frac{1}{4}}} + 1 \right) + 2 \arctan \left(\frac{\sqrt{2}x}{(a^4)^{\frac{1}{4}}} - 1 \right) \right)}{256(a^4)^{\frac{1}{4}}} - \frac{1}{a^{12}}$	124

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a^4+x^4)^3,x,method=_RETURNVERBOSE)

[Out] $-1/a^{12} * ((17/32*a^4*x^3+13/32*x^7)/(a^4+x^4)^2+45/256/(a^4)^{(1/4)}*2^{(1/2)}*(\ln((x^2-(a^4)^{(1/4)}*x*2^{(1/2)}+(a^4)^{(1/2)}))/(x^2+(a^4)^{(1/4)}*x*2^{(1/2)}+(a^4)^{(1/2)}))+2*\arctan(2^{(1/2)}/(a^4)^{(1/4)}*x+1)+2*\arctan(2^{(1/2)}/(a^4)^{(1/4)}*x-1)))-1/a^{12}/x$

Maxima [A]

time = 0.35, size = 147, normalized size = 0.94

$$\frac{32a^8 + 81a^4x^4 + 45x^8}{32(a^{20}x + 2a^{16}x^5 + a^{12}x^9)} - \frac{45 \left(\frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}(\sqrt{2}a+2x)}{2a}\right)}{a} + \frac{2\sqrt{2} \arctan\left(-\frac{\sqrt{2}(\sqrt{2}a-2x)}{2a}\right)}{a} - \frac{\sqrt{2} \log(\sqrt{2}ax+a^2+x^2)}{a} + \frac{\sqrt{2} \log(-\sqrt{2}ax+a^2+x^2)}{a} \right)}{256a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a^4+x^4)^3,x, algorithm="maxima")

[Out] $-1/32*(32*a^8 + 81*a^4*x^4 + 45*x^8)/(a^{20}*x + 2*a^{16}*x^5 + a^{12}*x^9) - 45/256*(2*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*(sqrt(2)*a + 2*x)/a)/a + 2*\text{sqrt}(2)*\arctan(-1/2*\text{sqrt}(2)*(sqrt(2)*a - 2*x)/a)/a - \text{sqrt}(2)*\log(\text{sqrt}(2)*a*x + a^2 + x^2)/a + \text{sqrt}(2)*\log(-\text{sqrt}(2)*a*x + a^2 + x^2)/a)/a^{12}$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(127) = 254.

time = 0.33, size = 338, normalized size = 2.15

$$\frac{256a^8 + 648a^4x^4 + 360x^8 - 180\sqrt{2}(a^{10} + 2a^6x^4 + a^2x^8)\text{arctan}\left(\frac{\sqrt{2}a^{\frac{1}{2}}x + \sqrt{2}a^{\frac{1}{2}}}{\sqrt{2}a^{\frac{1}{2}}x + a^{\frac{1}{2}}}\right) - 180\sqrt{2}(a^{10} + 2a^6x^4 + a^2x^8)\text{arctan}\left(\frac{-\sqrt{2}a^{\frac{1}{2}}x + \sqrt{2}a^{\frac{1}{2}}}{\sqrt{2}a^{\frac{1}{2}}x + a^{\frac{1}{2}}}\right) - 6\sqrt{2}(a^{10} + 2a^6x^4 + a^2x^8)\log\left(\frac{\sqrt{2}a^{\frac{1}{2}}x + a^{\frac{1}{2}}}{\sqrt{2}a^{\frac{1}{2}}x + a^{\frac{1}{2}}}\right) + 6\sqrt{2}(a^{10} + 2a^6x^4 + a^2x^8)\log\left(\frac{-\sqrt{2}a^{\frac{1}{2}}x + a^{\frac{1}{2}}}{\sqrt{2}a^{\frac{1}{2}}x + a^{\frac{1}{2}}}\right)}{256(a^{20}x + 2a^{16}x^5 + a^{12}x^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a^4+x^4)^3,x, algorithm="fricas")

[Out]
$$-1/256*(256*a^8 + 648*a^4*x^4 + 360*x^8 - 180*\sqrt{2}*(a^{20}*x + 2*a^{16}*x^5 + a^{12}*x^9)*(a^{-52})^{1/4}*\arctan(-\sqrt{2}*a^{12}*(a^{-52})^{1/4}*x + \sqrt{2})*\sqrt{\sqrt{2}*a^{40}*(a^{-52})^{3/4}*x + a^{28}*\sqrt{a^{-52}} + x^2}*a^{12}*(a^{-52})^{1/4} - 1) - 180*\sqrt{2}*(a^{20}*x + 2*a^{16}*x^5 + a^{12}*x^9)*(a^{-52})^{1/4}*\arctan(-\sqrt{2}*a^{12}*(a^{-52})^{1/4}*x + \sqrt{2})*\sqrt{-\sqrt{2}*a^{40}*(a^{-52})^{3/4}*x + a^{28}*\sqrt{a^{-52}} + x^2}*a^{12}*(a^{-52})^{1/4} + 1) - 45*\sqrt{2}*(a^{20}*x + 2*a^{16}*x^5 + a^{12}*x^9)*(a^{-52})^{1/4}*\log(\sqrt{2}*a^{40}*(a^{-52})^{3/4}*x + a^{28}*\sqrt{a^{-52}} + x^2) + 45*\sqrt{2}*(a^{20}*x + 2*a^{16}*x^5 + a^{12}*x^9)*(a^{-52})^{1/4}*\log(-\sqrt{2}*a^{40}*(a^{-52})^{3/4}*x + a^{28}*\sqrt{a^{-52}} + x^2))/(a^{20}*x + 2*a^{16}*x^5 + a^{12}*x^9)$$

Sympy [A]

time = 0.21, size = 66, normalized size = 0.42

$$\frac{-32a^8 - 81a^4x^4 - 45x^8}{32a^{20}x + 64a^{16}x^5 + 32a^{12}x^9} + \frac{\text{RootSum}\left(268435456t^4 + 4100625, \left(t \mapsto t \log\left(-\frac{2097152t^3a}{91125} + x\right)\right)\right)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(a**4+x**4)**3,x)

[Out]
$$(-32*a^{**8} - 81*a^{**4}*x^{**4} - 45*x^{**8})/(32*a^{**20}*x + 64*a^{**16}*x^{**5} + 32*a^{**12}*x^{**9}) + \text{RootSum}(268435456*_t^{**4} + 4100625, \text{Lambda}(_t, _t*\log(-2097152*_t^{**3}*a/91125 + x)))/a^{**13}$$

Giac [A]

time = 0.01, size = 215, normalized size = 1.37

$$-\frac{1}{a^{12}x} + \frac{-13x^7 - 17x^3a^4}{32a^{12}(x^4 + a^4)^2} - \frac{45|a|a^2 \ln(x^2 - \sqrt{2}|a|x + |a||a|)}{(128\sqrt{2})a^{16}} - \frac{45|a|a^2 \arctan\left(\frac{x - \frac{1}{2}\sqrt{2}|a|}{\frac{1}{2}\sqrt{2}|a|}\right)}{(64\sqrt{2})a^{16}} + \frac{45|a|a^2 \ln(x^2 + \sqrt{2}|a|x + |a||a|)}{128\sqrt{2}a^{16}} - \frac{45|a|a^2 \arctan\left(\frac{x + \frac{1}{2}\sqrt{2}|a|}{\frac{1}{2}\sqrt{2}|a|}\right)}{(64\sqrt{2})a^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a^4+x^4)^3,x)

[Out]
$$-45/128*\sqrt{2}*abs(a)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*abs(a) + 2*x)/abs(a))/a^{14} - 45/128*\sqrt{2}*abs(a)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*abs(a) - 2*x)/abs(a))/a^{14} + 45/256*\sqrt{2}*abs(a)*\log(\sqrt{2}*x*abs(a) + x^2 + abs(a)^2)/a^{14} - 45/256*\sqrt{2}*abs(a)*\log(-\sqrt{2}*x*abs(a) + x^2 + abs(a)^2)/a^{14} - 1/32*(17*a^4*x^3 + 13*x^7)/((a^4 + x^4)^2*a^{12}) - 1/(a^{12}*x)$$

Mupad [B]

time = 0.11, size = 76, normalized size = 0.48

$$\frac{45(-1)^{1/4} \operatorname{atanh}\left(\frac{(-1)^{1/4}x}{a}\right)}{64a^{13}} - \frac{45(-1)^{1/4} \operatorname{atan}\left(\frac{(-1)^{1/4}x}{a}\right)}{64a^{13}} - \frac{\frac{1}{a^4} + \frac{81x^4}{32a^8} + \frac{45x^8}{32a^{12}}}{a^8x + 2a^4x^5 + x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^4 + x^4)^3),x)
```

```
[Out] (45*(-1)^(1/4)*atanh((-1)^(1/4)*x/a))/(64*a^13) - (45*(-1)^(1/4)*atan((-1)^(1/4)*x/a))/(64*a^13) - (1/a^4 + (81*x^4)/(32*a^8) + (45*x^8)/(32*a^12))/(a^8*x + x^9 + 2*a^4*x^5)
```

$$3.175 \quad \int \frac{1}{x^3(a^4+x^4)^3} dx$$

Optimal. Leaf size=64

$$-\frac{15}{16a^{12}x^2} + \frac{1}{8a^4x^2(a^4+x^4)^2} + \frac{5}{16a^8x^2(a^4+x^4)} - \frac{15 \tan^{-1}\left(\frac{x^2}{a^2}\right)}{16a^{14}}$$

[Out] $-15/16/a^{12}/x^2+1/8/a^4/x^2/(a^4+x^4)^2+5/16/a^8/x^2/(a^4+x^4)-15/16*\arctan(x^2/a^2)/a^{14}$

Rubi [A]

time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {281, 296, 331, 209}

$$-\frac{15}{16a^{12}x^2} + \frac{1}{8a^4x^2(a^4+x^4)^2} - \frac{15 \tan^{-1}\left(\frac{x^2}{a^2}\right)}{16a^{14}} + \frac{5}{16a^8x^2(a^4+x^4)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a^4 + x^4)^3), x]

[Out] $-15/(16*a^{12}*x^2) + 1/(8*a^4*x^2*(a^4 + x^4)^2) + 5/(16*a^8*x^2*(a^4 + x^4)) - (15*ArcTan[x^2/a^2])/(16*a^{14})$

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 281

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 296

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(-(c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 331

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*c^n*(m + 1))], Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3 (a^4 + x^4)^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2 (a^4 + x^2)^3} dx, x, x^2 \right) \\
 &= \frac{1}{8a^4 x^2 (a^4 + x^4)^2} + \frac{5 \text{Subst} \left(\int \frac{1}{x^2 (a^4 + x^2)^2} dx, x, x^2 \right)}{8a^4} \\
 &= \frac{1}{8a^4 x^2 (a^4 + x^4)^2} + \frac{5}{16a^8 x^2 (a^4 + x^4)} + \frac{15 \text{Subst} \left(\int \frac{1}{x^2 (a^4 + x^2)} dx, x, x^2 \right)}{16a^8} \\
 &= -\frac{15}{16a^{12} x^2} + \frac{1}{8a^4 x^2 (a^4 + x^4)^2} + \frac{5}{16a^8 x^2 (a^4 + x^4)} - \frac{15 \text{Subst} \left(\int \frac{1}{a^4 + x^2} dx, x, x^2 \right)}{16a^{12}} \\
 &= -\frac{15}{16a^{12} x^2} + \frac{1}{8a^4 x^2 (a^4 + x^4)^2} + \frac{5}{16a^8 x^2 (a^4 + x^4)} - \frac{15 \tan^{-1} \left(\frac{x^2}{a^2} \right)}{16a^{14}}
 \end{aligned}$$

Mathematica [A]

time = 0.03, size = 75, normalized size = 1.17

$$\frac{-\frac{a^2(8a^8+25a^4x^4+15x^8)}{x^2(a^4+x^4)^2} + 15 \tan^{-1} \left(1 - \frac{\sqrt{2}x}{a} \right) + 15 \tan^{-1} \left(1 + \frac{\sqrt{2}x}{a} \right)}{16a^{14}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a^4 + x^4)^3), x]

[Out] (-(a^2*(8*a^8 + 25*a^4*x^4 + 15*x^8))/(x^2*(a^4 + x^4)^2)) + 15*ArcTan[1 - (Sqrt[2]*x)/a] + 15*ArcTan[1 + (Sqrt[2]*x)/a]/(16*a^14)

Mathics [C] Result contains complex when optimal does not.

time = 6.02, size = 93, normalized size = 1.45

$$\frac{-2a^2(8a^8 + 25a^4x^4 + 15x^8) + 15Ix^2(a^8 + 2a^4x^4 + x^8)(\text{Log}[-Ia^2 + x^2] - \text{Log}[Ia^2 + x^2])}{32a^{14}x^2(a^8 + 2a^4x^4 + x^8)}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x^3*(a^4 + x^4)^3),x]')

[Out] $(-2 a^2 (8 a^8 + 25 a^4 x^4 + 15 x^8) + 15 I x^2 (a^8 + 2 a^4 x^4 + x^8) (\text{Log}[-I a^2 + x^2] - \text{Log}[I a^2 + x^2])) / (32 a^{14} x^2 (a^8 + 2 a^4 x^4 + x^8))$

Maple [A]

time = 0.06, size = 53, normalized size = 0.83

method	result	size
default	$-\frac{\frac{9x^2a^4 + 7x^6}{(a^4 + x^4)^2} + \frac{15 \arctan\left(\frac{x^2}{a^2}\right)}{8a^2}}{2a^{12}} - \frac{1}{2a^{12}x^2}$	53
risch	$\frac{-\frac{15x^8}{16a^{12}} - \frac{25x^4}{16a^8} - \frac{1}{2a^4}}{x^2(a^4 + x^4)^2} + \frac{15 \left(\sum_{R=\text{RootOf}(a^{28} - Z^2 + 1)} -R \ln\left(\left(-5 - R^2 a^{28} - 4\right)x^2 - a^{16} - R\right)\right)}{32}$	76

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a^4+x^4)^3,x,method=_RETURNVERBOSE)

[Out] $-1/2/a^{12} * ((9/8*x^2*a^4 + 7/8*x^6)/(a^4+x^4)^2 + 15/8*\arctan(x^2/a^2)/a^2) - 1/2/a^{12}/x^2$

Maxima [A]

time = 0.35, size = 60, normalized size = 0.94

$$-\frac{8a^8 + 25a^4x^4 + 15x^8}{16(a^{20}x^2 + 2a^{16}x^6 + a^{12}x^{10})} - \frac{15 \arctan\left(\frac{x^2}{a^2}\right)}{16a^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a^4+x^4)^3,x, algorithm="maxima")

[Out] $-1/16*(8*a^8 + 25*a^4*x^4 + 15*x^8)/(a^{20}*x^2 + 2*a^{16}*x^6 + a^{12}*x^{10}) - 15/16*\arctan(x^2/a^2)/a^{14}$

Fricas [A]

time = 0.31, size = 78, normalized size = 1.22

$$\frac{8a^{10} + 25a^6x^4 + 15a^2x^8 + 15(a^8x^2 + 2a^4x^6 + x^{10}) \arctan\left(\frac{x^2}{a^2}\right)}{16(a^{22}x^2 + 2a^{18}x^6 + a^{14}x^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a^4+x^4)^3,x, algorithm="fricas")

[Out] $-1/16*(8*a^{10} + 25*a^6*x^4 + 15*a^2*x^8 + 15*(a^8*x^2 + 2*a^4*x^6 + x^{10})*\arctan(x^2/a^2))/(a^{22}*x^2 + 2*a^{18}*x^6 + a^{14}*x^{10})$

Sympy [C] Result contains complex when optimal does not.
time = 0.23, size = 78, normalized size = 1.22

$$\frac{-8a^8 - 25a^4x^4 - 15x^8}{16a^{20}x^2 + 32a^{16}x^6 + 16a^{12}x^{10}} + \frac{\frac{15i \log(-ia^2+x^2)}{32} - \frac{15i \log(ia^2+x^2)}{32}}{a^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(a**4+x**4)**3,x)`

[Out] $(-8*a^{**8} - 25*a^{**4}*x^{**4} - 15*x^{**8})/(16*a^{**20}*x^{**2} + 32*a^{**16}*x^{**6} + 16*a^{**12}*x^{**10}) + (15*I*\log(-I*a^{**2} + x^{**2})/32 - 15*I*\log(I*a^{**2} + x^{**2})/32)/a^{**14}$

Giac [A]

time = 0.00, size = 65, normalized size = 1.02

$$-\frac{1}{2a^{12}x^2} + \frac{-7x^6 - 9x^2a^4}{16a^{12}(x^4 + a^4)^2} - \frac{15 \arctan\left(\frac{x^2}{a^2}\right)}{2 \cdot 8a^{12}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(a^4+x^4)^3,x)`

[Out] $-1/16*(9*a^4*x^2 + 7*x^6)/((a^4 + x^4)^2*a^{12}) - 15/16*\arctan(x^2/a^2)/a^{14} - 1/2/(a^{12}*x^2)$

Mupad [B]

time = 0.22, size = 53, normalized size = 0.83

$$-\frac{15 \operatorname{atan}\left(\frac{x^2}{a^2}\right)}{16 a^{14}} - \frac{\frac{a^{10}}{2} + \frac{25 a^6 x^4}{16} + \frac{15 a^2 x^8}{16}}{a^{14} x^2 (a^4 + x^4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*(a^4 + x^4)^3),x)`

[Out] $-(15*\operatorname{atan}(x^2/a^2))/(16*a^{14}) - (a^{10}/2 + (15*a^2*x^8)/16 + (25*a^6*x^4)/16)/(a^{14}*x^2*(a^4 + x^4)^2)$

$$3.176 \quad \int \frac{x^{14}}{(3+2x^5)^3} dx$$

Optimal. Leaf size=39

$$-\frac{9}{80(3+2x^5)^2} + \frac{3}{20(3+2x^5)} + \frac{1}{40} \log(3+2x^5)$$

[Out] $-9/80/(2*x^5+3)^2+3/20/(2*x^5+3)+1/40*\ln(2*x^5+3)$

Rubi [A]

time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{3}{20(2x^5+3)} - \frac{9}{80(2x^5+3)^2} + \frac{1}{40} \log(2x^5+3)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{14}/(3+2*x^5)^3, x]$

[Out] $-9/(80*(3+2*x^5)^2) + 3/(20*(3+2*x^5)) + \text{Log}[3+2*x^5]/40$

Rule 45

$\text{Int}[(a_.) + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) || \text{LtQ}[9*m + 5*(n + 1), 0] || \text{GtQ}[m + n + 2, 0])$

Rule 272

$\text{Int}[(x_.)^{(m_.)*((a_.) + (b_.)*(x_.))^{(n_.))^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p, x}], x, x^n], x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{x^{14}}{(3+2x^5)^3} dx &= \frac{1}{5} \text{Subst} \left(\int \frac{x^2}{(3+2x)^3} dx, x, x^5 \right) \\ &= \frac{1}{5} \text{Subst} \left(\int \left(\frac{9}{4(3+2x)^3} - \frac{3}{2(3+2x)^2} + \frac{1}{4(3+2x)} \right) dx, x, x^5 \right) \\ &= -\frac{9}{80(3+2x^5)^2} + \frac{3}{20(3+2x^5)} + \frac{1}{40} \log(3+2x^5) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 33, normalized size = 0.85

$$\frac{1}{80} \left(\frac{3(9 + 8x^5)}{(3 + 2x^5)^2} + 2 \log(3 + 2x^5) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[x^14/(3 + 2*x^5)^3,x]``[Out] ((3*(9 + 8*x^5))/(3 + 2*x^5)^2 + 2*Log[3 + 2*x^5])/80`**Mathics [A]**

time = 3.43, size = 45, normalized size = 1.15

$$\frac{27 + 24x^5 + 2\text{Log}[3 + 2x^5](9 + 12x^5 + 4x^{10})}{720 + 960x^5 + 320x^{10}}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^14/(3 + 2*x^5)^3,x]')``[Out] (27 + 24 x ^ 5 + 2 Log[3 + 2 x ^ 5] (9 + 12 x ^ 5 + 4 x ^ 10)) / (80 (9 + 1 2 x ^ 5 + 4 x ^ 10))`**Maple [A]**

time = 0.05, size = 34, normalized size = 0.87

method	result	size
norman	$\frac{\frac{3x^5}{10} + \frac{27}{80}}{(2x^5+3)^2} + \frac{\ln(2x^5+3)}{40}$	29
risch	$\frac{\frac{3x^5}{10} + \frac{27}{80}}{(2x^5+3)^2} + \frac{\ln(2x^5+3)}{40}$	30
meijerg	$-\frac{x^5(6x^5+6)}{360(1+\frac{2x^5}{3})^2} + \frac{\ln(1+\frac{2x^5}{3})}{40}$	33
default	$-\frac{9}{80(2x^5+3)^2} + \frac{3}{20(2x^5+3)} + \frac{\ln(2x^5+3)}{40}$	34

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^14/(2*x^5+3)^3,x,method=_RETURNVERBOSE)``[Out] -9/80/(2*x^5+3)^2+3/20/(2*x^5+3)+1/40*ln(2*x^5+3)`**Maxima [A]**

time = 0.34, size = 34, normalized size = 0.87

$$\frac{3(8x^5 + 9)}{80(4x^{10} + 12x^5 + 9)} + \frac{1}{40} \log(2x^5 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁴/(2*x⁵+3)³,x, algorithm="maxima")

[Out] 3/80*(8*x⁵ + 9)/(4*x¹⁰ + 12*x⁵ + 9) + 1/40*log(2*x⁵ + 3)

Fricas [A]

time = 0.31, size = 45, normalized size = 1.15

$$\frac{24x^5 + 2(4x^{10} + 12x^5 + 9)\log(2x^5 + 3) + 27}{80(4x^{10} + 12x^5 + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁴/(2*x⁵+3)³,x, algorithm="fricas")

[Out] 1/80*(24*x⁵ + 2*(4*x¹⁰ + 12*x⁵ + 9)*log(2*x⁵ + 3) + 27)/(4*x¹⁰ + 12*x⁵ + 9)

Sympy [A]

time = 0.08, size = 27, normalized size = 0.69

$$\frac{24x^5 + 27}{320x^{10} + 960x^5 + 720} + \frac{\log(2x^5 + 3)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**14/(2*x**5+3)**3,x)

[Out] (24*x**5 + 27)/(320*x**10 + 960*x**5 + 720) + log(2*x**5 + 3)/40

Giac [A]

time = 0.02, size = 39, normalized size = 1.00

$$\frac{-3x^{10} - 3x^5}{20(2x^5 + 3)^2} + \frac{\ln|2x^5 + 3|}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁴/(2*x⁵+3)³,x)

[Out] -3/20*(x¹⁰ + x⁵)/(2*x⁵ + 3)² + 1/40*log(abs(2*x⁵ + 3))

Mupad [B]

time = 0.05, size = 29, normalized size = 0.74

$$\frac{\ln\left(x^5 + \frac{3}{2}\right)}{40} + \frac{\frac{3x^5}{40} + \frac{27}{320}}{x^{10} + 3x^5 + \frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹⁴/(2*x⁵ + 3)³,x)

[Out] log(x⁵ + 3/2)/40 + ((3*x⁵)/40 + 27/320)/(3*x⁵ + x¹⁰ + 9/4)

$$3.177 \quad \int \frac{x^6}{(3+2x^5)^3} dx$$

Optimal. Leaf size=319

$$-\frac{x^2}{20(3+2x^5)^2} + \frac{x^2}{150(3+2x^5)} - \frac{\sqrt{5+\sqrt{5}} \tan^{-1}\left(\sqrt{\frac{1}{5}(5+2\sqrt{5})} - \frac{2^{2^{7/10}}x}{\sqrt[5]{3}\sqrt{5-\sqrt{5}}}\right) \sqrt{5-\sqrt{5}}}{250 \cdot 2^{9/10} 3^{3/5}}$$

[Out] $-1/20*x^2/(2*x^5+3)^2+1/150*x^2/(2*x^5+3)-1/1500*\ln(3^{(1/5)+2^{(1/5)}*x})*2^{(3/5)*3^{(2/5)}+1/6000*\ln(2^{(3/5)*3^{(2/5)}+2*x^2-1/2*3^{(1/5)}*2^{(4/5)}*x*(5^{(1/2)+1})*(-5^{(1/2)+1}*2^{(3/5)*3^{(2/5)}+1/6000*\ln(2^{(3/5)*3^{(2/5)}+2*x^2-1/2*3^{(1/5)}*2^{(4/5)}*x*(-5^{(1/2)+1})*(5^{(1/2)+1}*2^{(3/5)*3^{(2/5)}-1/1500*\arctan(1/5*(25-10*5^{(1/2))}^{(1/2)+2/3*2^{(7/10)}*x*3^{(4/5)/(5+5^{(1/2))}^{(1/2)})*(5-5^{(1/2))}^{(1/2)}*2^{(1/10)}*3^{(2/5)+1/1500*\arctan(2/3*2^{(7/10)}*x*3^{(4/5)/(5-5^{(1/2))}^{(1/2)}-1/5*(25+10*5^{(1/2))}^{(1/2)})*(5+5^{(1/2))}^{(1/2)}*2^{(1/10)}*3^{(2/5)}$

Rubi [A]

time = 0.38, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$,

Rules used = {294, 296, 299, 648, 632, 210, 642, 31}

$$\frac{(1+\sqrt{5})\log\left(\frac{2^{2^{9/5}}x^2 - \frac{\sqrt[5]{3}(1-\sqrt{5})x}{2^{1/5}} + 3^{2/5}}{2^{1/5}}\right)}{1000 \cdot 2^{9/5} 3^{3/5}} + \frac{(1-\sqrt{5})\log\left(\frac{2^{2^{9/5}}x^2 - \frac{\sqrt[5]{3}(1+\sqrt{5})x}{2^{1/5}} + 3^{2/5}}{2^{1/5}}\right)}{1000 \cdot 2^{9/5} 3^{3/5}} + \frac{x^2}{150(2x^5+3)} - \frac{x^2}{20(2x^5+3)^2} - \frac{\log(\sqrt[5]{2}x+\sqrt[5]{3})}{250 \cdot 2^{9/5} 3^{3/5}} - \frac{\sqrt{5+\sqrt{5}} \tan^{-1}\left(\sqrt{\frac{1}{5}(5+2\sqrt{5})} - \frac{2^{2^{7/10}}x}{\sqrt[5]{3}\sqrt{5-\sqrt{5}}}\right)}{250 \cdot 2^{9/5} 3^{3/5}} - \frac{\sqrt{5-\sqrt{5}} \tan^{-1}\left(\sqrt{\frac{1}{5}(5-2\sqrt{5})} + \frac{2^{2^{7/10}}x}{\sqrt[5]{3}\sqrt{5+\sqrt{5}}}\right)}{250 \cdot 2^{9/5} 3^{3/5}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(3 + 2*x^5)^3,x]

[Out] $-1/20*x^2/(3+2*x^5)^2 + x^2/(150*(3+2*x^5)) - (\text{Sqrt}[5+\text{Sqrt}[5]]*\text{ArcTan}[\text{Sqrt}[(5+2*\text{Sqrt}[5])/5] - (2*2^{(7/10)}*x)/(3^{(1/5)}*\text{Sqrt}[5-\text{Sqrt}[5]])]/(250*2^{(9/10)}*3^{(3/5)}) - (\text{Sqrt}[5-\text{Sqrt}[5]]*\text{ArcTan}[\text{Sqrt}[(5-2*\text{Sqrt}[5])/5] + (2*2^{(7/10)}*x)/(3^{(1/5)}*\text{Sqrt}[5+\text{Sqrt}[5]])]/(250*2^{(9/10)}*3^{(3/5)}) - \text{Log}[3^{(1/5)} + 2^{(1/5)}*x]/(250*2^{(2/5)}*3^{(3/5)}) + ((1+\text{Sqrt}[5])*\text{Log}[3^{(2/5)} - (3^{(1/5)}*(1-\text{Sqrt}[5])*x)/2^{(4/5)} + 2^{(2/5)}*x^2)]/(1000*2^{(2/5)}*3^{(3/5)}) + ((1-\text{Sqrt}[5])*\text{Log}[3^{(2/5)} - (3^{(1/5)}*(1+\text{Sqrt}[5])*x)/2^{(4/5)} + 2^{(2/5)}*x^2)]/(1000*2^{(2/5)}*3^{(3/5)})$

Rule 31

Int[((a_) + (b_)*(x_)^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 210

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &

& (LtQ[a, 0] || LtQ[b, 0])

Rule 294

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 296

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(-(c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 299

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k - 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; (-r)^(m + 1)/(a*n*s^m)*Int[1/(r + s*x), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 1)/2}], x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
 \int \frac{x^6}{(3+2x^5)^3} dx &= -\frac{x^2}{20(3+2x^5)^2} + \frac{1}{10} \int \frac{x}{(3+2x^5)^2} dx \\
 &= -\frac{x^2}{20(3+2x^5)^2} + \frac{x^2}{150(3+2x^5)} + \frac{1}{50} \int \frac{x}{3+2x^5} dx \\
 &= -\frac{x^2}{20(3+2x^5)^2} + \frac{x^2}{150(3+2x^5)} - \frac{\int \frac{1}{\sqrt[5]{3} + \sqrt[5]{2} x} dx}{250\sqrt[5]{2} 3^{3/5}} + \frac{\int \frac{\frac{1}{4}\sqrt[5]{3} (1-\sqrt{5}) - \frac{(-1-\sqrt{5})x}{2 \cdot 2^{4/5}}}{3^{2/5} - \frac{\sqrt[5]{3} (1-\sqrt{5})x}{2^{4/5}} + 2^{2/5}x^2} dx}{125\sqrt[5]{2} 3^{3/5}} \\
 &= -\frac{x^2}{20(3+2x^5)^2} + \frac{x^2}{150(3+2x^5)} - \frac{\log(\sqrt[5]{3} + \sqrt[5]{2} x)}{250 \cdot 2^{2/5} 3^{3/5}} - \frac{\int \frac{1}{3^{2/5} - \frac{\sqrt[5]{3} (1-\sqrt{5})x}{2^{4/5}} + 2^{2/5}x^2} dx}{100\sqrt[5]{2} 3^{2/5} \sqrt{5}} \\
 &= -\frac{x^2}{20(3+2x^5)^2} + \frac{x^2}{150(3+2x^5)} - \frac{\log(\sqrt[5]{3} + \sqrt[5]{2} x)}{250 \cdot 2^{2/5} 3^{3/5}} + \frac{(1-\sqrt{5}) \log(2 \cdot 3^{2/5} - \sqrt[5]{6} x)}{1000 \cdot 2^{2/5} 3^{3/5}} \\
 &= -\frac{x^2}{20(3+2x^5)^2} + \frac{x^2}{150(3+2x^5)} - \frac{\tan^{-1}\left(\frac{\sqrt[5]{3} (1+\sqrt{5}) - 4\sqrt[5]{2} x}{\sqrt[5]{3} \sqrt{2(5-\sqrt{5})}}\right)}{25 \cdot 2^{9/10} 3^{3/5} \sqrt{5(5-\sqrt{5})}} - \frac{\tan^{-1}\left(\frac{\sqrt[5]{3} \sqrt{3-\sqrt{5}}}{\sqrt[5]{3} \sqrt{5-\sqrt{5}}}\right)}{25 \cdot 2^{9/10} 3^{3/5} \sqrt{5}}
 \end{aligned}$$

Mathematica [A]

time = 0.19, size = 293, normalized size = 0.92

$$\frac{-\frac{300x^2}{(3+2x^5)^2} + \frac{40x^2}{3+2x^5} - 4\sqrt[5]{2} 3^{2/5} \sqrt{5-\sqrt{5}} \tan^{-1}\left(\frac{-3+\sqrt[5]{3} + \sqrt[5]{2} x}{\sqrt[5]{3} \sqrt{2(5-\sqrt{5})}}\right) + 4\sqrt[5]{2} 3^{2/5} \sqrt{5+\sqrt{5}} \tan^{-1}\left(\frac{-3(1+\sqrt{5}) + \sqrt[5]{2} x}{\sqrt[5]{3} \sqrt{2(5-\sqrt{5})}}\right) - 4 \cdot 2^{9/10} 3^{3/5} \log(3 + \sqrt[5]{2} 3^{1/5} x) + 2^{9/10} 3^{3/5} (1 + \sqrt{5}) \log(3 + \sqrt[5]{2} (-1 + \sqrt{5}) x + 2^{2/5} 3^{1/5} x^2) - 2^{9/10} 3^{3/5} (-1 + \sqrt{5}) \log(3 - \sqrt[5]{2} (1 + \sqrt{5}) x + 2^{2/5} 3^{1/5} x^2)}{6000}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(3 + 2*x^5)^3,x]

[Out] ((-300*x^2)/(3 + 2*x^5)^2 + (40*x^2)/(3 + 2*x^5) - 4*2^(1/10)*3^(2/5)*Sqrt[5 - Sqrt[5]]*ArcTan[(-3 + 3*Sqrt[5] + 4*2^(1/5)*3^(4/5)*x)/(3*Sqrt[2*(5 + Sqrt[5])])] + 4*2^(1/10)*3^(2/5)*Sqrt[5 + Sqrt[5]]*ArcTan[(-3*(1 + Sqrt[5]) + 4*2^(1/5)*3^(4/5)*x)/(3*Sqrt[10 - 2*Sqrt[5]])] - 4*2^(3/5)*3^(2/5)*Log[3

$$+ 2^{(1/5)} * 3^{(4/5)} * x] + 2^{(3/5)} * 3^{(2/5)} * (1 + \text{Sqrt}[5]) * \text{Log}[3 + (3/2)^{(4/5)} * (-1 + \text{Sqrt}[5])] * x + 2^{(2/5)} * 3^{(3/5)} * x^2] - 2^{(3/5)} * 3^{(2/5)} * (-1 + \text{Sqrt}[5]) * \text{Log}[3 - (3/2)^{(4/5)} * (1 + \text{Sqrt}[5])] * x + 2^{(2/5)} * 3^{(3/5)} * x^2]) / 6000$$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.25, size = 65, normalized size = 0.20

$$\frac{x^2(-9 + 4x^5) + 300(9 + 12x^5 + 4x^{10}) \text{RootSum}[1 + 105468750000000\#1^5 \&, \text{Log}[x - 281250000\#1^3] \#1\&]}{2700 + 3600x^5 + 1200x^{10}}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x^6/(3 + 2*x^5)^3,x]')`

[Out] `(x ^ 2 (-9 + 4 x ^ 5) + 300 (9 + 12 x ^ 5 + 4 x ^ 10) RootSum[1 + 105468750000000 #1 ^ 5&, Log[x - 281250000 #1 ^ 3] #1&]) / (300 (9 + 12 x ^ 5 + 4 x ^ 10))`

Maple [A]

time = 0.10, size = 354, normalized size = 1.11

method	result
risch	$\frac{\frac{1}{75}x^7 - \frac{3}{100}x^2}{(2x^5+3)^2} + \frac{\left(\sum_{R=\text{RootOf}(2Z^5+3)} \frac{\ln(-R+x)}{-R^3} \right)}{500}$
meijerg	$108^{\frac{4}{5}} \left(-\frac{x^2 3^{\frac{3}{5}} 2^{\frac{2}{5}} \left(-\frac{28x^5}{3} + 21 \right)}{105 \left(1 + \frac{2x^5}{3} \right)^2} + \frac{2 \cdot 108^{\frac{1}{5}} x^2 \left(\frac{2^{\frac{3}{5}} 3^{\frac{2}{5}} \ln \left(1 + \frac{2^{\frac{1}{5}} 3^{\frac{4}{5}} (x^5)^{\frac{1}{5}}}{3} \right)}{2 (x^5)^{\frac{2}{5}}} - \frac{2^{\frac{3}{5}} 3^{\frac{2}{5}} \cos \left(\frac{2\pi}{5} \right) \ln \left(1 - \frac{2 \cos \left(\frac{\pi}{5} \right) 2^{\frac{1}{5}} 3^{\frac{4}{5}} (x^5)^{\frac{1}{5}}}{3} + \frac{2^{\frac{2}{5}} 3^{\frac{3}{5}} (x^5)^{\frac{2}{5}}}{3} \right)}{2 (x^5)^{\frac{2}{5}}} \right)}{2 (x^5)^{\frac{2}{5}}} \right)$
default	$\frac{\frac{1}{75}x^7 - \frac{3}{100}x^2}{(2x^5+3)^2} + \frac{48^{\frac{2}{5}} \ln \left(x \sqrt{5} 48^{\frac{1}{5}} - x 48^{\frac{1}{5}} + 48^{\frac{2}{5}} + 4x^2 \right) \sqrt{5}}{12000} + \frac{48^{\frac{2}{5}} \ln \left(x \sqrt{5} 48^{\frac{1}{5}} - x 48^{\frac{1}{5}} + 48^{\frac{2}{5}} + 4x^2 \right)}{12000} - \frac{48^{\frac{3}{5}} \sqrt{5} \arctan \left(\dots \right)}{12000}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(2*x^5+3)^3,x,method=_RETURNVERBOSE)`

[Out] `4*(1/300*x^7-3/400*x^2)/(2*x^5+3)^2+1/12000*48^(2/5)*ln(x*5^(1/2)*48^(1/5)-x*48^(1/5)+48^(2/5)+4*x^2)*5^(1/2)+1/12000*48^(2/5)*ln(x*5^(1/2)*48^(1/5)-x*48^(1/5)+48^(2/5)+4*x^2)-1/1500*48^(3/5)*5^(1/2)/(10*48^(2/5)+2*5^(1/2)*48`

$$\begin{aligned} & \sqrt[5]{x} \arctan\left(\frac{1}{10 \cdot 48 \sqrt[5]{x} + 2 \cdot 5 \sqrt[5]{x}} \cdot 48 \sqrt[5]{x}\right) \sqrt[5]{x} \sqrt[5]{48 \sqrt[5]{x} - 1} \\ & - \frac{1}{10 \cdot 48 \sqrt[5]{x} + 2 \cdot 5 \sqrt[5]{x}} \cdot 48 \sqrt[5]{x} \sqrt[5]{48 \sqrt[5]{x} - 1} + 8 \sqrt[5]{x} / (10 \cdot 48 \sqrt[5]{x} + 2 \cdot 5 \sqrt[5]{x}) \\ & \sqrt[5]{48 \sqrt[5]{x} - 1} + 1/12000 \cdot 48 \sqrt[5]{x} \cdot \ln(-x \cdot 5 \sqrt[5]{x} \cdot 48 \sqrt[5]{x} + 48 \sqrt[5]{x} - x \cdot 48 \sqrt[5]{x} \\ & + 4 \cdot x^2) - 1/12000 \cdot 48 \sqrt[5]{x} \cdot \ln(-x \cdot 5 \sqrt[5]{x} \cdot 48 \sqrt[5]{x} + 48 \sqrt[5]{x} - x \cdot 48 \sqrt[5]{x} + \\ & 4 \cdot x^2) \cdot 5 \sqrt[5]{x} + 1/1500 \cdot 48 \sqrt[5]{x} / (10 \cdot 48 \sqrt[5]{x} - 2 \cdot 5 \sqrt[5]{x}) \sqrt[5]{48 \sqrt[5]{x} - 1} \arctan \\ & \left(-\frac{1}{10 \cdot 48 \sqrt[5]{x} - 2 \cdot 5 \sqrt[5]{x}} \cdot 48 \sqrt[5]{x} \sqrt[5]{48 \sqrt[5]{x} - 1} - \frac{1}{10 \cdot 48 \sqrt[5]{x} - 2 \cdot 5 \sqrt[5]{x}} \cdot 48 \sqrt[5]{x} \sqrt[5]{48 \sqrt[5]{x} - 1} + 8 \sqrt[5]{x} / (10 \cdot 48 \sqrt[5]{x} - 2 \cdot 5 \sqrt[5]{x}) \sqrt[5]{48 \sqrt[5]{x} - 1} + 1/150 \cdot 48 \sqrt[5]{x} / (5 + 5 \sqrt[5]{x}) / (5 \sqrt[5]{x} - 5) \cdot \ln(48 \sqrt[5]{x} + 2 \cdot x) \right) \end{aligned}$$
Maxima [A]

time = 0.35, size = 335, normalized size = 1.05

$$\frac{3^{2t}(\sqrt{5}-5) \arctan\left(\frac{3^{2t}(\sqrt{5}+\sqrt{5} \cdot 3^{2t}-3^{2t})}{6\sqrt{2\sqrt{5}+10}}\right)}{750(\sqrt{5} \cdot 3^{2t}-3^{2t})\sqrt{2\sqrt{5}+10}} + \frac{3^{2t}(\sqrt{5}+5) \arctan\left(\frac{3^{2t}(\sqrt{5}-\sqrt{5} \cdot 3^{2t}-3^{2t})}{6\sqrt{2\sqrt{5}+10}}\right)}{750(\sqrt{5} \cdot 3^{2t}+3^{2t})\sqrt{2\sqrt{5}+10}} - \frac{1}{1500} \cdot 3^{2t} \log(2^{2t}x+3^t) + \frac{4x^2-9x^2}{300(4x^{10}+12x^5+9)} - \frac{\log(2 \cdot 2^{2t}x^2-x(\sqrt{5} \cdot 3^{2t}+3^{2t})+2 \cdot 3^t)}{250(\sqrt{5} \cdot 3^{2t}+3^{2t})} + \frac{\log(2 \cdot 2^{2t}x^2+x(\sqrt{5} \cdot 3^{2t}-3^{2t})+2 \cdot 3^t)}{250(\sqrt{5} \cdot 3^{2t}-3^{2t})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(2*x^5+3)^3,x, algorithm="maxima")

[Out] 1/750*3^(4/5)*2^(4/5)*(sqrt(5) - 5)*arctan(1/6*3^(4/5)*2^(4/5)*(4*2^(2/5)*x + sqrt(5)*3^(1/5)*2^(1/5) - 3^(1/5)*2^(1/5))/sqrt(2*sqrt(5) + 10))/((sqrt(5)*3^(2/5)*2^(1/5) - 3^(2/5)*2^(1/5))*sqrt(2*sqrt(5) + 10)) + 1/750*3^(4/5)*2^(4/5)*(sqrt(5) + 5)*arctan(1/6*3^(4/5)*2^(4/5)*(4*2^(2/5)*x - sqrt(5)*3^(1/5)*2^(1/5) - 3^(1/5)*2^(1/5))/sqrt(-2*sqrt(5) + 10))/((sqrt(5)*3^(2/5)*2^(1/5) + 3^(2/5)*2^(1/5))*sqrt(-2*sqrt(5) + 10)) - 1/1500*3^(2/5)*2^(3/5)*log(2^(1/5)*x + 3^(1/5)) + 1/300*(4*x^7 - 9*x^2)/(4*x^10 + 12*x^5 + 9) - 1/250*log(2*2^(2/5)*x^2 - x*(sqrt(5)*3^(1/5)*2^(1/5) + 3^(1/5)*2^(1/5)) + 2*3^(2/5))/(sqrt(5)*3^(3/5)*2^(2/5) + 3^(3/5)*2^(2/5)) + 1/250*log(2*2^(2/5)*x^2 + x*(sqrt(5)*3^(1/5)*2^(1/5) - 3^(1/5)*2^(1/5)) + 2*3^(2/5))/(sqrt(5)*3^(3/5)*2^(2/5) - 3^(3/5)*2^(2/5))

Fricas [C] Result contains complex when optimal does not.

time = 3.22, size = 1751, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(2*x^5+3)^3,x, algorithm="fricas")

[Out] 1/216000*(2880*x^7 - 2*108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5 + 9)*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)*log(-1/6912*108^(3/5)*(-1)^(2/5)*(108^(4/5)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(1/5)*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 1/64*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^3 - 1/6912*108^(4/5)*(-1)^(1/5)*(108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 4*108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) + 16*108^(3/5)*(-1)^(2/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) + 1/16*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 1/4*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*

$$\begin{aligned} & \text{sqrt}(-2*\text{sqrt}(5) + 10) + 1) + 108^{(2/5)}*(-1)^{(3/5)} + 6*x) - 2*108^{(4/5)}*(-1) \\ & ^{(1/5)}*(4*x^{10} + 12*x^5 + 9)*(\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1)*\log(1/ \\ & 384*108^{(2/5)}*(-1)^{(3/5)}*(\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1)^3 + x) + 8 \\ & *108^{(4/5)}*(-1)^{(1/5)}*(4*x^{10} + 12*x^5 + 9)*\log(-108^{(2/5)}*(-1)^{(3/5)} + 6*x \\ &) - 6480*x^2 + (108^{(4/5)}*(-1)^{(1/5)}*(4*x^{10} + 12*x^5 + 9)*(\text{sqrt}(5) + I*\text{sqrt} \\ & (-2*\text{sqrt}(5) + 10) + 1) + 108^{(4/5)}*(-1)^{(1/5)}*(4*x^{10} + 12*x^5 + 9)*(\text{sqrt}(\\ & 5) - I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1) - 4*108^{(4/5)}*(-1)^{(1/5)}*(4*x^{10} + 12*x^5 \\ & + 9) - 24*\text{sqrt}(3)*(4*x^{10} + 12*x^5 + 9)*\text{sqrt}(-1/864*108^{(4/5)}*(-1)^{(1/5)}*(\\ & 108^{(4/5)}*(-1)^{(1/5)}*(\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1) - 4*108^{(4/5)}* \\ & (-1)^{(1/5)}*(\text{sqrt}(5) + I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1) - 3/16*108^{(3/5)}*(-1)^{(\\ & 2/5)}*(\text{sqrt}(5) + I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1)^2 - 3/16*108^{(3/5)}*(-1)^{(2/5)}* \\ & (\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1)^2 + 1/2*108^{(3/5)}*(-1)^{(2/5)}*(\text{sqrt}(\\ & 5) - I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1) - 3*108^{(3/5)}*(-1)^{(2/5)})))*\log(1/768*108^{ \\ & (3/5)}*(-1)^{(2/5)}*(108^{(4/5)}*(-1)^{(1/5)}*(\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + \\ & 1) - 4*108^{(4/5)}*(-1)^{(1/5)}*(\text{sqrt}(5) + I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1)^2 + 1 \\ & /768*108^{(4/5)}*(-1)^{(1/5)}*(108^{(3/5)}*(-1)^{(2/5)}*(\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt}(5) \\ &) + 10) + 1)^2 - 4*108^{(3/5)}*(-1)^{(2/5)}*(\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt}(5) + 10) \\ & + 1) + 16*108^{(3/5)}*(-1)^{(2/5)}*(\text{sqrt}(5) + I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1) - 9 \\ & /16*108^{(2/5)}*(-1)^{(3/5)}*(\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1)^2 + 9/4*10 \\ & 8^{(2/5)}*(-1)^{(3/5)}*(\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1) + 1/3456*(108^{(4/ \\ & 5)}*(-1)^{(1/5)}*(108^{(4/5)}*\text{sqrt}(3)*(-1)^{(1/5)}*(\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt}(5) + \\ & 10) + 1) - 4*108^{(4/5)}*\text{sqrt}(3)*(-1)^{(1/5)}*(\text{sqrt}(5) + I*\text{sqrt}(-2*\text{sqrt}(5) + \\ & 10) + 1) - 432*108^{(3/5)}*\text{sqrt}(3)*(-1)^{(2/5)}*(\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt}(5) + \\ & 10) + 1))*\text{sqrt}(-1/864*108^{(4/5)}*(-1)^{(1/5)}*(108^{(4/5)}*(-1)^{(1/5)}*(\text{sqrt}(5) - \\ & I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1) - 4*108^{(4/5)}*(-1)^{(1/5)}*(\text{sqrt}(5) + I*\text{sqrt}(- \\ & 2*\text{sqrt}(5) + 10) + 1) - 3/16*108^{(3/5)}*(-1)^{(2/5)}*(\text{sqrt}(5) + I*\text{sqrt}(-2*\text{sqrt}(\\ & 5) + 10) + 1)^2 - 3/16*108^{(3/5)}*(-1)^{(2/5)}*(\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt}(5) + \\ & 10) + 1)^2 + 1/2*108^{(3/5)}*(-1)^{(2/5)}*(\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + \\ & 1) - 3*108^{(3/5)}*(-1)^{(2/5)})) + 108*x) + (108^{(4/5)}*(-1)^{(1/5)}*(4*x^{10} + 12* \\ & x^5 + 9)*(\text{sqrt}(5) + I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1) + 108^{(4/5)}*(-1)^{(1/5)}*(4* \\ & x^{10} + 12*x^5 + 9)*(\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1) - 4*108^{(4/5)}*(- \\ & 1)^{(1/5)}*(4*x^{10} + 12*x^5 + 9) + 24*\text{sqrt}(3)*(4*x^{10} + 12*x^5 + 9)*\text{sqrt}(-1/8 \\ & 64*108^{(4/5)}*(-1)^{(1/5)}*(108^{(4/5)}*(-1)^{(1/5)}*(\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt}(5) \\ & + 10) + 1) - 4*108^{(4/5)}*(-1)^{(1/5)}*(\text{sqrt}(5) + I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1) \\ &) - 3/16*108^{(3/5)}*(-1)^{(2/5)}*(\text{sqrt}(5) + I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1)^2 - 3 \\ & /16*108^{(3/5)}*(-1)^{(2/5)}*(\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1)^2 + 1/2*10 \\ & 8^{(3/5)}*(-1)^{(2/5)}*(\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1) - 3*108^{(3/5)}*(- \\ & 1)^{(2/5)})))*\log(1/768*108^{(3/5)}*(-1)^{(2/5)}*(108^{(4/5)}*(-1)^{(1/5)}*(\text{sqrt}(5) - \\ & I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1) - 4*108^{(4/5)}*(-1)^{(1/5)}*(\text{sqrt}(5) + I*\text{sqrt}(-2 \\ & *sqrt(5) + 10) + 1)^2 + 1/768*108^{(4/5)}*(-1)^{(1/5)}*(108^{(3/5)}*(-1)^{(2/5)}*(s \\ & qrt(5) - I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1)^2 - 4*108^{(3/5)}*(-1)^{(2/5)}*(\text{sqrt}(5) - \\ & I*\text{sqrt}(-2*\text{sqrt}(5) + 10) + 1) + 16*108^{(3/5)}*(-1)^{(2/5)}*(\text{sqrt}(5) + I*\text{sqrt}(\\ & -2*\text{sqrt}(5) + 10) + 1) - 9/16*108^{(2/5)}*(-1)^{(3/5)}*(\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt} \\ & (5) + 10) + 1)^2 + 9/4*108^{(2/5)}*(-1)^{(3/5)}*(\text{sqrt}(5) - I*\text{sqrt}(-2*\text{sqrt}(5) + \\ & 10) + 1) - 1/3456*(108^{(4/5)}*(-1)^{(1/5)}*(108^{(4/5)}*\text{sqrt}(3)*(-1)^{(1/5)}*(\text{sqrt}$$

(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*sqrt(3)*(-1)^(1/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) - 432*108^(3/5)*sqrt(3)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1))*sqrt(-1/864*108^(4/5)*(-1)^(1/5)*(108^(4/5)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(1/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) - 3/16*108^(3/5)*(-1)^(2/5)*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 3/16*108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 + 1/2*108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 3*108^(3/5)*(-1)^(2/5)) + 108*x))/(4*x^10 + 12*x^5 + 9)

Sympy [A]

time = 0.14, size = 37, normalized size = 0.12

$$\frac{4x^7 - 9x^2}{1200x^{10} + 3600x^5 + 2700} + \text{RootSum}(105468750000000t^5 + 1, (t \mapsto t \log(-281250000t^3 + x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(2*x**5+3)**3,x)

[Out] (4*x**7 - 9*x**2)/(1200*x**10 + 3600*x**5 + 2700) + RootSum(10546875000000*_t**5 + 1, Lambda(_t, _t*log(-281250000*_t**3 + x)))

Giac [A]

time = 0.15, size = 481, normalized size = 1.51

$$\frac{4x^7 - 9x^2}{1200x^{10} + 3600x^5 + 2700} + \text{RootSum}(10546875000000t^5 + 1, (t \mapsto t \log(-281250000t^3 + x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(2*x^5+3)^3,x)

[Out] 1/3000*(sqrt(5)*(3/2)^(2/5)*sqrt(2*sqrt(5) + 10) - (3/2)^(2/5)*sqrt(2*sqrt(5) + 10))*arctan(-2/3*(3/2)^(4/5)*((3/2)^(1/5)*(sqrt(5) - 1) + 4*x)/sqrt(2*sqrt(5) + 10)) - 1/3000*(sqrt(5)*(3/2)^(2/5)*sqrt(-2*sqrt(5) + 10) + (3/2)^(2/5)*sqrt(-2*sqrt(5) + 10))*arctan(2/3*(3/2)^(4/5)*((3/2)^(1/5)*(sqrt(5) + 1) - 4*x)/sqrt(-2*sqrt(5) + 10)) - 1/6000*((3/2)^(2/5)*(sqrt(5) - 5) + sqrt(5)*(3/2)^(2/5) + 3*(3/2)^(2/5))*log(x^2 - 1/2*x*(sqrt(5)*(3/2)^(1/5) + (3/2)^(1/5)) + (3/2)^(2/5)) + 1/6000*((3/2)^(2/5)*(sqrt(5) + 5) + sqrt(5)*(3/2)^(2/5) - 3*(3/2)^(2/5))*log(x^2 + 1/2*x*(sqrt(5)*(3/2)^(1/5) - (3/2)^(1/5)) + (3/2)^(2/5)) - 1/750*(3/2)^(2/5)*log(abs(x + (3/2)^(1/5))) + 1/300*(4*x^7 - 9*x^2)/(2*x^5 + 3)^2

Mupad [B]

time = 1.60, size = 285, normalized size = 0.89

$$\frac{4x^7 - 9x^2}{1200x^{10} + 3600x^5 + 2700} + \text{RootSum}(10546875000000t^5 + 1, (t \mapsto t \log(-281250000t^3 + x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^6/(2*x^5 + 3)^3,x)$

[Out] $(3^{2/5}*\log(x - (3^{1/5}*(2*2^{1/10})*(-5^{1/2} - 5)^{1/2} - 2^{3/5}*(5^{1/2} - 1))^3)/256)*(2*2^{1/10})*(-5^{1/2} - 5)^{1/2} - 2^{3/5}*(5^{1/2} - 1))/6000 - ((3*x^2)/400 - x^7/300)/(3*x^5 + x^{10} + 9/4) - (3^{2/5}*\log(x + (3^{1/5}*(2*2^{1/10})*(-5^{1/2} - 5)^{1/2} + 2^{3/5}*(5^{1/2} - 1))^3)/256)*(2*2^{1/10})*(-5^{1/2} - 5)^{1/2} + 2^{3/5}*(5^{1/2} - 1))/6000 - (72^{1/5})*\log(x + 72^{3/5}/12)/1500 + (3^{2/5}*\log(x - (3^{1/5}*(2^{3/5}*(5^{1/2} + 1) - 2*2^{1/10}*(5^{1/2} - 5)^{1/2}))^3)/256)*(2^{3/5}*(5^{1/2} + 1) - 2*2^{1/10}*(5^{1/2} - 5)^{1/2}))/6000 + (3^{2/5}*\log(x - (3^{1/5}*(2^{3/5}*(5^{1/2} + 1) + 2*2^{1/10}*(5^{1/2} - 5)^{1/2}))^3)/256)*(2^{3/5}*(5^{1/2} + 1) + 2*2^{1/10}*(5^{1/2} - 5)^{1/2}))/6000$

$$3.178 \quad \int \frac{9}{5x^2(3-2x^2)^3} dx$$

Optimal. Leaf size=59

$$-\frac{1}{8x} + \frac{3}{20x(3-2x^2)^2} + \frac{1}{8x(3-2x^2)} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{3}}x\right)}{4\sqrt{6}}$$

[Out] $-1/8/x+3/20/x/(-2*x^2+3)^2+1/8/x/(-2*x^2+3)+1/24*\operatorname{arctanh}(1/3*x*6^{(1/2)})*6^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {12, 296, 331, 212}

$$\frac{1}{8x(3-2x^2)} + \frac{3}{20x(3-2x^2)^2} - \frac{1}{8x} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{3}}x\right)}{4\sqrt{6}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[9/(5*x^2*(3-2*x^2)^3), x]$

[Out] $-1/8*1/x + 3/(20*x*(3-2*x^2)^2) + 1/(8*x*(3-2*x^2)) + \operatorname{ArcTanh}[\operatorname{Sqrt}[2/3]*x]/(4*\operatorname{Sqrt}[6])$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$ FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 212

$\operatorname{Int}[((a_*) + (b_*)(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 296

$\operatorname{Int}[(c_*)(x_)^{(m_*)}((a_*) + (b_*)(x_)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \operatorname{Simp}[(-(c*x)^{(m+1}))*((a+b*x^n)^{(p+1}))/((a*c*n*(p+1)))] + \operatorname{Dist}[(m+n*(p+1)+1)/((a*n*(p+1)))] + \operatorname{Int}[(c*x)^m*(a+b*x^n)^{(p+1)}, x], x] /;$ FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 331

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*c^n*(m + 1))], Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{9}{5x^2(3-2x^2)^3} dx &= \frac{9}{5} \int \frac{1}{x^2(3-2x^2)^3} dx \\
&= \frac{3}{20x(3-2x^2)^2} + \frac{3}{4} \int \frac{1}{x^2(3-2x^2)^2} dx \\
&= \frac{3}{20x(3-2x^2)^2} + \frac{1}{8x(3-2x^2)} + \frac{3}{8} \int \frac{1}{x^2(3-2x^2)} dx \\
&= -\frac{1}{8x} + \frac{3}{20x(3-2x^2)^2} + \frac{1}{8x(3-2x^2)} + \frac{1}{4} \int \frac{1}{3-2x^2} dx \\
&= -\frac{1}{8x} + \frac{3}{20x(3-2x^2)^2} + \frac{1}{8x(3-2x^2)} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{3}}x\right)}{4\sqrt{6}}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 65, normalized size = 1.10

$$\frac{1}{240} \left(-\frac{12(12-25x^2+10x^4)}{x(3-2x^2)^2} - 5\sqrt{6} \log(\sqrt{6}-2x) + 5\sqrt{6} \log(\sqrt{6}+2x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[9/(5*x^2*(3 - 2*x^2)^3),x]

[Out] ((-12*(12 - 25*x^2 + 10*x^4))/(x*(3 - 2*x^2)^2) - 5*Sqrt[6]*Log[Sqrt[6] - 2*x] + 5*Sqrt[6]*Log[Sqrt[6] + 2*x])/240

Mathics [A]

time = 2.25, size = 68, normalized size = 1.15

$$\frac{-144 + 5\sqrt{6}x(9 - 12x^2 + 4x^4) \left(\operatorname{Log}\left[\frac{\sqrt{6}}{2} + x\right] - \operatorname{Log}\left[-\frac{\sqrt{6}}{2} + x\right] \right) + 300x^2 - 120x^4}{240x(9 - 12x^2 + 4x^4)}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[9/(5*x^2*(3 - 2*x^2)^3),x]')`

[Out] $(-144 + 5 \sqrt{6} x (9 - 12 x^2 + 4 x^4) (\operatorname{Log}[\sqrt{6} / 2 + x] - \operatorname{Log}[-\sqrt{6} / 2 + x]) + 300 x^2 - 120 x^4) / (240 x (9 - 12 x^2 + 4 x^4))$

Maple [A]

time = 0.06, size = 39, normalized size = 0.66

method	result	size
default	$-\frac{1}{15x} - \frac{8(\frac{7}{16}x^3 - \frac{27}{32}x)}{15(2x^2-3)^2} + \frac{\operatorname{arctanh}\left(\frac{x\sqrt{6}}{3}\right)\sqrt{6}}{24}$	39
meijerg	$i\sqrt{6} \left(\frac{i\sqrt{6} \left(\frac{20}{3}x^4 - \frac{50}{3}x^2 + 8 \right)}{4x \left(-\frac{2x^2}{3} + 1 \right)^2} - \frac{15i \operatorname{arctanh}\left(\frac{x\sqrt{2}\sqrt{3}}{3}\right)}{2} \right)$	51
risch	$-\frac{\frac{1}{2}x^4 + \frac{5}{4}x^2 - \frac{3}{5}}{(2x^2-3)^2x} + \frac{\sqrt{6} \ln(2x + \sqrt{6})}{48} - \frac{\sqrt{6} \ln(2x - \sqrt{6})}{48}$	56

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(9/5/x^2/(-2*x^2+3)^3,x,method=_RETURNVERBOSE)`

[Out] $-1/15/x - 8/15*(7/16*x^3 - 27/32*x)/(2*x^2 - 3)^2 + 1/24*\operatorname{arctanh}(1/3*x*6^{(1/2)})*6^{(1/2)}$

Maxima [A]

time = 0.35, size = 56, normalized size = 0.95

$$-\frac{1}{48} \sqrt{6} \log\left(\frac{2x - \sqrt{6}}{2x + \sqrt{6}}\right) - \frac{10x^4 - 25x^2 + 12}{20(4x^5 - 12x^3 + 9x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(9/5/x^2/(-2*x^2+3)^3,x, algorithm="maxima")`

[Out] $-1/48*\operatorname{sqrt}(6)*\log((2*x - \operatorname{sqrt}(6))/(2*x + \operatorname{sqrt}(6))) - 1/20*(10*x^4 - 25*x^2 + 12)/(4*x^5 - 12*x^3 + 9*x)$

Fricas [A]

time = 0.31, size = 73, normalized size = 1.24

$$-\frac{120x^4 - 5\sqrt{6}(4x^5 - 12x^3 + 9x) \log\left(\frac{2x^2 + 2\sqrt{6}x + 3}{2x^2 - 3}\right) - 300x^2 + 144}{240(4x^5 - 12x^3 + 9x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(9/5/x^2/(-2*x^2+3)^3,x, algorithm="fricas")

[Out] -1/240*(120*x^4 - 5*sqrt(6)*(4*x^5 - 12*x^3 + 9*x)*log((2*x^2 + 2*sqrt(6)*x + 3)/(2*x^2 - 3)) - 300*x^2 + 144)/(4*x^5 - 12*x^3 + 9*x)

Sympy [A]

time = 0.08, size = 58, normalized size = 0.98

$$-\frac{9 \cdot (10x^4 - 25x^2 + 12)}{720x^5 - 2160x^3 + 1620x} - \frac{\sqrt{6} \log\left(x - \frac{\sqrt{6}}{2}\right)}{48} + \frac{\sqrt{6} \log\left(x + \frac{\sqrt{6}}{2}\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(9/5/x**2/(-2*x**2+3)**3,x)

[Out] -9*(10*x**4 - 25*x**2 + 12)/(720*x**5 - 2160*x**3 + 1620*x) - sqrt(6)*log(x - sqrt(6)/2)/48 + sqrt(6)*log(x + sqrt(6)/2)/48

Giac [A]

time = 0.00, size = 73, normalized size = 1.24

$$\frac{9}{5} \left(-\frac{1}{27x} - \frac{14x^3 - 27x}{108(2x^2 - 3)^2} - \frac{5 \ln \left| \frac{4x - 2\sqrt{6}}{4x + 2\sqrt{6}} \right|}{2 \cdot 36\sqrt{6}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(9/5/x^2/(-2*x^2+3)^3,x)

[Out] -1/48*sqrt(6)*log(abs((2*x - sqrt(6))/(2*x + sqrt(6)))) - 1/60*(14*x^3 - 27*x)/(2*x^2 - 3)^2 - 1/15/x

Mupad [B]

time = 0.24, size = 41, normalized size = 0.69

$$\frac{\sqrt{6} \operatorname{atanh}\left(\frac{\sqrt{6}x}{3}\right)}{24} - \frac{\frac{x^4}{8} - \frac{5x^2}{16} + \frac{3}{20}}{x^5 - 3x^3 + \frac{9x}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-9/(5*x^2*(2*x^2 - 3)^3),x)

[Out] (6^(1/2)*atanh((6^(1/2)*x)/3))/24 - (x^4/8 - (5*x^2)/16 + 3/20)/((9*x)/4 - 3*x^3 + x^5)

$$3.179 \quad \int \frac{4+3x^4}{x^2(1+x^2)^3} dx$$

Optimal. Leaf size=36

$$-\frac{4}{x} - \frac{7x}{4(1+x^2)^2} - \frac{25x}{8(1+x^2)} - \frac{57}{8} \tan^{-1}(x)$$

[Out] $-4/x - 7/4*x/(x^2+1)^2 - 25/8*x/(x^2+1) - 57/8*\arctan(x)$

Rubi [A]

time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1274, 467, 464, 209}

$$-\frac{25x}{8(x^2+1)} - \frac{7x}{4(x^2+1)^2} - \frac{4}{x} - \frac{57}{8} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(4 + 3*x^4)/(x^2*(1 + x^2)^3), x]$

[Out] $-4/x - (7*x)/(4*(1 + x^2)^2) - (25*x)/(8*(1 + x^2)) - (57*\text{ArcTan}[x])/8$

Rule 209

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))*\text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 464

$\text{Int}[(e_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})), x_Symbol] \rightarrow \text{Simp}[c*(e*x)^{(m+1)}*((a + b*x^n)^{(p+1)}/(a*e^{(m+1)})), x] + \text{Dist}[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), \text{Int}[(e*x)^{(m+n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ (\text{IntegerQ}[n] \ || \ \text{GtQ}[e, 0]) \ \&\& \ ((\text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[m, -1]) \ || \ (\text{LtQ}[n, 0] \ \&\& \ \text{GtQ}[m+n, -1])) \ \&\& \ !\text{ILtQ}[p, -1]$

Rule 467

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^2)^{(p_)}*((c_ + (d_)*(x_)^2)), x_Symbol] \rightarrow \text{Simp}[(-a)^{(m/2-1)}*(b*c - a*d)*x*((a + b*x^2)^{(p+1)}/(2*b^{(m/2+1)}*(p+1))), x] + \text{Dist}[1/(2*b^{(m/2+1)}*(p+1)), \text{Int}[x^m*(a + b*x^2)^{(p+1)}*\text{ExpandToSum}[2*b*(p+1)*\text{Together}[(b^{(m/2)}*(c + d*x^2) - (-a)^{(m/2-1)}*(b*c - a*d)*x^{-(m+2)}]/(a + b*x^2)] - ((-a)^{(m/2-1)}*(b*c - a*d))/x^m, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{ILtQ}[m/2$

, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1274

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Sym
bol] := Simp[(-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*x*((d + e*x^2)^(q + 1)/(2*e^(
2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m
*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*2*(-d)^(-m/2 + 1
)*e^(2*p)*(q + 1)*(a + c*x^4)^p - ((c*d^2 + a*e^2)^p/(e^(m/2)*x^m))*(d + e*
(2*q + 3)*x^2)], x], x] /; FreeQ[{a, c, d, e}, x] && IGtQ[p, 0] && ILt
Q[q, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{4 + 3x^4}{x^2(1+x^2)^3} dx &= -\frac{7x}{4(1+x^2)^2} - \frac{1}{4} \int \frac{-16 + 9x^2}{x^2(1+x^2)^2} dx \\ &= -\frac{7x}{4(1+x^2)^2} - \frac{25x}{8(1+x^2)} + \frac{1}{8} \int \frac{32 - 25x^2}{x^2(1+x^2)} dx \\ &= -\frac{4}{x} - \frac{7x}{4(1+x^2)^2} - \frac{25x}{8(1+x^2)} - \frac{57}{8} \int \frac{1}{1+x^2} dx \\ &= -\frac{4}{x} - \frac{7x}{4(1+x^2)^2} - \frac{25x}{8(1+x^2)} - \frac{57}{8} \tan^{-1}(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 33, normalized size = 0.92

$$-\frac{32 + 103x^2 + 57x^4}{8x(1+x^2)^2} - \frac{57}{8} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 3*x^4)/(x^2*(1 + x^2)^3), x]

[Out] -1/8*(32 + 103*x^2 + 57*x^4)/(x*(1 + x^2)^2) - (57*ArcTan[x])/8

Mathics [A]

time = 2.05, size = 44, normalized size = 1.22

$$\frac{-32 - 57x \operatorname{ArcTan}[x] (1 + 2x^2 + x^4) - 103x^2 - 57x^4}{8x(1 + 2x^2 + x^4)}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(3*x^4 + 4)/(x^2*(x^2 + 1)^3),x]')`

[Out] $(-32 - 57 x \operatorname{ArcTan}[x] (1 + 2 x^2 + x^4) - 103 x^2 - 57 x^4) / (8 x (1 + 2 x^2 + x^4))$

Maple [A]

time = 0.04, size = 29, normalized size = 0.81

method	result	size
default	$-\frac{4}{x} - \frac{\frac{25}{8}x^3 + \frac{39}{8}x}{(x^2+1)^2} - \frac{57 \arctan(x)}{8}$	29
risch	$\frac{-\frac{57}{8}x^4 - \frac{103}{8}x^2 - 4}{(x^2+1)^2 x} - \frac{57 \arctan(x)}{8}$	29
meijerg	$-\frac{15x^4 + 25x^2 + 8}{2x(x^2+1)^2} - \frac{57 \arctan(x)}{8} - \frac{x(-3x^2+3)}{8(x^2+1)^2}$	47

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^4+4)/x^2/(x^2+1)^3,x,method=_RETURNVERBOSE)`

[Out] $-4/x - (25/8*x^3 + 39/8*x)/(x^2+1)^2 - 57/8*\arctan(x)$

Maxima [A]

time = 0.34, size = 31, normalized size = 0.86

$$-\frac{57x^4 + 103x^2 + 32}{8(x^5 + 2x^3 + x)} - \frac{57}{8} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="maxima")`

[Out] $-1/8*(57*x^4 + 103*x^2 + 32)/(x^5 + 2*x^3 + x) - 57/8*\arctan(x)$

Fricas [A]

time = 0.31, size = 40, normalized size = 1.11

$$-\frac{57x^4 + 103x^2 + 57(x^5 + 2x^3 + x) \arctan(x) + 32}{8(x^5 + 2x^3 + x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="fricas")`

[Out] $-1/8*(57*x^4 + 103*x^2 + 57*(x^5 + 2*x^3 + x)*\arctan(x) + 32)/(x^5 + 2*x^3 + x)$

Sympy [A]

time = 0.07, size = 32, normalized size = 0.89

$$\frac{-57x^4 - 103x^2 - 32}{8x^5 + 16x^3 + 8x} - \frac{57 \operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**4+4)/x**2/(x**2+1)**3,x)

[Out] (-57*x**4 - 103*x**2 - 32)/(8*x**5 + 16*x**3 + 8*x) - 57*atan(x)/8

Giac [A]

time = 0.00, size = 34, normalized size = 0.94

$$-\frac{4}{x} + \frac{-25x^3 - 39x}{8(x^2 + 1)^2} - \frac{57}{8} \arctan x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x)

[Out] -1/8*(25*x^3 + 39*x)/(x^2 + 1)^2 - 4/x - 57/8*arctan(x)

Mupad [B]

time = 0.00, size = 29, normalized size = 0.81

$$-\frac{57 \operatorname{atan}(x)}{8} - \frac{\frac{57x^4}{8} + \frac{103x^2}{8} + 4}{x(x^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^4 + 4)/(x^2*(x^2 + 1)^3),x)

[Out] - (57*atan(x))/8 - ((103*x^2)/8 + (57*x^4)/8 + 4)/(x*(x^2 + 1)^2)

$$3.180 \quad \int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5} dx$$

Optimal. Leaf size=38

$$-\frac{3}{2(1-x)^2} + \frac{2}{1-x} + \frac{1}{1+x} + \log(1-x) - 2\log(1+x)$$

[Out] -3/2/(1-x)^2+2/(1-x)+1/(1+x)+ln(1-x)-2*ln(1+x)

Rubi [A]

time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {2099}

$$\frac{2}{1-x} + \frac{1}{x+1} - \frac{3}{2(1-x)^2} + \log(1-x) - 2\log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(5 - 3*x + 6*x^2 + 5*x^3 - x^4)/(-1 + x + 2*x^2 - 2*x^3 - x^4 + x^5), x]

[Out] -3/(2*(1 - x)^2) + 2/(1 - x) + (1 + x)^(-1) + Log[1 - x] - 2*Log[1 + x]

Rule 2099

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned} \int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5} dx &= \int \left(\frac{3}{(-1+x)^3} + \frac{2}{(-1+x)^2} + \frac{1}{-1+x} - \frac{1}{(1+x)^2} - \frac{2}{1+x} \right) dx \\ &= -\frac{3}{2(1-x)^2} + \frac{2}{1-x} + \frac{1}{1+x} + \log(1-x) - 2\log(1+x) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 32, normalized size = 0.84

$$-\frac{3}{2(-1+x)^2} - \frac{2}{-1+x} + \frac{1}{1+x} + \log(-1+x) - 2\log(1+x)$$

Antiderivative was successfully verified.

[In] Integrate[(5 - 3*x + 6*x^2 + 5*x^3 - x^4)/(-1 + x + 2*x^2 - 2*x^3 - x^4 + x^5), x]

[Out] $-3/(2*(-1+x)^2) - 2/(-1+x) + (1+x)^{-1} + \text{Log}[-1+x] - 2*\text{Log}[1+x]$

Mathics [A]

time = 2.14, size = 51, normalized size = 1.34

$$\frac{\frac{3}{2} - \frac{7x}{2} - x^2 + (1-x-x^2+x^3)(\text{Log}[-1+x] - 2\text{Log}[1+x])}{1-x-x^2+x^3}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(5 - 3*x + 6*x^2 + 5*x^3 - x^4)/(x^5 - x^4 - 2*x^3 + 2*x^2 + x - 1),x]')`

[Out] $(3/2 - 7x/2 - x^2 + (1-x-x^2+x^3)(\text{Log}[-1+x] - 2\text{Log}[1+x])) / (1-x-x^2+x^3)$

Maple [A]

time = 0.03, size = 31, normalized size = 0.82

method	result	size
default	$\ln(-1+x) - \frac{3}{2(-1+x)^2} - \frac{2}{-1+x} + \frac{1}{1+x} - 2\ln(1+x)$	31
norman	$\frac{-x^2 - \frac{7}{2}x + \frac{3}{2}}{(-1+x)^2(1+x)} - 2\ln(1+x) + \ln(-1+x)$	33
risch	$\frac{-x^2 - \frac{7}{2}x + \frac{3}{2}}{x^3 - x^2 - x + 1} - 2\ln(1+x) + \ln(-1+x)$	38

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x,method=_RETURNVERBOSE)`

[Out] $\ln(-1+x) - 3/2/(-1+x)^2 - 2/(-1+x) + 1/(1+x) - 2*\ln(1+x)$

Maxima [A]

time = 0.26, size = 38, normalized size = 1.00

$$-\frac{2x^2 + 7x - 3}{2(x^3 - x^2 - x + 1)} - 2\log(x+1) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x, algorithm="maxima")`

[Out] $-1/2*(2*x^2 + 7*x - 3)/(x^3 - x^2 - x + 1) - 2*\log(x+1) + \log(x-1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(32) = 64.

time = 0.31, size = 65, normalized size = 1.71

$$\frac{2x^2 + 4(x^3 - x^2 - x + 1)\log(x+1) - 2(x^3 - x^2 - x + 1)\log(x-1) + 7x - 3}{2(x^3 - x^2 - x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x, algorithm="fricas")

[Out] -1/2*(2*x^2 + 4*(x^3 - x^2 - x + 1)*log(x + 1) - 2*(x^3 - x^2 - x + 1)*log(x - 1) + 7*x - 3)/(x^3 - x^2 - x + 1)

Sympy [A]

time = 0.07, size = 36, normalized size = 0.95

$$-\frac{2x^2 + 7x - 3}{2x^3 - 2x^2 - 2x + 2} + \log(x - 1) - 2\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+5*x**3+6*x**2-3*x+5)/(x**5-x**4-2*x**3+2*x**2+x-1),x)

[Out] -(2*x**2 + 7*x - 3)/(2*x**3 - 2*x**2 - 2*x + 2) + log(x - 1) - 2*log(x + 1)

Giac [A]

time = 0.00, size = 40, normalized size = 1.05

$$-2\ln|x + 1| + \ln|x - 1| + \frac{\frac{1}{2}(-2x^2 - 7x + 3)}{(x - 1)^2(x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x)

[Out] -1/2*(2*x^2 + 7*x - 3)/((x + 1)*(x - 1)^2) - 2*log(abs(x + 1)) + log(abs(x - 1))

Mupad [B]

time = 0.07, size = 33, normalized size = 0.87

$$\ln(x - 1) - 2\ln(x + 1) + \frac{x^2 + \frac{7x}{2} - \frac{3}{2}}{-x^3 + x^2 + x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((6*x^2 - 3*x + 5*x^3 - x^4 + 5)/(x + 2*x^2 - 2*x^3 - x^4 + x^5 - 1),x)

[Out] log(x - 1) - 2*log(x + 1) + ((7*x)/2 + x^2 - 3/2)/(x + x^2 - x^3 - 1)

$$3.181 \quad \int \frac{1+x^2}{x(1+x^3)^2} dx$$

Optimal. Leaf size=64

$$\frac{x(x-x^2)}{3(1+x^3)} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{4}{9}\log(1+x) - \frac{5}{18}\log(1-x+x^2)$$

[Out] 1/3*x*(-x^2+x)/(x^3+1)+ln(x)-4/9*ln(1+x)-5/18*ln(x^2-x+1)-1/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)

Rubi [A]

time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1843, 1848, 648, 632, 210, 642}

$$-\frac{5}{18}\log(x^2-x+1) + \frac{x(x-x^2)}{3(x^3+1)} + \log(x) - \frac{4}{9}\log(x+1) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(x*(1 + x^3)^2), x]

[Out] (x*(x - x^2))/(3*(1 + x^3)) - ArcTan[(1 - 2*x)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - (4*Log[1 + x])/9 - (5*Log[1 - x + x^2])/18

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1843

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
+ 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x] + Simp[(-x)*R
*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; Fr
eeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_)^(m_.)))/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{x(1+x^3)^2} dx &= \frac{x(x-x^2)}{3(1+x^3)} - \frac{1}{3} \int \frac{-3-x^2}{x(1+x^3)} dx \\
&= \frac{x(x-x^2)}{3(1+x^3)} - \frac{1}{3} \int \left(-\frac{3}{x} + \frac{4}{3(1+x)} + \frac{-4+5x}{3(1-x+x^2)} \right) dx \\
&= \frac{x(x-x^2)}{3(1+x^3)} + \log(x) - \frac{4}{9} \log(1+x) - \frac{1}{9} \int \frac{-4+5x}{1-x+x^2} dx \\
&= \frac{x(x-x^2)}{3(1+x^3)} + \log(x) - \frac{4}{9} \log(1+x) + \frac{1}{6} \int \frac{1}{1-x+x^2} dx - \frac{5}{18} \int \frac{-1+2x}{1-x+x^2} dx \\
&= \frac{x(x-x^2)}{3(1+x^3)} + \log(x) - \frac{4}{9} \log(1+x) - \frac{5}{18} \log(1-x+x^2) - \frac{1}{3} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x \right) \\
&= \frac{x(x-x^2)}{3(1+x^3)} - \frac{\tan^{-1} \left(\frac{1-2x}{\sqrt{3}} \right)}{3\sqrt{3}} + \log(x) - \frac{4}{9} \log(1+x) - \frac{5}{18} \log(1-x+x^2)
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 65, normalized size = 1.02

$$\frac{1}{18} \left(\frac{6(1+x^2)}{1+x^3} + 2\sqrt{3} \tan^{-1} \left(\frac{-1+2x}{\sqrt{3}} \right) + 18 \log(x) - 2 \log(1+x) + \log(1-x+x^2) - 6 \log(1+x^3) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(x*(1 + x^3)^2),x]

[Out] ((6*(1 + x^2))/(1 + x^3) + 2*sqrt[3]*ArcTan[(-1 + 2*x)/sqrt[3]] + 18*Log[x] - 2*Log[1 + x] + Log[1 - x + x^2] - 6*Log[1 + x^3])/18

Mathics [A]

time = 2.23, size = 60, normalized size = 0.94

$$\frac{6 + 6x^2 + (1 + x^3) \left(-8\text{Log}[1 + x] - 5\text{Log}[1 - x + x^2] + 2\sqrt{3} \text{ArcTan} \left[\frac{\sqrt{3}(-1+2x)}{3} \right] + 18\text{Log}[x] \right)}{18 + 18x^3}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(x^2 + 1)/(x*(x^3 + 1)^2),x]')

[Out] (6 + 6 x ^ 2 + (1 + x ^ 3) (-8 Log[1 + x] - 5 Log[1 - x + x ^ 2] + 2 Sqrt[3] ArcTan[Sqrt[3] (-1 + 2 x) / 3] + 18 Log[x])) / (18 (1 + x ^ 3))

Maple [A]

time = 0.06, size = 61, normalized size = 0.95

method	result
risch	$\frac{\frac{x^2}{3} + \frac{1}{3}}{x^3+1} - \frac{5 \ln(4x^2-4x+4)}{18} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9} - \frac{4 \ln(1+x)}{9} + \ln(x)$
default	$\ln(x) + \frac{2}{9(1+x)} - \frac{4 \ln(1+x)}{9} - \frac{-1-x}{9(x^2-x+1)} - \frac{5 \ln(x^2-x+1)}{18} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$
meijerg	$\frac{x^2}{3x^3+3} - \frac{x^2 \ln\left(1+(x^3)^{\frac{1}{3}}\right)}{9(x^3)^{\frac{2}{3}}} + \frac{x^2 \ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{18(x^3)^{\frac{2}{3}}} + \frac{x^2 \sqrt{3} \arctan\left(\frac{\sqrt{3} (x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{9(x^3)^{\frac{2}{3}}} + \frac{1}{3} + \ln(x) - \frac{2x^3}{3(2x^3+2)} -$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/x/(x^3+1)^2,x,method=_RETURNVERBOSE)

[Out] ln(x)+2/9/(1+x)-4/9*ln(1+x)-1/9*(-1-x)/(x^2-x+1)-5/18*ln(x^2-x+1)+1/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))

Maxima [A]

time = 0.40, size = 50, normalized size = 0.78

$$\frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{x^2 + 1}{3(x^3 + 1)} - \frac{5}{18} \log(x^2 - x + 1) - \frac{4}{9} \log(x + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/x/(x^3+1)^2,x, algorithm="maxima")

[Out] $\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{3}\frac{(x^2+1)}{(x^3+1)} - \frac{5}{18}\log(x^2-x+1) - \frac{4}{9}\log(x+1) + \log(x)$

Fricas [A]

time = 0.32, size = 73, normalized size = 1.14

$$\frac{2\sqrt{3}(x^3+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + 6x^2 - 5(x^3+1)\log(x^2-x+1) - 8(x^3+1)\log(x+1) + 18(x^3+1)\log(x) + 6}{18(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/x/(x^3+1)^2,x, algorithm="fricas")

[Out] $\frac{1}{18}(2\sqrt{3}(x^3+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + 6x^2 - 5(x^3+1)\log(x^2-x+1) - 8(x^3+1)\log(x+1) + 18(x^3+1)\log(x) + 6)/(x^3+1)$

Sympy [A]

time = 0.11, size = 60, normalized size = 0.94

$$\frac{x^2+1}{3x^3+3} + \log(x) - \frac{4\log(x+1)}{9} - \frac{5\log(x^2-x+1)}{18} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x-\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/x/(x**3+1)**2,x)

[Out] $(x^2+1)/(3x^3+3) + \log(x) - 4\log(x+1)/9 - 5\log(x^2-x+1)/18 + \sqrt{3}\operatorname{atan}(2\sqrt{3}x/3 - \sqrt{3}/3)/9$

Giac [A]

time = 0.00, size = 68, normalized size = 1.06

$$\ln|x| - \frac{4}{9}\ln|x+1| - \frac{5}{18}\ln(x^2-x+1) + \frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\frac{1}{3}(x^2+1)}{(x+1)(x^2-x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/x/(x^3+1)^2,x)

[Out] $\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{3}\frac{(x^2+1)}{(x^3+1)} - \frac{5}{18}\log(x^2-x+1) - \frac{4}{9}\log(\operatorname{abs}(x+1)) + \log(\operatorname{abs}(x))$

Mupad [B]

time = 0.10, size = 63, normalized size = 0.98

$$\ln(x) - \frac{4\ln(x+1)}{9} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\operatorname{li}}{2}\right) \left(\frac{5}{18} + \frac{\sqrt{3}\operatorname{li}}{18}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\operatorname{li}}{2}\right) \left(-\frac{5}{18} + \frac{\sqrt{3}\operatorname{li}}{18}\right) + \frac{\frac{x^2}{3} + \frac{1}{3}}{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x*(x^3 + 1)^2),x)
```

```
[Out] log(x) - (4*log(x + 1))/9 - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/18  
+ 5/18) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/18 - 5/18) + (x^2/3 +  
1/3)/(x^3 + 1)
```


$$3.182 \quad \int \frac{-2-3x+x^2}{(1+x)^2(1+x+x^2)^2} dx$$

Optimal. Leaf size=63

$$-\frac{2}{1+x} - \frac{7+5x}{3(1+x+x^2)} - \frac{25 \tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \log(1+x) + \frac{1}{2} \log(1+x+x^2)$$

[Out] $-2/(1+x)+1/3*(-7-5*x)/(x^2+x+1)-\ln(1+x)+1/2*\ln(x^2+x+1)-25/9*\arctan(1/3*(1+2*x)*3^{(1/2)})*3^{(1/2)}$

Rubi [A]

time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1660, 1642, 648, 632, 210, 642}

$$-\frac{5x+7}{3(x^2+x+1)} + \frac{1}{2} \log(x^2+x+1) - \frac{2}{x+1} - \log(x+1) - \frac{25 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-2 - 3*x + x^2)/((1 + x)^2*(1 + x + x^2)^2), x]$

[Out] $-2/(1 + x) - (7 + 5*x)/(3*(1 + x + x^2)) - (25*\text{ArcTan}[(1 + 2*x)/\text{Sqrt}[3]])/(3*\text{Sqrt}[3]) - \text{Log}[1 + x] + \text{Log}[1 + x + x^2]/2$

Rule 210

$\text{Int}[(a_. + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1}*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 632

$\text{Int}[(a_. + (b_.)*(x_) + (c_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /;$ FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

$\text{Int}[(d_. + (e_.)*(x_))/((a_. + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /;$ FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{-2 - 3x + x^2}{(1+x)^2(1+x+x^2)^2} dx &= -\frac{7+5x}{3(1+x+x^2)} + \frac{1}{3} \int \frac{-8-19x-5x^2}{(1+x)^2(1+x+x^2)} dx \\
 &= -\frac{7+5x}{3(1+x+x^2)} + \frac{1}{3} \int \left(\frac{6}{(1+x)^2} - \frac{3}{1+x} + \frac{-11+3x}{1+x+x^2} \right) dx \\
 &= -\frac{2}{1+x} - \frac{7+5x}{3(1+x+x^2)} - \log(1+x) + \frac{1}{3} \int \frac{-11+3x}{1+x+x^2} dx \\
 &= -\frac{2}{1+x} - \frac{7+5x}{3(1+x+x^2)} - \log(1+x) + \frac{1}{2} \int \frac{1+2x}{1+x+x^2} dx - \frac{25}{6} \int \frac{1}{1+x+x^2} dx \\
 &= -\frac{2}{1+x} - \frac{7+5x}{3(1+x+x^2)} - \log(1+x) + \frac{1}{2} \log(1+x+x^2) + \frac{25}{3} \text{Subst} \left(\int \frac{1}{1+x+x^2} dx \right) \\
 &= -\frac{2}{1+x} - \frac{7+5x}{3(1+x+x^2)} - \frac{25 \tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)}{3\sqrt{3}} - \log(1+x) + \frac{1}{2} \log(1+x+x^2)
 \end{aligned}$$

Mathematica [A]

time = 0.02, size = 63, normalized size = 1.00

$$-\frac{2}{1+x} - \frac{7+5x}{3(1+x+x^2)} - \frac{25 \tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \log(1+x) + \frac{1}{2} \log(1+x+x^2)$$

Antiderivative was successfully verified.

[In] Integrate[(-2 - 3*x + x^2)/((1 + x)^2*(1 + x + x^2)^2), x]

[Out] -2/(1 + x) - (7 + 5*x)/(3*(1 + x + x^2)) - (25*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - Log[1 + x] + Log[1 + x + x^2]/2

Mathics [A]

time = 2.25, size = 73, normalized size = 1.16

$$\frac{-78 - 108x - 66x^2 + (1 + 2x + 2x^2 + x^3) \left(-50\sqrt{3} \operatorname{ArcTan}\left[\frac{\sqrt{3}(1+2x)}{3}\right] - 18\operatorname{Log}[1+x] + 9\operatorname{Log}[1+x+x^2] \right)}{18 + 36x + 36x^2 + 18x^3}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(x^2 - 3*x - 2)/((x + 1)^2*(x^2 + x + 1)^2), x]')

[Out] (-78 - 108 x - 66 x ^ 2 + (1 + 2 x + 2 x ^ 2 + x ^ 3) (-50 Sqrt[3] ArcTan[Sqrt[3] (1 + 2 x) / 3] - 18 Log[1 + x] + 9 Log[1 + x + x ^ 2])) / (18 (1 + 2 x + 2 x ^ 2 + x ^ 3))

Maple [A]

time = 0.11, size = 54, normalized size = 0.86

method	result	size
default	$-\frac{2}{1+x} - \ln(1+x) + \frac{-\frac{5x}{3} - \frac{7}{3}}{x^2+x+1} + \frac{\ln(x^2+x+1)}{2} - \frac{25 \arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}$	54
risch	$\frac{-\frac{11}{3}x^2 - 6x - \frac{13}{3}}{(x^2+x+1)(1+x)} + \frac{\ln(4x^2+4x+4)}{2} - \frac{25 \arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9} - \ln(1+x)$	61

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x,method=_RETURNVERBOSE)

[Out] -2/(1+x)-ln(1+x)+(-5/3*x-7/3)/(x^2+x+1)+1/2*ln(x^2+x+1)-25/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)

Maxima [A]

time = 0.34, size = 59, normalized size = 0.94

$$-\frac{25}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) - \frac{11x^2 + 18x + 13}{3(x^3 + 2x^2 + 2x + 1)} + \frac{1}{2} \log(x^2 + x + 1) - \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x, algorithm="maxima")

[Out] -25/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*(11*x^2 + 18*x + 13)/(x^3 + 2*x^2 + 2*x + 1) + 1/2*log(x^2 + x + 1) - log(x + 1)

Fricas [A]

time = 0.31, size = 97, normalized size = 1.54

$$\frac{50\sqrt{3}(x^3+2x^2+2x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+66x^2-9(x^3+2x^2+2x+1)\log(x^2+x+1)+18(x^3+2x^2+2x+1)\log(x+1)+108x+78}{18(x^3+2x^2+2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x, algorithm="fricas")

[Out] -1/18*(50*sqrt(3)*(x^3 + 2*x^2 + 2*x + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) + 6*6*x^2 - 9*(x^3 + 2*x^2 + 2*x + 1)*log(x^2 + x + 1) + 18*(x^3 + 2*x^2 + 2*x + 1)*log(x + 1) + 108*x + 78)/(x^3 + 2*x^2 + 2*x + 1)

Sympy [A]

time = 0.10, size = 68, normalized size = 1.08

$$\frac{-11x^2 - 18x - 13}{3x^3 + 6x^2 + 6x + 3} - \log(x + 1) + \frac{\log(x^2 + x + 1)}{2} - \frac{25\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-3*x-2)/(1+x)**2/(x**2+x+1)**2,x)

[Out] (-11*x**2 - 18*x - 13)/(3*x**3 + 6*x**2 + 6*x + 3) - log(x + 1) + log(x**2 + x + 1)/2 - 25*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9

Giac [A]

time = 0.00, size = 70, normalized size = 1.11

$$-\ln|x+1| + \frac{\ln(x^2+x+1)}{2} - \frac{25\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{-11x^2-18x-13}{3(x^3+2x^2+2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x)

[Out] -25/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*(11*x^2 + 18*x + 13)/(x^3 + 2*x^2 + 2*x + 1) + 1/2*log(x^2 + x + 1) - log(abs(x + 1))

Mupad [B]

time = 0.25, size = 73, normalized size = 1.16

$$-\ln(x+1) - \frac{\frac{11x^2}{3} + 6x + \frac{13}{3}}{x^3 + 2x^2 + 2x + 1} + \ln\left(x + \frac{1}{2} - \frac{\sqrt{3} \operatorname{li}}{2}\right) \left(\frac{1}{2} + \frac{\sqrt{3} \operatorname{25i}}{18}\right) - \ln\left(x + \frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2}\right) \left(-\frac{1}{2} + \frac{\sqrt{3} \operatorname{25i}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - x^2 + 2)/((x + 1)^2*(x + x^2 + 1)^2),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*25i)/18 + 1/2) - (6*x + (11*x^2)/3  
+ 13/3)/(2*x + 2*x^2 + x^3 + 1) - log(x + 1) - log(x + (3^(1/2)*1i)/2 + 1/2  
)*((3^(1/2)*25i)/18 - 1/2)
```

$$3.183 \quad \int \frac{1}{(1-4x)^3(2-3x)} dx$$

Optimal. Leaf size=43

$$\frac{1}{10(1-4x)^2} - \frac{3}{25(1-4x)} - \frac{9}{125} \log(1-4x) + \frac{9}{125} \log(2-3x)$$

[Out] 1/10/(1-4*x)^2-3/25/(1-4*x)-9/125*ln(1-4*x)+9/125*ln(2-3*x)

Rubi [A]

time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {46}

$$-\frac{3}{25(1-4x)} + \frac{1}{10(1-4x)^2} - \frac{9}{125} \log(1-4x) + \frac{9}{125} \log(2-3x)$$

Antiderivative was successfully verified.

[In] Int[1/((1 - 4*x)^3*(2 - 3*x)), x]

[Out] 1/(10*(1 - 4*x)^2) - 3/(25*(1 - 4*x)) - (9*Log[1 - 4*x])/125 + (9*Log[2 - 3*x])/125

Rule 46

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(1-4x)^3(2-3x)} dx &= \int \left(\frac{27}{125(-2+3x)} - \frac{4}{5(-1+4x)^3} - \frac{12}{25(-1+4x)^2} - \frac{36}{125(-1+4x)} \right) dx \\ &= \frac{1}{10(1-4x)^2} - \frac{3}{25(1-4x)} - \frac{9}{125} \log(1-4x) + \frac{9}{125} \log(2-3x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 46, normalized size = 1.07

$$\frac{-5 + 120x + 18(1-4x)^2 \log(8-12x) - 18(1-4x)^2 \log(-1+4x)}{250(1-4x)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 - 4*x)^3*(2 - 3*x)),x]

[Out] $(-5 + 120x + 18(1 - 4x)^2 \text{Log}[8 - 12x] - 18(1 - 4x)^2 \text{Log}[-1 + 4x]) / (250(1 - 4x)^2)$

Mathics [A]

time = 4.28, size = 42, normalized size = 0.98

$$\frac{-5 + 120x + 18(1 - 8x + 16x^2) (\text{Log}[-\frac{2}{3} + x] - \text{Log}[-\frac{1}{4} + x])}{250 - 2000x + 4000x^2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/((2 - 3*x)*(1 - 4*x)^3),x]')

[Out] $(-5 + 120x + 18(1 - 8x + 16x^2) (\text{Log}[-2/3 + x] - \text{Log}[-1/4 + x])) / (250(1 - 8x + 16x^2))$

Maple [A]

time = 0.06, size = 36, normalized size = 0.84

method	result	size
risch	$\frac{\frac{12x - 1}{25} - \frac{1}{50}}{(-1+4x)^2} + \frac{9 \ln(-2+3x)}{125} - \frac{9 \ln(-1+4x)}{125}$	32
norman	$\frac{\frac{8}{25}x + \frac{8}{25}x^2}{(-1+4x)^2} + \frac{9 \ln(-2+3x)}{125} - \frac{9 \ln(-1+4x)}{125}$	35
default	$\frac{1}{10(-1+4x)^2} + \frac{3}{25(-1+4x)} - \frac{9 \ln(-1+4x)}{125} + \frac{9 \ln(-2+3x)}{125}$	36

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-4*x)^3/(2-3*x),x,method=_RETURNVERBOSE)

[Out] $1/10/(-1+4*x)^2+3/25/(-1+4*x)-9/125*\ln(-1+4*x)+9/125*\ln(-2+3*x)$

Maxima [A]

time = 0.26, size = 36, normalized size = 0.84

$$\frac{24x - 1}{50(16x^2 - 8x + 1)} - \frac{9}{125} \log(4x - 1) + \frac{9}{125} \log(3x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-4*x)^3/(2-3*x),x, algorithm="maxima")

[Out] $1/50*(24*x - 1)/(16*x^2 - 8*x + 1) - 9/125*\log(4*x - 1) + 9/125*\log(3*x - 2)$

Fricas [A]

time = 0.31, size = 55, normalized size = 1.28

$$-\frac{18(16x^2 - 8x + 1) \log(4x - 1) - 18(16x^2 - 8x + 1) \log(3x - 2) - 120x + 5}{250(16x^2 - 8x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-4*x)^3/(2-3*x),x, algorithm="fricas")

[Out] -1/250*(18*(16*x^2 - 8*x + 1)*log(4*x - 1) - 18*(16*x^2 - 8*x + 1)*log(3*x - 2) - 120*x + 5)/(16*x^2 - 8*x + 1)

Sympy [A]

time = 0.07, size = 34, normalized size = 0.79

$$\frac{24x - 1}{800x^2 - 400x + 50} + \frac{9 \log\left(x - \frac{2}{3}\right)}{125} - \frac{9 \log\left(x - \frac{1}{4}\right)}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-4*x)**3/(2-3*x),x)

[Out] (24*x - 1)/(800*x**2 - 400*x + 50) + 9*log(x - 2/3)/125 - 9*log(x - 1/4)/125

Giac [A]

time = 0.00, size = 43, normalized size = 1.00

$$\frac{9}{125} \ln|3x - 2| - \frac{9}{125} \ln|4x - 1| + \frac{\frac{1}{125} \left(60x - \frac{5}{2}\right)}{(4x - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-4*x)^3/(2-3*x),x)

[Out] 1/50*(24*x - 1)/(4*x - 1)^2 - 9/125*log(abs(4*x - 1)) + 9/125*log(abs(3*x - 2))

Mupad [B]

time = 0.05, size = 25, normalized size = 0.58

$$\frac{\frac{3x}{100} - \frac{1}{800}}{x^2 - \frac{x}{2} + \frac{1}{16}} - \frac{18 \operatorname{atanh}\left(\frac{24x}{5} - \frac{11}{5}\right)}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((3*x - 2)*(4*x - 1)^3),x)

[Out] ((3*x)/100 - 1/800)/(x^2 - x/2 + 1/16) - (18*atanh((24*x)/5 - 11/5))/125

$$3.184 \quad \int \frac{x^3}{(2-5x^2)^7} dx$$

Optimal. Leaf size=27

$$\frac{1}{150(2-5x^2)^6} - \frac{1}{250(2-5x^2)^5}$$

[Out] 1/150/(-5*x^2+2)^6-1/250/(-5*x^2+2)^5

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{1}{150(2-5x^2)^6} - \frac{1}{250(2-5x^2)^5}$$

Antiderivative was successfully verified.

[In] Int[x^3/(2 - 5*x^2)^7,x]

[Out] 1/(150*(2 - 5*x^2)^6) - 1/(250*(2 - 5*x^2)^5)

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(2-5x^2)^7} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x}{(2-5x)^7} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(-\frac{2}{5(-2+5x)^7} - \frac{1}{5(-2+5x)^6} \right) dx, x, x^2 \right) \\ &= \frac{1}{150(2-5x^2)^6} - \frac{1}{250(2-5x^2)^5} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 20, normalized size = 0.74

$$\frac{-1 + 15x^2}{750(2 - 5x^2)^6}$$

Antiderivative was successfully verified.

`[In] Integrate[x^3/(2 - 5*x^2)^7,x]``[Out] (-1 + 15*x^2)/(750*(2 - 5*x^2)^6)`**Mathics [A]**

time = 2.08, size = 43, normalized size = 1.59

$$\frac{-1 + 15x^2}{48000 - 720000x^2 + 4500000x^4 - 15000000x^6 + 28125000x^8 - 28125000x^{10} + 11718750x^{12}}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^3/(2 - 5*x^2)^7,x]')``[Out] (-1 + 15 x ^ 2) / (750 (64 - 960 x ^ 2 + 6000 x ^ 4 - 20000 x ^ 6 + 37500 x ^ 8 - 37500 x ^ 10 + 15625 x ^ 12))`**Maple [A]**

time = 0.06, size = 24, normalized size = 0.89

method	result	size
gospers	$\frac{15x^2-1}{750(5x^2-2)^6}$	19
risch	$\frac{\frac{x^2}{50} - \frac{1}{750}}{(5x^2-2)^6}$	19
default	$\frac{1}{150(5x^2-2)^6} + \frac{1}{250(5x^2-2)^5}$	24
norman	$\frac{-\frac{25}{32}x^{10} - \frac{5}{12}x^6 + \frac{1}{8}x^4 + \frac{25}{32}x^8 + \frac{125}{384}x^{12}}{(5x^2-2)^6}$	37
meijerg	$\frac{x^4 \left(\frac{625}{16}x^8 - \frac{375}{4}x^6 + \frac{375}{4}x^4 - 50x^2 + 15 \right)}{7680 \left(1 - \frac{5x^2}{2} \right)^6}$	37

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^3/(-5*x^2+2)^7,x,method=_RETURNVERBOSE)``[Out] 1/150/(5*x^2-2)^6+1/250/(5*x^2-2)^5`**Maxima [A]**

time = 0.26, size = 43, normalized size = 1.59

$$\frac{15x^2 - 1}{750(15625x^{12} - 37500x^{10} + 37500x^8 - 20000x^6 + 6000x^4 - 960x^2 + 64)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x³/(-5*x²+2)⁷,x, algorithm="maxima")

[Out] 1/750*(15*x² - 1)/(15625*x¹² - 37500*x¹⁰ + 37500*x⁸ - 20000*x⁶ + 6000*x⁴ - 960*x² + 64)

Fricas [A]

time = 0.31, size = 43, normalized size = 1.59

$$\frac{15x^2 - 1}{750(15625x^{12} - 37500x^{10} + 37500x^8 - 20000x^6 + 6000x^4 - 960x^2 + 64)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x³/(-5*x²+2)⁷,x, algorithm="fricas")

[Out] 1/750*(15*x² - 1)/(15625*x¹² - 37500*x¹⁰ + 37500*x⁸ - 20000*x⁶ + 6000*x⁴ - 960*x² + 64)

Sympy [A]

time = 0.09, size = 39, normalized size = 1.44

$$\frac{1 - 15x^2}{11718750x^{12} - 28125000x^{10} + 28125000x^8 - 15000000x^6 + 4500000x^4 - 720000x^2 + 48000}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(-5*x**2+2)**7,x)

[Out] -(1 - 15*x**2)/(11718750*x**12 - 28125000*x**10 + 28125000*x**8 - 15000000*x**6 + 4500000*x**4 - 720000*x**2 + 48000)

Giac [A]

time = 0.00, size = 22, normalized size = 0.81

$$\frac{15x^2 - 1}{2 \cdot 375(5x^2 - 2)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x³/(-5*x²+2)⁷,x)

[Out] 1/750*(15*x² - 1)/(5*x² - 2)⁶

Mupad [B]

time = 0.11, size = 18, normalized size = 0.67

$$\frac{15x^2 - 1}{750(5x^2 - 2)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x³/(5*x² - 2)⁷,x)

[Out] (15*x² - 1)/(750*(5*x² - 2)⁶)

$$3.185 \quad \int \frac{x^7}{(2-5x^2)^3} dx$$

Optimal. Leaf size=46

$$-\frac{x^2}{250} + \frac{2}{625(2-5x^2)^2} - \frac{6}{625(2-5x^2)} - \frac{3}{625} \log(2-5x^2)$$

[Out] -1/250*x^2+2/625/(-5*x^2+2)^2-6/625/(-5*x^2+2)-3/625*ln(-5*x^2+2)

Rubi [A]

time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$-\frac{x^2}{250} - \frac{6}{625(2-5x^2)} + \frac{2}{625(2-5x^2)^2} - \frac{3}{625} \log(2-5x^2)$$

Antiderivative was successfully verified.

[In] Int[x^7/(2 - 5*x^2)^3,x]

[Out] -1/250*x^2 + 2/(625*(2 - 5*x^2)^2) - 6/(625*(2 - 5*x^2)) - (3*Log[2 - 5*x^2])/625

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^7}{(2-5x^2)^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^3}{(2-5x)^3} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(-\frac{1}{125} - \frac{8}{125(-2+5x)^3} - \frac{12}{125(-2+5x)^2} - \frac{6}{125(-2+5x)} \right) dx, x, x^2 \right) \\ &= -\frac{x^2}{250} + \frac{2}{625(2-5x^2)^2} - \frac{6}{625(2-5x^2)} - \frac{3}{625} \log(2-5x^2) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 44, normalized size = 0.96

$$\frac{12 - 150x^4 + 125x^6 + 6(2 - 5x^2)^2 \log(-2 + 5x^2)}{1250(2 - 5x^2)^2}$$

Antiderivative was successfully verified.

`[In] Integrate[x^7/(2 - 5*x^2)^3,x]``[Out] -1/1250*(12 - 150*x^4 + 125*x^6 + 6*(2 - 5*x^2)^2*Log[-2 + 5*x^2])/(2 - 5*x^2)^2`**Mathics [A]**

time = 2.05, size = 52, normalized size = 1.13

$$\frac{-20 + 60x^2 + (-4 + 20x^2 - 25x^4)(5x^2 + 6\text{Log}[-2 + 5x^2])}{5000 - 25000x^2 + 31250x^4}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^7/(2 - 5*x^2)^3,x]')``[Out] (-20 + 60 x ^ 2 + (-4 + 20 x ^ 2 - 25 x ^ 4) (5 x ^ 2 + 6 Log[-2 + 5 x ^ 2])) / (1250 (4 - 20 x ^ 2 + 25 x ^ 4))`**Maple [A]**

time = 0.05, size = 39, normalized size = 0.85

method	result	size
risch	$-\frac{x^2}{250} + \frac{\frac{6x^2}{125} - \frac{2}{125}}{(5x^2-2)^2} - \frac{3\ln(5x^2-2)}{625}$	35
norman	$-\frac{\frac{6}{125}x^2 + \frac{9}{50}x^4 - \frac{1}{10}x^6}{(5x^2-2)^2} - \frac{3\ln(5x^2-2)}{625}$	38
meijerg	$-\frac{x^2(25x^4-45x^2+12)}{1000(1-\frac{5x^2}{2})^2} - \frac{3\ln(1-\frac{5x^2}{2})}{625}$	38
default	$-\frac{x^2}{250} + \frac{2}{625(5x^2-2)^2} - \frac{3\ln(5x^2-2)}{625} + \frac{6}{625(5x^2-2)}$	39

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^7/(-5*x^2+2)^3,x,method=_RETURNVERBOSE)``[Out] -1/250*x^2+2/625/(5*x^2-2)^2-3/625*ln(5*x^2-2)+6/625/(5*x^2-2)`**Maxima [A]**

time = 0.26, size = 39, normalized size = 0.85

$$-\frac{1}{250}x^2 + \frac{2(3x^2 - 1)}{125(25x^4 - 20x^2 + 4)} - \frac{3}{625} \log(5x^2 - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(-5*x^2+2)^3,x, algorithm="maxima")

[Out] -1/250*x^2 + 2/125*(3*x^2 - 1)/(25*x^4 - 20*x^2 + 4) - 3/625*log(5*x^2 - 2)

Fricas [A]

time = 0.36, size = 55, normalized size = 1.20

$$-\frac{125x^6 - 100x^4 - 40x^2 + 6(25x^4 - 20x^2 + 4)\log(5x^2 - 2) + 20}{1250(25x^4 - 20x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(-5*x^2+2)^3,x, algorithm="fricas")

[Out] -1/1250*(125*x^6 - 100*x^4 - 40*x^2 + 6*(25*x^4 - 20*x^2 + 4)*log(5*x^2 - 2) + 20)/(25*x^4 - 20*x^2 + 4)

Sympy [A]

time = 0.06, size = 36, normalized size = 0.78

$$-\frac{x^2}{250} - \frac{2 - 6x^2}{3125x^4 - 2500x^2 + 500} - \frac{3\log(5x^2 - 2)}{625}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(-5*x**2+2)**3,x)

[Out] -x**2/250 - (2 - 6*x**2)/(3125*x**4 - 2500*x**2 + 500) - 3*log(5*x**2 - 2)/625

Giac [A]

time = 0.00, size = 47, normalized size = 1.02

$$-\frac{x^2}{250} + \frac{225x^4 - 120x^2 + 16}{1250(5x^2 - 2)^2} - \frac{3}{625} \ln|5x^2 - 2|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(-5*x^2+2)^3,x)

[Out] -1/250*x^2 + 1/1250*(225*x^4 - 120*x^2 + 16)/(5*x^2 - 2)^2 - 3/625*log(abs(5*x^2 - 2))

Mupad [B]

time = 0.19, size = 34, normalized size = 0.74

$$\frac{\frac{6x^2}{3125} - \frac{2}{3125}}{x^4 - \frac{4x^2}{5} + \frac{4}{25}} - \frac{3\ln\left(x^2 - \frac{2}{5}\right)}{625} - \frac{x^2}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x^7/(5*x^2 - 2)^3,x)

[Out] ((6*x^2)/3125 - 2/3125)/(x^4 - (4*x^2)/5 + 4/25) - (3*log(x^2 - 2/5))/625 - x^2/250

$$3.186 \quad \int \frac{1}{(-2+x)^3(1+x)^2} dx$$

Optimal. Leaf size=44

$$-\frac{1}{18(-2+x)^2} + \frac{2}{27(-2+x)} + \frac{1}{27(1+x)} + \frac{1}{27} \log(-2+x) - \frac{1}{27} \log(1+x)$$

[Out] -1/18/(-2+x)^2+2/27/(-2+x)+1/27/(1+x)+1/27*ln(-2+x)-1/27*ln(1+x)

Rubi [A]

time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{2}{27(2-x)} + \frac{1}{27(x+1)} - \frac{1}{18(2-x)^2} + \frac{1}{27} \log(2-x) - \frac{1}{27} \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[1/((-2 + x)^3*(1 + x)^2), x]

[Out] -1/18*1/(2 - x)^2 - 2/(27*(2 - x)) + 1/(27*(1 + x)) + Log[2 - x]/27 - Log[1 + x]/27

Rule 46

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(-2+x)^3(1+x)^2} dx &= \int \left(\frac{1}{9(-2+x)^3} - \frac{2}{27(-2+x)^2} + \frac{1}{27(-2+x)} - \frac{1}{27(1+x)^2} - \frac{1}{27(1+x)} \right) dx \\ &= -\frac{1}{18(2-x)^2} - \frac{2}{27(2-x)} + \frac{1}{27(1+x)} + \frac{1}{27} \log(2-x) - \frac{1}{27} \log(1+x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 39, normalized size = 0.89

$$\frac{1}{54} \left(\frac{3(-1-5x+2x^2)}{(-2+x)^2(1+x)} + 2 \log(-2+x) - 2 \log(1+x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((-2 + x)^3*(1 + x)^2),x]

[Out] ((3*(-1 - 5*x + 2*x^2))/((-2 + x)^2*(1 + x)) + 2*Log[-2 + x] - 2*Log[1 + x])/54

Mathics [A]

time = 2.10, size = 47, normalized size = 1.07

$$\frac{-3 - 15x + 6x^2 + 2(4 - 3x^2 + x^3)(\text{Log}[-2 + x] - \text{Log}[1 + x])}{216 - 162x^2 + 54x^3}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/((x - 2)^3*(x + 1)^2),x]')

[Out] (-3 - 15 x + 6 x ^ 2 + 2 (4 - 3 x ^ 2 + x ^ 3) (Log[-2 + x] - Log[1 + x])) / (54 (4 - 3 x ^ 2 + x ^ 3))

Maple [A]

time = 0.07, size = 35, normalized size = 0.80

method	result	size
default	$-\frac{1}{18(-2+x)^2} + \frac{2}{27(-2+x)} + \frac{1}{27+27x} + \frac{\ln(-2+x)}{27} - \frac{\ln(1+x)}{27}$	35
norman	$\frac{\frac{1}{9}x^2 - \frac{5}{18}x - \frac{1}{18}}{(-2+x)^2(1+x)} + \frac{\ln(-2+x)}{27} - \frac{\ln(1+x)}{27}$	35
risch	$\frac{\frac{1}{9}x^2 - \frac{5}{18}x - \frac{1}{18}}{(-2+x)^2(1+x)} + \frac{\ln(-2+x)}{27} - \frac{\ln(1+x)}{27}$	35

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-2+x)^3/(1+x)^2,x,method=_RETURNVERBOSE)

[Out] -1/18/(-2+x)^2+2/27/(-2+x)+1/27/(1+x)+1/27*ln(-2+x)-1/27*ln(1+x)

Maxima [A]

time = 0.26, size = 37, normalized size = 0.84

$$\frac{2x^2 - 5x - 1}{18(x^3 - 3x^2 + 4)} - \frac{1}{27} \log(x + 1) + \frac{1}{27} \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-2+x)^3/(1+x)^2,x, algorithm="maxima")

[Out] 1/18*(2*x^2 - 5*x - 1)/(x^3 - 3*x^2 + 4) - 1/27*log(x + 1) + 1/27*log(x - 2)

Fricas [A]

time = 0.31, size = 56, normalized size = 1.27

$$\frac{6x^2 - 2(x^3 - 3x^2 + 4) \log(x + 1) + 2(x^3 - 3x^2 + 4) \log(x - 2) - 15x - 3}{54(x^3 - 3x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-2+x)^3/(1+x)^2,x, algorithm="fricas")

[Out] 1/54*(6*x^2 - 2*(x^3 - 3*x^2 + 4)*log(x + 1) + 2*(x^3 - 3*x^2 + 4)*log(x - 2) - 15*x - 3)/(x^3 - 3*x^2 + 4)

Sympy [A]

time = 0.07, size = 34, normalized size = 0.77

$$\frac{2x^2 - 5x - 1}{18x^3 - 54x^2 + 72} + \frac{\log(x - 2)}{27} - \frac{\log(x + 1)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-2+x)**3/(1+x)**2,x)

[Out] (2*x**2 - 5*x - 1)/(18*x**3 - 54*x**2 + 72) + log(x - 2)/27 - log(x + 1)/27

Giac [A]

time = 0.00, size = 43, normalized size = 0.98

$$-\frac{\ln|x + 1|}{27} + \frac{\ln|x - 2|}{27} + \frac{\frac{1}{54}(6x^2 - 15x - 3)}{(x - 2)^2(x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-2+x)^3/(1+x)^2,x)

[Out] 1/18*(2*x^2 - 5*x - 1)/((x + 1)*(x - 2)^2) - 1/27*log(abs(x + 1)) + 1/27*log(abs(x - 2))

Mupad [B]

time = 0.05, size = 33, normalized size = 0.75

$$-\frac{2 \operatorname{atanh}\left(\frac{2x}{3} - \frac{1}{3}\right)}{27} - \frac{-\frac{x^2}{9} + \frac{5x}{18} + \frac{1}{18}}{x^3 - 3x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x + 1)^2*(x - 2)^3),x)

[Out] - (2*atanh((2*x)/3 - 1/3))/27 - ((5*x)/18 - x^2/9 + 1/18)/(x^3 - 3*x^2 + 4)

$$3.187 \quad \int \frac{1}{(2+x)^3(3+x)^4} dx$$

Optimal. Leaf size=54

$$-\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10 \log(2+x) - 10 \log(3+x)$$

[Out] $-1/2/(2+x)^2+4/(2+x)+1/3/(3+x)^3+3/2/(3+x)^2+6/(3+x)+10*\ln(2+x)-10*\ln(3+x)$

Rubi [A]

time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$\frac{4}{x+2} + \frac{6}{x+3} - \frac{1}{2(x+2)^2} + \frac{3}{2(x+3)^2} + \frac{1}{3(x+3)^3} + 10 \log(x+2) - 10 \log(x+3)$$

Antiderivative was successfully verified.

[In] Int[1/((2 + x)^3*(3 + x)^4), x]

[Out] $-1/2*1/(2 + x)^2 + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 + x) + 10*\text{Log}[2 + x] - 10*\text{Log}[3 + x]$

Rule 46

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\int \frac{1}{(2+x)^3(3+x)^4} dx = \int \left(\frac{1}{(2+x)^3} - \frac{4}{(2+x)^2} + \frac{10}{2+x} - \frac{1}{(3+x)^4} - \frac{3}{(3+x)^3} - \frac{6}{(3+x)^2} - \frac{10}{3+x} \right) dx$$

$$= -\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10 \log(2+x) - 10 \log(3+x)$$

Mathematica [A]

time = 0.00, size = 54, normalized size = 1.00

$$-\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10 \log(2+x) - 10 \log(3+x)$$

Antiderivative was successfully verified.

[In] Integrate[1/((2 + x)^3*(3 + x)^4),x]

[Out] $-1/2*1/(2 + x)^2 + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 + x) + 10*\text{Log}[2 + x] - 10*\text{Log}[3 + x]$

Mathics [A]

time = 2.46, size = 83, normalized size = 1.54

$$\frac{2627 + 4175x + 2450x^2 + 630x^3 + 60x^4 + 60(108 + 216x + 171x^2 + 67x^3 + 13x^4 + x^5)(\text{Log}[2 + x] - \text{Log}[3 + x])}{648 + 1296x + 1026x^2 + 402x^3 + 78x^4 + 6x^5}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/((x + 2)^3*(x + 3)^4),x]')

[Out] $(2627 + 4175x + 2450x^2 + 630x^3 + 60x^4 + 60(108 + 216x + 171x^2 + 67x^3 + 13x^4 + x^5)(\text{Log}[2 + x] - \text{Log}[3 + x])) / (6(108 + 216x + 171x^2 + 67x^3 + 13x^4 + x^5))$

Maple [A]

time = 0.00, size = 49, normalized size = 0.91

method	result	size
norman	$\frac{10x^4 + 105x^3 + \frac{1225}{3}x^2 + \frac{4175}{6}x + \frac{2627}{6}}{(2+x)^2(3+x)^3} + 10 \ln(2+x) - 10 \ln(3+x)$	45
risch	$\frac{10x^4 + 105x^3 + \frac{1225}{3}x^2 + \frac{4175}{6}x + \frac{2627}{6}}{(2+x)^2(3+x)^3} + 10 \ln(2+x) - 10 \ln(3+x)$	45
default	$-\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10 \ln(2+x) - 10 \ln(3+x)$	49

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2+x)^3/(3+x)^4,x,method=_RETURNVERBOSE)

[Out] $-1/2/(2+x)^2 + 4/(2+x) + 1/3/(3+x)^3 + 3/2/(3+x)^2 + 6/(3+x) + 10*\ln(2+x) - 10*\ln(3+x)$

Maxima [A]

time = 0.28, size = 60, normalized size = 1.11

$$\frac{60x^4 + 630x^3 + 2450x^2 + 4175x + 2627}{6(x^5 + 13x^4 + 67x^3 + 171x^2 + 216x + 108)} - 10 \log(x + 3) + 10 \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="maxima")

[Out] $1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108) - 10*\log(x + 3) + 10*\log(x + 2)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(48) = 96.

time = 0.31, size = 105, normalized size = 1.94

$$\frac{60x^4 + 630x^3 + 2450x^2 - 60(x^5 + 13x^4 + 67x^3 + 171x^2 + 216x + 108)\log(x + 3) + 60(x^5 + 13x^4 + 67x^3 + 171x^2 + 216x + 108)\log(x + 2) + 4175x + 2627}{6(x^5 + 13x^4 + 67x^3 + 171x^2 + 216x + 108)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="fricas")

[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 - 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108)*log(x + 3) + 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108)*log(x + 2) + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108)

Sympy [A]

time = 0.08, size = 58, normalized size = 1.07

$$\frac{60x^4 + 630x^3 + 2450x^2 + 4175x + 2627}{6x^5 + 78x^4 + 402x^3 + 1026x^2 + 1296x + 648} + 10 \log(x + 2) - 10 \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+x)**3/(3+x)**4,x)

[Out] (60*x**4 + 630*x**3 + 2450*x**2 + 4175*x + 2627)/(6*x**5 + 78*x**4 + 402*x**3 + 1026*x**2 + 1296*x + 648) + 10*log(x + 2) - 10*log(x + 3)

Giac [A]

time = 0.00, size = 52, normalized size = 0.96

$$10 \ln|x + 2| - 10 \ln|x + 3| + \frac{\frac{1}{6}(60x^4 + 630x^3 + 2450x^2 + 4175x + 2627)}{(x + 2)^2(x + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+x)^3/(3+x)^4,x)

[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/((x + 3)^3*(x + 2)^2) - 10*log(abs(x + 3)) + 10*log(abs(x + 2))

Mupad [B]

time = 0.00, size = 55, normalized size = 1.02

$$\frac{10x^4 + 105x^3 + \frac{1225x^2}{3} + \frac{4175x}{6} + \frac{2627}{6}}{x^5 + 13x^4 + 67x^3 + 171x^2 + 216x + 108} - 20 \operatorname{atanh}(2x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x + 2)^3*(x + 3)^4),x)

[Out] ((4175*x)/6 + (1225*x^2)/3 + 105*x^3 + 10*x^4 + 2627/6)/(216*x + 171*x^2 + 67*x^3 + 13*x^4 + x^5 + 108) - 20*atanh(2*x + 5)

$$3.188 \quad \int \frac{x^5}{(3+x)^2} dx$$

Optimal. Leaf size=36

$$-108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405 \log(3+x)$$

[Out] $-108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*\ln(3+x)$

Rubi [A]

time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$,

Rules used = {45}

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + \frac{243}{x+3} + 405 \log(x+3)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^5/(3+x)^2, x]$

[Out] $-108*x + (27*x^2)/2 - 2*x^3 + x^4/4 + 243/(3+x) + 405*\text{Log}[3+x]$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{x^5}{(3+x)^2} dx &= \int \left(-108 + 27x - 6x^2 + x^3 - \frac{243}{(3+x)^2} + \frac{405}{3+x} \right) dx \\ &= -108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405 \log(3+x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 36, normalized size = 1.00

$$\frac{1}{4} \left(-2079 - 432x + 54x^2 - 8x^3 + x^4 + \frac{972}{3+x} \right) + 405 \log(3+x)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(3 + x)^2,x]

[Out] $(-2079 - 432x + 54x^2 - 8x^3 + x^4 + 972/(3 + x))/4 + 405\text{Log}[3 + x]$

Mathics [A]

time = 1.85, size = 36, normalized size = 1.00

$$\frac{972 + (3 + x)(-432x + 54x^2 - 8x^3 + x^4 + 1620\text{Log}[3 + x])}{12 + 4x}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^5/(3 + x)^2,x]')

[Out] $(972 + (3 + x)(-432x + 54x^2 - 8x^3 + x^4 + 1620\text{Log}[3 + x])) / (4(3 + x))$

Maple [A]

time = 0.04, size = 33, normalized size = 0.92

method	result	size
default	$-108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405 \ln(3 + x)$	33
risch	$-108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405 \ln(3 + x)$	33
norman	$\frac{-\frac{135}{2}x^2 + \frac{15}{2}x^3 - \frac{5}{4}x^4 + \frac{1}{4}x^5 + 1215}{3+x} + 405 \ln(3 + x)$	36
meijerg	$-\frac{9x(-\frac{1}{27}x^4 + \frac{5}{27}x^3 - \frac{10}{9}x^2 + 10x + 60)}{4(1 + \frac{x}{3})} + 405 \ln(1 + \frac{x}{3})$	40

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(3+x)^2,x,method=_RETURNVERBOSE)

[Out] $-108x + 27/2x^2 - 2x^3 + 1/4x^4 + 243/(3+x) + 405\ln(3+x)$

Maxima [A]

time = 0.25, size = 32, normalized size = 0.89

$$\frac{1}{4}x^4 - 2x^3 + \frac{27}{2}x^2 - 108x + \frac{243}{x+3} + 405 \log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(3+x)^2,x, algorithm="maxima")

[Out] $1/4x^4 - 2x^3 + 27/2x^2 - 108x + 243/(x+3) + 405\log(x+3)$

Fricas [A]

time = 0.31, size = 39, normalized size = 1.08

$$\frac{x^5 - 5x^4 + 30x^3 - 270x^2 + 1620(x+3)\log(x+3) - 1296x + 972}{4(x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(3+x)^2,x, algorithm="fricas")`

[Out] $\frac{1}{4}(x^5 - 5x^4 + 30x^3 - 270x^2 + 1620(x + 3)\log(x + 3) - 1296x + 972)/(x + 3)$

Sympy [A]

time = 0.04, size = 31, normalized size = 0.86

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + 405 \log(x + 3) + \frac{243}{x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(3+x)**2,x)`

[Out] $x^{**4}/4 - 2*x^{**3} + 27*x^{**2}/2 - 108*x + 405*\log(x + 3) + 243/(x + 3)$

Giac [A]

time = 0.00, size = 38, normalized size = 1.06

$$\frac{1}{4}x^4 - 2x^3 + \frac{27}{2}x^2 - 108x + \frac{243}{x + 3} + 405 \ln|x + 3|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(3+x)^2,x)`

[Out] $\frac{1}{4}x^4 - 2x^3 + \frac{27}{2}x^2 - 108x + \frac{243}{(x + 3)} + 405*\log(\text{abs}(x + 3))$

Mupad [B]

time = 0.00, size = 32, normalized size = 0.89

$$405 \ln(x + 3) - 108x + \frac{243}{x + 3} + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(x + 3)^2,x)`

[Out] $405*\log(x + 3) - 108*x + 243/(x + 3) + (27*x^2)/2 - 2*x^3 + x^4/4$

3.189 $\int (b_1 + c_1 x) (a + 2bx + cx^2) dx$

Optimal. Leaf size=44

$$ab_1x + \frac{1}{2}(2bb_1 + ac_1)x^2 + \frac{1}{3}(b_1c + 2bc_1)x^3 + \frac{1}{4}cc_1x^4$$

[Out] a*b1*x+1/2*(a*c1+2*b*b1)*x^2+1/3*(2*b*c1+b1*c)*x^3+1/4*c*c1*x^4

Rubi [A]

time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {645}

$$\frac{1}{2}x^2(ac_1 + 2bb_1) + ab_1x + \frac{1}{3}x^3(2bc_1 + b_1c) + \frac{1}{4}cc_1x^4$$

Antiderivative was successfully verified.

[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2),x]

[Out] a*b1*x + ((2*b*b1 + a*c1)*x^2)/2 + ((b1*c + 2*b*c1)*x^3)/3 + (c*c1*x^4)/4

Rule 645

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || EqQ[a, 0])

Rubi steps

$$\begin{aligned} \int (b_1 + c_1 x) (a + 2bx + cx^2) dx &= \int (ab_1 + (2bb_1 + ac_1)x + (b_1c + 2bc_1)x^2 + cc_1x^3) dx \\ &= ab_1x + \frac{1}{2}(2bb_1 + ac_1)x^2 + \frac{1}{3}(b_1c + 2bc_1)x^3 + \frac{1}{4}cc_1x^4 \end{aligned}$$

Mathematica [A]

time = 0.01, size = 41, normalized size = 0.93

$$\frac{1}{12}x(6a(2b_1 + c_1x) + x(4b(3b_1 + 2c_1x) + cx(4b_1 + 3c_1x)))$$

Antiderivative was successfully verified.

[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2),x]

[Out] $(x*(6*a*(2*b1 + c1*x) + x*(4*b*(3*b1 + 2*c1*x) + c*x*(4*b1 + 3*c1*x))))/12$

Mathics [A]

time = 1.84, size = 37, normalized size = 0.84

$$x \left(ab1 + \frac{x(ac1 + 2bb1)}{2} + \frac{x^2(2bc1 + b1c)}{3} + \frac{cc1x^3}{4} \right)$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^1,x]')`

[Out] $x(a b1 + x(a c1 + 2 b b1) / 2 + x^2(2 b c1 + b1 c) / 3 + c c1 x^3 / 4)$

Maple [A]

time = 0.03, size = 39, normalized size = 0.89

method	result	size
norman	$\frac{cc1x^4}{4} + \left(\frac{2bc1}{3} + \frac{b1c}{3}\right)x^3 + \left(\frac{ac1}{2} + bb1\right)x^2 + ab1x$	38
default	$ab1x + \frac{(ac1+2bb1)x^2}{2} + \frac{(2bc1+b1c)x^3}{3} + \frac{cc1x^4}{4}$	39
gospers	$\frac{1}{4}cc1x^4 + \frac{2}{3}x^3bc1 + \frac{1}{3}x^3b1c + \frac{1}{2}x^2ac1 + x^2bb1 + ab1x$	40
risch	$\frac{1}{4}cc1x^4 + \frac{2}{3}x^3bc1 + \frac{1}{3}x^3b1c + \frac{1}{2}x^2ac1 + x^2bb1 + ab1x$	40

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c1*x+b1)*(c*x^2+2*b*x+a),x,method=_RETURNVERBOSE)`

[Out] $a*b1*x + 1/2*(a*c1 + 2*b*b1)*x^2 + 1/3*(2*b*c1 + b1*c)*x^3 + 1/4*c*c1*x^4$

Maxima [A]

time = 0.26, size = 38, normalized size = 0.86

$$\frac{1}{4}cc_1x^4 + \frac{1}{3}(b_1c + 2bc_1)x^3 + ab_1x + \frac{1}{2}(2bb_1 + ac_1)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c1*x+b1)*(c*x^2+2*b*x+a),x, algorithm="maxima")`

[Out] $1/4*c*c1*x^4 + 1/3*(b1*c + 2*b*c1)*x^3 + a*b1*x + 1/2*(2*b*b1 + a*c1)*x^2$

Fricas [A]

time = 0.31, size = 38, normalized size = 0.86

$$\frac{1}{4}cc_1x^4 + \frac{1}{3}(b_1c + 2bc_1)x^3 + ab_1x + \frac{1}{2}(2bb_1 + ac_1)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a),x, algorithm="fricas")

[Out] 1/4*c*c1*x^4 + 1/3*(b1*c + 2*b*c1)*x^3 + a*b1*x + 1/2*(2*b*b1 + a*c1)*x^2

Sympy [A]

time = 0.03, size = 39, normalized size = 0.89

$$ab_1x + \frac{cc_1x^4}{4} + x^3 \cdot \left(\frac{2bc_1}{3} + \frac{b_1c}{3} \right) + x^2 \left(\frac{ac_1}{2} + bb_1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a),x)

[Out] a*b1*x + c*c1*x**4/4 + x**3*(2*b*c1/3 + b1*c/3) + x**2*(a*c1/2 + b*b1)

Giac [A]

time = 0.00, size = 47, normalized size = 1.07

$$\frac{1}{4}x^4c_1c + \frac{2}{3}x^3c_1b + \frac{1}{3}x^3b_1c + \frac{1}{2}x^2c_1a + x^2b_1b + xb_1a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a),x)

[Out] 1/4*c*c1*x^4 + 1/3*b1*c*x^3 + 2/3*b*c1*x^3 + b*b1*x^2 + 1/2*a*c1*x^2 + a*b1*x

Mupad [B]

time = 0.05, size = 37, normalized size = 0.84

$$\frac{cc_1x^4}{4} + \left(\frac{2bc_1}{3} + \frac{b_1c}{3} \right) x^3 + \left(\frac{ac_1}{2} + bb_1 \right) x^2 + ab_1x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b1 + c1*x)*(a + 2*b*x + c*x^2),x)

[Out] x^2*((a*c1)/2 + b*b1) + x^3*((2*b*c1)/3 + (b1*c)/3) + a*b1*x + (c*c1*x^4)/4

3.190 $\int (b_1 + c_1 x) (a + 2bx + cx^2)^2 dx$

Optimal. Leaf size=96

$$a^2 b_1 x + \frac{1}{2} a (4bb_1 + ac_1) x^2 + \frac{2}{3} (2b^2 b_1 + ab_1 c + 2abc_1) x^3 + \frac{1}{2} (2bb_1 c + 2b^2 c_1 + acc_1) x^4 + \frac{1}{5} c (b_1 c + 4bc_1) x^5 + \frac{1}{6} c^2 c_1 x^6$$

[Out] $a^2 b_1 x + \frac{1}{2} a (4bb_1 + ac_1) x^2 + \frac{2}{3} (2b^2 b_1 + ab_1 c + 2abc_1) x^3 + \frac{1}{2} (2bb_1 c + 2b^2 c_1 + acc_1) x^4 + \frac{1}{5} c (b_1 c + 4bc_1) x^5 + \frac{1}{6} c^2 c_1 x^6$

Rubi [A]

time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {645}

$$a^2 b_1 x + \frac{1}{2} x^4 (acc_1 + 2b^2 c_1 + 2bb_1 c) + \frac{2}{3} x^3 (2abc_1 + ab_1 c + 2b^2 b_1) + \frac{1}{2} a x^2 (ac_1 + 4bb_1) + \frac{1}{5} c x^5 (4bc_1 + b_1 c) + \frac{1}{6} c^2 c_1 x^6$$

Antiderivative was successfully verified.

[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x]

[Out] $a^2 b_1 x + (a(4bb_1 + ac_1)x^2)/2 + (2(2b^2 b_1 + ab_1 c + 2abc_1)x^3)/3 + ((2bb_1 c + 2b^2 c_1 + acc_1)x^4)/2 + (c(b_1 c + 4bc_1)x^5)/5 + (c^2 c_1 x^6)/6$

Rule 645

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || EqQ[a, 0])

Rubi steps

$$\begin{aligned} \int (b_1 + c_1 x) (a + 2bx + cx^2)^2 dx &= \int (a^2 b_1 + a(4bb_1 + ac_1)x + 2(2b^2 b_1 + ab_1 c + 2abc_1)x^2 + 2(2bb_1 c + 2b^2 c_1 + acc_1)x^3 + (2bb_1 c + 2b^2 c_1 + acc_1)x^4) dx \\ &= a^2 b_1 x + \frac{1}{2} a (4bb_1 + ac_1) x^2 + \frac{2}{3} (2b^2 b_1 + ab_1 c + 2abc_1) x^3 + \frac{1}{2} (2bb_1 c + 2b^2 c_1 + acc_1) x^4 \end{aligned}$$

Mathematica [A]

time = 0.02, size = 91, normalized size = 0.95

$$\frac{1}{30} x (15a^2(2b_1 + c_1 x) + 5ax(4b(3b_1 + 2c_1 x) + cx(4b_1 + 3c_1 x)) + x^2(10b^2(4b_1 + 3c_1 x) + 6bcx(5b_1 + 4c_1 x) + c^2 x^2(6b_1 + 5c_1 x)))$$

Antiderivative was successfully verified.

[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x]

[Out] (x*(15*a^2*(2*b1 + c1*x) + 5*a*x*(4*b*(3*b1 + 2*c1*x) + c*x*(4*b1 + 3*c1*x)) + x^2*(10*b^2*(4*b1 + 3*c1*x) + 6*b*c*x*(5*b1 + 4*c1*x) + c^2*x^2*(6*b1 + 5*c1*x))))/30

Mathics [A]

time = 2.38, size = 87, normalized size = 0.91

$$\frac{x(30a^2b_1 + 15ax(ac_1 + 4bb_1) + 20x^2(2abc_1 + ab_1c + 2b^2b_1) + 15x^3(acc_1 + 2b^2c_1 + 2bb_1c) + 6cx^4(4bc_1 + b_1c) + 5c^2c_1x^5)}{30}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x]')

[Out] x (30 a ^ 2 b1 + 15 a x (a c1 + 4 b b1) + 20 x ^ 2 (2 a b c1 + a b1 c + 2 b ^ 2 b1) + 15 x ^ 3 (a c c1 + 2 b ^ 2 c1 + 2 b b1 c) + 6 c x ^ 4 (4 b c1 + b1 c) + 5 c ^ 2 c1 x ^ 5) / 30

Maple [A]

time = 0.10, size = 95, normalized size = 0.99

method	result
norman	$\frac{c^2 c_1 x^6}{6} + \left(\frac{4}{5} c_1 b c + \frac{1}{5} b_1 c^2\right) x^5 + \left(\frac{1}{2} a c c_1 + b^2 c_1 + b b_1 c\right) x^4 + \left(\frac{4}{3} a b c_1 + \frac{2}{3} a b_1 c + \frac{4}{3} b^2 b_1\right) x^3 + \left(\frac{1}{2} c_1 a^2 + 4 b_1 a b\right) x^2 + a^2 b_1 x$
default	$\frac{c^2 c_1 x^6}{6} + \frac{(4 c_1 b c + b_1 c^2) x^5}{5} + \frac{(4 b b_1 c + c_1 (2 a c + 4 b^2)) x^4}{4} + \frac{(b_1 (2 a c + 4 b^2) + 4 a b c_1) x^3}{3} + \frac{(c_1 a^2 + 4 b_1 a b) x^2}{2} + a^2 b_1 x$
gospers	$\frac{1}{6} c^2 c_1 x^6 + \frac{4}{5} x^5 c_1 b c + \frac{1}{5} x^5 b_1 c^2 + \frac{1}{2} x^4 a c c_1 + x^4 b^2 c_1 + x^4 b b_1 c + \frac{4}{3} x^3 a b c_1 + \frac{2}{3} x^3 a b_1 c + \frac{4}{3} x^3 b^2 b_1$
risch	$\frac{1}{6} c^2 c_1 x^6 + \frac{4}{5} x^5 c_1 b c + \frac{1}{5} x^5 b_1 c^2 + \frac{1}{2} x^4 a c c_1 + x^4 b^2 c_1 + x^4 b b_1 c + \frac{4}{3} x^3 a b c_1 + \frac{2}{3} x^3 a b_1 c + \frac{4}{3} x^3 b^2 b_1$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c1*x+b1)*(c*x^2+2*b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/6*c^2*c1*x^6+1/5*(4*b*c*c1+b1*c^2)*x^5+1/4*(4*b*b1*c+c1*(2*a*c+4*b^2))*x^4+1/3*(b1*(2*a*c+4*b^2)+4*a*b*c1)*x^3+1/2*(a^2*c1+4*a*b*b1)*x^2+a^2*b1*x

Maxima [A]

time = 0.26, size = 91, normalized size = 0.95

$$\frac{1}{6} c^2 c_1 x^6 + \frac{1}{5} (b_1 c^2 + 4 b c c_1) x^5 + \frac{1}{2} (2 b b_1 c + (2 b^2 + a c) c_1) x^4 + a^2 b_1 x + \frac{2}{3} (2 b^2 b_1 + a b_1 c + 2 a b c_1) x^3 + \frac{1}{2} (4 a b b_1 + a^2 c_1) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^2,x, algorithm="maxima")

[Out] 1/6*c^2*c1*x^6 + 1/5*(b1*c^2 + 4*b*c*c1)*x^5 + 1/2*(2*b*b1*c + (2*b^2 + a*c)*c1)*x^4 + a^2*b1*x + 2/3*(2*b^2*b1 + a*b1*c + 2*a*b*c1)*x^3 + 1/2*(4*a*b*b1 + a^2*c1)*x^2

Fricas [A]

time = 0.30, size = 91, normalized size = 0.95

$$\frac{1}{6}c^2c_1x^6 + \frac{1}{5}(b_1c^2 + 4bcc_1)x^5 + \frac{1}{2}(2bb_1c + (2b^2 + ac)c_1)x^4 + a^2b_1x + \frac{2}{3}(2b^2b_1 + ab_1c + 2abc_1)x^3 + \frac{1}{2}(4abb_1 + a^2c_1)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^2,x, algorithm="fricas")

[Out] 1/6*c^2*c1*x^6 + 1/5*(b1*c^2 + 4*b*c*c1)*x^5 + 1/2*(2*b*b1*c + (2*b^2 + a*c)*c1)*x^4 + a^2*b1*x + 2/3*(2*b^2*b1 + a*b1*c + 2*a*b*c1)*x^3 + 1/2*(4*a*b*b1 + a^2*c1)*x^2

Sympy [A]

time = 0.04, size = 100, normalized size = 1.04

$$a^2b_1x + \frac{c^2c_1x^6}{6} + x^5 \cdot \left(\frac{4bcc_1}{5} + \frac{b_1c^2}{5} \right) + x^4 \left(\frac{acc_1}{2} + b^2c_1 + bb_1c \right) + x^3 \cdot \left(\frac{4abc_1}{3} + \frac{2ab_1c}{3} + \frac{4b^2b_1}{3} \right) + x^2 \left(\frac{a^2c_1}{2} + 2abb_1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a)**2,x)

[Out] a**2*b1*x + c**2*c1*x**6/6 + x**5*(4*b*c*c1/5 + b1*c**2/5) + x**4*(a*c*c1/2 + b**2*c1 + b*b1*c) + x**3*(4*a*b*c1/3 + 2*a*b1*c/3 + 4*b**2*b1/3) + x**2*(a**2*c1/2 + 2*a*b*b1)

Giac [A]

time = 0.00, size = 114, normalized size = 1.19

$$\frac{1}{6}x^6c_1c^2 + \frac{4}{5}x^5c_1cb + \frac{1}{5}x^5b_1c^2 + \frac{1}{2}x^4c_1ca + x^4c_1b^2 + x^4b_1cb + \frac{4}{3}x^3c_1ba + \frac{2}{3}x^3b_1ca + \frac{4}{3}x^3b_1b^2 + \frac{1}{2}x^2c_1a^2 + 2x^2b_1ba + xb_1a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^2,x)

[Out] 1/6*c^2*c1*x^6 + 1/5*b1*c^2*x^5 + 4/5*b*c*c1*x^5 + b*b1*c*x^4 + b^2*c1*x^4 + 1/2*a*c*c1*x^4 + 4/3*b^2*b1*x^3 + 2/3*a*b1*c*x^3 + 4/3*a*b*c1*x^3 + 2*a*b*b1*x^2 + 1/2*a^2*c1*x^2 + a^2*b1*x

Mupad [B]

time = 0.19, size = 88, normalized size = 0.92

$$x^3 \left(\frac{4b_1b^2}{3} + \frac{4ac_1b}{3} + \frac{2ab_1c}{3} \right) + x^4 \left(c_1b^2 + b_1cb + \frac{ac_1}{2} \right) + x^2 \left(\frac{c_1a^2}{2} + 2bb_1a \right) + x^5 \left(\frac{b_1c^2}{5} + \frac{4bc_1c}{5} \right) + \frac{c^2c_1x^6}{6} + a^2b_1x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x)

[Out] x^3*((4*b^2*b1)/3 + (4*a*b*c1)/3 + (2*a*b1*c)/3) + x^4*(b^2*c1 + (a*c*c1)/2 + b*b1*c) + x^2*((a^2*c1)/2 + 2*a*b*b1) + x^5*((b1*c^2)/5 + (4*b*c*c1)/5) + (c^2*c1*x^6)/6 + a^2*b1*x

3.191 $\int (b1 + c1x) (a + 2bx + cx^2)^3 dx$

Optimal. Leaf size=167

$$a^3b1x + \frac{1}{2}a^2(6bb1+ac1)x^2 + a(4b^2b1 + ab1c + 2abc1)x^3 + \frac{1}{4}(8b^3b1 + 12abb1c + 12ab^2c1 + 3a^2cc1)x^4 + \frac{1}{5}(12a^3b1c + 12a^2b1c^2 + 12ab1c^3 + 8b^2b1c^2 + 8ab1c^3 + 4a^2c1c^2)x^5 + \frac{1}{6}c^2x^6 + \frac{1}{7}c^3x^7$$

[Out] $a^3b1x + \frac{1}{2}a^2(6bb1+ac1)x^2 + a(4b^2b1 + ab1c + 2abc1)x^3 + \frac{1}{4}(8b^3b1 + 12abb1c + 12ab^2c1 + 3a^2cc1)x^4 + \frac{1}{5}(12a^3b1c + 12a^2b1c^2 + 12ab1c^3 + 8b^2b1c^2 + 8ab1c^3 + 4a^2c1c^2)x^5 + \frac{1}{6}c^2x^6 + \frac{1}{7}c^3x^7$

Rubi [A]

time = 0.12, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {645}

$$a^3b1x + \frac{1}{4}a^4(3a^2cc1 + 12ab^2c1 + 12abb1c + 8b^3b1) + \frac{1}{2}a^2x^2(ac1 + 6bb1) + \frac{1}{2}cx^6(ac1 + 4b^2c1 + 2bb1c) + ax^3(2abc1 + ab1c + 4b^2b1) + \frac{1}{5}x^5(12abcc1 + 3ab1c^2 + 8b^2c1 + 12b^2b1c) + \frac{1}{7}c^2x^7(6bc1 + b1c) + \frac{1}{8}c^3c1x^8$$

Antiderivative was successfully verified.

[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x]

[Out] $a^3b1x + (a^2(6*b*b1 + a*c1)*x^2)/2 + a*(4*b^2*b1 + a*b1*c + 2*a*b*c1)*x^3 + ((8*b^3*b1 + 12*a*b*b1*c + 12*a*b^2*c1 + 3*a^2*c*c1)*x^4)/4 + ((12*b^2*b1*c + 3*a*b1*c^2 + 8*b^3*c1 + 12*a*b*c*c1)*x^5)/5 + (c*(2*b*b1*c + 4*b^2*c1 + a*c*c1)*x^6)/2 + (c^2*(b1*c + 6*b*c1)*x^7)/7 + (c^3*c1*x^8)/8$

Rule 645

Int[((d._) + (e._)*(x_))*((a._) + (b._)*(x_) + (c._)*(x_)^2)^(p._), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || EqQ[a, 0])

Rubi steps

$$\int (b1 + c1x) (a + 2bx + cx^2)^3 dx = \int (a^3b1 + a^2(6bb1 + ac1)x + 3a(4b^2b1 + ab1c + 2abc1)x^2 + (8b^3b1 + 12abb1c + 12ab^2c1 + 3a^2cc1)x^3 + \frac{1}{2}a^2(6bb1 + ac1)x^2 + a(4b^2b1 + ab1c + 2abc1)x^3 + \frac{1}{4}(8b^3b1 + 12abb1c + 12ab^2c1 + 3a^2cc1)x^4 + \frac{1}{5}(12a^3b1c + 12a^2b1c^2 + 12ab1c^3 + 8b^2b1c^2 + 8ab1c^3 + 4a^2c1c^2)x^5 + \frac{1}{6}c^2x^6 + \frac{1}{7}c^3x^7) dx$$

Mathematica [A]

time = 0.02, size = 167, normalized size = 1.00

$$a^3b1x + \frac{1}{2}a^2(6bb1 + ac1)x^2 + a(4b^2b1 + ab1c + 2abc1)x^3 + \frac{1}{4}(8b^3b1 + 12abb1c + 12ab^2c1 + 3a^2cc1)x^4 + \frac{1}{5}(12a^3b1c + 12a^2b1c^2 + 12ab1c^3 + 8b^2b1c^2 + 8ab1c^3 + 4a^2c1c^2)x^5 + \frac{1}{6}c^2(6bc1 + b1c)x^6 + \frac{1}{7}c^3c1x^7$$

Antiderivative was successfully verified.

[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x]

[Out] $a^3 b_1 x + (a^2 (6 b_1 b_1 + a c_1) x^2) / 2 + a (4 b_1^2 b_1 + a b_1 c_1 + 2 a b_1 c_1) x^3 + ((8 b_1^3 b_1 + 12 a b_1 b_1 c_1 + 12 a b_1^2 c_1 + 3 a^2 c_1 c_1) x^4) / 4 + ((12 b_1^2 b_1 c_1 + 3 a b_1 c_1^2 + 8 b_1^3 c_1 + 12 a b_1 c_1 c_1) x^5) / 5 + (c_1 (2 b_1 b_1 c_1 + 4 b_1^2 c_1 + a c_1 c_1) x^6) / 2 + (c_1^2 (b_1 c_1 + 6 b_1 c_1) x^7) / 7 + (c_1^3 c_1 x^8) / 8$

Mathics [A]

time = 2.80, size = 154, normalized size = 0.92

$$x \left(a^3 b_1 + \frac{a^2 x (a c_1 + 6 b_1 b_1)}{2} + a x^2 (2 a b_1 c_1 + a b_1 c_1 + 4 b_1^2 b_1) + x^3 (3 a^2 c_1 c_1 + 12 a b_1^2 c_1 + 12 a b_1 b_1 c_1 + 8 b_1^3 b_1) / 4 + x^4 (12 a b_1 c_1 c_1 + 3 a b_1 c_1^2 + 8 b_1^3 c_1 + 12 b_1^2 b_1 c_1) / 5 + c_1 x^5 (a c_1 c_1 + 4 b_1^2 c_1 + 2 b_1 b_1 c_1) / 2 + c_1^2 x^6 (6 b_1 c_1 + b_1 c_1) / 7 + c_1^3 c_1 x^7 / 8 \right)$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x]')

[Out] $x (a^3 b_1 + a^2 x (a c_1 + 6 b_1 b_1) / 2 + a x^2 (2 a b_1 c_1 + a b_1 c_1 + 4 b_1^2 b_1) + x^3 (3 a^2 c_1 c_1 + 12 a b_1^2 c_1 + 12 a b_1 b_1 c_1 + 8 b_1^3 b_1) / 4 + x^4 (12 a b_1 c_1 c_1 + 3 a b_1 c_1^2 + 8 b_1^3 c_1 + 12 b_1^2 b_1 c_1) / 5 + c_1 x^5 (a c_1 c_1 + 4 b_1^2 c_1 + 2 b_1 b_1 c_1) / 2 + c_1^2 x^6 (6 b_1 c_1 + b_1 c_1) / 7 + c_1^3 c_1 x^7 / 8)$

Maple [A]

time = 0.08, size = 237, normalized size = 1.42

method	result
norman	$\frac{c^3 c_1 x^8}{8} + (\frac{6}{7} c_1 b c^2 + \frac{1}{7} b_1 c^3) x^7 + (\frac{1}{2} a c^2 c_1 + 2 b^2 c c_1 + b_1 b c^2) x^6 + (\frac{12}{5} a b c c_1 + \frac{3}{5} a b_1 c^2 + \frac{8}{5} b^3 c^2)$
gospers	$\frac{1}{8} c^3 c_1 x^8 + \frac{6}{7} x^7 c_1 b c^2 + \frac{1}{7} x^7 b_1 c^3 + \frac{1}{2} x^6 a c^2 c_1 + 2 x^6 b^2 c c_1 + x^6 b_1 b c^2 + \frac{12}{5} x^5 a b c c_1 + \frac{3}{5} x^5 a b_1 c^2$
risch	$\frac{1}{8} c^3 c_1 x^8 + \frac{6}{7} x^7 c_1 b c^2 + \frac{1}{7} x^7 b_1 c^3 + \frac{1}{2} x^6 a c^2 c_1 + 2 x^6 b^2 c c_1 + x^6 b_1 b c^2 + \frac{12}{5} x^5 a b c c_1 + \frac{3}{5} x^5 a b_1 c^2$
default	$\frac{c^3 c_1 x^8}{8} + \frac{(6 c_1 b c^2 + b_1 c^3) x^7}{7} + \frac{(6 b_1 b c^2 + c_1 (c^2 a + 8 b^2 c + c(2 a c + 4 b^2))) x^6}{6} + \frac{(b_1 (c^2 a + 8 b^2 c + c(2 a c + 4 b^2)) + c_1 (8 a b c + 2 b(2 a c + 4 b^2))) x^5}{5}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c1*x+b1)*(c*x^2+2*b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] $1/8 * c^3 * c_1 * x^8 + 1/7 * (6 * b * c^2 * c_1 + b_1 * c^3) * x^7 + 1/6 * (6 * b_1 * b * c^2 + c_1 * (c^2 * a + 8 * b^2 * c + c * (2 * a * c + 4 * b^2))) * x^6 + 1/5 * (b_1 * (c^2 * a + 8 * b^2 * c + c * (2 * a * c + 4 * b^2)) + c_1 * (8 * a * b * c + 2 * b * (2 * a * c + 4 * b^2))) * x^5 + 1/4 * (b_1 * (8 * a * b * c + 2 * b * (2 * a * c + 4 * b^2)) + c_1 * (a * (2 * a * c + 4 * b^2) + 8 * a * b^2 + a^2 * c)) * x^4 + 1/3 * (b_1 * (a * (2 * a * c + 4 * b^2) + 8 * a * b^2 + a^2 * c) + 6 * c_1 * a^2 * b) * x^3 + 1/2 * (a^3 * c_1 + 6 * a^2 * b * b_1) * x^2 + a^3 * b_1 * x$

Maxima [A]

time = 0.28, size = 171, normalized size = 1.02

$$\frac{1}{8} c^3 c_1 x^8 + \frac{1}{7} (b_1 c^3 + 6 b c^2 c_1) x^7 + \frac{1}{2} (2 b b_1 c^2 + (4 b^2 c + a c^2) c_1) x^6 + \frac{1}{5} (12 b^2 b_1 c + 3 a b_1 c^2 + 4 (2 b^3 + 3 a b c) c_1) x^5 + a^3 b_1 x + \frac{1}{4} (8 b^3 b_1 + 12 a b b_1 c + 3 (4 a b^2 + a^2 c) c_1) x^4 + (4 a b^2 b_1 + a^2 b_1 c + 2 a^2 b c_1) x^3 + \frac{1}{2} (6 a^2 b b_1 + a^3 c_1) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^3,x, algorithm="maxima")

[Out] $\frac{1}{8}c^3c_1x^8 + \frac{1}{7}(b_1c^3 + 6bc^2c_1)x^7 + \frac{1}{2}(2b^2b_1c^2 + (4b^2c + ac^2)c_1)x^6 + \frac{1}{5}(12b^2b_1c + 3a^2b_1c^2 + 4(2b^3 + 3a^2b^2c)c_1)x^5 + a^3b_1c^2x + \frac{1}{4}(8b^3b_1 + 12a^2b^2b_1c + 3(4a^2b^2 + a^2c)c_1)x^4 + (4a^2b^2b_1 + a^2b_1c + 2a^2b^2c_1)x^3 + \frac{1}{2}(6a^2b^2b_1 + a^3c_1)x^2$

Fricas [A]

time = 0.32, size = 171, normalized size = 1.02

$$\frac{1}{8}c^3c_1x^8 + \frac{1}{7}(b_1c^3 + 6bc^2c_1)x^7 + \frac{1}{2}(2bb_1c^2 + (4b^2c + ac^2)c_1)x^6 + \frac{1}{5}(12b^2b_1c + 3ab_1c^2 + 4(2b^3 + 3abc)c_1)x^5 + a^3b_1c^2x + \frac{1}{4}(8b^3b_1 + 12abb_1c + 3(4ab^2 + a^2c)c_1)x^4 + (4ab^2b_1 + a^2b_1c + 2a^2bc_1)x^3 + \frac{1}{2}(6a^2bb_1 + a^3c_1)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{8}c^3c_1x^8 + \frac{1}{7}(b_1c^3 + 6bc^2c_1)x^7 + \frac{1}{2}(2b^2b_1c^2 + (4b^2c + ac^2)c_1)x^6 + \frac{1}{5}(12b^2b_1c + 3a^2b_1c^2 + 4(2b^3 + 3a^2b^2c)c_1)x^5 + a^3b_1c^2x + \frac{1}{4}(8b^3b_1 + 12a^2b^2b_1c + 3(4a^2b^2 + a^2c)c_1)x^4 + (4a^2b^2b_1 + a^2b_1c + 2a^2b^2c_1)x^3 + \frac{1}{2}(6a^2b^2b_1 + a^3c_1)x^2$

Sympy [A]

time = 0.05, size = 189, normalized size = 1.13

$$a^3b_1c^2x + \frac{c^3c_1x^8}{8} + x^7 \cdot \left(\frac{6bc^2c_1}{7} + \frac{b_1c^3}{7} \right) + x^6 \left(\frac{ac^2c_1}{2} + 2b^2c_1 + bb_1c^2 \right) + x^5 \cdot \left(\frac{12abcc_1}{5} + \frac{3ab_1c^2}{5} + \frac{8b^3c_1}{5} + \frac{12b^2b_1c}{5} \right) + x^4 \cdot \left(\frac{3a^2cc_1}{4} + 3ab^2c_1 + 3abb_1c + 2b^3b_1 \right) + x^3 \cdot (2a^2bc_1 + a^2b_1c + 4ab^2b_1) + x^2 \left(\frac{a^3c_1}{2} + 3a^2bb_1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a)**3,x)

[Out] $a^{**3}b_1x + c^{**3}c_1x^{**8}/8 + x^{**7}(6b^2b_1c^2/7 + b_1c^{**3}/7) + x^{**6}(a^{**2}c_1/2 + 2b^{**2}c^2c_1 + b^2b_1c^2) + x^{**5}(12a^2b^2b_1c/5 + 3a^2b_1c^2/5 + 8b^{**3}c_1/5 + 12b^{**2}b_1c/5) + x^{**4}(3a^{**2}c^2c_1/4 + 3a^2b^2c_1 + 3a^2b^2b_1c + 2b^{**3}b_1) + x^{**3}(2a^{**2}b^2c_1 + a^{**2}b_1c + 4a^2b^2b_1) + x^{**2}(a^{**3}c_1/2 + 3a^{**2}b^2b_1)$

Giac [A]

time = 0.00, size = 208, normalized size = 1.25

$$\frac{1}{8}c^3c_1x^8 + \frac{6}{7}x^7c_1c^2b + \frac{1}{7}x^7b_1c^3 + \frac{1}{2}x^6c_1c^2a + 2x^6c_1cb^2 + x^6b_1c^2b + \frac{12}{5}x^5c_1cba + \frac{8}{5}x^5c_1b^3 + \frac{3}{5}x^5b_1c^2a + \frac{12}{5}x^4b_1cb^2 + \frac{3}{4}x^4c_1ca^2 + 3x^4c_1b^2a + 3x^4b_1cba + 2x^4b_1b^3 + 2x^3c_1ba^2 + x^3b_1ca^2 + 4x^3b_1b^2a + \frac{1}{2}x^2c_1a^3 + 3x^2b_1ba^2 + x^2b_1a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^3,x)

[Out] $\frac{1}{8}c^3c_1x^8 + \frac{1}{7}b_1c^3x^7 + \frac{6}{7}b^2b_1c^2x^7 + b^2b_1c^2x^6 + 2b^2c^2c_1x^6 + \frac{1}{2}a^2c^2c_1x^6 + \frac{12}{5}b^2b_1c^2x^5 + \frac{3}{5}a^2b_1c^2x^5 + \frac{8}{5}b^3c_1x^5 + \frac{12}{5}a^2b^2b_1c^2x^5 + 2b^3b_1x^4 + 3a^2b^2b_1c^2x^4 + 3a^2b^2c_1x^4$

$$4 + 3/4*a^2*c*c1*x^4 + 4*a*b^2*b1*x^3 + a^2*b1*c*x^3 + 2*a^2*b*c1*x^3 + 3*a^2*b*b1*x^2 + 1/2*a^3*c1*x^2 + a^3*b1*x$$

Mupad [B]

time = 0.22, size = 164, normalized size = 0.98

$$x^7 \left(\frac{b_1 c^3}{7} + \frac{6 b c_1 c^2}{7} \right) + x^6 (2 c_1 a^2 b + b_1 c a^2 + 4 b_1 a b^2) + x^5 \left(2 c_1 b^2 c + b_1 b c^2 + \frac{a c_1 c^2}{2} \right) + x^4 \left(\frac{3 c c_1 a^2}{4} + 3 c_1 a b^2 + 3 b_1 c a b + 2 b_1 b^3 \right) + x^3 \left(\frac{8 c_1 b^3}{5} + \frac{12 b_1 b^2 c}{5} + \frac{12 a c_1 b c}{5} + \frac{3 a b_1 c^2}{5} \right) + x^2 \left(\frac{c_1 a^3}{2} + 3 b b_1 a^2 \right) + \frac{c^3 c_1 x^8}{8} + a^3 b_1 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x)

[Out] x^7*((b1*c^3)/7 + (6*b*c^2*c1)/7) + x^3*(4*a*b^2*b1 + 2*a^2*b*c1 + a^2*b1*c) + x^6*((a*c^2*c1)/2 + b*b1*c^2 + 2*b^2*c*c1) + x^4*(2*b^3*b1 + 3*a*b^2*c1 + (3*a^2*c*c1)/4 + 3*a*b*b1*c) + x^5*((8*b^3*c1)/5 + (3*a*b1*c^2)/5 + (12*b^2*b1*c)/5 + (12*a*b*c*c1)/5) + x^2*((a^3*c1)/2 + 3*a^2*b*b1) + (c^3*c1*x^8)/8 + a^3*b1*x

3.192 $\int (b_1 + c_1 x) (a + 2bx + cx^2)^4 dx$

Optimal. Leaf size=263

$$a^4 b_1 x + \frac{1}{2} a^3 (8bb_1 + ac_1) x^2 + \frac{4}{3} a^2 (6b^2 b_1 + ab_1 c + 2abc_1) x^3 + a (8b^3 b_1 + 6abb_1 c + 6ab^2 c_1 + a^2 cc_1) x^4 + \frac{2}{5} (8b^4 b_1 + 4a^2 b^2 c_1) x^5 + \frac{2}{7} a^2 (6b^3 b_1 + 3ab^2 c_1) x^6 + \frac{2}{9} a^2 (6b^2 b_1 + ab_1 c) x^7 + \frac{2}{11} a^2 (6b b_1) x^8 + \frac{2}{13} a^2 (b_1) x^9 + \frac{2}{15} a^2 (0) x^{10}$$

[Out] $a^4 b_1 x + \frac{1}{2} a^3 (8bb_1 + ac_1) x^2 + \frac{4}{3} a^2 (6b^2 b_1 + ab_1 c + 2abc_1) x^3 + a (8b^3 b_1 + 6abb_1 c + 6ab^2 c_1 + a^2 cc_1) x^4 + \frac{2}{5} (8b^4 b_1 + 4a^2 b^2 c_1) x^5 + \frac{2}{7} a^2 (6b^3 b_1 + 3ab^2 c_1) x^6 + \frac{2}{9} a^2 (6b^2 b_1 + ab_1 c) x^7 + \frac{2}{11} a^2 (6b b_1) x^8 + \frac{2}{13} a^2 (b_1) x^9 + \frac{2}{15} a^2 (0) x^{10}$

Rubi [A]

time = 0.22, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {645}

$a^4 b_1 x + \frac{1}{2} a^3 (8bb_1 + ac_1) x^2 + \frac{4}{3} a^2 (6b^2 b_1 + ab_1 c + 2abc_1) x^3 + a (8b^3 b_1 + 6abb_1 c + 6ab^2 c_1 + a^2 cc_1) x^4 + \frac{2}{5} (8b^4 b_1 + 4a^2 b^2 c_1) x^5 + \frac{2}{7} a^2 (6b^3 b_1 + 3ab^2 c_1) x^6 + \frac{2}{9} a^2 (6b^2 b_1 + ab_1 c) x^7 + \frac{2}{11} a^2 (6b b_1) x^8 + \frac{2}{13} a^2 (b_1) x^9 + \frac{2}{15} a^2 (0) x^{10}$

Antiderivative was successfully verified.

[In] $\text{Int}[(b_1 + c_1 x) (a + 2bx + cx^2)^4, x]$

[Out] $a^4 b_1 x + (a^3 (8b b_1 + a c_1) x^2) / 2 + (4 a^2 (6 b^2 b_1 + a b_1 c + 2 a b c_1) x^3) / 3 + a (8 b^3 b_1 + 6 a b b_1 c + 6 a b^2 c_1 + a^2 c c_1) x^4 + (2 (8 b^4 b_1 + 24 a b^2 b_1 c + 3 a^2 b_1 c^2 + 16 a^2 b^3 c_1 + 12 a^2 b^2 c_1 c) x^5) / 5 + ((16 b^3 b_1 c + 12 a b^2 b_1 c^2 + 8 b^4 c_1 + 24 a b^2 c_1 c + 3 a^2 c^2 c_1) x^6) / 3 + (4 c (6 b^2 b_1 c + a b_1 c^2 + 8 b^3 c_1 + 6 a b c c_1) x^7) / 7 + (c^2 (2 b b_1 c + 6 b^2 c_1 + a c c_1) x^8) / 2 + (c^3 (b_1 c + 8 b c_1) x^9) / 9 + (c^4 c_1 x^{10}) / 10$

Rule 645

$\text{Int}[(d + e x) (a + b x + c x^2)^p, x] \text{ :> } \text{Int}[\text{ExpandIntegrand}[(d + e x) (a + b x + c x^2)^p, x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, e\}, x \text{ \&\& } \text{NeQ}[2 c d - b e, 0] \text{ \&\& } \text{IntegerQ}[p] \text{ \&\& } (\text{GtQ}[p, 0] \text{ || } \text{EqQ}[a, 0])$

Rubi steps

$$\int (b_1 + c_1 x) (a + 2bx + cx^2)^4 dx = \int (a^4 b_1 + a^3 (8bb_1 + ac_1) x + 4a^2 (6b^2 b_1 + ab_1 c + 2abc_1) x^2 + 4a (8b^3 b_1 + 6abb_1 c + 6ab^2 c_1 + a^2 cc_1) x^3 + \frac{2}{5} (8b^4 b_1 + 4a^2 b^2 c_1) x^4 + \frac{2}{7} a^2 (6b^3 b_1 + 3ab^2 c_1) x^5 + \frac{2}{9} a^2 (6b^2 b_1 + ab_1 c) x^6 + \frac{2}{11} a^2 (6b b_1) x^7 + \frac{2}{13} a^2 (b_1) x^8 + \frac{2}{15} a^2 (0) x^9) dx$$

$$= a^4 b_1 x + \frac{1}{2} a^3 (8bb_1 + ac_1) x^2 + \frac{4}{3} a^2 (6b^2 b_1 + ab_1 c + 2abc_1) x^3 + a (8b^3 b_1 + 6abb_1 c + 6ab^2 c_1 + a^2 cc_1) x^4 + \frac{2}{5} (8b^4 b_1 + 4a^2 b^2 c_1) x^5 + \frac{2}{7} a^2 (6b^3 b_1 + 3ab^2 c_1) x^6 + \frac{2}{9} a^2 (6b^2 b_1 + ab_1 c) x^7 + \frac{2}{11} a^2 (6b b_1) x^8 + \frac{2}{13} a^2 (b_1) x^9 + \frac{2}{15} a^2 (0) x^{10}$$

Mathematica [A]

time = 0.04, size = 263, normalized size = 1.00

$$a^4 b^2 x^2 + \frac{1}{2} a^2 (88 b^2 + a c^2) x^2 + \frac{4}{3} a^2 (6 b^2 b^2 + a b^2 c + 2 a b^2 c^2) x^2 + a (8 b^2 b^2 + 6 a b^2 c + 6 a^2 c^2 + a^2 c^2) x^2 + \frac{2}{5} (8 b^2 b^2 + 24 a b^2 b^2 c + 3 a^2 b^2 c^2 + 16 a b^2 c^2 + 12 a^2 c^2) x^2 + \frac{1}{3} (16 b^2 b^2 c + 12 a b^2 b^2 c^2 + 8 b^2 c^2 + 24 a b^2 c^2 + 3 a^2 c^2) x^2 + \frac{4}{9} (6 b^2 b^2 c + a b^2 c^2 + 8 b^2 c^2 + 6 a b^2 c^2) x^2 + \frac{1}{2} (20 b^2 c + 6 b^2 c^2 + a c^2) x^2 + \frac{1}{5} (b^2 c + 8 b^2 c^2) x^2 + \frac{1}{10} c^2 x^2$$

Antiderivative was successfully verified.

[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x]

[Out] $a^4 b^2 x + (a^3 (8 b^2 b^2 + a^2 c^2) x^2) / 2 + (4 a^2 (6 b^2 b^2 + a b^2 c + 2 a^2 b^2 c^2) x^3) / 3 + a (8 b^2 b^2 + 6 a b^2 b^2 c + 6 a^2 b^2 c^2 + a^2 c^2) x^4 + (2 (8 b^2 b^2 + 24 a b^2 b^2 c + 3 a^2 b^2 c^2 + 16 a b^2 c^2 + 12 a^2 b^2 c^2) x^5) / 5 + ((16 b^2 b^2 c + 12 a b^2 b^2 c^2 + 8 b^2 c^2 + 24 a b^2 c^2 + 3 a^2 c^2) x^6) / 3 + (4 c (6 b^2 b^2 c + a b^2 c^2 + 8 b^2 c^2 + 6 a b^2 c^2) x^7) / 7 + (c^2 (2 b^2 b^2 c + 6 b^2 c^2 + a^2 c^2) x^8) / 2 + (c^3 (b^2 c + 8 b^2 c^2) x^9) / 9 + (c^4 b^2 x^{10}) / 10$

Mathics [A]

time = 7.52, size = 247, normalized size = 0.94

$$\frac{(630 b^2 c^4 + 315 a^2 c^3 (a c^2 + 8 b^2 b^2) + a^2 x^2 (1680 a b^2 c^2 + 840 a b^2 c + 5040 b^2 b^2) + 630 a x^3 (a^2 c^2 c^2 + 6 a b^2 c^2 + 6 a b^2 b^2 c + 8 b^2 b^2 c^2) + x^4 (3024 a^2 b^2 c^2 c^2 + 756 a^2 b^2 c^2 + 4032 a b^2 b^2 c^2 + 6048 a b^2 b^2 c + 2016 b^2 b^2 c^2) + x^5 (630 a^2 c^2 c^2 c^2 + 5040 a b^2 c^2 c^2 + 2520 a b^2 b^2 c^2 + 1680 b^2 b^2 c^2 + 3360 b^2 b^2 c^2) + 360 c x^6 (6 a b^2 c^2 c^2 + a b^2 c^2 + 8 b^2 b^2 c^2 + 6 b^2 b^2 c^2) + c^2 x^7 (315 a c^2 c^2 + 1890 b^2 c^2 + 630 b^2 b^2 c^2) + c^3 x^8 (560 b^2 c^2 + 70 b^2 c^2) + 63 c^4 c^2 x^9) / 630$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x]')

[Out] $x (630 a^4 b^2 + 315 a^3 x (a c^2 + 8 b^2 b^2) + a^2 x^2 (1680 a b^2 c^2 + 840 a b^2 c + 5040 b^2 b^2) + 630 a x^3 (a^2 c^2 c^2 + 6 a b^2 c^2 + 6 a b^2 b^2 c + 8 b^2 b^2 c^2) + x^4 (3024 a^2 b^2 c^2 c^2 + 756 a^2 b^2 c^2 + 4032 a b^2 b^2 c^2 + 6048 a b^2 b^2 c + 2016 b^2 b^2 c^2) + x^5 (630 a^2 c^2 c^2 c^2 + 5040 a b^2 c^2 c^2 + 2520 a b^2 b^2 c^2 + 1680 b^2 b^2 c^2 + 3360 b^2 b^2 c^2) + 360 c x^6 (6 a b^2 c^2 c^2 + a b^2 c^2 + 8 b^2 b^2 c^2 + 6 b^2 b^2 c^2) + c^2 x^7 (315 a c^2 c^2 + 1890 b^2 c^2 + 630 b^2 b^2 c^2) + c^3 x^8 (560 b^2 c^2 + 70 b^2 c^2) + 63 c^4 c^2 x^9) / 630$

Maple [A]

time = 0.08, size = 363, normalized size = 1.38

method	result
norman	$\frac{c^4 c^2 x^{10}}{10} + \left(\frac{8}{9} c^2 b^2 c^3 + \frac{1}{9} b^2 c^4\right) x^9 + \left(\frac{1}{2} a c^3 c^2 + 3 b^2 c^2 c^2 + b^2 b^2 c^3\right) x^8 + \left(\frac{24}{7} a b^2 c^2 c^2 + \frac{4}{7} a b^2 c^3 + \frac{32}{7} b^2 c^2\right) x^7 + \left(\frac{1}{2} x^2 c^2 a^4 + \frac{8}{3} x^6 b^4 c^2 + \frac{16}{5} x^5 b^4 b^2 + \frac{1}{9} x^9 b^2 c^4 + \frac{1}{10} c^4 c^2 x^{10} + 8 x^6 a b^2 c^2 c^2 + 4 x^6 a b^2 b^2 c^2 + \frac{24}{5} x^5 a^2 b^2 c^2\right) x^6 + \left(\frac{1}{2} x^2 c^2 a^4 + \frac{8}{3} x^6 b^4 c^2 + \frac{16}{5} x^5 b^4 b^2 + \frac{1}{9} x^9 b^2 c^4 + \frac{1}{10} c^4 c^2 x^{10} + 8 x^6 a b^2 c^2 c^2 + 4 x^6 a b^2 b^2 c^2 + \frac{24}{5} x^5 a^2 b^2 c^2\right) x^5 + \left(\frac{c^4 c^2 x^{10}}{10} + \frac{(8 c^2 b^2 c^3 + b^2 c^4) x^9}{9} + \frac{(8 b^2 b^2 c^3 + c^2 (2(2 a c + 4 b^2) c^2 + 16 b^2 c^2)) x^8}{8} + \frac{(b^2 (2(2 a c + 4 b^2) c^2 + 16 b^2 c^2) + c^2 (8 a b^2 c^2 + 8(2 a c + 4 b^2) c^2)) x^7}{7} + \frac{c^3 (315 a c^2 c^2 + 1890 b^2 c^2 + 630 b^2 b^2 c^2) x^6}{630} + \frac{c^2 (560 b^2 c^2 + 70 b^2 c^2) x^5}{630} + \frac{63 c^4 c^2 x^4}{630}\right)$
gospers	
risch	
default	

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c1*x+b1)*(c*x^2+2*b*x+a)^4,x,method=_RETURNVERBOSE)

[Out] $\frac{1}{10}c^4c_1x^{10} + \frac{1}{9}(8b^3c^3c_1 + b_1c^4)x^9 + \frac{1}{8}(8b^4c^3 + c_1(2(2ac^2 + 4b^2)c^2 + 16b^2c^2))x^8 + \frac{1}{7}(b_1(2(2ac^2 + 4b^2)c^2 + 16b^2c^2) + c_1(8a^2b^2c^2 + 8(2ac^2 + 4b^2)b_1c))x^7 + \frac{1}{6}(b_1(8a^2b^2c^2 + 8(2ac^2 + 4b^2)b_1c) + c_1(2a^2c^2 + 32ab^2c + (2ac^2 + 4b^2)^2))x^6 + \frac{1}{5}(b_1(2a^2c^2 + 32ab^2c + (2ac^2 + 4b^2)^2) + c_1(8a^2b^2c + 8a^2b_1(2ac^2 + 4b^2)))x^5 + \frac{1}{4}(b_1(8a^2b^2c + 8a^2b_1(2ac^2 + 4b^2)) + c_1(2a^2(2ac^2 + 4b^2) + 16b^2a^2))x^4 + \frac{1}{3}(b_1(2a^2(2ac^2 + 4b^2) + 16b^2a^2) + 8c_1a^3b)x^3 + \frac{1}{2}(a^4c_1 + 8a^3b_1)x^2 + a^4b_1x$

Maxima [A]

time = 0.26, size = 273, normalized size = 1.04

$$\frac{1}{10}c_1a^{10} + \frac{1}{9}(b_1c^4 + 8b^3c^3c_1)x^9 + \frac{1}{8}(28b^4c^3 + (6b^2c^2 + a^2c_1)c_1)x^8 + \frac{1}{7}(6b^4b_1c^2 + ab_1c^2 + 2(4b^2c + 3ab^2c_1)x^2 + \frac{1}{3}(16b^2b_1c + 12ab_1c^2 + (8b^4 + 24ab^2c + 3a^2c^2)c_1)x^2 + a^2b_1c + \frac{2}{3}(8b^4 + 24ab^2b_1c + 3a^2b_1c^2 + 4(4ab^3 + 3a^2bc)c_1)x^2 + (8ab^4b_1 + 6a^2b_1b_1c + (6a^2b^2 + a^2c_1)c_1)x^2 + \frac{4}{3}(6a^2b^2b_1 + a^2b_1c + 2a^2b_1c_1)x^2 + \frac{1}{2}(8a^2b_1 + a^2c_1)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^4,x, algorithm="maxima")

[Out] $\frac{1}{10}c^4c_1x^{10} + \frac{1}{9}(b_1c^4 + 8b^3c^3c_1)x^9 + \frac{1}{2}(2b^3b_1c^3 + (6b^2c^2 + a^2c^3)c_1)x^8 + \frac{4}{7}(6b^2b_1c^2 + ab_1c^3 + 2(4b^3c + 3a^2b^2c^2)c_1)x^7 + \frac{1}{3}(16b^3b_1c + 12a^2b^2b_1c^2 + (8b^4 + 24a^2b^2c + 3a^2c^2)c_1)x^6 + a^4b_1x + \frac{2}{5}(8b^4b_1 + 24a^2b^2b_1c + 3a^2b_1c^2 + 4(4a^2b^3 + 3a^2b^2c)c_1)x^5 + (8a^2b^3b_1 + 6a^2b^2b_1c + (6a^2b^2 + a^3c)c_1)x^4 + \frac{4}{3}(6a^2b^2b_1 + a^3b_1c + 2a^3b^2c_1)x^3 + \frac{1}{2}(8a^3b^2b_1 + a^4c_1)x^2$

Fricas [A]

time = 0.30, size = 273, normalized size = 1.04

$$\frac{1}{10}c_1a^{10} + \frac{1}{9}(b_1c^4 + 8b^3c^3c_1)x^9 + \frac{1}{8}(28b^4c^3 + (6b^2c^2 + a^2c_1)c_1)x^8 + \frac{1}{7}(6b^4b_1c^2 + ab_1c^2 + 2(4b^2c + 3ab^2c_1)x^2 + \frac{1}{3}(16b^2b_1c + 12ab_1c^2 + (8b^4 + 24ab^2c + 3a^2c^2)c_1)x^2 + a^2b_1c + \frac{2}{3}(8b^4 + 24ab^2b_1c + 3a^2b_1c^2 + 4(4ab^3 + 3a^2bc)c_1)x^2 + (8ab^4b_1 + 6a^2b_1b_1c + (6a^2b^2 + a^2c_1)c_1)x^2 + \frac{4}{3}(6a^2b^2b_1 + a^2b_1c + 2a^2b_1c_1)x^2 + \frac{1}{2}(8a^2b_1 + a^2c_1)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^4,x, algorithm="fricas")

[Out] $\frac{1}{10}c^4c_1x^{10} + \frac{1}{9}(b_1c^4 + 8b^3c^3c_1)x^9 + \frac{1}{2}(2b^3b_1c^3 + (6b^2c^2 + a^2c^3)c_1)x^8 + \frac{4}{7}(6b^2b_1c^2 + ab_1c^3 + 2(4b^3c + 3a^2b^2c^2)c_1)x^7 + \frac{1}{3}(16b^3b_1c + 12a^2b^2b_1c^2 + (8b^4 + 24a^2b^2c + 3a^2c^2)c_1)x^6 + a^4b_1x + \frac{2}{5}(8b^4b_1 + 24a^2b^2b_1c + 3a^2b_1c^2 + 4(4a^2b^3 + 3a^2b^2c)c_1)x^5 + (8a^2b^3b_1 + 6a^2b^2b_1c + (6a^2b^2 + a^3c)c_1)x^4 + \frac{4}{3}(6a^2b^2b_1 + a^3b_1c + 2a^3b^2c_1)x^3 + \frac{1}{2}(8a^3b^2b_1 + a^4c_1)x^2$

Sympy [A]

time = 0.06, size = 313, normalized size = 1.19

$$a^4b_1x + \frac{a^4c_1x^2}{2} + x^3 \left(\frac{8a^2b_1}{3} + \frac{b_1c^2}{3} \right) + x^4 \left(\frac{4a^2c_1}{3} + 3b_1^2c_1 + 6b_1c^2 \right) + x^5 \left(\frac{24ab_1b_1c}{3} + \frac{4ab_1c^2}{3} + \frac{32b_1^2c_1}{3} + \frac{24b_1^2b_1c^2}{3} \right) + x^6 \left(a^2c_1^2 + 8a^2b_1^2c_1 + 4ab_1b_1c^2 + \frac{8b_1^4}{3} + \frac{16b_1^2b_1c}{3} \right) + x^7 \left(\frac{24a^2b_1b_1c}{3} + \frac{6a^2b_1b_1c^2}{3} + \frac{32ab_1^2c_1}{3} + \frac{8ab_1^2b_1c}{3} + \frac{16b_1^4}{3} \right) + x^8 \left(a^2c_1^2 + 6a^2b_1^2c_1 + 6a^2b_1b_1c + 8ab_1^4 \right) + x^9 \left(\frac{8c_1^2}{3} + \frac{4a^2b_1}{3} + 5a^2b_1b_1 \right) + x^{10} \left(\frac{c_1^2}{2} + 4a^2b_1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a)**4,x)

[Out] a**4*b1*x + c**4*c1*x**10/10 + x**9*(8*b*c**3*c1/9 + b1*c**4/9) + x**8*(a*c**3*c1/2 + 3*b**2*c**2*c1 + b*b1*c**3) + x**7*(24*a*b*c**2*c1/7 + 4*a*b1*c**3/7 + 32*b**3*c*c1/7 + 24*b**2*b1*c**2/7) + x**6*(a**2*c**2*c1 + 8*a*b**2*c*c1 + 4*a*b*b1*c**2 + 8*b**4*c1/3 + 16*b**3*b1*c/3) + x**5*(24*a**2*b*c*c1/5 + 6*a**2*b1*c**2/5 + 32*a*b**3*c1/5 + 48*a*b**2*b1*c/5 + 16*b**4*b1/5) + x**4*(a**3*c*c1 + 6*a**2*b**2*c1 + 6*a**2*b*b1*c + 8*a*b**3*b1) + x**3*(8*a**3*b*c1/3 + 4*a**3*b1*c/3 + 8*a**2*b**2*b1) + x**2*(a**4*c1/2 + 4*a**3*b*b1)

Giac [A]

time = 0.00, size = 343, normalized size = 1.30

$$\frac{1}{10}c^4c_1x^{10} + \frac{8}{9}b^3c^3c_1x^9 + \frac{4}{9}b^2c^2c_1x^8 + \frac{1}{2}a^3c^3c_1x^8 + \frac{24}{7}a^2b^3c^2c_1x^7 + \frac{24}{7}a^2b^2b_1c^2x^7 + \frac{16}{3}b^3b_1c^2x^6 + 4a^2b^3b_1c^2x^6 + \frac{8}{3}b^4c^2c_1x^6 + 8a^2b^2c^2c_1x^6 + a^2c^2c_1x^6 + \frac{16}{5}b^4b_1c^2x^5 + 48/5a^2b^2b_1c^2x^5 + 6/5a^2b^2c^2x^5 + 32/5a^2b^3c_1x^5 + 24/5a^2b^2c^2c_1x^5 + 8a^2b^3b_1c^2x^4 + 6a^2b^2b_1c^2x^4 + 6a^2b^2c_1x^4 + a^3c^2c_1x^4 + 8a^2b^2b_1c^2x^3 + 4/3a^3b_1c^2x^3 + 8/3a^3b^2c_1x^3 + 4a^3b^2b_1c^2x^2 + 1/2a^4c_1x^2 + a^4b_1c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^4,x)

[Out] 1/10*c^4*c1*x^10 + 1/9*b1*c^4*x^9 + 8/9*b*c^3*c1*x^9 + b*b1*c^3*x^8 + 3*b^2*c^2*c1*x^8 + 1/2*a*c^3*c1*x^8 + 24/7*b^2*b1*c^2*x^7 + 4/7*a*b1*c^3*x^7 + 32/7*b^3*c*c1*x^7 + 24/7*a*b*c^2*c1*x^7 + 16/3*b^3*b1*c*x^6 + 4*a*b*b1*c^2*x^6 + 8/3*b^4*c1*x^6 + 8*a*b^2*c*c1*x^6 + a^2*c^2*c1*x^6 + 16/5*b^4*b1*x^5 + 48/5*a*b^2*b1*c*x^5 + 6/5*a^2*b1*c^2*x^5 + 32/5*a*b^3*c1*x^5 + 24/5*a^2*b*c^2*c1*x^5 + 8*a*b^3*b1*x^4 + 6*a^2*b*b1*c*x^4 + 6*a^2*b^2*c1*x^4 + a^3*c*c1*x^4 + 8*a^2*b^2*b1*x^3 + 4/3*a^3*b1*c*x^3 + 8/3*a^3*b*c1*x^3 + 4*a^3*b*b1*x^2 + 1/2*a^4*c1*x^2 + a^4*b1*x

Mupad [B]

time = 0.26, size = 263, normalized size = 1.00

$$x^9 \left(\frac{b_1 c^4}{9} + \frac{8 b c^3 c_1}{9} \right) + x^8 \left(\frac{3 b^2 c^2 c_1}{3} + \frac{a c^3 c_1}{2} + b b_1 c^3 \right) + x^7 \left(\frac{16 b^3 b_1 c^2}{5} + \frac{6 a^2 b_1 c^2}{5} + \frac{32 a b^3 c_1}{5} + \frac{48 a b^2 b_1 c}{5} + \frac{24 a^2 b c^2 c_1}{5} \right) + x^6 \left(\frac{8 b^4 c_1}{3} + a^2 c^2 c_1 + \frac{16 b^3 b_1 c}{3} + 4 a b b_1 c^2 + 8 a b^2 c c_1 \right) + x^5 \left(\frac{6 a^2 b^2 b_1 c}{3} + \frac{8 a^2 b^2 c_1}{3} + \frac{16 b^3 b_1 c}{3} + \frac{24 a b^2 b_1 c}{3} + \frac{24 a^2 b c^2 c_1}{3} + \frac{4 a b c^2}{3} \right) + x^4 \left(\frac{8 a^2 b^2 b_1 c}{3} + \frac{4 a^2 b^2 c_1}{3} + \frac{6 a^2 b^2 b_1 c}{3} + \frac{6 a^2 b^2 c_1}{3} + \frac{8 a^2 b^2 b_1 c}{3} + \frac{8 a^2 b^2 c_1}{3} \right) + x^3 \left(\frac{8 a^2 b^2 b_1 c}{3} + \frac{4 a^2 b^2 c_1}{3} + \frac{6 a^2 b^2 b_1 c}{3} + \frac{6 a^2 b^2 c_1}{3} + \frac{8 a^2 b^2 b_1 c}{3} + \frac{8 a^2 b^2 c_1}{3} \right) + x^2 \left(\frac{1}{2} a^4 c_1 + a^4 b_1 c^2 \right) + a^4 b_1 c^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x)

[Out] x^9*((b1*c^4)/9 + (8*b*c^3*c1)/9) + x^8*(8*a^2*b^2*b1 + (8*a^3*b*c1)/3 + (4*a^3*b1*c)/3) + x^8*(3*b^2*c^2*c1 + (a*c^3*c1)/2 + b*b1*c^3) + x^7*((16*b^4*b1)/5 + (6*a^2*b1*c^2)/5 + (32*a*b^3*c1)/5 + (48*a*b^2*b1*c)/5 + (24*a^2*b*c*c1)/5) + x^6*((8*b^4*c1)/3 + a^2*c^2*c1 + (16*b^3*b1*c)/3 + 4*a*b*b1*c^2 + 8*a*b^2*c*c1) + x^4*(6*a^2*b^2*c1 + 8*a*b^3*b1 + a^3*c*c1 + 6*a^2*b*b1*c) + x^7*((24*b^2*b1*c^2)/7 + (4*a*b1*c^3)/7 + (32*b^3*c*c1)/7 + (24*a*b*c^2*c1)/7) + x^2*((a^4*c1)/2 + 4*a^3*b*b1) + (c^4*c1*x^10)/10 + a^4*b1*x

3.193 $\int (b_1 + c_1 x) (a + 2bx + cx^2)^n dx$

Optimal. Leaf size=159

$$\frac{c_1 (a + 2bx + cx^2)^{1+n}}{2c(1+n)} - \frac{2^n (b_1 c - bc_1) \left(\frac{-b - \sqrt{b^2 - ac} + cx}{\sqrt{b^2 - ac}} \right)^{-1-n} (a + 2bx + cx^2)^{1+n} {}_2F_1 \left(-n, 1+n; 2+n; \frac{b+cx+\sqrt{b^2-ac}}{2\sqrt{b^2-ac}} \right)}{c\sqrt{b^2-ac}(1+n)}$$

[Out] 1/2*c1*(c*x^2+2*b*x+a)^(1+n)/c/(1+n)-2^n*(-b*c1+b1*c)*(c*x^2+2*b*x+a)^(1+n)*hypergeom([-n, 1+n], [2+n], 1/2*(b+c*x+(-a*c+b^2)^(1/2))/(-a*c+b^2)^(1/2))*((-b-c*x+(-a*c+b^2)^(1/2))/(-a*c+b^2)^(1/2))^(-1-n)/c/(1+n)/(-a*c+b^2)^(1/2)

Rubi [A]

time = 0.06, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {654, 638}

$$\frac{c_1 (a + 2bx + cx^2)^{n+1}}{2c(n+1)} - \frac{2^n (b_1 c - bc_1) \left(\frac{-\sqrt{b^2 - ac} + b + cx}{\sqrt{b^2 - ac}} \right)^{-n-1} (a + 2bx + cx^2)^{n+1} {}_2F_1 \left(-n, n+1; n+2; \frac{b+cx+\sqrt{b^2-ac}}{2\sqrt{b^2-ac}} \right)}{c(n+1)\sqrt{b^2-ac}}$$

Antiderivative was successfully verified.

[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x]

[Out] (c1*(a + 2*b*x + c*x^2)^(1 + n))/(2*c*(1 + n)) - (2^n*(b1*c - b*c1)*(-(b - Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c]))^(-1 - n)*(a + 2*b*x + c*x^2)^(1 + n)*Hypergeometric2F1[-n, 1 + n, 2 + n, (b + Sqrt[b^2 - a*c] + c*x)/(2*Sqrt[b^2 - a*c])]/(c*Sqrt[b^2 - a*c]*(1 + n))

Rule 638

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[4*p]

Rule 654

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int (b1 + c1x) (a + 2bx + cx^2)^n dx = \frac{c1 (a + 2bx + cx^2)^{1+n}}{2c(1+n)} + \frac{(2b1c - 2bc1) \int (a + 2bx + cx^2)^n dx}{2c}$$

$$= \frac{c1 (a + 2bx + cx^2)^{1+n}}{2c(1+n)} - \frac{2^n (b1c - bc1) \left(-\frac{b - \sqrt{b^2 - ac} + cx}{\sqrt{b^2 - ac}} \right)^{-1-n}}{c\sqrt{b^2 - ac}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

time = 0.43, size = 267, normalized size = 1.68

$$\frac{1}{2}(a + x(2b + cx))^n \left(c1x^2 \left(\frac{b - \sqrt{b^2 - ac} + cx}{b - \sqrt{b^2 - ac}} \right)^{-n} \left(\frac{b + \sqrt{b^2 - ac} + cx}{b + \sqrt{b^2 - ac}} \right)^{-n} {}_2F_1 \left(2; -n, -n; 3; -\frac{cx}{b + \sqrt{b^2 - ac}}, \frac{cx}{-b + \sqrt{b^2 - ac}} \right) + \frac{2^{1+n} b1 (b - \sqrt{b^2 - ac} + cx) \left(\frac{b + \sqrt{b^2 - ac} + cx}{\sqrt{b^2 - ac}} \right)^{-n} {}_2F_1 \left(-n, 1 + n; 2 + n; \frac{-b + \sqrt{b^2 - ac} - cx}{2\sqrt{b^2 - ac}} \right)}{c(1+n)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x]

[Out] ((a + x*(2*b + c*x))^n*((c1*x^2*AppellF1[2, -n, -n, 3, -((c*x)/(b + Sqrt[b^2 - a*c])), (c*x)/(-b + Sqrt[b^2 - a*c])])]/((b - Sqrt[b^2 - a*c] + c*x)/(b - Sqrt[b^2 - a*c]))^n*((b + Sqrt[b^2 - a*c] + c*x)/(b + Sqrt[b^2 - a*c]))^n) + (2^(1 + n)*b1*(b - Sqrt[b^2 - a*c] + c*x)*Hypergeometric2F1[-n, 1 + n, 2 + n, (-b + Sqrt[b^2 - a*c] - c*x)/(2*Sqrt[b^2 - a*c])])/(c*(1 + n)*((b + Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c])^n))/2

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x]')

[Out] Timed out

Maple [F]

time = 0.06, size = 0, normalized size = 0.00

$$\int (c1x + b1) (cx^2 + 2bx + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c1*x+b1)*(c*x^2+2*b*x+a)^n,x)

[Out] `int((c1*x+b1)*(c*x^2+2*b*x+a)^n,x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c1*x+b1)*(c*x^2+2*b*x+a)^n,x, algorithm="maxima")`

[Out] `integrate((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)`

Fricas [F]

time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c1*x+b1)*(c*x^2+2*b*x+a)^n,x, algorithm="fricas")`

[Out] `integral((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (b_1 + c_1 x) (a + 2bx + cx^2)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c1*x+b1)*(c*x**2+2*b*x+a)**n,x)`

[Out] `Integral((b1 + c1*x)*(a + 2*b*x + c*x**2)**n, x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c1*x+b1)*(c*x^2+2*b*x+a)^n,x)`

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (b_1 + c_1 x) (cx^2 + 2bx + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x)`

[Out] `int((b1 + c1*x)*(a + 2*b*x + c*x^2)^n, x)`

$$3.194 \quad \int \frac{b1+c1x}{a+2bx+cx^2} dx$$

Optimal. Leaf size=65

$$-\frac{(b1c - bc1) \tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2 - ac}}\right)}{c\sqrt{b^2 - ac}} + \frac{c1 \log(a + 2bx + cx^2)}{2c}$$

[Out] 1/2*c1*ln(c*x^2+2*b*x+a)/c-(-b*c1+b1*c)*arctanh((c*x+b)/(-a*c+b^2)^(1/2))/c/(-a*c+b^2)^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {648, 632, 212, 642}

$$\frac{c1 \log(a + 2bx + cx^2)}{2c} - \frac{(b1c - bc1) \tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2 - ac}}\right)}{c\sqrt{b^2 - ac}}$$

Antiderivative was successfully verified.

[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2),x]

[Out] -(((b1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]]/(c*Sqrt[b^2 - a*c])) + (c1*Log[a + 2*b*x + c*x^2])/(2*c)

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x])/b], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{b_1 + c_1 x}{a + 2bx + cx^2} dx &= \frac{c_1 \int \frac{2b+2cx}{a+2bx+cx^2} dx}{2c} + \frac{(2b_1c - 2bc_1) \int \frac{1}{a+2bx+cx^2} dx}{2c} \\ &= \frac{c_1 \log(a + 2bx + cx^2)}{2c} - \frac{(2b_1c - 2bc_1) \text{Subst}\left(\int \frac{1}{4(b^2-ac)-x^2} dx, x, 2b + 2cx\right)}{c} \\ &= -\frac{(b_1c - bc_1) \tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{c\sqrt{b^2-ac}} + \frac{c_1 \log(a + 2bx + cx^2)}{2c} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 66, normalized size = 1.02

$$\frac{(b_1c - bc_1) \tan^{-1}\left(\frac{b+cx}{\sqrt{-b^2+ac}}\right)}{c\sqrt{-b^2+ac}} + \frac{c_1 \log(a + 2bx + cx^2)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2), x]
```

```
[Out] ((b1*c - b*c1)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/(c*Sqrt[-b^2 + a*c]) + (c1*Log[a + 2*b*x + c*x^2])/(2*c)
```

Mathics [A]

time = 6.21, size = 100, normalized size = 1.54

$$\frac{\text{Log}\left[\frac{b+cx+\sqrt{-ac+b^2}}{c}\right] \left(-b_1c + c_1\sqrt{-ac+b^2} + bc_1\right) + \text{Log}\left[\frac{b+cx-\sqrt{-ac+b^2}}{c}\right] \left(b_1c + c_1\sqrt{-ac+b^2} - bc_1\right)}{2c\sqrt{-ac+b^2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^1, x]')
```

```
[Out] (Log[(b + c x + Sqrt[-a c + b ^ 2]) / c] (-b1 c + c1 Sqrt[-a c + b ^ 2] + b c1) + Log[(b + c x - Sqrt[-a c + b ^ 2]) / c] (b1 c + c1 Sqrt[-a c + b ^ 2] - b c1)) / (2 c Sqrt[-a c + b ^ 2])
```

Maple [A]

time = 0.14, size = 63, normalized size = 0.97

method	result
default	$\frac{c_1 \ln(cx^2 + 2bx + a)}{2c} + \frac{\left(b_1 - \frac{c_1 b}{c}\right) \arctan\left(\frac{2cx + 2b}{2\sqrt{ac - b^2}}\right)}{\sqrt{ac - b^2}}$
risch	$\frac{\ln\left(-abcc_1 + ab_1 c^2 + b^3 c_1 - b^2 b_1 c - \sqrt{-(bc_1 - b_1 c)^2 (ac - b^2)} \quad cx - \sqrt{-(bc_1 - b_1 c)^2 (ac - b^2)} \quad b\right)_{ac_1}}{2ac - 2b^2} - \dots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)/(c*x^2+2*b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c1*ln(c*x^2+2*b*x+a)/c+(b1-c1*b/c)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see 'assume?' for more de
```

Fricas [A]

time = 0.32, size = 203, normalized size = 3.12

$$\left[\frac{(b^2 - ac)c_1 \log(cx^2 + 2bx + a) - \sqrt{b^2 - ac} (b_1 c - bc_1) \log\left(\frac{c^2 x^2 + 2bcx + 2b^2 - ac + 2\sqrt{b^2 - ac}(cx + b)}{c^2 + 2bx + a}\right)}{2(b^2 c - ac^2)}, \frac{(b^2 - ac)c_1 \log(cx^2 + 2bx + a) - 2\sqrt{-b^2 + ac} (b_1 c - bc_1) \arctan\left(\frac{-\sqrt{-b^2 + ac}(cx + b)}{b^2 - ac}\right)}{2(b^2 c - ac^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2 - a*c)*c1*log(c*x^2 + 2*b*x + a) - sqrt(b^2 - a*c)*(b1*c - b*c1)*log((c^2*x^2 + 2*b*c*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2 + 2*b*x + a)))/(b^2*c - a*c^2), 1/2*((b^2 - a*c)*c1*log(c*x^2 + 2*b*x + a) - 2*sqrt(-b^2 + a*c)*(b1*c - b*c1)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(b^2 - a*c)))/(b^2*c - a*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(53) = 106$

time = 0.42, size = 246, normalized size = 3.78

$$\left(\frac{c_1 - \sqrt{-ac + b^2} (bc_1 - b_1c)}{2c} \log\left(x + \frac{-2ac\left(\frac{c_1}{2c} - \frac{\sqrt{-ac + b^2} (bc_1 - b_1c)}{2c(ac - b^2)}\right) + ac_1 + 2b^2\left(\frac{c_1}{2c} - \frac{\sqrt{-ac + b^2} (bc_1 - b_1c)}{2c(ac - b^2)}\right) - bb_1}{bc_1 - b_1c}\right)\right) + \left(\frac{c_1 + \sqrt{-ac + b^2} (bc_1 - b_1c)}{2c} \log\left(x + \frac{-2ac\left(\frac{c_1}{2c} + \frac{\sqrt{-ac + b^2} (bc_1 - b_1c)}{2c(ac - b^2)}\right) + ac_1 + 2b^2\left(\frac{c_1}{2c} + \frac{\sqrt{-ac + b^2} (bc_1 - b_1c)}{2c(ac - b^2)}\right) - bb_1}{bc_1 - b_1c}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)/(c*x**2+2*b*x+a),x)

[Out] (c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2)))*log(x + (-2*a*c*(c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2))) + a*c1 + 2*b**2*(c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2))) - b*b1)/(b*c1 - b1*c)) + (c1/(2*c) + sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2)))*log(x + (-2*a*c*(c1/(2*c) + sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2))) + a*c1 + 2*b**2*(c1/(2*c) + sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2))) - b*b1)/(b*c1 - b1*c))

Giac [A]

time = 0.00, size = 66, normalized size = 1.02

$$\frac{c_1 \ln(x^2c + 2xb + a)}{2c} + \frac{(-2c_1b + 2b_1c) \arctan\left(\frac{b+cx}{\sqrt{-b^2 + ac}}\right)}{2c\sqrt{-b^2 + ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a),x)

[Out] 1/2*c1*log(c*x^2 + 2*b*x + a)/c + (b1*c - b*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/(sqrt(-b^2 + a*c)*c)

Mupad [B]

time = 0.27, size = 155, normalized size = 2.38

$$\frac{b_1 \operatorname{atan}\left(\frac{b}{\sqrt{ac - b^2}} + \frac{cx}{\sqrt{ac - b^2}}\right)}{\sqrt{ac - b^2}} - \frac{2b^2 c_1 \ln(cx^2 + 2bx + a)}{4ac^2 - 4b^2c} + \frac{2ac c_1 \ln(cx^2 + 2bx + a)}{4ac^2 - 4b^2c} - \frac{b c_1 \operatorname{atan}\left(\frac{b}{\sqrt{ac - b^2}} + \frac{cx}{\sqrt{ac - b^2}}\right)}{c\sqrt{ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b1 + c1*x)/(a + 2*b*x + c*x^2),x)

[Out] (b1*atan(b/(a*c - b^2)^(1/2) + (c*x)/(a*c - b^2)^(1/2)))/(a*c - b^2)^(1/2) - (2*b^2*c1*log(a + 2*b*x + c*x^2))/(4*a*c^2 - 4*b^2*c) + (2*a*c*c1*log(a + 2*b*x + c*x^2))/(4*a*c^2 - 4*b^2*c) - (b*c1*atan(b/(a*c - b^2)^(1/2) + (c*x)/(a*c - b^2)^(1/2)))/(c*(a*c - b^2)^(1/2))

$$3.195 \quad \int \frac{b_1 + c_1 x}{(a + 2bx + cx^2)^2} dx$$

Optimal. Leaf size=89

$$-\frac{bb_1 - ac_1 + (b_1c - bc_1)x}{2(b^2 - ac)(a + 2bx + cx^2)} + \frac{(b_1c - bc_1) \tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2 - ac}}\right)}{2(b^2 - ac)^{3/2}}$$

[Out] 1/2*(-b*b1+a*c1-(-b*c1+b1*c)*x)/(-a*c+b^2)/(c*x^2+2*b*x+a)+1/2*(-b*c1+b1*c)*arctanh((c*x+b)/(-a*c+b^2)^(1/2))/(-a*c+b^2)^(3/2)

Rubi [A]

time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {652, 632, 212}

$$\frac{(b_1c - bc_1) \tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2 - ac}}\right)}{2(b^2 - ac)^{3/2}} - \frac{-ac_1 + x(b_1c - bc_1) + bb_1}{2(b^2 - ac)(a + 2bx + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x]

[Out] -1/2*(b*b1 - a*c1 + (b1*c - b*c1)*x)/((b^2 - a*c)*(a + 2*b*x + c*x^2)) + ((b1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(2*(b^2 - a*c)^(3/2))

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 652

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rubi steps

$$\begin{aligned}
 \int \frac{b_1 + c_1 x}{(a + 2bx + cx^2)^2} dx &= -\frac{bb_1 - ac_1 + (b_1c - bc_1)x}{2(b^2 - ac)(a + 2bx + cx^2)} - \frac{(b_1c - bc_1) \int \frac{1}{a+2bx+cx^2} dx}{2(b^2 - ac)} \\
 &= -\frac{bb_1 - ac_1 + (b_1c - bc_1)x}{2(b^2 - ac)(a + 2bx + cx^2)} + \frac{(b_1c - bc_1) \text{Subst} \left(\int \frac{1}{4(b^2-ac)-x^2} dx, x, 2b + 2cx \right)}{b^2 - ac} \\
 &= -\frac{bb_1 - ac_1 + (b_1c - bc_1)x}{2(b^2 - ac)(a + 2bx + cx^2)} + \frac{(b_1c - bc_1) \tanh^{-1} \left(\frac{b+cx}{\sqrt{b^2 - ac}} \right)}{2(b^2 - ac)^{3/2}}
 \end{aligned}$$

Mathematica [A]

time = 0.05, size = 88, normalized size = 0.99

$$\frac{-bb_1 + ac_1 - b_1cx + bc_1x}{a + x(2b + cx)} + \frac{(-b_1c + bc_1) \tan^{-1} \left(\frac{b + cx}{\sqrt{-b^2 + ac}} \right)}{\sqrt{-b^2 + ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x]

[Out] ((-(b*b1) + a*c1 - b1*c*x + b*c1*x)/(a + x*(2*b + c*x)) + ((-(b1*c) + b*c1)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(2*(b^2 - a*c))

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 399 vs. 2(89) = 178.
time = 8.92, size = 377, normalized size = 4.24

$$\frac{-\frac{c_1}{a} + \frac{b_1}{a} - \frac{c_1(b_1 - b)}{a^2}}{a^2c - ab^2 + 2bx(ac - b^2) + cx^2(ac - b^2)} \left(\frac{\text{Log} \left[\frac{x^2(c_1 - b_1) \sqrt{-\frac{1}{(ac - b^2)}} + a^2(c_1 - b_1) \sqrt{-\frac{1}{(ac - b^2)}} + c^2(c_1 - b_1) \sqrt{-\frac{1}{(ac - b^2)}}}{-(c_1 - b_1)} \right]}{1} - \text{Log} \left[\frac{x^2(c_1 - b_1) \sqrt{-\frac{1}{(ac - b^2)}} - a^2(c_1 - b_1) \sqrt{-\frac{1}{(ac - b^2)}} + c^2(c_1 - b_1) \sqrt{-\frac{1}{(ac - b^2)}}}{-(c_1 - b_1)} \right]}{1} \right) \sqrt{-\frac{1}{(ac - b^2)}}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x]')

[Out] (-a c1 / 2 + b b1 / 2 - x (b c1 - b1 c) / 2 + (a ^ 2 c - a b ^ 2 + 2 b x (a c - b ^ 2) + c x ^ 2 (a c - b ^ 2)) (b c1 - b1 c) (Log[(a ^ 2 c ^ 2 (-b c1 + b1 c) Sqrt[-1 / (a c - b ^ 2) ^ 3] + 2 a b ^ 2 c (b c1 - b1 c) Sqrt[-1 / (a c - b ^ 2) ^ 3] + b ^ 4 (-b c1 + b1 c) Sqrt[-1 / (a c - b ^ 2) ^ 3] + b ^ 2 c1 - b b1 c + c x (b c1 - b1 c)) / (c (b c1 - b1 c))] - Log[(a ^ 2 c ^ 2 (b c1 - b1 c) Sqrt[-1 / (a c - b ^ 2) ^ 3] - 2 a b ^ 2 c (b c1 - b1 c) Sqrt[-1 / (a c - b ^ 2) ^ 3] + b ^ 4 (b c1 - b1 c) Sqrt[-1 / (a c - b ^ 2) ^ 3] + b ^ 2 c1 - b b1 c + c x (b c1 - b1 c)) / (c (b c1 - b1 c))]) Sqrt[-1

$\frac{1}{4} \sqrt{4ac - b^2} \sqrt{4ac - b^2} \sqrt{4ac - b^2} / (a^2c - ab^2 + 2bx(a^2c - b^2) + cx^2)$

Maple [A]

time = 0.10, size = 103, normalized size = 1.16

method	result
default	$\frac{(-2bc1+2b1c)x+2bb1-2ac1}{(4ac-4b^2)(cx^2+2bx+a)} + \frac{(-2bc1+2b1c) \arctan\left(\frac{2cx+2b}{2\sqrt{ac-b^2}}\right)}{(4ac-4b^2)\sqrt{ac-b^2}}$
risch	$\frac{-\frac{(bc1-b1c)x}{2(ac-b^2)} - \frac{ac1-bb1}{2(ac-b^2)}}{cx^2+2bx+a} + \frac{\ln\left(\frac{(-c^2a+b^2c)x - (-ac+b^2)^{\frac{3}{2}} - abc + b^3}{4(-ac+b^2)^{\frac{3}{2}}}\right)bc1}{4(-ac+b^2)^{\frac{3}{2}}} - \frac{\ln\left(\frac{(-c^2a+b^2c)x - (-ac+b^2)^{\frac{3}{2}} - abc + b^3}{4(-ac+b^2)^{\frac{3}{2}}}\right)b1c}{4(-ac+b^2)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c1*x+b1)/(c*x^2+2*b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] $((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)+(-2*b*c1+2*b1*c)/(4*a*c-4*b^2)/(a*c-b^2)^{(1/2)*\arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^{(1/2)})}$

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see 'assume?' for more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(81) = 162.

time = 0.32, size = 447, normalized size = 5.02

$$\frac{2b^2c - 2ab^2c - (ab^2c - ab^2c + (b^2c - ab^2c)x^2 + 2(ab^2c - ab^2c)x\sqrt{b^2 - ac} \log\left(\frac{c^2x^2 + 2cx + a}{2\sqrt{b^2 - ac}}\right) - 2(ab^2c - ab^2c) - (b^2 - ab^2c)x}{4(ab^2 - 2a^2b^2c + a^2c^2 + (b^2c - 2ab^2c + a^2c^2)x^2 + 2(b^2 - 2ab^2c + a^2b^2c)x)} - \frac{b^2c - ab^2c - (ab^2c - ab^2c + (b^2c - ab^2c)x^2 + 2(ab^2c - ab^2c)x\sqrt{b^2 - ac} \arctan\left(\frac{\sqrt{b^2 - ac}x}{2\sqrt{b^2 - ac}}\right) - (ab^2c - ab^2c) + (b^2 - ab^2c)x}{2(ab^2 - 2a^2b^2c + a^2c^2 + (b^2c - 2ab^2c + a^2c^2)x^2 + 2(b^2 - 2ab^2c + a^2b^2c)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^2,x, algorithm="fricas")

[Out] $[-1/4*(2*b^3*b1 - 2*a*b*b1*c - (a*b1*c - a*b*c1 + (b1*c^2 - b*c*c1))*x^2 + 2*(b*b1*c - b^2*c1)*x]*\sqrt{b^2 - a*c}*\log((c^2*x^2 + 2*b*c*x + 2*b^2 - a*c + 2*\sqrt{b^2 - a*c}*(c*x + b))/(c*x^2 + 2*b*x + a)) - 2*(a*b^2 - a^2*c)*c1$

$$+ 2*(b^2*b1*c - a*b1*c^2 - (b^3 - a*b*c)*c1)*x)/(a*b^4 - 2*a^2*b^2*c + a^3*c^2 + (b^4*c - 2*a*b^2*c^2 + a^2*c^3)*x^2 + 2*(b^5 - 2*a*b^3*c + a^2*b*c^2)*x), -1/2*(b^3*b1 - a*b*b1*c - (a*b1*c - a*b*c1 + (b1*c^2 - b*c*c1)*x^2 + 2*(b*b1*c - b^2*c1)*x)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(b^2 - a*c)) - (a*b^2 - a^2*c)*c1 + (b^2*b1*c - a*b1*c^2 - (b^3 - a*b*c)*c1)*x)/(a*b^4 - 2*a^2*b^2*c + a^3*c^2 + (b^4*c - 2*a*b^2*c^2 + a^2*c^3)*x^2 + 2*(b^5 - 2*a*b^3*c + a^2*b*c^2)*x]$$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(75) = 150$

time = 0.56, size = 323, normalized size = 3.63

$$\frac{\sqrt{\frac{1}{(ac-b^2)^2}} \log\left(x + \frac{-a^2\sqrt{\frac{1}{(ac-b^2)^2}}(b_1-b_1c)+2ab^2\sqrt{\frac{1}{(ac-b^2)^2}}(b_1-b_1c)-a^2\sqrt{\frac{1}{(ac-b^2)^2}}(b_1-b_1c)+4b^2c_1-b_1c}{b_1-b_1c}}\right)}{4} - \frac{\sqrt{\frac{1}{(ac-b^2)^2}}(b_1-b_1c)\log\left(x + \frac{a^2\sqrt{\frac{1}{(ac-b^2)^2}}(b_1-b_1c)+2ab^2\sqrt{\frac{1}{(ac-b^2)^2}}(b_1-b_1c)+a^2\sqrt{\frac{1}{(ac-b^2)^2}}(b_1-b_1c)+4b^2c_1-b_1c}{b_1-b_1c}}\right)}{4} + \frac{-ac_1+bb_1+x(-b_1c+b_1c)}{2a^2c-2ab^2+x^2+(2ac^2-2b^2c)+x(4abc-4b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)/(c*x**2+2*b*x+a)**2,x)

[Out] $\sqrt{-1/(a*c - b**2)**3}*(b*c1 - b1*c)*\log(x + (-a**2*c**2*\sqrt{-1/(a*c - b**2)**3}*(b*c1 - b1*c) - b**4*\sqrt{-1/(a*c - b**2)**3}*(b*c1 - b1*c) + b**2*c1 - b*b1*c)/(b*c*c1 - b1*c**2))/4 - \sqrt{-1/(a*c - b**2)**3}*(b*c1 - b1*c)*\log(x + (a**2*c**2*\sqrt{-1/(a*c - b**2)**3}*(b*c1 - b1*c) - 2*a*b**2*c*\sqrt{-1/(a*c - b**2)**3}*(b*c1 - b1*c) + b**4*\sqrt{-1/(a*c - b**2)**3}*(b*c1 - b1*c) + b**2*c1 - b*b1*c)/(b*c*c1 - b1*c**2))/4 + (-a*c1 + b*b1 + x*(-b*c1 + b1*c))/(2*a**2*c - 2*a*b**2 + x**2*(2*a*c**2 - 2*b**2*c) + x*(4*a*b*c - 4*b**3))$

Giac [A]

time = 0.00, size = 95, normalized size = 1.07

$$\frac{xc_1b - xb_1c + c_1a - b_1b}{(-2ca + 2b^2)(x^2c + 2xb + a)} + \frac{(c_1b - b_1c) \arctan\left(\frac{b+cx}{\sqrt{-b^2 + ac}}\right)}{2(-ca + b^2)\sqrt{-b^2 + ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^2,x)

[Out] $-1/2*(b1*c - b*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^2 - a*c)*sqrt(-b^2 + a*c)) - 1/2*(b1*c*x - b*c1*x + b*b1 - a*c1)/((c*x^2 + 2*b*x + a)*(b^2 - a*c))$

Mupad [B]

time = 0.28, size = 159, normalized size = 1.79

$$\frac{\operatorname{atan}\left(\frac{2\left(\frac{(4b^3-4abc)(bc_1-b_1c)}{8(ac-b^2)^{5/2}} - \frac{cx(bc_1-b_1c)}{2(ac-b^2)^{3/2}}\right)(ac-b^2)}{bc_1-b_1c}\right)(bc_1-b_1c)}{2(ac-b^2)^{3/2}} - \frac{\frac{ac_1-bb_1}{2(ac-b^2)} + \frac{x(bc_1-b_1c)}{2(ac-b^2)}}{cx^2 + 2bx + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((b_1 + c_1 x)/(a + 2bx + cx^2)^2, x)$

[Out] $(\text{atan}((2*((4b^3 - 4abc)*(bc_1 - b_1c))/(8(a^2 - b^2)^{5/2}) - (cx*(bc_1 - b_1c))/(2(a^2 - b^2)^{3/2}))* (a^2 - b^2)/(bc_1 - b_1c))*(bc_1 - b_1c))/(2(a^2 - b^2)^{3/2}) - ((ac_1 - b_1b)/(2(a^2 - b^2)) + (x*(bc_1 - b_1c))/(2(a^2 - b^2)))/(a + 2bx + cx^2)$

$$3.196 \quad \int \frac{b_1 + c_1 x}{(a + 2bx + cx^2)^3} dx$$

Optimal. Leaf size=130

$$-\frac{bb_1 - ac_1 + (b_1c - bc_1)x}{4(b^2 - ac)(a + 2bx + cx^2)^2} + \frac{3(b_1c - bc_1)(b + cx)}{8(b^2 - ac)^2(a + 2bx + cx^2)} - \frac{3c(b_1c - bc_1) \tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2 - ac}}\right)}{8(b^2 - ac)^{5/2}}$$

[Out] 1/4*(-b*b1+a*c1-(-b*c1+b1*c)*x)/(-a*c+b^2)/(c*x^2+2*b*x+a)^2+3/8*(-b*c1+b1*c)*(c*x+b)/(-a*c+b^2)^2/(c*x^2+2*b*x+a)-3/8*c*(-b*c1+b1*c)*arctanh((c*x+b)/(-a*c+b^2)^(1/2))/(-a*c+b^2)^(5/2)

Rubi [A]

time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {652, 628, 632, 212}

$$\frac{3(b + cx)(b_1c - bc_1)}{8(b^2 - ac)^2(a + 2bx + cx^2)} - \frac{-ac_1 + x(b_1c - bc_1) + bb_1}{4(b^2 - ac)(a + 2bx + cx^2)^2} - \frac{3c(b_1c - bc_1) \tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2 - ac}}\right)}{8(b^2 - ac)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x]

[Out] -1/4*(b*b1 - a*c1 + (b1*c - b*c1)*x)/((b^2 - a*c)*(a + 2*b*x + c*x^2)^2) + (3*(b1*c - b*c1)*(b + c*x))/(8*(b^2 - a*c)^2*(a + 2*b*x + c*x^2)) - (3*c*(b1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(8*(b^2 - a*c)^(5/2))

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p + 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

Antiderivative was successfully verified.

[In] `mathics('Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x]')`

[Out]
$$\begin{aligned} & (-4 a^2 c c1 - 2 a b^2 c1 + 10 a b b1 c - 4 b^3 b1 + 3 c (a^4 c^2 - 2 a^3 b^2 c + a^2 b^4 + 4 a b x (a^2 c^2 - 2 a b^2 c + b^4) + 2 x^2 (a^3 c^3 - 3 a b^4 c + 2 b^6) + 4 b c x^3 (a^2 c^2 - 2 a b^2 c + b^4) + c^2 x^4 (a^2 c^2 - 2 a b^2 c + b^4)) (b c1 - b1 c) (\text{Log}[(a^3 c^3 (-b c1 + b1 c) \text{Sqrt}[-1 / (a c - b^2)]^5 + 3 a^2 b^2 c^2 (b c1 - b1 c) \text{Sqrt}[-1 / (a c - b^2)]^5 - 3 a b^4 c (b c1 - b1 c) \text{Sqrt}[-1 / (a c - b^2)]^5 + b^6 (b c1 - b1 c) \text{Sqrt}[-1 / (a c - b^2)]^5 + b^2 c1 - b b1 c + c x (b c1 - b1 c)) / (c (b c1 - b1 c))] - \text{Log}[(a^3 c^3 (b c1 - b1 c) \text{Sqrt}[-1 / (a c - b^2)]^5 + 3 a^2 b^2 c^2 (-b c1 + b1 c) \text{Sqrt}[-1 / (a c - b^2)]^5 + 3 a b^4 c (b c1 - b1 c) \text{Sqrt}[-1 / (a c - b^2)]^5 + b^6 (-b c1 + b1 c) \text{Sqrt}[-1 / (a c - b^2)]^5 + b^2 c1 - b b1 c + c x (b c1 - b1 c)) / (c (b c1 - b1 c))]) \text{Sqrt}[-1 / (a c - b^2)]^5 - 2 x (5 a b c c1 - 5 a b1 c^2 + 4 b^3 c1 - 4 b^2 b1 c) - 18 b c x^2 (b c1 - b1 c) + 6 c^2 x^3 (-b c1 + b1 c)) / (16 (a^4 c^2 - 2 a^3 b^2 c + a^2 b^4 + 4 a b x (a^2 c^2 - 2 a b^2 c + b^4) + 2 x^2 (a^3 c^3 - 3 a b^4 c + 2 b^6) + 4 b c x^3 (a^2 c^2 - 2 a b^2 c + b^4) + c^2 x^4 (a^2 c^2 - 2 a b^2 c + b^4))) \end{aligned}$$

Maple [A]

time = 0.11, size = 155, normalized size = 1.19

method	result
default	$\frac{(-2bc1+2b1c)x+2bb1-2ac1}{2(4ac-4b^2)(cx^2+2bx+a)^2} + \frac{3(-2bc1+2b1c) \left(\frac{2cx+2b}{(4ac-4b^2)(cx^2+2bx+a)} + \frac{2c \arctan\left(\frac{2cx+2b}{2\sqrt{ac-b^2}}\right)}{(4ac-4b^2)\sqrt{ac-b^2}} \right)}{2(4ac-4b^2)}$
risch	$\frac{-\frac{3c^2(bc1-b1c)x^3}{8(a^2c^2-2ab^2c+b^4)} - \frac{9bc(bc1-b1c)x^2}{8(a^2c^2-2ab^2c+b^4)} - \frac{(5ac+4b^2)(bc1-b1c)x}{8(a^2c^2-2ab^2c+b^4)} - \frac{2a^2cc1+ab^2c1-5abb1c+2b^3b1}{8(a^2c^2-2ab^2c+b^4)}}{(cx^2+2bx+a)^2} - \frac{3c \ln\left((a^2c^3-2ab^2c^2+b^4c)x - (-a^2c^3-2ab^2c^2+b^4c)\right)}{16(-ac)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c1*x+b1)/(c*x^2+2*b*x+a)^3,x,method=_RETURNVERBOSE)`

[Out]
$$\frac{1}{2} * \left(\frac{(-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1}{(4*a*c-4*b^2)} / (c*x^2+2*b*x+a)^2 + \frac{3}{2} * \frac{(-2*b*c1+2*b1*c)}{(4*a*c-4*b^2)} * \frac{(2*c*x+2*b)}{(4*a*c-4*b^2)} / (c*x^2+2*b*x+a) + 2 * \frac{c}{(4*a*c-4*b^2)} / (a*c-b^2)^{(1/2)} * \arctan\left(\frac{1}{2} * \frac{(2*c*x+2*b)}{(a*c-b^2)^{(1/2)}}\right) \right)$$

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* h elp (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see 'assume?' for more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(120) = 240.

time = 0.33, size = 1104, normalized size = 8.49

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/16*(4*b^5*b1 - 14*a*b^3*b1*c + 10*a^2*b*b1*c^2 - 6*(b^2*b1*c^3 - a*b1*c^4 - (b^3*c^2 - a*b*c^3)*c1)*x^3 - 18*(b^3*b1*c^2 - a*b*b1*c^3 - (b^4*c - a*b^2*c^2)*c1)*x^2 + 3*(a^2*b1*c^2 - a^2*b*c*c1 + (b1*c^4 - b*c^3*c1)*x^4 + 4*(b*b1*c^3 - b^2*c^2*c1)*x^3 + 2*(2*b^2*b1*c^2 + a*b1*c^3 - (2*b^3*c + a*b*c^2)*c1)*x^2 + 4*(a*b*b1*c^2 - a*b^2*c*c1)*x)*sqrt(b^2 - a*c)*log((c^2*x^2 + 2*b*c*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2 + 2*b*x + a) + 2*(a*b^4 + a^2*b^2*c - 2*a^3*c^2)*c1 - 2*(4*b^4*b1*c + a*b^2*b1*c^2 - 5*a^2*b1*c^3 - (4*b^5 + a*b^3*c - 5*a^2*b*c^2)*c1)*x)/(a^2*b^6 - 3*a^3*b^4*c + 3*a^4*b^2*c^2 - a^5*c^3 + (b^6*c^2 - 3*a*b^4*c^3 + 3*a^2*b^2*c^4 - a^3*c^5)*x^4 + 4*(b^7*c - 3*a*b^5*c^2 + 3*a^2*b^3*c^3 - a^3*b*c^4)*x^3 + 2*(2*b^8 - 5*a*b^6*c + 3*a^2*b^4*c^2 + a^3*b^2*c^3 - a^4*c^4)*x^2 + 4*(a*b^7 - 3*a^2*b^5*c + 3*a^3*b^3*c^2 - a^4*b*c^3)*x), -1/8*(2*b^5*b1 - 7*a*b^3*b1*c + 5*a^2*b*b1*c^2 - 3*(b^2*b1*c^3 - a*b1*c^4 - (b^3*c^2 - a*b*c^3)*c1)*x^3 - 9*(b^3*b1*c^2 - a*b*b1*c^3 - (b^4*c - a*b^2*c^2)*c1)*x^2 + 3*(a^2*b1*c^2 - a^2*b*c*c1 + (b1*c^4 - b*c^3*c1)*x^4 + 4*(b*b1*c^3 - b^2*c^2*c1)*x^3 + 2*(2*b^2*b1*c^2 + a*b1*c^3 - (2*b^3*c + a*b*c^2)*c1)*x^2 + 4*(a*b*b1*c^2 - a*b^2*c*c1)*x)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(b^2 - a*c)) + (a*b^4 + a^2*b^2*c - 2*a^3*c^2)*c1 - (4*b^4*b1*c + a*b^2*b1*c^2 - 5*a^2*b1*c^3 - (4*b^5 + a*b^3*c - 5*a^2*b*c^2)*c1)*x)/(a^2*b^6 - 3*a^3*b^4*c + 3*a^4*b^2*c^2 - a^5*c^3 + (b^6*c^2 - 3*a*b^4*c^3 + 3*a^2*b^2*c^4 - a^3*c^5)*x^4 + 4*(b^7*c - 3*a*b^5*c^2 + 3*a^2*b^3*c^3 - a^3*b*c^4)*x^3 + 2*(2*b^8 - 5*a*b^6*c + 3*a^2*b^4*c^2 + a^3*b^2*c^3 - a^4*c^4)*x^2 + 4*(a*b^7 - 3*a^2*b^5*c + 3*a^3*b^3*c^2 - a^4*b*c^3)*x)] \end{aligned}$$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 622 vs. 2(117) = 234

time = 1.03, size = 622, normalized size = 4.78

$$\int \frac{1}{(c x^2 + 2 b x + a)^3} dx = \frac{1}{2 \sqrt{a c}} \frac{1}{(c x^2 + 2 b x + a)^2} + \frac{1}{2 \sqrt{a c}} \frac{1}{(c x^2 + 2 b x + a)} + \frac{1}{2 \sqrt{a c}} \frac{1}{(c x^2 + 2 b x + a)^{3/2}} \arctan \left(\frac{2 c x + b}{\sqrt{a c}} \right) + \frac{1}{2 \sqrt{a c}} \frac{1}{(c x^2 + 2 b x + a)^{3/2}} \operatorname{arctanh} \left(\frac{2 c x + b}{\sqrt{a c}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)/(c*x**2+2*b*x+a)**3,x)

[Out] $3*c*\sqrt{-1/(a*c - b**2)**5}*(b*c1 - b1*c)*\log(x + (-3*a**3*c**4*\sqrt{-1/(a*c - b**2)**5}*(b*c1 - b1*c) + 9*a**2*b**2*c**3*\sqrt{-1/(a*c - b**2)**5}*(b*c1 - b1*c) - 9*a*b**4*c**2*\sqrt{-1/(a*c - b**2)**5}*(b*c1 - b1*c) + 3*b**6*c*\sqrt{-1/(a*c - b**2)**5}*(b*c1 - b1*c) + 3*b**2*c*c1 - 3*b*b1*c**2)/(3*b*c**2*c1 - 3*b1*c**3))/16 - 3*c*\sqrt{-1/(a*c - b**2)**5}*(b*c1 - b1*c)*\log(x + (3*a**3*c**4*\sqrt{-1/(a*c - b**2)**5}*(b*c1 - b1*c) - 9*a**2*b**2*c**3*\sqrt{-1/(a*c - b**2)**5}*(b*c1 - b1*c) + 9*a*b**4*c**2*\sqrt{-1/(a*c - b**2)**5}*(b*c1 - b1*c) - 3*b**6*c*\sqrt{-1/(a*c - b**2)**5}*(b*c1 - b1*c) + 3*b**2*c*c1 - 3*b*b1*c**2)/(3*b*c**2*c1 - 3*b1*c**3))/16 + (-2*a**2*c*c1 - a*b**2*c1 + 5*a*b*b1*c - 2*b**3*b1 + x**3*(-3*b*c**2*c1 + 3*b1*c**3) + x**2*(-9*b**2*c*c1 + 9*b*b1*c**2) + x*(-5*a*b*c*c1 + 5*a*b1*c**2 - 4*b**3*c1 + 4*b**2*b1*c))/(8*a**4*c**2 - 16*a**3*b**2*c + 8*a**2*b**4 + x**4*(8*a**2*c**4 - 16*a*b**2*c**3 + 8*b**4*c**2) + x**3*(32*a**2*b*c**3 - 64*a*b**3*c**2 + 32*b**5*c) + x**2*(16*a**3*c**3 - 48*a*b**4*c + 32*b**6) + x*(32*a**3*b*c**2 - 64*a**2*b**3*c + 32*a*b**5))$

Giac [A]

time = 0.00, size = 212, normalized size = 1.63

$$\frac{3x^3c_1c^2b - 3x^3b_1c^3 + 9x^2c_1cb^2 - 9x^2b_1c^2b + 5xc_1cba + 4xc_1b^3 - 5xb_1c^2a - 4xb_1cb^2 + 2c_1ca^2 + c_1b^2a - 5b_1cba + 2b_1b^3}{(-8c^2a^2 + 16cb^2a - 8b^4)(xc + 2xb + a)^2} + \frac{(3c_1cb - 3b_1c^2) \arctan\left(\frac{b+cx}{\sqrt{-b^2+ac}}\right)}{2(-4c^2a^2 + 8cb^2a - 4b^4)\sqrt{-b^2+ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^3,x)

[Out] $\frac{3}{8}*(b1*c^2 - b*c*c1)*\arctan((c*x + b)/\sqrt{-b^2 + a*c})/((b^4 - 2*a*b^2*c + a^2*c^2)*\sqrt{-b^2 + a*c}) + \frac{1}{8}*(3*b1*c^3*x^3 - 3*b*c^2*c1*x^3 + 9*b*b1*c^2*x^2 - 9*b^2*c*c1*x^2 + 4*b^2*b1*c*x + 5*a*b1*c^2*x - 4*b^3*c1*x - 5*a*b*c*c1*x - 2*b^3*b1 + 5*a*b*b1*c - a*b^2*c1 - 2*a^2*c*c1)/((b^4 - 2*a*b^2*c + a^2*c^2)*(c*x^2 + 2*b*x + a)^2)$

Mupad [B]

time = 0.50, size = 360, normalized size = 2.77

$$\frac{3c \operatorname{atan}\left(\frac{8\left(\frac{3c^2x(b_1-b_1c)}{8(a-c^2)^{5/2}} + \frac{3c(b_1-b_1c)(16a^2b^2c^2-32a^3c+16b^5)}{128(a-c^2)^{5/2}(a^2-2ab^2c+b^4)}\right)(a^2c^2-2ab^2c+b^4)}{3b_1c^2-3bcc_1}\right)(b_1-b_1c)}{8(ac-b^2)^{5/2}} - \frac{\frac{2cc_1a^2+c_1ab^2-5b_1cabb+2b_1b^3}{8(a^2c^2-2ab^2c+b^4)} + \frac{x(4b^2+5ac)(b_1-b_1c)}{8(a^2c^2-2ab^2c+b^4)} + \frac{3c^2x^3(b_1-b_1c)}{8(a^2c^2-2ab^2c+b^4)} + \frac{9bcx^2(b_1-b_1c)}{8(a^2c^2-2ab^2c+b^4)}}{a^2+x^2(4b^2+2ac)+c^2x^4+4abx+4bcx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x)

[Out] $(3*c*\operatorname{atan}((8*((3*c^2*x*(b*c1 - b1*c))/(8*(a*c - b^2)^(5/2)) + (3*c*(b*c1 - b1*c)*(16*b^5 + 16*a^2*b*c^2 - 32*a*b^3*c))/(128*(a*c - b^2)^(5/2)*(b^4 + a$

$$\begin{aligned}
& \left((a^2c^2 - 2ab^2c) \right) (b^4 + a^2c^2 - 2ab^2c) / (3b^2c^2 - 3b^2c^2) \cdot (b^2c^2 - 2ab^2c) \\
& \cdot (b^2c^2 - 2ab^2c) / (8(ac - b^2)^{5/2}) - ((2b^3b_1 + ab^2c_1 + 2a^2c^2c_1 - 5ab^2b_1c) / (8(b^4 + a^2c^2 - 2ab^2c)) + (x(5ac + 4b^2)(b^2c_1 - b_1c) / (8(b^4 + a^2c^2 - 2ab^2c)) + (3c^2x^3(b^2c_1 - b_1c) / (8(b^4 + a^2c^2 - 2ab^2c)) + (9b^2cx^2(b^2c_1 - b_1c) / (8(b^4 + a^2c^2 - 2ab^2c)))) / (a^2 + x^2(2ac + 4b^2) + c^2x^4 + 4abx + 4b^2cx^3)
\end{aligned}$$

$$3.197 \quad \int \frac{b1+c1x}{(a+2bx+cx^2)^4} dx$$

Optimal. Leaf size=173

$$\frac{bb1 - ac1 + (b1c - bc1)x}{6(b^2 - ac)(a + 2bx + cx^2)^3} + \frac{5(b1c - bc1)(b + cx)}{24(b^2 - ac)^2(a + 2bx + cx^2)^2} - \frac{5c(b1c - bc1)(b + cx)}{16(b^2 - ac)^3(a + 2bx + cx^2)} + \frac{5c^2(b1c - bc1)}{16(b^2 - ac)^4}$$

[Out] 1/6*(-b*b1+a*c1-(-b*c1+b1*c)*x)/(-a*c+b^2)/(c*x^2+2*b*x+a)^3+5/24*(-b*c1+b1*c)*(c*x+b)/(-a*c+b^2)^2/(c*x^2+2*b*x+a)^2-5/16*c*(-b*c1+b1*c)*(c*x+b)/(-a*c+b^2)^3/(c*x^2+2*b*x+a)+5/16*c^2*(-b*c1+b1*c)*arctanh((c*x+b)/(-a*c+b^2)^(1/2))/(-a*c+b^2)^(7/2)

Rubi [A]

time = 0.07, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {652, 628, 632, 212}

$$\frac{5c^2(b1c - bc1) \tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2 - ac}}\right)}{16(b^2 - ac)^{7/2}} - \frac{5c(b + cx)(b1c - bc1)}{16(b^2 - ac)^3(a + 2bx + cx^2)} + \frac{5(b + cx)(b1c - bc1)}{24(b^2 - ac)^2(a + 2bx + cx^2)^2} - \frac{-ac1 + x(b1c - bc1) + bb1}{6(b^2 - ac)(a + 2bx + cx^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^4, x]

[Out] -1/6*(b*b1 - a*c1 + (b1*c - b*c1)*x)/((b^2 - a*c)*(a + 2*b*x + c*x^2)^3) + (5*(b1*c - b*c1)*(b + c*x))/(24*(b^2 - a*c)^2*(a + 2*b*x + c*x^2)^2) - (5*c*(b1*c - b*c1)*(b + c*x))/(16*(b^2 - a*c)^3*(a + 2*b*x + c*x^2)) + (5*c^2*(b1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]]/(16*(b^2 - a*c)^(7/2))

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p + 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 632


```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c))*(a + b*x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned} \int \frac{b_1 + c_1 x}{(a + 2bx + cx^2)^4} dx &= -\frac{bb_1 - ac_1 + (b_1c - bc_1)x}{6(b^2 - ac)(a + 2bx + cx^2)^3} - \frac{(5(b_1c - bc_1)) \int \frac{1}{(a + 2bx + cx^2)^3} dx}{6(b^2 - ac)} \\ &= -\frac{bb_1 - ac_1 + (b_1c - bc_1)x}{6(b^2 - ac)(a + 2bx + cx^2)^3} + \frac{5(b_1c - bc_1)(b + cx)}{24(b^2 - ac)^2(a + 2bx + cx^2)^2} + \frac{5c(b_1c - bc_1)}{8(b^2 - ac)^3} \\ &= -\frac{bb_1 - ac_1 + (b_1c - bc_1)x}{6(b^2 - ac)(a + 2bx + cx^2)^3} + \frac{5(b_1c - bc_1)(b + cx)}{24(b^2 - ac)^2(a + 2bx + cx^2)^2} - \frac{5c(b_1c - bc_1)}{16(b^2 - ac)^3} \\ &= -\frac{bb_1 - ac_1 + (b_1c - bc_1)x}{6(b^2 - ac)(a + 2bx + cx^2)^3} + \frac{5(b_1c - bc_1)(b + cx)}{24(b^2 - ac)^2(a + 2bx + cx^2)^2} - \frac{5c(b_1c - bc_1)}{16(b^2 - ac)^3} \\ &= -\frac{bb_1 - ac_1 + (b_1c - bc_1)x}{6(b^2 - ac)(a + 2bx + cx^2)^3} + \frac{5(b_1c - bc_1)(b + cx)}{24(b^2 - ac)^2(a + 2bx + cx^2)^2} - \frac{5c(b_1c - bc_1)}{16(b^2 - ac)^3} \end{aligned}$$

Mathematica [A]

time = 0.14, size = 168, normalized size = 0.97

$$\frac{\frac{8(b^2 - ac)^2(-bb_1 + ac_1 - b_1cx + bc_1x)}{(a + x(2b + cx))^3} - \frac{10(b^2 - ac)(-b_1c + bc_1)(b + cx)}{(a + x(2b + cx))^2} + \frac{15c(-b_1c + bc_1)(b + cx)}{a + x(2b + cx)} + \frac{15c^2(-b_1c + bc_1) \tan^{-1}\left(\frac{b + cx}{\sqrt{-b^2 + ac}}\right)}{\sqrt{-b^2 + ac}}}{48(b^2 - ac)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^4, x]
```

```
[Out] ((8*(b^2 - a*c)^2*(-(b*b1) + a*c1 - b1*c*x + b*c1*x))/(a + x*(2*b + c*x))^3 - (10*(b^2 - a*c)*(-(b1*c) + b*c1)*(b + c*x))/(a + x*(2*b + c*x))^2 + (15*c*(-(b1*c) + b*c1)*(b + c*x))/(a + x*(2*b + c*x)) + (15*c^2*(-(b1*c) + b*c1)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(48*(b^2 - a*c)^3)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 1239 vs. $2(173) = 346$.
time = 33.00, size = 1215, normalized size = 7.02

Antiderivative was successfully verified.

[In] `mathics('Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x]')`

[Out]
$$\begin{aligned} & (-16 a^3 c^2 c_1 - 18 a^2 b^2 c c_1 + 66 a^2 b b_1 c^2 + 4 a b^4 c_1 - 52 a b^3 b_1 c + 16 b^5 b_1 - 6 x (11 a^2 b c^2 c_1 - 11 a^2 b \\ & 1 c^3 + 18 a b^3 c c_1 - 18 a b^2 b_1 c^2 - 4 b^5 c_1 + 4 b^4 b_1 c) - 60 b c x^2 (4 a b c c_1 - 4 a b_1 c^2 + b^3 c_1 - b^2 b_1 c) + 15 c \\ & ^2 (a^6 c^3 - 3 a^5 b^2 c^2 + 3 a^4 b^4 c - a^3 b^6 + 6 a^2 b x (a^3 c^3 - 3 a^2 b^2 c^2 + 3 a b^4 c - b^6) + 3 a \\ & x^2 (a^4 c^4 + a^3 b^2 c^3 - 9 a^2 b^4 c^2 + 11 a b^6 c - 4 b^8) + 4 b x^3 (3 a^4 c^4 - 7 a^3 b^2 c^3 + 3 a^2 b^4 c^2 + 3 a b^6 c - 2 b^8) + 3 c x^4 (a^4 c^4 + a^3 b^2 c^3 \\ & - 9 a^2 b^4 c^2 + 11 a b^6 c - 4 b^8) + 6 b c^2 x^5 (a^3 c^3 - 3 a^2 b^2 c^2 + 3 a b^4 c - b^6) + c^3 x^6 (a^3 c^3 - 3 a^2 b^2 c^2 + 3 a b^4 c - b^6)) (b c_1 - b_1 c) (\text{Log}[(a^4 c^4 (-b c_1 + b_1 c) \text{Sqrt}[-1 / (a c - b^2)^7] + 4 a^3 b^2 c^3 (b c_1 - b_1 c) \text{Sqrt}[-1 / (a c - b^2)^7] + 6 a^2 b^4 c^2 (-b c_1 + b_1 c) \text{Sqrt}[-1 / (a c - b^2)^7] + 4 a b^6 c (b c_1 - b_1 c) \text{Sqrt}[-1 / (a c - b^2)^7] + b^8 (-b c_1 + b_1 c) \text{Sqrt}[-1 / (a c - b^2)^7] + b^2 c_1 - b b_1 c + c x (b c_1 - b_1 c)) / (c (b c_1 - b_1 c))] - \text{Log}[(a^4 c^4 (b c_1 - b_1 c) \text{Sqrt}[-1 / (a c - b^2)^7] + 4 a^3 b^2 c^3 (-b c_1 + b_1 c) \text{Sqrt}[-1 / (a c - b^2)^7] + 6 a^2 b^4 c^2 (b c_1 - b_1 c) \text{Sqrt}[-1 / (a c - b^2)^7] - 4 a b^6 c (b c_1 - b_1 c) \text{Sqrt}[-1 / (a c - b^2)^7] + b^8 (b c_1 - b_1 c) \text{Sqrt}[-1 / (a c - b^2)^7] + b^2 c_1 - b b_1 c + c x (b c_1 - b_1 c)) / (c (b c_1 - b_1 c))] \text{Sqrt}[-1 / (a c - b^2)^7] + 20 c^2 x^3 (-4 a b c c_1 + 4 a b_1 c^2 - 11 b^3 c_1 + 11 b^2 b_1 c) - 150 b c^3 x^4 (b c_1 - b_1 c) + 30 c^4 x^5 (-b c_1 + b_1 c)) / (96 (a^6 c^3 - 3 a^5 b^2 c^2 + 3 a^4 b^4 c - a^3 b^6 + 6 a^2 b x (a^3 c^3 - 3 a^2 b^2 c^2 + 3 a b^4 c - b^6) + 3 a x^2 (a^4 c^4 + a^3 b^2 c^3 - 9 a^2 b^4 c^2 + 11 a b^6 c - 4 b^8) + 4 b x^3 (3 a^4 c^4 - 7 a^3 b^2 c^3 + 3 a^2 b^4 c^2 + 3 a b^6 c - 2 b^8) + 3 c x^4 (a^4 c^4 + a^3 b^2 c^3 - 9 a^2 b^4 c^2 + 11 a b^6 c - 4 b^8) + 6 b c^2 x^5 (a^3 c^3 - 3 a^2 b^2 c^2 + 3 a b^4 c - b^6) + c^3 x^6 (a^3 c^3 - 3 a^2 b^2 c^2 + 3 a b^4 c - b^6))) \end{aligned}$$

Maple [A]

time = 0.12, size = 206, normalized size = 1.19

method	result
--------	--------

default	$5(-2bc1+2b1c) \left(\frac{2cx+2b}{2(4ac-4b^2)(cx^2+2bx+a)^2} + \frac{3c \left(\frac{2cx+2b}{(4ac-4b^2)(cx^2+2bx+a)} + \frac{2c \arctan\left(\frac{2cx+2b}{2\sqrt{ac-b^2}}\right)}{(4ac-4b^2)\sqrt{ac-b^2}} \right)}{4ac-4b^2} \right)$
risch	$\frac{(-2bc1+2b1c)x+2bb1-2ac1}{3(4ac-4b^2)(cx^2+2bx+a)^3} + \frac{5c^4(bc1-b1c)x^5}{16(c^3a^3-3b^2c^2a^2+3b^4ca-b^6)} - \frac{25c^3(bc1-b1c)bx^4}{16(c^3a^3-3b^2c^2a^2+3b^4ca-b^6)} - \frac{5(4ac+11b^2)c^2(bc1-b1c)x^3}{24(c^3a^3-3b^2c^2a^2+3b^4ca-b^6)} - \frac{5b(4ac+b^2)c(bc1-b1c)x^2}{8(c^3a^3-3b^2c^2a^2+3b^4ca-b^6)} - \frac{(11a^2bc^2)}{(cx^2+2bx+a)^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c1*x+b1)/(c*x^2+2*b*x+a)^4,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{3} * ((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)^3 + 5/3 * (-2*b*c1+2*b1*c)/(4*a*c-4*b^2) * (1/2 * (2*c*x+2*b)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)^2 + 3*c/(4*a*c-4*b^2) * ((2*c*x+2*b)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a) + 2*c/(4*a*c-4*b^2)/(a*c-b^2)^(1/2) * \arctan(1/2 * (2*c*x+2*b)/(a*c-b^2)^(1/2))))$

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c1*x+b1)/(c*x^2+2*b*x+a)^4,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see 'assume?' for more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 962 vs. 2(161) = 322.

time = 0.35, size = 1950, normalized size = 11.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c1*x+b1)/(c*x^2+2*b*x+a)^4,x, algorithm="fricas")`

[Out] $[-1/96 * (16*b^7*b1 - 68*a*b^5*b1*c + 118*a^2*b^3*b1*c^2 - 66*a^3*b*b1*c^3 + 30*(b^2*b1*c^5 - a*b1*c^6 - (b^3*c^4 - a*b*c^5)*c1)*x^5 + 150*(b^3*b1*c^4 - a*b*b1*c^5 - (b^4*c^3 - a*b^2*c^4)*c1)*x^4 + 20*(11*b^4*b1*c^3 - 7*a*b^2*b$

$$\begin{aligned}
& 1*c^4 - 4*a^2*b^1*c^5 - (11*b^5*c^2 - 7*a*b^3*c^3 - 4*a^2*b*c^4)*c1)*x^3 + 6 \\
& 0*(b^5*b^1*c^2 + 3*a*b^3*b^1*c^3 - 4*a^2*b*b^1*c^4 - (b^6*c + 3*a*b^4*c^2 - 4* \\
& a^2*b^2*c^3)*c1)*x^2 - 15*(a^3*b^1*c^3 - a^3*b*c^2*c1 + (b^1*c^6 - b*c^5*c1)* \\
& x^6 + 6*(b*b^1*c^5 - b^2*c^4*c1)*x^5 + 3*(4*b^2*b^1*c^4 + a*b^1*c^5 - (4*b^3*c \\
& ^3 + a*b*c^4)*c1)*x^4 + 4*(2*b^3*b^1*c^3 + 3*a*b*b^1*c^4 - (2*b^4*c^2 + 3*a*b \\
& ^2*c^3)*c1)*x^3 + 3*(4*a*b^2*b^1*c^3 + a^2*b^1*c^4 - (4*a*b^3*c^2 + a^2*b*c^3 \\
&)*c1)*x^2 + 6*(a^2*b*b^1*c^3 - a^2*b^2*c^2*c1)*x)*sqrt(b^2 - a*c)*log((c^2*x \\
& ^2 + 2*b*c*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2 + 2*b*x + \\
& a)) + 2*(2*a*b^6 - 11*a^2*b^4*c + a^3*b^2*c^2 + 8*a^4*c^3)*c1 - 6*(4*b^6*b1 \\
& *c - 22*a*b^4*b1*c^2 + 7*a^2*b^2*b1*c^3 + 11*a^3*b1*c^4 - (4*b^7 - 22*a*b^5 \\
& *c + 7*a^2*b^3*c^2 + 11*a^3*b*c^3)*c1)*x)/(a^3*b^8 - 4*a^4*b^6*c + 6*a^5*b^ \\
& 4*c^2 - 4*a^6*b^2*c^3 + a^7*c^4 + (b^8*c^3 - 4*a*b^6*c^4 + 6*a^2*b^4*c^5 - \\
& 4*a^3*b^2*c^6 + a^4*c^7)*x^6 + 6*(b^9*c^2 - 4*a*b^7*c^3 + 6*a^2*b^5*c^4 - 4 \\
& *a^3*b^3*c^5 + a^4*b*c^6)*x^5 + 3*(4*b^10*c - 15*a*b^8*c^2 + 20*a^2*b^6*c^3 - \\
& 10*a^3*b^4*c^4 + a^5*c^6)*x^4 + 4*(2*b^11 - 5*a*b^9*c + 10*a^3*b^5*c^3 - \\
& 10*a^4*b^3*c^4 + 3*a^5*b*c^5)*x^3 + 3*(4*a*b^10 - 15*a^2*b^8*c + 20*a^3*b^ \\
& 6*c^2 - 10*a^4*b^4*c^3 + a^6*c^5)*x^2 + 6*(a^2*b^9 - 4*a^3*b^7*c + 6*a^4*b^ \\
& 5*c^2 - 4*a^5*b^3*c^3 + a^6*b*c^4)*x), -1/48*(8*b^7*b1 - 34*a*b^5*b1*c + 59 \\
& *a^2*b^3*b1*c^2 - 33*a^3*b*b1*c^3 + 15*(b^2*b1*c^5 - a*b1*c^6 - (b^3*c^4 - \\
& a*b*c^5)*c1)*x^5 + 75*(b^3*b1*c^4 - a*b*b1*c^5 - (b^4*c^3 - a*b^2*c^4)*c1)* \\
& x^4 + 10*(11*b^4*b1*c^3 - 7*a*b^2*b1*c^4 - 4*a^2*b1*c^5 - (11*b^5*c^2 - 7*a \\
& *b^3*c^3 - 4*a^2*b*c^4)*c1)*x^3 + 30*(b^5*b1*c^2 + 3*a*b^3*b1*c^3 - 4*a^2*b \\
& *b1*c^4 - (b^6*c + 3*a*b^4*c^2 - 4*a^2*b^2*c^3)*c1)*x^2 - 15*(a^3*b1*c^3 - \\
& a^3*b*c^2*c1 + (b^1*c^6 - b*c^5*c1)*x^6 + 6*(b*b^1*c^5 - b^2*c^4*c1)*x^5 + 3* \\
& (4*b^2*b^1*c^4 + a*b^1*c^5 - (4*b^3*c^3 + a*b*c^4)*c1)*x^4 + 4*(2*b^3*b^1*c^3 \\
& + 3*a*b*b^1*c^4 - (2*b^4*c^2 + 3*a*b^2*c^3)*c1)*x^3 + 3*(4*a*b^2*b^1*c^3 + a^ \\
& 2*b^1*c^4 - (4*a*b^3*c^2 + a^2*b*c^3)*c1)*x^2 + 6*(a^2*b*b^1*c^3 - a^2*b^2*c^ \\
& 2*c1)*x)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(b^2 - a*c)) + \\
& (2*a*b^6 - 11*a^2*b^4*c + a^3*b^2*c^2 + 8*a^4*c^3)*c1 - 3*(4*b^6*b1*c - 22 \\
& *a*b^4*b1*c^2 + 7*a^2*b^2*b1*c^3 + 11*a^3*b1*c^4 - (4*b^7 - 22*a*b^5*c + 7* \\
& a^2*b^3*c^2 + 11*a^3*b*c^3)*c1)*x)/(a^3*b^8 - 4*a^4*b^6*c + 6*a^5*b^4*c^2 - \\
& 4*a^6*b^2*c^3 + a^7*c^4 + (b^8*c^3 - 4*a*b^6*c^4 + 6*a^2*b^4*c^5 - 4*a^3*b^ \\
& ^2*c^6 + a^4*c^7)*x^6 + 6*(b^9*c^2 - 4*a*b^7*c^3 + 6*a^2*b^5*c^4 - 4*a^3*b^ \\
& 3*c^5 + a^4*b*c^6)*x^5 + 3*(4*b^10*c - 15*a*b^8*c^2 + 20*a^2*b^6*c^3 - 10*a \\
& ^3*b^4*c^4 + a^5*c^6)*x^4 + 4*(2*b^11 - 5*a*b^9*c + 10*a^3*b^5*c^3 - 10*a^4 \\
& *b^3*c^4 + 3*a^5*b*c^5)*x^3 + 3*(4*a*b^10 - 15*a^2*b^8*c + 20*a^3*b^6*c^2 - \\
& 10*a^4*b^4*c^3 + a^6*c^5)*x^2 + 6*(a^2*b^9 - 4*a^3*b^7*c + 6*a^4*b^5*c^2 - \\
& 4*a^5*b^3*c^3 + a^6*b*c^4)*x)]
\end{aligned}$$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(158) = 316$

time = 1.71, size = 1027, normalized size = 5.94



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)/(c*x**2+2*b*x+a)**4,x)

[Out] $5*c**2*\sqrt{-1/(a*c - b**2)**7}*(b*c1 - b1*c)*\log(x + (-5*a**4*c**6*\sqrt{-1/(a*c - b**2)**7}*(b*c1 - b1*c) + 20*a**3*b**2*c**5*\sqrt{-1/(a*c - b**2)**7}*(b*c1 - b1*c) - 30*a**2*b**4*c**4*\sqrt{-1/(a*c - b**2)**7}*(b*c1 - b1*c) + 20*a*b**6*c**3*\sqrt{-1/(a*c - b**2)**7}*(b*c1 - b1*c) - 5*b**8*c**2*\sqrt{-1/(a*c - b**2)**7}*(b*c1 - b1*c) + 5*b**2*c**2*c1 - 5*b*b1*c**3)/(5*b*c**3*c1 - 5*b1*c**4))/32 - 5*c**2*\sqrt{-1/(a*c - b**2)**7}*(b*c1 - b1*c)*\log(x + (5*a**4*c**6*\sqrt{-1/(a*c - b**2)**7}*(b*c1 - b1*c) - 20*a**3*b**2*c**5*\sqrt{-1/(a*c - b**2)**7}*(b*c1 - b1*c) + 30*a**2*b**4*c**4*\sqrt{-1/(a*c - b**2)**7}*(b*c1 - b1*c) - 20*a*b**6*c**3*\sqrt{-1/(a*c - b**2)**7}*(b*c1 - b1*c) + 5*b**8*c**2*\sqrt{-1/(a*c - b**2)**7}*(b*c1 - b1*c) + 5*b**2*c**2*c1 - 5*b*b1*c**3)/(5*b*c**3*c1 - 5*b1*c**4))/32 + (-8*a**3*c**2*c1 - 9*a**2*b**2*c*c1 + 33*a**2*b*b1*c**2 + 2*a*b**4*c1 - 26*a*b**3*b1*c + 8*b**5*b1 + x**5*(-15*b*c**4*c1 + 15*b1*c**5) + x**4*(-75*b**2*c**3*c1 + 75*b*b1*c**4) + x**3*(-40*a*b*c**3*c1 + 40*a*b1*c**4 - 110*b**3*c**2*c1 + 110*b**2*b1*c**3) + x**2*(-120*a*b**2*c**2*c1 + 120*a*b*b1*c**3 - 30*b**4*c*c1 + 30*b**3*b1*c**2) + x*(-33*a**2*b*c**2*c1 + 33*a**2*b1*c**3 - 54*a*b**3*c*c1 + 54*a*b**2*b1*c**2 + 12*b**5*c1 - 12*b**4*b1*c))/(48*a**6*c**3 - 144*a**5*b**2*c**2 + 144*a**4*b**4*c - 48*a**3*b**6 + x**6*(48*a**3*c**6 - 144*a**2*b**2*c**5 + 144*a*b**4*c**4 - 48*b**6*c**3) + x**5*(288*a**3*b*c**5 - 864*a**2*b**3*c**4 + 864*a*b**5*c**3 - 288*b**7*c**2) + x**4*(144*a**4*c**5 + 144*a**3*b**2*c**4 - 1296*a**2*b**4*c**3 + 1584*a*b**6*c**2 - 576*b**8*c) + x**3*(576*a**4*b*c**4 - 1344*a**3*b**3*c**3 + 576*a**2*b**5*c**2 + 576*a*b**7*c - 384*b**9) + x**2*(144*a**5*c**4 + 144*a**4*b**2*c**3 - 1296*a**3*b**4*c**2 + 1584*a**2*b**6*c - 576*a*b**8) + x*(288*a**5*b*c**3 - 864*a**4*b**3*c**2 + 864*a**3*b**5*c - 288*a**2*b**7))$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(161) = 322.

time = 0.00, size = 387, normalized size = 2.24

$$\frac{-15a^2c^6 + 15a^2b^2c^2 - 75a^2c^2b^2 + 75a^2b^2c^2 - 40a^2c^2b^2 + 110a^2c^2b^2 + 40a^2b^2c^2 + 110a^2b^2c^2 - 120a^2c^2b^2 - 30a^2c^2b^2 + 120a^2b^2c^2 + 30a^2b^2c^2 - 33a^2c^2b^2 - 54a^2c^2b^2 + 12a^2c^2b^2 + 33a^2b^2c^2 + 54a^2b^2c^2 - 12a^2c^2b^2 - 8c^2a^2 - 9c^2a^2 + 2c^2b^2 + 33b^2c^2 - 26b^2c^2 + 8b^2}{(48c^6 - 144c^5a^2 + 144c^4a^2 - 48b^6) (c^2 + 2ab + a^2)} \arctan\left(\frac{bx}{\sqrt{-b^2 + ac}}\right) + \frac{(5c^2b - 5b^2) \arctan\left(\frac{bx}{\sqrt{-b^2 + ac}}\right)}{2(-8c^6 + 24c^5a^2 - 24c^4a^2 + 8b^6) \sqrt{-b^2 + ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^4,x)

[Out] $-5/16*(b1*c^3 - b*c^2*c1)*\arctan((c*x + b)/\sqrt{-b^2 + a*c})/((b^6 - 3*a*b^4*c + 3*a^2*b^2*c^2 - a^3*c^3)*\sqrt{-b^2 + a*c}) - 1/48*(15*b1*c^5*x^5 - 15*b*c^4*c1*x^5 + 75*b*b1*c^4*x^4 - 75*b^2*c^3*c1*x^4 + 110*b^2*b1*c^3*x^3 + 40*a*b1*c^4*x^3 - 110*b^3*c^2*c1*x^3 - 40*a*b*c^3*c1*x^3 + 30*b^3*b1*c^2*x^2 + 120*a*b*b1*c^3*x^2 - 30*b^4*c*c1*x^2 - 120*a*b^2*c^2*c1*x^2 - 12*b^4*b1*c*c*x + 54*a*b^2*b1*c^2*x + 33*a^2*b1*c^3*x + 12*b^5*c1*x - 54*a*b^3*c*c1*x - 33*a^2*b*c^2*c1*x + 8*b^5*b1 - 26*a*b^3*b1*c + 33*a^2*b*b1*c^2 + 2*a*b^4*c1 - 9*a^2*b^2*c*c1 - 8*a^3*c^2*c1)/((b^6 - 3*a*b^4*c + 3*a^2*b^2*c^2 - a^3*c^3)*(c*x^2 + 2*b*x + a)^3)$

Mupad [B]

time = 0.71, size = 640, normalized size = 3.70

$$\frac{5c^2 \operatorname{atan}\left(\frac{\sqrt{a^2+2bx+c^2} \sqrt{b^2+12ac} \sqrt{3c^2+12ab} \sqrt{12b^2c+3ac^2} + a^2 + c^2x^2 + 6bx^2 + 6a^2bx}{16(a-c-b)^{7/2}}\right)}{x^3(8b^3+12ab^2c) + x^2(12ab^2+3a^2c) + x(3ac^2+12ab^2) + a^3 + c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x)

[Out] ((5*c^4*x^5*(b*c1 - b1*c))/(16*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)) - (8*b^5*b1 - 8*a^3*c^2*c1 + 2*a*b^4*c1 - 26*a*b^3*b1*c + 33*a^2*b*b1*c^2 - 9*a^2*b^2*c*c1)/(48*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)) + (x*(b*c1 - b1*c)*(11*a^2*c^2 - 4*b^4 + 18*a*b^2*c))/(16*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)) + (5*c*x^3*(4*a*c^2 + 11*b^2*c)*(b*c1 - b1*c))/(24*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)) + (5*c*x^2*(b^3 + 4*a*b*c)*(b*c1 - b1*c))/(8*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)) + (25*b*c^3*x^4*(b*c1 - b1*c))/(16*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)))/(x^3*(8*b^3 + 12*a*b^2*c) + x^2*(12*a*b^2 + 3*a^2*c) + x^4*(3*a*c^2 + 12*b^2*c) + a^3 + c^3*x^6 + 6*b*c^2*x^5 + 6*a^2*b*x) - (5*c^2*atan((16*((5*c^3*x*(b*c1 - b1*c))/(16*(a*c - b^2))^(7/2)) + (5*c^2*(b*c1 - b1*c)*(32*b^7 - 32*a^3*b*c^3 + 96*a^2*b^3*c^2 - 96*a*b^5*c))/(512*(a*c - b^2))^(7/2)*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)))*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c))/(5*b1*c^3 - 5*b*c^2*c1)*(b*c1 - b1*c))/(16*(a*c - b^2))^(7/2))

3.198 $\int (b_1 + c_1 x) (a + 2bx + cx^2)^{-n} dx$

Optimal. Leaf size=169

$$\frac{c_1 (a + 2bx + cx^2)^{1-n}}{2c(1-n)} - \frac{2^{-n}(b_1c - bc_1) \left(\frac{-b - \sqrt{b^2 - ac} + cx}{\sqrt{b^2 - ac}} \right)^{-1+n} (a + 2bx + cx^2)^{1-n} {}_2F_1 \left(1 - n, n; 2 - n; \frac{b + cx + \sqrt{b^2 - ac}}{2\sqrt{b^2 - ac}} \right)}{c\sqrt{b^2 - ac} (1-n)}$$

[Out] 1/2*c1*(c*x^2+2*b*x+a)^(1-n)/c/(1-n)-(-b*c1+b1*c)*(c*x^2+2*b*x+a)^(1-n)*hypergeom([n, 1-n], [2-n], 1/2*(b+c*x+(-a*c+b^2)^(1/2))/(-a*c+b^2)^(1/2))*((-b-c*x+(-a*c+b^2)^(1/2))/(-a*c+b^2)^(1/2))^(1+n)/(2^n)/c/(1-n)/(-a*c+b^2)^(1/2)

Rubi [A]

time = 0.05, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {654, 638}

$$\frac{c_1 (a + 2bx + cx^2)^{1-n}}{2c(1-n)} - \frac{2^{-n}(b_1c - bc_1) \left(\frac{-\sqrt{b^2 - ac} + b + cx}{\sqrt{b^2 - ac}} \right)^{n-1} (a + 2bx + cx^2)^{1-n} {}_2F_1 \left(1 - n, n; 2 - n; \frac{b + cx + \sqrt{b^2 - ac}}{2\sqrt{b^2 - ac}} \right)}{c(1-n)\sqrt{b^2 - ac}}$$

Antiderivative was successfully verified.

[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x]

[Out] (c1*(a + 2*b*x + c*x^2)^(1 - n))/(2*c*(1 - n)) - ((b1*c - b*c1)*(-(b - Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c]))^(1 + n)*(a + 2*b*x + c*x^2)^(1 - n)*Hypergeometric2F1[1 - n, n, 2 - n, (b + Sqrt[b^2 - a*c] + c*x)/(2*Sqrt[b^2 - a*c])]/(2^n*c*Sqrt[b^2 - a*c]*(1 - n))

Rule 638

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1))*((q - b - 2*c*x)/(2*q))^(p + 1))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[4*p]

Rule 654

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int (b_1 + c_1 x) (a + 2bx + cx^2)^{-n} dx = \frac{c_1 (a + 2bx + cx^2)^{1-n}}{2c(1-n)} + \frac{(2b_1c - 2bc_1) \int (a + 2bx + cx^2)^{-n} dx}{2c}$$

$$= \frac{c_1 (a + 2bx + cx^2)^{1-n}}{2c(1-n)} - \frac{2^{-n}(b_1c - bc_1) \left(-\frac{b - \sqrt{b^2 - ac} + cx}{\sqrt{b^2 - ac}} \right)^{-1+n}}{c\sqrt{b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

time = 0.52, size = 264, normalized size = 1.56

$$\frac{1}{2}(a + x(2b + cx))^{-n} \left(c_1 x^2 \left(\frac{b - \sqrt{b^2 - ac} + cx}{b - \sqrt{b^2 - ac}} \right)^n \left(\frac{b + \sqrt{b^2 - ac} + cx}{b + \sqrt{b^2 - ac}} \right)^n F_1 \left(2, n, n; 3; -\frac{cx}{b + \sqrt{b^2 - ac}}, \frac{cx}{-b + \sqrt{b^2 - ac}} \right) - \frac{2^{1-n} b_1 (b - \sqrt{b^2 - ac} + cx) \left(\frac{b - \sqrt{b^2 - ac} + cx}{\sqrt{b^2 - ac}} \right)^n {}_2F_1 \left(1 - n, n; 2 - n; \frac{b - \sqrt{b^2 - ac} - cx}{2\sqrt{b^2 - ac}} \right)}{c(-1+n)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x]

[Out] (c1*x^2*((b - Sqrt[b^2 - a*c] + c*x)/(b - Sqrt[b^2 - a*c]))^n*((b + Sqrt[b^2 - a*c] + c*x)/(b + Sqrt[b^2 - a*c]))^n*AppellF1[2, n, n, 3, -((c*x)/(b + Sqrt[b^2 - a*c])), (c*x)/(-b + Sqrt[b^2 - a*c])] - (2^(1 - n)*b1*(b - Sqrt[b^2 - a*c] + c*x)*((b + Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c])^n*Hypergeometric2F1[1 - n, n, 2 - n, (-b + Sqrt[b^2 - a*c] - c*x)/(2*Sqrt[b^2 - a*c])])/(c*(-1 + n)))/(2*(a + x*(2*b + c*x))^n)

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x]')

[Out] Timed out

Maple [F]

time = 0.06, size = 0, normalized size = 0.00

$$\int (c_1 x + b_1) (cx^2 + 2bx + a)^{-n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c1*x+b1)/((c*x^2+2*b*x+a)^n),x)

[Out] $\text{int}((c1*x+b1)/((c*x^2+2*b*x+a)^n), x)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((c1*x+b1)/((c*x^2+2*b*x+a)^n), x, \text{algorithm}="maxima")$

[Out] $\text{integrate}((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)$

Fricas [F]

time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((c1*x+b1)/((c*x^2+2*b*x+a)^n), x, \text{algorithm}="fricas")$

[Out] $\text{integral}((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((c1*x+b1)/((c*x**2+2*b*x+a)**n), x)$

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((c1*x+b1)/((c*x^2+2*b*x+a)^n), x)$

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b_1 + c_1 x}{(c x^2 + 2 b x + a)^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((b1 + c1*x)/(a + 2*b*x + c*x^2)^n, x)$

[Out] $\text{int}((b1 + c1*x)/(a + 2*b*x + c*x^2)^n, x)$

3.199 $\int \frac{x}{3+6x+2x^2} dx$

Optimal. Leaf size=49

$$\frac{1}{4} (1 - \sqrt{3}) \log(3 - \sqrt{3} + 2x) + \frac{1}{4} (1 + \sqrt{3}) \log(3 + \sqrt{3} + 2x)$$

[Out] 1/4*ln(3+2*x-3^(1/2))*(1-3^(1/2))+1/4*ln(3+2*x+3^(1/2))*(1+3^(1/2))

Rubi [A]

time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {646, 31}

$$\frac{1}{4} (1 - \sqrt{3}) \log(2x - \sqrt{3} + 3) + \frac{1}{4} (1 + \sqrt{3}) \log(2x + \sqrt{3} + 3)$$

Antiderivative was successfully verified.

[In] Int[x/(3 + 6*x + 2*x^2),x]

[Out] ((1 - Sqrt[3])*Log[3 - Sqrt[3] + 2*x])/4 + ((1 + Sqrt[3])*Log[3 + Sqrt[3] + 2*x])/4

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 646

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{x}{3+6x+2x^2} dx &= \frac{1}{2} (1 - \sqrt{3}) \int \frac{1}{3 - \sqrt{3} + 2x} dx + \frac{1}{2} (1 + \sqrt{3}) \int \frac{1}{3 + \sqrt{3} + 2x} dx \\ &= \frac{1}{4} (1 - \sqrt{3}) \log(3 - \sqrt{3} + 2x) + \frac{1}{4} (1 + \sqrt{3}) \log(3 + \sqrt{3} + 2x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 44, normalized size = 0.90

$$\frac{1}{4} \left(- \left((-1 + \sqrt{3}) \log(-3 + \sqrt{3} - 2x) \right) + (1 + \sqrt{3}) \log(3 + \sqrt{3} + 2x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/(3 + 6*x + 2*x^2),x]

[Out] $(-((-1 + \sqrt{3}) \cdot \text{Log}[-3 + \sqrt{3} - 2x]) + (1 + \sqrt{3}) \cdot \text{Log}[3 + \sqrt{3} + 2x])/4$

Mathics [A]

time = 2.65, size = 35, normalized size = 0.71

$$\frac{(1 + \sqrt{3}) \text{Log}\left[\frac{3}{2} + \frac{\sqrt{3}}{2} + x\right]}{4} + \frac{(1 - \sqrt{3}) \text{Log}\left[\frac{3}{2} - \frac{\sqrt{3}}{2} + x\right]}{4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x/(3 + 6*x + 2*x^2),x]')

[Out] $(1 + \sqrt{3}) \text{Log}[3 / 2 + \sqrt{3} / 2 + x] / 4 + (1 - \sqrt{3}) \text{Log}[3 / 2 - \sqrt{3} / 2 + x] / 4$

Maple [A]

time = 0.08, size = 31, normalized size = 0.63

method	result	size
default	$\frac{\ln(2x^2+6x+3)}{4} + \frac{\sqrt{3} \operatorname{arctanh}\left(\frac{(4x+6)\sqrt{3}}{6}\right)}{2}$	31
risch	$\frac{\ln(3+2x+\sqrt{3})}{4} + \frac{\ln(3+2x+\sqrt{3})\sqrt{3}}{4} + \frac{\ln(3+2x-\sqrt{3})}{4} - \frac{\ln(3+2x-\sqrt{3})\sqrt{3}}{4}$	56

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(2*x^2+6*x+3),x,method=_RETURNVERBOSE)

[Out] $1/4 \cdot \ln(2x^2+6x+3) + 1/2 \cdot 3^{(1/2)} \cdot \operatorname{arctanh}(1/6 \cdot (4x+6) \cdot 3^{(1/2)})$

Maxima [A]

time = 0.36, size = 41, normalized size = 0.84

$$-\frac{1}{4} \sqrt{3} \log\left(\frac{2x - \sqrt{3} + 3}{2x + \sqrt{3} + 3}\right) + \frac{1}{4} \log(2x^2 + 6x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2*x^2+6*x+3),x, algorithm="maxima")

[Out] $-1/4 \cdot \sqrt{3} \cdot \log((2x - \sqrt{3} + 3)/(2x + \sqrt{3} + 3)) + 1/4 \cdot \log(2x^2 + 6x + 3)$

Fricas [A]

time = 0.36, size = 52, normalized size = 1.06

$$\frac{1}{4} \sqrt{3} \log \left(\frac{2x^2 + \sqrt{3}(2x+3) + 6x+6}{2x^2 + 6x+3} \right) + \frac{1}{4} \log(2x^2 + 6x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x/(2*x^2+6*x+3),x, algorithm="fricas")`

```
[Out] 1/4*sqrt(3)*log((2*x^2 + sqrt(3)*(2*x + 3) + 6*x + 6)/(2*x^2 + 6*x + 3)) +
1/4*log(2*x^2 + 6*x + 3)
```

Sympy [A]

time = 0.06, size = 46, normalized size = 0.94

$$\left(\frac{1}{4} - \frac{\sqrt{3}}{4} \right) \log \left(x - \frac{\sqrt{3}}{2} + \frac{3}{2} \right) + \left(\frac{1}{4} + \frac{\sqrt{3}}{4} \right) \log \left(x + \frac{\sqrt{3}}{2} + \frac{3}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x/(2*x**2+6*x+3),x)`

```
[Out] (1/4 - sqrt(3)/4)*log(x - sqrt(3)/2 + 3/2) + (1/4 + sqrt(3)/4)*log(x + sqrt
(3)/2 + 3/2)
```

Giac [A]

time = 0.00, size = 57, normalized size = 1.16

$$\frac{\ln|2x^2 + 6x + 3|}{4} - \frac{3 \ln \left| \frac{4x+6-2\sqrt{3}}{4x+6+2\sqrt{3}} \right|}{2 \cdot 2\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x/(2*x^2+6*x+3),x)`

```
[Out] -1/4*sqrt(3)*log(abs((2*x - sqrt(3) + 3)/(2*x + sqrt(3) + 3))) + 1/4*log(ab
s(2*x^2 + 6*x + 3))
```

Mupad [B]

time = 0.17, size = 36, normalized size = 0.73

$$\ln \left(x + \frac{\sqrt{3}}{2} + \frac{3}{2} \right) \left(\frac{\sqrt{3}}{4} + \frac{1}{4} \right) - \ln \left(x - \frac{\sqrt{3}}{2} + \frac{3}{2} \right) \left(\frac{\sqrt{3}}{4} - \frac{1}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x/(6*x + 2*x^2 + 3),x)`

```
[Out] log(x + 3^(1/2)/2 + 3/2)*(3^(1/2)/4 + 1/4) - log(x - 3^(1/2)/2 + 3/2)*(3^(1
/2)/4 - 1/4)
```

$$3.200 \quad \int \frac{-3+2x}{(3+6x+2x^2)^3} dx$$

Optimal. Leaf size=61

$$\frac{5+4x}{4(3+6x+2x^2)^2} - \frac{3+2x}{2(3+6x+2x^2)} + \frac{\tanh^{-1}\left(\frac{3+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] 1/4*(5+4*x)/(2*x^2+6*x+3)^2+1/2*(-3-2*x)/(2*x^2+6*x+3)+1/3*arctanh(1/3*(3+2*x)*3^(1/2))*3^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {652, 628, 632, 212}

$$-\frac{2x+3}{2(2x^2+6x+3)} + \frac{4x+5}{4(2x^2+6x+3)^2} + \frac{\tanh^{-1}\left(\frac{2x+3}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-3 + 2*x)/(3 + 6*x + 2*x^2)^3,x]

[Out] (5 + 4*x)/(4*(3 + 6*x + 2*x^2)^2) - (3 + 2*x)/(2*(3 + 6*x + 2*x^2)) + ArcTanh[(3 + 2*x)/Sqrt[3]]/Sqrt[3]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p + 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*x
+ c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned} \int \frac{-3 + 2x}{(3 + 6x + 2x^2)^3} dx &= \frac{5 + 4x}{4(3 + 6x + 2x^2)^2} + 3 \int \frac{1}{(3 + 6x + 2x^2)^2} dx \\ &= \frac{5 + 4x}{4(3 + 6x + 2x^2)^2} - \frac{3 + 2x}{2(3 + 6x + 2x^2)} - \int \frac{1}{3 + 6x + 2x^2} dx \\ &= \frac{5 + 4x}{4(3 + 6x + 2x^2)^2} - \frac{3 + 2x}{2(3 + 6x + 2x^2)} + 2 \text{Subst} \left(\int \frac{1}{12 - x^2} dx, x, 6 + 4x \right) \\ &= \frac{5 + 4x}{4(3 + 6x + 2x^2)^2} - \frac{3 + 2x}{2(3 + 6x + 2x^2)} + \frac{\tanh^{-1} \left(\frac{3+2x}{\sqrt{3}} \right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 70, normalized size = 1.15

$$\frac{1}{12} \left(-\frac{3(13 + 44x + 36x^2 + 8x^3)}{(3 + 6x + 2x^2)^2} - 2\sqrt{3} \log(-3 + \sqrt{3} - 2x) + 2\sqrt{3} \log(3 + \sqrt{3} + 2x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 2*x)/(3 + 6*x + 2*x^2)^3, x]
```

```
[Out] ((-3*(13 + 44*x + 36*x^2 + 8*x^3))/(3 + 6*x + 2*x^2)^2 - 2*Sqrt[3]*Log[-3 +
Sqrt[3] - 2*x] + 2*Sqrt[3]*Log[3 + Sqrt[3] + 2*x])/12
```

Mathics [A]

time = 2.27, size = 85, normalized size = 1.39

$$\frac{-39 - 132x - 108x^2 - 24x^3 + 2\sqrt{3}(9 + 36x + 48x^2 + 24x^3 + 4x^4) \left(\text{Log} \left[\frac{3}{2} + \frac{\sqrt{3}}{2} + x \right] - \text{Log} \left[\frac{3}{2} - \frac{\sqrt{3}}{2} + x \right] \right)}{108 + 432x + 576x^2 + 288x^3 + 48x^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(2*x - 3)/(3 + 6*x + 2*x^2)^3, x]')
```

[Out] $(-39 - 132x - 108x^2 - 24x^3 + 2\sqrt{3}(9 + 36x + 48x^2 + 24x^3 + 4x^4) (\log[3/2 + \sqrt{3}/2 + x] - \log[3/2 - \sqrt{3}/2 + x])) / (12(9 + 36x + 48x^2 + 24x^3 + 4x^4))$

Maple [A]

time = 0.07, size = 56, normalized size = 0.92

method	result	size
default	$-\frac{-24x-30}{24(2x^2+6x+3)^2} - \frac{4x+6}{4(2x^2+6x+3)} + \frac{\sqrt{3} \operatorname{arctanh}\left(\frac{(4x+6)\sqrt{3}}{6}\right)}{3}$	56
risch	$\frac{-2x^3-9x^2-11x-\frac{13}{4}}{(2x^2+6x+3)^2} + \frac{\ln(3+2x+\sqrt{3})\sqrt{3}}{6} - \frac{\ln(3+2x-\sqrt{3})\sqrt{3}}{6}$	61

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x-3)/(2*x^2+6*x+3)^3,x,method=_RETURNVERBOSE)`

[Out] $-1/24*(-24*x-30)/(2*x^2+6*x+3)^2-1/4*(4*x+6)/(2*x^2+6*x+3)+1/3*3^{(1/2)}*\operatorname{arctanh}(1/6*(4*x+6)*3^{(1/2)})$

Maxima [A]

time = 0.34, size = 67, normalized size = 1.10

$$-\frac{1}{6}\sqrt{3}\log\left(\frac{2x-\sqrt{3}+3}{2x+\sqrt{3}+3}\right) - \frac{8x^3+36x^2+44x+13}{4(4x^4+24x^3+48x^2+36x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3+2*x)/(2*x^2+6*x+3)^3,x, algorithm="maxima")`

[Out] $-1/6*\sqrt{3}*\log((2*x - \sqrt{3} + 3)/(2*x + \sqrt{3} + 3)) - 1/4*(8*x^3 + 36*x^2 + 44*x + 13)/(4*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)$

Fricas [A]

time = 0.33, size = 97, normalized size = 1.59

$$\frac{24x^3 - 2\sqrt{3}(4x^4 + 24x^3 + 48x^2 + 36x + 9)\log\left(\frac{2x^2+\sqrt{3}(2x+3)+6x+6}{2x^2+6x+3}\right) + 108x^2 + 132x + 39}{12(4x^4 + 24x^3 + 48x^2 + 36x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3+2*x)/(2*x^2+6*x+3)^3,x, algorithm="fricas")`

[Out] $-1/12*(24*x^3 - 2*\sqrt{3}*(4*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)*\log((2*x^2 + \sqrt{3}*(2*x + 3) + 6*x + 6)/(2*x^2 + 6*x + 3)) + 108*x^2 + 132*x + 39)/(4*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)$

Sympy [A]

time = 0.09, size = 76, normalized size = 1.25

$$\frac{-8x^3 - 36x^2 - 44x - 13}{16x^4 + 96x^3 + 192x^2 + 144x + 36} - \frac{\sqrt{3} \log\left(x - \frac{\sqrt{3}}{2} + \frac{3}{2}\right)}{6} + \frac{\sqrt{3} \log\left(x + \frac{\sqrt{3}}{2} + \frac{3}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+2*x)/(2*x**2+6*x+3)**3,x)**[Out]** (-8*x**3 - 36*x**2 - 44*x - 13)/(16*x**4 + 96*x**3 + 192*x**2 + 144*x + 36) - sqrt(3)*log(x - sqrt(3)/2 + 3/2)/6 + sqrt(3)*log(x + sqrt(3)/2 + 3/2)/6**Giac [A]**

time = 0.00, size = 72, normalized size = 1.18

$$-\frac{8x^3 + 36x^2 + 44x + 13}{4(2x^2 + 6x + 3)^2} - \frac{\ln\left|\frac{4x+6-2\sqrt{3}}{4x+6+2\sqrt{3}}\right|}{2\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+2*x)/(2*x^2+6*x+3)^3,x)**[Out]** -1/6*sqrt(3)*log(abs((2*x - sqrt(3) + 3)/(2*x + sqrt(3) + 3))) - 1/4*(8*x^3 + 36*x^2 + 44*x + 13)/(2*x^2 + 6*x + 3)^2**Mupad [B]**

time = 0.21, size = 53, normalized size = 0.87

$$\frac{\sqrt{3} \operatorname{atanh}\left(\sqrt{3} \left(\frac{2x}{3} + 1\right)\right)}{3} - \frac{\frac{x^3}{2} + \frac{9x^2}{4} + \frac{11x}{4} + \frac{13}{16}}{x^4 + 6x^3 + 12x^2 + 9x + \frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x - 3)/(6*x + 2*x^2 + 3)^3,x)**[Out]** (3^(1/2)*atanh(3^(1/2)*((2*x)/3 + 1)))/3 - ((11*x)/4 + (9*x^2)/4 + x^3/2 + 13/16)/(9*x + 12*x^2 + 6*x^3 + x^4 + 9/4)

$$3.201 \quad \int \frac{-1+x}{(4+5x+x^2)^2} dx$$

Optimal. Leaf size=36

$$\frac{13+7x}{9(4+5x+x^2)} + \frac{7}{27} \log(1+x) - \frac{7}{27} \log(4+x)$$

[Out] 1/9*(13+7*x)/(x^2+5*x+4)+7/27*ln(1+x)-7/27*ln(4+x)

Rubi [A]

time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {652, 630, 31}

$$\frac{7x+13}{9(x^2+5x+4)} + \frac{7}{27} \log(x+1) - \frac{7}{27} \log(x+4)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x)/(4 + 5*x + x^2)^2,x]

[Out] (13 + 7*x)/(9*(4 + 5*x + x^2)) + (7*Log[1 + x])/27 - (7*Log[4 + x])/27

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 630

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 652

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rubi steps

$$\begin{aligned} \int \frac{-1+x}{(4+5x+x^2)^2} dx &= \frac{13+7x}{9(4+5x+x^2)} + \frac{7}{9} \int \frac{1}{4+5x+x^2} dx \\ &= \frac{13+7x}{9(4+5x+x^2)} + \frac{7}{27} \int \frac{1}{1+x} dx - \frac{7}{27} \int \frac{1}{4+x} dx \\ &= \frac{13+7x}{9(4+5x+x^2)} + \frac{7}{27} \log(1+x) - \frac{7}{27} \log(4+x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 33, normalized size = 0.92

$$\frac{1}{27} \left(\frac{39+21x}{4+5x+x^2} + 7 \log(1+x) - 7 \log(4+x) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(-1 + x)/(4 + 5*x + x^2)^2, x]``[Out] ((39 + 21*x)/(4 + 5*x + x^2) + 7*Log[1 + x] - 7*Log[4 + x])/27`**Mathics [A]**

time = 2.00, size = 38, normalized size = 1.06

$$\frac{39 + 21x + 7(4 + 5x + x^2)(\text{Log}[1 + x] - \text{Log}[4 + x])}{108 + 135x + 27x^2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(x - 1)/(x^2 + 5*x + 4)^2, x]')``[Out] (39 + 21 x + 7 (4 + 5 x + x ^ 2) (Log[1 + x] - Log[4 + x])) / (27 (4 + 5 x + x ^ 2))`**Maple [A]**

time = 0.07, size = 28, normalized size = 0.78

method	result	size
default	$\frac{5}{9(4+x)} - \frac{7 \ln(4+x)}{27} + \frac{2}{9(1+x)} + \frac{7 \ln(1+x)}{27}$	28
norman	$\frac{\frac{7x}{9} + \frac{13}{9}}{x^2+5x+4} + \frac{7 \ln(1+x)}{27} - \frac{7 \ln(4+x)}{27}$	30
risch	$\frac{\frac{7x}{9} + \frac{13}{9}}{x^2+5x+4} + \frac{7 \ln(1+x)}{27} - \frac{7 \ln(4+x)}{27}$	30

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((-1+x)/(x^2+5*x+4)^2, x, method=_RETURNVERBOSE)`

[Out] $5/9/(4+x) - 7/27*\ln(4+x) + 2/9/(1+x) + 7/27*\ln(1+x)$

Maxima [A]

time = 0.25, size = 30, normalized size = 0.83

$$\frac{7x + 13}{9(x^2 + 5x + 4)} - \frac{7}{27} \log(x + 4) + \frac{7}{27} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+x)/(x^2+5*x+4)^2,x, algorithm="maxima")`

[Out] $1/9*(7*x + 13)/(x^2 + 5*x + 4) - 7/27*\log(x + 4) + 7/27*\log(x + 1)$

Fricas [A]

time = 0.32, size = 45, normalized size = 1.25

$$-\frac{7(x^2 + 5x + 4)\log(x + 4) - 7(x^2 + 5x + 4)\log(x + 1) - 21x - 39}{27(x^2 + 5x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+x)/(x^2+5*x+4)^2,x, algorithm="fricas")`

[Out] $-1/27*(7*(x^2 + 5*x + 4)*\log(x + 4) - 7*(x^2 + 5*x + 4)*\log(x + 1) - 21*x - 39)/(x^2 + 5*x + 4)$

Sympy [A]

time = 0.06, size = 31, normalized size = 0.86

$$\frac{7x + 13}{9x^2 + 45x + 36} + \frac{7\log(x + 1)}{27} - \frac{7\log(x + 4)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+x)/(x**2+5*x+4)**2,x)`

[Out] $(7*x + 13)/(9*x**2 + 45*x + 36) + 7*\log(x + 1)/27 - 7*\log(x + 4)/27$

Giac [A]

time = 0.00, size = 36, normalized size = 1.00

$$\frac{7}{27} \ln|x + 1| - \frac{7}{27} \ln|x + 4| + \frac{7x + 13}{9(x^2 + 5x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+x)/(x^2+5*x+4)^2,x)`

[Out] $1/9*(7*x + 13)/(x^2 + 5*x + 4) - 7/27*\log(\text{abs}(x + 4)) + 7/27*\log(\text{abs}(x + 1))$

Mupad [B]

time = 0.05, size = 25, normalized size = 0.69

$$\frac{\frac{7x}{9} + \frac{13}{9}}{x^2 + 5x + 4} - \frac{14 \operatorname{atanh}\left(\frac{2x}{3} + \frac{5}{3}\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x - 1)/(5*x + x^2 + 4)^2,x)`

[Out] `((7*x)/9 + 13/9)/(5*x + x^2 + 4) - (14*atanh((2*x)/3 + 5/3))/27`

$$3.202 \quad \int \frac{1}{(2+3x+x^2)^5} dx$$

Optimal. Leaf size=87

$$\frac{-3-2x}{4(2+3x+x^2)^4} + \frac{7(3+2x)}{6(2+3x+x^2)^3} - \frac{35(3+2x)}{6(2+3x+x^2)^2} + \frac{35(3+2x)}{2+3x+x^2} + 70 \log(1+x) - 70 \log(2+x)$$

[Out] 1/4*(-3-2*x)/(x^2+3*x+2)^4+7/6*(3+2*x)/(x^2+3*x+2)^3-35/6*(3+2*x)/(x^2+3*x+2)^2+35*(3+2*x)/(x^2+3*x+2)+70*ln(1+x)-70*ln(2+x)

Rubi [A]

time = 0.01, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {628, 630, 31}

$$\frac{35(2x+3)}{x^2+3x+2} - \frac{35(2x+3)}{6(x^2+3x+2)^2} + \frac{7(2x+3)}{6(x^2+3x+2)^3} - \frac{2x+3}{4(x^2+3x+2)^4} + 70 \log(x+1) - 70 \log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + x^2)^(-5), x]

[Out] -1/4*(3 + 2*x)/(2 + 3*x + x^2)^4 + (7*(3 + 2*x))/(6*(2 + 3*x + x^2)^3) - (35*(3 + 2*x))/(6*(2 + 3*x + x^2)^2) + (35*(3 + 2*x))/(2 + 3*x + x^2) + 70*Log[1 + x] - 70*Log[2 + x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 628

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p + 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 630

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2+3x+x^2)^5} dx &= -\frac{3+2x}{4(2+3x+x^2)^4} - \frac{7}{2} \int \frac{1}{(2+3x+x^2)^4} dx \\
&= -\frac{3+2x}{4(2+3x+x^2)^4} + \frac{7(3+2x)}{6(2+3x+x^2)^3} + \frac{35}{3} \int \frac{1}{(2+3x+x^2)^3} dx \\
&= -\frac{3+2x}{4(2+3x+x^2)^4} + \frac{7(3+2x)}{6(2+3x+x^2)^3} - \frac{35(3+2x)}{6(2+3x+x^2)^2} - 35 \int \frac{1}{(2+3x+x^2)^2} dx \\
&= -\frac{3+2x}{4(2+3x+x^2)^4} + \frac{7(3+2x)}{6(2+3x+x^2)^3} - \frac{35(3+2x)}{6(2+3x+x^2)^2} + \frac{35(3+2x)}{2+3x+x^2} + 70 \int \frac{1}{2+3x+x^2} dx \\
&= -\frac{3+2x}{4(2+3x+x^2)^4} + \frac{7(3+2x)}{6(2+3x+x^2)^3} - \frac{35(3+2x)}{6(2+3x+x^2)^2} + \frac{35(3+2x)}{2+3x+x^2} + 70 \int \frac{1}{1+x} dx \\
&= -\frac{3+2x}{4(2+3x+x^2)^4} + \frac{7(3+2x)}{6(2+3x+x^2)^3} - \frac{35(3+2x)}{6(2+3x+x^2)^2} + \frac{35(3+2x)}{2+3x+x^2} + 70 \log(1+x)
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 87, normalized size = 1.00

$$\frac{-3-2x}{4(2+3x+x^2)^4} + \frac{7(3+2x)}{6(2+3x+x^2)^3} - \frac{35(3+2x)}{6(2+3x+x^2)^2} + \frac{35(3+2x)}{2+3x+x^2} + 70 \log(1+x) - 70 \log(2+x)$$

Antiderivative was successfully verified.

`[In] Integrate[(2 + 3*x + x^2)^(-5), x]`

```
[Out] (-3 - 2*x)/(4*(2 + 3*x + x^2)^4) + (7*(3 + 2*x))/(6*(2 + 3*x + x^2)^3) - (35*(3 + 2*x))/(6*(2 + 3*x + x^2)^2) + (35*(3 + 2*x))/(2 + 3*x + x^2) + 70*Log[1 + x] - 70*Log[2 + x]
```

Mathics [A]

time = 2.45, size = 128, normalized size = 1.47

$$\frac{9315 + 49176x + 109116x^2 + 131768x^3 + 93450x^4 + 38920x^5 + 8820x^6 + 840x^7 + 840(16 + 96x + 248x^2 + 360x^3 + 321x^4 + 180x^5 + 62x^6 + 12x^7 + x^8)(\log[1+x] - \log[2+x])}{192 + 1152x + 2976x^2 + 4320x^3 + 3852x^4 + 2160x^5 + 744x^6 + 144x^7 + 12x^8}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^2 + 3*x + 2)^5, x]')`

```
[Out] (9315 + 49176 x + 109116 x ^ 2 + 131768 x ^ 3 + 93450 x ^ 4 + 38920 x ^ 5 + 8820 x ^ 6 + 840 x ^ 7 + 840 (16 + 96 x + 248 x ^ 2 + 360 x ^ 3 + 321 x ^ 4 + 180 x ^ 5 + 62 x ^ 6 + 12 x ^ 7 + x ^ 8) (Log[1 + x] - Log[2 + x])) / (12 (16 + 96 x + 248 x ^ 2 + 360 x ^ 3 + 321 x ^ 4 + 180 x ^ 5 + 62 x ^ 6 + 12 x ^ 7 + x ^ 8))
```

Maple [A]

time = 0.07, size = 70, normalized size = 0.80

method	result
norman	$\frac{4098x+70x^7+735x^6+\frac{9730}{3}x^5+9093x^2+\frac{15575}{2}x^4+\frac{32942}{3}x^3+\frac{3105}{4}}{(x^2+3x+2)^4} + 70 \ln(1+x) - 70 \ln(2+x)$
risch	$\frac{4098x+70x^7+735x^6+\frac{9730}{3}x^5+9093x^2+\frac{15575}{2}x^4+\frac{32942}{3}x^3+\frac{3105}{4}}{(x^2+3x+2)^4} + 70 \ln(1+x) - 70 \ln(2+x)$
default	$\frac{1}{4(2+x)^4} + \frac{5}{3(2+x)^3} + \frac{15}{2(2+x)^2} + \frac{35}{2+x} - 70 \ln(2+x) - \frac{1}{4(1+x)^4} + \frac{5}{3(1+x)^3} - \frac{15}{2(1+x)^2} + \frac{35}{1+x} + 70 \ln(1+x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2+3*x+2)^5,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{4(2+x)^4} + \frac{5}{3(2+x)^3} + \frac{15}{2(2+x)^2} + \frac{35}{2+x} - 70 \ln(2+x) - \frac{1}{4(1+x)^4} + \frac{5}{3(1+x)^3} - \frac{15}{2(1+x)^2} + \frac{35}{1+x} + 70 \ln(1+x)$

Maxima [A]

time = 0.26, size = 90, normalized size = 1.03

$\frac{840x^7 + 8820x^6 + 38920x^5 + 93450x^4 + 131768x^3 + 109116x^2 + 49176x + 9315}{12(x^8 + 12x^7 + 62x^6 + 180x^5 + 321x^4 + 360x^3 + 248x^2 + 96x + 16)} - 70 \log(x+2) + 70 \log(x+1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2+3*x+2)^5,x, algorithm="maxima")`

[Out] $\frac{1}{12} \frac{(840x^7 + 8820x^6 + 38920x^5 + 93450x^4 + 131768x^3 + 109116x^2 + 49176x + 9315)}{(x^8 + 12x^7 + 62x^6 + 180x^5 + 321x^4 + 360x^3 + 248x^2 + 96x + 16)} - 70 \log(x+2) + 70 \log(x+1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(81) = 162.

time = 0.32, size = 165, normalized size = 1.90

$\frac{840x^7 + 8820x^6 + 38920x^5 + 93450x^4 + 131768x^3 + 109116x^2 + 49176x + 9315}{12(x^8 + 12x^7 + 62x^6 + 180x^5 + 321x^4 + 360x^3 + 248x^2 + 96x + 16)} - 70 \log(x+2) + 70 \log(x+1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2+3*x+2)^5,x, algorithm="fricas")`

[Out] $\frac{1}{12} \frac{(840x^7 + 8820x^6 + 38920x^5 + 93450x^4 + 131768x^3 + 109116x^2 + 49176x + 9315)}{(x^8 + 12x^7 + 62x^6 + 180x^5 + 321x^4 + 360x^3 + 248x^2 + 96x + 16)} - 70 \log(x+2) + 70 \log(x+1)$

Sympy [A]

time = 0.10, size = 88, normalized size = 1.01

$\frac{840x^7 + 8820x^6 + 38920x^5 + 93450x^4 + 131768x^3 + 109116x^2 + 49176x + 9315}{12x^8 + 144x^7 + 744x^6 + 2160x^5 + 3852x^4 + 4320x^3 + 2976x^2 + 1152x + 192} + 70 \log(x+1) - 70 \log(x+2)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+3*x+2)**5,x)

[Out] (840*x**7 + 8820*x**6 + 38920*x**5 + 93450*x**4 + 131768*x**3 + 109116*x**2 + 49176*x + 9315)/(12*x**8 + 144*x**7 + 744*x**6 + 2160*x**5 + 3852*x**4 + 4320*x**3 + 2976*x**2 + 1152*x + 192) + 70*log(x + 1) - 70*log(x + 2)

Giac [A]

time = 0.00, size = 65, normalized size = 0.75

$$70 \ln|x+1| - 70 \ln|x+2| + \frac{840x^7 + 8820x^6 + 38920x^5 + 93450x^4 + 131768x^3 + 109116x^2 + 49176x + 9315}{12(x^2 + 3x + 2)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+3*x+2)^5,x)

[Out] 1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 109116*x^2 + 49176*x + 9315)/(x^2 + 3*x + 2)^4 - 70*log(abs(x + 2)) + 70*log(abs(x + 1))

Mupad [B]

time = 0.09, size = 65, normalized size = 0.75

$$70 \ln\left(\frac{x+1}{x+2}\right) + 70\left(x + \frac{3}{2}\right) \left(\frac{1}{x^2 + 3x + 2} - \frac{1}{6(x^2 + 3x + 2)^2} + \frac{1}{30(x^2 + 3x + 2)^3} - \frac{1}{140(x^2 + 3x + 2)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x + x^2 + 2)^5,x)

[Out] 70*log((x + 1)/(x + 2)) + 70*(x + 3/2)*(1/(3*x + x^2 + 2) - 1/(6*(3*x + x^2 + 2)^2) + 1/(30*(3*x + x^2 + 2)^3) - 1/(140*(3*x + x^2 + 2)^4))

$$3.203 \quad \int \frac{1}{x^3(7-6x+2x^2)^2} dx$$

Optimal. Leaf size=81

$$\frac{1}{490x^2} - \frac{69}{1715x} - \frac{2-3x}{35x^2(7-6x+2x^2)} - \frac{234 \tan^{-1}\left(\frac{3-2x}{\sqrt{5}}\right)}{12005\sqrt{5}} + \frac{80 \log(x)}{2401} - \frac{40 \log(7-6x+2x^2)}{2401}$$

[Out] $-1/490/x^2 - 69/1715/x + 1/35 * (-2 + 3*x) / x^2 / (2*x^2 - 6*x + 7) + 80/2401 * \ln(x) - 40/2401 * \ln(2*x^2 - 6*x + 7) - 234/60025 * \arctan(1/5 * (3 - 2*x) * 5^{(1/2)}) * 5^{(1/2)}$

Rubi [A]

time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {754, 814, 648, 632, 210, 642}

$$-\frac{2-3x}{35x^2(2x^2-6x+7)} - \frac{1}{490x^2} - \frac{40 \log(2x^2-6x+7)}{2401} - \frac{69}{1715x} + \frac{80 \log(x)}{2401} - \frac{234 \tan^{-1}\left(\frac{3-2x}{\sqrt{5}}\right)}{12005\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(7 - 6*x + 2*x^2)^2),x]

[Out] $-1/490 * 1/x^2 - 69/(1715*x) - (2 - 3*x)/(35*x^2*(7 - 6*x + 2*x^2)) - (234 * \text{ArcTan}[(3 - 2*x)/\text{Sqrt}[5]])/(12005 * \text{Sqrt}[5]) + (80 * \text{Log}[x])/2401 - (40 * \text{Log}[7 - 6*x + 2*x^2])/2401$

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_)^m)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p, x_Symbol] := Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int((((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3(7-6x+2x^2)^2} dx &= -\frac{2-3x}{35x^2(7-6x+2x^2)} + \frac{1}{140} \int \frac{4+36x}{x^3(7-6x+2x^2)} dx \\
 &= -\frac{2-3x}{35x^2(7-6x+2x^2)} + \frac{1}{140} \int \left(\frac{4}{7x^3} + \frac{276}{49x^2} + \frac{1600}{343x} - \frac{8(-717+400x)}{343(7-6x+2x^2)} \right) dx \\
 &= -\frac{1}{490x^2} - \frac{69}{1715x} - \frac{2-3x}{35x^2(7-6x+2x^2)} + \frac{80 \log(x)}{2401} - \frac{2 \int \frac{-717+400x}{7-6x+2x^2} dx}{12005} \\
 &= -\frac{1}{490x^2} - \frac{69}{1715x} - \frac{2-3x}{35x^2(7-6x+2x^2)} + \frac{80 \log(x)}{2401} - \frac{40 \int \frac{-6+4x}{7-6x+2x^2} dx}{2401} + \frac{234}{2401} \\
 &= -\frac{1}{490x^2} - \frac{69}{1715x} - \frac{2-3x}{35x^2(7-6x+2x^2)} + \frac{80 \log(x)}{2401} - \frac{40 \log(7-6x+2x^2)}{2401} \\
 &= -\frac{1}{490x^2} - \frac{69}{1715x} - \frac{2-3x}{35x^2(7-6x+2x^2)} - \frac{234 \tan^{-1}\left(\frac{3-2x}{\sqrt{5}}\right)}{12005\sqrt{5}} + \frac{80 \log(x)}{2401} - \frac{40}{2401}
 \end{aligned}$$

Mathematica [A]

time = 0.02, size = 70, normalized size = 0.86

$$\frac{-\frac{1225}{x^2} - \frac{4200}{x} - \frac{140(-41+9x)}{7-6x+2x^2} + 468\sqrt{5} \tan^{-1}\left(\frac{-3+2x}{\sqrt{5}}\right) + 4000 \log(x) - 2000 \log(7-6x+2x^2)}{120050}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(7 - 6*x + 2*x^2)^2),x]

[Out] (-1225/x^2 - 4200/x - (140*(-41 + 9*x))/(7 - 6*x + 2*x^2) + 468*sqrt[5]*ArcTan[(-3 + 2*x)/sqrt[5]] + 4000*Log[x] - 2000*Log[7 - 6*x + 2*x^2])/120050

Mathics [A]

time = 2.37, size = 79, normalized size = 0.98

$$\frac{-8575 - 22050x + 4x^2(7 - 6x + 2x^2) \left(-500 \operatorname{Log}\left[\frac{7}{2} - 3x + x^2\right] + 117\sqrt{5} \operatorname{ArcTan}\left[\frac{\sqrt{5}(-3+2x)}{5}\right] + 1000 \operatorname{Log}[x] \right) + 28490x^2 - 9660x^3}{120050x^2(7 - 6x + 2x^2)}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x^3*(7 - 6*x + 2*x^2)^2),x]')

[Out] (-8575 - 22050 x + 4 x ^ 2 (7 - 6 x + 2 x ^ 2) (-500 Log[7 / 2 - 3 x + x ^ 2] + 117 Sqrt[5] ArcTan[Sqrt[5] (-3 + 2 x) / 5] + 1000 Log[x]) + 28490 x ^ 2 - 9660 x ^ 3) / (120050 x ^ 2 (7 - 6 x + 2 x ^ 2))

Maple [A]

time = 0.19, size = 62, normalized size = 0.77

method	result	size
default	$-\frac{4\left(\frac{63x}{20} - \frac{287}{20}\right)}{2401(x^2 - 3x + \frac{7}{2})} - \frac{40 \ln(2x^2 - 6x + 7)}{2401} + \frac{234\sqrt{5} \arctan\left(\frac{(4x-6)\sqrt{5}}{10}\right)}{60025} - \frac{1}{98x^2} - \frac{12}{343x} + \frac{80 \ln(x)}{2401}$	62
risch	$-\frac{\frac{138}{1715}x^3 + \frac{407}{1715}x^2 - \frac{9}{49}x - \frac{1}{14}}{x^2(2x^2 - 6x + 7)} - \frac{40 \ln(4x^2 - 12x + 14)}{2401} + \frac{234\sqrt{5} \arctan\left(\frac{(2x-3)\sqrt{5}}{5}\right)}{60025} + \frac{80 \ln(x)}{2401}$	67

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(2*x^2-6*x+7)^2,x,method=_RETURNVERBOSE)

[Out] -4/2401*(63/20*x-287/20)/(x^2-3*x+7/2)-40/2401*ln(2*x^2-6*x+7)+234/60025*5^(1/2)*arctan(1/10*(4*x-6)*5^(1/2))-1/98/x^2-12/343/x+80/2401*ln(x)

Maxima [A]

time = 0.36, size = 69, normalized size = 0.85

$$\frac{234}{60025} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} (2x - 3)\right) - \frac{276x^3 - 814x^2 + 630x + 245}{3430(2x^4 - 6x^3 + 7x^2)} - \frac{40}{2401} \log(2x^2 - 6x + 7) + \frac{80}{2401} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(2*x^2-6*x+7)^2,x, algorithm="maxima")

[Out] 234/60025*sqrt(5)*arctan(1/5*sqrt(5)*(2*x - 3)) - 1/3430*(276*x^3 - 814*x^2 + 630*x + 245)/(2*x^4 - 6*x^3 + 7*x^2) - 40/2401*log(2*x^2 - 6*x + 7) + 80/2401*log(x)

Fricas [A]

time = 0.31, size = 116, normalized size = 1.43

$$\frac{9660x^3 - 468\sqrt{5}(2x^4 - 6x^3 + 7x^2)\arctan\left(\frac{1}{5}\sqrt{5}(2x - 3)\right) - 28490x^2 + 2000(2x^4 - 6x^3 + 7x^2)\log(2x^2 - 6x + 7) - 4000(2x^4 - 6x^3 + 7x^2)\log(x) + 22050x + 8575}{120050(2x^4 - 6x^3 + 7x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(2*x^2-6*x+7)^2,x, algorithm="fricas")

[Out] -1/120050*(9660*x^3 - 468*sqrt(5)*(2*x^4 - 6*x^3 + 7*x^2)*arctan(1/5*sqrt(5)*(2*x - 3)) - 28490*x^2 + 2000*(2*x^4 - 6*x^3 + 7*x^2)*log(2*x^2 - 6*x + 7) - 4000*(2*x^4 - 6*x^3 + 7*x^2)*log(x) + 22050*x + 8575)/(2*x^4 - 6*x^3 + 7*x^2)

Sympy [A]

time = 0.11, size = 80, normalized size = 0.99

$$\frac{80\log(x)}{2401} - \frac{40\log\left(x^2 - 3x + \frac{7}{2}\right)}{2401} + \frac{234\sqrt{5}\operatorname{atan}\left(\frac{2\sqrt{5}x - 3\sqrt{5}}{5}\right)}{60025} + \frac{-276x^3 + 814x^2 - 630x - 245}{6860x^4 - 20580x^3 + 24010x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(2*x**2-6*x+7)**2,x)

[Out] 80*log(x)/2401 - 40*log(x**2 - 3*x + 7/2)/2401 + 234*sqrt(5)*atan(2*sqrt(5)*x/5 - 3*sqrt(5)/5)/60025 + (-276*x**3 + 814*x**2 - 630*x - 245)/(6860*x**4 - 20580*x**3 + 24010*x**2)

Giac [A]

time = 0.00, size = 85, normalized size = 1.05

$$-\frac{40}{2401}\ln(2x^2 - 6x + 7) + \frac{468\arctan\left(\frac{2x-3}{\sqrt{5}}\right)}{2 \cdot 12005\sqrt{5}} + \frac{80}{2401}\ln|x| + \frac{\frac{1}{24010}(-1932x^3 + 5698x^2 - 4410x - 1715)}{x^2(2x^2 - 6x + 7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(2*x^2-6*x+7)^2,x)

[Out] 234/60025*sqrt(5)*arctan(1/5*sqrt(5)*(2*x - 3)) - 1/3430*(276*x^3 - 814*x^2 + 630*x + 245)/((2*x^2 - 6*x + 7)*x^2) - 40/2401*log(2*x^2 - 6*x + 7) + 80/2401*log(abs(x))

Mupad [B]

time = 0.11, size = 77, normalized size = 0.95

$$\frac{80 \ln(x)}{2401} - \frac{\frac{69x^3}{1715} - \frac{407x^2}{3430} + \frac{9x}{98} + \frac{1}{28}}{x^4 - 3x^3 + \frac{7x^2}{2}} - \ln\left(x - \frac{3}{2} - \frac{\sqrt{5} \operatorname{li}}{2}\right) \left(\frac{40}{2401} + \frac{\sqrt{5} \operatorname{li}}{60025}\right) + \ln\left(x - \frac{3}{2} + \frac{\sqrt{5} \operatorname{li}}{2}\right) \left(-\frac{40}{2401} + \frac{\sqrt{5} \operatorname{li}}{60025}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(2*x^2 - 6*x + 7)^2),x)

[Out] (80*log(x))/2401 - ((9*x)/98 - (407*x^2)/3430 + (69*x^3)/1715 + 1/28)/((7*x^2)/2 - 3*x^3 + x^4) - log(x - (5^(1/2)*1i)/2 - 3/2)*((5^(1/2)*117i)/60025 + 40/2401) + log(x + (5^(1/2)*1i)/2 - 3/2)*((5^(1/2)*117i)/60025 - 40/2401)

$$3.204 \quad \int \frac{x^9}{(2+3x+x^2)^5} dx$$

Optimal. Leaf size=104

$$735x + \frac{x^8(4+3x)}{4(2+3x+x^2)^4} - \frac{x^6(110+81x)}{12(2+3x+x^2)^3} + \frac{x^4(184+135x)}{2(2+3x+x^2)^2} - \frac{x^2(2206+1593x)}{2(2+3x+x^2)} - 1471 \log(1+x) + 1472 \log(2+x)$$

[Out] 735*x+1/4*x^8*(4+3*x)/(x^2+3*x+2)^4-1/12*x^6*(110+81*x)/(x^2+3*x+2)^3+1/2*x^4*(184+135*x)/(x^2+3*x+2)^2-1/2*x^2*(2206+1593*x)/(x^2+3*x+2)-1471*ln(1+x)+1472*ln(2+x)

Rubi [A]

time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {752, 832, 787, 646, 31}

$$-\frac{(1593x+2206)x^2}{2(x^2+3x+2)} + \frac{(3x+4)x^8}{4(x^2+3x+2)^4} - \frac{(81x+110)x^6}{12(x^2+3x+2)^3} + \frac{(135x+184)x^4}{2(x^2+3x+2)^2} + 735x - 1471 \log(x+1) + 1472 \log(x+2)$$

Antiderivative was successfully verified.

[In] Int[x^9/(2 + 3*x + x^2)^5,x]

[Out] 735*x + (x^8*(4 + 3*x))/(4*(2 + 3*x + x^2)^4) - (x^6*(110 + 81*x))/(12*(2 + 3*x + x^2)^3) + (x^4*(184 + 135*x))/(2*(2 + 3*x + x^2)^2) - (x^2*(2206 + 1593*x))/(2*(2 + 3*x + x^2)) - 1471*Log[1 + x] + 1472*Log[2 + x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 646

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 752

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m-1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x + c*x^2)^(p+1)/((p+1)*(b^2 - 4*a*c))), x] + Dist[1/((p+1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m-2)*Simp[e*(2*a*e*(m-1) + b*d*(2*p - m + 4)) - 2*c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x]

```
1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4)) + e*(b^2*e*g*(m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) || !ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{(2+3x+x^2)^5} dx &= \frac{x^8(4+3x)}{4(2+3x+x^2)^4} - \frac{1}{4} \int \frac{x^7(32+3x)}{(2+3x+x^2)^4} dx \\
&= \frac{x^8(4+3x)}{4(2+3x+x^2)^4} - \frac{x^6(110+81x)}{12(2+3x+x^2)^3} - \frac{1}{12} \int \frac{(-660-72x)x^5}{(2+3x+x^2)^3} dx \\
&= \frac{x^8(4+3x)}{4(2+3x+x^2)^4} - \frac{x^6(110+81x)}{12(2+3x+x^2)^3} + \frac{x^4(184+135x)}{2(2+3x+x^2)^2} - \frac{1}{24} \int \frac{x^3(8832+1476x)}{(2+3x+x^2)^2} dx \\
&= \frac{x^8(4+3x)}{4(2+3x+x^2)^4} - \frac{x^6(110+81x)}{12(2+3x+x^2)^3} + \frac{x^4(184+135x)}{2(2+3x+x^2)^2} - \frac{x^2(2206+1593x)}{2(2+3x+x^2)} - \frac{1}{24} \int \frac{x^2(8832+1476x)}{(2+3x+x^2)} dx \\
&= 735x + \frac{x^8(4+3x)}{4(2+3x+x^2)^4} - \frac{x^6(110+81x)}{12(2+3x+x^2)^3} + \frac{x^4(184+135x)}{2(2+3x+x^2)^2} - \frac{x^2(2206+1593x)}{2(2+3x+x^2)} \\
&= 735x + \frac{x^8(4+3x)}{4(2+3x+x^2)^4} - \frac{x^6(110+81x)}{12(2+3x+x^2)^3} + \frac{x^4(184+135x)}{2(2+3x+x^2)^2} - \frac{x^2(2206+1593x)}{2(2+3x+x^2)} \\
&= 735x + \frac{x^8(4+3x)}{4(2+3x+x^2)^4} - \frac{x^6(110+81x)}{12(2+3x+x^2)^3} + \frac{x^4(184+135x)}{2(2+3x+x^2)^2} - \frac{x^2(2206+1593x)}{2(2+3x+x^2)}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 87, normalized size = 0.84

$$\frac{514 + 513x}{4(2+3x+x^2)^4} + \frac{415 + 1998x}{12(2+3x+x^2)^3} + \frac{3(451 + 456x)}{4(2+3x+x^2)^2} - \frac{2(1114 + 729x)}{2+3x+x^2} - 1471 \log(1+x) + 1472 \log(2+x)$$

Antiderivative was successfully verified.

`[In] Integrate[x^9/(2 + 3*x + x^2)^5,x]`

```
[Out] (514 + 513*x)/(4*(2 + 3*x + x^2)^4) + (415 + 1998*x)/(12*(2 + 3*x + x^2)^3)
+ (3*(451 + 456*x))/(4*(2 + 3*x + x^2)^2) - (2*(1114 + 729*x))/(2 + 3*x +
x^2) - 1471*Log[1 + x] + 1472*Log[2 + x]
```

Mathics [A]

time = 2.70, size = 130, normalized size = 1.25

$$\frac{-195280 - 1030560x - 2286008x^2 - 2759400x^3 - 1955853x^4 - 813888x^5 - 184200x^6 - 17496x^7 + 12(16 + 96x + 248x^2 + 360x^3 + 321x^4 + 180x^5 + 62x^6 + 12x^7 + x^8)(-1471 \log[1+x] + 1472 \log[2+x])}{192 + 1152x + 2976x^2 + 4320x^3 + 3852x^4 + 2160x^5 + 744x^6 + 144x^7 + 12x^8}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^9/(x^2 + 3*x + 2)^5,x]')`

```
[Out] (-195280 - 1030560 x - 2286008 x ^ 2 - 2759400 x ^ 3 - 1955853 x ^ 4 - 8138
88 x ^ 5 - 184200 x ^ 6 - 17496 x ^ 7 + 12 (16 + 96 x + 248 x ^ 2 + 360 x ^
3 + 321 x ^ 4 + 180 x ^ 5 + 62 x ^ 6 + 12 x ^ 7 + x ^ 8) (-1471 Log[1 + x]
+ 1472 Log[2 + x])) / (12 (16 + 96 x + 248 x ^ 2 + 360 x ^ 3 + 321 x ^ 4 +
180 x ^ 5 + 62 x ^ 6 + 12 x ^ 7 + x ^ 8))
```


Maple [A]

time = 0.07, size = 70, normalized size = 0.67

method	result
norman	$\frac{-229950x^3 - 85880x - 67824x^5 - 15350x^6 - 1458x^7 - \frac{651951}{4}x^4 - \frac{571502}{3}x^2 - \frac{48820}{3}}{(x^2+3x+2)^4} - 1471 \ln(1+x) + 1472 \ln(2+x)$
risch	$\frac{-229950x^3 - 85880x - 67824x^5 - 15350x^6 - 1458x^7 - \frac{651951}{4}x^4 - \frac{571502}{3}x^2 - \frac{48820}{3}}{(x^2+3x+2)^4} - 1471 \ln(1+x) + 1472 \ln(2+x)$
default	$-\frac{128}{(2+x)^4} - \frac{256}{3(2+x)^3} - \frac{384}{(2+x)^2} - \frac{1024}{2+x} + 1472 \ln(2+x) + \frac{1}{4(1+x)^4} - \frac{14}{3(1+x)^3} + \frac{48}{(1+x)^2} - \frac{434}{1+x} - 1471 \ln(1+x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(x^2+3*x+2)^5,x,method=_RETURNVERBOSE)**[Out]** -128/(2+x)^4-256/3/(2+x)^3-384/(2+x)^2-1024/(2+x)+1472*ln(2+x)+1/4/(1+x)^4-14/3/(1+x)^3+48/(1+x)^2-434/(1+x)-1471*ln(1+x)**Maxima [A]**

time = 0.26, size = 90, normalized size = 0.87

$$\frac{-17496x^7 + 184200x^6 + 813888x^5 + 1955853x^4 + 2759400x^3 + 2286008x^2 + 1030560x + 195280}{12(x^8 + 12x^7 + 62x^6 + 180x^5 + 321x^4 + 360x^3 + 248x^2 + 96x + 16)} + 1472 \log(x+2) - 1471 \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^2+3*x+2)^5,x, algorithm="maxima")**[Out]** -1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 2759400*x^3 + 2286008*x^2 + 1030560*x + 195280)/(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 16) + 1472*log(x + 2) - 1471*log(x + 1)**Fricas [A]**

time = 0.30, size = 165, normalized size = 1.59

$$\frac{-17496x^7 + 184200x^6 + 813888x^5 + 1955853x^4 + 2759400x^3 + 2286008x^2 - 17664(x^8 + 12x^7 + 62x^6 + 180x^5 + 321x^4 + 360x^3 + 248x^2 + 96x + 16)\log(x+2) + 17652(x^8 + 12x^7 + 62x^6 + 180x^5 + 321x^4 + 360x^3 + 248x^2 + 96x + 16)\log(x+1) + 1030560x + 195280}{12(x^8 + 12x^7 + 62x^6 + 180x^5 + 321x^4 + 360x^3 + 248x^2 + 96x + 16)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^2+3*x+2)^5,x, algorithm="fricas")**[Out]** -1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 2759400*x^3 + 2286008*x^2 - 17664*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)*log(x + 2) + 17652*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)*log(x + 1) + 1030560*x + 195280)/(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)**Sympy [A]**

time = 0.10, size = 90, normalized size = 0.87

$$\frac{-17496x^7 - 184200x^6 - 813888x^5 - 1955853x^4 - 2759400x^3 - 2286008x^2 - 1030560x - 195280}{12x^8 + 144x^7 + 744x^6 + 2160x^5 + 3852x^4 + 4320x^3 + 2976x^2 + 1152x + 192} - 1471 \log(x+1) + 1472 \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**9/(x**2+3*x+2)**5,x)

[Out] (-17496*x**7 - 184200*x**6 - 813888*x**5 - 1955853*x**4 - 2759400*x**3 - 2286008*x**2 - 1030560*x - 195280)/(12*x**8 + 144*x**7 + 744*x**6 + 2160*x**5 + 3852*x**4 + 4320*x**3 + 2976*x**2 + 1152*x + 192) - 1471*log(x + 1) + 1472*log(x + 2)

Giac [A]

time = 0.00, size = 74, normalized size = 0.71

$$-1471 \ln|x+1| + 1472 \ln|x+2| + \frac{\frac{1}{24}(-34992x^7 - 368400x^6 - 1627776x^5 - 3911706x^4 - 5518800x^3 - 4572016x^2 - 2061120x - 390560)}{(x+1)^4(x+2)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^2+3*x+2)^5,x)

[Out] -1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 2759400*x^3 + 2286008*x^2 + 1030560*x + 195280)/((x + 2)^4*(x + 1)^4) + 1472*log(abs(x + 2)) - 1471*log(abs(x + 1))

Mupad [B]

time = 0.21, size = 90, normalized size = 0.87

$$1472 \ln(x+2) - 1471 \ln(x+1) - \frac{1458x^7 + 15350x^6 + 67824x^5 + \frac{651951x^4}{4} + 229950x^3 + \frac{571502x^2}{3} + 85880x + \frac{48820}{3}}{x^8 + 12x^7 + 62x^6 + 180x^5 + 321x^4 + 360x^3 + 248x^2 + 96x + 16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(3*x + x^2 + 2)^5,x)

[Out] 1472*log(x + 2) - 1471*log(x + 1) - (85880*x + (571502*x^2)/3 + 229950*x^3 + (651951*x^4)/4 + 67824*x^5 + 15350*x^6 + 1458*x^7 + 48820/3)/(96*x + 248*x^2 + 360*x^3 + 321*x^4 + 180*x^5 + 62*x^6 + 12*x^7 + x^8 + 16)

$$3.205 \quad \int \frac{(1+2x)^2}{(3+5x+2x^2)^5} dx$$

Optimal. Leaf size=102

$$\frac{(1+2x)(7+6x)}{4(3+5x+2x^2)^4} + \frac{73+62x}{3(3+5x+2x^2)^3} - \frac{155(5+4x)}{3(3+5x+2x^2)^2} + \frac{620(5+4x)}{3+5x+2x^2} + 2480 \log(1+x) - 2480 \log(3+2x)$$

[Out] $1/4*(1+2*x)*(7+6*x)/(2*x^2+5*x+3)^4+1/3*(73+62*x)/(2*x^2+5*x+3)^3-155/3*(5+4*x)/(2*x^2+5*x+3)^2+620*(5+4*x)/(2*x^2+5*x+3)+2480*\ln(1+x)-2480*\ln(3+2*x)$

Rubi [A]

time = 0.02, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {752, 652, 628, 630, 31}

$$\frac{620(4x+5)}{2x^2+5x+3} - \frac{155(4x+5)}{3(2x^2+5x+3)^2} + \frac{62x+73}{3(2x^2+5x+3)^3} + \frac{(2x+1)(6x+7)}{4(2x^2+5x+3)^4} + 2480 \log(x+1) - 2480 \log(2x+3)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)^2/(3 + 5*x + 2*x^2)^5, x]

[Out] $((1+2*x)*(7+6*x))/(4*(3+5*x+2*x^2)^4) + (73+62*x)/(3*(3+5*x+2*x^2)^3) - (155*(5+4*x))/(3*(3+5*x+2*x^2)^2) + (620*(5+4*x))/(3+5*x+2*x^2) + 2480*\text{Log}[1+x] - 2480*\text{Log}[3+2*x]$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 628

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(b + 2*c*x)*((a + b*x + c*x^2)^(p+1)/((p+1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p+3)/((p+1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 630

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*x
+ c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] := Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)^2}{(3+5x+2x^2)^5} dx &= \frac{(1+2x)(7+6x)}{4(3+5x+2x^2)^4} - \frac{1}{4} \int \frac{-28-72x}{(3+5x+2x^2)^4} dx \\
&= \frac{(1+2x)(7+6x)}{4(3+5x+2x^2)^4} + \frac{73+62x}{3(3+5x+2x^2)^3} + \frac{310}{3} \int \frac{1}{(3+5x+2x^2)^3} dx \\
&= \frac{(1+2x)(7+6x)}{4(3+5x+2x^2)^4} + \frac{73+62x}{3(3+5x+2x^2)^3} - \frac{155(5+4x)}{3(3+5x+2x^2)^2} - 620 \int \frac{1}{(3+5x+2x^2)} dx \\
&= \frac{(1+2x)(7+6x)}{4(3+5x+2x^2)^4} + \frac{73+62x}{3(3+5x+2x^2)^3} - \frac{155(5+4x)}{3(3+5x+2x^2)^2} + \frac{620(5+4x)}{3+5x+2x^2} + 2480 \log(2(1+x)) \\
&= \frac{(1+2x)(7+6x)}{4(3+5x+2x^2)^4} + \frac{73+62x}{3(3+5x+2x^2)^3} - \frac{155(5+4x)}{3(3+5x+2x^2)^2} + \frac{620(5+4x)}{3+5x+2x^2} + 4960 \log(2(1+x)) \\
&= \frac{(1+2x)(7+6x)}{4(3+5x+2x^2)^4} + \frac{73+62x}{3(3+5x+2x^2)^3} - \frac{155(5+4x)}{3(3+5x+2x^2)^2} + \frac{620(5+4x)}{3+5x+2x^2} + 2480 \log(2(1+x))
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 99, normalized size = 0.97

$$-\frac{11+10x}{4(3+5x+2x^2)^4} + \frac{31(5+4x)}{6(3+5x+2x^2)^3} - \frac{155(5+4x)}{3(3+5x+2x^2)^2} + \frac{620(5+4x)}{3+5x+2x^2} + 2480 \log(2(1+x)) - 2480 \log(3+2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^2/(3 + 5*x + 2*x^2)^5,x]
```

[Out] $-1/4*(11 + 10*x)/(3 + 5*x + 2*x^2)^4 + (31*(5 + 4*x))/(6*(3 + 5*x + 2*x^2)^3) - (155*(5 + 4*x))/(3*(3 + 5*x + 2*x^2)^2) + (620*(5 + 4*x))/(3 + 5*x + 2*x^2) + 2480*\text{Log}[2*(1 + x)] - 2480*\text{Log}[3 + 2*x]$

Mathics [A]

time = 3.53, size = 132, normalized size = 1.29

$$\frac{977397 + 5712464x + 14209160x^2 + 19495776x^3 + 15934000x^4 + 7757440x^5 + 2083200x^6 + 238080x^7 + 29760(81 + 540x + 1566x^2 + 2580x^3 + 2641x^4 + 1720x^5 + 696x^6 + 160x^7 + 16x^8)(\text{Log}[1 + x] - \text{Log}[\frac{3}{2} + x])}{972 + 6480x + 18792x^2 + 30960x^3 + 31692x^4 + 20640x^5 + 8352x^6 + 1920x^7 + 192x^8}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(1 + 2*x)^2/(3 + 5*x + 2*x^2)^5,x]')`

[Out] $(977397 + 5712464 x + 14209160 x^2 + 19495776 x^3 + 15934000 x^4 + 7757440 x^5 + 2083200 x^6 + 238080 x^7 + 29760 (81 + 540 x + 1566 x^2 + 2580 x^3 + 2641 x^4 + 1720 x^5 + 696 x^6 + 160 x^7 + 16 x^8) (\text{Log}[1 + x] - \text{Log}[3/2 + x])) / (12 (81 + 540 x + 1566 x^2 + 2580 x^3 + 2641 x^4 + 1720 x^5 + 696 x^6 + 160 x^7 + 16 x^8))$

Maple [A]

time = 0.08, size = 80, normalized size = 0.78

method	result
norman	$\frac{173600x^6 + \frac{1428116}{3}x + \frac{3552290}{3}x^2 + 19840x^7 + \frac{1939360}{3}x^5 + 1624648x^3 + \frac{3983500}{3}x^4 + \frac{325799}{4}}{(2x^2+5x+3)^4} + 2480 \ln(1+x) - 2480 \ln(3-x)$
risch	$\frac{173600x^6 + \frac{1428116}{3}x + \frac{3552290}{3}x^2 + 19840x^7 + \frac{1939360}{3}x^5 + 1624648x^3 + \frac{3983500}{3}x^4 + \frac{325799}{4}}{(2x^2+5x+3)^4} + 2480 \ln(1+x) - 2480 \ln(3-x)$
default	$-\frac{1}{4(1+x)^4} + \frac{14}{3(1+x)^3} - \frac{52}{(1+x)^2} + \frac{560}{1+x} + 2480 \ln(1+x) + \frac{16}{(3+2x)^4} + \frac{256}{3(3+2x)^3} + \frac{328}{(3+2x)^2} + \frac{1360}{3+2x} - 2480 \ln(3+2x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+2*x)^2/(2*x^2+5*x+3)^5,x,method=_RETURNVERBOSE)`

[Out] $-1/4/(1+x)^4 + 14/3/(1+x)^3 - 52/(1+x)^2 + 560/(1+x) + 2480*\ln(1+x) + 16/(3+2*x)^4 + 256/3/(3+2*x)^3 + 328/(3+2*x)^2 + 1360/(3+2*x) - 2480*\ln(3+2*x)$

Maxima [A]

time = 0.26, size = 94, normalized size = 0.92

$$\frac{238080x^7 + 2083200x^6 + 7757440x^5 + 15934000x^4 + 19495776x^3 + 14209160x^2 + 5712464x + 977397}{12(16x^8 + 160x^7 + 696x^6 + 1720x^5 + 2641x^4 + 2580x^3 + 1566x^2 + 540x + 81)} - 2480 \log(2x + 3) + 2480 \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)^2/(2*x^2+5*x+3)^5,x, algorithm="maxima")`

[Out] $1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 19495776*x^3 + 14209160*x^2 + 5712464*x + 977397)/(16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5 + 2641*x^4 + 2580*x^3 + 1566*x^2 + 540*x + 81) - 2480*\log(2*x + 3) + 2480*\log(x + 1)$

Fricas [A]

time = 0.30, size = 173, normalized size = 1.70

$$\frac{238080x^7 + 2083200x^6 + 7757440x^5 + 15934000x^4 + 19495776x^3 + 14209160x^2 - 29760(16x^8 + 160x^7 + 696x^6 + 1720x^5 + 2641x^4 + 2580x^3 + 1566x^2 + 540x + 81)\log(2x + 3) + 29760(16x^8 + 160x^7 + 696x^6 + 1720x^5 + 2641x^4 + 2580x^3 + 1566x^2 + 540x + 81)\log(x + 1) + 5712464x + 977397}{12(16x^8 + 160x^7 + 696x^6 + 1720x^5 + 2641x^4 + 2580x^3 + 1566x^2 + 540x + 81)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)^2/(2*x^2+5*x+3)^5,x, algorithm="fricas")

[Out] 1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 19495776*x^3 + 14209160*x^2 - 29760*(16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5 + 2641*x^4 + 2580*x^3 + 1566*x^2 + 540*x + 81)*log(2*x + 3) + 29760*(16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5 + 2641*x^4 + 2580*x^3 + 1566*x^2 + 540*x + 81)*log(x + 1) + 5712464*x + 977397)/(16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5 + 2641*x^4 + 2580*x^3 + 1566*x^2 + 540*x + 81)

Sympy [A]

time = 0.12, size = 90, normalized size = 0.88

$$\frac{238080x^7 + 2083200x^6 + 7757440x^5 + 15934000x^4 + 19495776x^3 + 14209160x^2 + 5712464x + 977397}{192x^8 + 1920x^7 + 8352x^6 + 20640x^5 + 31692x^4 + 30960x^3 + 18792x^2 + 6480x + 972} + 2480\log(x + 1) - 2480\log\left(x + \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)**2/(2*x**2+5*x+3)**5,x)

[Out] (238080*x**7 + 2083200*x**6 + 7757440*x**5 + 15934000*x**4 + 19495776*x**3 + 14209160*x**2 + 5712464*x + 977397)/(192*x**8 + 1920*x**7 + 8352*x**6 + 20640*x**5 + 31692*x**4 + 30960*x**3 + 18792*x**2 + 6480*x + 972) + 2480*log(x + 1) - 2480*log(x + 3/2)

Giac [A]

time = 0.00, size = 69, normalized size = 0.68

$$2480\ln|x + 1| - 2480\ln|2x + 3| + \frac{238080x^7 + 2083200x^6 + 7757440x^5 + 15934000x^4 + 19495776x^3 + 14209160x^2 + 5712464x + 977397}{12(2x^2 + 5x + 3)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)^2/(2*x^2+5*x+3)^5,x)

[Out] 1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 19495776*x^3 + 14209160*x^2 + 5712464*x + 977397)/(2*x^2 + 5*x + 3)^4 - 2480*log(abs(2*x + 3)) + 2480*log(abs(x + 1))

Mupad [B]

time = 0.21, size = 85, normalized size = 0.83

$$\frac{1240x^7 + 10850x^6 + \frac{121210x^5}{3} + \frac{995875x^4}{12} + \frac{203081x^3}{2} + \frac{1776145x^2}{24} + \frac{357029x}{12} + \frac{325799}{64}}{x^8 + 10x^7 + \frac{87x^6}{2} + \frac{215x^5}{2} + \frac{2641x^4}{16} + \frac{645x^3}{4} + \frac{783x^2}{8} + \frac{135x}{4} + \frac{81}{16}} - 4960\operatorname{atanh}(4x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((2x + 1)^2/(5x + 2x^2 + 3)^5, x)$

[Out] $((357029x)/12 + (1776145x^2)/24 + (203081x^3)/2 + (995875x^4)/12 + (121210x^5)/3 + 10850x^6 + 1240x^7 + 325799/64)/((135x)/4 + (783x^2)/8 + (645x^3)/4 + (2641x^4)/16 + (215x^5)/2 + (87x^6)/2 + 10x^7 + x^8 + 81/16) - 4960*\text{atanh}(4x + 5)$

3.206

$$\int \frac{(a-bx^2)^3}{x^7} dx$$

Optimal. Leaf size=40

$$-\frac{a^3}{6x^6} + \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} - b^3 \log(x)$$

[Out] $-1/6*a^3/x^6+3/4*a^2*b/x^4-3/2*a*b^2/x^2-b^3*\ln(x)$

Rubi [A]

time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {272, 45}

$$-\frac{a^3}{6x^6} + \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} - b^3 \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a - b*x^2)^3/x^7, x]$

[Out] $-1/6*a^3/x^6 + (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) - b^3*\text{Log}[x]$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

$\text{Int}[(x_.)^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p, x}], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{(a-bx^2)^3}{x^7} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(a-bx)^3}{x^4} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{a^3}{x^4} - \frac{3a^2b}{x^3} + \frac{3ab^2}{x^2} - \frac{b^3}{x} \right) dx, x, x^2 \right) \\ &= -\frac{a^3}{6x^6} + \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} - b^3 \log(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 40, normalized size = 1.00

$$-\frac{a^3}{6x^6} + \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} - b^3 \log(x)$$

Antiderivative was successfully verified.

`[In] Integrate[(a - b*x^2)^3/x^7,x]``[Out] -1/6*a^3/x^6 + (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) - b^3*Log[x]`**Mathics [A]**

time = 2.05, size = 34, normalized size = 0.85

$$-\frac{a^3}{6x^6} + \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} - b^3 \text{Log}[x]$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(a - b*x^2)^3/x^7,x]')``[Out] -a ^ 3 / (6 x ^ 6) + 3 a ^ 2 b / (4 x ^ 4) - 3 a b ^ 2 / (2 x ^ 2) - b ^ 3 Log[x]`**Maple [A]**

time = 0.06, size = 35, normalized size = 0.88

method	result	size
default	$-\frac{a^3}{6x^6} + \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} - b^3 \ln(x)$	35
norman	$-\frac{\frac{1}{6}a^3 - \frac{3}{2}ab^2x^4 + \frac{3}{4}a^2bx^2}{x^6} - b^3 \ln(x)$	37
risch	$-\frac{\frac{1}{6}a^3 - \frac{3}{2}ab^2x^4 + \frac{3}{4}a^2bx^2}{x^6} - b^3 \ln(x)$	37

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((-b*x^2+a)^3/x^7,x,method=_RETURNVERBOSE)``[Out] -1/6*a^3/x^6+3/4*a^2*b/x^4-3/2*a*b^2/x^2-b^3*ln(x)`**Maxima [A]**

time = 0.26, size = 39, normalized size = 0.98

$$-\frac{1}{2} b^3 \log(x^2) - \frac{18 ab^2 x^4 - 9 a^2 b x^2 + 2 a^3}{12 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((-b*x^2+a)^3/x^7,x, algorithm="maxima")`

[Out] $-1/2*b^3*\log(x^2) - 1/12*(18*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3)/x^6$

Fricas [A]

time = 0.34, size = 39, normalized size = 0.98

$$\frac{12b^3x^6 \log(x) + 18ab^2x^4 - 9a^2bx^2 + 2a^3}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2+a)^3/x^7,x, algorithm="fricas")`

[Out] $-1/12*(12*b^3*x^6*\log(x) + 18*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3)/x^6$

Sympy [A]

time = 0.12, size = 37, normalized size = 0.92

$$-b^3 \log(x) - \frac{2a^3 - 9a^2bx^2 + 18ab^2x^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**2+a)**3/x**7,x)`

[Out] $-b**3*\log(x) - (2*a**3 - 9*a**2*b*x**2 + 18*a*b**2*x**4)/(12*x**6)$

Giac [A]

time = 0.00, size = 53, normalized size = 1.32

$$\frac{11x^6b^3 - 18x^4b^2a + 9x^2ba^2 - 2a^3}{12x^6} - \frac{1}{2}b^3 \ln(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2+a)^3/x^7,x)`

[Out] $-1/2*b^3*\log(x^2) + 1/12*(11*b^3*x^6 - 18*a*b^2*x^4 + 9*a^2*b*x^2 - 2*a^3)/x^6$

Mupad [B]

time = 0.20, size = 37, normalized size = 0.92

$$-b^3 \ln(x) - \frac{\frac{a^3}{6} - \frac{3a^2bx^2}{4} + \frac{3ab^2x^4}{2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a - b*x^2)^3/x^7,x)`

[Out] $-b^3*\log(x) - (a^3/6 - (3*a^2*b*x^2)/4 + (3*a*b^2*x^4)/2)/x^6$

3.207

$$\int \frac{x^{13}}{(a^4+x^4)^5} dx$$

Optimal. Leaf size=83

$$-\frac{x^{10}}{16(a^4+x^4)^4} - \frac{5x^6}{96(a^4+x^4)^3} - \frac{5x^2}{128(a^4+x^4)^2} + \frac{5x^2}{256a^4(a^4+x^4)} + \frac{5 \tan^{-1}\left(\frac{x^2}{a^2}\right)}{256a^6}$$

[Out] -1/16*x^10/(a^4+x^4)^4-5/96*x^6/(a^4+x^4)^3-5/128*x^2/(a^4+x^4)^2+5/256*x^2/a^4/(a^4+x^4)+5/256*arctan(x^2/a^2)/a^6

Rubi [A]

time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {281, 294, 205, 209}

$$-\frac{x^{10}}{16(a^4+x^4)^4} - \frac{5x^6}{96(a^4+x^4)^3} + \frac{5x^2}{256a^4(a^4+x^4)} - \frac{5x^2}{128(a^4+x^4)^2} + \frac{5 \tan^{-1}\left(\frac{x^2}{a^2}\right)}{256a^6}$$

Antiderivative was successfully verified.

[In] Int[x^13/(a^4 + x^4)^5,x]

[Out] -1/16*x^10/(a^4 + x^4)^4 - (5*x^6)/(96*(a^4 + x^4)^3) - (5*x^2)/(128*(a^4 + x^4)^2) + (5*x^2)/(256*a^4*(a^4 + x^4)) + (5*ArcTan[x^2/a^2])/(256*a^6)

Rule 205

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 281

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13}}{(a^4 + x^4)^5} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^6}{(a^4 + x^2)^5} dx, x, x^2 \right) \\
&= -\frac{x^{10}}{16(a^4 + x^4)^4} + \frac{5}{16} \text{Subst} \left(\int \frac{x^4}{(a^4 + x^2)^4} dx, x, x^2 \right) \\
&= -\frac{x^{10}}{16(a^4 + x^4)^4} - \frac{5x^6}{96(a^4 + x^4)^3} + \frac{5}{32} \text{Subst} \left(\int \frac{x^2}{(a^4 + x^2)^3} dx, x, x^2 \right) \\
&= -\frac{x^{10}}{16(a^4 + x^4)^4} - \frac{5x^6}{96(a^4 + x^4)^3} - \frac{5x^2}{128(a^4 + x^4)^2} + \frac{5}{128} \text{Subst} \left(\int \frac{1}{(a^4 + x^2)^2} dx, x, x^2 \right) \\
&= -\frac{x^{10}}{16(a^4 + x^4)^4} - \frac{5x^6}{96(a^4 + x^4)^3} - \frac{5x^2}{128(a^4 + x^4)^2} + \frac{5x^2}{256a^4(a^4 + x^4)} + \frac{5 \text{Subst} \left(\int \frac{1}{a^4 + x^2} dx, x, x^2 \right)}{256a^4} \\
&= -\frac{x^{10}}{16(a^4 + x^4)^4} - \frac{5x^6}{96(a^4 + x^4)^3} - \frac{5x^2}{128(a^4 + x^4)^2} + \frac{5x^2}{256a^4(a^4 + x^4)} + \frac{5 \tan^{-1} \left(\frac{x^2}{a^2} \right)}{256a^6}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 62, normalized size = 0.75

$$\frac{-\frac{a^2 x^2 (15a^{12} + 55a^8 x^4 + 73a^4 x^8 - 15x^{12})}{(a^4 + x^4)^4} + 15 \tan^{-1} \left(\frac{x^2}{a^2} \right)}{768a^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^13/(a^4 + x^4)^5, x]

[Out] (-((a^2*x^2*(15*a^12 + 55*a^8*x^4 + 73*a^4*x^8 - 15*x^12))/(a^4 + x^4)^4) + 15*ArcTan[x^2/a^2])/(768*a^6)

Mathics [C] Result contains complex when optimal does not.

time = 12.58, size = 130, normalized size = 1.57

$$\frac{2a^2 x^2 (-15a^{12} - 55a^8 x^4 - 73a^4 x^8 + 15x^{12}) - 15I(a^{16} + 4a^{12}x^4 + 6a^8x^8 + 4a^4x^{12} + x^{16})(\text{Log}[-Ia^2 + x^2] - \text{Log}[Ia^2 + x^2])}{1536a^6(a^{16} + 4a^{12}x^4 + 6a^8x^8 + 4a^4x^{12} + x^{16})}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x^13/(a^4 + x^4)^5,x]')`

[Out] $(2 a^2 x^2 (-15 a^{12} - 55 a^8 x^4 - 73 a^4 x^8 + 15 x^{12}) - 15 I (a^{16} + 4 a^{12} x^4 + 6 a^8 x^8 + 4 a^4 x^{12} + x^{16}) (\text{Log}[-I a^2 + x^2] - \text{Log}[I a^2 + x^2])) / (1536 a^6 (a^{16} + 4 a^{12} x^4 + 6 a^8 x^8 + 4 a^4 x^{12} + x^{16}))$

Maple [A]

time = 0.07, size = 56, normalized size = 0.67

method	result	size
risch	$\frac{-\frac{5a^8x^2}{256} - \frac{55a^4x^6}{768} - \frac{73x^{10}}{768} + \frac{5x^{14}}{256a^4}}{(a^4+x^4)^4} + \frac{5 \arctan\left(\frac{x^2}{a^2}\right)}{256a^6}$	55
default	$\frac{\frac{5x^{14}}{128a^4} - \frac{73x^{10}}{384} - \frac{55a^4x^6}{384} - \frac{5a^8x^2}{128}}{2(a^4+x^4)^4} + \frac{5 \arctan\left(\frac{x^2}{a^2}\right)}{256a^6}$	56

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^13/(a^4+x^4)^5,x,method=_RETURNVERBOSE)`

[Out] $1/2*(5/128/a^4*x^14-73/384*x^10-55/384*a^4*x^6-5/128*a^8*x^2)/(a^4+x^4)^4+5/256*\arctan(x^2/a^2)/a^6$

Maxima [A]

time = 0.36, size = 83, normalized size = 1.00

$$-\frac{15 a^{12} x^2 + 55 a^8 x^6 + 73 a^4 x^{10} - 15 x^{14}}{768 (a^{20} + 4 a^{16} x^4 + 6 a^{12} x^8 + 4 a^8 x^{12} + a^4 x^{16})} + \frac{5 \arctan\left(\frac{x^2}{a^2}\right)}{256 a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^13/(a^4+x^4)^5,x, algorithm="maxima")`

[Out] $-1/768*(15*a^{12}*x^2 + 55*a^8*x^6 + 73*a^4*x^{10} - 15*x^{14})/(a^{20} + 4*a^{16}*x^4 + 6*a^{12}*x^8 + 4*a^8*x^{12} + a^4*x^{16}) + 5/256*\arctan(x^2/a^2)/a^6$

Fricas [A]

time = 0.32, size = 113, normalized size = 1.36

$$\frac{15 a^{14} x^2 + 55 a^{10} x^6 + 73 a^6 x^{10} - 15 a^2 x^{14} - 15 (a^{16} + 4 a^{12} x^4 + 6 a^8 x^8 + 4 a^4 x^{12} + x^{16}) \arctan\left(\frac{x^2}{a^2}\right)}{768 (a^{22} + 4 a^{18} x^4 + 6 a^{14} x^8 + 4 a^{10} x^{12} + a^6 x^{16})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^13/(a^4+x^4)^5,x, algorithm="fricas")`

[Out] $-1/768*(15*a^{14}*x^2 + 55*a^{10}*x^6 + 73*a^6*x^{10} - 15*a^2*x^{14} - 15*(a^{16} + 4*a^{12}*x^4 + 6*a^8*x^8 + 4*a^4*x^{12} + x^{16})*\arctan(x^2/a^2))/(a^{22} + 4*a^{18}*x^4 + 6*a^{14}*x^8 + 4*a^{10}*x^{12} + a^6*x^{16})$

Sympy [C] Result contains complex when optimal does not.
time = 0.37, size = 102, normalized size = 1.23

$$\frac{-15a^{12}x^2 - 55a^8x^6 - 73a^4x^{10} + 15x^{14}}{768a^{20} + 3072a^{16}x^4 + 4608a^{12}x^8 + 3072a^8x^{12} + 768a^4x^{16}} + \frac{-\frac{5i \log(-ia^2+x^2)}{512} + \frac{5i \log(ia^2+x^2)}{512}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**13/(a**4+x**4)**5,x)`

[Out] $(-15*a^{12}*x^2 - 55*a^8*x^6 - 73*a^4*x^{10} + 15*x^{14})/(768*a^{20} + 3072*a^{16}*x^4 + 4608*a^{12}*x^8 + 3072*a^8*x^{12} + 768*a^4*x^{16}) + (-5*I*\log(-I*a^2 + x^2)/512 + 5*I*\log(I*a^2 + x^2)/512)/a^6$

Giac [A]

time = 0.00, size = 71, normalized size = 0.86

$$\frac{15x^{14} - 73x^{10}a^4 - 55x^6a^8 - 15x^2a^{12}}{768a^4(x^4 + a^4)^4} + \frac{5 \arctan\left(\frac{x^2}{a^2}\right)}{2 \cdot 128a^4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^13/(a^4+x^4)^5,x)`

[Out] $5/256*\arctan(x^2/a^2)/a^6 - 1/768*(15*a^{12}*x^2 + 55*a^8*x^6 + 73*a^4*x^{10} - 15*x^{14})/((a^4 + x^4)^4*a^4)$

Mupad [B]

time = 0.15, size = 79, normalized size = 0.95

$$\frac{5 \operatorname{atan}\left(\frac{x^2}{a^2}\right)}{256 a^6} - \frac{\frac{73 x^{10}}{768} + \frac{55 a^4 x^6}{768} + \frac{5 a^8 x^2}{256} - \frac{5 x^{14}}{256 a^4}}{a^{16} + 4 a^{12} x^4 + 6 a^8 x^8 + 4 a^4 x^{12} + x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^13/(a^4 + x^4)^5,x)`

[Out] $(5*\operatorname{atan}(x^2/a^2))/(256*a^6) - ((73*x^{10})/768 + (55*a^4*x^6)/768 + (5*a^8*x^2)/256 - (5*x^{14})/(256*a^4))/(a^{16} + x^{16} + 4*a^4*x^{12} + 6*a^8*x^8 + 4*a^{12}*x^4)$

$$3.208 \quad \int (2\sqrt{x} - x)^2 x^{3/2} (1 + x^2) dx$$

Optimal. Leaf size=49

$$\frac{8x^{7/2}}{7} - x^4 + \frac{2x^{9/2}}{9} + \frac{8x^{11/2}}{11} - \frac{2x^6}{3} + \frac{2x^{13/2}}{13}$$

[Out] 8/7*x^(7/2)-x^4+2/9*x^(9/2)+8/11*x^(11/2)-2/3*x^6+2/13*x^(13/2)

Rubi [A]

time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1598, 1834, 272, 45}

$$\frac{2x^{13/2}}{13} + \frac{8x^{11/2}}{11} + \frac{2x^{9/2}}{9} + \frac{8x^{7/2}}{7} - \frac{2x^6}{3} - x^4$$

Antiderivative was successfully verified.

[In] Int[(2*Sqrt[x] - x)^2*x^(3/2)*(1 + x^2),x]

[Out] (8*x^(7/2))/7 - x^4 + (2*x^(9/2))/9 + (8*x^(11/2))/11 - (2*x^6)/3 + (2*x^(13/2))/13

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1598

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1834

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,

n}], x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned}
 \int (2\sqrt{x} - x)^2 x^{3/2} (1 + x^2) dx &= \int (2 - \sqrt{x})^2 x^{5/2} (1 + x^2) dx \\
 &= \int \left((-2 + \sqrt{x})^2 x^{5/2} + (-2 + \sqrt{x})^2 x^{9/2} \right) dx \\
 &= \int (-2 + \sqrt{x})^2 x^{5/2} dx + \int (-2 + \sqrt{x})^2 x^{9/2} dx \\
 &= 2\text{Subst} \left(\int (-2 + x)^2 x^6 dx, x, \sqrt{x} \right) + 2\text{Subst} \left(\int (-2 + x)^2 x^{10} dx, x, \sqrt{x} \right) \\
 &= 2\text{Subst} \left(\int (4x^6 - 4x^7 + x^8) dx, x, \sqrt{x} \right) + 2\text{Subst} \left(\int (4x^{10} - 4x^{11} + x^{12}) dx, x, \sqrt{x} \right) \\
 &= \frac{8x^{7/2}}{7} - x^4 + \frac{2x^{9/2}}{9} + \frac{8x^{11/2}}{11} - \frac{2x^6}{3} + \frac{2x^{13/2}}{13}
 \end{aligned}$$

Mathematica [A]

time = 0.02, size = 43, normalized size = 0.88

$$\frac{10296x^{7/2} - 9009x^4 + 2002x^{9/2} + 6552x^{11/2} - 6006x^6 + 1386x^{13/2}}{9009}$$

Antiderivative was successfully verified.

[In] Integrate[(2*Sqrt[x] - x)^2*x^(3/2)*(1 + x^2),x]

[Out] (10296*x^(7/2) - 9009*x^4 + 2002*x^(9/2) + 6552*x^(11/2) - 6006*x^6 + 1386*x^(13/2))/9009

Mathics [A]

time = 2.65, size = 31, normalized size = 0.63

$$\frac{8x^{\frac{7}{2}}}{7} - x^4 + \frac{2x^{\frac{9}{2}}}{9} + \frac{8x^{\frac{11}{2}}}{11} - \frac{2x^6}{3} + \frac{2x^{\frac{13}{2}}}{13}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^(3/2)*(1 + x^2)*(2*Sqrt[x] - x)^2,x]')

[Out] 8 x ^ (7 / 2) / 7 - x ^ 4 + 2 x ^ (9 / 2) / 9 + 8 x ^ (11 / 2) / 11 - 2 x ^ 6 / 3 + 2 x ^ (13 / 2) / 13

Maple [A]

time = 0.07, size = 32, normalized size = 0.65

method	result	size
derivativedivides	$\frac{8x^{\frac{7}{2}}}{7} - x^4 + \frac{2x^{\frac{9}{2}}}{9} + \frac{8x^{\frac{11}{2}}}{11} - \frac{2x^6}{3} + \frac{2x^{\frac{13}{2}}}{13}$	32
default	$\frac{8x^{\frac{7}{2}}}{7} - x^4 + \frac{2x^{\frac{9}{2}}}{9} + \frac{8x^{\frac{11}{2}}}{11} - \frac{2x^6}{3} + \frac{2x^{\frac{13}{2}}}{13}$	32
trager	$-\frac{(2x^5+2x^4+5x^3+5x^2+5x+5)(-1+x)}{3} + \frac{2x^{\frac{7}{2}}(693x^3+3276x^2+1001x+5148)}{9009}$	52

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x,method=_RETURNVERBOSE)`

[Out] $8/7*x^{(7/2)}-x^4+2/9*x^{(9/2)}+8/11*x^{(11/2)}-2/3*x^6+2/13*x^{(13/2)}$

Maxima [A]

time = 0.26, size = 31, normalized size = 0.63

$$\frac{2}{13}x^{\frac{13}{2}} - \frac{2}{3}x^6 + \frac{8}{11}x^{\frac{11}{2}} + \frac{2}{9}x^{\frac{9}{2}} - x^4 + \frac{8}{7}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x, algorithm="maxima")`

[Out] $2/13*x^{(13/2)} - 2/3*x^6 + 8/11*x^{(11/2)} + 2/9*x^{(9/2)} - x^4 + 8/7*x^{(7/2)}$

Fricas [A]

time = 0.38, size = 37, normalized size = 0.76

$$-\frac{2}{3}x^6 - x^4 + \frac{2}{9009}(693x^6 + 3276x^5 + 1001x^4 + 5148x^3)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x, algorithm="fricas")`

[Out] $-2/3*x^6 - x^4 + 2/9009*(693*x^6 + 3276*x^5 + 1001*x^4 + 5148*x^3)*\text{sqrt}(x)$

Sympy [A]

time = 0.32, size = 42, normalized size = 0.86

$$\frac{2x^{\frac{13}{2}}}{13} + \frac{8x^{\frac{11}{2}}}{11} + \frac{2x^{\frac{9}{2}}}{9} + \frac{8x^{\frac{7}{2}}}{7} - \frac{2x^6}{3} - x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(3/2)*(x**2+1)*(-x+2*x**(1/2))**2,x)`

[Out] $2*x^{(13/2)}/13 + 8*x^{(11/2)}/11 + 2*x^{(9/2)}/9 + 8*x^{(7/2)}/7 - 2*x^{**6}/3 - x^{**4}$

Giac [A]

time = 0.00, size = 65, normalized size = 1.33

$$\frac{2}{13}\sqrt{x}x^6 - \frac{4}{6}x^6 + \frac{8}{11}\sqrt{x}x^5 + \frac{2}{9}\sqrt{x}x^4 - \frac{4}{4}x^4 + \frac{8}{7}\sqrt{x}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x)``[Out] 2/13*x^(13/2) - 2/3*x^6 + 8/11*x^(11/2) + 2/9*x^(9/2) - x^4 + 8/7*x^(7/2)`**Mupad [B]**

time = 0.19, size = 31, normalized size = 0.63

$$\frac{8x^{7/2}}{7} - \frac{2x^6}{3} - x^4 + \frac{2x^{9/2}}{9} + \frac{8x^{11/2}}{11} + \frac{2x^{13/2}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^(3/2)*(x - 2*x^(1/2))^2*(x^2 + 1),x)``[Out] (8*x^(7/2))/7 - (2*x^6)/3 - x^4 + (2*x^(9/2))/9 + (8*x^(11/2))/11 + (2*x^(13/2))/13`

$$3.209 \quad \int \left(-3x^{3/5} + x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3} + 4x^{3/2}\right) dx$$

Optimal. Leaf size=55

$$-\frac{45x^{43/15}}{43} + \frac{360x^{37/10}}{37} + \frac{60x^{113/30}}{113} - \frac{120x^{23/5}}{23} - \frac{x^{14/3}}{14} + \frac{8x^{11/2}}{11}$$

[Out] $-45/43*x^{(43/15)}+360/37*x^{(37/10)}+60/113*x^{(113/30)}-120/23*x^{(23/5)}-1/14*x^{(14/3)}+8/11*x^{(11/2)}$

Rubi [A]

time = 0.15, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1607, 1598, 1834}

$$\frac{8x^{11/2}}{11} - \frac{x^{14/3}}{14} - \frac{120x^{23/5}}{23} + \frac{60x^{113/30}}{113} + \frac{360x^{37/10}}{37} - \frac{45x^{43/15}}{43}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-3*x^{(3/5)} + x^{(3/2)})^2*(-1/3*x^{(2/3)} + 4*x^{(3/2)}), x]$

[Out] $(-45*x^{(43/15)})/43 + (360*x^{(37/10)})/37 + (60*x^{(113/30)})/113 - (120*x^{(23/5)})/23 - x^{(14/3)}/14 + (8*x^{(11/2)})/11$

Rule 1598

$\text{Int}[(u_.)*(x_.)^{(m_.)}*((a_.)*(x_.)^{(p_.)} + (b_.)*(x_.)^{(q_.)})^{(n_.)}, x_Symbol]$
 $\rightarrow \text{Int}[u*x^{(m+n*p)}*(a+b*x^{(q-p)})^n, x] /;$ FreeQ[{a, b, m, p, q}, x]
 && IntegerQ[n] && PosQ[q - p]

Rule 1607

$\text{Int}[(u_.)*((a_.)*(x_.)^{(p_.)} + (b_.)*(x_.)^{(q_.)})^{(n_.)}, x_Symbol] \rightarrow \text{Int}[u*x^{(n*p)}*(a+b*x^{(q-p)})^n, x] /;$ FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1834

$\text{Int}[(Pq_)*((c_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow$
 $\text{Int}[\text{ExpandIntegrand}[(c*x)^m*Pq*(a+b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, m, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned}
\int (-3x^{3/5} + x^{3/2})^2 \left(-\frac{x^{2/3}}{3} + 4x^{3/2} \right) dx &= \int (-3 + x^{9/10})^2 x^{6/5} \left(-\frac{x^{2/3}}{3} + 4x^{3/2} \right) dx \\
&= \int \left(-\frac{1}{3} + 4x^{5/6} \right) (-3 + x^{9/10})^2 x^{28/15} dx \\
&= 30 \text{Subst} \left(\int x^{85} \left(-\frac{1}{3} + 4x^{25} \right) (-3 + x^{27})^2 dx, x, \sqrt[30]{x} \right) \\
&= 30 \text{Subst} \left(\int \left(-3x^{85} + 36x^{110} + 2x^{112} - 24x^{137} - \frac{x^{139}}{3} + 4x^{164} \right) dx, x, \sqrt[30]{x} \right) \\
&= -\frac{45x^{43/15}}{43} + \frac{360x^{37/10}}{37} + \frac{60x^{113/30}}{113} - \frac{120x^{23/5}}{23} - \frac{x^{14/3}}{14} + \frac{8x^{11/2}}{11}
\end{aligned}$$

Mathematica [A]

time = 0.11, size = 55, normalized size = 1.00

$$-\frac{45x^{43/15}}{43} + \frac{360x^{37/10}}{37} + \frac{60x^{113/30}}{113} - \frac{120x^{23/5}}{23} - \frac{x^{14/3}}{14} + \frac{8x^{11/2}}{11}$$

Antiderivative was successfully verified.

`[In] Integrate[(-3*x^(3/5) + x^(3/2))^2*(-1/3*x^(2/3) + 4*x^(3/2)),x]``[Out] (-45*x^(43/15))/43 + (360*x^(37/10))/37 + (60*x^(113/30))/113 - (120*x^(23/5))/23 - x^(14/3)/14 + (8*x^(11/2))/11`**Mathics [A]**

time = 4.35, size = 31, normalized size = 0.56

$$-\frac{45x^{43/15}}{43} + \frac{360x^{37/10}}{37} + \frac{60x^{113/30}}{113} - \frac{120x^{23/5}}{23} - \frac{x^{14/3}}{14} + \frac{8x^{11/2}}{11}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(x^(3/2) - 3*x^(3/5))^2*(4*x^(3/2) - (1/3)*x^(2/3)),x]')``[Out] -45 x ^ (43 / 15) / 43 + 360 x ^ (37 / 10) / 37 + 60 x ^ (113 / 30) / 113 - 120 x ^ (23 / 5) / 23 - x ^ (14 / 3) / 14 + 8 x ^ (11 / 2) / 11`**Maple [A]**

time = 0.07, size = 32, normalized size = 0.58

method	result	size
derivativedivides	$-\frac{45x^{43/15}}{43} + \frac{360x^{37/10}}{37} + \frac{60x^{113/30}}{113} - \frac{120x^{23/5}}{23} - \frac{x^{14/3}}{14} + \frac{8x^{11/2}}{11}$	32

default	$-\frac{45x^{43}}{43} + \frac{360x^{37}}{37} + \frac{60x^{113}}{113} - \frac{120x^{23}}{23} - \frac{x^{14}}{14} + \frac{8x^{11}}{11}$	32
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x,method=_RETURNVERBOSE)`

[Out] $-45/43*x^{43/15}+360/37*x^{37/10}+60/113*x^{113/30}-120/23*x^{23/5}-1/14*x^{14/3}+8/11*x^{11/2}$

Maxima [A]

time = 0.26, size = 31, normalized size = 0.56

$$\frac{8}{11}x^{\frac{11}{2}} - \frac{1}{14}x^{\frac{14}{3}} - \frac{120}{23}x^{\frac{23}{5}} + \frac{60}{113}x^{\frac{113}{30}} + \frac{360}{37}x^{\frac{37}{10}} - \frac{45}{43}x^{\frac{43}{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x, algorithm="maxima")`

[Out] $8/11*x^{11/2} - 1/14*x^{14/3} - 120/23*x^{23/5} + 60/113*x^{113/30} + 360/37*x^{37/10} - 45/43*x^{43/15}$

Fricas [A]

time = 0.30, size = 31, normalized size = 0.56

$$\frac{8}{11}x^{\frac{11}{2}} - \frac{1}{14}x^{\frac{14}{3}} - \frac{120}{23}x^{\frac{23}{5}} + \frac{60}{113}x^{\frac{113}{30}} + \frac{360}{37}x^{\frac{37}{10}} - \frac{45}{43}x^{\frac{43}{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x, algorithm="fricas")`

[Out] $8/11*x^{11/2} - 1/14*x^{14/3} - 120/23*x^{23/5} + 60/113*x^{113/30} + 360/37*x^{37/10} - 45/43*x^{43/15}$

Sympy [A]

time = 0.87, size = 48, normalized size = 0.87

$$\frac{60x^{113}}{113} - \frac{45x^{43}}{43} + \frac{360x^{37}}{37} - \frac{120x^{23}}{23} - \frac{x^{14}}{14} + \frac{8x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3*x**(3/5)+x**(3/2))**2*(-1/3*x**(2/3)+4*x**(3/2)),x)`

[Out] $60*x^{113/30}/113 - 45*x^{43/15}/43 + 360*x^{37/10}/37 - 120*x^{23/5}/23 - x^{14/3}/14 + 8*x^{11/2}/11$

Giac [A]

time = 0.00, size = 120, normalized size = 2.18

$$\frac{\frac{120}{55}x^{\frac{1}{10}}\left(x^{\frac{1}{5}}\right)^2x^5 - \frac{360}{23}\left(x^{\frac{1}{5}}\right)^3x^4 - \frac{15}{70}x^{\frac{1}{15}}\left(x^{\frac{1}{5}}\right)^3x^4 + \frac{1080}{37}x^{\frac{1}{10}}\left(x^{\frac{1}{5}}\right)^3x^3 + \frac{180}{113}x^{\frac{1}{6}}\left(x^{\frac{1}{5}}\right)^3x^3 - \frac{135}{43}x^{\frac{1}{15}}\left(x^{\frac{1}{5}}\right)^4x^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x)**[Out]** 8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/30) + 360/37*x^(37/10) - 45/43*x^(43/15)**Mupad [B]**

time = 0.07, size = 31, normalized size = 0.56

$$\frac{8x^{11/2}}{11} - \frac{x^{14/3}}{14} - \frac{120x^{23/5}}{23} + \frac{360x^{37/10}}{37} - \frac{45x^{43/15}}{43} + \frac{60x^{113/30}}{113}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^(3/2) - 3*x^(3/5))^2*(x^(2/3)/3 - 4*x^(3/2)),x)**[Out]** (8*x^(11/2))/11 - x^(14/3)/14 - (120*x^(23/5))/23 + (360*x^(37/10))/37 - (45*x^(43/15))/43 + (60*x^(113/30))/113

$$3.210 \quad \int \frac{1}{1 + \sqrt{1 + x}} dx$$

Optimal. Leaf size=22

$$2\sqrt{1+x} - 2\log(1 + \sqrt{1+x})$$

[Out] -2*ln(1+(1+x)^(1/2))+2*(1+x)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {253, 196, 45}

$$2\sqrt{x+1} - 2\log(\sqrt{x+1} + 1)$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[1 + x])^(-1), x]

[Out] 2*Sqrt[1 + x] - 2*Log[1 + Sqrt[1 + x]]

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 196

Int[((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(1/n - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]

Rule 253

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{1 + \sqrt{1+x}} dx &= \text{Subst} \left(\int \frac{1}{1 + \sqrt{x}} dx, x, 1+x \right) \\
&= 2\text{Subst} \left(\int \frac{x}{1+x} dx, x, \sqrt{1+x} \right) \\
&= 2\text{Subst} \left(\int \left(1 + \frac{1}{-1-x} \right) dx, x, \sqrt{1+x} \right) \\
&= 2\sqrt{1+x} - 2\log(1 + \sqrt{1+x})
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 22, normalized size = 1.00

$$2\sqrt{1+x} - 2\log(1 + \sqrt{1+x})$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + Sqrt[1 + x])^(-1), x]``[Out] 2*Sqrt[1 + x] - 2*Log[1 + Sqrt[1 + x]]`**Mathics [A]**

time = 8.00, size = 18, normalized size = 0.82

$$-2\text{Log}[1 + \sqrt{1+x}] + 2\sqrt{1+x}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(1 + Sqrt[1 + x]), x]')``[Out] -2 Log[1 + Sqrt[1 + x]] + 2 Sqrt[1 + x]`**Maple [A]**

time = 0.04, size = 31, normalized size = 1.41

method	result	size
derivativedivides	$-2 \ln(1 + \sqrt{1+x}) + 2\sqrt{1+x}$	19
trager	$2\sqrt{1+x} - \ln(2\sqrt{1+x} + 2 + x)$	22
default	$2\sqrt{1+x} + \ln(-1 + \sqrt{1+x}) - \ln(1 + \sqrt{1+x}) - \ln(x)$	31
meijerg	$\frac{-4\sqrt{\pi} + 4\sqrt{\pi} \sqrt{1+x} - 4\sqrt{\pi} \ln\left(\frac{1}{2} + \frac{\sqrt{1+x}}{2}\right)}{2\sqrt{\pi}}$	37

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1+(1+x)^(1/2)),x,method=_RETURNVERBOSE)`

[Out] $2*(1+x)^{(1/2)}+\ln(-1+(1+x)^{(1/2)})-\ln(1+(1+x)^{(1/2)})-\ln(x)$

Maxima [A]

time = 0.26, size = 18, normalized size = 0.82

$$2\sqrt{x+1} - 2\log(\sqrt{x+1} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+(1+x)^(1/2)),x, algorithm="maxima")`

[Out] $2*\text{sqrt}(x + 1) - 2*\log(\text{sqrt}(x + 1) + 1)$

Fricas [A]

time = 0.30, size = 18, normalized size = 0.82

$$2\sqrt{x+1} - 2\log(\sqrt{x+1} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+(1+x)^(1/2)),x, algorithm="fricas")`

[Out] $2*\text{sqrt}(x + 1) - 2*\log(\text{sqrt}(x + 1) + 1)$

Sympy [A]

time = 0.06, size = 19, normalized size = 0.86

$$2\sqrt{x+1} - 2\log(\sqrt{x+1} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+(1+x)**(1/2)),x)`

[Out] $2*\text{sqrt}(x + 1) - 2*\log(\text{sqrt}(x + 1) + 1)$

Giac [A]

time = 0.00, size = 21, normalized size = 0.95

$$2\left(\sqrt{x+1} - \ln(\sqrt{x+1} + 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+(1+x)^(1/2)),x)`

[Out] $2*\sqrt{x + 1} - 2*\log(\sqrt{x + 1} + 1)$

Mupad [B]

time = 0.13, size = 18, normalized size = 0.82

$$2\sqrt{x+1} - 2\ln(\sqrt{x+1} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/((x + 1)^{(1/2)} + 1), x)$

[Out] $2*(x + 1)^{(1/2)} - 2*\log((x + 1)^{(1/2)} + 1)$

$$3.211 \quad \int \frac{x}{1 + \sqrt{1 + x}} dx$$

Optimal. Leaf size=15

$$-x + \frac{2}{3}(1+x)^{3/2}$$

[Out] $-x + 2/3*(1+x)^{(3/2)}$

Rubi [A]

time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {378}

$$\frac{2}{3}(x+1)^{3/2} - x$$

Antiderivative was successfully verified.

[In] `Int[x/(1 + Sqrt[1 + x]),x]`

[Out] $-x + (2*(1 + x)^{(3/2)})/3$

Rule 378

`Int[((a_) + (b_)*(v_)^(n_))^(p_)*(x_)^(m_), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]`

Rubi steps

$$\begin{aligned} \int \frac{x}{1 + \sqrt{1 + x}} dx &= \text{Subst} \left(\int (-1 + \sqrt{x}) dx, x, 1 + x \right) \\ &= -x + \frac{2}{3}(1+x)^{3/2} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 18, normalized size = 1.20

$$\frac{1}{3}(1+x) \left(-3 + 2\sqrt{1+x} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[x/(1 + Sqrt[1 + x]),x]`

[Out] $((1 + x)*(-3 + 2*\text{Sqrt}[1 + x]))/3$

Mathics [A]

time = 7.06, size = 19, normalized size = 1.27

$$-x + \frac{2x\sqrt{1+x}}{3} + \frac{2\sqrt{1+x}}{3}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x/(1 + Sqrt[1 + x]),x]')`

[Out] $-x + 2 x \text{ Sqrt}[1 + x] / 3 + 2 \text{ Sqrt}[1 + x] / 3$

Maple [A]

time = 0.01, size = 13, normalized size = 0.87

method	result	size
derivativedivides	$\frac{2(1+x)^{\frac{3}{2}}}{3} - 1 - x$	13
default	$\frac{2(1+x)^{\frac{3}{2}}}{3} - 1 - x$	13
trager	$-x + \left(\frac{2}{3} + \frac{2x}{3}\right) \sqrt{1+x}$	16
meijerg	$\frac{-\frac{\sqrt{\pi}}{6} (12x+8) + \frac{\sqrt{\pi}}{6} (8+8x) \sqrt{1+x}}{2\sqrt{\pi}}$	32

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(1+(1+x)^(1/2)),x,method=_RETURNVERBOSE)`

[Out] $2/3*(1+x)^(3/2)-1-x$

Maxima [A]

time = 0.36, size = 12, normalized size = 0.80

$$\frac{2}{3}(x+1)^{\frac{3}{2}} - x - 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(1+(1+x)^(1/2)),x, algorithm="maxima")`

[Out] $2/3*(x + 1)^(3/2) - x - 1$

Fricas [A]

time = 0.30, size = 11, normalized size = 0.73

$$\frac{2}{3}(x+1)^{\frac{3}{2}} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(1+(1+x)^(1/2)),x, algorithm="fricas")`

[Out] $\frac{2}{3}(x + 1)^{3/2} - x$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(10) = 20
time = 0.46, size = 22, normalized size = 1.47

$$\frac{2x\sqrt{x+1}}{3} - x + \frac{2\sqrt{x+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(1+(1+x)**(1/2)),x)`

[Out] $2*x*\text{sqrt}(x + 1)/3 - x + 2*\text{sqrt}(x + 1)/3$

Giac [A]

time = 0.00, size = 23, normalized size = 1.53

$$2 \left(\frac{1}{3} \sqrt{x+1} (x+1) - \frac{x+1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(1+(1+x)^(1/2)),x)`

[Out] $\frac{2}{3}(x + 1)^{3/2} - x - 1$

Mupad [B]

time = 0.03, size = 11, normalized size = 0.73

$$\frac{2(x+1)^{3/2}}{3} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/((x + 1)^(1/2) + 1),x)`

[Out] $(2*(x + 1)^{3/2})/3 - x$

$$3.212 \quad \int \frac{1 + \sqrt{1+x}}{-1 + \sqrt{1+x}} dx$$

Optimal. Leaf size=25

$$x + 4\sqrt{1+x} + 4\log(1 - \sqrt{1+x})$$

[Out] x+4*ln(1-(1+x)^(1/2))+4*(1+x)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {442, 383, 78}

$$x + 4\sqrt{x+1} + 4\log(1 - \sqrt{x+1})$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[1 + x])/(-1 + Sqrt[1 + x]),x]

[Out] x + 4*Sqrt[1 + x] + 4*Log[1 - Sqrt[1 + x]]

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x]
;/; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0])
|| EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
:> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]
;/; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 442

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symbol]
:> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x, u], x]
;/; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1 + \sqrt{1+x}}{-1 + \sqrt{1+x}} dx &= \text{Subst} \left(\int \frac{1 + \sqrt{x}}{-1 + \sqrt{x}} dx, x, 1+x \right) \\
&= 2\text{Subst} \left(\int \frac{x(1+x)}{-1+x} dx, x, \sqrt{1+x} \right) \\
&= 2\text{Subst} \left(\int \left(2 + \frac{2}{-1+x} + x \right) dx, x, \sqrt{1+x} \right) \\
&= x + 4\sqrt{1+x} + 4 \log \left(1 - \sqrt{1+x} \right)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 24, normalized size = 0.96

$$1 + x + 4\sqrt{1+x} + 4 \log \left(-1 + \sqrt{1+x} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + Sqrt[1 + x])/(-1 + Sqrt[1 + x]), x]``[Out] 1 + x + 4*Sqrt[1 + x] + 4*Log[-1 + Sqrt[1 + x]]`**Mathics [A]**

time = 4.88, size = 19, normalized size = 0.76

$$x + 4\text{Log} \left[-1 + \sqrt{1+x} \right] + 4\sqrt{1+x}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(Sqrt[1 + x] + 1)/(Sqrt[1 + x] - 1), x]')``[Out] x + 4 Log[-1 + Sqrt[1 + x]] + 4 Sqrt[1 + x]`**Maple [A]**

time = 0.05, size = 21, normalized size = 0.84

method	result	size
derivativedivides	$1 + x + 4\sqrt{1+x} + 4 \ln \left(-1 + \sqrt{1+x} \right)$	21
default	$1 + x + 4\sqrt{1+x} + 4 \ln \left(-1 + \sqrt{1+x} \right)$	21
trager	$-1 + x + 4\sqrt{1+x} + 2 \ln \left(2\sqrt{1+x} - 2 - x \right)$	26

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)), x, method=_RETURNVERBOSE)`

[Out] $1+x+4*(1+x)^{(1/2)}+4*\ln(-1+(1+x)^{(1/2)})$

Maxima [A]

time = 0.26, size = 20, normalized size = 0.80

$$x + 4\sqrt{x+1} + 4\log(\sqrt{x+1} - 1) + 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x, algorithm="maxima")`

[Out] $x + 4*\sqrt{x + 1} + 4*\log(\sqrt{x + 1} - 1) + 1$

Fricas [A]

time = 0.30, size = 19, normalized size = 0.76

$$x + 4\sqrt{x+1} + 4\log(\sqrt{x+1} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x, algorithm="fricas")`

[Out] $x + 4*\sqrt{x + 1} + 4*\log(\sqrt{x + 1} - 1)$

Sympy [A]

time = 0.07, size = 20, normalized size = 0.80

$$x + 4\sqrt{x+1} + 4\log(\sqrt{x+1} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+(1+x)**(1/2))/(-1+(1+x)**(1/2)),x)`

[Out] $x + 4*\sqrt{x + 1} + 4*\log(\sqrt{x + 1} - 1)$

Giac [A]

time = 0.00, size = 25, normalized size = 1.00

$$x + 1 + 4\sqrt{x+1} + 4\ln|\sqrt{x+1} - 1|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x)`

[Out] $x + 4*\sqrt{x + 1} + 4*\log(\text{abs}(\sqrt{x + 1} - 1)) + 1$

Mupad [B]

time = 0.22, size = 19, normalized size = 0.76

$$x + 4\ln(\sqrt{x+1} - 1) + 4\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x + 1)^(1/2) + 1)/((x + 1)^(1/2) - 1),x)`

[Out] $x + 4*\log((x + 1)^{(1/2)} - 1) + 4*(x + 1)^{(1/2)}$

$$3.213 \quad \int \frac{1}{-\sqrt{1+x} + (1+x)^{2/3}} dx$$

Optimal. Leaf size=33

$$6\sqrt[6]{1+x} + 3\sqrt[3]{1+x} + 6 \log \left(1 - \sqrt[6]{1+x} \right)$$

[Out] 6*(1+x)^(1/6)+3*(1+x)^(1/3)+6*ln(1-(1+x)^(1/6))

Rubi [A]

time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2037, 1607, 272, 45}

$$3\sqrt[3]{x+1} + 6\sqrt[6]{x+1} + 6 \log \left(1 - \sqrt[6]{x+1} \right)$$

Antiderivative was successfully verified.

[In] Int[(-Sqrt[1 + x] + (1 + x)^(2/3))^(−1), x]

[Out] 6*(1 + x)^(1/6) + 3*(1 + x)^(1/3) + 6*Log[1 - (1 + x)^(1/6)]

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1607

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 2037

Int[((a_.)*(u_)^(j_.) + (b_.)*(u_)^(n_.))^(p_), x_Symbol] := Dist[1/Coefficient[u, x, 1], Subst[Int[(a*x^j + b*x^n)^p, x], x, u], x] /; FreeQ[{a, b, j, n, p}, x] && LinearQ[u, x] && NeQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{-\sqrt{1+x} + (1+x)^{2/3}} dx &= \text{Subst} \left(\int \frac{1}{-\sqrt{x} + x^{2/3}} dx, x, 1+x \right) \\
&= \text{Subst} \left(\int \frac{1}{(-1 + \sqrt[6]{x}) \sqrt{x}} dx, x, 1+x \right) \\
&= 6 \text{Subst} \left(\int \frac{x^2}{-1+x} dx, x, \sqrt[6]{1+x} \right) \\
&= 6 \text{Subst} \left(\int \left(1 + \frac{1}{-1+x} + x \right) dx, x, \sqrt[6]{1+x} \right) \\
&= 6\sqrt[6]{1+x} + 3\sqrt[3]{1+x} + 6 \log \left(1 - \sqrt[6]{1+x} \right)
\end{aligned}$$

Mathematica [A]

time = 0.31, size = 31, normalized size = 0.94

$$3 \left(2\sqrt[6]{1+x} + \sqrt[3]{1+x} + 2 \log \left(-1 + \sqrt[6]{1+x} \right) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(-Sqrt[1 + x] + (1 + x)^(2/3))^(−1), x]``[Out] 3*(2*(1 + x)^(1/6) + (1 + x)^(1/3) + 2*Log[-1 + (1 + x)^(1/6)])`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/((1 + x)^(2/3) - Sqrt[1 + x]), x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(27) = 54.

time = 0.03, size = 111, normalized size = 3.36

method	result
derivativedivides	$3(1+x)^{\frac{1}{3}} + 6(1+x)^{\frac{1}{6}} + 6 \ln \left(-1 + (1+x)^{\frac{1}{6}} \right)$

default	$6(1+x)^{\frac{1}{6}} + 3(1+x)^{\frac{1}{3}} + \ln(x) + 2\ln\left(-1 + (1+x)^{\frac{1}{6}}\right) - \ln\left((1+x)^{\frac{1}{3}} + (1+x)^{\frac{1}{6}} + 1\right)$
---------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((1+x)^(2/3)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)`

[Out] $6*(1+x)^{(1/6)}+3*(1+x)^{(1/3)}+\ln(x)+2*\ln(-1+(1+x)^{(1/6)})-\ln((1+x)^{(1/3)}+(1+x)^{(1/6)}+1)-2*\ln((1+x)^{(1/6)}+1)+\ln((1+x)^{(1/3)}-(1+x)^{(1/6)}+1)-\ln(1+(1+x)^{(1/2)}))+\ln(-1+(1+x)^{(1/2}))+2*\ln(-1+(1+x)^{(1/3)}))-\ln((1+x)^{(2/3)}+(1+x)^{(1/3)}+1)$

Maxima [A]

time = 0.26, size = 25, normalized size = 0.76

$$3(x+1)^{\frac{1}{3}} + 6(x+1)^{\frac{1}{6}} + 6 \log\left((x+1)^{\frac{1}{6}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((1+x)^(2/3)-(1+x)^(1/2)),x, algorithm="maxima")`

[Out] $3*(x+1)^{(1/3)} + 6*(x+1)^{(1/6)} + 6*\log((x+1)^{(1/6)} - 1)$

Fricas [A]

time = 0.31, size = 25, normalized size = 0.76

$$3(x+1)^{\frac{1}{3}} + 6(x+1)^{\frac{1}{6}} + 6 \log\left((x+1)^{\frac{1}{6}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((1+x)^(2/3)-(1+x)^(1/2)),x, algorithm="fricas")`

[Out] $3*(x+1)^{(1/3)} + 6*(x+1)^{(1/6)} + 6*\log((x+1)^{(1/6)} - 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+1)^{\frac{2}{3}} - \sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((1+x)**(2/3)-(1+x)**(1/2)),x)`

[Out] `Integral(1/((x+1)**(2/3) - sqrt(x+1)), x)`

Giac [A]

time = 0.00, size = 31, normalized size = 0.94

$$6 \left(\frac{(x+1)^{\frac{1}{3}}}{2} + (x+1)^{\frac{1}{6}} + \ln\left|(x+1)^{\frac{1}{6}} - 1\right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((1+x)^(2/3)-(1+x)^(1/2)),x)`

[Out] $3*(x + 1)^{1/3} + 6*(x + 1)^{1/6} + 6*\log(\text{abs}((x + 1)^{1/6} - 1))$

Mupad [B]

time = 0.24, size = 25, normalized size = 0.76

$$6 \ln \left((x + 1)^{1/6} - 1 \right) + 3(x + 1)^{1/3} + 6(x + 1)^{1/6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/((x + 1)^(1/2) - (x + 1)^(2/3)),x)`

[Out] $6*\log((x + 1)^{1/6} - 1) + 3*(x + 1)^{1/3} + 6*(x + 1)^{1/6}$

$$3.214 \quad \int \frac{\sqrt[3]{1 + \sqrt[4]{x}}}{\sqrt{x}} dx$$

Optimal. Leaf size=29

$$-3(1 + \sqrt[4]{x})^{4/3} + \frac{12}{7}(1 + \sqrt[4]{x})^{7/3}$$

[Out] $-3*(1+x^{(1/4)})^{(4/3)}+12/7*(1+x^{(1/4)})^{(7/3)}$

Rubi [A]

time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {272, 45}

$$\frac{12}{7}(\sqrt[4]{x} + 1)^{7/3} - 3(\sqrt[4]{x} + 1)^{4/3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + x^{(1/4)})^{(1/3)}/\text{Sqrt}[x], x]$

[Out] $-3*(1 + x^{(1/4)})^{(4/3)} + (12*(1 + x^{(1/4)})^{(7/3)})/7$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

$\text{Int}[(x_.)^{(m_.)*((a_) + (b_.)*(x_.)^{(n_.))^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt[3]{1 + \sqrt[4]{x}}}{\sqrt{x}} dx &= 4\text{Subst} \left(\int x \sqrt[3]{1 + x} dx, x, \sqrt[4]{x} \right) \\ &= 4\text{Subst} \left(\int \left(-\sqrt[3]{1 + x} + (1 + x)^{4/3} \right) dx, x, \sqrt[4]{x} \right) \\ &= -3(1 + \sqrt[4]{x})^{4/3} + \frac{12}{7}(1 + \sqrt[4]{x})^{7/3} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 24, normalized size = 0.83

$$\frac{3}{7} (1 + \sqrt[4]{x})^{4/3} (-3 + 4\sqrt[4]{x})$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + x^(1/4))^(1/3)/Sqrt[x], x]``[Out] (3*(1 + x^(1/4))^(4/3)*(-3 + 4*x^(1/4)))/7`**Mathics [B]** Leaf count is larger than twice the leaf count of optimal. 92 vs. 2(29) = 58.

time = 3.15, size = 64, normalized size = 2.21

$$\frac{3 \left(-3x \left(1 + x^{\frac{1}{4}}\right)^{\frac{1}{3}} + 3x - 2x^{\frac{5}{4}} \left(1 + x^{\frac{1}{4}}\right)^{\frac{1}{3}} + 3x^{\frac{5}{4}} + 5x^{\frac{3}{2}} \left(1 + x^{\frac{1}{4}}\right)^{\frac{1}{3}} + 4x^{\frac{7}{4}} \left(1 + x^{\frac{1}{4}}\right)^{\frac{1}{3}} \right)}{7x + 7x^{\frac{5}{4}}}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(1 + x^(1/4))^(1/3)/Sqrt[x], x]')`

```
[Out] 3 (-3 x (1 + x ^ (1 / 4)) ^ (1 / 3) + 3 x - 2 x ^ (5 / 4) (1 + x ^ (1 / 4))
^ (1 / 3) + 3 x ^ (5 / 4) + 5 x ^ (3 / 2) (1 + x ^ (1 / 4)) ^ (1 / 3) + 4
x ^ (7 / 4) (1 + x ^ (1 / 4)) ^ (1 / 3)) / (7 (x + x ^ (5 / 4)))
```

Maple [A]

time = 0.05, size = 20, normalized size = 0.69

method	result	size
meijerg	$2\sqrt{x}$ hypergeom $\left(\left[-\frac{1}{3}, 2\right], [3], -x^{\frac{1}{4}}\right)$	17
derivativedivides	$-3 \left(1 + x^{\frac{1}{4}}\right)^{\frac{4}{3}} + \frac{12(1+x^{\frac{1}{4}})^{\frac{7}{3}}}{7}$	20
default	$-3 \left(1 + x^{\frac{1}{4}}\right)^{\frac{4}{3}} + \frac{12(1+x^{\frac{1}{4}})^{\frac{7}{3}}}{7}$	20

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((1+x^(1/4))^(1/3)/x^(1/2), x, method=_RETURNVERBOSE)``[Out] -3*(1+x^(1/4))^(4/3)+12/7*(1+x^(1/4))^(7/3)`**Maxima [A]**

time = 0.26, size = 19, normalized size = 0.66

$$\frac{12}{7} \left(x^{\frac{1}{4}} + 1\right)^{\frac{7}{3}} - 3 \left(x^{\frac{1}{4}} + 1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x^(1/4))^(1/3)/x^(1/2),x, algorithm="maxima")`

[Out] $12/7*(x^{1/4} + 1)^{7/3} - 3*(x^{1/4} + 1)^{4/3}$

Fricas [A]

time = 0.32, size = 19, normalized size = 0.66

$$\frac{3}{7} \left(4\sqrt{x} + x^{\frac{1}{4}} - 3 \right) \left(x^{\frac{1}{4}} + 1 \right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x^(1/4))^(1/3)/x^(1/2),x, algorithm="fricas")`

[Out] $3/7*(4*\sqrt{x} + x^{1/4} - 3)*(x^{1/4} + 1)^{1/3}$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(24) = 48

time = 0.67, size = 134, normalized size = 4.62

$$\frac{12x^{\frac{7}{4}}\sqrt[3]{\sqrt[4]{x} + 1}}{7x^{\frac{5}{4}} + 7x} - \frac{6x^{\frac{5}{4}}\sqrt[3]{\sqrt[4]{x} + 1}}{7x^{\frac{5}{4}} + 7x} + \frac{9x^{\frac{5}{4}}}{7x^{\frac{5}{4}} + 7x} + \frac{15x^{\frac{3}{2}}\sqrt[3]{\sqrt[4]{x} + 1}}{7x^{\frac{5}{4}} + 7x} - \frac{9x\sqrt[3]{\sqrt[4]{x} + 1}}{7x^{\frac{5}{4}} + 7x} + \frac{9x}{7x^{\frac{5}{4}} + 7x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x**(1/4))**(1/3)/x**(1/2),x)`

[Out] $12*x^{7/4}*(x^{1/4} + 1)^{1/3}/(7*x^{5/4} + 7*x) - 6*x^{5/4}*(x^{1/4} + 1)^{1/3}/(7*x^{5/4} + 7*x) + 9*x^{5/4}/(7*x^{5/4} + 7*x) + 15*x^{3/2}*(x^{1/4} + 1)^{1/3}/(7*x^{5/4} + 7*x) - 9*x*(x^{1/4} + 1)^{1/3}/(7*x^{5/4} + 7*x) + 9*x/(7*x^{5/4} + 7*x)$

Giac [A]

time = 0.00, size = 46, normalized size = 1.59

$$\frac{12}{7} \left(x^{\frac{1}{4}} + 1 \right)^{\frac{1}{3}} \left(x^{\frac{1}{4}} + 1 \right)^2 - 3 \left(x^{\frac{1}{4}} + 1 \right)^{\frac{1}{3}} \left(x^{\frac{1}{4}} + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x^(1/4))^(1/3)/x^(1/2),x)`

[Out] $12/7*(x^{1/4} + 1)^{7/3} - 3*(x^{1/4} + 1)^{4/3}$

Mupad [B]

time = 0.54, size = 16, normalized size = 0.55

$$\frac{3 \left(x^{1/4} + 1 \right)^{4/3} \left(4x^{1/4} - 3 \right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(1/4) + 1)^(1/3)/x^(1/2),x)`

[Out] $(3*(x^{1/4} + 1)^{4/3}*(4*x^{1/4} - 3))/7$

3.215 $\int \frac{1}{x^3(1+x)^{3/2}} dx$

Optimal. Leaf size=52

$$\frac{15}{4\sqrt{1+x}} - \frac{1}{2x^2\sqrt{1+x}} + \frac{5}{4x\sqrt{1+x}} - \frac{15}{4} \tanh^{-1}(\sqrt{1+x})$$

[Out] -15/4*arctanh((1+x)^(1/2))+15/4/(1+x)^(1/2)-1/2/x^2/(1+x)^(1/2)+5/4/x/(1+x)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$,

Rules used = {44, 53, 65, 213}

$$-\frac{1}{2x^2\sqrt{x+1}} + \frac{5}{4x\sqrt{x+1}} + \frac{15}{4\sqrt{x+1}} - \frac{15}{4} \tanh^{-1}(\sqrt{x+1})$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(1+x)^(3/2)),x]

[Out] 15/(4*Sqrt[1+x]) - 1/(2*x^2*Sqrt[1+x]) + 5/(4*x*Sqrt[1+x]) - (15*ArcTanh[Sqrt[1+x]])/4

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
```


[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 213

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3(1+x)^{3/2}} dx &= \frac{2}{x^2\sqrt{1+x}} + 5 \int \frac{1}{x^3\sqrt{1+x}} dx \\
 &= \frac{2}{x^2\sqrt{1+x}} - \frac{5\sqrt{1+x}}{2x^2} - \frac{15}{4} \int \frac{1}{x^2\sqrt{1+x}} dx \\
 &= \frac{2}{x^2\sqrt{1+x}} - \frac{5\sqrt{1+x}}{2x^2} + \frac{15\sqrt{1+x}}{4x} + \frac{15}{8} \int \frac{1}{x\sqrt{1+x}} dx \\
 &= \frac{2}{x^2\sqrt{1+x}} - \frac{5\sqrt{1+x}}{2x^2} + \frac{15\sqrt{1+x}}{4x} + \frac{15}{4} \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1+x} \right) \\
 &= \frac{2}{x^2\sqrt{1+x}} - \frac{5\sqrt{1+x}}{2x^2} + \frac{15\sqrt{1+x}}{4x} - \frac{15}{4} \tanh^{-1}(\sqrt{1+x})
 \end{aligned}$$

Mathematica [A]

time = 0.04, size = 36, normalized size = 0.69

$$\frac{1}{4} \left(\frac{-2 + 5x + 15x^2}{x^2\sqrt{1+x}} - 15 \tanh^{-1}(\sqrt{1+x}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(1+x)^(3/2)),x]

[Out] ((-2 + 5*x + 15*x^2)/(x^2*Sqrt[1 + x]) - 15*ArcTanh[Sqrt[1 + x]])/4

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 28.73, size = 1410, normalized size = 27.12

result too large to display

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x^3*(1+x)^(3/2)),x]')

```

[Out] Piecewise[{{-15 I / 8 Pi - 1 / (2 x ^ 2 Sqrt[1 + x]) + 5 / (4 x Sqrt[1 + x]
) - 15 ArcCoth[Sqrt[1 + x]] / 4 + 15 / (4 Sqrt[1 + x]), Abs[1 + x] > 1}}, -
1050 ArcTanh[Sqrt[1 + x]] (1 + x) ^ (9 / 2) / (-224 (1 + x) ^ (7 / 2) - 224
(1 + x) ^ (11 / 2) - 32 (1 + x) ^ (3 / 2) - 32 (1 + x) ^ (15 / 2) + 4 Sqrt
[1 + x] + 4 (1 + x) ^ (17 / 2) + 112 (1 + x) ^ (5 / 2) + 112 (1 + x) ^ (13
/ 2) + 280 (1 + x) ^ (9 / 2)) - 723 (1 + x) ^ 5 / (-224 (1 + x) ^ (7 / 2) -
224 (1 + x) ^ (11 / 2) - 32 (1 + x) ^ (3 / 2) - 32 (1 + x) ^ (15 / 2) + 4
Sqrt[1 + x] + 4 (1 + x) ^ (17 / 2) + 112 (1 + x) ^ (5 / 2) + 112 (1 + x) ^
(13 / 2) + 280 (1 + x) ^ (9 / 2)) - 625 (1 + x) ^ 3 / (-224 (1 + x) ^ (7 /
2) - 224 (1 + x) ^ (11 / 2) - 32 (1 + x) ^ (3 / 2) - 32 (1 + x) ^ (15 / 2)
+ 4 Sqrt[1 + x] + 4 (1 + x) ^ (17 / 2) + 112 (1 + x) ^ (5 / 2) + 112 (1 + x
) ^ (13 / 2) + 280 (1 + x) ^ (9 / 2)) - 420 ArcTanh[Sqrt[1 + x]] (1 + x) ^
(5 / 2) / (-224 (1 + x) ^ (7 / 2) - 224 (1 + x) ^ (11 / 2) - 32 (1 + x) ^ (
3 / 2) - 32 (1 + x) ^ (15 / 2) + 4 Sqrt[1 + x] + 4 (1 + x) ^ (17 / 2) + 112
(1 + x) ^ (5 / 2) + 112 (1 + x) ^ (13 / 2) + 280 (1 + x) ^ (9 / 2)) - 420
ArcTanh[Sqrt[1 + x]] (1 + x) ^ (13 / 2) / (-224 (1 + x) ^ (7 / 2) - 224 (1
+ x) ^ (11 / 2) - 32 (1 + x) ^ (3 / 2) - 32 (1 + x) ^ (15 / 2) + 4 Sqrt[1 +
x] + 4 (1 + x) ^ (17 / 2) + 112 (1 + x) ^ (5 / 2) + 112 (1 + x) ^ (13 / 2)
+ 280 (1 + x) ^ (9 / 2)) - 115 (1 + x) ^ 7 / (-224 (1 + x) ^ (7 / 2) - 224
(1 + x) ^ (11 / 2) - 32 (1 + x) ^ (3 / 2) - 32 (1 + x) ^ (15 / 2) + 4 Sqrt
[1 + x] + 4 (1 + x) ^ (17 / 2) + 112 (1 + x) ^ (5 / 2) + 112 (1 + x) ^ (13
/ 2) + 280 (1 + x) ^ (9 / 2)) - 73 (1 + x) / (-224 (1 + x) ^ (7 / 2) - 224
(1 + x) ^ (11 / 2) - 32 (1 + x) ^ (3 / 2) - 32 (1 + x) ^ (15 / 2) + 4 Sqrt[
1 + x] + 4 (1 + x) ^ (17 / 2) + 112 (1 + x) ^ (5 / 2) + 112 (1 + x) ^ (13 /
2) + 280 (1 + x) ^ (9 / 2)) - 15 ArcTanh[Sqrt[1 + x]] Sqrt[1 + x] / (-224
(1 + x) ^ (7 / 2) - 224 (1 + x) ^ (11 / 2) - 32 (1 + x) ^ (3 / 2) - 32 (1 +
x) ^ (15 / 2) + 4 Sqrt[1 + x] + 4 (1 + x) ^ (17 / 2) + 112 (1 + x) ^ (5 /
2) + 112 (1 + x) ^ (13 / 2) + 280 (1 + x) ^ (9 / 2)) - 15 ArcTanh[Sqrt[1 +
x]] (1 + x) ^ (17 / 2) / (-224 (1 + x) ^ (7 / 2) - 224 (1 + x) ^ (11 / 2) -
32 (1 + x) ^ (3 / 2) - 32 (1 + x) ^ (15 / 2) + 4 Sqrt[1 + x] + 4 (1 + x) ^
(17 / 2) + 112 (1 + x) ^ (5 / 2) + 112 (1 + x) ^ (13 / 2) + 280 (1 + x) ^
(9 / 2)) + 8 / (-224 (1 + x) ^ (7 / 2) - 224 (1 + x) ^ (11 / 2) - 32 (1 + x
) ^ (3 / 2) - 32 (1 + x) ^ (15 / 2) + 4 Sqrt[1 + x] + 4 (1 + x) ^ (17 / 2)
+ 112 (1 + x) ^ (5 / 2) + 112 (1 + x) ^ (13 / 2) + 280 (1 + x) ^ (9 / 2)) +
15 (1 + x) ^ 8 / (-224 (1 + x) ^ (7 / 2) - 224 (1 + x) ^ (11 / 2) - 32 (1
+ x) ^ (3 / 2) - 32 (1 + x) ^ (15 / 2) + 4 Sqrt[1 + x] + 4 (1 + x) ^ (17 /
2) + 112 (1 + x) ^ (5 / 2) + 112 (1 + x) ^ (13 / 2) + 280 (1 + x) ^ (9 / 2)
) + 120 ArcTanh[Sqrt[1 + x]] (1 + x) ^ (3 / 2) / (-224 (1 + x) ^ (7 / 2) -
224 (1 + x) ^ (11 / 2) - 32 (1 + x) ^ (3 / 2) - 32 (1 + x) ^ (15 / 2) + 4 S
qrt[1 + x] + 4 (1 + x) ^ (17 / 2) + 112 (1 + x) ^ (5 / 2) + 112 (1 + x) ^ (
13 / 2) + 280 (1 + x) ^ (9 / 2)) + 120 ArcTanh[Sqrt[1 + x]] (1 + x) ^ (15 /
2) / (-224 (1 + x) ^ (7 / 2) - 224 (1 + x) ^ (11 / 2) - 32 (1 + x) ^ (3 /
2) - 32 (1 + x) ^ (15 / 2) + 4 Sqrt[1 + x] + 4 (1 + x) ^ (17 / 2) + 112 (1
+ x) ^ (5 / 2) + 112 (1 + x) ^ (13 / 2) + 280 (1 + x) ^ (9 / 2)) + 285 (1 +
x) ^ 2 / (-224 (1 + x) ^ (7 / 2) - 224 (1 + x) ^ (11 / 2) - 32 (1 + x) ^ (

```

$$\begin{aligned} & 3 / 2) - 32 (1 + x)^{(15 / 2)} + 4 \operatorname{Sqrt}[1 + x] + 4 (1 + x)^{(17 / 2)} + 112 \\ & (1 + x)^{(5 / 2)} + 112 (1 + x)^{(13 / 2)} + 280 (1 + x)^{(9 / 2)} + 383 \\ & (1 + x)^6 / (-224 (1 + x)^{(7 / 2)} - 224 (1 + x)^{(11 / 2)} - 32 (1 + x) \\ & ^{(3 / 2)} - 32 (1 + x)^{(15 / 2)} + 4 \operatorname{Sqrt}[1 + x] + 4 (1 + x)^{(17 / 2)} + \\ & 112 (1 + x)^{(5 / 2)} + 112 (1 + x)^{(13 / 2)} + 280 (1 + x)^{(9 / 2)} + \\ & 840 \operatorname{ArcTanh}[\operatorname{Sqrt}[1 + x]] (1 + x)^{(7 / 2)} / (-224 (1 + x)^{(7 / 2)} - 224 \\ & (1 + x)^{(11 / 2)} - 32 (1 + x)^{(3 / 2)} - 32 (1 + x)^{(15 / 2)} + 4 \operatorname{Sqrt}[\\ & 1 + x] + 4 (1 + x)^{(17 / 2)} + 112 (1 + x)^{(5 / 2)} + 112 (1 + x)^{(13 / \\ & 2)} + 280 (1 + x)^{(9 / 2)} + 840 \operatorname{ArcTanh}[\operatorname{Sqrt}[1 + x]] (1 + x)^{(11 / 2)} \\ & / (-224 (1 + x)^{(7 / 2)} - 224 (1 + x)^{(11 / 2)} - 32 (1 + x)^{(3 / 2)} - \\ & 32 (1 + x)^{(15 / 2)} + 4 \operatorname{Sqrt}[1 + x] + 4 (1 + x)^{(17 / 2)} + 112 (1 + x) \\ & ^{(5 / 2)} + 112 (1 + x)^{(13 / 2)} + 280 (1 + x)^{(9 / 2)} + 845 (1 + x) \\ & ^4 / (-224 (1 + x)^{(7 / 2)} - 224 (1 + x)^{(11 / 2)} - 32 (1 + x)^{(3 / \\ & 2)} - 32 (1 + x)^{(15 / 2)} + 4 \operatorname{Sqrt}[1 + x] + 4 (1 + x)^{(17 / 2)} + 112 (1 \\ & + x)^{(5 / 2)} + 112 (1 + x)^{(13 / 2)} + 280 (1 + x)^{(9 / 2)}))] \end{aligned}$$

Maple [A]

time = 0.07, size = 73, normalized size = 1.40

method	result
risch	$\frac{15x^2+5x-2}{4\sqrt{1+x}x^2} - \frac{15 \operatorname{arctanh}\left(\sqrt{1+x}\right)}{4}$
trager	$\frac{15x^2+5x-2}{4\sqrt{1+x}x^2} + \frac{15 \ln\left(\frac{2\sqrt{1+x}-2-x}{x}\right)}{8}$
derivativedivides	$-\frac{1}{8(-1+\sqrt{1+x})^2} + \frac{7}{8(-1+\sqrt{1+x})} + \frac{15 \ln(-1+\sqrt{1+x})}{8} + \frac{2}{\sqrt{1+x}} + \frac{1}{8(1+\sqrt{1+x})}$
default	$-\frac{1}{8(-1+\sqrt{1+x})^2} + \frac{7}{8(-1+\sqrt{1+x})} + \frac{15 \ln(-1+\sqrt{1+x})}{8} + \frac{2}{\sqrt{1+x}} + \frac{1}{8(1+\sqrt{1+x})}$
meijerg	$\frac{-\frac{\sqrt{\pi}}{2x^2} + \frac{3\sqrt{\pi}}{2x} + \frac{15\left(\frac{47}{30} - 2\ln(2) + \ln(x)\right)\sqrt{\pi}}{8} + \frac{\sqrt{\pi}(-47x^2-24x+8)}{16x^2} - \frac{\sqrt{\pi}(-60x^2-20x+8)}{16x^2\sqrt{1+x}} - \frac{15\sqrt{\pi} \ln\left(\frac{1}{2} + \frac{\sqrt{1+x}}{2}\right)}{4}}{\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(1+x)^(3/2),x,method=_RETURNVERBOSE)`

[Out] $-1/8/(-1+(1+x)^{(1/2)})^2+7/8/(-1+(1+x)^{(1/2)})+15/8*\ln(-1+(1+x)^{(1/2)})+2/(1+x)^{(1/2)}+1/8/(1+(1+x)^{(1/2)})^2+7/8/(1+(1+x)^{(1/2)})-15/8*\ln(1+(1+x)^{(1/2)})$

Maxima [A]

time = 0.27, size = 55, normalized size = 1.06

$$\frac{15(x+1)^2 - 25x - 17}{4\left((x+1)^{\frac{5}{2}} - 2(x+1)^{\frac{3}{2}} + \sqrt{x+1}\right)} - \frac{15}{8} \log\left(\sqrt{x+1} + 1\right) + \frac{15}{8} \log\left(\sqrt{x+1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(1+x)^(3/2),x, algorithm="maxima")

[Out] 1/4*(15*(x + 1)^2 - 25*x - 17)/((x + 1)^(5/2) - 2*(x + 1)^(3/2) + sqrt(x + 1)) - 15/8*log(sqrt(x + 1) + 1) + 15/8*log(sqrt(x + 1) - 1)

Fricas [A]

time = 0.32, size = 63, normalized size = 1.21

$$\frac{15(x^3 + x^2) \log\left(\sqrt{x+1} + 1\right) - 15(x^3 + x^2) \log\left(\sqrt{x+1} - 1\right) - 2(15x^2 + 5x - 2)\sqrt{x+1}}{8(x^3 + x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(1+x)^(3/2),x, algorithm="fricas")

[Out] -1/8*(15*(x^3 + x^2)*log(sqrt(x + 1) + 1) - 15*(x^3 + x^2)*log(sqrt(x + 1) - 1) - 2*(15*x^2 + 5*x - 2)*sqrt(x + 1))/(x^3 + x^2)

Sympy [C] Result contains complex when optimal does not.

time = 1.58, size = 3966, normalized size = 76.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(1+x)**(3/2),x)

[Out] Piecewise((-30*(x + 1)**(17/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 15*I*pi*(x + 1)**(17/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 240*(x + 1)**(15/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 120*I*pi*(x + 1)**(15/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 840*(x + 1)**(13/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 420*I*pi*(x + 1)**

$$\begin{aligned}
& (13/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448* \\
& (x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5 \\
& /2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 1680*(x + 1)**(11/2)*acoth(sqrt(\\
& x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448 \\
& *(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(\\
& 5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 840*I*pi*(x + 1)**(11/2)/(8*(x \\
& + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/ \\
& 2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + \\
& 1)**(3/2) + 8*sqrt(x + 1)) - 2100*(x + 1)**(9/2)*acoth(sqrt(x + 1))/(8*(x \\
& + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/ \\
& 2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + \\
& 1)**(3/2) + 8*sqrt(x + 1)) - 1050*I*pi*(x + 1)**(9/2)/(8*(x + 1)**(17/2) - \\
& 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + \\
& 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8 \\
& *sqrt(x + 1)) + 1680*(x + 1)**(7/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - \\
& 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + \\
& 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8 \\
& *sqrt(x + 1)) + 840*I*pi*(x + 1)**(7/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(1 \\
& 5/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448 \\
& *(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - \\
& 840*(x + 1)**(5/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15 \\
& /2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448* \\
& (x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - \\
& 420*I*pi*(x + 1)**(5/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + \\
& 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) \\
& + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 240*(x + 1)**(3 \\
& /2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1 \\
&)** (13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + \\
& 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 120*I*pi*(x + 1) \\
& ** (3/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448 \\
& *(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(\\
& 5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 30*sqrt(x + 1)*acoth(sqrt(x + 1 \\
&))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + \\
& 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) \\
& - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 15*I*pi*sqrt(x + 1)/(8*(x + 1)**(17/ \\
& 2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(\\
& x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) \\
& + 8*sqrt(x + 1)) + 30*(x + 1)**8/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + \\
& 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + \\
& 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 230*(\\
& x + 1)**7/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 4 \\
& 48*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)* \\
& *(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 766*(x + 1)**6/(8*(x + 1)**(1 \\
& 7/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560 \\
& *(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/
\end{aligned}$$


```
(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) -
115*(x + 1)**7/(4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2)
) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x
+ 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) + 383*(x + 1)**6/(4*(x + 1
)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2)
+ 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)
)**(3/2) + 4*sqrt(x + 1)) - 723*(x + 1)**5/(4*(x + 1)**(17/2) - 32*(x + 1)**
(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 2
24*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1))
+ 845*(x + 1)**4/(4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(1
3/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*
(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) - 625*(x + 1)**3/(4*(x
+ 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/
2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x +
1)**(3/2) + 4*sqrt(x + 1)) + 285*(x + 1)**2/(4*(x + 1)**(17/2) - 32*(x + 1)
)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2)
- 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x +
1)) - 73*(x + 1)/(4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13
/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(
x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) + 8/(4*(x + 1)**(17/2) -
32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x +
1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4
*sqrt(x + 1)), True))
```

Giac [A]

time = 0.00, size = 71, normalized size = 1.37

$$2 \left(\frac{1}{\sqrt{x+1}} - \frac{-7\sqrt{x+1}(x+1) + 9\sqrt{x+1}}{8x^2} + \frac{15}{16} \ln \left| \sqrt{x+1} - 1 \right| - \frac{15}{16} \ln \left(\sqrt{x+1} + 1 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(1+x)^(3/2), x)

[Out] 2/sqrt(x + 1) + 1/4*(7*(x + 1)^(3/2) - 9*sqrt(x + 1))/x^2 - 15/8*log(sqrt(x + 1) + 1) + 15/8*log(abs(sqrt(x + 1) - 1))

Mupad [B]

time = 0.05, size = 43, normalized size = 0.83

$$-\frac{15 \operatorname{atanh} \left(\sqrt{x+1} \right)}{4} - \frac{\frac{25x}{4} - \frac{15(x+1)^2}{4} + \frac{17}{4}}{\sqrt{x+1} - 2(x+1)^{3/2} + (x+1)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(x + 1)^(3/2)), x)

[Out] - (15*atanh((x + 1)^(1/2)))/4 - ((25*x)/4 - (15*(x + 1)^2)/4 + 17/4)/((x + 1)^(1/2) - 2*(x + 1)^(3/2) + (x + 1)^(5/2))

3.216 $\int \frac{1}{(1-x)^{7/2}x^5} dx$

Optimal. Leaf size=118

$$\frac{3003}{320(1-x)^{5/2}} + \frac{1001}{64(1-x)^{3/2}} + \frac{3003}{64\sqrt{1-x}} - \frac{1}{4(1-x)^{5/2}x^4} - \frac{13}{24(1-x)^{5/2}x^3} - \frac{143}{96(1-x)^{5/2}x^2} - \frac{429}{64(1-x)^{5/2}x}$$

[Out] 3003/320/(1-x)^(5/2)+1001/64/(1-x)^(3/2)-1/4/(1-x)^(5/2)/x^4-13/24/(1-x)^(5/2)/x^3-143/96/(1-x)^(5/2)/x^2-429/64/(1-x)^(5/2)/x-3003/64*arctanh((1-x)^(1/2))+3003/64/(1-x)^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {44, 53, 65, 212}

$$-\frac{1}{4(1-x)^{5/2}x^4} - \frac{13}{24(1-x)^{5/2}x^3} - \frac{143}{96(1-x)^{5/2}x^2} + \frac{3003}{64\sqrt{1-x}} - \frac{429}{64(1-x)^{5/2}x} + \frac{1001}{64(1-x)^{3/2}} + \frac{3003}{320(1-x)^{5/2}} - \frac{3003}{64} \tanh^{-1}(\sqrt{1-x})$$

Antiderivative was successfully verified.

[In] Int[1/((1-x)^(7/2)*x^5),x]

[Out] 3003/(320*(1-x)^(5/2)) + 1001/(64*(1-x)^(3/2)) + 3003/(64*sqrt[1-x]) - 1/(4*(1-x)^(5/2)*x^4) - 13/(24*(1-x)^(5/2)*x^3) - 143/(96*(1-x)^(5/2)*x^2) - 429/(64*(1-x)^(5/2)*x) - (3003*ArcTanh[Sqrt[1-x]])/64

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```


$d*(x^p/b)^n, x], x, (a + b*x)^{1/p}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 212

$\text{Int}[(a_+ + (b_-)*(x_-)^2)^{-1}, x_Symbol] \text{:> Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]))*\text{ArcTanh}[\text{Rt}[-b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{1}{(1-x)^{7/2}x^5} dx &= \frac{2}{5(1-x)^{5/2}x^4} + \frac{13}{5} \int \frac{1}{(1-x)^{5/2}x^5} dx \\ &= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{143}{15} \int \frac{1}{(1-x)^{3/2}x^5} dx \\ &= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}x^4} + \frac{429}{5} \int \frac{1}{\sqrt{1-x}x^5} dx \\ &= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}x^4} - \frac{429\sqrt{1-x}}{20x^4} + \frac{3003}{40} \int \frac{1}{\sqrt{1-x}x^4} dx \\ &= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}x^4} - \frac{429\sqrt{1-x}}{20x^4} - \frac{1001\sqrt{1-x}}{40x^3} + \frac{1001}{40} \int \frac{1}{\sqrt{1-x}x^3} dx \\ &= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}x^4} - \frac{429\sqrt{1-x}}{20x^4} - \frac{1001\sqrt{1-x}}{40x^3} - \frac{1001}{40} \int \frac{1}{\sqrt{1-x}x^2} dx \\ &= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}x^4} - \frac{429\sqrt{1-x}}{20x^4} - \frac{1001\sqrt{1-x}}{40x^3} - \frac{1001}{40} \int \frac{1}{\sqrt{1-x}x} dx \\ &= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}x^4} - \frac{429\sqrt{1-x}}{20x^4} - \frac{1001\sqrt{1-x}}{40x^3} - \frac{1001}{40} \int \frac{1}{\sqrt{1-x}} dx \\ &= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}x^4} - \frac{429\sqrt{1-x}}{20x^4} - \frac{1001\sqrt{1-x}}{40x^3} - \frac{1001}{40} \int \frac{1}{\sqrt{1-x}} dx \end{aligned}$$

Mathematica [A]

time = 0.06, size = 70, normalized size = 0.59

$$\frac{240 + 520x + 1430x^2 + 6435x^3 - 69069x^4 + 105105x^5 - 45045x^6 + 45045(1-x)^{5/2}x^4 \tanh^{-1}(\sqrt{1-x})}{960(1-x)^{5/2}x^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1-x)^(7/2)*x^5),x]

[Out] $-1/960*(240 + 520*x + 1430*x^2 + 6435*x^3 - 69069*x^4 + 105105*x^5 - 45045*x^6 + 45045*(1 - x)^{(5/2)}*x^4*\text{ArcTanh}[\text{Sqrt}[1 - x]])/((1 - x)^{(5/2)}*x^4)$

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[1/(x^5*(1 - x)^(7/2)),x]')`

[Out] Timed out

Maple [A]

time = 0.10, size = 157, normalized size = 1.33

method	result
risch	$\frac{45045x^6 - 105105x^5 + 69069x^4 - 6435x^3 - 1430x^2 - 520x - 240}{960(-1+x)^2\sqrt{1-x}x^4} - \frac{3003 \operatorname{arctanh}(\sqrt{1-x})}{64}$
trager	$-\frac{(45045x^6 - 105105x^5 + 69069x^4 - 6435x^3 - 1430x^2 - 520x - 240)\sqrt{1-x}}{960(-1+x)^3x^4} + \frac{3003 \ln\left(\frac{-2+x+2\sqrt{1-x}}{x}\right)}{128}$
meijerg	$-\frac{\sqrt{\pi}}{4x^4} - \frac{7\sqrt{\pi}}{6x^3} - \frac{63\sqrt{\pi}}{16x^2} - \frac{231\sqrt{\pi}}{16x} + \frac{3003\left(\frac{329177}{180180} - 2\ln(2) + \ln(x) + i\pi\right)\sqrt{\pi}}{128} + \frac{\sqrt{\pi}\left(-329177x^4 + 110880x^3 + 30240x^2 + 8960x - 1008\right)}{7680x^4}$
derivativedivides	$\frac{2}{5(1-x)^{\frac{5}{2}}} + \frac{10}{3(1-x)^{\frac{3}{2}}} + \frac{30}{\sqrt{1-x}} - \frac{1}{64(\sqrt{1-x}-1)^4} + \frac{17}{96(\sqrt{1-x}-1)^3} - \frac{159}{128(\sqrt{1-x}-1)^2}$
default	$\frac{2}{5(1-x)^{\frac{5}{2}}} + \frac{10}{3(1-x)^{\frac{3}{2}}} + \frac{30}{\sqrt{1-x}} - \frac{1}{64(\sqrt{1-x}-1)^4} + \frac{17}{96(\sqrt{1-x}-1)^3} - \frac{159}{128(\sqrt{1-x}-1)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1-x)^(7/2)/x^5,x,method=_RETURNVERBOSE)`

[Out] $2/5/(1-x)^{(5/2)} + 10/3/(1-x)^{(3/2)} + 30/(1-x)^{(1/2)} - 1/64/((1-x)^{(1/2)}-1)^4 + 17/96/((1-x)^{(1/2)}-1)^3 - 159/128/((1-x)^{(1/2)}-1)^2 + 1083/128/((1-x)^{(1/2)}-1) + 3003/128*\ln((1-x)^{(1/2)}-1) + 1/64/((1-x)^{(1/2)}+1)^4 + 17/96/((1-x)^{(1/2)}+1)^3 + 159/128/((1-x)^{(1/2)}+1)^2 + 1083/128/((1-x)^{(1/2)}+1) - 3003/128*\ln((1-x)^{(1/2)}+1)$

Maxima [A]

time = 0.32, size = 111, normalized size = 0.94

$$\frac{45045(x-1)^6 + 165165(x-1)^5 + 219219(x-1)^4 + 119691(x-1)^3 + 18304(x-1)^2 - 1664x + 2048}{960\left((-x+1)^{\frac{3}{2}} - 4(-x+1)^{\frac{1}{2}} + 6(-x+1)^{\frac{3}{2}} - 4(-x+1)^{\frac{1}{2}} + (-x+1)^{\frac{5}{2}}\right)} - \frac{3003}{128} \log(\sqrt{-x+1} + 1) + \frac{3003}{128} \log(\sqrt{-x+1} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)^(7/2)/x^5,x, algorithm="maxima")

[Out] $\frac{1}{960} \cdot (45045 \cdot (x-1)^6 + 165165 \cdot (x-1)^5 + 219219 \cdot (x-1)^4 + 119691 \cdot (x-1)^3 + 18304 \cdot (x-1)^2 - 1664 \cdot x + 2048) / ((-x+1)^{13/2} - 4 \cdot (-x+1)^{11/2}) + 6 \cdot (-x+1)^{9/2} - 4 \cdot (-x+1)^{7/2} + (-x+1)^{5/2} - 3003/128 \cdot \log(\sqrt{-x+1} + 1) + 3003/128 \cdot \log(\sqrt{-x+1} - 1)$

Fricas [A]

time = 0.30, size = 125, normalized size = 1.06

$$\frac{45045(x^7 - 3x^6 + 3x^5 - x^4) \log(\sqrt{-x+1} + 1) - 45045(x^7 - 3x^6 + 3x^5 - x^4) \log(\sqrt{-x+1} - 1) + 2(45045x^6 - 105105x^5 + 69069x^4 - 6435x^3 - 1430x^2 - 520x - 240)\sqrt{-x+1}}{1920(x^7 - 3x^6 + 3x^5 - x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)^(7/2)/x^5,x, algorithm="fricas")

[Out] $\frac{-1}{1920} \cdot (45045 \cdot (x^7 - 3x^6 + 3x^5 - x^4) \cdot \log(\sqrt{-x+1} + 1) - 45045 \cdot (x^7 - 3x^6 + 3x^5 - x^4) \cdot \log(\sqrt{-x+1} - 1) + 2 \cdot (45045 \cdot x^6 - 105105 \cdot x^5 + 69069 \cdot x^4 - 6435 \cdot x^3 - 1430 \cdot x^2 - 520 \cdot x - 240) \cdot \sqrt{-x+1}) / (x^7 - 3x^6 + 3x^5 - x^4)$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)**(7/2)/x**5,x)

[Out] Timed out

Giac [A]

time = 0.00, size = 139, normalized size = 1.18

$$2 \left(\frac{3249\sqrt{-x+1}(-x+1)^3 - 10633\sqrt{-x+1}(-x+1)^2 + 11767\sqrt{-x+1}(-x+1) - 4431\sqrt{-x+1}}{384(-x)^4} - \frac{-225(-x+1)^2 - 25(-x+1) - 3}{15\sqrt{-x+1}(-x+1)^2} + \frac{3003}{256} \ln|\sqrt{-x+1} - 1| - \frac{3003}{256} \ln(\sqrt{-x+1} + 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)^(7/2)/x^5,x)

[Out] $\frac{2}{15} \cdot (225 \cdot (x-1)^2 - 25 \cdot x + 28) / ((x-1)^2 \cdot \sqrt{-x+1}) - \frac{1}{192} \cdot (3249 \cdot (x-1)^3 \cdot \sqrt{-x+1} + 10633 \cdot (x-1)^2 \cdot \sqrt{-x+1} - 11767 \cdot (-x+1)^{3/2} + 4431 \cdot \sqrt{-x+1}) / x^4 - \frac{3003}{128} \cdot \log(\sqrt{-x+1} + 1) + \frac{3003}{128} \cdot \log(\text{abs}(\sqrt{-x+1} - 1))$

Mupad [B]

time = 0.21, size = 96, normalized size = 0.81

$$\frac{\frac{286(x-1)^2}{15} - \frac{26x}{15} + \frac{39897(x-1)^3}{320} + \frac{73073(x-1)^4}{320} + \frac{11011(x-1)^5}{64} + \frac{3003(x-1)^6}{64} + \frac{32}{15}}{(1-x)^{5/2} - 4(1-x)^{7/2} + 6(1-x)^{9/2} - 4(1-x)^{11/2} + (1-x)^{13/2}} - \frac{3003 \operatorname{atanh}(\sqrt{1-x})}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(1 - x)^(7/2)),x)
```

```
[Out] ((286*(x - 1)^2)/15 - (26*x)/15 + (39897*(x - 1)^3)/320 + (73073*(x - 1)^4)/320 + (11011*(x - 1)^5)/64 + (3003*(x - 1)^6)/64 + 32/15)/((1 - x)^(5/2) - 4*(1 - x)^(7/2) + 6*(1 - x)^(9/2) - 4*(1 - x)^(11/2) + (1 - x)^(13/2)) - (3003*atanh((1 - x)^(1/2)))/64
```

$$3.217 \quad \int \frac{1}{(-1+x)^{2/3}x^5} dx$$

Optimal. Leaf size=104

$$\frac{\sqrt[3]{-1+x}}{4x^4} + \frac{11\sqrt[3]{-1+x}}{36x^3} + \frac{11\sqrt[3]{-1+x}}{27x^2} + \frac{55\sqrt[3]{-1+x}}{81x} - \frac{110 \tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x}}{\sqrt{3}}\right)}{81\sqrt{3}} + \frac{55}{81} \log(1 + \sqrt[3]{-1+x})$$

[Out] 1/4*(-1+x)^(1/3)/x^4+11/36*(-1+x)^(1/3)/x^3+11/27*(-1+x)^(1/3)/x^2+55/81*(-1+x)^(1/3)/x+55/81*ln(1+(-1+x)^(1/3))-55/243*ln(x)-110/243*arctan(1/3*(1-2*(-1+x)^(1/3))*3^(1/2))*3^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {44, 60, 632, 210, 31}

$$\frac{\sqrt[3]{x-1}}{4x^4} + \frac{11\sqrt[3]{x-1}}{36x^3} + \frac{11\sqrt[3]{x-1}}{27x^2} + \frac{55\sqrt[3]{x-1}}{81x} + \frac{55}{81} \log(\sqrt[3]{x-1} + 1) - \frac{55 \log(x)}{243} - \frac{110 \tan^{-1}\left(\frac{1-2\sqrt[3]{x-1}}{\sqrt{3}}\right)}{81\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/((-1 + x)^(2/3)*x^5), x]

[Out] (-1 + x)^(1/3)/(4*x^4) + (11*(-1 + x)^(1/3))/(36*x^3) + (11*(-1 + x)^(1/3))/(27*x^2) + (55*(-1 + x)^(1/3))/(81*x) - (110*ArcTan[(1 - 2*(-1 + x)^(1/3))/Sqrt[3]])/(81*Sqrt[3]) + (55*Log[1 + (-1 + x)^(1/3)])/81 - (55*Log[x])/243

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 44

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && LtQ[n, 0]

Rule 60

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x

]]) /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 210

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & (LtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(-1+x)^{2/3}x^5} dx &= \frac{\sqrt[3]{-1+x}}{4x^4} + \frac{11}{12} \int \frac{1}{(-1+x)^{2/3}x^4} dx \\
 &= \frac{\sqrt[3]{-1+x}}{4x^4} + \frac{11\sqrt[3]{-1+x}}{36x^3} + \frac{22}{27} \int \frac{1}{(-1+x)^{2/3}x^3} dx \\
 &= \frac{\sqrt[3]{-1+x}}{4x^4} + \frac{11\sqrt[3]{-1+x}}{36x^3} + \frac{11\sqrt[3]{-1+x}}{27x^2} + \frac{55}{81} \int \frac{1}{(-1+x)^{2/3}x^2} dx \\
 &= \frac{\sqrt[3]{-1+x}}{4x^4} + \frac{11\sqrt[3]{-1+x}}{36x^3} + \frac{11\sqrt[3]{-1+x}}{27x^2} + \frac{55\sqrt[3]{-1+x}}{81x} + \frac{110}{243} \int \frac{1}{(-1+x)^{2/3}x} dx \\
 &= \frac{\sqrt[3]{-1+x}}{4x^4} + \frac{11\sqrt[3]{-1+x}}{36x^3} + \frac{11\sqrt[3]{-1+x}}{27x^2} + \frac{55\sqrt[3]{-1+x}}{81x} - \frac{55 \log(x)}{243} + \frac{55}{81} \text{Subst} \left(\frac{1}{x}, \sqrt[3]{-1+x} \right) \\
 &= \frac{\sqrt[3]{-1+x}}{4x^4} + \frac{11\sqrt[3]{-1+x}}{36x^3} + \frac{11\sqrt[3]{-1+x}}{27x^2} + \frac{55\sqrt[3]{-1+x}}{81x} + \frac{55}{81} \log(1 + \sqrt[3]{-1+x}) \\
 &= \frac{\sqrt[3]{-1+x}}{4x^4} + \frac{11\sqrt[3]{-1+x}}{36x^3} + \frac{11\sqrt[3]{-1+x}}{27x^2} + \frac{55\sqrt[3]{-1+x}}{81x} - \frac{110 \tan^{-1} \left(\frac{1-2\sqrt[3]{-1+x}}{\sqrt{3}} \right)}{81\sqrt{3}}
 \end{aligned}$$

Mathematica [A]

time = 0.15, size = 90, normalized size = 0.87

$$\frac{1}{972} \left(\frac{3\sqrt[3]{-1+x} (81 + 99x + 132x^2 + 220x^3)}{x^4} - 440\sqrt{3} \tan^{-1} \left(\frac{1-2\sqrt[3]{-1+x}}{\sqrt{3}} \right) + 440 \log(1 + \sqrt[3]{-1+x}) - 220 \log(1 - \sqrt[3]{-1+x} + (-1+x)^{2/3}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((-1 + x)^(2/3)*x^5), x]

[Out] $((3*(-1+x)^{(1/3)}*(81+99*x+132*x^2+220*x^3))/x^4-440*\text{Sqrt}[3]*\text{ArcTan}[(1-2*(-1+x)^{(1/3)})/\text{Sqrt}[3]]+440*\text{Log}[1+(-1+x)^{(1/3)}]-220*\text{Log}[1-(-1+x)^{(1/3)}+(-1+x)^{(2/3)}])/972$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 155.10, size = 95, normalized size = 0.91

$$\frac{(-1+x)^{\frac{1}{3}}}{4x^4} + \frac{11(-1+x)^{\frac{1}{3}}}{36x^3} + \frac{11(-1+x)^{\frac{1}{3}}}{27x^2} + \frac{55(-1+x)^{\frac{1}{3}}}{81x} - \frac{\text{Log}\left[1-(-1+x)^{\frac{1}{3}}\text{exp_polar}\left[\frac{5}{3}\text{Pi}\right]\right]}{243} - \frac{\text{Log}\left[1-(-1+x)^{\frac{1}{3}}\text{exp_polar}\left[\frac{5}{3}\text{Pi}\right]\right]}{243} + \frac{110\text{Log}\left[1-(-1+x)^{\frac{1}{3}}\text{exp_polar}\left[\text{IPi}\right]\right]}{243}$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[1/(x^5*(x - 1)^(2/3)),x]')`

[Out] $(-1+x)^{(1/3)}/(4x^4) + 11(-1+x)^{(1/3)}/(36x^3) + 11(-1+x)^{(1/3)}/(27x^2) + 55(-1+x)^{(1/3)}/(81x) - 110(-1+x)^{(1/3)}\text{Log}[1-(-1+x)^{(1/3)}\text{exp_polar}[5\text{I}/3\text{Pi}]]/243 + 110(-1+x)^{(2/3)}\text{Log}[1-(-1+x)^{(1/3)}\text{exp_polar}[\text{I}/3\text{Pi}]]/243 + 110\text{Log}[1-(-1+x)^{(1/3)}\text{exp_polar}[\text{I}\text{Pi}]]/243$

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(75) = 150.

time = 0.31, size = 158, normalized size = 1.52

method	result
meijerg	$(-\text{signum}(-1+x))^{\frac{2}{3}} \left(-\frac{\Gamma(\frac{2}{3})}{4x^4} - \frac{2\Gamma(\frac{2}{3})}{9x^3} - \frac{5\Gamma(\frac{2}{3})}{18x^2} - \frac{40\Gamma(\frac{2}{3})}{81x} + \frac{110 \left(\frac{877}{1320} + \frac{\pi\sqrt{3}}{6} - \frac{3\ln(3)}{2} + \ln(x) + i\pi \right) \Gamma(\frac{2}{3})}{243} + \frac{308\Gamma(\frac{2}{3})x \text{hypergeom}\left(\left[1, \frac{2}{3}\right], \left[1, \frac{4}{3}\right], -\frac{1}{x}\right)}{243} \right)$
risch	$\frac{220x^4 - 88x^3 - 33x^2 - 18x - 81}{324x^4(-1+x)^{\frac{2}{3}}} + \frac{110(-\text{signum}(-1+x))^{\frac{2}{3}} \left(\left(\frac{\pi\sqrt{3}}{6} - \frac{3\ln(3)}{2} + \ln(x) + i\pi \right) \Gamma(\frac{2}{3}) + \frac{2\Gamma(\frac{2}{3})x \text{hypergeom}\left(\left[1, \frac{2}{3}\right], \left[1, \frac{4}{3}\right], -\frac{1}{x}\right)}{3} \right)}{243\Gamma(\frac{2}{3})\text{signum}(-1+x)^{\frac{2}{3}}}$
derivativedivides	$-\frac{-75(-1+x)^{\frac{7}{3}} + 190(-1+x)^2 - 350(-1+x)^{\frac{5}{3}} + \frac{1157(-1+x)^{\frac{4}{3}}}{4} + \frac{149}{4} - 138x - 116(-1+x)^{\frac{2}{3}} + 137(-1+x)^{\frac{1}{3}}}{243\left((-1+x)^{\frac{2}{3}} - (-1+x)^{\frac{1}{3}} + 1\right)^4} - \frac{55\ln\left((-1+x)^{\frac{1}{3}}\right)}{243}$
default	$-\frac{-75(-1+x)^{\frac{7}{3}} + 190(-1+x)^2 - 350(-1+x)^{\frac{5}{3}} + \frac{1157(-1+x)^{\frac{4}{3}}}{4} + \frac{149}{4} - 138x - 116(-1+x)^{\frac{2}{3}} + 137(-1+x)^{\frac{1}{3}}}{243\left((-1+x)^{\frac{2}{3}} - (-1+x)^{\frac{1}{3}} + 1\right)^4} - \frac{55\ln\left((-1+x)^{\frac{1}{3}}\right)}{243}$
trager	$\frac{(220x^3 + 132x^2 + 99x + 81)(-1+x)^{\frac{1}{3}}}{324x^4} - \frac{110\ln\left(\frac{72(-1+x)^{\frac{2}{3}}\text{RootOf}\left(2304_Z^2 + 48_Z + 1\right) - 1152\text{RootOf}\left(2304_Z^2 + 48_Z + 1\right)}{2304_Z^2 + 48_Z + 1}\right)}{2304_Z^2 + 48_Z + 1}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-1+x)^(2/3)/x^5,x,method=_RETURNVERBOSE)`

[Out]
$$-1/243*(-75*(-1+x)^{(7/3)}+190*(-1+x)^2-350*(-1+x)^{(5/3)}+1157/4*(-1+x)^{(4/3)}+149/4-138*x-116*(-1+x)^{(2/3)}+137*(-1+x)^{(1/3)})/((-1+x)^{(2/3)}-(-1+x)^{(1/3)}+1)^4-55/243*\ln((-1+x)^{(2/3)}-(-1+x)^{(1/3)}+1)+110/243*3^{(1/2)}*\arctan(1/3*(2*(-1+x)^{(1/3)}-1)*3^{(1/2)})-1/324/(1+(-1+x)^{(1/3)})^4-5/243/(1+(-1+x)^{(1/3)})^3-20/243/(1+(-1+x)^{(1/3)})^2-25/81/(1+(-1+x)^{(1/3)})+110/243*\ln(1+(-1+x)^{(1/3)})$$

Maxima [A]

time = 0.34, size = 105, normalized size = 1.01

$$\frac{110}{243} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2(x-1)^{\frac{1}{3}} - 1)\right) + \frac{220(x-1)^{\frac{10}{3}} + 792(x-1)^{\frac{7}{3}} + 1023(x-1)^{\frac{4}{3}} + 532(x-1)^{\frac{1}{3}}}{324((x-1)^4 + 4(x-1)^3 + 6(x-1)^2 + 4x - 3)} - \frac{55}{243} \log((x-1)^{\frac{2}{3}} - (x-1)^{\frac{1}{3}} + 1) + \frac{110}{243} \log((x-1)^{\frac{1}{3}} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1+x)^(2/3)/x^5,x, algorithm="maxima")`

[Out]
$$110/243*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*(x-1)^{(1/3)}-1))+1/324*(220*(x-1)^{(10/3)}+792*(x-1)^{(7/3)}+1023*(x-1)^{(4/3)}+532*(x-1)^{(1/3)})/((x-1)^4+4*(x-1)^3+6*(x-1)^2+4*x-3)-55/243*\log((x-1)^{(2/3)}-(x-1)^{(1/3)}+1)+110/243*\log((x-1)^{(1/3)}+1)$$

Fricas [A]

time = 0.31, size = 86, normalized size = 0.83

$$\frac{440 \sqrt{3} x^4 \arctan\left(\frac{2}{3} \sqrt{3} (x-1)^{\frac{1}{3}} - \frac{1}{3} \sqrt{3}\right) - 220 x^4 \log\left((x-1)^{\frac{2}{3}} - (x-1)^{\frac{1}{3}} + 1\right) + 440 x^4 \log\left((x-1)^{\frac{1}{3}} + 1\right) + 3(220 x^3 + 132 x^2 + 99 x + 81)(x-1)^{\frac{1}{3}}}{972 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1+x)^(2/3)/x^5,x, algorithm="fricas")`

[Out]
$$1/972*(440*\sqrt{3}*x^4*\arctan(2/3*\sqrt{3}*(x-1)^{(1/3)}-1/3*\sqrt{3})-220*x^4*\log((x-1)^{(2/3)}-(x-1)^{(1/3)}+1)+440*x^4*\log((x-1)^{(1/3)}+1)+3*(220*x^3+132*x^2+99*x+81)*(x-1)^{(1/3)})/x^4$$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1+x)**(2/3)/x**5,x)`

[Out] Timed out

Giac [A]

time = 0.00, size = 130, normalized size = 1.25

$$3 \left(\frac{110}{729} \ln|(x-1)^{\frac{1}{3}} + 1| - \frac{55}{729} \ln\left(\left((x-1)^{\frac{1}{3}}\right)^2 - (x-1)^{\frac{1}{3}} + 1\right) + \frac{\frac{1}{243} \cdot 110 \arctan\left(\frac{2(x-1)^{\frac{1}{3}} - 1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\frac{1}{972} (220(x-1)^{\frac{1}{3}}(x-1)^3 + 792(x-1)^{\frac{1}{3}}(x-1)^2 + 1023(x-1)^{\frac{1}{3}}(x-1) + 532(x-1)^{\frac{1}{3}})}{x^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+x)^(2/3)/x^5,x)

[Out] $110/243*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*(x - 1)^{(1/3)} - 1)) + 1/324*(220*(x - 1)^{(10/3)} + 792*(x - 1)^{(7/3)} + 1023*(x - 1)^{(4/3)} + 532*(x - 1)^{(1/3)})/x^4 - 55/243*\log((x - 1)^{(2/3)} - (x - 1)^{(1/3)} + 1) + 110/243*\log(\text{abs}((x - 1)^{(1/3)} + 1))$

Mupad [B]

time = 0.21, size = 120, normalized size = 1.15

$$\frac{110 \ln\left(\frac{12100(x-1)^{1/3} + 12100}{6561}\right)}{243} + \frac{133(x-1)^{1/3} + \frac{341(x-1)^{4/3}}{108} + \frac{22(x-1)^{7/3}}{9} + \frac{55(x-1)^{10/3}}{81}}{4x + 6(x-1)^2 + 4(x-1)^3 + (x-1)^4 - 3} - \ln\left(\frac{55}{27} - \frac{110(x-1)^{1/3}}{27} + \frac{\sqrt{3} 55i}{27}\right) \left(\frac{55}{243} + \frac{\sqrt{3} 55i}{243}\right) + \ln\left(\frac{110(x-1)^{1/3}}{27} - \frac{55}{27} + \frac{\sqrt{3} 55i}{27}\right) \left(-\frac{55}{243} + \frac{\sqrt{3} 55i}{243}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(x - 1)^(2/3)),x)

[Out] $(110*\log((12100*(x - 1)^{(1/3)})/6561 + 12100/6561))/243 + ((133*(x - 1)^{(1/3)})/81 + (341*(x - 1)^{(4/3)})/108 + (22*(x - 1)^{(7/3)})/9 + (55*(x - 1)^{(10/3)})/81)/(4*x + 6*(x - 1)^2 + 4*(x - 1)^3 + (x - 1)^4 - 3) - \log((3^{(1/2)}*55i)/27 - (110*(x - 1)^{(1/3)})/27 + 55/27)*((3^{(1/2)}*55i)/243 + 55/243) + \log((110*(x - 1)^{(1/3)})/27 + (3^{(1/2)}*55i)/27 - 55/27)*((3^{(1/2)}*55i)/243 - 55/243)$

$$3.218 \quad \int \sqrt{\frac{1-x}{1+x}} dx$$

Optimal. Leaf size=38

$$\sqrt{\frac{1-x}{1+x}} (1+x) - 2 \tan^{-1} \left(\sqrt{\frac{1-x}{1+x}} \right)$$

[Out] $-2*\arctan(((1-x)/(1+x))^(1/2))+(1+x)*((1-x)/(1+x))^(1/2)$

Rubi [A]

time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1979, 294, 210}

$$\sqrt{\frac{1-x}{x+1}} (x+1) - 2 \tan^{-1} \left(\sqrt{\frac{1-x}{x+1}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(1 - x)/(1 + x)], x]

[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 294

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !IntegerQ[m + n*(p + 1) + 1]/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1979

Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_.) + (d_.)*(x_)^(n_.)))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x^(q*(p + 1) - 1)*((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1), x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x] /; FreeQ[{a, b, c, d, e}, x] && FractionQ[p] && IntegerQ[1/n]

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{1-x}{1+x}} dx &= - \left(4 \text{Subst} \left(\int \frac{x^2}{(-1-x^2)^2} dx, x, \sqrt{\frac{1-x}{1+x}} \right) \right) \\
&= \sqrt{\frac{1-x}{1+x}} (1+x) + 2 \text{Subst} \left(\int \frac{1}{-1-x^2} dx, x, \sqrt{\frac{1-x}{1+x}} \right) \\
&= \sqrt{\frac{1-x}{1+x}} (1+x) - 2 \tan^{-1} \left(\sqrt{\frac{1-x}{1+x}} \right)
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 66, normalized size = 1.74

$$\frac{\sqrt{\frac{1-x}{1+x}} \left(\sqrt{1-x} (1+x) + 2\sqrt{1+x} \tan^{-1} \left(\frac{\sqrt{1+x}}{\sqrt{1-x}} \right) \right)}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

`[In] Integrate[Sqrt[(1 - x)/(1 + x)], x]``[Out] (Sqrt[(1 - x)/(1 + x)]*(Sqrt[1 - x]*(1 + x) + 2*Sqrt[1 + x]*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]))/Sqrt[1 - x]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Sqrt[(1 - x)/(1 + x)], x]')``[Out] cought exception: maximum recursion depth exceeded in comparison`**Maple [A]**

time = 0.08, size = 39, normalized size = 1.03

method	result
default	$\frac{\sqrt{-\frac{-1+x}{1+x}} (1+x) \left(\sqrt{-x^2+1} + \arcsin(x) \right)}{\sqrt{-(1+x)(-1+x)}}$
risch	$(1+x) \sqrt{-\frac{-1+x}{1+x}} - \frac{\arcsin(x) \sqrt{-\frac{-1+x}{1+x}} \sqrt{-(1+x)(-1+x)}}{-1+x}$

trager	$(1+x) \sqrt{-\frac{-1+x}{1+x}} + \text{RootOf}(-Z^2+1) \ln\left(\text{RootOf}(-Z^2+1) \sqrt{-\frac{-1+x}{1+x}} x + \text{RootOf}(-Z^2+1)\right)$
--------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((1-x)/(1+x))^(1/2),x,method=_RETURNVERBOSE)`

[Out] `(-(-1+x)/(1+x))^(1/2)*(1+x)/(-(-1+x)*(-1+x))^(1/2)*((-x^2+1)^(1/2)+arcsin(x))`

Maxima [A]

time = 0.34, size = 43, normalized size = 1.13

$$-\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1} - 2 \arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((1-x)/(1+x))^(1/2),x, algorithm="maxima")`

[Out] `-2*sqrt(-(x-1)/(x+1))/((x-1)/(x+1)-1)-2*arctan(sqrt(-(x-1)/(x+1)))`

Fricas [A]

time = 0.30, size = 32, normalized size = 0.84

$$(x+1)\sqrt{-\frac{x-1}{x+1}} - 2 \arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((1-x)/(1+x))^(1/2),x, algorithm="fricas")`

[Out] `(x+1)*sqrt(-(x-1)/(x+1))-2*arctan(sqrt(-(x-1)/(x+1)))`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{1-x}{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((1-x)/(1+x))**(1/2),x)`

[Out] `Integral(sqrt((1-x)/(x+1)), x)`

Giac [A]

time = 0.00, size = 31, normalized size = 0.82

$$\sqrt{-x^2 + 1} \operatorname{sign}(x + 1) + \operatorname{sign}(x + 1) \arcsin x + \frac{1}{2} \pi \operatorname{sign}(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)/(1+x))^(1/2),x)**[Out]** 1/2*pi*sgn(x + 1) + arcsin(x)*sgn(x + 1) + sqrt(-x^2 + 1)*sgn(x + 1)**Mupad [B]**

time = 0.19, size = 43, normalized size = 1.13

$$-2 \operatorname{atan} \left(\sqrt{-\frac{x-1}{x+1}} \right) - \frac{2 \sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x - 1)/(x + 1))^(1/2),x)**[Out]** - 2*atan((-x - 1)/(x + 1))^(1/2)) - (2*(-x - 1)/(x + 1))^(1/2))/((x - 1)/(x + 1) - 1)

$$3.219 \quad \int x \sqrt{\frac{-a+x}{b-x}} dx$$

Optimal. Leaf size=92

$$\frac{1}{4}(a-5b)(b-x)\sqrt{\frac{-a+x}{b-x}} + \frac{1}{2}(b-x)^2\sqrt{\frac{-a+x}{b-x}} - \frac{1}{4}(a-b)(a+3b)\tan^{-1}\left(\sqrt{\frac{-a+x}{b-x}}\right)$$

[Out] $-1/4*(a-b)*(a+3*b)*\arctan(((a+x)/(b-x))^{(1/2)})+1/4*(a-5*b)*(b-x)*((a+x)/(b-x))^{(1/2)}+1/2*(b-x)^2*((a+x)/(b-x))^{(1/2)}$

Rubi [A]

time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$,

Rules used = {1980, 466, 393, 209}

$$\frac{1}{2}(b-x)^2\sqrt{-\frac{a-x}{b-x}} + \frac{1}{4}(a-5b)(b-x)\sqrt{-\frac{a-x}{b-x}} - \frac{1}{4}(a-b)(a+3b)\tan^{-1}\left(\sqrt{-\frac{a-x}{b-x}}\right)$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[(-a + x)/(b - x)], x]

[Out] $((a - 5*b)*\text{Sqrt}[-((a - x)/(b - x))]*(b - x))/4 + (\text{Sqrt}[-((a - x)/(b - x))]*(b - x)^2)/2 - ((a - b)*(a + 3*b)*\text{ArcTan}[\text{Sqrt}[-((a - x)/(b - x))]])/4$

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 393

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(-b*c - a*d)*x*((a + b*x^n)^(p+1)/(a*b*n*(p+1))), x] - Dist[(a*d - b*c*(n*(p+1) + 1))/(a*b*n*(p+1)), Int[(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 466

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p+1)/(2*b^(m/2 + 1)*(p+1))), x] + Dist[1/(2*b^(m/2 + 1)*(p+1)), Int[(a + b*x^2)^(p+1)*ExpandToSum[2*b*(p+1)*x^2*Together[(b^(m/2)*x^(m-2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F

reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1980

Int[(x_)^(m_)*(((e_)*((a_) + (b_)*(x_)))/((c_) + (d_)*(x_)))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(p + 1) - 1)*((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)], x], x, (e*((a + b*x)/(c + d*x)))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int x \sqrt{\frac{-a+x}{b-x}} dx &= - \left((2(a-b)) \text{Subst} \left(\int \frac{x^2 (a+bx^2)}{(1+x^2)^3} dx, x, \sqrt{\frac{-a+x}{b-x}} \right) \right) \\ &= \frac{1}{2} \sqrt{\frac{-a-x}{b-x}} (b-x)^2 - \frac{1}{2} (-a+b) \text{Subst} \left(\int \frac{-a+b-4bx^2}{(1+x^2)^2} dx, x, \sqrt{\frac{-a+x}{b-x}} \right) \\ &= \frac{1}{4} (a-5b) \sqrt{\frac{-a-x}{b-x}} (b-x) + \frac{1}{2} \sqrt{\frac{-a-x}{b-x}} (b-x)^2 - \frac{1}{4} ((a-b)(a+3b)) \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt{\frac{-a+x}{b-x}} \right) \\ &= \frac{1}{4} (a-5b) \sqrt{\frac{-a-x}{b-x}} (b-x) + \frac{1}{2} \sqrt{\frac{-a-x}{b-x}} (b-x)^2 - \frac{1}{4} (a-b)(a+3b) \tan^{-1} \left(\sqrt{\frac{-a+x}{b-x}} \right) \end{aligned}$$

Mathematica [A]

time = 0.16, size = 99, normalized size = 1.08

$$\frac{\sqrt{\frac{-a+x}{b-x}} \left((a-3b-2x)(b-x)\sqrt{-a+x} + (-a^2-2ab+3b^2)\sqrt{b-x} \tan^{-1} \left(\frac{\sqrt{-a+x}}{\sqrt{b-x}} \right) \right)}{4\sqrt{-a+x}}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[(-a + x)/(b - x)], x]

[Out] (Sqrt[(-a + x)/(b - x)]*((a - 3*b - 2*x)*(b - x)*Sqrt[-a + x] + (-a^2 - 2*a*b + 3*b^2)*Sqrt[b - x]*ArcTan[Sqrt[-a + x]/Sqrt[b - x]])/(4*Sqrt[-a + x])

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[Sqrt[(x - a)/(b - x)]*x,x]')`

[Out] Timed out

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(80) = 160.

time = 0.05, size = 196, normalized size = 2.13

method	result
risch	$\frac{(a-3b-2x)(b-x)\sqrt{-\frac{a-x}{b-x}}\sqrt{-(b-x)(a-x)}}{4\sqrt{-(-b+x)(-a+x)}} + \frac{\left(\arctan\left(\frac{x-\frac{b}{2}-\frac{a}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)\right)^{ab} - 3\arctan\left(\frac{\sqrt{-ab+(a+b)x-x^2}}{\sqrt{-ab+(a+b)x-x^2}}\right)}{\dots}$
default	$\frac{\sqrt{-\frac{a-x}{b-x}}(b-x)\left(\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+ax+bx-x^2}}\right)\right)^{a^2+2b} \arctan\left(\frac{a+b-2x}{2\sqrt{-ab+ax+bx-x^2}}\right)^{a-3} \arctan\left(\frac{\sqrt{-ab+(a+b)x-x^2}}{2\sqrt{-ab+(a+b)x-x^2}}\right)}{8\sqrt{-(b-x)(-a+x)}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*((-a+x)/(b-x))^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{8}*(-(a-x)/(b-x))^{(1/2)}*(b-x)*(\arctan(1/2*(a+b-2*x)/(-a*b+a*x+b*x-x^2))^{(1/2)})*a^2+2*b*\arctan(1/2*(a+b-2*x)/(-a*b+a*x+b*x-x^2))^{(1/2)}*a-3*\arctan(1/2*(a+b-2*x)/(-a*b+a*x+b*x-x^2))^{(1/2)}*b^2+2*(-a*b+a*x+b*x-x^2)^{(1/2)}*a-6*(-a*b+a*x+b*x-x^2)^{(1/2)}*b-4*(-a*b+a*x+b*x-x^2)^{(1/2)}*x)/(-(b-x)*(a-x))^{(1/2)}$

Maxima [A]

time = 0.35, size = 130, normalized size = 1.41

$$-\frac{1}{4}(a^2+2ab-3b^2)\arctan\left(\sqrt{\frac{a-x}{b-x}}\right) - \frac{(a^2-6ab+5b^2)\left(-\frac{a-x}{b-x}\right)^{\frac{3}{2}} - (a^2+2ab-3b^2)\sqrt{\frac{a-x}{b-x}}}{4\left(\frac{(a-x)^2}{(b-x)^2} - \frac{2(a-x)}{b-x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*((-a+x)/(b-x))^(1/2),x, algorithm="maxima")`

[Out] $-1/4*(a^2+2*a*b-3*b^2)*\arctan(\text{sqrt}(-(a-x)/(b-x))) - 1/4*((a^2-6*a*b+5*b^2)*(-(a-x)/(b-x))^{(3/2)} - (a^2+2*a*b-3*b^2)*\text{sqrt}(-(a-x)/(b-x)))/((a-x)^2/(b-x)^2 - 2*(a-x)/(b-x) + 1)$

Fricas [A]

time = 0.38, size = 73, normalized size = 0.79

$$-\frac{1}{4}(a^2+2ab-3b^2)\arctan\left(\sqrt{\frac{a-x}{b-x}}\right) + \frac{1}{4}(ab-3b^2-(a-b)x+2x^2)\sqrt{\frac{a-x}{b-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((-a+x)/(b-x))^(1/2),x, algorithm="fricas")

[Out] $-1/4*(a^2 + 2*a*b - 3*b^2)*\arctan(\sqrt{-(a-x)/(b-x)}) + 1/4*(a*b - 3*b^2 - (a-b)*x + 2*x^2)*\sqrt{-(a-x)/(b-x)}$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{\frac{-a+x}{b-x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((-a+x)/(b-x))^(1/2),x)

[Out] Integral(x*sqrt((-a + x)/(b - x)), x)

Giac [A]

time = 0.02, size = 113, normalized size = 1.23

$$2 \left(\frac{1}{4} x \operatorname{sign}(-b+x) - \frac{2a \operatorname{sign}(-b+x) - 6b \operatorname{sign}(-b+x)}{16} \right) \sqrt{-x^2 - ab + ax + bx} - \frac{1}{2} \left(-\frac{1}{4} a^2 \operatorname{sign}(-b+x) + \frac{3}{4} b^2 \operatorname{sign}(-b+x) - \frac{1}{2} ab \operatorname{sign}(-b+x) \right) \operatorname{sign}(-a+b) \arcsin\left(\frac{a+b-2x}{a-b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((-a+x)/(b-x))^(1/2),x)

[Out] $1/8*(a^2*\operatorname{sgn}(-b+x) + 2*a*b*\operatorname{sgn}(-b+x) - 3*b^2*\operatorname{sgn}(-b+x))*\arcsin((a+b-2*x)/(a-b))*\operatorname{sgn}(-a+b) - 1/4*\sqrt{-a*b+a*x+b*x-x^2}*(a*\operatorname{sgn}(-b+x) - 3*b*\operatorname{sgn}(-b+x) - 2*x*\operatorname{sgn}(-b+x))$

Mupad [B]

time = 0.34, size = 140, normalized size = 1.52

$$\frac{\sqrt{-\frac{a-x}{b-x}} \left(\frac{a^2 1i}{4} + \frac{ab 1i}{2} - \frac{b^2 3i}{4} \right) 1i - \left(-\frac{a-x}{b-x} \right)^{3/2} \left(\frac{a^2 1i}{4} - \frac{ab 3i}{2} + \frac{b^2 5i}{4} \right) 1i}{\frac{(a-x)^2}{(b-x)^2} - \frac{2(a-x)}{b-x} + 1} - \frac{\operatorname{atan}\left(\sqrt{-\frac{a-x}{b-x}}\right) (a-b) (a+3b)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-(a-x)/(b-x))^(1/2),x)

[Out] $-((-(a-x)/(b-x))^(1/2)*((a*b*1i)/2 + (a^2*1i)/4 - (b^2*3i)/4)*1i - (-(a-x)/(b-x))^(3/2)*((a^2*1i)/4 - (a*b*3i)/2 + (b^2*5i)/4)*1i)/((a-x)^2/(b-x)^2 - (2*(a-x))/(b-x) + 1) - (\operatorname{atan}((-(a-x)/(b-x))^(1/2))*(a-b)*(a+3*b))/4$

$$3.220 \quad \int \frac{\sqrt{-5+x} \sqrt{3+x}}{(-1+x)(-25+x^2)} dx$$

Optimal. Leaf size=54

$$\frac{1}{6} \tan^{-1} \left(\frac{1}{4} \sqrt{-5+x} \sqrt{3+x} \right) + \frac{\tanh^{-1} \left(\frac{\sqrt{5} \sqrt{3+x}}{\sqrt{-5+x}} \right)}{3\sqrt{5}}$$

[Out] 1/6*arctan(1/4*(-5+x)^(1/2)*(3+x)^(1/2))+1/15*arctanh(5^(1/2)*(3+x)^(1/2)/(-5+x)^(1/2))*5^(1/2)

Rubi [A]

time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1600, 184, 94, 209, 95, 212}

$$\frac{1}{6} \tan^{-1} \left(\frac{1}{4} \sqrt{x-5} \sqrt{x+3} \right) + \frac{\tanh^{-1} \left(\frac{\sqrt{5} \sqrt{x+3}}{\sqrt{x-5}} \right)}{3\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-5 + x]*Sqrt[3 + x])/((-1 + x)*(-25 + x^2)), x]

[Out] ArcTan[(Sqrt[-5 + x]*Sqrt[3 + x])/4]/6 + ArcTanh[(Sqrt[5]*Sqrt[3 + x])/Sqrt[-5 + x]]/(3*Sqrt[5])

Rule 94

Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - f*(b*c + a*d), 0]

Rule 95

Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 184

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[

$(e + f*x)^{(p - 1)}*((g + h*x)^q/(a + b*x)), x], x] - \text{Dist}[(d*e - c*f)/(b*c - a*d), \text{Int}[(e + f*x)^{(p - 1)}*((g + h*x)^q/(c + d*x)), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, q\}, x] \&\& \text{LtQ}[0, p, 1]$

Rule 209

$\text{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] :> \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))*\text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{GtQ}[b, 0])$

Rule 212

$\text{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] :> \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]))*\text{ArcTanh}[\text{Rt}[-b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 1600

$\text{Int}[(u_.)*(P_x)^{(p_.)}*(Q_x)^{(q_.)}, x_Symbol] :> \text{Int}[u*\text{PolynomialQuotient}[P_x, Q_x, x]^p*Q_x^{(p + q)}, x] /; \text{FreeQ}[q, x] \&\& \text{PolyQ}[P_x, x] \&\& \text{PolyQ}[Q_x, x] \&\& \text{EqQ}[\text{PolynomialRemainder}[P_x, Q_x, x], 0] \&\& \text{IntegerQ}[p] \&\& \text{LtQ}[p*q, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{-5+x} \sqrt{3+x}}{(-1+x)(-25+x^2)} dx &= \int \frac{\sqrt{3+x}}{\sqrt{-5+x}(-1+x)(5+x)} dx \\ &= \frac{1}{3} \int \frac{1}{\sqrt{-5+x} \sqrt{3+x} (5+x)} dx + \frac{2}{3} \int \frac{1}{\sqrt{-5+x}(-1+x)\sqrt{3+x}} dx \\ &= \frac{2}{3} \text{Subst} \left(\int \frac{1}{2-10x^2} dx, x, \frac{\sqrt{3+x}}{\sqrt{-5+x}} \right) + \frac{2}{3} \text{Subst} \left(\int \frac{1}{16+x^2} dx, x, \sqrt{-5-x} \right) \\ &= \frac{1}{6} \tan^{-1} \left(\frac{1}{4} \sqrt{-5+x} \sqrt{3+x} \right) + \frac{\tanh^{-1} \left(\frac{\sqrt{5} \sqrt{3+x}}{\sqrt{-5+x}} \right)}{3\sqrt{5}} \end{aligned}$$

Mathematica [A]

time = 0.07, size = 47, normalized size = 0.87

$$\frac{1}{15} \left(-5 \tan^{-1} \left(\frac{1}{\sqrt{\frac{-5+x}{3+x}}} \right) + \sqrt{5} \tanh^{-1} \left(\frac{\sqrt{5}}{\sqrt{\frac{-5+x}{3+x}}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-5 + x]*Sqrt[3 + x])/((-1 + x)*(-25 + x^2)),x]

[Out] (-5*ArcTan[1/Sqrt[(-5 + x)/(3 + x)]] + Sqrt[5]*ArcTanh[Sqrt[5]/Sqrt[(-5 + x)/(3 + x)]])/15

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sqrt[x - 5]*(Sqrt[x + 3])/((x - 1)*(x^2 - 25)),x]')

[Out] Timed out

Maple [A]

time = 0.09, size = 64, normalized size = 1.19

method	result	size
default	$\frac{\sqrt{x-5} \sqrt{3+x} \left(\sqrt{5} \operatorname{arctanh}\left(\frac{(5+3x)\sqrt{5}}{5\sqrt{x^2-2x-15}}\right) - 5 \operatorname{arctan}\left(\frac{4}{\sqrt{x^2-2x-15}}\right) \right)}{30\sqrt{x^2-2x-15}}$	64

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-5)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x,method=_RETURNVERBOSE)

[Out] 1/30*(x-5)^(1/2)*(3+x)^(1/2)*(5^(1/2)*arctanh(1/5*(5+3*x)*5^(1/2)/(x^2-2*x-15)^(1/2))-5*arctan(4/(x^2-2*x-15)^(1/2)))/(x^2-2*x-15)^(1/2)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x, algorithm="maxima")

[Out] integrate(sqrt(x + 3)*sqrt(x - 5)/((x^2 - 25)*(x - 1)), x)

Fricas [A]

time = 0.38, size = 65, normalized size = 1.20

$$\frac{1}{30} \sqrt{5} \log \left(\frac{\sqrt{x+3} \sqrt{x-5} (3\sqrt{5} + 5) + \sqrt{5} (3x+5) + 9x+15}{x+5} \right) + \frac{1}{3} \operatorname{arctan} \left(\frac{1}{4} \sqrt{x+3} \sqrt{x-5} - \frac{1}{4} x + \frac{1}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x, algorithm="fricas")
[Out] 1/30*sqrt(5)*log((sqrt(x + 3)*sqrt(x - 5)*(3*sqrt(5) + 5) + sqrt(5)*(3*x + 5) + 9*x + 15)/(x + 5)) + 1/3*arctan(1/4*sqrt(x + 3)*sqrt(x - 5) - 1/4*x + 1/4)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+3}}{\sqrt{x-5}(x-1)(x+5)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+x)**(1/2)*(3+x)**(1/2)/(-1+x)/(x**2-25),x)
[Out] Integral(sqrt(x + 3)/(sqrt(x - 5)*(x - 1)*(x + 5)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(36) = 72.
time = 0.01, size = 101, normalized size = 1.87

$$4 \left(\frac{\arctan\left(\frac{(\sqrt{x+3}-\sqrt{x-5})^2}{8}\right)}{12} - \frac{\ln\left(\frac{2(\sqrt{x+3}-\sqrt{x-5})^2+24-8\sqrt{5}}{2(\sqrt{x+3}-\sqrt{x-5})^2+24+8\sqrt{5}}\right)}{24\sqrt{5}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x)
[Out] -1/30*sqrt(5)*log(((sqrt(x + 3) - sqrt(x - 5))^2 - 4*sqrt(5) + 12)/((sqrt(x + 3) - sqrt(x - 5))^2 + 4*sqrt(5) + 12)) - 1/3*arctan(1/8*(sqrt(x + 3) - sqrt(x - 5))^2)
```

Mupad [B]

time = 0.61, size = 95, normalized size = 1.76

$$\frac{\operatorname{atan}\left(\frac{\sqrt{x+3}\sqrt{x-5}-2\sqrt{2}\sqrt{x-5}}{x-2\sqrt{2}\sqrt{x+3}+3}\right)}{3} - \frac{\sqrt{5}\operatorname{atanh}\left(-\frac{\sqrt{5}\sqrt{x+3}\sqrt{x-5}-2\sqrt{2}\sqrt{5}\sqrt{x-5}}{5x-10\sqrt{2}\sqrt{x+3}+15}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 3)^(1/2)*(x - 5)^(1/2))/((x^2 - 25)*(x - 1)),x)
[Out] atan(((x + 3)^(1/2)*(x - 5)^(1/2) - 2*2^(1/2)*(x - 5)^(1/2))/(x - 2*2^(1/2)*(x + 3)^(1/2) + 3))/3 - (5^(1/2)*atanh(-(5^(1/2)*(x + 3)^(1/2)*(x - 5)^(1/2) - 2*2^(1/2)*5^(1/2)*(x - 5)^(1/2))/(5*x - 10*2^(1/2)*(x + 3)^(1/2) + 15)))/15
```

$$3.221 \quad \int \frac{x^2 \sqrt{1+x} \sqrt[4]{1-x^2}}{\sqrt{1-x} \left(\sqrt{1-x} - \sqrt{1+x} \right)} dx$$

Optimal. Leaf size=304

$$\frac{5}{16}(1-x)^{3/4} \sqrt[4]{1+x} - \frac{1}{16} \sqrt[4]{1-x} (1+x)^{3/4} + \frac{1}{24}(1-x)^{5/4}(1+x)^{3/4} + \frac{7(1-x^2)^{5/4}}{24\sqrt{1-x}} + \frac{x(1-x^2)^{5/4}}{6\sqrt{1-x}} + \frac{1}{6}\sqrt{1+x} \quad (1)$$

[Out] 5/16*(1-x)^(3/4)*(1+x)^(1/4)-1/16*(1-x)^(1/4)*(1+x)^(3/4)+1/24*(1-x)^(5/4)*(1+x)^(3/4)+3/16*arctan(-1+(1-x)^(1/4)*2^(1/2)/(1+x)^(1/4))*2^(1/2)+3/16*arctan(1+(1-x)^(1/4)*2^(1/2)/(1+x)^(1/4))*2^(1/2)+1/16*ln(1-(1-x)^(1/4)*2^(1/2)/(1+x)^(1/4)+(1-x)^(1/2)/(1+x)^(1/2))*2^(1/2)-1/16*ln(1+(1-x)^(1/4)*2^(1/2)/(1+x)^(1/4)+(1-x)^(1/2)/(1+x)^(1/2))*2^(1/2)+7/24*(-x^2+1)^(5/4)/(1-x)^(1/2)+1/6*x*(-x^2+1)^(5/4)/(1-x)^(1/2)+1/6*(-x^2+1)^(5/4)*(1+x)^(1/2)

Rubi [A]

time = 0.49, antiderivative size = 319, normalized size of antiderivative = 1.05, number of steps used = 33, number of rules used = 16, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2128, 809, 689, 52, 65, 246, 217, 1179, 642, 1176, 631, 210, 1647, 807, 338, 303}

$$\frac{(1-x^2)^{5/4}}{3(1-x)^{3/2}} + \frac{1}{6}\sqrt{x+1}(1-x)^{5/4} + \frac{1}{6}(1-x)^{5/4}(x+1)^{3/4} + \frac{1}{24}(1-x)^{5/4}(x+1)^{3/4} - \frac{1}{16}\sqrt[4]{1-x}(x+1)^{3/4} + \frac{5}{24}(1-x)^{5/4}\sqrt{x+1} - \frac{5}{48}(1-x)^{5/4}\sqrt{x+1} + \frac{\log\left(\frac{\sqrt{1-x}-\sqrt{2}\sqrt{1-x}+1}{\sqrt{x+1}}\right)}{8\sqrt{2}} - \frac{\log\left(\frac{\sqrt{1-x}+\sqrt{2}\sqrt{1-x}+1}{\sqrt{x+1}}\right)}{8\sqrt{2}} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{1-x}}{\sqrt{x+1}}\right)}{8\sqrt{2}} + \frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt{1-x}}{\sqrt{x+1}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Sqrt[1+x]*(1-x^2)^(1/4))/(Sqrt[1-x]*(Sqrt[1-x]-Sqrt[1+x])),x]

[Out] (-5*(1-x)^(3/4)*(1+x)^(1/4))/48 + (5*(1-x)^(7/4)*(1+x)^(1/4))/24 - ((1-x)^(1/4)*(1+x)^(3/4))/16 + ((1-x)^(5/4)*(1+x)^(3/4))/24 + ((1-x)^(7/4)*(1+x)^(5/4))/6 + (Sqrt[1+x]*(1-x^2)^(5/4))/6 + (1-x^2)^(9/4)/(3*(1-x)^(3/2)) - (3*ArcTan[1-(Sqrt[2]*(1-x)^(1/4))/(1+x)^(1/4)])/((8*Sqrt[2])) + (3*ArcTan[1+(Sqrt[2]*(1-x)^(1/4))/(1+x)^(1/4)])/((8*Sqrt[2])) + Log[1+Sqrt[1-x]/Sqrt[1+x] - (Sqrt[2]*(1-x)^(1/4))/(1+x)^(1/4)]/(8*Sqrt[2]) - Log[1+Sqrt[1-x]/Sqrt[1+x] + (Sqrt[2]*(1-x)^(1/4))/(1+x)^(1/4)]/(8*Sqrt[2])

Rule 52

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*(b*c - a*d)/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n

+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
 {p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
 [b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
 ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(
 -1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
 & (LtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]
]}, s = Denominator[Rt[a/b, 2]]], Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b
 }, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
 AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 246

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[a^(p + 1/n), Subst[Int
 [1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
 n]

Rule 303

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b,
 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
 & AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 338

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
 ^(-1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
 ^(-1)] && IntegersQ[m, p + (m + 1)/n]

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 689

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] && !IGtQ[m, 0]
```

Rule 807

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0]
```

Rule 809

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))), x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179


```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_)*(x_)^(m_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol]
:= Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 2128

```
Int[(u_)/((e_)*Sqrt[(a_) + (b_)*(x_)] + (f_)*Sqrt[(c_) + (d_)*(x_)]),
x_Symbol] := Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \sqrt{1+x} \sqrt[4]{1-x^2}}{\sqrt{1-x} (\sqrt{1-x} - \sqrt{1+x})} dx &= - \left(\frac{1}{2} \int x \sqrt{1+x} \sqrt[4]{1-x^2} dx \right) - \frac{1}{2} \int \frac{x(1+x) \sqrt[4]{1-x^2}}{\sqrt{1-x}} dx \\
&= \frac{1}{6} \sqrt{1+x} (1-x^2)^{5/4} - \frac{1}{12} \int \sqrt{1+x} \sqrt[4]{1-x^2} dx - \frac{1}{2} \int \frac{x(1-x)}{(1-x)} dx \\
&= \frac{1}{6} \sqrt{1+x} (1-x^2)^{5/4} + \frac{(1-x^2)^{9/4}}{3(1-x)^{3/2}} - \frac{1}{12} \int \sqrt[4]{1-x} (1+x)^{3/4} dx \\
&= \frac{1}{24} (1-x)^{5/4} (1+x)^{3/4} + \frac{1}{6} \sqrt{1+x} (1-x^2)^{5/4} + \frac{(1-x^2)^{9/4}}{3(1-x)^{3/2}} - \frac{1}{16} \\
&= -\frac{1}{16} \sqrt[4]{1-x} (1+x)^{3/4} + \frac{1}{24} (1-x)^{5/4} (1+x)^{3/4} + \frac{1}{6} (1-x)^{7/4} (1+x) \\
&= \frac{5}{24} (1-x)^{7/4} \sqrt[4]{1+x} - \frac{1}{16} \sqrt[4]{1-x} (1+x)^{3/4} + \frac{1}{24} (1-x)^{5/4} (1+x) \\
&= -\frac{5}{48} (1-x)^{3/4} \sqrt[4]{1+x} + \frac{5}{24} (1-x)^{7/4} \sqrt[4]{1+x} - \frac{1}{16} \sqrt[4]{1-x} (1+x) \\
&= -\frac{5}{48} (1-x)^{3/4} \sqrt[4]{1+x} + \frac{5}{24} (1-x)^{7/4} \sqrt[4]{1+x} - \frac{1}{16} \sqrt[4]{1-x} (1+x) \\
&= -\frac{5}{48} (1-x)^{3/4} \sqrt[4]{1+x} + \frac{5}{24} (1-x)^{7/4} \sqrt[4]{1+x} - \frac{1}{16} \sqrt[4]{1-x} (1+x) \\
&= -\frac{5}{48} (1-x)^{3/4} \sqrt[4]{1+x} + \frac{5}{24} (1-x)^{7/4} \sqrt[4]{1+x} - \frac{1}{16} \sqrt[4]{1-x} (1+x) \\
&= -\frac{5}{48} (1-x)^{3/4} \sqrt[4]{1+x} + \frac{5}{24} (1-x)^{7/4} \sqrt[4]{1+x} - \frac{1}{16} \sqrt[4]{1-x} (1+x) \\
&= -\frac{5}{48} (1-x)^{3/4} \sqrt[4]{1+x} + \frac{5}{24} (1-x)^{7/4} \sqrt[4]{1+x} - \frac{1}{16} \sqrt[4]{1-x} (1+x) \\
&= -\frac{5}{48} (1-x)^{3/4} \sqrt[4]{1+x} + \frac{5}{24} (1-x)^{7/4} \sqrt[4]{1+x} - \frac{1}{16} \sqrt[4]{1-x} (1+x)
\end{aligned}$$

Mathematica [A]

time = 15.86, size = 195, normalized size = 0.64

$$\frac{\sqrt{1+x} \left((-1+x)\sqrt{1+x} (7-8x^3+29\sqrt{1-x^2}+2x^2(-5+4\sqrt{1-x^2})+x(5+22\sqrt{1-x^2})) + 3\sqrt{-1+x} (\sqrt{-1+x}\sqrt{1+x}-5\sqrt{1-x^2}) \tan^{-1} \left(\sqrt{\frac{-1+x}{1+x}} \right) + 3\sqrt{-1+x} (\sqrt{-1+x}\sqrt{1+x}+5\sqrt{1-x^2}) \tanh^{-1} \left(\sqrt{\frac{-1+x}{1+x}} \right) \right)}{48(1-x^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*Sqrt[1 + x]*(1 - x^2)^(1/4))/(Sqrt[1 - x]*(Sqrt[1 - x] - Sqrt[1 + x])), x]

[Out] -1/48*((1 + x)^(1/4)*((-1 + x)*(1 + x)^(1/4)*(7 - 8*x^3 + 29*Sqrt[1 - x^2] + 2*x^2*(-5 + 4*Sqrt[1 - x^2]) + x*(5 + 22*Sqrt[1 - x^2])) + 3*(-1 + x)^(1/4)*(Sqrt[-1 + x]*Sqrt[1 + x] - 5*Sqrt[1 - x^2])*ArcTan[((-1 + x)/(1 + x))^(1/4)] + 3*(-1 + x)^(1/4)*(Sqrt[-1 + x]*Sqrt[1 + x] + 5*Sqrt[1 - x^2])*ArcTanh[((-1 + x)/(1 + x))^(1/4)]))/(1 - x^2)^(3/4)

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x^2*(1 - x^2)^(1/4)*Sqrt[1 + x]/(Sqrt[1 - x]*(Sqrt[1 - x] - Sqrt[1 + x])), x']')

[Out] Timed out

Maple [F]

time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2 (-x^2 + 1)^{\frac{1}{4}} \sqrt{1+x}}{\sqrt{1-x} (\sqrt{1-x} - \sqrt{1+x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/2)), x)

[Out] int(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/2)), x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/2)), x, algorithm="maxima")

[Out] -integrate((-x^2 + 1)^(1/4)*sqrt(x + 1)*x^2/(sqrt(-x + 1)*(sqrt(x + 1) - sqrt(-x + 1))), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(219) = 438.

time = 0.38, size = 577, normalized size = 1.90

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/48*(8*x^2 + 2*x - 7)*(-x^2 + 1)^{(1/4)}*\text{sqrt}(x + 1) + 1/48*(8*x^2 + 22*x + 29)*(-x^2 + 1)^{(1/4)}*\text{sqrt}(-x + 1) - 1/16*\text{sqrt}(2)*\text{arctan}((\text{sqrt}(2)*(x + 1)*\text{sqrt}(\text{sqrt}(2)*(-x^2 + 1)^{(1/4)}*\text{sqrt}(x + 1) + x + \text{sqrt}(-x^2 + 1) + 1)/(x + 1)) - \text{sqrt}(2)*(-x^2 + 1)^{(1/4)}*\text{sqrt}(x + 1) - x - 1)/(x + 1)) - 1/16*\text{sqrt}(2)*\text{arctan}((\text{sqrt}(2)*(x + 1)*\text{sqrt}(-(\text{sqrt}(2)*(-x^2 + 1)^{(1/4)}*\text{sqrt}(x + 1) - x - \text{sqrt}(-x^2 + 1) - 1)/(x + 1)) - \text{sqrt}(2)*(-x^2 + 1)^{(1/4)}*\text{sqrt}(x + 1) + x + 1)/(x + 1)) - 5/16*\text{sqrt}(2)*\text{arctan}((\text{sqrt}(2)*(x - 1)*\text{sqrt}(\text{sqrt}(2)*(-x^2 + 1)^{(1/4)}*\text{sqrt}(-x + 1) + x - \text{sqrt}(-x^2 + 1) - 1)/(x - 1)) - \text{sqrt}(2)*(-x^2 + 1)^{(1/4)}*\text{sqrt}(-x + 1) - x + 1)/(x - 1)) - 5/16*\text{sqrt}(2)*\text{arctan}((\text{sqrt}(2)*(x - 1)*\text{sqrt}(-(\text{sqrt}(2)*(-x^2 + 1)^{(1/4)}*\text{sqrt}(-x + 1) - x + \text{sqrt}(-x^2 + 1) + 1)/(x - 1)) - \text{sqrt}(2)*(-x^2 + 1)^{(1/4)}*\text{sqrt}(-x + 1) + x - 1)/(x - 1)) + 1/64*\text{sqrt}(2)*\log(4*(\text{sqrt}(2)*(-x^2 + 1)^{(1/4)}*\text{sqrt}(x + 1) + x + \text{sqrt}(-x^2 + 1) + 1)/(x + 1)) - 1/64*\text{sqrt}(2)*\log(-4*(\text{sqrt}(2)*(-x^2 + 1)^{(1/4)}*\text{sqrt}(x + 1) - x - \text{sqrt}(-x^2 + 1) - 1)/(x + 1)) + 5/64*\text{sqrt}(2)*\log(4*(\text{sqrt}(2)*(-x^2 + 1)^{(1/4)}*\text{sqrt}(-x + 1) + x - \text{sqrt}(-x^2 + 1) - 1)/(x - 1)) - 5/64*\text{sqrt}(2)*\log(-4*(\text{sqrt}(2)*(-x^2 + 1)^{(1/4)}*\text{sqrt}(-x + 1) - x + \text{sqrt}(-x^2 + 1) + 1)/(x - 1)) \end{aligned}$$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(-x**2+1)**(1/4)*(1+x)**(1/2)/(1-x)**(1/2)/((1-x)**(1/2)-(1+x)**(1/2)),x)

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/2)),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^2 (1-x^2)^{1/4} \sqrt{x+1}}{(\sqrt{x+1} - \sqrt{1-x}) \sqrt{1-x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2*(1-x^2)^(1/4)*(x+1)^(1/2))/(((x+1)^(1/2)-(1-x)^(1/2))*(1-x)^(1/2)),x)

[Out] -int((x^2*(1-x^2)^(1/4)*(x+1)^(1/2))/(((x+1)^(1/2)-(1-x)^(1/2))*(1-x)^(1/2)), x)

$$3.222 \quad \int \frac{\sqrt{1-x} x(1+x)^{2/3}}{-(1-x)^{5/6} \sqrt[3]{1+x} + (1-x)^{2/3} \sqrt{1+x}} dx$$

Optimal. Leaf size=292

$$-\frac{1}{12}(1-3x)(1-x)^{2/3} \sqrt[3]{1+x} + \frac{1}{4} \sqrt{1-x} x \sqrt{1+x} - \frac{1}{4}(1-x)(3+x) + \frac{1}{12} \sqrt[3]{1-x} (1+x)^{2/3} (1+3x) + \frac{1}{12} \sqrt[6]{1-x}$$

[Out] -1/12*(1-3*x)*(1-x)^(2/3)*(1+x)^(1/3)-1/4*(1-x)*(3+x)+1/12*(1-x)^(1/3)*(1+x)^(2/3)*(1+3*x)+1/12*(1-x)^(1/6)*(1+x)^(5/6)*(2+3*x)-1/12*(1-x)^(5/6)*(1+x)^(1/6)*(10+3*x)+1/6*arctan((1+x)^(1/6)/(1-x)^(1/6))-5/6*arctan(((1-x)^(1/3)-(1+x)^(1/3))/(1-x)^(1/6)/(1+x)^(1/6))-4/9*arctan(1/3*((1-x)^(1/3)-2*(1+x)^(1/3))/(1-x)^(1/3)*3^(1/2))*3^(1/2)+1/18*arctanh(((1-x)^(1/6)*(1+x)^(1/6))*3^(1/2)/((1-x)^(1/3)+(1+x)^(1/3)))*3^(1/2)+1/4*x*(1-x)^(1/2)*(1+x)^(1/2)

Rubi [A]

time = 1.18, antiderivative size = 522, normalized size of antiderivative = 1.79, number of steps used = 46, number of rules used = 21, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$,

Rules used = {6820, 6874, 52, 62, 531, 201, 222, 689, 904, 65, 246, 215, 648, 632, 210, 642, 209, 26, 21, 338, 301}

$\frac{1}{12} \sqrt[3]{1-x} (1+x)^{2/3} (1+3x) + \frac{1}{4} \sqrt{1-x} x \sqrt{1+x} - \frac{1}{4} (1-x)(3+x) + \frac{1}{12} \sqrt[3]{1-x} (1+x)^{2/3} (1+3x) + \frac{1}{12} \sqrt[6]{1-x}$

Warning: Unable to verify antiderivative.

[In] Int[(Sqrt[1 - x]*x*(1 + x)^(2/3))/(-((1 - x)^(5/6)*(1 + x)^(1/3)) + (1 - x)^(2/3)*Sqrt[1 + x]),x]

[Out] x/2 + x^2/4 - (7*(1 - x)^(5/6)*(1 + x)^(1/6))/12 + ((1 - x)^(2/3)*(1 + x)^(1/3))/6 - ((1 - x)^(5/3)*(1 + x)^(1/3))/4 + ((1 - x)^(1/3)*(1 + x)^(2/3))/3 - ((1 - x)^(4/3)*(1 + x)^(2/3))/4 + (5*(1 - x)^(1/6)*(1 + x)^(5/6))/12 - ((1 - x)^(7/6)*(1 + x)^(5/6))/4 - ((1 - x)^(5/6)*(1 + x)^(7/6))/4 + (x*Sqrt[1 - x^2])/4 + ArcSin[x]/4 - (2*ArcTan[(1 - x)^(1/6)/(1 + x)^(1/6)])/3 + (2*ArcTan[1/Sqrt[3] - (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))]/(3*Sqrt[3]) + ArcTan[Sqrt[3] - (2*(1 - x)^(1/6))/(1 + x)^(1/6)])/3 - ArcTan[Sqrt[3] + (2*(1 - x)^(1/6))/(1 + x)^(1/6)])/3 - (2*ArcTan[1/Sqrt[3] - (2*(1 + x)^(1/3))/(Sqrt[3]*(1 - x)^(1/3))]/(3*Sqrt[3]) - Log[1 - x]/9 + Log[1 + x]/9 + Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3)])/3 - Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3) - (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(12*Sqrt[3]) + Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3) + (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(12*Sqrt[3]) - Log[1 + (1 + x)^(1/3)/(1 - x)^(1/3)])/3

Rule 21

Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] := Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]

&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplifierQ[c + d*x, a + b*x])

Rule 26

Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(m_.)*((c_.) + (d_.)*(x_)^(j_.))^(p_.), x_Symbol] := Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && GtQ[a, 0] && LtQ[d, 0]

Rule 52

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 62

Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 201

Int[((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Simp[x*((a + b*x^n)^p/(n*p + 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 209

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 215

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a/b]

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 246

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[a^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/n]

Rule 301

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k - 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]

Rule 338

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[a^(p + (m + 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2

$^{-1}] \&\& \text{IntegersQ}[m, p + (m + 1)/n]$

Rule 531

$\text{Int}[(u_)*((c_)+(d_)*(x_)^{(n_}))^{(q_)*((a1_)+(b1_)*(x_)^{(non2_}))^{(p_)*((a2_)+(b2_)*(x_)^{(non2_}))^{(p_)}}, x_Symbol] \rightarrow \text{Int}[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x] /;$ FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 632

$\text{Int}[(a_)+(b_)*(x_)+(c_)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /;$ FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

$\text{Int}[(d_)+(e_)*(x_)]/[(a_)+(b_)*(x_)+(c_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /;$ FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

$\text{Int}[(d_)+(e_)*(x_)]/[(a_)+(b_)*(x_)+(c_)*(x_)^2], x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 689

$\text{Int}[(d_)+(e_)*(x_)]^{(m_)*((a_)+(c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Int}[(d + e*x)^{(m + p)}*(a/d + (c/e)*x)^p, x] /;$ FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] && !IGtQ[m, 0]

Rule 904

$\text{Int}[(d_)+(e_)*(x_)]^{(m_)*((f_)+(g_)*(x_))^{(n_)*((a_)+(c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Int}[(d + e*x)^{(m + p)}*(f + g*x)^n*(a/d + (c/e)*x)^p, x] /;$ FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] && !IGtQ[m, 0] && !IGtQ[n, 0]

Rule 6820

```
Int[u_, x_Symbol] :=> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl  
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]  
]
```

Rubi steps

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 29.55, size = 455, normalized size = 1.56

$$\frac{1}{12} \sqrt{1-x} \left((1-3x)^{1/3} - \frac{2\sqrt{1-x} \operatorname{arctanh}\left(\frac{x}{\sqrt{1-x}}\right)}{\sqrt{1-x}} - 3\sqrt{1-x} \operatorname{arctanh}\left(\frac{x}{\sqrt{1-x}}\right) - (1+3x)\sqrt{1-x} - \frac{2+3x\sqrt{1-x}}{\sqrt{1-x}} - \frac{2+3x\sqrt{1-x}}{\sqrt{1-x}} - 4x^{2/3} \operatorname{arctanh}\left(\frac{1}{3} \sqrt{\frac{1+x}{1-x}}\right) \right) + \frac{1}{12} \left(\tan^{-1}(x) + 14 \tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right) + 7(1+x^2) \tan^{-1}\left(\frac{(1-x)\sqrt{1-x}}{2\sqrt{1+x}}\right) + 7(1-x^2) \tan^{-1}\left(\frac{(1+x)\sqrt{1-x}}{2\sqrt{1+x}}\right) + 8\sqrt{2} \tan^{-1}\left(\frac{1-\sqrt{1-x}}{\sqrt{2}}\right) - \frac{2\sqrt{1-x} \operatorname{arctanh}\left(\frac{x}{\sqrt{1-x}}\right) \operatorname{arctanh}\left(\frac{x}{\sqrt{1+x}}\right)}{\sqrt{1-x} \sqrt{1+x}} - \operatorname{arctanh}\left(\frac{x}{\sqrt{1-x}}\right) + \operatorname{arctanh}\left(\frac{x}{\sqrt{1+x}}\right) - \operatorname{arctanh}\left(\frac{x}{\sqrt{1-x}}\right) + \operatorname{arctanh}\left(\frac{x}{\sqrt{1+x}}\right) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - x]*x*(1 + x)^(2/3))/(-(1 - x)^(5/6)*(1 + x)^(1/3)) + (1 - x)^(2/3)*Sqrt[1 + x]), x]
```

```
[Out] -1/12*((1 + x)^(1/3)*((1 - 3*x)*(1 - x)^(2/3) - (3*(1 - x)^(1/3)*x*(2 + x)) / (1 - x^2)^(1/3) - 3*(1 - x)^(1/3)*x*(1 - x^2)^(1/6) - (1 + 3*x)*(1 - x^2)^(1/3) - ((2 + 3*x)*Sqrt[1 - x^2])) / (1 - x)^(1/3) + ((10 + 3*x)*(1 - x^2)^(5/6)) / (1 + x) - 4*2^(2/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (1 + x)/2]) + (9*ArcSin[x] + 14*ArcTan[(1 + x)^(1/3)/(1 - x^2)^(1/6)] + 7*(1 + I*Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*(1 + x)^(1/3))/(2*(1 - x^2)^(1/6))] + 7*(1 - I*Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*(1 + x)^(1/3))/(2*(1 - x^2)^(1/6))] + 8*Sqrt[3]*ArcTan[(1 - (2*(1 - x^2)^(1/3)))/(1 + x)^(2/3)]/Sqrt[3]] - (15*2^(5/6)*Sqrt[1 - x^2]*Hypergeometric2F1[1/6, 1/6, 7/6, (1 - x)/2]) / ((1 - x)^(1/3)*Sqrt[1 + x]) - 8*Log[(1 + x)^(2/3) + (1 - x^2)^(1/3)] + 4*Log[(1 + x)^(1/3) + x*(1 + x)^(1/3) - (1 + x)^(2/3)*(1 - x^2)^(1/3) + (1 - x^2)^(2/3)]/36
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x*(1 + x)^(2/3)*Sqrt[1 - x]/(Sqrt[1 + x]*(1 - x)^(2/3) - (1 + x)^(1/3)*(1 - x)^(5/6)), x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5006 deep
```

Maple [F]

time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x(1+x)^{\frac{2}{3}} \sqrt{1-x}}{- (1-x)^{\frac{5}{6}} (1+x)^{\frac{1}{3}} + (1-x)^{\frac{2}{3}} \sqrt{1+x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(1+x)^(1/2)), x)
```

```
[Out] int(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(1+x)^(1/2)), x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)^(2/3)*x*sqrt(-x + 1)/(sqrt(x + 1)*(-x + 1)^(2/3) - (x + 1)^(1/3)*(-x + 1)^(5/6)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 865 vs. 2(220) = 440.

time = 0.37, size = 865, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*x^2 + 1/12*(3*x + 2)*(x + 1)^(5/6)*(-x + 1)^(1/6) + 1/12*(3*x + 1)*(x + 1)^(2/3)*(-x + 1)^(1/3) + 1/4*sqrt(x + 1)*x*sqrt(-x + 1) + 1/12*(3*x - 1)*(x + 1)^(1/3)*(-x + 1)^(2/3) - 1/12*(3*x + 10)*(x + 1)^(1/6)*(-x + 1)^(5/6) - 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*(x + 1) - 2*sqrt(3)*(x + 1)^(2/3)*(-x + 1)^(1/3))/(x + 1)) - 2/9*sqrt(3)*arctan(1/3*(sqrt(3)*(x - 1) + 2*sqrt(3)*(x + 1)^(1/3)*(-x + 1)^(2/3))/(x - 1)) - 5/72*sqrt(3)*log(100*(sqrt(3)*(x + 1)^(5/6)*(-x + 1)^(1/6) + x + (x + 1)^(2/3)*(-x + 1)^(1/3) + 1)/(x + 1)) + 5/72*sqrt(3)*log(-100*(sqrt(3)*(x + 1)^(5/6)*(-x + 1)^(1/6) - x - (x + 1)^(2/3)*(-x + 1)^(1/3) - 1)/(x + 1)) - 7/72*sqrt(3)*log(196*(sqrt(3)*(x + 1)^(1/6)*(-x + 1)^(5/6) + x - (x + 1)^(1/3)*(-x + 1)^(2/3) - 1)/(x - 1)) + 7/72*sqrt(3)*log(-196*(sqrt(3)*(x + 1)^(1/6)*(-x + 1)^(5/6) - x + (x + 1)^(1/3)*(-x + 1)^(2/3) + 1)/(x - 1)) + 1/2*x + 5/18*arctan(-(sqrt(3)*(x + 1) - 2*(x + 1)*sqrt((sqrt(3)*(x + 1)^(5/6)*(-x + 1)^(1/6) + x + (x + 1)^(2/3)*(-x + 1)^(1/3) + 1)/(x + 1)) + 2*(x + 1)^(5/6)*(-x + 1)^(1/6))/(x + 1)) + 5/18*arctan((sqrt(3)*(x + 1) + 2*(x + 1)*sqrt(-(sqrt(3)*(x + 1)^(5/6)*(-x + 1)^(1/6) - x - (x + 1)^(2/3)*(-x + 1)^(1/3) - 1)/(x + 1)) - 2*(x + 1)^(5/6)*(-x + 1)^(1/6))/(x + 1)) + 7/18*arctan(-(sqrt(3)*(x - 1) - 2*(x - 1)*sqrt((sqrt(3)*(x + 1)^(1/6)*(-x + 1)^(5/6) + x - (x + 1)^(1/3)*(-x + 1)^(2/3) - 1)/(x - 1)) + 2*(x + 1)^(1/6)*(-x + 1)^(5/6))/(x - 1)) + 7/18*arctan((sqrt(3)*(x - 1) + 2*(x - 1)*sqrt(-(sqrt(3)*(x + 1)^(1/6)*(-x + 1)^(5/6) - x + (x + 1)^(1/3)*(-x + 1)^(2/3) + 1)/(x - 1)) - 2*(x + 1)^(1/6)*(-x + 1)^(5/6))/(x - 1)) - 5/18*arctan((-x + 1)^(1/6)/(x + 1)^(1/6)) - 7/18*arctan((x + 1)^(1/6)*(-x + 1)^(5/6)/(x - 1)) - 1/2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - 2/9*log((x + (x + 1)^(2/3)*(-x + 1)^(1/3) + 1)/(x + 1)) + 1/9*log((x - (x + 1)^(2/3)*(-x + 1)^(1/3) - 1)/(x + 1))
```

$1)^{(2/3)}*(-x + 1)^{(1/3)} + (x + 1)^{(1/3)}*(-x + 1)^{(2/3)} + 1)/(x + 1)) - 1/9*$
 $\log((x - (x + 1)^{(2/3)}*(-x + 1)^{(1/3)} + (x + 1)^{(1/3)}*(-x + 1)^{(2/3)} - 1)/($
 $x - 1)) + 2/9*\log(-(x - (x + 1)^{(1/3)}*(-x + 1)^{(2/3)} - 1)/(x - 1))$

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(1+x)**(2/3)*(1-x)**(1/2)/(-(1-x)**(5/6)*(1+x)**(1/3)+(1-x)**(2/3)*(1+x)**(1/2)),x)

[Out] Exception raised: SystemError

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(1+x)^(1/2)),x)

[Out] Timed out

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \sqrt{1-x} (x+1)^{2/3}}{(1-x)^{2/3} \sqrt{x+1} - (1-x)^{5/6} (x+1)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(1 - x)^(1/2)*(x + 1)^(2/3))/((1 - x)^(2/3)*(x + 1)^(1/2) - (1 - x)^(5/6)*(x + 1)^(1/3)),x)

[Out] int((x*(1 - x)^(1/2)*(x + 1)^(2/3))/((1 - x)^(2/3)*(x + 1)^(1/2) - (1 - x)^(5/6)*(x + 1)^(1/3)), x)

$$3.223 \quad \int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}} dx$$

Optimal. Leaf size=25

$$-\frac{3(-1+x)(1+x)}{2\sqrt[3]{(-1+x)^4(1+x)^2}}$$

[Out] $-3/2*(-1+x)*(1+x)/((-1+x)^4*(1+x)^2)^{(1/3)}$

Rubi [A]

time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6851, 37}

$$\frac{3(1-x)(x+1)}{2\sqrt[3]{(1-x)^4(x+1)^2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[((-1+x)^4*(1+x)^2)^{-1/3}, x]$

[Out] $(3*(1-x)*(1+x))/(2*((1-x)^4*(1+x)^2)^{(1/3)})$

Rule 37

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)*((c + d*x)^{(n + 1)/((b*c - a*d)*(m + 1))}), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[m + n + 2, 0] \&\& \text{NeQ}[m, -1]$

Rule 6851

$\text{Int}[(u_.)*((a_.)*(v_.)^{(m_.)*(w_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[a^{\text{IntPart}[p]}*(a*v^m*w^n)^{\text{FracPart}[p]}/(v^{(m*\text{FracPart}[p])}*w^{(n*\text{FracPart}[p])}), \text{Int}[u*v^{(m*p)}*w^{(n*p)}, x], x] /; \text{FreeQ}\{a, m, n, p\}, x] \&\& !\text{IntegerQ}[p] \&\& !\text{FreeQ}[v, x] \&\& !\text{FreeQ}[w, x]$

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}} dx &= \frac{((-1+x)^{4/3}(1+x)^{2/3}) \int \frac{1}{(-1+x)^{4/3}(1+x)^{2/3}} dx}{\sqrt[3]{(-1+x)^4(1+x)^2}} \\ &= \frac{3(1-x)(1+x)}{2\sqrt[3]{(1-x)^4(1+x)^2}} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 25, normalized size = 1.00

$$-\frac{3(-1+x)(1+x)}{2\sqrt[3]{(-1+x)^4(1+x)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((-1 + x)^4*(1 + x)^2)^(-1/3),x]

[Out] (-3*(-1 + x)*(1 + x))/(2*((-1 + x)^4*(1 + x)^2)^(1/3))

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((x + 1)^2*(x - 1)^4)^(1/3),x]')

[Out] cought exception: maximum recursion depth exceeded in comparison

Maple [A]

time = 0.02, size = 22, normalized size = 0.88

method	result	size
gospers	$-\frac{3(-1+x)(1+x)}{2((-1+x)^4(1+x)^2)^{\frac{1}{3}}}$	22
risch	$-\frac{3(-1+x)(1+x)}{2((-1+x)^4(1+x)^2)^{\frac{1}{3}}}$	22
trager	$-\frac{3(x^6-2x^5-x^4+4x^3-x^2-2x+1)^{\frac{2}{3}}}{2(1+x)(-1+x)^3}$	43

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((-1+x)^4*(1+x)^2)^(1/3),x,method=_RETURNVERBOSE)

[Out] -3/2*(-1+x)*(1+x)/((-1+x)^4*(1+x)^2)^(1/3)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)^4*(1+x)^2)^(1/3),x, algorithm="maxima")

[Out] integrate(((x + 1)^2*(x - 1)^4)^(-1/3), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(21) = 42.

time = 0.30, size = 47, normalized size = 1.88

$$-\frac{3(x^6 - 2x^5 - x^4 + 4x^3 - x^2 - 2x + 1)^{\frac{2}{3}}}{2(x^4 - 2x^3 + 2x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)^4*(1+x)^2)^(1/3),x, algorithm="fricas")

[Out] -3/2*(x^6 - 2*x^5 - x^4 + 4*x^3 - x^2 - 2*x + 1)^(2/3)/(x^4 - 2*x^3 + 2*x - 1)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(x-1)^4(x+1)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)**4*(1+x)**2)**(1/3),x)

[Out] Integral(((x - 1)**4*(x + 1)**2)**(-1/3), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)^4*(1+x)^2)^(1/3),x)

[Out] Could not integrate

Mupad [B]

time = 0.24, size = 25, normalized size = 1.00

$$-\frac{3((x-1)^4(x+1)^2)^{\frac{2}{3}}}{2(x-1)^3(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x - 1)^4*(x + 1)^2)^(1/3),x)

[Out] -(3*((x - 1)^4*(x + 1)^2)^(2/3))/(2*(x - 1)^3*(x + 1))

$$3.224 \quad \int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}} dx$$

Optimal. Leaf size=25

$$\frac{4(-1+x)(2+x)}{3\sqrt[4]{(-1+x)^3(2+x)^5}}$$

[Out] 4/3*(-1+x)*(2+x)/((-1+x)^3*(2+x)^5)^(1/4)

Rubi [A]

time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6851, 37}

$$-\frac{4(1-x)(x+2)}{3\sqrt[4]{-(1-x)^3(x+2)^5}}$$

Antiderivative was successfully verified.

[In] Int[((-1 + x)^3*(2 + x)^5)^(-1/4), x]

[Out] (-4*(1 - x)*(2 + x))/(3*(-((1 - x)^3*(2 + x)^5))^(1/4))

Rule 37

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 6851

Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}} dx &= \frac{((-1+x)^{3/4}(2+x)^{5/4}) \int \frac{1}{(-1+x)^{3/4}(2+x)^{5/4}} dx}{\sqrt[4]{(-1+x)^3(2+x)^5}} \\ &= -\frac{4(1-x)(2+x)}{3\sqrt[4]{-(1-x)^3(2+x)^5}} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 25, normalized size = 1.00

$$\frac{4(-1+x)(2+x)}{3\sqrt[4]{(-1+x)^3(2+x)^5}}$$

Antiderivative was successfully verified.

[In] Integrate[((-1 + x)^3*(2 + x)^5)^(1/4), x]

[Out] (4*(-1 + x)*(2 + x))/(3*((-1 + x)^3*(2 + x)^5)^(1/4))

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((x - 1)^3*(x + 2)^5)^(1/4), x]')

[Out] cought exception: maximum recursion depth exceeded in comparison

Maple [A]

time = 0.03, size = 22, normalized size = 0.88

method	result	size
gospers	$\frac{4(-1+x)(2+x)}{3((-1+x)^3(2+x)^5)^{\frac{1}{4}}}$	22
risch	$\frac{4(-1+x)(2+x)}{3((-1+x)^3(2+x)^5)^{\frac{1}{4}}}$	22
trager	$\frac{4(x^8+7x^7+13x^6-11x^5-50x^4-8x^3+64x^2+16x-32)^{\frac{3}{4}}}{3(-1+x)^2(2+x)^4}$	53

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((-1+x)^3*(2+x)^5)^(1/4), x, method=_RETURNVERBOSE)

[Out] 4/3*(-1+x)*(2+x)/((-1+x)^3*(2+x)^5)^(1/4)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)^3*(2+x)^5)^(1/4), x, algorithm="maxima")

[Out] integrate(((x + 2)^5*(x - 1)^3)^(-1/4), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(21) = 42.

time = 0.30, size = 69, normalized size = 2.76

$$\frac{4(x^8 + 7x^7 + 13x^6 - 11x^5 - 50x^4 - 8x^3 + 64x^2 + 16x - 32)^{\frac{3}{4}}}{3(x^6 + 6x^5 + 9x^4 - 8x^3 - 24x^2 + 16)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)^3*(2+x)^5)^(1/4),x, algorithm="fricas")

[Out] 4/3*(x^8 + 7*x^7 + 13*x^6 - 11*x^5 - 50*x^4 - 8*x^3 + 64*x^2 + 16*x - 32)^(3/4)/(x^6 + 6*x^5 + 9*x^4 - 8*x^3 - 24*x^2 + 16)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)**3*(2+x)**5)**(1/4),x)

[Out] Integral(((x - 1)**3*(x + 2)**5)**(-1/4), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)^3*(2+x)^5)^(1/4),x)

[Out] Could not integrate

Mupad [B]

time = 0.26, size = 25, normalized size = 1.00

$$\frac{4((x-1)^3(x+2)^5)^{3/4}}{3(x-1)^2(x+2)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x - 1)^3*(x + 2)^5)^(1/4),x)

[Out] (4*((x - 1)^3*(x + 2)^5)^(3/4))/(3*(x - 1)^2*(x + 2)^4)

$$3.225 \quad \int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}} dx$$

Optimal. Leaf size=53

$$-\frac{3(-1+x)(1+x)}{8\sqrt[3]{(-1+x)^7(1+x)^2}} + \frac{9(-1+x)^2(1+x)}{16\sqrt[3]{(-1+x)^7(1+x)^2}}$$

[Out] $-3/8*(-1+x)*(1+x)/((-1+x)^7*(1+x)^2)^{(1/3)}+9/16*(-1+x)^2*(1+x)/((-1+x)^7*(1+x)^2)^{(1/3)}$

Rubi [A]

time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$,

Rules used = {6851, 47, 37}

$$\frac{9(x+1)(1-x)^2}{16\sqrt[3]{-(1-x)^7(x+1)^2}} + \frac{3(x+1)(1-x)}{8\sqrt[3]{-(1-x)^7(x+1)^2}}$$

Antiderivative was successfully verified.

[In] Int[$((-1+x)^7*(1+x)^2)^{-1/3}, x]$

[Out] $(3*(1-x)*(1+x))/(8*(-((1-x)^7*(1+x)^2))^{1/3}) + (9*(1-x)^2*(1+x))/(16*(-((1-x)^7*(1+x)^2))^{1/3})$

Rule 37

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(Simplify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + n + 2] && !IntegerQ[m] && !IntegerQ[n] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n])) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])

Rule 6851

Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^

`(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x]`

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}} dx &= \frac{((-1+x)^{7/3}(1+x)^{2/3}) \int \frac{1}{(-1+x)^{7/3}(1+x)^{2/3}} dx}{\sqrt[3]{(-1+x)^7(1+x)^2}} \\ &= \frac{3(1-x)(1+x)}{8\sqrt[3]{-(1-x)^7(1+x)^2}} - \frac{(3(-1+x)^{7/3}(1+x)^{2/3}) \int \frac{1}{(-1+x)^{4/3}(1+x)^{2/3}} dx}{8\sqrt[3]{(-1+x)^7(1+x)^2}} \\ &= \frac{3(1-x)(1+x)}{8\sqrt[3]{-(1-x)^7(1+x)^2}} + \frac{9(1-x)^2(1+x)}{16\sqrt[3]{-(1-x)^7(1+x)^2}} \end{aligned}$$

Mathematica [A]

time = 0.06, size = 30, normalized size = 0.57

$$\frac{3(-1+x)(1+x)(-5+3x)}{16\sqrt[3]{(-1+x)^7(1+x)^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[((-1 + x)^7*(1 + x)^2)^(-1/3), x]`

[Out] `(3*(-1 + x)*(1 + x)*(-5 + 3*x))/(16*((-1 + x)^7*(1 + x)^2)^(1/3))`

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[1/((x + 1)^2*(x - 1)^7)^(1/3), x]')`

[Out] cought exception: maximum recursion depth exceeded in comparison

Maple [A]

time = 0.02, size = 27, normalized size = 0.51

method	result	size
gosper	$\frac{3(1+x)(-1+x)(3x-5)}{16((-1+x)^7(1+x)^2)^{\frac{1}{3}}}$	27

risch	$\frac{3(-1+x)(3x^2-2x-5)}{16((-1+x)^7(1+x)^2)^{\frac{1}{3}}}$	29
trager	$\frac{3(3x-5)(x^9-5x^8+8x^7-14x^5+14x^4-8x^2+5x-1)^{\frac{2}{3}}}{16(-1+x)^6(1+x)}$	53

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((-1+x)^7*(1+x)^2)^(1/3),x,method=_RETURNVERBOSE)`

[Out] `3/16*(1+x)*(-1+x)*(3*x-5)/((-1+x)^7*(1+x)^2)^(1/3)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-1+x)^7*(1+x)^2)^(1/3),x, algorithm="maxima")`

[Out] `integrate(((x + 1)^2*(x - 1)^7)^(-1/3), x)`

Fricas [A]

time = 0.30, size = 77, normalized size = 1.45

$$\frac{3(x^9 - 5x^8 + 8x^7 - 14x^5 + 14x^4 - 8x^2 + 5x - 1)^{\frac{2}{3}}(3x - 5)}{16(x^7 - 5x^6 + 9x^5 - 5x^4 - 5x^3 + 9x^2 - 5x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-1+x)^7*(1+x)^2)^(1/3),x, algorithm="fricas")`

[Out] `3/16*(x^9 - 5*x^8 + 8*x^7 - 14*x^5 + 14*x^4 - 8*x^2 + 5*x - 1)^(2/3)*(3*x - 5)/(x^7 - 5*x^6 + 9*x^5 - 5*x^4 - 5*x^3 + 9*x^2 - 5*x + 1)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(x-1)^7(x+1)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-1+x)**7*(1+x)**2)**(1/3),x)`

[Out] `Integral(((x - 1)**7*(x + 1)**2)**(-1/3), x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-1+x)^7*(1+x)^2)^(1/3),x)`

[Out] Could not integrate

Mupad [B]

time = 0.23, size = 30, normalized size = 0.57

$$\frac{3(3x - 5)((x - 1)^7(x + 1)^2)^{2/3}}{16(x - 1)^6(x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((x - 1)^7*(x + 1)^2)^(1/3),x)`

[Out] `(3*(3*x - 5)*((x - 1)^7*(x + 1)^2)^(2/3))/(16*(x - 1)^6*(x + 1))`

$$3.226 \quad \int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}} dx$$

Optimal. Leaf size=67

$$\sqrt{3} \tan^{-1} \left(\frac{1 + \frac{2(-1+x)}{\sqrt[3]{(-1+x)^2(1+x)}}}{\sqrt{3}} \right) - \frac{1}{2} \log(1+x) - \frac{3}{2} \log \left(1 - \frac{-1+x}{\sqrt[3]{(-1+x)^2(1+x)}} \right)$$

[Out] $-1/2*\ln(1+x)-3/2*\ln(1+(1-x)/((-1+x)^2*(1+x))^{(1/3)})+\arctan(1/3*(1+2*(-1+x)/((-1+x)^2*(1+x))^{(1/3)})*3^{(1/2)})*3^{(1/2)}$

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 188 vs. $2(67) = 134$.
time = 0.08, antiderivative size = 188, normalized size of antiderivative = 2.81, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$,
Rules used = {2092, 2089, 62}

$$\frac{(3-3x)^{2/3} \sqrt[3]{x+1} \log\left(-\frac{8}{3}(x-1)\right)}{2 \cdot 3^{2/3} \sqrt[3]{x^3-x^2-x+1}} - \frac{\sqrt[3]{3} (3-3x)^{2/3} \sqrt[3]{x+1} \log\left(\frac{\sqrt[3]{3} \sqrt[3]{x+1}}{\sqrt[3]{3-3x}} + 1\right)}{2 \sqrt[3]{x^3-x^2-x+1}} - \frac{(3-3x)^{2/3} \sqrt[3]{x+1} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2 \sqrt[3]{x+1}}{\sqrt[3]{3} \sqrt[3]{3-3x}}\right)}{\sqrt[6]{3} \sqrt[3]{x^3-x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Int[((-1 + x)^2*(1 + x))^(-1/3), x]

[Out] $-(((3-3*x)^{(2/3)}*(1+x)^{(1/3)}*\text{ArcTan}[1/\text{Sqrt}[3] - (2*(1+x)^{(1/3)})]/(3^{(1/6)}*(3-3*x)^{(1/3)})))/(3^{(1/6)}*(1-x-x^2+x^3)^{(1/3)}) - ((3-3*x)^{(2/3)}*(1+x)^{(1/3)}*\text{Log}[(-8*(-1+x))/3])/(2*3^{(2/3)}*(1-x-x^2+x^3)^{(1/3)}) - (3^{(1/3)}*(3-3*x)^{(2/3)}*(1+x)^{(1/3)}*\text{Log}[1+(3^{(1/3)}*(1+x)^{(1/3)})]/(3-3*x)^{(1/3)})/(2*(1-x-x^2+x^3)^{(1/3)})$

Rule 62

Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3))]], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]

Rule 2089

Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x + d*x^3)^(p)/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] && !IntegerQ[p]

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] := With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}} dx = \text{Subst} \left(\int \frac{1}{\sqrt[3]{\frac{16}{27} - \frac{4x}{3} + x^3}} dx, x, -\frac{1}{3} + x \right)$$

$$= \frac{\left(4 \cdot 2^{2/3} (1-x)^{2/3} \sqrt[3]{1+x} \right) \text{Subst} \left(\int \frac{1}{\left(\frac{16}{9} - \frac{8x}{3}\right)^{2/3} \sqrt[3]{\frac{16}{9} + \frac{4x}{3}}} dx, x, -\frac{1}{3} + x \right)}{3 \sqrt[3]{1-x-x^2+x^3}}$$

$$= -\frac{\sqrt{3} (1-x)^{2/3} \sqrt[3]{1+x} \tan^{-1} \left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{1+x}}{\sqrt{3} \sqrt[3]{1-x}} \right)}{\sqrt[3]{1-x-x^2+x^3}} - \frac{(1-x)^{2/3} \sqrt[3]{1+x} \log \left(\frac{1-x}{1+x} \right)}{2 \sqrt[3]{1-x-x^2+x^3}}$$

Mathematica [A]

time = 0.07, size = 110, normalized size = 1.64

$$\frac{(-1+x)^{2/3} \sqrt[3]{1+x} \left(2\sqrt{3} \tan^{-1} \left(\frac{1+2\sqrt[3]{\frac{-1+x}{1+x}}}{\sqrt{3}} \right) - 2 \log \left(-1 + \sqrt[3]{\frac{-1+x}{1+x}} \right) + \log \left(1 + \sqrt[3]{\frac{-1+x}{1+x}} + \left(\frac{-1+x}{1+x} \right)^{2/3} \right) \right)}{2 \sqrt[3]{(-1+x)^2(1+x)}}$$

Antiderivative was successfully verified.

[In] Integrate[((-1 + x)^2*(1 + x))^(-1/3), x]

[Out] ((-1 + x)^(2/3)*(1 + x)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + 2*((-1 + x)/(1 + x))^(1/3)]/Sqrt[3]) - 2*Log[-1 + ((-1 + x)/(1 + x))^(1/3)] + Log[1 + ((-1 + x)/(1 + x))^(1/3) + ((-1 + x)/(1 + x))^(2/3)])/(2*((-1 + x)^2*(1 + x))^(1/3))

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[1/((x - 1)^2*(x + 1))^(1/3),x]')`

[Out] caught exception: maximum recursion depth exceeded in comparison

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.25, size = 370, normalized size = 5.52

method	result
trager	$-\ln\left(\frac{4\operatorname{RootOf}(_Z^2-_Z+1)^2x^2+3\operatorname{RootOf}(_Z^2-_Z+1)(x^3-x^2-x+1)^{\frac{2}{3}}-3\operatorname{RootOf}(_Z^2-_Z+1)(x^3-x^2-x+1)^{\frac{1}{3}}}{\dots}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((-1+x)^2*(1+x))^(1/3),x,method=_RETURNVERBOSE)`

[Out]
$$-\ln\left(\frac{4\operatorname{RootOf}(_Z^2-_Z+1)^2x^2+3\operatorname{RootOf}(_Z^2-_Z+1)(x^3-x^2-x+1)^{\frac{2}{3}}-3\operatorname{RootOf}(_Z^2-_Z+1)(x^3-x^2-x+1)^{\frac{1}{3}}}{\dots}\right)$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-1+x)^2*(1+x))^(1/3),x, algorithm="maxima")`

[Out] `integrate(((x + 1)*(x - 1)^2)^(-1/3), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(56) = 112.

time = 0.31, size = 128, normalized size = 1.91

$$-\sqrt{3} \arctan\left(\frac{\sqrt{3}(x-1)+2\sqrt{3}(x^3-x^2-x+1)^{\frac{1}{3}}}{3(x-1)}\right) + \frac{1}{2} \log\left(\frac{x^2+(x^3-x^2-x+1)^{\frac{1}{3}}(x-1)-2x+(x^3-x^2-x+1)^{\frac{2}{3}}+1}{x^2-2x+1}\right) - \log\left(\frac{x-(x^3-x^2-x+1)^{\frac{1}{3}}-1}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-1+x)^2*(1+x))^(1/3),x, algorithm="fricas")`

[Out]
$$-\sqrt{3} \arctan\left(\frac{\sqrt{3}(x-1)+2\sqrt{3}(x^3-x^2-x+1)^{\frac{1}{3}}}{3(x-1)}\right) + \frac{1}{2} \log\left(\frac{x^2+(x^3-x^2-x+1)^{\frac{1}{3}}(x-1)-2x+(x^3-x^2-x+1)^{\frac{2}{3}}+1}{x^2-2x+1}\right) - \log\left(\frac{x-(x^3-x^2-x+1)^{\frac{1}{3}}-1}{x-1}\right)$$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(x-1)^2(x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)**2*(1+x))**(1/3),x)

[Out] Integral(((x - 1)**2*(x + 1))**(-1/3), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)^2*(1+x))^(1/3),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{((x-1)^2(x+1))^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x - 1)^2*(x + 1))^(1/3),x)

[Out] int(1/((x - 1)^2*(x + 1))^(1/3), x)

$$3.227 \quad \int \frac{\frac{1}{x} + x}{\sqrt{(-2+x)(1+x)^3}} dx$$

Optimal. Leaf size=122

$$\frac{4(-2+x)(1+x)}{3\sqrt{(-2+x)(1+x)^3}} + \frac{2\sqrt{-2+x}(1+x)^{3/2} \sinh^{-1}\left(\frac{\sqrt{-2+x}}{\sqrt{3}}\right)}{\sqrt{(-2+x)(1+x)^3}} - \frac{\sqrt{2}\sqrt{-2+x}(1+x)^{3/2} \tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{-2+x}}\right)}{\sqrt{(-2+x)(1+x)^3}}$$

[Out] $-4/3*(-2+x)*(1+x)/((-2+x)*(1+x)^3)^{(1/2)}+2*(1+x)^{(3/2)*\operatorname{arcsinh}(1/3*(-2+x)^{(1/2)*3^{(1/2)}}*(-2+x)^{(1/2)/((-2+x)*(1+x)^3)^{(1/2)}-(1+x)^{(3/2)*\operatorname{arctan}(2^{(1/2)}*(1+x)^{(1/2)/(-2+x)^{(1/2)})*2^{(1/2)}*(-2+x)^{(1/2)/((-2+x)*(1+x)^3)^{(1/2)}}$

Rubi [A]

time = 0.21, antiderivative size = 133, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1607, 6851, 1628, 21, 132, 56, 221, 95, 210}

$$\frac{4(2-x)(x+1)}{3\sqrt{-((2-x)(x+1)^3)}} - \frac{\sqrt{2}\sqrt{x-2}(x+1)^{3/2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{x+1}}{\sqrt{x-2}}\right)}{\sqrt{-((2-x)(x+1)^3)}} + \frac{2\sqrt{x-2}(x+1)^{3/2} \sinh^{-1}\left(\frac{\sqrt{x-2}}{\sqrt{3}}\right)}{\sqrt{-((2-x)(x+1)^3)}}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1) + x)/Sqrt[(-2 + x)*(1 + x)^3], x]

[Out] $(4*(2-x)*(1+x))/(3*\operatorname{Sqrt}[-((2-x)*(1+x)^3)]) + (2*\operatorname{Sqrt}[-2+x]*(1+x)^{(3/2)*\operatorname{ArcSinh}[\operatorname{Sqrt}[-2+x]/\operatorname{Sqrt}[3]]})/\operatorname{Sqrt}[-((2-x)*(1+x)^3)] - (\operatorname{Sqrt}[2]*\operatorname{Sqrt}[-2+x]*(1+x)^{(3/2)*\operatorname{ArcTan}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[1+x])/\operatorname{Sqrt}[-2+x]]})/\operatorname{Sqrt}[-((2-x)*(1+x)^3)]$

Rule 21

Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] := Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])

Rule 56

Int[1/(Sqrt[(a_) + (b_)*(x_)]*Sqrt[(c_) + (d_)*(x_)]), x_Symbol] := Dist[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]

Rule 95

Int[(((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_))/((e_) + (f_)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)

```
- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x)^(m - 1), x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m, 0] || SumSimplerQ[m, -1] || !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^p, x_Symbol] := Dist[a^IntPart[p]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
```

$(m*p)*w^{(n*p)}, x], x] /; \text{FreeQ}\{a, m, n, p\}, x] \&\& \text{IntegerQ}[p] \&\& \text{FreeQ}[v, x] \&\& \text{FreeQ}[w, x]$

Rubi steps

$$\begin{aligned}
 \int \frac{\frac{1}{x} + x}{\sqrt{(-2+x)(1+x)^3}} dx &= \int \frac{1+x^2}{x\sqrt{(-2+x)(1+x)^3}} dx \\
 &= \frac{(\sqrt{-2+x}(1+x)^{3/2}) \int \frac{1+x^2}{\sqrt{-2+x} x(1+x)^{3/2}} dx}{\sqrt{(-2+x)(1+x)^3}} \\
 &= \frac{4(2-x)(1+x)}{3\sqrt{-(2-x)(1+x)^3}} - \frac{(2\sqrt{-2+x}(1+x)^{3/2}) \int \frac{-\frac{3}{2}-\frac{3x}{2}}{\sqrt{-2+x} x\sqrt{1+x}} dx}{3\sqrt{(-2+x)(1+x)^3}} \\
 &= \frac{4(2-x)(1+x)}{3\sqrt{-(2-x)(1+x)^3}} + \frac{(\sqrt{-2+x}(1+x)^{3/2}) \int \frac{\sqrt{1+x}}{\sqrt{-2+x} x} dx}{\sqrt{(-2+x)(1+x)^3}} \\
 &= \frac{4(2-x)(1+x)}{3\sqrt{-(2-x)(1+x)^3}} + \frac{(\sqrt{-2+x}(1+x)^{3/2}) \int \frac{1}{\sqrt{-2+x} \sqrt{1+x}} dx}{\sqrt{(-2+x)(1+x)^3}} + \dots \\
 &= \frac{4(2-x)(1+x)}{3\sqrt{-(2-x)(1+x)^3}} + \frac{(2\sqrt{-2+x}(1+x)^{3/2}) \text{Subst}\left(\int \frac{1}{-1-2x^2} dx, x, \frac{\sqrt{1+x}}{\sqrt{-2+x}}\right)}{\sqrt{(-2+x)(1+x)^3}} \\
 &= \frac{4(2-x)(1+x)}{3\sqrt{-(2-x)(1+x)^3}} + \frac{2\sqrt{-2+x}(1+x)^{3/2} \sinh^{-1}\left(\frac{\sqrt{-2+x}}{\sqrt{3}}\right)}{\sqrt{-(2-x)(1+x)^3}} - \frac{\sqrt{2}}{\sqrt{3}} \dots
 \end{aligned}$$

Mathematica [A]

time = 0.07, size = 96, normalized size = 0.79

$$\frac{(1+x) \left(-8 + 4x - 3\sqrt{2} \sqrt{-2+x} \sqrt{1+x} \tan^{-1} \left(\frac{\sqrt{-2+x}}{\sqrt{2}} \right) - 6\sqrt{-2+x} \sqrt{1+x} \tanh^{-1} \left(\sqrt{\frac{-2+x}{1+x}} \right) \right)}{3\sqrt{(-2+x)(1+x)^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1) + x)/Sqrt[(-2 + x)*(1 + x)^3], x]

[Out] -1/3*((1 + x)*(-8 + 4*x - 3*Sqrt[2]*Sqrt[-2 + x]*Sqrt[1 + x]*ArcTan[Sqrt[(-2 + x)/(1 + x)]/Sqrt[2]] - 6*Sqrt[-2 + x]*Sqrt[1 + x]*ArcTanh[Sqrt[(-2 + x)/(1 + x)]]))/Sqrt[(-2 + x)*(1 + x)^3]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(x + 1/x)/Sqrt[(x + 1)^3*(x - 2)],x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.12, size = 118, normalized size = 0.97

method	result
risch	$-\frac{4(-2+x)(1+x)}{3\sqrt{(-2+x)(1+x)^3}} + \frac{\left(\ln\left(x-\frac{1}{2}+\sqrt{x^2-x-2}\right) + \frac{\sqrt{2} \arctan\left(\frac{(-4-x)\sqrt{2}}{4\sqrt{x^2-x-2}}\right)}{2} \right) (1+x) \sqrt{(1+x)(-2+x)}}{\sqrt{(-2+x)(1+x)^3}}$
default	$\frac{\left(-3\sqrt{2} \arctan\left(\frac{(4+x)\sqrt{2}}{4\sqrt{x^2-x-2}}\right) x + 6 \ln\left(x-\frac{1}{2}+\sqrt{x^2-x-2}\right) x - 3\sqrt{2} \arctan\left(\frac{(4+x)\sqrt{2}}{4\sqrt{x^2-x-2}}\right) + 6 \ln\left(x-\frac{1}{2}+\sqrt{x^2-x-2}\right)\right) \sqrt{(-2+x)(1+x)^3}}{6\sqrt{(-2+x)(1+x)^3}}$
trager	$-\frac{4\sqrt{x^4+x^3-3x^2-5x-2}}{3(1+x)^2} + \frac{\text{RootOf}(-Z^2+2) \ln\left(\frac{\text{RootOf}(-Z^2+2)x^2+5\text{RootOf}(-Z^2+2)x+4\sqrt{x^4+x^3-3x^2-5x-2}}{x(1+x)}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x,method=_RETURNVERBOSE)`

```
[Out] 1/6*(-3*2^(1/2)*arctan(1/4*(4+x)*2^(1/2)/(x^2-x-2)^(1/2))*x+6*ln(x-1/2+(x^2-x-2)^(1/2))*x-3*2^(1/2)*arctan(1/4*(4+x)*2^(1/2)/(x^2-x-2)^(1/2))+6*ln(x-1/2+(x^2-x-2)^(1/2))-8*(x^2-x-2)^(1/2)*((1+x)*(-2+x))^(1/2)/((-2+x)*(1+x)^3)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x, algorithm="maxima")``[Out] integrate((x + 1/x)/sqrt((x + 1)^3*(x - 2)), x)`

Fricas [A]

time = 0.33, size = 142, normalized size = 1.16

$$\frac{3\sqrt{2}(x^2+2x+1)\arctan\left(\frac{-\sqrt{2}(x^2+x)-\sqrt{2}\sqrt{x^4+x^3-3x^2-5x-2}}{2(x+1)}\right)-4x^2-3(x^2+2x+1)\log\left(\frac{-2x^2+x-2\sqrt{x^4+x^3-3x^2-5x-2}-1}{x+1}\right)-8x-4\sqrt{x^4+x^3-3x^2-5x-2}-4}{3(x^2+2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x, algorithm="fricas")

[Out] 1/3*(3*sqrt(2)*(x^2 + 2*x + 1)*arctan(-1/2*(sqrt(2)*(x^2 + x) - sqrt(2)*sqrt(x^4 + x^3 - 3*x^2 - 5*x - 2))/(x + 1)) - 4*x^2 - 3*(x^2 + 2*x + 1)*log(-(2*x^2 + x - 2*sqrt(x^4 + x^3 - 3*x^2 - 5*x - 2) - 1)/(x + 1)) - 8*x - 4*sqrt(x^4 + x^3 - 3*x^2 - 5*x - 2) - 4)/(x^2 + 2*x + 1)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 1}{x\sqrt{(x-2)(x+1)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/x+x)/((-2+x)*(1+x)**3)**(1/2),x)**[Out]** Integral((x**2 + 1)/(x*sqrt((x - 2)*(x + 1)**3)), x)**Giac [A]**

time = 0.03, size = 89, normalized size = 0.73

$$2\left(\frac{\arctan\left(\frac{-x+\sqrt{x^2-x-2}}{\sqrt{2}}\right)}{\sqrt{2}\operatorname{sign}(x+1)} - \frac{\ln\left|2\left(-x+\sqrt{x^2-x-2}\right)+1\right|}{2\operatorname{sign}(x+1)} + \frac{2}{\left(-x+\sqrt{x^2-x-2}-1\right)\operatorname{sign}(x+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x)

[Out] sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 - x - 2)))/sgn(x + 1) - log(abs(-2*x + 2*sqrt(x^2 - x - 2) + 1))/sgn(x + 1) - 4/((x - sqrt(x^2 - x - 2) + 1)*sgn(x + 1))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x + \frac{1}{x}}{\sqrt{(x+1)^3(x-2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 1/x)/((x + 1)^3*(x - 2))^(1/2),x)**[Out]** int((x + 1/x)/((x + 1)^3*(x - 2))^(1/2), x)

$$3.228 \quad \int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2} dx$$

Optimal. Leaf size=150

$$-\frac{\sqrt[3]{(-1+x)^2(1+x)}}{x} - \frac{\tan^{-1}\left(\frac{1 - \frac{2(-1+x)}{\sqrt[3]{(-1+x)^2(1+x)}}}{\sqrt{3}}\right)}{\sqrt{3}} - \sqrt{3} \tan^{-1}\left(\frac{1 + \frac{2(-1+x)}{\sqrt[3]{(-1+x)^2(1+x)}}}{\sqrt{3}}\right) + \frac{\log(x)}{6}$$

[Out] $-\left((-1+x)^2(1+x)\right)^{1/3}/x+1/6*\ln(x)-2/3*\ln(1+x)-3/2*\ln(1+(-1+x)/((-1+x)^2(1+x))^{1/3})-1/2*\ln(1+(-1+x)/((-1+x)^2(1+x))^{1/3})-1/3*\arctan(1/3*(1-2*(-1+x)/((-1+x)^2(1+x))^{1/3})*3^{1/2})*3^{1/2}-\arctan(1/3*(1+2*(-1+x)/((-1+x)^2(1+x))^{1/3})*3^{1/2})*3^{1/2}$

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 404 vs. $2(150) = 300$. time = 0.21, antiderivative size = 404, normalized size of antiderivative = 2.69, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2106, 2102, 99, 163, 62, 93}

$$\frac{\sqrt[3]{x^2-x+1}}{x} + \frac{\sqrt[3]{x^2-x+1} \log(x)}{2\sqrt{3}(3-3x)^{3/2}\sqrt{x+1}} - \frac{3^{3/2}\sqrt[3]{x^2-x+1} \log\left(\frac{4x+1}{3}\right)}{2(3-3x)^{3/2}\sqrt{x+1}} - \frac{3^{3/2}\sqrt[3]{x^2-x+1} \log\left(\frac{\sqrt{3}-3x}{\sqrt{3}\sqrt{x+1}}+1\right)}{2(3-3x)^{3/2}\sqrt{x+1}} - \frac{3^{3/2}\sqrt[3]{x^2-x+1} \log\left(\frac{3}{2}\sqrt[3]{\sqrt{3}-3x} - \frac{3\sqrt[3]{x+1}}{\sqrt{3}}\right)}{2(3-3x)^{3/2}\sqrt{x+1}} - \frac{3\sqrt{3}\sqrt[3]{x^2-x+1} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt{3}-3x}{3\sqrt{x+1}}\right)}{(3-3x)^{3/2}\sqrt{x+1}} - \frac{\sqrt{3}\sqrt[3]{x^2-x+1} \tan^{-1}\left(\frac{2\sqrt{3}-3x}{3\sqrt{x+1}} + \frac{1}{\sqrt{3}}\right)}{(3-3x)^{3/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

[In] Int[$((-1+x)^2(1+x))^{1/3}/x^2, x]$

[Out] $-\left((1-x-x^2+x^3)^{1/3}/x - (3*3^{1/6}*(1-x-x^2+x^3)^{1/3}*\text{ArcTan}[1/\text{Sqrt}[3] - (2*(3-3*x)^{1/3})/(3^{5/6}*(1+x)^{1/3})])/(3-3*x)^{2/3}*(1+x)^{1/3} - (3^{1/6}*(1-x-x^2+x^3)^{1/3}*\text{ArcTan}[1/\text{Sqrt}[3] + (2*(3-3*x)^{1/3})/(3^{5/6}*(1+x)^{1/3})])/(3-3*x)^{2/3}*(1+x)^{1/3} + ((1-x-x^2+x^3)^{1/3}*\text{Log}[x])/(2*3^{1/3}*(3-3*x)^{2/3}*(1+x)^{1/3}) - (3^{2/3}*(1-x-x^2+x^3)^{1/3}*\text{Log}[(4*(1+x))/3])/(2*(3-3*x)^{2/3}*(1+x)^{1/3}) - (3*3^{2/3}*(1-x-x^2+x^3)^{1/3}*\text{Log}[1+(3-3*x)^{1/3}/(3^{1/3}*(1+x)^{1/3})])/(2*(3-3*x)^{2/3}*(1+x)^{1/3}) - (3^{2/3}*(1-x-x^2+x^3)^{1/3}*\text{Log}[(2/3)^{2/3}*(3-3*x)^{1/3} - (2^{2/3}*(1+x)^{1/3})/3^{1/3}])/(2*(3-3*x)^{2/3}*(1+x)^{1/3})$

Rule 62

Int[$1/(((a_.) + (b_.)*(x_))^{1/3}*((c_.) + (d_.)*(x_))^{2/3}), x_Symbol] :>$
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*x)^{1/3}/(Sqrt[3]*(c + d*x)^{1/3}))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*x)^{1/3}/(c + d*x)^{1/3}) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]

Rule 93

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] := With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqr
t[3])*q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))
]/(d*e - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*
(Log[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x]]) /; FreeQ[{
a, b, c, d, e, f}, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p/(b*(
m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
))))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*(e + f*x)^p/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 2102

```
Int[((e_.) + (f_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] := Dist[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int
[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d, e
, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] && !IntegerQ[p]
```

Rule 2106

```
Int[(P3_)^(p_)*((e_.) + (f_.)*(x_))^(m_), x_Symbol] := With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2} dx &= \text{Subst} \left(\int \frac{\sqrt[3]{\frac{16}{27} - \frac{4x}{3} + x^3}}{\left(\frac{1}{3} + x\right)^2} dx, x, -\frac{1}{3} + x \right) \\
&= \frac{\left(3\sqrt[3]{1-x-x^2+x^3}\right) \text{Subst} \left(\int \frac{\left(\frac{16}{9} - \frac{8x}{3}\right)^{2/3} \sqrt[3]{\frac{16}{9} + \frac{4x}{3}}}{\left(\frac{1}{3} + x\right)^2} dx, x, -\frac{1}{3} + x \right)}{4 \cdot 2^{2/3} (1-x)^{2/3} \sqrt[3]{1+x}} \\
&= -\frac{\sqrt[3]{1-x-x^2+x^3}}{x} + \frac{\left(3\sqrt[3]{1-x-x^2+x^3}\right) \text{Subst} \left(\int \frac{-\frac{64}{27} - \frac{32x}{9}}{\sqrt[3]{\frac{16}{9} - \frac{8x}{3}} \left(\frac{1}{3} + x\right) \left(\frac{16}{9} + \frac{4x}{3}\right)} dx, x, -\frac{1}{3} + x \right)}{4 \cdot 2^{2/3} (1-x)^{2/3} \sqrt[3]{1+x}} \\
&= -\frac{\sqrt[3]{1-x-x^2+x^3}}{x} - \frac{\left(4\sqrt[3]{2} \sqrt[3]{1-x-x^2+x^3}\right) \text{Subst} \left(\int \frac{1}{\sqrt[3]{\frac{16}{9} - \frac{8x}{3}} \left(\frac{1}{3} + x\right)} dx, x, -\frac{1}{3} + x \right)}{9(1-x)^{2/3} \sqrt[3]{1+x}} \\
&= -\frac{\sqrt[3]{1-x-x^2+x^3}}{x} - \frac{\sqrt[3]{3} \sqrt[3]{1-x-x^2+x^3} \tan^{-1} \left(\frac{1}{\sqrt[3]{3}} - \frac{2\sqrt[3]{1-x}}{\sqrt[3]{3} \sqrt[3]{1+x}} \right)}{(1-x)^{2/3} \sqrt[3]{1+x}}
\end{aligned}$$

Mathematica [A]

time = 0.33, size = 229, normalized size = 1.53

$$\frac{(-1+x)^{4/3}(1+x)^{2/3} \left(18(-1+x)^{2/3} \sqrt[3]{1+x} - 6\sqrt{3} x \tan^{-1} \left(\frac{\sqrt[3]{-1+x}}{\sqrt[3]{1+x}} \right) - 18\sqrt{3} x \tan^{-1} \left(\frac{\sqrt[3]{-1+x}}{\sqrt[3]{1+x}} \right) - 10x \log \left(\frac{x}{-1+x} \right) - 3x \log \left(1 + \frac{-1}{\sqrt[3]{1+x}} - \frac{\sqrt[3]{-1+x}}{\sqrt[3]{1+x}} \right) + 28x \log \left(-1 + \frac{\sqrt[3]{-1+x}}{\sqrt[3]{1+x}} \right) + 6x \log \left(1 + \frac{\sqrt[3]{-1+x}}{\sqrt[3]{1+x}} \right) + x \log \left(1 + \frac{1}{\sqrt[3]{1+x}} + \frac{\sqrt[3]{-1+x}}{\sqrt[3]{1+x}} \right) \right)}{18x(-1+x)^{2/3}(1+x)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[((-1 + x)^2*(1 + x))^(1/3)/x^2,x]

[Out] -1/18*((-1 + x)^(4/3)*(1 + x)^(2/3)*(18*(-1 + x)^(2/3)*(1 + x)^(1/3) - 6*Sqrt[3]*x*ArcTan[(1 - 2/((-1 + x)/(1 + x))^(1/3)]/Sqrt[3]] - 18*Sqrt[3]*x*ArcTan[(1 + 2/((-1 + x)/(1 + x))^(1/3)]/Sqrt[3]] - 10*x*Log[2/(-1 + x)] - 3*x*Log[1 + ((-1 + x)/(1 + x))^(-2/3) - ((-1 + x)/(1 + x))^(-1/3)] + 28*x*Log[-1 + ((-1 + x)/(1 + x))^(-1/3)] + 6*x*Log[1 + ((-1 + x)/(1 + x))^(-1/3)] + x*Log[1 + ((-1 + x)/(1 + x))^(-2/3) + ((-1 + x)/(1 + x))^(-1/3)]))/(x*(-1 + x)^2*(1 + x)^(2/3))

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[((x - 1)^2*(x + 1))^(1/3)/x^2,x']')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 2.97, size = 1247, normalized size = 8.31

method	result	size
risch	Expression too large to display	1247
trager	Expression too large to display	2055

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(((1-x)^2*(1+x))^(1/3)/x^2,x,method=_RETURNVERBOSE)`

```
[Out] -((1-x)^2*(1+x))^(1/3)/x+(-1/3*ln(-(-157880368143+288529720857*x+426276993
9861*x^5-1955796480*RootOf(_Z^2-3*_Z+9)^2*x^3-21459433600*RootOf(_Z^2-3*_Z+
9)^2*x^2-108655360*RootOf(_Z^2-3*_Z+9)^2+2933694720*RootOf(_Z^2-3*_Z+9)^2*x
^5+4262769939861*x^4-2395436537574*x^2-2841846626574*x^3+2933694720*RootOf(
_Z^2-3*_Z+9)^2*x^4-1755589628511*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)*x^
3+1412122229127*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x^4-585196542837*Ro
otOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)*x^2+941414819418*RootOf(_Z^2-3*_Z+9)*
(x^3+x^2-x-1)^(1/3)*x^3+195065514279*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3
)*x-627609879612*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x^2-104601646602*R
ootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x+334666315224*RootOf(_Z^2-3*_Z+9)*x
^5+1030402198152*(x^3+x^2-x-1)^(2/3)*x^3-5266768885533*(x^3+x^2-x-1)^(1/3)*
x^4+343467399384*(x^3+x^2-x-1)^(2/3)*x^2-3511179257022*(x^3+x^2-x-1)^(1/3)*
x^3+65021838093*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)-114489133128*(x^3+x
^2-x-1)^(2/3)*x+2340786171348*(x^3+x^2-x-1)^(1/3)*x^2+52300823301*RootOf(_Z
^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)+390131028558*(x^3+x^2-x-1)^(1/3)*x-223110876
816*RootOf(_Z^2-3*_Z+9)*x^3-477460395840*RootOf(_Z^2-3*_Z+9)*x^2-2667445677
36*RootOf(_Z^2-3*_Z+9)*x+334666315224*RootOf(_Z^2-3*_Z+9)*x^4-12395048712*R
ootOf(_Z^2-3*_Z+9)-195065514279*(x^3+x^2-x-1)^(1/3)-38163044376*(x^3+x^2-x-
1)^(2/3)-19612292480*RootOf(_Z^2-3*_Z+9)^2*x)/x/(1+x))+1/9*RootOf(_Z^2-3*_Z
+9)*ln(- (33401336760+117256110840*x-901836092520*x^5-9523415232*RootOf(_Z^2
-3*_Z+9)^2*x^3-104493028240*RootOf(_Z^2-3*_Z+9)^2*x^2-529078624*RootOf(_Z^2
-3*_Z+9)^2+14285122848*RootOf(_Z^2-3*_Z+9)^2*x^5-901836092520*x^4+685078835
760*x^2+601224061680*x^3+14285122848*RootOf(_Z^2-3*_Z+9)^2*x^4+175558962851
1*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)*x^3-343467399384*RootOf(_Z^2-3*_Z
```

+9)*(x^3+x^2-x-1)^(1/3)*x^4+585196542837*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)*x^2-228978266256*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x^3-195065514279*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)*x+152652177504*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x^2+25442029584*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x-1454977597671*RootOf(_Z^2-3*_Z+9)*x^5-4236366687381*(x^3+x^2-x-1)^(2/3)*x^3+5266768885533*(x^3+x^2-x-1)^(1/3)*x^4-1412122229127*(x^3+x^2-x-1)^(2/3)*x^2+3511179257022*(x^3+x^2-x-1)^(1/3)*x^3-65021838093*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)+470707409709*(x^3+x^2-x-1)^(2/3)*x-2340786171348*(x^3+x^2-x-1)^(1/3)*x^2-12721014792*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)-390131028558*(x^3+x^2-x-1)^(1/3)*x+969985065114*RootOf(_Z^2-3*_Z+9)*x^3+1047579629778*RootOf(_Z^2-3*_Z+9)*x^2+131482623837*RootOf(_Z^2-3*_Z+9)*x-1454977597671*RootOf(_Z^2-3*_Z+9)*x^4+53888059173*RootOf(_Z^2-3*_Z+9)+195065514279*(x^3+x^2-x-1)^(1/3)+156902469903*(x^3+x^2-x-1)^(2/3)-95498691632*RootOf(_Z^2-3*_Z+9)^2*x)/x/(1+x)))*((-1+x)^2*(1+x))^(1/3)*((-1+x)*(1+x)^2)^(1/3)/(-1+x)/(1+x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((−1+x)^2*(1+x))^(1/3)/x^2,x, algorithm="maxima")

[Out] integrate(((x + 1)*(x - 1)^2)^(1/3)/x^2, x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(126) = 252.

time = 0.34, size = 280, normalized size = 1.87

$$\frac{6\sqrt{3}x \arctan\left(\frac{\sqrt{3}(x-1)+\sqrt{3}(x^2-x+1)^{\frac{1}{3}}}{3(x-1)}\right) - 2\sqrt{3}x \arctan\left(\frac{-\sqrt{3}(x-1)-2\sqrt{3}(x^2-x+1)^{\frac{1}{3}}}{3(x-1)}\right) + 3x \log\left(\frac{x^2+(x^2-x+1)^{\frac{1}{3}}(x-1)-2x+(x^2-x+1)^{\frac{2}{3}}}{x^2-2x+1}\right) + x \log\left(\frac{x^2-(x^2-x+1)^{\frac{1}{3}}(x-1)-2x+(x^2-x+1)^{\frac{2}{3}}}{x^2-2x+1}\right) - 2x \log\left(\frac{x+(x^2-x+1)^{\frac{1}{3}}}{x-1}\right) - 6x \log\left(\frac{-x-(x^2-x+1)^{\frac{1}{3}}}{x-1}\right) - 6(x^3-x^2-x+1)^{\frac{1}{3}}}{6x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((−1+x)^2*(1+x))^(1/3)/x^2,x, algorithm="fricas")

[Out] 1/6*(6*sqrt(3)*x*arctan(1/3*(sqrt(3)*(x - 1) + 2*sqrt(3)*(x^3 - x^2 - x + 1)^(1/3)))/(x - 1)) - 2*sqrt(3)*x*arctan(-1/3*(sqrt(3)*(x - 1) - 2*sqrt(3)*(x^3 - x^2 - x + 1)^(1/3)))/(x - 1)) + 3*x*log((x^2 + (x^3 - x^2 - x + 1)^(1/3))*(x - 1) - 2*x + (x^3 - x^2 - x + 1)^(2/3) + 1)/(x^2 - 2*x + 1)) + x*log((x^2 - (x^3 - x^2 - x + 1)^(1/3))*(x - 1) - 2*x + (x^3 - x^2 - x + 1)^(2/3) + 1)/(x^2 - 2*x + 1)) - 2*x*log((x + (x^3 - x^2 - x + 1)^(1/3) - 1)/(x - 1)) - 6*x*log(-(x - (x^3 - x^2 - x + 1)^(1/3) - 1)/(x - 1)) - 6*(x^3 - x^2 - x + 1)^(1/3))/x

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{(x-1)^2(x+1)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((−1+x)**2*(1+x))**(1/3)/x**2,x)

[Out] Integral(((x - 1)**2*(x + 1))**(1/3)/x**2, x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((−1+x)^2*(1+x))^(1/3)/x^2,x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{((x - 1)^2 (x + 1))^{1/3}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x - 1)^2*(x + 1))^(1/3)/x^2,x)

[Out] int(((x - 1)^2*(x + 1))^(1/3)/x^2, x)

$$3.229 \quad \int \frac{1}{(-3-2x+x^2)^{5/2}} dx$$

Optimal. Leaf size=43

$$\frac{1-x}{12(-3-2x+x^2)^{3/2}} - \frac{1-x}{24\sqrt{-3-2x+x^2}}$$

[Out] 1/12*(1-x)/(x^2-2*x-3)^(3/2)+1/24*(-1+x)/(x^2-2*x-3)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {628, 627}

$$\frac{1-x}{12(x^2-2x-3)^{3/2}} - \frac{1-x}{24\sqrt{x^2-2x-3}}$$

Antiderivative was successfully verified.

[In] Int[(-3 - 2*x + x^2)^(-5/2), x]

[Out] (1 - x)/(12*(-3 - 2*x + x^2)^(3/2)) - (1 - x)/(24*sqrt[-3 - 2*x + x^2])

Rule 627

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] := Simp[-2*((b + 2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p + 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rubi steps

$$\begin{aligned} \int \frac{1}{(-3-2x+x^2)^{5/2}} dx &= \frac{1-x}{12(-3-2x+x^2)^{3/2}} - \frac{1}{6} \int \frac{1}{(-3-2x+x^2)^{3/2}} dx \\ &= \frac{1-x}{12(-3-2x+x^2)^{3/2}} - \frac{1-x}{24\sqrt{-3-2x+x^2}} \end{aligned}$$

Mathematica [A]

time = 0.15, size = 39, normalized size = 0.91

$$\frac{\sqrt{-3 - 2x + x^2} (5 - 3x - 3x^2 + x^3)}{24(-3 + x)^2(1 + x)^2}$$

Antiderivative was successfully verified.

`[In] Integrate[(-3 - 2*x + x^2)^(-5/2), x]``[Out] (Sqrt[-3 - 2*x + x^2]*(5 - 3*x - 3*x^2 + x^3))/(24*(-3 + x)^2*(1 + x)^2)`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x^2 - 2*x - 3)^(5/2), x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.08, size = 36, normalized size = 0.84

method	result	size
trager	$\frac{x^3 - 3x^2 - 3x + 5}{24(x^2 - 2x - 3)^{\frac{3}{2}}}$	26
risch	$\frac{x^3 - 3x^2 - 3x + 5}{24(x^2 - 2x - 3)^{\frac{3}{2}}}$	26
gospers	$\frac{(1+x)(-3+x)(x^3 - 3x^2 - 3x + 5)}{24(x^2 - 2x - 3)^{\frac{5}{2}}}$	32
default	$-\frac{2x-2}{24(x^2-2x-3)^{\frac{3}{2}}} + \frac{2x-2}{48\sqrt{x^2-2x-3}}$	36

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(x^2-2*x-3)^(5/2), x, method=_RETURNVERBOSE)``[Out] -1/24*(2*x-2)/(x^2-2*x-3)^(3/2)+1/48*(2*x-2)/(x^2-2*x-3)^(1/2)`**Maxima [A]**

time = 0.25, size = 51, normalized size = 1.19

$$\frac{x}{24\sqrt{x^2 - 2x - 3}} - \frac{1}{24\sqrt{x^2 - 2x - 3}} - \frac{x}{12(x^2 - 2x - 3)^{\frac{3}{2}}} + \frac{1}{12(x^2 - 2x - 3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-2*x-3)^(5/2),x, algorithm="maxima")

[Out] 1/24*x/sqrt(x^2 - 2*x - 3) - 1/24/sqrt(x^2 - 2*x - 3) - 1/12*x/(x^2 - 2*x - 3)^(3/2) + 1/12/(x^2 - 2*x - 3)^(3/2)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(31) = 62.

time = 0.31, size = 64, normalized size = 1.49

$$\frac{x^4 - 4x^3 - 2x^2 + (x^3 - 3x^2 - 3x + 5)\sqrt{x^2 - 2x - 3} + 12x + 9}{24(x^4 - 4x^3 - 2x^2 + 12x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-2*x-3)^(5/2),x, algorithm="fricas")

[Out] 1/24*(x^4 - 4*x^3 - 2*x^2 + (x^3 - 3*x^2 - 3*x + 5)*sqrt(x^2 - 2*x - 3) + 12*x + 9)/(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2 - 2x - 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2-2*x-3)**(5/2),x)

[Out] Integral((x**2 - 2*x - 3)**(-5/2), x)

Giac [A]

time = 0.00, size = 48, normalized size = 1.12

$$\frac{2\left(\left(\frac{x}{48} - \frac{1}{16}\right)x - \frac{1}{16}\right)x + \frac{5}{48}\sqrt{x^2 - 2x - 3}}{(x^2 - 2x - 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-2*x-3)^(5/2),x)

[Out] 1/24*((x - 3)*x - 3)*x + 5)/(x^2 - 2*x - 3)^(3/2)

Mupad [B]

time = 0.05, size = 27, normalized size = 0.63

$$\frac{(4x - 4)(-8x^2 + 16x + 40)}{768(x^2 - 2x - 3)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2 - 2*x - 3)^(5/2),x)

[Out] -((4*x - 4)*(16*x - 8*x^2 + 40))/(768*(x^2 - 2*x - 3)^(3/2))

$$3.230 \quad \int \frac{1}{\sqrt{9 + 3x - 5x^2 + x^3}} dx$$

Optimal. Leaf size=42

$$\frac{(3-x)\sqrt{1+x} \tanh^{-1}\left(\frac{\sqrt{1+x}}{2}\right)}{\sqrt{9+3x-5x^2+x^3}}$$

[Out] (3-x)*arctanh(1/2*(1+x)^(1/2))*(1+x)^(1/2)/(x^3-5*x^2+3*x+9)^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2092, 2089, 65, 212}

$$\frac{(3-x)\sqrt{x+1} \tanh^{-1}\left(\frac{\sqrt{x+1}}{2}\right)}{\sqrt{x^3-5x^2+3x+9}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[9 + 3*x - 5*x^2 + x^3],x]

[Out] ((3 - x)*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/Sqrt[9 + 3*x - 5*x^2 + x^3]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 212

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2089

Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] := Dist[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] && !IntegerQ[p]

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] := With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{9 + 3x - 5x^2 + x^3}} dx &= \text{Subst} \left(\int \frac{1}{\sqrt{\frac{128}{27} - \frac{16x}{3} + x^3}} dx, x, -\frac{5}{3} + x \right) \\ &= \frac{\left(128(3-x)\sqrt{1+x}\right) \text{Subst} \left(\int \frac{1}{\sqrt{\frac{128}{9} + \frac{16x}{3}}} dx, x, -\frac{5}{3} + x \right)}{3\sqrt{3} \sqrt{9 + 3x - 5x^2 + x^3}} \\ &= \frac{\left(16(3-x)\sqrt{1+x}\right) \text{Subst} \left(\int \frac{1}{\frac{128}{3} - 2x^2} dx, x, \frac{4\sqrt{1+x}}{\sqrt{3}} \right)}{\sqrt{3} \sqrt{9 + 3x - 5x^2 + x^3}} \\ &= \frac{(3-x)\sqrt{1+x} \tanh^{-1} \left(\frac{\sqrt{1+x}}{2} \right)}{\sqrt{9 + 3x - 5x^2 + x^3}} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 37, normalized size = 0.88

$$\frac{(-3+x)\sqrt{1+x} \tanh^{-1} \left(\frac{\sqrt{1+x}}{2} \right)}{\sqrt{(-3+x)^2(1+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 + 3*x - 5*x^2 + x^3], x]
```

```
[Out] -((( -3 + x)*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/Sqrt[(-3 + x)^2*(1 + x)]
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[1/Sqrt[x^3 - 5*x^2 + 3*x + 9],x]')`

[Out] caught exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.05, size = 45, normalized size = 1.07

method	result	size
trager	$-\frac{\ln\left(\frac{x^2+4\sqrt{x^3-5x^2+3x+9}+2x-15}{(-3+x)^2}\right)}{2}$	35
default	$\frac{(-3+x)\sqrt{1+x}\left(\ln(\sqrt{1+x}-2)-\ln(\sqrt{1+x}+2)\right)}{2\sqrt{x^3-5x^2+3x+9}}$	45

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3-5*x^2+3*x+9)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{2}*(-3+x)*(1+x)^{(1/2)}*(\ln((1+x)^{(1/2)}-2)-\ln((1+x)^{(1/2)}+2))/(x^3-5*x^2+3*x+9)^{(1/2)}$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^3-5*x^2+3*x+9)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/sqrt(x^3 - 5*x^2 + 3*x + 9), x)`

Fricas [A]

time = 0.31, size = 62, normalized size = 1.48

$$-\frac{1}{2} \log\left(\frac{2x + \sqrt{x^3 - 5x^2 + 3x + 9} - 6}{x - 3}\right) + \frac{1}{2} \log\left(-\frac{2x - \sqrt{x^3 - 5x^2 + 3x + 9} - 6}{x - 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^3-5*x^2+3*x+9)^(1/2),x, algorithm="fricas")`

[Out] $-\frac{1}{2}*\log((2*x + \text{sqrt}(x^3 - 5*x^2 + 3*x + 9) - 6)/(x - 3)) + \frac{1}{2}*\log(-(2*x - \text{sqrt}(x^3 - 5*x^2 + 3*x + 9) - 6)/(x - 3))$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3 - 5x^2 + 3x + 9}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**3-5*x**2+3*x+9)**(1/2),x)

[Out] Integral(1/sqrt(x**3 - 5*x**2 + 3*x + 9), x)

Giac [A]

time = 0.00, size = 41, normalized size = 0.98

$$2 \left(-\frac{\ln(\sqrt{x+1} + 2)}{4\operatorname{sign}(x-3)} + \frac{\ln|\sqrt{x+1} - 2|}{4\operatorname{sign}(x-3)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/2),x)

[Out] -1/2*log(sqrt(x + 1) + 2)/sgn(x - 3) + 1/2*log(abs(sqrt(x + 1) - 2))/sgn(x - 3)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^3 - 5x^2 + 3x + 9}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x - 5*x^2 + x^3 + 9)^(1/2),x)

[Out] int(1/(3*x - 5*x^2 + x^3 + 9)^(1/2), x)

$$3.231 \quad \int \frac{1}{(9+3x-5x^2+x^3)^{3/2}} dx$$

Optimal. Leaf size=139

$$\frac{(3-x)(1+x)}{8(9+3x-5x^2+x^3)^{3/2}} + \frac{5(3-x)^2(1+x)}{64(9+3x-5x^2+x^3)^{3/2}} - \frac{15(3-x)^3(1+x)}{256(9+3x-5x^2+x^3)^{3/2}} + \frac{15(3-x)^3(1+x)^{3/2} \operatorname{arctanh}\left(\frac{1+x}{2}\right)}{512(9+3x-5x^2+x^3)^{3/2}}$$

[Out] 1/8*(3-x)*(1+x)/(x^3-5*x^2+3*x+9)^(3/2)+5/64*(3-x)^2*(1+x)/(x^3-5*x^2+3*x+9)^(3/2)-15/256*(3-x)^3*(1+x)/(x^3-5*x^2+3*x+9)^(3/2)+15/512*(3-x)^3*(1+x)^(3/2)*arctanh(1/2*(1+x)^(1/2))/(x^3-5*x^2+3*x+9)^(3/2)

Rubi [A]

time = 0.08, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2092, 2089, 44, 53, 65, 212}

$$-\frac{15(x+1)(3-x)^3}{256(x^3-5x^2+3x+9)^{3/2}} + \frac{5(x+1)(3-x)^2}{64(x^3-5x^2+3x+9)^{3/2}} + \frac{(x+1)(3-x)}{8(x^3-5x^2+3x+9)^{3/2}} + \frac{15(x+1)^{3/2}(3-x)^3 \tanh^{-1}\left(\frac{\sqrt{x+1}}{2}\right)}{512(x^3-5x^2+3x+9)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(9 + 3*x - 5*x^2 + x^3)^(-3/2), x]

[Out] ((3 - x)*(1 + x))/(8*(9 + 3*x - 5*x^2 + x^3)^(3/2)) + (5*(3 - x)^2*(1 + x))/(64*(9 + 3*x - 5*x^2 + x^3)^(3/2)) - (15*(3 - x)^3*(1 + x))/(256*(9 + 3*x - 5*x^2 + x^3)^(3/2)) + (15*(3 - x)^3*(1 + x)^(3/2)*ArcTanh[Sqrt[1 + x]/2])/(512*(9 + 3*x - 5*x^2 + x^3)^(3/2))

Rule 44

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && LtQ[n, 0]

Rule 53

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && !ntLinearQ[a, b, c, d, m, n, x]

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2089

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] := Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] := With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(9 + 3x - 5x^2 + x^3)^{3/2}} dx &= \text{Subst} \left(\int \frac{1}{\left(\frac{128}{27} - \frac{16x}{3} + x^3\right)^{3/2}} dx, x, -\frac{5}{3} + x \right) \\
&= \frac{(2097152(3 - x)^3(1 + x)^{3/2}) \text{Subst} \left(\int \frac{1}{\left(\frac{128}{9} - \frac{32x}{3}\right)^3 \left(\frac{128}{9} + \frac{16x}{3}\right)^{3/2}} dx, x, -\frac{5}{3} + x \right)}{81\sqrt{3} (9 + 3x - 5x^2 + x^3)^{3/2}} \\
&= \frac{(3 - x)(1 + x)}{8(9 + 3x - 5x^2 + x^3)^{3/2}} + \frac{(20480(3 - x)^3(1 + x)^{3/2}) \text{Subst} \left(\int \frac{1}{\left(\frac{128}{9} - \frac{32x}{3}\right)^2 \left(\frac{128}{9} + \frac{16x}{3}\right)^{3/2}} dx, x, -\frac{5}{3} + x \right)}{27\sqrt{3} (9 + 3x - 5x^2 + x^3)^{3/2}} \\
&= \frac{(3 - x)(1 + x)}{8(9 + 3x - 5x^2 + x^3)^{3/2}} + \frac{5(3 - x)^2(1 + x)}{64(9 + 3x - 5x^2 + x^3)^{3/2}} + \frac{(80(3 - x)^3(1 + x)) \text{Subst} \left(\int \frac{1}{\left(\frac{128}{9} - \frac{32x}{3}\right) \left(\frac{128}{9} + \frac{16x}{3}\right)^{3/2}} dx, x, -\frac{5}{3} + x \right)}{27\sqrt{3} (9 + 3x - 5x^2 + x^3)^{3/2}} \\
&= \frac{(3 - x)(1 + x)}{8(9 + 3x - 5x^2 + x^3)^{3/2}} + \frac{5(3 - x)^2(1 + x)}{64(9 + 3x - 5x^2 + x^3)^{3/2}} - \frac{15(3 - x)^3(1 + x)}{256(9 + 3x - 5x^2 + x^3)^{3/2}} \\
&= \frac{(3 - x)(1 + x)}{8(9 + 3x - 5x^2 + x^3)^{3/2}} + \frac{5(3 - x)^2(1 + x)}{64(9 + 3x - 5x^2 + x^3)^{3/2}} - \frac{15(3 - x)^3(1 + x)}{256(9 + 3x - 5x^2 + x^3)^{3/2}} \\
&= \frac{(3 - x)(1 + x)}{8(9 + 3x - 5x^2 + x^3)^{3/2}} + \frac{5(3 - x)^2(1 + x)}{64(9 + 3x - 5x^2 + x^3)^{3/2}} - \frac{15(3 - x)^3(1 + x)}{256(9 + 3x - 5x^2 + x^3)^{3/2}}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 58, normalized size = 0.42

$$\frac{86 - 140x + 30x^2 - 15(-3 + x)^2\sqrt{1 + x} \tanh^{-1} \left(\frac{\sqrt{1 + x}}{2} \right)}{512(-3 + x)\sqrt{(-3 + x)^2(1 + x)}}$$

Antiderivative was successfully verified.

`[In] Integrate[(9 + 3*x - 5*x^2 + x^3)^(-3/2), x]``[Out] (86 - 140*x + 30*x^2 - 15*(-3 + x)^2*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/(512*(-3 + x)*Sqrt[(-3 + x)^2*(1 + x)])`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[1/(x^3 - 5*x^2 + 3*x + 9)^(3/2),x]')`

[Out] `cought exception: maximum recursion depth exceeded`

Maple [A]

time = 0.06, size = 144, normalized size = 1.04

method	result
risch	$\frac{15x^2 - 70x + 43}{256(-3+x)\sqrt{(1+x)(-3+x)^2}} + \frac{\left(\frac{15 \ln(\sqrt{1+x}-2)}{1024} - \frac{15 \ln(\sqrt{1+x}+2)}{1024}\right) \sqrt{1+x} (-3+x)}{\sqrt{(1+x)(-3+x)^2}}$
trager	$\frac{(15x^2 - 70x + 43)\sqrt{x^3 - 5x^2 + 3x + 9}}{256(-3+x)^3(1+x)} - \frac{15 \ln\left(\frac{x^2 + 4\sqrt{x^3 - 5x^2 + 3x + 9}}{(-3+x)^2} + 2x - 15\right)}{1024}$
default	$\frac{(-3+x)^3(1+x)\left(15(1+x)^{\frac{5}{2}} \ln(\sqrt{1+x}-2) - 15(1+x)^{\frac{5}{2}} \ln(\sqrt{1+x}+2) - 120(1+x)^{\frac{3}{2}} \ln(\sqrt{1+x}-2) + 120(1+x)^{\frac{3}{2}} \ln(\sqrt{1+x}+2)\right)}{1024(x^3 - 5x^2 + 3x + 9)^{\frac{3}{2}} (\sqrt{1+x}-2)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3-5*x^2+3*x+9)^(3/2),x,method=_RETURNVERBOSE)`

[Out] `1/1024*(-3+x)^3*(1+x)*(15*(1+x)^(5/2)*ln((1+x)^(1/2)-2)-15*(1+x)^(5/2)*ln((1+x)^(1/2)+2)-120*(1+x)^(3/2)*ln((1+x)^(1/2)-2)+120*(1+x)^(3/2)*ln((1+x)^(1/2)+2)+240*ln((1+x)^(1/2)-2)*(1+x)^(1/2)-240*ln((1+x)^(1/2)+2)*(1+x)^(1/2)+60*x^2-280*x+172)/(x^3-5*x^2+3*x+9)^(3/2)/((1+x)^(1/2)-2)^2/((1+x)^(1/2)+2)^2`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x, algorithm="maxima")`

[Out] `integrate((x^3 - 5*x^2 + 3*x + 9)^(-3/2), x)`

Fricas [A]

time = 0.32, size = 138, normalized size = 0.99

$$\frac{15(x^4 - 8x^3 + 18x^2 - 27) \log\left(\frac{2x + \sqrt{x^3 - 5x^2 + 3x + 9} - 6}{x-3}\right) - 15(x^4 - 8x^3 + 18x^2 - 27) \log\left(\frac{-2x - \sqrt{x^3 - 5x^2 + 3x + 9} - 6}{x-3}\right) - 4\sqrt{x^3 - 5x^2 + 3x + 9}(15x^2 - 70x + 43)}{1024(x^4 - 8x^3 + 18x^2 - 27)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x, algorithm="fricas")

[Out] -1/1024*(15*(x^4 - 8*x^3 + 18*x^2 - 27)*log((2*x + sqrt(x^3 - 5*x^2 + 3*x + 9) - 6)/(x - 3)) - 15*(x^4 - 8*x^3 + 18*x^2 - 27)*log(-(2*x - sqrt(x^3 - 5*x^2 + 3*x + 9) - 6)/(x - 3)) - 4*sqrt(x^3 - 5*x^2 + 3*x + 9)*(15*x^2 - 70*x + 43))/(x^4 - 8*x^3 + 18*x^2 - 27)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 - 5x^2 + 3x + 9)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**3-5*x**2+3*x+9)**(3/2),x)

[Out] Integral((x**3 - 5*x**2 + 3*x + 9)**(-3/2), x)

Giac [A]

time = 0.01, size = 97, normalized size = 0.70

$$2 \left(-\frac{-7\sqrt{x+1}(x+1) + 36\sqrt{x+1}}{512((x-3)^2 \operatorname{sign}(x-3))} - \frac{15 \ln(\sqrt{x+1} + 2)}{2048 \operatorname{sign}(x-3)} + \frac{15 \ln|\sqrt{x+1} - 2|}{2048 \operatorname{sign}(x-3)} + \frac{1}{64(\sqrt{x+1} \operatorname{sign}(x-3))} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x)

[Out] -15/1024*log(sqrt(x + 1) + 2)/sgn(x - 3) + 15/1024*log(abs(sqrt(x + 1) - 2)/sgn(x - 3) + 1/32/(sqrt(x + 1)*sgn(x - 3))) + 1/256*(7*(x + 1)^(3/2) - 36*sqrt(x + 1))/((x - 3)^2*sgn(x - 3))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x^3 - 5x^2 + 3x + 9)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x - 5*x^2 + x^3 + 9)^(3/2),x)

[Out] int(1/(3*x - 5*x^2 + x^3 + 9)^(3/2), x)

$$3.232 \quad \int \frac{1}{\sqrt[3]{9 + 3x - 5x^2 + x^3}} dx$$

Optimal. Leaf size=75

$$\sqrt{3} \tan^{-1} \left(\frac{1 + \frac{2(-3+x)}{\sqrt[3]{9 + 3x - 5x^2 + x^3}}}{\sqrt{3}} \right) - \frac{1}{2} \log(1+x) - \frac{3}{2} \log \left(1 - \frac{-3+x}{\sqrt[3]{9 + 3x - 5x^2 + x^3}} \right)$$

[Out] $-1/2*\ln(1+x)-3/2*\ln(1+(3-x)/(x^3-5*x^2+3*x+9)^{(1/3)})+\arctan(1/3*(1+2*(-3+x)/(x^3-5*x^2+3*x+9)^{(1/3}))*3^{(1/2)})*3^{(1/2)}$

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 188 vs. $2(75) = 150$.
time = 0.08, antiderivative size = 188, normalized size of antiderivative = 2.51, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$,
Rules used = {2092, 2089, 62}

$$-\frac{(9-3x)^{2/3}\sqrt[3]{x+1} \log\left(-\frac{32}{3}(x-3)\right)}{2 \cdot 3^{2/3}\sqrt[3]{x^3-5x^2+3x+9}} - \frac{\sqrt[3]{3}(9-3x)^{2/3}\sqrt[3]{x+1} \log\left(\frac{\sqrt[3]{3}\sqrt[3]{x+1}}{\sqrt[3]{9-3x}}+1\right)}{2\sqrt[3]{x^3-5x^2+3x+9}} - \frac{(9-3x)^{2/3}\sqrt[3]{x+1} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{x+1}}{\sqrt[3]{3}\sqrt[3]{9-3x}}\right)}{\sqrt[3]{3}\sqrt[3]{x^3-5x^2+3x+9}}$$

Antiderivative was successfully verified.

[In] Int[(9 + 3*x - 5*x^2 + x^3)^(-1/3), x]

[Out] $-(((9 - 3*x)^{(2/3)}*(1 + x)^{(1/3)}*ArcTan[1/Sqrt[3] - (2*(1 + x)^{(1/3)})/(3^{(1/6)}*(9 - 3*x)^{(1/3)})])/(3^{(1/6)}*(9 + 3*x - 5*x^2 + x^3)^{(1/3)}) - ((9 - 3*x)^{(2/3)}*(1 + x)^{(1/3)}*Log[(-32*(-3 + x))/3])/(2*3^{(2/3)}*(9 + 3*x - 5*x^2 + x^3)^{(1/3)}) - (3^{(1/3)}*(9 - 3*x)^{(2/3)}*(1 + x)^{(1/3)}*Log[1 + (3^{(1/3)}*(1 + x)^{(1/3)})/(9 - 3*x)^{(1/3)})])/(2*(9 + 3*x - 5*x^2 + x^3)^{(1/3)})$

Rule 62

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :=
  With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 2089

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] := Dist[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] && IntegerQ[p]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] := With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c
```

$d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] \&\& PolyQ[P3, x, 3]$

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}} dx &= \text{Subst} \left(\int \frac{1}{\sqrt[3]{\frac{128}{27} - \frac{16x}{3} + x^3}} dx, x, -\frac{5}{3} + x \right) \\ &= \frac{\left(16 \cdot 2^{2/3} (3-x)^{2/3} \sqrt[3]{1+x}\right) \text{Subst} \left(\int \frac{1}{\left(\frac{128}{9} - \frac{32x}{3}\right)^{2/3} \sqrt[3]{\frac{128}{9} + \frac{16x}{3}}} dx, x, -\frac{5}{3} + x \right)}{3\sqrt[3]{9+3x-5x^2+x^3}} \\ &= -\frac{\sqrt{3} (3-x)^{2/3} \sqrt[3]{1+x} \tan^{-1} \left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{1+x}}{\sqrt{3}\sqrt[3]{3-x}} \right)}{\sqrt[3]{9+3x-5x^2+x^3}} - \frac{(3-x)^{2/3} \sqrt[3]{1+x}}{2\sqrt[3]{9+3x-5x^2+x^3}} \end{aligned}$$

Mathematica [A]

time = 0.07, size = 110, normalized size = 1.47

$$\frac{(-3+x)^{2/3} \sqrt[3]{1+x} \left(2\sqrt{3} \tan^{-1} \left(\frac{1+2\sqrt[3]{\frac{-3+x}{1+x}}}{\sqrt{3}} \right) - 2 \log \left(-1 + \sqrt[3]{\frac{-3+x}{1+x}} \right) + \log \left(1 + \sqrt[3]{\frac{-3+x}{1+x}} + \left(\frac{-3+x}{1+x} \right)^{2/3} \right) \right)}{2\sqrt[3]{(-3+x)^2(1+x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(9 + 3*x - 5*x^2 + x^3)^(-1/3), x]

[Out] ((-3 + x)^(2/3)*(1 + x)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + 2*((-3 + x)/(1 + x))^(1/3))/Sqrt[3]] - 2*Log[-1 + ((-3 + x)/(1 + x))^(1/3)] + Log[1 + ((-3 + x)/(1 + x))^(1/3) + ((-3 + x)/(1 + x))^(2/3)]))/(2*((-3 + x)^2*(1 + x))^(1/3))

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x^3 - 5*x^2 + 3*x + 9)^(1/3), x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.24, size = 670, normalized size = 8.93

method	result
trager	$\text{RootOf}\left(_Z^2-3_Z+9\right) \ln\left(-\frac{20 \text{RootOf}\left(_Z^2-3_Z+9\right)^2 x^2+27 \text{RootOf}\left(_Z^2-3_Z+9\right)\left(x^3-5 x^2+3 x+9\right)^{\frac{2}{3}}+27 \text{RootOf}\left(_Z^2-3_Z+9\right)}{\dots}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3-5*x^2+3*x+9)^(1/3),x,method=_RETURNVERBOSE)`

[Out]
$$\frac{1}{3} \text{RootOf}\left(_Z^2-3_Z+9\right) \ln\left(-\left(20 \text{RootOf}\left(_Z^2-3_Z+9\right)^2 x^2+27 \text{RootOf}\left(_Z^2-3_Z+9\right)\left(x^3-5 x^2+3 x+9\right)^{\frac{2}{3}}+27 \text{RootOf}\left(_Z^2-3_Z+9\right)\right)\right) \left(x^3-5 x^2+3 x+9\right)^{\frac{2}{3}}+27 \text{RootOf}\left(_Z^2-3_Z+9\right)\left(x^3-5 x^2+3 x+9\right)^{\frac{1}{3}}\right) \cdot x-60 \text{RootOf}\left(_Z^2-3_Z+9\right)^2 x-33 \text{RootOf}\left(_Z^2-3_Z+9\right) x^2-216\left(x^3-5 x^2+3 x+9\right)^{\frac{2}{3}}-81 \text{RootOf}\left(_Z^2-3_Z+9\right)\left(x^3-5 x^2+3 x+9\right)^{\frac{1}{3}}-216\left(x^3-5 x^2+3 x+9\right)^{\frac{1}{3}} x-6 \text{RootOf}\left(_Z^2-3_Z+9\right) x-36 x^2+648\left(x^3-5 x^2+3 x+9\right)^{\frac{1}{3}}+315 \text{RootOf}\left(_Z^2-3_Z+9\right)+360 x-756\right) /(-3+x)-\frac{1}{3} \ln\left(\left(-20 \text{RootOf}\left(_Z^2-3_Z+9\right)^2 x^2+27 \text{RootOf}\left(_Z^2-3_Z+9\right)\left(x^3-5 x^2+3 x+9\right)^{\frac{2}{3}}+27 \text{RootOf}\left(_Z^2-3_Z+9\right)\right)\right) \left(x^3-5 x^2+3 x+9\right)^{\frac{1}{3}}\right) \cdot x+60 \text{RootOf}\left(_Z^2-3_Z+9\right)^2 x+87 \text{RootOf}\left(_Z^2-3_Z+9\right) x^2+135\left(x^3-5 x^2+3 x+9\right)^{\frac{2}{3}}-81 \text{RootOf}\left(_Z^2-3_Z+9\right)\left(x^3-5 x^2+3 x+9\right)^{\frac{1}{3}}+135\left(x^3-5 x^2+3 x+9\right)^{\frac{1}{3}} x-366 \text{RootOf}\left(_Z^2-3_Z+9\right) x-45 x^2-405\left(x^3-5 x^2+3 x+9\right)^{\frac{1}{3}}+315 \text{RootOf}\left(_Z^2-3_Z+9\right)+198 x-189\right) /(-3+x) \cdot \text{RootOf}\left(_Z^2-3_Z+9\right)+\ln\left(\left(-20 \text{RootOf}\left(_Z^2-3_Z+9\right)^2 x^2+27 \text{RootOf}\left(_Z^2-3_Z+9\right)\left(x^3-5 x^2+3 x+9\right)^{\frac{2}{3}}+27 \text{RootOf}\left(_Z^2-3_Z+9\right)\right)\right) \left(x^3-5 x^2+3 x+9\right)^{\frac{1}{3}}\right) \cdot x+60 \text{RootOf}\left(_Z^2-3_Z+9\right)^2 x+87 \text{RootOf}\left(_Z^2-3_Z+9\right) x^2+135\left(x^3-5 x^2+3 x+9\right)^{\frac{2}{3}}-81 \text{RootOf}\left(_Z^2-3_Z+9\right)\left(x^3-5 x^2+3 x+9\right)^{\frac{1}{3}}+135\left(x^3-5 x^2+3 x+9\right)^{\frac{1}{3}} x-366 \text{RootOf}\left(_Z^2-3_Z+9\right) x-45 x^2-405\left(x^3-5 x^2+3 x+9\right)^{\frac{1}{3}}+315 \text{RootOf}\left(_Z^2-3_Z+9\right)+198 x-189\right) /(-3+x)$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^3-5*x^2+3*x+9)^(1/3),x, algorithm="maxima")`

[Out] `integrate((x^3 - 5*x^2 + 3*x + 9)^(-1/3), x)`

Fricas [A]

time = 0.32, size = 128, normalized size = 1.71

$$-\sqrt{3} \arctan\left(\frac{\sqrt{3}(x-3)+2\sqrt{3}(x^3-5x^2+3x+9)^{\frac{1}{3}}}{3(x-3)}\right) + \frac{1}{2} \log\left(\frac{x^2+(x^3-5x^2+3x+9)^{\frac{1}{3}}(x-3)-6x+(x^3-5x^2+3x+9)^{\frac{2}{3}}+9}{x^2-6x+9}\right) - \log\left(\frac{x-(x^3-5x^2+3x+9)^{\frac{1}{3}}-3}{x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/3),x, algorithm="fricas")

[Out] $-\sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (x-3) + 2 \sqrt{3} (x^3 - 5x^2 + 3x + 9)^{1/3}\right) / (x-3) + \frac{1}{2} \log\left(\frac{(x^2 + (x^3 - 5x^2 + 3x + 9)^{1/3})(x-3) - 6x + (x^3 - 5x^2 + 3x + 9)^{2/3} + 9}{(x^2 - 6x + 9)}\right) - \log\left(\frac{-(x - (x^3 - 5x^2 + 3x + 9)^{1/3} - 3)}{(x-3)}\right)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x^3 - 5x^2 + 3x + 9}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**3-5*x**2+3*x+9)**(1/3),x)

[Out] Integral((x**3 - 5*x**2 + 3*x + 9)**(-1/3), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/3),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x^3 - 5x^2 + 3x + 9)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x - 5*x^2 + x^3 + 9)^(1/3),x)

[Out] int(1/(3*x - 5*x^2 + x^3 + 9)^(1/3), x)

$$3.233 \quad \int \frac{1}{(9+3x-5x^2+x^3)^{2/3}} dx$$

Optimal. Leaf size=29

$$\frac{3(3-x)(1+x)}{4(9+3x-5x^2+x^3)^{2/3}}$$

[Out] $3/4*(3-x)*(1+x)/(x^3-5*x^2+3*x+9)^{(2/3)}$

Rubi [A]

time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2092, 2089, 37}

$$\frac{3(3-x)(x+1)}{4(x^3-5x^2+3x+9)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(9 + 3*x - 5*x^2 + x^3)^(-2/3), x]

[Out] (3*(3 - x)*(1 + x))/(4*(9 + 3*x - 5*x^2 + x^3)^(2/3))

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2089

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] := Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] := With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(9 + 3x - 5x^2 + x^3)^{2/3}} dx &= \text{Subst} \left(\int \frac{1}{\left(\frac{128}{27} - \frac{16x}{3} + x^3\right)^{2/3}} dx, x, -\frac{5}{3} + x \right) \\ &= \frac{\left(512\sqrt[3]{2} (3-x)^{4/3}(1+x)^{2/3}\right) \text{Subst} \left(\int \frac{1}{\left(\frac{128}{9} - \frac{32x}{3}\right)^{4/3} \left(\frac{128}{9} + \frac{16x}{3}\right)^{2/3}} dx, x, -\frac{5}{3} + x \right)}{9(9 + 3x - 5x^2 + x^3)^{2/3}} \\ &= \frac{3(3-x)(1+x)}{4(9 + 3x - 5x^2 + x^3)^{2/3}} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 23, normalized size = 0.79

$$-\frac{3(-3+x)(1+x)}{4((-3+x)^2(1+x))^{2/3}}$$

Antiderivative was successfully verified.

`[In] Integrate[(9 + 3*x - 5*x^2 + x^3)^(-2/3), x]``[Out] (-3*(-3 + x)*(1 + x))/(4*((-3 + x)^2*(1 + x))^(2/3))`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x^3 - 5*x^2 + 3*x + 9)^(2/3), x]')``[Out] cought exception: maximum recursion depth exceeded in comparison`**Maple [A]**

time = 0.02, size = 20, normalized size = 0.69

method	result	size
risch	$-\frac{3(-3+x)(1+x)}{4((1+x)(-3+x)^2)^{2/3}}$	20
trager	$-\frac{3(x^3-5x^2+3x+9)^{1/3}}{4(-3+x)}$	23
gospers	$-\frac{3(1+x)(-3+x)}{4(x^3-5x^2+3x+9)^{2/3}}$	24

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3-5*x^2+3*x+9)^(2/3),x,method=_RETURNVERBOSE)`

[Out] `-3/4/((1+x)*(-3+x)^2)^(2/3)*(-3+x)*(1+x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^3-5*x^2+3*x+9)^(2/3),x, algorithm="maxima")`

[Out] `integrate((x^3 - 5*x^2 + 3*x + 9)^(-2/3), x)`

Fricas [A]

time = 0.31, size = 22, normalized size = 0.76

$$-\frac{3(x^3 - 5x^2 + 3x + 9)^{\frac{1}{3}}}{4(x - 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^3-5*x^2+3*x+9)^(2/3),x, algorithm="fricas")`

[Out] `-3/4*(x^3 - 5*x^2 + 3*x + 9)^(1/3)/(x - 3)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 - 5x^2 + 3x + 9)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**3-5*x**2+3*x+9)**(2/3),x)`

[Out] `Integral((x**3 - 5*x**2 + 3*x + 9)**(-2/3), x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-5*x^2+3*x+9)^(2/3),x)

[Out] Could not integrate

Mupad [B]

time = 0.05, size = 24, normalized size = 0.83

$$-\frac{3(x^3 - 5x^2 + 3x + 9)^{1/3}}{4(x - 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x - 5*x^2 + x^3 + 9)^(2/3),x)

[Out] -(3*(3*x - 5*x^2 + x^3 + 9)^(1/3))/(4*(x - 3))

$$3.234 \quad \int \frac{1}{(9+3x-5x^2+x^3)^{4/3}} dx$$

Optimal. Leaf size=92

$$\frac{3(3-x)(1+x)}{20(9+3x-5x^2+x^3)^{4/3}} + \frac{9(3-x)^2(1+x)}{80(9+3x-5x^2+x^3)^{4/3}} - \frac{27(3-x)^3(1+x)}{320(9+3x-5x^2+x^3)^{4/3}}$$

[Out] $3/20*(3-x)*(1+x)/(x^3-5*x^2+3*x+9)^{(4/3)}+9/80*(3-x)^2*(1+x)/(x^3-5*x^2+3*x+9)^{(4/3)}-27/320*(3-x)^3*(1+x)/(x^3-5*x^2+3*x+9)^{(4/3)}$

Rubi [A]

time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2092, 2089, 47, 37}

$$-\frac{27(x+1)(3-x)^3}{320(x^3-5x^2+3x+9)^{4/3}} + \frac{9(x+1)(3-x)^2}{80(x^3-5x^2+3x+9)^{4/3}} + \frac{3(x+1)(3-x)}{20(x^3-5x^2+3x+9)^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[(9 + 3*x - 5*x^2 + x^3)^(-4/3), x]

[Out] $(3*(3-x)*(1+x))/(20*(9+3*x-5*x^2+x^3)^{(4/3)})+(9*(3-x)^2*(1+x))/(80*(9+3*x-5*x^2+x^3)^{(4/3)})-(27*(3-x)^3*(1+x))/(320*(9+3*x-5*x^2+x^3)^{(4/3)})$

Rule 37

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(Simplify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])

Rule 2089

Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] := Dist[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x]

$x)^{(2*p)}, x], x] /; \text{FreeQ}\{a, b, d, p\}, x\} \&\& \text{EqQ}[4*b^3 + 27*a^2*d, 0] \&\& !\text{IntegerQ}[p]$

Rule 2092

$\text{Int}[(P3_)^{(p_)}, x_Symbol] \text{:> With}\{a = \text{Coeff}[P3, x, 0], b = \text{Coeff}[P3, x, 1], c = \text{Coeff}[P3, x, 2], d = \text{Coeff}[P3, x, 3]\}, \text{Subst}[\text{Int}[\text{Simp}[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^{(p)}, x], x, x + c/(3*d)] /; \text{NeQ}[c, 0] /; \text{FreeQ}[p, x] \&\& \text{PolyQ}[P3, x, 3]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(9 + 3x - 5x^2 + x^3)^{4/3}} dx &= \text{Subst} \left(\int \frac{1}{\left(\frac{128}{27} - \frac{16x}{3} + x^3\right)^{4/3}} dx, x, -\frac{5}{3} + x \right) \\ &= \frac{(262144 \cdot 2^{2/3} (3 - x)^{8/3} (1 + x)^{4/3}) \text{Subst} \left(\int \frac{1}{\left(\frac{128}{9} - \frac{32x}{3}\right)^{8/3} \left(\frac{128}{9} + \frac{16x}{3}\right)^{4/3}} dx, x, -\frac{5}{3} \right)}{81 (9 + 3x - 5x^2 + x^3)^{4/3}} \\ &= \frac{3(3 - x)(1 + x)}{20 (9 + 3x - 5x^2 + x^3)^{4/3}} + \frac{(4096 \cdot 2^{2/3} (3 - x)^{8/3} (1 + x)^{4/3}) \text{Subst} \left(\int \frac{1}{\left(\frac{128}{9}\right)^{8/3}} dx, x, -\frac{5}{3} \right)}{45 (9 + 3x - 5x^2 + x^3)^{4/3}} \\ &= \frac{3(3 - x)(1 + x)}{20 (9 + 3x - 5x^2 + x^3)^{4/3}} + \frac{9(3 - x)^2(1 + x)}{80 (9 + 3x - 5x^2 + x^3)^{4/3}} + \frac{(16 \cdot 2^{2/3} (3 - x)^{8/3})}{320 (9 + 3x - 5x^2 + x^3)^{4/3}} \\ &= \frac{3(3 - x)(1 + x)}{20 (9 + 3x - 5x^2 + x^3)^{4/3}} + \frac{9(3 - x)^2(1 + x)}{80 (9 + 3x - 5x^2 + x^3)^{4/3}} - \frac{27(3 - x)^3(1 + x)}{320 (9 + 3x - 5x^2 + x^3)^{4/3}} \end{aligned}$$

Mathematica [A]

time = 0.04, size = 32, normalized size = 0.35

$$\frac{3(29 - 42x + 9x^2)}{320(-3 + x)\sqrt[3]{(-3 + x)^2(1 + x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(9 + 3*x - 5*x^2 + x^3)^(-4/3), x]

[Out] (3*(29 - 42*x + 9*x^2))/(320*(-3 + x)*((-3 + x)^2*(1 + x))^(1/3))

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[1/(x^3 - 5*x^2 + 3*x + 9)^(4/3),x]')`

[Out] `cought exception: maximum recursion depth exceeded`

Maple [A]

time = 0.02, size = 29, normalized size = 0.32

method	result	size
risch	$\frac{\frac{27}{320}x^2 - \frac{63}{160}x + \frac{87}{320}}{(-3+x)\left((1+x)(-3+x)^2\right)^{\frac{1}{3}}}$	29
gospers	$\frac{3(1+x)(-3+x)(9x^2-42x+29)}{320(x^3-5x^2+3x+9)^{\frac{4}{3}}}$	34
trager	$\frac{3(9x^2-42x+29)(x^3-5x^2+3x+9)^{\frac{2}{3}}}{320(-3+x)^3(1+x)}$	38

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3-5*x^2+3*x+9)^(4/3),x,method=_RETURNVERBOSE)`

[Out] `3/320*(9*x^2-42*x+29)/(-3+x)/((1+x)*(-3+x)^2)^(1/3)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^3-5*x^2+3*x+9)^(4/3),x, algorithm="maxima")`

[Out] `integrate((x^3 - 5*x^2 + 3*x + 9)^(-4/3), x)`

Fricas [A]

time = 0.31, size = 44, normalized size = 0.48

$$\frac{3(x^3 - 5x^2 + 3x + 9)^{\frac{2}{3}}(9x^2 - 42x + 29)}{320(x^4 - 8x^3 + 18x^2 - 27)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^3-5*x^2+3*x+9)^(4/3),x, algorithm="fricas")`

[Out] `3/320*(x^3 - 5*x^2 + 3*x + 9)^(2/3)*(9*x^2 - 42*x + 29)/(x^4 - 8*x^3 + 18*x^2 - 27)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 - 5x^2 + 3x + 9)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**3-5*x**2+3*x+9)**(4/3),x)`

[Out] `Integral((x**3 - 5*x**2 + 3*x + 9)**(-4/3), x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^3-5*x^2+3*x+9)^(4/3),x)`

[Out] Could not integrate

Mupad [B]

time = 0.08, size = 37, normalized size = 0.40

$$\frac{3 (9x^2 - 42x + 29) (x^3 - 5x^2 + 3x + 9)^{2/3}}{320 (x + 1) (x - 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*x - 5*x^2 + x^3 + 9)^(4/3),x)`

[Out] `(3*(9*x^2 - 42*x + 29)*(3*x - 5*x^2 + x^3 + 9)^(2/3))/(320*(x + 1)*(x - 3)^3)`

$$3.235 \quad \int \frac{1}{\sqrt{4 + 3x - 2x^2}} dx$$

Optimal. Leaf size=19

$$-\frac{\sin^{-1}\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

[Out] -1/2*arcsin(1/41*(3-4*x)*41^(1/2))*2^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {633, 222}

$$-\frac{\sin^{-1}\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[4 + 3*x - 2*x^2], x]

[Out] -(ArcSin[(3 - 4*x)/Sqrt[41]]/Sqrt[2])

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 633

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*(-4*(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\int \frac{1}{\sqrt{4 + 3x - 2x^2}} dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1 - \frac{x^2}{41}}} dx, x, 3 - 4x\right)}{\sqrt{82}} = -\frac{\sin^{-1}\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

Mathematica [A]

time = 0.06, size = 32, normalized size = 1.68

$$\sqrt{2} \tan^{-1} \left(\frac{\sqrt{2} x}{-2 + \sqrt{4 + 3x - 2x^2}} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[1/Sqrt[4 + 3*x - 2*x^2], x]``[Out] Sqrt[2]*ArcTan[(Sqrt[2]*x)/(-2 + Sqrt[4 + 3*x - 2*x^2])]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/Sqrt[4 + 3*x - 2*x^2], x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [A]**

time = 0.14, size = 15, normalized size = 0.79

method	result	size
default	$\frac{\sqrt{2} \arcsin\left(\frac{4\sqrt{41}\left(x-\frac{3}{4}\right)}{41}\right)}{2}$	15
trager	$\frac{\text{RootOf}(_Z^2+2) \ln\left(-4\text{RootOf}(_Z^2+2)x+4\sqrt{-2x^2+3x+4}+3\text{RootOf}(_Z^2+2)\right)}{2}$	42

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(-2*x^2+3*x+4)^(1/2), x, method=_RETURNVERBOSE)``[Out] 1/2*2^(1/2)*arcsin(4/41*41^(1/2)*(x-3/4))`**Maxima [A]**

time = 0.36, size = 16, normalized size = 0.84

$$-\frac{1}{2} \sqrt{2} \arcsin \left(-\frac{1}{41} \sqrt{41} (4x - 3) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(-2*x^2+3*x+4)^(1/2), x, algorithm="maxima")`

[Out] $-1/2*\sqrt{2}*\arcsin(-1/41*\sqrt{41}*(4*x - 3))$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(16) = 32$.

time = 0.32, size = 33, normalized size = 1.74

$$-\sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{-2x^2 + 3x + 4} - 2\sqrt{2}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-2*x^2+3*x+4)^(1/2),x, algorithm="fricas")`

[Out] $-\sqrt{2}*\arctan(1/2*(\sqrt{2}*\sqrt{-2*x^2 + 3*x + 4} - 2*\sqrt{2}))/x$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^2 + 3x + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-2*x**2+3*x+4)**(1/2),x)`

[Out] $\text{Integral}(1/\sqrt{-2*x**2 + 3*x + 4}, x)$

Giac [A]

time = 0.00, size = 21, normalized size = 1.11

$$\frac{1}{2}\sqrt{2} \arcsin\left(\frac{4x - 3}{\sqrt{41}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-2*x^2+3*x+4)^(1/2),x)`

[Out] $1/2*\sqrt{2}*\arcsin(1/41*\sqrt{41}*(4*x - 3))$

Mupad [B]

time = 0.20, size = 16, normalized size = 0.84

$$\frac{\sqrt{2} \operatorname{asin}\left(\frac{\sqrt{41}(4x-3)}{41}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*x - 2*x^2 + 4)^(1/2),x)`

[Out] $(2^(1/2)*\operatorname{asin}((41^(1/2)*(4*x - 3))/41))/2$

$$3.236 \quad \int \frac{1}{\sqrt{-3 + 4x - x^2}} dx$$

Optimal. Leaf size=8

$$-\sin^{-1}(2 - x)$$

[Out] arcsin(-2+x)

Rubi [A]

time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {633, 222}

$$-\sin^{-1}(2 - x)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-3 + 4*x - x^2],x]

[Out] -ArcSin[2 - x]

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 633

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c)], x]^p, x], x, b + 2*c*x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-3 + 4x - x^2}} dx &= - \left(\frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1 - \frac{x^2}{4}}} dx, x, 4 - 2x \right) \right) \\ &= -\sin^{-1}(2 - x) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(8) = 16. time = 0.05, size = 23, normalized size = 2.88

$$-2 \tan^{-1} \left(\frac{\sqrt{-3 + 4x - x^2}}{-1 + x} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 4*x - x^2],x]
```

```
[Out] -2*ArcTan[Sqrt[-3 + 4*x - x^2]/(-1 + x)]
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/Sqrt[-3 + 4*x - x^2],x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]

time = 0.11, size = 5, normalized size = 0.62

method	result	size
default	$\arcsin(-2 + x)$	5
trager	$\text{RootOf}(_Z^2 + 1) \ln(-\text{RootOf}(_Z^2 + 1) x + \sqrt{-x^2 + 4x - 3}) + 2 \text{RootOf}(_Z^2 + 1)$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+4*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsin(-2+x)
```

Maxima [A]

time = 0.34, size = 8, normalized size = 1.00

$$- \arcsin(-x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-x + 2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(4) = 8.

time = 0.40, size = 29, normalized size = 3.62

$$- \arctan\left(\frac{\sqrt{-x^2 + 4x - 3}(x - 2)}{x^2 - 4x + 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+4*x-3)^(1/2),x, algorithm="fricas")

[Out] -arctan(sqrt(-x^2 + 4*x - 3)*(x - 2)/(x^2 - 4*x + 3))

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2 + 4x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**2+4*x-3)**(1/2),x)

[Out] Integral(1/sqrt(-x**2 + 4*x - 3), x)

Giac [A]

time = 0.00, size = 4, normalized size = 0.50

$$\arcsin(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+4*x-3)^(1/2),x)

[Out] arcsin(x - 2)

Mupad [B]

time = 0.18, size = 4, normalized size = 0.50

$$\operatorname{asin}(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x - x^2 - 3)^(1/2),x)

[Out] asin(x - 2)

$$3.237 \quad \int \frac{1}{\sqrt{-2 - 5x - 3x^2}} dx$$

Optimal. Leaf size=12

$$\frac{\sin^{-1}(5 + 6x)}{\sqrt{3}}$$

[Out] 1/3*arcsin(5+6*x)*3^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {633, 222}

$$\frac{\sin^{-1}(6x + 5)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-2 - 5*x - 3*x^2],x]

[Out] ArcSin[5 + 6*x]/Sqrt[3]

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 633

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*(-4*(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\int \frac{1}{\sqrt{-2 - 5x - 3x^2}} dx = \frac{\text{Subst} \left(\int \frac{1}{\sqrt{1 - x^2}} dx, x, -5 - 6x \right)}{\sqrt{3}} = \frac{\sin^{-1}(5 + 6x)}{\sqrt{3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

time = 0.06, size = 33, normalized size = 2.75

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{-2 - 5x - 3x^2}}{\sqrt{3}(1+x)} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-2 - 5*x - 3*x^2],x]

[Out] (-2*ArcTan[Sqrt[-2 - 5*x - 3*x^2]/(Sqrt[3]*(1 + x))])/Sqrt[3]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/Sqrt[-2 - 5*x - 3*x^2],x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [A]

time = 0.12, size = 12, normalized size = 1.00

method	result	size
default	$\frac{\arcsin(6x+5)\sqrt{3}}{3}$	12
trager	$\frac{\text{RootOf}(_Z^2+3) \ln\left(-6 \text{RootOf}(_Z^2+3)x+6\sqrt{-3x^2-5x-2}-5 \text{RootOf}(_Z^2+3)\right)}{3}$	42

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-3*x^2-5*x-2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3*arcsin(6*x+5)*3^(1/2)

Maxima [A]

time = 0.35, size = 11, normalized size = 0.92

$$\frac{1}{3} \sqrt{3} \arcsin(6x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2-5*x-2)^(1/2),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*arcsin(6*x + 5)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(11) = 22.

time = 0.30, size = 40, normalized size = 3.33

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^2-5x-2}(6x+5)}{6(3x^2+5x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2-5*x-2)^(1/2),x, algorithm="fricas")

[Out] -1/3*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 - 5*x - 2)*(6*x + 5)/(3*x^2 + 5*x + 2))

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^2-5x-2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x**2-5*x-2)**(1/2),x)

[Out] Integral(1/sqrt(-3*x**2 - 5*x - 2), x)

Giac [A]

time = 0.00, size = 14, normalized size = 1.17

$$\frac{1}{3}\sqrt{3}\arcsin(6x+5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2-5*x-2)^(1/2),x)

[Out] 1/3*sqrt(3)*arcsin(6*x + 5)

Mupad [B]

time = 0.22, size = 11, normalized size = 0.92

$$\frac{\sqrt{3}\operatorname{asin}(6x+5)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-5*x - 3*x^2 - 2)^(1/2),x)

[Out] (3^(1/2)*asin(6*x + 5))/3

$$3.238 \quad \int \frac{1}{\sqrt{1-x^2} (4+x^2)} dx$$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}}$$

[Out] 1/10*arctan(1/2*x*5^(1/2)/(-x^2+1)^(1/2))*5^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {385, 209}

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 - x^2]*(4 + x^2)),x]

[Out] ArcTan[(Sqrt[5]*x)/(2*Sqrt[1 - x^2])]/(2*Sqrt[5])

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1-x^2} (4+x^2)} dx &= \text{Subst} \left(\int \frac{1}{4+5x^2} dx, x, \frac{x}{\sqrt{1-x^2}} \right) \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}} \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.08, size = 42, normalized size = 1.35

$$\frac{i \tanh^{-1} \left(\frac{4+x^2+ix\sqrt{1-x^2}}{2\sqrt{5}} \right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[1 - x^2]*(4 + x^2)),x]

[Out] ((-1/2*I)*ArcTanh[(4 + x^2 + I*x*Sqrt[1 - x^2])/(2*Sqrt[5])])/Sqrt[5]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((x^2 + 4)*Sqrt[1 - x^2]),x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.09, size = 29, normalized size = 0.94

method	result	size
default	$\frac{\sqrt{5} \arctan\left(\frac{\sqrt{5} \sqrt{-x^2+1} x}{2x^2-2}\right)}{10}$	29
trager	$\frac{\text{RootOf}(-Z^2+5) \ln\left(\frac{9 \text{RootOf}(-Z^2+5) x^2+20x \sqrt{-x^2+1} -4 \text{RootOf}(-Z^2+5)}{x^2+4}\right)}{20}$	50

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+4)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/10*5^(1/2)*arctan(1/2*5^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+4)/(-x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((x^2 + 4)*sqrt(-x^2 + 1)), x)

Fricas [A]

time = 0.33, size = 23, normalized size = 0.74

$$-\frac{1}{10} \sqrt{5} \arctan \left(\frac{2 \sqrt{5} \sqrt{-x^2 + 1}}{5x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+4)/(-x^2+1)^(1/2),x, algorithm="fricas")

[Out] -1/10*sqrt(5)*arctan(2/5*sqrt(5)*sqrt(-x^2 + 1)/x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}(x^2+4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+4)/(-x**2+1)**(1/2),x)

[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*(x**2 + 4)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(21) = 42.

time = 0.00, size = 70, normalized size = 2.26

$$\frac{-\frac{1}{2} \pi \operatorname{sign}(x) - \arctan \left(\frac{2x \left(\left(-\frac{-2\sqrt{-x^2+1}+2}{2x} \right)^2 - 1 \right)}{\sqrt{5} \left(-2\sqrt{-x^2+1}+2 \right)} \right)}{2\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+4)/(-x^2+1)^(1/2),x)

[Out] 1/20*sqrt(5)*(pi*sgn(x) + 2*arctan(-1/5*sqrt(5)*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1)))

Mupad [B]

time = 0.52, size = 79, normalized size = 2.55

$$\frac{\sqrt{5} \ln \left(\frac{\sqrt{5} \frac{(-1+x 2i) \operatorname{li} - \sqrt{1-x^2} \operatorname{li}}{5}}{x-2i} \right) \operatorname{li}}{20} - \frac{\sqrt{5} \ln \left(\frac{\sqrt{5} \frac{(1+x 2i) \operatorname{li} + \sqrt{1-x^2} \operatorname{li}}{5}}{x+2i} \right) \operatorname{li}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(x^2 + 4)),x)
```

```
[Out] (5^(1/2)*log(((5^(1/2)*(x*2i - 1)*1i)/5 - (1 - x^2)^(1/2)*1i)/(x - 2i))*1i)/20 - (5^(1/2)*log(((5^(1/2)*(x*2i + 1)*1i)/5 + (1 - x^2)^(1/2)*1i)/(x + 2i))*1i)/20
```

$$3.239 \quad \int \frac{1}{(4+x^2)\sqrt{1+4x^2}} dx$$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{15}x}{2\sqrt{1+4x^2}}\right)}{2\sqrt{15}}$$

[Out] 1/30*arctanh(1/2*x*15^(1/2)/(4*x^2+1)^(1/2))*15^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{15}x}{2\sqrt{4x^2+1}}\right)}{2\sqrt{15}}$$

Antiderivative was successfully verified.

[In] Int[1/((4 + x^2)*Sqrt[1 + 4*x^2]),x]

[Out] ArcTanh[(Sqrt[15]*x)/(2*Sqrt[1 + 4*x^2])]/(2*Sqrt[15])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned} \int \frac{1}{(4+x^2)\sqrt{1+4x^2}} dx &= \text{Subst} \left(\int \frac{1}{4-15x^2} dx, x, \frac{x}{\sqrt{1+4x^2}} \right) \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{15}x}{2\sqrt{1+4x^2}}\right)}{2\sqrt{15}} \end{aligned}$$

Mathematica [A]

time = 0.08, size = 40, normalized size = 1.29

$$\frac{\tanh^{-1}\left(\frac{8+2x^2-x\sqrt{1+4x^2}}{2\sqrt{15}}\right)}{2\sqrt{15}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/((4 + x^2)*Sqrt[1 + 4*x^2]),x]``[Out] ArcTanh[(8 + 2*x^2 - x*Sqrt[1 + 4*x^2])/(2*Sqrt[15])]/(2*Sqrt[15])`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/((x^2 + 4)*Sqrt[4*x^2 + 1]),x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.09, size = 22, normalized size = 0.71

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{15}}{2\sqrt{4x^2+1}}\right)\sqrt{15}}{30}$	22
trager	$\frac{\operatorname{RootOf}(-Z^2-15) \ln\left(\frac{31 \operatorname{RootOf}(-Z^2-15) x^2+60\sqrt{4x^2+1} x+4 \operatorname{RootOf}(-Z^2-15)}{x^2+4}\right)}{60}$	50

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(x^2+4)/(4*x^2+1)^(1/2),x,method=_RETURNVERBOSE)``[Out] 1/30*arctanh(1/2*x*15^(1/2)/(4*x^2+1)^(1/2))*15^(1/2)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(x^2+4)/(4*x^2+1)^(1/2),x, algorithm="maxima")`

[Out] integrate(1/(sqrt(4*x^2 + 1)*(x^2 + 4)), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(21) = 42.

time = 0.32, size = 54, normalized size = 1.74

$$\frac{1}{60} \sqrt{15} \log \left(\frac{961 x^2 + 8 \sqrt{15} (31 x^2 + 4) + 4 \sqrt{4 x^2 + 1} (31 \sqrt{15} x + 120 x) + 124}{x^2 + 4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+4)/(4*x^2+1)^(1/2),x, algorithm="fricas")

[Out] 1/60*sqrt(15)*log((961*x^2 + 8*sqrt(15)*(31*x^2 + 4) + 4*sqrt(4*x^2 + 1)*(31*sqrt(15)*x + 120*x) + 124)/(x^2 + 4))

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2 + 4) \sqrt{4x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+4)/(4*x**2+1)**(1/2),x)

[Out] Integral(1/((x**2 + 4)*sqrt(4*x**2 + 1)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(21) = 42.

time = 0.00, size = 72, normalized size = 2.32

$$\frac{\ln \left(\frac{2(\sqrt{4x^2 + 1} - 2x)^2 + 62 - 16\sqrt{15}}{2(\sqrt{4x^2 + 1} - 2x)^2 + 62 + 16\sqrt{15}} \right)}{4\sqrt{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+4)/(4*x^2+1)^(1/2),x)

[Out] -1/60*sqrt(15)*log(((2*x - sqrt(4*x^2 + 1))^2 - 8*sqrt(15) + 31)/((2*x - sqrt(4*x^2 + 1))^2 + 8*sqrt(15) + 31))

Mupad [B]

time = 0.48, size = 61, normalized size = 1.97

$$\frac{\sqrt{15} \left(\ln(x - 2i) - \ln \left(x + \frac{\sqrt{15} \sqrt{x^2 + \frac{1}{4}}}{4} - \frac{1}{8}i \right) \right)}{60} + \frac{\sqrt{15} \left(\ln(x + 2i) - \ln \left(x - \frac{\sqrt{15} \sqrt{x^2 + \frac{1}{4}}}{4} + \frac{1}{8}i \right) \right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 4)*(4*x^2 + 1)^(1/2)),x)
```

```
[Out] (15^(1/2)*(log(x + 2i) - log(x - (15^(1/2)*(x^2 + 1/4)^(1/2))/4 + 1i/8)))/6  
0 - (15^(1/2)*(log(x - 2i) - log(x + (15^(1/2)*(x^2 + 1/4)^(1/2))/4 - 1i/8)  
))/60
```


$$3.240 \quad \int \frac{x}{(3-x^2)\sqrt{5-x^2}} dx$$

Optimal. Leaf size=24

$$\frac{\tanh^{-1}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

[Out] 1/2*arctanh(1/2*(-x^2+5)^(1/2)*2^(1/2))*2^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {455, 65, 213}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x/((3 - x^2)*Sqrt[5 - x^2]),x]

[Out] ArcTanh[Sqrt[5 - x^2]/Sqrt[2]]/Sqrt[2]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 455

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{x}{(3-x^2)\sqrt{5-x^2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(3-x)\sqrt{5-x}} dx, x, x^2 \right) \\ &= -\text{Subst} \left(\int \frac{1}{-2+x^2} dx, x, \sqrt{5-x^2} \right) \\ &= \frac{\tanh^{-1} \left(\frac{\sqrt{5-x^2}}{\sqrt{2}} \right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 24, normalized size = 1.00

$$\frac{\tanh^{-1} \left(\frac{\sqrt{5-x^2}}{\sqrt{2}} \right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

`[In] Integrate[x/((3 - x^2)*Sqrt[5 - x^2]),x]``[Out] ArcTanh[Sqrt[5 - x^2]/Sqrt[2]]/Sqrt[2]`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 5.62, size = 63, normalized size = 2.62

$$\text{Piecewise} \left[\left[\left[\left[\frac{\sqrt{2} \text{ArcCoth} \left[\frac{\sqrt{2}}{\sqrt{5-x^2}} \right]}{2}, \frac{1}{-5+x^2} < -\frac{1}{2} \right], \left[\frac{\sqrt{2} \text{ArcTanh} \left[\frac{\sqrt{2}}{\sqrt{5-x^2}} \right]}{2}, \frac{1}{-5+x^2} > -\frac{1}{2} \right], \{0, \text{True}\} \right] \right] \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[x/((3 - x^2)*Sqrt[5 - x^2]),x]')``[Out] Piecewise[{{Sqrt[2] ArcCoth[Sqrt[2] / Sqrt[5 - x ^ 2]] / 2, 1 / (-5 + x ^ 2) < -1 / 2}, {Sqrt[2] ArcTanh[Sqrt[2] / Sqrt[5 - x ^ 2]] / 2, 1 / (-5 + x ^ 2) > -1 / 2}, {0, True}}]`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(20) = 40.

time = 0.12, size = 100, normalized size = 4.17

method	result
--------	--------

trager	$\frac{\text{RootOf}(-Z^2-2) \ln\left(\frac{\text{RootOf}(-Z^2-2)^{x^2-7} \text{RootOf}(-Z^2-2)^{-4} \sqrt{-x^2+5}}{x^2-3}\right)}{4}$
default	$\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{(4-2\sqrt{3})(x-\sqrt{3})\sqrt{2}}{4\sqrt{-(x-\sqrt{3})^2-2\sqrt{3}(x-\sqrt{3})+2}}\right)}{4} + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{(4+2\sqrt{3})(x+\sqrt{3})\sqrt{2}}{4\sqrt{-(x+\sqrt{3})^2+2\sqrt{3}(x+\sqrt{3})+2}}\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(-x^2+3)/(-x^2+5)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{4} \cdot 2^{(1/2)} \cdot \operatorname{arctanh}\left(\frac{1}{4} \cdot (4-2 \cdot 3^{(1/2)}) \cdot (x-3^{(1/2)})\right) \cdot 2^{(1/2)} / \left(-\left(x-3^{(1/2)}\right)^2-2 \cdot 3^{(1/2)} \cdot \left(x-3^{(1/2)}\right)+2\right)^{(1/2)} + \frac{1}{4} \cdot 2^{(1/2)} \cdot \operatorname{arctanh}\left(\frac{1}{4} \cdot (4+2 \cdot 3^{(1/2)}) \cdot (x+3^{(1/2)})\right) \cdot 2^{(1/2)} / \left(-\left(x+3^{(1/2)}\right)^2+2 \cdot 3^{(1/2)} \cdot \left(x+3^{(1/2)}\right)+2\right)^{(1/2)}$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(20) = 40$.

time = 0.35, size = 112, normalized size = 4.67

$$\frac{1}{12} \sqrt{3} \left(\sqrt{3} \sqrt{2} \log \left(\sqrt{3} + \frac{2\sqrt{2}\sqrt{-x^2+5}}{|2x+2\sqrt{3}|} + \frac{4}{|2x+2\sqrt{3}|} \right) + \sqrt{3} \sqrt{2} \log \left(-\sqrt{3} + \frac{2\sqrt{2}\sqrt{-x^2+5}}{|2x-2\sqrt{3}|} + \frac{4}{|2x-2\sqrt{3}|} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-x^2+3)/(-x^2+5)^(1/2),x, algorithm="maxima")`

[Out] $\frac{1}{12} \cdot \sqrt{3} \cdot (\sqrt{3} \cdot \sqrt{2} \cdot \log(\sqrt{3} + \frac{2 \cdot \sqrt{2} \cdot \sqrt{-x^2+5}}{|2x+2\sqrt{3}|} + \frac{4}{|2x+2\sqrt{3}|}) + \sqrt{3} \cdot \sqrt{2} \cdot \log(-\sqrt{3} + \frac{2 \cdot \sqrt{2} \cdot \sqrt{-x^2+5}}{|2x-2\sqrt{3}|} + \frac{4}{|2x-2\sqrt{3}|}))$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.

time = 0.32, size = 48, normalized size = 2.00

$$\frac{1}{8} \sqrt{2} \log \left(\frac{x^4 - 4\sqrt{2}(x^2-7)\sqrt{-x^2+5} - 22x^2 + 89}{x^4 - 6x^2 + 9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-x^2+3)/(-x^2+5)^(1/2),x, algorithm="fricas")`

[Out] $\frac{1}{8} \cdot \sqrt{2} \cdot \log((x^4 - 4 \cdot \sqrt{2} \cdot (x^2 - 7) \cdot \sqrt{-x^2 + 5} - 22 \cdot x^2 + 89) / (x^4 - 6 \cdot x^2 + 9))$

Sympy [A]

time = 2.31, size = 61, normalized size = 2.54

$$-\begin{cases} \frac{\sqrt{2} \operatorname{acoth}\left(\frac{\sqrt{2}}{\sqrt{5-x^2}}\right)}{2} & \text{for } \frac{1}{5-x^2} > \frac{1}{2} \\ \frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2}}{\sqrt{5-x^2}}\right)}{2} & \text{for } \frac{1}{5-x^2} < \frac{1}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x/(-x**2+3)/(-x**2+5)**(1/2),x)``[Out] -Piecewise((-sqrt(2)*acoth(sqrt(2)/sqrt(5 - x**2))/2, 1/(5 - x**2) > 1/2), (-sqrt(2)*atanh(sqrt(2)/sqrt(5 - x**2))/2, 1/(5 - x**2) < 1/2))`**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.

time = 0.00, size = 54, normalized size = 2.25

$$-\frac{1}{4}\sqrt{2} \ln \left| \sqrt{-x^2+5} - \sqrt{2} \right| + \frac{1}{4}\sqrt{2} \ln \left(\sqrt{-x^2+5} + \sqrt{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x/(-x^2+3)/(-x^2+5)^(1/2),x)``[Out] 1/4*sqrt(2)*log(sqrt(2) + sqrt(-x^2 + 5)) - 1/4*sqrt(2)*log(abs(-sqrt(2) + sqrt(-x^2 + 5)))`**Mupad [B]**

time = 0.79, size = 78, normalized size = 3.25

$$\frac{\sqrt{2} \left(\ln \left(\frac{\frac{\sqrt{2} (\sqrt{3}^{x+5})^{1i}}{2} + \sqrt{5-x^2}^{1i}}{x+\sqrt{3}} \right) + \ln \left(\frac{\frac{\sqrt{2} (\sqrt{3}^{x-5})^{1i}}{2} - \sqrt{5-x^2}^{1i}}{x-\sqrt{3}} \right) \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(-x/((x^2 - 3)*(5 - x^2)^(1/2)),x)``[Out] (2^(1/2)*(log(((2^(1/2)*(3^(1/2)*x + 5)*1i)/2 + (5 - x^2)^(1/2)*1i)/(x + 3^(1/2)))) + log(((2^(1/2)*(3^(1/2)*x - 5)*1i)/2 - (5 - x^2)^(1/2)*1i)/(x - 3^(1/2))))/4`

$$3.241 \quad \int \frac{x}{\sqrt{3-x^2} (5-x^2)} dx$$

Optimal. Leaf size=25

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

[Out] $-1/2*\arctan(1/2*(-x^2+3)^{(1/2)}*2^{(1/2)})*2^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {455, 65, 209}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[3 - x^2]*(5 - x^2)),x]

[Out] -(ArcTan[Sqrt[3 - x^2]/Sqrt[2]]/Sqrt[2])

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 455

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{3-x^2} (5-x^2)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{3-x} (5-x)} dx, x, x^2 \right) \\ &= -\text{Subst} \left(\int \frac{1}{2+x^2} dx, x, \sqrt{3-x^2} \right) \\ &= -\frac{\tan^{-1} \left(\frac{\sqrt{3-x^2}}{\sqrt{2}} \right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 25, normalized size = 1.00

$$-\frac{\tan^{-1} \left(\frac{\sqrt{3-x^2}}{\sqrt{2}} \right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

`[In] Integrate[x/(Sqrt[3 - x^2]*(5 - x^2)),x]``[Out] -(ArcTan[Sqrt[3 - x^2]/Sqrt[2]]/Sqrt[2])`**Mathics [A]**

time = 3.96, size = 20, normalized size = 0.80

$$-\frac{\sqrt{2} \text{ArcTan} \left[\frac{\sqrt{2} \sqrt{3-x^2}}{2} \right]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x/((5 - x^2)*Sqrt[3 - x^2]),x]')``[Out] -Sqrt[2] ArcTan[Sqrt[2] Sqrt[3 - x ^ 2] / 2] / 2`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(20) = 40.

time = 0.13, size = 100, normalized size = 4.00

method	result
trager	$\frac{\text{RootOf}(-Z^2+2) \ln \left(\frac{\text{RootOf}(-Z^2+2)^{x^2} - \text{RootOf}(-Z^2+2)^{-4} \sqrt{-x^2+3}}{x^2-5} \right)}{4}$

default	$\frac{\sqrt{2} \arctan\left(\frac{(-4-2\sqrt{5})(x-\sqrt{5})\sqrt{2}}{4\sqrt{-(x-\sqrt{5})^2-2\sqrt{5}(x-\sqrt{5})-2}}\right)}{4}$	$\frac{\sqrt{2} \arctan\left(\frac{(-4+2\sqrt{5})(x+\sqrt{5})\sqrt{2}}{4\sqrt{-(x+\sqrt{5})^2+2\sqrt{5}(x+\sqrt{5})-2}}\right)}{4}$
---------	---	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(-x^2+5)/(-x^2+3)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/4*2^{(1/2)}*\arctan(1/4*(-4-2*5^{(1/2)}*(x-5^{(1/2)}))*2^{(1/2)}/(-(x-5^{(1/2)})^2-2*5^{(1/2)}*(x-5^{(1/2)})-2)^{(1/2)})-1/4*2^{(1/2)}*\arctan(1/4*(-4+2*5^{(1/2)}*(x+5^{(1/2)}))*2^{(1/2)}/(-(x+5^{(1/2)})^2+2*5^{(1/2)}*(x+5^{(1/2)})-2)^{(1/2)})$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(20) = 40$.

time = 0.35, size = 101, normalized size = 4.04

$$-\frac{1}{20}\sqrt{5}\left(\sqrt{5}\sqrt{2}\arcsin\left(\frac{2\sqrt{5}\sqrt{3}x}{3|2x+2\sqrt{5}|}+\frac{2\sqrt{3}}{|2x+2\sqrt{5}|}\right)-\sqrt{5}\sqrt{2}\arcsin\left(\frac{2\sqrt{5}\sqrt{3}x}{3|2x-2\sqrt{5}|}-\frac{2\sqrt{3}}{|2x-2\sqrt{5}|}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-x^2+5)/(-x^2+3)^(1/2),x, algorithm="maxima")`

[Out] $-1/20*\sqrt{5}*(\sqrt{5}*\sqrt{2}*\arcsin(2/3*\sqrt{5}*\sqrt{3}*x/\text{abs}(2*x + 2*\sqrt{5})) + 2*\sqrt{3}/\text{abs}(2*x + 2*\sqrt{5})) - \sqrt{5}*\sqrt{2}*\arcsin(2/3*\sqrt{5}*\sqrt{3}*x/\text{abs}(2*x - 2*\sqrt{5}) - 2*\sqrt{3}/\text{abs}(2*x - 2*\sqrt{5}))$

Fricas [A]

time = 0.31, size = 32, normalized size = 1.28

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}(x^2-1)\sqrt{-x^2+3}}{4(x^2-3)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-x^2+5)/(-x^2+3)^(1/2),x, algorithm="fricas")`

[Out] $-1/4*\sqrt{2}*\arctan(1/4*\sqrt{2}*(x^2-1)*\sqrt{-x^2+3}/(x^2-3))$

Sympy [A]

time = 2.29, size = 24, normalized size = 0.96

$$-\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{3-x^2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x**2+5)/(-x**2+3)**(1/2),x)

[Out] -sqrt(2)*atan(sqrt(2)*sqrt(3 - x**2)/2)/2

Giac [A]

time = 0.00, size = 27, normalized size = 1.08

$$-\frac{1}{2}\sqrt{2} \arctan\left(\frac{\sqrt{-x^2+3}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^2+5)/(-x^2+3)^(1/2),x)

[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 3))

Mupad [B]

time = 0.76, size = 83, normalized size = 3.32

$$\frac{\sqrt{2} \ln\left(\frac{\frac{\sqrt{2}(\sqrt{5}^{x+3})}{2} + \sqrt{3-x^2} \cdot 1i}{x+\sqrt{5}}\right) \cdot 1i}{4} - \frac{\sqrt{2} \ln\left(\frac{\frac{\sqrt{2}(\sqrt{5}^{x-3})}{2} - \sqrt{3-x^2} \cdot 1i}{x-\sqrt{5}}\right) \cdot 1i}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x/((3 - x^2)^(1/2)*(x^2 - 5)),x)

[Out] - (2^(1/2)*log(((2^(1/2)*(5^(1/2)*x + 3))/2 + (3 - x^2)^(1/2)*1i)/(x + 5^(1/2))) * 1i)/4 - (2^(1/2)*log(((2^(1/2)*(5^(1/2)*x - 3))/2 - (3 - x^2)^(1/2)*1i)/(x - 5^(1/2))) * 1i)/4

$$3.242 \quad \int \frac{1}{\sqrt{2+x^2}(-1+x^4)} dx$$

Optimal. Leaf size=43

$$-\frac{1}{2} \tan^{-1} \left(\frac{x}{\sqrt{2+x^2}} \right) - \frac{\tanh^{-1} \left(\frac{\sqrt{3}x}{\sqrt{2+x^2}} \right)}{2\sqrt{3}}$$

[Out] $-1/2*\arctan(x/(x^2+2)^{(1/2)})-1/6*\operatorname{arctanh}(x*3^{(1/2)}/(x^2+2)^{(1/2}))*3^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1189, 385, 212, 209}

$$-\frac{1}{2} \tan^{-1} \left(\frac{x}{\sqrt{x^2+2}} \right) - \frac{\tanh^{-1} \left(\frac{\sqrt{3}x}{\sqrt{x^2+2}} \right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[2 + x^2]*(-1 + x^4)),x]

[Out] $-1/2*\operatorname{ArcTan}[x/\operatorname{Sqrt}[2 + x^2]] - \operatorname{ArcTanh}[(\operatorname{Sqrt}[3]*x)/\operatorname{Sqrt}[2 + x^2]]/(2*\operatorname{Sqrt}[3])$

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 1189

Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{r = Rt[(-a)*c, 2]}, Dist[-c/(2*r), Int[(d + e*x^2)^q/(r - c*x^2), x], x] - Di

```
st[c/(2*r), Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e, q}, x] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[q]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{2+x^2}(-1+x^4)} dx &= -\left(\frac{1}{2} \int \frac{1}{(1-x^2)\sqrt{2+x^2}} dx\right) - \frac{1}{2} \int \frac{1}{(1+x^2)\sqrt{2+x^2}} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{1-3x^2} dx, x, \frac{x}{\sqrt{2+x^2}}\right)\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \frac{x}{\sqrt{2+x^2}}\right) \\ &= -\frac{1}{2} \tan^{-1}\left(\frac{x}{\sqrt{2+x^2}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{2+x^2}}\right)}{2\sqrt{3}} \end{aligned}$$

Mathematica [A]

time = 0.10, size = 57, normalized size = 1.33

$$\frac{1}{6} \left(3 \tan^{-1}\left(1 + x^2 - x\sqrt{2+x^2}\right) - \sqrt{3} \tanh^{-1}\left(\frac{1-x^2+x\sqrt{2+x^2}}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + x^2]*(-1 + x^4)),x]
```

```
[Out] (3*ArcTan[1 + x^2 - x*Sqrt[2 + x^2]] - Sqrt[3]*ArcTanh[(1 - x^2 + x*Sqrt[2 + x^2])/Sqrt[3]])/6
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((x^4 - 1)*Sqrt[x^2 + 2]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(31) = 62.

time = 0.15, size = 70, normalized size = 1.63

method	result
--------	--------

default	$\frac{\sqrt{3} \operatorname{arctanh}\left(\frac{(2x+4)\sqrt{3}}{6\sqrt{(-1+x)^2+1+2x}}\right) + \sqrt{3} \operatorname{arctanh}\left(\frac{(-2x+4)\sqrt{3}}{6\sqrt{(1+x)^2+1-2x}}\right) - \operatorname{arctan}\left(\frac{x}{\sqrt{x^2+2}}\right)}{12}$
trager	$\frac{\operatorname{RootOf}(-Z^2+1) \ln\left(\frac{\sqrt{x^2+2} \operatorname{RootOf}(-Z^2+1)}{x^2+1}\right) - \operatorname{RootOf}(-Z^2-3) \ln\left(\frac{2 \operatorname{RootOf}(-Z^2-3) x^2 + 3\sqrt{x^2+2}}{(1+x)(-1+x)}\right)}{4 \cdot 12}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4-1)/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/12*3^{(1/2)}*\operatorname{arctanh}(1/6*(2*x+4)*3^{(1/2)}/((-1+x)^2+1+2*x)^{(1/2)})+1/12*3^{(1/2)}*\operatorname{arctanh}(1/6*(-2*x+4)*3^{(1/2)}/((1+x)^2+1-2*x)^{(1/2)})-1/2*\operatorname{arctan}(x/(x^2+2)^{(1/2)})$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4-1)/(x^2+2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/((x^4 - 1)*sqrt(x^2 + 2)), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(31) = 62.

time = 0.31, size = 72, normalized size = 1.67

$$\frac{1}{12} \sqrt{3} \log\left(\frac{4x^2 - \sqrt{3}(2x^2 + 1) - \sqrt{x^2 + 2}(2\sqrt{3}x - 3x) + 2}{x^2 - 1}\right) - \frac{1}{2} \operatorname{arctan}\left(-x^2 + \sqrt{x^2 + 2}x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4-1)/(x^2+2)^(1/2),x, algorithm="fricas")`

[Out] $1/12*\sqrt{3}*\log((4*x^2 - \sqrt{3}*(2*x^2 + 1) - \sqrt{x^2 + 2}*(2*\sqrt{3}*x - 3*x) + 2)/(x^2 - 1)) - 1/2*\operatorname{arctan}(-x^2 + \sqrt{x^2 + 2}*x - 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x-1)(x+1)(x^2+1)\sqrt{x^2+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**4-1)/(x**2+2)**(1/2),x)

[Out] Integral(1/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(x**2 + 2)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(31) = 62.
time = 0.01, size = 91, normalized size = 2.12

$$-8 \left(\frac{\arctan \left(\frac{(\sqrt{x^2 + 2} - x)^2}{2} \right)}{16} + \frac{\ln \left(\left| \frac{2(\sqrt{x^2 + 2} - x)^2 - 8 - 4\sqrt{3}}{2(\sqrt{x^2 + 2} - x)^2 - 8 + 4\sqrt{3}} \right| \right)}{32\sqrt{3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4-1)/(x^2+2)^(1/2),x)

[Out] -1/12*sqrt(3)*log(abs(2*(x - sqrt(x^2 + 2))^2 - 4*sqrt(3) - 8)/abs(2*(x - sqrt(x^2 + 2))^2 + 4*sqrt(3) - 8)) + 1/2*arctan(1/2*(x - sqrt(x^2 + 2))^2)

Mupad [B]

time = 0.11, size = 107, normalized size = 2.49

$$\frac{\sqrt{3} (\ln(x-1) - \ln(x + \sqrt{3} \sqrt{x^2+2} + 2))}{12} - \frac{\sqrt{3} (\ln(x+1) - \ln(\sqrt{3} \sqrt{x^2+2} - x + 2))}{12} + \frac{\ln(\sqrt{x^2+2} + 2 - x \operatorname{li})}{4} - \frac{\ln(\sqrt{x^2+2} + 2 + x \operatorname{li})}{4} + \frac{\ln(x-i) \operatorname{li}}{4} - \frac{\ln(x+i) \operatorname{li}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^2 + 2)^(1/2)*(x^4 - 1)),x)

[Out] (log((x^2 + 2)^(1/2) - x*1i + 2)*1i)/4 - (log(x*1i + (x^2 + 2)^(1/2) + 2)*1i)/4 + (log(x - 1i)*1i)/4 - (log(x + 1i)*1i)/4 + (3^(1/2)*(log(x - 1) - log(x + 3^(1/2)*(x^2 + 2)^(1/2) + 2)))/12 - (3^(1/2)*(log(x + 1) - log(3^(1/2)*(x^2 + 2)^(1/2) - x + 2)))/12

$$3.243 \quad \int \frac{x}{(-1+x^2) \sqrt{4+2x+x^2}} dx$$

Optimal. Leaf size=62

$$-\frac{\tanh^{-1}\left(\frac{5+2x}{\sqrt{7}\sqrt{4+2x+x^2}}\right)}{2\sqrt{7}} - \frac{\tanh^{-1}\left(\frac{\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

[Out] -1/6*arctanh(1/3*(x^2+2*x+4)^(1/2)*3^(1/2))*3^(1/2)-1/14*arctanh(1/7*(5+2*x)*7^(1/2)/(x^2+2*x+4)^(1/2))*7^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1047, 738, 212, 702, 213}

$$-\frac{\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x/((-1 + x^2)*Sqrt[4 + 2*x + x^2]),x]

[Out] -1/2*ArcTanh[(5 + 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])]/Sqrt[7] - ArcTanh[Sqrt[4 + 2*x + x^2]/Sqrt[3]]/(2*Sqrt[3])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 702

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0]

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:= Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:= With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}} dx &= \frac{1}{2} \int \frac{1}{(-1+x)\sqrt{4+2x+x^2}} dx + \frac{1}{2} \int \frac{1}{(1+x)\sqrt{4+2x+x^2}} dx \\ &= 2 \operatorname{Subst} \left(\int \frac{1}{-12+4x^2} dx, x, \sqrt{4+2x+x^2} \right) - \operatorname{Subst} \left(\int \frac{1}{28-x^2} dx, x, \sqrt{4+2x+x^2} \right) \\ &= -\frac{\tanh^{-1} \left(\frac{10+4x}{2\sqrt{7}\sqrt{4+2x+x^2}} \right)}{2\sqrt{7}} - \frac{\tanh^{-1} \left(\frac{\sqrt{4+2x+x^2}}{\sqrt{3}} \right)}{2\sqrt{3}} \end{aligned}$$

Mathematica [A]

time = 0.14, size = 62, normalized size = 1.00

$$\frac{\tanh^{-1} \left(\frac{1+x-\sqrt{4+2x+x^2}}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{\tanh^{-1} \left(\frac{1-x+\sqrt{4+2x+x^2}}{\sqrt{7}} \right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-1 + x^2)*Sqrt[4 + 2*x + x^2]), x]
```

```
[Out] ArcTanh[(1 + x - Sqrt[4 + 2*x + x^2])/Sqrt[3]]/Sqrt[3] - ArcTanh[(1 - x + Sqrt[4 + 2*x + x^2])/Sqrt[7]]/Sqrt[7]
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[x/((x^2 - 1)*Sqrt[x^2 + 2*x + 4]),x']')`

[Out] caught exception: maximum recursion depth exceeded

Maple [A]

time = 0.19, size = 49, normalized size = 0.79

method	result
default	$\frac{\sqrt{7} \operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{14} - \frac{\sqrt{3} \operatorname{arctanh}\left(\frac{\sqrt{3}}{\sqrt{(1+x)^2+3}}\right)}{6}$
trager	$-\frac{\operatorname{RootOf}(-Z^2-3) \ln\left(\frac{\sqrt{x^2+2x+4} + \operatorname{RootOf}(-Z^2-3)}{1+x}\right)}{6} + \frac{\operatorname{RootOf}(-Z^2-7) \ln\left(-\frac{-2\operatorname{RootOf}(-Z^2-7)x+7\sqrt{x^2+2x+4}}{-1}\right)}{14}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(x^2-1)/(x^2+2*x+4)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/14*7^{(1/2)}*\operatorname{arctanh}(1/14*(10+4*x)*7^{(1/2)/((-1+x)^2+3+4*x)^{(1/2)})-1/6*3^{(1/2)}*\operatorname{arctanh}(3^{(1/2)/((1+x)^2+3)^{(1/2)})$

Maxima [A]

time = 0.36, size = 54, normalized size = 0.87

$$-\frac{1}{14} \sqrt{7} \operatorname{arsinh}\left(\frac{4\sqrt{3}x}{3|2x-2|} + \frac{10\sqrt{3}}{3|2x-2|}\right) - \frac{1}{6} \sqrt{3} \operatorname{arsinh}\left(\frac{2\sqrt{3}}{|2x+2|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^2-1)/(x^2+2*x+4)^(1/2),x, algorithm="maxima")`

[Out] $-1/14*\operatorname{sqrt}(7)*\operatorname{arcsinh}(4/3*\operatorname{sqrt}(3)*x/\operatorname{abs}(2*x-2)+10/3*\operatorname{sqrt}(3)/\operatorname{abs}(2*x-2))-1/6*\operatorname{sqrt}(3)*\operatorname{arcsinh}(2*\operatorname{sqrt}(3)/\operatorname{abs}(2*x+2))$

Fricas [A]

time = 0.36, size = 74, normalized size = 1.19

$$\frac{1}{14} \sqrt{7} \log\left(\frac{\sqrt{7}(2x+5) + \sqrt{x^2+2x+4}(2\sqrt{7}-7) - 4x - 10}{x-1}\right) + \frac{1}{6} \sqrt{3} \log\left(-\frac{\sqrt{3} - \sqrt{x^2+2x+4}}{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^2-1)/(x^2+2*x+4)^(1/2),x, algorithm="fricas")`

[Out] $1/14*\sqrt{7}*\log((\sqrt{7})*(2*x + 5) + \sqrt{x^2 + 2*x + 4}*(2*\sqrt{7} - 7) - 4*x - 10)/(x - 1)) + 1/6*\sqrt{3}*\log(-(\sqrt{3}) - \sqrt{x^2 + 2*x + 4})/(x + 1))$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x-1)(x+1)\sqrt{x^2+2x+4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x**2-1)/(x**2+2*x+4)**(1/2),x)`

[Out] `Integral(x/((x - 1)*(x + 1)*sqrt(x**2 + 2*x + 4)), x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(48) = 96.

time = 0.01, size = 136, normalized size = 2.19

$$2 \left(\frac{\ln \left(\frac{2(\sqrt{x^2+2x+4}-x)^{-2-2\sqrt{3}}}{2(\sqrt{x^2+2x+4}-x)^{-2+2\sqrt{3}}} \right)}{4\sqrt{3}} + \frac{\ln \left(\frac{2(\sqrt{x^2+2x+4}-x)^{+2-2\sqrt{7}}}{2(\sqrt{x^2+2x+4}-x)^{+2+2\sqrt{7}}} \right)}{4\sqrt{7}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^2-1)/(x^2+2*x+4)^(1/2),x)`

[Out] $1/14*\sqrt{7}*\log(\text{abs}(-2*x - 2*\sqrt{7}) + 2*\sqrt{x^2 + 2*x + 4} + 2)/\text{abs}(-2*x + 2*\sqrt{7} + 2*\sqrt{x^2 + 2*x + 4} + 2)) + 1/6*\sqrt{3}*\log(-1/2*\text{abs}(-2*x - 2*\sqrt{3}) + 2*\sqrt{x^2 + 2*x + 4} - 2)/(x - \sqrt{3}) - \sqrt{x^2 + 2*x + 4} + 1))$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{(x^2-1)\sqrt{x^2+2x+4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/((x^2 - 1)*(2*x + x^2 + 4)^(1/2)),x)`

[Out] `int(x/((x^2 - 1)*(2*x + x^2 + 4)^(1/2)), x)`

$$3.244 \quad \int \frac{1}{\sqrt{5+2x+x^2}(-8+x^3)} dx$$

Optimal. Leaf size=82

$$-\frac{\tan^{-1}\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right)}{4\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{7+3x}{\sqrt{13}\sqrt{5+2x+x^2}}\right)}{12\sqrt{13}} + \frac{1}{12} \tanh^{-1}\left(\sqrt{5+2x+x^2}\right)$$

[Out] 1/12*arctanh((x^2+2*x+5)^(1/2))-1/12*arctan(1/3*(1+x)*3^(1/2)/(x^2+2*x+5)^(1/2))*3^(1/2)-1/156*arctanh(1/13*(7+3*x)*13^(1/2)/(x^2+2*x+5)^(1/2))*13^(1/2)

Rubi [A]

time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2099, 738, 212, 1039, 996, 210, 1038}

$$-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{4\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}} + \frac{1}{12} \tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[5 + 2*x + x^2]*(-8 + x^3)),x]

[Out] -1/4*ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])]/Sqrt[3] - ArcTanh[(7 + 3*x)/(Sqrt[13]*Sqrt[5 + 2*x + x^2])]/(12*Sqrt[13]) + ArcTanh[Sqrt[5 + 2*x + x^2]]/12

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 738

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,

$d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[2*c*d - b*e, 0]$

Rule 996

$\text{Int}[1/((a_) + (b_)*(x_) + (c_)*(x_)^2)*\text{Sqrt}[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] \rightarrow \text{Dist}[-2*e, \text{Subst}[\text{Int}[1/(e*(b*e - 4*a*f) - (b*d - a*e)*x^2), x], x, (e + 2*f*x)/\text{Sqrt}[d + e*x + f*x^2]], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{EqQ}[c*e - b*f, 0]$

Rule 1038

$\text{Int}(((g_) + (h_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2)*\text{Sqrt}[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] \rightarrow \text{Dist}[-2*g, \text{Subst}[\text{Int}[1/(b*d - a*e - b*x^2), x], x, \text{Sqrt}[d + e*x + f*x^2]], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, g, h\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{EqQ}[c*e - b*f, 0] \&\& \text{EqQ}[h*e - 2*g*f, 0]$

Rule 1039

$\text{Int}(((g_) + (h_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2)*\text{Sqrt}[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] \rightarrow \text{Dist}[-(h*e - 2*g*f)/(2*f), \text{Int}[1/(a + b*x + c*x^2)*\text{Sqrt}[d + e*x + f*x^2]), x], x] + \text{Dist}[h/(2*f), \text{Int}[(e + 2*f*x)/(a + b*x + c*x^2)*\text{Sqrt}[d + e*x + f*x^2]), x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, g, h\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{EqQ}[c*e - b*f, 0] \&\& \text{NeQ}[h*e - 2*g*f, 0]$

Rule 2099

$\text{Int}[(P_)^{(p_)}*(Q_)^{(q_)}, x_Symbol] \rightarrow \text{With}\{PP = \text{Factor}[P]\}, \text{Int}[\text{ExpandIntegrand}[PP^p*Q^q, x], x] /;$ $!\text{SumQ}[\text{NonfreeFactors}[PP, x]] /;$ $\text{FreeQ}[q, x] \&\& \text{PolyQ}[P, x] \&\& \text{PolyQ}[Q, x] \&\& \text{IntegerQ}[p] \&\& \text{NeQ}[P, x]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{5+2x+x^2}(-8+x^3)} dx &= \int \left(\frac{1}{12(-2+x)\sqrt{5+2x+x^2}} + \frac{-4-x}{12(4+2x+x^2)\sqrt{5+2x+x^2}} \right) dx \\
&= \frac{1}{12} \int \frac{1}{(-2+x)\sqrt{5+2x+x^2}} dx + \frac{1}{12} \int \frac{-4-x}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx \\
&= -\left(\frac{1}{24} \int \frac{2+2x}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx \right) - \frac{1}{6} \text{Subst} \left(\int \frac{1}{52-x^2} dx, x, \sqrt{5+2x+x^2} \right) \\
&= -\frac{\tanh^{-1} \left(\frac{7+3x}{\sqrt{13}\sqrt{5+2x+x^2}} \right)}{12\sqrt{13}} + \frac{1}{6} \text{Subst} \left(\int \frac{1}{2-2x^2} dx, x, \sqrt{5+2x+x^2} \right) \\
&= -\frac{\tan^{-1} \left(\frac{2+2x}{2\sqrt{3}\sqrt{5+2x+x^2}} \right)}{4\sqrt{3}} - \frac{\tanh^{-1} \left(\frac{7+3x}{\sqrt{13}\sqrt{5+2x+x^2}} \right)}{12\sqrt{13}} + \frac{1}{12} \ln \left| \frac{2-x+\sqrt{5+2x+x^2}}{2+x+\sqrt{5+2x+x^2}} \right|
\end{aligned}$$

Mathematica [A]

time = 0.22, size = 90, normalized size = 1.10

$$\frac{1}{156} \left(13\sqrt{3} \tan^{-1} \left(\frac{4+2x+x^2-(1+x)\sqrt{5+2x+x^2}}{\sqrt{3}} \right) + 13 \tanh^{-1} \left(\sqrt{5+2x+x^2} \right) - 2\sqrt{13} \tanh^{-1} \left(\frac{2-x+\sqrt{5+2x+x^2}}{\sqrt{13}} \right) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[1/(Sqrt[5 + 2*x + x^2]*(-8 + x^3)), x]`

```
[Out] (13*Sqrt[3]*ArcTan[(4 + 2*x + x^2 - (1 + x)*Sqrt[5 + 2*x + x^2])/Sqrt[3]] +
13*ArcTanh[Sqrt[5 + 2*x + x^2]] - 2*Sqrt[13]*ArcTanh[(2 - x + Sqrt[5 + 2*x
+ x^2])/Sqrt[13]])/156
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/((x^3 - 8)*Sqrt[x^2 + 2*x + 5]), x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.36, size = 69, normalized size = 0.84

method	result
--------	--------

default	$\frac{\sqrt{13} \operatorname{arctanh}\left(\frac{(14+6x)\sqrt{13}}{26\sqrt{(-2+x)^2+1+6x}}\right)}{156} + \frac{\operatorname{arctanh}\left(\sqrt{x^2+2x+5}\right)}{12} - \frac{\sqrt{3} \operatorname{arctan}\left(\frac{\sqrt{3}^{(2+2x)}}{6\sqrt{x^2+2x+5}}\right)}{12}$
trager	$\frac{\ln\left(-\frac{5760 \operatorname{RootOf}\left(144_Z^2+12_Z+1\right)^2 x+252\sqrt{x^2+2x+5} \operatorname{RootOf}\left(144_Z^2+12_Z+1\right)+348 \operatorname{RootOf}\left(144_Z^2+12_Z+1\right) x}{6 \operatorname{RootOf}\left(144_Z^2+12_Z+1\right) x-1}\right)}{12}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3-8)/(x^2+2*x+5)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/156*13^{(1/2)}*\operatorname{arctanh}(1/26*(14+6*x)*13^{(1/2)/((-2+x)^2+1+6*x)^{(1/2)})+1/12*\operatorname{arctanh}((x^2+2*x+5)^{(1/2)})-1/12*3^{(1/2)}*\operatorname{arctan}(1/6*3^{(1/2)/(x^2+2*x+5)^{(1/2)}}*(2+2*x))$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^3-8)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/((x^3 - 8)*sqrt(x^2 + 2*x + 5)), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(64) = 128.

time = 0.32, size = 151, normalized size = 1.84

$$\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(x+2)+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right)-\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right)+\frac{1}{156}\sqrt{13}\log\left(\frac{\sqrt{13}(3x+7)+\sqrt{x^2+2x+5}(3\sqrt{13}-13)-9x-21}{x-2}\right)-\frac{1}{24}\log(x^2-\sqrt{x^2+2x+5}(x+2)+3x+6)+\frac{1}{24}\log(x^2-\sqrt{x^2+2x+5}x+x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^3-8)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")`

[Out] $1/12*\sqrt{3}*\operatorname{arctan}(-1/3*\sqrt{3}*(x+2)+1/3*\sqrt{3}*\sqrt{x^2+2*x+5})-1/12*\sqrt{3}*\operatorname{arctan}(-1/3*\sqrt{3}*x+1/3*\sqrt{3}*\sqrt{x^2+2*x+5})+1/156*\sqrt{13}*\log((\sqrt{13}*(3*x+7)+\sqrt{x^2+2*x+5}*(3*\sqrt{13}-13)-9*x-21)/(x-2))-1/24*\log(x^2-\sqrt{x^2+2*x+5}*(x+2)+3*x+6)+1/24*\log(x^2-\sqrt{x^2+2*x+5}*x+x+4)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x-2)(x^2+2x+4)\sqrt{x^2+2x+5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**3-8)/(x**2+2*x+5)**(1/2),x)

[Out] Integral(1/((x - 2)*(x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(64) = 128.

time = 0.01, size = 201, normalized size = 2.45

$$2 \left(\frac{\ln \left(\frac{(\sqrt{x^2+2x+5}-x)^2+3}{48} \right)}{48} - \frac{\arctan \left(\frac{\sqrt{x^2+2x+5}-x}{\sqrt{3}} \right)}{8\sqrt{3}} - \frac{\ln \left(\frac{(\sqrt{x^2+2x+5}-x)^2-4(\sqrt{x^2+2x+5}-x)+7}{48} \right)}{48} + \frac{\arctan \left(\frac{\sqrt{x^2+2x+5}-x-2}{8\sqrt{3}} \right)}{8\sqrt{3}} + \frac{\ln \left(\frac{2(\sqrt{x^2+2x+5}-x)+4-2\sqrt{13}}{2(\sqrt{x^2+2x+5}-x)+4+2\sqrt{13}} \right)}{24\sqrt{13}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-8)/(x^2+2*x+5)^(1/2),x)

[Out] 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) - 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5))) + 1/156*sqrt(13)*log(abs(-2*x - 2*sqrt(13) + 2*sqrt(x^2 + 2*x + 5) + 4)/abs(-2*x + 2*sqrt(13) + 2*sqrt(x^2 + 2*x + 5) + 4)) - 1/24*log((x - sqrt(x^2 + 2*x + 5))^2 + 4*x - 4*sqrt(x^2 + 2*x + 5) + 7) + 1/24*log((x - sqrt(x^2 + 2*x + 5))^2 + 3)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x^3 - 8) \sqrt{x^2 + 2x + 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^3 - 8)*(2*x + x^2 + 5)^(1/2)),x)

[Out] int(1/((x^3 - 8)*(2*x + x^2 + 5)^(1/2)), x)

$$3.245 \quad \int \frac{x}{(4+x+x^2) \sqrt{5+4x+4x^2}} dx$$

Optimal. Leaf size=63

$$\frac{\tan^{-1}\left(\frac{\sqrt{5+4x+4x^2}}{\sqrt{11}}\right)}{\sqrt{11}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{11}{15}}^{(1+2x)}}{\sqrt{5+4x+4x^2}}\right)}{\sqrt{165}}$$

[Out] 1/11*arctan(1/11*(4*x^2+4*x+5)^(1/2)*11^(1/2))*11^(1/2)-1/165*arctanh(1/15*(1+2*x)*165^(1/2)/(4*x^2+4*x+5)^(1/2))*165^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1039, 996, 213, 1038, 210}

$$\frac{\tan^{-1}\left(\frac{\sqrt{4x^2+4x+5}}{\sqrt{11}}\right)}{\sqrt{11}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{11}{15}}^{(2x+1)}}{\sqrt{4x^2+4x+5}}\right)}{\sqrt{165}}$$

Antiderivative was successfully verified.

[In] Int[x/((4 + x + x^2)*Sqrt[5 + 4*x + 4*x^2]),x]

[Out] ArcTan[Sqrt[5 + 4*x + 4*x^2]/Sqrt[11]]/Sqrt[11] - ArcTanh[(Sqrt[11/15]*(1 + 2*x))/Sqrt[5 + 4*x + 4*x^2]]/Sqrt[165]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 996

Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)

*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0]

Rule 1038

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*g, Subst[Int[1/(b*d - a*e - b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && EqQ[h*e - 2*g*f, 0]

Rule 1039

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-(h*e - 2*g*f)/(2*f), Int[1/(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]], x], x] + Dist[h/(2*f), Int[(e + 2*f*x)/(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && NeQ[h*e - 2*g*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}} dx &= \frac{1}{8} \int \frac{4+8x}{(4+x+x^2)\sqrt{5+4x+4x^2}} dx - \frac{1}{2} \int \frac{1}{(4+x+x^2)\sqrt{5+4x+4x^2}} dx \\ &= 4 \text{Subst} \left(\int \frac{1}{-240+11x^2} dx, x, \frac{4+8x}{\sqrt{5+4x+4x^2}} \right) - \text{Subst} \left(\int \frac{1}{-11x^2+4x+4} dx, x, \frac{4+8x}{\sqrt{5+4x+4x^2}} \right) \\ &= \frac{\tan^{-1} \left(\frac{\sqrt{5+4x+4x^2}}{\sqrt{11}} \right)}{\sqrt{11}} - \frac{\tanh^{-1} \left(\frac{\sqrt{\frac{11}{15}}(1+2x)}{\sqrt{5+4x+4x^2}} \right)}{\sqrt{165}} \end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.12, size = 102, normalized size = 1.62

$$\frac{1}{2} \text{RootSum} \left[69 - 108\#1 + 58\#1^2 - 4\#1^3 + \#1^4 \&, \frac{-5 \log(-2x + \sqrt{5+4x+4x^2} - \#1) + \log(-2x + \sqrt{5+4x+4x^2} - \#1) \#1^2}{-27 + 29\#1 - 3\#1^2 + \#1^3} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x/((4 + x + x^2)*Sqrt[5 + 4*x + 4*x^2]), x]

```
[Out] RootSum[69 - 108*#1 + 58*#1^2 - 4*#1^3 + #1^4 & , (-5*Log[-2*x + Sqrt[5 + 4*x + 4*x^2] - #1] + Log[-2*x + Sqrt[5 + 4*x + 4*x^2] - #1]*#1^2)/(-27 + 29*#1 - 3*#1^2 + #1^3) & ]/2
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/((x^2 + x + 4)*Sqrt[4*x^2 + 4*x + 5]),x]')
```

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.49, size = 53, normalized size = 0.84

method	result
default	$\frac{\arctan\left(\frac{\sqrt{4x^2 + 4x + 5} \sqrt{11}}{11}\right) \sqrt{11}}{11} - \frac{\sqrt{165} \operatorname{arctanh}\left(\frac{\sqrt{165} (8x+4)}{60\sqrt{4x^2 + 4x + 5}}\right)}{165}$
trager	$165 \ln\left(\frac{8276400 \operatorname{RootOf}\left(27225_Z^4 + 1155_Z^2 + 16\right)^5}{x + 385770 \operatorname{RootOf}\left(27225_Z^4 + 1155_Z^2 + 16\right)^3} x + 9405 \sqrt{4x^2 + 4x + 5} \operatorname{RootOf}\left(27225_Z^4 + 1155_Z^2 + 16\right)^3}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*arctan(1/11*(4*x^2+4*x+5)^(1/2)*11^(1/2))*11^(1/2)-1/165*165^(1/2)*arc tanh(1/60*165^(1/2)*(8*x+4)/(4*x^2+4*x+5)^(1/2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(4*x^2 + 4*x + 5)*(x^2 + x + 4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(52) = 104.

time = 0.34, size = 307, normalized size = 4.87

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x, algorithm="fricas")

[Out] 2/165*sqrt(165)*sqrt(15)*arctan(1/60*sqrt(2)*sqrt(4*x^2 - sqrt(4*x^2 + 4*x + 5)*(2*x + 1) + 4*x - sqrt(165) + 16)*(sqrt(165)*sqrt(15) + 15*sqrt(15)) + 1/60*sqrt(165)*sqrt(15)*(2*x + 1) - 1/60*sqrt(4*x^2 + 4*x + 5)*(sqrt(165)*sqrt(15) + 15*sqrt(15)) + 1/4*sqrt(15)*(2*x + 1)) + 2/165*sqrt(165)*sqrt(15)*arctan(1/60*sqrt(2)*sqrt(4*x^2 - sqrt(4*x^2 + 4*x + 5)*(2*x + 1) + 4*x + sqrt(165) + 16)*(sqrt(165)*sqrt(15) - 15*sqrt(15)) + 1/60*sqrt(165)*sqrt(15)*(2*x + 1) - 1/60*sqrt(4*x^2 + 4*x + 5)*(sqrt(165)*sqrt(15) - 15*sqrt(15)) - 1/4*sqrt(15)*(2*x + 1)) - 1/330*sqrt(165)*log(460800*x^2 - 115200*sqrt(4*x^2 + 4*x + 5)*(2*x + 1) + 460800*x + 115200*sqrt(165) + 1843200) + 1/330*sqrt(165)*log(460800*x^2 - 115200*sqrt(4*x^2 + 4*x + 5)*(2*x + 1) + 460800*x - 115200*sqrt(165) + 1843200)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x^2 + x + 4) \sqrt{4x^2 + 4x + 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x**2+x+4)/(4*x**2+4*x+5)**(1/2),x)

[Out] Integral(x/((x**2 + x + 4)*sqrt(4*x**2 + 4*x + 5)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(52) = 104.

time = 0.02, size = 292, normalized size = 4.63

$i \left(\frac{1}{200} \sqrt{165} \ln \left(\frac{300(\sqrt{4x^2+4x+5}-2x)-300}{300(\sqrt{4x^2+4x+5}-2x)-300} + \frac{300\sqrt{11}-300\sqrt{15}}{300\sqrt{11}-300\sqrt{15}} \right) - \frac{1}{330} \sqrt{165} \sqrt{11} \arctan \left(\frac{-1(\sqrt{4x^2+4x+5}-2x)+1}{4\sqrt{11}-4\sqrt{15}} \right) - \frac{1}{200} \sqrt{165} \ln \left(\frac{300(\sqrt{4x^2+4x+5}-2x)-300}{300(\sqrt{4x^2+4x+5}-2x)-300} - \frac{300\sqrt{11}-300\sqrt{15}}{300\sqrt{11}-300\sqrt{15}} \right) + \frac{1}{330} \sqrt{165} \sqrt{11} \arctan \left(\frac{\sqrt{4x^2+4x+5}-2x+1}{-\sqrt{11}-\sqrt{15}} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x)

[Out] 1/165*sqrt(165)*sqrt(15)*arctan(-(2*x - sqrt(4*x^2 + 4*x + 5) + 1)/(sqrt(15) + sqrt(11))) - 1/165*sqrt(165)*sqrt(15)*arctan(-(2*x - sqrt(4*x^2 + 4*x + 5) + 1)/(sqrt(15) - sqrt(11))) - 1/330*sqrt(165)*log(90000*(2*x - sqrt(4*x^2 + 4*x + 5) + 1)^2 + 90000*(sqrt(15) + sqrt(11))^2) + 1/330*sqrt(165)*log(90000*(2*x - sqrt(4*x^2 + 4*x + 5) + 1)^2 + 90000*(sqrt(15) - sqrt(11))^2)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sqrt{4x^2 + 4x + 5} (x^2 + x + 4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((4*x + 4*x^2 + 5)^(1/2)*(x + x^2 + 4)),x)
```

```
[Out] int(x/((4*x + 4*x^2 + 5)^(1/2)*(x + x^2 + 4)), x)
```

$$3.246 \quad \int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}} dx$$

Optimal. Leaf size=56

$$-2\sqrt{2} \tan^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x+x^2}}\right) + \sqrt{2} \tanh^{-1}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x+x^2}}\right)$$

[Out] $-2*\arctan(1/2*(1-x)*2^{(1/2)/(x^2+x+1)^{(1/2)})}*2^{(1/2)}+\operatorname{arctanh}(1/2*(1+x)*2^{(1/2)/(x^2+x+1)^{(1/2)})}*2^{(1/2)}$

Rubi [A]

time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1050, 1044, 213, 209}

$$\sqrt{2} \tanh^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+x+1}}\right) - 2\sqrt{2} \tan^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(3+x)/((1+x^2)*\text{Sqrt}[1+x+x^2]),x]$

[Out] $-2*\text{Sqrt}[2]*\text{ArcTan}[(1-x)/(\text{Sqrt}[2]*\text{Sqrt}[1+x+x^2])] + \text{Sqrt}[2]*\text{ArcTanh}[(1+x)/(\text{Sqrt}[2]*\text{Sqrt}[1+x+x^2])]$

Rule 209

$\text{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))*\text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 213

$\text{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[b, 2])^{-1})*\text{ArcTanh}[\text{Rt}[b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 1044

$\text{Int}[(g_+ + (h_+)*(x_+))/((a_+ + (c_+)*(x_+)^2)*\text{Sqrt}[(d_+ + (e_+)*(x_+) + (f_+)*(x_+)^2]), x_Symbol] \rightarrow \text{Dist}[-2*a*g*h, \text{Subst}[\text{Int}[1/\text{Simp}[2*a^2*g*h*c + a*e*x^2, x], x], x, \text{Simp}[a*h - g*c*x, x]/\text{Sqrt}[d + e*x + f*x^2]], x] /; \text{FreeQ}\{a, c, d, e, f, g, h\}, x \ \&\& \ \text{EqQ}[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]$

Rule 1050

$\text{Int}[(g_+ + (h_+)*(x_+))/((a_+ + (c_+)*(x_+)^2)*\text{Sqrt}[(d_+ + (e_+)*(x_+) + (f_+)*(x_+)^2]), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[(c*d - a*f)^2 + a*c*e^2, 2]\}, \text{Dist}$

```
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a +
c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rubi steps

$$\begin{aligned} \int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}} dx &= -\left(\frac{1}{2} \int \frac{-4-4x}{(1+x^2)\sqrt{1+x+x^2}} dx\right) + \frac{1}{2} \int \frac{2-2x}{(1+x^2)\sqrt{1+x+x^2}} dx \\ &= 4\text{Subst}\left(\int \frac{1}{-8+x^2} dx, x, \frac{-2-2x}{\sqrt{1+x+x^2}}\right) + 16\text{Subst}\left(\int \frac{1}{32+x^2} dx, x, \frac{-2-2x}{\sqrt{1+x+x^2}}\right) \\ &= -2\sqrt{2} \tan^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x+x^2}}\right) + \sqrt{2} \tanh^{-1}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x+x^2}}\right) \end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.12, size = 103, normalized size = 1.84

$$\frac{1}{2}\text{RootSum}\left[2-4\#1+2\#1^2+\#1^4\&, \frac{2\log(-x+\sqrt{1+x+x^2}-\#1)-6\log(-x+\sqrt{1+x+x^2}-\#1)\#1+\log(-x+\sqrt{1+x+x^2}-\#1)\#1^2}{-1+\#1+\#1^3}\& \right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x)/((1 + x^2)*Sqrt[1 + x + x^2]), x]
```

```
[Out] RootSum[2 - 4*#1 + 2*#1^2 + #1^4 & , (2*Log[-x + Sqrt[1 + x + x^2] - #1] -
6*Log[-x + Sqrt[1 + x + x^2] - #1]*#1 + Log[-x + Sqrt[1 + x + x^2] - #1]*#1
^2)/(-1 + #1 + #1^3) & ]/2
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x + 3)/((x^2 + 1)*Sqrt[x^2 + x + 1]), x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(46) = 92.

time = 0.45, size = 128, normalized size = 2.29

method	result
default	$\frac{\sqrt{\frac{(-1+x)^2}{(-1-x)^2} + 3} \sqrt{2} \left(\operatorname{arctanh} \left(\frac{\sqrt{\frac{(-1+x)^2}{(-1-x)^2} + 3} \sqrt{2}}{2} \right) - 2 \operatorname{arctan} \left(\frac{\sqrt{2} (-1+x)}{\sqrt{\frac{(-1+x)^2}{(-1-x)^2} + 3} (-1-x)} \right) \right)}{\sqrt{\frac{(-1+x)^2}{(-1-x)^2} + 3} \left(\frac{-1+x}{-1-x} + 1 \right)}$
trager	$-\frac{2 \ln \left(\frac{-12 \operatorname{RootOf}(4Z^4 + 12Z^2 + 25)^5 x - 172 \operatorname{RootOf}(4Z^4 + 12Z^2 + 25)^3 x + 320 \sqrt{x^2 + x + 1} \operatorname{RootOf}(4Z^4 + 12Z^2 + 25)}{2x \operatorname{RootOf}(4Z^4 + 12Z^2 + 25)} \right)}{2x \operatorname{RootOf}(4Z^4 + 12Z^2 + 25)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $((-1+x)^2/(-1-x)^2+3)^{(1/2)}*2^{(1/2)}*(\operatorname{arctanh}(1/2*((-1+x)^2/(-1-x)^2+3)^{(1/2)})*2^{(1/2)}-2*\operatorname{arctan}(2^{(1/2)/((-1+x)^2/(-1-x)^2+3)^{(1/2)}*(-1+x)/(-1-x)))/(((-1+x)^2/(-1-x)^2+3)/((-1+x)/(-1-x)+1)^2)^{(1/2)/((-1+x)/(-1-x)+1)}$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x, algorithm="maxima")`

[Out] `integrate((x + 3)/(sqrt(x^2 + x + 1)*(x^2 + 1)), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(44) = 88$.

time = 0.33, size = 303, normalized size = 5.41

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x, algorithm="fricas")`

[Out] $4/5*\sqrt{10}*\sqrt{5}*\arctan(1/25*\sqrt{5}*\sqrt{(\sqrt{10}*\sqrt{5}*(x-1) + 10*x^2 - \sqrt{x^2+x+1}*(\sqrt{10}*\sqrt{5} + 10*x) + 5*x + 15)*(\sqrt{10}*\sqrt{5} + 10)} + 1/5*\sqrt{10}*\sqrt{5}*(x+1) - 1/5*\sqrt{x^2+x+1}*(\sqrt{10}*\sqrt{5} + 10) + 2*x + 1) + 4/5*\sqrt{10}*\sqrt{5}*\arctan(1/5*\sqrt{10}*\sqrt{5}*(x+1) + 1/50*\sqrt{-20*\sqrt{10}*\sqrt{5}*(x-1) + 200*x^2 + 20*\sqrt{x^2+x+1}})$

+ x + 1)*(sqrt(10)*sqrt(5) - 10*x) + 100*x + 300)*(sqrt(10)*sqrt(5) - 10) - 1/5*sqrt(x^2 + x + 1)*(sqrt(10)*sqrt(5) - 10) - 2*x - 1) - 1/10*sqrt(10)*sqrt(5)*log(20*sqrt(10)*sqrt(5)*(x - 1) + 200*x^2 - 20*sqrt(x^2 + x + 1)*(sqrt(10)*sqrt(5) + 10*x) + 100*x + 300) + 1/10*sqrt(10)*sqrt(5)*log(-20*sqrt(10)*sqrt(5)*(x - 1) + 200*x^2 + 20*sqrt(x^2 + x + 1)*(sqrt(10)*sqrt(5) - 10*x) + 100*x + 300)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + 3}{(x^2 + 1) \sqrt{x^2 + x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+x)/(x**2+1)/(x**2+x+1)**(1/2),x)

[Out] Integral((x + 3)/((x**2 + 1)*sqrt(x**2 + x + 1)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(44) = 88.

time = 0.01, size = 259, normalized size = 4.62

$i \left(\frac{1}{2} \sqrt{2} \ln \left(\frac{(\sqrt{x^2+x+1}-x+\sqrt{2}+1)(\sqrt{x^2+x+1}-x+\sqrt{2}+1)+(\sqrt{x^2+x+1}-x-1)(\sqrt{x^2+x+1}-x-1)}{(\sqrt{x^2+x+1}-x+\sqrt{2}+1)(\sqrt{x^2+x+1}-x-1)} \right) + \sqrt{2} \left(\frac{1}{2} + \arctan \left(\frac{2\sqrt{2}-2+(-2\sqrt{2}+2)(\sqrt{x^2+x+1}-2)}{3} \right) \right) - \frac{1}{2} \sqrt{2} \ln \left(\frac{(\sqrt{x^2+x+1}-x-\sqrt{2}+1)(\sqrt{x^2+x+1}-x-\sqrt{2}+1)+(\sqrt{x^2+x+1}-x-1)(\sqrt{x^2+x+1}-x-1)}{(\sqrt{x^2+x+1}-x-\sqrt{2}+1)(\sqrt{x^2+x+1}-x-1)} \right) - \sqrt{2} \left(\frac{1}{2} + \arctan \left(\frac{-2\sqrt{2}-2+(-2\sqrt{2}+2)(\sqrt{x^2+x+1}-2)}{3} \right) \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x)

[Out] -1/2*sqrt(2)*(pi + 4*arctan(-(x - sqrt(x^2 + x + 1))*(sqrt(2) + 2) - sqrt(2) - 1)) + 1/2*sqrt(2)*(pi + 4*arctan((x - sqrt(x^2 + x + 1))*(sqrt(2) - 2) + sqrt(2) - 1)) - 1/2*sqrt(2)*log((x + sqrt(2) - sqrt(x^2 + x + 1) - 1)^2 + (x - sqrt(x^2 + x + 1) + 1)^2) + 1/2*sqrt(2)*log((x - sqrt(2) - sqrt(x^2 + x + 1) - 1)^2 + (x - sqrt(x^2 + x + 1) + 1)^2)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x + 3}{(x^2 + 1) \sqrt{x^2 + x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3)/((x^2 + 1)*(x + x^2 + 1)^(1/2)),x)

[Out] int((x + 3)/((x^2 + 1)*(x + x^2 + 1)^(1/2)), x)

$$3.247 \quad \int \frac{1+2x}{\sqrt{-1+6x+x^2} (4+4x+3x^2)} dx$$

Optimal. Leaf size=70

$$-\frac{5 \tan^{-1} \left(\frac{\sqrt{\frac{7}{2}}^{(2-x)}}{2\sqrt{-1+6x+x^2}} \right)}{6\sqrt{14}} - \frac{\tanh^{-1} \left(\frac{\sqrt{7}^{(1+x)}}{\sqrt{-1+6x+x^2}} \right)}{3\sqrt{7}}$$

[Out] $-1/21*\operatorname{arctanh}((1+x)*7^{(1/2)}/(x^2+6*x-1)^{(1/2)})*7^{(1/2)}-5/84*\operatorname{arctan}(1/4*(2-x)*7^{(1/2)}*2^{(1/2)}/(x^2+6*x-1)^{(1/2)})*14^{(1/2)}$

Rubi [A]

time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1049, 1043, 213, 209}

$$-\frac{5 \tan^{-1} \left(\frac{\sqrt{\frac{7}{2}}^{(2-x)}}{2\sqrt{x^2+6x-1}} \right)}{6\sqrt{14}} - \frac{\tanh^{-1} \left(\frac{\sqrt{7}^{(x+1)}}{\sqrt{x^2+6x-1}} \right)}{3\sqrt{7}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(1+2*x)/(\operatorname{Sqrt}[-1+6*x+x^2]*(4+4*x+3*x^2)),x]$

[Out] $(-5*\operatorname{ArcTan}[(\operatorname{Sqrt}[7/2]*(2-x))/(2*\operatorname{Sqrt}[-1+6*x+x^2])])/(6*\operatorname{Sqrt}[14]) - \operatorname{ArcTanh}[(\operatorname{Sqrt}[7]*(1+x))/\operatorname{Sqrt}[-1+6*x+x^2]]/(3*\operatorname{Sqrt}[7])$

Rule 209

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))*\operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{PosQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{GtQ}[b, 0])$

Rule 213

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2])^{-1})*\operatorname{ArcTanh}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{LtQ}[a, 0] \ || \ \operatorname{GtQ}[b, 0])$

Rule 1043

$\operatorname{Int}[(g_+ + (h_+)*(x_+))/((a_+ + (b_+)*(x_+) + (c_+)*(x_+)^2)*\operatorname{Sqrt}[(d_+ + (e_+)*(x_+) + (f_+)*(x_+)^2])], x_Symbol] \rightarrow \operatorname{Dist}[-2*g*(g*b - 2*a*h), \operatorname{Subst}[\operatorname{In}$

```
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2} (4+4x+3x^2)} dx = - \left(\frac{1}{42} \int \frac{-70-70x}{\sqrt{-1+6x+x^2} (4+4x+3x^2)} dx \right) + \frac{1}{42} \int \frac{1}{\sqrt{-1+6x+x^2}} dx$$

$$= - \left(\frac{896}{3} \text{Subst} \left(\int \frac{1}{-200704+28x^2} dx, x, \frac{-224-224x}{\sqrt{-1+6x+x^2}} \right) \right) - \frac{2}{3\sqrt{7}}$$

$$= - \frac{5 \tan^{-1} \left(\frac{\sqrt{\frac{7}{2}} (2-x)}{2\sqrt{-1+6x+x^2}} \right)}{6\sqrt{14}} - \frac{\tanh^{-1} \left(\frac{\sqrt{7}(1+x)}{\sqrt{-1+6x+x^2}} \right)}{3\sqrt{7}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.17, size = 123, normalized size = 1.76

$$\text{RootSum} \left[171 - 104\#1 + 46\#1^2 - 8\#1^3 + 3\#1^4 \&, \frac{4 \log(-x + \sqrt{-1+6x+x^2} - \#1) - \log(-x + \sqrt{-1+6x+x^2} - \#1) \#1 + \log(-x + \sqrt{-1+6x+x^2} - \#1) \#1^2}{-26 + 23\#1 - 6\#1^2 + 3\#1^3} \& \right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/(Sqrt[-1 + 6*x + x^2]*(4 + 4*x + 3*x^2)), x]
```

```
[Out] RootSum[171 - 104*#1 + 46*#1^2 - 8*#1^3 + 3*#1^4 & , (4*Log[-x + Sqrt[-1 +
6*x + x^2] - #1] - Log[-x + Sqrt[-1 + 6*x + x^2] - #1]*#1 + Log[-x + Sqrt[-
1 + 6*x + x^2] - #1]*#1^2)/(-26 + 23*#1 - 6*#1^2 + 3*#1^3) & ]
```


Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(2*x + 1)/((3*x^2 + 4*x + 4)*Sqrt[x^2 + 6*x - 1]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(53) = 106.

time = 0.63, size = 158, normalized size = 2.26

method	result
default	$\frac{\sqrt{-\frac{6(-2+x)^2}{(-1-x)^2} + 15} \left(4\sqrt{7} \operatorname{arctanh} \left(\frac{\sqrt{-\frac{6(-2+x)^2}{(-1-x)^2} + 15} \sqrt{7}}{21} \right) - 5\sqrt{14} \operatorname{arctan} \left(\frac{\sqrt{14} \sqrt{-\frac{6(-2+x)^2}{(-1-x)^2} + 15}}{4 \left(\frac{2(-2+x)^2}{(-1-x)^2} - 5 \right) (-1-x)} \right) \right)}{84 \sqrt{-\frac{3 \left(\frac{2(-2+x)^2}{(-1-x)^2} - 5 \right)}{\left(\frac{-2+x}{-1-x} + 1 \right)^2}} \left(\frac{-2+x}{-1-x} + 1 \right)}$
trager	$-\operatorname{RootOf} \left(451584_Z^4 + 7616_Z^2 + 121 \right) \ln \left(\frac{1568802816x \operatorname{RootOf} \left(451584_Z^4 + 7616_Z^2 + 121 \right)^5 + 6019776R}{\dots} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/84*(-6*(-2+x)^2/(-1-x)^2+15)^(1/2)*(4*7^(1/2)*arctanh(1/21*(-6*(-2+x)^2/(-1-x)^2+15)^(1/2)*7^(1/2))-5*14^(1/2)*arctan(1/4*14^(1/2)*(-6*(-2+x)^2/(-1-x)^2+15)^(1/2)/(2*(-2+x)^2/(-1-x)^2-5)*(-2+x)/(-1-x)))/(-3*(2*(-2+x)^2/(-1-x)^2-5)/((-2+x)/(-1-x)+1)^2)^(1/2)/((-2+x)/(-1-x)+1)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)/((3*x^2 + 4*x + 4)*sqrt(x^2 + 6*x - 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(51) = 102$.
time = 0.32, size = 311, normalized size = 4.44

(\frac{1}{84}\sqrt{14}\sqrt{2}\log(13068\sqrt{14}\sqrt{2}(x-2)+78408x^2-13068\sqrt{x^2+6x-1}(\sqrt{14}\sqrt{2}+6x+4)+287496x+287496)-1/84\sqrt{14}\sqrt{2}\log(-13068\sqrt{14}\sqrt{2}(x-2)+78408x^2+13068\sqrt{x^2+6x-1}(\sqrt{14}\sqrt{2}-6x-4)+287496x+287496))-5/42\sqrt{14}\arctan(1/24\sqrt{3}\sqrt{\sqrt{14}\sqrt{2}(x-2)+6x^2-\sqrt{x^2+6x-1}(\sqrt{14}\sqrt{2}+6x+4)+22x+22})(\sqrt{14}+\sqrt{2})+1/8\sqrt{2}(x+3)+1/8\sqrt{14}(x+1)-1/8\sqrt{x^2+6x-1}(\sqrt{14}+\sqrt{2}))-5/42\sqrt{14}\arctan(-1/8\sqrt{2}(x+3)+1/8\sqrt{14}(x+1)+1/1584\sqrt{-13068\sqrt{14}\sqrt{2}(x-2)+78408x^2+13068\sqrt{x^2+6x-1}(\sqrt{14}\sqrt{2}-6x-4)+287496x+287496})(\sqrt{14}-\sqrt{2})-1/8\sqrt{x^2+6x-1}(\sqrt{14}-\sqrt{2}))

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{84}\sqrt{14}\sqrt{2}\log(13068\sqrt{14}\sqrt{2}(x-2)+78408x^2-13068\sqrt{x^2+6x-1}(\sqrt{14}\sqrt{2}+6x+4)+287496x+287496)-1/84\sqrt{14}\sqrt{2}\log(-13068\sqrt{14}\sqrt{2}(x-2)+78408x^2+13068\sqrt{x^2+6x-1}(\sqrt{14}\sqrt{2}-6x-4)+287496x+287496))-5/42\sqrt{14}\arctan(1/24\sqrt{3}\sqrt{\sqrt{14}\sqrt{2}(x-2)+6x^2-\sqrt{x^2+6x-1}(\sqrt{14}\sqrt{2}+6x+4)+22x+22})(\sqrt{14}+\sqrt{2})+1/8\sqrt{2}(x+3)+1/8\sqrt{14}(x+1)-1/8\sqrt{x^2+6x-1}(\sqrt{14}+\sqrt{2}))-5/42\sqrt{14}\arctan(-1/8\sqrt{2}(x+3)+1/8\sqrt{14}(x+1)+1/1584\sqrt{-13068\sqrt{14}\sqrt{2}(x-2)+78408x^2+13068\sqrt{x^2+6x-1}(\sqrt{14}\sqrt{2}-6x-4)+287496x+287496})(\sqrt{14}-\sqrt{2})-1/8\sqrt{x^2+6x-1}(\sqrt{14}-\sqrt{2}))$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\sqrt{x^2+6x-1} \cdot (3x^2+4x+4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(3*x**2+4*x+4)/(x**2+6*x-1)**(1/2),x)

[Out] Integral((2*x + 1)/(sqrt(x**2 + 6*x - 1)*(3*x**2 + 4*x + 4)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(51) = 102$.

time = 0.02, size = 545, normalized size = 7.79

(-\frac{5}{84}\sqrt{7}\sqrt{2}(\arctan(2)+\arctan(1/8(x-\sqrt{x^2+6x-1})(\sqrt{14}+\sqrt{2}))+1/8\sqrt{14}+3/8\sqrt{2}))+5/84\sqrt{7}\sqrt{2}(\arctan(1/2)+\arctan(-1/8(x-\sqrt{x^2+6x-1})(\sqrt{14}-\sqrt{2}))-1/8\sqrt{14}+3/8\sqrt{2}))+1/42\sqrt{7}\log(4(4\sqrt{7}\sqrt{2}+3x+2\sqrt{x^2+6x-1}))

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x)

[Out] $-\frac{5}{84}\sqrt{7}\sqrt{2}(\arctan(2)+\arctan(1/8(x-\sqrt{x^2+6x-1})(\sqrt{14}+\sqrt{2}))+1/8\sqrt{14}+3/8\sqrt{2}))+5/84\sqrt{7}\sqrt{2}(\arctan(1/2)+\arctan(-1/8(x-\sqrt{x^2+6x-1})(\sqrt{14}-\sqrt{2}))-1/8\sqrt{14}+3/8\sqrt{2}))+1/42\sqrt{7}\log(4(4\sqrt{7}\sqrt{2}+3x+2\sqrt{x^2+6x-1}))$

```

sqrt(7) - 4*sqrt(2) - 3*sqrt(x^2 + 6*x - 1) + 2)^2 + 16*(sqrt(7)*sqrt(2) -
3*x - sqrt(7) - sqrt(2) + 3*sqrt(x^2 + 6*x - 1) - 2)^2) - 1/42*sqrt(7)*log(
4*(4*sqrt(7)*sqrt(2) + 3*x - sqrt(7) + 4*sqrt(2) - 3*sqrt(x^2 + 6*x - 1) +
2)^2 + 16*(sqrt(7)*sqrt(2) - 3*x + sqrt(7) + sqrt(2) + 3*sqrt(x^2 + 6*x - 1
) - 2)^2)

```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2x + 1}{\sqrt{x^2 + 6x - 1} (3x^2 + 4x + 4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 1)/((6*x + x^2 - 1)^(1/2)*(4*x + 3*x^2 + 4)),x)

[Out] int((2*x + 1)/((6*x + x^2 - 1)^(1/2)*(4*x + 3*x^2 + 4)), x)

$$3.248 \quad \int \frac{B+Ax}{(17-18x+5x^2) \sqrt{13-22x+10x^2}} dx$$

Optimal. Leaf size=80

$$\frac{(2A+B) \tan^{-1} \left(\frac{\sqrt{35} (2-x)}{\sqrt{13-22x+10x^2}} \right)}{\sqrt{35}} - \frac{(A+B) \tanh^{-1} \left(\frac{\sqrt{35} (1-x)}{2\sqrt{13-22x+10x^2}} \right)}{2\sqrt{35}}$$

[Out] $-1/35*(2*A+B)*\arctan((2-x)*35^{(1/2)}/(10*x^2-22*x+13)^{(1/2)})*35^{(1/2)}-1/70*(A+B)*\operatorname{arctanh}(1/2*(1-x)*35^{(1/2)}/(10*x^2-22*x+13)^{(1/2)})*35^{(1/2)}$

Rubi [A]

time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1049, 1043, 212, 210}

$$\frac{(2A+B) \tan^{-1} \left(\frac{\sqrt{35} (2-x)}{\sqrt{10x^2-22x+13}} \right)}{\sqrt{35}} - \frac{(A+B) \tanh^{-1} \left(\frac{\sqrt{35} (-x(A+B)+A+B)}{2\sqrt{10x^2-22x+13} (A+B)} \right)}{2\sqrt{35}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(B + A*x)/((17 - 18*x + 5*x^2)*\text{Sqrt}[13 - 22*x + 10*x^2]), x]$

[Out] $-\left(\frac{(2*A + B)*\text{ArcTan}[\text{Sqrt}[35]*(2 - x)]/\text{Sqrt}[13 - 22*x + 10*x^2]}{\text{Sqrt}[35]} - \frac{(A + B)*\text{ArcTanh}[\text{Sqrt}[35]*(A + B - (A + B)*x)]/(2*(A + B)*\text{Sqrt}[13 - 22*x + 10*x^2])}{2*\text{Sqrt}[35]}\right)$

Rule 210

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 212

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]))*\text{ArcTanh}[\text{Rt}[-b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 1043

$\text{Int}[(g_ + (h_)*(x_))/((a_ + (b_)*(x_) + (c_)*(x_)^2)*\text{Sqrt}[(d_ + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] \rightarrow \text{Dist}[-2*g*(g*b - 2*a*h), \text{Subst}[\text{Int}[1/\text{Simp}[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], \text{Simp}[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/\text{Sqrt}[d + e*x + f*x^2], x] /; \text{FreeQ}\{a, b,$

c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]

Rule 1049

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{B + Ax}{(17 - 18x + 5x^2) \sqrt{13 - 22x + 10x^2}} dx &= \frac{1}{70} \int \frac{140(A + B) - 70(A + B)x}{(17 - 18x + 5x^2) \sqrt{13 - 22x + 10x^2}} dx - \frac{1}{70} \int \frac{1}{(17 - 18x + 5x^2) \sqrt{13 - 22x + 10x^2}} dx \\ &= (560(A + B)^2) \text{Subst} \left(\int \frac{1}{313600(A + B)^2 - 140x^2} dx, x, \frac{17 - 18x + 5x^2}{313600(A + B)^2 - 140x^2} \right) \\ &= -\frac{(2A + B) \tan^{-1} \left(\frac{\sqrt{35} (2-x)}{\sqrt{13 - 22x + 10x^2}} \right)}{\sqrt{35}} - \frac{(A + B) \tanh^{-1} \left(\frac{17 - 18x + 5x^2}{313600(A + B)^2 - 140x^2} \right)}{\sqrt{35}} \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.83, size = 124, normalized size = 1.55

$$\frac{((1 + 4i)A + (1 + 2i)B) \tanh^{-1} \left(\frac{(4-i)\sqrt{10} - (2-i)\sqrt{10}x + (2-i)\sqrt{13 - 22x + 10x^2}}{\sqrt{35}} \right) + ((1 - 4i)A + (1 - 2i)B) \tanh^{-1} \left(\frac{(4+i)\sqrt{10} - (2+i)\sqrt{10}x + (2+i)\sqrt{13 - 22x + 10x^2}}{\sqrt{35}} \right)}{2\sqrt{35}}$$

Antiderivative was successfully verified.

[In] Integrate[(B + A*x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]

[Out] (((1 + 4*I)*A + (1 + 2*I)*B)*ArcTanh[(((4 - I)*Sqrt[10] - (2 - I)*Sqrt[10]*x + (2 - I)*Sqrt[13 - 22*x + 10*x^2])/Sqrt[35]] + ((1 - 4*I)*A + (1 - 2*I)*B)*ArcTanh[(((4 + I)*Sqrt[10] - (2 + I)*Sqrt[10]*x + (2 + I)*Sqrt[13 - 22*x + 10*x^2])/Sqrt[35]])/(2*Sqrt[35])

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(A*x + B)/((5*x^2 - 18*x + 17)*Sqrt[10*x^2 - 22*x + 13]),x]')`

[Out] `cought exception: maximum recursion depth exceeded`

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(64) = 128.

time = 0.21, size = 192, normalized size = 2.40

method	result
default	$\frac{\sqrt{\frac{(-2+x)^2}{(1-x)^2} + 9} \sqrt{35} \left(\operatorname{arctanh} \left(\frac{2 \sqrt{\frac{(-2+x)^2}{(1-x)^2} + 9} \sqrt{35}}{35} \right) A - 4 \arctan \left(\frac{\sqrt{35} (-2+x)}{\sqrt{\frac{(-2+x)^2}{(1-x)^2} + 9} (1-x)} \right) A + \operatorname{arctanh} \left(\frac{2 \sqrt{\frac{(-2+x)^2}{(1-x)^2} + 9}}{\frac{(-2+x)}{1-x} + 1} \right) \right)}{70 \sqrt{\frac{(-2+x)^2}{(1-x)^2} + 9} \left(\frac{-2+x}{1-x} + 1 \right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `1/70*((-2+x)^2/(1-x)^2+9)^(1/2)*35^(1/2)*(arctanh(2/35*((-2+x)^2/(1-x)^2+9)^(1/2)*35^(1/2))*A-4*arctan(35^(1/2)/((-2+x)^2/(1-x)^2+9)^(1/2)*(-2+x)/(1-x))*A+arctanh(2/35*((-2+x)^2/(1-x)^2+9)^(1/2)*35^(1/2))*B-2*arctan(35^(1/2)/((-2+x)^2/(1-x)^2+9)^(1/2)*(-2+x)/(1-x))*B)/(((2+x)^2/(1-x)^2+9)/((-2+x)/(1-x)+1))^2)^(1/2)/((-2+x)/(1-x)+1)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="maxima")`

[Out] `integrate((A*x + B)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)), x)`

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Ax + B}{(5x^2 - 18x + 17) \sqrt{10x^2 - 22x + 13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A*x+B)/(5*x**2-18*x+17)/(10*x**2-22*x+13)**(1/2), x)

[Out] Integral((A*x + B)/((5*x**2 - 18*x + 17)*sqrt(10*x**2 - 22*x + 13)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(61) = 122.

time = 0.10, size = 1186, normalized size = 14.82

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2), x)

[Out] $\frac{2}{35}\sqrt{35}(2A^2 + 3AB + B^2)\sqrt{A^2 + 2AB + B^2}(\arctan(3) + \arctan(-\frac{5(\sqrt{10}x - \sqrt{10x^2 - 22x + 13})(300\sqrt{14} - 1129) - 7658\sqrt{35} + 14361\sqrt{10})}{2329\sqrt{35} - 4358\sqrt{10}})) + \frac{2}{35}\sqrt{35}(2A^2 + 3AB + B^2)\sqrt{A^2 + 2AB + B^2}(\arctan(\frac{1}{7}) + \arctan(-\frac{5(\sqrt{10}x - \sqrt{10x^2 - 22x + 13})(62556\sqrt{14} + 245977) - 1617962\sqrt{35} - 3089577\sqrt{10})}{496201\sqrt{35} + 929846\sqrt{10}})) + \frac{1}{140}\sqrt{35}\sqrt{A^2 + 2AB + B^2}\log(25(546\sqrt{14}(\sqrt{10}x - \sqrt{10x^2 - 22x + 13}) + 2807\sqrt{10}x - 234\sqrt{35}\sqrt{14} - 1014\sqrt{14}\sqrt{10} - 1203\sqrt{35} - 5213\sqrt{10} - 2807\sqrt{10x^2 - 22x + 13})^2 + 25(78\sqrt{14}(\sqrt{10}x - \sqrt{10x^2 - 22x + 13}) + 401\sqrt{10}x + 48\sqrt{35}\sqrt{14} + 208\sqrt{14}\sqrt{10} + 141\sqrt{35} + 611\sqrt{10} - 401\sqrt{10x^2 - 22x + 13})^2) - \frac{1}{140}\sqrt{35}\sqrt{A^2 + 2AB + B^2}\log(625(18\sqrt{14}(\sqrt{10}x - \sqrt{10x^2 - 22x + 13}) - 75\sqrt{10}x + 8\sqrt{35}\sqrt{14} - 24\sqrt{14}\sqrt{10} - 37\sqrt{35} + 111\sqrt{10} + 75\sqrt{10x^2 - 22x + 13})^2 + 625(6\sqrt{14}(\sqrt{10}x - \sqrt{10x^2 - 22x + 13}) - 25\sqrt{10}x + 6\sqrt{35}\sqrt{14} - 18\sqrt{14}\sqrt{10} - 25\sqrt{35} + 75\sqrt{10} + 25\sqrt{10x^2 - 22x + 13})^2)$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B + Ax}{(5x^2 - 18x + 17) \sqrt{10x^2 - 22x + 13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B + A*x)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)),x)
```

```
[Out] int((B + A*x)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)), x)
```


$$3.249 \quad \int \frac{-2+x}{(17-18x+5x^2) \sqrt{13-22x+10x^2}} dx$$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{13-22x+10x^2}}\right)}{2\sqrt{35}}$$

[Out] 1/70*arctanh(1/2*(1-x)*35^(1/2)/(10*x^2-22*x+13)^(1/2))*35^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1043, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{10x^2-22x+13}}\right)}{2\sqrt{35}}$$

Antiderivative was successfully verified.

[In] Int[(-2 + x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]

[Out] ArcTanh[(Sqrt[35]*(1 - x))/(2*Sqrt[13 - 22*x + 10*x^2])]/(2*Sqrt[35])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1043

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]

Rubi steps

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}} dx = 8 \text{Subst} \left(\int \frac{1}{64-140x^2} dx, x, \frac{2-2x}{\sqrt{13-22x+10x^2}} \right)$$

$$= \frac{\tanh^{-1} \left(\frac{\sqrt{35}(1-x)}{2\sqrt{13-22x+10x^2}} \right)}{2\sqrt{35}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 85 vs. 2(38) = 76.

time = 0.36, size = 85, normalized size = 2.24

$$\frac{\tanh^{-1} \left(\frac{-135+145x-50x^2+\sqrt{10}(-9+5x)\sqrt{13-22x+10x^2}}{-20\sqrt{14}+10\sqrt{14}x-2\sqrt{35}\sqrt{13-22x+10x^2}} \right)}{2\sqrt{35}}$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]), x]

[Out] -1/2*ArcTanh[(-135 + 145*x - 50*x^2 + Sqrt[10]*(-9 + 5*x)*Sqrt[13 - 22*x + 10*x^2])/(-20*Sqrt[14] + 10*Sqrt[14]*x - 2*Sqrt[35]*Sqrt[13 - 22*x + 10*x^2])]/Sqrt[35]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(x - 2)/((5*x^2 - 18*x + 17)*Sqrt[10*x^2 - 22*x + 13]), x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(28) = 56.

time = 0.18, size = 94, normalized size = 2.47

method	result
trager	$\text{RootOf}(_Z^2 - 35) \ln \left(-\frac{-75 \text{RootOf}(_Z^2 - 35) x^2 + 140 \sqrt{10x^2 - 22x + 13} x + 158 \text{RootOf}(_Z^2 - 35) x - 140 \sqrt{10x^2 - 22x + 13}}{5x^2 - 18x + 17} \right)$

default	$\frac{\sqrt{\frac{(-2+x)^2}{(1-x)^2} + 9} \sqrt{35} \operatorname{arctanh}\left(\frac{2\sqrt{\frac{(-2+x)^2}{(1-x)^2} + 9} \sqrt{35}}{35}\right)}{70\sqrt{\frac{\frac{(-2+x)^2}{(1-x)^2} + 9}{\left(\frac{-2+x}{1-x} + 1\right)^2}} \left(\frac{-2+x}{1-x} + 1\right)}$
---------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/70/\left(\left(\frac{-2+x}{1-x}\right)^2+9\right)/\left(\frac{-2+x}{1-x}+1\right)^2)^{(1/2)}/\left(\frac{-2+x}{1-x}+1\right)*\left(\frac{-2+x}{1-x}\right)^2/\left(\frac{-2+x}{1-x}\right)^2+9)^{(1/2)}*35^{(1/2)}*\operatorname{arctanh}\left(\frac{2}{35}\left(\frac{-2+x}{1-x}\right)^2/\left(\frac{-2+x}{1-x}\right)^2+9\right)^{(1/2)}*35^{(1/2)}$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="maxima")`

[Out] `integrate((x - 2)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(26) = 52.

time = 0.30, size = 81, normalized size = 2.13

$$\frac{1}{280} \sqrt{35} \log\left(\frac{11225x^4 - 47220x^3 - 8\sqrt{35}(75x^3 - 233x^2 + 245x - 87)\sqrt{10x^2 - 22x + 13} + 75534x^2 - 54372x + 14849}{25x^4 - 180x^3 + 494x^2 - 612x + 289}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="fricas")`

[Out] $\frac{1}{280}\sqrt{35}*\log\left(\frac{11225*x^4 - 47220*x^3 - 8*\sqrt{35}*(75*x^3 - 233*x^2 + 245*x - 87)*\sqrt{10*x^2 - 22*x + 13} + 75534*x^2 - 54372*x + 14849}{(25*x^4 - 180*x^3 + 494*x^2 - 612*x + 289)}\right)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - 2}{(5x^2 - 18x + 17)\sqrt{10x^2 - 22x + 13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2+x)/(5*x**2-18*x+17)/(10*x**2-22*x+13)**(1/2), x)`

[Out] `Integral((x - 2)/((5*x**2 - 18*x + 17)*sqrt(10*x**2 - 22*x + 13)), x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(26) = 52.

time = 6.00, size = 293, normalized size = 7.71

⋮

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2), x)`

[Out] `1/140*sqrt(35)*log(abs(21875000000*sqrt(14)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13))^2 + 82031250000*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13))^2 - 91875000000*sqrt(35)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) - 172812500000*sqrt(10)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) + 240625000000*sqrt(14) + 913281250000)) - 1/140*sqrt(35)*log(abs(-21875000000*sqrt(14)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13))^2 + 82031250000*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13))^2 + 91875000000*sqrt(35)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) - 172812500000*sqrt(10)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) - 240625000000*sqrt(14) + 913281250000))`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x - 2}{(5x^2 - 18x + 17) \sqrt{10x^2 - 22x + 13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x - 2)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)), x)`

[Out] `int((x - 2)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)), x)`

3.250 $\int x^4 \sqrt{5 - x^2} dx$

Optimal. Leaf size=65

$$-\frac{25}{16}x\sqrt{5-x^2} - \frac{5}{24}x^3\sqrt{5-x^2} + \frac{1}{6}x^5\sqrt{5-x^2} + \frac{125}{16}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

[Out] 125/16*arcsin(1/5*x*5^(1/2))-25/16*x*(-x^2+5)^(1/2)-5/24*x^3*(-x^2+5)^(1/2)+1/6*x^5*(-x^2+5)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {285, 327, 222}

$$-\frac{25}{16}\sqrt{5-x^2}x + \frac{1}{6}\sqrt{5-x^2}x^5 - \frac{5}{24}\sqrt{5-x^2}x^3 + \frac{125}{16}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^4*Sqrt[5 - x^2], x]

[Out] (-25*x*Sqrt[5 - x^2])/16 - (5*x^3*Sqrt[5 - x^2])/24 + (x^5*Sqrt[5 - x^2])/6 + (125*ArcSin[x/Sqrt[5]])/16

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 285

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m+1)*((a+b*x^n)^p/(c*(m+n*p+1))), x] + Dist[a*n*(p/(m+n*p+1)), Int[(c*x)^m*(a+b*x^n)^(p-1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && GtQ[p, 0] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 327

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n-1)*(c*x)^(m-n+1)*((a+b*x^n)^(p+1)/(b*(m+n*p+1))), x] - Dist[a*c^n*((m-n+1)/(b*(m+n*p+1))), Int[(c*x)^(m-n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned}
\int x^4 \sqrt{5-x^2} \, dx &= \frac{1}{6} x^5 \sqrt{5-x^2} + \frac{5}{6} \int \frac{x^4}{\sqrt{5-x^2}} \, dx \\
&= -\frac{5}{24} x^3 \sqrt{5-x^2} + \frac{1}{6} x^5 \sqrt{5-x^2} + \frac{25}{8} \int \frac{x^2}{\sqrt{5-x^2}} \, dx \\
&= -\frac{25}{16} x \sqrt{5-x^2} - \frac{5}{24} x^3 \sqrt{5-x^2} + \frac{1}{6} x^5 \sqrt{5-x^2} + \frac{125}{16} \int \frac{1}{\sqrt{5-x^2}} \, dx \\
&= -\frac{25}{16} x \sqrt{5-x^2} - \frac{5}{24} x^3 \sqrt{5-x^2} + \frac{1}{6} x^5 \sqrt{5-x^2} + \frac{125}{16} \sin^{-1} \left(\frac{x}{\sqrt{5}} \right)
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 47, normalized size = 0.72

$$\frac{1}{48} x \sqrt{5-x^2} (-75 - 10x^2 + 8x^4) + \frac{125}{16} \tan^{-1} \left(\frac{x}{\sqrt{5-x^2}} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[x^4*sqrt[5 - x^2], x]``[Out] (x*sqrt[5 - x^2]*(-75 - 10*x^2 + 8*x^4))/48 + (125*ArcTan[x/sqrt[5 - x^2]])/16`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 6.50, size = 117, normalized size = 1.80

$$\text{Piecewise} \left[\left\{ \left\{ \frac{I \left(375x - 25x^3 - 50x^5 + 8x^7 - 375 \text{ArcCosh} \left[\frac{\sqrt{5}x}{5} \right] \sqrt{-5+x^2} \right)}{48\sqrt{-5+x^2}}, \text{Abs}[x^2] > 5 \right\} \right\}, \frac{-125x}{16\sqrt{5-x^2}} + \frac{25x^3}{48\sqrt{5-x^2}} + \frac{25x^5}{24\sqrt{5-x^2}} - \frac{x^7}{6\sqrt{5-x^2}} + \frac{125 \text{ArcSin} \left[\frac{\sqrt{5}x}{5} \right]}{16} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[x^4*sqrt[5 - x^2], x]')`

```
[Out] Piecewise[{{I / 48 (375 x - 25 x ^ 3 - 50 x ^ 5 + 8 x ^ 7 - 375 ArcCosh[Sqr
t[5] x / 5] Sqrt[-5 + x ^ 2]) / Sqrt[-5 + x ^ 2], Abs[x ^ 2] > 5}}, -125 x
/ (16 Sqrt[5 - x ^ 2]) + 25 x ^ 3 / (48 Sqrt[5 - x ^ 2]) + 25 x ^ 5 / (24 S
qrt[5 - x ^ 2]) - x ^ 7 / (6 Sqrt[5 - x ^ 2]) + 125 ArcSin[Sqrt[5] x / 5] /
16]
```

Maple [A]

time = 0.08, size = 49, normalized size = 0.75

method	result	size
--------	--------	------

risch	$-\frac{x(8x^4-10x^2-75)(x^2-5)}{48\sqrt{-x^2+5}} + \frac{125 \arcsin\left(\frac{x\sqrt{5}}{5}\right)}{16}$	40
default	$-\frac{x^3(-x^2+5)^{\frac{3}{2}}}{6} - \frac{5x(-x^2+5)^{\frac{3}{2}}}{8} + \frac{25x\sqrt{-x^2+5}}{16} + \frac{125 \arcsin\left(\frac{x\sqrt{5}}{5}\right)}{16}$	49
meijerg	$\frac{125i \left(\frac{i\sqrt{\pi} x\sqrt{5} \left(-\frac{8}{5}x^4+2x^2+15\right) \sqrt{-\frac{x^2}{5}+1} - i\sqrt{\pi} \arcsin\left(\frac{x\sqrt{5}}{5}\right)}{300} \right)}{4\sqrt{\pi}}$	52
trager	$\frac{x(8x^4-10x^2-75)\sqrt{-x^2+5}}{48} + \frac{125 \operatorname{RootOf}(-Z^2+1) \ln\left(\operatorname{RootOf}(-Z^2+1)\sqrt{-x^2+5}+x\right)}{16}$	53

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(-x^2+5)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `-1/6*x^3*(-x^2+5)^(3/2)-5/8*x*(-x^2+5)^(3/2)+25/16*x*(-x^2+5)^(1/2)+125/16*arcsin(1/5*x*5^(1/2))`

Maxima [A]

time = 0.35, size = 48, normalized size = 0.74

$$-\frac{1}{6}(-x^2+5)^{\frac{3}{2}}x^3 - \frac{5}{8}(-x^2+5)^{\frac{3}{2}}x + \frac{25}{16}\sqrt{-x^2+5}x + \frac{125}{16}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(-x^2+5)^(1/2),x, algorithm="maxima")`

[Out] `-1/6*(-x^2+5)^(3/2)*x^3 - 5/8*(-x^2+5)^(3/2)*x + 25/16*sqrt(-x^2+5)*x + 125/16*arcsin(1/5*sqrt(5)*x)`

Fricas [A]

time = 0.30, size = 42, normalized size = 0.65

$$\frac{1}{48}(8x^5 - 10x^3 - 75x)\sqrt{-x^2+5} - \frac{125}{16}\arctan\left(\frac{\sqrt{-x^2+5}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(-x^2+5)^(1/2),x, algorithm="fricas")`

[Out] `1/48*(8*x^5 - 10*x^3 - 75*x)*sqrt(-x^2+5) - 125/16*arctan(sqrt(-x^2+5)/x)`

Sympy [A]

time = 4.50, size = 153, normalized size = 2.35

$$\left\{ \begin{array}{l} \frac{ix^7}{6\sqrt{x^2-5}} - \frac{25ix^5}{24\sqrt{x^2-5}} - \frac{25ix^3}{48\sqrt{x^2-5}} + \frac{125ix}{16\sqrt{x^2-5}} - \frac{125i \operatorname{acosh}\left(\frac{\sqrt{5}x}{5}\right)}{16} \quad \text{for } |x^2| > 5 \\ -\frac{x^7}{6\sqrt{5-x^2}} + \frac{25x^5}{24\sqrt{5-x^2}} + \frac{25x^3}{48\sqrt{5-x^2}} - \frac{125x}{16\sqrt{5-x^2}} + \frac{125 \operatorname{asin}\left(\frac{\sqrt{5}x}{5}\right)}{16} \quad \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(-x**2+5)**(1/2),x)

[Out] Piecewise((I*x**7/(6*sqrt(x**2 - 5)) - 25*I*x**5/(24*sqrt(x**2 - 5)) - 25*I*x**3/(48*sqrt(x**2 - 5)) + 125*I*x/(16*sqrt(x**2 - 5)) - 125*I*acosh(sqrt(5)*x/5)/16, Abs(x**2) > 5), (-x**7/(6*sqrt(5 - x**2)) + 25*x**5/(24*sqrt(5 - x**2)) + 25*x**3/(48*sqrt(5 - x**2)) - 125*x/(16*sqrt(5 - x**2)) + 125*asin(sqrt(5)*x/5)/16, True))

Giac [A]

time = 0.00, size = 45, normalized size = 0.69

$$2 \left(\left(\frac{1}{12}xx - \frac{5}{48} \right) xx - \frac{25}{32} \right) x \sqrt{-x^2 + 5} + \frac{125}{16} \arcsin \left(\frac{x}{\sqrt{5}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(-x^2+5)^(1/2),x)

[Out] 1/48*(2*(4*x^2 - 5)*x^2 - 75)*sqrt(-x^2 + 5)*x + 125/16*arcsin(1/5*sqrt(5)*x)

Mupad [B]

time = 0.03, size = 35, normalized size = 0.54

$$\frac{125 \operatorname{asin}\left(\frac{\sqrt{5}x}{5}\right)}{16} - \sqrt{5-x^2} \left(-\frac{x^5}{6} + \frac{5x^3}{24} + \frac{25x}{16} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(5 - x^2)^(1/2),x)

[Out] (125*asin((5^(1/2)*x)/5))/16 - (5 - x^2)^(1/2)*((25*x)/16 + (5*x^3)/24 - x^5/6)

$$3.251 \quad \int \frac{1}{x^6 \sqrt{2+x^2}} dx$$

Optimal. Leaf size=49

$$-\frac{\sqrt{2+x^2}}{10x^5} + \frac{\sqrt{2+x^2}}{15x^3} - \frac{\sqrt{2+x^2}}{15x}$$

[Out] $-1/10*(x^2+2)^{(1/2)}/x^5+1/15*(x^2+2)^{(1/2)}/x^3-1/15/x*(x^2+2)^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$,

Rules used = {277, 270}

$$-\frac{\sqrt{x^2+2}}{15x} - \frac{\sqrt{x^2+2}}{10x^5} + \frac{\sqrt{x^2+2}}{15x^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*Sqrt[2 + x^2]),x]

[Out] $-1/10*\text{Sqrt}[2 + x^2]/x^5 + \text{Sqrt}[2 + x^2]/(15*x^3) - \text{Sqrt}[2 + x^2]/(15*x)$

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 277

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^6 \sqrt{2+x^2}} dx &= -\frac{\sqrt{2+x^2}}{10x^5} - \frac{2}{5} \int \frac{1}{x^4 \sqrt{2+x^2}} dx \\ &= -\frac{\sqrt{2+x^2}}{10x^5} + \frac{\sqrt{2+x^2}}{15x^3} + \frac{2}{15} \int \frac{1}{x^2 \sqrt{2+x^2}} dx \\ &= -\frac{\sqrt{2+x^2}}{10x^5} + \frac{\sqrt{2+x^2}}{15x^3} - \frac{\sqrt{2+x^2}}{15x} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 28, normalized size = 0.57

$$\frac{\sqrt{2+x^2}(-3+2x^2-2x^4)}{30x^5}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x^6*Sqrt[2 + x^2]),x]``[Out] (Sqrt[2 + x^2]*(-3 + 2*x^2 - 2*x^4))/(30*x^5)`**Mathics [A]**

time = 3.16, size = 28, normalized size = 0.57

$$\frac{(-3 + 2x^2 - 2x^4) \sqrt{\frac{2+x^2}{x^2}}}{30x^4}$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x^6*Sqrt[x^2 + 2]),x]')``[Out] (-3 + 2 x ^ 2 - 2 x ^ 4) Sqrt[(2 + x ^ 2) / x ^ 2] / (30 x ^ 4)`**Maple [A]**

time = 0.06, size = 38, normalized size = 0.78

method	result	size
gospers	$-\frac{\sqrt{x^2+2}(2x^4-2x^2+3)}{30x^5}$	25
trager	$-\frac{\sqrt{x^2+2}(2x^4-2x^2+3)}{30x^5}$	25
meijerg	$-\frac{\sqrt{2}(\frac{2}{3}x^4-\frac{2}{3}x^2+1)\sqrt{1+\frac{x^2}{2}}}{10x^5}$	30
risch	$-\frac{2x^6+2x^4-x^2+6}{30x^5\sqrt{x^2+2}}$	30
default	$-\frac{\sqrt{x^2+2}}{10x^5} + \frac{\sqrt{x^2+2}}{15x^3} - \frac{\sqrt{x^2+2}}{15x}$	38

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/x^6/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)``[Out] -1/10*(x^2+2)^(1/2)/x^5+1/15*(x^2+2)^(1/2)/x^3-1/15/x*(x^2+2)^(1/2)`**Maxima [A]**

time = 0.35, size = 37, normalized size = 0.76

$$-\frac{\sqrt{x^2+2}}{15x} + \frac{\sqrt{x^2+2}}{15x^3} - \frac{\sqrt{x^2+2}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^2+2)^(1/2),x, algorithm="maxima")

[Out] -1/15*sqrt(x^2 + 2)/x + 1/15*sqrt(x^2 + 2)/x^3 - 1/10*sqrt(x^2 + 2)/x^5

Fricas [A]

time = 0.31, size = 31, normalized size = 0.63

$$-\frac{2x^5 + (2x^4 - 2x^2 + 3)\sqrt{x^2 + 2}}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^2+2)^(1/2),x, algorithm="fricas")

[Out] -1/30*(2*x^5 + (2*x^4 - 2*x^2 + 3)*sqrt(x^2 + 2))/x^5

Sympy [A]

time = 1.46, size = 41, normalized size = 0.84

$$-\frac{\sqrt{1 + \frac{2}{x^2}}}{15} + \frac{\sqrt{1 + \frac{2}{x^2}}}{15x^2} - \frac{\sqrt{1 + \frac{2}{x^2}}}{10x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**6/(x**2+2)**(1/2),x)

[Out] -sqrt(1 + 2/x**2)/15 + sqrt(1 + 2/x**2)/(15*x**2) - sqrt(1 + 2/x**2)/(10*x**4)

Giac [A]

time = 0.00, size = 59, normalized size = 1.20

$$-\frac{64 \left(-5 \left(\sqrt{x^2 + 2} - x \right)^4 + 5 \left(\sqrt{x^2 + 2} - x \right)^2 - 2 \right)}{30 \left(\left(\sqrt{x^2 + 2} - x \right)^2 - 2 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^2+2)^(1/2),x)

[Out] 32/15*(5*(x - sqrt(x^2 + 2))^4 - 5*(x - sqrt(x^2 + 2))^2 + 2)/((x - sqrt(x^2 + 2))^2 - 2)^5

Mupad [B]

time = 0.03, size = 25, normalized size = 0.51

$$-\sqrt{x^2 + 2} \left(\frac{1}{15x} - \frac{1}{15x^3} + \frac{1}{10x^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^2 + 2)^(1/2)),x)
```

```
[Out] -(x^2 + 2)^(1/2)*(1/(15*x) - 1/(15*x^3) + 1/(10*x^5))
```

$$3.252 \quad \int \frac{1}{(3+2x^2)^{7/2}} dx$$

Optimal. Leaf size=49

$$\frac{x}{15(3+2x^2)^{5/2}} + \frac{4x}{135(3+2x^2)^{3/2}} + \frac{8x}{405\sqrt{3+2x^2}}$$

[Out] 1/15*x/(2*x^2+3)^(5/2)+4/135*x/(2*x^2+3)^(3/2)+8/405*x/(2*x^2+3)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$,

Rules used = {198, 197}

$$\frac{8x}{405\sqrt{2x^2+3}} + \frac{4x}{135(2x^2+3)^{3/2}} + \frac{x}{15(2x^2+3)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(3 + 2*x^2)^(-7/2), x]

[Out] x/(15*(3 + 2*x^2)^(5/2)) + (4*x)/(135*(3 + 2*x^2)^(3/2)) + (8*x)/(405*sqrt[3 + 2*x^2])

Rule 197

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 198

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{(3+2x^2)^{7/2}} dx &= \frac{x}{15(3+2x^2)^{5/2}} + \frac{4}{15} \int \frac{1}{(3+2x^2)^{5/2}} dx \\ &= \frac{x}{15(3+2x^2)^{5/2}} + \frac{4x}{135(3+2x^2)^{3/2}} + \frac{8}{135} \int \frac{1}{(3+2x^2)^{3/2}} dx \\ &= \frac{x}{15(3+2x^2)^{5/2}} + \frac{4x}{135(3+2x^2)^{3/2}} + \frac{8x}{405\sqrt{3+2x^2}} \end{aligned}$$

Mathematica [A]

time = 0.04, size = 28, normalized size = 0.57

$$\frac{x(135 + 120x^2 + 32x^4)}{405(3 + 2x^2)^{5/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(3 + 2*x^2)^(-7/2), x]``[Out] (x*(135 + 120*x^2 + 32*x^4))/(405*(3 + 2*x^2)^(5/2))`**Mathics [A]**

time = 6.85, size = 38, normalized size = 0.78

$$\frac{x(135 + 120x^2 + 32x^4)}{405\sqrt{3 + 2x^2}(9 + 12x^2 + 4x^4)}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(2*x^2 + 3)^(7/2), x]')``[Out] x (135 + 120 x ^ 2 + 32 x ^ 4) / (405 Sqrt[3 + 2 x ^ 2] (9 + 12 x ^ 2 + 4 x ^ 4))`**Maple [A]**

time = 0.05, size = 38, normalized size = 0.78

method	result	size
gospers	$\frac{x(32x^4+120x^2+135)}{405(2x^2+3)^{\frac{5}{2}}}$	25
trager	$\frac{x(32x^4+120x^2+135)}{405(2x^2+3)^{\frac{5}{2}}}$	25
risch	$\frac{x(32x^4+120x^2+135)}{405(2x^2+3)^{\frac{5}{2}}}$	25
meijerg	$\frac{\sqrt{3} x(\frac{32}{9}x^4 + \frac{40}{3}x^2 + 15)}{1215(1 + \frac{2x^2}{3})^{\frac{5}{2}}}$	28
default	$\frac{x}{15(2x^2+3)^{\frac{5}{2}}} + \frac{4x}{135(2x^2+3)^{\frac{3}{2}}} + \frac{8x}{405\sqrt{2x^2+3}}$	38

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(2*x^2+3)^(7/2), x, method=_RETURNVERBOSE)``[Out] 1/15*x/(2*x^2+3)^(5/2)+4/135*x/(2*x^2+3)^(3/2)+8/405*x/(2*x^2+3)^(1/2)`**Maxima [A]**

time = 0.25, size = 37, normalized size = 0.76

$$\frac{8x}{405\sqrt{2x^2+3}} + \frac{4x}{135(2x^2+3)^{\frac{3}{2}}} + \frac{x}{15(2x^2+3)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2+3)^(7/2),x, algorithm="maxima")`

[Out] $8/405*x/\sqrt{2*x^2 + 3} + 4/135*x/(2*x^2 + 3)^{(3/2)} + 1/15*x/(2*x^2 + 3)^{(5/2)}$

Fricas [A]

time = 0.31, size = 44, normalized size = 0.90

$$\frac{(32x^5 + 120x^3 + 135x)\sqrt{2x^2 + 3}}{405(8x^6 + 36x^4 + 54x^2 + 27)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2+3)^(7/2),x, algorithm="fricas")`

[Out] $1/405*(32*x^5 + 120*x^3 + 135*x)*\sqrt{2*x^2 + 3}/(8*x^6 + 36*x^4 + 54*x^2 + 27)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(42) = 84$

time = 3.58, size = 139, normalized size = 2.84

$$\frac{32x^5}{1620x^4\sqrt{2x^2+3} + 4860x^2\sqrt{2x^2+3} + 3645\sqrt{2x^2+3}} + \frac{120x^3}{1620x^4\sqrt{2x^2+3} + 4860x^2\sqrt{2x^2+3} + 3645\sqrt{2x^2+3}} + \frac{135x}{1620x^4\sqrt{2x^2+3} + 4860x^2\sqrt{2x^2+3} + 3645\sqrt{2x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x**2+3)**(7/2),x)`

[Out] $32*x**5/(1620*x**4*\sqrt{2*x**2 + 3} + 4860*x**2*\sqrt{2*x**2 + 3} + 3645*\sqrt{2*x**2 + 3}) + 120*x**3/(1620*x**4*\sqrt{2*x**2 + 3} + 4860*x**2*\sqrt{2*x**2 + 3} + 3645*\sqrt{2*x**2 + 3}) + 135*x/(1620*x**4*\sqrt{2*x**2 + 3} + 4860*x**2*\sqrt{2*x**2 + 3} + 3645*\sqrt{2*x**2 + 3})$

Giac [A]

time = 0.00, size = 41, normalized size = 0.84

$$\frac{2 \left(\left(\frac{16}{405}xx + \frac{4}{27} \right) xx + \frac{1}{6} \right) x\sqrt{2x^2 + 3}}{(2x^2 + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2+3)^(7/2),x)`

[Out] $1/405*(8*(4*x^2 + 15)*x^2 + 135)*x/(2*x^2 + 3)^{(5/2)}$

Mupad [B]

time = 0.05, size = 187, normalized size = 3.82

$$\frac{2\sqrt{2}\sqrt{x^2+\frac{3}{2}}}{405\left(x-\frac{\sqrt{6}11}{2}\right)} + \frac{2\sqrt{2}\sqrt{x^2+\frac{3}{2}}}{405\left(x+\frac{\sqrt{6}11}{2}\right)} + \frac{\sqrt{2}\sqrt{x^2+\frac{3}{2}}}{1440\left(-x^3+\frac{3i\sqrt{6}x^2}{2}+\frac{9x}{2}-\frac{\sqrt{6}3i}{4}\right)} + \frac{\sqrt{2}\sqrt{x^2+\frac{3}{2}}}{1440\left(-x^3-\frac{3i\sqrt{6}x^2}{2}+\frac{9x}{2}+\frac{\sqrt{6}3i}{4}\right)} + \frac{\sqrt{2}\sqrt{6}\sqrt{x^2+\frac{3}{2}}19i}{25920\left(x^2+1i\sqrt{6}x-\frac{3}{2}\right)} + \frac{\sqrt{2}\sqrt{6}\sqrt{x^2+\frac{3}{2}}19i}{25920\left(-x^2+1i\sqrt{6}x+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*x^2 + 3)^(7/2),x)`

[Out] $(2 \cdot 2^{1/2} \cdot (x^2 + 3/2)^{1/2}) / (405 \cdot (x - (6^{1/2} \cdot 1i) / 2)) + (2 \cdot 2^{1/2} \cdot (x^2 + 3/2)^{1/2}) / (405 \cdot (x + (6^{1/2} \cdot 1i) / 2)) + (2^{1/2} \cdot (x^2 + 3/2)^{1/2}) / (1440 \cdot ((9 \cdot x) / 2 - (6^{1/2} \cdot 3i) / 4 + (6^{1/2} \cdot x^2 \cdot 3i) / 2 - x^3)) + (2^{1/2} \cdot (x^2 + 3/2)^{1/2}) / (1440 \cdot ((9 \cdot x) / 2 + (6^{1/2} \cdot 3i) / 4 - (6^{1/2} \cdot x^2 \cdot 3i) / 2 - x^3)) + (2^{1/2} \cdot 6^{1/2} \cdot (x^2 + 3/2)^{1/2} \cdot 19i) / (25920 \cdot (6^{1/2} \cdot x \cdot 1i + x^2 - 3/2)) + (2^{1/2} \cdot 6^{1/2} \cdot (x^2 + 3/2)^{1/2} \cdot 19i) / (25920 \cdot (6^{1/2} \cdot x \cdot 1i - x^2 + 3/2))$

$$3.253 \quad \int \frac{x}{1+x^2+a\sqrt{1+x^2}} dx$$

Optimal. Leaf size=12

$$\log\left(a + \sqrt{1+x^2}\right)$$

[Out] ln(a+(x^2+1)^(1/2))

Rubi [A]

time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2186, 31}

$$\log\left(a + \sqrt{x^2 + 1}\right)$$

Antiderivative was successfully verified.

[In] Int[x/(1 + x^2 + a*Sqrt[1 + x^2]),x]

[Out] Log[a + Sqrt[1 + x^2]]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2186

Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]) , x_Symbol] := Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a + b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[(m + 1)/n]

Rubi steps

$$\begin{aligned} \int \frac{x}{1+x^2+a\sqrt{1+x^2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+x+a\sqrt{1+x}} dx, x, x^2 \right) \\ &= \text{Subst} \left(\int \frac{1}{a+x} dx, x, \sqrt{1+x^2} \right) \\ &= \log\left(a + \sqrt{1+x^2}\right) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 12, normalized size = 1.00

$$\log\left(a + \sqrt{1+x^2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 + x^2 + a*Sqrt[1 + x^2]),x]

[Out] Log[a + Sqrt[1 + x^2]]

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 54 vs. $2(12) = 24$.
time = 3.34, size = 42, normalized size = 3.50

$$-\frac{\text{Log}\left[-\sqrt{1+x^2}\right]}{2} + \frac{\text{Log}\left[1+a\sqrt{1+x^2}+x^2\right]}{2} + \frac{\text{Log}\left[a+\sqrt{1+x^2}\right]}{2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x/(1 + x^2 + a*Sqrt[1 + x^2]),x]')

[Out] -Log[-Sqrt[1 + x ^ 2]] / 2 + Log[1 + a Sqrt[1 + x ^ 2] + x ^ 2] / 2 + Log[a + Sqrt[1 + x ^ 2]] / 2

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(10) = 20$.
time = 0.03, size = 328, normalized size = 27.33

method	result
default	$-\frac{\sqrt{\left(x + \sqrt{(1+a)(a-1)}\right)^2 - 2\sqrt{(1+a)(a-1)}\left(x + \sqrt{(1+a)(a-1)}\right) + a^2}}{2a} + a \ln\left(\frac{2a^2 - 2}{\dots}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(1+x^2+a*(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)

[Out]
$$-1/2/a*((x+((1+a)*(a-1))^(1/2))^2-2*((1+a)*(a-1))^(1/2)*(x+((1+a)*(a-1))^(1/2))+a^2)^(1/2)+1/2*a/(a^2)^(1/2)*\ln((2*a^2-2*((1+a)*(a-1))^(1/2)*(x+((1+a)*(a-1))^(1/2))+2*((1+a)*(a-1))^(1/2)*(x+((1+a)*(a-1))^(1/2))+a^2)^(1/2))/(x+((1+a)*(a-1))^(1/2)))-1/2/a*((x-((1+a)*(a-1))^(1/2))^2+2*((1+a)*(a-1))^(1/2)*(x-((1+a)*(a-1))^(1/2))+a^2)^(1/2)+1/2*a/(a^2)^(1/2)*\ln((2*a^2+2*((1+a)*(a-1))^(1/2)*(x-((1+a)*(a-1))^(1/2))+2*((1+a)*(a-1))^(1/2)*(x-((1+a)*(a-1))^(1/2))+a^2)^(1/2))/(x-((1+a)*(a-1))^(1/2)))+1/a*(x^2+1)^(1/2)+1/2*\ln(-a^2+x^2+1)$$

Maxima [A]

time = 0.26, size = 10, normalized size = 0.83

$$\log\left(a + \sqrt{x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x^2+a*(x^2+1)^(1/2)),x, algorithm="maxima")

[Out] log(a + sqrt(x^2 + 1))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(10) = 20.

time = 0.31, size = 62, normalized size = 5.17

$$\frac{1}{2} \log(-a^2 + x^2 + 1) - \frac{1}{2} \log(ax + x^2 - \sqrt{x^2 + 1}(a + x) + 1) + \frac{1}{2} \log(-ax + x^2 + \sqrt{x^2 + 1}(a - x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x^2+a*(x^2+1)^(1/2)),x, algorithm="fricas")

[Out] 1/2*log(-a^2 + x^2 + 1) - 1/2*log(a*x + x^2 - sqrt(x^2 + 1)*(a + x) + 1) + 1/2*log(-a*x + x^2 + sqrt(x^2 + 1)*(a - x) + 1)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(10) = 20

time = 1.21, size = 53, normalized size = 4.42

$$-\frac{a \left(-\frac{\log(2a+2\sqrt{x^2+1})}{a} + \frac{\log(-2\sqrt{x^2+1})}{a} \right)}{2} + \frac{\log(a\sqrt{x^2+1} + x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x**2+a*(x**2+1)**(1/2)),x)

[Out] -a*(-log(2*a + 2*sqrt(x**2 + 1))/a + log(-2*sqrt(x**2 + 1))/a)/2 + log(a*sqrt(x**2 + 1) + x**2 + 1)/2

Giac [A]

time = 0.00, size = 13, normalized size = 1.08

$$\ln \left| \sqrt{x^2 + 1} + a \right|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x^2+a*(x^2+1)^(1/2)),x)

[Out] log(abs(a + sqrt(x^2 + 1)))

Mupad [B]

time = 0.27, size = 154, normalized size = 12.83

$$\frac{\ln(x + \sqrt{a-1}\sqrt{a+1})}{2} + \frac{\ln(x - \sqrt{a-1}\sqrt{a+1})}{2} - \frac{a(\ln(x + \sqrt{a-1}\sqrt{a+1}) - \ln(\sqrt{x^2+1}\sqrt{a^2 - x\sqrt{a-1}\sqrt{a+1} + 1))}{2\sqrt{(a-1)(a+1)+1}} - \frac{a(\ln(x - \sqrt{a-1}\sqrt{a+1}) - \ln(\sqrt{x^2+1}\sqrt{a^2 + x\sqrt{a-1}\sqrt{a+1} + 1))}{2\sqrt{(a-1)(a+1)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x/(a*(x^2 + 1)^{(1/2)} + x^2 + 1),x)$

[Out] $\log(x + (a - 1)^{(1/2)}*(a + 1)^{(1/2)})/2 + \log(x - (a - 1)^{(1/2)}*(a + 1)^{(1/2)})/2 - (a*(\log(x + (a - 1)^{(1/2)}*(a + 1)^{(1/2)}) - \log((x^2 + 1)^{(1/2)}*(a^2)^{(1/2) - x*(a - 1)^{(1/2)}*(a + 1)^{(1/2)} + 1)))/(2*((a - 1)*(a + 1) + 1)^{(1/2)}) - (a*(\log(x - (a - 1)^{(1/2)}*(a + 1)^{(1/2)}) - \log((x^2 + 1)^{(1/2)}*(a^2)^{(1/2) + x*(a - 1)^{(1/2)}*(a + 1)^{(1/2)} + 1)))/(2*((a - 1)*(a + 1) + 1)^{(1/2)})$

$$3.254 \quad \int \frac{1-x+x^2}{(1+x^2)^{3/2}} dx$$

Optimal. Leaf size=12

$$\frac{1}{\sqrt{1+x^2}} + \sinh^{-1}(x)$$

[Out] arcsinh(x)+1/(x^2+1)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1828, 221}

$$\frac{1}{\sqrt{x^2+1}} + \sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - x + x^2)/(1 + x^2)^(3/2), x]

[Out] 1/Sqrt[1 + x^2] + ArcSinh[x]

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 1828

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{1-x+x^2}{(1+x^2)^{3/2}} dx &= \frac{1}{\sqrt{1+x^2}} + \int \frac{1}{\sqrt{1+x^2}} dx \\ &= \frac{1}{\sqrt{1+x^2}} + \sinh^{-1}(x) \end{aligned}$$

Mathematica [A]

time = 0.10, size = 22, normalized size = 1.83

$$\frac{1}{\sqrt{1+x^2}} + \tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

Antiderivative was successfully verified.

`[In] Integrate[(1 - x + x^2)/(1 + x^2)^(3/2),x]``[Out] 1/Sqrt[1 + x^2] + ArcTanh[x/Sqrt[1 + x^2]]`**Mathics [A]**

time = 7.55, size = 10, normalized size = 0.83

$$\text{ArcSinh}[x] + \frac{1}{\sqrt{1+x^2}}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(x^2 - x + 1)/((1 + x^2)*Sqrt[1 + x^2]),x]')``[Out] ArcSinh[x] + 1 / Sqrt[1 + x ^ 2]`**Maple [A]**

time = 0.06, size = 11, normalized size = 0.92

method	result	size
default	$\text{arcsinh}(x) + \frac{1}{\sqrt{x^2+1}}$	11
risch	$\text{arcsinh}(x) + \frac{1}{\sqrt{x^2+1}}$	11
trager	$\frac{1}{\sqrt{x^2+1}} + \ln(x + \sqrt{x^2+1})$	19
meijerg	$\frac{x}{\sqrt{x^2+1}} + \frac{-\frac{\sqrt{\pi} x}{\sqrt{x^2+1}} + \sqrt{\pi} \text{arcsinh}(x)}{\sqrt{\pi}} - \frac{\sqrt{\pi} - \frac{\sqrt{\pi}}{\sqrt{x^2+1}}}{\sqrt{\pi}}$	56

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((x^2-x+1)/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)``[Out] arcsinh(x)+1/(x^2+1)^(1/2)`**Maxima [A]**

time = 0.33, size = 10, normalized size = 0.83

$$\frac{1}{\sqrt{x^2+1}} + \text{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-x+1)/(x^2+1)^(3/2),x, algorithm="maxima")

[Out] 1/sqrt(x^2 + 1) + arcsinh(x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(10) = 20.

time = 0.30, size = 37, normalized size = 3.08

$$\frac{(x^2 + 1) \log(-x + \sqrt{x^2 + 1}) - \sqrt{x^2 + 1}}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-x+1)/(x^2+1)^(3/2),x, algorithm="fricas")

[Out] -((x^2 + 1)*log(-x + sqrt(x^2 + 1)) - sqrt(x^2 + 1))/(x^2 + 1)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24

time = 5.50, size = 29, normalized size = 2.42

$$\frac{x^2 \operatorname{asinh}(x)}{x^2 + 1} + \frac{\operatorname{asinh}(x)}{x^2 + 1} + \frac{1}{\sqrt{x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-x+1)/(x**2+1)**(3/2),x)

[Out] x**2*asinh(x)/(x**2 + 1) + asinh(x)/(x**2 + 1) + 1/sqrt(x**2 + 1)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(10) = 20.
time = 0.00, size = 31, normalized size = 2.58

$$\frac{\sqrt{x^2 + 1}}{x^2 + 1} - \ln(\sqrt{x^2 + 1} - x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-x+1)/(x^2+1)^(3/2),x)

[Out] 1/sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))

Mupad [B]

time = 0.19, size = 24, normalized size = 2.00

$$\frac{\operatorname{asinh}(x) + x^2 \operatorname{asinh}(x) + \sqrt{x^2 + 1}}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - x + 1)/(x^2 + 1)^(3/2),x)

[Out] (asinh(x) + x^2*asinh(x) + (x^2 + 1)^(1/2))/(x^2 + 1)

$$3.255 \quad \int \frac{\sqrt{1+x^2}}{2+x^2} dx$$

Optimal. Leaf size=27

$$\sinh^{-1}(x) - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{1+x^2}}\right)}{\sqrt{2}}$$

[Out] arcsinh(x)-1/2*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {399, 221, 385, 212}

$$\sinh^{-1}(x) - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^2]/(2 + x^2), x]

[Out] ArcSinh[x] - ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])]/Sqrt[2]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 399

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[b/d, Int[(a + b*x^n)^(p-1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^n)^(p-1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*

d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1+x^2}}{2+x^2} dx &= \int \frac{1}{\sqrt{1+x^2}} dx - \int \frac{1}{\sqrt{1+x^2}(2+x^2)} dx \\ &= \sinh^{-1}(x) - \text{Subst} \left(\int \frac{1}{2-x^2} dx, x, \frac{x}{\sqrt{1+x^2}} \right) \\ &= \sinh^{-1}(x) - \frac{\tanh^{-1} \left(\frac{x}{\sqrt{2}\sqrt{1+x^2}} \right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A]

time = 0.07, size = 44, normalized size = 1.63

$$\tanh^{-1} \left(\frac{x}{\sqrt{1+x^2}} \right) - \frac{\tanh^{-1} \left(\frac{2+x^2-x\sqrt{1+x^2}}{\sqrt{2}} \right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + x^2]/(2 + x^2), x]

[Out] ArcTanh[x/Sqrt[1 + x^2]] - ArcTanh[(2 + x^2 - x*Sqrt[1 + x^2])/Sqrt[2]]/Sqrt[2]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sqrt[1 + x^2]/(2 + x^2), x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.10, size = 23, normalized size = 0.85

method	result	size
--------	--------	------

default	$\operatorname{arcsinh}(x) - \frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2\sqrt{x^2+1}}\right)\sqrt{2}}{2}$	23
trager	$\ln(x + \sqrt{x^2+1}) - \frac{\operatorname{RootOf}(-Z^2-2) \ln\left(\frac{3\operatorname{RootOf}(-Z^2-2)x^2+4x\sqrt{x^2+1}+2\operatorname{RootOf}(-Z^2-2)}{x^2+2}\right)}{4}$	59

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+1)^(1/2)/(x^2+2),x,method=_RETURNVERBOSE)`

[Out] `arcsinh(x)-1/2*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)^(1/2)/(x^2+2),x, algorithm="maxima")`

[Out] `sqrt(x^2 + 1)*x/(x^2 + 2) + integrate(sqrt(x^2 + 1)*x^4/(x^6 + 5*x^4 + 8*x^2 + 4), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(22) = 44.

time = 0.31, size = 67, normalized size = 2.48

$$\frac{1}{4}\sqrt{2}\log\left(\frac{9x^2 - 2\sqrt{2}(3x^2 + 2) - 2\sqrt{x^2 + 1}(3\sqrt{2}x - 4x) + 6}{x^2 + 2}\right) - \log(-x + \sqrt{x^2 + 1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)^(1/2)/(x^2+2),x, algorithm="fricas")`

[Out] `1/4*sqrt(2)*log((9*x^2 - 2*sqrt(2)*(3*x^2 + 2) - 2*sqrt(x^2 + 1)*(3*sqrt(2)*x - 4*x) + 6)/(x^2 + 2)) - log(-x + sqrt(x^2 + 1))`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{x^2+2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)**(1/2)/(x**2+2),x)

[Out] Integral(sqrt(x**2 + 1)/(x**2 + 2), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(22) = 44.
time = 0.00, size = 83, normalized size = 3.07

$$2 \left(-\frac{\ln(\sqrt{x^2+1} - x)}{2} + \frac{\ln\left(\frac{2(\sqrt{x^2+1} - x)^2 + 6 - 4\sqrt{2}}{2(\sqrt{x^2+1} - x)^2 + 6 + 4\sqrt{2}}\right)}{4\sqrt{2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^(1/2)/(x^2+2),x)

[Out] 1/4*sqrt(2)*log(((x - sqrt(x^2 + 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 + 1))^2 + 2*sqrt(2) + 3)) - log(-x + sqrt(x^2 + 1))

Mupad [B]

time = 0.17, size = 77, normalized size = 2.85

$$\operatorname{asinh}(x) + \frac{\sqrt{2} \left(\ln(x - \sqrt{2} i) - \ln(1 + \sqrt{2} x i + \sqrt{x^2 + 1} i) \right)}{4} - \frac{\sqrt{2} \left(\ln(x + \sqrt{2} i) - \ln(1 - \sqrt{2} x i + \sqrt{x^2 + 1} i) \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)^(1/2)/(x^2 + 2),x)

[Out] asinh(x) + (2^(1/2)*(log(x - 2^(1/2)*1i) - log(2^(1/2)*x*1i + (x^2 + 1)^(1/2)*1i + 1))/4 - (2^(1/2)*(log(x + 2^(1/2)*1i) - log((x^2 + 1)^(1/2)*1i - 2^(1/2)*x*1i + 1))/4

$$3.256 \quad \int \frac{1}{\sqrt{1+x^2} (2+x^2)^2} dx$$

Optimal. Leaf size=48

$$-\frac{x\sqrt{1+x^2}}{4(2+x^2)} + \frac{3 \tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{1+x^2}}\right)}{4\sqrt{2}}$$

[Out] 3/8*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)-1/4*x*(x^2+1)^(1/2)/(x^2+2)

Rubi [A]

time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {390, 385, 212}

$$\frac{3 \tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{4\sqrt{2}} - \frac{x\sqrt{x^2+1}}{4(x^2+2)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 + x^2]*(2 + x^2)^2),x]

[Out] -1/4*(x*Sqrt[1 + x^2])/(2 + x^2) + (3*ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])])/(4*Sqrt[2])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(-b)*x*(a + b*x^n)^(p+1)*((c + d*x^n)^(q+1)/(a*n*(p+1)*(b*c - a*d))), x] + Dist[(b*c + n*(p+1)*(b*c - a*d))/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p+q+2) + 1, 0] && (LtQ[p, -1] || !L

tQ[q, -1]) && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1+x^2} (2+x^2)^2} dx &= -\frac{x\sqrt{1+x^2}}{4(2+x^2)} + \frac{3}{4} \int \frac{1}{\sqrt{1+x^2} (2+x^2)} dx \\ &= -\frac{x\sqrt{1+x^2}}{4(2+x^2)} + \frac{3}{4} \text{Subst} \left(\int \frac{1}{2-x^2} dx, x, \frac{x}{\sqrt{1+x^2}} \right) \\ &= -\frac{x\sqrt{1+x^2}}{4(2+x^2)} + \frac{3 \tanh^{-1} \left(\frac{x}{\sqrt{2}\sqrt{1+x^2}} \right)}{4\sqrt{2}} \end{aligned}$$

Mathematica [A]

time = 0.09, size = 55, normalized size = 1.15

$$\frac{1}{8} \left(-\frac{2x\sqrt{1+x^2}}{2+x^2} + 3\sqrt{2} \tanh^{-1} \left(\frac{2+x^2-x\sqrt{1+x^2}}{\sqrt{2}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[1 + x^2]*(2 + x^2)^2), x]

[Out] ((-2*x*Sqrt[1 + x^2])/(2 + x^2) + 3*Sqrt[2]*ArcTanh[(2 + x^2 - x*Sqrt[1 + x^2])/Sqrt[2]])/8

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((2 + x^2)^2*Sqrt[1 + x^2]), x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [A]

time = 0.11, size = 46, normalized size = 0.96

method	result	size
risch	$\frac{3 \operatorname{arctanh} \left(\frac{x\sqrt{2}}{2\sqrt{x^2+1}} \right) \sqrt{2}}{8} - \frac{x\sqrt{x^2+1}}{4(x^2+2)}$	38

default	$\frac{x}{4\sqrt{x^2+1} \left(\frac{x^2}{x^2+1}-2\right)} + \frac{3 \operatorname{arctanh}\left(\frac{x\sqrt{2}}{2\sqrt{x^2+1}}\right)\sqrt{2}}{8}$	46
trager	$-\frac{x\sqrt{x^2+1}}{4(x^2+2)} + \frac{3 \operatorname{RootOf}(-Z^2-2) \ln\left(\frac{3 \operatorname{RootOf}(-Z^2-2)^{x^2+4x}\sqrt{x^2+1} + 2 \operatorname{RootOf}(-Z^2-2)}{x^2+2}\right)}{16}$	66

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2+2)^2/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $1/4*x/(x^2+1)^(1/2)/(x^2/(x^2+1)-2)+3/8*\operatorname{arctanh}(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2+2)^2/(x^2+1)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/((x^2 + 2)^2*sqrt(x^2 + 1)), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(37) = 74.

time = 0.30, size = 83, normalized size = 1.73

$$\frac{3\sqrt{2}(x^2+2)\log\left(\frac{9x^{2+2}\sqrt{2}(3x^2+2)+2\sqrt{x^2+1}(3\sqrt{2}x+4x)+6}{x^2+2}\right) - 4x^2 - 4\sqrt{x^2+1}x - 8}{16(x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2+2)^2/(x^2+1)^(1/2),x, algorithm="fricas")`

[Out] $1/16*(3*\sqrt{2}*(x^2+2)*\log((9*x^2+2*\sqrt{2}*(3*x^2+2)+2*\sqrt{x^2+1}*(3*\sqrt{2}*x+4*x)+6)/(x^2+2))-4*x^2-4*\sqrt{x^2+1}*x-8)/(x^2+2)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2+1}(x^2+2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2+2)**2/(x**2+1)**(1/2),x)`

[Out] `Integral(1/(sqrt(x**2 + 1)*(x**2 + 2)**2), x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(37) = 74.

time = 0.00, size = 122, normalized size = 2.54

$$-8 \left(\frac{3 \left(\sqrt{x^2 + 1} - x \right)^2 + 1}{16 \left(\left(\sqrt{x^2 + 1} - x \right)^4 + 6 \left(\sqrt{x^2 + 1} - x \right)^2 + 1 \right)} + \frac{3 \ln \left(\frac{2 \left(\sqrt{x^2 + 1} - x \right)^2 + 6 - 4\sqrt{2}}{2 \left(\sqrt{x^2 + 1} - x \right)^2 + 6 + 4\sqrt{2}} \right)}{64\sqrt{2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2+2)^2/(x^2+1)^(1/2),x)`

[Out] `-3/16*sqrt(2)*log(((x - sqrt(x^2 + 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 + 1))^2 + 2*sqrt(2) + 3)) - 1/2*(3*(x - sqrt(x^2 + 1))^2 + 1)/((x - sqrt(x^2 + 1))^4 + 6*(x - sqrt(x^2 + 1))^2 + 1)`

Mupad [B]

time = 0.09, size = 117, normalized size = 2.44

$$-\frac{3\sqrt{2} \left(\ln(x - \sqrt{2} \operatorname{li}) - \ln(1 + \sqrt{2} x \operatorname{li} + \sqrt{x^2 + 1} \operatorname{li}) \right)}{16} + \frac{3\sqrt{2} \left(\ln(x + \sqrt{2} \operatorname{li}) - \ln(1 - \sqrt{2} x \operatorname{li} + \sqrt{x^2 + 1} \operatorname{li}) \right)}{16} - \frac{\sqrt{x^2 + 1}}{8(x - \sqrt{2} \operatorname{li})} - \frac{\sqrt{x^2 + 1}}{8(x + \sqrt{2} \operatorname{li})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((x^2 + 1)^(1/2)*(x^2 + 2)^2),x)`

[Out] `(3*2^(1/2)*(log(x + 2^(1/2)*1i) - log((x^2 + 1)^(1/2)*1i - 2^(1/2)*x*1i + 1)))/16 - (3*2^(1/2)*(log(x - 2^(1/2)*1i) - log(2^(1/2)*x*1i + (x^2 + 1)^(1/2)*1i + 1))/16 - (x^2 + 1)^(1/2)/(8*(x - 2^(1/2)*1i)) - (x^2 + 1)^(1/2)/(8*(x + 2^(1/2)*1i))`

$$3.257 \quad \int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}} dx$$

Optimal. Leaf size=41

$$\tanh^{-1}\left(\frac{x}{\sqrt{-2+x^2}}\right) - \sqrt{\frac{3}{2}} \tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{-2+x^2}}\right)$$

[Out] arctanh(x/(x^2-2)^(1/2))-1/2*arctanh(1/3*x*6^(1/2)/(x^2-2)^(1/2))*6^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {494, 223, 212, 385, 213}

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-2}}\right) - \sqrt{\frac{3}{2}} \tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{x^2-2}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^2/((-6 + x^2)*Sqrt[-2 + x^2]),x]

[Out] ArcTanh[x/Sqrt[-2 + x^2]] - Sqrt[3/2]*ArcTanh[(Sqrt[2/3]*x)/Sqrt[-2 + x^2]]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 223

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 385


```
Int[((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 494

```
Int[(((e_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_))^(q_))/((a_) + (b_)*(x_)^(
n_)), x_Symbol] := Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(-6 + x^2)\sqrt{-2 + x^2}} dx &= 6 \int \frac{1}{(-6 + x^2)\sqrt{-2 + x^2}} dx + \int \frac{1}{\sqrt{-2 + x^2}} dx \\ &= 6 \operatorname{Subst} \left(\int \frac{1}{-6 + 4x^2} dx, x, \frac{x}{\sqrt{-2 + x^2}} \right) + \operatorname{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \frac{x}{\sqrt{-2 + x^2}} \right) \\ &= \tanh^{-1} \left(\frac{x}{\sqrt{-2 + x^2}} \right) - \sqrt{\frac{3}{2}} \tanh^{-1} \left(\frac{\sqrt{\frac{2}{3}} x}{\sqrt{-2 + x^2}} \right) \end{aligned}$$

Mathematica [A]

time = 0.12, size = 50, normalized size = 1.22

$$\tanh^{-1} \left(\frac{x}{\sqrt{-2 + x^2}} \right) - \sqrt{\frac{3}{2}} \tanh^{-1} \left(\frac{6 - x^2 + x\sqrt{-2 + x^2}}{2\sqrt{6}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-6 + x^2)*Sqrt[-2 + x^2]), x]
```

```
[Out] ArcTanh[x/Sqrt[-2 + x^2]] - Sqrt[3/2]*ArcTanh[(6 - x^2 + x*Sqrt[-2 + x^2])/
(2*Sqrt[6])]
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[x^2/((x^2 - 6)*Sqrt[x^2 - 2]),x]')`

[Out] `cought exception: maximum recursion depth exceeded while calling a Python object`

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(30) = 60$.

time = 0.16, size = 100, normalized size = 2.44

method	result
trager	$-\ln(x - \sqrt{x^2 - 2}) - \frac{\text{RootOf}(-Z^2 - 6) \ln\left(\frac{5 \text{RootOf}(-Z^2 - 6) x^2 + 12 \sqrt{x^2 - 2} x - 6 \text{RootOf}(-Z^2 - 6)}{x^2 - 6}\right)}{4}$
default	$\ln(x + \sqrt{x^2 - 2}) - \frac{\sqrt{6} \operatorname{arctanh}\left(\frac{8 + 2\sqrt{6}(x - \sqrt{6})}{4\sqrt{(x - \sqrt{6})^2 + 2\sqrt{6}(x - \sqrt{6}) + 4}}\right)}{4} + \frac{\sqrt{6} \operatorname{arctanh}\left(\frac{8 - 2\sqrt{6}(x + \sqrt{6})}{4\sqrt{(x + \sqrt{6})^2 - 2\sqrt{6}(x + \sqrt{6}) + 4}}\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(x^2-6)/(x^2-2)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `ln(x+(x^2-2)^(1/2))-1/4*6^(1/2)*arctanh(1/4*(8+2*6^(1/2)*(x-6^(1/2)))/((x-6^(1/2))^2+2*6^(1/2)*(x-6^(1/2))+4)^(1/2))+1/4*6^(1/2)*arctanh(1/4*(8-2*6^(1/2)*(x+6^(1/2)))/((x+6^(1/2))^2-2*6^(1/2)*(x+6^(1/2))+4)^(1/2))`

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(30) = 60$.

time = 0.35, size = 107, normalized size = 2.61

$$\frac{1}{12} \sqrt{6} \left(2\sqrt{6} \log(x + \sqrt{x^2 - 2}) - 3 \log\left(\sqrt{6} + \frac{4\sqrt{x^2 - 2}}{|2x - 2\sqrt{6}|} + \frac{8}{|2x - 2\sqrt{6}|}\right) + 3 \log\left(-\sqrt{6} + \frac{4\sqrt{x^2 - 2}}{|2x + 2\sqrt{6}|} + \frac{8}{|2x + 2\sqrt{6}|}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(x^2-6)/(x^2-2)^(1/2),x, algorithm="maxima")`

[Out] `1/12*sqrt(6)*(2*sqrt(6)*log(x + sqrt(x^2 - 2)) - 3*log(sqrt(6) + 4*sqrt(x^2 - 2)/abs(2*x - 2*sqrt(6)) + 8/abs(2*x - 2*sqrt(6))) + 3*log(-sqrt(6) + 4*sqrt(x^2 - 2)/abs(2*x + 2*sqrt(6)) + 8/abs(2*x + 2*sqrt(6))))`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(30) = 60$.

time = 0.29, size = 77, normalized size = 1.88

$$\frac{1}{4} \sqrt{3} \sqrt{2} \log\left(-\frac{2\sqrt{3}\sqrt{2}(5x^2 - 6) - 25x^2 + 2(5\sqrt{3}\sqrt{2}x - 12x)\sqrt{x^2 - 2} + 30}{x^2 - 6}\right) - \log(-x + \sqrt{x^2 - 2})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(x^2-6)/(x^2-2)^(1/2),x, algorithm="fricas")`

[Out] $\frac{1}{4}\sqrt{3}\sqrt{2}\log(-2\sqrt{3}\sqrt{2}(5x^2 - 6) - 25x^2 + 2(5\sqrt{3}\sqrt{2}x - 12x)\sqrt{x^2 - 2} + 30)/(x^2 - 6)) - \log(-x + \sqrt{x^2 - 2})$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^2 - 6)\sqrt{x^2 - 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(x**2-6)/(x**2-2)**(1/2),x)`

[Out] `Integral(x**2/((x**2 - 6)*sqrt(x**2 - 2)), x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(30) = 60$.
time = 0.01, size = 89, normalized size = 2.17

$$2 \left(-\frac{\ln\left(\left(\sqrt{x^2 - 2} - x\right)^2\right)}{4} - \frac{3 \ln\left(\frac{\left|2\left(\sqrt{x^2 - 2} - x\right)^2 - 20 - 8\sqrt{6}\right|}{\left|2\left(\sqrt{x^2 - 2} - x\right)^2 - 20 + 8\sqrt{6}\right|}\right)}{4\sqrt{6}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(x^2-6)/(x^2-2)^(1/2),x)`

[Out] $-\frac{1}{4}\sqrt{6}\log(\text{abs}(2(x - \sqrt{x^2 - 2}))^2 - 8\sqrt{6} - 20)/\text{abs}(2(x - \sqrt{x^2 - 2}))^2 + 8\sqrt{6} - 20) - \frac{1}{2}\log((x - \sqrt{x^2 - 2})^2)$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{x^2 - 2}(x^2 - 6)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((x^2 - 2)^(1/2)*(x^2 - 6)),x)`

[Out] `int(x^2/((x^2 - 2)^(1/2)*(x^2 - 6)), x)`

$$3.258 \quad \int \frac{5+x^2}{\sqrt{1-x^2} (1+x^2)^2} dx$$

Optimal. Leaf size=47

$$\frac{x\sqrt{1-x^2}}{1+x^2} + 2\sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)$$

[Out] 2*arctan(x*2^(1/2)/(-x^2+1)^(1/2))*2^(1/2)+x*(-x^2+1)^(1/2)/(x^2+1)

Rubi [A]

time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {541, 12, 385, 209}

$$\frac{\sqrt{1-x^2}x}{x^2+1} + 2\sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(5 + x^2)/(Sqrt[1 - x^2]*(1 + x^2)^2),x]

[Out] (x*Sqrt[1 - x^2])/(1 + x^2) + 2*Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 541

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(-b*e - a*f)*x*(a + b*x^n)^(p+1)*((c + d*x^n)^(q+1)/(a*n*(b*c - a*d)*(p+1))), x] + Dist[1/(a*n*(b*c - a*d)*(p+1)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b

$c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] \&\& LtQ[p, -1]$

Rubi steps

$$\begin{aligned} \int \frac{5 + x^2}{\sqrt{1 - x^2} (1 + x^2)^2} dx &= \frac{x\sqrt{1 - x^2}}{1 + x^2} - \frac{1}{4} \int -\frac{16}{\sqrt{1 - x^2} (1 + x^2)} dx \\ &= \frac{x\sqrt{1 - x^2}}{1 + x^2} + 4 \int \frac{1}{\sqrt{1 - x^2} (1 + x^2)} dx \\ &= \frac{x\sqrt{1 - x^2}}{1 + x^2} + 4 \text{Subst} \left(\int \frac{1}{1 + 2x^2} dx, x, \frac{x}{\sqrt{1 - x^2}} \right) \\ &= \frac{x\sqrt{1 - x^2}}{1 + x^2} + 2\sqrt{2} \tan^{-1} \left(\frac{\sqrt{2} x}{\sqrt{1 - x^2}} \right) \end{aligned}$$

Mathematica [A]

time = 0.14, size = 47, normalized size = 1.00

$$\frac{x\sqrt{1 - x^2}}{1 + x^2} + 2\sqrt{2} \tan^{-1} \left(\frac{\sqrt{2} x}{\sqrt{1 - x^2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x^2)/(Sqrt[1 - x^2]*(1 + x^2)^2),x]

[Out] (x*Sqrt[1 - x^2])/(1 + x^2) + 2*Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(x^2 + 5)/((1 + x^2)^2*Sqrt[1 - x^2]),x]')

[Out] Timed out

Maple [A]

time = 0.13, size = 70, normalized size = 1.49

method	result	size
--------	--------	------

risch	$-\frac{x(x^2-1)}{(x^2+1)\sqrt{-x^2+1}} - 2\sqrt{2} \arctan\left(\frac{x\sqrt{-x^2+1}\sqrt{2}}{x^2-1}\right)$	53
trager	$\frac{x\sqrt{-x^2+1}}{x^2+1} + \text{RootOf}(_Z^2+2) \ln\left(\frac{-3\text{RootOf}(_Z^2+2)x^2+4x\sqrt{-x^2+1}+\text{RootOf}(_Z^2+2)}{x^2+1}\right)$	66
default	$-\frac{x\sqrt{-x^2+1}}{2(x^2-1)\left(\frac{x^2(-x^2+1)}{(x^2-1)^2}+\frac{1}{2}\right)} - 2\sqrt{2} \arctan\left(\frac{x\sqrt{-x^2+1}\sqrt{2}}{x^2-1}\right)$	70

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `-1/2*x/(x^2-1)*(-x^2+1)^(1/2)/(x^2/(x^2-1)^2*(-x^2+1)+1/2)-2*2^(1/2)*arctan(x/(x^2-1)*(-x^2+1)^(1/2)*2^(1/2))`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x, algorithm="maxima")`

[Out] `integrate((x^2 + 5)/((x^2 + 1)^2*sqrt(-x^2 + 1)), x)`

Fricas [A]

time = 0.30, size = 50, normalized size = 1.06

$$\frac{2\sqrt{2}(x^2+1)\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}}{2x}\right) - \sqrt{-x^2+1}x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x, algorithm="fricas")`

[Out] `-(2*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 1)/x) - sqrt(-x^2 + 1)*x)/(x^2 + 1)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 5}{\sqrt{-(x-1)(x+1)}(x^2+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+5)/(x**2+1)**2/(-x**2+1)**(1/2), x)

[Out] Integral((x**2 + 5)/(sqrt(-(x - 1)*(x + 1))*(x**2 + 1)**2), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(39) = 78.

time = 0.01, size = 158, normalized size = 3.36

$$\frac{2 \left(\frac{2x}{-2\sqrt{-x^2+1}+2} - \frac{-2\sqrt{-x^2+1}+2}{2x} \right)}{\left(\frac{2x}{-2\sqrt{-x^2+1}+2} - \frac{-2\sqrt{-x^2+1}+2}{2x} \right)^2 + 8} - \frac{4 \left(-\frac{1}{2}\pi \operatorname{sign}(x) - \arctan \left(\frac{x \left(\left(\frac{-2\sqrt{-x^2+1}+2}{2x} \right)^2 - 1 \right)}{\sqrt{2} \left(-2\sqrt{-x^2+1}+2 \right)} \right) \right)}{\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2), x)

[Out] sqrt(2)*(pi*sgn(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - 2*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)^2 + 8)

Mupad [B]

time = 0.13, size = 115, normalized size = 2.45

$$\sqrt{2} \ln \left(\frac{\frac{\sqrt{2} (-1+x \operatorname{li}) \operatorname{li}}{2} - \sqrt{1-x^2} \operatorname{li}}{x-i} \right) \operatorname{li} - \sqrt{2} \ln \left(\frac{\frac{\sqrt{2} (1+x \operatorname{li}) \operatorname{li}}{2} + \sqrt{1-x^2} \operatorname{li}}{x+\operatorname{li}} \right) \operatorname{li} + \frac{\sqrt{1-x^2}}{2(x-i)} + \frac{\sqrt{1-x^2}}{2(x+\operatorname{li})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 5)/((1 - x^2)^(1/2)*(x^2 + 1)^2), x)

[Out] 2^(1/2)*log(((2^(1/2)*(x*1i - 1)*1i)/2 - (1 - x^2)^(1/2)*1i)/(x - 1i))*1i - 2^(1/2)*log(((2^(1/2)*(x*1i + 1)*1i)/2 + (1 - x^2)^(1/2)*1i)/(x + 1i))*1i + (1 - x^2)^(1/2)/(2*(x - 1i)) + (1 - x^2)^(1/2)/(2*(x + 1i))

$$3.259 \quad \int \frac{4x - \sqrt{1-x^2}}{5 + \sqrt{1-x^2}} dx$$

Optimal. Leaf size=88

$$-x - 4\sqrt{1-x^2} + 5 \sin^{-1}(x) + \frac{25 \tan^{-1}\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}} - \frac{25 \tan^{-1}\left(\frac{5x}{2\sqrt{6}\sqrt{1-x^2}}\right)}{2\sqrt{6}} + 20 \log\left(5 + \sqrt{1-x^2}\right)$$

[Out] $-x+5*\arcsin(x)+20*\ln(5+(-x^2+1)^{(1/2)})+25/12*\arctan(1/12*x*6^{(1/2)})*6^{(1/2)}$
 $-25/12*\arctan(5/12*x*6^{(1/2)} / (-x^2+1)^{(1/2)})*6^{(1/2)}-4*(-x^2+1)^{(1/2)}$

Rubi [A]

time = 0.15, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6874, 1605, 196, 45, 6872, 209, 399, 222, 385}

$$-4\sqrt{1-x^2} + 20 \log\left(\sqrt{1-x^2} + 5\right) - \frac{25 \tan^{-1}\left(\frac{5x}{2\sqrt{6}\sqrt{1-x^2}}\right)}{2\sqrt{6}} - x + 5 \sin^{-1}(x) + \frac{25 \tan^{-1}\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(4*x - Sqrt[1 - x^2])/(5 + Sqrt[1 - x^2]),x]

[Out] $-x - 4*\text{Sqrt}[1 - x^2] + 5*\text{ArcSin}[x] + (25*\text{ArcTan}[x/(2*\text{Sqrt}[6])])/(2*\text{Sqrt}[6])$
 $- (25*\text{ArcTan}[(5*x)/(2*\text{Sqrt}[6]*\text{Sqrt}[1 - x^2]))/(2*\text{Sqrt}[6]) + 20*\text{Log}[5 + \text{Sqrt}[1 - x^2]]$

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 196

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(1/n - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] := With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
PolyQ[Qr, x]
```

Rule 6872

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rule 6874

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{4x - \sqrt{1-x^2}}{5 + \sqrt{1-x^2}} dx &= \int \left(\frac{4x}{5 + \sqrt{1-x^2}} - \frac{\sqrt{1-x^2}}{5 + \sqrt{1-x^2}} \right) dx \\
&= 4 \int \frac{x}{5 + \sqrt{1-x^2}} dx - \int \frac{\sqrt{1-x^2}}{5 + \sqrt{1-x^2}} dx \\
&= - \left(2 \text{Subst} \left(\int \frac{1}{5 + \sqrt{x}} dx, x, 1-x^2 \right) \right) - \int \left(1 - \frac{5}{5 + \sqrt{1-x^2}} \right) dx \\
&= -x - 4 \text{Subst} \left(\int \frac{x}{5+x} dx, x, \sqrt{1-x^2} \right) + 5 \int \frac{1}{5 + \sqrt{1-x^2}} dx \\
&= -x - 4 \text{Subst} \left(\int \left(1 - \frac{5}{5+x} \right) dx, x, \sqrt{1-x^2} \right) + 5 \int \left(\frac{5}{24+x^2} - \frac{\sqrt{1-x^2}}{24+x^2} \right) dx \\
&= -x - 4\sqrt{1-x^2} + 20 \log(5 + \sqrt{1-x^2}) - 5 \int \frac{\sqrt{1-x^2}}{24+x^2} dx + 25 \int \frac{1}{24+x^2} dx \\
&= -x - 4\sqrt{1-x^2} + \frac{25 \tan^{-1} \left(\frac{x}{2\sqrt{6}} \right)}{2\sqrt{6}} + 20 \log(5 + \sqrt{1-x^2}) + 5 \int \frac{1}{\sqrt{1-x^2}} dx \\
&= -x - 4\sqrt{1-x^2} + 5 \sin^{-1}(x) + \frac{25 \tan^{-1} \left(\frac{x}{2\sqrt{6}} \right)}{2\sqrt{6}} + 20 \log(5 + \sqrt{1-x^2}) - 125 \text{S} \\
&= -x - 4\sqrt{1-x^2} + 5 \sin^{-1}(x) + \frac{25 \tan^{-1} \left(\frac{x}{2\sqrt{6}} \right)}{2\sqrt{6}} - \frac{25 \tan^{-1} \left(\frac{5x}{2\sqrt{6} \sqrt{1-x^2}} \right)}{2\sqrt{6}} + \dots
\end{aligned}$$

Mathematica [A]

time = 0.21, size = 108, normalized size = 1.23

$$-x - 4\sqrt{1-x^2} + 10 \tan^{-1} \left(\frac{x}{-1 + \sqrt{1-x^2}} \right) - \frac{25 \tan^{-1} \left(\frac{\sqrt{\frac{3}{2}} x}{-1 + \sqrt{1-x^2}} \right)}{\sqrt{6}} - 20 \log(-1 + \sqrt{1-x^2}) + 20 \log(-4 - x^2 + 4\sqrt{1-x^2})$$

Antiderivative was successfully verified.

`[In] Integrate[(4*x - Sqrt[1 - x^2])/(5 + Sqrt[1 - x^2]), x]`

```
[Out] -x - 4*Sqrt[1 - x^2] + 10*ArcTan[x/(-1 + Sqrt[1 - x^2])] - (25*ArcTan[(Sqrt[3/2]*x)/(-1 + Sqrt[1 - x^2])])/Sqrt[6] - 20*Log[-1 + Sqrt[1 - x^2]] + 20*Log[-4 - x^2 + 4*Sqrt[1 - x^2]]
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(4*x - Sqrt[1 - x^2])/(5 + Sqrt[1 - x^2]),x]')`

[Out] `cought exception: maximum recursion depth exceeded`

Maple [A]

time = 0.44, size = 82, normalized size = 0.93

method	result
default	$\frac{25 \arctan\left(\frac{x\sqrt{6}}{12}\right)\sqrt{6}}{12} + 10 \ln(x^2 + 24) - x + 5 \arcsin(x) + \frac{25\sqrt{6} \arctan\left(\frac{5\sqrt{6}\sqrt{-x^2+1}}{12(x^2-1)}\right)}{12} - 4\sqrt{6} \operatorname{arctanh}\left(\frac{1}{5}\sqrt{-x^2+1}\right)$
trager	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)`

[Out] `25/12*arctan(1/12*x*6^(1/2))*6^(1/2)+10*ln(x^2+24)-x+5*arcsin(x)+25/12*6^(1/2)*arctan(5/12*6^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)-4*(-x^2+1)^(1/2)+20*arctanh(1/5*(-x^2+1)^(1/2))`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x, algorithm="maxima")`

[Out] `-x - 4*sqrt(-x^2 + 1) + 5*integrate(1/(sqrt(x + 1)*sqrt(-x + 1) + 5), x) + 20*log(sqrt(-x^2 + 1) + 5)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(66) = 132.

time = 0.33, size = 160, normalized size = 1.82

$$\frac{25}{12}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}x\right) + \frac{25}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\sqrt{-x^2+1}-\sqrt{6}}{2x}\right) + \frac{25}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\sqrt{-x^2+1}-\sqrt{6}}{3x}\right) - x - 4\sqrt{-x^2+1} - 10\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + 10\log(x^2+24) - 10\log\left(\frac{-x^2+6\sqrt{-x^2+1}-6}{x^2}\right) + 10\log\left(\frac{x^2-4\sqrt{-x^2+1}+4}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x, algorithm="fricas")`

[Out] `25/12*sqrt(6)*arctan(1/12*sqrt(6)*x) + 25/12*sqrt(6)*arctan(1/2*(sqrt(6)*sqrt(-x^2 + 1) - sqrt(6))/x) + 25/12*sqrt(6)*arctan(1/3*(sqrt(6)*sqrt(-x^2 + 1) - sqrt(6))/x) - x - 4*sqrt(-x^2 + 1) - 10*arctan((sqrt(-x^2 + 1) - 1)/x) + 10*log(x^2 + 24) - 10*log(-x^2 + 6*sqrt(-x^2 + 1) - 6)/x^2 + 10*log((x^2 - 4*sqrt(-x^2 + 1) + 4)/x^2)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x - \sqrt{1-x^2}}{\sqrt{1-x^2} + 5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x-(-x**2+1)**(1/2))/(5+(-x**2+1)**(1/2)),x)**[Out]** Integral((4*x - sqrt(1 - x**2))/(sqrt(1 - x**2) + 5), x)**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(66) = 132.

time = 0.02, size = 185, normalized size = 2.10

$$10 \ln(x^2 + 24) + \frac{50 \arctan\left(\frac{x}{2\sqrt{6}}\right)}{4\sqrt{6}} - x - 4\sqrt{-x^2 + 1} - \frac{25 \arctan\left(\frac{-2\sqrt{-x^2 + 1} + 2}{\sqrt{6}x}\right)}{2\sqrt{6}} - \frac{25 \arctan\left(\frac{3(-2\sqrt{-x^2 + 1} + 2)}{2(\sqrt{6}x)}\right)}{2\sqrt{6}} + 5 \arcsin x + 10 \ln\left(2\left(\frac{-2\sqrt{-x^2 + 1} + 2}{2x}\right)^2 + 3\right) - 10 \ln\left(3\left(\frac{-2\sqrt{-x^2 + 1} + 2}{2x}\right)^2 + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x)

[Out] 25/12*sqrt(6)*arctan(1/12*sqrt(6)*x) - 25/12*sqrt(6)*arctan(-1/3*sqrt(6)*(sqrt(-x^2 + 1) - 1)/x) - 25/12*sqrt(6)*arctan(-1/2*sqrt(6)*(sqrt(-x^2 + 1) - 1)/x) - x - 4*sqrt(-x^2 + 1) + 5*arcsin(x) + 10*log(x^2 + 24) - 10*log(3*(sqrt(-x^2 + 1) - 1)^2/x^2 + 2) + 10*log(2*(sqrt(-x^2 + 1) - 1)^2/x^2 + 3)

Mupad [B]

time = 0.38, size = 159, normalized size = 1.81

$$5 \operatorname{asin}(x) - x - 4\sqrt{1-x^2} - \frac{\sqrt{24} \ln\left(\frac{\sqrt{6}x + \sqrt{1-x^2} + 1}{x - \sqrt{6}i}\right) (125 + \sqrt{24} 100i)}{240} - \frac{\sqrt{24} \ln\left(\frac{-\sqrt{24}x + \sqrt{1-x^2} + 1}{x + \sqrt{24}i}\right) (-125 + \sqrt{24} 100i)}{240} - \frac{\sqrt{24} \ln(x - \sqrt{6} 2i) (25 + \sqrt{24} 20i)}{48} - \frac{\sqrt{24} \ln(x + \sqrt{24} i) (-25 + \sqrt{24} 20i)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x - (1 - x^2)^(1/2))/((1 - x^2)^(1/2) + 5),x)

[Out] 5*asin(x) - x - 4*(1 - x^2)^(1/2) - (24^(1/2)*log(((2*6^(1/2)*x)/5 + (1 - x^2)^(1/2)*1i + 1i/5)/(x - 6^(1/2)*2i))*(24^(1/2)*100i + 125)*1i)/240 - (24^(1/2)*log(((1 - x^2)^(1/2)*1i - (24^(1/2)*x)/5 + 1i/5)/(x + 24^(1/2)*1i))*(24^(1/2)*100i - 125)*1i)/240 - (24^(1/2)*log(x - 6^(1/2)*2i)*(24^(1/2)*20i + 25)*1i)/48 - (24^(1/2)*log(x + 24^(1/2)*1i)*(24^(1/2)*20i - 25)*1i)/48

$$3.260 \quad \int \frac{x^2 \left(2 - \sqrt{1 + x^2}\right)}{\sqrt{1 + x^2} \left(1 - x^3 + (1 + x^2)^{3/2}\right)} dx$$

Optimal. Leaf size=136

$$\frac{8x}{9} - \frac{x^2}{6} + \frac{8\sqrt{1+x^2}}{9} - \frac{1}{6}x\sqrt{1+x^2} - \frac{41}{54}\sinh^{-1}(x) + \frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{1+3x}{2\sqrt{2}}\right) + \frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right)$$

[Out] 8/9*x-1/6*x^2-41/54*arcsinh(x)+7/27*arctanh(1/2*(1-x)/(x^2+1)^(1/2))-7/54*ln(3*x^2+2*x+3)+4/27*arctan(1/4*(1+3*x)*2^(1/2))*2^(1/2)+4/27*arctan(1/2*(1+x)*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)+8/9*(x^2+1)^(1/2)-1/6*x*(x^2+1)^(1/2)

Rubi [A]

time = 0.95, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 14, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6874, 201, 221, 648, 632, 210, 642, 1034, 12, 1095, 1051, 1045, 212, 267}

$$-\frac{x^2}{6} - \frac{1}{6}\sqrt{x^2+1}x + \frac{8\sqrt{x^2+1}}{9} - \frac{7}{54}\log(3x^2+2x+3) + \frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right) + \frac{7}{27}\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+1}}\right) + \frac{8x}{9} + \frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{3x+1}{2\sqrt{2}}\right) - \frac{41}{54}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x^2*(2 - Sqrt[1 + x^2]))/(Sqrt[1 + x^2]*(1 - x^3 + (1 + x^2)^(3/2))),x]

[Out] (8*x)/9 - x^2/6 + (8*Sqrt[1 + x^2])/9 - (x*Sqrt[1 + x^2])/6 - (41*ArcSinh[x])/54 + (4*Sqrt[2]*ArcTan[(1 + 3*x)/(2*Sqrt[2])])/27 + (4*Sqrt[2]*ArcTan[(1 + x)/(Sqrt[2]*Sqrt[1 + x^2])])/27 + (7*ArcTanh[(1 - x)/(2*Sqrt[1 + x^2])])/27 - (7*Log[3 + 2*x + 3*x^2])/54

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 201

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x*((a + b*x^n)^p/(n*p + 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &

& (LtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 221

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 267

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 632

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1034

Int[((g_) + (h_)*(x_))*((a_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[h*(a + c*x^2)^p*((d + e*x + f*x^2)^(q + 1)/(2*f*(p + q + 1))), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N

eQ[p + q + 1, 0]

Rule 1045

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - b*d*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + f*x^2]], x] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[b*h^2*d - 2*g*h*(c*d - a*f) - g^2*b*f, 0]
```

Rule 1051

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 + b^2*d*f, 2]}, Dist[1/(2*q), Int[Simp[h*b*d - g*(c*d - a*f - q) + (h*(c*d - a*f + q) + g*b*f)*x, x]/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*b*d - g*(c*d - a*f + q) + (h*(c*d - a*f - q) + g*b*f)*x, x]/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1095

```
Int[((A_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6874

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (2 - \sqrt{1+x^2})}{\sqrt{1+x^2} (1-x^3 + (1+x^2)^{3/2})} dx &= \int \left(-\frac{x^2}{1-x^3 + \sqrt{1+x^2} + x^2\sqrt{1+x^2}} - \frac{2x^2}{\sqrt{1+x^2} (-1+x^3 - (1+x^2)^{3/2})} \right) dx \\
&= - \left(2 \int \frac{x^2}{\sqrt{1+x^2} (-1+x^3 - (1+x^2)^{3/2})} dx \right) - \int \frac{2x^2}{1-x^3 + \sqrt{1+x^2}} dx \\
&= - \left(2 \int \left(-\frac{1}{3} + \frac{2}{9\sqrt{1+x^2}} - \frac{x}{3\sqrt{1+x^2}} + \frac{2x}{3(3+2x+3x^2)} + \frac{1}{9} \right) dx \right) \\
&= \frac{8x}{9} - \frac{x^2}{6} - \frac{1}{9} \int \frac{-3-5x}{3+2x+3x^2} dx - \frac{2}{9} \int \frac{3+5x}{\sqrt{1+x^2} (3+2x+3x^2)} dx \\
&= \frac{8x}{9} - \frac{x^2}{6} + \frac{8\sqrt{1+x^2}}{9} - \frac{1}{6} x\sqrt{1+x^2} - \frac{4}{9} \sinh^{-1}(x) + \frac{1}{18} \int \frac{1}{\sqrt{1-x^2}} dx \\
&= \frac{8x}{9} - \frac{x^2}{6} + \frac{8\sqrt{1+x^2}}{9} - \frac{1}{6} x\sqrt{1+x^2} - \frac{11}{18} \sinh^{-1}(x) - \frac{7}{54} \log(3+2x+3x^2) \\
&= \frac{8x}{9} - \frac{x^2}{6} + \frac{8\sqrt{1+x^2}}{9} - \frac{1}{6} x\sqrt{1+x^2} - \frac{11}{18} \sinh^{-1}(x) + \frac{4}{27} \sqrt{2} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) \\
&= \frac{8x}{9} - \frac{x^2}{6} + \frac{8\sqrt{1+x^2}}{9} - \frac{1}{6} x\sqrt{1+x^2} - \frac{41}{54} \sinh^{-1}(x) + \frac{4}{27} \sqrt{2} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) \\
&= \frac{8x}{9} - \frac{x^2}{6} + \frac{8\sqrt{1+x^2}}{9} - \frac{1}{6} x\sqrt{1+x^2} - \frac{41}{54} \sinh^{-1}(x) + \frac{4}{27} \sqrt{2} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) \\
&= \frac{8x}{9} - \frac{x^2}{6} + \frac{8\sqrt{1+x^2}}{9} - \frac{1}{6} x\sqrt{1+x^2} - \frac{41}{54} \sinh^{-1}(x) + \frac{4}{27} \sqrt{2} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right)
\end{aligned}$$

Mathematica [A]

time = 0.33, size = 104, normalized size = 0.76

$$\frac{1}{54} \left(48x - 9x^2 + 48\sqrt{1+x^2} - 9x\sqrt{1+x^2} + 16\sqrt{2} \tan^{-1} \left(\frac{1+x-\sqrt{1+x^2}}{\sqrt{2}} \right) - 55 \tanh^{-1} \left(\frac{x}{\sqrt{1+x^2}} \right) - 14 \log(-2-x-x^2+(1+x)\sqrt{1+x^2}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(2 - Sqrt[1 + x^2]))/(Sqrt[1 + x^2]*(1 - x^3 + (1 + x^2)^(3/2))),x]

[Out] (48*x - 9*x^2 + 48*Sqrt[1 + x^2] - 9*x*Sqrt[1 + x^2] + 16*Sqrt[2]*ArcTan[(1 + x - Sqrt[1 + x^2])/Sqrt[2]] - 55*ArcTanh[x/Sqrt[1 + x^2]] - 14*Log[-2 - x - x^2 + (1 + x)*Sqrt[1 + x^2]])/54

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(2 - Sqrt[x^2 + 1])*(x^2/(Sqrt[x^2 + 1]*((x^2 + 1)^(3/2) - x^3 + 1))),x]')
```

[Out] Timed out

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 655 vs. 2(99) = 198.

time = 0.03, size = 656, normalized size = 4.82

$$-\frac{x^2}{6} + \frac{8x}{9} - \frac{7 \ln(3x^2 + 2x + 3)}{54} + \frac{4\sqrt{2} \arctan\left(\frac{(6x+2)\sqrt{2}}{8}\right)}{27} - \frac{41 \operatorname{arcsinh}(x)}{54} + \frac{\sqrt{2} \sqrt{\frac{2(1+x)^2}{(1-x)^2} + 2}}{\left(\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x)
```

```
[Out] -1/6*x^2+8/9*x-7/54*ln(3*x^2+2*x+3)+4/27*2^(1/2)*arctan(1/8*(6*x+2)*2^(1/2))-41/54*arcsinh(x)+1/12*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(2^(1/2)*arctan(1/2*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/(((1+x)^2/(1-x)^2+1)*(1+x)/(1-x))-5*arctanh((2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/((1+x)/(1-x)+1)^2)^(1/2)/((1+x)/(1-x)+1)-3/8*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(2^(1/2)*arctan(1/2*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/(((1+x)^2/(1-x)^2+1)*(1+x)/(1-x))-arctanh((2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/((1+x)/(1-x)+1)^2)^(1/2)/((1+x)/(1-x)+1)-1/6*x*(x^2+1)^(1/2)+8/9*(x^2+1)^(1/2)+1/216*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(13*2^(1/2)*arctan(1/2*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/(((1+x)^2/(1-x)^2+1)*(1+x)/(1-x))+43*arctanh((2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/((1+x)/(1-x)+1)^2)^(1/2)/((1+x)/(1-x)+1)+1/36*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(11*2^(1/2)*arctan(1/2*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/(((1+x)^2/(1-x)^2+1)*(1+x)/(1-x))-arctanh((2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/((1+x)/(1-x)+1)^2)^(1/2)/((1+x)/(1-x)+1)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x, algorithm="maxima")

[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x) + integrate(-1/2*(3*x^10 - 4*x^9 + 5*x^8 - 2*x^7 + 15*x^6 + 6*x^5 + 9*x^4)/(2*x^13 + 7*x^11 - 4*x^10 + 11*x^9 - 11*x^8 + 13*x^7 - 13*x^6 + 11*x^5 - 11*x^4 + 4*x^3 - 7*x^2 - 2*(x^12 + 3*x^10 - 2*x^9 + 3*x^8 - 6*x^7 + 2*x^6 - 6*x^5 + 3*x^4 - 2*x^3 + 3*x^2 + 1)*sqrt(x^2 + 1) - 2), x) + 1/6*log(x^2 + x + 1) + 1/6*log(x - 1)

Fricas [A]

time = 0.31, size = 170, normalized size = 1.25

$$\frac{1}{6}x^2 - \frac{1}{18}\sqrt{x^2+1}(3x-16) + \frac{4}{27}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}(3x+1)\right) + \frac{4}{27}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}(3x-1) + \frac{3}{2}\sqrt{2}\sqrt{x^2+1}\right) - \frac{1}{27}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}(x+1) + \frac{1}{2}\sqrt{2}\sqrt{x^2+1}\right) + \frac{8}{9}x + \frac{7}{54}\log(3x^2 - \sqrt{x^2+1}(3x-1) - x + 2) - \frac{7}{54}\log(3x^2 + 2x + 3) - \frac{7}{54}\log(x^2 - \sqrt{x^2+1}(x+1) + x + 2) + \frac{41}{54}\log(-x + \sqrt{x^2+1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x, algorithm="fricas")

[Out] -1/6*x^2 - 1/18*sqrt(x^2 + 1)*(3*x - 16) + 4/27*sqrt(2)*arctan(1/4*sqrt(2)*(3*x + 1)) + 4/27*sqrt(2)*arctan(-1/2*sqrt(2)*(3*x - 1) + 3/2*sqrt(2)*sqrt(x^2 + 1)) - 4/27*sqrt(2)*arctan(-1/2*sqrt(2)*(x + 1) + 1/2*sqrt(2)*sqrt(x^2 + 1)) + 8/9*x + 7/54*log(3*x^2 - sqrt(x^2 + 1)*(3*x - 1) - x + 2) - 7/54*log(3*x^2 + 2*x + 3) - 7/54*log(x^2 - sqrt(x^2 + 1)*(x + 1) + x + 2) + 41/54*log(-x + sqrt(x^2 + 1))

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(2-(x**2+1)**(1/2))/(1-x**3+(x**2+1)**(3/2))/(x**2+1)**(1/2),x)

[Out] Timed out

Giac [A]

time = 0.02, size = 237, normalized size = 1.74

$$-\frac{7}{54}\ln(3x^2 + 2x + 3) + \frac{32\arctan\left(\frac{\sqrt{2}x}{4-27\sqrt{2}}\right)}{4-27\sqrt{2}} + \frac{-\frac{3}{9}x^2 + 8x}{9} + 2\left(\frac{4}{9} - \frac{x}{12}\right)\sqrt{x^2+1} + 2\left(\frac{41}{108}\ln(\sqrt{x^2+1}-x) - \frac{7}{108}\ln\left(\left(\sqrt{x^2+1}-x\right)^2 - 2(\sqrt{x^2+1}-x) + 3\right) - \frac{4\arctan\left(\frac{\sqrt{x^2+1}-x}{\sqrt{2}}\right)}{27\sqrt{2}} + \frac{7}{108}\ln\left(3(\sqrt{x^2+1}-x)^2 + 2(\sqrt{x^2+1}-x) + 1\right) + \frac{4\arctan\left(\frac{3(\sqrt{x^2+1}-x)+1}{\sqrt{2}}\right)}{27\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x)

[Out] $-1/6*x^2 - 1/18*\sqrt{x^2 + 1}*(3*x - 16) + 4/27*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(3*x - 3*\sqrt{x^2 + 1} - 1)) + 4/27*\sqrt{2}*\arctan(1/4*\sqrt{2}*(3*x + 1)) - 4/27*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(x - \sqrt{x^2 + 1} + 1)) + 8/9*x + 7/54*\log(3*(x - \sqrt{x^2 + 1})^2 - 2*x + 2*\sqrt{x^2 + 1} + 1) - 7/54*\log((x - \sqrt{x^2 + 1})^2 + 2*x - 2*\sqrt{x^2 + 1} + 3) - 7/54*\log(3*x^2 + 2*x + 3) + 4/54*\log(-x + \sqrt{x^2 + 1})$

Mupad [B]

time = 0.62, size = 216, normalized size = 1.59

$$\frac{8x}{9} - \frac{41 \operatorname{asinh}(x)}{54} - \left(\frac{x}{6} - \frac{8}{9}\right) \sqrt{x^2 + 1} - \frac{\sqrt{x^2 + 1} \ln\left(\frac{x+1-\sqrt{x^2+1}}{x}\right) \left(\frac{8}{9} + \frac{\sqrt{x^2+1}}{x}\right)}{8} + \frac{\sqrt{x^2 + 1} \ln\left(\frac{x+1+\sqrt{x^2+1}}{x}\right) \left(\frac{8}{9} + \frac{\sqrt{x^2+1}}{x}\right)}{8} + \frac{\sqrt{x^2 + 1} \left(\ln\left(\frac{x+1+\sqrt{x^2+1}}{x}\right) - \ln\left(\frac{1+\sqrt{x^2+1}}{1-\sqrt{x^2+1}}\right)\right)}{8 \sqrt{\left(\frac{1}{3} + \frac{\sqrt{x^2+1}}{3}\right)^2 + 1}} + \frac{\sqrt{x^2 + 1} \left(\ln\left(\frac{x+1-\sqrt{x^2+1}}{x}\right) - \ln\left(\frac{1-\sqrt{x^2+1}}{1+\sqrt{x^2+1}}\right)\right)}{8 \sqrt{\left(\frac{1}{3} + \frac{\sqrt{x^2+1}}{3}\right)^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}(-(x^2*(x^2 + 1)^{(1/2)} - 2))/((x^2 + 1)^{(1/2)}*((x^2 + 1)^{(3/2)} - x^3 + 1)), x$

[Out] $(8*x)/9 - (41*\operatorname{asinh}(x))/54 - (x/6 - 8/9)*(x^2 + 1)^{(1/2)} - x^2/6 + (2^{(1/2)})*\log(x - (2^{(1/2)}*2i)/3 + 1/3)*((2^{(1/2)}*14i)/27 - 16/27)*1i)/8 + (2^{(1/2)})*\log(x + (2^{(1/2)}*2i)/3 + 1/3)*((2^{(1/2)}*14i)/27 + 16/27)*1i)/8 + (2^{(1/2)})*((2^{(1/2)}*44i)/81 + 4/81)*(\log(x + (2^{(1/2)}*2i)/3 + 1/3) - \log((2^{(1/2)}*1i)/3 + 2/3)*(x^2 + 1)^{(1/2)} - x/3 - (2^{(1/2)}*x*2i)/3 + 1))*1i)/(8*((2^{(1/2)}*2i)/3 + 1/3)^2 + 1)^{(1/2)} + (2^{(1/2)})*((2^{(1/2)}*44i)/81 - 4/81)*(\log(x - (2^{(1/2)}*2i)/3 + 1/3) - \log((2^{(1/2)}*x*2i)/3 - ((2^{(1/2)}*1i)/3 - 2/3)*(x^2 + 1)^{(1/2)} - x/3 + 1))*1i)/(8*((2^{(1/2)}*2i)/3 - 1/3)^2 + 1)^{(1/2)}$

3.261 $\int x \sqrt{2rx - x^2} dx$

Optimal. Leaf size=64

$$-\frac{1}{2}r(r-x)\sqrt{2rx-x^2} - \frac{1}{3}(2rx-x^2)^{3/2} + r^3 \tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)$$

[Out] -1/3*(2*r*x-x^2)^(3/2)+r^3*arctan(x/(2*r*x-x^2)^(1/2))-1/2*r*(r-x)*(2*r*x-x^2)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {654, 626, 634, 209}

$$r^3 \tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right) - \frac{1}{2}r(r-x)\sqrt{2rx-x^2} - \frac{1}{3}(2rx-x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[2*r*x - x^2],x]

[Out] -1/2*(r*(r - x)*Sqrt[2*r*x - x^2]) - (2*r*x - x^2)^(3/2)/3 + r^3*ArcTan[x/Sqrt[2*r*x - x^2]]

Rule 209

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 626

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(b + 2*c*x)*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 634

Int[1/Sqrt[(b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 654

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]

&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
 \int x\sqrt{2rx-x^2} dx &= -\frac{1}{3}(2rx-x^2)^{3/2} + r \int \sqrt{2rx-x^2} dx \\
 &= -\frac{1}{2}r(r-x)\sqrt{2rx-x^2} - \frac{1}{3}(2rx-x^2)^{3/2} + \frac{1}{2}r^3 \int \frac{1}{\sqrt{2rx-x^2}} dx \\
 &= -\frac{1}{2}r(r-x)\sqrt{2rx-x^2} - \frac{1}{3}(2rx-x^2)^{3/2} + r^3 \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \frac{x}{\sqrt{2rx-x^2}} \right) \\
 &= -\frac{1}{2}r(r-x)\sqrt{2rx-x^2} - \frac{1}{3}(2rx-x^2)^{3/2} + r^3 \tan^{-1} \left(\frac{x}{\sqrt{2rx-x^2}} \right)
 \end{aligned}$$

Mathematica [A]

time = 0.09, size = 66, normalized size = 1.03

$$\frac{1}{6} \sqrt{-x(-2r+x)} \left(-3r^2 - rx + 2x^2 - \frac{6r^3 \tanh^{-1} \left(\frac{\sqrt{x}}{\sqrt{-2r+x}} \right)}{\sqrt{x} \sqrt{-2r+x}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[2*r*x - x^2],x]

[Out] (Sqrt[-(x*(-2*r + x))]*(-3*r^2 - r*x + 2*x^2 - (6*r^3*ArcTanh[Sqrt[x]/Sqrt[-2*r + x]])/(Sqrt[x]*Sqrt[-2*r + x])))/6

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x*Sqrt[2*r*x - x^2],x]')

[Out] cought exception: maximum recursion depth exceeded in comparison

Maple [A]

time = 0.07, size = 64, normalized size = 1.00

method	result	size
--------	--------	------

risch	$-\frac{(3r^2+rx-2x^2)x(2r-x)}{6\sqrt{-x(-2r+x)}} + \frac{r^3 \arctan\left(\frac{x-r}{\sqrt{2rx-x^2}}\right)}{2}$	60
default	$-\frac{(2rx-x^2)^{\frac{3}{2}}}{3} + r \left(-\frac{(2r-2x)\sqrt{2rx-x^2}}{4} + \frac{r^2 \arctan\left(\frac{x-r}{\sqrt{2rx-x^2}}\right)}{2} \right)$	64

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(2*r*x-x^2)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/3*(2*r*x-x^2)^{(3/2)}+r*(-1/4*(2*r-2*x)*(2*r*x-x^2)^{(1/2)}+1/2*r^2*\arctan((x-r)/(2*r*x-x^2)^{(1/2)}))$

Maxima [A]

time = 0.43, size = 63, normalized size = 0.98

$$-\frac{1}{2}r^3 \arcsin\left(\frac{r-x}{r}\right) - \frac{1}{2}\sqrt{2rx-x^2}r^2 + \frac{1}{2}\sqrt{2rx-x^2}rx - \frac{1}{3}(2rx-x^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*r*x-x^2)^(1/2),x, algorithm="maxima")`

[Out] $-1/2*r^3*\arcsin((r-x)/r) - 1/2*\sqrt{2*r*x-x^2}*r^2 + 1/2*\sqrt{2*r*x-x^2}*r*x - 1/3*(2*r*x-x^2)^{(3/2)}$

Fricas [A]

time = 0.32, size = 51, normalized size = 0.80

$$-r^3 \arctan\left(\frac{\sqrt{2rx-x^2}}{x}\right) - \frac{1}{6}(3r^2+rx-2x^2)\sqrt{2rx-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*r*x-x^2)^(1/2),x, algorithm="fricas")`

[Out] $-r^3*\arctan(\sqrt{2*r*x-x^2}/x) - 1/6*(3*r^2+r*x-2*x^2)*\sqrt{2*r*x-x^2}$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{-x(-2r+x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*r*x-x**2)**(1/2),x)

[Out] Integral(x*sqrt(-x*(-2*r + x)), x)

Giac [A]

time = 0.00, size = 54, normalized size = 0.84

$$2 \left(\left(\frac{x}{6} - \frac{r}{12} \right) x - \frac{3}{12} r^2 \right) \sqrt{2rx - x^2} - \frac{1}{2} r^3 \operatorname{sign}(r) \arcsin \left(\frac{r - x}{r} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*r*x-x^2)^(1/2),x)

[Out] -1/2*r^3*arcsin((r - x)/r)*sgn(r) - 1/6*(3*r^2 + (r - 2*x)*x)*sqrt(2*r*x - x^2)

Mupad [B]

time = 0.10, size = 56, normalized size = 0.88

$$-\frac{\sqrt{2rx - x^2} (12r^2 + 4rx - 8x^2)}{24} - \frac{r^3 \ln \left(x - r - \sqrt{x(2r - x)} \right) \operatorname{li}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*r*x - x^2)^(1/2),x)

[Out] - ((2*r*x - x^2)^(1/2)*(4*r*x + 12*r^2 - 8*x^2))/24 - (r^3*log(x - r - (x*(2*r - x))^(1/2)*1i)*1i)/2

3.262 $\int x^2 \sqrt{2rx - x^2} dx$

Optimal. Leaf size=89

$$-\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2} - \frac{5}{12}r(2rx-x^2)^{3/2} - \frac{1}{4}x(2rx-x^2)^{3/2} + \frac{5}{4}r^4 \tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)$$

[Out] $-5/12*r*(2*r*x-x^2)^{(3/2)}-1/4*x*(2*r*x-x^2)^{(3/2)}+5/4*r^4*\arctan(x/(2*r*x-x^2)^{(1/2)})-5/8*r^2*(r-x)*(2*r*x-x^2)^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {684, 654, 626, 634, 209}

$$\frac{5}{4}r^4 \tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right) - \frac{5}{8}r^2(r-x)\sqrt{2rx-x^2} - \frac{5}{12}r(2rx-x^2)^{3/2} - \frac{1}{4}x(2rx-x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*\text{Sqrt}[2*r*x - x^2], x]$

[Out] $(-5*r^2*(r-x)*\text{Sqrt}[2*r*x - x^2])/8 - (5*r*(2*r*x - x^2)^{(3/2)})/12 - (x*(2*r*x - x^2)^{(3/2)})/4 + (5*r^4*\text{ArcTan}[x/\text{Sqrt}[2*r*x - x^2]])/4$

Rule 209

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))*\text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 626

$\text{Int}[(a + (b \cdot x) + (c \cdot x)^2)^p, x_Symbol] \rightarrow \text{Simp}[(b + 2*c*x)*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - \text{Dist}[p*((b^2 - 4*a*c)/(2*c*(2*p + 1))), \text{Int}[(a + b*x + c*x^2)^{p-1}, x], x] /; \text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{IntegerQ}[4*p]$

Rule 634

$\text{Int}[1/\text{Sqrt}[(b \cdot x) + (c \cdot x)^2], x_Symbol] \rightarrow \text{Dist}[2, \text{Subst}[\text{Int}[1/(1 - c*x^2), x], x, x/\text{Sqrt}[b*x + c*x^2]], x] /; \text{FreeQ}\{b, c, x\}$

Rule 654

$\text{Int}[(d + (e \cdot x))*((a + (b \cdot x) + (c \cdot x)^2)^p), x_Symbol] \rightarrow \text{Simp}[e*((a + b*x + c*x^2)^{p+1}/(2*c*(p+1))), x] + \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, p, x\}$

&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 684

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]

Rubi steps

$$\begin{aligned}
 \int x^2 \sqrt{2rx - x^2} dx &= -\frac{1}{4}x(2rx - x^2)^{3/2} + \frac{1}{4}(5r) \int x\sqrt{2rx - x^2} dx \\
 &= -\frac{5}{12}r(2rx - x^2)^{3/2} - \frac{1}{4}x(2rx - x^2)^{3/2} + \frac{1}{4}(5r^2) \int \sqrt{2rx - x^2} dx \\
 &= -\frac{5}{8}r^2(r - x)\sqrt{2rx - x^2} - \frac{5}{12}r(2rx - x^2)^{3/2} - \frac{1}{4}x(2rx - x^2)^{3/2} + \frac{1}{8}(5r^4) \int \frac{1}{\sqrt{2r - x}} dx \\
 &= -\frac{5}{8}r^2(r - x)\sqrt{2rx - x^2} - \frac{5}{12}r(2rx - x^2)^{3/2} - \frac{1}{4}x(2rx - x^2)^{3/2} + \frac{1}{4}(5r^4) \operatorname{Subst}\left(\frac{1}{\sqrt{2r - x}}, -x\right) \\
 &= -\frac{5}{8}r^2(r - x)\sqrt{2rx - x^2} - \frac{5}{12}r(2rx - x^2)^{3/2} - \frac{1}{4}x(2rx - x^2)^{3/2} + \frac{5}{4}r^4 \tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-2r + x}}\right)
 \end{aligned}$$

Mathematica [A]

time = 0.09, size = 74, normalized size = 0.83

$$\frac{1}{24} \sqrt{-x(-2r + x)} \left(-15r^3 - 5r^2x - 2rx^2 + 6x^3 - \frac{30r^4 \tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{-2r + x}}\right)}{\sqrt{x} \sqrt{-2r + x}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[2*r*x - x^2], x]

[Out] (Sqrt[-(x*(-2*r + x))]*(-15*r^3 - 5*r^2*x - 2*r*x^2 + 6*x^3 - (30*r^4*ArcTanh[Sqrt[x]/Sqrt[-2*r + x]])/(Sqrt[x]*Sqrt[-2*r + x])))/24

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[x^2*sqrt[2*r*x - x^2],x]')`

[Out] `cought exception: maximum recursion depth exceeded`

Maple [A]

time = 0.07, size = 83, normalized size = 0.93

method	result	size
risch	$-\frac{(15r^3+5r^2x+2rx^2-6x^3)x(2r-x)}{24\sqrt{-x(-2r+x)}} + \frac{5r^4 \arctan\left(\frac{x-r}{\sqrt{2rx-x^2}}\right)}{8}$	69
default	$-\frac{x(2rx-x^2)^{\frac{3}{2}}}{4} + \frac{5r \left(-\frac{(2rx-x^2)^{\frac{3}{2}}}{3} + r \left(-\frac{(2r-2x)\sqrt{2rx-x^2}}{4} + \frac{r^2 \arctan\left(\frac{x-r}{\sqrt{2rx-x^2}}\right)}{2} \right) \right)}{4}$	83

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(2*r*x-x^2)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `-1/4*x*(2*r*x-x^2)^(3/2)+5/4*r*(-1/3*(2*r*x-x^2)^(3/2)+r*(-1/4*(2*r-2*x)*(2*r*x-x^2)^(1/2)+1/2*r^2*arctan((x-r)/(2*r*x-x^2)^(1/2))))`

Maxima [A]

time = 0.37, size = 81, normalized size = 0.91

$$-\frac{5}{8}r^4 \arcsin\left(\frac{r-x}{r}\right) - \frac{5}{8}\sqrt{2rx-x^2}r^3 + \frac{5}{8}\sqrt{2rx-x^2}r^2x - \frac{5}{12}(2rx-x^2)^{\frac{3}{2}}r - \frac{1}{4}(2rx-x^2)^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(2*r*x-x^2)^(1/2),x, algorithm="maxima")`

[Out] `-5/8*r^4*arcsin((r-x)/r) - 5/8*sqrt(2*r*x - x^2)*r^3 + 5/8*sqrt(2*r*x - x^2)*r^2*x - 5/12*(2*r*x - x^2)^(3/2)*r - 1/4*(2*r*x - x^2)^(3/2)*x`

Fricas [A]

time = 0.31, size = 60, normalized size = 0.67

$$-\frac{5}{4}r^4 \arctan\left(\frac{\sqrt{2rx-x^2}}{x}\right) - \frac{1}{24}(15r^3+5r^2x+2rx^2-6x^3)\sqrt{2rx-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(2*r*x-x^2)^(1/2),x, algorithm="fricas")`

[Out] $-5/4*r^4*\arctan(\sqrt{2*r*x - x^2}/x) - 1/24*(15*r^3 + 5*r^2*x + 2*r*x^2 - 6*x^3)*\sqrt{2*r*x - x^2}$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{-x(-2r+x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(2*r*x-x**2)**(1/2),x)`

[Out] `Integral(x**2*sqrt(-x*(-2*r + x)), x)`

Giac [A]

time = 0.01, size = 67, normalized size = 0.75

$$2 \left(\left(\left(\frac{x}{8} - \frac{2}{48}r \right) x - \frac{5}{48}r^2 \right) x - \frac{15}{48}r^3 \right) \sqrt{2rx - x^2} - \frac{5}{8}r^4 \operatorname{sign}(r) \arcsin\left(\frac{r-x}{r}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(2*r*x-x^2)^(1/2),x)`

[Out] $-5/8*r^4*\arcsin((r-x)/r)*\operatorname{sgn}(r) - 1/24*(15*r^3 + (5*r^2 + 2*(r-3*x))*x)*\sqrt{2*r*x - x^2}$

Mupad [B]

time = 0.32, size = 75, normalized size = 0.84

$$\frac{x(2rx - x^2)^{3/2}}{4} - \frac{5r \left(\frac{\sqrt{2rx - x^2} (12r^2 + 4rx - 8x^2)}{24} + \frac{r^3 \ln(x-r-\sqrt{x(2r-x)}) \operatorname{li}(1)}{2} \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(2*r*x - x^2)^(1/2),x)`

[Out] $-(x*(2*r*x - x^2)^{(3/2)})/4 - (5*r*((2*r*x - x^2)^{(1/2)}*(4*r*x + 12*r^2 - 8*x^2))/24 + (r^3*\log(x - r - (x*(2*r - x))^{(1/2)}*1i)*1i)/2)/4$

3.263 $\int x^3 \sqrt{2rx - x^2} dx$

Optimal. Leaf size=113

$$-\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2} - \frac{7}{12}r^2(2rx-x^2)^{3/2} - \frac{7}{20}rx(2rx-x^2)^{3/2} - \frac{1}{5}x^2(2rx-x^2)^{3/2} + \frac{7}{4}r^5 \tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)$$

[Out] $-7/12*r^2*(2*r*x-x^2)^{(3/2)}-7/20*r*x*(2*r*x-x^2)^{(3/2)}-1/5*x^2*(2*r*x-x^2)^{(3/2)}+7/4*r^5*\arctan(x/(2*r*x-x^2)^{(1/2)})-7/8*r^3*(r-x)*(2*r*x-x^2)^{(1/2)}$

Rubi [A]

time = 0.03, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {684, 654, 626, 634, 209}

$$\frac{7}{4}r^5 \tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right) - \frac{7}{8}r^3(r-x)\sqrt{2rx-x^2} - \frac{7}{12}r^2(2rx-x^2)^{3/2} - \frac{7}{20}rx(2rx-x^2)^{3/2} - \frac{1}{5}x^2(2rx-x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3*\text{Sqrt}[2*r*x - x^2], x]$

[Out] $(-7*r^3*(r-x)*\text{Sqrt}[2*r*x - x^2])/8 - (7*r^2*(2*r*x - x^2)^{(3/2)})/12 - (7*r*x*(2*r*x - x^2)^{(3/2)})/20 - (x^2*(2*r*x - x^2)^{(3/2)})/5 + (7*r^5*\text{ArcTan}[x/\text{Sqrt}[2*r*x - x^2]])/4$

Rule 209

$\text{Int}[(a + (b_*)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))*\text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 626

$\text{Int}[(a + (b_*)*(x_) + (c_*)*(x_)^2)^{(p)}, x_Symbol] \rightarrow \text{Simp}[(b + 2*c*x)*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - \text{Dist}[p*((b^2 - 4*a*c)/(2*c*(2*p + 1))), \text{Int}[(a + b*x + c*x^2)^{(p-1)}, x], x] /; \text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{IntegerQ}[4*p]$

Rule 634

$\text{Int}[1/\text{Sqrt}[(b_*)*(x_) + (c_*)*(x_)^2], x_Symbol] \rightarrow \text{Dist}[2, \text{Subst}[\text{Int}[1/(1 - c*x^2), x], x, x/\text{Sqrt}[b*x + c*x^2]], x] /; \text{FreeQ}\{b, c, x\}$

Rule 654

$\text{Int}[(d + (e_*)*(x_))*((a + (b_*)*(x_) + (c_*)*(x_)^2)^{(p)}, x_Symbol] \rightarrow \text{Simp}[e*((a + b*x + c*x^2)^{(p+1)})/(2*c*(p+1)), x] + \text{Dist}[(2*c*d - b$

$*e)/(2*c)$, Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 684

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]

Rubi steps

$$\begin{aligned}
 \int x^3 \sqrt{2rx - x^2} dx &= -\frac{1}{5}x^2 (2rx - x^2)^{3/2} + \frac{1}{5}(7r) \int x^2 \sqrt{2rx - x^2} dx \\
 &= -\frac{7}{20}rx (2rx - x^2)^{3/2} - \frac{1}{5}x^2 (2rx - x^2)^{3/2} + \frac{1}{4}(7r^2) \int x \sqrt{2rx - x^2} dx \\
 &= -\frac{7}{12}r^2 (2rx - x^2)^{3/2} - \frac{7}{20}rx (2rx - x^2)^{3/2} - \frac{1}{5}x^2 (2rx - x^2)^{3/2} + \frac{1}{4}(7r^3) \int \sqrt{2rx - x^2} dx \\
 &= -\frac{7}{8}r^3(r - x)\sqrt{2rx - x^2} - \frac{7}{12}r^2 (2rx - x^2)^{3/2} - \frac{7}{20}rx (2rx - x^2)^{3/2} - \frac{1}{5}x^2 (2rx - x^2)^{3/2} \\
 &= -\frac{7}{8}r^3(r - x)\sqrt{2rx - x^2} - \frac{7}{12}r^2 (2rx - x^2)^{3/2} - \frac{7}{20}rx (2rx - x^2)^{3/2} - \frac{1}{5}x^2 (2rx - x^2)^{3/2} \\
 &= -\frac{7}{8}r^3(r - x)\sqrt{2rx - x^2} - \frac{7}{12}r^2 (2rx - x^2)^{3/2} - \frac{7}{20}rx (2rx - x^2)^{3/2} - \frac{1}{5}x^2 (2rx - x^2)^{3/2}
 \end{aligned}$$

Mathematica [A]

time = 0.10, size = 82, normalized size = 0.73

$$\frac{1}{120} \sqrt{-x(-2r + x)} \left(-105r^4 - 35r^3x - 14r^2x^2 - 6rx^3 + 24x^4 - \frac{210r^5 \tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{-2r + x}}\right)}{\sqrt{x} \sqrt{-2r + x}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Sqrt[2*r*x - x^2], x]

[Out] (Sqrt[-(x*(-2*r + x))]*(-105*r^4 - 35*r^3*x - 14*r^2*x^2 - 6*r*x^3 + 24*x^4 - (210*r^5*ArcTanh[Sqrt[x]/Sqrt[-2*r + x]]))/(Sqrt[x]*Sqrt[-2*r + x]))/120

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3*Sqrt[2*r*x - x^2],x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python object
```

Maple [A]

time = 0.07, size = 104, normalized size = 0.92

method	result	size
risch	$-\frac{(105r^4+35r^3x+14r^2x^2+6rx^3-24x^4)x(2r-x)}{120\sqrt{-x(-2r+x)}} + \frac{7r^5 \arctan\left(\frac{x-r}{\sqrt{2rx-x^2}}\right)}{8}$	77
default	$-\frac{x^2(2rx-x^2)^{\frac{3}{2}}}{5} + \frac{7r \left(-\frac{x(2rx-x^2)^{\frac{3}{2}}}{4} + \frac{5r \left(-\frac{(2rx-x^2)^{\frac{3}{2}}}{3} + r \left(-\frac{(2r-2x)\sqrt{2rx-x^2}}{4} + \frac{r^2 \arctan\left(\frac{x-r}{\sqrt{2rx-x^2}}\right)}{2} \right) \right)}{4} \right)}{5}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*r*x-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*x^2*(2*r*x-x^2)^(3/2)+7/5*r*(-1/4*x*(2*r*x-x^2)^(3/2)+5/4*r*(-1/3*(2*r*x-x^2)^(3/2)+r*(-1/4*(2*r-2*x)*(2*r*x-x^2)^(1/2)+1/2*r^2*arctan((x-r)/(2*r*x-x^2)^(1/2))))
```

Maxima [A]

time = 0.35, size = 101, normalized size = 0.89

$$-\frac{7}{8}r^5 \arcsin\left(\frac{r-x}{r}\right) - \frac{7}{8}\sqrt{2rx-x^2}r^4 + \frac{7}{8}\sqrt{2rx-x^2}r^3x - \frac{7}{12}(2rx-x^2)^{\frac{3}{2}}r^2 - \frac{7}{20}(2rx-x^2)^{\frac{3}{2}}rx - \frac{1}{5}(2rx-x^2)^{\frac{3}{2}}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*r*x-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -7/8*r^5*arcsin((r-x)/r) - 7/8*sqrt(2*r*x - x^2)*r^4 + 7/8*sqrt(2*r*x - x^2)*r^3*x - 7/12*(2*r*x - x^2)^(3/2)*r^2 - 7/20*(2*r*x - x^2)^(3/2)*r*x - 1/5*(2*r*x - x^2)^(3/2)*x^2
```

Fricas [A]

time = 0.31, size = 68, normalized size = 0.60

$$-\frac{7}{4}r^5 \arctan\left(\frac{\sqrt{2rx-x^2}}{x}\right) - \frac{1}{120}(105r^4 + 35r^3x + 14r^2x^2 + 6rx^3 - 24x^4)\sqrt{2rx-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(2*r*x-x^2)^(1/2),x, algorithm="fricas")

[Out] -7/4*r^5*arctan(sqrt(2*r*x - x^2)/x) - 1/120*(105*r^4 + 35*r^3*x + 14*r^2*x^2 + 6*r*x^3 - 24*x^4)*sqrt(2*r*x - x^2)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{-x(-2r+x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(2*r*x-x**2)**(1/2),x)

[Out] Integral(x**3*sqrt(-x*(-2*r + x)), x)

Giac [A]

time = 0.01, size = 78, normalized size = 0.69

$$2\left(\left(\left(\left(\frac{x}{10} - \frac{6}{240}r\right)x - \frac{14}{240}r^2\right)x - \frac{35}{240}r^3\right)x - \frac{105}{240}r^4\right)\sqrt{2rx-x^2} - \frac{7}{8}r^5 \operatorname{sign}(r) \arcsin\left(\frac{r-x}{r}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(2*r*x-x^2)^(1/2),x)

[Out] -7/8*r^5*arcsin((r-x)/r)*sgn(r) - 1/120*(105*r^4 + (35*r^3 + 2*(7*r^2 + 3*(r-4*x)*x)*x)*x)*sqrt(2*r*x - x^2)

Mupad [B]

time = 0.14, size = 96, normalized size = 0.85

$$\frac{7r \left(\frac{x(2rx-x^2)^{3/2}}{4} + \frac{5r \left(\frac{\sqrt{2rx-x^2} (12r^2+4rx-8x^2)}{24} + \frac{r^3 \ln\left(\frac{x-r-\sqrt{x(2r-x)}}{2}\right)}{2} \right)}{4} \right)}{5} - \frac{x^2(2rx-x^2)^{3/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(2*r*x - x^2)^(1/2),x)

[Out] - (7*r*((x*(2*r*x - x^2)^(3/2))/4 + (5*r*((2*r*x - x^2)^(1/2)*(4*r*x + 12*r^2 - 8*x^2))/24 + (r^3*log(x - r - (x*(2*r - x))^(1/2)*1i)*1i)/2))/4)/5 - (x^2*(2*r*x - x^2)^(3/2))/5

$$3.264 \quad \int \frac{1}{(-1+x^2) \sqrt{2x+x^2}} dx$$

Optimal. Leaf size=49

$$-\frac{1}{2} \tan^{-1}(\sqrt{2x+x^2}) - \frac{\tanh^{-1}\left(\frac{1+2x}{\sqrt{3}\sqrt{2x+x^2}}\right)}{2\sqrt{3}}$$

[Out] -1/2*arctan((x^2+2*x)^(1/2))-1/6*arctanh(1/3*(1+2*x)*3^(1/2)/(x^2+2*x)^(1/2))*3^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {998, 702, 210, 738, 212}

$$-\frac{1}{2} \tan^{-1}(\sqrt{x^2+2x}) - \frac{\tanh^{-1}\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+2x}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/((-1 + x^2)*Sqrt[2*x + x^2]),x]

[Out] -1/2*ArcTan[Sqrt[2*x + x^2]] - ArcTanh[(1 + 2*x)/(Sqrt[3]*Sqrt[2*x + x^2])]/(2*Sqrt[3])

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 702

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0]

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 998

```
Int[1/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:> Dist[1/2, Int[1/((a - Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
+ Dist[1/2, Int[1/((a + Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f}, x]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(-1+x^2)\sqrt{2x+x^2}} dx &= \frac{1}{2} \int \frac{1}{(-1-x)\sqrt{2x+x^2}} dx + \frac{1}{2} \int \frac{1}{(-1+x)\sqrt{2x+x^2}} dx \\ &= 2 \operatorname{Subst} \left(\int \frac{1}{-4-4x^2} dx, x, \sqrt{2x+x^2} \right) - \operatorname{Subst} \left(\int \frac{1}{12-x^2} dx, x, \frac{2+4x}{\sqrt{2x+x^2}} \right) \\ &= -\frac{1}{2} \tan^{-1} \left(\sqrt{2x+x^2} \right) - \frac{\tanh^{-1} \left(\frac{2+4x}{2\sqrt{3}\sqrt{2x+x^2}} \right)}{2\sqrt{3}} \end{aligned}$$

Mathematica [A]

time = 0.11, size = 78, normalized size = 1.59

$$\frac{\sqrt{x} \sqrt{2+x} \left(3 \tan^{-1} \left(1+x-\sqrt{x} \sqrt{2+x} \right) - \sqrt{3} \tanh^{-1} \left(\frac{1-x+\sqrt{x} \sqrt{2+x}}{\sqrt{3}} \right) \right)}{3\sqrt{x(2+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x^2)*Sqrt[2*x + x^2]), x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + x]*(3*ArcTan[1 + x - Sqrt[x]*Sqrt[2 + x]] - Sqrt[3]*ArcTanh[(1 - x + Sqrt[x]*Sqrt[2 + x])/Sqrt[3]]))/(3*Sqrt[x*(2 + x)])
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[1/((x^2 - 1)*Sqrt[2*x + x^2]),x]')`

[Out] `cought exception: maximum recursion depth exceeded`

Maple [A]

time = 0.15, size = 42, normalized size = 0.86

method	result
default	$-\frac{\sqrt{3} \operatorname{arctanh}\left(\frac{(2+4x)\sqrt{3}}{6\sqrt{(-1+x)^2-1+4x}}\right)}{6} + \frac{\operatorname{arctan}\left(\frac{1}{\sqrt{(1+x)^2-1}}\right)}{2}$
trager	$\frac{\operatorname{RootOf}(_Z^2-3) \ln\left(\frac{-2\operatorname{RootOf}(_Z^2-3)x+3\sqrt{x^2+2x}-\operatorname{RootOf}(_Z^2-3)}{-1+x}\right)}{6} + \frac{\operatorname{RootOf}(_Z^2+1) \ln\left(\frac{-\operatorname{RootOf}(_Z^2+1)}{1+x}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2-1)/(x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `-1/6*3^(1/2)*arctanh(1/6*(2+4*x)*3^(1/2)/((-1+x)^2-1+4*x)^(1/2))+1/2*arctan(1/((1+x)^2-1)^(1/2))`

Maxima [A]

time = 0.34, size = 54, normalized size = 1.10

$$-\frac{1}{6}\sqrt{3} \log\left(\frac{2\sqrt{3}\sqrt{x^2+2x}}{|2x-2|} + \frac{6}{|2x-2|} + 2\right) + \frac{1}{2} \arcsin\left(\frac{2}{|2x+2|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2-1)/(x^2+2*x)^(1/2),x, algorithm="maxima")`

[Out] `-1/6*sqrt(3)*log(2*sqrt(3)*sqrt(x^2+2*x)/abs(2*x-2)+6/abs(2*x-2)+2)+1/2*arcsin(2/abs(2*x+2))`

Fricas [A]

time = 0.32, size = 62, normalized size = 1.27

$$\frac{1}{6}\sqrt{3} \log\left(\frac{\sqrt{3}(2x+1)+\sqrt{x^2+2x}(2\sqrt{3}-3)-4x-2}{x-1}\right) - \operatorname{arctan}\left(-x+\sqrt{x^2+2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2-1)/(x^2+2*x)^(1/2),x, algorithm="fricas")`

[Out] `1/6*sqrt(3)*log(-sqrt(3)*(2*x+1)+sqrt(x^2+2*x)*(2*sqrt(3)-3)-4*x-2)/(x-1))-arctan(-x+sqrt(x^2+2*x)-1)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x(x+2)}(x-1)(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2-1)/(x**2+2*x)**(1/2),x)**[Out]** Integral(1/(sqrt(x*(x + 2))*(x - 1)*(x + 1)), x)**Giac [A]**

time = 0.01, size = 88, normalized size = 1.80

$$2 \left(-\frac{\arctan\left(\sqrt{x^2+2x}-x-1\right)}{2} + \frac{\ln\left(\frac{\left|2\left(\sqrt{x^2+2x}-x\right)+2-2\sqrt{3}\right|}{\left|2\left(\sqrt{x^2+2x}-x\right)+2+2\sqrt{3}\right|}\right)}{4\sqrt{3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-1)/(x^2+2*x)^(1/2),x)**[Out]** 1/6*sqrt(3)*log(abs(-2*x - 2*sqrt(3) + 2*sqrt(x^2 + 2*x) + 2)/abs(-2*x + 2*sqrt(3) + 2*sqrt(x^2 + 2*x) + 2)) - arctan(-x + sqrt(x^2 + 2*x) - 1)**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2+2x}(x^2-1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2*x + x^2)^(1/2)*(x^2 - 1)),x)**[Out]** int(1/((2*x + x^2)^(1/2)*(x^2 - 1)), x)

$$3.265 \quad \int \frac{-2+3x}{(1+x)^3 \sqrt{2x-x^2}} dx$$

Optimal. Leaf size=79

$$-\frac{5\sqrt{2x-x^2}}{6(1+x)^2} - \frac{2\sqrt{2x-x^2}}{3(1+x)} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}\sqrt{2x-x^2}}\right)}{2\sqrt{3}}$$

[Out] 1/6*arctan(1/3*(1-2*x)*3^(1/2)/(-x^2+2*x)^(1/2))*3^(1/2)-5/6*(-x^2+2*x)^(1/2)/(1+x)^2-2/3*(-x^2+2*x)^(1/2)/(1+x)

Rubi [A]

time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {848, 820, 738, 210}

$$-\frac{2\sqrt{2x-x^2}}{3(x+1)} - \frac{5\sqrt{2x-x^2}}{6(x+1)^2} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}\sqrt{2x-x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-2 + 3*x)/((1 + x)^3*Sqrt[2*x - x^2]),x]

[Out] (-5*Sqrt[2*x - x^2])/(6*(1 + x)^2) - (2*Sqrt[2*x - x^2])/(3*(1 + x)) + ArcTan[(1 - 2*x)/(Sqrt[3]*Sqrt[2*x - x^2])]/(2*Sqrt[3])

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 738

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 820

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]

&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 848

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rubi steps

$$\begin{aligned} \int \frac{-2 + 3x}{(1+x)^3 \sqrt{2x-x^2}} dx &= -\frac{5\sqrt{2x-x^2}}{6(1+x)^2} + \frac{1}{6} \int \frac{-7+5x}{(1+x)^2 \sqrt{2x-x^2}} dx \\ &= -\frac{5\sqrt{2x-x^2}}{6(1+x)^2} - \frac{2\sqrt{2x-x^2}}{3(1+x)} - \frac{1}{2} \int \frac{1}{(1+x)\sqrt{2x-x^2}} dx \\ &= -\frac{5\sqrt{2x-x^2}}{6(1+x)^2} - \frac{2\sqrt{2x-x^2}}{3(1+x)} + \text{Subst} \left(\int \frac{1}{-12-x^2} dx, x, \frac{-2+4x}{\sqrt{2x-x^2}} \right) \\ &= -\frac{5\sqrt{2x-x^2}}{6(1+x)^2} - \frac{2\sqrt{2x-x^2}}{3(1+x)} - \frac{\tan^{-1} \left(\frac{-2+4x}{2\sqrt{3}\sqrt{2x-x^2}} \right)}{2\sqrt{3}} \end{aligned}$$

Mathematica [A]

time = 0.20, size = 78, normalized size = 0.99

$$\frac{x(-18 + x + 4x^2) - 2\sqrt{3}\sqrt{-2+x}\sqrt{x}(1+x)^2 \tanh^{-1}\left(\frac{1-\sqrt{-2+x}\sqrt{x+x}}{\sqrt{3}}\right)}{6\sqrt{-((-2+x)x)}(1+x)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + 3*x)/((1 + x)^3*Sqrt[2*x - x^2]), x]

[Out] (x*(-18 + x + 4*x^2) - 2*Sqrt[3]*Sqrt[-2 + x]*Sqrt[x]*(1 + x)^2*ArcTanh[(1 - Sqrt[-2 + x]*Sqrt[x] + x)/Sqrt[3]])/(6*Sqrt[-((-2 + x)*x)]*(1 + x)^2)

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(3*x - 2)/((x + 1)^3*Sqrt[2*x - x^2]),x]')`

[Out] Timed out

Maple [A]

time = 0.11, size = 74, normalized size = 0.94

method	result
risch	$\frac{x(-2+x)(4x+9)}{6(1+x)^2 \sqrt{-x(-2+x)}} - \frac{\sqrt{3} \arctan\left(\frac{(-2+4x)\sqrt{3}}{6\sqrt{-(1+x)^2+1+4x}}\right)}{6}$
trager	$-\frac{(4x+9)\sqrt{-x^2+2x}}{6(1+x)^2} - \frac{\text{RootOf}(-Z^2+3) \ln\left(\frac{-2\text{RootOf}(-Z^2+3)x+3\sqrt{-x^2+2x}+\text{RootOf}(-Z^2+3)}{1+x}\right)}{6}$
default	$-\frac{5\sqrt{-(1+x)^2+1+4x}}{6(1+x)^2} - \frac{2\sqrt{-(1+x)^2+1+4x}}{3(1+x)} - \frac{\sqrt{3} \arctan\left(\frac{(-2+4x)\sqrt{3}}{6\sqrt{-(1+x)^2+1+4x}}\right)}{6}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-5/6/(1+x)^2*(-(1+x)^2+1+4x)^{(1/2)}-2/3/(1+x)*(-(1+x)^2+1+4x)^{(1/2)}-1/6*3^{(1/2)}*\arctan(1/6*(-2+4*x)*3^{(1/2)/(-(1+x)^2+1+4*x)^{(1/2)})$

Maxima [A]

time = 0.35, size = 66, normalized size = 0.84

$$-\frac{1}{6}\sqrt{3} \arcsin\left(\frac{2x}{|x+1|} - \frac{1}{|x+1|}\right) - \frac{5\sqrt{-x^2+2x}}{6(x^2+2x+1)} - \frac{2\sqrt{-x^2+2x}}{3(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x, algorithm="maxima")`

[Out] $-1/6*\text{sqrt}(3)*\arcsin(2*x/\text{abs}(x+1) - 1/\text{abs}(x+1)) - 5/6*\text{sqrt}(-x^2+2*x)/(x^2+2*x+1) - 2/3*\text{sqrt}(-x^2+2*x)/(x+1)$

Fricas [A]

time = 0.31, size = 64, normalized size = 0.81

$$\frac{2\sqrt{3}(x^2+2x+1) \arctan\left(\frac{\sqrt{3}\sqrt{-x^2+2x}}{3x}\right) - \sqrt{-x^2+2x}(4x+9)}{6(x^2+2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x, algorithm="fricas")

[Out] 1/6*(2*sqrt(3)*(x^2 + 2*x + 1)*arctan(1/3*sqrt(3)*sqrt(-x^2 + 2*x)/x) - sqrt(-x^2 + 2*x)*(4*x + 9))/(x^2 + 2*x + 1)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x - 2}{\sqrt{-x(x - 2)} (x + 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+3*x)/(1+x)**3/(-x**2+2*x)**(1/2),x)

[Out] Integral((3*x - 2)/(sqrt(-x*(x - 2))*(x + 1)**3), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(64) = 128.

time = 0.00, size = 192, normalized size = 2.43

$$\frac{18 \left(\frac{-2\sqrt{-x^2 + 2x + 2}}{-2x + 2} \right)^3 - 39 \left(\frac{-2\sqrt{-x^2 + 2x + 2}}{-2x + 2} \right)^2 - 26 + \frac{34 \left(\frac{-2\sqrt{-x^2 + 2x + 2}}{-2x + 2} \right)}{-2x + 2}}{24 \left(\left(\frac{-2\sqrt{-x^2 + 2x + 2}}{-2x + 2} \right)^2 + 1 - \frac{-2\sqrt{-x^2 + 2x + 2}}{-2x + 2} \right)^2} + \frac{\arctan \left(\frac{-1 + \frac{2 \left(\frac{-2\sqrt{-x^2 + 2x + 2}}{-2x + 2} \right)}{\sqrt{3}}}{\sqrt{3}} \right)}{\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x)

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 + 2*x) - 1)/(x - 1) - 1)) + 1/24*(34*(sqrt(-x^2 + 2*x) - 1)/(x - 1) - 39*(sqrt(-x^2 + 2*x) - 1)^2/(x - 1)^2 + 18*(sqrt(-x^2 + 2*x) - 1)^3/(x - 1)^3 - 26)/((sqrt(-x^2 + 2*x) - 1)/(x - 1) - (sqrt(-x^2 + 2*x) - 1)^2/(x - 1)^2 - 1)^2

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3x - 2}{\sqrt{2x - x^2} (x + 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x - 2)/((2*x - x^2)^(1/2)*(x + 1)^3),x)

[Out] int((3*x - 2)/((2*x - x^2)^(1/2)*(x + 1)^3), x)

$$3.266 \quad \int \frac{1}{\sqrt{1+x+x^2}} dx$$

Optimal. Leaf size=12

$$\sinh^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)$$

[Out] arcsinh(1/3*(1+2*x)*3^(1/2))

Rubi [A]

time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {633, 221}

$$\sinh^{-1} \left(\frac{2x+1}{\sqrt{3}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 + x + x^2], x]

[Out] ArcSinh[(1 + 2*x)/Sqrt[3]]

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 633

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*(-4*(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\int \frac{1}{\sqrt{1+x+x^2}} dx = \frac{\text{Subst} \left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}} dx, x, 1+2x \right)}{\sqrt{3}} = \sinh^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)$$

Mathematica [A]

time = 0.05, size = 20, normalized size = 1.67

$$-\log\left(-1 - 2x + 2\sqrt{1 + x + x^2}\right)$$

Antiderivative was successfully verified.

`[In] Integrate[1/Sqrt[1 + x + x^2],x]``[Out] -Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/Sqrt[1 + x + x^2],x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [A]**

time = 0.13, size = 10, normalized size = 0.83

method	result	size
default	$\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)$	10
trager	$-\ln\left(2\sqrt{x^2+x+1}-1-2x\right)$	19

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)``[Out] arcsinh(2/3*3^(1/2)*(x+1/2))`**Maxima [A]**

time = 0.43, size = 11, normalized size = 0.92

$$\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(x^2+x+1)^(1/2),x, algorithm="maxima")``[Out] arcsinh(1/3*sqrt(3)*(2*x + 1))`

Fricas [A]

time = 0.30, size = 18, normalized size = 1.50

$$-\log\left(-2x + 2\sqrt{x^2 + x + 1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+x+1)^(1/2),x, algorithm="fricas")

[Out] -log(-2*x + 2*sqrt(x^2 + x + 1) - 1)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2 + x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+x+1)**(1/2),x)

[Out] Integral(1/sqrt(x**2 + x + 1), x)

Giac [A]

time = 0.00, size = 19, normalized size = 1.58

$$-\ln\left(2\left(\sqrt{x^2 + x + 1} - x\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+x+1)^(1/2),x)

[Out] -log(-2*x + 2*sqrt(x^2 + x + 1) - 1)

Mupad [B]

time = 0.24, size = 12, normalized size = 1.00

$$\ln\left(x + \sqrt{x^2 + x + 1} + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x + x^2 + 1)^(1/2),x)

[Out] log(x + (x + x^2 + 1)^(1/2) + 1/2)

$$3.267 \quad \int \frac{x^3}{\sqrt{1+x+x^2}} dx$$

Optimal. Leaf size=53

$$\frac{1}{3}x^2\sqrt{1+x+x^2} - \frac{1}{24}(1+10x)\sqrt{1+x+x^2} + \frac{7}{16}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)$$

[Out] 7/16*arcsinh(1/3*(1+2*x)*3^(1/2))+1/3*x^2*(x^2+x+1)^(1/2)-1/24*(1+10*x)*(x^2+x+1)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {756, 793, 633, 221}

$$\frac{1}{3}\sqrt{x^2+x+1}x^2 - \frac{1}{24}(10x+1)\sqrt{x^2+x+1} + \frac{7}{16}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[1 + x + x^2],x]

[Out] (x^2*Sqrt[1 + x + x^2])/3 - ((1 + 10*x)*Sqrt[1 + x + x^2])/24 + (7*ArcSinh[(1 + 2*x)/Sqrt[3]])/16

Rule 221

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 633

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*(-4*(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 756

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(-b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) - 2*c*e*g*(p + 1)*x)*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{\sqrt{1+x+x^2}} dx &= \frac{1}{3} x^2 \sqrt{1+x+x^2} + \frac{1}{3} \int \frac{(-2 - \frac{5x}{2})x}{\sqrt{1+x+x^2}} dx \\ &= \frac{1}{3} x^2 \sqrt{1+x+x^2} - \frac{1}{24} (1+10x) \sqrt{1+x+x^2} + \frac{7}{16} \int \frac{1}{\sqrt{1+x+x^2}} dx \\ &= \frac{1}{3} x^2 \sqrt{1+x+x^2} - \frac{1}{24} (1+10x) \sqrt{1+x+x^2} + \frac{7 \operatorname{Subst} \left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}} dx, x, 1+2x \right)}{16\sqrt{3}} \\ &= \frac{1}{3} x^2 \sqrt{1+x+x^2} - \frac{1}{24} (1+10x) \sqrt{1+x+x^2} + \frac{7}{16} \sinh^{-1} \left(\frac{1+2x}{\sqrt{3}} \right) \end{aligned}$$

Mathematica [A]

time = 0.06, size = 47, normalized size = 0.89

$$\frac{1}{24} \sqrt{1+x+x^2} (-1-10x+8x^2) - \frac{7}{16} \log(-1-2x+2\sqrt{1+x+x^2})$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[1 + x + x^2], x]
```

```
[Out] (Sqrt[1 + x + x^2]*(-1 - 10*x + 8*x^2))/24 - (7*Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]])/16
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3/Sqrt[1 + x + x^2], x]')
```

[Out] cought exception: maximum recursion depth exceeded in comparison

Maple [A]

time = 0.12, size = 47, normalized size = 0.89

method	result	size
risch	$\frac{(8x^2-10x-1)\sqrt{x^2+x+1}}{24} + \frac{7 \operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{16}$	33
trager	$\left(\frac{1}{3}x^2 - \frac{5}{12}x - \frac{1}{24}\right)\sqrt{x^2+x+1} + \frac{7 \ln\left(1+2x+2\sqrt{x^2+x+1}\right)}{16}$	39
default	$\frac{x^2\sqrt{x^2+x+1}}{3} - \frac{5x\sqrt{x^2+x+1}}{12} - \frac{\sqrt{x^2+x+1}}{24} + \frac{7 \operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{16}$	47

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $1/3*x^2*(x^2+x+1)^{(1/2)}-5/12*x*(x^2+x+1)^{(1/2)}-1/24*(x^2+x+1)^{(1/2)}+7/16*\operatorname{arcsinh}(2/3*3^{(1/2)}*(x+1/2))$

Maxima [A]

time = 0.36, size = 48, normalized size = 0.91

$$\frac{1}{3} \sqrt{x^2+x+1} x^2 - \frac{5}{12} \sqrt{x^2+x+1} x - \frac{1}{24} \sqrt{x^2+x+1} + \frac{7}{16} \operatorname{arsinh}\left(\frac{1}{3} \sqrt{3} (2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(x^2+x+1)^(1/2),x, algorithm="maxima")`

[Out] $1/3*\operatorname{sqrt}(x^2+x+1)*x^2 - 5/12*\operatorname{sqrt}(x^2+x+1)*x - 1/24*\operatorname{sqrt}(x^2+x+1) + 7/16*\operatorname{arcsinh}(1/3*\operatorname{sqrt}(3)*(2*x+1))$

Fricas [A]

time = 0.32, size = 39, normalized size = 0.74

$$\frac{1}{24} (8x^2 - 10x - 1)\sqrt{x^2+x+1} - \frac{7}{16} \log\left(-2x + 2\sqrt{x^2+x+1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(x^2+x+1)^(1/2),x, algorithm="fricas")`

[Out] $1/24*(8*x^2 - 10*x - 1)*\operatorname{sqrt}(x^2+x+1) - 7/16*\log(-2*x + 2*\operatorname{sqrt}(x^2+x+1) - 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2 + x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(x**2+x+1)**(1/2),x)**[Out]** Integral(x**3/sqrt(x**2 + x + 1), x)**Giac [A]**

time = 0.00, size = 51, normalized size = 0.96

$$2 \left(\left(\frac{x}{6} - \frac{5}{24} \right) x - \frac{1}{48} \right) \sqrt{x^2 + x + 1} - \frac{7}{16} \ln \left(2 \left(\sqrt{x^2 + x + 1} - x \right) - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^2+x+1)^(1/2),x)**[Out]** 1/24*(2*(4*x - 5)*x - 1)*sqrt(x^2 + x + 1) - 7/16*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\sqrt{x^2 + x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x + x^2 + 1)^(1/2),x)**[Out]** int(x^3/(x + x^2 + 1)^(1/2), x)

$$3.268 \quad \int \frac{1}{(1+x+x^2)^{3/2}} dx$$

Optimal. Leaf size=19

$$\frac{2(1+2x)}{3\sqrt{1+x+x^2}}$$

[Out] $2/3*(1+2*x)/(x^2+x+1)^{(1/2)}$

Rubi [A]

time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {627}

$$\frac{2(2x+1)}{3\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2)^(-3/2), x]

[Out] (2*(1 + 2*x))/(3*Sqrt[1 + x + x^2])

Rule 627

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \frac{1}{(1+x+x^2)^{3/2}} dx = \frac{2(1+2x)}{3\sqrt{1+x+x^2}}$$

Mathematica [A]

time = 0.08, size = 19, normalized size = 1.00

$$\frac{2(1+2x)}{3\sqrt{1+x+x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2)^(-3/2), x]

[Out] (2*(1 + 2*x))/(3*Sqrt[1 + x + x^2])

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(1 + x + x^2)^(3/2),x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.10, size = 16, normalized size = 0.84

method	result	size
gospers	$\frac{\frac{2}{3} + \frac{4x}{3}}{\sqrt{x^2 + x + 1}}$	16
default	$\frac{\frac{2}{3} + \frac{4x}{3}}{\sqrt{x^2 + x + 1}}$	16
trager	$\frac{\frac{2}{3} + \frac{4x}{3}}{\sqrt{x^2 + x + 1}}$	16
risch	$\frac{\frac{2}{3} + \frac{4x}{3}}{\sqrt{x^2 + x + 1}}$	16

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)``[Out] 2/3*(1+2*x)/(x^2+x+1)^(1/2)`**Maxima [A]**

time = 0.26, size = 22, normalized size = 1.16

$$\frac{4x}{3\sqrt{x^2 + x + 1}} + \frac{2}{3\sqrt{x^2 + x + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(x^2+x+1)^(3/2),x, algorithm="maxima")``[Out] 4/3*x/sqrt(x^2 + x + 1) + 2/3/sqrt(x^2 + x + 1)`**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(15) = 30.

time = 0.32, size = 34, normalized size = 1.79

$$\frac{2 \left(2x^2 + \sqrt{x^2 + x + 1} (2x + 1) + 2x + 2 \right)}{3(x^2 + x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+x+1)^(3/2),x, algorithm="fricas")

[Out] 2/3*(2*x^2 + sqrt(x^2 + x + 1)*(2*x + 1) + 2*x + 2)/(x^2 + x + 1)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2 + x + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+x+1)**(3/2),x)

[Out] Integral((x**2 + x + 1)**(-3/2), x)

Giac [A]

time = 0.01, size = 27, normalized size = 1.42

$$\frac{2 \left(\frac{2}{3}x + \frac{1}{3} \right) \sqrt{x^2 + x + 1}}{x^2 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+x+1)^(3/2),x)

[Out] 2/3*(2*x + 1)/sqrt(x^2 + x + 1)

Mupad [B]

time = 0.03, size = 13, normalized size = 0.68

$$\frac{4 \left(x + \frac{1}{2} \right)}{3 \sqrt{x^2 + x + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x + x^2 + 1)^(3/2),x)

[Out] (4*(x + 1/2))/(3*(x + x^2 + 1)^(1/2))

$$3.269 \quad \int \frac{x}{(1+x+x^2)^{3/2}} dx$$

Optimal. Leaf size=17

$$-\frac{2(2+x)}{3\sqrt{1+x+x^2}}$$

[Out] -2/3*(2+x)/(x^2+x+1)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {650}

$$-\frac{2(x+2)}{3\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

[In] Int[x/(1 + x + x^2)^(3/2), x]

[Out] (-2*(2 + x))/(3*Sqrt[1 + x + x^2])

Rule 650

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \frac{x}{(1+x+x^2)^{3/2}} dx = -\frac{2(2+x)}{3\sqrt{1+x+x^2}}$$

Mathematica [A]

time = 0.08, size = 17, normalized size = 1.00

$$-\frac{2(2+x)}{3\sqrt{1+x+x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 + x + x^2)^(3/2), x]

[Out] (-2*(2 + x))/(3*Sqrt[1 + x + x^2])

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[x/(1 + x + x^2)^(3/2),x]')``[Out] cought exception: maximum recursion depth exceeded in comparison`**Maple [A]**

time = 0.11, size = 27, normalized size = 1.59

method	result	size
gospers	$-\frac{2(2+x)}{3\sqrt{x^2+x+1}}$	14
trager	$-\frac{2(2+x)}{3\sqrt{x^2+x+1}}$	14
risch	$-\frac{2(2+x)}{3\sqrt{x^2+x+1}}$	14
default	$-\frac{1}{\sqrt{x^2+x+1}} - \frac{1+2x}{3\sqrt{x^2+x+1}}$	27

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)``[Out] -1/(x^2+x+1)^(1/2)-1/3*(1+2*x)/(x^2+x+1)^(1/2)`**Maxima [A]**

time = 0.25, size = 22, normalized size = 1.29

$$-\frac{2x}{3\sqrt{x^2+x+1}} - \frac{4}{3\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x/(x^2+x+1)^(3/2),x, algorithm="maxima")``[Out] -2/3*x/sqrt(x^2 + x + 1) - 4/3/sqrt(x^2 + x + 1)`**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(13) = 26.

time = 0.30, size = 28, normalized size = 1.65

$$-\frac{2\left(x^2 + \sqrt{x^2+x+1}(x+2) + x+1\right)}{3(x^2+x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2+x+1)^(3/2),x, algorithm="fricas")

[Out] -2/3*(x^2 + sqrt(x^2 + x + 1)*(x + 2) + x + 1)/(x^2 + x + 1)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x^2 + x + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x**2+x+1)**(3/2),x)

[Out] Integral(x/(x**2 + x + 1)**(3/2), x)

Giac [A]

time = 0.00, size = 29, normalized size = 1.71

$$\frac{2\left(-\frac{2}{3} - \frac{x}{3}\right)\sqrt{x^2 + x + 1}}{x^2 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2+x+1)^(3/2),x)

[Out] -2/3*(x + 2)/sqrt(x^2 + x + 1)

Mupad [B]

time = 0.02, size = 15, normalized size = 0.88

$$-\frac{2x + 4}{3\sqrt{x^2 + x + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x + x^2 + 1)^(3/2),x)

[Out] -(2*x + 4)/(3*(x + x^2 + 1)^(1/2))

$$3.270 \quad \int \frac{x^3}{(1+x+x^2)^{3/2}} dx$$

Optimal. Leaf size=56

$$-\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}} + \frac{1}{3}(5+2x)\sqrt{1+x+x^2} - \frac{3}{2}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)$$

[Out] $-3/2*\operatorname{arcsinh}(1/3*(1+2*x)*3^{(1/2)})-2/3*x^2*(2+x)/(x^2+x+1)^{(1/2)}+1/3*(5+2*x)*(x^2+x+1)^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {752, 793, 633, 221}

$$-\frac{2(x+2)x^2}{3\sqrt{x^2+x+1}} + \frac{1}{3}(2x+5)\sqrt{x^2+x+1} - \frac{3}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3/(1+x+x^2)^{(3/2)}, x]$

[Out] $(-2*x^2*(2+x))/(3*\operatorname{Sqrt}[1+x+x^2]) + ((5+2*x)*\operatorname{Sqrt}[1+x+x^2])/3 - (3*\operatorname{ArcSinh}[(1+2*x)/\operatorname{Sqrt}[3]])/2$

Rule 221

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_.) + (b_.)*(x_)^2], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[\operatorname{Rt}[b, 2]*(x/\operatorname{Sqrt}[a])]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}[\{a, b\}, x] \ \&\& \operatorname{GtQ}[a, 0] \ \&\& \operatorname{PosQ}[b]$

Rule 633

$\operatorname{Int}[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Dist}[1/(2*c*(-4*(c/(b^2 - 4*a*c)))^p), \operatorname{Subst}[\operatorname{Int}[\operatorname{Simp}[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; \operatorname{FreeQ}[\{a, b, c, p\}, x] \ \&\& \operatorname{GtQ}[4*a - b^2/c, 0]$

Rule 752

$\operatorname{Int}[((d_.) + (e_.)*(x_))^{(m_)}*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(d + e*x)^{(m-1)}*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x + c*x^2)^{(p+1})/((p+1)*(b^2 - 4*a*c))), x] + \operatorname{Dist}[1/((p+1)*(b^2 - 4*a*c)), \operatorname{Int}[(d + e*x)^{(m-2)}*\operatorname{Simp}[e*(2*a*e*(m-1) + b*d*(2*p - m + 4)) - 2*c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^{(p+1)}, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \ \&\& \operatorname{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \operatorname{NeQ}[2*c*d - b*e, 0] \ \&\& \operatorname{LtQ}[p, -1] \ \&\& \operatorname{GtQ}[m, 1] \ \&\& \operatorname{IntQuadraticQ}[a, b, c, d, e, m, p, x]$

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(-b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) - 2*c*e*g*(p + 1)*x)*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(1+x+x^2)^{3/2}} dx &= -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}} + \frac{2}{3} \int \frac{x(4+2x)}{\sqrt{1+x+x^2}} dx \\ &= -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}} + \frac{1}{3}(5+2x)\sqrt{1+x+x^2} - \frac{3}{2} \int \frac{1}{\sqrt{1+x+x^2}} dx \\ &= -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}} + \frac{1}{3}(5+2x)\sqrt{1+x+x^2} - \frac{1}{2}\sqrt{3} \operatorname{Subst} \left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}} dx, x, 1+x \right) \\ &= -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}} + \frac{1}{3}(5+2x)\sqrt{1+x+x^2} - \frac{3}{2} \sinh^{-1} \left(\frac{1+2x}{\sqrt{3}} \right) \end{aligned}$$

Mathematica [A]

time = 0.10, size = 47, normalized size = 0.84

$$\frac{5+7x+3x^2}{3\sqrt{1+x+x^2}} + \frac{3}{2} \log \left(-1 - 2x + 2\sqrt{1+x+x^2} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1+x+x^2)^(3/2),x]
```

```
[Out] (5 + 7*x + 3*x^2)/(3*Sqrt[1 + x + x^2]) + (3*Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]])/2
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3/(1+x+x^2)^(3/2),x]')
```

[Out] cought exception: maximum recursion depth exceeded

Maple [A]

time = 0.12, size = 61, normalized size = 1.09

method	result	size
risch	$\frac{3x^2+7x+5}{3\sqrt{x^2+x+1}} - \frac{3 \operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{2}$	33
trager	$\frac{3x^2+7x+5}{3\sqrt{x^2+x+1}} + \frac{3 \ln\left(2\sqrt{x^2+x+1}-1-2x\right)}{2}$	40
default	$\frac{x^2}{\sqrt{x^2+x+1}} + \frac{3x}{2\sqrt{x^2+x+1}} + \frac{5}{4\sqrt{x^2+x+1}} + \frac{\frac{5}{12} + \frac{5x}{6}}{\sqrt{x^2+x+1}} - \frac{3 \operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{2}$	61

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)`

[Out] $x^2/(x^2+x+1)^{(1/2)}+3/2*x/(x^2+x+1)^{(1/2)}+5/4/(x^2+x+1)^{(1/2)}+5/12*(1+2*x)/(x^2+x+1)^{(1/2)}-3/2*\operatorname{arcsinh}(2/3*3^{(1/2)}*(x+1/2))$

Maxima [A]

time = 0.35, size = 47, normalized size = 0.84

$$\frac{x^2}{\sqrt{x^2+x+1}} + \frac{7x}{3\sqrt{x^2+x+1}} + \frac{5}{3\sqrt{x^2+x+1}} - \frac{3}{2} \operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(x^2+x+1)^(3/2),x, algorithm="maxima")`

[Out] $x^2/\operatorname{sqrt}(x^2+x+1) + 7/3*x/\operatorname{sqrt}(x^2+x+1) + 5/3/\operatorname{sqrt}(x^2+x+1) - 3/2*\operatorname{arcsinh}(1/3*\operatorname{sqrt}(3)*(2*x+1))$

Fricas [A]

time = 0.31, size = 64, normalized size = 1.14

$$\frac{19x^2 + 18(x^2 + x + 1) \log\left(-2x + 2\sqrt{x^2 + x + 1} - 1\right) + 4(3x^2 + 7x + 5)\sqrt{x^2 + x + 1} + 19x + 19}{12(x^2 + x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(x^2+x+1)^(3/2),x, algorithm="fricas")`

[Out] $1/12*(19*x^2 + 18*(x^2 + x + 1)*\log(-2*x + 2*\operatorname{sqrt}(x^2 + x + 1) - 1) + 4*(3*x^2 + 7*x + 5)*\operatorname{sqrt}(x^2 + x + 1) + 19*x + 19)/(x^2 + x + 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(x^2 + x + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(x**2+x+1)**(3/2),x)**[Out]** Integral(x**3/(x**2 + x + 1)**(3/2), x)**Giac [A]**

time = 0.00, size = 58, normalized size = 1.04

$$\frac{2\left(\left(\frac{x}{2} + \frac{7}{6}\right)x + \frac{5}{6}\right)\sqrt{x^2 + x + 1}}{x^2 + x + 1} + \frac{3}{2}\ln\left(2\left(\sqrt{x^2 + x + 1} - x\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^2+x+1)^(3/2),x)**[Out]** 1/3*((3*x + 7)*x + 5)/sqrt(x^2 + x + 1) + 3/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{(x^2 + x + 1)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x + x^2 + 1)^(3/2),x)**[Out]** int(x^3/(x + x^2 + 1)^(3/2), x)

3.271 $\int x^2 \sqrt{1+x+x^2} dx$

Optimal. Leaf size=65

$$\frac{1}{64}(1+2x)\sqrt{1+x+x^2} - \frac{5}{24}(1+x+x^2)^{3/2} + \frac{1}{4}x(1+x+x^2)^{3/2} + \frac{3}{128}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)$$

[Out] $-5/24*(x^2+x+1)^{(3/2)}+1/4*x*(x^2+x+1)^{(3/2)}+3/128*\operatorname{arcsinh}(1/3*(1+2*x)*3^{(1/2)})+1/64*(1+2*x)*(x^2+x+1)^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {756, 654, 626, 633, 221}

$$\frac{1}{4}x(x^2+x+1)^{3/2} - \frac{5}{24}(x^2+x+1)^{3/2} + \frac{1}{64}(2x+1)\sqrt{x^2+x+1} + \frac{3}{128}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2*\operatorname{Sqrt}[1+x+x^2],x]$

[Out] $((1+2*x)*\operatorname{Sqrt}[1+x+x^2])/64 - (5*(1+x+x^2)^{(3/2)})/24 + (x*(1+x+x^2)^{(3/2)})/4 + (3*\operatorname{ArcSinh}[(1+2*x)/\operatorname{Sqrt}[3]])/128$

Rule 221

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_.) + (b_.)*(x_)^2], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[\operatorname{Rt}[b, 2]*(x/\operatorname{Sqrt}[a])]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}[\{a, b\}, x] \ \&\& \operatorname{GtQ}[a, 0] \ \&\& \operatorname{PosQ}[b]$

Rule 626

$\operatorname{Int}[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \operatorname{Simp}[(b + 2*c*x)*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - \operatorname{Dist}[p*((b^2 - 4*a*c)/(2*c*(2*p + 1))), \operatorname{Int}[(a + b*x + c*x^2)^{(p-1)}, x], x] /; \operatorname{FreeQ}[\{a, b, c\}, x] \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \ \&\& \operatorname{GtQ}[p, 0] \ \&\& \operatorname{IntegerQ}[4*p]$

Rule 633

$\operatorname{Int}[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \operatorname{Dist}[1/(2*c*(-4*c/(b^2 - 4*a*c)))^p, \operatorname{Subst}[\operatorname{Int}[\operatorname{Simp}[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; \operatorname{FreeQ}[\{a, b, c, p\}, x] \ \&\& \operatorname{GtQ}[4*a - b^2/c, 0]$

Rule 654

$\operatorname{Int}[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \operatorname{Simp}[e*((a + b*x + c*x^2)^{(p+1)})/(2*c*(p+1)), x] + \operatorname{Dist}[(2*c*d - b*e)/(2*c), \operatorname{Int}[(a + b*x + c*x^2)^p, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, p\}, x]$

&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 756

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rubi steps

$$\begin{aligned}
 \int x^2 \sqrt{1+x+x^2} dx &= \frac{1}{4}x(1+x+x^2)^{3/2} + \frac{1}{4} \int \left(-1 - \frac{5x}{2}\right) \sqrt{1+x+x^2} dx \\
 &= -\frac{5}{24}(1+x+x^2)^{3/2} + \frac{1}{4}x(1+x+x^2)^{3/2} + \frac{1}{16} \int \sqrt{1+x+x^2} dx \\
 &= \frac{1}{64}(1+2x)\sqrt{1+x+x^2} - \frac{5}{24}(1+x+x^2)^{3/2} + \frac{1}{4}x(1+x+x^2)^{3/2} + \frac{3}{128} \int \frac{1}{\sqrt{1+x+x^2}} dx \\
 &= \frac{1}{64}(1+2x)\sqrt{1+x+x^2} - \frac{5}{24}(1+x+x^2)^{3/2} + \frac{1}{4}x(1+x+x^2)^{3/2} + \frac{1}{128}\sqrt{3} \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-u}} du, 2x+1, x\right) \\
 &= \frac{1}{64}(1+2x)\sqrt{1+x+x^2} - \frac{5}{24}(1+x+x^2)^{3/2} + \frac{1}{4}x(1+x+x^2)^{3/2} + \frac{3}{128} \sinh^{-1}\left(\frac{2x+1}{\sqrt{1+x+x^2}}\right)
 \end{aligned}$$

Mathematica [A]

time = 0.06, size = 52, normalized size = 0.80

$$\frac{1}{192} \sqrt{1+x+x^2} (-37+14x+8x^2+48x^3) - \frac{3}{128} \log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[1 + x + x^2], x]

[Out] (Sqrt[1 + x + x^2]*(-37 + 14*x + 8*x^2 + 48*x^3))/192 - (3*Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]])/128

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

caught exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[x^2*sqrt[1 + x + x^2],x]')`

[Out] caught exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.12, size = 49, normalized size = 0.75

method	result	size
risch	$\frac{(48x^3+8x^2+14x-37)\sqrt{x^2+x+1}}{192} + \frac{3 \operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{128}$	38
trager	$\left(\frac{1}{4}x^3 + \frac{1}{24}x^2 + \frac{7}{96}x - \frac{37}{192}\right)\sqrt{x^2+x+1} - \frac{3 \ln\left(2\sqrt{x^2+x+1} - 1 - 2x\right)}{128}$	44
default	$\frac{x(x^2+x+1)^{\frac{3}{2}}}{4} - \frac{5(x^2+x+1)^{\frac{3}{2}}}{24} + \frac{(1+2x)\sqrt{x^2+x+1}}{64} + \frac{3 \operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{128}$	49

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{4}x(x^2+x+1)^{\frac{3}{2}} - \frac{5}{24}(x^2+x+1)^{\frac{3}{2}} + \frac{1}{64}(1+2x)(x^2+x+1)^{\frac{1}{2}} + \frac{3}{128} \operatorname{arcsinh}\left(\frac{2\sqrt{3}\sqrt{x^2+x+1}}{3}\right)$

Maxima [A]

time = 0.41, size = 56, normalized size = 0.86

$$\frac{1}{4}(x^2+x+1)^{\frac{3}{2}}x - \frac{5}{24}(x^2+x+1)^{\frac{3}{2}} + \frac{1}{32}\sqrt{x^2+x+1}x + \frac{1}{64}\sqrt{x^2+x+1} + \frac{3}{128} \operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(x^2+x+1)^(1/2),x, algorithm="maxima")`

[Out] $\frac{1}{4}(x^2+x+1)^{\frac{3}{2}}x - \frac{5}{24}(x^2+x+1)^{\frac{3}{2}} + \frac{1}{32}\sqrt{x^2+x+1}x + \frac{1}{64}\sqrt{x^2+x+1} + \frac{3}{128}\operatorname{arcsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$

Fricas [A]

time = 0.32, size = 44, normalized size = 0.68

$$\frac{1}{192}(48x^3+8x^2+14x-37)\sqrt{x^2+x+1} - \frac{3}{128} \log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(x^2+x+1)^(1/2),x, algorithm="fricas")`

[Out] $1/192*(48*x^3 + 8*x^2 + 14*x - 37)*\sqrt{x^2 + x + 1} - 3/128*\log(-2*x + 2*\sqrt{x^2 + x + 1} - 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{x^2 + x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(x**2+x+1)**(1/2),x)`

[Out] `Integral(x**2*sqrt(x**2 + x + 1), x)`

Giac [A]

time = 0.00, size = 57, normalized size = 0.88

$$2 \left(\left(\left(\frac{x}{8} + \frac{1}{48} \right) x + \frac{7}{192} \right) x - \frac{37}{384} \right) \sqrt{x^2 + x + 1} - \frac{3}{128} \ln \left(2 \left(\sqrt{x^2 + x + 1} - x \right) - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(x^2+x+1)^(1/2),x)`

[Out] $1/192*(2*(4*(6*x + 1)*x + 7)*x - 37)*\sqrt{x^2 + x + 1} - 3/128*\log(-2*x + 2*\sqrt{x^2 + x + 1} - 1)$

Mupad [B]

time = 0.13, size = 61, normalized size = 0.94

$$\frac{3 \ln \left(x + \sqrt{x^2 + x + 1} + \frac{1}{2} \right)}{128} - \frac{\left(\frac{x}{2} + \frac{1}{4} \right) \sqrt{x^2 + x + 1}}{4} - \frac{5 (8x^2 + 2x + 5) \sqrt{x^2 + x + 1}}{192} + \frac{x(x^2 + x + 1)^{3/2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(x + x^2 + 1)^(1/2),x)`

[Out] $(3*\log(x + (x + x^2 + 1)^{(1/2)} + 1/2))/128 - ((x/2 + 1/4)*(x + x^2 + 1)^{(1/2}))/4 - (5*(2*x + 8*x^2 + 5)*(x + x^2 + 1)^{(1/2}))/192 + (x*(x + x^2 + 1)^{(3/2}))/4$

3.272 $\int (1 + x + x^2)^{3/2} dx$

Optimal. Leaf size=55

$$\frac{9}{64}(1+2x)\sqrt{1+x+x^2} + \frac{1}{8}(1+2x)(1+x+x^2)^{3/2} + \frac{27}{128}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)$$

[Out] 1/8*(1+2*x)*(x^2+x+1)^(3/2)+27/128*arcsinh(1/3*(1+2*x)*3^(1/2))+9/64*(1+2*x)*(x^2+x+1)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {626, 633, 221}

$$\frac{1}{8}(2x+1)(x^2+x+1)^{3/2} + \frac{9}{64}(2x+1)\sqrt{x^2+x+1} + \frac{27}{128}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2)^(3/2), x]

[Out] (9*(1 + 2*x)*Sqrt[1 + x + x^2])/64 + ((1 + 2*x)*(1 + x + x^2)^(3/2))/8 + (2*7*ArcSinh[(1 + 2*x)/Sqrt[3]])/128

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 626

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 633

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned}
\int (1+x+x^2)^{3/2} dx &= \frac{1}{8}(1+2x)(1+x+x^2)^{3/2} + \frac{9}{16} \int \sqrt{1+x+x^2} dx \\
&= \frac{9}{64}(1+2x)\sqrt{1+x+x^2} + \frac{1}{8}(1+2x)(1+x+x^2)^{3/2} + \frac{27}{128} \int \frac{1}{\sqrt{1+x+x^2}} dx \\
&= \frac{9}{64}(1+2x)\sqrt{1+x+x^2} + \frac{1}{8}(1+2x)(1+x+x^2)^{3/2} + \frac{1}{128} (9\sqrt{3}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{1+x+x^2}} dx \right) \\
&= \frac{9}{64}(1+2x)\sqrt{1+x+x^2} + \frac{1}{8}(1+2x)(1+x+x^2)^{3/2} + \frac{27}{128} \sinh^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 52, normalized size = 0.95

$$\frac{1}{64} \sqrt{1+x+x^2} (17+42x+24x^2+16x^3) - \frac{27}{128} \log(-1-2x+2\sqrt{1+x+x^2})$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + x + x^2)^(3/2), x]``[Out] (Sqrt[1 + x + x^2]*(17 + 42*x + 24*x^2 + 16*x^3))/64 - (27*Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]])/128`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(1 + x + x^2)^(3/2), x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.11, size = 43, normalized size = 0.78

method	result	size
risch	$\frac{(16x^3+24x^2+42x+17)\sqrt{x^2+x+1}}{64} + \frac{27 \operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{128}$	38

default	$\frac{(1+2x)(x^2+x+1)^{\frac{3}{2}}}{8} + \frac{9(1+2x)\sqrt{x^2+x+1}}{64} + \frac{27 \operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{128}$	43
trager	$\left(\frac{1}{4}x^3 + \frac{3}{8}x^2 + \frac{21}{32}x + \frac{17}{64}\right)\sqrt{x^2+x+1} + \frac{27 \ln\left(1+2x+2\sqrt{x^2+x+1}\right)}{128}$	44

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)`

[Out] $1/8*(1+2*x)*(x^2+x+1)^{(3/2)}+9/64*(1+2*x)*(x^2+x+1)^{(1/2)}+27/128*\operatorname{arcsinh}(2/3*3^{(1/2)}*(x+1/2))$

Maxima [A]

time = 0.34, size = 56, normalized size = 1.02

$$\frac{1}{4}(x^2+x+1)^{\frac{3}{2}}x + \frac{1}{8}(x^2+x+1)^{\frac{3}{2}} + \frac{9}{32}\sqrt{x^2+x+1}x + \frac{9}{64}\sqrt{x^2+x+1} + \frac{27}{128}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+x+1)^(3/2),x, algorithm="maxima")`

[Out] $1/4*(x^2+x+1)^{(3/2)}*x + 1/8*(x^2+x+1)^{(3/2)} + 9/32*\operatorname{sqrt}(x^2+x+1)*x + 9/64*\operatorname{sqrt}(x^2+x+1) + 27/128*\operatorname{arcsinh}(1/3*\operatorname{sqrt}(3)*(2*x+1))$

Fricas [A]

time = 0.31, size = 44, normalized size = 0.80

$$\frac{1}{64}(16x^3 + 24x^2 + 42x + 17)\sqrt{x^2+x+1} - \frac{27}{128}\log\left(-2x + 2\sqrt{x^2+x+1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+x+1)^(3/2),x, algorithm="fricas")`

[Out] $1/64*(16*x^3 + 24*x^2 + 42*x + 17)*\operatorname{sqrt}(x^2+x+1) - 27/128*\log(-2*x + 2*\operatorname{sqrt}(x^2+x+1) - 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2+x+1)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+x+1)**(3/2),x)`

[Out] `Integral((x**2 + x + 1)**(3/2), x)`

Giac [A]

time = 0.00, size = 59, normalized size = 1.07

$$2 \left(\left(\left(\frac{x}{8} + \frac{3}{16} \right) x + \frac{21}{64} \right) x + \frac{17}{128} \right) \sqrt{x^2 + x + 1} - \frac{27}{128} \ln \left(2 \left(\sqrt{x^2 + x + 1} - x \right) - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)^(3/2),x)**[Out]** 1/64*(2*(4*(2*x + 3)*x + 21)*x + 17)*sqrt(x^2 + x + 1) - 27/128*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)**Mupad [B]**

time = 0.21, size = 43, normalized size = 0.78

$$\frac{27 \ln \left(x + \sqrt{x^2 + x + 1} + \frac{1}{2} \right)}{128} + \frac{\left(x + \frac{1}{2} \right) (x^2 + x + 1)^{3/2}}{4} + \frac{9 \left(\frac{x}{2} + \frac{1}{4} \right) \sqrt{x^2 + x + 1}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^2 + 1)^(3/2),x)**[Out]** (27*log(x + (x + x^2 + 1)^(1/2) + 1/2))/128 + ((x + 1/2)*(x + x^2 + 1)^(3/2))/4 + (9*(x/2 + 1/4)*(x + x^2 + 1)^(1/2))/16

3.273 $\int (1 + x + x^2)^{5/2} dx$

Optimal. Leaf size=74

$$\frac{45}{512}(1+2x)\sqrt{1+x+x^2} + \frac{5}{64}(1+2x)(1+x+x^2)^{3/2} + \frac{1}{12}(1+2x)(1+x+x^2)^{5/2} + \frac{135 \sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{1024}$$

[Out] 5/64*(1+2*x)*(x^2+x+1)^(3/2)+1/12*(1+2*x)*(x^2+x+1)^(5/2)+135/1024*arcsinh(1/3*(1+2*x)*3^(1/2))+45/512*(1+2*x)*(x^2+x+1)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {626, 633, 221}

$$\frac{1}{12}(2x+1)(x^2+x+1)^{5/2} + \frac{5}{64}(2x+1)(x^2+x+1)^{3/2} + \frac{45}{512}(2x+1)\sqrt{x^2+x+1} + \frac{135 \sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{1024}$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2)^(5/2), x]

[Out] (45*(1 + 2*x)*Sqrt[1 + x + x^2])/512 + (5*(1 + 2*x)*(1 + x + x^2)^(3/2))/64 + ((1 + 2*x)*(1 + x + x^2)^(5/2))/12 + (135*ArcSinh[(1 + 2*x)/Sqrt[3]])/1024

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 626

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 633

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*c/(b^2 - 4*a*c)))^p, Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned}
\int (1+x+x^2)^{5/2} dx &= \frac{1}{12}(1+2x)(1+x+x^2)^{5/2} + \frac{5}{8} \int (1+x+x^2)^{3/2} dx \\
&= \frac{5}{64}(1+2x)(1+x+x^2)^{3/2} + \frac{1}{12}(1+2x)(1+x+x^2)^{5/2} + \frac{45}{128} \int \sqrt{1+x+x^2} dx \\
&= \frac{45}{512}(1+2x)\sqrt{1+x+x^2} + \frac{5}{64}(1+2x)(1+x+x^2)^{3/2} + \frac{1}{12}(1+2x)(1+x+x^2)^{5/2} \\
&= \frac{45}{512}(1+2x)\sqrt{1+x+x^2} + \frac{5}{64}(1+2x)(1+x+x^2)^{3/2} + \frac{1}{12}(1+2x)(1+x+x^2)^{5/2} \\
&= \frac{45}{512}(1+2x)\sqrt{1+x+x^2} + \frac{5}{64}(1+2x)(1+x+x^2)^{3/2} + \frac{1}{12}(1+2x)(1+x+x^2)^{5/2}
\end{aligned}$$

Mathematica [A]

time = 0.17, size = 62, normalized size = 0.84

$$\frac{\sqrt{1+x+x^2} (383 + 1142x + 1256x^2 + 1264x^3 + 640x^4 + 256x^5)}{1536} - \frac{135 \log(-1 - 2x + 2\sqrt{1+x+x^2})}{1024}$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + x + x^2)^(5/2), x]`

```
[Out] (Sqrt[1 + x + x^2]*(383 + 1142*x + 1256*x^2 + 1264*x^3 + 640*x^4 + 256*x^5)
)/1536 - (135*Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]])/1024
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(1 + x + x^2)^(5/2), x]')`

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]

time = 0.12, size = 58, normalized size = 0.78

method	result	size
risch	$\frac{(256x^5+640x^4+1264x^3+1256x^2+1142x+383)\sqrt{x^2+x+1}}{1536} + \frac{135 \operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{1024}$	48
trager	$\left(\frac{1}{6}x^5 + \frac{5}{12}x^4 + \frac{79}{96}x^3 + \frac{157}{192}x^2 + \frac{571}{768}x + \frac{383}{1536}\right)\sqrt{x^2+x+1} + \frac{135 \ln\left(1+2x+2\sqrt{x^2+x+1}\right)}{1024}$	54
default	$\frac{(1+2x)(x^2+x+1)^{\frac{5}{2}}}{12} + \frac{5(1+2x)(x^2+x+1)^{\frac{3}{2}}}{64} + \frac{45(1+2x)\sqrt{x^2+x+1}}{512} + \frac{135 \operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{1024}$	58

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+x+1)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{12}(1+2x)(x^2+x+1)^{5/2} + \frac{5}{64}(1+2x)(x^2+x+1)^{3/2} + \frac{45}{512}(1+2x)(x^2+x+1)^{1/2} + \frac{135}{1024}\operatorname{arcsinh}\left(\frac{2\sqrt{3}\sqrt{x^2+x+1}}{3}\right)$

Maxima [A]

time = 0.35, size = 77, normalized size = 1.04

$$\frac{1}{6}(x^2+x+1)^{5/2}x + \frac{1}{12}(x^2+x+1)^{5/2} + \frac{5}{32}(x^2+x+1)^{3/2}x + \frac{5}{64}(x^2+x+1)^{3/2} + \frac{45}{256}\sqrt{x^2+x+1}x + \frac{45}{512}\sqrt{x^2+x+1} + \frac{135}{1024}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+x+1)^(5/2),x, algorithm="maxima")`

[Out] $\frac{1}{6}(x^2+x+1)^{5/2}x + \frac{1}{12}(x^2+x+1)^{5/2} + \frac{5}{32}(x^2+x+1)^{3/2}x + \frac{5}{64}(x^2+x+1)^{3/2} + \frac{45}{256}\sqrt{x^2+x+1}x + \frac{45}{512}\sqrt{x^2+x+1} + \frac{135}{1024}\operatorname{arcsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$

Fricas [A]

time = 0.31, size = 54, normalized size = 0.73

$$\frac{1}{1536}(256x^5+640x^4+1264x^3+1256x^2+1142x+383)\sqrt{x^2+x+1} - \frac{135}{1024}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+x+1)^(5/2),x, algorithm="fricas")`

[Out] $\frac{1}{1536}(256x^5+640x^4+1264x^3+1256x^2+1142x+383)\sqrt{x^2+x+1} - \frac{135}{1024}\log(-2x+2\sqrt{x^2+x+1}-1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2+x+1)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+x+1)**(5/2),x)

[Out] Integral((x**2 + x + 1)**(5/2), x)

Giac [A]

time = 0.00, size = 73, normalized size = 0.99

$$2 \left(\left(\left(\left(\left(\frac{x}{12} + \frac{5}{24} \right) x + \frac{79}{192} \right) x + \frac{157}{384} \right) x + \frac{571}{1536} \right) x + \frac{383}{3072} \right) \sqrt{x^2 + x + 1} - \frac{135}{1024} \ln \left(2 \left(\sqrt{x^2 + x + 1} - x \right) - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)^(5/2),x)

[Out] 1/1536*(2*(4*(2*(8*(2*x + 5)*x + 79)*x + 157)*x + 571)*x + 383)*sqrt(x^2 + x + 1) - 135/1024*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)

Mupad [B]

time = 0.07, size = 56, normalized size = 0.76

$$\frac{135 \ln \left(x + \sqrt{x^2 + x + 1} + \frac{1}{2} \right)}{1024} + \frac{5 \left(x + \frac{1}{2} \right) \left(x^2 + x + 1 \right)^{3/2}}{32} + \frac{\left(x + \frac{1}{2} \right) \left(x^2 + x + 1 \right)^{5/2}}{6} + \frac{45 \left(\frac{x}{2} + \frac{1}{4} \right) \sqrt{x^2 + x + 1}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^2 + 1)^(5/2),x)

[Out] (135*log(x + (x + x^2 + 1)^(1/2) + 1/2))/1024 + (5*(x + 1/2)*(x + x^2 + 1)^(3/2))/32 + ((x + 1/2)*(x + x^2 + 1)^(5/2))/6 + (45*(x/2 + 1/4)*(x + x^2 + 1)^(1/2))/128

$$3.274 \quad \int \frac{1}{x^2 \sqrt{1+x+x^2}} dx$$

Optimal. Leaf size=38

$$-\frac{\sqrt{1+x+x^2}}{x} + \frac{1}{2} \tanh^{-1} \left(\frac{2+x}{2\sqrt{1+x+x^2}} \right)$$

[Out] $1/2*\operatorname{arctanh}(1/2*(2+x)/(x^2+x+1)^{(1/2)})-(x^2+x+1)^{(1/2)}/x$

Rubi [A]

time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {744, 738, 212}

$$\frac{1}{2} \tanh^{-1} \left(\frac{x+2}{2\sqrt{x^2+x+1}} \right) - \frac{\sqrt{x^2+x+1}}{x}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[1/(x^2*\operatorname{Sqrt}[1+x+x^2]),x]$

[Out] $-(\operatorname{Sqrt}[1+x+x^2]/x) + \operatorname{ArcTanh}[(2+x)/(2*\operatorname{Sqrt}[1+x+x^2])]/2$

Rule 212

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (Gt Q[a, 0] || LtQ[b, 0])

Rule 738

$\operatorname{Int}[1/(((d_ + (e_)*(x_))*\operatorname{Sqrt}[(a_ + (b_)*(x_ + (c_)*(x_)^2)]), x_Symbol] := \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/\operatorname{Sqrt}[a + b*x + c*x^2]], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 744

$\operatorname{Int}[(d_ + (e_)*(x_))^m*(a_ + (b_)*(x_ + (c_)*(x_)^2)^p), x_Symbol] := \operatorname{Simp}[e*(d + e*x)^{m+1}*((a + b*x + c*x^2)^{p+1}/((m+1)*(c*d^2 - b*d*e + a*e^2))), x] + \operatorname{Dist}[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)), \operatorname{Int}[(d + e*x)^{m+1}*(a + b*x + c*x^2)^p, x], x] /;$ FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \sqrt{1+x+x^2}} dx &= -\frac{\sqrt{1+x+x^2}}{x} - \frac{1}{2} \int \frac{1}{x \sqrt{1+x+x^2}} dx \\
&= -\frac{\sqrt{1+x+x^2}}{x} + \text{Subst} \left(\int \frac{1}{4-x^2} dx, x, \frac{2+x}{\sqrt{1+x+x^2}} \right) \\
&= -\frac{\sqrt{1+x+x^2}}{x} + \frac{1}{2} \tanh^{-1} \left(\frac{2+x}{2\sqrt{1+x+x^2}} \right)
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 33, normalized size = 0.87

$$-\frac{\sqrt{1+x+x^2}}{x} - \tanh^{-1} \left(x - \sqrt{1+x+x^2} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x^2*Sqrt[1 + x + x^2]),x]``[Out] -(Sqrt[1 + x + x^2]/x) - ArcTanh[x - Sqrt[1 + x + x^2]]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x^2*Sqrt[1 + x + x^2]),x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [A]**

time = 0.12, size = 31, normalized size = 0.82

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{2} - \frac{\sqrt{x^2+x+1}}{x}$	31
risch	$\frac{\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{2} - \frac{\sqrt{x^2+x+1}}{x}$	31
trager	$-\frac{\sqrt{x^2+x+1}}{x} + \frac{\ln\left(\frac{2\sqrt{x^2+x+1}+2+x}{x}\right)}{2}$	35

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $1/2*\operatorname{arctanh}(1/2*(2+x)/(x^2+x+1)^(1/2))-(x^2+x+1)^(1/2)/x$

Maxima [A]

time = 0.35, size = 37, normalized size = 0.97

$$-\frac{\sqrt{x^2+x+1}}{x} + \frac{1}{2} \operatorname{arsinh}\left(\frac{\sqrt{3}x}{3|x|} + \frac{2\sqrt{3}}{3|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(x^2+x+1)^(1/2),x, algorithm="maxima")`

[Out] $-\operatorname{sqrt}(x^2+x+1)/x + 1/2*\operatorname{arcsinh}(1/3*\operatorname{sqrt}(3)*x/\operatorname{abs}(x) + 2/3*\operatorname{sqrt}(3)/\operatorname{abs}(x))$

Fricas [A]

time = 0.31, size = 52, normalized size = 1.37

$$\frac{x \log\left(-x + \sqrt{x^2+x+1} + 1\right) - x \log\left(-x + \sqrt{x^2+x+1} - 1\right) - 2x - 2\sqrt{x^2+x+1}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(x^2+x+1)^(1/2),x, algorithm="fricas")`

[Out] $1/2*(x*\log(-x + \operatorname{sqrt}(x^2+x+1) + 1) - x*\log(-x + \operatorname{sqrt}(x^2+x+1) - 1) - 2*x - 2*\operatorname{sqrt}(x^2+x+1))/x$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{x^2+x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(x**2+x+1)**(1/2),x)`

[Out] `Integral(1/(x**2*sqrt(x**2+x+1)),x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(30) = 60.

time = 0.00, size = 77, normalized size = 2.03

$$2 \left(-\frac{\ln\left|\sqrt{x^2+x+1} - x - 1\right|}{4} + \frac{\ln\left|\sqrt{x^2+x+1} - x + 1\right|}{4} + \frac{-\sqrt{x^2+x+1} + x + 2}{2 \left(\left(\sqrt{x^2+x+1} - x \right)^2 - 1 \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(x^2+x+1)^(1/2),x)`

[Out] $(x - \sqrt{x^2 + x + 1} + 2)/((x - \sqrt{x^2 + x + 1})^2 - 1) + 1/2 \cdot \log(\text{abs}(-x + \sqrt{x^2 + x + 1} + 1)) - 1/2 \cdot \log(\text{abs}(-x + \sqrt{x^2 + x + 1} - 1))$

Mupad [B]

time = 0.03, size = 31, normalized size = 0.82

$$\frac{\operatorname{atanh}\left(\frac{\frac{x}{2}+1}{\sqrt{x^2+x+1}}\right)}{2} - \frac{\sqrt{x^2+x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(x + x^2 + 1)^(1/2)),x)`

[Out] $\operatorname{atanh}((x/2 + 1)/(x + x^2 + 1)^{(1/2)})/2 - (x + x^2 + 1)^{(1/2)}/x$

$$3.275 \quad \int \frac{1}{x^3 \sqrt{1+x+x^2}} dx$$

Optimal. Leaf size=57

$$-\frac{\sqrt{1+x+x^2}}{2x^2} + \frac{3\sqrt{1+x+x^2}}{4x} + \frac{1}{8} \tanh^{-1}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)$$

[Out] 1/8*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))-1/2*(x^2+x+1)^(1/2)/x^2+3/4*(x^2+x+1)^(1/2)/x

Rubi [A]

time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {758, 820, 738, 212}

$$\frac{3\sqrt{x^2+x+1}}{4x} - \frac{\sqrt{x^2+x+1}}{2x^2} + \frac{1}{8} \tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*Sqrt[1+x+x^2]),x]

[Out] -1/2*Sqrt[1+x+x^2]/x^2 + (3*Sqrt[1+x+x^2])/(4*x) + ArcTanh[(2+x)/(2*Sqrt[1+x+x^2])]/8

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 738

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 758

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[e*(d + e*x)^(m+1)*((a + b*x + c*x^2)^(p+1)/((m+1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m+1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m+1)*Simp[c*d*(m+1) - b*e*(m+p+2) - c*e*(m+2*p+3)*x, x]*(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS

```
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 \sqrt{1+x+x^2}} dx &= -\frac{\sqrt{1+x+x^2}}{2x^2} - \frac{1}{2} \int \frac{\frac{3}{2} + x}{x^2 \sqrt{1+x+x^2}} dx \\ &= -\frac{\sqrt{1+x+x^2}}{2x^2} + \frac{3\sqrt{1+x+x^2}}{4x} - \frac{1}{8} \int \frac{1}{x\sqrt{1+x+x^2}} dx \\ &= -\frac{\sqrt{1+x+x^2}}{2x^2} + \frac{3\sqrt{1+x+x^2}}{4x} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{4-x^2} dx, x, \frac{2+x}{\sqrt{1+x+x^2}} \right) \\ &= -\frac{\sqrt{1+x+x^2}}{2x^2} + \frac{3\sqrt{1+x+x^2}}{4x} + \frac{1}{8} \tanh^{-1} \left(\frac{2+x}{2\sqrt{1+x+x^2}} \right) \end{aligned}$$

Mathematica [A]

time = 0.07, size = 42, normalized size = 0.74

$$\frac{(-2 + 3x)\sqrt{1+x+x^2}}{4x^2} - \frac{1}{4} \tanh^{-1} \left(x - \sqrt{1+x+x^2} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[1 + x + x^2]),x]
```

```
[Out] ((-2 + 3*x)*Sqrt[1 + x + x^2])/(4*x^2) - ArcTanh[x - Sqrt[1 + x + x^2]]/4
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^3*Sqrt[1 + x + x^2]),x]')
```

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.12, size = 44, normalized size = 0.77

method	result	size
trager	$\frac{(-2+3x)\sqrt{x^2+x+1}}{4x^2} + \frac{\ln\left(\frac{{}_2\sqrt{x^2+x+1}^{2+x}}{x}\right)}{8}$	40
risch	$\frac{3x^3+x^2+x-2}{4x^2\sqrt{x^2+x+1}} + \frac{\operatorname{arctanh}\left(\frac{2+x}{{}_2\sqrt{x^2+x+1}}\right)}{8}$	42
default	$\frac{\operatorname{arctanh}\left(\frac{2+x}{{}_2\sqrt{x^2+x+1}}\right)}{8} - \frac{\sqrt{x^2+x+1}}{2x^2} + \frac{3\sqrt{x^2+x+1}}{4x}$	44

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $1/8*\operatorname{arctanh}(1/2*(2+x)/(x^2+x+1)^(1/2))-1/2*(x^2+x+1)^(1/2)/x^2+3/4*(x^2+x+1)^(1/2)/x$

Maxima [A]

time = 0.34, size = 50, normalized size = 0.88

$$\frac{3\sqrt{x^2+x+1}}{4x} - \frac{\sqrt{x^2+x+1}}{2x^2} + \frac{1}{8} \operatorname{arsinh}\left(\frac{\sqrt{3}x}{3|x|} + \frac{2\sqrt{3}}{3|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="maxima")`

[Out] $3/4*\operatorname{sqrt}(x^2+x+1)/x - 1/2*\operatorname{sqrt}(x^2+x+1)/x^2 + 1/8*\operatorname{arcsinh}(1/3*\operatorname{sqrt}(3)*x/\operatorname{abs}(x) + 2/3*\operatorname{sqrt}(3)/\operatorname{abs}(x))$

Fricas [A]

time = 0.30, size = 63, normalized size = 1.11

$$\frac{x^2 \log\left(-x + \sqrt{x^2+x+1} + 1\right) - x^2 \log\left(-x + \sqrt{x^2+x+1} - 1\right) + 6x^2 + 2\sqrt{x^2+x+1}(3x-2)}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="fricas")`

[Out] $1/8*(x^2*\log(-x + \operatorname{sqrt}(x^2+x+1) + 1) - x^2*\log(-x + \operatorname{sqrt}(x^2+x+1) - 1) + 6*x^2 + 2*\operatorname{sqrt}(x^2+x+1)*(3*x - 2))/x^2$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt{x^2 + x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(x**2+x+1)**(1/2),x)

[Out] Integral(1/(x**3*sqrt(x**2 + x + 1)), x)

Giac [A]

time = 0.01, size = 98, normalized size = 1.72

$$2 \left(-\frac{\ln|\sqrt{x^2+x+1}-x-1|}{16} + \frac{\ln|\sqrt{x^2+x+1}-x+1|}{16} - \frac{(\sqrt{x^2+x+1}-x)^3 + 9(\sqrt{x^2+x+1}-x) - 8}{8 \left((\sqrt{x^2+x+1}-x)^2 - 1 \right)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^2+x+1)^(1/2),x)

[Out] 1/4*((x - sqrt(x^2 + x + 1))^3 + 9*x - 9*sqrt(x^2 + x + 1) + 8)/((x - sqrt(x^2 + x + 1))^2 - 1)^2 + 1/8*log(abs(-x + sqrt(x^2 + x + 1) + 1)) - 1/8*log(abs(-x + sqrt(x^2 + x + 1) - 1))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3 \sqrt{x^2 + x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(x + x^2 + 1)^(1/2)),x)

[Out] int(1/(x^3*(x + x^2 + 1)^(1/2)), x)

$$3.276 \quad \int \frac{1}{x^2(1+x+x^2)^{3/2}} dx$$

Optimal. Leaf size=62

$$\frac{2(1-x)}{3x\sqrt{1+x+x^2}} - \frac{5\sqrt{1+x+x^2}}{3x} + \frac{3}{2} \tanh^{-1}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)$$

[Out] 3/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))+2/3*(1-x)/x/(x^2+x+1)^(1/2)-5/3*(x^2+x+1)^(1/2)/x

Rubi [A]

time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {754, 820, 738, 212}

$$\frac{2(1-x)}{3x\sqrt{x^2+x+1}} - \frac{5\sqrt{x^2+x+1}}{3x} + \frac{3}{2} \tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(1+x+x^2)^(3/2)),x]

[Out] (2*(1-x))/(3*x*Sqrt[1+x+x^2]) - (5*Sqrt[1+x+x^2])/(3*x) + (3*ArcTanh[(2+x)/(2*Sqrt[1+x+x^2])])/2

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 738

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 754

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(d + e*x)^(m+1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*((a + b*x + c*x^2)^(p+1)/((p+1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((p+1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p+1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4

```
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2(1+x+x^2)^{3/2}} dx &= \frac{2(1-x)}{3x\sqrt{1+x+x^2}} + \frac{2}{3} \int \frac{\frac{5}{2}-x}{x^2\sqrt{1+x+x^2}} dx \\ &= \frac{2(1-x)}{3x\sqrt{1+x+x^2}} - \frac{5\sqrt{1+x+x^2}}{3x} - \frac{3}{2} \int \frac{1}{x\sqrt{1+x+x^2}} dx \\ &= \frac{2(1-x)}{3x\sqrt{1+x+x^2}} - \frac{5\sqrt{1+x+x^2}}{3x} + 3 \text{Subst} \left(\int \frac{1}{4-x^2} dx, x, \frac{2+x}{\sqrt{1+x+x^2}} \right) \\ &= \frac{2(1-x)}{3x\sqrt{1+x+x^2}} - \frac{5\sqrt{1+x+x^2}}{3x} + \frac{3}{2} \tanh^{-1} \left(\frac{2+x}{2\sqrt{1+x+x^2}} \right) \end{aligned}$$

Mathematica [A]

time = 0.12, size = 45, normalized size = 0.73

$$\frac{-3 - 7x - 5x^2}{3x\sqrt{1+x+x^2}} - 3 \tanh^{-1} \left(x - \sqrt{1+x+x^2} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1+x+x^2)^(3/2)),x]
```

```
[Out] (-3 - 7*x - 5*x^2)/(3*x*Sqrt[1 + x + x^2]) - 3*ArcTanh[x - Sqrt[1 + x + x^2]]
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[1/(x^2*(1 + x + x^2)^(3/2)),x]')`

[Out] `cought exception: maximum recursion depth exceeded`

Maple [A]

time = 0.12, size = 56, normalized size = 0.90

method	result	size
risch	$-\frac{5x^2+7x+3}{3x\sqrt{x^2+x+1}} + \frac{3 \operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{2}$	41
trager	$-\frac{5x^2+7x+3}{3x\sqrt{x^2+x+1}} + \frac{3 \ln\left(\frac{2\sqrt{x^2+x+1}+2+x}{x}\right)}{2}$	45
default	$-\frac{1}{x\sqrt{x^2+x+1}} - \frac{3}{2\sqrt{x^2+x+1}} - \frac{5(1+2x)}{6\sqrt{x^2+x+1}} + \frac{3 \operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{2}$	56

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)`

[Out] `-1/x/(x^2+x+1)^(1/2)-3/2/(x^2+x+1)^(1/2)-5/6*(1+2*x)/(x^2+x+1)^(1/2)+3/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))`

Maxima [A]

time = 0.41, size = 58, normalized size = 0.94

$$-\frac{5x}{3\sqrt{x^2+x+1}} - \frac{7}{3\sqrt{x^2+x+1}} - \frac{1}{\sqrt{x^2+x+1}x} + \frac{3}{2} \operatorname{arsinh}\left(\frac{\sqrt{3}x}{3|x|} + \frac{2\sqrt{3}}{3|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(x^2+x+1)^(3/2),x, algorithm="maxima")`

[Out] `-5/3*x/sqrt(x^2 + x + 1) - 7/3/sqrt(x^2 + x + 1) - 1/(sqrt(x^2 + x + 1)*x) + 3/2*arcsinh(1/3*sqrt(3)*x/abs(x) + 2/3*sqrt(3)/abs(x))`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(46) = 92.

time = 0.30, size = 94, normalized size = 1.52

$$\frac{10x^3 + 10x^2 - 9(x^3 + x^2 + x) \log(-x + \sqrt{x^2 + x + 1} + 1) + 9(x^3 + x^2 + x) \log(-x + \sqrt{x^2 + x + 1} - 1) + 2(5x^2 + 7x + 3)\sqrt{x^2 + x + 1} + 10x}{6(x^3 + x^2 + x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(x^2+x+1)^(3/2),x, algorithm="fricas")`

[Out] $-1/6*(10*x^3 + 10*x^2 - 9*(x^3 + x^2 + x)*\log(-x + \sqrt{x^2 + x + 1}) + 1) + 9*(x^3 + x^2 + x)*\log(-x + \sqrt{x^2 + x + 1}) - 1) + 2*(5*x^2 + 7*x + 3)*\sqrt{x^2 + x + 1} + 10*x)/(x^3 + x^2 + x)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (x^2 + x + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(x**2+x+1)**(3/2),x)`

[Out] `Integral(1/(x**2*(x**2 + x + 1)**(3/2)), x)`

Giac [A]

time = 0.00, size = 109, normalized size = 1.76

$$\frac{2\left(-\frac{2}{3}-\frac{x}{3}\right)\sqrt{x^2+x+1}}{x^2+x+1} + 2\left(-\frac{3}{4}\ln|\sqrt{x^2+x+1}-x-1| + \frac{3}{4}\ln|\sqrt{x^2+x+1}-x+1| - \frac{\sqrt{x^2+x+1}-x-2}{2\left(\left(\sqrt{x^2+x+1}-x\right)^2-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(x^2+x+1)^(3/2),x)`

[Out] $-2/3*(x + 2)/\sqrt{x^2 + x + 1} + (x - \sqrt{x^2 + x + 1} + 2)/((x - \sqrt{x^2 + x + 1})^2 - 1) + 3/2*\log(\text{abs}(-x + \sqrt{x^2 + x + 1}) + 1) - 3/2*\log(\text{abs}(-x + \sqrt{x^2 + x + 1}) - 1)$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2 (x^2 + x + 1)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(x + x^2 + 1)^(3/2)),x)`

[Out] `int(1/(x^2*(x + x^2 + 1)^(3/2)), x)`

$$3.277 \quad \int \frac{1}{x^3(1+x+x^2)^{3/2}} dx$$

Optimal. Leaf size=79

$$\frac{2(1-x)}{3x^2\sqrt{1+x+x^2}} - \frac{7\sqrt{1+x+x^2}}{6x^2} + \frac{37\sqrt{1+x+x^2}}{12x} - \frac{3}{8} \tanh^{-1}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)$$

[Out] $-3/8*\operatorname{arctanh}(1/2*(2+x)/(x^2+x+1)^{(1/2)})+2/3*(1-x)/x^2/(x^2+x+1)^{(1/2)}-7/6*(x^2+x+1)^{(1/2)}/x^2+37/12*(x^2+x+1)^{(1/2)}/x$

Rubi [A]

time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {754, 848, 820, 738, 212}

$$\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}} + \frac{37\sqrt{x^2+x+1}}{12x} - \frac{7\sqrt{x^2+x+1}}{6x^2} - \frac{3}{8} \tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

[In] `Int[1/(x^3*(1+x+x^2)^(3/2)),x]`

[Out] $(2*(1-x))/(3*x^2*\operatorname{Sqrt}[1+x+x^2]) - (7*\operatorname{Sqrt}[1+x+x^2])/(6*x^2) + (37*\operatorname{Sqrt}[1+x+x^2])/(12*x) - (3*\operatorname{ArcTanh}[(2+x)/(2*\operatorname{Sqrt}[1+x+x^2])])/8$

Rule 212

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 738

`Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]`

Rule 754

`Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(d + e*x)^(m+1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*((a + b*x + c*x^2)^(p+1))/((p+1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p+1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p+1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4`

*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 820

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(-e*f - d*g)*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 848

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3(1+x+x^2)^{3/2}} dx &= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}} + \frac{2}{3} \int \frac{\frac{7}{2}-2x}{x^3\sqrt{1+x+x^2}} dx \\ &= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}} - \frac{7\sqrt{1+x+x^2}}{6x^2} - \frac{1}{3} \int \frac{\frac{37}{4} + \frac{7x}{2}}{x^2\sqrt{1+x+x^2}} dx \\ &= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}} - \frac{7\sqrt{1+x+x^2}}{6x^2} + \frac{37\sqrt{1+x+x^2}}{12x} + \frac{3}{8} \int \frac{1}{x\sqrt{1+x+x^2}} dx \\ &= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}} - \frac{7\sqrt{1+x+x^2}}{6x^2} + \frac{37\sqrt{1+x+x^2}}{12x} - \frac{3}{4} \text{Subst} \left(\int \frac{1}{4-x^2} dx, \right. \\ &= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}} - \frac{7\sqrt{1+x+x^2}}{6x^2} + \frac{37\sqrt{1+x+x^2}}{12x} - \frac{3}{8} \tanh^{-1} \left(\frac{2+x}{2\sqrt{1+x+x^2}} \right) \end{aligned}$$

Mathematica [A]

time = 0.13, size = 52, normalized size = 0.66

$$\frac{-6 + 15x + 23x^2 + 37x^3}{12x^2\sqrt{1+x+x^2}} + \frac{3}{4} \tanh^{-1} \left(x - \sqrt{1+x+x^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(1 + x + x^2)^(3/2)),x]

[Out] $(-6 + 15x + 23x^2 + 37x^3)/(12x^2\sqrt{1 + x + x^2}) + (3\text{ArcTanh}[x - \sqrt{1 + x + x^2}])/4$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x^3*(1 + x + x^2)^(3/2)),x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [A]

time = 0.12, size = 69, normalized size = 0.87

method	result
risch	$\frac{37x^3+23x^2+15x-6}{12\sqrt{x^2+x+1}} \frac{1}{x^2} - \frac{3 \operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}$
trager	$\frac{37x^3+23x^2+15x-6}{12\sqrt{x^2+x+1}} \frac{1}{x^2} - \frac{3 \ln\left(\frac{2\sqrt{x^2+x+1}+2+x}{x}\right)}{8}$
default	$-\frac{1}{2x^2\sqrt{x^2+x+1}} + \frac{5}{4x\sqrt{x^2+x+1}} + \frac{3}{8\sqrt{x^2+x+1}} + \frac{\frac{37}{24} + \frac{37x}{12}}{\sqrt{x^2+x+1}} - \frac{3 \operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)

[Out] $-1/2/x^2/(x^2+x+1)^(1/2)+5/4/x/(x^2+x+1)^(1/2)+3/8/(x^2+x+1)^(1/2)+37/24*(1+2*x)/(x^2+x+1)^(1/2)-3/8*\operatorname{arctanh}(1/2*(2+x)/(x^2+x+1)^(1/2))$

Maxima [A]

time = 0.35, size = 71, normalized size = 0.90

$$\frac{37x}{12\sqrt{x^2+x+1}} + \frac{23}{12\sqrt{x^2+x+1}} + \frac{5}{4\sqrt{x^2+x+1}} \frac{1}{x} - \frac{1}{2\sqrt{x^2+x+1}} \frac{1}{x^2} - \frac{3}{8} \operatorname{arsinh}\left(\frac{\sqrt{3}x}{3|x|} + \frac{2\sqrt{3}}{3|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^2+x+1)^(3/2),x, algorithm="maxima")

[Out] $37/12*x/\sqrt{x^2 + x + 1} + 23/12/\sqrt{x^2 + x + 1} + 5/4/(\sqrt{x^2 + x + 1}) * x) - 1/2/(\sqrt{x^2 + x + 1} * x^2) - 3/8 * \operatorname{arcsinh}(1/3 * \sqrt{3} * x / \operatorname{abs}(x) + 2/3 * \sqrt{3} / \operatorname{abs}(x))$

Fricas [A]

time = 0.30, size = 107, normalized size = 1.35

$$\frac{74x^4 + 74x^3 + 74x^2 - 9(x^4 + x^3 + x^2) \log(-x + \sqrt{x^2 + x + 1} + 1) + 9(x^4 + x^3 + x^2) \log(-x + \sqrt{x^2 + x + 1} - 1) + 2(37x^3 + 23x^2 + 15x - 6)\sqrt{x^2 + x + 1}}{24(x^4 + x^3 + x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(x^2+x+1)^(3/2),x, algorithm="fricas")`

[Out] $1/24 * (74 * x^4 + 74 * x^3 + 74 * x^2 - 9 * (x^4 + x^3 + x^2) * \log(-x + \sqrt{x^2 + x + 1} + 1) + 9 * (x^4 + x^3 + x^2) * \log(-x + \sqrt{x^2 + x + 1} - 1) + 2 * (37 * x^3 + 23 * x^2 + 15 * x - 6) * \sqrt{x^2 + x + 1}) / (x^4 + x^3 + x^2)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (x^2 + x + 1)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(x**2+x+1)**(3/2),x)`

[Out] `Integral(1/(x**3*(x**2 + x + 1)**(3/2)), x)`

Giac [A]

time = 0.01, size = 147, normalized size = 1.86

$$\frac{2 \left(\frac{2}{3}x + \frac{1}{3} \right) \sqrt{x^2 + x + 1}}{x^2 + x + 1} + 2 \left(\frac{3}{16} \ln \left| \sqrt{x^2 + x + 1} - x - 1 \right| - \frac{3}{16} \ln \left| \sqrt{x^2 + x + 1} - x + 1 \right| + \frac{3 \left(\sqrt{x^2 + x + 1} - x \right)^3 - 8 \left(\sqrt{x^2 + x + 1} - x \right)^2 - 13 \left(\sqrt{x^2 + x + 1} - x \right) + 16}{8 \left(\left(\sqrt{x^2 + x + 1} - x \right)^2 - 1 \right)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(x^2+x+1)^(3/2),x)`

[Out] $2/3 * (2 * x + 1) / \sqrt{x^2 + x + 1} - 1/4 * (3 * (x - \sqrt{x^2 + x + 1})^3 + 8 * (x - \sqrt{x^2 + x + 1})^2 - 13 * x + 13 * \sqrt{x^2 + x + 1} - 16) / ((x - \sqrt{x^2 + x + 1})^2 - 1)^2 - 3/8 * \log(\operatorname{abs}(-x + \sqrt{x^2 + x + 1} + 1)) + 3/8 * \log(\operatorname{abs}(-x + \sqrt{x^2 + x + 1} - 1))$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 (x^2 + x + 1)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*(x + x^2 + 1)^(3/2)),x)`

[Out] `int(1/(x^3*(x + x^2 + 1)^(3/2)), x)`

$$3.278 \quad \int \frac{1}{(1+x)\sqrt{1+x+x^2}} dx$$

Optimal. Leaf size=22

$$-\tanh^{-1}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right)$$

[Out] -arctanh(1/2*(1-x)/(x^2+x+1)^(1/2))

Rubi [A]

time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {738, 212}

$$-\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/((1+x)*Sqrt[1+x+x^2]),x]

[Out] -ArcTanh[(1-x)/(2*Sqrt[1+x+x^2])]

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 738

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+x)\sqrt{1+x+x^2}} dx &= -\left(2\text{Subst}\left(\int \frac{1}{4-x^2} dx, x, \frac{1-x}{\sqrt{1+x+x^2}}\right)\right) \\ &= -\tanh^{-1}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right) \end{aligned}$$

Mathematica [A]

time = 0.07, size = 18, normalized size = 0.82

$$2 \tanh^{-1}\left(1+x-\sqrt{1+x+x^2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x)*Sqrt[1 + x + x^2]),x]

[Out] 2*ArcTanh[1 + x - Sqrt[1 + x + x^2]]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((x + 1)*Sqrt[1 + x + x^2]),x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [A]

time = 0.13, size = 22, normalized size = 1.00

method	result	size
default	$-\operatorname{arctanh}\left(\frac{1-x}{2\sqrt{(1+x)^2-x}}\right)$	22
trager	$-\ln\left(\frac{2\sqrt{x^2+x+1}+1-x}{1+x}\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+x)/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -arctanh(1/2*(1-x)/((1+x)^2-x)^(1/2))

Maxima [A]

time = 0.34, size = 25, normalized size = 1.14

$$\operatorname{arsinh}\left(\frac{\sqrt{3}x}{3|x+1|} - \frac{\sqrt{3}}{3|x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x^2+x+1)^(1/2),x, algorithm="maxima")

[Out] arcsinh(1/3*sqrt(3)*x/abs(x + 1) - 1/3*sqrt(3)/abs(x + 1))

Fricas [A]

time = 0.29, size = 30, normalized size = 1.36

$$-\log\left(-x + \sqrt{x^2 + x + 1}\right) + \log\left(-x + \sqrt{x^2 + x + 1} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x)/(x^2+x+1)^(1/2),x, algorithm="fricas")`

[Out] `-log(-x + sqrt(x^2 + x + 1)) + log(-x + sqrt(x^2 + x + 1) - 2)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+1)\sqrt{x^2+x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x)/(x**2+x+1)**(1/2),x)`

[Out] `Integral(1/((x + 1)*sqrt(x**2 + x + 1)), x)`

Giac [A]

time = 0.00, size = 41, normalized size = 1.86

$$2 \left(-\frac{\ln \left| \sqrt{x^2 + x + 1} - x \right|}{2} + \frac{\ln \left| \sqrt{x^2 + x + 1} - x - 2 \right|}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x)/(x^2+x+1)^(1/2),x)`

[Out] `-log(abs(-x + sqrt(x^2 + x + 1))) + log(abs(-x + sqrt(x^2 + x + 1) - 2))`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{(x+1)\sqrt{x^2+x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((x + 1)*(x + x^2 + 1)^(1/2)),x)`

[Out] `int(1/((x + 1)*(x + x^2 + 1)^(1/2)), x)`

$$3.279 \quad \int \frac{1}{\sqrt{4+2x+x^2}(-x+x^3)} dx$$

Optimal. Leaf size=86

$$\frac{1}{2} \tanh^{-1} \left(\frac{4+x}{2\sqrt{4+2x+x^2}} \right) - \frac{\tanh^{-1} \left(\frac{5+2x}{\sqrt{7}\sqrt{4+2x+x^2}} \right)}{2\sqrt{7}} - \frac{\tanh^{-1} \left(\frac{\sqrt{4+2x+x^2}}{\sqrt{3}} \right)}{2\sqrt{3}}$$

[Out] 1/2*arctanh(1/2*(4+x)/(x^2+2*x+4)^(1/2))-1/6*arctanh(1/3*(x^2+2*x+4)^(1/2)*3^(1/2))*3^(1/2)-1/14*arctanh(1/7*(5+2*x)*7^(1/2)/(x^2+2*x+4)^(1/2))*7^(1/2)

Rubi [A]

time = 0.18, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1607, 6857, 738, 212, 1047, 702, 213}

$$\frac{1}{2} \tanh^{-1} \left(\frac{x+4}{2\sqrt{x^2+2x+4}} \right) - \frac{\tanh^{-1} \left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}} \right)}{2\sqrt{7}} - \frac{\tanh^{-1} \left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}} \right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[4 + 2*x + x^2]*(-x + x^3)),x]

[Out] ArcTanh[(4 + x)/(2*Sqrt[4 + 2*x + x^2])]/2 - ArcTanh[(5 + 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])]/(2*Sqrt[7]) - ArcTanh[Sqrt[4 + 2*x + x^2]/Sqrt[3]]/(2*Sqrt[3])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 702

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E

qQ[2*c*d - b*e, 0]

Rule 738

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1047

Int[((g_.) + (h_.)*(x_))/((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]

Rule 1607

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 6857

Int[(u_)/((a_) + (b_.)*(x_)^n), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{4+2x+x^2}(-x+x^3)} dx &= \int \frac{1}{x(-1+x^2)\sqrt{4+2x+x^2}} dx \\
&= \int \left(-\frac{1}{x\sqrt{4+2x+x^2}} + \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}} \right) dx \\
&= -\int \frac{1}{x\sqrt{4+2x+x^2}} dx + \int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}} dx \\
&= \frac{1}{2} \int \frac{1}{(-1+x)\sqrt{4+2x+x^2}} dx + \frac{1}{2} \int \frac{1}{(1+x)\sqrt{4+2x+x^2}} dx + 2\text{Subst} \\
&= \frac{1}{2} \tanh^{-1} \left(\frac{4+x}{2\sqrt{4+2x+x^2}} \right) + 2\text{Subst} \left(\int \frac{1}{-12+4x^2} dx, x, \sqrt{4+2x+x^2} \right) \\
&= \frac{1}{2} \tanh^{-1} \left(\frac{4+x}{2\sqrt{4+2x+x^2}} \right) - \frac{\tanh^{-1} \left(\frac{10+4x}{2\sqrt{7}\sqrt{4+2x+x^2}} \right)}{2\sqrt{7}} - \frac{\tanh^{-1}}{2\sqrt{7}}
\end{aligned}$$

Mathematica [A]

time = 0.17, size = 85, normalized size = 0.99

$$-\tanh^{-1} \left(\frac{1}{2} (x - \sqrt{4+2x+x^2}) \right) + \frac{\tanh^{-1} \left(\frac{1+x-\sqrt{4+2x+x^2}}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{\tanh^{-1} \left(\frac{1-x+\sqrt{4+2x+x^2}}{\sqrt{7}} \right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[4 + 2*x + x^2]*(-x + x^3)),x]
```

```
[Out] -ArcTanh[(x - Sqrt[4 + 2*x + x^2])/2] + ArcTanh[(1 + x - Sqrt[4 + 2*x + x^2])
]/Sqrt[3]]/Sqrt[3] - ArcTanh[(1 - x + Sqrt[4 + 2*x + x^2])/Sqrt[7]]/Sqrt[7]
]
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((x^3 - x)*Sqrt[x^2 + 2*x + 4]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]

time = 0.24, size = 69, normalized size = 0.80

method	result
default	$\frac{\operatorname{arctanh}\left(\frac{8+2x}{4\sqrt{x^2+2x+4}}\right)}{2} - \frac{\sqrt{7} \operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{14} - \frac{\sqrt{3} \operatorname{arctanh}\left(\frac{\sqrt{3}}{\sqrt{(1+x)^2+3}}\right)}{6}$
trager	$\frac{\ln\left(\frac{2\sqrt{x^2+2x+4}}{x} + 4+x\right)}{2} - \frac{\operatorname{RootOf}(-Z^2-3) \ln\left(\frac{\sqrt{x^2+2x+4} + \operatorname{RootOf}(-Z^2-3)}{1+x}\right)}{6} - \frac{\operatorname{RootOf}(-Z^2-7) \ln\left(\dots\right)}{6}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3-x)/(x^2+2*x+4)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{2} \operatorname{arctanh}\left(\frac{1}{4} \frac{8+2x}{\sqrt{x^2+2x+4}}\right) - \frac{1}{14} \sqrt{7} \operatorname{arctanh}\left(\frac{1}{14} \frac{(10+4x)\sqrt{7}}{\sqrt{(-1+x)^2+3+4x}}\right) - \frac{1}{6} \sqrt{3} \operatorname{arctanh}\left(\frac{3}{\sqrt{(1+x)^2+3}}\right)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^3-x)/(x^2+2*x+4)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/((x^3 - x)*sqrt(x^2 + 2*x + 4)), x)`

Fricas [A]

time = 0.31, size = 110, normalized size = 1.28

$$\frac{1}{14} \sqrt{7} \log\left(\frac{\sqrt{7}(2x+5) + \sqrt{x^2+2x+4}(2\sqrt{7}-7) - 4x - 10}{x-1}\right) + \frac{1}{6} \sqrt{3} \log\left(\frac{-\sqrt{3} - \sqrt{x^2+2x+4}}{x+1}\right) + \frac{1}{2} \log(-x + \sqrt{x^2+2x+4} + 2) - \frac{1}{2} \log(-x + \sqrt{x^2+2x+4} - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^3-x)/(x^2+2*x+4)^(1/2),x, algorithm="fricas")`

[Out] $\frac{1}{14} \sqrt{7} \log((\sqrt{7}(2x+5) + \sqrt{x^2+2x+4}(2\sqrt{7}-7) - 4x - 10)/(x-1)) + \frac{1}{6} \sqrt{3} \log(-(\sqrt{3} - \sqrt{x^2+2x+4})/(x+1)) + \frac{1}{2} \log(-x + \sqrt{x^2+2x+4} + 2) - \frac{1}{2} \log(-x + \sqrt{x^2+2x+4} - 2)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(x-1)(x+1)\sqrt{x^2+2x+4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**3-x)/(x**2+2*x+4)**(1/2),x)

[Out] Integral(1/(x*(x - 1)*(x + 1)*sqrt(x**2 + 2*x + 4)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(66) = 132.

time = 0.02, size = 179, normalized size = 2.08

$$2 \left(-\frac{\ln|\sqrt{x^2+2x+4}-x-2|}{4} + \frac{\ln|\sqrt{x^2+2x+4}-x+2|}{4} + \frac{\ln\left(\frac{\left|\frac{2(\sqrt{x^2+2x+4}-x)-2-2\sqrt{3}}{2(\sqrt{x^2+2x+4}-x)-2+2\sqrt{3}}\right|}{4\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\ln\left(\frac{\left|\frac{2(\sqrt{x^2+2x+4}-x)+2-2\sqrt{7}}{2(\sqrt{x^2+2x+4}-x)+2+2\sqrt{7}}\right|}{4\sqrt{7}}\right)}{4\sqrt{7}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-x)/(x^2+2*x+4)^(1/2),x)

[Out] 1/14*sqrt(7)*log(abs(-2*x - 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)/abs(-2*x + 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)) + 1/6*sqrt(3)*log(-1/2*abs(-2*x - 2*sqrt(3) + 2*sqrt(x^2 + 2*x + 4) - 2)/(x - sqrt(3) - sqrt(x^2 + 2*x + 4) + 1)) + 1/2*log(abs(-x + sqrt(x^2 + 2*x + 4) + 2)) - 1/2*log(abs(-x + sqrt(x^2 + 2*x + 4) - 2))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{(x-x^3)\sqrt{x^2+2x+4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/((x - x^3)*(2*x + x^2 + 4)^(1/2)),x)

[Out] -int(1/((x - x^3)*(2*x + x^2 + 4)^(1/2)), x)

$$3.280 \quad \int \frac{\sqrt{4 + 2x + x^2}}{(-1+x)^2} dx$$

Optimal. Leaf size=62

$$\frac{\sqrt{4 + 2x + x^2}}{1 - x} + \sinh^{-1}\left(\frac{1 + x}{\sqrt{3}}\right) - \frac{2 \tanh^{-1}\left(\frac{5 + 2x}{\sqrt{7} \sqrt{4 + 2x + x^2}}\right)}{\sqrt{7}}$$

[Out] arcsinh(1/3*(1+x)*3^(1/2))-2/7*arctanh(1/7*(5+2*x)*7^(1/2)/(x^2+2*x+4)^(1/2))*7^(1/2)+(x^2+2*x+4)^(1/2)/(1-x)

Rubi [A]

time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {746, 857, 633, 221, 738, 212}

$$\frac{\sqrt{x^2 + 2x + 4}}{1 - x} - \frac{2 \tanh^{-1}\left(\frac{2x+5}{\sqrt{7} \sqrt{x^2 + 2x + 4}}\right)}{\sqrt{7}} + \sinh^{-1}\left(\frac{x + 1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[4 + 2*x + x^2]/(-1 + x)^2,x]

[Out] Sqrt[4 + 2*x + x^2]/(1 - x) + ArcSinh[(1 + x)/Sqrt[3]] - (2*ArcTanh[(5 + 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])])/Sqrt[7]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 633

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*(-4*(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Dist[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2} dx &= \frac{\sqrt{4+2x+x^2}}{1-x} + \frac{1}{2} \int \frac{2+2x}{(-1+x)\sqrt{4+2x+x^2}} dx \\
&= \frac{\sqrt{4+2x+x^2}}{1-x} + 2 \int \frac{1}{(-1+x)\sqrt{4+2x+x^2}} dx + \int \frac{1}{\sqrt{4+2x+x^2}} dx \\
&= \frac{\sqrt{4+2x+x^2}}{1-x} - 4 \operatorname{Subst} \left(\int \frac{1}{28-x^2} dx, x, \frac{10+4x}{\sqrt{4+2x+x^2}} \right) + \operatorname{Subst} \left(\int \frac{1}{\sqrt{1+\frac{x^2}{12}}} dx, x, \frac{10+4x}{\sqrt{4+2x+x^2}} \right) \\
&= \frac{\sqrt{4+2x+x^2}}{1-x} + \sinh^{-1} \left(\frac{1+x}{\sqrt{3}} \right) - \frac{2 \tanh^{-1} \left(\frac{5+2x}{\sqrt{7}\sqrt{4+2x+x^2}} \right)}{\sqrt{7}}
\end{aligned}$$

Mathematica [A]

time = 0.17, size = 71, normalized size = 1.15

$$-\frac{\sqrt{4+2x+x^2}}{-1+x} - \frac{4 \tanh^{-1}\left(\frac{1-x+\sqrt{4+2x+x^2}}{\sqrt{7}}\right)}{\sqrt{7}} - \log\left(-1-x+\sqrt{4+2x+x^2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[4 + 2*x + x^2]/(-1 + x)^2,x]

[Out] -(Sqrt[4 + 2*x + x^2]/(-1 + x)) - (4*ArcTanh[(1 - x + Sqrt[4 + 2*x + x^2])/Sqrt[7]])/Sqrt[7] - Log[-1 - x + Sqrt[4 + 2*x + x^2]]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sqrt[x^2 + 2*x + 4]/(x - 1)^2,x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.19, size = 91, normalized size = 1.47

method	result
risch	$-\frac{\sqrt{x^2+2x+4}}{-1+x} + \operatorname{arcsinh}\left(\frac{(1+x)\sqrt{3}}{3}\right) - \frac{2\sqrt{7} \operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{7}$
trager	$-\frac{\sqrt{x^2+2x+4}}{-1+x} + \ln(x+1+\sqrt{x^2+2x+4}) + \frac{2\operatorname{RootOf}(_Z^2-7) \ln\left(\frac{-2\operatorname{RootOf}(_Z^2-7)x+7\sqrt{x^2+2x+4}}{-1+x}\right)}{7}$
default	$-\frac{((-1+x)^2+3+4x)^{\frac{3}{2}}}{7(-1+x)} + \frac{2\sqrt{(-1+x)^2+3+4x}}{7} + \operatorname{arcsinh}\left(\frac{(1+x)\sqrt{3}}{3}\right) - \frac{2\sqrt{7} \operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{7}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+2*x+4)^(1/2)/(-1+x)^2,x,method=_RETURNVERBOSE)

[Out] -1/7/(-1+x)*((-1+x)^2+3+4*x)^(3/2)+2/7*((-1+x)^2+3+4*x)^(1/2)+arcsinh(1/3*(1+x)*3^(1/2))-2/7*7^(1/2)*arctanh(1/14*(10+4*x)*7^(1/2)/((-1+x)^2+3+4*x)^(1/2))+1/14*(2+2*x)*((-1+x)^2+3+4*x)^(1/2)

Maxima [A]

time = 0.35, size = 61, normalized size = 0.98

$$-\frac{2}{7}\sqrt{7}\operatorname{arsinh}\left(\frac{2\sqrt{3}x}{3|x-1|}+\frac{5\sqrt{3}}{3|x-1|}\right)-\frac{\sqrt{x^2+2x+4}}{x-1}+\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((x^2+2*x+4)^(1/2)/(-1+x)^2,x, algorithm="maxima")`

```
[Out] -2/7*sqrt(7)*arcsinh(2/3*sqrt(3)*x/abs(x - 1) + 5/3*sqrt(3)/abs(x - 1)) - s
qrt(x^2 + 2*x + 4)/(x - 1) + arcsinh(1/3*sqrt(3)*x + 1/3*sqrt(3))
```

Fricas [A]

time = 0.30, size = 92, normalized size = 1.48

$$\frac{2\sqrt{7}(x-1)\log\left(\frac{\sqrt{7}^{(2x+5)+\sqrt{x^2+2x+4}}(2\sqrt{7}-7)^{-4x-10}}{x-1}\right)-7(x-1)\log(-x+\sqrt{x^2+2x+4}-1)-7x-7\sqrt{x^2+2x+4}+7}{7(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((x^2+2*x+4)^(1/2)/(-1+x)^2,x, algorithm="fricas")`

```
[Out] 1/7*(2*sqrt(7)*(x - 1)*log((sqrt(7)*(2*x + 5) + sqrt(x^2 + 2*x + 4))*(2*sqrt
(7) - 7) - 4*x - 10)/(x - 1)) - 7*(x - 1)*log(-x + sqrt(x^2 + 2*x + 4) - 1)
- 7*x - 7*sqrt(x^2 + 2*x + 4) + 7)/(x - 1)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+2x+4}}{(x-1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((x**2+2*x+4)**(1/2)/(-1+x)**2,x)``[Out] Integral(sqrt(x**2 + 2*x + 4)/(x - 1)**2, x)`**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(53) = 106.

time = 0.01, size = 148, normalized size = 2.39

$$2\left(-\frac{\ln(\sqrt{x^2+2x+4}-x-1)}{2}+\frac{\ln\left(\left|\frac{2(\sqrt{x^2+2x+4}-x)+2-2\sqrt{7}}{2(\sqrt{x^2+2x+4}-x)+2+2\sqrt{7}}\right|\right)}{\sqrt{7}}+\frac{2(\sqrt{x^2+2x+4}-x)-5}{-(\sqrt{x^2+2x+4}-x)^2-2(\sqrt{x^2+2x+4}-x)+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2*x+4)^(1/2)/(-1+x)^2,x)

[Out] 2/7*sqrt(7)*log(abs(-2*x - 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)/abs(-2*x + 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)) + 2*(2*x - 2*sqrt(x^2 + 2*x + 4) + 5)/((x - sqrt(x^2 + 2*x + 4))^2 - 2*x + 2*sqrt(x^2 + 2*x + 4) - 6) - log(-x + sqrt(x^2 + 2*x + 4) - 1)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x^2 + 2x + 4}}{(x - 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + x^2 + 4)^(1/2)/(x - 1)^2,x)

[Out] int((2*x + x^2 + 4)^(1/2)/(x - 1)^2, x)

$$3.281 \quad \int \frac{3+2x}{(3+2x+x^2)^2 \sqrt{4+2x+x^2}} dx$$

Optimal. Leaf size=76

$$-\frac{(3-x)\sqrt{4+2x+x^2}}{4(3+2x+x^2)} - \frac{\tan^{-1}\left(\frac{1+x}{\sqrt{2}\sqrt{4+2x+x^2}}\right)}{4\sqrt{2}} + \tanh^{-1}\left(\sqrt{4+2x+x^2}\right)$$

[Out] arctanh((x^2+2*x+4)^(1/2))-1/8*arctan(1/2*(1+x)*2^(1/2)/(x^2+2*x+4)^(1/2))*2^(1/2)-1/4*(3-x)*(x^2+2*x+4)^(1/2)/(x^2+2*x+3)

Rubi [A]

time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1030, 1039, 996, 210, 1038, 212}

$$-\frac{\sqrt{x^2+2x+4}(3-x)}{4(x^2+2x+3)} - \frac{\tan^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+2x+4}}\right)}{4\sqrt{2}} + \tanh^{-1}\left(\sqrt{x^2+2x+4}\right)$$

Antiderivative was successfully verified.

[In] Int[(3 + 2*x)/((3 + 2*x + x^2)^2*Sqrt[4 + 2*x + x^2]),x]

[Out] -1/4*((3 - x)*Sqrt[4 + 2*x + x^2])/(3 + 2*x + x^2) - ArcTan[(1 + x)/(Sqrt[2]*Sqrt[4 + 2*x + x^2])]/(4*Sqrt[2]) + ArcTanh[Sqrt[4 + 2*x + x^2]]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 996

Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0]

Rule 1030

```

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*c*d - 2*a*c*e + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c*e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1])

```

Rule 1038

```

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g, Subst[Int[1/(b*d - a*e - b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && EqQ[h*e - 2*g*f, 0]

```

Rule 1039

```

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-(h*e - 2*g*f)/(2*f), Int[1/(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2], x], x] + Dist[h/(2*f), Int[(e + 2*f*x)/(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && NeQ[h*e - 2*g*f, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{3+2x}{(3+2x+x^2)^2 \sqrt{4+2x+x^2}} dx &= -\frac{(3-x)\sqrt{4+2x+x^2}}{4(3+2x+x^2)} + \frac{1}{8} \int \frac{-10-8x}{(3+2x+x^2)\sqrt{4+2x+x^2}} dx \\
&= -\frac{(3-x)\sqrt{4+2x+x^2}}{4(3+2x+x^2)} - \frac{1}{4} \int \frac{1}{(3+2x+x^2)\sqrt{4+2x+x^2}} dx - \frac{1}{2} \\
&= -\frac{(3-x)\sqrt{4+2x+x^2}}{4(3+2x+x^2)} + 2\text{Subst}\left(\int \frac{1}{2-2x^2} dx, x, \sqrt{4+2x+x^2}\right) \\
&= -\frac{(3-x)\sqrt{4+2x+x^2}}{4(3+2x+x^2)} - \frac{\tan^{-1}\left(\frac{2+2x}{2\sqrt{2}\sqrt{4+2x+x^2}}\right)}{4\sqrt{2}} + \tanh^{-1}\left(\frac{2+2x}{2\sqrt{2}\sqrt{4+2x+x^2}}\right)
\end{aligned}$$

Mathematica [A]

time = 0.42, size = 84, normalized size = 1.11

$$\frac{1}{8} \left(\frac{2(-3+x)\sqrt{4+2x+x^2}}{3+2x+x^2} + \sqrt{2} \tan^{-1} \left(\frac{3+2x+x^2 - (1+x)\sqrt{4+2x+x^2}}{\sqrt{2}} \right) \right) + \tanh^{-1}(\sqrt{4+2x+x^2})$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 2*x)/((3 + 2*x + x^2)^2*Sqrt[4 + 2*x + x^2]),x]

[Out] ((2*(-3 + x)*Sqrt[4 + 2*x + x^2])/(3 + 2*x + x^2) + Sqrt[2]*ArcTan[(3 + 2*x + x^2 - (1 + x)*Sqrt[4 + 2*x + x^2])/Sqrt[2]])/8 + ArcTanh[Sqrt[4 + 2*x + x^2]]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(2*x + 3)/((x^2 + 2*x + 3)^2*Sqrt[x^2 + 2*x + 4]),x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [A]

time = 0.56, size = 123, normalized size = 1.62

method	result
risch	$\frac{(-3+x)\sqrt{x^2+2x+4}}{4x^2+8x+12} + \operatorname{arctanh}(\sqrt{x^2+2x+4}) - \frac{\sqrt{2} \operatorname{arctan}\left(\frac{\sqrt{2}^{(2+2x)}}{4\sqrt{x^2+2x+4}}\right)}{8}$

default	$-\frac{1}{2(\sqrt{x^2+2x+4}+1)} + \frac{\ln(\sqrt{x^2+2x+4}+1)}{2} - \frac{1}{2(\sqrt{x^2+2x+4}-1)} - \frac{\ln(\sqrt{x^2+2x+4}-1)}{2}$
trager	$\frac{(-3+x)\sqrt{x^2+2x+4}}{4x^2+8x+12} + 3 \operatorname{RootOf}(384_Z^2 - 128_Z + 11) \ln\left(-\frac{-16128 \operatorname{RootOf}(384_Z^2 - 128_Z + 11)}{\dots}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$-1/2/((x^2+2*x+4)^(1/2)+1)+1/2*\ln((x^2+2*x+4)^(1/2)+1)-1/2/((x^2+2*x+4)^(1/2)-1)-1/2*\ln((x^2+2*x+4)^(1/2)-1)+3/4*(1+x)/(x^2+2*x+4)^(1/2)/((1+x)^2/(x^2+2*x+4)+2)-1/8*\arctan(1/2*(1+x)*2^(1/2)/(x^2+2*x+4)^(1/2))*2^(1/2)$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x, algorithm="maxima")`

[Out] `integrate((2*x + 3)/(sqrt(x^2 + 2*x + 4)*(x^2 + 2*x + 3)^2), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(61) = 122.

time = 0.31, size = 174, normalized size = 2.29

$$\frac{\sqrt{2}(x^2+2x+3)\arctan\left(-\frac{1}{2}\sqrt{2}(x+2)+\frac{1}{2}\sqrt{2}\sqrt{x^2+2x+4}\right)-\sqrt{2}(x^2+2x+3)\arctan\left(-\frac{1}{2}\sqrt{2}x+\frac{1}{2}\sqrt{2}\sqrt{x^2+2x+4}\right)+2x^3-4(x^2+2x+3)\log(x^2-\sqrt{x^2+2x+4}(x+2)+3x+5)+4(x^2+2x+3)\log(x^2-\sqrt{x^2+2x+4}x+x+3)+2\sqrt{x^2+2x+4}(x-3)+4x+6}{8(x^2+2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x, algorithm="fricas")`

[Out]
$$1/8*(\sqrt{2}*(x^2+2*x+3)*\arctan(-1/2*\sqrt{2}*(x+2)+1/2*\sqrt{2}*\sqrt{x^2+2*x+4})-\sqrt{2}*(x^2+2*x+3)*\arctan(-1/2*\sqrt{2}*x+1/2*\sqrt{2}*\sqrt{x^2+2*x+4}))+2*x^2-4*(x^2+2*x+3)*\log(x^2-\sqrt{x^2+2*x+4}*(x+2)+3*x+5)+4*(x^2+2*x+3)*\log(x^2-\sqrt{x^2+2*x+4}x+x+3)+2*\sqrt{x^2+2*x+4}*(x-3)+4*x+6)/(x^2+2*x+3)$$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+3}{(x^2+2x+3)^2 \sqrt{x^2+2x+4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(x**2+2*x+3)**2/(x**2+2*x+4)**(1/2), x)

[Out] Integral((2*x + 3)/((x**2 + 2*x + 3)**2*sqrt(x**2 + 2*x + 4)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(61) = 122.

time = 0.01, size = 273, normalized size = 3.59

$$\frac{1}{2} \left(\frac{\ln\left(\frac{(\sqrt{x^2+2x+4}-x)^2+2}{4}\right) - \arctan\left(\frac{\sqrt{x^2+2x+4}-x}{\sqrt{2}}\right)}{4} - \frac{\ln\left(\frac{(\sqrt{x^2+2x+4}-x)^2-4(\sqrt{x^2+2x+4}-x)+6}{4}\right) + \arctan\left(\frac{\sqrt{x^2+2x+4}-x}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{-4(\sqrt{x^2+2x+4}-x)^3+13(\sqrt{x^2+2x+4}-x)^2-26(\sqrt{x^2+2x+4}-x)+26}{4((\sqrt{x^2+2x+4}-x)^4-4(\sqrt{x^2+2x+4}-x)^3+8(\sqrt{x^2+2x+4}-x)^2-8(\sqrt{x^2+2x+4}-x)+12)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2), x)

[Out] 1/8*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 2*x + 4) + 2)) - 1/8*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 2*x + 4))) + 1/2*(4*(x - sqrt(x^2 + 2*x + 4))^3 + 13*(x - sqrt(x^2 + 2*x + 4))^2 + 26*x - 26*sqrt(x^2 + 2*x + 4) + 26)/((x - sqrt(x^2 + 2*x + 4))^4 + 4*(x - sqrt(x^2 + 2*x + 4))^3 + 8*(x - sqrt(x^2 + 2*x + 4))^2 + 8*x - 8*sqrt(x^2 + 2*x + 4) + 12) - 1/2*log((x - sqrt(x^2 + 2*x + 4))^2 + 4*x - 4*sqrt(x^2 + 2*x + 4) + 6) + 1/2*log((x - sqrt(x^2 + 2*x + 4))^2 + 2)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2x + 3}{(x^2 + 2x + 3)^2 \sqrt{x^2 + 2x + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 3)/((2*x + x^2 + 3)^2*(2*x + x^2 + 4)^(1/2)), x)

[Out] int((2*x + 3)/((2*x + x^2 + 3)^2*(2*x + x^2 + 4)^(1/2)), x)

$$3.282 \quad \int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}(-3+x+2x^2)} dx$$

Optimal. Leaf size=36

$$\sqrt{-3+2x+x^2} + \frac{\sqrt{-3+2x+x^2}}{2(1-x)}$$

[Out] $(x^2+2*x-3)^{(1/2)}+1/2*(x^2+2*x-3)^{(1/2)}/(1-x)$

Rubi [A]

time = 0.07, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1607, 1600, 1652, 664}

$$\frac{\sqrt{x^2+2x-3}}{2(1-x)} + \sqrt{x^2+2x-3}$$

Antiderivative was successfully verified.

[In] Int[(3*x^2 + 2*x^3)/(Sqrt[-3 + 2*x + x^2]*(-3 + x + 2*x^2)),x]

[Out] Sqrt[-3 + 2*x + x^2] + Sqrt[-3 + 2*x + x^2]/(2*(1 - x))

Rule 664

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]

Rule 1600

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 1607

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1652

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S

```
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{3x^2 + 2x^3}{\sqrt{-3 + 2x + x^2} (-3 + x + 2x^2)} dx &= \int \frac{x^2(3 + 2x)}{\sqrt{-3 + 2x + x^2} (-3 + x + 2x^2)} dx \\ &= \int \frac{x^2}{(-1 + x)\sqrt{-3 + 2x + x^2}} dx \\ &= \sqrt{-3 + 2x + x^2} + \int \frac{1}{(-1 + x)\sqrt{-3 + 2x + x^2}} dx \\ &= \sqrt{-3 + 2x + x^2} + \frac{\sqrt{-3 + 2x + x^2}}{2(1 - x)} \end{aligned}$$

Mathematica [A]

time = 0.07, size = 26, normalized size = 0.72

$$\frac{(-3 + 2x)\sqrt{-3 + 2x + x^2}}{2(-1 + x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x^2 + 2*x^3)/(Sqrt[-3 + 2*x + x^2]*(-3 + x + 2*x^2)),x]
```

```
[Out] ((-3 + 2*x)*Sqrt[-3 + 2*x + x^2])/(2*(-1 + x))
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(2*x^3 + 3*x^2)/((2*x^2 + x - 3)*Sqrt[x^2 + 2*x - 3]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```


Maple [A]

time = 0.11, size = 31, normalized size = 0.86

method	result	size
gospers	$\frac{(2x-3)(3+x)}{2\sqrt{x^2+2x-3}}$	21
trager	$\frac{(2x-3)\sqrt{x^2+2x-3}}{2x-2}$	23
risch	$\frac{2x^2+3x-9}{2\sqrt{x^2+2x-3}}$	23
default	$\sqrt{x^2+2x-3} - \frac{\sqrt{(-1+x)^2-4+4x}}{2(-1+x)}$	31

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x,method=_RETURNVERBOSE)`[Out] $(x^2+2x-3)^{1/2}-1/2/(-1+x)*((-1+x)^2-4+4x)^{1/2}$ **Maxima [A]**

time = 0.34, size = 28, normalized size = 0.78

$$\sqrt{x^2+2x-3} - \frac{\sqrt{x^2+2x-3}}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x, algorithm="maxima")`[Out] $\text{sqrt}(x^2+2x-3) - 1/2*\text{sqrt}(x^2+2x-3)/(x-1)$ **Fricas [A]**

time = 0.30, size = 22, normalized size = 0.61

$$\frac{\sqrt{x^2+2x-3}(2x-3)}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x, algorithm="fricas")`[Out] $1/2*\text{sqrt}(x^2+2x-3)*(2x-3)/(x-1)$ **Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{(x-1)(x+3)}(x-1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3*x**2)/(2*x**2+x-3)/(x**2+2*x-3)**(1/2),x)

[Out] Integral(x**2/(sqrt((x - 1)*(x + 3))*(x - 1)), x)

Giac [A]

time = 0.00, size = 33, normalized size = 0.92

$$\sqrt{x^2 + 2x - 3} - \frac{2}{\sqrt{x^2 + 2x - 3} - x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x)

[Out] sqrt(x^2 + 2*x - 3) + 2/(x - sqrt(x^2 + 2*x - 3) - 1)

Mupad [B]

time = 0.27, size = 19, normalized size = 0.53

$$\frac{\left(x - \frac{3}{2}\right) \sqrt{x^2 + 2x - 3}}{x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 + 2*x^3)/((x + 2*x^2 - 3)*(2*x + x^2 - 3)^(1/2)),x)

[Out] ((x - 3/2)*(2*x + x^2 - 3)^(1/2))/(x - 1)

$$3.283 \quad \int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}} dx$$

Optimal. Leaf size=87

$$-\frac{7}{4}\sqrt{2+x+x^2} + \frac{1}{2}x\sqrt{2+x+x^2} - \frac{1}{8}\sinh^{-1}\left(\frac{1+2x}{\sqrt{7}}\right) + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}\sqrt{2+x+x^2}}\right)}{\sqrt{3}} - \tanh^{-1}\left(\sqrt{2+x+x^2}\right)$$

[Out] $-1/8*\operatorname{arcsinh}(1/7*(1+2*x)*7^{(1/2)})-\operatorname{arctanh}((x^2+x+2)^{(1/2)})+1/3*\operatorname{arctan}(1/3*(1+2*x)*3^{(1/2)/(x^2+x+2)^{(1/2)})}*3^{(1/2)}-7/4*(x^2+x+2)^{(1/2)}+1/2*x*(x^2+x+2)^{(1/2)}$

Rubi [A]

time = 0.13, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6860, 654, 633, 221, 756, 1039, 996, 210, 1038, 212}

$$\frac{1}{2}\sqrt{x^2+x+2}x - \frac{7}{4}\sqrt{x^2+x+2} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+x+2}}\right)}{\sqrt{3}} - \tanh^{-1}\left(\sqrt{x^2+x+2}\right) - \frac{1}{8}\sinh^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(1+x^4)/((1+x+x^2)*\operatorname{Sqrt}[2+x+x^2]),x]$

[Out] $(-7*\operatorname{Sqrt}[2+x+x^2])/4 + (x*\operatorname{Sqrt}[2+x+x^2])/2 - \operatorname{ArcSinh}[(1+2*x)/\operatorname{Sqrt}[7]]/8 + \operatorname{ArcTan}[(1+2*x)/(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[2+x+x^2])]/\operatorname{Sqrt}[3] - \operatorname{ArcTanh}[\operatorname{Sqrt}[2+x+x^2]]$

Rule 210

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(-(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1})*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{PosQ}[a/b] \ \&\& (\operatorname{LtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 212

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 221

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_+ + (b_+)*(x_+)^2], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[\operatorname{Rt}[b, 2]*(x/\operatorname{Sqrt}[a])]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{GtQ}[a, 0] \ \&\& \operatorname{PosQ}[b]$

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] := Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 996

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1038

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g, Subst[Int[1/(b*d - a*e -
b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
EqQ[h*e - 2*g*f, 0]
```

Rule 1039

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
```

- b*f, 0] && NeQ[h*e - 2*g*f, 0]

Rule 6860

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}} dx &= \int \left(-\frac{x}{\sqrt{2+x+x^2}} + \frac{x^2}{\sqrt{2+x+x^2}} + \frac{1+x}{(1+x+x^2)\sqrt{2+x+x^2}} \right) dx \\ &= -\int \frac{x}{\sqrt{2+x+x^2}} dx + \int \frac{x^2}{\sqrt{2+x+x^2}} dx + \int \frac{1+x}{(1+x+x^2)\sqrt{2+x+x^2}} dx \\ &= -\sqrt{2+x+x^2} + \frac{1}{2}x\sqrt{2+x+x^2} + \frac{1}{2} \int \frac{1}{\sqrt{2+x+x^2}} dx + \frac{1}{2} \int \frac{-}{\sqrt{2+x+x^2}} dx \end{aligned}$$

Subst

$$= -\frac{7}{4}\sqrt{2+x+x^2} + \frac{1}{2}x\sqrt{2+x+x^2} - \frac{5}{8} \int \frac{1}{\sqrt{2+x+x^2}} dx + \dots$$

$$= -\frac{7}{4}\sqrt{2+x+x^2} + \frac{1}{2}x\sqrt{2+x+x^2} + \frac{1}{2} \sinh^{-1} \left(\frac{1+2x}{\sqrt{7}} \right) + \frac{\tan^{-1} \left(\frac{-}{\sqrt{7}} \right)}{\dots}$$

$$= -\frac{7}{4}\sqrt{2+x+x^2} + \frac{1}{2}x\sqrt{2+x+x^2} - \frac{1}{8} \sinh^{-1} \left(\frac{1+2x}{\sqrt{7}} \right) + \frac{\tan^{-1} \left(\frac{-}{\sqrt{7}} \right)}{\dots}$$

Mathematica [A]

time = 0.34, size = 95, normalized size = 1.09

$$-\frac{\tan^{-1} \left(\frac{2+2x+2x^2-(1+2x)\sqrt{2+x+x^2}}{\sqrt{3}} \right)}{\sqrt{3}} - \tanh^{-1} \left(\sqrt{2+x+x^2} \right) + \frac{1}{8} \left(2(-7+2x)\sqrt{2+x+x^2} + \log \left(-1-2x+2\sqrt{2+x+x^2} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^4)/((1 + x + x^2)*Sqrt[2 + x + x^2]),x]

```
[Out] -(ArcTan[(2 + 2*x + 2*x^2 - (1 + 2*x)*Sqrt[2 + x + x^2])/Sqrt[3]]/Sqrt[3])
- ArcTanh[Sqrt[2 + x + x^2]] + (2*(-7 + 2*x)*Sqrt[2 + x + x^2] + Log[-1 - 2
*x + 2*Sqrt[2 + x + x^2]])/8
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^4 + 1)/((x^2 + x + 1)*Sqrt[x^2 + x + 2]),x]')
```

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 1.54, size = 69, normalized size = 0.79

method	result
risch	$\frac{(2x-7)\sqrt{x^2+x+2}}{4} - \frac{\operatorname{arcsinh}\left(\frac{2\sqrt{7}\left(x+\frac{1}{2}\right)}{7}\right)}{8} - \operatorname{arctanh}\left(\sqrt{x^2+x+2}\right) + \frac{\operatorname{arctan}\left(\frac{(1+2x)\sqrt{3}}{3\sqrt{x^2+x+2}}\right)\sqrt{3}}{3}$
default	$\frac{x\sqrt{x^2+x+2}}{2} - \frac{7\sqrt{x^2+x+2}}{4} - \frac{\operatorname{arcsinh}\left(\frac{2\sqrt{7}\left(x+\frac{1}{2}\right)}{7}\right)}{8} - \operatorname{arctanh}\left(\sqrt{x^2+x+2}\right) + \frac{\operatorname{arctan}\left(\frac{(1+2x)\sqrt{3}}{3\sqrt{x^2+x+2}}\right)\sqrt{3}}{3}$
trager	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(x^2+x+2)^(1/2)-7/4*(x^2+x+2)^(1/2)-1/8*arcsinh(2/7*7^(1/2)*(x+1/2))-
arctanh((x^2+x+2)^(1/2))+1/3*arctan(1/3*(1+2*x)*3^(1/2)/(x^2+x+2)^(1/2))*3^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(sqrt(x^2 + x + 2)*(x^2 + x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(70) = 140.

time = 0.32, size = 147, normalized size = 1.69

$\frac{1}{4}\sqrt{x^2+x+2}(2x-7) - \frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(2x+3) + \frac{2}{3}\sqrt{3}\sqrt{x^2+x+2}\right) + \frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(2x-1) + \frac{2}{3}\sqrt{3}\sqrt{x^2+x+2}\right) + \frac{1}{2}\log\left(2x^2 - \sqrt{x^2+x+2}(2x+3) + 4x+5\right) - \frac{1}{2}\log\left(2x^2 - \sqrt{x^2+x+2}(2x-1) + 3\right) + \frac{1}{8}\log\left(-2x+2\sqrt{x^2+x+2}-1\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x, algorithm="fricas")

[Out] 1/4*sqrt(x^2 + x + 2)*(2*x - 7) - 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x + 3) + 2/3*sqrt(3)*sqrt(x^2 + x + 2)) + 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - 1) + 2/3*sqrt(3)*sqrt(x^2 + x + 2)) + 1/2*log(2*x^2 - sqrt(x^2 + x + 2)*(2*x + 3) + 4*x + 5) - 1/2*log(2*x^2 - sqrt(x^2 + x + 2)*(2*x - 1) + 3) + 1/8*log(-2*x + 2*sqrt(x^2 + x + 2) - 1)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 + 1}{(x^2 + x + 1) \sqrt{x^2 + x + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+1)/(x**2+x+1)/(x**2+x+2)**(1/2),x)

[Out] Integral((x**4 + 1)/((x**2 + x + 1)*sqrt(x**2 + x + 2)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(70) = 140.

time = 0.01, size = 186, normalized size = 2.14

$$2 \left(\frac{x}{4} - \frac{7}{8} \right) \sqrt{x^2 + x + 2} + 2 \left(\frac{\ln(2(\sqrt{x^2 + x + 2} - x) - 1)}{16} + \frac{\ln((\sqrt{x^2 + x + 2} - x)^2 - 3(\sqrt{x^2 + x + 2} - x) + 3)}{4} - \frac{\arctan\left(\frac{2(\sqrt{x^2 + x + 2} - x) - 3}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\ln(\sqrt{x^2 + x + 2} - x + (\sqrt{x^2 + x + 2} - x)^2 + 1)}{4} + \frac{\arctan\left(\frac{2(\sqrt{x^2 + x + 2} - x) + 1}{\sqrt{3}}\right)}{2\sqrt{3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x)

[Out] 1/4*sqrt(x^2 + x + 2)*(2*x - 7) - 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - 2*sqrt(x^2 + x + 2) + 3)) + 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - 2*sqrt(x^2 + x + 2) - 1)) + 1/2*log((x - sqrt(x^2 + x + 2))^2 + 3*x - 3*sqrt(x^2 + x + 2) + 3) - 1/2*log((x - sqrt(x^2 + x + 2))^2 - x + sqrt(x^2 + x + 2) + 1) + 1/8*log(-2*x + 2*sqrt(x^2 + x + 2) - 1)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4 + 1}{(x^2 + x + 1) \sqrt{x^2 + x + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4 + 1)/((x + x^2 + 1)*(x + x^2 + 2)^(1/2)),x)

[Out] int((x^4 + 1)/((x + x^2 + 1)*(x + x^2 + 2)^(1/2)), x)

$$3.284 \quad \int \frac{1}{(4+2x+x^2)^{7/2}} dx$$

Optimal. Leaf size=58

$$\frac{1+x}{15(4+2x+x^2)^{5/2}} + \frac{4(1+x)}{135(4+2x+x^2)^{3/2}} + \frac{8(1+x)}{405\sqrt{4+2x+x^2}}$$

[Out] 1/15*(1+x)/(x^2+2*x+4)^(5/2)+4/135*(1+x)/(x^2+2*x+4)^(3/2)+8/405*(1+x)/(x^2+2*x+4)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {628, 627}

$$\frac{8(x+1)}{405\sqrt{x^2+2x+4}} + \frac{4(x+1)}{135(x^2+2x+4)^{3/2}} + \frac{x+1}{15(x^2+2x+4)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(4 + 2*x + x^2)^(-7/2), x]

[Out] (1 + x)/(15*(4 + 2*x + x^2)^(5/2)) + (4*(1 + x))/(135*(4 + 2*x + x^2)^(3/2)) + (8*(1 + x))/(405*Sqrt[4 + 2*x + x^2])

Rule 627

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] := Simp[-2*((b + 2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p + 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(4+2x+x^2)^{7/2}} dx &= \frac{1+x}{15(4+2x+x^2)^{5/2}} + \frac{4}{15} \int \frac{1}{(4+2x+x^2)^{5/2}} dx \\
&= \frac{1+x}{15(4+2x+x^2)^{5/2}} + \frac{4(1+x)}{135(4+2x+x^2)^{3/2}} + \frac{8}{135} \int \frac{1}{(4+2x+x^2)^{3/2}} dx \\
&= \frac{1+x}{15(4+2x+x^2)^{5/2}} + \frac{4(1+x)}{135(4+2x+x^2)^{3/2}} + \frac{8(1+x)}{405\sqrt{4+2x+x^2}}
\end{aligned}$$

Mathematica [A]

time = 0.21, size = 39, normalized size = 0.67

$$\frac{(1+x)(203+152x+108x^2+32x^3+8x^4)}{405(4+2x+x^2)^{5/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(4 + 2*x + x^2)^(-7/2), x]``[Out] ((1 + x)*(203 + 152*x + 108*x^2 + 32*x^3 + 8*x^4))/(405*(4 + 2*x + x^2)^(5/2))`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x^2 + 2*x + 4)^(7/2), x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [A]**

time = 0.13, size = 53, normalized size = 0.91

method	result	size
gospers	$\frac{8x^5+40x^4+140x^3+260x^2+355x+203}{405(x^2+2x+4)^{5/2}}$	38
trager	$\frac{8x^5+40x^4+140x^3+260x^2+355x+203}{405(x^2+2x+4)^{5/2}}$	38
risch	$\frac{8x^5+40x^4+140x^3+260x^2+355x+203}{405(x^2+2x+4)^{5/2}}$	38

default	$\frac{2+2x}{30(x^2+2x+4)^{\frac{5}{2}}} + \frac{\frac{4}{135} + \frac{4x}{135}}{(x^2+2x+4)^{\frac{3}{2}}} + \frac{\frac{8}{405} + \frac{8x}{405}}{\sqrt{x^2 + 2x + 4}}$	53
---------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2+2*x+4)^(7/2),x,method=_RETURNVERBOSE)`

[Out] $1/30*(2+2*x)/(x^2+2*x+4)^(5/2)+2/135*(2+2*x)/(x^2+2*x+4)^(3/2)+4/405*(2+2*x)/(x^2+2*x+4)^(1/2)$

Maxima [A]

time = 0.26, size = 76, normalized size = 1.31

$$\frac{8x}{405\sqrt{x^2+2x+4}} + \frac{8}{405\sqrt{x^2+2x+4}} + \frac{4x}{135(x^2+2x+4)^{\frac{3}{2}}} + \frac{4}{135(x^2+2x+4)^{\frac{3}{2}}} + \frac{x}{15(x^2+2x+4)^{\frac{5}{2}}} + \frac{1}{15(x^2+2x+4)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2+2*x+4)^(7/2),x, algorithm="maxima")`

[Out] $8/405*x/\sqrt{x^2 + 2*x + 4} + 8/405/\sqrt{x^2 + 2*x + 4} + 4/135*x/(x^2 + 2*x + 4)^(3/2) + 4/135/(x^2 + 2*x + 4)^(3/2) + 1/15*x/(x^2 + 2*x + 4)^(5/2) + 1/15/(x^2 + 2*x + 4)^(5/2)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(46) = 92$.

time = 0.30, size = 98, normalized size = 1.69

$$\frac{8x^6 + 48x^5 + 192x^4 + 448x^3 + 768x^2 + (8x^5 + 40x^4 + 140x^3 + 260x^2 + 355x + 203)\sqrt{x^2 + 2x + 4} + 768x + 512}{405(x^6 + 6x^5 + 24x^4 + 56x^3 + 96x^2 + 96x + 64)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2+2*x+4)^(7/2),x, algorithm="fricas")`

[Out] $1/405*(8*x^6 + 48*x^5 + 192*x^4 + 448*x^3 + 768*x^2 + (8*x^5 + 40*x^4 + 140*x^3 + 260*x^2 + 355*x + 203)*\sqrt{x^2 + 2*x + 4} + 768*x + 512)/(x^6 + 6*x^5 + 24*x^4 + 56*x^3 + 96*x^2 + 96*x + 64)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2 + 2x + 4)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2+2*x+4)**(7/2),x)`

[Out] `Integral((x**2 + 2*x + 4)**(-7/2), x)`

Giac [A]

time = 0.01, size = 63, normalized size = 1.09

$$\frac{2 \left(\left(\left(\left(\frac{4}{405}x + \frac{4}{81} \right) x + \frac{14}{81} \right) x + \frac{26}{81} \right) x + \frac{71}{162} \right) x + \frac{203}{810} \right) \sqrt{x^2 + 2x + 4}}{(x^2 + 2x + 4)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+2*x+4)^(7/2),x)**[Out]** 1/405*((4*((2*(x + 5)*x + 35)*x + 65)*x + 355)*x + 203)/(x^2 + 2*x + 4)^(5/2)**Mupad [B]**

time = 0.23, size = 69, normalized size = 1.19

$$\frac{51x + 8x(x^2 + 2x + 4)^2 + 8(x^2 + 2x + 4)^2 + 12x^2 + 12x(x^2 + 2x + 4) + 75}{(x^2 + 2x + 4)^{3/2} (405x^2 + 810x + 1620)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x + x^2 + 4)^(7/2),x)**[Out]** (51*x + 8*x*(2*x + x^2 + 4)^2 + 8*(2*x + x^2 + 4)^2 + 12*x^2 + 12*x*(2*x + x^2 + 4) + 75)/((2*x + x^2 + 4)^(3/2)*(810*x + 405*x^2 + 1620))

$$3.285 \quad \int \frac{1}{(1+8x+3x^2)^{5/2}} dx$$

Optimal. Leaf size=47

$$-\frac{4+3x}{39(1+8x+3x^2)^{3/2}} + \frac{2(4+3x)}{169\sqrt{1+8x+3x^2}}$$

[Out] 1/39*(-4-3*x)/(3*x^2+8*x+1)^(3/2)+2/169*(4+3*x)/(3*x^2+8*x+1)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {628, 627}

$$\frac{2(3x+4)}{169\sqrt{3x^2+8x+1}} - \frac{3x+4}{39(3x^2+8x+1)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 8*x + 3*x^2)^(-5/2), x]

[Out] -1/39*(4 + 3*x)/(1 + 8*x + 3*x^2)^(3/2) + (2*(4 + 3*x))/(169*sqrt[1 + 8*x + 3*x^2])

Rule 627

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] := Simp[-2*((b + 2*c*x)/((b^2 - 4*a*c)*sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p + 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+8x+3x^2)^{5/2}} dx &= -\frac{4+3x}{39(1+8x+3x^2)^{3/2}} - \frac{2}{13} \int \frac{1}{(1+8x+3x^2)^{3/2}} dx \\ &= -\frac{4+3x}{39(1+8x+3x^2)^{3/2}} + \frac{2(4+3x)}{169\sqrt{1+8x+3x^2}} \end{aligned}$$

Mathematica [A]

time = 0.14, size = 33, normalized size = 0.70

$$\frac{(4 + 3x)(-7 + 48x + 18x^2)}{507(1 + 8x + 3x^2)^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + 8*x + 3*x^2)^(-5/2), x]``[Out] ((4 + 3*x)*(-7 + 48*x + 18*x^2))/(507*(1 + 8*x + 3*x^2)^(3/2))`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(3*x^2 + 8*x + 1)^(5/2), x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [A]**

time = 0.07, size = 40, normalized size = 0.85

method	result	size
gosper	$\frac{54x^3 + 216x^2 + 171x - 28}{507(3x^2 + 8x + 1)^{3/2}}$	30
trager	$\frac{54x^3 + 216x^2 + 171x - 28}{507(3x^2 + 8x + 1)^{3/2}}$	30
risch	$\frac{54x^3 + 216x^2 + 171x - 28}{507(3x^2 + 8x + 1)^{3/2}}$	30
default	$-\frac{6x+8}{78(3x^2+8x+1)^{3/2}} + \frac{6x+8}{169\sqrt{3x^2+8x+1}}$	40

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(3*x^2+8*x+1)^(5/2), x, method=_RETURNVERBOSE)``[Out] -1/78*(6*x+8)/(3*x^2+8*x+1)^(3/2)+1/169*(6*x+8)/(3*x^2+8*x+1)^(1/2)`**Maxima [A]**

time = 0.27, size = 59, normalized size = 1.26

$$\frac{6x}{169\sqrt{3x^2+8x+1}} + \frac{8}{169\sqrt{3x^2+8x+1}} - \frac{x}{13(3x^2+8x+1)^{3/2}} - \frac{4}{39(3x^2+8x+1)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+8*x+1)^(5/2),x, algorithm="maxima")

[Out] 6/169*x/sqrt(3*x^2 + 8*x + 1) + 8/169/sqrt(3*x^2 + 8*x + 1) - 1/13*x/(3*x^2 + 8*x + 1)^(3/2) - 4/39/(3*x^2 + 8*x + 1)^(3/2)

Fricas [A]

time = 0.29, size = 73, normalized size = 1.55

$$\frac{252x^4 + 1344x^3 + 1960x^2 - (54x^3 + 216x^2 + 171x - 28)\sqrt{3x^2 + 8x + 1} + 448x + 28}{507(9x^4 + 48x^3 + 70x^2 + 16x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+8*x+1)^(5/2),x, algorithm="fricas")

[Out] -1/507*(252*x^4 + 1344*x^3 + 1960*x^2 - (54*x^3 + 216*x^2 + 171*x - 28)*sqrt(3*x^2 + 8*x + 1) + 448*x + 28)/(9*x^4 + 48*x^3 + 70*x^2 + 16*x + 1)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3x^2 + 8x + 1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x**2+8*x+1)**(5/2),x)

[Out] Integral((3*x**2 + 8*x + 1)**(-5/2), x)

Giac [A]

time = 0.01, size = 53, normalized size = 1.13

$$\frac{2 \left(\left(\frac{9}{169}x + \frac{36}{169} \right) x + \frac{57}{338} \right) x - \frac{14}{507} \sqrt{3x^2 + 8x + 1}}{(3x^2 + 8x + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+8*x+1)^(5/2),x)

[Out] 1/507*(9*(6*(x + 4)*x + 19)*x - 28)/(3*x^2 + 8*x + 1)^(3/2)

Mupad [B]

time = 0.05, size = 29, normalized size = 0.62

$$\frac{(12x + 16)(72x^2 + 192x - 28)}{8112(3x^2 + 8x + 1)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*x + 3*x^2 + 1)^(5/2),x)

[Out] ((12*x + 16)*(192*x + 72*x^2 - 28))/(8112*(8*x + 3*x^2 + 1)^(3/2))

$$3.286 \quad \int \frac{1}{(5+4x-3x^2)^{5/2}} dx$$

Optimal. Leaf size=47

$$-\frac{2-3x}{57(5+4x-3x^2)^{3/2}} - \frac{2(2-3x)}{361\sqrt{5+4x-3x^2}}$$

[Out] 1/57*(-2+3*x)/(-3*x^2+4*x+5)^(3/2)-2/361*(2-3*x)/(-3*x^2+4*x+5)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {628, 627}

$$-\frac{2(2-3x)}{361\sqrt{-3x^2+4x+5}} - \frac{2-3x}{57(-3x^2+4x+5)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(5 + 4*x - 3*x^2)^(-5/2), x]

[Out] -1/57*(2 - 3*x)/(5 + 4*x - 3*x^2)^(3/2) - (2*(2 - 3*x))/(361*Sqrt[5 + 4*x - 3*x^2])

Rule 627

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[-2*((b + 2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p + 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rubi steps

$$\begin{aligned} \int \frac{1}{(5+4x-3x^2)^{5/2}} dx &= -\frac{2-3x}{57(5+4x-3x^2)^{3/2}} + \frac{2}{19} \int \frac{1}{(5+4x-3x^2)^{3/2}} dx \\ &= -\frac{2-3x}{57(5+4x-3x^2)^{3/2}} - \frac{2(2-3x)}{361\sqrt{5+4x-3x^2}} \end{aligned}$$

Mathematica [A]

time = 0.15, size = 33, normalized size = 0.70

$$\frac{-98 + 99x + 108x^2 - 54x^3}{1083(5 + 4x - 3x^2)^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(5 + 4*x - 3*x^2)^(-5/2), x]``[Out] (-98 + 99*x + 108*x^2 - 54*x^3)/(1083*(5 + 4*x - 3*x^2)^(3/2))`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(5 + 4*x - 3*x^2)^(5/2), x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.12, size = 40, normalized size = 0.85

method	result	size
gospers	$-\frac{54x^3 - 108x^2 - 99x + 98}{1083(-3x^2 + 4x + 5)^{3/2}}$	30
default	$-\frac{-6x + 4}{114(-3x^2 + 4x + 5)^{3/2}} - \frac{-6x + 4}{361\sqrt{-3x^2 + 4x + 5}}$	40
trager	$-\frac{(54x^3 - 108x^2 - 99x + 98)\sqrt{-3x^2 + 4x + 5}}{1083(3x^2 - 4x - 5)^2}$	42
risch	$\frac{54x^3 - 108x^2 - 99x + 98}{1083(3x^2 - 4x - 5)\sqrt{-3x^2 + 4x + 5}}$	42

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(-3*x^2+4*x+5)^(5/2), x, method=_RETURNVERBOSE)``[Out] -1/114*(-6*x+4)/(-3*x^2+4*x+5)^(3/2)-1/361*(-6*x+4)/(-3*x^2+4*x+5)^(1/2)`**Maxima [A]**

time = 0.29, size = 59, normalized size = 1.26

$$\frac{6x}{361\sqrt{-3x^2 + 4x + 5}} - \frac{4}{361\sqrt{-3x^2 + 4x + 5}} + \frac{x}{19(-3x^2 + 4x + 5)^{3/2}} - \frac{2}{57(-3x^2 + 4x + 5)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+4*x+5)^(5/2),x, algorithm="maxima")

[Out] $\frac{6}{361}x/\sqrt{-3x^2 + 4x + 5} - \frac{4}{361}/\sqrt{-3x^2 + 4x + 5} + \frac{1}{19}x/(-3x^2 + 4x + 5)^{3/2} - \frac{2}{57}/(-3x^2 + 4x + 5)^{3/2}$

Fricas [A]

time = 0.30, size = 51, normalized size = 1.09

$$\frac{(54x^3 - 108x^2 - 99x + 98)\sqrt{-3x^2 + 4x + 5}}{1083(9x^4 - 24x^3 - 14x^2 + 40x + 25)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+4*x+5)^(5/2),x, algorithm="fricas")

[Out] $-\frac{1}{1083}(54x^3 - 108x^2 - 99x + 98)\sqrt{-3x^2 + 4x + 5}/(9x^4 - 24x^3 - 14x^2 + 40x + 25)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-3x^2 + 4x + 5)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x**2+4*x+5)**(5/2),x)

[Out] Integral((-3*x**2 + 4*x + 5)**(-5/2), x)

Giac [A]

time = 0.01, size = 55, normalized size = 1.17

$$\frac{2\left(\left(\left(-\frac{9}{361}x + \frac{18}{361}\right)x + \frac{33}{722}\right)x - \frac{49}{1083}\right)\sqrt{-3x^2 + 4x + 5}}{(-3x^2 + 4x + 5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+4*x+5)^(5/2),x)

[Out] $-\frac{1}{1083}(9(6(x - 2)x - 11)x + 98)\sqrt{-3x^2 + 4x + 5}/(3x^2 - 4x - 5)^2$

Mupad [B]

time = 0.20, size = 29, normalized size = 0.62

$$\frac{(12x - 8)(-72x^2 + 96x + 196)}{17328(-3x^2 + 4x + 5)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x - 3*x^2 + 5)^(5/2),x)

[Out] $((12x - 8)(96x - 72x^2 + 196))/(17328(4x - 3x^2 + 5)^{3/2})$

$$3.287 \quad \int \frac{1}{1 + \sqrt{2 + 2x + x^2}} dx$$

Optimal. Leaf size=29

$$\frac{1}{1+x} - \frac{\sqrt{2+2x+x^2}}{1+x} + \sinh^{-1}(1+x)$$

[Out] 1/(1+x)+arcsinh(1+x)-(x^2+2*x+2)^(1/2)/(1+x)

Rubi [A]

time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6874, 698, 633, 221}

$$-\frac{\sqrt{x^2+2x+2}}{x+1} + \frac{1}{x+1} + \sinh^{-1}(x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[2 + 2*x + x^2])^(-1), x]

[Out] (1 + x)^(-1) - Sqrt[2 + 2*x + x^2]/(1 + x) + ArcSinh[1 + x]

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 633

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*(-4*(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 698

Int[((d_) + (e_.)*(x_)^(m_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Dist[b*(p/(d*e*(m + 1))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] && !(IntegerQ[m/2] && LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]

Rule 6874

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{1 + \sqrt{2 + 2x + x^2}} dx &= \int \left(-\frac{1}{(1+x)^2} + \frac{\sqrt{2+2x+x^2}}{(1+x)^2} \right) dx \\
 &= \frac{1}{1+x} + \int \frac{\sqrt{2+2x+x^2}}{(1+x)^2} dx \\
 &= \frac{1}{1+x} - \frac{\sqrt{2+2x+x^2}}{1+x} + \int \frac{1}{\sqrt{2+2x+x^2}} dx \\
 &= \frac{1}{1+x} - \frac{\sqrt{2+2x+x^2}}{1+x} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1+\frac{x^2}{4}}} dx, x, 2+2x \right) \\
 &= \frac{1}{1+x} - \frac{\sqrt{2+2x+x^2}}{1+x} + \sinh^{-1}(1+x)
 \end{aligned}$$

Mathematica [A]

time = 0.22, size = 43, normalized size = 1.48

$$\frac{-1 + \sqrt{2 + 2x + x^2} + (1 + x) \log(-1 - x + \sqrt{2 + 2x + x^2})}{1 + x}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sqrt[2 + 2*x + x^2])^(-1), x]

[Out] -((-1 + Sqrt[2 + 2*x + x^2] + (1 + x)*Log[-1 - x + Sqrt[2 + 2*x + x^2]])/(1 + x))

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(1 + Sqrt[x^2 + 2*x + 2]), x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.05, size = 40, normalized size = 1.38

method	result	size
default	$-\frac{((1+x)^2+1)^{\frac{3}{2}}}{1+x} + (1+x)\sqrt{(1+x)^2+1} + \operatorname{arcsinh}(1+x) + \frac{1}{1+x}$	40
trager	$-\frac{x}{1+x} - \frac{\sqrt{x^2+2x+2}}{1+x} - \ln(\sqrt{x^2+2x+2} - 1 - x)$	45

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1+(x^2+2*x+2)^(1/2)),x,method=_RETURNVERBOSE)`

[Out] `-1/(1+x)*((1+x)^2+1)^(3/2)+(1+x)*((1+x)^2+1)^(1/2)+arcsinh(1+x)+1/(1+x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+(x^2+2*x+2)^(1/2)),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(x^2 + 2*x + 2) + 1), x)`

Fricas [A]

time = 0.30, size = 39, normalized size = 1.34

$$-\frac{(x+1)\log(-x+\sqrt{x^2+2x+2}-1)+x+\sqrt{x^2+2x+2}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+(x^2+2*x+2)^(1/2)),x, algorithm="fricas")`

[Out] `-((x+1)*log(-x+sqrt(x^2+2*x+2)-1)+x+sqrt(x^2+2*x+2))/(x+1)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2+2x+2}+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+(x**2+2*x+2)**(1/2)),x)`

[Out] `Integral(1/(sqrt(x**2 + 2*x + 2) + 1), x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.
time = 0.00, size = 66, normalized size = 2.28

$$\frac{1}{x+1} + 2 \left(\frac{1}{\left(\sqrt{x^2+2x+2}-x\right)^2 - 2\left(\sqrt{x^2+2x+2}-x\right)} - \frac{\ln\left(\sqrt{x^2+2x+2}-x-1\right)}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(x^2+2*x+2)^(1/2)),x)

[Out] 2/((x - sqrt(x^2 + 2*x + 2))^2 + 2*x - 2*sqrt(x^2 + 2*x + 2)) + 1/(x + 1) - log(-x + sqrt(x^2 + 2*x + 2) - 1)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\frac{1}{x+1} + \int \frac{\sqrt{x^2+2x+2}}{(x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2*x + x^2 + 2)^(1/2) + 1),x)

[Out] 1/(x + 1) + int((2*x + x^2 + 2)^(1/2)/(x + 1)^2, x)

$$3.288 \quad \int \frac{1}{x + \sqrt{1 + x + x^2}} dx$$

Optimal. Leaf size=45

$$-x + \sqrt{1 + x + x^2} - \frac{3}{2} \sinh^{-1} \left(\frac{1 + 2x}{\sqrt{3}} \right) + 2 \log \left(x + \sqrt{1 + x + x^2} \right)$$

[Out] $-x - 3/2 * \operatorname{arcsinh}(1/3 * (1 + 2 * x) * 3^{(1/2)}) + 2 * \ln(x + (x^2 + x + 1)^{(1/2)}) + (x^2 + x + 1)^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2141, 907}

$$\frac{3}{2 \left(2 \left(\sqrt{x^2 + x + 1} + x \right) + 1 \right)} + 2 \log \left(\sqrt{x^2 + x + 1} + x \right) - \frac{3}{2} \log \left(2 \left(\sqrt{x^2 + x + 1} + x \right) + 1 \right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x + \operatorname{Sqrt}[1 + x + x^2])^{-1}, x]$

[Out] $3/(2*(1 + 2*(x + \operatorname{Sqrt}[1 + x + x^2]))) + 2*\operatorname{Log}[x + \operatorname{Sqrt}[1 + x + x^2]] - (3*\operatorname{Log}[1 + 2*(x + \operatorname{Sqrt}[1 + x + x^2])])/2$

Rule 907

$\operatorname{Int}[(d + (e * x)^m * (f + (g * x)^n) * (a + (b * x) + (c * x)^2)^p], x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(d + e * x)^m * (f + g * x)^n * (a + b * x + c * x^2)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e * f - d * g, 0] && NeQ[b^2 - 4 * a * c, 0] && NeQ[c * d^2 - b * d * e + a * e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 2141

$\operatorname{Int}[(g + (h * (d + (e * x) + (f * \operatorname{Sqrt}[a + (b * x) + (c * x)^2])^n))^p], x_Symbol] \rightarrow \operatorname{Dist}[2, \operatorname{Subst}[\operatorname{Int}[(g + h * x^n)^p * (d^2 * e - (b * d - a * e) * f^2 - (2 * d * e - b * f^2) * x + e * x^2) / (-2 * d * e + b * f^2 + 2 * e * x)^2], x], x, d + e * x + f * \operatorname{Sqrt}[a + b * x + c * x^2], x] /;$ FreeQ[{a, b, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c * f^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{1}{x + \sqrt{1+x+x^2}} dx &= 2\text{Subst} \left(\int \frac{1+x+x^2}{x(1+2x)^2} dx, x, x + \sqrt{1+x+x^2} \right) \\ &= 2\text{Subst} \left(\int \left(\frac{1}{x} - \frac{3}{2(1+2x)^2} - \frac{3}{2(1+2x)} \right) dx, x, x + \sqrt{1+x+x^2} \right) \\ &= \frac{3}{2(1+2(x+\sqrt{1+x+x^2}))} + 2\log(x + \sqrt{1+x+x^2}) - \frac{3}{2}\log(1+2(x+\sqrt{1+x+x^2})) \end{aligned}$$

Mathematica [A]

time = 0.08, size = 54, normalized size = 1.20

$$-x + \sqrt{1+x+x^2} + 2\log(-2-x+\sqrt{1+x+x^2}) - \frac{1}{2}\log(-1-2x+2\sqrt{1+x+x^2})$$

Antiderivative was successfully verified.

`[In] Integrate[(x + Sqrt[1 + x + x^2])^(-1), x]``[Out] -x + Sqrt[1 + x + x^2] + 2*Log[-2 - x + Sqrt[1 + x + x^2]] - Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]]/2`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

caught exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x + Sqrt[1 + x + x^2]), x]')``[Out] caught exception: maximum recursion depth exceeded while calling a Python object`**Maple [A]**

time = 0.06, size = 52, normalized size = 1.16

method	result	size
default	$\sqrt{(1+x)^2 - x} - \frac{\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{2} - \operatorname{arctanh}\left(\frac{1-x}{2\sqrt{(1+x)^2 - x}}\right) - x + \ln(1+x)$	52
trager	$\sqrt{x^2 + x + 1} - x + \frac{\ln\left(2x^2\sqrt{x^2 + x + 1} - 2x^3 + 8x\sqrt{x^2 + x + 1} - 9x^2 + 14\sqrt{x^2 + x + 1} - 12x - 13\right)}{2}$	65

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x+(x^2+x+1)^(1/2)),x,method=_RETURNVERBOSE)`

[Out] $((1+x)^2-x)^{1/2}-1/2*\operatorname{arcsinh}(2/3*3^{1/2}*(x+1/2))-\operatorname{arctanh}(1/2*(1-x)/((1+x)^2-x)^{1/2})-x+\ln(1+x)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x+(x^2+x+1)^(1/2)),x, algorithm="maxima")`

[Out] `integrate(1/(x + sqrt(x^2 + x + 1)), x)`

Fricas [A]

time = 0.30, size = 63, normalized size = 1.40

$-x + \sqrt{x^2 + x + 1} + \log(x + 1) - \log(-x + \sqrt{x^2 + x + 1}) + \log(-x + \sqrt{x^2 + x + 1} - 2) + \frac{1}{2} \log(-2x + 2\sqrt{x^2 + x + 1} - 1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x+(x^2+x+1)^(1/2)),x, algorithm="fricas")`

[Out] $-x + \sqrt{x^2 + x + 1} + \log(x + 1) - \log(-x + \sqrt{x^2 + x + 1}) + \log(-x + \sqrt{x^2 + x + 1} - 2) + 1/2*\log(-2*x + 2*\sqrt{x^2 + x + 1} - 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x + \sqrt{x^2 + x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x+(x**2+x+1)**(1/2)),x)`

[Out] `Integral(1/(x + sqrt(x**2 + x + 1)), x)`

Giac [A]

time = 0.00, size = 80, normalized size = 1.78

$-x + \ln|x + 1| + \sqrt{x^2 + x + 1} + 2 \left(-\frac{\ln|\sqrt{x^2 + x + 1} - x|}{2} + \frac{\ln|\sqrt{x^2 + x + 1} - x - 2|}{2} + \frac{\ln(2(\sqrt{x^2 + x + 1} - x) - 1)}{4} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(x^2+x+1)^(1/2)),x)

[Out] -x + sqrt(x^2 + x + 1) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) + log(abs(x + 1)) - log(abs(-x + sqrt(x^2 + x + 1))) + log(abs(-x + sqrt(x^2 + x + 1) - 2))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\ln(x+1) - x + \int \frac{\sqrt{x^2+x+1}}{x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x + (x + x^2 + 1)^(1/2)),x)

[Out] log(x + 1) - x + int((x + x^2 + 1)^(1/2)/(x + 1), x)

$$3.289 \quad \int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}} dx$$

Optimal. Leaf size=79

$$-\frac{x^3}{9} - \frac{x^4}{6} + \frac{1}{96}(1+2x)\sqrt{1+x+x^2} - \frac{5}{36}(1+x+x^2)^{3/2} + \frac{1}{6}x(1+x+x^2)^{3/2} + \frac{1}{64}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)$$

[Out] $-1/9*x^3-1/6*x^4-5/36*(x^2+x+1)^{(3/2)}+1/6*x*(x^2+x+1)^{(3/2)}+1/64*\operatorname{arcsinh}(1/3*(1+2*x)*3^{(1/2)})+1/96*(1+2*x)*(x^2+x+1)^{(1/2)}$

Rubi [A]

time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6874, 756, 654, 626, 633, 221}

$$-\frac{x^4}{6} - \frac{x^3}{9} + \frac{1}{6}(x^2+x+1)^{3/2}x - \frac{5}{36}(x^2+x+1)^{3/2} + \frac{1}{96}(2x+1)\sqrt{x^2+x+1} + \frac{1}{64}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2/(1+2*x+2*\operatorname{Sqrt}[1+x+x^2]),x]$

[Out] $-1/9*x^3 - x^4/6 + ((1+2*x)*\operatorname{Sqrt}[1+x+x^2])/96 - (5*(1+x+x^2)^{(3/2)})/36 + (x*(1+x+x^2)^{(3/2)})/6 + \operatorname{ArcSinh}[(1+2*x)/\operatorname{Sqrt}[3]]/64$

Rule 221

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_)+(b_)*(x_)^2], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[\operatorname{Rt}[b, 2]*(x/\operatorname{Sqrt}[a])]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{GtQ}[a, 0] \ \&\& \operatorname{PosQ}[b]$

Rule 626

$\operatorname{Int}[(a_)+(b_)*(x_)+(c_)*(x_)^2]^{(p_)}, x_Symbol] \rightarrow \operatorname{Simp}[(b+2*c*x)*((a+b*x+c*x^2)^p/(2*c*(2*p+1))), x] - \operatorname{Dist}[p*((b^2-4*a*c)/(2*c*(2*p+1))), \operatorname{Int}[(a+b*x+c*x^2)^{(p-1)}, x], x] /; \operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{NeQ}[b^2-4*a*c, 0] \ \&\& \operatorname{GtQ}[p, 0] \ \&\& \operatorname{IntegerQ}[4*p]$

Rule 633

$\operatorname{Int}[(a_)+(b_)*(x_)+(c_)*(x_)^2]^{(p_)}, x_Symbol] \rightarrow \operatorname{Dist}[1/(2*c*(-4*(c/(b^2-4*a*c)))^p), \operatorname{Subst}[\operatorname{Int}[\operatorname{Simp}[1-x^2/(b^2-4*a*c), x]^p, x], x, b+2*c*x], x] /; \operatorname{FreeQ}\{a, b, c, p\}, x \ \&\& \operatorname{GtQ}[4*a-b^2/c, 0]$

Rule 654

$\operatorname{Int}[(d_)+(e_)*(x_))*((a_)+(b_)*(x_)+(c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \operatorname{Simp}[e*((a+b*x+c*x^2)^{(p+1)})/(2*c*(p+1)), x] + \operatorname{Dist}[(2*c*d-b$

*e)/(2*c), Int[(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 756

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 6874

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{1 + 2x + 2\sqrt{1 + x + x^2}} dx &= \int \left(-\frac{x^2}{3} - \frac{2x^3}{3} + \frac{2}{3}x^2\sqrt{1 + x + x^2} \right) dx \\
 &= -\frac{x^3}{9} - \frac{x^4}{6} + \frac{2}{3} \int x^2\sqrt{1 + x + x^2} dx \\
 &= -\frac{x^3}{9} - \frac{x^4}{6} + \frac{1}{6}x(1 + x + x^2)^{3/2} + \frac{1}{6} \int \left(-1 - \frac{5x}{2} \right) \sqrt{1 + x + x^2} dx \\
 &= -\frac{x^3}{9} - \frac{x^4}{6} - \frac{5}{36}(1 + x + x^2)^{3/2} + \frac{1}{6}x(1 + x + x^2)^{3/2} + \frac{1}{24} \int \sqrt{1 + x + x^2} dx \\
 &= -\frac{x^3}{9} - \frac{x^4}{6} + \frac{1}{96}(1 + 2x)\sqrt{1 + x + x^2} - \frac{5}{36}(1 + x + x^2)^{3/2} + \frac{1}{6}x(1 + x + x^2)^{3/2} \\
 &= -\frac{x^3}{9} - \frac{x^4}{6} + \frac{1}{96}(1 + 2x)\sqrt{1 + x + x^2} - \frac{5}{36}(1 + x + x^2)^{3/2} + \frac{1}{6}x(1 + x + x^2)^{3/2} \\
 &= -\frac{x^3}{9} - \frac{x^4}{6} + \frac{1}{96}(1 + 2x)\sqrt{1 + x + x^2} - \frac{5}{36}(1 + x + x^2)^{3/2} + \frac{1}{6}x(1 + x + x^2)^{3/2}
 \end{aligned}$$

Mathematica [A]

time = 0.13, size = 64, normalized size = 0.81

$$-\frac{1}{18}x^3(2+3x) + \frac{1}{288}\sqrt{1+x+x^2}(-37+14x+8x^2+48x^3) - \frac{1}{64}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(1+2*x+2*Sqrt[1+x+x^2]),x]

[Out] -1/18*(x^3*(2+3*x)) + (Sqrt[1+x+x^2]*(-37+14*x+8*x^2+48*x^3))/288 - Log[-1-2*x+2*Sqrt[1+x+x^2]]/64

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x^2/(2*x+1+2*Sqrt[1+x+x^2]),x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.04, size = 59, normalized size = 0.75

method	result	size
trager	$-\frac{(2+3x)x^3}{18} + \frac{(\frac{1}{2}x^3 + \frac{1}{12}x^2 + \frac{7}{48}x - \frac{37}{96})\sqrt{x^2+x+1}}{3} + \frac{\ln\left(1+2x+2\sqrt{x^2+x+1}\right)}{64}$	55
default	$-\frac{x^3}{9} - \frac{x^4}{6} + \frac{x(x^2+x+1)^{\frac{3}{2}}}{6} - \frac{5(x^2+x+1)^{\frac{3}{2}}}{36} + \frac{(1+2x)\sqrt{x^2+x+1}}{96} + \frac{\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{64}$	59

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x,method=_RETURNVERBOSE)

[Out] -1/9*x^3-1/6*x^4+1/6*x*(x^2+x+1)^(3/2)-5/36*(x^2+x+1)^(3/2)+1/96*(1+2*x)*(x^2+x+1)^(1/2)+1/64*arcsinh(2/3*3^(1/2)*(x+1/2))

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x, algorithm="maxima")

[Out] integrate(x^2/(2*x + 2*sqrt(x^2 + x + 1) + 1), x)

Fricas [A]

time = 0.30, size = 54, normalized size = 0.68

$$-\frac{1}{6}x^4 - \frac{1}{9}x^3 + \frac{1}{288}(48x^3 + 8x^2 + 14x - 37)\sqrt{x^2 + x + 1} - \frac{1}{64}\log(-2x + 2\sqrt{x^2 + x + 1} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x, algorithm="fricas")

[Out] -1/6*x^4 - 1/9*x^3 + 1/288*(48*x^3 + 8*x^2 + 14*x - 37)*sqrt(x^2 + x + 1) - 1/64*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{2x + 2\sqrt{x^2 + x + 1} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(1+2*x+2*(x**2+x+1)**(1/2)),x)

[Out] Integral(x**2/(2*x + 2*sqrt(x**2 + x + 1) + 1), x)

Giac [A]

time = 0.00, size = 76, normalized size = 0.96

$$-\frac{\frac{1}{2}x^4 - \frac{1}{3}x^3}{3} + 2\left(\left(\left(\frac{x}{12} + \frac{1}{72}\right)x + \frac{7}{288}\right)x - \frac{37}{576}\right)\sqrt{x^2 + x + 1} - \frac{\ln\left(2\left(\sqrt{x^2 + x + 1} - x\right) - 1\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x)

[Out] -1/6*x^4 - 1/9*x^3 + 1/288*(2*(4*(6*x + 1)*x + 7)*x - 37)*sqrt(x^2 + x + 1) - 1/64*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)

Mupad [B]

time = 0.07, size = 71, normalized size = 0.90

$$\frac{\ln\left(x + \sqrt{x^2 + x + 1} + \frac{1}{2}\right)}{64} - \frac{\left(\frac{x}{2} + \frac{1}{4}\right)\sqrt{x^2 + x + 1}}{6} - \frac{x^3}{9} - \frac{x^4}{6} - \frac{5(8x^2 + 2x + 5)\sqrt{x^2 + x + 1}}{288} + \frac{x(x^2 + x + 1)^{3/2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(2*x + 2*(x + x^2 + 1)^(1/2) + 1),x)

[Out] log(x + (x + x^2 + 1)^(1/2) + 1/2)/64 - ((x/2 + 1/4)*(x + x^2 + 1)^(1/2))/6 - x^3/9 - x^4/6 - (5*(2*x + 8*x^2 + 5)*(x + x^2 + 1)^(1/2))/288 + (x*(x + x^2 + 1)^(3/2))/6

$$3.290 \quad \int \frac{-3x + \sqrt{1+x+x^2}}{-1 + \sqrt{1+x+x^2}} dx$$

Optimal. Leaf size=80

$$x - 3\sqrt{1+x+x^2} + \frac{5}{2} \sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) + 4 \tanh^{-1}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right) - \tanh^{-1}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right) + \log(x) - 4$$

[Out] $x + 5/2 * \operatorname{arcsinh}(1/3 * (1 + 2 * x) * 3^{(1/2)}) + 4 * \operatorname{arctanh}(1/2 * (1 - x) / (x^2 + x + 1)^{(1/2)}) - \operatorname{arctanh}(1/2 * (2 + x) / (x^2 + x + 1)^{(1/2)}) + \ln(x) - 4 * \ln(1 + x) - 3 * (x^2 + x + 1)^{(1/2)}$

Rubi [A]

time = 0.22, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {6874, 748, 857, 633, 221, 738, 212, 6872}

$$-3\sqrt{x^2+x+1} + 4 \tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right) - \tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right) + x + \log(x) - 4 \log(x+1) + \frac{5}{2} \sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(-3 * x + \operatorname{Sqrt}[1 + x + x^2]) / (-1 + \operatorname{Sqrt}[1 + x + x^2]), x]$

[Out] $x - 3 * \operatorname{Sqrt}[1 + x + x^2] + (5 * \operatorname{ArcSinh}[(1 + 2 * x) / \operatorname{Sqrt}[3]]) / 2 + 4 * \operatorname{ArcTanh}[(1 - x) / (2 * \operatorname{Sqrt}[1 + x + x^2])] - \operatorname{ArcTanh}[(2 + x) / (2 * \operatorname{Sqrt}[1 + x + x^2])] + \operatorname{Log}[x] - 4 * \operatorname{Log}[1 + x]$

Rule 212

$\operatorname{Int}[(a_) + (b_.) * (x_)^2]^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1 / (\operatorname{Rt}[a, 2] * \operatorname{Rt}[-b, 2])) * \operatorname{ArcTanh}[\operatorname{Rt}[-b, 2] * (x / \operatorname{Rt}[a, 2])], x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 221

$\operatorname{Int}[1 / \operatorname{Sqrt}[(a_) + (b_.) * (x_)^2], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[\operatorname{Rt}[b, 2] * (x / \operatorname{Sqrt}[a])] / \operatorname{Rt}[b, 2], x] /;$ FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 633

$\operatorname{Int}[(a_.) + (b_.) * (x_) + (c_.) * (x_)^2]^{(p_)}, x_Symbol] \rightarrow \operatorname{Dist}[1 / (2 * c * (-4 * (c / (b^2 - 4 * a * c)))^p), \operatorname{Subst}[\operatorname{Int}[\operatorname{Simp}[1 - x^2 / (b^2 - 4 * a * c), x]^p, x], x, b + 2 * c * x], x] /;$ FreeQ[{a, b, c, p}, x] && GtQ[4 * a - b^2 / c, 0]

Rule 738

$\operatorname{Int}[1 / (((d_.) + (e_.) * (x_)) * \operatorname{Sqrt}[(a_.) + (b_.) * (x_) + (c_.) * (x_)^2]), x_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1 / (4 * c * d^2 - 4 * b * d * e + 4 * a * e^2 - x^2), x], x, (2$

$*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 748

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 857

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 6872

Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[PolynomialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && PolynomialInQ[v, u, x]

Rule 6874

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int \frac{-3x + \sqrt{1+x+x^2}}{-1 + \sqrt{1+x+x^2}} dx &= \int \left(-\frac{3x}{-1 + \sqrt{1+x+x^2}} + \frac{\sqrt{1+x+x^2}}{-1 + \sqrt{1+x+x^2}} \right) dx \\
&= - \left(3 \int \frac{x}{-1 + \sqrt{1+x+x^2}} dx \right) + \int \frac{\sqrt{1+x+x^2}}{-1 + \sqrt{1+x+x^2}} dx \\
&= - \left(3 \int \left(\frac{1}{1+x} + \frac{\sqrt{1+x+x^2}}{1+x} \right) dx \right) + \int \left(1 + \frac{1}{-1 + \sqrt{1+x+x^2}} \right) dx \\
&= x - 3 \log(1+x) - 3 \int \frac{\sqrt{1+x+x^2}}{1+x} dx + \int \frac{1}{-1 + \sqrt{1+x+x^2}} dx \\
&= x - 3\sqrt{1+x+x^2} - 3 \log(1+x) + \frac{3}{2} \int \frac{-1+x}{(1+x)\sqrt{1+x+x^2}} dx + \int \left(\frac{1}{-1 + \sqrt{1+x+x^2}} \right) dx \\
&= x - 3\sqrt{1+x+x^2} + \log(x) - 4 \log(1+x) + \frac{3}{2} \int \frac{1}{\sqrt{1+x+x^2}} dx - 3 \int \frac{1}{1+x} dx \\
&= x - 3\sqrt{1+x+x^2} + \log(x) - 4 \log(1+x) - \frac{1}{2} \int \frac{-2-x}{x\sqrt{1+x+x^2}} dx + \frac{1}{2} \int \frac{1}{\sqrt{1+x+x^2}} dx \\
&= x - 3\sqrt{1+x+x^2} + \frac{3}{2} \sinh^{-1} \left(\frac{1+2x}{\sqrt{3}} \right) + 3 \tanh^{-1} \left(\frac{1-x}{2\sqrt{1+x+x^2}} \right) + \log(x) \\
&= x - 3\sqrt{1+x+x^2} + \frac{3}{2} \sinh^{-1} \left(\frac{1+2x}{\sqrt{3}} \right) + 3 \tanh^{-1} \left(\frac{1-x}{2\sqrt{1+x+x^2}} \right) + \log(x) \\
&= x - 3\sqrt{1+x+x^2} + \frac{5}{2} \sinh^{-1} \left(\frac{1+2x}{\sqrt{3}} \right) + 4 \tanh^{-1} \left(\frac{1-x}{2\sqrt{1+x+x^2}} \right) - \tan^{-1} \left(\frac{1-x}{\sqrt{1+x+x^2}} \right)
\end{aligned}$$

Mathematica [A]

time = 0.15, size = 72, normalized size = 0.90

$$x - 3\sqrt{1+x+x^2} - 8 \log(-2-x + \sqrt{1+x+x^2}) + 2 \log(-1-x + \sqrt{1+x+x^2}) + \frac{1}{2} \log(-1-2x + 2\sqrt{1+x+x^2})$$

Antiderivative was successfully verified.

`[In] Integrate[(-3*x + Sqrt[1 + x + x^2])/(-1 + Sqrt[1 + x + x^2]),x]`

```
[Out] x - 3*Sqrt[1 + x + x^2] - 8*Log[-2 - x + Sqrt[1 + x + x^2]] + 2*Log[-1 - x + Sqrt[1 + x + x^2]] + Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]]/2
```


Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Sqrt[1 + x + x^2] - 3*x)/(Sqrt[1 + x + x^2] - 1),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python object
```

Maple [A]

time = 0.24, size = 80, normalized size = 1.00

method	result
default	$\ln(x) - 4 \ln(1+x) + x - 4\sqrt{(1+x)^2 - x} + \frac{5 \operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{2} + 4 \operatorname{arctanh}\left(\frac{1-x}{2\sqrt{(1+x)^2 - x}}\right)$
trager	$-1 + x - 3\sqrt{x^2 + x + 1} + \frac{\ln\left(\frac{-8-96x+1790544x^5+3865870x^6+458x^2\sqrt{x^2+x+1}+512x^{15}+8\sqrt{x^2+x+1}+445}{\dots}\right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-4*ln(1+x)+x-4*((1+x)^2-x)^(1/2)+5/2*arcsinh(2/3*3^(1/2)*(x+1/2))+4*arctanh(1/2*(1-x)/((1+x)^2-x)^(1/2))+(x^2+x+1)^(1/2)-arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 3/4*x^2 + 1/2*x + integrate(-1/2*(3*x^3 + 2*x^2 - x)/(x^2 + x - 2*sqrt(x^2 + x + 1) + 2), x)
```

Fricas [A]

time = 0.32, size = 99, normalized size = 1.24

$$x - 3\sqrt{x^2 + x + 1} - 4 \log(x+1) + \log(x) - \log(-x + \sqrt{x^2 + x + 1} + 1) + 4 \log(-x + \sqrt{x^2 + x + 1}) + \log(-x + \sqrt{x^2 + x + 1} - 1) - 4 \log(-x + \sqrt{x^2 + x + 1} - 2) - \frac{5}{2} \log(-2x + 2\sqrt{x^2 + x + 1} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x, algorithm="fricas")

[Out] x - 3*sqrt(x^2 + x + 1) - 4*log(x + 1) + log(x) - log(-x + sqrt(x^2 + x + 1) + 1) + 4*log(-x + sqrt(x^2 + x + 1)) + log(-x + sqrt(x^2 + x + 1) - 1) - 4*log(-x + sqrt(x^2 + x + 1) - 2) - 5/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{3x}{\sqrt{x^2+x+1}-1} dx - \int \left(-\frac{\sqrt{x^2+x+1}}{\sqrt{x^2+x+1}-1} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x+(x**2+x+1)**(1/2))/(-1+(x**2+x+1)**(1/2)),x)

[Out] -Integral(3*x/(sqrt(x**2 + x + 1) - 1), x) - Integral(-sqrt(x**2 + x + 1)/(sqrt(x**2 + x + 1) - 1), x)

Giac [A]

time = 0.02, size = 126, normalized size = 1.58

$$x + \ln|x| - 4\ln|x+1| - 3\sqrt{x^2+x+1} + 2 \left(2\ln|\sqrt{x^2+x+1}-x| - 2\ln|\sqrt{x^2+x+1}-x-2| + \frac{\ln|\sqrt{x^2+x+1}-x-1|}{2} - \frac{\ln|\sqrt{x^2+x+1}-x+1|}{2} - \frac{5}{4}\ln(2(\sqrt{x^2+x+1}-x)-1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x)

[Out] x - 3*sqrt(x^2 + x + 1) - 5/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) - 4*log(abs(x + 1)) + log(abs(x)) - log(abs(-x + sqrt(x^2 + x + 1) + 1)) + 4*log(abs(-x + sqrt(x^2 + x + 1))) + log(abs(-x + sqrt(x^2 + x + 1) - 1)) - 4*log(abs(-x + sqrt(x^2 + x + 1) - 2))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$x - 4 \ln(x + 1) + \ln(x) - \int \frac{(3x - 1) \sqrt{x^2 + x + 1}}{x(x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3*x - (x + x^2 + 1)^(1/2))/((x + x^2 + 1)^(1/2) - 1),x)

[Out] x - 4*log(x + 1) + log(x) - int(((3*x - 1)*(x + x^2 + 1)^(1/2))/(x*(x + 1)), x)

$$3.291 \quad \int \frac{1+x}{-\sqrt{1+x+x^2} + \sqrt{4+2x+x^2}} dx$$

Optimal. Leaf size=158

$$-2\sqrt{1+x+x^2} + \frac{1}{4}(1+2x)\sqrt{1+x+x^2} - 2\sqrt{4+2x+x^2} + \frac{1}{2}(1+x)\sqrt{4+2x+x^2} + \frac{11}{2} \sinh^{-1}\left(\frac{1+x}{\sqrt{3}}\right) + \dots$$

[Out] 11/2*arcsinh(1/3*(1+x)*3^(1/2))+43/8*arcsinh(1/3*(1+2*x)*3^(1/2))-2*arctanh(1/14*(1+5*x)*7^(1/2)/(x^2+x+1)^(1/2))*7^(1/2)+2*arctanh(1/7*(1-2*x)*7^(1/2)/(x^2+2*x+4)^(1/2))*7^(1/2)-2*(x^2+x+1)^(1/2)+1/4*(1+2*x)*(x^2+x+1)^(1/2)-2*(x^2+2*x+4)^(1/2)+1/2*(1+x)*(x^2+2*x+4)^(1/2)

Rubi [A]

time = 0.34, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {6874, 748, 857, 633, 221, 738, 212, 626}

$$\frac{1}{2}\sqrt{x^2+2x+4}(x+1) + \frac{1}{4}(2x+1)\sqrt{x^2+x+1} - 2\sqrt{x^2+x+1} - 2\sqrt{x^2+2x+4} - 2\sqrt{7} \tanh^{-1}\left(\frac{5x+1}{2\sqrt{7}\sqrt{x^2+x+1}}\right) + 2\sqrt{7} \tanh^{-1}\left(\frac{1-2x}{\sqrt{7}\sqrt{x^2+2x+4}}\right) + \frac{11}{2} \sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right) + \frac{43}{8} \sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/(-Sqrt[1 + x + x^2] + Sqrt[4 + 2*x + x^2]), x]

[Out] -2*Sqrt[1 + x + x^2] + ((1 + 2*x)*Sqrt[1 + x + x^2])/4 - 2*Sqrt[4 + 2*x + x^2] + ((1 + x)*Sqrt[4 + 2*x + x^2])/2 + (11*ArcSinh[(1 + x)/Sqrt[3]])/2 + (43*ArcSinh[(1 + 2*x)/Sqrt[3]])/8 - 2*Sqrt[7]*ArcTanh[(1 + 5*x)/(2*Sqrt[7]*Sqrt[1 + x + x^2])] + 2*Sqrt[7]*ArcTanh[(1 - 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 626

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(b + 2*c*x)*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] := Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
& !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x}{-\sqrt{1+x+x^2} + \sqrt{4+2x+x^2}} dx &= \int \left(-\frac{1}{\sqrt{1+x+x^2} - \sqrt{4+2x+x^2}} - \frac{x}{\sqrt{1+x+x^2} - \sqrt{4+2x+x^2}} \right) dx \\
&= -\int \frac{1}{\sqrt{1+x+x^2} - \sqrt{4+2x+x^2}} dx - \int \frac{x}{\sqrt{1+x+x^2} - \sqrt{4+2x+x^2}} dx \\
&= -\int \left(-\frac{\sqrt{1+x+x^2}}{3+x} - \frac{\sqrt{4+2x+x^2}}{3+x} \right) dx - \int \left(-\sqrt{1+x+x^2} - \sqrt{4+2x+x^2} \right) dx \\
&= -\left(3 \int \frac{\sqrt{1+x+x^2}}{3+x} dx \right) - 3 \int \frac{\sqrt{4+2x+x^2}}{3+x} dx + \int \sqrt{1+x+x^2} dx + \int \sqrt{4+2x+x^2} dx \\
&= -2\sqrt{1+x+x^2} + \frac{1}{4}(1+2x)\sqrt{1+x+x^2} - 2\sqrt{4+2x+x^2} + \frac{1}{4}(1+2x)\sqrt{4+2x+x^2} \\
&= -2\sqrt{1+x+x^2} + \frac{1}{4}(1+2x)\sqrt{1+x+x^2} - 2\sqrt{4+2x+x^2} + \frac{1}{4}(1+2x)\sqrt{4+2x+x^2} \\
&= -2\sqrt{1+x+x^2} + \frac{1}{4}(1+2x)\sqrt{1+x+x^2} - 2\sqrt{4+2x+x^2} + \frac{1}{4}(1+2x)\sqrt{4+2x+x^2}
\end{aligned}$$

Mathematica [A]

time = 5.22, size = 159, normalized size = 1.01

$$\frac{1}{8} \left(-14\sqrt{1+x+x^2} + 4x\sqrt{1+x+x^2} - 12\sqrt{4+2x+x^2} + 4x\sqrt{4+2x+x^2} - 32\sqrt{7} \tanh^{-1} \left(\frac{3+x-\sqrt{1+x+x^2}}{\sqrt{7}} \right) - 32\sqrt{7} \tanh^{-1} \left(\frac{3+x-\sqrt{4+2x+x^2}}{\sqrt{7}} \right) - 43 \log(-1-2x+2\sqrt{1+x+x^2}) - 44 \log(-1-x+\sqrt{4+2x+x^2}) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + x)/(-Sqrt[1 + x + x^2] + Sqrt[4 + 2*x + x^2]), x]`

```
[Out] (-14*Sqrt[1 + x + x^2] + 4*x*Sqrt[1 + x + x^2] - 12*Sqrt[4 + 2*x + x^2] + 4*x*Sqrt[4 + 2*x + x^2] - 32*Sqrt[7]*ArcTanh[(3 + x - Sqrt[1 + x + x^2])/Sqrt[7]] - 32*Sqrt[7]*ArcTanh[(3 + x - Sqrt[4 + 2*x + x^2])/Sqrt[7]] - 43*Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]] - 44*Log[-1 - x + Sqrt[4 + 2*x + x^2]])/8
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(x + 1)/(Sqrt[x^2 + 2*x + 4] - Sqrt[x^2 + x + 1]),x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [A]

time = 0.01, size = 140, normalized size = 0.89

method	result
default	$-2\sqrt{(3+x)^2 - 5x - 8} + \frac{43 \operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{8} + 2\sqrt{7} \operatorname{arctanh}\left(\frac{(-1-5x)\sqrt{7}}{14\sqrt{(3+x)^2 - 5x - 8}}\right) - 2$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x,method=_RETURNVERBOSE)

[Out] -2*((3+x)^2-5*x-8)^(1/2)+43/8*arcsinh(2/3*3^(1/2)*(x+1/2))+2*7^(1/2)*arctanh(1/14*(-1-5*x)*7^(1/2)/((3+x)^2-5*x-8)^(1/2))-2*((3+x)^2-4*x-5)^(1/2)+11/2*arcsinh(1/3*(1+x)*3^(1/2))+2*7^(1/2)*arctanh(1/14*(2-4*x)*7^(1/2)/((3+x)^2-4*x-5)^(1/2))+1/4*(1+2*x)*(x^2+x+1)^(1/2)+1/4*(2+2*x)*(x^2+2*x+4)^(1/2)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x, algorithm="maxima")

[Out] integrate((x + 1)/(sqrt(x^2 + 2*x + 4) - sqrt(x^2 + x + 1)), x)

Fricas [A]

time = 0.31, size = 155, normalized size = 0.98

$$\frac{1}{4}\sqrt{x^2+x+1}(2x-7)+\frac{1}{2}\sqrt{x^2+2x+4}(x-3)+2\sqrt{7}\log\left(\frac{2\sqrt{7}(5x+1)+2\sqrt{x^2+x+1}(5\sqrt{7}-14)-25x-5}{x+3}\right)+2\sqrt{7}\log\left(\frac{\sqrt{7}(2x-1)+\sqrt{x^2+2x+4}(2\sqrt{7}-7)-4x+2}{x+3}\right)-\frac{11}{2}\log(-x+\sqrt{x^2+2x+4})-\frac{43}{8}\log(-2x+2\sqrt{x^2+x+1}-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x, algorithm="fricas")

[Out] 1/4*sqrt(x^2 + x + 1)*(2*x - 7) + 1/2*sqrt(x^2 + 2*x + 4)*(x - 3) + 2*sqrt(7)*log((2*sqrt(7)*(5*x + 1) + 2*sqrt(x^2 + x + 1)*(5*sqrt(7) - 14) - 25*x - 5)/(x + 3)) + 2*sqrt(7)*log((sqrt(7)*(2*x - 1) + sqrt(x^2 + 2*x + 4)*(2*sqrt(7) - 7) - 4*x + 2)/(x + 3)) - 11/2*log(-x + sqrt(x^2 + 2*x + 4)) - 43/8*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + 1}{-\sqrt{x^2 + x + 1} + \sqrt{x^2 + 2x + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-(x**2+x+1)**(1/2)+(x**2+2*x+4)**(1/2)),x)

[Out] Integral((x + 1)/(-sqrt(x**2 + x + 1) + sqrt(x**2 + 2*x + 4)), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x + 1}{\sqrt{x^2 + x + 1} - \sqrt{x^2 + 2x + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x + 1)/((x + x^2 + 1)^(1/2) - (2*x + x^2 + 4)^(1/2)),x)

[Out] int(-(x + 1)/((x + x^2 + 1)^(1/2) - (2*x + x^2 + 4)^(1/2)), x)

$$3.292 \quad \int \frac{1}{\sqrt{-1+x} x^3} dx$$

Optimal. Leaf size=41

$$\frac{\sqrt{-1+x}}{2x^2} + \frac{3\sqrt{-1+x}}{4x} + \frac{3}{4} \tan^{-1}(\sqrt{-1+x})$$

[Out] 3/4*arctan((-1+x)^(1/2))+1/2*(-1+x)^(1/2)/x^2+3/4*(-1+x)^(1/2)/x

Rubi [A]

time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {44, 65, 209}

$$\frac{\sqrt{x-1}}{2x^2} + \frac{3\sqrt{x-1}}{4x} + \frac{3}{4} \tan^{-1}(\sqrt{x-1})$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-1 + x]*x^3),x]

[Out] Sqrt[-1 + x]/(2*x^2) + (3*Sqrt[-1 + x])/(4*x) + (3*ArcTan[Sqrt[-1 + x]])/4

Rule 44

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && LtQ[n, 0]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1+x} x^3} dx &= \frac{\sqrt{-1+x}}{2x^2} + \frac{3}{4} \int \frac{1}{\sqrt{-1+x} x^2} dx \\
&= \frac{\sqrt{-1+x}}{2x^2} + \frac{3\sqrt{-1+x}}{4x} + \frac{3}{8} \int \frac{1}{\sqrt{-1+x} x} dx \\
&= \frac{\sqrt{-1+x}}{2x^2} + \frac{3\sqrt{-1+x}}{4x} + \frac{3}{4} \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt{-1+x} \right) \\
&= \frac{\sqrt{-1+x}}{2x^2} + \frac{3\sqrt{-1+x}}{4x} + \frac{3}{4} \tan^{-1}(\sqrt{-1+x})
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 31, normalized size = 0.76

$$\frac{1}{4} \left(\frac{\sqrt{-1+x} (2+3x)}{x^2} + 3 \tan^{-1}(\sqrt{-1+x}) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[1/(Sqrt[-1 + x]*x^3), x]``[Out] ((Sqrt[-1 + x]*(2 + 3*x))/x^2 + 3*ArcTan[Sqrt[-1 + x]])/4`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 3.93, size = 120, normalized size = 2.93

$$\text{Piecewise} \left[\left\{ \left\{ \frac{I \left(2x(1-x) + x^2(1-x) + 3x^3(-1+x) + 3x^{\frac{3}{2}} \text{ArcCosh} \left[\frac{1}{\sqrt{x}} \right] \left(\frac{1-x}{x} \right)^{\frac{3}{2}} \right)}{4x^{\frac{9}{2}} \left(\frac{1-x}{x} \right)^{\frac{3}{2}}}, \frac{1}{\text{Abs}[x]} > 1 \right\} \right\}, -\frac{1}{2x^{\frac{5}{2}} \sqrt{1-\frac{1}{x}}} - \frac{1}{4x^{\frac{3}{2}} \sqrt{1-\frac{1}{x}}} + \frac{3}{4\sqrt{x} \sqrt{1-\frac{1}{x}}} - \frac{3 \text{ArcSin} \left[\frac{1}{\sqrt{x}} \right]}{4} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x^3*Sqrt[x - 1]), x]')`

```
[Out] Piecewise[{{I / 4 (2 x (1 - x) + x ^ 2 (1 - x) + 3 x ^ 3 (-1 + x) + 3 x ^ (
9 / 2) ArcCosh[1 / Sqrt[x]] ((1 - x) / x) ^ (3 / 2)) / (x ^ (9 / 2) ((1 - x)
) / x) ^ (3 / 2)), 1 / Abs[x] > 1}}, -1 / (2 x ^ (5 / 2) Sqrt[1 - 1 / x]) -
1 / (4 x ^ (3 / 2) Sqrt[1 - 1 / x]) + 3 / (4 Sqrt[x] Sqrt[1 - 1 / x]) - 3
ArcSin[1 / Sqrt[x]] / 4]
```

Maple [A]

time = 0.11, size = 30, normalized size = 0.73

method	result
--------	--------

derivativdivides	$\frac{3 \arctan(\sqrt{-1+x})}{4} + \frac{\sqrt{-1+x}}{2x^2} + \frac{3\sqrt{-1+x}}{4x}$
default	$\frac{3 \arctan(\sqrt{-1+x})}{4} + \frac{\sqrt{-1+x}}{2x^2} + \frac{3\sqrt{-1+x}}{4x}$
risch	$\frac{3x^2-x-2}{4x^2\sqrt{-1+x}} + \frac{3 \arctan(\sqrt{-1+x})}{4}$
trager	$\frac{(2+3x)\sqrt{-1+x}}{4x^2} + \frac{3 \operatorname{RootOf}(-Z^2+1) \ln\left(\frac{2 \operatorname{RootOf}(-Z^2+1)\sqrt{-1+x}+x-2}{x}\right)}{8}$
meijerg	$\frac{\sqrt{-\operatorname{signum}(-1+x)} \left(-\frac{\sqrt{\pi}}{2x^2} - \frac{\sqrt{\pi}}{2x} + \frac{3\left(\frac{7}{6}-2\ln(2)+\ln(x+i\pi)\right)\sqrt{\pi}}{8} + \frac{\sqrt{\pi}(-7x^2+8x+8)}{16x^2} - \frac{\sqrt{\pi}(12x+8)\sqrt{\pi}}{16x^2} \right)}{\sqrt{\pi} \sqrt{\operatorname{signum}(-1+x)}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(-1+x)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $3/4*\arctan((-1+x)^(1/2))+1/2*(-1+x)^(1/2)/x^2+3/4*(-1+x)^(1/2)/x$

Maxima [A]

time = 0.34, size = 38, normalized size = 0.93

$$\frac{3(x-1)^{\frac{3}{2}} + 5\sqrt{x-1}}{4((x-1)^2 + 2x-1)} + \frac{3}{4} \arctan(\sqrt{x-1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(-1+x)^(1/2),x, algorithm="maxima")`

[Out] $1/4*(3*(x-1)^(3/2) + 5*\sqrt{x-1})/((x-1)^2 + 2*x-1) + 3/4*\arctan(\sqrt{x-1})$

Fricas [A]

time = 0.30, size = 28, normalized size = 0.68

$$\frac{3x^2 \arctan(\sqrt{x-1}) + (3x+2)\sqrt{x-1}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(-1+x)^(1/2),x, algorithm="fricas")`

[Out] $1/4*(3*x^2*\arctan(\sqrt{x-1}) + (3*x+2)*\sqrt{x-1})/x^2$

Sympy [A]

time = 1.95, size = 131, normalized size = 3.20

$$\left\{ \begin{array}{l} \frac{3i \operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{4} - \frac{3i}{4\sqrt{x} \sqrt{-1 + \frac{1}{x}}} + \frac{i}{4x^{\frac{3}{2}} \sqrt{-1 + \frac{1}{x}}} + \frac{i}{2x^{\frac{5}{2}} \sqrt{-1 + \frac{1}{x}}} \quad \text{for } \frac{1}{|x|} > 1 \\ -\frac{3 \operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)}{4} + \frac{3}{4\sqrt{x} \sqrt{1 - \frac{1}{x}}} - \frac{1}{4x^{\frac{3}{2}} \sqrt{1 - \frac{1}{x}}} - \frac{1}{2x^{\frac{5}{2}} \sqrt{1 - \frac{1}{x}}} \quad \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(-1+x)**(1/2),x)

[Out] Piecewise((3*I*acosh(1/sqrt(x))/4 - 3*I/(4*sqrt(x)*sqrt(-1 + 1/x)) + I/(4*x**(3/2)*sqrt(-1 + 1/x)) + I/(2*x**(5/2)*sqrt(-1 + 1/x)), 1/Abs(x) > 1), (-3*asin(1/sqrt(x))/4 + 3/(4*sqrt(x)*sqrt(1 - 1/x)) - 1/(4*x**(3/2)*sqrt(1 - 1/x)) - 1/(2*x**(5/2)*sqrt(1 - 1/x)), True))

Giac [A]

time = 0.00, size = 44, normalized size = 1.07

$$2 \left(\frac{3\sqrt{x-1}(x-1) + 5\sqrt{x-1}}{8x^2} + \frac{3}{8} \arctan(\sqrt{x-1}) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(-1+x)^(1/2),x)

[Out] 1/4*(3*(x - 1)^(3/2) + 5*sqrt(x - 1))/x^2 + 3/4*arctan(sqrt(x - 1))

Mupad [B]

time = 0.04, size = 29, normalized size = 0.71

$$\frac{3 \operatorname{atan}(\sqrt{x-1})}{4} + \frac{3\sqrt{x-1}}{4x} + \frac{\sqrt{x-1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(x - 1)^(1/2)),x)

[Out] (3*atan((x - 1)^(1/2)))/4 + (3*(x - 1)^(1/2))/(4*x) + (x - 1)^(1/2)/(2*x^2)

$$3.293 \quad \int \frac{1}{\left(1 - \frac{3}{x}\right)^{4/3} x^2} dx$$

Optimal. Leaf size=13

$$-\frac{1}{\sqrt[3]{1 - \frac{3}{x}}}$$

[Out] -1/(1-3/x)^(1/3)

Rubi [A]

time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {267}

$$-\frac{1}{\sqrt[3]{1 - \frac{3}{x}}}$$

Antiderivative was successfully verified.

[In] Int[1/((1 - 3/x)^(4/3)*x^2), x]

[Out] -(1 - 3/x)^(-1/3)

Rule 267

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int \frac{1}{\left(1 - \frac{3}{x}\right)^{4/3} x^2} dx = -\frac{1}{\sqrt[3]{1 - \frac{3}{x}}}$$

Mathematica [A]

time = 0.02, size = 13, normalized size = 1.00

$$-\frac{1}{\sqrt[3]{\frac{-3 + x}{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 - 3/x)^(4/3)*x^2),x]

[Out] -((-3 + x)/x)^(-1/3)

Mathics [A]

time = 1.78, size = 11, normalized size = 0.85

$$-\frac{1}{\left(\frac{-3+x}{x}\right)^{\frac{1}{3}}}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x^2*(1 - 3/x)^(4/3)),x]')

[Out] -1 / ((-3 + x) / x) ^ (1 / 3)

Maple [A]

time = 0.02, size = 12, normalized size = 0.92

method	result	size
derivativdivides	$-\frac{1}{\left(1-\frac{3}{x}\right)^{\frac{1}{3}}}$	12
default	$-\frac{1}{\left(1-\frac{3}{x}\right)^{\frac{1}{3}}}$	12
risch	$-\frac{1}{\left(\frac{-3+x}{x}\right)^{\frac{1}{3}}}$	12
gospers	$-\frac{-3+x}{x\left(\frac{-3+x}{x}\right)^{\frac{4}{3}}}$	18
trager	$-\frac{x\left(\frac{-3-x}{x}\right)^{\frac{2}{3}}}{-3+x}$	21

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-3/x)^(4/3)/x^2,x,method=_RETURNVERBOSE)

[Out] -1/(1-3/x)^(1/3)

Maxima [A]

time = 0.27, size = 11, normalized size = 0.85

$$-\frac{1}{\left(-\frac{3}{x} + 1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-3/x)^(4/3)/x^2,x, algorithm="maxima")

[Out] -1/(-3/x + 1)^(1/3)

Fricas [A]

time = 0.29, size = 17, normalized size = 1.31

$$-\frac{x \left(\frac{x-3}{x}\right)^{\frac{2}{3}}}{x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-3/x)^(4/3)/x^2,x, algorithm="fricas")**[Out]** -x*((x - 3)/x)^(2/3)/(x - 3)**Sympy [A]**

time = 0.27, size = 10, normalized size = 0.77

$$-\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-3/x)**(4/3)/x**2,x)**[Out]** -1/(1 - 3/x)**(1/3)**Giac [A]**

time = 0.00, size = 12, normalized size = 0.92

$$-\frac{1}{\left(\frac{x-3}{x}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-3/x)^(4/3)/x^2,x)**[Out]** -1/((x - 3)/x)^(1/3)**Mupad [B]**

time = 0.46, size = 11, normalized size = 0.85

$$-\frac{1}{\left(1-\frac{3}{x}\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(1 - 3/x)^(4/3)),x)**[Out]** -1/(1 - 3/x)^(1/3)

$$3.294 \quad \int \frac{(-1+3x)^{4/3}}{x^2} dx$$

Optimal. Leaf size=71

$$12\sqrt[3]{-1+3x} - \frac{(-1+3x)^{4/3}}{x} + 4\sqrt{3} \tan^{-1} \left(\frac{1-2\sqrt[3]{-1+3x}}{\sqrt{3}} \right) + 2\log(x) - 6\log(1+\sqrt[3]{-1+3x})$$

[Out] 12*(-1+3*x)^(1/3)-(-1+3*x)^(4/3)/x+2*ln(x)-6*ln(1+(-1+3*x)^(1/3))+4*arctan(1/3*(1-2*(-1+3*x)^(1/3))*3^(1/2))*3^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {43, 52, 60, 632, 210, 31}

$$-\frac{(3x-1)^{4/3}}{x} + 12\sqrt[3]{3x-1} + 2\log(x) - 6\log(\sqrt[3]{3x-1} + 1) + 4\sqrt{3} \tan^{-1} \left(\frac{1-2\sqrt[3]{3x-1}}{\sqrt{3}} \right)$$

Antiderivative was successfully verified.

[In] Int[(-1 + 3*x)^(4/3)/x^2,x]

[Out] 12*(-1 + 3*x)^(1/3) - (-1 + 3*x)^(4/3)/x + 4*Sqrt[3]*ArcTan[(1 - 2*(-1 + 3*x)^(1/3))/Sqrt[3]] + 2*Log[x] - 6*Log[1 + (-1 + 3*x)^(1/3)]

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 43

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && GtQ[n, 0]

Rule 52

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)
]], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
]]) /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)
)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+3x)^{4/3}}{x^2} dx &= -\frac{(-1+3x)^{4/3}}{x} + 4 \int \frac{\sqrt[3]{-1+3x}}{x} dx \\
&= 12\sqrt[3]{-1+3x} - \frac{(-1+3x)^{4/3}}{x} - 4 \int \frac{1}{x(-1+3x)^{2/3}} dx \\
&= 12\sqrt[3]{-1+3x} - \frac{(-1+3x)^{4/3}}{x} + 2 \log(x) - 6 \operatorname{Subst} \left(\int \frac{1}{1+x} dx, x, \sqrt[3]{-1+3x} \right) - 6S \\
&= 12\sqrt[3]{-1+3x} - \frac{(-1+3x)^{4/3}}{x} + 2 \log(x) - 6 \log(1 + \sqrt[3]{-1+3x}) + 12 \operatorname{Subst} \left(\int \frac{1}{-3} \right) \\
&= 12\sqrt[3]{-1+3x} - \frac{(-1+3x)^{4/3}}{x} + 4\sqrt{3} \tan^{-1} \left(\frac{1 - 2\sqrt[3]{-1+3x}}{\sqrt{3}} \right) + 2 \log(x) - 6 \log(1
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 85, normalized size = 1.20

$$\frac{\sqrt[3]{-1+3x}(1+9x)}{x} + 4\sqrt{3} \tan^{-1} \left(\frac{1 - 2\sqrt[3]{-1+3x}}{\sqrt{3}} \right) - 4 \log(1 + \sqrt[3]{-1+3x}) + 2 \log(1 - \sqrt[3]{-1+3x}) + (-1+3x)^{2/3}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 3*x)^(4/3)/x^2, x]

[Out] $((-1 + 3x)^{1/3} * (1 + 9x)) / x + 4 * \text{Sqrt}[3] * \text{ArcTan}[(1 - 2 * (-1 + 3x)^{1/3}) / \text{Sqrt}[3]] - 4 * \text{Log}[1 + (-1 + 3x)^{1/3}] + 2 * \text{Log}[1 - (-1 + 3x)^{1/3} + (-1 + 3x)^{2/3}]$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 6.51, size = 81, normalized size = 1.14

$$\frac{(-1 + 3x)^{\frac{1}{3}}}{x} - 4 \text{Log} \left[1 - (-1 + 3x)^{\frac{1}{3}} \exp_{\text{polar}}[I \text{Pi}] \right] - \text{Log} \left[1 - (-1 + 3x)^{\frac{1}{3}} \exp_{\text{polar}} \left[\frac{I}{3} \text{Pi} \right] \right] - \text{Log} \left[1 - (-1 + 3x)^{\frac{1}{3}} \exp_{\text{polar}} \left[\frac{5I}{3} \text{Pi} \right] \right] + 9 (-1 + 3x)^{\frac{1}{3}}$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(3*x - 1)^(4/3)/x^2,x]')`

[Out] $(-1 + 3x)^{1/3} / x - 4 (-1)^{2/3} \text{Log}[1 - (-1 + 3x)^{1/3} \exp_{\text{polar}}[I / 3 \text{Pi}]] - 4 \text{Log}[1 - (-1 + 3x)^{1/3} \exp_{\text{polar}}[I \text{Pi}]] + 4 (-1)^{1/3} \text{Log}[1 - (-1 + 3x)^{1/3} \exp_{\text{polar}}[5 I / 3 \text{Pi}]] + 9 (-1 + 3x)^{1/3}$

Maple [A]

time = 0.42, size = 109, normalized size = 1.54

method	result
meijerg	$\frac{4 \text{signum}(x - \frac{1}{3})^{\frac{4}{3}} \left(\frac{3\Gamma(\frac{2}{3})}{4x} + 3 \left(2 + \frac{\pi\sqrt{3}}{6} - \frac{\ln(3)}{2} + \ln(x) + i\pi \right) \Gamma(\frac{2}{3}) - \frac{3\Gamma(\frac{2}{3})x \text{hypergeom}(\left[\frac{2}{3}, 1, 1\right], [2, 3], 3x)}{2} \right)}{3\Gamma(\frac{2}{3})(-\text{signum}(x - \frac{1}{3}))^{\frac{4}{3}}}$
derivativedivides	$9(3x - 1)^{\frac{1}{3}} + \frac{1 + (3x - 1)^{\frac{1}{3}}}{(3x - 1)^{\frac{2}{3}} - (3x - 1)^{\frac{1}{3}} + 1} + 2 \ln \left((3x - 1)^{\frac{2}{3}} - (3x - 1)^{\frac{1}{3}} + 1 \right) - 4\sqrt{3} \arctan \left(\dots \right)$
default	$9(3x - 1)^{\frac{1}{3}} + \frac{1 + (3x - 1)^{\frac{1}{3}}}{(3x - 1)^{\frac{2}{3}} - (3x - 1)^{\frac{1}{3}} + 1} + 2 \ln \left((3x - 1)^{\frac{2}{3}} - (3x - 1)^{\frac{1}{3}} + 1 \right) - 4\sqrt{3} \arctan \left(\dots \right)$
risch	$\frac{(3x - 1)^{\frac{1}{3}}}{x} + \frac{\left(\frac{4(3x - 1)^{\frac{2}{3}}(-\text{signum}(x - \frac{1}{3}))^{\frac{2}{3}} \left(\left(\frac{\pi\sqrt{3}}{6} - \frac{\ln(3)}{2} + \ln(x) + i\pi \right) \Gamma(\frac{2}{3}) + 2\Gamma(\frac{2}{3})x \text{hypergeom}(\left[1, 1, \frac{5}{3}\right], [2, 2], 3x) \right)}{(3x - 1)^2 \Gamma(\frac{2}{3}) \text{signum}(x - \frac{1}{3})^{\frac{2}{3}}} \right)}{(3x - 1)^{\frac{2}{3}}}$
trager	$\frac{(1 + 9x)(3x - 1)^{\frac{1}{3}}}{x} - 4 \ln \left(\frac{\text{RootOf}(_Z^2 - _Z + 1)^2 x + \text{RootOf}(_Z^2 - _Z + 1)(3x - 1)^{\frac{2}{3}} - \text{RootOf}(_Z^2 - _Z + 1)}{x} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x-1)^(4/3)/x^2,x,method=_RETURNVERBOSE)`

[Out] $9 * (3 * x - 1)^{1/3} + (1 + (3 * x - 1)^{1/3}) / ((3 * x - 1)^{2/3} - (3 * x - 1)^{1/3} + 1) + 2 * \ln((3 * x - 1)^{2/3} - (3 * x - 1)^{1/3} + 1) - 4 * 3^{1/2} * \arctan(1/3 * (2 * (3 * x - 1)^{1/3} - 1) * 3^{1/2}) - 1 / (1 + (3 * x - 1)^{1/3}) - 4 * \ln(1 + (3 * x - 1)^{1/3})$

Maxima [A]

time = 0.34, size = 76, normalized size = 1.07

$$-4\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}\left(2(3x-1)^{\frac{1}{3}}-1\right)\right) + 9(3x-1)^{\frac{1}{3}} + \frac{(3x-1)^{\frac{1}{3}}}{x} + 2\log\left((3x-1)^{\frac{2}{3}}-(3x-1)^{\frac{1}{3}}+1\right) - 4\log\left((3x-1)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+3*x)^(4/3)/x^2,x, algorithm="maxima")

[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(3*x - 1)^(1/3) - 1)) + 9*(3*x - 1)^(1/3) + (3*x - 1)^(1/3)/x + 2*log((3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) - 4*log((3*x - 1)^(1/3) + 1)

Fricas [A]

time = 0.32, size = 80, normalized size = 1.13

$$\frac{4\sqrt{3}x \arctan\left(\frac{2}{3}\sqrt{3}(3x-1)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) - 2x \log\left((3x-1)^{\frac{2}{3}} - (3x-1)^{\frac{1}{3}} + 1\right) + 4x \log\left((3x-1)^{\frac{1}{3}} + 1\right) - (9x+1)(3x-1)^{\frac{1}{3}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+3*x)^(4/3)/x^2,x, algorithm="fricas")

[Out] -(4*sqrt(3)*x*arctan(2/3*sqrt(3)*(3*x - 1)^(1/3) - 1/3*sqrt(3)) - 2*x*log((3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) + 4*x*log((3*x - 1)^(1/3) + 1) - (9*x + 1)*(3*x - 1)^(1/3))/x

Sympy [C] Result contains complex when optimal does not.

time = 1.27, size = 541, normalized size = 7.62

$$\frac{\frac{189\sqrt{3}\Gamma\left(\frac{7}{3}\right)\Gamma\left(\frac{10}{3}\right)}{9(x-1)^2\Gamma\left(\frac{7}{3}\right)\Gamma\left(\frac{10}{3}\right)} + \frac{84\sqrt{3}\sqrt{x-\frac{1}{3}}\Gamma\left(\frac{7}{3}\right)}{9(x-1)\Gamma\left(\frac{7}{3}\right)\Gamma\left(\frac{10}{3}\right)} + \frac{84(x-1)\log\left(-\sqrt{3}\sqrt{x-\frac{1}{3}}e^{2\pi i}\right)\Gamma\left(\frac{7}{3}\right)}{9(x-1)\Gamma\left(\frac{7}{3}\right)\Gamma\left(\frac{10}{3}\right)} - \frac{84(x-1)e^{2\pi i}\log\left(-\sqrt{3}\sqrt{x-\frac{1}{3}}e^{2\pi i}\right)\Gamma\left(\frac{7}{3}\right)}{9(x-1)\Gamma\left(\frac{7}{3}\right)\Gamma\left(\frac{10}{3}\right)} + \frac{84(x-1)e^{4\pi i}\log\left(-\sqrt{3}\sqrt{x-\frac{1}{3}}e^{4\pi i}\right)\Gamma\left(\frac{7}{3}\right)}{9(x-1)\Gamma\left(\frac{7}{3}\right)\Gamma\left(\frac{10}{3}\right)} - \frac{28\log\left(-\sqrt{3}\sqrt{x-\frac{1}{3}}e^{2\pi i}\right)\Gamma\left(\frac{7}{3}\right)}{9(x-1)\Gamma\left(\frac{7}{3}\right)\Gamma\left(\frac{10}{3}\right)} - \frac{28e^{2\pi i}\log\left(-\sqrt{3}\sqrt{x-\frac{1}{3}}e^{2\pi i}\right)\Gamma\left(\frac{7}{3}\right)}{9(x-1)\Gamma\left(\frac{7}{3}\right)\Gamma\left(\frac{10}{3}\right)} - \frac{28e^{4\pi i}\log\left(-\sqrt{3}\sqrt{x-\frac{1}{3}}e^{4\pi i}\right)\Gamma\left(\frac{7}{3}\right)}{9(x-1)\Gamma\left(\frac{7}{3}\right)\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+3*x)**(4/3)/x**2,x)

[Out] 189*3**(1/3)*(x - 1/3)**(4/3)*exp(I*pi/3)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*gamma(10/3)) + 84*3**(1/3)*(x - 1/3)**(1/3)*exp(I*pi/3)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*gamma(10/3)) + 84*(x - 1/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(I*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*gamma(10/3)) - 84*(x - 1/3)*exp(I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(I*pi) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*gamma(10/3)) + 84*(x - 1/3)*exp(2*I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*gamma(10/3)) + 28*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(I*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*gamma(10/3)) - 28*exp(I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(I*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*gamma(10/3)) - 28*exp(I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(I*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*gamma(10/3))

i) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*gamma(10/3)) + 28*exp(2*I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*gamma(10/3))

Giac [A]

time = 0.00, size = 100, normalized size = 1.41

$$9(3x-1)^{\frac{1}{3}} + \frac{\frac{1}{3} \cdot 3(3x-1)^{\frac{1}{3}}}{x} - 4 \ln \left| (3x-1)^{\frac{1}{3}} + 1 \right| + 2 \ln \left(\left((3x-1)^{\frac{1}{3}} \right)^2 - (3x-1)^{\frac{1}{3}} + 1 \right) - \frac{12 \arctan \left(\frac{2(3x-1)^{\frac{1}{3}} - 1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+3*x)^(4/3)/x^2,x)

[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(3*x - 1)^(1/3) - 1)) + 9*(3*x - 1)^(1/3) + (3*x - 1)^(1/3)/x + 2*log((3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) - 4*log(abs((3*x - 1)^(1/3) + 1))

Mupad [B]

time = 0.21, size = 90, normalized size = 1.27

$$9(3x-1)^{1/3} - 4 \ln \left(144(3x-1)^{1/3} + 144 \right) + \frac{(3x-1)^{1/3}}{x} + \ln \left(18 - 36(3x-1)^{1/3} + \sqrt{3} 18i \right) \left(2 + \sqrt{3} 2i \right) - \ln \left(36(3x-1)^{1/3} - 18 + \sqrt{3} 18i \right) \left(-2 + \sqrt{3} 2i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x - 1)^(4/3)/x^2,x)

[Out] 9*(3*x - 1)^(1/3) - 4*log(144*(3*x - 1)^(1/3) + 144) + (3*x - 1)^(1/3)/x + log(3^(1/2)*18i - 36*(3*x - 1)^(1/3) + 18)*(3^(1/2)*2i + 2) - log(3^(1/2)*18i + 36*(3*x - 1)^(1/3) - 18)*(3^(1/2)*2i - 2)

3.295 $\int (4 - 3x)^{4/3} x^2 dx$

Optimal. Leaf size=40

$$-\frac{16}{63}(4-3x)^{7/3} + \frac{4}{45}(4-3x)^{10/3} - \frac{1}{117}(4-3x)^{13/3}$$

[Out] $-16/63*(4-3*x)^{(7/3)}+4/45*(4-3*x)^{(10/3)}-1/117*(4-3*x)^{(13/3)}$

Rubi [A]

time = 0.00, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$-\frac{1}{117}(4-3x)^{13/3} + \frac{4}{45}(4-3x)^{10/3} - \frac{16}{63}(4-3x)^{7/3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(4 - 3*x)^{(4/3)}*x^2, x]$

[Out] $(-16*(4 - 3*x)^{(7/3)})/63 + (4*(4 - 3*x)^{(10/3)})/45 - (4 - 3*x)^{(13/3)}/117$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int (4 - 3x)^{4/3} x^2 dx &= \int \left(\frac{16}{9}(4 - 3x)^{4/3} - \frac{8}{9}(4 - 3x)^{7/3} + \frac{1}{9}(4 - 3x)^{10/3} \right) dx \\ &= -\frac{16}{63}(4 - 3x)^{7/3} + \frac{4}{45}(4 - 3x)^{10/3} - \frac{1}{117}(4 - 3x)^{13/3} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 23, normalized size = 0.58

$$-\frac{1}{455}(4-3x)^{7/3}(16+28x+35x^2)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(4 - 3*x)^{(4/3)}*x^2, x]$

[Out] $-1/455*((4 - 3*x)^{(7/3)}*(16 + 28*x + 35*x^2))$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.

time = 2.93, size = 95, normalized size = 2.38

Piecewise $\left\{ \left\{ -\frac{1^{\frac{1}{3}}(-256 - 64x - 32x^2 + 588x^3 - 315x^4)(-4 + 3x)^{\frac{1}{3}}}{455}, \text{Abs}[x] > \frac{4}{3} \right\}, \left[-\frac{64x(4 - 3x)^{\frac{1}{3}}}{455} - \frac{32x^2(4 - 3x)^{\frac{1}{3}}}{455} + \frac{84x^3(4 - 3x)^{\frac{1}{3}}}{65} - \frac{9x^4(4 - 3x)^{\frac{1}{3}}}{13} - \frac{256(4 - 3x)^{\frac{1}{3}}}{455} \right] \right\}$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(4 - 3*x)^(4/3)*x^2,x]')`

[Out] `Piecewise[{{-1^(1/3)(-256 - 64x - 32x^2 + 588x^3 - 315x^4)(-4 + 3x)^(1/3)/455, Abs[x] > 4/3}}, -64x(4 - 3x)^(1/3)/455 - 32x^2(4 - 3x)^(1/3)/455 + 84x^3(4 - 3x)^(1/3)/65 - 9x^4(4 - 3x)^(1/3)/13 - 256(4 - 3x)^(1/3)/455]`

Maple [A]

time = 0.07, size = 29, normalized size = 0.72

method	result	size
meijerg	$\frac{4 \cdot 2^{\frac{2}{3}} x^3 \text{hypergeom}\left(\left[-\frac{4}{3}, 3\right], \left[4, \frac{3x}{4}\right]\right)}{3}$	18
gosper	$-\frac{(35x^2 + 28x + 16)(4 - 3x)^{\frac{7}{3}}}{455}$	20
derivativedivides	$-\frac{16(4 - 3x)^{\frac{7}{3}}}{63} + \frac{4(4 - 3x)^{\frac{10}{3}}}{45} - \frac{(4 - 3x)^{\frac{13}{3}}}{117}$	29
default	$-\frac{16(4 - 3x)^{\frac{7}{3}}}{63} + \frac{4(4 - 3x)^{\frac{10}{3}}}{45} - \frac{(4 - 3x)^{\frac{13}{3}}}{117}$	29
trager	$\left(-\frac{9}{13}x^4 + \frac{84}{65}x^3 - \frac{32}{455}x^2 - \frac{64}{455}x - \frac{256}{455}\right)(4 - 3x)^{\frac{1}{3}}$	29
risch	$\frac{(315x^4 - 588x^3 + 32x^2 + 64x + 256)(-4 + 3x)}{455(4 - 3x)^{\frac{2}{3}}}$	35

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4-3*x)^(4/3)*x^2,x,method=_RETURNVERBOSE)`

[Out] $-16/63*(4-3*x)^{(7/3)}+4/45*(4-3*x)^{(10/3)}-1/117*(4-3*x)^{(13/3)}$

Maxima [A]

time = 0.26, size = 28, normalized size = 0.70

$$-\frac{1}{117}(-3x + 4)^{\frac{13}{3}} + \frac{4}{45}(-3x + 4)^{\frac{10}{3}} - \frac{16}{63}(-3x + 4)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4-3*x)^(4/3)*x^2,x, algorithm="maxima")`

[Out] $-1/117*(-3*x + 4)^{(13/3)} + 4/45*(-3*x + 4)^{(10/3)} - 16/63*(-3*x + 4)^{(7/3)}$

Fricas [A]

time = 0.30, size = 29, normalized size = 0.72

$$-\frac{1}{455} (315x^4 - 588x^3 + 32x^2 + 64x + 256)(-3x + 4)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4-3*x)^(4/3)*x^2,x, algorithm="fricas")`

[Out] $-1/455*(315*x^4 - 588*x^3 + 32*x^2 + 64*x + 256)*(-3*x + 4)^{(1/3)}$

Sympy [C] Result contains complex when optimal does not.

time = 0.84, size = 178, normalized size = 4.45

$$\begin{cases} -\frac{9x^4\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{13} + \frac{84x^3\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{65} - \frac{32x^2\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{455} - \frac{64x\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{455} - \frac{256\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{455} & \text{for } |x| > \frac{4}{3} \\ -\frac{9x^4\sqrt[3]{4-3x}}{13} + \frac{84x^3\sqrt[3]{4-3x}}{65} - \frac{32x^2\sqrt[3]{4-3x}}{455} - \frac{64x\sqrt[3]{4-3x}}{455} - \frac{256\sqrt[3]{4-3x}}{455} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4-3*x)**(4/3)*x**2,x)`

[Out] `Piecewise((-9*x**4*(3*x - 4)**(1/3)*exp(I*pi/3)/13 + 84*x**3*(3*x - 4)**(1/3)*exp(I*pi/3)/65 - 32*x**2*(3*x - 4)**(1/3)*exp(I*pi/3)/455 - 64*x*(3*x - 4)**(1/3)*exp(I*pi/3)/455 - 256*(3*x - 4)**(1/3)*exp(I*pi/3)/455, Abs(x) > 4/3), (-9*x**4*(4 - 3*x)**(1/3)/13 + 84*x**3*(4 - 3*x)**(1/3)/65 - 32*x**2*(4 - 3*x)**(1/3)/455 - 64*x*(4 - 3*x)**(1/3)/455 - 256*(4 - 3*x)**(1/3)/455, True))`

Giac [A]

time = 0.00, size = 242, normalized size = 6.05

$$\frac{9}{81} \left(\frac{1}{15} (-3x+4)^4 (-3x+4)^4 - \frac{8}{5} (-3x+4)^4 (-3x+4)^3 + \frac{96}{7} (-3x+4)^4 (-3x+4)^2 - 64 (-3x+4)^4 (-3x+4) + 256 (-3x+4)^4 \right) + \frac{24}{27} \left(\frac{1}{15} (-3x+4)^4 (-3x+4)^3 + \frac{12}{7} (-3x+4)^4 (-3x+4)^2 - 12 (-3x+4)^4 (-3x+4) + 64 (-3x+4)^4 \right) - \frac{16}{9} \left(\frac{1}{7} (-3x+4)^4 (-3x+4)^2 - 2 (-3x+4)^4 (-3x+4) + 16 (-3x+4)^4 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4-3*x)^(4/3)*x^2,x)`

[Out] $-1/117*(3*x - 4)^4*(-3*x + 4)^{(1/3)} - 4/45*(3*x - 4)^3*(-3*x + 4)^{(1/3)} - 16/63*(3*x - 4)^2*(-3*x + 4)^{(1/3)}$

Mupad [B]

time = 0.20, size = 23, normalized size = 0.58

$$-\frac{(4-3x)^{7/3} (1092x + 35(3x-4)^2 - 416)}{4095}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(4 - 3*x)^(4/3),x)`

[Out] $-((4 - 3*x)^{(7/3})*(1092*x + 35*(3*x - 4)^2 - 416))/4095$

$$3.296 \quad \int \frac{(1-2\sqrt[3]{x})^{3/4}}{x} dx$$

Optimal. Leaf size=48

$$4(1-2\sqrt[3]{x})^{3/4} + 6 \tan^{-1} \left(\sqrt[4]{1-2\sqrt[3]{x}} \right) - 6 \tanh^{-1} \left(\sqrt[4]{1-2\sqrt[3]{x}} \right)$$

[Out] 4*(1-2*x^(1/3))^(3/4)+6*arctan((1-2*x^(1/3))^(1/4))-6*arctanh((1-2*x^(1/3))^(1/4))

Rubi [A]

time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {272, 52, 65, 304, 209, 212}

$$4(1-2\sqrt[3]{x})^{3/4} + 6 \tan^{-1} \left(\sqrt[4]{1-2\sqrt[3]{x}} \right) - 6 \tanh^{-1} \left(\sqrt[4]{1-2\sqrt[3]{x}} \right)$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*x^(1/3))^(3/4)/x,x]

[Out] 4*(1 - 2*x^(1/3))^(3/4) + 6*ArcTan[(1 - 2*x^(1/3))^(1/4)] - 6*ArcTanh[(1 - 2*x^(1/3))^(1/4)]

Rule 52

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 209

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 304

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(1 - 2\sqrt[3]{x})^{3/4}}{x} dx &= 3 \text{Subst} \left(\int \frac{(1 - 2x)^{3/4}}{x} dx, x, \sqrt[3]{x} \right) \\
 &= 4(1 - 2\sqrt[3]{x})^{3/4} + 3 \text{Subst} \left(\int \frac{1}{\sqrt[4]{1 - 2x} x} dx, x, \sqrt[3]{x} \right) \\
 &= 4(1 - 2\sqrt[3]{x})^{3/4} - 6 \text{Subst} \left(\int \frac{x^2}{\frac{1}{2} - \frac{x^4}{2}} dx, x, \sqrt[4]{1 - 2\sqrt[3]{x}} \right) \\
 &= 4(1 - 2\sqrt[3]{x})^{3/4} - 6 \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \sqrt[4]{1 - 2\sqrt[3]{x}} \right) + 6 \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \sqrt[4]{1 - 2\sqrt[3]{x}} \right) \\
 &= 4(1 - 2\sqrt[3]{x})^{3/4} + 6 \tan^{-1} \left(\sqrt[4]{1 - 2\sqrt[3]{x}} \right) - 6 \tanh^{-1} \left(\sqrt[4]{1 - 2\sqrt[3]{x}} \right)
 \end{aligned}$$

Mathematica [A]

time = 0.03, size = 48, normalized size = 1.00

$$4(1 - 2\sqrt[3]{x})^{3/4} + 6 \tan^{-1} \left(\sqrt[4]{1 - 2\sqrt[3]{x}} \right) - 6 \tanh^{-1} \left(\sqrt[4]{1 - 2\sqrt[3]{x}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*x^(1/3))^(3/4)/x,x]

[Out] 4*(1 - 2*x^(1/3))^(3/4) + 6*ArcTan[(1 - 2*x^(1/3))^(1/4)] - 6*ArcTanh[(1 - 2*x^(1/3))^(1/4)]

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.80, size = 25, normalized size = 0.52

$$-3 - \frac{2^{\frac{3}{4}} x^{\frac{1}{4}} \text{Gamma}\left[-\frac{3}{4}\right] \text{hyper}\left[\left\{-\frac{3}{4}, -\frac{3}{4}\right\}, \left\{\frac{1}{4}\right\}, \frac{1}{2x^{\frac{1}{3}}}\right]}{\text{Gamma}\left[\frac{1}{4}\right]}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(1 - 2*x^(1/3))^(3/4)/x,x]')

[Out] -3 - 2^(3/4) x^(1/4) Gamma[-3/4] hyper[{-3/4, -3/4}, {1/4}, 1/(2 x^(1/3))] / Gamma[1/4]

Maple [A]

time = 0.07, size = 53, normalized size = 1.10

method	result
derivativedivides	$4 \left(1 - 2x^{\frac{1}{3}}\right)^{\frac{3}{4}} + 3 \ln \left(\left(1 - 2x^{\frac{1}{3}}\right)^{\frac{1}{4}} - 1\right) - 3 \ln \left(\left(1 - 2x^{\frac{1}{3}}\right)^{\frac{1}{4}} + 1\right) + 6 \arctan \left(\left(1 - 2x^{\frac{1}{3}}\right)^{\frac{1}{4}}\right)$
default	$4 \left(1 - 2x^{\frac{1}{3}}\right)^{\frac{3}{4}} + 3 \ln \left(\left(1 - 2x^{\frac{1}{3}}\right)^{\frac{1}{4}} - 1\right) - 3 \ln \left(\left(1 - 2x^{\frac{1}{3}}\right)^{\frac{1}{4}} + 1\right) + 6 \arctan \left(\left(1 - 2x^{\frac{1}{3}}\right)^{\frac{1}{4}}\right)$
meijerg	$-\frac{9\sqrt{2} \Gamma\left(\frac{3}{4}\right) \left(-\frac{4\left(\frac{4}{3} - 2 \ln(2) - \frac{\pi}{2} + \frac{\ln(x)}{3} + i\pi\right) \pi \sqrt{2}}{3\Gamma\left(\frac{3}{4}\right)} + \frac{2\pi\sqrt{2} x^{\frac{1}{3}} \text{hypergeom}\left(\left[\frac{1}{4}, 1, 1\right], [2, 2], 2x^{\frac{1}{3}}\right)}{\Gamma\left(\frac{3}{4}\right)}\right)}{8\pi}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-2*x^(1/3))^(3/4)/x,x,method=_RETURNVERBOSE)

[Out] 4*(1-2*x^(1/3))^(3/4)+3*ln((1-2*x^(1/3))^(1/4)-1)-3*ln((1-2*x^(1/3))^(1/4)+1)+6*arctan((1-2*x^(1/3))^(1/4))

Maxima [A]

time = 0.34, size = 52, normalized size = 1.08

$$4 \left(-2x^{\frac{1}{3}} + 1\right)^{\frac{3}{4}} + 6 \arctan \left(\left(-2x^{\frac{1}{3}} + 1\right)^{\frac{1}{4}}\right) - 3 \log \left(\left(-2x^{\frac{1}{3}} + 1\right)^{\frac{1}{4}} + 1\right) + 3 \log \left(\left(-2x^{\frac{1}{3}} + 1\right)^{\frac{1}{4}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-2*x^(1/3))^(3/4)/x,x, algorithm="maxima")

[Out] $4*(-2*x^{1/3} + 1)^{3/4} + 6*\arctan((-2*x^{1/3} + 1)^{1/4}) - 3*\log((-2*x^{1/3} + 1)^{1/4} + 1) + 3*\log((-2*x^{1/3} + 1)^{1/4} - 1)$

Fricas [A]

time = 0.31, size = 52, normalized size = 1.08

$$4\left(-2x^{\frac{1}{3}} + 1\right)^{\frac{3}{4}} + 6 \arctan\left(\left(-2x^{\frac{1}{3}} + 1\right)^{\frac{1}{4}}\right) - 3 \log\left(\left(-2x^{\frac{1}{3}} + 1\right)^{\frac{1}{4}} + 1\right) + 3 \log\left(\left(-2x^{\frac{1}{3}} + 1\right)^{\frac{1}{4}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-2*x^(1/3))^(3/4)/x,x, algorithm="fricas")`

[Out] $4*(-2*x^{1/3} + 1)^{3/4} + 6*\arctan((-2*x^{1/3} + 1)^{1/4}) - 3*\log((-2*x^{1/3} + 1)^{1/4} + 1) + 3*\log((-2*x^{1/3} + 1)^{1/4} - 1)$

Sympy [C] Result contains complex when optimal does not.

time = 1.14, size = 51, normalized size = 1.06

$$\frac{3 \cdot 2^{\frac{3}{4}} \sqrt[4]{x} e^{\frac{3i\pi}{4}} \Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, -\frac{3}{4} \\ \frac{1}{4} \end{matrix} \middle| \frac{1}{2\sqrt[3]{x}}\right)}{\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-2*x**(1/3))**(3/4)/x,x)`

[Out] $-3*2^{3/4}*x^{1/4}*\exp(3*I*\pi/4)*\gamma(-3/4)*\text{hyper}((-3/4, -3/4), (1/4,), 1/(2*x^{1/3}))/\gamma(1/4)$

Giac [A]

time = 0.01, size = 77, normalized size = 1.60

$$3 \ln\left|\left(-2x^{\frac{1}{3}} + 1\right)^{\frac{1}{4}} - 1\right| - 3 \ln\left|\left(-2x^{\frac{1}{3}} + 1\right)^{\frac{1}{4}} + 1\right| + 6 \arctan\left(\left(-2x^{\frac{1}{3}} + 1\right)^{\frac{1}{4}}\right) + 4\left(\left(-2x^{\frac{1}{3}} + 1\right)^{\frac{1}{4}}\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-2*x^(1/3))^(3/4)/x,x)`

[Out] $4*(-2*x^{1/3} + 1)^{3/4} + 6*\arctan((-2*x^{1/3} + 1)^{1/4}) - 3*\log(\text{abs}((-2*x^{1/3} + 1)^{1/4} + 1)) + 3*\log(\text{abs}((-2*x^{1/3} + 1)^{1/4} - 1))$

Mupad [B]

time = 0.87, size = 36, normalized size = 0.75

$$6 \operatorname{atan}\left(\left(1 - 2x^{1/3}\right)^{1/4}\right) - 6 \operatorname{atanh}\left(\left(1 - 2x^{1/3}\right)^{1/4}\right) + 4\left(1 - 2x^{1/3}\right)^{3/4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1 - 2*x^(1/3))^(3/4)/x,x)`

[Out] $6*\operatorname{atan}((1 - 2*x^{1/3})^{1/4}) - 6*\operatorname{atanh}((1 - 2*x^{1/3})^{1/4}) + 4*(1 - 2*x^{1/3})^{3/4}$

$$3.297 \quad \int \frac{x}{(3-2\sqrt{x})^{3/4}} dx$$

Optimal. Leaf size=69

$$-\frac{27}{2} \sqrt[4]{3-2\sqrt{x}} + \frac{27}{10} (3-2\sqrt{x})^{5/4} - \frac{1}{2} (3-2\sqrt{x})^{9/4} + \frac{1}{26} (3-2\sqrt{x})^{13/4}$$

[Out] $-27/2*(3-2*x^{(1/2)})^{(1/4)}+27/10*(3-2*x^{(1/2)})^{(5/4)}-1/2*(3-2*x^{(1/2)})^{(9/4)}+1/26*(3-2*x^{(1/2)})^{(13/4)}$

Rubi [A]

time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$,

Rules used = {272, 45}

$$\frac{1}{26} (3-2\sqrt{x})^{13/4} - \frac{1}{2} (3-2\sqrt{x})^{9/4} + \frac{27}{10} (3-2\sqrt{x})^{5/4} - \frac{27}{2} \sqrt[4]{3-2\sqrt{x}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x/(3 - 2*\text{Sqrt}[x])^{(3/4)}, x]$

[Out] $(-27*(3 - 2*\text{Sqrt}[x])^{(1/4)})/2 + (27*(3 - 2*\text{Sqrt}[x])^{(5/4)})/10 - (3 - 2*\text{Sqrt}[x])^{(9/4)}/2 + (3 - 2*\text{Sqrt}[x])^{(13/4)}/26$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) || \text{LtQ}[9*m + 5*(n + 1), 0] || \text{GtQ}[m + n + 2, 0])$

Rule 272

$\text{Int}[(x_.)^{(m_.)*((a_) + (b_.)*(x_.)^{(n_.))^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{x}{(3-2\sqrt{x})^{3/4}} dx &= 2\text{Subst} \left(\int \frac{x^3}{(3-2x)^{3/4}} dx, x, \sqrt{x} \right) \\ &= 2\text{Subst} \left(\int \left(\frac{27}{8(3-2x)^{3/4}} - \frac{27}{8} \sqrt[4]{3-2x} + \frac{9}{8}(3-2x)^{5/4} - \frac{1}{8}(3-2x)^{9/4} \right) dx, x, \sqrt{x} \right) \\ &= -\frac{27}{2} \sqrt[4]{3-2\sqrt{x}} + \frac{27}{10} (3-2\sqrt{x})^{5/4} - \frac{1}{2} (3-2\sqrt{x})^{9/4} + \frac{1}{26} (3-2\sqrt{x})^{13/4} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 36, normalized size = 0.52

$$-\frac{4}{65} \sqrt[4]{3-2\sqrt{x}} (144 + 24\sqrt{x} + 10x + 5x^{3/2})$$

Antiderivative was successfully verified.

[In] Integrate[x/(3 - 2*Sqrt[x])^(3/4), x]

[Out] (-4*(3 - 2*Sqrt[x])^(1/4)*(144 + 24*Sqrt[x] + 10*x + 5*x^(3/2)))/65

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.

time = 13.25, size = 1506, normalized size = 21.83

result too large to display

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x/(3 - 2*Sqrt[x])^(3/4), x]')

[Out] Piecewise[{{4 (-104976 -1 ^ (1 / 4) x ^ 8 (-3 + 2 Sqrt[x]) ^ (1 / 4) + 104976 3 ^ (1 / 4) x ^ 8 - 419904 3 ^ (1 / 4) x ^ (17 / 2) + 402408 -1 ^ (1 / 4) x ^ (17 / 2) (-3 + 2 Sqrt[x]) ^ (1 / 4) - 637146 -1 ^ (1 / 4) x ^ 9 (-3 + 2 Sqrt[x]) ^ (1 / 4) + 699840 3 ^ (1 / 4) x ^ 9 - 622080 3 ^ (1 / 4) x ^ (19 / 2) + 530955 -1 ^ (1 / 4) x ^ (19 / 2) (-3 + 2 Sqrt[x]) ^ (1 / 4) - 241380 -1 ^ (1 / 4) x ^ 10 (-3 + 2 Sqrt[x]) ^ (1 / 4) + 311040 3 ^ (1 / 4) x ^ 10 - 82944 3 ^ (1 / 4) x ^ (21 / 2) + 50004 -1 ^ (1 / 4) x ^ (21 / 2) (-3 + 2 Sqrt[x]) ^ (1 / 4) + 4608 -1 ^ (1 / 4) x ^ 11 (-3 + 2 Sqrt[x]) ^ (1 / 4) + 9216 3 ^ (1 / 4) x ^ 11 - 6576 -1 ^ (1 / 4) x ^ (23 / 2) (-3 + 2 Sqrt[x]) ^ (1 / 4) + 2240 -1 ^ (1 / 4) x ^ 12 (-3 + 2 Sqrt[x]) ^ (1 / 4) - 320 -1 ^ (1 / 4) x ^ (25 / 2) (-3 + 2 Sqrt[x]) ^ (1 / 4)) / (65 (729 x ^ 8 - 2916 x ^ (17 / 2) + 4860 x ^ 9 - 4320 x ^ (19 / 2) + 2160 x ^ 10 - 576 x ^ (21 / 2) + 64 x ^ 11)), Abs[Sqrt[x]] > 3 / 2}}, -419904 3 ^ (1 / 4) x ^ 8 (3 - 2 Sqrt[x]) ^ (1 / 4) / (47385 3 ^ (1 / 4) x ^ 8 - 189540 3 ^ (1 / 4) x ^ (17 / 2) + 315900 3 ^ (1 / 4) x ^ 9 - 280800 3 ^ (1 / 4) x ^ (19 / 2) + 140400 3 ^ (1 / 4) x ^ 10 - 37440 3 ^ (1 / 4) x ^ (21 / 2) + 4160 3 ^ (1 / 4) x ^ 11) + 419904 Sqrt[3] x ^ 8 / (47385 3 ^ (1 / 4) x ^ 8 - 189540 3 ^ (1 / 4) x ^ (17 / 2) + 315900 3 ^ (1 / 4) x ^ 9 - 280800 3 ^ (1 / 4) x ^ (19 / 2) + 140400 3 ^ (1 / 4) x ^ 10 - 37440 3 ^ (1 / 4) x ^ (21 / 2) + 4160 3 ^ (1 / 4) x ^ 11) - 1679616 Sqrt[3] x ^ (17 / 2) / (47385 3 ^ (1 / 4) x ^ 8 - 189540 3 ^ (1 / 4) x ^ (17 / 2) + 315900 3 ^ (1 / 4) x ^ 9 - 280800 3 ^ (1 / 4) x ^ (19 / 2) + 140400 3 ^ (1 / 4) x ^ 10 - 37440 3 ^ (1 / 4) x ^ (21 / 2) + 4160 3 ^ (1 / 4) x ^ 11) + 1609632 3 ^ (1 / 4) x ^ (17 / 2) (3 - 2 Sqrt[x]) ^ (1 / 4) / (47385 3 ^ (1 / 4) x ^ 8 - 189540 3 ^ (1 / 4) x ^ (17 / 2) + 315900 3 ^ (1 / 4) x ^ 9 - 280800 3 ^ (1 / 4) x ^ (19 / 2) + 140400 3 ^ (1 / 4) x ^ 10 - 37440 3 ^ (1 / 4) x ^ (21 / 2) + 4160 3 ^ (1 / 4) x ^ 11) - 2548584 3 ^ (1 / 4) x ^ 9 (3 - 2 Sqrt[x]) ^ (1 / 4) / (47385 3 ^ (1 / 4) x ^ 8 - 189540 3 ^ (1 / 4) x ^ (17 / 2) + 315900 3 ^ (1 / 4) x ^ 9 - 280800 3 ^ (1 / 4) x ^ (19 / 2) + 140400 3 ^ (1 / 4) x ^ 10 - 37440 3 ^ (1 / 4) x ^ (21 / 2) + 4160 3 ^ (1 / 4) x ^ 11)

Maple [A]

time = 0.06, size = 46, normalized size = 0.67

method	result	size
meijerg	$\frac{3^{\frac{1}{4}} x^2 \operatorname{hypergeom}\left(\left[\frac{3}{4}, 4\right], [5], \frac{2\sqrt{x}}{3}\right)}{6}$	20
derivativedivides	$-\frac{27(3-2\sqrt{x})^{\frac{1}{4}}}{2} + \frac{27(3-2\sqrt{x})^{\frac{5}{4}}}{10} - \frac{(3-2\sqrt{x})^{\frac{9}{4}}}{2} + \frac{(3-2\sqrt{x})^{\frac{13}{4}}}{26}$	46
default	$-\frac{27(3-2\sqrt{x})^{\frac{1}{4}}}{2} + \frac{27(3-2\sqrt{x})^{\frac{5}{4}}}{10} - \frac{(3-2\sqrt{x})^{\frac{9}{4}}}{2} + \frac{(3-2\sqrt{x})^{\frac{13}{4}}}{26}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3-2*x^(1/2))^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -27/2*(3-2*x^(1/2))^(1/4)+27/10*(3-2*x^(1/2))^(5/4)-1/2*(3-2*x^(1/2))^(9/4)
+1/26*(3-2*x^(1/2))^(13/4)
```

Maxima [A]

time = 0.26, size = 45, normalized size = 0.65

$$\frac{1}{26} (-2\sqrt{x} + 3)^{\frac{13}{4}} - \frac{1}{2} (-2\sqrt{x} + 3)^{\frac{9}{4}} + \frac{27}{10} (-2\sqrt{x} + 3)^{\frac{5}{4}} - \frac{27}{2} (-2\sqrt{x} + 3)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3-2*x^(1/2))^(3/4),x, algorithm="maxima")
```

```
[Out] 1/26*(-2*sqrt(x) + 3)^(13/4) - 1/2*(-2*sqrt(x) + 3)^(9/4) + 27/10*(-2*sqrt(x)
+ 3)^(5/4) - 27/2*(-2*sqrt(x) + 3)^(1/4)
```

Fricas [A]

time = 0.31, size = 25, normalized size = 0.36

$$-\frac{4}{65} ((5x + 24)\sqrt{x} + 10x + 144)(-2\sqrt{x} + 3)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3-2*x^(1/2))^(3/4),x, algorithm="fricas")
```

```
[Out] -4/65*((5*x + 24)*sqrt(x) + 10*x + 144)*(-2*sqrt(x) + 3)^(1/4)
```

Sympy [C] Result contains complex when optimal does not.

time = 1.12, size = 3303, normalized size = 47.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(3-2*x**(1/2))**(3/4),x)

[Out] Piecewise((1280*3**(1/4)*x**(25/2)*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 26304*3**(1/4)*x**(23/2)*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 200016*3**(1/4)*x**(21/2)*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 331776*sqrt(3)*x**(21/2)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 2123820*3**(1/4)*x**(19/2)*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 2488320*sqrt(3)*x**(19/2)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 1609632*3**(1/4)*x**(17/2)*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 1679616*sqrt(3)*x**(17/2)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 8960*3**(1/4)*x**12*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 18432*3**(1/4)*x**11*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 36864*sqrt(3)*x**11/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 965520*3**(1/4)*x**10*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 1244160*sqrt(3)*x**10/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 2548584*3**(1/4)*x**9*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 2799360*sqrt(3)*x**9/(-37440*3**(1/4)*x**(21/2) - 280800*3**

$1/4)*x^{19/2} - 189540*3^{1/4}*x^{17/2} + 4160*3^{1/4}*x^{11} + 140400*3^{1/4}*x^{10} + 315900*3^{1/4}*x^9 + 47385*3^{1/4}*x^8) + 419904*3^{1/4}*x^8*(2*\sqrt{x} - 3)^{1/4}*\exp(-3*I*\pi/4)/(-37440*3^{1/4}*x^{21/2} - 280800*3^{1/4}*x^{19/2} - 189540*3^{1/4}*x^{17/2} + 4160*3^{1/4}*x^{11} + 140400*3^{1/4}*x^{10} + 315900*3^{1/4}*x^9 + 47385*3^{1/4}*x^8) + 419904*\sqrt{3}*x^8/(-37440*3^{1/4}*x^{21/2} - 280800*3^{1/4}*x^{19/2} - 189540*3^{1/4}*x^{17/2} + 4160*3^{1/4}*x^{11} + 140400*3^{1/4}*x^{10} + 315900*3^{1/4}*x^9 + 47385*3^{1/4}*x^8), \text{Abs}(\sqrt{x}) > 3/2), (-1280*3^{1/4}*x^{25/2}*(3 - 2*\sqrt{x})^{1/4}/(-37440*3^{1/4}*x^{21/2} - 280800*3^{1/4}*x^{19/2} - 189540*3^{1/4}*x^{17/2} + 4160*3^{1/4}*x^{11} + 140400*3^{1/4}*x^{10} + 315900*3^{1/4}*x^9 + 47385*3^{1/4}*x^8) - 26304*3^{1/4}*x^{23/2}*(3 - 2*\sqrt{x})^{1/4}/(-37440*3^{1/4}*x^{21/2} - 280800*3^{1/4}*x^{19/2} - 189540*3^{1/4}*x^{17/2} + 4160*3^{1/4}*x^{11} + 140400*3^{1/4}*x^{10} + 315900*3^{1/4}*x^9 + 47385*3^{1/4}*x^8) + 200016*3^{1/4}*x^{21/2}*(3 - 2*\sqrt{x})^{1/4}/(-37440*3^{1/4}*x^{21/2} - 280800*3^{1/4}*x^{19/2} - 189540*3^{1/4}*x^{17/2} + 4160*3^{1/4}*x^{11} + 140400*3^{1/4}*x^{10} + 315900*3^{1/4}*x^9 + 47385*3^{1/4}*x^8) - 331776*\sqrt{3}*x^{21/2}/(-37440*3^{1/4}*x^{21/2} - 280800*3^{1/4}*x^{19/2} - 189540*3^{1/4}*x^{17/2} + 4160*3^{1/4}*x^{11} + 140400*3^{1/4}*x^{10} + 315900*3^{1/4}*x^9 + 47385*3^{1/4}*x^8) + 2123820*3^{1/4}*x^{19/2}*(3 - 2*\sqrt{x})^{1/4}/(-37440*3^{1/4}*x^{21/2} - 280800*3^{1/4}*x^{19/2} - 189540*3^{1/4}*x^{17/2} + 4160*3^{1/4}*x^{11} + 140400*3^{1/4}*x^{10} + 315900*3^{1/4}*x^9 + 47385*3^{1/4}*x^8) - 2488320*\sqrt{3}*x^{19/2}/(-37440*3^{1/4}*x^{21/2} - 280800*3^{1/4}*x^{19/2} - 189540*3^{1/4}*x^{17/2} + 4160*3^{1/4}*x^{11} + 140400*3^{1/4}*x^{10} + 315900*3^{1/4}*x^9 + 47385*3^{1/4}*x^8) + 1609632*3^{1/4}*x^{17/2}*(3 - 2*\sqrt{x})^{1/4}/(-37440*3^{1/4}*x^{21/2} - 280800*3^{1/4}*x^{19/2} - 189540*3^{1/4}*x^{17/2} + 4160*3^{1/4}*x^{11} + 140400*3^{1/4}*x^{10} + 315900*3^{1/4}*x^9 + 47385*3^{1/4}*x^8) - 1679616*\sqrt{3}*x^{17/2}/(-37440*3^{1/4}*x^{21/2} - 280800*3^{1/4}*x^{19/2} - 189540*3^{1/4}*x^{17/2} + 4160*3^{1/4}*x^{11} + 140400*3^{1/4}*x^{10} + 315900*3^{1/4}*x^9 + 47385*3^{1/4}*x^8) + 8960*3^{1/4}*x^{12}*(3 - 2*\sqrt{x})^{1/4}/(-37440*3^{1/4}*x^{21/2} - 280800*3^{1/4}*x^{19/2} - 189540*3^{1/4}*x^{17/2} + 4160*3^{1/4}*x^{11} + 140400*3^{1/4}*x^{10} + 315900*3^{1/4}*x^9 + 47385*3^{1/4}*x^8) + 18432*3^{1/4}*x^{11}*(3 - 2*\sqrt{x})^{1/4}/(-37440*3^{1/4}*x^{21/2} - 280800*3^{1/4}*x^{19/2} - 189540*3^{1/4}*x^{17/2} + 4160*3^{1/4}*x^{11} + 140400*3^{1/4}*x^{10} + 315900*3^{1/4}*x^9 + 47385*3^{1/4}*x^8) + 36864*\sqrt{3}*x^{11}/(-37440*3^{1/4}*x^{21/2} - 280800*3^{1/4}*x^{19/2} - 189540*3^{1/4}*x^{17/2} + 4160*3^{1/4}*x^{11} + 140400*3^{1/4}*x^{10} + 315900*3^{1/4}*x^9 + 47385*3^{1/4}*x^8) - 965520*3^{1/4}*x^{10}*(3 - 2*\sqrt{x})^{1/4}/(-37440*3^{1/4}*x^{21/2} - 280800*3^{1/4}*x^{19/2} - 189540*3^{1/4}*x^{17/2} + 4160*3^{1/4}*x^{11} + 140400*3^{1/4}*x^{10} + 315900*3^{1/4}*x^9 + 47385*3^{1/4}*x^8) + 1244160*\sqrt{3}*x^{10}/(-37440*3^{1/4}*x^{21/2} - 280800*3^{1/4}*x^{19/2} - 189540*3^{1/4}*x^{17/2} + 4160*3^{1/4}*x^{11} + 140400*3^{1/4}*x^{10} + 315900*3^{1/4}*x^9 + 47385*3^{1/4}*x^8)$


```

1/4)*x**8) - 2548584*3**(1/4)*x**9*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*
x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3*
*(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4
)*x**8) + 2799360*sqrt(3)*x**9/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)
*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1
/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 419904*3**(1/4)*x
**8*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**
(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*
x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 419904*sqrt(3)*x**8/(
-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**
(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9
+ 47385*3**(1/4)*x**8), True))

```

Giac [A]

time = 0.00, size = 108, normalized size = 1.57

$$-\frac{4}{8} \left(-\frac{1}{13} (-2\sqrt{x} + 3)^{\frac{1}{4}} (-2\sqrt{x} + 3)^3 + (-2\sqrt{x} + 3)^{\frac{1}{4}} (-2\sqrt{x} + 3)^2 - \frac{27}{5} (-2\sqrt{x} + 3)^{\frac{1}{4}} (-2\sqrt{x} + 3) + 27 (-2\sqrt{x} + 3)^{\frac{1}{4}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(3-2*x^(1/2))^(3/4),x)

[Out] -1/26*(2*sqrt(x) - 3)^3*(-2*sqrt(x) + 3)^(1/4) - 1/2*(2*sqrt(x) - 3)^2*(-2*sqrt(x) + 3)^(1/4) + 27/10*(-2*sqrt(x) + 3)^(5/4) - 27/2*(-2*sqrt(x) + 3)^(1/4)

Mupad [B]

time = 0.30, size = 45, normalized size = 0.65

$$\frac{27(3-2\sqrt{x})^{5/4}}{10} - \frac{27(3-2\sqrt{x})^{1/4}}{2} - \frac{(3-2\sqrt{x})^{9/4}}{2} + \frac{(3-2\sqrt{x})^{13/4}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(3 - 2*x^(1/2))^(3/4),x)

[Out] (27*(3 - 2*x^(1/2))^(5/4))/10 - (27*(3 - 2*x^(1/2))^(1/4))/2 - (3 - 2*x^(1/2))^(9/4)/2 + (3 - 2*x^(1/2))^(13/4)/26

$$3.298 \quad \int \frac{(-1+2\sqrt{x})^{5/4}}{x^2} dx$$

Optimal. Leaf size=193

$$\frac{(-1+2\sqrt{x})^{5/4}}{x} - \frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}} - \frac{5 \tan^{-1}\left(1 - \sqrt{2} \sqrt[4]{-1+2\sqrt{x}}\right)}{2\sqrt{2}} + \frac{5 \tan^{-1}\left(1 + \sqrt{2} \sqrt[4]{-1+2\sqrt{x}}\right)}{2\sqrt{2}}$$

[Out] 5/4*arctan(-1+2^(1/2)*(-1+2*x^(1/2))^(1/4))*2^(1/2)+5/4*arctan(1+2^(1/2)*(-1+2*x^(1/2))^(1/4))*2^(1/2)-5/8*ln(1-2^(1/2)*(-1+2*x^(1/2))^(1/4)+(-1+2*x^(1/2))^(1/2))*2^(1/2)+5/8*ln(1+2^(1/2)*(-1+2*x^(1/2))^(1/4)+(-1+2*x^(1/2))^(1/2))*2^(1/2)-5/2*(-1+2*x^(1/2))^(1/4)/x^(1/2)-(-1+2*x^(1/2))^(5/4)/x

Rubi [A]

time = 0.07, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {272, 43, 65, 217, 1179, 642, 1176, 631, 210}

$$\frac{(2\sqrt{x}-1)^{5/4}}{x} - \frac{5\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{x}} - \frac{5 \log\left(\sqrt{2\sqrt{x}-1} - \sqrt{2}\sqrt[4]{2\sqrt{x}-1} + 1\right)}{4\sqrt{2}} + \frac{5 \log\left(\sqrt{2\sqrt{x}-1} + \sqrt{2}\sqrt[4]{2\sqrt{x}-1} + 1\right)}{4\sqrt{2}} - \frac{5 \tan^{-1}\left(1 - \sqrt{2}\sqrt[4]{2\sqrt{x}-1}\right)}{2\sqrt{2}} + \frac{5 \tan^{-1}\left(\sqrt{2}\sqrt[4]{2\sqrt{x}-1} + 1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + 2*Sqrt[x])^(5/4)/x^2,x]

[Out] -((-1 + 2*Sqrt[x])^(5/4)/x) - (5*(-1 + 2*Sqrt[x])^(1/4))/(2*Sqrt[x]) - (5*ArcTan[1 - Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4)])/(2*Sqrt[2]) + (5*ArcTan[1 + Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4)])/(2*Sqrt[2]) - (5*Log[1 - Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4) + Sqrt[-1 + 2*Sqrt[x]]])/(4*Sqrt[2]) + (5*Log[1 + Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4) + Sqrt[-1 + 2*Sqrt[x]]])/(4*Sqrt[2])

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && GtQ[n, 0]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 631

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1176

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1179

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rubi steps

$$\begin{aligned}
\int \frac{(-1 + 2\sqrt{x})^{5/4}}{x^2} dx &= 2\text{Subst} \left(\int \frac{(-1 + 2x)^{5/4}}{x^3} dx, x, \sqrt{x} \right) \\
&= -\frac{(-1 + 2\sqrt{x})^{5/4}}{x} + \frac{5}{2}\text{Subst} \left(\int \frac{\sqrt[4]{-1 + 2x}}{x^2} dx, x, \sqrt{x} \right) \\
&= -\frac{(-1 + 2\sqrt{x})^{5/4}}{x} - \frac{5\sqrt[4]{-1 + 2\sqrt{x}}}{2\sqrt{x}} + \frac{5}{4}\text{Subst} \left(\int \frac{1}{x(-1 + 2x)^{3/4}} dx, x, \sqrt{x} \right) \\
&= -\frac{(-1 + 2\sqrt{x})^{5/4}}{x} - \frac{5\sqrt[4]{-1 + 2\sqrt{x}}}{2\sqrt{x}} + \frac{5}{2}\text{Subst} \left(\int \frac{1}{\frac{1}{2} + \frac{x^4}{2}} dx, x, \sqrt[4]{-1 + 2\sqrt{x}} \right) \\
&= -\frac{(-1 + 2\sqrt{x})^{5/4}}{x} - \frac{5\sqrt[4]{-1 + 2\sqrt{x}}}{2\sqrt{x}} + \frac{5}{4}\text{Subst} \left(\int \frac{1 - x^2}{\frac{1}{2} + \frac{x^4}{2}} dx, x, \sqrt[4]{-1 + 2\sqrt{x}} \right) \\
&= -\frac{(-1 + 2\sqrt{x})^{5/4}}{x} - \frac{5\sqrt[4]{-1 + 2\sqrt{x}}}{2\sqrt{x}} + \frac{5}{4}\text{Subst} \left(\int \frac{1}{1 - \sqrt{2}x + x^2} dx, x, \sqrt[4]{-1 + 2\sqrt{x}} \right) \\
&= -\frac{(-1 + 2\sqrt{x})^{5/4}}{x} - \frac{5\sqrt[4]{-1 + 2\sqrt{x}}}{2\sqrt{x}} - \frac{5 \log \left(1 - \sqrt{2} \sqrt[4]{-1 + 2\sqrt{x}} + \sqrt{-1 + 2\sqrt{x}} \right)}{4\sqrt{2}} \\
&= -\frac{(-1 + 2\sqrt{x})^{5/4}}{x} - \frac{5\sqrt[4]{-1 + 2\sqrt{x}}}{2\sqrt{x}} - \frac{5 \tan^{-1} \left(1 - \sqrt{2} \sqrt[4]{-1 + 2\sqrt{x}} \right)}{2\sqrt{2}} + \frac{5 \tan^{-1} \left(1 + \sqrt{2} \sqrt[4]{-1 + 2\sqrt{x}} \right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A]

time = 0.21, size = 120, normalized size = 0.62

$$\frac{2(2 - 9\sqrt{x})\sqrt[4]{-1 + 2\sqrt{x}} + 5\sqrt{2}x \tan^{-1} \left(\frac{-1 + \sqrt{-1 + 2\sqrt{x}}}{\sqrt{2}\sqrt[4]{-1 + 2\sqrt{x}}} \right) + 5\sqrt{2}x \tanh^{-1} \left(\frac{\sqrt{2}\sqrt[4]{-1 + 2\sqrt{x}}}{1 + \sqrt{-1 + 2\sqrt{x}}} \right)}{4x}$$

Antiderivative was successfully verified.

`[In] Integrate[(-1 + 2*Sqrt[x])^(5/4)/x^2, x]`

```
[Out] (2*(2 - 9*Sqrt[x])*(-1 + 2*Sqrt[x])^(1/4) + 5*Sqrt[2]*x*ArcTan[(-1 + Sqrt[-1 + 2*Sqrt[x]])/(Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4))] + 5*Sqrt[2]*x*ArcTanh[(Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4))/(1 + Sqrt[-1 + 2*Sqrt[x]])])/(4*x)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 4.82, size = 23, normalized size = 0.12

$$\frac{-162^{\frac{1}{4}} \operatorname{hyper}\left[\left\{-\frac{5}{4}, \frac{3}{4}\right\}, \left\{\frac{7}{4}\right\}, \frac{\exp_{\text{polar}}[2i\pi]}{2\sqrt{x}}\right]}{3x^{\frac{3}{8}}}$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(2*Sqrt[x] - 1)^(5/4)/x^2,x]')`

[Out] `-16 2 ^ (1 / 4) hyper[{-5 / 4, 3 / 4}, {7 / 4}, exp_polar[2 I Pi] / (2 Sqrt [x])] / (3 x ^ (3 / 8))`

Maple [A]

time = 0.08, size = 125, normalized size = 0.65

method	result
meijerg	$5 \operatorname{signum}(-1+2\sqrt{x})^{\frac{5}{4}} \left(-\frac{2\Gamma(\frac{3}{4})}{5x} + \frac{2\Gamma(\frac{3}{4})}{\sqrt{x}} + \frac{(-2\ln(2) + \frac{\pi}{2} - \frac{3}{2} + \frac{\ln(x)}{2} + i\pi)\Gamma(\frac{3}{4})}{2} + \frac{\Gamma(\frac{3}{4})\sqrt{x}}{4} \operatorname{hypergeom}\left(\left[1, 1, \frac{7}{4}\right], [2, 4], 2\sqrt{x}\right) \right)$
derivativedivides	$\frac{9(-1+2\sqrt{x})^{\frac{5}{4}}}{4} - \frac{5(-1+2\sqrt{x})^{\frac{1}{4}}}{4} + \frac{5\sqrt{2} \left(\ln\left(\frac{1+\sqrt{2}(-1+2\sqrt{x})^{\frac{1}{4}} + \sqrt{-1+2\sqrt{x}}}{1-\sqrt{2}(-1+2\sqrt{x})^{\frac{1}{4}} + \sqrt{-1+2\sqrt{x}}}\right) + 2\arctan\left(\frac{1+\sqrt{2}(-1+2\sqrt{x})^{\frac{1}{4}} + \sqrt{-1+2\sqrt{x}}}{1-\sqrt{2}(-1+2\sqrt{x})^{\frac{1}{4}} + \sqrt{-1+2\sqrt{x}}}\right) \right)}{8}$
default	$\frac{9(-1+2\sqrt{x})^{\frac{5}{4}}}{4} - \frac{5(-1+2\sqrt{x})^{\frac{1}{4}}}{4} + \frac{5\sqrt{2} \left(\ln\left(\frac{1+\sqrt{2}(-1+2\sqrt{x})^{\frac{1}{4}} + \sqrt{-1+2\sqrt{x}}}{1-\sqrt{2}(-1+2\sqrt{x})^{\frac{1}{4}} + \sqrt{-1+2\sqrt{x}}}\right) + 2\arctan\left(\frac{1+\sqrt{2}(-1+2\sqrt{x})^{\frac{1}{4}} + \sqrt{-1+2\sqrt{x}}}{1-\sqrt{2}(-1+2\sqrt{x})^{\frac{1}{4}} + \sqrt{-1+2\sqrt{x}}}\right) \right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-1+2*x^(1/2))^(5/4)/x^2,x,method=_RETURNVERBOSE)`

[Out] `8*(-9/32*(-1+2*x^(1/2))^(5/4)-5/32*(-1+2*x^(1/2))^(1/4))/x+5/8*2^(1/2)*(ln((1+2^(1/2)*(-1+2*x^(1/2))^(1/4)+(-1+2*x^(1/2))^(1/2))/(1-2^(1/2)*(-1+2*x^(1/2))^(1/4)+(-1+2*x^(1/2))^(1/2)))+2*arctan(1+2^(1/2)*(-1+2*x^(1/2))^(1/4))+2*arctan(-1+2^(1/2)*(-1+2*x^(1/2))^(1/4)))`

Maxima [A]

time = 0.34, size = 157, normalized size = 0.81

$$\frac{5}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2}+2(2\sqrt{x}-1)^{\frac{1}{4}})\right) + \frac{5}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2}-2(2\sqrt{x}-1)^{\frac{1}{4}})\right) + \frac{5}{8}\sqrt{2}\log\left(\sqrt{2}(2\sqrt{x}-1)^{\frac{1}{4}} + \sqrt{2\sqrt{x}-1} + 1\right) - \frac{5}{8}\sqrt{2}\log\left(-\sqrt{2}(2\sqrt{x}-1)^{\frac{1}{4}} + \sqrt{2\sqrt{x}-1} + 1\right) - \frac{9(2\sqrt{x}-1)^{\frac{5}{4}} + 5(2\sqrt{x}-1)^{\frac{1}{4}}}{(2\sqrt{x}-1)^{\frac{7}{4}} + 4\sqrt{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+2*x^(1/2))^(5/4)/x^2,x, algorithm="maxima")`

[Out] $5/4*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2*(2*\sqrt{x} - 1)^{(1/4)})) + 5/4*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2*(2*\sqrt{x} - 1)^{(1/4)})) + 5/8*\sqrt{2}*\log(\sqrt{2}*(2*\sqrt{x} - 1)^{(1/4)} + \sqrt{2*\sqrt{x} - 1} + 1) - 5/8*\sqrt{2}*\log(-\sqrt{2}*(2*\sqrt{x} - 1)^{(1/4)} + \sqrt{2*\sqrt{x} - 1} + 1) - (9*(2*\sqrt{x} - 1)^{(5/4)} + 5*(2*\sqrt{x} - 1)^{(1/4)})/((2*\sqrt{x} - 1)^2 + 4*\sqrt{x} - 1)$

Fricas [A]

time = 0.32, size = 202, normalized size = 1.05

$$\frac{20\sqrt{2}x\arctan\left(\frac{\sqrt{2}\sqrt{2\sqrt{x}-1}+\sqrt{2\sqrt{x}-1}-\sqrt{2}\sqrt{2\sqrt{x}-1}}{1}\right)+20\sqrt{2}x\arctan\left(\frac{\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}-1}+4\sqrt{2\sqrt{x}-1}-\sqrt{2}\sqrt{2\sqrt{x}-1}}{1}\right)-5\sqrt{2}x\log\left(4\sqrt{2}\sqrt{2\sqrt{x}-1}+4\sqrt{2\sqrt{x}-1}+4\right)+5\sqrt{2}x\log\left(-4\sqrt{2}\sqrt{2\sqrt{x}-1}+4\sqrt{2\sqrt{x}-1}+4\right)+4(9\sqrt{x}-2)(2\sqrt{x}-1)^{\frac{5}{4}}}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+2*x^(1/2))^(5/4)/x^2,x, algorithm="fricas")`

[Out] $-1/8*(20*\sqrt{2}*x*\arctan(\sqrt{2}*\sqrt{2*\sqrt{x} - 1}) + \sqrt{2*\sqrt{x} - 1} + 1) - \sqrt{2}*(2*\sqrt{x} - 1)^{(1/4)} - 1) + 20*\sqrt{2}*x*\arctan(1/2*\sqrt{2}*\sqrt{-4*\sqrt{2}*(2*\sqrt{x} - 1)^{(1/4)} + 4*\sqrt{2*\sqrt{x} - 1} + 4} - \sqrt{2}*(2*\sqrt{x} - 1)^{(1/4)} + 1) - 5*\sqrt{2}*x*\log(4*\sqrt{2}*(2*\sqrt{x} - 1)^{(1/4)} + 4*\sqrt{2*\sqrt{x} - 1} + 4) + 5*\sqrt{2}*x*\log(-4*\sqrt{2}*(2*\sqrt{x} - 1)^{(1/4)} + 4*\sqrt{2*\sqrt{x} - 1} + 4) + 4*(9*\sqrt{x} - 2)*(2*\sqrt{x} - 1)^{(1/4)})/x$

Sympy [C] Result contains complex when optimal does not.

time = 3.18, size = 44, normalized size = 0.23

$$\frac{4 \cdot \sqrt[4]{2} \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{3}{4} \\ \frac{7}{4} \end{matrix} \middle| \frac{e^{2i\pi}}{2\sqrt{x}}\right)}{x^{\frac{3}{8}} \Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+2*x**(1/2))**(5/4)/x**2,x)`

[Out] $-4*2^{1/4}*\gamma(3/4)*\text{hyper}((-5/4, 3/4), (7/4,), \exp_polar(2*I*\pi)/(2*\sqrt{x}))/x^{3/8}*\gamma(7/4)$

Giac [A]

time = 0.01, size = 225, normalized size = 1.17

$$-\frac{5}{8}\sqrt{2}\ln\left(\sqrt{2\sqrt{x}-1}-\sqrt{2}\sqrt{2\sqrt{x}-1}+1\right)+\frac{5}{4}\sqrt{2}\arctan\left(\frac{2\left((2\sqrt{x}-1)^{\frac{1}{4}}-\frac{\sqrt{2}}{2}\right)}{\sqrt{2}}\right)+\frac{5}{8}\sqrt{2}\ln\left(\sqrt{2\sqrt{x}-1}+\sqrt{2}\sqrt{2\sqrt{x}-1}+1\right)+\frac{5}{4}\sqrt{2}\arctan\left(\frac{2\left((2\sqrt{x}-1)^{\frac{1}{4}}+\frac{\sqrt{2}}{2}\right)}{\sqrt{2}}\right)+\frac{-9(2\sqrt{x}-1)^{\frac{5}{4}}(2\sqrt{x}-1)-5(2\sqrt{x}-1)^{\frac{1}{4}}}{(2\sqrt{x})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+2*x^(1/2))^(5/4)/x^2,x)`

[Out] $5/4*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2*(2*\sqrt{x} - 1)^{1/4})) + 5/4*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2*(2*\sqrt{x} - 1)^{1/4})) + 5/8*\sqrt{2}*\log(\sqrt{2}*(2*\sqrt{x} - 1)^{1/4} + \sqrt{2*\sqrt{x} - 1} + 1) - 5/8*\sqrt{2}*\log(-\sqrt{2}*(2*\sqrt{x} - 1)^{1/4} + \sqrt{2*\sqrt{x} - 1} + 1) - 1/4*(9*(2*\sqrt{x} - 1)^{5/4} + 5*(2*\sqrt{x} - 1)^{1/4})/x$

Mupad [B]

time = 1.28, size = 77, normalized size = 0.40

$$-\frac{5(2\sqrt{x}-1)^{1/4}}{4x} - \frac{9(2\sqrt{x}-1)^{5/4}}{4x} + \sqrt{2} \operatorname{atan}\left(\sqrt{2}(2\sqrt{x}-1)^{1/4}\left(\frac{1}{2} - \frac{1}{2}i\right)\right)\left(\frac{5}{4} + \frac{5}{4}i\right) + \sqrt{2} \operatorname{atan}\left(\sqrt{2}(2\sqrt{x}-1)^{1/4}\left(\frac{1}{2} + \frac{1}{2}i\right)\right)\left(\frac{5}{4} - \frac{5}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((2*x^{1/2} - 1)^{5/4}/x^2, x)$

[Out] $2^{1/2}*\operatorname{atan}(2^{1/2}*(2*x^{1/2} - 1)^{1/4}*(1/2 - 1i/2))*(5/4 + 5i/4) - (9*(2*x^{1/2} - 1)^{5/4})/(4*x) - (5*(2*x^{1/2} - 1)^{1/4})/(4*x) + 2^{1/2}*\operatorname{atan}(2^{1/2}*(2*x^{1/2} - 1)^{1/4}*(1/2 + 1i/2))*(5/4 - 5i/4)$

3.299 $\int x^6 \sqrt[3]{1+x^7} dx$

Optimal. Leaf size=13

$$\frac{3}{28} (1+x^7)^{4/3}$$

[Out] 3/28*(x^7+1)^(4/3)

Rubi [A]

time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$\frac{3}{28} (x^7 + 1)^{4/3}$$

Antiderivative was successfully verified.

[In] Int[x^6*(1 + x^7)^(1/3),x]

[Out] (3*(1 + x^7)^(4/3))/28

Rule 267

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int x^6 \sqrt[3]{1+x^7} dx = \frac{3}{28} (1+x^7)^{4/3}$$

Mathematica [A]

time = 0.01, size = 13, normalized size = 1.00

$$\frac{3}{28} (1+x^7)^{4/3}$$

Antiderivative was successfully verified.

[In] Integrate[x^6*(1 + x^7)^(1/3),x]

[Out] (3*(1 + x^7)^(4/3))/28

Mathics [A]

time = 1.77, size = 9, normalized size = 0.69

$$\frac{3(1+x^7)^{\frac{4}{3}}}{28}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(x^7 + 1)^(1/3)*x^6,x]')`

[Out] $3(1 + x^7)^{4/3} / 28$

Maple [A]

time = 0.07, size = 10, normalized size = 0.77

method	result	size
derivativdivides	$\frac{3(x^7+1)^{\frac{4}{3}}}{28}$	10
default	$\frac{3(x^7+1)^{\frac{4}{3}}}{28}$	10
risch	$\frac{3(x^7+1)^{\frac{4}{3}}}{28}$	10
trager	$\left(\frac{3}{28} + \frac{3x^7}{28}\right) (x^7 + 1)^{\frac{1}{3}}$	16
meijerg	$\frac{x^7 \text{ hypergeom}\left(\left[-\frac{1}{3}, 1\right], [2], -x^7\right)}{7}$	17
gospers	$\frac{3(1+x)(x^6-x^5+x^4-x^3+x^2-x+1)(x^7+1)^{\frac{1}{3}}}{28}$	37

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*(x^7+1)^(1/3),x,method=_RETURNVERBOSE)`

[Out] $3/28*(x^7+1)^{4/3}$

Maxima [A]

time = 0.25, size = 9, normalized size = 0.69

$$\frac{3}{28} (x^7 + 1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(x^7+1)^(1/3),x, algorithm="maxima")`

[Out] $3/28*(x^7 + 1)^{4/3}$

Fricas [A]

time = 0.30, size = 9, normalized size = 0.69

$$\frac{3}{28} (x^7 + 1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(x^7+1)^(1/3),x, algorithm="fricas")`

[Out] $3/28*(x^7 + 1)^{4/3}$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$

time = 0.14, size = 26, normalized size = 2.00

$$\frac{3x^7\sqrt[3]{x^7+1}}{28} + \frac{3\sqrt[3]{x^7+1}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6*(x**7+1)**(1/3),x)`

[Out] $3*x**7*(x**7 + 1)**(1/3)/28 + 3*(x**7 + 1)**(1/3)/28$

Giac [A]

time = 0.00, size = 20, normalized size = 1.54

$$\frac{3(x^7 + 1)^{\frac{1}{3}}(x^7 + 1)}{7 \cdot 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(x^7+1)^(1/3),x)`

[Out] $3/28*(x^7 + 1)^{4/3}$

Mupad [B]

time = 0.29, size = 9, normalized size = 0.69

$$\frac{3(x^7 + 1)^{4/3}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*(x^7 + 1)^(1/3),x)`

[Out] $(3*(x^7 + 1)^{4/3})/28$

$$3.300 \quad \int \frac{x^6}{(1+x^7)^{5/3}} dx$$

Optimal. Leaf size=13

$$-\frac{3}{14(1+x^7)^{2/3}}$$

[Out] -3/14/(x^7+1)^(2/3)

Rubi [A]

time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$-\frac{3}{14(x^7+1)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(1+x^7)^(5/3),x]

[Out] -3/(14*(1+x^7)^(2/3))

Rule 267

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int \frac{x^6}{(1+x^7)^{5/3}} dx = -\frac{3}{14(1+x^7)^{2/3}}$$

Mathematica [A]

time = 0.01, size = 13, normalized size = 1.00

$$-\frac{3}{14(1+x^7)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(1+x^7)^(5/3),x]

[Out] -3/(14*(1+x^7)^(2/3))

Mathics [A]

time = 1.71, size = 9, normalized size = 0.69

$$\frac{-3}{14(1+x^7)^{\frac{2}{3}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^6/(x^7 + 1)^(5/3),x]')
```

```
[Out] -3 / (14 (1 + x ^ 7) ^ (2 / 3))
```

Maple [A]

time = 0.06, size = 10, normalized size = 0.77

method	result	size
derivativedivides	$-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$	10
default	$-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$	10
trager	$-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$	10
risch	$-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$	10
meijerg	$\frac{x^7 \operatorname{hypergeom}\left(\left[1, \frac{5}{3}\right], [2], -x^7\right)}{7}$	17
gospers	$-\frac{3(1+x)(x^6-x^5+x^4-x^3+x^2-x+1)}{14(x^7+1)^{\frac{5}{3}}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^7+1)^(5/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/14/(x^7+1)^(2/3)
```

Maxima [A]

time = 0.26, size = 9, normalized size = 0.69

$$-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^7+1)^(5/3),x, algorithm="maxima")
```

```
[Out] -3/14/(x^7 + 1)^(2/3)
```

Fricas [A]

time = 0.29, size = 9, normalized size = 0.69

$$-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(x^7+1)^(5/3),x, algorithm="fricas")`

[Out] $-3/14/(x^7 + 1)^{(2/3)}$

Sympy [A]

time = 0.22, size = 12, normalized size = 0.92

$$-\frac{3}{14(x^7 + 1)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(x**7+1)**(5/3),x)`

[Out] $-3/(14*(x**7 + 1)**(2/3))$

Giac [A]

time = 0.00, size = 18, normalized size = 1.38

$$-\frac{\frac{1}{14} \cdot 3}{\left((x^7 + 1)^{\frac{1}{3}}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(x^7+1)^(5/3),x)`

[Out] $-3/14/(x^7 + 1)^{(2/3)}$

Mupad [B]

time = 0.32, size = 9, normalized size = 0.69

$$-\frac{3}{14(x^7 + 1)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(x^7 + 1)^(5/3),x)`

[Out] $-3/(14*(x^7 + 1)^{(2/3)})$

$$3.301 \quad \int \frac{1}{x(-27+2x^7)^{2/3}} dx$$

Optimal. Leaf size=59

$$-\frac{\tan^{-1}\left(\frac{3-2\sqrt[3]{-27+2x^7}}{3\sqrt{3}}\right)}{21\sqrt{3}} - \frac{\log(x)}{18} + \frac{1}{42} \log\left(3 + \sqrt[3]{-27+2x^7}\right)$$

[Out] -1/18*ln(x)+1/42*ln(3+(2*x^7-27)^(1/3))-1/63*arctan(1/9*(3-2*(2*x^7-27)^(1/3))*3^(1/2))*3^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 60, 632, 210, 31}

$$\frac{1}{42} \log\left(\sqrt[3]{2x^7-27} + 3\right) - \frac{\tan^{-1}\left(\frac{3-2\sqrt[3]{2x^7-27}}{3\sqrt{3}}\right)}{21\sqrt{3}} - \frac{\log(x)}{18}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(-27 + 2*x^7)^(2/3)),x]

[Out] -1/21*ArcTan[(3 - 2*(-27 + 2*x^7)^(1/3))/(3*Sqrt[3])]/Sqrt[3] - Log[x]/18 + Log[3 + (-27 + 2*x^7)^(1/3)]/42

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 60

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(n-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 272

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[
Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x(-27 + 2x^7)^{2/3}} dx &= \frac{1}{7} \text{Subst} \left(\int \frac{1}{x(-27 + 2x)^{2/3}} dx, x, x^7 \right) \\ &= -\frac{\log(x)}{18} + \frac{1}{42} \text{Subst} \left(\int \frac{1}{3 + x} dx, x, \sqrt[3]{-27 + 2x^7} \right) + \frac{1}{14} \text{Subst} \left(\int \frac{1}{9 - 3x + x^2} dx, x, \sqrt[3]{-27 + 2x^7} \right) \\ &= -\frac{\log(x)}{18} + \frac{1}{42} \log \left(3 + \sqrt[3]{-27 + 2x^7} \right) - \frac{1}{7} \text{Subst} \left(\int \frac{1}{-27 - x^2} dx, x, -3 + 2\sqrt[3]{-27 + 2x^7} \right) \\ &= -\frac{\tan^{-1} \left(\frac{3 - 2\sqrt[3]{-27 + 2x^7}}{3\sqrt{3}} \right)}{21\sqrt{3}} - \frac{\log(x)}{18} + \frac{1}{42} \log \left(3 + \sqrt[3]{-27 + 2x^7} \right) \end{aligned}$$

Mathematica [A]

time = 0.05, size = 82, normalized size = 1.39

$$\frac{1}{126} \left(-2\sqrt{3} \tan^{-1} \left(\frac{3 - 2\sqrt[3]{-27 + 2x^7}}{3\sqrt{3}} \right) + 2 \log \left(3 + \sqrt[3]{-27 + 2x^7} \right) - \log \left(9 - 3\sqrt[3]{-27 + 2x^7} + (-27 + 2x^7)^{2/3} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-27 + 2*x^7)^(2/3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(3 - 2*(-27 + 2*x^7)^(1/3))/(3*Sqrt[3]]) + 2*Log[3 + (-2
7 + 2*x^7)^(1/3)] - Log[9 - 3*(-27 + 2*x^7)^(1/3) + (-27 + 2*x^7)^(2/3)])/1
26
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.13, size = 23, normalized size = 0.39

$$\frac{-32^{\frac{1}{3}} \text{hyper} \left[\left\{ \frac{2}{3}, \frac{2}{3} \right\}, \left\{ \frac{5}{3} \right\}, \frac{27 \exp_{\text{polar}}[2i\text{Pi}]}{2x^7} \right]}{28x^{\frac{14}{3}}}$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[1/(x*(2*x^7 - 27)^(2/3)),x]')`

[Out] $-3 \cdot 2^{1/3} \operatorname{hyper}[\{2/3, 2/3\}, \{5/3\}, 27 \exp_{\text{polar}}[2 \text{ I Pi}] / (2 x^7)] / (28 x^{14/3})$

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 6.23, size = 74, normalized size = 1.25

method	result
meijerg	$\frac{\left(-\operatorname{signum}\left(-1+\frac{2x^7}{27}\right)\right)^{\frac{2}{3}} \left(\left(\frac{\pi\sqrt{3}}{6} - \frac{9\ln(3)}{2} + 7\ln(x) + \ln(2) + i\pi \right) \Gamma\left(\frac{2}{3}\right) + \frac{4\Gamma\left(\frac{2}{3}\right)x^7 \operatorname{hypergeom}\left(\left[1, 1, \frac{5}{3}\right], [2, 2], \frac{2x^7}{27}\right)}{81} \right)}{63\operatorname{signum}\left(-1+\frac{2x^7}{27}\right)^{\frac{2}{3}} \Gamma\left(\frac{2}{3}\right)}$
trager	$\operatorname{RootOf}\left(81_Z^2 + 9_Z + 1\right) \ln\left(-\frac{757355840490254039191854 \operatorname{RootOf}\left(81_Z^2 + 9_Z + 1\right)^2 x^7 + 48949965800622396478998 \operatorname{RootOf}\left(81_Z^2 + 9_Z + 1\right)}{\dots}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(2*x^7-27)^(2/3),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{63} \operatorname{signum}\left(-1+\frac{2x^7}{27}\right)^{\frac{2}{3}} \left(-\operatorname{signum}\left(-1+\frac{2x^7}{27}\right) \left(\frac{1}{6} \pi \sqrt{3} - 9 \ln(3) + 7 \ln(x) + \ln(2) + i \pi \right) \Gamma\left(\frac{2}{3}\right) + \frac{4}{81} \Gamma\left(\frac{2}{3}\right) x^7 \operatorname{hypergeom}\left(\left[1, 1, \frac{5}{3}\right], [2, 2], \frac{2x^7}{27}\right) \right) / \Gamma\left(\frac{2}{3}\right)$

Maxima [A]

time = 0.35, size = 64, normalized size = 1.08

$$\frac{1}{63} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3} \left(2(2x^7 - 27)^{\frac{1}{3}} - 3\right)\right) - \frac{1}{126} \log\left((2x^7 - 27)^{\frac{2}{3}} - 3(2x^7 - 27)^{\frac{1}{3}} + 9\right) + \frac{1}{63} \log\left((2x^7 - 27)^{\frac{1}{3}} + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(2*x^7-27)^(2/3),x, algorithm="maxima")`

[Out] $\frac{1}{63} \sqrt{3} \arctan\left(\frac{1}{9} \sqrt{3} \left(2(2x^7 - 27)^{\frac{1}{3}} - 3\right)\right) - \frac{1}{126} \log\left((2x^7 - 27)^{\frac{2}{3}} - 3(2x^7 - 27)^{\frac{1}{3}} + 9\right) + \frac{1}{63} \log\left((2x^7 - 27)^{\frac{1}{3}} + 3\right)$

Fricas [A]

time = 0.30, size = 66, normalized size = 1.12

$$\frac{1}{63} \sqrt{3} \arctan\left(\frac{2}{9} \sqrt{3} (2x^7 - 27)^{\frac{1}{3}} - \frac{1}{3} \sqrt{3}\right) - \frac{1}{126} \log\left((2x^7 - 27)^{\frac{2}{3}} - 3(2x^7 - 27)^{\frac{1}{3}} + 9\right) + \frac{1}{63} \log\left((2x^7 - 27)^{\frac{1}{3}} + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(2*x^7-27)^(2/3),x, algorithm="fricas")`

[Out] $\frac{1}{63}\sqrt{3}\arctan\left(\frac{2}{9}\sqrt{3}\left(2x^7 - 27\right)^{1/3} - \frac{1}{3}\sqrt{3}\right) - \frac{1}{126}\log\left(\left(2x^7 - 27\right)^{2/3} - 3\left(2x^7 - 27\right)^{1/3} + 9\right) + \frac{1}{63}\log\left(\left(2x^7 - 27\right)^{1/3} + 3\right)$

Sympy [C] Result contains complex when optimal does not.

time = 0.48, size = 42, normalized size = 0.71

$$-\frac{\sqrt[3]{2}\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{2}{3} \middle| \frac{27e^{2i\pi}}{2x^7}\right)}{14x^{\frac{14}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(2*x**7-27)**(2/3),x)`

[Out] $-2^{1/3}\Gamma(2/3)\operatorname{hyper}\left(\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, 27\exp_{\text{polar}}(2I\pi)/(2x^{**7})\right)/(14x^{14/3}\Gamma(5/3))$

Giac [A]

time = 0.00, size = 124, normalized size = 2.10

$$\frac{3}{7}\left(-\frac{\ln\left(\left((2x^7 - 27)^{1/3}\right)^2 - 27^{1/3}(2x^7 - 27)^{1/3} + 27^{2/3}\cdot 27^{1/3}\right)}{54} + \frac{1}{27}\sqrt{3}\arctan\left(\frac{2\left((2x^7 - 27)^{1/3} - \frac{27^{1/3}}{2}\right)}{\sqrt{3}\cdot 27^{1/3}}\right) + \frac{1}{81}\cdot 27^{1/3}\ln\left|(2x^7 - 27)^{1/3} + 27^{1/3}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(2*x^7-27)^(2/3),x)`

[Out] $\frac{1}{63}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\left(2\left(2x^7 - 27\right)^{1/3} - 3\right)\right) - \frac{1}{126}\log\left(\left(2x^7 - 27\right)^{2/3} - 3\left(2x^7 - 27\right)^{1/3} + 9\right) + \frac{1}{63}\log\left(\left|2x^7 - 27\right|^{1/3} + 3\right)$

Mupad [B]

time = 0.46, size = 76, normalized size = 1.29

$$\frac{\ln\left(\frac{(2x^7-27)^{1/3}}{49} + \frac{3}{49}\right)}{63} - \ln\left(\frac{27}{14} - \frac{9(2x^7-27)^{1/3}}{7} + \frac{\sqrt{3}27i}{14}\right)\left(\frac{1}{126} + \frac{\sqrt{3}i}{126}\right) + \ln\left(\frac{9(2x^7-27)^{1/3}}{7} - \frac{27}{14} + \frac{\sqrt{3}27i}{14}\right)\left(-\frac{1}{126} + \frac{\sqrt{3}i}{126}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(2*x^7 - 27)^(2/3)),x)`

[Out] $\log\left(\frac{(2x^7 - 27)^{1/3}/49 + 3/49}{63}\right) - \log\left(\frac{(3^{1/2}27i)/14 - (9(2x^7 - 27)^{1/3})/7 + 27/14}{(3^{1/2}i)/126 + 1/126}\right) + \log\left(\frac{(3^{1/2}27i)/14 + (9(2x^7 - 27)^{1/3})/7 - 27/14}{(3^{1/2}i)/126 - 1/126}\right)$

3.302

$$\int \frac{(1+x^7)^{2/3}}{x^8} dx$$

Optimal. Leaf size=70

$$-\frac{(1+x^7)^{2/3}}{7x^7} + \frac{2 \tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^7}}{\sqrt{3}}\right)}{7\sqrt{3}} - \frac{\log(x)}{3} + \frac{1}{7} \log\left(1 - \sqrt[3]{1+x^7}\right)$$

[Out] -1/7*(x^7+1)^(2/3)/x^7-1/3*ln(x)+1/7*ln(1-(x^7+1)^(1/3))+2/21*arctan(1/3*(1+2*(x^7+1)^(1/3))*3^(1/2))*3^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {272, 43, 57, 632, 210, 31}

$$-\frac{(x^7+1)^{2/3}}{7x^7} + \frac{1}{7} \log\left(1 - \sqrt[3]{x^7+1}\right) + \frac{2 \tan^{-1}\left(\frac{2\sqrt[3]{x^7+1}+1}{\sqrt{3}}\right)}{7\sqrt{3}} - \frac{\log(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^7)^(2/3)/x^8, x]

[Out] -1/7*(1 + x^7)^(2/3)/x^7 + (2*ArcTan[(1 + 2*(1 + x^7)^(1/3))/Sqrt[3]])/(7*Sqrt[3]) - Log[x]/3 + Log[1 - (1 + x^7)^(1/3)]/7

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && GtQ[n, 0]

Rule 57

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(1+x^7)^{2/3}}{x^8} dx &= \frac{1}{7} \text{Subst} \left(\int \frac{(1+x)^{2/3}}{x^2} dx, x, x^7 \right) \\
 &= -\frac{(1+x^7)^{2/3}}{7x^7} + \frac{2}{21} \text{Subst} \left(\int \frac{1}{x^3 \sqrt[3]{1+x}} dx, x, x^7 \right) \\
 &= -\frac{(1+x^7)^{2/3}}{7x^7} - \frac{\log(x)}{3} - \frac{1}{7} \text{Subst} \left(\int \frac{1}{1-x} dx, x, \sqrt[3]{1+x^7} \right) + \frac{1}{7} \text{Subst} \left(\int \frac{1}{1+x+x^2} dx, x, \sqrt[3]{1+x^7} \right) \\
 &= -\frac{(1+x^7)^{2/3}}{7x^7} - \frac{\log(x)}{3} + \frac{1}{7} \log \left(1 - \sqrt[3]{1+x^7} \right) - \frac{2}{7} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 + 2\sqrt[3]{1+x^7} \right) \\
 &= -\frac{(1+x^7)^{2/3}}{7x^7} + \frac{2 \tan^{-1} \left(\frac{1+2\sqrt[3]{1+x^7}}{\sqrt{3}} \right)}{7\sqrt{3}} - \frac{\log(x)}{3} + \frac{1}{7} \log \left(1 - \sqrt[3]{1+x^7} \right)
 \end{aligned}$$

Mathematica [A]

time = 0.08, size = 83, normalized size = 1.19

$$\frac{1}{21} \left(-\frac{3(1+x^7)^{2/3}}{x^7} + 2\sqrt{3} \tan^{-1} \left(\frac{1+2\sqrt[3]{1+x^7}}{\sqrt{3}} \right) + 2 \log \left(-1 + \sqrt[3]{1+x^7} \right) - \log \left(1 + \sqrt[3]{1+x^7} + (1+x^7)^{2/3} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^7)^(2/3)/x^8, x]

[Out] $((-3*(1 + x^7)^{(2/3)})/x^7 + 2*\text{Sqrt}[3]*\text{ArcTan}[(1 + 2*(1 + x^7)^{(1/3)})/\text{Sqrt}[3]] + 2*\text{Log}[-1 + (1 + x^7)^{(1/3)}] - \text{Log}[1 + (1 + x^7)^{(1/3)} + (1 + x^7)^{(2/3)}])/21$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.38, size = 19, normalized size = 0.27

$$\frac{-3\text{hyper}\left[\left\{-\frac{2}{3}, \frac{1}{3}\right\}, \left\{\frac{4}{3}\right\}, \frac{\text{exp_polar}[i\text{Pi}]}{x^7}\right]}{7x^{\frac{7}{3}}}$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(x^7 + 1)^(2/3)/x^8,x]')`

[Out] `-3 hyper[{-2 / 3, 1 / 3}, {4 / 3}, exp_polar[I Pi] / x ^ 7] / (7 x ^ (7 / 3))`

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.

time = 6.23, size = 76, normalized size = 1.09

method	result
meijerg	$\frac{\sqrt{3} \Gamma\left(\frac{2}{3}\right) \left(\frac{\pi \sqrt{3}}{\Gamma\left(\frac{2}{3}\right) x^7} - \frac{2 \left(-\frac{\pi \sqrt{3}}{6} - \frac{3 \ln(3)}{2} - 1 + 7 \ln(x) \right) \pi \sqrt{3}}{3 \Gamma\left(\frac{2}{3}\right)} + \frac{\pi \sqrt{3} x^7 \text{hypergeom}\left(\left[1, 1, \frac{4}{3}\right], [2, 3], -x^7\right)}{9 \Gamma\left(\frac{2}{3}\right)} \right)}{21 \pi}$
risch	$-\frac{(x^7+1)^{\frac{2}{3}}}{7x^7} + \frac{\sqrt{3} \Gamma\left(\frac{2}{3}\right) \left(\frac{2 \left(-\frac{\pi \sqrt{3}}{6} - \frac{3 \ln(3)}{2} + 7 \ln(x) \right) \pi \sqrt{3}}{3 \Gamma\left(\frac{2}{3}\right)} - \frac{2 \pi \sqrt{3} x^7 \text{hypergeom}\left(\left[1, 1, \frac{4}{3}\right], [2, 2], -x^7\right)}{9 \Gamma\left(\frac{2}{3}\right)} \right)}{21 \pi}$
trager	$-\frac{(x^7+1)^{\frac{2}{3}}}{7x^7} + \frac{2 \ln\left(-\frac{3914010 \text{RootOf}\left(9_Z^2+3_Z+1\right)^2 x^7 - 2502441 \text{RootOf}\left(9_Z^2+3_Z+1\right) x^7 - 71266x^7 + 6095754 (x^7+1)^{\frac{2}{3}} \text{RootOf}\left(9_Z^2+3_Z+1\right)}{\dots} \right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^7+1)^(2/3)/x^8,x,method=_RETURNVERBOSE)`

[Out] `-1/21/Pi*3^(1/2)*GAMMA(2/3)*(Pi*3^(1/2)/GAMMA(2/3)/x^7-2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)-1+7*ln(x))*Pi*3^(1/2)/GAMMA(2/3)+1/9*Pi*3^(1/2)/GAMMA(2/3)*x^7*hypergeom([1,1,4/3],[2,3],-x^7))`

Maxima [A]

time = 0.34, size = 66, normalized size = 0.94

$$\frac{2}{21} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2(x^7+1)^{\frac{1}{3}}+1\right)\right) - \frac{(x^7+1)^{\frac{2}{3}}}{7x^7} - \frac{1}{21} \log\left((x^7+1)^{\frac{2}{3}}+(x^7+1)^{\frac{1}{3}}+1\right) + \frac{2}{21} \log\left((x^7+1)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^7+1)^(2/3)/x^8,x, algorithm="maxima")

[Out] $\frac{2}{21}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\right)\left(2(x^7+1)^{1/3}+1\right) - \frac{1}{7}(x^7+1)^{2/3}/x^7 - \frac{1}{21}\log\left((x^7+1)^{2/3}+(x^7+1)^{1/3}+1\right) + \frac{2}{21}\log\left((x^7+1)^{1/3}-1\right)$

Fricas [A]

time = 0.30, size = 79, normalized size = 1.13

$$\frac{2\sqrt{3}x^7\arctan\left(\frac{2}{3}\sqrt{3}(x^7+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right) - x^7\log\left((x^7+1)^{\frac{2}{3}}+(x^7+1)^{\frac{1}{3}}+1\right) + 2x^7\log\left((x^7+1)^{\frac{1}{3}}-1\right) - 3(x^7+1)^{\frac{2}{3}}}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^7+1)^(2/3)/x^8,x, algorithm="fricas")

[Out] $\frac{1}{21}\left(2\sqrt{3}x^7\arctan\left(\frac{2}{3}\sqrt{3}\right)\left(x^7+1\right)^{1/3} + \frac{1}{3}\sqrt{3}\right) - x^7\log\left((x^7+1)^{2/3}+(x^7+1)^{1/3}+1\right) + 2x^7\log\left((x^7+1)^{1/3}-1\right) - 3(x^7+1)^{2/3}/x^7$

Sympy [C] Result contains complex when optimal does not.

time = 0.79, size = 34, normalized size = 0.49

$$\frac{\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{e^{i\pi}}{x^7}\right)}{7x^{\frac{7}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**7+1)**(2/3)/x**8,x)

[Out] $-\text{gamma}(1/3)\text{hyper}\left(-2/3, 1/3, (4/3,)\right) \exp_polar(I\pi)/x^{**7}/(7*x^{**}(7/3)*\text{gamma}(4/3))$

Giac [A]

time = 0.00, size = 94, normalized size = 1.34

$$\frac{\frac{2}{3}\ln\left|(x^7+1)^{\frac{1}{3}}-1\right| - \frac{\ln\left(\left((x^7+1)^{\frac{1}{3}}\right)^2+(x^7+1)^{\frac{1}{3}}+1\right)}{3} + \frac{2\arctan\left(\frac{2(x^7+1)^{\frac{1}{3}}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\left((x^7+1)^{\frac{1}{3}}\right)^2}{x^7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^7+1)^(2/3)/x^8,x)

[Out] $2/21*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*(x^7 + 1)^{(1/3)} + 1)) - 1/7*(x^7 + 1)^{(2/3)}/x^7 - 1/21*\log((x^7 + 1)^{(2/3)} + (x^7 + 1)^{(1/3)} + 1) + 2/21*\log(\text{abs}((x^7 + 1)^{(1/3)} - 1))$

Mupad [B]

time = 0.37, size = 92, normalized size = 1.31

$$\frac{2 \ln\left(\frac{4(x^7+1)^{1/3}}{49} - \frac{4}{49}\right)}{21} + \ln\left(\frac{4(x^7+1)^{1/3}}{49} - 9\left(-\frac{1}{21} + \frac{\sqrt{3} \text{li}}{21}\right)^2\right) \left(-\frac{1}{21} + \frac{\sqrt{3} \text{li}}{21}\right) - \ln\left(\frac{4(x^7+1)^{1/3}}{49} - 9\left(\frac{1}{21} + \frac{\sqrt{3} \text{li}}{21}\right)^2\right) \left(\frac{1}{21} + \frac{\sqrt{3} \text{li}}{21}\right) - \frac{(x^7+1)^{2/3}}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^7 + 1)^{(2/3)}/x^8, x)$

[Out] $(2*\log((4*(x^7 + 1)^{(1/3)})/49 - 4/49))/21 + \log((4*(x^7 + 1)^{(1/3)})/49 - 9*((3^{(1/2)}*1i)/21 - 1/21)^2)*((3^{(1/2)}*1i)/21 - 1/21) - \log((4*(x^7 + 1)^{(1/3)})/49 - 9*((3^{(1/2)}*1i)/21 + 1/21)^2)*((3^{(1/2)}*1i)/21 + 1/21) - (x^7 + 1)^{(2/3)}/(7*x^7)$

3.303

$$\int \frac{\sqrt[4]{3 + 4x^4}}{x^2} dx$$

Optimal. Leaf size=68

$$-\frac{\sqrt[4]{3 + 4x^4}}{x} - \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{3 + 4x^4}}\right)}{\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{3 + 4x^4}}\right)}{\sqrt{2}}$$

[Out] $-(4*x^4+3)^{(1/4)}/x-1/2*\arctan(x*2^{(1/2)}/(4*x^4+3)^{(1/4)})*2^{(1/2)}+1/2*\arctan$
 $h(x*2^{(1/2)}/(4*x^4+3)^{(1/4)})*2^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {283, 338, 304, 209, 212}

$$-\frac{\sqrt[4]{4x^4 + 3}}{x} - \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4 + 3}}\right)}{\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4 + 3}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(3 + 4*x^4)^(1/4)/x^2,x]

[Out] $-((3 + 4*x^4)^{(1/4)}/x) - \text{ArcTan}[(\text{Sqrt}[2]*x)/(3 + 4*x^4)^{(1/4)}]/\text{Sqrt}[2] + \text{ArcTanh}[(\text{Sqrt}[2]*x)/(3 + 4*x^4)^{(1/4)}]/\text{Sqrt}[2]$

Rule 209

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 283

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-a/b,
  2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a
/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[a^(p + (m +
  1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt[4]{3+4x^4}}{x^2} dx &= -\frac{\sqrt[4]{3+4x^4}}{x} + 4 \int \frac{x^2}{(3+4x^4)^{3/4}} dx \\ &= -\frac{\sqrt[4]{3+4x^4}}{x} + 4 \operatorname{Subst} \left(\int \frac{x^2}{1-4x^4} dx, x, \frac{x}{\sqrt[4]{3+4x^4}} \right) \\ &= -\frac{\sqrt[4]{3+4x^4}}{x} + \operatorname{Subst} \left(\int \frac{1}{1-2x^2} dx, x, \frac{x}{\sqrt[4]{3+4x^4}} \right) - \operatorname{Subst} \left(\int \frac{1}{1+2x^2} dx, x, \frac{x}{\sqrt[4]{3+4x^4}} \right) \\ &= -\frac{\sqrt[4]{3+4x^4}}{x} - \frac{\tan^{-1} \left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}} \right)}{\sqrt{2}} + \frac{\tanh^{-1} \left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}} \right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A]

time = 0.13, size = 68, normalized size = 1.00

$$-\frac{\sqrt[4]{3+4x^4}}{x} - \frac{\tan^{-1} \left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}} \right)}{\sqrt{2}} + \frac{\tanh^{-1} \left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}} \right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x^4)^(1/4)/x^2, x]
```

```
[Out] -((3 + 4*x^4)^(1/4)/x) - ArcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2] + Ar
cTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.21, size = 29, normalized size = 0.43

$$\frac{3^{\frac{1}{4}} \text{Gamma}\left[-\frac{1}{4}\right] \text{hyper}\left[\left\{-\frac{1}{4}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \frac{4x^4 \exp_{\text{polar}}[i\text{Pi}]}{3}\right]}{4x \text{Gamma}\left[\frac{3}{4}\right]}$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(3 + 4*x^4)^(1/4)/x^2,x]')`

[Out] `3^(1/4) Gamma[-1/4] hyper[{-1/4, -1/4}, {3/4}, 4 x^4 exp_polar[I Pi] / 3] / (4 x Gamma[3/4])`

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 2.44, size = 20, normalized size = 0.29

method	result
meijerg	$-\frac{3^{\frac{1}{4}} \text{hypergeom}\left(\left[-\frac{1}{4}, -\frac{1}{4}\right], \left[\frac{3}{4}\right], -\frac{4x^4}{3}\right)}{x}$
risch	$-\frac{(4x^4+3)^{\frac{1}{4}}}{x} + \frac{4 \cdot 3^{\frac{1}{4}} x^3 \text{hypergeom}\left(\left[\frac{3}{4}, \frac{3}{4}\right], \left[\frac{7}{4}\right], -\frac{4x^4}{3}\right)}{9}$
trager	$-\frac{(4x^4+3)^{\frac{1}{4}}}{x} + \frac{\text{RootOf}(_Z^2+2) \ln\left(-4\sqrt{4x^4+3} \text{RootOf}(_Z^2+2)x^2+8\text{RootOf}(_Z^2+2)x^4-4(4x^4+3)^{\frac{3}{4}}x+8x^3(4x^4+3)^{\frac{3}{4}}\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^4+3)^(1/4)/x^2,x,method=_RETURNVERBOSE)`

[Out] `-3^(1/4)/x*hypergeom([-1/4, -1/4], [3/4], -4/3*x^4)`

Maxima [A]

time = 0.35, size = 83, normalized size = 1.22

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{\sqrt{2}(4x^4+3)^{\frac{1}{4}}}{2x}\right) - \frac{1}{4} \sqrt{2} \log\left(-\frac{\sqrt{2} - \frac{(4x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2} + \frac{(4x^4+3)^{\frac{1}{4}}}{x}}\right) - \frac{(4x^4+3)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^4+3)^(1/4)/x^2,x, algorithm="maxima")`

[Out] `1/2*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 1/4*sqrt(2)*log(-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) - (4*x^4 + 3)^(1/4)/x`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(55) = 110.

time = 2.26, size = 146, normalized size = 2.15

$$\frac{2\sqrt{2}x \arctan\left(\frac{\sqrt{2}(4x^4+3)^{\frac{1}{4}}}{2x}\right) - \sqrt{2}x \log\left(-\frac{\sqrt{2} - \frac{(4x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2} + \frac{(4x^4+3)^{\frac{1}{4}}}{x}}\right) - (4x^4+3)^{\frac{1}{4}}}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+3)^(1/4)/x^2,x, algorithm="fricas")

[Out] $-\frac{1}{8} \cdot (2\sqrt{2}) \cdot x \cdot \arctan\left(\frac{4\sqrt{2}}{3} \cdot (4x^4+3)^{1/4}\right) \cdot x^3 + \frac{2}{3} \sqrt{2} \cdot (4x^4+3)^{3/4} \cdot x - \sqrt{2} \cdot x \cdot \log(-256x^8 - 192x^4 - 4\sqrt{2} \cdot (16x^5 + 3x) \cdot (4x^4+3)^{3/4} - 8\sqrt{2} \cdot (16x^7 + 9x^3) \cdot (4x^4+3)^{1/4} - 16 \cdot (8x^6 + 3x^2) \cdot \sqrt{4x^4+3} - 9) + 8 \cdot (4x^4+3)^{1/4})/x$

Sympy [C] Result contains complex when optimal does not.

time = 0.56, size = 41, normalized size = 0.60

$$\frac{\sqrt[4]{3} \Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix} \middle| \frac{4x^4 e^{i\pi}}{3}\right)}{4x \Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**4+3)**(1/4)/x**2,x)

[Out] $3^{1/4} \cdot \gamma(-1/4) \cdot \text{hyper}((-1/4, -1/4), (3/4,), 4x^{**4} \cdot \exp(\text{polar}(I \cdot \pi)/3)/ (4x \cdot \gamma(3/4)))$

Giac [A]

time = 0.01, size = 104, normalized size = 1.53

$$-\frac{(4x^4+3)^{1/4}}{x} + \frac{\arctan\left(\frac{(4x^4+3)^{1/4}}{\sqrt{2}x}\right)}{\sqrt{2}} - \frac{\ln\left(\frac{-2\sqrt{2} + \frac{2(4x^4+3)^{1/4}}{x}}{2\sqrt{2} + \frac{2(4x^4+3)^{1/4}}{x}}\right)}{2\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+3)^(1/4)/x^2,x)

[Out] $\frac{1}{2} \sqrt{2} \cdot \arctan\left(\frac{1}{2} \sqrt{2} \cdot (4x^4+3)^{1/4}/x\right) - \frac{1}{4} \sqrt{2} \cdot \log\left(-\frac{\sqrt{2} - (4x^4+3)^{1/4}/x}{\sqrt{2} + (4x^4+3)^{1/4}/x}\right) - \frac{(4x^4+3)^{1/4}}{x}$

Mupad [B]

time = 0.42, size = 18, normalized size = 0.26

$$-\frac{3^{1/4} {}_2F_1\left(-\frac{1}{4}, -\frac{1}{4}; \frac{3}{4}; -\frac{4x^4}{3}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^4 + 3)^(1/4)/x^2,x)

[Out] $-(3^{1/4} \cdot \text{hypergeom}([-1/4, -1/4], 3/4, -(4x^4)/3))/x$

3.304 $\int x^2 (3 + 4x^4)^{5/4} dx$

Optimal. Leaf size=93

$$\frac{15}{32}x^3\sqrt[4]{3+4x^4} + \frac{1}{8}x^3(3+4x^4)^{5/4} - \frac{45 \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}} + \frac{45 \tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}$$

[Out] $15/32*x^3*(4*x^4+3)^{(1/4)}+1/8*x^3*(4*x^4+3)^{(5/4)}-45/256*\arctan(x*2^{(1/2)}/(4*x^4+3)^{(1/4)})*2^{(1/2)}+45/256*\operatorname{arctanh}(x*2^{(1/2)}/(4*x^4+3)^{(1/4)})*2^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,

Rules used = {285, 338, 304, 209, 212}

$$-\frac{45 \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{128\sqrt{2}} + \frac{45 \tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{128\sqrt{2}} + \frac{1}{8}(4x^4+3)^{5/4}x^3 + \frac{15}{32}\sqrt[4]{4x^4+3}x^3$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*(3 + 4*x^4)^{(5/4)}, x]$

[Out] $(15*x^3*(3 + 4*x^4)^{(1/4)})/32 + (x^3*(3 + 4*x^4)^{(5/4)})/8 - (45*\text{ArcTan}[(\text{Sqrt}[2]*x)/(3 + 4*x^4)^{(1/4)}])/(128*\text{Sqrt}[2]) + (45*\text{ArcTanh}[(\text{Sqrt}[2]*x)/(3 + 4*x^4)^{(1/4)}])/(128*\text{Sqrt}[2])$

Rule 209

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))*\text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 212

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]))*\text{ArcTanh}[\text{Rt}[-b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 285

$\text{Int}[(c_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}), x_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)}*((a + b*x^n)^p/(c*(m+n*p+1))), x] + \text{Dist}[a*n*(p/(m+n*p+1)), \text{Int}[(c*x)^m*(a + b*x^n)^{(p-1)}, x], x] /; \text{FreeQ}\{a, b, c, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{NeQ}[m+n*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 304

Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 338

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[a^(p + (m + 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegersQ[m, p + (m + 1)/n]

Rubi steps

$$\begin{aligned}
 \int x^2 (3 + 4x^4)^{5/4} dx &= \frac{1}{8}x^3 (3 + 4x^4)^{5/4} + \frac{15}{8} \int x^2 \sqrt[4]{3 + 4x^4} dx \\
 &= \frac{15}{32}x^3 \sqrt[4]{3 + 4x^4} + \frac{1}{8}x^3 (3 + 4x^4)^{5/4} + \frac{45}{32} \int \frac{x^2}{(3 + 4x^4)^{3/4}} dx \\
 &= \frac{15}{32}x^3 \sqrt[4]{3 + 4x^4} + \frac{1}{8}x^3 (3 + 4x^4)^{5/4} + \frac{45}{32} \text{Subst} \left(\int \frac{x^2}{1 - 4x^4} dx, x, \frac{x}{\sqrt[4]{3 + 4x^4}} \right) \\
 &= \frac{15}{32}x^3 \sqrt[4]{3 + 4x^4} + \frac{1}{8}x^3 (3 + 4x^4)^{5/4} + \frac{45}{128} \text{Subst} \left(\int \frac{1}{1 - 2x^2} dx, x, \frac{x}{\sqrt[4]{3 + 4x^4}} \right) - \frac{45}{128} \int \frac{1}{1 - 2x^2} dx \\
 &= \frac{15}{32}x^3 \sqrt[4]{3 + 4x^4} + \frac{1}{8}x^3 (3 + 4x^4)^{5/4} - \frac{45 \tan^{-1} \left(\frac{\sqrt{2} x}{\sqrt[4]{3 + 4x^4}} \right)}{128\sqrt{2}} + \frac{45 \tanh^{-1} \left(\frac{\sqrt{2} x}{\sqrt[4]{3 + 4x^4}} \right)}{128\sqrt{2}}
 \end{aligned}$$

Mathematica [A]

time = 0.22, size = 82, normalized size = 0.88

$$\frac{1}{32}x^3 \sqrt[4]{3 + 4x^4} (27 + 16x^4) - \frac{45 \tan^{-1} \left(\frac{\sqrt{2} x}{\sqrt[4]{3 + 4x^4}} \right)}{128\sqrt{2}} + \frac{45 \tanh^{-1} \left(\frac{\sqrt{2} x}{\sqrt[4]{3 + 4x^4}} \right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(3 + 4*x^4)^(5/4),x]

[Out] (x^3*(3 + 4*x^4)^(1/4)*(27 + 16*x^4))/32 - (45*ArcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(128*Sqrt[2]) + (45*ArcTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(128*Sqrt[2])

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.65, size = 22, normalized size = 0.24

$$3^{\frac{1}{4}} x^3 \text{hyper} \left[\left\{ -\frac{5}{4}, \frac{3}{4} \right\}, \left\{ \frac{7}{4} \right\}, \frac{4x^4 \exp_{\text{polar}} [IPi]}{3} \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[x^2*(3 + 4*x^4)^(5/4), x]')`

[Out] $3^{\frac{1}{4}} x^3 \text{hyper} \left[\left\{ -\frac{5}{4}, \frac{3}{4} \right\}, \left\{ \frac{7}{4} \right\}, \frac{4x^4 \exp_{\text{polar}} [IPi]}{3} \right]$

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.

time = 2.11, size = 19, normalized size = 0.20

method	result
meijerg	$3^{\frac{1}{4}} x^3 \text{hypergeom} \left(\left[-\frac{5}{4}, \frac{3}{4} \right], \left[\frac{7}{4} \right], -\frac{4x^4}{3} \right)$
risch	$\frac{x^3 (16x^4 + 27) (4x^4 + 3)^{\frac{1}{4}}}{32} + \frac{5 \cdot 3^{\frac{1}{4}} x^3 \text{hypergeom} \left(\left[\frac{3}{4}, \frac{3}{4} \right], \left[\frac{7}{4} \right], -\frac{4x^4}{3} \right)}{32}$
trager	$\frac{x^3 (16x^4 + 27) (4x^4 + 3)^{\frac{1}{4}}}{32} + \frac{45 \text{RootOf}(_Z^2 - 2) \ln \left(4 \text{RootOf}(_Z^2 - 2) \sqrt{4x^4 + 3} x^2 + 8 \text{RootOf}(_Z^2 - 2) x^4 + 4(4x^4 + 3)^{\frac{3}{4}} \right)}{512}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(4*x^4+3)^(5/4), x, method=_RETURNVERBOSE)`

[Out] $3^{\frac{1}{4}} x^3 \text{hypergeom} \left(\left[-\frac{5}{4}, \frac{3}{4} \right], \left[\frac{7}{4} \right], -\frac{4}{3} x^4 \right)$

Maxima [A]

time = 0.34, size = 130, normalized size = 1.40

$$\frac{45}{256} \sqrt{2} \arctan \left(\frac{\sqrt{2} (4x^4 + 3)^{\frac{1}{4}}}{2x} \right) - \frac{45}{512} \sqrt{2} \log \left(-\frac{\sqrt{2} - \frac{(4x^4 + 3)^{\frac{1}{4}}}{x}}{\sqrt{2} + \frac{(4x^4 + 3)^{\frac{1}{4}}}{x}} \right) + \frac{9 \left(\frac{20(4x^4 + 3)^{\frac{1}{4}}}{x} - \frac{9(4x^4 + 3)^{\frac{5}{4}}}{x^5} \right)}{32 \left(\frac{8(4x^4 + 3)}{x^4} - \frac{(4x^4 + 3)^2}{x^8} - 16 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(4*x^4+3)^(5/4), x, algorithm="maxima")`

[Out] $45/256 \sqrt{2} \arctan(1/2 \sqrt{2} (4x^4 + 3)^{\frac{1}{4}} / x) - 45/512 \sqrt{2} \log \left(-(\sqrt{2} - (4x^4 + 3)^{\frac{1}{4}} / x) / (\sqrt{2} + (4x^4 + 3)^{\frac{1}{4}} / x) \right) + 9/32 * \left(20 * (4x^4 + 3)^{\frac{1}{4}} / x - 9 * (4x^4 + 3)^{\frac{5}{4}} / x^5 \right) / (8 * (4x^4 + 3) / x^4 - (4x^4 + 3)^2 / x^8 - 16)$

Fricas [A]

time = 0.31, size = 105, normalized size = 1.13

$$\frac{45}{256} \sqrt{2} \arctan\left(\frac{\sqrt{2}(4x^4+3)^{\frac{1}{4}}}{2x}\right) + \frac{45}{512} \sqrt{2} \log\left(8x^4 + 4\sqrt{2}(4x^4+3)^{\frac{1}{4}}x^3 + 4\sqrt{4x^4+3}x^2 + 2\sqrt{2}(4x^4+3)^{\frac{3}{4}}x + 3\right) + \frac{1}{32}(16x^7 + 27x^3)(4x^4+3)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(4*x^4+3)^(5/4),x, algorithm="fricas")

[Out] 45/256*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 45/512*sqrt(2)*log(8*x^4 + 4*sqrt(2)*(4*x^4 + 3)^(1/4)*x^3 + 4*sqrt(4*x^4 + 3)*x^2 + 2*sqrt(2)*(4*x^4 + 3)^(3/4)*x + 3) + 1/32*(16*x^7 + 27*x^3)*(4*x^4 + 3)^(1/4)

Sympy [C] Result contains complex when optimal does not.

time = 0.97, size = 41, normalized size = 0.44

$$\frac{3 \cdot \sqrt[4]{3} x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{3}{4} \\ \frac{7}{4} \end{matrix} \middle| \frac{4x^4 e^{i\pi}}{3}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(4*x**4+3)**(5/4),x)

[Out] 3*3**(1/4)*x**3*gamma(3/4)*hyper((-5/4, 3/4), (7/4,), 4*x**4*exp_polar(I*pi)/3)/(4*gamma(7/4))

Giac [A]

time = 0.01, size = 144, normalized size = 1.55

$$\frac{-\frac{1}{288}x^8 \left(-\frac{540(4x^4+3)^{\frac{1}{4}}}{x} + \frac{243\left(4+\frac{3}{x^4}\right)(4x^4+3)^{\frac{1}{4}}}{x} \right) - \frac{135 \arctan\left(\frac{(4x^4+3)^{\frac{1}{4}}}{\sqrt{2}x}\right)}{128\sqrt{2}} + \frac{135 \ln\left(\frac{-2\sqrt{2} + \frac{2(4x^4+3)^{\frac{1}{4}}}{x}}{2\sqrt{2} + \frac{2(4x^4+3)^{\frac{1}{4}}}{x}}\right)}{256\sqrt{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(4*x^4+3)^(5/4),x)

[Out] 1/32*x^8*(9*(4*x^4 + 3)^(1/4)*(3/x^4 + 4)/x - 20*(4*x^4 + 3)^(1/4)/x) + 45/256*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 45/512*sqrt(2)*log(-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 (4x^4 + 3)^{5/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(4*x^4 + 3)^(5/4),x)`

[Out] `int(x^2*(4*x^4 + 3)^(5/4), x)`

3.305 $\int x^6 \sqrt[4]{3 + 4x^4} dx$

Optimal. Leaf size=93

$$\frac{3}{128}x^3\sqrt[4]{3+4x^4} + \frac{1}{8}x^7\sqrt[4]{3+4x^4} + \frac{27 \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}} - \frac{27 \tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}}$$

[Out] $3/128*x^3*(4*x^4+3)^{(1/4)}+1/8*x^7*(4*x^4+3)^{(1/4)}+27/1024*\arctan(x*2^{(1/2)}/(4*x^4+3)^{(1/4)})*2^{(1/2)}-27/1024*\operatorname{arctanh}(x*2^{(1/2)}/(4*x^4+3)^{(1/4)})*2^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$,

Rules used = {285, 327, 338, 304, 209, 212}

$$\frac{27 \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{512\sqrt{2}} - \frac{27 \tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{512\sqrt{2}} + \frac{1}{8}\sqrt[4]{4x^4+3}x^7 + \frac{3}{128}\sqrt[4]{4x^4+3}x^3$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^6*(3 + 4*x^4)^{(1/4)}, x]$

[Out] $(3*x^3*(3 + 4*x^4)^{(1/4)})/128 + (x^7*(3 + 4*x^4)^{(1/4)})/8 + (27*\text{ArcTan}[(\text{Sqrt}[2]*x)/(3 + 4*x^4)^{(1/4)}])/(512*\text{Sqrt}[2]) - (27*\text{ArcTanh}[(\text{Sqrt}[2]*x)/(3 + 4*x^4)^{(1/4)}])/(512*\text{Sqrt}[2])$

Rule 209

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))*\text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 212

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]))*\text{ArcTanh}[\text{Rt}[-b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 285

$\text{Int}[(c_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}), x_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)}*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + \text{Dist}[a*n*(p/(m + n*p + 1)), \text{Int}[(c*x)^m*(a + b*x^n)^{(p-1)}, x], x] /; \text{FreeQ}\{a, b, c, m\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{NeQ}[m + n*p + 1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-a/b,
  2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n
- 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[a^(p + (m +
1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int x^6 \sqrt[4]{3+4x^4} dx &= \frac{1}{8} x^7 \sqrt[4]{3+4x^4} + \frac{3}{8} \int \frac{x^6}{(3+4x^4)^{3/4}} dx \\
&= \frac{3}{128} x^3 \sqrt[4]{3+4x^4} + \frac{1}{8} x^7 \sqrt[4]{3+4x^4} - \frac{27}{128} \int \frac{x^2}{(3+4x^4)^{3/4}} dx \\
&= \frac{3}{128} x^3 \sqrt[4]{3+4x^4} + \frac{1}{8} x^7 \sqrt[4]{3+4x^4} - \frac{27}{128} \text{Subst} \left(\int \frac{x^2}{1-4x^4} dx, x, \frac{x}{\sqrt[4]{3+4x^4}} \right) \\
&= \frac{3}{128} x^3 \sqrt[4]{3+4x^4} + \frac{1}{8} x^7 \sqrt[4]{3+4x^4} - \frac{27}{512} \text{Subst} \left(\int \frac{1}{1-2x^2} dx, x, \frac{x}{\sqrt[4]{3+4x^4}} \right) + \frac{27}{512} \\
&= \frac{3}{128} x^3 \sqrt[4]{3+4x^4} + \frac{1}{8} x^7 \sqrt[4]{3+4x^4} + \frac{27 \tan^{-1} \left(\frac{\sqrt{2} x}{\sqrt[4]{3+4x^4}} \right)}{512 \sqrt{2}} - \frac{27 \tanh^{-1} \left(\frac{\sqrt{2} x}{\sqrt[4]{3+4x^4}} \right)}{512 \sqrt{2}}
\end{aligned}$$

Mathematica [A]

time = 0.21, size = 82, normalized size = 0.88

$$\frac{1}{128} x^3 \sqrt[4]{3+4x^4} (3+16x^4) + \frac{27 \tan^{-1} \left(\frac{\sqrt{2} x}{\sqrt[4]{3+4x^4}} \right)}{512 \sqrt{2}} - \frac{27 \tanh^{-1} \left(\frac{\sqrt{2} x}{\sqrt[4]{3+4x^4}} \right)}{512 \sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6*(3 + 4*x^4)^(1/4),x]

[Out] (x^3*(3 + 4*x^4)^(1/4)*(3 + 16*x^4))/128 + (27*ArcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(512*Sqrt[2]) - (27*ArcTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(512*Sqrt[2])

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.76, size = 23, normalized size = 0.25

$$\frac{3^{\frac{1}{4}} x^7 \text{hyper} \left[\left\{ -\frac{1}{4}, \frac{7}{4} \right\}, \left\{ \frac{11}{4} \right\}, \frac{4x^4 \exp_{\text{polar}}[i\text{Pi}]}{3} \right]}{7}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x^6*(3 + 4*x^4)^(1/4),x]')

[Out] 3^(1/4) x^7 hyper[{-1/4, 7/4}, {11/4}, 4 x^4 exp_polar[i Pi] / 3] / 7

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.

time = 2.08, size = 20, normalized size = 0.22

method	result
meijerg	$\frac{3^{\frac{1}{4}} x^7 \text{hypergeom} \left(\left[-\frac{1}{4}, \frac{7}{4} \right], \left[\frac{11}{4} \right], -\frac{4x^4}{3} \right)}{7}$
risch	$\frac{x^3 (16x^4 + 3) (4x^4 + 3)^{\frac{1}{4}}}{128} - \frac{3 \cdot 3^{\frac{1}{4}} x^3 \text{hypergeom} \left(\left[\frac{3}{4}, \frac{3}{4} \right], \left[\frac{7}{4} \right], -\frac{4x^4}{3} \right)}{128}$
trager	$\frac{x^3 (16x^4 + 3) (4x^4 + 3)^{\frac{1}{4}}}{128} - \frac{27 \text{RootOf}(_Z^2 - 2) \ln \left(4 \text{RootOf}(_Z^2 - 2) \sqrt{4x^4 + 3} x^2 + 8 \text{RootOf}(_Z^2 - 2) x^4 + 4(4x^4 + 3)^{\frac{3}{4}} x + \dots \right)}{2048}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(4*x^4+3)^(1/4),x,method=_RETURNVERBOSE)

[Out] 1/7*3^(1/4)*x^7*hypergeom([-1/4,7/4],[11/4],-4/3*x^4)

Maxima [A]

time = 0.36, size = 129, normalized size = 1.39

$$-\frac{27}{1024} \sqrt{2} \arctan \left(\frac{\sqrt{2} (4x^4 + 3)^{\frac{1}{4}}}{2x} \right) + \frac{27}{2048} \sqrt{2} \log \left(-\frac{\sqrt{2} - \frac{(4x^4 + 3)^{\frac{1}{4}}}{x}}{\sqrt{2} + \frac{(4x^4 + 3)^{\frac{1}{4}}}{x}} \right) - \frac{9 \left(\frac{12(4x^4 + 3)^{\frac{1}{4}}}{x} + \frac{(4x^4 + 3)^{\frac{5}{4}}}{x^5} \right)}{128 \left(\frac{8(4x^4 + 3)}{x^4} - \frac{(4x^4 + 3)^2}{x^8} - 16 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(4*x^4+3)^(1/4),x, algorithm="maxima")

[Out] $-27/1024*\sqrt{2}*\arctan(1/2*\sqrt{2}*(4*x^4 + 3)^{(1/4)}/x) + 27/2048*\sqrt{2}*\log(-(\sqrt{2} - (4*x^4 + 3)^{(1/4)}/x)/(\sqrt{2} + (4*x^4 + 3)^{(1/4)}/x)) - 9/128*(12*(4*x^4 + 3)^{(1/4)}/x + (4*x^4 + 3)^{(5/4)}/x^5)/(8*(4*x^4 + 3)/x^4 - (4*x^4 + 3)^2/x^8 - 16)$

Fricas [A]

time = 0.34, size = 105, normalized size = 1.13

$$-\frac{27}{1024}\sqrt{2}\arctan\left(\frac{\sqrt{2}(4x^4+3)^{\frac{1}{4}}}{2x}\right) + \frac{27}{2048}\sqrt{2}\log\left(8x^4 - 4\sqrt{2}(4x^4+3)^{\frac{1}{4}}x^3 + 4\sqrt{4x^4+3}x^2 - 2\sqrt{2}(4x^4+3)^{\frac{3}{4}}x + 3\right) + \frac{1}{128}(16x^7 + 3x^3)(4x^4+3)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(4*x^4+3)^(1/4),x, algorithm="fricas")

[Out] $-27/1024*\sqrt{2}*\arctan(1/2*\sqrt{2}*(4*x^4 + 3)^{(1/4)}/x) + 27/2048*\sqrt{2}*\log(8*x^4 - 4*\sqrt{2}*(4*x^4 + 3)^{(1/4)}*x^3 + 4*\sqrt{4*x^4 + 3}*x^2 - 2*\sqrt{2}*(4*x^4 + 3)^{(3/4)}*x + 3) + 1/128*(16*x^7 + 3*x^3)*(4*x^4 + 3)^{(1/4)}$

Sympy [C] Result contains complex when optimal does not.

time = 1.03, size = 39, normalized size = 0.42

$$\frac{\sqrt[4]{3} x^7 \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{7}{4} \\ \frac{11}{4} \end{matrix} \middle| \frac{4x^4 e^{i\pi}}{3}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(4*x**4+3)**(1/4),x)

[Out] $3**{(1/4)}*x**7*\text{gamma}(7/4)*\text{hyper}((-1/4, 7/4), (11/4,), 4*x**4*\text{exp_polar}(I*\text{pi})/3)/(4*\text{gamma}(11/4))$

Giac [A]

time = 0.01, size = 143, normalized size = 1.54

$$\frac{-\frac{1}{1152}x^8\left(\frac{324(4x^4+3)^{\frac{1}{4}}}{x} + \frac{27\left(4+\frac{3}{x^4}\right)(4x^4+3)^{\frac{1}{4}}}{x}\right) + \frac{81\arctan\left(\frac{(4x^4+3)^{\frac{1}{4}}}{\sqrt{2}x}\right)}{512\sqrt{2}} - \frac{81\ln\left(\frac{-2\sqrt{2} + \frac{2(4x^4+3)^{\frac{1}{4}}}{x}}{2\sqrt{2} + \frac{2(4x^4+3)^{\frac{1}{4}}}{x}}\right)}{1024\sqrt{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(4*x^4+3)^(1/4),x)

[Out] $1/128*x^8*((4*x^4 + 3)^{(1/4)}*(3/x^4 + 4)/x + 12*(4*x^4 + 3)^{(1/4)}/x) - 27/1024*\sqrt{2}*\arctan(1/2*\sqrt{2}*(4*x^4 + 3)^{(1/4)}/x) + 27/2048*\sqrt{2}*\log(-(\sqrt{2} - (4*x^4 + 3)^{(1/4)}/x)/(\sqrt{2} + (4*x^4 + 3)^{(1/4)}/x))$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^6 (4x^4 + 3)^{1/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(4*x^4 + 3)^(1/4), x)

[Out] int(x^6*(4*x^4 + 3)^(1/4), x)

3.306 $\int \sqrt[3]{x(1-x^2)} dx$

Optimal. Leaf size=93

$$\frac{1}{2}x\sqrt[3]{x(1-x^2)} + \frac{\tan^{-1}\left(\frac{2x-\sqrt[3]{x(1-x^2)}}{\sqrt{3}\sqrt[3]{x(1-x^2)}}\right)}{2\sqrt{3}} + \frac{\log(x)}{12} - \frac{1}{4}\log\left(x + \sqrt[3]{x(1-x^2)}\right)$$

[Out] $1/2*x*(x*(-x^2+1))^{(1/3)}+1/12*\ln(x)-1/4*\ln(x+(x*(-x^2+1))^{(1/3)})+1/6*\arctan(1/3*(2*x-(x*(-x^2+1))^{(1/3)})/(x*(-x^2+1))^{(1/3)}*3^{(1/2)})*3^{(1/2)}$

Rubi [A]

time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.39, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2004, 2029, 2057, 335, 281, 337}

$$\frac{1}{2}\sqrt[3]{x-x^3}x - \frac{(1-x^2)^{2/3}x^{2/3}\log\left(x^{2/3} + \sqrt[3]{1-x^2}\right)}{4(x-x^3)^{2/3}} - \frac{(1-x^2)^{2/3}x^{2/3}\tan^{-1}\left(\frac{1-\frac{2x^{2/3}}{\sqrt[3]{1-x^2}}}{\sqrt{3}}\right)}{2\sqrt{3}(x-x^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*(1-x^2))^{(1/3)}, x]$

[Out] $(x*(x-x^3)^{(1/3)})/2 - (x^{(2/3)}*(1-x^2)^{(2/3)}*\text{ArcTan}[(1-(2*x^{(2/3)}))/(1-x^2)^{(1/3)})/\text{Sqrt}[3]])/(2*\text{Sqrt}[3]*(x-x^3)^{(2/3)}) - (x^{(2/3)}*(1-x^2)^{(2/3)}*\text{Log}[x^{(2/3)} + (1-x^2)^{(1/3)}])/(4*(x-x^3)^{(2/3)})$

Rule 281

$\text{Int}[(x_)^{(m_*)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] := \text{With}[\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{(m+1)/k-1}*(a + b*x^{(n/k)})^{(p)}, x], x, x^{(k)}, x] /; k \neq 1] /; \text{FreeQ}[\{a, b, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$

Rule 335

$\text{Int}[(c_)*(x_)^{(m_*)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] := \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m+1)-1}*(a + b*(x^{(k*n)/c})^{(p)}, x], x, (c*x)^{(1/k)}, x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{Fractio}[\text{ractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 337

$\text{Int}[(x_)/((a_) + (b_)*(x_)^3)^{(2/3)}, x_Symbol] := \text{With}[\{q = \text{Rt}[b, 3]\}, \text{Simp}[-\text{ArcTan}[(1 + 2*q*(x/(a + b*x^3)^{(1/3)}))/\text{Sqrt}[3]]/(\text{Sqrt}[3]*q^2), x] - \text{Simp}[\text{Log}[q*x - (a + b*x^3)^{(1/3)}]/(2*q^2), x]] /; \text{FreeQ}[\{a, b\}, x]$

Rule 2004

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2029

Int[((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[x*((a*x^j + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && !IntegerQ[p] && LtQ[0, j, n] && GtQ[p, 0] && NeQ[n*p + 1, 0]

Rule 2057

Int[((c_)*(x_)^(m_))*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(FracPart[m] + j*FracPart[p]))*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rubi steps

$$\begin{aligned}
\int \sqrt[3]{x(1-x^2)} dx &= \int \sqrt[3]{x-x^3} dx \\
&= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{1}{3} \int \frac{x}{(x-x^3)^{2/3}} dx \\
&= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{(x^{2/3}(1-x^2)^{2/3}) \int \frac{\sqrt[3]{x}}{(1-x^2)^{2/3}} dx}{3(x-x^3)^{2/3}} \\
&= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{(x^{2/3}(1-x^2)^{2/3}) \text{Subst}\left(\int \frac{x^3}{(1-x^6)^{2/3}} dx, x, \sqrt[3]{x}\right)}{(x-x^3)^{2/3}} \\
&= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{(x^{2/3}(1-x^2)^{2/3}) \text{Subst}\left(\int \frac{x}{(1-x^3)^{2/3}} dx, x, x^{2/3}\right)}{2(x-x^3)^{2/3}} \\
&= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{(x^{2/3}(1-x^2)^{2/3}) \text{Subst}\left(\int \frac{x}{1+x^3} dx, x, \frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{2(x-x^3)^{2/3}} \\
&= \frac{1}{2}x\sqrt[3]{x-x^3} - \frac{(x^{2/3}(1-x^2)^{2/3}) \text{Subst}\left(\int \frac{1}{1+x} dx, x, \frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{6(x-x^3)^{2/3}} + \frac{(x^{2/3}(1-x^2)^{2/3}) \text{Subst}\left(\int \frac{1}{1-x} dx, x, \frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{6(x-x^3)^{2/3}} \\
&= \frac{1}{2}x\sqrt[3]{x-x^3} - \frac{x^{2/3}(1-x^2)^{2/3} \log\left(1 + \frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{6(x-x^3)^{2/3}} + \frac{(x^{2/3}(1-x^2)^{2/3}) \text{Subst}\left(\int \frac{1}{1-x} dx, x, \frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{12(x-x^3)^{2/3}} \\
&= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{x^{2/3}(1-x^2)^{2/3} \log\left(1 + \frac{x^{4/3}}{(1-x^2)^{2/3}} - \frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{12(x-x^3)^{2/3}} - \frac{x^{2/3}(1-x^2)^{2/3} \log\left(1 + \frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{6(x-x^3)^{2/3}} \\
&= \frac{1}{2}x\sqrt[3]{x-x^3} - \frac{x^{2/3}(1-x^2)^{2/3} \tan^{-1}\left(\frac{1 - \frac{2x^{2/3}}{\sqrt[3]{1-x^2}}}{\sqrt{3}}\right)}{2\sqrt{3}(x-x^3)^{2/3}} + \frac{x^{2/3}(1-x^2)^{2/3} \log\left(1 + \frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{12(x-x^3)^{2/3}}
\end{aligned}$$

Mathematica [A]

time = 0.44, size = 137, normalized size = 1.47

$$\frac{\sqrt[3]{x-x^3} \left(6x^{4/3} \sqrt[3]{-1+x^2} + 2\sqrt{3} \tan^{-1}\left(\frac{\sqrt{3} x^{2/3}}{x^{2/3} + 2\sqrt[3]{-1+x^2}}\right) + 2\log\left(-x^{2/3} + \sqrt[3]{-1+x^2}\right) - \log\left(x^{4/3} + x^{2/3} \sqrt[3]{-1+x^2} + (-1+x^2)^{2/3}\right) \right)}{12\sqrt[3]{x} \sqrt[3]{-1+x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(1 - x^2))^(1/3), x]

[Out] $((x - x^3)^{1/3} * (6 * x^{4/3} * (-1 + x^2)^{1/3} + 2 * \sqrt{3} * \text{ArcTan}[(\sqrt{3} * x^{2/3}) / (x^{2/3} + 2 * (-1 + x^2)^{1/3})] + 2 * \text{Log}[-x^{2/3} + (-1 + x^2)^{1/3}] - \text{Log}[x^{4/3} + x^{2/3} * (-1 + x^2)^{1/3} + (-1 + x^2)^{2/3}])) / (12 * x^{1/3} * (-1 + x^2)^{1/3})$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(x*(1 - x^2))^(1/3),x]')`

[Out] cought exception: maximum recursion depth exceeded

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.

time = 2.05, size = 15, normalized size = 0.16

method	result
meijerg	$\frac{3x^{4/3} \text{hypergeom}([\frac{-1}{3}, \frac{2}{3}], [\frac{5}{3}], x^2)}{4}$
trager	$\frac{x(-x^3+x)^{1/3}}{2} + \frac{\text{RootOf}(9_Z^2-3_Z+1) \ln(1395 \text{RootOf}(9_Z^2-3_Z+1)^2 x^2 - 6768 \text{RootOf}(9_Z^2-3_Z+1)(-x^3+x)^{2/3} + 4104 \text{RootOf}(9_Z^2-3_Z+1)^2)}{\text{RootOf}(9_Z^2-3_Z+1)}$
risch	$\frac{x(-x(x^2-1))^{1/3}}{2} + \frac{\ln\left(\frac{35x^4 \text{RootOf}(-_Z^2+6_Z+36)^2 + 1956 \text{RootOf}(-_Z^2+6_Z+36)x^4 - 175x^2 \text{RootOf}(-_Z^2+6_Z+36)^2 + 4104 \text{RootOf}(-_Z^2+6_Z+36)}{\text{RootOf}(-_Z^2+6_Z+36)}\right)}{\text{RootOf}(-_Z^2+6_Z+36)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(-x^2+1))^(1/3),x,method=_RETURNVERBOSE)`

[Out] $3/4 * x^{4/3} * \text{hypergeom}([-1/3, 2/3], [5/3], x^2)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x*(-x^2+1))^(1/3),x, algorithm="maxima")`

[Out] `integrate((-x^2 - 1)*x)^(1/3), x)`

Fricas [A]

time = 0.43, size = 99, normalized size = 1.06

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{44032959556\sqrt{3}(-x^3+x)^{\frac{1}{3}}x-\sqrt{3}(16754327161x^2-2707204793)+10524305234\sqrt{3}(-x^3+x)^{\frac{2}{3}}}{81835897185x^2-1102302937}\right)+\frac{1}{2}(-x^3+x)^{\frac{1}{3}}x-\frac{1}{12}\log\left(3(-x^3+x)^{\frac{1}{3}}x+3(-x^3+x)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x*(-x^2+1))^(1/3),x, algorithm="fricas")

[Out] $-\frac{1}{6}\sqrt{3}\arctan\left(\frac{44032959556\sqrt{3}(-x^3+x)^{\frac{1}{3}}x-\sqrt{3}(16754327161x^2-2707204793)+10524305234\sqrt{3}(-x^3+x)^{\frac{2}{3}}}{81835897185x^2-1102302937}\right)+\frac{1}{2}(-x^3+x)^{\frac{1}{3}}x-\frac{1}{12}\log\left(3(-x^3+x)^{\frac{1}{3}}x+3(-x^3+x)^{\frac{2}{3}}+1\right)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{x(1-x^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x*(-x**2+1))**(1/3),x)**[Out]** Integral((x*(1-x**2))**(1/3), x)**Giac [A]**

time = 0.00, size = 97, normalized size = 1.04

$$\frac{-\frac{\ln\left|\left(\frac{1}{x}\right)^2-1\right|^{\frac{1}{3}}+1}{3} + \frac{\ln\left(\left(\left(\frac{1}{x}\right)^2-1\right)^{\frac{1}{3}}\right)^2 - \left(\left(\frac{1}{x}\right)^2-1\right)^{\frac{1}{3}}+1}{6} - \frac{\arctan\left(\frac{2\left(\left(\frac{1}{x}\right)^2-1\right)^{\frac{1}{3}}-1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\left(\left(\frac{1}{x}\right)^2-1\right)^{\frac{1}{3}}}{\left(\frac{1}{x}\right)^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x*(-x^2+1))^(1/3),x)

[Out] $\frac{1}{2}x^2(1/x^2-1)^{\frac{1}{3}} - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2(1/x^2-1)^{\frac{1}{3}}-1)\right) + \frac{1}{12}\log\left(\frac{1}{3}(1/x^2-1)^{\frac{2}{3}} - (1/x^2-1)^{\frac{1}{3}} + 1\right) - \frac{1}{6}\log\left(\frac{1}{3}\left|\frac{1}{x^2-1}\right|^{\frac{1}{3}} + 1\right)$

Mupad [B]

time = 0.37, size = 29, normalized size = 0.31

$$\frac{3x(x-x^3)^{1/3} {}_2F_1\left(-\frac{1}{3}, \frac{2}{3}; \frac{5}{3}; x^2\right)}{4(1-x^2)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x*(x^2-1))^(1/3),x)**[Out]** (3*x*(x-x^3)^(1/3)*hypergeom([-1/3, 2/3], 5/3, x^2))/(4*(1-x^2)^(1/3))

3.307 $\int \sqrt{(1 + \sqrt[3]{x})x} dx$

Optimal. Leaf size=126

$$\frac{7}{64} \sqrt{(1 + \sqrt[3]{x})x} - \frac{21 \sqrt{(1 + \sqrt[3]{x})x}}{128 \sqrt[3]{x}} - \frac{7}{80} \sqrt[3]{x} \sqrt{(1 + \sqrt[3]{x})x} + \frac{3}{40} x^{2/3} \sqrt{(1 + \sqrt[3]{x})x} + \frac{3}{5} x \sqrt{(1 + \sqrt[3]{x})x}$$

[Out] 21/128*arctanh(x^(2/3)/((1+x^(1/3))*x)^(1/2))+7/64*((1+x^(1/3))*x)^(1/2)-21/128*((1+x^(1/3))*x)^(1/2)/x^(1/3)-7/80*x^(1/3)*((1+x^(1/3))*x)^(1/2)+3/40*x^(2/3)*((1+x^(1/3))*x)^(1/2)+3/5*x*((1+x^(1/3))*x)^(1/2)

Rubi [A]

time = 0.07, antiderivative size = 114, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2004, 2029, 2049, 2035, 2054, 212}

$$\frac{3}{5} \sqrt{x^{4/3} + x} x + \frac{3}{40} \sqrt{x^{4/3} + x} x^{2/3} - \frac{7}{80} \sqrt{x^{4/3} + x} \sqrt[3]{x} + \frac{7}{64} \sqrt{x^{4/3} + x} - \frac{21 \sqrt{x^{4/3} + x}}{128 \sqrt[3]{x}} + \frac{21}{128} \tanh^{-1} \left(\frac{x^{2/3}}{\sqrt{x^{4/3} + x}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(1 + x^(1/3))*x], x]

[Out] (7*Sqrt[x + x^(4/3)]/64 - (21*Sqrt[x + x^(4/3)]/(128*x^(1/3)) - (7*x^(1/3)*Sqrt[x + x^(4/3)]/80 + (3*x^(2/3)*Sqrt[x + x^(4/3)]/40 + (3*x*Sqrt[x + x^(4/3)]/5 + (21*ArcTanh[x^(2/3)/Sqrt[x + x^(4/3)]])/128

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2004

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rule 2029

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Simp[x*((a*x^j + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && !IntegerQ[p] && LtQ[0, j, n] && GtQ[p, 0] && NeQ[n*p + 1, 0]

Rule 2035

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] := Simp[-2*(Sqrt
[a*x^j + b*x^n]/(b*(n - 2)*x^(n - 1))), x] - Dist[a*((2*n - j - 2)/(b*(n -
2))), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
LtQ[2*(n - 1), j, n]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] := Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] := Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{(1 + \sqrt[3]{x})x} \, dx &= \int \sqrt{x + x^{4/3}} \, dx \\
&= \frac{3}{5}x\sqrt{x + x^{4/3}} + \frac{1}{10} \int \frac{x}{\sqrt{x + x^{4/3}}} \, dx \\
&= \frac{3}{40}x^{2/3}\sqrt{x + x^{4/3}} + \frac{3}{5}x\sqrt{x + x^{4/3}} - \frac{7}{80} \int \frac{x^{2/3}}{\sqrt{x + x^{4/3}}} \, dx \\
&= -\frac{7}{80}\sqrt[3]{x}\sqrt{x + x^{4/3}} + \frac{3}{40}x^{2/3}\sqrt{x + x^{4/3}} + \frac{3}{5}x\sqrt{x + x^{4/3}} + \frac{7}{96} \int \frac{\sqrt[3]{x}}{\sqrt{x + x^{4/3}}} \, dx \\
&= \frac{7}{64}\sqrt{x + x^{4/3}} - \frac{7}{80}\sqrt[3]{x}\sqrt{x + x^{4/3}} + \frac{3}{40}x^{2/3}\sqrt{x + x^{4/3}} + \frac{3}{5}x\sqrt{x + x^{4/3}} - \frac{7}{128} \int \frac{\sqrt[3]{x}}{\sqrt{x + x^{4/3}}} \, dx \\
&= \frac{7}{64}\sqrt{x + x^{4/3}} - \frac{21\sqrt{x + x^{4/3}}}{128\sqrt[3]{x}} - \frac{7}{80}\sqrt[3]{x}\sqrt{x + x^{4/3}} + \frac{3}{40}x^{2/3}\sqrt{x + x^{4/3}} + \frac{3}{5}x\sqrt{x + x^{4/3}} - \frac{7}{128} \int \frac{\sqrt[3]{x}}{\sqrt{x + x^{4/3}}} \, dx \\
&= \frac{7}{64}\sqrt{x + x^{4/3}} - \frac{21\sqrt{x + x^{4/3}}}{128\sqrt[3]{x}} - \frac{7}{80}\sqrt[3]{x}\sqrt{x + x^{4/3}} + \frac{3}{40}x^{2/3}\sqrt{x + x^{4/3}} + \frac{3}{5}x\sqrt{x + x^{4/3}} - \frac{7}{128} \int \frac{\sqrt[3]{x}}{\sqrt{x + x^{4/3}}} \, dx \\
&= \frac{7}{64}\sqrt{x + x^{4/3}} - \frac{21\sqrt{x + x^{4/3}}}{128\sqrt[3]{x}} - \frac{7}{80}\sqrt[3]{x}\sqrt{x + x^{4/3}} + \frac{3}{40}x^{2/3}\sqrt{x + x^{4/3}} + \frac{3}{5}x\sqrt{x + x^{4/3}} - \frac{7}{128} \int \frac{\sqrt[3]{x}}{\sqrt{x + x^{4/3}}} \, dx
\end{aligned}$$

Mathematica [A]

time = 0.12, size = 69, normalized size = 0.55

$$\frac{\sqrt{x + x^{4/3}} (-105 + 70\sqrt[3]{x} - 56x^{2/3} + 48x + 384x^{4/3})}{640\sqrt[3]{x}} + \frac{21}{128} \tanh^{-1}\left(\frac{x^{2/3}}{\sqrt{x + x^{4/3}}}\right)$$

Antiderivative was successfully verified.

`[In] Integrate[Sqrt[(1 + x^(1/3))*x], x]`

```
[Out] (Sqrt[x + x^(4/3)]*(-105 + 70*x^(1/3) - 56*x^(2/3) + 48*x + 384*x^(4/3)))/(
640*x^(1/3)) + (21*ArcTanh[x^(2/3)/Sqrt[x + x^(4/3)]])/128
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Sqrt[x*(1 + x^(1/3))], x]')`

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]

time = 0.06, size = 108, normalized size = 0.86

method	result
meijerg	$\frac{3 \left(\frac{\sqrt{\pi} x^{\frac{1}{6}} (-1152x^{\frac{4}{3}} - 144x + 168x^{\frac{2}{3}} - 210x^{\frac{1}{3}} + 315) \sqrt{x^{\frac{1}{3}} + 1}}{2880} - \frac{7\sqrt{\pi} \operatorname{arcsinh}\left(x^{\frac{1}{6}}\right)}{64} \right)}{2\sqrt{\pi}}$
derivativedivides	$\frac{\sqrt{\left(x^{\frac{1}{3}} + 1\right) x} \left(768x^{\frac{2}{3}} \left(x^{\frac{2}{3}} + x^{\frac{1}{3}}\right)^{\frac{3}{2}} - 672x^{\frac{1}{3}} \left(x^{\frac{2}{3}} + x^{\frac{1}{3}}\right)^{\frac{3}{2}} + 560 \left(x^{\frac{2}{3}} + x^{\frac{1}{3}}\right)^{\frac{3}{2}} - 420 \sqrt{x^{\frac{2}{3}} + x^{\frac{1}{3}}} x^{\frac{1}{3}} - 210 \sqrt{x^{\frac{2}{3}} + x^{\frac{1}{3}}} \right)}{1280x^{\frac{1}{3}} \sqrt{\left(x^{\frac{1}{3}} + 1\right) x^{\frac{1}{3}}}}$
default	$\frac{\sqrt{\left(x^{\frac{1}{3}} + 1\right) x} \left(768x^{\frac{2}{3}} \left(x^{\frac{2}{3}} + x^{\frac{1}{3}}\right)^{\frac{3}{2}} - 672x^{\frac{1}{3}} \left(x^{\frac{2}{3}} + x^{\frac{1}{3}}\right)^{\frac{3}{2}} + 560 \left(x^{\frac{2}{3}} + x^{\frac{1}{3}}\right)^{\frac{3}{2}} - 420 \sqrt{x^{\frac{2}{3}} + x^{\frac{1}{3}}} x^{\frac{1}{3}} - 210 \sqrt{x^{\frac{2}{3}} + x^{\frac{1}{3}}} \right)}{1280x^{\frac{1}{3}} \sqrt{\left(x^{\frac{1}{3}} + 1\right) x^{\frac{1}{3}}}}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(((x^(1/3)+1)*x)^(1/2), x, method=_RETURNVERBOSE)`

[Out] $\frac{1}{1280} * ((x^{1/3} + 1) * x)^{1/2} * (768 * x^{2/3} * (x^{2/3} + x^{1/3})^{3/2} - 672 * x^{1/3} * (x^{2/3} + x^{1/3})^{3/2} + 560 * (x^{2/3} + x^{1/3})^{3/2} - 420 * (x^{2/3} + x^{1/3})^{1/2} * x^{1/3} - 210 * (x^{2/3} + x^{1/3})^{1/2} + 105 * \ln(1/2 + x^{1/3} + (x^{2/3} + x^{1/3})^{1/2})) / x^{1/3} / ((x^{1/3} + 1) * x^{1/3})^{1/2}$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((1+x^(1/3))*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(x*(x^(1/3) + 1)), x)`

Fricas [A]

time = 43.78, size = 87, normalized size = 0.69

$$\frac{35x \log\left(\frac{32x^2 + 48x^{5/3} + 2(16x^{4/3} + 16x + 3x^{2/3})\sqrt{x^{4/3} + x + 18x^{1/3} + x}}{x}\right) + 2(384x^2 + 3(16x - 35)x^{2/3} - 56x^{4/3} + 70x)\sqrt{x^{4/3} + x}}{1280x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((1+x^(1/3))*x)^(1/2),x, algorithm="fricas")`

[Out] $\frac{1}{1280} * (35 * x * \log((32 * x^2 + 48 * x^{5/3} + 2 * (16 * x^{4/3} + 16 * x + 3 * x^{2/3})) * \sqrt{x^{4/3} + x + 18 * x^{1/3} + x}) / x + 2 * (384 * x^2 + 3 * (16 * x - 35) * x^{2/3} - 56 * x^{4/3} + 70 * x) * \sqrt{x^{4/3} + x}) / x$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x(\sqrt[3]{x} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((1+x**(1/3))*x)**(1/2),x)`

[Out] `Integral(sqrt(x*(x**(1/3) + 1)), x)`

Giac [A]

time = 0.01, size = 103, normalized size = 0.82

$$3 \operatorname{sign}(x) \left(2 \sqrt{\left(x^{1/3}\right)^2 + x^{1/3}} \left(x^{1/3} \left(x^{1/3} \left(\frac{x^{1/3}}{10} + \frac{1}{80} \right) - \frac{7}{480} \right) + \frac{7}{384} \right) - \frac{7}{256} \right) - \frac{7}{256} \ln \left| 2 \left(-x^{1/3} + \sqrt{\left(x^{1/3}\right)^2 + x^{1/3}} \right) - 1 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x^(1/3))*x)^(1/2),x)

[Out] 1/1280*(2*(2*(4*(6*x^(1/3))*(8*x^(1/3) + 1) - 7)*x^(1/3) + 35)*x^(1/3) - 105)*sqrt(x^(2/3) + x^(1/3)) - 105*log(abs(2*sqrt(x^(2/3) + x^(1/3)) - 2*x^(1/3) - 1))*sgn(x)

Mupad [B]

time = 0.29, size = 27, normalized size = 0.21

$$\frac{2x \sqrt{x + x^{4/3}} {}_2F_1\left(-\frac{1}{2}, \frac{9}{2}; \frac{11}{2}; -x^{1/3}\right)}{3 \sqrt{x^{1/3} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(x^(1/3) + 1))^(1/2),x)

[Out] (2*x*(x + x^(4/3))^(1/2)*hypergeom([-1/2, 9/2], 11/2, -x^(1/3)))/(3*(x^(1/3) + 1)^(1/2))

$$3.308 \quad \int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}} dx$$

Optimal. Leaf size=34

$$-\frac{\tanh^{-1}\left(\frac{1+2x^4}{\sqrt{3}\sqrt{1+2x^8}}\right)}{4\sqrt{3}}$$

[Out] -1/12*arctanh(1/3*(2*x^4+1)*3^(1/2)/(2*x^8+1)^(1/2))*3^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1483, 739, 212}

$$-\frac{\tanh^{-1}\left(\frac{2x^4+1}{\sqrt{3}\sqrt{2x^8+1}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((-1 + x^4)*Sqrt[1 + 2*x^8]),x]

[Out] -1/4*ArcTanh[(1 + 2*x^4)/(Sqrt[3]*Sqrt[1 + 2*x^8])]/Sqrt[3]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 739

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1483

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{(-1+x)\sqrt{1+2x^2}} dx, x, x^4 \right) \\ &= - \left(\frac{1}{4} \text{Subst} \left(\int \frac{1}{3-x^2} dx, x, \frac{1+2x^4}{\sqrt{1+2x^8}} \right) \right) \\ &= - \frac{\tanh^{-1} \left(\frac{1+2x^4}{\sqrt{3}\sqrt{1+2x^8}} \right)}{4\sqrt{3}} \end{aligned}$$

Mathematica [A]

time = 0.16, size = 41, normalized size = 1.21

$$-\frac{\tanh^{-1} \left(\frac{1}{3} \left(\sqrt{6} - \sqrt{6} x^4 + \sqrt{3 + 6x^8} \right) \right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

`[In] Integrate[x^3/((-1 + x^4)*Sqrt[1 + 2*x^8]),x]``[Out] -1/2*ArcTanh[(Sqrt[6] - Sqrt[6]*x^4 + Sqrt[3 + 6*x^8])/3]/Sqrt[3]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[x^3/((x^4 - 1)*Sqrt[2*x^8 + 1]),x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 0.15, size = 60, normalized size = 1.76

method	result	size
trager	$\frac{\text{RootOf}(_Z^2 - 3) \ln \left(\frac{-2 \text{RootOf}(_Z^2 - 3) x^4 + 3 \sqrt{2x^8 + 1} - \text{RootOf}(_Z^2 - 3)}{(-1+x)(1+x)(x^2+1)} \right)}{12}$	60

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^3/(x^4-1)/(2*x^8+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{12} \sqrt[3]{-Z^2-3} \ln\left(\frac{-(-2\sqrt[3]{-Z^2-3})x^4+3(2x^8+1)^{1/2}-\sqrt[3]{-Z^2-3}}{(-1+x)/(1+x)/(x^2+1)}\right)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(x^4-1)/(2*x^8+1)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^3/(sqrt(2*x^8 + 1)*(x^4 - 1)), x)`

Fricas [A]

time = 0.31, size = 49, normalized size = 1.44

$$\frac{1}{12} \sqrt{3} \log\left(\frac{2x^4 - \sqrt{3}(2x^4 + 1) - \sqrt{2x^8 + 1}(\sqrt{3} - 3) + 1}{x^4 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(x^4-1)/(2*x^8+1)^(1/2),x, algorithm="fricas")`

[Out] $\frac{1}{12} \sqrt{3} \log\left(\frac{(2x^4 - \sqrt{3})(2x^4 + 1) - \sqrt{2x^8 + 1}(\sqrt{3} - 3) + 1}{x^4 - 1}\right)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(x-1)(x+1)(x^2+1)\sqrt{2x^8+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(x**4-1)/(2*x**8+1)**(1/2),x)`

[Out] `Integral(x**3/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(2*x**8 + 1)), x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(27) = 54$.
time = 0.02, size = 92, normalized size = 2.71

$$\frac{\ln\left(\frac{\left|2(\sqrt{2x^8+1}-\sqrt{2}x^4)+2\sqrt{2}-2\sqrt{3}\right|}{2(\sqrt{2x^8+1}-\sqrt{2}x^4)+2\sqrt{2}+2\sqrt{3}}\right)}{4\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^4-1)/(2*x^8+1)^(1/2),x)

[Out] 1/12*sqrt(3)*log(-1/2*abs(-2*sqrt(2)*x^4 - 2*sqrt(3) + 2*sqrt(2) + 2*sqrt(2*x^8 + 1)))/(sqrt(2)*x^4 - sqrt(3) - sqrt(2) - sqrt(2*x^8 + 1))

Mupad [B]

time = 0.52, size = 35, normalized size = 1.03

$$\frac{\sqrt{3} \left(\ln \left(x^4 + \frac{\sqrt{2} \sqrt{3} \sqrt{x^8 + \frac{1}{2}}}{2} + \frac{1}{2} \right) - \ln(x^4 - 1) \right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((x^4 - 1)*(2*x^8 + 1)^(1/2)),x)

[Out] -(3^(1/2)*(log(x^4 + (2^(1/2)*3^(1/2)*(x^8 + 1/2)^(1/2))/2 + 1/2) - log(x^4 - 1)))/12

3.309 $\int x^9 \sqrt{1 + x^5 + x^{10}} dx$

Optimal. Leaf size=58

$$-\frac{1}{40} (1 + 2x^5) \sqrt{1 + x^5 + x^{10}} + \frac{1}{15} (1 + x^5 + x^{10})^{3/2} - \frac{3}{80} \sinh^{-1} \left(\frac{1 + 2x^5}{\sqrt{3}} \right)$$

[Out] 1/15*(x^10+x^5+1)^(3/2)-3/80*arcsinh(1/3*(2*x^5+1)*3^(1/2))-1/40*(2*x^5+1)*(x^10+x^5+1)^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1371, 654, 626, 633, 221}

$$-\frac{3}{80} \sinh^{-1} \left(\frac{2x^5 + 1}{\sqrt{3}} \right) + \frac{1}{15} (x^{10} + x^5 + 1)^{3/2} - \frac{1}{40} (2x^5 + 1) \sqrt{x^{10} + x^5 + 1}$$

Antiderivative was successfully verified.

[In] Int[x^9*Sqrt[1 + x^5 + x^10],x]

[Out] -1/40*((1 + 2*x^5)*Sqrt[1 + x^5 + x^10]) + (1 + x^5 + x^10)^(3/2)/15 - (3*ArcSinh[(1 + 2*x^5)/Sqrt[3]])/80

Rule 221

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 626

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x) * ((a + b*x + c*x^2)^p / (2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c) / (2*c*(2*p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 633

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 654

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[e*((a + b*x + c*x^2)^(p + 1) / (2*c*(p + 1))), x] + Dist[(2*c*d - b*e) / (2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]

&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int x^9 \sqrt{1+x^5+x^{10}} dx &= \frac{1}{5} \text{Subst} \left(\int x \sqrt{1+x+x^2} dx, x, x^5 \right) \\
 &= \frac{1}{15} (1+x^5+x^{10})^{3/2} - \frac{1}{10} \text{Subst} \left(\int \sqrt{1+x+x^2} dx, x, x^5 \right) \\
 &= -\frac{1}{40} (1+2x^5) \sqrt{1+x^5+x^{10}} + \frac{1}{15} (1+x^5+x^{10})^{3/2} - \frac{3}{80} \text{Subst} \left(\int \frac{1}{\sqrt{1+x+x^2}} dx, x, x^5 \right) \\
 &= -\frac{1}{40} (1+2x^5) \sqrt{1+x^5+x^{10}} + \frac{1}{15} (1+x^5+x^{10})^{3/2} - \frac{1}{80} \sqrt{3} \text{Subst} \left(\int \frac{1}{\sqrt{1+x+x^2}} dx, x, x^5 \right) \\
 &= -\frac{1}{40} (1+2x^5) \sqrt{1+x^5+x^{10}} + \frac{1}{15} (1+x^5+x^{10})^{3/2} - \frac{3}{80} \sinh^{-1} \left(\frac{1+2x^5}{\sqrt{3}} \right)
 \end{aligned}$$

Mathematica [A]

time = 0.07, size = 55, normalized size = 0.95

$$\frac{1}{120} \sqrt{1+x^5+x^{10}} (5+2x^5+8x^{10}) + \frac{3}{80} \log \left(-1-2x^5+2\sqrt{1+x^5+x^{10}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^9*Sqrt[1 + x^5 + x^10],x]

[Out] (Sqrt[1 + x^5 + x^10]*(5 + 2*x^5 + 8*x^10))/120 + (3*Log[-1 - 2*x^5 + 2*Sqrt[1 + x^5 + x^10]])/80

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[x^9*Sqrt[1 + x^5 + x^10],x]')`

[Out] Timed out

Maple [A]

time = 0.23, size = 47, normalized size = 0.81

method	result	size
trager	$\left(\frac{1}{15}x^{10} + \frac{1}{60}x^5 + \frac{1}{24}\right) \sqrt{x^{10} + x^5 + 1} - \frac{3 \ln\left(2x^5 + 2\sqrt{x^{10} + x^5 + 1} + 1\right)}{80}$	47
risch	$\frac{(8x^{10} + 2x^5 + 5)\sqrt{x^{10} + x^5 + 1}}{120} + \frac{3 \ln\left(-2x^5 + 2\sqrt{x^{10} + x^5 + 1} - 1\right)}{80}$	48

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9*(x^10+x^5+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $(1/15*x^{10} + 1/60*x^5 + 1/24)*(x^{10} + x^5 + 1)^{(1/2)} - 3/80*\ln(2*x^5 + 2*(x^{10} + x^5 + 1)^{(1/2)} + 1)$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9*(x^10+x^5+1)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(x^10 + x^5 + 1)*x^9, x)`

Fricas [A]

time = 0.30, size = 47, normalized size = 0.81

$$\frac{1}{120} (8x^{10} + 2x^5 + 5)\sqrt{x^{10} + x^5 + 1} + \frac{3}{80} \log\left(-2x^5 + 2\sqrt{x^{10} + x^5 + 1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9*(x^10+x^5+1)^(1/2),x, algorithm="fricas")`

[Out] $1/120*(8*x^{10} + 2*x^5 + 5)*\text{sqrt}(x^{10} + x^5 + 1) + 3/80*\log(-2*x^5 + 2*\text{sqrt}(x^{10} + x^5 + 1) - 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^9 \sqrt{(x^2 + x + 1)(x^8 - x^7 + x^5 - x^4 + x^3 - x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**9*(x**10+x**5+1)**(1/2),x)

[Out] Integral(x**9*sqrt((x**2 + x + 1)*(x**8 - x**7 + x**5 - x**4 + x**3 - x + 1)), x)

Giac [A]

time = 0.01, size = 62, normalized size = 1.07

$$\frac{2 \left(\left(\frac{x^5}{6} + \frac{1}{24} \right) x^5 + \frac{5}{48} \right) \sqrt{x^{10} + x^5 + 1} + \frac{3}{16} \ln \left(2 \left(\sqrt{x^{10} + x^5 + 1} - x^5 \right) - 1 \right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(x^10+x^5+1)^(1/2),x)

[Out] 1/120*sqrt(x^10 + x^5 + 1)*(2*(4*x^5 + 1)*x^5 + 5) + 3/80*log(-2*x^5 + 2*sqrt(x^10 + x^5 + 1) - 1)

Mupad [B]

time = 0.29, size = 43, normalized size = 0.74

$$\frac{\sqrt{x^{10} + x^5 + 1} (8x^{10} + 2x^5 + 5)}{120} - \frac{3 \ln \left(\sqrt{x^{10} + x^5 + 1} + x^5 + \frac{1}{2} \right)}{80}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9*(x^5 + x^10 + 1)^(1/2),x)

[Out] ((x^5 + x^10 + 1)^(1/2)*(2*x^5 + 8*x^10 + 5))/120 - (3*log((x^5 + x^10 + 1)^(1/2) + x^5 + 1/2))/80

$$3.310 \quad \int \frac{1}{x^5 \sqrt{4 + 2x^2 + x^4}} dx$$

Optimal. Leaf size=71

$$-\frac{\sqrt{4 + 2x^2 + x^4}}{16x^4} + \frac{3\sqrt{4 + 2x^2 + x^4}}{64x^2} + \frac{1}{128} \tanh^{-1} \left(\frac{4 + x^2}{2\sqrt{4 + 2x^2 + x^4}} \right)$$

[Out] 1/128*arctanh(1/2*(x^2+4)/(x^4+2*x^2+4)^(1/2))-1/16*(x^4+2*x^2+4)^(1/2)/x^4+3/64*(x^4+2*x^2+4)^(1/2)/x^2

Rubi [A]

time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1128, 758, 820, 738, 212}

$$\frac{3\sqrt{x^4 + 2x^2 + 4}}{64x^2} - \frac{\sqrt{x^4 + 2x^2 + 4}}{16x^4} + \frac{1}{128} \tanh^{-1} \left(\frac{x^2 + 4}{2\sqrt{x^4 + 2x^2 + 4}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*Sqrt[4 + 2*x^2 + x^4]),x]

[Out] -1/16*Sqrt[4 + 2*x^2 + x^4]/x^4 + (3*Sqrt[4 + 2*x^2 + x^4])/(64*x^2) + ArcTanh[(4 + x^2)/(2*Sqrt[4 + 2*x^2 + x^4])]/128

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 738

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 758

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x, x]*(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS

```
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^5 \sqrt{4 + 2x^2 + x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^3 \sqrt{4 + 2x + x^2}} dx, x, x^2 \right) \\ &= -\frac{\sqrt{4 + 2x^2 + x^4}}{16x^4} - \frac{1}{16} \text{Subst} \left(\int \frac{3 + x}{x^2 \sqrt{4 + 2x + x^2}} dx, x, x^2 \right) \\ &= -\frac{\sqrt{4 + 2x^2 + x^4}}{16x^4} + \frac{3\sqrt{4 + 2x^2 + x^4}}{64x^2} - \frac{1}{64} \text{Subst} \left(\int \frac{1}{x \sqrt{4 + 2x + x^2}} dx, x, x^2 \right) \\ &= -\frac{\sqrt{4 + 2x^2 + x^4}}{16x^4} + \frac{3\sqrt{4 + 2x^2 + x^4}}{64x^2} + \frac{1}{32} \text{Subst} \left(\int \frac{1}{16 - x^2} dx, x, \frac{2(4 + x^2)}{\sqrt{4 + 2x^2 + x^4}} \right) \\ &= -\frac{\sqrt{4 + 2x^2 + x^4}}{16x^4} + \frac{3\sqrt{4 + 2x^2 + x^4}}{64x^2} + \frac{1}{128} \tanh^{-1} \left(\frac{4 + x^2}{2\sqrt{4 + 2x^2 + x^4}} \right) \end{aligned}$$

Mathematica [A]

time = 0.09, size = 57, normalized size = 0.80

$$\frac{1}{64} \left(\frac{(-4 + 3x^2) \sqrt{4 + 2x^2 + x^4}}{x^4} - \tanh^{-1} \left(\frac{1}{2} \left(x^2 - \sqrt{4 + 2x^2 + x^4} \right) \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[4 + 2*x^2 + x^4]),x]
```


[Out] $(((-4 + 3x^2)\sqrt{4 + 2x^2 + x^4})/x^4 - \text{ArcTanh}[(x^2 - \sqrt{4 + 2x^2 + x^4})/2])/64$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

caught exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[1/(x^5*Sqrt[4 + 2*x^2 + x^4]),x]')`

[Out] caught exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.12, size = 60, normalized size = 0.85

method	result	size
trager	$\frac{(3x^2-4)\sqrt{x^4+2x^2+4}}{64x^4} - \frac{\ln\left(\frac{-x^2+2\sqrt{x^4+2x^2+4}-4}{x^2}\right)}{128}$	54
default	$-\frac{\sqrt{x^4+2x^2+4}}{16x^4} + \frac{3\sqrt{x^4+2x^2+4}}{64x^2} + \frac{\text{arctanh}\left(\frac{2x^2+8}{4\sqrt{x^4+2x^2+4}}\right)}{128}$	60
risch	$\frac{3x^6+2x^4+4x^2-16}{64x^4\sqrt{x^4+2x^2+4}} + \frac{\text{arctanh}\left(\frac{2x^2+8}{4\sqrt{x^4+2x^2+4}}\right)}{128}$	60
elliptic	$-\frac{\sqrt{x^4+2x^2+4}}{16x^4} + \frac{3\sqrt{x^4+2x^2+4}}{64x^2} + \frac{\text{arctanh}\left(\frac{2x^2+8}{4\sqrt{x^4+2x^2+4}}\right)}{128}$	60

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^5/(x^4+2*x^2+4)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/16*(x^4+2*x^2+4)^(1/2)/x^4+3/64*(x^4+2*x^2+4)^(1/2)/x^2+1/128*\text{arctanh}(1/4*(2*x^2+8)/(x^4+2*x^2+4)^(1/2))$

Maxima [A]

time = 0.34, size = 52, normalized size = 0.73

$$\frac{3\sqrt{x^4+2x^2+4}}{64x^2} - \frac{\sqrt{x^4+2x^2+4}}{16x^4} + \frac{1}{128} \text{arsinh}\left(\frac{1}{3}\sqrt{3} + \frac{4\sqrt{3}}{3x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^5/(x^4+2*x^2+4)^(1/2),x, algorithm="maxima")`

[Out] $\frac{3}{64}\sqrt{x^4 + 2x^2 + 4}/x^2 - \frac{1}{16}\sqrt{x^4 + 2x^2 + 4}/x^4 + \frac{1}{128}\operatorname{arc}\sinh\left(\frac{1}{3}\sqrt{3} + \frac{4}{3}\sqrt{3}/x^2\right)$

Fricas [A]

time = 0.31, size = 81, normalized size = 1.14

$$\frac{x^4 \log\left(-x^2 + \sqrt{x^4 + 2x^2 + 4} + 2\right) - x^4 \log\left(-x^2 + \sqrt{x^4 + 2x^2 + 4} - 2\right) + 6x^4 + 2\sqrt{x^4 + 2x^2 + 4}(3x^2 - 4)}{128x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^5/(x^4+2*x^2+4)^(1/2),x, algorithm="fricas")`

[Out] $\frac{1}{128}(x^4 \log(-x^2 + \sqrt{x^4 + 2x^2 + 4}) + 2) - x^4 \log(-x^2 + \sqrt{x^4 + 2x^2 + 4} - 2) + 6x^4 + 2\sqrt{x^4 + 2x^2 + 4}(3x^2 - 4)/x^4$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \sqrt{x^4 + 2x^2 + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(x**4+2*x**2+4)**(1/2),x)`

[Out] `Integral(1/(x**5*sqrt(x**4 + 2*x**2 + 4)), x)`

Giac [A]

time = 0.01, size = 124, normalized size = 1.75

$$\frac{\ln\left(-\sqrt{x^4 + 2x^2 + 4} + x^2 + 2\right)}{128} + \frac{\ln\left(\sqrt{x^4 + 2x^2 + 4} - x^2 + 2\right)}{128} - \frac{\left(\sqrt{x^4 + 2x^2 + 4} - x^2\right)^3 + 36\left(\sqrt{x^4 + 2x^2 + 4} - x^2\right) - 64}{32\left(\left(\sqrt{x^4 + 2x^2 + 4} - x^2\right)^2 - 4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^5/(x^4+2*x^2+4)^(1/2),x)`

[Out] $\frac{1}{32}((x^2 - \sqrt{x^4 + 2x^2 + 4})^3 + 36x^2 - 36\sqrt{x^4 + 2x^2 + 4} + 64)/((x^2 - \sqrt{x^4 + 2x^2 + 4})^2 - 4)^2 - \frac{1}{128}\log(x^2 - \sqrt{x^4 + 2x^2 + 4} + 2) + \frac{1}{128}\log(-x^2 + \sqrt{x^4 + 2x^2 + 4} + 2)$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5 \sqrt{x^4 + 2x^2 + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^5*(2*x^2 + x^4 + 4)^(1/2)),x)`

[Out] `int(1/(x^5*(2*x^2 + x^4 + 4)^(1/2)), x)`

$$3.311 \quad \int \frac{-1+x^2}{x \sqrt{1+3x^2+x^4}} dx$$

Optimal. Leaf size=21

$$\tanh^{-1} \left(\frac{1+x^2}{\sqrt{1+3x^2+x^4}} \right)$$

[Out] arctanh((x^2+1)/(x^4+3*x^2+1)^(1/2))

Rubi [A]

time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1265, 852, 212}

$$\tanh^{-1} \left(\frac{x^2+1}{\sqrt{x^4+3x^2+1}} \right)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)/(x*Sqrt[1 + 3*x^2 + x^4]),x]

[Out] ArcTanh[(1 + x^2)/Sqrt[1 + 3*x^2 + x^4]]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 852

Int[((f_) + (g_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[4*f*((a - d)/(b*d - a*e)), Subst[Int[1/(4*(a - d) - x^2), x], x, (2*(a - d) + (b - e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[4*c*(a - d) - (b - e)^2, 0] && EqQ[e*f*(b - e) - 2*g*(b*d - a*e), 0] && NeQ[b*d - a*e, 0]

Rule 1265

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{-1+x}{x\sqrt{1+3x+x^2}} dx, x, x^2 \right) \\ &= 2 \text{Subst} \left(\int \frac{1}{4-x^2} dx, x, \frac{2(1+x^2)}{\sqrt{1+3x^2+x^4}} \right) \\ &= \tanh^{-1} \left(\frac{1+x^2}{\sqrt{1+3x^2+x^4}} \right) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 52 vs. $2(21) = 42$.

time = 0.08, size = 52, normalized size = 2.48

$$-\tanh^{-1} \left(x^2 - \sqrt{1+3x^2+x^4} \right) - \frac{1}{2} \log \left(-3 - 2x^2 + 2\sqrt{1+3x^2+x^4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^2)/(x*Sqrt[1 + 3*x^2 + x^4]),x]

[Out] -ArcTanh[x^2 - Sqrt[1 + 3*x^2 + x^4]] - Log[-3 - 2*x^2 + 2*Sqrt[1 + 3*x^2 + x^4]]/2

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(x^2 - 1)/(x*Sqrt[1 + 3*x^2 + x^4]),x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(19) = 38$.

time = 0.10, size = 46, normalized size = 2.19

method	result	size
trager	$\ln \left(\frac{x^2 + \sqrt{x^4 + 3x^2 + 1} + 1}{x} \right)$	23
default	$\frac{\ln \left(x^2 + \frac{3}{2} + \sqrt{x^4 + 3x^2 + 1} \right)}{2} + \frac{\operatorname{arctanh} \left(\frac{3x^2 + 2}{2\sqrt{x^4 + 3x^2 + 1}} \right)}{2}$	46

elliptic	$\frac{\ln\left(x^2 + \frac{3}{2} + \sqrt{x^4 + 3x^2 + 1}\right)}{2} + \frac{\operatorname{arctanh}\left(\frac{3x^2 + 2}{2\sqrt{x^4 + 3x^2 + 1}}\right)}{2}$	46
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{2} \ln(x^2 + 3/2 + (x^4 + 3x^2 + 1)^{1/2}) + 1/2 \operatorname{arctanh}(1/2 * (3x^2 + 2) / (x^4 + 3x^2 + 1)^{1/2})$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(19) = 38.

time = 0.26, size = 52, normalized size = 2.48

$$\frac{1}{2} \log\left(2x^2 + 2\sqrt{x^4 + 3x^2 + 1} + 3\right) + \frac{1}{2} \log\left(\frac{2\sqrt{x^4 + 3x^2 + 1}}{x^2} + \frac{2}{x^2} + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x, algorithm="maxima")`

[Out] $\frac{1}{2} \log(2x^2 + 2\sqrt{x^4 + 3x^2 + 1} + 3) + \frac{1}{2} \log(2\sqrt{x^4 + 3x^2 + 1} / x^2 + 2/x^2 + 3)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(19) = 38.

time = 0.31, size = 59, normalized size = 2.81

$$-\frac{1}{2} \log\left(4x^4 + 11x^2 - \sqrt{x^4 + 3x^2 + 1}(4x^2 + 5) + 5\right) + \frac{1}{2} \log\left(-x^2 + \sqrt{x^4 + 3x^2 + 1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x, algorithm="fricas")`

[Out] $-1/2 \log(4x^4 + 11x^2 - \sqrt{x^4 + 3x^2 + 1}(4x^2 + 5) + 5) + 1/2 \log(-x^2 + \sqrt{x^4 + 3x^2 + 1} + 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{x\sqrt{x^4 + 3x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2-1)/x/(x**4+3*x**2+1)**(1/2),x)`

[Out] `Integral((x - 1)*(x + 1)/(x*sqrt(x**4 + 3*x**2 + 1)), x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(19) = 38$.
time = 0.01, size = 79, normalized size = 3.76

$$-\frac{\ln\left(\sqrt{x^4+3x^2+1}-x^2-1\right)}{2} + \frac{\ln\left(\sqrt{x^4+3x^2+1}-x^2+1\right)}{2} - \frac{\ln\left(-2\left(\sqrt{x^4+3x^2+1}-x^2\right)+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x)

[Out] $-\frac{1}{2}\log(2x^2 - 2\sqrt{x^4 + 3x^2 + 1} + 3) + \frac{1}{2}\log(-x^2 + \sqrt{x^4 + 3x^2 + 1} + 1) - \frac{1}{2}\log(-x^2 + \sqrt{x^4 + 3x^2 + 1} - 1)$

Mupad [B]

time = 0.81, size = 49, normalized size = 2.33

$$\frac{\ln\left(\frac{1}{x^2}\right)}{2} + \frac{\ln\left(\sqrt{x^4+3x^2+1}+x^2+\frac{3}{2}\right)}{2} + \frac{\ln\left(\frac{2\sqrt{x^4+3x^2+1}}{3}+x^2+\frac{2}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - 1)/(x*(3*x^2 + x^4 + 1)^(1/2)),x)

[Out] $\log(1/x^2)/2 + \log((3x^2 + x^4 + 1)^{1/2} + x^2 + 3/2)/2 + \log((2*(3x^2 + x^4 + 1)^{1/2})/3 + x^2 + 2/3)/2$

$$3.312 \quad \int (-3x + 2x^3) (-3x^2 + x^4)^{3/5} dx$$

Optimal. Leaf size=17

$$\frac{5}{16} (-3x^2 + x^4)^{8/5}$$

[Out] 5/16*(x^4-3*x^2)^(8/5)

Rubi [A]

time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1602}

$$\frac{5}{16} (x^4 - 3x^2)^{8/5}$$

Antiderivative was successfully verified.

[In] Int[(-3*x + 2*x^3)*(-3*x^2 + x^4)^(3/5),x]

[Out] (5*(-3*x^2 + x^4)^(8/5))/16

Rule 1602

Int[(Pp_)*(Qq_)^(m_), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq, x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (-3x + 2x^3) (-3x^2 + x^4)^{3/5} dx = \frac{5}{16} (-3x^2 + x^4)^{8/5}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 2 in optimal.

time = 10.05, size = 75, normalized size = 4.41

$$\frac{5(x^2(-3+x^2))^{3/5} \left(-39x^2 {}_2F_1 \left(-\frac{3}{5}, \frac{8}{5}; \frac{13}{5}; \frac{x^2}{3} \right) + 16x^4 {}_2F_1 \left(-\frac{3}{5}, \frac{13}{5}; \frac{18}{5}; \frac{x^2}{3} \right) \right)}{208 \left(1 - \frac{x^2}{3} \right)^{3/5}}$$

Antiderivative was successfully verified.

[In] Integrate[(-3*x + 2*x^3)*(-3*x^2 + x^4)^(3/5),x]

[Out] $(5*(x^2*(-3 + x^2))^{(3/5)}*(-39*x^2*\text{Hypergeometric2F1}[-3/5, 8/5, 13/5, x^{2/3}] + 16*x^4*\text{Hypergeometric2F1}[-3/5, 13/5, 18/5, x^{2/3}]))/(208*(1 - x^{2/3})^{(3/5)})$

Mathics [A]

time = 2.10, size = 13, normalized size = 0.76

$$\frac{5(x^2(-3 + x^2))^{8/5}}{16}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(x^4 - 3*x^2)^(3/5)*(2*x^3 - 3*x),x]')`

[Out] $5(x^2(-3 + x^2))^{(8/5)} / 16$

Maple [A]

time = 0.09, size = 14, normalized size = 0.82

method	result	size
default	$\frac{5(x^4-3x^2)^{8/5}}{16}$	14
gospers	$\frac{5(x^4-3x^2)^{3/5}x^2(x^2-3)}{16}$	22
trager	$\frac{5(x^4-3x^2)^{3/5}x^2(x^2-3)}{16}$	22
risch	$\frac{5x^2(x^2(x^2-3))^{3/5}(x^2-3)}{16}$	22
meijerg	$\frac{5 \cdot 3^{3/5} \operatorname{signum}\left(-1 + \frac{x^2}{3}\right)^{3/5} x^{26} \operatorname{hypergeom}\left(\left[-\frac{3}{5}, \frac{13}{5}\right], \left[\frac{18}{5}, \frac{x^2}{3}\right]\right)}{13 \left(-\operatorname{signum}\left(-1 + \frac{x^2}{3}\right)\right)^{3/5}} - \frac{15 \cdot 3^{3/5} \operatorname{signum}\left(-1 + \frac{x^2}{3}\right)^{3/5} x^{16} \operatorname{hypergeom}\left(\left[-\frac{3}{5}, \frac{8}{5}\right], \left[\frac{13}{5}, \frac{x^2}{3}\right]\right)}{16 \left(-\operatorname{signum}\left(-1 + \frac{x^2}{3}\right)\right)^{3/5}}$	84

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x,method=_RETURNVERBOSE)`

[Out] $5/16*(x^4-3*x^2)^{(8/5)}$

Maxima [A]

time = 0.27, size = 13, normalized size = 0.76

$$\frac{5}{16} (x^4 - 3x^2)^{8/5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x, algorithm="maxima")`

[Out] $5/16*(x^4 - 3*x^2)^{(8/5)}$

Fricas [A]

time = 0.30, size = 13, normalized size = 0.76

$$\frac{5}{16} (x^4 - 3x^2)^{\frac{8}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x, algorithm="fricas")

[Out] 5/16*(x^4 - 3*x^2)^(8/5)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(14) = 28

time = 0.24, size = 36, normalized size = 2.12

$$\frac{5x^4 (x^4 - 3x^2)^{\frac{3}{5}}}{16} - \frac{15x^2 (x^4 - 3x^2)^{\frac{3}{5}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3-3*x)*(x**4-3*x**2)**(3/5),x)

[Out] 5*x**4*(x**4 - 3*x**2)**(3/5)/16 - 15*x**2*(x**4 - 3*x**2)**(3/5)/16

Giac [A]

time = 0.00, size = 30, normalized size = 1.76

$$\frac{5}{16} \left((x^4 - 3x^2)^{\frac{1}{5}} \right)^3 (x^4 - 3x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x)

[Out] 5/16*(x^4 - 3*x^2)^(8/5)

Mupad [B]

time = 0.28, size = 21, normalized size = 1.24

$$\frac{5x^2 (x^2 - 3) (x^4 - 3x^2)^{3/5}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3*x - 2*x^3)*(x^4 - 3*x^2)^(3/5),x)

[Out] (5*x^2*(x^2 - 3)*(x^4 - 3*x^2)^(3/5))/16

$$3.313 \quad \int \frac{-2x^5 + 3x^8 - x^2(-1 + 3x^3)^{2/3}}{(-1 + 3x^3)^{3/4}} dx$$

Optimal. Leaf size=46

$$-\frac{4}{27} \sqrt[4]{-1 + 3x^3} - \frac{4}{33} (-1 + 3x^3)^{11/12} + \frac{4}{243} (-1 + 3x^3)^{9/4}$$

[Out] $-4/27*(3*x^3-1)^{(1/4)}-4/33*(3*x^3-1)^{(11/12)}+4/243*(3*x^3-1)^{(9/4)}$

Rubi [A]

time = 0.12, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {6874, 272, 45, 267}

$$\frac{4}{243} (3x^3 - 1)^{9/4} - \frac{4}{33} (3x^3 - 1)^{11/12} - \frac{4}{27} \sqrt[4]{3x^3 - 1}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-2*x^5 + 3*x^8 - x^2*(-1 + 3*x^3)^{(2/3)})/(-1 + 3*x^3)^{(3/4)}, x]$

[Out] $(-4*(-1 + 3*x^3)^{(1/4)})/27 - (4*(-1 + 3*x^3)^{(11/12)})/33 + (4*(-1 + 3*x^3)^{(9/4)})/243$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

Rule 267

$\text{Int}[(x_.)^{(m_.)*((a_.) + (b_.)*(x_.)^{(n_.))^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x^n)^{(p + 1)}/(b*n*(p + 1)), x] /; \text{FreeQ}\{a, b, m, n, p, x\} \ \&\& \ \text{EqQ}[m, n - 1] \ \&\& \ \text{NeQ}[p, -1]$

Rule 272

$\text{Int}[(x_.)^{(m_.)*((a_.) + (b_.)*(x_.)^{(n_.))^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p, x\} \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 6874

$\text{Int}[u_, x_Symbol] \rightarrow \text{With}\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]$

Rubi steps

$$\begin{aligned}
\int \frac{-2x^5 + 3x^8 - x^2(-1 + 3x^3)^{2/3}}{(-1 + 3x^3)^{3/4}} dx &= \int \left(-\frac{2x^5}{(-1 + 3x^3)^{3/4}} + \frac{3x^8}{(-1 + 3x^3)^{3/4}} - \frac{x^2}{\sqrt[12]{-1 + 3x^3}} \right) dx \\
&= -\left(2 \int \frac{x^5}{(-1 + 3x^3)^{3/4}} dx \right) + 3 \int \frac{x^8}{(-1 + 3x^3)^{3/4}} dx - \int \frac{x^2}{\sqrt[12]{-1 + 3x^3}} dx \\
&= -\frac{4}{33} (-1 + 3x^3)^{11/12} - \frac{2}{3} \text{Subst} \left(\int \frac{x}{(-1 + 3x)^{3/4}} dx, x, x^3 \right) + \text{Subst} \left(\int \frac{1}{\sqrt[12]{-1 + 3x^3}} dx, x, x^3 \right) \\
&= -\frac{4}{33} (-1 + 3x^3)^{11/12} - \frac{2}{3} \text{Subst} \left(\int \left(\frac{1}{3(-1 + 3x)^{3/4}} + \frac{1}{3} \sqrt[4]{-1 + 3x} \right) dx, x, x^3 \right) \\
&= -\frac{4}{27} \sqrt[4]{-1 + 3x^3} - \frac{4}{33} (-1 + 3x^3)^{11/12} + \frac{4}{243} (-1 + 3x^3)^{9/4}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 40, normalized size = 0.87

$$-\frac{4\sqrt[4]{-1 + 3x^3} \left(88 + 66x^3 - 99x^6 + 81(-1 + 3x^3)^{2/3} \right)}{2673}$$

Antiderivative was successfully verified.

`[In] Integrate[(-2*x^5 + 3*x^8 - x^2*(-1 + 3*x^3)^(2/3))/(-1 + 3*x^3)^(3/4), x]``[Out] (-4*(-1 + 3*x^3)^(1/4)*(88 + 66*x^3 - 99*x^6 + 81*(-1 + 3*x^3)^(2/3)))/2673`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(3*x^8 - 2*x^5 - x^2*(3*x^3 - 1)^(2/3))/(3*x^3 - 1)^(3/4), x]')``[Out] cought exception:`**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 2.

time = 0.10, size = 116, normalized size = 2.52

method	result
--------	--------

meijerg	$-\frac{(-\text{signum}(3x^3-1))^{\frac{3}{4}} x^6 \text{hypergeom}\left(\left[\frac{3}{4}, 2\right], [3], 3x^3\right)}{3\text{signum}(3x^3-1)^{\frac{3}{4}}} + \frac{(-\text{signum}(3x^3-1))^{\frac{3}{4}} x^9 \text{hypergeom}\left(\left[\frac{3}{4}, 3\right], [4], 3x^3\right)}{3\text{signum}(3x^3-1)^{\frac{3}{4}}} - \frac{(-\text{signum}(3x^3-1))^{\frac{3}{4}} x^{12} \text{hypergeom}\left(\left[\frac{3}{4}, 4\right], [5], 3x^3\right)}{3\text{signum}(3x^3-1)^{\frac{3}{4}}}$
---------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x,method=_RETURNVERBOSE)`

[Out]
$$-1/3/\text{signum}(3x^3-1)^{(3/4)} * (-\text{signum}(3x^3-1))^{(3/4)} * x^6 * \text{hypergeom}\left(\left[\frac{3}{4}, 2\right], [3], 3x^3\right) + 1/3/\text{signum}(3x^3-1)^{(3/4)} * (-\text{signum}(3x^3-1))^{(3/4)} * x^9 * \text{hypergeom}\left(\left[\frac{3}{4}, 3\right], [4], 3x^3\right) - 1/3/\text{signum}(3x^3-1)^{(1/12)} * (-\text{signum}(3x^3-1))^{(1/12)} * x^3 * \text{hypergeom}\left(\left[\frac{1}{12}, 1\right], [2], 3x^3\right)$$

Maxima [A]

time = 0.26, size = 34, normalized size = 0.74

$$\frac{4}{243} (3x^3 - 1)^{\frac{9}{4}} - \frac{4}{33} (3x^3 - 1)^{\frac{11}{12}} - \frac{4}{27} (3x^3 - 1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x, algorithm="maxima")`

[Out]
$$4/243*(3x^3 - 1)^{(9/4)} - 4/33*(3x^3 - 1)^{(11/12)} - 4/27*(3x^3 - 1)^{(1/4)}$$

Fricas [A]

time = 0.30, size = 35, normalized size = 0.76

$$\frac{4}{243} (9x^6 - 6x^3 - 8)(3x^3 - 1)^{\frac{1}{4}} - \frac{4}{33} (3x^3 - 1)^{\frac{11}{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x, algorithm="fricas")`

[Out]
$$4/243*(9x^6 - 6x^3 - 8)*(3x^3 - 1)^{(1/4)} - 4/33*(3x^3 - 1)^{(11/12)}$$

Sympy [C] Result contains complex when optimal does not.

time = 5.89, size = 221, normalized size = 4.80

$$-\frac{4(3x^3-1)^{\frac{11}{12}}}{33} - 2 \left(\begin{cases} \frac{4x^3\sqrt[3]{3x^3-1}}{45} + \frac{16\sqrt[3]{3x^3-1}}{135} & \text{for } |x^3| > \frac{1}{3} \\ -\frac{4x^3\sqrt[3]{1-3x^3}e^{-\frac{3i\pi}{4}}}{45} - \frac{16\sqrt[3]{1-3x^3}e^{-\frac{3i\pi}{4}}}{135} & \text{otherwise} \end{cases} \right) + 3 \left(\begin{cases} \frac{4x^6\sqrt[3]{3x^3-1}}{81} + \frac{32x^3\sqrt[3]{3x^3-1}}{1215} + \frac{128\sqrt[3]{3x^3-1}}{3645} & \text{for } |x^3| > \frac{1}{3} \\ \frac{4x^6\sqrt[3]{1-3x^3}e^{\frac{i\pi}{4}}}{81} + \frac{32x^3\sqrt[3]{1-3x^3}e^{\frac{i\pi}{4}}}{1215} + \frac{128\sqrt[3]{1-3x^3}e^{\frac{i\pi}{4}}}{3645} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x**5+3*x**8-x**2*(3*x**3-1)**(2/3))/(3*x**3-1)**(3/4),x)`

[Out] $-4*(3*x**3 - 1)**(11/12)/33 - 2*Piecewise((4*x**3*(3*x**3 - 1)**(1/4)/45 + 16*(3*x**3 - 1)**(1/4)/135, Abs(x**3) > 1/3), (-4*x**3*(1 - 3*x**3)**(1/4)*exp(-3*I*pi/4)/45 - 16*(1 - 3*x**3)**(1/4)*exp(-3*I*pi/4)/135, True)) + 3*Piecewise((4*x**6*(3*x**3 - 1)**(1/4)/81 + 32*x**3*(3*x**3 - 1)**(1/4)/1215 + 128*(3*x**3 - 1)**(1/4)/3645, Abs(x**3) > 1/3), (4*x**6*(1 - 3*x**3)**(1/4)*exp(I*pi/4)/81 + 32*x**3*(1 - 3*x**3)**(1/4)*exp(I*pi/4)/1215 + 128*(1 - 3*x**3)**(1/4)*exp(I*pi/4)/3645, True))$

Giac [A]

time = 0.00, size = 73, normalized size = 1.59

$$\frac{\frac{4}{27} (3x^3 - 1)^{\frac{1}{4}} (3x^3 - 1)^2 - \frac{12}{11} (3x^3 - 1)^{\frac{1}{4}} \left((3x^3 - 1)^{\frac{1}{3}} \right)^2 - \frac{4}{3} (3x^3 - 1)^{\frac{1}{4}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x)`

[Out] $4/243*(3*x^3 - 1)^{(9/4)} - 4/33*(3*x^3 - 1)^{(11/12)} - 4/27*(3*x^3 - 1)^{(1/4)}$

Mupad [B]

time = 0.37, size = 34, normalized size = 0.74

$$-(3x^3 - 1)^{1/4} \left(\frac{8x^3}{81} - \frac{4x^6}{27} + \frac{4(3x^3 - 1)^{2/3}}{33} + \frac{32}{243} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^2*(3*x^3 - 1)^(2/3) + 2*x^5 - 3*x^8)/(3*x^3 - 1)^(3/4),x)`

[Out] $-(3*x^3 - 1)^{(1/4)}*((8*x^3)/81 - (4*x^6)/27 + (4*(3*x^3 - 1)^(2/3))/33 + 32/243)$

$$3.314 \quad \int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}} dx$$

Optimal. Leaf size=78

$$-\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3^{5/6}} - \frac{\log(-1+x^3)}{6\sqrt[3]{3}} + \frac{\log(\sqrt[3]{3}x - \sqrt[3]{2+x^3})}{2\sqrt[3]{3}}$$

[Out] $-1/3*\arctan(1/3*(1+2*3^{(1/3)}*x/(x^3+2)^{(1/3)})*3^{(1/2)})*3^{(1/6)}-1/18*\ln(x^3-1)*3^{(2/3)}+1/6*\ln(3^{(1/3)}*x-(x^3+2)^{(1/3)})*3^{(2/3)}$

Rubi [A]

time = 0.01, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {384}

$$-\frac{\log(x^3-1)}{6\sqrt[3]{3}} + \frac{\log(\sqrt[3]{3}x - \sqrt[3]{x^3+2})}{2\sqrt[3]{3}} - \frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

[In] Int[1/((-1 + x^3)*(2 + x^3)^(1/3)),x]

[Out] $-(\text{ArcTan}[(1 + (2*3^{(1/3)}*x)/(2 + x^3)^{(1/3)})/\text{Sqrt}[3]]/3^{(5/6)}) - \text{Log}[-1 + x^3]/(6*3^{(1/3)}) + \text{Log}[3^{(1/3)}*x - (2 + x^3)^{(1/3)}]/(2*3^{(1/3)})$

Rule 384

Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x] + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}} dx &= \text{Subst} \left(\int \frac{1}{-1+3x^3} dx, x, \frac{x}{\sqrt[3]{2+x^3}} \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \frac{1}{-1+\sqrt[3]{3}x} dx, x, \frac{x}{\sqrt[3]{2+x^3}} \right) + \frac{1}{3} \text{Subst} \left(\int \frac{-2-\sqrt[3]{3}x}{1+\sqrt[3]{3}x+3^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{2+x^3}} \right) \\
&= \frac{\log \left(1 - \frac{\sqrt[3]{3}x}{\sqrt[3]{2+x^3}} \right)}{3\sqrt[3]{3}} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+\sqrt[3]{3}x+3^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{2+x^3}} \right) - \frac{1}{3} \text{Subst} \left(\int \frac{-2-\sqrt[3]{3}x}{1+\sqrt[3]{3}x+3^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{2+x^3}} \right) \\
&= \frac{\log \left(1 - \frac{\sqrt[3]{3}x}{\sqrt[3]{2+x^3}} \right)}{3\sqrt[3]{3}} - \frac{\log \left(1 + \frac{3^{2/3}x^2}{(2+x^3)^{2/3}} + \frac{\sqrt[3]{3}x}{\sqrt[3]{2+x^3}} \right)}{6\sqrt[3]{3}} + \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, \frac{x}{\sqrt[3]{2+x^3}} \right)}{3\sqrt[3]{3}} \\
&= -\frac{\tan^{-1} \left(\frac{1+\frac{2\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}}{\sqrt[3]{3}} \right)}{3^{5/6}} + \frac{\log \left(1 - \frac{\sqrt[3]{3}x}{\sqrt[3]{2+x^3}} \right)}{3\sqrt[3]{3}} - \frac{\log \left(1 + \frac{3^{2/3}x^2}{(2+x^3)^{2/3}} + \frac{\sqrt[3]{3}x}{\sqrt[3]{2+x^3}} \right)}{6\sqrt[3]{3}}
\end{aligned}$$

Mathematica [A]

time = 0.20, size = 110, normalized size = 1.41

$$\frac{-6 \tan^{-1} \left(\frac{3^{5/6}x}{\sqrt[3]{3}x+2\sqrt[3]{2+x^3}} \right) + \sqrt[3]{3} \left(2 \log \left(-3x + 3^{2/3}\sqrt[3]{2+x^3} \right) - \log \left(3x^2 + 3^{2/3}x\sqrt[3]{2+x^3} + \sqrt[3]{3} (2+x^3)^{2/3} \right) \right)}{6 \cdot 3^{5/6}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/((-1 + x^3)*(2 + x^3)^(1/3)),x]`

```
[Out] (-6*ArcTan[(3^(5/6)*x)/(3^(1/3)*x + 2*(2 + x^3)^(1/3))] + Sqrt[3]*(2*Log[-3*x + 3^(2/3)*(2 + x^3)^(1/3)] - Log[3*x^2 + 3^(2/3)*x*(2 + x^3)^(1/3) + 3^(1/3)*(2 + x^3)^(2/3)]))/(6*3^(5/6))
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/((x^3 - 1)*(x^3 + 2)^(1/3)),x]')``[Out] cought exception: maximum recursion depth exceeded in comparison`**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 1.70, size = 904, normalized size = 11.59

method	result	size
trager	Expression too large to display	904

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-1)/(x^3+2)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*ln(-(27*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3-9*x^3*RootOf(_Z^3-9)^3*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)-15*(x^3+2)^(2/3)*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x-45*(x^3+2)^(1/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*x^2+2*(x^3+2)^(1/3)*RootOf(_Z^3-9)^2*x^2-9*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3+3*RootOf(_Z^3-9)*x^3+6*x*(x^3+2)^(2/3)-18*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+6*RootOf(_Z^3-9))/(-1+x)/(x^2+x+1))*RootOf(_Z^3-9)-ln(-(27*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3-9*x^3*RootOf(_Z^3-9)^3*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)-15*(x^3+2)^(2/3)*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x-45*(x^3+2)^(1/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*x^2+2*(x^3+2)^(1/3)*RootOf(_Z^3-9)^2*x^2-9*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3+3*RootOf(_Z^3-9)*x^3+6*x*(x^3+2)^(2/3)-18*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+6*RootOf(_Z^3-9))/(-1+x)/(x^2+x+1))*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*ln(-(27*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+12*x^3*RootOf(_Z^3-9)^3*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+15*(x^3+2)^(2/3)*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x+45*(x^3+2)^(1/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*x^2+7*(x^3+2)^(1/3)*RootOf(_Z^3-9)^2*x^2+36*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3+16*RootOf(_Z^3-9)*x^3+21*x*(x^3+2)^(2/3)+18*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+8*RootOf(_Z^3-9))/(-1+x)/(x^2+x+1))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)/(x^3+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 2)^(1/3)*(x^3 - 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(59) = 118.

time = 1.58, size = 232, normalized size = 2.97

$$\frac{1}{27} \cdot 3^{\frac{1}{3}} \log\left(\frac{9 \cdot 3^{\frac{1}{3}}(x^2+2)^{\frac{1}{3}}x^2 - 2 \cdot 3^{\frac{1}{3}}(x^3-1) - 9(x^2+2)^{\frac{1}{3}}x}{x^3-1}\right) - \frac{1}{54} \cdot 3^{\frac{1}{3}} \log\left(\frac{3 \cdot 3^{\frac{1}{3}}(7x^4+2x)(x^2+2)^{\frac{1}{3}} + 3^{\frac{1}{3}}(31x^6+46x^3+4) + 9(5x^6+4x^2)(x^2+2)^{\frac{1}{3}}}{x^6-2x^3+1}\right) - \frac{1}{9} \cdot 3^{\frac{1}{3}} \arctan\left(\frac{3^{\frac{1}{3}}(12 \cdot 3^{\frac{1}{3}}(7x^7-5x^4-2x)(x^2+2)^{\frac{1}{3}} - 3^{\frac{1}{3}}(127x^9+402x^6+192x^3+8) - 18(31x^8+46x^5+4x^2)(x^2+2)^{\frac{1}{3}})}{3(251x^9+462x^6+24x^3-8)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-1)/(x^3+2)^(1/3),x, algorithm="fricas")

[Out] 1/27*3^(2/3)*log((9*3^(1/3)*(x^3 + 2)^(1/3)*x^2 - 2*3^(2/3)*(x^3 - 1) - 9*(x^3 + 2)^(2/3)*x)/(x^3 - 1)) - 1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 2*x)*(x^3 + 2)^(2/3) + 3^(1/3)*(31*x^6 + 46*x^3 + 4) + 9*(5*x^5 + 4*x^2)*(x^3 + 2)^(1/3))/(x^6 - 2*x^3 + 1)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)*(7*x^7 - 5*x^4 - 2*x)*(x^3 + 2)^(2/3) - 3^(1/3)*(127*x^9 + 402*x^6 + 192*x^3 + 8) - 18*(31*x^8 + 46*x^5 + 4*x^2)*(x^3 + 2)^(1/3))/(251*x^9 + 462*x^6 + 24*x^3 - 8))

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x-1)\sqrt[3]{x^3+2}(x^2+x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**3-1)/(x**3+2)**(1/3),x)

[Out] Integral(1/((x - 1)*(x**3 + 2)**(1/3)*(x**2 + x + 1)), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3-1)/(x^3+2)^(1/3),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x^3-1)(x^3+2)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^3 - 1)*(x^3 + 2)^(1/3)),x)

[Out] int(1/((x^3 - 1)*(x^3 + 2)^(1/3)), x)

$$3.315 \quad \int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}} dx$$

Optimal. Leaf size=141

$$-\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{2\sqrt{2}} - \frac{\log\left(1 + \frac{x^2}{\sqrt{2+x^4}} - \frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}} + \frac{\log\left(1 + \frac{x^2}{\sqrt{2+x^4}}\right)}{4\sqrt{2}}$$

[Out] 1/4*arctan(-1+x*2^(1/2)/(x^4+2)^(1/4))*2^(1/2)+1/4*arctan(1+x*2^(1/2)/(x^4+2)^(1/4))*2^(1/2)-1/8*ln(1-x*2^(1/2)/(x^4+2)^(1/4)+x^2/(x^4+2)^(1/2))*2^(1/2)+1/8*ln(1+x*2^(1/2)/(x^4+2)^(1/4)+x^2/(x^4+2)^(1/2))*2^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {385, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}x}{\sqrt{x^4+2}}\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+2}} + 1\right)}{2\sqrt{2}} - \frac{\log\left(-\frac{\sqrt{2}x}{\sqrt{x^4+2}} + \frac{x^2}{\sqrt{x^4+2}} + 1\right)}{4\sqrt{2}} + \frac{\log\left(\frac{\sqrt{2}x}{\sqrt{x^4+2}} + \frac{x^2}{\sqrt{x^4+2}} + 1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x^4)*(2 + x^4)^(1/4)),x]

[Out] -1/2*ArcTan[1 - (Sqrt[2]*x)/(2 + x^4)^(1/4)]/Sqrt[2] + ArcTan[1 + (Sqrt[2]*x)/(2 + x^4)^(1/4)]/(2*Sqrt[2]) - Log[1 + x^2/Sqrt[2 + x^4] - (Sqrt[2]*x)/(2 + x^4)^(1/4)]/(4*Sqrt[2]) + Log[1 + x^2/Sqrt[2 + x^4] + (Sqrt[2]*x)/(2 + x^4)^(1/4)]/(4*Sqrt[2])

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b

, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 631

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1176

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1179

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}} dx &= \text{Subst} \left(\int \frac{1}{1+x^4} dx, x, \frac{x}{\sqrt[4]{2+x^4}} \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1-x^2}{1+x^4} dx, x, \frac{x}{\sqrt[4]{2+x^4}} \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1+x^2}{1+x^4} dx, x, \frac{x}{\sqrt[4]{2+x^4}} \right) \\
&= \frac{1}{4} \text{Subst} \left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \frac{x}{\sqrt[4]{2+x^4}} \right) + \frac{1}{4} \text{Subst} \left(\int \frac{1}{1+\sqrt{2}x+x^2} dx, x, \frac{x}{\sqrt[4]{2+x^4}} \right) \\
&= -\frac{\log \left(1 + \frac{x^2}{\sqrt{2+x^4}} - \frac{\sqrt{2}x}{\sqrt[4]{2+x^4}} \right)}{4\sqrt{2}} + \frac{\log \left(1 + \frac{x^2}{\sqrt{2+x^4}} + \frac{\sqrt{2}x}{\sqrt[4]{2+x^4}} \right)}{4\sqrt{2}} + \frac{\text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt[4]{2+x^4}} \right)}{2\sqrt{2}} \\
&= -\frac{\tan^{-1} \left(1 - \frac{\sqrt{2}x}{\sqrt[4]{2+x^4}} \right)}{2\sqrt{2}} + \frac{\tan^{-1} \left(1 + \frac{\sqrt{2}x}{\sqrt[4]{2+x^4}} \right)}{2\sqrt{2}} - \frac{\log \left(1 + \frac{x^2}{\sqrt{2+x^4}} - \frac{\sqrt{2}x}{\sqrt[4]{2+x^4}} \right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A]

time = 0.16, size = 76, normalized size = 0.54

$$\frac{\tan^{-1} \left(\frac{\sqrt{2}x\sqrt[4]{2+x^4}}{-x^2+\sqrt{2+x^4}} \right) + \tanh^{-1} \left(\frac{\sqrt{2}x\sqrt[4]{2+x^4}}{x^2+\sqrt{2+x^4}} \right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/((1 + x^4)*(2 + x^4)^(1/4)),x]`

```
[Out] (ArcTan[(Sqrt[2]*x*(2 + x^4)^(1/4))/(-x^2 + Sqrt[2 + x^4])] + ArcTanh[(Sqrt[2]*x*(2 + x^4)^(1/4))/(x^2 + Sqrt[2 + x^4])])/(2*Sqrt[2])
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/((x^4 + 1)*(x^4 + 2)^(1/4)),x]')`

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python object
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 0.75, size = 150, normalized size = 1.06

method	result
trager	$\frac{\text{RootOf}(-Z^4+1)^3 \ln\left(\frac{(x^4+2)^{\frac{1}{4}} \text{RootOf}(-Z^4+1)^2 x^3 - \sqrt{x^4+2} \text{RootOf}(-Z^4+1) x^2 + (x^4+2)^{\frac{3}{4}} x + \text{RootOf}(-Z^4+1)^3}{x^4+1}\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1)/(x^4+2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*RootOf(_Z^4+1)^3*ln(((x^4+2)^(1/4)*RootOf(_Z^4+1)^2*x^3-(x^4+2)^(1/2)*R
ootOf(_Z^4+1)*x^2+(x^4+2)^(3/4)*x+RootOf(_Z^4+1)^3)/(x^4+1))-1/4*RootOf(_Z^
4+1)*ln(((x^4+2)^(1/2)*RootOf(_Z^4+1)^3*x^2-(x^4+2)^(1/4)*RootOf(_Z^4+1)^2*
x^3+(x^4+2)^(3/4)*x-RootOf(_Z^4+1))/(x^4+1))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(x^4+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 2)^(1/4)*(x^4 + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(104) = 208.

time = 2.48, size = 388, normalized size = 2.75

$$\frac{1}{4} \sqrt{2} \arctan\left(\frac{\sqrt{(x^4+2)^{3/4} - \sqrt{(x^4+2)^{5/4} - (x^4+2)^{3/4}}}}{\sqrt{(x^4+2)^{3/4} + \sqrt{(x^4+2)^{5/4} - (x^4+2)^{3/4}}}}\right) + \frac{1}{4} \sqrt{2} \arctan\left(\frac{\sqrt{(x^4+2)^{3/4} - \sqrt{(x^4+2)^{5/4} - (x^4+2)^{3/4}}}}{\sqrt{(x^4+2)^{3/4} + \sqrt{(x^4+2)^{5/4} - (x^4+2)^{3/4}}}}\right) - \frac{1}{16} \sqrt{2} \ln\left(\frac{(x^4+2)^{3/4} - \sqrt{(x^4+2)^{5/4} - (x^4+2)^{3/4}}}{(x^4+2)^{3/4} + \sqrt{(x^4+2)^{5/4} - (x^4+2)^{3/4}}}\right) + \frac{1}{16} \sqrt{2} \ln\left(\frac{(x^4+2)^{3/4} - \sqrt{(x^4+2)^{5/4} - (x^4+2)^{3/4}}}{(x^4+2)^{3/4} + \sqrt{(x^4+2)^{5/4} - (x^4+2)^{3/4}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(x^4+2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*(sqrt(2)*(x^4 + 2)^(3/4)*x^2 - sqrt(2)*(x^4 + 2)^(5/
4) - (2*x^5 - sqrt(2)*(x^4 + 2)^(3/4)*x^2 - sqrt(2)*(x^4 + 2)^(5/4) + 4*x)*
sqrt((x^4 + sqrt(2)*(x^4 + 2)^(1/4)*x^3 + 2*sqrt(x^4 + 2)*x^2 + sqrt(2)*(x^
4 + 2)^(3/4)*x + 1)/(x^4 + 1)))/(x^5 + 2*x)) + 1/4*sqrt(2)*arctan(1/2*(sqrt
(2)*(x^4 + 2)^(3/4)*x^2 - sqrt(2)*(x^4 + 2)^(5/4) + (2*x^5 + sqrt(2)*(x^4 +
2)^(3/4)*x^2 + sqrt(2)*(x^4 + 2)^(5/4) + 4*x)*sqrt((x^4 - sqrt(2)*(x^4 + 2
)^(1/4)*x^3 + 2*sqrt(x^4 + 2)*x^2 - sqrt(2)*(x^4 + 2)^(3/4)*x + 1)/(x^4 + 1
)))/(x^5 + 2*x)) + 1/16*sqrt(2)*log(4*(x^4 + sqrt(2)*(x^4 + 2)^(1/4)*x^3 +
2*sqrt(x^4 + 2)*x^2 + sqrt(2)*(x^4 + 2)^(3/4)*x + 1)/(x^4 + 1)) - 1/16*sqrt
(2)*log(4*(x^4 - sqrt(2)*(x^4 + 2)^(1/4)*x^3 + 2*sqrt(x^4 + 2)*x^2 - sqrt(2
)*(x^4 + 2)^(3/4)*x + 1)/(x^4 + 1))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^4 + 1) \sqrt[4]{x^4 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**4+1)/(x**4+2)**(1/4),x)

[Out] Integral(1/((x**4 + 1)*(x**4 + 2)**(1/4)), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+1)/(x^4+2)^(1/4),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x^4 + 1) (x^4 + 2)^{1/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^4 + 1)*(x^4 + 2)^(1/4)),x)

[Out] int(1/((x^4 + 1)*(x^4 + 2)^(1/4)), x)

$$3.316 \quad \int \frac{-1+x^3}{\sqrt[3]{2+x^3}} dx$$

Optimal. Leaf size=63

$$\frac{1}{3}x(2+x^3)^{2/3} - \frac{5 \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{5}{6} \log\left(-x + \sqrt[3]{2+x^3}\right)$$

[Out] 1/3*x*(x^3+2)^(2/3)+5/6*ln(-x+(x^3+2)^(1/3))-5/9*arctan(1/3*(1+2*x/(x^3+2)^(1/3))*3^(1/2))*3^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {396, 245}

$$\frac{1}{3}(x^3+2)^{2/3}x + \frac{5}{6} \log\left(\sqrt[3]{x^3+2} - x\right) - \frac{5 \tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^3)/(2 + x^3)^(1/3), x]

[Out] (x*(2 + x^3)^(2/3))/3 - (5*ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) + (5*Log[-x + (2 + x^3)^(1/3)])/6

Rule 245

Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] := Simp[ArcTan[(1 + 2*Rt[b, 3]*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

Rule 396

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[d*x*((a + b*x^n)^(p+1)/(b*(n*(p+1)+1))), x] - Dist[(a*d - b*c*(n*(p+1)+1))/(b*(n*(p+1)+1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p+1)+1, 0]

Rubi steps

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}} dx = \frac{1}{3}x(2+x^3)^{2/3} - \frac{5}{3} \int \frac{1}{\sqrt[3]{2+x^3}} dx$$

$$= \frac{1}{3}x(2+x^3)^{2/3} - \frac{5 \tan^{-1}\left(\frac{1+\sqrt[3]{\frac{2x}{2+x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{5}{6} \log\left(-x + \sqrt[3]{2+x^3}\right)$$

Mathematica [A]

time = 0.14, size = 90, normalized size = 1.43

$$\frac{1}{18} \left(6x(2+x^3)^{2/3} - 10\sqrt{3} \tan^{-1}\left(\frac{\sqrt{3}x}{x+2\sqrt[3]{2+x^3}}\right) + 10 \log\left(-x + \sqrt[3]{2+x^3}\right) - 5 \log\left(x^2 + x\sqrt[3]{2+x^3} + (2+x^3)^{2/3}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^3)/(2 + x^3)^(1/3), x]

[Out] (6*x*(2 + x^3)^(2/3) - 10*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(2 + x^3)^(1/3))] + 10*Log[-x + (2 + x^3)^(1/3)] - 5*Log[x^2 + x*(2 + x^3)^(1/3) + (2 + x^3)^(2/3)])/18

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 3.14, size = 43, normalized size = 0.68

$$\frac{2^{\frac{2}{3}}x \left(x^3 \text{hyper} \left[\left\{ \frac{1}{3}, \frac{4}{3} \right\}, \left\{ \frac{7}{3} \right\}, \frac{x^3 \exp_{\text{polar}}[i\text{Pi}]}{2} \right] - 4 \text{hyper} \left[\left\{ \frac{1}{3}, \frac{1}{3} \right\}, \left\{ \frac{4}{3} \right\}, \frac{x^3 \exp_{\text{polar}}[i\text{Pi}]}{2} \right] \right)}{8}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(x^3 - 1)/(x^3 + 2)^(1/3), x]')

[Out] 2^(2/3) x (x^3 hyper[{1/3, 4/3}, {7/3}, x^3 exp_polar[i Pi] / 2] - 4 hyper[{1/3, 1/3}, {4/3}, x^3 exp_polar[i Pi] / 2]) / 8

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.

time = 1.01, size = 29, normalized size = 0.46

method	result
risch	$\frac{x(x^3+2)^{\frac{2}{3}}}{3} - \frac{5 \cdot 2^{\frac{2}{3}} x \text{ hypergeom}\left(\left[\frac{1}{3}, \frac{1}{3}\right], \left[\frac{4}{3}\right], -\frac{x^3}{2}\right)}{6}$
meijerg	$-\frac{2^{\frac{2}{3}} x \text{ hypergeom}\left(\left[\frac{1}{3}, \frac{1}{3}\right], \left[\frac{4}{3}\right], -\frac{x^3}{2}\right)}{2} + \frac{2^{\frac{2}{3}} x^4 \text{ hypergeom}\left(\left[\frac{1}{3}, \frac{4}{3}\right], \left[\frac{7}{3}\right], -\frac{x^3}{2}\right)}{8}$

trager	$\frac{x(x^3+2)^{\frac{2}{3}}}{3} + \frac{5 \ln\left(-8 \operatorname{RootOf}\left(4_Z^2+2_Z+1\right)^2 x^3 - 6 \operatorname{RootOf}\left(4_Z^2+2_Z+1\right)(x^3+2)^{\frac{2}{3}} x + 2 \operatorname{RootOf}\left(4_Z^2+2_Z+1\right) x^3\right)}{9}$
--------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3-1)/(x^3+2)^(1/3),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{3}x(x^3+2)^{2/3} - \frac{5}{6}2^{2/3}x \operatorname{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{3}\right], \left[\frac{4}{3}\right], -\frac{1}{2}x^3\right)$

Maxima [A]

time = 0.34, size = 94, normalized size = 1.49

$$\frac{5}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2(x^3+2)^{\frac{1}{3}}}{x} + 1\right)\right) + \frac{2(x^3+2)^{\frac{2}{3}}}{3x^2\left(\frac{x^3+2}{x^3}-1\right)} - \frac{5}{18} \log\left(\frac{(x^3+2)^{\frac{1}{3}}}{x} + \frac{(x^3+2)^{\frac{2}{3}}}{x^2} + 1\right) + \frac{5}{9} \log\left(\frac{(x^3+2)^{\frac{1}{3}}}{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="maxima")`

[Out] $\frac{5}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2(x^3+2)^{1/3}}{x} + 1\right)\right) + \frac{2}{3}(x^3+2)^{2/3}/(x^2((x^3+2)/x^3-1)) - \frac{5}{18}\log((x^3+2)^{1/3}/x + (x^3+2)^{2/3}/x^2 + 1) + \frac{5}{9}\log((x^3+2)^{1/3}/x - 1)$

Fricas [A]

time = 0.30, size = 86, normalized size = 1.37

$$\frac{1}{3}(x^3+2)^{\frac{2}{3}}x + \frac{5}{9}\sqrt{3} \arctan\left(\frac{\sqrt{3}x + 2\sqrt{3}(x^3+2)^{\frac{1}{3}}}{3x}\right) + \frac{5}{9} \log\left(-\frac{x - (x^3+2)^{\frac{1}{3}}}{x}\right) - \frac{5}{18} \log\left(\frac{x^2 + (x^3+2)^{\frac{1}{3}}x + (x^3+2)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="fricas")`

[Out] $\frac{1}{3}(x^3+2)^{2/3}x + \frac{5}{9}\sqrt{3} \arctan\left(\frac{\sqrt{3}x + 2\sqrt{3}(x^3+2)^{1/3}}{3x}\right) + \frac{5}{9}\log\left(-\frac{x - (x^3+2)^{1/3}}{x}\right) - \frac{5}{18}\log\left(\frac{x^2 + (x^3+2)^{1/3}x + (x^3+2)^{2/3}}{x^2}\right)$

Sympy [C] Result contains complex when optimal does not.

time = 1.08, size = 71, normalized size = 1.13

$$\frac{2^{\frac{2}{3}}x^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{4}{3} \middle| \frac{x^3 e^{i\pi}}{2}\right)}{6\Gamma\left(\frac{7}{3}\right)} - \frac{2^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{3} \middle| \frac{x^3 e^{i\pi}}{2}\right)}{6\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3-1)/(x**3+2)**(1/3),x)`

[Out] $2^{2/3} x^4 \Gamma(4/3) \operatorname{hyper}((1/3, 4/3), (7/3,), x^3 \exp(\pi i/2)) / (6 \Gamma(7/3)) - 2^{2/3} x \Gamma(1/3) \operatorname{hyper}((1/3, 1/3), (4/3,), x^3 \exp(\pi i/2)) / (6 \Gamma(4/3))$

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3-1)/(x^3+2)^(1/3),x)`

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3 - 1}{(x^3 + 2)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3 - 1)/(x^3 + 2)^(1/3),x)`

[Out] `int((x^3 - 1)/(x^3 + 2)^(1/3), x)`

$$3.317 \quad \int \frac{(1+x^4)^{3/4}}{(2+x^4)^2} dx$$

Optimal. Leaf size=74

$$\frac{x(1+x^4)^{3/4}}{8(2+x^4)} + \frac{3 \tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16 \cdot 2^{3/4}} + \frac{3 \tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16 \cdot 2^{3/4}}$$

[Out] 1/8*x*(x^4+1)^(3/4)/(x^4+2)+3/32*arctan(1/2*x*2^(3/4)/(x^4+1)^(1/4))*2^(1/4)+3/32*arctanh(1/2*x*2^(3/4)/(x^4+1)^(1/4))*2^(1/4)

Rubi [A]

time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {386, 385, 218, 212, 209}

$$\frac{(x^4+1)^{3/4} x}{8(x^4+2)} + \frac{3 \tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{16 \cdot 2^{3/4}} + \frac{3 \tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{16 \cdot 2^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)^(3/4)/(2 + x^4)^2,x]

[Out] (x*(1 + x^4)^(3/4))/(8*(2 + x^4)) + (3*ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))])/(16*2^(3/4)) + (3*ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))])/(16*2^(3/4))

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 218

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(1+x^4)^{3/4}}{(2+x^4)^2} dx &= \frac{x(1+x^4)^{3/4}}{8(2+x^4)} + \frac{3}{8} \int \frac{1}{\sqrt[4]{1+x^4}(2+x^4)} dx \\ &= \frac{x(1+x^4)^{3/4}}{8(2+x^4)} + \frac{3}{8} \text{Subst} \left(\int \frac{1}{2-x^4} dx, x, \frac{x}{\sqrt[4]{1+x^4}} \right) \\ &= \frac{x(1+x^4)^{3/4}}{8(2+x^4)} + \frac{3 \text{Subst} \left(\int \frac{1}{\sqrt{2-x^2}} dx, x, \frac{x}{\sqrt[4]{1+x^4}} \right)}{16\sqrt{2}} + \frac{3 \text{Subst} \left(\int \frac{1}{\sqrt{2+x^2}} dx, x, \frac{x}{\sqrt[4]{1+x^4}} \right)}{16\sqrt{2}} \\ &= \frac{x(1+x^4)^{3/4}}{8(2+x^4)} + \frac{3 \tan^{-1} \left(\frac{x}{\sqrt[4]{2} \sqrt[4]{1+x^4}} \right)}{16 \cdot 2^{3/4}} + \frac{3 \tanh^{-1} \left(\frac{x}{\sqrt[4]{2} \sqrt[4]{1+x^4}} \right)}{16 \cdot 2^{3/4}} \end{aligned}$$

Mathematica [A]

time = 0.22, size = 74, normalized size = 1.00

$$\frac{x(1+x^4)^{3/4}}{8(2+x^4)} + \frac{3 \tan^{-1} \left(\frac{x}{\sqrt[4]{2} \sqrt[4]{1+x^4}} \right)}{16 \cdot 2^{3/4}} + \frac{3 \tanh^{-1} \left(\frac{x}{\sqrt[4]{2} \sqrt[4]{1+x^4}} \right)}{16 \cdot 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)^(3/4)/(2 + x^4)^2, x]
```

```
[Out] (x*(1 + x^4)^(3/4))/(8*(2 + x^4)) + (3*ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))])/(16*2^(3/4)) + (3*ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))])/(16*2^(3/4))
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(x^4 + 1)^(3/4)/(x^4 + 2)^2,x]')`

[Out] `cought exception: maximum recursion depth exceeded in comparison`

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.69, size = 227, normalized size = 3.07

method	result
risch	$\frac{(x^4+1)^{\frac{3}{4}}x}{8x^4+16} - \frac{3 \operatorname{RootOf}(-Z^4-2) \ln\left(\frac{2\sqrt{x^4+1} \operatorname{RootOf}(-Z^4-2)^3 x^{2-2}(x^4+1)^{\frac{1}{4}} \operatorname{RootOf}(-Z^4-2)^2 x^3+3 \operatorname{RootOf}(-Z^4-2)x}{x^4+2}\right)}{64}$
trager	$\frac{(x^4+1)^{\frac{3}{4}}x}{8x^4+16} - \frac{3 \operatorname{RootOf}(-Z^4-2) \ln\left(\frac{2\sqrt{x^4+1} \operatorname{RootOf}(-Z^4-2)^3 x^{2-2}(x^4+1)^{\frac{1}{4}} \operatorname{RootOf}(-Z^4-2)^2 x^3+3 \operatorname{RootOf}(-Z^4-2)x}{x^4+2}\right)}{64}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4+1)^(3/4)/(x^4+2)^2,x,method=_RETURNVERBOSE)`

[Out]
$$\frac{1}{8}x(x^4+1)^{3/4}/(x^4+2) - \frac{3}{64}\operatorname{RootOf}(-Z^4-2)\ln\left(\frac{2(x^4+1)^{1/2}\operatorname{RootOf}(-Z^4-2)^3x^2-2(x^4+1)^{1/4}\operatorname{RootOf}(-Z^4-2)^2x^3+3\operatorname{RootOf}(-Z^4-2)x^4-4(x^4+1)^{3/4}x+2\operatorname{RootOf}(-Z^4-2)}{(x^4+2)}\right) + \frac{3}{64}\operatorname{RootOf}(-Z^2+\operatorname{RootOf}(-Z^4-2)^2)\ln\left(\frac{2(x^4+1)^{1/2}\operatorname{RootOf}(-Z^4-2)^2\operatorname{RootOf}(-Z^2+\operatorname{RootOf}(-Z^4-2)^2)x^2+2(x^4+1)^{1/4}\operatorname{RootOf}(-Z^4-2)^2x^3-3\operatorname{RootOf}(-Z^2+\operatorname{RootOf}(-Z^4-2)^2)x^4-4(x^4+1)^{3/4}x-2\operatorname{RootOf}(-Z^2+\operatorname{RootOf}(-Z^4-2)^2)}{(x^4+2)}\right)$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+1)^(3/4)/(x^4+2)^2,x, algorithm="maxima")`

[Out] `integrate((x^4 + 1)^(3/4)/(x^4 + 2)^2, x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(56) = 112.

time = 3.64, size = 242, normalized size = 3.27

$$\frac{12 \cdot 8^{\frac{3}{4}}(x^2+2) \arctan\left(\frac{8^{\frac{3}{4}}(x^2+1)^{\frac{3}{4}}+8^{\frac{3}{4}}(x^2+1)^{\frac{3}{4}}\sqrt{x^4+1}}{2(x^2+2)}\right) - 3 \cdot 8^{\frac{3}{4}}(x^2+2) \log\left(\frac{8^{\frac{3}{4}}\sqrt{x^4+1}+8^{\frac{3}{4}}\sqrt{x^4+1}+8^{\frac{3}{4}}(3x^2+2)+16(x^2+1)^{\frac{3}{4}}}{x^4+2}\right) + 3 \cdot 8^{\frac{3}{4}}(x^2+2) \log\left(\frac{8^{\frac{3}{4}}\sqrt{x^4+1}+8^{\frac{3}{4}}\sqrt{x^4+1}+8^{\frac{3}{4}}(3x^2+2)+16(x^2+1)^{\frac{3}{4}}}{x^4+2}\right) - 64(x^2+1)^{\frac{3}{4}}x}{512(x^4+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)^(3/4)/(x^4+2)^2,x, algorithm="fricas")

[Out]
$$\frac{-1/512*(12*8^{3/4}*(x^4 + 2)*\arctan(-1/2*(8^{3/4}*(x^4 + 1)^{1/4}*x^3 + 4*8^{1/4}*(x^4 + 1)^{3/4}*x - 2^{1/4}*(8^{3/4}*\sqrt{x^4 + 1})*x^2 + 8^{1/4}*(3*x^4 + 2)))/(x^4 + 2) - 3*8^{3/4}*(x^4 + 2)*\log((8*\sqrt{2}*(x^4 + 1)^{1/4}*x^3 + 8*8^{1/4}*\sqrt{x^4 + 1})*x^2 + 8^{3/4}*(3*x^4 + 2) + 16*(x^4 + 1)^{3/4})*x)/(x^4 + 2) + 3*8^{3/4}*(x^4 + 2)*\log((8*\sqrt{2}*(x^4 + 1)^{1/4}*x^3 - 8*8^{1/4}*\sqrt{x^4 + 1})*x^2 - 8^{3/4}*(3*x^4 + 2) + 16*(x^4 + 1)^{3/4})*x)/(x^4 + 2) - 64*(x^4 + 1)^{3/4}*x)/(x^4 + 2)}$$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4 + 1)^{\frac{3}{4}}}{(x^4 + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+1)**(3/4)/(x**4+2)**2,x)

[Out] Integral((x**4 + 1)**(3/4)/(x**4 + 2)**2, x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)^(3/4)/(x^4+2)^2,x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^4 + 1)^{3/4}}{(x^4 + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4 + 1)^(3/4)/(x^4 + 2)^2,x)

[Out] int((x^4 + 1)^(3/4)/(x^4 + 2)^2, x)

$$3.318 \quad \int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}} dx$$

Optimal. Leaf size=48

$$-\frac{5x(-2+x^5)}{33(3+x^5)^{11/5}} + \frac{5x}{297(3+x^5)^{6/5}} + \frac{97x}{891\sqrt[5]{3+x^5}}$$

[Out] $-5/33*x*(x^5-2)/(x^5+3)^{(11/5)}+5/297*x/(x^5+3)^{(6/5)}+97/891*x/(x^5+3)^{(1/5)}$

Rubi [A]

time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$,

Rules used = {386, 197}

$$\frac{x(2-x^5)^2}{33(x^5+3)^{11/5}} + \frac{10x(2-x^5)}{297(x^5+3)^{6/5}} + \frac{100x}{891\sqrt[5]{x^5+3}}$$

Antiderivative was successfully verified.

[In] Int[(-2 + x^5)^2/(3 + x^5)^(16/5), x]

[Out] $(x*(2 - x^5)^2)/(33*(3 + x^5)^{(11/5)}) + (10*x*(2 - x^5))/(297*(3 + x^5)^{(6/5)}) + (100*x)/(891*(3 + x^5)^{(1/5)})$

Rule 197

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 386

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}} dx &= \frac{x(2-x^5)^2}{33(3+x^5)^{11/5}} - \frac{20}{33} \int \frac{-2+x^5}{(3+x^5)^{11/5}} dx \\ &= \frac{x(2-x^5)^2}{33(3+x^5)^{11/5}} + \frac{10x(2-x^5)}{297(3+x^5)^{6/5}} + \frac{100}{297} \int \frac{1}{(3+x^5)^{6/5}} dx \\ &= \frac{x(2-x^5)^2}{33(3+x^5)^{11/5}} + \frac{10x(2-x^5)}{297(3+x^5)^{6/5}} + \frac{100x}{891\sqrt[5]{3+x^5}} \end{aligned}$$

Mathematica [A]

time = 0.74, size = 26, normalized size = 0.54

$$\frac{x(1188 + 462x^5 + 97x^{10})}{891(3 + x^5)^{11/5}}$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + x^5)^2/(3 + x^5)^(16/5),x]

[Out] (x*(1188 + 462*x^5 + 97*x^10))/(891*(3 + x^5)^(11/5))

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(x^5 - 2)^2/((x^5 + 3)^3*(x^5 + 3)^(1/5)),x]')

[Out] Timed out

Maple [A]

time = 0.10, size = 23, normalized size = 0.48

method	result	size
gospers	$\frac{x(97x^{10}+462x^5+1188)}{891(x^5+3)^{\frac{11}{5}}}$	23
trager	$\frac{x(97x^{10}+462x^5+1188)}{891(x^5+3)^{\frac{11}{5}}}$	23
risch	$\frac{x(97x^{10}+462x^5+1188)}{891(x^5+3)^{\frac{11}{5}}}$	23
meijerg	$\frac{43^{\frac{4}{5}}x\left(\frac{25}{9}x^{10}+\frac{55}{3}x^5+33\right)}{2673\left(1+\frac{x^5}{3}\right)^{\frac{11}{5}}} + \frac{3^{\frac{4}{5}}x^{11}}{891\left(1+\frac{x^5}{3}\right)^{\frac{11}{5}}} - \frac{23^{\frac{4}{5}}x^6\left(11+\frac{5x^5}{3}\right)}{2673\left(1+\frac{x^5}{3}\right)^{\frac{11}{5}}}$	70

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5-2)^2/(x^5+3)^(16/5),x,method=_RETURNVERBOSE)

[Out] 1/891*x*(97*x^10+462*x^5+1188)/(x^5+3)^(11/5)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(36) = 72.

time = 0.29, size = 73, normalized size = 1.52

$$-\frac{4x^{11}\left(\frac{11(x^5+3)}{x^5} - \frac{33(x^5+3)^2}{x^{10}} - 3\right)}{891(x^5+3)^{\frac{11}{5}}} - \frac{2x^{11}\left(\frac{11(x^5+3)}{x^5} - 6\right)}{297(x^5+3)^{\frac{11}{5}}} + \frac{x^{11}}{33(x^5+3)^{\frac{11}{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^5-2)^2/(x^5+3)^(16/5),x, algorithm="maxima")`

[Out] $-4/891*x^{11}*(11*(x^5 + 3)/x^5 - 33*(x^5 + 3)^2/x^{10} - 3)/(x^5 + 3)^{(11/5)} - 2/297*x^{11}*(11*(x^5 + 3)/x^5 - 6)/(x^5 + 3)^{(11/5)} + 1/33*x^{11}/(x^5 + 3)^{(11/5)}$

Fricas [A]

time = 0.30, size = 40, normalized size = 0.83

$$\frac{(97x^{11} + 462x^6 + 1188x)(x^5 + 3)^{\frac{4}{5}}}{891(x^{15} + 9x^{10} + 27x^5 + 27)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^5-2)^2/(x^5+3)^(16/5),x, algorithm="fricas")`

[Out] $1/891*(97*x^{11} + 462*x^6 + 1188*x)*(x^5 + 3)^{(4/5)}/(x^{15} + 9*x^{10} + 27*x^5 + 27)$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**5-2)**2/(x**5+3)**(16/5),x)`

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^5-2)^2/(x^5+3)^(16/5),x)`

[Out] Could not integrate

Mupad [B]

time = 0.26, size = 23, normalized size = 0.48

$$\frac{97x^{11} + 462x^6 + 1188x}{891(x^5 + 3)^{11/5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5 - 2)^2/(x^5 + 3)^(16/5),x)`

[Out] $(1188*x + 462*x^6 + 97*x^{11})/(891*(x^5 + 3)^{(11/5)})$

$$3.319 \quad \int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}} dx$$

Optimal. Leaf size=90

$$\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}}{\sqrt{3}}\right)}{3^{5/6}} - \frac{\log(1-(1+x)^3)}{6\sqrt[3]{3}} + \frac{\log\left(\sqrt[3]{3}(1+x) - \sqrt[3]{2+(1+x)^3}\right)}{2\sqrt[3]{3}}$$

[Out] $-1/3*\arctan(1/3*(1+2*3^{(1/3)}*(1+x)/(2+(1+x)^3)^{(1/3)})*3^{(1/2)}*3^{(1/6)}-1/18*\ln(1-(1+x)^3)*3^{(2/3)}+1/6*\ln(3^{(1/3)}*(1+x)-(2+(1+x)^3)^{(1/3)})*3^{(2/3)}$

Rubi [A]

time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {443, 442, 384}

$$\frac{\log(1-(x+1)^3)}{6\sqrt[3]{3}} + \frac{\log\left(\sqrt[3]{3}(x+1) - \sqrt[3]{(x+1)^3+2}\right)}{2\sqrt[3]{3}} - \frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

[In] `Int[1/((3*x + 3*x^2 + x^3)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]`

[Out] $-(\text{ArcTan}[(1 + (2*3^{(1/3)}*(1 + x))/(2 + (1 + x)^3)^{(1/3)})/\text{Sqrt}[3]]/3^{(5/6)}) - \text{Log}[1 - (1 + x)^3]/(6*3^{(1/3)}) + \text{Log}[3^{(1/3)}*(1 + x) - (2 + (1 + x)^3)^{(1/3})]/(2*3^{(1/3)})$

Rule 384

`Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x] + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]`

Rule 442

`Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symbol] := Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u, x]`

Rule 443

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] := Int[NormalizePseudoBinomial[u, x]^p
*NormalizePseudoBinomial[v, x]^q, x] /; FreeQ[{p, q}, x] && PseudoBinomialP
airQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3x + 3x^2 + x^3) \sqrt[3]{3 + 3x + 3x^2 + x^3}} dx &= \int \frac{1}{(-1 + (1+x)^3) \sqrt[3]{2 + (1+x)^3}} dx \\
&= \text{Subst} \left(\int \frac{1}{(-1 + x^3) \sqrt[3]{2 + x^3}} dx, x, 1+x \right) \\
&= \text{Subst} \left(\int \frac{1}{-1 + 3x^3} dx, x, \frac{1+x}{\sqrt[3]{2 + (1+x)^3}} \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \frac{1}{-1 + \sqrt[3]{3} x} dx, x, \frac{1+x}{\sqrt[3]{2 + (1+x)^3}} \right) + \frac{1}{3} \text{Subst} \left(\int \frac{1}{-1 + \sqrt[3]{3} x} dx, x, \frac{1+x}{\sqrt[3]{2 + (1+x)^3}} \right) \\
&= \frac{\log \left(1 - \frac{\sqrt[3]{3} (1+x)}{\sqrt[3]{2 + (1+x)^3}} \right)}{3\sqrt[3]{3}} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{1 + \sqrt[3]{3} x + 3^{2/3} x^2} dx, x, \frac{1+x}{\sqrt[3]{2 + (1+x)^3}} \right) \\
&= \frac{\log \left(1 - \frac{\sqrt[3]{3} (1+x)}{\sqrt[3]{2 + (1+x)^3}} \right)}{3\sqrt[3]{3}} - \frac{\log \left(1 + \frac{3^{2/3} (1+x)^2}{(2 + (1+x)^3)^{2/3}} + \frac{\sqrt[3]{3}}{\sqrt[3]{2 + (1+x)^3}} \right)}{6\sqrt[3]{3}} \\
&= -\frac{\tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{3} (1+x)}{\sqrt[3]{2 + (1+x)^3}}}{\sqrt[3]{3}} \right)}{3^{5/6}} + \frac{\log \left(1 - \frac{\sqrt[3]{3} (1+x)}{\sqrt[3]{2 + (1+x)^3}} \right)}{3\sqrt[3]{3}}
\end{aligned}$$

Mathematica [A]

time = 0.34, size = 180, normalized size = 2.00

$$\frac{\tan^{-1} \left(\frac{\sqrt[3]{3} \sqrt[3]{3 + 3x + 3x^2 + x^3}}{2\sqrt[3]{3} + 2\sqrt[3]{3} x + \sqrt[3]{3} + 3x + 3x^2 + x^3} \right)}{3^{5/6}} + \frac{2 \log \left(\sqrt[3]{3} + \sqrt[3]{3} x - \sqrt[3]{3 + 3x + 3x^2 + x^3} \right) - \log \left(3^{2/3} + 2 \cdot 3^{2/3} x + 3^{2/3} x^2 + \sqrt[3]{3} (1+x) \sqrt[3]{3 + 3x + 3x^2 + x^3} + (3 + 3x + 3x^2 + x^3)^{2/3} \right)}{6\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3*x + 3*x^2 + x^3)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(Sqrt[3]*(3 + 3*x + 3*x^2 + x^3)^(1/3))/(2*3^(1/3) + 2*3^(1/3)*x + (3 + 3*x + 3*x^2 + x^3)^(1/3))]/3^(5/6) + (2*Log[3^(1/3) + 3^(1/3)*x - (3 +
```

$$\frac{3x + 3x^2 + x^3)^{1/3} - \text{Log}[3^{2/3} + 2 \cdot 3^{2/3}x + 3^{2/3}x^2 + 3^{1/3} \cdot (1+x) \cdot (3 + 3x + 3x^2 + x^3)^{1/3} + (3 + 3x + 3x^2 + x^3)^{2/3}]}{(6 \cdot 3^{1/3})}$$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((x^3 + 3*x^2 + 3*x)*(x^3 + 3*x^2 + 3*x + 3)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 8.54, size = 2515, normalized size = 27.94

method	result	size
trager	Expression too large to display	2515

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*ln((203214279*x^3*RootOf(_Z^3-9)^3*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+5196569877*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+11379999624*RootOf(_Z^3-9)*x+2837496903*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)+8512490709*(x^3+3*x^2+3*x+3)^(2/3)*x-1422499953*RootOf(_Z^3-9)^3*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)-36375989139*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2+114324537294*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+15322002984*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^(2/3)*x+4470714138*RootOf(_Z^3-9)+609642837*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x+15589709631*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x+3793333208*RootOf(_Z^3-9)*x^3+97002637704*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3+11379999624*RootOf(_Z^3-9)*x^2+291007913112*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2+291007913112*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x+609642837*RootOf(_Z^3-9)^3*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2+15589709631*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*x^2+15322002984*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^(2/3)+2837496903*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)*x^2+5674993806*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)*x+45966008952*RootOf(_Z^3-9)*RootOf(RootOf
```

$$\begin{aligned}
& (_Z^3-9)^2+9*_Z*\text{RootOf}(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^{(1/3)}+45966008952 \\
& *\text{RootOf}(_Z^3-9)*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{RootOf}(_Z^3-9)+81*_Z^2)*(x^3+3 \\
& *x^2+3*x+3)^{(1/3)}*x^2+91932017904*\text{RootOf}(_Z^3-9)*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9* \\
& _Z*\text{RootOf}(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^{(1/3)}*x+8512490709*(x^3+3*x^2+ \\
& 3*x+3)^{(2/3)}/x/(_Z^2+3*x+3))-1/9*\ln((374182374*x^3*\text{RootOf}(_Z^3-9)^3*\text{RootOf}(_Z^3-9) \\
& \text{RootOf}(_Z^3-9)^2+9*_Z*\text{RootOf}(_Z^3-9)+81*_Z^2)+5196569877*\text{RootOf}(\text{RootOf}(_Z^3-9) \\
& ^2+9*_Z*\text{RootOf}(_Z^3-9)+81*_Z^2)^2*\text{RootOf}(_Z^3-9)^2*x^3-19831665822*\text{RootOf} \\
& f(_Z^3-9)*x-2269837425*\text{RootOf}(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^{(1/3)}-6809512275* \\
& (x^3+3*x^2+3*x+3)^{(2/3)}*x-2619276618*\text{RootOf}(_Z^3-9)^3*\text{RootOf}(\text{RootOf}(_Z^3-9) \\
& ^2+9*_Z*\text{RootOf}(_Z^3-9)+81*_Z^2)-36375989139*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{Ro} \\
& otOf(_Z^3-9)+81*_Z^2)^2*\text{RootOf}(_Z^3-9)^2-150700526433*\text{RootOf}(\text{RootOf}(_Z^3-9) \\
& ^2+9*_Z*\text{RootOf}(_Z^3-9)+81*_Z^2)-15322002984*\text{RootOf}(_Z^3-9)^2*\text{RootOf}(\text{RootOf}(_Z^3-9) \\
& ^2+9*_Z*\text{RootOf}(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^{(2/3)}*x-1085128884 \\
& 6*\text{RootOf}(_Z^3-9)+1122547122*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{RootOf}(_Z^3-9)+81* \\
& _Z^2)*\text{RootOf}(_Z^3-9)^3*x+15589709631*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{RootOf}(_Z \\
& ^3-9)+81*_Z^2)^2*\text{RootOf}(_Z^3-9)^2*x-6610555274*\text{RootOf}(_Z^3-9)*x^3-918060678 \\
& 27*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{RootOf}(_Z^3-9)+81*_Z^2)*x^3-19831665822*\text{Ro} \\
& otOf(_Z^3-9)*x^2-275418203481*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{RootOf}(_Z^3-9)+81 \\
& *_Z^2)*x^2-275418203481*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{RootOf}(_Z^3-9)+81*_Z^2 \\
&)*x+1122547122*\text{RootOf}(_Z^3-9)^3*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{RootOf}(_Z^3-9) \\
& +81*_Z^2)*x^2+15589709631*\text{RootOf}(_Z^3-9)^2*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{Ro} \\
& otOf(_Z^3-9)+81*_Z^2)^2*x^2-15322002984*\text{RootOf}(_Z^3-9)^2*\text{RootOf}(\text{RootOf}(_Z^3- \\
& 9)^2+9*_Z*\text{RootOf}(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^{(2/3)}-2269837425*\text{RootOf} \\
& (_Z^3-9)^2*(x^3+3*x^2+3*x+3)^{(1/3)}*x^2-4539674850*\text{RootOf}(_Z^3-9)^2*(x^3+3*x \\
& ^2+3*x+3)^{(1/3)}*x-45966008952*\text{RootOf}(_Z^3-9)*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{R} \\
& ootOf(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^{(1/3)}-45966008952*\text{RootOf}(_Z^3-9)*\text{R} \\
& ootOf(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{RootOf}(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^{(1/3)} \\
& *x^2-91932017904*\text{RootOf}(_Z^3-9)*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{RootOf}(_Z^3-9) \\
& +81*_Z^2)*(x^3+3*x^2+3*x+3)^{(1/3)}*x-6809512275*(x^3+3*x^2+3*x+3)^{(2/3)}/x/(\\
& x^2+3*x+3))*\text{RootOf}(_Z^3-9)-\ln((374182374*x^3*\text{RootOf}(_Z^3-9)^3*\text{RootOf}(\text{RootOf} \\
& (_Z^3-9)^2+9*_Z*\text{RootOf}(_Z^3-9)+81*_Z^2)+5196569877*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+ \\
& 9*_Z*\text{RootOf}(_Z^3-9)+81*_Z^2)^2*\text{RootOf}(_Z^3-9)^2*x^3-19831665822*\text{RootOf}(_Z^3 \\
& -9)*x-2269837425*\text{RootOf}(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^{(1/3)}-6809512275*(x^3+3 \\
& *x^2+3*x+3)^{(2/3)}*x-2619276618*\text{RootOf}(_Z^3-9)^3*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9* \\
& _Z*\text{RootOf}(_Z^3-9)+81*_Z^2)-36375989139*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{RootOf}(_Z \\
& ^3-9)+81*_Z^2)^2*\text{RootOf}(_Z^3-9)^2-150700526433*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9* \\
& _Z*\text{RootOf}(_Z^3-9)+81*_Z^2)-15322002984*\text{RootOf}(_Z^3-9)^2*\text{RootOf}(\text{RootOf}(_Z^3-9) \\
&)^2+9*_Z*\text{RootOf}(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^{(2/3)}*x-10851288846*\text{Root} \\
& Of(_Z^3-9)+1122547122*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{RootOf}(_Z^3-9)+81*_Z^2)* \\
& \text{RootOf}(_Z^3-9)^3*x+15589709631*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{RootOf}(_Z^3-9)+ \\
& 81*_Z^2)^2*\text{RootOf}(_Z^3-9)^2*x-6610555274*\text{RootOf}(_Z^3-9)*x^3-91806067827*\text{Ro} \\
& otOf(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{RootOf}(_Z^3-9)+81*_Z^2)*x^3-19831665822*\text{RootOf}(_Z \\
& ^3-9)*x^2-275418203481*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{RootOf}(_Z^3-9)+81*_Z^2) \\
& *x^2-275418203481*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{RootOf}(_Z^3-9)+81*_Z^2)*x+11 \\
& 22547122*\text{RootOf}(_Z^3-9)^3*\text{RootOf}(\text{RootOf}(_Z^3-9)^2+9*_Z*\text{RootOf}(_Z^3-9)+81*_Z
\end{aligned}$$

```

^2)*x^2+15589709631*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z
^3-9)+81*_Z^2)^2*x^2-15322002984*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+9
*_Z*RootOf(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^(2/3)-2269837425*RootOf(_Z^3-
9)^2*(x^3+3*x^2+3*x+3)^(1/3)*x^2-4539674850*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x
+3)^(1/3)*x-45966008952*RootOf(_Z^3-9)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(
_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^(1/3)-45966008952*RootOf(_Z^3-9)*RootOf(
RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*(x^3+3*x^2+3*x+3)^(1/3)*x^2-9
1932017904*RootOf(_Z^3-9)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z
^2)*(x^3+3*x^2+3*x+3)^(1/3)*x-6809512275*(x^3+3*x^2+3*x+3)^(2/3))/x/(x^2+3*
x+3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)

```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^3 + 3*x^2 + 3*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(71) = 142$.

time = 5.50, size = 458, normalized size = 5.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 28*x^3 + 42*x^2 + 30*x + 9)*(x^3 + 3*
x^2 + 3*x + 3)^(2/3) + 3^(1/3)*(31*x^6 + 186*x^5 + 465*x^4 + 666*x^3 + 603*
x^2 + 324*x + 81) + 9*(5*x^5 + 25*x^4 + 50*x^3 + 54*x^2 + 33*x + 9)*(x^3 +
3*x^2 + 3*x + 3)^(1/3))/(x^6 + 6*x^5 + 15*x^4 + 18*x^3 + 9*x^2)) + 1/27*3^(
2/3)*log((2*3^(2/3)*(x^3 + 3*x^2 + 3*x) - 9*3^(1/3)*(x^3 + 3*x^2 + 3*x + 3)
^(1/3)*(x^2 + 2*x + 1) + 9*(x^3 + 3*x^2 + 3*x + 3)^(2/3)*(x + 1))/(x^3 + 3*
x^2 + 3*x)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)*(7*x^7 + 49*x^6 +
147*x^5 + 240*x^4 + 225*x^3 + 117*x^2 + 27*x)*(x^3 + 3*x^2 + 3*x + 3)^(2/3)
- 3^(1/3)*(127*x^9 + 1143*x^8 + 4572*x^7 + 11070*x^6 + 18414*x^5 + 22032*x
^4 + 18900*x^3 + 11178*x^2 + 4131*x + 729) - 18*(31*x^8 + 248*x^7 + 868*x^6
+ 1782*x^5 + 2400*x^4 + 2196*x^3 + 1332*x^2 + 486*x + 81)*(x^3 + 3*x^2 + 3
*x + 3)^(1/3))/(251*x^9 + 2259*x^8 + 9036*x^7 + 21546*x^6 + 34398*x^5 + 385
56*x^4 + 30348*x^3 + 16038*x^2 + 5103*x + 729))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(x^2 + 3x + 3)\sqrt[3]{x^3 + 3x^2 + 3x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**3+3*x**2+3*x)/(x**3+3*x**2+3*x+3)**(1/3),x)`

[Out] `Integral(1/(x*(x**2 + 3*x + 3)*(x**3 + 3*x**2 + 3*x + 3)**(1/3)), x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x)`

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x^3 + 3x^2 + 3x)(x^3 + 3x^2 + 3x + 3)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((3*x + 3*x^2 + x^3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)),x)`

[Out] `int(1/((3*x + 3*x^2 + x^3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)), x)`

$$3.320 \quad \int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}} dx$$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

[Out] 1/2*arctan(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1713, 209}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]

[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1713

Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}} dx &= \text{Subst} \left(\int \frac{1}{1+2x^2} dx, x, \frac{x}{\sqrt{1+x^4}} \right) \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 23, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]), x]``[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]), x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.13, size = 112, normalized size = 4.87

method	result
elliptic	$\frac{\arctan\left(\frac{\sqrt{x^4+1}\sqrt{2}}{2x}\right)\sqrt{2}}{2}$
trager	$\frac{\text{RootOf}(-Z^2+2)\ln\left(\frac{\text{RootOf}(-Z^2+2)_x+\sqrt{x^4+1}}{x^2+1}\right)}{2}$
default	$\frac{\sqrt{-ix^2+1}\sqrt{ix^2+1}\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right), i\right)}{\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)\sqrt{x^4+1}} - \frac{2(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\sqrt{ix^2+1}\text{EllipticPi}\left((-1)^{\frac{1}{4}}x\right)}{\sqrt{x^4+1}}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((-x^2+1)/(x^2+1)/(x^4+1)^(1/2), x, method=_RETURNVERBOSE)`

```
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)), I)-2*(-1)^(3/4)*(1-I*x^2)^(1/2)*
(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x, I, (-I)^(1/2)/(-1)^(1/4)
))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")``[Out] -integrate((x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)`**Fricas [A]**

time = 0.52, size = 18, normalized size = 0.78

$$\frac{1}{2} \sqrt{2} \arctan \left(\frac{\sqrt{2} x}{\sqrt{x^4 + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")``[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1))`**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2 \sqrt{x^4 + 1} + \sqrt{x^4 + 1}} dx - \int \left(-\frac{1}{x^2 \sqrt{x^4 + 1} + \sqrt{x^4 + 1}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((-x**2+1)/(x**2+1)/(x**4+1)**(1/2),x)``[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) + sqrt(x**4 + 1)), x) - Integral(-1/(x**2*sqrt(x**4 + 1) + sqrt(x**4 + 1)), x)`**Giac [F] N/A**

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x)``[Out] Could not integrate`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{x^2 - 1}{(x^2 + 1) \sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(-(x^2 - 1)/((x^2 + 1)*(x^4 + 1)^{(1/2)}), x)$

[Out] $-\text{int}((x^2 - 1)/((x^2 + 1)*(x^4 + 1)^{(1/2)}), x)$

$$3.321 \quad \int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}} dx$$

Optimal. Leaf size=23

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

[Out] 1/2*arctanh(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1713, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]

[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1713

Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}} dx &= \text{Subst} \left(\int \frac{1}{1-2x^2} dx, x, \frac{x}{\sqrt{1+x^4}} \right) \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 23, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]), x]``[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]), x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.15, size = 112, normalized size = 4.87

method	result
elliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^4+1}\sqrt{2}}{2x}\right)\sqrt{2}}{2}$
trager	$-\frac{\operatorname{RootOf}(-Z^2-2)\ln\left(\frac{\operatorname{RootOf}(-Z^2-2)x-\sqrt{x^4+1}}{(1+x)(-1+x)}\right)}{2}$
default	$-\frac{\sqrt{-ix^2+1}\sqrt{ix^2+1}\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)\sqrt{x^4+1}} - \frac{2(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\sqrt{ix^2+1}\operatorname{EllipticPi}\left((-1)^{\frac{1}{4}}x\right)}{\sqrt{x^4+1}}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((x^2+1)/(-x^2+1)/(x^4+1)^(1/2), x, method=_RETURNVERBOSE)`

```
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)), I)-2*(-1)^(3/4)*(1-I*x^2)^(1/2)*
(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x, -I, (-I)^(1/2)/(-1)^(1/4))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(18) = 36.

time = 0.33, size = 42, normalized size = 1.83

$$\frac{1}{4} \sqrt{2} \log \left(\frac{x^4 + 2 \sqrt{2} \sqrt{x^4 + 1} x + 2x^2 + 1}{x^4 - 2x^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^4 + 2*sqrt(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2 + 1))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2 \sqrt{x^4 + 1} - \sqrt{x^4 + 1}} dx - \int \frac{1}{x^2 \sqrt{x^4 + 1} - \sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(-x**2+1)/(x**4+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x) - Integral(1/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x)
```

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{x^2 + 1}{(x^2 - 1)\sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^2 + 1)/((x^2 - 1)*(x^4 + 1)^(1/2)), x)`

[Out] `int(-(x^2 + 1)/((x^2 - 1)*(x^4 + 1)^(1/2)), x)`

$$3.322 \quad \int \frac{1+x^2}{x\sqrt{1+x^4}} dx$$

Optimal. Leaf size=16

$$\tanh^{-1}\left(\frac{-1+x^2}{\sqrt{1+x^4}}\right)$$

[Out] arctanh((x^2-1)/(x^4+1)^(1/2))

Rubi [A]

time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1266, 858, 221, 272, 65, 213}

$$\frac{1}{2} \sinh^{-1}(x^2) - \frac{1}{2} \tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(x*Sqrt[1 + x^4]),x]

[Out] ArcSinh[x^2]/2 - ArcTanh[Sqrt[1 + x^4]]/2

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 858

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 1266

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{1+x^2}{x\sqrt{1+x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1+x}{x\sqrt{1+x^2}} dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1+x^2}} dx, x, x^2 \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{1+x^2}} dx, x, x^2 \right) \\
 &= \frac{1}{2} \sinh^{-1}(x^2) + \frac{1}{4} \text{Subst} \left(\int \frac{1}{x\sqrt{1+x}} dx, x, x^4 \right) \\
 &= \frac{1}{2} \sinh^{-1}(x^2) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1+x^4} \right) \\
 &= \frac{1}{2} \sinh^{-1}(x^2) - \frac{1}{2} \tanh^{-1}(\sqrt{1+x^4})
 \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.

time = 0.18, size = 35, normalized size = 2.19

$$\frac{1}{2} \tanh^{-1} \left(\frac{x^2}{\sqrt{1+x^4}} \right) - \tanh^{-1} \left(x^2 + \sqrt{1+x^4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(x*Sqrt[1 + x^4]), x]

[Out] ArcTanh[x^2/Sqrt[1 + x^4]]/2 - ArcTanh[x^2 + Sqrt[1 + x^4]]

Mathics [A]

time = 6.17, size = 13, normalized size = 0.81

$$-\frac{\text{ArcSinh} \left[\frac{1}{x^2} \right]}{2} + \frac{\text{ArcSinh} [x^2]}{2}$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(x^2 + 1)/(x*sqrt[1 + x^4]),x]')`

[Out] $-\text{ArcSinh}[1 / x^2] / 2 + \text{ArcSinh}[x^2] / 2$

Maple [A]

time = 0.11, size = 18, normalized size = 1.12

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2} + \frac{\operatorname{arcsinh}(x^2)}{2}$	18
elliptic	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2} + \frac{\operatorname{arcsinh}(x^2)}{2}$	18
trager	$-\ln\left(\frac{-x^2 + \sqrt{x^4+1} + 1}{x}\right)$	22
meijerg	$\frac{\operatorname{arcsinh}(x^2)}{2} + \frac{(-2\ln(2)+4\ln(x))\sqrt{\pi} - 2\sqrt{\pi} \ln\left(\frac{1}{2} + \frac{\sqrt{x^4+1}}{2}\right)}{4\sqrt{\pi}}$	44

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+1)/x/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/2*\operatorname{arctanh}(1/(x^4+1)^{(1/2)})+1/2*\operatorname{arcsinh}(x^2)$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(14) = 28.

time = 0.34, size = 57, normalized size = 3.56

$$-\frac{1}{4} \log\left(\sqrt{x^4+1} + 1\right) + \frac{1}{4} \log\left(\sqrt{x^4+1} - 1\right) + \frac{1}{4} \log\left(\frac{\sqrt{x^4+1}}{x^2} + 1\right) - \frac{1}{4} \log\left(\frac{\sqrt{x^4+1}}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)/x/(x^4+1)^(1/2),x, algorithm="maxima")`

[Out] $-1/4*\log(\operatorname{sqrt}(x^4 + 1) + 1) + 1/4*\log(\operatorname{sqrt}(x^4 + 1) - 1) + 1/4*\log(\operatorname{sqrt}(x^4 + 1)/x^2 + 1) - 1/4*\log(\operatorname{sqrt}(x^4 + 1)/x^2 - 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(14) = 28.

time = 0.30, size = 49, normalized size = 3.06

$$-\frac{1}{2} \log\left(2x^4 - x^2 - \sqrt{x^4+1}(2x^2 - 1) + 1\right) + \frac{1}{2} \log\left(-x^2 + \sqrt{x^4+1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/x/(x^4+1)^(1/2),x, algorithm="fricas")

[Out] $-1/2*\log(2*x^4 - x^2 - \sqrt{x^4 + 1}*(2*x^2 - 1) + 1) + 1/2*\log(-x^2 + \sqrt{x^4 + 1} - 1)$

Sympy [A]

time = 4.35, size = 14, normalized size = 0.88

$$-\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2} + \frac{\operatorname{asinh}(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/x/(x**4+1)**(1/2),x)

[Out] $-\operatorname{asinh}(x^{**(-2)})/2 + \operatorname{asinh}(x^{**2})/2$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(14) = 28$.

time = 0.00, size = 59, normalized size = 3.69

$$-\frac{\ln\left(\sqrt{x^4+1}-x^2\right)}{2} + \frac{\ln\left(-\sqrt{x^4+1}+x^2+1\right)}{2} - \frac{\ln\left(\sqrt{x^4+1}-x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/x/(x^4+1)^(1/2),x)

[Out] $1/2*\log(x^2 - \sqrt{x^4 + 1} + 1) - 1/2*\log(-x^2 + \sqrt{x^4 + 1} + 1) - 1/2*\log(-x^2 + \sqrt{x^4 + 1})$

Mupad [B]

time = 0.15, size = 17, normalized size = 1.06

$$\frac{\operatorname{asinh}(x^2)}{2} - \frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)/(x*(x^4 + 1)^(1/2)),x)

[Out] $\operatorname{asinh}(x^2)/2 - \operatorname{atanh}((x^4 + 1)^{(1/2)})/2$

$$3.323 \quad \int \frac{-1+x^2}{x\sqrt{1+x^4}} dx$$

Optimal. Leaf size=16

$$\tanh^{-1}\left(\frac{1+x^2}{\sqrt{1+x^4}}\right)$$

[Out] arctanh((x^2+1)/(x^4+1)^(1/2))

Rubi [A]

time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1266, 858, 221, 272, 65, 213}

$$\frac{1}{2} \tanh^{-1}\left(\sqrt{x^4+1}\right) + \frac{1}{2} \sinh^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)/(x*Sqrt[1 + x^4]),x]

[Out] ArcSinh[x^2]/2 + ArcTanh[Sqrt[1 + x^4]]/2

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1)*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 858

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 1266

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{-1+x^2}{x\sqrt{1+x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{-1+x}{x\sqrt{1+x^2}} dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1+x^2}} dx, x, x^2 \right) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{1+x^2}} dx, x, x^2 \right) \\
 &= \frac{1}{2} \sinh^{-1}(x^2) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{x\sqrt{1+x}} dx, x, x^4 \right) \\
 &= \frac{1}{2} \sinh^{-1}(x^2) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1+x^4} \right) \\
 &= \frac{1}{2} \sinh^{-1}(x^2) + \frac{1}{2} \tanh^{-1}(\sqrt{1+x^4})
 \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(16) = 32.

time = 0.06, size = 37, normalized size = 2.31

$$\frac{1}{2} \tanh^{-1} \left(\frac{x^2}{\sqrt{1+x^4}} \right) - \tanh^{-1} \left(x^2 - \sqrt{1+x^4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^2)/(x*Sqrt[1 + x^4]), x]

[Out] ArcTanh[x^2/Sqrt[1 + x^4]]/2 - ArcTanh[x^2 - Sqrt[1 + x^4]]

Mathics [A]

time = 4.25, size = 13, normalized size = 0.81

$$\frac{\text{ArcSinh} \left[\frac{1}{x^2} \right]}{2} + \frac{\text{ArcSinh} [x^2]}{2}$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(x^2 - 1)/(x*sqrt[1 + x^4]),x]')`

[Out] `ArcSinh[1 / x ^ 2] / 2 + ArcSinh[x ^ 2] / 2`

Maple [A]

time = 0.11, size = 18, normalized size = 1.12

method	result	size
default	$\frac{\operatorname{arcsinh}(x^2)}{2} + \frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4 + 1}}\right)}{2}$	18
trager	$\ln\left(\frac{x^2 + \sqrt{x^4 + 1} + 1}{x}\right)$	18
elliptic	$\frac{\operatorname{arcsinh}(x^2)}{2} + \frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4 + 1}}\right)}{2}$	18
meijerg	$\frac{\operatorname{arcsinh}(x^2)}{2} - \frac{(-2\ln(2) + 4\ln(x))\sqrt{\pi} - 2\sqrt{\pi} \ln\left(\frac{1}{2} + \frac{\sqrt{x^4 + 1}}{2}\right)}{4\sqrt{\pi}}$	44

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2-1)/x/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `1/2*arcsinh(x^2)+1/2*arctanh(1/(x^4+1)^(1/2))`

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(14) = 28.

time = 0.34, size = 57, normalized size = 3.56

$$\frac{1}{4} \log(\sqrt{x^4 + 1} + 1) - \frac{1}{4} \log(\sqrt{x^4 + 1} - 1) + \frac{1}{4} \log\left(\frac{\sqrt{x^4 + 1}}{x^2} + 1\right) - \frac{1}{4} \log\left(\frac{\sqrt{x^4 + 1}}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2-1)/x/(x^4+1)^(1/2),x, algorithm="maxima")`

[Out] `1/4*log(sqrt(x^4 + 1) + 1) - 1/4*log(sqrt(x^4 + 1) - 1) + 1/4*log(sqrt(x^4 + 1)/x^2 + 1) - 1/4*log(sqrt(x^4 + 1)/x^2 - 1)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(14) = 28.

time = 0.31, size = 47, normalized size = 2.94

$$-\frac{1}{2} \log\left(2x^4 + x^2 - \sqrt{x^4 + 1}(2x^2 + 1) + 1\right) + \frac{1}{2} \log\left(-x^2 + \sqrt{x^4 + 1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/x/(x^4+1)^(1/2),x, algorithm="fricas")

[Out] $-1/2*\log(2*x^4 + x^2 - \sqrt{x^4 + 1}*(2*x^2 + 1) + 1) + 1/2*\log(-x^2 + \sqrt{x^4 + 1} + 1)$

Sympy [A]

time = 2.58, size = 14, normalized size = 0.88

$$\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2} + \frac{\operatorname{asinh}(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-1)/x/(x**4+1)**(1/2),x)

[Out] $\operatorname{asinh}(x^{**(-2)})/2 + \operatorname{asinh}(x^{**2})/2$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(14) = 28$.
time = 0.00, size = 59, normalized size = 3.69

$$-\frac{\ln\left(\sqrt{x^4+1}-x^2\right)}{2} - \frac{\ln\left(-\sqrt{x^4+1}+x^2+1\right)}{2} + \frac{\ln\left(\sqrt{x^4+1}-x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/x/(x^4+1)^(1/2),x)

[Out] $-1/2*\log(x^2 - \sqrt{x^4 + 1} + 1) + 1/2*\log(-x^2 + \sqrt{x^4 + 1} + 1) - 1/2*\log(-x^2 + \sqrt{x^4 + 1})$

Mupad [B]

time = 0.23, size = 17, normalized size = 1.06

$$\frac{\operatorname{asinh}(x^2)}{2} + \frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - 1)/(x*(x^4 + 1)^(1/2)),x)

[Out] $\operatorname{asinh}(x^2)/2 + \operatorname{atanh}((x^4 + 1)^{(1/2)})/2$

$$3.324 \quad \int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}} dx$$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{1+x^2+x^4}}\right)}{\sqrt{3}}$$

[Out] 1/3*arctanh(x*3^(1/2)/(x^4+x^2+1)^(1/2))*3^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1712, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{x^4+x^2+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^2 + x^4]),x]

[Out] ArcTanh[(Sqrt[3]*x)/Sqrt[1 + x^2 + x^4]]/Sqrt[3]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1712

Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] := Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2), x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}} dx &= \text{Subst} \left(\int \frac{1}{1-3x^2} dx, x, \frac{x}{\sqrt{1+x^2+x^4}} \right) \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{1+x^2+x^4}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A]

time = 0.15, size = 26, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{1+x^2+x^4}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^2 + x^4]),x]``[Out] ArcTanh[(Sqrt[3]*x)/Sqrt[1 + x^2 + x^4]]/Sqrt[3]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^2 + x^4]),x]')``[Out] cought exception: maximum recursion depth exceeded in comparison`**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.27, size = 184, normalized size = 7.08

method	result
elliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^4+x^2+1}\sqrt{2}\sqrt{6}}{6x}\right)\sqrt{6}\sqrt{2}}{6}$
trager	$\frac{\operatorname{RootOf}(-Z^2-3)\ln\left(\frac{-\operatorname{RootOf}(-Z^2-3)x+\sqrt{x^4+x^2+1}}{(1+x)(-1+x)}\right)}{3}$
default	$\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\operatorname{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

```
[Out] -2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*El
```

$\text{lipticPi}((-1/2+1/2*I*3^{(1/2)})^{(1/2)}*x, 1/(-1/2+1/2*I*3^{(1/2)}), (-1/2-1/2*I*3^{(1/2)})^{(1/2)}/(-1/2+1/2*I*3^{(1/2)})^{(1/2)})$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")`

[Out] `-integrate((x^2 + 1)/(sqrt(x^4 + x^2 + 1)*(x^2 - 1)), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(21) = 42$.

time = 0.59, size = 45, normalized size = 1.73

$$\frac{1}{6} \sqrt{3} \log \left(\frac{x^4 + 2\sqrt{3}\sqrt{x^4 + x^2 + 1}x + 4x^2 + 1}{x^4 - 2x^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")`

[Out] `1/6*sqrt(3)*log((x^4 + 2*sqrt(3)*sqrt(x^4 + x^2 + 1)*x + 4*x^2 + 1)/(x^4 - 2*x^2 + 1))`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2\sqrt{x^4 + x^2 + 1} - \sqrt{x^4 + x^2 + 1}} dx - \int \frac{1}{x^2\sqrt{x^4 + x^2 + 1} - \sqrt{x^4 + x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+1)/(-x**2+1)/(x**4+x**2+1)**(1/2),x)`

[Out] `-Integral(x**2/(x**2*sqrt(x**4 + x**2 + 1) - sqrt(x**4 + x**2 + 1)), x) - Integral(1/(x**2*sqrt(x**4 + x**2 + 1) - sqrt(x**4 + x**2 + 1)), x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x)`

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{x^2 + 1}{(x^2 - 1) \sqrt{x^4 + x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^2 + 1)/((x^2 - 1)*(x^2 + x^4 + 1)^(1/2)), x)`

[Out] `int(-(x^2 + 1)/((x^2 - 1)*(x^2 + x^4 + 1)^(1/2)), x)`

$$3.325 \quad \int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}} dx$$

Optimal. Leaf size=15

$$\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)$$

[Out] arctan(x/(x^4+x^2+1)^(1/2))

Rubi [A]

time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1712, 209}

$$\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]

[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]

Rule 209

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1712

Int[((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]), x_Symbol] := Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2), x], x, x/Sqrt[a + b*x^2 + c*x^4], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}} dx &= \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right) \\ &= \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) \end{aligned}$$

Mathematica [A]

time = 0.12, size = 15, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]

[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.22, size = 188, normalized size = 12.53

method	result
elliptic	$-\arctan\left(\frac{\sqrt{x^4+x^2+1}}{x}\right)$
trager	$\text{RootOf}(-Z^2+1) \ln\left(-\frac{\text{RootOf}(-Z^2+1)x-\sqrt{x^4+x^2+1}}{x^2+1}\right)$
default	$-\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out]
$$-2/(-2+2*I*3^{(1/2)})^{(1/2)}*(1-(-1/2+1/2*I*3^{(1/2)})x^2)^{(1/2)}*(1-(-1/2-1/2*I*3^{(1/2)})x^2)^{(1/2)}/(x^4+x^2+1)^{(1/2)}*EllipticF(1/2*x*(-2+2*I*3^{(1/2)})^{(1/2)},1/2*(-2+2*I*3^{(1/2)})^{(1/2)})+2/(-1/2+1/2*I*3^{(1/2)})^{(1/2)}*(1+1/2*x^2-1/2*I*x^2*3^{(1/2)})^{(1/2)}*(1+1/2*x^2+1/2*I*x^2*3^{(1/2)})^{(1/2)}/(x^4+x^2+1)^{(1/2)}*EllipticPi((-1/2+1/2*I*3^{(1/2)})^{(1/2)}x,-1/(-1/2+1/2*I*3^{(1/2)}),(-1/2-1/2*I*3^{(1/2)})^{(1/2)}/(-1/2+1/2*I*3^{(1/2)})^{(1/2)})$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)

Fricas [A]

time = 0.33, size = 13, normalized size = 0.87

$$\arctan\left(\frac{x}{\sqrt{x^4 + x^2 + 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")

[Out] arctan(x/sqrt(x^4 + x^2 + 1))

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2\sqrt{x^4 + x^2 + 1} + \sqrt{x^4 + x^2 + 1}} dx - \int \left(-\frac{1}{x^2\sqrt{x^4 + x^2 + 1} + \sqrt{x^4 + x^2 + 1}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)/(x**2+1)/(x**4+x**2+1)**(1/2),x)

[Out] -Integral(x**2/(x**2*sqrt(x**4 + x**2 + 1) + sqrt(x**4 + x**2 + 1)), x) - Integral(-1/(x**2*sqrt(x**4 + x**2 + 1) + sqrt(x**4 + x**2 + 1)), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.07

$$-\int \frac{x^2 - 1}{(x^2 + 1)\sqrt{x^4 + x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 1)/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)

[Out] -int((x^2 - 1)/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)

$$3.326 \quad \int \frac{-1+x^4}{x^2 \sqrt{1+x^2+x^4}} dx$$

Optimal. Leaf size=16

$$\frac{\sqrt{1+x^2+x^4}}{x}$$

[Out] 1/x*(x^4+x^2+1)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1604}

$$\frac{\sqrt{x^4+x^2+1}}{x}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^4)/(x^2*sqrt[1 + x^2 + x^4]),x]

[Out] Sqrt[1 + x^2 + x^4]/x

Rule 1604

Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :-> With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq^(m + 1)*(Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]), x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr + (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]

Rubi steps

$$\int \frac{-1+x^4}{x^2 \sqrt{1+x^2+x^4}} dx = \frac{\sqrt{1+x^2+x^4}}{x}$$

Mathematica [A]

time = 0.25, size = 16, normalized size = 1.00

$$\frac{\sqrt{1+x^2+x^4}}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^4)/(x^2*Sqrt[1 + x^2 + x^4]),x]

[Out] Sqrt[1 + x^2 + x^4]/x

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(x^4 - 1)/(x^2*Sqrt[x^4 + x^2 + 1]),x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [A]

time = 0.04, size = 15, normalized size = 0.94

method	result	size
default	$\frac{\sqrt{x^4 + x^2 + 1}}{x}$	15
trager	$\frac{\sqrt{x^4 + x^2 + 1}}{x}$	15
risch	$\frac{\sqrt{x^4 + x^2 + 1}}{x}$	15
elliptic	$\frac{\sqrt{x^4 + x^2 + 1}}{x}$	15
gospers	$\frac{(x^2+x+1)(x^2-x+1)}{\sqrt{x^4 + x^2 + 1} x}$	29

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/x*(x^4+x^2+1)^(1/2)

Maxima [A]

time = 0.29, size = 22, normalized size = 1.38

$$\frac{\sqrt{x^2 + x + 1} \sqrt{x^2 - x + 1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")

[Out] sqrt(x^2 + x + 1)*sqrt(x^2 - x + 1)/x

Fricas [A]

time = 0.30, size = 14, normalized size = 0.88

$$\frac{\sqrt{x^4 + x^2 + 1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")

[Out] sqrt(x^4 + x^2 + 1)/x

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)(x^2+1)}{x^2 \sqrt{(x^2-x+1)(x^2+x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4-1)/x**2/(x**4+x**2+1)**(1/2),x)

[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)/(x**2*sqrt((x**2 - x + 1)*(x**2 + x + 1))), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x)

[Out] Could not integrate

Mupad [B]

time = 0.06, size = 14, normalized size = 0.88

$$\frac{\sqrt{x^4 + x^2 + 1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4 - 1)/(x^2*(x^2 + x^4 + 1)^(1/2)),x)

[Out] (x^2 + x^4 + 1)^(1/2)/x

$$3.327 \quad \int \frac{1-x^2}{(1+2ax+x^2) \sqrt{1+2ax+2bx^2+2ax^3+x^4}} dx$$

Optimal. Leaf size=74

$$\frac{\tan^{-1} \left(\frac{a+2(1+a^2-b)x+ax^2}{\sqrt{2} \sqrt{1-b} \sqrt{1+2ax+2bx^2+2ax^3+x^4}} \right)}{\sqrt{2} \sqrt{1-b}}$$

[Out] 1/2*arctan(1/2*(a+2*(a^2-b+1)*x+a*x^2)*2^(1/2)/(1-b)^(1/2)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2))*2^(1/2)/(1-b)^(1/2)

Rubi [A]

time = 0.12, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {2109}

$$\frac{\tan^{-1} \left(\frac{2x(a^2-b+1)+ax^2+a}{\sqrt{2} \sqrt{1-b} \sqrt{2ax^3+2ax+2bx^2+x^4+1}} \right)}{\sqrt{2} \sqrt{1-b}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/((1 + 2*a*x + x^2)*Sqrt[1 + 2*a*x + 2*b*x^2 + 2*a*x^3 + x^4]), x]

[Out] ArcTan[(a + 2*(1 + a^2 - b)*x + a*x^2)/(Sqrt[2]*Sqrt[1 - b]*Sqrt[1 + 2*a*x + 2*b*x^2 + 2*a*x^3 + x^4])]/(Sqrt[2]*Sqrt[1 - b])

Rule 2109

Int[((f_) + (g_)*(x_)^2)/(((d_) + (e_)*(x_) + (d_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2 + (b_)*(x_)^3 + (a_)*(x_)^4]), x_Symbol] :> Simp[a*(f/(d*Rt[a^2*(2*a - c), 2]))*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*Rt[a^2*(2*a - c), 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])], x] / ; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f + g, 0] && PosQ[a^2*(2*a - c)]

Rubi steps

$$\int \frac{1-x^2}{(1+2ax+x^2) \sqrt{1+2ax+2bx^2+2ax^3+x^4}} dx = \frac{\tan^{-1} \left(\frac{a+2(1+a^2-b)x+ax^2}{\sqrt{2} \sqrt{1-b} \sqrt{1+2ax+2bx^2+2ax^3+x^4}} \right)}{\sqrt{2} \sqrt{1-b}}$$

Mathematica [A]

time = 0.60, size = 65, normalized size = 0.88

$$\frac{\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{-1+b} x}{1+2ax+x^2-\sqrt{1+2bx^2+x^4+2a(x+x^3)}} \right)}{\sqrt{-1+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/((1 + 2*a*x + x^2)*Sqrt[1 + 2*a*x + 2*b*x^2 + 2*a*x^3 + x^4]), x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[-1 + b]*x)/(1 + 2*a*x + x^2 - Sqrt[1 + 2*b*x^2 + x^4 + 2*a*(x + x^3)])])/Sqrt[-1 + b])
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x^2)/((1 + 2*a*x + x^2)*Sqrt[1 + 2*a*x + 2*b*x^2 + 2*a*x^3 + x^4]), x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.17, size = 247419, normalized size = 3343.50

method	result	size
default	Expression too large to display	247419
elliptic	Expression too large to display	258804

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2), x, method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/(sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(2*a*x + x^2 + 1)), x)

Fricas [A]

time = 0.46, size = 252, normalized size = 3.41

$$\left[\frac{\sqrt{2} \log \left(\frac{4a^2b^2 + (a^2+2b-2)x^4 + 4a^2b + 2(2a^2+3b+4b^2+2)x^2 + a^2}{4a^2b^2 + x^4 + 2(2a^2+1)x^2 + 4ax + 1} \cdot \frac{2\sqrt{2}\sqrt{2ax^3+x^4+2bx^2+2ax+1}}{\sqrt{b-1}} \right)}{4\sqrt{b-1}} \cdot \frac{1}{2}\sqrt{2}\sqrt{\frac{1}{b-1}} \arctan \left(\frac{\sqrt{2}\sqrt{2ax^3+x^4+2bx^2+2ax+1}(b-1)\sqrt{\frac{1}{b-1}}}{ax^2+2(a^2-b+1)x+a} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x, algorithm="fricas")

[Out] [1/4*sqrt(2)*log((4*a^3*x^3 + (a^2 + 2*b - 2)*x^4 + 4*a^3*x + 2*(2*a^4 + 5*a^2 - 2*(2*a^2 + 3)*b + 4*b^2 + 2)*x^2 + a^2 - 2*sqrt(2)*sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*((a*b - a)*x^2 + a*b - 2*(a^2 - (a^2 + 2)*b + b^2 + 1)*x - a)/sqrt(b - 1) + 2*b - 2)/(4*a*x^3 + x^4 + 2*(2*a^2 + 1)*x^2 + 4*a*x + 1))/sqrt(b - 1), 1/2*sqrt(2)*sqrt(-1/(b - 1))*arctan(sqrt(2)*sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(b - 1)*sqrt(-1/(b - 1))/(a*x^2 + 2*(a^2 - b + 1)*x + a))]

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{2ax\sqrt{2ax^3+2ax+2bx^2+x^4+1} + x^2\sqrt{2ax^3+2ax+2bx^2+x^4+1} + \sqrt{2ax^3+2ax+2bx^2+x^4+1}} dx - \int \left(\frac{1}{2ax\sqrt{2ax^3+2ax+2bx^2+x^4+1} + x^2\sqrt{2ax^3+2ax+2bx^2+x^4+1} + \sqrt{2ax^3+2ax+2bx^2+x^4+1}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)/(2*a*x+x**2+1)/(2*a*x**3+x**4+2*b*x**2+2*a*x+1)**(1/2), x)

[Out] -Integral(x**2/(2*a*x*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + x**2*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1)), x) - Integral(-1/(2*a*x*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + x**2*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1)), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2 - 1}{(x^2 + 2ax + 1) \sqrt{x^4 + 2ax^3 + 2bx^2 + 2ax + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 1)/((2*a*x + x^2 + 1)*(2*a*x + 2*a*x^3 + 2*b*x^2 + x^4 + 1)^(1/2)),x)

[Out] -int((x^2 - 1)/((2*a*x + x^2 + 1)*(2*a*x + 2*a*x^3 + 2*b*x^2 + x^4 + 1)^(1/2)), x)

$$3.328 \quad \int \frac{1}{(1+x^4) \sqrt{-x^2 + \sqrt{1+x^4}}} dx$$

Optimal. Leaf size=22

$$\tan^{-1} \left(\frac{x}{\sqrt{-x^2 + \sqrt{1+x^4}}} \right)$$

[Out] arctan(x/(-x^2+(x^4+1)^(1/2))^(1/2))

Rubi [A]

time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2153, 209}

$$\tan^{-1} \left(\frac{x}{\sqrt{\sqrt{x^4+1} - x^2}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]

[Out] ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2153

Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)]), x_Symbol] := Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2/n]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+x^4) \sqrt{-x^2 + \sqrt{1+x^4}}} dx &= \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \frac{x}{\sqrt{-x^2 + \sqrt{1+x^4}}} \right) \\ &= \tan^{-1} \left(\frac{x}{\sqrt{-x^2 + \sqrt{1+x^4}}} \right) \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.21, size = 96, normalized size = 4.36

$$i \tanh^{-1} \left(\sqrt{2} + \sqrt{2} x^4 - ix^3 \sqrt{-x^2 + \sqrt{1+x^4}} + \frac{\sqrt{1+x^4} \left(-2x^2 + i\sqrt{2} x \sqrt{-x^2 + \sqrt{1+x^4}} \right)}{\sqrt{2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]

[Out] I*ArcTanh[Sqrt[2] + Sqrt[2]*x^4 - I*x^3*Sqrt[-x^2 + Sqrt[1 + x^4]] + (Sqrt[1 + x^4]*(-2*x^2 + I*Sqrt[2]*x*Sqrt[-x^2 + Sqrt[1 + x^4]]))/Sqrt[2]]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((1 + x^4)*Sqrt[Sqrt[1 + x^4] - x^2]),x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [F]

time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^4 + 1) \sqrt{-x^2 + \sqrt{x^4 + 1}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)

[Out] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(18) = 36.

time = 0.66, size = 62, normalized size = 2.82

$$-\frac{1}{4} \arctan \left(\frac{4 \left(10x^7 - 6x^3 + (7x^5 - x)\sqrt{x^4 + 1} \right) \sqrt{-x^2 + \sqrt{x^4 + 1}}}{17x^8 - 46x^4 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")

[Out] -1/4*arctan(4*(10*x^7 - 6*x^3 + (7*x^5 - x)*sqrt(x^4 + 1))*sqrt(-x^2 + sqrt(x^4 + 1))/(17*x^8 - 46*x^4 + 1))

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2 + \sqrt{x^4 + 1}} (x^4 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**4+1)/(-x**2+(x**4+1)**(1/2))**(1/2),x)

[Out] Integral(1/(sqrt(-x**2 + sqrt(x**4 + 1))*(x**4 + 1)), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{\sqrt{x^4 + 1} - x^2} (x^4 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)),x)

[Out] int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)), x)

$$3.329 \quad \int \frac{1}{(1+x^{2n}) \sqrt{-x^2 + (1+x^{2n})^{\frac{1}{n}}}} dx$$

Optimal. Leaf size=24

$$\tan^{-1} \left(\frac{x}{\sqrt{-x^2 + (1+x^{2n})^{\frac{1}{n}}}} \right)$$

[Out] arctan(x/(-x^2+(1+x^(2*n))^(1/n))^(1/2))

Rubi [A]

time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2153, 209}

$$\tan^{-1} \left(\frac{x}{\sqrt{(x^{2n} + 1)^{\frac{1}{n}} - x^2}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x^(2*n))*Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]),x]

[Out] ArcTan[x/Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]]

Rule 209

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2153

Int[1/(((a_) + (b_)*(x_)^(n_.))*Sqrt[(c_)*(x_)^2 + (d_)*((a_) + (b_)*(x_)^(n_.))^p]), x_Symbol] := Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2/n]

Rubi steps

$$\int \frac{1}{(1+x^{2n}) \sqrt{-x^2 + (1+x^{2n})^{\frac{1}{n}}}} dx = \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \frac{x}{\sqrt{-x^2 + (1+x^{2n})^{\frac{1}{n}}}} \right) \\ = \tan^{-1} \left(\frac{x}{\sqrt{-x^2 + (1+x^{2n})^{\frac{1}{n}}}} \right)$$

Mathematica [A]

time = 0.11, size = 26, normalized size = 1.08

$$\cot^{-1} \left(\frac{\sqrt{-x^2 + (1 + x^{2n})^{\frac{1}{n}}}}{x} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x^(2*n))*Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]),x]

[Out] ArcCot[Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]/x]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((1 + x^(2*n))*((1 + x^(2*n))^(1/n) - x^2)^(1/2)),x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [F]

time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(1 + x^{2n}) \sqrt{-x^2 + (1 + x^{2n})^{\frac{1}{n}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+x^(2*n)))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x)

[Out] int(1/(1+x^(2*n)))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x^(2*n)))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-x^2 + (x^(2*n) + 1)^(1/n))*(x^(2*n) + 1)), x)

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x^(2*n)))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2 + (x^{2n} + 1)^{\frac{1}{n}}} (x^{2n} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x**(2*n)))/(-x**2+(1+x**(2*n))**(1/n))**(1/2),x)`

[Out] `Integral(1/(sqrt(-x**2 + (x**(2*n) + 1)**(1/n))*(x**(2*n) + 1)), x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x^(2*n)))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x)`

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(x^{2n} + 1) \sqrt{(x^{2n} + 1)^{1/n} - x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((x^(2*n) + 1)*((x^(2*n) + 1)^(1/n) - x^2)^(1/2)),x)`

[Out] `int(1/((x^(2*n) + 1)*((x^(2*n) + 1)^(1/n) - x^2)^(1/2)), x)`

3.330 $\int \cos^2(x) dx$

Optimal. Leaf size=14

$$\frac{x}{2} + \frac{1}{2} \cos(x) \sin(x)$$

[Out] 1/2*x+1/2*cos(x)*sin(x)

Rubi [A]

time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2715, 8}

$$\frac{x}{2} + \frac{1}{2} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2,x]

[Out] x/2 + (Cos[x]*Sin[x])/2

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*SIN[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos^2(x) dx &= \frac{1}{2} \cos(x) \sin(x) + \frac{\int 1 dx}{2} \\ &= \frac{x}{2} + \frac{1}{2} \cos(x) \sin(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2} + \frac{1}{4} \sin(2x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^2,x]

[Out] x/2 + Sin[2*x]/4

Mathics [A]

time = 1.60, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin[2x]}{4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Cos[x]^2,x]')

[Out] x / 2 + Sin[2 x] / 4

Maple [A]

time = 0.01, size = 11, normalized size = 0.79

method	result	size
default	$\frac{x}{2} + \frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2} + \frac{\sin(2x)}{4}$	11
norman	$\frac{x(\tan^2(\frac{x}{2})) + \frac{x}{2} - (\tan^3(\frac{x}{2})) + \frac{x(\tan^4(\frac{x}{2}))}{2} + \tan(\frac{x}{2})}{(1+\tan^2(\frac{x}{2}))^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*x+1/2*cos(x)*sin(x)

Maxima [A]

time = 0.25, size = 10, normalized size = 0.71

$$\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2,x, algorithm="maxima")

[Out] 1/2*x + 1/4*sin(2*x)

Fricas [A]

time = 0.31, size = 10, normalized size = 0.71

$$\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2,x, algorithm="fricas")

[Out] 1/2*cos(x)*sin(x) + 1/2*x

Sympy [A]

time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(x) \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**2,x)

[Out] x/2 + sin(x)*cos(x)/2

Giac [A]

time = 0.00, size = 12, normalized size = 0.86

$$\frac{x}{2} + \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2,x)

[Out] 1/2*x + 1/4*sin(2*x)

Mupad [B]

time = 0.18, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2,x)

[Out] x/2 + sin(2*x)/4

3.331 $\int \cos^3(x) dx$

Optimal. Leaf size=11

$$\sin(x) - \frac{\sin^3(x)}{3}$$

[Out] `sin(x)-1/3*sin(x)^3`

Rubi [A]

time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2713}

$$\sin(x) - \frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

[In] `Int[Cos[x]^3,x]`

[Out] `Sin[x] - Sin[x]^3/3`

Rule 2713

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rubi steps

$$\begin{aligned} \int \cos^3(x) dx &= -\text{Subst} \left(\int (1 - x^2) dx, x, -\sin(x) \right) \\ &= \sin(x) - \frac{\sin^3(x)}{3} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 15, normalized size = 1.36

$$\frac{3 \sin(x)}{4} + \frac{1}{12} \sin(3x)$$

Antiderivative was successfully verified.

[In] `Integrate[Cos[x]^3,x]`

[Out] `(3*Ssin[x])/4 + Sin[3*x]/12`

Mathics [A]

time = 1.57, size = 9, normalized size = 0.82

$$\sin[x] - \frac{\sin[x]^3}{3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Cos[x]^3,x]')``[Out] Sin[x] - Sin[x]^3 / 3`**Maple [A]**

time = 0.06, size = 11, normalized size = 1.00

method	result	size
default	$\frac{(2+\cos^2(x)) \sin(x)}{3}$	11
risch	$\frac{3 \sin(x)}{4} + \frac{\sin(3x)}{12}$	12

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x)^3,x,method=_RETURNVERBOSE)``[Out] 1/3*(2+cos(x)^2)*sin(x)`**Maxima [A]**

time = 0.30, size = 9, normalized size = 0.82

$$-\frac{1}{3} \sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)^3,x, algorithm="maxima")``[Out] -1/3*sin(x)^3 + sin(x)`**Fricas [A]**

time = 0.31, size = 10, normalized size = 0.91

$$\frac{1}{3} (\cos(x)^2 + 2) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)^3,x, algorithm="fricas")``[Out] 1/3*(cos(x)^2 + 2)*sin(x)`

Sympy [A]

time = 0.03, size = 8, normalized size = 0.73

$$-\frac{\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**3,x)

[Out] -sin(x)**3/3 + sin(x)

Giac [A]

time = 0.00, size = 11, normalized size = 1.00

$$\sin x - \frac{\sin^3 x}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3,x)

[Out] -1/3*sin(x)^3 + sin(x)

Mupad [B]

time = 0.03, size = 9, normalized size = 0.82

$$\sin(x) - \frac{\sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^3,x)

[Out] sin(x) - sin(x)^3/3

3.332 $\int \sin^4(x) dx$

Optimal. Leaf size=24

$$\frac{3x}{8} - \frac{3}{8} \cos(x) \sin(x) - \frac{1}{4} \cos(x) \sin^3(x)$$

[Out] 3/8*x-3/8*cos(x)*sin(x)-1/4*cos(x)*sin(x)^3

Rubi [A]

time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2715, 8}

$$\frac{3x}{8} - \frac{1}{4} \sin^3(x) \cos(x) - \frac{3}{8} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^4,x]

[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*Sine[c + d*x])^(n - 1)/(d*n), x] + Dist[b^2*((n - 1)/n), Int[(b*Sine[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \sin^4(x) dx &= -\frac{1}{4} \cos(x) \sin^3(x) + \frac{3}{4} \int \sin^2(x) dx \\ &= -\frac{3}{8} \cos(x) \sin(x) - \frac{1}{4} \cos(x) \sin^3(x) + \frac{3}{8} \int 1 dx \\ &= \frac{3x}{8} - \frac{3}{8} \cos(x) \sin(x) - \frac{1}{4} \cos(x) \sin^3(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 22, normalized size = 0.92

$$\frac{3x}{8} - \frac{1}{4} \sin(2x) + \frac{1}{32} \sin(4x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^4,x]

[Out] (3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32

Mathics [A]

time = 1.72, size = 16, normalized size = 0.67

$$\frac{3x}{8} - \frac{\sin[2x]}{4} + \frac{\sin[4x]}{32}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Sin[x]^4,x]')

[Out] 3 x / 8 - Sin[2 x] / 4 + Sin[4 x] / 32

Maple [A]

time = 0.07, size = 18, normalized size = 0.75

method	result	size
risch	$\frac{3x}{8} + \frac{\sin(4x)}{32} - \frac{\sin(2x)}{4}$	17
default	$-\frac{(\sin^3(x) + \frac{3\sin(x)}{2})\cos(x)}{4} + \frac{3x}{8}$	18
norman	$\frac{3x}{8} - \frac{11(\tan^3(\frac{x}{2}))}{4} + \frac{11(\tan^5(\frac{x}{2}))}{4} + \frac{3(\tan^7(\frac{x}{2}))}{4} + \frac{3x(\tan^2(\frac{x}{2}))}{2} + \frac{9x(\tan^4(\frac{x}{2}))}{4} + \frac{3x(\tan^6(\frac{x}{2}))}{2} + \frac{3x(\tan^8(\frac{x}{2}))}{8} - \frac{3\tan(\frac{x}{2})}{4}$ $(1+\tan^2(\frac{x}{2}))^4$	82

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^4,x,method=_RETURNVERBOSE)

[Out] -1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x

Maxima [A]

time = 0.25, size = 16, normalized size = 0.67

$$\frac{3}{8}x + \frac{1}{32}\sin(4x) - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^4,x, algorithm="maxima")

[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)

Fricas [A]

time = 0.31, size = 19, normalized size = 0.79

$$\frac{1}{8}(2\cos(x)^3 - 5\cos(x))\sin(x) + \frac{3}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^4,x, algorithm="fricas")

[Out] 1/8*(2*cos(x)^3 - 5*cos(x))*sin(x) + 3/8*x

Sympy [A]

time = 0.03, size = 24, normalized size = 1.00

$$\frac{3x}{8} - \frac{\sin^3(x) \cos(x)}{4} - \frac{3 \sin(x) \cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**4,x)

[Out] 3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8

Giac [A]

time = 0.00, size = 21, normalized size = 0.88

$$\frac{3}{8}x - \frac{\sin(2x)}{4} + \frac{\sin(4x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^4,x)

[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)

Mupad [B]

time = 0.03, size = 16, normalized size = 0.67

$$\frac{3x}{8} - \frac{\sin(2x)}{4} + \frac{\sin(4x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^4,x)

[Out] (3*x)/8 - sin(2*x)/4 + sin(4*x)/32

3.333 $\int \cos^6(x) dx$

Optimal. Leaf size=34

$$\frac{5x}{16} + \frac{5}{16} \cos(x) \sin(x) + \frac{5}{24} \cos^3(x) \sin(x) + \frac{1}{6} \cos^5(x) \sin(x)$$

[Out] 5/16*x+5/16*cos(x)*sin(x)+5/24*cos(x)^3*sin(x)+1/6*cos(x)^5*sin(x)

Rubi [A]

time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$,

Rules used = {2715, 8}

$$\frac{5x}{16} + \frac{1}{6} \sin(x) \cos^5(x) + \frac{5}{24} \sin(x) \cos^3(x) + \frac{5}{16} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^6,x]

[Out] (5*x)/16 + (5*Cos[x]*Sin[x])/16 + (5*Cos[x]^3*Ssin[x])/24 + (Cos[x]^5*Ssin[x])/6

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Ssin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos^6(x) dx &= \frac{1}{6} \cos^5(x) \sin(x) + \frac{5}{6} \int \cos^4(x) dx \\ &= \frac{5}{24} \cos^3(x) \sin(x) + \frac{1}{6} \cos^5(x) \sin(x) + \frac{5}{8} \int \cos^2(x) dx \\ &= \frac{5}{16} \cos(x) \sin(x) + \frac{5}{24} \cos^3(x) \sin(x) + \frac{1}{6} \cos^5(x) \sin(x) + \frac{5 \int 1 dx}{16} \\ &= \frac{5x}{16} + \frac{5}{16} \cos(x) \sin(x) + \frac{5}{24} \cos^3(x) \sin(x) + \frac{1}{6} \cos^5(x) \sin(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 30, normalized size = 0.88

$$\frac{5x}{16} + \frac{15}{64} \sin(2x) + \frac{3}{64} \sin(4x) + \frac{1}{192} \sin(6x)$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[x]^6,x]``[Out] (5*x)/16 + (15*Sin[2*x])/64 + (3*Sin[4*x])/64 + Sin[6*x]/192`**Mathics [A]**

time = 1.83, size = 32, normalized size = 0.94

$$\frac{5x}{16} + \frac{(1 - \cos[2x])^2 \sin[2x]}{48} + \frac{13 \sin[4x]}{192} + \frac{5 \sin[2x]}{24}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Cos[x]^6,x]')``[Out] 5 x / 16 + (1 - Cos[2 x]) ^ 2 Sin[2 x] / 48 + 13 Sin[4 x] / 192 + 5 Sin[2 x] / 24`**Maple [A]**

time = 0.08, size = 24, normalized size = 0.71

method	result
risch	$\frac{5x}{16} + \frac{\sin(6x)}{192} + \frac{3 \sin(4x)}{64} + \frac{15 \sin(2x)}{64}$
default	$\frac{\left(\cos^5(x) + \frac{5(\cos^3(x))}{4} + \frac{15 \cos(x)}{8}\right) \sin(x)}{6} + \frac{5x}{16}$
norman	$\frac{\frac{5x}{16} - \frac{5(\tan^3(\frac{x}{2}))}{24} + \frac{15(\tan^5(\frac{x}{2}))}{4} - \frac{15(\tan^7(\frac{x}{2}))}{4} + \frac{5(\tan^9(\frac{x}{2}))}{24} - \frac{11(\tan^{11}(\frac{x}{2}))}{8} + \frac{15x(\tan^2(\frac{x}{2}))}{8} + \frac{75x(\tan^4(\frac{x}{2}))}{16} + \frac{25x(\tan^6(\frac{x}{2}))}{4} + \frac{75x(\tan^8(\frac{x}{2}))}{16}}{(1+\tan^2(\frac{x}{2}))^6}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x)^6,x,method=_RETURNVERBOSE)``[Out] 1/6*(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)+5/16*x`**Maxima [A]**

time = 0.25, size = 24, normalized size = 0.71

$$-\frac{1}{48} \sin(2x)^3 + \frac{5}{16} x + \frac{3}{64} \sin(4x) + \frac{1}{4} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^6,x, algorithm="maxima")

[Out] $-1/48*\sin(2*x)^3 + 5/16*x + 3/64*\sin(4*x) + 1/4*\sin(2*x)$

Fricas [A]

time = 0.31, size = 25, normalized size = 0.74

$$\frac{1}{48} (8 \cos(x)^5 + 10 \cos(x)^3 + 15 \cos(x)) \sin(x) + \frac{5}{16} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^6,x, algorithm="fricas")

[Out] $1/48*(8*\cos(x)^5 + 10*\cos(x)^3 + 15*\cos(x))*\sin(x) + 5/16*x$

Sympy [A]

time = 0.03, size = 36, normalized size = 1.06

$$\frac{5x}{16} + \frac{\sin(x) \cos^5(x)}{6} + \frac{5 \sin(x) \cos^3(x)}{24} + \frac{5 \sin(x) \cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**6,x)

[Out] $5*x/16 + \sin(x)*\cos(x)**5/6 + 5*\sin(x)*\cos(x)**3/24 + 5*\sin(x)*\cos(x)/16$

Giac [A]

time = 0.00, size = 31, normalized size = 0.91

$$\frac{5}{16}x + \frac{15}{64} \sin(2x) + \frac{3}{64} \sin(4x) + \frac{\sin(6x)}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^6,x)

[Out] $5/16*x + 1/192*\sin(6*x) + 3/64*\sin(4*x) + 15/64*\sin(2*x)$

Mupad [B]

time = 0.04, size = 22, normalized size = 0.65

$$\frac{5x}{16} + \frac{15 \sin(2x)}{64} + \frac{3 \sin(4x)}{64} + \frac{\sin(6x)}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^6,x)

[Out] $(5*x)/16 + (15*\sin(2*x))/64 + (3*\sin(4*x))/64 + \sin(6*x)/192$

3.334 $\int \sin^8(x) dx$

Optimal. Leaf size=44

$$\frac{35x}{128} - \frac{35}{128} \cos(x) \sin(x) - \frac{35}{192} \cos(x) \sin^3(x) - \frac{7}{48} \cos(x) \sin^5(x) - \frac{1}{8} \cos(x) \sin^7(x)$$

[Out] 35/128*x-35/128*cos(x)*sin(x)-35/192*cos(x)*sin(x)^3-7/48*cos(x)*sin(x)^5-1/8*cos(x)*sin(x)^7

Rubi [A]

time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2715, 8}

$$\frac{35x}{128} - \frac{1}{8} \sin^7(x) \cos(x) - \frac{7}{48} \sin^5(x) \cos(x) - \frac{35}{192} \sin^3(x) \cos(x) - \frac{35}{128} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^8,x]

[Out] (35*x)/128 - (35*Cos[x]*Sin[x])/128 - (35*Cos[x]*Sin[x]^3)/192 - (7*Cos[x]*Sin[x]^5)/48 - (Cos[x]*Sin[x]^7)/8

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*SIN[c + d*x])^(n - 1)/(d*n), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \sin^8(x) dx &= -\frac{1}{8} \cos(x) \sin^7(x) + \frac{7}{8} \int \sin^6(x) dx \\ &= -\frac{7}{48} \cos(x) \sin^5(x) - \frac{1}{8} \cos(x) \sin^7(x) + \frac{35}{48} \int \sin^4(x) dx \\ &= -\frac{35}{192} \cos(x) \sin^3(x) - \frac{7}{48} \cos(x) \sin^5(x) - \frac{1}{8} \cos(x) \sin^7(x) + \frac{35}{64} \int \sin^2(x) dx \\ &= -\frac{35}{128} \cos(x) \sin(x) - \frac{35}{192} \cos(x) \sin^3(x) - \frac{7}{48} \cos(x) \sin^5(x) - \frac{1}{8} \cos(x) \sin^7(x) + \frac{35}{128} \int 1 dx \\ &= \frac{35x}{128} - \frac{35}{128} \cos(x) \sin(x) - \frac{35}{192} \cos(x) \sin^3(x) - \frac{7}{48} \cos(x) \sin^5(x) - \frac{1}{8} \cos(x) \sin^7(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 38, normalized size = 0.86

$$\frac{35x}{128} - \frac{7}{32} \sin(2x) + \frac{7}{128} \sin(4x) - \frac{1}{96} \sin(6x) + \frac{\sin(8x)}{1024}$$

Antiderivative was successfully verified.

`[In] Integrate[Sin[x]^8,x]``[Out] (35*x)/128 - (7*Sin[2*x])/32 + (7*Sin[4*x])/128 - Sin[6*x]/96 + Sin[8*x]/1024`**Mathics [A]**

time = 1.88, size = 44, normalized size = 1.00

$$\frac{35x}{128} - \frac{35\sin[2x]}{192} - \frac{7(-1 + \cos[2x])^2 \sin[2x]}{384} + \frac{(-1 + \cos[2x])^3 \sin[2x]}{128} + \frac{35\sin[4x]}{1536}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sin[x]^8,x]')``[Out] 35 x / 128 - 35 Sin[2 x] / 192 - 7 (-1 + Cos[2 x]) ^ 2 Sin[2 x] / 384 + (-1 + Cos[2 x]) ^ 3 Sin[2 x] / 128 + 35 Sin[4 x] / 1536`**Maple [A]**

time = 0.09, size = 30, normalized size = 0.68

method	result
risch	$\frac{35x}{128} + \frac{\sin(8x)}{1024} - \frac{\sin(6x)}{96} + \frac{7\sin(4x)}{128} - \frac{7\sin(2x)}{32}$
default	$-\frac{\left(\sin^7(x) + \frac{7\sin^5(x)}{6} + \frac{35\sin^3(x)}{24} + \frac{35\sin(x)}{16}\right)\cos(x)}{8} + \frac{35x}{128}$
norman	$\frac{35x}{128} - \frac{35\tan\left(\frac{x}{2}\right)}{64} + \frac{245x\left(\tan^4\left(\frac{x}{2}\right)\right)}{32} + \frac{35x\left(\tan^2\left(\frac{x}{2}\right)\right)}{16} - \frac{805\left(\tan^3\left(\frac{x}{2}\right)\right)}{192} + \frac{35x\left(\tan^{14}\left(\frac{x}{2}\right)\right)}{16} + \frac{35x\left(\tan^{16}\left(\frac{x}{2}\right)\right)}{128} + \frac{245x\left(\tan^6\left(\frac{x}{2}\right)\right)}{16} + \frac{1225x\left(\tan^8\left(\frac{x}{2}\right)\right)}{64}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sin(x)^8,x,method=_RETURNVERBOSE)``[Out] -1/8*(sin(x)^7+7/6*sin(x)^5+35/24*sin(x)^3+35/16*sin(x))*cos(x)+35/128*x`**Maxima [A]**

time = 0.26, size = 30, normalized size = 0.68

$$\frac{1}{24} \sin(2x)^3 + \frac{35}{128} x + \frac{1}{1024} \sin(8x) + \frac{7}{128} \sin(4x) - \frac{1}{4} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^8,x, algorithm="maxima")

[Out] 1/24*sin(2*x)^3 + 35/128*x + 1/1024*sin(8*x) + 7/128*sin(4*x) - 1/4*sin(2*x)

Fricas [A]

time = 0.32, size = 31, normalized size = 0.70

$$\frac{1}{384} (48 \cos(x)^7 - 200 \cos(x)^5 + 326 \cos(x)^3 - 279 \cos(x)) \sin(x) + \frac{35}{128} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^8,x, algorithm="fricas")

[Out] 1/384*(48*cos(x)^7 - 200*cos(x)^5 + 326*cos(x)^3 - 279*cos(x))*sin(x) + 35/128*x

Sympy [A]

time = 0.03, size = 48, normalized size = 1.09

$$\frac{35x}{128} - \frac{\sin^7(x) \cos(x)}{8} - \frac{7 \sin^5(x) \cos(x)}{48} - \frac{35 \sin^3(x) \cos(x)}{192} - \frac{35 \sin(x) \cos(x)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**8,x)

[Out] 35*x/128 - sin(x)**7*cos(x)/8 - 7*sin(x)**5*cos(x)/48 - 35*sin(x)**3*cos(x)/192 - 35*sin(x)*cos(x)/128

Giac [A]

time = 0.00, size = 39, normalized size = 0.89

$$\frac{35}{128} x - \frac{7}{32} \sin(2x) + \frac{7}{128} \sin(4x) - \frac{\sin(6x)}{96} + \frac{\sin(8x)}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^8,x)

[Out] 35/128*x + 1/1024*sin(8*x) - 1/96*sin(6*x) + 7/128*sin(4*x) - 7/32*sin(2*x)

Mupad [B]

time = 0.03, size = 28, normalized size = 0.64

$$\frac{35x}{128} - \frac{7 \sin(2x)}{32} + \frac{7 \sin(4x)}{128} - \frac{\sin(6x)}{96} + \frac{\sin(8x)}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^8,x)

[Out] (35*x)/128 - (7*sin(2*x))/32 + (7*sin(4*x))/128 - sin(6*x)/96 + sin(8*x)/1024

3.335 $\int \cos^4\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$

Optimal. Leaf size=20

$$\frac{3x}{8} + \frac{\cos(x)}{2} - \frac{1}{8} \cos(x) \sin(x)$$

[Out] $3/8*x+1/2*\cos(x)-1/8*\cos(x)*\sin(x)$

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 64 vs. $2(20) = 40$.
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 3.20, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$,
Rules used = {2715, 8}

$$\frac{3x}{8} + \frac{1}{2} \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) \cos^3\left(\frac{x}{2} + \frac{\pi}{4}\right) + \frac{3}{4} \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) \cos\left(\frac{x}{2} + \frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[\text{Pi}/4 + x/2]^4, x]$

[Out] $(3*x)/8 + (3*\text{Cos}[\text{Pi}/4 + x/2]*\text{Sin}[\text{Pi}/4 + x/2])/4 + (\text{Cos}[\text{Pi}/4 + x/2]^3*\text{Sin}[\text{Pi}/4 + x/2])/2$

Rule 8

$\text{Int}[a_, x_Symbol] \text{ :> } \text{Simp}[a*x, x] \text{ /; } \text{FreeQ}[a, x]$

Rule 2715

$\text{Int}[(b_)*\sin[(c_)+(d_)*(x_)]^{(n_)}, x_Symbol] \text{ :> } \text{Simp}[(-b)*\text{Cos}[c + d*x]*((b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[b^2*((n-1)/n), \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] \text{ /; } \text{FreeQ}\{b, c, d, x\} \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rubi steps

$$\begin{aligned} \int \cos^4\left(\frac{\pi}{4} + \frac{x}{2}\right) dx &= \frac{1}{2} \cos^3\left(\frac{\pi}{4} + \frac{x}{2}\right) \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) + \frac{3}{4} \int \sin^2\left(\frac{\pi}{4} + \frac{x}{2}\right) dx \\ &= \frac{3}{4} \cos\left(\frac{\pi}{4} + \frac{x}{2}\right) \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) + \frac{1}{2} \cos^3\left(\frac{\pi}{4} + \frac{x}{2}\right) \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) + \frac{3 \int 1 dx}{8} \\ &= \frac{3x}{8} + \frac{3}{4} \cos\left(\frac{\pi}{4} + \frac{x}{2}\right) \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) + \frac{1}{2} \cos^3\left(\frac{\pi}{4} + \frac{x}{2}\right) \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 21, normalized size = 1.05

$$\frac{1}{16}(3\pi + 6x + 8 \cos(x) - 2 \cos(x) \sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[Pi/4 + x/2]^4,x]

[Out] (3*Pi + 6*x + 8*Cos[x] - 2*Cos[x]*Sin[x])/16

Mathics [A]

time = 2.67, size = 14, normalized size = 0.70

$$\frac{3x}{8} - \frac{\sin[2x]}{16} + \frac{\cos[x]}{2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Cos[Pi/4 + x/2]^4,x]')

[Out] 3 x / 8 - Sin[2 x] / 16 + Cos[x] / 2

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(14) = 28.

time = 0.09, size = 39, normalized size = 1.95

method	result
risch	$\frac{3x}{8} + \frac{\cos(x)}{2} - \frac{\sin(2x)}{16}$
derivativdivides	$\frac{\left(\cos^3\left(\frac{\pi}{4} + \frac{x}{2}\right) + \frac{3\cos\left(\frac{\pi}{4} + \frac{x}{2}\right)}{2}\right)\sin\left(\frac{\pi}{4} + \frac{x}{2}\right)}{2} + \frac{3\pi}{16} + \frac{3x}{8}$
default	$\frac{\left(\cos^3\left(\frac{\pi}{4} + \frac{x}{2}\right) + \frac{3\cos\left(\frac{\pi}{4} + \frac{x}{2}\right)}{2}\right)\sin\left(\frac{\pi}{4} + \frac{x}{2}\right)}{2} + \frac{3\pi}{16} + \frac{3x}{8}$
norman	$\frac{\frac{3x}{8} - \frac{3(\tan^3(\frac{\pi}{8} + \frac{x}{4}))}{2} + \frac{3(\tan^5(\frac{\pi}{8} + \frac{x}{4}))}{2} - \frac{5(\tan^7(\frac{\pi}{8} + \frac{x}{4}))}{2} + \frac{3x(\tan^2(\frac{\pi}{8} + \frac{x}{4}))}{2} + \frac{9x(\tan^4(\frac{\pi}{8} + \frac{x}{4}))}{4} + \frac{3x(\tan^6(\frac{\pi}{8} + \frac{x}{4}))}{2} + \frac{3x(\tan^8(\frac{\pi}{8} + \frac{x}{4}))}{2}}{(1 + \tan^2(\frac{\pi}{8} + \frac{x}{4}))^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/4*Pi+1/2*x)^4,x,method=_RETURNVERBOSE)

[Out] 1/2*(cos(1/4*Pi+1/2*x)^3+3/2*cos(1/4*Pi+1/2*x))*sin(1/4*Pi+1/2*x)+3/16*Pi+3/8*x

Maxima [A]

time = 0.26, size = 23, normalized size = 1.15

$$\frac{3}{16}\pi + \frac{3}{8}x + \frac{1}{16}\sin(\pi + 2x) + \frac{1}{2}\sin\left(\frac{1}{2}\pi + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/4*pi+1/2*x)^4,x, algorithm="maxima")

[Out] $3/16\pi + 3/8x + 1/16\sin(\pi + 2x) + 1/2\sin(1/2\pi + x)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.

time = 0.31, size = 37, normalized size = 1.85

$$\frac{1}{4} \left(2 \cos \left(\frac{1}{4} \pi + \frac{1}{2} x \right)^3 + 3 \cos \left(\frac{1}{4} \pi + \frac{1}{2} x \right) \right) \sin \left(\frac{1}{4} \pi + \frac{1}{2} x \right) + \frac{3}{8} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/4*pi+1/2*x)^4,x, algorithm="fricas")`

[Out] $1/4*(2*\cos(1/4*\pi + 1/2*x)^3 + 3*\cos(1/4*\pi + 1/2*x))*\sin(1/4*\pi + 1/2*x) + 3/8*x$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(17) = 34$

time = 0.16, size = 99, normalized size = 4.95

$$\frac{3x \sin^4 \left(\frac{x}{2} + \frac{\pi}{4} \right)}{8} + \frac{3x \sin^2 \left(\frac{x}{2} + \frac{\pi}{4} \right) \cos^2 \left(\frac{x}{2} + \frac{\pi}{4} \right)}{4} + \frac{3x \cos^4 \left(\frac{x}{2} + \frac{\pi}{4} \right)}{8} + \frac{3 \sin^3 \left(\frac{x}{2} + \frac{\pi}{4} \right) \cos \left(\frac{x}{2} + \frac{\pi}{4} \right)}{4} + \frac{5 \sin \left(\frac{x}{2} + \frac{\pi}{4} \right) \cos^3 \left(\frac{x}{2} + \frac{\pi}{4} \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/4*pi+1/2*x)**4,x)`

[Out] $3*x*\sin(x/2 + \pi/4)**4/8 + 3*x*\sin(x/2 + \pi/4)**2*\cos(x/2 + \pi/4)**2/4 + 3*x*\cos(x/2 + \pi/4)**4/8 + 3*\sin(x/2 + \pi/4)**3*\cos(x/2 + \pi/4)/4 + 5*\sin(x/2 + \pi/4)*\cos(x/2 + \pi/4)**3/4$

Giac [A]

time = 0.00, size = 21, normalized size = 1.05

$$2 \left(\frac{3}{16} x + \frac{\cos x}{4} - \frac{\sin(2x)}{32} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/4*pi+1/2*x)^4,x)`

[Out] $3/8*x + 1/2*\cos(x) - 1/16*\sin(2*x)$

Mupad [B]

time = 0.27, size = 20, normalized size = 1.00

$$\frac{3x}{8} + \frac{\sin(\Pi + 2x)}{16} + \frac{\sin\left(\frac{\Pi}{2} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(Pi/4 + x/2)^4,x)`

[Out] $(3*x)/8 + \sin(\pi + 2*x)/16 + \sin(\pi/2 + x)/2$

3.336 $\int -\sin^3\left(\frac{\pi}{12} - 3x\right) dx$

Optimal. Leaf size=31

$$-\frac{1}{3}\cos\left(\frac{\pi}{12} - 3x\right) + \frac{1}{9}\cos^3\left(\frac{\pi}{12} - 3x\right)$$

[Out] -1/3*sin(5/12*Pi+3*x)+1/9*sin(5/12*Pi+3*x)^3

Rubi [A]

time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2713}

$$\frac{1}{9}\cos^3\left(\frac{\pi}{12} - 3x\right) - \frac{1}{3}\cos\left(\frac{\pi}{12} - 3x\right)$$

Antiderivative was successfully verified.

[In] Int[-Sin[Pi/12 - 3*x]^3,x]

[Out] -1/3*Cos[Pi/12 - 3*x] + Cos[Pi/12 - 3*x]^3/9

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned}\int -\sin^3\left(\frac{\pi}{12} - 3x\right) dx &= -\left(\frac{1}{3}\text{Subst}\left(\int (1 - x^2) dx, x, \cos\left(\frac{\pi}{12} - 3x\right)\right)\right) \\ &= -\frac{1}{3}\cos\left(\frac{\pi}{12} - 3x\right) + \frac{1}{9}\cos^3\left(\frac{\pi}{12} - 3x\right)\end{aligned}$$

Mathematica [A]

time = 0.01, size = 31, normalized size = 1.00

$$-\frac{1}{4}\cos\left(\frac{\pi}{12} - 3x\right) + \frac{1}{36}\cos\left(3\left(\frac{\pi}{12} - 3x\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[-Sin[Pi/12 - 3*x]^3,x]

[Out] -1/4*Cos[Pi/12 - 3*x] + Cos[3*(Pi/12 - 3*x)]/36

Mathics [A]

time = 2.27, size = 35, normalized size = 1.13

$$\frac{\left(-3\cos\left[\frac{5\pi}{12} + 3x\right]^2 - 2\sin\left[\frac{5\pi}{12} + 3x\right]^2\right)\sin\left[\frac{5\pi}{12} + 3x\right]}{9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sin[3*x - Pi/12]^3,x]')
```

```
[Out] (-3 Cos[5 Pi / 12 + 3 x] ^ 2 - 2 Sin[5 Pi / 12 + 3 x] ^ 2) Sin[5 Pi / 12 + 3 x] / 9
```

Maple [A]

time = 0.27, size = 23, normalized size = 0.74

method	result	size
risch	$\frac{\sin\left(\frac{\pi}{4}+9x\right)}{36} - \frac{\sin\left(\frac{5\pi}{12}+3x\right)}{4}$	22
derivativedivides	$-\frac{(2+\cos^2\left(\frac{5\pi}{12}+3x\right))\sin\left(\frac{5\pi}{12}+3x\right)}{9}$	23
default	$-\frac{(2+\cos^2\left(\frac{5\pi}{12}+3x\right))\sin\left(\frac{5\pi}{12}+3x\right)}{9}$	23
norman	$\frac{4\left(\tan^3\left(\frac{5\pi}{24}+\frac{3x}{2}\right)\right)}{9} - \frac{2\left(\tan^5\left(\frac{5\pi}{24}+\frac{3x}{2}\right)\right)}{3} - \frac{2\tan\left(\frac{5\pi}{24}+\frac{3x}{2}\right)}{3}$ $\frac{\quad}{\left(1+\tan^2\left(\frac{5\pi}{24}+\frac{3x}{2}\right)\right)^3}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(5/12*Pi+3*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*(2+cos(5/12*Pi+3*x)^2)*sin(5/12*Pi+3*x)
```

Maxima [A]

time = 0.25, size = 23, normalized size = 0.74

$$\frac{1}{9}\sin\left(\frac{5}{12}\pi + 3x\right)^3 - \frac{1}{3}\sin\left(\frac{5}{12}\pi + 3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(5/12*pi+3*x)^3,x, algorithm="maxima")
```

```
[Out] 1/9*sin(5/12*pi + 3*x)^3 - 1/3*sin(5/12*pi + 3*x)
```

Fricas [A]

time = 0.31, size = 22, normalized size = 0.71

$$-\frac{1}{9}\left(\cos\left(\frac{5}{12}\pi + 3x\right)^2 + 2\right)\sin\left(\frac{5}{12}\pi + 3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-cos(5/12*pi+3*x)^3,x, algorithm="fricas")

[Out] -1/9*(cos(5/12*pi + 3*x)^2 + 2)*sin(5/12*pi + 3*x)

Sympy [A]

time = 0.11, size = 39, normalized size = 1.26

$$-\frac{2 \sin^3\left(3x + \frac{5\pi}{12}\right)}{9} - \frac{\sin\left(3x + \frac{5\pi}{12}\right) \cos^2\left(3x + \frac{5\pi}{12}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-cos(5/12*pi+3*x)**3,x)

[Out] -2*sin(3*x + 5*pi/12)**3/9 - sin(3*x + 5*pi/12)*cos(3*x + 5*pi/12)**2/3

Giac [A]

time = 0.03, size = 31, normalized size = 1.00

$$-\frac{\sin\left(\frac{5}{12}\pi + 3x\right) - \frac{\sin^3\left(\frac{5}{12}\pi + 3x\right)}{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-cos(5/12*pi+3*x)^3,x)

[Out] 1/9*sin(5/12*pi + 3*x)^3 - 1/3*sin(5/12*pi + 3*x)

Mupad [B]

time = 0.25, size = 22, normalized size = 0.71

$$\frac{\sin\left(\frac{5\Pi}{12} + 3x\right) \left(\sin\left(\frac{5\Pi}{12} + 3x\right)^2 - 3\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-cos((5*Pi)/12 + 3*x)^3,x)

[Out] (sin((5*Pi)/12 + 3*x)*(sin((5*Pi)/12 + 3*x)^2 - 3))/9

3.337 $\int \csc^6(x) dx$

Optimal. Leaf size=21

$$-\cot(x) - \frac{2 \cot^3(x)}{3} - \frac{\cot^5(x)}{5}$$

[Out] $-\cot(x) - 2/3 * \cot(x)^3 - 1/5 * \cot(x)^5$

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3852}

$$-\frac{1}{5} \cot^5(x) - \frac{2 \cot^3(x)}{3} - \cot(x)$$

Antiderivative was successfully verified.

[In] Int[Csc[x]^6,x]

[Out] -Cot[x] - (2*Cot[x]^3)/3 - Cot[x]^5/5

Rule 3852

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int \csc^6(x) dx &= -\text{Subst} \left(\int (1 + 2x^2 + x^4) dx, x, \cot(x) \right) \\ &= -\cot(x) - \frac{2 \cot^3(x)}{3} - \frac{\cot^5(x)}{5} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 27, normalized size = 1.29

$$-\frac{8 \cot(x)}{15} - \frac{4}{15} \cot(x) \csc^2(x) - \frac{1}{5} \cot(x) \csc^4(x)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^6,x]

[Out] (-8*Cot[x])/15 - (4*Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5

Mathics [A]

time = 1.79, size = 23, normalized size = 1.10

$$\frac{-8}{15 \tan [x]} - \frac{4 \cos [x]}{15 \sin [x]^3} - \frac{\cos [x]}{5 \sin [x]^5}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/Sin[x]^6,x]')``[Out] -8 / (15 Tan[x]) - 4 Cos[x] / (15 Sin[x] ^ 3) - Cos[x] / (5 Sin[x] ^ 5)`**Maple [A]**

time = 0.08, size = 18, normalized size = 0.86

method	result	size
default	$\left(-\frac{8}{15} - \frac{\csc^4(x)}{5} - \frac{4(\csc^2(x))}{15}\right) \cot(x)$	18
risch	$-\frac{16i(10e^{4ix} - 5e^{2ix} + 1)}{15(e^{2ix} - 1)^5}$	29
norman	$-\frac{1}{160} - \frac{5(\tan^2(\frac{x}{2}))}{96} - \frac{5(\tan^4(\frac{x}{2}))}{16} + \frac{5(\tan^6(\frac{x}{2}))}{16} + \frac{5(\tan^8(\frac{x}{2}))}{96} + \frac{(\tan^{10}(\frac{x}{2}))}{160}$ $\frac{1}{\tan(\frac{x}{2})^5}$	50

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/sin(x)^6,x,method=_RETURNVERBOSE)``[Out] (-8/15-1/5*csc(x)^4-4/15*csc(x)^2)*cot(x)`**Maxima [A]**

time = 0.25, size = 20, normalized size = 0.95

$$-\frac{15 \tan (x)^4 + 10 \tan (x)^2 + 3}{15 \tan (x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/sin(x)^6,x, algorithm="maxima")``[Out] -1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5`**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

time = 0.31, size = 37, normalized size = 1.76

$$-\frac{8 \cos (x)^5 - 20 \cos (x)^3 + 15 \cos (x)}{15 (\cos (x)^4 - 2 \cos (x)^2 + 1) \sin (x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(x)^6,x, algorithm="fricas")

[Out] $-1/15*(8*\cos(x)^5 - 20*\cos(x)^3 + 15*\cos(x))/((\cos(x)^4 - 2*\cos(x)^2 + 1)*\sin(x))$

Sympy [A]

time = 0.03, size = 32, normalized size = 1.52

$$-\frac{8 \cos(x)}{15 \sin(x)} - \frac{4 \cos(x)}{15 \sin^3(x)} - \frac{\cos(x)}{5 \sin^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(x)**6,x)

[Out] $-8*\cos(x)/(15*\sin(x)) - 4*\cos(x)/(15*\sin(x)**3) - \cos(x)/(5*\sin(x)**5)$

Giac [A]

time = 0.00, size = 25, normalized size = 1.19

$$\frac{2(-15 \tan^4 x - 10 \tan^2 x - 3)}{30 \tan^5 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(x)^6,x)

[Out] $-1/15*(15*\tan(x)^4 + 10*\tan(x)^2 + 3)/\tan(x)^5$

Mupad [B]

time = 0.20, size = 27, normalized size = 1.29

$$-\frac{8 \cos(x) \sin(x)^4 + 4 \cos(x) \sin(x)^2 + 3 \cos(x)}{15 \sin(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sin(x)^6,x)

[Out] $-(3*\cos(x) + 4*\cos(x)*\sin(x)^2 + 8*\cos(x)*\sin(x)^4)/(15*\sin(x)^5)$

3.338 $\int \csc^7(x) dx$

Optimal. Leaf size=36

$$-\frac{5}{16} \tanh^{-1}(\cos(x)) - \frac{5}{16} \cot(x) \csc(x) - \frac{5}{24} \cot(x) \csc^3(x) - \frac{1}{6} \cot(x) \csc^5(x)$$

[Out] -5/16*arctanh(cos(x))-5/16*cot(x)*csc(x)-5/24*cot(x)*csc(x)^3-1/6*cot(x)*csc(x)^5

Rubi [A]

time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3853, 3855}

$$-\frac{5}{16} \tanh^{-1}(\cos(x)) - \frac{1}{6} \cot(x) \csc^5(x) - \frac{5}{24} \cot(x) \csc^3(x) - \frac{5}{16} \cot(x) \csc(x)$$

Antiderivative was successfully verified.

[In] Int[Csc[x]^7,x]

[Out] (-5*ArcTanh[Cos[x]])/16 - (5*Cot[x]*Csc[x])/16 - (5*Cot[x]*Csc[x]^3)/24 - (Cot[x]*Csc[x]^5)/6

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n], x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \csc^7(x) dx &= -\frac{1}{6} \cot(x) \csc^5(x) + \frac{5}{6} \int \csc^5(x) dx \\ &= -\frac{5}{24} \cot(x) \csc^3(x) - \frac{1}{6} \cot(x) \csc^5(x) + \frac{5}{8} \int \csc^3(x) dx \\ &= -\frac{5}{16} \cot(x) \csc(x) - \frac{5}{24} \cot(x) \csc^3(x) - \frac{1}{6} \cot(x) \csc^5(x) + \frac{5}{16} \int \csc(x) dx \\ &= -\frac{5}{16} \tanh^{-1}(\cos(x)) - \frac{5}{16} \cot(x) \csc(x) - \frac{5}{24} \cot(x) \csc^3(x) - \frac{1}{6} \cot(x) \csc^5(x) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 95 vs. $2(36) = 72$.

time = 0.01, size = 95, normalized size = 2.64

$$-\frac{5}{64} \csc^2\left(\frac{x}{2}\right) - \frac{1}{64} \csc^4\left(\frac{x}{2}\right) - \frac{1}{384} \csc^6\left(\frac{x}{2}\right) - \frac{5}{16} \log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{5}{16} \log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{5}{64} \sec^2\left(\frac{x}{2}\right) + \frac{1}{64} \sec^4\left(\frac{x}{2}\right) + \frac{1}{384} \sec^6\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^7,x]

[Out] $(-5*\text{Csc}[x/2]^2)/64 - \text{Csc}[x/2]^4/64 - \text{Csc}[x/2]^6/384 - (5*\text{Log}[\text{Cos}[x/2]])/16 + (5*\text{Log}[\text{Sin}[x/2]])/16 + (5*\text{Sec}[x/2]^2)/64 + \text{Sec}[x/2]^4/64 + \text{Sec}[x/2]^6/384$

Mathics [A]

time = 2.37, size = 44, normalized size = 1.22

$$\frac{-2\text{Cos}[x] (8 + 10\text{Sin}[x]^2 + 15\text{Sin}[x]^4) + 15 (\text{Log}[-1 + \text{Cos}[x]] - \text{Log}[1 + \text{Cos}[x]]) \text{Sin}[x]^6}{96\text{Sin}[x]^6}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Csc[x]^7,x]')

[Out] $(-2 \text{Cos}[x] (8 + 10 \text{Sin}[x]^2 + 15 \text{Sin}[x]^4) + 15 (\text{Log}[-1 + \text{Cos}[x]] - \text{Log}[1 + \text{Cos}[x]]) \text{Sin}[x]^6) / (96 \text{Sin}[x]^6)$

Maple [A]

time = 0.09, size = 32, normalized size = 0.89

method	result	size
default	$\left(-\frac{\csc^5(x)}{6} - \frac{5 \csc^3(x)}{24} - \frac{5 \csc(x)}{16}\right) \cot(x) + \frac{5 \ln(\csc(x) - \cot(x))}{16}$	32
norman	$\frac{-\frac{1}{384} - \frac{3(\tan^2(\frac{x}{2}))}{128} - \frac{15(\tan^4(\frac{x}{2}))}{128} + \frac{15(\tan^8(\frac{x}{2}))}{128} + \frac{3(\tan^{10}(\frac{x}{2}))}{128} + \frac{(\tan^{12}(\frac{x}{2}))}{384}}{\tan(\frac{x}{2})^6} + \frac{5 \ln(\tan(\frac{x}{2}))}{16}$	58
risch	$\frac{15 e^{11ix} - 85 e^{9ix} + 198 e^{7ix} + 198 e^{5ix} - 85 e^{3ix} + 15 e^{ix}}{24(e^{2ix} - 1)^6} + \frac{5 \ln(e^{ix} - 1)}{16} - \frac{5 \ln(1 + e^{ix})}{16}$	76

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)^7,x,method=_RETURNVERBOSE)

[Out] $(-1/6*\text{csc}(x)^5 - 5/24*\text{csc}(x)^3 - 5/16*\text{csc}(x))*\text{cot}(x) + 5/16*\ln(\text{csc}(x) - \text{cot}(x))$

Maxima [A]

time = 0.25, size = 54, normalized size = 1.50

$$\frac{15 \cos(x)^5 - 40 \cos(x)^3 + 33 \cos(x)}{48 (\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1)} - \frac{5}{32} \log(\cos(x) + 1) + \frac{5}{32} \log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^7,x, algorithm="maxima")

[Out] $1/48*(15*\cos(x)^5 - 40*\cos(x)^3 + 33*\cos(x))/(\cos(x)^6 - 3*\cos(x)^4 + 3*\cos(x)^2 - 1) - 5/32*\log(\cos(x) + 1) + 5/32*\log(\cos(x) - 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(28) = 56.

time = 0.32, size = 93, normalized size = 2.58

$$\frac{30 \cos(x)^5 - 80 \cos(x)^3 - 15 (\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 15 (\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 66 \cos(x)}{96 (\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^7,x, algorithm="fricas")

[Out] $1/96*(30*\cos(x)^5 - 80*\cos(x)^3 - 15*(\cos(x)^6 - 3*\cos(x)^4 + 3*\cos(x)^2 - 1)*\log(1/2*\cos(x) + 1/2) + 15*(\cos(x)^6 - 3*\cos(x)^4 + 3*\cos(x)^2 - 1)*\log(-1/2*\cos(x) + 1/2) + 66*\cos(x))/(\cos(x)^6 - 3*\cos(x)^4 + 3*\cos(x)^2 - 1)$

Sympy [A]

time = 0.08, size = 60, normalized size = 1.67

$$-\frac{-15 \cos^5(x) + 40 \cos^3(x) - 33 \cos(x)}{48 \cos^6(x) - 144 \cos^4(x) + 144 \cos^2(x) - 48} + \frac{5 \log(\cos(x) - 1)}{32} - \frac{5 \log(\cos(x) + 1)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)**7,x)

[Out] $-(-15*\cos(x)**5 + 40*\cos(x)**3 - 33*\cos(x))/(48*\cos(x)**6 - 144*\cos(x)**4 + 144*\cos(x)**2 - 48) + 5*\log(\cos(x) - 1)/32 - 5*\log(\cos(x) + 1)/32$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(28) = 56.

time = 0.00, size = 133, normalized size = 3.69

$$2 \left(\frac{\frac{65536}{3} \left(\frac{1-\cos x}{1+\cos x}\right)^3 + 196608 \left(\frac{1-\cos x}{1+\cos x}\right)^2 + \frac{983040(1-\cos x)}{1+\cos x}}{16777216} + \frac{-110 \left(\frac{1-\cos x}{1+\cos x}\right)^3 - 45 \left(\frac{1-\cos x}{1+\cos x}\right)^2 - \frac{9(1-\cos x)}{1+\cos x} - 1}{768 \left(\frac{1-\cos x}{1+\cos x}\right)^3} + \frac{5}{64} \ln\left(\frac{1-\cos x}{1+\cos x}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^7,x)

[Out] $-1/384*(9*(\cos(x) - 1)/(\cos(x) + 1) - 45*(\cos(x) - 1)^2/(\cos(x) + 1)^2 + 110*(\cos(x) - 1)^3/(\cos(x) + 1)^3 - 1)*(\cos(x) + 1)^3/(\cos(x) - 1)^3 - 15/128*(\cos(x) - 1)/(\cos(x) + 1) + 3/128*(\cos(x) - 1)^2/(\cos(x) + 1)^2 - 1/384*(\cos(x) - 1)^3/(\cos(x) + 1)^3 + 5/32*\log(-(\cos(x) - 1)/(\cos(x) + 1)))$

Mupad [B]

time = 0.25, size = 44, normalized size = 1.22

$$\frac{\frac{5 \cos(x)^5}{16} - \frac{5 \cos(x)^3}{6} + \frac{11 \cos(x)}{16}}{\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1} - \frac{5 \operatorname{atanh}(\cos(x))}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/sin(x)^7,x)`

[Out] `((11*cos(x))/16 - (5*cos(x)^3)/6 + (5*cos(x)^5)/16)/(3*cos(x)^2 - 3*cos(x)^4 + cos(x)^6 - 1) - (5*atanh(cos(x)))/16`

3.339 $\int \sec^{12}(x) dx$

Optimal. Leaf size=41

$$\tan(x) + \frac{5 \tan^3(x)}{3} + 2 \tan^5(x) + \frac{10 \tan^7(x)}{7} + \frac{5 \tan^9(x)}{9} + \frac{\tan^{11}(x)}{11}$$

[Out] $\tan(x)+5/3*\tan(x)^3+2*\tan(x)^5+10/7*\tan(x)^7+5/9*\tan(x)^9+1/11*\tan(x)^{11}$

Rubi [A]

time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3852}

$$\frac{\tan^{11}(x)}{11} + \frac{5 \tan^9(x)}{9} + \frac{10 \tan^7(x)}{7} + 2 \tan^5(x) + \frac{5 \tan^3(x)}{3} + \tan(x)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^12,x]

[Out] Tan[x] + (5*Tan[x]^3)/3 + 2*Tan[x]^5 + (10*Tan[x]^7)/7 + (5*Tan[x]^9)/9 + Tan[x]^11/11

Rule 3852

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int \sec^{12}(x) dx &= -\text{Subst} \left(\int (1 + 5x^2 + 10x^4 + 10x^6 + 5x^8 + x^{10}) dx, x, -\tan(x) \right) \\ &= \tan(x) + \frac{5 \tan^3(x)}{3} + 2 \tan^5(x) + \frac{10 \tan^7(x)}{7} + \frac{5 \tan^9(x)}{9} + \frac{\tan^{11}(x)}{11} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 57, normalized size = 1.39

$$\frac{256 \tan(x)}{693} + \frac{128}{693} \sec^2(x) \tan(x) + \frac{32}{231} \sec^4(x) \tan(x) + \frac{80}{693} \sec^6(x) \tan(x) + \frac{10}{99} \sec^8(x) \tan(x) + \frac{1}{11} \sec^{10}(x) \tan(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^12,x]

[Out] $(256*\text{Tan}[x])/693 + (128*\text{Sec}[x]^2*\text{Tan}[x])/693 + (32*\text{Sec}[x]^4*\text{Tan}[x])/231 + (80*\text{Sec}[x]^6*\text{Tan}[x])/693 + (10*\text{Sec}[x]^8*\text{Tan}[x])/99 + (\text{Sec}[x]^{10}*\text{Tan}[x])/11$

Mathics [A]

time = 2.01, size = 40, normalized size = 0.98

$$\frac{(213 - 230\text{Sin}[x]^2 + 80\text{Sin}[x]^4 + 96\text{Cos}[x]^6 + 128\text{Cos}[x]^8 + 256\text{Cos}[x]^{10}) \text{Sin}[x]}{693\text{Cos}[x]^{11}}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[1/Cos[x]^12,x]')`

[Out] $(213 - 230 \text{Sin}[x]^2 + 80 \text{Sin}[x]^4 + 96 \text{Cos}[x]^6 + 128 \text{Cos}[x]^8 + 256 \text{Cos}[x]^{10}) \text{Sin}[x] / (693 \text{Cos}[x]^{11})$

Maple [A]

time = 0.08, size = 37, normalized size = 0.90

method	result	size
default	$-\left(-\frac{256}{693} - \frac{(\sec^{10}(x))}{11} - \frac{10(\sec^8(x))}{99} - \frac{80(\sec^6(x))}{693} - \frac{32(\sec^4(x))}{231} - \frac{128(\sec^2(x))}{693}\right) \tan(x)$	37
risch	$\frac{512i(462 e^{10ix} + 330 e^{8ix} + 165 e^{6ix} + 55 e^{4ix} + 11 e^{2ix} + 1)}{693(e^{2ix} + 1)^{11}}$	50

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/cos(x)^12,x,method=_RETURNVERBOSE)`

[Out] $-(-256/693 - 1/11*\sec(x)^{10} - 10/99*\sec(x)^8 - 80/693*\sec(x)^6 - 32/231*\sec(x)^4 - 128/693*\sec(x)^2)*\tan(x)$

Maxima [A]

time = 0.26, size = 33, normalized size = 0.80

$$\frac{1}{11} \tan(x)^{11} + \frac{5}{9} \tan(x)^9 + \frac{10}{7} \tan(x)^7 + 2 \tan(x)^5 + \frac{5}{3} \tan(x)^3 + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/cos(x)^12,x, algorithm="maxima")`

[Out] $1/11*\tan(x)^{11} + 5/9*\tan(x)^9 + 10/7*\tan(x)^7 + 2*\tan(x)^5 + 5/3*\tan(x)^3 + \tan(x)$

Fricas [A]

time = 0.32, size = 40, normalized size = 0.98

$$\frac{(256 \cos(x)^{10} + 128 \cos(x)^8 + 96 \cos(x)^6 + 80 \cos(x)^4 + 70 \cos(x)^2 + 63) \sin(x)}{693 \cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(x)^12,x, algorithm="fricas")

[Out] 1/693*(256*cos(x)^10 + 128*cos(x)^8 + 96*cos(x)^6 + 80*cos(x)^4 + 70*cos(x)^2 + 63)*sin(x)/cos(x)^11

Sympy [A]

time = 0.03, size = 66, normalized size = 1.61

$$\frac{256 \sin(x)}{693 \cos(x)} + \frac{128 \sin(x)}{693 \cos^3(x)} + \frac{32 \sin(x)}{231 \cos^5(x)} + \frac{80 \sin(x)}{693 \cos^7(x)} + \frac{10 \sin(x)}{99 \cos^9(x)} + \frac{\sin(x)}{11 \cos^{11}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(x)**12,x)

[Out] 256*sin(x)/(693*cos(x)) + 128*sin(x)/(693*cos(x)**3) + 32*sin(x)/(231*cos(x)**5) + 80*sin(x)/(693*cos(x)**7) + 10*sin(x)/(99*cos(x)**9) + sin(x)/(11*cos(x)**11)

Giac [A]

time = 0.00, size = 45, normalized size = 1.10

$$\frac{2}{2} \left(\frac{1}{11} \tan^{11} x + \frac{5}{9} \tan^9 x + \frac{10}{7} \tan^7 x + 2 \tan^5 x + \frac{5}{3} \tan^3 x + \tan x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(x)^12,x)

[Out] 1/11*tan(x)^11 + 5/9*tan(x)^9 + 10/7*tan(x)^7 + 2*tan(x)^5 + 5/3*tan(x)^3 + tan(x)

Mupad [B]

time = 0.21, size = 51, normalized size = 1.24

$$\frac{256 \sin(x) \cos(x)^{10} + 128 \sin(x) \cos(x)^8 + 96 \sin(x) \cos(x)^6 + 80 \sin(x) \cos(x)^4 + 70 \sin(x) \cos(x)^2 + 63 \sin(x)}{693 \cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(x)^12,x)

[Out] (63*sin(x) + 70*cos(x)^2*sin(x) + 80*cos(x)^4*sin(x) + 96*cos(x)^6*sin(x) + 128*cos(x)^8*sin(x) + 256*cos(x)^10*sin(x))/(693*cos(x)^11)

3.340 $\int \sec^3\left(\frac{\pi}{4} + 3x\right) dx$

Optimal. Leaf size=40

$$\frac{1}{6} \tanh^{-1}\left(\sin\left(\frac{\pi}{4} + 3x\right)\right) + \frac{1}{6} \sec\left(\frac{\pi}{4} + 3x\right) \tan\left(\frac{\pi}{4} + 3x\right)$$

[Out] 1/6*arctanh(sin(1/4*Pi+3*x))+1/6*sec(1/4*Pi+3*x)*tan(1/4*Pi+3*x)

Rubi [A]

time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3853, 3855}

$$\frac{1}{6} \tanh^{-1}\left(\sin\left(3x + \frac{\pi}{4}\right)\right) + \frac{1}{6} \tan\left(3x + \frac{\pi}{4}\right) \sec\left(3x + \frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

[In] Int[Sec[Pi/4 + 3*x]^3,x]

[Out] ArcTanh[Sin[Pi/4 + 3*x]]/6 + (Sec[Pi/4 + 3*x]*Tan[Pi/4 + 3*x])/6

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n], x_Symbol] :> Simp[(-b)*Cos[c + d*x]*(b*Csc[c + d*x])^(n-1)/(d*(n-1)), x] + Dist[b^2*((n-2)/(n-1)), Int[(b*Csc[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] & IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \sec^3\left(\frac{\pi}{4} + 3x\right) dx &= \frac{1}{6} \sec\left(\frac{\pi}{4} + 3x\right) \tan\left(\frac{\pi}{4} + 3x\right) + \frac{1}{2} \int \csc\left(\frac{\pi}{4} - 3x\right) dx \\ &= \frac{1}{6} \tanh^{-1}\left(\sin\left(\frac{\pi}{4} + 3x\right)\right) + \frac{1}{6} \sec\left(\frac{\pi}{4} + 3x\right) \tan\left(\frac{\pi}{4} + 3x\right) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 40, normalized size = 1.00

$$\frac{1}{6} \tanh^{-1}\left(\sin\left(\frac{\pi}{4} + 3x\right)\right) + \frac{1}{6} \sec\left(\frac{\pi}{4} + 3x\right) \tan\left(\frac{\pi}{4} + 3x\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[Pi/4 + 3*x]^3,x]

[Out] ArcTanh[Sin[Pi/4 + 3*x]]/6 + (Sec[Pi/4 + 3*x]*Tan[Pi/4 + 3*x])/6

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 224 vs. 2(40) = 80.
time = 9.73, size = 166, normalized size = 4.15

$$\frac{\text{Log}\left[1 + \text{Tan}\left[\frac{\text{Pi}}{8} + \frac{3x}{2}\right]\right] - 2\text{Log}\left[1 + \text{Tan}\left[\frac{\text{Pi}}{8} + \frac{3x}{2}\right]\right] \text{Tan}\left[\frac{\text{Pi}}{8} + \frac{3x}{2}\right]^2 - \text{Log}\left[-1 + \text{Tan}\left[\frac{\text{Pi}}{8} + \frac{3x}{2}\right]\right] - \text{Log}\left[-1 + \text{Tan}\left[\frac{\text{Pi}}{8} + \frac{3x}{2}\right]\right] \text{Tan}\left[\frac{\text{Pi}}{8} + \frac{3x}{2}\right]^4 + 2\text{Log}\left[-1 + \text{Tan}\left[\frac{\text{Pi}}{8} + \frac{3x}{2}\right]\right] \text{Tan}\left[\frac{\text{Pi}}{8} + \frac{3x}{2}\right]^2 + 2\text{Tan}\left[\frac{\text{Pi}}{8} + \frac{3x}{2}\right]^3 + 2\text{Tan}\left[\frac{\text{Pi}}{8} + \frac{3x}{2}\right] + \text{Log}\left[1 + \text{Tan}\left[\frac{\text{Pi}}{8} + \frac{3x}{2}\right]\right] \text{Tan}\left[\frac{\text{Pi}}{8} + \frac{3x}{2}\right]^4}{6 - 12\text{Tan}\left[\frac{\text{Pi}}{8} + \frac{3x}{2}\right]^2 + 6\text{Tan}\left[\frac{\text{Pi}}{8} + \frac{3x}{2}\right]^4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/Cos[Pi/4 + 3*x]^3,x]')

[Out] (Log[1 + Tan[Pi / 8 + 3 x / 2]] - 2 Log[1 + Tan[Pi / 8 + 3 x / 2]] Tan[Pi / 8 + 3 x / 2] ^ 2 - Log[-1 + Tan[Pi / 8 + 3 x / 2]] - Log[-1 + Tan[Pi / 8 + 3 x / 2]] Tan[Pi / 8 + 3 x / 2] ^ 4 + 2 Log[-1 + Tan[Pi / 8 + 3 x / 2]] Tan[Pi / 8 + 3 x / 2] ^ 2 + 2 Tan[Pi / 8 + 3 x / 2] ^ 3 + 2 Tan[Pi / 8 + 3 x / 2] + Log[1 + Tan[Pi / 8 + 3 x / 2]] Tan[Pi / 8 + 3 x / 2] ^ 4) / (6 (1 + Tan[Pi / 8 + 3 x / 2] ^ 4 - 2 Tan[Pi / 8 + 3 x / 2] ^ 2))

Maple [A]

time = 0.33, size = 40, normalized size = 1.00

method	result	size
derivativedivides	$\frac{\sec(\frac{\pi}{4}+3x) \tan(\frac{\pi}{4}+3x)}{6} + \frac{\ln(\sec(\frac{\pi}{4}+3x)+\tan(\frac{\pi}{4}+3x))}{6}$	40
default	$\frac{\sec(\frac{\pi}{4}+3x) \tan(\frac{\pi}{4}+3x)}{6} + \frac{\ln(\sec(\frac{\pi}{4}+3x)+\tan(\frac{\pi}{4}+3x))}{6}$	40
norman	$\frac{\frac{\tan^3(\frac{\pi}{8}+\frac{3x}{2})}{3} + \frac{\tan(\frac{\pi}{8}+\frac{3x}{2})}{3}}{(\tan^2(\frac{\pi}{8}+\frac{3x}{2})-1)^2} - \frac{\ln(\tan(\frac{\pi}{8}+\frac{3x}{2})-1)}{6} + \frac{\ln(\tan(\frac{\pi}{8}+\frac{3x}{2})+1)}{6}$	66
risch	$-\frac{i((-1)^{\frac{3}{4}}e^{9ix}-(-1)^{\frac{1}{4}}e^{3ix})}{3(ie^{6ix}+1)^2} + \frac{\ln(e^{\frac{i(\pi+12x)}{4}}+i)}{6} - \frac{\ln(e^{\frac{i(\pi+12x)}{4}}-i)}{6}$	67

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(1/4*Pi+3*x)^3,x,method=_RETURNVERBOSE)

[Out] 1/6*sec(1/4*Pi+3*x)*tan(1/4*Pi+3*x)+1/6*ln(sec(1/4*Pi+3*x)+tan(1/4*Pi+3*x))

Maxima [A]

time = 0.25, size = 51, normalized size = 1.28

$$-\frac{\sin\left(\frac{1}{4}\pi + 3x\right)}{6\left(\sin\left(\frac{1}{4}\pi + 3x\right)^2 - 1\right)} + \frac{1}{12} \log\left(\sin\left(\frac{1}{4}\pi + 3x\right) + 1\right) - \frac{1}{12} \log\left(\sin\left(\frac{1}{4}\pi + 3x\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(1/4*pi+3*x)^3,x, algorithm="maxima")

[Out] -1/6*sin(1/4*pi + 3*x)/(sin(1/4*pi + 3*x)^2 - 1) + 1/12*log(sin(1/4*pi + 3*x) + 1) - 1/12*log(sin(1/4*pi + 3*x) - 1)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(30) = 60.

time = 0.33, size = 70, normalized size = 1.75

$$\frac{\cos\left(\frac{1}{4}\pi + 3x\right)^2 \log\left(\sin\left(\frac{1}{4}\pi + 3x\right) + 1\right) - \cos\left(\frac{1}{4}\pi + 3x\right)^2 \log\left(-\sin\left(\frac{1}{4}\pi + 3x\right) + 1\right) + 2 \sin\left(\frac{1}{4}\pi + 3x\right)}{12 \cos\left(\frac{1}{4}\pi + 3x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(1/4*pi+3*x)^3,x, algorithm="fricas")

[Out] 1/12*(cos(1/4*pi + 3*x)^2*log(sin(1/4*pi + 3*x) + 1) - cos(1/4*pi + 3*x)^2*log(-sin(1/4*pi + 3*x) + 1) + 2*sin(1/4*pi + 3*x))/cos(1/4*pi + 3*x)^2

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(29) = 58

time = 0.55, size = 388, normalized size = 9.70

$$\frac{\frac{\log(\tan(\frac{x}{4}) - 1)\tan^2(\frac{x}{4})}{6\tan^2(\frac{x}{4}) - 12\tan^4(\frac{x}{4}) + 6} + \frac{2\log(\tan(\frac{x}{4}) - 1)\tan^2(\frac{x}{4})}{6\tan^2(\frac{x}{4}) - 12\tan^4(\frac{x}{4}) + 6} - \frac{\log(\tan(\frac{x}{4}) - 1)}{6\tan^2(\frac{x}{4}) - 12\tan^4(\frac{x}{4}) + 6} + \frac{\log(\tan(\frac{x}{4}) + 1)\tan^2(\frac{x}{4})}{6\tan^2(\frac{x}{4}) - 12\tan^4(\frac{x}{4}) + 6} - \frac{2\log(\tan(\frac{x}{4}) + 1)\tan^2(\frac{x}{4})}{6\tan^2(\frac{x}{4}) - 12\tan^4(\frac{x}{4}) + 6} + \frac{\log(\tan(\frac{x}{4}) + 1)}{6\tan^2(\frac{x}{4}) - 12\tan^4(\frac{x}{4}) + 6} + \frac{2\tan^2(\frac{x}{4})}{6\tan^2(\frac{x}{4}) - 12\tan^4(\frac{x}{4}) + 6} + \frac{2\tan(\frac{x}{4})}{6\tan^2(\frac{x}{4}) - 12\tan^4(\frac{x}{4}) + 6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(1/4*pi+3*x)**3,x)

[Out] -log(tan(3*x/2 + pi/8) - 1)*tan(3*x/2 + pi/8)**4/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + 2*log(tan(3*x/2 + pi/8) - 1)*tan(3*x/2 + pi/8)**2/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) - log(tan(3*x/2 + pi/8) - 1)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + log(tan(3*x/2 + pi/8) + 1)*tan(3*x/2 + pi/8)**4/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) - 2*log(tan(3*x/2 + pi/8) + 1)*tan(3*x/2 + pi/8)**2/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + log(tan(3*x/2 + pi/8) + 1)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + 2*tan(3*x/2 + pi/8)**3/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + 2*tan(3*x/2 + pi/8)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6)

Giac [A]

time = 0.01, size = 64, normalized size = 1.60

$$\frac{2}{3} \left(-\frac{\ln\left(-\sin\left(\frac{\pi+12x}{4}\right) + 1\right)}{8} + \frac{\ln\left(\sin\left(\frac{\pi+12x}{4}\right) + 1\right)}{8} - \frac{\sin\left(\frac{\pi+12x}{4}\right)}{4\left(\sin^2\left(\frac{\pi+12x}{4}\right) - 1\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(1/4*pi+3*x)^3,x)

[Out] $-1/6*\sin(1/4*\pi + 3*x)/(\sin(1/4*\pi + 3*x)^2 - 1) + 1/12*\log(\sin(1/4*\pi + 3*x) + 1) - 1/12*\log(-\sin(1/4*\pi + 3*x) + 1)$

Mupad [B]

time = 0.62, size = 35, normalized size = 0.88

$$\frac{\ln\left(\tan\left(\frac{\pi}{8} + \frac{3x}{2} + \frac{\pi}{4}\right)\right)}{6} + \frac{\tan\left(\frac{\pi}{4} + 3x\right)}{6 \cos\left(\frac{\pi}{4} + 3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/\cos(\pi/4 + 3*x)^3, x)$

[Out] $\log(\tan(\pi/8 + (3*x)/2 + \pi/4))/6 + \tan(\pi/4 + 3*x)/(6*\cos(\pi/4 + 3*x))$

3.341 $\int \tan^6(x) dx$

Optimal. Leaf size=22

$$-x + \tan(x) - \frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5}$$

[Out] $-x + \tan(x) - 1/3 * \tan(x)^3 + 1/5 * \tan(x)^5$

Rubi [A]

time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 8}

$$-x + \frac{\tan^5(x)}{5} - \frac{\tan^3(x)}{3} + \tan(x)$$

Antiderivative was successfully verified.

[In] Int[Tan[x]^6,x]

[Out] $-x + \text{Tan}[x] - \text{Tan}[x]^3/3 + \text{Tan}[x]^5/5$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3554

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[b*((b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int \tan^6(x) dx &= \frac{\tan^5(x)}{5} - \int \tan^4(x) dx \\ &= -\frac{1}{3} \tan^3(x) + \frac{\tan^5(x)}{5} + \int \tan^2(x) dx \\ &= \tan(x) - \frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5} - \int 1 dx \\ &= -x + \tan(x) - \frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 30, normalized size = 1.36

$$-x + \frac{23 \tan(x)}{15} - \frac{11}{15} \sec^2(x) \tan(x) + \frac{1}{5} \sec^4(x) \tan(x)$$

Antiderivative was successfully verified.

[In] Integrate[Tan[x]^6,x]

[Out] $-x + (23*\text{Tan}[x])/15 - (11*\text{Sec}[x]^2*\text{Tan}[x])/15 + (\text{Sec}[x]^4*\text{Tan}[x])/5$

Mathics [A]

time = 1.75, size = 18, normalized size = 0.82

$$-x + \text{Tan}[x] - \frac{\text{Tan}[x]^3}{3} + \frac{\text{Tan}[x]^5}{5}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Tan[x]^6,x]')

[Out] $-x + \text{Tan}[x] - \text{Tan}[x]^3 / 3 + \text{Tan}[x]^5 / 5$

Maple [A]

time = 0.01, size = 21, normalized size = 0.95

method	result	size
norman	$-x + \tan(x) - \frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5}$	19
derivativedivides	$\frac{\tan^5(x)}{5} - \frac{\tan^3(x)}{3} + \tan(x) - \arctan(\tan(x))$	21
default	$\frac{\tan^5(x)}{5} - \frac{\tan^3(x)}{3} + \tan(x) - \arctan(\tan(x))$	21
risch	$-x + \frac{2i(45e^{8ix} + 90e^{6ix} + 140e^{4ix} + 70e^{2ix} + 23)}{15(e^{2ix} + 1)^5}$	47

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)^6,x,method=_RETURNVERBOSE)

[Out] $1/5*\tan(x)^5 - 1/3*\tan(x)^3 + \tan(x) - \arctan(\tan(x))$

Maxima [A]

time = 0.34, size = 18, normalized size = 0.82

$$\frac{1}{5} \tan(x)^5 - \frac{1}{3} \tan(x)^3 - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)^6,x, algorithm="maxima")

[Out] $1/5*\tan(x)^5 - 1/3*\tan(x)^3 - x + \tan(x)$

Fricas [A]

time = 0.31, size = 18, normalized size = 0.82

$$\frac{1}{5} \tan(x)^5 - \frac{1}{3} \tan(x)^3 - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)^6,x, algorithm="fricas")

[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)

Sympy [A]

time = 0.03, size = 31, normalized size = 1.41

$$-x + \frac{\sin^5(x)}{5 \cos^5(x)} - \frac{\sin^3(x)}{3 \cos^3(x)} + \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)**6,x)

[Out] -x + sin(x)**5/(5*cos(x)**5) - sin(x)**3/(3*cos(x)**3) + sin(x)/cos(x)

Giac [A]

time = 0.00, size = 20, normalized size = 0.91

$$\frac{\tan^5 x}{5} - \frac{\tan^3 x}{3} + \tan x - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)^6,x)

[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)

Mupad [B]

time = 0.03, size = 18, normalized size = 0.82

$$\frac{\tan(x)^5}{5} - \frac{\tan(x)^3}{3} + \tan(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)^6,x)

[Out] tan(x) - x - tan(x)^3/3 + tan(x)^5/5

3.342 $\int \cot^5(x) dx$

Optimal. Leaf size=20

$$\frac{\cot^2(x)}{2} - \frac{\cot^4(x)}{4} + \log(\sin(x))$$

[Out] 1/2*cot(x)^2-1/4*cot(x)^4+ln(sin(x))

Rubi [A]

time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 3556}

$$-\frac{1}{4} \cot^4(x) + \frac{\cot^2(x)}{2} + \log(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cot[x]^5,x]

[Out] Cot[x]^2/2 - Cot[x]^4/4 + Log[Sin[x]]

Rule 3554

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3556

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \cot^5(x) dx &= -\frac{1}{4} \cot^4(x) - \int \cot^3(x) dx \\ &= \frac{\cot^2(x)}{2} - \frac{\cot^4(x)}{4} + \int \cot(x) dx \\ &= \frac{\cot^2(x)}{2} - \frac{\cot^4(x)}{4} + \log(\sin(x)) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 16, normalized size = 0.80

$$\csc^2(x) - \frac{\csc^4(x)}{4} + \log(\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]^5,x]

[Out] Csc[x]^2 - Csc[x]^4/4 + Log[Sin[x]]

Mathics [A]

time = 1.72, size = 14, normalized size = 0.70

$$\text{Log}[\text{Sin}[x]] + \frac{1}{\text{Sin}[x]^2} - \frac{1}{4\text{Sin}[x]^4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/Tan[x]^5,x]')

[Out] Log[Sin[x]] + 1 / Sin[x] ^ 2 - 1 / (4 Sin[x] ^ 4)

Maple [A]

time = 0.03, size = 26, normalized size = 1.30

method	result	size
derivativedivides	$-\frac{\ln(1+\tan^2(x))}{2} - \frac{1}{4\tan(x)^4} + \ln(\tan(x)) + \frac{1}{2\tan(x)^2}$	26
default	$-\frac{\ln(1+\tan^2(x))}{2} - \frac{1}{4\tan(x)^4} + \ln(\tan(x)) + \frac{1}{2\tan(x)^2}$	26
norman	$-\frac{\frac{1}{4} + \frac{\tan^2(x)}{2}}{\tan(x)^4} - \frac{\ln(1+\tan^2(x))}{2} + \ln(\tan(x))$	27
risch	$-ix - \frac{4(e^{6ix} - e^{4ix} + e^{2ix})}{(e^{2ix} - 1)^4} + \ln(e^{2ix} - 1)$	43

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/tan(x)^5,x,method=_RETURNVERBOSE)

[Out] -1/2*ln(1+tan(x)^2)-1/4/tan(x)^4+ln(tan(x))+1/2/tan(x)^2

Maxima [A]

time = 0.26, size = 22, normalized size = 1.10

$$\frac{4 \sin(x)^2 - 1}{4 \sin(x)^4} + \frac{1}{2} \log(\sin(x)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(x)^5,x, algorithm="maxima")

[Out] 1/4*(4*sin(x)^2 - 1)/sin(x)^4 + 1/2*log(sin(x)^2)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(16) = 32$.

time = 0.31, size = 40, normalized size = 2.00

$$\frac{2 \log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right) \tan(x)^4 + 3 \tan(x)^4 + 2 \tan(x)^2 - 1}{4 \tan(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(x)^5,x, algorithm="fricas")

[Out] 1/4*(2*log(tan(x)^2/(tan(x)^2 + 1))*tan(x)^4 + 3*tan(x)^4 + 2*tan(x)^2 - 1)/tan(x)^4

Sympy [A]

time = 0.05, size = 19, normalized size = 0.95

$$\frac{4 \sin^2(x) - 1}{4 \sin^4(x)} + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(x)**5,x)

[Out] (4*sin(x)**2 - 1)/(4*sin(x)**4) + log(sin(x))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(16) = 32$.

time = 0.00, size = 43, normalized size = 2.15

$$-\frac{\ln(\tan^2 x + 1)}{2} + \frac{\ln(\tan^2 x)}{2} + \frac{-3 \tan^4 x + 2 \tan^2 x - 1}{4 \tan^4 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(x)^5,x)

[Out] -1/4*(3*tan(x)^4 - 2*tan(x)^2 + 1)/tan(x)^4 - 1/2*log(tan(x)^2 + 1) + 1/2*log(tan(x)^2)

Mupad [B]

time = 0.27, size = 26, normalized size = 1.30

$$\ln(\tan(x)) - \frac{\ln(\tan(x)^2 + 1)}{2} + \frac{\frac{\tan(x)^2}{2} - \frac{1}{4}}{\tan(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/tan(x)^5,x)

[Out] log(tan(x)) - log(tan(x)^2 + 1)/2 + (tan(x)^2/2 - 1/4)/tan(x)^4

3.343 $\int \cot^4\left(\frac{\pi}{4} + \frac{x}{3}\right) dx$

Optimal. Leaf size=32

$$x + 3 \cot\left(\frac{\pi}{4} + \frac{x}{3}\right) - \cot^3\left(\frac{\pi}{4} + \frac{x}{3}\right)$$

[Out] x+3*cot(1/4*Pi+1/3*x)-cot(1/4*Pi+1/3*x)^3

Rubi [A]

time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3554, 8}

$$x - \cot^3\left(\frac{x}{3} + \frac{\pi}{4}\right) + 3 \cot\left(\frac{x}{3} + \frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

[In] Int[Cot[Pi/4 + x/3]^4,x]

[Out] x + 3*Cot[Pi/4 + x/3] - Cot[Pi/4 + x/3]^3

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3554

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[b*((b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int \cot^4\left(\frac{\pi}{4} + \frac{x}{3}\right) dx &= -\cot^3\left(\frac{\pi}{4} + \frac{x}{3}\right) - \int \tan^2\left(\frac{\pi}{4} - \frac{x}{3}\right) dx \\ &= 3 \cot\left(\frac{\pi}{4} + \frac{x}{3}\right) - \cot^3\left(\frac{\pi}{4} + \frac{x}{3}\right) + \int 1 dx \\ &= x + 3 \cot\left(\frac{\pi}{4} + \frac{x}{3}\right) - \cot^3\left(\frac{\pi}{4} + \frac{x}{3}\right) \end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.01, size = 40, normalized size = 1.25

$$-\cot^3\left(\frac{\pi}{4} + \frac{x}{3}\right) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; -\tan^2\left(\frac{\pi}{4} + \frac{x}{3}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[Pi/4 + x/3]^4,x]

[Out] -(Cot[Pi/4 + x/3]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[Pi/4 + x/3]^2])

Mathics [A]

time = 2.01, size = 26, normalized size = 0.81

$$x - \frac{1}{\tan\left[\frac{\pi}{4} + \frac{x}{3}\right]^3} + \frac{3}{\tan\left[\frac{\pi}{4} + \frac{x}{3}\right]}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Cot[x/3 - 3*Pi/4]^4,x]')

[Out] x - 1 / Tan[Pi / 4 + x / 3] ^ 3 + 3 / Tan[Pi / 4 + x / 3]

Maple [A]

time = 0.05, size = 38, normalized size = 1.19

method	result	size
derivativdivides	$-\left(\cot^3\left(\frac{\pi}{4} + \frac{x}{3}\right)\right) + 3 \cot\left(\frac{\pi}{4} + \frac{x}{3}\right) - \frac{3\pi}{2} + 3 \operatorname{arccot}\left(\cot\left(\frac{\pi}{4} + \frac{x}{3}\right)\right)$	38
default	$-\left(\cot^3\left(\frac{\pi}{4} + \frac{x}{3}\right)\right) + 3 \cot\left(\frac{\pi}{4} + \frac{x}{3}\right) - \frac{3\pi}{2} + 3 \operatorname{arccot}\left(\cot\left(\frac{\pi}{4} + \frac{x}{3}\right)\right)$	38
norman	$\frac{-1+x(\tan^3(\frac{\pi}{4}+\frac{x}{3}))+3(\tan^2(\frac{\pi}{4}+\frac{x}{3}))}{\tan(\frac{\pi}{4}+\frac{x}{3})^3}$	38
risch	$x + \frac{4i(-3e^{\frac{4ix}{3}} - 3ie^{\frac{2ix}{3}} + 2)}{\left(e^{\frac{i(3\pi+4x)}{6}} - 1\right)^3}$	38

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(1/4*Pi+1/3*x)^4,x,method=_RETURNVERBOSE)

[Out] -cot(1/4*Pi+1/3*x)^3+3*cot(1/4*Pi+1/3*x)-3/2*Pi+3*arccot(cot(1/4*Pi+1/3*x))

Maxima [A]

time = 0.36, size = 30, normalized size = 0.94

$$\frac{3}{4}\pi + x + \frac{3 \tan\left(\frac{1}{4}\pi + \frac{1}{3}x\right)^2 - 1}{\tan\left(\frac{1}{4}\pi + \frac{1}{3}x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(1/4*pi+1/3*x)^4,x, algorithm="maxima")

[Out] 3/4*pi + x + (3*tan(1/4*pi + 1/3*x)^2 - 1)/tan(1/4*pi + 1/3*x)^3

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(24) = 48$.

time = 0.33, size = 70, normalized size = 2.19

$$\frac{4 \cos\left(\frac{1}{2}\pi + \frac{2}{3}x\right)^2 + \left(x \cos\left(\frac{1}{2}\pi + \frac{2}{3}x\right) - x\right) \sin\left(\frac{1}{2}\pi + \frac{2}{3}x\right) + 2 \cos\left(\frac{1}{2}\pi + \frac{2}{3}x\right) - 2}{\left(\cos\left(\frac{1}{2}\pi + \frac{2}{3}x\right) - 1\right) \sin\left(\frac{1}{2}\pi + \frac{2}{3}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(1/4*pi+1/3*x)^4,x, algorithm="fricas")

[Out] (4*cos(1/2*pi + 2/3*x)^2 + (x*cos(1/2*pi + 2/3*x) - x)*sin(1/2*pi + 2/3*x) + 2*cos(1/2*pi + 2/3*x) - 2)/((cos(1/2*pi + 2/3*x) - 1)*sin(1/2*pi + 2/3*x))

Sympy [A]

time = 0.08, size = 20, normalized size = 0.62

$$x - \cot^3\left(\frac{x}{3} + \frac{\pi}{4}\right) + 3 \cot\left(\frac{x}{3} + \frac{\pi}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(1/4*pi+1/3*x)**4,x)

[Out] x - cot(x/3 + pi/4)**3 + 3*cot(x/3 + pi/4)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(24) = 48$.
time = 0.02, size = 92, normalized size = 2.88

$$6 \left(\frac{\frac{256}{3} \tan^3\left(\frac{\frac{x}{3} + \frac{\pi}{4}}{2}\right) - 1280 \tan\left(\frac{\frac{x}{3} + \frac{\pi}{4}}{2}\right)}{4096} + \frac{15 \tan^2\left(\frac{\frac{x}{3} + \frac{\pi}{4}}{2}\right) - 1}{48 \tan^3\left(\frac{\frac{x}{3} + \frac{\pi}{4}}{2}\right)} + \frac{\frac{x}{3} + \frac{\pi}{4}}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(1/4*pi+1/3*x)^4,x)

[Out] 3/4*pi + 1/8*tan(1/8*pi + 1/6*x)^3 + x + 1/8*(15*tan(1/8*pi + 1/6*x)^2 - 1)/tan(1/8*pi + 1/6*x)^3 - 15/8*tan(1/8*pi + 1/6*x)

Mupad [B]

time = 0.09, size = 24, normalized size = 0.75

$$-\cot\left(\frac{\Pi}{4} + \frac{x}{3}\right)^3 + 3 \cot\left(\frac{\Pi}{4} + \frac{x}{3}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(Pi/4 + x/3)^4,x)

[Out] x + 3*cot(Pi/4 + x/3) - cot(Pi/4 + x/3)^3

3.344 $\int \cos^6(x) \sin^4(x) dx$

Optimal. Leaf size=56

$$\frac{3x}{256} + \frac{3}{256} \cos(x) \sin(x) + \frac{1}{128} \cos^3(x) \sin(x) + \frac{1}{160} \cos^5(x) \sin(x) - \frac{3}{80} \cos^7(x) \sin(x) - \frac{1}{10} \cos^7(x) \sin^3(x)$$

[Out] 3/256*x+3/256*cos(x)*sin(x)+1/128*cos(x)^3*sin(x)+1/160*cos(x)^5*sin(x)-3/80*cos(x)^7*sin(x)-1/10*cos(x)^7*sin(x)^3

Rubi [A]

time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2648, 2715, 8}

$$\frac{3x}{256} - \frac{1}{10} \sin^3(x) \cos^7(x) - \frac{3}{80} \sin(x) \cos^7(x) + \frac{1}{160} \sin(x) \cos^5(x) + \frac{1}{128} \sin(x) \cos^3(x) + \frac{3}{256} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^6*Sin[x]^4,x]

[Out] (3*x)/256 + (3*Cos[x]*Sin[x])/256 + (Cos[x]^3*Sin[x])/128 + (Cos[x]^5*Sin[x])/160 - (3*Cos[x]^7*Sin[x])/80 - (Cos[x]^7*Sin[x]^3)/10

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2648

Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^n]*((a_.)*sin[(e_.) + (f_.)*(x_)])^m, x_Symbol] := Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m - 1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^n, x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
\int \cos^6(x) \sin^4(x) dx &= -\frac{1}{10} \cos^7(x) \sin^3(x) + \frac{3}{10} \int \cos^6(x) \sin^2(x) dx \\
&= -\frac{3}{80} \cos^7(x) \sin(x) - \frac{1}{10} \cos^7(x) \sin^3(x) + \frac{3}{80} \int \cos^6(x) dx \\
&= \frac{1}{160} \cos^5(x) \sin(x) - \frac{3}{80} \cos^7(x) \sin(x) - \frac{1}{10} \cos^7(x) \sin^3(x) + \frac{1}{32} \int \cos^4(x) dx \\
&= \frac{1}{128} \cos^3(x) \sin(x) + \frac{1}{160} \cos^5(x) \sin(x) - \frac{3}{80} \cos^7(x) \sin(x) - \frac{1}{10} \cos^7(x) \sin^3(x) + \frac{1}{32} \int \cos^2(x) dx \\
&= \frac{3}{256} \cos(x) \sin(x) + \frac{1}{128} \cos^3(x) \sin(x) + \frac{1}{160} \cos^5(x) \sin(x) - \frac{3}{80} \cos^7(x) \sin(x) - \frac{1}{32} \int \cos(x) dx \\
&= \frac{3x}{256} + \frac{3}{256} \cos(x) \sin(x) + \frac{1}{128} \cos^3(x) \sin(x) + \frac{1}{160} \cos^5(x) \sin(x) - \frac{3}{80} \cos^7(x) \sin(x) - \frac{1}{32} \sin(x) + C
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 46, normalized size = 0.82

$$\frac{3x}{256} + \frac{1}{512} \sin(2x) - \frac{1}{256} \sin(4x) - \frac{\sin(6x)}{1024} + \frac{\sin(8x)}{2048} + \frac{\sin(10x)}{5120}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[x]^6*Sin[x]^4,x]``[Out] (3*x)/256 + Sin[2*x]/512 - Sin[4*x]/256 - Sin[6*x]/1024 + Sin[8*x]/2048 + Sin[10*x]/5120`**Mathics [A]**

time = 1.96, size = 64, normalized size = 1.14

$$\frac{3x}{256} - \frac{21(1 - \cos[2x])^3 \sin[2x]}{1280} - \frac{\sin[2x]}{128} + \frac{\sin[4x]}{1024} + \frac{(1 - \cos[2x])^4 \sin[2x]}{320} + \frac{31(1 - \cos[2x])^2 \sin[2x]}{1280}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sin[x]^4*Cos[x]^6,x]')``[Out] 3 x / 256 - 21 (1 - Cos[2 x]) ^ 3 Sin[2 x] / 1280 - Sin[2 x] / 128 + Sin[4 x] / 1024 + (1 - Cos[2 x]) ^ 4 Sin[2 x] / 320 + 31 (1 - Cos[2 x]) ^ 2 Sin[2 x] / 1280`**Maple [A]**

time = 0.08, size = 42, normalized size = 0.75

method	result	size
--------	--------	------

risch	$\frac{3x}{256} + \frac{\sin(10x)}{5120} + \frac{\sin(8x)}{2048} - \frac{\sin(6x)}{1024} - \frac{\sin(4x)}{256} + \frac{\sin(2x)}{512}$	35
default	$-\frac{(\cos^7(x))(\sin^3(x))}{10} - \frac{3(\cos^7(x))\sin(x)}{80} + \frac{\left(\cos^5(x) + \frac{5(\cos^3(x))}{4} + \frac{15\cos(x)}{8}\right)\sin(x)}{160} + \frac{3x}{256}$	42

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)^6*sin(x)^4,x,method=_RETURNVERBOSE)`

[Out] $-1/10*\cos(x)^7*\sin(x)^3 - 3/80*\cos(x)^7*\sin(x) + 1/160*(\cos(x)^5 + 5/4*\cos(x)^3 + 5/8*\cos(x))*\sin(x) + 3/256*x$

Maxima [A]

time = 0.26, size = 24, normalized size = 0.43

$$\frac{1}{320} \sin(2x)^5 + \frac{3}{256} x + \frac{1}{2048} \sin(8x) - \frac{1}{256} \sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^6*sin(x)^4,x, algorithm="maxima")`

[Out] $1/320*\sin(2*x)^5 + 3/256*x + 1/2048*\sin(8*x) - 1/256*\sin(4*x)$

Fricas [A]

time = 0.32, size = 37, normalized size = 0.66

$$\frac{1}{1280} (128 \cos(x)^9 - 176 \cos(x)^7 + 8 \cos(x)^5 + 10 \cos(x)^3 + 15 \cos(x)) \sin(x) + \frac{3}{256} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^6*sin(x)^4,x, algorithm="fricas")`

[Out] $1/1280*(128*\cos(x)^9 - 176*\cos(x)^7 + 8*\cos(x)^5 + 10*\cos(x)^3 + 15*\cos(x))*\sin(x) + 3/256*x$

Sympy [A]

time = 0.03, size = 56, normalized size = 1.00

$$\frac{3x}{256} + \frac{\sin(x)\cos^9(x)}{10} - \frac{11\sin(x)\cos^7(x)}{80} + \frac{\sin(x)\cos^5(x)}{160} + \frac{\sin(x)\cos^3(x)}{128} + \frac{3\sin(x)\cos(x)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)**6*sin(x)**4,x)`

[Out] $3*x/256 + \sin(x)*\cos(x)**9/10 - 11*\sin(x)*\cos(x)**7/80 + \sin(x)*\cos(x)**5/160 + \sin(x)*\cos(x)**3/128 + 3*\sin(x)*\cos(x)/256$

Giac [A]

time = 0.00, size = 43, normalized size = 0.77

$$\frac{3}{256}x + \frac{\sin(2x)}{512} - \frac{\sin(4x)}{256} - \frac{\sin(6x)}{1024} + \frac{\sin(8x)}{2048} + \frac{\sin(10x)}{5120}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^6*sin(x)^4,x)**[Out]** 3/256*x + 1/5120*sin(10*x) + 1/2048*sin(8*x) - 1/1024*sin(6*x) - 1/256*sin(4*x) + 1/512*sin(2*x)**Mupad [B]**

time = 0.04, size = 38, normalized size = 0.68

$$\left(\frac{\cos(x)^5}{10} + \frac{\cos(x)^3}{16} + \frac{\cos(x)}{32} \right) \sin(x)^5 + \frac{3x}{256} - \frac{\sin(2x)}{128} + \frac{\sin(4x)}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^6*sin(x)^4,x)**[Out]** (3*x)/256 - sin(2*x)/128 + sin(4*x)/1024 + sin(x)^5*(cos(x)/32 + cos(x)^3/16 + cos(x)^5/10)

3.345 $\int \cos^6(x) \sin^7(x) dx$

Optimal. Leaf size=33

$$-\frac{1}{7} \cos^7(x) + \frac{\cos^9(x)}{3} - \frac{3 \cos^{11}(x)}{11} + \frac{\cos^{13}(x)}{13}$$

[Out] $-1/7*\cos(x)^7+1/3*\cos(x)^9-3/11*\cos(x)^{11}+1/13*\cos(x)^{13}$

Rubi [A]

time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2645, 276}

$$\frac{\cos^{13}(x)}{13} - \frac{3 \cos^{11}(x)}{11} + \frac{\cos^9(x)}{3} - \frac{\cos^7(x)}{7}$$

Antiderivative was successfully verified.

[In] `Int[Cos[x]^6*Sin[x]^7,x]`

[Out] $-1/7*\text{Cos}[x]^7 + \text{Cos}[x]^9/3 - (3*\text{Cos}[x]^{11})/11 + \text{Cos}[x]^{13}/13$

Rule 276

`Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[Exp andIntegrand[(c*x)^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]`

Rule 2645

`Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Cos[e + f*x], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])`

Rubi steps

$$\begin{aligned} \int \cos^6(x) \sin^7(x) dx &= -\text{Subst} \left(\int x^6 (1 - x^2)^3 dx, x, \cos(x) \right) \\ &= -\text{Subst} \left(\int (x^6 - 3x^8 + 3x^{10} - x^{12}) dx, x, \cos(x) \right) \\ &= -\frac{1}{7} \cos^7(x) + \frac{\cos^9(x)}{3} - \frac{3 \cos^{11}(x)}{11} + \frac{\cos^{13}(x)}{13} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 55, normalized size = 1.67

$$-\frac{5 \cos(x)}{1024} - \frac{5 \cos(3x)}{4096} + \frac{3 \cos(5x)}{4096} + \frac{3 \cos(7x)}{14336} - \frac{\cos(9x)}{6144} - \frac{\cos(11x)}{45056} + \frac{\cos(13x)}{53248}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[x]^6*Sin[x]^7,x]`

```
[Out] (-5*Cos[x])/1024 - (5*Cos[3*x])/4096 + (3*Cos[5*x])/4096 + (3*Cos[7*x])/14336 - Cos[9*x]/6144 - Cos[11*x]/45056 + Cos[13*x]/53248
```

Mathics [A]

time = 1.76, size = 26, normalized size = 0.79

$$\frac{(-429 - 819 \cos[x]^4 + 231 \cos[x]^6 + 1001 \cos[x]^2) \cos[x]^7}{3003}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sin[x]^7*cos[x]^6,x]')`

```
[Out] (-429 - 819 Cos[x]^4 + 231 Cos[x]^6 + 1001 Cos[x]^2) Cos[x]^7 / 3003
```

Maple [A]

time = 0.03, size = 38, normalized size = 1.15

method	result	size
default	$-\frac{(\cos^7(x))(\sin^6(x))}{13} - \frac{6(\sin^4(x))(\cos^7(x))}{143} - \frac{8(\sin^2(x))(\cos^7(x))}{429} - \frac{16(\cos^7(x))}{3003}$	38
risch	$-\frac{5 \cos(x)}{1024} + \frac{\cos(13x)}{53248} - \frac{\cos(11x)}{45056} - \frac{\cos(9x)}{6144} + \frac{3 \cos(7x)}{14336} + \frac{3 \cos(5x)}{4096} - \frac{5 \cos(3x)}{4096}$	42

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x)^6*sin(x)^7,x,method=_RETURNVERBOSE)`

```
[Out] -1/13*cos(x)^7*sin(x)^6-6/143*sin(x)^4*cos(x)^7-8/429*sin(x)^2*cos(x)^7-16/3003*cos(x)^7
```

Maxima [A]

time = 0.26, size = 25, normalized size = 0.76

$$\frac{1}{13} \cos(x)^{13} - \frac{3}{11} \cos(x)^{11} + \frac{1}{3} \cos(x)^9 - \frac{1}{7} \cos(x)^7$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)^6*sin(x)^7,x, algorithm="maxima")`

[Out] $1/13*\cos(x)^{13} - 3/11*\cos(x)^{11} + 1/3*\cos(x)^9 - 1/7*\cos(x)^7$

Fricas [A]

time = 0.33, size = 25, normalized size = 0.76

$$\frac{1}{13} \cos(x)^{13} - \frac{3}{11} \cos(x)^{11} + \frac{1}{3} \cos(x)^9 - \frac{1}{7} \cos(x)^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^6*sin(x)^7,x, algorithm="fricas")`

[Out] $1/13*\cos(x)^{13} - 3/11*\cos(x)^{11} + 1/3*\cos(x)^9 - 1/7*\cos(x)^7$

Sympy [A]

time = 0.03, size = 27, normalized size = 0.82

$$\frac{\cos^{13}(x)}{13} - \frac{3 \cos^{11}(x)}{11} + \frac{\cos^9(x)}{3} - \frac{\cos^7(x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)**6*sin(x)**7,x)`

[Out] $\cos(x)**13/13 - 3*\cos(x)**11/11 + \cos(x)**9/3 - \cos(x)**7/7$

Giac [A]

time = 0.00, size = 33, normalized size = 1.00

$$\frac{\cos^{13} x}{13} - \frac{3}{11} \cos^{11} x + \frac{3}{9} \cos^9 x - \frac{\cos^7 x}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^6*sin(x)^7,x)`

[Out] $1/13*\cos(x)^{13} - 3/11*\cos(x)^{11} + 1/3*\cos(x)^9 - 1/7*\cos(x)^7$

Mupad [B]

time = 0.20, size = 25, normalized size = 0.76

$$\frac{\cos(x)^{13}}{13} - \frac{3 \cos(x)^{11}}{11} + \frac{\cos(x)^9}{3} - \frac{\cos(x)^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)^6*sin(x)^7,x)`

[Out] $\cos(x)^9/3 - \cos(x)^7/7 - (3*\cos(x)^{11})/11 + \cos(x)^{13}/13$

3.346 $\int \sin^{10}(x) \tan(x) dx$

Optimal. Leaf size=46

$$\frac{5 \cos^2(x)}{2} - \frac{5 \cos^4(x)}{2} + \frac{5 \cos^6(x)}{3} - \frac{5 \cos^8(x)}{8} + \frac{\cos^{10}(x)}{10} - \log(\cos(x))$$

[Out] 5/2*cos(x)^2-5/2*cos(x)^4+5/3*cos(x)^6-5/8*cos(x)^8+1/10*cos(x)^10-ln(cos(x))

Rubi [A]

time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2670, 272, 45}

$$\frac{\cos^{10}(x)}{10} - \frac{5 \cos^8(x)}{8} + \frac{5 \cos^6(x)}{3} - \frac{5 \cos^4(x)}{2} + \frac{5 \cos^2(x)}{2} - \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^10*Tan[x],x]

[Out] (5*Cos[x]^2)/2 - (5*Cos[x]^4)/2 + (5*Cos[x]^6)/3 - (5*Cos[x]^8)/8 + Cos[x]^10/10 - Log[Cos[x]]

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 2670

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]

Rubi steps

$$\begin{aligned}
\int \sin^{10}(x) \tan(x) dx &= -\text{Subst} \left(\int \frac{(1-x^2)^5}{x} dx, x, \cos(x) \right) \\
&= - \left(\frac{1}{2} \text{Subst} \left(\int \frac{(1-x)^5}{x} dx, x, \cos^2(x) \right) \right) \\
&= - \left(\frac{1}{2} \text{Subst} \left(\int \left(-5 + \frac{1}{x} + 10x - 10x^2 + 5x^3 - x^4 \right) dx, x, \cos^2(x) \right) \right) \\
&= \frac{5 \cos^2(x)}{2} - \frac{5 \cos^4(x)}{2} + \frac{5 \cos^6(x)}{3} - \frac{5 \cos^8(x)}{8} + \frac{\cos^{10}(x)}{10} - \log(\cos(x))
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 46, normalized size = 1.00

$$\frac{5 \cos^2(x)}{2} - \frac{5 \cos^4(x)}{2} + \frac{5 \cos^6(x)}{3} - \frac{5 \cos^8(x)}{8} + \frac{\cos^{10}(x)}{10} - \log(\cos(x))$$

Antiderivative was successfully verified.

`[In] Integrate[Sin[x]^10*Tan[x], x]`

```
[Out] (5*Cos[x]^2)/2 - (5*Cos[x]^4)/2 + (5*Cos[x]^6)/3 - (5*Cos[x]^8)/8 + Cos[x]^10/10 - Log[Cos[x]]
```

Mathics [A]

time = 1.86, size = 36, normalized size = 0.78

$$-\frac{5 \cos^4[x]}{2} - \text{Log}[\cos[x]] - \frac{5 \cos^8[x]}{8} + \frac{\cos^{10}[x]}{10} + \frac{5 \cos^6[x]}{3} + \frac{5 \cos^2[x]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sin[x]^11/Cos[x], x]')`

```
[Out] -5 Cos[x]^4 / 2 - Log[Cos[x]] - 5 Cos[x]^8 / 8 + Cos[x]^10 / 10 + 5 Cos[x]^6 / 3 + 5 Cos[x]^2 / 2
```

Maple [A]

time = 0.04, size = 37, normalized size = 0.80

method	result	size
default	$-\frac{\sin^{10}(x)}{10} - \frac{\sin^8(x)}{8} - \frac{\sin^6(x)}{6} - \frac{\sin^4(x)}{4} - \frac{\sin^2(x)}{2} - \ln(\cos(x))$	37
risch	$ix + \frac{281 e^{2ix}}{1024} + \frac{281 e^{-2ix}}{1024} - \ln(e^{2ix} + 1) + \frac{\cos(10x)}{5120} - \frac{3 \cos(8x)}{1024} + \frac{67 \cos(6x)}{3072} - \frac{29 \cos(4x)}{256}$	54

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)^11/cos(x),x,method=_RETURNVERBOSE)`

[Out] $-1/10*\sin(x)^{10}-1/8*\sin(x)^8-1/6*\sin(x)^6-1/4*\sin(x)^4-1/2*\sin(x)^2-\ln(\cos(x))$

Maxima [A]

time = 0.26, size = 40, normalized size = 0.87

$$-\frac{1}{10} \sin(x)^{10} - \frac{1}{8} \sin(x)^8 - \frac{1}{6} \sin(x)^6 - \frac{1}{4} \sin(x)^4 - \frac{1}{2} \sin(x)^2 - \frac{1}{2} \log(\sin(x)^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)^11/cos(x),x, algorithm="maxima")`

[Out] $-1/10*\sin(x)^{10} - 1/8*\sin(x)^8 - 1/6*\sin(x)^6 - 1/4*\sin(x)^4 - 1/2*\sin(x)^2 - 1/2*\log(\sin(x)^2 - 1)$

Fricas [A]

time = 0.34, size = 38, normalized size = 0.83

$$\frac{1}{10} \cos(x)^{10} - \frac{5}{8} \cos(x)^8 + \frac{5}{3} \cos(x)^6 - \frac{5}{2} \cos(x)^4 + \frac{5}{2} \cos(x)^2 - \log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)^11/cos(x),x, algorithm="fricas")`

[Out] $1/10*\cos(x)^{10} - 5/8*\cos(x)^8 + 5/3*\cos(x)^6 - 5/2*\cos(x)^4 + 5/2*\cos(x)^2 - \log(-\cos(x))$

Sympy [A]

time = 0.05, size = 44, normalized size = 0.96

$$-\log(\cos(x)) + \frac{\cos^{10}(x)}{10} - \frac{5\cos^8(x)}{8} + \frac{5\cos^6(x)}{3} - \frac{5\cos^4(x)}{2} + \frac{5\cos^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)**11/cos(x),x)`

[Out] $-\log(\cos(x)) + \cos(x)**10/10 - 5*\cos(x)**8/8 + 5*\cos(x)**6/3 - 5*\cos(x)**4/2 + 5*\cos(x)**2/2$

Giac [A]

time = 0.00, size = 52, normalized size = 1.13

$$\frac{\frac{16}{5} \sin^{10} x + 4 \sin^8 x + \frac{16}{3} \sin^6 x + 8 \sin^4 x + 16 \sin^2 x}{32} - \frac{\ln(-\sin^2 x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)^11/cos(x),x)`

[Out] $-1/10*\sin(x)^{10} - 1/8*\sin(x)^8 - 1/6*\sin(x)^6 - 1/4*\sin(x)^4 - 1/2*\sin(x)^2 - 1/2*\log(-\sin(x)^2 + 1)$

Mupad [B]

time = 0.04, size = 36, normalized size = 0.78

$$-\frac{\sin(x)^{10}}{10} - \frac{\sin(x)^8}{8} - \frac{\sin(x)^6}{6} - \frac{\sin(x)^4}{4} - \frac{\sin(x)^2}{2} - \ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)^11/cos(x),x)`

[Out] $-\log(\cos(x)) - \sin(x)^2/2 - \sin(x)^4/4 - \sin(x)^6/6 - \sin(x)^8/8 - \sin(x)^{10}/10$

3.347 $\int \csc^6(x) \sec^6(x) dx$

Optimal. Leaf size=41

$$-10 \cot(x) - \frac{5 \cot^3(x)}{3} - \frac{\cot^5(x)}{5} + 10 \tan(x) + \frac{5 \tan^3(x)}{3} + \frac{\tan^5(x)}{5}$$

[Out] -10*cot(x)-5/3*cot(x)^3-1/5*cot(x)^5+10*tan(x)+5/3*tan(x)^3+1/5*tan(x)^5

Rubi [A]

time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2700, 276}

$$\frac{\tan^5(x)}{5} + \frac{5 \tan^3(x)}{3} + 10 \tan(x) - \frac{1}{5} \cot^5(x) - \frac{5 \cot^3(x)}{3} - 10 \cot(x)$$

Antiderivative was successfully verified.

[In] Int[Csc[x]^6*Sec[x]^6,x]

[Out] -10*Cot[x] - (5*Cot[x]^3)/3 - Cot[x]^5/5 + 10*Tan[x] + (5*Tan[x]^3)/3 + Tan[x]^5/5

Rule 276

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2700

Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]

Rubi steps

$$\begin{aligned} \int \csc^6(x) \sec^6(x) dx &= \text{Subst} \left(\int \frac{(1+x^2)^5}{x^6} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \left(10 + \frac{1}{x^6} + \frac{5}{x^4} + \frac{10}{x^2} + 5x^2 + x^4 \right) dx, x, \tan(x) \right) \\ &= -10 \cot(x) - \frac{5 \cot^3(x)}{3} - \frac{\cot^5(x)}{5} + 10 \tan(x) + \frac{5 \tan^3(x)}{3} + \frac{\tan^5(x)}{5} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 53, normalized size = 1.29

$$-\frac{128 \cot(x)}{15} - \frac{19}{15} \cot(x) \csc^2(x) - \frac{1}{5} \cot(x) \csc^4(x) + \frac{128 \tan(x)}{15} + \frac{19}{15} \sec^2(x) \tan(x) + \frac{1}{5} \sec^4(x) \tan(x)$$

Antiderivative was successfully verified.

`[In] Integrate[Csc[x]^6*Sec[x]^6,x]`

```
[Out] (-128*Cot[x])/15 - (19*Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5 + (128*Tan[x])/15 + (19*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

Mathics [A]

time = 2.04, size = 39, normalized size = 0.95

$$\frac{32(-32 \cos[x]^3 \sin[x]^3 - 8 \sin[2x]^5 - 3 \sin[2x])}{15 \sin[2x]^5 \tan[2x]}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(Sin[x]^6*Cos[x]^6),x]')`

```
[Out] 32(-32 Cos[x]^3 Sin[x]^3 - 8 Sin[2 x]^5 - 3 Sin[2 x]) / (15 Sin[2 x]^5 Tan[2 x])
```

Maple [A]

time = 0.08, size = 56, normalized size = 1.37

method	result	size
risch	$-\frac{512i(10e^{8ix}-5e^{4ix}+1)}{15(e^{2ix}-1)^5(e^{2ix}+1)^5}$	38
default	$\frac{1}{5 \sin(x)^5 \cos(x)^5} - \frac{2}{5 \sin(x)^5 \cos(x)^3} + \frac{16}{15 \sin(x)^3 \cos(x)^3} - \frac{32}{15 \sin(x)^3 \cos(x)} + \frac{128}{15 \cos(x) \sin(x)} - \frac{256 \cot(x)}{15}$	56

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/cos(x)^6/sin(x)^6,x,method=_RETURNVERBOSE)`

```
[Out] 1/5/sin(x)^5/cos(x)^5-2/5/sin(x)^5/cos(x)^3+16/15/sin(x)^3/cos(x)^3-32/15/sin(x)^3/cos(x)+128/15/cos(x)/sin(x)-256/15*cot(x)
```

Maxima [A]

time = 0.26, size = 37, normalized size = 0.90

$$\frac{1}{5} \tan(x)^5 + \frac{5}{3} \tan(x)^3 - \frac{150 \tan(x)^4 + 25 \tan(x)^2 + 3}{15 \tan(x)^5} + 10 \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(x)^6/sin(x)^6,x, algorithm="maxima")

[Out] $1/5*\tan(x)^5 + 5/3*\tan(x)^3 - 1/15*(150*\tan(x)^4 + 25*\tan(x)^2 + 3)/\tan(x)^5 + 10*\tan(x)$

Fricas [A]

time = 0.32, size = 55, normalized size = 1.34

$$-\frac{256 \cos(x)^{10} - 640 \cos(x)^8 + 480 \cos(x)^6 - 80 \cos(x)^4 - 10 \cos(x)^2 - 3}{15 (\cos(x)^9 - 2 \cos(x)^7 + \cos(x)^5) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(x)^6/sin(x)^6,x, algorithm="fricas")

[Out] $-1/15*(256*\cos(x)^{10} - 640*\cos(x)^8 + 480*\cos(x)^6 - 80*\cos(x)^4 - 10*\cos(x)^2 - 3)/((\cos(x)^9 - 2*\cos(x)^7 + \cos(x)^5)*\sin(x))$

Sympy [A]

time = 0.03, size = 44, normalized size = 1.07

$$-\frac{256 \cos(2x)}{15 \sin(2x)} - \frac{128 \cos(2x)}{15 \sin^3(2x)} - \frac{32 \cos(2x)}{5 \sin^5(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(x)**6/sin(x)**6,x)

[Out] $-256*\cos(2*x)/(15*\sin(2*x)) - 128*\cos(2*x)/(15*\sin(2*x)**3) - 32*\cos(2*x)/(5*\sin(2*x)**5)$

Giac [A]

time = 0.00, size = 31, normalized size = 0.76

$$\frac{2(-240 \tan^4(2x) - 160 \tan^2(2x) - 48)}{15 \tan^5(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(x)^6/sin(x)^6,x)

[Out] $-32/15*(15*\tan(2*x)^4 + 10*\tan(2*x)^2 + 3)/\tan(2*x)^5$

Mupad [B]

time = 0.12, size = 27, normalized size = 0.66

$$-\frac{32 \left(\frac{\cos(2x)}{3} - \frac{\cos(6x)}{6} + \frac{\cos(10x)}{30} \right)}{\sin(2x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^6*sin(x)^6),x)

[Out] $-(32*(\cos(2*x)/3 - \cos(6*x)/6 + \cos(10*x)/30))/\sin(2*x)^5$

3.348 $\int \cos^2(x) \sin^2(x) dx$

Optimal. Leaf size=24

$$\frac{x}{8} + \frac{1}{8} \cos(x) \sin(x) - \frac{1}{4} \cos^3(x) \sin(x)$$

[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)

Rubi [A]

time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2648, 2715, 8}

$$\frac{x}{8} - \frac{1}{4} \sin(x) \cos^3(x) + \frac{1}{8} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2*Sin[x]^2,x]

[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2648

Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^n_]*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m - 1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^n_, x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
 \int \cos^2(x) \sin^2(x) dx &= -\frac{1}{4} \cos^3(x) \sin(x) + \frac{1}{4} \int \cos^2(x) dx \\
 &= \frac{1}{8} \cos(x) \sin(x) - \frac{1}{4} \cos^3(x) \sin(x) + \frac{\int 1 dx}{8} \\
 &= \frac{x}{8} + \frac{1}{8} \cos(x) \sin(x) - \frac{1}{4} \cos^3(x) \sin(x)
 \end{aligned}$$

Mathematica [A]

time = 0.00, size = 14, normalized size = 0.58

$$\frac{x}{8} - \frac{1}{32} \sin(4x)$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[x]^2*Sin[x]^2,x]``[Out] x/8 - Sin[4*x]/32`**Mathics [A]**

time = 1.65, size = 10, normalized size = 0.42

$$\frac{x}{8} - \frac{\text{Sin}[4x]}{32}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sin[x]^2*Cos[x]^2,x]')``[Out] x / 8 - Sin[4 x] / 32`**Maple [A]**

time = 0.03, size = 19, normalized size = 0.79

method	result	size
risch	$\frac{x}{8} - \frac{\sin(4x)}{32}$	11
default	$\frac{x}{8} + \frac{\cos(x) \sin(x)}{8} - \frac{(\cos^3(x) \sin(x))}{4}$	19
norman	$\frac{x}{8} + \frac{7(\tan^3(\frac{x}{2}))}{4} - \frac{7(\tan^5(\frac{x}{2}))}{4} + \frac{(\tan^7(\frac{x}{2}))}{4} + \frac{x(\tan^2(\frac{x}{2}))}{2} + \frac{3x(\tan^4(\frac{x}{2}))}{4} + \frac{x(\tan^6(\frac{x}{2}))}{2} + \frac{x(\tan^8(\frac{x}{2}))}{8} - \frac{\tan(\frac{x}{2})}{4}$ $(1+\tan^2(\frac{x}{2}))^4$	82

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x)^2*sin(x)^2,x,method=_RETURNVERBOSE)``[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)`

Maxima [A]

time = 0.25, size = 10, normalized size = 0.42

$$\frac{1}{8}x - \frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")``[Out] 1/8*x - 1/32*sin(4*x)`**Fricas [A]**

time = 0.31, size = 19, normalized size = 0.79

$$-\frac{1}{8}(2\cos(x)^3 - \cos(x))\sin(x) + \frac{1}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")``[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x`**Sympy [A]**

time = 0.03, size = 14, normalized size = 0.58

$$\frac{x}{8} - \frac{\sin(2x)\cos(2x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)**2*sin(x)**2,x)``[Out] x/8 - sin(2*x)*cos(2*x)/16`**Giac [A]**

time = 0.00, size = 13, normalized size = 0.54

$$\frac{x}{8} - \frac{\sin(4x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)^2*sin(x)^2,x)``[Out] 1/8*x - 1/32*sin(4*x)`**Mupad [B]**

time = 0.04, size = 18, normalized size = 0.75

$$\frac{\cos(x)\sin(x)^3}{4} - \frac{\cos(x)\sin(x)}{8} + \frac{x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x)^2*sin(x)^2,x)``[Out] x/8 - (cos(x)*sin(x))/8 + (cos(x)*sin(x)^3)/4`

3.349 $\int \cos^4(x) \sin^4(x) dx$

Optimal. Leaf size=46

$$\frac{3x}{128} + \frac{3}{128} \cos(x) \sin(x) + \frac{1}{64} \cos^3(x) \sin(x) - \frac{1}{16} \cos^5(x) \sin(x) - \frac{1}{8} \cos^5(x) \sin^3(x)$$

[Out] 3/128*x+3/128*cos(x)*sin(x)+1/64*cos(x)^3*sin(x)-1/16*cos(x)^5*sin(x)-1/8*cos(x)^5*sin(x)^3

Rubi [A]

time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2648, 2715, 8}

$$\frac{3x}{128} - \frac{1}{8} \sin^3(x) \cos^5(x) - \frac{1}{16} \sin(x) \cos^5(x) + \frac{1}{64} \sin(x) \cos^3(x) + \frac{3}{128} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^4*Sin[x]^4,x]

[Out] (3*x)/128 + (3*Cos[x]*Sin[x])/128 + (Cos[x]^3*Sin[x])/64 - (Cos[x]^5*Sin[x])/16 - (Cos[x]^5*Sin[x]^3)/8

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2648

Int[(cos[(e_) + (f_)*(x_)]*(b_.))^n_)*((a_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m - 1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^n_, x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
\int \cos^4(x) \sin^4(x) dx &= -\frac{1}{8} \cos^5(x) \sin^3(x) + \frac{3}{8} \int \cos^4(x) \sin^2(x) dx \\
&= -\frac{1}{16} \cos^5(x) \sin(x) - \frac{1}{8} \cos^5(x) \sin^3(x) + \frac{1}{16} \int \cos^4(x) dx \\
&= \frac{1}{64} \cos^3(x) \sin(x) - \frac{1}{16} \cos^5(x) \sin(x) - \frac{1}{8} \cos^5(x) \sin^3(x) + \frac{3}{64} \int \cos^2(x) dx \\
&= \frac{3}{128} \cos(x) \sin(x) + \frac{1}{64} \cos^3(x) \sin(x) - \frac{1}{16} \cos^5(x) \sin(x) - \frac{1}{8} \cos^5(x) \sin^3(x) + \frac{3}{128} \int 1 dx \\
&= \frac{3x}{128} + \frac{3}{128} \cos(x) \sin(x) + \frac{1}{64} \cos^3(x) \sin(x) - \frac{1}{16} \cos^5(x) \sin(x) - \frac{1}{8} \cos^5(x) \sin^3(x)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 22, normalized size = 0.48

$$\frac{3x}{128} - \frac{1}{128} \sin(4x) + \frac{\sin(8x)}{1024}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[x]^4*Sin[x]^4,x]``[Out] (3*x)/128 - Sin[4*x]/128 + Sin[8*x]/1024`**Mathics [A]**

time = 1.79, size = 22, normalized size = 0.48

$$\frac{3x}{128} - \frac{\cos[2x] \sin[2x]^3}{128} - \frac{3 \sin[4x]}{512}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sin[x]^4*Cos[x]^4,x]')``[Out] 3 x / 128 - Cos[2 x] Sin[2 x]^3 / 128 - 3 Sin[4 x] / 512`**Maple [A]**

time = 0.08, size = 36, normalized size = 0.78

method	result
risch	$\frac{3x}{128} + \frac{\sin(8x)}{1024} - \frac{\sin(4x)}{128}$
default	$-\frac{(\cos^5(x))(\sin^3(x))}{8} - \frac{(\cos^5(x)) \sin(x)}{16} + \frac{(\cos^3(x) + \frac{3 \cos(x)}{2}) \sin(x)}{64} + \frac{3x}{128}$

norman	$\frac{3x}{128} - \frac{3 \tan(\frac{x}{2})}{64} + \frac{21x(\tan^4(\frac{x}{2}))}{32} + \frac{3x(\tan^2(\frac{x}{2}))}{16} - \frac{23(\tan^3(\frac{x}{2}))}{64} + \frac{3x(\tan^{14}(\frac{x}{2}))}{16} + \frac{3x(\tan^{16}(\frac{x}{2}))}{128} + \frac{21x(\tan^6(\frac{x}{2}))}{16} + \frac{105x(\tan^8(\frac{x}{2}))}{64} + \frac{33x}{(1+\tan^2(\frac{x}{2}))}$
--------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)^4*sin(x)^4,x,method=_RETURNVERBOSE)`

[Out] `-1/8*cos(x)^5*sin(x)^3-1/16*cos(x)^5*sin(x)+1/64*(cos(x)^3+3/2*cos(x))*sin(x)+3/128*x`

Maxima [A]

time = 0.26, size = 16, normalized size = 0.35

$$\frac{3}{128}x + \frac{1}{1024}\sin(8x) - \frac{1}{128}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^4*sin(x)^4,x, algorithm="maxima")`

[Out] `3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)`

Fricas [A]

time = 0.31, size = 31, normalized size = 0.67

$$\frac{1}{128}(16 \cos(x)^7 - 24 \cos(x)^5 + 2 \cos(x)^3 + 3 \cos(x)) \sin(x) + \frac{3}{128}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^4*sin(x)^4,x, algorithm="fricas")`

[Out] `1/128*(16*cos(x)^7 - 24*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 3/128*x`

Sympy [A]

time = 0.03, size = 31, normalized size = 0.67

$$\frac{3x}{128} - \frac{\sin^3(2x)\cos(2x)}{128} - \frac{3\sin(2x)\cos(2x)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)**4*sin(x)**4,x)`

[Out] `3*x/128 - sin(2*x)**3*cos(2*x)/128 - 3*sin(2*x)*cos(2*x)/256`

Giac [A]

time = 0.00, size = 21, normalized size = 0.46

$$\frac{3}{128}x - \frac{\sin(4x)}{128} + \frac{\sin(8x)}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^4*sin(x)^4,x)`

[Out] `3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)`

Mupad [B]

time = 0.04, size = 32, normalized size = 0.70

$$\left(\frac{\cos(x)^3}{8} + \frac{\cos(x)}{16} \right) \sin(x)^5 + \frac{3x}{128} - \frac{\sin(2x)}{64} + \frac{\sin(4x)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)^4*sin(x)^4,x)`

[Out] `(3*x)/128 - sin(2*x)/64 + sin(4*x)/512 + sin(x)^5*(cos(x)/16 + cos(x)^3/8)`

3.350 $\int \cos^6(x) \sin^6(x) dx$

Optimal. Leaf size=68

$$\frac{5x}{1024} + \frac{5 \cos(x) \sin(x)}{1024} + \frac{5 \cos^3(x) \sin(x)}{1536} + \frac{1}{384} \cos^5(x) \sin(x) - \frac{1}{64} \cos^7(x) \sin(x) - \frac{1}{24} \cos^7(x) \sin^3(x) - \frac{1}{12} \cos^7(x) \sin^5(x)$$

[Out] 5/1024*x+5/1024*cos(x)*sin(x)+5/1536*cos(x)^3*sin(x)+1/384*cos(x)^5*sin(x)-1/64*cos(x)^7*sin(x)-1/24*cos(x)^7*sin(x)^3-1/12*cos(x)^7*sin(x)^5

Rubi [A]

time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2648, 2715, 8}

$$\frac{5x}{1024} - \frac{1}{12} \sin^5(x) \cos^7(x) - \frac{1}{24} \sin^3(x) \cos^7(x) - \frac{1}{64} \sin(x) \cos^7(x) + \frac{1}{384} \sin(x) \cos^5(x) + \frac{5 \sin(x) \cos^3(x)}{1536} + \frac{5 \sin(x) \cos(x)}{1024}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^6*Sin[x]^6,x]

[Out] (5*x)/1024 + (5*Cos[x]*Sin[x])/1024 + (5*Cos[x]^3*Sin[x])/1536 + (Cos[x]^5*Sin[x])/384 - (Cos[x]^7*Sin[x])/64 - (Cos[x]^7*Sin[x]^3)/24 - (Cos[x]^7*Sin[x]^5)/12

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2648

Int[(cos[(e_.) + (f_.)*(x_.)]*(b_.))^n]*((a_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m - 1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^n, x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
\int \cos^6(x) \sin^6(x) dx &= -\frac{1}{12} \cos^7(x) \sin^5(x) + \frac{5}{12} \int \cos^6(x) \sin^4(x) dx \\
&= -\frac{1}{24} \cos^7(x) \sin^3(x) - \frac{1}{12} \cos^7(x) \sin^5(x) + \frac{1}{8} \int \cos^6(x) \sin^2(x) dx \\
&= -\frac{1}{64} \cos^7(x) \sin(x) - \frac{1}{24} \cos^7(x) \sin^3(x) - \frac{1}{12} \cos^7(x) \sin^5(x) + \frac{1}{64} \int \cos^6(x) dx \\
&= \frac{1}{384} \cos^5(x) \sin(x) - \frac{1}{64} \cos^7(x) \sin(x) - \frac{1}{24} \cos^7(x) \sin^3(x) - \frac{1}{12} \cos^7(x) \sin^5(x) + \frac{5}{384} \int \cos^4(x) dx \\
&= \frac{5 \cos^3(x) \sin(x)}{1536} + \frac{1}{384} \cos^5(x) \sin(x) - \frac{1}{64} \cos^7(x) \sin(x) - \frac{1}{24} \cos^7(x) \sin^3(x) - \frac{1}{12} \cos^7(x) \sin^5(x) + \frac{5}{384} \int \cos^2(x) dx \\
&= \frac{5 \cos(x) \sin(x)}{1024} + \frac{5 \cos^3(x) \sin(x)}{1536} + \frac{1}{384} \cos^5(x) \sin(x) - \frac{1}{64} \cos^7(x) \sin(x) - \frac{1}{24} \cos^7(x) \sin^3(x) - \frac{1}{12} \cos^7(x) \sin^5(x) \\
&= \frac{5x}{1024} + \frac{5 \cos(x) \sin(x)}{1024} + \frac{5 \cos^3(x) \sin(x)}{1536} + \frac{1}{384} \cos^5(x) \sin(x) - \frac{1}{64} \cos^7(x) \sin(x) - \frac{1}{24} \cos^7(x) \sin^3(x) - \frac{1}{12} \cos^7(x) \sin^5(x)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 30, normalized size = 0.44

$$\frac{5x}{1024} - \frac{15 \sin(4x)}{8192} + \frac{3 \sin(8x)}{8192} - \frac{\sin(12x)}{24576}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[x]^6*Sin[x]^6,x]``[Out] (5*x)/1024 - (15*Sin[4*x])/8192 + (3*Sin[8*x])/8192 - Sin[12*x]/24576`**Mathics [A]**

time = 1.95, size = 34, normalized size = 0.50

$$\frac{5x}{1024} - \frac{5 \cos[2x] \sin[2x]^3}{3072} - \frac{\cos[2x] \sin[2x]^5}{768} - \frac{5 \sin[4x]}{4096}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sin[x]^6*Cos[x]^6,x]')``[Out] 5 x / 1024 - 5 Cos[2 x] Sin[2 x] ^ 3 / 3072 - Cos[2 x] Sin[2 x] ^ 5 / 768 - 5 Sin[4 x] / 4096`**Maple [A]**

time = 0.08, size = 52, normalized size = 0.76

method	result	size

risch	$\frac{5x}{1024} - \frac{\sin(12x)}{24576} + \frac{3\sin(8x)}{8192} - \frac{15\sin(4x)}{8192}$	23
default	$-\frac{(\cos^7(x))(\sin^5(x))}{12} - \frac{(\cos^7(x))(\sin^3(x))}{24} - \frac{(\cos^7(x))\sin(x)}{64} + \frac{\left(\cos^5(x) + \frac{5(\cos^3(x))}{4} + \frac{15\cos(x)}{8}\right)\sin(x)}{384} + \frac{5x}{1024}$	52

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)^6*sin(x)^6,x,method=_RETURNVERBOSE)`

[Out] $-1/12*\cos(x)^7*\sin(x)^5-1/24*\cos(x)^7*\sin(x)^3-1/64*\cos(x)^7*\sin(x)+1/384*(\cos(x)^5+5/4*\cos(x)^3+15/8*\cos(x))*\sin(x)+5/1024*x$

Maxima [A]

time = 0.27, size = 24, normalized size = 0.35

$$\frac{1}{6144} \sin(4x)^3 + \frac{5}{1024} x + \frac{3}{8192} \sin(8x) - \frac{1}{512} \sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^6*sin(x)^6,x, algorithm="maxima")`

[Out] $1/6144*\sin(4*x)^3 + 5/1024*x + 3/8192*\sin(8*x) - 1/512*\sin(4*x)$

Fricas [A]

time = 0.31, size = 43, normalized size = 0.63

$$-\frac{1}{3072} (256 \cos(x)^{11} - 640 \cos(x)^9 + 432 \cos(x)^7 - 8 \cos(x)^5 - 10 \cos(x)^3 - 15 \cos(x)) \sin(x) + \frac{5}{1024} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^6*sin(x)^6,x, algorithm="fricas")`

[Out] $-1/3072*(256*\cos(x)^{11} - 640*\cos(x)^9 + 432*\cos(x)^7 - 8*\cos(x)^5 - 10*\cos(x)^3 - 15*\cos(x))*\sin(x) + 5/1024*x$

Sympy [A]

time = 0.03, size = 46, normalized size = 0.68

$$\frac{5x}{1024} - \frac{\sin^5(2x)\cos(2x)}{768} - \frac{5\sin^3(2x)\cos(2x)}{3072} - \frac{5\sin(2x)\cos(2x)}{2048}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)**6*sin(x)**6,x)`

[Out] $5*x/1024 - \sin(2*x)**5*\cos(2*x)/768 - 5*\sin(2*x)**3*\cos(2*x)/3072 - 5*\sin(2*x)*\cos(2*x)/2048$

Giac [A]

time = 0.00, size = 32, normalized size = 0.47

$$\frac{5}{1024}x - \frac{15}{8192}\sin(4x) + \frac{3}{8192}\sin(8x) - \frac{\sin(12x)}{24576}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^6*sin(x)^6,x)

[Out] 5/1024*x - 1/24576*sin(12*x) + 3/8192*sin(8*x) - 15/8192*sin(4*x)

Mupad [B]

time = 0.03, size = 44, normalized size = 0.65

$$\left(\frac{\cos(x)^5}{12} + \frac{\cos(x)^3}{24} + \frac{\cos(x)}{64}\right)\sin(x)^7 + \frac{5x}{1024} - \frac{15\sin(2x)}{4096} + \frac{3\sin(4x)}{4096} - \frac{\sin(6x)}{12288}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^6*sin(x)^6,x)

[Out] (5*x)/1024 - (15*sin(2*x))/4096 + (3*sin(4*x))/4096 - sin(6*x)/12288 + sin(x)^7*(cos(x)/64 + cos(x)^3/24 + cos(x)^5/12)

3.351 $\int \cos^8(x) \sin^8(x) dx$

Optimal. Leaf size=90

$$\frac{35x}{32768} + \frac{35 \cos(x) \sin(x)}{32768} + \frac{35 \cos^3(x) \sin(x)}{49152} + \frac{7 \cos^5(x) \sin(x)}{12288} + \frac{\cos^7(x) \sin(x)}{2048} - \frac{1}{256} \cos^9(x) \sin(x) - \frac{5}{384} \cos^9(x)$$

[Out] 35/32768*x+35/32768*cos(x)*sin(x)+35/49152*cos(x)^3*sin(x)+7/12288*cos(x)^5*sin(x)+1/2048*cos(x)^7*sin(x)-1/256*cos(x)^9*sin(x)-5/384*cos(x)^9*sin(x)^3-1/32*cos(x)^9*sin(x)^5-1/16*cos(x)^9*sin(x)^7

Rubi [A]

time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2648, 2715, 8}

$$\frac{35x}{32768} - \frac{1}{16} \sin^7(x) \cos^9(x) - \frac{1}{32} \sin^5(x) \cos^9(x) - \frac{5}{384} \sin^3(x) \cos^9(x) - \frac{1}{256} \sin(x) \cos^9(x) + \frac{\sin(x) \cos^7(x)}{2048} + \frac{7 \sin(x) \cos^5(x)}{12288} + \frac{35 \sin(x) \cos^3(x)}{49152} + \frac{35 \sin(x) \cos(x)}{32768}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^8*Sin[x]^8,x]

[Out] (35*x)/32768 + (35*Cos[x]*Sin[x])/32768 + (35*Cos[x]^3*Sin[x])/49152 + (7*Cos[x]^5*Sin[x])/12288 + (Cos[x]^7*Sin[x])/2048 - (Cos[x]^9*Sin[x])/256 - (5*Cos[x]^9*Sin[x]^3)/384 - (Cos[x]^9*Sin[x]^5)/32 - (Cos[x]^9*Sin[x]^7)/16

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2648

Int[(cos[(e_.) + (f_.)*(x_.)]*(b_.))^n]*((a_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m - 1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^n, x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
\int \cos^8(x) \sin^8(x) dx &= -\frac{1}{16} \cos^9(x) \sin^7(x) + \frac{7}{16} \int \cos^8(x) \sin^6(x) dx \\
&= -\frac{1}{32} \cos^9(x) \sin^5(x) - \frac{1}{16} \cos^9(x) \sin^7(x) + \frac{5}{32} \int \cos^8(x) \sin^4(x) dx \\
&= -\frac{5}{384} \cos^9(x) \sin^3(x) - \frac{1}{32} \cos^9(x) \sin^5(x) - \frac{1}{16} \cos^9(x) \sin^7(x) + \frac{5}{128} \int \cos^8(x) \sin^2(x) dx \\
&= -\frac{1}{256} \cos^9(x) \sin(x) - \frac{5}{384} \cos^9(x) \sin^3(x) - \frac{1}{32} \cos^9(x) \sin^5(x) - \frac{1}{16} \cos^9(x) \sin^7(x) \\
&= \frac{\cos^7(x) \sin(x)}{2048} - \frac{1}{256} \cos^9(x) \sin(x) - \frac{5}{384} \cos^9(x) \sin^3(x) - \frac{1}{32} \cos^9(x) \sin^5(x) - \frac{1}{16} \cos^9(x) \sin^7(x) \\
&= \frac{7 \cos^5(x) \sin(x)}{12288} + \frac{\cos^7(x) \sin(x)}{2048} - \frac{1}{256} \cos^9(x) \sin(x) - \frac{5}{384} \cos^9(x) \sin^3(x) - \frac{1}{32} \cos^9(x) \sin^5(x) \\
&= \frac{35 \cos^3(x) \sin(x)}{49152} + \frac{7 \cos^5(x) \sin(x)}{12288} + \frac{\cos^7(x) \sin(x)}{2048} - \frac{1}{256} \cos^9(x) \sin(x) - \frac{5}{384} \cos^9(x) \sin^3(x) \\
&= \frac{35 \cos(x) \sin(x)}{32768} + \frac{35 \cos^3(x) \sin(x)}{49152} + \frac{7 \cos^5(x) \sin(x)}{12288} + \frac{\cos^7(x) \sin(x)}{2048} - \frac{1}{256} \cos^9(x) \sin(x) \\
&= \frac{35x}{32768} + \frac{35 \cos(x) \sin(x)}{32768} + \frac{35 \cos^3(x) \sin(x)}{49152} + \frac{7 \cos^5(x) \sin(x)}{12288} + \frac{\cos^7(x) \sin(x)}{2048} - \frac{1}{256} \cos^9(x) \sin(x)
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 38, normalized size = 0.42

$$\frac{35x}{32768} - \frac{7 \sin(4x)}{16384} + \frac{7 \sin(8x)}{65536} - \frac{\sin(12x)}{49152} + \frac{\sin(16x)}{524288}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[x]^8*Sin[x]^8,x]``[Out] (35*x)/32768 - (7*Sin[4*x])/16384 + (7*Sin[8*x])/65536 - Sin[12*x]/49152 + Sin[16*x]/524288`**Mathics [A]**

time = 2.09, size = 46, normalized size = 0.51

$$\frac{35x}{32768} - \frac{35 \cos[2x] \sin[2x]^3}{98304} - \frac{7 \cos[2x] \sin[2x]^5}{24576} - \frac{35 \sin[4x]}{131072} - \frac{\cos[2x] \sin[2x]^7}{4096}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sin[x]^8*Cos[x]^8,x]')``[Out] 35 x / 32768 - 35 Cos[2 x] Sin[2 x]^3 / 98304 - 7 Cos[2 x] Sin[2 x]^5 / 24576 - 35 Sin[4 x] / 131072 - Cos[2 x] Sin[2 x]^7 / 4096`

Maple [A]

time = 0.08, size = 68, normalized size = 0.76

method	result
risch	$\frac{35x}{32768} + \frac{\sin(16x)}{524288} - \frac{\sin(12x)}{49152} + \frac{7\sin(8x)}{65536} - \frac{7\sin(4x)}{16384}$
default	$-\frac{(\cos^9(x))(\sin^7(x))}{16} - \frac{(\cos^9(x))(\sin^5(x))}{32} - \frac{5(\cos^9(x))(\sin^3(x))}{384} - \frac{(\cos^9(x))\sin(x)}{256} + \frac{\left(\cos^7(x) + \frac{7(\cos^5(x))}{6} + \frac{35(\cos^3(x))}{24}\right)}{2048}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x)^8*sin(x)^8,x,method=_RETURNVERBOSE)`

```
[Out] -1/16*cos(x)^9*sin(x)^7-1/32*cos(x)^9*sin(x)^5-5/384*cos(x)^9*sin(x)^3-1/25
6*cos(x)^9*sin(x)+1/2048*(cos(x)^7+7/6*cos(x)^5+35/24*cos(x)^3+35/16*cos(x)
)*sin(x)+35/32768*x
```

Maxima [A]

time = 0.27, size = 30, normalized size = 0.33

$$\frac{1}{12288} \sin(4x)^3 + \frac{35}{32768} x + \frac{1}{524288} \sin(16x) + \frac{7}{65536} \sin(8x) - \frac{1}{2048} \sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)^8*sin(x)^8,x, algorithm="maxima")`

```
[Out] 1/12288*sin(4*x)^3 + 35/32768*x + 1/524288*sin(16*x) + 7/65536*sin(8*x) - 1
/2048*sin(4*x)
```

Fricas [A]

time = 0.34, size = 55, normalized size = 0.61

$$\frac{1}{98304} (6144 \cos(x)^{15} - 21504 \cos(x)^{13} + 25856 \cos(x)^{11} - 10880 \cos(x)^9 + 48 \cos(x)^7 + 56 \cos(x)^5 + 70 \cos(x)^3 + 105 \cos(x)) \sin(x) + \frac{35}{32768} x$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)^8*sin(x)^8,x, algorithm="fricas")`

```
[Out] 1/98304*(6144*cos(x)^15 - 21504*cos(x)^13 + 25856*cos(x)^11 - 10880*cos(x)^
9 + 48*cos(x)^7 + 56*cos(x)^5 + 70*cos(x)^3 + 105*cos(x))*sin(x) + 35/32768
*x
```

Sympy [A]

time = 0.03, size = 61, normalized size = 0.68

$$\frac{35x}{32768} - \frac{\sin^7(2x) \cos(2x)}{4096} - \frac{7 \sin^5(2x) \cos(2x)}{24576} - \frac{35 \sin^3(2x) \cos(2x)}{98304} - \frac{35 \sin(2x) \cos(2x)}{65536}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**8*sin(x)**8,x)

[Out] 35*x/32768 - sin(2*x)**7*cos(2*x)/4096 - 7*sin(2*x)**5*cos(2*x)/24576 - 35*sin(2*x)**3*cos(2*x)/98304 - 35*sin(2*x)*cos(2*x)/65536

Giac [A]

time = 0.00, size = 39, normalized size = 0.43

$$\frac{35}{32768}x - \frac{7}{16384}\sin(4x) + \frac{7}{65536}\sin(8x) - \frac{\sin(12x)}{49152} + \frac{\sin(16x)}{524288}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^8*sin(x)^8,x)

[Out] 35/32768*x + 1/524288*sin(16*x) - 1/49152*sin(12*x) + 7/65536*sin(8*x) - 7/16384*sin(4*x)

Mupad [B]

time = 0.04, size = 56, normalized size = 0.62

$$\left(\frac{\cos(x)^7}{16} + \frac{\cos(x)^5}{32} + \frac{5\cos(x)^3}{384} + \frac{\cos(x)}{256}\right)\sin(x)^9 + \frac{35x}{32768} - \frac{7\sin(2x)}{8192} + \frac{7\sin(4x)}{32768} - \frac{\sin(6x)}{24576} + \frac{\sin(8x)}{262144}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^8*sin(x)^8,x)

[Out] (35*x)/32768 - (7*sin(2*x))/8192 + (7*sin(4*x))/32768 - sin(6*x)/24576 + sin(8*x)/262144 + sin(x)^9*(cos(x)/256 + (5*cos(x)^3)/384 + cos(x)^5/32 + cos(x)^7/16)

3.352 $\int \cos^{2m}(x) \sin^{2m}(x) dx$

Optimal. Leaf size=68

$$\frac{\cos^{-1+2m}(x) \cos^2(x)^{\frac{1}{2}-m} {}_2F_1\left(\frac{1}{2}(1-2m), \frac{1}{2}(1+2m); \frac{1}{2}(3+2m); \sin^2(x)\right) \sin^{1+2m}(x)}{1+2m}$$

[Out] $\cos(x)^{-1+2*m}*(\cos(x)^2)^{(1/2-m)}*\text{hypergeom}([1/2+m, 1/2-m], [3/2+m], \sin(x)^2)*\sin(x)^{(1+2*m)}/(1+2*m)$

Rubi [A]

time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2657}

$$\frac{\sin^{2m+1}(x) \cos^{2m-1}(x) \cos^2(x)^{\frac{1}{2}-m} {}_2F_1\left(\frac{1}{2}(1-2m), \frac{1}{2}(2m+1); \frac{1}{2}(2m+3); \sin^2(x)\right)}{2m+1}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[x]^{(2*m)}*\text{Sin}[x]^{(2*m)}, x]$

[Out] $(\text{Cos}[x]^{-1+2*m}*(\text{Cos}[x]^2)^{(1/2-m)}*\text{Hypergeometric2F1}[(1-2*m)/2, (1+2*m)/2, (3+2*m)/2, \text{Sin}[x]^2]*\text{Sin}[x]^{(1+2*m)})/(1+2*m)$

Rule 2657

$\text{Int}[(\cos[(e_.) + (f_.)*(x_.)]*(b_.)^{(n_.)}*((a_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}, x_Symbol] :> \text{Simp}[b^{(2*\text{IntPart}[(n-1)/2]+1)}*(b*\text{Cos}[e+f*x])^{(2*\text{FracPart}[(n-1)/2])}*((a*\text{Sin}[e+f*x])^{(m+1)})/(a*f*(m+1)*(\text{Cos}[e+f*x]^2)^{\text{FracPart}[(n-1)/2])}*\text{Hypergeometric2F1}[(1+m)/2, (1-n)/2, (3+m)/2, \text{Sin}[e+f*x]^2], x] /; \text{FreeQ}\{a, b, e, f, m, n\}, x]$

Rubi steps

$$\int \cos^{2m}(x) \sin^{2m}(x) dx = \frac{\cos^{-1+2m}(x) \cos^2(x)^{\frac{1}{2}-m} {}_2F_1\left(\frac{1}{2}(1-2m), \frac{1}{2}(1+2m); \frac{1}{2}(3+2m); \sin^2(x)\right) \sin^{1+2m}(x)}{1+2m}$$

Mathematica [A]

time = 0.04, size = 58, normalized size = 0.85

$$\frac{\cos^{-1+2m}(x) \cos^2(x)^{\frac{1}{2}-m} {}_2F_1\left(\frac{1}{2}-m, \frac{1}{2}+m; \frac{3}{2}+m; \sin^2(x)\right) \sin^{1+2m}(x)}{1+2m}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^(2*m)*Sin[x]^(2*m),x]

[Out] (Cos[x]^(-1 + 2*m)*(Cos[x]^2)^(1/2 - m)*Hypergeometric2F1[1/2 - m, 1/2 + m, 3/2 + m, Sin[x]^2]*Sin[x]^(1 + 2*m))/(1 + 2*m)

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sin[x]^(2*m)*Cos[x]^(2*m),x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [F]

time = 0.02, size = 0, normalized size = 0.00

$$\int (\cos^{2m}(x)) (\sin^{2m}(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^(2*m)*sin(x)^(2*m),x)

[Out] int(cos(x)^(2*m)*sin(x)^(2*m),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="maxima")

[Out] integrate(cos(x)^(2*m)*sin(x)^(2*m), x)

Fricas [F]

time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="fricas")

[Out] integral(cos(x)^(2*m)*sin(x)^(2*m), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{2m}(x) \cos^{2m}(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**(2*m)*sin(x)**(2*m), x)

[Out] Integral(sin(x)**(2*m)*cos(x)**(2*m), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^(2*m)*sin(x)^(2*m), x)

[Out] Could not integrate

Mupad [B]

time = 0.75, size = 52, normalized size = 0.76

$$-\frac{\cos(x)^{2m+1} \sin(x)^{2m+1} {}_2F_1\left(\frac{1}{2} - m, m + \frac{1}{2}; m + \frac{3}{2}; \cos(x)^2\right)}{(2m+1) (\sin(x)^2)^{m+\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^(2*m)*sin(x)^(2*m), x)

[Out] -(cos(x)^(2*m + 1)*sin(x)^(2*m + 1)*hypergeom([1/2 - m, m + 1/2], m + 3/2, cos(x)^2))/((2*m + 1)*(sin(x)^2)^(m + 1/2))

3.353 $\int \csc^3\left(\frac{\pi}{4} + 2x\right) \sec\left(\frac{\pi}{4} + 2x\right) dx$

Optimal. Leaf size=32

$$-\frac{1}{4} \cot^2\left(\frac{\pi}{4} + 2x\right) + \frac{1}{2} \log\left(\tan\left(\frac{\pi}{4} + 2x\right)\right)$$

[Out] $-1/4*\cot(1/4*Pi+2*x)^2+1/2*\ln(\tan(1/4*Pi+2*x))$

Rubi [A]

time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2700, 14}

$$\frac{1}{2} \log\left(\tan\left(2x + \frac{\pi}{4}\right)\right) - \frac{1}{4} \cot^2\left(2x + \frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[\text{Pi}/4 + 2*x]^3*\text{Sec}[\text{Pi}/4 + 2*x], x]$

[Out] $-1/4*\text{Cot}[\text{Pi}/4 + 2*x]^2 + \text{Log}[\text{Tan}[\text{Pi}/4 + 2*x]]/2$

Rule 14

$\text{Int}[(u_*)((c_)*(x_))^{(m_)}], x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^{m*u}, x], x] /;$ FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)+(b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rule 2700

$\text{Int}[\text{csc}[(e_)+(f_)*(x_)]^{(m_)}*\text{sec}[(e_)+(f_)*(x_)]^{(n_)}], x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(1+x^2)^{(m+n)/2-1}/x^m, x], x, \text{Tan}[e+f*x]], x] /;$ FreeQ[{e, f}, x] && IntegersQ[m, n, (m+n)/2]

Rubi steps

$$\begin{aligned} \int \csc^3\left(\frac{\pi}{4} + 2x\right) \sec\left(\frac{\pi}{4} + 2x\right) dx &= \frac{1}{2} \text{Subst}\left(\int \frac{1+x^2}{x^3} dx, x, \tan\left(\frac{\pi}{4} + 2x\right)\right) \\ &= \frac{1}{2} \text{Subst}\left(\int \left(\frac{1}{x^3} + \frac{1}{x}\right) dx, x, \tan\left(\frac{\pi}{4} + 2x\right)\right) \\ &= -\frac{1}{4} \cot^2\left(\frac{\pi}{4} + 2x\right) + \frac{1}{2} \log\left(\tan\left(\frac{\pi}{4} + 2x\right)\right) \end{aligned}$$

Mathematica [A]

time = 0.04, size = 45, normalized size = 1.41

$$\frac{1}{4} \left(-\csc^2 \left(\frac{\pi}{4} + 2x \right) - 2 \log \left(\cos \left(\frac{1}{4}(\pi + 8x) \right) \right) + 2 \log \left(\sin \left(\frac{\pi}{4} + 2x \right) \right) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[Csc[Pi/4 + 2*x]^3*Sec[Pi/4 + 2*x],x]``[Out] (-Csc[Pi/4 + 2*x]^2 - 2*Log[Cos[(Pi + 8*x)/4]] + 2*Log[Sin[Pi/4 + 2*x]])/4`**Mathics [B]** Leaf count is larger than twice the leaf count of optimal. 72 vs. 2(32) = 64.

time = 5.01, size = 52, normalized size = 1.62

$$\frac{\text{Log} \left[-1 + \text{Tan} \left[\frac{\text{Pi}}{8} + x \right] \right]}{2} - \frac{\text{Log} \left[1 + \text{Tan} \left[\frac{\text{Pi}}{8} + x \right] \right]}{2} - \frac{1}{16 \text{Tan} \left[\frac{\text{Pi}}{8} + x \right]^2} - \frac{\text{Tan} \left[\frac{\text{Pi}}{8} + x \right]^2}{16} + \frac{\text{Log} \left[\text{Tan} \left[\frac{\text{Pi}}{8} + x \right] \right]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(Sin[Pi/4 + 2*x]^3*Cos[Pi/4 + 2*x]),x]')``[Out] -Log[-1 + Tan[Pi / 8 + x]] / 2 - Log[1 + Tan[Pi / 8 + x]] / 2 - 1 / (16 Tan[Pi / 8 + x] ^ 2) - Tan[Pi / 8 + x] ^ 2 / 16 + Log[Tan[Pi / 8 + x]] / 2`**Maple [A]**

time = 0.06, size = 25, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{1}{4 \sin(\frac{\pi}{4} + 2x)^2} + \frac{\ln(\tan(\frac{\pi}{4} + 2x))}{2}$	25
default	$-\frac{1}{4 \sin(\frac{\pi}{4} + 2x)^2} + \frac{\ln(\tan(\frac{\pi}{4} + 2x))}{2}$	25
risch	$\frac{ie^{4ix}}{(ie^{4ix}-1)^2} + \frac{\ln(ie^{4ix}-1)}{2} - \frac{\ln(ie^{4ix}+1)}{2}$	48
norman	$-\frac{1}{16} - \frac{(\tan^4(\frac{\pi}{8} + x))}{16} + \frac{\ln(\tan(\frac{\pi}{8} + x))}{2} - \frac{\ln(\tan(\frac{\pi}{8} + x) - 1)}{2} - \frac{\ln(\tan(\frac{\pi}{8} + x) + 1)}{2}$	54

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/cos(1/4*Pi+2*x)/sin(1/4*Pi+2*x)^3,x,method=_RETURNVERBOSE)``[Out] -1/4/sin(1/4*Pi+2*x)^2+1/2*ln(tan(1/4*Pi+2*x))`**Maxima [A]**

time = 0.25, size = 41, normalized size = 1.28

$$-\frac{1}{4 \sin \left(\frac{1}{4} \pi + 2x \right)^2} - \frac{1}{4} \log \left(\sin \left(\frac{1}{4} \pi + 2x \right)^2 - 1 \right) + \frac{1}{4} \log \left(\sin \left(\frac{1}{4} \pi + 2x \right)^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)^3,x, algorithm="maxima")

[Out] -1/4/sin(1/4*pi + 2*x)^2 - 1/4*log(sin(1/4*pi + 2*x)^2 - 1) + 1/4*log(sin(1/4*pi + 2*x)^2)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(24) = 48.

time = 0.33, size = 71, normalized size = 2.22

$$\frac{\left(\cos\left(\frac{1}{4}\pi + 2x\right)^2 - 1\right) \log\left(\cos\left(\frac{1}{4}\pi + 2x\right)^2\right) - \left(\cos\left(\frac{1}{4}\pi + 2x\right)^2 - 1\right) \log\left(-\frac{1}{4}\cos\left(\frac{1}{4}\pi + 2x\right)^2 + \frac{1}{4}\right) - 1}{4\left(\cos\left(\frac{1}{4}\pi + 2x\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)^3,x, algorithm="fricas")

[Out] -1/4*((cos(1/4*pi + 2*x)^2 - 1)*log(cos(1/4*pi + 2*x)^2) - (cos(1/4*pi + 2*x)^2 - 1)*log(-1/4*cos(1/4*pi + 2*x)^2 + 1/4) - 1)/(cos(1/4*pi + 2*x)^2 - 1)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(22) = 44

time = 0.63, size = 54, normalized size = 1.69

$$-\frac{\log\left(\tan\left(x + \frac{\pi}{8}\right) - 1\right)}{2} - \frac{\log\left(\tan\left(x + \frac{\pi}{8}\right) + 1\right)}{2} + \frac{\log\left(\tan\left(x + \frac{\pi}{8}\right)\right)}{2} - \frac{\tan^2\left(x + \frac{\pi}{8}\right)}{16} - \frac{1}{16\tan^2\left(x + \frac{\pi}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)**3,x)

[Out] -log(tan(x + pi/8) - 1)/2 - log(tan(x + pi/8) + 1)/2 + log(tan(x + pi/8))/2 - tan(x + pi/8)**2/16 - 1/(16*tan(x + pi/8)**2)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(24) = 48.

time = 0.03, size = 149, normalized size = 4.66

$$-\frac{1 - \cos\left(\frac{\pi+8x}{4}\right)}{\left(1 + \cos\left(\frac{\pi+8x}{4}\right)\right) \cdot 16} + \frac{-\frac{4(1 - \cos\left(\frac{\pi+8x}{4}\right))}{1 + \cos\left(\frac{\pi+8x}{4}\right)} - 1}{\frac{16(1 - \cos\left(\frac{\pi+8x}{4}\right))}{1 + \cos\left(\frac{\pi+8x}{4}\right)}} - \frac{\ln\left|\frac{1 - \cos\left(\frac{\pi+8x}{4}\right)}{1 + \cos\left(\frac{\pi+8x}{4}\right)} - 1\right|}{2} + \frac{\ln\left(\frac{1 - \cos\left(\frac{\pi+8x}{4}\right)}{1 + \cos\left(\frac{\pi+8x}{4}\right)}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)^3,x)

[Out] -1/16*(4*(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1) - 1)*(cos(1/4*pi + 2*x) + 1)/(cos(1/4*pi + 2*x) - 1) + 1/16*(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1)

```
pi + 2*x) + 1) + 1/4*log(-(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1))
- 1/2*log(abs(-(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1) - 1))
```

Mupad [B]

time = 0.28, size = 24, normalized size = 0.75

$$\frac{\ln\left(\tan\left(\frac{\pi}{4} + 2x\right)\right)}{2} - \frac{1}{4\sin\left(\frac{\pi}{4} + 2x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(Pi/4 + 2*x)*sin(Pi/4 + 2*x)^3),x)
```

```
[Out] log(tan(Pi/4 + 2*x))/2 - 1/(4*sin(Pi/4 + 2*x)^2)
```

3.354 $\int \sec^2(x) \tan^2(x) dx$

Optimal. Leaf size=8

$$\frac{\tan^3(x)}{3}$$

[Out] 1/3*tan(x)^3

Rubi [A]

time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2687, 30}

$$\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2*Tan[x]^2,x]

[Out] Tan[x]^3/3

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2687

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rubi steps

$$\begin{aligned} \int \sec^2(x) \tan^2(x) dx &= \text{Subst} \left(\int x^2 dx, x, \tan(x) \right) \\ &= \frac{\tan^3(x)}{3} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 8, normalized size = 1.00

$$\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2*Tan[x]^2,x]

[Out] Tan[x]^3/3

Mathics [A]

time = 1.64, size = 6, normalized size = 0.75

$$\frac{\text{Tan}[x]^3}{3}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Tan[x]^2*Sec[x]^2,x]')

[Out] Tan[x] ^ 3 / 3

Maple [A]

time = 0.04, size = 11, normalized size = 1.38

method	result	size
default	$\frac{\sin^3(x)}{3 \cos(x)^3}$	11
risch	$-\frac{2i(3e^{4ix}+1)}{3(e^{2ix}+1)^3}$	22

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2*tan(x)^2,x,method=_RETURNVERBOSE)

[Out] 1/3*sin(x)^3/cos(x)^3

Maxima [A]

time = 0.25, size = 6, normalized size = 0.75

$$\frac{1}{3} \tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*tan(x)^2,x, algorithm="maxima")

[Out] 1/3*tan(x)^3

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.
time = 0.31, size = 14, normalized size = 1.75

$$-\frac{(\cos(x)^2 - 1) \sin(x)}{3 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)^2*tan(x)^2,x, algorithm="fricas")`

[Out] $-1/3*(\cos(x)^2 - 1)*\sin(x)/\cos(x)^3$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(5) = 10$

time = 0.03, size = 17, normalized size = 2.12

$$-\frac{\sin(x)}{3\cos(x)} + \frac{\sin(x)}{3\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)**2*tan(x)**2,x)`

[Out] $-\sin(x)/(3*\cos(x)) + \sin(x)/(3*\cos(x)**3)$

Giac [A]

time = 0.00, size = 7, normalized size = 0.88

$$\frac{\tan^3 x}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)^2*tan(x)^2,x)`

[Out] $1/3*\tan(x)^3$

Mupad [B]

time = 0.03, size = 6, normalized size = 0.75

$$\frac{\tan(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(x)^2/cos(x)^2,x)`

[Out] $\tan(x)^3/3$

3.355 $\int \cot^3(x) \csc(x) dx$

Optimal. Leaf size=11

$$\csc(x) - \frac{\csc^3(x)}{3}$$

[Out] $\csc(x) - 1/3 * \csc(x)^3$

Rubi [A]

time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2686}

$$\csc(x) - \frac{\csc^3(x)}{3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[x]^3 * \text{Csc}[x], x]$

[Out] $\text{Csc}[x] - \text{Csc}[x]^3/3$

Rule 2686

$\text{Int}[(a_*) * \sec[(e_*) + (f_*) * (x_*)]^{(m_*)} * ((b_*) * \tan[(e_*) + (f_*) * (x_*)])^{(n_*)}, x_Symbol] :> \text{Dist}[a/f, \text{Subst}[\text{Int}[(a*x)^{(m-1)} * (-1 + x^2)^{((n-1)/2)}, x], x, \text{Sec}[e + f*x]], x] /; \text{FreeQ}\{a, e, f, m\}, x \ \&\& \ \text{IntegerQ}[(n-1)/2] \ \&\& \ !(\text{IntegerQ}[m/2] \ \&\& \ \text{LtQ}[0, m, n + 1])$

Rubi steps

$$\begin{aligned} \int \cot^3(x) \csc(x) dx &= -\text{Subst} \left(\int (-1 + x^2) dx, x, \csc(x) \right) \\ &= \csc(x) - \frac{\csc^3(x)}{3} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 11, normalized size = 1.00

$$\csc(x) - \frac{\csc^3(x)}{3}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Cot}[x]^3 * \text{Csc}[x], x]$

[Out] $\text{Csc}[x] - \text{Csc}[x]^3/3$

Mathics [A]

time = 1.63, size = 14, normalized size = 1.27

$$\frac{3 - \frac{1}{\text{Sin}[x]^2}}{3\text{Sin}[x]}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[Cot[x]^3*Csc[x],x]')`

[Out] $(3 - 1 / \text{Sin}[x]^2) / (3 \text{Sin}[x])$

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(9) = 18$.

time = 0.07, size = 32, normalized size = 2.91

method	result	size
default	$-\frac{\cos^4(x)}{3\sin(x)^3} + \frac{\cos^4(x)}{3\sin(x)} + \frac{(2+\cos^2(x))\sin(x)}{3}$	32
risch	$\frac{2i(3e^{5ix}-2e^{3ix}+3e^{ix})}{3(e^{2ix}-1)^3}$	35

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(x)^3*csc(x),x,method=_RETURNVERBOSE)`

[Out] $-1/3/\sin(x)^3*\cos(x)^4+1/3/\sin(x)*\cos(x)^4+1/3*(2+\cos(x)^2)*\sin(x)$

Maxima [A]

time = 0.26, size = 14, normalized size = 1.27

$$\frac{3 \sin(x)^2 - 1}{3 \sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)^3*csc(x),x, algorithm="maxima")`

[Out] $1/3*(3*\sin(x)^2 - 1)/\sin(x)^3$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(9) = 18$.

time = 0.40, size = 22, normalized size = 2.00

$$\frac{3 \cos(x)^2 - 2}{3 (\cos(x)^2 - 1) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^3*csc(x),x, algorithm="fricas")

[Out] 1/3*(3*cos(x)^2 - 2)/((cos(x)^2 - 1)*sin(x))

Sympy [A]

time = 0.05, size = 15, normalized size = 1.36

$$-\frac{1 - 3 \sin^2(x)}{3 \sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)**3*csc(x),x)

[Out] -(1 - 3*sin(x)**2)/(3*sin(x)**3)

Giac [A]

time = 0.00, size = 16, normalized size = 1.45

$$\frac{3 \sin^2 x - 1}{3 \sin^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^3*csc(x),x)

[Out] 1/3*(3*sin(x)^2 - 1)/sin(x)^3

Mupad [B]

time = 0.32, size = 11, normalized size = 1.00

$$\frac{\sin(x)^2 - \frac{1}{3}}{\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)^3/sin(x),x)

[Out] (sin(x)^2 - 1/3)/sin(x)^3

3.356 $\int \sec^3(x) \tan(x) dx$

Optimal. Leaf size=8

$$\frac{\sec^3(x)}{3}$$

[Out] 1/3*sec(x)^3

Rubi [A]

time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2686, 30}

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^3*Tan[x],x]

[Out] Sec[x]^3/3

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2686

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rubi steps

$$\begin{aligned} \int \sec^3(x) \tan(x) dx &= \text{Subst} \left(\int x^2 dx, x, \sec(x) \right) \\ &= \frac{\sec^3(x)}{3} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 8, normalized size = 1.00

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^3*Tan[x],x]

[Out] Sec[x]^3/3

Mathics [A]

time = 1.55, size = 6, normalized size = 0.75

$$\frac{1}{3\cos[x]^3}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Tan[x]*Sec[x]^3,x]')

[Out] 1 / (3 Cos[x] ^ 3)

Maple [A]

time = 0.02, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\frac{(\sec^3(x))}{3}$	7
default	$\frac{(\sec^3(x))}{3}$	7
risch	$\frac{8e^{3ix}}{3(e^{2ix}+1)^3}$	17

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^3*tan(x),x,method=_RETURNVERBOSE)

[Out] 1/3*sec(x)^3

Maxima [A]

time = 0.25, size = 6, normalized size = 0.75

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^3*tan(x),x, algorithm="maxima")

[Out] 1/3/cos(x)^3

Fricas [A]

time = 0.31, size = 6, normalized size = 0.75

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)^3*tan(x),x, algorithm="fricas")`

[Out] $1/3/\cos(x)^3$

Sympy [A]

time = 0.03, size = 7, normalized size = 0.88

$$\frac{1}{3 \cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)**3*tan(x),x)`

[Out] $1/(3*\cos(x)**3)$

Giac [A]

time = 0.00, size = 8, normalized size = 1.00

$$\frac{1}{3 \cos^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)^3*tan(x),x)`

[Out] $1/3/\cos(x)^3$

Mupad [B]

time = 0.32, size = 6, normalized size = 0.75

$$\frac{1}{3 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(x)/cos(x)^3,x)`

[Out] $1/(3*\cos(x)^3)$

3.357 $\int \cot^2(x) \csc^3(x) dx$

Optimal. Leaf size=26

$$\frac{1}{8} \tanh^{-1}(\cos(x)) + \frac{1}{8} \cot(x) \csc(x) - \frac{1}{4} \cot(x) \csc^3(x)$$

[Out] 1/8*arctanh(cos(x))+1/8*cot(x)*csc(x)-1/4*cot(x)*csc(x)^3

Rubi [A]

time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2691, 3853, 3855}

$$\frac{1}{8} \tanh^{-1}(\cos(x)) - \frac{1}{4} \cot(x) \csc^3(x) + \frac{1}{8} \cot(x) \csc(x)$$

Antiderivative was successfully verified.

[In] Int[Cot[x]^2*Csc[x]^3,x]

[Out] ArcTanh[Cos[x]]/8 + (Cot[x]*Csc[x])/8 - (Cot[x]*Csc[x]^3)/4

Rule 2691

Int[((a_.)*sec[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \cot^2(x) \csc^3(x) dx &= -\frac{1}{4} \cot(x) \csc^3(x) - \frac{1}{4} \int \csc^3(x) dx \\
&= \frac{1}{8} \cot(x) \csc(x) - \frac{1}{4} \cot(x) \csc^3(x) - \frac{1}{8} \int \csc(x) dx \\
&= \frac{1}{8} \tanh^{-1}(\cos(x)) + \frac{1}{8} \cot(x) \csc(x) - \frac{1}{4} \cot(x) \csc^3(x)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 71 vs. 2(26) = 52.

time = 0.01, size = 71, normalized size = 2.73

$$\frac{1}{32} \csc^2\left(\frac{x}{2}\right) - \frac{1}{64} \csc^4\left(\frac{x}{2}\right) + \frac{1}{8} \log\left(\cos\left(\frac{x}{2}\right)\right) - \frac{1}{8} \log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{32} \sec^2\left(\frac{x}{2}\right) + \frac{1}{64} \sec^4\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]^2*Csc[x]^3,x]

[Out] Csc[x/2]^2/32 - Csc[x/2]^4/64 + Log[Cos[x/2]]/8 - Log[Sin[x/2]]/8 - Sec[x/2]^2/32 + Sec[x/2]^4/64

Mathics [A]

time = 2.11, size = 31, normalized size = 1.19

$$-\frac{\cos[x]}{4\sin[x]^4} - \frac{\log[-1 + \cos[x]]}{16} + \frac{\log[1 + \cos[x]]}{16} + \frac{\cos[x]}{8\sin[x]^2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Cot[x]^2*Csc[x]^3,x]')

[Out] -Cos[x] / (4 Sin[x]^4) - Log[-1 + Cos[x]] / 16 + Log[1 + Cos[x]] / 16 + Cos[x] / (8 Sin[x]^2)

Maple [A]

time = 0.05, size = 36, normalized size = 1.38

method	result	size
default	$-\frac{\cos^3(x)}{4\sin(x)^4} - \frac{\cos^3(x)}{8\sin(x)^2} - \frac{\cos(x)}{8} - \frac{\ln(\csc(x)-\cot(x))}{8}$	36
risch	$-\frac{e^{7ix}+7e^{5ix}+7e^{3ix}+e^{ix}}{4(e^{2ix}-1)^4} + \frac{\ln(1+e^{ix})}{8} - \frac{\ln(e^{ix}-1)}{8}$	58

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)^2*csc(x)^3,x,method=_RETURNVERBOSE)

[Out] $-1/4*\cos(x)^3/\sin(x)^4-1/8/\sin(x)^2*\cos(x)^3-1/8*\cos(x)-1/8*\ln(\csc(x)-\cot(x))$

Maxima [A]

time = 0.25, size = 38, normalized size = 1.46

$$-\frac{\cos(x)^3 + \cos(x)}{8(\cos(x)^4 - 2\cos(x)^2 + 1)} + \frac{1}{16} \log(\cos(x) + 1) - \frac{1}{16} \log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)^2*csc(x)^3,x, algorithm="maxima")`

[Out] $-1/8*(\cos(x)^3 + \cos(x))/(\cos(x)^4 - 2*\cos(x)^2 + 1) + 1/16*\log(\cos(x) + 1) - 1/16*\log(\cos(x) - 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(20) = 40.

time = 0.33, size = 68, normalized size = 2.62

$$\frac{2\cos(x)^3 - (\cos(x)^4 - 2\cos(x)^2 + 1)\log(\frac{1}{2}\cos(x) + \frac{1}{2}) + (\cos(x)^4 - 2\cos(x)^2 + 1)\log(-\frac{1}{2}\cos(x) + \frac{1}{2}) + 2\cos(x)}{16(\cos(x)^4 - 2\cos(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)^2*csc(x)^3,x, algorithm="fricas")`

[Out] $-1/16*(2*\cos(x)^3 - (\cos(x)^4 - 2*\cos(x)^2 + 1)*\log(1/2*\cos(x) + 1/2) + (\cos(x)^4 - 2*\cos(x)^2 + 1)*\log(-1/2*\cos(x) + 1/2) + 2*\cos(x))/(\cos(x)^4 - 2*\cos(x)^2 + 1)$

Sympy [A]

time = 0.07, size = 41, normalized size = 1.58

$$\frac{-\cos^3(x) - \cos(x)}{8\cos^4(x) - 16\cos^2(x) + 8} - \frac{\log(\cos(x) - 1)}{16} + \frac{\log(\cos(x) + 1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)**2*csc(x)**3,x)`

[Out] $(-\cos(x)**3 - \cos(x))/(8*\cos(x)**4 - 16*\cos(x)**2 + 8) - \log(\cos(x) - 1)/16 + \log(\cos(x) + 1)/16$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(20) = 40.

time = 0.00, size = 47, normalized size = 1.81

$$-\frac{\ln|\cos x + \frac{1}{\cos x} - 2|}{32} + \frac{\ln|\cos x + \frac{1}{\cos x} + 2|}{32} - \frac{\cos x + \frac{1}{\cos x}}{8\left(\left(\cos x + \frac{1}{\cos x}\right)^2 - 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)^2*csc(x)^3,x)`

[Out] $-1/8*(1/\cos(x) + \cos(x))/((1/\cos(x) + \cos(x))^2 - 4) + 1/32*\log(\text{abs}(1/\cos(x) + \cos(x) + 2)) - 1/32*\log(\text{abs}(1/\cos(x) + \cos(x) - 2))$

Mupad [B]

time = 0.32, size = 24, normalized size = 0.92

$$\frac{\tan\left(\frac{x}{2}\right)^4}{64} - \frac{1}{64 \tan\left(\frac{x}{2}\right)^4} - \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(x)^2/sin(x)^3,x)`

[Out] $\tan(x/2)^4/64 - 1/(64*\tan(x/2)^4) - \log(\tan(x/2))/8$

3.358 $\int \cot^3(x) \csc^4(x) dx$

Optimal. Leaf size=17

$$\frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

[Out] 1/4*csc(x)^4-1/6*csc(x)^6

Rubi [A]

time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2686, 14}

$$\frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

Antiderivative was successfully verified.

[In] Int[Cot[x]^3*Csc[x]^4,x]

[Out] Csc[x]^4/4 - Csc[x]^6/6

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_
+ (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_)*sec[(e_) + (f_)*(x_)]^(m_))*((b_)*tan[(e_) + (f_)*(x_)]^(
n_)), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \int \cot^3(x) \csc^4(x) dx &= -\text{Subst} \left(\int x^3 (-1 + x^2) dx, x, \csc(x) \right) \\ &= -\text{Subst} \left(\int (-x^3 + x^5) dx, x, \csc(x) \right) \\ &= \frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 17, normalized size = 1.00

$$\frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

Antiderivative was successfully verified.

`[In] Integrate[Cot[x]^3*Csc[x]^4,x]``[Out] Csc[x]^4/4 - Csc[x]^6/6`**Mathics [A]**

time = 1.64, size = 14, normalized size = 0.82

$$\frac{-2 + 3\sin[x]^2}{12\sin[x]^6}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Cos[x]^3/Sin[x]^7,x]')``[Out] (-2 + 3 Sin[x]^2) / (12 Sin[x]^6)`**Maple [A]**

time = 0.04, size = 22, normalized size = 1.29

method	result	size
default	$-\frac{\cos^4(x)}{6\sin(x)^6} - \frac{\cos^4(x)}{12\sin(x)^4}$	22
norman	$-\frac{1}{384} + \frac{3(\tan^4(\frac{x}{2}))}{128} + \frac{3(\tan^8(\frac{x}{2}))}{128} - \frac{(\tan^{12}(\frac{x}{2}))}{384}$ $\frac{\tan(\frac{x}{2})^6}{\tan(\frac{x}{2})^6}$	34
risch	$\frac{4e^{8ix} + \frac{8e^{6ix}}{3} + 4e^{4ix}}{(e^{2ix} - 1)^6}$	34

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x)^3/sin(x)^7,x,method=_RETURNVERBOSE)``[Out] -1/6/sin(x)^6*cos(x)^4-1/12/sin(x)^4*cos(x)^4`**Maxima [A]**

time = 0.25, size = 14, normalized size = 0.82

$$\frac{3\sin(x)^2 - 2}{12\sin(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="maxima")

[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(13) = 26.

time = 0.31, size = 30, normalized size = 1.76

$$\frac{3 \cos(x)^2 - 1}{12 (\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="fricas")

[Out] 1/12*(3*cos(x)^2 - 1)/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)

Sympy [A]

time = 0.05, size = 15, normalized size = 0.88

$$-\frac{2 - 3 \sin^2(x)}{12 \sin^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**3/sin(x)**7,x)

[Out] -(2 - 3*sin(x)**2)/(12*sin(x)**6)

Giac [A]

time = 0.00, size = 16, normalized size = 0.94

$$\frac{3 \sin^2 x - 2}{12 \sin^6 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3/sin(x)^7,x)

[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6

Mupad [B]

time = 0.07, size = 13, normalized size = 0.76

$$\frac{\frac{\sin(x)^2}{4} - \frac{1}{6}}{\sin(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^3/sin(x)^7,x)

[Out] (sin(x)^2/4 - 1/6)/sin(x)^6

3.359 $\int \sec^{\frac{13}{2}}(x) \sin^5(x) dx$

Optimal. Leaf size=31

$$\frac{2}{3} \sec^{\frac{3}{2}}(x) - \frac{4}{7} \sec^{\frac{7}{2}}(x) + \frac{2}{11} \sec^{\frac{11}{2}}(x)$$

[Out] $2/3*\sec(x)^{(3/2)}-4/7*\sec(x)^{(7/2)}+2/11*\sec(x)^{(11/2)}$

Rubi [A]

time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2702, 276}

$$\frac{2}{11} \sec^{\frac{11}{2}}(x) - \frac{4}{7} \sec^{\frac{7}{2}}(x) + \frac{2}{3} \sec^{\frac{3}{2}}(x)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^(13/2)*Sin[x]^5,x]

[Out] (2*Sec[x]^(3/2))/3 - (4*Sec[x]^(7/2))/7 + (2*Sec[x]^(11/2))/11

Rule 276

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2702

Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

Rubi steps

$$\begin{aligned} \int \sec^{\frac{13}{2}}(x) \sin^5(x) dx &= \text{Subst} \left(\int \sqrt{x} (-1 + x^2)^2 dx, x, \sec(x) \right) \\ &= \text{Subst} \left(\int (\sqrt{x} - 2x^{5/2} + x^{9/2}) dx, x, \sec(x) \right) \\ &= \frac{2}{3} \sec^{\frac{3}{2}}(x) - \frac{4}{7} \sec^{\frac{7}{2}}(x) + \frac{2}{11} \sec^{\frac{11}{2}}(x) \end{aligned}$$

Mathematica [A]

time = 0.03, size = 24, normalized size = 0.77

$$\frac{1}{924}(135 + 44 \cos(2x) + 77 \cos(4x)) \sec^{\frac{11}{2}}(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^(13/2)*Sin[x]^5,x]

[Out] ((135 + 44*Cos[2*x] + 77*Cos[4*x])*Sec[x]^(11/2))/924

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Tan[x]^5*Sec[x]^(3/2),x]')

[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(19) = 38.

time = 0.11, size = 49, normalized size = 1.58

method	result	size
default	$\frac{\frac{32(\sin^8(\frac{x}{2}))}{3} - \frac{64(\sin^6(\frac{x}{2}))}{3} + \frac{96(\sin^4(\frac{x}{2}))}{7} - \frac{64(\sin^2(\frac{x}{2}))}{21} + \frac{64}{231}}{(-2(\sin^2(\frac{x}{2}))+1)^{\frac{11}{2}}}$	49

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^(3/2)*tan(x)^5,x,method=_RETURNVERBOSE)

[Out] 32/231/(-2*sin(1/2*x)^2+1)^(11/2)*(77*sin(1/2*x)^8-154*sin(1/2*x)^6+99*sin(1/2*x)^4-22*sin(1/2*x)^2+2)

Maxima [A]

time = 0.27, size = 19, normalized size = 0.61

$$\frac{2}{3 \cos(x)^{\frac{3}{2}}} - \frac{4}{7 \cos(x)^{\frac{7}{2}}} + \frac{2}{11 \cos(x)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="maxima")

[Out] 2/3/cos(x)^(3/2) - 4/7/cos(x)^(7/2) + 2/11/cos(x)^(11/2)

Fricas [A]

time = 0.32, size = 20, normalized size = 0.65

$$\frac{2(77 \cos(x)^4 - 66 \cos(x)^2 + 21)}{231 \cos(x)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="fricas")``[Out] 2/231*(77*cos(x)^4 - 66*cos(x)^2 + 21)/cos(x)^(11/2)`**Sympy [A]**

time = 16.08, size = 39, normalized size = 1.26

$$\frac{2 \tan^4(x) \sec^{\frac{3}{2}}(x)}{11} - \frac{16 \tan^2(x) \sec^{\frac{3}{2}}(x)}{77} + \frac{64 \sec^{\frac{3}{2}}(x)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sec(x)**(3/2)*tan(x)**5,x)``[Out] 2*tan(x)**4*sec(x)**(3/2)/11 - 16*tan(x)**2*sec(x)**(3/2)/77 + 64*sec(x)**(3/2)/231`**Giac [A]**

time = 0.00, size = 30, normalized size = 0.97

$$\frac{154 \cos^4 x - 132 \cos^2 x + 42}{231 \sqrt{\cos x} \cos^5 x}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sec(x)^(3/2)*tan(x)^5,x)``[Out] 2/231*(77*cos(x)^4 - 66*cos(x)^2 + 21)/cos(x)^(11/2)`**Mupad [B]**

time = 1.12, size = 22, normalized size = 0.71

$$\frac{2 \left(\frac{1}{\cos(x)}\right)^{11/2} (77 \cos(x)^4 - 66 \cos(x)^2 + 21)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(tan(x)^5*(1/cos(x))^(3/2),x)``[Out] (2*(1/cos(x))^(11/2)*(77*cos(x)^4 - 66*cos(x)^2 + 21))/231`

3.360 $\int \sec^4(x) \tan^{\frac{3}{2}}(x) dx$

Optimal. Leaf size=21

$$\frac{2}{5} \tan^{\frac{5}{2}}(x) + \frac{2}{9} \tan^{\frac{9}{2}}(x)$$

[Out] $2/5*\tan(x)^{(5/2)}+2/9*\tan(x)^{(9/2)}$

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2687, 14}

$$\frac{2}{9} \tan^{\frac{9}{2}}(x) + \frac{2}{5} \tan^{\frac{5}{2}}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[x]^4*\text{Tan}[x]^{(3/2)},x]$

[Out] $(2*\text{Tan}[x]^{(5/2)})/5 + (2*\text{Tan}[x]^{(9/2)})/9$

Rule 14

$\text{Int}[(u_*)((c_*)(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}[\{c, m\}, x] \ \&\& \ \text{SumQ}[u] \ \&\& \ !\text{LinearQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (a_ + (b_)*(v_)) /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{InverseFunctionQ}[v]]$

Rule 2687

$\text{Int}[\text{sec}[(e_.) + (f_.)*(x_)]^{(m_)}*((b_.)*\text{tan}[(e_.) + (f_.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(b*x)^n*(1 + x^2)^{(m/2 - 1)}, x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{b, e, f, n\}, x] \ \&\& \ \text{IntegerQ}[m/2] \ \&\& \ !(\text{IntegerQ}[(n - 1)/2] \ \&\& \ \text{LtQ}[0, n, m - 1])$

Rubi steps

$$\begin{aligned} \int \sec^4(x) \tan^{\frac{3}{2}}(x) dx &= \text{Subst} \left(\int x^{3/2} (1 + x^2) dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int (x^{3/2} + x^{7/2}) dx, x, \tan(x) \right) \\ &= \frac{2}{5} \tan^{\frac{5}{2}}(x) + \frac{2}{9} \tan^{\frac{9}{2}}(x) \end{aligned}$$

Mathematica [A]

time = 0.03, size = 22, normalized size = 1.05

$$\frac{2}{45}(7 + 2 \cos(2x)) \sec^2(x) \tan^{\frac{5}{2}}(x)$$

Antiderivative was successfully verified.

`[In] Integrate[Sec[x]^4*Tan[x]^(3/2),x]``[Out] (2*(7 + 2*Cos[2*x])*Sec[x]^2*Tan[x]^(5/2))/45`**Mathics [F(-1)]**

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Tan[x]^(3/2)*Sec[x]^4,x]')``[Out] Timed out`**Maple [A]**

time = 0.28, size = 26, normalized size = 1.24

method	result	size
default	$\frac{2(4(\cos^2(x))+5)\sin(x)\left(\frac{\sin(x)}{\cos(x)}\right)^{\frac{3}{2}}}{45\cos(x)^3}$	26

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sec(x)^4*tan(x)^(3/2),x,method=_RETURNVERBOSE)``[Out] 2/45*(4*cos(x)^2+5)*sin(x)*(sin(x)/cos(x))^(3/2)/cos(x)^3`**Maxima [A]**

time = 0.25, size = 13, normalized size = 0.62

$$\frac{2}{9} \tan(x)^{\frac{9}{2}} + \frac{2}{5} \tan(x)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="maxima")``[Out] 2/9*tan(x)^(9/2) + 2/5*tan(x)^(5/2)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.
time = 0.34, size = 27, normalized size = 1.29

$$-\frac{2(4\cos(x)^4 + \cos(x)^2 - 5)\sqrt{\frac{\sin(x)}{\cos(x)}}}{45\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="fricas")

[Out] -2/45*(4*cos(x)^4 + cos(x)^2 - 5)*sqrt(sin(x)/cos(x))/cos(x)^4

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \tan^{\frac{3}{2}}(x) \sec^4(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**4*tan(x)**(3/2),x)

[Out] Integral(tan(x)**(3/2)*sec(x)**4, x)

Giac [A]

time = 0.00, size = 29, normalized size = 1.38

$$2\left(\frac{1}{9}\sqrt{\tan x} \tan^4 x + \frac{1}{5}\sqrt{\tan x} \tan^2 x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^4*tan(x)^(3/2),x)

[Out] 2/9*tan(x)^(9/2) + 2/5*tan(x)^(5/2)

Mupad [B]

time = 1.15, size = 32, normalized size = 1.52

$$-\frac{4\sqrt{\sin(2x)}(2\cos(2x)^2 + 5\cos(2x) - 7)}{45(\cos(2x) + 1)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)^(3/2)/cos(x)^4,x)

[Out] -(4*sin(2*x)^(1/2)*(5*cos(2*x) + 2*cos(2*x)^2 - 7))/(45*(cos(2*x) + 1)^(5/2))

3.361 $\int \cot^4(x) \csc^3(x) dx$

Optimal. Leaf size=38

$$-\frac{1}{16} \tanh^{-1}(\cos(x)) - \frac{1}{16} \cot(x) \csc(x) + \frac{1}{8} \cot(x) \csc^3(x) - \frac{1}{6} \cot^3(x) \csc^3(x)$$

[Out] -1/16*arctanh(cos(x))-1/16*cot(x)*csc(x)+1/8*cot(x)*csc(x)^3-1/6*cot(x)^3*csc(x)^3

Rubi [A]

time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2691, 3853, 3855}

$$-\frac{1}{16} \tanh^{-1}(\cos(x)) - \frac{1}{6} \cot^3(x) \csc^3(x) + \frac{1}{8} \cot(x) \csc^3(x) - \frac{1}{16} \cot(x) \csc(x)$$

Antiderivative was successfully verified.

[In] Int[Cot[x]^4*Csc[x]^3,x]

[Out] -1/16*ArcTanh[Cos[x]] - (Cot[x]*Csc[x])/16 + (Cot[x]*Csc[x]^3)/8 - (Cot[x]^3*Csc[x]^3)/6

Rule 2691

Int[((a_.)*sec[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \cot^4(x) \csc^3(x) dx &= -\frac{1}{6} \cot^3(x) \csc^3(x) - \frac{1}{2} \int \cot^2(x) \csc^3(x) dx \\
&= \frac{1}{8} \cot(x) \csc^3(x) - \frac{1}{6} \cot^3(x) \csc^3(x) + \frac{1}{8} \int \csc^3(x) dx \\
&= -\frac{1}{16} \cot(x) \csc(x) + \frac{1}{8} \cot(x) \csc^3(x) - \frac{1}{6} \cot^3(x) \csc^3(x) + \frac{1}{16} \int \csc(x) dx \\
&= -\frac{1}{16} \tanh^{-1}(\cos(x)) - \frac{1}{16} \cot(x) \csc(x) + \frac{1}{8} \cot(x) \csc^3(x) - \frac{1}{6} \cot^3(x) \csc^3(x)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 95 vs. 2(38) = 76.

time = 0.01, size = 95, normalized size = 2.50

$$-\frac{1}{64} \csc^2\left(\frac{x}{2}\right) + \frac{1}{64} \csc^4\left(\frac{x}{2}\right) - \frac{1}{384} \csc^6\left(\frac{x}{2}\right) - \frac{1}{16} \log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{1}{16} \log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{1}{64} \sec^2\left(\frac{x}{2}\right) - \frac{1}{64} \sec^4\left(\frac{x}{2}\right) + \frac{1}{384} \sec^6\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]^4*Csc[x]^3,x]

[Out] -1/64*Csc[x/2]^2 + Csc[x/2]^4/64 - Csc[x/2]^6/384 - Log[Cos[x/2]]/16 + Log[Sin[x/2]]/16 + Sec[x/2]^2/64 - Sec[x/2]^4/64 + Sec[x/2]^6/384

Mathics [A]

time = 2.67, size = 44, normalized size = 1.16

$$\frac{-2\cos[x](-3 + 3\cos[x]^4 + 8\cos[x]^2) + 3(\log[-1 + \cos[x]] - \log[1 + \cos[x]])\sin[x]^6}{96\sin[x]^6}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Cot[x]^4*Csc[x]^3,x]')

[Out] (-2 Cos[x] (-3 + 3 Cos[x]^4 + 8 Cos[x]^2) + 3 (Log[-1 + Cos[x]] - Log[1 + Cos[x]]) Sin[x]^6) / (96 Sin[x]^6)

Maple [A]

time = 0.05, size = 52, normalized size = 1.37

method	result	size
default	$-\frac{\cos^5(x)}{6\sin(x)^6} - \frac{\cos^5(x)}{24\sin(x)^4} + \frac{\cos^5(x)}{48\sin(x)^2} + \frac{\cos^3(x)}{48} + \frac{\cos(x)}{16} + \frac{\ln(\csc(x)-\cot(x))}{16}$	52
risch	$\frac{3e^{11ix} + 47e^{9ix} + 78e^{7ix} + 78e^{5ix} + 47e^{3ix} + 3e^{ix}}{24(e^{2ix}-1)^6} + \frac{\ln(e^{ix}-1)}{16} - \frac{\ln(1+e^{ix})}{16}$	76

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(x)^4*csc(x)^3,x,method=_RETURNVERBOSE)`

[Out] $-1/6/\sin(x)^6*\cos(x)^5-1/24/\sin(x)^4*\cos(x)^5+1/48/\sin(x)^2*\cos(x)^5+1/48*\cos(x)^3+1/16*\cos(x)+1/16*\ln(\csc(x)-\cot(x))$

Maxima [A]

time = 0.25, size = 54, normalized size = 1.42

$$\frac{3 \cos(x)^5 + 8 \cos(x)^3 - 3 \cos(x)}{48 (\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1)} - \frac{1}{32} \log(\cos(x) + 1) + \frac{1}{32} \log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)^4*csc(x)^3,x, algorithm="maxima")`

[Out] $1/48*(3*\cos(x)^5 + 8*\cos(x)^3 - 3*\cos(x))/(\cos(x)^6 - 3*\cos(x)^4 + 3*\cos(x)^2 - 1) - 1/32*\log(\cos(x) + 1) + 1/32*\log(\cos(x) - 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(30) = 60$.

time = 0.32, size = 93, normalized size = 2.45

$$\frac{6 \cos(x)^5 + 16 \cos(x)^3 - 3 (\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 3 (\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) - 6 \cos(x)}{96 (\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)^4*csc(x)^3,x, algorithm="fricas")`

[Out] $1/96*(6*\cos(x)^5 + 16*\cos(x)^3 - 3*(\cos(x)^6 - 3*\cos(x)^4 + 3*\cos(x)^2 - 1)*\log(1/2*\cos(x) + 1/2) + 3*(\cos(x)^6 - 3*\cos(x)^4 + 3*\cos(x)^2 - 1)*\log(-1/2*\cos(x) + 1/2) - 6*\cos(x))/(\cos(x)^6 - 3*\cos(x)^4 + 3*\cos(x)^2 - 1)$

Sympy [A]

time = 0.08, size = 56, normalized size = 1.47

$$-\frac{-3 \cos^5(x) - 8 \cos^3(x) + 3 \cos(x)}{48 \cos^6(x) - 144 \cos^4(x) + 144 \cos^2(x) - 48} + \frac{\log(\cos(x) - 1)}{32} - \frac{\log(\cos(x) + 1)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)**4*csc(x)**3,x)`

[Out] $-(-3*\cos(x)**5 - 8*\cos(x)**3 + 3*\cos(x))/(48*\cos(x)**6 - 144*\cos(x)**4 + 144*\cos(x)**2 - 48) + \log(\cos(x) - 1)/32 - \log(\cos(x) + 1)/32$

Giac [A]

time = 0.00, size = 51, normalized size = 1.34

$$\frac{\ln(-\cos x + 1)}{32} - \frac{\ln(\cos x + 1)}{32} - \frac{-3\cos^5 x - 8\cos^3 x + 3\cos x}{48(\cos^2 x - 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^4*csc(x)^3,x)

[Out] 1/48*(3*cos(x)^5 + 8*cos(x)^3 - 3*cos(x))/(cos(x)^2 - 1)^3 - 1/32*log(cos(x) + 1) + 1/32*log(-cos(x) + 1)

Mupad [B]

time = 0.28, size = 57, normalized size = 1.50

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{16} + \frac{\frac{\tan\left(\frac{x}{2}\right)^4}{128} + \frac{\tan\left(\frac{x}{2}\right)^2}{128} - \frac{1}{384}}{\tan\left(\frac{x}{2}\right)^6} - \frac{\tan\left(\frac{x}{2}\right)^2}{128} - \frac{\tan\left(\frac{x}{2}\right)^4}{128} + \frac{\tan\left(\frac{x}{2}\right)^6}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)^4/sin(x)^3,x)

[Out] log(tan(x/2))/16 + (tan(x/2)^2/128 + tan(x/2)^4/128 - 1/384)/tan(x/2)^6 - tan(x/2)^2/128 - tan(x/2)^4/128 + tan(x/2)^6/384

3.362 $\int \sec^3\left(\frac{\pi}{4} + \frac{x}{2}\right) \tan^2\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$

Optimal. Leaf size=76

$$-\frac{1}{4} \tanh^{-1}\left(\sin\left(\frac{\pi}{4} + \frac{x}{2}\right)\right) - \frac{1}{4} \sec\left(\frac{\pi}{4} + \frac{x}{2}\right) \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + \frac{1}{2} \sec^3\left(\frac{\pi}{4} + \frac{x}{2}\right) \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

[Out] -1/4*arctanh(sin(1/4*Pi+1/2*x))-1/4*sec(1/4*Pi+1/2*x)*tan(1/4*Pi+1/2*x)+1/2*sec(1/4*Pi+1/2*x)^3*tan(1/4*Pi+1/2*x)

Rubi [A]

time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2691, 3853, 3855}

$$-\frac{1}{4} \tanh^{-1}\left(\sin\left(\frac{x}{2} + \frac{\pi}{4}\right)\right) + \frac{1}{2} \tan\left(\frac{x}{2} + \frac{\pi}{4}\right) \sec^3\left(\frac{x}{2} + \frac{\pi}{4}\right) - \frac{1}{4} \tan\left(\frac{x}{2} + \frac{\pi}{4}\right) \sec\left(\frac{x}{2} + \frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

[In] Int[Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2]^2,x]

[Out] -1/4*ArcTanh[Sin[Pi/4 + x/2]] - (Sec[Pi/4 + x/2]*Tan[Pi/4 + x/2])/4 + (Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2])/2

Rule 2691

Int[((a_.)*sec[(e_.) + (f_.)*(x_.)]^(m_.))*((b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \sec^3\left(\frac{\pi}{4} + \frac{x}{2}\right) \tan^2\left(\frac{\pi}{4} + \frac{x}{2}\right) dx &= \frac{1}{2} \sec^3\left(\frac{\pi}{4} + \frac{x}{2}\right) \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) - \frac{1}{4} \int \sec^3\left(\frac{\pi}{4} + \frac{x}{2}\right) dx \\ &= -\frac{1}{4} \sec\left(\frac{\pi}{4} + \frac{x}{2}\right) \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + \frac{1}{2} \sec^3\left(\frac{\pi}{4} + \frac{x}{2}\right) \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) - \frac{1}{8} \int \sec^3\left(\frac{\pi}{4} + \frac{x}{2}\right) dx \\ &= -\frac{1}{4} \tanh^{-1}\left(\sin\left(\frac{\pi}{4} + \frac{x}{2}\right)\right) - \frac{1}{4} \sec\left(\frac{\pi}{4} + \frac{x}{2}\right) \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + \frac{1}{2} \sec^3\left(\frac{\pi}{4} + \frac{x}{2}\right) \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) \end{aligned}$$

Mathematica [A]

time = 0.05, size = 74, normalized size = 0.97

$$-\frac{1}{4} \tanh^{-1}\left(\sin\left(\frac{\pi}{4} + \frac{x}{2}\right)\right) - \frac{1}{4} \sec^2\left(\frac{1}{4}(\pi + 2x)\right) \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) + \frac{1}{2} \sec^4\left(\frac{1}{4}(\pi + 2x)\right) \sin\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

Antiderivative was successfully verified.

`[In] Integrate[Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2]^2,x]``[Out] -1/4*ArcTanh[Sin[Pi/4 + x/2]] - (Sec[(Pi + 2*x)/4]^2*Sin[Pi/4 + x/2])/4 + (Sec[(Pi + 2*x)/4]^4*Sin[Pi/4 + x/2])/2`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Tan[Pi/4 + x/2]^2*Sec[Pi/4 + x/2]^3,x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.38, size = 76, normalized size = 1.00

method	result	size
derivativedivides	$\frac{\sin^3\left(\frac{\pi}{4} + \frac{x}{2}\right)}{2 \cos\left(\frac{\pi}{4} + \frac{x}{2}\right)^4} + \frac{\sin^3\left(\frac{\pi}{4} + \frac{x}{2}\right)}{4 \cos\left(\frac{\pi}{4} + \frac{x}{2}\right)^2} + \frac{\sin\left(\frac{\pi}{4} + \frac{x}{2}\right)}{4} - \frac{\ln\left(\sec\left(\frac{\pi}{4} + \frac{x}{2}\right) + \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right)}{4}$	76
default	$\frac{\sin^3\left(\frac{\pi}{4} + \frac{x}{2}\right)}{2 \cos\left(\frac{\pi}{4} + \frac{x}{2}\right)^4} + \frac{\sin^3\left(\frac{\pi}{4} + \frac{x}{2}\right)}{4 \cos\left(\frac{\pi}{4} + \frac{x}{2}\right)^2} + \frac{\sin\left(\frac{\pi}{4} + \frac{x}{2}\right)}{4} - \frac{\ln\left(\sec\left(\frac{\pi}{4} + \frac{x}{2}\right) + \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right)}{4}$	76
risch	$\frac{i\left(-(-1)^{\frac{3}{4}} e^{\frac{7ix}{2}} + 7(-1)^{\frac{1}{4}} e^{\frac{5ix}{2}} + 7(-1)^{\frac{3}{4}} e^{\frac{3ix}{2}} - (-1)^{\frac{1}{4}} e^{\frac{ix}{2}}\right)}{2(ie^{ix} + 1)^4} + \frac{\ln\left(e^{\frac{i(\pi+2x)}{4}} - i\right)}{4} - \frac{\ln\left(e^{\frac{i(\pi+2x)}{4}} + i\right)}{4}$	88

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sec(1/4*Pi+1/2*x)^3*tan(1/4*Pi+1/2*x)^2,x,method=_RETURNVERBOSE)`

[Out] $1/2*\sin(1/4*\pi+1/2*x)^3/\cos(1/4*\pi+1/2*x)^4+1/4*\sin(1/4*\pi+1/2*x)^3/\cos(1/4*\pi+1/2*x)^2+1/4*\sin(1/4*\pi+1/2*x)-1/4*\ln(\sec(1/4*\pi+1/2*x)+\tan(1/4*\pi+1/2*x))$

Maxima [A]

time = 0.34, size = 74, normalized size = 0.97

$$\frac{\sin\left(\frac{1}{4}\pi + \frac{1}{2}x\right)^3 + \sin\left(\frac{1}{4}\pi + \frac{1}{2}x\right)}{4\left(\sin\left(\frac{1}{4}\pi + \frac{1}{2}x\right)^4 - 2\sin\left(\frac{1}{4}\pi + \frac{1}{2}x\right)^2 + 1\right)} - \frac{1}{8}\log\left(\sin\left(\frac{1}{4}\pi + \frac{1}{2}x\right) + 1\right) + \frac{1}{8}\log\left(\sin\left(\frac{1}{4}\pi + \frac{1}{2}x\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(1/4*pi+1/2*x)^3*tan(1/4*pi+1/2*x)^2,x, algorithm="maxima")`

[Out] $1/4*(\sin(1/4*\pi + 1/2*x)^3 + \sin(1/4*\pi + 1/2*x))/(\sin(1/4*\pi + 1/2*x)^4 - 2*\sin(1/4*\pi + 1/2*x)^2 + 1) - 1/8*\log(\sin(1/4*\pi + 1/2*x) + 1) + 1/8*\log(\sin(1/4*\pi + 1/2*x) - 1)$

Fricas [A]

time = 0.33, size = 82, normalized size = 1.08

$$\frac{\cos\left(\frac{1}{4}\pi + \frac{1}{2}x\right)^4 \log\left(\sin\left(\frac{1}{4}\pi + \frac{1}{2}x\right) + 1\right) - \cos\left(\frac{1}{4}\pi + \frac{1}{2}x\right)^4 \log\left(-\sin\left(\frac{1}{4}\pi + \frac{1}{2}x\right) + 1\right) + 2\left(\cos\left(\frac{1}{4}\pi + \frac{1}{2}x\right)^2 - 2\right)\sin\left(\frac{1}{4}\pi + \frac{1}{2}x\right)}{8\cos\left(\frac{1}{4}\pi + \frac{1}{2}x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(1/4*pi+1/2*x)^3*tan(1/4*pi+1/2*x)^2,x, algorithm="fricas")`

[Out] $-1/8*(\cos(1/4*\pi + 1/2*x)^4*\log(\sin(1/4*\pi + 1/2*x) + 1) - \cos(1/4*\pi + 1/2*x)^4*\log(-\sin(1/4*\pi + 1/2*x) + 1) + 2*(\cos(1/4*\pi + 1/2*x)^2 - 2)*\sin(1/4*\pi + 1/2*x))/\cos(1/4*\pi + 1/2*x)^4$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \tan^2\left(\frac{x}{2} + \frac{\pi}{4}\right) \sec^3\left(\frac{x}{2} + \frac{\pi}{4}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(1/4*pi+1/2*x)**3*tan(1/4*pi+1/2*x)**2,x)`

[Out] `Integral(tan(x/2 + pi/4)**2*sec(x/2 + pi/4)**3, x)`

Giac [A]

time = 0.07, size = 109, normalized size = 1.43

$$4\left(\frac{\ln\left|\sin\left(\frac{\pi+2x}{4}\right) + \frac{1}{\sin\left(\frac{\pi+2x}{4}\right)} - 2\right|}{64} - \frac{\ln\left|\sin\left(\frac{\pi+2x}{4}\right) + \frac{1}{\sin\left(\frac{\pi+2x}{4}\right)} + 2\right|}{64} - \frac{-\sin\left(\frac{\pi+2x}{4}\right) - \frac{1}{\sin\left(\frac{\pi+2x}{4}\right)}}{16\left(\left(\sin\left(\frac{\pi+2x}{4}\right) + \frac{1}{\sin\left(\frac{\pi+2x}{4}\right)}\right)^2 - 4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(1/4*pi+1/2*x)^3*tan(1/4*pi+1/2*x)^2,x)

[Out] 1/4*(1/sin(1/4*pi + 1/2*x) + sin(1/4*pi + 1/2*x))/((1/sin(1/4*pi + 1/2*x) + sin(1/4*pi + 1/2*x))^2 - 4) - 1/16*log(abs(1/sin(1/4*pi + 1/2*x) + sin(1/4*pi + 1/2*x) + 2)) + 1/16*log(abs(1/sin(1/4*pi + 1/2*x) + sin(1/4*pi + 1/2*x) - 2))

Mupad [B]

time = 6.38, size = 75, normalized size = 0.99

$$\frac{2 \left(\frac{\tan\left(\frac{\pi}{8} + \frac{x}{4}\right)^7}{4} + \frac{7 \tan\left(\frac{\pi}{8} + \frac{x}{4}\right)^5}{4} + \frac{7 \tan\left(\frac{\pi}{8} + \frac{x}{4}\right)^3}{4} + \frac{\tan\left(\frac{\pi}{8} + \frac{x}{4}\right)}{4} \right)}{\left(\tan\left(\frac{\pi}{8} + \frac{x}{4}\right)^2 - 1 \right)^4} - \frac{\operatorname{atanh}\left(\tan\left(\frac{\pi}{8} + \frac{x}{4}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(Pi/4 + x/2)^2/cos(Pi/4 + x/2)^3,x)

[Out] (2*(tan(Pi/8 + x/4)/4 + (7*tan(Pi/8 + x/4)^3)/4 + (7*tan(Pi/8 + x/4)^5)/4 + tan(Pi/8 + x/4)^7/4))/(tan(Pi/8 + x/4)^2 - 1)^4 - atanh(tan(Pi/8 + x/4))/2

3.363 $\int (1 + \cot^3(x)) (a \sec^2(x) - \sin(2x))^2 dx$

Optimal. Leaf size=88

$$\frac{x}{2} + 4ax + 2 \cos^2(x) + \cos^4(x) + 4a \cot(x) - \frac{1}{2} a^2 \cot^2(x) + (4-a)a \log(\cos(x)) + (4+a^2) \log(\sin(x)) + \frac{1}{2} \cos(x) \sin(x)$$

[Out] 1/2*x+4*a*x+2*cos(x)^2+cos(x)^4+4*a*cot(x)-1/2*a^2*cot(x)^2+(4-a)*a*ln(cos(x))+(a^2+4)*ln(sin(x))+1/2*cos(x)*sin(x)-cos(x)^3*sin(x)+a^2*tan(x)+1/3*a^2*tan(x)^3

Rubi [A]

time = 0.37, antiderivative size = 84, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1819, 1816, 649, 209, 266}

$$\frac{1}{3} a^2 \tan^3(x) + a^2 \tan(x) - \frac{1}{2} a^2 \cot^2(x) + (a^2 + 4) \log(\tan(x)) + \frac{1}{2} (8a + 1)x + 4a \cot(x) + 4(a + 1) \log(\cos(x)) + \cos^4(x)(1 - \tan(x)) + \frac{1}{2} \cos^2(x)(\tan(x) + 4)$$

Antiderivative was successfully verified.

[In] Int[(1 + Cot[x]^3)*(a*Sec[x]^2 - Sin[2*x])^2,x]

[Out] ((1 + 8*a)*x)/2 + 4*a*Cot[x] - (a^2*Cot[x]^2)/2 + 4*(1 + a)*Log[Cos[x]] + (4 + a^2)*Log[Tan[x]] + Cos[x]^4*(1 - Tan[x]) + a^2*Tan[x] + (a^2*Tan[x]^3)/3 + (Cos[x]^2*(4 + Tan[x]))/2

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 649

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[(-a)*c]

Rule 1816

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]

&& PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1819

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (1 + \cot^3(x)) (a \sec^2(x) - \sin(2x))^2 dx &= \text{Subst} \left(\int \frac{(1 + x^3) (a - 2x + 2ax^2 + ax^4)^2}{x^3 (1 + x^2)^3} dx, x, \tan(x) \right) \\
&= \cos^4(x)(1 - \tan(x)) - \frac{1}{4} \text{Subst} \left(\int \frac{-4a^2 + 16ax - 4(4 + 3a^2)}{x^3 (1 + x^2)^3} dx, x, \tan(x) \right) \\
&= \cos^4(x)(1 - \tan(x)) + \frac{1}{2} \cos^2(x)(4 + \tan(x)) + \frac{1}{8} \text{Subst} \left(\int \frac{8a^2 - 16ax + 4(4 + 3a^2)}{x^3 (1 + x^2)^3} dx, x, \tan(x) \right) \\
&= \cos^4(x)(1 - \tan(x)) + \frac{1}{2} \cos^2(x)(4 + \tan(x)) + \frac{1}{8} \text{Subst} \left(\int \frac{8a^2 - 16ax + 4(4 + 3a^2)}{x^3 (1 + x^2)^3} dx, x, \tan(x) \right) \\
&= 4a \cot(x) - \frac{1}{2} a^2 \cot^2(x) + (4 + a^2) \log(\tan(x)) + \cos^4(x)(1 - \tan(x)) \\
&= 4a \cot(x) - \frac{1}{2} a^2 \cot^2(x) + (4 + a^2) \log(\tan(x)) + \cos^4(x)(1 - \tan(x)) \\
&= \frac{1}{2} (1 + 8a)x + 4a \cot(x) - \frac{1}{2} a^2 \cot^2(x) + 4(1 + a) \log(\cos(x)) - \frac{1}{2} a^2 \cot^2(x)
\end{aligned}$$

Mathematica [A]

time = 1.36, size = 127, normalized size = 1.44

$$\frac{2 \cos^3(x) \sin(x) (-a \sec^2(x) + \sin(2x))^2 (-96a \cot^2(x) - 8a^2(2 + \cos(2x)) \sec^2(x) - 3 \cot(x) (4x + 32ax + 12 \cos(2x) + \cos(4x) - 4a^2 \csc^2(x) + 32a \log(\cos(x)) - 8a^2 \log(\cos(x)) + 32 \log(\sin(x)) + 8a^2 \log(\sin(x)) - \sin(4x)))}{3(-4a + 2 \sin(2x) + \sin(4x))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Cot[x]^3)*(a*Sec[x]^2 - Sin[2*x])^2,x]

[Out] (-2*Cos[x]^3*Sin[x]*(-(a*Sec[x]^2) + Sin[2*x])^2*(-96*a*Cot[x]^2 - 8*a^2*(2 + Cos[2*x])*Sec[x]^2 - 3*Cot[x]*(4*x + 32*a*x + 12*Cos[2*x] + Cos[4*x] - 4*a^2*Csc[x]^2 + 32*a*Log[Cos[x]] - 8*a^2*Log[Cos[x]] + 32*Log[Sin[x]] + 8*a^2*Log[Sin[x]] - Sin[4*x])))/(3*(-4*a + 2*Sin[2*x] + Sin[4*x])^2)

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(a*Sec[x]^2 - Sin[2*x])^2*(Cot[x]^3 + 1),x]')``[Out] Exception raised: SystemError >> excessive stack use: stack is 8009 deep`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(80) = 160.

time = 0.34, size = 210, normalized size = 2.39

method	result
default	$\frac{2(\cos^8(x))}{\sin(x)^2} + 2(\cos^6(x)) + \cos^4(x) + 2(\cos^2(x)) + 4\ln(\sin(x)) - \frac{4(\cos^7(x))}{\sin(x)} - 4\left(\cos^5(x) + \frac{5(\cos^3(x))}{4}\right)$
risch	$\frac{x}{2} + \frac{ie^{4ix}}{16} + 4ax - \frac{ie^{-4ix}}{16} + \frac{e^{4ix}}{16} - 4ix + \frac{3e^{2ix}}{4} + \frac{3e^{-2ix}}{4} + \frac{e^{-4ix}}{16} - 4iax + \frac{2a(12ie^{8ix} + 3ae^{8ix} + 6iae^{6ix} + 24ie^{6ix})}{4}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x,method=_RETURNVERBOSE)`

```
[Out] 2/sin(x)^2*cos(x)^8+2*cos(x)^6+cos(x)^4+2*cos(x)^2+4*ln(sin(x))-4/sin(x)*cos(x)^7-4*(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)+1/2*x-2/sin(x)^2*cos(x)^6+8/sin(x)*cos(x)^5+8*(cos(x)^3+3/2*cos(x))*sin(x)-4*a*(-1/2*cot(x)^2-ln(sin(x)))-4*a*(-x-cot(x))-4*cot(x)-4*a/sin(x)^2+a^2*(-1/2/sin(x)^2+ln(tan(x)))-a^2*(1/cos(x)/sin(x)-2*cot(x))-4*a*(-1/2/sin(x)^2+ln(tan(x)))+a^2*(1/3/sin(x)/cos(x)^3+4/3/cos(x)/sin(x)-8/3*cot(x))
```

Maxima [A]

time = 0.35, size = 115, normalized size = 1.31

$$\frac{1}{3}(\tan(x)^3 + 3\tan(x))a^2 - \frac{1}{2}a^2\left(\frac{1}{\sin(x)^2} + \log(\sin(x)^2 - 1) - \log(\sin(x)^2)\right) + 4a\left(x + \frac{1}{\tan(x)}\right) + 2a\log(-\sin(x)^2 + 1) + \frac{1}{2}x + \frac{1}{8}\cos(4x) + \frac{3}{2}\cos(2x) + 2\log(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1) + 2\log(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1) - \frac{1}{8}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x, algorithm="maxima")`

```
[Out] 1/3*(tan(x)^3 + 3*tan(x))*a^2 - 1/2*a^2*(1/sin(x)^2 + log(sin(x)^2 - 1) - log(sin(x)^2)) + 4*a*(x + 1/tan(x)) + 2*a*log(-sin(x)^2 + 1) + 1/2*x + 1/8*cos(4*x) + 3/2*cos(2*x) + 2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 1/8*sin(4*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(79) = 158.

time = 0.47, size = 178, normalized size = 2.02

$$\frac{24\cos^9(x) + 24\cos^7(x) + 3(4(8a+1)x - 27)\cos^5(x) + 3(4a^2 - 4(8a+1)x + 11)\cos^3(x) - 12((a^2 - 4a)\cos(x)^5 - (a^2 - 4a)\cos(x)^3) \log(\cos(x)^2) + 12((a^2 + 4)\cos(x)^5 - (a^2 + 4)\cos(x)^3) \log(-\frac{1}{2}\cos(x)^2 + \frac{1}{2}) - 4(6\cos(x)^8 - 9\cos(x)^6 - (4a^2 - 24a - 3)\cos(x)^4 + 2a^2\cos(x)^2 + 2a^2)\sin(x)}{24(\cos(x)^3 - \cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x, algorithm="fricas")

[Out] $\frac{1}{24}(24\cos(x)^9 + 24\cos(x)^7 + 3(4(8a + 1)x - 27)\cos(x)^5 + 3(4a^2 - 4(8a + 1)x + 11)\cos(x)^3 - 12((a^2 - 4a)\cos(x)^5 - (a^2 - 4a)\cos(x)^3)\log(\cos(x)^2) + 12((a^2 + 4)\cos(x)^5 - (a^2 + 4)\cos(x)^3)\log(-\frac{1}{4}\cos(x)^2 + \frac{1}{4}) - 4(6\cos(x)^8 - 9\cos(x)^6 - (4a^2 - 24a - 3)\cos(x)^4 + 2a^2\cos(x)^2 + 2a^2)\sin(x))/(\cos(x)^5 - \cos(x)^3)$

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cot(x)**3)*(a*sec(x)**2-sin(2*x))**2,x)

[Out] Exception raised: SystemError

Giac [A]

time = 0.01, size = 166, normalized size = 1.89

$$2 \left(\frac{\frac{4}{3}\tan^3 x \cdot a^2 + 4\tan x \cdot a^2}{8} + \frac{-\tan^6 x \cdot a^2 + 4\tan^4 x \cdot a + 8\tan^2 x \cdot a + \tan^5 x \cdot 3\tan^4 x \cdot a^2 + 8\tan^3 x \cdot a + 4\tan^2 x + 16\tan^3 x \cdot a - \tan^3 x \cdot 3\tan^2 x \cdot a^2 + 4\tan^2 x \cdot a + 6\tan^2 x + 8\tan x \cdot a - a^2}{4(\tan^3 x + \tan x)} + \frac{1}{2}(a^2 + 4)\ln|\tan x| + (-a - 1)\ln(\tan^2 x + 1) + \frac{1}{4}(8a + 1)x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x)

[Out] $\frac{1}{3}a^2\tan(x)^3 + a^2\tan(x) + \frac{1}{2}(8a + 1)x - 2(a + 1)\log(\tan(x)^2 + 1) + (a^2 + 4)\log(\text{abs}(\tan(x))) - \frac{1}{2}(a^2\tan(x)^6 - 4a\tan(x)^6 + 3a^2\tan(x)^4 - 8a\tan(x)^5 - 8a\tan(x)^4 - \tan(x)^5 + 3a^2\tan(x)^2 - 16a\tan(x)^3 - 4\tan(x)^4 - 4a\tan(x)^2 + \tan(x)^3 + a^2 - 8a\tan(x) - 6\tan(x)^2)/(\tan(x)^3 + \tan(x))^2$

Mupad [B]

time = 0.38, size = 133, normalized size = 1.51

$$a^2 \tan(x) - \frac{\tan(x)^4 \left(\frac{a^2}{2} - 2 \right) - 4a \tan(x) + \frac{a^2}{2} - \tan(x)^5 \left(4a + \frac{1}{2} \right) - \tan(x)^3 \left(8a - \frac{1}{2} \right) + \tan(x)^2 (a^2 - 3)}{\tan(x)^6 + 2\tan(x)^4 + \tan(x)^2} - \ln(\tan(x) - i) \left(a(2 + 2i) + 2 + \frac{1}{4}i \right) - \ln(\tan(x) + i) \left(a(2 - 2i) + 2 - \frac{1}{4}i \right) + \frac{a^2 \tan(x)^3}{3} + \ln(\tan(x)) (a^2 + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(x)^3 + 1)*(sin(2*x) - a/cos(x)^2)^2,x)

[Out] $a^2\tan(x) - (\tan(x)^4(a^2/2 - 2) - 4a\tan(x) + a^2/2 - \tan(x)^5(4a + 1/2) - \tan(x)^3(8a - 1/2) + \tan(x)^2(a^2 - 3))/(\tan(x)^2 + 2\tan(x)^4 + \tan(x)^6) - \log(\tan(x) - i)(a(2 + 2i) + (2 + 1i/4)) - \log(\tan(x) + i)(a(2 - 2i) + (2 - 1i/4)) + (a^2\tan(x)^3)/3 + \log(\tan(x))(a^2 + 4)$

$$3.364 \quad \int (4 - 3 \cos(x)) \left(1 - \frac{\sin(x)}{2}\right)^4 dx$$

Optimal. Leaf size=70

$$\frac{227x}{32} + 10 \cos(x) - 3 \cos^2(x) - \frac{2 \cos^3(x)}{3} - 3 \sin(x) - \frac{99}{32} \cos(x) \sin(x) - \frac{3 \sin^3(x)}{2} - \frac{1}{16} \cos(x) \sin^3(x) + \frac{3 \sin^4(x)}{8} - \dots$$

[Out] 227/32*x+10*cos(x)-3*cos(x)^2-2/3*cos(x)^3-3*sin(x)-99/32*cos(x)*sin(x)-3/2*sin(x)^3-1/16*cos(x)*sin(x)^3+3/8*sin(x)^4-3/80*sin(x)^5

Rubi [A]

time = 0.10, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4486, 2717, 2747, 2748, 2715, 8, 655}

$$\frac{227x}{32} - \frac{3}{80} \sin^5(x) + \frac{3 \sin^4(x)}{8} - \frac{3 \sin^3(x)}{2} - 3 \sin(x) - \frac{2 \cos^3(x)}{3} - 3 \cos^2(x) + 10 \cos(x) - \frac{1}{16} \sin^3(x) \cos(x) - \frac{99}{32} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[(4 - 3*Cos[x])*(1 - Sin[x]/2)^4,x]

[Out] (227*x)/32 + 10*Cos[x] - 3*Cos[x]^2 - (2*Cos[x]^3)/3 - 3*Sin[x] - (99*Cos[x]*Sin[x])/32 - (3*Sin[x]^3)/2 - (Cos[x]*Sin[x]^3)/16 + (3*Sin[x]^4)/8 - (3*Sin[x]^5)/80

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 655

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[e*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1)/(d*n), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_.)]^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /; !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}
\int (4 - 3 \cos(x)) \left(1 - \frac{\sin(x)}{2}\right)^4 dx &= \int \left(4 - 3 \cos(x) + 2(-4 + 3 \cos(x)) \sin(x) - \frac{3}{2}(-4 + 3 \cos(x)) \sin^2(x)\right. \\
&= 4x - \frac{1}{16} \int (-4 + 3 \cos(x)) \sin^4(x) dx + \frac{1}{2} \int (-4 + 3 \cos(x)) \sin^3(x) dx \\
&= 4x - 3 \sin(x) - \frac{3 \sin^3(x)}{2} - \frac{3 \sin^5(x)}{80} - \frac{1}{54} \text{Subst} \left(\int (-4 + x) (9 - x) dx \right) \\
&= 4x + 8 \cos(x) - 3 \cos^2(x) - 3 \sin(x) - 3 \cos(x) \sin(x) - \frac{3 \sin^3(x)}{2} - \frac{3 \sin^5(x)}{80} \\
&= 7x + 10 \cos(x) - 3 \cos^2(x) - \frac{2 \cos^3(x)}{3} - 3 \sin(x) - \frac{99}{32} \cos(x) \sin(x) \\
&= \frac{227x}{32} + 10 \cos(x) - 3 \cos^2(x) - \frac{2 \cos^3(x)}{3} - 3 \sin(x) - \frac{99}{32} \cos(x) \sin(x)
\end{aligned}$$

Mathematica [A]

time = 0.05, size = 74, normalized size = 1.06

$$\frac{227x}{32} + \frac{19 \cos(x)}{2} - \frac{27}{16} \cos(2x) - \frac{1}{6} \cos(3x) + \frac{3}{64} \cos(4x) - \frac{531 \sin(x)}{128} - \frac{25}{16} \sin(2x) + \frac{99}{256} \sin(3x) + \frac{1}{128} \sin(4x) - \frac{3 \sin(5x)}{1280}$$

Antiderivative was successfully verified.

[In] Integrate[(4 - 3*Cos[x])*(1 - Sin[x]/2)^4,x]

[Out] $(227*x)/32 + (19*\cos[x])/2 - (27*\cos[2*x])/16 - \cos[3*x]/6 + (3*\cos[4*x])/64 - (531*\sin[x])/128 - (25*\sin[2*x])/16 + (99*\sin[3*x])/256 + \sin[4*x]/128 - (3*\sin[5*x])/1280$

Mathics [A]

time = 3.05, size = 65, normalized size = 0.93

$$-\frac{3}{2} + \frac{227x}{32} - \frac{33\sin[x]}{8} - \frac{25\sin[2x]}{16} - \frac{3\cos[2x]}{2} - \frac{\cos[3x]}{6} - \frac{3(-1 + \cos[2x])^2 \sin[x]}{320} + \frac{\sin[4x]}{128} + \frac{3(-1 + \cos[2x])^2}{32} + \frac{3\sin[3x]}{8} + \frac{19\cos[x]}{2}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(1 - 1/2*Sin[x])^4*(4 - 3*Cos[x]),x]')`

[Out] $-3 / 2 + 227 x / 32 - 33 \sin[x] / 8 - 25 \sin[2 x] / 16 - 3 \cos[2 x] / 2 - \cos[3 x] / 6 - 3 (-1 + \cos[2 x])^2 \sin[x] / 320 + \sin[4 x] / 128 + 3 (-1 + \cos[2 x])^2 / 32 + 3 \sin[3 x] / 8 + 19 \cos[x] / 2$

Maple [A]

time = 0.15, size = 66, normalized size = 0.94

method	result
risch	$\frac{227x}{32} + \frac{19\cos(x)}{2} - \frac{531\sin(x)}{128} - \frac{3\sin(5x)}{1280} + \frac{3\cos(4x)}{64} + \frac{\sin(4x)}{128} - \frac{\cos(3x)}{6} + \frac{99\sin(3x)}{256} - \frac{27\cos(2x)}{16} - \frac{25\sin(2x)}{16}$
default	$\frac{227x}{32} + 8\cos(x) - 3\cos(x)\sin(x) + \frac{2(2+\sin^2(x))\cos(x)}{3} - \frac{(\sin^3(x) + \frac{3\sin(x)}{2})\cos(x)}{16} - 3\sin(x) - 3(\cos^2(x) - \frac{1}{2})$
norman	$\frac{11(\tan^2(\frac{x}{2})) - \frac{151(\tan^8(\frac{x}{2}))}{3} - \frac{128(\tan^6(\frac{x}{2}))}{3} - \frac{47(\tan^{10}(\frac{x}{2}))}{3} + \frac{227x}{32} - \frac{391(\tan^3(\frac{x}{2}))}{8} - \frac{306(\tan^5(\frac{x}{2}))}{5} - \frac{185(\tan^7(\frac{x}{2}))}{8} + \frac{3(\tan^9(\frac{x}{2}))}{16} + \frac{1}{(1+\tan^2(\frac{x}{2}))^5}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4-3*cos(x))*(1-1/2*sin(x))^4,x,method=_RETURNVERBOSE)`

[Out] $227/32*x+8*\cos(x)-3*\cos(x)*\sin(x)+2/3*(2+\sin(x)^2)*\cos(x)-1/16*(\sin(x)^3+3/2*\sin(x))*\cos(x)-3*\sin(x)-3*\cos(x)^2-3/2*\sin(x)^3+3/8*\sin(x)^4-3/80*\sin(x)^5$

Maxima [A]

time = 0.26, size = 54, normalized size = 0.77

$$-\frac{3}{80}\sin(x)^5 + \frac{3}{8}\sin(x)^4 - \frac{2}{3}\cos(x)^3 - \frac{3}{2}\sin(x)^3 - 3\cos(x)^2 + \frac{227}{32}x + 10\cos(x) + \frac{1}{128}\sin(4x) - \frac{25}{16}\sin(2x) - 3\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4-3*cos(x))*(1-1/2*sin(x))^4,x, algorithm="maxima")`

[Out] $-3/80*\sin(x)^5 + 3/8*\sin(x)^4 - 2/3*\cos(x)^3 - 3/2*\sin(x)^3 - 3*\cos(x)^2 + 227/32*x + 10*\cos(x) + 1/128*\sin(4*x) - 25/16*\sin(2*x) - 3*\sin(x)$

Fricas [A]

time = 0.32, size = 54, normalized size = 0.77

$$\frac{3}{8} \cos(x)^4 - \frac{2}{3} \cos(x)^3 - \frac{15}{4} \cos(x)^2 - \frac{1}{160} (6 \cos(x)^4 - 10 \cos(x)^3 - 252 \cos(x)^2 + 505 \cos(x) + 726) \sin(x) + \frac{227}{32} x + 10 \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4-3*cos(x))*(1-1/2*sin(x))^4,x, algorithm="fricas")**[Out]** 3/8*cos(x)^4 - 2/3*cos(x)^3 - 15/4*cos(x)^2 - 1/160*(6*cos(x)^4 - 10*cos(x)^3 - 252*cos(x)^2 + 505*cos(x) + 726)*sin(x) + 227/32*x + 10*cos(x)**Sympy [A]**

time = 0.27, size = 148, normalized size = 2.11

$$\frac{3x \sin^4(x)}{32} + \frac{3x \sin^2(x) \cos^2(x)}{16} + 3x \sin^2(x) + \frac{3x \cos^4(x)}{32} + 3x \cos^2(x) + 4x - \frac{3 \sin^5(x)}{80} + \frac{3 \sin^4(x)}{8} - \frac{5 \sin^3(x) \cos(x)}{32} - \frac{3 \sin^2(x)}{2} + 2 \sin^2(x) \cos(x) - \frac{3 \sin(x) \cos^3(x)}{32} - 3 \sin(x) \cos(x) - 3 \sin(x) + \frac{4 \cos^3(x)}{3} - 3 \cos^2(x) + 8 \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4-3*cos(x))*(1-1/2*sin(x))**4,x)**[Out]** 3*x*sin(x)**4/32 + 3*x*sin(x)**2*cos(x)**2/16 + 3*x*sin(x)**2 + 3*x*cos(x)**4/32 + 3*x*cos(x)**2 + 4*x - 3*sin(x)**5/80 + 3*sin(x)**4/8 - 5*sin(x)**3*cos(x)/32 - 3*sin(x)**3/2 + 2*sin(x)**2*cos(x) - 3*sin(x)*cos(x)**3/32 - 3*sin(x)*cos(x) - 3*sin(x) + 4*cos(x)**3/3 - 3*cos(x)**2 + 8*cos(x)**Giac [A]**

time = 0.00, size = 100, normalized size = 1.43

$$-\frac{531}{128} \sin x + \frac{99}{256} \sin(3x) - \frac{27}{16} \cos(2x) - \frac{3}{1280} \sin(5x) + \frac{3}{64} \cos(4x) + \frac{83}{32} x - \frac{21}{16} \sin(2x) + \frac{11}{2} \cos x + \frac{\sin(4x)}{128} - \frac{\cos(3x)}{6} + \frac{x}{2} - \frac{\sin(2x)}{4} + 2 \cos x + 2 \cos x + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4-3*cos(x))*(1-1/2*sin(x))^4,x)**[Out]** 227/32*x + 3/64*cos(4*x) - 1/6*cos(3*x) - 27/16*cos(2*x) + 19/2*cos(x) - 3/1280*sin(5*x) + 1/128*sin(4*x) + 99/256*sin(3*x) - 25/16*sin(2*x) - 531/128*sin(x)**Mupad [B]**

time = 0.38, size = 94, normalized size = 1.34

$$-\frac{6 \sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right)^9}{5} + 6 \cos\left(\frac{x}{2}\right)^8 + \frac{17 \sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right)^7}{5} - \frac{52 \cos\left(\frac{x}{2}\right)^6}{3} + \frac{93 \sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right)^5}{10} + 2 \cos\left(\frac{x}{2}\right)^4 - \frac{191 \sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right)^3}{8} + 28 \cos\left(\frac{x}{2}\right)^2 + \frac{3 \sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right)}{16} + \frac{227x}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3*cos(x) - 4)*(sin(x)/2 - 1)^4,x)**[Out]** (227*x)/32 - (191*cos(x/2)^3*sin(x/2))/8 + (93*cos(x/2)^5*sin(x/2))/10 + (17*cos(x/2)^7*sin(x/2))/5 - (6*cos(x/2)^9*sin(x/2))/5 + 28*cos(x/2)^2 + 2*cos(x/2)^4 - (52*cos(x/2)^6)/3 + 6*cos(x/2)^8 + (3*cos(x/2)*sin(x/2))/16

3.365 $\int \left(\frac{1}{2} - 3 \cot(x)\right) (3 - 2 \cot(x))^3 dx$

Optimal. Leaf size=33

$$-\frac{285x}{2} + 5(3 - 2 \cot(x))^2 + (3 - 2 \cot(x))^3 - 42 \cot(x) + 4 \log(\sin(x))$$

[Out] $-285/2*x+5*(3-2*\cot(x))^2+(3-2*\cot(x))^3-42*\cot(x)+4*\ln(\sin(x))$

Rubi [A]

time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3609, 3606, 3556}

$$-\frac{285x}{2} + (3 - 2 \cot(x))^3 + 5(3 - 2 \cot(x))^2 - 42 \cot(x) + 4 \log(\sin(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1/2 - 3*\text{Cot}[x])*(3 - 2*\text{Cot}[x])^3, x]$

[Out] $(-285*x)/2 + 5*(3 - 2*\text{Cot}[x])^2 + (3 - 2*\text{Cot}[x])^3 - 42*\text{Cot}[x] + 4*\text{Log}[\text{Sin}[x]]$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x_Symbol] \text{ :> } \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] \text{ /; } \text{FreeQ}\{c, d\}, x]$

Rule 3606

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)], x_Symbol] \text{ :> } \text{Simp}[(a*c - b*d)*x, x] + (\text{Dist}[b*c + a*d, \text{Int}[\text{Tan}[e + f*x], x], x] + \text{Simp}[b*d*(\text{Tan}[e + f*x]/f), x]) \text{ /; } \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[b*c + a*d, 0]$

Rule 3609

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)}*(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)], x_Symbol] \text{ :> } \text{Simp}[d*((a + b*\text{Tan}[e + f*x])^m/(f*m)), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m - 1)}*\text{Simp}[a*c - b*d + (b*c + a*d)*\text{Tan}[e + f*x], x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{GtQ}[m, 0]$

Rubi steps

$$\begin{aligned}
\int \left(\frac{1}{2} - 3 \cot(x) \right) (3 - 2 \cot(x))^3 dx &= (3 - 2 \cot(x))^3 + \int \left(-\frac{9}{2} - 10 \cot(x) \right) (3 - 2 \cot(x))^2 dx \\
&= 5(3 - 2 \cot(x))^2 + (3 - 2 \cot(x))^3 + \int \left(-\frac{67}{2} - 21 \cot(x) \right) (3 - 2 \cot(x)) dx \\
&= -\frac{285x}{2} + 5(3 - 2 \cot(x))^2 + (3 - 2 \cot(x))^3 - 42 \cot(x) + 4 \int \cot(x) dx \\
&= -\frac{285x}{2} + 5(3 - 2 \cot(x))^2 + (3 - 2 \cot(x))^3 - 42 \cot(x) + 4 \log(\sin(x))
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 29, normalized size = 0.88

$$-\frac{285x}{2} - 148 \cot(x) + 56 \csc^2(x) - 8 \cot(x) \csc^2(x) + 4 \log(\sin(x))$$

Antiderivative was successfully verified.

`[In] Integrate[(1/2 - 3*Cot[x])*(3 - 2*Cot[x])^3, x]``[Out] (-285*x)/2 - 148*Cot[x] + 56*Csc[x]^2 - 8*Cot[x]*Csc[x]^2 + 4*Log[Sin[x]]`**Mathics [A]**

time = 2.18, size = 34, normalized size = 1.03

$$\frac{-285x}{2} - \frac{156}{\tan[x]} - \frac{8}{\tan[x]^3} - 2 \operatorname{Log} \left[\frac{1}{\cos[x]^2} \right] + 4 \operatorname{Log}[\tan[x]] + \frac{56}{\tan[x]^2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(3 - 2*Cot[x])^3*(1/2 - 3*Cot[x]), x]')``[Out] -285 x / 2 - 156 / Tan[x] - 8 / Tan[x]^3 - 2 Log[1 / Cos[x]^2] + 4 Log[Tan[x]] + 56 / Tan[x]^2`**Maple [A]**

time = 0.04, size = 35, normalized size = 1.06

method	result
derivativedivides	$-8(\cot^3(x)) + 56(\cot^2(x)) - 156 \cot(x) - 2 \ln(\cot^2(x) + 1) + \frac{285\pi}{4} - \frac{285 \operatorname{arccot}(\cot(x))}{2}$
default	$-8(\cot^3(x)) + 56(\cot^2(x)) - 156 \cot(x) - 2 \ln(\cot^2(x) + 1) + \frac{285\pi}{4} - \frac{285 \operatorname{arccot}(\cot(x))}{2}$
norman	$\frac{-8 - 156(\tan^2(x)) - \frac{285x(\tan^3(x))}{2} + 56 \tan(x)}{\tan(x)^3} + 4 \ln(\tan(x)) - 2 \ln(1 + \tan^2(x))$

risch	$-\frac{285x}{2} - 4ix + \frac{(-\frac{224}{1873} - \frac{264i}{1873})(1873e^{4ix} - 1260ie^{2ix} - 3358e^{2ix} + 1221 + 1036i)}{(e^{2ix} - 1)^3} + 4 \ln(e^{2ix} - 1)$
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/2-3*cot(x))*(3-2*cot(x))^3,x,method=_RETURNVERBOSE)`

[Out] $-8*\cot(x)^3+56*\cot(x)^2-156*\cot(x)-2*\ln(\cot(x)^2+1)+285/4*\text{Pi}-285/2*\text{arccot}(\cot(x))$

Maxima [A]

time = 0.35, size = 36, normalized size = 1.09

$$-\frac{285}{2}x - \frac{4(39 \tan(x)^2 - 14 \tan(x) + 2)}{\tan(x)^3} - 2 \log(\tan(x)^2 + 1) + 4 \log(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/2-3*cot(x))*(3-2*cot(x))^3,x, algorithm="maxima")`

[Out] $-285/2*x - 4*(39*\tan(x)^2 - 14*\tan(x) + 2)/\tan(x)^3 - 2*\log(\tan(x)^2 + 1) + 4*\log(\tan(x))$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(33) = 66$.

time = 0.33, size = 71, normalized size = 2.15

$$\frac{4(\cos(2x) - 1) \log(-\frac{1}{2} \cos(2x) + \frac{1}{2}) \sin(2x) - 296 \cos(2x)^2 - (285x \cos(2x) - 285x + 224) \sin(2x) + 32 \cos(2x) + 328}{2(\cos(2x) - 1) \sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/2-3*cot(x))*(3-2*cot(x))^3,x, algorithm="fricas")`

[Out] $1/2*(4*(\cos(2*x) - 1)*\log(-1/2*\cos(2*x) + 1/2)*\sin(2*x) - 296*\cos(2*x)^2 - (285*x*\cos(2*x) - 285*x + 224)*\sin(2*x) + 32*\cos(2*x) + 328)/((\cos(2*x) - 1)*\sin(2*x))$

Sympy [A]

time = 0.24, size = 39, normalized size = 1.18

$$-\frac{285x}{2} - 2 \log(\tan^2(x) + 1) + 4 \log(\tan(x)) - \frac{156}{\tan(x)} + \frac{56}{\tan^2(x)} - \frac{8}{\tan^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/2-3*cot(x))*(3-2*cot(x))**3,x)`

[Out] $-285*x/2 - 2*\log(\tan(x)**2 + 1) + 4*\log(\tan(x)) - 156/\tan(x) + 56/\tan(x)**2 - 8/\tan(x)**3$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(33) = 66$.
time = 0.01, size = 101, normalized size = 3.06

$$2 \left(\frac{4 \tan^3 \left(\frac{x}{2} \right) + 56 \tan^2 \left(\frac{x}{2} \right) + 300 \tan \left(\frac{x}{2} \right) + \frac{-22 \tan^3 \left(\frac{x}{2} \right) - 225 \tan^2 \left(\frac{x}{2} \right) + 42 \tan \left(\frac{x}{2} \right) - 3}{6 \tan^3 \left(\frac{x}{2} \right)} - 2 \ln \left(\tan^2 \left(\frac{x}{2} \right) + 1 \right) - \frac{285x}{2 \cdot 2} + 2 \ln \left| \tan \left(\frac{x}{2} \right) \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/2-3*cot(x))*(3-2*cot(x))^3,x)

[Out] $\tan(1/2*x)^3 + 14*\tan(1/2*x)^2 - 285/2*x - 1/3*(22*\tan(1/2*x)^3 + 225*\tan(1/2*x)^2 - 42*\tan(1/2*x) + 3)/\tan(1/2*x)^3 - 4*\log(\tan(1/2*x)^2 + 1) + 4*\log(\text{abs}(\tan(1/2*x))) + 75*\tan(1/2*x)$

Mupad [B]

time = 0.52, size = 75, normalized size = 2.27

$$x \left(-\frac{285}{2} - 4i \right) + 4 \ln \left(e^{x \cdot 2i} - 1 \right) + \frac{64i}{3e^{x \cdot 2i} - 3e^{x \cdot 4i} + e^{x \cdot 6i} - 1} + \frac{-224 + 96i}{1 + e^{x \cdot 4i} - 2e^{x \cdot 2i}} + \frac{-224 - 264i}{e^{x \cdot 2i} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*cot(x) - 3)^3*(3*cot(x) - 1/2),x)

[Out] $4*\log(\exp(x*2i) - 1) - x*(285/2 + 4i) + 64i/(3*\exp(x*2i) - 3*\exp(x*4i) + \exp(x*6i) - 1) - (224 - 96i)/(\exp(x*4i) - 2*\exp(x*2i) + 1) - (224 + 264i)/(\exp(x*2i) - 1)$

3.366 $\int \cos(5x) \sec^5(x) dx$

Optimal. Leaf size=16

$$16x - 15 \tan(x) + \frac{5 \tan^3(x)}{3}$$

[Out] 16*x-15*tan(x)+5/3*tan(x)^3

Rubi [A]

time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1167, 209}

$$16x + \frac{5 \tan^3(x)}{3} - 15 \tan(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[5*x]*Sec[x]^5,x]

[Out] 16*x - 15*Tan[x] + (5*Tan[x]^3)/3

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1167

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int \cos(5x) \sec^5(x) dx &= \text{Subst} \left(\int \frac{1 - 10x^2 + 5x^4}{1 + x^2} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \left(-15 + 5x^2 + \frac{16}{1 + x^2} \right) dx, x, \tan(x) \right) \\ &= -15 \tan(x) + \frac{5 \tan^3(x)}{3} + 16 \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \tan(x) \right) \\ &= 16x - 15 \tan(x) + \frac{5 \tan^3(x)}{3} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 20, normalized size = 1.25

$$16x - \frac{50 \tan(x)}{3} + \frac{5}{3} \sec^2(x) \tan(x)$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[5*x]*Sec[x]^5,x]``[Out] 16*x - (50*Tan[x])/3 + (5*Sec[x]^2*Tan[x])/3`**Mathics [A]**

time = 11.94, size = 14, normalized size = 0.88

$$16x - 15 \operatorname{Tan}[x] + \frac{5 \operatorname{Tan}[x]^3}{3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Cos[5*x]/Cos[x]^5,x]')``[Out] 16 x - 15 Tan[x] + 5 Tan[x]^3 / 3`**Maple [A]**

time = 0.12, size = 21, normalized size = 1.31

method	result	size
default	$16x - 5 \left(-\frac{2}{3} - \frac{\sec^2(x)}{3} \right) \tan(x) - 20 \tan(x)$	21
risch	$16x - \frac{20i(6e^{4ix} + 9e^{2ix} + 5)}{3(e^{2ix} + 1)^3}$	33

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(5*x)/cos(x)^5,x,method=_RETURNVERBOSE)``[Out] 16*x-5*(-2/3-1/3*sec(x)^2)*tan(x)-20*tan(x)`**Maxima [A]**

time = 0.35, size = 14, normalized size = 0.88

$$\frac{5}{3} \tan(x)^3 + 16x - 15 \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(5*x)/cos(x)^5,x, algorithm="maxima")``[Out] 5/3*tan(x)^3 + 16*x - 15*tan(x)`

Fricas [A]

time = 0.33, size = 26, normalized size = 1.62

$$\frac{48x \cos(x)^3 - 5(10 \cos(x)^2 - 1) \sin(x)}{3 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(5*x)/cos(x)^5,x, algorithm="fricas")``[Out] 1/3*(48*x*cos(x)^3 - 5*(10*cos(x)^2 - 1)*sin(x))/cos(x)^3`**Sympy [A]**

time = 10.30, size = 24, normalized size = 1.50

$$16x - \frac{20 \sin(x)}{\cos(x)} + \frac{5 \tan^3(x)}{3} + 5 \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(5*x)/cos(x)**5,x)``[Out] 16*x - 20*sin(x)/cos(x) + 5*tan(x)**3/3 + 5*tan(x)`**Giac [A]**

time = 0.00, size = 23, normalized size = 1.44

$$2 \left(\frac{\frac{20}{3} \tan^3 x - 60 \tan x}{8} + 8x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(5*x)/cos(x)^5,x)``[Out] 5/3*tan(x)^3 + 16*x - 15*tan(x)`**Mupad [B]**

time = 0.31, size = 26, normalized size = 1.62

$$\frac{48x \cos(x)^3 - 50 \sin(x) \cos(x)^2 + 5 \sin(x)}{3 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(5*x)/cos(x)^5,x)``[Out] (5*sin(x) + 48*x*cos(x)^3 - 50*cos(x)^2*sin(x))/(3*cos(x)^3)`

3.367 $\int \cos(4x) \sec(x) dx$

Optimal. Leaf size=12

$$\tanh^{-1}(\sin(x)) - \frac{8 \sin^3(x)}{3}$$

[Out] arctanh(sin(x))-8/3*sin(x)^3

Rubi [A]

time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4449, 1167, 212}

$$\tanh^{-1}(\sin(x)) - \frac{8 \sin^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[Cos[4*x]*Sec[x],x]

[Out] ArcTanh[Sin[x]] - (8*Sin[x]^3)/3

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1167

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 4449

Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))]^(n_), x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned}
\int \cos(4x) \sec(x) dx &= \text{Subst} \left(\int \frac{1 - 8x^2 + 8x^4}{1 - x^2} dx, x, \sin(x) \right) \\
&= \text{Subst} \left(\int \left(-8x^2 + \frac{1}{1 - x^2} \right) dx, x, \sin(x) \right) \\
&= -\frac{8}{3} \sin^3(x) + \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \sin(x) \right) \\
&= \tanh^{-1}(\sin(x)) - \frac{8 \sin^3(x)}{3}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 12, normalized size = 1.00

$$\tanh^{-1}(\sin(x)) - \frac{8 \sin^3(x)}{3}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[4*x]*Sec[x],x]``[Out] ArcTanh[Sin[x]] - (8*Sin[x]^3)/3`**Mathics [B]** Leaf count is larger than twice the leaf count of optimal. 27 vs. 2(12) = 24.
time = 2.75, size = 21, normalized size = 1.75

$$\frac{-8 \sin[x]^3}{3} - \frac{\text{Log}[-1 + \sin[x]]}{2} + \frac{\text{Log}[1 + \sin[x]]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Cos[4*x]/Cos[x],x]')``[Out] -8 Sin[x]^3 / 3 - Log[-1 + Sin[x]] / 2 + Log[1 + Sin[x]] / 2`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

time = 0.08, size = 22, normalized size = 1.83

method	result	size
default	$\ln(\sec(x) + \tan(x)) + \frac{8(2 + \cos^2(x)) \sin(x)}{3} - 8 \sin(x)$	22
risch	$i e^{ix} - i e^{-ix} + \ln(e^{ix} + i) - \ln(e^{ix} - i) + \frac{2 \sin(3x)}{3}$	44

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(4*x)/cos(x),x,method=_RETURNVERBOSE)`

[Out] $\ln(\sec(x)+\tan(x))+8/3*(2+\cos(x)^2)*\sin(x)-8*\sin(x)$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.

time = 0.27, size = 21, normalized size = 1.75

$$-\frac{8}{3} \sin(x)^3 + \frac{1}{2} \log(\sin(x) + 1) - \frac{1}{2} \log(\sin(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(4*x)/cos(x),x, algorithm="maxima")`

[Out] $-8/3*\sin(x)^3 + 1/2*\log(\sin(x) + 1) - 1/2*\log(\sin(x) - 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

time = 0.32, size = 27, normalized size = 2.25

$$\frac{8}{3} (\cos(x)^2 - 1) \sin(x) + \frac{1}{2} \log(\sin(x) + 1) - \frac{1}{2} \log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(4*x)/cos(x),x, algorithm="fricas")`

[Out] $8/3*(\cos(x)^2 - 1)*\sin(x) + 1/2*\log(\sin(x) + 1) - 1/2*\log(-\sin(x) + 1)$

Sympy [A]

time = 0.82, size = 24, normalized size = 2.00

$$-\frac{\log(\sin(x) - 1)}{2} + \frac{\log(\sin(x) + 1)}{2} - \frac{8 \sin^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(4*x)/cos(x),x)`

[Out] $-\log(\sin(x) - 1)/2 + \log(\sin(x) + 1)/2 - 8*\sin(x)**3/3$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.
time = 0.00, size = 32, normalized size = 2.67

$$2 \left(-\frac{\ln(-\sin x + 1)}{4} + \frac{\ln(\sin x + 1)}{4} - \frac{1}{8} \cdot \frac{32}{3} \sin^3 x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(4*x)/cos(x),x)`

[Out] $-8/3*\sin(x)^3 + 1/2*\log(\sin(x) + 1) - 1/2*\log(-\sin(x) + 1)$

Mupad [B]

time = 0.27, size = 10, normalized size = 0.83

$$\operatorname{atanh}(\sin(x)) - \frac{8 \sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(4*x)/cos(x),x)`

[Out] `atanh(sin(x)) - (8*sin(x)^3)/3`

3.368 $\int \cos(x) \cos(4x) dx$

Optimal. Leaf size=17

$$\frac{1}{6} \sin(3x) + \frac{1}{10} \sin(5x)$$

[Out] 1/6*sin(3*x)+1/10*sin(5*x)

Rubi [A]

time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4368}

$$\frac{1}{6} \sin(3x) + \frac{1}{10} \sin(5x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cos[4*x],x]

[Out] Sin[3*x]/6 + Sin[5*x]/10

Rule 4368

Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\int \cos(x) \cos(4x) dx = \frac{1}{6} \sin(3x) + \frac{1}{10} \sin(5x)$$

Mathematica [A]

time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6} \sin(3x) + \frac{1}{10} \sin(5x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cos[4*x],x]

[Out] Sin[3*x]/6 + Sin[5*x]/10

Mathics [A]

time = 2.05, size = 18, normalized size = 1.06

$$\frac{(15 - 40\sin[x]^2 + 24\sin[x]^4) \sin[x]}{15}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[Cos[4*x]*Cos[x],x]')`

[Out] $(15 - 40 \sin[x]^2 + 24 \sin[x]^4) \sin[x] / 15$

Maple [A]

time = 0.07, size = 14, normalized size = 0.82

method	result	size
default	$\frac{\sin(3x)}{6} + \frac{\sin(5x)}{10}$	14
risch	$\frac{\sin(3x)}{6} + \frac{\sin(5x)}{10}$	14
norman	$\frac{-\frac{8 \tan(2x) \left(\tan^2\left(\frac{x}{2}\right)\right)}{15} + \frac{2 \left(\tan^2(2x)\right) \tan\left(\frac{x}{2}\right)}{15} + \frac{8 \tan(2x)}{15} - \frac{2 \tan\left(\frac{x}{2}\right)}{15}}{(1 + \tan^2\left(\frac{x}{2}\right))(1 + \tan^2(2x))}$	59

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*cos(4*x),x,method=_RETURNVERBOSE)`

[Out] $1/6*\sin(3*x)+1/10*\sin(5*x)$

Maxima [A]

time = 0.25, size = 13, normalized size = 0.76

$$\frac{1}{10} \sin(5x) + \frac{1}{6} \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(4*x),x, algorithm="maxima")`

[Out] $1/10*\sin(5*x) + 1/6*\sin(3*x)$

Fricas [A]

time = 0.33, size = 18, normalized size = 1.06

$$\frac{1}{15} (24 \cos(x)^4 - 8 \cos(x)^2 - 1) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(4*x),x, algorithm="fricas")`

[Out] $1/15*(24*\cos(x)^4 - 8*\cos(x)^2 - 1)*\sin(x)$

Sympy [A]

time = 0.14, size = 20, normalized size = 1.18

$$-\frac{\sin(x) \cos(4x)}{15} + \frac{4 \sin(4x) \cos(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(4*x),x)`

[Out] `-sin(x)*cos(4*x)/15 + 4*sin(4*x)*cos(x)/15`

Giac [A]

time = 0.00, size = 15, normalized size = 0.88

$$\frac{\sin(3x)}{6} + \frac{\sin(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(4*x),x)`

[Out] `1/10*sin(5*x) + 1/6*sin(3*x)`

Mupad [B]

time = 0.19, size = 13, normalized size = 0.76

$$\frac{\sin(3x)}{6} + \frac{\sin(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(4*x)*cos(x),x)`

[Out] `sin(3*x)/6 + sin(5*x)/10`

3.369 $\int \cos(4x) \sec^5(x) dx$

Optimal. Leaf size=26

$$\frac{35}{8} \tanh^{-1}(\sin(x)) - \frac{29}{8} \sec(x) \tan(x) + \frac{1}{4} \sec^3(x) \tan(x)$$

[Out] 35/8*arctanh(sin(x))-29/8*sec(x)*tan(x)+1/4*sec(x)^3*tan(x)

Rubi [A]

time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4449, 1171, 393, 212}

$$\frac{35}{8} \tanh^{-1}(\sin(x)) + \frac{1}{4} \tan(x) \sec^3(x) - \frac{29}{8} \tan(x) \sec(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[4*x]*Sec[x]^5,x]

[Out] (35*ArcTanh[Sin[x]])/8 - (29*Sec[x]*Tan[x])/8 + (Sec[x]^3*Tan[x])/4

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 393

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(-b*c - a*d)*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 1171

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 4449


```
Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))]^(n_), x_Symbol] := With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\int \cos(4x) \sec^5(x) dx &= \text{Subst} \left(\int \frac{1 - 8x^2 + 8x^4}{(1 - x^2)^3} dx, x, \sin(x) \right) \\
&= \frac{1}{4} \sec^3(x) \tan(x) - \frac{1}{4} \text{Subst} \left(\int \frac{-3 + 32x^2}{(1 - x^2)^2} dx, x, \sin(x) \right) \\
&= -\frac{29}{8} \sec(x) \tan(x) + \frac{1}{4} \sec^3(x) \tan(x) + \frac{35}{8} \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \sin(x) \right) \\
&= \frac{35}{8} \tanh^{-1}(\sin(x)) - \frac{29}{8} \sec(x) \tan(x) + \frac{1}{4} \sec^3(x) \tan(x)
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 26, normalized size = 1.00

$$\frac{1}{8} (35 \tanh^{-1}(\sin(x)) - 27 \sec^3(x) \tan(x) + 29 \sec(x) \tan^3(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[4*x]*Sec[x]^5,x]
```

```
[Out] (35*ArcTanh[Sin[x]] - 27*Sec[x]^3*Tan[x] + 29*Sec[x]*Tan[x]^3)/8
```

Mathics [A]

time = 12.67, size = 46, normalized size = 1.77

$$\frac{-64 \cos^2[x] \sin[x] + 2(5 - 3 \sin^2[x]) \sin[x] + 35(\log[1 + \sin[x]] - \log[-1 + \sin[x]]) \cos^4[x]}{16 \cos^4[x]}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Cos[4*x]/Cos[x]^5,x]')
```

```
[Out] (-64 Cos[x]^2 Sin[x] + 2(5 - 3 Sin[x]^2) Sin[x] + 35(Log[1 + Sin[x]] -
Log[-1 + Sin[x]]) Cos[x]^4) / (16 Cos[x]^4)
```

Maple [A]

time = 0.11, size = 31, normalized size = 1.19

method	result	size
default	$-\left(-\frac{\sec^3(x)}{4} - \frac{3\sec(x)}{8}\right)\tan(x) + \frac{35\ln(\sec(x)+\tan(x))}{8} - 4\sec(x)\tan(x)$	31
risch	$\frac{i(29e^{7ix}+21e^{5ix}-21e^{3ix}-29e^{ix})}{4(e^{2ix}+1)^4} - \frac{35\ln(e^{ix}-i)}{8} + \frac{35\ln(e^{ix}+i)}{8}$	65

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(4*x)/cos(x)^5,x,method=_RETURNVERBOSE)`

[Out] $-(1/4*\sec(x)^3-3/8*\sec(x))*\tan(x)+35/8*\ln(\sec(x)+\tan(x))-4*\sec(x)*\tan(x)$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(20) = 40$.

time = 0.25, size = 54, normalized size = 2.08

$$\frac{5\sin(x)^3 - 3\sin(x)}{8(\sin(x)^4 - 2\sin(x)^2 + 1)} + \frac{3\sin(x)}{\sin(x)^2 - 1} + \frac{35}{16}\log(\sin(x) + 1) - \frac{35}{16}\log(\sin(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(4*x)/cos(x)^5,x, algorithm="maxima")`

[Out] $1/8*(5*\sin(x)^3 - 3*\sin(x))/(\sin(x)^4 - 2*\sin(x)^2 + 1) + 3*\sin(x)/(\sin(x)^2 - 1) + 35/16*\log(\sin(x) + 1) - 35/16*\log(\sin(x) - 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(20) = 40$.

time = 0.35, size = 43, normalized size = 1.65

$$\frac{35\cos(x)^4\log(\sin(x) + 1) - 35\cos(x)^4\log(-\sin(x) + 1) - 2(29\cos(x)^2 - 2)\sin(x)}{16\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(4*x)/cos(x)^5,x, algorithm="fricas")`

[Out] $1/16*(35*\cos(x)^4*\log(\sin(x) + 1) - 35*\cos(x)^4*\log(-\sin(x) + 1) - 2*(29*\cos(x)^2 - 2)*\sin(x))/\cos(x)^4$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(27) = 54$

time = 10.24, size = 75, normalized size = 2.88

$$-\frac{35\log(\sin(x) - 1)}{16} + \frac{35\log(\sin(x) + 1)}{16} - \frac{3\sin^3(x)}{8\sin^4(x) - 16\sin^2(x) + 8} + \frac{5\sin(x)}{8\sin^4(x) - 16\sin^2(x) + 8} + \frac{8\sin(x)}{2\sin^2(x) - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(4*x)/cos(x)**5,x)

[Out] $-35 \cdot \log(\sin(x) - 1)/16 + 35 \cdot \log(\sin(x) + 1)/16 - 3 \cdot \sin(x)^3 / (8 \cdot \sin(x)^4 - 16 \cdot \sin(x)^2 + 8) + 5 \cdot \sin(x) / (8 \cdot \sin(x)^4 - 16 \cdot \sin(x)^2 + 8) + 8 \cdot \sin(x) / (2 \cdot \sin(x)^2 - 2)$

Giac [A]

time = 0.00, size = 47, normalized size = 1.81

$$2 \left(-\frac{35}{32} \ln(-\sin x + 1) + \frac{35}{32} \ln(\sin x + 1) - \frac{-29 \sin^3 x + 27 \sin x}{16 (\sin^2 x - 1)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(4*x)/cos(x)^5,x)

[Out] $1/8 \cdot (29 \cdot \sin(x)^3 - 27 \cdot \sin(x)) / (\sin(x)^2 - 1)^2 + 35/16 \cdot \log(\sin(x) + 1) - 35/16 \cdot \log(-\sin(x) + 1)$

Mupad [B]

time = 0.23, size = 33, normalized size = 1.27

$$\frac{35 \operatorname{atanh}(\sin(x))}{8} - \frac{\frac{27 \sin(x)}{8} - \frac{29 \sin(x)^3}{8}}{\sin(x)^4 - 2 \sin(x)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(4*x)/cos(x)^5,x)

[Out] $(35 \cdot \operatorname{atanh}(\sin(x))) / 8 - ((27 \cdot \sin(x)) / 8 - (29 \cdot \sin(x)^3) / 8) / (\sin(x)^4 - 2 \cdot \sin(x)^2 + 1)$

3.370 $\int \cos^4(x) \cos(4x) dx$

Optimal. Leaf size=38

$$\frac{x}{16} + \frac{1}{8} \sin(2x) + \frac{3}{32} \sin(4x) + \frac{1}{24} \sin(6x) + \frac{1}{128} \sin(8x)$$

[Out] 1/16*x+1/8*sin(2*x)+3/32*sin(4*x)+1/24*sin(6*x)+1/128*sin(8*x)

Rubi [A]

time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$,

Rules used = {4439, 2717}

$$\frac{x}{16} + \frac{1}{8} \sin(2x) + \frac{3}{32} \sin(4x) + \frac{1}{24} \sin(6x) + \frac{1}{128} \sin(8x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^4*Cos[4*x],x]

[Out] x/16 + Sin[2*x]/8 + (3*Sin[4*x])/32 + Sin[6*x]/24 + Sin[8*x]/128

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 4439

Int[(F_)[(a_.) + (b_.)*(x_.)]^(p_.)*(G_)[(c_.) + (d_.)*(x_.)]^(q_.), x_Symbol] := Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /;
FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \cos^4(x) \cos(4x) dx &= \int \left(\frac{1}{16} + \frac{1}{4} \cos(2x) + \frac{3}{8} \cos(4x) + \frac{1}{4} \cos(6x) + \frac{1}{16} \cos(8x) \right) dx \\ &= \frac{x}{16} + \frac{1}{16} \int \cos(8x) dx + \frac{1}{4} \int \cos(2x) dx + \frac{1}{4} \int \cos(6x) dx + \frac{3}{8} \int \cos(4x) dx \\ &= \frac{x}{16} + \frac{1}{8} \sin(2x) + \frac{3}{32} \sin(4x) + \frac{1}{24} \sin(6x) + \frac{1}{128} \sin(8x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 38, normalized size = 1.00

$$\frac{x}{16} + \frac{1}{8} \sin(2x) + \frac{3}{32} \sin(4x) + \frac{1}{24} \sin(6x) + \frac{1}{128} \sin(8x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^4*Cos[4*x],x]

[Out] x/16 + Sin[2*x]/8 + (3*Sin[4*x])/32 + Sin[6*x]/24 + Sin[8*x]/128

Mathics [A]

time = 4.98, size = 48, normalized size = 1.26

$$\frac{x}{16} - \frac{5\sin[2x]}{24} - \frac{(1 - \cos[2x])^3 \sin[2x]}{16} + \frac{65\sin[4x]}{192} + \frac{17(1 - \cos[2x])^2 \sin[2x]}{48}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Cos[4*x]*Cos[x]^4,x]')

[Out] x / 16 - 5 Sin[2 x] / 24 - (1 - Cos[2 x]) ^ 3 Sin[2 x] / 16 + 65 Sin[4 x] / 192 + 17 (1 - Cos[2 x]) ^ 2 Sin[2 x] / 48

Maple [A]

time = 0.09, size = 29, normalized size = 0.76

method	result	size
default	$\frac{x}{16} + \frac{\sin(2x)}{8} + \frac{3\sin(4x)}{32} + \frac{\sin(6x)}{24} + \frac{\sin(8x)}{128}$	29
risch	$\frac{x}{16} + \frac{\sin(2x)}{8} + \frac{3\sin(4x)}{32} + \frac{\sin(6x)}{24} + \frac{\sin(8x)}{128}$	29

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^4*cos(4*x),x,method=_RETURNVERBOSE)

[Out] 1/16*x+1/8*sin(2*x)+3/32*sin(4*x)+1/24*sin(6*x)+1/128*sin(8*x)

Maxima [A]

time = 0.28, size = 30, normalized size = 0.79

$$-\frac{1}{6} \sin(2x)^3 + \frac{1}{16} x + \frac{1}{128} \sin(8x) + \frac{3}{32} \sin(4x) + \frac{1}{4} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^4*cos(4*x),x, algorithm="maxima")

[Out] -1/6*sin(2*x)^3 + 1/16*x + 1/128*sin(8*x) + 3/32*sin(4*x) + 1/4*sin(2*x)

Fricas [A]

time = 0.34, size = 31, normalized size = 0.82

$$\frac{1}{48} (48 \cos(x)^7 - 8 \cos(x)^5 + 2 \cos(x)^3 + 3 \cos(x)) \sin(x) + \frac{1}{16} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^4*cos(4*x),x, algorithm="fricas")

[Out] 1/48*(48*cos(x)^7 - 8*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 1/16*x

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(31) = 62

time = 2.29, size = 139, normalized size = 3.66

$$\frac{x \sin^4(x) \cos(4x)}{16} - \frac{x \sin^3(x) \sin(4x) \cos(x)}{4} - \frac{3x \sin^2(x) \cos^2(x) \cos(4x)}{8} + \frac{x \sin(x) \sin(4x) \cos^3(x)}{4} + \frac{x \cos^4(x) \cos(4x)}{16} - \frac{\sin^4(x) \sin(4x)}{24} - \frac{5 \sin^3(x) \cos(x) \cos(4x)}{48} - \frac{11 \sin(x) \cos^3(x) \cos(4x)}{48} + \frac{7 \sin(4x) \cos^4(x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**4*cos(4*x),x)

[Out] x*sin(x)**4*cos(4*x)/16 - x*sin(x)**3*sin(4*x)*cos(x)/4 - 3*x*sin(x)**2*cos(x)**2*cos(4*x)/8 + x*sin(x)*sin(4*x)*cos(x)**3/4 + x*cos(x)**4*cos(4*x)/16 - sin(x)**4*sin(4*x)/24 - 5*sin(x)**3*cos(x)*cos(4*x)/48 - 11*sin(x)*cos(x)**3*cos(4*x)/48 + 7*sin(4*x)*cos(x)**4/24

Giac [A]

time = 0.00, size = 35, normalized size = 0.92

$$\frac{x}{16} + \frac{\sin(2x)}{8} + \frac{3}{32} \sin(4x) + \frac{\sin(6x)}{24} + \frac{\sin(8x)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^4*cos(4*x),x)

[Out] 1/16*x + 1/128*sin(8*x) + 1/24*sin(6*x) + 3/32*sin(4*x) + 1/8*sin(2*x)

Mupad [B]

time = 0.30, size = 36, normalized size = 0.95

$$\frac{x}{16} + \frac{\frac{\tan(x)^7}{16} + \frac{11 \tan(x)^5}{48} + \frac{5 \tan(x)^3}{48} + \frac{15 \tan(x)}{16}}{(\tan(x)^2 + 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(4*x)*cos(x)^4,x)

[Out] x/16 + ((15*tan(x))/16 + (5*tan(x)^3)/48 + (11*tan(x)^5)/48 + tan(x)^7/16)/(tan(x)^2 + 1)^4

3.371 $\int \cos(5x) \csc^5(x) dx$

Optimal. Leaf size=20

$$6 \csc^2(x) - \frac{\csc^4(x)}{4} + 16 \log(\sin(x))$$

[Out] 6*csc(x)^2-1/4*csc(x)^4+16*ln(sin(x))

Rubi [A]

time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4451, 1261, 712}

$$-\frac{1}{4} \csc^4(x) + 6 \csc^2(x) + 16 \log(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[5*x]*Csc[x]^5,x]

[Out] 6*Csc[x]^2 - Csc[x]^4/4 + 16*Log[Sin[x]]

Rule 712

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rule 1261

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rule 4451

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned}
\int \cos(5x) \csc^5(x) dx &= -\text{Subst} \left(\int \frac{x(5 - 20x^2 + 16x^4)}{(1 - x^2)^3} dx, x, \cos(x) \right) \\
&= -\left(\frac{1}{2} \text{Subst} \left(\int \frac{5 - 20x + 16x^2}{(1 - x)^3} dx, x, \cos^2(x) \right) \right) \\
&= -\left(\frac{1}{2} \text{Subst} \left(\int \left(-\frac{1}{(-1 + x)^3} - \frac{12}{(-1 + x)^2} - \frac{16}{-1 + x} \right) dx, x, \cos^2(x) \right) \right) \\
&= 6 \csc^2(x) - \frac{\csc^4(x)}{4} + 16 \log(\sin(x))
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 20, normalized size = 1.00

$$6 \csc^2(x) - \frac{\csc^4(x)}{4} + 16 \log(\sin(x))$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[5*x]*Csc[x]^5,x]``[Out] 6*Csc[x]^2 - Csc[x]^4/4 + 16*Log[Sin[x]]`**Mathics [A]**

time = 15.82, size = 20, normalized size = 1.00

$$-\frac{1}{4\text{Sin}[x]^4} + \frac{6}{\text{Sin}[x]^2} + 8\text{Log}[\text{Sin}[x]^2]$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Cos[5*x]/Sin[x]^5,x]')``[Out] -1 / (4 Sin[x]^4) + 6 / Sin[x]^2 + 8 Log[Sin[x]^2]`**Maple [A]**

time = 0.08, size = 35, normalized size = 1.75

method	result	size
default	$\frac{5(\cos^4(x))}{\sin(x)^4} - \frac{5}{4\sin(x)^4} - 4(\cot^4(x)) + 8(\cot^2(x)) + 16 \ln(\sin(x))$	35
risch	$-16ix - \frac{4(6e^{6ix} - 11e^{4ix} + 6e^{2ix})}{(e^{2ix} - 1)^4} + 16 \ln(e^{2ix} - 1)$	49

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(5*x)/sin(x)^5,x,method=_RETURNVERBOSE)`

[Out] $5/\sin(x)^4 \cos(x)^4 - 5/4/\sin(x)^4 - 4 \cot(x)^4 + 8 \cot(x)^2 + 16 \ln(\sin(x))$

Maxima [A]

time = 0.28, size = 33, normalized size = 1.65

$$\frac{5}{\sin(x)^2} + \frac{4 \sin(x)^2 - 1}{4 \sin(x)^4} + \frac{11}{2} \log(\sin(x)^2) + 5 \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(5*x)/sin(x)^5,x, algorithm="maxima")`

[Out] $5/\sin(x)^2 + 1/4*(4*\sin(x)^2 - 1)/\sin(x)^4 + 11/2*\log(\sin(x)^2) + 5*\log(\sin(x))$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(18) = 36.

time = 0.33, size = 43, normalized size = 2.15

$$-\frac{24 \cos(x)^2 - 64 (\cos(x)^4 - 2 \cos(x)^2 + 1) \log\left(\frac{1}{2} \sin(x)\right) - 23}{4 (\cos(x)^4 - 2 \cos(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(5*x)/sin(x)^5,x, algorithm="fricas")`

[Out] $-1/4*(24*\cos(x)^2 - 64*(\cos(x)^4 - 2*\cos(x)^2 + 1)*\log(1/2*\sin(x)) - 23)/(\cos(x)^4 - 2*\cos(x)^2 + 1)$

Sympy [A]

time = 13.29, size = 22, normalized size = 1.10

$$8 \log(\sin^2(x)) + \frac{6}{\sin^2(x)} - \frac{1}{4 \sin^4(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(5*x)/sin(x)**5,x)`

[Out] $8*\log(\sin(x)**2) + 6/\sin(x)**2 - 1/(4*\sin(x)**4)$

Giac [A]

time = 0.00, size = 23, normalized size = 1.15

$$\frac{24 \sin^2 x - 1}{4 \sin^4 x} + 16 \ln |\sin x|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(5*x)/sin(x)^5,x)`

[Out] $\frac{1}{4}*(24*\sin(x)^2 - 1)/\sin(x)^4 + 16*\log(\text{abs}(\sin(x)))$

Mupad [B]

time = 0.07, size = 21, normalized size = 1.05

$$8 \ln(\sin(x)^2) + \frac{6 \sin(x)^2 - \frac{1}{4}}{\sin(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(5*x)/sin(x)^5,x)`

[Out] $8*\log(\sin(x)^2) + (6*\sin(x)^2 - 1/4)/\sin(x)^4$

3.372 $\int \csc^4(x) \sin(4x) dx$

Optimal. Leaf size=12

$$-2 \csc^2(x) - 8 \log(\sin(x))$$

[Out] $-2*\csc(x)^2-8*\ln(\sin(x))$

Rubi [A]

time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$,

Rules used = {14}

$$-2 \csc^2(x) - 8 \log(\sin(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[x]^4*\text{Sin}[4*x], x]$

[Out] $-2*\text{Csc}[x]^2 - 8*\text{Log}[\text{Sin}[x]]$

Rule 14

$\text{Int}[(u_*)((c_)*(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}\{c, m\}, x] \ \&\& \ \text{SumQ}[u] \ \&\& \ !\text{LinearQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (a_)+ (b_.)*(v_)] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{InverseFunctionQ}[v]$

Rubi steps

$$\begin{aligned} \int \csc^4(x) \sin(4x) dx &= \text{Subst} \left(\int \frac{4 - 8x^2}{x^3} dx, x, \sin(x) \right) \\ &= \text{Subst} \left(\int \left(\frac{4}{x^3} - \frac{8}{x} \right) dx, x, \sin(x) \right) \\ &= -2 \csc^2(x) - 8 \log(\sin(x)) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 12, normalized size = 1.00

$$-2 \csc^2(x) - 8 \log(\sin(x))$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Csc}[x]^4*\text{Sin}[4*x], x]$

[Out] $-2*\text{Csc}[x]^2 - 8*\text{Log}[\text{Sin}[x]]$

Mathics [A]

time = 5.14, size = 12, normalized size = 1.00

$$-8\text{Log}[\text{Sin}[x]] - \frac{2}{\text{Sin}[x]^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sin[4*x]/Sin[x]^4,x]')
```

```
[Out] -8 Log[Sin[x]] - 2 / Sin[x]^2
```

Maple [A]

time = 0.08, size = 19, normalized size = 1.58

method	result	size
default	$\frac{2}{\sin(x)^2} - 4(\cot^2(x)) - 8 \ln(\sin(x))$	19
risch	$8ix + \frac{8e^{2ix}}{(e^{2ix}-1)^2} - 8 \ln(e^{2ix} - 1)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(4*x)/sin(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/sin(x)^2-4*cot(x)^2-8*ln(sin(x))
```

Maxima [A]

time = 0.26, size = 19, normalized size = 1.58

$$-\frac{2}{\sin(x)^2} - 2 \log(\sin(x)^2) - 4 \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(4*x)/sin(x)^4,x, algorithm="maxima")
```

```
[Out] -2/sin(x)^2 - 2*log(sin(x)^2) - 4*log(sin(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.

time = 0.34, size = 25, normalized size = 2.08

$$-\frac{2(4(\cos(x)^2 - 1) \log(\frac{1}{2} \sin(x)) - 1)}{\cos(x)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(4*x)/sin(x)^4,x, algorithm="fricas")
```

[Out] $-2*(4*(\cos(x)^2 - 1)*\log(1/2*\sin(x)) - 1)/(\cos(x)^2 - 1)$

Sympy [A]

time = 3.32, size = 14, normalized size = 1.17

$$-8 \log(\sin(x)) - \frac{2}{\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(4*x)/sin(x)**4,x)`

[Out] $-8*\log(\sin(x)) - 2/\sin(x)**2$

Giac [A]

time = 0.00, size = 16, normalized size = 1.33

$$-\frac{2}{\sin^2 x} - 8 \ln |\sin x|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(4*x)/sin(x)^4,x)`

[Out] $-2/\sin(x)^2 - 8*\log(\text{abs}(\sin(x)))$

Mupad [B]

time = 0.30, size = 35, normalized size = 2.92

$$8 \ln \left(\tan \left(\frac{x}{2} \right)^2 + 1 \right) - 8 \ln \left(\tan \left(\frac{x}{2} \right) \right) - \frac{1}{2 \tan \left(\frac{x}{2} \right)^2} - \frac{\tan \left(\frac{x}{2} \right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(4*x)/sin(x)^4,x)`

[Out] $8*\log(\tan(x/2)^2 + 1) - 8*\log(\tan(x/2)) - 1/(2*\tan(x/2)^2) - \tan(x/2)^2/2$

3.373 $\int \frac{\cot(x)}{2+\sin(2x)} dx$

Optimal. Leaf size=64

$$-\frac{x}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-2\cos^2(x)}{2+\sqrt{3}+2\cos(x)\sin(x)}\right)}{2\sqrt{3}} + \frac{1}{2}\log(\sin(x)) - \frac{1}{4}\log(1+\cos(x)\sin(x))$$

[Out] $1/2*\ln(\sin(x))-1/4*\ln(1+\cos(x)*\sin(x))-1/6*x*3^{(1/2)}+1/6*\arctan((1-2*\cos(x)^2)/(2+2*\cos(x)*\sin(x)+3^{(1/2)}))*3^{(1/2)}$

Rubi [A]

time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {719, 29, 648, 632, 210, 642}

$$-\frac{x}{2\sqrt{3}} - \frac{1}{4}\log(\tan^2(x) + \tan(x) + 1) + \frac{1}{2}\log(\tan(x)) + \frac{\tan^{-1}\left(\frac{1-2\cos^2(x)}{2\sin(x)\cos(x)+\sqrt{3}+2}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] `Int[Cot[x]/(2 + Sin[2*x]), x]`

[Out] $-1/2*x/\text{Sqrt}[3] + \text{ArcTan}[(1 - 2*\text{Cos}[x]^2)/(2 + \text{Sqrt}[3] + 2*\text{Cos}[x]*\text{Sin}[x])]/(2*\text{Sqrt}[3]) + \text{Log}[\text{Tan}[x]]/2 - \text{Log}[1 + \text{Tan}[x] + \text{Tan}[x]^2]/4$

Rule 29

`Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]`

Rule 210

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 632

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 642

`Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d},`

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\cot(x)}{2 + \sin(2x)} dx &= \text{Subst} \left(\int \frac{1}{x(2 + 2x + 2x^2)} dx, x, \tan(x) \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x} dx, x, \tan(x) \right) + \frac{1}{2} \text{Subst} \left(\int \frac{-2 - 2x}{2 + 2x + 2x^2} dx, x, \tan(x) \right) \\
 &= \frac{1}{2} \log(\tan(x)) - \frac{1}{4} \text{Subst} \left(\int \frac{2 + 4x}{2 + 2x + 2x^2} dx, x, \tan(x) \right) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{2 + 2x + 2x^2} dx, x, \tan(x) \right) \\
 &= \frac{1}{2} \log(\tan(x)) - \frac{1}{4} \log(1 + \tan(x) + \tan^2(x)) + \text{Subst} \left(\int \frac{1}{-12 - x^2} dx, x, 2 + 4 \tan(x) \right) \\
 &= -\frac{x}{2\sqrt{3}} + \frac{\tan^{-1} \left(\frac{1 - 2 \cos^2(x)}{2 + \sqrt{3} + 2 \cos(x) \sin(x)} \right)}{2\sqrt{3}} + \frac{1}{2} \log(\tan(x)) - \frac{1}{4} \log(1 + \tan(x) + \tan^2(x))
 \end{aligned}$$

Mathematica [A]

time = 0.03, size = 39, normalized size = 0.61

$$\frac{1}{12} \left(-2\sqrt{3} \tan^{-1} \left(\frac{1 + 2 \tan(x)}{\sqrt{3}} \right) + 6 \log(\sin(x)) - 3 \log(2 + \sin(2x)) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(2 + Sin[2*x]),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 + 2*Tan[x])/Sqrt[3]] + 6*Log[Sin[x]] - 3*Log[2 + Sin[2*x]])/12
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Cos[x]/(Sin[x]*(2 + Sin[2*x])),x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.15, size = 35, normalized size = 0.55

method	result
default	$-\frac{\ln(1+\tan(x)+\tan^2(x))}{4} - \frac{\sqrt{3} \arctan\left(\frac{(2\tan(x)+1)\sqrt{3}}{3}\right)}{6} + \frac{\ln(\tan(x))}{2}$
risch	$\frac{\ln(e^{2ix}-1)}{2} - \frac{\ln(e^{2ix}-i\sqrt{3}+2i)}{4} + \frac{i \ln(e^{2ix}-i\sqrt{3}+2i)\sqrt{3}}{12} - \frac{\ln(e^{2ix}+i\sqrt{3}+2i)}{4} - \frac{i \ln(e^{2ix}+i\sqrt{3}+2i)\sqrt{3}}{12}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x)/sin(x)/(2+sin(2*x)),x,method=_RETURNVERBOSE)``[Out] -1/4*ln(1+tan(x)+tan(x)^2)-1/6*3^(1/2)*arctan(1/3*(2*tan(x)+1)*3^(1/2))+1/2*ln(tan(x))`**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(51) = 102.

time = 0.37, size = 208, normalized size = 3.25

$$-\frac{1}{24}\sqrt{3}\left(\sqrt{3}\log(-2(4\sin(2x)+1)\cos(4x)+\cos(4x)^2+16\cos(2x)\sin(4x)+\sin(4x)^2+16\sin(2x)^2+8\sin(2x)+1)-2\sqrt{3}\log(\cos(x)^2+\sin(x)^2+2\cos(x)+1)-2\sqrt{3}\log(\cos(x)^2+\sin(x)^2-2\cos(x)+1)-2\arctan\left(\frac{2\sqrt{3}\cos(2x)}{\cos(2x)^2-2(\sqrt{3}-2)\sin(2x)+\sin(2x)^2-4\sqrt{3}+7}\right)-\frac{\cos(2x)^2+\sin(2x)^2+4\sin(2x)+1}{\cos(2x)^2-2(\sqrt{3}-2)\sin(2x)+\sin(2x)^2-4\sqrt{3}+7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)/sin(x)/(2+sin(2*x)),x, algorithm="maxima")`

```
[Out] -1/24*sqrt(3)*(sqrt(3)*log(-2*(4*sin(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 16*cos(2*x)^2 + 8*cos(2*x)*sin(4*x) + sin(4*x)^2 + 16*sin(2*x)^2 + 8*sin(2*x) + 1) - 2*sqrt(3)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 2*sqrt(3)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*arctan2(2*sqrt(3)*cos(2*x)/(cos(2*x)^2 - 2*(sqrt(3) - 2)*sin(2*x) + sin(2*x)^2 - 4*sqrt(3) + 7), (cos(2*x)^2 + sin(2*x)^2 + 4*sin(2*x) + 1)/(cos(2*x)^2 - 2*(sqrt(3) - 2)*sin(2*x) + sin(2*x)^2 - 4*sqrt(3) + 7))
```

Fricas [A]

time = 0.33, size = 64, normalized size = 1.00

$$-\frac{1}{12}\sqrt{3} \arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)+\sqrt{3}}{3(2\cos(x)^2-1)}\right) - \frac{1}{8}\log(-\cos(x)^4+\cos(x)^2+2\cos(x)\sin(x)+1) + \frac{1}{4}\log\left(-\frac{1}{4}\cos(x)^2+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/sin(x)/(2+sin(2*x)),x, algorithm="fricas")

[Out] $-1/12*\sqrt{3}*\arctan(1/3*(4*\sqrt{3}*\cos(x)*\sin(x) + \sqrt{3}))/ (2*\cos(x)^2 - 1) - 1/8*\log(-\cos(x)^4 + \cos(x)^2 + 2*\cos(x)*\sin(x) + 1) + 1/4*\log(-1/4*\cos(x)^2 + 1/4)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{(\sin(2x) + 2)\sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/sin(x)/(2+sin(2*x)),x)

[Out] Integral(cos(x)/((sin(2*x) + 2)*sin(x)), x)

Giac [A]

time = 0.01, size = 89, normalized size = 1.39

$$2 \left(\frac{\ln|\tan x|}{4} - \frac{\ln(\tan^2 x + \tan x + 1)}{8} - \frac{\arctan\left(\frac{-\sqrt{3}\sin(2x) + \cos(2x) + 2\sin(2x) + 1}{\sqrt{3}\cos(2x) + \sqrt{3} - 2\cos(2x) + \sin(2x) + 2}\right) + x}{4\sqrt{3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/sin(x)/(2+sin(2*x)),x)

[Out] $-1/6*\sqrt{3}*(x + \arctan(-(\sqrt{3}*\sin(2*x) - \cos(2*x) - 2*\sin(2*x) - 1)/(\sqrt{3}*\cos(2*x) + \sqrt{3} - 2*\cos(2*x) + \sin(2*x) + 2))) - 1/4*\log(\tan(x)^2 + \tan(x) + 1) + 1/2*\log(\text{abs}(\tan(x)))$

Mupad [B]

time = 0.34, size = 47, normalized size = 0.73

$$\frac{\ln(\tan(x))}{2} + \ln\left(\tan(x) + \frac{1}{2} - \frac{\sqrt{3} \operatorname{li}}{2}\right) \left(-\frac{1}{4} + \frac{\sqrt{3} \operatorname{li}}{12}\right) - \ln\left(\tan(x) + \frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2}\right) \left(\frac{1}{4} + \frac{\sqrt{3} \operatorname{li}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(sin(x)*(sin(2*x) + 2)),x)

[Out] $\log(\tan(x))/2 + \log(\tan(x) - (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*1i)/12 - 1/4) - \log(\tan(x) + (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*1i)/12 + 1/4)$

3.374 $\int \cos(x) \cot(x) \sec(3x) dx$

Optimal. Leaf size=11

$$-\frac{1}{2} \log(-4 + \csc^2(x))$$

[Out] -1/2*ln(-4+csc(x)^2)

Rubi [A]

time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.55, number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {4441, 272, 36, 31, 29}

$$\log(\sin(x)) - \frac{1}{2} \log(1 - 4 \sin^2(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cot[x]*Sec[3*x],x]

[Out] Log[Sin[x]] - Log[1 - 4*Sin[x]^2]/2

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 272

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4441

Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))], x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]

]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned}
 \int \cos(x) \cot(x) \sec(3x) dx &= \text{Subst} \left(\int \frac{1}{x(1-4x^2)} dx, x, \sin(x) \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-4x)x} dx, x, \sin^2(x) \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x} dx, x, \sin^2(x) \right) + 2 \text{Subst} \left(\int \frac{1}{1-4x} dx, x, \sin^2(x) \right) \\
 &= \log(\sin(x)) - \frac{1}{2} \log(1-4\sin^2(x))
 \end{aligned}$$

Mathematica [A]

time = 0.01, size = 17, normalized size = 1.55

$$\log(\sin(x)) - \frac{1}{2} \log(1-4\sin^2(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cot[x]*Sec[3*x],x]

[Out] Log[Sin[x]] - Log[1 - 4*Sin[x]^2]/2

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Cos[x]^2/(Sin[x]*Cos[3*x]),x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(9) = 18.

time = 0.09, size = 27, normalized size = 2.45

method	result	size
default	$\frac{\ln(1+\cos(x))}{2} - \frac{\ln(4(\cos^2(x))-3)}{2} + \frac{\ln(\cos(x)-1)}{2}$	27
risch	$\ln(e^{2ix} - 1) - \frac{\ln(e^{4ix} - e^{2ix} + 1)}{2}$	27

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)^2/cos(3*x)/sin(x),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{2} \ln(1+\cos(x)) - \frac{1}{2} \ln(4\cos(x)^2 - 3) + \frac{1}{2} \ln(\cos(x) - 1)$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(9) = 18$.

time = 0.26, size = 92, normalized size = 8.36

$$-\frac{1}{4} \log(-2(\cos(2x) - 1)\cos(4x) + \cos(4x)^2 + \cos(2x)^2 + \sin(4x)^2 - 2\sin(4x)\sin(2x) + \sin(2x)^2 - 2\cos(2x) + 1) + \frac{1}{2} \log(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1) + \frac{1}{2} \log(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^2/cos(3*x)/sin(x),x, algorithm="maxima")`

[Out] $-\frac{1}{4} \log(-2(\cos(2x) - 1)\cos(4x) + \cos(4x)^2 + \cos(2x)^2 + \sin(4x)^2 - 2\sin(4x)\sin(2x) + \sin(2x)^2 - 2\cos(2x) + 1) + \frac{1}{2} \log(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1) + \frac{1}{2} \log(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1)$

Fricas [A]

time = 0.33, size = 17, normalized size = 1.55

$$-\frac{1}{2} \log(4\cos(x)^2 - 3) + \log\left(\frac{1}{2} \sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^2/cos(3*x)/sin(x),x, algorithm="fricas")`

[Out] $-\frac{1}{2} \log(4\cos(x)^2 - 3) + \log(1/2 \sin(x))$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(x)}{\sin(x)\cos(3x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)**2/cos(3*x)/sin(x),x)`

[Out] `Integral(cos(x)**2/(sin(x)*cos(3*x)), x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(9) = 18$.

time = 0.00, size = 26, normalized size = 2.36

$$\frac{\ln(-\cos^2 x + 1)}{2} - \frac{\ln|4\cos^2 x - 3|}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^2/cos(3*x)/sin(x),x)`

[Out] `1/2*log(-cos(x)^2 + 1) - 1/2*log(abs(4*cos(x)^2 - 3))`

Mupad [B]

time = 0.62, size = 25, normalized size = 2.27

$$\ln\left(\tan\left(\frac{x}{2}\right)\right) - \frac{\ln\left(\tan\left(\frac{x}{2}\right)^4 - 14\tan\left(\frac{x}{2}\right)^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)^2/(cos(3*x)*sin(x)),x)`

[Out] `log(tan(x/2)) - log(tan(x/2)^4 - 14*tan(x/2)^2 + 1)/2`

$$3.375 \quad \int \frac{\sin(2x)}{\cos^4(x) + \sin^4(x)} dx$$

Optimal. Leaf size=7

$$-\tan^{-1}(\cos(2x))$$

[Out] -arctan(cos(2*x))

Rubi [A]

time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {12, 1121, 631, 210}

$$-\tan^{-1}(\cos(2x))$$

Antiderivative was successfully verified.

[In] Int[Sin[2*x]/(Cos[x]^4 + Sin[x]^4),x]

[Out] -ArcTan[Cos[2*x]]

Rule 12

Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :=> Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 631

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :=> With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1121

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :=> Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rubi steps

$$\begin{aligned}
\int \frac{\sin(2x)}{\cos^4(x) + \sin^4(x)} dx &= \text{Subst} \left(\int \frac{2x}{1 - 2x^2 + 2x^4} dx, x, \sin(x) \right) \\
&= 2\text{Subst} \left(\int \frac{x}{1 - 2x^2 + 2x^4} dx, x, \sin(x) \right) \\
&= \text{Subst} \left(\int \frac{1}{1 - 2x + 2x^2} dx, x, \sin^2(x) \right) \\
&= \text{Subst} \left(\int \frac{1}{-1 - x^2} dx, x, 1 - 2\sin^2(x) \right) \\
&= -\tan^{-1}(1 - 2\sin^2(x))
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 7, normalized size = 1.00

$$-\tan^{-1}(\cos(2x))$$

Antiderivative was successfully verified.

`[In] Integrate[Sin[2*x]/(Cos[x]^4 + Sin[x]^4), x]``[Out] -ArcTan[Cos[2*x]]`**Mathics [F(-1)]**

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Sin[2*x]/(Cos[x]^4 + Sin[x]^4), x]')``[Out] Timed out`**Maple [A]**

time = 0.10, size = 12, normalized size = 1.71

method	result	size
derivativedivides	$-\arctan(2(\cos^2(x)) - 1)$	12
default	$-\arctan(2(\cos^2(x)) - 1)$	12
risch	$-\frac{i \ln(e^{4ix} + 2ie^{2ix} + 1)}{2} + \frac{i \ln(e^{4ix} - 2ie^{2ix} + 1)}{2}$	40

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sin(2*x)/(sin(x)^4+cos(x)^4), x, method=_RETURNVERBOSE)`

[Out] $-\arctan(2\cos(x)^2-1)$

Maxima [A]

time = 0.39, size = 9, normalized size = 1.29

$$\arctan(2\sin(x)^2-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(2*x)/(cos(x)^4+sin(x)^4),x, algorithm="maxima")`

[Out] $\arctan(2\sin(x)^2-1)$

Fricas [A]

time = 0.32, size = 11, normalized size = 1.57

$$-\arctan(2\cos(x)^2-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(2*x)/(cos(x)^4+sin(x)^4),x, algorithm="fricas")`

[Out] $-\arctan(2\cos(x)^2-1)$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(2*x)/(cos(x)**4+sin(x)**4),x)`

[Out] Timed out

Giac [A]

time = 0.00, size = 9, normalized size = 1.29

$$\arctan(2\sin^2 x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(2*x)/(cos(x)^4+sin(x)^4),x)`

[Out] $\arctan(2\sin(x)^2-1)$

Mupad [B]

time = 0.27, size = 5, normalized size = 0.71

$$\operatorname{atan}(\tan(x)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(2*x)/(cos(x)^4 + sin(x)^4),x)`

[Out] $\operatorname{atan}(\tan(x)^2)$

$$3.376 \quad \int \frac{1}{4 + \sqrt{3} \cos(x) + \sin(x)} dx$$

Optimal. Leaf size=53

$$\frac{x}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\cos(x) - \sqrt{3} \sin(x)}{2(2 + \sqrt{3}) + \sqrt{3} \cos(x) + \sin(x)}\right)}{\sqrt{3}}$$

[Out] 1/6*x*3^(1/2)+1/3*arctan((cos(x)-sin(x)*3^(1/2))/(sin(x)+cos(x)*3^(1/2)+4+2*3^(1/2)))*3^(1/2)

Rubi [A]

time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.57, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3203, 631, 210}

$$\frac{x}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{(3-4\sqrt{3})\sin(x) + (4-\sqrt{3})\cos(x)}{(4-\sqrt{3})\sin(x) - ((3-4\sqrt{3})\cos(x) + 2(5+2\sqrt{3}))}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(4 + Sqrt[3]*Cos[x] + Sin[x])^(-1), x]

[Out] x/(2*Sqrt[3]) + ArcTan[((4 - Sqrt[3])*Cos[x] + (3 - 4*Sqrt[3])*Sin[x])/(2*(5 + 2*Sqrt[3]) - (3 - 4*Sqrt[3])*Cos[x] + (4 - Sqrt[3])*Sin[x])]/Sqrt[3]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 631

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3203

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/

2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{4 + \sqrt{3} \cos(x) + \sin(x)} dx &= 2\text{Subst} \left(\int \frac{1}{4 + \sqrt{3} + 2x + (4 - \sqrt{3})x^2} dx, x, \tan\left(\frac{x}{2}\right) \right) \\ &= - \left(2\text{Subst} \left(\int \frac{1}{-12 - x^2} dx, x, 1 + (4 - \sqrt{3}) \tan\left(\frac{x}{2}\right) \right) \right) \\ &= \frac{x}{2\sqrt{3}} + \frac{\tan^{-1} \left(\frac{(4 - \sqrt{3}) \cos(x) + (3 - 4\sqrt{3}) \sin(x)}{2(5 + 2\sqrt{3}) - (3 - 4\sqrt{3}) \cos(x) + (4 - \sqrt{3}) \sin(x)} \right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A]

time = 0.04, size = 33, normalized size = 0.62

$$\frac{\tan^{-1} \left(\frac{-1 + (-4 + \sqrt{3}) \tan\left(\frac{x}{2}\right)}{2\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(4 + Sqrt[3]*Cos[x] + Sin[x])^(-1), x]

[Out] -(ArcTan[(-1 + (-4 + Sqrt[3])*Tan[x/2])/(2*Sqrt[3])]/Sqrt[3])

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 9.95, size = 39, normalized size = 0.74

$$\frac{\sqrt{3} \left(\text{PiFloor} \left[-\frac{1}{2} + \frac{x}{2\text{Pi}} \right] + \text{ArcTan} \left[\frac{\sqrt{3}}{6} - \frac{\text{Tan}[\frac{x}{2}]}{2} + \frac{2\sqrt{3} \text{Tan}[\frac{x}{2}]}{3} \right] \right)}{3}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(4 + Sqrt[3]*Cos[x] + Sin[x]), x]')

[Out] Sqrt[3] (Pi Floor[-1 / 2 + x / (2 Pi)] + ArcTan[Sqrt[3] / 6 - Tan[x / 2] / 2 + 2 Sqrt[3] Tan[x / 2] / 3]) / 3

Maple [A]

time = 0.11, size = 43, normalized size = 0.81

method	result	size
default	$-\frac{52 \arctan\left(\frac{26 \tan\left(\frac{x}{2}\right) + 2\sqrt{3} + 8}{16\sqrt{3} + 12}\right)}{(\sqrt{3} - 4)(16\sqrt{3} + 12)}$	43
risch	$-\frac{i\sqrt{3} \ln\left(-\frac{i\sqrt{3}}{2} + \sqrt{3} - \frac{3}{2} + i + e^{ix}\right)}{6} + \frac{i\sqrt{3} \ln\left(e^{ix} + \sqrt{3} + \frac{3}{2} + i + \frac{i\sqrt{3}}{2}\right)}{6}$	52

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(4+sin(x)+3^(1/2)*cos(x)),x,method=_RETURNVERBOSE)`

[Out] `-52/(3^(1/2)-4)/(16*3^(1/2)+12)*arctan((26*tan(1/2*x)+2*3^(1/2)+8)/(16*3^(1/2)+12))`

Maxima [A]

time = 0.35, size = 27, normalized size = 0.51

$$-\frac{1}{3}\sqrt{3} \arctan\left(\frac{1}{6}\sqrt{3}\left(\frac{(\sqrt{3}-4)\sin(x)}{\cos(x)+1}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(4+sin(x)+cos(x)*3^(1/2)),x, algorithm="maxima")`

[Out] `-1/3*sqrt(3)*arctan(1/6*sqrt(3)*((sqrt(3)-4)*sin(x)/(cos(x)+1)-1))`

Fricas [A]

time = 0.36, size = 38, normalized size = 0.72

$$\frac{1}{6}\sqrt{3} \arctan\left(\frac{2\left(\left(4\sqrt{3}\cos(x)+3\right)\sin(x)+\sqrt{3}\cos(x)+3\right)}{3\left(4\cos(x)^2-3\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(4+sin(x)+cos(x)*3^(1/2)),x, algorithm="fricas")`

[Out] `1/6*sqrt(3)*arctan(2/3*((4*sqrt(3)*cos(x)+3)*sin(x)+sqrt(3)*cos(x)+3)/(4*cos(x)^2-3))`

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(48) = 96

time = 4.90, size = 107, normalized size = 2.02

$$\frac{13906891405206808\sqrt{3}\left(\operatorname{atan}\left(-\frac{\tan\left(\frac{x}{2}\right)}{2}+\frac{2\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}+\frac{\sqrt{3}}{6}\right)+\pi\left[\frac{x-\frac{\pi}{2}}{\pi}\right]\right)}{-41720674215620424+24087442555831531\sqrt{3}}+\frac{24087442555831531\left(\operatorname{atan}\left(-\frac{\tan\left(\frac{x}{2}\right)}{2}+\frac{2\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}+\frac{\sqrt{3}}{6}\right)+\pi\left[\frac{x-\frac{\pi}{2}}{\pi}\right]\right)}{-41720674215620424+24087442555831531\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4+sin(x)+cos(x)*3**(1/2)),x)

[Out] -13906891405206808*sqrt(3)*(atan(-tan(x/2)/2 + 2*sqrt(3)*tan(x/2)/3 + sqrt(3)/6) + pi*floor((x/2 - pi/2)/pi))/(-41720674215620424 + 24087442555831531*sqrt(3)) + 24087442555831531*(atan(-tan(x/2)/2 + 2*sqrt(3)*tan(x/2)/3 + sqrt(3)/6) + pi*floor((x/2 - pi/2)/pi))/(-41720674215620424 + 24087442555831531*sqrt(3))

Giac [A]

time = 0.00, size = 97, normalized size = 1.83

$$\frac{2 \left(2\sqrt{3} + 8 \right) \left(\arctan \left(\frac{\sqrt{3} \cos x - 8\sqrt{3} \sin x + \sqrt{3} + 4 \cos x + 7 \sin x + 4}{8\sqrt{3} \cos x + \sqrt{3} \sin x + 8\sqrt{3} - 7 \cos x + 4 \sin x + 19} \right) + \frac{x}{2} \right)}{16\sqrt{3} + 12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4+sin(x)+cos(x)*3^(1/2)),x)

[Out] 1/2*(x + 2*arctan((sqrt(3)*cos(x) - 8*sqrt(3)*sin(x) + sqrt(3) + 4*cos(x) + 7*sin(x) + 4)/(8*sqrt(3)*cos(x) + sqrt(3)*sin(x) + 8*sqrt(3) - 7*cos(x) + 4*sin(x) + 19)))*(sqrt(3) + 4)/(4*sqrt(3) + 3)

Mupad [B]

time = 0.21, size = 23, normalized size = 0.43

$$\frac{\sqrt{12} \operatorname{atan} \left(\frac{\sqrt{12} \left(\tan \left(\frac{x}{2} \right) \left(\sqrt{3} - 4 \right) - 1 \right)}{12} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x) + 3^(1/2)*cos(x) + 4),x)

[Out] -(12^(1/2)*atan((12^(1/2)*(tan(x/2)*(3^(1/2) - 4) - 1))/12))/6

$$3.377 \quad \int \frac{1}{3+4 \cos(x)+4 \sin(x)} dx$$

Optimal. Leaf size=33

$$\frac{\tanh^{-1}\left(\frac{\sqrt{23}(\cos(x)-\sin(x))}{8+3\cos(x)+3\sin(x)}\right)}{\sqrt{23}}$$

[Out] $-1/23*\operatorname{arctanh}((\cos(x)-\sin(x))*23^{(1/2)}/(8+3*\cos(x)+3*\sin(x)))*23^{(1/2)}$

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 94 vs. $2(33) = 66$.
time = 0.05, antiderivative size = 94, normalized size of antiderivative = 2.85, number of
steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$,
Rules used = {3203, 632, 212}

$$\frac{\log\left(\frac{\sqrt{23}\sin(x)-4\sin(x)-4\sqrt{23}\cos(x)+19\cos(x)+4(5-\sqrt{23})}{2\sqrt{23}}\right)}{2\sqrt{23}} - \frac{\log\left(\frac{-\sqrt{23}\sin(x)-4\sin(x)+4\sqrt{23}\cos(x)+19\cos(x)+4(5+\sqrt{23})}{2\sqrt{23}}\right)}{2\sqrt{23}}$$

Antiderivative was successfully verified.

[In] `Int[(3 + 4*Cos[x] + 4*Sin[x])^(-1), x]`

[Out] $-1/2*\operatorname{Log}[4*(5 + \operatorname{Sqrt}[23]) + 19*\operatorname{Cos}[x] + 4*\operatorname{Sqrt}[23]*\operatorname{Cos}[x] - 4*\operatorname{Sin}[x] - \operatorname{Sqrt}[23]*\operatorname{Sin}[x]]/\operatorname{Sqrt}[23] + \operatorname{Log}[4*(5 - \operatorname{Sqrt}[23]) + 19*\operatorname{Cos}[x] - 4*\operatorname{Sqrt}[23]*\operatorname{Cos}[x] - 4*\operatorname{Sin}[x] + \operatorname{Sqrt}[23]*\operatorname{Sin}[x]]/(2*\operatorname{Sqrt}[23])$

Rule 212

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 632

`Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 3203

`Int[(cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)])^(-1), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]`

Rubi steps

$$\begin{aligned} \int \frac{1}{3 + 4 \cos(x) + 4 \sin(x)} dx &= 2 \text{Subst} \left(\int \frac{1}{7 + 8x - x^2} dx, x, \tan\left(\frac{x}{2}\right) \right) \\ &= - \left(4 \text{Subst} \left(\int \frac{1}{92 - x^2} dx, x, 8 - 2 \tan\left(\frac{x}{2}\right) \right) \right) \\ &= - \frac{\log\left(4\left(5 + \sqrt{23}\right) + 19 \cos(x) + 4\sqrt{23} \cos(x) - 4 \sin(x) - \sqrt{23} \sin(x)\right)}{2\sqrt{23}} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 22, normalized size = 0.67

$$\frac{2 \tanh^{-1} \left(\frac{-4 + \tan\left(\frac{x}{2}\right)}{\sqrt{23}} \right)}{\sqrt{23}}$$

Antiderivative was successfully verified.

`[In] Integrate[(3 + 4*Cos[x] + 4*Sin[x])^(-1),x]``[Out] (2*ArcTanh[(-4 + Tan[x/2])/Sqrt[23]])/Sqrt[23]`**Mathics [A]**

time = 2.42, size = 30, normalized size = 0.91

$$\frac{\sqrt{23} \left(\text{Log} \left[-4 + \sqrt{23} + \text{Tan} \left[\frac{x}{2} \right] \right] - \text{Log} \left[-4 - \sqrt{23} + \text{Tan} \left[\frac{x}{2} \right] \right] \right)}{23}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(3 + 4*Cos[x] + 4*Sin[x]),x]')``[Out] Sqrt[23] (Log[-4 + Sqrt[23] + Tan[x / 2]] - Log[-4 - Sqrt[23] + Tan[x / 2]]) / 23`**Maple [A]**

time = 0.07, size = 20, normalized size = 0.61

method	result	size
default	$\frac{2\sqrt{23} \operatorname{arctanh} \left(\frac{(2 \tan(\frac{x}{2}) - 8) \sqrt{23}}{46} \right)}{23}$	20
risch	$\frac{\sqrt{23} \ln \left(e^{ix + \frac{3}{8} + \frac{3i}{8}} - \frac{\sqrt{23}}{8} + i \frac{\sqrt{23}}{8} \right)}{23} - \frac{\sqrt{23} \ln \left(e^{ix + \frac{3}{8} + \frac{3i}{8}} + \frac{\sqrt{23}}{8} - i \frac{\sqrt{23}}{8} \right)}{23}$	54

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3+4*cos(x)+4*sin(x)),x,method=_RETURNVERBOSE)`

[Out] $2/23*23^{(1/2)}*\operatorname{arctanh}(1/46*(2*\tan(1/2*x)-8)*23^{(1/2)})$

Maxima [A]

time = 0.33, size = 39, normalized size = 1.18

$$-\frac{1}{23}\sqrt{23}\log\left(-\frac{\sqrt{23}-\frac{\sin(x)}{\cos(x)+1}+4}{\sqrt{23}+\frac{\sin(x)}{\cos(x)+1}-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3+4*cos(x)+4*sin(x)),x, algorithm="maxima")`

[Out] $-1/23*\sqrt{23}*\log(-(\sqrt{23}-\sin(x)/(\cos(x)+1)+4)/(\sqrt{23}+\sin(x)/(\cos(x)+1)-4))$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(29) = 58.

time = 0.33, size = 66, normalized size = 2.00

$$\frac{1}{46}\sqrt{23}\log\left(-\frac{6\sqrt{23}\cos(x)^2+8(\sqrt{23}-3)\cos(x)-2(4\sqrt{23}-7\cos(x)+12)\sin(x)-3\sqrt{23}-48}{8(4\cos(x)+3)\sin(x)+24\cos(x)+25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3+4*cos(x)+4*sin(x)),x, algorithm="fricas")`

[Out] $1/46*\sqrt{23}*\log(-6*\sqrt{23}*\cos(x)^2+8*(\sqrt{23}-3)*\cos(x)-2*(4*\sqrt{23}-7*\cos(x)+12)*\sin(x)-3*\sqrt{23}-48)/(8*(4*\cos(x)+3)*\sin(x)+24*\cos(x)+25))$

Sympy [A]

time = 0.32, size = 39, normalized size = 1.18

$$\frac{\sqrt{23}\log\left(\tan\left(\frac{x}{2}\right)-4+\sqrt{23}\right)}{23}-\frac{\sqrt{23}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{23}-4\right)}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3+4*cos(x)+4*sin(x)),x)`

[Out] $\sqrt{23}*\log(\tan(x/2)-4+\sqrt{23})/23-\sqrt{23}*\log(\tan(x/2)-\sqrt{23}-4)/23$

Giac [A]

time = 0.00, size = 49, normalized size = 1.48

$$\frac{2 \ln \left(\frac{\left| 2 \tan\left(\frac{x}{2}\right) - 8 - 2\sqrt{23} \right|}{\left| 2 \tan\left(\frac{x}{2}\right) - 8 + 2\sqrt{23} \right|} \right)}{2\sqrt{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(3+4*cos(x)+4*sin(x)),x)``[Out] -1/23*sqrt(23)*log(abs(-2*sqrt(23) + 2*tan(1/2*x) - 8)/abs(2*sqrt(23) + 2*tan(1/2*x) - 8))`**Mupad [B]**

time = 0.08, size = 17, normalized size = 0.52

$$\frac{2\sqrt{23} \operatorname{atanh}\left(\frac{\sqrt{23}(\tan(\frac{x}{2})-4)}{23}\right)}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(4*cos(x) + 4*sin(x) + 3),x)``[Out] (2*23^(1/2)*atanh((23^(1/2)*(tan(x/2) - 4))/23))/23`

$$3.378 \quad \int \frac{1}{4-3 \cos^2(x)+5 \sin^2(x)} dx$$

Optimal. Leaf size=27

$$\frac{x}{3} + \frac{1}{3} \tan^{-1} \left(\frac{2 \cos(x) \sin(x)}{1 + 2 \sin^2(x)} \right)$$

[Out] 1/3*x+1/3*arctan(2*cos(x)*sin(x)/(1+2*sin(x)^2))

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {209}

$$\frac{x}{3} + \frac{1}{3} \tan^{-1} \left(\frac{2 \sin(x) \cos(x)}{2 \sin^2(x) + 1} \right)$$

Antiderivative was successfully verified.

[In] Int[(4 - 3*Cos[x]^2 + 5*Sin[x]^2)^(-1),x]

[Out] x/3 + ArcTan[(2*Cos[x]*Sin[x])/(1 + 2*Sin[x]^2)]/3

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{4-3 \cos^2(x)+5 \sin^2(x)} dx &= \text{Subst} \left(\int \frac{1}{1+9x^2} dx, x, \tan(x) \right) \\ &= \frac{x}{3} + \frac{1}{3} \tan^{-1} \left(\frac{2 \cos(x) \sin(x)}{1 + 2 \sin^2(x)} \right) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 9, normalized size = 0.33

$$\frac{1}{3} \tan^{-1}(3 \tan(x))$$

Antiderivative was successfully verified.

[In] Integrate[(4 - 3*Cos[x]^2 + 5*Sin[x]^2)^(-1),x]

[Out] ArcTan[3*Tan[x]]/3

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 9.80, size = 169, normalized size = 6.26

$$\left(\frac{768398401}{3} - 181113240\sqrt{2}\right) \left(\sqrt{17-12\sqrt{2}} \left(4478554083\text{PiFloor}\left[-\frac{1}{2} + \frac{x}{2\text{Pi}}\right] + 4478554083\text{ArcTan}\left[\frac{\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{17-12\sqrt{2}}}\right]\right) + \sqrt{17+12\sqrt{2}} \left(131836323\text{PiFloor}\left[-\frac{1}{2} + \frac{x}{2\text{Pi}}\right] + 131836323\text{ArcTan}\left[\frac{\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{17+12\sqrt{2}}}\right]\right) + \sqrt{34-24\sqrt{2}} \left(3166815962\text{PiFloor}\left[-\frac{1}{2} + \frac{x}{2\text{Pi}}\right] + 3166815962\text{ArcTan}\left[\frac{\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{17-12\sqrt{2}}}\right]\right) + \sqrt{34+24\sqrt{2}} \left(93222358\text{PiFloor}\left[-\frac{1}{2} + \frac{x}{2\text{Pi}}\right] + 93222358\text{ArcTan}\left[\frac{\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{17+12\sqrt{2}}}\right]\right)\right)$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(4 - 3*Cos[x]^2 + 5*Sin[x]^2),x]')

[Out] (768398401 / 3 - 181113240 Sqrt[2]) (Sqrt[17 - 12 Sqrt[2]] (4478554083 Pi Floor[-1 / 2 + x / (2 Pi)] + 4478554083 ArcTan[Tan[x / 2] / Sqrt[17 - 12 Sqrt[2]]]) + Sqrt[17 + 12 Sqrt[2]] (131836323 Pi Floor[-1 / 2 + x / (2 Pi)] + 131836323 ArcTan[Tan[x / 2] / Sqrt[17 + 12 Sqrt[2]]]) + Sqrt[34 - 24 Sqrt[2]] (3166815962 Pi Floor[-1 / 2 + x / (2 Pi)] + 3166815962 ArcTan[Tan[x / 2] / Sqrt[17 - 12 Sqrt[2]]]) + Sqrt[34 + 24 Sqrt[2]] (93222358 Pi Floor[-1 / 2 + x / (2 Pi)] + 93222358 ArcTan[Tan[x / 2] / Sqrt[17 + 12 Sqrt[2]]]))

Maple [A]

time = 0.06, size = 8, normalized size = 0.30

method	result	size
default	$\frac{\arctan(3 \tan(x))}{3}$	8
risch	$-\frac{i \ln(e^{2ix} - \frac{1}{2})}{6} + \frac{i \ln(e^{2ix} - 2)}{6}$	24

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4-3*cos(x)^2+5*sin(x)^2),x,method=_RETURNVERBOSE)

[Out] 1/3*arctan(3*tan(x))

Maxima [A]

time = 0.34, size = 7, normalized size = 0.26

$$\frac{1}{3} \arctan(3 \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-3*cos(x)^2+5*sin(x)^2),x, algorithm="maxima")

[Out] 1/3*arctan(3*tan(x))

Fricas [A]

time = 0.32, size = 21, normalized size = 0.78

$$-\frac{1}{6} \arctan\left(\frac{10 \cos(x)^2 - 9}{6 \cos(x) \sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-3*cos(x)^2+5*sin(x)^2),x, algorithm="fricas")

[Out] -1/6*arctan(1/6*(10*cos(x)^2 - 9)/(cos(x)*sin(x)))

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(22) = 44

time = 5.09, size = 219, normalized size = 8.11

$$\frac{447854083\sqrt{17-12\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{17-12\sqrt{2}}}\right)+\pi\left\lfloor\frac{x}{2\pi}\right\rfloor\right)}{2305195203+1630019160\sqrt{2}}+\frac{3166815962\sqrt{17-12\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{17-12\sqrt{2}}}\right)+\pi\left\lfloor\frac{x}{2\pi}\right\rfloor\right)}{2305195203+1630019160\sqrt{2}}+\frac{131836323\sqrt{12\sqrt{2}+17}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{12\sqrt{2}+17}}\right)+\pi\left\lfloor\frac{x}{2\pi}\right\rfloor\right)}{2305195203+1630019160\sqrt{2}}+\frac{93222358\sqrt{2}\sqrt{12\sqrt{2}+17}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{12\sqrt{2}+17}}\right)+\pi\left\lfloor\frac{x}{2\pi}\right\rfloor\right)}{2305195203+1630019160\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-3*cos(x)**2+5*sin(x)**2),x)

[Out] 4478554083*sqrt(17 - 12*sqrt(2))*(atan(tan(x/2)/sqrt(17 - 12*sqrt(2)))) + pi *floor((x/2 - pi/2)/pi))/(2305195203 + 1630019160*sqrt(2)) + 3166815962*sqrt(2)*sqrt(17 - 12*sqrt(2))*(atan(tan(x/2)/sqrt(17 - 12*sqrt(2)))) + pi*floor((x/2 - pi/2)/pi))/(2305195203 + 1630019160*sqrt(2)) + 131836323*sqrt(12*sqrt(2) + 17)*(atan(tan(x/2)/sqrt(12*sqrt(2) + 17))) + pi*floor((x/2 - pi/2)/pi))/(2305195203 + 1630019160*sqrt(2)) + 93222358*sqrt(2)*sqrt(12*sqrt(2) + 17)*(atan(tan(x/2)/sqrt(12*sqrt(2) + 17))) + pi*floor((x/2 - pi/2)/pi))/(2305195203 + 1630019160*sqrt(2))

Giac [A]

time = 0.00, size = 20, normalized size = 0.74

$$\frac{2}{6}\left(\arctan\left(-\frac{\sin(2x)}{\cos(2x)-2}\right)+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-3*cos(x)^2+5*sin(x)^2),x)

[Out] 1/3*x + 1/3*arctan(-sin(2*x)/(cos(2*x) - 2))

Mupad [B]

time = 0.28, size = 16, normalized size = 0.59

$$\frac{x}{3}-\frac{\operatorname{atan}(\tan(x))}{3}+\frac{\operatorname{atan}(3\tan(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*sin(x)^2 - 3*cos(x)^2 + 4),x)

[Out] x/3 - atan(tan(x))/3 + atan(3*tan(x))/3

$$3.379 \quad \int \frac{1}{4+4 \cot(x)+\tan(x)} dx$$

Optimal. Leaf size=28

$$\frac{4x}{25} - \frac{3}{25} \log(2 \cos(x) + \sin(x)) + \frac{2}{5(2 + \tan(x))}$$

[Out] 4/25*x-3/25*ln(2*cos(x)+sin(x))+2/5/(2+tan(x))

Rubi [A]

time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {815, 649, 209, 266}

$$\frac{4x}{25} + \frac{2}{5(\tan(x) + 2)} - \frac{3}{25} \log(\sin(x) + 2 \cos(x))$$

Antiderivative was successfully verified.

[In] Int[(4 + 4*Cot[x] + Tan[x])^(-1),x]

[Out] (4*x)/25 - (3*Log[2*Cos[x] + Sin[x]])/25 + 2/(5*(2 + Tan[x]))

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 649

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[(-a)*c]

Rule 815

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{1}{4 + 4 \cot(x) + \tan(x)} dx &= \text{Subst} \left(\int \frac{x}{(2+x)^2(1+x^2)} dx, x, \tan(x) \right) \\
&= \text{Subst} \left(\int \left(-\frac{2}{5(2+x)^2} - \frac{3}{25(2+x)} + \frac{4+3x}{25(1+x^2)} \right) dx, x, \tan(x) \right) \\
&= -\frac{3}{25} \log(2 + \tan(x)) + \frac{2}{5(2 + \tan(x))} + \frac{1}{25} \text{Subst} \left(\int \frac{4+3x}{1+x^2} dx, x, \tan(x) \right) \\
&= -\frac{3}{25} \log(2 + \tan(x)) + \frac{2}{5(2 + \tan(x))} + \frac{3}{25} \text{Subst} \left(\int \frac{x}{1+x^2} dx, x, \tan(x) \right) \\
&= \frac{4x}{25} - \frac{3}{25} \log(\cos(x)) - \frac{3}{25} \log(2 + \tan(x)) + \frac{2}{5(2 + \tan(x))}
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 41, normalized size = 1.46

$$\frac{-5 + 4x + \cot(x)(8x - 6 \log(2 \cos(x) + \sin(x))) - 3 \log(2 \cos(x) + \sin(x))}{25 + 50 \cot(x)}$$

Antiderivative was successfully verified.

`[In] Integrate[(4 + 4*Cot[x] + Tan[x])^(-1), x]``[Out] (-5 + 4*x + Cot[x]*(8*x - 6*Log[2*Cos[x] + Sin[x]]) - 3*Log[2*Cos[x] + Sin[x]])/(25 + 50*Cot[x])`**Mathics [A]**

time = 3.03, size = 50, normalized size = 1.79

$$\frac{20 + 8x \tan[x] + 16x - 12 \log[2 + \tan[x]] - 6 \log[2 + \tan[x]] \tan[x] + 3 \log\left[\frac{1}{\cos[x]^2}\right] \tan[x] + 6 \log\left[\frac{1}{\cos[x]^2}\right]}{100 + 50 \tan[x]}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(4 + Tan[x] + 4*Cot[x]), x]')``[Out] (20 + 8 x Tan[x] + 16 x - 12 Log[2 + Tan[x]] - 6 Log[2 + Tan[x]] Tan[x] + 3 Log[1 / Cos[x]^2] Tan[x] + 6 Log[1 / Cos[x]^2]) / (50 (2 + Tan[x]))`**Maple [A]**

time = 0.11, size = 31, normalized size = 1.11

method	result	size
default	$\frac{2}{5(2+\tan(x))} - \frac{3 \ln(2+\tan(x))}{25} + \frac{3 \ln(1+\tan^2(x))}{50} + \frac{4 \arctan(\tan(x))}{25}$	31

norman	$\frac{\frac{8x}{25} + \frac{4x \tan(x)}{25} + \frac{2}{5}}{2 + \tan(x)} - \frac{3 \ln(2 + \tan(x))}{25} + \frac{3 \ln(1 + \tan^2(x))}{50}$	35
risch	$\frac{4x}{25} + \frac{3ix}{25} + \frac{16}{25(5e^{2ix} + 3 + 4i)} - \frac{12i}{25(5e^{2ix} + 3 + 4i)} - \frac{3 \ln(e^{2ix} + \frac{3}{5} + \frac{4i}{5})}{25}$	52

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(4+4*cot(x)+tan(x)),x,method=_RETURNVERBOSE)`

[Out] $2/5/(2+\tan(x))-3/25*\ln(2+\tan(x))+3/50*\ln(1+\tan(x)^2)+4/25*\arctan(\tan(x))$

Maxima [A]

time = 0.34, size = 28, normalized size = 1.00

$$\frac{4}{25}x + \frac{2}{5(\tan(x) + 2)} + \frac{3}{50} \log(\tan(x)^2 + 1) - \frac{3}{25} \log(\tan(x) + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(4+4*cot(x)+tan(x)),x, algorithm="maxima")`

[Out] $4/25*x + 2/5/(\tan(x) + 2) + 3/50*\log(\tan(x)^2 + 1) - 3/25*\log(\tan(x) + 2)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(22) = 44.

time = 0.32, size = 46, normalized size = 1.64

$$\frac{3(\tan(x) + 2) \log\left(\frac{\tan(x)^2 + 4 \tan(x) + 4}{\tan(x)^2 + 1}\right) - 8(x - 1)\tan(x) - 16x - 4}{50(\tan(x) + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(4+4*cot(x)+tan(x)),x, algorithm="fricas")`

[Out] $-1/50*(3*(\tan(x) + 2)*\log((\tan(x)^2 + 4*\tan(x) + 4)/(\tan(x)^2 + 1)) - 8*(x - 1)*\tan(x) - 16*x - 4)/(\tan(x) + 2)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(26) = 52

time = 0.24, size = 102, normalized size = 3.64

$$\frac{8x \tan(x)}{50 \tan(x) + 100} + \frac{16x}{50 \tan(x) + 100} - \frac{6 \log(\tan(x) + 2) \tan(x)}{50 \tan(x) + 100} - \frac{12 \log(\tan(x) + 2)}{50 \tan(x) + 100} + \frac{3 \log(\tan^2(x) + 1) \tan(x)}{50 \tan(x) + 100} + \frac{6 \log(\tan^2(x) + 1)}{50 \tan(x) + 100} + \frac{20}{50 \tan(x) + 100}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(4+4*cot(x)+tan(x)),x)`

[Out] $8*x*\tan(x)/(50*\tan(x) + 100) + 16*x/(50*\tan(x) + 100) - 6*\log(\tan(x) + 2)*\tan(x)/(50*\tan(x) + 100) - 12*\log(\tan(x) + 2)/(50*\tan(x) + 100) + 3*\log(\tan(x)^2 + 1)*\tan(x)/(50*\tan(x) + 100) + 6*\log(\tan(x)^2 + 1)/(50*\tan(x) + 100) + 20/(50*\tan(x) + 100)$

$x^{**2} + 1) * \tan(x) / (50 * \tan(x) + 100) + 6 * \log(\tan(x)^{**2} + 1) / (50 * \tan(x) + 100) + 20 / (50 * \tan(x) + 100)$

Giac [A]

time = 0.00, size = 36, normalized size = 1.29

$$\frac{3}{50} \ln(\tan^2 x + 1) + \frac{4}{25} x - \frac{3}{25} \ln|\tan x + 2| + \frac{2}{5(\tan x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4+4*cot(x)+tan(x)),x)

[Out] 4/25*x + 2/5/(tan(x) + 2) + 3/50*log(tan(x)^2 + 1) - 3/25*log(abs(tan(x) + 2))

Mupad [B]

time = 0.32, size = 38, normalized size = 1.36

$$\frac{2}{5(\tan(x) + 2)} - \frac{3 \ln(\tan(x) + 2)}{25} + \ln(\tan(x) - i) \left(\frac{3}{50} - \frac{2}{25}i \right) + \ln(\tan(x) + 1i) \left(\frac{3}{50} + \frac{2}{25}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*cot(x) + tan(x) + 4),x)

[Out] log(tan(x) - 1i)*(3/50 - 2i/25) - (3*log(tan(x) + 2))/25 + log(tan(x) + 1i)*(3/50 + 2i/25) + 2/(5*(tan(x) + 2))

$$3.380 \quad \int \frac{1}{(2 \sec(x) + \sin(x))^2} dx$$

Optimal. Leaf size=67

$$\frac{8x}{15\sqrt{15}} - \frac{8 \tan^{-1} \left(\frac{1-2 \cos^2(x)}{4+\sqrt{15}+2 \cos(x) \sin(x)} \right)}{15\sqrt{15}} + \frac{1+4 \tan(x)}{15(2+\tan(x)+2 \tan^2(x))}$$

[Out] 8/225*x*15^(1/2)-8/225*arctan((1-2*cos(x)^2)/(4+2*cos(x)*sin(x)+15^(1/2)))*15^(1/2)+1/15*(1+4*tan(x))/(2+tan(x)+2*tan(x)^2)

Rubi [A]

time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {628, 632, 210}

$$\frac{8x}{15\sqrt{15}} + \frac{4 \tan(x) + 1}{15(2 \tan^2(x) + \tan(x) + 2)} - \frac{8 \tan^{-1} \left(\frac{1-2 \cos^2(x)}{2 \sin(x) \cos(x) + \sqrt{15} + 4} \right)}{15\sqrt{15}}$$

Antiderivative was successfully verified.

[In] Int[(2*Sec[x] + Sin[x])^(-2),x]

[Out] (8*x)/(15*sqrt[15]) - (8*ArcTan[(1 - 2*Cos[x]^2)/(4 + sqrt[15] + 2*Cos[x]*Sin[x])])/(15*sqrt[15]) + (1 + 4*Tan[x])/(15*(2 + Tan[x] + 2*Tan[x]^2))

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p + 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2 \sec(x) + \sin(x))^2} dx &= \text{Subst} \left(\int \frac{1}{(2 + x + 2x^2)^2} dx, x, \tan(x) \right) \\
&= \frac{1 + 4 \tan(x)}{15 (2 + \tan(x) + 2 \tan^2(x))} + \frac{4}{15} \text{Subst} \left(\int \frac{1}{2 + x + 2x^2} dx, x, \tan(x) \right) \\
&= \frac{1 + 4 \tan(x)}{15 (2 + \tan(x) + 2 \tan^2(x))} - \frac{8}{15} \text{Subst} \left(\int \frac{1}{-15 - x^2} dx, x, 1 + 4 \tan(x) \right) \\
&= \frac{8x}{15\sqrt{15}} - \frac{8 \tan^{-1} \left(\frac{1 - 2 \cos^2(x)}{4 + \sqrt{15} + 2 \cos(x) \sin(x)} \right)}{15\sqrt{15}} + \frac{1 + 4 \tan(x)}{15 (2 + \tan(x) + 2 \tan^2(x))}
\end{aligned}$$

Mathematica [A]

time = 0.08, size = 58, normalized size = 0.87

$$\frac{\sec^2(x)(4 + \sin(2x)) \left(15(-15 + \cos(2x)) + 8\sqrt{15} \tan^{-1} \left(\frac{1+4 \tan(x)}{\sqrt{15}} \right) (4 + \sin(2x)) \right)}{900(2 \sec(x) + \sin(x))^2}$$

Antiderivative was successfully verified.

`[In] Integrate[(2*Sec[x] + Sin[x])^(-2),x]`

```
[Out] (Sec[x]^2*(4 + Sin[2*x])*(15*(-15 + Cos[2*x]) + 8*Sqrt[15]*ArcTan[(1 + 4*Tan[x])/Sqrt[15]]*(4 + Sin[2*x])))/(900*(2*Sec[x] + Sin[x])^2)
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(Sin[x] + 2*Sec[x])^2,x]')``[Out] Timed out`**Maple [A]**

time = 0.12, size = 39, normalized size = 0.58

method	result	size
default	$\frac{1+4 \tan(x)}{30+15 \tan(x)+30(\tan^2(x))} + \frac{8\sqrt{15} \arctan\left(\frac{(1+4 \tan(x))\sqrt{15}}{15}\right)}{225}$	39

risch	$\frac{\left(\frac{8}{3615} - \frac{2i}{241}\right)(241e^{2ix} - 15 + 4i)}{e^{4ix} + 8ie^{2ix} - 1} + \frac{4i\sqrt{15} \ln\left(e^{2ix} + i\sqrt{15} + 4i\right)}{225} - \frac{4i\sqrt{15} \ln\left(e^{2ix} - i\sqrt{15} + 4i\right)}{225}$	76
-------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*sec(x)+sin(x))^2,x,method=_RETURNVERBOSE)`

[Out] $1/15*(1+4*\tan(x))/(2+\tan(x)+2*\tan(x)^2)+8/225*15^{(1/2)}*\arctan(1/15*(1+4*\tan(x))*15^{(1/2)})$

Maxima [A]

time = 0.33, size = 38, normalized size = 0.57

$$\frac{8}{225} \sqrt{15} \arctan\left(\frac{1}{15} \sqrt{15} (4 \tan(x) + 1)\right) + \frac{4 \tan(x) + 1}{15 (2 \tan(x)^2 + \tan(x) + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*sec(x)+sin(x))^2,x, algorithm="maxima")`

[Out] $8/225*\sqrt{15}*\arctan(1/15*\sqrt{15}*(4*\tan(x) + 1)) + 1/15*(4*\tan(x) + 1)/(2*\tan(x)^2 + \tan(x) + 2)$

Fricas [A]

time = 0.32, size = 61, normalized size = 0.91

$$\frac{4 \left(\sqrt{15} \cos(x) \sin(x) + 2 \sqrt{15} \right) \arctan\left(\frac{8 \sqrt{15} \cos(x) \sin(x) + \sqrt{15}}{15 (2 \cos(x)^2 - 1)} \right) + 15 \cos(x)^2 - 120}{225 (\cos(x) \sin(x) + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*sec(x)+sin(x))^2,x, algorithm="fricas")`

[Out] $1/225*(4*(\sqrt{15}*\cos(x)*\sin(x) + 2*\sqrt{15}))*\arctan(1/15*(8*\sqrt{15}*\cos(x)*\sin(x) + \sqrt{15}))/2*\cos(x)^2 - 1) + 15*\cos(x)^2 - 120)/(\cos(x)*\sin(x) + 2)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\sin(x) + 2 \sec(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*sec(x)+sin(x))**2,x)`

[Out] `Integral((sin(x) + 2*sec(x))**(-2), x)`

Giac [A]

time = 0.00, size = 89, normalized size = 1.33

$$2 \left(\frac{4 \tan x + 1}{30 (2 \tan^2 x + \tan x + 2)} + \frac{4 \left(\arctan \left(\frac{-\sqrt{15} \sin(2x) + \cos(2x) + 4 \sin(2x) + 1}{\sqrt{15} \cos(2x) + \sqrt{15} - 4 \cos(2x) + \sin(2x) + 4} \right) + x \right)}{15 \sqrt{15}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*sec(x)+sin(x))^2,x)

[Out] 8/225*sqrt(15)*(x + arctan(-(sqrt(15)*sin(2*x) - cos(2*x) - 4*sin(2*x) - 1)/(sqrt(15)*cos(2*x) + sqrt(15) - 4*cos(2*x) + sin(2*x) + 4))) + 1/15*(4*tan(x) + 1)/(2*tan(x)^2 + tan(x) + 2)

Mupad [B]

time = 0.43, size = 120, normalized size = 1.79

$$\frac{4 \sqrt{15} \left(2 \operatorname{atan} \left(\frac{2 \sqrt{15} \tan(\frac{x}{2})^3}{15} - \frac{2 \sqrt{15} \tan(\frac{x}{2})^2}{15} + \frac{2 \sqrt{15} \tan(\frac{x}{2})}{5} + \frac{\sqrt{15}}{15} \right) - 2 \operatorname{atan} \left(\frac{\sqrt{15}}{15} - \frac{2 \sqrt{15} \tan(\frac{x}{2})}{15} \right) \right)}{225} - \frac{\frac{7 \tan(\frac{x}{2})^3}{30} + \frac{2 \tan(\frac{x}{2})^2}{15} - \frac{7 \tan(\frac{x}{2})}{30}}{\tan(\frac{x}{2})^4 - \tan(\frac{x}{2})^3 + 2 \tan(\frac{x}{2})^2 + \tan(\frac{x}{2}) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x) + 2/cos(x))^2,x)

[Out] (4*15^(1/2)*(2*atan((2*15^(1/2)*tan(x/2))/5 + 15^(1/2)/15 - (2*15^(1/2)*tan(x/2)^2)/15 + (2*15^(1/2)*tan(x/2)^3)/15) - 2*atan(15^(1/2)/15 - (2*15^(1/2)*tan(x/2))/15))/225 - ((2*tan(x/2)^2)/15 - (7*tan(x/2))/30 + (7*tan(x/2)^3)/30)/(tan(x/2) + 2*tan(x/2)^2 - tan(x/2)^3 + tan(x/2)^4 + 1)

$$3.381 \quad \int \frac{1}{(\cos(x) + 2 \sec(x))^2} dx$$

Optimal. Leaf size=55

$$\frac{x}{6\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{2+\sqrt{6}+\cos^2(x)}\right)}{6\sqrt{6}} + \frac{\tan(x)}{6(3+2\tan^2(x))}$$

[Out] 1/36*x*6^(1/2)-1/36*arctan(cos(x)*sin(x)/(2+cos(x)^2+6^(1/2)))*6^(1/2)+1/6*tan(x)/(3+2*tan(x)^2)

Rubi [A]

time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {205, 209}

$$\frac{x}{6\sqrt{6}} + \frac{\tan(x)}{6(2\tan^2(x) + 3)} - \frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x) + \sqrt{6} + 2}\right)}{6\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x] + 2*Sec[x])^(-2), x]

[Out] x/(6*Sqrt[6]) - ArcTan[(Cos[x]*Sin[x])/(2 + Sqrt[6] + Cos[x]^2)]/(6*Sqrt[6]) + Tan[x]/(6*(3 + 2*Tan[x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{1}{(\cos(x) + 2 \sec(x))^2} dx &= \text{Subst} \left(\int \frac{1}{(3 + 2x^2)^2} dx, x, \tan(x) \right) \\
&= \frac{\tan(x)}{6(3 + 2 \tan^2(x))} + \frac{1}{6} \text{Subst} \left(\int \frac{1}{3 + 2x^2} dx, x, \tan(x) \right) \\
&= \frac{x}{6\sqrt{6}} - \frac{\tan^{-1} \left(\frac{\cos(x) \sin(x)}{2 + \sqrt{6} + \cos^2(x)} \right)}{6\sqrt{6}} + \frac{\tan(x)}{6(3 + 2 \tan^2(x))}
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 54, normalized size = 0.98

$$\frac{(5 + \cos(2x)) \sec^4(x) \left(\sqrt{6} \tan^{-1} \left(\sqrt{\frac{2}{3}} \tan(x) \right) (5 + \cos(2x)) + 6 \sin(2x) \right)}{144 (1 + 2 \sec^2(x))^2}$$

Antiderivative was successfully verified.

`[In] Integrate[(Cos[x] + 2*Sec[x])^(-2), x]`

```
[Out] ((5 + Cos[2*x])*Sec[x]^4*(Sqrt[6]*ArcTan[Sqrt[2/3]*Tan[x]]*(5 + Cos[2*x]) +
6*Sin[2*x]))/(144*(1 + 2*Sec[x]^2)^2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 12.50, size = 69, normalized size = 1.25

$$\frac{2\sqrt{6} \left(\text{PiFloor} \left[-\frac{1}{2} + \frac{x}{\text{Pi}} \right] + \text{ArcTan} \left[\frac{\sqrt{6} \text{Tan}[x]}{3} \right] \right) \text{Tan}[x]^2 + 3\sqrt{6} \left(\text{PiFloor} \left[-\frac{1}{2} + \frac{x}{\text{Pi}} \right] + \text{ArcTan} \left[\frac{\sqrt{6} \text{Tan}[x]}{3} \right] \right) + 6\text{Tan}[x]}{108 + 72\text{Tan}[x]^2}$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(Cos[x] + 2*Sec[x])^2, x]')`

```
[Out] (2 Sqrt[6] (Pi Floor[-1 / 2 + x / Pi] + ArcTan[Sqrt[6] Tan[x] / 3]) Tan[x]
^ 2 + 3 Sqrt[6] (Pi Floor[-1 / 2 + x / Pi] + ArcTan[Sqrt[6] Tan[x] / 3]) +
6 Tan[x]) / (36 (3 + 2 Tan[x] ^ 2))
```

Maple [A]

time = 0.08, size = 29, normalized size = 0.53

method	result	size
--------	--------	------

default	$\frac{\tan(x)}{18+12(\tan^2(x))} + \frac{\sqrt{6} \arctan\left(\frac{\tan(x)\sqrt{6}}{3}\right)}{36}$	29
risch	$\frac{i(5e^{2ix}+1)}{3e^{4ix}+30e^{2ix}+3} + \frac{i\sqrt{6} \ln(e^{2ix}+2\sqrt{6}+5)}{72} - \frac{i\sqrt{6} \ln(e^{2ix}-2\sqrt{6}+5)}{72}$	68

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cos(x)+2*sec(x))^2,x,method=_RETURNVERBOSE)`

[Out] $1/6*\tan(x)/(3+2*\tan(x)^2)+1/36*6^{(1/2)}*\arctan(1/3*\tan(x)*6^{(1/2)})$

Maxima [A]

time = 0.34, size = 28, normalized size = 0.51

$$\frac{1}{36} \sqrt{6} \arctan\left(\frac{1}{3} \sqrt{6} \tan(x)\right) + \frac{\tan(x)}{6(2 \tan(x)^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(cos(x)+2*sec(x))^2,x, algorithm="maxima")`

[Out] $1/36*\sqrt{6}*\arctan(1/3*\sqrt{6}*\tan(x)) + 1/6*\tan(x)/(2*\tan(x)^2 + 3)$

Fricas [A]

time = 0.33, size = 58, normalized size = 1.05

$$\frac{\left(\sqrt{6} \cos(x)^2 + 2\sqrt{6}\right) \arctan\left(\frac{5\sqrt{6} \cos(x)^2 - 2\sqrt{6}}{12 \cos(x) \sin(x)}\right) - 12 \cos(x) \sin(x)}{72(\cos(x)^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(cos(x)+2*sec(x))^2,x, algorithm="fricas")`

[Out] $-1/72*((\sqrt{6}*\cos(x)^2 + 2*\sqrt{6})*\arctan(1/12*(5*\sqrt{6}*\cos(x)^2 - 2*\sqrt{6}))/(\cos(x)*\sin(x))) - 12*\cos(x)*\sin(x)/(\cos(x)^2 + 2)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\cos(x) + 2 \sec(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(cos(x)+2*sec(x))**2,x)`

[Out] Integral((cos(x) + 2*sec(x))**(-2), x)

Giac [A]

time = 0.00, size = 74, normalized size = 1.35

$$2 \left(\frac{\tan x}{12(2 \tan^2 x + 3)} + \frac{\arctan \left(\frac{-\sqrt{6} \sin(2x) + 2 \sin(2x)}{\sqrt{6} \cos(2x) + \sqrt{6} - 2 \cos(2x) + 2} \right) + x}{2 \cdot 6 \sqrt{6}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)+2*sec(x))^2,x)

[Out] 1/36*sqrt(6)*(x + arctan(-(sqrt(6)*sin(2*x) - 2*sin(2*x))/(sqrt(6)*cos(2*x) + sqrt(6) - 2*cos(2*x) + 2))) + 1/6*tan(x)/(2*tan(x)^2 + 3)

Mupad [B]

time = 0.39, size = 77, normalized size = 1.40

$$\frac{\sqrt{6} \left(2 \operatorname{atan} \left(\frac{\sqrt{6} \tan(\frac{x}{2})^3}{4} + \frac{5 \sqrt{6} \tan(\frac{x}{2})}{12} \right) + 2 \operatorname{atan} \left(\frac{\sqrt{6} \tan(\frac{x}{2})}{4} \right) \right)}{72} + \frac{\frac{\tan(\frac{x}{2})}{9} - \frac{\tan(\frac{x}{2})^3}{9}}{\tan(\frac{x}{2})^4 + \frac{2 \tan(\frac{x}{2})^2}{3} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x) + 2/cos(x))^2,x)

[Out] (6^(1/2)*(2*atan((5*6^(1/2)*tan(x/2))/12 + (6^(1/2)*tan(x/2)^3)/4) + 2*atan((6^(1/2)*tan(x/2))/4)))/72 + (tan(x/2)/9 - tan(x/2)^3/9)/((2*tan(x/2)^2)/3 + tan(x/2)^4 + 1)

$$3.382 \quad \int \frac{5 - \tan(x) - 6 \tan^2(x)}{(1 + 3 \tan(x))^3} dx$$

Optimal. Leaf size=42

$$-\frac{67x}{250} - \frac{28}{125} \log(\cos(x) + 3 \sin(x)) - \frac{7}{10(1 + 3 \tan(x))^2} - \frac{29}{50(1 + 3 \tan(x))}$$

[Out] -67/250*x-28/125*ln(cos(x)+3*sin(x))-7/10/(1+3*tan(x))^2-29/50/(1+3*tan(x))

Rubi [A]

time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3709, 3610, 3612, 3611}

$$-\frac{67x}{250} - \frac{29}{50(3 \tan(x) + 1)} - \frac{7}{10(3 \tan(x) + 1)^2} - \frac{28}{125} \log(3 \sin(x) + \cos(x))$$

Antiderivative was successfully verified.

[In] Int[(5 - Tan[x] - 6*Tan[x]^2)/(1 + 3*Tan[x])^3,x]

[Out] (-67*x)/250 - (28*Log[Cos[x] + 3*Sin[x]])/125 - 7/(10*(1 + 3*Tan[x])^2) - 29/(50*(1 + 3*Tan[x]))

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a*c + b*d, 0]
```


Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)^2), x_Symbol] := Simp[(A*b^2 -
a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{5 - \tan(x) - 6 \tan^2(x)}{(1 + 3 \tan(x))^3} dx &= -\frac{7}{10(1 + 3 \tan(x))^2} + \frac{1}{10} \int \frac{8 - 34 \tan(x)}{(1 + 3 \tan(x))^2} dx \\ &= -\frac{7}{10(1 + 3 \tan(x))^2} - \frac{29}{50(1 + 3 \tan(x))} + \frac{1}{100} \int \frac{-94 - 58 \tan(x)}{1 + 3 \tan(x)} dx \\ &= -\frac{67x}{250} - \frac{7}{10(1 + 3 \tan(x))^2} - \frac{29}{50(1 + 3 \tan(x))} - \frac{28}{125} \int \frac{3 - \tan(x)}{1 + 3 \tan(x)} dx \\ &= -\frac{67x}{250} - \frac{28}{125} \log(\cos(x) + 3 \sin(x)) - \frac{7}{10(1 + 3 \tan(x))^2} - \frac{29}{50(1 + 3 \tan(x))} \end{aligned}$$

Mathematica [A]

time = 0.14, size = 70, normalized size = 1.67

$$\frac{-1305 + 670x + 560 \log(\cos(x) + 3 \sin(x)) - 4 \cos(2x)(-405 + 134x + 112 \log(\cos(x) + 3 \sin(x))) + 6(-90 + 67x + 56 \log(\cos(x) + 3 \sin(x))) \sin(2x)}{500(\cos(x) + 3 \sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - Tan[x] - 6*Tan[x]^2)/(1 + 3*Tan[x])^3,x]
```

```
[Out] -1/500*(-1305 + 670*x + 560*Log[Cos[x] + 3*Sin[x]] - 4*Cos[2*x]*(-405 + 134
*x + 112*Log[Cos[x] + 3*Sin[x]]) + 6*(-90 + 67*x + 56*Log[Cos[x] + 3*Sin[x]
])*Sin[2*x])/(Cos[x] + 3*Sin[x])^2
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 93 vs. 2(42) = 84.

time = 4.66, size = 91, normalized size = 2.17

$$\frac{-320 - 603x \tan^2[x] - 402x \tan[x] - 67x - 504 \log[1 + 3 \tan[x]] \tan^2[x] - 435 \tan[x] - 336 \log[1 + 3 \tan[x]] \tan[x] - 56 \log[1 + 3 \tan[x]] + 28 \log\left[\frac{1}{\cos[2x]}\right] + 168 \log\left[\frac{1}{\cos[x]}\right] \tan[x] + 252 \log\left[\frac{1}{\cos[x]}\right] \tan^2[x]}{250(1 + 3 \tan[x])^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(5 - Tan[x] - 6*Tan[x]^2)/(1 + 3*Tan[x])^3,x]')
```

```
[Out] (-320 - 603 x Tan[x]^2 - 402 x Tan[x] - 67 x - 504 Log[1 + 3 Tan[x]] Tan[
x]^2 - 435 Tan[x] - 336 Log[1 + 3 Tan[x]] Tan[x] - 56 Log[1 + 3 Tan[x]] +
```

28 Log[1 / Cos[x] ^ 2] + 168 Log[1 / Cos[x] ^ 2] Tan[x] + 252 Log[1 / Cos[x] ^ 2] Tan[x] ^ 2) / (250 (1 + 3 Tan[x]) ^ 2)

Maple [A]

time = 0.05, size = 45, normalized size = 1.07

method	result	size
derivativedivides	$\frac{14 \ln(1+\tan^2(x))}{125} - \frac{67 \arctan(\tan(x))}{250} - \frac{7}{10(1+3 \tan(x))^2} - \frac{29}{50(1+3 \tan(x))} - \frac{28 \ln(1+3 \tan(x))}{125}$	45
default	$\frac{14 \ln(1+\tan^2(x))}{125} - \frac{67 \arctan(\tan(x))}{250} - \frac{7}{10(1+3 \tan(x))^2} - \frac{29}{50(1+3 \tan(x))} - \frac{28 \ln(1+3 \tan(x))}{125}$	45
risch	$-\frac{67x}{250} + \frac{28ix}{125} + \frac{(-\frac{36}{24125} - \frac{621i}{48250})(965 e^{2ix} - 324 + 768i)}{(5 e^{2ix} - 4 + 3i)^2} - \frac{28 \ln(e^{2ix} - \frac{4}{5} + \frac{3i}{5})}{125}$	49
norman	$\frac{\frac{297 \tan(x)}{50} + \frac{288(\tan^2(x))}{25} - \frac{67x}{250} - \frac{201x \tan(x)}{125} - \frac{603x(\tan^2(x))}{250}}{(1+3 \tan(x))^2} - \frac{28 \ln(1+3 \tan(x))}{125} + \frac{14 \ln(1+\tan^2(x))}{125}$	55

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x,method=_RETURNVERBOSE)

[Out] 14/125*ln(1+tan(x)^2)-67/250*arctan(tan(x))-7/10/(1+3*tan(x))^2-29/50/(1+3*tan(x))-28/125*ln(1+3*tan(x))

Maxima [A]

time = 0.34, size = 44, normalized size = 1.05

$$-\frac{67}{250}x - \frac{87 \tan(x) + 64}{50(9 \tan(x)^2 + 6 \tan(x) + 1)} + \frac{14}{125} \log(\tan(x)^2 + 1) - \frac{28}{125} \log(3 \tan(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x, algorithm="maxima")

[Out] -67/250*x - 1/50*(87*tan(x) + 64)/(9*tan(x)^2 + 6*tan(x) + 1) + 14/125*log(tan(x)^2 + 1) - 28/125*log(3*tan(x) + 1)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(34) = 68.

time = 0.33, size = 77, normalized size = 1.83

$$\frac{9(134x - 1)\tan(x)^2 + 56(9 \tan(x)^2 + 6 \tan(x) + 1) \log\left(\frac{9 \tan(x)^2 + 6 \tan(x) + 1}{\tan(x)^2 + 1}\right) + 12(67x + 72)\tan(x) + 134x + 639}{500(9 \tan(x)^2 + 6 \tan(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x, algorithm="fricas")

[Out] -1/500*(9*(134*x - 1)*tan(x)^2 + 56*(9*tan(x)^2 + 6*tan(x) + 1)*log((9*tan(x)^2 + 6*tan(x) + 1)/(tan(x)^2 + 1)) + 12*(67*x + 72)*tan(x) + 134*x + 639)/(9*tan(x)^2 + 6*tan(x) + 1)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(39) = 78$
time = 0.26, size = 252, normalized size = 6.00

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5-tan(x)-6*tan(x)**2)/(1+3*tan(x))**3,x)

[Out]
$$\begin{aligned} & -603*x*\tan(x)**2/(2250*\tan(x)**2 + 1500*\tan(x) + 250) - 402*x*\tan(x)/(2250* \\ & \tan(x)**2 + 1500*\tan(x) + 250) - 67*x/(2250*\tan(x)**2 + 1500*\tan(x) + 250) \\ & - 504*\log(3*\tan(x) + 1)*\tan(x)**2/(2250*\tan(x)**2 + 1500*\tan(x) + 250) - 33 \\ & 6*\log(3*\tan(x) + 1)*\tan(x)/(2250*\tan(x)**2 + 1500*\tan(x) + 250) - 56*\log(3* \\ & \tan(x) + 1)/(2250*\tan(x)**2 + 1500*\tan(x) + 250) + 252*\log(\tan(x)**2 + 1)*\tan \\ & \tan(x)**2/(2250*\tan(x)**2 + 1500*\tan(x) + 250) + 168*\log(\tan(x)**2 + 1)*\tan \\ & (x)/(2250*\tan(x)**2 + 1500*\tan(x) + 250) + 28*\log(\tan(x)**2 + 1)/(2250*\tan(x) \\ &)**2 + 1500*\tan(x) + 250) - 435*\tan(x)/(2250*\tan(x)**2 + 1500*\tan(x) + 250) \\ & - 320/(2250*\tan(x)**2 + 1500*\tan(x) + 250) \end{aligned}$$

Giac [A]

time = 0.00, size = 48, normalized size = 1.14

$$\frac{14}{125} \ln(\tan^2 x + 1) - \frac{67}{250}x - \frac{28}{125} \ln|3 \tan x + 1| + \frac{-435 \tan x - 320}{250(3 \tan x + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x)

[Out]
$$\begin{aligned} & -67/250*x - 1/50*(87*\tan(x) + 64)/(3*\tan(x) + 1)^2 + 14/125*\log(\tan(x)^2 + \\ & 1) - 28/125*\log(\text{abs}(3*\tan(x) + 1)) \end{aligned}$$

Mupad [B]

time = 0.28, size = 48, normalized size = 1.14

$$-\frac{28 \ln\left(\tan(x) + \frac{1}{3}\right)}{125} - \frac{\frac{29 \tan(x)}{150} + \frac{32}{225}}{\tan(x)^2 + \frac{2 \tan(x)}{3} + \frac{1}{9}} + \ln(\tan(x) - i) \left(\frac{14}{125} + \frac{67}{500}i\right) + \ln(\tan(x) + i) \left(\frac{14}{125} - \frac{67}{500}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(tan(x) + 6*tan(x)^2 - 5)/(3*tan(x) + 1)^3,x)

[Out]
$$\begin{aligned} & \log(\tan(x) - 1i)*(14/125 + 67i/500) - (28*\log(\tan(x) + 1/3))/125 + \log(\tan(x) \\ & + 1i)*(14/125 - 67i/500) - ((29*\tan(x))/150 + 32/225)/((2*\tan(x))/3 + \tan \\ & \tan(x)^2 + 1/9) \end{aligned}$$

3.383 $\int \cos^2(x) \sec(3x) dx$

Optimal. Leaf size=9

$$\frac{1}{2} \tanh^{-1}(2 \sin(x))$$

[Out] 1/2*arctanh(2*sin(x))

Rubi [A]

time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {212}

$$\frac{1}{2} \tanh^{-1}(2 \sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2*Sec[3*x],x]

[Out] ArcTanh[2*Sin[x]]/2

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \cos^2(x) \sec(3x) dx &= \text{Subst} \left(\int \frac{1}{1 - 4x^2} dx, x, \sin(x) \right) \\ &= \frac{1}{2} \tanh^{-1}(2 \sin(x)) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 9, normalized size = 1.00

$$\frac{1}{2} \tanh^{-1}(2 \sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^2*Sec[3*x],x]

[Out] ArcTanh[2*Sin[x]]/2

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 85 vs. $2(9) = 18$.
time = 4.79, size = 69, normalized size = 7.67

$$-\frac{\text{Log}\left[-1 + \text{Tan}\left[\frac{x}{2}\right]\right]}{6} - \frac{\text{Log}\left[-1 + \text{Sin}[3x]\right]}{12} - \frac{\text{Log}\left[\frac{1-2\text{Sin}[x]}{1+\text{Cos}[x]}\right]}{12} + \frac{\text{Log}\left[1 + \text{Sin}[3x]\right]}{12} + \frac{\text{Log}\left[\frac{1+2\text{Sin}[x]}{1+\text{Cos}[x]}\right]}{12} + \frac{\text{Log}\left[1 + \text{Tan}\left[\frac{x}{2}\right]\right]}{6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Cos[x]^2/Cos[3*x],x]')
```

```
[Out] -Log[-1 + Tan[x / 2]] / 6 - Log[-1 + Sin[3 x]] / 12 - Log[(1 - 2 Sin[x]) / (1 + Cos[x])] / 12 + Log[1 + Sin[3 x]] / 12 + Log[(1 + 2 Sin[x]) / (1 + Cos[x])] / 12 + Log[1 + Tan[x / 2]] / 6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.
time = 0.08, size = 20, normalized size = 2.22

method	result	size
default	$\frac{\ln(1+2\sin(x))}{4} - \frac{\ln(2\sin(x)-1)}{4}$	20
risch	$-\frac{\ln(-ie^{ix}+e^{2ix}-1)}{4} + \frac{\ln(ie^{ix}+e^{2ix}-1)}{4}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/cos(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(1+2*sin(x))-1/4*ln(2*sin(x)-1)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/cos(3*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)^2/cos(3*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.
time = 0.32, size = 19, normalized size = 2.11

$$\frac{1}{4} \log(2 \sin(x) + 1) - \frac{1}{4} \log(-2 \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/cos(3*x),x, algorithm="fricas")
```

[Out] $1/4*\log(2*\sin(x) + 1) - 1/4*\log(-2*\sin(x) + 1)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(7) = 14$

time = 2.17, size = 76, normalized size = 8.44

$$-\frac{\log(\sin(3x) - 1)}{12} + \frac{\log(\sin(3x) + 1)}{12} - \frac{\log(\tan(\frac{x}{2}) - 1)}{6} + \frac{\log(\tan(\frac{x}{2}) + 1)}{6} - \frac{\log(\tan^2(\frac{x}{2}) - 4\tan(\frac{x}{2}) + 1)}{12} + \frac{\log(\tan^2(\frac{x}{2}) + 4\tan(\frac{x}{2}) + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)**2/cos(3*x),x)`

[Out] $-\log(\sin(3x) - 1)/12 + \log(\sin(3x) + 1)/12 - \log(\tan(x/2) - 1)/6 + \log(\tan(x/2) + 1)/6 - \log(\tan(x/2)**2 - 4*\tan(x/2) + 1)/12 + \log(\tan(x/2)**2 + 4*\tan(x/2) + 1)/12$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(7) = 14$.
time = 0.00, size = 26, normalized size = 2.89

$$2 \left(-\frac{\ln |2 \sin x - 1|}{8} + \frac{\ln |2 \sin x + 1|}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^2/cos(3*x),x)`

[Out] $1/4*\log(\text{abs}(2*\sin(x) + 1)) - 1/4*\log(\text{abs}(2*\sin(x) - 1))$

Mupad [B]

time = 0.36, size = 7, normalized size = 0.78

$$\frac{\operatorname{atanh}(2 \sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)^2/cos(3*x),x)`

[Out] $\operatorname{atanh}(2*\sin(x))/2$

3.384 $\int \sec(2x) \sin(x) dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\sqrt{2} \cos(x)\right)}{\sqrt{2}}$$

[Out] 1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4442, 213}

$$\frac{\tanh^{-1}\left(\sqrt{2} \cos(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sec[2*x]*Sin[x],x]

[Out] ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2]

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 4442

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned} \int \sec(2x) \sin(x) dx &= -\text{Subst} \left(\int \frac{1}{-1 + 2x^2} dx, x, \cos(x) \right) \\ &= \frac{\tanh^{-1}\left(\sqrt{2} \cos(x)\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.24, size = 174, normalized size = 11.60

$$\frac{2i \tan^{-1} \left(\frac{\cos(\frac{x}{2}) - (-1 + \sqrt{2}) \sin(\frac{x}{2})}{(1 + \sqrt{2}) \cos(\frac{x}{2}) - \sin(\frac{x}{2})} \right) - 2i \tan^{-1} \left(\frac{\cos(\frac{x}{2}) - (1 + \sqrt{2}) \sin(\frac{x}{2})}{(-1 + \sqrt{2}) \cos(\frac{x}{2}) - \sin(\frac{x}{2})} \right) + 4 \tanh^{-1} \left(\sqrt{2} + \tan \left(\frac{x}{2} \right) \right) - \log \left(2 - \sqrt{2} \cos(x) - \sqrt{2} \sin(x) \right) + \log \left(2 + \sqrt{2} \cos(x) - \sqrt{2} \sin(x) \right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2*x]*Sin[x],x]

[Out] ((2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2])*Cos[x/2] - Sin[x/2])] - (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*Sin[x/2])/((-1 + Sqrt[2])*Cos[x/2] - Sin[x/2])] + 4*ArcTanh[Sqrt[2] + Tan[x/2]] - Log[2 - Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]] + Log[2 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]])/(4*Sqrt[2])

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sin[x]/Cos[2*x],x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.06, size = 13, normalized size = 0.87

method	result	size
default	$\frac{\operatorname{arctanh}(\cos(x)\sqrt{2})\sqrt{2}}{2}$	13
risch	$\frac{\sqrt{2} \ln(e^{2ix} + \sqrt{2} e^{ix} + 1)}{4} - \frac{\sqrt{2} \ln(e^{2ix} - \sqrt{2} e^{ix} + 1)}{4}$	47

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/cos(2*x),x,method=_RETURNVERBOSE)

[Out] 1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(12) = 24.

time = 0.34, size = 129, normalized size = 8.60

$$\frac{1}{8}\sqrt{2} \log(2\sqrt{2} \sin(2x) \sin(x) + 2(\sqrt{2} \cos(x) + 1) \cos(2x) + \cos(2x)^2 + 2 \cos(x)^2 + \sin(2x)^2 + 2 \sin(x)^2 + 2\sqrt{2} \cos(x) + 1) - \frac{1}{8}\sqrt{2} \log(-2\sqrt{2} \sin(2x) \sin(x) - 2(\sqrt{2} \cos(x) - 1) \cos(2x) + \cos(2x)^2 + 2 \cos(x)^2 + \sin(2x)^2 + 2 \sin(x)^2 - 2\sqrt{2} \cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/cos(2*x),x, algorithm="maxima")`

[Out] $\frac{1}{8}\sqrt{2}\log(2\sqrt{2}\sin(2x)\sin(x) + 2(\sqrt{2}\cos(x) + 1)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 1) - \frac{1}{8}\sqrt{2}\log(-2\sqrt{2}\sin(2x)\sin(x) - 2(\sqrt{2}\cos(x) - 1)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

time = 0.33, size = 33, normalized size = 2.20

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2 + 2\sqrt{2}\cos(x) + 1}{2\cos(x)^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/cos(2*x),x, algorithm="fricas")`

[Out] $\frac{1}{4}\sqrt{2}\log(-2\cos(x)^2 + 2\sqrt{2}\cos(x) + 1)/(2\cos(x)^2 - 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{\cos(2x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/cos(2*x),x)`

[Out] `Integral(sin(x)/cos(2*x), x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(12) = 24.

time = 0.01, size = 60, normalized size = 4.00

$$\frac{2\ln\left(\frac{\left|\frac{2(1-\cos x)}{1+\cos x}-6-4\sqrt{2}\right|}{\left|\frac{2(1-\cos x)}{1+\cos x}-6+4\sqrt{2}\right|}\right)}{4\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/cos(2*x),x)`

[Out] $\frac{1}{4}\sqrt{2}\log(\text{abs}(-4\sqrt{2} - 2(\cos(x) - 1)/(\cos(x) + 1) - 6)/\text{abs}(4\sqrt{2} - 2(\cos(x) - 1)/(\cos(x) + 1) - 6))$

Mupad [B]

time = 0.13, size = 12, normalized size = 0.80

$$\frac{\sqrt{2} \operatorname{atanh}\left(\sqrt{2} \cos(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/cos(2*x),x)`

[Out] `(2^(1/2)*atanh(2^(1/2)*cos(x)))/2`

3.385 $\int \sec(2x) \sin^2(x) dx$

Optimal. Leaf size=17

$$-\frac{x}{2} + \frac{1}{4} \tanh^{-1}(2 \cos(x) \sin(x))$$

[Out] -1/2*x+1/4*arctanh(2*cos(x)*sin(x))

Rubi [A]

time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {304, 209, 212}

$$\frac{1}{4} \tanh^{-1}(2 \sin(x) \cos(x)) - \frac{x}{2}$$

Antiderivative was successfully verified.

[In] Int[Sec[2*x]*Sin[x]^2,x]

[Out] -1/2*x + ArcTanh[2*Cos[x]*Sin[x]]/4

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 304

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rubi steps

$$\begin{aligned} \int \sec(2x) \sin^2(x) dx &= \text{Subst} \left(\int \frac{x^2}{1-x^4} dx, x, \tan(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \tan(x) \right) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \tan(x) \right) \\ &= -\frac{x}{2} + \frac{1}{4} \tanh^{-1}(2 \cos(x) \sin(x)) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 28, normalized size = 1.65

$$-\frac{x}{2} - \frac{1}{4} \log(\cos(x) - \sin(x)) + \frac{1}{4} \log(\cos(x) + \sin(x))$$

Antiderivative was successfully verified.

`[In] Integrate[Sec[2*x]*Sin[x]^2,x]``[Out] -1/2*x - Log[Cos[x] - Sin[x]]/4 + Log[Cos[x] + Sin[x]]/4`**Mathics [A]**

time = 2.41, size = 22, normalized size = 1.29

$$-\frac{x}{2} - \frac{\text{Log}[-1 + \text{Sin}[2x]]}{8} + \frac{\text{Log}[1 + \text{Sin}[2x]]}{8}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sin[x]^2/Cos[2*x],x]')``[Out] -x / 2 - Log[-1 + Sin[2 x]] / 8 + Log[1 + Sin[2 x]] / 8`**Maple [A]**

time = 0.07, size = 21, normalized size = 1.24

method	result	size
default	$-\frac{\arctan(\tan(x))}{2} + \frac{\ln(\tan(x)+1)}{4} - \frac{\ln(-1+\tan(x))}{4}$	21
risch	$-\frac{x}{2} - \frac{\ln(e^{2ix}-i)}{4} + \frac{\ln(e^{2ix}+i)}{4}$	27

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sin(x)^2/cos(2*x),x,method=_RETURNVERBOSE)``[Out] -1/2*arctan(tan(x))+1/4*ln(tan(x)+1)-1/4*ln(-1+tan(x))`

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(13) = 26.

time = 0.35, size = 128, normalized size = 7.53

$$-\frac{1}{2}x - \frac{1}{8}\log(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2) + \frac{1}{8}\log(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2) + \frac{1}{8}\log(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2) - \frac{1}{8}\log(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/cos(2*x),x, algorithm="maxima")

[Out] $-1/2*x - 1/8*\log(2*\cos(x)^2 + 2*\sin(x)^2 + 2*\sqrt{2}*\cos(x) + 2*\sqrt{2}*\sin(x) + 2) + 1/8*\log(2*\cos(x)^2 + 2*\sin(x)^2 + 2*\sqrt{2}*\cos(x) - 2*\sqrt{2}*\sin(x) + 2) + 1/8*\log(2*\cos(x)^2 + 2*\sin(x)^2 - 2*\sqrt{2}*\cos(x) + 2*\sqrt{2}*\sin(x) + 2) - 1/8*\log(2*\cos(x)^2 + 2*\sin(x)^2 - 2*\sqrt{2}*\cos(x) - 2*\sqrt{2}*\sin(x) + 2)$

Fricas [A]

time = 0.32, size = 26, normalized size = 1.53

$$-\frac{1}{2}x + \frac{1}{8}\log(2\cos(x)\sin(x) + 1) - \frac{1}{8}\log(-2\cos(x)\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/cos(2*x),x, algorithm="fricas")

[Out] $-1/2*x + 1/8*\log(2*\cos(x)*\sin(x) + 1) - 1/8*\log(-2*\cos(x)*\sin(x) + 1)$

Sympy [A]

time = 0.56, size = 22, normalized size = 1.29

$$-\frac{x}{2} - \frac{\log(\sin(2x) - 1)}{8} + \frac{\log(\sin(2x) + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**2/cos(2*x),x)

[Out] $-x/2 - \log(\sin(2*x) - 1)/8 + \log(\sin(2*x) + 1)/8$

Giac [A]

time = 0.00, size = 27, normalized size = 1.59

$$2\left(-\frac{\ln|\tan x - 1|}{8} + \frac{\ln|\tan x + 1|}{8} - \frac{x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/cos(2*x),x)

[Out] $-1/2*x + 1/4*\log(\text{abs}(\tan(x) + 1)) - 1/4*\log(\text{abs}(\tan(x) - 1))$

Mupad [B]

time = 0.24, size = 9, normalized size = 0.53

$$\frac{\operatorname{atanh}(\tan(x))}{2} - \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)^2/cos(2*x),x)`

[Out] `atanh(tan(x))/2 - x/2`

3.386 $\int \sec(3x) \sin^3(x) dx$

Optimal. Leaf size=21

$$\frac{1}{3} \log(\cos(x)) - \frac{1}{24} \log(3 - 4 \cos^2(x))$$

[Out] 1/3*ln(cos(x))-1/24*ln(3-4*cos(x)^2)

Rubi [A]

time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4451, 457, 78}

$$\frac{1}{3} \log(\cos(x)) - \frac{1}{24} \log(3 - 4 \cos^2(x))$$

Antiderivative was successfully verified.

[In] Int[Sec[3*x]*Sin[x]^3,x]

[Out] Log[Cos[x]]/3 - Log[3 - 4*Cos[x]^2]/24

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 457

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4451

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned}
\int \sec(3x) \sin^3(x) dx &= -\text{Subst} \left(\int \frac{-1+x^2}{x(3-4x^2)} dx, x, \cos(x) \right) \\
&= - \left(\frac{1}{2} \text{Subst} \left(\int \frac{-1+x}{(3-4x)x} dx, x, \cos^2(x) \right) \right) \\
&= - \left(\frac{1}{2} \text{Subst} \left(\int \left(-\frac{1}{3x} + \frac{1}{3(-3+4x)} \right) dx, x, \cos^2(x) \right) \right) \\
&= \frac{1}{3} \log(\cos(x)) - \frac{1}{24} \log(3-4\cos^2(x))
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 21, normalized size = 1.00

$$\frac{1}{3} \log(\cos(x)) - \frac{1}{24} \log(1 - 4 \sin^2(x))$$

Antiderivative was successfully verified.

`[In] Integrate[Sec[3*x]*Sin[x]^3,x]``[Out] Log[Cos[x]]/3 - Log[1 - 4*Sin[x]^2]/24`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Sin[x]^3/Cos[3*x],x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.11, size = 18, normalized size = 0.86

method	result	size
default	$-\frac{\ln(4(\cos^2(x))-3)}{24} + \frac{\ln(\cos(x))}{3}$	18
risch	$-\frac{ix}{4} + \frac{\ln(e^{2ix}+1)}{3} - \frac{\ln(e^{4ix}-e^{2ix}+1)}{24}$	33

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sin(x)^3/cos(3*x),x,method=_RETURNVERBOSE)``[Out] -1/24*ln(4*cos(x)^2-3)+1/3*ln(cos(x))`

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(17) = 34$.
time = 0.37, size = 81, normalized size = 3.86

$$-\frac{1}{48} \log(-2(\cos(2x) - 1)\cos(4x) + \cos(4x)^2 + \cos(2x)^2 + \sin(4x)^2 - 2\sin(4x)\sin(2x) + \sin(2x)^2 - 2\cos(2x) + 1) + \frac{1}{6} \log(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/cos(3*x),x, algorithm="maxima")

[Out] $-1/48*\log(-2*(\cos(2*x) - 1)*\cos(4*x) + \cos(4*x)^2 + \cos(2*x)^2 + \sin(4*x)^2 - 2*\sin(4*x)*\sin(2*x) + \sin(2*x)^2 - 2*\cos(2*x) + 1) + 1/6*\log(\cos(2*x)^2 + \sin(2*x)^2 + 2*\cos(2*x) + 1)$

Fricas [A]

time = 0.34, size = 19, normalized size = 0.90

$$-\frac{1}{24} \log(4 \cos(x)^2 - 3) + \frac{1}{3} \log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/cos(3*x),x, algorithm="fricas")

[Out] $-1/24*\log(4*\cos(x)^2 - 3) + 1/3*\log(-\cos(x))$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(x)}{\cos(3x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**3/cos(3*x),x)

[Out] Integral(sin(x)**3/cos(3*x), x)

Giac [A]

time = 0.00, size = 26, normalized size = 1.24

$$\frac{\ln(-\sin^2 x + 1)}{6} - \frac{\ln|4 \sin^2 x - 1|}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/cos(3*x),x)

[Out] $1/6*\log(-\sin(x)^2 + 1) - 1/24*\log(\text{abs}(4*\sin(x)^2 - 1))$

Mupad [B]

time = 0.12, size = 15, normalized size = 0.71

$$\frac{\ln(\cos(x))}{3} - \frac{\ln(\cos(x)^2 - \frac{3}{4})}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)^3/cos(3*x),x)`

[Out] `log(cos(x))/3 - log(cos(x)^2 - 3/4)/24`

3.387 $\int \cos(x) \csc(3x) dx$

Optimal. Leaf size=21

$$\frac{1}{3} \log(\sin(x)) - \frac{1}{6} \log(3 - 4 \sin^2(x))$$

[Out] 1/3*ln(sin(x))-1/6*ln(3-4*sin(x)^2)

Rubi [A]

time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {4441, 272, 36, 31, 29}

$$\frac{1}{3} \log(\sin(x)) - \frac{1}{6} \log(3 - 4 \sin^2(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Csc[3*x],x]

[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 272

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4441

Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))], x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]

```
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
 \int \cos(x) \csc(3x) dx &= \text{Subst} \left(\int \frac{1}{x(3-4x^2)} dx, x, \sin(x) \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(3-4x)x} dx, x, \sin^2(x) \right) \\
 &= \frac{1}{6} \text{Subst} \left(\int \frac{1}{x} dx, x, \sin^2(x) \right) + \frac{2}{3} \text{Subst} \left(\int \frac{1}{3-4x} dx, x, \sin^2(x) \right) \\
 &= \frac{1}{3} \log(\sin(x)) - \frac{1}{6} \log(3-4\sin^2(x))
 \end{aligned}$$

Mathematica [A]

time = 0.01, size = 21, normalized size = 1.00

$$\frac{1}{3} \log(\sin(x)) - \frac{1}{6} \log(3-4\sin^2(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[3*x], x]
```

```
[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

Mathics [A]

time = 2.38, size = 17, normalized size = 0.81

$$-\frac{\text{Log}[-3 + 4\text{Sin}[x]^2]}{6} + \frac{\text{Log}[\text{Sin}[x]]}{3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Cos[x]/Sin[3*x], x]')
```

```
[Out] -Log[-3 + 4 Sin[x]^2] / 6 + Log[Sin[x]] / 3
```

Maple [A]

time = 0.08, size = 34, normalized size = 1.62

method	result	size
risch	$\frac{\ln(e^{2ix}-1)}{3} - \frac{\ln(e^{4ix}+e^{2ix}+1)}{6}$	27
default	$\frac{\ln(\cos(x)-1)}{6} - \frac{\ln(1+2\cos(x))}{6} - \frac{\ln(2\cos(x)-1)}{6} + \frac{\ln(1+\cos(x))}{6}$	34

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)/sin(3*x),x,method=_RETURNVERBOSE)`

[Out] $1/6*\ln(\cos(x)-1)-1/6*\ln(1+2*\cos(x))-1/6*\ln(2*\cos(x)-1)+1/6*\ln(1+\cos(x))$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(17) = 34$.

time = 0.34, size = 129, normalized size = 6.14

$$-\frac{1}{12} \log(2(\cos(x)+1)\cos(2x) + \cos(2x)^2 + \cos(x)^2 + \sin(2x)^2 + 2\sin(2x)\sin(x) + \sin(x)^2 + 2\cos(x) + 1) - \frac{1}{12} \log(-2(\cos(x)-1)\cos(2x) + \cos(2x)^2 + \cos(x)^2 + \sin(2x)^2 - 2\sin(2x)\sin(x) + \sin(x)^2 - 2\cos(x) + 1) + \frac{1}{6} \log(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1) + \frac{1}{6} \log(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/sin(3*x),x, algorithm="maxima")`

[Out] $-1/12*\log(2*(\cos(x) + 1)*\cos(2*x) + \cos(2*x)^2 + \cos(x)^2 + \sin(2*x)^2 + 2*\sin(2*x)*\sin(x) + \sin(x)^2 + 2*\cos(x) + 1) - 1/12*\log(-2*(\cos(x) - 1)*\cos(2*x) + \cos(2*x)^2 + \cos(x)^2 + \sin(2*x)^2 - 2*\sin(2*x)*\sin(x) + \sin(x)^2 - 2*\cos(x) + 1) + 1/6*\log(\cos(x)^2 + \sin(x)^2 + 2*\cos(x) + 1) + 1/6*\log(\cos(x)^2 + \sin(x)^2 - 2*\cos(x) + 1)$

Fricas [A]

time = 0.32, size = 19, normalized size = 0.90

$$-\frac{1}{6} \log(4 \cos(x)^2 - 1) + \frac{1}{3} \log\left(\frac{1}{2} \sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/sin(3*x),x, algorithm="fricas")`

[Out] $-1/6*\log(4*\cos(x)^2 - 1) + 1/3*\log(1/2*\sin(x))$

Sympy [A]

time = 0.57, size = 17, normalized size = 0.81

$$-\frac{\log(4 \sin^2(x) - 3)}{6} + \frac{\log(\sin(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/sin(3*x),x)`

[Out] $-\log(4*\sin(x)**2 - 3)/6 + \log(\sin(x))/3$

Giac [A]

time = 0.00, size = 26, normalized size = 1.24

$$\frac{\ln(-\cos^2 x + 1)}{6} - \frac{\ln|4 \cos^2 x - 1|}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(3*x),x)
```

```
[Out] 1/6*log(-cos(x)^2 + 1) - 1/6*log(abs(4*cos(x)^2 - 1))
```

Mupad [B]

time = 0.26, size = 17, normalized size = 0.81

$$\frac{\ln(\sin(x))}{3} - \frac{\ln\left(\frac{1}{4} - \cos(x)^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(3*x),x)
```

```
[Out] log(sin(x))/3 - log(1/4 - cos(x)^2)/6
```

3.388 $\int \csc(4x) \sin(x) dx$

Optimal. Leaf size=26

$$-\frac{1}{4} \tanh^{-1}(\sin(x)) + \frac{\tanh^{-1}(\sqrt{2} \sin(x))}{2\sqrt{2}}$$

[Out] $-1/4*\operatorname{arctanh}(\sin(x))+1/4*\operatorname{arctanh}(\sin(x)*2^{(1/2)})*2^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1107, 213}

$$\frac{\tanh^{-1}(\sqrt{2} \sin(x))}{2\sqrt{2}} - \frac{1}{4} \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csc}[4*x]*\operatorname{Sin}[x], x]$

[Out] $-1/4*\operatorname{ArcTanh}[\operatorname{Sin}[x]] + \operatorname{ArcTanh}[\operatorname{Sqrt}[2]*\operatorname{Sin}[x]]/(2*\operatorname{Sqrt}[2])$

Rule 213

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2])^{-1})*\operatorname{ArcTanh}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{LtQ}[a, 0] \ || \ \operatorname{GtQ}[b, 0])$

Rule 1107

$\operatorname{Int}[(a_ + (b_)*(x_)^2 + (c_)*(x_)^4)^{-1}, x_Symbol] := \operatorname{With}\{q = \operatorname{Rt}[b^2 - 4*a*c, 2]\}, \operatorname{Dist}[c/q, \operatorname{Int}[1/(b/2 - q/2 + c*x^2), x], x] - \operatorname{Dist}[c/q, \operatorname{Int}[1/(b/2 + q/2 + c*x^2), x], x] /;$ $\operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \ \&\& \operatorname{PosQ}[b^2 - 4*a*c]$

Rubi steps

$$\begin{aligned} \int \csc(4x) \sin(x) dx &= \operatorname{Subst} \left(\int \frac{1}{4 - 12x^2 + 8x^4} dx, x, \sin(x) \right) \\ &= 2\operatorname{Subst} \left(\int \frac{1}{-8 + 8x^2} dx, x, \sin(x) \right) - 2\operatorname{Subst} \left(\int \frac{1}{-4 + 8x^2} dx, x, \sin(x) \right) \\ &= -\frac{1}{4} \tanh^{-1}(\sin(x)) + \frac{\tanh^{-1}(\sqrt{2} \sin(x))}{2\sqrt{2}} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 26, normalized size = 1.00

$$-\frac{1}{4} \tanh^{-1}(\sin(x)) + \frac{\tanh^{-1}(\sqrt{2} \sin(x))}{2\sqrt{2}}$$

Antiderivative was successfully verified.

`[In] Integrate[Csc[4*x]*Sin[x],x]``[Out] -1/4*ArcTanh[Sin[x]] + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])`**Mathics [B]** Leaf count is larger than twice the leaf count of optimal. 123 vs. $2(26) = 52$.

time = 8.43, size = 83, normalized size = 3.19

$$-\frac{\text{Log}\left[1 + \text{Tan}\left[\frac{x}{2}\right]\right]}{4} - \frac{\sqrt{2} \text{Log}\left[-1 - \sqrt{2} + \text{Tan}\left[\frac{x}{2}\right]\right]}{8} - \frac{\sqrt{2} \text{Log}\left[1 - \sqrt{2} + \text{Tan}\left[\frac{x}{2}\right]\right]}{8} + \frac{\sqrt{2} \text{Log}\left[-1 + \sqrt{2} + \text{Tan}\left[\frac{x}{2}\right]\right]}{8} + \frac{\sqrt{2} \text{Log}\left[1 + \sqrt{2} + \text{Tan}\left[\frac{x}{2}\right]\right]}{8} + \frac{\text{Log}\left[-1 + \text{Tan}\left[\frac{x}{2}\right]\right]}{4}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sin[x]/Sin[4*x],x]')`

```
[Out] -Log[1 + Tan[x / 2]] / 4 - Sqrt[2] Log[-1 - Sqrt[2] + Tan[x / 2]] / 8 - Sqrt[2] Log[1 - Sqrt[2] + Tan[x / 2]] / 8 + Sqrt[2] Log[-1 + Sqrt[2] + Tan[x / 2]] / 8 + Sqrt[2] Log[1 + Sqrt[2] + Tan[x / 2]] / 8 + Log[-1 + Tan[x / 2]] / 4
```

Maple [A]

time = 0.10, size = 28, normalized size = 1.08

method	result	size
default	$-\frac{\ln(\sin(x)+1)}{8} + \frac{\arctanh(\sin(x)\sqrt{2})\sqrt{2}}{4} + \frac{\ln(-1+\sin(x))}{8}$	28
risch	$\frac{\ln(e^{ix}-i)}{4} - \frac{\ln(e^{ix}+i)}{4} + \frac{\sqrt{2} \ln(e^{2ix}+i\sqrt{2}e^{ix}-1)}{8} - \frac{\sqrt{2} \ln(e^{2ix}-i\sqrt{2}e^{ix}-1)}{8}$	72

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sin(x)/sin(4*x),x,method=_RETURNVERBOSE)``[Out] -1/8*ln(sin(x)+1)+1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)+1/8*ln(-1+sin(x))`**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(18) = 36$.

time = 0.36, size = 171, normalized size = 6.58

$$\frac{1}{16} \sqrt{2} \log(2 \cos(x)^2 + 2 \sin(x)^2 + 2 \sqrt{2} \cos(x) + 2 \sqrt{2} \sin(x) + 2) - \frac{1}{16} \sqrt{2} \log(2 \cos(x)^2 + 2 \sin(x)^2 + 2 \sqrt{2} \cos(x) - 2 \sqrt{2} \sin(x) + 2) + \frac{1}{16} \sqrt{2} \log(2 \cos(x)^2 + 2 \sin(x)^2 - 2 \sqrt{2} \cos(x) + 2 \sqrt{2} \sin(x) + 2) - \frac{1}{16} \sqrt{2} \log(2 \cos(x)^2 + 2 \sin(x)^2 - 2 \sqrt{2} \cos(x) - 2 \sqrt{2} \sin(x) + 2) - \frac{1}{8} \log(\cos(x)^2 + \sin(x)^2 + 2 \sin(x) + 1) + \frac{1}{8} \log(\cos(x)^2 + \sin(x)^2 - 2 \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/sin(4*x),x, algorithm="maxima")

[Out] $\frac{1}{16}\sqrt{2}\log(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2) - \frac{1}{16}\sqrt{2}\log(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2) + \frac{1}{16}\sqrt{2}\log(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2) - \frac{1}{16}\sqrt{2}\log(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2) - \frac{1}{8}\log(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1) + \frac{1}{8}\log(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(18) = 36.

time = 0.35, size = 50, normalized size = 1.92

$$\frac{1}{8}\sqrt{2}\log\left(\frac{2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/sin(4*x),x, algorithm="fricas")

[Out] $\frac{1}{8}\sqrt{2}\log(-(2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3)/(2\cos(x)^2 - 1)) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(22) = 44

time = 3.49, size = 294, normalized size = 11.31

$$\frac{27720\sqrt{2}\log(\tan(\frac{x}{2}) - 1)}{110880\sqrt{2} + 156808} - \frac{39202\log(\tan(\frac{x}{2}) - 1)}{110880\sqrt{2} + 156808} - \frac{39202\log(\tan(\frac{x}{2}) + 1)}{110880\sqrt{2} + 156808} + \frac{27720\sqrt{2}\log(\tan(\frac{x}{2}) + 1)}{110880\sqrt{2} + 156808} + \frac{27720\log(\tan(\frac{x}{2}) - 1 + \sqrt{2})}{110880\sqrt{2} + 156808} + \frac{19601\sqrt{2}\log(\tan(\frac{x}{2}) - 1 + \sqrt{2})}{110880\sqrt{2} + 156808} + \frac{27720\log(\tan(\frac{x}{2}) + 1 + \sqrt{2})}{110880\sqrt{2} + 156808} + \frac{19601\sqrt{2}\log(\tan(\frac{x}{2}) + 1 + \sqrt{2})}{110880\sqrt{2} + 156808} - \frac{19601\sqrt{2}\log(\tan(\frac{x}{2}) - \sqrt{2} - 1)}{110880\sqrt{2} + 156808} - \frac{27720\log(\tan(\frac{x}{2}) - \sqrt{2} - 1)}{110880\sqrt{2} + 156808} - \frac{19601\sqrt{2}\log(\tan(\frac{x}{2}) - \sqrt{2} + 1)}{110880\sqrt{2} + 156808} - \frac{27720\log(\tan(\frac{x}{2}) - \sqrt{2} + 1)}{110880\sqrt{2} + 156808}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/sin(4*x),x)

[Out] $27720\sqrt{2}\log(\tan(x/2) - 1)/(110880\sqrt{2} + 156808) + 39202\log(\tan(x/2) - 1)/(110880\sqrt{2} + 156808) - 39202\log(\tan(x/2) + 1)/(110880\sqrt{2} + 156808) - 27720\sqrt{2}\log(\tan(x/2) + 1)/(110880\sqrt{2} + 156808) + 27720\log(\tan(x/2) - 1 + \sqrt{2})/(110880\sqrt{2} + 156808) + 19601\sqrt{2}\log(\tan(x/2) - 1 + \sqrt{2})/(110880\sqrt{2} + 156808) + 27720\log(\tan(x/2) + 1 + \sqrt{2})/(110880\sqrt{2} + 156808) + 19601\sqrt{2}\log(\tan(x/2) + 1 + \sqrt{2})/(110880\sqrt{2} + 156808) - 19601\sqrt{2}\log(\tan(x/2) - \sqrt{2} - 1)/(110880\sqrt{2} + 156808) - 27720\log(\tan(x/2) - \sqrt{2} - 1)/(110880\sqrt{2} + 156808) - 19601\sqrt{2}\log(\tan(x/2) - \sqrt{2} + 1)/(110880\sqrt{2} + 156808) - 27720\log(\tan(x/2) - \sqrt{2} + 1)/(110880\sqrt{2} + 156808)$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(18) = 36.
time = 0.00, size = 59, normalized size = 2.27

$$\frac{\ln(-\sin x + 1)}{8} - \frac{\ln(\sin x + 1)}{8} - \frac{\ln\left(\frac{|4\sin x - 2\sqrt{2}|}{|4\sin x + 2\sqrt{2}|}\right)}{4\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/sin(4*x),x)

[Out] $-1/8*\sqrt{2}*\log(\text{abs}(-2*\sqrt{2} + 4*\sin(x))/\text{abs}(2*\sqrt{2} + 4*\sin(x))) - 1/8*\log(\sin(x) + 1) + 1/8*\log(-\sin(x) + 1)$

Mupad [B]

time = 0.47, size = 27, normalized size = 1.04

$$\frac{\sqrt{2} \operatorname{atanh}\left(\sqrt{2} \sin(x)\right)}{4} - \frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/sin(4*x),x)

[Out] $(2^{(1/2)}*\operatorname{atanh}(2^{(1/2)}*\sin(x)))/4 - \operatorname{atanh}(\sin(x/2)/\cos(x/2))/2$

3.389 $\int \csc(4x) \sin^3(x) dx$

Optimal. Leaf size=26

$$-\frac{1}{4} \tanh^{-1}(\sin(x)) + \frac{\tanh^{-1}(\sqrt{2} \sin(x))}{4\sqrt{2}}$$

[Out] $-1/4*\operatorname{arctanh}(\sin(x))+1/8*\operatorname{arctanh}(\sin(x)*2^{(1/2)})*2^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1144, 213}

$$\frac{\tanh^{-1}(\sqrt{2} \sin(x))}{4\sqrt{2}} - \frac{1}{4} \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csc}[4*x]*\operatorname{Sin}[x]^3, x]$

[Out] $-1/4*\operatorname{ArcTanh}[\operatorname{Sin}[x]] + \operatorname{ArcTanh}[\operatorname{Sqrt}[2]*\operatorname{Sin}[x]]/(4*\operatorname{Sqrt}[2])$

Rule 213

$\operatorname{Int}[(a_+ + (b_+)(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2])^{-1})*\operatorname{ArcTanh}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1144

$\operatorname{Int}[(d_+)(x_+)^m / ((a_+ + (b_+)(x_+)^2 + (c_+)(x_+)^4), x_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[b^2 - 4*a*c, 2]\}, \operatorname{Dist}[(d^2/2)*(b/q + 1), \operatorname{Int}[(d*x)^{m-2} / (b/2 + q/2 + c*x^2), x], x] - \operatorname{Dist}[(d^2/2)*(b/q - 1), \operatorname{Int}[(d*x)^{m-2} / (b/2 - q/2 + c*x^2), x], x] /;$ FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 2]

Rubi steps

$$\begin{aligned} \int \csc(4x) \sin^3(x) dx &= \operatorname{Subst} \left(\int \frac{x^2}{4 - 12x^2 + 8x^4} dx, x, \sin(x) \right) \\ &= 2\operatorname{Subst} \left(\int \frac{1}{-8 + 8x^2} dx, x, \sin(x) \right) - \operatorname{Subst} \left(\int \frac{1}{-4 + 8x^2} dx, x, \sin(x) \right) \\ &= -\frac{1}{4} \tanh^{-1}(\sin(x)) + \frac{\tanh^{-1}(\sqrt{2} \sin(x))}{4\sqrt{2}} \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.21, size = 218, normalized size = 8.38

$$\frac{-2i \tan^{-1} \left(\frac{\cos(\frac{x}{2}) - (-1 + \sqrt{2}) \sin(\frac{x}{2})}{(1 + \sqrt{2}) \cos(\frac{x}{2}) - \sin(\frac{x}{2})} \right) - 2i \tan^{-1} \left(\frac{\cos(\frac{x}{2}) - (1 + \sqrt{2}) \sin(\frac{x}{2})}{(-1 + \sqrt{2}) \cos(\frac{x}{2}) - \sin(\frac{x}{2})} \right) + 4\sqrt{2} \log(\cos(\frac{x}{2}) - \sin(\frac{x}{2})) - 4\sqrt{2} \log(\cos(\frac{x}{2}) + \sin(\frac{x}{2})) + 2 \log(\sqrt{2} + 2 \sin(x)) - \log(2 - \sqrt{2} \cos(x) - \sqrt{2} \sin(x)) - \log(2 + \sqrt{2} \cos(x) - \sqrt{2} \sin(x))}{16\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[4*x]*Sin[x]^3,x]

[Out] ((-2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2])*Cos[x/2] - Sin[x/2])] - (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*Sin[x/2])/((-1 + Sqrt[2])*Cos[x/2] - Sin[x/2])] + 4*Sqrt[2]*Log[Cos[x/2] - Sin[x/2]] - 4*Sqrt[2]*Log[Cos[x/2] + Sin[x/2]] + 2*Log[Sqrt[2] + 2*Sin[x]] - Log[2 - Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]] - Log[2 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]])/(16*Sqrt[2])

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 123 vs. 2(26) = 52.

time = 13.08, size = 83, normalized size = 3.19

$$-\frac{\text{Log}\left[1 + \text{Tan}\left[\frac{x}{2}\right]\right]}{4} - \frac{\sqrt{2} \text{Log}\left[-1 - \sqrt{2} + \text{Tan}\left[\frac{x}{2}\right]\right]}{16} - \frac{\sqrt{2} \text{Log}\left[1 - \sqrt{2} + \text{Tan}\left[\frac{x}{2}\right]\right]}{16} + \frac{\sqrt{2} \text{Log}\left[-1 + \sqrt{2} + \text{Tan}\left[\frac{x}{2}\right]\right]}{16} + \frac{\sqrt{2} \text{Log}\left[1 + \sqrt{2} + \text{Tan}\left[\frac{x}{2}\right]\right]}{16} + \frac{\text{Log}\left[-1 + \text{Tan}\left[\frac{x}{2}\right]\right]}{4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Sin[x]^3/Sin[4*x],x]')

[Out] -Log[1 + Tan[x / 2]] / 4 - Sqrt[2] Log[-1 - Sqrt[2] + Tan[x / 2]] / 16 - Sqrt[2] Log[1 - Sqrt[2] + Tan[x / 2]] / 16 + Sqrt[2] Log[-1 + Sqrt[2] + Tan[x / 2]] / 16 + Sqrt[2] Log[1 + Sqrt[2] + Tan[x / 2]] / 16 + Log[-1 + Tan[x / 2]] / 4

Maple [A]

time = 0.10, size = 28, normalized size = 1.08

method	result	size
default	$\frac{\text{arctanh}\left(\frac{\sin(x)\sqrt{2}}{8}\right)\sqrt{2}}{8} + \frac{\ln(-1+\sin(x))}{8} - \frac{\ln(\sin(x)+1)}{8}$	28
risch	$-\frac{\ln(e^{ix}+i)}{4} + \frac{\ln(e^{ix}-i)}{4} - \frac{\sqrt{2} \ln(e^{2ix}-i\sqrt{2}e^{ix}-1)}{16} + \frac{\sqrt{2} \ln(e^{2ix}+i\sqrt{2}e^{ix}-1)}{16}$	72

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^3/sin(4*x),x,method=_RETURNVERBOSE)

[Out] 1/8*arctanh(sin(x)*2^(1/2))*2^(1/2)+1/8*ln(-1+sin(x))-1/8*ln(sin(x)+1)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(18) = 36.

time = 0.35, size = 171, normalized size = 6.58

$$\frac{1}{8}\sqrt{2} \log(2 \cos(x)^2 + 2 \sin(x)^2 + 2\sqrt{2} \cos(x) + 2\sqrt{2} \sin(x) + 2) - \frac{1}{8}\sqrt{2} \log(2 \cos(x)^2 + 2 \sin(x)^2 + 2\sqrt{2} \cos(x) - 2\sqrt{2} \sin(x) + 2) + \frac{1}{8}\sqrt{2} \log(2 \cos(x)^2 + 2 \sin(x)^2 - 2\sqrt{2} \cos(x) + 2\sqrt{2} \sin(x) + 2) - \frac{1}{8}\sqrt{2} \log(2 \cos(x)^2 + 2 \sin(x)^2 - 2\sqrt{2} \cos(x) - 2\sqrt{2} \sin(x) + 2) - \frac{1}{8} \log(\cos(x)^2 + \sin(x)^2 + 2 \sin(x) + 1) + \frac{1}{8} \log(\cos(x)^2 + \sin(x)^2 - 2 \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/sin(4*x),x, algorithm="maxima")

[Out] $\frac{1}{32}\sqrt{2}\log(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2})\sin(x) + 2) - \frac{1}{32}\sqrt{2}\log(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2) + \frac{1}{32}\sqrt{2}\log(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2) - \frac{1}{32}\sqrt{2}\log(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2) - \frac{1}{8}\log(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1) + \frac{1}{8}\log(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.

time = 0.37, size = 50, normalized size = 1.92

$$\frac{1}{16}\sqrt{2}\log\left(-\frac{2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/sin(4*x),x, algorithm="fricas")

[Out] $\frac{1}{16}\sqrt{2}\log(-(2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3)/(2\cos(x)^2 - 1)) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(22) = 44$

time = 7.80, size = 294, normalized size = 11.31

.....

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**3/sin(4*x),x)

[Out] $4093147632754948\log(\tan(x/2) - 1)/(16372590531019792 + 11577169790074752\sqrt{2}) + 2894292447518688\sqrt{2}\log(\tan(x/2) - 1)/(16372590531019792 + 11577169790074752\sqrt{2}) - 4093147632754948\log(\tan(x/2) + 1)/(16372590531019792 + 11577169790074752\sqrt{2}) - 2894292447518688\sqrt{2}\log(\tan(x/2) + 1)/(16372590531019792 + 11577169790074752\sqrt{2}) + 1447146223759344\log(\tan(x/2) - 1 + \sqrt{2})/(16372590531019792 + 11577169790074752\sqrt{2}) + 1023286908188737\sqrt{2}\log(\tan(x/2) - 1 + \sqrt{2})/(16372590531019792 + 11577169790074752\sqrt{2}) + 1447146223759344\log(\tan(x/2) + 1 + \sqrt{2})/(16372590531019792 + 11577169790074752\sqrt{2}) + 1023286908188737\sqrt{2}\log(\tan(x/2) + 1 + \sqrt{2})/(16372590531019792 + 11577169790074752\sqrt{2}) - 1447146223759344\log(\tan(x/2) - \sqrt{2} - 1)/(16372590531019792 + 11577169790074752\sqrt{2}) - 1023286908188737\sqrt{2}\log(\tan(x/2) - \sqrt{2} - 1)/(16372590531019792 + 11577169790074752\sqrt{2}) - 1447146223759344\log(\tan(x/2) - \sqrt{2} - 1)/(16372590531019792 + 11577169790074752\sqrt{2}) - 1023286908188737\sqrt{2}\log(\tan(x/2) - \sqrt{2} - 1)/(16372590531019792 + 11577169790074752\sqrt{2}) - 1447146223759344\log(\tan(x/2) - \sqrt{2} - 1)/(16372590531019792 + 11577169790074752\sqrt{2})$

$x/2) - \sqrt{2} + 1)/(16372590531019792 + 11577169790074752*\sqrt{2}) - 10232$
 $86908188737*\sqrt{2}*\log(\tan(x/2) - \sqrt{2} + 1)/(16372590531019792 + 115771$
 $69790074752*\sqrt{2})$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.
time = 0.00, size = 59, normalized size = 2.27

$$\frac{\ln(-\sin x + 1)}{8} - \frac{\ln(\sin x + 1)}{8} - \frac{\ln\left(\left|\frac{4\sin x - 2\sqrt{2}}{4\sin x + 2\sqrt{2}}\right|\right)}{8\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/sin(4*x),x)

[Out] $-1/16*\sqrt{2}*\log(\text{abs}(-2*\sqrt{2} + 4*\sin(x))/\text{abs}(2*\sqrt{2} + 4*\sin(x))) - 1$
 $/8*\log(\sin(x) + 1) + 1/8*\log(-\sin(x) + 1)$

Mupad [B]

time = 0.29, size = 27, normalized size = 1.04

$$\frac{\sqrt{2} \operatorname{atanh}(\sqrt{2} \sin(x))}{8} - \frac{\operatorname{atanh}\left(\frac{\sin(\frac{x}{2})}{\cos(\frac{x}{2})}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^3/sin(4*x),x)

[Out] $(2^{(1/2)}*\operatorname{atanh}(2^{(1/2)}*\sin(x)))/8 - \operatorname{atanh}(\sin(x/2)/\cos(x/2))/2$

3.390 $\int \sqrt{1 + \sin(2x)} dx$

Optimal. Leaf size=16

$$-\frac{\cos(2x)}{\sqrt{1 + \sin(2x)}}$$

[Out] $-\cos(2*x)/(1+\sin(2*x))^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2725}

$$-\frac{\cos(2x)}{\sqrt{\sin(2x) + 1}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[1 + \text{Sin}[2*x]], x]$

[Out] $-(\text{Cos}[2*x]/\text{Sqrt}[1 + \text{Sin}[2*x]])$

Rule 2725

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \text{ :> Simp}[-2*b*(\text{Cos}[c + d*x]/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]])), x] \text{ /; FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{Eq} \text{Q}[a^2 - b^2, 0]$

Rubi steps

$$\int \sqrt{1 + \sin(2x)} dx = -\frac{\cos(2x)}{\sqrt{1 + \sin(2x)}}$$

Mathematica [A]

time = 0.01, size = 25, normalized size = 1.56

$$\frac{(-\cos(x) + \sin(x))\sqrt{1 + \sin(2x)}}{\cos(x) + \sin(x)}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Sqrt}[1 + \text{Sin}[2*x]], x]$

[Out] $((-\text{Cos}[x] + \text{Sin}[x])* \text{Sqrt}[1 + \text{Sin}[2*x]])/(\text{Cos}[x] + \text{Sin}[x])$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[1 + Sin[2*x]],x]')
```

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.10, size = 22, normalized size = 1.38

method	result	size
default	$\frac{(\sin(2x)-1)\sqrt{1+\sin(2x)}}{\cos(2x)}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (sin(2*x)-1)*(1+sin(2*x))^(1/2)/cos(2*x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(2*x) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(14) = 28.

time = 0.38, size = 34, normalized size = 2.12

$$-\frac{(\cos(2x) - \sin(2x) + 1)\sqrt{\sin(2x) + 1}}{\cos(2x) + \sin(2x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] -(cos(2*x) - sin(2*x) + 1)*sqrt(sin(2*x) + 1)/(cos(2*x) + sin(2*x) + 1)
```


Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin(2x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sin(2*x))**(1/2),x)

[Out] Integral(sqrt(sin(2*x) + 1), x)

Giac [A]

time = 0.00, size = 23, normalized size = 1.44

$$\sqrt{2} \operatorname{sign}\left(\cos\left(-\frac{\pi}{4} + x\right)\right) \sin\left(-\frac{\pi}{4} + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sin(2*x))^(1/2),x)

[Out] sqrt(2)*sgn(cos(-1/4*pi + x))*sin(-1/4*pi + x)

Mupad [B]

time = 0.23, size = 21, normalized size = 1.31

$$\frac{(\sin(2x) - 1) \sqrt{\sin(2x) + 1}}{\cos(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(2*x) + 1)^(1/2),x)

[Out] ((sin(2*x) - 1)*(sin(2*x) + 1)^(1/2))/cos(2*x)

3.391 $\int \sqrt{1 - \sin(2x)} dx$

Optimal. Leaf size=17

$$\frac{\cos(2x)}{\sqrt{1 - \sin(2x)}}$$

[Out] $\cos(2*x)/(1-\sin(2*x))^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2725}

$$\frac{\cos(2x)}{\sqrt{1 - \sin(2x)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[1 - \text{Sin}[2*x]], x]$

[Out] $\text{Cos}[2*x]/\text{Sqrt}[1 - \text{Sin}[2*x]]$

Rule 2725

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \text{ :> } \text{Simp}[-2*b*(\text{Cos}[c + d*x]/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]])), x] \text{ /; } \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rubi steps

$$\int \sqrt{1 - \sin(2x)} dx = \frac{\cos(2x)}{\sqrt{1 - \sin(2x)}}$$

Mathematica [A]

time = 0.01, size = 27, normalized size = 1.59

$$\frac{(\cos(x) + \sin(x))\sqrt{1 - \sin(2x)}}{\cos(x) - \sin(x)}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Sqrt}[1 - \text{Sin}[2*x]], x]$

[Out] $((\text{Cos}[x] + \text{Sin}[x])*\text{Sqrt}[1 - \text{Sin}[2*x]])/(\text{Cos}[x] - \text{Sin}[x]))$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Sqrt[1 - Sin[2*x]],x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.10, size = 31, normalized size = 1.82

method	result	size
default	$-\frac{(\sin(2x)-1)(1+\sin(2x))}{\cos(2x)\sqrt{1-\sin(2x)}}$	31

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((1-sin(2*x))^(1/2),x,method=_RETURNVERBOSE)``[Out] -(sin(2*x)-1)*(1+sin(2*x))/cos(2*x)/(1-sin(2*x))^(1/2)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((1-sin(2*x))^(1/2),x, algorithm="maxima")``[Out] integrate(sqrt(-sin(2*x) + 1), x)`**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(15) = 30.

time = 0.31, size = 35, normalized size = 2.06

$$\frac{(\cos(2x) + \sin(2x) + 1)\sqrt{-\sin(2x) + 1}}{\cos(2x) - \sin(2x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((1-sin(2*x))^(1/2),x, algorithm="fricas")``[Out] (cos(2*x) + sin(2*x) + 1)*sqrt(-sin(2*x) + 1)/(cos(2*x) - sin(2*x) + 1)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1 - \sin(2x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sin(2*x))**(1/2),x)

[Out] Integral(sqrt(1 - sin(2*x)), x)

Giac [A]

time = 0.00, size = 39, normalized size = 2.29

$$\sqrt{2} \left(\operatorname{sign} \left(\sin \left(\frac{2}{2}x - \frac{\pi}{4} \right) \right) - \operatorname{sign} \left(\sin \left(-\frac{\pi}{4} + x \right) \right) \cos \left(-\frac{\pi}{4} + x \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sin(2*x))^(1/2),x)

[Out] -sqrt(2)*(cos(-1/4*pi + x)*sgn(sin(-1/4*pi + x)) - sgn(sin(-1/4*pi + x)))

Mupad [B]

time = 0.22, size = 23, normalized size = 1.35

$$\frac{\sqrt{1 - \sin(2x)} (\sin(2x) + 1)}{\cos(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - sin(2*x))^(1/2),x)

[Out] ((1 - sin(2*x))^(1/2)*(sin(2*x) + 1))/cos(2*x)

$$3.392 \quad \int \frac{1}{\sqrt{1 + \cos(2x)}} dx$$

Optimal. Leaf size=27

$$\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1+\cos(2x)}}\right)}{\sqrt{2}}$$

[Out] 1/2*arctanh(1/2*sin(2*x)*2^(1/2)/(1+cos(2*x))^(1/2))*2^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2728, 212}

$$\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{\cos(2x)+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 + Cos[2*x]],x]

[Out] ArcTanh[Sin[2*x]/(Sqrt[2]*Sqrt[1 + Cos[2*x]])]/Sqrt[2]

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1 + \cos(2x)}} dx &= -\text{Subst} \left(\int \frac{1}{2 - x^2} dx, x, -\frac{\sin(2x)}{\sqrt{1 + \cos(2x)}} \right) \\ &= \frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1+\cos(2x)}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 47, normalized size = 1.74

$$-\frac{\cos(x) \left(\log \left(\cos \left(\frac{x}{2} \right) - \sin \left(\frac{x}{2} \right) \right) - \log \left(\cos \left(\frac{x}{2} \right) + \sin \left(\frac{x}{2} \right) \right) \right)}{\sqrt{1 + \cos(2x)}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/Sqrt[1 + Cos[2*x]],x]``[Out] -((Cos[x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]))/Sqrt[1 + Cos[2*x]])`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/Sqrt[1 + Cos[2*x]],x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 0.05, size = 9, normalized size = 0.33

method	result	size
default	$\frac{\sqrt{2} \operatorname{am}^{-1}(x 1)}{2}$	9
risch	$\frac{\sqrt{2} \ln(e^{ix+i} \cos(x))}{\sqrt{(e^{2ix} + 1)^2 e^{-2ix}}} - \frac{\sqrt{2} \ln(e^{ix-i} \cos(x))}{\sqrt{(e^{2ix} + 1)^2 e^{-2ix}}}$	67

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(cos(2*x)+1)^(1/2),x,method=_RETURNVERBOSE)``[Out] 1/2*2^(1/2)*InverseJacobiAM(x,1)`**Maxima [A]**

time = 0.39, size = 41, normalized size = 1.52

$$\frac{1}{4} \sqrt{2} \log(\cos(x)^2 + \sin(x)^2 + 2 \sin(x) + 1) - \frac{1}{4} \sqrt{2} \log(\cos(x)^2 + \sin(x)^2 - 2 \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(1+cos(2*x))^(1/2),x, algorithm="maxima")`

[Out] $\frac{1}{4}\sqrt{2}\log(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1) - \frac{1}{4}\sqrt{2}\log(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(23) = 46.

time = 0.32, size = 55, normalized size = 2.04

$$\frac{1}{4}\sqrt{2}\log\left(\frac{-\cos(2x)^2 - 2\sqrt{2}\sqrt{\cos(2x)+1}\sin(2x) - 2\cos(2x) - 3}{\cos(2x)^2 + 2\cos(2x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+cos(2*x))^(1/2),x, algorithm="fricas")`

[Out] $\frac{1}{4}\sqrt{2}\log(-(\cos(2x))^2 - 2\sqrt{2}\sqrt{\cos(2x)+1}\sin(2x) - 2\cos(2x) - 3)/(\cos(2x)^2 + 2\cos(2x) + 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos(2x)+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+cos(2*x))**(1/2),x)`

[Out] `Integral(1/sqrt(cos(2*x) + 1), x)`

Giac [A]

time = 0.00, size = 42, normalized size = 1.56

$$\frac{2\left(\frac{\ln|\sin x + \frac{1}{\sin x} + 2|}{8\text{sign}(\cos x)} - \frac{\ln|\sin x + \frac{1}{\sin x} - 2|}{8\text{sign}(\cos x)}\right)}{\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+cos(2*x))^(1/2),x)`

[Out] $\frac{1}{8}\sqrt{2}(\log(\text{abs}(1/\sin(x) + \sin(x) + 2))/\text{sgn}(\cos(x)) - \log(\text{abs}(1/\sin(x) + \sin(x) - 2))/\text{sgn}(\cos(x)))$

Mupad [B]

time = 0.05, size = 13, normalized size = 0.48

$$\frac{\sqrt{2}\operatorname{asinh}\left(\frac{\sin(x)}{\cos(x)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cos(2*x) + 1)^(1/2),x)`

[Out] $(2^{1/2}\operatorname{asinh}(\sin(x)/\cos(x)))/2$

$$3.393 \quad \int \frac{1}{\sqrt{1 - \cos(2x)}} dx$$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1-\cos(2x)}}\right)}{\sqrt{2}}$$

[Out] -1/2*arctanh(1/2*sin(2*x)*2^(1/2)/(1-cos(2*x))^(1/2))*2^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2728, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1-\cos(2x)}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 - Cos[2*x]],x]

[Out] -(ArcTanh[Sin[2*x]/(Sqrt[2]*Sqrt[1 - Cos[2*x]])]/Sqrt[2])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1 - \cos(2x)}} dx &= -\text{Subst} \left(\int \frac{1}{2 - x^2} dx, x, \frac{\sin(2x)}{\sqrt{1 - \cos(2x)}} \right) \\ &= -\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1-\cos(2x)}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 33, normalized size = 1.10

$$\frac{(\log(\cos(\frac{x}{2})) - \log(\sin(\frac{x}{2}))) \sin(x)}{\sqrt{1 - \cos(2x)}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/Sqrt[1 - Cos[2*x]],x]``[Out] -((Log[Cos[x/2]] - Log[Sin[x/2]])*Sin[x])/Sqrt[1 - Cos[2*x]]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/Sqrt[1 - Cos[2*x]],x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [A]**

time = 0.08, size = 17, normalized size = 0.57

method	result	size
default	$-\frac{\sin(x) \operatorname{arctanh}(\cos(x)) \sqrt{2}}{\sqrt{2 - 2 \cos(2x)}}$	17
risch	$\frac{\sqrt{2} \ln(e^{ix} - 1) \sin(x)}{\sqrt{-(e^{2ix} - 1)^2 e^{-2ix}}} - \frac{\sqrt{2} \ln(1 + e^{ix}) \sin(x)}{\sqrt{-(e^{2ix} - 1)^2 e^{-2ix}}}$	67

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(1-cos(2*x))^(1/2),x,method=_RETURNVERBOSE)``[Out] -1/2*sin(x)*arctanh(cos(x))*2^(1/2)/(sin(x)^2)^(1/2)`**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(25) = 50.

time = 0.40, size = 101, normalized size = 3.37

$$-\frac{1}{4} \sqrt{2} \log\left(\cos\left(\frac{1}{2} \arctan(\sin(2x), \cos(2x))\right) + \sin\left(\frac{1}{2} \arctan(\sin(2x), \cos(2x))\right) + 2 \cos\left(\frac{1}{2} \arctan(\sin(2x), \cos(2x))\right) + 1\right) + \frac{1}{4} \sqrt{2} \log\left(\cos\left(\frac{1}{2} \arctan(\sin(2x), \cos(2x))\right) + \sin\left(\frac{1}{2} \arctan(\sin(2x), \cos(2x))\right) - 2 \cos\left(\frac{1}{2} \arctan(\sin(2x), \cos(2x))\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(1-cos(2*x))^(1/2),x, algorithm="maxima")`

[Out] $-1/4*\sqrt{2}*\log(\cos(1/2*\arctan2(\sin(2*x), \cos(2*x)))^2 + \sin(1/2*\arctan2(\sin(2*x), \cos(2*x)))^2 + 2*\cos(1/2*\arctan2(\sin(2*x), \cos(2*x))) + 1) + 1/4*\sqrt{2}*\log(\cos(1/2*\arctan2(\sin(2*x), \cos(2*x)))^2 + \sin(1/2*\arctan2(\sin(2*x), \cos(2*x)))^2 - 2*\cos(1/2*\arctan2(\sin(2*x), \cos(2*x))) + 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(25) = 50.

time = 0.32, size = 58, normalized size = 1.93

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{(\cos(2x)+3)\sin(2x)-2\left(\sqrt{2}\cos(2x)+\sqrt{2}\right)\sqrt{-\cos(2x)+1}}{(\cos(2x)-1)\sin(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-cos(2*x))^(1/2),x, algorithm="fricas")`

[Out] $1/4*\sqrt{2}*\log(-((\cos(2*x)+3)*\sin(2*x)-2*(\sqrt{2}*\cos(2*x)+\sqrt{2}))*\sqrt{-\cos(2*x)+1})/((\cos(2*x)-1)*\sin(2*x))$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-\cos(2x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-cos(2*x))**(1/2),x)`

[Out] `Integral(1/sqrt(1-cos(2*x)), x)`

Giac [A]

time = 0.00, size = 18, normalized size = 0.60

$$\frac{\ln\left|\tan\left(\frac{x}{2}\right)\right|}{\sqrt{2}\operatorname{sign}(\sin x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-cos(2*x))^(1/2),x)`

[Out] $1/2*\sqrt{2}*\log(\operatorname{abs}(\tan(1/2*x)))/\operatorname{sgn}(\sin(x))$

Mupad [B]

time = 0.24, size = 28, normalized size = 0.93

$$\frac{\sqrt{2}\sin(2x)\operatorname{atanh}\left(\sqrt{\cos(x)^2}\right)}{2\sqrt{1-\cos(2x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1 - cos(2*x))^(1/2),x)`

[Out] $-(2^{1/2} \sin(2x) \operatorname{atanh}(\cos(x)^2)^{1/2}) / (2(1 - \cos(2x)^2)^{1/2})$

$$3.394 \quad \int \frac{1}{(1-\cos(3x))^{3/2}} dx$$

Optimal. Leaf size=53

$$-\frac{\tanh^{-1}\left(\frac{\sin(3x)}{\sqrt{2}\sqrt{1-\cos(3x)}}\right)}{6\sqrt{2}} - \frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}$$

[Out] -1/6*sin(3*x)/(1-cos(3*x))^(3/2)-1/12*arctanh(1/2*sin(3*x)*2^(1/2)/(1-cos(3*x))^(1/2))*2^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2729, 2728, 212}

$$-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sin(3x)}{\sqrt{2}\sqrt{1-\cos(3x)}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Cos[3*x])^(-3/2), x]

[Out] -1/6*ArcTanh[Sin[3*x]/(Sqrt[2]*Sqrt[1 - Cos[3*x]])]/Sqrt[2] - Sin[3*x]/(6*(1 - Cos[3*x])^(3/2))

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2729

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[b*Cos[c + d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1 - \cos(3x))^{3/2}} dx &= -\frac{\sin(3x)}{6(1 - \cos(3x))^{3/2}} + \frac{1}{4} \int \frac{1}{\sqrt{1 - \cos(3x)}} dx \\
&= -\frac{\sin(3x)}{6(1 - \cos(3x))^{3/2}} - \frac{1}{6} \text{Subst} \left(\int \frac{1}{2 - x^2} dx, x, \frac{\sin(3x)}{\sqrt{1 - \cos(3x)}} \right) \\
&= -\frac{\tanh^{-1} \left(\frac{\sin(3x)}{\sqrt{2} \sqrt{1 - \cos(3x)}} \right)}{6\sqrt{2}} - \frac{\sin(3x)}{6(1 - \cos(3x))^{3/2}}
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 61, normalized size = 1.15

$$-\frac{(\csc^2(\frac{3x}{4}) + 4 \log(\cos(\frac{3x}{4})) - 4 \log(\sin(\frac{3x}{4})) - \sec^2(\frac{3x}{4})) \sin^3(\frac{3x}{2})}{12(1 - \cos(3x))^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(1 - Cos[3*x])^(-3/2), x]`

```
[Out] -1/12*((Csc[(3*x)/4]^2 + 4*Log[Cos[(3*x)/4]] - 4*Log[Sin[(3*x)/4]] - Sec[(3*x)/4]^2)*Sin[(3*x)/2]^3)/(1 - Cos[3*x])^(3/2)
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(1 - Cos[3*x])^(3/2), x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.09, size = 52, normalized size = 0.98

method	result	size
default	$-\frac{\left(\frac{\cos(\frac{3x}{2})}{2} + \frac{(\ln(\cos(\frac{3x}{2})+1) - \ln(\cos(\frac{3x}{2})-1))(\sin^2(\frac{3x}{2}))}{4}\right)\sqrt{2}}{3 \sin(\frac{3x}{2}) \sqrt{2 - 2 \cos(3x)}}$	52

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(1-cos(3*x))^(3/2), x, method=_RETURNVERBOSE)`

[Out] $-1/6*(1/2*\cos(3/2*x)+1/4*(\ln(\cos(3/2*x)+1)-\ln(\cos(3/2*x)-1))*\sin(3/2*x)^2)/\sin(3/2*x)*2^{(1/2)}/(\sin(3/2*x)^2)^{(1/2)}$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(42) = 84.

time = 0.40, size = 433, normalized size = 8.17

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-cos(3*x))^(3/2),x, algorithm="maxima")`

[Out] $1/12*(4*(\sin(6*x) - 2*\sin(3*x))*\cos(3/2*\pi + 3/2*\arctan2(\sin(3*x), \cos(3*x))) - 4*(\sin(6*x) - 2*\sin(3*x))*\cos(1/2*\pi + 1/2*\arctan2(\sin(3*x), \cos(3*x))) + (2*(2*\cos(3*x) - 1)*\cos(6*x) - \cos(6*x)^2 - 4*\cos(3*x)^2 - \sin(6*x)^2 + 4*\sin(6*x)*\sin(3*x) - 4*\sin(3*x)^2 + 4*\cos(3*x) - 1)*\log(\cos(1/2*\arctan2(\sin(3*x), \cos(3*x)))^2 + \sin(1/2*\arctan2(\sin(3*x), \cos(3*x)))^2 + 2*\cos(1/2*\arctan2(\sin(3*x), \cos(3*x)))) + 1) - (2*(2*\cos(3*x) - 1)*\cos(6*x) - \cos(6*x)^2 - 4*\cos(3*x)^2 - \sin(6*x)^2 + 4*\sin(6*x)*\sin(3*x) - 4*\sin(3*x)^2 + 4*\cos(3*x) - 1)*\log(\cos(1/2*\arctan2(\sin(3*x), \cos(3*x)))^2 + \sin(1/2*\arctan2(\sin(3*x), \cos(3*x)))^2 - 2*\cos(1/2*\arctan2(\sin(3*x), \cos(3*x)))) + 1) - 4*(\cos(6*x) - 2*\cos(3*x) + 1)*\sin(3/2*\pi + 3/2*\arctan2(\sin(3*x), \cos(3*x))) + 4*(\cos(6*x) - 2*\cos(3*x) + 1)*\sin(1/2*\pi + 1/2*\arctan2(\sin(3*x), \cos(3*x))))/(sqrt(2)*\cos(6*x)^2 + 4*sqrt(2)*\cos(3*x)^2 + sqrt(2)*\sin(6*x)^2 - 4*sqrt(2)*\sin(6*x)*\sin(3*x) + 4*sqrt(2)*\sin(3*x)^2 - 2*(2*sqrt(2)*\cos(3*x) - sqrt(2))*\cos(6*x) - 4*sqrt(2)*\cos(3*x) + sqrt(2))$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(42) = 84.

time = 0.32, size = 107, normalized size = 2.02

$$\frac{(\sqrt{2} \cos(3x) - \sqrt{2}) \log\left(-\frac{(\cos(3x)+3)\sin(3x)-2(\sqrt{2}\cos(3x)+\sqrt{2})\sqrt{-\cos(3x)+1}}{(\cos(3x)-1)\sin(3x)}\right) \sin(3x) + 4(\cos(3x)+1)\sqrt{-\cos(3x)+1}}{24(\cos(3x)-1)\sin(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-cos(3*x))^(3/2),x, algorithm="fricas")`

[Out] $1/24*((\sqrt{2}*\cos(3*x) - \sqrt{2})*\log(-((\cos(3*x) + 3)*\sin(3*x) - 2*(\sqrt{2}*\cos(3*x) + \sqrt{2}))*\sqrt{-\cos(3*x) + 1}))/((\cos(3*x) - 1)*\sin(3*x))*\sin(3*x) + 4*(\cos(3*x) + 1)*\sqrt{-\cos(3*x) + 1}))/((\cos(3*x) - 1)*\sin(3*x))$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(1 - \cos(3x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-cos(3*x))**(3/2),x)`

[Out] `Integral((1 - cos(3*x))**(-3/2), x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(42) = 84$.
time = 0.01, size = 123, normalized size = 2.32

$$4 \frac{\left(\frac{-\cos\left(\frac{3}{2}x\right)+1}{32\left(\cos\left(\frac{3}{2}x\right)+1\right)\text{sign}\left(\sin\left(\frac{3}{2}x\right)\right)} + \frac{\ln\left(\frac{-\cos\left(\frac{3}{2}x\right)+1}{\cos\left(\frac{3}{2}x\right)+1}\right)}{16\text{sign}\left(\sin\left(\frac{3}{2}x\right)\right)} + \frac{\left(\cos\left(\frac{3}{2}x\right)+1\right)\left(-\frac{2\left(-\cos\left(\frac{3}{2}x\right)+1\right)}{\cos\left(\frac{3}{2}x\right)+1}-1\right)}{32\left(-\cos\left(\frac{3}{2}x\right)+1\right)\text{sign}\left(\sin\left(\frac{3}{2}x\right)\right)} \right)}{\sqrt{2} \cdot 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-cos(3*x))^(3/2),x)`

[Out] `-1/48*sqrt(2)*((2*(cos(3/2*x) - 1)/(cos(3/2*x) + 1) - 1)*(cos(3/2*x) + 1)/((cos(3/2*x) - 1)*sgn(sin(3/2*x))) - 2*log(-(cos(3/2*x) - 1)/(cos(3/2*x) + 1)))/sgn(sin(3/2*x)) + (cos(3/2*x) - 1)/((cos(3/2*x) + 1)*sgn(sin(3/2*x)))`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(1 - \cos(3x))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1 - cos(3*x))^(3/2),x)`

[Out] `int(1/(1 - cos(3*x))^(3/2), x)`

3.395 $\int \left(1 - \sin\left(\frac{2x}{3}\right)\right)^{5/2} dx$

Optimal. Leaf size=73

$$\frac{32 \cos\left(\frac{2x}{3}\right)}{5 \sqrt{1 - \sin\left(\frac{2x}{3}\right)}} + \frac{8}{5} \cos\left(\frac{2x}{3}\right) \sqrt{1 - \sin\left(\frac{2x}{3}\right)} + \frac{3}{5} \cos\left(\frac{2x}{3}\right) \left(1 - \sin\left(\frac{2x}{3}\right)\right)^{3/2}$$

[Out] 3/5*cos(2/3*x)*(1-sin(2/3*x))^(3/2)+32/5*cos(2/3*x)/(1-sin(2/3*x))^(1/2)+8/5*cos(2/3*x)*(1-sin(2/3*x))^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2726, 2725}

$$\frac{3}{5} \left(1 - \sin\left(\frac{2x}{3}\right)\right)^{3/2} \cos\left(\frac{2x}{3}\right) + \frac{8}{5} \sqrt{1 - \sin\left(\frac{2x}{3}\right)} \cos\left(\frac{2x}{3}\right) + \frac{32 \cos\left(\frac{2x}{3}\right)}{5 \sqrt{1 - \sin\left(\frac{2x}{3}\right)}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sin[(2*x)/3])^(5/2), x]

[Out] (32*Cos[(2*x)/3])/(5*Sqrt[1 - Sin[(2*x)/3]]) + (8*Cos[(2*x)/3]*Sqrt[1 - Sin[(2*x)/3]])/5 + (3*Cos[(2*x)/3]*(1 - Sin[(2*x)/3])^(3/2))/5

Rule 2725

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2726

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n), Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0]

Rubi steps

$$\begin{aligned}
\int \left(1 - \sin\left(\frac{2x}{3}\right)\right)^{5/2} dx &= \frac{3}{5} \cos\left(\frac{2x}{3}\right) \left(1 - \sin\left(\frac{2x}{3}\right)\right)^{3/2} + \frac{8}{5} \int \left(1 - \sin\left(\frac{2x}{3}\right)\right)^{3/2} dx \\
&= \frac{8}{5} \cos\left(\frac{2x}{3}\right) \sqrt{1 - \sin\left(\frac{2x}{3}\right)} + \frac{3}{5} \cos\left(\frac{2x}{3}\right) \left(1 - \sin\left(\frac{2x}{3}\right)\right)^{3/2} + \frac{32}{15} \int \sqrt{1 - \sin\left(\frac{2x}{3}\right)} dx \\
&= \frac{32 \cos\left(\frac{2x}{3}\right)}{5 \sqrt{1 - \sin\left(\frac{2x}{3}\right)}} + \frac{8}{5} \cos\left(\frac{2x}{3}\right) \sqrt{1 - \sin\left(\frac{2x}{3}\right)} + \frac{3}{5} \cos\left(\frac{2x}{3}\right) \left(1 - \sin\left(\frac{2x}{3}\right)\right)^{3/2}
\end{aligned}$$

Mathematica [A]

time = 0.11, size = 76, normalized size = 1.04

$$\frac{\left(1 - \sin\left(\frac{2x}{3}\right)\right)^{5/2} \left(150 \cos\left(\frac{x}{3}\right) + 25 \cos(x) - 3 \cos\left(\frac{5x}{3}\right) + 150 \sin\left(\frac{x}{3}\right) - 25 \sin(x) - 3 \sin\left(\frac{5x}{3}\right)\right)}{20 \left(\cos\left(\frac{x}{3}\right) - \sin\left(\frac{x}{3}\right)\right)^5}$$

Antiderivative was successfully verified.

`[In] Integrate[(1 - Sin[(2*x)/3])^(5/2), x]`

```
[Out] ((1 - Sin[(2*x)/3])^(5/2)*(150*Cos[x/3] + 25*Cos[x] - 3*Cos[(5*x)/3] + 150*Sin[x/3] - 25*Sin[x] - 3*Sin[(5*x)/3]))/(20*(Cos[x/3] - Sin[x/3])^5)
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(1 - Sin[2*(x/3)])^(5/2), x]')``[Out] Timed out`**Maple [A]**

time = 0.18, size = 47, normalized size = 0.64

method	result	size
default	$-\frac{(-1 + \sin(\frac{2x}{3}))(\sin(\frac{2x}{3}) + 1)(3(\sin^2(\frac{2x}{3})) - 14\sin(\frac{2x}{3}) + 43)}{5 \cos(\frac{2x}{3}) \sqrt{1 - \sin(\frac{2x}{3})}}$	47

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((1-sin(2/3*x))^(5/2), x, method=_RETURNVERBOSE)`

[Out] $-1/5*(-1+\sin(2/3*x))*(\sin(2/3*x)+1)*(3*\sin(2/3*x)^2-14*\sin(2/3*x)+43)/\cos(2/3*x)/(1-\sin(2/3*x))^{1/2}$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-sin(2/3*x))^(5/2),x, algorithm="maxima")`

[Out] `integrate((-sin(2/3*x) + 1)^(5/2), x)`

Fricas [A]

time = 0.61, size = 71, normalized size = 0.97

$$\frac{\left(3 \cos\left(\frac{2}{3}x\right)^3 - 11 \cos\left(\frac{2}{3}x\right)^2 + \left(3 \cos\left(\frac{2}{3}x\right)^2 + 14 \cos\left(\frac{2}{3}x\right) - 32\right) \sin\left(\frac{2}{3}x\right) - 46 \cos\left(\frac{2}{3}x\right) - 32\right) \sqrt{-\sin\left(\frac{2}{3}x\right) + 1}}{5 \left(\cos\left(\frac{2}{3}x\right) - \sin\left(\frac{2}{3}x\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-sin(2/3*x))^(5/2),x, algorithm="fricas")`

[Out] $-1/5*(3*\cos(2/3*x)^3 - 11*\cos(2/3*x)^2 + (3*\cos(2/3*x)^2 + 14*\cos(2/3*x) - 32)*\sin(2/3*x) - 46*\cos(2/3*x) - 32)*\sqrt{-\sin(2/3*x) + 1}/(\cos(2/3*x) - \sin(2/3*x) + 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1 - \sin\left(\frac{2x}{3}\right)\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-sin(2/3*x))**(5/2),x)`

[Out] `Integral((1 - sin(2*x/3))**(5/2), x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(49) = 98.

time = 0.01, size = 161, normalized size = 2.21

$$\sqrt{2} \left(\frac{32}{5} \operatorname{sign}\left(\sin\left(\frac{2x}{3} - \frac{\pi}{4}\right)\right) + 3 \left(-\operatorname{sign}\left(\sin\left(-\frac{\pi}{4} + \frac{x}{3}\right)\right) \sin\left(\frac{\pi + \frac{4x}{3}}{4}\right) - \frac{1}{3} \operatorname{sign}\left(\sin\left(-\frac{\pi}{4} + \frac{x}{3}\right)\right) \sin\left(\frac{\pi - 4x}{4}\right) - \frac{1}{12} \operatorname{sign}\left(\sin\left(-\frac{\pi}{4} + \frac{x}{3}\right)\right) \cos\left(\frac{\pi + 4x}{4}\right) - \frac{3}{2} \operatorname{sign}\left(\sin\left(-\frac{\pi}{4} + \frac{x}{3}\right)\right) \cos\left(\frac{\pi - \frac{4x}{3}}{4}\right) + \frac{1}{20} \operatorname{sign}\left(\sin\left(-\frac{\pi}{4} + \frac{x}{3}\right)\right) \cos\left(\frac{\pi - \frac{4x}{3}}{4}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-sin(2/3*x))^(5/2),x)`

```
[Out] -1/20*sqrt(2)*(5*cos(1/4*pi + x)*sgn(sin(-1/4*pi + 1/3*x)) + 90*cos(1/4*pi
- 1/3*x)*sgn(sin(-1/4*pi + 1/3*x)) - 3*cos(1/4*pi - 5/3*x)*sgn(sin(-1/4*pi
+ 1/3*x)) + 60*sgn(sin(-1/4*pi + 1/3*x))*sin(1/4*pi + 1/3*x) + 20*sgn(sin(-
1/4*pi + 1/3*x))*sin(1/4*pi - x) - 128*sgn(sin(-1/4*pi + 1/3*x)))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(1 - \sin\left(\frac{2x}{3}\right) \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - sin((2*x)/3))^(5/2), x)
```

```
[Out] int((1 - sin((2*x)/3))^(5/2), x)
```

$$3.396 \quad \int \frac{\cos(x) \left(-\cos^2(x) + 2\sqrt[4]{1+2\sin(x)} \right)}{(1+2\sin(x))^{3/2}} dx$$

Optimal. Leaf size=55

$$\frac{3}{4\sqrt{1+2\sin(x)}} - \frac{4}{\sqrt[4]{1+2\sin(x)}} - \frac{1}{2}\sqrt{1+2\sin(x)} + \frac{1}{12}(1+2\sin(x))^{3/2}$$

[Out] $-4/(1+2*\sin(x))^{(1/4)}+1/12*(1+2*\sin(x))^{(3/2)}+3/4/(1+2*\sin(x))^{(1/2)}-1/2*(1+2*\sin(x))^{(1/2)}$

Rubi [A]

time = 0.09, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4441, 14}

$$\frac{1}{12}(2\sin(x)+1)^{3/2} - \frac{1}{2}\sqrt{2\sin(x)+1} - \frac{4}{\sqrt[4]{2\sin(x)+1}} + \frac{3}{4\sqrt{2\sin(x)+1}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[x]*(-\text{Cos}[x]^2 + 2*(1 + 2*\text{Sin}[x])^{(1/4)})) / (1 + 2*\text{Sin}[x])^{(3/2)}, x]$

[Out] $3/(4*\text{Sqrt}[1 + 2*\text{Sin}[x]]) - 4/(1 + 2*\text{Sin}[x])^{(1/4)} - \text{Sqrt}[1 + 2*\text{Sin}[x]]/2 + (1 + 2*\text{Sin}[x])^{(3/2)}/12$

Rule 14

$\text{Int}[(u_*)*((c_*)*(x_*)^{(m_*)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /;$ FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 4441

$\text{Int}[(u_*)*(F_)[(c_*)*((a_*) + (b_*)*(x_*)]), x_Symbol] \rightarrow \text{With}\{d = \text{FreeFactors}[\text{Sin}[c*(a + b*x)], x]\}, \text{Dist}[d/(b*c), \text{Subst}[\text{Int}[\text{SubstFor}[1, \text{Sin}[c*(a + b*x)]]/d, u, x], x], x, \text{Sin}[c*(a + b*x)]/d, x] /;$ FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x) \left(-\cos^2(x) + 2\sqrt[4]{1+2\sin(x)} \right)}{(1+2\sin(x))^{3/2}} dx &= \text{Subst} \left(\int \frac{-1+x^2+2\sqrt[4]{1+2x}}{(1+2x)^{3/2}} dx, x, \sin(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{-3+8x-2x^4+x^8}{x^3} dx, x, \sqrt[4]{1+2\sin(x)} \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(-\frac{3}{x^3} + \frac{8}{x^2} - 2x + x^5 \right) dx, x, \sqrt[4]{1+2\sin(x)} \right) \\
&= \frac{3}{4\sqrt[4]{1+2\sin(x)}} - \frac{4}{\sqrt[4]{1+2\sin(x)}} - \frac{1}{2}\sqrt{1+2\sin(x)} + \frac{1}{10}
\end{aligned}$$

Mathematica [A]

time = 0.05, size = 36, normalized size = 0.65

$$\frac{-3 + \cos(2x) + 4\sin(x) + 24\sqrt[4]{1+2\sin(x)}}{6\sqrt[4]{1+2\sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*(-Cos[x]^2 + 2*(1 + 2*Sin[x])^(1/4)))/(1 + 2*Sin[x])^(3/2), x]
```

```
[Out] -1/6*(-3 + Cos[2*x] + 4*Sin[x] + 24*(1 + 2*Sin[x])^(1/4))/Sqrt[1 + 2*Sin[x]]
```

Mathics [A]

time = 35.79, size = 27, normalized size = 0.49

$$\frac{-4}{(1+2\sin[x])^{1/4}} + \frac{(-1+\sin[x])^2}{3\sqrt{1+2\sin[x]}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(2*(1 + 2*Sin[x])^(1/4) - Cos[x]^2)/(1 + 2*Sin[x])^(3/2) *Cos[x], x]')
```

```
[Out] -4 / (1 + 2 Sin[x]) ^ (1 / 4) + (-1 + Sin[x]) ^ 2 / (3 Sqrt[1 + 2 Sin[x]])
```

Maple [A]

time = 0.69, size = 31, normalized size = 0.56

method	result	size
--------	--------	------

default	$\frac{\sin^2(x) - 2\sin(x) - 12(1 + 2\sin(x))^{\frac{1}{4}} + 1}{3\sqrt{1 + 2\sin(x)}}$	31
---------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x,method=_RETURNVERBOSE)`

[Out] $1/3/(1+2*\sin(x))^{1/2}*(\sin(x)^2-2*\sin(x)-12*(1+2*\sin(x))^{1/4}+1)$

Maxima [A]

time = 0.26, size = 43, normalized size = 0.78

$$\frac{1}{12} (2 \sin(x) + 1)^{\frac{3}{2}} - \frac{16 (2 \sin(x) + 1)^{\frac{1}{4}} - 3}{4 \sqrt{2 \sin(x) + 1}} - \frac{1}{2} \sqrt{2 \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x, algorithm="maxima")`

[Out] $1/12*(2*\sin(x) + 1)^{3/2} - 1/4*(16*(2*\sin(x) + 1)^{1/4} - 3)/\sqrt{2*\sin(x) + 1} - 1/2*\sqrt{2*\sin(x) + 1}$

Fricas [A]

time = 0.31, size = 40, normalized size = 0.73

$$-\frac{(\cos(x)^2 + 2 \sin(x) - 2) \sqrt{2 \sin(x) + 1} + 12 (2 \sin(x) + 1)^{\frac{3}{4}}}{3 (2 \sin(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x, algorithm="fricas")`

[Out] $-1/3*((\cos(x)^2 + 2*\sin(x) - 2)*\sqrt{2*\sin(x) + 1} + 12*(2*\sin(x) + 1)^{3/4})/(2*\sin(x) + 1)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(48) = 96$

time = 43.34, size = 230, normalized size = 4.18

$$\frac{4(2\sin(x)+1)^{\frac{1}{2}}\sin^2(x)}{6\sqrt{2\sin(x)+1}\sin(x)+3\sqrt{2\sin(x)+1}} - \frac{2(2\sin(x)+1)^{\frac{1}{2}}\sin(x)}{6\sqrt{2\sin(x)+1}\sin(x)+3\sqrt{2\sin(x)+1}} + \frac{3(2\sin(x)+1)^{\frac{1}{2}}\cos^2(x)}{6\sqrt{2\sin(x)+1}\sin(x)+3\sqrt{2\sin(x)+1}} - \frac{2(2\sin(x)+1)^{\frac{1}{2}}}{6\sqrt{2\sin(x)+1}\sin(x)+3\sqrt{2\sin(x)+1}} - \frac{24\sin(x)}{6\sqrt{2\sin(x)+1}\sin(x)+3\sqrt{2\sin(x)+1}} - \frac{12}{6\sqrt{2\sin(x)+1}\sin(x)+3\sqrt{2\sin(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(-cos(x)**2+2*(1+2*sin(x))**(1/4))/(1+2*sin(x))**(3/2),x)`

[Out] $4*(2*\sin(x) + 1)**(3/4)*\sin(x)**2/(6*(2*\sin(x) + 1)**(1/4)*\sin(x) + 3*(2*\sin(x) + 1)**(1/4)) - 2*(2*\sin(x) + 1)**(3/4)*\sin(x)/(6*(2*\sin(x) + 1)**(1/4)*\sin(x) + 3*(2*\sin(x) + 1)**(1/4)) + 3*(2*\sin(x) + 1)**(3/4)*\cos(x)**2/(6*(2*\sin(x) + 1)**(1/4)*\sin(x) + 3*(2*\sin(x) + 1)**(1/4)) - 2*(2*\sin(x) + 1)**(3/4)/(6*(2*\sin(x) + 1)**(1/4)*\sin(x) + 3*(2*\sin(x) + 1)**(1/4)) - 24*\sin(x)/(6*(2*\sin(x) + 1)**(1/4)*\sin(x) + 3*(2*\sin(x) + 1)**(1/4)) - 12/(6*(2*\sin(x) + 1)**(1/4)*\sin(x) + 3*(2*\sin(x) + 1)**(1/4))$

Giac [A]

time = 0.01, size = 67, normalized size = 1.22

$$\frac{\frac{16}{3}\sqrt{2\sin x + 1}(2\sin x + 1) - 32\sqrt{2\sin x + 1}}{64} + \frac{-16(2\sin x + 1)^{\frac{1}{4}} + 3}{4\sqrt{2\sin x + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x)`

[Out] $1/12*(2*\sin(x) + 1)^{(3/2)} - 1/4*(16*(2*\sin(x) + 1)^{(1/4)} - 3)/\sqrt{2*\sin(x) + 1} - 1/2*\sqrt{2*\sin(x) + 1}$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$-\int -\frac{\cos(x) \left(2(2\sin(x) + 1)^{1/4} - \cos(x)^2\right)}{(2\sin(x) + 1)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(x)*(2*(2*sin(x) + 1)^(1/4) - cos(x)^2))/(2*sin(x) + 1)^(3/2),x)`

[Out] $-\text{int}(-(\cos(x)*(2*(2*\sin(x) + 1)^{(1/4)} - \cos(x)^2))/(2*\sin(x) + 1)^{(3/2)}, x)$

3.397 $\int \sqrt{\tan(x)} dx$

Optimal. Leaf size=98

$$-\frac{\tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(x)}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1 + \sqrt{2} \sqrt{\tan(x)}\right)}{\sqrt{2}} + \frac{\log\left(1 - \sqrt{2} \sqrt{\tan(x)} + \tan(x)\right)}{2\sqrt{2}} - \frac{\log\left(1 + \sqrt{2} \sqrt{\tan(x)} + \tan(x)\right)}{2\sqrt{2}}$$

[Out] 1/2*arctan(-1+2^(1/2)*tan(x)^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/2)*tan(x)^(1/2))*2^(1/2)+1/4*ln(1-2^(1/2)*tan(x)^(1/2)+tan(x))*2^(1/2)-1/4*ln(1+2^(1/2)*tan(x)^(1/2)+tan(x))*2^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(x)}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\sqrt{2} \sqrt{\tan(x)} + 1\right)}{\sqrt{2}} + \frac{\log\left(\tan(x) - \sqrt{2} \sqrt{\tan(x)} + 1\right)}{2\sqrt{2}} - \frac{\log\left(\tan(x) + \sqrt{2} \sqrt{\tan(x)} + 1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Tan[x]], x]

[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2])

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 303

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 335

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\tan(x)} \, dx &= \text{Subst} \left(\int \frac{\sqrt{x}}{1+x^2} \, dx, x, \tan(x) \right) \\
&= 2 \text{Subst} \left(\int \frac{x^2}{1+x^4} \, dx, x, \sqrt{\tan(x)} \right) \\
&= -\text{Subst} \left(\int \frac{1-x^2}{1+x^4} \, dx, x, \sqrt{\tan(x)} \right) + \text{Subst} \left(\int \frac{1+x^2}{1+x^4} \, dx, x, \sqrt{\tan(x)} \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-\sqrt{2}x+x^2} \, dx, x, \sqrt{\tan(x)} \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+\sqrt{2}x+x^2} \, dx, x, \sqrt{\tan(x)} \right) \\
&= \frac{\log \left(1 - \sqrt{2} \sqrt{\tan(x)} + \tan(x) \right)}{2\sqrt{2}} - \frac{\log \left(1 + \sqrt{2} \sqrt{\tan(x)} + \tan(x) \right)}{2\sqrt{2}} + \text{Subst} \left(\int \frac{1}{-1-x^2} \, dx, x, \sqrt{\tan(x)} \right) \\
&= -\frac{\tan^{-1} \left(1 - \sqrt{2} \sqrt{\tan(x)} \right)}{\sqrt{2}} + \frac{\tan^{-1} \left(1 + \sqrt{2} \sqrt{\tan(x)} \right)}{\sqrt{2}} + \frac{\log \left(1 - \sqrt{2} \sqrt{\tan(x)} + \tan(x) \right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.01, size = 24, normalized size = 0.24

$$\frac{2}{3} {}_2F_1 \left(\frac{3}{4}, 1; \frac{7}{4}; -\tan^2(x) \right) \tan^{\frac{3}{2}}(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Tan[x]],x]

[Out] (2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[x]^2]*Tan[x]^(3/2))/3

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sqrt[Tan[x]],x]')

[Out] cought exception: maximum recursion depth exceeded in comparison

Maple [A]

time = 0.05, size = 49, normalized size = 0.50

method	result
lookup	$\frac{\left(\sqrt{\tan(x)}\cos(x)\sqrt{2}\arccos(\cos(x)-\sin(x))\right)}{2\sqrt{\cos(x)\sin(x)}} - \frac{\sqrt{2}\ln\left(\cos(x)+\sqrt{2}\left(\sqrt{\tan(x)}\cos(x)+\sin(x)\right)\right)}{2}$
default	$\frac{\left(\sqrt{\tan(x)}\cos(x)\sqrt{2}\arccos(\cos(x)-\sin(x))\right)}{2\sqrt{\cos(x)\sin(x)}} - \frac{\sqrt{2}\ln\left(\cos(x)+\sqrt{2}\left(\sqrt{\tan(x)}\cos(x)+\sin(x)\right)\right)}{2}$
derivativedivides	$\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\left(\sqrt{\tan(x)}+\tan(x)\right)}{1+\sqrt{2}\left(\sqrt{\tan(x)}+\tan(x)\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(x)}\right)\right)+2\arctan\left(-1+\sqrt{2}\left(\sqrt{\tan(x)}\right)\right)\right)$

4

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(x)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $1/2*\tan(x)^{(1/2)}/(\cos(x)*\sin(x))^{(1/2)}*\cos(x)*2^{(1/2)}*\arccos(\cos(x)-\sin(x)) - 1/2*2^{(1/2)}*\ln(\cos(x)+2^{(1/2)}*\tan(x)^{(1/2)}*\cos(x)+\sin(x))$

Maxima [A]

time = 0.36, size = 80, normalized size = 0.82

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)^(1/2),x, algorithm="maxima")`

[Out] $1/2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2*\sqrt{\tan(x)})) + 1/2*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2*\sqrt{\tan(x)})) - 1/4*\sqrt{2}*\log(\sqrt{2}*\sqrt{\tan(x)} + \tan(x) + 1) + 1/4*\sqrt{2}*\log(-\sqrt{2}*\sqrt{\tan(x)} + \tan(x) + 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(70) = 140.

time = 0.31, size = 180, normalized size = 1.84

$$-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{\sqrt{\frac{\sin(x)}{\cos(x)}\cos(x)+\cos(x)+\sin(x)}}{\cos(x)}}-\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}-1\right)-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{-\frac{\sqrt{\frac{\sin(x)}{\cos(x)}\cos(x)-\cos(x)-\sin(x)}}{\cos(x)}}-\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}+1\right)-\frac{1}{4}\sqrt{2}\log\left(\frac{4\left(\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}\cos(x)+\cos(x)+\sin(x)}\right)}{\cos(x)}\right)+\frac{1}{4}\sqrt{2}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}\cos(x)-\cos(x)-\sin(x)}\right)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)^(1/2),x, algorithm="fricas")`

[Out] $-\sqrt{2}*\arctan(\sqrt{2}*\sqrt{(\sqrt{2}*\sqrt{\sin(x)/\cos(x)}*\cos(x)+\cos(x)+\sin(x))/\cos(x)}) - \sqrt{2}*\sqrt{\sin(x)/\cos(x)} - 1 - \sqrt{2}*\arctan(\sqrt{2}*\sqrt{-(\sqrt{2}*\sqrt{\sin(x)/\cos(x)}*\cos(x)-\cos(x)-\sin(x))/\cos(x)}) - \sqrt{2}*\sqrt{\sin(x)/\cos(x)} + 1 - 1/4*\sqrt{2}*\log(4*(\sqrt{2}*\sqrt{\sin(x)/\cos(x)})$

$(x) \cdot \cos(x) + \cos(x) + \sin(x)) / \cos(x) + 1/4 \cdot \sqrt{2} \cdot \log(-4 \cdot (\sqrt{2} \cdot \sqrt{\sin(x) / \cos(x)}) \cdot \cos(x) - \cos(x) - \sin(x)) / \cos(x))$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)**(1/2), x)

[Out] Integral(sqrt(tan(x)), x)

Giac [A]

time = 0.00, size = 120, normalized size = 1.22

$$2 \left(\frac{1}{8} \sqrt{2} \ln(\tan x - \sqrt{2} \sqrt{\tan x} + 1) + \frac{1}{4} \sqrt{2} \arctan\left(\frac{2(\sqrt{\tan x} - \frac{\sqrt{2}}{2})}{\sqrt{2}}\right) - \frac{1}{8} \sqrt{2} \ln(\tan x + \sqrt{2} \sqrt{\tan x} + 1) + \frac{1}{4} \sqrt{2} \arctan\left(\frac{2(\sqrt{\tan x} + \frac{\sqrt{2}}{2})}{\sqrt{2}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)^(1/2), x)

[Out] $1/2 \cdot \sqrt{2} \cdot \arctan(1/2 \cdot \sqrt{2} \cdot (\sqrt{2} + 2 \cdot \sqrt{\tan(x)})) + 1/2 \cdot \sqrt{2} \cdot \arctan(-1/2 \cdot \sqrt{2} \cdot (\sqrt{2} - 2 \cdot \sqrt{\tan(x)})) - 1/4 \cdot \sqrt{2} \cdot \log(\sqrt{2} \cdot \sqrt{\tan(x)} + \tan(x) + 1) + 1/4 \cdot \sqrt{2} \cdot \log(-\sqrt{2} \cdot \sqrt{\tan(x)} + \tan(x) + 1)$

Mupad [B]

time = 0.13, size = 65, normalized size = 0.66

$$\frac{\sqrt{2} \left(\ln(\sqrt{2} \sqrt{\tan(x)} - \tan(x) - 1) - \ln(\tan(x) + \sqrt{2} \sqrt{\tan(x)} + 1) \right)}{4} + \frac{\sqrt{2} \left(\operatorname{atan}(\sqrt{2} \sqrt{\tan(x)} - 1) + \operatorname{atan}(\sqrt{2} \sqrt{\tan(x)} + 1) \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)^(1/2), x)

[Out] $(2^{(1/2)} \cdot (\log(2^{(1/2)} \cdot \tan(x)^{(1/2)} - \tan(x) - 1) - \log(\tan(x) + 2^{(1/2)} \cdot \tan(x)^{(1/2)} + 1))) / 4 + (2^{(1/2)} \cdot (\operatorname{atan}(2^{(1/2)} \cdot \tan(x)^{(1/2)} - 1) + \operatorname{atan}(2^{(1/2)} \cdot \tan(x)^{(1/2)} + 1))) / 2$

$$3.398 \quad \int \frac{1}{\sqrt[3]{\tan(5x)}} dx$$

Optimal. Leaf size=57

$$-\frac{1}{10}\sqrt{3} \tan^{-1}\left(\frac{1-2\tan^{\frac{2}{3}}(5x)}{\sqrt{3}}\right) + \frac{3}{20} \log\left(1+\tan^{\frac{2}{3}}(5x)\right) - \frac{1}{20} \log\left(1+\tan^2(5x)\right)$$

[Out] 3/20*ln(1+tan(5*x)^(2/3))-1/20*ln(1+tan(5*x)^2)-1/10*arctan(1/3*(1-2*tan(5*x)^(2/3))*3^(1/2))*3^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.125$, Rules used = {3557, 335, 281, 206, 31, 648, 632, 210, 642}

$$-\frac{1}{10}\sqrt{3} \tan^{-1}\left(\frac{1-2\tan^{\frac{2}{3}}(5x)}{\sqrt{3}}\right) + \frac{1}{10} \log\left(\tan^{\frac{2}{3}}(5x)+1\right) - \frac{1}{20} \log\left(\tan^{\frac{4}{3}}(5x)-\tan^{\frac{2}{3}}(5x)+1\right)$$

Antiderivative was successfully verified.

[In] Int[Tan[5*x]^(-1/3),x]

[Out] -1/10*(Sqrt[3]*ArcTan[(1-2*Tan[5*x]^(2/3))/Sqrt[3]]) + Log[1+Tan[5*x]^(2/3)]/10 - Log[1-Tan[5*x]^(2/3)+Tan[5*x]^(4/3)]/20

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(n-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

Rule 281

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x], x]

x^k , x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 335

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n))^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 3557

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{\tan(5x)}} dx &= \frac{1}{5} \text{Subst} \left(\int \frac{1}{\sqrt[3]{x} (1+x^2)} dx, x, \tan(5x) \right) \\
&= \frac{3}{5} \text{Subst} \left(\int \frac{x}{1+x^6} dx, x, \sqrt[3]{\tan(5x)} \right) \\
&= \frac{3}{10} \text{Subst} \left(\int \frac{1}{1+x^3} dx, x, \tan^{\frac{2}{3}}(5x) \right) \\
&= \frac{1}{10} \text{Subst} \left(\int \frac{1}{1+x} dx, x, \tan^{\frac{2}{3}}(5x) \right) + \frac{1}{10} \text{Subst} \left(\int \frac{2-x}{1-x+x^2} dx, x, \tan^{\frac{2}{3}}(5x) \right) \\
&= \frac{1}{10} \log \left(1 + \tan^{\frac{2}{3}}(5x) \right) - \frac{1}{20} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, \tan^{\frac{2}{3}}(5x) \right) + \frac{3}{20} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, \tan^{\frac{2}{3}}(5x) \right) \\
&= \frac{1}{10} \log \left(1 + \tan^{\frac{2}{3}}(5x) \right) - \frac{1}{20} \log \left(1 - \tan^{\frac{2}{3}}(5x) + \tan^{\frac{4}{3}}(5x) \right) - \frac{3}{10} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, \tan^{\frac{2}{3}}(5x) \right) \\
&= -\frac{1}{10} \sqrt{3} \tan^{-1} \left(\frac{1 - 2 \tan^{\frac{2}{3}}(5x)}{\sqrt{3}} \right) + \frac{1}{10} \log \left(1 + \tan^{\frac{2}{3}}(5x) \right) - \frac{1}{20} \log \left(1 - \tan^{\frac{2}{3}}(5x) + \tan^{\frac{4}{3}}(5x) \right)
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 69, normalized size = 1.21

$$\frac{1}{10} \sqrt{3} \tan^{-1} \left(\frac{-1 + 2 \tan^{\frac{2}{3}}(5x)}{\sqrt{3}} \right) + \frac{1}{10} \log \left(1 + \tan^{\frac{2}{3}}(5x) \right) - \frac{1}{20} \log \left(1 - \tan^{\frac{2}{3}}(5x) + \tan^{\frac{4}{3}}(5x) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[Tan[5*x]^(-1/3),x]`

```
[Out] (Sqrt[3]*ArcTan[(-1 + 2*Tan[5*x]^(2/3))/Sqrt[3]])/10 + Log[1 + Tan[5*x]^(2/3)]/10 - Log[1 - Tan[5*x]^(2/3) + Tan[5*x]^(4/3)]/20
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/Tan[5*x]^(1/3),x]')``[Out] cought exception: maximum recursion depth exceeded in comparison`**Maple [A]**

time = 0.06, size = 53, normalized size = 0.93

method	result	size
derivativedivides	$\frac{\ln(1+\tan^{\frac{2}{3}}(5x))}{10} - \frac{\ln(1-(\tan^{\frac{2}{3}}(5x))+\tan^{\frac{4}{3}}(5x))}{20} + \frac{\sqrt{3} \arctan\left(\frac{(2(\tan^{\frac{2}{3}}(5x))-1)\sqrt{3}}{3}\right)}{10}$	53
default	$\frac{\ln(1+\tan^{\frac{2}{3}}(5x))}{10} - \frac{\ln(1-(\tan^{\frac{2}{3}}(5x))+\tan^{\frac{4}{3}}(5x))}{20} + \frac{\sqrt{3} \arctan\left(\frac{(2(\tan^{\frac{2}{3}}(5x))-1)\sqrt{3}}{3}\right)}{10}$	53

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/tan(5*x)^(1/3),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{10} \ln(1+\tan(5x)^{2/3}) - \frac{1}{20} \ln(1-\tan(5x)^{2/3}+\tan(5x)^{4/3}) + \frac{1}{10} 3^{1/2} (1/2) * \arctan(1/3 * (2 * \tan(5x)^{2/3} - 1) * 3^{1/2})$

Maxima [A]

time = 0.35, size = 52, normalized size = 0.91

$$\frac{1}{10} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 \tan(5x)^{\frac{2}{3}} - 1)\right) - \frac{1}{20} \log(\tan(5x)^{\frac{4}{3}} - \tan(5x)^{\frac{2}{3}} + 1) + \frac{1}{10} \log(\tan(5x)^{\frac{2}{3}} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/tan(5*x)^(1/3),x, algorithm="maxima")`

[Out] $\frac{1}{10} \sqrt{3} * \arctan(1/3 * \sqrt{3} * (2 * \tan(5x)^{2/3} - 1)) - \frac{1}{20} * \log(\tan(5x)^{4/3} - \tan(5x)^{2/3} + 1) + \frac{1}{10} * \log(\tan(5x)^{2/3} + 1)$

Fricas [A]

time = 0.31, size = 54, normalized size = 0.95

$$\frac{1}{10} \sqrt{3} \arctan\left(\frac{2}{3} \sqrt{3} \tan(5x)^{\frac{2}{3}} - \frac{1}{3} \sqrt{3}\right) - \frac{1}{20} \log(\tan(5x)^{\frac{4}{3}} - \tan(5x)^{\frac{2}{3}} + 1) + \frac{1}{10} \log(\tan(5x)^{\frac{2}{3}} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/tan(5*x)^(1/3),x, algorithm="fricas")`

[Out] $\frac{1}{10} \sqrt{3} * \arctan(2/3 * \sqrt{3} * \tan(5x)^{2/3} - 1/3 * \sqrt{3}) - \frac{1}{20} * \log(\tan(5x)^{4/3} - \tan(5x)^{2/3} + 1) + \frac{1}{10} * \log(\tan(5x)^{2/3} + 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{\tan(5x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(5*x)**(1/3),x)

[Out] Integral(tan(5*x)**(-1/3), x)

Giac [A]

time = 0.04, size = 83, normalized size = 1.46

$$\frac{3}{2} \left(-\frac{\ln \left(\tan(5x)^{\frac{1}{3}} \tan(5x) - \left(\tan(5x)^{\frac{1}{3}} \right)^2 + 1 \right)}{30} + \frac{1}{15} \sqrt{3} \arctan \left(\frac{2 \left(\left(\tan(5x)^{\frac{1}{3}} \right)^2 - \frac{1}{2} \right)}{\sqrt{3}} \right) + \frac{\ln \left(\left(\tan(5x)^{\frac{1}{3}} \right)^2 + 1 \right)}{15} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/tan(5*x)^(1/3),x)

[Out] 1/10*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(5*x)^(2/3) - 1)) - 1/20*log(tan(5*x)^(4/3) - tan(5*x)^(2/3) + 1) + 1/10*log(tan(5*x)^(2/3) + 1)

Mupad [B]

time = 0.69, size = 67, normalized size = 1.18

$$\frac{\ln \left(\frac{81 \tan(5x)^{2/3} + 81}{10} \right) - \ln \left(81 - 162 \tan(5x)^{2/3} + \sqrt{3} 81i \right) \left(\frac{1}{20} + \frac{\sqrt{3} 1i}{20} \right) + \ln \left(162 \tan(5x)^{2/3} - 81 + \sqrt{3} 81i \right) \left(-\frac{1}{20} + \frac{\sqrt{3} 1i}{20} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/tan(5*x)^(1/3),x)

[Out] log(81*tan(5*x)^(2/3) + 81)/10 - log(3^(1/2)*81i - 162*tan(5*x)^(2/3) + 81) * ((3^(1/2)*1i)/20 + 1/20) + log(3^(1/2)*81i + 162*tan(5*x)^(2/3) - 81) * ((3^(1/2)*1i)/20 - 1/20)

$$3.399 \quad \int \frac{1}{(4+3 \tan(2x))^{3/2}} dx$$

Optimal. Leaf size=87

$$-\frac{9 \tan^{-1}\left(\frac{1-3 \tan(2x)}{\sqrt{2} \sqrt{4+3 \tan(2x)}}\right)}{250\sqrt{2}} + \frac{13 \tanh^{-1}\left(\frac{3+\tan(2x)}{\sqrt{2} \sqrt{4+3 \tan(2x)}}\right)}{250\sqrt{2}} - \frac{3}{25\sqrt{4+3 \tan(2x)}}$$

[Out] -9/500*arctan(1/2*(1-3*tan(2*x))*2^(1/2)/(4+3*tan(2*x))^(1/2))*2^(1/2)+13/500*arctanh(1/2*(3+tan(2*x))*2^(1/2)/(4+3*tan(2*x))^(1/2))*2^(1/2)-3/25/(4+3*tan(2*x))^(1/2)

Rubi [A]

time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3564, 3617, 3616, 209, 213}

$$-\frac{9 \tan^{-1}\left(\frac{1-3 \tan(2x)}{\sqrt{2} \sqrt{3 \tan(2x) + 4}}\right)}{250\sqrt{2}} - \frac{3}{25\sqrt{3 \tan(2x) + 4}} + \frac{13 \tanh^{-1}\left(\frac{\tan(2x)+3}{\sqrt{2} \sqrt{3 \tan(2x) + 4}}\right)}{250\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(4 + 3*Tan[2*x])^(-3/2), x]

[Out] (-9*ArcTan[(1 - 3*Tan[2*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[2*x]])]/(250*Sqrt[2]) + (13*ArcTanh[(3 + Tan[2*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[2*x]])]/(250*Sqrt[2]) - 3/(25*Sqrt[4 + 3*Tan[2*x]]))

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 3564

Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[b*((a + b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,

b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]

Rule 3616

Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 + x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]

Rule 3617

Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]], x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(4 + 3 \tan(2x))^{3/2}} dx &= -\frac{3}{25 \sqrt{4 + 3 \tan(2x)}} + \frac{1}{25} \int \frac{4 - 3 \tan(2x)}{\sqrt{4 + 3 \tan(2x)}} dx \\
 &= -\frac{3}{25 \sqrt{4 + 3 \tan(2x)}} + \frac{1}{250} \int \frac{27 + 9 \tan(2x)}{\sqrt{4 + 3 \tan(2x)}} dx - \frac{1}{250} \int \frac{-13 + 39 \tan(2x)}{\sqrt{4 + 3 \tan(2x)}} dx \\
 &= -\frac{3}{25 \sqrt{4 + 3 \tan(2x)}} - \frac{81}{250} \text{Subst} \left(\int \frac{1}{162 + x^2} dx, x, \frac{9 - 27 \tan(2x)}{\sqrt{4 + 3 \tan(2x)}} \right) + \frac{15}{250} \int \frac{1}{\sqrt{4 + 3 \tan(2x)}} dx \\
 &= -\frac{9 \tan^{-1} \left(\frac{1 - 3 \tan(2x)}{\sqrt{2} \sqrt{4 + 3 \tan(2x)}} \right)}{250 \sqrt{2}} + \frac{13 \tanh^{-1} \left(\frac{3 + \tan(2x)}{\sqrt{2} \sqrt{4 + 3 \tan(2x)}} \right)}{250 \sqrt{2}} - \frac{15}{250} \int \frac{1}{\sqrt{4 + 3 \tan(2x)}} dx
 \end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.07, size = 73, normalized size = 0.84

$$\frac{(3 + 4i) {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \left(\frac{4}{25} - \frac{3i}{25} \right) (4 + 3 \tan(2x)) \right) + (3 - 4i) {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \left(\frac{4}{25} + \frac{3i}{25} \right) (4 + 3 \tan(2x)) \right)}{50 \sqrt{4 + 3 \tan(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*Tan[2*x])^(-3/2),x]
```

```
[Out] -1/50*((3 + 4*I)*Hypergeometric2F1[-1/2, 1, 1/2, (4/25 - (3*I)/25)*(4 + 3*Tan[2*x]]) + (3 - 4*I)*Hypergeometric2F1[-1/2, 1, 1/2, (4/25 + (3*I)/25)*(4 + 3*Tan[2*x])])/Sqrt[4 + 3*Tan[2*x]]
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(4 + 3*Tan[2*x])^(3/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python object
```

Maple [A]

time = 0.13, size = 130, normalized size = 1.49

method	result
derivativedivides	$-\frac{13\sqrt{2} \ln\left(9+3\tan(2x)-3\sqrt{4+3\tan(2x)}\sqrt{2}\right)}{1000} + \frac{9\sqrt{2} \arctan\left(\frac{\left(2\sqrt{4+3\tan(2x)}-3\sqrt{2}\right)\sqrt{2}}{2}\right)}{500}$
default	$-\frac{13\sqrt{2} \ln\left(9+3\tan(2x)-3\sqrt{4+3\tan(2x)}\sqrt{2}\right)}{1000} + \frac{9\sqrt{2} \arctan\left(\frac{\left(2\sqrt{4+3\tan(2x)}-3\sqrt{2}\right)\sqrt{2}}{2}\right)}{500}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+3*tan(2*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -13/1000*2^(1/2)*ln(9+3*tan(2*x)-3*(4+3*tan(2*x))^(1/2)*2^(1/2))+9/500*2^(1/2)*arctan(1/2*(2*(4+3*tan(2*x))^(1/2)-3*2^(1/2))*2^(1/2))+13/1000*2^(1/2)*ln(9+3*tan(2*x)+3*(4+3*tan(2*x))^(1/2)*2^(1/2))+9/500*2^(1/2)*arctan(1/2*(2*(4+3*tan(2*x))^(1/2)+3*2^(1/2))*2^(1/2))-3/25/(4+3*tan(2*x))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3213 vs. 2(69) = 138.

time = 0.58, size = 3213, normalized size = 36.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+3*tan(2*x))^(3/2),x, algorithm="maxima")
```

```

[Out] -1/18000*(2000*(3*cos(4*x) + sin(4*x))*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(
8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^3 + 2000*
(3*cos(4*x) + sin(4*x))*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*
x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))*sin(1/2*arctan2(-3*cos(8
*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4
))^2 - 2000*(cos(4*x) - 3*sin(4*x) - 3)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin
(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^3 - 80*(
48*cos(4*x) + 25*sin(4*x) - 27)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) +
8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4)) - 80*(25*(cos(4*
x) - 3*sin(4*x) - 3)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x)
+ 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 - 25*cos(4*x) + 48*sin(4*
x) + 75)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8
*x) + 8*cos(4*x) + 3*sin(8*x) + 4)) + 9*(18*(sqrt(2)*cos(1/2*arctan2(-3*cos
(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) +
4))^2 + sqrt(2)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3,
4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2)*arctan2(1/3*25^(1/4)*(25*cos
(4*x)^4 + 25*sin(4*x)^4 + 64*cos(4*x)^3 + 2*(25*cos(4*x)^2 + 32*cos(4*x) +
25)*sin(4*x)^2 + 48*sin(4*x)^3 + 78*cos(4*x)^2 + 48*(cos(4*x)^2 + 2*cos(4*x
) + 1)*sin(4*x) + 64*cos(4*x) + 25)^(1/4)*sin(1/2*arctan2(-8/3*cos(4*x)^2 +
2/9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x), 7/9*cos(4*
x)^2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*cos(4*x) + 25/
9)) + cos(4*x) - 4/3*sin(4*x), 1/3*25^(1/4)*(25*cos(4*x)^4 + 25*sin(4*x)^4
+ 64*cos(4*x)^3 + 2*(25*cos(4*x)^2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(
4*x)^3 + 78*cos(4*x)^2 + 48*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos
(4*x) + 25)^(1/4)*cos(1/2*arctan2(-8/3*cos(4*x)^2 + 2/9*(7*cos(4*x) + 16)*s
in(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x), 7/9*cos(4*x)^2 + 8/3*(2*cos(4*x) +
1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*cos(4*x) + 25/9)) - 4/3*cos(4*x) - sin
(4*x) - 4/3) + 18*(sqrt(2)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin
(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 + sqrt(2)*sin(1/2*a
rctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) +
3*sin(8*x) + 4))^2)*arctan2(2/3*4^(1/4)*(4*cos(4*x)^4 + 4*sin(4*x)^4 + 16*
cos(4*x)^3 + (8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^2 + 12*sin(4*x)^3 +
33*cos(4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 34*cos(4*x) +
13)^(1/4)*sin(1/2*arctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8/3*cos(4*x) + 8/3
, 16/9*cos(4*x)^2 - 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(4*x) + 16/9))
+ 4/3*sin(4*x) + 1, 2/3*4^(1/4)*(4*cos(4*x)^4 + 4*sin(4*x)^4 + 16*cos(4*x)
^3 + (8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^2 + 12*sin(4*x)^3 + 33*cos(
4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 34*cos(4*x) + 13)^(1/4
)*cos(1/2*arctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8/3*cos(4*x) + 8/3, 16/9*c
os(4*x)^2 - 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(4*x) + 16/9)) + 4/3*c
os(4*x) + 4/3) + 13*(sqrt(2)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*s
in(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 + sqrt(2)*sin(1/2
*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x)
+ 3*sin(8*x) + 4))^2)*log(-2/9*25^(1/4)*(25*cos(4*x)^4 + 25*sin(4*x)^4 + 6
4*cos(4*x)^3 + 2*(25*cos(4*x)^2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4*x

```

$$\begin{aligned}
&)^3 + 78\cos(4x)^2 + 48(\cos(4x)^2 + 2\cos(4x) + 1)\sin(4x) + 64\cos(4x \\
&x) + 25)^{(1/4)}(4\cos(4x) + 3\sin(4x) + 4)\cos(1/2\arctan2(-8/3\cos(4x)^2 \\
&+ 2/9(7\cos(4x) + 16)\sin(4x) + 8/3\sin(4x)^2 - 8/3\cos(4x), 7/9\cos \\
&(4x)^2 + 8/3(2\cos(4x) + 1)\sin(4x) - 7/9\sin(4x)^2 + 32/9\cos(4x) + \\
&25/9)) + 5/9\sqrt{25\cos(4x)^4 + 25\sin(4x)^4 + 64\cos(4x)^3 + 2(25\cos \\
&(4x)^2 + 32\cos(4x) + 25)\sin(4x)^2 + 48\sin(4x)^3 + 78\cos(4x)^2 + 48 \\
&(\cos(4x)^2 + 2\cos(4x) + 1)\sin(4x) + 64\cos(4x) + 25)\cos(1/2\arctan2 \\
&(-8/3\cos(4x)^2 + 2/9(7\cos(4x) + 16)\sin(4x) + 8/3\sin(4x)^2 - 8/3\cos \\
&(4x), 7/9\cos(4x)^2 + 8/3(2\cos(4x) + 1)\sin(4x) - 7/9\sin(4x)^2 + 3 \\
&2/9\cos(4x) + 25/9))^2 + 2/925^{(1/4)}(25\cos(4x)^4 + 25\sin(4x)^4 + 64 \\
&\cos(4x)^3 + 2(25\cos(4x)^2 + 32\cos(4x) + 25)\sin(4x)^2 + 48\sin(4x)^3 \\
&+ 78\cos(4x)^2 + 48(\cos(4x)^2 + 2\cos(4x) + 1)\sin(4x) + 64\cos(4x) \\
&+ 25)^{(1/4)}(3\cos(4x) - 4\sin(4x))\sin(1/2\arctan2(-8/3\cos(4x)^2 + 2/ \\
&9(7\cos(4x) + 16)\sin(4x) + 8/3\sin(4x)^2 - 8/3\cos(4x), 7/9\cos(4x)^2 \\
&+ 8/3(2\cos(4x) + 1)\sin(4x) - 7/9\sin(4x)^2 + 32/9\cos(4x) + 25/9)) \\
&+ 5/9\sqrt{25\cos(4x)^4 + 25\sin(4x)^4 + 64\cos(4x)^3 + 2(25\cos(4x)^2 \\
&+ 32\cos(4x) + 25)\sin(4x)^2 + 48\sin(4x)^3 + 78\cos(4x)^2 + 48(\cos \\
&(4x)^2 + 2\cos(4x) + 1)\sin(4x) + 64\cos(4x) + 25)\sin(1/2\arctan2(-8/3 \\
&\cos(4x)^2 + 2/9(7\cos(4x) + 16)\sin(4x) + 8/3\sin(4x)^2 - 8/3\cos(4x) \\
&, 7/9\cos(4x)^2 + 8/3(2\cos(4x) + 1)\sin(4x) - 7/9\sin(4x)^2 + 32/9\cos \\
&(4x) + 25/9))^2 + 25/9\cos(4x)^2 + 25/9\sin(4x)^2 + 32/9\cos(4x) + 8/3 \\
&\sin(4x) + 16/9) - 13(\sqrt{2}\cos(1/2\arctan2(-3\cos(8x) + 4\sin(8x) + \\
&8\sin(4x) + 3, 4\cos(8x) + 8\cos(4x) + 3\sin(8x) + 4))^2 + \sqrt{2}\sin(\\
&1/2\arctan2(-3\cos(8x) + 4\sin(8x) + 8\sin(4x) + 3, 4\cos(8x) + 8\cos(4 \\
&x) + 3\sin(8x) + 4))^2)\log(16/94^{(1/4)}(4\cos(4x)^4 + 4\sin(4x)^4 + 1 \\
&6\cos(4x)^3 + (8\cos(4x)^2 + 16\cos(4x) + 17)\sin(4x)^2 + 12\sin(4x)^3 \\
&+ 33\cos(4x)^2 + 12(\cos(4x)^2 + 2\cos(4x) + 1)\sin(4x) + 34\cos(4x) \\
&+ 13)^{(1/4)}(\cos(4x) + 1)\cos(1/2\arctan2(32/9(\cos(4x) + 1)\sin(4x) + 8 \\
&/3\cos(4x) + 8/3, 16/9\cos(4x)^2 - 16/9\sin(4x)^2 + 32/9\cos(4x) - 8/3 \\
&\sin(4x) + 16/9)) + 8/9\sqrt{4\cos(4x)^4 + 4\sin(4x)^4 + 16\cos(4x)^3 + \\
&(8\cos(4x)^2 + 16\cos(4x) + 17)\sin(4x)^2 + 12\sin(4x)^3 + 33\cos(4x)^2 \\
&+ 12(\cos(4x)^2 + 2\cos(4x) + 1)\sin(4x) + 34\cos(4x) + 13)\cos(1/2\ar \\
&\text{ctan2}(32/9(\cos(4x) + 1)\sin(4x) + 8/3\cos(4x) + 8/3, 16/9\cos(4x)^2 - \\
&16/9\sin(4x)^2 + 32/9\cos(4x) - 8/3\sin(4x) + 16/9))^2 + 4/94^{(1/4)}(4 \\
&\cos(4x)^4 + 4\sin(4x)^4 + 16\cos(4x)^3 + (8\cos(4x)^2 + 16\cos(4x) + \\
&17)\sin(4x)^2 + 12\sin(4x)^3 + 33\cos(4x)^2 + 12(\cos(4x)^2 + 2\cos(4x) \\
&+ 1)\sin(4x) + 34\cos(4x) + 13)^{(1/4)}(4\sin(4x) + 3)\sin(1/2\arctan2(\\
&32/9(\cos(4x) + 1)\sin(4x) + 8/3\cos(4x) + 8/3, 16/9\cos(4x)^2 - 16/9\sin \\
&(4x)^2 + 32/9\cos(4x) - 8/3\sin(4x) + 16/9)) + 8/9\sqrt{4\cos(4x)^4 + \\
&4\sin(4x)^4 + 16\cos(4x)^3 + (8\cos(4x)^2 + 16\cos(4x) + 17)\sin(4x)^2 \\
&+ 12\sin(4x)^3 + 33\cos(4x)^2 + 12(\cos(4x)^2 + 2\cos(4x) + 1)\sin(4x) \\
&+ 34\cos(4x) + 13)\sin(1/2\arctan2(32/9(\cos(4x) + 1)\sin(4x) + 8/3\cos \\
&(4x) + 8/3, 16/9\cos(4x)^2 - 16/9\sin(4x)^2 + 32/9\cos(4x) - 8/3\sin(4 \\
&x) + 16/9))^2 + 16/9\cos(4x)^2 + 16/9\sin(4x)^2 + 32/9\cos(4x) + 8/3\sin \\
&(4x) + 25/9))(2(32\cos(4x) - 24\sin(4x) + 7)\cos(8x) + 25\cos(8x))^2
\end{aligned}$$

$$2 + 64\cos(4x)^2 + 16(3\cos(4x) + 4\sin(4x) + 3)\sin(8x) + 25\sin(8x)^2 + 64\sin(4x)^2 + 64\cos(4x) + 48\sin(4x) + 25)^{(1/4)} / ((2(32\cos(4x) - 24\sin(4x) + 7)\cos(8x) + 25\cos(8x)^2 + 64\cos(4x)^2 + 16(3\cos(4x) + 4\sin(4x) + 3)\sin(8x) + 25\sin(8x)^2 + 64\sin(4x)^2 + 64\cos(4x) + 48\sin(4x) + 25)^{(1/4)} (\cos(1/2\arctan2(-3\cos(8x) + 4\sin(8x) + 8\sin(4x) + 3, 4\cos(8x) + 8\cos(4x) + 3\sin(8x) + 4))^2 + \sin(1/2\arctan2(-3\cos(8x) + 4\sin(8x) + 8\sin(4x) + 3, 4\cos(8x) + 8\cos(4x) + 3\sin(8x) + 4))^2))$$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(69) = 138.

time = 0.57, size = 541, normalized size = 6.22

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4+3*tan(2*x))^(3/2),x, algorithm="fricas")

[Out]
$$-1/5000*(36*(7*\sqrt{10}*\sqrt{5}*\cos(2*x)^2 + 24*\sqrt{10}*\sqrt{5}*\cos(2*x)*\sin(2*x) + 9*\sqrt{10}*\sqrt{5})*\arctan(1/25*\sqrt{15}*\sqrt{10}*\sqrt{5}*\sqrt{((\sqrt{10}*\sqrt{5}*\sqrt{((4*\cos(2*x) + 3*\sin(2*x))/\cos(2*x))*\cos(2*x) + 15*\cos(2*x) + 5*\sin(2*x))/\cos(2*x)) - 1/5*\sqrt{10}*\sqrt{5}*\sqrt{((4*\cos(2*x) + 3*\sin(2*x))/\cos(2*x)) - 3) + 36*(7*\sqrt{10}*\sqrt{5}*\cos(2*x)^2 + 24*\sqrt{10}*\sqrt{5}*\cos(2*x)*\sin(2*x) + 9*\sqrt{10}*\sqrt{5})*\arctan(1/25*\sqrt{15}*\sqrt{10}*\sqrt{5}*\sqrt{-(\sqrt{10}*\sqrt{5}*\sqrt{((4*\cos(2*x) + 3*\sin(2*x))/\cos(2*x))*\cos(2*x) - 15*\cos(2*x) - 5*\sin(2*x))/\cos(2*x)) - 1/5*\sqrt{10}*\sqrt{5}*\sqrt{((4*\cos(2*x) + 3*\sin(2*x))/\cos(2*x)) + 3) - 13*(7*\sqrt{10}*\sqrt{5}*\cos(2*x)^2 + 24*\sqrt{10}*\sqrt{5}*\cos(2*x)*\sin(2*x) + 9*\sqrt{10}*\sqrt{5})*\log(9375*(\sqrt{10}*\sqrt{5}*\sqrt{((4*\cos(2*x) + 3*\sin(2*x))/\cos(2*x))*\cos(2*x) + 15*\cos(2*x) + 5*\sin(2*x))/\cos(2*x)) + 13*(7*\sqrt{10}*\sqrt{5}*\cos(2*x)^2 + 24*\sqrt{10}*\sqrt{5}*\cos(2*x)*\sin(2*x) + 9*\sqrt{10}*\sqrt{5})*\log(-9375*(\sqrt{10}*\sqrt{5}*\sqrt{((4*\cos(2*x) + 3*\sin(2*x))/\cos(2*x))*\cos(2*x) - 15*\cos(2*x) - 5*\sin(2*x))/\cos(2*x)) + 600*(4*\cos(2*x)^2 + 3*\cos(2*x)*\sin(2*x))*\sqrt{((4*\cos(2*x) + 3*\sin(2*x))/\cos(2*x))})/(7*\cos(2*x)^2 + 24*\cos(2*x)*\sin(2*x) + 9)$$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3 \tan(2x) + 4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4+3*tan(2*x))**(3/2),x)

[Out] Integral((3*tan(2*x) + 4)**(-3/2), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(69) = 138.

time = 0.01, size = 234, normalized size = 2.69

$$-\frac{13}{1000}\sqrt{2}\ln\left(3\tan(2x)+4-\frac{3}{5}\cdot 25^{\frac{1}{4}}\sqrt{10}\sqrt{3\tan(2x)+4}+25^{\frac{1}{4}}\cdot 25^{\frac{1}{4}}\right)+\frac{9}{500}\sqrt{2}\arctan\left(\frac{10\left(\sqrt{3\tan(2x)+4}-\frac{3}{5}\sqrt{10}\cdot 25^{\frac{1}{4}}\right)}{\sqrt{10}\cdot 25^{\frac{1}{4}}}\right)+\frac{13}{1000}\sqrt{2}\ln\left(3\tan(2x)+4+\frac{3}{5}\cdot 25^{\frac{1}{4}}\sqrt{10}\sqrt{3\tan(2x)+4}+25^{\frac{1}{4}}\cdot 25^{\frac{1}{4}}\right)+\frac{9}{500}\sqrt{2}\arctan\left(\frac{10\left(\sqrt{3\tan(2x)+4}+\frac{3}{5}\sqrt{10}\cdot 25^{\frac{1}{4}}\right)}{\sqrt{10}\cdot 25^{\frac{1}{4}}}\right)-\frac{3}{25\sqrt{3\tan(2x)+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4+3*tan(2*x))^(3/2),x)

[Out] 9/500*sqrt(2)*arctan(1/250*25^(3/4)*sqrt(10)*(3*25^(1/4)*sqrt(10) + 10*sqrt(3*tan(2*x) + 4))) + 9/500*sqrt(2)*arctan(-1/250*25^(3/4)*sqrt(10)*(3*25^(1/4)*sqrt(10) - 10*sqrt(3*tan(2*x) + 4))) + 13/1000*sqrt(2)*log(3/5*25^(1/4)*sqrt(10)*sqrt(3*tan(2*x) + 4) + 3*tan(2*x) + 9) - 13/1000*sqrt(2)*log(-3/5*25^(1/4)*sqrt(10)*sqrt(3*tan(2*x) + 4) + 3*tan(2*x) + 9) - 3/25/sqrt(3*tan(2*x) + 4)

Mupad [B]

time = 0.44, size = 63, normalized size = 0.72

$$-\frac{3}{25\sqrt{3\tan(2x)+4}}+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{3\tan(2x)+4}\left(\frac{1}{10}-\frac{3}{10}i\right)\right)\left(\frac{9}{500}+\frac{13}{500}i\right)+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{3\tan(2x)+4}\left(\frac{1}{10}+\frac{3}{10}i\right)\right)\left(\frac{9}{500}-\frac{13}{500}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*tan(2*x) + 4)^(3/2),x)

[Out] 2^(1/2)*atan(2^(1/2)*(3*tan(2*x) + 4)^(1/2)*(1/10 - 3i/10))*(9/500 + 13i/500) + 2^(1/2)*atan(2^(1/2)*(3*tan(2*x) + 4)^(1/2)*(1/10 + 3i/10))*(9/500 - 13i/500) - 3/(25*(3*tan(2*x) + 4)^(1/2))

$$3.400 \quad \int \frac{\sec^2(x) \left(-\sqrt{4-3\tan(x)} + 3\tan(x) \right)}{(4-3\tan(x))^{3/2}} dx$$

Optimal. Leaf size=40

$$\frac{1}{3} \log(4-3\tan(x)) + \frac{8}{3\sqrt{4-3\tan(x)}} + \frac{2}{3} \sqrt{4-3\tan(x)}$$

[Out] 1/3*ln(4-3*tan(x))+8/3/(4-3*tan(x))^(1/2)+2/3*(4-3*tan(x))^(1/2)

Rubi [A]

time = 0.09, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$,

Rules used = {4427, 45}

$$\frac{2}{3} \sqrt{4-3\tan(x)} + \frac{8}{3\sqrt{4-3\tan(x)}} + \frac{1}{3} \log(4-3\tan(x))$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2*(-Sqrt[4 - 3*Tan[x]] + 3*Tan[x]))/(4 - 3*Tan[x])^(3/2),x]

[Out] Log[4 - 3*Tan[x]]/3 + 8/(3*Sqrt[4 - 3*Tan[x]]) + (2*Sqrt[4 - 3*Tan[x]])/3

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] := With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]]/d, u, x], x], x, Tan[c*(a + b*x)]/d, x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x) \left(-\sqrt{4 - 3 \tan(x)} + 3 \tan(x) \right)}{(4 - 3 \tan(x))^{3/2}} dx &= \text{Subst} \left(\int \left(\frac{3x}{(4 - 3x)^{3/2}} + \frac{1}{-4 + 3x} \right) dx, x, \tan(x) \right) \\
&= \frac{1}{3} \log(4 - 3 \tan(x)) + 3 \text{Subst} \left(\int \frac{x}{(4 - 3x)^{3/2}} dx, x, \tan(x) \right) \\
&= \frac{1}{3} \log(4 - 3 \tan(x)) + 3 \text{Subst} \left(\int \left(\frac{4}{3(4 - 3x)^{3/2}} - \frac{1}{3\sqrt{4 - 3x}} \right) dx, x, \tan(x) \right) \\
&= \frac{1}{3} \log(4 - 3 \tan(x)) + \frac{8}{3\sqrt{4 - 3 \tan(x)}} + \frac{2}{3} \sqrt{4 - 3 \tan(x)}
\end{aligned}$$

Mathematica [A]

time = 0.91, size = 38, normalized size = 0.95

$$\frac{16 + \log(4 - 3 \tan(x)) \sqrt{4 - 3 \tan(x)} - 6 \tan(x)}{3 \sqrt{4 - 3 \tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(-Sqrt[4 - 3*Tan[x]] + 3*Tan[x]))/(4 - 3*Tan[x])^(3/2), x]
```

```
[Out] (16 + Log[4 - 3*Tan[x]]*Sqrt[4 - 3*Tan[x]] - 6*Tan[x])/(3*Sqrt[4 - 3*Tan[x]])
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(3*Tan[x] - Sqrt[4 - 3*Tan[x]])/(Cos[x]^2*(4 - 3*Tan[x])^(3/2)), x]')
```

[Out] Timed out

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(30) = 60.

time = 0.47, size = 219, normalized size = 5.48

method	result
--------	--------

default	$(\cos(x)-1)^2(1+\cos(x))^2 \left(16 \sqrt{\frac{4 \cos(x)-3 \sin(x)}{\cos(x)}} \cos(x) - 4 \cos(x) \ln\left(-\frac{-1+\cos(x)-\sin(x)}{\sin(x)}\right) + 4 \cos(x) \ln\left(-\frac{\sin(x)-2+2 \cos(x)}{\sin(x)}\right) - 4 \cos(x) \ln\left(-\frac{\sin(x)+2-2 \cos(x)}{\sin(x)}\right) \right)$
---------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x,method=_RE
TURNVERBOSE)`

[Out] $\frac{1}{3}(\cos(x)-1)^2(1+\cos(x))^2(16\sqrt{\frac{4\cos(x)-3\sin(x)}{\cos(x)}}\cos(x)-4\cos(x)\ln(-\frac{-1+\cos(x)-\sin(x)}{\sin(x)})+4\cos(x)\ln(-\frac{\sin(x)-2+2\cos(x)}{\sin(x)})-4\cos(x)\ln(-\frac{\sin(x)+2-2\cos(x)}{\sin(x)}))$

Maxima [A]

time = 0.26, size = 30, normalized size = 0.75

$$\frac{2}{3} \sqrt{-3 \tan(x) + 4} + \frac{8}{3 \sqrt{-3 \tan(x) + 4}} + \frac{1}{3} \log(-3 \tan(x) + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x,alg
orithm="maxima")`

[Out] $2/3\sqrt{-3\tan(x) + 4} + 8/3/\sqrt{-3\tan(x) + 4} + 1/3\log(-3\tan(x) + 4)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(30) = 60.

time = 0.32, size = 82, normalized size = 2.05

$$\frac{(4 \cos(x) - 3 \sin(x)) \log\left(\frac{7}{4} \cos(x)^2 - 6 \cos(x) \sin(x) + \frac{9}{4}\right) - (4 \cos(x) - 3 \sin(x)) \log(\cos(x)^2) + 4 \sqrt{\frac{4 \cos(x) - 3 \sin(x)}{\cos(x)}} (8 \cos(x) - 3 \sin(x))}{6 (4 \cos(x) - 3 \sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x,alg
orithm="fricas")`

[Out] $\frac{1}{6}((4\cos(x) - 3\sin(x))\log(7/4\cos(x)^2 - 6\cos(x)\sin(x) + 9/4) - (4\cos(x) - 3\sin(x))\log(\cos(x)^2) + 4\sqrt{(4\cos(x) - 3\sin(x))/\cos(x)}(8\cos(x) - 3\sin(x)))/(4\cos(x) - 3\sin(x))$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{4-3 \tan(x)}}{-3 \sqrt{4-3 \tan(x)} \cos^2(x) \tan(x) + 4 \sqrt{4-3 \tan(x)} \cos^2(x)} dx - \int \left(-\frac{3 \tan(x)}{-3 \sqrt{4-3 \tan(x)} \cos^2(x) \tan(x) + 4 \sqrt{4-3 \tan(x)} \cos^2(x)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4-3*tan(x))**(1/2)+3*tan(x))/cos(x)**2/(4-3*tan(x))**(3/2),x)

[Out] -Integral(sqrt(4 - 3*tan(x))/(-3*sqrt(4 - 3*tan(x))*cos(x)**2*tan(x) + 4*sqrt(4 - 3*tan(x))*cos(x)**2), x) - Integral(-3*tan(x)/(-3*sqrt(4 - 3*tan(x))*cos(x)**2*tan(x) + 4*sqrt(4 - 3*tan(x))*cos(x)**2), x)

Giac [A]

time = 0.01, size = 44, normalized size = 1.10

$$\frac{2}{3}\sqrt{-3\tan x + 4} + \frac{8}{3\sqrt{-3\tan x + 4}} + \frac{\ln|-3\tan x + 4|}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x)

[Out] 2/3*sqrt(-3*tan(x) + 4) + 8/3/sqrt(-3*tan(x) + 4) + 1/3*log(abs(-3*tan(x) + 4))

Mupad [B]

time = 1.42, size = 105, normalized size = 2.62

$$\frac{\ln(e^{x2i}(-\frac{16}{3}-4i)-\frac{16}{3}+4i)}{3} - \frac{\ln(e^{x2i}(\frac{16}{3}-4i)+\frac{16}{3}-4i)}{3} + \frac{2e^{x1i}\cos(x)\left(\frac{32e^{x1i}\cos(x)}{3}-4e^{x1i}\sin(x)\right)\sqrt{4-\frac{3\sin(x)}{\cos(x)}}}{8e^{x2i}+8\cos(2x)e^{x2i}-6\sin(2x)e^{x2i}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*tan(x) - (4 - 3*tan(x))^(1/2))/(cos(x)^2*(4 - 3*tan(x))^(3/2)),x)

[Out] log(-exp(x*2i)*(16/3 + 4i) - (16/3 - 4i))/3 - log(exp(x*2i)*(16/3 - 4i) + (16/3 - 4i))/3 + (2*exp(x*1i)*cos(x)*((32*exp(x*1i)*cos(x))/3 - 4*exp(x*1i)*sin(x))*(4 - (3*sin(x))/cos(x))^(1/2))/(8*exp(x*2i) + 8*cos(2*x)*exp(x*2i) - 6*sin(2*x)*exp(x*2i))

$$3.401 \quad \int \frac{\tan(x)}{\left(-1 + \sqrt{\tan(x)}\right)^2} dx$$

Optimal. Leaf size=84

$$-\frac{x}{2} + \frac{\tan^{-1}\left(\frac{1-\tan(x)}{\sqrt{2}\sqrt{\tan(x)}}\right)}{\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{1+\tan(x)}{\sqrt{2}\sqrt{\tan(x)}}\right)}{\sqrt{2}} + \frac{1}{2} \log(\cos(x)) + \log\left(1 - \sqrt{\tan(x)}\right) + \frac{1}{1 - \sqrt{\tan(x)}}$$

[Out] $-1/2*x + 1/2*\ln(\cos(x)) + \ln(1 - \tan(x)^{(1/2)}) + 1/2*\arctan(1/2*(1 - \tan(x))*2^{(1/2)}/\tan(x)^{(1/2)}) * 2^{(1/2)} + 1/2*\operatorname{arctanh}(1/2*(1 + \tan(x))*2^{(1/2)}/\tan(x)^{(1/2)}) * 2^{(1/2)} + 1/(1 - \tan(x)^{(1/2)})$

Rubi [A]

time = 0.22, antiderivative size = 133, normalized size of antiderivative = 1.58, number of steps used = 19, number of rules used = 13, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3751, 6857, 1845, 303, 1176, 631, 210, 1179, 642, 1262, 649, 209, 266}

$$-\frac{x}{2} + \frac{\tan^{-1}\left(\frac{1 - \sqrt{2}\sqrt{\tan(x)}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\tan(x)} + 1}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{1}{1 - \sqrt{\tan(x)}} + \log\left(1 - \sqrt{\tan(x)}\right) - \frac{\log\left(\frac{\tan(x) - \sqrt{2}\sqrt{\tan(x)} + 1}{2\sqrt{2}}\right)}{2\sqrt{2}} + \frac{\log\left(\frac{\tan(x) + \sqrt{2}\sqrt{\tan(x)} + 1}{2\sqrt{2}}\right)}{2\sqrt{2}} + \frac{1}{2} \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Tan[x]/(-1 + Sqrt[Tan[x]])^2,x]

[Out] $-1/2*x + \operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[x]]]/\operatorname{Sqrt}[2] - \operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[x]]]/\operatorname{Sqrt}[2] + \operatorname{Log}[\operatorname{Cos}[x]]/2 + \operatorname{Log}[1 - \operatorname{Sqrt}[\operatorname{Tan}[x]]] - \operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[x]] + \operatorname{Tan}[x]]/(2*\operatorname{Sqrt}[2]) + \operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[x]] + \operatorname{Tan}[x]]/(2*\operatorname{Sqrt}[2]) + (1 - \operatorname{Sqrt}[\operatorname{Tan}[x]])^{(-1)}$

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2]))^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 266

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1845

```
Int[((Pq_)*((c_)*(x_)^(m_)))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}], Int[v, x] /; SumQ[v] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 3751

```
Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) +
(f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)], x], x, c*(Tan[e + f*x]/ff)], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(x)}{(-1 + \sqrt{\tan(x)})^2} dx &= \text{Subst} \left(\int \frac{x}{(-1 + \sqrt{x})^2 (1 + x^2)} dx, x, \tan(x) \right) \\
&= 2\text{Subst} \left(\int \frac{x^3}{(-1 + x)^2 (1 + x^4)} dx, x, \sqrt{\tan(x)} \right) \\
&= 2\text{Subst} \left(\int \left(\frac{1}{2(-1 + x)^2} + \frac{1}{2(-1 + x)} - \frac{x(1 + x)^2}{2(1 + x^4)} \right) dx, x, \sqrt{\tan(x)} \right) \\
&= \log \left(1 - \sqrt{\tan(x)} \right) + \frac{1}{1 - \sqrt{\tan(x)}} - \text{Subst} \left(\int \frac{x(1 + x)^2}{1 + x^4} dx, x, \sqrt{\tan(x)} \right) \\
&= \log \left(1 - \sqrt{\tan(x)} \right) + \frac{1}{1 - \sqrt{\tan(x)}} - \text{Subst} \left(\int \left(\frac{2x^2}{1 + x^4} + \frac{x(1 + x^2)}{1 + x^4} \right) dx, x, \sqrt{\tan(x)} \right) \\
&= \log \left(1 - \sqrt{\tan(x)} \right) + \frac{1}{1 - \sqrt{\tan(x)}} - 2\text{Subst} \left(\int \frac{x^2}{1 + x^4} dx, x, \sqrt{\tan(x)} \right) \\
&= \log \left(1 - \sqrt{\tan(x)} \right) + \frac{1}{1 - \sqrt{\tan(x)}} - \frac{1}{2} \text{Subst} \left(\int \frac{1 + x}{1 + x^2} dx, x, \tan(x) \right) + \text{Subst} \left(\int \frac{x}{1 + x^2} dx, x, \tan(x) \right) \\
&= \log \left(1 - \sqrt{\tan(x)} \right) + \frac{1}{1 - \sqrt{\tan(x)}} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \tan(x) \right) - \frac{1}{2} \text{Subst} \left(\int \frac{x}{1 + x^2} dx, x, \tan(x) \right) \\
&= -\frac{x}{2} + \frac{1}{2} \log(\cos(x)) + \log \left(1 - \sqrt{\tan(x)} \right) - \frac{\log \left(1 - \sqrt{2} \sqrt{\tan(x)} + \tan(x) \right)}{2\sqrt{2}} \\
&= -\frac{x}{2} + \frac{\tan^{-1} \left(1 - \sqrt{2} \sqrt{\tan(x)} \right)}{\sqrt{2}} - \frac{\tan^{-1} \left(1 + \sqrt{2} \sqrt{\tan(x)} \right)}{\sqrt{2}} + \frac{1}{2} \log(\cos(x))
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.18, size = 62, normalized size = 0.74

$$-\frac{1}{2} \tan^{-1}(\tan(x)) + \frac{1}{2} \log(\cos(x)) + \log \left(1 - \sqrt{\tan(x)} \right) + \frac{1}{1 - \sqrt{\tan(x)}} - \frac{2}{3} {}_2F_1 \left(\frac{3}{4}, 1; \frac{7}{4}; -\tan^2(x) \right) \tan^{\frac{3}{2}}(x)$$

Antiderivative was successfully verified.

[In] Integrate[Tan[x]/(-1 + Sqrt[Tan[x]])^2,x]

[Out] -1/2*ArcTan[Tan[x]] + Log[Cos[x]]/2 + Log[1 - Sqrt[Tan[x]]] + (1 - Sqrt[Tan[x]])^(-1) - (2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[x]^2]*Tan[x]^(3/2))/3

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Tan[x]/(Sqrt[Tan[x]] - 1)^2,x]')
```

[Out] caught exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.05, size = 94, normalized size = 1.12

method	result
derivativedivides	$-\frac{1}{-1+\sqrt{\tan(x)}} + \ln(-1 + \sqrt{\tan(x)}) - \frac{\arctan(\tan(x))}{2} - \frac{\sqrt{2}}{\sqrt{2}} \left(\ln \left(\frac{1-\sqrt{2}(\sqrt{\tan(x)}+\tan(x))}{1+\sqrt{2}(\sqrt{\tan(x)}+\tan(x))} \right) \right)$
default	$-\frac{1}{-1+\sqrt{\tan(x)}} + \ln(-1 + \sqrt{\tan(x)}) - \frac{\arctan(\tan(x))}{2} - \frac{\sqrt{2}}{\sqrt{2}} \left(\ln \left(\frac{1-\sqrt{2}(\sqrt{\tan(x)}+\tan(x))}{1+\sqrt{2}(\sqrt{\tan(x)}+\tan(x))} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(-1+tan(x)^(1/2))^2,x,method=_RETURNVERBOSE)
```

[Out] $-1/(-1+\tan(x)^{(1/2)})+\ln(-1+\tan(x)^{(1/2)})-1/2*\arctan(\tan(x))-1/4*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(x)^{(1/2)}+\tan(x))/(1+2^{(1/2)}*\tan(x)^{(1/2)}+\tan(x)))+2*\arctan(1+2^{(1/2)}*\tan(x)^{(1/2)}+2*\arctan(-1+2^{(1/2)}*\tan(x)^{(1/2)}))-1/4*\ln(1+\tan(x)^2)$

Maxima [A]

time = 0.36, size = 117, normalized size = 1.39

$$\frac{1}{4}\sqrt{2}(\sqrt{2}-2)\arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2}+2\sqrt{\tan(x)})\right)-\frac{1}{4}\sqrt{2}(\sqrt{2}+2)\arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2}-2\sqrt{\tan(x)})\right)-\frac{1}{8}\sqrt{2}(\sqrt{2}-2)\log(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1)-\frac{1}{8}\sqrt{2}(\sqrt{2}+2)\log(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1)-\frac{1}{\sqrt{\tan(x)}-1}+\log(\sqrt{\tan(x)}-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-1+tan(x)^(1/2))^2,x, algorithm="maxima")
```

[Out] $1/4*\sqrt{2}*(\sqrt{2}-2)*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2*\sqrt{\tan(x)}))-1/4*\sqrt{2}*(\sqrt{2}+2)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2*\sqrt{\tan(x)}))-1/8*\sqrt{2}*(\sqrt{2}-2)*\log(\sqrt{2}*\sqrt{\tan(x)}+\tan(x)+1)-1/8*\sqrt{2}*(\sqrt{2}+2)*\log(-\sqrt{2}*\sqrt{\tan(x)}+\tan(x)+1)-1/(\sqrt{\tan(x)}-1)+\log(\sqrt{\tan(x)}-1)$

Fricas [C] Result contains complex when optimal does not.

time = 1.25, size = 603, normalized size = 7.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)/(-1+tan(x)^(1/2))^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/8*(2*(2*\sqrt{-1} - I + 1)*(tan(x) - 1)*\log(-1/2*(2*\sqrt{-1} - I + 1)^2*(\\ & 4*(-1)^{1/4} + 2*I + 1) - (2*(-1)^{1/4} + I + 1)^3 - ((2*(-1)^{1/4} + I + 1 \\ &)^2 - 8*(-1)^{1/4} - 4*I - 3)*(2*\sqrt{-1} - I + 1) + 4*(2*(-1)^{1/4} + I + \\ & 1)^2 + 6*\sqrt{tan(x)} - 16*(-1)^{1/4} - 8*I - 9) + 2*(2*(-1)^{1/4} + I + 1) \\ & *(tan(x) - 1)*\log((2*(-1)^{1/4} + I + 1)^3 - 7/2*(2*(-1)^{1/4} + I + 1)^2 + \\ & 6*\sqrt{tan(x)} + 14*(-1)^{1/4} + 7*I + 14) - ((2*\sqrt{-1} - I + 1)*(tan(x) \\ & - 1) + (2*(-1)^{1/4} + I + 1)*(tan(x) - 1) - 4*\sqrt{-3/16*(2*\sqrt{-1} - I \\ & + 1)^2 - 3/16*(2*(-1)^{1/4} + I + 1)^2 - 1/8*(2*\sqrt{-1} - I + 1)*(2*(-1)^{1/4} \\ & + I - 3) + (-1)^{1/4} + 1/2*I - 1/2)*(tan(x) - 1) - 4*tan(x) + 4)*\log(\\ & 1/4*(2*\sqrt{-1} - I + 1)^2*(4*(-1)^{1/4} + 2*I + 1) + 1/2*((2*(-1)^{1/4} + \\ & I + 1)^2 - 8*(-1)^{1/4} - 4*I - 3)*(2*\sqrt{-1} - I + 1) - 1/4*(2*(-1)^{1/4} \\ & + I + 1)^2 + \sqrt{-3/16*(2*\sqrt{-1} - I + 1)^2 - 3/16*(2*(-1)^{1/4} + I + \\ & 1)^2 - 1/8*(2*\sqrt{-1} - I + 1)*(2*(-1)^{1/4} + I - 3) + (-1)^{1/4} + 1/2*I \\ & - 1/2)*((2*\sqrt{-1} - I + 1)*(4*(-1)^{1/4} + 2*I + 1) - 2*(-1)^{1/4} - I + \\ & 1) + 6*\sqrt{tan(x)} + (-1)^{1/4} + 1/2*I - 5/2) - ((2*\sqrt{-1} - I + 1)*(tan(x) \\ & - 1) + (2*(-1)^{1/4} + I + 1)*(tan(x) - 1) + 4*\sqrt{-3/16*(2*\sqrt{-1} - I \\ & + 1)^2 - 3/16*(2*(-1)^{1/4} + I + 1)^2 - 1/8*(2*\sqrt{-1} - I + 1)*(2*(-1)^{1/4} \\ & + I - 3) + (-1)^{1/4} + 1/2*I - 1/2)*(tan(x) - 1) - 4*tan(x) + 4) \\ & *\log(1/4*(2*\sqrt{-1} - I + 1)^2*(4*(-1)^{1/4} + 2*I + 1) + 1/2*((2*(-1)^{1/4} \\ & + I + 1)^2 - 8*(-1)^{1/4} - 4*I - 3)*(2*\sqrt{-1} - I + 1) - 1/4*(2*(-1)^{1/4} \\ & + I + 1)^2 - \sqrt{-3/16*(2*\sqrt{-1} - I + 1)^2 - 3/16*(2*(-1)^{1/4} + I + \\ & 1)^2 - 1/8*(2*\sqrt{-1} - I + 1)*(2*(-1)^{1/4} + I - 3) + (-1)^{1/4} + \\ & 1/2*I - 1/2)*((2*\sqrt{-1} - I + 1)*(4*(-1)^{1/4} + 2*I + 1) - 2*(-1)^{1/4} \\ & - I + 1) + 6*\sqrt{tan(x)} + (-1)^{1/4} + 1/2*I - 5/2) - 8*(tan(x) - 1)*\log(\\ & \sqrt{tan(x)} - 1) + 8*\sqrt{tan(x)} + 8)/(tan(x) - 1) \end{aligned}$$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{(\sqrt{\tan(x)} - 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)/(-1+tan(x)**(1/2))**2,x)

[Out] Integral(tan(x)/(sqrt(tan(x)) - 1)**2, x)

Giac [A]

time = 0.01, size = 155, normalized size = 1.85

$$-\frac{1}{4}\sqrt{2} \ln(\tan x - \sqrt{2}\sqrt{\tan x} + 1) + \frac{1}{2}(-\sqrt{2} - 1) \arctan\left(\frac{2(\sqrt{\tan x} - \frac{\sqrt{2}}{2})}{\sqrt{2}}\right) + \frac{1}{4}\sqrt{2} \ln(\tan x + \sqrt{2}\sqrt{\tan x} + 1) + \frac{1}{2}(-\sqrt{2} + 1) \arctan\left(\frac{2(\sqrt{\tan x} + \frac{\sqrt{2}}{2})}{\sqrt{2}}\right) - \frac{\ln(\tan^2 x + 1)}{4} + \ln|\sqrt{\tan x} - 1| - \frac{1}{\sqrt{\tan x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)/(-1+tan(x)^(1/2))^2,x)

[Out] $-1/2*(\sqrt{2} - 1)*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2*\sqrt{\tan(x)})) - 1/2*(\sqrt{2} + 1)*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2*\sqrt{\tan(x)})) + 1/4*\sqrt{2}*\log(\sqrt{2}*\sqrt{\tan(x)} + \tan(x) + 1) - 1/4*\sqrt{2}*\log(-\sqrt{2}*\sqrt{\tan(x)} + \tan(x) + 1) - 1/(\sqrt{\tan(x)} - 1) - 1/4*\log(\tan(x)^2 + 1) + \log(\text{abs}(\sqrt{\tan(x)} - 1))$

Mupad [B]

time = 1.27, size = 228, normalized size = 2.71

$\ln\left(\frac{612\sqrt{6017}-612}{\sqrt{6017}-1}\right) + \sum_{k=1}^4 \ln\left(\sqrt{6017} + \cos\left(\frac{\pi}{2} + \frac{\pi}{16} + k\right)\sqrt{6017} + \cos\left(\frac{\pi}{2} + \frac{\pi}{16} + k\right)\sqrt{6017} + \cos\left(\frac{\pi}{2} + \frac{\pi}{16} + k\right)\sqrt{6017} + \cos\left(\frac{\pi}{2} + \frac{\pi}{16} + k\right)\sqrt{6017} + 32\cos\left(\frac{\pi}{2} + \frac{\pi}{16} + k\right) - 384\cos\left(\frac{\pi}{2} + \frac{\pi}{16} + k\right) - 256\cos\left(\frac{\pi}{2} + \frac{\pi}{16} + k\right) - \cos\left(\frac{\pi}{2} + \frac{\pi}{16} + k\right)\sqrt{6017} + \cos\left(\frac{\pi}{2} + \frac{\pi}{16} + k\right)\sqrt{6017} + \cos\left(\frac{\pi}{2} + \frac{\pi}{16} + k\right)\sqrt{6017} + \cos\left(\frac{\pi}{2} + \frac{\pi}{16} + k\right)\sqrt{6017}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)/(tan(x)^(1/2) - 1)^2,x)

[Out] $\log(612*\tan(x)^{(1/2)} - 612) - 1/(\tan(x)^{(1/2)} - 1) + \text{symsum}(\log(4*\tan(x)^{(1/2)} + 80*\text{root}(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^2*\tan(x)^{(1/2)} + 448*\text{root}(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^3*\tan(x)^{(1/2)} + 128*\text{root}(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^4*\tan(x)^{(1/2)} + 32*\text{root}(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^2 - 384*\text{root}(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^3 - 256*\text{root}(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^4 - 48*\text{root}(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)*\tan(x)^{(1/2)} - 4)*\text{root}(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k), k, 1, 4)$

$$3.402 \quad \int \frac{\sin(x)}{\sqrt{\sin(2x)}} dx$$

Optimal. Leaf size=31

$$-\frac{1}{2} \sin^{-1}(\cos(x) - \sin(x)) - \frac{1}{2} \log \left(\cos(x) + \sin(x) + \sqrt{\sin(2x)} \right)$$

[Out] -1/2*arcsin(cos(x)-sin(x))-1/2*ln(cos(x)+sin(x)+sin(2*x)^(1/2))

Rubi [A]

time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4391}

$$-\frac{1}{2} \sin^{-1}(\cos(x) - \sin(x)) - \frac{1}{2} \log \left(\sin(x) + \sqrt{\sin(2x)} + \cos(x) \right)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/Sqrt[Sin[2*x]],x]

[Out] -1/2*ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2

Rule 4391

Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2]

Rubi steps

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}} dx = -\frac{1}{2} \sin^{-1}(\cos(x) - \sin(x)) - \frac{1}{2} \log \left(\cos(x) + \sin(x) + \sqrt{\sin(2x)} \right)$$

Mathematica [A]

time = 0.03, size = 31, normalized size = 1.00

$$\frac{1}{2} \left(-\sin^{-1}(\cos(x) - \sin(x)) - \log \left(\cos(x) + \sin(x) + \sqrt{\sin(2x)} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/Sqrt[Sin[2*x]],x]

[Out] $(-\text{ArcSin}[\text{Cos}[x] - \text{Sin}[x]] - \text{Log}[\text{Cos}[x] + \text{Sin}[x] + \text{Sqrt}[\text{Sin}[2*x]]])/2$

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[Sin[x]/Sqrt[Sin[2*x]],x]')`

[Out] Timed out

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.15, size = 266, normalized size = 8.58

method	result
default	$-\frac{\sqrt{-\frac{\tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1}} (\tan^2(\frac{x}{2})-1) \left(2\sqrt{1+\tan(\frac{x}{2})} \sqrt{-2\tan(\frac{x}{2})+2} \sqrt{-\tan(\frac{x}{2})} \text{EllipticE}\left(\sqrt{1+\tan(\frac{x}{2})}\right) \right)}{1}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$-1/2*(-\tan(1/2*x)/(\tan(1/2*x)^2-1))^(1/2)*(\tan(1/2*x)^2-1)*(2*(1+\tan(1/2*x))^(1/2)*(-2*\tan(1/2*x)+2)^(1/2)*(-\tan(1/2*x))^(1/2)*\text{EllipticE}((1+\tan(1/2*x))^(1/2),1/2*2^(1/2))*\tan(1/2*x)^2-(1+\tan(1/2*x))^(1/2)*(-2*\tan(1/2*x)+2)^(1/2)*(-\tan(1/2*x))^(1/2)*\text{EllipticF}((1+\tan(1/2*x))^(1/2),1/2*2^(1/2))*\tan(1/2*x)^2+2*(1+\tan(1/2*x))^(1/2)*(-2*\tan(1/2*x)+2)^(1/2)*(-\tan(1/2*x))^(1/2)*\text{EllipticE}((1+\tan(1/2*x))^(1/2),1/2*2^(1/2))-(1+\tan(1/2*x))^(1/2)*(-2*\tan(1/2*x)+2)^(1/2)*(-\tan(1/2*x))^(1/2)*\text{EllipticF}((1+\tan(1/2*x))^(1/2),1/2*2^(1/2))+2*\tan(1/2*x)^4-2*\tan(1/2*x)^2)/(\tan(1/2*x)*(\tan(1/2*x)^2-1))^(1/2)/(1+\tan(1/2*x)^2)/(\tan(1/2*x)^3-\tan(1/2*x))^(1/2)$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/sin(2*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sin(x)/sqrt(sin(2*x)), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(25) = 50$.

time = 0.32, size = 137, normalized size = 4.42

$$\frac{1}{4} \arctan\left(\frac{-\sqrt{2} \sqrt{\cos(x)\sin(x)}(\cos(x) - \sin(x)) + \cos(x)\sin(x)}{\cos(x)^2 + 2\cos(x)\sin(x) - 1}\right) - \frac{1}{4} \arctan\left(\frac{2\sqrt{2} \sqrt{\cos(x)\sin(x)} - \cos(x) - \sin(x)}{\cos(x) - \sin(x)}\right) + \frac{1}{8} \log\left(\frac{-32\cos(x)^4 + 4\sqrt{2}(4\cos(x)^3 - (4\cos(x)^2 + 1)\sin(x) - 5\cos(x))\sqrt{\cos(x)\sin(x)} + 32\cos(x)^2 + 16\cos(x)\sin(x) + 1}{1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/sin(2*x)^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{4} \arctan\left(\frac{-\sqrt{2} \sqrt{\cos(x) \sin(x)} (\cos(x) - \sin(x)) + \cos(x) \sin(x)}{\cos(x)^2 + 2 \cos(x) \sin(x) - 1}\right) - \frac{1}{4} \arctan\left(\frac{-(2 \sqrt{2} \sqrt{\cos(x) \sin(x)} - \cos(x) - \sin(x))}{\cos(x) - \sin(x)}\right) + \frac{1}{8} \log(-32 \cos(x)^4 + 4 \sqrt{2} (4 \cos(x)^3 - (4 \cos(x)^2 + 1) \sin(x) - 5 \cos(x)) \sqrt{\cos(x) \sin(x)} + 32 \cos(x)^2 + 16 \cos(x) \sin(x) + 1)$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/sin(2*x)**(1/2),x)

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/sin(2*x)^(1/2),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/sin(2*x)^(1/2),x)

[Out] int(sin(x)/sin(2*x)^(1/2), x)

$$3.403 \quad \int \frac{\cos(x)}{\sqrt{\sin(2x)}} dx$$

Optimal. Leaf size=31

$$-\frac{1}{2} \sin^{-1}(\cos(x) - \sin(x)) + \frac{1}{2} \log \left(\cos(x) + \sin(x) + \sqrt{\sin(2x)} \right)$$

[Out] $-1/2*\arcsin(\cos(x)-\sin(x))+1/2*\ln(\cos(x)+\sin(x)+\sin(2*x)^{(1/2)})$

Rubi [A]

time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4390}

$$\frac{1}{2} \log \left(\sin(x) + \sqrt{\sin(2x)} + \cos(x) \right) - \frac{1}{2} \sin^{-1}(\cos(x) - \sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/Sqrt[Sin[2*x]],x]

[Out] $-1/2*\text{ArcSin}[\text{Cos}[x] - \text{Sin}[x]] + \text{Log}[\text{Cos}[x] + \text{Sin}[x] + \text{Sqrt}[\text{Sin}[2*x]]]/2$

Rule 4390

Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2]

Rubi steps

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}} dx = -\frac{1}{2} \sin^{-1}(\cos(x) - \sin(x)) + \frac{1}{2} \log \left(\cos(x) + \sin(x) + \sqrt{\sin(2x)} \right)$$

Mathematica [A]

time = 0.02, size = 29, normalized size = 0.94

$$\frac{1}{2} \left(-\sin^{-1}(\cos(x) - \sin(x)) + \log \left(\cos(x) + \sin(x) + \sqrt{\sin(2x)} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/Sqrt[Sin[2*x]],x]

[Out] $(-\text{ArcSin}[\text{Cos}[x] - \text{Sin}[x]] + \text{Log}[\text{Cos}[x] + \text{Sin}[x] + \text{Sqrt}[\text{Sin}[2*x]]])/2$

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[Cos[x]/Sqrt[Sin[2*x]],x]')`

[Out] Timed out

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.11, size = 98, normalized size = 3.16

method	result
default	$\frac{\sqrt{-\frac{\tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1}} (\tan^2(\frac{x}{2})-1) \sqrt{1+\tan(\frac{x}{2})} \sqrt{-2\tan(\frac{x}{2})+2} \sqrt{-\tan(\frac{x}{2})} \text{EllipticF}\left(\sqrt{1+\tan(\frac{x}{2})}, \frac{\sqrt{\tan(\frac{x}{2})}}{\sqrt{1+\tan(\frac{x}{2})}}\right)}{\sqrt{\tan(\frac{x}{2})} (\tan^2(\frac{x}{2})-1) \sqrt{\tan^3(\frac{x}{2})-\tan(\frac{x}{2})}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $(-\tan(1/2*x)/(\tan(1/2*x)^2-1))^{1/2} * (\tan(1/2*x)^2-1)/(\tan(1/2*x) * (\tan(1/2*x)^2-1))^{1/2} * (1+\tan(1/2*x))^{1/2} * (-2*\tan(1/2*x)+2)^{1/2} * (-\tan(1/2*x))^{1/2} / (\tan(1/2*x)^3-\tan(1/2*x))^{1/2} * \text{EllipticF}((1+\tan(1/2*x))^{1/2}, 1/2*2^{1/2})$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(cos(x)/sqrt(sin(2*x)), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(25) = 50.

time = 0.34, size = 137, normalized size = 4.42

$$\frac{1}{4} \arctan\left(\frac{-\sqrt{2} \sqrt{\cos(x) \sin(x)} (\cos(x) - \sin(x)) + \cos(x) \sin(x)}{\cos(x)^2 + 2 \cos(x) \sin(x) - 1}\right) - \frac{1}{4} \arctan\left(\frac{-2\sqrt{2} \sqrt{\cos(x) \sin(x)} - \cos(x) - \sin(x)}{\cos(x) - \sin(x)}\right) - \frac{1}{8} \log\left(\frac{-32 \cos(x)^4 + 4\sqrt{2}(4 \cos(x)^3 - (4 \cos(x)^2 + 1) \sin(x) - 5 \cos(x)) \sqrt{\cos(x) \sin(x)} + 32 \cos(x)^2 + 16 \cos(x) \sin(x) + 1}{\cos(x) - \sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{4}\arctan\left(\frac{-\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x) - \sin(x)) + \cos(x)\sin(x)}{\cos(x)^2 + 2\cos(x)\sin(x) - 1}\right) - \frac{1}{4}\arctan\left(\frac{-(2\sqrt{2}\sqrt{\cos(x)\sin(x)} - \cos(x) - \sin(x))}{\cos(x) - \sin(x)}\right) - \frac{1}{8}\log(-32\cos(x)^4 + 4\sqrt{2}(2)(4\cos(x)^3 - (4\cos(x)^2 + 1)\sin(x) - 5\cos(x))\sqrt{\cos(x)\sin(x)} + 32\cos(x)^2 + 16\cos(x)\sin(x) + 1)$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/sin(2*x)**(1/2),x)

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/sin(2*x)^(1/2),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/sin(2*x)^(1/2),x)

[Out] int(cos(x)/sin(2*x)^(1/2), x)

3.404 $\int \sin(x) \sqrt{\sin(2x)} dx$

Optimal. Leaf size=45

$$-\frac{1}{4} \sin^{-1}(\cos(x) - \sin(x)) + \frac{1}{4} \log(\cos(x) + \sin(x) + \sqrt{\sin(2x)}) - \frac{1}{2} \cos(x) \sqrt{\sin(2x)}$$

[Out] -1/4*arcsin(cos(x)-sin(x))+1/4*ln(cos(x)+sin(x)+sin(2*x)^(1/2))-1/2*cos(x)*sin(2*x)^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4387, 4390}

$$-\frac{1}{4} \sin^{-1}(\cos(x) - \sin(x)) - \frac{1}{2} \sqrt{\sin(2x)} \cos(x) + \frac{1}{4} \log(\sin(x) + \sqrt{\sin(2x)} + \cos(x))$$

Antiderivative was successfully verified.

[In] Int[Sin[x]*Sqrt[Sin[2*x]],x]

[Out] -1/4*ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/4 - (Cos[x]*Sqrt[Sin[2*x]])/2

Rule 4387

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
  :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*
(g/(2*p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned} \int \sin(x) \sqrt{\sin(2x)} dx &= -\frac{1}{2} \cos(x) \sqrt{\sin(2x)} + \frac{1}{2} \int \frac{\cos(x)}{\sqrt{\sin(2x)}} dx \\ &= -\frac{1}{4} \sin^{-1}(\cos(x) - \sin(x)) + \frac{1}{4} \log(\cos(x) + \sin(x) + \sqrt{\sin(2x)}) - \frac{1}{2} \cos(x) \sqrt{\sin(2x)} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 41, normalized size = 0.91

$$\frac{1}{4} \left(-\sin^{-1}(\cos(x) - \sin(x)) + \log \left(\cos(x) + \sin(x) + \sqrt{\sin(2x)} \right) - 2 \cos(x) \sqrt{\sin(2x)} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[Sin[x]*Sqrt[Sin[2*x]],x]`

```
[Out] (-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]) - 2*Cos[x]
]*Sqrt[Sin[2*x]]/4
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Sqrt[Sin[2*x]]*Sin[x],x]')``[Out] Timed out`**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.10, size = 171, normalized size = 3.80

method	result
default	$\sqrt{-\frac{\tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1}} (\tan^2(\frac{x}{2})-1) \left(\sqrt{1+\tan(\frac{x}{2})} \sqrt{-2\tan(\frac{x}{2})+2} \sqrt{-\tan(\frac{x}{2})} \operatorname{EllipticF}\left(\sqrt{1+\tan(\frac{x}{2})}, \frac{x}{2}\right) - \sqrt{\tan(\frac{x}{2})} (\tan^2(\frac{x}{2}) - \dots \right)$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sin(x)*sin(2*x)^(1/2),x,method=_RETURNVERBOSE)`

```
[Out] (-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*((1+tan(1/2*x))^(1/2)
*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2)
,1/2*2^(1/2))*tan(1/2*x)^2+(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-t
an(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))+2*tan(1/2*x)^3
-2*tan(1/2*x))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)/(tan(1/2*x)^3-tan(1/2*x)
)^(1/2)/(1+tan(1/2*x)^2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(2*x)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(sin(2*x))*sin(x), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(35) = 70.

time = 0.33, size = 151, normalized size = 3.36

$$-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}\cos(x) + \frac{1}{8}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right) - \frac{1}{8}\arctan\left(\frac{-2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right) - \frac{1}{16}\log\left(\frac{-32\cos(x)^4+4\sqrt{2}(4\cos(x)^3-(4\cos(x)^2+1)\sin(x)-5\cos(x))\sqrt{\cos(x)\sin(x)}+32\cos(x)^2+16\cos(x)\sin(x)+1)}{\dots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(2*x)^(1/2),x, algorithm="fricas")

[Out] -1/2*sqrt(2)*sqrt(cos(x)*sin(x))*cos(x) + 1/8*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x))/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/8*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) - 1/16*log(-32*cos(x)^4 + 4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1)

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(2*x)**(1/2),x)

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(2*x)^(1/2),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sin(2x)} \sin(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)^(1/2)*sin(x),x)

[Out] int(sin(2*x)^(1/2)*sin(x), x)

3.405 $\int (\cos(x) - \sin(x)) \sqrt{\sin(2x)} dx$

Optimal. Leaf size=47

$$-\frac{1}{2} \log(\cos(x) + \sin(x) + \sqrt{\sin(2x)}) + \frac{1}{2} \cos(x) \sqrt{\sin(2x)} + \frac{1}{2} \sin(x) \sqrt{\sin(2x)}$$

[Out] $-1/2*\ln(\cos(x)+\sin(x)+\sin(2*x)^{(1/2)})+1/2*\cos(x)*\sin(2*x)^{(1/2)}+1/2*\sin(x)*\sin(2*x)^{(1/2)}$

Rubi [A]

time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {4486, 4386, 4391, 4387, 4390}

$$\frac{1}{2} \sin(x) \sqrt{\sin(2x)} + \frac{1}{2} \sqrt{\sin(2x)} \cos(x) - \frac{1}{2} \log(\sin(x) + \sqrt{\sin(2x)} + \cos(x))$$

Antiderivative was successfully verified.

[In] `Int[(Cos[x] - Sin[x])*Sqrt[Sin[2*x]],x]`

[Out] $-1/2*\text{Log}[\text{Cos}[x] + \text{Sin}[x] + \text{Sqrt}[\text{Sin}[2*x]]] + (\text{Cos}[x]*\text{Sqrt}[\text{Sin}[2*x]])/2 + (\text{Sin}[x]*\text{Sqrt}[\text{Sin}[2*x]])/2$

Rule 4386

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
:> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*(
g/(2*p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4387

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
:> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*(
g/(2*p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
a*d, 0] && EqQ[d/b, 2]
```

Rule 4391

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
a*d, 0] && EqQ[d/b, 2]
```

Rule 4486

```
Int[u_, x_Symbol] := With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
!InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned} \int (\cos(x) - \sin(x)) \sqrt{\sin(2x)} \, dx &= \int (\cos(x) \sqrt{\sin(2x)} - \sin(x) \sqrt{\sin(2x)}) \, dx \\ &= \int \cos(x) \sqrt{\sin(2x)} \, dx - \int \sin(x) \sqrt{\sin(2x)} \, dx \\ &= \frac{1}{2} \cos(x) \sqrt{\sin(2x)} + \frac{1}{2} \sin(x) \sqrt{\sin(2x)} - \frac{1}{2} \int \frac{\cos(x)}{\sqrt{\sin(2x)}} \, dx + \frac{1}{2} \int \frac{\sin(x)}{\sqrt{\sin(2x)}} \, dx \\ &= -\frac{1}{2} \log(\cos(x) + \sin(x) + \sqrt{\sin(2x)}) + \frac{1}{2} \cos(x) \sqrt{\sin(2x)} + \frac{1}{2} \sin(x) \sqrt{\sin(2x)} \end{aligned}$$

Mathematica [A]

time = 0.04, size = 43, normalized size = 0.91

$$\frac{1}{2} \left(-\log(\cos(x) + \sin(x) + \sqrt{\sin(2x)}) + \cos(x) \sqrt{\sin(2x)} + \sin(x) \sqrt{\sin(2x)} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] - Sin[x])*Sqrt[Sin[2*x]], x]
```

```
[Out] (-Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]] + Cos[x]*Sqrt[Sin[2*x]] + Sin[x]*Sqrt[Sin[2*x]])/2
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Cos[x] - Sin[x])*Sqrt[Sin[2*x]], x]')
```

[Out] Timed out

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.18, size = 443, normalized size = 9.43

method	result
default	$-\sqrt{-\frac{\tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1}} (\tan^2(\frac{x}{2})-1) \left(3\sqrt{(1+\tan(\frac{x}{2})) (\tan(\frac{x}{2})-1) \tan(\frac{x}{2})} \sqrt{1+\tan(\frac{x}{2})} \sqrt{-2\tan(\frac{x}{2})} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(x)-sin(x))*sin(2*x)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -(-\tan(1/2*x)/(\tan(1/2*x)^2-1))^{(1/2)} * (\tan(1/2*x)^2-1) * (3*((1+\tan(1/2*x)) * \\ & \tan(1/2*x)-1) * \tan(1/2*x))^{(1/2)} * (1+\tan(1/2*x))^{(1/2)} * (-2*\tan(1/2*x)+2)^{(1/2)} \\ & * (-\tan(1/2*x))^{(1/2)} * \text{EllipticF}((1+\tan(1/2*x))^{(1/2)}, 1/2*2^{(1/2)}) * \tan(1/2*x) \\ &)^2 - 4*((1+\tan(1/2*x)) * (\tan(1/2*x)-1) * \tan(1/2*x))^{(1/2)} * (1+\tan(1/2*x))^{(1/2)} \\ & * (-2*\tan(1/2*x)+2)^{(1/2)} * (-\tan(1/2*x))^{(1/2)} * \text{EllipticE}((1+\tan(1/2*x))^{(1/2)}, \\ & 1/2*2^{(1/2)}) * \tan(1/2*x)^2 + 3*(1+\tan(1/2*x))^{(1/2)} * (-2*\tan(1/2*x)+2)^{(1/2)} * \\ & (-\tan(1/2*x))^{(1/2)} * \text{EllipticF}((1+\tan(1/2*x))^{(1/2)}, 1/2*2^{(1/2)}) * ((1+\tan(1/2*x) \\ & x)) * (\tan(1/2*x)-1) * \tan(1/2*x))^{(1/2)} - 4*((1+\tan(1/2*x)) * (\tan(1/2*x)-1) * \tan(1 \\ & /2*x))^{(1/2)} * (1+\tan(1/2*x))^{(1/2)} * (-2*\tan(1/2*x)+2)^{(1/2)} * (-\tan(1/2*x))^{(1/2)} \\ & * \text{EllipticE}((1+\tan(1/2*x))^{(1/2)}, 1/2*2^{(1/2)}) - 4*(\tan(1/2*x)^3 - \tan(1/2*x))^{(1/2)} \\ & * \tan(1/2*x)^4 + 2*((1+\tan(1/2*x)) * (\tan(1/2*x)-1) * \tan(1/2*x))^{(1/2)} * \tan(1 \\ & /2*x)^3 - 4*(\tan(1/2*x)^3 - \tan(1/2*x))^{(1/2)} * \tan(1/2*x)^2 - 2*((1+\tan(1/2*x)) * (\tan \\ & an(1/2*x)-1) * \tan(1/2*x))^{(1/2)} * \tan(1/2*x)) / (\tan(1/2*x) * (\tan(1/2*x)^2-1))^{(1 \\ & /2)} / ((1+\tan(1/2*x)) * (\tan(1/2*x)-1) * \tan(1/2*x))^{(1/2)} / (\tan(1/2*x)^3 - \tan(1/2*x \\ & x))^{(1/2)} / (1+\tan(1/2*x)^2) \end{aligned}$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((cos(x)-sin(x))*sin(2*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate((cos(x) - sin(x))*sqrt(sin(2*x)), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(35) = 70.

time = 0.32, size = 76, normalized size = 1.62

$$\frac{1}{2} \sqrt{2} \sqrt{\cos(x) \sin(x)} (\cos(x) + \sin(x)) + \frac{1}{8} \log \left(-32 \cos(x)^4 + 4\sqrt{2} (4 \cos(x)^3 - (4 \cos(x)^2 + 1) \sin(x) - 5 \cos(x)) \sqrt{\cos(x) \sin(x)} + 32 \cos(x)^2 + 16 \cos(x) \sin(x) + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)-sin(x))*sin(2*x)^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x) + \sin(x)) + \frac{1}{8}\log(-32\cos(x)^4 + 4\sqrt{2}(4\cos(x)^3 - (4\cos(x)^2 + 1)\sin(x) - 5\cos(x))\sqrt{\cos(x)\sin(x)} + 32\cos(x)^2 + 16\cos(x)\sin(x) + 1)$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)-sin(x))*sin(2*x)**(1/2),x)

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)-sin(x))*sin(2*x)^(1/2),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sin(2x)} (\cos(x) - \sin(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)^(1/2)*(cos(x) - sin(x)),x)

[Out] int(sin(2*x)^(1/2)*(cos(x) - sin(x)), x)

$$3.406 \quad \int \frac{\sin^7(x)}{\sin^2(2x)} dx$$

Optimal. Leaf size=61

$$-\frac{1}{16} \sin^{-1}(\cos(x) - \sin(x)) + \frac{1}{16} \log\left(\cos(x) + \sin(x) + \sqrt{\sin(2x)}\right) + \frac{\sin^5(x)}{5 \sin^{5/2}(2x)} - \frac{\sin(x)}{4 \sqrt{\sin(2x)}}$$

[Out] -1/16*arcsin(cos(x)-sin(x))+1/16*ln(cos(x)+sin(x)+sin(2*x)^(1/2))+1/5*sin(x)^5/sin(2*x)^(5/2)-1/4*sin(x)/sin(2*x)^(1/2)

Rubi [A]

time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4379, 4393, 4390}

$$\frac{\sin^5(x)}{5 \sin^{5/2}(2x)} - \frac{\sin(x)}{4 \sqrt{\sin(2x)}} - \frac{1}{16} \sin^{-1}(\cos(x) - \sin(x)) + \frac{1}{16} \log\left(\sin(x) + \sqrt{\sin(2x)} + \cos(x)\right)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^7/Sin[2*x]^(7/2),x]

[Out] -1/16*ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/16 + Sin[x]^5/(5*Sin[2*x]^(5/2)) - Sin[x]/(4*Sqrt[Sin[2*x]])

Rule 4379

Int[((e_)*sin[(a_.) + (b_.)*(x_)]))^(m_)*((g_)*sin[(c_.) + (d_.)*(x_)]))^(p_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Dist[e^4*((m + p - 1)/(4*g^2*(p + 1))), Int[(e*Sin[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && GtQ[m, 2] && LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]

Rule 4390

Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2]

Rule 4393

Int[((g_)*sin[(c_.) + (d_.)*(x_)]))^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] &

& IntegerQ[2*p]

Rubi steps

$$\begin{aligned}
 \int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)} dx &= \frac{\sin^5(x)}{5 \sin^{\frac{5}{2}}(2x)} - \frac{1}{4} \int \frac{\sin^3(x)}{\sin^{\frac{3}{2}}(2x)} dx \\
 &= \frac{\sin^5(x)}{5 \sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4 \sqrt{\sin(2x)}} + \frac{1}{16} \int \csc(x) \sqrt{\sin(2x)} dx \\
 &= \frac{\sin^5(x)}{5 \sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4 \sqrt{\sin(2x)}} + \frac{1}{8} \int \frac{\cos(x)}{\sqrt{\sin(2x)}} dx \\
 &= -\frac{1}{16} \sin^{-1}(\cos(x) - \sin(x)) + \frac{1}{16} \log\left(\cos(x) + \sin(x) + \sqrt{\sin(2x)}\right) + \frac{\sin^5(x)}{5 \sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4 \sqrt{\sin(2x)}}
 \end{aligned}$$

Mathematica [A]

time = 0.06, size = 50, normalized size = 0.82

$$\frac{1}{80} \left(5 \left(-\sin^{-1}(\cos(x) - \sin(x)) + \log\left(\cos(x) + \sin(x) + \sqrt{\sin(2x)}\right) \right) + 2 \sec(x) (-6 + \sec^2(x)) \sqrt{\sin(2x)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^7/Sin[2*x]^(7/2),x]

[Out] (5*(-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]) + 2*Sec[x]*(-6 + Sec[x]^2)*Sqrt[Sin[2*x]])/80

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sin[x]^7/Sin[2*x]^(7/2),x]')

[Out] Timed out

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.14, size = 510, normalized size = 8.36

method	result
--------	--------

default	$\sqrt{-\frac{\tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1}} (\tan^2(\frac{x}{2})-1) \left(5\sqrt{1+\tan(\frac{x}{2})} \sqrt{-2\tan(\frac{x}{2})+2} \sqrt{-\tan(\frac{x}{2})} \operatorname{EllipticF}\left(\sqrt{1+\tan(\frac{x}{2})}\right) \right)$
---------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)^7/sin(2*x)^(7/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{2688}(-\tan(1/2*x)/(\tan(1/2*x)^2-1))^{1/2}(\tan(1/2*x)^2-1)^{1/2}(5(1+\tan(1/2*x))^{1/2}(-2*\tan(1/2*x)+2)^{1/2}(-\tan(1/2*x))^{1/2}\operatorname{EllipticF}((1+\tan(1/2*x))^{1/2},1/2*2^{1/2})*\tan(1/2*x)^{14}+35(1+\tan(1/2*x))^{1/2}(-2*\tan(1/2*x)+2)^{1/2}(-\tan(1/2*x))^{1/2}\operatorname{EllipticF}((1+\tan(1/2*x))^{1/2},1/2*2^{1/2})*\tan(1/2*x)^{12}+10*\tan(1/2*x)^{15}+105(1+\tan(1/2*x))^{1/2}(-2*\tan(1/2*x)+2)^{1/2}(-\tan(1/2*x))^{1/2}\operatorname{EllipticF}((1+\tan(1/2*x))^{1/2},1/2*2^{1/2})*\tan(1/2*x)^{10}+66*\tan(1/2*x)^{13}+175(1+\tan(1/2*x))^{1/2}(-2*\tan(1/2*x)+2)^{1/2}(-\tan(1/2*x))^{1/2}\operatorname{EllipticF}((1+\tan(1/2*x))^{1/2},1/2*2^{1/2})*\tan(1/2*x)^8-1014*\tan(1/2*x)^{11}+175(1+\tan(1/2*x))^{1/2}(-2*\tan(1/2*x)+2)^{1/2}(-\tan(1/2*x))^{1/2}\operatorname{EllipticF}((1+\tan(1/2*x))^{1/2},1/2*2^{1/2})*\tan(1/2*x)^6+2002*\tan(1/2*x)^9+105(1+\tan(1/2*x))^{1/2}(-2*\tan(1/2*x)+2)^{1/2}(-\tan(1/2*x))^{1/2}\operatorname{EllipticF}((1+\tan(1/2*x))^{1/2},1/2*2^{1/2})*\tan(1/2*x)^4-2002*\tan(1/2*x)^7+35(1+\tan(1/2*x))^{1/2}(-2*\tan(1/2*x)+2)^{1/2}(-\tan(1/2*x))^{1/2}\operatorname{EllipticF}((1+\tan(1/2*x))^{1/2},1/2*2^{1/2})*\tan(1/2*x)^2+1014*\tan(1/2*x)^5+5(1+\tan(1/2*x))^{1/2}(-2*\tan(1/2*x)+2)^{1/2}(-\tan(1/2*x))^{1/2}\operatorname{EllipticF}((1+\tan(1/2*x))^{1/2},1/2*2^{1/2})-66*\tan(1/2*x)^3-10*\tan(1/2*x))/(\tan(1/2*x)*(\tan(1/2*x)^2-1))^{1/2}/(1+\tan(1/2*x)^2)^{7/2}/(\tan(1/2*x)^3-\tan(1/2*x))^{1/2}$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="maxima")`

[Out] `integrate(sin(x)^7/sin(2*x)^(7/2), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(47) = 94.

time = 0.33, size = 181, normalized size = 2.97

$$\frac{10 \arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)}\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)}\right) \cos(x)^3 - 10 \arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)}\sin(x)}{\cos(x)^2-2\cos(x)\sin(x)}\right) \cos(x)^3 - 5 \cos(x)^3 \log\left(-32 \cos(x)^4 + 4\sqrt{2}(4 \cos(x)^2 - (4 \cos(x)^2 + 1) \sin(x) - 5 \cos(x))\sqrt{\cos(x)\sin(x)} + 32 \cos(x)^2 + 16 \cos(x)\sin(x) + 1\right) - 48 \cos(x)^3 - 8\sqrt{2}(6 \cos(x)^2 - 1)\sqrt{\cos(x)\sin(x)}}{320 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="fricas")

[Out] $\frac{1}{320} \cdot (10 \cdot \arctan(-\sqrt{2} \cdot \sqrt{\cos(x) \sin(x)}) \cdot (\cos(x) - \sin(x)) + \cos(x) \sin(x)) / (\cos(x)^2 + 2 \cos(x) \sin(x) - 1)) \cdot \cos(x)^3 - 10 \cdot \arctan(-(2 \sqrt{2}) \cdot \sqrt{\cos(x) \sin(x)} - \cos(x) - \sin(x)) / (\cos(x) - \sin(x))) \cdot \cos(x)^3 - 5 \cos(x)^3 \log(-32 \cos(x)^4 + 4 \sqrt{2} (4 \cos(x)^3 - (4 \cos(x)^2 + 1) \sin(x) - 5 \cos(x)) \sqrt{\cos(x) \sin(x)} + 32 \cos(x)^2 + 16 \cos(x) \sin(x) + 1) - 48 \cos(x)^3 - 8 \sqrt{2} (6 \cos(x)^2 - 1) \sqrt{\cos(x) \sin(x)}) / \cos(x)^3$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**7/sin(2*x)**(7/2),x)

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^7/sin(2*x)^(7/2),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(x)^7}{\sin(2x)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^7/sin(2*x)^(7/2),x)

[Out] int(sin(x)^7/sin(2*x)^(7/2), x)

$$3.407 \quad \int \frac{\cos^7(x)}{\sin^2(2x)} dx$$

Optimal. Leaf size=61

$$-\frac{1}{16} \sin^{-1}(\cos(x) - \sin(x)) - \frac{1}{16} \log\left(\cos(x) + \sin(x) + \sqrt{\sin(2x)}\right) - \frac{\cos^5(x)}{5 \sin^{5/2}(2x)} + \frac{\cos(x)}{4 \sqrt{\sin(2x)}}$$

[Out] $-1/16*\arcsin(\cos(x)-\sin(x))-1/16*\ln(\cos(x)+\sin(x)+\sin(2*x)^{(1/2)})-1/5*\cos(x)^5/\sin(2*x)^{(5/2)}+1/4*\cos(x)/\sin(2*x)^{(1/2)}$

Rubi [A]

time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4378, 4392, 4391}

$$-\frac{\cos^5(x)}{5 \sin^{5/2}(2x)} - \frac{1}{16} \sin^{-1}(\cos(x) - \sin(x)) + \frac{\cos(x)}{4 \sqrt{\sin(2x)}} - \frac{1}{16} \log\left(\sin(x) + \sqrt{\sin(2x)} + \cos(x)\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[x]^7/\text{Sin}[2*x]^{(7/2)}, x]$

[Out] $-1/16*\text{ArcSin}[\text{Cos}[x] - \text{Sin}[x]] - \text{Log}[\text{Cos}[x] + \text{Sin}[x] + \text{Sqrt}[\text{Sin}[2*x]]]/16 - \text{Cos}[x]^5/(5*\text{Sin}[2*x]^{(5/2)}) + \text{Cos}[x]/(4*\text{Sqrt}[\text{Sin}[2*x]])$

Rule 4378

$\text{Int}[(\cos[(a_.) + (b_.)*(x_.)]*(e_.))^{(m_.)}*((g_.)*\sin[(c_.) + (d_.)*(x_.)])^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[e^{2*(e*\cos[a + b*x])^{(m-2)}}*((g*\sin[c + d*x])^{(p+1)})/(2*b*g*(p+1)), x] + \text{Dist}[e^{4*((m+p-1)/(4*g^2*(p+1))}, \text{Int}[(e*\cos[a + b*x])^{(m-4)}*(g*\sin[c + d*x])^{(p+2)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, g\}, x] \&\& \text{EqQ}[b*c - a*d, 0] \&\& \text{EqQ}[d/b, 2] \&\& !\text{IntegerQ}[p] \&\& \text{GtQ}[m, 2] \&\& \text{LtQ}[p, -1] \&\& (\text{GtQ}[m, 3] || \text{EqQ}[p, -3/2]) \&\& \text{IntegersQ}[2*m, 2*p]$

Rule 4391

$\text{Int}[\sin[(a_.) + (b_.)*(x_.)]/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[-\text{ArcSin}[\cos[a + b*x] - \sin[a + b*x]]/d, x] - \text{Simp}[\text{Log}[\cos[a + b*x] + \sin[a + b*x] + \text{Sqrt}[\sin[c + d*x]]]/d, x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{EqQ}[b*c - a*d, 0] \&\& \text{EqQ}[d/b, 2]$

Rule 4392

$\text{Int}[(g_.)*\sin[(c_.) + (d_.)*(x_.)]^{(p_.)}/\cos[(a_.) + (b_.)*(x_.)], x_Symbol] \rightarrow \text{Dist}[2*g, \text{Int}[\sin[a + b*x]*(g*\sin[c + d*x])^{(p-1)}, x], x] /; \text{FreeQ}\{a, b, c, d, g, p\}, x] \&\& \text{EqQ}[b*c - a*d, 0] \&\& \text{EqQ}[d/b, 2] \&\& !\text{IntegerQ}[p] \&$

& IntegerQ[2*p]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)} dx &= -\frac{\cos^5(x)}{5 \sin^{\frac{5}{2}}(2x)} - \frac{1}{4} \int \frac{\cos^3(x)}{\sin^{\frac{3}{2}}(2x)} dx \\
 &= -\frac{\cos^5(x)}{5 \sin^{\frac{5}{2}}(2x)} + \frac{\cos(x)}{4 \sqrt{\sin(2x)}} + \frac{1}{16} \int \sec(x) \sqrt{\sin(2x)} dx \\
 &= -\frac{\cos^5(x)}{5 \sin^{\frac{5}{2}}(2x)} + \frac{\cos(x)}{4 \sqrt{\sin(2x)}} + \frac{1}{8} \int \frac{\sin(x)}{\sqrt{\sin(2x)}} dx \\
 &= -\frac{1}{16} \sin^{-1}(\cos(x) - \sin(x)) - \frac{1}{16} \log\left(\cos(x) + \sin(x) + \sqrt{\sin(2x)}\right) - \frac{\cos^5(x)}{5 \sin^{\frac{5}{2}}(2x)} + \frac{\cos(x)}{4 \sqrt{\sin(2x)}}
 \end{aligned}$$

Mathematica [A]

time = 0.05, size = 56, normalized size = 0.92

$$\frac{1}{16} \left(-\sin^{-1}(\cos(x) - \sin(x)) - \log\left(\cos(x) + \sin(x) + \sqrt{\sin(2x)}\right) \right) + \left(\frac{3 \csc(x)}{20} - \frac{\csc^3(x)}{40} \right) \sqrt{\sin(2x)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^7/Sin[2*x]^(7/2),x]

[Out] (-ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/16 + ((3 *Csc[x])/20 - Csc[x]^3/40)*Sqrt[Sin[2*x]]

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Cos[x]^7/Sin[2*x]^(7/2),x]')

[Out] Timed out

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.16, size = 1108, normalized size = 18.16

method	result	size
default	Expression too large to display	1108

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="maxima")

[Out] integrate(cos(x)^7/sin(2*x)^(7/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(47) = 94.

time = 0.33, size = 205, normalized size = 3.36

$$\frac{10(\cos(x)^2 - 1)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(2x)\sin(x)}}{\cos(x) - \sin(x)}\right)\sin(x) - 10(\cos(x)^2 - 1)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(2x)\sin(x)}}{\cos(x) - \sin(x)}\right)\sin(x) + 5(\cos(x)^2 - 1)\log\left(\frac{-32\cos(x)^4 + 4\sqrt{2}(4\cos(x)^2 - (4\cos(x)^2 + 1)\sin(x) - 5\cos(x))\sqrt{\cos(2x)\sin(x)} + 32\cos(x)^2 + 16\cos(x)\sin(x) + 1}{320(\cos(x)^2 - 1)\sin(x)}\right)\sin(x) + 8\sqrt{2}(6\cos(x)^2 - 5)\sqrt{\cos(2x)\sin(x)} + 48(\cos(x)^2 - 1)\sin(x)}{320(\cos(x)^2 - 1)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="fricas")

[Out] $\frac{1}{320} \cdot (10 \cdot (\cos(x)^2 - 1) \cdot \arctan(-\sqrt{2} \cdot \sqrt{\cos(x) \cdot \sin(x)}) \cdot (\cos(x) - \sin(x)) + \cos(x) \cdot \sin(x)) / (\cos(x)^2 + 2 \cdot \cos(x) \cdot \sin(x) - 1)) \cdot \sin(x) - 10 \cdot (\cos(x)^2 - 1) \cdot \arctan(-(2 \cdot \sqrt{2}) \cdot \sqrt{\cos(x) \cdot \sin(x)} - \cos(x) - \sin(x)) / (\cos(x) - \sin(x))) \cdot \sin(x) + 5 \cdot (\cos(x)^2 - 1) \cdot \log(-32 \cdot \cos(x)^4 + 4 \cdot \sqrt{2} \cdot (4 \cdot \cos(x)^2 - (4 \cdot \cos(x)^2 + 1) \cdot \sin(x) - 5 \cdot \cos(x)) \cdot \sqrt{\cos(x) \cdot \sin(x)} + 32 \cdot \cos(x)^2 + 16 \cdot \cos(x) \cdot \sin(x) + 1) \cdot \sin(x) + 8 \cdot \sqrt{2} \cdot (6 \cdot \cos(x)^2 - 5) \cdot \sqrt{\cos(x) \cdot \sin(x)} + 48 \cdot (\cos(x)^2 - 1) \cdot \sin(x)) / ((\cos(x)^2 - 1) \cdot \sin(x))$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**7/sin(2*x)**(7/2),x)

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^7/sin(2*x)^(7/2),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(x)^7}{\sin(2x)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/sin(2*x)^(7/2),x)
```

```
[Out] int(cos(x)^7/sin(2*x)^(7/2), x)
```

3.408 $\int \csc^5(x) \sin^{\frac{3}{2}}(2x) dx$

Optimal. Leaf size=16

$$-\frac{1}{5} \csc^5(x) \sin^{\frac{5}{2}}(2x)$$

[Out] -1/5*csc(x)^5*sin(2*x)^(5/2)

Rubi [A]

time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4377}

$$-\frac{1}{5} \sin^{\frac{5}{2}}(2x) \csc^5(x)$$

Antiderivative was successfully verified.

[In] Int[Csc[x]^5*Sin[2*x]^(3/2),x]

[Out] -1/5*(Csc[x]^5*Sin[2*x]^(5/2))

Rule 4377

Int[((e_.)*sin[(a_.) + (b_.)*(x_.)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_.)])^(p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]

Rubi steps

$$\int \csc^5(x) \sin^{\frac{3}{2}}(2x) dx = -\frac{1}{5} \csc^5(x) \sin^{\frac{5}{2}}(2x)$$

Mathematica [A]

time = 0.02, size = 16, normalized size = 1.00

$$-\frac{1}{5} \csc^5(x) \sin^{\frac{5}{2}}(2x)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^5*Sin[2*x]^(3/2),x]

[Out] -1/5*(Csc[x]^5*Sin[2*x]^(5/2))

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Sin[2*x]^(3/2)/Sin[x]^5,x]')``[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep`**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.16, size = 508, normalized size = 31.75

method	result
default	$\sqrt{-\frac{\tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1}} \left(96 \sqrt{1 + \tan(\frac{x}{2})} \sqrt{-2 \tan(\frac{x}{2}) + 2} \sqrt{-\tan(\frac{x}{2})} \operatorname{EllipticE}\left(\sqrt{1 + \tan(\frac{x}{2})}, \frac{\sqrt{2}}{2}\right) \sqrt{\dots} \right)$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sin(2*x)^(3/2)/sin(x)^5,x,method=_RETURNVERBOSE)`

```
[Out] 1/5*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(96*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^2-48*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^2-((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^6+28*tan(1/2*x)^4*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)+40*tan(1/2*x)^4*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)+tan(1/2*x)^4*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)-28*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*((1+tan(1/2*x))*((tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^2+((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^2-(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="maxima")

[Out] integrate(sin(2*x)^(3/2)/sin(x)^5, x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(12) = 24.

time = 0.47, size = 39, normalized size = 2.44

$$\frac{4 \left(\sqrt{2} \sqrt{\cos(x) \sin(x)} \cos(x)^2 + (\cos(x)^2 - 1) \sin(x) \right)}{5 (\cos(x)^2 - 1) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="fricas")

[Out] 4/5*(sqrt(2)*sqrt(cos(x)*sin(x))*cos(x)^2 + (cos(x)^2 - 1)*sin(x))/((cos(x)^2 - 1)*sin(x))

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)**(3/2)/sin(x)**5,x)

[Out] Exception raised: SystemError

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)^(3/2)/sin(x)^5,x)

[Out] Could not integrate

Mupad [B]

time = 0.56, size = 18, normalized size = 1.12

$$\frac{4 \sqrt{\sin(2x)} (\sin(x)^2 - 1)}{5 \sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)^(3/2)/sin(x)^5,x)

[Out] (4*sin(2*x)^(1/2)*(sin(x)^2 - 1))/(5*sin(x)^3)

$$3.409 \quad \int \frac{\sec^3(x)}{\sqrt{\sin(2x)}} dx$$

Optimal. Leaf size=31

$$\frac{4}{5} \sec(x) \sqrt{\sin(2x)} + \frac{1}{5} \sec^3(x) \sqrt{\sin(2x)}$$

[Out] 4/5*sec(x)*sin(2*x)^(1/2)+1/5*sec(x)^3*sin(2*x)^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4384, 4376}

$$\frac{1}{5} \sqrt{\sin(2x)} \sec^3(x) + \frac{4}{5} \sqrt{\sin(2x)} \sec(x)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^3/Sqrt[Sin[2*x]],x]

[Out] (4*Sec[x]*Sqrt[Sin[2*x]])/5 + (Sec[x]^3*Sqrt[Sin[2*x]])/5

Rule 4376

Int[(cos[(a_.) + (b_.)*(x_.)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_.)])^(p_.), x_Symbol] :> Simp[(-e*cos[a + b*x])^m*((g*sin[c + d*x])^(p + 1)/(b*g*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]

Rule 4384

Int[(cos[(a_.) + (b_.)*(x_.)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_.)])^(p_.), x_Symbol] :> Simp[(-e*cos[a + b*x])^m*((g*sin[c + d*x])^(p + 1)/(2*b*g*(m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*cos[a + b*x])^(m + 2)*(g*sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && LtQ[m, -1] && NeQ[m + 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegerQ[2*m, 2*p]

Rubi steps

$$\begin{aligned} \int \frac{\sec^3(x)}{\sqrt{\sin(2x)}} dx &= \frac{1}{5} \sec^3(x) \sqrt{\sin(2x)} + \frac{4}{5} \int \frac{\sec(x)}{\sqrt{\sin(2x)}} dx \\ &= \frac{4}{5} \sec(x) \sqrt{\sin(2x)} + \frac{1}{5} \sec^3(x) \sqrt{\sin(2x)} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 20, normalized size = 0.65

$$\frac{1}{5} \sec(x) (4 + \sec^2(x)) \sqrt{\sin(2x)}$$

Antiderivative was successfully verified.

`[In] Integrate[Sec[x]^3/Sqrt[Sin[2*x]],x]``[Out] (Sec[x]*(4 + Sec[x]^2)*Sqrt[Sin[2*x]])/5`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(Cos[x]^3*Sqrt[Sin[2*x]]),x]')``[Out] Exception raised: SystemError >> excessive stack use: stack is 3881 deep`**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.12, size = 286, normalized size = 9.23

method	result
default	$\frac{\sqrt{-\frac{\tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1}} (\tan^2(\frac{x}{2})-1) \left(5 \sqrt{1 + \tan(\frac{x}{2})} \sqrt{-2 \tan(\frac{x}{2}) + 2} \sqrt{-\tan(\frac{x}{2})} \operatorname{EllipticF}\left(\sqrt{1 + \tan(\frac{x}{2})}, \right. \right.$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/cos(x)^3/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)`

```
[Out] 1/12*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(5*(1+tan(1/2*x))
)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x)
)^(1/2),1/2*2^(1/2))*tan(1/2*x)^6+15*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)
^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(
1/2*x)^4-14*tan(1/2*x)^7+15*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-
tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^2+
2*tan(1/2*x)^5+5*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x)
)^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))-2*tan(1/2*x)^3+14*tan(1/
2*x)/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)/(1+tan(1/2*x)^2)^3/(tan(1/2*x)^3-
tan(1/2*x))^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/cos(x)^3/sin(2*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(cos(x)^3*sqrt(sin(2*x))), x)`

Fricas [A]

time = 0.32, size = 32, normalized size = 1.03

$$\frac{4 \cos(x)^3 + \sqrt{2} (4 \cos(x)^2 + 1) \sqrt{\cos(x) \sin(x)}}{5 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/cos(x)^3/sin(2*x)^(1/2),x, algorithm="fricas")`

[Out] `1/5*(4*cos(x)^3 + sqrt(2)*(4*cos(x)^2 + 1)*sqrt(cos(x)*sin(x)))/cos(x)^3`

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/cos(x)**3/sin(2*x)**(1/2),x)`

[Out] Exception raised: SystemError

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/cos(x)^3/sin(2*x)^(1/2),x)`

[Out] Could not integrate

Mupad [B]

time = 0.39, size = 20, normalized size = 0.65

$$\frac{\sqrt{\sin(2x)} (2 \cos(2x) + 3)}{5 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(sin(2*x)^(1/2)*cos(x)^3),x)`

[Out] `(sin(2*x)^(1/2)*(2*cos(2*x) + 3))/(5*cos(x)^3)`

$$3.410 \quad \int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)} dx$$

Optimal. Leaf size=29

$$-\frac{2 \cos(x)}{3 \sin^{\frac{3}{2}}(2x)} + \frac{4 \sin(x)}{3 \sqrt{\sin(2x)}}$$

[Out] $-2/3*\cos(x)/\sin(2*x)^{(3/2)}+4/3*\sin(x)/\sin(2*x)^{(1/2)}$

Rubi [A]

time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4393, 4388, 4377}

$$\frac{4 \sin(x)}{3 \sqrt{\sin(2x)}} - \frac{2 \cos(x)}{3 \sin^{\frac{3}{2}}(2x)}$$

Antiderivative was successfully verified.

[In] Int[Csc[x]/Sin[2*x]^(3/2),x]

[Out] $(-2*\cos[x])/(3*\sin[2*x]^(3/2)) + (4*\sin[x])/(3*\text{Sqrt}[\sin[2*x]])$

Rule 4377

Int[((e_.)*sin[(a_.) + (b_.)*(x_.)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_.)])^(p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]

Rule 4388

Int[cos[(a_.) + (b_.)*(x_.)]*((g_.)*sin[(c_.) + (d_.)*(x_.)])^(p_), x_Symbol] :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Dist[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]

Rule 4393

Int[((g_.)*sin[(c_.) + (d_.)*(x_.)])^(p_)/sin[(a_.) + (b_.)*(x_.)], x_Symbol] :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && IntegerQ[2*p]

Rubi steps

$$\begin{aligned}
\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)} dx &= 2 \int \frac{\cos(x)}{\sin^{\frac{5}{2}}(2x)} dx \\
&= -\frac{2 \cos(x)}{3 \sin^{\frac{3}{2}}(2x)} + \frac{4}{3} \int \frac{\sin(x)}{\sin^{\frac{3}{2}}(2x)} dx \\
&= -\frac{2 \cos(x)}{3 \sin^{\frac{3}{2}}(2x)} + \frac{4 \sin(x)}{3 \sqrt{\sin(2x)}}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 24, normalized size = 0.83

$$\left(-\frac{1}{6} \cot(x) \csc(x) + \frac{\sec(x)}{2}\right) \sqrt{\sin(2x)}$$

Antiderivative was successfully verified.

`[In] Integrate[Csc[x]/Sin[2*x]^(3/2),x]``[Out] (-1/6*(Cot[x]*Csc[x]) + Sec[x]/2)*Sqrt[Sin[2*x]]`**Mathics [F(-1)]**

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(Sin[x]*Sin[2*x]^(3/2)),x]')``[Out] Timed out`**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.09, size = 121, normalized size = 4.17

method	result
default	$-\frac{\sqrt{-\frac{\tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1}} (\tan^2(\frac{x}{2})-1) \left(2 \sqrt{1+\tan(\frac{x}{2})} \sqrt{-2 \tan(\frac{x}{2})+2} \sqrt{-\tan(\frac{x}{2})} \operatorname{EllipticF}\left(\sqrt{1+\tan(\frac{x}{2})}\right)\right)}{12 \tan(\frac{x}{2}) \sqrt{\tan(\frac{x}{2}) (\tan^2(\frac{x}{2})-1)} \sqrt{\tan^3(\frac{x}{2})-\tan(\frac{x}{2})}}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/sin(x)/sin(2*x)^(3/2),x,method=_RETURNVERBOSE)`
`[Out] -1/12*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/tan(1/2*x)*(2*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((`

$1 + \tan(1/2*x)^{(1/2)}, 1/2*2^{(1/2)} * \tan(1/2*x) - \tan(1/2*x)^{4+1} / (\tan(1/2*x) * (\tan(1/2*x)^{2-1})^{(1/2)} / (\tan(1/2*x)^3 - \tan(1/2*x))^{(1/2)}$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(x)/sin(2*x)^(3/2),x, algorithm="maxima")

[Out] integrate(1/(sin(2*x)^(3/2)*sin(x)), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(21) = 42$.

time = 0.32, size = 43, normalized size = 1.48

$$\frac{4 \cos(x)^3 + \sqrt{2} (4 \cos(x)^2 - 3) \sqrt{\cos(x) \sin(x)} - 4 \cos(x)}{6 (\cos(x)^3 - \cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(x)/sin(2*x)^(3/2),x, algorithm="fricas")

[Out] $1/6 * (4 * \cos(x)^3 + \sqrt{2} * (4 * \cos(x)^2 - 3) * \sqrt{\cos(x) * \sin(x)} - 4 * \cos(x)) / (\cos(x)^3 - \cos(x))$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(x)/sin(2*x)**(3/2),x)

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(x)/sin(2*x)^(3/2),x)

[Out] Could not integrate

Mupad [B]

time = 0.43, size = 29, normalized size = 1.00

$$-\frac{\sqrt{\sin(2x)} (2 \cos(2x) - 1)}{6 (\cos(x) - \cos(x)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(sin(2*x)^(3/2)*sin(x)),x)`

[Out] `-(sin(2*x)^(1/2)*(2*cos(2*x) - 1))/(6*(cos(x) - cos(x)^3))`

$$3.411 \quad \int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)} dx$$

Optimal. Leaf size=68

$$\frac{33}{32} \tanh^{-1}\left(\frac{1}{2} \sec(x) \sqrt{\sin(2x)}\right) - \frac{9 \cos(x)}{16 \sqrt{\sin(2x)}} - \frac{5 \cos(x) \cot(x)}{24 \sqrt{\sin(2x)}} + \frac{\cos(x) \cot^2(x)}{20 \sqrt{\sin(2x)}}$$

[Out] 33/32*arctanh(1/2*sin(2*x)^(1/2)/cos(x))-9/16*cos(x)/sin(2*x)^(1/2)-5/24*cos(x)*cot(x)/sin(2*x)^(1/2)+1/20*cos(x)*cot(x)^2/sin(2*x)^(1/2)

Rubi [A]

time = 0.55, antiderivative size = 95, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4475, 1633, 65, 213}

$$\frac{\cos^5(x)}{5 \sin^{\frac{5}{2}}(2x)} - \frac{5 \sin(x) \cos^4(x)}{6 \sin^{\frac{5}{2}}(2x)} - \frac{9 \sin^2(x) \cos^3(x)}{4 \sin^{\frac{5}{2}}(2x)} + \frac{33 \sin^5(x) \tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2} \sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]^3*(Cos[2*x] - 3*Tan[x]))/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]

[Out] Cos[x]^5/(5*Sin[2*x]^(5/2)) - (5*Cos[x]^4*Sin[x])/(6*Sin[2*x]^(5/2)) - (9*Cos[x]^3*Sin[x]^2)/(4*Sin[2*x]^(5/2)) + (33*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]]*Sin[x]^5)/(4*Sqrt[2]*Sin[2*x]^(5/2)*Tan[x]^(5/2))

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1)*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1633

Int[((Px_)*((c_.) + (d_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_)), x_Symbol] := Int[ExpandIntegrand[1/Sqrt[c + d*x], Px*((c + d*x)^(n + 1/2)/(a + b*x)), x],

$x]$ /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0] && GtQ[Expon[Px, x], 2]

Rule 4475

Int[(u_)*((c_)*sin[v_])^(m_), x_Symbol] := With[{w = FunctionOfTrig[u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x]}, Dist[(c*SIN[v])^m*((c*Tan[v/2])^m/Sin[v/2]^(2*m)), Int[u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x], x] /; !FalseQ[w] && FunctionOfQ[NonfreeFactors[Tan[w], x], u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] && !SumQ[u] && InverseFunctionFreeQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos^3(x)(\cos(2x) - 3 \tan(x))}{(\sin^2(x) - \sin(2x)) \sin^{\frac{5}{2}}(2x)} dx &= \frac{\sin^5(x) \int \frac{\csc^2(x)(\cos(2x) - 3 \tan(x))}{(\sin^2(x) - \sin(2x)) \sqrt{\tan(x)}} dx}{\sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)} \\ &= \frac{\sin^5(x) \text{Subst} \left(\int \frac{-1+3x+x^2+3x^3}{(2-x)x^{7/2}} dx, x, \tan(x) \right)}{\sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)} \\ &= \frac{\sin^5(x) \text{Subst} \left(\int \left(-\frac{1}{2x^{7/2}} + \frac{5}{4x^{5/2}} + \frac{9}{8x^{3/2}} - \frac{33}{8(-2+x)\sqrt{x}} \right) dx, x, \tan(x) \right)}{\sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)} \\ &= \frac{\cos^5(x)}{5 \sin^{\frac{5}{2}}(2x)} - \frac{5 \cos^4(x) \sin(x)}{6 \sin^{\frac{5}{2}}(2x)} - \frac{9 \cos^3(x) \sin^2(x)}{4 \sin^{\frac{5}{2}}(2x)} - \frac{(33 \sin^5(x)) \text{Subst}}{8 \sin^{\frac{5}{2}}(2x)} \\ &= \frac{\cos^5(x)}{5 \sin^{\frac{5}{2}}(2x)} - \frac{5 \cos^4(x) \sin(x)}{6 \sin^{\frac{5}{2}}(2x)} - \frac{9 \cos^3(x) \sin^2(x)}{4 \sin^{\frac{5}{2}}(2x)} - \frac{(33 \sin^5(x)) \text{Subst}}{4 \sin^{\frac{5}{2}}(2x)} \\ &= \frac{\cos^5(x)}{5 \sin^{\frac{5}{2}}(2x)} - \frac{5 \cos^4(x) \sin(x)}{6 \sin^{\frac{5}{2}}(2x)} - \frac{9 \cos^3(x) \sin^2(x)}{4 \sin^{\frac{5}{2}}(2x)} + \frac{33 \tanh^{-1} \left(\frac{\sqrt{\tan(x)}}{\sqrt{2}} \right)}{4\sqrt{2} \sin^{\frac{5}{2}}(2x)} \end{aligned}$$

Mathematica [A]

time = 1.05, size = 111, normalized size = 1.63

$$\cos(x) \sqrt{\sin(2x)} \left(\frac{\frac{1}{15} \csc(x) (-147 - 50 \cot(x) + 12 \csc^2(x)) + \frac{33 \tan^{-1} \left(\frac{\sqrt{\tan\left(\frac{x}{2}\right)}}{\sqrt{-1 + \tan^2\left(\frac{x}{2}\right)}} \right) \sqrt{\frac{\cos(x)}{2 + 2 \cos(x)} \sec(x)}}{\sqrt{\tan\left(\frac{x}{2}\right)}}}{16(\cos(x) + \cos(3x) - 6 \sin(x))} \right) (\cos(2x) - 3 \tan(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*(Cos[2*x] - 3*Tan[x]))/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]
```

```
[Out] (Cos[x]*Sqrt[Sin[2*x]]*((Csc[x]*(-147 - 50*Cot[x] + 12*Csc[x]^2))/15 + (33*ArcTan[Sqrt[Tan[x/2]]/Sqrt[-1 + Tan[x/2]^2]]*Sqrt[-(Cos[x]/(2 + 2*Cos[x]))]*Sec[x])/Sqrt[Tan[x/2]]*(Cos[2*x] - 3*Tan[x]))/(16*(Cos[x] + Cos[3*x] - 6*Sin[x])))
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Cos[2*x] - 3*Tan[x])*(Cos[x]^3/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2))),x]')
```

```
[Out] Timed out
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 0.36, size = 761, normalized size = 11.19

method	result	size
default	Expression too large to display	761

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3840*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(932*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^2-3024*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^2+24*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^6+3*sum((34*_alpha^3+13*_alpha^2+34*_alpha-21)*(_alpha^3+2*_alpha-3)*(1+tan(1/2*x))^(1/2)*(-tan(1/2*x)+1)^(1/2)*(-tan(1/2*x))^(1/2)/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),-1/4*_alpha^3-1/2*_alpha+3/4,1/2*2^(1/2)),_alpha=RootOf(_Z^4+_Z^3+2*_Z^2-_Z+1))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*2^(1/2)*tan(1/2*x)^2+200*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^5-552*tan(1/2*x)^4*(tan(1/2
```

```
*x)^3-tan(1/2*x))^(1/2)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)-19
20*tan(1/2*x)^4*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^3-tan(1/2*x
))^^(1/2)-24*tan(1/2*x)^4*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((1+tan(1/2*x)
)*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)+552*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*((1+
tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^2-24*((1+tan(1/2*x)
)*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(
1/2*x)^2-200*tan(1/2*x)*((1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2)*(t
an(1/2*x)*(tan(1/2*x)^2-1))^(1/2)+24*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((
1+tan(1/2*x))*(tan(1/2*x)-1)*tan(1/2*x))^(1/2))/((1+tan(1/2*x))*(tan(1/2*x)
-1)*tan(1/2*x))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

Maxima [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x
, algorithm="maxima")
```

[Out] Timed out

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(52) = 104.

time = 0.33, size = 136, normalized size = 2.00

```
495 (cos(x)^2 - 1) log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x)) + 1/2*cos(x) + 3/2*sin(x)) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2*sin(x) - 495 (cos(x)^2 - 1) log(1/2*cos(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(x)*sin(x) + 1/2*sin(x) + 4*sqrt(2)*(147*cos(x)^2 - 50*cos(x)*sin(x) - 135)*sqrt(cos(x)*sin(x)) + 388*(cos(x)^2 - 1)*sin(x)) / (1920*(cos(x)^2 - 1)*sin(x))
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x
, algorithm="fricas")
```

```
[Out] -1/1920*(495*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x)
+ 3*sin(x)) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2*sin(x) - 495*(cos(x)^
2 - 1)*log(1/2*cos(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(
x)*sin(x) + 1/2*sin(x) + 4*sqrt(2)*(147*cos(x)^2 - 50*cos(x)*sin(x) - 135)
*sqrt(cos(x)*sin(x)) + 388*(cos(x)^2 - 1)*sin(x))/((cos(x)^2 - 1)*sin(x))
```

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*(cos(2*x)-3*tan(x))/(sin(x)**2-sin(2*x))/sin(2*x)**(5/2
),x)
```

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x
)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\cos(x)^3 (\cos(2x) - 3 \tan(x))}{\sin(2x)^{5/2} (\sin(2x) - \sin(x)^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(cos(x)^3*(cos(2*x) - 3*tan(x)))/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)),x)

[Out] int(-(cos(x)^3*(cos(2*x) - 3*tan(x)))/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)), x)

3.412 $\int \sqrt{\sec^4(x) \tan(x)} dx$

Optimal. Leaf size=19

$$\frac{2}{3} \cos(x) \sin(x) \sqrt{\sec^4(x) \tan(x)}$$

[Out] 2/3*cos(x)*sin(x)*(sec(x)^4*tan(x))^(1/2)

Rubi [A]

time = 0.07, antiderivative size = 29, normalized size of antiderivative = 1.53, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2024, 1968, 1264, 30}

$$\frac{2 \tan^2(x) \sec^2(x)}{3 \sqrt{\tan^5(x) + 2 \tan^3(x) + \tan(x)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[x]^4*Tan[x]],x]

[Out] (2*Sec[x]^2*Tan[x]^2)/(3*Sqrt[Tan[x] + 2*Tan[x]^3 + Tan[x]^5])

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 1264

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]

Rule 1968

Int[(x_)^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(k_) + (c_)*(x_)^(n_))^(p_)*((A_) + (B_)*(x_)^(q_)), x_Symbol] := Dist[(a*x^j + b*x^k + c*x^n)^p/(x^(j*p)*(a + b*x^(k - j) + c*x^(2*(k - j)))^p), Int[x^(m + j*p)*(A + B*x^(k - j))*(a + b*x^(k - j) + c*x^(2*(k - j)))^p, x], x] /; FreeQ[{a, b, c, A, B, j, k, m, p}, x] && EqQ[q, k - j] && EqQ[n, 2*k - j] && !IntegerQ[p] && PosQ[k - j]

Rule 2024

Int[(u_)^(p_)*((f_)*(x_))^(m_)*(z_), x_Symbol] := Int[(f*x)^m*ExpandToSum[z, x]*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p}, x] && BinomialQ[z, x] &

& GeneralizedTrinomialQ[u, x] && EqQ[BinomialDegree[z, x] - GeneralizedTrinomialDegree[u, x], 0] && !(BinomialMatchQ[z, x] && GeneralizedTrinomialMatchQ[u, x])

Rubi steps

$$\begin{aligned}
 \int \sqrt{\sec^4(x) \tan(x)} \, dx &= \text{Subst} \left(\int \frac{x(1+x^2)}{\sqrt{x(1+x^2)^2}} \, dx, x, \tan(x) \right) \\
 &= \text{Subst} \left(\int \frac{x(1+x^2)}{\sqrt{x+2x^3+x^5}} \, dx, x, \tan(x) \right) \\
 &= \frac{\left(\sqrt{\tan(x)} \sqrt{1+2\tan^2(x)+\tan^4(x)} \right) \text{Subst} \left(\int \frac{\sqrt{x}(1+x^2)}{\sqrt{1+2x^2+x^4}} \, dx, x, \tan(x) \right)}{\sqrt{\tan(x)+2\tan^3(x)+\tan^5(x)}} \\
 &= \frac{\left(\sec^2(x) \sqrt{\tan(x)} \right) \text{Subst} \left(\int \sqrt{x} \, dx, x, \tan(x) \right)}{\sqrt{\tan(x)+2\tan^3(x)+\tan^5(x)}} \\
 &= \frac{2 \sec^2(x) \tan^2(x)}{3 \sqrt{\tan(x)+2\tan^3(x)+\tan^5(x)}}
 \end{aligned}$$

Mathematica [A]

time = 0.01, size = 19, normalized size = 1.00

$$\frac{2}{3} \cos(x) \sin(x) \sqrt{\sec^4(x) \tan(x)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[x]^4*Tan[x]],x]

[Out] (2*Cos[x]*Sin[x]*Sqrt[Sec[x]^4*Tan[x]])/3

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sqrt[Sin[x]/Cos[x]^5],x]')

[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep

Maple [A]

time = 0.20, size = 16, normalized size = 0.84

method	result	size
default	$\frac{2 \cos(x) \sin(x) \sqrt{\frac{\sin(x)}{\cos(x)^5}}}{3}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)/cos(x)^5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*cos(x)*sin(x)*(sin(x)/cos(x)^5)^(1/2)
```

Maxima [A]

time = 0.34, size = 6, normalized size = 0.32

$$\frac{2}{3} \tan(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)/cos(x)^5)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*tan(x)^(3/2)
```

Fricas [A]

time = 0.31, size = 15, normalized size = 0.79

$$\frac{2}{3} \sqrt{\frac{\sin(x)}{\cos(x)^5}} \cos(x) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)/cos(x)^5)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(sin(x)/cos(x)^5)*cos(x)*sin(x)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)/cos(x)**5)**(1/2),x)
```

```
[Out] Exception raised: SystemError
```

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)/cos(x)^5)^(1/2),x)

[Out] Could not integrate

Mupad [B]

time = 0.51, size = 15, normalized size = 0.79

$$\frac{\sin(2x) \sqrt{\frac{\sin(x)}{\cos(x)^5}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x)/cos(x)^5)^(1/2),x)

[Out] (sin(2*x)*(sin(x)/cos(x)^5)^(1/2))/3

3.413 $\int \sqrt{\sin^4(x) \tan(x)} dx$

Optimal. Leaf size=92

$$\frac{3 \tan^{-1} \left(\frac{(1 - \cot(x)) \csc^2(x) \sqrt{\sin^4(x) \tan(x)}}{\sqrt{2}} \right)}{4\sqrt{2}} + \frac{3 \log \left(\cos(x) + \sin(x) - \sqrt{2} \cot(x) \csc(x) \sqrt{\sin^4(x) \tan(x)} \right)}{4\sqrt{2}}$$

[Out] 3/8*arctan(1/2*(1-cot(x))*csc(x)^2*(sin(x)^4*tan(x))^(1/2)*2^(1/2))*2^(1/2)+3/8*ln(cos(x)+sin(x)-cot(x)*csc(x)*2^(1/2)*(sin(x)^4*tan(x))^(1/2))*2^(1/2)-1/2*cot(x)*(sin(x)^4*tan(x))^(1/2)

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 204 vs. 2(92) = 184. time = 0.15, antiderivative size = 204, normalized size of antiderivative = 2.22, number of steps used = 13, number of rules used = 9, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {6851, 294, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{1}{2} \cot(x) \sqrt{\sin^4(x) \tan(x)} - \frac{3 \tan^{-1} \left(\frac{(1 - \sqrt{2} \sqrt{\tan(x)}) \sec^2(x) \sqrt{\sin^4(x) \tan(x)}}{4\sqrt{2} \tan^{\frac{3}{2}}(x)} \right)}{4\sqrt{2} \tan^{\frac{3}{2}}(x)} + \frac{3 \tan^{-1} \left(\frac{(\sqrt{2} \sqrt{\tan(x)} + 1) \sec^2(x) \sqrt{\sin^4(x) \tan(x)}}{4\sqrt{2} \tan^{\frac{3}{2}}(x)} \right)}{4\sqrt{2} \tan^{\frac{3}{2}}(x)} + \frac{3 \sec^2(x) \log \left(\frac{\tan(x) - \sqrt{2} \sqrt{\tan(x)} + 1}{\sqrt{\sin^4(x) \tan(x)}} \right)}{8\sqrt{2} \tan^{\frac{3}{2}}(x)} - \frac{3 \sec^2(x) \log \left(\frac{\tan(x) + \sqrt{2} \sqrt{\tan(x)} + 1}{\sqrt{\sin^4(x) \tan(x)}} \right)}{8\sqrt{2} \tan^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sin[x]^4*Tan[x]], x]

[Out] -1/2*(Cot[x]*Sqrt[Sin[x]^4*Tan[x]]) - (3*ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Sin[x]^4*Tan[x]])/(4*Sqrt[2]*Tan[x]^(5/2)) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Sin[x]^4*Tan[x]])/(4*Sqrt[2]*Tan[x]^(5/2)) + (3*Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^2*Sqrt[Sin[x]^4*Tan[x]])/(8*Sqrt[2]*Tan[x]^(5/2)) - (3*Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^2*Sqrt[Sin[x]^4*Tan[x]])/(8*Sqrt[2]*Tan[x]^(5/2))

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 294

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n-1)*(c*x)^(m-n+1)*((a+b*x^n)^(p+1)/(b*n*(p+1))), x] - Dist[c^n*((m-n+1)/(b*n*(p+1))), Int[(c*x)^(m-n)*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && ! LtQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 303

```
Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6851

```
Int[(u_)*((a_)*(v_)^(m_)*(w_)^(n_))^(p_), x_Symbol] := Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}
 \int \sqrt{\sin^4(x) \tan(x)} dx &= \text{Subst} \left(\int \frac{\sqrt{\frac{x^5}{(1+x^2)^2}}}{1+x^2} dx, x, \tan(x) \right) \\
 &= \frac{\left(\sec^2(x) \sqrt{\sin^4(x) \tan(x)} \right) \text{Subst} \left(\int \frac{x^{5/2}}{(1+x^2)^2} dx, x, \tan(x) \right)}{\tan^{\frac{5}{2}}(x)} \\
 &= -\frac{1}{2} \cot(x) \sqrt{\sin^4(x) \tan(x)} + \frac{\left(3 \sec^2(x) \sqrt{\sin^4(x) \tan(x)} \right) \text{Subst} \left(\int \frac{\sqrt{x}}{1+x^2} dx, x, \tan(x) \right)}{4 \tan^{\frac{5}{2}}(x)} \\
 &= -\frac{1}{2} \cot(x) \sqrt{\sin^4(x) \tan(x)} + \frac{\left(3 \sec^2(x) \sqrt{\sin^4(x) \tan(x)} \right) \text{Subst} \left(\int \frac{x^2}{1+x^4} dx, x, \tan(x) \right)}{2 \tan^{\frac{5}{2}}(x)} \\
 &= -\frac{1}{2} \cot(x) \sqrt{\sin^4(x) \tan(x)} - \frac{\left(3 \sec^2(x) \sqrt{\sin^4(x) \tan(x)} \right) \text{Subst} \left(\int \frac{1-x^2}{1+x^4} dx, x, \tan(x) \right)}{4 \tan^{\frac{5}{2}}(x)} \\
 &= -\frac{1}{2} \cot(x) \sqrt{\sin^4(x) \tan(x)} + \frac{\left(3 \sec^2(x) \sqrt{\sin^4(x) \tan(x)} \right) \text{Subst} \left(\int \frac{1}{1-\sqrt{2} x+x^2} dx, x, \tan(x) \right)}{8 \tan^{\frac{5}{2}}(x)} \\
 &= -\frac{1}{2} \cot(x) \sqrt{\sin^4(x) \tan(x)} + \frac{3 \log \left(1 - \sqrt{2} \sqrt{\tan(x)} + \tan(x) \right) \sec^2(x) \sqrt{\sin^4(x) \tan(x)}}{8 \sqrt{2} \tan^{\frac{5}{2}}(x)} \\
 &= -\frac{1}{2} \cot(x) \sqrt{\sin^4(x) \tan(x)} - \frac{3 \tan^{-1} \left(1 - \sqrt{2} \sqrt{\tan(x)} \right) \sec^2(x) \sqrt{\sin^4(x) \tan(x)}}{4 \sqrt{2} \tan^{\frac{5}{2}}(x)}
 \end{aligned}$$

Mathematica [A]

time = 0.05, size = 66, normalized size = 0.72

$$-\frac{1}{8} \csc^3(x) \left(3 \sin^{-1}(\cos(x) - \sin(x)) + 3 \log(\cos(x) + \sin(x) + \sqrt{\sin(2x)}) + 2 \sin(x) \sqrt{\sin(2x)} \right) \sqrt{\sin(2x)} \sqrt{\sin^4(x) \tan(x)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sin[x]^4*Tan[x]],x]

[Out] -1/8*(Csc[x]^3*(3*ArcSin[Cos[x] - Sin[x]] + 3*Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]) + 2*Sin[x]*Sqrt[Sin[2*x]])*Sqrt[Sin[2*x]]*Sqrt[Sin[x]^4*Tan[x]])

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[Sqrt[Sin[x]^5/Cos[x]],x]')`

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.26, size = 318, normalized size = 3.46

method	result
default	$-\frac{\sqrt{32} (\cos(x)-1) \left(3i \sqrt{\frac{\cos(x)-1}{\sin(x)}} \sqrt{\frac{-1+\cos(x)+\sin(x)}{\sin(x)}} \sqrt{\frac{-1+\cos(x)-\sin(x)}{\sin(x)}} \operatorname{EllipticPi}\left(\sqrt{\frac{-1+\cos(x)-\sin(x)}{\sin(x)}}, \frac{1}{2}\right) \right)}{1}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((sin(x)^5/cos(x))^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$-1/32 \cdot 32^{1/2} \cdot (\cos(x)-1) \cdot (3i \cdot ((\cos(x)-1)/\sin(x))^{1/2} \cdot ((-1+\cos(x)+\sin(x))/\sin(x))^{1/2} \cdot (-(-1+\cos(x)-\sin(x))/\sin(x))^{1/2} \cdot \operatorname{EllipticPi}((-(-1+\cos(x)-\sin(x))/\sin(x))/\sin(x))^{1/2}, 1/2-1/2i, 1/2 \cdot 2^{1/2})) - 3i \cdot ((\cos(x)-1)/\sin(x))^{1/2} \cdot ((-1+\cos(x)+\sin(x))/\sin(x))^{1/2} \cdot (-(-1+\cos(x)-\sin(x))/\sin(x))^{1/2} \cdot \operatorname{EllipticPi}((-(-1+\cos(x)-\sin(x))/\sin(x))/\sin(x))^{1/2}, 1/2+1/2i, 1/2 \cdot 2^{1/2})) - 3 \cdot ((\cos(x)-1)/\sin(x))^{1/2} \cdot ((-1+\cos(x)+\sin(x))/\sin(x))^{1/2} \cdot (-(-1+\cos(x)-\sin(x))/\sin(x))^{1/2} \cdot \operatorname{EllipticPi}((-(-1+\cos(x)-\sin(x))/\sin(x))/\sin(x))^{1/2}, 1/2-1/2i, 1/2 \cdot 2^{1/2})) - 3 \cdot ((\cos(x)-1)/\sin(x))^{1/2} \cdot ((-1+\cos(x)+\sin(x))/\sin(x))^{1/2} \cdot (-(-1+\cos(x)-\sin(x))/\sin(x))^{1/2} \cdot \operatorname{EllipticPi}((-(-1+\cos(x)-\sin(x))/\sin(x))/\sin(x))^{1/2}, 1/2+1/2i, 1/2 \cdot 2^{1/2})) + 2 \cdot \cos(x)^2 \cdot 2^{1/2} - 2 \cdot \cos(x) \cdot 2^{1/2} \cdot (1+\cos(x))^2 \cdot (\sin(x)^5/\cos(x))^{1/2} / \sin(x)^5$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((sin(x)^5/cos(x))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(sin(x)^5/cos(x)), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1006 vs. 2(71) = 142.

time = 29.40, size = 1006, normalized size = 10.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^5/cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(16*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x))*cos(x)*sin(x) - 6*
(sqrt(2)*cos(x)^2 - sqrt(2))*arctan(1/2*(2*cos(x)^4 - 4*cos(x)^2 - 2*(cos(x)
)^3 - cos(x))*sin(x) + sqrt(2)*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(
x))*sqrt((cos(x)^2 + 4*(cos(x)^3 - cos(x))*sin(x) + 2*(sqrt(2)*cos(x)^2 + s
qrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) - 1)/
(cos(x)^2 - 1)) - sqrt(2)*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) +
2)/(cos(x)^4 - 2*cos(x)^2 + (cos(x)^3 - cos(x))*sin(x) + 1)) - 6*(sqrt(2)*
cos(x)^2 - sqrt(2))*arctan(-1/2*(2*cos(x)^4 - 4*cos(x)^2 - 2*(cos(x)^3 - co
s(x))*sin(x) - sqrt(2)*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x))*sqrt
((cos(x)^2 + 4*(cos(x)^3 - cos(x))*sin(x) - 2*(sqrt(2)*cos(x)^2 + sqrt(2)*c
os(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) - 1)/(cos(x)^
2 - 1)) + sqrt(2)*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) + 2)/(cos
(x)^4 - 2*cos(x)^2 + (cos(x)^3 - cos(x))*sin(x) + 1)) - 6*(sqrt(2)*cos(x)^2
- sqrt(2))*arctan(((sqrt(2)*cos(x)^2 - sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)
^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) - (2*cos(x)^4 - 3*cos(x)^2 - (sqrt(2)*c
os(x)^2 - sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/co
s(x)) + 1)*sqrt((cos(x)^2 + 4*(cos(x)^3 - cos(x))*sin(x) + 2*(sqrt(2)*cos(x)
)^2 + sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)
) - 1)/(cos(x)^2 - 1)))/(cos(x)^2 - 2*(cos(x)^3 - cos(x))*sin(x) - 1)) - 6*
(sqrt(2)*cos(x)^2 - sqrt(2))*arctan(((sqrt(2)*cos(x)^2 - sqrt(2)*cos(x)*sin
(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) + (2*cos(x)^4 - 3*cos(
x)^2 + (sqrt(2)*cos(x)^2 - sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)
^2 + 1)*sin(x)/cos(x)) + 1)*sqrt((cos(x)^2 + 4*(cos(x)^3 - cos(x))*sin(x) -
2*(sqrt(2)*cos(x)^2 + sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 +
1)*sin(x)/cos(x)) - 1)/(cos(x)^2 - 1)))/(cos(x)^2 - 2*(cos(x)^3 - cos(x))*
sin(x) - 1)) + 3*(sqrt(2)*cos(x)^2 - sqrt(2))*log((cos(x)^2 + 4*(cos(x)^3 -
cos(x))*sin(x) + 2*(sqrt(2)*cos(x)^2 + sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)
^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) - 1)/(cos(x)^2 - 1)) - 3*(sqrt(2)*cos(x)
)^2 - sqrt(2))*log((cos(x)^2 + 4*(cos(x)^3 - cos(x))*sin(x) - 2*(sqrt(2)*co
s(x)^2 + sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos
(x)) - 1)/(cos(x)^2 - 1)))/(cos(x)^2 - 1)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{\sin^5(x)}{\cos(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)**5/cos(x))**(1/2),x)
```

[Out] Integral(sqrt(sin(x)**5/cos(x)), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)^5/cos(x))^(1/2),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{\sin(x)^5}{\cos(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x)^5/cos(x))^(1/2),x)

[Out] int((sin(x)^5/cos(x))^(1/2), x)

3.414 $\int \sqrt[3]{\sec^{12}(x) \tan^2(x)} dx$

Optimal. Leaf size=47

$$\frac{3}{5} \cos^3(x) \sin(x) \sqrt[3]{\sec^{12}(x) \tan^2(x)} + \frac{3}{11} \cos(x) \sin^3(x) \sqrt[3]{\sec^{12}(x) \tan^2(x)}$$

[Out] 3/5*cos(x)^3*sin(x)*(sec(x)^12*tan(x)^2)^(1/3)+3/11*cos(x)*sin(x)^3*(sec(x)^12*tan(x)^2)^(1/3)

Rubi [A]

time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1986, 15, 14}

$$\frac{3}{5} \sin(x) \cos^3(x) \sqrt[3]{\tan^2(x) \sec^{12}(x)} + \frac{3}{11} \sin^3(x) \cos(x) \sqrt[3]{\tan^2(x) \sec^{12}(x)}$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^12*Tan[x]^2)^(1/3),x]

[Out] (3*Cos[x]^3*Sin[x]*(Sec[x]^12*Tan[x]^2)^(1/3))/5 + (3*Cos[x]*Sin[x]^3*(Sec[x]^12*Tan[x]^2)^(1/3))/11

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 15

Int[(u_)*((a_)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^FracPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 1986

Int[(u_)*((e_)*((a_)+(b_)*(x_)^(n_))^(q_)*((c_)+(d_)*(x_)^(n_))^(r_))^(p_), x_Symbol] :> Dist[Simp[(e*(a+b*x^n)^q*(c+d*x^n)^r)^p/((a+b*x^n)^(p*q)*(c+d*x^n)^(p*r))], Int[u*(a+b*x^n)^(p*q)*(c+d*x^n)^(p*r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]

Rubi steps

$$\begin{aligned}
\int \sqrt[3]{\sec^{12}(x) \tan^2(x)} dx &= \text{Subst} \left(\int \frac{\sqrt[3]{x^2 (1+x^2)^6}}{1+x^2} dx, x, \tan(x) \right) \\
&= \frac{\left(\cos^4(x) \sqrt[3]{\sec^{12}(x) \tan^2(x)} \right) \text{Subst} \left(\int x^{2/3} (1+x^2) dx, x, \tan(x) \right)}{\tan^{2/3}(x)} \\
&= \frac{\left(\cos^4(x) \sqrt[3]{\sec^{12}(x) \tan^2(x)} \right) \text{Subst} \left(\int (x^{2/3} + x^{8/3}) dx, x, \tan(x) \right)}{\tan^{2/3}(x)} \\
&= \frac{3}{5} \cos^3(x) \sin(x) \sqrt[3]{\sec^{12}(x) \tan^2(x)} + \frac{3}{11} \cos(x) \sin^3(x) \sqrt[3]{\sec^{12}(x) \tan^2(x)}
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 63, normalized size = 1.34

$$\frac{3 \cos(x) \sin(x) \sqrt[3]{\sec^{12}(x) \tan^2(x)} \left(-3 + 8(-\tan^2(x))^{5/6} + 3 \cos(2x) \left(-1 + (-\tan^2(x))^{5/6} \right) \right)}{55 (-\tan^2(x))^{5/6}}$$

Antiderivative was successfully verified.

`[In] Integrate[(Sec[x]^12*Tan[x]^2)^(1/3), x]`

```
[Out] (3*Cos[x]*Sin[x]*(Sec[x]^12*Tan[x]^2)^(1/3)*(-3 + 8*(-Tan[x]^2)^(5/6) + 3*Cos[2*x]*(-1 + (-Tan[x]^2)^(5/6))))/(55*(-Tan[x]^2)^(5/6))
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(Sin[x]^2/Cos[x]^14)^(1/3), x]')``[Out] Timed out`**Maple [F]**

time = 0.38, size = 0, normalized size = 0.00

$$\int \left(\frac{\sin^2(x)}{\cos(x)^{14}} \right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((sin(x)^2/cos(x)^14)^(1/3),x)`

[Out] `int((sin(x)^2/cos(x)^14)^(1/3),x)`

Maxima [A]

time = 0.43, size = 13, normalized size = 0.28

$$\frac{3}{11} \tan(x)^{\frac{11}{3}} + \frac{3}{5} \tan(x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="maxima")`

[Out] `3/11*tan(x)^(11/3) + 3/5*tan(x)^(5/3)`

Fricas [A]

time = 0.31, size = 29, normalized size = 0.62

$$\frac{3}{55} (6 \cos(x)^3 + 5 \cos(x)) \left(-\frac{\cos(x)^2 - 1}{\cos(x)^{14}} \right)^{\frac{1}{3}} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="fricas")`

[Out] `3/55*(6*cos(x)^3 + 5*cos(x))*(-(cos(x)^2 - 1)/cos(x)^14)^(1/3)*sin(x)`

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((sin(x)**2/cos(x)**14)**(1/3),x)`

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((sin(x)^2/cos(x)^14)^(1/3),x)`

[Out] Could not integrate

Mupad [B]

time = 3.94, size = 32, normalized size = 0.68

$$\frac{6 \sin(2x) (1 - \cos(2x))^{1/3} (3 \cos(2x) + 8)}{55 (\cos(2x) + 1)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x)^2/cos(x)^14)^(1/3),x)

[Out] (6*sin(2*x)*(1 - cos(2*x))^(1/3)*(3*cos(2*x) + 8))/(55*(cos(2*x) + 1)^(7/3))

$$3.415 \quad \int \frac{1}{\sqrt[4]{\cos^{11}(x) \sin^{13}(x)}} dx$$

Optimal. Leaf size=70

$$-\frac{4 \cos^5(x) \sin(x)}{9 \sqrt[4]{\cos^{11}(x) \sin^{13}(x)}} - \frac{8 \cos^3(x) \sin^3(x)}{\sqrt[4]{\cos^{11}(x) \sin^{13}(x)}} + \frac{4 \cos(x) \sin^5(x)}{7 \sqrt[4]{\cos^{11}(x) \sin^{13}(x)}}$$

[Out] $-4/9*\cos(x)^5*\sin(x)/(\cos(x)^{11}*\sin(x)^{13})^{(1/4)}-8*\cos(x)^3*\sin(x)^3/(\cos(x)^{11}*\sin(x)^{13})^{(1/4)}+4/7*\cos(x)*\sin(x)^5/(\cos(x)^{11}*\sin(x)^{13})^{(1/4)}$

Rubi [A]

time = 0.12, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6851, 276}

$$\frac{4 \sin^5(x) \cos(x)}{7 \sqrt[4]{\sin^{13}(x) \cos^{11}(x)}} - \frac{4 \sin(x) \cos^5(x)}{9 \sqrt[4]{\sin^{13}(x) \cos^{11}(x)}} - \frac{8 \sin^3(x) \cos^3(x)}{\sqrt[4]{\sin^{13}(x) \cos^{11}(x)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[x]^{11}*\text{Sin}[x]^{13})^{(-1/4)}, x]$

[Out] $(-4*\text{Cos}[x]^5*\text{Sin}[x])/ (9*(\text{Cos}[x]^{11}*\text{Sin}[x]^{13})^{(1/4)}) - (8*\text{Cos}[x]^3*\text{Sin}[x]^3)/(\text{Cos}[x]^{11}*\text{Sin}[x]^{13})^{(1/4)} + (4*\text{Cos}[x]*\text{Sin}[x]^5)/(7*(\text{Cos}[x]^{11}*\text{Sin}[x]^{13})^{(1/4)})$

Rule 276

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Int}[\text{Exp andIntegrand}[(c*x)^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, m, n, x\} \&\& \text{IGtQ}[p, 0]$

Rule 6851

$\text{Int}[(u_*)*((a_*)*(v_*)^{(m_*)}*(w_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Dist}[a^{\text{IntPart}[p]}]*(a*v^m*w^n)^{\text{FracPart}[p]}/(v^{(m*\text{FracPart}[p])}*w^{(n*\text{FracPart}[p])}), \text{Int}[u*v^{(m*p)}*w^{(n*p)}, x], x] /; \text{FreeQ}\{a, m, n, p\}, x \&\& \text{!IntegerQ}[p] \&\& \text{!FreeQ}[v, x] \&\& \text{!FreeQ}[w, x]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[4]{\cos^{11}(x) \sin^{13}(x)}} dx &= \text{Subst} \left(\int \frac{1}{\sqrt[4]{\frac{x^{13}}{(1+x^2)^{12}} (1+x^2)}} dx, x, \tan(x) \right) \\
&= \frac{\left(\cos^6(x) \tan^{\frac{13}{4}}(x) \right) \text{Subst} \left(\int \frac{(1+x^2)^2}{x^{13/4}} dx, x, \tan(x) \right)}{\sqrt[4]{\cos^{11}(x) \sin^{13}(x)}} \\
&= \frac{\left(\cos^6(x) \tan^{\frac{13}{4}}(x) \right) \text{Subst} \left(\int \left(\frac{1}{x^{13/4}} + \frac{2}{x^{5/4}} + x^{3/4} \right) dx, x, \tan(x) \right)}{\sqrt[4]{\cos^{11}(x) \sin^{13}(x)}} \\
&= -\frac{4 \cos^5(x) \sin(x)}{9 \sqrt[4]{\cos^{11}(x) \sin^{13}(x)}} - \frac{8 \cos^3(x) \sin^3(x)}{\sqrt[4]{\cos^{11}(x) \sin^{13}(x)}} + \frac{4 \cos(x) \sin^5(x)}{7 \sqrt[4]{\cos^{11}(x) \sin^{13}(x)}}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 35, normalized size = 0.50

$$-\frac{4 \cos(x)(15 + 8 \cos(2x) - 16 \cos(4x)) \sin(x)}{63 \sqrt[4]{\cos^{11}(x) \sin^{13}(x)}}$$

Antiderivative was successfully verified.

`[In] Integrate[(Cos[x]^11*Sin[x]^13)^(-1/4), x]``[Out] (-4*Cos[x]*(15 + 8*Cos[2*x] - 16*Cos[4*x])*Sin[x])/((63*(Cos[x]^11*Sin[x]^13)^(1/4))`**Mathics [F(-1)]**

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(Sin[x]^13*Cos[x]^11)^(1/4), x]')``[Out] Timed out`**Maple [F]**

time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{((\cos^{11}(x))(\sin^{13}(x)))^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cos(x)^11*sin(x)^13)^(1/4),x)`

[Out] `int(1/(cos(x)^11*sin(x)^13)^(1/4),x)`

Maxima [A]

time = 0.36, size = 77, normalized size = 1.10

$$\frac{4}{23} \tan(x)^{\frac{23}{4}} + \frac{8}{15} \tan(x)^{\frac{15}{4}} + \frac{4}{7} \tan(x)^{\frac{7}{4}} - \frac{4(35 \tan(x)^7 + 161 \tan(x)^5 + 345 \tan(x)^3 - 805 \tan(x))}{805 \tan(x)^{\frac{5}{4}}} + \frac{4(21 \tan(x)^7 + 135 \tan(x)^5 - 945 \tan(x)^3 - 35 \tan(x))}{315 \tan(x)^{\frac{13}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(cos(x)^11*sin(x)^13)^(1/4),x, algorithm="maxima")`

[Out] `4/23*tan(x)^(23/4) + 8/15*tan(x)^(15/4) + 4/7*tan(x)^(7/4) - 4/805*(35*tan(x)^7 + 161*tan(x)^5 + 345*tan(x)^3 - 805*tan(x))/tan(x)^(5/4) + 4/315*(21*tan(x)^7 + 135*tan(x)^5 - 945*tan(x)^3 - 35*tan(x))/tan(x)^(13/4)`

Fricas [A]

time = 0.32, size = 101, normalized size = 1.44

$$\frac{4(128 \cos(x)^4 - 144 \cos(x)^2 + 9)((\cos(x)^{23} - 6 \cos(x)^{21} + 15 \cos(x)^{19} - 20 \cos(x)^{17} + 15 \cos(x)^{15} - 6 \cos(x)^{13} + \cos(x)^{11}) \sin(x))^{\frac{3}{4}}}{63(\cos(x)^{22} - 6 \cos(x)^{20} + 15 \cos(x)^{18} - 20 \cos(x)^{16} + 15 \cos(x)^{14} - 6 \cos(x)^{12} + \cos(x)^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(cos(x)^11*sin(x)^13)^(1/4),x, algorithm="fricas")`

[Out] `4/63*(128*cos(x)^4 - 144*cos(x)^2 + 9)*((cos(x)^23 - 6*cos(x)^21 + 15*cos(x)^19 - 20*cos(x)^17 + 15*cos(x)^15 - 6*cos(x)^13 + cos(x)^11)*sin(x))^(3/4) / (cos(x)^22 - 6*cos(x)^20 + 15*cos(x)^18 - 20*cos(x)^16 + 15*cos(x)^14 - 6*cos(x)^12 + cos(x)^10)`

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(cos(x)**11*sin(x)**13)**(1/4),x)`

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^11*sin(x)^13)^(1/4),x)

[Out] Could not integrate

Mupad [B]

time = 3.55, size = 110, normalized size = 1.57

$$\frac{2^{3/4} (-32 \cos(2x)^2 + 8 \cos(2x) + 31) (924 \sin(2x) - 132 \sin(4x) - 660 \sin(6x) + 165 \sin(8x) + 330 \sin(10x) - 110 \sin(12x) - 110 \sin(14x) + 44 \sin(16x) + 22 \sin(18x) - 10 \sin(20x) - 2 \sin(22x) + \sin(24x))^{3/4}}{2016 (\cos(2x) - 1)^6 (\cos(2x) + 1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^11*sin(x)^13)^(1/4),x)

[Out] $-(2^{3/4} * (8 * \cos(2*x) - 32 * \cos(2*x)^2 + 31) * (924 * \sin(2*x) - 132 * \sin(4*x) - 660 * \sin(6*x) + 165 * \sin(8*x) + 330 * \sin(10*x) - 110 * \sin(12*x) - 110 * \sin(14*x) + 44 * \sin(16*x) + 22 * \sin(18*x) - 10 * \sin(20*x) - 2 * \sin(22*x) + \sin(24*x))^{3/4}) / (2016 * (\cos(2*x) - 1)^6 * (\cos(2*x) + 1)^5)$

$$3.416 \quad \int \frac{\cos(2x) - \sqrt{\sin(2x)}}{\sqrt{\cos^3(x) \sin(x)}} dx$$

Optimal. Leaf size=108

$$-\sqrt{2} \log\left(\cos(x) + \sin(x) - \sqrt{2} \sec(x) \sqrt{\cos^3(x) \sin(x)}\right) - \frac{\sin^{-1}(\cos(x) - \sin(x)) \cos(x) \sqrt{\sin(2x)}}{\sqrt{\cos^3(x) \sin(x)}} - \tan(x)$$

[Out] $-\ln(\cos(x) + \sin(x) - \sec(x) \cdot 2^{1/2} \cdot (\cos(x)^3 \sin(x))^{1/2}) \cdot 2^{1/2} - \sin(2x) / (\cos(x)^3 \sin(x))^{1/2} - \arcsin(\cos(x) - \sin(x)) \cdot \cos(x) \cdot \sin(2x)^{1/2} / (\cos(x)^3 \sin(x))^{1/2} - \operatorname{arctanh}(\sin(x)) \cdot \cos(x) \cdot \sin(2x)^{1/2} / (\cos(x)^3 \sin(x))^{1/2}$

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 234 vs. $2(108) = 216$. time = 0.98, antiderivative size = 234, normalized size of antiderivative = 2.17, number of steps used = 27, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6851, 6857, 221, 335, 217, 1179, 642, 1176, 631, 210, 327}

$$-\frac{2 \sec^2(x) \sqrt{\sin(x) \cos^3(x)}}{\sqrt{\tan(x)}} - \frac{\sqrt{2} \tan^{-1}(1 - \sqrt{2} \sqrt{\tan(x)}) \sec^2(x) \sqrt{\sin(x) \cos^3(x)}}{\sqrt{\tan(x)}} - \frac{\sqrt{2} \tan^{-1}(\sqrt{2} \sqrt{\tan(x)} + 1) \sec^2(x) \sqrt{\sin(x) \cos^3(x)}}{\sqrt{\tan(x)}} - \frac{\sec^2(x) \log(\tan(x) - \sqrt{2} \sqrt{\tan(x)} + 1) \sqrt{\sin(x) \cos^3(x)}}{\sqrt{2} \sqrt{\tan(x)}} + \frac{\sec^2(x) \log(\tan(x) + \sqrt{2} \sqrt{\tan(x)} + 1) \sqrt{\sin(x) \cos^3(x)}}{\sqrt{2} \sqrt{\tan(x)}} - \sqrt{2} \cos(x) \sec^4(x)^{1/2} \sqrt{\sin(x) \cos^3(x)} \sqrt{\sin(x) \cos^3(x)} \operatorname{sinh}^{-1}(\tan(x))$$

Antiderivative was successfully verified.

[In] `Int[(Cos[2*x] - Sqrt[Sin[2*x]])/Sqrt[Cos[x]^3*Sin[x]], x]`

[Out] $-2 \cdot \operatorname{Sec}[x]^2 \cdot \operatorname{Sqrt}[\operatorname{Cos}[x]^3 \cdot \operatorname{Sin}[x]] - \operatorname{Sqrt}[2] \cdot \operatorname{ArcSinh}[\operatorname{Tan}[x]] \cdot \operatorname{Cot}[x] \cdot (\operatorname{Sec}[x]^2)^{3/2} \cdot \operatorname{Sqrt}[\operatorname{Cos}[x] \cdot \operatorname{Sin}[x]] \cdot \operatorname{Sqrt}[\operatorname{Cos}[x]^3 \cdot \operatorname{Sin}[x]] - (\operatorname{Sqrt}[2] \cdot \operatorname{ArcTan}[1 - \operatorname{Sqrt}[2] \cdot \operatorname{Sqrt}[\operatorname{Tan}[x]]]) \cdot \operatorname{Sec}[x]^2 \cdot \operatorname{Sqrt}[\operatorname{Cos}[x]^3 \cdot \operatorname{Sin}[x]] / \operatorname{Sqrt}[\operatorname{Tan}[x]] + (\operatorname{Sqrt}[2] \cdot \operatorname{ArcTan}[1 + \operatorname{Sqrt}[2] \cdot \operatorname{Sqrt}[\operatorname{Tan}[x]]]) \cdot \operatorname{Sec}[x]^2 \cdot \operatorname{Sqrt}[\operatorname{Cos}[x]^3 \cdot \operatorname{Sin}[x]] / \operatorname{Sqrt}[\operatorname{Tan}[x]] - (\operatorname{Log}[1 - \operatorname{Sqrt}[2] \cdot \operatorname{Sqrt}[\operatorname{Tan}[x]] + \operatorname{Tan}[x]] \cdot \operatorname{Sec}[x]^2 \cdot \operatorname{Sqrt}[\operatorname{Cos}[x]^3 \cdot \operatorname{Sin}[x]]) / (\operatorname{Sqrt}[2] \cdot \operatorname{Sqrt}[\operatorname{Tan}[x]]) + (\operatorname{Log}[1 + \operatorname{Sqrt}[2] \cdot \operatorname{Sqrt}[\operatorname{Tan}[x]] + \operatorname{Tan}[x]] \cdot \operatorname{Sec}[x]^2 \cdot \operatorname{Sqrt}[\operatorname{Cos}[x]^3 \cdot \operatorname{Sin}[x]]) / (\operatorname{Sqrt}[2] \cdot \operatorname{Sqrt}[\operatorname{Tan}[x]])$

Rule 210

`Int[((a_) + (b_) * (x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2] * Rt[-b, 2])^(-1) * ArcTan[Rt[-b, 2] * (x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 217

`Int[((a_) + (b_) * (x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))`

Rule 221

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 327

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[a*c^n*(m - n + 1)/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 335

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 631

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1176

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1179

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^p_, x_Symbol] := Dist[a^IntPart[p]
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(2x) - \sqrt{\sin(2x)}}{\sqrt{\cos^3(x) \sin(x)}} dx &= \text{Subst} \left(\int \frac{\sqrt{\frac{x}{(1+x^2)^2}} \left(1 - x^2 - \frac{x}{\sqrt{2+2x^2}} \right)}{x} dx, x, \tan(x) \right) \\
&= \frac{\left(\sec^2(x) \sqrt{\cos^3(x) \sin(x)} \right) \text{Subst} \left(\int \frac{1-x^2 - \frac{x}{\sqrt{2+2x^2}}}{\sqrt{x}(1+x^2)} dx, x, \tan(x) \right)}{\sqrt{\tan(x)}} \\
&= \frac{\left(\sec^2(x) \sqrt{\cos^3(x) \sin(x)} \right) \text{Subst} \left(\int \left(-\frac{\sqrt{2} \sqrt{\frac{x}{1+x^2}}}{\sqrt{x}} + \frac{1}{\sqrt{x}(1+x^2)} - \frac{x^{3/2}}{1+x^2} \right) dx, x, \tan(x) \right)}{\sqrt{\tan(x)}} \\
&= \frac{\left(\sec^2(x) \sqrt{\cos^3(x) \sin(x)} \right) \text{Subst} \left(\int \frac{1}{\sqrt{x}(1+x^2)} dx, x, \tan(x) \right)}{\sqrt{\tan(x)}} - \left(\sec^2(x) \sqrt{\cos^3(x) \sin(x)} \right) \frac{x^{3/2}}{1+x^2} \\
&= -2 \sec^2(x) \sqrt{\cos^3(x) \sin(x)} - \left(\sqrt{2} \cot(x) \sec^2(x)^{3/2} \sqrt{\cos(x) \sin(x)} \sqrt{\cos^3(x) \sin(x)} \right) \\
&= -2 \sec^2(x) \sqrt{\cos^3(x) \sin(x)} - \sqrt{2} \sinh^{-1}(\tan(x)) \cot(x) \sec^2(x)^{3/2} \sqrt{\cos(x) \sin(x)} \\
&= -2 \sec^2(x) \sqrt{\cos^3(x) \sin(x)} - \sqrt{2} \sinh^{-1}(\tan(x)) \cot(x) \sec^2(x)^{3/2} \sqrt{\cos(x) \sin(x)} \\
&= -2 \sec^2(x) \sqrt{\cos^3(x) \sin(x)} - \sqrt{2} \sinh^{-1}(\tan(x)) \cot(x) \sec^2(x)^{3/2} \sqrt{\cos(x) \sin(x)} \\
&= -2 \sec^2(x) \sqrt{\cos^3(x) \sin(x)} - \sqrt{2} \sinh^{-1}(\tan(x)) \cot(x) \sec^2(x)^{3/2} \sqrt{\cos(x) \sin(x)} \\
&= -2 \sec^2(x) \sqrt{\cos^3(x) \sin(x)} - \sqrt{2} \sinh^{-1}(\tan(x)) \cot(x) \sec^2(x)^{3/2} \sqrt{\cos(x) \sin(x)}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.18, size = 105, normalized size = 0.97

$$\frac{-4 \cos^3(x) {}_2F_1\left(\frac{3}{4}, \frac{3}{4}; \frac{7}{4}; \cos^2(x)\right) \sin(x) - 3 \cos(x) \sqrt{\sin^2(x)} \left(2 \sin(x) + (-\log(\cos(\frac{x}{2}) - \sin(\frac{x}{2})) + \log(\cos(\frac{x}{2}) + \sin(\frac{x}{2}))) \sqrt{\sin(2x)}\right)}{3 \sqrt{\cos^3(x) \sin(x)} \sqrt{\sin^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[2*x] - Sqrt[Sin[2*x]])/Sqrt[Cos[x]^3*Sin[x]], x]

[Out] (-4*Cos[x]^3*Hypergeometric2F1[3/4, 3/4, 7/4, Cos[x]^2*Sin[x] - 3*Cos[x]*(Sin[x]^2)^(1/4)*(2*Sin[x] + (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])*Sqrt[Sin[2*x]]))/(3*Sqrt[Cos[x]^3*Sin[x]]*(Sin[x]^2)^(1/4))

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(Cos[2*x] - Sqrt[Sin[2*x]])/Sqrt[Sin[x]*Cos[x]^3], x]')

[Out] Timed out

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.52, size = 247, normalized size = 2.29

method	result
default	$-\frac{2 \cos(x) \sin(x)}{\sqrt{(\cos^3(x) \sin(x))}} + \frac{2\sqrt{2} \operatorname{arctanh}\left(\frac{\cos(x)-1}{\sin(x)}\right) \cos(x) \sqrt{\cos(x) \sin(x)}}{\sqrt{(\cos^3(x) \sin(x))}} - \frac{\sqrt{2}}{\sqrt{(\cos^3(x) \sin(x))}} \left(i \operatorname{EllipticPi}\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2), x, method=_RETURNVERBOSE)

[Out] -2*cos(x)*sin(x)/(cos(x)^3*sin(x))^(1/2)+2*2^(1/2)*arctanh((cos(x)-1)/sin(x))*cos(x)*(cos(x)*sin(x))^(1/2)/(cos(x)^3*sin(x))^(1/2)-2^(1/2)*(I*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2), 1/2-1/2*I, 1/2*2^(1/2))-I*EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2), 1/2+1/2*I, 1/2*2^(1/2))+EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2), 1/2-1/2*I, 1/2*2^(1/2))+EllipticPi((-(-1+cos(x)-sin(x))/sin(x))^(1/2), 1/2+1/2*I, 1/2*2^(1/2))-2*EllipticF((-(-1+cos(x)-sin(x))/sin(x))^(1/2), 1/2*2^(1/2)))*cos(x)*sin(x)^2*(-(-1+cos(x)-sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((cos(x)-1)/sin(x))^(1/2)/(cos(x)-1)/(cos(x)^3*sin(x))^(1/2)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x, algorithm="maxima")

[Out] $\frac{1}{2}\sqrt{2}\int 2\left(\left(\cos(4x) + 1\right)\cos\left(\frac{1}{2}\arctan 2\left(\sin(x), -\cos(x) + 1\right)\right) - \sin(4x)\sin\left(\frac{1}{2}\arctan 2\left(\sin(x), -\cos(x) + 1\right)\right)\right)\cos\left(\frac{1}{2}\arctan 2\left(\sin(x), \cos(x) + 1\right)\right) + \left(\cos\left(\frac{1}{2}\arctan 2\left(\sin(x), -\cos(x) + 1\right)\right)\sin(4x) + \left(\cos(4x) + 1\right)\sin\left(\frac{1}{2}\arctan 2\left(\sin(x), -\cos(x) + 1\right)\right)\right)\sin\left(\frac{1}{2}\arctan 2\left(\sin(x), \cos(x) + 1\right)\right)\cos\left(\frac{3}{2}\arctan 2\left(\sin(2x), \cos(2x) + 1\right)\right) + \left(\cos\left(\frac{1}{2}\arctan 2\left(\sin(x), -\cos(x) + 1\right)\right)\sin(4x) + \left(\cos(4x) + 1\right)\sin\left(\frac{1}{2}\arctan 2\left(\sin(x), -\cos(x) + 1\right)\right)\right)\cos\left(\frac{1}{2}\arctan 2\left(\sin(x), \cos(x) + 1\right)\right) - \left(\cos(4x) + 1\right)\cos\left(\frac{1}{2}\arctan 2\left(\sin(x), -\cos(x) + 1\right)\right) - \sin(4x)\sin\left(\frac{1}{2}\arctan 2\left(\sin(x), -\cos(x) + 1\right)\right)\right)\sin\left(\frac{1}{2}\arctan 2\left(\sin(x), \cos(x) + 1\right)\right)\sin\left(\frac{3}{2}\arctan 2\left(\sin(2x), \cos(2x) + 1\right)\right)\right)/\left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right)^{3/4}\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right)^{1/4}\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)^{1/4}, x - \frac{1}{2}\sqrt{2}\int -2\left(\left(\cos\left(\frac{1}{2}\arctan 2\left(\sin(x), -\cos(x) + 1\right)\right)\sin(4x) + \left(\cos(4x) + 1\right)\sin\left(\frac{1}{2}\arctan 2\left(\sin(x), -\cos(x) + 1\right)\right)\right)\cos\left(\frac{1}{2}\arctan 2\left(\sin(x), \cos(x) + 1\right)\right) - \left(\cos(4x) + 1\right)\cos\left(\frac{1}{2}\arctan 2\left(\sin(x), -\cos(x) + 1\right)\right) - \sin(4x)\sin\left(\frac{1}{2}\arctan 2\left(\sin(x), -\cos(x) + 1\right)\right)\right)\sin\left(\frac{1}{2}\arctan 2\left(\sin(x), \cos(x) + 1\right)\right)\cos\left(\frac{3}{2}\arctan 2\left(\sin(2x), \cos(2x) + 1\right)\right) - \left(\left(\cos(4x) + 1\right)\cos\left(\frac{1}{2}\arctan 2\left(\sin(x), -\cos(x) + 1\right)\right) - \sin(4x)\sin\left(\frac{1}{2}\arctan 2\left(\sin(x), -\cos(x) + 1\right)\right)\right)\cos\left(\frac{1}{2}\arctan 2\left(\sin(x), \cos(x) + 1\right)\right) + \left(\cos\left(\frac{1}{2}\arctan 2\left(\sin(x), -\cos(x) + 1\right)\right)\sin(4x) + \left(\cos(4x) + 1\right)\sin\left(\frac{1}{2}\arctan 2\left(\sin(x), -\cos(x) + 1\right)\right)\right)\sin\left(\frac{1}{2}\arctan 2\left(\sin(x), \cos(x) + 1\right)\right)\sin\left(\frac{3}{2}\arctan 2\left(\sin(2x), \cos(2x) + 1\right)\right)\right)/\left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right)^{3/4}\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right)^{1/4}\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)^{1/4}, x - \frac{1}{2}\sqrt{2}\log(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1) + \frac{1}{2}\sqrt{2}\log(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 611 vs. 2(92) = 184.

time = 4.71, size = 611, normalized size = 5.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x, algorithm="fricas")

[Out] $-\frac{1}{4}(2\sqrt{2})\arctan\left(-\frac{1}{2}(2\cos(x))^4 - 2\cos(x)^3\sin(x) - 2\cos(x)^2 - \sqrt{2}\sqrt{\cos(x)^3\sin(x)}\right)\sqrt{(4\cos(x)^2\sin(x) + 2\sqrt{2})\sqrt{\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1}}$

$$\begin{aligned} & x^3 \sin(x) (\cos(x) + \sin(x)) + \cos(x) / \cos(x) - \sqrt{2} \sqrt{\cos(x)^3 \sin(x)} / (\cos(x)^4 + \cos(x)^3 \sin(x) - \cos(x)^2) * \cos(x)^2 + 2 \sqrt{2} \arctan \\ & (1/2 * (2 \cos(x)^4 - 2 \cos(x)^3 \sin(x) - 2 \cos(x)^2 + \sqrt{2} \sqrt{\cos(x)^3 \sin(x)}) * \sqrt{(4 \cos(x)^2 \sin(x) - 2 \sqrt{2} \sqrt{\cos(x)^3 \sin(x)} * (\cos(x) + \sin(x)) + \cos(x)) / \cos(x)} + \sqrt{2} \sqrt{\cos(x)^3 \sin(x)}) / (\cos(x)^4 + \cos(x)^3 \sin(x) - \cos(x)^2) * \cos(x)^2 - 2 \sqrt{2} \arctan(-1/2 * (\sqrt{2} \sqrt{\cos(x)^3 \sin(x)} * (\cos(x) - \sin(x)) + (2 \cos(x)^2 \sin(x) - \sqrt{2} \sqrt{\cos(x)^3 \sin(x)} * (\cos(x) + \sin(x))) * \sqrt{(4 \cos(x)^2 \sin(x) + 2 \sqrt{2} \sqrt{\cos(x)^3 \sin(x)} * (\cos(x) + \sin(x)) + \cos(x)) / \cos(x)}) / (\cos(x)^2 \sin(x))) * \cos(x)^2 - 2 \sqrt{2} \arctan(-1/2 * (\sqrt{2} \sqrt{\cos(x)^3 \sin(x)} * (\cos(x) - \sin(x)) - (2 \cos(x)^2 \sin(x) + \sqrt{2} \sqrt{\cos(x)^3 \sin(x)} * (\cos(x) + \sin(x))) * \sqrt{(4 \cos(x)^2 \sin(x) - 2 \sqrt{2} \sqrt{\cos(x)^3 \sin(x)} * (\cos(x) + \sin(x)) + \cos(x)) / \cos(x)}) / (\cos(x)^2 \sin(x))) * \cos(x)^2 - \sqrt{2} \cos(x)^2 \log((4 \cos(x)^2 \sin(x) + 2 \sqrt{2} \sqrt{\cos(x)^3 \sin(x)} * (\cos(x) + \sin(x)) + \cos(x)) / \cos(x)) + \sqrt{2} \cos(x)^2 \log((4 \cos(x)^2 \sin(x) - 2 \sqrt{2} \sqrt{\cos(x)^3 \sin(x)} * (\cos(x) + \sin(x)) + \cos(x)) / \cos(x)) - \sqrt{2} \cos(x)^2 \log((\cos(x)^6 - 8 \cos(x)^4 + 4 \sqrt{2} \sqrt{\cos(x)^3 \sin(x)} * (\cos(x)^2 - 2) \sqrt{2} \sqrt{\cos(x) \sin(x)} + 8 \cos(x)^2) / \cos(x)^6) + 8 \sqrt{2} \sqrt{\cos(x)^3 \sin(x)}) / \cos(x)^2 \end{aligned}$$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(2*x)-sin(2*x)**(1/2))/(cos(x)**3*sin(x))**(1/2),x)

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(2x) - \sqrt{\sin(2x)}}{\sqrt{\cos(x)^3 \sin(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(2*x) - sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x)

[Out] int((cos(2*x) - sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2), x)

$$3.417 \quad \int \frac{\sqrt{\cos(x) \sin^3(x)} - 2 \sin(2x)}{-\sqrt{\cos^3(x) \sin(x)} + \sqrt{\tan(x)}} dx$$

Optimal. Leaf size=364

$$-2\sqrt{2} \coth^{-1} \left(\frac{\cos(x)(\cos(x) + \sin(x))}{\sqrt{2} \sqrt{\cos^3(x) \sin(x)}} \right) + \sqrt[4]{2} \coth^{-1} \left(\frac{\cos(x) (\sqrt{2} \cos(x) + \sin(x))}{2^{3/4} \sqrt{\cos^3(x) \sin(x)}} \right) - \sqrt[4]{2} \coth^{-1} \left(\frac{\sqrt{2}}{2^{3/4}} \right)$$

```
[Out] 2^(1/4)*arccoth(1/2*cos(x)*(sin(x)+cos(x)*2^(1/2))*2^(1/4)/(cos(x)^3*sin(x)
)^(1/2))-2^(1/4)*arccoth(1/2*(2^(1/2)+tan(x))*2^(1/4)/tan(x)^(1/2))+2^(1/4)
*arctan(1/2*cos(x)*(-sin(x)+cos(x)*2^(1/2))*2^(1/4)/(cos(x)^3*sin(x))^(1/2)
)-2^(1/4)*arctan(1/2*(2^(1/2)-tan(x))*2^(1/4)/tan(x)^(1/2))-2*arccoth(1/2*c
os(x)*(cos(x)+sin(x))*2^(1/2)/(cos(x)^3*sin(x))^(1/2))*2^(1/2)-2*arctan(1/2
*cos(x)*(cos(x)-sin(x))*2^(1/2)/(cos(x)^3*sin(x))^(1/2))*2^(1/2)+4*csc(x)*s
ec(x)*(cos(x)^3*sin(x))^(1/2)+1/4*csc(x)^2*ln(1+cos(x)^2)*sec(x)^2*(cos(x)^
3*sin(x))^(1/2)*(cos(x)*sin(x)^3)^(1/2)+1/2*csc(x)^2*ln(sin(x))*sec(x)^2*(c
os(x)^3*sin(x))^(1/2)*(cos(x)*sin(x)^3)^(1/2)+4/tan(x)^(1/2)-1/4*csc(x)^2*l
n(1+cos(x)^2)*(cos(x)*sin(x)^3)^(1/2)*tan(x)^(1/2)+1/2*csc(x)^2*ln(sin(x))*
(cos(x)*sin(x)^3)^(1/2)*tan(x)^(1/2)
```

Rubi [A]

time = 2.89, antiderivative size = 665, normalized size of antiderivative = 1.83, number of steps used = 66, number of rules used = 21, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.512$,

Rules used = {6857, 6874, 6851, 331, 335, 303, 1176, 631, 210, 1179, 642, 477, 493, 6865, 15, 29, 272, 36, 31, 266, 455}

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[Cos[x]*Sin[x]^3] - 2*Sin[2*x])/(-Sqrt[Cos[x]^3*Ssin[x]] + Sqrt[Tan
[x]]) ,x]
```

```
[Out] -(2^(1/4)*ArcTan[1 - 2^(1/4)*Sqrt[Tan[x]]]) + 2^(1/4)*ArcTan[1 + 2^(1/4)*Sq
rt[Tan[x]]] + Log[Sqrt[2] - 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]/2^(3/4) - Log[Sq
rt[2] + 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]/2^(3/4) + 4*Csc[x]*Sec[x]*Sqrt[Cos[x
]^3*Ssin[x]] - (Csc[x]^2*Log[Sec[x]^2]*Sec[x]^2*Sqrt[Cos[x]^3*Ssin[x]]*Sqrt[C
os[x]*Sin[x]^3])/2 + Csc[x]^2*Log[Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Ssin
[x]]*Sqrt[Cos[x]*Sin[x]^3] + (Csc[x]^2*Log[2 + Tan[x]^2]*Sec[x]^2*Sqrt[Cos[x
]^3*Ssin[x]]*Sqrt[Cos[x]*Sin[x]^3])/4 + (Log[Tan[x]]*Sec[x]^2*Sqrt[Cos[x]*Si
n[x]^3])/(2*Tan[x]^(3/2)) - (Log[2 + Tan[x]^2]*Sec[x]^2*Sqrt[Cos[x]*Sin[x]^
3])/(4*Tan[x]^(3/2)) + 4/Sqrt[Tan[x]] + (2^(1/4)*ArcTan[1 - 2^(1/4)*Sqrt[Ta
n[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Ssin[x]])/Sqrt[Tan[x]] - (2^(1/4)*ArcTan[1 + 2
^(1/4)*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Ssin[x]])/Sqrt[Tan[x]] - (2*Sqrt
```

$$\begin{aligned} & [2] * \text{ArcTan}[1 - \text{Sqrt}[2] * \text{Sqrt}[\text{Tan}[x]]] * \text{Sec}[x]^2 * \text{Sqrt}[\text{Cos}[x]^3 * \text{Sin}[x]] / \text{Sqrt}[\text{Tan}[x]] \\ & + (2 * \text{Sqrt}[2] * \text{ArcTan}[1 + \text{Sqrt}[2] * \text{Sqrt}[\text{Tan}[x]]] * \text{Sec}[x]^2 * \text{Sqrt}[\text{Cos}[x]^3 * \text{Sin}[x]]) / \text{Sqrt}[\text{Tan}[x]] \\ & + (\text{Sqrt}[2] * \text{Log}[1 - \text{Sqrt}[2] * \text{Sqrt}[\text{Tan}[x]] + \text{Tan}[x]] * \text{Sec}[x]^2 * \text{Sqrt}[\text{Cos}[x]^3 * \text{Sin}[x]]) / \text{Sqrt}[\text{Tan}[x]] \\ & - (\text{Sqrt}[2] * \text{Log}[1 + \text{Sqrt}[2] * \text{Sqrt}[\text{Tan}[x]] + \text{Tan}[x]] * \text{Sec}[x]^2 * \text{Sqrt}[\text{Cos}[x]^3 * \text{Sin}[x]]) / \text{Sqrt}[\text{Tan}[x]] \\ & - (\text{Log}[\text{Sqrt}[2] - 2^{(3/4)} * \text{Sqrt}[\text{Tan}[x]] + \text{Tan}[x]] * \text{Sec}[x]^2 * \text{Sqrt}[\text{Cos}[x]^3 * \text{Sin}[x]]) / (2^{(3/4)} * \text{Sqrt}[\text{Tan}[x]]) \\ & + (\text{Log}[\text{Sqrt}[2] + 2^{(3/4)} * \text{Sqrt}[\text{Tan}[x]] + \text{Tan}[x]] * \text{Sec}[x]^2 * \text{Sqrt}[\text{Cos}[x]^3 * \text{Sin}[x]]) / (2^{(3/4)} * \text{Sqrt}[\text{Tan}[x]]) \end{aligned}$$
Rule 15

$$\text{Int}[(u_.) * ((a_.) * (x_)^{(n_)})^{(m_)}, x_Symbol] \rightarrow \text{Dist}[a^{\text{IntPart}[m]} * ((a * x^n)^{\text{FracPart}[m]} / x^{(n * \text{FracPart}[m])}), \text{Int}[u * x^{(m * n)}, x], x] /; \text{FreeQ}\{a, m, n\}, x \&\& \text{IntegerQ}[m]$$
Rule 29

$$\text{Int}[(x_)^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[x], x]$$
Rule 31

$$\text{Int}[(a_.) + (b_.) * (x_)^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b * x, x]] / b, x] /; \text{FreeQ}\{a, b\}, x$$
Rule 36

$$\text{Int}[1 / (((a_.) + (b_.) * (x_)) * ((c_.) + (d_.) * (x_))), x_Symbol] \rightarrow \text{Dist}[b / (b * c - a * d), \text{Int}[1 / (a + b * x), x], x] - \text{Dist}[d / (b * c - a * d), \text{Int}[1 / (c + d * x), x], x] /; \text{FreeQ}\{a, b, c, d\}, x \&\& \text{NeQ}[b * c - a * d, 0]$$
Rule 210

$$\text{Int}[(a_.) + (b_.) * (x_)^2)^{(-1)}, x_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2] * \text{Rt}[-b, 2])^{(-1)} * \text{ArcTan}[\text{Rt}[-b, 2] * (x / \text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b\}, x \&\& \text{PosQ}[a / b] \& \& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$$
Rule 266

$$\text{Int}[(x_)^{(m_.)} / ((a_.) + (b_.) * (x_)^{(n_)}), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b * x^n, x]] / (b * n), x] /; \text{FreeQ}\{a, b, m, n\}, x \&\& \text{EqQ}[m, n - 1]$$
Rule 272

$$\text{Int}[(x_)^{(m_.)} * ((a_.) + (b_.) * (x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1 / n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1) / n] - 1) * (a + b * x)^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x \&\& \text{IntegerQ}[\text{Simplify}[(m + 1) / n]]$$

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 335

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rule 477

```
Int[((e_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
+ 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 493

```
Int[((e_.)*(x_)^(m_.))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
x_Symbol] := Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6851

```
Int[(u_)*((a_)*(v_)^(m_)*(w_)^(n_))^(p_), x_Symbol] := Dist[a^IntPart[p]*(a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))], Int[u*v^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x]
```

Rule 6857

```
Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] := With[{k = Denominator[m]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cos(x) \sin^3(x)} - 2 \sin(2x)}{-\sqrt{\cos^3(x) \sin(x)} + \sqrt{\tan(x)}} dx &= \text{Subst} \left(\int \frac{\sqrt{\frac{x^3}{(1+x^2)^2}} - \frac{4x}{1+x^2}}{(1+x^2) \left(\sqrt{x} - \sqrt{\frac{x}{(1+x^2)^2}} \right)} dx, x, \tan(x) \right) \\
&= \text{Subst} \left(\int \left(\frac{4x}{(1+x^2)^2 \left(-\sqrt{x} + \sqrt{\frac{x}{(1+x^2)^2}} \right)} - \frac{\sqrt{\frac{x}{(1+x^2)^2}}}{(1+x^2) \left(-\sqrt{x} + \sqrt{\frac{x}{(1+x^2)^2}} \right)} \right) dx, x, \tan(x) \right) \\
&= 4 \text{Subst} \left(\int \frac{x}{(1+x^2)^2 \left(-\sqrt{x} + \sqrt{\frac{x}{(1+x^2)^2}} \right)} dx, x, \tan(x) \right) - \text{Subst} \left(\int \frac{\sqrt{\frac{x}{(1+x^2)^2}}}{(1+x^2) \left(-\sqrt{x} + \sqrt{\frac{x}{(1+x^2)^2}} \right)} dx, x, \tan(x) \right) \\
&= 4 \text{Subst} \left(\int \left(-\frac{1}{2x^{3/2}} - \frac{\sqrt{\frac{x}{(1+x^2)^2}}}{2x^2} + \frac{\sqrt{x}}{2(2+x^2)} + \frac{\sqrt{\frac{x}{(1+x^2)^2}}}{2(2+x^2)} \right) dx, x, \tan(x) \right) \\
&= \frac{4}{\sqrt{\tan(x)}} - 2 \text{Subst} \left(\int \frac{\sqrt{\frac{x}{(1+x^2)^2}}}{x^2} dx, x, \tan(x) \right) + 2 \text{Subst} \left(\int \frac{1}{2+x^2} dx, x, \tan(x) \right) \\
&\quad \left(2 \sec^2(x) \sqrt{\tan(x)} \right) \\
&= \frac{4}{\sqrt{\tan(x)}} + 4 \text{Subst} \left(\int \frac{x^2}{2+x^4} dx, x, \sqrt{\tan(x)} \right) - \frac{2 \log \left(\sqrt{2} - 2^{3/4} \sqrt{\tan(x)} + \tan(x) \right)}{\sqrt{\tan(x)}} \\
&= 4 \csc(x) \sec(x) \sqrt{\cos^3(x) \sin(x)} + \frac{4}{\sqrt{\tan(x)}} - 2 \text{Subst} \left(\int \frac{\sqrt{2} - 2^{3/4} \sqrt{\tan(x)}}{2+x^2} dx, x, \sqrt{\tan(x)} \right) \\
&= 4 \csc(x) \sec(x) \sqrt{\cos^3(x) \sin(x)} + \frac{4}{\sqrt{\tan(x)}} + \frac{\text{Subst} \left(\int \frac{2^{3/4} - \sqrt{2}}{-\sqrt{2} - 2^{3/4} \sqrt{\tan(x)}} dx, x, \sqrt{\tan(x)} \right)}{\sqrt{\tan(x)}} \\
&\quad \frac{\log \left(\sqrt{2} - 2^{3/4} \sqrt{\tan(x)} + \tan(x) \right)}{\sqrt{\tan(x)}} - \frac{\log \left(\sqrt{2} + 2^{3/4} \sqrt{\tan(x)} + \tan(x) \right)}{\sqrt{\tan(x)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 7.97, size = 434, normalized size = 1.19

$$\frac{\text{ArcCsc}\left(\frac{\sqrt{\cos(x)}}{\sqrt{2\cos(x)+1}}\right) - \frac{\text{ArcCsc}\left(\frac{\sqrt{2\cos(x)+1}}{\sqrt{2\cos(x)+1}}\right) - 2\log\left(\frac{\sqrt{2\cos(x)+1}}{\sqrt{2\cos(x)+1}}\right) - \log\left(\frac{\sqrt{2\cos(x)+1}}{\sqrt{2\cos(x)+1}}\right) + \frac{\text{ArcCsc}\left(\frac{\sqrt{2\cos(x)+1}}{\sqrt{2\cos(x)+1}}\right)}{\sqrt{2\cos(x)+1}}}{\sqrt{2\cos(x)+1}} \cdot \left((4+4i)\tan^{-1}\left(\frac{\sqrt{2\cos(x)+1}}{\sqrt{-1+\tan(x/2)}}\right) - (4-4i)\tan^{-1}\left(\frac{\sqrt{2\cos(x)+1}}{\sqrt{-1+\tan(x/2)}}\right) + 2^{3/4}\tan^{-1}\left(\frac{\sqrt{2\cos(x)+1}\sqrt{-1+\tan(x/2)}}{\sqrt{2\cos(x)+1}}\right) + 2^{3/4}\tan^{-1}\left(\frac{\sqrt{2\cos(x)+1}\sqrt{-1+\tan(x/2)}}{\sqrt{2\cos(x)+1}}\right) \right) \cos^2(x) \sqrt{\cos(x)} (-1+\tan(x/2))^{3/4} + \frac{1}{\sqrt{2\cos(x)+1}} + \frac{1}{2}\cos^2(x) \left(2\log\left(\frac{\sqrt{2\cos(x)+1}}{\sqrt{2\cos(x)+1}}\right) - \log\left(\frac{\sqrt{2\cos(x)+1}}{\sqrt{2\cos(x)+1}}\right) + \frac{\text{ArcCsc}\left(\frac{\sqrt{2\cos(x)+1}}{\sqrt{2\cos(x)+1}}\right)}{\sqrt{2\cos(x)+1}} \right) \sqrt{2\cos(x)+1} + \frac{\sqrt{2\cos(x)+1} \cos^2(x) \sqrt{\cos(x)} \left(\frac{1}{2} + \frac{\text{ArcCsc}\left(\frac{\sqrt{2\cos(x)+1}}{\sqrt{2\cos(x)+1}}\right)}{\sqrt{2\cos(x)+1}} \right) \tan^2(x)}{\sqrt{2\cos(x)+1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[Cos[x]*Sin[x]^3] - 2*Sin[2*x])/(-Sqrt[Cos[x]^3*Sin[x]] + Sqrt[Tan[x]]), x]

[Out] 4*Csc[x]*Sec[x]*Sqrt[Cos[x]^3*Sin[x]] - (Cos[x]*Csc[x/2]*(4*Log[Sec[x/2]^2] - 2*Log[Tan[x/2]] - Log[1 + Tan[x/2]^4])*Sec[x/2]*Sqrt[Cos[x]*Sin[x]^3])/(8*Sqrt[Cos[x]^3*Sin[x]]) + (((4 + 4*I)*ArcTan[((1 + I)*Sqrt[Tan[x/2]])/Sqrt[-1 + Tan[x/2]^2]] - (4 - 4*I)*ArcTan[((1/2 + I/2)*Sqrt[-1 + Tan[x/2]^2])/Sqrt[Tan[x/2]]] + 2^(3/4)*ArcTan[(2*2^(3/4)*Cos[x/2]^2*Sqrt[Tan[x/2]]*Sqrt[-1 + Tan[x/2]^2])/(2*Cos[x] + Sqrt[2]*Sin[x])) + 2^(3/4)*ArcTanh[(2^(3/4)*Sqrt[Tan[x/2]]*Sqrt[-1 + Tan[x/2]^2])/(-1 + Sqrt[2]*Tan[x/2] + Tan[x/2]^2)]*Cos[x/2]^4*Sqrt[Tan[x/2]]*(-1 + Tan[x/2]^2)^(3/2)/Sqrt[Cos[x]^3*Sin[x]] + 4/Sqrt[Tan[x]] + (Csc[x]^2*(2*Log[Tan[x]] - Log[2 + Tan[x]^2])*Sqrt[Cos[x]*Sin[x]^3]*Sqrt[Tan[x]])/4 + (4*Sqrt[2]*(Cos[x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (2*Sin[x]^2)/(3 + Cos[2*x])]*Tan[x]^(3/2))/(3*(3 + Cos[2*x])^(3/4))

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(Sqrt[Sin[x]^3*Cos[x]] - 2*Sin[2*x])/(Sqrt[Tan[x]] - Sqrt[Sin[x]*Cos[x]^3]), x]')

[Out] Timed out

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 9.42, size = 22968, normalized size = 63.10

method	result	size
default	Expression too large to display	22968

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+tan(x)^(1/2)), x, method=_RETURNVERBOSE)

[Out] result too large to display

$(2)\cos(x) + \sqrt{2}\sin(3x) + 2\sqrt{2}\sin(2x) + \sqrt{2}\sin(x))\cos(4x) - (\sqrt{2}\cos(3x) + 2\sqrt{2}\cos(2x) + \sqrt{2}\cos(x) - \sqrt{2}\sin(3x) - 2\sqrt{2}\sin(2x) - \sqrt{2}\sin(x))\sin(4x) - \sqrt{2}\cos(3x) - 2\sqrt{2}\cos(2x) - \sqrt{2}\cos(x) - \sqrt{2}\sin(3x) - 2\sqrt{2}\sin(2x) - \sqrt{2}\sin(x))\cos(1/2\arctan2(\sin(x), -\cos(x) + 1)) - ((\sqrt{2}\cos(3x) + 2\sqrt{2}\cos(2x) + \sqrt{2}\cos(x) - \sqrt{2}\sin(3x) - 2\sqrt{2}\sin(2x) - \sqrt{2}\sin(x))\cos(4x) + (\sqrt{2}\cos(3x) + 2\sqrt{2}\cos(2x) + \sqrt{2}\cos(x) + \sqrt{2}\sin(3x) + 2\sqrt{2}\sin(2x) + \sqrt{2}\sin(x))\sin(4x) - \sqrt{2}\cos(3x) - 2\sqrt{2}\cos(2x) - \sqrt{2}\cos(x) + \sqrt{2}\sin(3x) + 2\sqrt{2}\sin(2x) + \sqrt{2}\sin(x))\sin(1/2\arctan2(\sin(x), -\cos(x) + 1)))\sin(1/2\arctan2(\sin(x), \cos(x) + 1)))\sin(1/2\arctan2(\sin(2x), \cos(2x) + 1)))/((((2*(2*\cos(2*x) + \cos(x))*\cos(3*x) + \cos(3*x)^2 + 4*\cos(2*x)^2 + 4*\cos(2*x)*\cos(x) + \cos(x)^2 + 2*(2*\sin(2*x) + \sin(x))*\sin(3*x) + \sin(3*x)^2 + 4*\sin(2*x)^2 + 4*\sin(2*x)*\sin(x) + \sin(x)^2)*\cos(1/2\arctan2(\sin(x), -\cos(x) + 1))^2 + (2*(2*\cos(2*x) + \cos(x))*\cos(3*x) + \cos(3*x)^2 + 4*\cos(2*x)^2 + 4*\cos(2*x)*\cos(x) + \cos(x)^2 + 2*(2*\sin(2*x) + \sin(x))*\sin(3*x) + \sin(3*x)^2 + 4*\sin(2*x)^2 + 4*\sin(2*x)*\sin(x) + \sin(x)^2)*\sin(1/2\arctan2(\sin(x), -\cos(x) + 1))^2)*\cos(1/2\arctan2(\sin(x), \cos(x) + 1))^2 + ((2*(2*\cos(2*x) + \cos(x))*\cos(3*x) + \cos(3*x)^2 + 4*\cos(2*x)^2 + 4*\cos(2*x)*\cos(x) + \cos(x)^2 + 2*(2*\sin(2*x) + \sin(x))*\sin(3*x) + \sin(3*x)^2 + 4*\sin(2*x)^2 + 4*\sin(2*x)*\sin(x) + \sin(x)^2)*\cos(1/2\arctan2(\sin(x), -\cos(x) + 1))^2 + (2*(2*\cos(2*x) + \cos(x))*\cos(3*x) + \cos(3*x)^2 + 4*\cos(2*x)^2 + 4*\cos(2*x)*\cos(x) + \cos(x)^2 + 2*(2*\sin(2*x) + \sin(x))*\sin(3*x) + \sin(3*x)^2 + 4*\sin(2*x)^2 + 4*\sin(2*x)*\sin(x) + \sin(x)^2)*\sin(1/2\arctan2(\sin(x), -\cos(x) + 1))^2)*\sin(1/2\arctan2(\sin(x), \cos(x) + 1))^2*(\cos(x)^2 + \sin(x)^2 + 2*\cos(x) + 1)^{(1/4)}*(\cos(x)^2 + \sin(x)^2 - 2*\cos(x) + 1)^{(1/4)}), x) + 2*\integrate(1/4*(\cos(2*x)^2 + \sin(2*x)^2 + 2*\cos(2*x) + 1)^{(1/4)}*(((\sqrt{2}\cos(3*x) - 2*\sqrt{2}\cos(2*x) + \sqrt{2}\cos(x) + \sqrt{2}\sin(3*x) - 2*\sqrt{2}\sin(2*x) + \sqrt{2}\sin(x))\cos(4*x) - (\sqrt{2}\cos(3*x) - 2*\sqrt{2}\cos(2*x) + \sqrt{2}\cos(x) - \sqrt{2}\sin(3*x) + 2*\sqrt{2}\sin(2*x) - \sqrt{2}\sin(x))\sin(4*x) - \sqrt{2}\cos(3*x) + 2*\sqrt{2}\cos(2*x) - \sqrt{2}\cos(x) - \sqrt{2}\sin(3*x) + 2*\sqrt{2}\sin(2*x) - \sqrt{2}\sin(x))\cos(1/2\arctan2(\sin(x), -\cos(x) + 1)) - ((\sqrt{2}\cos(3*x) - 2*\sqrt{2}\cos(2*x) + \sqrt{2}\cos(x) - \sqrt{2}\sin(3*x) + 2*\sqrt{2}\sin(2*x) - \sqrt{2}\sin(x))\cos(4*x) + (\sqrt{2}\cos(3*x) - 2*\sqrt{2}\cos(2*x) + \sqrt{2}\cos(x) + \sqrt{2}\sin(3*x) - 2*\sqrt{2}\sin(2*x) + \sqrt{2}\sin(x))\sin(4*x) - \sqrt{2}\cos(3*x) + 2*\sqrt{2}\cos(2*x) - \sqrt{2}\cos(x) + \sqrt{2}\sin(3*x) - 2*\sqrt{2}\sin(2*x) + \sqrt{2}\sin(x))\sin(1/2\arctan2(\sin(x), -\cos(x) + 1)))\cos(1/2\arctan2(\sin(x), \cos(x) + 1)) + (((\sqrt{2}\cos(3*x) - 2*\sqrt{2}\cos(2*x) + \sqrt{2}\cos(x) - \sqrt{2}\sin(3*x) + 2*\sqrt{2}\sin(2*x) - \sqrt{2}\sin(x))\cos(4*x) + (\sqrt{2}\cos(3*x) - 2*\sqrt{2}\cos(2*x) + \sqrt{2}\cos(x) + \sqrt{2}\sin(3*x) - 2*\sqrt{2}\sin(2*x) + \sqrt{2}\sin(x))\sin(4*x) - \sqrt{2}\cos(3*x) + 2*\sqrt{2}\cos(2*x) - \sqrt{2}\cos(x) + \sqrt{2}\sin(3*x) - 2*\sqrt{2}\sin(2*x) + \sqrt{2}\sin(x))\cos(1/2\arctan2(\sin(x), -\cos(x) + 1)) + ((\sqrt{2}\cos(3*x) - 2*\sqrt{2}\cos(2*x) + \sqrt{2}\cos(x) + \sqrt{2}\sin(3*x) - 2*\sqrt{2}\sin(2*x) + \sqrt{2}\sin(x))\cos($

$$\begin{aligned}
&4x) - (\sqrt{2}\cos(3x) - 2\sqrt{2}\cos(2x) + \sqrt{2}\cos(x) - \sqrt{2}\sin(3x) + 2\sqrt{2}\sin(2x) - \sqrt{2}\sin(x))\sin(4x) - \sqrt{2}\cos(3x) + \\
&2\sqrt{2}\cos(2x) - \sqrt{2}\cos(x) - \sqrt{2}\sin(3x) + 2\sqrt{2}\sin(2x) \\
&) - \sqrt{2}\sin(x)\sin(1/2\arctan2(\sin(x), -\cos(x) + 1)))\sin(1/2\arctan2(\sin(x), \cos(x) + 1)))\cos(1/2\arctan2(\sin(2x), \cos(2x) + 1)) - (((\sqrt{2} \\
&)\cos(3x) - 2\sqrt{2}\cos(2x) + \sqrt{2}\cos(x) - \sqrt{2}\sin(3x) + 2\sqrt{2}\sin(2x) - \sqrt{2}\sin(x))\cos(4x) + (\sqrt{2}\cos(3x) - 2\sqrt{2}\cos(2x) + \sqrt{2}\cos(x) + \sqrt{2}\sin(3x) - 2\sqrt{2}\sin(2x) + \sqrt{2}\sin(x))\cos(1/2\arctan2(\sin(x), -\cos(x) + 1)) + ((\sqrt{2}\cos(3x) - 2\sqrt{2}\cos(2x) + \sqrt{2}\cos(x) + \sqrt{2}\sin(3x) - 2\sqrt{2}\sin(2x) + \sqrt{2}\sin(x))\cos(4x) - (\sqrt{2}\cos(3x) - 2\sqrt{2}\cos(2x) + \sqrt{2}\cos(x) - \sqrt{2}\sin(3x) + 2\sqrt{2}\sin(2x) - \sqrt{2}\sin(x))\sin(4x) - \sqrt{2}\cos(3x) + 2\sqrt{2}\cos(2x) - \sqrt{2}\cos(x) - \sqrt{2}\sin(3x) + 2\sqrt{2}\sin(2x) - \sqrt{2}\sin(x))\sin(1/2\arctan2(\sin(x), -\cos(x) + 1)))\cos(1/2\arctan2(\sin(x), \cos(x) + 1)) - (((\sqrt{2}\cos(3x) - 2\sqrt{2}\cos(2x) + \sqrt{2}\cos(x) + \sqrt{2}\sin(3x) - 2\sqrt{2}\sin(2x) + \sqrt{2}\sin(x))\cos(4x) - (\sqrt{2}\cos(3x) - 2\sqrt{2}\cos(2x) + \sqrt{2}\cos(x) - \sqrt{2}\sin(3x) + 2\sqrt{2}\sin(2x) - \sqrt{2}\sin(x))\sin(4x) - \sqrt{2}\cos(3x) + 2\sqrt{2}\cos(2x) - \sqrt{2}\cos(x) - \sqrt{2}\sin(3x) + 2\sqrt{2}\sin(2x) - \sqrt{2}\sin(x))\cos(1/2\arctan2(\sin(x), -\cos(x) + 1)) - (((\sqrt{2}\cos(3x) - 2\sqrt{2}\cos(2x) + \sqrt{2}\cos(x) - \sqrt{2}\sin(3x) + 2\sqrt{2}\sin(2x) - \sqrt{2}\sin(x))\cos(4x) + (\sqrt{2}\cos(3x) - 2\sqrt{2}\cos(2x) + \sqrt{2}\cos(x) + \sqrt{2}\sin(3x) - 2\sqrt{2}\sin(2x) + \sqrt{2}\sin(x))\sin(4x) - \sqrt{2}\cos(3x) + 2\sqrt{2}\cos(2x) - \sqrt{2}\cos(x) + \sqrt{2}\sin(3x) - 2\sqrt{2}\sin(2x) + \sqrt{2}\sin(x))\sin(1/2\arctan2(\sin(x), -\cos(x) + 1)))\sin(1/2\arctan2(\sin(x), \cos(x) + 1)))\sin(1/2\arctan2(\sin(2x), \cos(2x) + 1))) / (((2*(2*\cos(2x) - \cos(x))*\cos(3x) - \cos(3x)^2 - 4*\cos(2x)^2 + 4*\cos(2x)*\cos(x) - \cos(x)^2 + 2*(2*\sin(2x) - \sin(x))*\sin(3x) - \sin(3x)^2 - 4*\sin(2x)^2 + 4*\sin(2x)*\sin(x) - \sin(x)^2)*\cos(1/2\arctan2(\sin(x), -\cos(x) + 1))^2 + (2*(2*\cos(2x) - \cos(x))*\cos(3x) - \cos(3x)^2 - 4*\cos(2x)^2 + 4*\cos(2x)*\cos(x) - \cos(x)^2 + 2*(2*\sin(2x) - \sin(x))*\sin(3x) - \sin(3x)^2 - 4*\sin(2x)^2 + 4*\sin(2x)*\sin(x) - \sin(x)^2)*\sin(1/2\arctan2(\sin(x), -\cos(x) + 1))^2*\cos(1/2\arctan2(\sin(x), \cos(x) + 1))^2 + ((2*(2*\cos(2x) - \cos(x))*\cos(3x) - \cos(3x)^2 - 4*\cos(2x)^2 + 4*\cos(2x)*\cos(x) - \cos(x)^2 + 2*(2*\sin(2x) - \sin(x))*\sin(3x) - \sin(3x)^2 - 4*\sin(2x)^2 + 4*\sin(2x)*\sin(x) - \sin(x)^2)*\cos(1/2\arctan2(\sin(x), -\cos(x) + 1))^2 + (2*(2*\cos(2x) - \cos(x))*\cos(3x) - \cos(3x)^2 - 4*\cos(2x)^2 + 4*\cos(2x)*\cos(x) - \cos(x)^2 + 2*(2*\sin(2x) - \sin(x))*\sin(3x) - \sin(3x)^2 - 4*\sin(2x)^2 + 4*\sin(2x)*\sin(x) - \sin(x)^2)*\sin(1/2\arctan2(\sin(x), -\cos(x) + 1))^2*\sin(1/2\arctan2(\sin(x), \cos(x) + 1))^2*(\cos(x)^2 + \sin(x)^2 + 2*\cos(x) + 1)^(1/4))*(\cos(x)^2 + \sin(x)^2 - 2*\cos(x) + 1)^(1/4)), x) + 1/2*log(\cos(x)^2 + \sin(x)^2 + 2*\cos(x) + 1) + 1/2*log(\cos(x)^2 + \sin(x)^2 - 2*\cos(x) + 1)
\end{aligned}$$

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+tan(x)^(1/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: not invertible
```

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*sin(2*x)+(cos(x)*sin(x)**3)**(1/2))/(-(cos(x)**3*sin(x))**(1/2)+tan(x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+tan(x)^(1/2)),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{2 \sin(2x) - \sqrt{\cos(x) \sin(x)^3}}{\sqrt{\tan(x)} - \sqrt{\cos(x)^3 \sin(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*sin(2*x) - (cos(x)*sin(x)^3)^(1/2))/(tan(x)^(1/2) - (cos(x)^3*sin(x))^(1/2)),x)
```

```
[Out] -int((2*sin(2*x) - (cos(x)*sin(x)^3)^(1/2))/(tan(x)^(1/2) - (cos(x)^3*sin(x))^(1/2)), x)
```

$$3.418 \quad \int \frac{-3 \tan(x) + \sqrt[3]{\sec^6(x) \tan(x)}}{(\cos^5(x) \sin(x))^{2/3}} dx$$

Optimal. Leaf size=125

$$-\frac{9 \sin^4(x)}{10 (\cos^5(x) \sin(x))^{2/3}} - \frac{9}{4} \sec^8(x) (\cos^5(x) \sin(x))^{4/3} + \frac{3}{2} \sqrt[3]{\cos^5(x) \sin(x)} \sqrt[3]{\sec^6(x) \tan(x)} + \frac{3}{4} \sqrt[3]{\cos^5(x) \sin(x)}$$

[Out] $-9/10*\sin(x)^4/(\cos(x)^5*\sin(x))^{(2/3)}-9/4*\sec(x)^8*(\cos(x)^5*\sin(x))^{(4/3)}+3/2*(\cos(x)^5*\sin(x))^{(1/3)}*(\sec(x)^6*\tan(x))^{(1/3)}+3/4*(\cos(x)^5*\sin(x))^{(1/3)}*\tan(x)^2*(\sec(x)^6*\tan(x))^{(1/3)}+3/14*(\cos(x)^5*\sin(x))^{(1/3)}*\tan(x)^4*(\sec(x)^6*\tan(x))^{(1/3)}$

Rubi [A]

time = 0.63, antiderivative size = 141, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6851, 6865, 6874, 14}

$$-\frac{9 \sin^4(x)}{10 (\sin(x) \cos^5(x))^{2/3}} - \frac{9 \sin^2(x) \cos^2(x)}{4 (\sin(x) \cos^5(x))^{2/3}} + \frac{3 \sin^5(x) \cos(x) \sqrt[3]{\tan(x) \sec^6(x)}}{14 (\sin(x) \cos^5(x))^{2/3}} + \frac{3 \sin(x) \cos^5(x) \sqrt[3]{\tan(x) \sec^6(x)}}{2 (\sin(x) \cos^5(x))^{2/3}} + \frac{3 \sin^3(x) \cos^3(x) \sqrt[3]{\tan(x) \sec^6(x)}}{4 (\sin(x) \cos^5(x))^{2/3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-3*\text{Tan}[x] + (\text{Sec}[x]^6*\text{Tan}[x])^{(1/3)})/(\text{Cos}[x]^5*\text{Sin}[x])^{(2/3)}, x]$

[Out] $(-9*\text{Cos}[x]^2*\text{Sin}[x]^2)/(4*(\text{Cos}[x]^5*\text{Sin}[x])^{(2/3)}) - (9*\text{Sin}[x]^4)/(10*(\text{Cos}[x]^5*\text{Sin}[x])^{(2/3)}) + (3*\text{Cos}[x]^5*\text{Sin}[x]*(\text{Sec}[x]^6*\text{Tan}[x])^{(1/3)})/(2*(\text{Cos}[x]^5*\text{Sin}[x])^{(2/3)}) + (3*\text{Cos}[x]^3*\text{Sin}[x]^3*(\text{Sec}[x]^6*\text{Tan}[x])^{(1/3)})/(4*(\text{Cos}[x]^5*\text{Sin}[x])^{(2/3)}) + (3*\text{Cos}[x]*\text{Sin}[x]^5*(\text{Sec}[x]^6*\text{Tan}[x])^{(1/3)})/(14*(\text{Cos}[x]^5*\text{Sin}[x])^{(2/3)})$

Rule 14

$\text{Int}[(u_*)*((c_*)(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}\{c, m\}, x \ \&\& \ \text{SumQ}[u] \ \&\& \ !\text{LinearQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (a_*) + (b_*)(v_)] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{InverseFunctionQ}[v]$

Rule 6851

$\text{Int}[(u_*)*((a_*)(v_))^{(m_.)}*(w_))^{(n_.)}^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[a^{\text{IntPart}[p]} * ((a*v^m*w^n)^{\text{FracPart}[p]} / (v^{(m*\text{FracPart}[p])} * w^{(n*\text{FracPart}[p])}))], \text{Int}[u*v^{(m*p)}*w^{(n*p)}, x], x] /; \text{FreeQ}\{a, m, n, p\}, x \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ !\text{FreeQ}[v, x] \ \&\& \ !\text{FreeQ}[w, x]$

Rule 6865

$\text{Int}[(u_*)(x_))^{(m_.)}, x_Symbol] \rightarrow \text{With}\{k = \text{Denominator}[m]\}, \text{Dist}[k, \text{Subst}[\text{Int}[x^{(k*(m+1)-1)}*(u /. x \rightarrow x^k), x], x, x^{(1/k)}], x] /; \text{FractionQ}[m]$

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3 \tan(x) + \sqrt[3]{\sec^6(x) \tan(x)}}{(\cos^5(x) \sin(x))^{2/3}} dx &= \text{Subst} \left(\int \frac{-3x + \sqrt[3]{x(1+x^2)^3}}{\left(\frac{x}{(1+x^2)^3}\right)^{2/3} (1+x^2)} dx, x, \tan(x) \right) \\
&= \frac{(\cos^4(x) \tan^{\frac{2}{3}}(x)) \text{Subst} \left(\int \frac{(1+x^2) \left(-3x + \sqrt[3]{x(1+x^2)^3}\right)}{x^{2/3}} dx, x, \tan(x) \right)}{(\cos^5(x) \sin(x))^{2/3}} \\
&= \frac{(3 \cos^4(x) \tan^{\frac{2}{3}}(x)) \text{Subst} \left(\int (1+x^6) \left(-3x^3 + \sqrt[3]{x^3(1+x^6)^3}\right) dx \right)}{(\cos^5(x) \sin(x))^{2/3}} \\
&= \frac{(3 \cos^4(x) \tan^{\frac{2}{3}}(x)) \text{Subst} \left(\int \left(-3x^3 + \sqrt[3]{x^3(1+x^6)^3} - x^6 \left(3x^3 - \sqrt[3]{x^3(1+x^6)^3}\right)\right) dx \right)}{(\cos^5(x) \sin(x))^{2/3}} \\
&= -\frac{9 \cos^2(x) \sin^2(x)}{4 (\cos^5(x) \sin(x))^{2/3}} + \frac{(3 \cos^4(x) \tan^{\frac{2}{3}}(x)) \text{Subst} \left(\int \sqrt[3]{x^3(1+x^6)^3} dx \right)}{(\cos^5(x) \sin(x))^{2/3}} \\
&= -\frac{9 \cos^2(x) \sin^2(x)}{4 (\cos^5(x) \sin(x))^{2/3}} - \frac{(3 \cos^4(x) \tan^{\frac{2}{3}}(x)) \text{Subst} \left(\int \left(3x^9 - x^6 \sqrt[3]{x^3(1+x^6)^3}\right) dx \right)}{(\cos^5(x) \sin(x))^{2/3}} \\
&= -\frac{9 \cos^2(x) \sin^2(x)}{4 (\cos^5(x) \sin(x))^{2/3}} - \frac{9 \sin^4(x)}{10 (\cos^5(x) \sin(x))^{2/3}} + \frac{(3 \cos^4(x) \tan^{\frac{2}{3}}(x)) \text{Subst} \left(\int \sqrt[3]{x^3(1+x^6)^3} dx \right)}{(\cos^5(x) \sin(x))^{2/3}} \\
&= -\frac{9 \cos^2(x) \sin^2(x)}{4 (\cos^5(x) \sin(x))^{2/3}} - \frac{9 \sin^4(x)}{10 (\cos^5(x) \sin(x))^{2/3}} + \frac{3 \cos^5(x) \sin(x) \sqrt[3]{\sec^6(x) \tan(x)}}{2 (\cos^5(x) \sin(x))^{2/3}} \\
&= -\frac{9 \cos^2(x) \sin^2(x)}{4 (\cos^5(x) \sin(x))^{2/3}} - \frac{9 \sin^4(x)}{10 (\cos^5(x) \sin(x))^{2/3}} + \frac{3 \cos^5(x) \sin(x) \sqrt[3]{\sec^6(x) \tan(x)}}{2 (\cos^5(x) \sin(x))^{2/3}} \\
&= -\frac{9 \cos^2(x) \sin^2(x)}{4 (\cos^5(x) \sin(x))^{2/3}} - \frac{9 \sin^4(x)}{10 (\cos^5(x) \sin(x))^{2/3}} + \frac{3 \cos^5(x) \sin(x) \sqrt[3]{\sec^6(x) \tan(x)}}{2 (\cos^5(x) \sin(x))^{2/3}}
\end{aligned}$$

Mathematica [A]

time = 0.23, size = 58, normalized size = 0.46

$$\frac{3 \sin(x) \left(924 \sin(x) + 252 \sin(3x) - 5(158 \cos(x) + 57 \cos(3x) + 9 \cos(5x)) \sqrt[3]{\sec^6(x) \tan(x)} \right)}{2240 (\cos^5(x) \sin(x))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(-3*Tan[x] + (Sec[x]^6*Tan[x])^(1/3))/(Cos[x]^5*Sin[x])^(2/3), x]

[Out] (-3*Sin[x]*(924*Sin[x] + 252*Sin[3*x] - 5*(158*Cos[x] + 57*Cos[3*x] + 9*Cos[5*x])*(Sec[x]^6*Tan[x])^(1/3)))/(2240*(Cos[x]^5*Sin[x])^(2/3))

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[((Sin[x]/Cos[x]^7)^(1/3) - 3*Tan[x])/(Sin[x]*Cos[x]^5)^(2/3), x]')

[Out] Timed out

Maple [F]

time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{\sin(x)}{\cos(x)^7} \right)^{\frac{1}{3}} - 3 \tan(x)}{((\cos^5(x)) \sin(x))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3), x)

[Out] int(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3), x)

Maxima [A]

time = 0.38, size = 60, normalized size = 0.48

$$-\frac{3}{20} \tan(x)^{\frac{20}{3}} - \frac{3}{7} \tan(x)^{\frac{14}{3}} - \frac{9}{10} \tan(x)^{\frac{10}{3}} - \frac{3}{8} \tan(x)^{\frac{8}{3}} - \frac{9}{4} \tan(x)^{\frac{4}{3}} + \frac{3(14 \tan(x)^7 + 60 \tan(x)^5 + 105 \tan(x)^3 + 140 \tan(x))}{280 \tan(x)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3), x, algorithm="maxima")

[Out] $-3/20*\tan(x)^{(20/3)} - 3/7*\tan(x)^{(14/3)} - 9/10*\tan(x)^{(10/3)} - 3/8*\tan(x)^{(8/3)} - 9/4*\tan(x)^{(4/3)} + 3/280*(14*\tan(x)^7 + 60*\tan(x)^5 + 105*\tan(x)^3 + 140*\tan(x))/\tan(x)^{(1/3)}$

Fricas [A]

time = 0.32, size = 56, normalized size = 0.45

$$\frac{3 (\cos(x)^5 \sin(x))^{\frac{1}{3}} \left(21 (3 \cos(x)^2 + 2) \sin(x) - 5 (9 \cos(x)^5 + 3 \cos(x)^3 + 2 \cos(x)) \left(\frac{\sin(x)}{\cos(x)^7} \right)^{\frac{1}{3}} \right)}{140 \cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, algorithm="fricas")`

[Out] $-3/140*(\cos(x)^5*\sin(x))^{(1/3)}*(21*(3*\cos(x)^2 + 2)*\sin(x) - 5*(9*\cos(x)^5 + 3*\cos(x)^3 + 2*\cos(x))*(\sin(x)/\cos(x)^7)^{(1/3)})/\cos(x)^5$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((sin(x)/cos(x)**7)**(1/3)-3*tan(x))/(cos(x)**5*sin(x))**(2/3),x)`

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x)`

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{3 \tan(x) - \left(\frac{\sin(x)}{\cos(x)^7} \right)^{1/3}}{(\cos(x)^5 \sin(x))^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(3*tan(x) - (sin(x)/cos(x)^7)^(1/3))/(cos(x)^5*sin(x))^(2/3),x)`

[Out] `int(-(3*tan(x) - (sin(x)/cos(x)^7)^(1/3))/(cos(x)^5*sin(x))^(2/3), x)`

3.419 $\int (1 + 2 \cos^2(x))^{5/2} \sin(x) dx$

Optimal. Leaf size=73

$$-\frac{5 \sinh^{-1}(\sqrt{2} \cos(x))}{16\sqrt{2}} - \frac{5}{16} \cos(x) \sqrt{1 + 2 \cos^2(x)} - \frac{5}{24} \cos(x) (1 + 2 \cos^2(x))^{3/2} - \frac{1}{6} \cos(x) (1 + 2 \cos^2(x))^{5/2}$$

[Out] $-5/24*\cos(x)*(1+2*\cos(x)^2)^{(3/2)}-1/6*\cos(x)*(1+2*\cos(x)^2)^{(5/2)}-5/32*\operatorname{arcsinh}(\cos(x)*2^{(1/2)})*2^{(1/2)}-5/16*\cos(x)*(1+2*\cos(x)^2)^{(1/2)}$

Rubi [A]

time = 0.03, antiderivative size = 67, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3269, 201, 221}

$$-\frac{1}{6} \cos(x)(\cos(2x) + 2)^{5/2} - \frac{5}{24} \cos(x)(\cos(2x) + 2)^{3/2} - \frac{5}{16} \cos(x) \sqrt{\cos(2x) + 2} - \frac{5 \sinh^{-1}(\sqrt{2} \cos(x))}{16\sqrt{2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(1 + 2*\operatorname{Cos}[x]^2)^{(5/2)}*\operatorname{Sin}[x], x]$

[Out] $(-5*\operatorname{ArcSinh}[\operatorname{Sqrt}[2]*\operatorname{Cos}[x]])/(16*\operatorname{Sqrt}[2]) - (5*\operatorname{Cos}[x]*\operatorname{Sqrt}[2 + \operatorname{Cos}[2*x]])/16 - (5*\operatorname{Cos}[x]*(2 + \operatorname{Cos}[2*x])^{(3/2)})/24 - (\operatorname{Cos}[x]*(2 + \operatorname{Cos}[2*x])^{(5/2)})/6$

Rule 201

$\operatorname{Int}[(a + (b \cdot x)^n)^p, x_Symbol] \rightarrow \operatorname{Simp}[x * ((a + b * x^n)^p / (n * p + 1)), x] + \operatorname{Dist}[a * n * (p / (n * p + 1)), \operatorname{Int}[(a + b * x^n)^{p-1}, x], x] /;$ FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 221

$\operatorname{Int}[1/\operatorname{Sqrt}[(a + (b \cdot x)^2)], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[\operatorname{Rt}[b, 2] * (x/\operatorname{Sqrt}[a])]/\operatorname{Rt}[b, 2], x] /;$ FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 3269

$\operatorname{Int}[\cos[(e + (f \cdot x)^m) * ((a + (b \cdot x) \sin[(e + (f \cdot x)^m])^2)^{p-1}), x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Sin}[e + f * x], x]\}, \operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2 * x^2)^{((m-1)/2)} * (a + b * ff^2 * x^2)^p, x], x, \operatorname{Sin}[e + f * x]/ff], x] /;$ FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m-1)/2]

Rubi steps

$$\begin{aligned}
\int (1 + 2 \cos^2(x))^{5/2} \sin(x) dx &= -\text{Subst} \left(\int (1 + 2x^2)^{5/2} dx, x, \cos(x) \right) \\
&= -\frac{1}{6} \cos(x)(2 + \cos(2x))^{5/2} - \frac{5}{6} \text{Subst} \left(\int (1 + 2x^2)^{3/2} dx, x, \cos(x) \right) \\
&= -\frac{5}{24} \cos(x)(2 + \cos(2x))^{3/2} - \frac{1}{6} \cos(x)(2 + \cos(2x))^{5/2} - \frac{5}{8} \text{Subst} \left(\int \sqrt{1 + 2x^2} dx, x, \cos(x) \right) \\
&= -\frac{5}{16} \cos(x) \sqrt{2 + \cos(2x)} - \frac{5}{24} \cos(x)(2 + \cos(2x))^{3/2} - \frac{1}{6} \cos(x)(2 + \cos(2x))^{5/2} \\
&= -\frac{5 \sinh^{-1}(\sqrt{2} \cos(x))}{16\sqrt{2}} - \frac{5}{16} \cos(x) \sqrt{2 + \cos(2x)} - \frac{5}{24} \cos(x)(2 + \cos(2x))^{3/2}
\end{aligned}$$

Mathematica [A]

time = 0.08, size = 61, normalized size = 0.84

$$\frac{1}{96} \left(-2\sqrt{2 + \cos(2x)} (92 \cos(x) + 23 \cos(3x) + 2 \cos(5x)) - 15\sqrt{2} \log \left(\sqrt{2} \cos(x) + \sqrt{2 + \cos(2x)} \right) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + 2*Cos[x]^2)^(5/2)*Sin[x], x]`

```
[Out] (-2*Sqrt[2 + Cos[2*x]]*(92*Cos[x] + 23*Cos[3*x] + 2*Cos[5*x]) - 15*Sqrt[2]*
Log[Sqrt[2]*Cos[x] + Sqrt[2 + Cos[2*x]]])/96
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(2*Cos[x]^2 + 1)^(5/2)*Sin[x], x]')`

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Maple [A]

time = 0.04, size = 56, normalized size = 0.77

method	result
derivativedivides	$-\frac{5 \cos(x)(1+2(\cos^2(x)))^{\frac{3}{2}}}{24} - \frac{\cos(x)(1+2(\cos^2(x)))^{\frac{5}{2}}}{6} - \frac{5 \operatorname{arcsinh}(\cos(x)\sqrt{2})\sqrt{2}}{32} - \frac{5 \cos(x)\sqrt{1+2(\cos^2(x))}}{16}$

default	$-\frac{5 \cos(x)(1+2(\cos^2(x)))^{\frac{3}{2}}}{24} - \frac{\cos(x)(1+2(\cos^2(x)))^{\frac{5}{2}}}{6} - \frac{5 \operatorname{arcsinh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{32} - \frac{5 \cos(x)\sqrt{1+2(\cos^2(x))}}{16}$
---------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+2*cos(x)^2)^(5/2)*sin(x),x,method=_RETURNVERBOSE)`

[Out] $-5/24*\cos(x)*(1+2*\cos(x)^2)^{(3/2)}-1/6*\cos(x)*(1+2*\cos(x)^2)^{(5/2)}-5/32*\operatorname{arcsinh}(\cos(x)*2^{(1/2)})*2^{(1/2)}-5/16*\cos(x)*(1+2*\cos(x)^2)^{(1/2)}$

Maxima [A]

time = 0.32, size = 55, normalized size = 0.75

$$-\frac{1}{6}(2\cos(x)^2+1)^{\frac{5}{2}}\cos(x) - \frac{5}{24}(2\cos(x)^2+1)^{\frac{3}{2}}\cos(x) - \frac{5}{32}\sqrt{2}\operatorname{arsinh}\left(\sqrt{2}\cos(x)\right) - \frac{5}{16}\sqrt{2\cos(x)^2+1}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*cos(x)^2)^(5/2)*sin(x),x, algorithm="maxima")`

[Out] $-1/6*(2*\cos(x)^2+1)^{(5/2)}*\cos(x) - 5/24*(2*\cos(x)^2+1)^{(3/2)}*\cos(x) - 5/32*\operatorname{sqrt}(2)*\operatorname{arcsinh}(\operatorname{sqrt}(2)*\cos(x)) - 5/16*\operatorname{sqrt}(2*\cos(x)^2+1)*\cos(x)$

Fricas [A]

time = 0.35, size = 108, normalized size = 1.48

$$-\frac{1}{48}(32\cos(x)^5+52\cos(x)^3+33\cos(x))\sqrt{2\cos(x)^2+1} + \frac{5}{256}\sqrt{2}\log\left(2048\cos(x)^8+2048\cos(x)^6+640\cos(x)^4+64\cos(x)^2-8(128\sqrt{2}\cos(x)^7+96\sqrt{2}\cos(x)^5+20\sqrt{2}\cos(x)^3+\sqrt{2}\cos(x))\sqrt{2\cos(x)^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*cos(x)^2)^(5/2)*sin(x),x, algorithm="fricas")`

[Out] $-1/48*(32*\cos(x)^5+52*\cos(x)^3+33*\cos(x))*\operatorname{sqrt}(2*\cos(x)^2+1) + 5/256*\operatorname{sqrt}(2)*\log(2048*\cos(x)^8+2048*\cos(x)^6+640*\cos(x)^4+64*\cos(x)^2-8*(128*\operatorname{sqrt}(2)*\cos(x)^7+96*\operatorname{sqrt}(2)*\cos(x)^5+20*\operatorname{sqrt}(2)*\cos(x)^3+\operatorname{sqrt}(2)*\cos(x))*\operatorname{sqrt}(2*\cos(x)^2+1)+1)$

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*cos(x)**2)**(5/2)*sin(x),x)`

[Out] Exception raised: SystemError

Giac [A]

time = 0.01, size = 73, normalized size = 1.00

$$2\left(\left(-\frac{13}{24}-\frac{1}{3}\cos x\cdot\cos x\right)\cos x\cdot\cos x - \frac{11}{32}\right)\cos x\cdot\sqrt{2\cos^2 x+1} + \frac{5\ln\left(\sqrt{2\cos^2 x+1}-\sqrt{2}\cos x\right)}{16\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*cos(x)^2)^(5/2)*sin(x),x)

[Out] -1/48*(4*(8*cos(x)^2 + 13)*cos(x)^2 + 33)*sqrt(2*cos(x)^2 + 1)*cos(x) + 5/3
2*sqrt(2)*log(-sqrt(2)*cos(x) + sqrt(2*cos(x)^2 + 1))

Mupad [B]

time = 0.13, size = 43, normalized size = 0.59

$$-\frac{5\sqrt{2}\operatorname{asinh}\left(\sqrt{2}\cos(x)\right)}{32} - \frac{\sqrt{2}\sqrt{\cos(x)^2 + \frac{1}{2}}\left(\frac{4\cos(x)^5}{3} + \frac{13\cos(x)^3}{6} + \frac{11\cos(x)}{8}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)*(2*cos(x)^2 + 1)^(5/2),x)

[Out] - (5*2^(1/2)*asinh(2^(1/2)*cos(x)))/32 - (2^(1/2)*(cos(x)^2 + 1/2)^(1/2)*((
11*cos(x))/8 + (13*cos(x)^3)/6 + (4*cos(x)^5)/3))/2

$$3.420 \quad \int \cos(x) (5 \cos^2(x) + \sin^2(x))^{5/2} dx$$

Optimal. Leaf size=69

$$\frac{625}{32} \sin^{-1} \left(\frac{2 \sin(x)}{\sqrt{5}} \right) + \frac{125}{16} \sin(x) \sqrt{5 - 4 \sin^2(x)} + \frac{25}{24} \sin(x) (5 - 4 \sin^2(x))^{3/2} + \frac{1}{6} \sin(x) (5 - 4 \sin^2(x))^{5/2}$$

[Out] 625/32*arcsin(2/5*sin(x)*5^(1/2))+25/24*sin(x)*(5-4*sin(x)^2)^(3/2)+1/6*sin(x)*(5-4*sin(x)^2)^(5/2)+125/16*sin(x)*(5-4*sin(x)^2)^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$,

Rules used = {4441, 201, 222}

$$\frac{625}{32} \sin^{-1} \left(\frac{2 \sin(x)}{\sqrt{5}} \right) + \frac{1}{6} \sin(x) (5 - 4 \sin^2(x))^{5/2} + \frac{25}{24} \sin(x) (5 - 4 \sin^2(x))^{3/2} + \frac{125}{16} \sin(x) \sqrt{5 - 4 \sin^2(x)}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*(5*Cos[x]^2 + Sin[x]^2)^(5/2),x]

[Out] (625*ArcSin[(2*Sin[x])/Sqrt[5]]/32 + (125*Sin[x]*Sqrt[5 - 4*Sin[x]^2])/16 + (25*Sin[x]*(5 - 4*Sin[x]^2)^(3/2))/24 + (Sin[x]*(5 - 4*Sin[x]^2)^(5/2))/6

Rule 201

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x*((a + b*x^n)^p/(n*p + 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 4441

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned}
\int \cos(x) (5 \cos^2(x) + \sin^2(x))^{5/2} dx &= \text{Subst} \left(\int (5 - 4x^2)^{5/2} dx, x, \sin(x) \right) \\
&= \frac{1}{6} \sin(x) (5 - 4 \sin^2(x))^{5/2} + \frac{25}{6} \text{Subst} \left(\int (5 - 4x^2)^{3/2} dx, x, \sin(x) \right) \\
&= \frac{25}{24} \sin(x) (5 - 4 \sin^2(x))^{3/2} + \frac{1}{6} \sin(x) (5 - 4 \sin^2(x))^{5/2} + \frac{125}{8} \text{Subst} \left(\int (5 - 4x^2)^{1/2} dx, x, \sin(x) \right) \\
&= \frac{125}{16} \sin(x) \sqrt{5 - 4 \sin^2(x)} + \frac{25}{24} \sin(x) (5 - 4 \sin^2(x))^{3/2} + \frac{1}{6} \sin(x) (5 - 4 \sin^2(x))^{5/2} \\
&= \frac{625}{32} \sin^{-1} \left(\frac{2 \sin(x)}{\sqrt{5}} \right) + \frac{125}{16} \sin(x) \sqrt{5 - 4 \sin^2(x)} + \frac{25}{24} \sin(x) (5 - 4 \sin^2(x))^{3/2}
\end{aligned}$$

Mathematica [A]

time = 0.05, size = 48, normalized size = 0.70

$$\frac{1}{96} \left(1875 \sin^{-1} \left(\frac{2 \sin(x)}{\sqrt{5}} \right) + 2 \sqrt{3 + 2 \cos(2x)} (515 \sin(x) + 90 \sin(3x) + 8 \sin(5x)) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[x]*(5*Cos[x]^2 + Sin[x]^2)^(5/2),x]``[Out] (1875*ArcSin[(2*Sin[x])/Sqrt[5]] + 2*Sqrt[3 + 2*Cos[2*x]]*(515*Sin[x] + 90*Sin[3*x] + 8*Sin[5*x]))/96`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(5*Cos[x]^2 + Sin[x]^2)^(5/2)*Cos[x],x]')``[Out] Exception raised: SystemError >> excessive stack use: stack is 4846 deep`**Maple [A]**

time = 0.16, size = 103, normalized size = 1.49

method	result
default	$\frac{\sqrt{(4(\cos^2(x) + 1)(\sin^2(x)))} \left(512 \sqrt{-4(\sin^4(x) + 5(\sin^2(x)))} (\sin^4(x)) - 2080 \sqrt{-4(\sin^4(x) + 5(\sin^2(x)))} (\sin^2(x)) + 192 \sin(x) \sqrt{4(\cos^2(x) + 1)} \right)}{96}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{192} * ((4 * \cos(x)^2 + 1) * \sin(x)^2)^{(1/2)} * (512 * (-4 * \sin(x)^4 + 5 * \sin(x)^2)^{(1/2)} * \sin(x)^4 - 2080 * (-4 * \sin(x)^4 + 5 * \sin(x)^2)^{(1/2)} * \sin(x)^2 + 3300 * (-4 * \sin(x)^4 + 5 * \sin(x)^2)^{(1/2)} + 1875 * \arcsin(-1 + 8/5 * \sin(x)^2)) / \sin(x) / (4 * \cos(x)^2 + 1)^{(1/2)}$

Maxima [A]

time = 0.32, size = 53, normalized size = 0.77

$$\frac{1}{6} (-4 \sin(x)^2 + 5)^{\frac{5}{2}} \sin(x) + \frac{25}{24} (-4 \sin(x)^2 + 5)^{\frac{3}{2}} \sin(x) + \frac{125}{16} \sqrt{-4 \sin(x)^2 + 5} \sin(x) + \frac{625}{32} \arcsin\left(\frac{2}{5} \sqrt{5} \sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x, algorithm="maxima")`

[Out] $\frac{1}{6} * (-4 * \sin(x)^2 + 5)^{(5/2)} * \sin(x) + \frac{25}{24} * (-4 * \sin(x)^2 + 5)^{(3/2)} * \sin(x) + \frac{125}{16} * \sqrt{-4 * \sin(x)^2 + 5} * \sin(x) + \frac{625}{32} * \arcsin(2/5 * \sqrt{5} * \sin(x))$

Fricas [A]

time = 0.33, size = 88, normalized size = 1.28

$$\frac{1}{48} (128 \cos(x)^4 + 264 \cos(x)^2 + 433) \sqrt{4 \cos(x)^2 + 1} \sin(x) + \frac{625}{64} \arctan\left(\frac{4(8 \cos(x)^2 - 3) \sqrt{4 \cos(x)^2 + 1} \sin(x) - 25 \cos(x) \sin(x)}{64 \cos(x)^4 - 23 \cos(x)^2 - 16}\right) + \frac{625}{64} \arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x, algorithm="fricas")`

[Out] $\frac{1}{48} * (128 * \cos(x)^4 + 264 * \cos(x)^2 + 433) * \sqrt{4 * \cos(x)^2 + 1} * \sin(x) + \frac{625}{64} * \arctan((4 * (8 * \cos(x)^2 - 3) * \sqrt{4 * \cos(x)^2 + 1} * \sin(x) - 25 * \cos(x) * \sin(x)) / (64 * \cos(x)^4 - 23 * \cos(x)^2 - 16)) + \frac{625}{64} * \arctan(\sin(x) / \cos(x))$

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(5*cos(x)**2+sin(x)**2)**(5/2),x)`

[Out] Exception raised: SystemError

Giac [A]

time = 0.01, size = 56, normalized size = 0.81

$$2 \left(\left(\frac{4}{3} \sin x \cdot \sin x - \frac{65}{12} \right) \sin x \cdot \sin x + \frac{275}{32} \right) \sin x \cdot \sqrt{-4 \sin^2 x + 5} + \frac{625}{32} \arcsin\left(\frac{2 \sin x}{\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x)`

[Out] $\frac{1}{48}*(8*(16*\sin(x)^2 - 65)*\sin(x)^2 + 825)*\sqrt{-4*\sin(x)^2 + 5}*\sin(x) + 625/32*\arcsin(2/5*\sqrt{5}*\sin(x))$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(x) (5 \cos(x)^2 + \sin(x)^2)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*(5*cos(x)^2 + sin(x)^2)^(5/2),x)`

[Out] `int(cos(x)*(5*cos(x)^2 + sin(x)^2)^(5/2), x)`

$$3.421 \quad \int \cos(x) (-\cos^2(x) - 5\sin^2(x))^{3/2} dx$$

Optimal. Leaf size=58

$$\frac{3}{16} \tan^{-1} \left(\frac{2 \sin(x)}{\sqrt{-1 - 4 \sin^2(x)}} \right) - \frac{3}{8} \sin(x) \sqrt{-1 - 4 \sin^2(x)} + \frac{1}{4} \sin(x) (-1 - 4 \sin^2(x))^{3/2}$$

[Out] 3/16*arctan(2*sin(x)/(-1-4*sin(x)^2)^(1/2))+1/4*sin(x)*(-1-4*sin(x)^2)^(3/2)-3/8*sin(x)*(-1-4*sin(x)^2)^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4441, 201, 223, 209}

$$\frac{1}{4} \sin(x) (-4 \sin^2(x) - 1)^{3/2} - \frac{3}{8} \sin(x) \sqrt{-4 \sin^2(x) - 1} + \frac{3}{16} \tan^{-1} \left(\frac{2 \sin(x)}{\sqrt{-4 \sin^2(x) - 1}} \right)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*(-Cos[x]^2 - 5*Sin[x]^2)^(3/2),x]

[Out] (3*ArcTan[(2*Sin[x])/Sqrt[-1 - 4*Sin[x]^2]])/16 - (3*Sin[x]*Sqrt[-1 - 4*Sin[x]^2])/8 + (Sin[x]*(-1 - 4*Sin[x]^2)^(3/2))/4

Rule 201

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x*((a + b*x^n)^p/(n*p + 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 223

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
 \int \cos(x) (-\cos^2(x) - 5 \sin^2(x))^{3/2} dx &= \text{Subst} \left(\int (-1 - 4x^2)^{3/2} dx, x, \sin(x) \right) \\
 &= \frac{1}{4} \sin(x) (-1 - 4 \sin^2(x))^{3/2} - \frac{3}{4} \text{Subst} \left(\int \sqrt{-1 - 4x^2} dx, x, \sin(x) \right) \\
 &= -\frac{3}{8} \sin(x) \sqrt{-1 - 4 \sin^2(x)} + \frac{1}{4} \sin(x) (-1 - 4 \sin^2(x))^{3/2} + \frac{3}{8} \text{Subst} \left(\int \sqrt{-1 - 4x^2} dx, x, \sin(x) \right) \\
 &= -\frac{3}{8} \sin(x) \sqrt{-1 - 4 \sin^2(x)} + \frac{1}{4} \sin(x) (-1 - 4 \sin^2(x))^{3/2} + \frac{3}{8} \text{Subst} \left(\int \sqrt{-1 - 4x^2} dx, x, \sin(x) \right) \\
 &= \frac{3}{16} \tan^{-1} \left(\frac{2 \sin(x)}{\sqrt{-1 - 4 \sin^2(x)}} \right) - \frac{3}{8} \sin(x) \sqrt{-1 - 4 \sin^2(x)} + \frac{1}{4} \sin(x) (-1 - 4 \sin^2(x))^{3/2}
 \end{aligned}$$

Mathematica [A]

time = 0.05, size = 61, normalized size = 1.05

$$\frac{\sqrt{-3 + 2 \cos(2x)} \left(-3 \sinh^{-1}(2 \sin(x)) + 2 \sqrt{3 - 2 \cos(2x)} (-11 \sin(x) + 2 \sin(3x)) \right)}{16 \sqrt{1 + 4 \sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(-Cos[x]^2 - 5*Sin[x]^2)^(3/2),x]
```

```
[Out] (Sqrt[-3 + 2*Cos[2*x]]*(-3*ArcSinh[2*Sin[x]] + 2*Sqrt[3 - 2*Cos[2*x]]*(-11*Sin[x] + 2*Sin[3*x])))/(16*Sqrt[1 + 4*Sin[x]^2])
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(-Cos[x]^2 - 5*Sin[x]^2)^(3/2)*Cos[x],x]')
```

[Out] Timed out

Maple [A]

time = 0.18, size = 82, normalized size = 1.41

method	result
default	$\frac{\sqrt{(4(\cos^2(x)) - 5)(\sin^2(x))} \left(-32\sqrt{-4(\sin^4(x)) - (\sin^2(x))} (\sin^2(x))^{-20} \sqrt{-4(\sin^4(x)) - (\sin^2(x))} \right)}{32\sin(x)\sqrt{4(\cos^2(x)) - 5}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)`

[Out]
$$\frac{1}{32} * ((4 * \cos(x)^2 - 5) * \sin(x)^2)^{(1/2)} * (-32 * (-4 * \sin(x)^4 - \sin(x)^2)^{(1/2)} * \sin(x)^2 - 20 * (-4 * \sin(x)^4 - \sin(x)^2)^{(1/2)} + 3 * \arcsin(8 * \sin(x)^2 + 1)) / \sin(x) / (4 * \cos(x)^2 - 5)^{(1/2)}$$

Maxima [C] Result contains complex when optimal does not.

time = 0.34, size = 36, normalized size = 0.62

$$\frac{1}{4} (-4 \sin(x)^2 - 1)^{\frac{3}{2}} \sin(x) - \frac{3}{8} \sqrt{-4 \sin(x)^2 - 1} \sin(x) - \frac{3}{16} i \operatorname{arsinh}(2 \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x, algorithm="maxima")`

[Out]
$$\frac{1}{4} * (-4 * \sin(x)^2 - 1)^{(3/2)} * \sin(x) - \frac{3}{8} * \sqrt{-4 * \sin(x)^2 - 1} * \sin(x) - \frac{3}{16} * I * \operatorname{arsinh}(2 * \sin(x))$$

Fricas [C] Result contains complex when optimal does not.

time = 0.33, size = 123, normalized size = 2.12

$$\frac{1}{128} \left(12i e^{4ix} \log \left(-\frac{1}{2} \sqrt{e^{4ix} - 3e^{2ix} + 1} (4e^{2ix} - 5) + 2e^{4ix} - \frac{11}{2} e^{2ix} + \frac{5}{2} \right) - 12i e^{4ix} \log \left(\sqrt{e^{4ix} - 3e^{2ix} + 1} - e^{2ix} - 1 \right) - 8(2i e^{6ix} - 11i e^{4ix} + 11i e^{2ix} - 2i) \sqrt{e^{4ix} - 3e^{2ix} + 1} - 145i e^{4ix} \right) e^{-4ix}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x, algorithm="fricas")`

[Out]
$$\frac{1}{128} * (12 * I * e^{(4 * I * x)} * \log(-1/2 * \sqrt{e^{(4 * I * x)} - 3 * e^{(2 * I * x)} + 1} * (4 * e^{(2 * I * x)} - 5) + 2 * e^{(4 * I * x)} - 11/2 * e^{(2 * I * x)} + 5/2) - 12 * I * e^{(4 * I * x)} * \log(\sqrt{e^{(4 * I * x)} - 3 * e^{(2 * I * x)} + 1} - e^{(2 * I * x)} - 1) - 8 * (2 * I * e^{(6 * I * x)} - 11 * I * e^{(4 * I * x)} + 11 * I * e^{(2 * I * x)} - 2 * I) * \sqrt{e^{(4 * I * x)} - 3 * e^{(2 * I * x)} + 1} - 145 * I * e^{(4 * I * x)}) * e^{(-4 * I * x)}$$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(-cos(x)**2-5*sin(x)**2)**(3/2),x)`

[Out] Timed out

Giac [C] Result contains complex when optimal does not.

time = 0.01, size = 55, normalized size = 0.95

$$i \left(2 \left(-\frac{5}{16} - \frac{1}{2} \sin x \cdot \sin x \right) \sin x \cdot \sqrt{4 \sin^2 x + 1} + \frac{3}{16} \ln \left(\sqrt{4 \sin^2 x + 1} - 2 \sin x \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x)`

[Out] `-1/8*sqrt(4*sin(x)^2 + 1)*(8*I*sin(x)^2 + 5*I)*sin(x) + 3/16*I*log(sqrt(4*sin(x)^2 + 1) - 2*sin(x))`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \cos(x) (-\cos(x)^2 - 5 \sin(x)^2)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*(-cos(x)^2 - 5*sin(x)^2)^(3/2),x)`

[Out] `int(cos(x)*(-cos(x)^2 - 5*sin(x)^2)^(3/2), x)`

$$3.422 \quad \int \frac{\sin(x)}{(5 \cos^2(x) - 2 \sin^2(x))^{7/2}} dx$$

Optimal. Leaf size=55

$$\frac{\cos(x)}{10(-2 + 7 \cos^2(x))^{5/2}} - \frac{\cos(x)}{15(-2 + 7 \cos^2(x))^{3/2}} + \frac{\cos(x)}{15\sqrt{-2 + 7 \cos^2(x)}}$$

[Out] 1/10*cos(x)/(-2+7*cos(x)^2)^(5/2)-1/15*cos(x)/(-2+7*cos(x)^2)^(3/2)+1/15*cos(x)/(-2+7*cos(x)^2)^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4442, 198, 197}

$$\frac{\cos(x)}{15\sqrt{7 \cos^2(x) - 2}} - \frac{\cos(x)}{15(7 \cos^2(x) - 2)^{3/2}} + \frac{\cos(x)}{10(7 \cos^2(x) - 2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(5*Cos[x]^2 - 2*Sin[x]^2)^(7/2),x]

[Out] Cos[x]/(10*(-2 + 7*Cos[x]^2)^(5/2)) - Cos[x]/(15*(-2 + 7*Cos[x]^2)^(3/2)) + Cos[x]/(15*Sqrt[-2 + 7*Cos[x]^2])

Rule 197

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 198

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

Rule 4442

Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))], x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned}
\int \frac{\sin(x)}{(5 \cos^2(x) - 2 \sin^2(x))^{7/2}} dx &= -\text{Subst} \left(\int \frac{1}{(-2 + 7x^2)^{7/2}} dx, x, \cos(x) \right) \\
&= \frac{\cos(x)}{10(-2 + 7 \cos^2(x))^{5/2}} + \frac{2}{5} \text{Subst} \left(\int \frac{1}{(-2 + 7x^2)^{5/2}} dx, x, \cos(x) \right) \\
&= \frac{\cos(x)}{10(-2 + 7 \cos^2(x))^{5/2}} - \frac{\cos(x)}{15(-2 + 7 \cos^2(x))^{3/2}} - \frac{2}{15} \text{Subst} \left(\int \frac{1}{(-2 + 7x^2)} dx, x, \cos(x) \right) \\
&= \frac{\cos(x)}{10(-2 + 7 \cos^2(x))^{5/2}} - \frac{\cos(x)}{15(-2 + 7 \cos^2(x))^{3/2}} + \frac{\cos(x)}{15 \sqrt{-2 + 7 \cos^2(x)}}
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 37, normalized size = 0.67

$$\frac{\cos(x)(67 + 56 \cos(2x) + 49 \cos(4x))}{15\sqrt{2} (3 + 7 \cos(2x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(5*Cos[x]^2 - 2*Sin[x]^2)^(7/2),x]
```

```
[Out] (Cos[x]*(67 + 56*Cos[2*x] + 49*Cos[4*x]))/(15*Sqrt[2]*(3 + 7*Cos[2*x])^(5/2))
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sin[x]/(5*Cos[x]^2 - 2*Sin[x]^2)^(7/2),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5987 deep
```

Maple [A]

time = 0.09, size = 44, normalized size = 0.80

method	result	size
derivativedivides	$\frac{\cos(x)}{10(-2+7(\cos^2(x)))^{5/2}} - \frac{\cos(x)}{15(-2+7(\cos^2(x)))^{3/2}} + \frac{\cos(x)}{15\sqrt{-2+7(\cos^2(x))}}$	44
default	$\frac{\cos(x)}{10(-2+7(\cos^2(x)))^{5/2}} - \frac{\cos(x)}{15(-2+7(\cos^2(x)))^{3/2}} + \frac{\cos(x)}{15\sqrt{-2+7(\cos^2(x))}}$	44

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{10} \cos(x) / (-2+7 \cos(x)^2)^{(5/2)} - \frac{1}{15} \cos(x) / (-2+7 \cos(x)^2)^{(3/2)} + \frac{1}{15} \cos(x) / (-2+7 \cos(x)^2)^{(1/2)}$

Maxima [A]

time = 0.24, size = 43, normalized size = 0.78

$$\frac{\cos(x)}{15 \sqrt{7 \cos(x)^2 - 2}} - \frac{\cos(x)}{15 (7 \cos(x)^2 - 2)^{\frac{3}{2}}} + \frac{\cos(x)}{10 (7 \cos(x)^2 - 2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x, algorithm="maxima")`

[Out] $\frac{1}{15} \cos(x) / \sqrt{7 \cos(x)^2 - 2} - \frac{1}{15} \cos(x) / (7 \cos(x)^2 - 2)^{(3/2)} + \frac{1}{10} \cos(x) / (7 \cos(x)^2 - 2)^{(5/2)}$

Fricas [A]

time = 0.35, size = 51, normalized size = 0.93

$$\frac{(98 \cos(x)^5 - 70 \cos(x)^3 + 15 \cos(x)) \sqrt{7 \cos(x)^2 - 2}}{30 (343 \cos(x)^6 - 294 \cos(x)^4 + 84 \cos(x)^2 - 8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x, algorithm="fricas")`

[Out] $\frac{1}{30} (98 \cos(x)^5 - 70 \cos(x)^3 + 15 \cos(x)) \sqrt{7 \cos(x)^2 - 2} / (343 \cos(x)^6 - 294 \cos(x)^4 + 84 \cos(x)^2 - 8)$

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/(5*cos(x)**2-2*sin(x)**2)**(7/2),x)`

[Out] Exception raised: SystemError

Giac [A]

time = 0.01, size = 50, normalized size = 0.91

$$\frac{2 \left(\left(-\frac{49}{30} \cos x \cdot \cos x + \frac{7}{6} \right) \cos x \cdot \cos x - \frac{1}{4} \right) \cos x \cdot \sqrt{7 \cos^2 x - 2}}{(7 \cos^2 x - 2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x)`

[Out] $1/30*(14*(7*\cos(x)^2 - 5)*\cos(x)^2 + 15)*\cos(x)/(7*\cos(x)^2 - 2)^(5/2)$

Mupad [B]

time = 0.66, size = 28, normalized size = 0.51

$$\frac{\cos(x) (98 \cos(x)^4 - 70 \cos(x)^2 + 15)}{30 (7 \cos(x)^2 - 2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/(5*cos(x)^2 - 2*sin(x)^2)^(7/2),x)`

[Out] $(\cos(x)*(98*\cos(x)^4 - 70*\cos(x)^2 + 15))/(30*(7*\cos(x)^2 - 2)^(5/2))$

$$3.423 \quad \int \frac{\cos(x) \cos(2x)}{(2-5 \sin^2(x))^{3/2}} dx$$

Optimal. Leaf size=39

$$\frac{2 \sin^{-1} \left(\sqrt{\frac{5}{2}} \sin(x) \right)}{5\sqrt{5}} + \frac{\sin(x)}{10\sqrt{2-5 \sin^2(x)}}$$

[Out] 2/25*arcsin(1/2*sin(x)*10^(1/2))*5^(1/2)+1/10*sin(x)/(2-5*sin(x)^2)^(1/2)

Rubi [A]

time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4441, 393, 222}

$$\frac{2 \sin^{-1} \left(\sqrt{\frac{5}{2}} \sin(x) \right)}{5\sqrt{5}} + \frac{\sin(x)}{10\sqrt{2-5 \sin^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]*Cos[2*x])/(2 - 5*Sin[x]^2)^(3/2), x]

[Out] (2*ArcSin[Sqrt[5/2]*Sin[x]])/(5*Sqrt[5]) + Sin[x]/(10*Sqrt[2 - 5*Sin[x]^2])

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 393

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 4441

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x) \cos(2x)}{(2 - 5 \sin^2(x))^{3/2}} dx &= \text{Subst} \left(\int \frac{1 - 2x^2}{(2 - 5x^2)^{3/2}} dx, x, \sin(x) \right) \\
&= \frac{\sin(x)}{10\sqrt{2 - 5 \sin^2(x)}} + \frac{2}{5} \text{Subst} \left(\int \frac{1}{\sqrt{2 - 5x^2}} dx, x, \sin(x) \right) \\
&= \frac{2 \sin^{-1} \left(\sqrt{\frac{5}{2}} \sin(x) \right)}{5\sqrt{5}} + \frac{\sin(x)}{10\sqrt{2 - 5 \sin^2(x)}}
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 39, normalized size = 1.00

$$\frac{1}{50} \left(4\sqrt{5} \sin^{-1} \left(\sqrt{\frac{5}{2}} \sin(x) \right) + \frac{5 \sin(x)}{\sqrt{2 - 5 \sin^2(x)}} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(Cos[x]*Cos[2*x])/(2 - 5*Sin[x]^2)^(3/2), x]``[Out] (4*Sqrt[5]*ArcSin[Sqrt[5/2]*Sin[x]] + (5*Sin[x])/Sqrt[2 - 5*Sin[x]^2])/50`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Cos[2*x]*(Cos[x]/(2 - 5*Sin[x]^2)^(3/2)), x]')``[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(28) = 56$.

time = 0.18, size = 58, normalized size = 1.49

method	result	size
default	$\frac{20 \arcsin\left(\frac{\sin(x)\sqrt{10}}{2}\right) \sqrt{5} (\cos^2(x) + 5 \sin(x) \sqrt{5 (\cos^2(x)) - 3} - 12 \arcsin\left(\frac{\sin(x)\sqrt{10}}{2}\right) \sqrt{5}}{250(\cos^2(x)) - 150}}$	58

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)`

[Out] $1/50/(5*\cos(x)^2-3)*(20*\arcsin(1/2*\sin(x)*10^{1/2})*5^{1/2}*\cos(x)^2+5*\sin(x)*(5*\cos(x)^2-3)^{1/2}-12*\arcsin(1/2*\sin(x)*10^{1/2})*5^{1/2})$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 716 vs. 2(28) = 56.

time = 0.39, size = 716, normalized size = 18.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x, algorithm="maxima")`

[Out] $1/50*(5*\cos(1/2*\arctan2(5*\sin(4*x) - 2*\sin(2*x), 5*\cos(4*x) - 2*\cos(2*x) + 5))*\sin(2*x) - 5*(\cos(2*x) - 1)*\sin(1/2*\arctan2(5*\sin(4*x) - 2*\sin(2*x), 5*\cos(4*x) - 2*\cos(2*x) + 5)) + 2*(-10*(2*\cos(2*x) - 5)*\cos(4*x) + 25*\cos(4*x))^2 + 4*\cos(2*x)^2 + 25*\sin(4*x)^2 - 20*\sin(4*x)*\sin(2*x) + 4*\sin(2*x)^2 - 20*\cos(2*x) + 25)^{1/4}*(\sqrt{5}*\arctan2(1/12*\sqrt{6}*(\sqrt{6}*(25/36)^{1/4})*(25*\cos(2*x)^4 + 25*\sin(2*x)^4 - 20*\cos(2*x)^3 + 2*(25*\cos(2*x)^2 - 10*\cos(2*x) - 23)*\sin(2*x)^2 + 54*\cos(2*x)^2 - 20*\cos(2*x) + 25)^{1/4}*\sin(1/2*\arctan2(5/12*(5*\cos(2*x) - 1)*\sin(2*x), 25/24*\cos(2*x)^2 - 25/24*\sin(2*x)^2 - 5/12*\cos(2*x) + 25/24)) + 5*\sin(2*x)), 5/12*\sqrt{6}*\cos(2*x) + 1/2*(25/36)^{1/4}*(25*\cos(2*x)^4 + 25*\sin(2*x)^4 - 20*\cos(2*x)^3 + 2*(25*\cos(2*x)^2 - 10*\cos(2*x) - 23)*\sin(2*x)^2 + 54*\cos(2*x)^2 - 20*\cos(2*x) + 25)^{1/4}*\cos(1/2*\arctan2(5/12*(5*\cos(2*x) - 1)*\sin(2*x), 25/24*\cos(2*x)^2 - 25/24*\sin(2*x)^2 - 5/12*\cos(2*x) + 25/24)) - 1/12*\sqrt{6})) + \sqrt{5}*\arctan2(1/12*\sqrt{6}*(\sqrt{6}*(1/36)^{1/4}*(\cos(2*x)^4 + \sin(2*x)^4 - 20*\cos(2*x)^3 + 2*(\cos(2*x)^2 - 10*\cos(2*x) + 1)*\sin(2*x)^2 + 198*\cos(2*x)^2 - 980*\cos(2*x) + 2401)^{1/4}*\sin(1/2*\arctan2(1/12*(\cos(2*x) - 5)*\sin(2*x), 1/24*\cos(2*x)^2 - 1/24*\sin(2*x)^2 - 5/12*\cos(2*x) + 49/24)) + \sin(2*x)), 1/12*\sqrt{6}*\cos(2*x) + 1/2*(1/36)^{1/4}*(\cos(2*x)^4 + \sin(2*x)^4 - 20*\cos(2*x)^3 + 2*(\cos(2*x)^2 - 10*\cos(2*x) + 1)*\sin(2*x)^2 + 198*\cos(2*x)^2 - 980*\cos(2*x) + 2401)^{1/4})*\cos(1/2*\arctan2(1/12*(\cos(2*x) - 5)*\sin(2*x), 1/24*\cos(2*x)^2 - 1/24*\sin(2*x)^2 - 5/12*\cos(2*x) + 49/24)) - 5/12*\sqrt{6}))))/(-10*(2*\cos(2*x) - 5)*\cos(4*x) + 25*\cos(4*x)^2 + 4*\cos(2*x)^2 + 25*\sin(4*x)^2 - 20*\sin(4*x)*\sin(2*x) + 4*\sin(2*x)^2 - 20*\cos(2*x) + 25)^{1/4}$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(28) = 56.

time = 0.33, size = 100, normalized size = 2.56

$$\frac{\left(5\sqrt{5}\cos(x)^2 - 3\sqrt{5}\right)\arctan\left(\frac{\left(50\sqrt{5}\cos(x)^4 - 80\sqrt{5}\cos(x)^2 + 31\sqrt{5}\right)\sqrt{5\cos(x)^2 - 3}}{10\left(25\cos(x)^4 - 35\cos(x)^2 + 12\right)\sin(x)}\right) - 5\sqrt{5\cos(x)^2 - 3}\sin(x)}{50\left(5\cos(x)^2 - 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x, algorithm="fricas")

[Out] -1/50*((5*sqrt(5)*cos(x)^2 - 3*sqrt(5))*arctan(1/10*(50*sqrt(5)*cos(x)^4 - 80*sqrt(5)*cos(x)^2 + 31*sqrt(5))*sqrt(5*cos(x)^2 - 3)/((25*cos(x)^4 - 35*cos(x)^2 + 12)*sin(x))) - 5*sqrt(5*cos(x)^2 - 3)*sin(x))/(5*cos(x)^2 - 3)

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(2*x)/(2-5*sin(x)**2)**(3/2),x)

[Out] Exception raised: SystemError

Giac [A]

time = 0.02, size = 49, normalized size = 1.26

$$\frac{\sin x \cdot \sqrt{-5 \sin^2 x + 2}}{10(-5 \sin^2 x + 2)} + \frac{2}{25} \sqrt{5} \arcsin\left(\frac{5 \sin x}{\sqrt{10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x)

[Out] 2/25*sqrt(5)*arcsin(1/2*sqrt(10)*sin(x)) - 1/10*sqrt(-5*sin(x)^2 + 2)*sin(x)/(5*sin(x)^2 - 2)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos(2x) \cos(x)}{(2 - 5 \sin(x)^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(2*x)*cos(x))/(2 - 5*sin(x)^2)^(3/2),x)

[Out] int((cos(2*x)*cos(x))/(2 - 5*sin(x)^2)^(3/2), x)

$$3.424 \quad \int \frac{\sin(5x)}{(5 \cos^2(x) + 9 \sin^2(x))^{5/2}} dx$$

Optimal. Leaf size=48

$$-\frac{1}{2} \sin^{-1} \left(\frac{2 \cos(x)}{3} \right) - \frac{55 \cos(x)}{27 (9 - 4 \cos^2(x))^{3/2}} + \frac{295 \cos(x)}{243 \sqrt{9 - 4 \cos^2(x)}}$$

[Out] $-1/2*\arcsin(2/3*\cos(x))-55/27*\cos(x)/(9-4*\cos(x)^2)^{(3/2)}+295/243*\cos(x)/(9-4*\cos(x)^2)^{(1/2)}$

Rubi [A]

time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1171, 393, 222}

$$\frac{295 \cos(x)}{243 \sqrt{9 - 4 \cos^2(x)}} - \frac{55 \cos(x)}{27 (9 - 4 \cos^2(x))^{3/2}} - \frac{1}{2} \sin^{-1} \left(\frac{2 \cos(x)}{3} \right)$$

Antiderivative was successfully verified.

[In] Int[Sin[5*x]/(5*Cos[x]^2 + 9*Sin[x]^2)^(5/2), x]

[Out] $-1/2*\text{ArcSin}[(2*\text{Cos}[x])/3] - (55*\text{Cos}[x])/(27*(9 - 4*\text{Cos}[x]^2)^{(3/2)}) + (295*\text{Cos}[x])/(243*\text{Sqrt}[9 - 4*\text{Cos}[x]^2])$

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 393

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 1171

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x}, R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0], Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2

- b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{\sin(5x)}{(5 \cos^2(x) + 9 \sin^2(x))^{5/2}} dx &= -\text{Subst} \left(\int \frac{1 - 12x^2 + 16x^4}{(9 - 4x^2)^{5/2}} dx, x, \cos(x) \right) \\
 &= -\frac{55 \cos(x)}{27 (9 - 4 \cos^2(x))^{3/2}} + \frac{1}{27} \text{Subst} \left(\int \frac{52 + 108x^2}{(9 - 4x^2)^{3/2}} dx, x, \cos(x) \right) \\
 &= -\frac{55 \cos(x)}{27 (9 - 4 \cos^2(x))^{3/2}} + \frac{295 \cos(x)}{243 \sqrt{9 - 4 \cos^2(x)}} - \text{Subst} \left(\int \frac{1}{\sqrt{9 - 4x^2}} dx, \right. \\
 &= -\frac{1}{2} \sin^{-1} \left(\frac{2 \cos(x)}{3} \right) - \frac{55 \cos(x)}{27 (9 - 4 \cos^2(x))^{3/2}} + \frac{295 \cos(x)}{243 \sqrt{9 - 4 \cos^2(x)}}
 \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.20, size = 63, normalized size = 1.31

$$\frac{2550 \cos(x) - 590 \cos(3x) + 243i(7 - 2 \cos(2x))^{3/2} \log \left(2i \cos(x) + \sqrt{7 - 2 \cos(2x)} \right)}{486(7 - 2 \cos(2x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[5*x]/(5*Cos[x]^2 + 9*Sin[x]^2)^(5/2),x]

[Out] (2550*Cos[x] - 590*Cos[3*x] + (243*I)*(7 - 2*Cos[2*x])^(3/2)*Log[(2*I)*Cos[x] + Sqrt[7 - 2*Cos[2*x]])/(486*(7 - 2*Cos[2*x])^(3/2))

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sin[5*x]/(5*Cos[x]^2 + 9*Sin[x]^2)^(5/2),x]')

[Out] Timed out

Maple [A]

time = 0.12, size = 53, normalized size = 1.10

method	result	size
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derivativedivides	$-\frac{4(\cos^3(x))}{3(9-4(\cos^2(x)))^{\frac{3}{2}}} + \frac{214 \cos(x)}{243 \sqrt{9-4(\cos^2(x))}} - \frac{\arcsin\left(\frac{2 \cos(x)}{3}\right)}{2} + \frac{26 \cos(x)}{27(9-4(\cos^2(x)))^{\frac{3}{2}}}$	53
default	$-\frac{4(\cos^3(x))}{3(9-4(\cos^2(x)))^{\frac{3}{2}}} + \frac{214 \cos(x)}{243 \sqrt{9-4(\cos^2(x))}} - \frac{\arcsin\left(\frac{2 \cos(x)}{3}\right)}{2} + \frac{26 \cos(x)}{27(9-4(\cos^2(x)))^{\frac{3}{2}}}$	53

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $-4/3*\cos(x)^3/(9-4*\cos(x)^2)^(3/2)+214/243*\cos(x)/(9-4*\cos(x)^2)^(1/2)-1/2*\arcsin(2/3*\cos(x))+26/27*\cos(x)/(9-4*\cos(x)^2)^(3/2)$

Maxima [A]

time = 0.33, size = 69, normalized size = 1.44

$$-2 \left(\frac{2 \cos(x)^2}{(-4 \cos(x)^2 + 9)^{\frac{3}{2}}} - \frac{3}{(-4 \cos(x)^2 + 9)^{\frac{3}{2}}} \right) \cos(x) + \frac{52 \cos(x)}{243 \sqrt{-4 \cos(x)^2 + 9}} + \frac{26 \cos(x)}{27 (-4 \cos(x)^2 + 9)^{\frac{3}{2}}} - \frac{1}{2} \arcsin\left(\frac{2}{3} \cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x, algorithm="maxima")`

[Out] $-2*(2*\cos(x)^2/(-4*\cos(x)^2 + 9)^(3/2) - 3/(-4*\cos(x)^2 + 9)^(3/2))*\cos(x) + 52/243*\cos(x)/\sqrt{-4*\cos(x)^2 + 9} + 26/27*\cos(x)/(-4*\cos(x)^2 + 9)^(3/2) - 1/2*\arcsin(2/3*\cos(x))$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(36) = 72.

time = 0.33, size = 131, normalized size = 2.73

$$\frac{243(16 \cos(x)^4 - 72 \cos(x)^2 + 81) \arctan\left(-\frac{81 \cos(x) \sin(x) - 4(8 \cos(x)^3 - 9 \cos(x)) \sqrt{-4 \cos(x)^2 + 9}}{64 \cos(x)^4 - 225 \cos(x)^2 + 81}\right) - 243(16 \cos(x)^4 - 72 \cos(x)^2 + 81) \arctan\left(\frac{\sin(x)}{\cos(x)}\right) - 80(59 \cos(x)^3 - 108 \cos(x)) \sqrt{-4 \cos(x)^2 + 9}}{972(16 \cos(x)^4 - 72 \cos(x)^2 + 81)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x, algorithm="fricas")`

[Out] $1/972*(243*(16*\cos(x)^4 - 72*\cos(x)^2 + 81)*\arctan(-81*\cos(x)*\sin(x) - 4*(8*\cos(x)^3 - 9*\cos(x))*\sqrt{-4*\cos(x)^2 + 9})/(64*\cos(x)^4 - 225*\cos(x)^2 + 81) - 243*(16*\cos(x)^4 - 72*\cos(x)^2 + 81)*\arctan(\sin(x)/\cos(x)) - 80*(59*\cos(x)^3 - 108*\cos(x))*\sqrt{-4*\cos(x)^2 + 9})/(16*\cos(x)^4 - 72*\cos(x)^2 + 81)$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(5*x)/(5*cos(x)**2+9*sin(x)**2)**(5/2),x)

[Out] Timed out

Giac [A]

time = 0.02, size = 54, normalized size = 1.12

$$\frac{2 \left(-\frac{590}{243} \cos x \cdot \cos x + \frac{40}{9} \right) \cos x \cdot \sqrt{-4 \cos^2 x + 9}}{(-4 \cos^2 x + 9)^2} - \frac{\arcsin \left(\frac{2}{3} \cos x \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x)

[Out] -20/243*(59*cos(x)^2 - 108)*sqrt(-4*cos(x)^2 + 9)*cos(x)/(4*cos(x)^2 - 9)^2 - 1/2*arcsin(2/3*cos(x))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(5x)}{(5 \cos(x)^2 + 9 \sin(x)^2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(5*x)/(5*cos(x)^2 + 9*sin(x)^2)^(5/2),x)

[Out] int(sin(5*x)/(5*cos(x)^2 + 9*sin(x)^2)^(5/2), x)

$$3.425 \quad \int \frac{\cos(x) \cos(2x) \sin(3x)}{(-5+4 \sin^2(x))^{5/2}} dx$$

Optimal. Leaf size=49

$$-\frac{1}{4(-5+4 \sin^2(x))^{3/2}} - \frac{5}{8\sqrt{-5+4 \sin^2(x)}} + \frac{1}{8}\sqrt{-5+4 \sin^2(x)}$$

[Out] $-1/4/(-5+4*\sin(x)^2)^{(3/2)}-5/8/(-5+4*\sin(x)^2)^{(1/2)}+1/8*(-5+4*\sin(x)^2)^{(1/2)}$

Rubi [A]

time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4441, 1261, 712}

$$\frac{1}{8}\sqrt{4 \sin^2(x) - 5} - \frac{5}{8\sqrt{4 \sin^2(x) - 5}} - \frac{1}{4(4 \sin^2(x) - 5)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]*Cos[2*x]*Sin[3*x])/(-5 + 4*Sin[x]^2)^(5/2), x]

[Out] $-1/4*1/(-5 + 4*\sin[x]^2)^{(3/2)} - 5/(8*\text{Sqrt}[-5 + 4*\sin[x]^2]) + \text{Sqrt}[-5 + 4*\sin[x]^2]/8$

Rule 712

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rule 1261

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rule 4441

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x) \cos(2x) \sin(3x)}{(-5 + 4 \sin^2(x))^{5/2}} dx &= \text{Subst} \left(\int \frac{x(3 - 10x^2 + 8x^4)}{(-5 + 4x^2)^{5/2}} dx, x, \sin(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{3 - 10x + 8x^2}{(-5 + 4x)^{5/2}} dx, x, \sin^2(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{3}{(-5 + 4x)^{5/2}} + \frac{5}{2(-5 + 4x)^{3/2}} + \frac{1}{2\sqrt{-5 + 4x}} \right) dx, x, \sin^2(x) \right) \\
&= -\frac{1}{4(-5 + 4 \sin^2(x))^{3/2}} - \frac{5}{8\sqrt{-5 + 4 \sin^2(x)}} + \frac{1}{8} \sqrt{-5 + 4 \sin^2(x)}
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 28, normalized size = 0.57

$$\frac{12 + 11 \cos(2x) + \cos(4x)}{4(-5 + 4 \sin^2(x))^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(Cos[x]*Cos[2*x]*Sin[3*x])/(-5 + 4*Sin[x]^2)^(5/2), x]``[Out] (12 + 11*Cos[2*x] + Cos[4*x])/(4*(-5 + 4*Sin[x]^2)^(3/2))`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Cos[x]*Cos[2*x]*Sin[3*x]/(4*Sin[x]^2 - 5)^(5/2), x]')``[Out] Exception raised: SystemError >> excessive stack use: stack is 6436 deep`**Maple [A]**

time = 0.08, size = 46, normalized size = 0.94

method	result	size
derivativedivides	$\frac{2(\cos^4(x))}{(-4(\cos^2(x))-1)^{\frac{3}{2}}} + \frac{7(\cos^2(x))}{2(-4(\cos^2(x))-1)^{\frac{3}{2}}} + \frac{1}{2(-4(\cos^2(x))-1)^{\frac{3}{2}}}$	46
default	$\frac{2(\cos^4(x))}{(-4(\cos^2(x))-1)^{\frac{3}{2}}} + \frac{7(\cos^2(x))}{2(-4(\cos^2(x))-1)^{\frac{3}{2}}} + \frac{1}{2(-4(\cos^2(x))-1)^{\frac{3}{2}}}$	46

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $2*\cos(x)^4/(-4*\cos(x)^2-1)^{(3/2)}+7/2*\cos(x)^2/(-4*\cos(x)^2-1)^{(3/2)}+1/2/(-4*\cos(x)^2-1)^{(3/2)}$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(37) = 74$.

time = 0.28, size = 192, normalized size = 3.92

$$\frac{(\cos(11x) + 14\cos(9x) + 58\cos(7x) + 94\cos(5x) + 58\cos(3x) + 15\cos(x))\cos\left(\frac{5}{2}\arctan\left(\frac{\sin(4x) + 3\sin(2x)}{-\cos(4x) - 3\cos(2x) - 1}\right) - \frac{(\sin(11x) + 14\sin(9x) + 58\sin(7x) + 94\sin(5x) + 58\sin(3x) + 13\sin(x))\sin\left(\frac{5}{2}\arctan\left(\frac{\sin(4x) + 3\sin(2x)}{-\cos(4x) - 3\cos(2x) - 1}\right)\right)}{8(2(3\cos(2x) + 1)\cos(4x) + \cos(4x)^2 + 9\cos(2x)^2 + \sin(4x)^2 + 6\sin(4x)\sin(2x) + 9\sin(2x)^2 + 6\cos(2x) + 1)^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x, algorithm="maxima")`

[Out] $-1/8*((\cos(11*x) + 14*\cos(9*x) + 58*\cos(7*x) + 94*\cos(5*x) + 58*\cos(3*x) + 15*\cos(x))*\cos(5/2*\arctan2(\sin(4*x) + 3*\sin(2*x), -\cos(4*x) - 3*\cos(2*x) - 1)) - (\sin(11*x) + 14*\sin(9*x) + 58*\sin(7*x) + 94*\sin(5*x) + 58*\sin(3*x) + 13*\sin(x))*\sin(5/2*\arctan2(\sin(4*x) + 3*\sin(2*x), -\cos(4*x) - 3*\cos(2*x) - 1)))/(2*(3*\cos(2*x) + 1)*\cos(4*x) + \cos(4*x)^2 + 9*\cos(2*x)^2 + \sin(4*x)^2 + 6*\sin(4*x)*\sin(2*x) + 9*\sin(2*x)^2 + 6*\cos(2*x) + 1)^{5/4}$

Fricas [A]

time = 0.33, size = 1, normalized size = 0.02

0

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x, algorithm="fricas")`

[Out] 0

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)**2)**(5/2),x)`

[Out] Exception raised: SystemError

Giac [A]

time = 0.00, size = 54, normalized size = 1.10

$$\frac{\sqrt{4 \sin^2 x - 5}}{8} + \frac{-5(4 \sin^2 x - 5) - 2}{8\sqrt{4 \sin^2 x - 5}(4 \sin^2 x - 5)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x)``[Out] 1/8*sqrt(4*sin(x)^2 - 5) - 1/8*(20*sin(x)^2 - 23)/(4*sin(x)^2 - 5)^(3/2)`**Mupad [B]**

time = 0.63, size = 28, normalized size = 0.57

$$\frac{2 \cos(2x)^2 + 11 \cos(2x) + 11}{4(-2 \cos(2x) - 3)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((cos(2*x)*sin(3*x)*cos(x))/(4*sin(x)^2 - 5)^(5/2),x)``[Out] (11*cos(2*x) + 2*cos(2*x)^2 + 11)/(4*(- 2*cos(2*x) - 3)^(3/2))`

$$3.426 \quad \int \frac{\csc^2(x)(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x))}{\sqrt{-5+\sin^2(x)}} dx$$

Optimal. Leaf size=111

$$2 \tan^{-1} \left(\frac{\cos(x)}{\sqrt{-5+\sin^2(x)}} \right) - \frac{\tan^{-1} \left(\frac{\sqrt{5} \cos(x)}{\sqrt{-5+\sin^2(x)}} \right)}{\sqrt{5}} - \frac{2 \tan^{-1} \left(\frac{\sqrt{-5+\sin^2(x)}}{\sqrt{5}} \right)}{\sqrt{5}} - 2 \tanh^{-1} \left(\frac{\sin(x)}{\sqrt{-5+\sin^2(x)}} \right)$$

[Out] 2*arctan(cos(x)/(-5+sin(x)^2)^(1/2))-2*arctanh(sin(x)/(-5+sin(x)^2)^(1/2))-1/5*arctan(cos(x)*5^(1/2)/(-5+sin(x)^2)^(1/2))*5^(1/2)-2/5*arctan(1/5*(-5+sin(x)^2)^(1/2)*5^(1/2))*5^(1/2)+2*(-5+sin(x)^2)^(1/2)+2/5*(-5+sin(x)^2)^(1/2)/sin(x)

Rubi [A]

time = 0.37, antiderivative size = 119, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 13, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {4486, 4441, 462, 223, 212, 4451, 6857, 209, 267, 1024, 385, 455, 65}

$$2\sqrt{-\cos^2(x)-4} - 2 \tanh^{-1} \left(\frac{\sin(x)}{\sqrt{\sin^2(x)-5}} \right) + 2 \tan^{-1} \left(\frac{\cos(x)}{\sqrt{-\cos^2(x)-4}} \right) - \frac{\tan^{-1} \left(\frac{\sqrt{5} \cos(x)}{\sqrt{-\cos^2(x)-4}} \right)}{\sqrt{5}} - \frac{2 \tan^{-1} \left(\frac{\sqrt{-\cos^2(x)-4}}{\sqrt{5}} \right)}{\sqrt{5}} + \frac{2}{5} \sqrt{\sin^2(x)-5} \csc(x)$$

Antiderivative was successfully verified.

[In] Int[(Csc[x]^2*(-2*Cos[x]^3*(-1+Sin[x]))+Cos[2*x]*Sin[x])/Sqrt[-5+Sin[x]^2],x]

[Out] 2*ArcTan[Cos[x]/Sqrt[-4-Cos[x]^2]]-ArcTan[(Sqrt[5]*Cos[x])/Sqrt[-4-Cos[x]^2]]/Sqrt[5]-2*ArcTan[Sqrt[-4-Cos[x]^2]/Sqrt[5]]/Sqrt[5]-2*ArcTanh[Sin[x]/Sqrt[-5+Sin[x]^2]]+2*Sqrt[-4-Cos[x]^2]+(2*Csc[x]*Sqrt[-5+Sin[x]^2])/5

Rule 65

Int[((a_.)+(b_.)*(x_)^(m_))*((c_.)+(d_.)*(x_)^(n_)),x_Symbol]:>With[{p=Denominator[m]},Dist[p/b,Subst[Int[x^(p*(m+1)-1)*(c-a*(d/b)+d*(x^p/b))^n,x],x,(a+b*x)^(1/p)],x] /;FreeQ[{a,b,c,d},x]&&NeQ[b*c-a*d,0]&&LtQ[-1,m,0]&&LeQ[-1,n,0]&&LeQ[Denominator[n],Denominator[m]]&&IntLinearQ[a,b,c,d,m,n,x]

Rule 209

Int[((a_.)+(b_.)*(x_)^2)^(-1),x_Symbol]:>Simp[(1/(Rt[a,2]*Rt[b,2]))*ArcTan[Rt[b,2]*(x/Rt[a,2])],x] /;FreeQ[{a,b},x]&&PosQ[a/b]&&(GtQ[a,0]||GtQ[b,0])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 223

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 267

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 455

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 462

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))), x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1]))

Rule 1024

Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h, Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}, x]

Rule 4441

```
Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 4451

```
Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))]^(n_), x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 4486

```
Int[u_, x_Symbol] := With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /; !InertTrigFreeQ[u]
```

Rule 6857

```
Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(x) (-2 \cos^3(x)(-1 + \sin(x)) + \cos(2x) \sin(x))}{\sqrt{-5 + \sin^2(x)}} dx &= \int \left(\frac{2 \cos(x) \cot^2(x)}{\sqrt{-5 + \sin^2(x)}} + \frac{(-2 \cos^3(x) + \cos(2x))}{\sqrt{-5 + \sin^2(x)}} \right) dx \\
&= 2 \int \frac{\cos(x) \cot^2(x)}{\sqrt{-5 + \sin^2(x)}} dx + \int \frac{(-2 \cos^3(x) + \cos(2x))}{\sqrt{-5 + \sin^2(x)}} dx \\
&= 2 \text{Subst} \left(\int \frac{1 - x^2}{x^2 \sqrt{-5 + x^2}} dx, x, \sin(x) \right) - \text{Subst} \left(\int \frac{1}{\sqrt{-5 + x^2}} dx, x, \sin(x) \right) \\
&= \frac{2}{5} \csc(x) \sqrt{-5 + \sin^2(x)} - 2 \text{Subst} \left(\int \frac{1}{\sqrt{-5 + x^2}} dx, x, \sin(x) \right) \\
&= \frac{2}{5} \csc(x) \sqrt{-5 + \sin^2(x)} + 2 \text{Subst} \left(\int \frac{1}{\sqrt{-4 - x^2}} dx, x, \sin(x) \right) \\
&= -2 \tanh^{-1} \left(\frac{\sin(x)}{\sqrt{-5 + \sin^2(x)}} \right) + 2 \sqrt{-4 - \cos^2(x)} \\
&= 2 \tan^{-1} \left(\frac{\cos(x)}{\sqrt{-4 - \cos^2(x)}} \right) - 2 \tanh^{-1} \left(\frac{\sin(x)}{\sqrt{-5 + \sin^2(x)}} \right) \\
&= 2 \tan^{-1} \left(\frac{\cos(x)}{\sqrt{-4 - \cos^2(x)}} \right) - \frac{\tan^{-1} \left(\frac{\sqrt{5} \cos(x)}{\sqrt{-4 - \cos^2(x)}} \right)}{\sqrt{5}} \\
&= 2 \tan^{-1} \left(\frac{\cos(x)}{\sqrt{-4 - \cos^2(x)}} \right) - \frac{\tan^{-1} \left(\frac{\sqrt{5} \cos(x)}{\sqrt{-4 - \cos^2(x)}} \right)}{\sqrt{5}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 1.10, size = 295, normalized size = 2.66

$$\frac{2\sqrt{2}(-2\cos^3(x) + \cos(2x) + 2\cos^2(x)\cos(x)) \left(18 + 2\cos(2x) + 20\sqrt{2} \tanh^{-1} \left(\frac{2\sqrt{2}\cos(x)}{\sqrt{-(9 + \cos(2x))\sec^2(x/2)}} \right) \right) \csc^2(x) \sqrt{-(9 + \cos(2x))\sec^2(x/2)} \sin(x) + 85 \sin(x) + \sqrt{10} \tan^{-1} \left(\frac{\sqrt{10}\cos(x)}{\sqrt{-9 - \cos(2x)}} \right) \sqrt{-9 - \cos(2x)} \sin(x) + 2\sqrt{10} \tan^{-1} \left(\frac{\sqrt{-9 - \cos(2x)}}{\sqrt{10}} \right) \sqrt{-9 - \cos(2x)} \sin(x) + 10\sqrt{2} \sqrt{-9 - \cos(2x)} \log \left(\frac{\sqrt{2}\cos(x) + \sqrt{-9 - \cos(2x)}}{\sqrt{10}} \right) \sin(x) + 5 \sin(3x)}{5\sqrt{-9 - \cos(2x)}(-4\cos(x) - 2\cos(3x) + 2\sin(x) + 2\sin(2x) - 2\sin(3x) + \sin(4x))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Csc[x]^2*(-2*Cos[x]^3*(-1 + Sin[x])) + Cos[2*x]*Sin[x])/Sqrt[-5 + Sin[x]^2], x]

[Out] (2*Sqrt[2]*(-2*Cos[x]^3 + Cos[2*x] + 2*Cos[x]^2*Cot[x])*(18 + 2*Cos[2*x] + 20*Sqrt[2]*ArcTanh[(2*Sqrt[2]*Tan[x/2])/Sqrt[-((9 + Cos[2*x])*Sec[x/2]^4)]])


```
*Cos[x/2]^3*Sqrt[-((9 + Cos[2*x])*Sec[x/2]^4)]*Sin[x/2] + 85*Sin[x] + Sqrt[10]*ArcTan[(Sqrt[10]*Cos[x])/Sqrt[-9 - Cos[2*x]]]*Sqrt[-9 - Cos[2*x]]*Sin[x] + 2*Sqrt[10]*ArcTan[Sqrt[-9 - Cos[2*x]]/Sqrt[10]]*Sqrt[-9 - Cos[2*x]]*Sin[x] + (10*I)*Sqrt[2]*Sqrt[-9 - Cos[2*x]]*Log[I*Sqrt[2]*Cos[x] + Sqrt[-9 - Cos[2*x]]]*Sin[x] + 5*Sin[3*x])/(5*Sqrt[-9 - Cos[2*x]]*(-6*Cos[x] - 2*Cos[3*x] + 2*Sin[x] + 2*Sin[2*x] - 2*Sin[3*x] + Sin[4*x]))
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Sin[x]*Cos[2*x] - 2*(Sin[x] - 1)*Cos[x]^3)/(Sin[x]^2*Sqrt[Sin[x]^2 - 5]),x]')
```

[Out] Timed out

Maple [A]

time = 0.30, size = 131, normalized size = 1.18

method	result
default	$-2 \ln \left(\sin(x) + \sqrt{-5 + \sin^2(x)} \right) + 2\sqrt{-5 + \sin^2(x)} + \frac{2\sqrt{5} \arctan\left(\frac{\sqrt{5}}{\sqrt{-5 + \sin^2(x)}}\right)}{5} + 2\sqrt{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)^(1/2), x,method=_RETURNVERBOSE)
```

```
[Out] -2*ln(sin(x)+(-5+sin(x)^2)^(1/2))+2*(-5+sin(x)^2)^(1/2)+2/5*5^(1/2)*arctan(1/(-5+sin(x)^2)^(1/2)*5^(1/2))+2/5*(-5+sin(x)^2)^(1/2)/sin(x)-1/10*((-5+sin(x)^2)*cos(x)^2)^(1/2)*(-5^(1/2)*arctan(1/5*(3*sin(x)^2-5)*5^(1/2)/(-cos(x)^4-4*cos(x)^2)^(1/2))-10*arcsin(1+1/2*cos(x)^2))/cos(x)/(-5+sin(x)^2)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.

time = 0.35, size = 115, normalized size = 1.04

$$\frac{2}{5}\sqrt{5} \arcsin\left(\frac{\sqrt{5}}{|\sin(x)|}\right) - \frac{1}{10}i\sqrt{5} \operatorname{arsinh}\left(\frac{\cos(x)}{2(\cos(x)+1)} - \frac{2}{\cos(x)+1}\right) - \frac{1}{10}i\sqrt{5} \operatorname{arsinh}\left(-\frac{\cos(x)}{2(\cos(x)-1)} - \frac{2}{\cos(x)-1}\right) + 2\sqrt{\sin(x)^2-5} + \frac{2\sqrt{\sin(x)^2-5}}{5\sin(x)} - 2i \operatorname{arsinh}\left(\frac{1}{2}\cos(x)\right) - 2\log\left(2\sqrt{\sin(x)^2-5} + 2\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*sqrt(5)*arcsin(sqrt(5)/abs(sin(x))) - 1/10*I*sqrt(5)*arcsinh(1/2*cos(x))/(cos(x) + 1) - 2/(cos(x) + 1) - 1/10*I*sqrt(5)*arcsinh(-1/2*cos(x))/(cos(x)
```

) - 1) - 2/(cos(x) - 1) + 2*sqrt(sin(x)^2 - 5) + 2/5*sqrt(sin(x)^2 - 5)/sin(x) - 2*I*arcsinh(1/2*cos(x)) - 2*log(2*sqrt(sin(x)^2 - 5) + 2*sin(x))

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x))^2^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*cos(x)**3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)**2/(-5+sin(x)**2)**(1/2),x)

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x))^2^(1/2),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(2x) \sin(x) - 2 \cos(x)^3 (\sin(x) - 1)}{\sin(x)^2 \sqrt{\sin(x)^2 - 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(2*x)*sin(x) - 2*cos(x)^3*(sin(x) - 1))/(sin(x)^2*(sin(x)^2 - 5)^(1/2)),x)

[Out] int((cos(2*x)*sin(x) - 2*cos(x)^3*(sin(x) - 1))/(sin(x)^2*(sin(x)^2 - 5)^(1/2)), x)

$$3.427 \quad \int \frac{\cos(3x)}{-\sqrt{-1 + 8 \cos^2(x)} + \sqrt{3 \cos^2(x) - \sin^2(x)}} dx$$

Optimal. Leaf size=112

$$\frac{5 \sin^{-1} \left(2 \sqrt{\frac{2}{7}} \sin(x) \right)}{4\sqrt{2}} + \frac{3}{4} \sin^{-1} \left(\frac{2 \sin(x)}{\sqrt{3}} \right) - \frac{3}{4} \tan^{-1} \left(\frac{\sin(x)}{\sqrt{-1 + 4 \cos^2(x)}} \right) - \frac{3}{4} \tan^{-1} \left(\frac{\sin(x)}{\sqrt{-1 + 8 \cos^2(x)}} \right)$$

[Out] 3/4*arcsin(2/3*sin(x)*3^(1/2))-3/4*arctan(sin(x)/(-1+4*cos(x)^2)^(1/2))-3/4*arctan(sin(x)/(-1+8*cos(x)^2)^(1/2))+5/8*arcsin(2/7*sin(x)*14^(1/2))*2^(1/2)-1/2*sin(x)*(-1+4*cos(x)^2)^(1/2)-1/2*sin(x)*(-1+8*cos(x)^2)^(1/2)

Rubi [A]

time = 0.28, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6874, 399, 222, 385, 210, 201}

$$\frac{5 \sin^{-1} \left(2 \sqrt{\frac{2}{7}} \sin(x) \right)}{4\sqrt{2}} + \frac{3}{4} \sin^{-1} \left(\frac{2 \sin(x)}{\sqrt{3}} \right) - \frac{1}{2} \sin(x) \sqrt{7 - 8 \sin^2(x)} - \frac{1}{2} \sin(x) \sqrt{3 - 4 \sin^2(x)} - \frac{3}{4} \tan^{-1} \left(\frac{\sin(x)}{\sqrt{7 - 8 \sin^2(x)}} \right) - \frac{3}{4} \tan^{-1} \left(\frac{\sin(x)}{\sqrt{3 - 4 \sin^2(x)}} \right)$$

Antiderivative was successfully verified.

[In] Int[Cos[3*x]/(-Sqrt[-1 + 8*Cos[x]^2] + Sqrt[3*Cos[x]^2 - Sin[x]^2]),x]

[Out] (5*ArcSin[2*Sqrt[2/7]*Sin[x]])/(4*Sqrt[2]) + (3*ArcSin[(2*Sin[x])/Sqrt[3]])/4 - (3*ArcTan[Sin[x]/Sqrt[7 - 8*Sin[x]^2]])/4 - (3*ArcTan[Sin[x]/Sqrt[3 - 4*Sin[x]^2]])/4 - (Sin[x]*Sqrt[7 - 8*Sin[x]^2])/2 - (Sin[x]*Sqrt[3 - 4*Sin[x]^2])/2

Rule 201

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x*((a + b*x^n)^p/(n*p + 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 6874

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)} + \sqrt{3\cos^2(x) - \sin^2(x)}} dx &= \text{Subst} \left(\int \frac{-1+4x^2}{\sqrt{7-8x^2} - \sqrt{3-4x^2}} dx, x, \sin(x) \right) \\
&= \text{Subst} \left(\int \left(-\frac{1}{\sqrt{7-8x^2} - \sqrt{3-4x^2}} + \frac{x^2}{\sqrt{7-8x^2}} \right) dx, x, \sin(x) \right) \\
&= 4 \text{Subst} \left(\int \frac{x^2}{\sqrt{7-8x^2} - \sqrt{3-4x^2}} dx, x, \sin(x) \right) - \\
&= 4 \text{Subst} \left(\int \left(-\frac{1}{4}\sqrt{7-8x^2} - \frac{1}{4}\sqrt{3-4x^2} - \frac{\sqrt{7-8x^2}}{4(-1+x^2)} \right) dx, x, \sin(x) \right) \\
&= \frac{1}{4} \text{Subst} \left(\int \frac{\sqrt{7-8x^2}}{-1+x^2} dx, x, \sin(x) \right) + \frac{1}{4} \text{Subst} \left(\int \frac{\sqrt{3-4x^2}}{-1+x^2} dx, x, \sin(x) \right) \\
&= -\frac{1}{2} \sin(x) \sqrt{7-8\sin^2(x)} - \frac{1}{2} \sin(x) \sqrt{3-4\sin^2(x)} \\
&= -\frac{11 \sin^{-1} \left(2\sqrt{\frac{2}{7}} \sin(x) \right)}{4\sqrt{2}} + 2\sqrt{2} \sin^{-1} \left(2\sqrt{\frac{2}{7}} \sin(x) \right) \\
&= -\frac{11 \sin^{-1} \left(2\sqrt{\frac{2}{7}} \sin(x) \right)}{4\sqrt{2}} + 2\sqrt{2} \sin^{-1} \left(2\sqrt{\frac{2}{7}} \sin(x) \right)
\end{aligned}$$

Mathematica [A]

time = 0.22, size = 156, normalized size = 1.39

$$\frac{1}{8} \left(5\sqrt{2} \sin^{-1} \left(2\sqrt{\frac{2}{7}} \sin(x) \right) + 6 \sin^{-1} \left(\frac{2\sin(x)}{\sqrt{3}} \right) + 3 \tan^{-1} \left(\frac{7-8\sin(x)}{\sqrt{3+4\cos(2x)}} \right) + 3 \tan^{-1} \left(\frac{3-4\sin(x)}{\sqrt{1+2\cos(2x)}} \right) - 3 \tan^{-1} \left(\frac{3+4\sin(x)}{\sqrt{1+2\cos(2x)}} \right) - 3 \tan^{-1} \left(\frac{7+8\sin(x)}{\sqrt{3+4\cos(2x)}} \right) - 4\sqrt{1+2\cos(2x)} \sin(x) - 4\sqrt{3+4\cos(2x)} \sin(x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]/(-Sqrt[-1 + 8*Cos[x]^2] + Sqrt[3*Cos[x]^2 - Sin[x]^2]),x]
```

```
[Out] (5*Sqrt[2]*ArcSin[2*Sqrt[2/7]*Sin[x]] + 6*ArcSin[(2*Sin[x])/Sqrt[3]] + 3*ArcTan[(7 - 8*Sin[x])/Sqrt[3 + 4*Cos[2*x]]] + 3*ArcTan[(3 - 4*Sin[x])/Sqrt[1 + 2*Cos[2*x]]] - 3*ArcTan[(3 + 4*Sin[x])/Sqrt[1 + 2*Cos[2*x]]] - 3*ArcTan[(7 + 8*Sin[x])/Sqrt[3 + 4*Cos[2*x]]] - 4*Sqrt[1 + 2*Cos[2*x]]*Sin[x] - 4*Sqrt[3 + 4*Cos[2*x]]*Sin[x])/8
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Cos[3*x]/(Sqrt[3*Cos[x]^2 - Sin[x]^2] - Sqrt[8*Cos[x]^2 - 1]),x]')

[Out] caught exception: maximum recursion depth exceeded while calling a Python object

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(3x)}{-\sqrt{-1 + 8(\cos^2(x))} + \sqrt{3(\cos^2(x)) - (\sin^2(x))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x)

[Out] int(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x)

Maxima [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x, algorithm="maxima")

[Out] Timed out

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(84) = 168.

time = 0.38, size = 195, normalized size = 1.74

$$\frac{5}{32} \sqrt{2} \arctan\left(\frac{(512\sqrt{2}\cos(x)^4 - 576\sqrt{2}\cos(x)^2 + 113\sqrt{2})\sqrt{8\cos(x)^2 - 1}}{16(128\cos(x)^4 - 88\cos(x)^2 + 9)\sin(x)}\right) - \frac{1}{2}\sqrt{8\cos(x)^2 - 1}\sin(x) - \frac{1}{2}\sqrt{4\cos(x)^2 - 1}\sin(x) + \frac{3}{8}\arctan\left(\frac{4(8\cos(x)^2 - 5)\sqrt{4\cos(x)^2 - 1}\sin(x) - 9\cos(x)\sin(x)}{64\cos(x)^2 - 71\cos(x) + 16}\right) + \frac{3}{8}\arctan\left(\frac{\sin(x)}{\cos(x)}\right) + \frac{3}{8}\arctan\left(\frac{9\cos(x)^2 - 2}{2\sqrt{8\cos(x)^2 - 1}\sin(x)}\right) + \frac{3}{4}\arctan\left(\frac{\sqrt{4\cos(x)^2 - 1}}{\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x, algorithm="fricas")

[Out] -5/32*sqrt(2)*arctan(1/16*(512*sqrt(2)*cos(x)^4 - 576*sqrt(2)*cos(x)^2 + 113*sqrt(2))*sqrt(8*cos(x)^2 - 1)/((128*cos(x)^4 - 88*cos(x)^2 + 9)*sin(x))) - 1/2*sqrt(8*cos(x)^2 - 1)*sin(x) - 1/2*sqrt(4*cos(x)^2 - 1)*sin(x) + 3/8*arctan((4*(8*cos(x)^2 - 5)*sqrt(4*cos(x)^2 - 1)*sin(x) - 9*cos(x)*sin(x))/(64*cos(x)^2 - 71*cos(x) + 16)) + 3/8*arctan(sin(x)/cos(x)) + 3/8*arctan(1/

$2*(9*\cos(x)^2 - 2)/(\sqrt{8*\cos(x)^2 - 1}*\sin(x)) + 3/4*\arctan(\sqrt{4*\cos(x)^2 - 1}/\sin(x))$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(3x)}{\sqrt{-\sin^2(x) + 3\cos^2(x)} - \sqrt{8\cos^2(x) - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)/(-(-1+8*cos(x)**2)**(1/2)+(3*cos(x)**2-sin(x)**2)**(1/2)),x)

[Out] Integral(cos(3*x)/(sqrt(-sin(x)**2 + 3*cos(x)**2) - sqrt(8*cos(x)**2 - 1)), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$-\int -\frac{\cos(3x)}{\sqrt{3\cos(x)^2 - \sin(x)^2} - \sqrt{8\cos(x)^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(3*x)/((3*cos(x)^2 - sin(x)^2)^(1/2) - (8*cos(x)^2 - 1)^(1/2)),x)

[Out] -int(-cos(3*x)/((3*cos(x)^2 - sin(x)^2)^(1/2) - (8*cos(x)^2 - 1)^(1/2)), x)

3.428 $\int (2 - 3 \sin^2(x))^{3/5} \sin(4x) dx$

Optimal. Leaf size=33

$$\frac{5}{36} (2 - 3 \sin^2(x))^{8/5} - \frac{20}{117} (2 - 3 \sin^2(x))^{13/5}$$

[Out] 5/36*(2-3*sin(x)^2)^(8/5)-20/117*(2-3*sin(x)^2)^(13/5)

Rubi [A]

time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {12, 455, 45}

$$\frac{5}{36} (2 - 3 \sin^2(x))^{8/5} - \frac{20}{117} (2 - 3 \sin^2(x))^{13/5}$$

Antiderivative was successfully verified.

[In] Int[(2 - 3*Sin[x]^2)^(3/5)*Sin[4*x],x]

[Out] (5*(2 - 3*Sin[x]^2)^(8/5))/36 - (20*(2 - 3*Sin[x]^2)^(13/5))/117

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 455

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
\int (2 - 3 \sin^2(x))^{3/5} \sin(4x) dx &= \text{Subst} \left(\int 4x (2 - 3x^2)^{3/5} (1 - 2x^2) dx, x, \sin(x) \right) \\
&= 4 \text{Subst} \left(\int x (2 - 3x^2)^{3/5} (1 - 2x^2) dx, x, \sin(x) \right) \\
&= 2 \text{Subst} \left(\int (2 - 3x)^{3/5} (1 - 2x) dx, x, \sin^2(x) \right) \\
&= 2 \text{Subst} \left(\int \left(-\frac{1}{3}(2 - 3x)^{3/5} + \frac{2}{3}(2 - 3x)^{8/5} \right) dx, x, \sin^2(x) \right) \\
&= \frac{5}{36} (2 - 3 \sin^2(x))^{8/5} - \frac{20}{117} (2 - 3 \sin^2(x))^{13/5}
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 29, normalized size = 0.88

$$-\frac{5(1 + 3 \cos(2x))^{8/5}(-5 + 24 \cos(2x))}{936 2^{3/5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*Sin[x]^2)^(3/5)*Sin[4*x],x]
```

```
[Out] (-5*(1 + 3*Cos[2*x])^(8/5)*(-5 + 24*Cos[2*x]))/(936*2^(3/5))
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(2 - 3*Sin[x]^2)^(3/5)*Sin[4*x],x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8008 deep
```

Maple [A]

time = 0.14, size = 38, normalized size = 1.15

method	result	size
default	$\frac{5(3(\cos^2(x)) - 1)^{\frac{8}{5}}}{12} - \frac{20\left(\frac{1}{2} + \frac{3\cos(2x)}{2}\right)^{\frac{13}{5}}}{117} - \frac{5\left(\frac{1}{2} + \frac{3\cos(2x)}{2}\right)^{\frac{8}{5}}}{18}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*sin(x)^2)^(3/5)*sin(4*x),x,method=_RETURNVERBOSE)
```

[Out] $5/12*(3*\cos(x)^2-1)^{(8/5)}-20/117*(1/2+3/2*\cos(2*x))^{(13/5)}-5/18*(1/2+3/2*\cos(2*x))^{(8/5)}$

Maxima [A]

time = 0.24, size = 25, normalized size = 0.76

$$-\frac{20}{117}(-3\sin(x)^2+2)^{\frac{13}{5}}+\frac{5}{36}(-3\sin(x)^2+2)^{\frac{8}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2-3*sin(x)^2)^(3/5)*sin(4*x),x, algorithm="maxima")`

[Out] $-20/117*(-3*\sin(x)^2+2)^{(13/5)}+5/36*(-3*\sin(x)^2+2)^{(8/5)}$

Fricas [A]

time = 0.35, size = 26, normalized size = 0.79

$$-\frac{5}{468}(144\cos(x)^4-135\cos(x)^2+29)(3\cos(x)^2-1)^{\frac{3}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2-3*sin(x)^2)^(3/5)*sin(4*x),x, algorithm="fricas")`

[Out] $-5/468*(144*\cos(x)^4-135*\cos(x)^2+29)*(3*\cos(x)^2-1)^{(3/5)}$

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2-3*sin(x)**2)**(3/5)*sin(4*x),x)`

[Out] Exception raised: SystemError

Giac [A]

time = 0.01, size = 169, normalized size = 5.12

$$\frac{4}{3}\left(\frac{10}{3}\left(\frac{1}{8}\left(-3\sin^2x+2\right)^{\frac{3}{5}}\left(-3\sin^2x+2\right)-\frac{2}{3}\left(-3\sin^2x+2\right)^{\frac{3}{5}}\right)+\frac{15}{9}\left(\frac{1}{13}\left(-3\sin^2x+2\right)^{\frac{3}{5}}\left(-3\sin^2x+2\right)^2-\frac{1}{2}\left(-3\sin^2x+2\right)^{\frac{3}{5}}\left(-3\sin^2x+2\right)+\frac{4}{3}\left(-3\sin^2x+2\right)^{\frac{3}{5}}\right)\right)-\frac{10}{24}\left(-3\sin^2x+2\right)^{\frac{3}{5}}\left(-3\sin^2x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2-3*sin(x)^2)^(3/5)*sin(4*x),x)`

[Out] $-20/117*(3*\sin(x)^2-2)^2*(-3*\sin(x)^2+2)^{(3/5)}+5/36*(-3*\sin(x)^2+2)^{(8/5)}$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \sin(4x) (2 - 3 \sin(x)^2)^{3/5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(4*x)*(2 - 3*sin(x)^2)^(3/5),x)`

[Out] `int(sin(4*x)*(2 - 3*sin(x)^2)^(3/5), x)`

3.429 $\int \cos(x) \sqrt{\cos(2x)} dx$

Optimal. Leaf size=33

$$\frac{\sin^{-1}\left(\frac{\sqrt{2}\sin(x)}{2\sqrt{2}}\right)}{2\sqrt{2}} + \frac{1}{2}\sqrt{\cos(2x)}\sin(x)$$

[Out] 1/4*arcsin(sin(x)*2^(1/2))*2^(1/2)+1/2*sin(x)*cos(2*x)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4441, 201, 222}

$$\frac{\sin^{-1}\left(\frac{\sqrt{2}\sin(x)}{2\sqrt{2}}\right)}{2\sqrt{2}} + \frac{1}{2}\sin(x)\sqrt{\cos(2x)}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sqrt[Cos[2*x]],x]

[Out] ArcSin[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + (Sqrt[Cos[2*x]]*Sin[x])/2

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x*((a + b*x^n)^p/(n*p + 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\int \cos(x) \sqrt{\cos(2x)} \, dx &= \text{Subst} \left(\int \sqrt{1-2x^2} \, dx, x, \sin(x) \right) \\
&= \frac{1}{2} \sqrt{\cos(2x)} \sin(x) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1-2x^2}} \, dx, x, \sin(x) \right) \\
&= \frac{\sin^{-1}(\sqrt{2} \sin(x))}{2\sqrt{2}} + \frac{1}{2} \sqrt{\cos(2x)} \sin(x)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 32, normalized size = 0.97

$$\frac{1}{4} \left(\sqrt{2} \sin^{-1}(\sqrt{2} \sin(x)) + 2 \sqrt{\cos(2x)} \sin(x) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[x]*Sqrt[Cos[2*x]],x]``[Out] (Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] + 2*Sqrt[Cos[2*x]]*Sin[x])/4`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Sqrt[Cos[2*x]]*Cos[x],x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(23) = 46.

time = 0.15, size = 62, normalized size = 1.88

method	result	size
default	$-\frac{\sqrt{(2(\cos^2(x)) - 1)(\sin^2(x))} \left(-\sqrt{2} \arcsin(4(\sin^2(x)) - 1) - 4 \sqrt{-2(\sin^4(x)) + \sin^2(x)} \right)}{8 \sin(x) \sqrt{2(\cos^2(x)) - 1}}$	62

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x)*cos(2*x)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/8*((2*\cos(x)^2-1)*\sin(x)^2)^{(1/2)}*(-2^{(1/2)}*\arcsin(4*\sin(x)^2-1)-4*(-2*\sin(x)^4+\sin(x)^2)^{(1/2)})/\sin(x)/(2*\cos(x)^2-1)^{(1/2)}$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(23) = 46$.

time = 0.39, size = 488, normalized size = 14.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(2*x)^(1/2),x, algorithm="maxima")`

[Out] $1/16*\sqrt{2}*(2*(\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1)^{(1/4)}*(\cos(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1))*\sin(2*x) - (\cos(2*x) - 1)*\sin(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1))) + \arctan2(-(\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1)^{(1/4)}*(\cos(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1))*\sin(2*x) - \cos(2*x)*\sin(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1))), (\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1)^{(1/4)}*(\cos(2*x)*\cos(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1)) + \sin(2*x)*\sin(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1)))) + 1 - \arctan2(-(\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1)^{(1/4)}*(\cos(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1))*\sin(2*x) - \cos(2*x)*\sin(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1))), (\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1)^{(1/4)}*(\cos(2*x)*\cos(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1)) + \sin(2*x)*\sin(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1)))) - 1) - \arctan2((\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1)^{(1/4)}*\sin(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1))), (\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1)^{(1/4)}*\cos(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1)) + 1) + \arctan2((\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1)^{(1/4)}*\sin(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1))), (\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1)^{(1/4)}*\cos(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1)) + 1)) - 1))$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(23) = 46$.

time = 0.33, size = 77, normalized size = 2.33

$$-\frac{1}{16}\sqrt{2}\arctan\left(\frac{(32\sqrt{2}\cos(x)^4-48\sqrt{2}\cos(x)^2+17\sqrt{2})\sqrt{2\cos(x)^2-1}}{8(8\cos(x)^4-10\cos(x)^2+3)\sin(x)}\right)+\frac{1}{2}\sqrt{2\cos(x)^2-1}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(2*x)^(1/2),x, algorithm="fricas")`

[Out] $-1/16*\sqrt{2}*\arctan(1/8*(32*\sqrt{2}*\cos(x)^4 - 48*\sqrt{2}*\cos(x)^2 + 17*\sqrt{2}))*\sqrt{2*\cos(x)^2 - 1}/((8*\cos(x)^4 - 10*\cos(x)^2 + 3)*\sin(x)) + 1/2*\sqrt{2*\cos(x)^2 - 1}*\sin(x)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\sqrt{\cos(2x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(2*x)**(1/2),x)`

[Out] `Integral(cos(x)*sqrt(cos(2*x)), x)`

Giac [A]

time = 0.02, size = 36, normalized size = 1.09

$$\frac{1}{2} \sin x \cdot \sqrt{-2 \sin^2 x + 1} + \frac{1}{4} \sqrt{2} \arcsin(\sqrt{2} \sin x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(2*x)^(1/2),x)`

[Out] `1/4*sqrt(2)*arcsin(sqrt(2)*sin(x)) + 1/2*sqrt(-2*sin(x)^2 + 1)*sin(x)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{\cos(2x)} \cos(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(2*x)^(1/2)*cos(x),x)`

[Out] `int(cos(2*x)^(1/2)*cos(x), x)`

3.430 $\int \cos^3(2x) \sin(x) dx$

Optimal. Leaf size=55

$$-\frac{3 \tanh^{-1}\left(\frac{\sqrt{2} \cos(x)}{\sqrt{\cos(2x)}}\right)}{8\sqrt{2}} + \frac{3}{8} \cos(x) \sqrt{\cos(2x)} - \frac{1}{4} \cos(x) \cos^{\frac{3}{2}}(2x)$$

[Out] $-1/4*\cos(x)*\cos(2*x)^{(3/2)}-3/16*\operatorname{arctanh}(\cos(x)*2^{(1/2)}/\cos(2*x)^{(1/2)})*2^{(1/2)}+3/8*\cos(x)*\cos(2*x)^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4442, 201, 223, 212}

$$-\frac{1}{4} \cos(x) \cos^{\frac{3}{2}}(2x) + \frac{3}{8} \cos(x) \sqrt{\cos(2x)} - \frac{3 \tanh^{-1}\left(\frac{\sqrt{2} \cos(x)}{\sqrt{\cos(2x)}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] `Int[Cos[2*x]^(3/2)*Sin[x],x]`

[Out] $(-3*\operatorname{ArcTanh}[(\operatorname{Sqrt}[2]*\operatorname{Cos}[x])/\operatorname{Sqrt}[\operatorname{Cos}[2*x]]])/(8*\operatorname{Sqrt}[2]) + (3*\operatorname{Cos}[x]*\operatorname{Sqrt}[\operatorname{Cos}[2*x]])/8 - (\operatorname{Cos}[x]*\operatorname{Cos}[2*x]^{(3/2)})/4$

Rule 201

`Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x*((a + b*x^n)^p/(n*p + 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])`

Rule 212

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 223

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

Rule 4442


```
Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}
 \int \cos^{\frac{3}{2}}(2x) \sin(x) dx &= -\text{Subst} \left(\int (-1 + 2x^2)^{3/2} dx, x, \cos(x) \right) \\
 &= -\frac{1}{4} \cos(x) \cos^{\frac{3}{2}}(2x) + \frac{3}{4} \text{Subst} \left(\int \sqrt{-1 + 2x^2} dx, x, \cos(x) \right) \\
 &= \frac{3}{8} \cos(x) \sqrt{\cos(2x)} - \frac{1}{4} \cos(x) \cos^{\frac{3}{2}}(2x) - \frac{3}{8} \text{Subst} \left(\int \frac{1}{\sqrt{-1 + 2x^2}} dx, x, \cos(x) \right) \\
 &= \frac{3}{8} \cos(x) \sqrt{\cos(2x)} - \frac{1}{4} \cos(x) \cos^{\frac{3}{2}}(2x) - \frac{3}{8} \text{Subst} \left(\int \frac{1}{1 - 2x^2} dx, x, \frac{\cos(x)}{\sqrt{\cos(2x)}} \right) \\
 &= -\frac{3 \tanh^{-1} \left(\frac{\sqrt{2} \cos(x)}{\sqrt{\cos(2x)}} \right)}{8\sqrt{2}} + \frac{3}{8} \cos(x) \sqrt{\cos(2x)} - \frac{1}{4} \cos(x) \cos^{\frac{3}{2}}(2x)
 \end{aligned}$$

Mathematica [A]

time = 0.07, size = 49, normalized size = 0.89

$$-\frac{1}{8} \sqrt{\cos(2x)} (-2 \cos(x) + \cos(3x)) - \frac{3 \log \left(\sqrt{2} \cos(x) + \sqrt{\cos(2x)} \right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]^(3/2)*Sin[x],x]
```

```
[Out] -1/8*(Sqrt[Cos[2*x]]*(-2*Cos[x] + Cos[3*x])) - (3*Log[Sqrt[2]*Cos[x] + Sqrt[Cos[2*x]]])/(8*Sqrt[2])
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Cos[2*x]^(3/2)*Sin[x],x]')
```

```
[Out] Timed out
```

Maple [A]

time = 0.09, size = 55, normalized size = 1.00

method	result
default	$-\frac{(\cos^3(x))\sqrt{2(\cos^2(x)) - 1}}{2} + \frac{5\cos(x)\sqrt{2(\cos^2(x)) - 1}}{8} - \frac{3\ln(\cos(x)\sqrt{2} + \sqrt{2(\cos^2(x)) - 1})\sqrt{2}}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^(3/2)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*cos(x)^3*(2*cos(x)^2-1)^(1/2)+5/8*cos(x)*(2*cos(x)^2-1)^(1/2)-3/16*ln(
cos(x)*2^(1/2)+(2*cos(x)^2-1)^(1/2))*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 790 vs. 2(39) = 78.

time = 0.38, size = 790, normalized size = 14.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/128*sqrt(2)*(4*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(((cos(4
*x) - 2)*cos(1/2*arctan2(sin(4*x), cos(4*x))) + sin(4*x)*sin(1/2*arctan2(si
n(4*x), cos(4*x))) + cos(4*x) - 2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))
- (cos(1/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x) - (cos(4*x) - 2)*sin(1/2*
arctan2(sin(4*x), cos(4*x))) - sin(4*x))*sin(1/2*arctan2(sin(4*x), cos(4*x)
+ 1)))) + 3*log(sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arct
an2(sin(4*x), cos(4*x) + 1))^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x)
+ 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + 2*(cos(4*x)^2 + sin(4*x)^
2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) - 3
*log(sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x)
), cos(4*x) + 1))^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*sin(1/
2*arctan2(sin(4*x), cos(4*x) + 1))^2 - 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4
*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) + 3*log(((cos(
1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2
)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(1/2*arctan2(sin(4*x), c
os(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2)*sin(1/2*arctan2(sin(4
*x), cos(4*x) + 1))^2)*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1) + 2*(
cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x),
cos(4*x) + 1))*cos(1/2*arctan2(sin(4*x), cos(4*x))) + sin(1/2*arctan2(sin(4
*x), cos(4*x) + 1))*sin(1/2*arctan2(sin(4*x), cos(4*x)))) + 1) - 3*log(((co
s(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))
^2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(1/2*arctan2(sin(4*x),
cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2)*sin(1/2*arctan2(sin
(4*x), cos(4*x) + 1))^2)*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1) - 2
```

$(\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1)^{1/4} (\cos(1/2 \arctan(2 \sin(4x), \cos(4x) + 1)) \cos(1/2 \arctan(2 \sin(4x), \cos(4x))) + \sin(1/2 \arctan(2 \sin(4x), \cos(4x)))) + 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(39) = 78.

time = 0.32, size = 103, normalized size = 1.87

$$-\frac{1}{8} (4 \cos(x)^3 - 5 \cos(x)) \sqrt{2 \cos(x)^2 - 1} + \frac{3}{128} \sqrt{2} \log \left(2048 \cos(x)^8 - 2048 \cos(x)^6 + 640 \cos(x)^4 - 64 \cos(x)^2 - 8 (128 \sqrt{2} \cos(x)^7 - 96 \sqrt{2} \cos(x)^5 + 20 \sqrt{2} \cos(x)^3 - \sqrt{2} \cos(x)) \sqrt{2 \cos(x)^2 - 1} + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="fricas")

[Out] $-1/8*(4*\cos(x)^3 - 5*\cos(x))*\sqrt{2*\cos(x)^2 - 1} + 3/128*\sqrt{2}*\log(2048*\cos(x)^8 - 2048*\cos(x)^6 + 640*\cos(x)^4 - 64*\cos(x)^2 - 8*(128*\sqrt{2}*\cos(x)^7 - 96*\sqrt{2}*\cos(x)^5 + 20*\sqrt{2}*\cos(x)^3 - \sqrt{2}*\cos(x))*\sqrt{2*\cos(x)^2 - 1} + 1)$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)**(3/2)*sin(x),x)

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)^(3/2)*sin(x),x)

[Out] Could not integrate

Mupad [B]

time = 0.42, size = 29, normalized size = 0.53

$$-\frac{\cos(2x)^{3/2} \cos(x) {}_2F_1\left(-\frac{3}{2}, \frac{1}{2}; \frac{3}{2}; \cos(2x) + 1\right)}{(-\cos(2x))^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)^(3/2)*sin(x),x)

[Out] $-(\cos(2x)^{3/2}*\cos(x)*\text{hypergeom}([-3/2, 1/2], 3/2, \cos(2x) + 1))/(-\cos(2x))^{3/2}$

$$3.431 \quad \int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)} dx$$

Optimal. Leaf size=16

$$-\frac{\cos(3x)}{3 \cos^{\frac{3}{2}}(2x)}$$

[Out] -1/3*cos(3*x)/cos(2*x)^(3/2)

Rubi [A]

time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4416}

$$-\frac{\cos(3x)}{3 \cos^{\frac{3}{2}}(2x)}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/Cos[2*x]^(5/2),x]

[Out] -1/3*Cos[3*x]/Cos[2*x]^(3/2)

Rule 4416

```
Int[(cos[(a_.) + (b_.)*(x_.)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_.)], x_Symbol
] := Simp[(- (m + 2)) * (e * Cos[a + b*x])^(m + 1) * (Cos[(m + 1) * (a + b*x)] / (d * e *
(m + 1))), x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d
/b, Abs[m + 2]]
```

Rubi steps

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)} dx = -\frac{\cos(3x)}{3 \cos^{\frac{3}{2}}(2x)}$$

Mathematica [A]

time = 0.03, size = 16, normalized size = 1.00

$$-\frac{\cos(3x)}{3 \cos^{\frac{3}{2}}(2x)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/Cos[2*x]^(5/2),x]

[Out] $-1/3*\text{Cos}[3*x]/\text{Cos}[2*x]^{(3/2)}$

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[Sin[x]/Cos[2*x]^(5/2),x]')`

[Out] Timed out

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(12) = 24.

time = 0.15, size = 39, normalized size = 2.44

method	result	size
default	$\frac{\sqrt{1 - 2(\sin^2(x))} \cos(x)(4(\sin^2(x))-1)}{12(\sin^4(x))-12(\sin^2(x))+3}$	39

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/cos(2*x)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $1/3/(4*\sin(x)^4-4*\sin(x)^2+1)*(1-2*\sin(x)^2)^{(1/2)}*\cos(x)*(4*\sin(x)^2-1)$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(12) = 24.

time = 0.34, size = 90, normalized size = 5.62

$$\frac{\sqrt{2} \sin\left(\frac{3}{2} \arctan(\sin(4x), \cos(4x) + 1)\right) \sin\left(\frac{3}{2} \arctan(\sin(4x), \cos(4x))\right) + \left(\sqrt{2} \cos\left(\frac{3}{2} \arctan(\sin(4x), \cos(4x))\right) + \sqrt{2}\right) \cos\left(\frac{3}{2} \arctan(\sin(4x), \cos(4x) + 1)\right)}{3(\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="maxima")`

[Out] $-1/3*(\text{sqrt}(2)*\sin(3/2*\arctan2(\sin(4*x), \cos(4*x) + 1))*\sin(3/2*\arctan2(\sin(4*x), \cos(4*x))) + (\text{sqrt}(2)*\cos(3/2*\arctan2(\sin(4*x), \cos(4*x))) + \text{sqrt}(2))*\cos(3/2*\arctan2(\sin(4*x), \cos(4*x) + 1)))/(\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1)^{(3/4)}$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(12) = 24.

time = 0.32, size = 39, normalized size = 2.44

$$\frac{(4 \cos(x)^3 - 3 \cos(x)) \sqrt{2 \cos(x)^2 - 1}}{3(4 \cos(x)^4 - 4 \cos(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="fricas")`

[Out] $-1/3*(4*\cos(x)^3 - 3*\cos(x))*\sqrt{2*\cos(x)^2 - 1}/(4*\cos(x)^4 - 4*\cos(x)^2 + 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/cos(2*x)**(5/2),x)`

[Out] `Integral(sin(x)/cos(2*x)**(5/2), x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(12) = 24$.

time = 0.06, size = 88, normalized size = 5.50

$$\frac{4 \cdot 2 \left(\left(\left(\frac{\tan^2\left(\frac{x}{2}\right)}{24} - \frac{5}{8} \right) \tan^2\left(\frac{x}{2}\right) + \frac{5}{8} \right) \tan^2\left(\frac{x}{2}\right) - \frac{1}{24} \right) \sqrt{\tan^4\left(\frac{x}{2}\right) - 6 \tan^2\left(\frac{x}{2}\right) + 1}}{\left(\tan^4\left(\frac{x}{2}\right) - 6 \tan^2\left(\frac{x}{2}\right) + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/cos(2*x)^(5/2),x)`

[Out] $1/3*((\tan(1/2*x)^2 - 15)*\tan(1/2*x)^2 + 15)*\tan(1/2*x)^2 - 1)/(\tan(1/2*x)^4 - 6*\tan(1/2*x)^2 + 1)^{(3/2)}$

Mupad [B]

time = 0.35, size = 12, normalized size = 0.75

$$-\frac{\cos(3x)}{3 \cos(2x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/cos(2*x)^(5/2),x)`

[Out] $-\cos(3*x)/(3*\cos(2*x)^{(3/2)})$

3.432 $\int \cos^{\frac{3}{2}}(2x) \sec^3(x) dx$

Optimal. Leaf size=49

$$2\sqrt{2} \sin^{-1}(\sqrt{2} \sin(x)) - \frac{5}{2} \tan^{-1}\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right) - \frac{1}{2} \sqrt{\cos(2x)} \sec(x) \tan(x)$$

[Out] $-5/2*\arctan(\sin(x)/\cos(2*x)^{(1/2)})+2*\arcsin(\sin(x)*2^{(1/2)})*2^{(1/2)}-1/2*\sec(x)*\cos(2*x)^{(1/2)}*\tan(x)$

Rubi [A]

time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {4449, 424, 537, 222, 385, 209}

$$2\sqrt{2} \sin^{-1}(\sqrt{2} \sin(x)) - \frac{5}{2} \tan^{-1}\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right) - \frac{1}{2} \sqrt{\cos(2x)} \tan(x) \sec(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[2*x]^(3/2)*Sec[x]^3,x]

[Out] $2*\text{Sqrt}[2]*\text{ArcSin}[\text{Sqrt}[2]*\text{Sin}[x]] - (5*\text{ArcTan}[\text{Sin}[x]/\text{Sqrt}[\text{Cos}[2*x]]])/2 - (\text{Sqrt}[\text{Cos}[2*x]]*\text{Sec}[x]*\text{Tan}[x])/2$

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 424

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(a*d - c*b)*x*(a + b*x^n)^(p+1)*((c + d*x^n)^(q-1)/(a*b*n*(p+1))), x] - Dist[1/(a*b*n*(p+1)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^(q-1), x]]

```
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
+ q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^n)*Sqrt[(c_) + (d_)*(x
_)^n]], x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4449

```
Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^(n - 1)/2, Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(2x) \sec^3(x) dx &= \text{Subst} \left(\int \frac{(1 - 2x^2)^{3/2}}{(1 - x^2)^2} dx, x, \sin(x) \right) \\
&= -\frac{1}{2} \sqrt{\cos(2x)} \sec(x) \tan(x) - \frac{1}{2} \text{Subst} \left(\int \frac{-3 + 8x^2}{\sqrt{1 - 2x^2} (1 - x^2)} dx, x, \sin(x) \right) \\
&= -\frac{1}{2} \sqrt{\cos(2x)} \sec(x) \tan(x) - \frac{5}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1 - 2x^2} (1 - x^2)} dx, x, \sin(x) \right) + 4S \\
&= 2\sqrt{2} \sin^{-1} \left(\sqrt{2} \sin(x) \right) - \frac{1}{2} \sqrt{\cos(2x)} \sec(x) \tan(x) - \frac{5}{2} \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \right. \\
&= 2\sqrt{2} \sin^{-1} \left(\sqrt{2} \sin(x) \right) - \frac{5}{2} \tan^{-1} \left(\frac{\sin(x)}{\sqrt{\cos(2x)}} \right) - \frac{1}{2} \sqrt{\cos(2x)} \sec(x) \tan(x)
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 49, normalized size = 1.00

$$\frac{1}{2} \left(4\sqrt{2} \sin^{-1} \left(\sqrt{2} \sin(x) \right) - 5 \tan^{-1} \left(\frac{\sin(x)}{\sqrt{\cos(2x)}} \right) - \sqrt{\cos(2x)} \sec(x) \tan(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[2*x]^(3/2)*Sec[x]^3,x]

[Out] (4*sqrt(2)*ArcSin[Sqrt[2]*Sin[x]] - 5*ArcTan[Sin[x]/Sqrt[Cos[2*x]]] - Sqrt[Cos[2*x]]*Sec[x]*Tan[x])/2

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Cos[2*x]^(3/2)/Cos[x]^3,x]')

[Out] Timed out

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(37) = 74.

time = 0.16, size = 100, normalized size = 2.04

method	result
default	$\frac{\sqrt{2(\cos^2(x) - 1)(\sin^2(x))} \left(4\sqrt{2} \arcsin(4(\cos^2(x) - 3)(\cos^2(x) - 5) \arctan\left(\frac{3(\cos^2(x) - 2)}{2\sqrt{-2(\sin^4(x) + \sin^2(x))}}\right) \right)}{4 \cos(x)^2 \sin(x) \sqrt{2(\cos^2(x) - 1)}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)^(3/2)/cos(x)^3,x,method=_RETURNVERBOSE)

[Out] -1/4*((2*cos(x)^2-1)*sin(x)^2)^(1/2)*(4*2^(1/2)*arcsin(4*cos(x)^2-3)*cos(x)^2-5*arctan(1/2*(3*cos(x)^2-2)/(-2*sin(x)^4+sin(x)^2)^(1/2))*cos(x)^2+2*(-2)*sin(x)^4+sin(x)^2)^(1/2))/cos(x)^2/sin(x)/(2*cos(x)^2-1)^(1/2)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="maxima")

[Out] integrate(cos(2*x)^(3/2)/cos(x)^3, x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(37) = 74.

time = 0.34, size = 118, normalized size = 2.41

$$\frac{2\sqrt{2} \arctan\left(\frac{(32\sqrt{2}\cos(x)^4 - 48\sqrt{2}\cos(x)^2 + 17\sqrt{2})\sqrt{2\cos(x)^2 - 1}}{8(8\cos(x)^4 - 10\cos(x)^2 + 3)\sin(x)}\right) \cos(x)^2 - 5 \arctan\left(\frac{3\cos(x)^2 - 2}{2\sqrt{2\cos(x)^2 - 1}\sin(x)}\right) \cos(x)^2 + 2\sqrt{2\cos(x)^2 - 1}\sin(x)}{4\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="fricas")

[Out]
$$-1/4*(2*\sqrt{2}*\arctan(1/8*(32*\sqrt{2}*\cos(x)^4 - 48*\sqrt{2}*\cos(x)^2 + 17*\sqrt{2})*\sqrt{2*\cos(x)^2 - 1})/((8*\cos(x)^4 - 10*\cos(x)^2 + 3)*\sin(x)))*\cos(x)^2 - 5*\arctan(1/2*(3*\cos(x)^2 - 2)/(\sqrt{2*\cos(x)^2 - 1}*\sin(x)))*\cos(x)^2 + 2*\sqrt{2*\cos(x)^2 - 1}*\sin(x))/\cos(x)^2$$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)**(3/2)/cos(x)**3,x)

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)^(3/2)/cos(x)^3,x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(2x)^{3/2}}{\cos(x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)^(3/2)/cos(x)^3,x)

[Out] int(cos(2*x)^(3/2)/cos(x)^3, x)

$$3.433 \quad \int \frac{\sin^2(x)(3\sin^3(x) - \cos(x)\sin(4x))}{\cos^{\frac{7}{2}}(2x)} dx$$

Optimal. Leaf size=87

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{\sqrt{2}} - \frac{11\cos(x)}{20\cos^{\frac{3}{2}}(2x)} - \frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)} + \frac{63\cos(x)}{20\sqrt{\cos(2x)}} + \frac{3\cos(x)\sin^2(x)}{10\cos^{\frac{5}{2}}(2x)}$$

[Out] $-11/20*\cos(x)/\cos(2*x)^{(3/2)}-2/3*\cos(x)^3/\cos(2*x)^{(3/2)}+3/10*\cos(x)*\sin(x)^2/\cos(2*x)^{(5/2)}-1/2*\operatorname{arctanh}(\cos(x)*2^{(1/2)}/\cos(2*x)^{(1/2)})*2^{(1/2)}+63/20*\cos(x)/\cos(2*x)^{(1/2)}$

Rubi [A]

time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {4462, 12, 463, 294, 223, 212, 4451, 386, 197}

$$-\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)} + \frac{13\cos(x)}{5\sqrt{\cos(2x)}} + \frac{3\sin^4(x)\cos(x)}{5\cos^{\frac{5}{2}}(2x)} - \frac{4\sin^2(x)\cos(x)}{5\cos^{\frac{3}{2}}(2x)} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sin}[x]^2*(3*\operatorname{Sin}[x]^3 - \operatorname{Cos}[x]*\operatorname{Sin}[4*x]))/\operatorname{Cos}[2*x]^{(7/2)}, x]$

[Out] $-(\operatorname{ArcTanh}[(\operatorname{Sqrt}[2]*\operatorname{Cos}[x])/\operatorname{Sqrt}[\operatorname{Cos}[2*x]]]/\operatorname{Sqrt}[2]) - (2*\operatorname{Cos}[x]^3)/(3*\operatorname{Cos}[2*x]^{(3/2)}) + (13*\operatorname{Cos}[x])/(5*\operatorname{Sqrt}[\operatorname{Cos}[2*x]]) - (4*\operatorname{Cos}[x]*\operatorname{Sin}[x]^2)/(5*\operatorname{Cos}[2*x]^{(3/2)}) + (3*\operatorname{Cos}[x]*\operatorname{Sin}[x]^4)/(5*\operatorname{Cos}[2*x]^{(5/2)})$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& \operatorname{!Match} Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 197

$\operatorname{Int}[((a_) + (b_.)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \operatorname{Simp}[x*((a + b*x^n)^{(p + 1)}/a), x] /; \operatorname{FreeQ}\{a, b, n, p\}, x \&\& \operatorname{EqQ}[1/n + p + 1, 0]$

Rule 212

$\operatorname{Int}[((a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{Gt} Q[a, 0] \operatorname{||} \operatorname{Lt} Q[b, 0])$

Rule 223

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 294

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 386

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

Rule 463

Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*e*(m + 1))), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && NeQ[m, -1]

Rule 4451

Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))]^(n_), x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^2)^(n - 1)/2], Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])

Rule 4462

Int[(u_)*((v_) + (d_)*(F_)[(c_)*((a_) + (b_)*(x_))]^(n_)), x_Symbol] := With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] + Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c*(a + b*x)]/e, u, x] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(x) (3 \sin^3(x) - \cos(x) \sin(4x))}{\cos^{\frac{7}{2}}(2x)} dx &= 3 \int \frac{\sin^5(x)}{\cos^{\frac{7}{2}}(2x)} dx - \int \frac{\cos(x) \sin^2(x) \sin(4x)}{\cos^{\frac{7}{2}}(2x)} dx \\
&= - \left(3 \text{Subst} \left(\int \frac{(1-x^2)^2}{(-1+2x^2)^{7/2}} dx, x, \cos(x) \right) \right) + \text{Subst} \left(\int \frac{4x}{(-1+2x^2)^{5/2}} dx, x, \cos(x) \right) \\
&= \frac{3 \cos(x) \sin^4(x)}{5 \cos^{\frac{5}{2}}(2x)} + \frac{12}{5} \text{Subst} \left(\int \frac{1-x^2}{(-1+2x^2)^{5/2}} dx, x, \cos(x) \right) + \\
&= -\frac{2 \cos^3(x)}{3 \cos^{\frac{3}{2}}(2x)} - \frac{4 \cos(x) \sin^2(x)}{5 \cos^{\frac{3}{2}}(2x)} + \frac{3 \cos(x) \sin^4(x)}{5 \cos^{\frac{5}{2}}(2x)} - \frac{8}{5} \text{Subst} \left(\int \frac{x}{(-1+2x^2)^{3/2}} dx, x, \cos(x) \right) \\
&= -\frac{2 \cos^3(x)}{3 \cos^{\frac{3}{2}}(2x)} + \frac{13 \cos(x)}{5 \sqrt{\cos(2x)}} - \frac{4 \cos(x) \sin^2(x)}{5 \cos^{\frac{3}{2}}(2x)} + \frac{3 \cos(x) \sin^4(x)}{5 \cos^{\frac{5}{2}}(2x)} \\
&= -\frac{2 \cos^3(x)}{3 \cos^{\frac{3}{2}}(2x)} + \frac{13 \cos(x)}{5 \sqrt{\cos(2x)}} - \frac{4 \cos(x) \sin^2(x)}{5 \cos^{\frac{3}{2}}(2x)} + \frac{3 \cos(x) \sin^4(x)}{5 \cos^{\frac{5}{2}}(2x)} \\
&\quad - \frac{\tanh^{-1} \left(\frac{\sqrt{2} \cos(x)}{\sqrt{\cos(2x)}} \right)}{\sqrt{2}} - \frac{2 \cos^3(x)}{3 \cos^{\frac{3}{2}}(2x)} + \frac{13 \cos(x)}{5 \sqrt{\cos(2x)}} - \frac{4 \cos(x) \sin^2(x)}{5 \cos^{\frac{3}{2}}(2x)} + \frac{3 \cos(x) \sin^4(x)}{5 \cos^{\frac{5}{2}}(2x)}
\end{aligned}$$

Mathematica [A]

time = 0.13, size = 62, normalized size = 0.71

$$\frac{250 \cos(x) + 45 \cos(3x) + 169 \cos(5x) - 120 \sqrt{2} \cos^{\frac{5}{2}}(2x) \log \left(\sqrt{2} \cos(x) + \sqrt{\cos(2x)} \right)}{240 \cos^{\frac{5}{2}}(2x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x]^2*(3*SIN[x]^3 - Cos[x]*Sin[4*x]))/Cos[2*x]^(7/2),x]
```

```
[Out] (250*Cos[x] + 45*Cos[3*x] + 169*Cos[5*x] - 120*Sqrt[2]*Cos[2*x]^(5/2)*Log[Sqrt[2]*Cos[x] + Sqrt[Cos[2*x]])/(240*Cos[2*x]^(5/2))
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(3*SIN[x]^3 - Cos[x]*Sin[4*x])/(Csc[x]^2*Cos[2*x]^(7/2)),x]')
```

[Out] Timed out

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(65) = 130$.

time = 0.31, size = 180, normalized size = 2.07

method	result
default	$-\frac{120 \ln(\cos(x)\sqrt{2} + \sqrt{1 - 2(\sin^2(x))})\sqrt{2} (\sin^6(x)) + 338 \sqrt{1 - 2(\sin^2(x))} \cos(x)(\sin^4(x)) - 180 \ln(\cos(x)\sqrt{2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x,method=_RETURNVE
RBOSE)`

[Out]
$$-1/30/(8*\sin(x)^6-12*\sin(x)^4+6*\sin(x)^2-1)*(120*\ln(\cos(x)*2^{(1/2)}+(1-2*\sin(x)^2)^{(1/2)})*2^{(1/2)}*\sin(x)^6+338*(1-2*\sin(x)^2)^{(1/2)}*\cos(x)*\sin(x)^4-180*\ln(\cos(x)*2^{(1/2)}+(1-2*\sin(x)^2)^{(1/2)})*2^{(1/2)}*\sin(x)^4-276*(1-2*\sin(x)^2)^{(1/2)}*\sin(x)^2*\cos(x)+90*\ln(\cos(x)*2^{(1/2)}+(1-2*\sin(x)^2)^{(1/2)})*2^{(1/2)}*\sin(x)^2+58*(1-2*\sin(x)^2)^{(1/2)}*\cos(x)-15*\ln(\cos(x)*2^{(1/2)}+(1-2*\sin(x)^2)^{(1/2)})*2^{(1/2)})$$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1359 vs. $2(65) = 130$.

time = 0.44, size = 1359, normalized size = 15.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x, algorithm
="maxima")`

[Out]
$$\begin{aligned} & 1/48*(4*(4*\sqrt{2}*\sin(4*x)*\sin(5/2*\arctan2(\sin(4*x), \cos(4*x))) + 4*(\sqrt{2}*\cos(4*x) + \sqrt{2})*\cos(5/2*\arctan2(\sin(4*x), \cos(4*x))) + 3*\sqrt{2}*\cos(8*x) + 7*\sqrt{2}*\cos(4*x) + 4*\sqrt{2})*\cos(5/2*\arctan2(\sin(4*x), \cos(4*x) + 1)) + 12*\sqrt{2}*\sqrt{\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1}*\cos(3/2*\arctan2(\sin(4*x), \cos(4*x) + 1)) - 12*(\sqrt{2}*\cos(4*x)^2 + \sqrt{2}*\sin(4*x)^2 + 2*\sqrt{2}*\cos(4*x) + \sqrt{2})*\cos(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1)) - 4*(4*\sqrt{2}*\cos(5/2*\arctan2(\sin(4*x), \cos(4*x)))*\sin(4*x) - 4*(\sqrt{2}*\cos(4*x) + \sqrt{2})*\sin(5/2*\arctan2(\sin(4*x), \cos(4*x))) - 3*\sqrt{2}*\sin(8*x) - 7*\sqrt{2}*\sin(4*x))*\sin(5/2*\arctan2(\sin(4*x), \cos(4*x) + 1)) - 3*(\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1)^{(1/4)}*((\sqrt{2}*\cos(4*x)^2 + \sqrt{2}*\sin(4*x)^2 + 2*\sqrt{2}*\cos(4*x) + \sqrt{2})*\log(\sqrt{\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1}*\cos(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1))^2 + \sqrt{\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1}*\sin(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1))^2 + 2*(\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1)^{(1/4)}*\cos(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1)) + 1) - (\sqrt{2}*\cos(4*x)^2 + \sqrt{2}*\sin(4*x)^2 + 2* \end{aligned}$$

$$\begin{aligned} & \sqrt{2} \cos(4x) + \sqrt{2} \log(\sqrt{\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1} \cdot \cos(1/2 \arctan 2(\sin(4x), \cos(4x) + 1))^2 + \sqrt{\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1} \cdot \sin(1/2 \arctan 2(\sin(4x), \cos(4x) + 1))^2 - 2(\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1)^{1/4} \cdot \cos(1/2 \arctan 2(\sin(4x), \cos(4x) + 1)) + 1) + (\sqrt{2} \cos(4x)^2 + \sqrt{2} \sin(4x)^2 + 2\sqrt{2} \cos(4x) + \sqrt{2}) \cdot \log(((\cos(1/2 \arctan 2(\sin(4x), \cos(4x)))^2 + \sin(1/2 \arctan 2(\sin(4x), \cos(4x))))^2 \cdot \cos(1/2 \arctan 2(\sin(4x), \cos(4x) + 1))^2 + (\cos(1/2 \arctan 2(\sin(4x), \cos(4x)))^2 + \sin(1/2 \arctan 2(\sin(4x), \cos(4x)))^2) \cdot \sin(1/2 \arctan 2(\sin(4x), \cos(4x) + 1))^2) \cdot \sqrt{\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1} + 2(\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1)^{1/4} \cdot (\cos(1/2 \arctan 2(\sin(4x), \cos(4x) + 1)) \cdot \cos(1/2 \arctan 2(\sin(4x), \cos(4x)))) + \sin(1/2 \arctan 2(\sin(4x), \cos(4x) + 1)) \cdot \sin(1/2 \arctan 2(\sin(4x), \cos(4x)))) + 1) - (\sqrt{2} \cos(4x)^2 + \sqrt{2} \sin(4x)^2 + 2\sqrt{2} \cos(4x) + \sqrt{2}) \cdot \log(((\cos(1/2 \arctan 2(\sin(4x), \cos(4x)))^2 + \sin(1/2 \arctan 2(\sin(4x), \cos(4x))))^2 \cdot \cos(1/2 \arctan 2(\sin(4x), \cos(4x) + 1))^2 + (\cos(1/2 \arctan 2(\sin(4x), \cos(4x)))^2 + \sin(1/2 \arctan 2(\sin(4x), \cos(4x)))^2) \cdot \sin(1/2 \arctan 2(\sin(4x), \cos(4x) + 1))^2) \cdot \sqrt{\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1} - 2(\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1)^{1/4} \cdot (\cos(1/2 \arctan 2(\sin(4x), \cos(4x) + 1)) \cdot \cos(1/2 \arctan 2(\sin(4x), \cos(4x)))) + \sin(1/2 \arctan 2(\sin(4x), \cos(4x) + 1)) \cdot \sin(1/2 \arctan 2(\sin(4x), \cos(4x)))) + 1)) / (\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1)^{5/4} + 1/20 \cdot (((15 \cos(8x) + 70 \cos(4x) + 43) \cdot \cos(5/2 \arctan 2(\sin(4x), \cos(4x)))) + 5 \cdot (3 \sin(8x) + 14 \sin(4x)) \cdot \sin(5/2 \arctan 2(\sin(4x), \cos(4x)))) - 12) \cdot \cos(5/2 \arctan 2(\sin(4x), \cos(4x) + 1)) + 15 \cdot (\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1) \cdot \cos(1/2 \arctan 2(\sin(4x), \cos(4x) + 1)) - (5 \cdot (3 \sin(8x) + 14 \sin(4x)) \cdot \cos(5/2 \arctan 2(\sin(4x), \cos(4x)))) - (15 \cos(8x) + 70 \cos(4x) + 43) \cdot \sin(5/2 \arctan 2(\sin(4x), \cos(4x)))) \cdot \sin(5/2 \arctan 2(\sin(4x), \cos(4x) + 1)) + 40 \sqrt{\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1} \cdot \cos(3/2 \arctan 2(\sin(4x), \cos(4x) + 1))) / ((\sqrt{2} \cos(4x)^2 + \sqrt{2} \sin(4x)^2 + 2\sqrt{2} \cos(4x) + \sqrt{2}) \cdot (\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1)^{1/4}) \end{aligned}$$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(65) = 130.

time = 0.34, size = 163, normalized size = 1.87

$$\frac{15 \left(8 \sqrt{2} \cos(x)^8 - 12 \sqrt{2} \cos(x)^6 + 6 \sqrt{2} \cos(x)^4 - \sqrt{2} \right) \log \left(2048 \cos(x)^8 - 2048 \cos(x)^6 + 640 \cos(x)^4 - 64 \cos(x)^2 - 8 \left(128 \sqrt{2} \cos(x)^7 - 96 \sqrt{2} \cos(x)^5 + 20 \sqrt{2} \cos(x)^3 - \sqrt{2} \cos(x) \right) \sqrt{2 \cos(x)^2 - 1} + 1 \right) + 16 \left(169 \cos(x)^5 - 200 \cos(x)^3 + 60 \cos(x) \right) \sqrt{2 \cos(x)^2 - 1}}{240 \left(8 \cos(x)^8 - 12 \cos(x)^6 + 6 \cos(x)^4 - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x, algorithm="fricas")

[Out] 1/240*(15*(8*sqrt(2)*cos(x)^6 - 12*sqrt(2)*cos(x)^4 + 6*sqrt(2)*cos(x)^2 - sqrt(2))*log(2048*cos(x)^8 - 2048*cos(x)^6 + 640*cos(x)^4 - 64*cos(x)^2 - 8*(128*sqrt(2)*cos(x)^7 - 96*sqrt(2)*cos(x)^5 + 20*sqrt(2)*cos(x)^3 - sqrt(2)*cos(x))*sqrt(2*cos(x)^2 - 1) + 1) + 16*(169*cos(x)^5 - 200*cos(x)^3 + 60*cos(x))*sqrt(2*cos(x)^2 - 1)/(8*cos(x)^6 - 12*cos(x)^4 + 6*cos(x)^2 - 1)

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*sin(x)**3-cos(x)*sin(4*x))/cos(2*x)**(7/2)/csc(x)**2,x)**[Out]** Timed out**Giac [A]**

time = 0.02, size = 79, normalized size = 0.91

$$\frac{2 \left(\left(\frac{169}{30} \cos x \cdot \cos x - \frac{20}{3} \right) \cos x \cdot \cos x + 2 \right) \cos x \cdot \sqrt{2 \cos^2 x - 1}}{(2 \cos^2 x - 1)^3} + \frac{\ln \left| \sqrt{2 \cos^2 x - 1} - \sqrt{2} \cos x \right|}{\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x)**[Out]** 1/2*sqrt(2)*log(abs(-sqrt(2)*cos(x) + sqrt(2*cos(x)^2 - 1))) + 1/15*((169*cos(x)^2 - 200)*cos(x)^2 + 60)*cos(x)/(2*cos(x)^2 - 1)^(5/2)**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(x)^2 (3 \sin(x)^3 - \sin(4x) \cos(x))}{\cos(2x)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x)^2*(3*sin(x)^3 - sin(4*x)*cos(x)))/cos(2*x)^(7/2),x)**[Out]** int((sin(x)^2*(3*sin(x)^3 - sin(4*x)*cos(x)))/cos(2*x)^(7/2), x)

3.434 $\int (4 - 5 \sec^2(x))^{3/2} dx$

Optimal. Leaf size=68

$$8 \tan^{-1} \left(\frac{2 \tan(x)}{\sqrt{-1 - 5 \tan^2(x)}} \right) - \frac{7}{2} \sqrt{5} \tan^{-1} \left(\frac{\sqrt{5} \tan(x)}{\sqrt{-1 - 5 \tan^2(x)}} \right) - \frac{5}{2} \tan(x) \sqrt{-1 - 5 \tan^2(x)}$$

[Out] 8*arctan(2*tan(x)/(-1-5*tan(x)^2)^(1/2))-7/2*arctan(5^(1/2)*tan(x)/(-1-5*tan(x)^2)^(1/2))*5^(1/2)-5/2*(-1-5*tan(x)^2)^(1/2)*tan(x)

Rubi [A]

time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4213, 427, 537, 223, 209, 385}

$$-\frac{5}{2} \tan(x) \sqrt{-5 \tan^2(x) - 1} + 8 \tan^{-1} \left(\frac{2 \tan(x)}{\sqrt{-5 \tan^2(x) - 1}} \right) - \frac{7}{2} \sqrt{5} \tan^{-1} \left(\frac{\sqrt{5} \tan(x)}{\sqrt{-5 \tan^2(x) - 1}} \right)$$

Antiderivative was successfully verified.

[In] Int[(4 - 5*Sec[x]^2)^(3/2), x]

[Out] 8*ArcTan[(2*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]] - (7*Sqrt[5]*ArcTan[(Sqrt[5]*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]])/2 - (5*Tan[x]*Sqrt[-1 - 5*Tan[x]^2])/2

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 223

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 427

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x
_)^(n_)], x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4213

```
Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)^2]^p), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int (4 - 5 \sec^2(x))^{3/2} dx &= \text{Subst} \left(\int \frac{(-1 - 5x^2)^{3/2}}{1 + x^2} dx, x, \tan(x) \right) \\
&= -\frac{5}{2} \tan(x) \sqrt{-1 - 5 \tan^2(x)} + \frac{1}{2} \text{Subst} \left(\int \frac{-3 - 35x^2}{\sqrt{-1 - 5x^2} (1 + x^2)} dx, x, \tan(x) \right) \\
&= -\frac{5}{2} \tan(x) \sqrt{-1 - 5 \tan^2(x)} + 16 \text{Subst} \left(\int \frac{1}{\sqrt{-1 - 5x^2} (1 + x^2)} dx, x, \tan(x) \right) \\
&= -\frac{5}{2} \tan(x) \sqrt{-1 - 5 \tan^2(x)} + 16 \text{Subst} \left(\int \frac{1}{1 + 4x^2} dx, x, \frac{\tan(x)}{\sqrt{-1 - 5 \tan^2(x)}} \right) \\
&= 8 \tan^{-1} \left(\frac{2 \tan(x)}{\sqrt{-1 - 5 \tan^2(x)}} \right) - \frac{7}{2} \sqrt{5} \tan^{-1} \left(\frac{\sqrt{5} \tan(x)}{\sqrt{-1 - 5 \tan^2(x)}} \right) - \frac{5}{2} \tan(x)
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.14, size = 115, normalized size = 1.69

$$\frac{(-5 + 4 \cos^2(x)) \sec(x) \sqrt{4 - 5 \sec^2(x)} \left(7\sqrt{5} \tan^{-1} \left(\frac{\sqrt{5} \sin(x)}{\sqrt{-3 + 2 \cos(2x)}} \right) \cos^2(x) + 16i \cos^2(x) \log \left(\sqrt{-3 + 2 \cos(2x)} + 2i \sin(x) \right) + 5 \sqrt{-3 + 2 \cos(2x)} \sin(x) \right)}{2(-3 + 2 \cos(2x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(4 - 5*Sec[x]^2)^(3/2),x]

[Out]
$$-1/2*((-5 + 4*\cos[x]^2)*\sec[x]*\sqrt{4 - 5*\sec[x]^2}*(7*\sqrt{5}*\operatorname{ArcTan}[\sqrt{5}*\sin[x]]/\sqrt{-3 + 2*\cos[2*x]})*\cos[x]^2 + (16*I)*\cos[x]^2*\log[\sqrt{-3 + 2*\cos[2*x]}] + (2*I)*\sin[x]] + 5*\sqrt{-3 + 2*\cos[2*x]}*\sin[x])/(-3 + 2*\cos[2*x])^(3/2)$$

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(4 - 5*Sec[x]^2)^(3/2),x]')

[Out] Timed out

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.46, size = 754, normalized size = 11.09

method	result	size
default	Expression too large to display	754

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4-5*sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out]
$$-1/2*I/(-9-4*5^{(1/2)})^{(1/2)}/(2+5^{(1/2)})*(64*I*\cos(x)^2*\sin(x)*2^{(1/2)}*(-2*(2*\cos(x)*5^{(1/2)}+4*\cos(x)-2*5^{(1/2)}-5)/(1+\cos(x)))^{(1/2)}*((2*\cos(x)*5^{(1/2)}-4*\cos(x)-2*5^{(1/2)}+5)/(1+\cos(x)))^{(1/2)}*\operatorname{EllipticPi}((-9-4*5^{(1/2)})^{(1/2)}*(\cos(x)-1)/\sin(x),1/(9+4*5^{(1/2)}),(-9+4*5^{(1/2)})^{(1/2)}/(-9-4*5^{(1/2)})^{(1/2)})*5^{(1/2)}+3*I*\cos(x)^2*\sin(x)*2^{(1/2)}*(-2*(2*\cos(x)*5^{(1/2)}+4*\cos(x)-2*5^{(1/2)}-5)/(1+\cos(x)))^{(1/2)}*((2*\cos(x)*5^{(1/2)}-4*\cos(x)-2*5^{(1/2)}+5)/(1+\cos(x)))^{(1/2)}*\operatorname{EllipticF}(I*(\cos(x)-1)*(2+5^{(1/2)})/\sin(x),9-4*5^{(1/2)})*5^{(1/2)}-70*I*\cos(x)^2*\sin(x)*2^{(1/2)}*(-2*(2*\cos(x)*5^{(1/2)}+4*\cos(x)-2*5^{(1/2)}-5)/(1+\cos(x)))^{(1/2)}*((2*\cos(x)*5^{(1/2)}-4*\cos(x)-2*5^{(1/2)}+5)/(1+\cos(x)))^{(1/2)}*\operatorname{EllipticPi}((-9-4*5^{(1/2)})^{(1/2)}*(\cos(x)-1)/\sin(x),-1/(9+4*5^{(1/2)}),(-9+4*5^{(1/2)})^{(1/2)}/(-9-4*5^{(1/2)})^{(1/2)})*5^{(1/2)}+128*I*\cos(x)^2*\sin(x)*2^{(1/2)}*(-2*(2*\cos(x)*5^{(1/2)}+4*\cos(x)-2*5^{(1/2)}-5)/(1+\cos(x)))^{(1/2)}*((2*\cos(x)*5^{(1/2)}-4*\cos(x)-2*5^{(1/2)}+5)/(1+\cos(x)))^{(1/2)}*\operatorname{EllipticPi}((-9-4*5^{(1/2)})^{(1/2)}*(\cos(x)-1)/\sin(x),1/(9+4*5^{(1/2)}),(-9+4*5^{(1/2)})^{(1/2)}/(-9-4*5^{(1/2)})^{(1/2)})+6*I*\cos(x)^2*\sin(x)*2^{(1/2)}*(-2*(2*\cos(x)*5^{(1/2)}+4*\cos(x)-2*5^{(1/2)}-5)/(1+\cos(x)))^{(1/2)}*((2*\cos(x)*5^{(1/2)}-4*\cos(x)-2*5^{(1/2)}+5)/(1+\cos(x)))^{(1/2)}*\operatorname{EllipticF}(I*(\cos(x)-1)*(2+5^{(1/2)})/\sin(x),9-4*5^{(1/2)})-140*I*\cos(x)^2*\sin(x)*2^{(1/2)}*(-2*(2*\cos(x)*5^{(1/2)}+4*\cos(x)-2*5^{(1/2)}-5)/(1+\cos(x)))^{(1/2)}*((2*\cos(x)*5^{(1/2)}-4*\cos(x)-2*5^{(1/2)}+5)/(1+\cos(x)))^{(1/2)}*((2*\cos(x)*5^{(1/2)}+4*\cos(x)-2*5^{(1/2)}-5)/(1+\cos(x)))^{(1/2)}*((2*\cos(x)*5^{(1/2)}-4*\cos(x)-2*5^{(1/2)}+5)/(1+\cos(x)))^{(1/2)}$$

$s(x) \cdot 5^{1/2} - 4 \cos(x) - 2 \cdot 5^{1/2} + 5 / (1 + \cos(x))^{1/2} \cdot \text{EllipticPi}((-9 - 4 \cdot 5^{1/2})^{1/2} \cdot (\cos(x) - 1) / \sin(x), -1 / (9 + 4 \cdot 5^{1/2}), (-9 + 4 \cdot 5^{1/2})^{1/2} / (-9 - 4 \cdot 5^{1/2})^{1/2}) + 80 \cos(x)^3 \cdot 5^{1/2} + 180 \cos(x)^3 - 80 \cos(x)^2 \cdot 5^{1/2} - 180 \cos(x)^2 - 100 \cos(x) \cdot 5^{1/2} - 225 \cos(x) + 100 \cdot 5^{1/2} + 225) \cdot \cos(x) \cdot \sin(x) \cdot ((4 \cos(x)^2 - 5) / \cos(x)^2)^{3/2} / (\cos(x) - 1) / (4 \cos(x)^2 - 5)^2$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4-5*sec(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((-5*sec(x)^2 + 4)^(3/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(54) = 108.

time = 0.33, size = 130, normalized size = 1.91

$$7\sqrt{5} \arctan\left(\frac{\sqrt{5} \sqrt{\frac{4 \cos(x)^2 - 5}{\cos(x)^2}} \cos(x)}{5 \sin(x)}\right) \cos(x) + 8 \arctan\left(\frac{4(8 \cos(x)^3 - 9 \cos(x)) \sqrt{\frac{4 \cos(x)^2 - 5}{\cos(x)^2}} \sin(x) + \cos(x) \sin(x)}{64 \cos(x)^4 - 143 \cos(x)^2 + 80}\right) \cos(x) - 8 \arctan\left(\frac{\sin(x)}{\cos(x)}\right) \cos(x) - 5 \sqrt{\frac{4 \cos(x)^2 - 5}{\cos(x)^2}} \sin(x)$$

2 cos(x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4-5*sec(x)^2)^(3/2),x, algorithm="fricas")

[Out] 1/2*(7*sqrt(5)*arctan(1/5*sqrt(5)*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*cos(x)/sin(x))*cos(x) + 8*arctan((4*(8*cos(x)^3 - 9*cos(x))*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*sin(x) + cos(x)*sin(x))/(64*cos(x)^4 - 143*cos(x)^2 + 80))*cos(x) - 8*arctan(sin(x)/cos(x))*cos(x) - 5*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*sin(x))/cos(x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (4 - 5 \sec^2(x))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4-5*sec(x)**2)**(3/2),x)

[Out] Integral((4 - 5*sec(x)**2)**(3/2), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-5*sec(x)^2)^(3/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(4 - \frac{5}{\cos(x)^2}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4 - 5/cos(x)^2)^(3/2),x)
```

```
[Out] int((4 - 5/cos(x)^2)^(3/2), x)
```

$$3.435 \quad \int \frac{1}{(4-5 \sec^2(x))^{3/2}} dx$$

Optimal. Leaf size=40

$$\frac{1}{8} \tan^{-1} \left(\frac{2 \tan(x)}{\sqrt{-1-5 \tan^2(x)}} \right) - \frac{5 \tan(x)}{4 \sqrt{-1-5 \tan^2(x)}}$$

[Out] 1/8*arctan(2*tan(x)/(-1-5*tan(x)^2)^(1/2))-5/4*tan(x)/(-1-5*tan(x)^2)^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4213, 390, 385, 209}

$$\frac{1}{8} \tan^{-1} \left(\frac{2 \tan(x)}{\sqrt{-5 \tan^2(x) - 1}} \right) - \frac{5 \tan(x)}{4 \sqrt{-5 \tan^2(x) - 1}}$$

Antiderivative was successfully verified.

[In] Int[(4 - 5*Sec[x]^2)^(-3/2), x]

[Out] ArcTan[(2*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]]/8 - (5*Tan[x])/(4*Sqrt[-1 - 5*Tan[x]^2])

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(-b)*x*(a + b*x^n)^(p+1)*((c + d*x^n)^(q+1)/(a*n*(p+1)*(b*c - a*d))), x] + Dist[(b*c + n*(p+1)*(b*c - a*d))/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p+q+2)+1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

Rule 4213

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] & & NeQ[a + b, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(4 - 5 \sec^2(x))^{3/2}} dx &= \text{Subst} \left(\int \frac{1}{(-1 - 5x^2)^{3/2} (1 + x^2)} dx, x, \tan(x) \right) \\
 &= -\frac{5 \tan(x)}{4 \sqrt{-1 - 5 \tan^2(x)}} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{\sqrt{-1 - 5x^2} (1 + x^2)} dx, x, \tan(x) \right) \\
 &= -\frac{5 \tan(x)}{4 \sqrt{-1 - 5 \tan^2(x)}} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{1 + 4x^2} dx, x, \frac{\tan(x)}{\sqrt{-1 - 5 \tan^2(x)}} \right) \\
 &= \frac{1}{8} \tan^{-1} \left(\frac{2 \tan(x)}{\sqrt{-1 - 5 \tan^2(x)}} \right) - \frac{5 \tan(x)}{4 \sqrt{-1 - 5 \tan^2(x)}}
 \end{aligned}$$

Mathematica [A]

time = 0.08, size = 79, normalized size = 1.98

$$\frac{(-3 + 2 \cos(2x))^{3/2} \sec^3(x) \left(\sinh^{-1}(2 \sin(x))(-3 + 2 \cos(2x)) + 10 \sqrt{3 - 2 \cos(2x)} \sin(x) \right)}{8 (4 - 5 \sec^2(x))^{3/2} \sqrt{-(1 + 4 \sin^2(x))^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(4 - 5*Sec[x]^2)^(-3/2), x]

[Out] -1/8*((-3 + 2*Cos[2*x])^(3/2)*Sec[x]^3*(ArcSinh[2*Sin[x]]*(-3 + 2*Cos[2*x]) + 10*Sqrt[3 - 2*Cos[2*x]]*Sin[x]))/((4 - 5*Sec[x]^2)^(3/2)*Sqrt[-(1 + 4*Sin[x]^2)^2])

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[1/(4 - 5*Sec[x]^2)^(3/2),x]')`

[Out] Timed out

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.44, size = 473, normalized size = 11.82

method	result
default	$\frac{i(4(\cos^2(x))-5) \left(2i \sin(x) \sqrt{2} \operatorname{EllipticPi} \left(\frac{\sqrt{-9-4\sqrt{5}} (\cos(x)-1)}{\sin(x)}, \frac{1}{9+4\sqrt{5}}, \frac{\sqrt{-9+4\sqrt{5}}}{\sqrt{-9-4\sqrt{5}}} \right) \sqrt{5} \sqrt{-\frac{2(2\cos(x)-1)}{9+4\sqrt{5}}} \right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(4-5*sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)`

[Out]
$$-1/4*I/(-9-4*5^{(1/2)})^{(1/2)}/(2+5^{(1/2)})*(4*\cos(x)^2-5)*(2*I*\sin(x)*2^{(1/2)}* \operatorname{EllipticPi}((-9-4*5^{(1/2)})^{(1/2)}*(\cos(x)-1)/\sin(x),1/(9+4*5^{(1/2)})),(-9+4*5^{(1/2)})^{(1/2)}/(-9-4*5^{(1/2)})^{(1/2)})*5^{(1/2)}*(-2*(2*\cos(x)*5^{(1/2)}+4*\cos(x)-2*5^{(1/2)}-5)/(1+\cos(x)))^{(1/2)}*((2*\cos(x)*5^{(1/2)}-4*\cos(x)-2*5^{(1/2)}+5)/(1+\cos(x)))^{(1/2)}-I*\sin(x)*2^{(1/2)}*\operatorname{EllipticF}(I*(\cos(x)-1)*(2+5^{(1/2)})/\sin(x),9-4*5^{(1/2)})*5^{(1/2)}*(-2*(2*\cos(x)*5^{(1/2)}+4*\cos(x)-2*5^{(1/2)}-5)/(1+\cos(x)))^{(1/2)}*((2*\cos(x)*5^{(1/2)}-4*\cos(x)-2*5^{(1/2)}+5)/(1+\cos(x)))^{(1/2)}+4*I*\sin(x)*2^{(1/2)}*\operatorname{EllipticPi}((-9-4*5^{(1/2)})^{(1/2)}*(\cos(x)-1)/\sin(x),1/(9+4*5^{(1/2)})),(-9+4*5^{(1/2)})^{(1/2)}/(-9-4*5^{(1/2)})^{(1/2)})*(-2*(2*\cos(x)*5^{(1/2)}+4*\cos(x)-2*5^{(1/2)}-5)/(1+\cos(x)))^{(1/2)}*((2*\cos(x)*5^{(1/2)}-4*\cos(x)-2*5^{(1/2)}+5)/(1+\cos(x)))^{(1/2)}-2*I*\sin(x)*2^{(1/2)}*\operatorname{EllipticF}(I*(\cos(x)-1)*(2+5^{(1/2)})/\sin(x),9-4*5^{(1/2)})*(-2*(2*\cos(x)*5^{(1/2)}+4*\cos(x)-2*5^{(1/2)}-5)/(1+\cos(x)))^{(1/2)}*((2*\cos(x)*5^{(1/2)}-4*\cos(x)-2*5^{(1/2)}+5)/(1+\cos(x)))^{(1/2)}+20*\cos(x)*5^{(1/2)}+45*\cos(x)-20*5^{(1/2)}-45)*\sin(x)/(\cos(x)-1)/\cos(x)^3/((4*\cos(x)^2-5)/\cos(x)^2)^{(3/2)}$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(4-5*sec(x)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((-5*sec(x)^2 + 4)^(-3/2), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(32) = 64$.

time = 0.32, size = 115, normalized size = 2.88

$$20 \sqrt{\frac{4 \cos(x)^2 - 5}{\cos(x)^2}} \cos(x) \sin(x) - (4 \cos(x)^2 - 5) \arctan\left(\frac{4(8 \cos(x)^3 - 9 \cos(x)) \sqrt{\frac{4 \cos(x)^2 - 5}{\cos(x)^2}} \sin(x) + \cos(x) \sin(x)}{64 \cos(x)^4 - 143 \cos(x)^2 + 80}\right) + (4 \cos(x)^2 - 5) \arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

$$16(4 \cos(x)^2 - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-5*sec(x)^2)^(3/2),x, algorithm="fricas")

[Out] -1/16*(20*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*cos(x)*sin(x) - (4*cos(x)^2 - 5)*arctan((4*(8*cos(x)^3 - 9*cos(x))*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*sin(x) + cos(x)*sin(x))/(64*cos(x)^4 - 143*cos(x)^2 + 80)) + (4*cos(x)^2 - 5)*arctan(sin(x)/cos(x)))/(4*cos(x)^2 - 5)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(4 - 5 \sec^2(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-5*sec(x)**2)**(3/2),x)

[Out] Integral((4 - 5*sec(x)**2)**(-3/2), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-5*sec(x)^2)^(3/2),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(4 - \frac{5}{\cos(x)^2}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4 - 5/cos(x)^2)^(3/2),x)

[Out] int(1/(4 - 5/cos(x)^2)^(3/2), x)

$$3.436 \quad \int \frac{-2 \cot^2(x) + \sin(x)}{(1 + 5 \tan^2(x))^{3/2}} dx$$

Optimal. Leaf size=94

$$-\frac{1}{4} \tanh^{-1} \left(\frac{2 \tan(x)}{\sqrt{1 + 5 \tan^2(x)}} \right) - \frac{\cos(x)}{4 \sqrt{1 + 5 \tan^2(x)}} - \frac{5 \cot(x)}{2 \sqrt{1 + 5 \tan^2(x)}} - \frac{1}{8} \cos(x) \sqrt{1 + 5 \tan^2(x)} + \frac{9}{2} \cot(x)$$

[Out] -1/4*arctanh(2*tan(x)/(1+5*tan(x)^2)^(1/2))-1/4*cos(x)/(1+5*tan(x)^2)^(1/2)
-5/2*cot(x)/(1+5*tan(x)^2)^(1/2)-1/8*cos(x)*(1+5*tan(x)^2)^(1/2)+9/2*cot(x)
*(1+5*tan(x)^2)^(1/2)

Rubi [A]

time = 0.11, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {4462, 12, 3751, 483, 597, 385, 212, 3745, 277, 197}

$$-\frac{5 \sec(x)}{8 \sqrt{5 \sec^2(x) - 4}} + \frac{\cos(x)}{4 \sqrt{5 \sec^2(x) - 4}} - \frac{1}{4} \tanh^{-1} \left(\frac{2 \tan(x)}{\sqrt{5 \tan^2(x) + 1}} \right) + \frac{9}{2} \sqrt{5 \tan^2(x) + 1} \cot(x) - \frac{5 \cot(x)}{2 \sqrt{5 \tan^2(x) + 1}}$$

Antiderivative was successfully verified.

[In] Int[(-2*Cot[x]^2 + Sin[x])/(1 + 5*Tan[x]^2)^(3/2), x]

[Out] -1/4*ArcTanh[(2*Tan[x])/Sqrt[1 + 5*Tan[x]^2]] + Cos[x]/(4*Sqrt[-4 + 5*Sec[x]^2]) - (5*Sec[x])/(8*Sqrt[-4 + 5*Sec[x]^2]) - (5*Cot[x])/(2*Sqrt[1 + 5*Tan[x]^2]) + (9*Cot[x]*Sqrt[1 + 5*Tan[x]^2])/2

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 197

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] := Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
+ 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
- 1)/2]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

alQ[n]))

Rule 4462

```
Int[(u_)*((v_) + (d_)*(F_)[(c_)*((a_) + (b_)*(x_))]^(n_)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}
\int \frac{-2 \cot^2(x) + \sin(x)}{(1 + 5 \tan^2(x))^{3/2}} dx &= \int -\frac{2 \cot^2(x)}{(1 + 5 \tan^2(x))^{3/2}} dx + \int \frac{\sin(x)}{(1 + 5 \tan^2(x))^{3/2}} dx \\
&= -\left(2 \int \frac{\cot^2(x)}{(1 + 5 \tan^2(x))^{3/2}} dx\right) + \text{Subst}\left(\int \frac{1}{x^2(-4 + 5x^2)^{3/2}} dx, x, \sec(x)\right) \\
&= \frac{\cos(x)}{4\sqrt{-4 + 5 \sec^2(x)}} - 2\text{Subst}\left(\int \frac{1}{x^2(1 + x^2)(1 + 5x^2)^{3/2}} dx, x, \tan(x)\right) + \frac{5}{2}S \\
&= \frac{\cos(x)}{4\sqrt{-4 + 5 \sec^2(x)}} - \frac{5 \sec(x)}{8\sqrt{-4 + 5 \sec^2(x)}} - \frac{5 \cot(x)}{2\sqrt{1 + 5 \tan^2(x)}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(1 + x^2)(1 + 5x^2)^{3/2}} dx, x, \tan(x)\right) \\
&= \frac{\cos(x)}{4\sqrt{-4 + 5 \sec^2(x)}} - \frac{5 \sec(x)}{8\sqrt{-4 + 5 \sec^2(x)}} - \frac{5 \cot(x)}{2\sqrt{1 + 5 \tan^2(x)}} + \frac{9}{2} \cot(x) \sqrt{1 + 5 \tan^2(x)} \\
&= \frac{\cos(x)}{4\sqrt{-4 + 5 \sec^2(x)}} - \frac{5 \sec(x)}{8\sqrt{-4 + 5 \sec^2(x)}} - \frac{5 \cot(x)}{2\sqrt{1 + 5 \tan^2(x)}} + \frac{9}{2} \cot(x) \sqrt{1 + 5 \tan^2(x)} \\
&= -\frac{1}{4} \tanh^{-1}\left(\frac{2 \tan(x)}{\sqrt{1 + 5 \tan^2(x)}}\right) + \frac{\cos(x)}{4\sqrt{-4 + 5 \sec^2(x)}} - \frac{5 \sec(x)}{8\sqrt{-4 + 5 \sec^2(x)}}
\end{aligned}$$

Mathematica [A]

time = 0.35, size = 131, normalized size = 1.39

$$\frac{(-3 + 2 \cos(2x))^{3/2} (-1 + 2 \cot^2(x) \csc(x)) \sin^2(x) \left(-2 \sinh^{-1}(2 \sin(x)) (4 + \csc^2(x)) + (-2 + 164 \csc(x) - 3 \csc^2(x) + 16 \csc^3(x)) \sqrt{1 + 4 \sin^2(x)}\right) \tan(x)}{2\sqrt{-(3 - 2 \cos(2x))^2 (5 + \cot^2(x)) (4 + 4 \cos(2x) - 3 \sin(x) + \sin(3x))} \sqrt{1 + 5 \tan^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(-2*Cot[x]^2 + Sin[x])/(1 + 5*Tan[x]^2)^(3/2), x]

```
[Out] -1/2*((-3 + 2*Cos[2*x])^(3/2)*(-1 + 2*Cot[x]^2*Csc[x])*Sin[x]^2*(-2*ArcSinh
[2*Sin[x]]*(4 + Csc[x]^2) + (-2 + 164*Csc[x] - 3*Csc[x]^2 + 16*Csc[x]^3)*Sq
rt[1 + 4*Sin[x]^2])*Tan[x])/(Sqrt[-(3 - 2*Cos[2*x])^2]*(5 + Cot[x]^2)*(4 +
4*Cos[2*x] - 3*Sin[x] + Sin[3*x])*Sqrt[1 + 5*Tan[x]^2])
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Sin[x] - 2*Cot[x]^2)/(1 + 5*Tan[x]^2)^(3/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 1.36, size = 975, normalized size = 10.37

method	result	size
default	Expression too large to display	975

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I/(-9+4*5^(1/2))^(1/2)/(2+5^(1/2))^2/(-2+5^(1/2))^2/(4*cos(x)^2-5)^2*(4
*I*cos(x)*EllipticF(I*(cos(x)-1)*(-2+5^(1/2))/sin(x),9+4*5^(1/2))*2^(1/2)*(
(2*cos(x)*5^(1/2)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*(-2*(2*cos(x)*5^(
1/2)+4*cos(x)-2*5^(1/2)-5)/(1+cos(x)))^(1/2)*sin(x)-8*I*EllipticPi((-9+4*5^(
1/2))^(1/2)*(cos(x)-1)/sin(x),-1/(-9+4*5^(1/2)),(-9-4*5^(1/2))^(1/2)/(-9+4
*5^(1/2))^(1/2))*2^(1/2)*((2*cos(x)*5^(1/2)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)
))^1/2)*(-2*(2*cos(x)*5^(1/2)+4*cos(x)-2*5^(1/2)-5)/(1+cos(x)))^(1/2)*sin(
x)+4*I*EllipticF(I*(cos(x)-1)*(-2+5^(1/2))/sin(x),9+4*5^(1/2))*2^(1/2)*((2*
cos(x)*5^(1/2)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*(-2*(2*cos(x)*5^(1/2
)+4*cos(x)-2*5^(1/2)-5)/(1+cos(x)))^(1/2)*sin(x)-3*I*cos(x)*arctanh(1/2*(-1
6)^(1/2)*cos(x)*(cos(x)-1)/sin(x)^2/(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2))*5
^(1/2)*(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*sin(x)-8*I*2^(1/2)*((2*cos(x)*5
^(1/2)-4*cos(x)-2*5^(1/2)+5)/(1+cos(x)))^(1/2)*(-2*(2*cos(x)*5^(1/2)+4*cos(
x)-2*5^(1/2)-5)/(1+cos(x)))^(1/2)*EllipticPi((-9+4*5^(1/2))^(1/2)*(cos(x)-1
)/sin(x),-1/(-9+4*5^(1/2)),(-9-4*5^(1/2))^(1/2)/(-9+4*5^(1/2))^(1/2))*sin(x
)*cos(x)-3*cos(x)*sin(x)*arctan(2*cos(x)*(cos(x)-1)/sin(x)^2/(-(4*cos(x)^2-
5)/(1+cos(x))^2)^(1/2))*5^(1/2)*(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)+6*I*co
s(x)*arctanh(1/2*(-16)^(1/2)*cos(x)*(cos(x)-1)/sin(x)^2/(-(4*cos(x)^2-5)/(1
+cos(x))^2)^(1/2))*(-4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*sin(x)-3*I*arctanh(
1/2*(-16)^(1/2)*cos(x)*(cos(x)-1)/sin(x)^2/(-(4*cos(x)^2-5)/(1+cos(x))^2)^(
1/2))*5^(1/2)*(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*sin(x)+2*cos(x)^2*sin(x)
```

```
*5^(1/2)+6*cos(x)*sin(x)*(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*arctan(2*cos(x)*
(cos(x)-1)/sin(x)^2/(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2))-3*sin(x)*arctan(2*cos(x)*
(cos(x)-1)/sin(x)^2/(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2))*5^(1/2)
)*(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)+6*I*arctanh(1/2*(-16)^(1/2)*cos(x)*
(cos(x)-1)/sin(x)^2/(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2))*(-(4*cos(x)^2-5)/(
1+cos(x))^2)^(1/2)*sin(x)-4*cos(x)^2*sin(x)-164*cos(x)^2*5^(1/2)+6*arctan(2
*cos(x)*cos(x)-1)/sin(x)^2/(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2))*(-(4*cos(x)
)^2-5)/(1+cos(x))^2)^(1/2)*sin(x)+328*cos(x)^2-5*sin(x)*5^(1/2)+10*sin(x)+
180*5^(1/2)-360)*cos(x)^3*(-(4*cos(x)^2-5)/cos(x)^2)^(3/2)/sin(x)
```

Maxima [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x, algorithm="maxima")
```

[Out] Timed out

Fricas [A]

time = 0.37, size = 97, normalized size = 1.03

$$\frac{2(4\cos(x)^2 - 5)\log\left(\sqrt{\frac{4\cos(x)^2 - 5}{\cos(x)^2}}\cos(x) - 2\sin(x)\right)\sin(x) + (164\cos(x)^3 - (2\cos(x)^3 - 5\cos(x))\sin(x) - 180\cos(x))\sqrt{\frac{4\cos(x)^2 - 5}{\cos(x)^2}}}{8(4\cos(x)^2 - 5)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(4*cos(x)^2 - 5)*log(sqrt(-(4*cos(x)^2 - 5)/cos(x)^2)*cos(x) - 2*sin(x))*sin(x) + (164*cos(x)^3 - (2*cos(x)^3 - 5*cos(x))*sin(x) - 180*cos(x))*sqrt(-(4*cos(x)^2 - 5)/cos(x)^2))/((4*cos(x)^2 - 5)*sin(x))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\sin(x)}{5\sqrt{5\tan^2(x)+1}\tan^2(x)+\sqrt{5\tan^2(x)+1}}\right)dx - \int\frac{2\cot^2(x)}{5\sqrt{5\tan^2(x)+1}\tan^2(x)+\sqrt{5\tan^2(x)+1}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cot(x)**2+sin(x))/(1+5*tan(x)**2)**(3/2),x)
```

```
[Out] -Integral(-sin(x)/(5*sqrt(5*tan(x)**2 + 1)*tan(x)**2 + sqrt(5*tan(x)**2 + 1)), x) - Integral(2*cot(x)**2/(5*sqrt(5*tan(x)**2 + 1)*tan(x)**2 + sqrt(5*tan(x)**2 + 1)), x)
```

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(x) - 2 \cot(x)^2}{(5 \tan(x)^2 + 1)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x) - 2*cot(x)^2)/(5*tan(x)^2 + 1)^(3/2),x)

[Out] int((sin(x) - 2*cot(x)^2)/(5*tan(x)^2 + 1)^(3/2), x)

$$3.437 \quad \int \frac{(-3 + \cos(2x)) \sec^4(x)}{\sqrt{4 - \cot^2(x)}} dx$$

Optimal. Leaf size=39

$$-\frac{2}{3} \sqrt{4 - \cot^2(x)} \tan(x) - \frac{1}{3} \sqrt{4 - \cot^2(x)} \tan^3(x)$$

[Out] $-2/3*(4-\cot(x)^2)^{(1/2)}*\tan(x)-1/3*(4-\cot(x)^2)^{(1/2)}*\tan(x)^3$

Rubi [A]

time = 0.09, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {12, 445, 464, 197}

$$-\frac{1}{3} \tan^3(x) \sqrt{4 - \cot^2(x)} - \frac{2}{3} \tan(x) \sqrt{4 - \cot^2(x)}$$

Antiderivative was successfully verified.

[In] `Int[((-3 + Cos[2*x])*Sec[x]^4)/Sqrt[4 - Cot[x]^2], x]`

[Out] `(-2*Sqrt[4 - Cot[x]^2]*Tan[x])/3 - (Sqrt[4 - Cot[x]^2]*Tan[x]^3)/3`

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 197

`Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]`

Rule 445

`Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !IntegerQ[p])`

Rule 464

`Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]`

Rubi steps

$$\begin{aligned}
 \int \frac{(-3 + \cos(2x)) \sec^4(x)}{\sqrt{4 - \cot^2(x)}} dx &= \text{Subst} \left(\int \frac{2(-1 - 2x^2)}{\sqrt{4 - \frac{1}{x^2}}} dx, x, \tan(x) \right) \\
 &= 2 \text{Subst} \left(\int \frac{-1 - 2x^2}{\sqrt{4 - \frac{1}{x^2}}} dx, x, \tan(x) \right) \\
 &= 2 \text{Subst} \left(\int \frac{(-2 - \frac{1}{x^2}) x^2}{\sqrt{4 - \frac{1}{x^2}}} dx, x, \tan(x) \right) \\
 &= -\frac{1}{3} \sqrt{4 - \cot^2(x)} \tan^3(x) - \frac{8}{3} \text{Subst} \left(\int \frac{1}{\sqrt{4 - \frac{1}{x^2}}} dx, x, \tan(x) \right) \\
 &= -\frac{2}{3} \sqrt{4 - \cot^2(x)} \tan(x) - \frac{1}{3} \sqrt{4 - \cot^2(x)} \tan^3(x)
 \end{aligned}$$

Mathematica [A]

time = 0.05, size = 36, normalized size = 0.92

$$\frac{(3 + \cos(2x))(-3 + 5 \cos(2x)) \csc(x) \sec^3(x)}{12 \sqrt{4 - \cot^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[((-3 + Cos[2*x])*Sec[x]^4)/Sqrt[4 - Cot[x]^2], x]

[Out] ((3 + Cos[2*x])*(-3 + 5*Cos[2*x])*Csc[x]*Sec[x]^3)/(12*Sqrt[4 - Cot[x]^2])

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(Cos[2*x] - 3)/(Cos[x]^4*Sqrt[4 - Cot[x]^2]), x]')

[Out] Timed out

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.

time = 0.54, size = 64, normalized size = 1.64

method	result	size
default	$-\frac{(5(\cos^2(x))+2)\sqrt{-\frac{5(\cos^2(x))-4}{\sin(x)^2}}\sin(x)\sqrt{4}}{12\cos(x)^3} + \frac{\sqrt{4}\sin(x)\sqrt{-\frac{5(\cos^2(x))-4}{\sin(x)^2}}}{4\cos(x)}$	64

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/12*(5*\cos(x)^2+2)*(-5*\cos(x)^2-4)/\sin(x)^2)^{(1/2)}*\sin(x)*4^{(1/2)}/\cos(x)^3+1/4*4^{(1/2)}*\sin(x)*(-5*\cos(x)^2-4)/\sin(x)^2)^{(1/2)}/\cos(x)$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.

time = 0.26, size = 63, normalized size = 1.62

$$-\frac{1}{48}\left(-\frac{1}{\tan(x)^2}+4\right)^{\frac{3}{2}}\tan(x)^3+\frac{3}{16}\sqrt{-\frac{1}{\tan(x)^2}+4}\tan(x)-\frac{8\tan(x)^4+26\tan(x)^2-7}{8\sqrt{2\tan(x)+1}\sqrt{2\tan(x)-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x,algorithm="maxima")`

[Out] $-1/48*(-1/\tan(x)^2+4)^{(3/2)}*\tan(x)^3+3/16*\sqrt{-1/\tan(x)^2+4}*\tan(x)-1/8*(8*\tan(x)^4+26*\tan(x)^2-7)/(\sqrt{2*\tan(x)+1}*\sqrt{2*\tan(x)-1})$

Fricas [A]

time = 0.32, size = 33, normalized size = 0.85

$$-\frac{(\cos(x)^2+1)\sqrt{\frac{5\cos(x)^2-4}{\cos(x)^2-1}}\sin(x)}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x,algorithm="fricas")`

[Out] $-1/3*(\cos(x)^2+1)*\sqrt{(5*\cos(x)^2-4)/(\cos(x)^2-1)}*\sin(x)/\cos(x)^3$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(2x) - 3}{\sqrt{-(\cot(x) - 2)(\cot(x) + 2)} \cos^4(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3+cos(2*x))/cos(x)**4/(4-cot(x)**2)**(1/2), x)`

[Out] `Integral((cos(2*x) - 3)/(sqrt(-(cot(x) - 2)*(cot(x) + 2))*cos(x)**4), x)`

Giac [C] Result contains complex when optimal does not.

time = 0.02, size = 193, normalized size = 4.95

$$\frac{\frac{128000}{3} \left(\frac{-2\sqrt{5}\sqrt{-5\cos^2 x + 4} + 4\sqrt{5}}{10\cos x} \right)^3 - \frac{17920 \left(-2\sqrt{5}\sqrt{-5\cos^2 x + 4} + 4\sqrt{5} \right)}{\cos x}}{163840\sqrt{5}} + \frac{-34406400 \left(\frac{-2\sqrt{5}\sqrt{-5\cos^2 x + 4} + 4\sqrt{5}}{10\cos x} \right)^2 - 8192000}{31457280 \left(\sqrt{5} \left(\frac{-2\sqrt{5}\sqrt{-5\cos^2 x + 4} + 4\sqrt{5}}{10\cos x} \right)^3 \right)}$$

$$\frac{2}{3} \operatorname{isgn}(\sin x) - \frac{\operatorname{sign}(\sin x)}{\operatorname{sign}(\sin x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2), x)`

[Out] `-1/2400*(sqrt(5)*((sqrt(5)*sqrt(-5*cos(x)^2 + 4) - 2*sqrt(5))^3/cos(x)^3 + 105*(sqrt(5)*sqrt(-5*cos(x)^2 + 4) - 2*sqrt(5))/cos(x)) - 125*sqrt(5)*(21*(sqrt(5)*sqrt(-5*cos(x)^2 + 4) - 2*sqrt(5))^2/cos(x)^2 + 125)*cos(x)^3/(sqrt(5)*sqrt(-5*cos(x)^2 + 4) - 2*sqrt(5))^3)/sgn(sin(x)) + 2/3*I*sgn(sin(x))`

Mupad [B]

time = 0.76, size = 20, normalized size = 0.51

$$-\frac{\tan(x) (\tan(x)^2 + 2) \sqrt{4 - \frac{1}{\tan(x)^2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(2*x) - 3)/(cos(x)^4*(4 - cot(x)^2)^(1/2)), x)`

[Out] `-(tan(x)*(tan(x)^2 + 2)*(4 - 1/tan(x)^2)^(1/2))/3`

$$3.438 \quad \int \frac{(3+\sin^2(x)) \tan^3(x)}{(-2+\cos^2(x))(5-4\sec^2(x))^{3/2}} dx$$

Optimal. Leaf size=73

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{3}}\right)}{6\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{5\sqrt{5}} - \frac{2}{15\sqrt{5-4\sec^2(x)}}$$

[Out] -1/18*arctanh(1/3*(5-4*sec(x)^2)^(1/2)*3^(1/2))*3^(1/2)-1/25*arctanh(1/5*(5-4*sec(x)^2)^(1/2)*5^(1/2))*5^(1/2)-2/15/(5-4*sec(x)^2)^(1/2)

Rubi [A]

time = 0.71, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {4458, 6857, 267, 272, 53, 65, 212, 528, 457, 87, 162, 213}

$$-\frac{2}{15\sqrt{5-4\sec^2(x)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{3}}\right)}{6\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[((3 + Sin[x]^2)*Tan[x]^3)/((-2 + Cos[x]^2)*(5 - 4*Sec[x]^2)^(3/2)), x]

[Out] -1/6*ArcTanh[Sqrt[5 - 4*Sec[x]^2]/Sqrt[3]]/Sqrt[3] - ArcTanh[Sqrt[5 - 4*Sec[x]^2]/Sqrt[5]]/(5*Sqrt[5]) - 2/(15*Sqrt[5 - 4*Sec[x]^2])

Rule 53

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 87

Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] := Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
 x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
 , f}, x] && LtQ[p, -1]

Rule 162

Int((((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
 ((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
 f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 212

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
 ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
 Q[a, 0] || LtQ[b, 0])

Rule 213

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-
 1)*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])

Rule 267

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)
 ^p/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
 Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
 , m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 457

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
 *(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
 b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] := Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !I
ntegerQ[p])
```

Rule 4458

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] := With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Dist[-(b*c*d^(n - 1))^(-1), Subst[Int[SubstF
or[(1 - d^2*x^2)^((n - 1)/2)/x^n, Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(
a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b,
c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(3 + \sin^2(x)) \tan^3(x)}{(-2 + \cos^2(x)) (5 - 4 \sec^2(x))^{3/2}} dx &= -\text{Subst} \left(\int \frac{(1 - x^2) (4 - x^2)}{(5 - \frac{4}{x^2})^{3/2} x^3 (-2 + x^2)} dx, x, \cos(x) \right) \\
&= -\text{Subst} \left(\int \left(-\frac{2}{(5 - \frac{4}{x^2})^{3/2} x^3} + \frac{3}{2(5 - \frac{4}{x^2})^{3/2} x} - \frac{x}{2(5 - \frac{4}{x^2})^{3/2}} \right) dx, x, \cos(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x}{(5 - \frac{4}{x^2})^{3/2} (-2 + x^2)} dx, x, \cos(x) \right) - \frac{3}{2} \text{Subst} \left(\int \frac{x}{(5 - \frac{4}{x^2})^{3/2}} dx, x, \cos(x) \right) \\
&= -\frac{1}{2\sqrt{5 - 4 \sec^2(x)}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{(5 - \frac{4}{x^2})^{3/2} (1 - \frac{2}{x^2}) x} dx, x, \cos(x) \right) \\
&= -\frac{1}{5\sqrt{5 - 4 \sec^2(x)}} + \frac{3}{20} \text{Subst} \left(\int \frac{1}{\sqrt{5 - 4x} x} dx, x, \sec^2(x) \right) \\
&= -\frac{2}{15\sqrt{5 - 4 \sec^2(x)}} + \frac{1}{120} \text{Subst} \left(\int \frac{-6 - 8x}{\sqrt{5 - 4x} (1 - 2x)x} dx, x, \sec^2(x) \right) \\
&= -\frac{3 \tanh^{-1} \left(\frac{\sqrt{5 - 4 \sec^2(x)}}{\sqrt{5}} \right)}{10\sqrt{5}} - \frac{2}{15\sqrt{5 - 4 \sec^2(x)}} - \frac{1}{20} \text{Subst} \left(\int \frac{1}{\sqrt{5 - 4x} x} dx, x, \sec^2(x) \right) \\
&= -\frac{3 \tanh^{-1} \left(\frac{\sqrt{5 - 4 \sec^2(x)}}{\sqrt{5}} \right)}{10\sqrt{5}} - \frac{2}{15\sqrt{5 - 4 \sec^2(x)}} + \frac{1}{40} \text{Subst} \left(\int \frac{1}{\sqrt{5 - 4x} x} dx, x, \sec^2(x) \right) \\
&= -\frac{\tanh^{-1} \left(\frac{\sqrt{5 - 4 \sec^2(x)}}{\sqrt{3}} \right)}{6\sqrt{3}} - \frac{\tanh^{-1} \left(\frac{\sqrt{5 - 4 \sec^2(x)}}{\sqrt{5}} \right)}{5\sqrt{5}} - \frac{1}{10\sqrt{5 - 4 \sec^2(x)}}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 255 vs. 2(73) = 146.

time = 3.77, size = 255, normalized size = 3.49

$$\frac{15 \tanh^{-1} \left(\frac{\sqrt{-3 + 5 \cos(2x)}}{\sqrt{6} \sqrt{\cos^2(x)}} \right) \sqrt{-9 + 15 \cos(2x)} \sin^2(x) - 2 \left(9\sqrt{5} \sqrt{-3 + 5 \cos(2x)} \left(\log(10 \sin^2(x)) - \log \left(5 \left(-\sqrt{-3 + 5 \cos(2x)} + \cos(2x) \sqrt{-3 + 5 \cos(2x)} + \sqrt{10} \sqrt{\sin^2(x)} \sqrt{\sin^2(2x)} \right) \right) \right) \sin^2(x) + 15\sqrt{2} \sqrt{\sin^2(x)} \sqrt{\sin^2(2x)} + 10 \tanh^{-1} \left(\frac{\sqrt{6} \sin(x)}{\sqrt{-3 + 5 \cos(2x)}} \right) \sqrt{-9 + 15 \cos(2x)} \sec(x) \sqrt{\sin^2(x)} \sqrt{\sin^2(2x)}}{225 \sqrt{10 - 8 \sec^2(x)} \sqrt{\sin^2(x)} \sqrt{\sin^2(2x)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((3 + Sin[x]^2)*Tan[x]^3)/((-2 + Cos[x]^2)*(5 - 4*Sec[x]^2)^(3/2)), x]

[Out] (15*ArcTanh[Sqrt[-3 + 5*Cos[2*x]]/(Sqrt[6]*Sqrt[Cos[x]^2])]*Sqrt[-9 + 15*Cos[2*x]]*Sin[x]^2 - 2*(9*Sqrt[5]*Sqrt[-3 + 5*Cos[2*x]]*(Log[10*Sin[x]^2] - Log[5*(-Sqrt[-3 + 5*Cos[2*x]] + Cos[2*x]*Sqrt[-3 + 5*Cos[2*x]] + Sqrt[10]*Sqrt[Sin[x]^2]*Sqrt[Sin[2*x]^2]))*Sin[x]^2 + 15*Sqrt[2]*Sqrt[Sin[x]^2]*Sqrt[

```
Sin[2*x]^2] + 10*ArcTanh[(Sqrt[6]*Cos[x])/Sqrt[-3 + 5*Cos[2*x]]]*Sqrt[-9 +
15*Cos[2*x]]*Sec[x]*Sqrt[Sin[x]^2]*Sqrt[Sin[2*x]^2]))/(225*Sqrt[10 - 8*Sec[
x]^2]*Sqrt[Sin[x]^2]*Sqrt[Sin[2*x]^2])
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(3 + Sin[x]^2)*Tan[x]^3/((Cos[x]^2 - 2)*(5 - 4*Sec[x]^2)
^(3/2)),x]')
```

[Out] Timed out

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1610 vs. $2(55) = 110$.

time = 1.53, size = 1611, normalized size = 22.07

method	result	size
default	Expression too large to display	1611

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+sin(x)^2)*tan(x)^3/(cos(x)^2-2)/(5-4*sec(x)^2)^(3/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] -3/5/(-5+2*5^(1/2))/(5+2*5^(1/2))/(-6+2*5^(1/2)+2^(1/2))/(6-2*5^(1/2)+2^(1/
2))/(-6+2*5^(1/2)+2^(1/2))/(2*3^(1/2)+6^(1/2))/(-6-2*5^(1/2)+2^(1/2))/(2*3^(
1/2)-6^(1/2))*(5*cos(x)^2-4)*(50*cos(x)*3^(1/2)*2^(1/2))*((5*cos(x)^2-4)/(1+
cos(x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2)))*4^(1/2)*(cos(x)-1)*(5*cos(x)
*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((5*cos(x)^2-4)/(1+cos(x))^2)^(1/
2))+50*cos(x)*3^(1/2)*2^(1/2))*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)*arctanh(
1/2/(2*3^(1/2)-6^(1/2)))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(
1/2)-4)/sin(x)^2/((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2))-25*cos(x)*2^(1/2)*6^(
1/2))*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2)))*4
^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((5*cos
(x)^2-4)/(1+cos(x))^2)^(1/2))+25*cos(x)*2^(1/2)*6^(1/2))*((5*cos(x)^2-4)/(1+
cos(x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)-6^(1/2)))*4^(1/2)*(cos(x)-1)*(5*cos(x)
*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin(x)^2/((5*cos(x)^2-4)/(1+cos(x))^2)^(1/
2))+100*cos(x)*3^(1/2))*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)*arctanh(1/2/(2*
3^(1/2)+6^(1/2)))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4
)/sin(x)^2/((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2))-100*cos(x)*3^(1/2))*((5*cos(x)
^2-4)/(1+cos(x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)-6^(1/2)))*4^(1/2)*(cos(x)
-1)*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin(x)^2/((5*cos(x)^2-4)/(1+co
s(x))^2)^(1/2))-50*cos(x)*6^(1/2))*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)*arcta
```



```

nh(1/2/(2*3^(1/2)+6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)+10*cos(x)+4
*2^(1/2)+4)/sin(x)^2/((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2))-50*cos(x)*6^(1/2)
*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)-6^(1/2))*4^(1/2)
)*(cos(x)-1)*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin(x)^2/((5*cos(x)^2
-4)/(1+cos(x))^2)^(1/2))+72*cos(x)*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)*5^(1
/2)*arctanh(1/2*5^(1/2)*cos(x)*4^(1/2)*(cos(x)-1)/sin(x)^2/((5*cos(x)^2-4)/
(1+cos(x))^2)^(1/2))+50*3^(1/2)*2^(1/2)*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)
*arctanh(1/2/(2*3^(1/2)+6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)+10*co
s(x)+4*2^(1/2)+4)/sin(x)^2/((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2))+50*3^(1/2)*
2^(1/2)*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)-6^(1/2))
)*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin(x)^2/((5*c
os(x)^2-4)/(1+cos(x))^2)^(1/2))-25*2^(1/2)*((5*cos(x)^2-4)/(1+cos(x))^2)^(1
/2)*6^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^
(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2))+
25*2^(1/2)*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)*6^(1/2)*arctanh(1/2/(2*3^(1/
2)-6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin
(x)^2/((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2))+100*3^(1/2)*((5*cos(x)^2-4)/(1+c
os(x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)
)*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((5*cos(x)^2-4)/(1+cos(x))^2)^(1/
2))-100*3^(1/2)*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)*arctanh(1/2/(2*3^(1/2)-
6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin(x)
^2/((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2))-50*((5*cos(x)^2-4)/(1+cos(x))^2)^(1
/2)*6^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^
(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2))-
50*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)*6^(1/2)*arctanh(1/2/(2*3^(1/2)-6^(1/
2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin(x)^2/((
5*cos(x)^2-4)/(1+cos(x))^2)^(1/2))+72*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)*a
rctanh(1/2*5^(1/2)*cos(x)*4^(1/2)*(cos(x)-1)/sin(x)^2/((5*cos(x)^2-4)/(1+co
s(x))^2)^(1/2))*5^(1/2)-240*cos(x))/cos(x)^3/((5*cos(x)^2-4)/cos(x)^2)^(3/2)
)

```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x, algor
ithm="maxima")

[Out] integrate((sin(x)^2 + 3)*tan(x)^3/((cos(x)^2 - 2)*(-4*sec(x)^2 + 5)^(3/2)),
x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs.
2(55) = 110.

time = 0.39, size = 257, normalized size = 3.52

$$\frac{480 \sqrt{\frac{2 \cos(x)^2 - 4}{\cos(x)^2}} \cos(x)^2 - 18 (5 \sqrt{5} \cos(x)^2 - 4 \sqrt{5}) \log\left(\frac{625 \cos(x)^8 - 1000 \cos(x)^6 + 500 \cos(x)^4 - 80 \cos(x)^2 - (125 \sqrt{5} \cos(x)^8 - 150 \sqrt{5} \cos(x)^6 + 50 \sqrt{5} \cos(x)^4 - 4 \sqrt{5} \cos(x)^2) \sqrt{\frac{5 \cos(x)^2 - 4}{\cos(x)^2} + 2}}{3600 (5 \cos(x)^2 - 4)}\right) - 25 (5 \sqrt{3} \cos(x)^2 - 4 \sqrt{3}) \log\left(\frac{1921 \cos(x)^8 - 3464 \cos(x)^6 + 2040 \cos(x)^4 - 416 \cos(x)^2 - 8 (62 \sqrt{3} \cos(x)^8 - 87 \sqrt{3} \cos(x)^6 + 36 \sqrt{3} \cos(x)^4 - 4 \sqrt{3} \cos(x)^2) \sqrt{\frac{5 \cos(x)^2 - 4}{\cos(x)^2} + 16}}{\cos(x)^8 - 8 \cos(x)^6 + 24 \cos(x)^4 - 32 \cos(x)^2 + 16}\right)}{3600 (5 \cos(x)^2 - 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x, algorithm="fricas")

[Out] -1/3600*(480*sqrt((5*cos(x)^2 - 4)/cos(x)^2)*cos(x)^2 - 18*(5*sqrt(5)*cos(x)^2 - 4*sqrt(5))*log(625*cos(x)^8 - 1000*cos(x)^6 + 500*cos(x)^4 - 80*cos(x)^2 - (125*sqrt(5)*cos(x)^8 - 150*sqrt(5)*cos(x)^6 + 50*sqrt(5)*cos(x)^4 - 4*sqrt(5)*cos(x)^2)*sqrt((5*cos(x)^2 - 4)/cos(x)^2) + 2) - 25*(5*sqrt(3)*cos(x)^2 - 4*sqrt(3))*log((1921*cos(x)^8 - 3464*cos(x)^6 + 2040*cos(x)^4 - 416*cos(x)^2 - 8*(62*sqrt(3)*cos(x)^8 - 87*sqrt(3)*cos(x)^6 + 36*sqrt(3)*cos(x)^4 - 4*sqrt(3)*cos(x)^2)*sqrt((5*cos(x)^2 - 4)/cos(x)^2) + 16)/(cos(x)^8 - 8*cos(x)^6 + 24*cos(x)^4 - 32*cos(x)^2 + 16)))/(5*cos(x)^2 - 4)

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+sin(x)**2)*tan(x)**3/(-2+cos(x)**2)/(5-4*sec(x)**2)**(3/2),x)

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(55) = 110.

time = 0.05, size = 161, normalized size = 2.21

$$\frac{-\frac{2\sqrt{5}\cos^2 x - 4}{15(5\cos^2 x - 4)} \cos x + 2 \left(\frac{\ln\left(\left(\sqrt{5}\cos^2 x - 4 - \sqrt{5}\cos x\right)^2\right)}{20\sqrt{5}} - \frac{\sqrt{5} \ln\left(\frac{-2\left(\sqrt{5}\cos^2 x - 4 - \sqrt{5}\cos x\right)^2 + 8\sqrt{15} + 32}{2\left(\sqrt{5}\cos^2 x - 4 - \sqrt{5}\cos x\right)^2 + 8\sqrt{15} - 32}\right)}{24\sqrt{15}} \right)}{\text{sign}(\cos x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x)

[Out] -1/900*(5*sqrt(15)*sqrt(5)*log(-2*((sqrt(5)*cos(x) - sqrt(5*cos(x)^2 - 4))^2 - 4*sqrt(15) - 16)/abs(2*(sqrt(5)*cos(x) - sqrt(5*cos(x)^2 - 4))^2 + 8*sqrt(15) - 32)) - 18*sqrt(5)*log((sqrt(5)*cos(x) - sqrt(5*cos(x)^2 - 4))^2) + 120*cos(x)/sqrt(5*cos(x)^2 - 4))/sgn(cos(x))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x)^3 (\sin(x)^2 + 3)}{(\cos(x)^2 - 2) \left(5 - \frac{4}{\cos(x)^2}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(x)^3*(sin(x)^2 + 3))/((cos(x)^2 - 2)*(5 - 4/cos(x)^2)^(3/2)),x)

[Out] int((tan(x)^3*(sin(x)^2 + 3))/((cos(x)^2 - 2)*(5 - 4/cos(x)^2)^(3/2)), x)

$$3.439 \quad \int \frac{\csc^2(x) \left(\sec^2(x) - 3 \tan(x) \sqrt{4 \sec^2(x) + 5 \tan^2(x)} \right)}{(4 \sec^2(x) + 5 \tan^2(x))^{3/2}} dx$$

Optimal. Leaf size=57

$$-\frac{3}{4} \log(\tan(x)) + \frac{3}{8} \log(4 + 9 \tan^2(x)) - \frac{\cot(x)}{4 \sqrt{4 + 9 \tan^2(x)}} - \frac{7 \tan(x)}{8 \sqrt{4 + 9 \tan^2(x)}}$$

[Out] $-3/4*\ln(\tan(x))+3/8*\ln(4+9*\tan(x)^2)-1/4*\cot(x)/(4+9*\tan(x)^2)^{(1/2)}-7/8*\tan(x)/(4+9*\tan(x)^2)^{(1/2)}$

Rubi [A]

time = 0.53, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {6874, 197, 277, 272, 36, 29, 31}

$$-\frac{7 \tan(x)}{8 \sqrt{9 \tan^2(x) + 4}} + \frac{3}{8} \log(9 \tan^2(x) + 4) - \frac{3}{4} \log(\tan(x)) - \frac{\cot(x)}{4 \sqrt{9 \tan^2(x) + 4}}$$

Antiderivative was successfully verified.

[In] `Int[(Csc[x]^2*(Sec[x]^2 - 3*Tan[x]*Sqrt[4*Sec[x]^2 + 5*Tan[x]^2]))/(4*Sec[x]^2 + 5*Tan[x]^2)^(3/2),x]`

[Out] $(-3*\text{Log}[\text{Tan}[x]])/4 + (3*\text{Log}[4 + 9*\text{Tan}[x]^2])/8 - \text{Cot}[x]/(4*\text{Sqrt}[4 + 9*\text{Tan}[x]^2]) - (7*\text{Tan}[x])/(8*\text{Sqrt}[4 + 9*\text{Tan}[x]^2])$

Rule 29

`Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]`

Rule 31

`Int[((a_) + (b_.)*(x_)^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rule 36

`Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]`

Rule 197

`Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]`

Rule 272

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6874

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\csc^2(x) \left(\sec^2(x) - 3 \tan(x) \sqrt{4 \sec^2(x) + 5 \tan^2(x)} \right)}{(4 \sec^2(x) + 5 \tan^2(x))^{3/2}} dx = \text{Subst} \left(\int \frac{1 + x^2 - 3x \sqrt{4 + 9x^2}}{x^2 (4 + 9x^2)^{3/2}} dx, x, \tan(x) \right)$$

$$= \text{Subst} \left(\int \left(\frac{1}{(4 + 9x^2)^{3/2}} + \frac{1}{x^2 (4 + 9x^2)^{3/2}} \right) dx, x, \tan(x) \right)$$

$$= - \left(3 \text{Subst} \left(\int \frac{1}{x (4 + 9x^2)} dx, x, \tan(x) \right) \right)$$

$$= - \frac{\cot(x)}{4 \sqrt{4 + 9 \tan^2(x)}} + \frac{\tan(x)}{4 \sqrt{4 + 9 \tan^2(x)}} -$$

$$= - \frac{\cot(x)}{4 \sqrt{4 + 9 \tan^2(x)}} - \frac{7 \tan(x)}{8 \sqrt{4 + 9 \tan^2(x)}} -$$

$$= -\frac{3}{4} \log(\tan(x)) + \frac{3}{8} \log(4 + 9 \tan^2(x)) - \frac{1}{4}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 116 vs. 2(57) = 114.

time = 0.63, size = 116, normalized size = 2.04

$$\frac{5 \cot(x) + 6 \sqrt{\frac{13 - 5 \cos(2x)}{1 + \cos(2x)}} \log\left(1 + 7 \tan^2\left(\frac{x}{2}\right) + \tan^4\left(\frac{x}{2}\right)\right) - 9 \csc(x) \sec(x) - 5 \tan(x) - 6\sqrt{2} \log\left(\tan\left(\frac{x}{2}\right)\right) \sqrt{-5 + 13 \sec^2(x) + 5 \tan^2(x)}}{16 \sqrt{\frac{13 - 5 \cos(2x)}{1 + \cos(2x)}}}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x]^2*(Sec[x]^2 - 3*Tan[x]*Sqrt[4*Sec[x]^2 + 5*Tan[x]^2]))/(4*Sec[x]^2 + 5*Tan[x]^2)^(3/2), x]

[Out] (5*Cot[x] + 6*Sqrt[(13 - 5*Cos[2*x])/(1 + Cos[2*x])]*Log[1 + 7*Tan[x/2]^2 + Tan[x/2]^4] - 9*Csc[x]*Sec[x] - 5*Tan[x] - 6*Sqrt[2]*Log[Tan[x/2]]*Sqrt[-5 + 13*Sec[x]^2 + 5*Tan[x]^2])/(16*Sqrt[(13 - 5*Cos[2*x])/(1 + Cos[2*x])])

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(Sec[x]^2 - 3*Tan[x]*Sqrt[4*Sec[x]^2 + 5*Tan[x]^2])/(Sin[x]^2*(4*Sec[x]^2 + 5*Tan[x]^2)^(3/2))], x)')

[Out] Exception raised: SystemError >> excessive stack use: stack is 4372 deep

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(45) = 90.

time = 0.67, size = 117, normalized size = 2.05

method	result
default	$\frac{-3(\cos^3(x) \sin(x) \left(-\frac{5(\cos^2(x)-9)}{\cos(x)^2}\right)^{\frac{3}{2}} \ln\left(-\frac{5(\cos^2(x)-9)}{(1+\cos(x))^2}\right) + 6(\cos^3(x) \sin(x) \left(-\frac{5(\cos^2(x)-9)}{\cos(x)^2}\right)^{\frac{3}{2}} \ln\left(-\frac{\cos(x)-1}{\sin(x)}\right) + 25(\cos^4(x) - 8 \cos(x)^3 \sin(x) \left(-\frac{5(\cos^2(x)-9)}{\cos(x)^2}\right)^{\frac{3}{2}})}{8 \cos(x)^3 \sin(x) \left(-\frac{5(\cos^2(x)-9)}{\cos(x)^2}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec(x)^2+5*tan(x)^2)^(3/2), x, method=_RETURNVERBOSE)

[Out] -1/8*(-3*cos(x)^3*sin(x)*(-(5*cos(x)^2-9)/cos(x)^2)^(3/2)*ln(-(5*cos(x)^2-9)/(1+cos(x))^2)+6*cos(x)^3*sin(x)*(-(5*cos(x)^2-9)/cos(x)^2)^(3/2)*ln(-(cos(x)-1)/sin(x))+25*cos(x)^4-80*cos(x)^2+63)/cos(x)^3/sin(x)/(-(5*cos(x)^2-9)/cos(x)^2)^(3/2)

Maxima [A]

time = 0.33, size = 47, normalized size = 0.82

$$-\frac{7 \tan(x)}{8 \sqrt{9 \tan(x)^2 + 4}} - \frac{1}{4 \sqrt{9 \tan(x)^2 + 4} \tan(x)} + \frac{3}{8} \log(9 \tan(x)^2 + 4) - \frac{3}{4} \log(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec(x)^2+5*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -7/8*tan(x)/sqrt(9*tan(x)^2 + 4) - 1/4/(sqrt(9*tan(x)^2 + 4)*tan(x)) + 3/8*log(9*tan(x)^2 + 4) - 3/4*log(tan(x))
```

Fricas [A]

time = 0.33, size = 84, normalized size = 1.47

$$\frac{3(5 \cos(x)^2 - 9) \log\left(-\frac{5}{4} \cos(x)^2 + \frac{9}{4}\right) \sin(x) - 6(5 \cos(x)^2 - 9) \log\left(\frac{1}{2} \sin(x)\right) \sin(x) - (5 \cos(x)^3 - 7 \cos(x)) \sqrt{\frac{5 \cos(x)^2 - 9}{\cos(x)^2}}}{8(5 \cos(x)^2 - 9) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec(x)^2+5*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*(5*cos(x)^2 - 9)*log(-5/4*cos(x)^2 + 9/4)*sin(x) - 6*(5*cos(x)^2 - 9)*log(1/2*sin(x))*sin(x) - (5*cos(x)^3 - 7*cos(x))*sqrt(-(5*cos(x)^2 - 9)/cos(x)^2))/((5*cos(x)^2 - 9)*sin(x))
```

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)**2-3*(4*sec(x)**2+5*tan(x)**2)**(1/2)*tan(x))/sin(x)**2/(4*sec(x)**2+5*tan(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec(x)^2+5*tan(x)^2)^(3/2),x)

[Out] Could not integrate

Mupad [B]

time = 1.47, size = 113, normalized size = 1.98

$$\frac{3 \ln((\cos(2x) + \sin(2x) i) (5 \cos(2x) - 13))}{8} - \frac{3 \ln(\cos(2x) 852930i - 852930 \sin(2x) - 852930i)}{4} - \frac{\frac{18 \sin(2x) \sqrt{13 - 5 \cos(2x)}}{\sqrt{\cos(2x) + 1}} - \frac{5 \sin(4x) \sqrt{13 - 5 \cos(2x)}}{\sqrt{\cos(2x) + 1}}}{80 \cos(2x)^2 - 288 \cos(2x) + 208}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/cos(x)^2 - 3*tan(x)*(4/cos(x)^2 + 5*tan(x)^2)^(1/2))/(sin(x)^2*(4/cos(x)^2 + 5*tan(x)^2)^(3/2)),x)

[Out] (3*log((cos(2*x) + sin(2*x)*1i)*(5*cos(2*x) - 13)))/8 - (3*log(cos(2*x)*852930i - 852930*sin(2*x) - 852930i))/4 - ((18*sin(2*x)*(13 - 5*cos(2*x))^(1/2))/(cos(2*x) + 1)^(1/2) - (5*sin(4*x)*(13 - 5*cos(2*x))^(1/2))/(cos(2*x) + 1)^(1/2))/(80*cos(2*x)^2 - 288*cos(2*x) + 208)

3.440 $\int \tan(x) (1 + 5 \tan^2(x))^{5/2} dx$

Optimal. Leaf size=66

$$-32 \tan^{-1} \left(\frac{1}{2} \sqrt{1 + 5 \tan^2(x)} \right) + 16 \sqrt{1 + 5 \tan^2(x)} - \frac{4}{3} (1 + 5 \tan^2(x))^{3/2} + \frac{1}{5} (1 + 5 \tan^2(x))^{5/2}$$

[Out] $-32*\arctan(1/2*(1+5*\tan(x)^2)^{(1/2)})+16*(1+5*\tan(x)^2)^{(1/2)}-4/3*(1+5*\tan(x)^2)^{(3/2)}+1/5*(1+5*\tan(x)^2)^{(5/2)}$

Rubi [A]

time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3751, 455, 52, 65, 209}

$$\frac{1}{5} (5 \tan^2(x) + 1)^{5/2} - \frac{4}{3} (5 \tan^2(x) + 1)^{3/2} + 16 \sqrt{5 \tan^2(x) + 1} - 32 \tan^{-1} \left(\frac{1}{2} \sqrt{5 \tan^2(x) + 1} \right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Tan}[x]*(1 + 5*\text{Tan}[x]^2)^{(5/2)}, x]$

[Out] $-32*\text{ArcTan}[\text{Sqrt}[1 + 5*\text{Tan}[x]^2]/2] + 16*\text{Sqrt}[1 + 5*\text{Tan}[x]^2] - (4*(1 + 5*\text{Tan}[x]^2)^{(3/2)})/3 + (1 + 5*\text{Tan}[x]^2)^{(5/2)}/5$

Rule 52

$\text{Int}[(a_. + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + n + 1))), x] + \text{Dist}[n*((b*c - a*d)/(b*(m + n + 1))), \text{Int}[(a + b*x)^m*(c + d*x)^{(n - 1)}, x], x] /;$ FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 65

$\text{Int}[(a_. + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^{(1/p)}], x]] /;$ FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 209

$\text{Int}[(a_. + (b_.)*(x_)^2)^{(-1)}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))*\text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rule 3751

```
Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) +
(f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)], x], x, c*(Tan[e + f*x]/ff)], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \tan(x) (1 + 5 \tan^2(x))^{5/2} dx &= \text{Subst} \left(\int \frac{x (1 + 5x^2)^{5/2}}{1 + x^2} dx, x, \tan(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{(1 + 5x)^{5/2}}{1 + x} dx, x, \tan^2(x) \right) \\
&= \frac{1}{5} (1 + 5 \tan^2(x))^{5/2} - 2 \text{Subst} \left(\int \frac{(1 + 5x)^{3/2}}{1 + x} dx, x, \tan^2(x) \right) \\
&= -\frac{4}{3} (1 + 5 \tan^2(x))^{3/2} + \frac{1}{5} (1 + 5 \tan^2(x))^{5/2} + 8 \text{Subst} \left(\int \frac{\sqrt{1 + 5x}}{1 + x} dx, \right. \\
&= 16 \sqrt{1 + 5 \tan^2(x)} - \frac{4}{3} (1 + 5 \tan^2(x))^{3/2} + \frac{1}{5} (1 + 5 \tan^2(x))^{5/2} - 32 \text{Subst} \\
&= 16 \sqrt{1 + 5 \tan^2(x)} - \frac{4}{3} (1 + 5 \tan^2(x))^{3/2} + \frac{1}{5} (1 + 5 \tan^2(x))^{5/2} - \frac{64}{5} \text{Subst} \\
&= -32 \tan^{-1} \left(\frac{1}{2} \sqrt{1 + 5 \tan^2(x)} \right) + 16 \sqrt{1 + 5 \tan^2(x)} - \frac{4}{3} (1 + 5 \tan^2(x))
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.05, size = 49, normalized size = 0.74

$$\frac{5\sqrt{5} {}_2F_1\left(-\frac{5}{2}, -\frac{5}{2}; -\frac{3}{2}; \frac{4\cos^2(x)}{5}\right) (1 + 5 \tan^2(x))^{5/2}}{(3 - 2 \cos(2x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]*(1 + 5*Tan[x]^2)^(5/2),x]
```

```
[Out] (5*sqrt[5]*Hypergeometric2F1[-5/2, -5/2, -3/2, (4*cos[x]^2)/5]*(1 + 5*Tan[x]^2)^(5/2))/(3 - 2*cos[2*x])^(5/2)
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + 5*Tan[x]^2)^(5/2)*Tan[x],x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]

time = 0.05, size = 61, normalized size = 0.92

method	result
derivativedivides	$5 \tan^4(x) \sqrt{1 + 5 \tan^2(x)} - \frac{14 \tan^2(x) \sqrt{1 + 5 \tan^2(x)}}{3} + \frac{223 \sqrt{1 + 5 \tan^2(x)}}{15}$
default	$5 \tan^4(x) \sqrt{1 + 5 \tan^2(x)} - \frac{14 \tan^2(x) \sqrt{1 + 5 \tan^2(x)}}{3} + \frac{223 \sqrt{1 + 5 \tan^2(x)}}{15}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(1+5*tan(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5*tan(x)^4*(1+5*tan(x)^2)^(1/2)-14/3*tan(x)^2*(1+5*tan(x)^2)^(1/2)+223/15*(1+5*tan(x)^2)^(1/2)-32*arctan(1/2*(1+5*tan(x)^2)^(1/2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1+5*tan(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((5*tan(x)^2 + 1)^(5/2)*tan(x), x)
```

Fricas [A]

time = 0.32, size = 50, normalized size = 0.76

$$\frac{1}{15} (75 \tan(x)^4 - 70 \tan(x)^2 + 223) \sqrt{5 \tan(x)^2 + 1} - 16 \arctan \left(\frac{5 \tan(x)^2 - 3}{4 \sqrt{5 \tan(x)^2 + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)*(1+5*tan(x)^2)^(5/2),x, algorithm="fricas")

[Out] 1/15*(75*tan(x)^4 - 70*tan(x)^2 + 223)*sqrt(5*tan(x)^2 + 1) - 16*arctan(1/4*(5*tan(x)^2 - 3)/sqrt(5*tan(x)^2 + 1))

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (5 \tan^2(x) + 1)^{\frac{5}{2}} \tan(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)*(1+5*tan(x)**2)**(5/2),x)

[Out] Integral((5*tan(x)**2 + 1)**(5/2)*tan(x), x)

Giac [A]

time = 0.01, size = 84, normalized size = 1.27

$$\frac{1}{5} \sqrt{5 \tan^2 x + 1} (5 \tan^2 x + 1)^2 - \frac{4}{3} \sqrt{5 \tan^2 x + 1} (5 \tan^2 x + 1) + 16 \sqrt{5 \tan^2 x + 1} - 32 \arctan\left(\frac{\sqrt{5 \tan^2 x + 1}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)*(1+5*tan(x)^2)^(5/2),x)

[Out] 1/5*(5*tan(x)^2 + 1)^(5/2) - 4/3*(5*tan(x)^2 + 1)^(3/2) + 16*sqrt(5*tan(x)^2 + 1) - 32*arctan(1/2*sqrt(5*tan(x)^2 + 1))

Mupad [B]

time = 1.57, size = 90, normalized size = 1.36

$$\frac{\sqrt{5} \sqrt{\tan(x)^2 + \frac{1}{5}} \left(25 \tan(x)^4 - \frac{70 \tan(x)^2}{3} + \frac{223}{3}\right)}{5} - \ln\left(\tan(x) - \frac{2\sqrt{5} \sqrt{\tan(x)^2 + \frac{1}{5}}}{5} + \frac{1}{5}i\right) 16i - \ln\left(\tan(x) + \frac{2\sqrt{5} \sqrt{\tan(x)^2 + \frac{1}{5}}}{5} - \frac{1}{5}i\right) 16i + \ln(\tan(x) - i) 16i + \ln(\tan(x) + i) 16i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)*(5*tan(x)^2 + 1)^(5/2),x)

[Out] log(tan(x) - 1i)*16i - log(tan(x) + (2*5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/5 - 1i/5)*16i - log(tan(x) - (2*5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/5 + 1i/5)*16i + log(tan(x) + 1i)*16i + (5^(1/2)*(tan(x)^2 + 1/5)^(1/2)*(25*tan(x)^4 - (70*tan(x)^2)/3 + 223/3))/5

$$3.441 \quad \int \frac{\tan(x)}{(1+5 \tan^2(x))^{5/2}} dx$$

Optimal. Leaf size=54

$$\frac{1}{32} \tan^{-1} \left(\frac{1}{2} \sqrt{1 + 5 \tan^2(x)} \right) - \frac{1}{12 (1 + 5 \tan^2(x))^{3/2}} + \frac{1}{16 \sqrt{1 + 5 \tan^2(x)}}$$

[Out] 1/32*arctan(1/2*(1+5*tan(x)^2)^(1/2))+1/16/(1+5*tan(x)^2)^(1/2)-1/12/(1+5*tan(x)^2)^(3/2)

Rubi [A]

time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3751, 455, 53, 65, 209}

$$\frac{1}{16 \sqrt{5 \tan^2(x) + 1}} - \frac{1}{12 (5 \tan^2(x) + 1)^{3/2}} + \frac{1}{32} \tan^{-1} \left(\frac{1}{2} \sqrt{5 \tan^2(x) + 1} \right)$$

Antiderivative was successfully verified.

[In] Int[Tan[x]/(1 + 5*Tan[x]^2)^(5/2), x]

[Out] ArcTan[Sqrt[1 + 5*Tan[x]^2]/2]/32 - 1/(12*(1 + 5*Tan[x]^2)^(3/2)) + 1/(16*Sqrt[1 + 5*Tan[x]^2])

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

, 0] || GtQ[b, 0])

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rule 3751

```
Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) +
(f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)], x], x, c*(Tan[e + f*x]/ff)], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
 \int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}} dx &= \text{Subst} \left(\int \frac{x}{(1+x^2)(1+5x^2)^{5/2}} dx, x, \tan(x) \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1+x)(1+5x)^{5/2}} dx, x, \tan^2(x) \right) \\
 &= -\frac{1}{12(1+5\tan^2(x))^{3/2}} - \frac{1}{8} \text{Subst} \left(\int \frac{1}{(1+x)(1+5x)^{3/2}} dx, x, \tan^2(x) \right) \\
 &= -\frac{1}{12(1+5\tan^2(x))^{3/2}} + \frac{1}{16\sqrt{1+5\tan^2(x)}} + \frac{1}{32} \text{Subst} \left(\int \frac{1}{(1+x)\sqrt{1+5x}} dx \right) \\
 &= -\frac{1}{12(1+5\tan^2(x))^{3/2}} + \frac{1}{16\sqrt{1+5\tan^2(x)}} + \frac{1}{80} \text{Subst} \left(\int \frac{1}{\frac{4}{5} + \frac{x^2}{5}} dx, x, \sqrt{1+5x} \right) \\
 &= \frac{1}{32} \tan^{-1} \left(\frac{1}{2} \sqrt{1+5\tan^2(x)} \right) - \frac{1}{12(1+5\tan^2(x))^{3/2}} + \frac{1}{16\sqrt{1+5\tan^2(x)}}
 \end{aligned}$$

Mathematica [A]

time = 0.26, size = 71, normalized size = 1.31

$$\frac{(-3 + 2 \cos(2x)) \left(-6 \cos(x) + 8 \cos(3x) - 3(-3 + 2 \cos(2x))^{3/2} \log \left(2 \cos(x) + \sqrt{-3 + 2 \cos(2x)} \right) \right) \sec^5(x)}{96(1+5\tan^2(x))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[x]/(1 + 5*Tan[x]^2)^(5/2), x]

[Out] ((-3 + 2*Cos[2*x])*(-6*Cos[x] + 8*Cos[3*x] - 3*(-3 + 2*Cos[2*x])^(3/2)*Log[2*Cos[x] + Sqrt[-3 + 2*Cos[2*x]])*Sec[x]^5)/(96*(1 + 5*Tan[x]^2)^(5/2))

Mathics [A]

time = 5.42, size = 40, normalized size = 0.74

$$-\frac{1}{12(1+5\tan[x]^2)^{\frac{3}{2}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{1+5\tan[x]^2}}{2}\right]}{32} + \frac{1}{16\sqrt{1+5\tan[x]^2}}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Tan[x]/(1 + 5*Tan[x]^2)^(5/2), x]')

[Out] -1 / (12 (1 + 5 Tan[x]^2)^(3/2)) + ArcTan[Sqrt[1 + 5 Tan[x]^2] / 2] / 32 + 1 / (16 Sqrt[1 + 5 Tan[x]^2])

Maple [A]

time = 0.05, size = 41, normalized size = 0.76

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sqrt{1+5(\tan^2(x))}}{2}\right)}{32} + \frac{1}{16\sqrt{1+5(\tan^2(x))}} - \frac{1}{12(1+5(\tan^2(x)))^{\frac{3}{2}}}$	41
default	$\frac{\arctan\left(\frac{\sqrt{1+5(\tan^2(x))}}{2}\right)}{32} + \frac{1}{16\sqrt{1+5(\tan^2(x))}} - \frac{1}{12(1+5(\tan^2(x)))^{\frac{3}{2}}}$	41

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)/(1+5*tan(x)^2)^(5/2), x, method=_RETURNVERBOSE)

[Out] 1/32*arctan(1/2*(1+5*tan(x)^2)^(1/2))+1/16/(1+5*tan(x)^2)^(1/2)-1/12/(1+5*tan(x)^2)^(3/2)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)/(1+5*tan(x)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(tan(x)/(5*tan(x)^2 + 1)^(5/2), x)

Fricas [A]

time = 0.33, size = 76, normalized size = 1.41

$$\frac{3(25 \tan(x)^4 + 10 \tan(x)^2 + 1) \arctan\left(\frac{5 \tan(x)^2 - 3}{4 \sqrt{5 \tan(x)^2 + 1}}\right) + 4(15 \tan(x)^2 - 1) \sqrt{5 \tan(x)^2 + 1}}{192(25 \tan(x)^4 + 10 \tan(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)/(1+5*tan(x)^2)^(5/2),x, algorithm="fricas")

[Out] 1/192*(3*(25*tan(x)^4 + 10*tan(x)^2 + 1)*arctan(1/4*(5*tan(x)^2 - 3)/sqrt(5*tan(x)^2 + 1)) + 4*(15*tan(x)^2 - 1)*sqrt(5*tan(x)^2 + 1))/(25*tan(x)^4 + 10*tan(x)^2 + 1)

Sympy [A]

time = 3.47, size = 46, normalized size = 0.85

$$\frac{\operatorname{atan}\left(\frac{\sqrt{5 \tan^2(x) + 1}}{2}\right)}{32} + \frac{1}{16 \sqrt{5 \tan^2(x) + 1}} - \frac{1}{12 (5 \tan^2(x) + 1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)/(1+5*tan(x)**2)**(5/2),x)

[Out] atan(sqrt(5*tan(x)**2 + 1)/2)/32 + 1/(16*sqrt(5*tan(x)**2 + 1)) - 1/(12*(5*tan(x)**2 + 1)**(3/2))

Giac [A]

time = 0.01, size = 57, normalized size = 1.06

$$\frac{3(5 \tan^2 x + 1) - 4}{48 \sqrt{5 \tan^2 x + 1} (5 \tan^2 x + 1)} + \frac{\arctan\left(\frac{\sqrt{5 \tan^2 x + 1}}{2}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)/(1+5*tan(x)^2)^(5/2),x)

[Out] 1/48*(15*tan(x)^2 - 1)/(5*tan(x)^2 + 1)^(3/2) + 1/32*arctan(1/2*sqrt(5*tan(x)^2 + 1))

Mupad [B]

time = 0.51, size = 172, normalized size = 3.19

$$\frac{\ln\left(\tan(x) - \frac{\sqrt{5}\sqrt{\tan(x)^2 + \frac{1}{5}}}{5} + \frac{1}{5}i\right)}{64} + \frac{\ln\left(\tan(x) + \frac{\sqrt{5}\sqrt{\tan(x)^2 + \frac{1}{5}}}{5} - \frac{1}{5}i\right)}{64} - \frac{\ln(\tan(x) - i)}{64} - \frac{\ln(\tan(x) + i)}{64} - \frac{\sqrt{\tan(x)^2 + \frac{1}{5}}}{96\left(\tan(x) - \frac{\sqrt{5}i}{5}\right)} + \frac{\sqrt{\tan(x)^2 + \frac{1}{5}}}{96\left(\tan(x) + \frac{\sqrt{5}i}{5}\right)} + \frac{\sqrt{5}\sqrt{\tan(x)^2 + \frac{1}{5}}}{240\left(\tan(x)^2 + \frac{2\sqrt{5}\tan(x)}{5} - \frac{1}{5}\right)} - \frac{\sqrt{5}\sqrt{\tan(x)^2 + \frac{1}{5}}}{240\left(-\tan(x)^2 + \frac{2\sqrt{5}\tan(x)}{5} + \frac{1}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(x)/(5*tan(x)^2 + 1)^(5/2),x)`

[Out] $(\log(\tan(x) - (2*5^{(1/2)}*(\tan(x)^2 + 1/5)^{(1/2))}/5 + 1i/5)*1i)/64 + (\log(\tan(x) + (2*5^{(1/2)}*(\tan(x)^2 + 1/5)^{(1/2))}/5 - 1i/5)*1i)/64 - (\log(\tan(x) - 1i)*1i)/64 - (\log(\tan(x) + 1i)*1i)/64 - ((\tan(x)^2 + 1/5)^{(1/2)}*1i)/(96*(\tan(x) - (5^{(1/2)}*1i)/5)) + ((\tan(x)^2 + 1/5)^{(1/2)}*1i)/(96*(\tan(x) + (5^{(1/2)}*1i)/5)) + (5^{(1/2)}*(\tan(x)^2 + 1/5)^{(1/2))}/(240*(\tan(x)^2 + (5^{(1/2)}*\tan(x)*2i)/5 - 1/5)) - (5^{(1/2)}*(\tan(x)^2 + 1/5)^{(1/2))}/(240*((5^{(1/2)}*\tan(x)*2i)/5 - \tan(x)^2 + 1/5))$

$$3.442 \quad \int \frac{\tan(x)}{\sqrt[3]{a^3 + b^3 \tan^2(x)}} dx$$

Optimal. Leaf size=133

$$\frac{\sqrt{3} \tan^{-1} \left(\frac{1 + \frac{\sqrt[3]{a^3 + b^3 \tan^2(x)}}{\sqrt[3]{a^3 - b^3}}}{\sqrt{3}} \right)}{2\sqrt[3]{a^3 - b^3}} + \frac{\log(\cos(x))}{2\sqrt[3]{a^3 - b^3}} + \frac{3 \log \left(\sqrt[3]{a^3 - b^3} - \sqrt[3]{a^3 + b^3 \tan^2(x)} \right)}{4\sqrt[3]{a^3 - b^3}}$$

[Out] 1/2*ln(cos(x))/(a^3-b^3)^(1/3)+3/4*ln((a^3-b^3)^(1/3)-(a^3+b^3*tan(x)^2)^(1/3))/(a^3-b^3)^(1/3)+1/2*arctan(1/3*(1+2*(a^3+b^3*tan(x)^2)^(1/3)/(a^3-b^3)^(1/3))*3^(1/2))*3^(1/2)/(a^3-b^3)^(1/3)

Rubi [A]

time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3751, 455, 57, 631, 210, 31}

$$\frac{\sqrt{3} \tan^{-1} \left(\frac{\frac{\sqrt[3]{a^3 + b^3 \tan^2(x)}}{\sqrt[3]{a^3 - b^3}} + 1}{\sqrt{3}} \right)}{2\sqrt[3]{a^3 - b^3}} + \frac{3 \log \left(\sqrt[3]{a^3 - b^3} - \sqrt[3]{a^3 + b^3 \tan^2(x)} \right)}{4\sqrt[3]{a^3 - b^3}} + \frac{\log(\cos(x))}{2\sqrt[3]{a^3 - b^3}}$$

Antiderivative was successfully verified.

[In] Int[Tan[x]/(a^3 + b^3*Tan[x]^2)^(1/3), x]

[Out] (Sqrt[3]*ArcTan[(1 + (2*(a^3 + b^3*Tan[x]^2)^(1/3))/(a^3 - b^3)^(1/3))/Sqrt[3]])/(2*(a^3 - b^3)^(1/3)) + Log[Cos[x]]/(2*(a^3 - b^3)^(1/3)) + (3*Log[(a^3 - b^3)^(1/3) - (a^3 + b^3*Tan[x]^2)^(1/3)])/(4*(a^3 - b^3)^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 57

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]) /;

FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 210

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 455

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 631

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3751

Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)], x], x, c*(Tan[e + f*x]/ff)], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \frac{\tan(x)}{\sqrt[3]{a^3 + b^3 \tan^2(x)}} dx &= \text{Subst} \left(\int \frac{x}{(1+x^2) \sqrt[3]{a^3 + b^3 x^2}} dx, x, \tan(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1+x) \sqrt[3]{a^3 + b^3 x}} dx, x, \tan^2(x) \right) \\
&= \frac{\log(\cos(x))}{2\sqrt[3]{a^3 - b^3}} + \frac{3}{4} \text{Subst} \left(\int \frac{1}{(a^3 - b^3)^{2/3} + \sqrt[3]{a^3 - b^3} x + x^2} dx, x, \sqrt[3]{a^3 + b^3 \tan^2(x)} \right) \\
&= \frac{\log(\cos(x))}{2\sqrt[3]{a^3 - b^3}} + \frac{3 \log \left(\sqrt[3]{a^3 - b^3} - \sqrt[3]{a^3 + b^3 \tan^2(x)} \right)}{4\sqrt[3]{a^3 - b^3}} - \frac{3 \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x \right)}{2\sqrt[3]{a^3 - b^3}} \\
&= \frac{\sqrt{3} \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{a^3 + b^3 \tan^2(x)}}{\sqrt[3]{a^3 - b^3}}}{\sqrt{3}} \right)}{2\sqrt[3]{a^3 - b^3}} + \frac{\log(\cos(x))}{2\sqrt[3]{a^3 - b^3}} + \frac{3 \log \left(\sqrt[3]{a^3 - b^3} - \sqrt[3]{a^3 + b^3 \tan^2(x)} \right)}{4\sqrt[3]{a^3 - b^3}}
\end{aligned}$$

Mathematica [A]

time = 0.09, size = 105, normalized size = 0.79

$$\frac{2\sqrt{3} \tan^{-1} \left(\frac{1 + \frac{2\sqrt[3]{a^3 + b^3 \tan^2(x)}}{\sqrt[3]{a^3 - b^3}}}{\sqrt{3}} \right) + 2 \log(\cos(x)) + 3 \log \left(\sqrt[3]{a^3 - b^3} - \sqrt[3]{a^3 + b^3 \tan^2(x)} \right)}{4\sqrt[3]{a^3 - b^3}}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[x]/(a^3 + b^3*Tan[x]^2)^(1/3), x]

[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a^3 + b^3*Tan[x]^2)^(1/3))/(a^3 - b^3)^(1/3))/Sqrt[3]] + 2*Log[Cos[x]] + 3*Log[(a^3 - b^3)^(1/3) - (a^3 + b^3*Tan[x]^2)^(1/3)])/ (4*(a^3 - b^3)^(1/3))

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[Tan[x]/(a^3 + b^3*Tan[x]^2)^(1/3),x]')`

[Out] caught exception: maximum recursion depth exceeded

Maple [F]

time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{(a^3 + b^3 (\tan^2(x)))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x)`

[Out] `int(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x, algorithm="maxima")`

[Out] `integrate(tan(x)/(b^3*tan(x)^2 + a^3)^(1/3), x)`

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x, algorithm="fricas")`

[Out] Timed out

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\sqrt[3]{a^3 + b^3 \tan^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)/(a**3+b**3*tan(x)**2)**(1/3),x)`

[Out] `Integral(tan(x)/(a**3 + b**3*tan(x)**2)**(1/3), x)`

Giac [A]

time = 0.02, size = 274, normalized size = 2.06

$$3 \left(\frac{(a^3 - b^3)^2 \ln \left((a^3 + b^3 \tan^2 x)^{\frac{1}{3}} + (a^3 - b^3)^{\frac{1}{3}} (a^3 + b^3 \tan^2 x)^{\frac{1}{3}} + (a^3 - b^3)^{\frac{1}{3}} (a^3 - b^3)^{\frac{1}{3}} \right)}{12a^3 - 12b^3} - \frac{4 (a^3 - b^3)^2 \arctan \left(\frac{2 \left((a^3 + b^3 \tan^2 x)^{\frac{1}{3}} + \frac{(a^3 - b^3)^{\frac{1}{3}}}{2} \right)}{\sqrt{3} (a^3 - b^3)^{\frac{1}{3}}} \right)}{-8\sqrt{3} a^3 + 8\sqrt{3} b^3} - \frac{\frac{1}{3} (a^3 - b^3)^{\frac{1}{3}} (a^3 - b^3)^{\frac{1}{3}} \ln \left((a^3 + b^3 \tan^2 x)^{\frac{1}{3}} - (a^3 - b^3)^{\frac{1}{3}} \right)}{2b^3 - 2a^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x)

[Out] $3/2*(a^3 - b^3)^{2/3}*\arctan(1/3*\sqrt{3}*(2*(b^3*\tan(x)^2 + a^3)^{1/3} + (a^3 - b^3)^{1/3}))/((a^3 - b^3)^{1/3})*(\sqrt{3}*a^3 - \sqrt{3}*b^3) - 1/4*\log((b^3*\tan(x)^2 + a^3)^{2/3} + (b^3*\tan(x)^2 + a^3)^{1/3}*(a^3 - b^3)^{1/3} + (a^3 - b^3)^{2/3}))/((a^3 - b^3)^{1/3} + 1/2*\log(\text{abs}((b^3*\tan(x)^2 + a^3)^{1/3} - (a^3 - b^3)^{1/3}))/((a^3 - b^3)^{1/3}))$

Mupad [B]

time = 1.45, size = 250, normalized size = 1.88

$$\frac{\ln \left(\frac{9(a^3 + b^3 \tan(x)^2)^{1/3}}{4} - \frac{9a^3 - 9b^3}{4(a-b)^{2/3}(a^2 + ab + b^2)^{2/3}} \right)}{2(a-b)^{1/3}(a^2 + ab + b^2)^{1/3}} + \frac{\ln \left(\frac{9(a^3 + b^3 \tan(x)^2)^{1/3}}{4} - \frac{(-1 + \sqrt{3} \text{li})^2 (9a^3 - 9b^3)}{16(a-b)^{2/3}(a^2 + ab + b^2)^{2/3}} \right) (-1 + \sqrt{3} \text{li})}{4(a-b)^{1/3}(a^2 + ab + b^2)^{1/3}} - \frac{\ln \left(\frac{9(a^3 + b^3 \tan(x)^2)^{1/3}}{4} - \frac{(1 + \sqrt{3} \text{li})^2 (9a^3 - 9b^3)}{16(a-b)^{2/3}(a^2 + ab + b^2)^{2/3}} \right) (1 + \sqrt{3} \text{li})}{4(a-b)^{1/3}(a^2 + ab + b^2)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)/(b^3*tan(x)^2 + a^3)^(1/3),x)

[Out] $\log((9*(b^3*\tan(x)^2 + a^3)^{1/3}))/4 - (9*a^3 - 9*b^3)/(4*(a - b)^{2/3}*(a*b + a^2 + b^2)^{2/3}))/((2*(a - b)^{1/3}*(a*b + a^2 + b^2)^{1/3})) + (\log((9*(b^3*\tan(x)^2 + a^3)^{1/3}))/4 - ((3^{1/2}*1i - 1)^2*(9*a^3 - 9*b^3))/(16*(a - b)^{2/3}*(a*b + a^2 + b^2)^{2/3}))*((3^{1/2}*1i - 1))/(4*(a - b)^{1/3}*(a*b + a^2 + b^2)^{1/3}) - (\log((9*(b^3*\tan(x)^2 + a^3)^{1/3}))/4 - ((3^{1/2}*1i + 1)^2*(9*a^3 - 9*b^3))/(16*(a - b)^{2/3}*(a*b + a^2 + b^2)^{2/3}))*((3^{1/2}*1i + 1))/(4*(a - b)^{1/3}*(a*b + a^2 + b^2)^{1/3})$

3.443 $\int \tan(x) (1 - 7 \tan^2(x))^{2/3} dx$

Optimal. Leaf size=69

$$2\sqrt{3} \tan^{-1} \left(\frac{1 + \sqrt[3]{1 - 7 \tan^2(x)}}{\sqrt{3}} \right) + 2 \log(\cos(x)) + 3 \log \left(2 - \sqrt[3]{1 - 7 \tan^2(x)} \right) + \frac{3}{4} (1 - 7 \tan^2(x))^{2/3}$$

[Out] 2*ln(cos(x))+3*ln(2-(1-7*tan(x)^2)^(1/3))+2*arctan(1/3*(1+(1-7*tan(x)^2)^(1/3))*3^(1/2))*3^(1/2)+3/4*(1-7*tan(x)^2)^(2/3)

Rubi [A]

time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3751, 455, 52, 57, 632, 210, 31}

$$\frac{3}{4} (1 - 7 \tan^2(x))^{2/3} + 2\sqrt{3} \tan^{-1} \left(\frac{\sqrt[3]{1 - 7 \tan^2(x)} + 1}{\sqrt{3}} \right) + 3 \log \left(2 - \sqrt[3]{1 - 7 \tan^2(x)} \right) + 2 \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Tan[x]*(1 - 7*Tan[x]^2)^(2/3),x]

[Out] 2*sqrt[3]*ArcTan[(1 + (1 - 7*Tan[x]^2)^(1/3))/sqrt[3]] + 2*Log[Cos[x]] + 3*Log[2 - (1 - 7*Tan[x]^2)^(1/3)] + (3*(1 - 7*Tan[x]^2)^(2/3))/4

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 52

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 57

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]) /;

FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 210

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 455

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3751

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)], x], x, c*(Tan[e + f*x]/ff)], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \tan(x) (1 - 7 \tan^2(x))^{2/3} dx &= \text{Subst} \left(\int \frac{x (1 - 7x^2)^{2/3}}{1 + x^2} dx, x, \tan(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{(1 - 7x)^{2/3}}{1 + x} dx, x, \tan^2(x) \right) \\
&= \frac{3}{4} (1 - 7 \tan^2(x))^{2/3} + 4 \text{Subst} \left(\int \frac{1}{\sqrt[3]{1 - 7x} (1 + x)} dx, x, \tan^2(x) \right) \\
&= 2 \log(\cos(x)) + \frac{3}{4} (1 - 7 \tan^2(x))^{2/3} - 3 \text{Subst} \left(\int \frac{1}{2 - x} dx, x, \sqrt[3]{1 - 7 \tan^2(x)} \right) \\
&= 2 \log(\cos(x)) + 3 \log \left(2 - \sqrt[3]{1 - 7 \tan^2(x)} \right) + \frac{3}{4} (1 - 7 \tan^2(x))^{2/3} - 12 \log(2) \\
&= 2\sqrt{3} \tan^{-1} \left(\frac{1 + \sqrt[3]{1 - 7 \tan^2(x)}}{\sqrt{3}} \right) + 2 \log(\cos(x)) + 3 \log \left(2 - \sqrt[3]{1 - 7 \tan^2(x)} \right)
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.06, size = 42, normalized size = 0.61

$$-\frac{3}{4} \left(-1 + {}_2F_1 \left(\frac{2}{3}, 1; \frac{5}{3}; \frac{1}{8} (-3 + 4 \cos(2x)) \sec^2(x) \right) \right) (1 - 7 \tan^2(x))^{2/3}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[x]*(1 - 7*Tan[x]^2)^(2/3),x]

[Out] (-3*(-1 + Hypergeometric2F1[2/3, 1, 5/3, ((-3 + 4*Cos[2*x])*Sec[x]^2)/8]))*(1 - 7*Tan[x]^2)^(2/3)/4

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(1 - 7*Tan[x]^2)^(2/3)*Tan[x],x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [F]

time = 0.03, size = 0, normalized size = 0.00

$$\int \tan(x) (1 - 7 (\tan^2(x)))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(x)*(1-7*tan(x)^2)^(2/3),x)`

[Out] `int(tan(x)*(1-7*tan(x)^2)^(2/3),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)*(1-7*tan(x)^2)^(2/3),x, algorithm="maxima")`

[Out] `integrate((-7*tan(x)^2 + 1)^(2/3)*tan(x), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(58) = 116.

time = 0.67, size = 117, normalized size = 1.70

$$2\sqrt{3} \arctan\left(\frac{7\sqrt{3}\tan(x)^2 + 4\sqrt{3}(-7\tan(x)^2 + 1)^{\frac{2}{3}} - 16\sqrt{3}(-7\tan(x)^2 + 1)^{\frac{1}{3}} - \sqrt{3}}{7\tan(x)^2 - 65}\right) + \frac{3}{4}(-7\tan(x)^2 + 1)^{\frac{2}{3}} + \log\left(\frac{7\tan(x)^2 + 6(-7\tan(x)^2 + 1)^{\frac{2}{3}} - 12(-7\tan(x)^2 + 1)^{\frac{1}{3}} + 7}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)*(1-7*tan(x)^2)^(2/3),x, algorithm="fricas")`

[Out] `2*sqrt(3)*arctan((7*sqrt(3)*tan(x)^2 + 4*sqrt(3)*(-7*tan(x)^2 + 1)^(2/3) - 16*sqrt(3)*(-7*tan(x)^2 + 1)^(1/3) - sqrt(3))/(7*tan(x)^2 - 65)) + 3/4*(-7*tan(x)^2 + 1)^(2/3) + log((7*tan(x)^2 + 6*(-7*tan(x)^2 + 1)^(2/3) - 12*(-7*tan(x)^2 + 1)^(1/3) + 7)/(tan(x)^2 + 1))`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (1 - 7 \tan^2(x))^{\frac{2}{3}} \tan(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)*(1-7*tan(x)**2)**(2/3),x)`

[Out] `Integral((1 - 7*tan(x)**2)**(2/3)*tan(x), x)`

Giac [A]

time = 0.01, size = 104, normalized size = 1.51

$$2\ln\left|(-7\tan^2x + 1)^{\frac{1}{3}} - 2\right| - \ln\left(\left((-7\tan^2x + 1)^{\frac{1}{3}}\right)^2 + 2(-7\tan^2x + 1)^{\frac{1}{3}} + 4\right) + \frac{6\arctan\left(\frac{(-7\tan^2x + 1)^{\frac{1}{3}} + 1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{4}\left((-7\tan^2x + 1)^{\frac{1}{3}}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)*(1-7*tan(x)^2)^(2/3),x)

[Out] $2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left((-7\tan(x)^2 + 1)^{1/3} + 1\right)\right) + \frac{3}{4}(-7\tan(x)^2 + 1)^{2/3} - \log\left((-7\tan(x)^2 + 1)^{2/3} + 2(-7\tan(x)^2 + 1)^{1/3} + 4\right) + 2\log\left(\text{abs}\left((-7\tan(x)^2 + 1)^{1/3} - 2\right)\right)$

Mupad [B]

time = 0.72, size = 101, normalized size = 1.46

$$2 \ln\left(144(1-7\tan(x)^2)^{1/3}-288\right) + \frac{3(1-7\tan(x)^2)^{2/3}}{4} + \ln\left(144(1-7\tan(x)^2)^{1/3}-72(-1+\sqrt{3}i)^2\right)(-1+\sqrt{3}i) - \ln\left(144(1-7\tan(x)^2)^{1/3}-72(1+\sqrt{3}i)^2\right)(1+\sqrt{3}i)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)*(1 - 7*tan(x)^2)^(2/3),x)

[Out] $2\log(144*(1 - 7*\tan(x)^2)^{1/3} - 288) + (3*(1 - 7*\tan(x)^2)^{2/3})/4 + \log(144*(1 - 7*\tan(x)^2)^{1/3} - 72*(3^{1/2}*1i - 1)^2)*(3^{1/2}*1i - 1) - \log(144*(1 - 7*\tan(x)^2)^{1/3} - 72*(3^{1/2}*1i + 1)^2)*(3^{1/2}*1i + 1)$

$$3.444 \quad \int \frac{\cot(x)}{\sqrt[4]{a^4 + b^4 \csc^2(x)}} dx$$

Optimal. Leaf size=52

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4 + b^4 \csc^2(x)}}{a}\right)}{a} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4 + b^4 \csc^2(x)}}{a}\right)}{a}$$

[Out] $-\arctan((a^4+b^4*\csc(x)^2)^{(1/4)}/a)/a+\operatorname{arctanh}((a^4+b^4*\csc(x)^2)^{(1/4)}/a)/a$

Rubi [A]

time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {4224, 272, 65, 304, 209, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4 + b^4 \csc^2(x)}}{a}\right)}{a} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4 + b^4 \csc^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[x]/(a^4 + b^4*\text{Csc}[x]^2)^{(1/4)}, x]$

[Out] $-(\text{ArcTan}[(a^4 + b^4*\text{Csc}[x]^2)^{(1/4)}/a])/a + \text{ArcTanh}[(a^4 + b^4*\text{Csc}[x]^2)^{(1/4)}/a]/a$

Rule 65

$\text{Int}[(a_. + (b_.)*(x_)^m)*((c_.) + (d_.)*(x_)^n), x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)*(c-a*(d/b)+d*(x^p/b))^n}, x], x, (a+b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 209

$\text{Int}[(a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))*\text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{GtQ}[b, 0])$

Rule 212

$\text{Int}[(a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]))*\text{ArcTanh}[\text{Rt}[-b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 272

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-a/b,
2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a
/b, 0]
```

Rule 4224

```
Int[((a_) + (b_)*((c_)*sec[(e_) + (f_)*(x_)])^(n_))^(p_)*tan[(e_) + (
f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x], x],
x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
- 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)}{\sqrt[4]{a^4 + b^4 \csc^2(x)}} dx &= -\text{Subst} \left(\int \frac{1}{x \sqrt[4]{a^4 + b^4 x^2}} dx, x, \csc(x) \right) \\
&= - \left(\frac{1}{2} \text{Subst} \left(\int \frac{1}{x \sqrt[4]{a^4 + b^4 x}} dx, x, \csc^2(x) \right) \right) \\
&= - \frac{2 \text{Subst} \left(\int \frac{x^2}{-\frac{a^4}{b^4} + \frac{x^4}{b^4}} dx, x, \sqrt[4]{a^4 + b^4 \csc^2(x)} \right)}{b^4} \\
&= \text{Subst} \left(\int \frac{1}{a^2 - x^2} dx, x, \sqrt[4]{a^4 + b^4 \csc^2(x)} \right) - \text{Subst} \left(\int \frac{1}{a^2 + x^2} dx, x, \sqrt[4]{a^4 + b^4 \csc^2(x)} \right) \\
&= - \frac{\tan^{-1} \left(\frac{\sqrt[4]{a^4 + b^4 \csc^2(x)}}{a} \right)}{a} + \frac{\tanh^{-1} \left(\frac{\sqrt[4]{a^4 + b^4 \csc^2(x)}}{a} \right)}{a}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 256 vs. 2(52) = 104.

time = 0.19, size = 256, normalized size = 4.92

$$\frac{\sqrt[4]{-a^4 - 2b^4 + a^4 \cos(2x)} \left(-2 \tan^{-1} \left(1 - \frac{\sqrt{2} a \sqrt{\sin(x)}}{\sqrt{-b^4 - a^4 \sin^2(x)}} \right) + 2 \tan^{-1} \left(1 + \frac{\sqrt{2} a \sqrt{\sin(x)}}{\sqrt{-b^4 - a^4 \sin^2(x)}} \right) - \log \left(1 + \frac{a^2 \sin(x)}{\sqrt{-b^4 - a^4 \sin^2(x)}} - \frac{\sqrt{2} a \sqrt{\sin(x)}}{\sqrt{-b^4 - a^4 \sin^2(x)}} \right) + \log \left(1 + \frac{a^2 \sin(x)}{\sqrt{-b^4 - a^4 \sin^2(x)}} + \frac{\sqrt{2} a \sqrt{\sin(x)}}{\sqrt{-b^4 - a^4 \sin^2(x)}} \right) \right)}{2^{3/4} a \sqrt[4]{a^4 + b^4 \csc^2(x)} \sqrt{\sin(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]/(a^4 + b^4*Csc[x]^2)^(1/4),x]

[Out]
$$\frac{((-a^4 - 2b^4 + a^4 \cos[2x])^{1/4} (-2 \operatorname{ArcTan}[1 - (\sqrt{2} a \sqrt{\sin[x]})] / (-b^4 - a^4 \sin[x]^2)^{1/4}) + 2 \operatorname{ArcTan}[1 + (\sqrt{2} a \sqrt{\sin[x]})] / (-b^4 - a^4 \sin[x]^2)^{1/4}) - \operatorname{Log}[1 + (a^2 \sin[x]) / \sqrt{-b^4 - a^4 \sin[x]^2}] - (\sqrt{2} a \sqrt{\sin[x]}) / (-b^4 - a^4 \sin[x]^2)^{1/4} + \operatorname{Log}[1 + (a^2 \sin[x]) / \sqrt{-b^4 - a^4 \sin[x]^2}] + (\sqrt{2} a \sqrt{\sin[x]}) / (-b^4 - a^4 \sin[x]^2)^{1/4})}{(2 \cdot 2^{3/4} a (a^4 + b^4 \operatorname{Csc}[x]^2)^{1/4} \sqrt{\sin[x]})}$$

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Cot[x]/(a^4 + b^4*Csc[x]^2)^(1/4),x]')

[Out] Timed out

Maple [F]

time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{(a^4 + b^4 (\csc^2(x)))^{1/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x)

[Out] int(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x)

Maxima [A]

time = 0.34, size = 71, normalized size = 1.37

$$-\frac{\arctan\left(\frac{\left(a^4 + \frac{b^4}{\sin(x)^2}\right)^{1/4}}{a}\right)}{a} + \frac{\log\left(a + \left(a^4 + \frac{b^4}{\sin(x)^2}\right)^{1/4}\right)}{2a} - \frac{\log\left(-a + \left(a^4 + \frac{b^4}{\sin(x)^2}\right)^{1/4}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x, algorithm="maxima")

[Out]
$$-\arctan\left(\frac{(a^4 + b^4/\sin(x)^2)^{1/4}}{a}\right)/a + 1/2 \cdot \log(a + (a^4 + b^4/\sin(x)^2)^{1/4})/a - 1/2 \cdot \log(-a + (a^4 + b^4/\sin(x)^2)^{1/4})/a$$

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 + b^4 \csc^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(a**4+b**4*csc(x)**2)**(1/4),x)

[Out] Integral(cot(x)/(a**4 + b**4*csc(x)**2)**(1/4), x)

Giac [A]

time = 0.10, size = 84, normalized size = 1.62

$$-2 \left(\frac{\ln \left| \left(a^4 + \frac{b^4}{\sin^2 x} \right)^{\frac{1}{4}} - a \right|}{4a} - \frac{\ln \left| \left(a^4 + \frac{b^4}{\sin^2 x} \right)^{\frac{1}{4}} + a \right|}{4a} + \frac{\arctan \left(\frac{\left(a^4 + \frac{b^4}{\sin^2 x} \right)^{\frac{1}{4}}}{a} \right)}{2a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x)

[Out] -arctan((a^4 + b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(abs(a + (a^4 + b^4/sin(x)^2)^(1/4)))/a - 1/2*log(abs(-a + (a^4 + b^4/sin(x)^2)^(1/4)))/a

Mupad [B]

time = 0.41, size = 46, normalized size = 0.88

$$\frac{\operatorname{atan} \left(\frac{\left(\frac{b^4}{\sin(x)^2} + a^4 \right)^{1/4}}{a} \right) - \operatorname{atanh} \left(\frac{\left(\frac{b^4}{\sin(x)^2} + a^4 \right)^{1/4}}{a} \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)/(b^4/sin(x)^2 + a^4)^(1/4),x)

[Out] -(atan((b^4/sin(x)^2 + a^4)^(1/4)/a) - atanh((b^4/sin(x)^2 + a^4)^(1/4)/a))/a

$$3.445 \quad \int \frac{\cot(x)}{\sqrt[4]{a^4 - b^4 \csc^2(x)}} dx$$

Optimal. Leaf size=54

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4 - b^4 \csc^2(x)}}{a}\right)}{a} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4 - b^4 \csc^2(x)}}{a}\right)}{a}$$

[Out] $-\arctan((a^4 - b^4 \csc(x)^2)^{1/4}/a)/a + \operatorname{arctanh}((a^4 - b^4 \csc(x)^2)^{1/4}/a)/a$

Rubi [A]

time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4224, 272, 65, 304, 209, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4 - b^4 \csc^2(x)}}{a}\right)}{a} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4 - b^4 \csc^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[x]/(a^4 - b^4 \text{Csc}[x]^2)^{1/4}, x]$

[Out] $-(\text{ArcTan}[(a^4 - b^4 \text{Csc}[x]^2)^{1/4}/a])/a + \text{ArcTanh}[(a^4 - b^4 \text{Csc}[x]^2)^{1/4}/a]/a$

Rule 65

$\text{Int}[(a_. + (b_.)(x_)^m)((c_. + (d_.)(x_)^n), x_Symbol] :> \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p(m+1)-1)}(c - a(d/b) + d(x^p/b))^n, x], x, (a + b*x)^{1/p}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 209

$\text{Int}[(a_) + (b_.)(x_)^2)^{-1}, x_Symbol] :> \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))*\text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] \|\ \text{GtQ}[b, 0])$

Rule 212

$\text{Int}[(a_) + (b_.)(x_)^2)^{-1}, x_Symbol] :> \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]))*\text{ArcTanh}[\text{Rt}[-b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \|\ \text{LtQ}[b, 0])$

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-a/b,
2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a
/b, 0]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x], x],
x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
- 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)}{\sqrt[4]{a^4 - b^4 \csc^2(x)}} dx &= -\text{Subst} \left(\int \frac{1}{x \sqrt[4]{a^4 - b^4 x^2}} dx, x, \csc(x) \right) \\
&= - \left(\frac{1}{2} \text{Subst} \left(\int \frac{1}{x \sqrt[4]{a^4 - b^4 x}} dx, x, \csc^2(x) \right) \right) \\
&= \frac{2 \text{Subst} \left(\int \frac{x^2}{\frac{a^4}{b^4} - \frac{x^4}{b^4}} dx, x, \sqrt[4]{a^4 - b^4 \csc^2(x)} \right)}{b^4} \\
&= \text{Subst} \left(\int \frac{1}{a^2 - x^2} dx, x, \sqrt[4]{a^4 - b^4 \csc^2(x)} \right) - \text{Subst} \left(\int \frac{1}{a^2 + x^2} dx, x, \sqrt[4]{a^4 - b^4 \csc^2(x)} \right) \\
&= -\frac{\tan^{-1} \left(\frac{\sqrt[4]{a^4 - b^4 \csc^2(x)}}{a} \right)}{a} + \frac{\tanh^{-1} \left(\frac{\sqrt[4]{a^4 - b^4 \csc^2(x)}}{a} \right)}{a}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 245 vs. 2(54) = 108.

time = 0.18, size = 245, normalized size = 4.54

$$\frac{\sqrt{-a^4 + 2b^4 + a^4 \cos(2x)} \left(-2 \tan^{-1} \left(1 - \frac{\sqrt{2} a \sqrt{\sin(x)}}{\sqrt[4]{b^4 - a^4 \sin^2(x)}} \right) + 2 \tan^{-1} \left(1 + \frac{\sqrt{2} a \sqrt{\sin(x)}}{\sqrt[4]{b^4 - a^4 \sin^2(x)}} \right) - \log \left(1 + \frac{a^2 \sin(x)}{\sqrt[4]{b^4 - a^4 \sin^2(x)}} - \frac{\sqrt{2} a \sqrt{\sin(x)}}{\sqrt[4]{b^4 - a^4 \sin^2(x)}} \right) + \log \left(1 + \frac{a^2 \sin(x)}{\sqrt[4]{b^4 - a^4 \sin^2(x)}} + \frac{\sqrt{2} a \sqrt{\sin(x)}}{\sqrt[4]{b^4 - a^4 \sin^2(x)}} \right) \right)}{2^{23/4} a \sqrt[4]{a^4 - b^4 \csc^2(x)} \sqrt{\sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a^4 - b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] ((-a^4 + 2*b^4 + a^4*Cos[2*x])^(1/4)*(-2*ArcTan[1 - (Sqrt[2]*a*Sqrt[Sin[x]])]/(b^4 - a^4*Sin[x]^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*a*Sqrt[Sin[x]])]/(b^4 - a^4*Sin[x]^2)^(1/4)] - Log[1 + (a^2*Sin[x])/Sqrt[b^4 - a^4*Sin[x]^2] - (Sqrt[2]*a*Sqrt[Sin[x]])/(b^4 - a^4*Sin[x]^2)^(1/4)] + Log[1 + (a^2*Sin[x])/Sqrt[b^4 - a^4*Sin[x]^2] + (Sqrt[2]*a*Sqrt[Sin[x]])/(b^4 - a^4*Sin[x]^2)^(1/4)))/(2*2^(3/4)*a*(a^4 - b^4*Csc[x]^2)^(1/4)*Sqrt[Sin[x]])
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Cot[x]/(a^4 - b^4*Csc[x]^2)^(1/4),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Maple [F]

time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{(a^4 - b^4 (\csc^2(x)))^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x)
```

```
[Out] int(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x)
```

Maxima [A]

time = 0.34, size = 74, normalized size = 1.37

$$-\frac{\arctan\left(\frac{\left(a^4 - \frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a} + \frac{\log\left(a + \left(a^4 - \frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2a} - \frac{\log\left(-a + \left(a^4 - \frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x, algorithm="maxima")
```

```
[Out] -arctan((a^4 - b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(a + (a^4 - b^4/sin(x)^2)^(1/4))/a - 1/2*log(-a + (a^4 - b^4/sin(x)^2)^(1/4))/a
```

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt[4]{(a^2 - b^2 \csc(x))(a^2 + b^2 \csc(x))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(a**4-b**4*csc(x)**2)**(1/4),x)

[Out] Integral(cot(x)/((a**2 - b**2*csc(x))*(a**2 + b**2*csc(x)))**(1/4), x)

Giac [A]

time = 0.09, size = 87, normalized size = 1.61

$$-2 \left(\frac{\ln \left| \left(a^4 - \frac{b^4}{\sin^2 x} \right)^{\frac{1}{4}} - a \right|}{4a} - \frac{\ln \left| \left(a^4 - \frac{b^4}{\sin^2 x} \right)^{\frac{1}{4}} + a \right|}{4a} + \frac{\arctan \left(\frac{\left(a^4 - \frac{b^4}{\sin^2 x} \right)^{\frac{1}{4}}}{a} \right)}{2a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x)

[Out] -arctan((a^4 - b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(abs(a + (a^4 - b^4/sin(x)^2)^(1/4)))/a - 1/2*log(abs(-a + (a^4 - b^4/sin(x)^2)^(1/4)))/a

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot(x)}{\left(a^4 - \frac{b^4}{\sin(x)^2} \right)^{1/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)/(a^4 - b^4/sin(x)^2)^(1/4),x)

[Out] int(cot(x)/(a^4 - b^4/sin(x)^2)^(1/4), x)

$$3.446 \quad \int \frac{\sec^2(x) \tan(x) \left(\sqrt[3]{1 - 3 \sec^2(x)} \sin^2(x) + 3 \tan^2(x) \right)}{(1 - 3 \sec^2(x))^{5/6} \left(1 - \sqrt{1 - 3 \sec^2(x)} \right)} dx$$

Optimal. Leaf size=133

$$\sqrt{3} \tan^{-1} \left(\frac{1 + 2 \sqrt[6]{1 - 3 \sec^2(x)}}{\sqrt{3}} \right) + \frac{1}{4} \log(\sec^2(x)) - \frac{3}{2} \log \left(1 - \sqrt[6]{1 - 3 \sec^2(x)} \right) + \frac{1}{3} \log \left(1 - \sqrt{1 - 3 \sec^2(x)} \right)$$

[Out] $\frac{1}{4} \ln(\sec(x)^2) - \frac{3}{2} \ln(1 - (1 - 3 \sec(x)^2)^{1/6}) + \frac{1}{3} \ln(1 - (1 - 3 \sec(x)^2)^{1/2}) - (1 - 3 \sec(x)^2)^{1/6} - \frac{1}{4} (1 - 3 \sec(x)^2)^{2/3} + \arctan(1/3 * (1 + 2 * (1 - 3 \sec(x)^2)^{1/6})) * 3^{1/2} * 3^{1/2} + 1/2 / (1 - (1 - 3 \sec(x)^2)^{1/2})$

Rubi [A]

time = 3.19, antiderivative size = 174, normalized size of antiderivative = 1.31, number of steps used = 29, number of rules used = 16, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}} = 0.262$, Rules used = {4446, 6874, 6816, 267, 6829, 348, 59, 632, 210, 31, 6820, 272, 43, 65, 212, 25}

$$\frac{\cos^2(x)}{6} - \frac{1}{4} (1 - 3 \sec^2(x))^{2/3} - \sqrt{1 - 3 \sec^2(x)} - \frac{3}{2} \log(1 - \sqrt[6]{1 - 3 \sec^2(x)}) + \frac{1}{2} \log(1 - \sqrt{1 - 3 \sec^2(x)}) + \frac{1}{6} \cos^2(x) \sqrt{1 - 3 \sec^2(x)} + \sqrt{3} \tan^{-1} \left(\frac{2 \sqrt[6]{1 - 3 \sec^2(x)} + 1}{\sqrt{3}} \right) + \frac{1}{2} \tanh^{-1}(\sqrt{1 - 3 \sec^2(x)}) + \frac{1}{3} \log(1 - \sqrt{(3 - \cos^2(x)) \sec^2(x)})$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2*Tan[x]*((1 - 3*Sec[x]^2)^(1/3)*Sin[x]^2 + 3*Tan[x]^2))/((1 - 3*Sec[x]^2)^(5/6)*(1 - Sqrt[1 - 3*Sec[x]^2])),x]

[Out] Sqrt[3]*ArcTan[(1 + 2*(1 - 3*Sec[x]^2)^(1/6))/Sqrt[3]] + ArcTanh[Sqrt[1 - 3*Sec[x]^2]]/2 + Cos[x]^2/6 + Log[1 - Sqrt[-((3 - Cos[x]^2)*Sec[x]^2)]]/3 - (3*Log[1 - (1 - 3*Sec[x]^2)^(1/6)])/2 + Log[1 - Sqrt[1 - 3*Sec[x]^2]]/2 - (1 - 3*Sec[x]^2)^(1/6) + (Cos[x]^2*Sqrt[1 - 3*Sec[x]^2])/6 - (1 - 3*Sec[x]^2)^(2/3)/4

Rule 25

Int[(u_)*((a_) + (b_)*(x_)^(n_))^(m_)*((c_) + (d_)*(x_)^(q_))^(p_), x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d, 0] && !(IntegerQ[m] && NegQ[n])

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && GtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(
-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4446

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rule 6816

```
Int[(u_)/(y_), x_Symbol] := With[{q = DerivativeDivides[y, u, x]}, Simp[q*Log[RemoveContent[y, x]], x] /; !FalseQ[q]]
```

Rule 6820

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifierIntegrandQ[v, u, x]]
```

Rule 6829

```
Int[(u_.)*(v_)^(m_.)*((a_.) + (b_.)*(y_)^(n_))^(p_.), x_Symbol] := Module[{q, r}, Dist[q*r, Subst[Int[x^m*(a + b*x^n)^p, x], x, y], x] /; !FalseQ[r = Divides[y^m, v^m, x]] && !FalseQ[q = DerivativeDivides[y, u, x]]] /; FreeQ[{a, b, m, n, p}, x]
```

Rule 6874

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x) \tan(x) \left(\sqrt[3]{1 - 3 \sec^2(x)} \sin^2(x) + 3 \tan^2(x) \right)}{(1 - 3 \sec^2(x))^{5/6} \left(1 - \sqrt{1 - 3 \sec^2(x)} \right)} dx &= -\text{Subst} \left(\int \frac{(1 - x^2) \left(3 + \sqrt[3]{1 - \frac{3}{x^2}} x^2 \right)}{\left(1 - \sqrt{1 - \frac{3}{x^2}} \right) \left(1 - \frac{3}{x^2} \right)^{5/6} x} \right) \\
&= -\text{Subst} \left(\int \left(\frac{-3 - x^2 \sqrt[3]{\frac{-3 + x^2}{x^2}}}{\left(1 - \frac{3}{x^2} \right)^{5/6} x^5 \left(-1 + \sqrt{\frac{-3 + x^2}{x^2}} \right)} \right) \right) \\
&= -\text{Subst} \left(\int \frac{-3 - x^2 \sqrt[3]{\frac{-3 + x^2}{x^2}}}{\left(1 - \frac{3}{x^2} \right)^{5/6} x^5 \left(-1 + \sqrt{\frac{-3 + x^2}{x^2}} \right)} \right) \\
&= -\text{Subst} \left(\int \left(\frac{1}{\sqrt{1 - \frac{3}{x^2}} x^3 \left(1 - \sqrt{\frac{-3 + x^2}{x^2}} \right)} \right) \right) \\
&= 3\text{Subst} \left(\int \frac{1}{\left(1 - \frac{3}{x^2} \right)^{5/6} x^5 \left(-1 + \sqrt{\frac{-3 + x^2}{x^2}} \right)} \right) \\
&= \frac{1}{3} \log \left(1 - \sqrt{-(3 - \cos^2(x)) \sec^2(x)} \right) - \frac{1}{2} \text{S} \\
&= \frac{1}{3} \log \left(1 - \sqrt{-(3 - \cos^2(x)) \sec^2(x)} \right) - \text{Su} \\
&= \frac{\cos^2(x)}{6} + \frac{1}{3} \log \left(1 - \sqrt{-(3 - \cos^2(x)) \sec^2(x)} \right) \\
&= \frac{\cos^2(x)}{6} - \frac{3}{2} \log \left(1 - \sqrt[6]{-(3 - \cos^2(x)) \sec^2(x)} \right) \\
&= \sqrt{3} \tan^{-1} \left(\frac{1 + 2 \sqrt[6]{-(3 - \cos^2(x)) \sec^2(x)}}{\sqrt{3}} \right) \\
&= \sqrt{3} \tan^{-1} \left(\frac{1 + 2 \sqrt[6]{-(3 - \cos^2(x)) \sec^2(x)}}{\sqrt{3}} \right)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 370 vs. 2(133) = 266.

time = 54.12, size = 370, normalized size = 2.78

$$\frac{(-1 + \sqrt{1 - 3 \sec^2(x)}) \left(\frac{(-1 + \sqrt{1 - 3 \sec^2(x)}) + \sin(x) \sqrt{3 \tan^2(x) - 2}}{\sqrt{3 \tan^2(x) - 2}} + \sin(x) \sqrt{3 \tan^2(x) - 2} + \sin(x) \sqrt{3 \tan^2(x) - 2} - \sin(x) \sqrt{3 \tan^2(x) - 2} - \sin(x) \sqrt{3 \tan^2(x) - 2} - \sin(x) \sqrt{3 \tan^2(x) - 2} + \sin(x) \sqrt{3 \tan^2(x) - 2} \right)}{(-1 + \sqrt{1 - 3 \sec^2(x)}) \sqrt{3 \tan^2(x) - 2} + \sin(x) \sqrt{3 \tan^2(x) - 2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*Tan[x]*((1 - 3*Sec[x]^2)^(1/3)*Sin[x]^2 + 3*Tan[x]^2))/
((1 - 3*Sec[x]^2)^(5/6)*(1 - Sqrt[1 - 3*Sec[x]^2])),x]
```

```
[Out] -1/6*((3 + Cos[x]^2*(1 - 3*Sec[x]^2)^(1/3))*(6 - 12*Log[-1 + (-2 - 3*Tan[x]^2)^(1/6)] +
6*Log[1 + (-2 - 3*Tan[x]^2)^(1/6) + (-2 - 3*Tan[x]^2)^(1/3)] +
Log[-1 + Sqrt[-2 - 3*Tan[x]^2]] + 3*Log[1 + Sqrt[-2 - 3*Tan[x]^2]] - 18*(-
2 - 3*Tan[x]^2)^(1/6) - 9*Tan[x]^2*(-2 - 3*Tan[x]^2)^(1/6) + 12*Log[-1 + (-
2 - 3*Tan[x]^2)^(1/6)]*Sqrt[-2 - 3*Tan[x]^2] - 6*Log[1 + (-2 - 3*Tan[x]^2)^(1/6) +
(-2 - 3*Tan[x]^2)^(1/3)]*Sqrt[-2 - 3*Tan[x]^2] - Log[-1 + Sqrt[-2 -
3*Tan[x]^2]]*Sqrt[-2 - 3*Tan[x]^2] - 3*Log[1 + Sqrt[-2 - 3*Tan[x]^2]]*Sqrt
[-2 - 3*Tan[x]^2] + 9*(-2 - 3*Tan[x]^2)^(2/3) - 12*Sqrt[3]*ArcTan[(1 + 2*(-
2 - 3*Tan[x]^2)^(1/6))/Sqrt[3]]*(-1 + Sqrt[-2 - 3*Tan[x]^2])))/((6 + (-2 -
3*Tan[x]^2)^(1/3) + Cos[2*x]*(-2 - 3*Tan[x]^2)^(1/3))*(-1 + Sqrt[-2 - 3*Tan
[x]^2]))
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(3*Tan[x]^2 + Sin[x]^2*(1 - 3*Sec[x]^2)^(1/3))/(Cos[x]^2
*(1 - 3*Sec[x]^2)^(5/6)*(1 - Sqrt[1 - 3*Sec[x]^2]))*Tan[x],x]')
```

```
[Out] Timed out
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x) \left((1 - 3(\sec^2(x)))^{1/3} (\sin^2(x)) + 3(\tan^2(x)) \right)}{\cos(x)^2 (1 - 3(\sec^2(x)))^{5/6} \left(1 - \sqrt{1 - 3(\sec^2(x))} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*sec(x)^
2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x)
```

```
[Out] int(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*sec(x)^
2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x)
```


Maxima [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*sec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x, algorithm="maxima")
```

[Out] Timed out

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*sec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x, algorithm="fricas")
```

[Out] Exception raised: TypeError >> Error detected within library code: Curve not irreducible after change of variable 0 -> infinity

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*((1-3*sec(x)**2)**(1/3)*sin(x)**2+3*tan(x)**2)/cos(x)**2/(1-3*sec(x)**2)**(5/6)/(1-(1-3*sec(x)**2)**(1/2)),x)
```

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*sec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x)
```

[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\tan(x) \left(\sin(x)^2 \left(1 - \frac{3}{\cos(x)^2} \right)^{1/3} + 3 \tan(x)^2 \right)}{\cos(x)^2 \left(\sqrt{1 - \frac{3}{\cos(x)^2}} - 1 \right) \left(1 - \frac{3}{\cos(x)^2} \right)^{5/6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(tan(x)*(sin(x)^2*(1 - 3/cos(x)^2)^(1/3) + 3*tan(x)^2))/(cos(x)^2*((1 - 3/cos(x)^2)^(1/2) - 1)*(1 - 3/cos(x)^2)^(5/6)),x)
```

```
[Out] -int((tan(x)*(sin(x)^2*(1 - 3/cos(x)^2)^(1/3) + 3*tan(x)^2))/(cos(x)^2*((1 - 3/cos(x)^2)^(1/2) - 1)*(1 - 3/cos(x)^2)^(5/6)), x)
```

$$3.447 \quad \int \frac{\sec^2(x)(-\cos(2x)+2\tan^2(x))}{(\tan(x)\tan(2x))^{3/2}} dx$$

Optimal. Leaf size=100

$$2 \tanh^{-1} \left(\frac{\tan(x)}{\sqrt{\tan(x)\tan(2x)}} \right) - \frac{11 \tanh^{-1} \left(\frac{\sqrt{2} \tan(x)}{\sqrt{\tan(x)\tan(2x)}} \right)}{4\sqrt{2}} + \frac{\tan(x)}{2(\tan(x)\tan(2x))^{3/2}} + \frac{2 \tan^3(x)}{3(\tan(x)\tan(2x))^{3/2}}$$

[Out] 2*arctanh(tan(x)/(tan(x)*tan(2*x))^(1/2))-11/8*arctanh(2^(1/2)*tan(x)/(tan(x)*tan(2*x))^(1/2))*2^(1/2)+3/4*tan(x)/(tan(x)*tan(2*x))^(1/2)+1/2*tan(x)/(tan(x)*tan(2*x))^(3/2)+2/3*tan(x)^3/(tan(x)*tan(2*x))^(3/2)

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 208 vs. 2(100) = 200. time = 0.65, antiderivative size = 208, normalized size of antiderivative = 2.08, number of steps used = 22, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {4482, 12, 1986, 15, 6857, 272, 43, 52, 65, 209, 455}

$$\frac{(1-\tan^2(x))\tan(x)}{3\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{3\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} - \frac{11\tan^{-1}\left(\frac{\sqrt{\tan^2(x)-1}}{\sqrt{1-\tan^2(x)}}\right)\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{\tan^2(x)-1}} + \frac{2\tan^{-1}\left(\frac{\sqrt{\tan^2(x)-1}}{\sqrt{2}}\right)\tan(x)}{\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{\tan^2(x)-1}} + \frac{(1-\tan^2(x))\cot(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2*(-Cos[2*x] + 2*Tan[x]^2))/(Tan[x]*Tan[2*x])^(3/2), x]

[Out] (3*Tan[x])/(4*Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]) + (Cot[x]*(1 - Tan[x]^2))/(4*Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]) + (Tan[x]*(1 - Tan[x]^2))/(3*Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]) - (11*ArcTan[Sqrt[-1 + Tan[x]^2]]*Tan[x])/(4*Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]*Sqrt[-1 + Tan[x]^2]) + (2*ArcTan[Sqrt[-1 + Tan[x]^2]/Sqrt[2]]*Tan[x])/(Sqrt[Tan[x]^2/(1 - Tan[x]^2)]*Sqrt[-1 + Tan[x]^2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[a^IntPart[m]*((a*x^n)^FracPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^(
r_.))^(p_), x_Symbol] := Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
```

), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]

Rule 4482

Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rule 6857

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x) (-\cos(2x) + 2 \tan^2(x))}{(\tan(x) \tan(2x))^{3/2}} dx &= \int \frac{\sec^2(x) (-\cos(2x) + 2 \tan^2(x))}{(-1 + \sec(2x))^{3/2}} dx \\
&= \text{Subst} \left(\int \frac{(1-x^2)(-1+3x^2+2x^4)}{2\sqrt{2} x^2 \sqrt{\frac{x^2}{1-x^2}} (1+x^2)} dx, x, \tan(x) \right) \\
&= \frac{\text{Subst} \left(\int \frac{(1-x^2)(-1+3x^2+2x^4)}{x^2 \sqrt{\frac{x^2}{1-x^2}} (1+x^2)} dx, x, \tan(x) \right)}{2\sqrt{2}} \\
&= \frac{\tan(x) \text{Subst} \left(\int \frac{(1-x^2)^{3/2} (-1+3x^2+2x^4)}{x^3 (1+x^2)} dx, x, \tan(x) \right)}{2\sqrt{2} \sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}} \sqrt{1-\tan^2(x)}} \\
&= \frac{\tan(x) \text{Subst} \left(\int \left(-\frac{(1-x^2)^{3/2}}{x^3} + \frac{4(1-x^2)^{3/2}}{x} - \frac{2x(1-x^2)^{3/2}}{1+x^2} \right) dx, x, \tan(x) \right)}{2\sqrt{2} \sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}} \sqrt{1-\tan^2(x)}} \\
&= \frac{\tan(x) \text{Subst} \left(\int \frac{(1-x^2)^{3/2}}{x^3} dx, x, \tan(x) \right)}{2\sqrt{2} \sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}} \sqrt{1-\tan^2(x)}} - \frac{\tan(x) \text{Subst} \left(\int \frac{x(1-x^2)^{3/2}}{1+x^2} dx, x, \tan(x) \right)}{\sqrt{2} \sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}} \sqrt{1-\tan^2(x)}} \\
&= \frac{\tan(x) \text{Subst} \left(\int \frac{(1-x)^{3/2}}{x^2} dx, x, \tan^2(x) \right)}{4\sqrt{2} \sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}} \sqrt{1-\tan^2(x)}} - \frac{\tan(x) \text{Subst} \left(\int \frac{(1-x)^{3/2}}{1+x} dx, x, \tan^2(x) \right)}{2\sqrt{2} \sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}} \sqrt{1-\tan^2(x)}} \\
&= \frac{\cot(x) (1-\tan^2(x))}{4\sqrt{2} \sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} - \frac{\tan(x) (1-\tan^2(x))}{3\sqrt{2} \sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\sqrt{2} \tan(x) (1-\tan^2(x))}{3\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} \\
&= \frac{3 \tan(x)}{4\sqrt{2} \sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\cot(x) (1-\tan^2(x))}{4\sqrt{2} \sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} - \frac{\tan(x) (1-\tan^2(x))}{3\sqrt{2} \sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} \\
&= \frac{3 \tan(x)}{4\sqrt{2} \sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\cot(x) (1-\tan^2(x))}{4\sqrt{2} \sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} - \frac{\tan(x) (1-\tan^2(x))}{3\sqrt{2} \sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}}
\end{aligned}$$

Mathematica [A]

time = 3.06, size = 169, normalized size = 1.69

$$\frac{(-\cos(2x) + 2 \tan^2(x)) \left(\frac{4\sqrt{2} \left(-2 \tanh^{-1} \left(\sqrt{\frac{\cos(2x)}{1 + \cos(2x)}} \right) + \sqrt{2} \tanh^{-1} \left(\sqrt{1 - \tan^2(x)} \right) \right) \cos(2x) \tan(x)}{\sqrt{1 - \tan^2(x)}} - 3 \tan^{-1} \left(\sqrt{-1 + \tan^2(x)} \right) \cos(x) \sin(x) \sqrt{-1 + \tan^2(x)} + \frac{1}{3} (-3 \cot(x) - 4 \cos(x) \sin(x) + (5 + 9 \cos(2x)) \tan^3(x)) \tan^2(2x) \right)}{2(-3 + 6 \cos(2x) + \cos(4x))(\tan(x) \tan(2x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2*(-Cos[2*x] + 2*Tan[x]^2))/(Tan[x]*Tan[2*x])^(3/2), x]

[Out] ((-Cos[2*x] + 2*Tan[x]^2)*((4*Sqrt[2]*(-2*ArcTanh[Sqrt[Cos[2*x]/(1 + Cos[2*x])]]) + Sqrt[2]*ArcTanh[Sqrt[1 - Tan[x]^2]])*Cos[2*x]*Tan[x])/Sqrt[1 - Tan[x]^2] - 3*ArcTan[Sqrt[-1 + Tan[x]^2]]*Cos[x]*Sin[x]*Sqrt[-1 + Tan[x]^2] + (-3*Cot[x] - 4*Cos[x]*Sin[x] + (5 + 9*Cos[2*x])*Tan[x]^3)/3)*Tan[2*x]^2)/(2*(-3 + 6*Cos[2*x] + Cos[4*x])*(Tan[x]*Tan[2*x])^(3/2))

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(2*Tan[x]^2 - Cos[2*x])/(Cos[x]^2*(Tan[x]*Tan[2*x])^(3/2)), x]')

[Out] Timed out

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. 2(78) = 156.

time = 0.64, size = 559, normalized size = 5.59

method	result	size
default	Expression too large to display	559

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2), x, method=_RETURNVERBOSE)

[Out] 1/96*2^(1/2)*4^(1/2)*(cos(x)-1)^2*(48*cos(x)^4*arctanh(1/2*2^(1/2)*cos(x))*4^(1/2)*(cos(x)-1)/((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)/sin(x)^2)*2^(1/2)-22*cos(x)^4*((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)-201*cos(x)^4*ln(-2*(cos(x)^2*(2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)-2*cos(x)^2+cos(x)-((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)+1)/sin(x)^2)+168*cos(x)^4*ln(-4*(cos(x)^2*(2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)-2*cos(x)^2+cos(x)-((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)+1)/sin(x)^2)-33*cos(x)^4*arctanh(1/2*4^(1/2)*(2*cos(x)^2-3*cos(x)+1)/((2*cos(x)

$$\begin{aligned} & ^2-1)/(1+\cos(x))^2)^{(1/2)}/\sin(x)^2)-48*\cos(x)^3*\operatorname{arctanh}(1/2*2^{(1/2)}*\cos(x)* \\ & 4^{(1/2)}*(\cos(x)-1)/((2*\cos(x)^2-1)/(1+\cos(x))^2)^{(1/2)}/\sin(x)^2)*2^{(1/2)}+20 \\ & 1*\cos(x)^3*\ln(-2*(\cos(x)^2*((2*\cos(x)^2-1)/(1+\cos(x))^2)^{(1/2)}-2*\cos(x)^2+\cos(x)- \\ & ((2*\cos(x)^2-1)/(1+\cos(x))^2)^{(1/2)}+1)/\sin(x)^2)-168*\cos(x)^3*\ln(-4*(\\ & \cos(x)^2*((2*\cos(x)^2-1)/(1+\cos(x))^2)^{(1/2)}-2*\cos(x)^2+\cos(x)-((2*\cos(x)^2 \\ & -1)/(1+\cos(x))^2)^{(1/2)}+1)/\sin(x)^2)+33*\cos(x)^3*\operatorname{arctanh}(1/2*4^{(1/2)}*(2*\cos \\ & (x)^2-3*\cos(x)+1)/((2*\cos(x)^2-1)/(1+\cos(x))^2)^{(1/2)}/\sin(x)^2)+36*\cos(x)^2 \\ & *((2*\cos(x)^2-1)/(1+\cos(x))^2)^{(1/2)}-8*((2*\cos(x)^2-1)/(1+\cos(x))^2)^{(1/2)}) \\ & / \cos(x)^3/\sin(x)^3/(\sin(x)^2/(2*\cos(x)^2-1))^{(3/2)}/((2*\cos(x)^2-1)/(1+\cos(x) \\ &))^2)^{(3/2)} \end{aligned}$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x, algorithm="maxima")

[Out] integrate((2*tan(x)^2 - cos(2*x))/((tan(2*x)*tan(x))^(3/2)*cos(x)^2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(78) = 156.

time = 0.66, size = 271, normalized size = 2.71

$$\frac{24 (\cos(x)^2 - \cos(x)) \log\left(\frac{1 + \sqrt{2} (1 + \cos(x)^2 + \cos(x)) \sqrt{\frac{\cos(x)^2 - 1}{2 \cos(x)^2 - 1}}}{2 \cos(x)^2 - 1}\right) \sin(x) - 33 (\sqrt{2} \cos(x)^2 - \sqrt{2} \cos(x)^2) \log\left(\frac{1 + \sqrt{2} (1 + \sqrt{2} - 1) \cos(x) - (1 + \sqrt{2} - 1) \cos(x)}{2 \cos(x)^2 - 1}\right) \sin(x) - 2 \sqrt{2} (22 \cos(x)^2 - 47 \cos(x)^2 + 26 \cos(x)^2 - 4) \sqrt{\frac{\cos(x)^2 - 1}{2 \cos(x)^2 - 1}} - 44 (\cos(x)^2 - \cos(x)) \sin(x)}{48 (\cos(x)^2 - \cos(x)) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/48*(24*(\cos(x)^5 - \cos(x)^3)*\log(-(4*\sqrt{2})*(8*\cos(x)^5 - 6*\cos(x)^3 + \\ & \cos(x))*\sqrt{-(\cos(x)^2 - 1)/(2*\cos(x)^2 - 1)} - (32*\cos(x)^4 - 16*\cos(x)^2 \\ & + 1)*\sin(x))/\sin(x))*\sin(x) - 33*(\sqrt{2}*\cos(x)^5 - \sqrt{2}*\cos(x)^3)*\log \\ & (4*(\sqrt{2})*(2*(3*\sqrt{2} - 4)*\cos(x)^3 - (3*\sqrt{2} - 4)*\cos(x))*\sqrt{-(\cos(x)^2 - 1)/(2*\cos(x)^2 - 1)} \\ & + (3*(2*\sqrt{2} - 3)*\cos(x)^2 - 2*\sqrt{2} + 3)*\sin(x))/((\cos(x)^2 - 1)*\sin(x))*\sin(x) - 2*\sqrt{2}*(22*\cos(x)^6 - 47*\cos \\ & (x)^4 + 26*\cos(x)^2 - 4)*\sqrt{-(\cos(x)^2 - 1)/(2*\cos(x)^2 - 1)} - 44*(\cos(x)^5 - \cos(x)^3)*\sin(x))/((\cos(x)^5 - \cos(x)^3)*\sin(x)) \end{aligned}$$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-cos(2*x)+2*tan(x)**2)/cos(x)**2/(tan(x)*tan(2*x))**(3/2),x)`

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(78) = 156.

time = 0.05, size = 230, normalized size = 2.30

$$\frac{-\frac{16}{3}\sqrt{-\tan^2 x + 1}(-\tan^2 x + 1) - 8\sqrt{-\tan^2 x + 1}}{16\sqrt{2} \operatorname{sign}(\tan x) \operatorname{sign}(\tan^2 x - 1)} + \frac{\ln\left(\frac{-2\sqrt{-\tan^2 x + 1} + 2\sqrt{2}}{2\sqrt{-\tan^2 x + 1} + 2\sqrt{2}}\right)}{\operatorname{sign}(\tan x) \operatorname{sign}(\tan^2 x - 1)} + \frac{11 \ln(\sqrt{-\tan^2 x + 1} + 1)}{8\sqrt{2} \operatorname{sign}(\tan x) \operatorname{sign}(\tan^2 x - 1)} - \frac{11 \ln(-\sqrt{-\tan^2 x + 1} + 1)}{8\sqrt{2} \operatorname{sign}(\tan x) \operatorname{sign}(\tan^2 x - 1)} - \frac{\sqrt{2} \sqrt{-\tan^2 x + 1}}{8 \tan^2 x \cdot \operatorname{sign}(\tan x) \operatorname{sign}(\tan^2 x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x)`

[Out]
$$-1/12*\sqrt{2}*(2*(-\tan(x)^2 + 1)^{(3/2)} + 3*\sqrt{-\tan(x)^2 + 1})/(\operatorname{sgn}(\tan(x)^2 - 1)*\operatorname{sgn}(\tan(x))) + 11/16*\sqrt{2}*\log(\sqrt{-\tan(x)^2 + 1} + 1)/(\operatorname{sgn}(\tan(x)^2 - 1)*\operatorname{sgn}(\tan(x))) - 11/16*\sqrt{2}*\log(-\sqrt{-\tan(x)^2 + 1} + 1)/(\operatorname{sgn}(\tan(x)^2 - 1)*\operatorname{sgn}(\tan(x))) + \log((\sqrt{2} - \sqrt{-\tan(x)^2 + 1})/(\sqrt{2} + \sqrt{-\tan(x)^2 + 1}))/(\operatorname{sgn}(\tan(x)^2 - 1)*\operatorname{sgn}(\tan(x))) - 1/8*\sqrt{2}*\sqrt{-\tan(x)^2 + 1}/(\operatorname{sgn}(\tan(x)^2 - 1)*\operatorname{sgn}(\tan(x))*\tan(x)^2)$$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\cos(2x) - 2 \tan(x)^2}{\cos(x)^2 (\tan(2x) \tan(x))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-cos(2*x) - 2*tan(x)^2)/(cos(x)^2*(tan(2*x)*tan(x))^(3/2)),x)`

[Out] `-int((cos(2*x) - 2*tan(x)^2)/(cos(x)^2*(tan(2*x)*tan(x))^(3/2)), x)`

$$3.448 \quad \int \frac{\tan(x)}{(a^3 - b^3 \cos^n(x))^{4/3}} dx$$

Optimal. Leaf size=112

$$\frac{\sqrt{3} \tan^{-1} \left(\frac{a + 2\sqrt[3]{a^3 - b^3 \cos^n(x)}}{\sqrt{3} a} \right)}{a^{4n}} - \frac{3}{a^3 n \sqrt[3]{a^3 - b^3 \cos^n(x)}} + \frac{\log(\cos(x))}{2a^4} - \frac{3 \log \left(a - \sqrt[3]{a^3 - b^3 \cos^n(x)} \right)}{2a^{4n}}$$

[Out] $-3/a^3/n/(a^3-b^3*\cos(x)^n)^{(1/3)}+1/2*\ln(\cos(x))/a^4-3/2*\ln(a-(a^3-b^3*\cos(x)^n)^{(1/3))}/a^4/n-\arctan(1/3*(a+2*(a^3-b^3*\cos(x)^n)^{(1/3))}/a^3^{(1/2)})*3^{(1/2)}/a^4/n$

Rubi [A]

time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3309, 272, 53, 57, 631, 210, 31}

$$\frac{\log(\cos(x))}{2a^4} - \frac{3}{a^3 n \sqrt[3]{a^3 - b^3 \cos^n(x)}} - \frac{3 \log \left(a - \sqrt[3]{a^3 - b^3 \cos^n(x)} \right)}{2a^{4n}} - \frac{\sqrt{3} \tan^{-1} \left(\frac{2\sqrt[3]{a^3 - b^3 \cos^n(x)} + a}{\sqrt{3} a} \right)}{a^{4n}}$$

Antiderivative was successfully verified.

[In] Int[Tan[x]/(a^3 - b^3*Cos[x]^n)^(4/3), x]

[Out] $-\left(\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{a + 2\sqrt[3]{a^3 - b^3 \cos^n(x)}}{\sqrt{3} a}\right]}{a^{4n}}\right) - \frac{3}{a^3 n \sqrt[3]{a^3 - b^3 \cos^n(x)}} + \frac{\log[\cos(x)]}{2a^4} - \frac{3 \log[a - \sqrt[3]{a^3 - b^3 \cos^n(x)}]}{2a^{4n}}$

Rule 31

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 53

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 57

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x]

] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]) /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 210

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])

Rule 272

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 631

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3309

Int[((a_) + (b_)*((c_)*sin[(e_) + (f_)*(x_)])^(n_))^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1)/2)], x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m - 1] && LtQ[(m - 1)/2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\tan(x)}{(a^3 - b^3 \cos^n(x))^{4/3}} dx &= -\text{Subst} \left(\int \frac{1}{x (a^3 - b^3 x^n)^{4/3}} dx, x, \cos(x) \right) \\
&= -\frac{\text{Subst} \left(\int \frac{1}{x (a^3 - b^3 x^n)^{4/3}} dx, x, \cos^n(x) \right)}{n} \\
&= -\frac{3}{a^3 n \sqrt[3]{a^3 - b^3 \cos^n(x)}} - \frac{\text{Subst} \left(\int \frac{1}{x \sqrt[3]{a^3 - b^3 x}} dx, x, \cos^n(x) \right)}{a^3 n} \\
&= -\frac{3}{a^3 n \sqrt[3]{a^3 - b^3 \cos^n(x)}} + \frac{\log(\cos(x))}{2a^4} + \frac{3 \text{Subst} \left(\int \frac{1}{a-x} dx, x, \sqrt[3]{a^3 - b^3 \cos^n(x)} \right)}{2a^4 n} \\
&= -\frac{3}{a^3 n \sqrt[3]{a^3 - b^3 \cos^n(x)}} + \frac{\log(\cos(x))}{2a^4} - \frac{3 \log \left(a - \sqrt[3]{a^3 - b^3 \cos^n(x)} \right)}{2a^4 n} + \frac{3 \text{Subst} \left(\int \frac{1}{a-x} dx, x, \sqrt[3]{a^3 - b^3 \cos^n(x)} \right)}{2a^4 n} \\
&= -\frac{\sqrt{3} \tan^{-1} \left(\frac{1 + \frac{2 \sqrt[3]{a^3 - b^3 \cos^n(x)}}{\sqrt{3}}}{\sqrt{3}} \right)}{a^4 n} - \frac{3}{a^3 n \sqrt[3]{a^3 - b^3 \cos^n(x)}} + \frac{\log(\cos(x))}{2a^4}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.03, size = 47, normalized size = 0.42

$$-\frac{{}_3F_1 \left(-\frac{1}{3}, 1, \frac{2}{3}; 1 - \frac{b^3 \cos^n(x)}{a^3} \right)}{a^3 n \sqrt[3]{a^3 - b^3 \cos^n(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[x]/(a^3 - b^3*Cos[x]^n)^(4/3),x]

[Out] (-3*Hypergeometric2F1[-1/3, 1, 2/3, 1 - (b^3*Cos[x]^n)/a^3])/(a^3*n*(a^3 - b^3*Cos[x]^n)^(1/3))

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Tan[x]/(a^3 - b^3*Cos[x]^n)^(4/3),x]')

[Out] Timed out

Maple [A]

time = 0.31, size = 127, normalized size = 1.13

method	result
derivativedivides	$\frac{\ln\left(\frac{a^2+a(a^3-b^3(\cos^n(x)))^{\frac{1}{3}}+(a^3-b^3(\cos^n(x)))^{\frac{2}{3}}}{2}\right)+\sqrt{3}\arctan\left(\frac{(a+2(a^3-b^3(\cos^n(x)))^{\frac{1}{3}})\sqrt{3}}{3a}\right)}{a^4} + \frac{\ln(a-(a^3-b^3(\cos^n(x)))^{\frac{1}{3}})}{a^4}$
default	$\frac{\ln\left(\frac{a^2+a(a^3-b^3(\cos^n(x)))^{\frac{1}{3}}+(a^3-b^3(\cos^n(x)))^{\frac{2}{3}}}{2}\right)+\sqrt{3}\arctan\left(\frac{(a+2(a^3-b^3(\cos^n(x)))^{\frac{1}{3}})\sqrt{3}}{3a}\right)}{a^4} + \frac{\ln(a-(a^3-b^3(\cos^n(x)))^{\frac{1}{3}})}{a^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x,method=_RETURNVERBOSE)`

[Out] $-1/n*(1/a^4*(-1/2*\ln(a^2+a*(a^3-b^3*\cos(x)^n)^{1/3}+(a^3-b^3*\cos(x)^n)^{2/3}))+3^{1/2}*arctan(1/3*(a+2*(a^3-b^3*\cos(x)^n)^{1/3})/a*3^{1/2}))+1/a^4*\ln(a-(a^3-b^3*\cos(x)^n)^{1/3}))+3/a^3/(a^3-b^3*\cos(x)^n)^{1/3})$

Maxima [A]

time = 0.33, size = 136, normalized size = 1.21

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2(-b^3\cos(x)^n+a^3)^{\frac{1}{3}})}{3a}\right)}{a^4n} + \frac{\log(a^2+(-b^3\cos(x)^n+a^3)^{\frac{1}{3}}a+(-b^3\cos(x)^n+a^3)^{\frac{2}{3}})}{2a^4n} - \frac{\log(-a+(-b^3\cos(x)^n+a^3)^{\frac{1}{3}})}{a^4n} - \frac{3}{(-b^3\cos(x)^n+a^3)^{\frac{1}{3}}a^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x, algorithm="maxima")`

[Out] $-\sqrt{3}*arctan(1/3*\sqrt{3}*(a+2*(-b^3*\cos(x)^n+a^3)^{1/3})/a)/(a^4*n) + 1/2*\log(a^2+(-b^3*\cos(x)^n+a^3)^{1/3}*a+(-b^3*\cos(x)^n+a^3)^{2/3})/(a^4*n) - \log(-a+(-b^3*\cos(x)^n+a^3)^{1/3})/(a^4*n) - 3/((-b^3*\cos(x)^n+a^3)^{1/3}*a^3*n)$

Fricas [A]

time = 0.34, size = 185, normalized size = 1.65

$$\frac{2(\sqrt{3}b^3\cos(x)^n-\sqrt{3}a^3)\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\frac{(-b^3\cos(x)^n+a^3)^{\frac{1}{3}}}{3a}}{3a}\right)-(b^3\cos(x)^n-a^3)\log(a^2+(-b^3\cos(x)^n+a^3)^{\frac{1}{3}}a+(-b^3\cos(x)^n+a^3)^{\frac{2}{3}})+2(b^3\cos(x)^n-a^3)\log(-a+(-b^3\cos(x)^n+a^3)^{\frac{1}{3}})-6(-b^3\cos(x)^n+a^3)^{\frac{1}{3}}a}{2(a^4b^3n\cos(x)^n-a^4n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x, algorithm="fricas")`

[Out]
$$-1/2*(2*(\sqrt{3})*b^3*\cos(x)^n - \sqrt{3}*a^3)*\arctan(1/3*(\sqrt{3})*a + 2*\sqrt{3})*(-b^3*\cos(x)^n + a^3)^{(1/3)}/a - (b^3*\cos(x)^n - a^3)*\log(a^2 + (-b^3*\cos(x)^n + a^3)^{(1/3)}*a + (-b^3*\cos(x)^n + a^3)^{(2/3)}) + 2*(b^3*\cos(x)^n - a^3)*\log(-a + (-b^3*\cos(x)^n + a^3)^{(1/3)}) - 6*(-b^3*\cos(x)^n + a^3)^{(2/3)}*a)/(a^4*b^3*n*\cos(x)^n - a^7*n)$$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{(a^3 - b^3 \cos^n(x))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)/(a**3-b**3*cos(x)**n)**(4/3), x)

[Out] Integral(tan(x)/(a**3 - b**3*cos(x)**n)**(4/3), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)/(a^3-b^3*cos(x)^n)^(4/3), x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x)}{(a^3 - b^3 \cos(x)^n)^{4/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)/(a^3 - b^3*cos(x)^n)^(4/3), x)

[Out] int(tan(x)/(a^3 - b^3*cos(x)^n)^(4/3), x)

3.449 $\int (1 + 2 \cos^9(x))^{5/6} \tan(x) dx$

Optimal. Leaf size=95

$$\frac{\tan^{-1}\left(\frac{1-\sqrt[3]{1+2\cos^9(x)}}{\sqrt{3}\sqrt[6]{1+2\cos^9(x)}}\right)}{3\sqrt{3}} + \frac{1}{3} \tanh^{-1}\left(\sqrt[6]{1+2\cos^9(x)}\right) - \frac{1}{9} \tanh^{-1}\left(\sqrt{1+2\cos^9(x)}\right) - \frac{2}{15} (1+2\cos^9(x))^{5/6}$$

[Out] 1/3*arctanh((1+2*cos(x)^9)^(1/6))-1/9*arctanh((1+2*cos(x)^9)^(1/2))-2/15*(1+2*cos(x)^9)^(5/6)+1/9*arctan(1/3*(1-(1+2*cos(x)^9)^(1/3))/(1+2*cos(x)^9)^(1/6)*3^(1/2))*3^(1/2)

Rubi [A]

time = 0.17, antiderivative size = 162, normalized size of antiderivative = 1.71, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3309, 272, 52, 65, 302, 648, 632, 210, 642, 212}

$$-\frac{2}{15}(2\cos^9(x)+1)^{5/6} - \frac{1}{18} \log\left(\sqrt[3]{2\cos^9(x)+1} - \sqrt[6]{2\cos^9(x)+1} + 1\right) + \frac{1}{18} \log\left(\sqrt[3]{2\cos^9(x)+1} + \sqrt[6]{2\cos^9(x)+1} + 1\right) + \frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{2\cos^9(x)+1}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{2\cos^9(x)+1}+1}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{2}{9} \tanh^{-1}\left(\sqrt[6]{2\cos^9(x)+1}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*Cos[x]^9)^(5/6)*Tan[x], x]

[Out] ArcTan[(1 - 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]]/(3*Sqrt[3]) - ArcTan[(1 + 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]]/(3*Sqrt[3]) + (2*ArcTanh[(1 + 2*Cos[x]^9)^(1/6)]/9 - (2*(1 + 2*Cos[x]^9)^(5/6))/15 - Log[1 - (1 + 2*Cos[x]^9)^(1/6) + (1 + 2*Cos[x]^9)^(1/3)]/18 + Log[1 + (1 + 2*Cos[x]^9)^(1/6) + (1 + 2*Cos[x]^9)^(1/3)]/18

Rule 52

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
+ 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
n - 1] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```


`[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 3309

`Int[((a_) + (b_)*((c_)*sin[(e_) + (f_)*(x_)])^(n_))^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^(m + 1)/2], x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && LtQ[(m - 1)/2, 0]`

Rubi steps

$$\begin{aligned}
 \int (1 + 2 \cos^9(x))^{5/6} \tan(x) dx &= -\text{Subst} \left(\int \frac{(1 + 2x^9)^{5/6}}{x} dx, x, \cos(x) \right) \\
 &= - \left(\frac{1}{9} \text{Subst} \left(\int \frac{(1 + 2x)^{5/6}}{x} dx, x, \cos^9(x) \right) \right) \\
 &= -\frac{2}{15} (1 + 2 \cos^9(x))^{5/6} - \frac{1}{9} \text{Subst} \left(\int \frac{1}{x \sqrt[6]{1 + 2x}} dx, x, \cos^9(x) \right) \\
 &= -\frac{2}{15} (1 + 2 \cos^9(x))^{5/6} - \frac{1}{3} \text{Subst} \left(\int \frac{x^4}{-\frac{1}{2} + \frac{x^6}{2}} dx, x, \sqrt[6]{1 + 2 \cos^9(x)} \right) \\
 &= -\frac{2}{15} (1 + 2 \cos^9(x))^{5/6} + \frac{2}{9} \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \sqrt[6]{1 + 2 \cos^9(x)} \right) + \frac{2}{9} \\
 &= \frac{2}{9} \tanh^{-1} \left(\sqrt[6]{1 + 2 \cos^9(x)} \right) - \frac{2}{15} (1 + 2 \cos^9(x))^{5/6} - \frac{1}{18} \text{Subst} \left(\int \frac{-1}{1 -} \right) \\
 &= \frac{2}{9} \tanh^{-1} \left(\sqrt[6]{1 + 2 \cos^9(x)} \right) - \frac{2}{15} (1 + 2 \cos^9(x))^{5/6} - \frac{1}{18} \log \left(1 - \sqrt[6]{1 +} \right) \\
 &= \frac{\tan^{-1} \left(\frac{1 - 2 \sqrt[6]{1 + 2 \cos^9(x)}}{\sqrt{3}} \right)}{3\sqrt{3}} - \frac{\tan^{-1} \left(\frac{1 + 2 \sqrt[6]{1 + 2 \cos^9(x)}}{\sqrt{3}} \right)}{3\sqrt{3}} + \frac{2}{9} \tanh^{-1}
 \end{aligned}$$

Mathematica [A]

time = 0.09, size = 154, normalized size = 1.62

$$\frac{1}{90} \left(10\sqrt{3} \tan^{-1} \left(\frac{1 - 2\sqrt[6]{1 + 2\cos^9(x)}}{\sqrt{3}} \right) - 10\sqrt{3} \tan^{-1} \left(\frac{1 + 2\sqrt[6]{1 + 2\cos^9(x)}}{\sqrt{3}} \right) + 20 \tanh^{-1} \left(\sqrt[6]{1 + 2\cos^9(x)} \right) - 12(1 + 2\cos^9(x))^{5/6} - 5 \log \left(1 - \sqrt[6]{1 + 2\cos^9(x)} + \sqrt[6]{1 + 2\cos^9(x)} \right) + 5 \log \left(1 + \sqrt[6]{1 + 2\cos^9(x)} + \sqrt[6]{1 + 2\cos^9(x)} \right) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + 2*Cos[x]^9)^(5/6)*Tan[x], x]`

`[Out] (10*Sqrt[3]*ArcTan[(1 - 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]] - 10*Sqrt[3]*ArcTan[(1 + 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]] + 20*ArcTanh[(1 + 2*Cos[x]^9)^(1/6)] - 12*(1 + 2*Cos[x]^9)^(5/6) - 5*Log[1 - Sqrt[6]{1 + 2*Cos[x]^9} + Sqrt[6]{1 + 2*Cos[x]^9}] + 5*Log[1 + Sqrt[6]{1 + 2*Cos[x]^9} + Sqrt[6]{1 + 2*Cos[x]^9}])`

1/6]] - 12*(1 + 2*cos[x]^9)^(5/6) - 5*Log[1 - (1 + 2*cos[x]^9)^(1/6) + (1 + 2*cos[x]^9)^(1/3)] + 5*Log[1 + (1 + 2*cos[x]^9)^(1/6) + (1 + 2*cos[x]^9)^(1/3)]/90

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(1 + 2*cos[x]^9)^(5/6)*Tan[x], x]')

[Out] Timed out

Maple [F]

time = 0.12, size = 0, normalized size = 0.00

$$\int (1 + 2(\cos^9(x)))^{\frac{5}{6}} \tan(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2*cos(x)^9)^(5/6)*tan(x), x)

[Out] int((1+2*cos(x)^9)^(5/6)*tan(x), x)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(72) = 144.

time = 0.34, size = 145, normalized size = 1.53

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2(2\cos(x)^9+1)^{\frac{1}{6}}+1)\right) - \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2(2\cos(x)^9+1)^{\frac{1}{6}}-1)\right) - \frac{2}{15}(2\cos(x)^9+1)^{\frac{1}{6}} + \frac{1}{18}\log((2\cos(x)^9+1)^{\frac{1}{3}}+(2\cos(x)^9+1)^{\frac{1}{6}}+1) - \frac{1}{18}\log((2\cos(x)^9+1)^{\frac{1}{3}}-(2\cos(x)^9+1)^{\frac{1}{6}}+1) + \frac{1}{9}\log((2\cos(x)^9+1)^{\frac{1}{6}}+1) - \frac{1}{9}\log((2\cos(x)^9+1)^{\frac{1}{6}}-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*cos(x)^9)^(5/6)*tan(x), x, algorithm="maxima")

[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) + 1)) - 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) - 1)) - 2/15*(2*cos(x)^9 + 1)^(5/6) + 1/18*log((2*cos(x)^9 + 1)^(1/3) + (2*cos(x)^9 + 1)^(1/6) + 1) - 1/18*log((2*cos(x)^9 + 1)^(1/3) - (2*cos(x)^9 + 1)^(1/6) + 1) + 1/9*log((2*cos(x)^9 + 1)^(1/6) + 1) - 1/9*log((2*cos(x)^9 + 1)^(1/6) - 1)

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*cos(x)^9)^(5/6)*tan(x),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*cos(x)**9)**(5/6)*tan(x),x)

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(72) = 144.

time = 0.02, size = 189, normalized size = 1.99

$$-\frac{2}{3} \left(\frac{\ln|(2\cos^9 x + 1)^{\frac{1}{3}} - 1|}{6} - \frac{\ln|(2\cos^9 x + 1)^{\frac{1}{3}} + 1|}{6} + \frac{\ln|(2\cos^9 x + 1)^{\frac{1}{3}} - (2\cos^9 x + 1)^{\frac{1}{6}} + 1|}{12} + \frac{\arctan\left(\frac{2(2\cos^9 x + 1)^{\frac{1}{3}} - 1}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\ln|(2\cos^9 x + 1)^{\frac{1}{3}} + (2\cos^9 x + 1)^{\frac{1}{6}} + 1|}{12} + \frac{\arctan\left(\frac{2(2\cos^9 x + 1)^{\frac{1}{3}} + 1}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{(2\cos^9 x + 1)^{\frac{5}{6}}}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*cos(x)^9)^(5/6)*tan(x),x)

[Out] $-1/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*(2*\cos(x)^9 + 1)^{(1/6)} + 1)) - 1/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*(2*\cos(x)^9 + 1)^{(1/6)} - 1)) - 2/15*(2*\cos(x)^9 + 1)^{(5/6)} + 1/18*\log((2*\cos(x)^9 + 1)^{(1/3)} + (2*\cos(x)^9 + 1)^{(1/6)} + 1) - 1/18*\log((2*\cos(x)^9 + 1)^{(1/3)} - (2*\cos(x)^9 + 1)^{(1/6)} + 1) + 1/9*\log((2*\cos(x)^9 + 1)^{(1/6)} + 1) - 1/9*\log(\text{abs}((2*\cos(x)^9 + 1)^{(1/6)} - 1))$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(x) (2 \cos(x)^9 + 1)^{5/6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)*(2*cos(x)^9 + 1)^(5/6),x)

[Out] int(tan(x)*(2*cos(x)^9 + 1)^(5/6), x)

$$3.450 \quad \int \frac{\cos(x) \sin^8(x)}{(2-5 \sin^3(x))^{4/3}} dx$$

Optimal. Leaf size=49

$$\frac{4}{125 \sqrt[3]{2-5 \sin^3(x)}} + \frac{2}{125} (2-5 \sin^3(x))^{2/3} - \frac{1}{625} (2-5 \sin^3(x))^{5/3}$$

[Out] 4/125/(2-5*sin(x)^3)^(1/3)+2/125*(2-5*sin(x)^3)^(2/3)-1/625*(2-5*sin(x)^3)^(5/3)

Rubi [A]

time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4419, 272, 45}

$$-\frac{1}{625} (2-5 \sin^3(x))^{5/3} + \frac{2}{125} (2-5 \sin^3(x))^{2/3} + \frac{4}{125 \sqrt[3]{2-5 \sin^3(x)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]*Sin[x]^8)/(2 - 5*Sin[x]^3)^(4/3),x]

[Out] 4/(125*(2 - 5*Sin[x]^3)^(1/3)) + (2*(2 - 5*Sin[x]^3)^(2/3))/125 - (2 - 5*Sin[x]^3)^(5/3)/625

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4419

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x) \sin^8(x)}{(2 - 5 \sin^3(x))^{4/3}} dx &= \text{Subst} \left(\int \frac{x^8}{(2 - 5x^3)^{4/3}} dx, x, \sin(x) \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{(2 - 5x)^{4/3}} dx, x, \sin^3(x) \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{4}{25(2 - 5x)^{4/3}} - \frac{4}{25\sqrt[3]{2 - 5x}} + \frac{1}{25}(2 - 5x)^{2/3} \right) dx, x, \sin^3(x) \right) \\
&= \frac{4}{125\sqrt[3]{2 - 5\sin^3(x)}} + \frac{2}{125} (2 - 5\sin^3(x))^{2/3} - \frac{1}{625} (2 - 5\sin^3(x))^{5/3}
\end{aligned}$$

Mathematica [A]

time = 0.32, size = 30, normalized size = 0.61

$$\frac{36 - 30 \sin^3(x) - 25 \sin^6(x)}{625 \sqrt[3]{2 - 5 \sin^3(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]*Sin[x]^8)/(2 - 5*Sin[x]^3)^(4/3), x]

[Out] (36 - 30*Sin[x]^3 - 25*Sin[x]^6)/(625*(2 - 5*Sin[x]^3)^(1/3))

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sin[x]^9*Cot[x]/(2 - 5*Sin[x]^3)^(4/3), x]')

[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep

Maple [F]

time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{\cot(x) (\sin^9(x))}{(2 - 5 (\sin^3(x)))^{4/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3), x)

[Out] `int(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x)`

Maxima [A]

time = 0.24, size = 37, normalized size = 0.76

$$-\frac{1}{625}(-5 \sin(x)^3 + 2)^{\frac{5}{3}} + \frac{2}{125}(-5 \sin(x)^3 + 2)^{\frac{2}{3}} + \frac{4}{125(-5 \sin(x)^3 + 2)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x, algorithm="maxima")`

[Out] `-1/625*(-5*sin(x)^3 + 2)^(5/3) + 2/125*(-5*sin(x)^3 + 2)^(2/3) + 4/125/(-5*sin(x)^3 + 2)^(1/3)`

Fricas [A]

time = 0.37, size = 46, normalized size = 0.94

$$\frac{25 \cos(x)^6 - 75 \cos(x)^4 + 75 \cos(x)^2 + 30(\cos(x)^2 - 1) \sin(x) + 11}{625(5(\cos(x)^2 - 1) \sin(x) + 2)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x, algorithm="fricas")`

[Out] `1/625*(25*cos(x)^6 - 75*cos(x)^4 + 75*cos(x)^2 + 30*(cos(x)^2 - 1)*sin(x) + 11)/(5*(cos(x)^2 - 1)*sin(x) + 2)^(1/3)`

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)*sin(x)**9/(2-5*sin(x)**3)**(4/3),x)`

[Out] Exception raised: SystemError

Giac [A]

time = 0.00, size = 67, normalized size = 1.37

$$\frac{-48828125 \left((-5 \sin^3 x + 2)^{\frac{1}{3}} \right)^2 (-5 \sin^3 x + 2) + 488281250 \left((-5 \sin^3 x + 2)^{\frac{1}{3}} \right)^2}{30517578125} + \frac{4}{125(-5 \sin^3 x + 2)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x)`

[Out] $-1/625*(-5*\sin(x)^3 + 2)^{(5/3)} + 2/125*(-5*\sin(x)^3 + 2)^{(2/3)} + 4/125/(-5*\sin(x)^3 + 2)^{(1/3)}$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot(x) \sin(x)^9}{(2 - 5 \sin(x)^3)^{4/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cot(x)*sin(x)^9)/(2 - 5*sin(x)^3)^(4/3), x)`

[Out] `int((cot(x)*sin(x)^9)/(2 - 5*sin(x)^3)^(4/3), x)`

$$3.451 \quad \int \frac{\sec^2(x) \tan(x) \left(1 + \sqrt[3]{1 - 8 \tan^2(x)}\right)}{(1 - 8 \tan^2(x))^{2/3}} dx$$

Optimal. Leaf size=20

$$-\frac{3}{32} \left(1 + \sqrt[3]{1 - 8 \tan^2(x)}\right)^2$$

[Out] -3/32*(1+(1-8*tan(x)^2)^(1/3))^2

Rubi [A]

time = 0.13, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {4427, 6818}

$$-\frac{3}{32} \left(\sqrt[3]{1 - 8 \tan^2(x)} + 1\right)^2$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2*Tan[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/3), x]

[Out] (-3*(1 + (1 - 8*Tan[x]^2)^(1/3))^2)/32

Rule 4427

Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))]^2, x_Symbol] := With[{d = FreeFactors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a + b*x)]]/d, u, x], x], x, Tan[c*(a + b*x)]/d, x] /; FunctionOfQ[Tan[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] || EqQ[F, sec])

Rule 6818

Int[(u_)*(y_)^(m_), x_Symbol] := With[{q = DerivativeDivides[y, u, x]}, Simp[q*(y^(m + 1)/(m + 1)), x] /; !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x) \tan(x) \left(1 + \sqrt[3]{1 - 8 \tan^2(x)}\right)}{(1 - 8 \tan^2(x))^{2/3}} dx &= \text{Subst} \left(\int \frac{x \left(1 + \sqrt[3]{1 - 8x^2}\right)}{(1 - 8x^2)^{2/3}} dx, x, \tan(x) \right) \\ &= -\frac{3}{32} \left(1 + \sqrt[3]{1 - 8 \tan^2(x)}\right)^2 \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.

time = 0.14, size = 42, normalized size = 2.10

$$\frac{3(-7 + 9 \cos(2x)) \sec^2(x) \left(2 + \sqrt[3]{1 - 8 \tan^2(x)}\right)}{64 (1 - 8 \tan^2(x))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2*Tan[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/3),x]

[Out] (-3*(-7 + 9*Cos[2*x])*Sec[x]^2*(2 + (1 - 8*Tan[x]^2)^(1/3)))/(64*(1 - 8*Tan[x]^2)^(2/3))

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[((1 + (1 - 8*Tan[x]^2)^(1/3)))/(Cos[x]^2*(1 - 8*Tan[x]^2)^(2/3))]*Tan[x],x]')

[Out] Timed out

Maple [A]

time = 0.05, size = 26, normalized size = 1.30

method	result	size
derivativedivides	$-\frac{3(1-8(\tan^2(x)))^{\frac{1}{3}}}{16} - \frac{3(1-8(\tan^2(x)))^{\frac{2}{3}}}{32}$	26
default	$-\frac{3(1-8(\tan^2(x)))^{\frac{1}{3}}}{16} - \frac{3(1-8(\tan^2(x)))^{\frac{2}{3}}}{32}$	26

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,method=_RETURNVERBOSE)

[Out] -3/16*(1-8*tan(x)^2)^(1/3)-3/32*(1-8*tan(x)^2)^(2/3)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(16) = 32.

time = 0.29, size = 86, normalized size = 4.30

$$\frac{3 \left(\frac{(9 \sin(x)^2 - 1)(3 \sin(x) - 1)^{\frac{1}{3}} (\sin(x) + 1)^{\frac{1}{3}} (\sin(x) - 1)^{\frac{1}{3}}}{(3 \sin(x) + 1)^{\frac{1}{3}}} + \frac{2(9 \sin(x)^2 - 1)(\sin(x) + 1)^{\frac{2}{3}} (\sin(x) - 1)^{\frac{2}{3}}}{(3 \sin(x) + 1)^{\frac{2}{3}}} \right)}{32 (\sin(x)^2 - 1)(3 \sin(x) - 1)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="maxima")`

[Out]
$$-3/32*((9*\sin(x)^2 - 1)*(3*\sin(x) - 1)^{(1/3)}*(\sin(x) + 1)^{(1/3)}*(\sin(x) - 1)^{(1/3)})/(3*\sin(x) + 1)^{(1/3)} + 2*(9*\sin(x)^2 - 1)*(\sin(x) + 1)^{(2/3)}*(\sin(x) - 1)^{(2/3)}/(3*\sin(x) + 1)^{(2/3)})/((\sin(x)^2 - 1)*(3*\sin(x) - 1)^{(2/3)})$$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.

time = 0.32, size = 35, normalized size = 1.75

$$-\frac{3}{32} \left(\frac{9 \cos(x)^2 - 8}{\cos(x)^2} \right)^{\frac{2}{3}} - \frac{3}{16} \left(\frac{9 \cos(x)^2 - 8}{\cos(x)^2} \right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="fricas")`

[Out]
$$-3/32*((9*\cos(x)^2 - 8)/\cos(x)^2)^{(2/3)} - 3/16*((9*\cos(x)^2 - 8)/\cos(x)^2)^{(1/3)}$$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt[3]{1 - 8 \tan^2(x)} + 1 \right) \tan(x)}{(1 - 8 \tan^2(x))^{\frac{2}{3}} \cos^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)*(1+(1-8*tan(x)**2)**(1/3))/cos(x)**2/(1-8*tan(x)**2)**(2/3),x)`

[Out] `Integral(((1 - 8*tan(x)**2)**(1/3) + 1)*tan(x)/((1 - 8*tan(x)**2)**(2/3)*cos(x)**2), x)`

Giac [A]

time = 0.03, size = 40, normalized size = 2.00

$$\frac{-\frac{3}{2} \left((-8 \tan^2 x + 1)^{\frac{1}{3}} \right)^2 - 3(-8 \tan^2 x + 1)^{\frac{1}{3}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x)`

[Out] $-3/32*(-8*\tan(x)^2 + 1)^{(2/3)} - 3/16*(-8*\tan(x)^2 + 1)^{(1/3)}$

Mupad [B]

time = 0.62, size = 43, normalized size = 2.15

$$-\frac{3 \left((18 \cos(x)^2 - 16)^{1/3} + 2 (2 \cos(x)^2)^{1/3} \right) (18 \cos(x)^2 - 16)^{1/3}}{32 (2 \cos(x)^2)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((tan(x)*((1 - 8*tan(x)^2)^(1/3) + 1))/(cos(x)^2*(1 - 8*tan(x)^2)^(2/3)),x)`

[Out] $-(3*((18*\cos(x)^2 - 16)^{(1/3)} + 2*(2*\cos(x)^2)^{(1/3}))* (18*\cos(x)^2 - 16)^{(1/3}))/ (32*(2*\cos(x)^2)^{(2/3}))$

$$3.452 \quad \int \frac{\csc(x) \sec(x) \left(1 + \sqrt[3]{1 - 8 \tan^2(x)}\right)}{(1 - 8 \tan^2(x))^{2/3}} dx$$

Optimal. Leaf size=27

$$-\log(\tan(x)) + \frac{3}{2} \log\left(1 - \sqrt[3]{1 - 8 \tan^2(x)}\right)$$

[Out] $-\ln(\tan(x)) + 3/2 * \ln(1 - (1 - 8 * \tan(x)^2)^{(1/3)})$

Rubi [A]

time = 0.58, antiderivative size = 35, normalized size of antiderivative = 1.30, number of steps used = 15, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {4451, 6857, 528, 455, 59, 632, 210, 31, 57}

$$\frac{3}{2} \log\left(1 - \sqrt[3]{9 - 8 \sec^2(x)}\right) - \frac{1}{2} \log(1 - \sec^2(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Csc}[x] * \text{Sec}[x] * (1 + (1 - 8 * \text{Tan}[x]^2)^{(1/3)})) / (1 - 8 * \text{Tan}[x]^2)^{(2/3)}, x]$

[Out] $-1/2 * \text{Log}[1 - \text{Sec}[x]^2] + (3 * \text{Log}[1 - (9 - 8 * \text{Sec}[x]^2)^{(1/3)}]) / 2$

Rule 31

$\text{Int}[(a + (b * x)^{-1}), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b * x, x]] / b, x] / ; \text{FreeQ}\{a, b, x\}$

Rule 57

$\text{Int}[1 / (((a + (b * x)) * ((c + (d * x))^{(1/3)})), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(b * c - a * d) / b, 3]\}, \text{Simp}[-\text{Log}[\text{RemoveContent}[a + b * x, x]] / (2 * b * q), x] + (\text{Dist}[3 / (2 * b), \text{Subst}[\text{Int}[1 / (q^2 + q * x + x^2), x], x, (c + d * x)^{(1/3)}], x] - \text{Dist}[3 / (2 * b * q), \text{Subst}[\text{Int}[1 / (q - x), x], x, (c + d * x)^{(1/3)}], x])] / ; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{PosQ}[(b * c - a * d) / b]$

Rule 59

$\text{Int}[1 / (((a + (b * x)) * ((c + (d * x))^{(2/3)})), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(b * c - a * d) / b, 3]\}, \text{Simp}[-\text{Log}[\text{RemoveContent}[a + b * x, x]] / (2 * b * q^2), x] + (-\text{Dist}[3 / (2 * b * q), \text{Subst}[\text{Int}[1 / (q^2 + q * x + x^2), x], x, (c + d * x)^{(1/3)}], x] - \text{Dist}[3 / (2 * b * q^2), \text{Subst}[\text{Int}[1 / (q - x), x], x, (c + d * x)^{(1/3)}], x])] / ; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{PosQ}[(b * c - a * d) / b]$

Rule 210

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rule 528

```
Int[(x_)^(m_)*((c_) + (d_)*(x_)^(mn_))^(q_)*((a_) + (b_)*(x_)^(n_))^(
p_), x_Symbol] := Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !I
ntegerQ[p])
```

Rule 632

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4451

```
Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))]^(n_), x_Symbol] := With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 6857

```
Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(x) \sec(x) \left(1 + \sqrt[3]{1 - 8 \tan^2(x)}\right)}{(1 - 8 \tan^2(x))^{2/3}} dx &= -\text{Subst} \left(\int \frac{1 + \sqrt[3]{9 - \frac{8}{x^2}}}{\left(9 - \frac{8}{x^2}\right)^{2/3} x (1 - x^2)} dx, x, \cos(x) \right) \\
&= -\text{Subst} \left(\int \left(-\frac{1}{\left(9 - \frac{8}{x^2}\right)^{2/3} x (-1 + x^2)} - \frac{1}{\sqrt[3]{9 - \frac{8}{x^2}} x (-1 + x^2)} \right) dx, x, \cos(x) \right) \\
&= \text{Subst} \left(\int \frac{1}{\left(9 - \frac{8}{x^2}\right)^{2/3} x (-1 + x^2)} dx, x, \cos(x) \right) + \text{Subst} \left(\int \frac{1}{\sqrt[3]{9 - \frac{8}{x^2}} x (-1 + x^2)} dx, x, \cos(x) \right) \\
&= \text{Subst} \left(\int \frac{1}{\left(9 - \frac{8}{x^2}\right)^{2/3} \left(1 - \frac{1}{x^2}\right) x^3} dx, x, \cos(x) \right) + \text{Subst} \left(\int \frac{1}{\sqrt[3]{9 - \frac{8}{x^2}} x (-1 + x^2)} dx, x, \cos(x) \right) \\
&= -\left(\frac{1}{2} \text{Subst} \left(\int \frac{1}{(9 - 8x)^{2/3} (1 - x)} dx, x, \sec^2(x) \right)\right) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt[3]{9 - 8 \sec^2(x)}} dx, x, \sec^2(x) \right) \\
&= -\log(\tan(x)) - 2 \left(\frac{3}{4} \text{Subst} \left(\int \frac{1}{1 - x} dx, x, \sqrt[3]{9 - 8 \sec^2(x)} \right)\right) \\
&= \frac{3}{2} \log \left(1 - \sqrt[3]{9 - 8 \sec^2(x)}\right) - \log(\tan(x))
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2512 vs. 2(27) = 54.
time = 45.13, size = 2512, normalized size = 93.04

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Csc[x]*Sec[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/3), x]

[Out] (2*Sqrt[3]*ArcTan[(1 + 2*(1 - 8*Tan[x]^2)^(1/3))/Sqrt[3]] + 2*Log[-1 + (1 - 8*Tan[x]^2)^(1/3)] - Log[1 + (1 - 8*Tan[x]^2)^(1/3) + (1 - 8*Tan[x]^2)^(2/3)])/4 + ((Cos[x]/(1 + Cos[x]))^(2/3)*(-7 + 9*Cos[2*x])^(4/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*(Cos[x]/(1 + Cos[x]))^(2/3))/((Cos[x]/(1 + Cos[x]))^(2/3) + (-7 + 9*Cos[2*x])*Sec[x/2]^4)^(1/3)] - Log[(2*(Cos[x]/(1 + Cos[x]))^(1/3) + 2*(Cos[x]/(1 + Cos[x]))^(2/3)*((-7 + 9*Cos[2*x])*Sec[x/2]^4)^(1/3) + ((-7 + 9*Cos[2*x])*Sec[x/2]^4)^(2/3))/2^(2/3) - (Cos[x]*Sec[x/2]^2)^(1/3)*Tan[x

$$\begin{aligned}
& /2]^2 + 2*\text{Log}[-(\text{Cos}[x]*\text{Sec}[x/2]^2)^{(2/3)} + (1 - 34*\text{Tan}[x/2]^2 + \text{Tan}[x/2]^4)^{(1/3)}] * \text{Sec}[x]^{(5/3)} / (2*((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4)^{(1/3)} * (-23*\text{Sin}[x] \\
& + 9*\text{Sin}[3*x]) * (((\text{Cos}[x]/(1 + \text{Cos}[x]))^{(2/3)} * (-7 + 9*\text{Cos}[2*x])^{(1/3)} * (2*\text{Sqr} \\
& \text{t}[3]*\text{ArcTan}[(\text{Sqrt}[3]*(\text{Cos}[x]/(1 + \text{Cos}[x]))^{(2/3)})/((\text{Cos}[x]/(1 + \text{Cos}[x]))^{(2} \\
& /3) + ((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4)^{(1/3)})) - \text{Log}[(2*(\text{Cos}[x]/(1 + \text{Cos}[x])) \\
& ^{(1/3)} + 2*(\text{Cos}[x]/(1 + \text{Cos}[x]))^{(2/3)} * ((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4)^{(1/3)} \\
& + ((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4)^{(2/3)})/2^{(2/3)} - (\text{Cos}[x] * \text{Sec}[x/2]^2)^{(1/3)} \\
&) * \text{Tan}[x/2]^2 + 2*\text{Log}[-(\text{Cos}[x]*\text{Sec}[x/2]^2)^{(2/3)} + (1 - 34*\text{Tan}[x/2]^2 + \text{Tan} \\
& [x/2]^4)^{(1/3)}] * \text{Sec}[x]^{(5/3)} * \text{Sin}[x] / (6*((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4)^{(1/} \\
& 3)) + ((-7 + 9*\text{Cos}[2*x])^{(1/3)} * (2*\text{Sqrt}[3]*\text{ArcTan}[(\text{Sqrt}[3]*(\text{Cos}[x]/(1 + \text{Cos}[\\
& x]))^{(2/3)})/((\text{Cos}[x]/(1 + \text{Cos}[x]))^{(2/3)} + ((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4)^{(\\
& 1/3)})) - \text{Log}[(2*(\text{Cos}[x]/(1 + \text{Cos}[x]))^{(1/3)} + 2*(\text{Cos}[x]/(1 + \text{Cos}[x]))^{(2/3)} \\
& * ((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4)^{(1/3)} + ((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4)^{(2/3} \\
&))/2^{(2/3)} - (\text{Cos}[x] * \text{Sec}[x/2]^2)^{(1/3)} * \text{Tan}[x/2]^2 + 2*\text{Log}[-(\text{Cos}[x]*\text{Sec}[x/2 \\
&]^2)^{(2/3)} + (1 - 34*\text{Tan}[x/2]^2 + \text{Tan}[x/2]^4)^{(1/3)}] * \text{Sec}[x]^{(2/3)} * ((\text{Cos}[x] \\
& * \text{Sin}[x]) / (1 + \text{Cos}[x])^2 - \text{Sin}[x] / (1 + \text{Cos}[x])) / (6*(\text{Cos}[x]/(1 + \text{Cos}[x]))^{(1 \\
& /3)} * ((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4)^{(1/3)} - (3*(\text{Cos}[x]/(1 + \text{Cos}[x]))^{(2/3)} * \\
& (2*\text{Sqrt}[3]*\text{ArcTan}[(\text{Sqrt}[3]*(\text{Cos}[x]/(1 + \text{Cos}[x]))^{(2/3)})/((\text{Cos}[x]/(1 + \text{Cos}[x] \\
&]))^{(2/3)} + ((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4)^{(1/3)})) - \text{Log}[(2*(\text{Cos}[x]/(1 + \text{Co} \\
& s[x]))^{(1/3)} + 2*(\text{Cos}[x]/(1 + \text{Cos}[x]))^{(2/3)} * ((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4) \\
& ^{(1/3)} + ((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4)^{(2/3)})/2^{(2/3)} - (\text{Cos}[x] * \text{Sec}[x/2]^2 \\
&)^{(1/3)} * \text{Tan}[x/2]^2 + 2*\text{Log}[-(\text{Cos}[x]*\text{Sec}[x/2]^2)^{(2/3)} + (1 - 34*\text{Tan}[x/2]^2 \\
& + \text{Tan}[x/2]^4)^{(1/3)}] * \text{Sec}[x]^{(2/3)} * \text{Sin}[2*x] / (2*(-7 + 9*\text{Cos}[2*x])^{(2/3)} * ((\\
& -7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4)^{(1/3)} - ((\text{Cos}[x]/(1 + \text{Cos}[x]))^{(2/3)} * (-7 + 9* \\
& \text{Cos}[2*x])^{(1/3)} * (2*\text{Sqrt}[3]*\text{ArcTan}[(\text{Sqrt}[3]*(\text{Cos}[x]/(1 + \text{Cos}[x]))^{(2/3)})/((\text{C} \\
& os[x]/(1 + \text{Cos}[x]))^{(2/3)} + ((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4)^{(1/3)})) - \text{Log}[(2 \\
& * (\text{Cos}[x]/(1 + \text{Cos}[x]))^{(1/3)} + 2*(\text{Cos}[x]/(1 + \text{Cos}[x]))^{(2/3)} * ((-7 + 9*\text{Cos}[2 \\
& *x]) * \text{Sec}[x/2]^4)^{(1/3)} + ((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4)^{(2/3)})/2^{(2/3)} - (\text{C} \\
& os[x] * \text{Sec}[x/2]^2)^{(1/3)} * \text{Tan}[x/2]^2 + 2*\text{Log}[-(\text{Cos}[x]*\text{Sec}[x/2]^2)^{(2/3)} + (1 \\
& - 34*\text{Tan}[x/2]^2 + \text{Tan}[x/2]^4)^{(1/3)}] * \text{Sec}[x]^{(2/3)} * (-18*\text{Sec}[x/2]^4 * \text{Sin}[2*x \\
&] + 2*(-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4 * \text{Tan}[x/2]) / (12*((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/ \\
& 2]^4)^{(4/3)} + ((\text{Cos}[x]/(1 + \text{Cos}[x]))^{(2/3)} * (-7 + 9*\text{Cos}[2*x])^{(1/3)} * \text{Sec}[x]^{ \\
& (2/3)} * ((2*((-2*(-\text{Sec}[x/2]^2 * \text{Sin}[x]) + \text{Cos}[x] * \text{Sec}[x/2]^2 * \text{Tan}[x/2])) / (3*(\text{Cos} \\
& [x] * \text{Sec}[x/2]^2)^{(1/3)} + (-34*\text{Sec}[x/2]^2 * \text{Tan}[x/2] + 2*\text{Sec}[x/2]^2 * \text{Tan}[x/2]^3 \\
&)) / (3*(1 - 34*\text{Tan}[x/2]^2 + \text{Tan}[x/2]^4)^{(2/3)})) / (-\text{Cos}[x] * \text{Sec}[x/2]^2)^{(2/3)} \\
& + (1 - 34*\text{Tan}[x/2]^2 + \text{Tan}[x/2]^4)^{(1/3)} + (2*\text{Sqrt}[3] * ((2*((\text{Cos}[x] * \text{Sin}[x]) \\
& / (1 + \text{Cos}[x])^2 - \text{Sin}[x] / (1 + \text{Cos}[x])) / (\text{Sqrt}[3] * (\text{Cos}[x]/(1 + \text{Cos}[x]))^{(1/3} \\
&) * ((\text{Cos}[x]/(1 + \text{Cos}[x]))^{(2/3)} + ((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4)^{(1/3)})) - (\\
& \text{Sqrt}[3] * (\text{Cos}[x]/(1 + \text{Cos}[x]))^{(2/3)} * ((2*((\text{Cos}[x] * \text{Sin}[x]) / (1 + \text{Cos}[x])^2 - \text{S} \\
& in[x] / (1 + \text{Cos}[x])) / (3*(\text{Cos}[x]/(1 + \text{Cos}[x]))^{(1/3)} + (-18*\text{Sec}[x/2]^4 * \text{Sin}[\\
& 2*x] + 2*(-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4 * \text{Tan}[x/2]) / (3*((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x \\
& /2]^4)^{(2/3)})) / ((\text{Cos}[x]/(1 + \text{Cos}[x]))^{(2/3)} + ((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^ \\
& 4)^{(1/3)})^2) / (1 + (3*(\text{Cos}[x]/(1 + \text{Cos}[x]))^{(4/3)}) / ((\text{Cos}[x]/(1 + \text{Cos}[x]))^{(\\
& 2/3)} + ((-7 + 9*\text{Cos}[2*x]) * \text{Sec}[x/2]^4)^{(1/3)})^2) - (-\text{Sec}[x/2]^2 * (\text{Cos}[x] * \text{Sec} \\
& [x/2]^2)^{(1/3)} * \text{Tan}[x/2]) - (\text{Tan}[x/2]^2 * (-\text{Sec}[x/2]^2 * \text{Sin}[x]) + \text{Cos}[x] * \text{Sec}[x
\end{aligned}$$

$$\begin{aligned} & /2]^2 * \tan[x/2]) / (3 * (\cos[x] * \sec[x/2]^2)^{(2/3)}) + ((2 * ((\cos[x] * \sin[x]) / (1 + \cos[x]))^2 - \sin[x] / (1 + \cos[x]))) / (3 * (\cos[x] / (1 + \cos[x]))^{(2/3)}) + (4 * ((-7 + 9 * \cos[2*x]) * \sec[x/2]^4)^{(1/3)} * ((\cos[x] * \sin[x]) / (1 + \cos[x]))^2 - \sin[x] / (1 + \cos[x]))) / (3 * (\cos[x] / (1 + \cos[x]))^{(1/3)}) + (2 * (\cos[x] / (1 + \cos[x]))^{(2/3)} * (-18 * \sec[x/2]^4 * \sin[2*x] + 2 * (-7 + 9 * \cos[2*x]) * \sec[x/2]^4 * \tan[x/2])) / (3 * ((-7 + 9 * \cos[2*x]) * \sec[x/2]^4)^{(2/3)}) + (2 * (-18 * \sec[x/2]^4 * \sin[2*x] + 2 * (-7 + 9 * \cos[2*x]) * \sec[x/2]^4 * \tan[x/2])) / (3 * ((-7 + 9 * \cos[2*x]) * \sec[x/2]^4)^{(1/3)})) / 2^{(2/3)} / ((2 * (\cos[x] / (1 + \cos[x]))^{(1/3)} + 2 * (\cos[x] / (1 + \cos[x]))^{(2/3)} * ((-7 + 9 * \cos[2*x]) * \sec[x/2]^4)^{(1/3)} + ((-7 + 9 * \cos[2*x]) * \sec[x/2]^4)^{(2/3)) / 2^{(2/3)} - (\cos[x] * \sec[x/2]^2)^{(1/3)} * \tan[x/2]^2)) / (4 * ((-7 + 9 * \cos[2*x]) * \sec[x/2]^4)^{(1/3)}) * (1 - 8 * \tan[x]^2)^{(2/3)}) \end{aligned}$$

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[((1 + (1 - 8*Tan[x]^2)^(1/3)) / (Cos[x]^2 * (1 - 8*Tan[x]^2)^(2/3))) * Cot[x], x]')`

[Out] Timed out

Maple [F]

time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\cot(x) \left(1 + (1 - 8 \tan^2(x))^{1/3}\right)}{\cos(x)^2 (1 - 8 \tan^2(x))^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3), x)`

[Out] `int(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3), x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3), x, algorithm="maxima")`

[Out] `integrate(((1-8*tan(x)^2 + 1)^(1/3) + 1)*cot(x)/((1-8*tan(x)^2 + 1)^(2/3)*cos(x)^2), x)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(23) = 46.

time = 1.03, size = 93, normalized size = 3.44

$$-\frac{1}{2} \log \left(\frac{16 \left(145 \cos(x)^4 - 200 \cos(x)^2 + 3(11 \cos(x)^4 - 8 \cos(x)^2) \left(\frac{9 \cos(x)^2 - 8}{\cos(x)^2} \right)^{\frac{2}{3}} + 3(19 \cos(x)^4 - 16 \cos(x)^2) \left(\frac{9 \cos(x)^2 - 8}{\cos(x)^2} \right)^{\frac{1}{3}} + 64 \right)}{\cos(x)^4 - 2 \cos(x)^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3), x, algorithm="fricas")

[Out] -1/2*log(16*(145*cos(x)^4 - 200*cos(x)^2 + 3*(11*cos(x)^4 - 8*cos(x)^2)*((9*cos(x)^2 - 8)/cos(x)^2)^(2/3) + 3*(19*cos(x)^4 - 16*cos(x)^2)*((9*cos(x)^2 - 8)/cos(x)^2)^(1/3) + 64)/(cos(x)^4 - 2*cos(x)^2 + 1))

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt[3]{1 - 8 \tan^2(x)} + 1 \right) \cot(x)}{(1 - 8 \tan^2(x))^{\frac{2}{3}} \cos^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*(1+(1-8*tan(x)**2)**(1/3))/cos(x)**2/(1-8*tan(x)**2)**(2/3), x)

[Out] Integral(((1 - 8*tan(x)**2)**(1/3) + 1)*cot(x)/((1 - 8*tan(x)**2)**(2/3)*cos(x)**2), x)

Giac [A]

time = 0.05, size = 56, normalized size = 2.07

$$\ln \left| \left(-8 \left(\frac{1}{\cos x} \right)^2 + 9 \right)^{\frac{1}{3}} - 1 \right| - \frac{\ln \left(\left(\left(-8 \left(\frac{1}{\cos x} \right)^2 + 9 \right)^{\frac{1}{3}} \right)^2 + \left(-8 \left(\frac{1}{\cos x} \right)^2 + 9 \right)^{\frac{1}{3}} + 1 \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3), x)

[Out] -1/2*log((-8/cos(x)^2 + 9)^(2/3) + (-8/cos(x)^2 + 9)^(1/3) + 1) + log(abs((-8/cos(x)^2 + 9)^(1/3) - 1))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cot(x) \left((1 - 8 \tan(x)^2)^{1/3} + 1 \right)}{\cos(x)^2 (1 - 8 \tan(x)^2)^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)*((1 - 8*tan(x)^2)^(1/3) + 1))/(cos(x)^2*(1 - 8*tan(x)^2)^(2/3))  
,x)
```

```
[Out] int((cot(x)*((1 - 8*tan(x)^2)^(1/3) + 1))/(cos(x)^2*(1 - 8*tan(x)^2)^(2/3))  
, x)
```

$$3.453 \quad \int \frac{\left(5 \cos^2(x) - \sqrt{-1 + 5 \sin^2(x)}\right) \tan(x)}{\sqrt[4]{-1 + 5 \sin^2(x)} \left(2 + \sqrt{-1 + 5 \sin^2(x)}\right)} dx$$

Optimal. Leaf size=101

$$\frac{3 \tan^{-1}\left(\frac{\sqrt[4]{-1 + 5 \sin^2(x)}}{\sqrt{2}}\right) - \tanh^{-1}\left(\frac{\sqrt[4]{-1 + 5 \sin^2(x)}}{\sqrt{2}}\right)}{\sqrt{2}} + 2 \sqrt[4]{-1 + 5 \sin^2(x)} - \frac{\sqrt[4]{-1 + 5 \sin^2(x)}}{2 \left(2 + \sqrt{-1 + 5 \sin^2(x)}\right)}$$

[Out] $2*(-1+5*\sin(x)^2)^{(1/4)}-3/2*\arctan(1/2*(-1+5*\sin(x)^2)^{(1/4)}*2^{(1/2)})*2^{(1/2)}-1/4*\operatorname{arctanh}(1/2*(-1+5*\sin(x)^2)^{(1/4)}*2^{(1/2)})*2^{(1/2)}-1/2*(-1+5*\sin(x)^2)^{(1/4)}/(2+(-1+5*\sin(x)^2)^{(1/2)})$

Rubi [A]

time = 0.86, antiderivative size = 126, normalized size of antiderivative = 1.25, number of steps used = 14, number of rules used = 10, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {4446, 6874, 6829, 348, 52, 65, 209, 481, 536, 213}

$$2\sqrt[4]{4-5\cos^2(x)} - \frac{\sqrt[4]{4-5\cos^2(x)}}{2(\sqrt{4-5\cos^2(x)}+2)} - 2\sqrt{2} \tan^{-1}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right) + \frac{\tan^{-1}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\left(\left(5*\text{Cos}[x]^2 - \text{Sqrt}[-1 + 5*\text{Sin}[x]^2]\right)*\text{Tan}[x]\right)/\left(\left(-1 + 5*\text{Sin}[x]^2\right)^{(1/4)}*(2 + \text{Sqrt}[-1 + 5*\text{Sin}[x]^2])\right), x]$

[Out] $\text{ArcTan}[(4 - 5*\text{Cos}[x]^2)^{(1/4)}/\text{Sqrt}[2]]/\text{Sqrt}[2] - 2*\text{Sqrt}[2]*\text{ArcTan}[(4 - 5*\text{Cos}[x]^2)^{(1/4)}/\text{Sqrt}[2]] - \text{ArcTanh}[(4 - 5*\text{Cos}[x]^2)^{(1/4)}/\text{Sqrt}[2]]/(2*\text{Sqrt}[2]) + 2*(4 - 5*\text{Cos}[x]^2)^{(1/4)} - (4 - 5*\text{Cos}[x]^2)^{(1/4)}/(2*(2 + \text{Sqrt}[4 - 5*\text{Cos}[x]^2]))$

Rule 52

$\text{Int}[\left((a_.) + (b_.)*(x_.)\right)^{(m_.)}*\left((c_.) + (d_.)*(x_.)\right)^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*(c + d*x)^n/(b*(m + n + 1)), x] + \text{Dist}[n*((b*c - a*d)/(b*(m + n + 1))], \text{Int}[(a + b*x)^m*(c + d*x)^{(n - 1)}, x], x] /;$ FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 65

Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1)*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 348

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]

Rule 481

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 536

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 4446

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)]}, x], Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(

$a + b*x])/d, u, x], x], x, \text{Cos}[c*(a + b*x)]/d], x] /; \text{FunctionOfQ}[\text{Cos}[c*(a + b*x)]/d, u, x]] /; \text{FreeQ}\{a, b, c\}, x\} \&\& (\text{EqQ}[F, \text{Tan}] \mid\mid \text{EqQ}[F, \text{tan}])$

Rule 6829

$\text{Int}[(u_*)*(v_)^{(m_*)}*((a_*) + (b_*)*(y_)^{(n_)})^{(p_*)}, x_Symbol] \rightarrow \text{Module}\{q, r\}, \text{Dist}[q*r, \text{Subst}[\text{Int}[x^m*(a + b*x^n)^p, x], x, y], x] /; \text{!FalseQ}[r = \text{Divides}[y^m, v^m, x]] \&\& \text{!FalseQ}[q = \text{DerivativeDivides}[y, u, x]]] /; \text{FreeQ}\{a, b, m, n, p\}, x]$

Rule 6874

$\text{Int}[u_, x_Symbol] \rightarrow \text{With}\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]$
]

Rubi steps

$$\begin{aligned} \int \frac{\left(5 \cos^2(x) - \sqrt{-1 + 5 \sin^2(x)}\right) \tan(x)}{\sqrt[4]{-1 + 5 \sin^2(x)} \left(2 + \sqrt{-1 + 5 \sin^2(x)}\right)} dx &= -\text{Subst} \left(\int \frac{5x^2 - \sqrt{4 - 5x^2}}{\sqrt[4]{4 - 5x^2} \left(2x + x\sqrt{4 - 5x^2}\right)} dx, x, \cos(x) \right) \\ &= -\text{Subst} \left(\int \left(\frac{5x}{\sqrt[4]{4 - 5x^2} \left(2 + \sqrt{4 - 5x^2}\right)} - \frac{\sqrt[4]{4 - 5x^2}}{x \left(2 + \sqrt{4 - 5x^2}\right)} \right) dx, x, \cos(x) \right) \\ &= - \left(5 \text{Subst} \left(\int \frac{x}{\sqrt[4]{4 - 5x^2} \left(2 + \sqrt{4 - 5x^2}\right)} dx, x, \cos(x) \right) \right. \\ &\quad \left. + \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt[4]{4 - 5x^2}}{\left(2 + \sqrt{4 - 5x^2}\right) x} dx, x, \cos^2(x) \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{x} dx, x, \cos^2(x) \right) \right) \\ &= 2 \text{Subst} \left(\int \frac{x^4}{(-2 + x^2) (2 + x^2)^2} dx, x, \sqrt[4]{4 - 5 \cos^2(x)} \right) \\ &= 2 \sqrt[4]{4 - 5 \cos^2(x)} - \frac{\sqrt[4]{4 - 5 \cos^2(x)}}{2 \left(2 + \sqrt{4 - 5 \cos^2(x)}\right)} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{x} dx, x, \sqrt[4]{4 - 5 \cos^2(x)} \right) \\ &= 2 \sqrt[4]{4 - 5 \cos^2(x)} - \frac{\sqrt[4]{4 - 5 \cos^2(x)}}{2 \left(2 + \sqrt{4 - 5 \cos^2(x)}\right)} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{x} dx, x, \sqrt[4]{4 - 5 \cos^2(x)} \right) \\ &= \frac{\tan^{-1} \left(\frac{\sqrt[4]{4 - 5 \cos^2(x)}}{\sqrt{2}} \right)}{\sqrt{2}} - 2\sqrt{2} \tan^{-1} \left(\frac{\sqrt[4]{4 - 5 \cos^2(x)}}{\sqrt{2}} \right) \end{aligned}$$

Mathematica [A]

time = 0.26, size = 89, normalized size = 0.88

$$\frac{1}{4} \left(-6\sqrt{2} \tan^{-1} \left(\frac{\sqrt[4]{3-5\cos(2x)}}{2^{3/4}} \right) - \sqrt{2} \tanh^{-1} \left(\frac{\sqrt[4]{3-5\cos(2x)}}{2^{3/4}} \right) - 2\sqrt[4]{4-5\cos^2(x)} \left(-4 + \frac{1}{2 + \sqrt{4-5\cos^2(x)}} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((5*Cos[x]^2 - Sqrt[-1 + 5*Sin[x]^2])*Tan[x])/((-1 + 5*Sin[x]^2)^(1/4)*(2 + Sqrt[-1 + 5*Sin[x]^2])),x]
```

```
[Out] (-6*Sqrt[2]*ArcTan[(3 - 5*Cos[2*x])^(1/4)/2^(3/4)] - Sqrt[2]*ArcTanh[(3 - 5*Cos[2*x])^(1/4)/2^(3/4)] - 2*(4 - 5*Cos[x]^2)^(1/4)*(-4 + (2 + Sqrt[4 - 5*Cos[x]^2])^(-1)))/4
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(5*Cos[x]^2 - Sqrt[5*Sin[x]^2 - 1])/((5*Sin[x]^2 - 1)^(1/4)*(2 + Sqrt[5*Sin[x]^2 - 1]))*Tan[x],x]')
```

```
[Out] Timed out
```

Maple [F]

time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{\left(5(\cos^2(x)) - \sqrt{-1 + 5(\sin^2(x))} \right) \tan(x)}{(-1 + 5(\sin^2(x)))^{\frac{1}{4}} \left(2 + \sqrt{-1 + 5(\sin^2(x))} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(2+(-1+5*sin(x)^2)^(1/2)),x)
```

```
[Out] int((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(2+(-1+5*sin(x)^2)^(1/2)),x)
```

Maxima [A]

time = 0.33, size = 100, normalized size = 0.99

$$-\frac{3}{2}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}(5\sin(x)^2-1)^{\frac{1}{4}}\right) + \frac{1}{8}\sqrt{2} \log\left(-\frac{\sqrt{2}-(5\sin(x)^2-1)^{\frac{1}{4}}}{\sqrt{2}+(5\sin(x)^2-1)^{\frac{1}{4}}}\right) + 2(5\sin(x)^2-1)^{\frac{1}{4}} - \frac{(5\sin(x)^2-1)^{\frac{1}{4}}}{2\left(\sqrt{5\sin(x)^2-1}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(
2+(-1+5*sin(x)^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(5*sin(x)^2 - 1)^(1/4)) + 1/8*sqrt(2)*log(-
(sqrt(2) - (5*sin(x)^2 - 1)^(1/4))/(sqrt(2) + (5*sin(x)^2 - 1)^(1/4))) + 2*
(5*sin(x)^2 - 1)^(1/4) - 1/2*(5*sin(x)^2 - 1)^(1/4)/(sqrt(5*sin(x)^2 - 1) +
2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(81) = 162.

time = 43.32, size = 461, normalized size = 4.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(
2+(-1+5*sin(x)^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/160*(70*(5*sqrt(2)*cos(x)^4 - 4*sqrt(2)*cos(x)^2)*arctan(-2/5*((5*sqrt(2)
*cos(x)^2 - 4*sqrt(2))*(-5*cos(x)^2 + 4)^(3/4) - 2*sqrt(2)*(-5*cos(x)^2 + 4
)^(5/4)))/(5*cos(x)^4 - 4*cos(x)^2)) - 50*(5*sqrt(2)*cos(x)^4 - 4*sqrt(2)*co
s(x)^2)*arctan(2/5*(sqrt(2)*(-5*cos(x)^2 + 4)^(3/4) + 2*sqrt(2)*(-5*cos(x)^
2 + 4)^(1/4))/cos(x)^2) + 35*(5*sqrt(2)*cos(x)^4 - 4*sqrt(2)*cos(x)^2)*log(
-(125*cos(x)^6 - 1700*cos(x)^4 - 8*(15*sqrt(2)*cos(x)^2 - 16*sqrt(2))*(-5*c
os(x)^2 + 4)^(5/4) + 2560*cos(x)^2 + 4*(25*sqrt(2)*cos(x)^4 - 100*sqrt(2)*c
os(x)^2 + 64*sqrt(2))*(-5*cos(x)^2 + 4)^(3/4) - 16*(25*cos(x)^4 - 60*cos(x)
^2 + 32)*sqrt(-5*cos(x)^2 + 4) - 1024)/(5*cos(x)^6 - 4*cos(x)^4)) + 25*(5*s
qrt(2)*cos(x)^4 - 4*sqrt(2)*cos(x)^2)*log(-(25*cos(x)^4 - 320*cos(x)^2 - 4*
(5*sqrt(2)*cos(x)^2 - 16*sqrt(2))*(-5*cos(x)^2 + 4)^(3/4) - 16*(5*cos(x)^2
- 8)*sqrt(-5*cos(x)^2 + 4) - 8*(15*sqrt(2)*cos(x)^2 - 16*sqrt(2))*(-5*cos(x)
)^2 + 4)^(1/4) + 256)/cos(x)^4) + 16*(5*cos(x)^2 - 2*(10*cos(x)^2 - 1)*sqrt
(-5*cos(x)^2 + 4) - 4)*(-5*cos(x)^2 + 4)^(3/4))/(5*cos(x)^4 - 4*cos(x)^2)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sqrt{5 \sin^2(x) - 1} + 5 \cos^2(x)\right) \tan(x)}{\left(\sqrt{5 \sin^2(x) - 1} + 2\right) \sqrt[4]{5 \sin^2(x) - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*cos(x)**2-(-1+5*sin(x)**2)**(1/2))*tan(x)/(-1+5*sin(x)**2)**(1
/4)/(2+(-1+5*sin(x)**2)**(1/2)),x)
```

[Out] Integral((-sqrt(5*sin(x)**2 - 1) + 5*cos(x)**2)*tan(x)/((sqrt(5*sin(x)**2 - 1) + 2)*(5*sin(x)**2 - 1)**(1/4)), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(2+(-1+5*sin(x)^2)^(1/2)),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x) \left(5 \cos(x)^2 - \sqrt{5 \sin(x)^2 - 1} \right)}{(5 \sin(x)^2 - 1)^{1/4} \left(\sqrt{5 \sin(x)^2 - 1} + 2 \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(x)*(5*cos(x)^2 - (5*sin(x)^2 - 1)^(1/2)))/((5*sin(x)^2 - 1)^(1/4)*((5*sin(x)^2 - 1)^(1/2) + 2)),x)

[Out] int((tan(x)*(5*cos(x)^2 - (5*sin(x)^2 - 1)^(1/2)))/((5*sin(x)^2 - 1)^(1/4)*((5*sin(x)^2 - 1)^(1/2) + 2)), x)

3.454 $\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) dx$

Optimal. Leaf size=25

$$-\frac{3}{40} \cos^{\frac{5}{3}}(2x) - \frac{3}{64} \cos^{\frac{8}{3}}(2x)$$

[Out] $-3/40*\cos(2*x)^{(5/3)}-3/64*\cos(2*x)^{(8/3)}$

Rubi [A]

time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4442, 272, 45}

$$-\frac{3}{64} \cos^{\frac{8}{3}}(2x) - \frac{3}{40} \cos^{\frac{5}{3}}(2x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[x]^3*\text{Cos}[2*x]^{(2/3)}*\text{Sin}[x], x]$

[Out] $(-3*\text{Cos}[2*x]^{(5/3)})/40 - (3*\text{Cos}[2*x]^{(8/3)})/64$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

$\text{Int}[(x_.)^{(m_.)*((a_.) + (b_.)*(x_.)^{(n_.))^{(p_.)}, x_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n) - 1}*(a + b*x)^p, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4442

$\text{Int}[(u_.)*(F_.)[(c_.)*((a_.) + (b_.)*(x_.))], x_Symbol] := \text{With}[\{d = \text{FreeFactors}[\text{Cos}[c*(a + b*x)], x]\}, \text{Dist}[-d/(b*c), \text{Subst}[\text{Int}[\text{SubstFor}[1, \text{Cos}[c*(a + b*x)]]/d, u, x], x], x, \text{Cos}[c*(a + b*x)]/d, x] /;$ FunctionOfQ[Cos[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned}
\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) dx &= -\text{Subst} \left(\int x^3 (-1 + 2x^2)^{2/3} dx, x, \cos(x) \right) \\
&= - \left(\frac{1}{2} \text{Subst} \left(\int x(-1 + 2x)^{2/3} dx, x, \cos^2(x) \right) \right) \\
&= - \left(\frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{2}(-1 + 2x)^{2/3} + \frac{1}{2}(-1 + 2x)^{5/3} \right) dx, x, \cos^2(x) \right) \right) \\
&= -\frac{3}{40} (-1 + 2 \cos^2(x))^{5/3} - \frac{3}{64} (-1 + 2 \cos^2(x))^{8/3}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 4.37, size = 140, normalized size = 5.60

$$-\frac{3}{40} \cos^{\frac{5}{3}}(2x) - \frac{3e^{-6ix} \sqrt[3]{1 + e^{4ix}} \left((1 + e^{4ix})^{2/3} (1 + e^{8ix}) + 2e^{4ix} {}_2F_1 \left(-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}; -e^{4ix} \right) + e^{8ix} {}_2F_1 \left(\frac{1}{3}, \frac{2}{3}, \frac{5}{3}; -e^{4ix} \right) \right)}{256 \cdot 2^{2/3} \sqrt[3]{e^{-2ix} + e^{2ix}}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^3*Cos[2*x]^(2/3)*Sin[x],x]

[Out] (-3*Cos[2*x]^(5/3))/40 - (3*(1 + E^((4*I)*x))^(1/3)*((1 + E^((4*I)*x))^(2/3)*(1 + E^((8*I)*x)) + 2*E^((4*I)*x)*Hypergeometric2F1[-1/3, 1/3, 2/3, -E^((4*I)*x)] + E^((8*I)*x)*Hypergeometric2F1[1/3, 2/3, 5/3, -E^((4*I)*x)]))/(256*2^(2/3)*E^((6*I)*x)*(E^((-2*I)*x) + E^((2*I)*x))^(1/3))

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Cos[x]^4*Cos[2*x]^(2/3)*Tan[x],x]')

[Out] Timed out

Maple [F]

time = 0.13, size = 0, normalized size = 0.00

$$\int (\cos^4(x)) \left(\cos^{\frac{2}{3}}(2x) \right) \tan(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^4*cos(2*x)^(2/3)*tan(x),x)

[Out] `int(cos(x)^4*cos(2*x)^(2/3)*tan(x),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^4*cos(2*x)^(2/3)*tan(x),x, algorithm="maxima")`

[Out] `integrate(cos(2*x)^(2/3)*cos(x)^4*tan(x), x)`

Fricas [A]

time = 0.32, size = 26, normalized size = 1.04

$$-\frac{3}{320} (20 \cos(x)^4 - 4 \cos(x)^2 - 3) (2 \cos(x)^2 - 1)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^4*cos(2*x)^(2/3)*tan(x),x, algorithm="fricas")`

[Out] `-3/320*(20*cos(x)^4 - 4*cos(x)^2 - 3)*(2*cos(x)^2 - 1)^(2/3)`

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)**4*cos(2*x)**(2/3)*tan(x),x)`

[Out] Timed out

Giac [A]

time = 0.01, size = 128, normalized size = 5.12

$$\frac{-\frac{3}{2} \left(\frac{1}{5} \left((2 \cos^2 x - 1)^{\frac{1}{3}} \right)^2 (2 \cos^2 x - 1) + \frac{\left((2 \cos^2 x - 1)^{\frac{1}{3}} \right)^2}{2} \right) + \frac{6}{4} \left(\frac{1}{8} \left((2 \cos^2 x - 1)^{\frac{1}{3}} \right)^2 (2 \cos^2 x - 1)^2 + \frac{2}{5} \left((2 \cos^2 x - 1)^{\frac{1}{3}} \right)^2 (2 \cos^2 x - 1) + \frac{\left((2 \cos^2 x - 1)^{\frac{1}{3}} \right)^2}{2} \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^4*cos(2*x)^(2/3)*tan(x),x)`

[Out] `-3/64*(2*cos(x)^2 - 1)^(8/3) - 3/40*(2*cos(x)^2 - 1)^(5/3)`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.04

$$\int \cos(2x)^{2/3} \cos(x)^4 \tan(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(2*x)^(2/3)*cos(x)^4*tan(x),x)`

[Out] `int(cos(2*x)^(2/3)*cos(x)^4*tan(x), x)`

$$3.455 \quad \int \frac{\sin^6(x) \tan(x)}{\cos^{\frac{3}{4}}(2x)} dx$$

Optimal. Leaf size=102

$$\frac{\tan^{-1}\left(\frac{1-\sqrt{\cos(2x)}}{\sqrt{2}\sqrt[4]{\cos(2x)}}\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{1+\sqrt{\cos(2x)}}{\sqrt{2}\sqrt[4]{\cos(2x)}}\right)}{\sqrt{2}} + \frac{7}{4}\sqrt[4]{\cos(2x)} - \frac{1}{5}\cos^{\frac{5}{4}}(2x) + \frac{1}{36}\cos^{\frac{9}{4}}(2x)$$

[Out] $7/4*\cos(2*x)^{(1/4)}-1/5*\cos(2*x)^{(5/4)}+1/36*\cos(2*x)^{(9/4)}+1/2*\arctan(1/2*(1-\cos(2*x)^{(1/2)})/\cos(2*x)^{(1/4)}*2^{(1/2)})*2^{(1/2)}-1/2*\operatorname{arctanh}(1/2*(1+\cos(2*x)^{(1/2)})/\cos(2*x)^{(1/4)}*2^{(1/2)})*2^{(1/2)}$

Rubi [A]

time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.51, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {4446, 457, 90, 65, 217, 1179, 642, 1176, 631, 210}

$$\frac{1}{36}\cos^{\frac{9}{4}}(2x) - \frac{1}{5}\cos^{\frac{5}{4}}(2x) + \frac{7}{4}\sqrt[4]{\cos(2x)} + \frac{\log(\sqrt{\cos(2x)} - \sqrt{2}\sqrt[4]{\cos(2x)} + 1)}{2\sqrt{2}} - \frac{\log(\sqrt{\cos(2x)} + \sqrt{2}\sqrt[4]{\cos(2x)} + 1)}{2\sqrt{2}} + \frac{\tan^{-1}(1 - \sqrt{2}\sqrt[4]{\cos(2x)})}{\sqrt{2}} - \frac{\tan^{-1}(\sqrt{2}\sqrt[4]{\cos(2x)} + 1)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(Sin[x]^6*Tan[x])/Cos[2*x]^(3/4),x]

[Out] ArcTan[1 - Sqrt[2]*Cos[2*x]^(1/4)]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Cos[2*x]^(1/4)]/Sqrt[2] + (7*Cos[2*x]^(1/4))/4 - Cos[2*x]^(5/4)/5 + Cos[2*x]^(9/4)/36 + Log[1 - Sqrt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2*x]]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2*x]]]/(2*Sqrt[2])

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 90

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &

& (LtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[((a_) + (b_)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 457

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 631

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1176

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1179

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 4446

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^6(x) \tan(x)}{\cos^{\frac{3}{4}}(2x)} dx &= -\text{Subst} \left(\int \frac{(1-x^2)^3}{x(-1+2x^2)^{3/4}} dx, x, \cos(x) \right) \\
&= - \left(\frac{1}{2} \text{Subst} \left(\int \frac{(1-x)^3}{x(-1+2x)^{3/4}} dx, x, \cos^2(x) \right) \right) \\
&= - \left(\frac{1}{2} \text{Subst} \left(\int \left(-\frac{7}{4(-1+2x)^{3/4}} + \frac{1}{x(-1+2x)^{3/4}} + \sqrt[4]{-1+2x} - \frac{1}{4}(-1+2x)^{5/4} \right) dx, x, \cos^2(x) \right) \right) \\
&= \frac{7}{4} \sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5} (-1+2\cos^2(x))^{5/4} + \frac{1}{36} (-1+2\cos^2(x))^{9/4} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{x} dx, x, \cos^2(x) \right) \\
&= \frac{7}{4} \sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5} (-1+2\cos^2(x))^{5/4} + \frac{1}{36} (-1+2\cos^2(x))^{9/4} - \text{Subst} \left(\int \frac{1}{x} dx, x, \cos^2(x) \right) \\
&= \frac{7}{4} \sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5} (-1+2\cos^2(x))^{5/4} + \frac{1}{36} (-1+2\cos^2(x))^{9/4} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{x} dx, x, \cos^2(x) \right) \\
&= \frac{7}{4} \sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5} (-1+2\cos^2(x))^{5/4} + \frac{1}{36} (-1+2\cos^2(x))^{9/4} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{x} dx, x, \cos^2(x) \right) \\
&= \frac{7}{4} \sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5} (-1+2\cos^2(x))^{5/4} + \frac{1}{36} (-1+2\cos^2(x))^{9/4} + \frac{\log(1-\sqrt{2})}{\sqrt{2}} \\
&= \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+2\cos^2(x)}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+2\cos^2(x)}\right)}{\sqrt{2}} + \frac{7}{4}\sqrt[4]{-1}
\end{aligned}$$

Mathematica [A]

time = 0.08, size = 153, normalized size = 1.50

$$\frac{1}{360} \left(180\sqrt{2} \tan^{-1}\left(1-\sqrt{2}\sqrt[4]{\cos(2x)}\right) - 180\sqrt{2} \tan^{-1}\left(1+\sqrt{2}\sqrt[4]{\cos(2x)}\right) + 635\sqrt{\cos(2x)} - 72\cos^{\frac{5}{4}}(2x) + 5\sqrt[4]{\cos(2x)} \cos(4x) + 90\sqrt{2} \log\left(1-\sqrt{2}\sqrt[4]{\cos(2x)}+\sqrt{\cos(2x)}\right) - 90\sqrt{2} \log\left(1+\sqrt{2}\sqrt[4]{\cos(2x)}+\sqrt{\cos(2x)}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sin[x]^6*Tan[x])/Cos[2*x]^(3/4), x]

[Out] (180*Sqrt[2]*ArcTan[1 - Sqrt[2]*Cos[2*x]^(1/4)] - 180*Sqrt[2]*ArcTan[1 + Sqrt[2]*Cos[2*x]^(1/4)] + 635*Cos[2*x]^(1/4) - 72*Cos[2*x]^(5/4) + 5*Cos[2*x]

$\frac{1}{4} \cos(4x) + 90 \sqrt{2} \log[1 - \sqrt{2} \cos(2x)]^{1/4} + \sqrt{\cos(2x)}$
 $]- 90 \sqrt{2} \log[1 + \sqrt{2} \cos(2x)]^{1/4} + \sqrt{\cos(2x)}] / 360$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[Sin[x]^6*(Tan[x]/Cos[2*x]^(3/4)),x]')`

[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep

Maple [F]

time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{(\sin^6(x)) \tan(x)}{\cos(2x)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)^6*tan(x)/cos(2*x)^(3/4),x)`

[Out] `int(sin(x)^6*tan(x)/cos(2*x)^(3/4),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="maxima")`

[Out] `integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4), x)`

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**6*tan(x)/cos(2*x)**(3/4),x)

[Out] Exception raised: SystemError

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(75) = 150.

time = 0.01, size = 244, normalized size = 2.39

$$\frac{1}{4}\sqrt{2}\ln(\sqrt{-2\sin^2x+1}-\sqrt{2}(-2\sin^2x+1)^{1/4}+1)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{2((-2\sin^2x+1)^{1/4}-\frac{\sqrt{2}}{2})}{\sqrt{2}}\right)-\frac{1}{4}\sqrt{2}\ln(\sqrt{-2\sin^2x+1}+\sqrt{2}(-2\sin^2x+1)^{1/4}+1)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{2((-2\sin^2x+1)^{1/4}+\frac{\sqrt{2}}{2})}{\sqrt{2}}\right)+\frac{2039(-2\sin^2x+1)^{1/4}(-2\sin^2x+1)^{3/4}-2039(-2\sin^2x+1)^{1/4}(-2\sin^2x+1)+458752(-2\sin^2x+1)^{1/4}}{262144}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x)

[Out] 1/36*(2*sin(x)^2 - 1)^2*(-2*sin(x)^2 + 1)^(1/4) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(-2*sin(x)^2 + 1)^(1/4))) - 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(-2*sin(x)^2 + 1)^(1/4))) - 1/4*sqrt(2)*log(sqrt(2)*(-2*sin(x)^2 + 1)^(1/4) + sqrt(-2*sin(x)^2 + 1) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*(-2*sin(x)^2 + 1)^(1/4) + sqrt(-2*sin(x)^2 + 1) + 1) - 1/5*(-2*sin(x)^2 + 1)^(5/4) + 7/4*(-2*sin(x)^2 + 1)^(1/4)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(x)^6 \tan(x)}{\cos(2x)^{3/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x)^6*tan(x))/cos(2*x)^(3/4),x)

[Out] int((sin(x)^6*tan(x))/cos(2*x)^(3/4), x)

3.456 $\int \sqrt{\tan(x) \tan(2x)} dx$

Optimal. Leaf size=17

$$-\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right)$$

[Out] `-arctanh(tan(x)/(tan(x)*tan(2*x))^(1/2))`

Rubi [A]

time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4482, 3859, 213}

$$-\tanh^{-1}\left(\frac{\tan(2x)}{\sqrt{\sec(2x) - 1}}\right)$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[Tan[x]*Tan[2*x]],x]`

[Out] `-ArcTanh[Tan[2*x]/Sqrt[-1 + Sec[2*x]]]`

Rule 213

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

Rule 3859

`Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2*(b/d), Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

Rule 4482

`Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]`

Rubi steps

$$\begin{aligned}
\int \sqrt{\tan(x) \tan(2x)} \, dx &= \int \sqrt{-1 + \sec(2x)} \, dx \\
&= -\text{Subst} \left(\int \frac{1}{-1 + x^2} \, dx, x, -\frac{\tan(2x)}{\sqrt{-1 + \sec(2x)}} \right) \\
&= -\tanh^{-1} \left(\frac{\tan(2x)}{\sqrt{-1 + \sec(2x)}} \right)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 45 vs. 2(17) = 34.

time = 0.05, size = 45, normalized size = 2.65

$$-\frac{\tanh^{-1} \left(\frac{\sqrt{2} \cos(x)}{\sqrt{\cos(2x)}} \right) \sqrt{\cos(2x)} \csc(x) \sqrt{\tan(x) \tan(2x)}}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Tan[x]*Tan[2*x]],x]

[Out] -((ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]])*Sqrt[Cos[2*x]]*Csc[x]*Sqrt[Tan[x]*Tan[2*x]])/Sqrt[2])

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sqrt[Tan[x]*Tan[2*x]],x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(15) = 30.

time = 0.28, size = 88, normalized size = 5.18

method	result	size
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default	$\frac{\sqrt{4} \sqrt{\frac{1-\cos^2(x)}{2(\cos^2(x)-1)}} \sin(x) \sqrt{\frac{2(\cos^2(x)-1)}{(1+\cos(x))^2}} \operatorname{arctanh}\left(\frac{\sqrt{2} \cos(x) \sqrt{4} (\cos(x)-1)}{2 \sqrt{\frac{2(\cos^2(x)-1)}{(1+\cos(x))^2}} \sin(x)^2}\right)}{2(\cos(x)-1)}$	88
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Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(x)*tan(2*x))^(1/2),x,method=_RETURNVERBOSE)

[Out] $-1/2*4^{(1/2)*((1-\cos(x)^2)/(2*\cos(x)^2-1))^{(1/2)*\sin(x)*((2*\cos(x)^2-1)/(1+\cos(x))^2)^{(1/2)*\operatorname{arctanh}(1/2*2^{(1/2)*\cos(x)*4^{(1/2)*(\cos(x)-1)/((2*\cos(x)^2-1)/(1+\cos(x))^2)^{(1/2)/\sin(x)^2)/(\cos(x)-1}}$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(15) = 30.

time = 0.35, size = 259, normalized size = 15.24

$\frac{1}{2} \log\left(\frac{\tan(x)^3 - 2\sqrt{2}(\tan(x)^2 - 1)\sqrt{-\frac{\tan(x)^2}{\tan(x)^2 - 1}} - 3\tan(x)}{\tan(x)^3 + \tan(x)}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((tan(x)*tan(2*x))^(1/2),x, algorithm="maxima")

[Out] $1/4*\log(4*\sqrt{\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1}*\cos(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1))^2 + 4*\sqrt{\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1}*\sin(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1))^2 + 8*(\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1)^{(1/4)*\cos(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1)) + 4) - 1/4*\log(\cos(2*x)^2 + \sin(2*x)^2 + \sqrt{\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1}*(\cos(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1))^2 + \sin(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1))^2) + 2*(\cos(4*x)^2 + \sin(4*x)^2 + 2*\cos(4*x) + 1)^{(1/4)*(\cos(2*x)*\cos(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1)) + \sin(2*x)*\sin(1/2*\arctan2(\sin(4*x), \cos(4*x) + 1)))$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(15) = 30.

time = 0.36, size = 50, normalized size = 2.94

$$\frac{1}{2} \log\left(\frac{\tan(x)^3 - 2\sqrt{2}(\tan(x)^2 - 1)\sqrt{-\frac{\tan(x)^2}{\tan(x)^2 - 1}} - 3\tan(x)}{\tan(x)^3 + \tan(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((tan(x)*tan(2*x))^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{2} \log(-(\tan(x))^3 - 2\sqrt{2}(\tan(x))^2 - 1)\sqrt{-\tan(x)^2/(\tan(x)^2 - 1)} - 3\tan(x))/(\tan(x)^3 + \tan(x))$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan(x) \tan(2x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((tan(x)*tan(2*x))**(1/2),x)`

[Out] `Integral(sqrt(tan(x)*tan(2*x)), x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(15) = 30$.
time = 0.03, size = 111, normalized size = 6.53

$$-\sqrt{2} \left(\frac{1}{4} (-\sqrt{2} \ln(\sqrt{2} + 1) + \sqrt{2} \ln(\sqrt{2} - 1)) \operatorname{sign}(\tan x) - \left(\frac{1}{4} \sqrt{2} \ln(\sqrt{2} + \sqrt{-\tan^2 x + 1}) - \frac{1}{4} \sqrt{2} \ln(\sqrt{2} - \sqrt{-\tan^2 x + 1}) \right) \operatorname{sign}(\tan x) \operatorname{sign}(\tan^2 x - 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((tan(x)*tan(2*x))^(1/2),x)`

[Out] $\frac{1}{4}\sqrt{2} * ((\sqrt{2} * \log(\sqrt{2} + \sqrt{-\tan(x)^2 + 1}) - \sqrt{2} * \log(\sqrt{2} - \sqrt{-\tan(x)^2 + 1})) * \operatorname{sgn}(\tan(x)^2 - 1) * \operatorname{sgn}(\tan(x)) + (\sqrt{2} * \log(\sqrt{2} + 1) - \sqrt{2} * \log(\sqrt{2} - 1)) * \operatorname{sgn}(\tan(x)))$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.06

$$\int \sqrt{\tan(2x) \tan(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((tan(2*x)*tan(x))^(1/2),x)`

[Out] `int((tan(2*x)*tan(x))^(1/2), x)`

3.457 $\int \sqrt{\cot(2x) \tan(x)} dx$

Optimal. Leaf size=32

$$-\frac{\sin^{-1}(\tan(x))}{\sqrt{2}} + \tan^{-1}\left(\frac{\sqrt{2} \tan(x)}{\sqrt{1 - \tan^2(x)}}\right)$$

[Out] arctan(2^(1/2)*tan(x)/(1-tan(x)^2)^(1/2))-1/2*arcsin(tan(x))*2^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {12, 399, 222, 385, 209}

$$\tan^{-1}\left(\frac{\sqrt{2} \tan(x)}{\sqrt{1 - \tan^2(x)}}\right) - \frac{\sin^{-1}(\tan(x))}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cot[2*x]*Tan[x]],x]

[Out] -(ArcSin[Tan[x]]/Sqrt[2]) + ArcTan[(Sqrt[2]*Tan[x])/Sqrt[1 - Tan[x]^2]]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\cot(2x) \tan(x)} \, dx &= \text{Subst} \left(\int \frac{\sqrt{1-x^2}}{\sqrt{2} (1+x^2)} \, dx, x, \tan(x) \right) \\
&= \frac{\text{Subst} \left(\int \frac{\sqrt{1-x^2}}{1+x^2} \, dx, x, \tan(x) \right)}{\sqrt{2}} \\
&= -\frac{\text{Subst} \left(\int \frac{1}{\sqrt{1-x^2}} \, dx, x, \tan(x) \right)}{\sqrt{2}} + \sqrt{2} \text{Subst} \left(\int \frac{1}{\sqrt{1-x^2} (1+x^2)} \, dx, x, \tan(x) \right) \\
&= -\frac{\sin^{-1}(\tan(x))}{\sqrt{2}} + \sqrt{2} \text{Subst} \left(\int \frac{1}{1+2x^2} \, dx, x, \frac{\tan(x)}{\sqrt{1-\tan^2(x)}} \right) \\
&= -\frac{\sin^{-1}(\tan(x))}{\sqrt{2}} + \tan^{-1} \left(\frac{\sqrt{2} \tan(x)}{\sqrt{1-\tan^2(x)}} \right)
\end{aligned}$$

Mathematica [A]

time = 0.05, size = 52, normalized size = 1.62

$$\frac{\left(\sqrt{2} \sin^{-1} \left(\sqrt{2} \sin(x) \right) - \tan^{-1} \left(\frac{\sin(x)}{\sqrt{\cos(2x)}} \right) \right) \cos(x) \sqrt{\cot(2x) \tan(x)}}{\sqrt{\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[2*x]*Tan[x]], x]
```

```
[Out] ((Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - ArcTan[Sin[x]/Sqrt[Cos[2*x]]])*Cos[x]*Sqr
t[Cot[2*x]*Tan[x]])/Sqrt[Cos[2*x]]
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[Sqrt[Cot[2*x]/Cot[x]],x]')`

[Out] caught exception: maximum recursion depth exceeded

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.48, size = 242, normalized size = 7.56

method	result
default	$\sqrt{2} \left(4 \operatorname{EllipticPi} \left(\frac{\sqrt{3+2\sqrt{2}} \cos(x)-1}{\sin(x)}, -\frac{1}{3+2\sqrt{2}}, \frac{\sqrt{3-2\sqrt{2}}}{\sqrt{3+2\sqrt{2}}} \right) - \operatorname{EllipticF} \left(\frac{(\cos(x)-1)(1+\sqrt{2})}{\sin(x)}, 3-2\sqrt{2} \right) \right) -$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cot(2*x)/cot(x))^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{2} \cdot 2^{1/2} / (3+2 \cdot 2^{1/2})^{1/2} / (1+2^{1/2}) \cdot (4 \cdot \operatorname{EllipticPi}((3+2 \cdot 2^{1/2})^{1/2} / 2) \cdot (\cos(x)-1) / \sin(x), -1 / (3+2 \cdot 2^{1/2}), (3-2 \cdot 2^{1/2})^{1/2} / (3+2 \cdot 2^{1/2})^{1/2}) - \operatorname{EllipticF}((\cos(x)-1) \cdot (1+2^{1/2}) / \sin(x), 3-2 \cdot 2^{1/2}) - 2 \cdot \operatorname{EllipticPi}((3+2 \cdot 2^{1/2})^{1/2} / 2) \cdot (\cos(x)-1) / \sin(x), 1 / (3+2 \cdot 2^{1/2}), (3-2 \cdot 2^{1/2})^{1/2} / (3+2 \cdot 2^{1/2})^{1/2})) \cdot (2+2^{1/2}) \cdot \cos(x) \cdot \sin(x)^2 \cdot ((2 \cdot \cos(x)^2-1) / \cos(x)^2)^{1/2} \cdot (-2 \cdot (\cos(x) \cdot 2^{1/2}-2^{1/2}-2 \cdot \cos(x)+1) / (1+\cos(x)))^{1/2} \cdot ((\cos(x) \cdot 2^{1/2}+2 \cdot \cos(x)-2^{1/2}-1) / (1+\cos(x)))^{1/2} / (\cos(x)-1) / (2 \cdot \cos(x)^2-1)$

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((cot(2*x)/cot(x))^(1/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
sign: argument cannot be imaginary; found %i

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(26) = 52.

time = 0.34, size = 115, normalized size = 3.59

$$\frac{1}{4} \sqrt{2} \arctan \left(\frac{\sqrt{2} (3 \cos(2x)^2 + 2 \cos(2x) - 1) \sqrt{\frac{\cos(2x)}{\cos(2x) + 1}}}{4 \cos(2x) \sin(2x)} \right) - \frac{1}{2} \arctan \left(\frac{\sqrt{2} (2 \sqrt{2} \cos(2x)^2 + \sqrt{2} \cos(2x) - \sqrt{2}) \sqrt{\frac{\cos(2x)}{\cos(2x) + 1}}}{4 \cos(2x) \sin(2x)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(2*x)/cot(x))^(1/2),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*(3*cos(2*x)^2 + 2*cos(2*x) - 1)*sqrt(cos(2*x)/(cos(2*x) + 1))/(cos(2*x)*sin(2*x))) - 1/2*arctan(1/4*sqrt(2)*(2*sqrt(2)*cos(2*x)^2 + sqrt(2)*cos(2*x) - sqrt(2))*sqrt(cos(2*x)/(cos(2*x) + 1))/(cos(2*x)*sin(2*x)))

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{\cot(2x)}{\cot(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(2*x)/cot(x))**(1/2),x)

[Out] Integral(sqrt(cot(2*x)/cot(x)), x)

Giac [C] Result contains complex when optimal does not.

time = 0.05, size = 171, normalized size = 5.34

$$\frac{\frac{1}{2}(\pi - \sqrt{2} \arctan(-i) - \sqrt{2} \arctan(\sqrt{2}) - i \ln(2\sqrt{2} + 3)) \operatorname{sign}(\sin(2x)) + \left(\frac{\arctan\left(\frac{-1 + \frac{-2\sqrt{2}\sqrt{-2\cos^2 x + 3\cos^2 x - 1} + i}{\sqrt{2}}}{\sqrt{2}}\right) - \frac{\arcsin(4\cos^2 x - 3)}{2}\right) \operatorname{sign}(\cos x) + \frac{(i\sqrt{2} \ln(2i\sqrt{2} + 3) + 2\arctan(-i)) \operatorname{sign}(\cos x)}{2\sqrt{2}}}{\operatorname{sign}(\sin(2x)) \operatorname{sign}(\cos x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(2*x)/cot(x))^(1/2),x)

[Out] 1/2*(pi - sqrt(2)*arctan(-I) - sqrt(2)*arctan(sqrt(2)) - I*log(2*sqrt(2) + 3))*sgn(sin(2*x)) - 1/4*(sqrt(2)*(-I*sqrt(2)*log(2*I*sqrt(2) + 3*I) - 2*arctan(-I))*sgn(cos(x)) + 2*(sqrt(2)*arctan(1/4*sqrt(2)*(3*(2*sqrt(2)*sqrt(-2*cos(x)^4 + 3*cos(x)^2 - 1) - 1)/(4*cos(x)^2 - 3) - 1)) + arcsin(4*cos(x)^2 - 3))*sgn(cos(x)))/(sgn(cos(x))*sgn(sin(2*x)))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{\frac{\cot(2x)}{\cot(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(2*x)/cot(x))^(1/2),x)

[Out] int((cot(2*x)/cot(x))^(1/2), x)

$$3.458 \quad \int \frac{1}{x^5(5+x^2)} dx$$

Optimal. Leaf size=31

$$-\frac{1}{20x^4} + \frac{1}{50x^2} + \frac{\log(x)}{125} - \frac{1}{250} \log(5+x^2)$$

[Out] $-1/20/x^4+1/50/x^2+1/125*\ln(x)-1/250*\ln(x^2+5)$

Rubi [A]

time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {272, 46}

$$-\frac{1}{20x^4} + \frac{1}{50x^2} - \frac{1}{250} \log(x^2+5) + \frac{\log(x)}{125}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(5 + x^2)),x]

[Out] $-1/20*1/x^4 + 1/(50*x^2) + \text{Log}[x]/125 - \text{Log}[5 + x^2]/250$

Rule 46

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^5(5+x^2)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^3(5+x)} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{5x^3} - \frac{1}{25x^2} + \frac{1}{125x} - \frac{1}{125(5+x)} \right) dx, x, x^2 \right) \\ &= -\frac{1}{20x^4} + \frac{1}{50x^2} + \frac{\log(x)}{125} - \frac{1}{250} \log(5+x^2) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 31, normalized size = 1.00

$$-\frac{1}{20x^4} + \frac{1}{50x^2} + \frac{\log(x)}{125} - \frac{1}{250} \log(5 + x^2)$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x^5*(5 + x^2)),x]``[Out] -1/20*1/x^4 + 1/(50*x^2) + Log[x]/125 - Log[5 + x^2]/250`**Mathics [A]**

time = 1.72, size = 23, normalized size = 0.74

$$-\frac{1}{20x^4} + \frac{1}{50x^2} - \frac{\text{Log}[5 + x^2]}{250} + \frac{\text{Log}[x]}{125}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^5*(5 + x^2)),x]')``[Out] -1 / (20 x ^ 4) + 1 / (50 x ^ 2) - Log[5 + x ^ 2] / 250 + Log[x] / 125`**Maple [A]**

time = 0.08, size = 24, normalized size = 0.77

method	result	size
default	$-\frac{1}{20x^4} + \frac{1}{50x^2} + \frac{\ln(x)}{125} - \frac{\ln(x^2+5)}{250}$	24
norman	$-\frac{\frac{1}{20} + \frac{x^2}{50}}{x^4} + \frac{\ln(x)}{125} - \frac{\ln(x^2+5)}{250}$	25
risch	$-\frac{\frac{1}{20} + \frac{x^2}{50}}{x^4} + \frac{\ln(x)}{125} - \frac{\ln(x^2+5)}{250}$	25
meijerg	$-\frac{1}{20x^4} + \frac{1}{50x^2} + \frac{\ln(x)}{125} - \frac{\ln(5)}{250} - \frac{\ln\left(1 + \frac{x^2}{5}\right)}{250}$	30

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/x^5/(x^2+5),x,method=_RETURNVERBOSE)``[Out] -1/20/x^4+1/50/x^2+1/125*ln(x)-1/250*ln(x^2+5)`**Maxima [A]**

time = 0.25, size = 27, normalized size = 0.87

$$\frac{2x^2 - 5}{100x^4} - \frac{1}{250} \log(x^2 + 5) + \frac{1}{250} \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^2+5),x, algorithm="maxima")

[Out] 1/100*(2*x^2 - 5)/x^4 - 1/250*log(x^2 + 5) + 1/250*log(x^2)

Fricas [A]

time = 0.30, size = 30, normalized size = 0.97

$$-\frac{2x^4 \log(x^2 + 5) - 4x^4 \log(x) - 10x^2 + 25}{500x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^2+5),x, algorithm="fricas")

[Out] -1/500*(2*x^4*log(x^2 + 5) - 4*x^4*log(x) - 10*x^2 + 25)/x^4

Sympy [A]

time = 0.06, size = 24, normalized size = 0.77

$$\frac{\log(x)}{125} - \frac{\log(x^2 + 5)}{250} + \frac{2x^2 - 5}{100x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(x**2+5),x)

[Out] log(x)/125 - log(x**2 + 5)/250 + (2*x**2 - 5)/(100*x**4)

Giac [A]

time = 0.00, size = 38, normalized size = 1.23

$$-\frac{\ln(x^2 + 5)}{250} + \frac{\ln(x^2)}{250} + \frac{-3x^4 + 10x^2 - 25}{500x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^2+5),x)

[Out] -1/500*(3*x^4 - 10*x^2 + 25)/x^4 - 1/250*log(x^2 + 5) + 1/250*log(x^2)

Mupad [B]

time = 0.31, size = 24, normalized size = 0.77

$$\frac{\ln(x)}{125} - \frac{\ln(x^2 + 5)}{250} + \frac{\frac{x^2}{50} - \frac{1}{20}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(x^2 + 5)),x)

[Out] log(x)/125 - log(x^2 + 5)/250 + (x^2/50 - 1/20)/x^4

$$3.459 \quad \int \frac{1}{x^6(5+x^2)} dx$$

Optimal. Leaf size=39

$$-\frac{1}{25x^5} + \frac{1}{75x^3} - \frac{1}{125x} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

[Out] -1/25/x^5+1/75/x^3-1/125/x-1/625*arctan(1/5*x*5^(1/2))*5^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {331, 209}

$$-\frac{1}{25x^5} + \frac{1}{75x^3} - \frac{1}{125x} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*(5 + x^2)),x]

[Out] -1/25*1/x^5 + 1/(75*x^3) - 1/(125*x) - ArcTan[x/Sqrt[5]]/(125*Sqrt[5])

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 331

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m+1)*((a + b*x^n)^(p+1)/(a*c*(m+1))), x] - Dist[b*((m+n*(p+1)+1)/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6(5+x^2)} dx &= -\frac{1}{25x^5} - \frac{1}{5} \int \frac{1}{x^4(5+x^2)} dx \\
&= -\frac{1}{25x^5} + \frac{1}{75x^3} + \frac{1}{25} \int \frac{1}{x^2(5+x^2)} dx \\
&= -\frac{1}{25x^5} + \frac{1}{75x^3} - \frac{1}{125x} - \frac{1}{125} \int \frac{1}{5+x^2} dx \\
&= -\frac{1}{25x^5} + \frac{1}{75x^3} - \frac{1}{125x} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 39, normalized size = 1.00

$$-\frac{1}{25x^5} + \frac{1}{75x^3} - \frac{1}{125x} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x^6*(5 + x^2)),x]``[Out] -1/25*1/x^5 + 1/(75*x^3) - 1/(125*x) - ArcTan[x/Sqrt[5]]/(125*Sqrt[5])`**Mathics [A]**

time = 1.74, size = 28, normalized size = 0.72

$$-\frac{1}{25x^5} + \frac{1}{75x^3} - \frac{1}{125x} - \frac{\sqrt{5} \operatorname{ArcTan}\left[\frac{\sqrt{5}x}{5}\right]}{625}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^6*(5 + x^2)),x]')``[Out] -1 / (25 x ^ 5) + 1 / (75 x ^ 3) - 1 / (125 x) - Sqrt[5] ArcTan[Sqrt[5] x / 5] / 625`**Maple [A]**

time = 0.08, size = 29, normalized size = 0.74

method	result	size
default	$-\frac{1}{25x^5} + \frac{1}{75x^3} - \frac{1}{125x} - \frac{\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}}{625}$	29

risch	$\frac{-\frac{1}{125}x^4 + \frac{1}{75}x^2 - \frac{1}{25}}{x^5} - \frac{\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}}{625}$	30
meijerg	$\frac{\sqrt{5}\left(-\frac{2\sqrt{5}}{x} + \frac{10\sqrt{5}}{3x^3} - \frac{10\sqrt{5}}{x^5} - 2\arctan\left(\frac{x\sqrt{5}}{5}\right)\right)}{1250}$	40

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^6/(x^2+5),x,method=_RETURNVERBOSE)`

[Out] $-1/25/x^5 + 1/75/x^3 - 1/125/x - 1/625 \cdot \arctan(1/5 \cdot x \cdot 5^{(1/2)}) \cdot 5^{(1/2)}$

Maxima [A]

time = 0.34, size = 30, normalized size = 0.77

$$-\frac{1}{625} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} x\right) - \frac{3x^4 - 5x^2 + 15}{375x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^6/(x^2+5),x, algorithm="maxima")`

[Out] $-1/625 \cdot \sqrt{5} \cdot \arctan(1/5 \cdot \sqrt{5} \cdot x) - 1/375 \cdot (3 \cdot x^4 - 5 \cdot x^2 + 15) / x^5$

Fricas [A]

time = 0.30, size = 32, normalized size = 0.82

$$-\frac{3\sqrt{5}x^5 \arctan\left(\frac{1}{5}\sqrt{5}x\right) + 15x^4 - 25x^2 + 75}{1875x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^6/(x^2+5),x, algorithm="fricas")`

[Out] $-1/1875 \cdot (3 \cdot \sqrt{5} \cdot x^5 \cdot \arctan(1/5 \cdot \sqrt{5} \cdot x) + 15 \cdot x^4 - 25 \cdot x^2 + 75) / x^5$

Sympy [A]

time = 0.07, size = 32, normalized size = 0.82

$$-\frac{\sqrt{5} \operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)}{625} + \frac{-3x^4 + 5x^2 - 15}{375x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(x**2+5),x)`

[Out] $-\sqrt{5} \cdot \operatorname{atan}(\sqrt{5} \cdot x / 5) / 625 + (-3 \cdot x^{**4} + 5 \cdot x^{**2} - 15) / (375 \cdot x^{**5})$

Giac [A]

time = 0.00, size = 43, normalized size = 1.10

$$-\frac{3x^4 - 5x^2 + 15}{375x^5} - \frac{2 \arctan\left(\frac{x}{\sqrt{5}}\right)}{2 \cdot 125\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^2+5),x)**[Out]** -1/625*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/375*(3*x^4 - 5*x^2 + 15)/x^5**Mupad [B]**

time = 0.03, size = 30, normalized size = 0.77

$$-\frac{\frac{x^4}{125} - \frac{x^2}{75} + \frac{1}{25}}{x^5} - \frac{\sqrt{5} \operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)}{625}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6*(x^2 + 5)),x)**[Out]** - (x^4/125 - x^2/75 + 1/25)/x^5 - (5^(1/2)*atan((5^(1/2)*x)/5))/625

$$3.460 \quad \int \frac{1}{x(-4+x^2)^4} dx$$

Optimal. Leaf size=58

$$\frac{1}{24(4-x^2)^3} + \frac{1}{64(4-x^2)^2} + \frac{1}{128(4-x^2)} + \frac{\log(x)}{256} - \frac{1}{512} \log(4-x^2)$$

[Out] 1/24/(-x^2+4)^3+1/64/(-x^2+4)^2+1/128/(-x^2+4)+1/256*ln(x)-1/512*ln(-x^2+4)

Rubi [A]

time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {272, 46}

$$\frac{1}{128(4-x^2)} + \frac{1}{64(4-x^2)^2} + \frac{1}{24(4-x^2)^3} - \frac{1}{512} \log(4-x^2) + \frac{\log(x)}{256}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(-4 + x^2)^4),x]

[Out] 1/(24*(4 - x^2)^3) + 1/(64*(4 - x^2)^2) + 1/(128*(4 - x^2)) + Log[x]/256 - Log[4 - x^2]/512

Rule 46

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{1}{x(-4+x^2)^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(-4+x)^4 x} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{4(-4+x)^4} - \frac{1}{16(-4+x)^3} + \frac{1}{64(-4+x)^2} - \frac{1}{256(-4+x)} + \frac{1}{256x} \right) dx, x, x^2 \right) \\ &= \frac{1}{24(4-x^2)^3} + \frac{1}{64(4-x^2)^2} + \frac{1}{128(4-x^2)} + \frac{\log(x)}{256} - \frac{1}{512} \log(4-x^2) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 40, normalized size = 0.69

$$\frac{-\frac{4(88-30x^2+3x^4)}{(-4+x^2)^3} + 6 \log(x) - 3 \log(4-x^2)}{1536}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x*(-4 + x^2)^4), x]``[Out] ((-4*(88 - 30*x^2 + 3*x^4))/(-4 + x^2)^3 + 6*Log[x] - 3*Log[4 - x^2])/1536`**Mathics [A]**

time = 1.93, size = 61, normalized size = 1.05

$$\frac{-352 + 120x^2 - 12x^4 + 3(-64 + 48x^2 - 12x^4 + x^6)(-\text{Log}[-4 + x^2] + 2\text{Log}[x])}{-98304 + 73728x^2 - 18432x^4 + 1536x^6}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x*(x^2 - 4)^4), x]')``[Out] (-352 + 120 x ^ 2 - 12 x ^ 4 + 3 (-64 + 48 x ^ 2 - 12 x ^ 4 + x ^ 6) (-Log[-4 + x ^ 2] + 2 Log[x])) / (1536 (-64 + 48 x ^ 2 - 12 x ^ 4 + x ^ 6))`**Maple [A]**

time = 0.09, size = 60, normalized size = 1.03

method	result
risch	$\frac{-\frac{1}{128}x^4 + \frac{5}{64}x^2 - \frac{11}{48}}{(x^2-4)^3} + \frac{\ln(x)}{256} - \frac{\ln(x^2-4)}{512}$
norman	$\frac{-\frac{1}{128}x^4 + \frac{5}{64}x^2 - \frac{11}{48}}{(x^2-4)^3} + \frac{\ln(x)}{256} - \frac{\ln(-2+x)}{512} - \frac{\ln(2+x)}{512}$
meijerg	$\frac{11}{3072} + \frac{\ln(x)}{256} - \frac{\ln(2)}{256} + \frac{i\pi}{512} + \frac{x^2(\frac{11}{16}x^4 - \frac{27}{4}x^2 + 18)}{12288(1-\frac{x^2}{4})^3} - \frac{\ln(1-\frac{x^2}{4})}{512}$
default	$\frac{\ln(x)}{256} - \frac{1}{1536(-2+x)^3} + \frac{3}{2048(-2+x)^2} - \frac{11}{4096(-2+x)} - \frac{\ln(-2+x)}{512} + \frac{1}{1536(2+x)^3} + \frac{3}{2048(2+x)^2} + \frac{11}{4096(2+x)} - \frac{\ln(2)}{512}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/x/(x^2-4)^4,x,method=_RETURNVERBOSE)``[Out] 1/256*ln(x)-1/1536/(-2+x)^3+3/2048/(-2+x)^2-11/4096/(-2+x)-1/512*ln(-2+x)+1/1536/(2+x)^3+3/2048/(2+x)^2+11/4096/(2+x)-1/512*ln(2+x)`**Maxima [A]**

time = 0.25, size = 46, normalized size = 0.79

$$-\frac{3x^4 - 30x^2 + 88}{384(x^6 - 12x^4 + 48x^2 - 64)} - \frac{1}{512} \log(x^2 - 4) + \frac{1}{512} \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^2-4)^4,x, algorithm="maxima")

[Out] $-1/384*(3*x^4 - 30*x^2 + 88)/(x^6 - 12*x^4 + 48*x^2 - 64) - 1/512*\log(x^2 - 4) + 1/512*\log(x^2)$

Fricas [A]

time = 0.30, size = 73, normalized size = 1.26

$$\frac{12x^4 - 120x^2 + 3(x^6 - 12x^4 + 48x^2 - 64)\log(x^2 - 4) - 6(x^6 - 12x^4 + 48x^2 - 64)\log(x) + 352}{1536(x^6 - 12x^4 + 48x^2 - 64)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^2-4)^4,x, algorithm="fricas")

[Out] $-1/1536*(12*x^4 - 120*x^2 + 3*(x^6 - 12*x^4 + 48*x^2 - 64)*\log(x^2 - 4) - 6*(x^6 - 12*x^4 + 48*x^2 - 64)*\log(x) + 352)/(x^6 - 12*x^4 + 48*x^2 - 64)$

Sympy [A]

time = 0.08, size = 41, normalized size = 0.71

$$\frac{-3x^4 + 30x^2 - 88}{384x^6 - 4608x^4 + 18432x^2 - 24576} + \frac{\log(x)}{256} - \frac{\log(x^2 - 4)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x**2-4)**4,x)

[Out] $(-3*x**4 + 30*x**2 - 88)/(384*x**6 - 4608*x**4 + 18432*x**2 - 24576) + \log(x)/256 - \log(x**2 - 4)/512$

Giac [A]

time = 0.00, size = 48, normalized size = 0.83

$$\frac{\ln(x^2)}{512} - \frac{\ln|x^2 - 4|}{512} + \frac{11x^6 - 156x^4 + 768x^2 - 1408}{3072(x^2 - 4)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^2-4)^4,x)

[Out] $1/3072*(11*x^6 - 156*x^4 + 768*x^2 - 1408)/(x^2 - 4)^3 + 1/512*\log(x^2) - 1/512*\log(\text{abs}(x^2 - 4))$

Mupad [B]

time = 0.08, size = 44, normalized size = 0.76

$$\frac{\ln(x)}{256} - \frac{\ln(x^2 - 4)}{512} - \frac{\frac{x^4}{128} - \frac{5x^2}{64} + \frac{11}{48}}{x^6 - 12x^4 + 48x^2 - 64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 - 4)^4),x)
```

```
[Out] log(x)/256 - log(x^2 - 4)/512 - (x^4/128 - (5*x^2)/64 + 11/48)/(48*x^2 - 12  
*x^4 + x^6 - 64)
```

$$3.461 \quad \int \frac{1}{x(-2+x^2)^{5/2}} dx$$

Optimal. Leaf size=52

$$-\frac{1}{6(-2+x^2)^{3/2}} + \frac{1}{4\sqrt{-2+x^2}} + \frac{\tan^{-1}\left(\frac{\sqrt{-2+x^2}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

[Out] -1/6/(x^2-2)^(3/2)+1/8*arctan(1/2*(x^2-2)^(1/2)*2^(1/2))*2^(1/2)+1/4/(x^2-2)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {272, 53, 65, 209}

$$\frac{1}{4\sqrt{x^2-2}} - \frac{1}{6(x^2-2)^{3/2}} + \frac{\tan^{-1}\left(\frac{\sqrt{x^2-2}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(-2 + x^2)^(5/2)), x]

[Out] -1/6*1/(-2 + x^2)^(3/2) + 1/(4*Sqrt[-2 + x^2]) + ArcTan[Sqrt[-2 + x^2]/Sqrt[2]]/(4*Sqrt[2])

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[
n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(-2+x^2)^{5/2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(-2+x)^{5/2}x} dx, x, x^2 \right) \\
 &= -\frac{1}{6(-2+x^2)^{3/2}} - \frac{1}{4} \text{Subst} \left(\int \frac{1}{(-2+x)^{3/2}x} dx, x, x^2 \right) \\
 &= -\frac{1}{6(-2+x^2)^{3/2}} + \frac{1}{4\sqrt{-2+x^2}} + \frac{1}{8} \text{Subst} \left(\int \frac{1}{\sqrt{-2+x}x} dx, x, x^2 \right) \\
 &= -\frac{1}{6(-2+x^2)^{3/2}} + \frac{1}{4\sqrt{-2+x^2}} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{2+x^2} dx, x, \sqrt{-2+x^2} \right) \\
 &= -\frac{1}{6(-2+x^2)^{3/2}} + \frac{1}{4\sqrt{-2+x^2}} + \frac{\tan^{-1} \left(\frac{\sqrt{-2+x^2}}{\sqrt{2}} \right)}{4\sqrt{2}}
 \end{aligned}$$

Mathematica [A]

time = 0.04, size = 46, normalized size = 0.88

$$\frac{-8 + 3x^2}{12(-2 + x^2)^{3/2}} + \frac{\tan^{-1} \left(\frac{\sqrt{-2 + x^2}}{\sqrt{2}} \right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-2 + x^2)^(5/2)), x]
```

```
[Out] (-8 + 3*x^2)/(12*(-2 + x^2)^(3/2)) + ArcTan[Sqrt[-2 + x^2]/Sqrt[2]]/(4*Sqrt[2])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 6.71, size = 613, normalized size = 11.79

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x*(x^2 - 2)^(5/2)),x]')

[Out] Piecewise[{{(12 I Sqrt[2] x ^ 2 Log[x ^ 2] + 12 x ^ 2 Sqrt[-2 + x ^ 2] - 3 I Sqrt[2] x ^ 4 Log[x ^ 2] + 6 Sqrt[2] x ^ 4 (-ArcSin[Sqrt[2] / x] + I Log[x])) - 32 Sqrt[-2 + x ^ 2] - 12 I Sqrt[2] Log[x ^ 2] + 24 Sqrt[2] (-I x ^ 2 Log[x] + x ^ 2 ArcSin[Sqrt[2] / x] - ArcSin[Sqrt[2] / x] + I Log[x])) / (48 (4 - 4 x ^ 2 + x ^ 4)), Abs[x ^ 2] > 2}}, -24 I x ^ 2 Log[1 + Sqrt[1 - x ^ 2 / 2]] / (96 Sqrt[2] - 96 Sqrt[2] x ^ 2 + 24 Sqrt[2] x ^ 4) - 12 I x ^ 2 Log[2] / (96 Sqrt[2] - 96 Sqrt[2] x ^ 2 + 24 Sqrt[2] x ^ 4) + I 6 Sqrt[2] x ^ 2 Sqrt[2 - x ^ 2] / (96 Sqrt[2] - 96 Sqrt[2] x ^ 2 + 24 Sqrt[2] x ^ 4) + I 12 x ^ 2 Log[x ^ 2] / (96 Sqrt[2] - 96 Sqrt[2] x ^ 2 + 24 Sqrt[2] x ^ 4) - 3 I x ^ 4 Log[x ^ 2] / (96 Sqrt[2] - 96 Sqrt[2] x ^ 2 + 24 Sqrt[2] x ^ 4) + I 3 x ^ 4 Log[2] / (96 Sqrt[2] - 96 Sqrt[2] x ^ 2 + 24 Sqrt[2] x ^ 4) + I 6 x ^ 4 Log[1 + Sqrt[1 - x ^ 2 / 2]] / (96 Sqrt[2] - 96 Sqrt[2] x ^ 2 + 24 Sqrt[2] x ^ 4) - 12 Pi / (96 Sqrt[2] - 96 Sqrt[2] x ^ 2 + 24 Sqrt[2] x ^ 4) + 12 Pi x ^ 2 / (96 Sqrt[2] - 96 Sqrt[2] x ^ 2 + 24 Sqrt[2] x ^ 4) - 3 Pi x ^ 4 / (96 Sqrt[2] - 96 Sqrt[2] x ^ 2 + 24 Sqrt[2] x ^ 4) - 16 I Sqrt[2] Sqrt[2 - x ^ 2] / (96 Sqrt[2] - 96 Sqrt[2] x ^ 2 + 24 Sqrt[2] x ^ 4) - 12 I Log[x ^ 2] / (96 Sqrt[2] - 96 Sqrt[2] x ^ 2 + 24 Sqrt[2] x ^ 4) + I 12 Log[2] / (96 Sqrt[2] - 96 Sqrt[2] x ^ 2 + 24 Sqrt[2] x ^ 4) + I 24 Log[1 + Sqrt[1 - x ^ 2 / 2]] / (96 Sqrt[2] - 96 Sqrt[2] x ^ 2 + 24 Sqrt[2] x ^ 4)}

Maple [A]

time = 0.11, size = 37, normalized size = 0.71

method	result	si
risch	$\frac{3x^2-8}{12(x^2-2)^{\frac{3}{2}}} - \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}}{\sqrt{x^2-2}}\right)}{8}$	33
default	$-\frac{1}{6(x^2-2)^{\frac{3}{2}}} + \frac{1}{4\sqrt{x^2-2}} - \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}}{\sqrt{x^2-2}}\right)}{8}$	37
trager	$\frac{3x^2-8}{12(x^2-2)^{\frac{3}{2}}} - \frac{\text{RootOf}(-Z^2+2) \ln\left(\frac{\sqrt{x^2-2} - \text{RootOf}(-Z^2+2)}{x}\right)}{8}$	47
meijerg	$\frac{\sqrt{2} \left(-\text{signum}\left(-1+\frac{x^2}{2}\right)\right)^{\frac{5}{2}} \left(\frac{3\left(\frac{8}{3}-3\ln(2)+2\ln(x)+i\pi\right)\sqrt{\pi}}{4} - 2\sqrt{\pi} + \frac{\sqrt{\pi}(-6x^2+16)}{8\left(-\frac{x^2}{2}+1\right)^{\frac{3}{2}}}\right) - \frac{3\sqrt{\pi} \ln\left(\frac{1}{2} + \frac{\sqrt{-\frac{x^2}{2}+1}}{2}\right)}{2}}{12\sqrt{\pi} \text{signum}\left(-1+\frac{x^2}{2}\right)^{\frac{5}{2}}}$	96


```

*x**2*log(x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 24*x**2*
asin(sqrt(2)/x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 16*sqrt(
2)*sqrt(x**2 - 2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 24*I*1
og(x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 12*I*log(x**2)/(24
*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 24*asin(sqrt(2)/x)/(24*sqrt
(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)), Abs(x**2) > 2), (-3*I*x**4*log(x*
2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 6*I*x**4*log(sqrt(1
- x**2/2) + 1)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 3*pi*x**4
/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 3*I*x**4*log(2)/(24*sqrt
(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 6*sqrt(2)*I*x**2*sqrt(2 - x**2)
/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 12*I*x**2*log(x**2)/(24
*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 24*I*x**2*log(sqrt(1 - x**2
/2) + 1)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 12*pi*x**2/(24*
sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 12*I*x**2*log(2)/(24*sqrt(2)
*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 16*sqrt(2)*I*sqrt(2 - x**2)/(24*sqrt
(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 12*I*log(x**2)/(24*sqrt(2)*x**4
- 96*sqrt(2)*x**2 + 96*sqrt(2)) + 24*I*log(sqrt(1 - x**2/2) + 1)/(24*sqrt(
2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 12*pi/(24*sqrt(2)*x**4 - 96*sqrt(
2)*x**2 + 96*sqrt(2)) + 12*I*log(2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96
*sqrt(2)), True))

```

Giac [A]

time = 0.00, size = 55, normalized size = 1.06

$$\frac{3(x^2 - 2) - 2}{12\sqrt{x^2 - 2}(x^2 - 2)} + \frac{\arctan\left(\frac{\sqrt{x^2 - 2}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^2-2)^(5/2), x)

[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x^2 - 2)) + 1/12*(3*x^2 - 8)/(x^2 - 2)^(3/2)

Mupad [B]

time = 0.47, size = 34, normalized size = 0.65

$$\frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{x^2 - 2}}{2}\right)}{8} + \frac{\frac{x^2}{4} - \frac{2}{3}}{(x^2 - 2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(x^2 - 2)^(5/2)), x)

[Out] (2^(1/2)*atan((2^(1/2)*(x^2 - 2)^(1/2))/2))/8 + (x^2/4 - 2/3)/(x^2 - 2)^(3/2)

$$3.462 \quad \int \frac{(-10+x^2)^{5/2}}{x} dx$$

Optimal. Leaf size=61

$$100\sqrt{-10+x^2} - \frac{10}{3}(-10+x^2)^{3/2} + \frac{1}{5}(-10+x^2)^{5/2} - 100\sqrt{10} \tan^{-1}\left(\frac{\sqrt{-10+x^2}}{\sqrt{10}}\right)$$

[Out] -10/3*(x^2-10)^(3/2)+1/5*(x^2-10)^(5/2)-100*arctan(1/10*(x^2-10)^(1/2)*10^(1/2))*10^(1/2)+100*(x^2-10)^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {272, 52, 65, 209}

$$\frac{1}{5}(x^2-10)^{5/2} - \frac{10}{3}(x^2-10)^{3/2} + 100\sqrt{x^2-10} - 100\sqrt{10} \tan^{-1}\left(\frac{\sqrt{x^2-10}}{\sqrt{10}}\right)$$

Antiderivative was successfully verified.

[In] Int[(-10 + x^2)^(5/2)/x,x]

[Out] 100*Sqrt[-10 + x^2] - (10*(-10 + x^2)^(3/2))/3 + (-10 + x^2)^(5/2)/5 - 100*Sqrt[10]*ArcTan[Sqrt[-10 + x^2]/Sqrt[10]]

Rule 52

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-10 + x^2)^{5/2}}{x} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(-10 + x)^{5/2}}{x} dx, x, x^2 \right) \\
&= \frac{1}{5} (-10 + x^2)^{5/2} - 5 \text{Subst} \left(\int \frac{(-10 + x)^{3/2}}{x} dx, x, x^2 \right) \\
&= -\frac{10}{3} (-10 + x^2)^{3/2} + \frac{1}{5} (-10 + x^2)^{5/2} + 50 \text{Subst} \left(\int \frac{\sqrt{-10 + x}}{x} dx, x, x^2 \right) \\
&= 100\sqrt{-10 + x^2} - \frac{10}{3} (-10 + x^2)^{3/2} + \frac{1}{5} (-10 + x^2)^{5/2} - 500 \text{Subst} \left(\int \frac{1}{\sqrt{-10 + x}} dx, x, x^2 \right) \\
&= 100\sqrt{-10 + x^2} - \frac{10}{3} (-10 + x^2)^{3/2} + \frac{1}{5} (-10 + x^2)^{5/2} - 1000 \text{Subst} \left(\int \frac{1}{10 + x^2} dx, x, x^2 \right) \\
&= 100\sqrt{-10 + x^2} - \frac{10}{3} (-10 + x^2)^{3/2} + \frac{1}{5} (-10 + x^2)^{5/2} - 100\sqrt{10} \tan^{-1} \left(\frac{\sqrt{-10 + x^2}}{\sqrt{10}} \right)
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 49, normalized size = 0.80

$$\frac{1}{15} \sqrt{-10 + x^2} (2300 - 110x^2 + 3x^4) - 100\sqrt{10} \tan^{-1} \left(\frac{\sqrt{-10 + x^2}}{\sqrt{10}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-10 + x^2)^(5/2)/x,x]

[Out] (Sqrt[-10 + x^2]*(2300 - 110*x^2 + 3*x^4))/15 - 100*Sqrt[10]*ArcTan[Sqrt[-10 + x^2]/Sqrt[10]]

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 5.72, size = 138, normalized size = 2.26

Piecewise[{{{\frac{-22x^2\sqrt{-10+x^2}}{3} + \frac{x^4\sqrt{-10+x^2}}{5} - 100I\sqrt{10}\text{Log}[x] + 50I\sqrt{10}\text{Log}[x^2] + 100\sqrt{10}\text{ArcSin}\left[\frac{\sqrt{10}}{x}\right] + \frac{460\sqrt{-10+x^2}}{3}, \text{Abs}[x^2] > 10}}, \frac{-22Ix^2\sqrt{10-x^2}}{3} + \frac{Ix^4\sqrt{10-x^2}}{5} - 100I\sqrt{10}\text{Log}\left[1 + \sqrt{1 - \frac{x^2}{10}}\right] + I50\sqrt{10}\text{Log}[x^2] + \frac{I460\sqrt{10-x^2}}{3}}

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^2 - 10)^(5/2)/x,x]')
```

```
[Out] Piecewise[{{-22 x ^ 2 Sqrt[-10 + x ^ 2] / 3 + x ^ 4 Sqrt[-10 + x ^ 2] / 5 -
100 I Sqrt[10] Log[x] + 50 I Sqrt[10] Log[x ^ 2] + 100 Sqrt[10] ArcSin[Sqr
t[10] / x] + 460 Sqrt[-10 + x ^ 2] / 3, Abs[x ^ 2] > 10}}, -22 I x ^ 2 Sqrt
[10 - x ^ 2] / 3 + I x ^ 4 Sqrt[10 - x ^ 2] / 5 - 100 I Sqrt[10] Log[1 + Sq
rt[1 - x ^ 2 / 10]] + I 50 Sqrt[10] Log[x ^ 2] + I 460 Sqrt[10 - x ^ 2] / 3
]
```

Maple [A]

time = 0.14, size = 46, normalized size = 0.75

method	result
default	$\frac{(x^2-10)^{\frac{5}{2}}}{5} - \frac{10(x^2-10)^{\frac{3}{2}}}{3} + 100\sqrt{x^2-10} + 100\sqrt{10} \arctan\left(\frac{\sqrt{10}}{\sqrt{x^2-10}}\right)$
trager	$\left(\frac{1}{5}x^4 - \frac{22}{3}x^2 + \frac{460}{3}\right)\sqrt{x^2-10} - 100\text{RootOf}\left(_Z^2 + 10\right)\ln\left(\frac{\text{RootOf}\left(_Z^2 + 10\right) + \sqrt{x^2-10}}{x}\right)$
meijerg	$\frac{375\sqrt{2}\sqrt{5}\text{signum}\left(-1 + \frac{x^2}{10}\right)^{\frac{5}{2}} - \frac{8\left(\frac{46}{15} - 3\ln(2) + 2\ln(x) - \ln(5) + i\pi\right)\sqrt{\pi}}{15} + \frac{368\sqrt{\pi}}{225} - \frac{4\sqrt{\pi}\left(\frac{3}{25}x^4 - \frac{22}{5}x^2 + 92\right)\sqrt{1 - \frac{x^2}{10}}}{225}}{4\sqrt{\pi}\left(-\text{signum}\left(-1 + \frac{x^2}{10}\right)\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-10)^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(x^2-10)^(5/2)-10/3*(x^2-10)^(3/2)+100*(x^2-10)^(1/2)+100*10^(1/2)*arct
an(1/(x^2-10)^(1/2)*10^(1/2))
```

Maxima [A]

time = 0.32, size = 42, normalized size = 0.69

$$\frac{1}{5}(x^2-10)^{\frac{5}{2}} - \frac{10}{3}(x^2-10)^{\frac{3}{2}} + 100\sqrt{10} \arcsin\left(\frac{\sqrt{10}}{|x|}\right) + 100\sqrt{x^2-10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 1/5*(x^2 - 10)^(5/2) - 10/3*(x^2 - 10)^(3/2) + 100*sqrt(10)*arcsin(sqrt(10)
/abs(x)) + 100*sqrt(x^2 - 10)
```

Fricas [A]

time = 0.61, size = 47, normalized size = 0.77

$$\frac{1}{15} (3x^4 - 110x^2 + 2300) \sqrt{x^2 - 10} - 200 \sqrt{10} \arctan \left(-\frac{1}{10} \sqrt{10} x + \frac{1}{10} \sqrt{10} \sqrt{x^2 - 10} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((x^2-10)^(5/2)/x,x, algorithm="fricas")``[Out] 1/15*(3*x^4 - 110*x^2 + 2300)*sqrt(x^2 - 10) - 200*sqrt(10)*arctan(-1/10*sqrt(10)*x + 1/10*sqrt(10)*sqrt(x^2 - 10))`**Sympy [C] Result contains complex when optimal does not.**

time = 3.93, size = 165, normalized size = 2.70

$$\begin{cases} \frac{x^4 \sqrt{x^2 - 10}}{5} - \frac{22x^2 \sqrt{x^2 - 10}}{3} + \frac{460 \sqrt{x^2 - 10}}{3} - 100 \sqrt{10} i \log(x) + 50 \sqrt{10} i \log(x^2) + 100 \sqrt{10} \operatorname{asin} \left(\frac{\sqrt{10}}{x} \right) & \text{for } |x^2| > 10 \\ ix^4 \sqrt{10 - x^2} - \frac{22ix^2 \sqrt{10 - x^2}}{3} + \frac{460i \sqrt{10 - x^2}}{3} + 50 \sqrt{10} i \log(x^2) - 100 \sqrt{10} i \log \left(\sqrt{1 - \frac{x^2}{10}} + 1 \right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((x**2-10)**(5/2)/x,x)``[Out] Piecewise((x**4*sqrt(x**2 - 10)/5 - 22*x**2*sqrt(x**2 - 10)/3 + 460*sqrt(x**2 - 10)/3 - 100*sqrt(10)*I*log(x) + 50*sqrt(10)*I*log(x**2) + 100*sqrt(10)*asin(sqrt(10)/x), Abs(x**2) > 10), (I*x**4*sqrt(10 - x**2)/5 - 22*I*x**2*sqrt(10 - x**2)/3 + 460*I*sqrt(10 - x**2)/3 + 50*sqrt(10)*I*log(x**2) - 100*sqrt(10)*I*log(sqrt(1 - x**2/10) + 1), True))`**Giac [A]**

time = 0.00, size = 76, normalized size = 1.25

$$\frac{1}{5} \sqrt{x^2 - 10} (x^2 - 10)^2 - \frac{10}{3} \sqrt{x^2 - 10} (x^2 - 10) + 100 \sqrt{x^2 - 10} - \frac{1000 \arctan \left(\frac{\sqrt{x^2 - 10}}{\sqrt{10}} \right)}{\sqrt{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((x^2-10)^(5/2)/x,x)``[Out] 1/5*(x^2 - 10)^(5/2) - 10/3*(x^2 - 10)^(3/2) - 100*sqrt(10)*arctan(1/10*sqrt(10)*sqrt(x^2 - 10)) + 100*sqrt(x^2 - 10)`**Mupad [B]**

time = 0.47, size = 46, normalized size = 0.75

$$100 \sqrt{x^2 - 10} - 100 \sqrt{10} \operatorname{atan} \left(\frac{\sqrt{10} \sqrt{x^2 - 10}}{10} \right) - \frac{10 (x^2 - 10)^{3/2}}{3} + \frac{(x^2 - 10)^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 - 10)^(5/2)/x,x)`

[Out] $100*(x^2 - 10)^{(1/2)} - 100*10^{(1/2)}*\operatorname{atan}((10^{(1/2)}*(x^2 - 10)^{(1/2)})/10) - (10*(x^2 - 10)^{(3/2)})/3 + (x^2 - 10)^{(5/2)}/5$

3.463 $\int x^{1+2n} dx$

Optimal. Leaf size=16

$$\frac{x^{2(1+n)}}{2(1+n)}$$

[Out] $1/2*x^{(2+2*n)}/(1+n)$

Rubi [A]

time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {30}

$$\frac{x^{2(n+1)}}{2(n+1)}$$

Antiderivative was successfully verified.

[In] Int[x^(1 + 2*n), x]

[Out] x^(2*(1 + n))/(2*(1 + n))

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\int x^{1+2n} dx = \frac{x^{2(1+n)}}{2(1+n)}$$

Mathematica [A]

time = 0.00, size = 15, normalized size = 0.94

$$\frac{x^{2+2n}}{2+2n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(1 + 2*n), x]

[Out] x^(2 + 2*n)/(2 + 2*n)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.66, size = 22, normalized size = 1.38

$$\text{Piecewise} \left[\left\{ \left\{ \frac{x^{2+2n}}{2(1+n)}, n \neq -1 \right\} \right\}, \text{Log}[x] \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[x^(2*n + 1),x]')`

[Out] `Piecewise[{{x ^ (2 + 2 n) / (2 (1 + n)), n != -1}}, Log[x]]`

Maple [A]

time = 0.01, size = 16, normalized size = 1.00

method	result	size
gospers	$\frac{x^{2+2n}}{2+2n}$	15
default	$\frac{x^{2+2n}}{2+2n}$	16
risch	$\frac{x x^{1+2n}}{2+2n}$	16
norman	$\frac{x e^{(1+2n) \ln(x)}}{2+2n}$	18

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1+2*n),x,method=_RETURNVERBOSE)`

[Out] `x^(2+2*n)/(2+2*n)`

Maxima [A]

time = 0.26, size = 14, normalized size = 0.88

$$\frac{x^{2n+2}}{2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1+2*n),x, algorithm="maxima")`

[Out] `1/2*x^(2*n + 2)/(n + 1)`

Fricas [A]

time = 0.31, size = 15, normalized size = 0.94

$$\frac{x x^{2n+1}}{2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1+2*n),x, algorithm="fricas")

[Out] 1/2*x*x^(2*n + 1)/(n + 1)

Sympy [A]

time = 0.03, size = 15, normalized size = 0.94

$$\begin{cases} \frac{x^{2n+2}}{2n+2} & \text{for } n \neq -1 \\ \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1+2*n),x)

[Out] Piecewise((x**(2*n + 2)/(2*n + 2), Ne(n, -1)), (log(x), True))

Giac [A]

time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^{1+2n+1}}{1 + 2n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1+2*n),x)

[Out] 1/2*x^(2*n + 2)/(n + 1)

Mupad [B]

time = 0.46, size = 24, normalized size = 1.50

$$\begin{cases} \ln(x) & \text{if } n = -1 \\ \frac{x^{2n+2}}{2(n+1)} & \text{if } n \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n + 1),x)

[Out] piecewise(n == -1, log(x), n ~= -1, x^(2*n + 2)/(2*(n + 1)))

$$3.464 \quad \int \frac{x^7}{(-5+x^2)^3} dx$$

Optimal. Leaf size=46

$$\frac{x^2}{2} - \frac{125}{4(5-x^2)^2} + \frac{75}{2(5-x^2)} + \frac{15}{2} \log(5-x^2)$$

[Out] 1/2*x^2-125/4/(-x^2+5)^2+75/2/(-x^2+5)+15/2*ln(-x^2+5)

Rubi [A]

time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {272, 45}

$$\frac{x^2}{2} + \frac{75}{2(5-x^2)} - \frac{125}{4(5-x^2)^2} + \frac{15}{2} \log(5-x^2)$$

Antiderivative was successfully verified.

[In] Int[x^7/(-5 + x^2)^3,x]

[Out] x^2/2 - 125/(4*(5 - x^2)^2) + 75/(2*(5 - x^2)) + (15*Log[5 - x^2])/2

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^7}{(-5+x^2)^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^3}{(-5+x)^3} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(1 + \frac{125}{(-5+x)^3} + \frac{75}{(-5+x)^2} + \frac{15}{-5+x} \right) dx, x, x^2 \right) \\ &= \frac{x^2}{2} - \frac{125}{4(5-x^2)^2} + \frac{75}{2(5-x^2)} + \frac{15}{2} \log(5-x^2) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 36, normalized size = 0.78

$$\frac{1}{4} \left(2x^2 - \frac{125}{(-5 + x^2)^2} - \frac{150}{-5 + x^2} + 30 \log(-5 + x^2) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[x^7/(-5 + x^2)^3,x]``[Out] (2*x^2 - 125/(-5 + x^2)^2 - 150/(-5 + x^2) + 30*Log[-5 + x^2])/4`**Mathics [A]**

time = 1.86, size = 45, normalized size = 0.98

$$\frac{625 - 150x^2 + 2(25 - 10x^2 + x^4)(x^2 + 15\text{Log}[-5 + x^2])}{100 - 40x^2 + 4x^4}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^7/(x^2 - 5)^3,x]')``[Out] (625 - 150 x ^ 2 + 2 (25 - 10 x ^ 2 + x ^ 4) (x ^ 2 + 15 Log[-5 + x ^ 2])) / (4 (25 - 10 x ^ 2 + x ^ 4))`**Maple [A]**

time = 0.08, size = 33, normalized size = 0.72

method	result	size
norman	$\frac{-75x^2 + \frac{1}{2}x^6 + \frac{1125}{4}}{(x^2-5)^2} + \frac{15 \ln(x^2-5)}{2}$	30
risch	$\frac{x^2}{2} + \frac{-75x^2 + \frac{625}{4}}{(x^2-5)^2} + \frac{15 \ln(x^2-5)}{2}$	30
default	$\frac{x^2}{2} - \frac{125}{4(x^2-5)^2} + \frac{15 \ln(x^2-5)}{2} - \frac{75}{2(x^2-5)}$	33
meijerg	$\frac{x^2(\frac{4}{25}x^4 - \frac{18}{5}x^2 + 12)}{8(-\frac{x^2}{5} + 1)^2} + \frac{15 \ln(-\frac{x^2}{5} + 1)}{2}$	38

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^7/(x^2-5)^3,x,method=_RETURNVERBOSE)``[Out] 1/2*x^2-125/4/(x^2-5)^2+15/2*ln(x^2-5)-75/2/(x^2-5)`**Maxima [A]**

time = 0.24, size = 35, normalized size = 0.76

$$\frac{1}{2}x^2 - \frac{25(6x^2 - 25)}{4(x^4 - 10x^2 + 25)} + \frac{15}{2} \log(x^2 - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^2-5)^3,x, algorithm="maxima")

[Out] 1/2*x^2 - 25/4*(6*x^2 - 25)/(x^4 - 10*x^2 + 25) + 15/2*log(x^2 - 5)

Fricas [A]

time = 0.31, size = 49, normalized size = 1.07

$$\frac{2x^6 - 20x^4 - 100x^2 + 30(x^4 - 10x^2 + 25)\log(x^2 - 5) + 625}{4(x^4 - 10x^2 + 25)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^2-5)^3,x, algorithm="fricas")

[Out] 1/4*(2*x^6 - 20*x^4 - 100*x^2 + 30*(x^4 - 10*x^2 + 25)*log(x^2 - 5) + 625)/(x^4 - 10*x^2 + 25)

Sympy [A]

time = 0.06, size = 32, normalized size = 0.70

$$\frac{x^2}{2} + \frac{625 - 150x^2}{4x^4 - 40x^2 + 100} + \frac{15 \log(x^2 - 5)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(x**2-5)**3,x)

[Out] x**2/2 + (625 - 150*x**2)/(4*x**4 - 40*x**2 + 100) + 15*log(x**2 - 5)/2

Giac [A]

time = 0.00, size = 42, normalized size = 0.91

$$\frac{x^2}{2} + \frac{-45x^4 + 300x^2 - 500}{4(x^2 - 5)^2} + \frac{15}{2} \ln|x^2 - 5|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^2-5)^3,x)

[Out] 1/2*x^2 - 5/4*(9*x^4 - 60*x^2 + 100)/(x^2 - 5)^2 + 15/2*log(abs(x^2 - 5))

Mupad [B]

time = 0.06, size = 35, normalized size = 0.76

$$\frac{15 \ln(x^2 - 5)}{2} - \frac{\frac{75x^2}{2} - \frac{625}{4}}{x^4 - 10x^2 + 25} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(x^2 - 5)^3,x)

[Out] (15*log(x^2 - 5))/2 - ((75*x^2)/2 - 625/4)/(x^4 - 10*x^2 + 25) + x^2/2

$$3.465 \quad \int \frac{-4x^3 + 3x^5}{(-1 + x^2)^5} dx$$

Optimal. Leaf size=40

$$\frac{1}{8(1-x^2)^4} + \frac{1}{3(1-x^2)^3} - \frac{3}{4(1-x^2)^2}$$

[Out] 1/8/(-x^2+1)^4+1/3/(-x^2+1)^3-3/4/(-x^2+1)^2

Rubi [A]

time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1607, 457, 78}

$$-\frac{3}{4(1-x^2)^2} + \frac{1}{3(1-x^2)^3} + \frac{1}{8(1-x^2)^4}$$

Antiderivative was successfully verified.

[In] Int[(-4*x^3 + 3*x^5)/(-1 + x^2)^5,x]

[Out] 1/(8*(1 - x^2)^4) + 1/(3*(1 - x^2)^3) - 3/(4*(1 - x^2)^2)

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-4x^3 + 3x^5}{(-1 + x^2)^5} dx &= \int \frac{x^3(-4 + 3x^2)}{(-1 + x^2)^5} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x(-4 + 3x)}{(-1 + x)^5} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(-\frac{1}{(-1 + x)^5} + \frac{2}{(-1 + x)^4} + \frac{3}{(-1 + x)^3} \right) dx, x, x^2 \right) \\
&= \frac{1}{8(1 - x^2)^4} + \frac{1}{3(1 - x^2)^3} - \frac{3}{4(1 - x^2)^2}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 23, normalized size = 0.58

$$\frac{-7 + 28x^2 - 18x^4}{24(-1 + x^2)^4}$$

Antiderivative was successfully verified.

`[In] Integrate[(-4*x^3 + 3*x^5)/(-1 + x^2)^5, x]``[Out] (-7 + 28*x^2 - 18*x^4)/(24*(-1 + x^2)^4)`**Mathics [A]**

time = 1.82, size = 36, normalized size = 0.90

$$\frac{-7 + 28x^2 - 18x^4}{24 - 96x^2 + 144x^4 - 96x^6 + 24x^8}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(3*x^5 - 4*x^3)/(x^2 - 1)^5, x]')``[Out] (-7 + 28 x ^ 2 - 18 x ^ 4) / (24 (1 - 4 x ^ 2 + 6 x ^ 4 - 4 x ^ 6 + x ^ 8))`**Maple [A]**

time = 0.07, size = 58, normalized size = 1.45

method	result	size
norman	$\frac{-\frac{3}{4}x^4 + \frac{7}{6}x^2 - \frac{7}{24}}{(x^2-1)^4}$	21
risch	$\frac{-\frac{3}{4}x^4 + \frac{7}{6}x^2 - \frac{7}{24}}{(x^2-1)^4}$	21
gosper	$-\frac{18x^4 - 28x^2 + 7}{24(x^2-1)^4}$	22

meijerg	$-\frac{x^6(-x^2+4)}{8(-x^2+1)^4} + \frac{x^4(x^4-4x^2+6)}{6(-x^2+1)^4}$	47
default	$\frac{1}{128(-1+x)^4} - \frac{11}{192(-1+x)^3} - \frac{27}{256(-1+x)^2} + \frac{27}{256(-1+x)} + \frac{1}{128(1+x)^4} + \frac{11}{192(1+x)^3} - \frac{27}{256(1+x)^2} - \frac{27}{256(1+x)}$	58

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^5-4*x^3)/(x^2-1)^5,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{128}(-1+x)^{-4} - \frac{11}{192}(-1+x)^{-3} - \frac{27}{256}(-1+x)^{-2} + \frac{27}{256}(-1+x)^{-1} + \frac{1}{128}(1+x)^{-4} + \frac{11}{192}(1+x)^{-3} - \frac{27}{256}(1+x)^{-2} - \frac{27}{256}(1+x)^{-1}$

Maxima [A]

time = 0.24, size = 36, normalized size = 0.90

$$-\frac{18x^4 - 28x^2 + 7}{24(x^8 - 4x^6 + 6x^4 - 4x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^5-4*x^3)/(x^2-1)^5,x, algorithm="maxima")`

[Out] $-1/24*(18*x^4 - 28*x^2 + 7)/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)$

Fricas [A]

time = 0.30, size = 36, normalized size = 0.90

$$-\frac{18x^4 - 28x^2 + 7}{24(x^8 - 4x^6 + 6x^4 - 4x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^5-4*x^3)/(x^2-1)^5,x, algorithm="fricas")`

[Out] $-1/24*(18*x^4 - 28*x^2 + 7)/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)$

Sympy [A]

time = 0.07, size = 32, normalized size = 0.80

$$\frac{-18x^4 + 28x^2 - 7}{24x^8 - 96x^6 + 144x^4 - 96x^2 + 24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**5-4*x**3)/(x**2-1)**5,x)`

[Out] $(-18*x**4 + 28*x**2 - 7)/(24*x**8 - 96*x**6 + 144*x**4 - 96*x**2 + 24)$

Giac [A]

time = 0.00, size = 24, normalized size = 0.60

$$\frac{-18x^4 + 28x^2 - 7}{24(x^2 - 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^5-4*x^3)/(x^2-1)^5,x)`

[Out] `-1/24*(18*x^4 - 28*x^2 + 7)/(x^2 - 1)^4`

Mupad [B]

time = 0.07, size = 36, normalized size = 0.90

$$-\frac{\frac{3x^4}{4} - \frac{7x^2}{6} + \frac{7}{24}}{x^8 - 4x^6 + 6x^4 - 4x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(4*x^3 - 3*x^5)/(x^2 - 1)^5,x)`

[Out] `-((3*x^4)/4 - (7*x^2)/6 + 7/24)/(6*x^4 - 4*x^2 - 4*x^6 + x^8 + 1)`

3.466 $\int x^3 (1 + x^2)^{9/14} dx$

Optimal. Leaf size=27

$$-\frac{7}{23} (1 + x^2)^{23/14} + \frac{7}{37} (1 + x^2)^{37/14}$$

[Out] $-7/23*(x^2+1)^{(23/14)}+7/37*(x^2+1)^{(37/14)}$

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{7}{37} (x^2 + 1)^{37/14} - \frac{7}{23} (x^2 + 1)^{23/14}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3*(1 + x^2)^{(9/14)}, x]$

[Out] $(-7*(1 + x^2)^{(23/14)})/23 + (7*(1 + x^2)^{(37/14)})/37$

Rule 45

$\text{Int}[(a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

Rule 272

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p, x}], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int x^3 (1 + x^2)^{9/14} dx &= \frac{1}{2} \text{Subst} \left(\int x(1 + x)^{9/14} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int (-(1 + x)^{9/14} + (1 + x)^{23/14}) dx, x, x^2 \right) \\ &= -\frac{7}{23} (1 + x^2)^{23/14} + \frac{7}{37} (1 + x^2)^{37/14} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 25, normalized size = 0.93

$$\frac{7}{851} (1 + x^2)^{9/14} (-14 + 9x^2 + 23x^4)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(1 + x^2)^(9/14),x]

[Out] (7*(1 + x^2)^(9/14)*(-14 + 9*x^2 + 23*x^4))/851

Mathics [A]

time = 4.78, size = 21, normalized size = 0.78

$$\frac{7(-14 + 9x^2 + 23x^4)(1 + x^2)^{\frac{9}{14}}}{851}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(1 + x^2)^(9/14)*x^3,x]')

[Out] 7 (-14 + 9 x ^ 2 + 23 x ^ 4) (1 + x ^ 2) ^ (9 / 14) / 851

Maple [A]

time = 0.08, size = 17, normalized size = 0.63

method	result	size
gospers	$\frac{7(x^2+1)^{\frac{23}{14}}(23x^2-14)}{851}$	17
meijerg	$\frac{x^4 \operatorname{hypergeom}\left(\left[-\frac{9}{14}, 2\right], [3], -x^2\right)}{4}$	17
trager	$\left(\frac{7}{37}x^4 + \frac{63}{851}x^2 - \frac{98}{851}\right)(x^2+1)^{\frac{9}{14}}$	21
risch	$\frac{7(x^2+1)^{\frac{9}{14}}(23x^4+9x^2-14)}{851}$	22

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(x^2+1)^(9/14),x,method=_RETURNVERBOSE)

[Out] 7/851*(x^2+1)^(23/14)*(23*x^2-14)

Maxima [A]

time = 0.24, size = 19, normalized size = 0.70

$$\frac{7}{37} (x^2 + 1)^{\frac{37}{14}} - \frac{7}{23} (x^2 + 1)^{\frac{23}{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(x^2+1)^(9/14),x, algorithm="maxima")

[Out] 7/37*(x^2 + 1)^(37/14) - 7/23*(x^2 + 1)^(23/14)

Fricas [A]

time = 0.31, size = 21, normalized size = 0.78

$$\frac{7}{851} (23x^4 + 9x^2 - 14)(x^2 + 1)^{\frac{9}{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(x^2+1)^(9/14),x, algorithm="fricas")

[Out] 7/851*(23*x^4 + 9*x^2 - 14)*(x^2 + 1)^(9/14)

Sympy [A]

time = 3.96, size = 41, normalized size = 1.52

$$\frac{7x^4(x^2 + 1)^{\frac{9}{14}}}{37} + \frac{63x^2(x^2 + 1)^{\frac{9}{14}}}{851} - \frac{98(x^2 + 1)^{\frac{9}{14}}}{851}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(x**2+1)**(9/14),x)

[Out] 7*x**4*(x**2 + 1)**(9/14)/37 + 63*x**2*(x**2 + 1)**(9/14)/851 - 98*(x**2 + 1)**(9/14)/851

Giac [A]

time = 0.00, size = 34, normalized size = 1.26

$$7 \left(\frac{\left((x^2 + 1)^{\frac{1}{14}} \right)^{37}}{37} - \frac{\left((x^2 + 1)^{\frac{1}{14}} \right)^{23}}{23} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(x^2+1)^(9/14),x)

[Out] 7/37*(x^2 + 1)^(37/14) - 7/23*(x^2 + 1)^(23/14)

Mupad [B]

time = 0.38, size = 20, normalized size = 0.74

$$(x^2 + 1)^{9/14} \left(\frac{7x^4}{37} + \frac{63x^2}{851} - \frac{98}{851} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(x^2 + 1)^(9/14),x)

[Out] (x^2 + 1)^(9/14)*((63*x^2)/851 + (7*x^4)/37 - 98/851)

$$3.467 \quad \int \frac{x^5}{(-4+x^2)^{13/6}} dx$$

Optimal. Leaf size=38

$$-\frac{48}{7(-4+x^2)^{7/6}} - \frac{24}{\sqrt[6]{-4+x^2}} + \frac{3}{5}(-4+x^2)^{5/6}$$

[Out] $-48/7/(x^2-4)^{(7/6)}-24/(x^2-4)^{(1/6)}+3/5*(x^2-4)^{(5/6)}$

Rubi [A]

time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{3}{5}(x^2-4)^{5/6} - \frac{24}{\sqrt[6]{x^2-4}} - \frac{48}{7(x^2-4)^{7/6}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(-4 + x^2)^(13/6), x]

[Out] $-48/(7*(-4 + x^2)^{(7/6)}) - 24/(-4 + x^2)^{(1/6)} + (3*(-4 + x^2)^{(5/6)})/5$

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^5}{(-4+x^2)^{13/6}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(-4+x)^{13/6}} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{16}{(-4+x)^{13/6}} + \frac{8}{(-4+x)^{7/6}} + \frac{1}{\sqrt[6]{-4+x}} \right) dx, x, x^2 \right) \\ &= -\frac{48}{7(-4+x^2)^{7/6}} - \frac{24}{\sqrt[6]{-4+x^2}} + \frac{3}{5}(-4+x^2)^{5/6} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 25, normalized size = 0.66

$$\frac{3(1152 - 336x^2 + 7x^4)}{35(-4 + x^2)^{7/6}}$$

Antiderivative was successfully verified.

`[In] Integrate[x^5/(-4 + x^2)^(13/6), x]``[Out] (3*(1152 - 336*x^2 + 7*x^4))/(35*(-4 + x^2)^(7/6))`**Mathics [A]**

time = 3.23, size = 21, normalized size = 0.55

$$\frac{3(1152 - 336x^2 + 7x^4)}{35(-4 + x^2)^{7/6}}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^5/(x^2 - 4)^(13/6), x]')``[Out] 3 (1152 - 336 x ^ 2 + 7 x ^ 4) / (35 (-4 + x ^ 2) ^ (7 / 6))`**Maple [A]**

time = 0.08, size = 22, normalized size = 0.58

method	result	size
trager	$\frac{\frac{3}{5}x^4 - \frac{144}{5}x^2 + \frac{3456}{35}}{(x^2-4)^{7/6}}$	22
risch	$\frac{\frac{3}{5}x^4 - \frac{144}{5}x^2 + \frac{3456}{35}}{(x^2-4)^{7/6}}$	22
gospers	$\frac{3(-2+x)(2+x)(7x^4-336x^2+1152)}{35(x^2-4)^{13/6}}$	28
meijerg	$\frac{2^{2/3} \left(-\operatorname{signum}\left(-1 + \frac{x^2}{4}\right) \right)^{13/6} x^6 \operatorname{hypergeom}\left(\left[\frac{13}{6}, 3\right], [4], \frac{x^2}{4}\right)}{192 \operatorname{signum}\left(-1 + \frac{x^2}{4}\right)^{13/6}}$	42

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^5/(x^2-4)^(13/6), x, method=_RETURNVERBOSE)``[Out] 3/35*(7*x^4-336*x^2+1152)/(x^2-4)^(7/6)`**Maxima [A]**

time = 0.26, size = 28, normalized size = 0.74

$$\frac{3}{5}(x^2 - 4)^{5/6} - \frac{24}{(x^2 - 4)^{1/6}} - \frac{48}{7(x^2 - 4)^{7/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(x^2-4)^(13/6),x, algorithm="maxima")`

[Out] $3/5*(x^2 - 4)^{(5/6)} - 24/(x^2 - 4)^{(1/6)} - 48/7/(x^2 - 4)^{(7/6)}$

Fricas [A]

time = 0.31, size = 33, normalized size = 0.87

$$\frac{3(7x^4 - 336x^2 + 1152)(x^2 - 4)^{\frac{5}{6}}}{35(x^4 - 8x^2 + 16)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(x^2-4)^(13/6),x, algorithm="fricas")`

[Out] $3/35*(7*x^4 - 336*x^2 + 1152)*(x^2 - 4)^{(5/6)}/(x^4 - 8*x^2 + 16)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(32) = 64$

time = 1.08, size = 82, normalized size = 2.16

$$\frac{21x^4}{35x^2\sqrt[6]{x^2 - 4} - 140\sqrt[6]{x^2 - 4}} - \frac{1008x^2}{35x^2\sqrt[6]{x^2 - 4} - 140\sqrt[6]{x^2 - 4}} + \frac{3456}{35x^2\sqrt[6]{x^2 - 4} - 140\sqrt[6]{x^2 - 4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(x**2-4)**(13/6),x)`

[Out] $21*x**4/(35*x**2*(x**2 - 4)**(1/6) - 140*(x**2 - 4)**(1/6)) - 1008*x**2/(35*x**2*(x**2 - 4)**(1/6) - 140*(x**2 - 4)**(1/6)) + 3456/(35*x**2*(x**2 - 4)**(1/6) - 140*(x**2 - 4)**(1/6))$

Giac [A]

time = 0.00, size = 46, normalized size = 1.21

$$3 \left(\frac{\left((x^2 - 4)^{\frac{1}{6}} \right)^5}{5} + \frac{-56(x^2 - 4) - 16}{7(x^2 - 4)^{\frac{1}{6}}(x^2 - 4)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(x^2-4)^(13/6),x)`

[Out] $3/5*(x^2 - 4)^{(5/6)} - 24/7*(7*x^2 - 26)/(x^2 - 4)^{(7/6)}$

Mupad [B]

time = 0.45, size = 21, normalized size = 0.55

$$\frac{3(7x^4 - 336x^2 + 1152)}{35(x^2 - 4)^{7/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^2 - 4)^(13/6),x)
```

```
[Out] (3*(7*x^4 - 336*x^2 + 1152))/(35*(x^2 - 4)^(7/6))
```

$$3.468 \quad \int \frac{1}{(1+2x^2)^{5/2}} dx$$

Optimal. Leaf size=33

$$\frac{x}{3(1+2x^2)^{3/2}} + \frac{2x}{3\sqrt{1+2x^2}}$$

[Out] 1/3*x/(2*x^2+1)^(3/2)+2/3*x/(2*x^2+1)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {198, 197}

$$\frac{2x}{3\sqrt{2x^2+1}} + \frac{x}{3(2x^2+1)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2)^(-5/2), x]

[Out] x/(3*(1 + 2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 + 2*x^2])

Rule 197

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 198

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+2x^2)^{5/2}} dx &= \frac{x}{3(1+2x^2)^{3/2}} + \frac{2}{3} \int \frac{1}{(1+2x^2)^{3/2}} dx \\ &= \frac{x}{3(1+2x^2)^{3/2}} + \frac{2x}{3\sqrt{1+2x^2}} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 24, normalized size = 0.73

$$\frac{3x + 4x^3}{3(1+2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^2)^(-5/2), x]

[Out] (3*x + 4*x^3)/(3*(1 + 2*x^2)^(3/2))

Mathics [A]

time = 3.11, size = 19, normalized size = 0.58

$$\frac{x(3 + 4x^2)}{3(1 + 2x^2)^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(1+2*x^2)^(5/2), x]')

[Out] x (3 + 4 x ^ 2) / (3 (1 + 2 x ^ 2) ^ (3 / 2))

Maple [A]

time = 0.07, size = 26, normalized size = 0.79

method	result	size
gosper	$\frac{x(4x^2+3)}{3(2x^2+1)^{\frac{3}{2}}}$	20
trager	$\frac{x(4x^2+3)}{3(2x^2+1)^{\frac{3}{2}}}$	20
meijerg	$\frac{x(4x^2+3)}{3(2x^2+1)^{\frac{3}{2}}}$	20
risch	$\frac{x(4x^2+3)}{3(2x^2+1)^{\frac{3}{2}}}$	20
default	$\frac{x}{3(2x^2+1)^{\frac{3}{2}}} + \frac{2x}{3\sqrt{2x^2+1}}$	26

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x^2+1)^(5/2), x, method=_RETURNVERBOSE)

[Out] 1/3*x/(2*x^2+1)^(3/2)+2/3*x/(2*x^2+1)^(1/2)

Maxima [A]

time = 0.24, size = 25, normalized size = 0.76

$$\frac{2x}{3\sqrt{2x^2+1}} + \frac{x}{3(2x^2+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2+1)^(5/2), x, algorithm="maxima")

[Out] 2/3*x/sqrt(2*x^2 + 1) + 1/3*x/(2*x^2 + 1)^(3/2)

Fricas [A]

time = 0.31, size = 34, normalized size = 1.03

$$\frac{(4x^3 + 3x)\sqrt{2x^2 + 1}}{3(4x^4 + 4x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2+1)^(5/2),x, algorithm="fricas")**[Out]** 1/3*(4*x^3 + 3*x)*sqrt(2*x^2 + 1)/(4*x^4 + 4*x^2 + 1)**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(27) = 54

time = 0.97, size = 61, normalized size = 1.85

$$\frac{4x^3}{6x^2\sqrt{2x^2+1} + 3\sqrt{2x^2+1}} + \frac{3x}{6x^2\sqrt{2x^2+1} + 3\sqrt{2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x**2+1)**(5/2),x)**[Out]** 4*x**3/(6*x**2*sqrt(2*x**2 + 1) + 3*sqrt(2*x**2 + 1)) + 3*x/(6*x**2*sqrt(2*x**2 + 1) + 3*sqrt(2*x**2 + 1))**Giac [A]**

time = 0.00, size = 33, normalized size = 1.00

$$\frac{2\left(\frac{2}{3}xx + \frac{1}{2}\right)x\sqrt{2x^2 + 1}}{(2x^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2+1)^(5/2),x)**[Out]** 1/3*(4*x^2 + 3)*x/(2*x^2 + 1)^(3/2)**Mupad [B]**

time = 0.04, size = 99, normalized size = 3.00

$$\frac{\sqrt{x^2 + \frac{1}{2}} \operatorname{li}}{24 \left(-x^2 + \operatorname{li} \sqrt{2} x + \frac{1}{2}\right)} + \frac{\sqrt{x^2 + \frac{1}{2}} \operatorname{li}}{24 \left(x^2 + \operatorname{li} \sqrt{2} x - \frac{1}{2}\right)} + \frac{\sqrt{2} \sqrt{x^2 + \frac{1}{2}}}{6 \left(x - \frac{\sqrt{2} \operatorname{li}}{2}\right)} + \frac{\sqrt{2} \sqrt{x^2 + \frac{1}{2}}}{6 \left(x + \frac{\sqrt{2} \operatorname{li}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x^2 + 1)^(5/2),x)**[Out]** ((x^2 + 1/2)^(1/2)*1i)/(24*(2^(1/2)*x*1i - x^2 + 1/2)) + ((x^2 + 1/2)^(1/2)*1i)/(24*(2^(1/2)*x*1i + x^2 - 1/2)) + (2^(1/2)*(x^2 + 1/2)^(1/2))/(6*(x - (2^(1/2)*1i)/2)) + (2^(1/2)*(x^2 + 1/2)^(1/2))/(6*(x + (2^(1/2)*1i)/2))

$$3.469 \quad \int \frac{1}{(-1-2x+x^2)^{5/2}} dx$$

Optimal. Leaf size=43

$$\frac{1-x}{6(-1-2x+x^2)^{3/2}} - \frac{1-x}{6\sqrt{-1-2x+x^2}}$$

[Out] 1/6*(1-x)/(x^2-2*x-1)^(3/2)+1/6*(-1+x)/(x^2-2*x-1)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {628, 627}

$$\frac{1-x}{6(x^2-2x-1)^{3/2}} - \frac{1-x}{6\sqrt{x^2-2x-1}}$$

Antiderivative was successfully verified.

[In] Int[(-1 - 2*x + x^2)^(-5/2), x]

[Out] (1 - x)/(6*(-1 - 2*x + x^2)^(3/2)) - (1 - x)/(6*Sqrt[-1 - 2*x + x^2])

Rule 627

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] := Simp[-2*((b + 2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p + 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rubi steps

$$\begin{aligned} \int \frac{1}{(-1-2x+x^2)^{5/2}} dx &= \frac{1-x}{6(-1-2x+x^2)^{3/2}} - \frac{1}{3} \int \frac{1}{(-1-2x+x^2)^{3/2}} dx \\ &= \frac{1-x}{6(-1-2x+x^2)^{3/2}} - \frac{1-x}{6\sqrt{-1-2x+x^2}} \end{aligned}$$

Mathematica [A]

time = 0.15, size = 26, normalized size = 0.60

$$\frac{2 - 3x^2 + x^3}{6(-1 - 2x + x^2)^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(-1 - 2*x + x^2)^(-5/2), x]``[Out] (2 - 3*x^2 + x^3)/(6*(-1 - 2*x + x^2)^(3/2))`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x^2 - 2*x - 1)^(5/2), x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [A]**

time = 0.12, size = 36, normalized size = 0.84

method	result	size
gospers	$\frac{x^3 - 3x^2 + 2}{6(x^2 - 2x - 1)^{3/2}}$	23
trager	$\frac{x^3 - 3x^2 + 2}{6(x^2 - 2x - 1)^{3/2}}$	23
risch	$\frac{x^3 - 3x^2 + 2}{6(x^2 - 2x - 1)^{3/2}}$	23
default	$-\frac{2x-2}{12(x^2-2x-1)^{3/2}} + \frac{2x-2}{12\sqrt{x^2-2x-1}}$	36

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(x^2-2*x-1)^(5/2), x, method=_RETURNVERBOSE)``[Out] -1/12*(2*x-2)/(x^2-2*x-1)^(3/2)+1/12*(2*x-2)/(x^2-2*x-1)^(1/2)`**Maxima [A]**

time = 0.24, size = 51, normalized size = 1.19

$$\frac{x}{6\sqrt{x^2-2x-1}} - \frac{1}{6\sqrt{x^2-2x-1}} - \frac{x}{6(x^2-2x-1)^{3/2}} + \frac{1}{6(x^2-2x-1)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-2*x-1)^(5/2),x, algorithm="maxima")

[Out] 1/6*x/sqrt(x^2 - 2*x - 1) - 1/6/sqrt(x^2 - 2*x - 1) - 1/6*x/(x^2 - 2*x - 1)^(3/2) + 1/6/(x^2 - 2*x - 1)^(3/2)

Fricas [A]

time = 0.31, size = 61, normalized size = 1.42

$$\frac{x^4 - 4x^3 + 2x^2 + (x^3 - 3x^2 + 2)\sqrt{x^2 - 2x - 1} + 4x + 1}{6(x^4 - 4x^3 + 2x^2 + 4x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-2*x-1)^(5/2),x, algorithm="fricas")

[Out] 1/6*(x^4 - 4*x^3 + 2*x^2 + (x^3 - 3*x^2 + 2)*sqrt(x^2 - 2*x - 1) + 4*x + 1)/(x^4 - 4*x^3 + 2*x^2 + 4*x + 1)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2 - 2x - 1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2-2*x-1)**(5/2),x)

[Out] Integral((x**2 - 2*x - 1)**(-5/2), x)

Giac [A]

time = 0.00, size = 41, normalized size = 0.95

$$\frac{2\left(\left(\frac{x}{12} - \frac{1}{4}\right)xx + \frac{1}{6}\right)\sqrt{x^2 - 2x - 1}}{(x^2 - 2x - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-2*x-1)^(5/2),x)

[Out] 1/6*((x - 3)*x^2 + 2)/(x^2 - 2*x - 1)^(3/2)

Mupad [B]

time = 0.28, size = 22, normalized size = 0.51

$$\frac{x^3 - 3x^2 + 2}{6(x^2 - 2x - 1)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2 - 2*x - 1)^(5/2),x)

[Out] (x^3 - 3*x^2 + 2)/(6*(x^2 - 2*x - 1)^(3/2))

$$3.470 \quad \int \frac{1}{x^4(-8+x^2)^{3/2}} dx$$

Optimal. Leaf size=47

$$\frac{1}{24x^3\sqrt{-8+x^2}} + \frac{1}{48x\sqrt{-8+x^2}} - \frac{x}{192\sqrt{-8+x^2}}$$

[Out] 1/24/x^3/(x^2-8)^(1/2)+1/48/x/(x^2-8)^(1/2)-1/192*x/(x^2-8)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {277, 197}

$$-\frac{x}{192\sqrt{x^2-8}} + \frac{1}{48\sqrt{x^2-8}x} + \frac{1}{24\sqrt{x^2-8}x^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(-8 + x^2)^(3/2)), x]

[Out] 1/(24*x^3*Sqrt[-8 + x^2]) + 1/(48*x*Sqrt[-8 + x^2]) - x/(192*Sqrt[-8 + x^2])

Rule 197

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 277

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x^(m + 1)*((a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^4(-8+x^2)^{3/2}} dx &= \frac{1}{24x^3\sqrt{-8+x^2}} + \frac{1}{6} \int \frac{1}{x^2(-8+x^2)^{3/2}} dx \\ &= \frac{1}{24x^3\sqrt{-8+x^2}} + \frac{1}{48x\sqrt{-8+x^2}} + \frac{1}{24} \int \frac{1}{(-8+x^2)^{3/2}} dx \\ &= \frac{1}{24x^3\sqrt{-8+x^2}} + \frac{1}{48x\sqrt{-8+x^2}} - \frac{x}{192\sqrt{-8+x^2}} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 28, normalized size = 0.60

$$\frac{8 + 4x^2 - x^4}{192x^3\sqrt{-8 + x^2}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x^4*(-8 + x^2)^(3/2)),x]``[Out] (8 + 4*x^2 - x^4)/(192*x^3*Sqrt[-8 + x^2])`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.

time = 3.85, size = 127, normalized size = 2.70

$$\text{Piecewise} \left[\left\{ \left\{ \frac{I(8 + 4x^2 - x^4) \sqrt{\frac{8 - x^2}{x^2}}}{192x^2(-8 + x^2)}, \text{Abs}[x^2] > \frac{1}{8} \right\} \right\}, \frac{4x^2 \sqrt{1 - \frac{8}{x^2}}}{-1536x^2 + 192x^4} - \frac{x^4 \sqrt{1 - \frac{8}{x^2}}}{-1536x^2 + 192x^4} + \frac{8 \sqrt{1 - \frac{8}{x^2}}}{-1536x^2 + 192x^4} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x^4*(x^2 - 8)^(3/2)),x]')`
`[Out] Piecewise[{{I / 192 (8 + 4 x ^ 2 - x ^ 4) Sqrt[(8 - x ^ 2) / x ^ 2] / (x ^ 2 (-8 + x ^ 2)), 1 / Abs[x ^ 2] > 1 / 8}}, 4 x ^ 2 Sqrt[1 - 8 / x ^ 2] / (-1536 x ^ 2 + 192 x ^ 4) - x ^ 4 Sqrt[1 - 8 / x ^ 2] / (-1536 x ^ 2 + 192 x ^ 4) + 8 Sqrt[1 - 8 / x ^ 2] / (-1536 x ^ 2 + 192 x ^ 4)}`
Maple [A]

time = 0.08, size = 36, normalized size = 0.77

method	result	size
gosper	$-\frac{x^4 - 4x^2 - 8}{192x^3\sqrt{x^2 - 8}}$	23
trager	$-\frac{x^4 - 4x^2 - 8}{192x^3\sqrt{x^2 - 8}}$	23
risch	$-\frac{x^4 - 4x^2 - 8}{192x^3\sqrt{x^2 - 8}}$	23
default	$\frac{1}{24x^3\sqrt{x^2 - 8}} + \frac{1}{48x\sqrt{x^2 - 8}} - \frac{x}{192\sqrt{x^2 - 8}}$	36
meijerg	$-\frac{\sqrt{2} \left(-\text{signum}\left(-1 + \frac{x^2}{8}\right)\right)^{\frac{3}{2}} \left(-\frac{1}{8}x^4 + \frac{1}{2}x^2 + 1\right)}{96\text{signum}\left(-1 + \frac{x^2}{8}\right)^{\frac{3}{2}} x^3 \sqrt{1 - \frac{x^2}{8}}}$	52

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/x^4/(x^2-8)^(3/2),x,method=_RETURNVERBOSE)`

[Out] $1/24/x^3/(x^2-8)^{(1/2)}+1/48/x/(x^2-8)^{(1/2)}-1/192*x/(x^2-8)^{(1/2)}$

Maxima [A]

time = 0.33, size = 35, normalized size = 0.74

$$-\frac{x}{192\sqrt{x^2-8}} + \frac{1}{48\sqrt{x^2-8}x} + \frac{1}{24\sqrt{x^2-8}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(x^2-8)^(3/2),x, algorithm="maxima")`

[Out] $-1/192*x/\text{sqrt}(x^2 - 8) + 1/48/(\text{sqrt}(x^2 - 8)*x) + 1/24/(\text{sqrt}(x^2 - 8)*x^3)$

Fricas [A]

time = 0.31, size = 40, normalized size = 0.85

$$-\frac{x^5 - 8x^3 + (x^4 - 4x^2 - 8)\sqrt{x^2 - 8}}{192(x^5 - 8x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(x^2-8)^(3/2),x, algorithm="fricas")`

[Out] $-1/192*(x^5 - 8*x^3 + (x^4 - 4*x^2 - 8)*\text{sqrt}(x^2 - 8))/(x^5 - 8*x^3)$

Sympy [A]

time = 1.72, size = 153, normalized size = 3.26

$$\begin{cases} -\frac{ix^4\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{4ix^2\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{8i\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2} & \text{for } \frac{1}{|x^2|} > \frac{1}{8} \\ -\frac{x^4\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{4x^2\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{8\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(x**2-8)**(3/2),x)`

[Out] `Piecewise((-I*x**4*sqrt(-1 + 8/x**2)/(192*x**4 - 1536*x**2) + 4*I*x**2*sqrt(-1 + 8/x**2)/(192*x**4 - 1536*x**2) + 8*I*sqrt(-1 + 8/x**2)/(192*x**4 - 1536*x**2), 1/Abs(x**2) > 1/8), (-x**4*sqrt(1 - 8/x**2)/(192*x**4 - 1536*x**2) + 4*x**2*sqrt(1 - 8/x**2)/(192*x**4 - 1536*x**2) + 8*sqrt(1 - 8/x**2)/(192*x**4 - 1536*x**2), True))`

Giac [A]

time = 0.00, size = 81, normalized size = 1.72

$$-\frac{2x\sqrt{x^2-8}}{1024(x^2-8)} + \frac{2\left(-3\left(\sqrt{x^2-8}-x\right)^4-96\left(\sqrt{x^2-8}-x\right)^2-320\right)}{192\left(\left(\sqrt{x^2-8}-x\right)^2+8\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(x^2-8)^(3/2),x)`

[Out]
$$-1/512*x/\sqrt{x^2 - 8} - 1/96*(3*(x - \sqrt{x^2 - 8})^4 + 96*(x - \sqrt{x^2 - 8})^2 + 320)/((x - \sqrt{x^2 - 8})^2 + 8)^3$$

Mupad [B]

time = 0.45, size = 24, normalized size = 0.51

$$\frac{-x^4 + 4x^2 + 8}{192x^3\sqrt{x^2 - 8}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(x^2 - 8)^(3/2)),x)`

[Out]
$$(4*x^2 - x^4 + 8)/(192*x^3*(x^2 - 8)^(1/2))$$

$$3.471 \quad \int \frac{(5+x^2)^2}{x^{13/3}} dx$$

Optimal. Leaf size=28

$$-\frac{15}{2x^{10/3}} - \frac{15}{2x^{4/3}} + \frac{3x^{2/3}}{2}$$

[Out] $-15/2/x^{(10/3)}-15/2/x^{(4/3)}+3/2*x^{(2/3)}$

Rubi [A]

time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {276}

$$\frac{3x^{2/3}}{2} - \frac{15}{2x^{4/3}} - \frac{15}{2x^{10/3}}$$

Antiderivative was successfully verified.

[In] Int[(5 + x^2)^2/x^(13/3), x]

[Out] $-15/(2*x^{(10/3)}) - 15/(2*x^{(4/3)}) + (3*x^{(2/3)})/2$

Rule 276

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{(5+x^2)^2}{x^{13/3}} dx &= \int \left(\frac{25}{x^{13/3}} + \frac{10}{x^{7/3}} + \frac{1}{\sqrt[3]{x}} \right) dx \\ &= -\frac{15}{2x^{10/3}} - \frac{15}{2x^{4/3}} + \frac{3x^{2/3}}{2} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 19, normalized size = 0.68

$$\frac{3(-5 - 5x^2 + x^4)}{2x^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x^2)^2/x^(13/3), x]

[Out] $(3*(-5 - 5*x^2 + x^4))/(2*x^{(10/3)})$

Mathics [A]

time = 2.59, size = 15, normalized size = 0.54

$$\frac{3(-5 - 5x^2 + x^4)}{2x^{\frac{10}{3}}}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(x^2 + 5)^2/(x^4*x^(1/3)),x]')`

[Out] $3(-5 - 5x^2 + x^4)/(2x^{(10/3)})$

Maple [A]

time = 0.06, size = 17, normalized size = 0.61

method	result	size
gosper	$\frac{\frac{3}{2}x^4 - \frac{15}{2} - \frac{15}{2}x^2}{x^{\frac{10}{3}}}$	16
trager	$\frac{\frac{3}{2}x^4 - \frac{15}{2} - \frac{15}{2}x^2}{x^{\frac{10}{3}}}$	16
risch	$\frac{\frac{3}{2}x^4 - \frac{15}{2} - \frac{15}{2}x^2}{x^{\frac{10}{3}}}$	16
derivativdivides	$-\frac{15}{2x^{\frac{10}{3}}} - \frac{15}{2x^{\frac{4}{3}}} + \frac{3x^{\frac{2}{3}}}{2}$	17
default	$-\frac{15}{2x^{\frac{10}{3}}} - \frac{15}{2x^{\frac{4}{3}}} + \frac{3x^{\frac{2}{3}}}{2}$	17

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+5)^2/x^(13/3),x,method=_RETURNVERBOSE)`

[Out] $-15/2/x^{(10/3)} - 15/2/x^{(4/3)} + 3/2*x^{(2/3)}$

Maxima [A]

time = 0.24, size = 16, normalized size = 0.57

$$\frac{3}{2}x^{\frac{2}{3}} - \frac{15(x^2 + 1)}{2x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+5)^2/x^(13/3),x, algorithm="maxima")`

[Out] $3/2*x^{(2/3)} - 15/2*(x^2 + 1)/x^{(10/3)}$

Fricas [A]

time = 0.31, size = 15, normalized size = 0.54

$$\frac{3(x^4 - 5x^2 - 5)}{2x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+5)^2/x^(13/3),x, algorithm="fricas")`

[Out] $3/2*(x^4 - 5*x^2 - 5)/x^{(10/3)}$

Sympy [A]

time = 1.15, size = 24, normalized size = 0.86

$$\frac{3x^{\frac{2}{3}}}{2} - \frac{15}{2x^{\frac{4}{3}}} - \frac{15}{2x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+5)**2/x**(13/3),x)`

[Out] $3*x^{(2/3)}/2 - 15/(2*x^{(4/3)}) - 15/(2*x^{(10/3)})$

Giac [A]

time = 0.00, size = 34, normalized size = 1.21

$$\frac{3}{2} \left(x^{\frac{1}{3}} \right)^2 + \frac{\frac{1}{2}(-15x^2 - 15)}{x^{\frac{1}{3}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+5)^2/x^(13/3),x)`

[Out] $3/2*x^{(2/3)} - 15/2*(x^2 + 1)/x^{(10/3)}$

Mupad [B]

time = 0.27, size = 17, normalized size = 0.61

$$-\frac{-3x^4 + 15x^2 + 15}{2x^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 5)^2/x^(13/3),x)`

[Out] $-(15*x^2 - 3*x^4 + 15)/(2*x^{(10/3)})$

$$3.472 \quad \int \frac{1}{x^7(1+x^2)^3} dx$$

Optimal. Leaf size=52

$$-\frac{1}{6x^6} + \frac{3}{4x^4} - \frac{3}{x^2} - \frac{1}{4(1+x^2)^2} - \frac{2}{1+x^2} - 10 \log(x) + 5 \log(1+x^2)$$

[Out] $-1/6/x^6+3/4/x^4-3/x^2-1/4/(x^2+1)^2-2/(x^2+1)-10*\ln(x)+5*\ln(x^2+1)$

Rubi [A]

time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {272, 46}

$$-\frac{1}{6x^6} + \frac{3}{4x^4} - \frac{2}{x^2+1} - \frac{3}{x^2} - \frac{1}{4(x^2+1)^2} + 5 \log(x^2+1) - 10 \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^7*(1+x^2)^3),x]$

[Out] $-1/6*1/x^6 + 3/(4*x^4) - 3/x^2 - 1/(4*(1+x^2)^2) - 2/(1+x^2) - 10*\text{Log}[x] + 5*\text{Log}[1+x^2]$

Rule 46

$\text{Int}[(a_+ + (b_+)(x_+))^{(m_+)}((c_+ + (d_+)(x_+))^{(n_+)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{ILtQ}[m, 0] \&\& \text{IntegerQ}[n] \&\& !(\text{IGtQ}[n, 0] \&\& \text{LtQ}[m + n + 2, 0])$

Rule 272

$\text{Int}[(x_+)^{(m_+)}((a_+ + (b_+)(x_+))^{(n_+)})^{(p_+)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*x)^p, x}], x, x^n], x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{1}{x^7(1+x^2)^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^4(1+x)^3} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{x^4} - \frac{3}{x^3} + \frac{6}{x^2} - \frac{10}{x} + \frac{1}{(1+x)^3} + \frac{4}{(1+x)^2} + \frac{10}{1+x} \right) dx, x, x^2 \right) \\ &= -\frac{1}{6x^6} + \frac{3}{4x^4} - \frac{3}{x^2} - \frac{1}{4(1+x^2)^2} - \frac{2}{1+x^2} - 10 \log(x) + 5 \log(1+x^2) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 49, normalized size = 0.94

$$-\frac{2 - 5x^2 + 20x^4 + 90x^6 + 60x^8}{12x^6(1 + x^2)^2} - 10 \log(x) + 5 \log(1 + x^2)$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x^7*(1 + x^2)^3),x]`

```
[Out] -1/12*(2 - 5*x^2 + 20*x^4 + 90*x^6 + 60*x^8)/(x^6*(1 + x^2)^2) - 10*Log[x]
+ 5*Log[1 + x^2]
```

Mathics [A]

time = 2.04, size = 65, normalized size = 1.25

$$\frac{-2 + 5x^2 - 20x^4 - 90x^6 + 60x^6(1 + 2x^2 + x^4)(\text{Log}[1 + x^2] - 2\text{Log}[x]) - 60x^8}{12x^6(1 + 2x^2 + x^4)}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^7*(1 + x^2)^3),x]')`

```
[Out] (-2 + 5 x ^ 2 - 20 x ^ 4 - 90 x ^ 6 + 60 x ^ 6 (1 + 2 x ^ 2 + x ^ 4) (Log[1
+ x ^ 2] - 2 Log[x]) - 60 x ^ 8) / (12 x ^ 6 (1 + 2 x ^ 2 + x ^ 4))
```

Maple [A]

time = 0.08, size = 47, normalized size = 0.90

method	result	size
default	$-\frac{1}{6x^6} + \frac{3}{4x^4} - \frac{3}{x^2} - \frac{1}{4(x^2+1)^2} - \frac{2}{x^2+1} - 10 \ln(x) + 5 \ln(x^2 + 1)$	47
norman	$-\frac{\frac{1}{6} - 5x^8 - \frac{15}{2}x^6 + \frac{5}{12}x^2 - \frac{5}{3}x^4}{x^6(x^2+1)^2} - 10 \ln(x) + 5 \ln(x^2 + 1)$	47
risch	$-\frac{\frac{1}{6} - 5x^8 - \frac{15}{2}x^6 + \frac{5}{12}x^2 - \frac{5}{3}x^4}{x^6(x^2+1)^2} - 10 \ln(x) + 5 \ln(x^2 + 1)$	47
meijerg	$-\frac{1}{6x^6} + \frac{3}{4x^4} - \frac{3}{x^2} - \frac{9}{4} - 10 \ln(x) + \frac{x^2(9x^2+10)}{4(x^2+1)^2} + 5 \ln(x^2 + 1)$	49

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/x^7/(x^2+1)^3,x,method=_RETURNVERBOSE)`

```
[Out] -1/6/x^6+3/4/x^4-3/x^2-1/4/(x^2+1)^2-2/(x^2+1)-10*ln(x)+5*ln(x^2+1)
```

Maxima [A]

time = 0.24, size = 53, normalized size = 1.02

$$-\frac{60x^8 + 90x^6 + 20x^4 - 5x^2 + 2}{12(x^{10} + 2x^8 + x^6)} + 5 \log(x^2 + 1) - 5 \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^2+1)^3,x, algorithm="maxima")

[Out] $-1/12*(60*x^8 + 90*x^6 + 20*x^4 - 5*x^2 + 2)/(x^{10} + 2*x^8 + x^6) + 5*\log(x^2 + 1) - 5*\log(x^2)$

Fricas [A]

time = 0.31, size = 74, normalized size = 1.42

$$\frac{60x^8 + 90x^6 + 20x^4 - 5x^2 - 60(x^{10} + 2x^8 + x^6)\log(x^2 + 1) + 120(x^{10} + 2x^8 + x^6)\log(x) + 2}{12(x^{10} + 2x^8 + x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^2+1)^3,x, algorithm="fricas")

[Out] $-1/12*(60*x^8 + 90*x^6 + 20*x^4 - 5*x^2 - 60*(x^{10} + 2*x^8 + x^6)*\log(x^2 + 1) + 120*(x^{10} + 2*x^8 + x^6)*\log(x) + 2)/(x^{10} + 2*x^8 + x^6)$

Sympy [A]

time = 0.08, size = 49, normalized size = 0.94

$$-10\log(x) + 5\log(x^2 + 1) + \frac{-60x^8 - 90x^6 - 20x^4 + 5x^2 - 2}{12x^{10} + 24x^8 + 12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**7/(x**2+1)**3,x)

[Out] $-10*\log(x) + 5*\log(x**2 + 1) + (-60*x**8 - 90*x**6 - 20*x**4 + 5*x**2 - 2)/(12*x**10 + 24*x**8 + 12*x**6)$

Giac [A]

time = 0.00, size = 66, normalized size = 1.27

$$\frac{-30x^4 - 68x^2 - 39}{4(x^2 + 1)^2} + \frac{110x^6 - 36x^4 + 9x^2 - 2}{12x^6} + 5\ln(x^2 + 1) - 5\ln(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^2+1)^3,x)

[Out] $-1/4*(30*x^4 + 68*x^2 + 39)/(x^2 + 1)^2 + 1/12*(110*x^6 - 36*x^4 + 9*x^2 - 2)/x^6 + 5*\log(x^2 + 1) - 5*\log(x^2)$

Mupad [B]

time = 0.05, size = 51, normalized size = 0.98

$$5\ln(x^2 + 1) - 10\ln(x) - \frac{5x^8 + \frac{15x^6}{2} + \frac{5x^4}{3} - \frac{5x^2}{12} + \frac{1}{6}}{x^{10} + 2x^8 + x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^7*(x^2 + 1)^3),x)`

[Out] $5*\log(x^2 + 1) - 10*\log(x) - ((5*x^4)/3 - (5*x^2)/12 + (15*x^6)/2 + 5*x^8 + 1/6)/(x^6 + 2*x^8 + x^{10})$

$$3.473 \quad \int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}} dx$$

Optimal. Leaf size=25

$$-\frac{9\left(1+\frac{2}{x^2}\right)^{7/9}x}{10\sqrt{2+x^2}}$$

[Out] $-9/10*(1+2/x^2)^{(7/9)*x/(x^2+2)^{(1/2)}$

Rubi [A]

time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2016, 446, 270}

$$-\frac{9\left(\frac{2}{x^2}+1\right)^{7/9}x}{10\sqrt{x^2+2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2+x^2)/x^2]^{(7/9)}/(2+x^2)^{(3/2)}, x]$

[Out] $(-9*(1+2/x^2)^{(7/9)*x})/(10*\text{Sqrt}[2+x^2])$

Rule 270

$\text{Int}[(c_.*x_*)^{(m_*)}*((a_)+(b_.*x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)}*((a+b*x^n)^{(p+1)}/(a*c*(m+1))), x] /; \text{FreeQ}\{a, b, c, m, n, p\}, x] \ \&\& \ \text{EqQ}[(m+1)/n+p+1, 0] \ \&\& \ \text{NeQ}[m, -1]$

Rule 446

$\text{Int}[(c_)+(d_.*x_*)^{(mn_*)})^{(q_*)}*((a_)+(b_.*x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Dist}[x^{(n*\text{FracPart}[q])}*((c+d/x^n)^{\text{FracPart}[q]}/(d+c*x^n)^{\text{FracPart}[q]}), \text{Int}[(a+b*x^n)^p*((d+c*x^n)^q/x^{(n*q)}), x], x] /; \text{FreeQ}\{a, b, c, d, n, p, q\}, x] \ \&\& \ \text{EqQ}[mn, -n] \ \&\& \ !\text{IntegerQ}[q] \ \&\& \ !\text{IntegerQ}[p]$

Rule 2016

$\text{Int}[(u_*)^{(q_*)}*(v_*)^{(p_*)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandToSum}[u, x]^q*\text{ExpandToSum}[v, x]^p, x] /; \text{FreeQ}\{p, q\}, x] \ \&\& \ \text{BinomialQ}[u, x] \ \&\& \ \text{BinomialQ}[v, x] \ \&\& \ !(\text{BinomialMatchQ}[u, x] \ \&\& \ \text{BinomialMatchQ}[v, x])$

Rubi steps

$$\begin{aligned}
\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}} dx &= \int \frac{\left(1 + \frac{2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}} dx \\
&= \frac{\left(\left(1 + \frac{2}{x^2}\right)^{7/9} x^{14/9}\right) \int \frac{1}{x^{14/9}(2+x^2)^{13/18}} dx}{(2+x^2)^{7/9}} \\
&= -\frac{9\left(1 + \frac{2}{x^2}\right)^{7/9} x}{10\sqrt{2+x^2}}
\end{aligned}$$

Mathematica [A]

time = 6.36, size = 25, normalized size = 1.00

$$-\frac{9\left(1 + \frac{2}{x^2}\right)^{7/9} x}{10\sqrt{2+x^2}}$$

Antiderivative was successfully verified.

`[In] Integrate[((2 + x^2)/x^2)^(7/9)/(2 + x^2)^(3/2), x]``[Out] (-9*(1 + 2/x^2)^(7/9)*x)/(10*sqrt[2 + x^2])`**Mathics [F(-1)]**

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[((2 + x^2)/x^2)^(7/9)/(2 + x^2)^(3/2), x]')``[Out] Timed out`**Maple [A]**

time = 0.11, size = 22, normalized size = 0.88

method	result	size
gospers	$-\frac{9x\left(\frac{x^2+2}{x^2}\right)^{7/9}}{10\sqrt{x^2+2}}$	22
risch	$-\frac{9x\left(\frac{x^2+2}{x^2}\right)^{7/9}}{10\sqrt{x^2+2}}$	22

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x,method=_RETURNVERBOSE)`

[Out] $-9/10*x/(x^2+2)^{(1/2)}*((x^2+2)/x^2)^{(7/9)}$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x, algorithm="maxima")`

[Out] `integrate(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2), x)`

Fricas [A]

time = 0.46, size = 21, normalized size = 0.84

$$-\frac{9x\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{10\sqrt{x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x, algorithm="fricas")`

[Out] $-9/10*x*((x^2 + 2)/x^2)^{(7/9)}/\text{sqrt}(x^2 + 2)$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((x**2+2)/x**2)**(7/9)/(x**2+2)**(3/2),x)`

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x)`

[Out] Could not integrate

Mupad [B]

time = 0.45, size = 15, normalized size = 0.60

$$-\frac{9x(x^2+2)^{5/18}\left(\frac{1}{x^2}\right)^{7/9}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2), x)`

[Out] `-(9*x*(x^2 + 2)^(5/18)*(1/x^2)^(7/9))/10`

$$3.474 \quad \int \frac{x^4}{\left(\sqrt{10} - x^2\right)^{9/2}} dx$$

Optimal. Leaf size=50

$$\frac{x^5}{7\sqrt{10} \left(\sqrt{10} - x^2\right)^{7/2}} + \frac{x^5}{175 \left(\sqrt{10} - x^2\right)^{5/2}}$$

[Out] 1/70*x^5*10^(1/2)/(-x^2+10^(1/2))^(7/2)+1/175*x^5/(-x^2+10^(1/2))^(5/2)

Rubi [A]

time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {277, 270}

$$\frac{x^5}{5\sqrt{10} \left(\sqrt{10} - x^2\right)^{7/2}} - \frac{x^7}{175 \left(\sqrt{10} - x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(Sqrt[10] - x^2)^(9/2),x]

[Out] x^5/(5*Sqrt[10]*(Sqrt[10] - x^2)^(7/2)) - x^7/(175*(Sqrt[10] - x^2)^(7/2))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 277

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x^(m + 1)*((a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{x^4}{\left(\sqrt{10} - x^2\right)^{9/2}} dx &= \frac{x^5}{5\sqrt{10} \left(\sqrt{10} - x^2\right)^{7/2}} - \frac{1}{5} \sqrt{\frac{2}{5}} \int \frac{x^6}{\left(\sqrt{10} - x^2\right)^{9/2}} dx \\ &= \frac{x^5}{5\sqrt{10} \left(\sqrt{10} - x^2\right)^{7/2}} - \frac{x^7}{175 \left(\sqrt{10} - x^2\right)^{7/2}} \end{aligned}$$

Mathematica [A]

time = 0.19, size = 35, normalized size = 0.70

$$\frac{x^5 \left(-7\sqrt{10} + 2x^2 \right)}{350 \left(\sqrt{10} - x^2 \right)^{7/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[x^4/(Sqrt[10] - x^2)^(9/2),x]``[Out] -1/350*(x^5*(-7*Sqrt[10] + 2*x^2))/(Sqrt[10] - x^2)^(7/2)`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[x^4/(Sqrt[10] - x^2)^(9/2),x]')``[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(36) = 72.

time = 0.14, size = 121, normalized size = 2.42

method	result
gospers	$\frac{x^5 \left(-2x^2 + 7\sqrt{10} \right)}{350 \left(-x^2 + \sqrt{10} \right)^{7/2}}$
meijerg	$\frac{10^{3/4} x^5 \left(-\sqrt{2} \sqrt{5} x^2 + 7 \right)}{35000 \left(1 - \frac{\sqrt{10} x^2}{10} \right)^{7/2}}$
risch	$\frac{2x^7 - 7\sqrt{10} x^5}{350 \left(x^2 - \sqrt{10} \right)^3 \sqrt{-x^2 + \sqrt{10}}}$
trager	$-\frac{2\sqrt{10} \left(\sqrt{10} x^2 - 35 \right) x^5 \sqrt{-x^2 + \sqrt{10}}}{35 \left(\sqrt{10} x^2 - 10 \right)^4}$

default	$\frac{x^3}{4(-x^2 + \sqrt{10})^{\frac{7}{2}}} - \frac{3\sqrt{10}}{6(-x^2 + \sqrt{10})^{\frac{7}{2}}} + \frac{\sqrt{10}}{70(-x^2 + \sqrt{10})^{\frac{7}{2}}} + \frac{3\sqrt{10}}{50(-x^2 + \sqrt{10})^{\frac{5}{2}}} + \frac{2\sqrt{10}}{30(-x^2 + \sqrt{10})^{\frac{3}{2}}}$
---------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(-x^2+10^(1/2))^(9/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{4}x^3/(-x^2+10^{(1/2)})^{(7/2)}-3/4*10^{(1/2)}*(1/6*x/(-x^2+10^{(1/2)})^{(7/2)}-1/6*10^{(1/2)}*(1/70*x*10^{(1/2)}/(-x^2+10^{(1/2)})^{(7/2)}+3/35*10^{(1/2)}*(1/50*x*10^{(1/2)}/(-x^2+10^{(1/2)})^{(5/2)}+2/25*10^{(1/2)}*(1/30*x*10^{(1/2)}/(-x^2+10^{(1/2)})^{(3/2)}+1/15*x/(-x^2+10^{(1/2)})^{(1/2)}))$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(36) = 72.

time = 0.32, size = 79, normalized size = 1.58

$$\frac{x}{175\sqrt{-x^2 + \sqrt{10}}} + \frac{\sqrt{10}x}{350(-x^2 + \sqrt{10})^{\frac{3}{2}}} + \frac{x^3}{4(-x^2 + \sqrt{10})^{\frac{7}{2}}} + \frac{3x}{140(-x^2 + \sqrt{10})^{\frac{5}{2}}} - \frac{3\sqrt{10}x}{28(-x^2 + \sqrt{10})^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-x^2+10^(1/2))^(9/2),x, algorithm="maxima")`

[Out] $\frac{1}{175}x/\sqrt{-x^2 + \sqrt{10}} + \frac{1}{350}\sqrt{10}x/(-x^2 + \sqrt{10})^{3/2} + \frac{1}{4}x^3/(-x^2 + \sqrt{10})^{7/2} + \frac{3}{140}x/(-x^2 + \sqrt{10})^{5/2} - \frac{3}{28}\sqrt{10}x/(-x^2 + \sqrt{10})^{7/2}$

Fricas [A]

time = 0.31, size = 69, normalized size = 1.38

$$-\frac{\left(2x^{15} - 160x^{11} - 2600x^7 + \sqrt{10}(x^{13} - 340x^9 - 700x^5)\right)\sqrt{-x^2 + \sqrt{10}}}{350(x^{16} - 40x^{12} + 600x^8 - 4000x^4 + 10000)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-x^2+10^(1/2))^(9/2),x, algorithm="fricas")`

[Out] $-\frac{1}{350}(2x^{15} - 160x^{11} - 2600x^7 + \sqrt{10}(x^{13} - 340x^9 - 700x^5))\sqrt{-x^2 + \sqrt{10}}/(x^{16} - 40x^{12} + 600x^8 - 4000x^4 + 10000)$

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(-x**2+10**(1/2))**(9/2),x)`

[Out] Exception raised: SystemError

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(36) = 72.

time = 0.02, size = 134, normalized size = 2.68

$$\frac{112 \left(\frac{2x}{-2\sqrt{-x^2 + \sqrt{10}} + 2 \cdot 10^{\frac{1}{4}}} - \frac{-2\sqrt{-x^2 + \sqrt{10}} + 2 \cdot 10^{\frac{1}{4}}}{2x} \right)^2 + 320}{175 \left(\frac{2x}{-2\sqrt{-x^2 + \sqrt{10}} + 2 \cdot 10^{\frac{1}{4}}} - \frac{-2\sqrt{-x^2 + \sqrt{10}} + 2 \cdot 10^{\frac{1}{4}}}{2x} \right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-x^2+10^(1/2))^(9/2),x)`

[Out] $-\frac{16}{175}(7(x/(\sqrt{-x^2 + \sqrt{10}}) - 10^{1/4}) - (\sqrt{-x^2 + \sqrt{10}}) - 10^{1/4})/x^2 + 20)/(x/(\sqrt{-x^2 + \sqrt{10}}) - 10^{1/4}) - (\sqrt{-x^2 + \sqrt{10}}) - 10^{1/4})/x^7$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{(\sqrt{10} - x^2)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(10^(1/2) - x^2)^(9/2),x)
```

```
[Out] int(x^4/(10^(1/2) - x^2)^(9/2), x)
```


$$3.475 \quad \int \frac{x^2}{(3-x^2)^{3/2}} dx$$

Optimal. Leaf size=24

$$\frac{x}{\sqrt{3-x^2}} - \sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

[Out] `-arcsin(1/3*x*3^(1/2))+x/(-x^2+3)^(1/2)`

Rubi [A]

time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {294, 222}

$$\frac{x}{\sqrt{3-x^2}} - \sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] `Int[x^2/(3 - x^2)^(3/2),x]`

[Out] `x/Sqrt[3 - x^2] - ArcSin[x/Sqrt[3]]`

Rule 222

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

Rule 294

`Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n * ((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && ! LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]`

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(3-x^2)^{3/2}} dx &= \frac{x}{\sqrt{3-x^2}} - \int \frac{1}{\sqrt{3-x^2}} dx \\ &= \frac{x}{\sqrt{3-x^2}} - \sin^{-1}\left(\frac{x}{\sqrt{3}}\right) \end{aligned}$$

Mathematica [A]

time = 0.04, size = 30, normalized size = 1.25

$$\frac{x}{\sqrt{3-x^2}} - \tan^{-1}\left(\frac{x}{\sqrt{3-x^2}}\right)$$

Antiderivative was successfully verified.

`[In] Integrate[x^2/(3 - x^2)^(3/2),x]``[Out] x/Sqrt[3 - x^2] - ArcTan[x/Sqrt[3 - x^2]]`**Mathics [A]**

time = 2.00, size = 21, normalized size = 0.88

$$\frac{x}{\sqrt{3-x^2}} - \text{ArcSin}\left[\frac{\sqrt{3}x}{3}\right]$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^2/(3 - x^2)^(3/2),x]')``[Out] x / Sqrt[3 - x ^ 2] - ArcSin[Sqrt[3] x / 3]`**Maple [A]**

time = 0.10, size = 22, normalized size = 0.92

method	result	size
default	$-\arcsin\left(\frac{x\sqrt{3}}{3}\right) + \frac{x}{\sqrt{-x^2+3}}$	22
risch	$-\arcsin\left(\frac{x\sqrt{3}}{3}\right) + \frac{x}{\sqrt{-x^2+3}}$	22
meijerg	$\frac{i\left(-\frac{i\sqrt{\pi}x\sqrt{3}}{3\sqrt{-\frac{x^2}{3}+1}} + i\sqrt{\pi}\arcsin\left(\frac{x\sqrt{3}}{3}\right)\right)}{\sqrt{\pi}}$	40
trager	$-\frac{x\sqrt{-x^2+3}}{x^2-3} - \text{RootOf}(_Z^2+1)\ln(\text{RootOf}(_Z^2+1)\sqrt{-x^2+3}+x)$	48

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2/(-x^2+3)^(3/2),x,method=_RETURNVERBOSE)``[Out] -arcsin(1/3*x*3^(1/2))+x/(-x^2+3)^(1/2)`**Maxima [A]**

time = 0.33, size = 21, normalized size = 0.88

$$\frac{x}{\sqrt{-x^2+3}} - \arcsin\left(\frac{1}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-x^2+3)^(3/2),x, algorithm="maxima")`

[Out] `x/sqrt(-x^2 + 3) - arcsin(1/3*sqrt(3)*x)`

Fricas [A]

time = 0.31, size = 41, normalized size = 1.71

$$\frac{(x^2 - 3) \arctan\left(\frac{\sqrt{-x^2 + 3}}{x}\right) - \sqrt{-x^2 + 3} x}{x^2 - 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-x^2+3)^(3/2),x, algorithm="fricas")`

[Out] `((x^2 - 3)*arctan(sqrt(-x^2 + 3)/x) - sqrt(-x^2 + 3)*x)/(x^2 - 3)`

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(19) = 38

time = 0.22, size = 49, normalized size = 2.04

$$-\frac{x^2 \operatorname{asin}\left(\frac{\sqrt{3} x}{3}\right)}{x^2 - 3} - \frac{x \sqrt{3 - x^2}}{x^2 - 3} + \frac{3 \operatorname{asin}\left(\frac{\sqrt{3} x}{3}\right)}{x^2 - 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(-x**2+3)**(3/2),x)`

[Out] `-x**2*asin(sqrt(3)*x/3)/(x**2 - 3) - x*sqrt(3 - x**2)/(x**2 - 3) + 3*asin(sqrt(3)*x/3)/(x**2 - 3)`

Giac [A]

time = 0.00, size = 33, normalized size = 1.38

$$\frac{2x\sqrt{-x^2 + 3}}{2(-x^2 + 3)} - \arcsin\left(\frac{x}{\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-x^2+3)^(3/2),x)`

[Out] `-sqrt(-x^2 + 3)*x/(x^2 - 3) - arcsin(1/3*sqrt(3)*x)`

Mupad [B]

time = 0.30, size = 54, normalized size = 2.25

$$-\operatorname{asin}\left(\frac{\sqrt{3} x}{3}\right) - \frac{\sqrt{3 - x^2}}{2(x - \sqrt{3})} - \frac{\sqrt{3 - x^2}}{2(x + \sqrt{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3 - x^2)^(3/2),x)
```

```
[Out] - asin((3^(1/2)*x)/3) - (3 - x^2)^(1/2)/(2*(x - 3^(1/2))) - (3 - x^2)^(1/2)
/(2*(x + 3^(1/2)))
```

$$3.476 \quad \int \frac{(25-x^2)^{3/2}}{x^4} dx$$

Optimal. Leaf size=40

$$\frac{\sqrt{25-x^2}}{x} - \frac{(25-x^2)^{3/2}}{3x^3} + \sin^{-1}\left(\frac{x}{5}\right)$$

[Out] $-1/3*(-x^2+25)^{(3/2)}/x^3+\arcsin(1/5*x)+(-x^2+25)^{(1/2)}/x$

Rubi [A]

time = 0.00, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {283, 222}

$$\frac{\sqrt{25-x^2}}{x} - \frac{(25-x^2)^{3/2}}{3x^3} + \sin^{-1}\left(\frac{x}{5}\right)$$

Antiderivative was successfully verified.

[In] Int[(25 - x^2)^(3/2)/x^4,x]

[Out] Sqrt[25 - x^2]/x - (25 - x^2)^(3/2)/(3*x^3) + ArcSin[x/5]

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 283

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m+1)*((a+b*x^n)^p/(c*(m+1))), x] - Dist[b*n*(p/(c^n*(m+1))), Int[(c*x)^(m+n)*(a+b*x^n)^(p-1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m+n*p+n+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned} \int \frac{(25-x^2)^{3/2}}{x^4} dx &= -\frac{(25-x^2)^{3/2}}{3x^3} - \int \frac{\sqrt{25-x^2}}{x^2} dx \\ &= \frac{\sqrt{25-x^2}}{x} - \frac{(25-x^2)^{3/2}}{3x^3} + \int \frac{1}{\sqrt{25-x^2}} dx \\ &= \frac{\sqrt{25-x^2}}{x} - \frac{(25-x^2)^{3/2}}{3x^3} + \sin^{-1}\left(\frac{x}{5}\right) \end{aligned}$$

Mathematica [A]

time = 0.05, size = 46, normalized size = 1.15

$$\frac{\sqrt{25-x^2}(-25+4x^2)}{3x^3} - 2 \tan^{-1} \left(\frac{\sqrt{25-x^2}}{5+x} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(25 - x^2)^(3/2)/x^4,x]``[Out] (Sqrt[25 - x^2]*(-25 + 4*x^2))/(3*x^3) - 2*ArcTan[Sqrt[25 - x^2]/(5 + x)]`**Mathics [A]**

time = 2.12, size = 33, normalized size = 0.82

$$\frac{-25\sqrt{25-x^2}}{3x^3} + \frac{4\sqrt{25-x^2}}{3x} + \text{ArcSin} \left[\frac{x}{5} \right]$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(25 - x^2)^(3/2)/x^4,x]')``[Out] -25 Sqrt[25 - x ^ 2] / (3 x ^ 3) + 4 Sqrt[25 - x ^ 2] / (3 x) + ArcSin[x / 5]`**Maple [A]**

time = 0.11, size = 58, normalized size = 1.45

method	result	size
risch	$-\frac{4x^4-125x^2+625}{3x^3\sqrt{-x^2+25}} + \arcsin\left(\frac{x}{5}\right)$	32
meijerg	$3i \left(\frac{-\frac{1000i\sqrt{\pi} \left(1-\frac{4x^2}{25}\right) \sqrt{-\frac{x^2}{25}+1}}{9x^3} + \frac{8i\sqrt{\pi} \arcsin\left(\frac{x}{5}\right)}{3}}{8\sqrt{\pi}} \right)$	43
trager	$\frac{(4x^2-25)\sqrt{-x^2+25}}{3x^3} + \text{RootOf}(_Z^2+1) \ln(-\text{RootOf}(_Z^2+1)x + \sqrt{-x^2+25})$	50
default	$-\frac{(-x^2+25)^{\frac{5}{2}}}{75x^3} + \frac{2(-x^2+25)^{\frac{5}{2}}}{1875x} + \frac{2x(-x^2+25)^{\frac{3}{2}}}{1875} + \frac{\sqrt{-x^2+25}}{25}x + \arcsin\left(\frac{x}{5}\right)$	58

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((-x^2+25)^(3/2)/x^4,x,method=_RETURNVERBOSE)``[Out] -1/75/x^3*(-x^2+25)^(5/2)+2/1875/x*(-x^2+25)^(5/2)+2/1875*x*(-x^2+25)^(3/2)+1/25*(-x^2+25)^(1/2)*x+arcsin(1/5*x)`

Maxima [A]

time = 0.34, size = 45, normalized size = 1.12

$$\frac{1}{25} \sqrt{-x^2 + 25} x + \frac{2(-x^2 + 25)^{\frac{3}{2}}}{75x} - \frac{(-x^2 + 25)^{\frac{5}{2}}}{75x^3} + \arcsin\left(\frac{1}{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+25)^(3/2)/x^4,x, algorithm="maxima")

[Out] 1/25*sqrt(-x^2 + 25)*x + 2/75*(-x^2 + 25)^(3/2)/x - 1/75*(-x^2 + 25)^(5/2)/x^3 + arcsin(1/5*x)

Fricas [A]

time = 0.31, size = 45, normalized size = 1.12

$$\frac{6x^3 \arctan\left(\frac{\sqrt{-x^2 + 25} - 5}{x}\right) - (4x^2 - 25)\sqrt{-x^2 + 25}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+25)^(3/2)/x^4,x, algorithm="fricas")

[Out] -1/3*(6*x^3*arctan((sqrt(-x^2 + 25) - 5)/x) - (4*x^2 - 25)*sqrt(-x^2 + 25))/x^3

Sympy [A]

time = 0.51, size = 32, normalized size = 0.80

$$\operatorname{asin}\left(\frac{x}{5}\right) + \frac{4\sqrt{25 - x^2}}{3x} - \frac{25\sqrt{25 - x^2}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+25)**(3/2)/x**4,x)

[Out] asin(x/5) + 4*sqrt(25 - x**2)/(3*x) - 25*sqrt(25 - x**2)/(3*x**3)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(32) = 64.

time = 0.01, size = 113, normalized size = 2.82

$$\frac{-\frac{64}{3} \left(\frac{-2\sqrt{-x^2 + 25} + 10}{2x} \right)^3 - \frac{160(-2\sqrt{-x^2 + 25} + 10)}{x}}{512} + \frac{-15 \left(\frac{-2\sqrt{-x^2 + 25} + 10}{2x} \right)^2 + 1}{24 \left(\frac{-2\sqrt{-x^2 + 25} + 10}{2x} \right)^3} + \arcsin\left(\frac{x}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+25)^(3/2)/x^4,x)

[Out] $-1/24*x^3*(15*(\sqrt{-x^2 + 25} - 5)^2/x^2 - 1)/(\sqrt{-x^2 + 25} - 5)^3 + 5/8*(\sqrt{-x^2 + 25} - 5)/x - 1/24*(\sqrt{-x^2 + 25} - 5)^3/x^3 + \arcsin(1/5*x)$
)

Mupad [B]

time = 0.04, size = 33, normalized size = 0.82

$$\arcsin\left(\frac{x}{5}\right) + \frac{4\sqrt{25-x^2}}{3x} - \frac{25\sqrt{25-x^2}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((25 - x^2)^{(3/2)}/x^4, x)$

[Out] $\arcsin(x/5) + (4*(25 - x^2)^{(1/2)})/(3*x) - (25*(25 - x^2)^{(1/2)})/(3*x^3)$

$$3.477 \quad \int \frac{1}{(1-2x^2)^{7/2}} dx$$

Optimal. Leaf size=49

$$\frac{x}{5(1-2x^2)^{5/2}} + \frac{4x}{15(1-2x^2)^{3/2}} + \frac{8x}{15\sqrt{1-2x^2}}$$

[Out] 1/5*x/(-2*x^2+1)^(5/2)+4/15*x/(-2*x^2+1)^(3/2)+8/15*x/(-2*x^2+1)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$,

Rules used = {198, 197}

$$\frac{8x}{15\sqrt{1-2x^2}} + \frac{4x}{15(1-2x^2)^{3/2}} + \frac{x}{5(1-2x^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*x^2)^(-7/2), x]

[Out] x/(5*(1 - 2*x^2)^(5/2)) + (4*x)/(15*(1 - 2*x^2)^(3/2)) + (8*x)/(15*sqrt[1 - 2*x^2])

Rule 197

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 198

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1-2x^2)^{7/2}} dx &= \frac{x}{5(1-2x^2)^{5/2}} + \frac{4}{5} \int \frac{1}{(1-2x^2)^{5/2}} dx \\ &= \frac{x}{5(1-2x^2)^{5/2}} + \frac{4x}{15(1-2x^2)^{3/2}} + \frac{8}{15} \int \frac{1}{(1-2x^2)^{3/2}} dx \\ &= \frac{x}{5(1-2x^2)^{5/2}} + \frac{4x}{15(1-2x^2)^{3/2}} + \frac{8x}{15\sqrt{1-2x^2}} \end{aligned}$$

Mathematica [A]

time = 0.16, size = 28, normalized size = 0.57

$$\frac{x(15 - 40x^2 + 32x^4)}{15(1 - 2x^2)^{5/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(1 - 2*x^2)^(-7/2), x]``[Out] (x*(15 - 40*x^2 + 32*x^4))/(15*(1 - 2*x^2)^(5/2))`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.

time = 6.94, size = 187, normalized size = 3.82

$$\text{Piecewise}\left[\left\{\left\{\frac{Ix(-15 + 40x^2 - 32x^4)}{15\sqrt{-1 + 2x^2}(1 - 4x^2 + 4x^4)}, \text{Abs}[x^2] > \frac{1}{2}\right\}\right\}, \frac{15x}{-60x^2\sqrt{1 - 2x^2} + 60x^4\sqrt{1 - 2x^2} + 15\sqrt{1 - 2x^2}} - \frac{40x^3}{-60x^2\sqrt{1 - 2x^2} + 60x^4\sqrt{1 - 2x^2} + 15\sqrt{1 - 2x^2}} + \frac{32x^5}{-60x^2\sqrt{1 - 2x^2} + 60x^4\sqrt{1 - 2x^2} + 15\sqrt{1 - 2x^2}}\right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(1 - 2*x^2)^(7/2), x]')`
`[Out] Piecewise[{{I / 15 x (-15 + 40 x ^ 2 - 32 x ^ 4) / (Sqrt[-1 + 2 x ^ 2] (1 - 4 x ^ 2 + 4 x ^ 4)), Abs[x ^ 2] > 1 / 2}}, 15 x / (-60 x ^ 2 Sqrt[1 - 2 x ^ 2] + 60 x ^ 4 Sqrt[1 - 2 x ^ 2] + 15 Sqrt[1 - 2 x ^ 2]) - 40 x ^ 3 / (-60 x ^ 2 Sqrt[1 - 2 x ^ 2] + 60 x ^ 4 Sqrt[1 - 2 x ^ 2] + 15 Sqrt[1 - 2 x ^ 2]) + 32 x ^ 5 / (-60 x ^ 2 Sqrt[1 - 2 x ^ 2] + 60 x ^ 4 Sqrt[1 - 2 x ^ 2] + 15 Sqrt[1 - 2 x ^ 2])}]`
Maple [A]

time = 0.08, size = 38, normalized size = 0.78

method	result	size
gospers	$\frac{x(32x^4 - 40x^2 + 15)}{15(-2x^2 + 1)^{5/2}}$	25
meijerg	$\frac{x(32x^4 - 40x^2 + 15)}{15(-2x^2 + 1)^{5/2}}$	25
trager	$-\frac{(32x^4 - 40x^2 + 15)x\sqrt{-2x^2 + 1}}{15(2x^2 - 1)^3}$	34
risch	$\frac{x(32x^4 - 40x^2 + 15)}{15(2x^2 - 1)^2\sqrt{-2x^2 + 1}}$	34
default	$\frac{x}{5(-2x^2 + 1)^{5/2}} + \frac{4x}{15(-2x^2 + 1)^{3/2}} + \frac{8x}{15\sqrt{-2x^2 + 1}}$	38

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(-2*x^2+1)^(7/2), x, method=_RETURNVERBOSE)`

[Out] $1/5*x/(-2*x^2+1)^{(5/2)}+4/15*x/(-2*x^2+1)^{(3/2)}+8/15*x/(-2*x^2+1)^{(1/2)}$

Maxima [A]

time = 0.24, size = 37, normalized size = 0.76

$$\frac{8x}{15\sqrt{-2x^2+1}} + \frac{4x}{15(-2x^2+1)^{\frac{3}{2}}} + \frac{x}{5(-2x^2+1)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-2*x^2+1)^(7/2),x, algorithm="maxima")`

[Out] $8/15*x/\text{sqrt}(-2*x^2+1) + 4/15*x/(-2*x^2+1)^{(3/2)} + 1/5*x/(-2*x^2+1)^{(5/2)}$

Fricas [A]

time = 0.31, size = 44, normalized size = 0.90

$$\frac{(32x^5 - 40x^3 + 15x)\sqrt{-2x^2+1}}{15(8x^6 - 12x^4 + 6x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-2*x^2+1)^(7/2),x, algorithm="fricas")`

[Out] $-1/15*(32*x^5 - 40*x^3 + 15*x)*\text{sqrt}(-2*x^2+1)/(8*x^6 - 12*x^4 + 6*x^2 - 1)$

Sympy [C] Result contains complex when optimal does not.

time = 3.79, size = 291, normalized size = 5.94

$$\begin{cases} -\frac{32x^5}{60x^4\sqrt{2x^2-1}-60x^2\sqrt{2x^2-1}+15\sqrt{2x^2-1}} + \frac{40x^3}{60x^4\sqrt{2x^2-1}-60x^2\sqrt{2x^2-1}+15\sqrt{2x^2-1}} - \frac{15ix}{60x^4\sqrt{2x^2-1}-60x^2\sqrt{2x^2-1}+15\sqrt{2x^2-1}} & \text{for } |x^2| > \frac{1}{2} \\ \frac{32x^5}{60x^4\sqrt{1-2x^2}-60x^2\sqrt{1-2x^2}+15\sqrt{1-2x^2}} - \frac{40x^3}{60x^4\sqrt{1-2x^2}-60x^2\sqrt{1-2x^2}+15\sqrt{1-2x^2}} + \frac{15x}{60x^4\sqrt{1-2x^2}-60x^2\sqrt{1-2x^2}+15\sqrt{1-2x^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-2*x**2+1)**(7/2),x)`

[Out] `Piecewise((-32*I*x**5/(60*x**4*sqrt(2*x**2-1)-60*x**2*sqrt(2*x**2-1)+15*sqrt(2*x**2-1))+40*I*x**3/(60*x**4*sqrt(2*x**2-1)-60*x**2*sqrt(2*x**2-1)+15*sqrt(2*x**2-1))-15*I*x/(60*x**4*sqrt(2*x**2-1)-60*x**2*sqrt(2*x**2-1)+15*sqrt(2*x**2-1)), Abs(x**2) > 1/2), (32*x**5/(60*x**4*sqrt(1-2*x**2)-60*x**2*sqrt(1-2*x**2)+15*sqrt(1-2*x**2))-40*x**3/(60*x**4*sqrt(1-2*x**2)-60*x**2*sqrt(1-2*x**2)+15*sqrt(1-2*x**2))+15*x/(60*x**4*sqrt(1-2*x**2)-60*x**2*sqrt(1-2*x**2)+15*sqrt(1-2*x**2)), True))`

Giac [A]

time = 0.01, size = 43, normalized size = 0.88

$$\frac{2\left(\left(\frac{16}{15}xx - \frac{4}{3}\right)xx + \frac{1}{2}\right)x\sqrt{-2x^2+1}}{(-2x^2+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-2*x^2+1)^(7/2),x)

[Out] -1/15*(8*(4*x^2 - 5)*x^2 + 15)*sqrt(-2*x^2 + 1)*x/(2*x^2 - 1)^3

Mupad [B]

time = 0.28, size = 179, normalized size = 3.65

$$\frac{19\sqrt{\frac{1}{2}-x^2}}{480\left(x^2-\sqrt{2}x+\frac{1}{2}\right)} - \frac{19\sqrt{\frac{1}{2}-x^2}}{480\left(x^2+\sqrt{2}x+\frac{1}{2}\right)} - \frac{\sqrt{2}\sqrt{\frac{1}{2}-x^2}}{160\left(x^3-\frac{3\sqrt{2}}{2}x^2+\frac{3x}{2}-\frac{\sqrt{2}}{4}\right)} - \frac{\sqrt{2}\sqrt{\frac{1}{2}-x^2}}{160\left(x^3+\frac{3\sqrt{2}}{2}x^2+\frac{3x}{2}+\frac{\sqrt{2}}{4}\right)} - \frac{2\sqrt{2}\sqrt{\frac{1}{2}-x^2}}{15\left(x-\frac{\sqrt{2}}{2}\right)} - \frac{2\sqrt{2}\sqrt{\frac{1}{2}-x^2}}{15\left(x+\frac{\sqrt{2}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1 - 2*x^2)^(7/2),x)

[Out] (19*(1/2 - x^2)^(1/2))/(480*(x^2 - 2^(1/2)*x + 1/2)) - (19*(1/2 - x^2)^(1/2))/(480*(2^(1/2)*x + x^2 + 1/2)) - (2^(1/2)*(1/2 - x^2)^(1/2))/(160*((3*x)/2 - 2^(1/2)/4 - (3*2^(1/2)*x^2)/2 + x^3)) - (2^(1/2)*(1/2 - x^2)^(1/2))/(160*((3*x)/2 + 2^(1/2)/4 + (3*2^(1/2)*x^2)/2 + x^3)) - (2*2^(1/2)*(1/2 - x^2)^(1/2))/(15*(x - 2^(1/2)/2)) - (2*2^(1/2)*(1/2 - x^2)^(1/2))/(15*(x + 2^(1/2)/2))

$$3.478 \quad \int \frac{1}{(-7+6x-x^2)^{5/2}} dx$$

Optimal. Leaf size=47

$$-\frac{3-x}{6(-7+6x-x^2)^{3/2}} - \frac{3-x}{6\sqrt{-7+6x-x^2}}$$

[Out] 1/6*(-3+x)/(-x^2+6*x-7)^(3/2)+1/6*(-3+x)/(-x^2+6*x-7)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {628, 627}

$$-\frac{3-x}{6\sqrt{-x^2+6x-7}} - \frac{3-x}{6(-x^2+6x-7)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(-7 + 6*x - x^2)^(-5/2), x]

[Out] -1/6*(3 - x)/(-7 + 6*x - x^2)^(3/2) - (3 - x)/(6*sqrt[-7 + 6*x - x^2])

Rule 627

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 2*c*x)/(b^2 - 4*a*c)*sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p + 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rubi steps

$$\begin{aligned} \int \frac{1}{(-7+6x-x^2)^{5/2}} dx &= -\frac{3-x}{6(-7+6x-x^2)^{3/2}} + \frac{1}{3} \int \frac{1}{(-7+6x-x^2)^{3/2}} dx \\ &= -\frac{3-x}{6(-7+6x-x^2)^{3/2}} - \frac{3-x}{6\sqrt{-7+6x-x^2}} \end{aligned}$$

Mathematica [A]

time = 0.14, size = 31, normalized size = 0.66

$$-\frac{-18 + 24x - 9x^2 + x^3}{6(-7 + 6x - x^2)^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(-7 + 6*x - x^2)^(-5/2), x]``[Out] -1/6*(-18 + 24*x - 9*x^2 + x^3)/(-7 + 6*x - x^2)^(3/2)`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(-x^2 + 6*x - 7)^(5/2), x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.16, size = 40, normalized size = 0.85

method	result	size
gospers	$-\frac{x^3 - 9x^2 + 24x - 18}{6(-x^2 + 6x - 7)^{3/2}}$	28
trager	$-\frac{(x^3 - 9x^2 + 24x - 18)\sqrt{-x^2 + 6x - 7}}{6(x^2 - 6x + 7)^2}$	38
risch	$\frac{x^3 - 9x^2 + 24x - 18}{6(x^2 - 6x + 7)\sqrt{-x^2 + 6x - 7}}$	38
default	$-\frac{6 - 2x}{12(-x^2 + 6x - 7)^{3/2}} - \frac{6 - 2x}{12\sqrt{-x^2 + 6x - 7}}$	40

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(-x^2+6*x-7)^(5/2), x, method=_RETURNVERBOSE)``[Out] -1/12*(6-2*x)/(-x^2+6*x-7)^(3/2)-1/12*(6-2*x)/(-x^2+6*x-7)^(1/2)`**Maxima [A]**

time = 0.25, size = 59, normalized size = 1.26

$$\frac{x}{6\sqrt{-x^2 + 6x - 7}} - \frac{1}{2\sqrt{-x^2 + 6x - 7}} + \frac{x}{6(-x^2 + 6x - 7)^{3/2}} - \frac{1}{2(-x^2 + 6x - 7)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+6*x-7)^(5/2),x, algorithm="maxima")

[Out] $1/6*x/\sqrt{-x^2 + 6*x - 7} - 1/2/\sqrt{-x^2 + 6*x - 7} + 1/6*x/(-x^2 + 6*x - 7)^{(3/2)} - 1/2/(-x^2 + 6*x - 7)^{(3/2)}$

Fricas [A]

time = 0.32, size = 47, normalized size = 1.00

$$\frac{(x^3 - 9x^2 + 24x - 18)\sqrt{-x^2 + 6x - 7}}{6(x^4 - 12x^3 + 50x^2 - 84x + 49)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+6*x-7)^(5/2),x, algorithm="fricas")

[Out] $-1/6*(x^3 - 9*x^2 + 24*x - 18)*\sqrt{-x^2 + 6*x - 7}/(x^4 - 12*x^3 + 50*x^2 - 84*x + 49)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-x^2 + 6x - 7)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**2+6*x-7)**(5/2),x)

[Out] Integral((-x**2 + 6*x - 7)**(-5/2), x)

Giac [A]

time = 0.01, size = 48, normalized size = 1.02

$$\frac{2\left(\left(\frac{3}{4} - \frac{x}{12}\right)x - 2\right)x + \frac{3}{2}}{(-x^2 + 6x - 7)^2} \sqrt{-x^2 + 6x - 7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+6*x-7)^(5/2),x)

[Out] $-1/6*(((x - 9)*x + 24)*x - 18)*\sqrt{-x^2 + 6*x - 7}/(x^2 - 6*x + 7)^2$

Mupad [B]

time = 0.29, size = 29, normalized size = 0.62

$$\frac{(4x - 12)(8x^2 - 48x + 48)}{192(-x^2 + 6x - 7)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(6*x - x^2 - 7)^(5/2),x)

[Out] $-((4*x - 12)*(8*x^2 - 48*x + 48))/(192*(6*x - x^2 - 7)^{(3/2)})$

3.479 $\int (1 - 2x - 2x^2)^3 dx$

Optimal. Leaf size=36

$$x - 3x^2 + 2x^3 + 4x^4 - \frac{12x^5}{5} - 4x^6 - \frac{8x^7}{7}$$

[Out] x-3*x^2+2*x^3+4*x^4-12/5*x^5-4*x^6-8/7*x^7

Rubi [A]

time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {625}

$$-\frac{8x^7}{7} - 4x^6 - \frac{12x^5}{5} + 4x^4 + 2x^3 - 3x^2 + x$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*x - 2*x^2)^3, x]

[Out] x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7

Rule 625

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && (EqQ[a, 0] || !PerfectSquareQ[b^2 - 4*a*c])

Rubi steps

$$\begin{aligned} \int (1 - 2x - 2x^2)^3 dx &= \int (1 - 6x + 6x^2 + 16x^3 - 12x^4 - 24x^5 - 8x^6) dx \\ &= x - 3x^2 + 2x^3 + 4x^4 - \frac{12x^5}{5} - 4x^6 - \frac{8x^7}{7} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 36, normalized size = 1.00

$$x - 3x^2 + 2x^3 + 4x^4 - \frac{12x^5}{5} - 4x^6 - \frac{8x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*x - 2*x^2)^3, x]

[Out] $x - 3x^2 + 2x^3 + 4x^4 - (12x^5)/5 - 4x^6 - (8x^7)/7$

Mathics [A]

time = 1.66, size = 33, normalized size = 0.92

$$\frac{x(35 - 105x + 70x^2 + 140x^3 - 84x^4 - 140x^5 - 40x^6)}{35}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(-2*x^2 - 2*x + 1)^3,x]')`

[Out] $x(35 - 105x + 70x^2 + 140x^3 - 84x^4 - 140x^5 - 40x^6) / 35$

Maple [A]

time = 0.12, size = 33, normalized size = 0.92

method	result	size
default	$x - 3x^2 + 2x^3 + 4x^4 - \frac{12}{5}x^5 - 4x^6 - \frac{8}{7}x^7$	33
norman	$x - 3x^2 + 2x^3 + 4x^4 - \frac{12}{5}x^5 - 4x^6 - \frac{8}{7}x^7$	33
risch	$x - 3x^2 + 2x^3 + 4x^4 - \frac{12}{5}x^5 - 4x^6 - \frac{8}{7}x^7$	33
gosper	$-\frac{x(40x^6 + 140x^5 + 84x^4 - 140x^3 - 70x^2 + 105x - 35)}{35}$	34

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-2*x^2-2*x+1)^3,x,method=_RETURNVERBOSE)`

[Out] $x - 3x^2 + 2x^3 + 4x^4 - 12/5x^5 - 4x^6 - 8/7x^7$

Maxima [A]

time = 0.24, size = 32, normalized size = 0.89

$$-\frac{8}{7}x^7 - 4x^6 - \frac{12}{5}x^5 + 4x^4 + 2x^3 - 3x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2*x^2-2*x+1)^3,x, algorithm="maxima")`

[Out] $-8/7x^7 - 4x^6 - 12/5x^5 + 4x^4 + 2x^3 - 3x^2 + x$

Fricas [A]

time = 0.30, size = 32, normalized size = 0.89

$$-\frac{8}{7}x^7 - 4x^6 - \frac{12}{5}x^5 + 4x^4 + 2x^3 - 3x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2-2*x+1)^3,x, algorithm="fricas")

[Out] $-8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x$

Sympy [A]

time = 0.03, size = 34, normalized size = 0.94

$$-\frac{8x^7}{7} - 4x^6 - \frac{12x^5}{5} + 4x^4 + 2x^3 - 3x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x**2-2*x+1)**3,x)

[Out] $-8*x**7/7 - 4*x**6 - 12*x**5/5 + 4*x**4 + 2*x**3 - 3*x**2 + x$

Giac [A]

time = 0.00, size = 40, normalized size = 1.11

$$-\frac{8}{7}x^7 - 4x^6 - \frac{12}{5}x^5 + 4x^4 + 2x^3 - 3x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2-2*x+1)^3,x)

[Out] $-8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x$

Mupad [B]

time = 0.03, size = 32, normalized size = 0.89

$$-\frac{8x^7}{7} - 4x^6 - \frac{12x^5}{5} + 4x^4 + 2x^3 - 3x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2*x + 2*x^2 - 1)^3,x)

[Out] $x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7$

$$3.480 \quad \int (-1 + 5x) (-1 - x + x^2)^2 dx$$

Optimal. Leaf size=39

$$-x + \frac{3x^2}{2} + \frac{11x^3}{3} - \frac{3x^4}{4} - \frac{11x^5}{5} + \frac{5x^6}{6}$$

[Out] $-x+3/2*x^2+11/3*x^3-3/4*x^4-11/5*x^5+5/6*x^6$

Rubi [A]

time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$,

Rules used = {645}

$$\frac{5x^6}{6} - \frac{11x^5}{5} - \frac{3x^4}{4} + \frac{11x^3}{3} + \frac{3x^2}{2} - x$$

Antiderivative was successfully verified.

[In] Int[(-1 + 5*x)*(-1 - x + x^2)^2,x]

[Out] $-x + (3*x^2)/2 + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6$

Rule 645

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || EqQ[a, 0])

Rubi steps

$$\begin{aligned} \int (-1 + 5x) (-1 - x + x^2)^2 dx &= \int (-1 + 3x + 11x^2 - 3x^3 - 11x^4 + 5x^5) dx \\ &= -x + \frac{3x^2}{2} + \frac{11x^3}{3} - \frac{3x^4}{4} - \frac{11x^5}{5} + \frac{5x^6}{6} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 39, normalized size = 1.00

$$-x + \frac{3x^2}{2} + \frac{11x^3}{3} - \frac{3x^4}{4} - \frac{11x^5}{5} + \frac{5x^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 5*x)*(-1 - x + x^2)^2,x]

[Out] $-x + (3x^2)/2 + (11x^3)/3 - (3x^4)/4 - (11x^5)/5 + (5x^6)/6$

Mathics [A]

time = 1.65, size = 28, normalized size = 0.72

$$\frac{x(-60 + 90x + 220x^2 - 45x^3 - 132x^4 + 50x^5)}{60}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(x^2 - x - 1)^2*(5*x - 1),x]')`

[Out] $x(-60 + 90x + 220x^2 - 45x^3 - 132x^4 + 50x^5) / 60$

Maple [A]

time = 0.10, size = 30, normalized size = 0.77

method	result	size
gospser	$-x + \frac{3}{2}x^2 + \frac{11}{3}x^3 - \frac{3}{4}x^4 - \frac{11}{5}x^5 + \frac{5}{6}x^6$	30
default	$-x + \frac{3}{2}x^2 + \frac{11}{3}x^3 - \frac{3}{4}x^4 - \frac{11}{5}x^5 + \frac{5}{6}x^6$	30
norman	$-x + \frac{3}{2}x^2 + \frac{11}{3}x^3 - \frac{3}{4}x^4 - \frac{11}{5}x^5 + \frac{5}{6}x^6$	30
risch	$-x + \frac{3}{2}x^2 + \frac{11}{3}x^3 - \frac{3}{4}x^4 - \frac{11}{5}x^5 + \frac{5}{6}x^6$	30

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x-1)*(x^2-x-1)^2,x,method=_RETURNVERBOSE)`

[Out] $-x + 3/2x^2 + 11/3x^3 - 3/4x^4 - 11/5x^5 + 5/6x^6$

Maxima [A]

time = 0.26, size = 29, normalized size = 0.74

$$\frac{5}{6}x^6 - \frac{11}{5}x^5 - \frac{3}{4}x^4 + \frac{11}{3}x^3 + \frac{3}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+5*x)*(x^2-x-1)^2,x, algorithm="maxima")`

[Out] $5/6x^6 - 11/5x^5 - 3/4x^4 + 11/3x^3 + 3/2x^2 - x$

Fricas [A]

time = 0.29, size = 29, normalized size = 0.74

$$\frac{5}{6}x^6 - \frac{11}{5}x^5 - \frac{3}{4}x^4 + \frac{11}{3}x^3 + \frac{3}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+5*x)*(x^2-x-1)^2,x, algorithm="fricas")`

[Out] $5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x$

Sympy [A]

time = 0.03, size = 34, normalized size = 0.87

$$\frac{5x^6}{6} - \frac{11x^5}{5} - \frac{3x^4}{4} + \frac{11x^3}{3} + \frac{3x^2}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+5*x)*(x**2-x-1)**2,x)`

[Out] $5*x**6/6 - 11*x**5/5 - 3*x**4/4 + 11*x**3/3 + 3*x**2/2 - x$

Giac [A]

time = 0.00, size = 40, normalized size = 1.03

$$\frac{5}{6}x^6 - \frac{11}{5}x^5 - \frac{3}{4}x^4 + \frac{11}{3}x^3 + \frac{3}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+5*x)*(x^2-x-1)^2,x)`

[Out] $5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x$

Mupad [B]

time = 0.03, size = 29, normalized size = 0.74

$$\frac{5x^6}{6} - \frac{11x^5}{5} - \frac{3x^4}{4} + \frac{11x^3}{3} + \frac{3x^2}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x - 1)*(x - x^2 + 1)^2,x)`

[Out] $(3*x^2)/2 - x + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6$

$$3.481 \quad \int \frac{1+3x}{(1-8x+2x^2)^{5/2}} dx$$

Optimal. Leaf size=47

$$\frac{1-2x}{6(1-8x+2x^2)^{3/2}} - \frac{2(2-x)}{21\sqrt{1-8x+2x^2}}$$

[Out] $1/6*(1-2*x)/(2*x^2-8*x+1)^(3/2)-2/21*(2-x)/(2*x^2-8*x+1)^(1/2)$

Rubi [A]

time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {652, 627}

$$\frac{1-2x}{6(2x^2-8x+1)^{3/2}} - \frac{2(2-x)}{21\sqrt{2x^2-8x+1}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3*x)/(1 - 8*x + 2*x^2)^(5/2), x]

[Out] (1 - 2*x)/(6*(1 - 8*x + 2*x^2)^(3/2)) - (2*(2 - x))/(21*Sqrt[1 - 8*x + 2*x^2])

Rule 627

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] := Simp[-2*((b + 2*c*x)/(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 652

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rubi steps

$$\begin{aligned} \int \frac{1+3x}{(1-8x+2x^2)^{5/2}} dx &= \frac{1-2x}{6(1-8x+2x^2)^{3/2}} - \frac{2}{3} \int \frac{1}{(1-8x+2x^2)^{3/2}} dx \\ &= \frac{1-2x}{6(1-8x+2x^2)^{3/2}} - \frac{2(2-x)}{21\sqrt{1-8x+2x^2}} \end{aligned}$$

Mathematica [A]

time = 0.20, size = 33, normalized size = 0.70

$$\frac{-1 + 54x - 48x^2 + 8x^3}{42(1 - 8x + 2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3*x)/(1 - 8*x + 2*x^2)^(5/2), x]

[Out] (-1 + 54*x - 48*x^2 + 8*x^3)/(42*(1 - 8*x + 2*x^2)^(3/2))

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(3*x + 1)/(2*x^2 - 8*x + 1)^(5/2), x]')

[Out] Timed out

Maple [A]

time = 0.18, size = 54, normalized size = 1.15

method	result	size
gosper	$\frac{8x^3 - 48x^2 + 54x - 1}{42(2x^2 - 8x + 1)^{3/2}}$	30
trager	$\frac{8x^3 - 48x^2 + 54x - 1}{42(2x^2 - 8x + 1)^{3/2}}$	30
risch	$\frac{8x^3 - 48x^2 + 54x - 1}{42(2x^2 - 8x + 1)^{3/2}}$	30
default	$-\frac{4x-8}{12(2x^2-8x+1)^{3/2}} + \frac{4x-8}{42\sqrt{2x^2-8x+1}} - \frac{1}{2(2x^2-8x+1)^{3/2}}$	54

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+3*x)/(2*x^2-8*x+1)^(5/2), x, method=_RETURNVERBOSE)

[Out] -1/12*(4*x-8)/(2*x^2-8*x+1)^(3/2)+1/42*(4*x-8)/(2*x^2-8*x+1)^(1/2)-1/2/(2*x^2-8*x+1)^(3/2)

Maxima [A]

time = 0.25, size = 59, normalized size = 1.26

$$\frac{2x}{21\sqrt{2x^2-8x+1}} - \frac{4}{21\sqrt{2x^2-8x+1}} - \frac{x}{3(2x^2-8x+1)^{3/2}} + \frac{1}{6(2x^2-8x+1)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+3*x)/(2*x^2-8*x+1)^(5/2),x, algorithm="maxima")

[Out] $2/21*x/\sqrt{2*x^2 - 8*x + 1} - 4/21/\sqrt{2*x^2 - 8*x + 1} - 1/3*x/(2*x^2 - 8*x + 1)^{(3/2)} + 1/6/(2*x^2 - 8*x + 1)^{(3/2)}$

Fricas [A]

time = 0.31, size = 73, normalized size = 1.55

$$\frac{4x^4 - 32x^3 + 68x^2 - (8x^3 - 48x^2 + 54x - 1)\sqrt{2x^2 - 8x + 1} - 16x + 1}{42(4x^4 - 32x^3 + 68x^2 - 16x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+3*x)/(2*x^2-8*x+1)^(5/2),x, algorithm="fricas")

[Out] $-1/42*(4*x^4 - 32*x^3 + 68*x^2 - (8*x^3 - 48*x^2 + 54*x - 1)*\sqrt{2*x^2 - 8*x + 1} - 16*x + 1)/(4*x^4 - 32*x^3 + 68*x^2 - 16*x + 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x + 1}{(2x^2 - 8x + 1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+3*x)/(2*x**2-8*x+1)**(5/2),x)

[Out] Integral((3*x + 1)/(2*x**2 - 8*x + 1)**(5/2), x)

Giac [A]

time = 0.01, size = 54, normalized size = 1.15

$$\frac{2 \left(\left(\frac{2}{21}x - \frac{4}{7} \right) x + \frac{9}{14} \right) x - \frac{1}{84} \sqrt{2x^2 - 8x + 1}}{(2x^2 - 8x + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+3*x)/(2*x^2-8*x+1)^(5/2),x)

[Out] $1/42*(2*(4*(x - 6)*x + 27)*x - 1)/(2*x^2 - 8*x + 1)^{(3/2)}$

Mupad [B]

time = 0.36, size = 29, normalized size = 0.62

$$\frac{8x^3 - 48x^2 + 54x - 1}{42(2x^2 - 8x + 1)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x + 1)/(2*x^2 - 8*x + 1)^(5/2),x)

[Out] $(54*x - 48*x^2 + 8*x^3 - 1)/(42*(2*x^2 - 8*x + 1)^{(3/2)})$

$$3.482 \quad \int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}} dx$$

Optimal. Leaf size=45

$$-\frac{4(1+x)}{15(1+2x-4x^2)^{3/2}} - \frac{7+122x}{75\sqrt{1+2x-4x^2}}$$

[Out] $-4/15*(1+x)/(-4*x^2+2*x+1)^{(3/2)}+1/75*(-7-122*x)/(-4*x^2+2*x+1)^{(1/2)}$

Rubi [A]

time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1674, 650}

$$-\frac{4(x+1)}{15(-4x^2+2x+1)^{3/2}} - \frac{122x+7}{75\sqrt{-4x^2+2x+1}}$$

Antiderivative was successfully verified.

[In] Int[(-1 - 8*x + 8*x^3)/(1 + 2*x - 4*x^2)^(5/2), x]

[Out] $(-4*(1+x))/(15*(1+2*x-4*x^2)^{(3/2)}) - (7+122*x)/(75*\text{Sqrt}[1+2*x-4*x^2])$

Rule 650

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0]

Rule 1674

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p+1))/((p+1)*(b^2 - 4*a*c)), x] + Dist[1/((p+1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p+1)*ExpandToSum[(p+1)*(b^2 - 4*a*c)*Q - (2*p+3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rubi steps

$$\int \frac{-1 - 8x + 8x^3}{(1 + 2x - 4x^2)^{5/2}} dx = -\frac{4(1+x)}{15(1+2x-4x^2)^{3/2}} - \frac{1}{30} \int \frac{46+60x}{(1+2x-4x^2)^{3/2}} dx$$

$$= -\frac{4(1+x)}{15(1+2x-4x^2)^{3/2}} - \frac{7+122x}{75\sqrt{1+2x-4x^2}}$$

Mathematica [A]

time = 0.35, size = 33, normalized size = 0.73

$$-\frac{27 + 156x + 216x^2 - 488x^3}{75(1 + 2x - 4x^2)^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(-1 - 8*x + 8*x^3)/(1 + 2*x - 4*x^2)^(5/2), x]``[Out] -1/75*(27 + 156*x + 216*x^2 - 488*x^3)/(1 + 2*x - 4*x^2)^(3/2)`**Mathics [F(-1)]**

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(8*x^3 - 8*x - 1)/(1 + 2*x - 4*x^2)^(5/2), x]')``[Out] Timed out`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(37) = 74.

time = 0.14, size = 86, normalized size = 1.91

method	result	size
gospers	$\frac{488x^3 - 216x^2 - 156x - 27}{75(-4x^2 + 2x + 1)^{3/2}}$	30
trager	$\frac{(488x^3 - 216x^2 - 156x - 27)\sqrt{-4x^2 + 2x + 1}}{75(4x^2 - 2x - 1)^2}$	42
risch	$-\frac{488x^3 - 216x^2 - 156x - 27}{75(4x^2 - 2x - 1)\sqrt{-4x^2 + 2x + 1}}$	42
default	$\frac{2x^2}{(-4x^2 + 2x + 1)^{3/2}} - \frac{x}{4(-4x^2 + 2x + 1)^{3/2}} - \frac{49}{48(-4x^2 + 2x + 1)^{3/2}} + \frac{\frac{61}{240} - \frac{61x}{60}}{(-4x^2 + 2x + 1)^{3/2}} + \frac{\frac{61}{150} - \frac{122x}{75}}{\sqrt{-4x^2 + 2x + 1}}$	86

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $2x^2/(-4x^2+2x+1)^{(3/2)} - 1/4x/(-4x^2+2x+1)^{(3/2)} - 49/48/(-4x^2+2x+1)^{(3/2)} + 61/480(2-8x)/(-4x^2+2x+1)^{(3/2)} + 61/300(2-8x)/(-4x^2+2x+1)^{(1/2)}$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(37) = 74$.

time = 0.24, size = 76, normalized size = 1.69

$$-\frac{122x}{75\sqrt{-4x^2+2x+1}} + \frac{2x^2}{(-4x^2+2x+1)^{\frac{3}{2}}} + \frac{61}{150\sqrt{-4x^2+2x+1}} - \frac{19x}{15(-4x^2+2x+1)^{\frac{3}{2}}} - \frac{23}{30(-4x^2+2x+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x, algorithm="maxima")`

[Out] $-122/75x/\sqrt{-4x^2+2x+1} + 2x^2/(-4x^2+2x+1)^{(3/2)} + 61/150/\sqrt{-4x^2+2x+1} - 19/15x/(-4x^2+2x+1)^{(3/2)} - 23/30/(-4x^2+2x+1)^{(3/2)}$

Fricas [A]

time = 0.31, size = 73, normalized size = 1.62

$$\frac{432x^4 - 432x^3 - 108x^2 - (488x^3 - 216x^2 - 156x - 27)\sqrt{-4x^2+2x+1} + 108x + 27}{75(16x^4 - 16x^3 - 4x^2 + 4x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x, algorithm="fricas")`

[Out] $-1/75*(432x^4 - 432x^3 - 108x^2 - (488x^3 - 216x^2 - 156x - 27)*\sqrt{-4x^2+2x+1} + 108x + 27)/(16x^4 - 16x^3 - 4x^2 + 4x + 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{8x^3 - 8x - 1}{(-4x^2 + 2x + 1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((8*x**3-8*x-1)/(-4*x**2+2*x+1)**(5/2),x)`

[Out] `Integral((8*x**3 - 8*x - 1)/(-4*x**2 + 2*x + 1)**(5/2), x)`

Giac [A]

time = 0.01, size = 55, normalized size = 1.22

$$\frac{2\left(\left(\left(\frac{244}{75}x - \frac{36}{25}\right)x - \frac{26}{25}\right)x - \frac{9}{50}\right)\sqrt{-4x^2+2x+1}}{(-4x^2+2x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x)

[Out] 1/75*(4*(2*(61*x - 27)*x - 39)*x - 27)*sqrt(-4*x^2 + 2*x + 1)/(4*x^2 - 2*x - 1)^2

Mupad [B]

time = 0.19, size = 29, normalized size = 0.64

$$-\frac{-488x^3 + 216x^2 + 156x + 27}{75(-4x^2 + 2x + 1)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(8*x - 8*x^3 + 1)/(2*x - 4*x^2 + 1)^(5/2),x)

[Out] -(156*x + 216*x^2 - 488*x^3 + 27)/(75*(2*x - 4*x^2 + 1)^(3/2))

3.483 $\int x^2 \cos^5(x) dx$

Optimal. Leaf size=83

$$\frac{16}{15}x \cos(x) + \frac{8}{45}x \cos^3(x) + \frac{2}{25}x \cos^5(x) - \frac{298 \sin(x)}{225} + \frac{8}{15}x^2 \sin(x) + \frac{4}{15}x^2 \cos^2(x) \sin(x) + \frac{1}{5}x^2 \cos^4(x) \sin(x) +$$

[Out] 16/15*x*cos(x)+8/45*x*cos(x)^3+2/25*x*cos(x)^5-298/225*sin(x)+8/15*x^2*sin(x)+4/15*x^2*cos(x)^2*sin(x)+1/5*x^2*cos(x)^4*sin(x)+76/675*sin(x)^3-2/125*sin(x)^5

Rubi [A]

time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3392, 3377, 2717, 2713}

$$\frac{8}{15}x^2 \sin(x) + \frac{1}{5}x^2 \sin(x) \cos^4(x) + \frac{4}{15}x^2 \sin(x) \cos^2(x) - \frac{2 \sin^5(x)}{125} + \frac{76 \sin^3(x)}{675} - \frac{298 \sin(x)}{225} + \frac{2}{25}x \cos^5(x) + \frac{8}{45}x \cos^3(x) + \frac{16}{15}x \cos(x)$$

Antiderivative was successfully verified.

[In] Int[x^2*cos[x]^5,x]

[Out] (16*x*cos[x])/15 + (8*x*cos[x]^3)/45 + (2*x*cos[x]^5)/25 - (298*Sin[x])/225 + (8*x^2*sin[x])/15 + (4*x^2*cos[x]^2*sin[x])/15 + (x^2*cos[x]^4*sin[x])/5 + (76*Sin[x]^3)/675 - (2*Sin[x]^5)/125

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :=> Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :=> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3377

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :=> Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3392

Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=> Simp[d*m*(c + d*x)^(m - 1)*((b*Sine[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sine[e + f*x])^(n - 2), x], x] - Dist[d

```

^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

```

Rubi steps

$$\begin{aligned}
\int x^2 \cos^5(x) dx &= \frac{2}{25} x \cos^5(x) + \frac{1}{5} x^2 \cos^4(x) \sin(x) - \frac{2}{25} \int \cos^5(x) dx + \frac{4}{5} \int x^2 \cos^3(x) dx \\
&= \frac{8}{45} x \cos^3(x) + \frac{2}{25} x \cos^5(x) + \frac{4}{15} x^2 \cos^2(x) \sin(x) + \frac{1}{5} x^2 \cos^4(x) \sin(x) + \frac{2}{25} \text{Subst} \left(\int (1 - \right. \\
&= \frac{8}{45} x \cos^3(x) + \frac{2}{25} x \cos^5(x) - \frac{2 \sin(x)}{25} + \frac{8}{15} x^2 \sin(x) + \frac{4}{15} x^2 \cos^2(x) \sin(x) + \frac{1}{5} x^2 \cos^4(x) \sin(x) \\
&= \frac{16}{15} x \cos(x) + \frac{8}{45} x \cos^3(x) + \frac{2}{25} x \cos^5(x) - \frac{58 \sin(x)}{225} + \frac{8}{15} x^2 \sin(x) + \frac{4}{15} x^2 \cos^2(x) \sin(x) \\
&= \frac{16}{15} x \cos(x) + \frac{8}{45} x \cos^3(x) + \frac{2}{25} x \cos^5(x) - \frac{298 \sin(x)}{225} + \frac{8}{15} x^2 \sin(x) + \frac{4}{15} x^2 \cos^2(x) \sin(x)
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 67, normalized size = 0.81

$$\frac{5}{4} x \cos(x) + \frac{5}{72} x \cos(3x) + \frac{1}{200} x \cos(5x) + \frac{5}{8} (-2 + x^2) \sin(x) + \frac{5}{432} (-2 + 9x^2) \sin(3x) + \frac{(-2 + 25x^2) \sin(5x)}{2000}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Cos[x]^5,x]

[Out] (5*x*Cos[x])/4 + (5*x*Cos[3*x])/72 + (x*Cos[5*x])/200 + (5*(-2 + x^2)*Sin[x])/8 + (5*(-2 + 9*x^2)*Sin[3*x])/432 + ((-2 + 25*x^2)*Sin[5*x])/2000

Mathics [A]

time = 2.69, size = 66, normalized size = 0.80

$$\frac{-56x \cos[x] \sin[x]^4}{45} + \frac{298x \cos[x]^5}{225} + \frac{104x \cos[x] \sin[x]^2}{45} - \frac{2x^2 \sin[x]^3}{3} + \frac{x^2 \sin[x]^5}{5} + x^2 \sin[x] - \frac{298 \sin[x]}{225} - \frac{2 \sin[x]^5}{125} + \frac{76 \sin[x]^3}{675}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^2*Cos[x]^5,x]')

[Out] -56 x Cos[x] Sin[x]^4 / 45 + 298 x Cos[x]^5 / 225 + 104 x Cos[x] Sin[x]^2 / 45 - 2 x^2 Sin[x]^3 / 3 + x^2 Sin[x]^5 / 5 + x^2 Sin[x] - 298 Sin[x] / 225 - 2 Sin[x]^5 / 125 + 76 Sin[x]^3 / 675

Maple [A]

time = 0.14, size = 70, normalized size = 0.84

method	result
risch	$\frac{5x \cos(x)}{4} + \frac{5(x^2-2) \sin(x)}{8} + \frac{x \cos(5x)}{200} + \frac{(25x^2-2) \sin(5x)}{2000} + \frac{5x \cos(3x)}{72} + \frac{5(9x^2-2) \sin(3x)}{432}$
default	$\frac{x^2 \left(\frac{8}{3} + \cos^4(x) + \frac{4(\cos^2(x))}{3} \right) \sin(x)}{5} + \frac{2x(\cos^5(x))}{25} - \frac{2 \left(\frac{8}{3} + \cos^4(x) + \frac{4(\cos^2(x))}{3} \right) \sin(x)}{125} + \frac{8x(\cos^3(x))}{45} - \frac{8(2+\cos^2(x)) \sin(x)}{135}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cos(x)^5,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{5}x^2*(\frac{8}{3}+\cos(x)^4+\frac{4}{3}\cos(x)^2)*\sin(x)+\frac{2}{25}x*\cos(x)^5-\frac{2}{125}*(\frac{8}{3}+\cos(x)^4+\frac{4}{3}\cos(x)^2)*\sin(x)+\frac{8}{45}x*\cos(x)^3-\frac{8}{135}*(2+\cos(x)^2)*\sin(x)-\frac{16}{15}\sin(x)+\frac{16}{15}x*\cos(x)$

Maxima [A]

time = 0.24, size = 55, normalized size = 0.66

$$\frac{1}{200} x \cos(5x) + \frac{5}{72} x \cos(3x) + \frac{5}{4} x \cos(x) + \frac{1}{2000} (25x^2 - 2) \sin(5x) + \frac{5}{432} (9x^2 - 2) \sin(3x) + \frac{5}{8} (x^2 - 2) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(x)^5,x, algorithm="maxima")`

[Out] $\frac{1}{200}x*\cos(5*x) + \frac{5}{72}x*\cos(3*x) + \frac{5}{4}x*\cos(x) + \frac{1}{2000}*(25*x^2 - 2)*\sin(5*x) + \frac{5}{432}*(9*x^2 - 2)*\sin(3*x) + \frac{5}{8}*(x^2 - 2)*\sin(x)$

Fricas [A]

time = 0.32, size = 57, normalized size = 0.69

$$\frac{2}{25} x \cos(x)^5 + \frac{8}{45} x \cos(x)^3 + \frac{16}{15} x \cos(x) + \frac{1}{3375} (27 (25x^2 - 2) \cos(x)^4 + 4 (225x^2 - 68) \cos(x)^2 + 1800x^2 - 4144) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(x)^5,x, algorithm="fricas")`

[Out] $\frac{2}{25}x*\cos(x)^5 + \frac{8}{45}x*\cos(x)^3 + \frac{16}{15}x*\cos(x) + \frac{1}{3375}*(27*(25*x^2 - 2)*\cos(x)^4 + 4*(225*x^2 - 68)*\cos(x)^2 + 1800*x^2 - 4144)*\sin(x)$

Sympy [A]

time = 0.52, size = 112, normalized size = 1.35

$$\frac{8x^2 \sin^5(x)}{15} + \frac{4x^2 \sin^3(x) \cos^2(x)}{3} + x^2 \sin(x) \cos^4(x) + \frac{16x \sin^4(x) \cos(x)}{15} + \frac{104x \sin^2(x) \cos^3(x)}{45} + \frac{298x \cos^5(x)}{225} - \frac{4144 \sin^5(x)}{3375} - \frac{1712 \sin^3(x) \cos^2(x)}{675} - \frac{298 \sin(x) \cos^4(x)}{225}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*cos(x)**5,x)`

[Out] $8*x**2*\sin(x)**5/15 + 4*x**2*\sin(x)**3*\cos(x)**2/3 + x**2*\sin(x)*\cos(x)**4 + 16*x*\sin(x)**4*\cos(x)/15 + 104*x*\sin(x)**2*\cos(x)**3/45 + 298*x*\cos(x)**5$

$/225 - 4144*\sin(x)**5/3375 - 1712*\sin(x)**3*\cos(x)**2/675 - 298*\sin(x)*\cos(x)**4/225$

Giac [A]

time = 0.00, size = 72, normalized size = 0.87

$$\frac{10}{8}x \cos x - \frac{1}{8}(-5x^2 + 10) \sin x + \frac{30}{432}x \cos(3x) - \frac{1}{432}(-45x^2 + 10) \sin(3x) + \frac{10}{2000}x \cos(5x) - \frac{1}{2000}(-25x^2 + 2) \sin(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(x)^5,x)

[Out] $1/200*x*\cos(5*x) + 5/72*x*\cos(3*x) + 5/4*x*\cos(x) + 1/2000*(25*x^2 - 2)*\sin(5*x) + 5/432*(9*x^2 - 2)*\sin(3*x) + 5/8*(x^2 - 2)*\sin(x)$

Mupad [B]

time = 0.40, size = 69, normalized size = 0.83

$$\frac{8x \cos(x)^3}{45} - \frac{4144 \sin(x)}{3375} + \frac{2x \cos(x)^5}{25} + \frac{8x^2 \sin(x)}{15} - \frac{272 \cos(x)^2 \sin(x)}{3375} - \frac{2 \cos(x)^4 \sin(x)}{125} + \frac{16x \cos(x)}{15} + \frac{4x^2 \cos(x)^2 \sin(x)}{15} + \frac{x^2 \cos(x)^4 \sin(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cos(x)^5,x)

[Out] $(8*x*\cos(x)^3)/45 - (4144*\sin(x))/3375 + (2*x*\cos(x)^5)/25 + (8*x^2*\sin(x))/15 - (272*\cos(x)^2*\sin(x))/3375 - (2*\cos(x)^4*\sin(x))/125 + (16*x*\cos(x))/15 + (4*x^2*\cos(x)^2*\sin(x))/15 + (x^2*\cos(x)^4*\sin(x))/5$

3.484 $\int x^3 \sin^3(x) dx$

Optimal. Leaf size=73

$$\frac{40}{9}x \cos(x) - \frac{2}{3}x^3 \cos(x) - \frac{40 \sin(x)}{9} + 2x^2 \sin(x) + \frac{2}{9}x \cos(x) \sin^2(x) - \frac{1}{3}x^3 \cos(x) \sin^2(x) - \frac{2 \sin^3(x)}{27} + \frac{1}{3}x^2 \sin^3(x)$$

[Out] 40/9*x*cos(x)-2/3*x^3*cos(x)-40/9*sin(x)+2*x^2*sin(x)+2/9*x*cos(x)*sin(x)^2-1/3*x^3*cos(x)*sin(x)^2-2/27*sin(x)^3+1/3*x^2*sin(x)^3

Rubi [A]

time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3392, 3377, 2717, 3391}

$$-\frac{2}{3}x^3 \cos(x) - \frac{1}{3}x^3 \sin^2(x) \cos(x) + \frac{1}{3}x^2 \sin^3(x) + 2x^2 \sin(x) - \frac{2 \sin^3(x)}{27} - \frac{40 \sin(x)}{9} + \frac{40}{9}x \cos(x) + \frac{2}{9}x \sin^2(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[x^3*Sin[x]^3,x]

[Out] (40*x*Cos[x])/9 - (2*x^3*Cos[x])/3 - (40*Sin[x])/9 + 2*x^2*Sin[x] + (2*x*Cos[x]*Sin[x]^2)/9 - (x^3*Cos[x]*Sin[x]^2)/3 - (2*Sin[x]^3)/27 + (x^2*Sin[x]^3)/3

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3377

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3391

Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c + d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rule 3392

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist

```
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
 \int x^3 \sin^3(x) dx &= -\frac{1}{3}x^3 \cos(x) \sin^2(x) + \frac{1}{3}x^2 \sin^3(x) + \frac{2}{3} \int x^3 \sin(x) dx - \frac{2}{3} \int x \sin^3(x) dx \\
 &= -\frac{2}{3}x^3 \cos(x) + \frac{2}{9}x \cos(x) \sin^2(x) - \frac{1}{3}x^3 \cos(x) \sin^2(x) - \frac{2 \sin^3(x)}{27} + \frac{1}{3}x^2 \sin^3(x) - \frac{4}{9} \int x \sin^3(x) dx \\
 &= \frac{4}{9}x \cos(x) - \frac{2}{3}x^3 \cos(x) + 2x^2 \sin(x) + \frac{2}{9}x \cos(x) \sin^2(x) - \frac{1}{3}x^3 \cos(x) \sin^2(x) - \frac{2 \sin^3(x)}{27} \\
 &= \frac{40}{9}x \cos(x) - \frac{2}{3}x^3 \cos(x) - \frac{4 \sin(x)}{9} + 2x^2 \sin(x) + \frac{2}{9}x \cos(x) \sin^2(x) - \frac{1}{3}x^3 \cos(x) \sin^2(x) \\
 &= \frac{40}{9}x \cos(x) - \frac{2}{3}x^3 \cos(x) - \frac{40 \sin(x)}{9} + 2x^2 \sin(x) + \frac{2}{9}x \cos(x) \sin^2(x) - \frac{1}{3}x^3 \cos(x) \sin^2(x)
 \end{aligned}$$

Mathematica [A]

time = 0.06, size = 51, normalized size = 0.70

$$\frac{1}{108} (-81x(-6 + x^2) \cos(x) + 3x(-2 + 3x^2) \cos(3x) + 243(-2 + x^2) \sin(x) - (-2 + 9x^2) \sin(3x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[x]^3,x]
```

```
[Out] (-81*x*(-6 + x^2)*Cos[x] + 3*x*(-2 + 3*x^2)*Cos[3*x] + 243*(-2 + x^2)*Sin[x]
] - (-2 + 9*x^2)*Sin[3*x])/108
```

Mathics [A]

time = 2.42, size = 59, normalized size = 0.81

$$\frac{-2x \cos[x]^3}{9} + \frac{14x \cos[x]}{3} - \frac{x^2 \cos[x]^2 \sin[x]}{3} + \frac{7x^2 \sin[x]}{3} - x^3 \cos[x] + \frac{x^3 \cos[x]^3}{3} - \frac{122 \sin[x]}{27} + \frac{2 \cos[x]^2 \sin[x]}{27}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3*Sin[x]^3,x]')
```

```
[Out] -2 x Cos[x] ^ 3 / 9 + 14 x Cos[x] / 3 - x ^ 2 Cos[x] ^ 2 Sin[x] / 3 + 7 x ^
2 Sin[x] / 3 - x ^ 3 Cos[x] + x ^ 3 Cos[x] ^ 3 / 3 - 122 Sin[x] / 27 + 2 C
os[x] ^ 2 Sin[x] / 27
```

Maple [A]

time = 0.12, size = 57, normalized size = 0.78

method	result
risch	$\left(-\frac{3}{4}x^3 + \frac{9}{2}x\right) \cos(x) + \frac{9(x^2-2)\sin(x)}{4} + \left(\frac{1}{12}x^3 - \frac{1}{18}x\right) \cos(3x) - \frac{(9x^2-2)\sin(3x)}{108}$
default	$-\frac{x^3(2+\sin^2(x))\cos(x)}{3} + 2x^2\sin(x) - \frac{40\sin(x)}{9} + 4x\cos(x) + \frac{x^2(\sin^3(x))}{3} + \frac{2x(2+\sin^2(x))\cos(x)}{9} - \frac{2(\sin^3(x))}{27}$
norman	$\frac{\frac{40x}{9} - \frac{2x^3}{3} - \frac{496(\tan^3(\frac{x}{2}))}{27} - \frac{80(\tan^5(\frac{x}{2}))}{9} + \frac{16x(\tan^2(\frac{x}{2}))}{3} - \frac{16x(\tan^4(\frac{x}{2}))}{3} - \frac{40x(\tan^6(\frac{x}{2}))}{9} + 4x^2\tan(\frac{x}{2}) + \frac{32x^2(\tan^3(\frac{x}{2}))}{3} + 4x^2(\tan^5(\frac{x}{2}))}{(1+\tan^2(\frac{x}{2}))^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^3*(2+sin(x)^2)*cos(x)+2*x^2*sin(x)-40/9*sin(x)+4*x*cos(x)+1/3*x^2*si
n(x)^3+2/9*x*(2+sin(x)^2)*cos(x)-2/27*sin(x)^3
```

Maxima [A]

time = 0.25, size = 49, normalized size = 0.67

$$\frac{1}{36} (3x^3 - 2x) \cos(3x) - \frac{3}{4} (x^3 - 6x) \cos(x) - \frac{1}{108} (9x^2 - 2) \sin(3x) + \frac{9}{4} (x^2 - 2) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/36*(3*x^3 - 2*x)*cos(3*x) - 3/4*(x^3 - 6*x)*cos(x) - 1/108*(9*x^2 - 2)*si
n(3*x) + 9/4*(x^2 - 2)*sin(x)
```

Fricas [A]

time = 0.32, size = 52, normalized size = 0.71

$$\frac{1}{9} (3x^3 - 2x) \cos(x)^3 - \frac{1}{3} (3x^3 - 14x) \cos(x) - \frac{1}{27} ((9x^2 - 2) \cos(x)^2 - 63x^2 + 122) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/9*(3*x^3 - 2*x)*cos(x)^3 - 1/3*(3*x^3 - 14*x)*cos(x) - 1/27*((9*x^2 - 2)*
cos(x)^2 - 63*x^2 + 122)*sin(x)
```

Sympy [A]

time = 0.34, size = 92, normalized size = 1.26

$$-x^3 \sin^2(x) \cos(x) - \frac{2x^3 \cos^3(x)}{3} + \frac{7x^2 \sin^3(x)}{3} + 2x^2 \sin(x) \cos^2(x) + \frac{14x \sin^2(x) \cos(x)}{3} + \frac{40x \cos^3(x)}{9} - \frac{122 \sin^3(x)}{27} - \frac{40 \sin(x) \cos^2(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*sin(x)**3,x)

[Out] $-x^{**3}*\sin(x)**2*\cos(x) - 2*x^{**3}*\cos(x)**3/3 + 7*x^{**2}*\sin(x)**3/3 + 2*x^{**2}*\sin(x)*\cos(x)**2 + 14*x*\sin(x)**2*\cos(x)/3 + 40*x*\cos(x)**3/9 - 122*\sin(x)**3/27 - 40*\sin(x)*\cos(x)**2/9$

Giac [A]

time = 0.00, size = 60, normalized size = 0.82

$$\frac{1}{4}(-3x^3 + 18x)\cos x + \frac{1}{4}(9x^2 - 18)\sin x + \frac{1}{324}(27x^3 - 18x)\cos(3x) + \frac{1}{324}(-27x^2 + 6)\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*sin(x)^3,x)

[Out] $1/36*(3*x^3 - 2*x)*\cos(3*x) - 3/4*(x^3 - 6*x)*\cos(x) - 1/108*(9*x^2 - 2)*\sin(3*x) + 9/4*(x^2 - 2)*\sin(x)$

Mupad [B]

time = 0.33, size = 59, normalized size = 0.81

$$\frac{7x^2 \sin(x)}{3} - \frac{2x \cos(x)^3}{9} - x^3 \cos(x) - \frac{122 \sin(x)}{27} + \frac{x^3 \cos(x)^3}{3} + \frac{2 \cos(x)^2 \sin(x)}{27} + \frac{14x \cos(x)}{3} - \frac{x^2 \cos(x)^2 \sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*sin(x)^3,x)

[Out] $(7*x^2*\sin(x))/3 - (2*x*\cos(x)^3)/9 - x^3*\cos(x) - (122*\sin(x))/27 + (x^3*\cos(x)^3)/3 + (2*\cos(x)^2*\sin(x))/27 + (14*x*\cos(x))/3 - (x^2*\cos(x)^2*\sin(x))/3$

3.485 $\int x^2 \sin^6(x) dx$

Optimal. Leaf size=105

$$-\frac{245x}{1152} + \frac{5x^3}{48} + \frac{245 \cos(x) \sin(x)}{1152} - \frac{5}{16} x^2 \cos(x) \sin(x) + \frac{5}{16} x \sin^2(x) + \frac{65 \cos(x) \sin^3(x)}{1728} - \frac{5}{24} x^2 \cos(x) \sin^3(x) +$$

[Out] $-245/1152*x+5/48*x^3+245/1152*\cos(x)*\sin(x)-5/16*x^2*\cos(x)*\sin(x)+5/16*x*\sin(x)^2+65/1728*\cos(x)*\sin(x)^3-5/24*x^2*\cos(x)*\sin(x)^3+5/48*x*\sin(x)^4+1/108*\cos(x)*\sin(x)^5-1/6*x^2*\cos(x)*\sin(x)^5+1/18*x*\sin(x)^6$

Rubi [A]

time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3392, 30, 2715, 8}

$$\frac{5x^3}{48} - \frac{1}{6} x^2 \sin^5(x) \cos(x) - \frac{5}{24} x^2 \sin^3(x) \cos(x) - \frac{5}{16} x^2 \sin(x) \cos(x) - \frac{245x}{1152} + \frac{1}{18} x \sin^6(x) + \frac{5}{48} x \sin^4(x) + \frac{5}{16} x \sin^2(x) + \frac{1}{108} \sin^5(x) \cos(x) + \frac{65 \sin^3(x) \cos(x)}{1728} + \frac{245 \sin(x) \cos(x)}{1152}$$

Antiderivative was successfully verified.

[In] Int[x^2*Sin[x]^6,x]

[Out] $(-245*x)/1152 + (5*x^3)/48 + (245*\text{Cos}[x]*\text{Sin}[x])/1152 - (5*x^2*\text{Cos}[x]*\text{Sin}[x])/16 + (5*x*\text{Sin}[x]^2)/16 + (65*\text{Cos}[x]*\text{Sin}[x]^3)/1728 - (5*x^2*\text{Cos}[x]*\text{Sin}[x]^3)/24 + (5*x*\text{Sin}[x]^4)/48 + (\text{Cos}[x]*\text{Sin}[x]^5)/108 - (x^2*\text{Cos}[x]*\text{Sin}[x]^5)/6 + (x*\text{Sin}[x]^6)/18$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*SIN[c + d*x])^(n-1)/(d*n), x] + Dist[b^2*((n-1)/n), Int[(b*SIN[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3392

Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[d*m*(c + d*x)^(m-1)*((b*SIN[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n-1)/n), Int[(c + d*x)^m*(b*SIN[e + f*x])^(n-2), x], x] - Dist[d

```

^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n), x) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

```

Rubi steps

$$\begin{aligned}
\int x^2 \sin^6(x) dx &= -\frac{1}{6}x^2 \cos(x) \sin^5(x) + \frac{1}{18}x \sin^6(x) - \frac{1}{18} \int \sin^6(x) dx + \frac{5}{6} \int x^2 \sin^4(x) dx \\
&= -\frac{5}{24}x^2 \cos(x) \sin^3(x) + \frac{5}{48}x \sin^4(x) + \frac{1}{108} \cos(x) \sin^5(x) - \frac{1}{6}x^2 \cos(x) \sin^5(x) + \frac{1}{18}x \sin^6(x) \\
&= -\frac{5}{16}x^2 \cos(x) \sin(x) + \frac{5}{16}x \sin^2(x) + \frac{65 \cos(x) \sin^3(x)}{1728} - \frac{5}{24}x^2 \cos(x) \sin^3(x) + \frac{5}{48}x \sin^4(x) \\
&= \frac{5x^3}{48} + \frac{245 \cos(x) \sin(x)}{1152} - \frac{5}{16}x^2 \cos(x) \sin(x) + \frac{5}{16}x \sin^2(x) + \frac{65 \cos(x) \sin^3(x)}{1728} - \frac{5}{24}x^2 \cos(x) \sin^3(x) \\
&= -\frac{245x}{1152} + \frac{5x^3}{48} + \frac{245 \cos(x) \sin(x)}{1152} - \frac{5}{16}x^2 \cos(x) \sin(x) + \frac{5}{16}x \sin^2(x) + \frac{65 \cos(x) \sin^3(x)}{1728}
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 70, normalized size = 0.67

$$\frac{1440x^3 - 3240x \cos(2x) + 324x \cos(4x) - 24x \cos(6x) - 1620(-1 + 2x^2) \sin(2x) + 81(-1 + 8x^2) \sin(4x) - 4(-1 + 18x^2) \sin(6x)}{13824}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sin[x]^6,x]

[Out] (1440*x^3 - 3240*x*Cos[2*x] + 324*x*Cos[4*x] - 24*x*Cos[6*x] - 1620*(-1 + 2*x^2)*Sin[2*x] + 81*(-1 + 8*x^2)*Sin[4*x] - 4*(-1 + 18*x^2)*Sin[6*x])/13824

Mathics [A]

time = 3.48, size = 122, normalized size = 1.16

$$\frac{-125x \sin^2[x]}{384} - \frac{245x \cos[x]^6}{1152} - \frac{181x \sin^6[x]}{1152} + \frac{95x \sin^4[x]}{128} - \frac{5x^2 \cos[x] \sin^3[x]}{6} - \frac{5x^2 \cos[x]^3 \sin[x]}{16} + \frac{7x^2 \cos[x] \sin^5[x]}{48} - \frac{5x^3 \sin^4[x]}{16} + \frac{5x^3 \cos[x]^6}{48} + \frac{5x^3 \sin^2[x]^6}{48} + \frac{5x^3 \sin[x]^2}{16} - \frac{703 \cos[x] \sin^5[x]}{3456} + \frac{245 \cos[x]^5 \sin[x]}{1152} + \frac{25 \cos[x] \sin^3[x]}{54}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^2*Sin[x]^6,x]')

[Out] -125 x Sin[x]^2 / 384 - 245 x Cos[x]^6 / 1152 - 181 x Sin[x]^6 / 1152 + 95 x Sin[x]^4 / 128 - 5 x^2 Cos[x] Sin[x]^3 / 6 - 5 x^2 Cos[x]^3 Sin[x] / 16 + 7 x^2 Cos[x] Sin[x]^5 / 48 - 5 x^3 Sin[x]^4 / 16 + 5 x^3 Cos[x]^6 / 48 + 5 x^3 Sin[x]^6 / 48 + 5 x^3 Sin[x]^2 / 16 - 703 Cos[x] Sin[x]^5 / 3456 + 245 Cos[x]^5 Sin[x] / 1152 + 25 Cos[x] Sin[x]^3 / 54

Maple [A]

time = 0.17, size = 96, normalized size = 0.91

method	result
risch	$\frac{5x^3}{48} - \frac{x \cos(6x)}{576} - \frac{(18x^2-1) \sin(6x)}{3456} + \frac{3x \cos(4x)}{128} + \frac{3(8x^2-1) \sin(4x)}{512} - \frac{15x \cos(2x)}{64} - \frac{15(2x^2-1) \sin(2x)}{128}$
default	$x^2 \left(-\frac{\left(\sin^5(x) + \frac{5(\sin^3(x))}{4} + \frac{15 \sin(x)}{8} \right) \cos(x)}{6} + \frac{5x}{16} \right) + \frac{x(\sin^6(x))}{18} + \frac{\left(\sin^5(x) + \frac{5(\sin^3(x))}{4} + \frac{15 \sin(x)}{8} \right) \cos(x)}{108} + \frac{115x}{1152} +$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*sin(x)^6,x,method=_RETURNVERBOSE)

[Out] $x^2*(-1/6*(\sin(x)^5+5/4*\sin(x)^3+15/8*\sin(x))*\cos(x)+5/16*x)+1/18*x*\sin(x)^6+1/108*(\sin(x)^5+5/4*\sin(x)^3+15/8*\sin(x))*\cos(x)+115/1152*x+5/48*x*\sin(x)^4+5/192*(\sin(x)^3+3/2*\sin(x))*\cos(x)-5/16*x*\cos(x)^2+5/32*\cos(x)*\sin(x)-5/24*x^3$

Maxima [A]

time = 0.25, size = 66, normalized size = 0.63

$$\frac{5}{48} x^3 - \frac{1}{576} x \cos(6x) + \frac{3}{128} x \cos(4x) - \frac{15}{64} x \cos(2x) - \frac{1}{3456} (18x^2 - 1) \sin(6x) + \frac{3}{512} (8x^2 - 1) \sin(4x) - \frac{15}{128} (2x^2 - 1) \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sin(x)^6,x, algorithm="maxima")

[Out] $5/48*x^3 - 1/576*x*\cos(6*x) + 3/128*x*\cos(4*x) - 15/64*x*\cos(2*x) - 1/3456*(18*x^2 - 1)*\sin(6*x) + 3/512*(8*x^2 - 1)*\sin(4*x) - 15/128*(2*x^2 - 1)*\sin(2*x)$

Fricas [A]

time = 0.32, size = 72, normalized size = 0.69

$$-\frac{1}{18} x \cos(x)^6 + \frac{13}{48} x \cos(x)^4 + \frac{5}{48} x^3 - \frac{11}{16} x \cos(x)^2 - \frac{1}{3456} (32(18x^2 - 1) \cos(x)^5 - 2(936x^2 - 97) \cos(x)^3 + 3(792x^2 - 299) \cos(x)) \sin(x) + \frac{299}{1152} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sin(x)^6,x, algorithm="fricas")

[Out] $-1/18*x*\cos(x)^6 + 13/48*x*\cos(x)^4 + 5/48*x^3 - 11/16*x*\cos(x)^2 - 1/3456*(32*(18*x^2 - 1)*\cos(x)^5 - 2*(936*x^2 - 97)*\cos(x)^3 + 3*(792*x^2 - 299)*\cos(x))*\sin(x) + 299/1152*x$

Sympy [A]

time = 0.76, size = 192, normalized size = 1.83

$$\frac{5x^3 \sin^6(x)}{48} + \frac{5x^3 \sin^4(x) \cos^2(x)}{16} + \frac{5x^3 \sin^2(x) \cos^4(x)}{16} + \frac{5x^3 \cos^6(x)}{48} - \frac{11x^2 \sin^5(x) \cos(x)}{16} - \frac{5x^2 \sin^3(x) \cos^3(x)}{6} - \frac{5x^2 \sin(x) \cos^5(x)}{16} + \frac{299x \sin^4(x)}{1152} + \frac{35x \sin^2(x) \cos^2(x)}{384} - \frac{125x \sin^2(x) \cos^4(x)}{384} - \frac{245x \cos^6(x)}{1152} + \frac{299 \sin^5(x) \cos(x)}{1152} + \frac{25 \sin^3(x) \cos^3(x)}{54} + \frac{245 \sin(x) \cos^5(x)}{1152}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*sin(x)**6,x)

[Out] 5*x**3*sin(x)**6/48 + 5*x**3*sin(x)**4*cos(x)**2/16 + 5*x**3*sin(x)**2*cos(x)**4/16 + 5*x**3*cos(x)**6/48 - 11*x**2*sin(x)**5*cos(x)/16 - 5*x**2*sin(x)**3*cos(x)**3/6 - 5*x**2*sin(x)*cos(x)**5/16 + 299*x*sin(x)**6/1152 + 35*x*sin(x)**4*cos(x)**2/384 - 125*x*sin(x)**2*cos(x)**4/384 - 245*x*cos(x)**6/1152 + 299*sin(x)**5*cos(x)/1152 + 25*sin(x)**3*cos(x)**3/54 + 245*sin(x)*cos(x)**5/1152

Giac [A]

time = 0.00, size = 85, normalized size = 0.81

$$-\frac{60}{256}x \cos(2x) - \frac{1}{256}(60x^2 - 30) \sin(2x) + \frac{1}{16} \cdot \frac{5}{3}x^3 + \frac{24}{1024}x \cos(4x) - \frac{1}{1024}(-48x^2 + 6) \sin(4x) - \frac{12}{6912}x \cos(6x) - \frac{1}{6912}(36x^2 - 2) \sin(6x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sin(x)^6,x)

[Out] 5/48*x^3 - 1/576*x*cos(6*x) + 3/128*x*cos(4*x) - 15/64*x*cos(2*x) - 1/3456*(18*x^2 - 1)*sin(6*x) + 3/512*(8*x^2 - 1)*sin(4*x) - 15/128*(2*x^2 - 1)*sin(2*x)

Mupad [B]

time = 0.40, size = 88, normalized size = 0.84

$$\frac{15 \sin(2x)}{128} - \frac{3 \sin(4x)}{512} + \frac{\sin(6x)}{3456} - \frac{3x(2\sin(2x)^2 - 1)}{128} + \frac{x(2\sin(3x)^2 - 1)}{576} - \frac{15x^2 \sin(2x)}{64} + \frac{3x^2 \sin(4x)}{64} - \frac{x^2 \sin(6x)}{192} + \frac{5x^3}{48} + \frac{15x(2\sin(x)^2 - 1)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*sin(x)^6,x)

[Out] (15*sin(2*x))/128 - (3*sin(4*x))/512 + sin(6*x)/3456 - (3*x*(2*sin(2*x)^2 - 1))/128 + (x*(2*sin(3*x)^2 - 1))/576 - (15*x^2*sin(2*x))/64 + (3*x^2*sin(4*x))/64 - (x^2*sin(6*x))/192 + (5*x^3)/48 + (15*x*(2*sin(x)^2 - 1))/64

3.486 $\int x^2 \cos(x) \sin^2(x) dx$

Optimal. Leaf size=44

$$\frac{4}{9}x \cos(x) - \frac{4 \sin(x)}{9} + \frac{2}{9}x \cos(x) \sin^2(x) - \frac{2 \sin^3(x)}{27} + \frac{1}{3}x^2 \sin^3(x)$$

[Out] $4/9*x*\cos(x)-4/9*\sin(x)+2/9*x*\cos(x)*\sin(x)^2-2/27*\sin(x)^3+1/3*x^2*\sin(x)^3$

Rubi [A]

time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3524, 3391, 3377, 2717}

$$\frac{1}{3}x^2 \sin^3(x) - \frac{2 \sin^3(x)}{27} - \frac{4 \sin(x)}{9} + \frac{4}{9}x \cos(x) + \frac{2}{9}x \sin^2(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[x^2*Cos[x]*Sin[x]^2,x]

[Out] $(4*x*\cos[x])/9 - (4*\sin[x])/9 + (2*x*\cos[x]*\sin[x]^2)/9 - (2*\sin[x]^3)/27 + (x^2*\sin[x]^3)/3$

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3377

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3391

Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[d*((b*Ssin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c + d*x)*(b*Ssin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b*Ssin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rule 3524

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^(p_.), x_Symbol] := Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1))

)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
 \int x^2 \cos(x) \sin^2(x) dx &= \frac{1}{3}x^2 \sin^3(x) - \frac{2}{3} \int x \sin^3(x) dx \\
 &= \frac{2}{9}x \cos(x) \sin^2(x) - \frac{2 \sin^3(x)}{27} + \frac{1}{3}x^2 \sin^3(x) - \frac{4}{9} \int x \sin(x) dx \\
 &= \frac{4}{9}x \cos(x) + \frac{2}{9}x \cos(x) \sin^2(x) - \frac{2 \sin^3(x)}{27} + \frac{1}{3}x^2 \sin^3(x) - \frac{4}{9} \int \cos(x) dx \\
 &= \frac{4}{9}x \cos(x) - \frac{4 \sin(x)}{9} + \frac{2}{9}x \cos(x) \sin^2(x) - \frac{2 \sin^3(x)}{27} + \frac{1}{3}x^2 \sin^3(x)
 \end{aligned}$$

Mathematica [A]

time = 0.08, size = 39, normalized size = 0.89

$$\frac{1}{54} (27x \cos(x) - 3x \cos(3x) + (-26 + 9x^2 + (2 - 9x^2) \cos(2x)) \sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Cos[x]*Sin[x]^2,x]

[Out] (27*x*Cos[x] - 3*x*Cos[3*x] + (-26 + 9*x^2 + (2 - 9*x^2)*Cos[2*x])*Sin[x])/54

Mathics [A]

time = 2.08, size = 34, normalized size = 0.77

$$\frac{2x \cos[x] \sin[x]^2}{9} + \frac{4x \cos[x]}{9} + \frac{x^2 \sin[x]^3}{3} - \frac{4 \sin[x]}{9} - \frac{2 \sin[x]^3}{27}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^2*Sin[x]^2*Cos[x],x]')

[Out] 2 x Cos[x] Sin[x]^2 / 9 + 4 x Cos[x] / 9 + x^2 Sin[x]^3 / 3 - 4 Sin[x] / 9 - 2 Sin[x]^3 / 27

Maple [A]

time = 0.10, size = 32, normalized size = 0.73

method	result	size
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default	$\frac{x^2 \sin^3(x)}{3} + \frac{2x(2+\sin^2(x)) \cos(x)}{9} - \frac{2(\sin^3(x))}{27} - \frac{4 \sin(x)}{9}$	32
risch	$\frac{x \cos(x)}{2} + \frac{(x^2-2) \sin(x)}{4} - \frac{x \cos(3x)}{18} - \frac{(9x^2-2) \sin(3x)}{108}$	36
norman	$\frac{4x}{9} - \frac{64(\tan^3(\frac{x}{2}))}{27} - \frac{8(\tan^5(\frac{x}{2}))}{9} + \frac{4x(\tan^2(\frac{x}{2}))}{3} - \frac{4x(\tan^4(\frac{x}{2}))}{3} - \frac{4x(\tan^6(\frac{x}{2}))}{9} + \frac{8x^2(\tan^3(\frac{x}{2}))}{3} - \frac{8 \tan(\frac{x}{2})}{9}$ $(1+\tan^2(\frac{x}{2}))^3$	76

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cos(x)*sin(x)^2,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{3}x^2\sin(x)^3 + \frac{2}{9}x(2+\sin(x)^2)\cos(x) - \frac{2}{27}\sin(x)^3 - \frac{4}{9}\sin(x)$

Maxima [A]

time = 0.24, size = 35, normalized size = 0.80

$$-\frac{1}{18}x \cos(3x) + \frac{1}{2}x \cos(x) - \frac{1}{108}(9x^2 - 2)\sin(3x) + \frac{1}{4}(x^2 - 2)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(x)*sin(x)^2,x, algorithm="maxima")`

[Out] $-1/18*x*\cos(3*x) + 1/2*x*\cos(x) - 1/108*(9*x^2 - 2)*\sin(3*x) + 1/4*(x^2 - 2)*\sin(x)$

Fricas [A]

time = 0.32, size = 36, normalized size = 0.82

$$-\frac{2}{9}x \cos(x)^3 + \frac{2}{3}x \cos(x) - \frac{1}{27}((9x^2 - 2)\cos(x)^2 - 9x^2 + 14)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(x)*sin(x)^2,x, algorithm="fricas")`

[Out] $-2/9*x*\cos(x)^3 + 2/3*x*\cos(x) - 1/27*((9*x^2 - 2)*\cos(x)^2 - 9*x^2 + 14)*\sin(x)$

Sympy [A]

time = 0.24, size = 53, normalized size = 1.20

$$\frac{x^2 \sin^3(x)}{3} + \frac{2x \sin^2(x) \cos(x)}{3} + \frac{4x \cos^3(x)}{9} - \frac{14 \sin^3(x)}{27} - \frac{4 \sin(x) \cos^2(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*cos(x)*sin(x)**2,x)`

[Out] $x**2*\sin(x)**3/3 + 2*x*\sin(x)**2*\cos(x)/3 + 4*x*\cos(x)**3/9 - 14*\sin(x)**3/27 - 4*\sin(x)*\cos(x)**2/9$

Giac [A]

time = 0.00, size = 45, normalized size = 1.02

$$\frac{2}{4}x \cos x - \frac{1}{4}(-x^2 + 2) \sin x - \frac{6}{108}x \cos(3x) - \frac{1}{108}(9x^2 - 2) \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2*cos(x)*sin(x)^2,x)``[Out] -1/18*x*cos(3*x) + 1/2*x*cos(x) - 1/108*(9*x^2 - 2)*sin(3*x) + 1/4*(x^2 - 2)*sin(x)`**Mupad [B]**

time = 0.07, size = 40, normalized size = 0.91

$$\frac{x^2 \sin(x)^3}{3} + \frac{4x \cos(x)^3}{9} + \frac{2x \cos(x) \sin(x)^2}{3} - \frac{4 \cos(x)^2 \sin(x)}{9} - \frac{14 \sin(x)^3}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2*cos(x)*sin(x)^2,x)``[Out] (4*x*cos(x)^3)/9 - (14*sin(x)^3)/27 + (x^2*sin(x)^3)/3 - (4*cos(x)^2*sin(x))/9 + (2*x*cos(x)*sin(x)^2)/3`

3.487 $\int x \cos^2(x) \cot^2(x) dx$

Optimal. Leaf size=33

$$-\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x \cot(x) + \log(\sin(x)) - \frac{1}{2}x \cos(x) \sin(x)$$

[Out] $-3/4*x^2-1/4*\cos(x)^2-x*\cot(x)+\ln(\sin(x))-1/2*x*\cos(x)*\sin(x)$

Rubi [A]

time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4493, 3391, 30, 3801, 3556}

$$-\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x \cot(x) + \log(\sin(x)) - \frac{1}{2}x \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Cos}[x]^2*\text{Cot}[x]^2, x]$

[Out] $(-3*x^2)/4 - \text{Cos}[x]^2/4 - x*\text{Cot}[x] + \text{Log}[\text{Sin}[x]] - (x*\text{Cos}[x]*\text{Sin}[x])/2$

Rule 30

$\text{Int}[(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[x^{(m+1)}/(m+1), x] /; \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

Rule 3391

$\text{Int}[((c_.) + (d_.)*(x_))*((b_.)*\sin[(e_.) + (f_.)*(x_)])^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[d*((b*\sin[e + f*x])^n/(f^2*n^2)), x] + (\text{Dist}[b^2*((n-1)/n), \text{Int}[(c + d*x)*(b*\sin[e + f*x])^{(n-2)}, x], x] - \text{Simp}[b*(c + d*x)*\cos[e + f*x]*((b*\sin[e + f*x])^{(n-1)}/(f*n)), x]) /; \text{FreeQ}\{b, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[n, 1]$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_)], x_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3801

$\text{Int}[((c_.) + (d_.)*(x_))^{(m_.)*((b_.)*\tan[(e_.) + (f_.)*(x_)])^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[b*(c + d*x)^m*((b*\tan[e + f*x])^{(n-1)}/(f*(n-1))), x] + (-\text{Dist}[b*d*(m/(f*(n-1))), \text{Int}[(c + d*x)^{(m-1)}*(b*\tan[e + f*x])^{(n-1)}, x], x] - \text{Dist}[b^2, \text{Int}[(c + d*x)^m*(b*\tan[e + f*x])^{(n-2)}, x], x]) /; \text{FreeQ}\{b, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{GtQ}[m, 0]$

Rule 4493

$\text{Int}[\text{Cos}[(a_.) + (b_.)(x_.)]^{(n_.)} \text{Cot}[(a_.) + (b_.)(x_.)]^{(p_.)} ((c_.) + (d_.)(x_.))^{(m_.)}, x_Symbol] \text{ :> } -\text{Int}[(c + d*x)^m \text{Cos}[a + b*x]^n \text{Cot}[a + b*x]^{(p-2)}, x] + \text{Int}[(c + d*x)^m \text{Cos}[a + b*x]^{(n-2)} \text{Cot}[a + b*x]^p, x] \text{ /; FreeQ}\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$

Rubi steps

$$\begin{aligned} \int x \cos^2(x) \cot^2(x) dx &= - \int x \cos^2(x) dx + \int x \cot^2(x) dx \\ &= -\frac{1}{4} \cos^2(x) - x \cot(x) - \frac{1}{2} x \cos(x) \sin(x) - \frac{\int x dx}{2} - \int x dx + \int \cot(x) dx \\ &= -\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x \cot(x) + \log(\sin(x)) - \frac{1}{2} x \cos(x) \sin(x) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 33, normalized size = 1.00

$$-\frac{3x^2}{4} - \frac{1}{8} \cos(2x) - x \cot(x) + \log(\sin(x)) - \frac{1}{4} x \sin(2x)$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[x]^2*Cot[x]^2,x]

[Out] (-3*x^2)/4 - Cos[2*x]/8 - x*Cot[x] + Log[Sin[x]] - (x*Sin[2*x])/4

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 83 vs. 2(33) = 66.
time = 8.54, size = 67, normalized size = 2.03

$$\frac{-3x \cos[x] \sin[x]}{4} - \frac{x \cos\left[\frac{x}{2}\right]^5}{2 \sin\left[\frac{x}{2}\right]} + \frac{x \sin\left[\frac{x}{2}\right]^5}{2 \cos\left[\frac{x}{2}\right]} - \frac{3x^2}{4} + \text{Log}\left[\text{Tan}\left[\frac{x}{2}\right]\right] - \text{Log}[2] - \text{Log}\left[\frac{1}{1 + \cos[x]}\right] + \frac{\sin[x]^2}{4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x*Cos[x]^4/Sin[x]^2,x]')

[Out] -3 x Cos[x] Sin[x] / 4 - x Cos[x / 2] ^ 5 / (2 Sin[x / 2]) + x Sin[x / 2] ^ 5 / (2 Cos[x / 2]) - 3 x ^ 2 / 4 + Log[Tan[x / 2]] - Log[2] - Log[1 / (1 + Cos[x])] + Sin[x] ^ 2 / 4

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 60, normalized size = 1.82

method	result
risch	$-\frac{3x^2}{4} + \frac{i(i+2x)e^{2ix}}{16} - \frac{i(-i+2x)e^{-2ix}}{16} - 2ix - \frac{2ix}{e^{2ix}-1} + \ln(e^{2ix}-1)$
norman	$-\frac{\tan(\frac{x}{2})}{2} - \frac{(\tan^5(\frac{x}{2}))}{2} - \frac{x}{2} - \frac{3x(\tan^2(\frac{x}{2}))}{2} + \frac{3x(\tan^4(\frac{x}{2}))}{2} + \frac{x(\tan^6(\frac{x}{2}))}{2} - \frac{3x^2 \tan(\frac{x}{2})}{4} - \frac{3x^2(\tan^3(\frac{x}{2}))}{2} - \frac{3x^2(\tan^5(\frac{x}{2}))}{4} - \ln(1 + \frac{1}{(1+\tan^2(\frac{x}{2}))^2 \tan(\frac{x}{2})})$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cos(x)^4/sin(x)^2,x,method=_RETURNVERBOSE)`

[Out] $-3/4*x^2+1/16*I*(I+2*x)*\exp(2*I*x)-1/16*I*(-I+2*x)*\exp(-2*I*x)-2*I*x-2*I*x/(\exp(2*I*x)-1)+\ln(\exp(2*I*x)-1)$

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x)^4/sin(x)^2,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

Fricas [A]

time = 0.34, size = 45, normalized size = 1.36

$$\frac{4x \cos(x)^3 - 12x \cos(x) - (6x^2 + 2 \cos(x)^2 - 1) \sin(x) + 8 \log\left(\frac{1}{2} \sin(x)\right) \sin(x)}{8 \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x)^4/sin(x)^2,x, algorithm="fricas")`

[Out] $1/8*(4*x*cos(x)^3 - 12*x*cos(x) - (6*x^2 + 2*cos(x)^2 - 1)*sin(x) + 8*log(1/2*sin(x))*sin(x))/sin(x)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(32) = 64$

time = 0.82, size = 507, normalized size = 15.36

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x)**4/sin(x)**2,x)`

```
[Out] -3*x**2*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 6*x**2*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 3*x**2*tan(x/2)/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 2*x*tan(x/2)**6/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 6*x*tan(x/2)**4/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 6*x*tan(x/2)**2/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 2*x/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 4*log(tan(x/2)**2 + 1)*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 8*log(tan(x/2)**2 + 1)*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 4*log(tan(x/2)**2 + 1)*tan(x/2)/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 4*log(tan(x/2))*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 8*log(tan(x/2))*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 4*log(tan(x/2))*tan(x/2)/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 4*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(27) = 54$.
time = 0.01, size = 89, normalized size = 2.70

$$\frac{-6x^2 \tan^3 x - 6x^2 \tan x - 12x \tan^2 x - 8x + 4 \ln\left(\frac{4 \tan^2 x}{\tan^2 x + 1}\right) \tan^3 x + 4 \ln\left(\frac{4 \tan^2 x}{\tan^2 x + 1}\right) \tan x + \tan^3 x - \tan x}{8 \tan^3 x + 8 \tan x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^4/sin(x)^2,x)
```

```
[Out] -1/8*(6*x^2*tan(x)^3 - 4*log(4*tan(x)^2/(tan(x)^2 + 1))*tan(x)^3 + 6*x^2*tan(x) + 12*x*tan(x)^2 - tan(x)^3 - 4*log(4*tan(x)^2/(tan(x)^2 + 1))*tan(x) + 8*x + tan(x))/(tan(x)^3 + tan(x))
```

Mupad [B]

time = 0.49, size = 56, normalized size = 1.70

$$\ln(e^{x2i} - 1) - e^{-x2i} \left(\frac{1}{16} + \frac{x1i}{8}\right) + e^{x2i} \left(-\frac{1}{16} + \frac{x1i}{8}\right) - \frac{3x^2}{4} - x2i - \frac{x2i}{e^{x2i} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(x)^4)/sin(x)^2,x)
```

```
[Out] log(exp(x*2i) - 1) - x*2i - exp(-x*2i)*((x*1i)/8 + 1/16) + exp(x*2i)*((x*1i)/8 - 1/16) - (x*2i)/(exp(x*2i) - 1) - (3*x^2)/4
```


3.488 $\int x \sec(x) \tan^3(x) dx$

Optimal. Leaf size=30

$$\frac{5}{6} \tanh^{-1}(\sin(x)) - x \sec(x) + \frac{1}{3} x \sec^3(x) - \frac{1}{6} \sec(x) \tan(x)$$

[Out] 5/6*arctanh(sin(x))-x*sec(x)+1/3*x*sec(x)^3-1/6*sec(x)*tan(x)

Rubi [A]

time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2686, 4502, 3855, 3853}

$$\frac{1}{3} x \sec^3(x) - x \sec(x) + \frac{5}{6} \tanh^{-1}(\sin(x)) - \frac{1}{6} \tan(x) \sec(x)$$

Antiderivative was successfully verified.

[In] Int[x*Sec[x]*Tan[x]^3,x]

[Out] (5*ArcTanh[Sin[x]])/6 - x*Sec[x] + (x*Sec[x]^3)/3 - (Sec[x]*Tan[x])/6

Rule 2686

Int[((a_.)*sec[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)], x], x, Sec[e + f*x], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4502

Int[((c_.) + (d_.)*(x_.))^(m_.)*Sec[(a_.) + (b_.)*(x_.)]^(n_.)*Tan[(a_.) + (b_.)*(x_.)]^(p_.), x_Symbol] :> Module[{u = IntHide[Sec[a + b*x]^n*Tan[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m, 0] && (IntegerQ[n/2] || Int

egerQ[(p - 1)/2])

Rubi steps

$$\begin{aligned}
 \int x \sec(x) \tan^3(x) dx &= -x \sec(x) + \frac{1}{3} x \sec^3(x) - \int \left(-\sec(x) + \frac{\sec^3(x)}{3} \right) dx \\
 &= -x \sec(x) + \frac{1}{3} x \sec^3(x) - \frac{1}{3} \int \sec^3(x) dx + \int \sec(x) dx \\
 &= \tanh^{-1}(\sin(x)) - x \sec(x) + \frac{1}{3} x \sec^3(x) - \frac{1}{6} \sec(x) \tan(x) - \frac{1}{6} \int \sec(x) dx \\
 &= \frac{5}{6} \tanh^{-1}(\sin(x)) - x \sec(x) + \frac{1}{3} x \sec^3(x) - \frac{1}{6} \sec(x) \tan(x)
 \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 104 vs. 2(30) = 60.

time = 0.08, size = 104, normalized size = 3.47

$$-\frac{1}{24} \sec^3(x) (4x + 12x \cos(2x) + 5 \cos(3x) \log(\cos(\frac{x}{2}) - \sin(\frac{x}{2})) + 15 \cos(x) (\log(\cos(\frac{x}{2}) - \sin(\frac{x}{2})) - \log(\cos(\frac{x}{2}) + \sin(\frac{x}{2}))) - 5 \cos(3x) \log(\cos(\frac{x}{2}) + \sin(\frac{x}{2})) + 2 \sin(2x))$$

Antiderivative was successfully verified.

[In] Integrate[x*Sec[x]*Tan[x]^3,x]

[Out] -1/24*(Sec[x]^3*(4*x + 12*x*Cos[2*x] + 5*Cos[3*x]*Log[Cos[x/2] - Sin[x/2]] + 15*Cos[x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) - 5*Cos[3*x]*Log[Cos[x/2] + Sin[x/2]] + 2*Sin[2*x]))

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 215 vs. 2(30) = 60. time = 9.22, size = 171, normalized size = 5.70

$$\frac{-12x \tan^2\left(\frac{x}{2}\right) - 12x \tan\left(\frac{x}{2}\right)^4 + 4x + 4x \tan^6\left(\frac{x}{2}\right) - 15 \log\left(-1 + \tan\left(\frac{x}{2}\right)\right) \tan^2\left(\frac{x}{2}\right) - 15 \log\left(1 + \tan\left(\frac{x}{2}\right)\right) \tan^4\left(\frac{x}{2}\right) - 5 \log\left(-1 + \tan\left(\frac{x}{2}\right)\right) \tan\left(\frac{x}{2}\right)^6 - 5 \log\left(1 + \tan\left(\frac{x}{2}\right)\right) \tan^3\left(\frac{x}{2}\right) + 2 \tan\left(\frac{x}{2}\right) + 5 \log\left(-1 + \tan\left(\frac{x}{2}\right)\right) + 5 \log\left(1 + \tan\left(\frac{x}{2}\right)\right) \tan\left(\frac{x}{2}\right)^6 + 15 \log\left(-1 + \tan\left(\frac{x}{2}\right)\right) \tan\left(\frac{x}{2}\right)^4 + 15 \log\left(1 + \tan\left(\frac{x}{2}\right)\right) \tan\left(\frac{x}{2}\right)^2}{6 \left(-1 + \tan\left(\frac{x}{2}\right)\right)^3 \left(1 + \tan\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x*Sin[x]^3/Cos[x]^4,x]')

[Out] (-12 x Tan[x / 2]^2 - 12 x Tan[x / 2]^4 + 4 x + 4 x Tan[x / 2]^6 - 15 Log[-1 + Tan[x / 2]] Tan[x / 2]^2 - 15 Log[1 + Tan[x / 2]] Tan[x / 2]^4 - 5 Log[-1 + Tan[x / 2]] Tan[x / 2]^6 - 5 Log[1 + Tan[x / 2]] - 2 Tan[x / 2]^5 + 2 Tan[x / 2] + 5 Log[-1 + Tan[x / 2]] + 5 Log[1 + Tan[x / 2]] Tan[x / 2]^6 + 15 Log[-1 + Tan[x / 2]] Tan[x / 2]^4 + 15 Log[1 + Tan[x / 2]] Tan[x / 2]^2) / (6 (-1 + Tan[x / 2])^3 (1 + Tan[x / 2])^3)

Maple [A]

time = 0.20, size = 30, normalized size = 1.00

method	result	size
default	$-\frac{x}{\cos(x)} + \frac{5 \ln(\sec(x) + \tan(x))}{6} + \frac{x}{3 \cos(x)^3} - \frac{\sec(x) \tan(x)}{6}$	30
norman	$\frac{\frac{2x}{3} - \frac{(\tan^5(\frac{x}{2}))}{3} - 2x(\tan^2(\frac{x}{2})) - 2x(\tan^4(\frac{x}{2})) + \frac{2x(\tan^6(\frac{x}{2}))}{3} + \frac{\tan(\frac{x}{2})}{3}}{(\tan^2(\frac{x}{2}) - 1)^3} - \frac{5 \ln(\tan(\frac{x}{2}) - 1)}{6} + \frac{5 \ln(1 + \tan(\frac{x}{2}))}{6}$	76
risch	$-\frac{6x e^{5ix} + 4x e^{3ix} - i e^{5ix} + 6x e^{ix} + i e^{ix}}{3(e^{2ix} + 1)^3} + \frac{5 \ln(e^{ix} + i)}{6} - \frac{5 \ln(e^{ix} - i)}{6}$	76

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*sin(x)^3/cos(x)^4,x,method=_RETURNVERBOSE)`

[Out] $-x/\cos(x) + 5/6 * \ln(\sec(x) + \tan(x)) + 1/3 * x/\cos(x)^3 - 1/6 * \sec(x) * \tan(x)$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(24) = 48$.

time = 0.35, size = 619, normalized size = 20.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x)^3/cos(x)^4,x, algorithm="maxima")`

[Out] $-1/12 * (48 * x * \sin(3 * x) * \sin(2 * x) + 4 * (6 * x * \cos(5 * x) + 4 * x * \cos(3 * x) + 6 * x * \cos(x) + \sin(5 * x) - \sin(x)) * \cos(6 * x) + 12 * (6 * x * \cos(4 * x) + 6 * x * \cos(2 * x) + 2 * x - \sin(4 * x) - \sin(2 * x)) * \cos(5 * x) + 12 * (4 * x * \cos(3 * x) + 6 * x * \cos(x) - \sin(x)) * \cos(4 * x) + 16 * (3 * x * \cos(2 * x) + x) * \cos(3 * x) + 12 * (6 * x * \cos(x) - \sin(x)) * \cos(2 * x) + 24 * x * \cos(x) - 5 * (2 * (3 * \cos(4 * x) + 3 * \cos(2 * x) + 1) * \cos(6 * x) + \cos(6 * x)^2 + 6 * (3 * \cos(2 * x) + 1) * \cos(4 * x) + 9 * \cos(4 * x)^2 + 9 * \cos(2 * x)^2 + 6 * (\sin(4 * x) + \sin(2 * x)) * \sin(6 * x) + \sin(6 * x)^2 + 9 * \sin(4 * x)^2 + 18 * \sin(4 * x) * \sin(2 * x) + 9 * \sin(2 * x)^2 + 6 * \cos(2 * x) + 1) * \log(\cos(x)^2 + \sin(x)^2 + 2 * \sin(x) + 1) + 5 * (2 * (3 * \cos(4 * x) + 3 * \cos(2 * x) + 1) * \cos(6 * x) + \cos(6 * x)^2 + 6 * (3 * \cos(2 * x) + 1) * \cos(4 * x) + 9 * \cos(4 * x)^2 + 9 * \cos(2 * x)^2 + 6 * (\sin(4 * x) + \sin(2 * x)) * \sin(6 * x) + \sin(6 * x)^2 + 9 * \sin(4 * x)^2 + 18 * \sin(4 * x) * \sin(2 * x) + 9 * \sin(2 * x)^2 + 6 * \cos(2 * x) + 1) * \log(\cos(x)^2 + \sin(x)^2 - 2 * \sin(x) + 1) + 4 * (6 * x * \sin(5 * x) + 4 * x * \sin(3 * x) + 6 * x * \sin(x) - \cos(5 * x) + \cos(x)) * \sin(6 * x) + 4 * (18 * x * \sin(4 * x) + 18 * x * \sin(2 * x) + 3 * \cos(4 * x) + 3 * \cos(2 * x) + 1) * \sin(5 * x) + 12 * (4 * x * \sin(3 * x) + 6 * x * \sin(x) + \cos(x)) * \sin(4 * x) + 12 * (6 * x * \sin(x) + \cos(x)) * \sin(2 * x) - 4 * \sin(x)) / (2 * (3 * \cos(4 * x) + 3 * \cos(2 * x) + 1) * \cos(6 * x) + \cos(6 * x)^2 + 6 * (3 * \cos(2 * x) + 1) * \cos(4 * x) + 9 * \cos(4 * x)^2 + 9 * \cos(2 * x)^2 + 6 * (\sin(4 * x) + \sin(2 * x)) * \sin(6 * x) + \sin(6 * x)^2 + 9 * \sin(4 * x)^2 + 18 * \sin(4 * x) * \sin(2 * x) + 9 * \sin(2 * x)^2 + 6 * \cos(2 * x) + 1)$

Fricas [A]

time = 0.32, size = 47, normalized size = 1.57

$$\frac{5 \cos(x)^3 \log(\sin(x) + 1) - 5 \cos(x)^3 \log(-\sin(x) + 1) - 12 x \cos(x)^2 - 2 \cos(x) \sin(x) + 4 x}{12 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)^3/cos(x)^4,x, algorithm="fricas")

[Out] 1/12*(5*cos(x)^3*log(sin(x) + 1) - 5*cos(x)^3*log(-sin(x) + 1) - 12*x*cos(x)^2 - 2*cos(x)*sin(x) + 4*x)/cos(x)^3

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 551 vs. 2(29) = 58

time = 0.73, size = 551, normalized size = 18.37

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)**3/cos(x)**4,x)

[Out] 4*x*tan(x/2)**6/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 12*x*tan(x/2)**4/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 12*x*tan(x/2)**2/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 4*x/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 5*log(tan(x/2) - 1)*tan(x/2)**6/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 15*log(tan(x/2) - 1)*tan(x/2)**4/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 15*log(tan(x/2) - 1)*tan(x/2)**2/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 5*log(tan(x/2) - 1)/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 5*log(tan(x/2) + 1)*tan(x/2)**6/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 15*log(tan(x/2) + 1)*tan(x/2)**4/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 15*log(tan(x/2) + 1)*tan(x/2)**2/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 5*log(tan(x/2) + 1)/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 2*tan(x/2)**5/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 2*tan(x/2)/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(24) = 48.

time = 0.10, size = 391, normalized size = 13.03

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)^3/cos(x)^4,x)

[Out] 1/12*(8*x*tan(1/2*x)^6 + 5*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6 - 5*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6 - 24*x*tan(1/2*x)^4 - 15*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4 + 15*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4 - 4*tan(1/2*x)^5 - 24*

```
x*tan(1/2*x)^2 + 15*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 +
1))*tan(1/2*x)^2 - 15*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^
2 + 1))*tan(1/2*x)^2 + 8*x - 5*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan
(1/2*x)^2 + 1)) + 5*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 +
1)) + 4*tan(1/2*x))/(tan(1/2*x)^6 - 3*tan(1/2*x)^4 + 3*tan(1/2*x)^2 - 1)
```

Mupad [B]

time = 0.50, size = 35, normalized size = 1.17

$$\frac{x \cos(x)^2 - \frac{x}{3} + \frac{\sin(2x)}{12}}{\cos(x)^3} - \frac{\operatorname{atan}(\cos(x) + \sin(x) \operatorname{li} 5) 5i}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*sin(x)^3)/cos(x)^4,x)

[Out] - (atan(cos(x) + sin(x)*1i)*5i)/3 - (sin(2*x)/12 - x/3 + x*cos(x)^2)/cos(x)^3

3.489 $\int x \sec^2(x) \tan(x) dx$

Optimal. Leaf size=16

$$\frac{1}{2}x \sec^2(x) - \frac{\tan(x)}{2}$$

[Out] 1/2*x*sec(x)^2-1/2*tan(x)

Rubi [A]

time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3842, 3852, 8}

$$\frac{1}{2}x \sec^2(x) - \frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[x*Sec[x]^2*Tan[x],x]

[Out] (x*Sec[x]^2)/2 - Tan[x]/2

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3842

Int[(x_)^(m_)*Sec[(a_) + (b_)*(x_)^(n_)]^(p_)*Tan[(a_) + (b_)*(x_)^(n_)]^(q_), x_Symbol] := Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x] - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]

Rule 3852

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int x \sec^2(x) \tan(x) dx &= \frac{1}{2}x \sec^2(x) - \frac{1}{2} \int \sec^2(x) dx \\ &= \frac{1}{2}x \sec^2(x) + \frac{1}{2} \text{Subst}\left(\int 1 dx, x, -\tan(x)\right) \\ &= \frac{1}{2}x \sec^2(x) - \frac{\tan(x)}{2} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 16, normalized size = 1.00

$$\frac{1}{2}x \sec^2(x) - \frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sec[x]^2*Tan[x],x]**[Out]** (x*Sec[x]^2)/2 - Tan[x]/2**Mathics [B]** Leaf count is larger than twice the leaf count of optimal. 75 vs. 2(16) = 32.
time = 4.12, size = 61, normalized size = 3.81

$$-\frac{x \cos[2x]}{8 \cos[x]^2} + \frac{x}{8 \cos[x]^2} + \frac{x(-1 + \cos[x])^2}{8 \cos[x]^2} + \frac{x}{2(-1 + \tan[\frac{x}{2}])^2(1 + \tan[\frac{x}{2}])^2} - \frac{\sin[2x]}{4 \cos[x]^2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x*Sin[x]/Cos[x]^3,x]')**[Out]** -x Cos[2 x] / (8 Cos[x] ^ 2) + x / (8 Cos[x] ^ 2) + x (-1 + Cos[x]) ^ 2 / (8 Cos[x] ^ 2) + x / (2 (-1 + Tan[x / 2]) ^ 2 (1 + Tan[x / 2]) ^ 2) - Sin[2 x] / (4 Cos[x] ^ 2)**Maple [A]**

time = 0.10, size = 13, normalized size = 0.81

method	result	size
default	$\frac{x}{2 \cos(x)^2} - \frac{\tan(x)}{2}$	13
risch	$\frac{2x e^{2ix} - i e^{2ix} - i}{(e^{2ix} + 1)^2}$	30
norman	$\frac{\tan^3(\frac{x}{2}) + x(\tan^2(\frac{x}{2}) + \frac{x}{2} + \frac{x(\tan^4(\frac{x}{2}))}{2}) - \tan(\frac{x}{2})}{(\tan^2(\frac{x}{2}) - 1)^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(x)/cos(x)^3,x,method=_RETURNVERBOSE)**[Out]** 1/2*x/cos(x)^2-1/2*tan(x)**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(12) = 24.

time = 0.25, size = 132, normalized size = 8.25

$$\frac{4x \cos(2x)^2 + 4x \sin(2x)^2 + (2x \cos(2x) + \sin(2x)) \cos(4x) + 2x \cos(2x) + (2x \sin(2x) - \cos(2x) - 1) \sin(4x) - \sin(2x)}{2(2 \cos(2x) + 1) \cos(4x) + \cos(4x)^2 + 4 \cos(2x)^2 + \sin(4x)^2 + 4 \sin(4x) \sin(2x) + 4 \sin(2x)^2 + 4 \cos(2x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)/cos(x)^3,x, algorithm="maxima")

[Out] (4*x*cos(2*x)^2 + 4*x*sin(2*x)^2 + (2*x*cos(2*x) + sin(2*x))*cos(4*x) + 2*x*cos(2*x) + (2*x*sin(2*x) - cos(2*x) - 1)*sin(4*x) - sin(2*x))/(2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)

Fricas [A]

time = 0.31, size = 15, normalized size = 0.94

$$\frac{\cos(x) \sin(x) - x}{2 \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)/cos(x)^3,x, algorithm="fricas")

[Out] -1/2*(cos(x)*sin(x) - x)/cos(x)^2

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(12) = 24

time = 0.46, size = 128, normalized size = 8.00

$$\frac{x \tan^4\left(\frac{x}{2}\right)}{2 \tan^4\left(\frac{x}{2}\right) - 4 \tan^2\left(\frac{x}{2}\right) + 2} + \frac{2x \tan^2\left(\frac{x}{2}\right)}{2 \tan^4\left(\frac{x}{2}\right) - 4 \tan^2\left(\frac{x}{2}\right) + 2} + \frac{x}{2 \tan^4\left(\frac{x}{2}\right) - 4 \tan^2\left(\frac{x}{2}\right) + 2} + \frac{2 \tan^3\left(\frac{x}{2}\right)}{2 \tan^4\left(\frac{x}{2}\right) - 4 \tan^2\left(\frac{x}{2}\right) + 2} - \frac{2 \tan\left(\frac{x}{2}\right)}{2 \tan^4\left(\frac{x}{2}\right) - 4 \tan^2\left(\frac{x}{2}\right) + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)/cos(x)**3,x)

[Out] x*tan(x/2)**4/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) + 2*x*tan(x/2)**2/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) + x/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) + 2*tan(x/2)**3/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) - 2*tan(x/2)/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(12) = 24.

time = 0.00, size = 61, normalized size = 3.81

$$\frac{x \tan^4\left(\frac{x}{2}\right) + 2x \tan^2\left(\frac{x}{2}\right) + x + 2 \tan^3\left(\frac{x}{2}\right) - 2 \tan\left(\frac{x}{2}\right)}{2 \tan^4\left(\frac{x}{2}\right) - 4 \tan^2\left(\frac{x}{2}\right) + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)/cos(x)^3,x)

[Out] 1/2*(x*tan(1/2*x)^4 + 2*x*tan(1/2*x)^2 + 2*tan(1/2*x)^3 + x - 2*tan(1/2*x))/(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)

Mupad [B]

time = 0.35, size = 16, normalized size = 1.00

$$\frac{2x - \sin(2x)}{4\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*sin(x))/cos(x)^3,x)`

[Out] `(2*x - sin(2*x))/(4*cos(x)^2)`

3.490 $\int x \sin^2(x) \tan(x) dx$

Optimal. Leaf size=62

$$\frac{x}{4} + \frac{ix^2}{2} - x \log(1 + e^{2ix}) + \frac{1}{2}i\text{Li}_2(-e^{2ix}) - \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x\sin^2(x)$$

[Out] 1/4*x+1/2*I*x^2-x*ln(1+exp(2*I*x))+1/2*I*polylog(2,-exp(2*I*x))-1/4*cos(x)*sin(x)-1/2*x*sin(x)^2

Rubi [A]

time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {4492, 3524, 2715, 8, 3800, 2221, 2317, 2438}

$$\frac{1}{2}i\text{Li}_2(-e^{2ix}) + \frac{ix^2}{2} + \frac{x}{4} - x \log(1 + e^{2ix}) - \frac{1}{2}x\sin^2(x) - \frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

[In] Int[x*Sin[x]^2*Tan[x],x]

[Out] x/4 + (I/2)*x^2 - x*Log[1 + E^((2*I)*x)] + (I/2)*PolyLog[2, -E^((2*I)*x)] - (Cos[x]*Sin[x])/4 - (x*Sin[x]^2)/2

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2221

Int[(((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2317

Int[Log[(a_) + (b_)*((F_)^(e_)*((c_) + (d_)*(x_)))^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2438

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^(p_.), x_Symbol] := Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 4492

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
 \int x \sin^2(x) \tan(x) dx &= - \int x \cos(x) \sin(x) dx + \int x \tan(x) dx \\
 &= \frac{ix^2}{2} - \frac{1}{2}x \sin^2(x) - 2i \int \frac{e^{2ix}x}{1 + e^{2ix}} dx + \frac{1}{2} \int \sin^2(x) dx \\
 &= \frac{ix^2}{2} - x \log(1 + e^{2ix}) - \frac{1}{4} \cos(x) \sin(x) - \frac{1}{2}x \sin^2(x) + \frac{\int 1 dx}{4} + \int \log(1 + e^{2ix}) dx \\
 &= \frac{x}{4} + \frac{ix^2}{2} - x \log(1 + e^{2ix}) - \frac{1}{4} \cos(x) \sin(x) - \frac{1}{2}x \sin^2(x) - \frac{1}{2}i \text{Subst} \left(\int \frac{\log(1 + e^{2ix})}{x} dx \right) \\
 &= \frac{x}{4} + \frac{ix^2}{2} - x \log(1 + e^{2ix}) + \frac{1}{2}i \text{Li}_2(-e^{2ix}) - \frac{1}{4} \cos(x) \sin(x) - \frac{1}{2}x \sin^2(x)
 \end{aligned}$$

Mathematica [A]

time = 0.01, size = 57, normalized size = 0.92

$$\frac{ix^2}{2} + \frac{1}{4}x \cos(2x) - x \log(1 + e^{2ix}) + \frac{1}{2}i\text{Li}_2(-e^{2ix}) - \frac{1}{8}\sin(2x)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sin[x]^2*Tan[x],x]

[Out] (I/2)*x^2 + (x*Cos[2*x])/4 - x*Log[1 + E^((2*I)*x)] + (I/2)*PolyLog[2, -E^((2*I)*x)] - Sin[2*x]/8

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x*Sin[x]^3/Cos[x],x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.08, size = 57, normalized size = 0.92

method	result	size
risch	$\frac{ix^2}{2} + \frac{(i+2x)e^{2ix}}{16} + \frac{(-i+2x)e^{-2ix}}{16} - x \ln(e^{2ix} + 1) + \frac{i \text{polylog}(2, -e^{2ix})}{2}$	57

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(x)^3/cos(x),x,method=_RETURNVERBOSE)

[Out] 1/2*I*x^2+1/16*(I+2*x)*exp(2*I*x)+1/16*(-I+2*x)*exp(-2*I*x)-x*ln(exp(2*I*x)+1)+1/2*I*polylog(2,-exp(2*I*x))

Maxima [A]

time = 0.33, size = 66, normalized size = 1.06

$$\frac{1}{2}ix^2 - ix \arctan(\sin(2x), \cos(2x) + 1) + \frac{1}{4}x \cos(2x) - \frac{1}{2}x \log(\cos(2x)^2 + \sin(2x)^2 + 2 \cos(2x) + 1) + \frac{1}{2}i\text{Li}_2(-e^{2ix}) - \frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)^3/cos(x),x, algorithm="maxima")

[Out] 1/2*I*x^2 - I*x*arctan2(sin(2*x), cos(2*x) + 1) + 1/4*x*cos(2*x) - 1/2*x*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + 1/2*I*dilog(-e^(2*I*x)) - 1/8*sin(2*x)

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(41) = 82$.
time = 0.34, size = 113, normalized size = 1.82

$$\frac{1}{2}x \cos(x)^2 - \frac{1}{2}x \log(i \cos(x) + \sin(x) + 1) - \frac{1}{2}x \log(i \cos(x) - \sin(x) + 1) - \frac{1}{2}x \log(-i \cos(x) + \sin(x) + 1) - \frac{1}{2}x \log(-i \cos(x) - \sin(x) + 1) - \frac{1}{4} \cos(x) \sin(x) - \frac{1}{4}x - \frac{1}{2}i \operatorname{Li}_2(i \cos(x) + \sin(x)) + \frac{1}{2}i \operatorname{Li}_2(i \cos(x) - \sin(x)) + \frac{1}{2}i \operatorname{Li}_2(-i \cos(x) + \sin(x)) - \frac{1}{2}i \operatorname{Li}_2(-i \cos(x) - \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)^3/cos(x),x, algorithm="fricas")

[Out] $\frac{1}{2}x \cos(x)^2 - \frac{1}{2}x \log(i \cos(x) + \sin(x) + 1) - \frac{1}{2}x \log(i \cos(x) - \sin(x) + 1) - \frac{1}{2}x \log(-i \cos(x) + \sin(x) + 1) - \frac{1}{2}x \log(-i \cos(x) - \sin(x) + 1) - \frac{1}{4} \cos(x) \sin(x) - \frac{1}{4}x - \frac{1}{2}i \operatorname{dilog}(i \cos(x) + \sin(x)) + \frac{1}{2}i \operatorname{dilog}(i \cos(x) - \sin(x)) + \frac{1}{2}i \operatorname{dilog}(-i \cos(x) + \sin(x)) - \frac{1}{2}i \operatorname{dilog}(-i \cos(x) - \sin(x))$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin^3(x)}{\cos(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)**3/cos(x),x)

[Out] Integral(x*sin(x)**3/cos(x), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)^3/cos(x),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x \sin(x)^3}{\cos(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*sin(x)^3)/cos(x),x)

[Out] int((x*sin(x)^3)/cos(x), x)

3.491 $\int x \tan^3(x) dx$

Optimal. Leaf size=59

$$\frac{x}{2} - \frac{ix^2}{2} + x \log(1 + e^{2ix}) - \frac{1}{2}i\text{Li}_2(-e^{2ix}) - \frac{\tan(x)}{2} + \frac{1}{2}x \tan^2(x)$$

[Out] 1/2*x-1/2*I*x^2+x*ln(1+exp(2*I*x))-1/2*I*polylog(2,-exp(2*I*x))-1/2*tan(x)+1/2*x*tan(x)^2

Rubi [A]

time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {3801, 3554, 8, 3800, 2221, 2317, 2438}

$$-\frac{1}{2}i\text{Li}_2(-e^{2ix}) - \frac{ix^2}{2} + \frac{x}{2} + x \log(1 + e^{2ix}) + \frac{1}{2}x \tan^2(x) - \frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[x*Tan[x]^3,x]

[Out] x/2 - (I/2)*x^2 + x*Log[1 + E^((2*I)*x)] - (I/2)*PolyLog[2, -E^((2*I)*x)] - Tan[x]/2 + (x*Tan[x]^2)/2

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2221

Int[(((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2317

Int[Log[(a_) + (b_)*((F_)^(e_)*((c_) + (d_)*(x_)))^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2438

Int[Log[(c_)*((d_) + (e_)*(x_))^(n_)]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
+ f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] := Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x \tan^3(x) dx &= \frac{1}{2} x \tan^2(x) - \frac{1}{2} \int \tan^2(x) dx - \int x \tan(x) dx \\
&= -\frac{ix^2}{2} - \frac{\tan(x)}{2} + \frac{1}{2} x \tan^2(x) + 2i \int \frac{e^{2ix} x}{1 + e^{2ix}} dx + \frac{\int 1 dx}{2} \\
&= \frac{x}{2} - \frac{ix^2}{2} + x \log(1 + e^{2ix}) - \frac{\tan(x)}{2} + \frac{1}{2} x \tan^2(x) - \int \log(1 + e^{2ix}) dx \\
&= \frac{x}{2} - \frac{ix^2}{2} + x \log(1 + e^{2ix}) - \frac{\tan(x)}{2} + \frac{1}{2} x \tan^2(x) + \frac{1}{2} i \text{Subst} \left(\int \frac{\log(1 + x)}{x} dx, x, e^{2ix} \right) \\
&= \frac{x}{2} - \frac{ix^2}{2} + x \log(1 + e^{2ix}) - \frac{1}{2} i \text{Li}_2(-e^{2ix}) - \frac{\tan(x)}{2} + \frac{1}{2} x \tan^2(x)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 54, normalized size = 0.92

$$-\frac{ix^2}{2} + x \log(1 + e^{2ix}) - \frac{1}{2} i \text{Li}_2(-e^{2ix}) + \frac{1}{2} x \sec^2(x) - \frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Tan[x]^3,x]

[Out] $(-1/2*I)*x^2 + x*\text{Log}[1 + E^((2*I)*x)] - (I/2)*\text{PolyLog}[2, -E^((2*I)*x)] + (x*\text{Sec}[x]^2)/2 - \text{Tan}[x]/2$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[x*Sin[x]^3/Cos[x]^3,x]')`

[Out] cought exception: maximum recursion depth exceeded

Maple [A]

time = 0.27, size = 59, normalized size = 1.00

method	result	size
risch	$-\frac{ix^2}{2} + \frac{2xe^{2ix} - ie^{2ix} - i}{(e^{2ix} + 1)^2} + x \ln(e^{2ix} + 1) - \frac{i \text{polylog}(2, -e^{2ix})}{2}$	59

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*sin(x)^3/cos(x)^3,x,method=_RETURNVERBOSE)`

[Out] $-1/2*I*x^2 + (2*x*\exp(2*I*x) - I*\exp(2*I*x) - I) / (\exp(2*I*x) + 1)^2 + x*\ln(\exp(2*I*x) + 1) - 1/2*I*\text{polylog}(2, -\exp(2*I*x))$

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(38) = 76$.

time = 0.34, size = 210, normalized size = 3.56

$\frac{x^2 \cos(4x) + ix^2 \sin(4x) + x^2 - 2ix \cos(4x) + 2x \cos(2x) + ix \sin(4x) + 2ix \sin(2x) + x \arctan(\sin(2x), \cos(2x) + 1) + 2(x^2 + 2ix + 1) \cos(2x) + (\cos(4x) + 2 \cos(2x) + i \sin(4x) + 2i \sin(2x) + 1) \text{Li}_2(-e^{2ix}) - (-ix \cos(4x) - 2ix \cos(2x) + x \sin(4x) + 2ix \sin(2x) - ix) \log(\cos(2x)^2 + \sin(2x)^2 + 2 \cos(2x) + 1) + 2(i x^2 - 2ix + i \sin(2x) + 2i \cos(4x) - 4i \cos(2x) + 2 \sin(4x) + 4 \sin(2x) - 2)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x)^3/cos(x)^3,x, algorithm="maxima")`

[Out] $-(x^2*\cos(4*x) + I*x^2*\sin(4*x) + x^2 - 2*(x*\cos(4*x) + 2*x*\cos(2*x) + I*x*\sin(4*x) + 2*I*x*\sin(2*x) + x)*\arctan2(\sin(2*x), \cos(2*x) + 1) + 2*(x^2 + 2*I*x + 1)*\cos(2*x) + (\cos(4*x) + 2*\cos(2*x) + I*\sin(4*x) + 2*I*\sin(2*x) + 1)*\text{dilog}(-e^{(2*I*x)}) - (-I*x*\cos(4*x) - 2*I*x*\cos(2*x) + x*\sin(4*x) + 2*x*\sin(2*x) - I*x)*\log(\cos(2*x)^2 + \sin(2*x)^2 + 2*\cos(2*x) + 1) + 2*(I*x^2 - 2*x + I)*\sin(2*x) + 2)/(-2*I*\cos(4*x) - 4*I*\cos(2*x) + 2*\sin(4*x) + 4*\sin(2*x) - 2*I)$

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(38) = 76$.

time = 0.36, size = 138, normalized size = 2.34

$\frac{x \cos(x)^2 \log(i \cos(x) + \sin(x) + 1) + x \cos(x)^2 \log(i \cos(x) - \sin(x) + 1) + x \cos(x)^2 \log(-i \cos(x) + \sin(x) + 1) + x \cos(x)^2 \log(-i \cos(x) - \sin(x) + 1) + i \cos(x)^2 \text{Li}_2(i \cos(x) + \sin(x)) - i \cos(x)^2 \text{Li}_2(i \cos(x) - \sin(x)) - i \cos(x)^2 \text{Li}_2(-i \cos(x) + \sin(x)) + i \cos(x)^2 \text{Li}_2(-i \cos(x) - \sin(x)) - \cos(x) \sin(x) + x}{2 \cos(x)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x)^3/cos(x)^3,x, algorithm="fricas")`

[Out] $\frac{1}{2}(x\cos(x)^2\log(I\cos(x) + \sin(x) + 1) + x\cos(x)^2\log(I\cos(x) - \sin(x) + 1) + x\cos(x)^2\log(-I\cos(x) + \sin(x) + 1) + x\cos(x)^2\log(-I\cos(x) - \sin(x) + 1) + I\cos(x)^2\operatorname{dilog}(I\cos(x) + \sin(x)) - I\cos(x)^2\operatorname{dilog}(I\cos(x) - \sin(x)) - I\cos(x)^2\operatorname{dilog}(-I\cos(x) + \sin(x)) + I\cos(x)^2\operatorname{dilog}(-I\cos(x) - \sin(x)) - \cos(x)\sin(x) + x)/\cos(x)^2$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin^3(x)}{\cos^3(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x)**3/cos(x)**3,x)`

[Out] `Integral(x*sin(x)**3/cos(x)**3, x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x)^3/cos(x)^3,x)`

[Out] `Could not integrate`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x \sin(x)^3}{\cos(x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*sin(x)^3)/cos(x)^3,x)`

[Out] `int((x*sin(x)^3)/cos(x)^3, x)`

$$3.492 \quad \int \frac{2x + \sin(2x)}{(\cos(x) + x \sin(x))^2} dx$$

Optimal. Leaf size=12

$$\frac{2}{1 + \frac{\cot(x)}{x}}$$

[Out] 2/(1+cot(x)/x)

Rubi [A]

time = 0.07, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6843, 32}

$$\frac{2}{\frac{\cot(x)}{x} + 1}$$

Antiderivative was successfully verified.

[In] Int[(2*x + Sin[2*x])/(Cos[x] + x*Sin[x])^2,x]

[Out] 2/(1 + Cot[x]/x)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 6843

Int[(u_)*((a_.)*(v_)^(p_.) + (b_.)*(w_)^(q_.))^(m_.), x_Symbol] := With[{c = Simplify[u/(p*w*D[v, x] - q*v*D[w, x])]}, Dist[c*p, Subst[Int[(b + a*x^p)^m, x], x, v*w^(m*q + 1)], x] /; FreeQ[c, x] /; FreeQ[{a, b, m, p, q}, x] && EqQ[p + q*(m*p + 1), 0] && IntegerQ[p] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{2x + \sin(2x)}{(\cos(x) + x \sin(x))^2} dx &= - \left(2 \text{Subst} \left(\int \frac{1}{(1+x)^2} dx, x, \frac{\cot(x)}{x} \right) \right) \\ &= \frac{2}{1 + \frac{\cot(x)}{x}} \end{aligned}$$

Mathematica [A]

time = 0.15, size = 14, normalized size = 1.17

$$\frac{2x \sin(x)}{\cos(x) + x \sin(x)}$$

Antiderivative was successfully verified.

[In] Integrate[(2*x + Sin[2*x])/(Cos[x] + x*Sin[x])^2,x]

[Out] (2*x*Sin[x])/(Cos[x] + x*Sin[x])

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(2*x+Sin[2*x])/(x*Sin[x]+Cos[x])^2,x]')

[Out] Timed out

Maple [C] Result contains complex when optimal does not.

time = 0.87, size = 44, normalized size = 3.67

method	result	size
risch	$-\frac{2i}{x+i} - \frac{4ix}{(x+i)(xe^{2ix}-x+ie^{2ix}+i)}$	44

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x,method=_RETURNVERBOSE)

[Out] -2*I/(x+I)-4*I*x/(x+I)/(x*exp(2*I*x)-x+I*exp(2*I*x)+I)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(12) = 24$.

time = 0.34, size = 78, normalized size = 6.50

$$\frac{2(\cos(2x)^2 + 2x\sin(2x) + \sin(2x)^2 + 2\cos(2x) + 1)}{(x^2 + 1)\cos(2x)^2 + (x^2 + 1)\sin(2x)^2 + x^2 - 2(x^2 - 1)\cos(2x) + 4x\sin(2x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x, algorithm="maxima")

[Out] -2*(cos(2*x)^2 + 2*x*sin(2*x) + sin(2*x)^2 + 2*cos(2*x) + 1)/((x^2 + 1)*cos(2*x)^2 + (x^2 + 1)*sin(2*x)^2 + x^2 - 2*(x^2 - 1)*cos(2*x) + 4*x*sin(2*x) + 1)

Fricas [A]

time = 0.33, size = 13, normalized size = 1.08

$$\frac{2\cos(x)}{x\sin(x) + \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x, algorithm="fricas")

[Out] -2*cos(x)/(x*sin(x) + cos(x))

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x + \sin(2x)}{(x \sin(x) + \cos(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))**2,x)

[Out] Integral((2*x + sin(2*x))/(x*sin(x) + cos(x))**2, x)

Giac [A]

time = 0.01, size = 10, normalized size = 0.83

$$-\frac{2}{x \tan x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x)

[Out] -2/(x*tan(x) + 1)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{2x + \sin(2x)}{(\cos(x) + x \sin(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + sin(2*x))/(cos(x) + x*sin(x))^2,x)

[Out] int((2*x + sin(2*x))/(cos(x) + x*sin(x))^2, x)

$$3.493 \quad \int \frac{x^2}{(x \cos(x) - \sin(x))^2} dx$$

Optimal. Leaf size=20

$$-\cot(x) + \frac{x \csc(x)}{x \cos(x) - \sin(x)}$$

[Out] -cot(x)+x*csc(x)/(x*cos(x)-sin(x))

Rubi [A]

time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4690, 3852, 8}

$$\frac{x \csc(x)}{x \cos(x) - \sin(x)} - \cot(x)$$

Antiderivative was successfully verified.

[In] Int[x^2/(x*cos[x] - Sin[x])^2,x]

[Out] -Cot[x] + (x*Csc[x])/(x*cos[x] - Sin[x])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3852

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 4690

Int[(x_)^2/(Cos[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] := Simp[x/(a*d*Sin[a*x]*(c*Sin[a*x] + d*x*cos[a*x])), x] + Dist[1/d^2, Int[1/Sin[a*x]^2, x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(x \cos(x) - \sin(x))^2} dx &= \frac{x \csc(x)}{x \cos(x) - \sin(x)} + \int \csc^2(x) dx \\ &= \frac{x \csc(x)}{x \cos(x) - \sin(x)} - \text{Subst}\left(\int 1 dx, x, \cot(x)\right) \\ &= -\cot(x) + \frac{x \csc(x)}{x \cos(x) - \sin(x)} \end{aligned}$$

Mathematica [A]

time = 0.15, size = 19, normalized size = 0.95

$$\frac{\cos(x) + x \sin(x)}{x \cos(x) - \sin(x)}$$

Antiderivative was successfully verified.

`[In] Integrate[x^2/(x*Cos[x] - Sin[x])^2,x]``[Out] (Cos[x] + x*Sin[x])/(x*Cos[x] - Sin[x])`**Mathics [B]** Leaf count is larger than twice the leaf count of optimal. 100 vs. $2(20) = 40$.
time = 3.50, size = 86, normalized size = 4.30

$$\frac{-x \cos[x] + x \left(-x + x \tan\left[\frac{x}{2}\right]^2 + 2 \tan\left[\frac{x}{2}\right] \right) \sin[x] + \sin[x] + \frac{(-1 + \cos[x]) \left(-x + x \tan\left[\frac{x}{2}\right]^2 + 2 \tan\left[\frac{x}{2}\right] \right)}{2}}{\left(-x + x \tan\left[\frac{x}{2}\right]^2 + 2 \tan\left[\frac{x}{2}\right] \right) (x \cos[x] - \sin[x])}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(x/(x*Cos[x]-Sin[x]))^2,x]')``[Out] (-x Cos[x] + x (-x + x Tan[x / 2] ^ 2 + 2 Tan[x / 2]) Sin[x] + Sin[x] + (-1 + Cos[x]) (-x + x Tan[x / 2] ^ 2 + 2 Tan[x / 2]) / 2) / ((-x + x Tan[x / 2]] ^ 2 + 2 Tan[x / 2]) (x Cos[x] - Sin[x]))`**Maple [C]** Result contains complex when optimal does not.
time = 0.25, size = 29, normalized size = 1.45

method	result	size
risch	$\frac{2i(x-i)}{ie^{2ix} + x e^{2ix} - i + x}$	29
norman	$\frac{-1 + \tan^2\left(\frac{x}{2}\right) - 2x \tan\left(\frac{x}{2}\right)}{x \left(\tan^2\left(\frac{x}{2}\right) - x + 2 \tan\left(\frac{x}{2}\right) \right)}$	37

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2/(x*cos(x)-sin(x))^2,x,method=_RETURNVERBOSE)``[Out] 2*I*(x-I)/(I*exp(2*I*x)+x*exp(2*I*x)-I+x)`**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(20) = 40$.
time = 0.24, size = 69, normalized size = 3.45

$$\frac{2(2x \cos(2x) + (x^2 - 1) \sin(2x))}{(x^2 + 1) \cos(2x)^2 + (x^2 + 1) \sin(2x)^2 + x^2 + 2(x^2 - 1) \cos(2x) - 4x \sin(2x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x*cos(x)-sin(x))^2,x, algorithm="maxima")

[Out] 2*(2*x*cos(2*x) + (x^2 - 1)*sin(2*x))/((x^2 + 1)*cos(2*x)^2 + (x^2 + 1)*sin(2*x)^2 + x^2 + 2*(x^2 - 1)*cos(2*x) - 4*x*sin(2*x) + 1)

Fricas [A]

time = 0.32, size = 19, normalized size = 0.95

$$\frac{x \sin(x) + \cos(x)}{x \cos(x) - \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x*cos(x)-sin(x))^2,x, algorithm="fricas")

[Out] (x*sin(x) + cos(x))/(x*cos(x) - sin(x))

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(15) = 30

time = 0.79, size = 66, normalized size = 3.30

$$-\frac{2x \tan\left(\frac{x}{2}\right)}{x \tan^2\left(\frac{x}{2}\right) - x + 2 \tan\left(\frac{x}{2}\right)} + \frac{\tan^2\left(\frac{x}{2}\right)}{x \tan^2\left(\frac{x}{2}\right) - x + 2 \tan\left(\frac{x}{2}\right)} - \frac{1}{x \tan^2\left(\frac{x}{2}\right) - x + 2 \tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(x*cos(x)-sin(x))**2,x)

[Out] -2*x*tan(x/2)/(x*tan(x/2)**2 - x + 2*tan(x/2)) + tan(x/2)**2/(x*tan(x/2)**2 - x + 2*tan(x/2)) - 1/(x*tan(x/2)**2 - x + 2*tan(x/2))

Giac [A]

time = 0.00, size = 13, normalized size = 0.65

$$\frac{x \tan x + 1}{x - \tan x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x*cos(x)-sin(x))^2,x)

[Out] (x*tan(x) + 1)/(x - tan(x))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^2}{(\sin(x) - x \cos(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(sin(x) - x*cos(x))^2,x)

[Out] int(x^2/(sin(x) - x*cos(x))^2, x)

3.494 $\int a^{mx} b^{nx} dx$

Optimal. Leaf size=22

$$\frac{a^{mx} b^{nx}}{m \log(a) + n \log(b)}$$

[Out] $a^{(m*x)}*b^{(n*x)}/(m*\ln(a)+n*\ln(b))$

Rubi [A]

time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2325, 2225}

$$\frac{a^{mx} b^{nx}}{m \log(a) + n \log(b)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[a^{(m*x)}*b^{(n*x)}, x]$

[Out] $(a^{(m*x)}*b^{(n*x)})/(m*\text{Log}[a] + n*\text{Log}[b])$

Rule 2225

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x)))^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

Rule 2325

$\text{Int}[(u_)*(F_)^{(v_)}*(G_)^{(w_)}, x_Symbol] \rightarrow \text{With}\{z = v*\text{Log}[F] + w*\text{Log}[G]\}, \text{Int}[u*\text{NormalizeIntegrand}[E^z, x], x] /; \text{BinomialQ}[z, x] \|\| (\text{PolynomialQ}[z, x] \&\& \text{LeQ}[\text{Exponent}[z, x], 2]) /; \text{FreeQ}\{F, G\}, x]$

Rubi steps

$$\begin{aligned} \int a^{mx} b^{nx} dx &= \int e^{x(m \log(a) + n \log(b))} dx \\ &= \frac{a^{mx} b^{nx}}{m \log(a) + n \log(b)} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 22, normalized size = 1.00

$$\frac{a^{mx} b^{nx}}{m \log(a) + n \log(b)}$$

Antiderivative was successfully verified.

[In] Integrate[a^(m*x)*b^(n*x),x]

[Out] (a^(m*x)*b^(n*x))/(m*Log[a] + n*Log[b])

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.95, size = 51, normalized size = 2.32

$$\text{Piecewise} \left[\left\{ \left\{ \frac{a^{mx} b^{nx}}{m \text{Log}[a] + n \text{Log}[b]}, m \neq -\frac{n \text{Log}[b]}{\text{Log}[a]} \right\} \right\}, x E^{-nx \text{Log}[b]} b^{nx} \right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[a^(m*x)*b^(n*x),x]')

[Out] Piecewise[{{a ^ (m x) b ^ (n x) / (m Log[a] + n Log[b]), m != -n Log[b] / Log[a]}, x E ^ (-n x Log[b]) b ^ (n x)}

Maple [A]

time = 0.03, size = 23, normalized size = 1.05

method	result	size
gospers	$\frac{a^{mx} b^{nx}}{m \ln(a) + n \ln(b)}$	23
risch	$\frac{a^{mx} b^{nx}}{m \ln(a) + n \ln(b)}$	23
norman	$\frac{e^{mx \ln(a)} e^{nx \ln(b)}}{m \ln(a) + n \ln(b)}$	25
meijerg	$-\frac{1 - e^{xn \ln(b) \left(1 + \frac{m \ln(a)}{n \ln(b)}\right)}}{n \ln(b) \left(1 + \frac{m \ln(a)}{n \ln(b)}\right)}$	48

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a^(m*x)*b^(n*x),x,method=_RETURNVERBOSE)

[Out] a^(m*x)*b^(n*x)/(m*ln(a)+n*ln(b))

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^(m*x)*b^(n*x),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume((log(b)*n)/(log(a)*m)>0)', see 'assume?' f

Fricas [A]

time = 0.32, size = 22, normalized size = 1.00

$$\frac{a^{mx}b^{nx}}{m \log(a) + n \log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^(m*x)*b^(n*x),x, algorithm="fricas")

[Out] a^(m*x)*b^(n*x)/(m*log(a) + n*log(b))

Sympy [A]

time = 0.32, size = 42, normalized size = 1.91

$$\begin{cases} \frac{a^{mx}b^{nx}}{m \log(a) + n \log(b)} & \text{for } m \neq -\frac{n \log(b)}{\log(a)} \\ b^{nx} x e^{-nx \log(b)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a**(m*x)*b**(n*x),x)

[Out] Piecewise((a**(m*x)*b**(n*x)/(m*log(a) + n*log(b)), Ne(m, -n*log(b)/log(a))), (b**(n*x)*x*exp(-n*x*log(b)), True))

Giac [C] Result contains complex when optimal does not.

time = 0.00, size = 319, normalized size = 14.50

$$e^{(m \ln|a| + n \ln|b|)x} \left(\frac{2(2m \ln|a| + 2n \ln|b|) \cos\left(\frac{1}{2}\pi m(1 - \text{sign}(a)) + \frac{1}{2}\pi n(1 - \text{sign}(b))\right)x}{(2m \ln|a| + 2n \ln|b|)^2 + (-\pi \text{sign}(a) + \pi m - \pi \text{sign}(b) + \pi n)^2} - \frac{2(\pi \text{sign}(a) - \pi m + \pi \text{sign}(b) - \pi n) \sin\left(\frac{1}{2}\pi m(1 - \text{sign}(a)) + \frac{1}{2}\pi n(1 - \text{sign}(b))\right)x}{(2m \ln|a| + 2n \ln|b|)^2 + (-\pi \text{sign}(a) + \pi m - \pi \text{sign}(b) + \pi n)^2} \right) + \frac{e^{(m \ln|a| + n \ln|b|)x} \left(\frac{2m \cos\left(\frac{1}{2}\pi m(1 - \text{sign}(a)) + \frac{1}{2}\pi n(1 - \text{sign}(b))\right)x}{2m \ln|a| - \pi \text{sign}(a) + \pi m + 2n \ln|b| - \pi \text{sign}(b) + \pi n} - \frac{2m \sin\left(\frac{1}{2}\pi m(1 - \text{sign}(a)) + \frac{1}{2}\pi n(1 - \text{sign}(b))\right)x}{2m \ln|a| + \pi \text{sign}(a) - \pi m + 2n \ln|b| + \pi \text{sign}(b) - \pi n} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^(m*x)*b^(n*x),x)

[Out] 2*e^((m*log(abs(a)) + n*log(abs(b)))*x)*(2*(m*log(abs(a)) + n*log(abs(b))))*cos(-1/2*pi*m*x*sgn(a) - 1/2*pi*n*x*sgn(b) + 1/2*pi*m*x + 1/2*pi*n*x)/((pi*m*sgn(a) + pi*n*sgn(b) - pi*m - pi*n)^2 + 4*(m*log(abs(a)) + n*log(abs(b)))^2) - (pi*m*sgn(a) + pi*n*sgn(b) - pi*m - pi*n)*sin(-1/2*pi*m*x*sgn(a) - 1/2*pi*n*x*sgn(b) + 1/2*pi*m*x + 1/2*pi*n*x)/((pi*m*sgn(a) + pi*n*sgn(b) - pi*m - pi*n)^2 + 4*(m*log(abs(a)) + n*log(abs(b)))^2) - e^((m*log(abs(a)) + n*log(abs(b)))*x)*(e^(1/2*I*pi*m*x*sgn(a) + 1/2*I*pi*n*x*sgn(b) - 1/2*I*pi*m*x - 1/2*I*pi*n*x)/(I*pi*m*sgn(a) + I*pi*n*sgn(b) - I*pi*m - I*pi*n + 2*m*log(abs(a)) + 2*n*log(abs(b))) - e^(-1/2*I*pi*m*x*sgn(a) - 1/2*I*pi*n*x*sgn(b) + 1/2*I*pi*m*x + 1/2*I*pi*n*x)/(-I*pi*m*sgn(a) - I*pi*n*sgn(b) + I*pi*m + I*pi*n + 2*m*log(abs(a)) + 2*n*log(abs(b))))

Mupad [B]

time = 0.33, size = 22, normalized size = 1.00

$$\frac{a^{mx} b^{nx}}{m \ln(a) + n \ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a^(m*x)*b^(n*x),x)`

[Out] `(a^(m*x)*b^(n*x))/(m*log(a) + n*log(b))`

3.495 $\int a^{-x} b^{-x} (a^x - b^x)^2 dx$

Optimal. Leaf size=34

$$-2x + \frac{a^x b^{-x} - a^{-x} b^x}{\log(a) - \log(b)}$$

[Out] $-2*x + (a^x/(b^x) - b^x/(a^x))/(\ln(a) - \ln(b))$

Rubi [A]

time = 0.13, antiderivative size = 41, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2325, 6874, 2225, 8}

$$-\frac{a^{-x} b^x}{\log(a) - \log(b)} + \frac{a^x b^{-x}}{\log(a) - \log(b)} - 2x$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a^x - b^x)^2/(a^x b^x), x]$

[Out] $-2*x + a^x/(b^x*(\text{Log}[a] - \text{Log}[b])) - b^x/(a^x*(\text{Log}[a] - \text{Log}[b]))$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] \text{ /; FreeQ}[a, x]$

Rule 2225

$\text{Int}[(F_)^{((c_.) * ((a_.) + (b_.) * (x_)))}^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x)))^n / (b*c*n*\text{Log}[F])}, x] \text{ /; FreeQ}[\{F, a, b, c, n\}, x]$

Rule 2325

$\text{Int}[(u_.) * (F_)^{(v_)} * (G_)^{(w_)}, x_Symbol] \rightarrow \text{With}[\{z = v*\text{Log}[F] + w*\text{Log}[G]\}, \text{Int}[u*\text{NormalizeIntegrand}[E^z, x], x] \text{ /; BinomialQ}[z, x] \text{ || (PolynomialQ}[z, x] \text{ \&\& LeQ}[\text{Exponent}[z, x], 2]) \text{ /; FreeQ}[\{F, G\}, x]$

Rule 6874

$\text{Int}[u_, x_Symbol] \rightarrow \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] \text{ /; SumQ}[v]$
]

Rubi steps

$$\begin{aligned}
\int a^{-x} b^{-x} (a^x - b^x)^2 dx &= \int (a^x - b^x)^2 e^{-x(\log(a)+\log(b))} dx \\
&= \int (a^{2x} e^{-x(\log(a)+\log(b))} - 2a^x b^x e^{-x(\log(a)+\log(b))} + b^{2x} e^{-x(\log(a)+\log(b))}) dx \\
&= -\left(2 \int a^x b^x e^{-x(\log(a)+\log(b))} dx\right) + \int a^{2x} e^{-x(\log(a)+\log(b))} dx + \int b^{2x} e^{-x(\log(a)+\log(b))} dx \\
&= -(2 \int 1 dx) + \int e^{-x(\log(a)-\log(b))} dx + \int e^{x(\log(a)-\log(b))} dx \\
&= -2x + \frac{a^x b^{-x}}{\log(a) - \log(b)} - \frac{a^{-x} b^x}{\log(a) - \log(b)}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 46, normalized size = 1.35

$$-2x + \frac{e^{x(\log(a)-\log(b))}}{\log(a) - \log(b)} + \frac{e^{x(-\log(a)+\log(b))}}{-\log(a) + \log(b)}$$

Antiderivative was successfully verified.

`[In] Integrate[(a^x - b^x)^2/(a^x*b^x), x]``[Out] -2*x + E^(x*(Log[a] - Log[b]))/(Log[a] - Log[b]) + E^(x*(-Log[a] + Log[b]))/(-Log[a] + Log[b])`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: Invalid NaN comparison

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(a^x - b^x)^2/(a^x*b^x), x]')``[Out] cought exception: Invalid NaN comparison`**Maple [A]**

time = 0.04, size = 42, normalized size = 1.24

method	result	size
risch	$-2x + \frac{a^x b^{-x}}{\ln(a)-\ln(b)} - \frac{b^x a^{-x}}{\ln(a)-\ln(b)}$	42
norman	$\left(\frac{e^{2x \ln(a)}}{\ln(a)-\ln(b)} - \frac{e^{2x \ln(b)}}{\ln(a)-\ln(b)} - 2x e^{x \ln(a)} e^{x \ln(b)}\right) e^{-x \ln(a)} e^{-x \ln(b)}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^x-b^x)^2/(a^x)/(b^x),x,method=_RETURNVERBOSE)
```

```
[Out] -2*x+a^x/(b^x)/(ln(a)-ln(b))-b^x/(a^x)/(ln(a)-ln(b))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^x-b^x)^2/(a^x)/(b^x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-log(b)/log(a)>0)', see 'assume?' f
or more
```

Fricas [A]

time = 0.31, size = 52, normalized size = 1.53

$$\frac{2(x \log(a) - x \log(b))a^x b^x - a^{2x} + b^{2x}}{a^x b^x (\log(a) - \log(b))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^x-b^x)^2/(a^x)/(b^x),x, algorithm="fricas")
```

```
[Out] -(2*(x*log(a) - x*log(b))*a^x*b^x - a^(2*x) + b^(2*x))/(a^x*b^x*(log(a) - l
og(b)))
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**x-b**x)**2/(a**x)/(b**x),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^x-b^x)^2/(a^x)/(b^x),x)`

[Out] Could not integrate

Mupad [B]

time = 0.49, size = 34, normalized size = 1.00

$$\frac{\frac{a^x}{b^x} - \frac{b^x}{a^x}}{\ln(a) - \ln(b)} - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^x - b^x)^2/(a^x*b^x),x)`

[Out] `(a^x/b^x - b^x/a^x)/(log(a) - log(b)) - 2*x`

3.496 $\int (-e^{-x} + e^x) dx$

Optimal. Leaf size=9

$$e^{-x} + e^x$$

[Out] exp(-x)+exp(x)

Rubi [A]

time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2225}

$$e^{-x} + e^x$$

Antiderivative was successfully verified.

[In] Int[-E^(-x) + E^x,x]

[Out] E^(-x) + E^x

Rule 2225

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int (-e^{-x} + e^x) dx &= - \int e^{-x} dx + \int e^x dx \\ &= e^{-x} + e^x \end{aligned}$$

Mathematica [A]

time = 0.01, size = 9, normalized size = 1.00

$$e^{-x} + e^x$$

Antiderivative was successfully verified.

[In] Integrate[-E^(-x) + E^x,x]

[Out] E^(-x) + E^x

Mathics [A]

time = 1.61, size = 4, normalized size = 0.44

$$2\text{Cosh}[x]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(E^x - E^(-x))^1,x]')
```

```
[Out] 2 Cosh[x]
```

Maple [A]

time = 0.01, size = 8, normalized size = 0.89

method	result	size
derivativedivides	$e^{-x} + e^x$	8
default	$e^{-x} + e^x$	8
risch	$e^{-x} + e^x$	8
meijerg	$-2 + e^{-x} + e^x$	9
norman	$(1 + e^{2x})e^{-x}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/exp(x)+exp(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/exp(x)+exp(x)
```

Maxima [A]

time = 0.26, size = 7, normalized size = 0.78

$$e^{(-x)} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/exp(x)+exp(x),x, algorithm="maxima")
```

```
[Out] e^(-x) + e^x
```

Fricas [A]

time = 0.32, size = 11, normalized size = 1.22

$$(e^{(2x)} + 1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/exp(x)+exp(x),x, algorithm="fricas")
```

```
[Out] (e^(2*x) + 1)*e^(-x)
```

Sympy [A]

time = 0.05, size = 7, normalized size = 0.78

$$e^x + e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/exp(x)+exp(x),x)

[Out] exp(x) + exp(-x)

Giac [A]

time = 0.00, size = 6, normalized size = 0.67

$$e^{-x} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/exp(x)+exp(x),x)

[Out] e^x + 1/e^x

Mupad [B]

time = 0.05, size = 4, normalized size = 0.44

$$2 \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x) - exp(-x),x)

[Out] 2*cosh(x)

3.497 $\int (-e^{-x} + e^x)^2 dx$

Optimal. Leaf size=22

$$-\frac{1}{2}e^{-2x} + \frac{e^{2x}}{2} - 2x$$

[Out] -1/2/exp(2*x)+1/2*exp(2*x)-2*x

Rubi [A]

time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2320, 272, 45}

$$-2x - \frac{e^{-2x}}{2} + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

[In] Int[(-E^(-x) + E^x)^2,x]

[Out] -1/2*1/E^(2*x) + E^(2*x)/2 - 2*x

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int (-e^{-x} + e^x)^2 dx &= \text{Subst} \left(\int \frac{(1-x^2)^2}{x^3} dx, x, e^x \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{(1-x)^2}{x^2} dx, x, e^{2x} \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(1 + \frac{1}{x^2} - \frac{2}{x} \right) dx, x, e^{2x} \right) \\
&= -\frac{1}{2} e^{-2x} + \frac{e^{2x}}{2} - 2x
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 22, normalized size = 1.00

$$-\frac{1}{2}e^{-2x} + \frac{e^{2x}}{2} - 2x$$

Antiderivative was successfully verified.

`[In] Integrate[(-E^(-x) + E^x)^2, x]``[Out] -1/2*1/E^(2*x) + E^(2*x)/2 - 2*x`**Mathics [A]**

time = 1.71, size = 8, normalized size = 0.36

$$-2x + \text{Sinh}[2x]$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(E^x - E^(-x))^2, x]')``[Out] -2 x + Sinh[2 x]`**Maple [A]**

time = 0.02, size = 19, normalized size = 0.86

method	result	size
risch	$-2x + \frac{e^{2x}}{2} - \frac{e^{-2x}}{2}$	17
derivativedivides	$\frac{e^{2x}}{2} - \frac{e^{-2x}}{2} - 2 \ln(e^x)$	19
default	$\frac{e^{2x}}{2} - \frac{e^{-2x}}{2} - 2 \ln(e^x)$	19
norman	$\left(-\frac{1}{2} + \frac{e^{4x}}{2} - 2e^{2x}x\right) e^{-2x}$	21

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-1/exp(x)+exp(x))^2,x,method=_RETURNVERBOSE)`

[Out] `1/2*exp(x)^2-1/2/exp(x)^2-2*ln(exp(x))`

Maxima [A]

time = 0.24, size = 16, normalized size = 0.73

$$-2x + \frac{1}{2}e^{(2x)} - \frac{1}{2}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1/exp(x)+exp(x))^2,x, algorithm="maxima")`

[Out] `-2*x + 1/2*e^(2*x) - 1/2*e^(-2*x)`

Fricas [A]

time = 0.30, size = 21, normalized size = 0.95

$$-\frac{1}{2}(4xe^{(2x)} - e^{(4x)} + 1)e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1/exp(x)+exp(x))^2,x, algorithm="fricas")`

[Out] `-1/2*(4*x*e^(2*x) - e^(4*x) + 1)*e^(-2*x)`

Sympy [A]

time = 0.06, size = 17, normalized size = 0.77

$$-2x + \frac{e^{2x}}{2} - \frac{e^{-2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1/exp(x)+exp(x))**2,x)`

[Out] `-2*x + exp(2*x)/2 - exp(-2*x)/2`

Giac [A]

time = 0.00, size = 28, normalized size = 1.27

$$\frac{(e^x)^2}{2} + \frac{2(e^x)^2 - 1}{2(e^x)^2} - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1/exp(x)+exp(x))^2,x)

[Out] $1/2*e^{(2*x)} - 2*x + 1/2*(2*e^{(2*x)} - 1)/e^{(2*x)}$

Mupad [B]

time = 0.06, size = 8, normalized size = 0.36

$$\sinh(2x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(-x) - exp(x))^2,x)

[Out] $\sinh(2*x) - 2*x$

3.498 $\int (-e^{-x} + e^x)^3 dx$

Optimal. Leaf size=31

$$\frac{e^{-3x}}{3} - 3e^{-x} - 3e^x + \frac{e^{3x}}{3}$$

[Out] 1/3/exp(3*x)-3/exp(x)-3*exp(x)+1/3*exp(3*x)

Rubi [A]

time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2320, 276}

$$\frac{e^{-3x}}{3} - 3e^{-x} - 3e^x + \frac{e^{3x}}{3}$$

Antiderivative was successfully verified.

[In] Int[(-E^(-x) + E^x)^3,x]

[Out] 1/(3*E^(3*x)) - 3/E^x - 3*E^x + E^(3*x)/3

Rule 276

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2320

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\begin{aligned} \int (-e^{-x} + e^x)^3 dx &= \text{Subst} \left(\int \frac{(-1 + x^2)^3}{x^4} dx, x, e^x \right) \\ &= \text{Subst} \left(\int \left(-3 - \frac{1}{x^4} + \frac{3}{x^2} + x^2 \right) dx, x, e^x \right) \\ &= \frac{e^{-3x}}{3} - 3e^{-x} - 3e^x + \frac{e^{3x}}{3} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 30, normalized size = 0.97

$$\frac{1}{3}e^{-3x} (1 - 9e^{2x} - 9e^{4x} + e^{6x})$$

Antiderivative was successfully verified.

[In] Integrate[(-E^(-x) + E^x)^3, x]

[Out] (1 - 9*E^(2*x) - 9*E^(4*x) + E^(6*x))/(3*E^(3*x))

Mathics [A]

time = 1.78, size = 11, normalized size = 0.35

$$-6\text{Cosh}[x] + \frac{2\text{Cosh}[3x]}{3}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(E^x - E^(-x))^3, x]')

[Out] -6 Cosh[x] + 2 Cosh[3 x] / 3

Maple [A]

time = 0.03, size = 24, normalized size = 0.77

method	result	size
derivativeldivides	$\frac{e^{3x}}{3} - 3e^x - 3e^{-x} + \frac{e^{-3x}}{3}$	24
default	$\frac{e^{3x}}{3} - 3e^x - 3e^{-x} + \frac{e^{-3x}}{3}$	24
risch	$\frac{e^{3x}}{3} - 3e^x - 3e^{-x} + \frac{e^{-3x}}{3}$	24
meijerg	$\frac{16}{3} + \frac{e^{-3x}}{3} - 3e^{-x} - 3e^x + \frac{e^{3x}}{3}$	25
norman	$\left(\frac{1}{3} - 3e^{2x} - 3e^{4x} + \frac{e^{6x}}{3}\right)e^{-3x}$	26

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1/exp(x)+exp(x))^3,x,method=_RETURNVERBOSE)

[Out] 1/3*exp(x)^3-3*exp(x)-3/exp(x)+1/3/exp(x)^3

Maxima [A]

time = 0.25, size = 23, normalized size = 0.74

$$\frac{1}{3}e^{(3x)} - 3e^{(-x)} + \frac{1}{3}e^{(-3x)} - 3e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1/exp(x)+exp(x))^3,x, algorithm="maxima")

[Out] 1/3*e^(3*x) - 3*e^(-x) + 1/3*e^(-3*x) - 3*e^x

Fricas [A]

time = 0.59, size = 24, normalized size = 0.77

$$\frac{1}{3} (e^{6x} - 9e^{4x} - 9e^{2x} + 1)e^{-3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1/exp(x)+exp(x))^3,x, algorithm="fricas")

[Out] 1/3*(e^(6*x) - 9*e^(4*x) - 9*e^(2*x) + 1)*e^(-3*x)

Sympy [A]

time = 0.10, size = 24, normalized size = 0.77

$$\frac{e^{3x}}{3} - 3e^x - 3e^{-x} + \frac{e^{-3x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1/exp(x)+exp(x))**3,x)

[Out] exp(3*x)/3 - 3*exp(x) - 3*exp(-x) + exp(-3*x)/3

Giac [A]

time = 0.00, size = 30, normalized size = 0.97

$$\frac{(e^x)^3}{3} - 3e^x + \frac{-9(e^x)^2 + 1}{3(e^x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1/exp(x)+exp(x))^3,x)

[Out] 1/3*e^(3*x) - 3*e^x - 1/3*(9*e^(2*x) - 1)/e^(3*x)

Mupad [B]

time = 0.31, size = 23, normalized size = 0.74

$$\frac{e^{-3x}}{3} - 3e^{-x} + \frac{e^{3x}}{3} - 3e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(exp(-x) - exp(x))^3,x)

[Out] exp(-3*x)/3 - 3*exp(-x) + exp(3*x)/3 - 3*exp(x)

3.499 $\int (-e^{-x} + e^x)^4 dx$

Optimal. Leaf size=36

$$-\frac{1}{4}e^{-4x} + 2e^{-2x} - 2e^{2x} + \frac{e^{4x}}{4} + 6x$$

[Out] -1/4/exp(4*x)+2/exp(2*x)-2*exp(2*x)+1/4*exp(4*x)+6*x

Rubi [A]

time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2320, 272, 45}

$$6x - \frac{e^{-4x}}{4} + 2e^{-2x} - 2e^{2x} + \frac{e^{4x}}{4}$$

Antiderivative was successfully verified.

[In] Int[(-E^(-x) + E^x)^4, x]

[Out] -1/4*1/E^(4*x) + 2/E^(2*x) - 2*E^(2*x) + E^(4*x)/4 + 6*x

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int (-e^{-x} + e^x)^4 dx &= \text{Subst} \left(\int \frac{(1-x^2)^4}{x^5} dx, x, e^x \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{(1-x)^4}{x^3} dx, x, e^{2x} \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(-4 + \frac{1}{x^3} - \frac{4}{x^2} + \frac{6}{x} + x \right) dx, x, e^{2x} \right) \\
&= -\frac{1}{4} e^{-4x} + 2e^{-2x} - 2e^{2x} + \frac{e^{4x}}{4} + 6x
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 37, normalized size = 1.03

$$\frac{1}{4} e^{-4x} (-1 + 8e^{2x} - 8e^{6x} + e^{8x}) + 6 \log(e^x)$$

Antiderivative was successfully verified.

`[In] Integrate[(-E^(-x) + E^x)^4, x]``[Out] (-1 + 8*E^(2*x) - 8*E^(6*x) + E^(8*x))/(4*E^(4*x)) + 6*Log[E^x]`**Mathics [A]**

time = 1.81, size = 16, normalized size = 0.44

$$6x - 4\text{Sinh}[2x] + \frac{\text{Sinh}[4x]}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(E^x - E^(-x))^4, x]')``[Out] 6 x - 4 Sinh[2 x] + Sinh[4 x] / 2`**Maple [A]**

time = 0.03, size = 31, normalized size = 0.86

method	result	size
risch	$6x + \frac{e^{4x}}{4} - 2e^{2x} + 2e^{-2x} - \frac{e^{-4x}}{4}$	29
derivativedivides	$\frac{e^{4x}}{4} - 2e^{2x} + 2e^{-2x} - \frac{e^{-4x}}{4} + 6 \ln(e^x)$	31
default	$\frac{e^{4x}}{4} - 2e^{2x} + 2e^{-2x} - \frac{e^{-4x}}{4} + 6 \ln(e^x)$	31
norman	$\left(-\frac{1}{4} + 2e^{2x} - 2e^{6x} + \frac{e^{8x}}{4} + 6xe^{4x}\right)e^{-4x}$	33

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-1/exp(x)+exp(x))^4,x,method=_RETURNVERBOSE)`

[Out] `1/4*exp(x)^4-2*exp(x)^2+2/exp(x)^2-1/4/exp(x)^4+6*ln(exp(x))`

Maxima [A]

time = 0.24, size = 28, normalized size = 0.78

$$6x + \frac{1}{4}e^{4x} - 2e^{2x} + 2e^{-2x} - \frac{1}{4}e^{-4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1/exp(x)+exp(x))^4,x, algorithm="maxima")`

[Out] `6*x + 1/4*e^(4*x) - 2*e^(2*x) + 2*e^(-2*x) - 1/4*e^(-4*x)`

Fricas [A]

time = 0.31, size = 31, normalized size = 0.86

$$\frac{1}{4}(24xe^{4x} + e^{8x} - 8e^{6x} + 8e^{2x} - 1)e^{-4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1/exp(x)+exp(x))^4,x, algorithm="fricas")`

[Out] `1/4*(24*x*e^(4*x) + e^(8*x) - 8*e^(6*x) + 8*e^(2*x) - 1)*e^(-4*x)`

Sympy [A]

time = 0.08, size = 31, normalized size = 0.86

$$6x + \frac{e^{4x}}{4} - 2e^{2x} + 2e^{-2x} - \frac{e^{-4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1/exp(x)+exp(x))**4,x)`

[Out] `6*x + exp(4*x)/4 - 2*exp(2*x) + 2*exp(-2*x) - exp(-4*x)/4`

Giac [A]

time = 0.00, size = 42, normalized size = 1.17

$$\frac{(e^x)^4 - 8(e^x)^2}{4} + \frac{-18(e^x)^4 + 8(e^x)^2 - 1}{4(e^x)^4} + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1/exp(x)+exp(x))^4,x)

[Out] $1/4*e^{4*x} - 2*e^{2*x} + 6*x - 1/4*(18*e^{4*x} - 8*e^{2*x} + 1)/e^{4*x}$

Mupad [B]

time = 0.34, size = 28, normalized size = 0.78

$$6x + 2e^{-2x} - 2e^{2x} - \frac{e^{-4x}}{4} + \frac{e^{4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(-x) - exp(x))^4,x)

[Out] $6*x + 2*\exp(-2*x) - 2*\exp(2*x) - \exp(-4*x)/4 + \exp(4*x)/4$

3.500 $\int (-e^{-x} + e^x)^n dx$

Optimal. Leaf size=48

$$\frac{(-e^{-x} + e^x)^n (1 - e^{2x}) {}_2F_1\left(1, \frac{2+n}{2}; 1 - \frac{n}{2}; e^{2x}\right)}{n}$$

[Out] $-((-1/\exp(x)+\exp(x))^n*(1-\exp(2*x))*\text{hypergeom}([1, 1+1/2*n], [1-1/2*n], \exp(2*x))/n$

Rubi [A]

time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2320, 2057, 372, 371}

$$\frac{(e^x - e^{-x})^n (1 - e^{2x})^{-n} {}_2F_1\left(-n, -\frac{n}{2}; 1 - \frac{n}{2}; e^{2x}\right)}{n}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-E^{-x} + E^x)^n, x]$

[Out] $-(((E^{-x} + E^x)^n * \text{Hypergeometric2F1}[-n, -1/2*n, 1 - n/2, E^{(2*x)}]) / ((1 - E^{(2*x)})^n * n))$

Rule 371

$\text{Int}[(c_*)*(x_)^{(m_*)}*((a_*) + (b_*)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[a^p * ((c*x)^{(m+1}) / (c*(m+1))) * \text{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n + 1, (-b)*(x^n/a)], x] /;$ $\text{FreeQ}\{a, b, c, m, n, p\}, x \ \&\& \ !\text{IGtQ}[p, 0] \ \&\& \ (\text{ILtQ}[p, 0] \ || \ \text{GtQ}[a, 0])$

Rule 372

$\text{Int}[(c_*)*(x_)^{(m_*)}*((a_*) + (b_*)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[a^{\text{IntPart}[p]} * ((a + b*x^n)^{\text{FracPart}[p]} / (1 + b*(x^n/a))^{\text{FracPart}[p]}], \text{Int}[(c*x)^{m*(1 + b*(x^n/a))^p}, x], x] /;$ $\text{FreeQ}\{a, b, c, m, n, p\}, x \ \&\& \ !\text{IGtQ}[p, 0] \ \&\& \ !(\text{ILtQ}[p, 0] \ || \ \text{GtQ}[a, 0])$

Rule 2057

$\text{Int}[(c_*)*(x_)^{(m_*)}*((a_*)*(x_)^{(j_*)} + (b_*)*(x_)^{(n_*)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[c^{\text{IntPart}[m]} * (c*x)^{\text{FracPart}[m]} * ((a*x^j + b*x^n)^{\text{FracPart}[p]} / (x^{\text{FracPart}[m] + j*\text{FracPart}[p]} * (a + b*x^{(n-j)})^{\text{FracPart}[p]})), \text{Int}[x^{(m+j*p)} * (a + b*x^{(n-j)})^p, x], x] /;$ $\text{FreeQ}\{a, b, c, j, m, n, p\}, x \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{NeQ}[n, j] \ \&\& \ \text{PosQ}[n - j]$

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned} \int (-e^{-x} + e^x)^n dx &= \text{Subst} \left(\int \frac{\left(-\frac{1}{x} + x\right)^n}{x} dx, x, e^x \right) \\ &= \left((e^x)^n (-e^{-x} + e^x)^n (-1 + e^{2x})^{-n} \right) \text{Subst} \left(\int x^{-1-n} (-1 + x^2)^n dx, x, e^x \right) \\ &= \left((e^x)^n (-e^{-x} + e^x)^n (1 - e^{2x})^{-n} \right) \text{Subst} \left(\int x^{-1-n} (1 - x^2)^n dx, x, e^x \right) \\ &= -\frac{(-e^{-x} + e^x)^n (1 - e^{2x})^{-n} {}_2F_1\left(-n, -\frac{n}{2}; 1 - \frac{n}{2}; e^{2x}\right)}{n} \end{aligned}$$

Mathematica [A]

time = 0.05, size = 45, normalized size = 0.94

$$\frac{(-e^{-x} + e^x)^n (-1 + e^{2x}) {}_2F_1\left(1, 1 + \frac{n}{2}; 1 - \frac{n}{2}; e^{2x}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-E^(-x) + E^x)^n, x]
```

```
[Out] ((-E^(-x) + E^x)^n*(-1 + E^(2*x))*Hypergeometric2F1[1, 1 + n/2, 1 - n/2, E^(2*x)])/n
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(E^x - E^(-x))^n, x]')
```

```
[Out] Timed out
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int (-e^{-x} + e^x)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-1/exp(x)+exp(x))^n,x)`

[Out] `int((-1/exp(x)+exp(x))^n,x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1/exp(x)+exp(x))^n,x, algorithm="maxima")`

[Out] `integrate((-e^(-x) + e^x)^n, x)`

Fricas [F]

time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1/exp(x)+exp(x))^n,x, algorithm="fricas")`

[Out] `integral((-e^(-x) + e^x)^n, x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (e^x - e^{-x})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1/exp(x)+exp(x))**n,x)`

[Out] `Integral((exp(x) - exp(-x))**n, x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1/exp(x)+exp(x))^n,x)`

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int (e^x - e^{-x})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((exp(x) - exp(-x))^n, x)`

[Out] `int((exp(x) - exp(-x))^n, x)`

3.501 $\int (a^{-4x} - a^{2x})^3 dx$

Optimal. Leaf size=43

$$3x - \frac{a^{-12x}}{12 \log(a)} + \frac{a^{-6x}}{2 \log(a)} - \frac{a^{6x}}{6 \log(a)}$$

[Out] 3*x-1/12/(a^(12*x))/ln(a)+1/2/(a^(6*x))/ln(a)-1/6*a^(6*x)/ln(a)

Rubi [A]

time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2320, 272, 45}

$$-\frac{a^{-12x}}{12 \log(a)} + \frac{a^{-6x}}{2 \log(a)} - \frac{a^{6x}}{6 \log(a)} + 3x$$

Antiderivative was successfully verified.

[In] Int[(a^(-4*x) - a^(2*x))^3,x]

[Out] 3*x - 1/(12*a^(12*x)*Log[a]) + 1/(2*a^(6*x)*Log[a]) - a^(6*x)/(6*Log[a])

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int (a^{-4x} - a^{2x})^3 dx &= \frac{\text{Subst} \left(\int \frac{(1-x^3)^3}{x^7} dx, x, a^{2x} \right)}{2 \log(a)} \\
&= \frac{\text{Subst} \left(\int \frac{(1-x)^3}{x^3} dx, x, a^{6x} \right)}{6 \log(a)} \\
&= \frac{\text{Subst} \left(\int \left(-1 + \frac{1}{x^3} - \frac{3}{x^2} + \frac{3}{x} \right) dx, x, a^{6x} \right)}{6 \log(a)} \\
&= 3x - \frac{a^{-12x}}{12 \log(a)} + \frac{a^{-6x}}{2 \log(a)} - \frac{a^{6x}}{6 \log(a)}
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 34, normalized size = 0.79

$$-\frac{a^{-12x} - 6a^{-6x} + 2a^{6x} - 36 \log(a^x)}{12 \log(a)}$$

Antiderivative was successfully verified.

`[In] Integrate[(a^(-4*x) - a^(2*x))^3, x]``[Out] -1/12*(a^(-12*x) - 6/a^(6*x) + 2*a^(6*x) - 36*Log[a^x])/Log[a]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(a^(-4*x) - a^(2*x))^3, x]')``[Out] cought exception:`**Maple [A]**

time = 0.03, size = 44, normalized size = 1.02

method	result	size
risch	$3x - \frac{a^{6x}}{6 \ln(a)} + \frac{a^{-6x}}{2 \ln(a)} - \frac{a^{-12x}}{12 \ln(a)}$	44
norman	$\left(-\frac{1}{12 \ln(a)} + 3x e^{12x \ln(a)} + \frac{e^{6x \ln(a)}}{2 \ln(a)} - \frac{e^{18x \ln(a)}}{6 \ln(a)} \right) e^{-12x \ln(a)}$	56

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/(a^(4*x))-a^(2*x))^3,x,method=_RETURNVERBOSE)`

[Out] $3*x - 1/6/\ln(a)*(a^(2*x))^3 + 1/2/\ln(a)/(a^(2*x))^3 - 1/12/\ln(a)/(a^(2*x))^6$

Maxima [A]

time = 0.24, size = 41, normalized size = 0.95

$$3x - \frac{a^{6x}}{6 \log(a)} - \frac{1}{12 a^{12x} \log(a)} + \frac{1}{2 a^{6x} \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/(a^(4*x))-a^(2*x))^3,x, algorithm="maxima")`

[Out] $3*x - 1/6*a^(6*x)/\log(a) - 1/12/(a^(12*x)*\log(a)) + 1/2/(a^(6*x)*\log(a))$

Fricas [A]

time = 0.31, size = 39, normalized size = 0.91

$$\frac{36 a^{12x} x \log(a) - 2 a^{18x} + 6 a^{6x} - 1}{12 a^{12x} \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/(a^(4*x))-a^(2*x))^3,x, algorithm="fricas")`

[Out] $1/12*(36*a^(12*x)*x*\log(a) - 2*a^(18*x) + 6*a^(6*x) - 1)/(a^(12*x)*\log(a))$

Sympy [A]

time = 0.13, size = 53, normalized size = 1.23

$$3x + \begin{cases} \frac{-24a^{6x} \log(a)^2 + 72a^{-6x} \log(a)^2 - 12a^{-12x} \log(a)^2}{144 \log(a)^3} & \text{for } \log(a)^3 \neq 0 \\ -3x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/(a**(4*x))-a**(2*x))**3,x)`

[Out] $3*x + \text{Piecewise}((-24*a**(6*x)*\log(a)**2 + 72*\log(a)**2/a**(6*x) - 12*\log(a)**2/a**(12*x))/(144*\log(a)**3), \text{Ne}(\log(a)**3, 0)), (-3*x, \text{True}))$

Giac [A]

time = 0.01, size = 49, normalized size = 1.14

$$\frac{-\frac{(a^x)^6}{6} + \frac{-9(a^x)^{12} + 6(a^x)^6 - 1}{12(a^x)^{12}} + \frac{\ln((a^x)^6)}{2}}{\ln a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/(a^(4*x))-a^(2*x))^3,x)`

[Out] $-1/12*(2*a^{(6*x)} + (9*a^{(12*x)} - 6*a^{(6*x)} + 1)/a^{(12*x)} - 6*\log(a^{(6*x)}))/\log(a)$

Mupad [B]

time = 0.41, size = 41, normalized size = 0.95

$$3x + \frac{1}{2a^{6x} \ln(a)} - \frac{a^{6x}}{6 \ln(a)} - \frac{1}{12a^{12x} \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(-(a^{(2*x)} - 1/a^{(4*x)})^3, x)$

[Out] $3*x + 1/(2*a^{(6*x)}*\log(a)) - a^{(6*x)}/(6*\log(a)) - 1/(12*a^{(12*x)}*\log(a))$

3.502 $\int (a^{kx} + a^{lx}) dx$

Optimal. Leaf size=27

$$\frac{a^{kx}}{k \log(a)} + \frac{a^{lx}}{l \log(a)}$$

[Out] $a^{(k*x)}/k/\ln(a)+a^{(l*x)}/l/\ln(a)$

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2225}

$$\frac{a^{kx}}{k \log(a)} + \frac{a^{lx}}{l \log(a)}$$

Antiderivative was successfully verified.

[In] Int[a^(k*x) + a^(l*x),x]

[Out] a^(k*x)/(k*Log[a]) + a^(l*x)/(l*Log[a])

Rule 2225

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int (a^{kx} + a^{lx}) dx &= \int a^{kx} dx + \int a^{lx} dx \\ &= \frac{a^{kx}}{k \log(a)} + \frac{a^{lx}}{l \log(a)} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 27, normalized size = 1.00

$$\frac{a^{kx}}{k \log(a)} + \frac{a^{lx}}{l \log(a)}$$

Antiderivative was successfully verified.

[In] Integrate[a^(k*x) + a^(l*x),x]

[Out] a^(k*x)/(k*Log[a]) + a^(l*x)/(l*Log[a])

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.27, size = 91, normalized size = 3.37

Piecewise $\left[\left\{ \left\{ \frac{k a^{lx} + l a^{kx}}{kl \operatorname{Log}[a]}, k \operatorname{Log}[a] \neq 0 \&\& l \operatorname{Log}[a] \neq 0 \right\}, \left\{ \frac{a^{kx}}{k \operatorname{Log}[a]} + x, k \operatorname{Log}[a] \neq 0 \right\}, \left\{ \frac{a^{lx}}{l \operatorname{Log}[a]} + x, l \operatorname{Log}[a] \neq 0 \right\}, \{2x, \text{True}\} \right\} \right]$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(a^(k*x) + a^(l*x))^1,x]')`

[Out] `Piecewise[{{(k a^(1 x) + l a^(k x)) / (k l Log[a]), k Log[a] != 0 && l Log[a] != 0}, {a^(k x) / (k Log[a]) + x, k Log[a] != 0}, {a^(1 x) / (l Log[a]) + x, l Log[a] != 0}, {2 x, True}}]`

Maple [A]

time = 0.02, size = 28, normalized size = 1.04

method	result	size
default	$\frac{a^{kx}}{k \ln(a)} + \frac{a^{lx}}{l \ln(a)}$	28
risch	$\frac{a^{kx}}{k \ln(a)} + \frac{a^{lx}}{l \ln(a)}$	28
norman	$\frac{e^{kx \ln(a)}}{k \ln(a)} + \frac{e^{lx \ln(a)}}{l \ln(a)}$	30
meijerg	$-\frac{1-e^{kx \ln(a)}}{k \ln(a)} - \frac{1-e^{lx \ln(a)}}{l \ln(a)}$	40

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a^(k*x)+a^(l*x),x,method=_RETURNVERBOSE)`

[Out] `a^(k*x)/k/ln(a)+a^(l*x)/l/ln(a)`

Maxima [A]

time = 0.24, size = 27, normalized size = 1.00

$$\frac{a^{kx}}{k \log(a)} + \frac{a^{lx}}{l \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a^(k*x)+a^(l*x),x, algorithm="maxima")`

[Out] `a^(k*x)/(k*log(a)) + a^(l*x)/(l*log(a))`

Fricas [A]

time = 0.30, size = 26, normalized size = 0.96

$$\frac{a^{lx}k + a^{kx}l}{kl \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^(k*x)+a^(l*x),x, algorithm="fricas")

[Out] (a^(l*x)*k + a^(k*x)*l)/(k*l*log(a))

Sympy [A]

time = 0.08, size = 29, normalized size = 1.07

$$\left\{ \begin{array}{ll} \frac{a^{kx}}{k \log(a)} & \text{for } k \log(a) \neq 0 \\ x & \text{otherwise} \end{array} \right. + \left\{ \begin{array}{ll} \frac{a^{lx}}{l \log(a)} & \text{for } l \log(a) \neq 0 \\ x & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a**(k*x)+a**(l*x),x)

[Out] Piecewise((a**(k*x)/(k*log(a)), Ne(k*log(a), 0)), (x, True)) + Piecewise((a**(l*x)/(l*log(a)), Ne(l*log(a), 0)), (x, True))

Giac [A]

time = 0.00, size = 23, normalized size = 0.85

$$\frac{a^{kx}}{k \ln a} + \frac{a^{lx}}{l \ln a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^(k*x)+a^(l*x),x)

[Out] a^(k*x)/(k*log(a)) + a^(l*x)/(l*log(a))

Mupad [B]

time = 0.36, size = 26, normalized size = 0.96

$$\frac{a^{kx} l + a^{lx} k}{kl \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a^(k*x) + a^(l*x),x)

[Out] (a^(k*x)*l + a^(l*x)*k)/(k*l*log(a))

3.503 $\int (a^{kx} + a^{lx})^2 dx$

Optimal. Leaf size=53

$$\frac{a^{2kx}}{2k \log(a)} + \frac{a^{2lx}}{2l \log(a)} + \frac{2a^{(k+l)x}}{(k+l) \log(a)}$$

[Out] $1/2*a^{(2*k*x)}/k/\ln(a)+1/2*a^{(2*l*x)}/l/\ln(a)+2*a^{((k+l)*x)}/(k+l)/\ln(a)$

Rubi [A]

time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6874, 2225}

$$\frac{2a^{x(k+l)}}{\log(a)(k+l)} + \frac{a^{2kx}}{2k \log(a)} + \frac{a^{2lx}}{2l \log(a)}$$

Antiderivative was successfully verified.

[In] Int[(a^(k*x) + a^(l*x))^2,x]

[Out] $a^{(2*k*x)}/(2*k*\text{Log}[a]) + a^{(2*l*x)}/(2*l*\text{Log}[a]) + (2*a^{((k+l)*x)})/((k+l)*\text{Log}[a])$

Rule 2225

Int[((F_)^((c_)*(a_) + (b_)*(x_)))^(n_), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 6874

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int (a^{kx} + a^{lx})^2 dx &= \frac{\text{Subst} \left(\int (e^{kx} + e^{lx})^2 dx, x, x \log(a) \right)}{\log(a)} \\ &= \frac{\text{Subst} \left(\int (e^{2kx} + e^{2lx} + 2e^{(k+l)x}) dx, x, x \log(a) \right)}{\log(a)} \\ &= \frac{\text{Subst} \left(\int e^{2kx} dx, x, x \log(a) \right)}{\log(a)} + \frac{\text{Subst} \left(\int e^{2lx} dx, x, x \log(a) \right)}{\log(a)} + \frac{2 \text{Subst} \left(\int e^{(k+l)x} dx, x, x \log(a) \right)}{\log(a)} \\ &= \frac{a^{2kx}}{2k \log(a)} + \frac{a^{2lx}}{2l \log(a)} + \frac{2a^{(k+l)x}}{(k+l) \log(a)} \end{aligned}$$

Mathematica [A]

time = 0.04, size = 53, normalized size = 1.00

$$\frac{a^{2kx}}{2k \log(a)} + \frac{a^{2lx}}{2l \log(a)} + \frac{2a^{(k+l)x}}{(k+l) \log(a)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^(k*x) + a^(l*x))^2,x]**[Out]** a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) + (2*a^((k + l)*x))/((k + l)*Log[a])**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.70, size = 277, normalized size = 5.23

$$\text{Piecewise}\left[\left\{\left\{4x, a==1 \&\& k==1 \mid k==0 \&\& l==1 \mid l==0\right\}, \left\{\frac{x \log(a) + \frac{a^{2x}}{2} + 2a^{lx}}{l \log(a)}, k==0\right\}, \left\{-\frac{a^{-2x}}{2l \log(a)} + \frac{a^{2lx}}{2l \log(a)} + 2x, k== -1\right\}, \left\{\frac{kx \log(a) + \frac{a^{2lx}}{2} + 2a^{lx}}{k \log(a)}, l==0\right\}, \frac{k^2 a^{2lx}}{2k^2 l \log(a) + 2kl^2 \log(a)} + \frac{4kl a^{lx} a^{lx}}{2k^2 l \log(a) + 2kl^2 \log(a)} + \frac{kl a^{2lx}}{2k^2 l \log(a) + 2kl^2 \log(a)} + \frac{kl a^{2lx}}{2k^2 l \log(a) + 2kl^2 \log(a)} + \frac{P a^{2lx}}{2k^2 l \log(a) + 2kl^2 \log(a)}\right\}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a^(k*x) + a^(l*x))^2,x]')
[Out] Piecewise[{{4 x, a == 1 && a == 1 || k == 0 && a == 1 || l == 0}, {(1 x Log[a] + a^(2 l x) / 2 + 2 a^(1 x)) / (1 Log[a]), k == 0}, {-a^(-2 l x) / (2 l Log[a]) + a^(2 l x) / (2 l Log[a]) + 2 x, k == -1}, {(k x Log[a] + a^(2 k x) / 2 + 2 a^(k x)) / (k Log[a]), l == 0}}, k^2 a^(2 l x) / (2 k^2 l Log[a] + 2 k l^2 Log[a]) + 4 k l a^(k x) a^(l x) / (2 k^2 l Log[a] + 2 k l^2 Log[a]) + k l a^(2 k x) / (2 k^2 l Log[a] + 2 k l^2 Log[a]) + k l a^(2 l x) / (2 k^2 l Log[a] + 2 k l^2 Log[a]) + 1^2 a^(2 k x) / (2 k^2 l Log[a] + 2 k l^2 Log[a])}]
Maple [A]

time = 0.04, size = 55, normalized size = 1.04

method	result	size
risch	$\frac{a^{2kx}}{2k \ln(a)} + \frac{a^{2lx}}{2l \ln(a)} + \frac{2a^{kx} a^{lx}}{\ln(a)(k+l)}$	55
norman	$\frac{e^{2kx \ln(a)}}{2k \ln(a)} + \frac{e^{2lx \ln(a)}}{2l \ln(a)} + \frac{2e^{kx \ln(a)} e^{lx \ln(a)}}{\ln(a)(k+l)}$	59
meijerg	$-\frac{1-e^{2kx \ln(a)}}{2k \ln(a)} - \frac{2\left(1-e^{xl \ln(a)\left(1+\frac{k}{l}\right)}\right)}{l \ln(a)\left(1+\frac{k}{l}\right)} - \frac{1-e^{2lx \ln(a)}}{2l \ln(a)}$	77

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(k*x)+a^(l*x))^2,x,method=_RETURNVERBOSE)**[Out]** 1/2/k/ln(a)*(a^(k*x))^2+1/2/l/ln(a)*(a^(l*x))^2+2/ln(a)/(k+l)*a^(k*x)*a^(l*x)

Maxima [A]

time = 0.24, size = 51, normalized size = 0.96

$$\frac{2a^{kx+lx}}{(k+l)\log(a)} + \frac{a^{2kx}}{2k\log(a)} + \frac{a^{2lx}}{2l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(k*x)+a^(l*x))^2,x, algorithm="maxima")**[Out]** 2*a^(k*x + l*x)/((k + l)*log(a)) + 1/2*a^(2*k*x)/(k*log(a)) + 1/2*a^(2*l*x)/(l*log(a))**Fricas [A]**

time = 0.32, size = 62, normalized size = 1.17

$$\frac{4a^{kx}a^{lx}kl + (kl + l^2)a^{2kx} + (k^2 + kl)a^{2lx}}{2(k^2l + kl^2)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(k*x)+a^(l*x))^2,x, algorithm="fricas")**[Out]** 1/2*(4*a^(k*x)*a^(l*x)*k*l + (k*l + l^2)*a^(2*k*x) + (k^2 + k*l)*a^(2*l*x))/((k^2*l + k*l^2)*log(a))**Sympy [A]**

time = 0.52, size = 250, normalized size = 4.72

$$\left\{ \begin{array}{ll} 4x & \text{for } a = 1 \wedge (a = 1 \vee k = 0) \wedge (a = 1 \vee l = 0) \\ \frac{a^{2lx}}{2l\log(a)} + \frac{2a^{lx}}{l\log(a)} + x & \text{for } k = 0 \\ \frac{a^{2lx}}{2l\log(a)} + 2x - \frac{a^{-2lx}}{2l\log(a)} & \text{for } k = -l \\ \frac{a^{2kx}}{2k\log(a)} + \frac{2a^{kx}}{k\log(a)} + x & \text{for } l = 0 \\ \frac{a^{2kx}kl}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2kx}l^2}{2k^2l\log(a)+2kl^2\log(a)} + \frac{4a^{kx}a^{lx}kl}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2lx}k^2}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2lx}kl}{2k^2l\log(a)+2kl^2\log(a)} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**(k*x)+a**(l*x))**2,x)

[Out] Piecewise((4*x, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))), (a**(2*l*x)/(2*l*log(a)) + 2*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(2*l*x)/(2*l*log(a)) + 2*x - 1/(2*a**(2*l*x)*l*log(a)), Eq(k, -l)), (a**(2*k*x)/(2*k*log(a)) + 2*a**(k*x)/(k*log(a)) + x, Eq(l, 0)), (a**(2*k*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*k*x)*l**2/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + 4*a**(k*x)*a**(l*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*l*x)*k**2/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*l*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a)), True))

Giac [C] Result contains complex when optimal does not.

time = 0.02, size = 677, normalized size = 12.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(k*x)+a^(l*x))^2,x)

[Out] $(2*k*\cos(-\pi*k*x*\text{sgn}(a) + \pi*k*x)*\log(\text{abs}(a)))/(4*k^2*\log(\text{abs}(a))^2 + (\pi*k*\text{sgn}(a) - \pi*k)^2) - (\pi*k*\text{sgn}(a) - \pi*k)*\sin(-\pi*k*x*\text{sgn}(a) + \pi*k*x)/(4*k^2*\log(\text{abs}(a))^2 + (\pi*k*\text{sgn}(a) - \pi*k)^2)*\text{abs}(a)^(2*k*x) + (2*l*\cos(-\pi*l*x*\text{sgn}(a) + \pi*l*x)*\log(\text{abs}(a)))/(4*l^2*\log(\text{abs}(a))^2 + (\pi*l*\text{sgn}(a) - \pi*l)^2) - (\pi*l*\text{sgn}(a) - \pi*l)*\sin(-\pi*l*x*\text{sgn}(a) + \pi*l*x)/(4*l^2*\log(\text{abs}(a))^2 + (\pi*l*\text{sgn}(a) - \pi*l)^2)*\text{abs}(a)^(2*l*x) + 4*e^{((k*\log(\text{abs}(a)) + l*\log(\text{abs}(a)))x)}*(2*(k*\log(\text{abs}(a)) + l*\log(\text{abs}(a)))*\cos(-1/2*\pi*k*x*\text{sgn}(a) - 1/2*\pi*l*x*\text{sgn}(a) + 1/2*\pi*k*x + 1/2*\pi*l*x)/((\pi*k*\text{sgn}(a) + \pi*l*\text{sgn}(a) - \pi*k - \pi*l)^2 + 4*(k*\log(\text{abs}(a)) + l*\log(\text{abs}(a)))^2) - (\pi*k*\text{sgn}(a) + \pi*l*\text{sgn}(a) - \pi*k - \pi*l)*\sin(-1/2*\pi*k*x*\text{sgn}(a) - 1/2*\pi*l*x*\text{sgn}(a) + 1/2*\pi*k*x + 1/2*\pi*l*x)/((\pi*k*\text{sgn}(a) + \pi*l*\text{sgn}(a) - \pi*k - \pi*l)^2 + 4*(k*\log(\text{abs}(a)) + l*\log(\text{abs}(a)))^2)) - 1/2*I*\text{abs}(a)^(2*k*x)*(-I*e^{(I*\pi*k*x*\text{sgn}(a) - I*\pi*k*x)/(I*\pi*k*\text{sgn}(a) - I*\pi*k + 2*k*\log(\text{abs}(a)))} + I*e^{(-I*\pi*k*x*\text{sgn}(a) + I*\pi*k*x)/(-I*\pi*k*\text{sgn}(a) + I*\pi*k + 2*k*\log(\text{abs}(a)))}) - 2*e^{((k*\log(\text{abs}(a)) + l*\log(\text{abs}(a)))x)}*(e^{(1/2*I*\pi*k*x*\text{sgn}(a) + 1/2*I*\pi*l*x*\text{sgn}(a) - 1/2*I*\pi*k*x - 1/2*I*\pi*l*x)/(I*\pi*k*\text{sgn}(a) + I*\pi*l*\text{sgn}(a) - I*\pi*k - I*\pi*l + 2*k*\log(\text{abs}(a)) + 2*l*\log(\text{abs}(a)))} - e^{(-1/2*I*\pi*k*x*\text{sgn}(a) - 1/2*I*\pi*l*x*\text{sgn}(a) + 1/2*I*\pi*k*x + 1/2*I*\pi*l*x)/(-I*\pi*k*\text{sgn}(a) - I*\pi*l*\text{sgn}(a) + I*\pi*k + I*\pi*l + 2*k*\log(\text{abs}(a)) + 2*l*\log(\text{abs}(a)))}) - 1/2*I*\text{abs}(a)^(2*l*x)*(-I*e^{(I*\pi*l*x*\text{sgn}(a) - I*\pi*l*x)/(I*\pi*l*\text{sgn}(a) - I*\pi*l + 2*l*\log(\text{abs}(a)))} + I*e^{(-I*\pi*l*x*\text{sgn}(a) + I*\pi*l*x)/(-I*\pi*l*\text{sgn}(a) + I*\pi*l + 2*l*\log(\text{abs}(a)))})$

Mupad [B]

time = 0.38, size = 68, normalized size = 1.28

$$\frac{a^{2kx}}{2k \ln(a)} + \frac{\frac{a^{2lx}k^2}{2} + l \left(2a^{kx+lx}k + \frac{a^{2lx}k}{2} \right)}{kl \ln(a) (k+l)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(k*x) + a^(l*x))^2,x)

[Out] $a^{(2*k*x)}/(2*k*\log(a)) + ((a^{(2*l*x)}*k^2)/2 + 1*(2*a^{(k*x + l*x)}*k + (a^{(2*l*x)}*k)/2))/(k*l*\log(a)*(k + l))$

3.504 $\int (a^{kx} + a^{lx})^3 dx$

Optimal. Leaf size=79

$$\frac{a^{3kx}}{3k \log(a)} + \frac{a^{3lx}}{3l \log(a)} + \frac{3a^{(2k+l)x}}{(2k+l) \log(a)} + \frac{3a^{(k+2l)x}}{(k+2l) \log(a)}$$

[Out] $1/3*a^{(3*k*x)}/k/\ln(a)+1/3*a^{(3*l*x)}/l/\ln(a)+3*a^{((2*k+1)*x)}/(2*k+1)/\ln(a)+3*a^{((k+2*1)*x)}/(k+2*1)/\ln(a)$

Rubi [A]

time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6874, 2225}

$$\frac{3a^{x(2k+l)}}{\log(a)(2k+l)} + \frac{3a^{x(k+2l)}}{\log(a)(k+2l)} + \frac{a^{3kx}}{3k \log(a)} + \frac{a^{3lx}}{3l \log(a)}$$

Antiderivative was successfully verified.

[In] Int[(a^(k*x) + a^(l*x))^3, x]

[Out] $a^{(3*k*x)}/(3*k*\text{Log}[a]) + a^{(3*l*x)}/(3*l*\text{Log}[a]) + (3*a^{((2*k+1)*x)})/((2*k+1)*\text{Log}[a]) + (3*a^{((k+2*1)*x)})/((k+2*1)*\text{Log}[a])$

Rule 2225

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 6874

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int (a^{kx} + a^{lx})^3 dx &= \frac{\text{Subst} \left(\int (e^{kx} + e^{lx})^3 dx, x, x \log(a) \right)}{\log(a)} \\ &= \frac{\text{Subst} \left(\int (e^{3kx} + e^{3lx} + 3e^{(2k+l)x} + 3e^{(k+2l)x}) dx, x, x \log(a) \right)}{\log(a)} \\ &= \frac{\text{Subst} \left(\int e^{3kx} dx, x, x \log(a) \right)}{\log(a)} + \frac{\text{Subst} \left(\int e^{3lx} dx, x, x \log(a) \right)}{\log(a)} + \frac{3 \text{Subst} \left(\int e^{(2k+l)x} dx, x, x \log(a) \right)}{\log(a)} \\ &= \frac{a^{3kx}}{3k \log(a)} + \frac{a^{3lx}}{3l \log(a)} + \frac{3a^{(2k+l)x}}{(2k+l) \log(a)} + \frac{3a^{(k+2l)x}}{(k+2l) \log(a)} \end{aligned}$$

Mathematica [A]

time = 0.07, size = 65, normalized size = 0.82

$$\frac{\frac{a^{3kx}}{k} + \frac{a^{3lx}}{l} + \frac{9a^{(2k+l)x}}{2k+l} + \frac{9a^{(k+2l)x}}{k+2l}}{3 \log(a)}$$

Antiderivative was successfully verified.

`[In] Integrate[(a^(k*x) + a^(l*x))^3, x]`

```
[Out] (a^(3*k*x)/k + a^(3*l*x)/l + (9*a^((2*k + 1)*x))/(2*k + 1) + (9*a^((k + 2*l)*x))/(k + 2*l))/(3*Log[a])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 5.30, size = 636, normalized size = 8.05

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(a^(k*x) + a^(l*x))^3, x]')`

```
[Out] Piecewise[{{8 x, a == 1 && a == 1 || k == 0 && a == 1 || l == 0}, {(1 x Log[a] + a^(3 l x) / 3 + 3 a^(2 l x) / 2 + 3 a^(l x)) / (1 Log[a]), k == 0}, {(-1 / 6 + 3 l x Log[a] a^(6 l x) - a^(3 l x) + a^(9 l x) / 3) a^(-6 l x) / (1 Log[a]), k == -2 l}, {-2 a^(-3 l x / 2) / (3 l Log[a]) + a^(3 l x) / (3 l Log[a]) + 2 a^(3 l x / 2) / (1 Log[a]) + 3 x, l == -2 k}, {(k x Log[a] + a^(3 k x) / 3 + 3 a^(2 k x) / 2 + 3 a^(k x)) / (k Log[a]), l == 0}}, 2 k^3 a^(3 l x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a]) + 2 k^2 l a^(3 k x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a]) + 5 k^2 l a^(3 l x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a]) + 9 k^2 l a^(2 k x) a^(l x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a]) + 18 k^2 l a^(k x) a^(2 l x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a]) + 2 k l^2 a^(3 l x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a]) + 5 k l^2 a^(3 k x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a]) + 9 k l^2 a^(k x) a^(2 l x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a]) + 18 k l^2 a^(2 k x) a^(l x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a]) + 2 l^3 a^(3 k x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a])]
```

Maple [A]

time = 0.06, size = 84, normalized size = 1.06

method	result	size
--------	--------	------

risch	$\frac{a^{3kx}}{3k \ln(a)} + \frac{a^{3lx}}{3l \ln(a)} + \frac{3a^{kx} a^{2lx}}{\ln(a)(k+2l)} + \frac{3a^{2kx} a^{lx}}{\ln(a)(2k+l)}$	84
norman	$\frac{e^{3kx \ln(a)}}{3k \ln(a)} + \frac{e^{3lx \ln(a)}}{3l \ln(a)} + \frac{3e^{kx \ln(a)} e^{2lx \ln(a)}}{\ln(a)(k+2l)} + \frac{3e^{2kx \ln(a)} e^{lx \ln(a)}}{\ln(a)(2k+l)}$	90
meijerg	$-\frac{1-e^{3kx \ln(a)}}{3k \ln(a)} - \frac{3\left(1-e^{xk \ln(a)\left(2+\frac{l}{k}\right)}\right)}{k \ln(a)\left(2+\frac{l}{k}\right)} - \frac{3\left(1-e^{xl \ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{1}{1+\frac{k}{l}}\right)}\right)}{l \ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{1}{1+\frac{k}{l}}\right)} - \frac{1-e^{3lx \ln(a)}}{3l \ln(a)}$	136

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x)+a^(l*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/k/ln(a)*(a^(k*x))^3+1/3/l/ln(a)*(a^(l*x))^3+3/ln(a)/(k+2*l)*a^(k*x)*(a^(l*x))^2+3/ln(a)/(2*k+1)*(a^(k*x))^2*a^(l*x)
```

Maxima [A]

time = 0.25, size = 77, normalized size = 0.97

$$\frac{3a^{2kx+lx}}{(2k+l)\log(a)} + \frac{3a^{kx+2lx}}{(k+2l)\log(a)} + \frac{a^{3kx}}{3k\log(a)} + \frac{a^{3lx}}{3l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^3,x, algorithm="maxima")
```

```
[Out] 3*a^(2*k*x + l*x)/((2*k + 1)*log(a)) + 3*a^(k*x + 2*l*x)/((k + 2*l)*log(a)) + 1/3*a^(3*k*x)/(k*log(a)) + 1/3*a^(3*l*x)/(l*log(a))
```

Fricas [A]

time = 0.31, size = 130, normalized size = 1.65

$$\frac{9(2k^2l + kl^2)a^{kx}a^{2lx} + 9(k^2l + 2kl^2)a^{2kx}a^{lx} + (2k^2l + 5kl^2 + 2l^3)a^{3kx} + (2k^3 + 5k^2l + 2kl^2)a^{3lx}}{3(2k^3l + 5k^2l^2 + 2kl^3)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(9*(2*k^2*l + k*l^2)*a^(k*x)*a^(2*l*x) + 9*(k^2*l + 2*k*l^2)*a^(2*k*x)*a^(l*x) + (2*k^2*l + 5*k*l^2 + 2*l^3)*a^(3*k*x) + (2*k^3 + 5*k^2*l + 2*k*l^2)*a^(3*l*x))/((2*k^3*l + 5*k^2*l^2 + 2*k*l^3)*log(a))
```

Sympy [A]

time = 2.72, size = 665, normalized size = 8.42

```

(Sr
  9*(2*k**2*l + k*l**2)*a**(k*x)*a**(2*l*x) + 9*(k**2*l + 2*k*l**2)*a**(2*k*x)*a**(l*x) + (2*k**2*l + 5*k*l**2 + 2*l**3)*a**(3*k*x) + (2*k**3 + 5*k**2*l + 2*k*l**2)*a**(3*l*x)
  / (3*(2*k**3*l + 5*k**2*l**2 + 2*k*l**3)*log(a))
)

for a = 1 ^ (a - 1 + k = 0) ^ (a - 1 + l = 0)
for k = 0
for k = -2l
for k = -l
for l = 0
otherwise
```

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**(k*x)+a**(l*x))**3,x)

[Out] Piecewise((8*x, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(1, 0))), (a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(2*l*log(a)) + 3*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(3*l*x)/(3*l*log(a)) + 3*x - 1/(a**(3*l*x)*l*log(a)) - 1/(6*a**(6*l*x)*l*log(a)), Eq(k, -2*1)), (2*a**(3*l*x/2)/(l*log(a)) + a**(3*l*x)/(3*l*log(a)) + 3*x - 2/(3*a**(3*l*x/2)*l*log(a)), Eq(k, -1/2)), (a*(3*k*x)/(3*k*log(a)) + 3*a**(2*k*x)/(2*k*log(a)) + 3*a**(k*x)/(k*log(a)) + x, Eq(1, 0)), (2*a**(3*k*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 5*a**(3*k*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 2*a**(3*k*x)*l**3/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 9*a**(2*k*x)*a**(l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a**(2*k*x)*a**(l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a**(k*x)*a**(2*l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 9*a**(k*x)*a**(2*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 2*a**(3*l*x)*k**3/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 5*a**(3*l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 2*a**(3*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)), True))

Giac [C] Result contains complex when optimal does not.

time = 0.02, size = 1035, normalized size = 13.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(k*x)+a^(l*x))^3,x)

[Out]
$$\begin{aligned} & \frac{2}{3} * (2 * k * \cos(-3/2 * \pi * k * x * \operatorname{sgn}(a)) + 3/2 * \pi * k * x) * \log(\operatorname{abs}(a)) / (4 * k^2 * \log(\operatorname{abs}(a)))^2 \\ & + (\pi * k * \operatorname{sgn}(a) - \pi * k)^2 - (\pi * k * \operatorname{sgn}(a) - \pi * k) * \sin(-3/2 * \pi * k * x * \operatorname{sgn}(a)) \\ & + 3/2 * \pi * k * x) / (4 * k^2 * \log(\operatorname{abs}(a)))^2 + (\pi * k * \operatorname{sgn}(a) - \pi * k)^2) * \operatorname{abs}(a)^{(3 * k * x)} \\ & + 2/3 * (2 * l * \cos(-3/2 * \pi * l * x * \operatorname{sgn}(a)) + 3/2 * \pi * l * x) * \log(\operatorname{abs}(a)) / (4 * l^2 * \log(\operatorname{abs}(a)))^2 \\ & + (\pi * l * \operatorname{sgn}(a) - \pi * l)^2 - (\pi * l * \operatorname{sgn}(a) - \pi * l) * \sin(-3/2 * \pi * l * x * \operatorname{sgn}(a)) \\ & + 3/2 * \pi * l * x) / (4 * l^2 * \log(\operatorname{abs}(a)))^2 + (\pi * l * \operatorname{sgn}(a) - \pi * l)^2) * \operatorname{abs}(a)^{(3 * l * x)} \\ & + 6 * e^{((k * \log(\operatorname{abs}(a)) + 2 * l * \log(\operatorname{abs}(a))) * x) * (2 * (k * \log(\operatorname{abs}(a)) + 2 * l * \log(\operatorname{abs}(a)))) * \cos(-1/2 * \pi * k * x * \operatorname{sgn}(a) - \pi * l * x * \operatorname{sgn}(a) + 1/2 * \pi * k * x + \pi * l * x)} \\ & / ((\pi * k * \operatorname{sgn}(a) + 2 * \pi * l * \operatorname{sgn}(a) - \pi * k - 2 * \pi * l)^2 + 4 * (k * \log(\operatorname{abs}(a)) + 2 * l * \log(\operatorname{abs}(a))))^2) \\ & - (\pi * k * \operatorname{sgn}(a) + 2 * \pi * l * \operatorname{sgn}(a) - \pi * k - 2 * \pi * l) * \sin(-1/2 * \pi * k * x * \operatorname{sgn}(a) - \pi * l * x * \operatorname{sgn}(a) + 1/2 * \pi * k * x + \pi * l * x) / ((\pi * k * \operatorname{sgn}(a) + 2 * \pi * l * \operatorname{sgn}(a) - \pi * k - 2 * \pi * l)^2 + 4 * (k * \log(\operatorname{abs}(a)) + 2 * l * \log(\operatorname{abs}(a))))^2) \\ & + 6 * e^{((2 * k * \log(\operatorname{abs}(a)) + l * \log(\operatorname{abs}(a))) * x) * (2 * (2 * k * \log(\operatorname{abs}(a)) + l * \log(\operatorname{abs}(a)))) * \cos(-\pi * k * x * \operatorname{sgn}(a) - 1/2 * \pi * l * x * \operatorname{sgn}(a) + \pi * k * x + 1/2 * \pi * l * x) / ((2 * \pi * k * \operatorname{sgn}(a) + \pi * l * \operatorname{sgn}(a) - 2 * \pi * k - \pi * l)^2 + 4 * (2 * k * \log(\operatorname{abs}(a)) + l * \log(\operatorname{abs}(a))))^2) \\ & - (2 * \pi * k * \operatorname{sgn}(a) + \pi * l * \operatorname{sgn}(a) - 2 * \pi * k - \pi * l) * \sin(-\pi * k * x * \operatorname{sgn}(a) - 1/2 * \pi * \end{aligned}$$


```

*1*x*sgn(a) + pi*k*x + 1/2*pi*1*x)/((2*pi*k*sgn(a) + pi*1*sgn(a) - 2*pi*k -
pi*1)^2 + 4*(2*k*log(abs(a)) + 1*log(abs(a)))^2)) + I*abs(a)^(3*k*x)*(I*e^
(3/2*I*pi*k*x*sgn(a) - 3/2*I*pi*k*x)/(3*I*pi*k*sgn(a) - 3*I*pi*k + 6*k*log(
abs(a))) - I*e^(-3/2*I*pi*k*x*sgn(a) + 3/2*I*pi*k*x)/(-3*I*pi*k*sgn(a) + 3*
I*pi*k + 6*k*log(abs(a)))) - 3*e^((2*k*log(abs(a)) + 1*log(abs(a)))x)*(e^(
I*pi*k*x*sgn(a) + 1/2*I*pi*1*x*sgn(a) - I*pi*k*x - 1/2*I*pi*1*x)/(2*I*pi*k*
sgn(a) + I*pi*1*sgn(a) - 2*I*pi*k - I*pi*1 + 4*k*log(abs(a)) + 2*1*log(abs(
a))) - e^(-I*pi*k*x*sgn(a) - 1/2*I*pi*1*x*sgn(a) + I*pi*k*x + 1/2*I*pi*1*x)
/(-2*I*pi*k*sgn(a) - I*pi*1*sgn(a) + 2*I*pi*k + I*pi*1 + 4*k*log(abs(a)) +
2*1*log(abs(a)))) - 3*e^((k*log(abs(a)) + 2*1*log(abs(a)))x)*(e^(1/2*I*pi*
k*x*sgn(a) + I*pi*1*x*sgn(a) - 1/2*I*pi*k*x - I*pi*1*x)/(I*pi*k*sgn(a) + 2*
I*pi*1*sgn(a) - I*pi*k - 2*I*pi*1 + 2*k*log(abs(a)) + 4*1*log(abs(a))) - e^
(-1/2*I*pi*k*x*sgn(a) - I*pi*1*x*sgn(a) + 1/2*I*pi*k*x + I*pi*1*x)/(-I*pi*k
*sgn(a) - 2*I*pi*1*sgn(a) + I*pi*k + 2*I*pi*1 + 2*k*log(abs(a)) + 4*1*log(a
bs(a)))) + I*abs(a)^(3*1*x)*(I*e^(3/2*I*pi*1*x*sgn(a) - 3/2*I*pi*1*x)/(3*I*
pi*1*sgn(a) - 3*I*pi*1 + 6*1*log(abs(a))) - I*e^(-3/2*I*pi*1*x*sgn(a) + 3/2
*I*pi*1*x)/(-3*I*pi*1*sgn(a) + 3*I*pi*1 + 6*1*log(abs(a))))

```

Mupad [B]

time = 0.39, size = 81, normalized size = 1.03

$$\frac{3 a^{kx} a^{2lx}}{k \ln(a) + 2l \ln(a)} + \frac{3 a^{2kx} a^{lx}}{2k \ln(a) + l \ln(a)} + \frac{a^{3kx}}{3k \ln(a)} + \frac{a^{3lx}}{3l \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(k*x) + a^(1*x))^3,x)

[Out] (3*a^(k*x)*a^(2*1*x))/(k*log(a) + 2*1*log(a)) + (3*a^(2*k*x)*a^(1*x))/(2*k*log(a) + 1*log(a)) + a^(3*k*x)/(3*k*log(a)) + a^(3*1*x)/(3*1*log(a))

3.505 $\int (a^{kx} + a^{lx})^4 dx$

Optimal. Leaf size=98

$$\frac{a^{4kx}}{4k \log(a)} + \frac{a^{4lx}}{4l \log(a)} + \frac{3a^{2(k+l)x}}{(k+l) \log(a)} + \frac{4a^{(3k+l)x}}{(3k+l) \log(a)} + \frac{4a^{(k+3l)x}}{(k+3l) \log(a)}$$

[Out] $1/4*a^{(4*k*x)}/k/\ln(a)+1/4*a^{(4*l*x)}/l/\ln(a)+3*a^{(2*(k+l)*x)}/(k+l)/\ln(a)+4*a^{((3*k+l)*x)}/(3*k+l)/\ln(a)+4*a^{((k+3*l)*x)}/(k+3*l)/\ln(a)$

Rubi [A]

time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6874, 2225}

$$\frac{3a^{2x(k+l)}}{\log(a)(k+l)} + \frac{4a^{x(3k+l)}}{\log(a)(3k+l)} + \frac{4a^{x(k+3l)}}{\log(a)(k+3l)} + \frac{a^{4kx}}{4k \log(a)} + \frac{a^{4lx}}{4l \log(a)}$$

Antiderivative was successfully verified.

[In] `Int[(a^(k*x) + a^(l*x))^4, x]`

[Out] $a^{(4*k*x)}/(4*k*\text{Log}[a]) + a^{(4*l*x)}/(4*l*\text{Log}[a]) + (3*a^{(2*(k+1)*x)})/((k+1)*\text{Log}[a]) + (4*a^{((3*k+1)*x)})/((3*k+1)*\text{Log}[a]) + (4*a^{((k+3*1)*x)})/((k+3*1)*\text{Log}[a])$

Rule 2225

`Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]`

Rule 6874

`Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]`

Rubi steps

$$\begin{aligned} \int (a^{kx} + a^{lx})^4 dx &= \frac{\text{Subst}\left(\int (e^{kx} + e^{lx})^4 dx, x, x \log(a)\right)}{\log(a)} \\ &= \frac{\text{Subst}\left(\int (e^{4kx} + e^{4lx} + 6e^{2(k+l)x} + 4e^{(3k+l)x} + 4e^{(k+3l)x}) dx, x, x \log(a)\right)}{\log(a)} \\ &= \frac{\text{Subst}\left(\int e^{4kx} dx, x, x \log(a)\right)}{\log(a)} + \frac{\text{Subst}\left(\int e^{4lx} dx, x, x \log(a)\right)}{\log(a)} + \frac{4\text{Subst}\left(\int e^{(3k+l)x} dx, x, x \log(a)\right)}{\log(a)} \\ &= \frac{a^{4kx}}{4k \log(a)} + \frac{a^{4lx}}{4l \log(a)} + \frac{3a^{2(k+l)x}}{(k+l) \log(a)} + \frac{4a^{(3k+l)x}}{(3k+l) \log(a)} + \frac{4a^{(k+3l)x}}{(k+3l) \log(a)} \end{aligned}$$

Mathematica [A]

time = 0.07, size = 80, normalized size = 0.82

$$\frac{\frac{a^{4kx}}{k} + \frac{a^{4lx}}{l} + \frac{12a^{2(k+l)x}}{k+l} + \frac{16a^{(3k+l)x}}{3k+l} + \frac{16a^{(k+3l)x}}{k+3l}}{4 \log(a)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^(k*x) + a^(l*x))^4,x]

[Out] (a^(4*k*x)/k + a^(4*l*x)/l + (12*a^(2*(k + l)*x))/(k + l) + (16*a^((3*k + l)*x))/(3*k + l) + (16*a^((k + 3*l)*x))/(k + 3*l))/(4*Log[a])

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 23.85, size = 1211, normalized size = 12.36

result too large to display

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a^(k*x) + a^(l*x))^4,x]')

[Out] Piecewise[{{16 x, a == 1 && a == 1 || k == 0 && a == 1 || l == 0}, {(1 x Log[a] + a^(4 l x) / 4 + 4 a^(3 l x) / 3 + 3 a^(2 l x) + 4 a^(1 x)) / (1 Log[a]), k == 0}, {(-1 + 48 l x Log[a] a^(12 l x) - 18 a^(8 l x) - 6 a^(4 l x) + 3 a^(16 l x)) a^(-12 l x) / (12 l Log[a]), k == -3 l}, {(-1 + 24 l x Log[a] a^(4 l x) - 8 a^(2 l x) + (8 + a^(2 l x)) a^(6 l x)) a^(-4 l x) / (4 l Log[a]), k == -1}, {(-3 + 16 l x Log[a] a^(4 l x / 3) + a^(16 l x / 3) + 6 a^(4 l x) + 18 a^(8 l x / 3)) a^(-4 l x / 3) / (4 l Log[a]), l == -3 k}, {(k x Log[a] + a^(4 k x) / 4 + 4 a^(3 k x) / 3 + 3 a^(2 k x) + 4 a^(k x)) / (k Log[a]), l == 0}}, 3 k^4 a^(4 l x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a]) + 3 k^3 l a^(4 k x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a]) + 13 k^3 l a^(4 l x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a]) + 16 k^3 l a^(3 k x) a^(1 x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a]) + 36 k^3 l a^(2 k x) a^(2 l x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a]) + 48 k^3 l a^(k x) a^(3 l x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a]) + 13 k^2 l^2 a^(4 k x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a]) + 13 k^2 l^2 a^(4 l x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a]) + 64 k^2 l^2 a^(3 k x) a^(1 x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a]) + 12 k l^4 Log[a] + 64 k^2 l^2 a^(k x) a^(3 l x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a])

$$\begin{aligned} &] + 120 k^2 l^2 a^{(2kx)} a^{(2lx)} / (12 k^4 l \text{Log}[a] + 52 k^3 l^2 \text{Log}[a] + 52 k^2 l^3 \text{Log}[a] + 12 k l^4 \text{Log}[a]) + 3 k l^3 a^{(4lx)} / (12 k^4 l \text{Log}[a] + 52 k^3 l^2 \text{Log}[a] + 52 k^2 l^3 \text{Log}[a] + 12 k l^4 \text{Log}[a]) + 13 k l^3 a^{(4kx)} / (12 k^4 l \text{Log}[a] + 52 k^3 l^2 \text{Log}[a] + 52 k^2 l^3 \text{Log}[a] + 12 k l^4 \text{Log}[a]) + 16 k l^3 a^{(kx)} a^{(3lx)} / (12 k^4 l \text{Log}[a] + 52 k^3 l^2 \text{Log}[a] + 52 k^2 l^3 \text{Log}[a] + 12 k l^4 \text{Log}[a]) + 36 k l^3 a^{(2kx)} a^{(2lx)} / (12 k^4 l \text{Log}[a] + 52 k^3 l^2 \text{Log}[a] + 52 k^2 l^3 \text{Log}[a] + 12 k l^4 \text{Log}[a]) + 48 k l^3 a^{(3kx)} a^{(lx)} / (12 k^4 l \text{Log}[a] + 52 k^3 l^2 \text{Log}[a] + 52 k^2 l^3 \text{Log}[a] + 12 k l^4 \text{Log}[a]) + 3 l^4 a^{(4kx)} / (12 k^4 l \text{Log}[a] + 52 k^3 l^2 \text{Log}[a] + 52 k^2 l^3 \text{Log}[a] + 12 k l^4 \text{Log}[a]) \end{aligned}$$

Maple [A]

time = 0.04, size = 109, normalized size = 1.11

method	result
risch	$\frac{a^{4kx}}{4k \ln(a)} + \frac{4a^{3kx} a^{lx}}{\ln(a)(3k+l)} + \frac{3a^{2kx} a^{2lx}}{\ln(a)(k+l)} + \frac{4a^{kx} a^{3lx}}{\ln(a)(k+3l)} + \frac{a^{4lx}}{4l \ln(a)}$
meijerg	$-\frac{1-e^{4kx \ln(a)}}{4k \ln(a)} - \frac{4 \left(1 - e^{xl \ln(a) \left(1 + \frac{k}{l}\right) \left(1 + \frac{2k}{l(1+\frac{k}{l})}\right)} \right)}{l \ln(a) \left(1 + \frac{k}{l}\right) \left(1 + \frac{2k}{l(1+\frac{k}{l})}\right)} - \frac{3 \left(1 - e^{2xl \ln(a) \left(1 + \frac{k}{l}\right)} \right)}{l \ln(a) \left(1 + \frac{k}{l}\right)} - \frac{4 \left(1 - e^{xl \ln(a) \left(1 + \frac{k}{l}\right) \left(1 + \frac{2}{1+\frac{k}{l}}\right)} \right)}{l \ln(a) \left(1 + \frac{k}{l}\right) \left(1 + \frac{2}{1+\frac{k}{l}}\right)} - \frac{1-e^{4lx}}{4l \ln(a)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^(k*x)+a^(l*x))^4,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{4} \frac{1}{\ln(a)} \frac{1}{k} (a^{kx})^4 + 4 (a^{kx})^3 \frac{1}{\ln(a)} \frac{1}{(3k+1)} a^{lx} + 3 (a^{kx})^2 \frac{1}{\ln(a)} \frac{1}{(k+1)} (a^{lx})^2 + 4 a^{kx} \frac{1}{\ln(a)} \frac{1}{(k+3)} (a^{lx})^3 + \frac{1}{4} \frac{1}{\ln(a)} \frac{1}{l} (a^{lx})^4$

Maxima [A]

time = 0.25, size = 99, normalized size = 1.01

$$\frac{4 a^{3kx+lx}}{(3k+l) \log(a)} + \frac{4 a^{kx+3lx}}{(k+3l) \log(a)} + \frac{3 a^{2kx+2lx}}{(k+l) \log(a)} + \frac{a^{4kx}}{4k \log(a)} + \frac{a^{4lx}}{4l \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^(k*x)+a^(l*x))^4,x, algorithm="maxima")`

[Out] $4 a^{(3kx+lx)} / ((3k+1) \log(a)) + 4 a^{(kx+3lx)} / ((k+3) \log(a)) + 3 a^{(2kx+2lx)} / ((k+1) \log(a)) + \frac{1}{4} a^{(4kx)} / (k \log(a)) + \frac{1}{4} a^{(4lx)} / (l \log(a))$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(94) = 188.

time = 0.32, size = 205, normalized size = 2.09

$$\frac{16(3k^3l + 4k^2l^2 + kl^3)a^{kx}a^{3lx} + 12(3k^3l + 10k^2l^2 + 3kl^3)a^{2kx}a^{2lx} + 16(k^3l + 4k^2l^2 + 3kl^3)a^{3kx}a^{lx} + (3k^3l + 13k^2l^2 + 13kl^3 + 3l^4)a^{4kx} + (3k^4 + 13k^3l + 13k^2l^2 + 3kl^3)a^{4lx}}{4(3k^4l + 13k^3l^2 + 13k^2l^3 + 3kl^4)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(k*x)+a^(l*x))^4,x, algorithm="fricas")

[Out] $\frac{1}{4}*(16*(3*k^3*l + 4*k^2*l^2 + k*l^3)*a^{k*x}*a^{(3*l*x)} + 12*(3*k^3*l + 10*k^2*l^2 + 3*k*l^3)*a^{(2*k*x)}*a^{(2*l*x)} + 16*(k^3*l + 4*k^2*l^2 + 3*k*l^3)*a^{(3*k*x)}*a^{(l*x)} + (3*k^3*l + 13*k^2*l^2 + 13*k*l^3 + 3*l^4)*a^{(4*k*x)} + (3*k^4 + 13*k^3*l + 13*k^2*l^2 + 3*k*l^3)*a^{(4*l*x)})/((3*k^4*l + 13*k^3*l^2 + 13*k^2*l^3 + 3*k*l^4)*\log(a))$

Sympy [A]

time = 21.81, size = 1350, normalized size = 13.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**(k*x)+a**(l*x))**4,x)

[Out] Piecewise((16*x, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))), (a**(4*l*x)/(4*l*log(a)) + 4*a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(l*log(a)) + 4*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(4*l*x)/(4*l*log(a)) + 4*x - 3/(2*a**(4*l*x)*l*log(a)) - 1/(2*a**(8*l*x)*l*log(a)) - 1/(12*a**(12*l*x)*l*log(a)), Eq(k, -3*l)), (a**(4*l*x)/(4*l*log(a)) + 2*a**(2*l*x)/(l*log(a)) + 6*x - 2/(a**(2*l*x)*l*log(a)) - 1/(4*a**(4*l*x)*l*log(a)), Eq(k, -l)), (3*a**(8*l*x/3)/(2*l*log(a)) + 9*a**(4*l*x/3)/(2*l*log(a)) + a**(4*l*x)/(4*l*log(a)) + 4*x - 3/(4*a**(4*l*x/3)*l*log(a)), Eq(k, -l/3)), (a**(4*k*x)/(4*k*log(a)) + 4*a**(3*k*x)/(3*k*log(a)) + 3*a**(2*k*x)/(k*log(a)) + 4*a**(k*x)/(k*log(a)) + x, Eq(l, 0)), (3*a**(4*k*x)*k**3*l/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 13*a**(4*k*x)*k**2*l**2/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 13*a**(4*k*x)*k*l**3/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 3*a**(4*k*x)*l**4/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 16*a**(3*k*x)*a**(l*x)*k**3*l/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 64*a**(3*k*x)*a**(l*x)*k**2*l**2/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 48*a**(3*k*x)*a**(l*x)*k*l**3/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 36*a**(2*k*x)*a**(2*l*x)*k**3*l/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 120*a**(2*k*x)*a**(2*l*x)*k**2*l**2/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 36*a**(2*k*x)*a**(2*l*x)*k*l**3/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 48*a**(k*x)*a**(3*l*x)*k**3*l/(12*k**4*l*log(a) + 52

```
*k**3*1**2*log(a) + 52*k**2*1**3*log(a) + 12*k*1**4*log(a)) + 64*a**(k*x)*a
**(3*1*x)*k**2*1**2/(12*k**4*1*log(a) + 52*k**3*1**2*log(a) + 52*k**2*1**3*
log(a) + 12*k*1**4*log(a)) + 16*a**(k*x)*a**(3*1*x)*k*1**3/(12*k**4*1*log(a
) + 52*k**3*1**2*log(a) + 52*k**2*1**3*log(a) + 12*k*1**4*log(a)) + 3*a**(4
*1*x)*k**4/(12*k**4*1*log(a) + 52*k**3*1**2*log(a) + 52*k**2*1**3*log(a) +
12*k*1**4*log(a)) + 13*a**(4*1*x)*k**3*1/(12*k**4*1*log(a) + 52*k**3*1**2*1
og(a) + 52*k**2*1**3*log(a) + 12*k*1**4*log(a)) + 13*a**(4*1*x)*k**2*1**2/(
12*k**4*1*log(a) + 52*k**3*1**2*log(a) + 52*k**2*1**3*log(a) + 12*k*1**4*lo
g(a)) + 3*a**(4*1*x)*k*1**3/(12*k**4*1*log(a) + 52*k**3*1**2*log(a) + 52*k*
**2*1**3*log(a) + 12*k*1**4*log(a)), True))
```

Giac [C] Result contains complex when optimal does not.

time = 0.02, size = 1361, normalized size = 13.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(k*x)+a^(1*x))^4,x)

```
[Out] 1/2*(2*k*cos(-2*pi*k*x*sgn(a) + 2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2
+ (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-2*pi*k*x*sgn(a) + 2*pi
i*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(4*k*x) + 1/2
*(2*1*cos(-2*pi*1*x*sgn(a) + 2*pi*1*x)*log(abs(a))/(4*1^2*log(abs(a))^2 + (
pi*1*sgn(a) - pi*1)^2) - (pi*1*sgn(a) - pi*1)*sin(-2*pi*1*x*sgn(a) + 2*pi*1
*x)/(4*1^2*log(abs(a))^2 + (pi*1*sgn(a) - pi*1)^2))*abs(a)^(4*1*x) + 8*e^((
k*log(abs(a)) + 3*1*log(abs(a)))*x)*(2*(k*log(abs(a)) + 3*1*log(abs(a)))co
s(-1/2*pi*k*x*sgn(a) - 3/2*pi*1*x*sgn(a) + 1/2*pi*k*x + 3/2*pi*1*x)/((pi*k*
sgn(a) + 3*pi*1*sgn(a) - pi*k - 3*pi*1)^2 + 4*(k*log(abs(a)) + 3*1*log(abs(
a))))^2) - (pi*k*sgn(a) + 3*pi*1*sgn(a) - pi*k - 3*pi*1)*sin(-1/2*pi*k*x*sgn
(a) - 3/2*pi*1*x*sgn(a) + 1/2*pi*k*x + 3/2*pi*1*x)/((pi*k*sgn(a) + 3*pi*1*sg
n(a) - pi*k - 3*pi*1)^2 + 4*(k*log(abs(a)) + 3*1*log(abs(a))))^2) + 6*e^(2
*(k*log(abs(a)) + 1*log(abs(a)))*x)*(2*(k*log(abs(a)) + 1*log(abs(a)))cos(
-pi*k*x*sgn(a) - pi*1*x*sgn(a) + pi*k*x + pi*1*x)/((pi*k*sgn(a) + pi*1*sgn(
a) - pi*k - pi*1)^2 + 4*(k*log(abs(a)) + 1*log(abs(a))))^2) - (pi*k*sgn(a) +
pi*1*sgn(a) - pi*k - pi*1)*sin(-pi*k*x*sgn(a) - pi*1*x*sgn(a) + pi*k*x + p
i*1*x)/((pi*k*sgn(a) + pi*1*sgn(a) - pi*k - pi*1)^2 + 4*(k*log(abs(a)) + 1*
log(abs(a))))^2) + 8*e^((3*k*log(abs(a)) + 1*log(abs(a)))*x)*(2*(3*k*log(ab
s(a)) + 1*log(abs(a)))cos(-3/2*pi*k*x*sgn(a) - 1/2*pi*1*x*sgn(a) + 3/2*pi*
k*x + 1/2*pi*1*x)/((3*pi*k*sgn(a) + pi*1*sgn(a) - 3*pi*k - pi*1)^2 + 4*(3*k
*log(abs(a)) + 1*log(abs(a))))^2) - (3*pi*k*sgn(a) + pi*1*sgn(a) - 3*pi*k -
pi*1)*sin(-3/2*pi*k*x*sgn(a) - 1/2*pi*1*x*sgn(a) + 3/2*pi*k*x + 1/2*pi*1*x)
/((3*pi*k*sgn(a) + pi*1*sgn(a) - 3*pi*k - pi*1)^2 + 4*(3*k*log(abs(a)) + 1*
log(abs(a))))^2) - 1/2*I*abs(a)^(4*k*x)*(-I*e^(2*I*pi*k*x*sgn(a) - 2*I*pi*k
*x)/(2*I*pi*k*sgn(a) - 2*I*pi*k + 4*k*log(abs(a))) + I*e^(-2*I*pi*k*x*sgn(a)
) + 2*I*pi*k*x)/(-2*I*pi*k*sgn(a) + 2*I*pi*k + 4*k*log(abs(a)))) - 4*e^((3*
k*log(abs(a)) + 1*log(abs(a)))*x)*(e^(3/2*I*pi*k*x*sgn(a) + 1/2*I*pi*1*x*sg
```

$$\begin{aligned} & n(a) - 3/2 * I * \pi * k * x - 1/2 * I * \pi * l * x) / (3 * I * \pi * k * \operatorname{sgn}(a) + I * \pi * l * \operatorname{sgn}(a) - 3 * I * \\ & \pi * k - I * \pi * l + 6 * k * \log(\operatorname{abs}(a)) + 2 * l * \log(\operatorname{abs}(a))) - e^{(-3/2 * I * \pi * k * x * \operatorname{sgn}(a) \\ &) - 1/2 * I * \pi * l * x * \operatorname{sgn}(a) + 3/2 * I * \pi * k * x + 1/2 * I * \pi * l * x) / (-3 * I * \pi * k * \operatorname{sgn}(a) - \\ & I * \pi * l * \operatorname{sgn}(a) + 3 * I * \pi * k + I * \pi * l + 6 * k * \log(\operatorname{abs}(a)) + 2 * l * \log(\operatorname{abs}(a)))} - 3 \\ & * e^{(2 * (k * \log(\operatorname{abs}(a)) + l * \log(\operatorname{abs}(a))) * x) * (e^{(I * \pi * k * x * \operatorname{sgn}(a) + I * \pi * l * x * \operatorname{sgn}(\\ & a) - I * \pi * k * x - I * \pi * l * x) / (I * \pi * k * \operatorname{sgn}(a) + I * \pi * l * \operatorname{sgn}(a) - I * \pi * k - I * \pi * l \\ & + 2 * k * \log(\operatorname{abs}(a)) + 2 * l * \log(\operatorname{abs}(a)))} - e^{(-I * \pi * k * x * \operatorname{sgn}(a) - I * \pi * l * x * \operatorname{sgn}(\\ & a) + I * \pi * k * x + I * \pi * l * x) / (-I * \pi * k * \operatorname{sgn}(a) - I * \pi * l * \operatorname{sgn}(a) + I * \pi * k + I * \pi * l \\ & + 2 * k * \log(\operatorname{abs}(a)) + 2 * l * \log(\operatorname{abs}(a)))} - 4 * e^{((k * \log(\operatorname{abs}(a)) + 3 * l * \log(\operatorname{abs}(\\ & a))) * x) * (e^{(1/2 * I * \pi * k * x * \operatorname{sgn}(a) + 3/2 * I * \pi * l * x * \operatorname{sgn}(a) - 1/2 * I * \pi * k * x - 3/2 * \\ & I * \pi * l * x) / (I * \pi * k * \operatorname{sgn}(a) + 3 * I * \pi * l * \operatorname{sgn}(a) - I * \pi * k - 3 * I * \pi * l + 2 * k * \log(\operatorname{abs}(\\ & a)) + 6 * l * \log(\operatorname{abs}(a)))} - e^{(-1/2 * I * \pi * k * x * \operatorname{sgn}(a) - 3/2 * I * \pi * l * x * \operatorname{sgn}(a) + \\ & 1/2 * I * \pi * k * x + 3/2 * I * \pi * l * x) / (-I * \pi * k * \operatorname{sgn}(a) - 3 * I * \pi * l * \operatorname{sgn}(a) + I * \pi * k + 3 \\ & * I * \pi * l + 2 * k * \log(\operatorname{abs}(a)) + 6 * l * \log(\operatorname{abs}(a)))} - 1/2 * I * \operatorname{abs}(a)^{(4 * l * x) * (-I * e^{ \\ & (2 * I * \pi * l * x * \operatorname{sgn}(a) - 2 * I * \pi * l * x) / (2 * I * \pi * l * \operatorname{sgn}(a) - 2 * I * \pi * l + 4 * l * \log(\operatorname{abs}(\\ & a)))} + I * e^{(-2 * I * \pi * l * x * \operatorname{sgn}(a) + 2 * I * \pi * l * x) / (-2 * I * \pi * l * \operatorname{sgn}(a) + 2 * I * \pi * l + \\ & 4 * l * \log(\operatorname{abs}(a)))} \end{aligned}$$

Mupad [B]

time = 0.42, size = 106, normalized size = 1.08

$$\frac{3 a^{2 k x} a^{2 l x}}{k \ln(a) + l \ln(a)} + \frac{4 a^{k x} a^{3 l x}}{k \ln(a) + 3 l \ln(a)} + \frac{4 a^{3 k x} a^{l x}}{3 k \ln(a) + l \ln(a)} + \frac{a^{4 k x}}{4 k \ln(a)} + \frac{a^{4 l x}}{4 l \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(k*x) + a^(l*x))^4,x)

[Out] (3*a^(2*k*x)*a^(2*l*x))/(k*log(a) + l*log(a)) + (4*a^(k*x)*a^(3*l*x))/(k*log(a) + 3*l*log(a)) + (4*a^(3*k*x)*a^(l*x))/(3*k*log(a) + l*log(a)) + a^(4*k*x)/(4*k*log(a)) + a^(4*l*x)/(4*l*log(a))

3.506 $\int (a^{kx} + a^{lx})^n dx$

Optimal. Leaf size=72

$$\frac{(1 + a^{(k-l)x}) (a^{kx} + a^{lx})^n {}_2F_1\left(1, 1 + \frac{kn}{k-l}; 1 + \frac{ln}{k-l}; -a^{(k-l)x}\right)}{ln \log(a)}$$

[Out] (1+a^((k-1)*x))*(a^(k*x)+a^(l*x))^n*hypergeom([1, 1+k*n/(k-1)], [1+1*n/(k-1)], -a^((k-1)*x))/1/n/ln(a)

Rubi [A]

time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$,

Rules used = {2323, 2283}

$$\frac{(a^{-(x(k-l))} + 1)^{-n} (a^{kx} + a^{lx})^n {}_2F_1\left(-n, -\frac{kn}{k-l}; 1 - \frac{kn}{k-l}; -a^{-(k-l)x}\right)}{kn \log(a)}$$

Antiderivative was successfully verified.

[In] Int[(a^(k*x) + a^(l*x))^n, x]

[Out] ((a^(k*x) + a^(l*x))^n*Hypergeometric2F1[-n, -((k*n)/(k - 1)), 1 - (k*n)/(k - 1), -a^(-((k - 1)*x))])/((1 + a^(-((k - 1)*x)))^n*k*n*Log[a])

Rule 2283

Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x)))/(g*h*Log[G])*Hypergeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1, Simplify[(-b/a)*F^(e*(c + d*x))], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 2323

Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && !IntegerQ[n] && LinearQ[{v, w}, x]

Rubi steps

$$\begin{aligned} \int (a^{kx} + a^{lx})^n dx &= \left(a^{-knx} (1 + a^{-(k-l)x})^{-n} (a^{kx} + a^{lx})^n \right) \int a^{knx} (1 + a^{-(k-l)x})^n dx \\ &= \frac{(1 + a^{-(k-l)x})^{-n} (a^{kx} + a^{lx})^n {}_2F_1\left(-n, -\frac{kn}{k-l}; 1 - \frac{kn}{k-l}; -a^{-(k-l)x}\right)}{kn \log(a)} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 73, normalized size = 1.01

$$\frac{(a^{kx} + a^{lx})^n (1 + a^{(-k+l)x}) {}_2F_1\left(1, 1 + n + \frac{kn}{-k+l}; 1 + \frac{kn}{-k+l}; -a^{(-k+l)x}\right)}{kn \log(a)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^(k*x) + a^(l*x))^n,x]

[Out] ((a^(k*x) + a^(l*x))^n*(1 + a^((-k + 1)*x))*Hypergeometric2F1[1, 1 + n + (k*n)/(-k + 1), 1 + (k*n)/(-k + 1), -a^((-k + 1)*x)])/(k*n*Log[a])

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a^(k*x) + a^(l*x))^n,x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [F]

time = 0.03, size = 0, normalized size = 0.00

$$\int (a^{kx} + a^{lx})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(k*x)+a^(l*x))^n,x)

[Out] int((a^(k*x)+a^(l*x))^n,x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(k*x)+a^(l*x))^n,x, algorithm="maxima")

[Out] integrate((a^(k*x) + a^(l*x))^n, x)

Fricas [F]

time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(k*x)+a^(l*x))^n,x, algorithm="fricas")

[Out] integral((a^(k*x) + a^(l*x))^n, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a^{kx} + a^{lx})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**(k*x)+a**(l*x))**n,x)

[Out] Integral((a**(k*x) + a**(l*x))**n, x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(k*x)+a^(l*x))^n,x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a^{kx} + a^{lx})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(k*x) + a^(l*x))^n,x)

[Out] int((a^(k*x) + a^(l*x))^n, x)

3.507 $\int (a^{kx} - a^{lx}) dx$

Optimal. Leaf size=28

$$\frac{a^{kx}}{k \log(a)} - \frac{a^{lx}}{l \log(a)}$$

[Out] $a^{(k*x)/k/\ln(a)} - a^{(1*x)/1/\ln(a)}$

Rubi [A]

time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2225}

$$\frac{a^{kx}}{k \log(a)} - \frac{a^{lx}}{l \log(a)}$$

Antiderivative was successfully verified.

[In] Int[a^(k*x) - a^(1*x), x]

[Out] a^(k*x)/(k*Log[a]) - a^(1*x)/(1*Log[a])

Rule 2225

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int (a^{kx} - a^{lx}) dx &= \int a^{kx} dx - \int a^{lx} dx \\ &= \frac{a^{kx}}{k \log(a)} - \frac{a^{lx}}{l \log(a)} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 28, normalized size = 1.00

$$\frac{a^{kx}}{k \log(a)} - \frac{a^{lx}}{l \log(a)}$$

Antiderivative was successfully verified.

[In] Integrate[a^(k*x) - a^(1*x), x]

[Out] a^(k*x)/(k*Log[a]) - a^(1*x)/(1*Log[a])

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.35, size = 93, normalized size = 3.32

$$\text{Piecewise} \left[\left\{ \left\{ \frac{-ka^{lx} + la^{kx}}{kl \text{Log}[a]}, k \text{Log}[a] \neq 0 \&\& l \text{Log}[a] \neq 0 \right\}, \left\{ \frac{a^{kx}}{k \text{Log}[a]} - x, k \text{Log}[a] \neq 0 \right\}, \left\{ -\frac{a^{lx}}{l \text{Log}[a]} + x, l \text{Log}[a] \neq 0 \right\}, \{0, \text{True}\} \right\} \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(a^(k*x) - a^(l*x))^1,x]')`

[Out] `Piecewise[{{(-k a^(l x) + l a^(k x)) / (k l Log[a]), k Log[a] != 0 && l Log[a] != 0}, {a^(k x) / (k Log[a]) - x, k Log[a] != 0}, {-a^(l x) / (l Log[a]) + x, l Log[a] != 0}, {0, True}}]`

Maple [A]

time = 0.02, size = 29, normalized size = 1.04

method	result	size
default	$\frac{a^{kx}}{k \ln(a)} - \frac{a^{lx}}{l \ln(a)}$	29
risch	$\frac{a^{kx}}{k \ln(a)} - \frac{a^{lx}}{l \ln(a)}$	29
norman	$\frac{e^{kx \ln(a)}}{k \ln(a)} - \frac{e^{lx \ln(a)}}{l \ln(a)}$	31
meijerg	$-\frac{1-e^{kx \ln(a)}}{k \ln(a)} + \frac{1-e^{lx \ln(a)}}{l \ln(a)}$	39

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a^(k*x)-a^(l*x),x,method=_RETURNVERBOSE)`

[Out] `a^(k*x)/k/ln(a)-a^(l*x)/l/ln(a)`

Maxima [A]

time = 0.24, size = 28, normalized size = 1.00

$$\frac{a^{kx}}{k \log(a)} - \frac{a^{lx}}{l \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a^(k*x)-a^(l*x),x, algorithm="maxima")`

[Out] `a^(k*x)/(k*log(a)) - a^(l*x)/(l*log(a))`

Fricas [A]

time = 0.32, size = 28, normalized size = 1.00

$$-\frac{a^{lx}k - a^{kx}l}{kl \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^(k*x)-a^(l*x),x, algorithm="fricas")

[Out] $-(a^{(l*x)*k} - a^{(k*x)*l})/(k*l*log(a))$

Sympy [A]

time = 0.07, size = 29, normalized size = 1.04

$$\begin{cases} \frac{a^{kx}}{k \log(a)} & \text{for } k \log(a) \neq 0 \\ x & \text{otherwise} \end{cases} - \begin{cases} \frac{a^{lx}}{l \log(a)} & \text{for } l \log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a**(k*x)-a**(l*x),x)

[Out] Piecewise((a**(k*x)/(k*log(a)), Ne(k*log(a), 0)), (x, True)) - Piecewise((a**(l*x)/(l*log(a)), Ne(l*log(a), 0)), (x, True))

Giac [A]

time = 0.00, size = 24, normalized size = 0.86

$$\frac{a^{kx}}{k \ln a} - \frac{a^{lx}}{l \ln a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^(k*x)-a^(l*x),x)

[Out] $a^{(k*x)}/(k*log(a)) - a^{(l*x)}/(l*log(a))$

Mupad [B]

time = 0.31, size = 27, normalized size = 0.96

$$\frac{a^{kx} l - a^{lx} k}{kl \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a^(k*x) - a^(l*x),x)

[Out] $(a^{(k*x)*l} - a^{(l*x)*k})/(k*l*log(a))$

3.508 $\int (a^{kx} - a^{lx})^2 dx$

Optimal. Leaf size=53

$$\frac{a^{2kx}}{2k \log(a)} + \frac{a^{2lx}}{2l \log(a)} - \frac{2a^{(k+l)x}}{(k+l) \log(a)}$$

[Out] $1/2*a^{(2*k*x)}/k/\ln(a)+1/2*a^{(2*l*x)}/l/\ln(a)-2*a^{((k+l)*x)}/(k+l)/\ln(a)$

Rubi [A]

time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6874, 2225}

$$-\frac{2a^{x(k+l)}}{\log(a)(k+l)} + \frac{a^{2kx}}{2k \log(a)} + \frac{a^{2lx}}{2l \log(a)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a^{(k*x)} - a^{(l*x)})^2, x]$

[Out] $a^{(2*k*x)}/(2*k*\text{Log}[a]) + a^{(2*l*x)}/(2*l*\text{Log}[a]) - (2*a^{((k+l)*x)})/((k+l)*\text{Log}[a])$

Rule 2225

$\text{Int}[(F_)^{(c_*)*((a_*) + (b_*)(x_*))})^{(n_*)}, x_Symbol] \text{ :> Simp}[(F^{(c*(a + b*x)))^n}/(b*c*n*\text{Log}[F]), x] \text{ /; FreeQ}\{F, a, b, c, n\}, x]$

Rule 6874

$\text{Int}[u_, x_Symbol] \text{ :> With}\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] \text{ /; SumQ}[v]$

Rubi steps

$$\begin{aligned} \int (a^{kx} - a^{lx})^2 dx &= \frac{\text{Subst}\left(\int (e^{kx} - e^{lx})^2 dx, x, x \log(a)\right)}{\log(a)} \\ &= \frac{\text{Subst}\left(\int (e^{2kx} + e^{2lx} - 2e^{(k+l)x}) dx, x, x \log(a)\right)}{\log(a)} \\ &= \frac{\text{Subst}\left(\int e^{2kx} dx, x, x \log(a)\right)}{\log(a)} + \frac{\text{Subst}\left(\int e^{2lx} dx, x, x \log(a)\right)}{\log(a)} - \frac{2\text{Subst}\left(\int e^{(k+l)x} dx, x, x \log(a)\right)}{\log(a)} \\ &= \frac{a^{2kx}}{2k \log(a)} + \frac{a^{2lx}}{2l \log(a)} - \frac{2a^{(k+l)x}}{(k+l) \log(a)} \end{aligned}$$

Mathematica [A]

time = 0.04, size = 53, normalized size = 1.00

$$\frac{a^{2kx}}{2k \log(a)} + \frac{a^{2lx}}{2l \log(a)} - \frac{2a^{(k+l)x}}{(k+l) \log(a)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^(k*x) - a^(l*x))^2,x]**[Out]** a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) - (2*a^((k + l)*x))/((k + l)*Log[a])**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.75, size = 275, normalized size = 5.19

$$\text{Piecewise}\left[\left\{\left\{0, a == 1 \&\& k == 1 \mid k == 0 \&\& a == 1 \mid l == 0\right\}, \left\{\frac{l x \text{Log}[a] - 2a^{lx} + \frac{a^{2lx}}{2l}}{l \text{Log}[a]}, k == 0\right\}, \left\{-\frac{a^{-lx}}{2l \text{Log}[a]} + \frac{a^{2lx}}{2l \text{Log}[a]} - 2x, k == -1\right\}, \left\{\frac{k x \text{Log}[a] - 2a^{kx} + \frac{a^{2kx}}{2k}}{k \text{Log}[a]}, l == 0\right\}\right\}, \frac{k^2 a^{2lx}}{2k^2 l \text{Log}[a] + 2k l \text{Log}[a]} - \frac{4kl a^{lx}}{2k^2 l \text{Log}[a] + 2k l \text{Log}[a]} + \frac{k l a^{2lx}}{2k^2 l \text{Log}[a] + 2k l \text{Log}[a]} + \frac{k l a^{2lx}}{2k^2 l \text{Log}[a] + 2k l \text{Log}[a]} + \frac{l^2 a^{2lx}}{2k^2 l \text{Log}[a] + 2k l \text{Log}[a]}\right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a^(k*x) - a^(l*x))^2,x]')

[Out] Piecewise[{{0, a == 1 && a == 1 || k == 0 && a == 1 || l == 0}, {(1 x Log[a] - 2 a^(1 x) + a^(2 1 x) / 2) / (1 Log[a]), k == 0}, {-a^(-2 1 x) / (2 1 Log[a]) + a^(2 1 x) / (2 1 Log[a]) - 2 x, k == -1}, {(k x Log[a] - 2 a^(k x) + a^(2 k x) / 2) / (k Log[a]), l == 0}}, k^2 a^(2 1 x) / (2 k^2 1 Log[a] + 2 k 1^2 Log[a]) - 4 k 1 a^(k x) a^(1 x) / (2 k^2 1 Log[a] + 2 k 1^2 Log[a]) + k 1 a^(2 k x) / (2 k^2 1 Log[a] + 2 k 1^2 Log[a]) + k 1 a^(2 1 x) / (2 k^2 1 Log[a] + 2 k 1^2 Log[a]) + 1^2 a^(2 k x) / (2 k^2 1 Log[a] + 2 k 1^2 Log[a])}]

Maple [A]

time = 0.03, size = 55, normalized size = 1.04

method	result	size
risch	$\frac{a^{2kx}}{2k \ln(a)} + \frac{a^{2lx}}{2l \ln(a)} - \frac{2a^{kx} a^{lx}}{\ln(a)(k+l)}$	55
norman	$\frac{e^{2kx \ln(a)}}{2k \ln(a)} + \frac{e^{2lx \ln(a)}}{2l \ln(a)} - \frac{2e^{kx \ln(a)} e^{lx \ln(a)}}{\ln(a)(k+l)}$	59
meijerg	$-\frac{1-e^{2kx \ln(a)}}{2k \ln(a)} + \frac{2-2e^{xl \ln(a)} \left(1+\frac{k}{l}\right)}{l \ln(a) \left(1+\frac{k}{l}\right)} - \frac{1-e^{2lx \ln(a)}}{2l \ln(a)}$	77

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(k*x)-a^(l*x))^2,x,method=_RETURNVERBOSE)**[Out]** 1/2/k/ln(a)*(a^(k*x))^2+1/2/l/ln(a)*(a^(l*x))^2-2/ln(a)/(k+l)*a^(k*x)*a^(l*x)

Maxima [A]

time = 0.25, size = 51, normalized size = 0.96

$$-\frac{2a^{kx+lx}}{(k+l)\log(a)} + \frac{a^{2kx}}{2k\log(a)} + \frac{a^{2lx}}{2l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(k*x)-a^(l*x))^2,x, algorithm="maxima")**[Out]** -2*a^(k*x + l*x)/((k + l)*log(a)) + 1/2*a^(2*k*x)/(k*log(a)) + 1/2*a^(2*l*x)/(l*log(a))**Fricas [A]**

time = 0.33, size = 64, normalized size = 1.21

$$-\frac{4a^{kx}a^{lx}kl - (kl + l^2)a^{2kx} - (k^2 + kl)a^{2lx}}{2(k^2l + kl^2)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(k*x)-a^(l*x))^2,x, algorithm="fricas")**[Out]** -1/2*(4*a^(k*x)*a^(l*x)*k*l - (k*l + l^2)*a^(2*k*x) - (k^2 + k*l)*a^(2*l*x))/((k^2*l + k*l^2)*log(a))**Sympy [A]**

time = 0.53, size = 248, normalized size = 4.68

$$\begin{cases} 0 & \text{for } a = 1 \wedge (a = 1 \vee k = 0) \wedge (a = 1 \vee l = 0) \\ \frac{a^{2lx}}{2l\log(a)} - \frac{2a^{lx}}{l\log(a)} + x & \text{for } k = 0 \\ \frac{a^{2lx}}{2l\log(a)} - 2x - \frac{a^{-2lx}}{2l\log(a)} & \text{for } k = -l \\ \frac{a^{2kx}}{2k\log(a)} - \frac{2a^{kx}}{k\log(a)} + x & \text{for } l = 0 \\ \frac{a^{2kx}kl}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2kx}l^2}{2k^2l\log(a)+2kl^2\log(a)} - \frac{4a^{kx}a^{lx}kl}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2lx}k^2}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2lx}kl}{2k^2l\log(a)+2kl^2\log(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**(k*x)-a**(l*x))**2,x)**[Out]** Piecewise((0, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))), (a**(2*l*x)/(2*l*log(a)) - 2*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(2*l*x)/(2*l*log(a)) - 2*x - 1/(2*a**(2*l*x)*l*log(a)), Eq(k, -1)), (a**(2*k*x)/(2*k*log(a)) - 2*a**(k*x)/(k*log(a)) + x, Eq(l, 0)), (a**(2*k*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*k*x)*l**2/(2*k**2*l*log(a) + 2*k*l**2*log(a)) - 4*a**(k*x)*a**(l*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*l*x)*k**2/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*l*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a)), True))**Giac [C]** Result contains complex when optimal does not.

time = 0.01, size = 677, normalized size = 12.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(k*x)-a^(l*x))^2,x)

[Out] $(2*k*\cos(-\pi*k*x*\text{sgn}(a) + \pi*k*x)*\log(\text{abs}(a)))/(4*k^2*\log(\text{abs}(a))^2 + (\pi*k*\text{sgn}(a) - \pi*k)^2) - (\pi*k*\text{sgn}(a) - \pi*k)*\sin(-\pi*k*x*\text{sgn}(a) + \pi*k*x)/(4*k^2*\log(\text{abs}(a))^2 + (\pi*k*\text{sgn}(a) - \pi*k)^2)*\text{abs}(a)^{(2*k*x)} + (2*\cos(-\pi*l*x*\text{sgn}(a) + \pi*l*x)*\log(\text{abs}(a)))/(4*l^2*\log(\text{abs}(a))^2 + (\pi*l*\text{sgn}(a) - \pi*l)^2) - (\pi*l*\text{sgn}(a) - \pi*l)*\sin(-\pi*l*x*\text{sgn}(a) + \pi*l*x)/(4*l^2*\log(\text{abs}(a))^2 + (\pi*l*\text{sgn}(a) - \pi*l)^2)*\text{abs}(a)^{(2*l*x)} - 4*e^{((k*\log(\text{abs}(a)) + l*\log(\text{abs}(a)))*x)}*(2*(k*\log(\text{abs}(a)) + l*\log(\text{abs}(a)))*\cos(-1/2*\pi*k*x*\text{sgn}(a) - 1/2*\pi*l*x*\text{sgn}(a) + 1/2*\pi*k*x + 1/2*\pi*l*x)/((\pi*k*\text{sgn}(a) + \pi*l*\text{sgn}(a) - \pi*k - \pi*l)^2 + 4*(k*\log(\text{abs}(a)) + l*\log(\text{abs}(a)))^2) - (\pi*k*\text{sgn}(a) + \pi*l*\text{sgn}(a) - \pi*k - \pi*l)*\sin(-1/2*\pi*k*x*\text{sgn}(a) - 1/2*\pi*l*x*\text{sgn}(a) + 1/2*\pi*k*x + 1/2*\pi*l*x)/((\pi*k*\text{sgn}(a) + \pi*l*\text{sgn}(a) - \pi*k - \pi*l)^2 + 4*(k*\log(\text{abs}(a)) + l*\log(\text{abs}(a)))^2) - 1/2*I*\text{abs}(a)^{(2*k*x)}*(-I*e^{(I*\pi*k*x*\text{sgn}(a) - I*\pi*k*x)/(I*\pi*k*\text{sgn}(a) - I*\pi*k + 2*k*\log(\text{abs}(a)))} + I*e^{(-I*\pi*k*x*\text{sgn}(a) + I*\pi*k*x)/(-I*\pi*k*\text{sgn}(a) + I*\pi*k + 2*k*\log(\text{abs}(a)))}) + 2*e^{((k*\log(\text{abs}(a)) + l*\log(\text{abs}(a)))*x)}*(e^{(1/2*I*\pi*k*x*\text{sgn}(a) + 1/2*I*\pi*l*x*\text{sgn}(a) - 1/2*I*\pi*k*x - 1/2*I*\pi*l*x)/(I*\pi*k*\text{sgn}(a) + I*\pi*l*\text{sgn}(a) - I*\pi*k - I*\pi*l + 2*k*\log(\text{abs}(a)) + 2*l*\log(\text{abs}(a)))} - e^{(-1/2*I*\pi*k*x*\text{sgn}(a) - 1/2*I*\pi*l*x*\text{sgn}(a) + 1/2*I*\pi*k*x + 1/2*I*\pi*l*x)/(-I*\pi*k*\text{sgn}(a) - I*\pi*l*\text{sgn}(a) + I*\pi*k + I*\pi*l + 2*k*\log(\text{abs}(a)) + 2*l*\log(\text{abs}(a)))}) - 1/2*I*\text{abs}(a)^{(2*l*x)}*(-I*e^{(I*\pi*l*x*\text{sgn}(a) - I*\pi*l*x)/(I*\pi*l*\text{sgn}(a) - I*\pi*l + 2*l*\log(\text{abs}(a)))} + I*e^{(-I*\pi*l*x*\text{sgn}(a) + I*\pi*l*x)/(-I*\pi*l*\text{sgn}(a) + I*\pi*l + 2*l*\log(\text{abs}(a)))})$

Mupad [B]

time = 0.36, size = 69, normalized size = 1.30

$$\frac{a^{2kx}}{2k \ln(a)} + \frac{\frac{a^{2lx} k^2}{2} - l \left(2a^{kx+lx} k - \frac{a^{2lx} k}{2} \right)}{kl \ln(a) (k+l)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(k*x) - a^(l*x))^2,x)

[Out] $a^{(2*k*x)}/(2*k*\log(a)) + ((a^{(2*l*x)}*k^2)/2 - 1*(2*a^{(k*x + l*x)}*k - (a^{(2*l*x)}*k)/2))/(k*l*\log(a)*(k + l))$

3.509 $\int (a^{kx} - a^{lx})^3 dx$

Optimal. Leaf size=79

$$\frac{a^{3kx}}{3k \log(a)} - \frac{a^{3lx}}{3l \log(a)} - \frac{3a^{(2k+l)x}}{(2k+l) \log(a)} + \frac{3a^{(k+2l)x}}{(k+2l) \log(a)}$$

[Out] $1/3*a^{(3*k*x)}/k/\ln(a)-1/3*a^{(3*l*x)}/l/\ln(a)-3*a^{((2*k+1)*x)}/(2*k+1)/\ln(a)+3*a^{((k+2*l)*x)}/(k+2*l)/\ln(a)$

Rubi [A]

time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6874, 2225}

$$-\frac{3a^{x(2k+l)}}{\log(a)(2k+l)} + \frac{3a^{x(k+2l)}}{\log(a)(k+2l)} + \frac{a^{3kx}}{3k \log(a)} - \frac{a^{3lx}}{3l \log(a)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a^{(k*x)} - a^{(l*x)})^3, x]$

[Out] $a^{(3*k*x)}/(3*k*\text{Log}[a]) - a^{(3*l*x)}/(3*l*\text{Log}[a]) - (3*a^{((2*k+1)*x)})/((2*k+1)*\text{Log}[a]) + (3*a^{((k+2*l)*x)})/((k+2*l)*\text{Log}[a])$

Rule 2225

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))}^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x)))}^n/(b*c*n*\text{Log}[F]), x] /;$ FreeQ[{F, a, b, c, n}, x]

Rule 6874

$\text{Int}[u_, x_Symbol] \rightarrow \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

Rubi steps

$$\begin{aligned} \int (a^{kx} - a^{lx})^3 dx &= \frac{\text{Subst}\left(\int (e^{kx} - e^{lx})^3 dx, x, x \log(a)\right)}{\log(a)} \\ &= \frac{\text{Subst}\left(\int (e^{3kx} - e^{3lx} - 3e^{(2k+l)x} + 3e^{(k+2l)x}) dx, x, x \log(a)\right)}{\log(a)} \\ &= \frac{\text{Subst}\left(\int e^{3kx} dx, x, x \log(a)\right)}{\log(a)} - \frac{\text{Subst}\left(\int e^{3lx} dx, x, x \log(a)\right)}{\log(a)} - \frac{3\text{Subst}\left(\int e^{(2k+l)x} dx, x, x \log(a)\right)}{\log(a)} \\ &= \frac{a^{3kx}}{3k \log(a)} - \frac{a^{3lx}}{3l \log(a)} - \frac{3a^{(2k+l)x}}{(2k+l) \log(a)} + \frac{3a^{(k+2l)x}}{(k+2l) \log(a)} \end{aligned}$$

Mathematica [A]

time = 0.06, size = 66, normalized size = 0.84

$$\frac{\frac{a^{3kx}}{k} - \frac{a^{3lx}}{l} - \frac{9a^{(2k+l)x}}{2k+l} + \frac{9a^{(k+2l)x}}{k+2l}}{3 \log(a)}$$

Antiderivative was successfully verified.

`[In] Integrate[(a^(k*x) - a^(l*x))^3, x]`

```
[Out] (a^(3*k*x)/k - a^(3*l*x)/l - (9*a^((2*k + 1)*x))/(2*k + 1) + (9*a^((k + 2*l)*x))/(k + 2*l))/(3*Log[a])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 5.34, size = 633, normalized size = 8.01

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(a^(k*x) - a^(l*x))^3, x]')`

```
[Out] Piecewise[{{0, a == 1 && a == 1 || k == 0 && a == 1 || l == 0}, {(1 x Log[a] - 3 a^(1 x) - a^(3 l x) / 3 + 3 a^(2 l x) / 2) / (1 Log[a]), k == 0}, {(-1 / 6 + 3 l x Log[a] a^(6 l x) + a^(3 l x) - a^(9 l x) / 3) a^(-6 l x) / (1 Log[a]), k == -2 l}, {-2 a^(-3 l x / 2) / (3 l Log[a]) - a^(3 l x) / (3 l Log[a]) + 2 a^(3 l x / 2) / (1 Log[a]) - 3 x, l == -2 k}, {(-k x Log[a] - 3 a^(2 k x) / 2 + a^(3 k x) / 3 + 3 a^(k x)) / (k Log[a]), l == 0}}, -2 k^3 a^(3 l x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a]) - 9 k^2 l a^(2 k x) a^(1 x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a]) - 5 k^2 l a^(3 l x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a]) + 2 k^2 l a^(3 k x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a]) + 18 k^2 l a^(k x) a^(2 l x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a]) - 18 k l^2 a^(2 k x) a^(1 x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a]) - 2 k l^2 a^(3 l x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a]) + 5 k l^2 a^(3 k x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a]) + 9 k l^2 a^(k x) a^(2 l x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a]) + 2 l^3 a^(3 k x) / (6 k^3 l Log[a] + 15 k^2 l^2 Log[a] + 6 k l^3 Log[a])}]
```

Maple [A]

time = 0.04, size = 84, normalized size = 1.06

method	result	size
--------	--------	------

risch	$\frac{a^{3kx}}{3k \ln(a)} - \frac{a^{3lx}}{3l \ln(a)} + \frac{3a^{kx} a^{2lx}}{\ln(a)(k+2l)} - \frac{3a^{2kx} a^{lx}}{\ln(a)(2k+l)}$	84
norman	$\frac{e^{3kx \ln(a)}}{3k \ln(a)} - \frac{e^{3lx \ln(a)}}{3l \ln(a)} + \frac{3e^{kx \ln(a)} e^{2lx \ln(a)}}{\ln(a)(k+2l)} - \frac{3e^{2kx \ln(a)} e^{lx \ln(a)}}{\ln(a)(2k+l)}$	90
meijerg	$-\frac{1-e^{3kx \ln(a)}}{3k \ln(a)} + \frac{3-3e^{xk \ln(a)(2+\frac{k}{l})}}{k \ln(a)(2+\frac{l}{k})} - \frac{3 \left(1-e^{xl \ln(a)(1+\frac{k}{l})} \left(1+\frac{1}{1+\frac{l}{k}} \right) \right)}{l \ln(a)(1+\frac{k}{l}) \left(1+\frac{1}{1+\frac{l}{k}} \right)} + \frac{1-e^{3lx \ln(a)}}{3l \ln(a)}$	136

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^(k*x)-a^(l*x))^3,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{3} \frac{a^{3kx}}{k \ln(a)} - \frac{1}{3} \frac{a^{3lx}}{l \ln(a)} + \frac{3a^{kx} a^{2lx}}{\ln(a)(k+2l)} - \frac{3a^{2kx} a^{lx}}{\ln(a)(2k+l)}$

Maxima [A]

time = 0.25, size = 77, normalized size = 0.97

$$-\frac{3 a^{2 k x+l x}}{(2 k+l) \log (a)}+\frac{3 a^{k x+2 l x}}{(k+2 l) \log (a)}+\frac{a^{3 k x}}{3 k \log (a)}-\frac{a^{3 l x}}{3 l \log (a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^(k*x)-a^(l*x))^3,x, algorithm="maxima")`

[Out] $-3a^{(2kx + lx)}/((2k + l)\log(a)) + 3a^{(kx + 2lx)}/((k + 2l)\log(a)) + 1/3a^{(3kx)}/(k\log(a)) - 1/3a^{(3lx)}/(l\log(a))$

Fricas [A]

time = 0.31, size = 131, normalized size = 1.66

$$\frac{9(2k^2l + kl^2)a^{kx} a^{2lx} - 9(k^2l + 2kl^2)a^{2kx} a^{lx} + (2k^2l + 5kl^2 + 2l^3)a^{3kx} - (2k^3 + 5k^2l + 2kl^2)a^{3lx}}{3(2k^3l + 5k^2l^2 + 2kl^3)\log(a)}$$

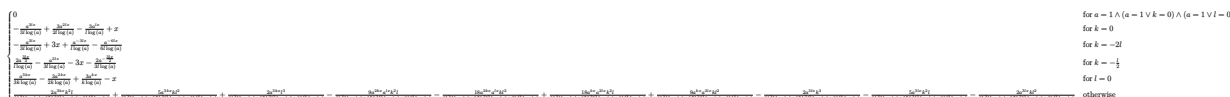
Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^(k*x)-a^(l*x))^3,x, algorithm="fricas")`

[Out] $\frac{1}{3} \frac{9(k^2l^2 + k^2l + kl^2)a^{kx} a^{2lx} - 9(k^2l + 2kl^2)a^{2kx} a^{lx} - (2k^3 + 5k^2l + 2kl^2)a^{3lx}}{(2k^3l + 5k^2l^2 + 2kl^3)\log(a)}$

Sympy [A]

time = 2.71, size = 663, normalized size = 8.39



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**(k*x)-a**(l*x))**3,x)

[Out] Piecewise((0, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))), (-a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(2*l*log(a)) - 3*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (-a**(3*l*x)/(3*l*log(a)) + 3*x + 1/(a**(3*l*x)*l*log(a)) - 1/(6*a**(6*l*x)*l*log(a)), Eq(k, -2*l)), (2*a**(3*l*x/2)/(l*log(a)) - a**(3*l*x)/(3*l*log(a)) - 3*x - 2/(3*a**(3*l*x/2)*l*log(a)), Eq(k, -1/2)), (a**(3*k*x)/(3*k*log(a)) - 3*a**(2*k*x)/(2*k*log(a)) + 3*a**(k*x)/(k*log(a)) - x, Eq(l, 0)), (2*a**(3*k*x)*k**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 5*a**(3*k*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 2*a**(3*k*x)*l**3/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) - 9*a**(2*k*x)*a**(l*x)*k**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) - 18*a**(2*k*x)*a**(l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a**(k*x)*a**(2*l*x)*k**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 9*a**(k*x)*a**(2*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) - 2*a**(3*l*x)*k**3/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) - 5*a**(3*l*x)*k**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) - 2*a**(3*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)), True))

Giac [C] Result contains complex when optimal does not.

time = 0.02, size = 1035, normalized size = 13.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(k*x)-a^(l*x))^3,x)

[Out]
$$\begin{aligned} & 2/3*(2*k*\cos(-3/2*\pi*k*x*\text{sgn}(a) + 3/2*\pi*k*x)*\log(\text{abs}(a)))/(4*k^2*\log(\text{abs}(a)))^2 + (\pi*k*\text{sgn}(a) - \pi*k)^2 - (\pi*k*\text{sgn}(a) - \pi*k)*\sin(-3/2*\pi*k*x*\text{sgn}(a) \\ & + 3/2*\pi*k*x)/(4*k^2*\log(\text{abs}(a))^2 + (\pi*k*\text{sgn}(a) - \pi*k)^2)*\text{abs}(a)^(3*k*x) - 2/3*(2*l*\cos(-3/2*\pi*l*x*\text{sgn}(a) + 3/2*\pi*l*x)*\log(\text{abs}(a)))/(4*l^2*\log(\text{abs}(a))^2 + (\pi*l*\text{sgn}(a) - \pi*l)^2 - (\pi*l*\text{sgn}(a) - \pi*l)*\sin(-3/2*\pi*l*x*\text{sgn}(a) + 3/2*\pi*l*x)/(4*l^2*\log(\text{abs}(a))^2 + (\pi*l*\text{sgn}(a) - \pi*l)^2)*\text{abs}(a)^(3*l*x) \\ & + 6*e^{((k*\log(\text{abs}(a)) + 2*l*\log(\text{abs}(a))))*x}*(2*(k*\log(\text{abs}(a)) + 2*l*\log(\text{abs}(a))))*\cos(-1/2*\pi*k*x*\text{sgn}(a) - \pi*l*x*\text{sgn}(a) + 1/2*\pi*k*x + \pi*l*x)/((\pi*k*\text{sgn}(a) + 2*\pi*l*\text{sgn}(a) - \pi*k - 2*\pi*l)^2 + 4*(k*\log(\text{abs}(a)) + 2*l*\log(\text{abs}(a))))^2 - (\pi*k*\text{sgn}(a) + 2*\pi*l*\text{sgn}(a) - \pi*k - 2*\pi*l)*\sin(-1/2*\pi*k*x*\text{sgn}(a) - \pi*l*x*\text{sgn}(a) + 1/2*\pi*k*x + \pi*l*x)/((\pi*k*\text{sgn}(a) + 2*\pi*l*\text{sgn}(a) - \pi*k - 2*\pi*l)^2 + 4*(k*\log(\text{abs}(a)) + 2*l*\log(\text{abs}(a))))^2) - 6*e^{((2*k*\log(\text{abs}(a)) + l*\log(\text{abs}(a))))*x}*(2*(2*k*\log(\text{abs}(a)) + l*\log(\text{abs}(a))))*\cos(-\pi*k*x*\text{sgn}(a) - 1/2*\pi*l*x*\text{sgn}(a) + \pi*k*x + 1/2*\pi*l*x)/((2*\pi*k*\text{sgn}(a) + \pi*l*\text{sgn}(a) - 2*\pi*k - \pi*l)^2 + 4*(2*k*\log(\text{abs}(a)) + l*\log(\text{abs}(a))))^2) - (2*\pi*k*\text{sgn}(a) + \pi*l*\text{sgn}(a) - 2*\pi*k - \pi*l)*\sin(-\pi*k*x*\text{sgn}(a) - 1/2*\pi \end{aligned}$$

```

*x*sgn(a) + pi*k*x + 1/2*pi*1*x)/((2*pi*k*sgn(a) + pi*1*sgn(a) - 2*pi*k -
pi*1)^2 + 4*(2*k*log(abs(a)) + 1*log(abs(a)))^2)) + I*abs(a)^(3*k*x)*(I*e^
(3/2*I*pi*k*x*sgn(a) - 3/2*I*pi*k*x)/(3*I*pi*k*sgn(a) - 3*I*pi*k + 6*k*log(
abs(a))) - I*e^(-3/2*I*pi*k*x*sgn(a) + 3/2*I*pi*k*x)/(-3*I*pi*k*sgn(a) + 3*
I*pi*k + 6*k*log(abs(a)))) + 3*e^((2*k*log(abs(a)) + 1*log(abs(a))))*x)*(e^(
I*pi*k*x*sgn(a) + 1/2*I*pi*1*x*sgn(a) - I*pi*k*x - 1/2*I*pi*1*x)/(2*I*pi*k*
sgn(a) + I*pi*1*sgn(a) - 2*I*pi*k - I*pi*1 + 4*k*log(abs(a)) + 2*1*log(abs(
a))) - e^(-I*pi*k*x*sgn(a) - 1/2*I*pi*1*x*sgn(a) + I*pi*k*x + 1/2*I*pi*1*x)
/(-2*I*pi*k*sgn(a) - I*pi*1*sgn(a) + 2*I*pi*k + I*pi*1 + 4*k*log(abs(a)) +
2*1*log(abs(a)))) - 3*e^((k*log(abs(a)) + 2*1*log(abs(a))))*x)*(e^(1/2*I*pi*
k*x*sgn(a) + I*pi*1*x*sgn(a) - 1/2*I*pi*k*x - I*pi*1*x)/(I*pi*k*sgn(a) + 2*
I*pi*1*sgn(a) - I*pi*k - 2*I*pi*1 + 2*k*log(abs(a)) + 4*1*log(abs(a))) - e^
(-1/2*I*pi*k*x*sgn(a) - I*pi*1*x*sgn(a) + 1/2*I*pi*k*x + I*pi*1*x)/(-I*pi*k
*sgn(a) - 2*I*pi*1*sgn(a) + I*pi*k + 2*I*pi*1 + 2*k*log(abs(a)) + 4*1*log(a
bs(a)))) + I*abs(a)^(3*1*x)*(-I*e^(3/2*I*pi*1*x*sgn(a) - 3/2*I*pi*1*x)/(3*I
*pi*1*sgn(a) - 3*I*pi*1 + 6*1*log(abs(a))) + I*e^(-3/2*I*pi*1*x*sgn(a) + 3/
2*I*pi*1*x)/(-3*I*pi*1*sgn(a) + 3*I*pi*1 + 6*1*log(abs(a))))

```

Mupad [B]

time = 0.36, size = 81, normalized size = 1.03

$$\frac{3a^{kx}a^{2lx}}{k \ln(a) + 2l \ln(a)} - \frac{3a^{2kx}a^{lx}}{2k \ln(a) + l \ln(a)} + \frac{a^{3kx}}{3k \ln(a)} - \frac{a^{3lx}}{3l \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(k*x) - a^(1*x))^3,x)

[Out] (3*a^(k*x)*a^(2*1*x))/(k*log(a) + 2*1*log(a)) - (3*a^(2*k*x)*a^(1*x))/(2*k*log(a) + 1*log(a)) + a^(3*k*x)/(3*k*log(a)) - a^(3*1*x)/(3*1*log(a))

3.510 $\int (a^{kx} - a^{lx})^4 dx$

Optimal. Leaf size=98

$$\frac{a^{4kx}}{4k \log(a)} + \frac{a^{4lx}}{4l \log(a)} + \frac{3a^{2(k+l)x}}{(k+l) \log(a)} - \frac{4a^{(3k+l)x}}{(3k+l) \log(a)} - \frac{4a^{(k+3l)x}}{(k+3l) \log(a)}$$

[Out] $1/4*a^{(4*k*x)}/k/\ln(a)+1/4*a^{(4*l*x)}/l/\ln(a)+3*a^{(2*(k+l)*x)}/(k+l)/\ln(a)-4*a^{((3*k+1)*x)}/(3*k+1)/\ln(a)-4*a^{((k+3*1)*x)}/(k+3*1)/\ln(a)$

Rubi [A]

time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$,

Rules used = {6874, 2225}

$$\frac{3a^{2x(k+l)}}{\log(a)(k+l)} - \frac{4a^{x(3k+l)}}{\log(a)(3k+l)} - \frac{4a^{x(k+3l)}}{\log(a)(k+3l)} + \frac{a^{4kx}}{4k \log(a)} + \frac{a^{4lx}}{4l \log(a)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a^{k*x} - a^{l*x})^4, x]$

[Out] $a^{(4*k*x)}/(4*k*\text{Log}[a]) + a^{(4*l*x)}/(4*l*\text{Log}[a]) + (3*a^{(2*(k+1)*x)})/((k+1)*\text{Log}[a]) - (4*a^{((3*k+1)*x)})/((3*k+1)*\text{Log}[a]) - (4*a^{((k+3*1)*x)})/((k+3*1)*\text{Log}[a])$

Rule 2225

$\text{Int}[(F_)^{((c_*)*((a_*) + (b_*)*(x_)))}^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x)))}^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

Rule 6874

$\text{Int}[u_, x_Symbol] \rightarrow \text{With}\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]$

Rubi steps

$$\begin{aligned} \int (a^{kx} - a^{lx})^4 dx &= \frac{\text{Subst} \left(\int (e^{kx} - e^{lx})^4 dx, x, x \log(a) \right)}{\log(a)} \\ &= \frac{\text{Subst} \left(\int (e^{4kx} + e^{4lx} + 6e^{2(k+l)x} - 4e^{(3k+l)x} - 4e^{(k+3l)x}) dx, x, x \log(a) \right)}{\log(a)} \\ &= \frac{\text{Subst} \left(\int e^{4kx} dx, x, x \log(a) \right)}{\log(a)} + \frac{\text{Subst} \left(\int e^{4lx} dx, x, x \log(a) \right)}{\log(a)} - \frac{4 \text{Subst} \left(\int e^{(3k+l)x} dx, x, x \log(a) \right)}{\log(a)} \\ &= \frac{a^{4kx}}{4k \log(a)} + \frac{a^{4lx}}{4l \log(a)} + \frac{3a^{2(k+l)x}}{(k+l) \log(a)} - \frac{4a^{(3k+l)x}}{(3k+l) \log(a)} - \frac{4a^{(k+3l)x}}{(k+3l) \log(a)} \end{aligned}$$

Mathematica [A]

time = 0.07, size = 80, normalized size = 0.82

$$\frac{\frac{a^{4kx}}{k} + \frac{a^{4lx}}{l} + \frac{12a^{2(k+l)x}}{k+l} - \frac{16a^{(3k+l)x}}{3k+l} - \frac{16a^{(k+3l)x}}{k+3l}}{4 \log(a)}$$

Antiderivative was successfully verified.

`[In] Integrate[(a^(k*x) - a^(l*x))^4, x]`

```
[Out] (a^(4*k*x)/k + a^(4*l*x)/l + (12*a^(2*(k + l)*x))/(k + l) - (16*a^((3*k + 1)*x))/(3*k + 1) - (16*a^((k + 3*l)*x))/(k + 3*l))/(4*Log[a])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 23.88, size = 1209, normalized size = 12.34

result too large to display

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(a^(k*x) - a^(l*x))^4, x]')`

```
[Out] Piecewise[{{0, a == 1 && a == 1 || k == 0 && a == 1 || l == 0}, {(1 x Log[a] - 4 a^(1 x) - 4 a^(3 l x) / 3 + a^(4 l x) / 4 + 3 a^(2 l x)) / (1 Log[a]), k == 0}, {(-1 - 48 l x Log[a] a^(12 l x) - 18 a^(8 l x) + 3 a^(16 l x) + 6 a^(4 l x)) a^(-12 l x) / (12 l Log[a]), k == -3 l}, {(-1 + 24 l x Log[a] a^(4 l x) + 8 a^(2 l x) + (-8 + a^(2 l x)) a^(6 l x)) a^(-4 l x) / (4 l Log[a]), k == -1}, {(-3 - 16 l x Log[a] a^(4 l x / 3) + a^(16 l x / 3) - 6 a^(4 l x) + 18 a^(8 l x / 3)) a^(-4 l x / 3) / (4 l Log[a]), l == -3 k}, {(k x Log[a] - 4 a^(k x) - 4 a^(3 k x) / 3 + a^(4 k x) / 4 + 3 a^(2 k x)) / (k Log[a]), l == 0}}, 3 k^4 a^(4 l x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a]) - 48 k^3 l a^(k x) a^(3 l x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a]) - 16 k^3 l a^(3 k x) a^(l x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a]) + 3 k^3 l a^(4 k x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a]) + 13 k^3 l a^(4 l x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a]) + 36 k^3 l a^(2 k x) a^(2 l x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a]) - 64 k^2 l^2 a^(3 k x) a^(l x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a]) - 64 k^2 l^2 a^(k x) a^(3 l x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a]) + 13 k^2 l^2 a^(4 k x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a]) + 13 k^2 l^2 a^(4 l x) / (12 k^4 l Log[a] + 52 k^3 l^2 Log[a] + 52 k^2 l^3 Log[a] + 12 k l^4 Log[a])
```


+ 120 k ^ 2 l ^ 2 a ^ (2 k x) a ^ (2 l x) / (12 k ^ 4 l Log[a] + 52 k ^ 3 l ^ 2 Log[a] + 52 k ^ 2 l ^ 3 Log[a] + 12 k l ^ 4 Log[a]) - 48 k l ^ 3 a ^ (3 k x) a ^ (l x) / (12 k ^ 4 l Log[a] + 52 k ^ 3 l ^ 2 Log[a] + 52 k ^ 2 l ^ 3 Log[a] + 12 k l ^ 4 Log[a]) - 16 k l ^ 3 a ^ (k x) a ^ (3 l x) / (12 k ^ 4 l Log[a] + 52 k ^ 3 l ^ 2 Log[a] + 52 k ^ 2 l ^ 3 Log[a] + 12 k l ^ 4 Log[a]) + 3 k l ^ 3 a ^ (4 l x) / (12 k ^ 4 l Log[a] + 52 k ^ 3 l ^ 2 Log[a] + 52 k ^ 2 l ^ 3 Log[a] + 12 k l ^ 4 Log[a]) + 52 k ^ 2 l ^ 3 Log[a] + 12 k l ^ 4 Log[a]) + 13 k l ^ 3 a ^ (4 k x) / (12 k ^ 4 l Log[a] + 52 k ^ 3 l ^ 2 Log[a] + 52 k ^ 2 l ^ 3 Log[a] + 12 k l ^ 4 Log[a]) + 36 k l ^ 3 a ^ (2 k x) a ^ (2 l x) / (12 k ^ 4 l Log[a] + 52 k ^ 3 l ^ 2 Log[a] + 52 k ^ 2 l ^ 3 Log[a] + 12 k l ^ 4 Log[a]) + 3 l ^ 4 a ^ (4 k x) / (12 k ^ 4 l Log[a] + 52 k ^ 3 l ^ 2 Log[a] + 52 k ^ 2 l ^ 3 Log[a] + 12 k l ^ 4 Log[a])]

Maple [A]

time = 0.03, size = 109, normalized size = 1.11

method	result
risch	$\frac{a^{4kx}}{4k \ln(a)} - \frac{4a^{3kx}a^{lx}}{\ln(a)(3k+l)} + \frac{3a^{2kx}a^{2lx}}{\ln(a)(k+l)} - \frac{4a^{kx}a^{3lx}}{\ln(a)(k+3l)} + \frac{a^{4lx}}{4l \ln(a)}$
meijerg	$-\frac{1-e^{4kx \ln(a)}}{4k \ln(a)} + \frac{4-4e^{xl \ln(a)(1+\frac{k}{l})} \left(1+\frac{2k}{l(1+\frac{k}{l})}\right)}{l \ln(a)(1+\frac{k}{l}) \left(1+\frac{2k}{l(1+\frac{k}{l})}\right)} - \frac{3 \left(1-e^{2xl \ln(a)(1+\frac{k}{l})}\right)}{l \ln(a)(1+\frac{k}{l})} + \frac{4-4e^{xl \ln(a)(1+\frac{k}{l})} \left(1+\frac{2}{1+\frac{k}{l}}\right)}{l \ln(a)(1+\frac{k}{l}) \left(1+\frac{2}{1+\frac{k}{l}}\right)} - \frac{1-e^{4lx \ln(a)}}{4l \ln(a)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(k*x)-a^(l*x))^4,x,method=_RETURNVERBOSE)

[Out] 1/4/ln(a)/k*(a^(k*x))^4-4*(a^(k*x))^3/ln(a)/(3*k+1)*a^(l*x)+3*(a^(k*x))^2/ln(a)/(k+1)*(a^(l*x))^2-4*a^(k*x)/ln(a)/(k+3*1)*(a^(l*x))^3+1/4/ln(a)/l*(a^(l*x))^4

Maxima [A]

time = 0.25, size = 99, normalized size = 1.01

$$-\frac{4a^{3kx+lx}}{(3k+l)\log(a)} - \frac{4a^{kx+3lx}}{(k+3l)\log(a)} + \frac{3a^{2kx+2lx}}{(k+l)\log(a)} + \frac{a^{4kx}}{4k\log(a)} + \frac{a^{4lx}}{4l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(k*x)-a^(l*x))^4,x, algorithm="maxima")

[Out] -4*a^(3*k*x + l*x)/((3*k + 1)*log(a)) - 4*a^(k*x + 3*l*x)/((k + 3*1)*log(a)) + 3*a^(2*k*x + 2*l*x)/((k + 1)*log(a)) + 1/4*a^(4*k*x)/(k*log(a)) + 1/4*a^(4*l*x)/(l*log(a))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(94) = 188.

time = 0.31, size = 207, normalized size = 2.11

$$\frac{16(3k^3l + 4k^2l^2 + kl^3)a^{kx}a^{3lx} - 12(3k^3l + 10k^2l^2 + 3kl^3)a^{2kx}a^{2lx} + 16(k^3l + 4k^2l^2 + 3kl^3)a^{kx}a^{lx} - (3k^3l + 13k^2l^2 + 13kl^3 + 3l^4)a^{4kx} - (3k^4 + 13k^3l + 13k^2l^2 + 3kl^3)a^{4lx}}{4(3k^4l + 13k^3l^2 + 13k^2l^3 + 3kl^4)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(k*x)-a^(l*x))^4,x, algorithm="fricas")

[Out]
$$-1/4*(16*(3*k^3*1 + 4*k^2*1^2 + k*1^3)*a^(k*x)*a^(3*1*x) - 12*(3*k^3*1 + 10*k^2*1^2 + 3*k*1^3)*a^(2*k*x)*a^(2*1*x) + 16*(k^3*1 + 4*k^2*1^2 + 3*k*1^3)*a^(3*k*x)*a^(1*x) - (3*k^3*1 + 13*k^2*1^2 + 13*k*1^3 + 3*1^4)*a^(4*k*x) - (3*k^4 + 13*k^3*1 + 13*k^2*1^2 + 3*k*1^3)*a^(4*1*x))/((3*k^4*1 + 13*k^3*1^2 + 13*k^2*1^3 + 3*k*1^4)*\log(a))$$

Sympy [A]

time = 21.99, size = 1348, normalized size = 13.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**(k*x)-a**(l*x))**4,x)

[Out] Piecewise((0, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(1, 0))), (a**(4*1*x)/(4*1*log(a)) - 4*a**(3*1*x)/(3*1*log(a)) + 3*a**(2*1*x)/(1*log(a)) - 4*a**(1*x)/(1*log(a)) + x, Eq(k, 0)), (a**(4*1*x)/(4*1*log(a)) - 4*x - 3/(2*a**(4*1*x)*1*log(a)) + 1/(2*a**(8*1*x)*1*log(a)) - 1/(12*a**(12*1*x)*1*log(a)), Eq(k, -3*1)), (a**(4*1*x)/(4*1*log(a)) - 2*a**(2*1*x)/(1*log(a)) + 6*x + 2/(a**(2*1*x)*1*log(a)) - 1/(4*a**(4*1*x)*1*log(a)), Eq(k, -1)), (-3*a**(8*1*x/3)/(2*1*log(a)) + 9*a**(4*1*x/3)/(2*1*log(a)) + a**(4*1*x)/(4*1*log(a)) - 4*x - 3/(4*a**(4*1*x/3)*1*log(a)), Eq(k, -1/3)), (a**(4*k*x)/(4*k*log(a)) - 4*a**(3*k*x)/(3*k*log(a)) + 3*a**(2*k*x)/(k*log(a)) - 4*a**(k*x)/(k*log(a)) + x, Eq(1, 0)), (3*a**(4*k*x)*k**3*1/(12*k**4*1*log(a) + 52*k**3*1**2*log(a) + 52*k**2*1**3*log(a) + 12*k*1**4*log(a)) + 13*a**(4*k*x)*k**2*1**2/(12*k**4*1*log(a) + 52*k**3*1**2*log(a) + 52*k**2*1**3*log(a) + 12*k*1**4*log(a)) + 13*a**(4*k*x)*k*1**3/(12*k**4*1*log(a) + 52*k**3*1**2*log(a) + 52*k**2*1**3*log(a) + 12*k*1**4*log(a)) + 3*a**(4*k*x)*1**4/(12*k**4*1*log(a) + 52*k**3*1**2*log(a) + 52*k**2*1**3*log(a) + 12*k*1**4*log(a)) - 16*a**(3*k*x)*a**(1*x)*k**3*1/(12*k**4*1*log(a) + 52*k**3*1**2*log(a) + 52*k**2*1**3*log(a) + 12*k*1**4*log(a)) - 64*a**(3*k*x)*a**(1*x)*k**2*1**2/(12*k**4*1*log(a) + 52*k**3*1**2*log(a) + 52*k**2*1**3*log(a) + 12*k*1**4*log(a)) - 48*a**(3*k*x)*a**(1*x)*k*1**3/(12*k**4*1*log(a) + 52*k**3*1**2*log(a) + 52*k**2*1**3*log(a) + 12*k*1**4*log(a)) + 36*a**(2*k*x)*a**(2*1*x)*k**3*1/(12*k**4*1*log(a) + 52*k**3*1**2*log(a) + 52*k**2*1**3*log(a) + 12*k*1**4*log(a)) + 120*a**(2*k*x)*a**(2*1*x)*k**2*1**2/(12*k**4*1*log(a) + 52*k**3*1**2*log(a) + 52*k**2*1**3*log(a) + 12*k*1**4*log(a)) + 36*a**(2*k*x)*a**(2*1*x)*k*1**3/(12*k**4*1*log(a) + 52*k**3*1**2*log(a) + 52*k**2*1**3*log(a) + 12*k*1**4*log(a)) - 48*a**(k*x)*a**(3*1*x)*k**3*1/(12*k**4*1*log(a) + 52*k

```

**3*1**2*log(a) + 52*k**2*1**3*log(a) + 12*k*1**4*log(a)) - 64*a**(k*x)*a**
(3*1*x)*k**2*1**2/(12*k**4*1*log(a) + 52*k**3*1**2*log(a) + 52*k**2*1**3*lo
g(a) + 12*k*1**4*log(a)) - 16*a**(k*x)*a**(3*1*x)*k*1**3/(12*k**4*1*log(a)
+ 52*k**3*1**2*log(a) + 52*k**2*1**3*log(a) + 12*k*1**4*log(a)) + 3*a**(4*1
*x)*k**4/(12*k**4*1*log(a) + 52*k**3*1**2*log(a) + 52*k**2*1**3*log(a) + 12
*k*1**4*log(a)) + 13*a**(4*1*x)*k**3*1/(12*k**4*1*log(a) + 52*k**3*1**2*log
(a) + 52*k**2*1**3*log(a) + 12*k*1**4*log(a)) + 13*a**(4*1*x)*k**2*1**2/(12
*k**4*1*log(a) + 52*k**3*1**2*log(a) + 52*k**2*1**3*log(a) + 12*k*1**4*log(
a)) + 3*a**(4*1*x)*k*1**3/(12*k**4*1*log(a) + 52*k**3*1**2*log(a) + 52*k**2
*1**3*log(a) + 12*k*1**4*log(a)), True))

```

Giac [C] Result contains complex when optimal does not.

time = 0.02, size = 1361, normalized size = 13.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(k*x)-a^(l*x))^4,x)

```

[Out] 1/2*(2*k*cos(-2*pi*k*x*sgn(a) + 2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2
+ (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-2*pi*k*x*sgn(a) + 2*p
i*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(4*k*x) + 1/2
*(2*1*cos(-2*pi*1*x*sgn(a) + 2*pi*1*x)*log(abs(a))/(4*1^2*log(abs(a))^2 + (
pi*1*sgn(a) - pi*1)^2) - (pi*1*sgn(a) - pi*1)*sin(-2*pi*1*x*sgn(a) + 2*pi*1
*x)/(4*1^2*log(abs(a))^2 + (pi*1*sgn(a) - pi*1)^2))*abs(a)^(4*1*x) - 8*e^((
k*log(abs(a)) + 3*1*log(abs(a)))*x)*(2*(k*log(abs(a)) + 3*1*log(abs(a)))*co
s(-1/2*pi*k*x*sgn(a) - 3/2*pi*1*x*sgn(a) + 1/2*pi*k*x + 3/2*pi*1*x)/((pi*k*
sgn(a) + 3*pi*1*sgn(a) - pi*k - 3*pi*1)^2 + 4*(k*log(abs(a)) + 3*1*log(abs(
a)))^2) - (pi*k*sgn(a) + 3*pi*1*sgn(a) - pi*k - 3*pi*1)*sin(-1/2*pi*k*x*sgn
(a) - 3/2*pi*1*x*sgn(a) + 1/2*pi*k*x + 3/2*pi*1*x)/((pi*k*sgn(a) + 3*pi*1*s
gn(a) - pi*k - 3*pi*1)^2 + 4*(k*log(abs(a)) + 3*1*log(abs(a)))^2)) + 6*e^(2
*(k*log(abs(a)) + 1*log(abs(a)))*x)*(2*(k*log(abs(a)) + 1*log(abs(a)))*cos(
-pi*k*x*sgn(a) - pi*1*x*sgn(a) + pi*k*x + pi*1*x)/((pi*k*sgn(a) + pi*1*sgn(
a) - pi*k - pi*1)^2 + 4*(k*log(abs(a)) + 1*log(abs(a)))^2) - (pi*k*sgn(a) +
pi*1*sgn(a) - pi*k - pi*1)*sin(-pi*k*x*sgn(a) - pi*1*x*sgn(a) + pi*k*x + p
i*1*x)/((pi*k*sgn(a) + pi*1*sgn(a) - pi*k - pi*1)^2 + 4*(k*log(abs(a)) + 1*
log(abs(a)))^2)) - 8*e^((3*k*log(abs(a)) + 1*log(abs(a)))*x)*(2*(3*k*log(ab
s(a)) + 1*log(abs(a)))*cos(-3/2*pi*k*x*sgn(a) - 1/2*pi*1*x*sgn(a) + 3/2*pi*
k*x + 1/2*pi*1*x)/((3*pi*k*sgn(a) + pi*1*sgn(a) - 3*pi*k - pi*1)^2 + 4*(3*k
*log(abs(a)) + 1*log(abs(a)))^2) - (3*pi*k*sgn(a) + pi*1*sgn(a) - 3*pi*k -
pi*1)*sin(-3/2*pi*k*x*sgn(a) - 1/2*pi*1*x*sgn(a) + 3/2*pi*k*x + 1/2*pi*1*x)
/((3*pi*k*sgn(a) + pi*1*sgn(a) - 3*pi*k - pi*1)^2 + 4*(3*k*log(abs(a)) + 1*
log(abs(a)))^2)) - 1/2*I*abs(a)^(4*k*x)*(-I*e^(2*I*pi*k*x*sgn(a) - 2*I*pi*k
*x)/(2*I*pi*k*sgn(a) - 2*I*pi*k + 4*k*log(abs(a))) + I*e^(-2*I*pi*k*x*sgn(a)
) + 2*I*pi*k*x)/(-2*I*pi*k*sgn(a) + 2*I*pi*k + 4*k*log(abs(a)))) + 4*e^((3*
k*log(abs(a)) + 1*log(abs(a)))*x)*(e^(3/2*I*pi*k*x*sgn(a) + 1/2*I*pi*1*x*sg

```

$$\begin{aligned} & n(a) - 3/2 * I * \pi * k * x - 1/2 * I * \pi * l * x) / (3 * I * \pi * k * \operatorname{sgn}(a) + I * \pi * l * \operatorname{sgn}(a) - 3 * I * \\ & \pi * k - I * \pi * l + 6 * k * \log(\operatorname{abs}(a)) + 2 * l * \log(\operatorname{abs}(a))) - e^{(-3/2 * I * \pi * k * x * \operatorname{sgn}(a) \\ &) - 1/2 * I * \pi * l * x * \operatorname{sgn}(a) + 3/2 * I * \pi * k * x + 1/2 * I * \pi * l * x) / (-3 * I * \pi * k * \operatorname{sgn}(a) - \\ & I * \pi * l * \operatorname{sgn}(a) + 3 * I * \pi * k + I * \pi * l + 6 * k * \log(\operatorname{abs}(a)) + 2 * l * \log(\operatorname{abs}(a)))} - 3 \\ & * e^{(2 * (k * \log(\operatorname{abs}(a)) + l * \log(\operatorname{abs}(a))) * x) * (e^{(I * \pi * k * x * \operatorname{sgn}(a) + I * \pi * l * x * \operatorname{sgn}(\\ & a) - I * \pi * k * x - I * \pi * l * x) / (I * \pi * k * \operatorname{sgn}(a) + I * \pi * l * \operatorname{sgn}(a) - I * \pi * k - I * \pi * l \\ & + 2 * k * \log(\operatorname{abs}(a)) + 2 * l * \log(\operatorname{abs}(a)))} - e^{(-I * \pi * k * x * \operatorname{sgn}(a) - I * \pi * l * x * \operatorname{sgn}(\\ & a) + I * \pi * k * x + I * \pi * l * x) / (-I * \pi * k * \operatorname{sgn}(a) - I * \pi * l * \operatorname{sgn}(a) + I * \pi * k + I * \pi * l \\ & + 2 * k * \log(\operatorname{abs}(a)) + 2 * l * \log(\operatorname{abs}(a)))} + 4 * e^{((k * \log(\operatorname{abs}(a)) + 3 * l * \log(\operatorname{abs}(\\ & a))) * x) * (e^{(1/2 * I * \pi * k * x * \operatorname{sgn}(a) + 3/2 * I * \pi * l * x * \operatorname{sgn}(a) - 1/2 * I * \pi * k * x - 3/2 * \\ & I * \pi * l * x) / (I * \pi * k * \operatorname{sgn}(a) + 3 * I * \pi * l * \operatorname{sgn}(a) - I * \pi * k - 3 * I * \pi * l + 2 * k * \log(\operatorname{abs}(\\ & a) + 6 * l * \log(\operatorname{abs}(a)))} - e^{(-1/2 * I * \pi * k * x * \operatorname{sgn}(a) - 3/2 * I * \pi * l * x * \operatorname{sgn}(a) + \\ & 1/2 * I * \pi * k * x + 3/2 * I * \pi * l * x) / (-I * \pi * k * \operatorname{sgn}(a) - 3 * I * \pi * l * \operatorname{sgn}(a) + I * \pi * k + 3 \\ & * I * \pi * l + 2 * k * \log(\operatorname{abs}(a)) + 6 * l * \log(\operatorname{abs}(a)))} - 1/2 * I * \operatorname{abs}(a)^{(4 * l * x) * (-I * e^{ \\ & (2 * I * \pi * l * x * \operatorname{sgn}(a) - 2 * I * \pi * l * x) / (2 * I * \pi * l * \operatorname{sgn}(a) - 2 * I * \pi * l + 4 * l * \log(\operatorname{abs}(\\ & a)))} + I * e^{(-2 * I * \pi * l * x * \operatorname{sgn}(a) + 2 * I * \pi * l * x) / (-2 * I * \pi * l * \operatorname{sgn}(a) + 2 * I * \pi * l + \\ & 4 * l * \log(\operatorname{abs}(a)))} \end{aligned}$$

Mupad [B]

time = 0.35, size = 106, normalized size = 1.08

$$\frac{3 a^{2 k x} a^{2 l x}}{k \ln (a)+l \ln (a)}-\frac{4 a^{k x} a^{3 l x}}{k \ln (a)+3 l \ln (a)}-\frac{4 a^{3 k x} a^{l x}}{3 k \ln (a)+l \ln (a)}+\frac{a^{4 k x}}{4 k \ln (a)}+\frac{a^{4 l x}}{4 l \ln (a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(k*x) - a^(l*x))^4,x)

[Out] (3*a^(2*k*x)*a^(2*l*x))/(k*log(a) + l*log(a)) - (4*a^(k*x)*a^(3*l*x))/(k*log(a) + 3*l*log(a)) - (4*a^(3*k*x)*a^(l*x))/(3*k*log(a) + l*log(a)) + a^(4*k*x)/(4*k*log(a)) + a^(4*l*x)/(4*l*log(a))

3.511 $\int (a^{kx} - a^{lx})^n dx$

Optimal. Leaf size=74

$$\frac{(1 - a^{(k-l)x}) (a^{kx} - a^{lx})^n {}_2F_1\left(1, 1 + \frac{kn}{k-l}; 1 + \frac{ln}{k-l}; a^{(k-l)x}\right)}{ln \log(a)}$$

[Out] (1-a^((k-1)*x))*(a^(k*x)-a^(l*x))^n*hypergeom([1, 1+k*n/(k-1)], [1+l*n/(k-1)], a^((k-1)*x))/1/n/ln(a)

Rubi [A]

time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$,

Rules used = {2323, 2283}

$$\frac{(1 - a^{-(x(k-l))})^{-n} (a^{kx} - a^{lx})^n {}_2F_1\left(-n, -\frac{kn}{k-l}; 1 - \frac{kn}{k-l}; a^{-((k-l)x)}\right)}{kn \log(a)}$$

Antiderivative was successfully verified.

[In] Int[(a^(k*x) - a^(l*x))^n, x]

[Out] ((a^(k*x) - a^(l*x))^n*Hypergeometric2F1[-n, -((k*n)/(k - 1)), 1 - (k*n)/(k - 1), a^(-((k - 1)*x))])/((1 - a^(-((k - 1)*x)))^n*k*n*Log[a])

Rule 2283

Int[((a_) + (b_)*(F_)^(e_)*((c_) + (d_)*(x_)))^(p_)*(G_)^(h_)*((f_ + (g_)*(x_)), x_Symbol] := Simp[a^p*(G^(h*(f + g*x)))/(g*h*Log[G])*Hypergeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1, Simplify[(-b/a)*F^(e*(c + d*x))], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 2323

Int[(u_)*((a_)*(F_)^(v_) + (b_)*(F_)^(w_))^(n_), x_Symbol] := Dist[(a*F^v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && !IntegerQ[n] && LinearQ[{v, w}, x]

Rubi steps

$$\begin{aligned} \int (a^{kx} - a^{lx})^n dx &= \left(a^{-knx} (1 - a^{-(k-l)x})^{-n} (a^{kx} - a^{lx})^n \right) \int a^{knx} (1 - a^{-(k-l)x})^n dx \\ &= \frac{(1 - a^{-(k-l)x})^{-n} (a^{kx} - a^{lx})^n {}_2F_1\left(-n, -\frac{kn}{k-l}; 1 - \frac{kn}{k-l}; a^{-(k-l)x}\right)}{kn \log(a)} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 75, normalized size = 1.01

$$\frac{(a^{kx} - a^{lx})^n (1 - a^{(-k+l)x}) {}_2F_1\left(1, 1 + n + \frac{kn}{-k+l}; 1 + \frac{kn}{-k+l}; a^{(-k+l)x}\right)}{kn \log(a)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^(k*x) - a^(l*x))^n, x]

[Out] ((a^(k*x) - a^(l*x))^n*(1 - a^((-k + 1)*x))*Hypergeometric2F1[1, 1 + n + (k*n)/(-k + 1), 1 + (k*n)/(-k + 1), a^((-k + 1)*x)])/(k*n*Log[a])

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a^(k*x) - a^(l*x))^n, x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [F]

time = 0.04, size = 0, normalized size = 0.00

$$\int (a^{kx} - a^{lx})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(k*x)-a^(l*x))^n, x)

[Out] int((a^(k*x)-a^(l*x))^n, x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(k*x)-a^(l*x))^n, x, algorithm="maxima")

[Out] integrate((a^(k*x) - a^(l*x))^n, x)

Fricas [F]

time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^(k*x)-a^(l*x))^n,x, algorithm="fricas")`

[Out] `integral((a^(k*x) - a^(l*x))^n, x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a^{kx} - a^{lx})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a**(k*x)-a**(l*x))**n,x)`

[Out] `Integral((a**(k*x) - a**(l*x))**n, x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^(k*x)-a^(l*x))^n,x)`

[Out] `Could not integrate`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a^{kx} - a^{lx})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^(k*x) - a^(l*x))^n,x)`

[Out] `int((a^(k*x) - a^(l*x))^n, x)`

3.512 $\int (1 + a^{mx}) dx$

Optimal. Leaf size=15

$$x + \frac{a^{mx}}{m \log(a)}$$

[Out] $x + a^{(m*x)}/m/\ln(a)$

Rubi [A]

time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2225}

$$\frac{a^{mx}}{m \log(a)} + x$$

Antiderivative was successfully verified.

[In] `Int[1 + a^(m*x), x]`

[Out] `x + a^(m*x)/(m*Log[a])`

Rule 2225

`Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]`

Rubi steps

$$\begin{aligned} \int (1 + a^{mx}) dx &= x + \int a^{mx} dx \\ &= x + \frac{a^{mx}}{m \log(a)} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 15, normalized size = 1.00

$$x + \frac{a^{mx}}{m \log(a)}$$

Antiderivative was successfully verified.

[In] `Integrate[1 + a^(m*x), x]`

[Out] `x + a^(m*x)/(m*Log[a])`

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.90, size = 29, normalized size = 1.93

$$\text{Piecewise} \left[\left\{ \left\{ \frac{a^{mx}}{m \text{Log}[a]} + x, m \text{Log}[a] \neq 0 \right\}, \{2x, \text{True}\} \right\} \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(1 + a^(m*x))^1,x]')`

[Out] `Piecewise[{{a ^ (m x) / (m Log[a]) + x, m Log[a] != 0}, {2 x, True}}]`

Maple [A]

time = 0.02, size = 16, normalized size = 1.07

method	result	size
default	$x + \frac{a^{mx}}{m \ln(a)}$	16
risch	$x + \frac{a^{mx}}{m \ln(a)}$	16
norman	$x + \frac{e^{mx \ln(a)}}{m \ln(a)}$	17
derivativedivides	$\frac{a^{mx} + \ln(a^{mx})}{m \ln(a)}$	21

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1+a^(m*x),x,method=_RETURNVERBOSE)`

[Out] `x+a^(m*x)/m/ln(a)`

Maxima [A]

time = 0.24, size = 15, normalized size = 1.00

$$x + \frac{a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1+a^(m*x),x, algorithm="maxima")`

[Out] `x + a^(m*x)/(m*log(a))`

Fricas [A]

time = 0.32, size = 19, normalized size = 1.27

$$\frac{mx \log(a) + a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1+a^(m*x),x, algorithm="fricas")

[Out] (m*x*log(a) + a^(m*x))/(m*log(a))

Sympy [A]

time = 0.05, size = 15, normalized size = 1.00

$$x + \begin{cases} \frac{a^{mx}}{m \log(a)} & \text{for } m \log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1+a**(m*x),x)

[Out] x + Piecewise((a**(m*x)/(m*log(a)), Ne(m*log(a), 0)), (x, True))

Giac [A]

time = 0.00, size = 13, normalized size = 0.87

$$x + \frac{a^{mx}}{m \ln a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1+a^(m*x),x)

[Out] x + a^(m*x)/(m*log(a))

Mupad [B]

time = 0.32, size = 15, normalized size = 1.00

$$x + \frac{a^{mx}}{m \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a^(m*x) + 1,x)

[Out] x + a^(m*x)/(m*log(a))

3.513 $\int (1 + a^{mx})^2 dx$

Optimal. Leaf size=33

$$x + \frac{2a^{mx}}{m \log(a)} + \frac{a^{2mx}}{2m \log(a)}$$

[Out] $x + 2*a^{(m*x)}/m/\ln(a) + 1/2*a^{(2*m*x)}/m/\ln(a)$

Rubi [A]

time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2320, 45}

$$\frac{2a^{mx}}{m \log(a)} + \frac{a^{2mx}}{2m \log(a)} + x$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + a^{(m*x)})^2, x]$

[Out] $x + (2*a^{(m*x)})/(m*\text{Log}[a]) + a^{(2*m*x)}/(2*m*\text{Log}[a])$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2320

$\text{Int}[u_, x_Symbol] := \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /;$ FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_.)*((a_.)*(v_.))^{(n_.)}^{(m_.)} /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^{((c_.)*((a_.) + (b_.)*x))* (F_.)[v_.]} /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\begin{aligned} \int (1 + a^{mx})^2 dx &= \frac{\text{Subst} \left(\int \frac{(1+x)^2}{x} dx, x, a^{mx} \right)}{m \log(a)} \\ &= \frac{\text{Subst} \left(\int \left(2 + \frac{1}{x} + x \right) dx, x, a^{mx} \right)}{m \log(a)} \\ &= x + \frac{2a^{mx}}{m \log(a)} + \frac{a^{2mx}}{2m \log(a)} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 35, normalized size = 1.06

$$\frac{\frac{a^{mx}(4+a^{mx})}{2m} + \frac{\log(a^{mx})}{m}}{\log(a)}$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + a^(m*x))^2, x]``[Out] ((a^(m*x)*(4 + a^(m*x)))/(2*m) + Log[a^(m*x)]/m)/Log[a]`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.23, size = 46, normalized size = 1.39

$$\text{Piecewise} \left[\left\{ \left\{ \frac{mx \text{Log}[a] + \frac{(4+a^{mx})a^{mx}}{2}}{m \text{Log}[a]}, m^2 \text{Log}[a]^2 \neq 0 \right\}, \{4x, \text{True}\} \right\} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(1 + a^(m*x))^2, x]')``[Out] Piecewise[{{(m x Log[a] + (4 + a^(m x)) a^(m x) / 2) / (m Log[a]), m^2 Log[a]^2 != 0}, {4 x, True}}`**Maple [A]**

time = 0.02, size = 32, normalized size = 0.97

method	result	size
derivativedivides	$\frac{\frac{a^{2mx}}{2} + 2a^{mx} + \ln(a^{mx})}{m \ln(a)}$	32
default	$\frac{\frac{a^{2mx}}{2} + 2a^{mx} + \ln(a^{mx})}{m \ln(a)}$	32
risch	$x + \frac{2a^{mx}}{m \ln(a)} + \frac{a^{2mx}}{2m \ln(a)}$	33
norman	$x + \frac{2e^{mx \ln(a)}}{m \ln(a)} + \frac{e^{2mx \ln(a)}}{2m \ln(a)}$	35

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((1+a^(m*x))^2,x,method=_RETURNVERBOSE)``[Out] 1/m/ln(a)*(1/2*(a^(m*x))^2+2*a^(m*x)+ln(a^(m*x)))`**Maxima [A]**

time = 0.24, size = 31, normalized size = 0.94

$$x + \frac{a^{2mx}}{2m \log(a)} + \frac{2a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+a^(m*x))^2,x, algorithm="maxima")

[Out] x + 1/2*a^(2*m*x)/(m*log(a)) + 2*a^(m*x)/(m*log(a))

Fricas [A]

time = 0.31, size = 29, normalized size = 0.88

$$\frac{2mx \log(a) + a^{2mx} + 4a^{mx}}{2m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+a^(m*x))^2,x, algorithm="fricas")

[Out] 1/2*(2*m*x*log(a) + a^(2*m*x) + 4*a^(m*x))/(m*log(a))

Sympy [A]

time = 0.06, size = 44, normalized size = 1.33

$$x + \begin{cases} \frac{a^{2mx}m \log(a) + 4a^{mx}m \log(a)}{2m^2 \log(a)^2} & \text{for } m^2 \log(a)^2 \neq 0 \\ 3x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+a**(m*x))**2,x)

[Out] x + Piecewise(((a**(2*m*x))*m*log(a) + 4*a**(m*x)*m*log(a))/(2*m**2*log(a)**2), Ne(m**2*log(a)**2, 0)), (3*x, True))

Giac [A]

time = 0.00, size = 30, normalized size = 0.91

$$\frac{\frac{(a^{mx})^2}{2} + 2a^{mx} + mx \ln |a|}{m \ln a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+a^(m*x))^2,x)

[Out] 1/2*(2*m*x*log(abs(a)) + a^(2*m*x) + 4*a^(m*x))/(m*log(a))

Mupad [B]

time = 0.32, size = 26, normalized size = 0.79

$$x + \frac{2a^{mx} + \frac{a^{2mx}}{2}}{m \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(m*x) + 1)^2,x)

[Out] x + (2*a^(m*x) + a^(2*m*x)/2)/(m*log(a))

3.514 $\int (1 + a^{mx})^3 dx$

Optimal. Leaf size=50

$$x + \frac{3a^{mx}}{m \log(a)} + \frac{3a^{2mx}}{2m \log(a)} + \frac{a^{3mx}}{3m \log(a)}$$

[Out] $x + 3a^{(m*x)}/m/\ln(a) + 3/2*a^{(2*m*x)}/m/\ln(a) + 1/3*a^{(3*m*x)}/m/\ln(a)$

Rubi [A]

time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2320, 45}

$$\frac{3a^{mx}}{m \log(a)} + \frac{3a^{2mx}}{2m \log(a)} + \frac{a^{3mx}}{3m \log(a)} + x$$

Antiderivative was successfully verified.

[In] Int[(1 + a^(m*x))^3,x]

[Out] $x + (3a^{(m*x)})/(m*\text{Log}[a]) + (3a^{(2*m*x)})/(2*m*\text{Log}[a]) + a^{(3*m*x)}/(3*m*\text{Log}[a])$

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned} \int (1 + a^{mx})^3 dx &= \frac{\text{Subst} \left(\int \frac{(1+x)^3}{x} dx, x, a^{mx} \right)}{m \log(a)} \\ &= \frac{\text{Subst} \left(\int \left(3 + \frac{1}{x} + 3x + x^2 \right) dx, x, a^{mx} \right)}{m \log(a)} \\ &= x + \frac{3a^{mx}}{m \log(a)} + \frac{3a^{2mx}}{2m \log(a)} + \frac{a^{3mx}}{3m \log(a)} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 45, normalized size = 0.90

$$\frac{\frac{a^{mx}(18+9a^{mx}+2a^{2mx})}{6m} + \frac{\log(a^{mx})}{m}}{\log(a)}$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + a^(m*x))^3, x]``[Out] ((a^(m*x)*(18 + 9*a^(m*x) + 2*a^(2*m*x)))/(6*m) + Log[a^(m*x)]/m)/Log[a]`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.59, size = 56, normalized size = 1.12

$$\text{Piecewise} \left[\left\{ \left\{ \frac{mx \text{Log}[a] + \frac{(18+2a^{2mx}+9a^{mx})a^{mx}}{6}}{m \text{Log}[a]}, m^3 \text{Log}[a]^3 \neq 0 \right\}, \{8x, \text{True}\} \right\} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(1 + a^(m*x))^3, x]')``[Out] Piecewise[{{(m x Log[a] + (18 + 2 a ^ (2 m x) + 9 a ^ (m x)) a ^ (m x) / 6) / (m Log[a]), m ^ 3 Log[a] ^ 3 != 0}, {8 x, True}}]`**Maple [A]**

time = 0.02, size = 41, normalized size = 0.82

method	result	size
derivativedivides	$\frac{\frac{a^{3mx}}{3} + \frac{3a^{2mx}}{2} + 3a^{mx} + \ln(a^{mx})}{m \ln(a)}$	41
default	$\frac{\frac{a^{3mx}}{3} + \frac{3a^{2mx}}{2} + 3a^{mx} + \ln(a^{mx})}{m \ln(a)}$	41
risch	$x + \frac{3a^{mx}}{m \ln(a)} + \frac{3a^{2mx}}{2m \ln(a)} + \frac{a^{3mx}}{3m \ln(a)}$	49

norman	$x + \frac{3e^{mx \ln(a)}}{m \ln(a)} + \frac{3e^{2mx \ln(a)}}{2m \ln(a)} + \frac{e^{3mx \ln(a)}}{3m \ln(a)}$	52
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+a^(m*x))^3,x,method=_RETURNVERBOSE)`

[Out] `1/m/ln(a)*(1/3*(a^(m*x))^3+3/2*(a^(m*x))^2+3*a^(m*x)+ln(a^(m*x)))`

Maxima [A]

time = 0.24, size = 46, normalized size = 0.92

$$x + \frac{a^{3mx}}{3m \log(a)} + \frac{3a^{2mx}}{2m \log(a)} + \frac{3a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+a^(m*x))^3,x, algorithm="maxima")`

[Out] `x + 1/3*a^(3*m*x)/(m*log(a)) + 3/2*a^(2*m*x)/(m*log(a)) + 3*a^(m*x)/(m*log(a))`

Fricas [A]

time = 0.31, size = 39, normalized size = 0.78

$$\frac{6mx \log(a) + 2a^{3mx} + 9a^{2mx} + 18a^{mx}}{6m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+a^(m*x))^3,x, algorithm="fricas")`

[Out] `1/6*(6*m*x*log(a) + 2*a^(3*m*x) + 9*a^(2*m*x) + 18*a^(m*x))/(m*log(a))`

Sympy [A]

time = 0.07, size = 70, normalized size = 1.40

$$x + \begin{cases} \frac{2a^{3mx}m^2 \log(a)^2 + 9a^{2mx}m^2 \log(a)^2 + 18a^{mx}m^2 \log(a)^2}{6m^3 \log(a)^3} & \text{for } m^3 \log(a)^3 \neq 0 \\ 7x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+a**(m*x))**3,x)`

[Out] `x + Piecewise(((2*a**(3*m*x)*m**2*log(a)**2 + 9*a**(2*m*x)*m**2*log(a)**2 + 18*a**(m*x)*m**2*log(a)**2)/(6*m**3*log(a)**3), Ne(m**3*log(a)**3, 0)), (7*x, True))`

Giac [A]

time = 0.00, size = 41, normalized size = 0.82

$$\frac{\frac{(a^{mx})^3}{3} + \frac{3}{2}(a^{mx})^2 + 3a^{mx} + mx \ln |a|}{m \ln a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+a^(m*x))^3,x)**[Out]** 1/6*(6*m*x*log(abs(a)) + 2*a^(3*m*x) + 9*a^(2*m*x) + 18*a^(m*x))/(m*log(a))**Mupad [B]**

time = 0.33, size = 34, normalized size = 0.68

$$x + \frac{3a^{mx} + \frac{3a^{2mx}}{2} + \frac{a^{3mx}}{3}}{m \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(m*x) + 1)^3,x)**[Out]** x + (3*a^(m*x) + (3*a^(2*m*x))/2 + a^(3*m*x)/3)/(m*log(a))

3.515 $\int (1 + a^{mx})^4 dx$

Optimal. Leaf size=65

$$x + \frac{4a^{mx}}{m \log(a)} + \frac{3a^{2mx}}{m \log(a)} + \frac{4a^{3mx}}{3m \log(a)} + \frac{a^{4mx}}{4m \log(a)}$$

[Out] $x+4*a^{(m*x)}/m/\ln(a)+3*a^{(2*m*x)}/m/\ln(a)+4/3*a^{(3*m*x)}/m/\ln(a)+1/4*a^{(4*m*x)}/m/\ln(a)$

Rubi [A]

time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2320, 45}

$$\frac{4a^{mx}}{m \log(a)} + \frac{3a^{2mx}}{m \log(a)} + \frac{4a^{3mx}}{3m \log(a)} + \frac{a^{4mx}}{4m \log(a)} + x$$

Antiderivative was successfully verified.

[In] Int[(1 + a^(m*x))^4, x]

[Out] $x + (4*a^{(m*x)})/(m*\text{Log}[a]) + (3*a^{(2*m*x)})/(m*\text{Log}[a]) + (4*a^{(3*m*x)})/(3*m*\text{Log}[a]) + a^{(4*m*x)}/(4*m*\text{Log}[a])$

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2320

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\begin{aligned}
\int (1 + a^{mx})^4 dx &= \frac{\text{Subst} \left(\int \frac{(1+x)^4}{x} dx, x, a^{mx} \right)}{m \log(a)} \\
&= \frac{\text{Subst} \left(\int \left(4 + \frac{1}{x} + 6x + 4x^2 + x^3 \right) dx, x, a^{mx} \right)}{m \log(a)} \\
&= x + \frac{4a^{mx}}{m \log(a)} + \frac{3a^{2mx}}{m \log(a)} + \frac{4a^{3mx}}{3m \log(a)} + \frac{a^{4mx}}{4m \log(a)}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 53, normalized size = 0.82

$$\frac{\frac{a^{mx}(48+36a^{mx}+16a^{2mx}+3a^{3mx})}{12m} + \frac{\log(a^{mx})}{m}}{\log(a)}$$

Antiderivative was successfully verified.

`[In] Integrate[(1 + a^(m*x))^4, x]``[Out] ((a^(m*x)*(48 + 36*a^(m*x) + 16*a^(2*m*x) + 3*a^(3*m*x)))/(12*m) + Log[a^(m*x)])/m)/Log[a]`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.84, size = 64, normalized size = 0.98

$$\text{Piecewise} \left[\left\{ \left\{ \frac{mx \text{Log}[a] + \frac{(48+3a^{3mx}+16a^{2mx}+36a^{mx})a^{mx}}{12}}{m \text{Log}[a]}, m^4 \text{Log}[a]^4 \neq 0 \right\}, \{16x, \text{True}\} \right\} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(1 + a^(m*x))^4, x]')``[Out] Piecewise[{{(m x Log[a] + (48 + 3 a^(3 m x) + 16 a^(2 m x) + 36 a^(m x)) a^(m x) / 12) / (m Log[a]), m^4 Log[a]^4 != 0}, {16 x, True}}]`**Maple [A]**

time = 0.02, size = 50, normalized size = 0.77

method	result	size
derivativedivides	$\frac{\frac{a^{4mx}}{4} + \frac{4a^{3mx}}{3} + 3a^{2mx} + 4a^{mx} + \ln(a^{mx})}{m \ln(a)}$	50
default	$\frac{\frac{a^{4mx}}{4} + \frac{4a^{3mx}}{3} + 3a^{2mx} + 4a^{mx} + \ln(a^{mx})}{m \ln(a)}$	50

risch	$x + \frac{4a^{mx}}{m \ln(a)} + \frac{3a^{2mx}}{m \ln(a)} + \frac{4a^{3mx}}{3m \ln(a)} + \frac{a^{4mx}}{4m \ln(a)}$	65
norman	$x + \frac{4e^{mx \ln(a)}}{m \ln(a)} + \frac{3e^{2mx \ln(a)}}{m \ln(a)} + \frac{4e^{3mx \ln(a)}}{3m \ln(a)} + \frac{e^{4mx \ln(a)}}{4m \ln(a)}$	69

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+a^(m*x))^4,x,method=_RETURNVERBOSE)`

[Out] $1/m/\ln(a)*(1/4*(a^(m*x))^4+4/3*(a^(m*x))^3+3*(a^(m*x))^2+4*a^(m*x)+\ln(a^(m*x)))$

Maxima [A]

time = 0.24, size = 61, normalized size = 0.94

$$x + \frac{a^{4mx}}{4m \log(a)} + \frac{4a^{3mx}}{3m \log(a)} + \frac{3a^{2mx}}{m \log(a)} + \frac{4a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+a^(m*x))^4,x, algorithm="maxima")`

[Out] $x + 1/4*a^(4*m*x)/(m*log(a)) + 4/3*a^(3*m*x)/(m*log(a)) + 3*a^(2*m*x)/(m*log(a)) + 4*a^(m*x)/(m*log(a))$

Fricas [A]

time = 0.30, size = 47, normalized size = 0.72

$$\frac{12mx \log(a) + 3a^{4mx} + 16a^{3mx} + 36a^{2mx} + 48a^{mx}}{12m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+a^(m*x))^4,x, algorithm="fricas")`

[Out] $1/12*(12*m*x*\log(a) + 3*a^(4*m*x) + 16*a^(3*m*x) + 36*a^(2*m*x) + 48*a^(m*x))/m*\log(a)$

Sympy [A]

time = 0.08, size = 87, normalized size = 1.34

$$x + \begin{cases} \frac{3a^{4mx}m^3 \log(a)^3 + 16a^{3mx}m^3 \log(a)^3 + 36a^{2mx}m^3 \log(a)^3 + 48a^{mx}m^3 \log(a)^3}{12m^4 \log(a)^4} & \text{for } m^4 \log(a)^4 \neq 0 \\ 15x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+a**(m*x))**4,x)`

[Out] $x + \text{Piecewise}(((3*a**(4*m*x))*m**3*\log(a)**3 + 16*a**(3*m*x))*m**3*\log(a)**3 + 36*a**(2*m*x))*m**3*\log(a)**3 + 48*a**(m*x))*m**3*\log(a)**3)/(12*m**4*\log(a)**4), \text{Ne}(m**4*\log(a)**4, 0)), (15*x, \text{True}))$

Giac [A]

time = 0.00, size = 50, normalized size = 0.77

$$\frac{\frac{(a^{mx})^4}{4} + \frac{4}{3} (a^{mx})^3 + 3 (a^{mx})^2 + 4a^{mx} + mx \ln |a|}{m \ln a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+a^(m*x))^4,x)**[Out]** 1/12*(12*m*x*log(abs(a)) + 3*a^(4*m*x) + 16*a^(3*m*x) + 36*a^(2*m*x) + 48*a^(m*x))/(m*log(a))**Mupad [B]**

time = 0.33, size = 42, normalized size = 0.65

$$x + \frac{4a^{mx} + 3a^{2mx} + \frac{4a^{3mx}}{3} + \frac{a^{4mx}}{4}}{m \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(m*x) + 1)^4,x)**[Out]** x + (4*a^(m*x) + 3*a^(2*m*x) + (4*a^(3*m*x))/3 + a^(4*m*x)/4)/(m*log(a))

3.516 $\int (1 + a^{mx})^n dx$

Optimal. Leaf size=40

$$-\frac{(1 + a^{mx})^{1+n} {}_2F_1(1, 1 + n; 2 + n; 1 + a^{mx})}{m(1 + n) \log(a)}$$

[Out] $-(1+a^{(m*x)})^{(1+n)}*\text{hypergeom}([1, 1+n], [2+n], 1+a^{(m*x)})/m/(1+n)/\ln(a)$

Rubi [A]

time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2320, 67}

$$-\frac{(a^{mx} + 1)^{n+1} {}_2F_1(1, n + 1; n + 2; a^{mx} + 1)}{m(n + 1) \log(a)}$$

Antiderivative was successfully verified.

[In] Int[(1 + a^(m*x))^n, x]

[Out] $-\left(\left(1 + a^{(m*x)}\right)^{(1 + n)}*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, 1 + a^{(m*x)}]\right)/\left(m*(1 + n)*\text{Log}[a]\right)$

Rule 67

Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((c + d*x)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 + d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-d/(b*c), 0])

Rule 2320

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\begin{aligned} \int (1 + a^{mx})^n dx &= \frac{\text{Subst}\left(\int \frac{(1+x)^n}{x} dx, x, a^{mx}\right)}{m \log(a)} \\ &= -\frac{(1 + a^{mx})^{1+n} {}_2F_1(1, 1 + n; 2 + n; 1 + a^{mx})}{m(1 + n) \log(a)} \end{aligned}$$

Mathematica [A]

time = 0.04, size = 40, normalized size = 1.00

$$\frac{(1 + a^{mx})^{1+n} {}_2F_1(1, 1 + n; 2 + n; 1 + a^{mx})}{m(1 + n) \log(a)}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + a^(m*x))^n,x]

[Out] -(((1 + a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + a^(m*x)]))/(m*(1 + n)*Log[a])

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(1 + a^(m*x))^n,x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int (1 + a^{mx})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+a^(m*x))^n,x)

[Out] int((1+a^(m*x))^n,x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+a^(m*x))^n,x, algorithm="maxima")

[Out] integrate((a^(m*x) + 1)^n, x)

Fricas [F]

time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+a^(m*x))^n,x, algorithm="fricas")

[Out] integral((a^(m*x) + 1)^n, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a^{mx} + 1)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+a**(m*x))**n,x)

[Out] Integral((a**(m*x) + 1)**n, x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+a^(m*x))^n,x)

[Out] Could not integrate

Mupad [B]

time = 0.31, size = 55, normalized size = 1.38

$$\frac{(a^{mx} + 1)^n {}_2F_1\left(-n, -n; 1 - n; -\frac{1}{a^{mx}}\right)}{m n \ln(a) \left(\frac{1}{a^{mx}} + 1\right)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(m*x) + 1)^n,x)

[Out] ((a^(m*x) + 1)^n*hypergeom([-n, -n], 1 - n, -1/a^(m*x)))/(m*n*log(a)*(1/a^(m*x) + 1)^n)

3.517 $\int (1 - a^{mx}) dx$

Optimal. Leaf size=16

$$x - \frac{a^{mx}}{m \log(a)}$$

[Out] $x - a^{(m*x)}/m/\ln(a)$

Rubi [A]

time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2225}

$$x - \frac{a^{mx}}{m \log(a)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1 - a^{(m*x)}, x]$

[Out] $x - a^{(m*x)}/(m*\text{Log}[a])$

Rule 2225

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))^{(n_.)}, x_Symbol] :> \text{Simp}[(F^{(c*(a + b*x)))^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

Rubi steps

$$\begin{aligned} \int (1 - a^{mx}) dx &= x - \int a^{mx} dx \\ &= x - \frac{a^{mx}}{m \log(a)} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 16, normalized size = 1.00

$$x - \frac{a^{mx}}{m \log(a)}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[1 - a^{(m*x)}, x]$

[Out] $x - a^{(m*x)}/(m*\text{Log}[a])$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.90, size = 28, normalized size = 1.75

$$\text{Piecewise} \left[\left\{ \left\{ -\frac{a^{mx}}{m \text{Log}[a]} + x, m \text{Log}[a] \neq 0 \right\}, \{0, \text{True}\} \right\} \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(1 - a^(m*x))^1,x]')`

[Out] `Piecewise[{{-a ^ (m x) / (m Log[a]) + x, m Log[a] != 0}, {0, True}}]`

Maple [A]

time = 0.02, size = 17, normalized size = 1.06

method	result	size
default	$x - \frac{a^{mx}}{m \ln(a)}$	17
risch	$x - \frac{a^{mx}}{m \ln(a)}$	17
norman	$x - \frac{e^{mx \ln(a)}}{m \ln(a)}$	18
derivativedivides	$\frac{-a^{mx} + \ln(a^{mx})}{m \ln(a)}$	23

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1-a^(m*x),x,method=_RETURNVERBOSE)`

[Out] `x-a^(m*x)/m/ln(a)`

Maxima [A]

time = 0.24, size = 16, normalized size = 1.00

$$x - \frac{a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1-a^(m*x),x, algorithm="maxima")`

[Out] `x - a^(m*x)/(m*log(a))`

Fricas [A]

time = 0.33, size = 21, normalized size = 1.31

$$\frac{mx \log(a) - a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-a^(m*x),x, algorithm="fricas")

[Out] (m*x*log(a) - a^(m*x))/(m*log(a))

Sympy [A]

time = 0.05, size = 19, normalized size = 1.19

$$x + \begin{cases} -\frac{a^{mx}}{m \log(a)} & \text{for } m \log(a) \neq 0 \\ -x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-a**(m*x),x)

[Out] x + Piecewise((-a**(m*x)/(m*log(a)), Ne(m*log(a), 0)), (-x, True))

Giac [A]

time = 0.00, size = 14, normalized size = 0.88

$$x - \frac{a^{mx}}{m \ln a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-a^(m*x),x)

[Out] x - a^(m*x)/(m*log(a))

Mupad [B]

time = 0.30, size = 16, normalized size = 1.00

$$x - \frac{a^{mx}}{m \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1 - a^(m*x),x)

[Out] x - a^(m*x)/(m*log(a))

3.518 $\int (1 - a^{mx})^2 dx$

Optimal. Leaf size=33

$$x - \frac{2a^{mx}}{m \log(a)} + \frac{a^{2mx}}{2m \log(a)}$$

[Out] $x - 2*a^{(m*x)}/m/\ln(a) + 1/2*a^{(2*m*x)}/m/\ln(a)$

Rubi [A]

time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2320, 45}

$$-\frac{2a^{mx}}{m \log(a)} + \frac{a^{2mx}}{2m \log(a)} + x$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 - a^{(m*x)})^2, x]$

[Out] $x - (2*a^{(m*x)})/(m*\text{Log}[a]) + a^{(2*m*x)}/(2*m*\text{Log}[a])$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2320

$\text{Int}[u_, x_Symbol] \rightarrow \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /;$ FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\begin{aligned} \int (1 - a^{mx})^2 dx &= \frac{\text{Subst} \left(\int \frac{(1-x)^2}{x} dx, x, a^{mx} \right)}{m \log(a)} \\ &= \frac{\text{Subst} \left(\int \left(-2 + \frac{1}{x} + x \right) dx, x, a^{mx} \right)}{m \log(a)} \\ &= x - \frac{2a^{mx}}{m \log(a)} + \frac{a^{2mx}}{2m \log(a)} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 35, normalized size = 1.06

$$\frac{\frac{a^{mx}(-4+a^{mx})}{2m} + \frac{\log(a^{mx})}{m}}{\log(a)}$$

Antiderivative was successfully verified.

`[In] Integrate[(1 - a^(m*x))^2,x]``[Out] ((a^(m*x)*(-4 + a^(m*x)))/(2*m) + Log[a^(m*x)]/m)/Log[a]`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.23, size = 44, normalized size = 1.33

$$\text{Piecewise} \left[\left\{ \left\{ \frac{mx \text{Log}[a] + \frac{(-4+a^{mx})a^{mx}}{2}}{m \text{Log}[a]}, m^2 \text{Log}[a]^2 \neq 0 \right\}, \{0, \text{True}\} \right\} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(1 - a^(m*x))^2,x]')``[Out] Piecewise[{{(m x Log[a] + (-4 + a^(m x)) a^(m x) / 2) / (m Log[a]), m^2 Log[a]^2 != 0}, {0, True}}`**Maple [A]**

time = 0.02, size = 32, normalized size = 0.97

method	result	size
derivativedivides	$\frac{\frac{a^{2mx}}{2} - 2a^{mx} + \ln(a^{mx})}{m \ln(a)}}$	32
default	$\frac{\frac{a^{2mx}}{2} - 2a^{mx} + \ln(a^{mx})}{m \ln(a)}}$	32
risch	$x - \frac{2a^{mx}}{m \ln(a)} + \frac{a^{2mx}}{2m \ln(a)}$	33
norman	$x - \frac{2e^{mx \ln(a)}}{m \ln(a)} + \frac{e^{2mx \ln(a)}}{2m \ln(a)}$	35

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((1-a^(m*x))^2,x,method=_RETURNVERBOSE)``[Out] 1/m/ln(a)*(1/2*(a^(m*x))^2-2*a^(m*x)+ln(a^(m*x)))`**Maxima [A]**

time = 0.24, size = 31, normalized size = 0.94

$$x + \frac{a^{2mx}}{2m \log(a)} - \frac{2a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-a^(m*x))^2,x, algorithm="maxima")

[Out] x + 1/2*a^(2*m*x)/(m*log(a)) - 2*a^(m*x)/(m*log(a))

Fricas [A]

time = 0.31, size = 29, normalized size = 0.88

$$\frac{2mx \log(a) + a^{2mx} - 4a^{mx}}{2m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-a^(m*x))^2,x, algorithm="fricas")

[Out] 1/2*(2*m*x*log(a) + a^(2*m*x) - 4*a^(m*x))/(m*log(a))

Sympy [A]

time = 0.06, size = 44, normalized size = 1.33

$$x + \begin{cases} \frac{a^{2mx} m \log(a) - 4a^{mx} m \log(a)}{2m^2 \log(a)^2} & \text{for } m^2 \log(a)^2 \neq 0 \\ -x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-a**(m*x))**2,x)

[Out] x + Piecewise(((a**(2*m*x)*m*log(a) - 4*a**(m*x)*m*log(a))/(2*m**2*log(a)**2), Ne(m**2*log(a)**2, 0)), (-x, True))

Giac [A]

time = 0.00, size = 31, normalized size = 0.94

$$\frac{\frac{(a^{mx})^2}{2} - 2a^{mx} + mx \ln |a|}{m \ln a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-a^(m*x))^2,x)

[Out] 1/2*(2*m*x*log(abs(a)) + a^(2*m*x) - 4*a^(m*x))/(m*log(a))

Mupad [B]

time = 0.30, size = 27, normalized size = 0.82

$$x - \frac{2a^{mx} - \frac{a^{2mx}}{2}}{m \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(m*x) - 1)^2,x)

[Out] x - (2*a^(m*x) - a^(2*m*x)/2)/(m*log(a))

3.519 $\int (1 - a^{mx})^3 dx$

Optimal. Leaf size=50

$$x - \frac{3a^{mx}}{m \log(a)} + \frac{3a^{2mx}}{2m \log(a)} - \frac{a^{3mx}}{3m \log(a)}$$

[Out] $x - 3a^{(m*x)}/m/\ln(a) + 3/2*a^{(2*m*x)}/m/\ln(a) - 1/3*a^{(3*m*x)}/m/\ln(a)$

Rubi [A]

time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2320, 45}

$$-\frac{3a^{mx}}{m \log(a)} + \frac{3a^{2mx}}{2m \log(a)} - \frac{a^{3mx}}{3m \log(a)} + x$$

Antiderivative was successfully verified.

[In] Int[(1 - a^(m*x))^3,x]

[Out] $x - (3*a^{(m*x)})/(m*\text{Log}[a]) + (3*a^{(2*m*x)})/(2*m*\text{Log}[a]) - a^{(3*m*x)}/(3*m*\text{Log}[a])$

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2320

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\begin{aligned}
\int (1 - a^{mx})^3 dx &= \frac{\text{Subst} \left(\int \frac{(1-x)^3}{x} dx, x, a^{mx} \right)}{m \log(a)} \\
&= \frac{\text{Subst} \left(\int \left(-3 + \frac{1}{x} + 3x - x^2 \right) dx, x, a^{mx} \right)}{m \log(a)} \\
&= x - \frac{3a^{mx}}{m \log(a)} + \frac{3a^{2mx}}{2m \log(a)} - \frac{a^{3mx}}{3m \log(a)}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 45, normalized size = 0.90

$$\frac{-\frac{a^{mx}(18-9a^{mx}+2a^{2mx})}{6m} + \frac{\log(a^{mx})}{m}}{\log(a)}$$

Antiderivative was successfully verified.

`[In] Integrate[(1 - a^(m*x))^3, x]``[Out] (-1/6*(a^(m*x)*(18 - 9*a^(m*x) + 2*a^(2*m*x)))/m + Log[a^(m*x)]/m)/Log[a]`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.62, size = 54, normalized size = 1.08

$$\text{Piecewise} \left[\left\{ \left\{ \frac{mx \text{Log}[a] + \frac{(-18-2a^{2mx}+9a^{mx})a^{mx}}{6}}{m \text{Log}[a]}, m^3 \text{Log}[a]^3 \neq 0 \right\}, \{0, \text{True}\} \right\} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(1 - a^(m*x))^3, x]')``[Out] Piecewise[{{(m x Log[a] + (-18 - 2 a ^ (2 m x) + 9 a ^ (m x)) a ^ (m x) / 6) / (m Log[a]), m ^ 3 Log[a] ^ 3 != 0}, {0, True}}`**Maple [A]**

time = 0.02, size = 41, normalized size = 0.82

method	result	size
derivativedivides	$\frac{-\frac{a^{3mx}}{3} + \frac{3a^{2mx}}{2} - 3a^{mx} + \ln(a^{mx})}{m \ln(a)}$	41
default	$\frac{-\frac{a^{3mx}}{3} + \frac{3a^{2mx}}{2} - 3a^{mx} + \ln(a^{mx})}{m \ln(a)}$	41
risch	$x - \frac{3a^{mx}}{m \ln(a)} + \frac{3a^{2mx}}{2m \ln(a)} - \frac{a^{3mx}}{3m \ln(a)}$	49

norman	$x - \frac{3e^{mx \ln(a)}}{m \ln(a)} + \frac{3e^{2mx \ln(a)}}{2m \ln(a)} - \frac{e^{3mx \ln(a)}}{3m \ln(a)}$	52
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1-a^(m*x))^3,x,method=_RETURNVERBOSE)`

[Out] `1/m/ln(a)*(-1/3*(a^(m*x))^3+3/2*(a^(m*x))^2-3*a^(m*x)+ln(a^(m*x)))`

Maxima [A]

time = 0.25, size = 46, normalized size = 0.92

$$x - \frac{a^{3mx}}{3m \log(a)} + \frac{3a^{2mx}}{2m \log(a)} - \frac{3a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-a^(m*x))^3,x, algorithm="maxima")`

[Out] `x - 1/3*a^(3*m*x)/(m*log(a)) + 3/2*a^(2*m*x)/(m*log(a)) - 3*a^(m*x)/(m*log(a))`

Fricas [A]

time = 0.31, size = 39, normalized size = 0.78

$$\frac{6mx \log(a) - 2a^{3mx} + 9a^{2mx} - 18a^{mx}}{6m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-a^(m*x))^3,x, algorithm="fricas")`

[Out] `1/6*(6*m*x*log(a) - 2*a^(3*m*x) + 9*a^(2*m*x) - 18*a^(m*x))/(m*log(a))`

Sympy [A]

time = 0.08, size = 70, normalized size = 1.40

$$x + \begin{cases} \frac{-2a^{3mx}m^2 \log(a)^2 + 9a^{2mx}m^2 \log(a)^2 - 18a^{mx}m^2 \log(a)^2}{6m^3 \log(a)^3} & \text{for } m^3 \log(a)^3 \neq 0 \\ -x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-a**(m*x))**3,x)`

[Out] `x + Piecewise(((-2*a**(3*m*x)*m**2*log(a)**2 + 9*a**(2*m*x)*m**2*log(a)**2 - 18*a**(m*x)*m**2*log(a)**2)/(6*m**3*log(a)**3), Ne(m**3*log(a)**3, 0)), (-x, True))`

Giac [A]

time = 0.00, size = 43, normalized size = 0.86

$$\frac{-\frac{(a^{mx})^3}{3} + \frac{3}{2}(a^{mx})^2 - 3a^{mx} + mx \ln |a|}{m \ln a}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((1-a^(m*x))^3,x)``[Out] 1/6*(6*m*x*log(abs(a)) - 2*a^(3*m*x) + 9*a^(2*m*x) - 18*a^(m*x))/(m*log(a))`**Mupad [B]**

time = 0.32, size = 35, normalized size = 0.70

$$x - \frac{3a^{mx} - \frac{3a^{2mx}}{2} + \frac{a^{3mx}}{3}}{m \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(-(a^(m*x) - 1)^3,x)``[Out] x - (3*a^(m*x) - (3*a^(2*m*x))/2 + a^(3*m*x)/3)/(m*log(a))`

3.520 $\int (1 - a^{mx})^4 dx$

Optimal. Leaf size=65

$$x - \frac{4a^{mx}}{m \log(a)} + \frac{3a^{2mx}}{m \log(a)} - \frac{4a^{3mx}}{3m \log(a)} + \frac{a^{4mx}}{4m \log(a)}$$

[Out] $x - 4a^{(m*x)}/m/\ln(a) + 3a^{(2*m*x)}/m/\ln(a) - 4/3a^{(3*m*x)}/m/\ln(a) + 1/4a^{(4*m*x)}/m/\ln(a)$

Rubi [A]

time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2320, 45}

$$-\frac{4a^{mx}}{m \log(a)} + \frac{3a^{2mx}}{m \log(a)} - \frac{4a^{3mx}}{3m \log(a)} + \frac{a^{4mx}}{4m \log(a)} + x$$

Antiderivative was successfully verified.

[In] Int[(1 - a^(m*x))^4, x]

[Out] $x - (4a^{(m*x)})/(m*\text{Log}[a]) + (3a^{(2*m*x)})/(m*\text{Log}[a]) - (4a^{(3*m*x)})/(3*m*\text{Log}[a]) + a^{(4*m*x)}/(4*m*\text{Log}[a])$

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2320

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\begin{aligned}
\int (1 - a^{mx})^4 dx &= \frac{\text{Subst} \left(\int \frac{(1-x)^4}{x} dx, x, a^{mx} \right)}{m \log(a)} \\
&= \frac{\text{Subst} \left(\int \left(-4 + \frac{1}{x} + 6x - 4x^2 + x^3 \right) dx, x, a^{mx} \right)}{m \log(a)} \\
&= x - \frac{4a^{mx}}{m \log(a)} + \frac{3a^{2mx}}{m \log(a)} - \frac{4a^{3mx}}{3m \log(a)} + \frac{a^{4mx}}{4m \log(a)}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 53, normalized size = 0.82

$$\frac{\frac{a^{mx}(-48 + 36a^{mx} - 16a^{2mx} + 3a^{3mx})}{12m} + \frac{\log(a^{mx})}{m}}{\log(a)}$$

Antiderivative was successfully verified.

`[In] Integrate[(1 - a^(m*x))^4, x]``[Out] ((a^(m*x)*(-48 + 36*a^(m*x) - 16*a^(2*m*x) + 3*a^(3*m*x)))/(12*m) + Log[a^(m*x)]/m)/Log[a]`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.86, size = 62, normalized size = 0.95

$$\text{Piecewise} \left[\left\{ \left\{ \frac{mx \text{Log}[a] + \frac{(-48 - 16a^{2mx} + 3a^{3mx} + 36a^{mx})a^{mx}}{12}}{m \text{Log}[a]}, m^4 \text{Log}[a]^4 \neq 0 \right\}, \{0, \text{True}\} \right\} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(1 - a^(m*x))^4, x]')``[Out] Piecewise[{{(m x Log[a] + (-48 - 16 a^(2 m x) + 3 a^(3 m x) + 36 a^(m x)) a^(m x) / 12) / (m Log[a]), m^4 Log[a]^4 != 0}, {0, True}}`**Maple [A]**

time = 0.04, size = 50, normalized size = 0.77

method	result	size
derivativedivides	$\frac{\frac{a^{4mx}}{4} - \frac{4a^{3mx}}{3} + 3a^{2mx} - 4a^{mx} + \ln(a^{mx})}{m \ln(a)}$	50
default	$\frac{\frac{a^{4mx}}{4} - \frac{4a^{3mx}}{3} + 3a^{2mx} - 4a^{mx} + \ln(a^{mx})}{m \ln(a)}$	50

risch	$x - \frac{4a^{mx}}{m \ln(a)} + \frac{3a^{2mx}}{m \ln(a)} - \frac{4a^{3mx}}{3m \ln(a)} + \frac{a^{4mx}}{4m \ln(a)}$	65
norman	$x - \frac{4e^{mx \ln(a)}}{m \ln(a)} + \frac{3e^{2mx \ln(a)}}{m \ln(a)} - \frac{4e^{3mx \ln(a)}}{3m \ln(a)} + \frac{e^{4mx \ln(a)}}{4m \ln(a)}$	69

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1-a^(m*x))^4,x,method=_RETURNVERBOSE)`

[Out] $1/m/\ln(a)*(1/4*(a^(m*x))^4-4/3*(a^(m*x))^3+3*(a^(m*x))^2-4*a^(m*x)+\ln(a^(m*x)))$

Maxima [A]

time = 0.26, size = 61, normalized size = 0.94

$$x + \frac{a^{4mx}}{4m \log(a)} - \frac{4a^{3mx}}{3m \log(a)} + \frac{3a^{2mx}}{m \log(a)} - \frac{4a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-a^(m*x))^4,x, algorithm="maxima")`

[Out] $x + 1/4*a^(4*m*x)/(m*log(a)) - 4/3*a^(3*m*x)/(m*log(a)) + 3*a^(2*m*x)/(m*log(a)) - 4*a^(m*x)/(m*log(a))$

Fricas [A]

time = 0.31, size = 47, normalized size = 0.72

$$\frac{12mx \log(a) + 3a^{4mx} - 16a^{3mx} + 36a^{2mx} - 48a^{mx}}{12m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-a^(m*x))^4,x, algorithm="fricas")`

[Out] $1/12*(12*m*x*log(a) + 3*a^(4*m*x) - 16*a^(3*m*x) + 36*a^(2*m*x) - 48*a^(m*x))/m*log(a)$

Sympy [A]

time = 0.09, size = 87, normalized size = 1.34

$$x + \begin{cases} \frac{3a^{4mx}m^3 \log(a)^3 - 16a^{3mx}m^3 \log(a)^3 + 36a^{2mx}m^3 \log(a)^3 - 48a^{mx}m^3 \log(a)^3}{12m^4 \log(a)^4} & \text{for } m^4 \log(a)^4 \neq 0 \\ -x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-a**(m*x))**4,x)`

[Out] $x + \text{Piecewise}(((3*a**(4*m*x))*m**3*\log(a)**3 - 16*a**(3*m*x))*m**3*\log(a)**3 + 36*a**(2*m*x))*m**3*\log(a)**3 - 48*a**(m*x))*m**3*\log(a)**3)/(12*m**4*\log(a)**4), \text{Ne}(m**4*\log(a)**4, 0)), (-x, \text{True}))$

Giac [A]

time = 0.00, size = 52, normalized size = 0.80

$$\frac{\frac{(a^{mx})^4}{4} - \frac{4}{3}(a^{mx})^3 + 3(a^{mx})^2 - 4a^{mx} + mx \ln |a|}{m \ln a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-a^(m*x))^4,x)**[Out]** 1/12*(12*m*x*log(abs(a)) + 3*a^(4*m*x) - 16*a^(3*m*x) + 36*a^(2*m*x) - 48*a^(m*x))/(m*log(a))**Mupad [B]**

time = 0.31, size = 43, normalized size = 0.66

$$x - \frac{4a^{mx} - 3a^{2mx} + \frac{4a^{3mx}}{3} - \frac{a^{4mx}}{4}}{m \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(m*x) - 1)^4,x)**[Out]** x - (4*a^(m*x) - 3*a^(2*m*x) + (4*a^(3*m*x))/3 - a^(4*m*x)/4)/(m*log(a))

3.521 $\int (1 - a^{mx})^n dx$

Optimal. Leaf size=44

$$-\frac{(1 - a^{mx})^{1+n} {}_2F_1(1, 1+n; 2+n; 1 - a^{mx})}{m(1+n)\log(a)}$$

[Out] $-(1-a^{m*x})^{(1+n)}*\text{hypergeom}([1, 1+n], [2+n], 1-a^{m*x})/m/(1+n)/\ln(a)$

Rubi [A]

time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2320, 67}

$$-\frac{(1 - a^{mx})^{n+1} {}_2F_1(1, n+1; n+2; 1 - a^{mx})}{m(n+1)\log(a)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 - a^{m*x})^n, x]$

[Out] $-\left(\left(\left(1 - a^{m*x}\right)^{(1+n)}*\text{Hypergeometric2F1}[1, 1+n, 2+n, 1 - a^{m*x}]\right)\right)/\left(m*(1+n)*\text{Log}[a]\right)$

Rule 67

$\text{Int}[(b*x)^m*((c) + (d*x)^n), x_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(n+1)}/(d*(n+1)*(-d/(b*c))^m)*\text{Hypergeometric2F1}[-m, n+1, n+2, 1 + d*(x/c)], x] /;$ FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-d/(b*c), 0])

Rule 2320

$\text{Int}[u, x_Symbol] \rightarrow \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /;$ FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\begin{aligned} \int (1 - a^{mx})^n dx &= \frac{\text{Subst}\left(\int \frac{(1-x)^n}{x} dx, x, a^{mx}\right)}{m \log(a)} \\ &= -\frac{(1 - a^{mx})^{1+n} {}_2F_1(1, 1+n; 2+n; 1 - a^{mx})}{m(1+n)\log(a)} \end{aligned}$$

Mathematica [A]

time = 0.04, size = 44, normalized size = 1.00

$$-\frac{(1 - a^{mx})^{1+n} {}_2F_1(1, 1 + n; 2 + n; 1 - a^{mx})}{m(1 + n) \log(a)}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - a^(m*x))^n, x]

[Out] -(((1 - a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 - a^(m*x)])/(m*(1 + n)*Log[a]))

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(1 - a^(m*x))^n, x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [F]

time = 0.02, size = 0, normalized size = 0.00

$$\int (1 - a^{mx})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-a^(m*x))^n, x)

[Out] int((1-a^(m*x))^n, x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-a^(m*x))^n, x, algorithm="maxima")

[Out] integrate((-a^(m*x) + 1)^n, x)

Fricas [F]

time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-a^(m*x))^n,x, algorithm="fricas")

[Out] integral((-a^(m*x) + 1)^n, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (1 - a^{mx})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-a**(m*x))**n,x)

[Out] Integral((1 - a**(m*x))**n, x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-a^(m*x))^n,x)

[Out] Could not integrate

Mupad [B]

time = 0.32, size = 57, normalized size = 1.30

$$\frac{(1 - a^{mx})^n {}_2F_1\left(-n, -n; 1 - n; \frac{1}{a^{mx}}\right)}{m n \ln(a) \left(1 - \frac{1}{a^{mx}}\right)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - a^(m*x))^n,x)

[Out] ((1 - a^(m*x))^n*hypergeom([-n, -n], 1 - n, 1/a^(m*x)))/(m*n*log(a)*(1 - 1/a^(m*x))^n)

3.522 $\int \frac{1}{b+ae^{nx}} dx$

Optimal. Leaf size=24

$$\frac{x}{b} - \frac{\log(b + ae^{nx})}{bn}$$

[Out] x/b-ln(b+a*exp(n*x))/b/n

Rubi [A]

time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2320, 36, 29, 31}

$$\frac{x}{b} - \frac{\log(ae^{nx} + b)}{bn}$$

Antiderivative was successfully verified.

[In] Int[(b + a*E^(n*x))^(-1),x]

[Out] x/b - Log[b + a*E^(n*x)]/(b*n)

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 2320

Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\begin{aligned} \int \frac{1}{b + ae^{nx}} dx &= \frac{\text{Subst} \left(\int \frac{1}{x(b+ax)} dx, x, e^{nx} \right)}{n} \\ &= \frac{\text{Subst} \left(\int \frac{1}{x} dx, x, e^{nx} \right)}{bn} - \frac{a \text{Subst} \left(\int \frac{1}{b+ax} dx, x, e^{nx} \right)}{bn} \\ &= \frac{x}{b} - \frac{\log(b + ae^{nx})}{bn} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 38, normalized size = 1.58

$$\frac{\log(e^{nx})}{bn} - \frac{\log(b^2n + abe^{nx}n)}{bn}$$

Antiderivative was successfully verified.

`[In] Integrate[(b + a*E^(n*x))^(-1), x]``[Out] Log[E^(n*x)]/(b*n) - Log[b^2*n + a*b*E^(n*x)*n]/(b*n)`**Mathics [A]**

time = 1.81, size = 25, normalized size = 1.04

$$\frac{nx - \text{Log} \left[\frac{b}{a} + E^{nx} \right]}{bn}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(a*E^(n*x) + b), x]')``[Out] (n x - Log[b / a + E ^ (n x)]) / (b n)`**Maple [A]**

time = 0.02, size = 29, normalized size = 1.21

method	result	size
norman	$\frac{x}{b} - \frac{\ln(b+ae^{nx})}{bn}$	24
risch	$\frac{x}{b} - \frac{\ln\left(e^{nx} + \frac{b}{a}\right)}{bn}$	26
derivativedivides	$\frac{-\frac{\ln(b+ae^{nx})}{b} + \frac{\ln(e^{nx})}{b}}{n}$	29
default	$\frac{-\frac{\ln(b+ae^{nx})}{b} + \frac{\ln(e^{nx})}{b}}{n}$	29

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b+a*exp(n*x)),x,method=_RETURNVERBOSE)`

[Out] `1/n*(-1/b*ln(b+a*exp(n*x))+1/b*ln(exp(n*x)))`

Maxima [A]

time = 0.24, size = 23, normalized size = 0.96

$$\frac{x}{b} - \frac{\log(ae^{nx} + b)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b+a*exp(n*x)),x, algorithm="maxima")`

[Out] `x/b - log(a*e^(n*x) + b)/(b*n)`

Fricas [A]

time = 0.31, size = 22, normalized size = 0.92

$$\frac{nx - \log(ae^{nx} + b)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b+a*exp(n*x)),x, algorithm="fricas")`

[Out] `(n*x - log(a*e^(n*x) + b))/(b*n)`

Sympy [A]

time = 0.06, size = 15, normalized size = 0.62

$$\frac{x}{b} - \frac{\log(e^{nx} + \frac{b}{a})}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b+a*exp(n*x)),x)`

[Out] `x/b - log(exp(n*x) + b/a)/(b*n)`

Giac [A]

time = 0.00, size = 26, normalized size = 1.08

$$\frac{\frac{nx}{b} - \frac{a \ln|e^{nx} a + b|}{ab}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b+a*exp(n*x)),x)

[Out] (n*x/b - log(abs(a*e^(n*x) + b))/b)/n

Mupad [B]

time = 0.33, size = 22, normalized size = 0.92

$$-\frac{\ln(b + a e^{n x}) - n x}{b n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b + a*exp(n*x)),x)

[Out] -(log(b + a*exp(n*x)) - n*x)/(b*n)

3.523 $\int \frac{e^x}{b+ae^{3x}} dx$

Optimal. Leaf size=100

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}e^x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}} + \frac{\log\left(\sqrt[3]{b} + \sqrt[3]{a}e^x\right)}{2\sqrt[3]{a}b^{2/3}} - \frac{\log(b+ae^{3x})}{6\sqrt[3]{a}b^{2/3}}$$

[Out] $1/2*\ln(b^{(1/3)+a^{(1/3)*\exp(x)}/a^{(1/3)}/b^{(2/3)}-1/6*\ln(b+a*\exp(3*x))/a^{(1/3)}/b^{(2/3)}-1/3*\arctan(1/3*(b^{(1/3)}-2*a^{(1/3)*\exp(x)}/b^{(1/3)*3^{(1/2)}})/a^{(1/3)}/b^{(2/3)*3^{(1/2)}})$

Rubi [A]

time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2281, 206, 31, 648, 631, 210, 642}

$$-\frac{\log\left(a^{2/3}e^{2x} - \sqrt[3]{a}\sqrt[3]{b}e^x + b^{2/3}\right)}{6\sqrt[3]{a}b^{2/3}} + \frac{\log\left(\sqrt[3]{a}e^x + \sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}e^x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[E^x/(b + a*E^(3*x)),x]

[Out] $-(\text{ArcTan}[(b^{(1/3)} - 2*a^{(1/3)*E^x}/(\text{Sqrt}[3]*b^{(1/3)})]/(\text{Sqrt}[3]*a^{(1/3)*b^{(2/3)}})) + \text{Log}[b^{(1/3)} + a^{(1/3)*E^x}/(3*a^{(1/3)*b^{(2/3)}}) - \text{Log}[b^{(2/3)} - a^{(1/3)*b^{(1/3)*E^x} + a^{(2/3)*E^{(2*x)}}]/(6*a^{(1/3)*b^{(2/3)}})$

Rule 31

Int[((a_) + (b_.)*(x_)^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2281

```
Int[((a_) + (b_)*(F_)^((e_)*((c_) + (d_)*(x_))))^(p_)*(G_)^((h_)*((f_) + (g_)*(x_))), x_Symbol] := With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)* (a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denominator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^x}{b + ae^{3x}} dx &= \text{Subst} \left(\int \frac{1}{b + ax^3} dx, x, e^x \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{\sqrt[3]{b} + \sqrt[3]{a} x} dx, x, e^x \right)}{3b^{2/3}} + \frac{\text{Subst} \left(\int \frac{2\sqrt[3]{b} - \sqrt[3]{a} x}{b^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + a^{2/3} x^2} dx, x, e^x \right)}{3b^{2/3}} \\
&= \frac{\log(\sqrt[3]{b} + \sqrt[3]{a} e^x)}{3\sqrt[3]{a} b^{2/3}} - \frac{\text{Subst} \left(\int \frac{-\sqrt[3]{a} \sqrt[3]{b} + 2a^{2/3} x}{b^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + a^{2/3} x^2} dx, x, e^x \right)}{6\sqrt[3]{a} b^{2/3}} + \frac{\text{Subst} \left(\int \frac{1}{b^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x} dx, x, e^x \right)}{2\sqrt[3]{b}} \\
&= \frac{\log(\sqrt[3]{b} + \sqrt[3]{a} e^x)}{3\sqrt[3]{a} b^{2/3}} - \frac{\log(b^{2/3} - \sqrt[3]{a} \sqrt[3]{b} e^x + a^{2/3} e^{2x})}{6\sqrt[3]{a} b^{2/3}} + \frac{\text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2\sqrt[3]{a}}{\sqrt[3]{b}} \right)}{\sqrt[3]{a} b^{2/3}} \\
&= -\frac{\tan^{-1} \left(\frac{1 - 2\sqrt[3]{a} e^x}{\sqrt[3]{b}} \right)}{\sqrt{3} \sqrt[3]{a} b^{2/3}} + \frac{\log(\sqrt[3]{b} + \sqrt[3]{a} e^x)}{3\sqrt[3]{a} b^{2/3}} - \frac{\log(b^{2/3} - \sqrt[3]{a} \sqrt[3]{b} e^x + a^{2/3} e^{2x})}{6\sqrt[3]{a} b^{2/3}}
\end{aligned}$$

Mathematica [A]

time = 0.08, size = 97, normalized size = 0.97

$$\frac{2\sqrt{3} \tan^{-1} \left(\frac{1 - 2\sqrt[3]{a} e^x}{\sqrt[3]{b}} \right) - 2 \log(\sqrt[3]{b} + \sqrt[3]{a} e^x) + \log(b^{2/3} - \sqrt[3]{a} \sqrt[3]{b} e^x + a^{2/3} e^{2x})}{6\sqrt[3]{a} b^{2/3}}$$

Antiderivative was successfully verified.

`[In] Integrate[E^x/(b + aE^(3*x)), x]`

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*E^x)/b^(1/3))/Sqrt[3]] - 2*Log[b^(1/3) + a^(1/3)*E^x] + Log[b^(2/3) - a^(1/3)*b^(1/3)*E^x + a^(2/3)*E^(2*x)])/(a^(1/3)*b^(2/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.91, size = 28, normalized size = 0.28

$$\text{RootSum}[-1 + 27ab^2\#1^3 \&, \text{Log}[3b\#1 + E^x] \#1 \&]$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[E^x/(aE^(3*x) + b), x]')`

[Out] RootSum[-1 + 27 a b ^ 2 #1 ^ 3&, Log[3 b #1 + E ^ x] #1&]

Maple [A]

time = 0.03, size = 95, normalized size = 0.95

method	result	size
risch	$\sum_{R=\text{RootOf}(27ab^2Z^3-1)} -R \ln(3bR + e^x)$	26
default	$\frac{\ln\left(e^x + \left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{\ln\left(e^{2x} - \left(\frac{b}{a}\right)^{\frac{1}{3}}e^x + \left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(\frac{2e^x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$	95

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(b+a*exp(3*x)),x,method=_RETURNVERBOSE)

[Out] 1/3/a/(b/a)^(2/3)*ln(exp(x)+(b/a)^(1/3))-1/6/a/(b/a)^(2/3)*ln(exp(x)^2-(b/a)^(1/3)*exp(x)+(b/a)^(2/3))+1/3/a/(b/a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(b/a)^(1/3)*exp(x)-1))

Maxima [A]

time = 0.33, size = 100, normalized size = 1.00

$$\frac{\sqrt{3} \arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2e^x\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{\log\left(-\left(\frac{b}{a}\right)^{\frac{1}{3}}e^x + \left(\frac{b}{a}\right)^{\frac{2}{3}} + e^{(2x)}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{\log\left(\left(\frac{b}{a}\right)^{\frac{1}{3}} + e^x\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(b+a*exp(3*x)),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*((b/a)^(1/3) - 2*e^x)/(b/a)^(1/3))/(a*(b/a)^(2/3)) - 1/6*log(-(b/a)^(1/3)*e^x + (b/a)^(2/3) + e^(2*x))/(a*(b/a)^(2/3)) + 1/3*log((b/a)^(1/3) + e^x)/(a*(b/a)^(2/3))

Fricas [A]

time = 0.33, size = 311, normalized size = 3.11

$$\left[\frac{3 \sqrt{\frac{1}{3}} ab \sqrt{\frac{-(ab)^2}{a}} \log\left(\frac{2 ab^{2/3} - 3 (ab)^{2/3} e^x - 3 \sqrt{\frac{1}{3}} (2 ab^{2/3} + (ab)^{2/3} e^x - (ab)^{2/3}) \sqrt{\frac{-(ab)^2}{a}}}{ab^2}\right) - (ab)^2 \log(ab e^{2x} - (ab)^2 e^x + (ab)^2 b) + 2 (ab)^2 \log(ab e^x + (ab)^2)}{6 ab^2}, \frac{6 \sqrt{\frac{1}{3}} ab \sqrt{\frac{-(ab)^2}{a}} \arctan\left(\frac{\sqrt{\frac{1}{3}} (2 (ab)^2 e^x - (ab)^2) \sqrt{\frac{-(ab)^2}{a}}}{3}\right) - (ab)^2 \log(ab e^{2x} - (ab)^2 e^x + (ab)^2 b) + 2 (ab)^2 \log(ab e^x + (ab)^2)}{6 ab^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(b+a*exp(3*x)),x, algorithm="fricas")

[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a*b^2)^(1/3)/a)*log((2*a*b*e^(3*x) - 3*(a*b^2)^(1/3)*b*e^x - b^2 + 3*sqrt(1/3)*(2*a*b*e^(2*x) + (a*b^2)^(2/3)*e^x - (a*b^2)^(1/3)*b)*sqrt(-(a*b^2)^(1/3)/a))/(a*e^(3*x) + b) - (a*b^2)^(2/3)*log(a*b*e^(2*x) - (a*b^2)^(2/3)*e^x + (a*b^2)^(1/3)*b) + 2*(a*b^2)^(2/3)*log(a*b*e^x + (a*b^2)^(2/3)))/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt((a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*(a*b^2)^(2/3)*e^x - (a*b^2)^(1/3)*b)*sqrt((a*b^2)^(1/3)/a)/b^2) - (a*b^2)^(2/3)*log(a*b*e^(2*x) - (a*b^2)^(2/3)*e^x + (a*b^2)^(1/3)*b) + 2*(a*b^2)^(2/3)*log(a*b*e^x + (a*b^2)^(2/3)))/(a*b^2)]

Sympy [A]

time = 0.10, size = 22, normalized size = 0.22

$$\text{RootSum}(27z^3ab^2 - 1, (i \mapsto i \log(3ib + e^x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(b+a*exp(3*x)),x)

[Out] RootSum(27*_z**3*a*b**2 - 1, Lambda(_i, _i*log(3*_i*b + exp(x))))

Giac [A]

time = 0.00, size = 142, normalized size = 1.42

$$\frac{\frac{1}{6}(-a^2b)^{\frac{1}{3}} \ln\left(\left(e^x\right)^2 + \left(-\frac{b}{a}\right)^{\frac{1}{3}} e^x + \left(-\frac{b}{a}\right)^{\frac{1}{3}} \left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{ab} + \frac{(-a^2b)^{\frac{1}{3}} \arctan\left(\frac{2\left(e^x + \left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{\sqrt{3}\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab} - \frac{\frac{1}{3}\left(-\frac{b}{a}\right)^{\frac{1}{3}} \ln\left|e^x - \left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(b+a*exp(3*x)),x)

[Out] -1/3*(-b/a)^(1/3)*log(abs(e^x - (-b/a)^(1/3)))/b + 1/3*sqrt(3)*(-a^2*b)^(1/3)*arctan(1/3*sqrt(3)*(2*e^x + (-b/a)^(1/3))/(-b/a)^(1/3))/(a*b) + 1/6*(-a^2*b)^(1/3)*log(e^x*(-b/a)^(1/3) + e^(2*x) + (-b/a)^(2/3))/(a*b)

Mupad [B]

time = 1.51, size = 104, normalized size = 1.04

$$\frac{\ln\left(\frac{b^{1/3}}{a^{7/3}} + \frac{e^x}{a^2}\right)}{3a^{1/3}b^{2/3}} + \frac{\ln\left(\frac{e^x}{a^2} + \frac{b^{1/3}(-1+\sqrt{3}i)}{2a^{7/3}}\right)(-1+\sqrt{3}i)}{6a^{1/3}b^{2/3}} - \frac{\ln\left(\frac{e^x}{a^2} - \frac{b^{1/3}(1+\sqrt{3}i)}{2a^{7/3}}\right)(1+\sqrt{3}i)}{6a^{1/3}b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(b + a*exp(3*x)),x)

[Out] log(b^(1/3)/a^(7/3) + exp(x)/a^2)/(3*a^(1/3)*b^(2/3)) + (log(exp(x)/a^2 + (b^(1/3)*(3^(1/2)*1i - 1))/(2*a^(7/3)))*(3^(1/2)*1i - 1))/(6*a^(1/3)*b^(2/3)) - (log(exp(x)/a^2 - (b^(1/3)*(3^(1/2)*1i + 1))/(2*a^(7/3)))*(3^(1/2)*1i + 1))/(6*a^(1/3)*b^(2/3))

3.524

$$\int \frac{-1+e^x}{1+e^x} dx$$

Optimal. Leaf size=12

$$-x + 2 \log(1 + e^x)$$

[Out] -x+2*ln(1+exp(x))

Rubi [A]

time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2320, 78}

$$2 \log(e^x + 1) - x$$

Antiderivative was successfully verified.

[In] Int[(-1 + E^x)/(1 + E^x), x]

[Out] -x + 2*Log[1 + E^x]

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned} \int \frac{-1+e^x}{1+e^x} dx &= \text{Subst} \left(\int \frac{-1+x}{x(1+x)} dx, x, e^x \right) \\ &= \text{Subst} \left(\int \left(-\frac{1}{x} + \frac{2}{1+x} \right) dx, x, e^x \right) \\ &= -x + 2 \log(1 + e^x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 15, normalized size = 1.25

$$-\log(e^x) + 2\log(1 + e^x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + E^x)/(1 + E^x),x]

[Out] -Log[E^x] + 2*Log[1 + E^x]

Mathics [A]

time = 1.67, size = 12, normalized size = 1.00

$$-x + 2\text{Log}[1 + E^x]$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(E^x - 1)/(E^x + 1),x]')

[Out] -x + 2 Log[1 + E ^ x]

Maple [A]

time = 0.01, size = 14, normalized size = 1.17

method	result	size
norman	$-x + 2 \ln(1 + e^x)$	12
risch	$-x + 2 \ln(1 + e^x)$	12
derivativedivides	$-\ln(e^x) + 2 \ln(1 + e^x)$	14
default	$-\ln(e^x) + 2 \ln(1 + e^x)$	14

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+exp(x))/(1+exp(x)),x,method=_RETURNVERBOSE)

[Out] -ln(exp(x))+2*ln(1+exp(x))

Maxima [A]

time = 0.25, size = 11, normalized size = 0.92

$$-x + 2 \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+exp(x))/(1+exp(x)),x, algorithm="maxima")

[Out] -x + 2*log(e^x + 1)

Fricas [A]

time = 0.32, size = 11, normalized size = 0.92

$$-x + 2 \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+exp(x))/(1+exp(x)),x, algorithm="fricas")

[Out] -x + 2*log(e^x + 1)

Sympy [A]

time = 0.04, size = 8, normalized size = 0.67

$$-x + 2 \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+exp(x))/(1+exp(x)),x)

[Out] -x + 2*log(exp(x) + 1)

Giac [A]

time = 0.00, size = 10, normalized size = 0.83

$$-x + 2 \ln(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+exp(x))/(1+exp(x)),x)

[Out] -x + 2*log(e^x + 1)

Mupad [B]

time = 0.05, size = 11, normalized size = 0.92

$$2 \ln(e^x + 1) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(x) - 1)/(exp(x) + 1),x)

[Out] 2*log(exp(x) + 1) - x

$$3.525 \quad \int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}} dx$$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{1-3e^{2x}}{\sqrt{2}}\right)}{6\sqrt{2}} + \frac{1}{12} \log(1-2e^{2x}+3e^{4x})$$

[Out] 1/12*ln(1-2*exp(2*x)+3*exp(4*x))-1/12*arctan(1/2*(1-3*exp(2*x))*2^(1/2))*2^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2320, 648, 632, 210, 642}

$$\frac{1}{12} \log(-2e^{2x} + 3e^{4x} + 1) - \frac{\tan^{-1}\left(\frac{1-3e^{2x}}{\sqrt{2}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[E^(4*x)/(1 - 2*E^(2*x) + 3*E^(4*x)),x]

[Out] -1/6*ArcTan[(1 - 3*E^(2*x))/Sqrt[2]]/Sqrt[2] + Log[1 - 2*E^(2*x) + 3*E^(4*x)]/12

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned} \int \frac{e^{4x}}{1 - 2e^{2x} + 3e^{4x}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x}{1 - 2x + 3x^2} dx, x, e^{2x} \right) \\ &= \frac{1}{12} \text{Subst} \left(\int \frac{-2 + 6x}{1 - 2x + 3x^2} dx, x, e^{2x} \right) + \frac{1}{6} \text{Subst} \left(\int \frac{1}{1 - 2x + 3x^2} dx, x, e^{2x} \right) \\ &= \frac{1}{12} \log(1 - 2e^{2x} + 3e^{4x}) - \frac{1}{3} \text{Subst} \left(\int \frac{1}{-8 - x^2} dx, x, -2 + 6e^{2x} \right) \\ &= -\frac{\tan^{-1} \left(\frac{1 - 3e^{2x}}{\sqrt{2}} \right)}{6\sqrt{2}} + \frac{1}{12} \log(1 - 2e^{2x} + 3e^{4x}) \end{aligned}$$

Mathematica [A]

time = 0.04, size = 45, normalized size = 0.96

$$\frac{1}{12} \left(-\sqrt{2} \tan^{-1} \left(\frac{1 - 3e^{2x}}{\sqrt{2}} \right) + \log(1 - 2e^{2x} + 3e^{4x}) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*x)/(1 - 2*E^(2*x) + 3*E^(4*x)), x]
```

```
[Out] (- (Sqrt[2]*ArcTan[(1 - 3*E^(2*x))/Sqrt[2]]) + Log[1 - 2*E^(2*x) + 3*E^(4*x)])/12
```

Mathics [C] Result contains complex when optimal does not.

time = 1.91, size = 30, normalized size = 0.64

$$\text{RootSum} [1 - 16\#1 + 96\#1^2 \&, \text{Log} [-1 + E^{2x} + 8\#1] \#1 \&]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[E^(4*x)/(3*E^(4*x) - 2*E^(2*x) + 1),x]')`

[Out] `RootSum[1 - 16 #1 + 96 #1 ^ 2&, Log[-1 + E ^ (2 x) + 8 #1] #1&]`

Maple [A]

time = 0.02, size = 38, normalized size = 0.81

method	result	size
default	$\frac{\ln(1-2e^{2x}+3e^{4x})}{12} + \frac{\sqrt{2} \arctan\left(\frac{(6e^{2x}-2)\sqrt{2}}{4}\right)}{12}$	38
risch	$\frac{\ln\left(e^{2x}-\frac{1}{3}+\frac{i\sqrt{2}}{3}\right)}{12} + \frac{i \ln\left(e^{2x}-\frac{1}{3}+\frac{i\sqrt{2}}{3}\right)\sqrt{2}}{24} + \frac{\ln\left(e^{2x}-\frac{1}{3}-\frac{i\sqrt{2}}{3}\right)}{12} - \frac{i \ln\left(e^{2x}-\frac{1}{3}-\frac{i\sqrt{2}}{3}\right)\sqrt{2}}{24}$	70

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x,method=_RETURNVERBOSE)`

[Out] `1/12*ln(1-2*exp(x)^2+3*exp(x)^4)+1/12*2^(1/2)*arctan(1/4*(6*exp(x)^2-2)*2^(1/2))`

Maxima [A]

time = 0.34, size = 37, normalized size = 0.79

$$\frac{1}{12} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} (3e^{(2x)} - 1)\right) + \frac{1}{12} \log(3e^{(4x)} - 2e^{(2x)} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x, algorithm="maxima")`

[Out] `1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*e^(2*x) - 1)) + 1/12*log(3*e^(4*x) - 2*e^(2*x) + 1)`

Fricas [A]

time = 0.31, size = 39, normalized size = 0.83

$$\frac{1}{12} \sqrt{2} \arctan\left(\frac{3}{2} \sqrt{2} e^{(2x)} - \frac{1}{2} \sqrt{2}\right) + \frac{1}{12} \log(3e^{(4x)} - 2e^{(2x)} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x, algorithm="fricas")`

[Out] `1/12*sqrt(2)*arctan(3/2*sqrt(2)*e^(2*x) - 1/2*sqrt(2)) + 1/12*log(3*e^(4*x) - 2*e^(2*x) + 1)`

Sympy [A]

time = 0.08, size = 22, normalized size = 0.47

$$\text{RootSum}(96z^2 - 16z + 1, (i \mapsto i \log(8i + e^{2x} - 1)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)), x)

[Out] RootSum(96*_z**2 - 16*_z + 1, Lambda(_i, _i*log(8*_i + exp(2*x) - 1)))

Giac [A]

time = 0.00, size = 50, normalized size = 1.06

$$\frac{\frac{\ln(3(e^{2x})^2 - 2e^{2x} + 1)}{6} + \frac{\arctan\left(\frac{3e^{2x} - 1}{\sqrt{2}}\right)}{3\sqrt{2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)), x)

[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*e^(2*x) - 1)) + 1/12*log(3*e^(4*x) - 2*e^(2*x) + 1)

Mupad [B]

time = 0.33, size = 39, normalized size = 0.83

$$\frac{\ln(3e^{4x} - 2e^{2x} + 1)}{12} - \frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}e^{2x}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(4*x)/(3*exp(4*x) - 2*exp(2*x) + 1), x)

[Out] log(3*exp(4*x) - 2*exp(2*x) + 1)/12 - (2^(1/2)*atan(2^(1/2)/2 - (3*2^(1/2)*exp(2*x))/2))/12

$$3.526 \quad \int \frac{e^x + e^{5x}}{-1 + e^x - e^{2x} + e^{3x}} dx$$

Optimal. Leaf size=39

$$e^x + \frac{e^{2x}}{2} - \tan^{-1}(e^x) + \log(1 - e^x) - \frac{1}{2} \log(1 + e^{2x})$$

[Out] exp(x)+1/2*exp(2*x)-arctan(exp(x))+ln(1-exp(x))-1/2*ln(1+exp(2*x))

Rubi [A]

time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2320, 2099, 649, 209, 266}

$$e^x + \frac{e^{2x}}{2} + \log(1 - e^x) - \frac{1}{2} \log(e^{2x} + 1) - \tan^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Int[(E^x + E^(5*x))/(-1 + E^x - E^(2*x) + E^(3*x)),x]

[Out] E^x + E^(2*x)/2 - ArcTan[E^x] + Log[1 - E^x] - Log[1 + E^(2*x)]/2

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 649

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[(-a)*c]

Rule 2099

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned} \int \frac{e^x + e^{5x}}{-1 + e^x - e^{2x} + e^{3x}} dx &= \text{Subst} \left(\int \frac{-1 - x^4}{1 - x + x^2 - x^3} dx, x, e^x \right) \\ &= \text{Subst} \left(\int \left(1 + \frac{1}{-1 + x} + x + \frac{-1 - x}{1 + x^2} \right) dx, x, e^x \right) \\ &= e^x + \frac{e^{2x}}{2} + \log(1 - e^x) + \text{Subst} \left(\int \frac{-1 - x}{1 + x^2} dx, x, e^x \right) \\ &= e^x + \frac{e^{2x}}{2} + \log(1 - e^x) - \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, e^x \right) - \text{Subst} \left(\int \frac{x}{1 + x^2} dx, x, e^x \right) \\ &= e^x + \frac{e^{2x}}{2} - \tan^{-1}(e^x) + \log(1 - e^x) - \frac{1}{2} \log(1 + e^{2x}) \end{aligned}$$

Mathematica [A]

time = 0.04, size = 37, normalized size = 0.95

$$e^x + \frac{e^{2x}}{2} - \tan^{-1}(e^x) + \log(-1 + e^x) - \frac{1}{2} \log(1 + e^{2x})$$

Antiderivative was successfully verified.

[In] Integrate[(E^x + E^(5*x))/(-1 + E^x - E^(2*x) + E^(3*x)), x]

[Out] E^x + E^(2*x)/2 - ArcTan[E^x] + Log[-1 + E^x] - Log[1 + E^(2*x)]/2

Mathics [C] Result contains complex when optimal does not.

time = 2.24, size = 51, normalized size = 1.31

$$\text{Log}[-1 + E^x] + E^x + \text{RootSum} \left[1 + 2\#1^2 + 2\#1\&, \text{Log} \left[\frac{-3}{5} + E^x - \frac{6\#1}{5} + \frac{4\#1^2}{5} \right] \#1\& \right] + \frac{E^{2x}}{2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(E^(5*x) + E^x)/(E^(3*x) - E^(2*x) + E^x - 1), x]')

[Out] Log[-1 + E^x] + E^x + RootSum[1 + 2 #1^2 + 2 #1&, Log[-3 / 5 + E^x - 6 #1 / 5 + 4 #1^2 / 5] #1&] + E^(2 x) / 2

Maple [A]

time = 0.05, size = 29, normalized size = 0.74

method	result	size
default	$\ln(-1 + e^x) - \frac{\ln(1+e^{2x})}{2} - \arctan(e^x) + \frac{e^{2x}}{2} + e^x$	29
risch	$\frac{e^{2x}}{2} + e^x + \ln(-1 + e^x) - \frac{\ln(e^x - i)}{2} + \frac{i \ln(e^x - i)}{2} - \frac{\ln(e^x + i)}{2} - \frac{i \ln(e^x + i)}{2}$	49
meijerg	$\frac{\left(\sum_{k=1}^{\infty} \frac{1 - e^{-x(3+k)}}{(3+k) \left(1 - \frac{1}{3+k}\right)} \right)}{2} - \frac{\left(\sum_{k=1}^{\infty} \frac{1 - e^{-x(2+k)}}{(2+k) \left(1 - \frac{1}{2+k}\right)} \right)}{2}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(-1+exp(x))-1/2*ln(1+exp(x)^2)-arctan(exp(x))+1/2*exp(x)^2+exp(x)
```

Maxima [A]

time = 0.33, size = 28, normalized size = 0.72

$$-\arctan(e^x) + \frac{1}{2} e^{(2x)} + e^x - \frac{1}{2} \log(e^{(2x)} + 1) + \log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x, algorithm="maxima")
```

```
[Out] -arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(e^x - 1)
```

Fricas [A]

time = 0.38, size = 28, normalized size = 0.72

$$-\arctan(e^x) + \frac{1}{2} e^{(2x)} + e^x - \frac{1}{2} \log(e^{(2x)} + 1) + \log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x, algorithm="fricas")
```

```
[Out] -arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(e^x - 1)
```

Sympy [A]

time = 0.11, size = 48, normalized size = 1.23

$$\frac{e^{2x}}{2} + e^x + \log(e^x - 1) + \text{RootSum} \left(2z^2 + 2z + 1, \left(i \mapsto i \log \left(\frac{4i^2}{5} - \frac{6i}{5} + e^x - \frac{3}{5} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x)

[Out] $\exp(2x)/2 + \exp(x) + \log(\exp(x) - 1) + \text{RootSum}(2z^2 + 2z + 1, \text{Lambda}(_i, _i \log(4_i^2/5 - 6_i/5 + \exp(x) - 3/5)))$

Giac [A]

time = 0.00, size = 31, normalized size = 0.79

$$\frac{(e^x)^2}{2} + e^x + \ln|e^x - 1| - \frac{\ln((e^x)^2 + 1)}{2} - \arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x)

[Out] $1/2 * e^{2x} + e^x - \arctan(e^x) - 1/2 * \log(e^{2x} + 1) + \log(\text{abs}(e^x - 1))$

Mupad [B]

time = 0.09, size = 28, normalized size = 0.72

$$\frac{e^{2x}}{2} - \frac{\ln(e^{2x} + 1)}{2} - \text{atan}(e^x) + \ln(e^x - 1) + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(exp(5*x) + exp(x))/(exp(2*x) - exp(3*x) - exp(x) + 1),x)

[Out] $\exp(2x)/2 - \log(\exp(2x) + 1)/2 - \text{atan}(\exp(x)) + \log(\exp(x) - 1) + \exp(x)$

$$3.527 \quad \int e^{nx} (a + be^{nx})^{r/s} dx$$

Optimal. Leaf size=30

$$\frac{(a + be^{nx})^{\frac{r+s}{s}} s}{bn(r + s)}$$

[Out] (a+b*exp(n*x))^(r+s)/s*s/b/n/(r+s)

Rubi [A]

time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2278, 32}

$$\frac{s(a + be^{nx})^{\frac{r+s}{s}}}{bn(r + s)}$$

Antiderivative was successfully verified.

[In] Int[E^(n*x)*(a + b*E^(n*x))^(r/s),x]

[Out] ((a + b*E^(n*x))^(r + s)/s)/(b*n*(r + s))

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2278

Int[((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(n_.)*((a_) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(p_.), x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]

Rubi steps

$$\begin{aligned} \int e^{nx} (a + be^{nx})^{r/s} dx &= \frac{\text{Subst} \left(\int (a + bx)^{r/s} dx, x, e^{nx} \right)}{n} \\ &= \frac{(a + be^{nx})^{\frac{r+s}{s}} s}{bn(r + s)} \end{aligned}$$

Mathematica [A]

time = 0.06, size = 30, normalized size = 1.00

$$\frac{(a + be^{nx})^{1 + \frac{r}{s}} s}{bnr + bns}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*x)*(a + b*E^(n*x))^(r/s),x]

[Out] ((a + b*E^(n*x))^(1 + r/s)*s)/(b*n*r + b*n*s)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.28, size = 144, normalized size = 4.80

Piecewise $\left[\left\{ \frac{x}{a}, b=0 \ \&\& \ n=0 \ \&\& \ r=-s \right\}, \left\{ \frac{E^{nx} a^{\frac{r}{s}}}{n}, b=0 \right\}, \left\{ x(a+b)^{\frac{r}{s}}, n=0 \right\}, \left\{ \frac{\text{Log}\left[\frac{a}{b} + E^{nx}\right]}{bn}, r=-s \right\} \right], \frac{as(a + bE^{nx})^{\frac{r}{s}}}{bnr + bns} + \frac{bsE^{nx}(a + bE^{nx})^{\frac{r}{s}}}{bnr + bns}$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a + b*E^(n*x))^(r/s)*E^(n*x),x]')

[Out] Piecewise[{{x / a, b == 0 && n == 0 && r == -s}, {E ^ (n x) a ^ (r / s) / n, b == 0}, {x (a + b) ^ (r / s), n == 0}, {Log[a / b + E ^ (n x)] / (b n), r == -s}}, a s (a + b E ^ (n x)) ^ (r / s) / (b n r + b n s) + b s E ^ (n x) (a + b E ^ (n x)) ^ (r / s) / (b n r + b n s)]

Maple [A]

time = 0.03, size = 33, normalized size = 1.10

method	result	size
derivativedivides	$\frac{(a+be^{nx})^{\frac{r}{s}+1}}{nb\left(\frac{r}{s}+1\right)}$	33
default	$\frac{(a+be^{nx})^{\frac{r}{s}+1}}{nb\left(\frac{r}{s}+1\right)}$	33
risch	$\frac{s(a+be^{nx})(a+be^{nx})^{\frac{r}{s}}}{bn(r+s)}$	36
norman	$\frac{se^{nx}e^{\frac{r \ln(a+be^{nx})}{s}}}{n(r+s)} + \frac{ase^{\frac{r \ln(a+be^{nx})}{s}}}{bn(r+s)}$	60

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*x)*(a+b*exp(n*x))^(r/s),x,method=_RETURNVERBOSE)

[Out] 1/n*(a+b*exp(n*x))^(r/s+1)/b/(r/s+1)

Maxima [A]

time = 0.25, size = 32, normalized size = 1.07

$$\frac{(be^{(n x)} + a)^{\frac{r}{s}+1}}{bn\left(\frac{r}{s} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*x)*(a+b*exp(n*x))^(r/s),x, algorithm="maxima")

[Out] $(b \cdot e^{n \cdot x} + a)^{(r/s + 1)} / (b \cdot n \cdot (r/s + 1))$

Fricas [A]

time = 0.32, size = 37, normalized size = 1.23

$$\frac{(bse^{nx} + as)(be^{nx} + a)^{\frac{r}{s}}}{bnr + bns}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*x)*(a+b*exp(n*x))^(r/s),x, algorithm="fricas")`

[Out] $(b \cdot s \cdot e^{n \cdot x} + a \cdot s) \cdot (b \cdot e^{n \cdot x} + a)^{(r/s)} / (b \cdot n \cdot r + b \cdot n \cdot s)$

Sympy [A]

time = 0.49, size = 94, normalized size = 3.13

$$\begin{cases} \frac{x}{a} & \text{for } b = 0 \wedge n = 0 \wedge r = -s \\ \frac{a^{\frac{r}{s}} e^{nx}}{n} & \text{for } b = 0 \\ x (a + b)^{\frac{r}{s}} & \text{for } n = 0 \\ \frac{\log(\frac{a}{b} + e^{nx})}{bn} & \text{for } r = -s \\ \frac{as(a+be^{nx})^{\frac{r}{s}}}{bnr+bns} + \frac{bs(a+be^{nx})^{\frac{r}{s}} e^{nx}}{bnr+bns} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*x)*(a+b*exp(n*x))**(r/s),x)`

[Out] `Piecewise((x/a, Eq(b, 0) & Eq(n, 0) & Eq(r, -s)), (a**(r/s)*exp(n*x)/n, Eq(b, 0)), (x*(a + b)**(r/s), Eq(n, 0)), (log(a/b + exp(n*x))/(b*n), Eq(r, -s)), (a*s*(a + b*exp(n*x))**(r/s)/(b*n*r + b*n*s) + b*s*(a + b*exp(n*x))**(r/s)*exp(n*x)/(b*n*r + b*n*s), True))`

Giac [A]

time = 0.00, size = 27, normalized size = 0.90

$$\frac{(a + be^{nx})^{\frac{r}{s}+1}}{bn \left(\frac{r}{s} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*x)*(a+b*exp(n*x))^(r/s),x)`

[Out] $(b \cdot e^{n \cdot x} + a)^{(r/s + 1)} / (b \cdot n \cdot (r/s + 1))$

Mupad [B]

time = 0.35, size = 29, normalized size = 0.97

$$\frac{s (a + be^{nx})^{\frac{r}{s}+1}}{bn (r + s)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*x)*(a + b*exp(n*x))^(r/s),x)
```

```
[Out] (s*(a + b*exp(n*x))^(r/s + 1))/(b*n*(r + s))
```

3.528 $\int \sqrt[4]{1 - 2e^{x/3}} dx$

Optimal. Leaf size=54

$$12\sqrt[4]{1 - 2e^{x/3}} - 6 \tan^{-1} \left(\sqrt[4]{1 - 2e^{x/3}} \right) - 6 \tanh^{-1} \left(\sqrt[4]{1 - 2e^{x/3}} \right)$$

[Out] 12*(1-2*exp(1/3*x))^(1/4)-6*arctan((1-2*exp(1/3*x))^(1/4))-6*arctanh((1-2*exp(1/3*x))^(1/4))

Rubi [A]

time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2320, 52, 65, 218, 212, 209}

$$12\sqrt[4]{1 - 2e^{x/3}} - 6 \tan^{-1} \left(\sqrt[4]{1 - 2e^{x/3}} \right) - 6 \tanh^{-1} \left(\sqrt[4]{1 - 2e^{x/3}} \right)$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*E^(x/3))^(1/4), x]

[Out] 12*(1 - 2*E^(x/3))^(1/4) - 6*ArcTan[(1 - 2*E^(x/3))^(1/4)] - 6*ArcTanh[(1 - 2*E^(x/3))^(1/4)]

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*(b*c - a*d)/(
b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[
m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b
, 0]
```

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int \sqrt[4]{1 - 2e^{x/3}} dx &= 3 \text{Subst} \left(\int \frac{\sqrt[4]{1 - 2x}}{x} dx, x, e^{x/3} \right) \\
&= 12 \sqrt[4]{1 - 2e^{x/3}} + 3 \text{Subst} \left(\int \frac{1}{(1 - 2x)^{3/4} x} dx, x, e^{x/3} \right) \\
&= 12 \sqrt[4]{1 - 2e^{x/3}} - 6 \text{Subst} \left(\int \frac{1}{\frac{1}{2} - \frac{x^4}{2}} dx, x, \sqrt[4]{1 - 2e^{x/3}} \right) \\
&= 12 \sqrt[4]{1 - 2e^{x/3}} - 6 \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \sqrt[4]{1 - 2e^{x/3}} \right) - 6 \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \right. \\
&= 12 \sqrt[4]{1 - 2e^{x/3}} - 6 \tan^{-1} \left(\sqrt[4]{1 - 2e^{x/3}} \right) - 6 \tanh^{-1} \left(\sqrt[4]{1 - 2e^{x/3}} \right)
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 54, normalized size = 1.00

$$12 \sqrt[4]{1 - 2e^{x/3}} - 6 \tan^{-1} \left(\sqrt[4]{1 - 2e^{x/3}} \right) - 6 \tanh^{-1} \left(\sqrt[4]{1 - 2e^{x/3}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*E^(x/3))^(1/4), x]

[Out] $12*(1 - 2*E^{(x/3)})^{(1/4)} - 6*ArcTan[(1 - 2*E^{(x/3)})^{(1/4)}] - 6*ArcTanh[(1 - 2*E^{(x/3)})^{(1/4)}]$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(1 - 2*E^(x/3))^(1/4),x]')`

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.05, size = 57, normalized size = 1.06

method	result
derivativedivides	$12(1 - 2e^{\frac{x}{3}})^{\frac{1}{4}} + 3 \ln\left((1 - 2e^{\frac{x}{3}})^{\frac{1}{4}} - 1\right) - 3 \ln\left((1 - 2e^{\frac{x}{3}})^{\frac{1}{4}} + 1\right) - 6 \arctan\left((1 - 2e^{\frac{x}{3}})^{\frac{1}{4}}\right)$
default	$12(1 - 2e^{\frac{x}{3}})^{\frac{1}{4}} + 3 \ln\left((1 - 2e^{\frac{x}{3}})^{\frac{1}{4}} - 1\right) - 3 \ln\left((1 - 2e^{\frac{x}{3}})^{\frac{1}{4}} + 1\right) - 6 \arctan\left((1 - 2e^{\frac{x}{3}})^{\frac{1}{4}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1-2*exp(1/3*x))^(1/4),x,method=_RETURNVERBOSE)`

[Out] $12*(1-2*\exp(1/3*x))^{(1/4)}+3*\ln((1-2*\exp(1/3*x))^{(1/4)}-1)-3*\ln((1-2*\exp(1/3*x))^{(1/4)}+1)-6*\arctan((1-2*\exp(1/3*x))^{(1/4)})$

Maxima [A]

time = 0.34, size = 56, normalized size = 1.04

$12\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}-6\arctan\left(\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}\right)-3\log\left(\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}+1\right)+3\log\left(\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}-1\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-2*exp(1/3*x))^(1/4),x, algorithm="maxima")`

[Out] $12*(-2*e^{(1/3*x)} + 1)^{(1/4)} - 6*\arctan((-2*e^{(1/3*x)} + 1)^{(1/4)}) - 3*\log((-2*e^{(1/3*x)} + 1)^{(1/4)} + 1) + 3*\log((-2*e^{(1/3*x)} + 1)^{(1/4)} - 1)$

Fricas [A]

time = 0.31, size = 56, normalized size = 1.04

$12\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}-6\arctan\left(\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}\right)-3\log\left(\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}+1\right)+3\log\left(\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}-1\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-2*exp(1/3*x))^(1/4),x, algorithm="fricas")

[Out] $12*(-2*e^{1/3*x} + 1)^{1/4} - 6*\arctan((-2*e^{1/3*x} + 1)^{1/4}) - 3*\log((-2*e^{1/3*x} + 1)^{1/4} + 1) + 3*\log((-2*e^{1/3*x} + 1)^{1/4} - 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{1 - 2e^{\frac{x}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-2*exp(1/3*x))**(1/4),x)

[Out] Integral((1 - 2*exp(x/3))**(1/4), x)

Giac [A]

time = 0.00, size = 79, normalized size = 1.46

$$-2 \cdot 3 \left(-\frac{\ln \left| (-2e^{\frac{x}{3}} + 1)^{\frac{1}{4}} - 1 \right|}{2} + \frac{\ln \left((-2e^{\frac{x}{3}} + 1)^{\frac{1}{4}} + 1 \right)}{2} + \arctan \left((-2e^{\frac{x}{3}} + 1)^{\frac{1}{4}} \right) - 2 \left(-2e^{\frac{x}{3}} + 1 \right)^{\frac{1}{4}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-2*exp(1/3*x))^(1/4),x)

[Out] $12*(-2*e^{1/3*x} + 1)^{1/4} - 6*\arctan((-2*e^{1/3*x} + 1)^{1/4}) - 3*\log((-2*e^{1/3*x} + 1)^{1/4} + 1) + 3*\log(\text{abs}((-2*e^{1/3*x} + 1)^{1/4} - 1))$

Mupad [B]

time = 0.35, size = 33, normalized size = 0.61

$$\frac{12(2 - 4e^{x/3})^{1/4} {}_2F_1\left(-\frac{1}{4}, -\frac{1}{4}; \frac{3}{4}; \frac{e^{-x/3}}{2}\right)}{(2 - e^{-x/3})^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - 2*exp(x/3))^(1/4),x)

[Out] $(12*(2 - 4*\exp(x/3))^{1/4}*\text{hypergeom}([-1/4, -1/4], 3/4, \exp(-x/3)/2))/(2 - \exp(-x/3))^{1/4}$

3.529 $\int (a + be^{nx})^{r/s} dx$

Optimal. Leaf size=59

$$\frac{(a + be^{nx})^{\frac{r+s}{s}} s {}_2F_1\left(1, \frac{r+s}{s}; 2 + \frac{r}{s}; 1 + \frac{be^{nx}}{a}\right)}{an(r+s)}$$

[Out] $-(a+b*\exp(n*x))^{((r+s)/s)}*s*\text{hypergeom}([1, (r+s)/s], [2+r/s], 1+b*\exp(n*x)/a)/a/n/(r+s)$

Rubi [A]

time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2320, 67}

$$\frac{s(a + be^{nx})^{\frac{r+s}{s}} {}_2F_1\left(1, \frac{r+s}{s}; \frac{r}{s} + 2; \frac{e^{nx}b}{a} + 1\right)}{an(r+s)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*E^{(n*x)})^{(r/s)}, x]$

[Out] $-\left(\left(a + b*E^{(n*x)}\right)^{((r+s)/s)}*s*\text{Hypergeometric2F1}\left[1, (r+s)/s, 2 + r/s, 1 + (b*E^{(n*x)})/a\right]/(a*n*(r+s))\right)$

Rule 67

$\text{Int}[(b*x)^m*((c) + (d*x)^n), x_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(n+1)}/(d*(n+1)*(-d/(b*c))^m)*\text{Hypergeometric2F1}[-m, n+1, n+2, 1 + d*(x/c)], x] /; \text{FreeQ}\{b, c, d, m, n\}, x \ \&\& \ !\text{IntegerQ}[n] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{GtQ}[-d/(b*c), 0])$

Rule 2320

$\text{Int}[u, x_Symbol] \rightarrow \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (w_)*((a_)*(v_)^{(n_)})^{(m_)} /; \text{FreeQ}\{a, m, n\}, x] \ \&\& \ \text{IntegerQ}[m*n] \ \&\& \ !\text{MatchQ}[u, E^{((c_)*((a_)+(b_)*x))* (F_)[v_]} /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{InverseFunctionQ}[F[x]]$

Rubi steps

$$\begin{aligned} \int (a + be^{nx})^{r/s} dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^{r/s}}{x} dx, x, e^{nx}\right)}{n} \\ &= \frac{(a + be^{nx})^{\frac{r+s}{s}} s {}_2F_1\left(1, \frac{r+s}{s}; 2 + \frac{r}{s}; 1 + \frac{be^{nx}}{a}\right)}{an(r+s)} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 59, normalized size = 1.00

$$\frac{(a + be^{nx})^{\frac{r+s}{s}} {}_2F_1\left(1, \frac{r+s}{s}; 2 + \frac{r}{s}; 1 + \frac{be^{nx}}{a}\right)}{an(r+s)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*E^(n*x))^(r/s),x]

[Out] -(((a + b*E^(n*x))^(r + s)/s)*s*Hypergeometric2F1[1, (r + s)/s, 2 + r/s, 1 + (b*E^(n*x))/a])/(a*n*(r + s))

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a + b*E^(n*x))^(r/s),x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int (a + be^{nx})^{\frac{r}{s}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*exp(n*x))^(r/s),x)

[Out] int((a+b*exp(n*x))^(r/s),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*exp(n*x))^(r/s),x, algorithm="maxima")

[Out] integrate((b*e^(n*x) + a)^(r/s), x)

Fricas [F]

time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*exp(n*x))^(r/s),x, algorithm="fricas")

[Out] integral((b*e^(n*x) + a)^(r/s), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + be^{nx})^{\frac{r}{s}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*exp(n*x))**(r/s),x)

[Out] Integral((a + b*exp(n*x))**(r/s), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*exp(n*x))^(r/s),x)

[Out] Could not integrate

Mupad [B]

time = 0.40, size = 75, normalized size = 1.27

$$\frac{s(a + be^{nx})^{r/s} {}_2F_1\left(-\frac{r}{s}, -\frac{r}{s}; 1 - \frac{r}{s}; -\frac{ae^{-nx}}{b}\right)}{nr\left(\frac{ae^{-nx}}{b} + 1\right)^{r/s}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*exp(n*x))^(r/s),x)

[Out] (s*(a + b*exp(n*x))^(r/s)*hypergeom([-r/s, -r/s], 1 - r/s, -(a*exp(-n*x))/b))/ (n*r*((a*exp(-n*x))/b + 1)^(r/s))

$$3.530 \quad \int \frac{e^x}{\sqrt{a^2 + e^{2x}}} dx$$

Optimal. Leaf size=18

$$\tanh^{-1} \left(\frac{e^x}{\sqrt{a^2 + e^{2x}}} \right)$$

[Out] arctanh(exp(x)/(a^2+exp(2*x))^(1/2))

Rubi [A]

time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2281, 223, 212}

$$\tanh^{-1} \left(\frac{e^x}{\sqrt{a^2 + e^{2x}}} \right)$$

Antiderivative was successfully verified.

[In] Int[E^x/Sqrt[a^2 + E^(2*x)],x]

[Out] ArcTanh[E^x/Sqrt[a^2 + E^(2*x)]]

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 223

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 2281

Int[((a_) + (b_)*(F_)^((e_)*((c_) + (d_)*(x_))))^(p_)*(G_)^((h_)*((f_) + (g_)*(x_))), x_Symbol] := With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denominator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

Rubi steps

$$\begin{aligned}
\int \frac{e^x}{\sqrt{a^2 + e^{2x}}} dx &= \text{Subst} \left(\int \frac{1}{\sqrt{a^2 + x^2}} dx, x, e^x \right) \\
&= \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \frac{e^x}{\sqrt{a^2 + e^{2x}}} \right) \\
&= \tanh^{-1} \left(\frac{e^x}{\sqrt{a^2 + e^{2x}}} \right)
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 18, normalized size = 1.00

$$\tanh^{-1} \left(\frac{e^x}{\sqrt{a^2 + e^{2x}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/Sqrt[a^2 + E^(2*x)], x]

[Out] ArcTanh[E^x/Sqrt[a^2 + E^(2*x)]]

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.94, size = 37, normalized size = 2.06

$$\text{Piecewise} \left[\left\{ \left\{ \text{ArcSinh} \left[E^x \sqrt{\frac{1}{a^2}} \right], a^2 > 0 \right\}, \left\{ \text{ArcCosh} \left[E^x \sqrt{-\frac{1}{a^2}} \right], a^2 < 0 \right\} \right\}, \text{Undefined} \right]$$

Antiderivative was successfully verified.

[In] mathics('Integrate[E^x/Sqrt[E^(2*x) + a^2], x]')

[Out] Piecewise[{{ArcSinh[E ^ x Sqrt[1 / a ^ 2]], a ^ 2 > 0}, {ArcCosh[E ^ x Sqrt[-1 / a ^ 2]], a ^ 2 < 0}}, Undefined]

Maple [A]

time = 0.02, size = 15, normalized size = 0.83

method	result	size
default	$\ln(e^x + \sqrt{a^2 + e^{2x}})$	15

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(a^2+exp(2*x))^(1/2), x, method=_RETURNVERBOSE)

[Out] ln(exp(x)+(a^2+exp(x)^2)^(1/2))

Maxima [A]

time = 0.24, size = 7, normalized size = 0.39

$$\operatorname{arsinh}\left(\frac{e^x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(a^2+exp(2*x))^(1/2),x, algorithm="maxima")

[Out] arcsinh(e^x/a)

Fricas [A]

time = 0.31, size = 18, normalized size = 1.00

$$-\log\left(\sqrt{a^2 + e^{(2x)}} - e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(a^2+exp(2*x))^(1/2),x, algorithm="fricas")

[Out] -log(sqrt(a^2 + e^(2*x)) - e^x)

Sympy [A]

time = 0.36, size = 31, normalized size = 1.72

$$\begin{cases} \operatorname{asinh}\left(\sqrt{\frac{1}{a^2}} e^x\right) & \text{for } a^2 > 0 \\ \operatorname{acosh}\left(\sqrt{-\frac{1}{a^2}} e^x\right) & \text{for } a^2 < 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(a**2+exp(2*x))**(1/2),x)

[Out] Piecewise((asinh(sqrt(a**(-2))*exp(x)), a**2 > 0), (acosh(sqrt(-1/a**2)*exp(x)), a**2 < 0))

Giac [A]

time = 0.00, size = 18, normalized size = 1.00

$$-\ln\left(\sqrt{a^2 + (e^x)^2} - e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(a^2+exp(2*x))^(1/2),x)

[Out] -log(-e^x + sqrt(a^2 + e^(2*x)))

Mupad [B]

time = 0.41, size = 14, normalized size = 0.78

$$\ln\left(e^x + \sqrt{a^2 + e^{2x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)/(exp(2*x) + a^2)^(1/2),x)`

[Out] `log(exp(x) + (exp(2*x) + a^2)^(1/2))`

$$3.531 \quad \int \frac{e^x}{\sqrt{-a^2 + e^{2x}}} dx$$

Optimal. Leaf size=20

$$\tanh^{-1} \left(\frac{e^x}{\sqrt{-a^2 + e^{2x}}} \right)$$

[Out] arctanh(exp(x)/(-a^2+exp(2*x))^(1/2))

Rubi [A]

time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2281, 223, 212}

$$\tanh^{-1} \left(\frac{e^x}{\sqrt{e^{2x} - a^2}} \right)$$

Antiderivative was successfully verified.

[In] Int[E^x/Sqrt[-a^2 + E^(2*x)], x]

[Out] ArcTanh[E^x/Sqrt[-a^2 + E^(2*x)]]

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 223

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 2281

Int[((a_) + (b_)*(F_)^((e_)*((c_) + (d_)*(x_))))^(p_)*(G_)^((h_)*((f_) + (g_)*(x_))), x_Symbol] := With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denominator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

Rubi steps

$$\begin{aligned}
\int \frac{e^x}{\sqrt{-a^2 + e^{2x}}} dx &= \text{Subst} \left(\int \frac{1}{\sqrt{-a^2 + x^2}} dx, x, e^x \right) \\
&= \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \frac{e^x}{\sqrt{-a^2 + e^{2x}}} \right) \\
&= \tanh^{-1} \left(\frac{e^x}{\sqrt{-a^2 + e^{2x}}} \right)
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 20, normalized size = 1.00

$$\tanh^{-1} \left(\frac{e^x}{\sqrt{-a^2 + e^{2x}}} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[E^x/Sqrt[-a^2 + E^(2*x)], x]``[Out] ArcTanh[E^x/Sqrt[-a^2 + E^(2*x)]]`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.96, size = 37, normalized size = 1.85

$$\text{Piecewise} \left[\left\{ \left\{ \text{ArcSinh} \left[E^x \sqrt{-\frac{1}{a^2}} \right], a^2 < 0 \right\}, \left\{ \text{ArcCosh} \left[E^x \sqrt{\frac{1}{a^2}} \right], a^2 > 0 \right\} \right\}, \text{Undefined} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[E^x/Sqrt[E^(2*x) - a^2], x]')``[Out] Piecewise[{{ArcSinh[E ^ x Sqrt[-1 / a ^ 2]], a ^ 2 < 0}, {ArcCosh[E ^ x Sqrt[1 / a ^ 2]], a ^ 2 > 0}}, Undefined]`**Maple [A]**

time = 0.02, size = 17, normalized size = 0.85

method	result	size
default	$\ln(e^x + \sqrt{-a^2 + e^{2x}})$	17

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(exp(x)/(-a^2+exp(2*x))^(1/2), x, method=_RETURNVERBOSE)``[Out] ln(exp(x)+(-a^2+exp(x)^2)^(1/2))`

Maxima [A]

time = 0.25, size = 20, normalized size = 1.00

$$\log \left(2 \sqrt{-a^2 + e^{(2x)}} + 2 e^x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)/(-a^2+exp(2*x))^(1/2),x, algorithm="maxima")``[Out] log(2*sqrt(-a^2 + e^(2*x)) + 2*e^x)`**Fricas [A]**

time = 0.52, size = 20, normalized size = 1.00

$$-\log \left(\sqrt{-a^2 + e^{(2x)}} - e^x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)/(-a^2+exp(2*x))^(1/2),x, algorithm="fricas")``[Out] -log(sqrt(-a^2 + e^(2*x)) - e^x)`**Sympy [A]**

time = 0.38, size = 31, normalized size = 1.55

$$\begin{cases} \operatorname{asinh} \left(\sqrt{-\frac{1}{a^2}} e^x \right) & \text{for } a^2 < 0 \\ \operatorname{acosh} \left(\sqrt{\frac{1}{a^2}} e^x \right) & \text{for } a^2 > 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)/(-a**2+exp(2*x))**(1/2),x)``[Out] Piecewise((asinh(sqrt(-1/a**2)*exp(x)), a**2 < 0), (acosh(sqrt(a**(-2))*exp(x)), a**2 > 0))`**Giac [A]**

time = 0.00, size = 19, normalized size = 0.95

$$-\ln \left(-\sqrt{-a^2 + (e^x)^2} + e^x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)/(-a^2+exp(2*x))^(1/2),x)``[Out] -log(e^x - sqrt(-a^2 + e^(2*x)))`

Mupad [B]

time = 0.41, size = 16, normalized size = 0.80

$$\ln\left(e^x + \sqrt{e^{2x} - a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)/(exp(2*x) - a^2)^(1/2),x)`

[Out] `log(exp(x) + (exp(2*x) - a^2)^(1/2))`

$$3.532 \quad \int \frac{e^{3x/4}}{(-2+e^{3x/4})\sqrt{-2+e^{3x/4}+e^{3x/2}}} dx$$

Optimal. Leaf size=40

$$\frac{2}{3} \tanh^{-1} \left(\frac{2 - 5e^{3x/4}}{4\sqrt{-2 + e^{3x/4} + e^{3x/2}}} \right)$$

[Out] 2/3*arctanh(1/4*(2-5*exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2))

Rubi [A]

time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2320, 738, 212}

$$\frac{2}{3} \tanh^{-1} \left(\frac{2 - 5e^{3x/4}}{4\sqrt{e^{3x/4} + e^{3x/2} - 2}} \right)$$

Antiderivative was successfully verified.

[In] Int[E^((3*x)/4)/((-2 + E^((3*x)/4))*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)]],x
]

[Out] (2*ArcTanh[(2 - 5*E^((3*x)/4))/(4*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)]])]/3

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 738

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 2320

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\begin{aligned}
\int \frac{e^{3x/4}}{(-2 + e^{3x/4}) \sqrt{-2 + e^{3x/4} + e^{3x/2}}} dx &= \frac{4}{3} \text{Subst} \left(\int \frac{1}{(-2 + x) \sqrt{-2 + x + x^2}} dx, x, e^{3x/4} \right) \\
&= - \left(\frac{8}{3} \text{Subst} \left(\int \frac{1}{16 - x^2} dx, x, \frac{-2 + 5e^{3x/4}}{\sqrt{-2 + e^{3x/4} + e^{3x/2}}} \right) \right) \\
&= \frac{2}{3} \tanh^{-1} \left(\frac{2 - 5e^{3x/4}}{4 \sqrt{-2 + e^{3x/4} + e^{3x/2}}} \right)
\end{aligned}$$

Mathematica [A]

time = 0.11, size = 42, normalized size = 1.05

$$-\frac{4}{3} \tanh^{-1} \left(1 - \frac{1}{2} e^{3x/4} + \frac{1}{2} \sqrt{-2 + e^{3x/4} + e^{3x/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[E^((3*x)/4)/((-2 + E^((3*x)/4))*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)]],x]

[Out] (-4*ArcTanh[1 - E^((3*x)/4)/2 + Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)]/2])/3

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[E^((3/4)*x)/((E^((3/4)*x) - 2)*Sqrt[E^((3/2)*x) + E^((3/4)*x) - 2]]),x')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{e^{\frac{3x}{4}}}{\left(-2 + e^{\frac{3x}{4}}\right) \sqrt{-2 + e^{\frac{3x}{4}} + e^{\frac{3x}{2}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x)`

[Out] `int(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x)`

Maxima [A]

time = 0.33, size = 39, normalized size = 0.98

$$-\frac{2}{3} \log \left(\frac{4 \sqrt{e^{(\frac{3}{2}x)} + e^{(\frac{3}{4}x)} - 2}}{|e^{(\frac{3}{4}x)} - 2|} + \frac{8}{|e^{(\frac{3}{4}x)} - 2|} + 5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x, algorithm="maxima")`

[Out] `-2/3*log(4*sqrt(e^(3/2*x) + e^(3/4*x) - 2)/abs(e^(3/4*x) - 2) + 8/abs(e^(3/4*x) - 2) + 5)`

Fricas [A]

time = 0.30, size = 46, normalized size = 1.15

$$-\frac{2}{3} \log \left(\sqrt{e^{(\frac{3}{2}x)} + e^{(\frac{3}{4}x)} - 2} - e^{(\frac{3}{4}x)} + 4 \right) + \frac{2}{3} \log \left(\sqrt{e^{(\frac{3}{2}x)} + e^{(\frac{3}{4}x)} - 2} - e^{(\frac{3}{4}x)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x, algorithm="fricas")`

[Out] `-2/3*log(sqrt(e^(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x) + 4) + 2/3*log(sqrt(e^(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x))`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\frac{3x}{4}}}{\left(e^{\frac{3x}{4}} - 2\right) \sqrt{e^{\frac{3x}{4}} + e^{\frac{3x}{2}} - 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x)`

[Out] `Integral(exp(3*x/4)/((exp(3*x/4) - 2)*sqrt(exp(3*x/4) + exp(3*x/2) - 2)), x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.

time = 0.01, size = 76, normalized size = 1.90

$$\frac{2}{3} \cdot 4 \left(\frac{\ln \left| \sqrt{(e^{\frac{x}{4}})^6 + (e^{\frac{x}{4}})^3 - 2} - (e^{\frac{x}{4}})^3 \right|}{4} - \frac{\ln \left| \sqrt{(e^{\frac{x}{4}})^6 + (e^{\frac{x}{4}})^3 - 2} - (e^{\frac{x}{4}})^3 + 4 \right|}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x)

[Out] -2/3*log(abs(-e^(3/4*x) + sqrt(e^(3/2*x) + e^(3/4*x) - 2) + 4)) + 2/3*log(abs(-e^(3/4*x) + sqrt(e^(3/2*x) + e^(3/4*x) - 2)))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^{\frac{3x}{4}}}{\left(e^{\frac{3x}{4}} - 2\right) \sqrt{e^{\frac{3x}{2}} + e^{\frac{3x}{4}} - 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp((3*x)/4)/((exp((3*x)/4) - 2)*(exp((3*x)/2) + exp((3*x)/4) - 2)^(1/2)),x)

[Out] int(exp((3*x)/4)/((exp((3*x)/4) - 2)*(exp((3*x)/2) + exp((3*x)/4) - 2)^(1/2)), x)

$$3.533 \quad \int e^{-2x} (-3 + e^{7x})^{2/3} dx$$

Optimal. Leaf size=37

$$\frac{1}{6} e^{-2x} (-3 + e^{7x})^{5/3} {}_2F_1\left(1, \frac{29}{21}; \frac{5}{7}; \frac{e^{7x}}{3}\right)$$

[Out] 1/6*(-3+exp(7*x))^(5/3)*hypergeom([1, 29/21], [5/7], 1/3*exp(7*x))/exp(2*x)

Rubi [A]

time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.54, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2281, 342, 372, 371}

$$\frac{3^{2/3} e^{-2x} (e^{7x} - 3)^{2/3} {}_2F_1\left(-\frac{2}{3}, -\frac{2}{7}; \frac{5}{7}; \frac{e^{7x}}{3}\right)}{2(3 - e^{7x})^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(-3 + E^(7*x))^(2/3)/E^(2*x), x]

[Out] -1/2*(3^(2/3)*(-3 + E^(7*x))^(2/3)*Hypergeometric2F1[-2/3, -2/7, 5/7, E^(7*x)/3])/(E^(2*x)*(3 - E^(7*x))^(2/3))

Rule 342

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && IntegerQ[m]

Rule 371

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p * ((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 372

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^(m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

Rule 2281

Int[((a_) + (b_)*(F_)^(e_)*((c_) + (d_)*(x_)))^(p_)*(G_)^(h_)*((f_) + (g_)*(x_)), x_Symbol] := With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log

```
[G]))], Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}
 \int e^{-2x} (-3 + e^{7x})^{2/3} dx &= -\text{Subst} \left(\int \left(-3 + \frac{1}{x^7} \right)^{2/3} x dx, x, e^{-x} \right) \\
 &= \text{Subst} \left(\int \frac{(-3 + x^7)^{2/3}}{x^3} dx, x, e^x \right) \\
 &= \frac{(-3 + e^{7x})^{2/3} \text{Subst} \left(\int \frac{(1 - \frac{x^7}{3})^{2/3}}{x^3} dx, x, e^x \right)}{(1 - \frac{e^{7x}}{3})^{2/3}} \\
 &= -\frac{3^{2/3} e^{-2x} (-3 + e^{7x})^{2/3} {}_2F_1 \left(-\frac{2}{3}, -\frac{2}{7}; \frac{5}{7}; \frac{e^{7x}}{3} \right)}{2 (3 - e^{7x})^{2/3}}
 \end{aligned}$$

Mathematica [A]

time = 0.03, size = 54, normalized size = 1.46

$$\frac{e^{-2x} (-3 + e^{7x})^{2/3} {}_2F_1 \left(-\frac{2}{3}, -\frac{2}{7}; \frac{5}{7}; \frac{e^{7x}}{3} \right)}{2 \left(1 - \frac{e^{7x}}{3} \right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + E^(7*x))^(2/3)/E^(2*x),x]
```

```
[Out] -1/2*((-3 + E^(7*x))^(2/3)*Hypergeometric2F1[-2/3, -2/7, 5/7, E^(7*x)/3])/
(E^(2*x)*(1 - E^(7*x)/3)^(2/3))
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(E^(7*x) - 3)^(2/3)/E^(2*x),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int (-3 + e^{7x})^{\frac{2}{3}} e^{-2x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-3+exp(7*x))^(2/3)/exp(2*x),x)

[Out] int((-3+exp(7*x))^(2/3)/exp(2*x),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+exp(7*x))^(2/3)/exp(2*x),x, algorithm="maxima")

[Out] integrate((e^(7*x) - 3)^(2/3)*e^(-2*x), x)

Fricas [F]

time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+exp(7*x))^(2/3)/exp(2*x),x, algorithm="fricas")

[Out] integral((e^(7*x) - 3)^(2/3)*e^(-2*x), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (e^{7x} - 3)^{\frac{2}{3}} e^{-2x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+exp(7*x))**(2/3)/exp(2*x),x)

[Out] Integral((exp(7*x) - 3)**(2/3)*exp(-2*x), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+exp(7*x))^(2/3)/exp(2*x),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int e^{-2x} (e^{7x} - 3)^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*x)*(exp(7*x) - 3)^(2/3),x)
```

```
[Out] int(exp(-2*x)*(exp(7*x) - 3)^(2/3), x)
```


$$3.534 \quad \int \frac{e^{2x}}{(3 - e^{x/2})^{3/4}} dx$$

Optimal. Leaf size=73

$$-216\sqrt[4]{3 - e^{x/2}} + \frac{216}{5} (3 - e^{x/2})^{5/4} - 8(3 - e^{x/2})^{9/4} + \frac{8}{13} (3 - e^{x/2})^{13/4}$$

[Out] -216*(3-exp(1/2*x))^(1/4)+216/5*(3-exp(1/2*x))^(5/4)-8*(3-exp(1/2*x))^(9/4)+8/13*(3-exp(1/2*x))^(13/4)

Rubi [A]

time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2280, 45}

$$\frac{8}{13} (3 - e^{x/2})^{13/4} - 8(3 - e^{x/2})^{9/4} + \frac{216}{5} (3 - e^{x/2})^{5/4} - 216\sqrt[4]{3 - e^{x/2}}$$

Antiderivative was successfully verified.

[In] Int[E^(2*x)/(3 - E^(x/2))^(3/4), x]

[Out] -216*(3 - E^(x/2))^(1/4) + (216*(3 - E^(x/2))^(5/4))/5 - 8*(3 - E^(x/2))^(9/4) + (8*(3 - E^(x/2))^(13/4))/13

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2280

Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log[F]))}], Dist[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])), Subst[Int[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/Denominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

Rubi steps

$$\begin{aligned}
\int \frac{e^{2x}}{(3 - e^{x/2})^{3/4}} dx &= 2\text{Subst} \left(\int \frac{x^3}{(3 - x)^{3/4}} dx, x, e^{x/2} \right) \\
&= 2\text{Subst} \left(\int \left(\frac{27}{(3 - x)^{3/4}} - 27\sqrt[4]{3 - x} + 9(3 - x)^{5/4} - (3 - x)^{9/4} \right) dx, x, e^{x/2} \right) \\
&= -216\sqrt[4]{3 - e^{x/2}} + \frac{216}{5} (3 - e^{x/2})^{5/4} - 8(3 - e^{x/2})^{9/4} + \frac{8}{13} (3 - e^{x/2})^{13/4}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 44, normalized size = 0.60

$$-\frac{8}{65} \sqrt[4]{3 - e^{x/2}} (1152 + 96e^{x/2} + 20e^x + 5e^{3x/2})$$

Antiderivative was successfully verified.

`[In] Integrate[E^(2*x)/(3 - E^(x/2))^(3/4), x]``[Out] (-8*(3 - E^(x/2))^(1/4)*(1152 + 96*E^(x/2) + 20*E^x + 5*E^((3*x)/2)))/65`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[E^(2*x)/(3 - E^(x/2))^(3/4), x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [A]**

time = 0.02, size = 37, normalized size = 0.51

method	result	size
risch	$\frac{8 \left(5e^{\frac{3x}{2}} + 20e^x + 96e^{\frac{x}{2}} + 1152 \right) \left(-3 + e^{\frac{x}{2}} \right)}{65 \left(3 - e^{\frac{x}{2}} \right)^{\frac{3}{4}}}$	37

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(exp(2*x)/(3-exp(1/2*x))^(3/4), x, method=_RETURNVERBOSE)``[Out] 8/65/(3-exp(1/2*x))^(3/4)*(5*exp(3/2*x)+20*exp(x)+96*exp(1/2*x)+1152)*(-3+exp(1/2*x))`

Maxima [A]

time = 0.24, size = 49, normalized size = 0.67

$$\frac{8}{13} \left(-e^{\frac{1}{2}x} + 3\right)^{\frac{13}{4}} - 8 \left(-e^{\frac{1}{2}x} + 3\right)^{\frac{9}{4}} + \frac{216}{5} \left(-e^{\frac{1}{2}x} + 3\right)^{\frac{5}{4}} - 216 \left(-e^{\frac{1}{2}x} + 3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)/(3-exp(1/2*x))^(3/4),x, algorithm="maxima")**[Out]** 8/13*(-e^(1/2*x) + 3)^(13/4) - 8*(-e^(1/2*x) + 3)^(9/4) + 216/5*(-e^(1/2*x) + 3)^(5/4) - 216*(-e^(1/2*x) + 3)^(1/4)**Fricas [A]**

time = 0.30, size = 30, normalized size = 0.41

$$-\frac{8}{65} \left(5e^{\frac{3}{2}x} + 96e^{\frac{1}{2}x} + 20e^x + 1152\right) \left(-e^{\frac{1}{2}x} + 3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)/(3-exp(1/2*x))^(3/4),x, algorithm="fricas")**[Out]** -8/65*(5*e^(3/2*x) + 96*e^(1/2*x) + 20*e^x + 1152)*(-e^(1/2*x) + 3)^(1/4)**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{2x}}{\left(3 - e^{\frac{x}{2}}\right)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)/(3-exp(1/2*x))**(3/4),x)**[Out]** Integral(exp(2*x)/(3 - exp(x/2))**(3/4), x)**Giac [A]**

time = 0.00, size = 93, normalized size = 1.27

$$-4 \cdot 2 \left(-\frac{1}{13} \left(-e^{\frac{x}{2}} + 3\right)^{\frac{1}{4}} \left(-e^{\frac{x}{2}} + 3\right)^3 + \left(-e^{\frac{x}{2}} + 3\right)^{\frac{1}{4}} \left(-e^{\frac{x}{2}} + 3\right)^2 - \frac{27}{5} \left(-e^{\frac{x}{2}} + 3\right)^{\frac{1}{4}} \left(-e^{\frac{x}{2}} + 3\right) + 27 \left(-e^{\frac{x}{2}} + 3\right)^{\frac{1}{4}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)/(3-exp(1/2*x))^(3/4),x)**[Out]** -8/13*(e^(1/2*x) - 3)^3*(-e^(1/2*x) + 3)^(1/4) - 8*(e^(1/2*x) - 3)^2*(-e^(1/2*x) + 3)^(1/4) + 216/5*(-e^(1/2*x) + 3)^(5/4) - 216*(-e^(1/2*x) + 3)^(1/4)

Mupad [B]

time = 0.11, size = 30, normalized size = 0.41

$$-(3 - e^{x/2})^{1/4} \left(\frac{768 e^{x/2}}{65} + \frac{8 e^{\frac{3x}{2}}}{13} + \frac{32 e^x}{13} + \frac{9216}{65} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(2*x)/(3 - exp(x/2))^(3/4), x)`

[Out] `-(3 - exp(x/2))^(1/4)*((768*exp(x/2))/65 + (8*exp((3*x)/2))/13 + (32*exp(x))/13 + 9216/65)`

3.535 $\int e^{-x/2} x^3 dx$

Optimal. Leaf size=44

$$-96e^{-x/2} - 48e^{-x/2}x - 12e^{-x/2}x^2 - 2e^{-x/2}x^3$$

[Out] $-96/\exp(1/2*x)-48*x/\exp(1/2*x)-12*x^2/\exp(1/2*x)-2*x^3/\exp(1/2*x)$

Rubi [A]

time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2207, 2225}

$$-2e^{-x/2}x^3 - 12e^{-x/2}x^2 - 48e^{-x/2}x - 96e^{-x/2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3/E^{(x/2)}, x]$

[Out] $-96/E^{(x/2)} - (48*x)/E^{(x/2)} - (12*x^2)/E^{(x/2)} - (2*x^3)/E^{(x/2)}$

Rule 2207

$\text{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))}^{(n_*)}*((c_*) + (d_*)*(x_*))^{(m_*)}, x_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*((b*F^{(g*(e + f*x)))^n/(f*g*n*\text{Log}[F])), x] - \text{Dist}[d*(m/(f*g*n*\text{Log}[F])), \text{Int}[(c + d*x)^{(m-1)}*(b*F^{(g*(e + f*x)))^n}, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0] \&\& \text{IntegerQ}[2*m] \&\& !\text{TrueQ}[\$UseGamma]$

Rule 2225

$\text{Int}[(F_*)^{((c_*)*((a_*) + (b_*)*(x_*)))}^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x))})^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

Rubi steps

$$\begin{aligned} \int e^{-x/2} x^3 dx &= -2e^{-x/2} x^3 + 6 \int e^{-x/2} x^2 dx \\ &= -12e^{-x/2} x^2 - 2e^{-x/2} x^3 + 24 \int e^{-x/2} x dx \\ &= -48e^{-x/2} x - 12e^{-x/2} x^2 - 2e^{-x/2} x^3 + 48 \int e^{-x/2} dx \\ &= -96e^{-x/2} - 48e^{-x/2} x - 12e^{-x/2} x^2 - 2e^{-x/2} x^3 \end{aligned}$$

Mathematica [A]

time = 0.01, size = 23, normalized size = 0.52

$$e^{-x/2} (-96 - 48x - 12x^2 - 2x^3)$$

Antiderivative was successfully verified.

[In] Integrate[x^3/E^(x/2), x]

[Out] (-96 - 48*x - 12*x^2 - 2*x^3)/E^(x/2)

Mathics [A]

time = 1.64, size = 22, normalized size = 0.50

$$2 (-48 - 24x - 6x^2 - x^3) E^{-\frac{x}{2}}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^3/E^(x/2), x]')

[Out] 2 (-48 - 24 x - 6 x ^ 2 - x ^ 3) E ^ (-x / 2)

Maple [A]

time = 0.02, size = 41, normalized size = 0.93

method	result	size
risch	$(-2x^3 - 12x^2 - 48x - 96) e^{-\frac{x}{2}}$	21
gospers	$-2(x^3 + 6x^2 + 24x + 48) e^{-\frac{x}{2}}$	22
norman	$(-2x^3 - 12x^2 - 48x - 96) e^{-\frac{x}{2}}$	23
meijerg	$96 - 4\left(\frac{1}{2}x^3 + 3x^2 + 12x + 24\right) e^{-\frac{x}{2}}$	24
derivativdivides	$-96 e^{-\frac{x}{2}} - 48x e^{-\frac{x}{2}} - 12x^2 e^{-\frac{x}{2}} - 2x^3 e^{-\frac{x}{2}}$	41
default	$-96 e^{-\frac{x}{2}} - 48x e^{-\frac{x}{2}} - 12x^2 e^{-\frac{x}{2}} - 2x^3 e^{-\frac{x}{2}}$	41

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/exp(1/2*x), x, method=_RETURNVERBOSE)

[Out] -96/exp(1/2*x)-48*x/exp(1/2*x)-12*x^2/exp(1/2*x)-2*x^3/exp(1/2*x)

Maxima [A]

time = 0.24, size = 19, normalized size = 0.43

$$-2(x^3 + 6x^2 + 24x + 48)e^{(-\frac{1}{2}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/exp(1/2*x), x, algorithm="maxima")

[Out] $-2*(x^3 + 6*x^2 + 24*x + 48)*e^{(-1/2*x)}$

Fricas [A]

time = 0.31, size = 19, normalized size = 0.43

$$-2(x^3 + 6x^2 + 24x + 48)e^{(-\frac{1}{2}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/exp(1/2*x),x, algorithm="fricas")`

[Out] $-2*(x^3 + 6*x^2 + 24*x + 48)*e^{(-1/2*x)}$

Sympy [A]

time = 0.05, size = 20, normalized size = 0.45

$$(-2x^3 - 12x^2 - 48x - 96)e^{-\frac{x}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/exp(1/2*x),x)`

[Out] $(-2*x**3 - 12*x**2 - 48*x - 96)*exp(-x/2)$

Giac [A]

time = 0.00, size = 32, normalized size = 0.73

$$2\left(-8\left(\frac{x}{2}\right)^3 - 24\left(\frac{x}{2}\right)^2 - 24x - 48\right)e^{-\frac{x}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/exp(1/2*x),x)`

[Out] $-2*(x^3 + 6*x^2 + 24*x + 48)/e^{(1/2*x)}$

Mupad [B]

time = 0.03, size = 21, normalized size = 0.48

$$-16e^{-\frac{x}{2}}\left(\frac{x^3}{8} + \frac{3x^2}{4} + 3x + 6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*exp(-x/2),x)`

[Out] $-16*exp(-x/2)*(3*x + (3*x^2)/4 + x^3/8 + 6)$

3.536 $\int \frac{e^{-x/2}}{x^3} dx$

Optimal. Leaf size=39

$$-\frac{e^{-x/2}}{2x^2} + \frac{e^{-x/2}}{4x} + \frac{\text{Ei}\left(-\frac{x}{2}\right)}{8}$$

[Out] $-1/2/\exp(1/2*x)/x^2+1/4/\exp(1/2*x)/x+1/8*\text{Ei}(-1/2*x)$

Rubi [A]

time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2208, 2209}

$$\frac{\text{Ei}\left(-\frac{x}{2}\right)}{8} - \frac{e^{-x/2}}{2x^2} + \frac{e^{-x/2}}{4x}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(E^{(x/2)*x^3}), x]$

[Out] $-1/2*1/(E^{(x/2)*x^2}) + 1/(4*E^{(x/2)*x}) + \text{ExpIntegralEi}[-1/2*x]/8$

Rule 2208

$\text{Int}[(b_*)*(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))^{(n_*)*((c_*) + (d_*)*(x_*))^{(m_*)}, x_Symbol] :> \text{Simp}[(c + d*x)^{(m + 1)}*((b*F^{(g*(e + f*x)))^n/(d*(m + 1)))]$, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[\$UseGamma]

Rule 2209

$\text{Int}[(F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))^{(c_*) + (d_*)*(x_*)}, x_Symbol] :> \text{Simp}[(F^{(g*(e - c*(f/d))})/d)*\text{ExpIntegralEi}[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]$

Rubi steps

$$\begin{aligned} \int \frac{e^{-x/2}}{x^3} dx &= -\frac{e^{-x/2}}{2x^2} - \frac{1}{4} \int \frac{e^{-x/2}}{x^2} dx \\ &= -\frac{e^{-x/2}}{2x^2} + \frac{e^{-x/2}}{4x} + \frac{1}{8} \int \frac{e^{-x/2}}{x} dx \\ &= -\frac{e^{-x/2}}{2x^2} + \frac{e^{-x/2}}{4x} + \frac{\text{Ei}\left(-\frac{x}{2}\right)}{8} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 26, normalized size = 0.67

$$\frac{1}{8} \left(\frac{2e^{-x/2}(-2+x)}{x^2} + \text{Ei} \left(-\frac{x}{2} \right) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[1/(E^(x/2)*x^3),x]``[Out] ((2*(-2 + x))/(E^(x/2)*x^2) + ExpIntegralEi[-1/2*x])/8`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

time = 2.41, size = 31, normalized size = 0.79

$$-\frac{E^{-\frac{x}{2}}}{2x^2} + \frac{E^{-\frac{x}{2}}}{4x} + \frac{\text{ExpIntegralEi} \left[\frac{x \exp_{\text{polar}}[i\text{Pi}]}{2} \right]}{8}$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x^3*E^(x/2)),x]')``[Out] -E^(-x/2)/(2*x^2) + E^(-x/2)/(4*x) + ExpIntegralEi[x exp_polar[I Pi]/2]/8`**Maple [A]**

time = 0.04, size = 31, normalized size = 0.79

method	result	size
risch	$-\frac{e^{-\frac{x}{2}}}{2x^2} + \frac{e^{-\frac{x}{2}}}{4x} - \frac{\text{expIntegral}(1, \frac{x}{2})}{8}$	27
derivativedivides	$-\frac{e^{-\frac{x}{2}}}{2x^2} + \frac{e^{-\frac{x}{2}}}{4x} - \frac{\text{expIntegral}(1, \frac{x}{2})}{8}$	31
default	$-\frac{e^{-\frac{x}{2}}}{2x^2} + \frac{e^{-\frac{x}{2}}}{4x} - \frac{\text{expIntegral}(1, \frac{x}{2})}{8}$	31
meijerg	$-\frac{1}{2x^2} + \frac{1}{2x} - \frac{3}{16} + \frac{\ln(x)}{8} - \frac{\ln(2)}{8} + \frac{9x^2 - 6x + 6}{12x^2} - \frac{(-\frac{3x}{2} + 3)e^{-\frac{x}{2}}}{6x^2} - \frac{\ln(\frac{x}{2})}{8} - \frac{\text{expIntegral}(1, \frac{x}{2})}{8}$	63

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/exp(1/2*x)/x^3,x,method=_RETURNVERBOSE)``[Out] -1/2/exp(1/2*x)/x^2+1/4/exp(1/2*x)/x-1/8*Ei(1,1/2*x)`**Maxima [A]**

time = 0.27, size = 7, normalized size = 0.18

$$-\frac{1}{4} \Gamma \left(-2, \frac{1}{2} x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(1/2*x)/x^3,x, algorithm="maxima")

[Out] -1/4*gamma(-2, 1/2*x)

Fricas [A]

time = 0.49, size = 23, normalized size = 0.59

$$\frac{x^2 \operatorname{Ei}\left(-\frac{1}{2}x\right) + 2(x-2)e^{\left(-\frac{1}{2}x\right)}}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(1/2*x)/x^3,x, algorithm="fricas")

[Out] 1/8*(x^2*Ei(-1/2*x) + 2*(x - 2)*e^(-1/2*x))/x^2

Sympy [C] Result contains complex when optimal does not.

time = 0.69, size = 32, normalized size = 0.82

$$\frac{\operatorname{Ei}\left(\frac{xe^{i\pi}}{2}\right)}{8} + \frac{e^{-\frac{x}{2}}}{4x} - \frac{e^{-\frac{x}{2}}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(1/2*x)/x**3,x)

[Out] Ei(x*exp_polar(I*pi)/2)/8 + exp(-x/2)/(4*x) - exp(-x/2)/(2*x**2)

Giac [A]

time = 0.00, size = 42, normalized size = 1.08

$$\frac{2\left(\left(\frac{x}{2}\right)^2 \operatorname{Ei}\left(-\frac{x}{2}\right) + \frac{1}{2}xe^{-\frac{x}{2}} - e^{-\frac{x}{2}}\right)}{16\left(\frac{x}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/exp(1/2*x)/x^3,x)

[Out] 1/8*(x^2*Ei(-1/2*x) + 2*x/e^(1/2*x) - 4/e^(1/2*x))/x^2

Mupad [B]

time = 0.27, size = 22, normalized size = 0.56

$$\frac{e^{-\frac{x}{2}}\left(\frac{1}{x} - \frac{2}{x^2}\right)}{4} - \frac{\operatorname{expint}\left(\frac{x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-x/2)/x^3,x)

[Out] (exp(-x/2)*(1/x - 2/x^2))/4 - expint(x/2)/8

3.537 $\int a^{3x} x^2 dx$

Optimal. Leaf size=44

$$\frac{2a^{3x}}{27 \log^3(a)} - \frac{2a^{3x}x}{9 \log^2(a)} + \frac{a^{3x}x^2}{3 \log(a)}$$

[Out] $2/27*a^{(3*x)}/\ln(a)^3-2/9*a^{(3*x)}*x/\ln(a)^2+1/3*a^{(3*x)}*x^2/\ln(a)$

Rubi [A]

time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2207, 2225}

$$\frac{x^2 a^{3x}}{3 \log(a)} + \frac{2a^{3x}}{27 \log^3(a)} - \frac{2xa^{3x}}{9 \log^2(a)}$$

Antiderivative was successfully verified.

[In] Int[a^(3*x)*x^2,x]

[Out] $(2*a^{(3*x)})/(27*\text{Log}[a]^3) - (2*a^{(3*x)}*x)/(9*\text{Log}[a]^2) + (a^{(3*x)}*x^2)/(3*\text{Log}[a])$

Rule 2207

Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[\$UseGamma]

Rule 2225

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int a^{3x} x^2 dx &= \frac{a^{3x} x^2}{3 \log(a)} - \frac{2 \int a^{3x} x dx}{3 \log(a)} \\ &= -\frac{2a^{3x}x}{9 \log^2(a)} + \frac{a^{3x}x^2}{3 \log(a)} + \frac{2 \int a^{3x} dx}{9 \log^2(a)} \\ &= \frac{2a^{3x}}{27 \log^3(a)} - \frac{2a^{3x}x}{9 \log^2(a)} + \frac{a^{3x}x^2}{3 \log(a)} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 29, normalized size = 0.66

$$\frac{a^{3x} (2 - 6x \log(a) + 9x^2 \log^2(a))}{27 \log^3(a)}$$

Antiderivative was successfully verified.

`[In] Integrate[a^(3*x)*x^2,x]``[Out] (a^(3*x)*(2 - 6*x*Log[a] + 9*x^2*Log[a]^2))/(27*Log[a]^3)`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.71, size = 41, normalized size = 0.93

$$\text{Piecewise} \left[\left\{ \left\{ \frac{(2 - 6x \text{Log}[a] + 9x^2 \text{Log}[a]^2) a^{3x}}{27 \text{Log}[a]^3}, \text{Log}[a]^3 \neq 0 \right\} \right\}, \frac{x^3}{3} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[x^2*a^(3*x),x]')``[Out] Piecewise[{{(2 - 6 x Log[a] + 9 x ^ 2 Log[a] ^ 2) a ^ (3 x) / (27 Log[a] ^ 3), Log[a] ^ 3 != 0}}, x ^ 3 / 3]`**Maple [A]**

time = 0.02, size = 28, normalized size = 0.64

method	result	size
gosper	$\frac{(9x^2 \ln(a)^2 - 6x \ln(a) + 2) a^{3x}}{27 \ln(a)^3}$	28
risch	$\frac{(9x^2 \ln(a)^2 - 6x \ln(a) + 2) a^{3x}}{27 \ln(a)^3}$	28
meijerg	$-\frac{2 - \frac{(27x^2 \ln(a)^2 - 18x \ln(a) + 6) e^{3x \ln(a)}}{3}}{27 \ln(a)^3}$	33
norman	$\frac{2 e^{3x \ln(a)}}{27 \ln(a)^3} - \frac{2x e^{3x \ln(a)}}{9 \ln(a)^2} + \frac{x^2 e^{3x \ln(a)}}{3 \ln(a)}$	42

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(a^(3*x)*x^2,x,method=_RETURNVERBOSE)``[Out] 1/27*(9*x^2*ln(a)^2-6*x*ln(a)+2)*a^(3*x)/ln(a)^3`**Maxima [A]**

time = 0.25, size = 27, normalized size = 0.61

$$\frac{(9x^2 \log(a)^2 - 6x \log(a) + 2) a^{3x}}{27 \log(a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^(3*x)*x^2,x, algorithm="maxima")

[Out] 1/27*(9*x^2*log(a)^2 - 6*x*log(a) + 2)*a^(3*x)/log(a)^3

Fricas [A]

time = 0.31, size = 27, normalized size = 0.61

$$\frac{(9x^2 \log(a)^2 - 6x \log(a) + 2)a^{3x}}{27 \log(a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^(3*x)*x^2,x, algorithm="fricas")

[Out] 1/27*(9*x^2*log(a)^2 - 6*x*log(a) + 2)*a^(3*x)/log(a)^3

Sympy [A]

time = 0.06, size = 37, normalized size = 0.84

$$\begin{cases} \frac{a^{3x}(9x^2 \log(a)^2 - 6x \log(a) + 2)}{27 \log(a)^3} & \text{for } \log(a)^3 \neq 0 \\ \frac{x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a**(3*x)*x**2,x)

[Out] Piecewise((a**(3*x)*(9*x**2*log(a)**2 - 6*x*log(a) + 2)/(27*log(a)**3), Ne(log(a)**3, 0)), (x**3/3, True))

Giac [C] Result contains complex when optimal does not.

time = 0.00, size = 1076, normalized size = 24.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a^(3*x)*x^2,x)

[Out] -1/27*((6*(3*pi*x^2*log(abs(a))*sgn(a) - 3*pi*x^2*log(abs(a)) - pi*x*sgn(a) + pi*x)*(pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log(abs(a))^2)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log(abs(a))^2)^2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(abs(a))^3)^2 - (9*pi^2*x^2*sgn(a) - 9*pi^2*x^2 + 18*x^2*log(abs(a))^2 - 12*x*log(abs(a)) + 4)*(3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(abs(a))^3)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log(abs(a))^2)^2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(abs(a))^3)^2))*cos(-3/2*pi*x*sgn(a) + 3/2*pi*x) - ((9*pi^2*x^2*sgn(a) - 9*pi^2*x^2 + 18*x^2*log(a

```

bs(a))^2 - 12*x*log(abs(a)) + 4)*(pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) -
pi^3 + 3*pi*log(abs(a))^2)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^
3 + 3*pi*log(abs(a))^2)^2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a))
+ 2*log(abs(a))^3)^2) + 6*(3*pi*x^2*log(abs(a))*sgn(a) - 3*pi*x^2*log(abs(
a)) - pi*x*sgn(a) + pi*x)*(3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) +
2*log(abs(a))^3)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log
(abs(a))^2)^2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(a
bs(a))^3)^2))*sin(-3/2*pi*x*sgn(a) + 3/2*pi*x))*abs(a)^(3*x) - 2*I*((-9*I*pi
i^2*x^2*sgn(a) + 18*pi*x^2*log(abs(a))*sgn(a) + 9*I*pi^2*x^2 - 18*pi*x^2*log
(abs(a)) - 18*I*x^2*log(abs(a))^2 - 6*pi*x*sgn(a) + 6*pi*x + 12*I*x*log(ab
s(a)) - 4*I)*e^(3/2*I*pi*x*sgn(a) - 3/2*I*pi*x)/(-108*I*pi^3*sgn(a) + 324*pi
i^2*log(abs(a))*sgn(a) + 324*I*pi*log(abs(a))^2*sgn(a) + 108*I*pi^3 - 324*pi
i^2*log(abs(a)) - 324*I*pi*log(abs(a))^2 + 216*log(abs(a))^3) - (-9*I*pi^2*x
^2*sgn(a) - 18*pi*x^2*log(abs(a))*sgn(a) + 9*I*pi^2*x^2 + 18*pi*x^2*log(ab
s(a)) - 18*I*x^2*log(abs(a))^2 + 6*pi*x*sgn(a) - 6*pi*x + 12*I*x*log(abs(a)
) - 4*I)*e^(-3/2*I*pi*x*sgn(a) + 3/2*I*pi*x)/(108*I*pi^3*sgn(a) + 324*pi^2*
log(abs(a))*sgn(a) - 324*I*pi*log(abs(a))^2*sgn(a) - 108*I*pi^3 - 324*pi^2*
log(abs(a)) + 324*I*pi*log(abs(a))^2 + 216*log(abs(a))^3))*abs(a)^(3*x)

```

Mupad [B]

time = 0.06, size = 27, normalized size = 0.61

$$\frac{a^{3x} (9x^2 \ln(a)^2 - 6x \ln(a) + 2)}{27 \ln(a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a^(3*x)*x^2,x)

[Out] (a^(3*x)*(9*x^2*log(a)^2 - 6*x*log(a) + 2))/(27*log(a)^3)

3.538 $\int e^{x^2} x (1 + x^2) dx$

Optimal. Leaf size=12

$$\frac{1}{2}e^{x^2} x^2$$

[Out] 1/2*exp(x^2)*x^2

Rubi [A]

time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2258, 2240, 2243}

$$\frac{1}{2}e^{x^2} x^2$$

Antiderivative was successfully verified.

[In] Int[E^x^2*x*(1 + x^2),x]

[Out] (E^x^2*x^2)/2

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n * Log[F])), x]
/; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*Log[F])), x]
- Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b*(c + d*x)^n), x], x]
/; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])
```

Rule 2258

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol]
:> Int[ExpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x]
/; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int e^{x^2} x (1 + x^2) dx &= \int (e^{x^2} x + e^{x^2} x^3) dx \\
&= \int e^{x^2} x dx + \int e^{x^2} x^3 dx \\
&= \frac{e^{x^2}}{2} + \frac{1}{2} e^{x^2} x^2 - \int e^{x^2} x dx \\
&= \frac{1}{2} e^{x^2} x^2
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 12, normalized size = 1.00

$$\frac{1}{2} e^{x^2} x^2$$

Antiderivative was successfully verified.

`[In] Integrate[E^x^2*x*(1 + x^2),x]``[Out] (E^x^2*x^2)/2`**Mathics [A]**

time = 1.57, size = 10, normalized size = 0.83

$$\frac{x^2 E^{x^2}}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x*(x^2 + 1)*E^x^2,x]')``[Out] x ^ 2 E ^ x ^ 2 / 2`**Maple [A]**

time = 0.02, size = 10, normalized size = 0.83

method	result	size
gospers	$\frac{e^{x^2} x^2}{2}$	10
derivativedivides	$\frac{e^{x^2} x^2}{2}$	10
default	$\frac{e^{x^2} x^2}{2}$	10
norman	$\frac{e^{x^2} x^2}{2}$	10

risch	$\frac{e^{x^2} x^2}{2}$	10
meijerg	$-\frac{(-2x^2+2)e^{x^2}}{4} + \frac{e^{x^2}}{2}$	21

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x^2)*x*(x^2+1),x,method=_RETURNVERBOSE)`

[Out] `1/2*exp(x^2)*x^2`

Maxima [A]

time = 0.25, size = 18, normalized size = 1.50

$$\frac{1}{2} (x^2 - 1)e^{(x^2)} + \frac{1}{2} e^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2)*x*(x^2+1),x, algorithm="maxima")`

[Out] `1/2*(x^2 - 1)*e^(x^2) + 1/2*e^(x^2)`

Fricas [A]

time = 0.31, size = 9, normalized size = 0.75

$$\frac{1}{2} x^2 e^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2)*x*(x^2+1),x, algorithm="fricas")`

[Out] `1/2*x^2*e^(x^2)`

Sympy [A]

time = 0.05, size = 8, normalized size = 0.67

$$\frac{x^2 e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x**2)*x*(x**2+1),x)`

[Out] `x**2*exp(x**2)/2`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(9) = 18.

time = 0.00, size = 18, normalized size = 1.50

$$\frac{(x^2 - 1)e^{x^2} + e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x*(x^2+1),x)
```

```
[Out] 1/2*(x^2 - 1)*e^(x^2) + 1/2*e^(x^2)
```

Mupad [B]

time = 0.05, size = 9, normalized size = 0.75

$$\frac{x^2 e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x^2)*(x^2 + 1),x)
```

```
[Out] (x^2*exp(x^2))/2
```

$$3.539 \quad \int \frac{x}{(e^{-x} + e^x)^2} dx$$

Optimal. Leaf size=32

$$\frac{x}{2} - \frac{x}{2(1 + e^{2x})} - \frac{1}{4} \log(1 + e^{2x})$$

[Out] 1/2*x-1/2*x/(1+exp(2*x))-1/4*ln(1+exp(2*x))

Rubi [A]

time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2321, 2222, 2320, 36, 29, 31}

$$-\frac{x}{2(e^{2x} + 1)} + \frac{x}{2} - \frac{1}{4} \log(e^{2x} + 1)$$

Antiderivative was successfully verified.

[In] Int[x/(E^(-x) + E^x)^2,x]

[Out] x/2 - x/(2*(1 + E^(2*x))) - Log[1 + E^(2*x)]/4

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_)*(x_))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 2222

Int[((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))^(p_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x))))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log[F]), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a + b*(F^(g*(e + f*x))))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x] && NeQ[p, -1]

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2321

```
Int[(u_)*((a_)*(F_)^(v_) + (b_)*(F_)^(w_))^(n_), x_Symbol] := Int[u*F^(n
*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ
[n, 0] && LinearQ[{v, w}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(e^{-x} + e^x)^2} dx &= \int \frac{e^{2x} x}{(1 + e^{2x})^2} dx \\
&= -\frac{x}{2(1 + e^{2x})} + \frac{1}{2} \int \frac{1}{1 + e^{2x}} dx \\
&= -\frac{x}{2(1 + e^{2x})} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{x(1 + x)} dx, x, e^{2x} \right) \\
&= -\frac{x}{2(1 + e^{2x})} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{x} dx, x, e^{2x} \right) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{1 + x} dx, x, e^{2x} \right) \\
&= \frac{x}{2} - \frac{x}{2(1 + e^{2x})} - \frac{1}{4} \log(1 + e^{2x})
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 31, normalized size = 0.97

$$\frac{e^{2x} x}{2 + 2e^{2x}} - \frac{1}{4} \log(1 + e^{2x})$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(E^(-x) + E^x)^2, x]
```

```
[Out] (E^(2*x)*x)/(2 + 2*E^(2*x)) - Log[1 + E^(2*x)]/4
```

Mathics [A]

time = 1.81, size = 40, normalized size = 1.25

$$\frac{-2x - \text{Log}[1 + E^{-2x}] - \text{Log}[1 + E^{-2x}] E^{2x}}{4 + 4E^{2x}}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x/(E^x + E^(-x))^2,x]')`

[Out] $(-2x - \text{Log}[1 + E^{(-2x)}] - \text{Log}[1 + E^{(-2x)}] E^{(2x)}) / (4(1 + E^{(2x)}))$

Maple [A]

time = 0.03, size = 26, normalized size = 0.81

method	result	size
risch	$\frac{x}{2} - \frac{x}{2(1+e^{2x})} - \frac{\ln(1+e^{2x})}{4}$	25
default	$-\frac{\ln(1+e^{2x})}{4} + \frac{e^{2x}x}{2+2e^{2x}}$	26
norman	$-\frac{\ln(1+e^{2x})}{4} + \frac{e^{2x}x}{2+2e^{2x}}$	26

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(exp(-x)+exp(x))^2,x,method=_RETURNVERBOSE)`

[Out] $-1/4*\ln(1+\exp(x)^2)+1/2*x*\exp(x)^2/(1+\exp(x)^2)$

Maxima [A]

time = 0.34, size = 25, normalized size = 0.78

$$\frac{xe^{(2x)}}{2(e^{(2x)} + 1)} - \frac{1}{4} \log(e^{(2x)} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(exp(-x)+exp(x))^2,x, algorithm="maxima")`

[Out] $1/2*x*e^{(2*x)}/(e^{(2*x)} + 1) - 1/4*\log(e^{(2*x)} + 1)$

Fricas [A]

time = 0.31, size = 33, normalized size = 1.03

$$\frac{2xe^{(2x)} - (e^{(2x)} + 1) \log(e^{(2x)} + 1)}{4(e^{(2x)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(exp(-x)+exp(x))^2,x, algorithm="fricas")`

[Out] $1/4*(2*x*e^{(2*x)} - (e^{(2*x)} + 1)*\log(e^{(2*x)} + 1))/(e^{(2*x)} + 1)$

Sympy [A]

time = 0.06, size = 24, normalized size = 0.75

$$-\frac{x}{2} + \frac{x}{2 + 2e^{-2x}} - \frac{\log(1 + e^{-2x})}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(exp(-x)+exp(x))*2,x)

[Out] -x/2 + x/(2 + 2*exp(-2*x)) - log(1 + exp(-2*x))/4

Giac [A]

time = 0.00, size = 39, normalized size = 1.22

$$\frac{2xe^{2x} - e^{2x} \ln(e^{2x} + 1) - \ln(e^{2x} + 1)}{4e^{2x} + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(exp(-x)+exp(x))^2,x)

[Out] 1/4*(2*e^(2*x)*x - e^(2*x)*log(e^(2*x) + 1) - log(e^(2*x) + 1))/(e^(2*x) + 1)

Mupad [B]

time = 0.33, size = 26, normalized size = 0.81

$$\frac{x e^{2x}}{2 (e^{2x} + 1)} - \frac{\ln(e^{2x} + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(exp(-x) + exp(x))^2,x)

[Out] (x*exp(2*x))/(2*(exp(2*x) + 1)) - log(exp(2*x) + 1)/4

$$3.540 \quad \int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}} dx$$

Optimal. Leaf size=15

$$e^x \sqrt{1-x^2}$$

[Out] exp(x)*(-x^2+1)^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2326}

$$e^x \sqrt{1-x^2}$$

Antiderivative was successfully verified.

[In] Int[(E^x*(1 - x - x^2))/Sqrt[1 - x^2],x]

[Out] E^x*Sqrt[1 - x^2]

Rule 2326

Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u, x]))}], Simp[F^u*z, x] /; EqQ[D[z, x], w*y] /; FreeQ[F, x]

Rubi steps

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}} dx = e^x \sqrt{1-x^2}$$

Mathematica [A]

time = 0.17, size = 15, normalized size = 1.00

$$e^x \sqrt{1-x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(E^x*(1 - x - x^2))/Sqrt[1 - x^2],x]

[Out] E^x*Sqrt[1 - x^2]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x - x^2)/Sqrt[1 - x^2]*E^x,x]')
```

```
[Out] caught exception: maximum recursion depth exceeded while calling a Python object
```

Maple [A]

time = 0.09, size = 20, normalized size = 1.33

method	result	size
gosper	$-\frac{e^x(1+x)(-1+x)}{\sqrt{-x^2+1}}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -exp(x)*(1+x)*(-1+x)/(-x^2+1)^(1/2)
```

Maxima [A]

time = 0.28, size = 21, normalized size = 1.40

$$-\frac{(x^2 - 1)e^x}{\sqrt{x + 1} \sqrt{-x + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(x^2 - 1)*e^x/(sqrt(x + 1)*sqrt(-x + 1))
```

Fricas [A]

time = 0.31, size = 12, normalized size = 0.80

$$\sqrt{-x^2 + 1} e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-x^2 + 1)*e^x
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{e^x}{\sqrt{1-x^2}} \right) dx - \int \frac{xe^x}{\sqrt{1-x^2}} dx - \int \frac{x^2e^x}{\sqrt{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*(-x**2-x+1)/(-x**2+1)**(1/2),x)

[Out] -Integral(-exp(x)/sqrt(1 - x**2), x) - Integral(x*exp(x)/sqrt(1 - x**2), x)
- Integral(x**2*exp(x)/sqrt(1 - x**2), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x)

[Out] Could not integrate

Mupad [B]

time = 0.45, size = 12, normalized size = 0.80

$$e^x \sqrt{1 - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(exp(x)*(x + x^2 - 1))/(1 - x^2)^(1/2),x)

[Out] exp(x)*(1 - x^2)^(1/2)

3.541 $\int e^{-3x} \cos(2x) dx$

Optimal. Leaf size=27

$$-\frac{3}{13}e^{-3x} \cos(2x) + \frac{2}{13}e^{-3x} \sin(2x)$$

[Out] $-3/13*\cos(2*x)/\exp(3*x)+2/13*\sin(2*x)/\exp(3*x)$

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4518}

$$\frac{2}{13}e^{-3x} \sin(2x) - \frac{3}{13}e^{-3x} \cos(2x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[2*x]/E^{(3*x)}, x]$

[Out] $(-3*\text{Cos}[2*x])/(13*E^{(3*x)}) + (2*\text{Sin}[2*x])/(13*E^{(3*x)})$

Rule 4518

$\text{Int}[\text{Cos}[(d_.) + (e_.)*(x_.)]*(F_)^{((c_.)*((a_.) + (b_.)*(x_.)))}, x_Symbol] := \text{Simp}[b*c*\text{Log}[F]*F^{(c*(a + b*x))*(\text{Cos}[d + e*x]/(e^2 + b^2*c^2*\text{Log}[F]^2))}, x] + \text{Simp}[e*F^{(c*(a + b*x))*(\text{Sin}[d + e*x]/(e^2 + b^2*c^2*\text{Log}[F]^2))}, x] /; \text{FreeQ}\{F, a, b, c, d, e\}, x] \&\& \text{NeQ}[e^2 + b^2*c^2*\text{Log}[F]^2, 0]$

Rubi steps

$$\int e^{-3x} \cos(2x) dx = -\frac{3}{13}e^{-3x} \cos(2x) + \frac{2}{13}e^{-3x} \sin(2x)$$

Mathematica [A]

time = 0.02, size = 22, normalized size = 0.81

$$\frac{1}{13}e^{-3x}(-3 \cos(2x) + 2 \sin(2x))$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Cos}[2*x]/E^{(3*x)}, x]$

[Out] $(-3*\text{Cos}[2*x] + 2*\text{Sin}[2*x])/(13*E^{(3*x)})$

Mathics [A]

time = 1.79, size = 20, normalized size = 0.74

$$\frac{(-3\cos[2x] + 2\sin[2x]) E^{-3x}}{13}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Cos[2*x]/E^(3*x),x]')
```

```
[Out] (-3 Cos[2 x] + 2 Sin[2 x]) E ^ (-3 x) / 13
```

Maple [A]

time = 0.04, size = 22, normalized size = 0.81

method	result	size
default	$-\frac{3e^{-3x}\cos(2x)}{13} + \frac{2e^{-3x}\sin(2x)}{13}$	22
norman	$\frac{\left(-\frac{3}{13} + \frac{3(\tan^2(x))}{13} + \frac{4\tan(x)}{13}\right)e^{-3x}}{1+\tan^2(x)}$	28
risch	$-\frac{3e^{(-3+2i)x}}{26} - \frac{ie^{(-3+2i)x}}{13} - \frac{3e^{(-3-2i)x}}{26} + \frac{ie^{(-3-2i)x}}{13}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)/exp(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] -3/13*exp(-3*x)*cos(2*x)+2/13*exp(-3*x)*sin(2*x)
```

Maxima [A]

time = 0.24, size = 19, normalized size = 0.70

$$-\frac{1}{13} (3 \cos(2x) - 2 \sin(2x)) e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/exp(3*x),x, algorithm="maxima")
```

```
[Out] -1/13*(3*cos(2*x) - 2*sin(2*x))*e^(-3*x)
```

Fricas [A]

time = 0.34, size = 21, normalized size = 0.78

$$-\frac{3}{13} \cos(2x) e^{(-3x)} + \frac{2}{13} e^{(-3x)} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/exp(3*x),x, algorithm="fricas")
```

[Out] $-3/13*\cos(2*x)*e^{(-3*x)} + 2/13*e^{(-3*x)}*\sin(2*x)$

Sympy [A]

time = 0.20, size = 26, normalized size = 0.96

$$\frac{2e^{-3x} \sin(2x)}{13} - \frac{3e^{-3x} \cos(2x)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(2*x)/exp(3*x), x)`

[Out] $2*\exp(-3*x)*\sin(2*x)/13 - 3*\exp(-3*x)*\cos(2*x)/13$

Giac [A]

time = 0.00, size = 24, normalized size = 0.89

$$e^{-3x} \left(-\frac{3}{13} \cos(2x) + \frac{2}{13} \sin(2x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(2*x)/exp(3*x), x)`

[Out] $-1/13*(3*\cos(2*x) - 2*\sin(2*x))/e^{(3*x)}$

Mupad [B]

time = 0.03, size = 19, normalized size = 0.70

$$-\frac{e^{-3x} (3 \cos(2x) - 2 \sin(2x))}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(2*x)*exp(-3*x), x)`

[Out] $-(\exp(-3*x)*(3*\cos(2*x) - 2*\sin(2*x)))/13$

$$3.542 \quad \int \frac{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}} dx$$

Optimal. Leaf size=35

$$-\frac{30 \cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}} + \frac{6 \sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

[Out] $-30/13*\cos(1/2*x)/\exp(x)^{(1/3)}+6/13*\sin(1/2*x)/\exp(x)^{(1/3)}$

Rubi [A]

time = 0.07, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2319, 6874, 4518, 4517}

$$\frac{6 \sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}} - \frac{30 \cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[x/2] + \text{Sin}[x/2])/(\text{E}^x)^{(1/3)}, x]$

[Out] $(-30*\text{Cos}[x/2])/(13*(\text{E}^x)^{(1/3)}) + (6*\text{Sin}[x/2])/(13*(\text{E}^x)^{(1/3)})$

Rule 2319

$\text{Int}[(u_*)*((a_*)*(F_)^{(v_)})^{(n_)}, x_Symbol] \rightarrow \text{Dist}[(a*F^v)^n/F^{(n*v)}, \text{Int}[u*F^{(n*v)}, x], x] /; \text{FreeQ}\{F, a, n\}, x] \&\& \text{!IntegerQ}[n]$

Rule 4517

$\text{Int}[(F_)^{((c_*)*((a_*) + (b_*)*(x_)))}*\text{Sin}[(d_*) + (e_*)*(x_)], x_Symbol] \rightarrow \text{Simp}[b*c*\text{Log}[F]*F^{(c*(a + b*x))}*(\text{Sin}[d + e*x]/(e^2 + b^2*c^2*\text{Log}[F]^2)), x] - \text{Simp}[e*F^{(c*(a + b*x))}*(\text{Cos}[d + e*x]/(e^2 + b^2*c^2*\text{Log}[F]^2)), x] /; \text{FreeQ}\{F, a, b, c, d, e\}, x] \&\& \text{NeQ}[e^2 + b^2*c^2*\text{Log}[F]^2, 0]$

Rule 4518

$\text{Int}[\text{Cos}[(d_*) + (e_*)*(x_)]*(F_)^{((c_*)*((a_*) + (b_*)*(x_)))}, x_Symbol] \rightarrow \text{Simp}[b*c*\text{Log}[F]*F^{(c*(a + b*x))}*(\text{Cos}[d + e*x]/(e^2 + b^2*c^2*\text{Log}[F]^2)), x] + \text{Simp}[e*F^{(c*(a + b*x))}*(\text{Sin}[d + e*x]/(e^2 + b^2*c^2*\text{Log}[F]^2)), x] /; \text{FreeQ}\{F, a, b, c, d, e\}, x] \&\& \text{NeQ}[e^2 + b^2*c^2*\text{Log}[F]^2, 0]$

Rule 6874

$\text{Int}[u_, x_Symbol] \rightarrow \text{With}\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]$

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}} dx &= \frac{e^{x/3} \int e^{-x/3} \left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right) dx}{\sqrt[3]{e^x}} \\
&= \frac{(6e^{x/3}) \text{Subst}\left(\int e^{-2x} (\cos(3x) + \sin(3x)) dx, x, \frac{x}{6}\right)}{\sqrt[3]{e^x}} \\
&= \frac{(6e^{x/3}) \text{Subst}\left(\int (e^{-2x} \cos(3x) + e^{-2x} \sin(3x)) dx, x, \frac{x}{6}\right)}{\sqrt[3]{e^x}} \\
&= \frac{(6e^{x/3}) \text{Subst}\left(\int e^{-2x} \cos(3x) dx, x, \frac{x}{6}\right)}{\sqrt[3]{e^x}} + \frac{(6e^{x/3}) \text{Subst}\left(\int e^{-2x} \sin(3x) dx, x, \frac{x}{6}\right)}{\sqrt[3]{e^x}} \\
&= -\frac{30 \cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}} + \frac{6 \sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 26, normalized size = 0.74

$$\frac{6 \left(-5 \cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right)}{13\sqrt[3]{e^x}}$$

Antiderivative was successfully verified.

`[In] Integrate[(Cos[x/2] + Sin[x/2])/(E^x)^(1/3), x]``[Out] (6*(-5*Cos[x/2] + Sin[x/2]))/(13*(E^x)^(1/3))`**Mathics** [A]

time = 1.84, size = 18, normalized size = 0.51

$$\frac{6 \left(\sin\left[\frac{x}{2}\right] - 5\cos\left[\frac{x}{2}\right]\right)}{13(E^x)^{\frac{1}{3}}}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(Sin[x/2] + Cos[x/2])/(E^x)^(1/3), x]')``[Out] 6 (Sin[x / 2] - 5 Cos[x / 2]) / (13 (E ^ x) ^ (1 / 3))`**Maple** [A]

time = 0.06, size = 22, normalized size = 0.63

method	result	size
--------	--------	------

default	$-\frac{30e^{-\frac{x}{3}}\cos\left(\frac{x}{2}\right)}{13} + \frac{6e^{-\frac{x}{3}}\sin\left(\frac{x}{2}\right)}{13}$	22
risch	$\frac{\left(-\frac{15}{169} - \frac{3i}{169}\right)\left((25-5i)\cos\left(\frac{x}{2}\right) + (-5+i)\sin\left(\frac{x}{2}\right)\right)}{\left(e^x\right)^{\frac{1}{3}}}$	26

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x,method=_RETURNVERBOSE)`

[Out] `-30/13*exp(-1/3*x)*cos(1/2*x)+6/13*exp(-1/3*x)*sin(1/2*x)`

Maxima [A]

time = 0.25, size = 39, normalized size = 1.11

$$-\frac{6}{13} \left(3 \cos\left(\frac{1}{2}x\right) + 2 \sin\left(\frac{1}{2}x\right) \right) e^{(-\frac{1}{3}x)} - \frac{6}{13} \left(2 \cos\left(\frac{1}{2}x\right) - 3 \sin\left(\frac{1}{2}x\right) \right) e^{(-\frac{1}{3}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x, algorithm="maxima")`

[Out] `-6/13*(3*cos(1/2*x) + 2*sin(1/2*x))*e^(-1/3*x) - 6/13*(2*cos(1/2*x) - 3*sin(1/2*x))*e^(-1/3*x)`

Fricas [A]

time = 0.30, size = 21, normalized size = 0.60

$$-\frac{30}{13} \cos\left(\frac{1}{2}x\right) e^{(-\frac{1}{3}x)} + \frac{6}{13} e^{(-\frac{1}{3}x)} \sin\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x, algorithm="fricas")`

[Out] `-30/13*cos(1/2*x)*e^(-1/3*x) + 6/13*e^(-1/3*x)*sin(1/2*x)`

Sympy [A]

time = 0.27, size = 29, normalized size = 0.83

$$\frac{6 \sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}} - \frac{30 \cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((cos(1/2*x)+sin(1/2*x))/exp(x)**(1/3),x)`

[Out] `6*sin(x/2)/(13*exp(x)**(1/3)) - 30*cos(x/2)/(13*exp(x)**(1/3))`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(21) = 42.
time = 0.00, size = 56, normalized size = 1.60

$$6 \left(e^{-\frac{x}{3}} \left(-\frac{2}{13} \cos\left(\frac{x}{2}\right) + \frac{3}{13} \sin\left(\frac{x}{2}\right) \right) + e^{-\frac{x}{3}} \left(-\frac{3}{13} \cos\left(\frac{x}{2}\right) - \frac{2}{13} \sin\left(\frac{x}{2}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x)

[Out] -6/13*(3*cos(1/2*x) + 2*sin(1/2*x))/e^(1/3*x) - 6/13*(2*cos(1/2*x) - 3*sin(1/2*x))/e^(1/3*x)

Mupad [B]

time = 0.10, size = 19, normalized size = 0.54

$$\frac{6 e^{-\frac{x}{3}} \left(5 \cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right) \right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x/2) + sin(x/2))/exp(x)^(1/3),x)

[Out] -(6*exp(-x/3)*(5*cos(x/2) - sin(x/2)))/13

$$3.543 \quad \int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}} dx$$

Optimal. Leaf size=57

$$-\frac{4 \cos\left(\frac{3x}{2}\right) \log(3)}{3\sqrt[4]{3^{3x}} (4 + \log^2(3))} + \frac{8 \sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}} (4 + \log^2(3))}$$

[Out] $-4/3*\cos(3/2*x)*\ln(3)/(3^(3*x))^(1/4)/(4+\ln(3)^2)+8/3*\sin(3/2*x)/(3^(3*x))^(1/4)/(4+\ln(3)^2)$

Rubi [A]

time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2319, 4518}

$$\frac{8 \sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}} (4 + \log^2(3))} - \frac{4 \log(3) \cos\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}} (4 + \log^2(3))}$$

Antiderivative was successfully verified.

[In] Int[Cos[(3*x)/2]/(3^(3*x))^(1/4), x]

[Out] $(-4*\cos[(3*x)/2]*\log[3])/(3*(3^(3*x))^(1/4)*(4 + \log[3]^2)) + (8*\sin[(3*x)/2])/(3*(3^(3*x))^(1/4)*(4 + \log[3]^2))$

Rule 2319

Int[(u_)*((a_)*(F_)^(v_))^(n_), x_Symbol] := Dist[(a*F^v)^n/F^(n*v), Int[u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] && !IntegerQ[n]

Rule 4518

Int[Cos[(d_)+(e_)*(x_)]*(F_)^((c_)*((a_)+(b_)*(x_))), x_Symbol] := Simp[b*c*Log[F]*F^(c*(a+b*x))*(Cos[d+e*x]/(e^2+b^2*c^2*Log[F]^2)), x] + Simp[e*F^(c*(a+b*x))*(Sin[d+e*x]/(e^2+b^2*c^2*Log[F]^2)), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2+b^2*c^2*Log[F]^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}} dx &= \frac{3^{3x/4} \int 3^{-3x/4} \cos\left(\frac{3x}{2}\right) dx}{\sqrt[4]{3^{3x}}} \\ &= -\frac{4 \cos\left(\frac{3x}{2}\right) \log(3)}{3\sqrt[4]{3^{3x}} (4 + \log^2(3))} + \frac{8 \sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}} (4 + \log^2(3))} \end{aligned}$$

Mathematica [A]

time = 0.05, size = 37, normalized size = 0.65

$$-\frac{4 \left(\cos \left(\frac{3x}{2} \right) \log(3) - 2 \sin \left(\frac{3x}{2} \right) \right)}{3 \sqrt[4]{27^x} (4 + \log^2(3))}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[(3*x)/2]/(3^(3*x))^(1/4),x]``[Out] (-4*(Cos[(3*x)/2]*Log[3] - 2*Sin[(3*x)/2]))/(3*(27^x)^(1/4)*(4 + Log[3]^2))`**Mathics [A]**

time = 2.65, size = 30, normalized size = 0.53

$$\frac{4 \left(-\cos \left[\frac{3x}{2} \right] \text{Log}[3] + 2 \sin \left[\frac{3x}{2} \right] \right)}{3 \left(4 + \text{Log}[3]^2 \right) (27^x)^{\frac{1}{4}}}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Cos[3*x/2]/(3^(3*x))^(1/4),x]')``[Out] 4 (-Cos[3 x / 2] Log[3] + 2 Sin[3 x / 2]) / (3 (4 + Log[3]^2) (27^x)^(1/4))`**Maple [C] Result contains complex when optimal does not.**

time = 0.05, size = 37, normalized size = 0.65

method	result	size
risch	$-\frac{2(2 \cos(\frac{3x}{2}) \ln(3) - 4 \sin(\frac{3x}{2}))}{3(2i + \ln(3))(-2i + \ln(3))(27^x)^{\frac{1}{4}}}$	37

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(3/2*x)/(3^(3*x))^(1/4),x,method=_RETURNVERBOSE)``[Out] -2/3/(2*I+ln(3))/(-2*I+ln(3))/(27^x)^(1/4)*(2*cos(3/2*x)*ln(3)-4*sin(3/2*x))`**Maxima [A]**

time = 0.33, size = 31, normalized size = 0.54

$$-\frac{4 \left(\cos \left(\frac{3}{2} x \right) \log(3) - 2 \sin \left(\frac{3}{2} x \right) \right)}{3 \left(\log(3)^2 + 4 \right) 3^{\frac{3}{4} x}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="maxima")`

[Out] $-4/3*(\cos(3/2*x)*\log(3) - 2*\sin(3/2*x))/((\log(3)^2 + 4)*3^{(3/4*x)})$

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

Sympy [A]

time = 0.57, size = 70, normalized size = 1.23

$$\frac{8 \sin\left(\frac{3x}{2}\right)}{3^{\frac{4}{3}3^{3x}} \log(3)^2 + 12^{\frac{4}{3}3^{3x}}} - \frac{4 \log(3) \cos\left(\frac{3x}{2}\right)}{3^{\frac{4}{3}3^{3x}} \log(3)^2 + 12^{\frac{4}{3}3^{3x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(3/2*x)/(3**(3*x))**(1/4),x)`

[Out] $8*\sin(3*x/2)/(3*(3**(3*x))**(1/4)*\log(3)**2 + 12*(3**(3*x))**(1/4)) - 4*\log(3)*\cos(3*x/2)/(3*(3**(3*x))**(1/4)*\log(3)**2 + 12*(3**(3*x))**(1/4))$

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(3/2*x)/(3^(3*x))^(1/4),x)`

[Out] Could not integrate

Mupad [B]

time = 0.04, size = 33, normalized size = 0.58

$$\frac{\frac{3 \sin\left(\frac{3x}{2}\right)}{2} - \frac{3 \cos\left(\frac{3x}{2}\right) \ln(3)}{4}}{3^{\frac{3x}{4}} \left(\frac{9 \ln(3)^2}{16} + \frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos((3*x)/2)/(3^(3*x))^(1/4),x)`

[Out] $((3*\sin((3*x)/2))/2 - (3*\cos((3*x)/2)*\log(3))/4)/(3^{((3*x)/4)*((9*\log(3)^2)/16 + 9/4)})$

3.544 $\int e^{mx} \cos^2(x) dx$

Optimal. Leaf size=54

$$\frac{2e^{mx}}{m(4+m^2)} + \frac{e^{mx}m \cos^2(x)}{4+m^2} + \frac{2e^{mx} \cos(x) \sin(x)}{4+m^2}$$

[Out] $2*\exp(m*x)/m/(m^2+4)+\exp(m*x)*m*\cos(x)^2/(m^2+4)+2*\exp(m*x)*\cos(x)*\sin(x)/(m^2+4)$

Rubi [A]

time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4520, 2225}

$$\frac{2e^{mx}}{m(m^2+4)} + \frac{me^{mx} \cos^2(x)}{m^2+4} + \frac{2e^{mx} \sin(x) \cos(x)}{m^2+4}$$

Antiderivative was successfully verified.

[In] Int[E^(m*x)*Cos[x]^2,x]

[Out] $(2*E^{(m*x)})/(m*(4+m^2)) + (E^{(m*x)*m*\cos[x]^2)/(4+m^2) + (2*E^{(m*x)*\cos[x]*\sin[x]})/(4+m^2)$

Rule 2225

Int[((F_)^((c_.)*((a_.)+(b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a+b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 4520

Int[Cos[(d_.)+(e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.)+(b_.)*(x_))), x_Symbol] :> Simp[b*c*Log[F]*F^(c*(a+b*x))*(Cos[d+e*x]^m/(e^2*m^2+b^2*c^2*Log[F]^2)), x] + (Dist[(m*(m-1)*e^2)/(e^2*m^2+b^2*c^2*Log[F]^2), Int[F^(c*(a+b*x))*Cos[d+e*x]^(m-2), x], x] + Simp[e*m*F^(c*(a+b*x))*Sin[d+e*x]*(Cos[d+e*x]^(m-1)/(e^2*m^2+b^2*c^2*Log[F]^2)), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*m^2+b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]

Rubi steps

$$\begin{aligned} \int e^{mx} \cos^2(x) dx &= \frac{e^{mx}m \cos^2(x)}{4+m^2} + \frac{2e^{mx} \cos(x) \sin(x)}{4+m^2} + \frac{2 \int e^{mx} dx}{4+m^2} \\ &= \frac{2e^{mx}}{m(4+m^2)} + \frac{e^{mx}m \cos^2(x)}{4+m^2} + \frac{2e^{mx} \cos(x) \sin(x)}{4+m^2} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 39, normalized size = 0.72

$$\frac{e^{mx} (4 + m^2 + m^2 \cos(2x) + 2m \sin(2x))}{2m (4 + m^2)}$$

Antiderivative was successfully verified.

[In] Integrate[E^(m*x)*Cos[x]^2,x]**[Out]** (E^(m*x)*(4 + m^2 + m^2*Cos[2*x] + 2*m*Sin[2*x]))/(2*m*(4 + m^2))**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 3.47, size = 178, normalized size = 3.30

$$\text{Piecewise}\left[\left\{\left\{\frac{x}{2} + \frac{\sin[2x]}{4}, m=0\right\}, \left\{\frac{(2I + 2Ix\sin[2x] + 2x\cos[2x] - 2I\cos[2x] + 3\sin[2x])E^{-2Ix}}{8}, m=-2I\right\}, \left\{\frac{(-2I - 2Ix\sin[2x] + 2x\cos[2x] + 2I\cos[2x] + 3\sin[2x])E^{2Ix}}{8}, m=2I\right\}, \frac{2m\cos[x]E^{mx}\sin[x]}{4m+m^3} + \frac{m^2E^{mx}\cos[x]^2}{4m+m^3} + \frac{2E^{mx}\cos[x]^2}{4m+m^3} + \frac{2E^{mx}\sin[x]^2}{4m+m^3}\right\}\right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[E^(m*x)*Cos[x]^2,x]')

[Out] Piecewise[{{x / 2 + Sin[2 x] / 4, m == 0}, {(2 I + 2 I x Sin[2 x] + 2 x Cos[2 x] - 2 I Cos[2 x] + 3 Sin[2 x]) E ^ (-2 I x) / 8, m == -2 I}, {(-2 I - 2 I x Sin[2 x] + 2 x Cos[2 x] + 2 I Cos[2 x] + 3 Sin[2 x]) E ^ (2 I x) / 8, m == 2 I}}, 2 m Cos[x] E ^ (m x) Sin[x] / (4 m + m ^ 3) + m ^ 2 E ^ (m x) C os[x] ^ 2 / (4 m + m ^ 3) + 2 E ^ (m x) Cos[x] ^ 2 / (4 m + m ^ 3) + 2 E ^ (m x) Sin[x] ^ 2 / (4 m + m ^ 3)]

Maple [A]

time = 0.07, size = 45, normalized size = 0.83

method	result	size
risch	$\frac{e^{mx}}{2m} + \frac{e^{(2i+m)x}}{8i+4m} + \frac{e^{x(m-2i)}}{4m-8i}$	41
default	$\frac{e^{mx}}{2m} + \frac{m e^{mx} \cos(2x)}{2m^2+8} + \frac{e^{mx} \sin(2x)}{m^2+4}$	45
norman	$\frac{(m^2+2)e^{mx}}{m(m^2+4)} + \frac{(m^2+2)e^{mx} \left(\tan^4\left(\frac{x}{2}\right)\right)}{m(m^2+4)} + \frac{4 e^{mx} \tan\left(\frac{x}{2}\right)}{m^2+4} - \frac{4 e^{mx} \left(\tan^3\left(\frac{x}{2}\right)\right)}{m^2+4} - \frac{2(m^2-2)e^{mx} \left(\tan^2\left(\frac{x}{2}\right)\right)}{m(m^2+4)}$ $\frac{\quad}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	122

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(m*x)*cos(x)^2,x,method=_RETURNVERBOSE)**[Out]** 1/2*exp(m*x)/m+1/2*m/(m^2+4)*exp(m*x)*cos(2*x)+1/(m^2+4)*exp(m*x)*sin(2*x)**Maxima [A]**

time = 0.24, size = 45, normalized size = 0.83

$$\frac{m^2 \cos(2x) e^{(mx)} + 2m e^{(mx)} \sin(2x) + (m^2 + 4) e^{(mx)}}{2(m^3 + 4m)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(m*x)*cos(x)^2,x, algorithm="maxima")

[Out] $1/2*(m^2*\cos(2*x)*e^{(m*x)} + 2*m*e^{(m*x)}*\sin(2*x) + (m^2 + 4)*e^{(m*x)})/(m^3 + 4*m)$

Fricas [A]

time = 0.32, size = 37, normalized size = 0.69

$$\frac{2 m \cos (x) e^{(m x)} \sin (x) + \left(m^2 \cos (x)^2 + 2\right) e^{(m x)}}{m^3 + 4 m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(m*x)*cos(x)^2,x, algorithm="fricas")

[Out] $(2*m*\cos(x)*e^{(m*x)}*\sin(x) + (m^2*\cos(x)^2 + 2)*e^{(m*x)})/(m^3 + 4*m)$

Sympy [A]

time = 0.44, size = 265, normalized size = 4.91

$$\begin{cases} \frac{x \sin^2(x)}{2} + \frac{x \cos^2(x)}{2} + \frac{\sin(x) \cos(x)}{2} & \text{for } m = 0 \\ -\frac{x e^{-2ix} \sin^2(x)}{4} + \frac{i x e^{-2ix} \sin(x) \cos(x)}{2} + \frac{x e^{-2ix} \cos^2(x)}{4} - \frac{e^{-2ix} \sin(x) \cos(x)}{4} + \frac{i e^{-2ix} \cos^2(x)}{2} & \text{for } m = -2i \\ -\frac{x e^{2ix} \sin^2(x)}{4} - \frac{i x e^{2ix} \sin(x) \cos(x)}{2} + \frac{x e^{2ix} \cos^2(x)}{4} - \frac{e^{2ix} \sin(x) \cos(x)}{4} - \frac{i e^{2ix} \cos^2(x)}{2} & \text{for } m = 2i \\ \frac{m^2 e^{mx} \cos^2(x)}{m^3+4m} + \frac{2m e^{mx} \sin(x) \cos(x)}{m^3+4m} + \frac{2e^{mx} \sin^2(x)}{m^3+4m} + \frac{2e^{mx} \cos^2(x)}{m^3+4m} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(m*x)*cos(x)**2,x)

[Out] Piecewise((x*sin(x)**2/2 + x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m, 0)), (-x*exp(-2*I*x)*sin(x)**2/4 + I*x*exp(-2*I*x)*sin(x)*cos(x)/2 + x*exp(-2*I*x)*cos(x)**2/4 - exp(-2*I*x)*sin(x)*cos(x)/4 + I*exp(-2*I*x)*cos(x)**2/2, Eq(m, -2*I)), (-x*exp(2*I*x)*sin(x)**2/4 - I*x*exp(2*I*x)*sin(x)*cos(x)/2 + x*exp(2*I*x)*cos(x)**2/4 - exp(2*I*x)*sin(x)*cos(x)/4 - I*exp(2*I*x)*cos(x)**2/2, Eq(m, 2*I)), (m**2*exp(m*x)*cos(x)**2/(m**3 + 4*m) + 2*m*exp(m*x)*sin(x)*cos(x)/(m**3 + 4*m) + 2*exp(m*x)*sin(x)**2/(m**3 + 4*m) + 2*exp(m*x)*cos(x)**2/(m**3 + 4*m), True))

Giac [A]

time = 0.00, size = 45, normalized size = 0.83

$$\frac{e^{mx}}{2m} + e^{mx} \left(\frac{2m \cos(2x)}{(2m)^2 + 16} + \frac{4 \sin(2x)}{(2m)^2 + 16} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(m*x)*cos(x)^2,x)

[Out] 1/2*e^(m*x)*(m*cos(2*x)/(m^2 + 4) + 2*sin(2*x)/(m^2 + 4)) + 1/2*e^(m*x)/m

Mupad [B]

time = 0.05, size = 37, normalized size = 0.69

$$\frac{e^{mx}}{2m} + \frac{e^{mx} (2 \sin(2x) + m \cos(2x))}{2(m^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(m*x)*cos(x)^2,x)

[Out] exp(m*x)/(2*m) + (exp(m*x)*(2*sin(2*x) + m*cos(2*x)))/(2*(m^2 + 4))

3.545 $\int e^{mx} \sin^3(x) dx$

Optimal. Leaf size=82

$$-\frac{6e^{mx} \cos(x)}{9 + 10m^2 + m^4} + \frac{6e^{mx} m \sin(x)}{9 + 10m^2 + m^4} - \frac{3e^{mx} \cos(x) \sin^2(x)}{9 + m^2} + \frac{e^{mx} m \sin^3(x)}{9 + m^2}$$

[Out] $-6*\exp(m*x)*\cos(x)/(m^4+10*m^2+9)+6*\exp(m*x)*m*\sin(x)/(m^4+10*m^2+9)-3*\exp(m*x)*\cos(x)*\sin(x)^2/(m^2+9)+\exp(m*x)*m*\sin(x)^3/(m^2+9)$

Rubi [A]

time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4519, 4517}

$$\frac{me^{mx} \sin^3(x)}{m^2 + 9} - \frac{3e^{mx} \sin^2(x) \cos(x)}{m^2 + 9} + \frac{6me^{mx} \sin(x)}{m^4 + 10m^2 + 9} - \frac{6e^{mx} \cos(x)}{m^4 + 10m^2 + 9}$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{(m*x)}*\text{Sin}[x]^3, x]$

[Out] $(-6*E^{(m*x)}*\text{Cos}[x])/(9 + 10*m^2 + m^4) + (6*E^{(m*x)}*m*\text{Sin}[x])/(9 + 10*m^2 + m^4) - (3*E^{(m*x)}*\text{Cos}[x]*\text{Sin}[x]^2)/(9 + m^2) + (E^{(m*x)}*m*\text{Sin}[x]^3)/(9 + m^2)$

Rule 4517

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))}*\text{Sin}[(d_.) + (e_.)*(x_)], x_Symbol] := \text{Simp}[b*c*\text{Log}[F]*F^{(c*(a + b*x))}*(\text{Sin}[d + e*x]/(e^2 + b^2*c^2*\text{Log}[F]^2)), x] - \text{Simp}[e*F^{(c*(a + b*x))}*(\text{Cos}[d + e*x]/(e^2 + b^2*c^2*\text{Log}[F]^2)), x] /; \text{FreeQ}\{F, a, b, c, d, e\}, x] \&\& \text{NeQ}[e^2 + b^2*c^2*\text{Log}[F]^2, 0]$

Rule 4519

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))}*\text{Sin}[(d_.) + (e_.)*(x_)]^{(n_)}, x_Symbol] := \text{Simp}[b*c*\text{Log}[F]*F^{(c*(a + b*x))}*(\text{Sin}[d + e*x]^{(n)}/(e^2*n^2 + b^2*c^2*\text{Log}[F]^2)), x] + (\text{Dist}[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*\text{Log}[F]^2), \text{Int}[F^{(c*(a + b*x))}*\text{Sin}[d + e*x]^{(n - 2)}, x], x] - \text{Simp}[e*n*F^{(c*(a + b*x))}*\text{Cos}[d + e*x]*(\text{Sin}[d + e*x]^{(n - 1)}/(e^2*n^2 + b^2*c^2*\text{Log}[F]^2)), x]) /; \text{FreeQ}\{F, a, b, c, d, e\}, x] \&\& \text{NeQ}[e^2*n^2 + b^2*c^2*\text{Log}[F]^2, 0] \&\& \text{GtQ}[n, 1]$

Rubi steps

$$\begin{aligned} \int e^{mx} \sin^3(x) dx &= -\frac{3e^{mx} \cos(x) \sin^2(x)}{9 + m^2} + \frac{e^{mx} m \sin^3(x)}{9 + m^2} + \frac{6 \int e^{mx} \sin(x) dx}{9 + m^2} \\ &= -\frac{6e^{mx} \cos(x)}{9 + 10m^2 + m^4} + \frac{6e^{mx} m \sin(x)}{9 + 10m^2 + m^4} - \frac{3e^{mx} \cos(x) \sin^2(x)}{9 + m^2} + \frac{e^{mx} m \sin^3(x)}{9 + m^2} \end{aligned}$$

Mathematica [A]

time = 0.12, size = 64, normalized size = 0.78

$$\frac{e^{mx} (-3(9 + m^2) \cos(x) + 3(1 + m^2) \cos(3x) - 2m(-13 - m^2 + (1 + m^2) \cos(2x)) \sin(x))}{4(9 + 10m^2 + m^4)}$$

Antiderivative was successfully verified.

`[In] Integrate[E^(m*x)*Sin[x]^3,x]`

```
[Out] (E^(m*x)*(-3*(9 + m^2)*Cos[x] + 3*(1 + m^2)*Cos[3*x] - 2*m*(-13 - m^2 + (1 + m^2)*Cos[2*x])*Sin[x]))/(4*(9 + 10*m^2 + m^4))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 6.48, size = 342, normalized size = 4.17

```
=====
```

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[E^(m*x)*Sin[x]^3,x]')`

```
[Out] Piecewise[{{(-9 x Sin[x] - 12 I x Cos[x] Sin[x]^2 + 3 I x Cos[x] + 12 x Sin[x]^3 + Cos[x] + 9 I Sin[x]^3 + 5 Cos[x] Sin[x]^2) E^(-3 I x) / 24, m == -3 I}, {(-3 I x Cos[x] + 3 x Sin[x] - 3 Cos[x] - 3 Cos[x] Sin[x]^2 - I Sin[x]^3) E^(-I x) / 8, m == -I}, {(3 I x Cos[x] + 3 x Sin[x] - 3 Cos[x] - 3 Cos[x] Sin[x]^2 + I Sin[x]^3) E^(I x) / 8, m == I}, {(-9 x Sin[x] - 3 I x Cos[x] + 12 I x Cos[x] Sin[x]^2 + 12 x Sin[x]^3 + Cos[x] - 9 I Sin[x]^3 + 5 Cos[x] Sin[x]^2) E^(3 I x) / 24, m == 3 I}}, 6 m E^(m x) Cos[x]^2 Sin[x] / (9 + 10 m^2 + m^4) + 7 m E^(m x) Sin[x]^3 / (9 + 10 m^2 + m^4) - 3 m^2 Cos[x] E^(m x) Sin[x]^2 / (9 + 10 m^2 + m^4) + m^3 E^(m x) Sin[x]^3 / (9 + 10 m^2 + m^4) - 9 Cos[x] E^(m x) Sin[x]^2 / (9 + 10 m^2 + m^4) - 6 E^(m x) Cos[x]^3 / (9 + 10 m^2 + m^4)]
```

Maple [A]

time = 0.10, size = 68, normalized size = 0.83

method	result
risch	$\frac{ie^{(3i+m)x}}{24i+8m} - \frac{3ie^{(i+m)x}}{8(i+m)} + \frac{3ie^{x(m-i)}}{8(m-i)} - \frac{ie^{x(m-3i)}}{8(m-3i)}$
default	$-\frac{3e^{mx} \cos(x)}{4(m^2+1)} + \frac{3me^{mx} \sin(x)}{4(m^2+1)} + \frac{3e^{mx} \cos(3x)}{4(m^2+9)} - \frac{me^{mx} \sin(3x)}{4(m^2+9)}$
norman	$\frac{-\frac{6e^{mx}}{m^4+10m^2+9} + \frac{6e^{mx} \left(\tan^6\left(\frac{x}{2}\right)\right)}{m^4+10m^2+9} + \frac{12me^{mx} \tan\left(\frac{x}{2}\right)}{m^4+10m^2+9} + \frac{12me^{mx} \left(\tan^5\left(\frac{x}{2}\right)\right)}{m^4+10m^2+9} - \frac{6(2m^2+3)e^{mx} \left(\tan^2\left(\frac{x}{2}\right)\right)}{m^4+10m^2+9} + \frac{6(2m^2+3)e^{mx} \left(\tan^4\left(\frac{x}{2}\right)\right)}{m^4+10m^2+9} + 8m}{(1+\tan^2\left(\frac{x}{2}\right))^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(m*x)*sin(x)^3,x,method=_RETURNVERBOSE)`

[Out] $-3/4/(m^2+1)*\exp(m*x)*\cos(x)+3/4*m/(m^2+1)*\exp(m*x)*\sin(x)+3/4/(m^2+9)*\exp(m*x)*\cos(3*x)-1/4*m/(m^2+9)*\exp(m*x)*\sin(3*x)$

Maxima [A]

time = 0.25, size = 73, normalized size = 0.89

$$\frac{3(m^2+1)\cos(3x)e^{mx} - 3(m^2+9)\cos(x)e^{mx} - (m^3+m)e^{mx}\sin(3x) + 3(m^3+9m)e^{mx}\sin(x)}{4(m^4+10m^2+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(m*x)*sin(x)^3,x, algorithm="maxima")`

[Out] $1/4*(3*(m^2+1)*\cos(3*x)*e^{m*x} - 3*(m^2+9)*\cos(x)*e^{m*x} - (m^3+m)*e^{m*x}*\sin(3*x) + 3*(m^3+9*m)*e^{m*x}*\sin(x))/(m^4+10*m^2+9)$

Fricas [A]

time = 0.32, size = 65, normalized size = 0.79

$$\frac{(m^3 - (m^3 + m)\cos(x)^2 + 7m)e^{mx}\sin(x) + 3((m^2 + 1)\cos(x)^3 - (m^2 + 3)\cos(x))e^{mx}}{m^4 + 10m^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(m*x)*sin(x)^3,x, algorithm="fricas")`

[Out] $((m^3 - (m^3 + m)\cos(x)^2 + 7m)*e^{m*x}*\sin(x) + 3*((m^2 + 1)\cos(x)^3 - (m^2 + 3)\cos(x))*e^{m*x})/(m^4 + 10*m^2 + 9)$

Sympy [A]

time = 1.39, size = 638, normalized size = 7.78

$$\left\{ \begin{array}{ll} \frac{xe^{-3ix}\sin^3(x) - 3ixe^{-3ix}\sin^2(x)\cos(x) - 3xe^{-3ix}\sin(x)\cos^2(x) + ixe^{-3ix}\cos^3(x) + \frac{7ie^{-3ix}\sin^3(x)}{24} + \frac{ie^{-3ix}\sin(x)\cos^2(x)}{4} + \frac{e^{-3ix}\cos^3(x)}{8}}{8} & \text{for } m = -3i \\ \frac{3xe^{-ix}\sin^3(x) - 3ixe^{-ix}\sin^2(x)\cos(x) + 3xe^{-ix}\sin(x)\cos^2(x) - 3ixe^{-ix}\cos^3(x) + \frac{5ie^{-ix}\sin^3(x)}{8} + \frac{3ie^{-ix}\sin(x)\cos^2(x)}{4} + \frac{3e^{-ix}\cos^3(x)}{8}}{8} & \text{for } m = -i \\ \frac{3xe^{ix}\sin^3(x) + 3ixe^{ix}\sin^2(x)\cos(x) + 3xe^{ix}\sin(x)\cos^2(x) + 3ixe^{ix}\cos^3(x) - \frac{5ie^{ix}\sin^3(x)}{8} - \frac{3ie^{ix}\sin(x)\cos^2(x)}{4} + \frac{3e^{ix}\cos^3(x)}{8}}{8} & \text{for } m = i \\ \frac{xe^{3ix}\sin^3(x) + 3ixe^{3ix}\sin^2(x)\cos(x) - 3xe^{3ix}\sin(x)\cos^2(x) - ixe^{3ix}\cos^3(x) - \frac{7ie^{3ix}\sin^3(x)}{24} - \frac{ie^{3ix}\sin(x)\cos^2(x)}{4} + \frac{e^{3ix}\cos^3(x)}{8}}{8} & \text{for } m = 3i \\ \frac{m^3e^{mx}\sin^3(x) - 3m^2e^{mx}\sin^2(x)\cos(x) + \frac{7me^{mx}\sin^3(x)}{m^4+10m^2+9} + \frac{6me^{mx}\sin(x)\cos^2(x)}{m^4+10m^2+9} - \frac{9e^{mx}\sin^2(x)\cos(x)}{m^4+10m^2+9} - \frac{6e^{mx}\cos^3(x)}{m^4+10m^2+9}}{m^4+10m^2+9} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(m*x)*sin(x)**3,x)`

[Out] $\text{Piecewise}((x*\exp(-3*I*x))*\sin(x)**3/8 - 3*I*x*\exp(-3*I*x)*\sin(x)**2*\cos(x)/8 - 3*x*\exp(-3*I*x)*\sin(x)*\cos(x)**2/8 + I*x*\exp(-3*I*x)*\cos(x)**3/8 + 7*I*e$

```

xp(-3*I*x)*sin(x)**3/24 + I*exp(-3*I*x)*sin(x)*cos(x)**2/4 + exp(-3*I*x)*co
s(x)**3/8, Eq(m, -3*I)), (3*x*exp(-I*x)*sin(x)**3/8 - 3*I*x*exp(-I*x)*sin(x)
)**2*cos(x)/8 + 3*x*exp(-I*x)*sin(x)*cos(x)**2/8 - 3*I*x*exp(-I*x)*cos(x)**
3/8 + 5*I*exp(-I*x)*sin(x)**3/8 + 3*I*exp(-I*x)*sin(x)*cos(x)**2/4 + 3*exp(
-I*x)*cos(x)**3/8, Eq(m, -I)), (3*x*exp(I*x)*sin(x)**3/8 + 3*I*x*exp(I*x)*s
in(x)**2*cos(x)/8 + 3*x*exp(I*x)*sin(x)*cos(x)**2/8 + 3*I*x*exp(I*x)*cos(x)
**3/8 - 5*I*exp(I*x)*sin(x)**3/8 - 3*I*exp(I*x)*sin(x)*cos(x)**2/4 + 3*exp(
I*x)*cos(x)**3/8, Eq(m, I)), (x*exp(3*I*x)*sin(x)**3/8 + 3*I*x*exp(3*I*x)*s
in(x)**2*cos(x)/8 - 3*x*exp(3*I*x)*sin(x)*cos(x)**2/8 - I*x*exp(3*I*x)*cos(
x)**3/8 - 7*I*exp(3*I*x)*sin(x)**3/24 - I*exp(3*I*x)*sin(x)*cos(x)**2/4 + e
xp(3*I*x)*cos(x)**3/8, Eq(m, 3*I)), (m**3*exp(m*x)*sin(x)**3/(m**4 + 10*m**
2 + 9) - 3*m**2*exp(m*x)*sin(x)**2*cos(x)/(m**4 + 10*m**2 + 9) + 7*m*exp(m*
x)*sin(x)**3/(m**4 + 10*m**2 + 9) + 6*m*exp(m*x)*sin(x)*cos(x)**2/(m**4 + 1
0*m**2 + 9) - 9*exp(m*x)*sin(x)**2*cos(x)/(m**4 + 10*m**2 + 9) - 6*exp(m*x)
*cos(x)**3/(m**4 + 10*m**2 + 9), True))

```

Giac [A]

time = 0.00, size = 71, normalized size = 0.87

$$e^{mx} \left(-\frac{4 \cdot 3 \cos x}{(4m)^2 + 16} + \frac{4 \cdot 3m \sin x}{(4m)^2 + 16} \right) + e^{mx} \left(\frac{12 \cos(3x)}{(4m)^2 + 144} - \frac{4m \sin(3x)}{(4m)^2 + 144} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(m*x)*sin(x)^3,x)

[Out] -1/4*e^(m*x)*(m*sin(3*x)/(m^2 + 9) - 3*cos(3*x)/(m^2 + 9)) + 3/4*e^(m*x)*(m*sin(x)/(m^2 + 1) - cos(x)/(m^2 + 1))

Mupad [B]

time = 0.06, size = 47, normalized size = 0.57

$$\frac{e^{mx} \left(\frac{3(\cos(x) - m \sin(x))}{m^2 + 1} - \frac{3 \cos(3x) - m \sin(3x)}{m^2 + 9} \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(m*x)*sin(x)^3,x)

[Out] -(exp(m*x)*((3*(cos(x) - m*sin(x)))/(m^2 + 1) - (3*cos(3*x) - m*sin(3*x))/(m^2 + 9)))/4

$$3.546 \quad \int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}} dx$$

Optimal. Leaf size=79

$$-\frac{48 \cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}} - \frac{2 \cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}} + \frac{32 \sin\left(\frac{x}{3}\right)}{65\sqrt{e^x}} + \frac{4 \cos^2\left(\frac{x}{3}\right) \sin\left(\frac{x}{3}\right)}{5\sqrt{e^x}}$$

[Out] $-48/65*\cos(1/3*x)/\exp(x)^{(1/2)}-2/5*\cos(1/3*x)^3/\exp(x)^{(1/2)}+32/65*\sin(1/3*x)/\exp(x)^{(1/2)}+4/5*\cos(1/3*x)^2*\sin(1/3*x)/\exp(x)^{(1/2)}$

Rubi [A]

time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2319, 4520, 4518}

$$\frac{32 \sin\left(\frac{x}{3}\right)}{65\sqrt{e^x}} - \frac{2 \cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}} - \frac{48 \cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}} + \frac{4 \sin\left(\frac{x}{3}\right) \cos^2\left(\frac{x}{3}\right)}{5\sqrt{e^x}}$$

Antiderivative was successfully verified.

[In] Int[Cos[x/3]^3/Sqrt[E^x], x]

[Out] $(-48*\text{Cos}[x/3])/(65*\text{Sqrt}[E^x]) - (2*\text{Cos}[x/3]^3)/(5*\text{Sqrt}[E^x]) + (32*\text{Sin}[x/3])/(65*\text{Sqrt}[E^x]) + (4*\text{Cos}[x/3]^2*\text{Sin}[x/3])/(5*\text{Sqrt}[E^x])$

Rule 2319

Int[(u_)*((a_)*(F_)^(v_))^(n_), x_Symbol] := Dist[(a*F^v)^n/F^(n*v), Int[u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] && !IntegerQ[n]

Rule 4518

Int[Cos[(d_)+(e_)*(x_)]*(F_)^((c_)*((a_)+(b_)*(x_))), x_Symbol] := Simp[b*c*Log[F]*F^(c*(a+b*x))*(Cos[d+e*x]/(e^2+b^2*c^2*Log[F]^2)), x] + Simp[e*F^(c*(a+b*x))*(Sin[d+e*x]/(e^2+b^2*c^2*Log[F]^2)), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2+b^2*c^2*Log[F]^2, 0]

Rule 4520

Int[Cos[(d_)+(e_)*(x_)]^(m_)*(F_)^((c_)*((a_)+(b_)*(x_))), x_Symbol] := Simp[b*c*Log[F]*F^(c*(a+b*x))*(Cos[d+e*x]^m/(e^2*m^2+b^2*c^2*Log[F]^2)), x] + (Dist[(m*(m-1)*e^2)/(e^2*m^2+b^2*c^2*Log[F]^2), Int[F^(c*(a+b*x))*Cos[d+e*x]^(m-2), x], x] + Simp[e*m*F^(c*(a+b*x))*Sin[d+e*x]*(Cos[d+e*x]^(m-1)/(e^2*m^2+b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*m^2+b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]

Rubi steps

$$\begin{aligned} \int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}} dx &= \frac{e^{x/2} \int e^{-x/2} \cos^3\left(\frac{x}{3}\right) dx}{\sqrt{e^x}} \\ &= -\frac{2 \cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}} + \frac{4 \cos^2\left(\frac{x}{3}\right) \sin\left(\frac{x}{3}\right)}{5\sqrt{e^x}} + \frac{(8e^{x/2}) \int e^{-x/2} \cos\left(\frac{x}{3}\right) dx}{15\sqrt{e^x}} \\ &= -\frac{48 \cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}} - \frac{2 \cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}} + \frac{32 \sin\left(\frac{x}{3}\right)}{65\sqrt{e^x}} + \frac{4 \cos^2\left(\frac{x}{3}\right) \sin\left(\frac{x}{3}\right)}{5\sqrt{e^x}} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 36, normalized size = 0.46

$$\frac{-135 \cos\left(\frac{x}{3}\right) - 13 \cos(x) + 90 \sin\left(\frac{x}{3}\right) + 26 \sin(x)}{130\sqrt{e^x}}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[x/3]^3/Sqrt[E^x], x]``[Out] (-135*Cos[x/3] - 13*Cos[x] + 90*Sin[x/3] + 26*Sin[x])/(130*Sqrt[E^x])`**Mathics [A]**

time = 2.45, size = 28, normalized size = 0.35

$$\frac{-135 \cos\left[\frac{x}{3}\right] - 13 \cos[x] + 26 \sin[x] + 90 \sin\left[\frac{x}{3}\right]}{130\sqrt{E^x}}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Cos[x/3]^3/Sqrt[E^x], x]')``[Out] (-135 Cos[x / 3] - 13 Cos[x] + 26 Sin[x] + 90 Sin[x / 3]) / (130 Sqrt[E ^ x])`**Maple [A]**

time = 0.09, size = 38, normalized size = 0.48

method	result	size
default	$-\frac{e^{-\frac{x}{2}} \cos(x)}{10} + \frac{e^{-\frac{x}{2}} \sin(x)}{5} - \frac{27 e^{-\frac{x}{2}} \cos\left(\frac{x}{3}\right)}{26} + \frac{9 e^{-\frac{x}{2}} \sin\left(\frac{x}{3}\right)}{13}$	38
risch	$\frac{\left(-\frac{1}{1300} - \frac{i}{650}\right)\left(-52ie^{-ix} + 65e^{ix} - 39e^{-ix} + (270 - 540i)\cos\left(\frac{x}{3}\right) + (-180 + 360i)\sin\left(\frac{x}{3}\right)\right)}{\sqrt{e^x}}$	48

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(1/3*x)^3/exp(x)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/10*\exp(-1/2*x)*\cos(x)+1/5*\exp(-1/2*x)*\sin(x)-27/26*\exp(-1/2*x)*\cos(1/3*x)+9/13*\exp(-1/2*x)*\sin(1/3*x)$

Maxima [A]

time = 0.25, size = 27, normalized size = 0.34

$$-\frac{1}{130} \left(135 \cos\left(\frac{1}{3}x\right) + 13 \cos(x) - 90 \sin\left(\frac{1}{3}x\right) - 26 \sin(x) \right) e^{(-\frac{1}{2}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/3*x)^3/exp(x)^(1/2),x, algorithm="maxima")`

[Out] $-1/130*(135*\cos(1/3*x) + 13*\cos(x) - 90*\sin(1/3*x) - 26*\sin(x))*e^{(-1/2*x)}$

Fricas [A]

time = 0.31, size = 42, normalized size = 0.53

$$\frac{4}{65} \left(13 \cos\left(\frac{1}{3}x\right)^2 + 8 \right) e^{(-\frac{1}{2}x)} \sin\left(\frac{1}{3}x\right) - \frac{2}{65} \left(13 \cos\left(\frac{1}{3}x\right)^3 + 24 \cos\left(\frac{1}{3}x\right) \right) e^{(-\frac{1}{2}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/3*x)^3/exp(x)^(1/2),x, algorithm="fricas")`

[Out] $4/65*(13*\cos(1/3*x)^2 + 8)*e^{(-1/2*x)}*\sin(1/3*x) - 2/65*(13*\cos(1/3*x)^3 + 24*\cos(1/3*x))*e^{(-1/2*x)}$

Sympy [A]

time = 0.54, size = 76, normalized size = 0.96

$$\frac{32 \sin^3\left(\frac{x}{3}\right)}{65 \sqrt{e^x}} - \frac{48 \sin^2\left(\frac{x}{3}\right) \cos\left(\frac{x}{3}\right)}{65 \sqrt{e^x}} + \frac{84 \sin\left(\frac{x}{3}\right) \cos^2\left(\frac{x}{3}\right)}{65 \sqrt{e^x}} - \frac{74 \cos^3\left(\frac{x}{3}\right)}{65 \sqrt{e^x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/3*x)**3/exp(x)**(1/2),x)`

[Out] $32*\sin(x/3)**3/(65*\sqrt{\exp(x)}) - 48*\sin(x/3)**2*\cos(x/3)/(65*\sqrt{\exp(x)}) + 84*\sin(x/3)*\cos(x/3)**2/(65*\sqrt{\exp(x)}) - 74*\cos(x/3)**3/(65*\sqrt{\exp(x)})$

Giac [A]

time = 0.00, size = 49, normalized size = 0.62

$$6 \left(e^{-\frac{x}{2}} \left(-\frac{9}{52} \cos\left(\frac{x}{3}\right) + \frac{3}{26} \sin\left(\frac{x}{3}\right) \right) + e^{-\frac{x}{2}} \left(-\frac{\cos x}{60} + \frac{\sin x}{30} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/3*x)^3/exp(x)^(1/2),x)`

[Out] $-9/26*(3*\cos(1/3*x) - 2*\sin(1/3*x))/e^{(1/2*x)} - 1/10*(\cos(x) - 2*\sin(x))/e^{(1/2*x)}$

Mupad [B]

time = 0.30, size = 39, normalized size = 0.49

$$e^{-\frac{x}{2}} \left(\frac{8 \cos\left(\frac{x}{3}\right)^3}{5} - \frac{16 \sin\left(\frac{x}{3}\right) \cos\left(\frac{x}{3}\right)^2}{5} + \frac{192 \cos\left(\frac{x}{3}\right)}{65} - \frac{128 \sin\left(\frac{x}{3}\right)}{65} \right)$$

4

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x/3)^3/exp(x)^(1/2),x)`

[Out] $-(\exp(-x/2)*((192*\cos(x/3))/65 - (128*\sin(x/3))/65 - (16*\cos(x/3)^2*\sin(x/3))/5 + (8*\cos(x/3)^3)/5))/4$

3.547 $\int e^{2x} \cos^2(x) \sin^2(x) dx$

Optimal. Leaf size=36

$$\frac{e^{2x}}{16} - \frac{1}{80}e^{2x} \cos(4x) - \frac{1}{40}e^{2x} \sin(4x)$$

[Out] 1/16*exp(2*x)-1/80*exp(2*x)*cos(4*x)-1/40*exp(2*x)*sin(4*x)

Rubi [A]

time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4557, 2225, 4518}

$$\frac{e^{2x}}{16} - \frac{1}{40}e^{2x} \sin(4x) - \frac{1}{80}e^{2x} \cos(4x)$$

Antiderivative was successfully verified.

[In] Int[E^(2*x)*Cos[x]^2*Sin[x]^2,x]

[Out] E^(2*x)/16 - (E^(2*x)*Cos[4*x])/80 - (E^(2*x)*Sin[4*x])/40

Rule 2225

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 4518

Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]

Rule 4557

Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)), Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}\int e^{2x} \cos^2(x) \sin^2(x) dx &= \int \left(\frac{e^{2x}}{8} - \frac{1}{8} e^{2x} \cos(4x) \right) dx \\ &= \frac{1}{8} \int e^{2x} dx - \frac{1}{8} \int e^{2x} \cos(4x) dx \\ &= \frac{e^{2x}}{16} - \frac{1}{80} e^{2x} \cos(4x) - \frac{1}{40} e^{2x} \sin(4x)\end{aligned}$$

Mathematica [A]

time = 0.03, size = 21, normalized size = 0.58

$$-\frac{1}{80} e^{2x} (-5 + \cos(4x) + 2 \sin(4x))$$

Antiderivative was successfully verified.

`[In] Integrate[E^(2*x)*Cos[x]^2*Sin[x]^2,x]``[Out] -1/80*(E^(2*x)*(-5 + Cos[4*x] + 2*Sin[4*x]))`**Mathics [A]**

time = 2.78, size = 46, normalized size = 1.28

$$\frac{\left(4 - 2\sqrt{2} \sin\left[\frac{\text{Pi}}{4} + 2x\right] - (1 - \cos[2x])^2 - \sin[4x] + 2\sin[2x]\right) E^{2x}}{40}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[E^(2*x)*Sin[x]^2*Cos[x]^2,x]')``[Out] (4 - 2 Sqrt[2] Sin[Pi / 4 + 2 x] - (1 - Cos[2 x]) ^ 2 - Sin[4 x] + 2 Sin[2 x]) E ^ (2 x) / 40`**Maple [A]**

time = 0.04, size = 28, normalized size = 0.78

method	result
default	$\frac{e^{2x}}{16} - \frac{e^{2x} \cos(4x)}{80} - \frac{e^{2x} \sin(4x)}{40}$
risch	$\frac{e^{2x}}{16} - \frac{e^{(2+4i)x}}{160} + \frac{ie^{(2+4i)x}}{80} - \frac{e^{(2-4i)x}}{160} - \frac{ie^{(2-4i)x}}{80}$
norman	$\frac{-\frac{e^{2x} \tan\left(\frac{x}{2}\right)}{5} + \frac{3e^{2x} \left(\tan^2\left(\frac{x}{2}\right)\right)}{5} + \frac{7e^{2x} \left(\tan^3\left(\frac{x}{2}\right)\right)}{5} - \frac{e^{2x} \left(\tan^4\left(\frac{x}{2}\right)\right)}{2} - \frac{7e^{2x} \left(\tan^5\left(\frac{x}{2}\right)\right)}{5} + \frac{3e^{2x} \left(\tan^6\left(\frac{x}{2}\right)\right)}{5} + \frac{e^{2x} \left(\tan^7\left(\frac{x}{2}\right)\right)}{5} + \frac{e^{2x} \left(\tan^8\left(\frac{x}{2}\right)\right)}{20}}{\left(1 + \tan^2\left(\frac{x}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(2*x)*cos(x)^2*sin(x)^2,x,method=_RETURNVERBOSE)`

[Out] $-\frac{1}{80}\exp(2x)\cos(4x)-\frac{1}{40}\exp(2x)\sin(4x)+\frac{1}{16}\exp(x)^2$

Maxima [A]

time = 0.24, size = 27, normalized size = 0.75

$$-\frac{1}{80}\cos(4x)e^{(2x)} - \frac{1}{40}e^{(2x)}\sin(4x) + \frac{1}{16}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(2*x)*cos(x)^2*sin(x)^2,x, algorithm="maxima")`

[Out] $-1/80*\cos(4*x)*e^{(2*x)} - 1/40*e^{(2*x)}*\sin(4*x) + 1/16*e^{(2*x)}$

Fricas [A]

time = 0.31, size = 40, normalized size = 1.11

$$-\frac{1}{10}(2\cos(x)^3 - \cos(x))e^{(2x)}\sin(x) - \frac{1}{20}(2\cos(x)^4 - 2\cos(x)^2 - 1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(2*x)*cos(x)^2*sin(x)^2,x, algorithm="fricas")`

[Out] $-1/10*(2*\cos(x)^3 - \cos(x))*e^{(2*x)}*\sin(x) - 1/20*(2*\cos(x)^4 - 2*\cos(x)^2 - 1)*e^{(2*x)}$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(29) = 58

time = 0.74, size = 70, normalized size = 1.94

$$\frac{e^{2x}\sin^4(x)}{20} + \frac{e^{2x}\sin^3(x)\cos(x)}{10} + \frac{e^{2x}\sin^2(x)\cos^2(x)}{5} - \frac{e^{2x}\sin(x)\cos^3(x)}{10} + \frac{e^{2x}\cos^4(x)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(2*x)*cos(x)**2*sin(x)**2,x)`

[Out] $\exp(2x)*\sin(x)**4/20 + \exp(2x)*\sin(x)**3*\cos(x)/10 + \exp(2x)*\sin(x)**2*\cos(x)**2/5 - \exp(2x)*\sin(x)*\cos(x)**3/10 + \exp(2x)*\cos(x)**4/20$

Giac [A]

time = 0.00, size = 30, normalized size = 0.83

$$\frac{e^{2x}}{16} + e^{2x} \left(-\frac{\cos(4x)}{80} - \frac{\sin(4x)}{40} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(2*x)*cos(x)^2*sin(x)^2,x)`

[Out] $-1/80*e^{(2*x)}*(\cos(4*x) + 2*\sin(4*x)) + 1/16*e^{(2*x)}$

Mupad [B]

time = 0.41, size = 18, normalized size = 0.50

$$-\frac{e^{2x} (\cos(4x) + 2 \sin(4x) - 5)}{80}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\exp(2*x)*\cos(x)^2*\sin(x)^2,x)$

[Out] $-(\exp(2*x)*(\cos(4*x) + 2*\sin(4*x) - 5))/80$

$$3.548 \quad \int e^{3x} \cos^2\left(\frac{3x}{2}\right) \sin^2\left(\frac{3x}{2}\right) dx$$

Optimal. Leaf size=36

$$\frac{e^{3x}}{24} - \frac{1}{120}e^{3x} \cos(6x) - \frac{1}{60}e^{3x} \sin(6x)$$

[Out] 1/24*exp(3*x)-1/120*exp(3*x)*cos(6*x)-1/60*exp(3*x)*sin(6*x)

Rubi [A]

time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4557, 2225, 4518}

$$\frac{e^{3x}}{24} - \frac{1}{60}e^{3x} \sin(6x) - \frac{1}{120}e^{3x} \cos(6x)$$

Antiderivative was successfully verified.

[In] Int[E^(3*x)*Cos[(3*x)/2]^2*Sin[(3*x)/2]^2,x]

[Out] E^(3*x)/24 - (E^(3*x)*Cos[6*x])/120 - (E^(3*x)*Sin[6*x])/60

Rule 2225

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 4518

Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]

Rule 4557

Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)), Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int e^{3x} \cos^2\left(\frac{3x}{2}\right) \sin^2\left(\frac{3x}{2}\right) dx &= \int \left(\frac{e^{3x}}{8} - \frac{1}{8}e^{3x} \cos(6x)\right) dx \\ &= \frac{1}{8} \int e^{3x} dx - \frac{1}{8} \int e^{3x} \cos(6x) dx \\ &= \frac{e^{3x}}{24} - \frac{1}{120}e^{3x} \cos(6x) - \frac{1}{60}e^{3x} \sin(6x) \end{aligned}$$

Mathematica [A]

time = 0.03, size = 21, normalized size = 0.58

$$-\frac{1}{120}e^{3x}(-5 + \cos(6x) + 2\sin(6x))$$

Antiderivative was successfully verified.

`[In] Integrate[E^(3*x)*Cos[(3*x)/2]^2*Sin[(3*x)/2]^2,x]``[Out] -1/120*(E^(3*x)*(-5 + Cos[6*x] + 2*Sin[6*x]))`**Mathics [A]**

time = 2.97, size = 21, normalized size = 0.58

$$\frac{\left(\frac{5}{2} - \sin[6x] - \frac{\cos[6x]}{2}\right) E^{3x}}{60}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[E^(3*x)*Sin[3*(x/2)]^2*Cos[3*(x/2)]^2,x']')``[Out] (5 / 2 - Sin[6 x] - Cos[6 x] / 2) E ^ (3 x) / 60`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(27) = 54.

time = 0.06, size = 63, normalized size = 1.75

method	result
risch	$\frac{e^{3x}}{24} - \frac{e^{(3+6i)x}}{240} + \frac{ie^{(3+6i)x}}{120} - \frac{e^{(3-6i)x}}{240} - \frac{ie^{(3-6i)x}}{120}$
default	$-\frac{4(3\cos(x)+6\sin(x))e^{3x}(\cos^5(x))}{45} + \frac{2(3\cos(x)+4\sin(x))e^{3x}(\cos^3(x))}{15} - \frac{(3\cos(x)+2\sin(x))e^{3x}\cos(x)}{20} + \frac{e^{3x}}{20}$
norman	$-\frac{2e^{3x}\tan\left(\frac{3x}{4}\right)}{15} + \frac{2e^{3x}\left(\tan^2\left(\frac{3x}{4}\right)\right)}{5} + \frac{14e^{3x}\left(\tan^3\left(\frac{3x}{4}\right)\right)}{15} - \frac{e^{3x}\left(\tan^4\left(\frac{3x}{4}\right)\right)}{3} - \frac{14e^{3x}\left(\tan^5\left(\frac{3x}{4}\right)\right)}{15} + \frac{2e^{3x}\left(\tan^6\left(\frac{3x}{4}\right)\right)}{5} + \frac{2e^{3x}\left(\tan^7\left(\frac{3x}{4}\right)\right)}{15} + \frac{e^{3x}}{(1+\tan^2\left(\frac{3x}{4}\right))^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x,method=_RETURNVERBOSE)

[Out] -4/45*(3*cos(x)+6*sin(x))*exp(3*x)*cos(x)^5+2/15*(3*cos(x)+4*sin(x))*exp(3*x)*cos(x)^3-1/20*(3*cos(x)+2*sin(x))*exp(3*x)*cos(x)+1/20*exp(x)^3

Maxima [A]

time = 0.25, size = 27, normalized size = 0.75

$$-\frac{1}{120} \cos(6x) e^{(3x)} - \frac{1}{60} e^{(3x)} \sin(6x) + \frac{1}{24} e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x, algorithm="maxima")

[Out] -1/120*cos(6*x)*e^(3*x) - 1/60*e^(3*x)*sin(6*x) + 1/24*e^(3*x)

Fricas [A]

time = 0.31, size = 50, normalized size = 1.39

$$-\frac{1}{15} \left(2 \cos\left(\frac{3}{2}x\right)^3 - \cos\left(\frac{3}{2}x\right) \right) e^{(3x)} \sin\left(\frac{3}{2}x\right) - \frac{1}{30} \left(2 \cos\left(\frac{3}{2}x\right)^4 - 2 \cos\left(\frac{3}{2}x\right)^2 - 1 \right) e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x, algorithm="fricas")

[Out] -1/15*(2*cos(3/2*x)^3 - cos(3/2*x))*e^(3*x)*sin(3/2*x) - 1/30*(2*cos(3/2*x)^4 - 2*cos(3/2*x)^2 - 1)*e^(3*x)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(29) = 58

time = 0.73, size = 99, normalized size = 2.75

$$\frac{e^{3x} \sin^4\left(\frac{3x}{2}\right)}{30} + \frac{e^{3x} \sin^3\left(\frac{3x}{2}\right) \cos\left(\frac{3x}{2}\right)}{15} + \frac{2e^{3x} \sin^2\left(\frac{3x}{2}\right) \cos^2\left(\frac{3x}{2}\right)}{15} - \frac{e^{3x} \sin\left(\frac{3x}{2}\right) \cos^3\left(\frac{3x}{2}\right)}{15} + \frac{e^{3x} \cos^4\left(\frac{3x}{2}\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3*x)*cos(3/2*x)**2*sin(3/2*x)**2,x)

[Out] exp(3*x)*sin(3*x/2)**4/30 + exp(3*x)*sin(3*x/2)**3*cos(3*x/2)/15 + 2*exp(3*x)*sin(3*x/2)**2*cos(3*x/2)**2/15 - exp(3*x)*sin(3*x/2)*cos(3*x/2)**3/15 + exp(3*x)*cos(3*x/2)**4/30

Giac [A]

time = 0.00, size = 34, normalized size = 0.94

$$\frac{2}{3} \left(\frac{e^{3x}}{16} + e^{3x} \left(-\frac{\cos(6x)}{80} - \frac{\sin(6x)}{40} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x)`

[Out] `-1/120*e^(3*x)*(cos(6*x) + 2*sin(6*x)) + 1/24*e^(3*x)`

Mupad [B]

time = 0.39, size = 18, normalized size = 0.50

$$\frac{e^{3x} (\cos(6x) + 2 \sin(6x) - 5)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos((3*x)/2)^2*sin((3*x)/2)^2*exp(3*x),x)`

[Out] `-(exp(3*x)*(cos(6*x) + 2*sin(6*x) - 5))/120`

3.549 $\int e^{mx} \tan^2(x) dx$

Optimal. Leaf size=58

$$-\frac{e^{mx}}{m} + \frac{4e^{(2i+m)x} {}_2F_1\left(2, 1 - \frac{im}{2}; 2 - \frac{im}{2}; -e^{2ix}\right)}{2i + m}$$

[Out] $-\exp(m*x)/m + 4*\exp((2*I+m)*x)*\text{hypergeom}([2, 1-1/2*I*m], [2-1/2*I*m], -\exp(2*I*x))/(2*I+m)$

Rubi [A]

time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.47, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4527, 2225, 2283}

$$\frac{4e^{mx} {}_2F_1\left(1, -\frac{im}{2}; 1 - \frac{im}{2}; -e^{2ix}\right)}{m} - \frac{4e^{mx} {}_2F_1\left(2, -\frac{im}{2}; 1 - \frac{im}{2}; -e^{2ix}\right)}{m} - \frac{e^{mx}}{m}$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{(m*x)}*\text{Tan}[x]^2, x]$

[Out] $-(E^{(m*x)}/m) + (4*E^{(m*x)}*\text{Hypergeometric2F1}[1, (-1/2*I)*m, 1 - (I/2)*m, -E^{((2*I)*x)}])/m - (4*E^{(m*x)}*\text{Hypergeometric2F1}[2, (-1/2*I)*m, 1 - (I/2)*m, -E^{((2*I)*x)}])/m$

Rule 2225

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))}^{(n_.)}, x_Symbol] \text{ :> } \text{Simp}[(F^{(c*(a + b*x))})^n/(b*c*n*\text{Log}[F]), x] \text{ /; } \text{FreeQ}\{F, a, b, c, n\}, x]$

Rule 2283

$\text{Int}[(a_.) + (b_.)*(F_)^{((e_.)*((c_.) + (d_.)*(x_)))}^{(p_)}*(G_)^{((h_.)*((f_.) + (g_.)*(x_)))}, x_Symbol] \text{ :> } \text{Simp}[a^p*(G^{(h*(f + g*x))})/(g*h*\text{Log}[G])*Hypergeometric2F1[-p, g*h*(\text{Log}[G]/(d*e*\text{Log}[F])), g*h*(\text{Log}[G]/(d*e*\text{Log}[F])) + 1, \text{Simplify}[(-b/a)*F^{(e*(c + d*x)}]), x] \text{ /; } \text{FreeQ}\{F, G, a, b, c, d, e, f, g, h, p\}, x] \ \&\& \ (\text{ILtQ}[p, 0] \ || \ \text{GtQ}[a, 0])$

Rule 4527

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))}*\text{Tan}[(d_.) + (e_.)*(x_)]^{(n_.)}, x_Symbol] \text{ :> } \text{Dist}[I^n, \text{Int}[\text{ExpandIntegrand}[F^{(c*(a + b*x))}*((1 - E^{(2*I*(d + e*x))})^n)/(1 + E^{(2*I*(d + e*x))})^n], x], x] \text{ /; } \text{FreeQ}\{F, a, b, c, d, e\}, x] \ \&\& \ \text{IntegerQ}[n]$

Rubi steps

$$\begin{aligned}
\int e^{mx} \tan^2(x) dx &= - \int \left(e^{mx} + \frac{4e^{mx}}{(1+e^{2ix})^2} - \frac{4e^{mx}}{1+e^{2ix}} \right) dx \\
&= - \left(4 \int \frac{e^{mx}}{(1+e^{2ix})^2} dx \right) + 4 \int \frac{e^{mx}}{1+e^{2ix}} dx - \int e^{mx} dx \\
&= -\frac{e^{mx}}{m} + \frac{4e^{mx} {}_2F_1\left(1, -\frac{im}{2}; 1 - \frac{im}{2}; -e^{2ix}\right)}{m} - \frac{4e^{mx} {}_2F_1\left(2, -\frac{im}{2}; 1 - \frac{im}{2}; -e^{2ix}\right)}{m}
\end{aligned}$$

Mathematica [A]

time = 0.17, size = 97, normalized size = 1.67

$$\frac{e^{mx} \left(-1 + \frac{ie^{2ix} m^2 {}_2F_1\left(1, 1 - \frac{im}{2}; 2 - \frac{im}{2}; -e^{2ix}\right)}{2i+m} - im {}_2F_1\left(1, -\frac{im}{2}; 1 - \frac{im}{2}; -e^{2ix}\right) + m \tan(x) \right)}{m}$$

Antiderivative was successfully verified.

`[In] Integrate[E^(m*x)*Tan[x]^2,x]`

```
[Out] (E^(m*x)*(-1 + (I*E^((2*I)*x))*m^2*Hypergeometric2F1[1, 1 - (I/2)*m, 2 - (I/2)*m, -E^((2*I)*x)])/(2*I + m) - I*m*Hypergeometric2F1[1, (-1/2*I)*m, 1 - (I/2)*m, -E^((2*I)*x)] + m*Tan[x])/m
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[E^(m*x)*Tan[x]^2,x]')`

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python object
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int e^{mx} (\tan^2(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(exp(m*x)*tan(x)^2,x)``[Out] int(exp(m*x)*tan(x)^2,x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(m*x)*tan(x)^2,x, algorithm="maxima")

[Out] $-(m^4 + 20m^2 + 64)\cos(4x)^2e^{mx} - 4(m^4 + 12m^2 - 64)\cos(2x)^2e^{mx} + (m^4 + 20m^2 + 64)e^{mx}\sin(4x)^2 - 4(m^4 + 12m^2 - 64)e^{mx}\sin(2x)^2 - 16(11m^2 - 16)\cos(2x)e^{mx} + 8(5m^3 - 16m)e^{mx}\sin(2x) + 2(8(m^2 + 16)\cos(2x)e^{mx} + 4(m^3 + 16m)e^{mx}\sin(2x) + (m^4 - 28m^2 + 64)e^{mx})\cos(4x) + (m^4 - 76m^2 + 64)e^{mx} - 16(m^6 + 20m^4 + (m^6 + 20m^4 + 64m^2)\cos(4x)^2 + 4(m^6 + 20m^4 + 64m^2)\cos(2x)^2 + (m^6 + 20m^4 + 64m^2)\sin(4x)^2 + 4(m^6 + 20m^4 + 64m^2)\sin(4x)\sin(2x) + 4(m^6 + 20m^4 + 64m^2)\sin(2x)^2 + 64m^2 + 2(m^6 + 20m^4 + 64m^2 + 2(m^6 + 20m^4 + 64m^2)\cos(2x))\cos(4x) + 4(m^6 + 20m^4 + 64m^2)\cos(2x))\int(-6m\cos(6x)e^{mx} + 18m\cos(4x)e^{mx} + 18m\cos(2x)e^{mx} - (m^2 - 8)e^{mx}\sin(6x) - 3(m^2 - 8)e^{mx}\sin(4x) - 3(m^2 - 8)e^{mx}\sin(2x) + 6me^{mx})/(m^4 + (m^4 + 20m^2 + 64)\cos(6x)^2 + 9(m^4 + 20m^2 + 64)\cos(4x)^2 + 9(m^4 + 20m^2 + 64)\cos(2x)^2 + (m^4 + 20m^2 + 64)\sin(6x)^2 + 9(m^4 + 20m^2 + 64)\sin(4x)^2 + 18(m^4 + 20m^2 + 64)\sin(4x)\sin(2x) + 9(m^4 + 20m^2 + 64)\sin(2x)^2 + 20m^2 + 2(m^4 + 20m^2 + 3(m^4 + 20m^2 + 64)\cos(4x) + 3(m^4 + 20m^2 + 64)\cos(2x) + 64)\cos(6x) + 6(m^4 + 20m^2 + 3(m^4 + 20m^2 + 64)\cos(2x) + 64)\cos(4x) + 6(m^4 + 20m^2 + 64)\cos(2x) + 6((m^4 + 20m^2 + 64)\sin(4x) + (m^4 + 20m^2 + 64)\sin(2x))\sin(6x) + 64, x) - 8((m^3 + 16m)\cos(2x)e^{mx} - 2(m^2 + 16)e^{mx}\sin(2x) - 2(m^3 - 8m)e^{mx})\sin(4x))/(m^5 + 20m^3 + (m^5 + 20m^3 + 64m)\cos(4x)^2 + 4(m^5 + 20m^3 + 64m)\cos(2x)^2 + (m^5 + 20m^3 + 64m)\sin(4x)^2 + 4(m^5 + 20m^3 + 64m)\sin(4x)\sin(2x) + 4(m^5 + 20m^3 + 64m)\sin(2x)^2 + 2(m^5 + 20m^3 + 2(m^5 + 20m^3 + 64m)\cos(2x) + 64m)\cos(4x) + 4(m^5 + 20m^3 + 64m)\cos(2x) + 64m)$

Fricas [F]

time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(m*x)*tan(x)^2,x, algorithm="fricas")**[Out]** integral(e^{m*x}*tan(x)^2, x)**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int e^{mx} \tan^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(m*x)*tan(x)**2,x)`

[Out] `Integral(exp(m*x)*tan(x)**2, x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(m*x)*tan(x)^2,x)`

[Out] `Could not integrate`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int e^{mx} \tan(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(m*x)*tan(x)^2,x)`

[Out] `int(exp(m*x)*tan(x)^2, x)`

3.550 $\int e^{mx} \csc^2(x) dx$

Optimal. Leaf size=45

$$-\frac{4e^{(2i+m)x} {}_2F_1\left(2, 1 - \frac{im}{2}; 2 - \frac{im}{2}; e^{2ix}\right)}{2i + m}$$

[Out] $-4*\exp((2*I+m)*x)*\text{hypergeom}([2, 1-1/2*I*m], [2-1/2*I*m], \exp(2*I*x))/(2*I+m)$

Rubi [A]

time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4538}

$$-\frac{4e^{(m+2i)x} {}_2F_1\left(2, 1 - \frac{im}{2}; 2 - \frac{im}{2}; e^{2ix}\right)}{m + 2i}$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{(m*x)}*\text{Csc}[x]^2, x]$

[Out] $(-4*E^{((2*I + m)*x)}*\text{Hypergeometric2F1}[2, 1 - (I/2)*m, 2 - (I/2)*m, E^{((2*I)*x)}])/(2*I + m)$

Rule 4538

$\text{Int}[\text{Csc}[(d_.) + (e_.)*(x_.)]^{(n_.)}*(F_.)^{((c_.)*((a_.) + (b_.)*(x_.)))}, x_Symbol] := \text{Simp}[(-2*I)^n * E^{(I*n*(d + e*x))} * (F^{(c*(a + b*x))}) / (I*e^n + b*c*\text{Log}[F]) * \text{Hypergeometric2F1}[n, n/2 - I*b*c*(\text{Log}[F]/(2*e)), 1 + n/2 - I*b*c*(\text{Log}[F]/(2*e)), E^{(2*I*(d + e*x))}], x] /; \text{FreeQ}\{F, a, b, c, d, e\}, x] \&\& \text{IntegerQ}[n]$

Rubi steps

$$\int e^{mx} \csc^2(x) dx = -\frac{4e^{(2i+m)x} {}_2F_1\left(2, 1 - \frac{im}{2}; 2 - \frac{im}{2}; e^{2ix}\right)}{2i + m}$$

Mathematica [A]

time = 0.14, size = 90, normalized size = 2.00

$$\frac{e^{mx} \left(e^{2ix} m {}_2F_1\left(1, 1 - \frac{im}{2}; 2 - \frac{im}{2}; e^{2ix}\right) + (2i + m) \left(-i \cot(x) + {}_2F_1\left(1, -\frac{im}{2}; 1 - \frac{im}{2}; e^{2ix}\right) \right) \right)}{-2 + im}$$

Antiderivative was successfully verified.

[In] Integrate[E^(m*x)*Csc[x]^2,x]

[Out] (E^(m*x)*(E^((2*I)*x)*m*Hypergeometric2F1[1, 1 - (I/2)*m, 2 - (I/2)*m, E^((2*I)*x)] + (2*I + m)*((-I)*Cot[x] + Hypergeometric2F1[1, (-1/2*I)*m, 1 - (I/2)*m, E^((2*I)*x)])))/(-2 + I*m)

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[E^(m*x)/Sin[x]^2,x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [F]

time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{e^{mx}}{\sin(x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(m*x)/sin(x)^2,x)

[Out] int(exp(m*x)/sin(x)^2,x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(m*x)/sin(x)^2,x, algorithm="maxima")

[Out] 4*(2*(m^3 + 16*m)*cos(2*x)^2*e^(m*x) + 2*(m^3 + 16*m)*e^(m*x)*sin(2*x)^2 - (m^3 + 64*m)*cos(2*x)*e^(m*x) + 2*(5*m^2 - 16)*e^(m*x)*sin(2*x) - ((m^3 + 16*m)*cos(2*x)*e^(m*x) - 2*(m^2 + 16)*e^(m*x)*sin(2*x) - 24*m*e^(m*x))*cos(4*x) + 24*m*e^(m*x) - 4*(m^5 + 20*m^3 + (m^5 + 20*m^3 + 64*m)*cos(4*x)^2 + 4*(m^5 + 20*m^3 + 64*m)*cos(2*x)^2 + (m^5 + 20*m^3 + 64*m)*sin(4*x)^2 - 4*(m^5 + 20*m^3 + 64*m)*sin(4*x)*sin(2*x) + 4*(m^5 + 20*m^3 + 64*m)*sin(2*x)^2 + 2*(m^5 + 20*m^3 - 2*(m^5 + 20*m^3 + 64*m)*cos(2*x) + 64*m)*cos(4*x) - 4*(m^5 + 20*m^3 + 64*m)*cos(2*x) + 64*m)*integrate(-(6*m*cos(6*x)*e^(m*x) - 18*m*cos(4*x)*e^(m*x) + 18*m*cos(2*x)*e^(m*x) - (m^2 - 8)*e^(m*x)*sin(6*x) + 3*(m^2 - 8)*e^(m*x)*sin(4*x) - 3*(m^2 - 8)*e^(m*x)*sin(2*x) - 6*m*e^(m*x))/ (m^4 + (m^4 + 20*m^2 + 64)*cos(6*x)^2 + 9*(m^4 + 20*m^2 + 64)*cos(4*x)^2 +

$9*(m^4 + 20*m^2 + 64)*\cos(2*x)^2 + (m^4 + 20*m^2 + 64)*\sin(6*x)^2 + 9*(m^4 + 20*m^2 + 64)*\sin(4*x)^2 - 18*(m^4 + 20*m^2 + 64)*\sin(4*x)*\sin(2*x) + 9*(m^4 + 20*m^2 + 64)*\sin(2*x)^2 + 20*m^2 - 2*(m^4 + 20*m^2 + 3*(m^4 + 20*m^2 + 64)*\cos(4*x) - 3*(m^4 + 20*m^2 + 64)*\cos(2*x) + 64)*\cos(6*x) + 6*(m^4 + 20*m^2 - 3*(m^4 + 20*m^2 + 64)*\cos(2*x) + 64)*\cos(4*x) - 6*(m^4 + 20*m^2 + 64)*\cos(2*x) - 6*((m^4 + 20*m^2 + 64)*\sin(4*x) - (m^4 + 20*m^2 + 64)*\sin(2*x))*\sin(6*x) + 64), x) - (2*(m^2 + 16)*\cos(2*x)*e^{m*x} + (m^3 + 16*m)*e^{m*x})*\sin(2*x) + 4*(m^2 - 8)*e^{m*x})*\sin(4*x))/(m^4 + (m^4 + 20*m^2 + 64)*\cos(4*x)^2 + 4*(m^4 + 20*m^2 + 64)*\cos(2*x)^2 + (m^4 + 20*m^2 + 64)*\sin(4*x)^2 - 4*(m^4 + 20*m^2 + 64)*\sin(4*x)*\sin(2*x) + 4*(m^4 + 20*m^2 + 64)*\sin(2*x)^2 + 20*m^2 + 2*(m^4 + 20*m^2 - 2*(m^4 + 20*m^2 + 64)*\cos(2*x) + 64)*\cos(4*x) - 4*(m^4 + 20*m^2 + 64)*\cos(2*x) + 64)$

Fricas [F]

time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(m*x)/sin(x)^2,x, algorithm="fricas")

[Out] integral(-e^{m*x}/(cos(x)^2 - 1), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{mx}}{\sin^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(m*x)/sin(x)**2,x)

[Out] Integral(exp(m*x)/sin(x)**2, x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(m*x)/sin(x)^2,x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^{mx}}{\sin(x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)/sin(x)^2,x)
```

```
[Out] int(exp(m*x)/sin(x)^2, x)
```

3.551 $\int e^{mx} \sec^3(x) dx$

Optimal. Leaf size=51

$$\frac{8e^{(3i+m)x} {}_2F_1\left(3, \frac{1}{2}(3-im); \frac{1}{2}(5-im); -e^{2ix}\right)}{3i+m}$$

[Out] $8*\exp((3*I+m)*x)*\text{hypergeom}([3, 3/2-1/2*I*m], [5/2-1/2*I*m], -\exp(2*I*x))/(3*I+m)$

Rubi [A]

time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.51, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4533, 4536}

$$(-m+i)(-e^{(m+i)x}) {}_2F_1\left(1, \frac{1}{2}(1-im); \frac{1}{2}(3-im); -e^{2ix}\right) - \frac{1}{2}me^{mx} \sec(x) + \frac{1}{2}e^{mx} \tan(x) \sec(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{(m*x)}*\text{Sec}[x]^3, x]$

[Out] $-(E^{((I+m)*x)}*(I-m)*\text{Hypergeometric2F1}[1, (1-I*m)/2, (3-I*m)/2, -E^{(2*I)*x}]) - (E^{(m*x)}*m*\text{Sec}[x])/2 + (E^{(m*x)}*\text{Sec}[x]*\text{Tan}[x])/2$

Rule 4533

$\text{Int}[(F_)^{((c_.)*((a_.)+(b_.)*(x_)))}*\text{Sec}[(d_.)+(e_.)*(x_)]^{(n_.)}, x_Symbol] :> \text{Simp}[(-b)*c*\text{Log}[F]*F^{(c*(a+b*x))*(\text{Sec}[d+e*x]^{(n-2)})/(e^{2*(n-1)}*(n-2))}, x] + (\text{Dist}[(e^{2*(n-2)}+b^2*c^2*\text{Log}[F]^2)/(e^{2*(n-1)}*(n-2))], \text{Int}[F^{(c*(a+b*x))*\text{Sec}[d+e*x]^{(n-2)}}, x], x] + \text{Simp}[F^{(c*(a+b*x))*\text{Sec}[d+e*x]^{(n-1)}*(\text{Sin}[d+e*x]/(e*(n-1)))}, x] /; \text{FreeQ}\{F, a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2*c^2*\text{Log}[F]^2 + e^{2*(n-2)}, 0] \&\& \text{GtQ}[n, 1] \&\& \text{NeQ}[n, 2]$

Rule 4536

$\text{Int}[(F_)^{((c_.)*((a_.)+(b_.)*(x_)))}*\text{Sec}[(d_.)+(e_.)*(x_)]^{(n_.)}, x_Symbol] :> \text{Simp}[2^n*E^{(I*n*(d+e*x))*F^{(c*(a+b*x))}/(I*e^n + b*c*\text{Log}[F])]*\text{Hypergeometric2F1}[n, n/2 - I*b*c*(\text{Log}[F]/(2*e)), 1 + n/2 - I*b*c*(\text{Log}[F]/(2*e)), -E^{(2*I*(d+e*x))}], x] /; \text{FreeQ}\{F, a, b, c, d, e\}, x] \&\& \text{IntegerQ}[n]$

Rubi steps

$$\begin{aligned} \int e^{mx} \sec^3(x) dx &= -\frac{1}{2}e^{mx} m \sec(x) + \frac{1}{2}e^{mx} \sec(x) \tan(x) + \frac{1}{2}(1+m^2) \int e^{mx} \sec(x) dx \\ &= -e^{(i+m)x}(i-m) {}_2F_1\left(1, \frac{1}{2}(1-im); \frac{1}{2}(3-im); -e^{2ix}\right) - \frac{1}{2}e^{mx} m \sec(x) + \frac{1}{2}e^{mx} \sec(x) \tan(x) \end{aligned}$$

Mathematica [A]

time = 0.04, size = 66, normalized size = 1.29

$$\frac{1}{2}e^{mx} \left(2e^{ix}(-i+m) {}_2F_1 \left(1, \frac{1}{2} - \frac{im}{2}; \frac{3}{2} - \frac{im}{2}; -e^{2ix} \right) + \sec(x)(-m + \tan(x)) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[E^(m*x)*Sec[x]^3,x]`

```
[Out] (E^(m*x)*(2*E^(I*x)*(-I + m)*Hypergeometric2F1[1, 1/2 - (I/2)*m, 3/2 - (I/2)*m, -E^((2*I)*x)] + Sec[x]*(-m + Tan[x]))/2
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[E^(m*x)/Cos[x]^3,x]')`

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python object
```

Maple [F]

time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{e^{mx}}{\cos(x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(exp(m*x)/cos(x)^3,x)`

```
[Out] int(exp(m*x)/cos(x)^3,x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(m*x)/cos(x)^3,x, algorithm="maxima")`

```
[Out] 8*(48*m*cos(x)*e^(m*x) + 6*(m^2 - 15)*e^(m*x)*sin(x) + ((m^3 + 25*m)*cos(3*x)*e^(m*x) + 48*m*cos(x)*e^(m*x) - 3*(m^2 + 25)*e^(m*x)*sin(3*x) + 6*(m^2 - 15)*e^(m*x)*sin(x))*cos(6*x) + 3*((m^3 + 25*m)*cos(3*x)*e^(m*x) + 48*m*cos(x)*e^(m*x) - 3*(m^2 + 25)*e^(m*x)*sin(3*x) + 6*(m^2 - 15)*e^(m*x)*sin(x))*
```

$$\begin{aligned}
& \cos(4*x) + (3*(m^3 + 25*m)*\cos(2*x)*e^{(m*x)} + 9*(m^2 + 25)*e^{(m*x)}*\sin(2*x) \\
& + (m^3 + 25*m)*e^{(m*x)})*\cos(3*x) + 18*(8*m*\cos(x)*e^{(m*x)} + (m^2 - 15)*e^{(m*x)}*\sin(x))*\cos(2*x) - 6*(m^4 + (m^4 + 34*m^2 + 225)*\cos(6*x)^2 + 9*(m^4 + 34*m^2 + 225)*\cos(4*x)^2 + 9*(m^4 + 34*m^2 + 225)*\cos(2*x)^2 + (m^4 + 34*m^2 + 225)*\sin(6*x)^2 + 9*(m^4 + 34*m^2 + 225)*\sin(4*x)^2 + 18*(m^4 + 34*m^2 + 225)*\sin(4*x)*\sin(2*x) + 9*(m^4 + 34*m^2 + 225)*\sin(2*x)^2 + 34*m^2 + 2*(m^4 + 34*m^2 + 3*(m^4 + 34*m^2 + 225)*\cos(4*x) + 3*(m^4 + 34*m^2 + 225)*\cos(2*x) + 225)*\cos(6*x) + 6*(m^4 + 34*m^2 + 3*(m^4 + 34*m^2 + 225)*\cos(2*x) + 225)*\cos(4*x) + 6*(m^4 + 34*m^2 + 225)*\cos(2*x) + 6*((m^4 + 34*m^2 + 225)*\sin(4*x) + (m^4 + 34*m^2 + 225)*\sin(2*x))*\sin(6*x) + 225)*\integrate(((m^2 - 15)*\cos(x)*e^{(m*x)} - 8*m*e^{(m*x)}*\sin(x) + ((m^2 - 15)*\cos(x)*e^{(m*x)} - 8*m*e^{(m*x)}*\sin(x))*\cos(8*x) + 4*((m^2 - 15)*\cos(x)*e^{(m*x)} - 8*m*e^{(m*x)}*\sin(x))*\cos(6*x) + 6*((m^2 - 15)*\cos(x)*e^{(m*x)} - 8*m*e^{(m*x)}*\sin(x))*\cos(4*x) + 4*((m^2 - 15)*\cos(x)*e^{(m*x)} - 8*m*e^{(m*x)}*\sin(x))*\cos(2*x) + (8*m*\cos(x)*e^{(m*x)} + (m^2 - 15)*e^{(m*x)}*\sin(x))*\sin(8*x) + 4*(8*m*\cos(x)*e^{(m*x)} + (m^2 - 15)*e^{(m*x)}*\sin(x))*\sin(6*x) + 6*(8*m*\cos(x)*e^{(m*x)} + (m^2 - 15)*e^{(m*x)}*\sin(x))*\sin(4*x) + 4*(8*m*\cos(x)*e^{(m*x)} + (m^2 - 15)*e^{(m*x)}*\sin(x))*\sin(2*x))/(m^4 + (m^4 + 34*m^2 + 225)*\cos(8*x)^2 + 16*(m^4 + 34*m^2 + 225)*\cos(6*x)^2 + 36*(m^4 + 34*m^2 + 225)*\cos(4*x)^2 + 16*(m^4 + 34*m^2 + 225)*\cos(2*x)^2 + (m^4 + 34*m^2 + 225)*\sin(8*x)^2 + 16*(m^4 + 34*m^2 + 225)*\sin(6*x)^2 + 36*(m^4 + 34*m^2 + 225)*\sin(4*x)^2 + 48*(m^4 + 34*m^2 + 225)*\sin(4*x)*\sin(2*x) + 16*(m^4 + 34*m^2 + 225)*\sin(2*x)^2 + 34*m^2 + 2*(m^4 + 34*m^2 + 4*(m^4 + 34*m^2 + 225)*\cos(6*x) + 6*(m^4 + 34*m^2 + 225)*\cos(4*x) + 4*(m^4 + 34*m^2 + 225)*\cos(2*x) + 225)*\cos(8*x) + 8*(m^4 + 34*m^2 + 6*(m^4 + 34*m^2 + 225)*\cos(4*x) + 4*(m^4 + 34*m^2 + 225)*\cos(2*x) + 225)*\cos(6*x) + 12*(m^4 + 34*m^2 + 4*(m^4 + 34*m^2 + 225)*\cos(2*x) + 225)*\cos(4*x) + 8*(m^4 + 34*m^2 + 225)*\cos(2*x) + 4*(2*(m^4 + 34*m^2 + 225)*\sin(6*x) + 3*(m^4 + 34*m^2 + 225)*\sin(4*x) + 2*(m^4 + 34*m^2 + 225)*\sin(2*x))*\sin(8*x) + 16*(3*(m^4 + 34*m^2 + 225)*\sin(4*x) + 2*(m^4 + 34*m^2 + 225)*\sin(2*x))*\sin(6*x) + 225), x) - 6*(m^5 + 34*m^3 + (m^5 + 34*m^3 + 225*m)*\cos(6*x)^2 + 9*(m^5 + 34*m^3 + 225*m)*\cos(4*x)^2 + 9*(m^5 + 34*m^3 + 225*m)*\cos(2*x)^2 + (m^5 + 34*m^3 + 225*m)*\sin(6*x)^2 + 9*(m^5 + 34*m^3 + 225*m)*\sin(4*x)^2 + 18*(m^5 + 34*m^3 + 225*m)*\sin(4*x)*\sin(2*x) + 9*(m^5 + 34*m^3 + 225*m)*\sin(2*x)^2 + 2*(m^5 + 34*m^3 + 3*(m^5 + 34*m^3 + 225*m)*\cos(4*x) + 3*(m^5 + 34*m^3 + 225*m)*\cos(2*x) + 225*m)*\cos(6*x) + 6*(m^5 + 34*m^3 + 3*(m^5 + 34*m^3 + 225*m)*\cos(2*x) + 225*m)*\cos(4*x) + 6*(m^5 + 34*m^3 + 225*m)*\cos(2*x) + 6*((m^5 + 34*m^3 + 225*m)*\sin(4*x) + (m^5 + 34*m^3 + 225*m)*\sin(2*x))*\sin(6*x) + 225*m)*\integrate((8*m*\cos(x)*e^{(m*x)} + (m^2 - 15)*e^{(m*x)}*\sin(x) + (8*m*\cos(x)*e^{(m*x)} + (m^2 - 15)*e^{(m*x)}*\sin(x))*\cos(8*x) + 4*(8*m*\cos(x)*e^{(m*x)} + (m^2 - 15)*e^{(m*x)}*\sin(x))*\cos(6*x) + 6*(8*m*\cos(x)*e^{(m*x)} + (m^2 - 15)*e^{(m*x)}*\sin(x))*\cos(4*x) + 4*(8*m*\cos(x)*e^{(m*x)} + (m^2 - 15)*e^{(m*x)}*\sin(x))*\cos(2*x) - ((m^2 - 15)*\cos(x)*e^{(m*x)} - 8*m*e^{(m*x)}*\sin(x))*\sin(8*x) - 4*((m^2 - 15)*\cos(x)*e^{(m*x)} - 8*m*e^{(m*x)}*\sin(x))*\sin(6*x) - 6*((m^2 - 15)*\cos(x)*e^{(m*x)} - 8*m*e^{(m*x)}*\sin(x))*\sin(4*x) - 4*((m^2 - 15)*\cos(x)*e^{(m*x)} - 8*m*e^{(m*x)}*\sin(x))*\sin(2*x))/(m^4 + (m^4 + 34*m^2 + 225)*\cos(8*x)^2 + 16*(m^4
\end{aligned}$$

+ 34*m^2 + 225)*cos(6*x)^2 + 36*(m^4 + 34*m^2 + 225)*cos(4*x)^2 + 16*(m^4 + 34*m^2 + 225)*cos(2*x)^2 + (m^4 + 34*m^2 + 225)*sin(8*x)^2 + 16*(m^4 + 34*m^2 + 225)*sin(6*x)^2 + 36*(m^4 + 34*m^2 + 225)*sin(4*x)^2 + 48*(m^4 + 34*m^2 + 225)*sin(4*x)*sin(2*x) + 16*(m^4 + 34*m^2 + 225)*sin(2*x)^2 + 34*m^2 + 2*(m^4 + 34*m^2 + 4*(m^4 + 34*m^2 + 225)*cos(6*x) + 6*(m^4 + 34*m^2 + 225)*cos(4*x) + 4*(m^4 + 34*m^2 + 225)*cos(2*x) + 225)*cos(8*x) + 8*(m^4 + 34*m^2 + 6*(m^4 + 34*m^2 + 225)*cos(4*x) + 4*(m^4 + 34*m^2 + 225)*cos(2*x) + 225)*cos(6*x) + 12*(m^4 + 34*m^2 + 4*(m^4 + 34*m^2 + 225)*cos(2*x) + 225)*cos(4*x) + 8*(m^4 + 34*m^2 + 225)*cos(2*x) + 4*(2*(m^4 + 34*m^2 + 225)*sin(6*x) + 3*(m^4 + 34*m^2 + 225)*sin(4*x) + 2*(m^4 + 34*m^2 + 225)*sin(2*x))*sin(8*x) + 16*(3*(m^4 + 34*m^2 + 225)*sin(4*x) + 2*(m^4 + 34*m^2 + 225)*sin(2*x))*sin(6*x) + 225), x) + (3*(m^2 + 25)*cos(3*x)*e^(m*x) - 6*(m^2 - 15)*cos(x)*e^(m*x) + (m^3 + 25*m)*e^(m*x)*sin(3*x) + 48*m*e^(m*x)*sin(x))*sin(6*x) + 3*(3*(m^2 + 25)*cos(3*x)*e^(m*x) - 6*(m^2 - 15)*cos(x)*e^(m*x) + (m^3 + 25*m)*e^(m*x)*sin(3*x) + 48*m*e^(m*x)*sin(x))*sin(4*x) - 3*(3*(m^2 + 25)*cos(2*x)*e^(m*x) - (m^3 + 25*m)*e^(m*x)*sin(2*x) + (m^2 + 25)*e^(m*x))*sin(3*x) - 18*((m^2 - 15)*cos(x)*e^(m*x) - 8*m*e^(m*x)*sin(x))*sin(2*x))/(m^4 + (m^4 + 34*m^2 + 225)*cos(6*x)^2 + 9*(m^4 + 34*m^2 + 225)*cos(4*x)^2 + 9*(m^4 + 34*m^2 + 225)*cos(2*x)^2 + (m^4 + 34*m^2 + 225)*sin(6*x)^2 + 9*(m^4 + 34*m^2 + 225)*sin(4*x)^2 + 18*(m^4 + 34*m^2 + 225)*sin(4*x)*sin(2*x) + 9*(m^4 + 34*m^2 + 225)*sin(2*x)^2 + 34*m^2 + 2*(m^4 + 34*m^2 + 3*(m^4 + 34*m^2 + 225)*cos(4*x) + 3*(m^4 + 34*m^2 + 225)*cos(2*x) + 225)*cos(6*x) + 6*(m^4 + 34*m^2 + 3*(m^4 + 34*m^2 + 225)*cos(2*x) + 225)*cos(4*x) + 6*(m^4 + 34*m^2 + 225)*cos(2*x) + 6*((m^4 + 34*m^2 + 225)*sin(4*x) + (m^4 + 34*m^2 + 225)*sin(2*x))*sin(6*x) + 225)

Fricas [F]

time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(m*x)/cos(x)^3,x, algorithm="fricas")

[Out] integral(e^(m*x)/cos(x)^3, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{mx}}{\cos^3(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(m*x)/cos(x)**3,x)

[Out] Integral(exp(m*x)/cos(x)**3, x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(m*x)/cos(x)^3,x)`

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^{mx}}{\cos(x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(m*x)/cos(x)^3,x)`

[Out] `int(exp(m*x)/cos(x)^3, x)`

$$3.552 \quad \int \frac{e^x}{1+\cos(x)} dx$$

Optimal. Leaf size=28

$$(1-i)e^{(1+i)x} {}_2F_1(1-i, 2; 2-i; -e^{ix})$$

[Out] (1-I)*exp((1+I)*x)*hypergeom([2, 1-I], [2-I], -exp(I*x))

Rubi [A]

time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4542, 4536}

$$(1-i)e^{(1+i)x} {}_2F_1(1-i, 2; 2-i; -e^{ix})$$

Antiderivative was successfully verified.

[In] Int[E^x/(1 + Cos[x]), x]

[Out] (1 - I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, -E^(I*x)]

Rule 4536

Int[(F_)^((c_)*((a_) + (b_)*(x_)))*Sec[(d_) + (e_)*(x_)]^(n_), x_Symbol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x)))/(I*e*n + b*c*Log[F])*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]

Rule 4542

Int[(Cos[(d_) + (e_)*(x_)]*(g_) + (f_)]^(n_)*(F_)^((c_)*((a_) + (b_)*(x_))), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && ILtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{e^x}{1+\cos(x)} dx &= \frac{1}{2} \int e^x \sec^2\left(\frac{x}{2}\right) dx \\ &= (1-i)e^{(1+i)x} {}_2F_1(1-i, 2; 2-i; -e^{ix}) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 28, normalized size = 1.00

$$(1-i)e^{(1+i)x} {}_2F_1(1-i, 2; 2-i; -e^{ix})$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(1 + Cos[x]),x]

[Out] (1 - I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, -E^(I*x)]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[E^x/(1 + Cos[x]),x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{e^x}{1 + \cos(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(1+cos(x)),x)

[Out] int(exp(x)/(1+cos(x)),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(1+cos(x)),x, algorithm="maxima")

[Out] -2*((cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)

Fricas [F]

time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(1+cos(x)),x, algorithm="fricas")

[Out] `integral(e^x/(cos(x) + 1), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x}{\cos(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+cos(x)),x)`

[Out] `Integral(exp(x)/(cos(x) + 1), x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+cos(x)),x)`

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{e^x}{\cos(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)/(cos(x) + 1),x)`

[Out] `int(exp(x)/(cos(x) + 1), x)`

3.553 $\int \frac{e^x}{1-\cos(x)} dx$

Optimal. Leaf size=26

$$(-1+i)e^{(1+i)x} {}_2F_1(1-i, 2; 2-i; e^{ix})$$

[Out] (-1+I)*exp((1+I)*x)*hypergeom([2, 1-I], [2-I], exp(I*x))

Rubi [A]

time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4543, 4538}

$$(-1+i)e^{(1+i)x} {}_2F_1(1-i, 2; 2-i; e^{ix})$$

Antiderivative was successfully verified.

[In] Int[E^x/(1 - Cos[x]),x]

[Out] (-1 + I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, E^(I*x)]

Rule 4538

Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] := Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x)))/(I*e*n + b*c*Log[F])]*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]

Rule 4543

Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] := Dist[2^n*f^n, Int[F^(c*(a + b*x))*Sin[d/2 + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f + g, 0] && ILtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{e^x}{1-\cos(x)} dx &= \frac{1}{2} \int e^x \csc^2\left(\frac{x}{2}\right) dx \\ &= (-1+i)e^{(1+i)x} {}_2F_1(1-i, 2; 2-i; e^{ix}) \end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 84 vs. $2(26) = 52$.

time = 0.05, size = 84, normalized size = 3.23

$$\frac{(1+i)e^x \sin\left(\frac{x}{2}\right) \left((1-i)\cos\left(\frac{x}{2}\right) + (1+i) {}_2F_1\left(-i, 1; 1-i; e^{ix}\right) \sin\left(\frac{x}{2}\right) + e^{ix} {}_2F_1\left(1, 1-i; 2-i; e^{ix}\right) \sin\left(\frac{x}{2}\right) \right)}{-1 + \cos(x)}$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(1 - Cos[x]),x]

[Out] ((1 + I)*E^x*Sin[x/2]*((1 - I)*Cos[x/2] + (1 + I)*Hypergeometric2F1[-I, 1, 1 - I, E^(I*x)]*Sin[x/2] + E^(I*x)*Hypergeometric2F1[1, 1 - I, 2 - I, E^(I*x)]*Sin[x/2]))/(-1 + Cos[x])

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[E^x/(1 - Cos[x]),x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [F]

time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{e^x}{1 - \cos(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(1-cos(x)),x)

[Out] int(exp(x)/(1-cos(x)),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(1-cos(x)),x, algorithm="maxima")

[Out] 2*((cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)

Fricas [F]

time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)/(1-cos(x)),x, algorithm="fricas")``[Out] integral(-e^x/(cos(x) - 1), x)`**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{e^x}{\cos(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)/(1-cos(x)),x)``[Out] -Integral(exp(x)/(cos(x) - 1), x)`**Giac [F] N/A**

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)/(1-cos(x)),x)``[Out] Could not integrate`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.04

$$- \int \frac{e^x}{\cos(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(-exp(x)/(cos(x) - 1),x)``[Out] -int(exp(x)/(cos(x) - 1), x)`

$$3.554 \quad \int \frac{e^x}{1+\sin(x)} dx$$

Optimal. Leaf size=30

$$(-1+i)e^{(1-i)x} {}_2F_1(1+i, 2; 2+i; -ie^{-ix})$$

[Out] (-1+I)*exp((1-I)*x)*hypergeom([2, 1+I], [2+I], -I/exp(I*x))

Rubi [A]

time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4541, 4535}

$$(-1+i)e^{(1-i)x} {}_2F_1(1+i, 2; 2+i; -ie^{-ix})$$

Antiderivative was successfully verified.

[In] Int[E^x/(1 + Sin[x]),x]

[Out] (-1 + I)*E^((1 - I)*x)*Hypergeometric2F1[1 + I, 2, 2 + I, (-I)/E^(I*x)]

Rule 4535

Int[(F_)^((c_)*((a_) + (b_)*(x_)))*Sec[(d_) + Pi*(k_) + (e_)*(x_)]^(n_), x_Symbol] :> Simp[2^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*(F^(c*(a + b*x)))/(I*e*n + b*c*Log[F])*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e)), (-E^(2*I*k*Pi))*E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]

Rule 4541

Int[(F_)^((c_)*((a_) + (b_)*(x_)))*((f_) + (g_)*Sin[(d_) + (e_)*(x_)])^(n_), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 - f*(Pi/(4*g)) + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f^2 - g^2, 0] && ILtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{e^x}{1+\sin(x)} dx &= \frac{1}{2} \int e^x \sec^2\left(\frac{\pi}{4} - \frac{x}{2}\right) dx \\ &= (-1+i)e^{(1-i)x} {}_2F_1(1+i, 2; 2+i; -ie^{-ix}) \end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 61 vs. 2(30) = 60.

time = 0.42, size = 61, normalized size = 2.03

$$\frac{2e^x \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)} - (1-i)(1-(1-i) {}_2F_1(-i, 1; 1-i; i \cos(x) - \sin(x)))(\cosh(x) + \sinh(x))$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(1 + Sin[x]),x]

[Out] (2*E^x*Sin[x/2])/(Cos[x/2] + Sin[x/2]) - (1 - I)*(1 - (1 - I)*Hypergeometric2F1[-I, 1, 1 - I, I*Cos[x] - Sin[x]])*(Cosh[x] + Sinh[x])

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[E^x/(1 + Sin[x]),x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [F]

time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{e^x}{\sin(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(sin(x)+1),x)

[Out] int(exp(x)/(sin(x)+1),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(1+sin(x)),x, algorithm="maxima")

[Out] -2*(cos(x)*e^x - (cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)*integrate(cos(x)*e^x/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)

Fricas [F]

time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+sin(x)),x, algorithm="fricas")`

[Out] `integral(e^x/(sin(x) + 1), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^x}{\sin(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+sin(x)),x)`

[Out] `Integral(exp(x)/(sin(x) + 1), x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+sin(x)),x)`

[Out] `Could not integrate`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{e^x}{\sin(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)/(sin(x) + 1),x)`

[Out] `int(exp(x)/(sin(x) + 1), x)`

3.555 $\int \frac{e^x}{1-\sin(x)} dx$

Optimal. Leaf size=30

$$(1+i)e^{(1+i)x} {}_2F_1(1-i, 2; 2-i; -ie^{ix})$$

[Out] (1+I)*exp((1+I)*x)*hypergeom([2, 1-I], [2-I], -I*exp(I*x))

Rubi [A]

time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4541, 4535}

$$(1+i)e^{(1+i)x} {}_2F_1(1-i, 2; 2-i; -ie^{ix})$$

Antiderivative was successfully verified.

[In] Int[E^x/(1 - Sin[x]),x]

[Out] (1 + I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, (-I)*E^(I*x)]

Rule 4535

Int[(F_)^((c_)*((a_) + (b_)*(x_)))*Sec[(d_) + Pi*(k_) + (e_)*(x_)]^(n_), x_Symbol] :> Simp[2^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*(F^(c*(a + b*x)))/(I*e*n + b*c*Log[F])*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e)), (-E^(2*I*k*Pi))*E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]

Rule 4541

Int[(F_)^((c_)*((a_) + (b_)*(x_)))*((f_) + (g_)*Sin[(d_) + (e_)*(x_)])^(n_), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 - f*(Pi/(4*g)) + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f^2 - g^2, 0] && ILtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{e^x}{1-\sin(x)} dx &= \frac{1}{2} \int e^x \sec^2\left(\frac{\pi}{4} + \frac{x}{2}\right) dx \\ &= (1+i)e^{(1+i)x} {}_2F_1(1-i, 2; 2-i; -ie^{ix}) \end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 61 vs. $2(30) = 60$.

time = 0.44, size = 61, normalized size = 2.03

$$\frac{2e^x \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)} + (1+i)(1 - (1+i) {}_2F_1(-i, 1; 1-i; -i \cos(x) + \sin(x)))(\cosh(x) + \sinh(x))$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(1 - Sin[x]),x]

[Out] (2*E^x*Sin[x/2])/(Cos[x/2] - Sin[x/2]) + (1 + I)*(1 - (1 + I)*Hypergeometric2F1[-I, 1, 1 - I, (-I)*Cos[x] + Sin[x]])*(Cosh[x] + Sinh[x])

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[E^x/(1 - Sin[x]),x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [F]

time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{e^x}{1 - \sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(1-sin(x)),x)

[Out] int(exp(x)/(1-sin(x)),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(1-sin(x)),x, algorithm="maxima")

[Out] 2*(cos(x)*e^x - (cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)*integrate(cos(x)*e^x/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)

Fricas [F]

time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(1-sin(x)),x, algorithm="fricas")

[Out] integral(-e^x/(sin(x) - 1), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{e^x}{\sin(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(1-sin(x)),x)

[Out] -Integral(exp(x)/(sin(x) - 1), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(1-sin(x)),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$- \int \frac{e^x}{\sin(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-exp(x)/(sin(x) - 1),x)

[Out] -int(exp(x)/(sin(x) - 1), x)

$$3.556 \quad \int \frac{e^x(1-\sin(x))}{1-\cos(x)} dx$$

Optimal. Leaf size=15

$$-\frac{e^x \sin(x)}{1 - \cos(x)}$$

[Out] -exp(x)*sin(x)/(1-cos(x))

Rubi [A]

time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2326}

$$-\frac{e^x \sin(x)}{1 - \cos(x)}$$

Antiderivative was successfully verified.

[In] Int[(E^x*(1 - Sin[x]))/(1 - Cos[x]),x]

[Out] -((E^x*Sin[x])/(1 - Cos[x]))

Rule 2326

Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :=> With[{z = v*(y/(Log[F]*D[u, x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]

Rubi steps

$$\int \frac{e^x(1 - \sin(x))}{1 - \cos(x)} dx = -\frac{e^x \sin(x)}{1 - \cos(x)}$$

Mathematica [A]

time = 0.15, size = 11, normalized size = 0.73

$$-e^x \cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(E^x*(1 - Sin[x]))/(1 - Cos[x]),x]

[Out] -(E^x*Cot[x/2])

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[E^x*((1 - Sin[x])/(1 - Cos[x])),x]')`

[Out] `cought exception: maximum recursion depth exceeded`

Maple [C] Result contains complex when optimal does not.
time = 0.07, size = 21, normalized size = 1.40

method	result	size
risch	$-ie^x - \frac{2ie^x}{e^{ix}-1}$	21
norman	$\frac{-e^x(\tan^2(\frac{x}{2})) - e^x}{(1+\tan^2(\frac{x}{2}))\tan(\frac{x}{2})}$	33

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)*(1-sin(x))/(1-cos(x)),x,method=_RETURNVERBOSE)`

[Out] `-I*exp(x)-2*I*exp(x)/(exp(I*x)-1)`

Maxima [A]

time = 0.29, size = 22, normalized size = 1.47

$$-\frac{2e^x \sin(x)}{\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*(1-sin(x))/(1-cos(x)),x, algorithm="maxima")`

[Out] `-2*e^x*sin(x)/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)`

Fricas [A]

time = 0.30, size = 12, normalized size = 0.80

$$-\frac{(\cos(x) + 1)e^x}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*(1-sin(x))/(1-cos(x)),x, algorithm="fricas")`

[Out] `-(cos(x) + 1)*e^x/sin(x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(\sin(x) - 1)e^x}{\cos(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*(1-sin(x))/(1-cos(x)),x)

[Out] Integral((sin(x) - 1)*exp(x)/(cos(x) - 1), x)

Giac [A]

time = 0.00, size = 10, normalized size = 0.67

$$-\frac{e^x}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*(1-sin(x))/(1-cos(x)),x)

[Out] -e^x/tan(1/2*x)

Mupad [B]

time = 0.39, size = 8, normalized size = 0.53

$$-\cot\left(\frac{x}{2}\right) e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(x)*(sin(x) - 1))/(cos(x) - 1),x)

[Out] -cot(x/2)*exp(x)

$$3.557 \quad \int \frac{e^x(1+\sin(x))}{1-\cos(x)} dx$$

Optimal. Leaf size=41

$$(-2 + 2i)e^{(1+i)x} {}_2F_1(1 - i, 2; 2 - i; e^{ix}) + \frac{e^x \sin(x)}{1 - \cos(x)}$$

[Out] (-2+2*I)*exp((1+I)*x)*hypergeom([2, 1-I], [2-I], exp(I*x))+exp(x)*sin(x)/(1-cos(x))

Rubi [A]

time = 0.07, antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4551, 4549, 4528, 2225, 2283, 2326}

$$-4ie^x {}_2F_1(-i, 1; 1 - i; e^{ix}) + 2ie^x - \frac{e^x \sin(x)}{1 - \cos(x)}$$

Antiderivative was successfully verified.

[In] Int[(E^x*(1 + Sin[x]))/(1 - Cos[x]),x]

[Out] (2*I)*E^x - (4*I)*E^x*Hypergeometric2F1[-I, 1, 1 - I, E^(I*x)] - (E^x*Sin[x])/(1 - Cos[x])

Rule 2225

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 2283

Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := Simp[a^p*(G^(h*(f + g*x)))/(g*h*Log[G])*Hypergeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1, Simplify[(-b/a)*F^(e*(c + d*x))], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 2326

Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] := With[{z = v*(y/(Log[F])*D[u, x])}], Simp[F^u*z, x] /; EqQ[D[z, x], w*y] /; FreeQ[F, x]

Rule 4528

Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] := Dist[(-I)^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d + e

$x))^{n/(1 - E^{(2I*(d + e*x))^{n}}), x], x], x] /; \text{FreeQ}\{F, a, b, c, d, e\}, x] \&\& \text{IntegerQ}[n]$

Rule 4549

$\text{Int}[(\text{Cos}[d_.] + (e_.)*(x_.))*(g_.) + (f_.))^{(n_.)*(F_.)^{((c_.)*((a_.) + (b_.)*(x_.)))*\text{Sin}[d_.] + (e_.)*(x_.)]^{(m_.)}, x_Symbol] :> \text{Dist}[f^n, \text{Int}[F^{c*(a + b*x)}*\text{Cot}[d/2 + e*(x/2)]^m, x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g\}, x] \&\& \text{EqQ}[f + g, 0] \&\& \text{IntegersQ}[m, n] \&\& \text{EqQ}[m + n, 0]$

Rule 4551

$\text{Int}[(F_.)^{((c_.)*((a_.) + (b_.)*(x_.)))*((h_.) + (i_.)*\text{Sin}[d_.] + (e_.)*(x_.))]/(\text{Cos}[d_.] + (e_.)*(x_.))*(g_.) + (f_.), x_Symbol] :> \text{Dist}[2*i, \text{Int}[F^{c*(a + b*x)}*(\text{Sin}[d + e*x]/(f + g*\text{Cos}[d + e*x])), x], x] + \text{Int}[F^{c*(a + b*x)}*((h - i*\text{Sin}[d + e*x])/(f + g*\text{Cos}[d + e*x])), x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, h, i\}, x] \&\& \text{EqQ}[f^2 - g^2, 0] \&\& \text{EqQ}[h^2 - i^2, 0] \&\& \text{EqQ}[g*h + f*i, 0]$

Rubi steps

$$\begin{aligned} \int \frac{e^x(1 + \sin(x))}{1 - \cos(x)} dx &= 2 \int \frac{e^x \sin(x)}{1 - \cos(x)} dx + \int \frac{e^x(1 - \sin(x))}{1 - \cos(x)} dx \\ &= -\frac{e^x \sin(x)}{1 - \cos(x)} + 2 \int e^x \cot\left(\frac{x}{2}\right) dx \\ &= -\frac{e^x \sin(x)}{1 - \cos(x)} - 2i \int \left(-e^x - \frac{2e^x}{-1 + e^{ix}}\right) dx \\ &= -\frac{e^x \sin(x)}{1 - \cos(x)} + 2i \int e^x dx + 4i \int \frac{e^x}{-1 + e^{ix}} dx \\ &= 2ie^x - 4ie^x {}_2F_1(-i, 1; 1 - i; e^{ix}) - \frac{e^x \sin(x)}{1 - \cos(x)} \end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 100 vs. $2(41) = 82$.

time = 0.15, size = 100, normalized size = 2.44

$$\frac{2e^x \sin\left(\frac{x}{2}\right) \left(\cos\left(\frac{x}{2}\right) + 2i {}_2F_1(-i, 1; 1 - i; e^{ix}) \sin\left(\frac{x}{2}\right) + (1 + i)e^{ix} {}_2F_1(1, 1 - i; 2 - i; e^{ix}) \sin\left(\frac{x}{2}\right)\right) (1 + \sin(x))}{(-1 + \cos(x)) \left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(E^x*(1 + Sin[x]))/(1 - Cos[x]), x]

```
[Out] (2*E^x*Sin[x/2]*(Cos[x/2] + (2*I)*Hypergeometric2F1[-I, 1, 1 - I, E^(I*x)]*
Sin[x/2] + (1 + I)*E^(I*x)*Hypergeometric2F1[1, 1 - I, 2 - I, E^(I*x)]*Sin[
x/2])*(1 + Sin[x]))/((-1 + Cos[x])*(Cos[x/2] + Sin[x/2])^2)
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[E^x*((1 + Sin[x])/(1 - Cos[x])),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]

time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{e^x (\sin(x) + 1)}{1 - \cos(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(sin(x)+1)/(1-cos(x)),x)
```

```
[Out] int(exp(x)*(sin(x)+1)/(1-cos(x)),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sin(x))/(1-cos(x)),x, algorithm="maxima")
```

```
[Out] 2*(2*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 +
sin(x)^2 - 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 - 2*cos(x)
+ 1)
```

Fricas [F]

time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sin(x))/(1-cos(x)),x, algorithm="fricas")
```

```
[Out] integral(-(e^x*sin(x) + e^x)/(cos(x) - 1), x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e^x}{\cos(x) - 1} dx - \int \frac{e^x \sin(x)}{\cos(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*(1+sin(x))/(1-cos(x)),x)**[Out]** -Integral(exp(x)/(cos(x) - 1), x) - Integral(exp(x)*sin(x)/(cos(x) - 1), x)**Giac [F]** N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*(1+sin(x))/(1-cos(x)),x)**[Out]** Could not integrate**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{e^x (\sin(x) + 1)}{\cos(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(exp(x)*(sin(x) + 1))/(cos(x) - 1),x)**[Out]** int(-(exp(x)*(sin(x) + 1))/(cos(x) - 1), x)

$$3.558 \quad \int \frac{e^x(1+\sin(x))}{1+\cos(x)} dx$$

Optimal. Leaf size=12

$$\frac{e^x \sin(x)}{1 + \cos(x)}$$

[Out] exp(x)*sin(x)/(1+cos(x))

Rubi [A]

time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2326}

$$\frac{e^x \sin(x)}{\cos(x) + 1}$$

Antiderivative was successfully verified.

[In] Int[(E^x*(1 + Sin[x]))/(1 + Cos[x]),x]

[Out] (E^x*Sin[x])/(1 + Cos[x])

Rule 2326

Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] := With[{z = v*(y/(Log[F]*D[u, x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]

Rubi steps

$$\int \frac{e^x(1 + \sin(x))}{1 + \cos(x)} dx = \frac{e^x \sin(x)}{1 + \cos(x)}$$

Mathematica [A]

time = 0.13, size = 10, normalized size = 0.83

$$e^x \tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(E^x*(1 + Sin[x]))/(1 + Cos[x]),x]

[Out] E^x*Tan[x/2]

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[E^x*((1 + Sin[x])/(1 + Cos[x])),x]')`

[Out] `cought exception: maximum recursion depth exceeded`

Maple [A]

time = 0.05, size = 8, normalized size = 0.67

method	result	size
norman	$e^x \tan\left(\frac{x}{2}\right)$	8
risch	$-ie^x + \frac{2ie^x}{1+e^{ix}}$	21

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)*(sin(x)+1)/(1+cos(x)),x,method=_RETURNVERBOSE)`

[Out] `exp(x)*tan(1/2*x)`

Maxima [A]

time = 0.30, size = 22, normalized size = 1.83

$$\frac{2 e^x \sin(x)}{\cos(x)^2 + \sin(x)^2 + 2 \cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*(1+sin(x))/(1+cos(x)),x, algorithm="maxima")`

[Out] `2*e^x*sin(x)/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)`

Fricas [A]

time = 0.31, size = 11, normalized size = 0.92

$$\frac{e^x \sin(x)}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*(1+sin(x))/(1+cos(x)),x, algorithm="fricas")`

[Out] `e^x*sin(x)/(cos(x) + 1)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(\sin(x) + 1) e^x}{\cos(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*(1+sin(x))/(1+cos(x)),x)

[Out] Integral((sin(x) + 1)*exp(x)/(cos(x) + 1), x)

Giac [A]

time = 0.00, size = 8, normalized size = 0.67

$$e^x \tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*(1+sin(x))/(1+cos(x)),x)

[Out] e^x*tan(1/2*x)

Mupad [B]

time = 0.37, size = 7, normalized size = 0.58

$$\tan\left(\frac{x}{2}\right) e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(x)*(sin(x) + 1))/(cos(x) + 1),x)

[Out] tan(x/2)*exp(x)

$$3.559 \quad \int \frac{e^x(1-\sin(x))}{1+\cos(x)} dx$$

Optimal. Leaf size=42

$$(2-2i)e^{(1+i)x} {}_2F_1(1-i, 2; 2-i; -e^{ix}) - \frac{e^x \sin(x)}{1+\cos(x)}$$

[Out] (2-2*I)*exp((1+I)*x)*hypergeom([2, 1-I], [2-I], -exp(I*x))-exp(x)*sin(x)/(1+cos(x))

Rubi [A]

time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4551, 4548, 4527, 2225, 2283, 2326}

$$-4ie^x {}_2F_1(-i, 1; 1-i; -e^{ix}) + 2ie^x + \frac{e^x \sin(x)}{\cos(x) + 1}$$

Antiderivative was successfully verified.

[In] Int[(E^x*(1 - Sin[x]))/(1 + Cos[x]),x]

[Out] (2*I)*E^x - (4*I)*E^x*Hypergeometric2F1[-I, 1, 1 - I, -E^(I*x)] + (E^x*Sin[x])/(1 + Cos[x])

Rule 2225

Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 2283

Int[((a_) + (b_)*(F_)^(e_)*((c_) + (d_)*(x_)))^(p_)*(G_)^(h_)*((f_) + (g_)*(x_)), x_Symbol] := Simp[a^p*(G^(h*(f + g*x)))/(g*h*Log[G])*Hypergeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1, Simplify[(-b/a)*F^(e*(c + d*x))], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 2326

Int[(y_)*(F_)^(u_)*((v_) + (w_)), x_Symbol] := With[{z = v*(y/(Log[F]*D[u, x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]

Rule 4527

Int[(F_)^((c_)*((a_) + (b_)*(x_)))*Tan[(d_) + (e_)*(x_)]^(n_), x_Symbol] := Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x))

)^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]

Rule 4548

Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_.))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] := Dist[f^n, Int[F^(c*(a + b*x))*Tan[d/2 + e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]

Rule 4551

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*((h_) + (i_.)*Sin[(d_.) + (e_.)*(x_)]))/((Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_.)), x_Symbol] := Dist[2*i, Int[F^(c*(a + b*x))*(Sin[d + e*x]/(f + g*Cos[d + e*x])), x], x] + Int[F^(c*(a + b*x))*((h - i*Sin[d + e*x])/(f + g*Cos[d + e*x])), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h + f*i, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{e^x(1 - \sin(x))}{1 + \cos(x)} dx &= - \left(2 \int \frac{e^x \sin(x)}{1 + \cos(x)} dx \right) + \int \frac{e^x(1 + \sin(x))}{1 + \cos(x)} dx \\
 &= \frac{e^x \sin(x)}{1 + \cos(x)} - 2 \int e^x \tan\left(\frac{x}{2}\right) dx \\
 &= \frac{e^x \sin(x)}{1 + \cos(x)} - 2i \int \left(-e^x + \frac{2e^x}{1 + e^{ix}} \right) dx \\
 &= \frac{e^x \sin(x)}{1 + \cos(x)} + 2i \int e^x dx - 4i \int \frac{e^x}{1 + e^{ix}} dx \\
 &= 2ie^x - 4ie^x {}_2F_1\left(-i, 1; 1 - i; -e^{ix}\right) + \frac{e^x \sin(x)}{1 + \cos(x)}
 \end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 87 vs. $2(42) = 84$.

time = 0.15, size = 87, normalized size = 2.07

$$\frac{2e^x \cos\left(\frac{x}{2}\right) \left(2i \cos\left(\frac{x}{2}\right) {}_2F_1\left(-i, 1; 1 - i; -e^{ix}\right) - (1 + i)e^{ix} \cos\left(\frac{x}{2}\right) {}_2F_1\left(1, 1 - i; 2 - i; -e^{ix}\right) - \sin\left(\frac{x}{2}\right)\right)}{1 + \cos(x)}$$

Antiderivative was successfully verified.

[In] Integrate[(E^x*(1 - Sin[x]))/(1 + Cos[x]),x]

```
[Out] (-2*E^x*cos[x/2]*((2*I)*cos[x/2]*Hypergeometric2F1[-I, 1, 1 - I, -E^(I*x)]
- (1 + I)*E^(I*x)*cos[x/2]*Hypergeometric2F1[1, 1 - I, 2 - I, -E^(I*x)] - S
in[x/2]))/(1 + Cos[x])
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

caught exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[E^x*((1 - Sin[x])/(1 + Cos[x])),x]')
```

```
[Out] caught exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]

time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{e^x (1 - \sin(x))}{1 + \cos(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1-sin(x))/(1+cos(x)),x)
```

```
[Out] int(exp(x)*(1-sin(x))/(1+cos(x)),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sin(x))/(1+cos(x)),x, algorithm="maxima")
```

```
[Out] -2*(2*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 +
sin(x)^2 + 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x)
+ 1)
```

Fricas [F]

time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sin(x))/(1+cos(x)),x, algorithm="fricas")
```

```
[Out] integral(-(e^x*sin(x) - e^x)/(cos(x) + 1), x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{e^x}{\cos(x) + 1} \right) dx - \int \frac{e^x \sin(x)}{\cos(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*(1-sin(x))/(1+cos(x)),x)**[Out]** -Integral(-exp(x)/(cos(x) + 1), x) - Integral(exp(x)*sin(x)/(cos(x) + 1), x)**Giac [F]** N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*(1-sin(x))/(1+cos(x)),x)**[Out]** Could not integrate**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{e^x (\sin(x) - 1)}{\cos(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(exp(x)*(sin(x) - 1))/(cos(x) + 1),x)**[Out]** -int((exp(x)*(sin(x) - 1))/(cos(x) + 1), x)

$$3.560 \quad \int \frac{e^x(1-\cos(x))}{1-\sin(x)} dx$$

Optimal. Leaf size=46

$$(2+2i)e^{(1+i)x} {}_2F_1(1-i, 2; 2-i; -ie^{ix}) - \frac{e^x \cos(x)}{1-\sin(x)}$$

[Out] (2+2*I)*exp((1+I)*x)*hypergeom([2, 1-I], [2-I], -I*exp(I*x))-exp(x)*cos(x)/(1-sin(x))

Rubi [A]

time = 0.08, antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4550, 4547, 4527, 2225, 2283, 2326}

$$-4ie^x {}_2F_1(-i, 1; 1-i; -ie^{ix}) + 2ie^x + \frac{e^x \cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

[In] Int[(E^x*(1 - Cos[x]))/(1 - Sin[x]),x]

[Out] (2*I)*E^x - (4*I)*E^x*Hypergeometric2F1[-I, 1, 1 - I, (-I)*E^(I*x)] + (E^x*Cos[x])/(1 - Sin[x])

Rule 2225

Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 2283

Int[((a_) + (b_)*(F_)^(e_)*((c_) + (d_)*(x_)))^(p_)*(G_)^(h_)*((f_) + (g_)*(x_)), x_Symbol] := Simp[a^p*(G^(h*(f + g*x)))/(g*h*Log[G])*Hypergeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1, Simplify[(-b/a)*F^(e*(c + d*x))], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 2326

Int[(y_)*(F_)^(u_)*((v_) + (w_)), x_Symbol] := With[{z = v*(y/(Log[F]*D[u, x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]

Rule 4527

Int[(F_)^((c_)*((a_) + (b_)*(x_)))*Tan[(d_) + (e_)*(x_)]^(n_), x_Symbol] := Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x))

)^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
&& IntegerQ[n]

Rule 4547

Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]^(n_.), x_Symbol] := Dist[g^n, Int[F^(c*(a + b*x))*Tan[f*(Pi/(4*g)) - d/2 - e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]

Rule 4550

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*(Cos[(d_.) + (e_.)*(x_)]*(i_.) + (h_)))/((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]), x_Symbol] := Dist[2*i, Int[F^(c*(a + b*x))*Cos[d + e*x]/(f + g*SIN[d + e*x]), x], x] + Int[F^(c*(a + b*x))*((h - i*cos[d + e*x])/(f + g*SIN[d + e*x])), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h - f*i, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{e^x(1 - \cos(x))}{1 - \sin(x)} dx &= - \left(2 \int \frac{e^x \cos(x)}{1 - \sin(x)} dx \right) + \int \frac{e^x(1 + \cos(x))}{1 - \sin(x)} dx \\
 &= \frac{e^x \cos(x)}{1 - \sin(x)} - 2 \int e^x \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) dx \\
 &= \frac{e^x \cos(x)}{1 - \sin(x)} - 2i \int \left(-e^x + \frac{2e^x}{1 + e^{2i(\frac{\pi}{4} + \frac{x}{2})}} \right) dx \\
 &= \frac{e^x \cos(x)}{1 - \sin(x)} + 2i \int e^x dx - 4i \int \frac{e^x}{1 + e^{2i(\frac{\pi}{4} + \frac{x}{2})}} dx \\
 &= 2ie^x - 4ie^x {}_2F_1(-i, 1; 1 - i; -ie^{ix}) + \frac{e^x \cos(x)}{1 - \sin(x)}
 \end{aligned}$$

Mathematica [A]

time = 0.50, size = 72, normalized size = 1.57

$$\frac{1}{2}(-1 + \cos(x)) \csc^2\left(\frac{x}{2}\right) \left(-\frac{e^x((1 - 2i) + (1 + 2i) \cot(\frac{x}{2}))}{-1 + \cot(\frac{x}{2})} + 4i {}_2F_1(-i, 1; 1 - i; -i \cos(x) + \sin(x))(\cosh(x) + \sinh(x)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(E^x*(1 - Cos[x]))/(1 - Sin[x]),x]

[Out] $((-1 + \cos[x]) \cdot \csc[x/2]^2 \cdot (-((e^x \cdot ((1 - 2I) + (1 + 2I) \cdot \cot[x/2])) / (-1 + \cot[x/2])) + (4I) \cdot \text{Hypergeometric2F1}[-I, 1, 1 - I, (-I) \cdot \cos[x] + \sin[x]] \cdot (\cos[x] + \sinh[x]))) / 2$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[E^x*((1 - Cos[x])/(1 - Sin[x])),x]')`

[Out] cought exception: maximum recursion depth exceeded

Maple [F]

time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{e^x (1 - \cos(x))}{1 - \sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)*(1-cos(x))/(1-sin(x)),x)`

[Out] `int(exp(x)*(1-cos(x))/(1-sin(x)),x)`

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*(1-cos(x))/(1-sin(x)),x, algorithm="maxima")`

[Out] $2 \cdot (\cos(x) \cdot e^x - 2 \cdot (\cos(x)^2 + \sin(x)^2 - 2 \cdot \sin(x) + 1) \cdot \text{integrate}(\cos(x) \cdot e^x / (\cos(x)^2 + \sin(x)^2 - 2 \cdot \sin(x) + 1), x)) / (\cos(x)^2 + \sin(x)^2 - 2 \cdot \sin(x) + 1)$

Fricas [F]

time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*(1-cos(x))/(1-sin(x)),x, algorithm="fricas")`

[Out] `integral((cos(x) - 1)*e^x/(sin(x) - 1), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(\cos(x) - 1) e^x}{\sin(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)*(1-cos(x))/(1-sin(x)),x)``[Out] Integral((cos(x) - 1)*exp(x)/(sin(x) - 1), x)`**Giac [F] N/A**

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)*(1-cos(x))/(1-sin(x)),x)``[Out] Could not integrate`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^x (\cos(x) - 1)}{\sin(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((exp(x)*(cos(x) - 1))/(sin(x) - 1),x)``[Out] int((exp(x)*(cos(x) - 1))/(sin(x) - 1), x)`

$$3.561 \quad \int \frac{e^x(1+\cos(x))}{1-\sin(x)} dx$$

Optimal. Leaf size=14

$$\frac{e^x \cos(x)}{1 - \sin(x)}$$

[Out] exp(x)*cos(x)/(1-sin(x))

Rubi [A]

time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2326}

$$\frac{e^x \cos(x)}{1 - \sin(x)}$$

Antiderivative was successfully verified.

[In] Int[(E^x*(1 + Cos[x]))/(1 - Sin[x]),x]

[Out] (E^x*Cos[x])/(1 - Sin[x])

Rule 2326

Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u, x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]

Rubi steps

$$\int \frac{e^x(1 + \cos(x))}{1 - \sin(x)} dx = \frac{e^x \cos(x)}{1 - \sin(x)}$$

Mathematica [A]

time = 0.05, size = 23, normalized size = 1.64

$$\frac{e^x \left(1 + \tan\left(\frac{x}{2}\right)\right)}{-1 + \tan\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(E^x*(1 + Cos[x]))/(1 - Sin[x]),x]

[Out] -((E^x*(1 + Tan[x/2]))/(-1 + Tan[x/2]))

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[E^x*((1 + Cos[x])/(1 - Sin[x])),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python object
```

Maple [C] Result contains complex when optimal does not.

time = 0.10, size = 21, normalized size = 1.50

method	result	size
risch	$-ie^x + \frac{2e^x}{e^{ix} - i}$	21
norman	$\frac{-e^x \tan(\frac{x}{2}) - e^x (\tan^2(\frac{x}{2})) - e^x (\tan^3(\frac{x}{2})) - e^x}{(1 + \tan^2(\frac{x}{2})) (\tan(\frac{x}{2}) - 1)}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1+cos(x))/(1-sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*exp(x)+2*exp(x)/(exp(I*x)-I)
```

Maxima [A]

time = 0.30, size = 22, normalized size = 1.57

$$\frac{2 \cos(x) e^x}{\cos(x)^2 + \sin(x)^2 - 2 \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1-sin(x)),x, algorithm="maxima")
```

```
[Out] 2*cos(x)*e^x/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [A]

time = 0.31, size = 24, normalized size = 1.71

$$\frac{(\cos(x) + 1)e^x + e^x \sin(x)}{\cos(x) - \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1-sin(x)),x, algorithm="fricas")
```

```
[Out] ((cos(x) + 1)*e^x + e^x*sin(x))/(cos(x) - sin(x) + 1)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e^x}{\sin(x) - 1} dx - \int \frac{e^x \cos(x)}{\sin(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*(1+cos(x))/(1-sin(x)),x)

[Out] -Integral(exp(x)/(sin(x) - 1), x) - Integral(exp(x)*cos(x)/(sin(x) - 1), x)

Giac [A]

time = 0.00, size = 22, normalized size = 1.57

$$\frac{-e^x \tan\left(\frac{x}{2}\right) - e^x}{\tan\left(\frac{x}{2}\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*(1+cos(x))/(1-sin(x)),x)

[Out] -(e^x*tan(1/2*x) + e^x)/(tan(1/2*x) - 1)

Mupad [B]

time = 0.37, size = 24, normalized size = 1.71

$$-\frac{e^x (-1 + e^{x \cdot 1i} 1i)}{e^{x \cdot 1i} - 1i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(exp(x)*(cos(x) + 1))/(sin(x) - 1),x)

[Out] -(exp(x)*(exp(x*1i)*1i - 1))/(exp(x*1i) - 1i)

$$3.562 \quad \int \frac{e^x(1+\cos(x))}{1+\sin(x)} dx$$

Optimal. Leaf size=43

$$(-2 - 2i)e^{(1+i)x} {}_2F_1(1 - i, 2; 2 - i; ie^{ix}) + \frac{e^x \cos(x)}{1 + \sin(x)}$$

[Out] $(-2-2*I)*\exp((1+I)*x)*\text{hypergeom}([2, 1-I], [2-I], I*\exp(I*x))+\exp(x)*\cos(x)/(1+\sin(x))$

Rubi [A]

time = 0.08, antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4550, 4547, 4527, 2225, 2283, 2326}

$$4ie^x {}_2F_1(i, 1; 1 + i; -ie^{-ix}) - 2ie^x - \frac{e^x \cos(x)}{\sin(x) + 1}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(E^x*(1 + \text{Cos}[x]))/(1 + \text{Sin}[x]), x]$

[Out] $(-2*I)*E^x + (4*I)*E^x*\text{Hypergeometric2F1}[I, 1, 1 + I, (-I)/E^{I*x}] - (E^x*\text{Cos}[x])/(1 + \text{Sin}[x])$

Rule 2225

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))}^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x)))})^n/(b*c*n*\text{Log}[F]), x] /;$ $\text{FreeQ}\{F, a, b, c, n\}, x]$

Rule 2283

$\text{Int}[(a_.) + (b_.)*(F_)^{((e_.)*((c_.) + (d_.)*(x_)))}^{(p_.)}*(G_)^{((h_.)*((f_.) + (g_.)*(x_)))}, x_Symbol] \rightarrow \text{Simp}[a^p*(G^{(h*(f + g*x))})/(g*h*\text{Log}[G])*\text{Hypergeometric2F1}[-p, g*h*(\text{Log}[G]/(d*e*\text{Log}[F])), g*h*(\text{Log}[G]/(d*e*\text{Log}[F])) + 1, \text{Simplify}[(-b/a)*F^{(e*(c + d*x))}], x] /;$ $\text{FreeQ}\{F, G, a, b, c, d, e, f, g, h, p\}, x \ \&\& \ (\text{ILtQ}[p, 0] \ || \ \text{GtQ}[a, 0])$

Rule 2326

$\text{Int}[(y_.)*(F_)^{(u_.)}*((v_.) + (w_)), x_Symbol] \rightarrow \text{With}\{z = v*(y/(\text{Log}[F]*D[u, x]))\}, \text{Simp}[F^u*z, x] /;$ $\text{EqQ}[D[z, x], w*y] /;$ $\text{FreeQ}[F, x]$

Rule 4527

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))}*\text{Tan}[(d_.) + (e_.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[I^n, \text{Int}[\text{ExpandIntegrand}[F^{(c*(a + b*x))}*(1 - E^{(2*I*(d + e*x))})], x]]$

)^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]

Rule 4547

Int[Cos[(d_.) + (e_.)*(x_.)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_.)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_.)])^(n_.), x_Symbol] :> Dist[g^n, Int[F^(c*(a + b*x))*Tan[f*(Pi/(4*g)) - d/2 - e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]

Rule 4550

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_.)))*(Cos[(d_.) + (e_.)*(x_.)]*(i_.) + (h_.)))/((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_.)]), x_Symbol] :> Dist[2*i, Int[F^(c*(a + b*x))*Cos[d + e*x]/(f + g*SIN[d + e*x]), x], x] + Int[F^(c*(a + b*x))*((h - i*cos[d + e*x])/(f + g*SIN[d + e*x])), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h - f*i, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{e^x(1 + \cos(x))}{1 + \sin(x)} dx &= 2 \int \frac{e^x \cos(x)}{1 + \sin(x)} dx + \int \frac{e^x(1 - \cos(x))}{1 + \sin(x)} dx \\
 &= -\frac{e^x \cos(x)}{1 + \sin(x)} + 2 \int e^x \tan\left(\frac{\pi}{4} - \frac{x}{2}\right) dx \\
 &= -\frac{e^x \cos(x)}{1 + \sin(x)} + 2i \int \left(-e^x + \frac{2e^x}{1 + e^{2i(\frac{\pi}{4} - \frac{x}{2})}}\right) dx \\
 &= -\frac{e^x \cos(x)}{1 + \sin(x)} - 2i \int e^x dx + 4i \int \frac{e^x}{1 + e^{2i(\frac{\pi}{4} - \frac{x}{2})}} dx \\
 &= -2ie^x + 4ie^x {}_2F_1\left(i, 1; 1 + i; -ie^{-ix}\right) - \frac{e^x \cos(x)}{1 + \sin(x)}
 \end{aligned}$$

Mathematica [A]

time = 0.13, size = 73, normalized size = 1.70

$$\frac{1}{2}(1 + \cos(x)) \sec^2\left(\frac{x}{2}\right) \left(-4i {}_2F_1(-i, 1; 1 - i; i \cos(x) - \sin(x))(\cosh(x) + \sinh(x)) + \frac{e^x((-1 + 2i) + (1 + 2i) \tan(\frac{x}{2}))}{1 + \tan(\frac{x}{2})}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(E^x*(1 + Cos[x]))/(1 + Sin[x]), x]

```
[Out] ((1 + Cos[x])*Sec[x/2]^2*((-4*I)*Hypergeometric2F1[-I, 1, 1 - I, I*Cos[x] -
Sin[x]]*(Cosh[x] + Sinh[x]) + (E^x*((-1 + 2*I) + (1 + 2*I)*Tan[x/2])))/(1 +
Tan[x/2]))/2
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[E^x*((1 + Cos[x])/(1 + Sin[x])),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]

time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{e^x (1 + \cos(x))}{\sin(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1+cos(x))/(sin(x)+1),x)
```

```
[Out] int(exp(x)*(1+cos(x))/(sin(x)+1),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1+sin(x)),x, algorithm="maxima")
```

```
[Out] -2*(cos(x)*e^x - 2*(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)*integrate(cos(x)*e^
x/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 + 2*sin(x)
+ 1)
```

Fricas [F]

time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1+sin(x)),x, algorithm="fricas")
```

```
[Out] integral((cos(x) + 1)*e^x/(sin(x) + 1), x)
```


Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(\cos(x) + 1) e^x}{\sin(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*(1+cos(x))/(1+sin(x)),x)**[Out]** Integral((cos(x) + 1)*exp(x)/(sin(x) + 1), x)**Giac [F] N/A**

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*(1+cos(x))/(1+sin(x)),x)**[Out]** Could not integrate**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^x (\cos(x) + 1)}{\sin(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(x)*(cos(x) + 1))/(sin(x) + 1),x)**[Out]** int((exp(x)*(cos(x) + 1))/(sin(x) + 1), x)

$$3.563 \quad \int \frac{e^x(1-\cos(x))}{1+\sin(x)} dx$$

Optimal. Leaf size=13

$$-\frac{e^x \cos(x)}{1 + \sin(x)}$$

[Out] -exp(x)*cos(x)/(1+sin(x))

Rubi [A]

time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2326}

$$-\frac{e^x \cos(x)}{\sin(x) + 1}$$

Antiderivative was successfully verified.

[In] Int[(E^x*(1 - Cos[x]))/(1 + Sin[x]),x]

[Out] -((E^x*Cos[x])/(1 + Sin[x]))

Rule 2326

Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] := With[{z = v*(y/(Log[F]*D[u, x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]

Rubi steps

$$\int \frac{e^x(1 - \cos(x))}{1 + \sin(x)} dx = -\frac{e^x \cos(x)}{1 + \sin(x)}$$

Mathematica [A]

time = 0.04, size = 23, normalized size = 1.77

$$-\frac{e^x \left(-1 + \cot\left(\frac{x}{2}\right)\right)}{1 + \cot\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(E^x*(1 - Cos[x]))/(1 + Sin[x]),x]

[Out] -((E^x*(-1 + Cot[x/2]))/(1 + Cot[x/2]))

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[E^x*((1 - Cos[x])/(1 + Sin[x])),x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [C]** Result contains complex when optimal does not.

time = 0.08, size = 21, normalized size = 1.62

method	result	size
risch	$-ie^x - \frac{2e^x}{e^{ix} + i}$	21
norman	$\frac{e^x \tan(\frac{x}{2}) + e^x (\tan^3(\frac{x}{2})) - e^x (\tan^2(\frac{x}{2})) - e^x}{(1 + \tan^2(\frac{x}{2}))(1 + \tan(\frac{x}{2}))}$	51

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(exp(x)*(1-cos(x))/(sin(x)+1),x,method=_RETURNVERBOSE)``[Out] -I*exp(x)-2*exp(x)/(exp(I*x)+I)`**Maxima [A]**

time = 0.30, size = 22, normalized size = 1.69

$$-\frac{2 \cos(x) e^x}{\cos(x)^2 + \sin(x)^2 + 2 \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)*(1-cos(x))/(1+sin(x)),x, algorithm="maxima")``[Out] -2*cos(x)*e^x/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)`**Fricas [A]**

time = 0.31, size = 24, normalized size = 1.85

$$-\frac{(\cos(x) + 1)e^x - e^x \sin(x)}{\cos(x) + \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)*(1-cos(x))/(1+sin(x)),x, algorithm="fricas")``[Out] -((cos(x) + 1)*e^x - e^x*sin(x))/(cos(x) + sin(x) + 1)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{e^x}{\sin(x) + 1} \right) dx - \int \frac{e^x \cos(x)}{\sin(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*(1-cos(x))/(1+sin(x)),x)**[Out]** -Integral(-exp(x)/(sin(x) + 1), x) - Integral(exp(x)*cos(x)/(sin(x) + 1), x)**Giac [A]**

time = 0.00, size = 21, normalized size = 1.62

$$\frac{e^x \tan\left(\frac{x}{2}\right) - e^x}{\tan\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*(1-cos(x))/(1+sin(x)),x)**[Out]** (e^x*tan(1/2*x) - e^x)/(tan(1/2*x) + 1)**Mupad [B]**

time = 0.42, size = 20, normalized size = 1.54

$$-e^x \operatorname{li} - \frac{2e^x}{e^{x \operatorname{li}} + \operatorname{li}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(exp(x)*(cos(x) - 1))/(sin(x) + 1),x)**[Out]** -exp(x)*li - (2*exp(x))/(exp(x*li) + li)

3.564 $\int e^x x \cos(x) dx$

Optimal. Leaf size=30

$$\frac{1}{2}e^x x \cos(x) - \frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x)$$

[Out] $1/2*\exp(x)*x*\cos(x)-1/2*\exp(x)*\sin(x)+1/2*\exp(x)*x*\sin(x)$

Rubi [A]

time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4518, 4554, 4517}

$$-\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

Antiderivative was successfully verified.

[In] Int[E^x*x*Cos[x],x]

[Out] (E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :=
  Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :=
  Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] := Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int e^x x \cos(x) dx &= \frac{1}{2} e^x x \cos(x) + \frac{1}{2} e^x x \sin(x) - \int \left(\frac{1}{2} e^x \cos(x) + \frac{1}{2} e^x \sin(x) \right) dx \\
&= \frac{1}{2} e^x x \cos(x) + \frac{1}{2} e^x x \sin(x) - \frac{1}{2} \int e^x \cos(x) dx - \frac{1}{2} \int e^x \sin(x) dx \\
&= \frac{1}{2} e^x x \cos(x) - \frac{1}{2} e^x \sin(x) + \frac{1}{2} e^x x \sin(x)
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 18, normalized size = 0.60

$$\frac{1}{2} e^x (x \cos(x) + (-1 + x) \sin(x))$$

Antiderivative was successfully verified.

`[In] Integrate[E^x*x*Cos[x], x]``[Out] (E^x*(x*Cos[x] + (-1 + x)*Sin[x]))/2`**Mathics [A]**

time = 1.86, size = 21, normalized size = 0.70

$$\frac{\left(\sqrt{2} x \sin\left[\frac{\text{Pi}}{4} + x\right] - \sin[x]\right) E^x}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x*E^x*Cos[x], x]')``[Out] (Sqrt[2] x Sin[Pi / 4 + x] - Sin[x]) E ^ x / 2`**Maple [A]**

time = 0.02, size = 20, normalized size = 0.67

method	result	size
default	$\frac{e^x x \cos(x)}{2} - \left(-\frac{x}{2} + \frac{1}{2}\right) e^x \sin(x)$	20
risch	$\left(\frac{1}{8} - \frac{i}{8}\right) (-1 + i + 2x) e^{(1+i)x} + \left(\frac{1}{8} + \frac{i}{8}\right) (-1 - i + 2x) e^{(1-i)x}$	36
norman	$\frac{e^x x \tan\left(\frac{x}{2}\right) + \frac{e^x x}{2} - e^x \tan\left(\frac{x}{2}\right) - \frac{e^x x \left(\tan^2\left(\frac{x}{2}\right)\right)}{2}}{1 + \tan^2\left(\frac{x}{2}\right)}$	45

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(exp(x)*x*cos(x), x, method=_RETURNVERBOSE)`

[Out] $1/2*\exp(x)*x*\cos(x)-(-1/2*x+1/2)*\exp(x)*\sin(x)$

Maxima [A]

time = 0.24, size = 17, normalized size = 0.57

$$\frac{1}{2}x \cos(x) e^x + \frac{1}{2}(x-1)e^x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x*cos(x),x, algorithm="maxima")`

[Out] $1/2*x*\cos(x)*e^x + 1/2*(x-1)*e^x*\sin(x)$

Fricas [A]

time = 0.31, size = 17, normalized size = 0.57

$$\frac{1}{2}x \cos(x) e^x + \frac{1}{2}(x-1)e^x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x*cos(x),x, algorithm="fricas")`

[Out] $1/2*x*\cos(x)*e^x + 1/2*(x-1)*e^x*\sin(x)$

Sympy [A]

time = 0.19, size = 27, normalized size = 0.90

$$\frac{x e^x \sin(x)}{2} + \frac{x e^x \cos(x)}{2} - \frac{e^x \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x*cos(x),x)`

[Out] $x*\exp(x)*\sin(x)/2 + x*\exp(x)*\cos(x)/2 - \exp(x)*\sin(x)/2$

Giac [A]

time = 0.00, size = 18, normalized size = 0.60

$$e^x \left(\frac{1}{2}x \cos x + \frac{1}{2}(x-1) \sin x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x*cos(x),x)`

[Out] $1/2*(x*\cos(x) + (x-1)*\sin(x))*e^x$

Mupad [B]

time = 0.07, size = 17, normalized size = 0.57

$$\frac{e^x (x \cos(x) - \sin(x) + x \sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x)*cos(x),x)
```

```
[Out] (exp(x)*(x*cos(x) - sin(x) + x*sin(x)))/2
```


3.565 $\int e^x x^2 \sin(x) dx$

Optimal. Leaf size=50

$$-\frac{1}{2}e^x \cos(x) + e^x x \cos(x) - \frac{1}{2}e^x x^2 \cos(x) - \frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x^2 \sin(x)$$

[Out] $-1/2*\exp(x)*\cos(x)+\exp(x)*x*\cos(x)-1/2*\exp(x)*x^2*\cos(x)-1/2*\exp(x)*\sin(x)+1/2*\exp(x)*x^2*\sin(x)$

Rubi [A]

time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {4517, 4553, 14, 4518, 4554}

$$\frac{1}{2}e^x x^2 \sin(x) - \frac{1}{2}e^x x^2 \cos(x) - \frac{1}{2}e^x \sin(x) + e^x x \cos(x) - \frac{1}{2}e^x \cos(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^x*x^2*\text{Sin}[x], x]$

[Out] $-1/2*(E^x*\text{Cos}[x]) + E^x*x*\text{Cos}[x] - (E^x*x^2*\text{Cos}[x])/2 - (E^x*\text{Sin}[x])/2 + (E^x*x^2*\text{Sin}[x])/2$

Rule 14

$\text{Int}[(u_*)*((c_*)*(x_*))^{(m_*)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}\{c, m\}, x \ \&\& \ \text{SumQ}[u] \ \&\& \ !\text{LinearQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (a_*) + (b_*)*(v_*)] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{InverseFunctionQ}[v]$

Rule 4517

$\text{Int}[(F_*)^{((c_*)*((a_*) + (b_*)*(x_*)))}*\text{Sin}[(d_*) + (e_*)*(x_*)], x_Symbol] \rightarrow \text{Simp}[b*c*\text{Log}[F]*F^{(c*(a + b*x))}*(\text{Sin}[d + e*x]/(e^2 + b^2*c^2*\text{Log}[F]^2)), x] - \text{Simp}[e*F^{(c*(a + b*x))}*(\text{Cos}[d + e*x]/(e^2 + b^2*c^2*\text{Log}[F]^2)), x] /; \text{FreeQ}\{F, a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[e^2 + b^2*c^2*\text{Log}[F]^2, 0]$

Rule 4518

$\text{Int}[\text{Cos}[(d_*) + (e_*)*(x_*)]*(F_*)^{((c_*)*((a_*) + (b_*)*(x_*)))}, x_Symbol] \rightarrow \text{Simp}[b*c*\text{Log}[F]*F^{(c*(a + b*x))}*(\text{Cos}[d + e*x]/(e^2 + b^2*c^2*\text{Log}[F]^2)), x] + \text{Simp}[e*F^{(c*(a + b*x))}*(\text{Sin}[d + e*x]/(e^2 + b^2*c^2*\text{Log}[F]^2)), x] /; \text{FreeQ}\{F, a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[e^2 + b^2*c^2*\text{Log}[F]^2, 0]$

Rule 4553

$\text{Int}[(F_*)^{((c_*)*((a_*) + (b_*)*(x_*)))}*((f_*)*(x_*))^{(m_*)}*\text{Sin}[(d_*) + (e_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Module}\{u = \text{IntHide}[F^{(c*(a + b*x))}*\text{Sin}[d + e*x]^{(n_*)}$

```
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int e^x x^2 \sin(x) dx &= -\frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x x^2 \sin(x) - 2 \int x \left(-\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x) \right) dx \\
&= -\frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x x^2 \sin(x) - 2 \int \left(-\frac{1}{2}e^x x \cos(x) + \frac{1}{2}e^x x \sin(x) \right) dx \\
&= -\frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x x^2 \sin(x) + \int e^x x \cos(x) dx - \int e^x x \sin(x) dx \\
&= e^x x \cos(x) - \frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x x^2 \sin(x) + \int \left(-\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x) \right) dx - \int \left(\frac{1}{2}e^x \cos(x) - \frac{1}{2}e^x \sin(x) \right) dx \\
&= e^x x \cos(x) - \frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x x^2 \sin(x) - 2 \left(\frac{1}{2} \int e^x \cos(x) dx \right) \\
&= e^x x \cos(x) - \frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x x^2 \sin(x) - 2 \left(\frac{1}{4}e^x \cos(x) + \frac{1}{4}e^x \sin(x) \right)
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 25, normalized size = 0.50

$$\frac{1}{2}e^x \left(-(-1 + x)^2 \cos(x) + (-1 + x^2) \sin(x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x^2*Sin[x],x]
```

```
[Out] (E^x*(-((-1 + x)^2*Cos[x]) + (-1 + x^2)*Sin[x]))/2
```

Mathics [A]

time = 2.14, size = 44, normalized size = 0.88

$$\frac{\sqrt{2} \left(x \cos \left[\frac{\text{Pi}}{4} + x \right] + x \sin \left[\frac{\text{Pi}}{4} + x \right] - x^2 \cos \left[\frac{\text{Pi}}{4} + x \right] - \sin \left[\frac{\text{Pi}}{4} + x \right] \right) E^x}{2}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x^2*E^x*Sin[x],x]')`

[Out] `Sqrt[2] (x Cos[Pi / 4 + x] + x Sin[Pi / 4 + x] - x ^ 2 Cos[Pi / 4 + x] - Sin[Pi / 4 + x]) E ^ x / 2`

Maple [A]

time = 0.04, size = 27, normalized size = 0.54

method	result	size
default	$\left(-\frac{1}{2}x^2 + x - \frac{1}{2}\right) e^x \cos(x) + \left(\frac{x^2}{2} - \frac{1}{2}\right) e^x \sin(x)$	27
risch	$\left(-\frac{1}{4} - \frac{i}{4}\right) (x^2 + ix - x - i) e^{(1+i)x} + \left(-\frac{1}{4} + \frac{i}{4}\right) (x^2 - ix - x + i) e^{(1-i)x}$	48
norman	$\frac{e^x x + e^x x^2 \tan\left(\frac{x}{2}\right) - \frac{e^x x^2}{2} - e^x \tan\left(\frac{x}{2}\right) + \frac{e^x \left(\tan^2\left(\frac{x}{2}\right)\right)}{2} - e^x x \left(\tan^2\left(\frac{x}{2}\right)\right) + \frac{e^x x^2 \left(\tan^2\left(\frac{x}{2}\right)\right)}{2} - \frac{e^x}{2}}{1 + \tan^2\left(\frac{x}{2}\right)}$	80

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)*x^2*sin(x),x,method=_RETURNVERBOSE)`

[Out] `(-1/2*x^2+x-1/2)*exp(x)*cos(x)+(1/2*x^2-1/2)*exp(x)*sin(x)`

Maxima [A]

time = 0.25, size = 26, normalized size = 0.52

$$-\frac{1}{2} (x^2 - 2x + 1) \cos(x) e^x + \frac{1}{2} (x^2 - 1) e^x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x^2*sin(x),x, algorithm="maxima")`

[Out] `-1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)`

Fricas [A]

time = 0.30, size = 26, normalized size = 0.52

$$-\frac{1}{2} (x^2 - 2x + 1) \cos(x) e^x + \frac{1}{2} (x^2 - 1) e^x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x^2*sin(x),x, algorithm="fricas")`

[Out] `-1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)`

Sympy [A]

time = 0.36, size = 48, normalized size = 0.96

$$\frac{x^2 e^x \sin(x)}{2} - \frac{x^2 e^x \cos(x)}{2} + x e^x \cos(x) - \frac{e^x \sin(x)}{2} - \frac{e^x \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*x**2*sin(x),x)

[Out] x**2*exp(x)*sin(x)/2 - x**2*exp(x)*cos(x)/2 + x*exp(x)*cos(x) - exp(x)*sin(x)/2 - exp(x)*cos(x)/2

Giac [A]

time = 0.00, size = 59, normalized size = 1.18

$$e^x \left(\left(-\frac{-2x+2}{4} - \frac{2x^2-2x}{4} \right) \cos x + \left(-\frac{-2x+2}{4} + \frac{2x^2-2x}{4} \right) \sin x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*x^2*sin(x),x)

[Out] -1/2*((x^2 - 2*x + 1)*cos(x) - (x^2 - 1)*sin(x))*e^x

Mupad [B]

time = 0.33, size = 21, normalized size = 0.42

$$\frac{e^x (x-1) (\cos(x) + \sin(x) - x \cos(x) + x \sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*exp(x)*sin(x),x)

[Out] (exp(x)*(x - 1)*(cos(x) + sin(x) - x*cos(x) + x*sin(x)))/2

3.566 $\int e^{-3x} x^2 \sin(x) dx$

Optimal. Leaf size=75

$$-\frac{13}{250}e^{-3x} \cos(x) - \frac{3}{25}e^{-3x}x \cos(x) - \frac{1}{10}e^{-3x}x^2 \cos(x) - \frac{9}{250}e^{-3x} \sin(x) - \frac{4}{25}e^{-3x}x \sin(x) - \frac{3}{10}e^{-3x}x^2 \sin(x)$$

[Out] $-13/250*\cos(x)/\exp(3*x)-3/25*x*\cos(x)/\exp(3*x)-1/10*x^2*\cos(x)/\exp(3*x)-9/250*\sin(x)/\exp(3*x)-4/25*x*\sin(x)/\exp(3*x)-3/10*x^2*\sin(x)/\exp(3*x)$

Rubi [A]

time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {4517, 4553, 14, 4518, 4554}

$$-\frac{3}{10}e^{-3x}x^2 \sin(x) - \frac{1}{10}e^{-3x}x^2 \cos(x) - \frac{4}{25}e^{-3x}x \sin(x) - \frac{9}{250}e^{-3x} \sin(x) - \frac{3}{25}e^{-3x}x \cos(x) - \frac{13}{250}e^{-3x} \cos(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^2*\text{Sin}[x])/E^{(3*x)},x]$

[Out] $(-13*\text{Cos}[x])/(250*E^{(3*x)}) - (3*x*\text{Cos}[x])/(25*E^{(3*x)}) - (x^2*\text{Cos}[x])/(10*E^{(3*x)}) - (9*\text{Sin}[x])/(250*E^{(3*x)}) - (4*x*\text{Sin}[x])/(25*E^{(3*x)}) - (3*x^2*\text{Sin}[x])/(10*E^{(3*x)})$

Rule 14

$\text{Int}[(u)*((c_.)*(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /;$ FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 4517

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))}*\text{Sin}[(d_.) + (e_.)*(x_)], x_Symbol] \rightarrow \text{Simp}[b*c*\text{Log}[F]*F^{(c*(a + b*x))}*(\text{Sin}[d + e*x]/(e^2 + b^2*c^2*\text{Log}[F]^2)), x] - \text{Simp}[e*F^{(c*(a + b*x))}*(\text{Cos}[d + e*x]/(e^2 + b^2*c^2*\text{Log}[F]^2)), x] /;$ FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]

Rule 4518

$\text{Int}[\text{Cos}[(d_.) + (e_.)*(x_)]*(F_)^{((c_.)*((a_.) + (b_.)*(x_)))}, x_Symbol] \rightarrow \text{Simp}[b*c*\text{Log}[F]*F^{(c*(a + b*x))}*(\text{Cos}[d + e*x]/(e^2 + b^2*c^2*\text{Log}[F]^2)), x] + \text{Simp}[e*F^{(c*(a + b*x))}*(\text{Sin}[d + e*x]/(e^2 + b^2*c^2*\text{Log}[F]^2)), x] /;$ FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]

Rule 4553

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))}*((f_.)*(x_))^{(m_.)}*\text{Sin}[(d_.) + (e_.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Module}[\{u = \text{IntHide}[F^{(c*(a + b*x))}*\text{Sin}[d + e*x]^$

$n, x\}$, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]

Rule 4554

Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int e^{-3x} x^2 \sin(x) dx &= -\frac{1}{10} e^{-3x} x^2 \cos(x) - \frac{3}{10} e^{-3x} x^2 \sin(x) - 2 \int x \left(-\frac{1}{10} e^{-3x} \cos(x) - \frac{3}{10} e^{-3x} \sin(x) \right) dx \\
 &= -\frac{1}{10} e^{-3x} x^2 \cos(x) - \frac{3}{10} e^{-3x} x^2 \sin(x) - 2 \int \left(-\frac{1}{10} e^{-3x} x \cos(x) - \frac{3}{10} e^{-3x} x \sin(x) \right) dx \\
 &= -\frac{1}{10} e^{-3x} x^2 \cos(x) - \frac{3}{10} e^{-3x} x^2 \sin(x) + \frac{1}{5} \int e^{-3x} x \cos(x) dx + \frac{3}{5} \int e^{-3x} x \sin(x) dx \\
 &= -\frac{3}{25} e^{-3x} x \cos(x) - \frac{1}{10} e^{-3x} x^2 \cos(x) - \frac{4}{25} e^{-3x} x \sin(x) - \frac{3}{10} e^{-3x} x^2 \sin(x) - \frac{1}{5} \int \left(-\frac{3}{10} e^{-3x} \cos(x) - \frac{4}{10} e^{-3x} \sin(x) \right) dx \\
 &= -\frac{3}{25} e^{-3x} x \cos(x) - \frac{1}{10} e^{-3x} x^2 \cos(x) - \frac{4}{25} e^{-3x} x \sin(x) - \frac{3}{10} e^{-3x} x^2 \sin(x) - \frac{1}{50} \int e^{-3x} dx \\
 &= -\frac{2}{125} e^{-3x} \cos(x) - \frac{3}{25} e^{-3x} x \cos(x) - \frac{1}{10} e^{-3x} x^2 \cos(x) - \frac{6}{125} e^{-3x} \sin(x) - \frac{4}{25} e^{-3x} x \sin(x)
 \end{aligned}$$

Mathematica [A]

time = 0.03, size = 38, normalized size = 0.51

$$\frac{1}{250} e^{-3x} \left(-((13 + 30x + 25x^2) \cos(x)) - (9 + 40x + 75x^2) \sin(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*Sin[x])/E^(3*x), x]

[Out] (-((13 + 30*x + 25*x^2)*Cos[x]) - (9 + 40*x + 75*x^2)*Sin[x])/(250*E^(3*x))

Mathics [A]

time = 2.26, size = 40, normalized size = 0.53

$$\frac{(30x \cos[x] + 40x \sin[x] + 25x^2 \cos[x] + 75x^2 \sin[x] + 9 \sin[x] + 13 \cos[x]) E^{-3x}}{250}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[x^2*(Sin[x]/E^(3*x)),x]')`

[Out] $-(30 x \cos[x] + 40 x \sin[x] + 25 x^2 \cos[x] + 75 x^2 \sin[x] + 9 \sin[x] + 13 \cos[x]) E^{-3 x} / 250$

Maple [A]

time = 0.05, size = 36, normalized size = 0.48

method	result
default	$\left(-\frac{1}{10}x^2 - \frac{3}{25}x - \frac{13}{250}\right) e^{-3x} \cos(x) + \left(-\frac{3}{10}x^2 - \frac{4}{25}x - \frac{9}{250}\right) e^{-3x} \sin(x)$
risch	$\left(-\frac{1}{500} + \frac{3i}{500}\right) (25x^2 + 5ix + 15x + 3i + 4) e^{(-3+i)x} + \left(-\frac{1}{500} - \frac{3i}{500}\right) (25x^2 - 5ix + 15x - 3i + 4) e^{(-3-i)x}$
norman	$\frac{\left(-\frac{13}{250} - \frac{3x}{25} - \frac{x^2}{10} + \frac{13 \tan^2\left(\frac{x}{2}\right)}{250} - \frac{8x \tan\left(\frac{x}{2}\right)}{25} + \frac{3x \tan^2\left(\frac{x}{2}\right)}{25} - \frac{3x^2 \tan\left(\frac{x}{2}\right)}{5} + \frac{x^2 \tan^2\left(\frac{x}{2}\right)}{10} - \frac{9 \tan\left(\frac{x}{2}\right)}{125}\right) e^{-3x}}{1 + \tan^2\left(\frac{x}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*sin(x)/exp(3*x),x,method=_RETURNVERBOSE)`

[Out] $(-1/10*x^2-3/25*x-13/250)*exp(-3*x)*cos(x)+(-3/10*x^2-4/25*x-9/250)*exp(-3*x)*sin(x)$

Maxima [A]

time = 0.25, size = 33, normalized size = 0.44

$$-\frac{1}{250} \left((25x^2 + 30x + 13) \cos(x) + (75x^2 + 40x + 9) \sin(x) \right) e^{-3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*sin(x)/exp(3*x),x, algorithm="maxima")`

[Out] $-1/250*((25*x^2 + 30*x + 13)*cos(x) + (75*x^2 + 40*x + 9)*sin(x))*e^{-3*x}$

Fricas [A]

time = 0.31, size = 37, normalized size = 0.49

$$-\frac{1}{250} (25x^2 + 30x + 13) \cos(x) e^{-3x} - \frac{1}{250} (75x^2 + 40x + 9) e^{-3x} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*sin(x)/exp(3*x),x, algorithm="fricas")`

[Out] $-1/250*(25*x^2 + 30*x + 13)*cos(x)*e^{-3*x} - 1/250*(75*x^2 + 40*x + 9)*e^{-3*x}*sin(x)$

Sympy [A]

time = 0.46, size = 80, normalized size = 1.07

$$\frac{3x^2 e^{-3x} \sin(x)}{10} - \frac{x^2 e^{-3x} \cos(x)}{10} - \frac{4x e^{-3x} \sin(x)}{25} - \frac{3x e^{-3x} \cos(x)}{25} - \frac{9e^{-3x} \sin(x)}{250} - \frac{13e^{-3x} \cos(x)}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*sin(x)/exp(3*x),x)

[Out] $-3*x**2*exp(-3*x)*sin(x)/10 - x**2*exp(-3*x)*cos(x)/10 - 4*x*exp(-3*x)*sin(x)/25 - 3*x*exp(-3*x)*cos(x)/25 - 9*exp(-3*x)*sin(x)/250 - 13*exp(-3*x)*cos(x)/250$

Giac [A]

time = 0.00, size = 74, normalized size = 0.99

$$e^{-3x} \left(\left(-\frac{13}{500} (8x^2 + 6x + 2) - \frac{9}{500} (-6x^2 - 2x) \right) \cos x + \left(-\frac{9}{500} (8x^2 + 6x + 2) + \frac{13}{500} (-6x^2 - 2x) \right) \sin x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sin(x)/exp(3*x),x)

[Out] $-1/250*((25*x^2 + 30*x + 13)*cos(x) + (75*x^2 + 40*x + 9)*sin(x))/e^(3*x)$

Mupad [B]

time = 0.11, size = 39, normalized size = 0.52

$$\frac{e^{-3x} (13 \cos(x) + 9 \sin(x) + 25x^2 \cos(x) + 75x^2 \sin(x) + 30x \cos(x) + 40x \sin(x))}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*exp(-3*x)*sin(x),x)

[Out] $-(exp(-3*x)*(13*cos(x) + 9*sin(x) + 25*x^2*cos(x) + 75*x^2*sin(x) + 30*x*cos(x) + 40*x*sin(x)))/250$

3.567 $\int e^{x/2} x^2 \cos^3(x) dx$

Optimal. Leaf size=187

$$-\frac{132}{125}e^{x/2} \cos(x) + \frac{18}{25}e^{x/2} x \cos(x) + \frac{48}{185}e^{x/2} x^2 \cos(x) + \frac{2}{37}e^{x/2} x^2 \cos^3(x) - \frac{428e^{x/2} \cos(3x)}{50653} + \frac{70e^{x/2} x \cos(3x)}{1369}$$

[Out] $-132/125*\exp(1/2*x)*\cos(x)+18/25*\exp(1/2*x)*x*\cos(x)+48/185*\exp(1/2*x)*x^2*\cos(x)+2/37*\exp(1/2*x)*x^2*\cos(x)^3-428/50653*\exp(1/2*x)*\cos(3*x)+70/1369*\exp(1/2*x)*x*\cos(3*x)-24/125*\exp(1/2*x)*\sin(x)-24/25*\exp(1/2*x)*x*\sin(x)+96/185*\exp(1/2*x)*x^2*\sin(x)+12/37*\exp(1/2*x)*x^2*\cos(x)^2*\sin(x)-792/50653*\exp(1/2*x)*\sin(3*x)-24/1369*\exp(1/2*x)*x*\sin(3*x)$

Rubi [A]

time = 0.29, antiderivative size = 253, normalized size of antiderivative = 1.35, number of steps used = 31, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {4520, 4518, 4554, 14, 4517, 4557, 4553, 4558}

$$\frac{96}{185}e^{x/2}x^2\sin(x) + \frac{2}{37}e^{x/2}x^2\cos^3(x) + \frac{48}{185}e^{x/2}x^2\cos(x) + \frac{12}{37}e^{x/2}x^2\sin(x)\cos^2(x) - \frac{1218672e^{x/2}\sin(x)}{6331625} - \frac{32556e^{x/2}\sin(x)}{34225} - \frac{816e^{x/2}\sin(3x)}{50653} - \frac{12e^{x/2}x\sin(3x)}{1369} + \frac{16e^{x/2}\cos^3(x)}{50653} - \frac{8e^{x/2}x\cos^3(x)}{1369} - \frac{6687696e^{x/2}\cos(x)}{6331625} + \frac{24792e^{x/2}x\cos(x)}{34225} - \frac{432e^{x/2}\cos(3x)}{50653} + \frac{72e^{x/2}x\cos(3x)}{1369} + \frac{96e^{x/2}\sin(x)\cos^2(x)}{50653} - \frac{48e^{x/2}x\sin(x)\cos^2(x)}{1369}$$

Antiderivative was successfully verified.

[In] Int[E^(x/2)*x^2*Cos[x]^3,x]

[Out] $(-6687696*E^{(x/2)}*Cos[x])/6331625 + (24792*E^{(x/2)}*x*Cos[x])/34225 + (48*E^{(x/2)}*x^2*Cos[x])/185 + (16*E^{(x/2)}*Cos[x]^3)/50653 - (8*E^{(x/2)}*x*Cos[x]^3)/1369 + (2*E^{(x/2)}*x^2*Cos[x]^3)/37 - (432*E^{(x/2)}*Cos[3*x])/50653 + (72*E^{(x/2)}*x*Cos[3*x])/1369 - (1218672*E^{(x/2)}*Sin[x])/6331625 - (32556*E^{(x/2)}*x*Sin[x])/34225 + (96*E^{(x/2)}*x^2*Sin[x])/185 + (96*E^{(x/2)}*Cos[x]^2*Sin[x])/50653 - (48*E^{(x/2)}*x*Cos[x]^2*Sin[x])/1369 + (12*E^{(x/2)}*x^2*Cos[x]^2*Sin[x])/37 - (816*E^{(x/2)}*Sin[3*x])/50653 - (12*E^{(x/2)}*x*Sin[3*x])/1369$

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)+(b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rule 4517

Int[(F_)^((c_)*((a_)+(b_)*(x_)))*Sin[(d_)+(e_)*(x_)], x_Symbol] := Simp[b*c*Log[F]*F^(c*(a+b*x))*(Sin[d+e*x]/(e^2+b^2*c^2*Log[F]^2)), x] - Simp[e*F^(c*(a+b*x))*(Cos[d+e*x]/(e^2+b^2*c^2*Log[F]^2)), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2+b^2*c^2*Log[F]^2, 0]

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :=
  Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4520

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :=
  Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] +
  (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] +
  Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)/(e^2*m^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
  a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_)*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :=
  Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_), x_Symbol] :=
  Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_), x_Symbol] :=
  Int[ExpandTrigReduce[F^(c*(a + b*x)), Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4558

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(x_)^(p_)*Sin[(d_.) + (e_.)*(x_)]^(m_), x_Symbol] :=
  Int[ExpandTrigReduce[x^p*F^(c*(a + b*x)), Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{x/2} x^2 \cos^3(x) dx &= \frac{48}{185} e^{x/2} x^2 \cos(x) + \frac{2}{37} e^{x/2} x^2 \cos^3(x) + \frac{96}{185} e^{x/2} x^2 \sin(x) + \frac{12}{37} e^{x/2} x^2 \cos^2(x) \sin(x) - \\
&= \frac{48}{185} e^{x/2} x^2 \cos(x) + \frac{2}{37} e^{x/2} x^2 \cos^3(x) + \frac{96}{185} e^{x/2} x^2 \sin(x) + \frac{12}{37} e^{x/2} x^2 \cos^2(x) \sin(x) - \\
&= \frac{48}{185} e^{x/2} x^2 \cos(x) + \frac{2}{37} e^{x/2} x^2 \cos^3(x) + \frac{96}{185} e^{x/2} x^2 \sin(x) + \frac{12}{37} e^{x/2} x^2 \cos^2(x) \sin(x) - \\
&= \frac{20352 e^{x/2} x \cos(x)}{34225} + \frac{48}{185} e^{x/2} x^2 \cos(x) - \frac{8 e^{x/2} x \cos^3(x)}{1369} + \frac{2}{37} e^{x/2} x^2 \cos^3(x) - \frac{30336}{12663250} \\
&= \frac{20352 e^{x/2} x \cos(x)}{34225} + \frac{48}{185} e^{x/2} x^2 \cos(x) - \frac{8 e^{x/2} x \cos^3(x)}{1369} + \frac{2}{37} e^{x/2} x^2 \cos^3(x) - \frac{30336}{12663250} \\
&= -\frac{48384 e^{x/2} \cos(x)}{171125} + \frac{24792 e^{x/2} x \cos(x)}{34225} + \frac{48}{185} e^{x/2} x^2 \cos(x) + \frac{16 e^{x/2} \cos^3(x)}{50653} - \frac{8 e^{x/2}}{12663250} \\
&= -\frac{1780608 e^{x/2} \cos(x)}{6331625} + \frac{24792 e^{x/2} x \cos(x)}{34225} + \frac{48}{185} e^{x/2} x^2 \cos(x) + \frac{16 e^{x/2} \cos^3(x)}{50653} - \frac{8 e^{x/2}}{12663250} \\
&= -\frac{2482128 e^{x/2} \cos(x)}{6331625} + \frac{24792 e^{x/2} x \cos(x)}{34225} + \frac{48}{185} e^{x/2} x^2 \cos(x) + \frac{16 e^{x/2} \cos^3(x)}{50653} - \frac{8 e^{x/2}}{12663250}
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 72, normalized size = 0.39

$$\frac{e^{x/2} (151959 (-88 + 60x + 25x^2) \cos(x) + 125 (-856 + 5180x + 1369x^2) \cos(3x) + 303918 (-8 - 40x + 25x^2) \sin(x) + 750 (-264 - 296x + 1369x^2) \sin(3x))}{12663250}$$

Antiderivative was successfully verified.

[In] Integrate[E^(x/2)*x^2*Cos[x]^3,x]

[Out] (E^(x/2)*(151959*(-88 + 60*x + 25*x^2)*Cos[x] + 125*(-856 + 5180*x + 1369*x^2)*Cos[3*x] + 303918*(-8 - 40*x + 25*x^2)*Sin[x] + 750*(-264 - 296*x + 1369*x^2)*Sin[3*x]))/12663250

Mathics [A]

time = 4.02, size = 90, normalized size = 0.48

$$\frac{2 (-3205680 x \sin[x] - 647500 x \cos[x] \sin[x]^2 + 222000 x \sin[x]^3 + 2441260 x \cos[x] - 1026750 x^2 \sin[x]^3 - 171125 x^2 \cos[x] \sin[x]^2 + 992525 x^2 \cos[x] + 2669550 x^2 \sin[x] - 3369848 \cos[x] - 756336 \sin[x] + 107000 \cos[x] \sin[x]^2 + 198000 \sin[x]^3) E^{x/2}}{6331625}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[E^(x/2)*x^2*Cos[x]^3,x]')

[Out] 2 (-3205680 x Sin[x] - 647500 x Cos[x] Sin[x]^2 + 222000 x Sin[x]^3 + 2441260 x Cos[x] - 1026750 x^2 Sin[x]^3 - 171125 x^2 Cos[x] Sin[x]^2

+ 992525 x ^ 2 Cos[x] + 2669550 x ^ 2 Sin[x] - 3369848 Cos[x] - 756336 Sin[x] + 107000 Cos[x] Sin[x] ^ 2 + 198000 Sin[x] ^ 3) E ^ (x / 2) / 6331625

Maple [A]

time = 0.11, size = 78, normalized size = 0.42

method	result
default	$\frac{\left(\frac{2}{37}x^2 + \frac{280}{1369}x - \frac{1712}{50653}\right)e^{\frac{x}{2}}\cos(3x) - \left(-\frac{12}{37}x^2 + \frac{96}{1369}x + \frac{3168}{50653}\right)e^{\frac{x}{2}}\sin(3x) + \frac{3\left(\frac{2}{5}x^2 + \frac{24}{25}x - \frac{176}{125}\right)e^{\frac{x}{2}}\cos(x) - 3\left(-\frac{4}{5}x^2 + \frac{32}{25}x + \frac{32}{125}\right)e^{\frac{x}{2}}\sin(x)}{4}$
risch	$\left(\frac{1}{202612} - \frac{3i}{101306}\right)(1369x^2 + 888ix - 148x - 96i - 280)e^{\left(\frac{1}{2}+3i\right)x} + \left(\frac{3}{500} - \frac{3i}{250}\right)(25x^2 + 40ix - 20x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(1/2*x)*x^2*cos(x)^3,x,method=_RETURNVERBOSE)

[Out] 1/4*(2/37*x^2+280/1369*x-1712/50653)*exp(1/2*x)*cos(3*x)-1/4*(-12/37*x^2+96/1369*x+3168/50653)*exp(1/2*x)*sin(3*x)+3/4*(2/5*x^2+24/25*x-176/125)*exp(1/2*x)*cos(x)-3/4*(-4/5*x^2+32/25*x+32/125)*exp(1/2*x)*sin(x)

Maxima [A]

time = 0.25, size = 77, normalized size = 0.41

$$\frac{1}{101306}(1369x^2 + 5180x - 856)\cos(3x)e^{\left(\frac{1}{2}\right)x} + \frac{3}{250}(25x^2 + 60x - 88)\cos(x)e^{\left(\frac{1}{2}\right)x} + \frac{3}{50653}(1369x^2 - 296x - 264)e^{\left(\frac{1}{2}\right)x}\sin(3x) + \frac{3}{125}(25x^2 - 40x - 8)e^{\left(\frac{1}{2}\right)x}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1/2*x)*x^2*cos(x)^3,x, algorithm="maxima")

[Out] 1/101306*(1369*x^2 + 5180*x - 856)*cos(3*x)*e^(1/2*x) + 3/250*(25*x^2 + 60*x - 88)*cos(x)*e^(1/2*x) + 3/50653*(1369*x^2 - 296*x - 264)*e^(1/2*x)*sin(3*x) + 3/125*(25*x^2 - 40*x - 8)*e^(1/2*x)*sin(x)

Fricas [A]

time = 0.34, size = 72, normalized size = 0.39

$$\frac{12}{6331625}(125(1369x^2 - 296x - 264)\cos(x)^2 + 273800x^2 - 497280x - 93056)e^{\left(\frac{1}{2}\right)x}\sin(x) + \frac{2}{6331625}(125(1369x^2 + 5180x - 856)\cos(x)^3 + 24(34225x^2 + 74740x - 135952)\cos(x))e^{\left(\frac{1}{2}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1/2*x)*x^2*cos(x)^3,x, algorithm="fricas")

[Out] 12/6331625*(125*(1369*x^2 - 296*x - 264)*cos(x)^2 + 273800*x^2 - 497280*x - 93056)*e^(1/2*x)*sin(x) + 2/6331625*(125*(1369*x^2 + 5180*x - 856)*cos(x)^3 + 24*(34225*x^2 + 74740*x - 135952)*cos(x))*e^(1/2*x)

Sympy [A]

time = 1.41, size = 202, normalized size = 1.08

$$\frac{96x^2e^{\frac{x}{2}}\sin^2(x)}{185} + \frac{48x^2e^{\frac{x}{2}}\sin^2(x)\cos(x)}{185} + \frac{156x^2e^{\frac{x}{2}}\sin(x)\cos^2(x)}{185} + \frac{58x^2e^{\frac{x}{2}}\cos^3(x)}{185} - \frac{32256xe^{\frac{x}{2}}\sin^3(x)}{34225} + \frac{19392xe^{\frac{x}{2}}\sin^2(x)\cos(x)}{34225} - \frac{34656xe^{\frac{x}{2}}\sin(x)\cos^2(x)}{34225} + \frac{26392xe^{\frac{x}{2}}\cos^3(x)}{34225} - \frac{1116672e^{\frac{x}{2}}\sin^3(x)}{6331625} - \frac{6525696e^{\frac{x}{2}}\sin^2(x)\cos(x)}{6331625} - \frac{1512672e^{\frac{x}{2}}\sin(x)\cos^2(x)}{6331625} - \frac{6739696e^{\frac{x}{2}}\cos^3(x)}{6331625}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1/2*x)*x**2*cos(x)**3,x)

[Out] 96*x**2*exp(x/2)*sin(x)**3/185 + 48*x**2*exp(x/2)*sin(x)**2*cos(x)/185 + 156*x**2*exp(x/2)*sin(x)*cos(x)**2/185 + 58*x**2*exp(x/2)*cos(x)**3/185 - 32256*x*exp(x/2)*sin(x)**3/34225 + 19392*x*exp(x/2)*sin(x)**2*cos(x)/34225 - 34656*x*exp(x/2)*sin(x)*cos(x)**2/34225 + 26392*x*exp(x/2)*cos(x)**3/34225 - 1116672*exp(x/2)*sin(x)**3/6331625 - 6525696*exp(x/2)*sin(x)**2*cos(x)/6331625 - 1512672*exp(x/2)*sin(x)*cos(x)**2/6331625 - 6739696*exp(x/2)*cos(x)**3/6331625

Giac [A]

time = 0.00, size = 185, normalized size = 0.99

$$2 \left(e^{\frac{x}{2}} \left(-\frac{11}{125} \left(-9 \left(\frac{x}{2} \right)^2 - 3x + 6 \right) - \frac{2}{125} \left(12 \left(\frac{x}{2} \right)^3 - 6x \right) \right) \cos x - \left(\frac{2}{125} \left(-9 \left(\frac{x}{2} \right)^2 - 3x + 6 \right) - \frac{11}{125} \left(12 \left(\frac{x}{2} \right)^3 - 6x \right) \right) \sin x + e^{\frac{x}{2}} \left(-\frac{107}{50653} \left(-35 \left(\frac{x}{2} \right)^2 - x + 2 \right) - \frac{198}{50653} \left(12 \left(\frac{x}{2} \right)^3 - 6x \right) \right) \cos(3x) - \left(\frac{198}{50653} \left(-35 \left(\frac{x}{2} \right)^2 - x + 2 \right) - \frac{107}{50653} \left(12 \left(\frac{x}{2} \right)^3 - 6x \right) \right) \sin(3x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1/2*x)*x^2*cos(x)^3,x)

[Out] 1/101306*((1369*x^2 + 5180*x - 856)*cos(3*x) + 6*(1369*x^2 - 296*x - 264)*sin(3*x))*e^(1/2*x) + 3/250*((25*x^2 + 60*x - 88)*cos(x) + 2*(25*x^2 - 40*x - 8)*sin(x))*e^(1/2*x)

Mupad [B]

time = 0.31, size = 83, normalized size = 0.44

$$\frac{e^{x/2} (107000 \cos(3x) + 198000 \sin(3x) + 13372392 \cos(x) + 2431344 \sin(x) - 647500 x \cos(3x) - 3798975 x^2 \cos(x) + 222000 x \sin(3x) - 7597950 x^2 \sin(x) - 171125 x^2 \cos(3x) - 1026750 x^2 \sin(3x) - 9117540 x \cos(x) + 12156720 x \sin(x))}{12663250}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*exp(x/2)*cos(x)^3,x)

[Out] -(exp(x/2)*(107000*cos(3*x) + 198000*sin(3*x) + 13372392*cos(x) + 2431344*sin(x) - 647500*x*cos(3*x) - 3798975*x^2*cos(x) + 222000*x*sin(3*x) - 7597950*x^2*sin(x) - 171125*x^2*cos(3*x) - 1026750*x^2*sin(3*x) - 9117540*x*cos(x) + 12156720*x*sin(x)))/12663250

3.568 $\int e^{2x} x^2 \sin(4x) dx$

Optimal. Leaf size=87

$$\frac{1}{250}e^{2x} \cos(4x) + \frac{2}{25}e^{2x} x \cos(4x) - \frac{1}{5}e^{2x} x^2 \cos(4x) - \frac{11}{500}e^{2x} \sin(4x) + \frac{3}{50}e^{2x} x \sin(4x) + \frac{1}{10}e^{2x} x^2 \sin(4x)$$

[Out] 1/250*exp(2*x)*cos(4*x)+2/25*exp(2*x)*x*cos(4*x)-1/5*exp(2*x)*x^2*cos(4*x)-11/500*exp(2*x)*sin(4*x)+3/50*exp(2*x)*x*sin(4*x)+1/10*exp(2*x)*x^2*sin(4*x)

Rubi [A]

time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {4517, 4553, 14, 4518, 4554}

$$\frac{1}{10}e^{2x} x^2 \sin(4x) - \frac{1}{5}e^{2x} x^2 \cos(4x) + \frac{3}{50}e^{2x} x \sin(4x) - \frac{11}{500}e^{2x} \sin(4x) + \frac{2}{25}e^{2x} x \cos(4x) + \frac{1}{250}e^{2x} \cos(4x)$$

Antiderivative was successfully verified.

[In] Int[E^(2*x)*x^2*Sin[4*x], x]

[Out] (E^(2*x)*Cos[4*x])/250 + (2*E^(2*x)*x*Cos[4*x])/25 - (E^(2*x)*x^2*Cos[4*x])/5 - (11*E^(2*x)*Sin[4*x])/500 + (3*E^(2*x)*x*Sin[4*x])/50 + (E^(2*x)*x^2*Sin[4*x])/10

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 4517

Int[(F_)^((c_)*((a_.) + (b_)*(x_)))*Sin[(d_.) + (e_)*(x_)], x_Symbol] := Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]

Rule 4518

Int[Cos[(d_.) + (e_)*(x_)]*(F_)^((c_)*((a_.) + (b_)*(x_))), x_Symbol] := Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]

Rule 4553

```
Int[(F_)^((c_)*(a_) + (b_)*(x_))*((f_)*(x_)^(m_)*Sin[(d_) + (e_)*
(x_)]^(n_), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_) + (e_)*(x_)]^(n_)*(F_)^((c_)*(a_) + (b_)*(x_))*((f_)*
(x_))^(m_), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{2x} x^2 \sin(4x) dx &= -\frac{1}{5} e^{2x} x^2 \cos(4x) + \frac{1}{10} e^{2x} x^2 \sin(4x) - 2 \int x \left(-\frac{1}{5} e^{2x} \cos(4x) + \frac{1}{10} e^{2x} \sin(4x) \right) dx \\
&= -\frac{1}{5} e^{2x} x^2 \cos(4x) + \frac{1}{10} e^{2x} x^2 \sin(4x) - 2 \int \left(-\frac{1}{5} e^{2x} x \cos(4x) + \frac{1}{10} e^{2x} x \sin(4x) \right) dx \\
&= -\frac{1}{5} e^{2x} x^2 \cos(4x) + \frac{1}{10} e^{2x} x^2 \sin(4x) - \frac{1}{5} \int e^{2x} x \sin(4x) dx + \frac{2}{5} \int e^{2x} x \cos(4x) dx \\
&= \frac{2}{25} e^{2x} x \cos(4x) - \frac{1}{5} e^{2x} x^2 \cos(4x) + \frac{3}{50} e^{2x} x \sin(4x) + \frac{1}{10} e^{2x} x^2 \sin(4x) + \frac{1}{5} \int \left(-\frac{1}{5} e^{2x} \right) dx \\
&= \frac{2}{25} e^{2x} x \cos(4x) - \frac{1}{5} e^{2x} x^2 \cos(4x) + \frac{3}{50} e^{2x} x \sin(4x) + \frac{1}{10} e^{2x} x^2 \sin(4x) + \frac{1}{50} \int e^{2x} \sin(4x) dx \\
&= \frac{3}{250} e^{2x} \cos(4x) + \frac{2}{25} e^{2x} x \cos(4x) - \frac{1}{5} e^{2x} x^2 \cos(4x) - \frac{3}{500} e^{2x} \sin(4x) + \frac{3}{50} e^{2x} x \sin(4x)
\end{aligned}$$

Mathematica [A]

time = 0.05, size = 40, normalized size = 0.46

$$\frac{1}{500} e^{2x} \left((2 + 40x - 100x^2) \cos(4x) + (-11 + 30x + 50x^2) \sin(4x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*x^2*Sin[4*x], x]
```

```
[Out] (E^(2*x)*((2 + 40*x - 100*x^2)*Cos[4*x] + (-11 + 30*x + 50*x^2)*Sin[4*x]))/
500
```

Mathics [A]

time = 2.50, size = 52, normalized size = 0.60

$$\frac{(30x \sin[4x] + 40x \cos[4x] - 100x^2 \cos[4x] + 50x^2 \sin[4x] - 11 \sin[4x] + 2 \cos[4x]) E^{2x}}{500}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[E^(2*x)*x^2*Sin[4*x],x]')`

[Out] $(30 x \sin[4 x] + 40 x \cos[4 x] - 100 x^2 \cos[4 x] + 50 x^2 \sin[4 x] - 1 \sin[4 x] + 2 \cos[4 x]) E^{(2 x)} / 500$

Maple [A]

time = 0.05, size = 40, normalized size = 0.46

method	result
default	$\left(-\frac{1}{5}x^2 + \frac{2}{25}x + \frac{1}{250}\right) e^{2x} \cos(4x) + \left(\frac{1}{10}x^2 + \frac{3}{50}x - \frac{11}{500}\right) e^{2x} \sin(4x)$
risch	$\left(-\frac{1}{500} - \frac{i}{1000}\right) (50x^2 + 20ix - 10x - 4i - 3) e^{(2+4i)x} + \left(-\frac{1}{500} + \frac{i}{1000}\right) (50x^2 - 20ix - 10x + 4i - 3) e^{(2-4i)x}$
norman	$\frac{2e^{2x}x - e^{2x}x^2}{25} - \frac{11e^{2x}\tan(2x)}{250} - \frac{e^{2x}(\tan^2(2x))}{250} + \frac{3e^{2x}x\tan(2x)}{25} - \frac{2e^{2x}x(\tan^2(2x))}{25} + \frac{e^{2x}x^2\tan(2x)}{5} + \frac{e^{2x}x^2(\tan^2(2x))}{5} + \frac{e^{2x}}{250}$ $1 + \tan^2(2x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(2*x)*x^2*sin(4*x),x,method=_RETURNVERBOSE)`

[Out] $(-1/5*x^2+2/25*x+1/250)*exp(2*x)*cos(4*x)+(1/10*x^2+3/50*x-11/500)*exp(2*x)*sin(4*x)$

Maxima [A]

time = 0.27, size = 41, normalized size = 0.47

$$-\frac{1}{250} (50x^2 - 20x - 1) \cos(4x) e^{(2x)} + \frac{1}{500} (50x^2 + 30x - 11) e^{(2x)} \sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(2*x)*x^2*sin(4*x),x, algorithm="maxima")`

[Out] $-1/250*(50*x^2 - 20*x - 1)*cos(4*x)*e^{(2*x)} + 1/500*(50*x^2 + 30*x - 11)*e^{(2*x)}*sin(4*x)$

Fricas [A]

time = 0.31, size = 41, normalized size = 0.47

$$-\frac{1}{250} (50x^2 - 20x - 1) \cos(4x) e^{(2x)} + \frac{1}{500} (50x^2 + 30x - 11) e^{(2x)} \sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(2*x)*x^2*sin(4*x),x, algorithm="fricas")`

[Out] $-1/250*(50*x^2 - 20*x - 1)*cos(4*x)*e^{(2*x)} + 1/500*(50*x^2 + 30*x - 11)*e^{(2*x)}*sin(4*x)$

Sympy [A]

time = 0.37, size = 85, normalized size = 0.98

$$\frac{x^2 e^{2x} \sin(4x)}{10} - \frac{x^2 e^{2x} \cos(4x)}{5} + \frac{3x e^{2x} \sin(4x)}{50} + \frac{2x e^{2x} \cos(4x)}{25} - \frac{11 e^{2x} \sin(4x)}{500} + \frac{e^{2x} \cos(4x)}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)*x**2*sin(4*x),x)

[Out] x**2*exp(2*x)*sin(4*x)/10 - x**2*exp(2*x)*cos(4*x)/5 + 3*x*exp(2*x)*sin(4*x)/50 + 2*x*exp(2*x)*cos(4*x)/25 - 11*exp(2*x)*sin(4*x)/500 + exp(2*x)*cos(4*x)/250

Giac [A]

time = 0.00, size = 89, normalized size = 1.02

$$\frac{1}{2} e^{2x} \left(\left(\frac{-3(2x)^2 - 4x + 2}{250} - \frac{11}{500} (4(2x)^2 - 8x) \right) \cos(4x) + \left(-\frac{11}{500} (-3(2x)^2 - 4x + 2) - \frac{4(2x)^2 - 8x}{250} \right) \sin(4x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)*x^2*sin(4*x),x)

[Out] -1/500*(2*(50*x^2 - 20*x - 1)*cos(4*x) - (50*x^2 + 30*x - 11)*sin(4*x))*e^(2*x)

Mupad [B]

time = 0.35, size = 51, normalized size = 0.59

$$\frac{e^{2x} (2 \cos(4x) - 11 \sin(4x) + 40x \cos(4x) + 30x \sin(4x) - 100x^2 \cos(4x) + 50x^2 \sin(4x))}{500}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*sin(4*x)*exp(2*x),x)

[Out] (exp(2*x)*(2*cos(4*x) - 11*sin(4*x) + 40*x*cos(4*x) + 30*x*sin(4*x) - 100*x^2*cos(4*x) + 50*x^2*sin(4*x)))/500

3.569 $\int e^{x/2} x^2 \cos(x) \sin^2(x) dx$

Optimal. Leaf size=185

$$-\frac{44}{125}e^{x/2} \cos(x) + \frac{6}{25}e^{x/2} x \cos(x) + \frac{1}{10}e^{x/2} x^2 \cos(x) + \frac{428e^{x/2} \cos(3x)}{50653} - \frac{70e^{x/2} x \cos(3x)}{1369} - \frac{1}{74}e^{x/2} x^2 \cos(3x) - \frac{8}{125}e^{x/2} \sin(x) + \frac{8}{25}e^{x/2} x \sin(x) - \frac{1}{5}e^{x/2} x^2 \sin(x) + \frac{792e^{x/2} \sin(3x)}{50653} - \frac{24e^{x/2} x \sin(3x)}{1369} + \frac{3}{37}e^{x/2} x^2 \sin(3x)$$

[Out] $-44/125*\exp(1/2*x)*\cos(x)+6/25*\exp(1/2*x)*x*\cos(x)+1/10*\exp(1/2*x)*x^2*\cos(x)+428/50653*\exp(1/2*x)*\cos(3*x)-70/1369*\exp(1/2*x)*x*\cos(3*x)-1/74*\exp(1/2*x)*x^2*\cos(3*x)-8/125*\exp(1/2*x)*\sin(x)-8/25*\exp(1/2*x)*x*\sin(x)+1/5*\exp(1/2*x)*x^2*\sin(x)+792/50653*\exp(1/2*x)*\sin(3*x)+24/1369*\exp(1/2*x)*x*\sin(3*x)-3/37*\exp(1/2*x)*x^2*\sin(3*x)$

Rubi [A]

time = 0.21, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$,

Rules used = {4558, 4518, 4554, 14, 4517, 4553}

$$\frac{1}{5}e^{x/2}x^2\sin(x) - \frac{3}{37}e^{x/2}x^2\sin(3x) + \frac{1}{10}e^{x/2}x^2\cos(x) - \frac{1}{74}e^{x/2}x^2\cos(3x) - \frac{8}{25}e^{x/2}x\sin(x) + \frac{24e^{x/2}x\sin(3x)}{1369} - \frac{8}{125}e^{x/2}\sin(x) + \frac{792e^{x/2}\sin(3x)}{50653} + \frac{6}{25}e^{x/2}x\cos(x) - \frac{70e^{x/2}x\cos(3x)}{1369} - \frac{44}{125}e^{x/2}\cos(x) + \frac{428e^{x/2}\cos(3x)}{50653}$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{(x/2)}*x^2*\text{Cos}[x]*\text{Sin}[x]^2, x]$

[Out] $(-44*E^{(x/2)}*\text{Cos}[x])/125 + (6*E^{(x/2)}*x*\text{Cos}[x])/25 + (E^{(x/2)}*x^2*\text{Cos}[x])/10 + (428*E^{(x/2)}*\text{Cos}[3*x])/50653 - (70*E^{(x/2)}*x*\text{Cos}[3*x])/1369 - (E^{(x/2)}*x^2*\text{Cos}[3*x])/74 - (8*E^{(x/2)}*\text{Sin}[x])/125 - (8*E^{(x/2)}*x*\text{Sin}[x])/25 + (E^{(x/2)}*x^2*\text{Sin}[x])/5 + (792*E^{(x/2)}*\text{Sin}[3*x])/50653 + (24*E^{(x/2)}*x*\text{Sin}[3*x])/1369 - (3*E^{(x/2)}*x^2*\text{Sin}[3*x])/37$

Rule 14

$\text{Int}[(u_)*((c_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}\{c, m\}, x] \ \&\& \ \text{SumQ}[u] \ \&\& \ !\text{LinearQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (a_ + (b_)*(v_)) /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{InverseFunctionQ}[v]$

Rule 4517

$\text{Int}[(F_)^{((c_)*((a_)+(b_)*(x_)))*\text{Sin}[(d_)+(e_)*(x_)]}, x_Symbol] \rightarrow \text{Simp}[b*c*\text{Log}[F]*F^{(c*(a+b*x))*(\text{Sin}[d+e*x]/(e^2+b^2*c^2*\text{Log}[F]^2))}, x] - \text{Simp}[e*F^{(c*(a+b*x))*(\text{Cos}[d+e*x]/(e^2+b^2*c^2*\text{Log}[F]^2))}, x] /; \text{FreeQ}\{F, a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[e^2+b^2*c^2*\text{Log}[F]^2, 0]$

Rule 4518

$\text{Int}[\text{Cos}[(d_)+(e_)*(x_)]*(F_)^{((c_)*((a_)+(b_)*(x_)))}, x_Symbol] \rightarrow \text{Simp}[b*c*\text{Log}[F]*F^{(c*(a+b*x))*(\text{Cos}[d+e*x]/(e^2+b^2*c^2*\text{Log}[F]^2))}, x] + \text{Simp}[e*F^{(c*(a+b*x))*(\text{Sin}[d+e*x]/(e^2+b^2*c^2*\text{Log}[F]^2))}, x] /; \text{FreeQ}\{F, a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[e^2+b^2*c^2*\text{Log}[F]^2, 0]$

FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]

Rule 4553

Int[(F_)^((c_)*((a_) + (b_)*(x_)))*((f_)*(x_))^(m_)*Sin[(d_) + (e_)*(x_)]^(n_), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]

Rule 4554

Int[Cos[(d_) + (e_)*(x_)]^(n_)*(F_)^((c_)*((a_) + (b_)*(x_)))*((f_)*(x_))^(m_), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]

Rule 4558

Int[Cos[(f_) + (g_)*(x_)]^(n_)*(F_)^((c_)*((a_) + (b_)*(x_)))*(x_)^(p_) * Sin[(d_) + (e_)*(x_)]^(m_), x_Symbol] :> Int[ExpandTrigReduce[x^p*F^(c*(a + b*x)), Sin[d + e*x]^m * Cos[f + g*x]^n, x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int e^{x/2} x^2 \cos(x) \sin^2(x) dx &= \int \left(\frac{1}{4} e^{x/2} x^2 \cos(x) - \frac{1}{4} e^{x/2} x^2 \cos(3x) \right) dx \\
 &= \frac{1}{4} \int e^{x/2} x^2 \cos(x) dx - \frac{1}{4} \int e^{x/2} x^2 \cos(3x) dx \\
 &= \frac{1}{10} e^{x/2} x^2 \cos(x) - \frac{1}{74} e^{x/2} x^2 \cos(3x) + \frac{1}{5} e^{x/2} x^2 \sin(x) - \frac{3}{37} e^{x/2} x^2 \sin(3x) - \frac{1}{2} \\
 &= \frac{1}{10} e^{x/2} x^2 \cos(x) - \frac{1}{74} e^{x/2} x^2 \cos(3x) + \frac{1}{5} e^{x/2} x^2 \sin(x) - \frac{3}{37} e^{x/2} x^2 \sin(3x) - \frac{1}{2} \\
 &= \frac{1}{10} e^{x/2} x^2 \cos(x) - \frac{1}{74} e^{x/2} x^2 \cos(3x) + \frac{1}{5} e^{x/2} x^2 \sin(x) - \frac{3}{37} e^{x/2} x^2 \sin(3x) + \frac{1}{37} \\
 &= \frac{6}{25} e^{x/2} x \cos(x) + \frac{1}{10} e^{x/2} x^2 \cos(x) - \frac{70 e^{x/2} x \cos(3x)}{1369} - \frac{1}{74} e^{x/2} x^2 \cos(3x) - \frac{8}{25} \\
 &= \frac{6}{25} e^{x/2} x \cos(x) + \frac{1}{10} e^{x/2} x^2 \cos(x) - \frac{70 e^{x/2} x \cos(3x)}{1369} - \frac{1}{74} e^{x/2} x^2 \cos(3x) - \frac{8}{25} \\
 &= -\frac{12}{125} e^{x/2} \cos(x) + \frac{6}{25} e^{x/2} x \cos(x) + \frac{1}{10} e^{x/2} x^2 \cos(x) + \frac{140 e^{x/2} \cos(3x)}{50653} - \frac{70 e^{x/2} x \cos(3x)}{50653}
 \end{aligned}$$

Mathematica [A]

time = 0.14, size = 76, normalized size = 0.41

$$\frac{e^{x/2} (50653 ((-88 + 60x + 25x^2) \cos(x) + 2(-8 - 40x + 25x^2) \sin(x)) - 125 ((-856 + 5180x + 1369x^2) \cos(3x) + 6(-264 - 296x + 1369x^2) \sin(3x)))}{12663250}$$

Antiderivative was successfully verified.

[In] Integrate[E^(x/2)*x^2*Cos[x]*Sin[x]^2,x]

[Out] (E^(x/2)*(50653*((-88 + 60*x + 25*x^2)*Cos[x] + 2*(-8 - 40*x + 25*x^2)*Sin[x]) - 125*((-856 + 5180*x + 1369*x^2)*Cos[3*x] + 6*(-264 - 296*x + 1369*x^2)*Sin[3*x]))/12663250

Mathics [A]

time = 4.01, size = 90, normalized size = 0.49

$$\frac{2(-846560x \sin[x] - 222000x \sin[x]^3 + 597920x \cos[x] + 647500x \cos[x] \sin[x]^2 - 136900x^2 \sin[x] + 171125x^2 \cos[x] \sin[x]^2 + 273800x^2 \cos[x] + 1026750x^2 \sin[x]^3 - 1087616 \cos[x] - 198000 \sin[x]^3 - 107000 \cos[x] \sin[x]^2 - 54112 \sin[x]) E^{x/2}}{6331625}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[E^(x/2)*x^2*Sin[x]^2*Cos[x],x]')

[Out] 2(-846560 x Sin[x] - 222000 x Sin[x]^3 + 597920 x Cos[x] + 647500 x Cos[x] Sin[x]^2 - 136900 x^2 Sin[x] + 171125 x^2 Cos[x] Sin[x]^2 + 273800 x^2 Cos[x] + 1026750 x^2 Sin[x]^3 - 1087616 Cos[x] - 198000 Sin[x]^3 - 107000 Cos[x] Sin[x]^2 - 54112 Sin[x]) E^(x/2) / 6331625

Maple [A]

time = 0.08, size = 78, normalized size = 0.42

method	result
default	$\frac{(\frac{2}{5}x^2 + \frac{24}{25}x - \frac{176}{125})e^{\frac{x}{2}} \cos(x)}{4} - \frac{(-\frac{4}{5}x^2 + \frac{32}{25}x + \frac{32}{125})e^{\frac{x}{2}} \sin(x)}{4} - \frac{(\frac{2}{37}x^2 + \frac{280}{1369}x - \frac{1712}{50653})e^{\frac{x}{2}} \cos(3x)}{4} + \frac{(-\frac{12}{37}x^2 + \frac{96}{1369}x + \frac{3168}{50653})e^{\frac{x}{2}} \sin(3x)}{4}$
risch	$(-\frac{1}{202612} + \frac{3i}{101306})(1369x^2 + 888ix - 148x - 96i - 280)e^{(\frac{1}{2}+3i)x} + (\frac{1}{500} - \frac{i}{250})(25x^2 + 40ix - 20)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x,method=_RETURNVERBOSE)

[Out] 1/4*(2/5*x^2+24/25*x-176/125)*exp(1/2*x)*cos(x)-1/4*(-4/5*x^2+32/25*x+32/125)*exp(1/2*x)*sin(x)-1/4*(2/37*x^2+280/1369*x-1712/50653)*exp(1/2*x)*cos(3*x)+1/4*(-12/37*x^2+96/1369*x+3168/50653)*exp(1/2*x)*sin(3*x)

Maxima [A]

time = 0.26, size = 77, normalized size = 0.42

$$-\frac{1}{101306} (1369x^2 + 5180x - 856) \cos(3x) e^{(\frac{1}{2}x)} + \frac{1}{250} (25x^2 + 60x - 88) \cos(x) e^{(\frac{1}{2}x)} - \frac{3}{50653} (1369x^2 - 296x - 264) e^{(\frac{1}{2}x)} \sin(3x) + \frac{1}{125} (25x^2 - 40x - 8) e^{(\frac{1}{2}x)} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x, algorithm="maxima")

[Out] -1/101306*(1369*x^2 + 5180*x - 856)*cos(3*x)*e^(1/2*x) + 1/250*(25*x^2 + 60*x - 88)*cos(x)*e^(1/2*x) - 3/50653*(1369*x^2 - 296*x - 264)*e^(1/2*x)*sin(3*x) + 1/125*(25*x^2 - 40*x - 8)*e^(1/2*x)*sin(x)

Fricas [A]

time = 0.31, size = 72, normalized size = 0.39

$$-\frac{4}{6331625} (375 (1369x^2 - 296x - 264) \cos(x)^2 - 444925x^2 + 534280x + 126056) e^{\frac{1}{2}x} \sin(x) - \frac{2}{6331625} (125 (1369x^2 + 5180x - 856) \cos(x)^3 - (444925x^2 + 1245420x - 1194616) \cos(x)) e^{\frac{1}{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x, algorithm="fricas")

[Out] -4/6331625*(375*(1369*x^2 - 296*x - 264)*cos(x)^2 - 444925*x^2 + 534280*x + 126056)*e^(1/2*x)*sin(x) - 2/6331625*(125*(1369*x^2 + 5180*x - 856)*cos(x)^3 - (444925*x^2 + 1245420*x - 1194616)*cos(x))*e^(1/2*x)

Sympy [A]

time = 1.40, size = 202, normalized size = 1.09

$$\frac{52x^2e^{\frac{x}{2}}\sin^3(x)}{185} + \frac{26x^2e^{\frac{x}{2}}\sin^2(x)\cos(x)}{185} - \frac{8x^2e^{\frac{x}{2}}\sin(x)\cos^2(x)}{185} + \frac{16x^2e^{\frac{x}{2}}\cos^3(x)}{185} - \frac{11552xe^{\frac{x}{2}}\sin^3(x)}{34225} + \frac{13464xe^{\frac{x}{2}}\sin^2(x)\cos(x)}{34225} - \frac{9152xe^{\frac{x}{2}}\sin(x)\cos^2(x)}{34225} + \frac{6464xe^{\frac{x}{2}}\cos^3(x)}{34225} - \frac{504224e^{\frac{x}{2}}\sin^3(x)}{6331625} - \frac{2389232e^{\frac{x}{2}}\sin^2(x)\cos(x)}{6331625} - \frac{108224e^{\frac{x}{2}}\sin(x)\cos^2(x)}{6331625} - \frac{2175232e^{\frac{x}{2}}\cos^3(x)}{6331625}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1/2*x)*x**2*cos(x)*sin(x)**2,x)

[Out] 52*x**2*exp(x/2)*sin(x)**3/185 + 26*x**2*exp(x/2)*sin(x)**2*cos(x)/185 - 8*x**2*exp(x/2)*sin(x)*cos(x)**2/185 + 16*x**2*exp(x/2)*cos(x)**3/185 - 11552*x*exp(x/2)*sin(x)**3/34225 + 13464*x*exp(x/2)*sin(x)**2*cos(x)/34225 - 9152*x*exp(x/2)*sin(x)*cos(x)**2/34225 + 6464*x*exp(x/2)*cos(x)**3/34225 - 504224*exp(x/2)*sin(x)**3/6331625 - 2389232*exp(x/2)*sin(x)**2*cos(x)/6331625 - 108224*exp(x/2)*sin(x)*cos(x)**2/6331625 - 2175232*exp(x/2)*cos(x)**3/6331625

Giac [A]

time = 0.00, size = 177, normalized size = 0.96

$$2 \left(e^{\frac{x}{2}} \left(\left(-\frac{11}{125} \left(-3 \left(\frac{x}{2} \right)^2 - x + 2 \right) - \frac{2}{125} \left(4 \left(\frac{x}{2} \right)^2 - 2x \right) \right) \cos x - \left(\frac{2}{125} \left(-3 \left(\frac{x}{2} \right)^2 - x + 2 \right) - \frac{11}{125} \left(4 \left(\frac{x}{2} \right)^2 - 2x \right) \right) \sin x \right) + e^{\frac{x}{2}} \left(\left(-\frac{107}{50653} \left(35 \left(\frac{x}{2} \right)^2 + x - 2 \right) - \frac{198}{50653} \left(-12 \left(\frac{x}{2} \right)^2 + 6x \right) \right) \cos(3x) - \left(\frac{198}{50653} \left(35 \left(\frac{x}{2} \right)^2 + x - 2 \right) - \frac{107}{50653} \left(-12 \left(\frac{x}{2} \right)^2 + 6x \right) \right) \sin(3x) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x)

[Out] -1/101306*((1369*x^2 + 5180*x - 856)*cos(3*x) + 6*(1369*x^2 - 296*x - 264)*sin(3*x))*e^(1/2*x) + 1/250*((25*x^2 + 60*x - 88)*cos(x) + 2*(25*x^2 - 40*x - 8)*sin(x))*e^(1/2*x)

Mupad [B]

time = 0.53, size = 83, normalized size = 0.45

$$\frac{e^{x/2} (107000 \cos(3x) + 198000 \sin(3x) - 4457464 \cos(x) - 810448 \sin(x) - 647500x \cos(3x) + 1266325x^2 \cos(x) + 222000x \sin(3x) + 2532650x^2 \sin(x) - 171125x^2 \cos(3x) - 1026750x^2 \sin(3x) + 3039180x \cos(x) - 4052240x \sin(x))}{12663250}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*exp(x/2)*cos(x)*sin(x)^2,x)`

[Out] `(exp(x/2)*(107000*cos(3*x) + 198000*sin(3*x) - 4457464*cos(x) - 810448*sin(x) - 647500*x*cos(3*x) + 1266325*x^2*cos(x) + 222000*x*sin(3*x) + 2532650*x^2*sin(x) - 171125*x^2*cos(3*x) - 1026750*x^2*sin(3*x) + 3039180*x*cos(x) - 4052240*x*sin(x)))/12663250`

3.570 $\int \cosh(x) dx$

Optimal. Leaf size=2

$\sinh(x)$

[Out] $\sinh(x)$

Rubi [A]

time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2717}

$\sinh(x)$

Antiderivative was successfully verified.

[In] Int[Cosh[x],x]

[Out] Sinh[x]

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rubi steps

$$\int \cosh(x) dx = \sinh(x)$$

Mathematica [A]

time = 0.00, size = 2, normalized size = 1.00

$\sinh(x)$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x],x]

[Out] Sinh[x]

Mathics [A]

time = 1.49, size = 2, normalized size = 1.00

Sinh[x]

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Cosh[x],x]')
```

```
[Out] Sinh[x]
```

Maple [A]

time = 0.01, size = 3, normalized size = 1.50

method	result	size
lookup	$\sinh(x)$	3
default	$\sinh(x)$	3
meijerg	$\sinh(x)$	3
risch	$-\frac{e^{-x}}{2} + \frac{e^x}{2}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x),x,method=_RETURNVERBOSE)
```

```
[Out] sinh(x)
```

Maxima [A]

time = 0.24, size = 2, normalized size = 1.00

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x),x, algorithm="maxima")
```

```
[Out] sinh(x)
```

Fricas [A]

time = 0.30, size = 2, normalized size = 1.00

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x),x, algorithm="fricas")
```

```
[Out] sinh(x)
```

Sympy [A]

time = 0.06, size = 2, normalized size = 1.00

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x),x)

[Out] sinh(x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(2) = 4.
time = 0.00, size = 10, normalized size = 5.00

$$\frac{e^x - \frac{1}{e^x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x),x)

[Out] 1/2*e^x - 1/2/e^x

Mupad [B]

time = 0.02, size = 2, normalized size = 1.00

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x),x)

[Out] sinh(x)

3.571 $\int \sinh(x) dx$

Optimal. Leaf size=2

$\cosh(x)$

[Out] $\cosh(x)$

Rubi [A]

time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2718}

$\cosh(x)$

Antiderivative was successfully verified.

[In] `Int[Sinh[x],x]`

[Out] `Cosh[x]`

Rule 2718

`Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\int \sinh(x) dx = \cosh(x)$$

Mathematica [A]

time = 0.00, size = 2, normalized size = 1.00

$\cosh(x)$

Antiderivative was successfully verified.

[In] `Integrate[Sinh[x],x]`

[Out] `Cosh[x]`

Mathics [A]

time = 1.49, size = 2, normalized size = 1.00

`Cosh[x]`

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sinh[x],x]')
```

```
[Out] Cosh[x]
```

Maple [A]

time = 0.01, size = 3, normalized size = 1.50

method	result	size
lookup	$\cosh(x)$	3
default	$\cosh(x)$	3
risch	$\frac{e^x}{2} + \frac{e^{-x}}{2}$	12
meijerg	$-\sqrt{\pi} \left(\frac{1}{\sqrt{\pi}} - \frac{\cosh(x)}{\sqrt{\pi}} \right)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x),x,method=_RETURNVERBOSE)
```

```
[Out] cosh(x)
```

Maxima [A]

time = 0.24, size = 2, normalized size = 1.00

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x),x, algorithm="maxima")
```

```
[Out] cosh(x)
```

Fricas [A]

time = 0.30, size = 2, normalized size = 1.00

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x),x, algorithm="fricas")
```

```
[Out] cosh(x)
```

Sympy [A]

time = 0.06, size = 2, normalized size = 1.00

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x),x)`

[Out] `cosh(x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(2) = 4$.
time = 0.00, size = 9, normalized size = 4.50

$$\frac{e^x + \frac{1}{e^x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x),x)`

[Out] `1/2*e^x + 1/2/e^x`

Mupad [B]

time = 0.02, size = 2, normalized size = 1.00

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(x),x)`

[Out] `cosh(x)`

3.572 $\int \tanh(x) dx$

Optimal. Leaf size=3

$$\log(\cosh(x))$$

[Out] $\ln(\cosh(x))$

Rubi [A]

time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556}

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

[In] `Int[Tanh[x], x]`

[Out] `Log[Cosh[x]]`

Rule 3556

`Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\int \tanh(x) dx = \log(\cosh(x))$$

Mathematica [A]

time = 0.00, size = 3, normalized size = 1.00

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

[In] `Integrate[Tanh[x], x]`

[Out] `Log[Cosh[x]]`

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 9 vs. 2(3) = 6. time = 1.77, size = 9, normalized size = 3.00

$$x - \text{Log}[1 + \text{Tanh}[x]]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[Tanh[x],x]')`

[Out] `x - Log[1 + Tanh[x]]`

Maple [A]

time = 0.01, size = 4, normalized size = 1.33

method	result	size
lookup	$\ln(\cosh(x))$	4
default	$\ln(\cosh(x))$	4
risch	$-x + \ln(1 + e^{2x})$	12
derivativedivides	$-\frac{\ln(\tanh(x)-1)}{2} - \frac{\ln(\tanh(x)+1)}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x),x,method=_RETURNVERBOSE)`

[Out] `ln(cosh(x))`

Maxima [A]

time = 0.25, size = 3, normalized size = 1.00

$$\log(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x),x, algorithm="maxima")`

[Out] `log(cosh(x))`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(3) = 6$.
time = 0.31, size = 18, normalized size = 6.00

$$-x + \log\left(\frac{2 \cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x),x, algorithm="fricas")`

[Out] `-x + log(2*cosh(x)/(cosh(x) - sinh(x)))`

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$.
time = 0.06, size = 7, normalized size = 2.33

$$x - \log(\tanh(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x),x)`

[Out] `x - log(tanh(x) + 1)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(3) = 6$.
time = 0.00, size = 10, normalized size = 3.33

$$-x + \ln(e^{2x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x),x)`

[Out] `-x + log(e^(2*x) + 1)`

Mupad [B]

time = 0.02, size = 3, normalized size = 1.00

$$\ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x),x)`

[Out] `log(cosh(x))`

3.573 $\int \coth(x) dx$

Optimal. Leaf size=3

$$\log(\sinh(x))$$

[Out] `ln(sinh(x))`

Rubi [A]

time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556}

$$\log(\sinh(x))$$

Antiderivative was successfully verified.

[In] `Int[Coth[x], x]`

[Out] `Log[Sinh[x]]`

Rule 3556

`Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\int \coth(x) dx = \log(\sinh(x))$$

Mathematica [A]

time = 0.00, size = 3, normalized size = 1.00

$$\log(\sinh(x))$$

Antiderivative was successfully verified.

[In] `Integrate[Coth[x], x]`

[Out] `Log[Sinh[x]]`

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 12 vs. $2(3) = 6$. time = 1.89, size = 12, normalized size = 4.00

$$x + \text{Log}[\text{Tanh}[x]] - \text{Log}[1 + \text{Tanh}[x]]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[Coth[x],x]')`

[Out] `x + Log[Tanh[x]] - Log[1 + Tanh[x]]`

Maple [A]

time = 0.01, size = 4, normalized size = 1.33

method	result	size
lookup	$\ln(\sinh(x))$	4
default	$\ln(\sinh(x))$	4
risch	$-x + \ln(e^{2x} - 1)$	12
derivativedivides	$-\frac{\ln(\coth(x)-1)}{2} - \frac{\ln(\coth(x)+1)}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x),x,method=_RETURNVERBOSE)`

[Out] `ln(sinh(x))`

Maxima [A]

time = 0.24, size = 3, normalized size = 1.00

$$\log(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x),x, algorithm="maxima")`

[Out] `log(sinh(x))`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(3) = 6$.
time = 0.32, size = 18, normalized size = 6.00

$$-x + \log\left(\frac{2 \sinh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x),x, algorithm="fricas")`

[Out] `-x + log(2*sinh(x)/(cosh(x) - sinh(x)))`

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(3) = 6$.
time = 0.16, size = 12, normalized size = 4.00

$$x - \log(\tanh(x) + 1) + \log(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x),x)`

[Out] `x - log(tanh(x) + 1) + log(tanh(x))`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(3) = 6$.
time = 0.00, size = 11, normalized size = 3.67

$$-x + \ln |e^{2x} - 1|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x),x)`

[Out] `-x + log(abs(e^(2*x) - 1))`

Mupad [B]

time = 0.03, size = 3, normalized size = 1.00

$$\ln(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x),x)`

[Out] `log(sinh(x))`

3.574 $\int \operatorname{sech}(x) dx$

Optimal. Leaf size=3

$$\tan^{-1}(\sinh(x))$$

[Out] arctan(sinh(x))

Rubi [A]

time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3855}

$$\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

[In] Int[Sech[x],x]

[Out] ArcTan[Sinh[x]]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \operatorname{sech}(x) dx = \tan^{-1}(\sinh(x))$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 9 vs. $2(3) = 6$. time = 0.00, size = 9, normalized size = 3.00

$$2 \tan^{-1} \left(\tanh \left(\frac{x}{2} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sech[x],x]

[Out] 2*ArcTan[Tanh[x/2]]

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 9 vs. $2(3) = 6$. time = 1.65, size = 7, normalized size = 2.33

$$2 \operatorname{ArcTan} \left[\operatorname{Tanh} \left[\frac{x}{2} \right] \right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sech[x],x]')
```

```
[Out] 2 ArcTan[Tanh[x / 2]]
```

Maple [A]

time = 0.01, size = 4, normalized size = 1.33

method	result	size
lookup	$\arctan(\sinh(x))$	4
default	$\arctan(\sinh(x))$	4
risch	$i \ln(e^x + i) - i \ln(e^x - i)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(sinh(x))
```

Maxima [A]

time = 0.24, size = 3, normalized size = 1.00

$$\arctan(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x),x, algorithm="maxima")
```

```
[Out] arctan(sinh(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8 vs. $2(3) = 6$.
time = 0.31, size = 8, normalized size = 2.67

$$2 \arctan(\cosh(x) + \sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x),x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(x) + sinh(x))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x),x)

[Out] Integral(sech(x), x)

Giac [A]

time = 0.00, size = 5, normalized size = 1.67

$$2 \arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x),x)

[Out] 2*arctan(e^x)

Mupad [B]

time = 0.02, size = 5, normalized size = 1.67

$$2 \operatorname{atan}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cosh(x),x)

[Out] 2*atan(exp(x))

3.575 $\int \operatorname{csch}(x) dx$

Optimal. Leaf size=5

$$-\tanh^{-1}(\cosh(x))$$

[Out] `-arctanh(cosh(x))`

Rubi [A]

time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3855}

$$-\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

[In] `Int[Csch[x], x]`

[Out] `-ArcTanh[Cosh[x]]`

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\int \operatorname{csch}(x) dx = -\tanh^{-1}(\cosh(x))$$

Mathematica [A]

time = 0.00, size = 7, normalized size = 1.40

$$\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] `Integrate[Csch[x], x]`

[Out] `Log[Tanh[x/2]]`

Mathics [A]

time = 1.63, size = 5, normalized size = 1.00

$$\operatorname{Log}\left[\operatorname{Tanh}\left[\frac{x}{2}\right]\right]$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[Csch[x],x]')`

[Out] `Log[Tanh[x / 2]]`

Maple [A]

time = 0.01, size = 6, normalized size = 1.20

method	result	size
lookup	$\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	6
default	$\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	6
risch	$\ln(-1 + e^x) - \ln(1 + e^x)$	14

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(x),x,method=_RETURNVERBOSE)`

[Out] `ln(tanh(1/2*x))`

Maxima [A]

time = 0.24, size = 5, normalized size = 1.00

$$\log\left(\tanh\left(\frac{1}{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(x),x, algorithm="maxima")`

[Out] `log(tanh(1/2*x))`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(5) = 10$.
time = 0.30, size = 17, normalized size = 3.40

$$-\log(\cosh(x) + \sinh(x) + 1) + \log(\cosh(x) + \sinh(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(x),x, algorithm="fricas")`

[Out] `-log(cosh(x) + sinh(x) + 1) + log(cosh(x) + sinh(x) - 1)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(x), x)`

[Out] `Integral(csch(x), x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(5) = 10$.
time = 0.00, size = 21, normalized size = 4.20

$$2 \left(\frac{\ln |e^x - 1|}{2} - \frac{\ln(e^x + 1)}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(x), x)`

[Out] `-log(e^x + 1) + log(abs(e^x - 1))`

Mupad [B]

time = 0.01, size = 5, normalized size = 1.00

$$\ln \left(\tanh \left(\frac{x}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/sinh(x), x)`

[Out] `log(tanh(x/2))`

3.576 $\int \cosh^2(x) dx$

Optimal. Leaf size=14

$$\frac{x}{2} + \frac{1}{2} \cosh(x) \sinh(x)$$

[Out] 1/2*x+1/2*cosh(x)*sinh(x)

Rubi [A]

time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2715, 8}

$$\frac{x}{2} + \frac{1}{2} \sinh(x) \cosh(x)$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^2,x]

[Out] x/2 + (Cosh[x]*Sinh[x])/2

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*SIN[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cosh^2(x) dx &= \frac{1}{2} \cosh(x) \sinh(x) + \frac{\int 1 dx}{2} \\ &= \frac{x}{2} + \frac{1}{2} \cosh(x) \sinh(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2} + \frac{1}{4} \sinh(2x)$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]^2,x]

[Out] x/2 + Sinh[2*x]/4

Mathics [A]

time = 1.83, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\text{Sinh}[2x]}{4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Cosh[x]^2,x]')

[Out] x / 2 + Sinh[2 x] / 4

Maple [A]

time = 0.02, size = 11, normalized size = 0.79

method	result	size
default	$\frac{x}{2} + \frac{\cosh(x)\sinh(x)}{2}$	11
risch	$\frac{x}{2} + \frac{e^{2x}}{8} - \frac{e^{-2x}}{8}$	17

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*x+1/2*cosh(x)*sinh(x)

Maxima [A]

time = 0.25, size = 16, normalized size = 1.14

$$\frac{1}{2}x + \frac{1}{8}e^{(2x)} - \frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^2,x, algorithm="maxima")

[Out] 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)

Fricas [A]

time = 0.33, size = 10, normalized size = 0.71

$$\frac{1}{2} \cosh(x) \sinh(x) + \frac{1}{2} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^2,x, algorithm="fricas")

[Out] $1/2*\cosh(x)*\sinh(x) + 1/2*x$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$

time = 0.08, size = 24, normalized size = 1.71

$$-\frac{x \sinh^2(x)}{2} + \frac{x \cosh^2(x)}{2} + \frac{\sinh(x) \cosh(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(x)**2,x)`

[Out] $-x*\sinh(x)**2/2 + x*\cosh(x)**2/2 + \sinh(x)*\cosh(x)/2$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(10) = 20$.
time = 0.00, size = 31, normalized size = 2.21

$$\frac{\frac{(e^x)^2}{2} + \frac{-2(e^x)^2-1}{2(e^x)^2} + 2x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(x)^2,x)`

[Out] $1/8*e^{(2*x)} + 1/2*x - 1/8*(2*e^{(2*x)} + 1)/e^{(2*x)}$

Mupad [B]

time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sinh(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(x)^2,x)`

[Out] $x/2 + \sinh(2*x)/4$

3.577 $\int \sinh^5(x) dx$

Optimal. Leaf size=19

$$\cosh(x) - \frac{2 \cosh^3(x)}{3} + \frac{\cosh^5(x)}{5}$$

[Out] $\cosh(x) - 2/3 * \cosh(x)^3 + 1/5 * \cosh(x)^5$

Rubi [A]

time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2713}

$$\frac{\cosh^5(x)}{5} - \frac{2 \cosh^3(x)}{3} + \cosh(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[x]^5, x]$

[Out] $\text{Cosh}[x] - (2 * \text{Cosh}[x]^3) / 3 + \text{Cosh}[x]^5 / 5$

Rule 2713

$\text{Int}[\sin[(c_.) + (d_.)(x_)]^{(n_)}, x_Symbol] \text{ :> Dist}[-d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{((n - 1)/2)}, x], x], x, \text{Cos}[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x] \&\& \text{IGtQ}[(n - 1)/2, 0]$

Rubi steps

$$\begin{aligned} \int \sinh^5(x) dx &= \text{Subst} \left(\int (1 - 2x^2 + x^4) dx, x, \cosh(x) \right) \\ &= \cosh(x) - \frac{2 \cosh^3(x)}{3} + \frac{\cosh^5(x)}{5} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 23, normalized size = 1.21

$$\frac{5 \cosh(x)}{8} - \frac{5}{48} \cosh(3x) + \frac{1}{80} \cosh(5x)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Sinh}[x]^5, x]$

[Out] $(5 * \text{Cosh}[x]) / 8 - (5 * \text{Cosh}[3 * x]) / 48 + \text{Cosh}[5 * x] / 80$

Mathics [A]

time = 2.02, size = 18, normalized size = 0.95

$$\frac{\text{Cosh}[x] (8 - 4\text{Sinh}[x]^2 + 3\text{Sinh}[x]^4)}{15}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Sinh[x]^5,x]')``[Out] Cosh[x] (8 - 4 Sinh[x] ^ 2 + 3 Sinh[x] ^ 4) / 15`**Maple [A]**

time = 0.08, size = 18, normalized size = 0.95

method	result	size
default	$\left(\frac{8}{15} + \frac{\sinh^4(x)}{5} - \frac{4\sinh^2(x)}{15}\right) \cosh(x)$	18
risch	$\frac{e^{5x}}{160} - \frac{5e^{3x}}{96} + \frac{5e^x}{16} + \frac{5e^{-x}}{16} - \frac{5e^{-3x}}{96} + \frac{e^{-5x}}{160}$	36

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sinh(x)^5,x,method=_RETURNVERBOSE)``[Out] (8/15+1/5*sinh(x)^4-4/15*sinh(x)^2)*cosh(x)`**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.

time = 0.24, size = 35, normalized size = 1.84

$$\frac{1}{160} e^{(5x)} - \frac{5}{96} e^{(3x)} + \frac{5}{16} e^{(-x)} - \frac{5}{96} e^{(-3x)} + \frac{1}{160} e^{(-5x)} + \frac{5}{16} e^x$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sinh(x)^5,x, algorithm="maxima")``[Out] 1/160*e^(5*x) - 5/96*e^(3*x) + 5/16*e^(-x) - 5/96*e^(-3*x) + 1/160*e^(-5*x) + 5/16*e^x`**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(15) = 30$.

time = 0.31, size = 42, normalized size = 2.21

$$\frac{1}{80} \cosh(x)^5 + \frac{1}{16} \cosh(x) \sinh(x)^4 - \frac{5}{48} \cosh(x)^3 + \frac{1}{16} (2 \cosh(x)^3 - 5 \cosh(x)) \sinh(x)^2 + \frac{5}{8} \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(sinh(x)^5,x, algorithm="fricas")`

[Out] $1/80*\cosh(x)^5 + 1/16*\cosh(x)*\sinh(x)^4 - 5/48*\cosh(x)^3 + 1/16*(2*\cosh(x))^3 - 5*\cosh(x))*\sinh(x)^2 + 5/8*\cosh(x)$

Sympy [A]

time = 0.23, size = 29, normalized size = 1.53

$$\sinh^4(x) \cosh(x) - \frac{4 \sinh^2(x) \cosh^3(x)}{3} + \frac{8 \cosh^5(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x)**5,x)`

[Out] $\sinh(x)**4*\cosh(x) - 4*\sinh(x)**2*\cosh(x)**3/3 + 8*\cosh(x)**5/15$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(15) = 30$.

time = 0.00, size = 47, normalized size = 2.47

$$\frac{\frac{(e^x)^5}{5} - \frac{5}{3}(e^x)^3 + 10e^x + \frac{150(e^x)^4 - 25(e^x)^2 + 3}{15(e^x)^5}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x)^5,x)`

[Out] $1/160*e^{(5*x)} - 5/96*e^{(3*x)} + 5/16*e^x + 1/480*(150*e^{(4*x)} - 25*e^{(2*x)} + 3)/e^{(5*x)}$

Mupad [B]

time = 0.03, size = 15, normalized size = 0.79

$$\frac{\cosh(x)^5}{5} - \frac{2 \cosh(x)^3}{3} + \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(x)^5,x)`

[Out] $\cosh(x) - (2*\cosh(x)^3)/3 + \cosh(x)^5/5$

3.578 $\int \tanh^4(x) dx$

Optimal. Leaf size=14

$$x - \tanh(x) - \frac{\tanh^3(x)}{3}$$

[Out] x-tanh(x)-1/3*tanh(x)^3

Rubi [A]

time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 8}

$$x - \frac{1}{3} \tanh^3(x) - \tanh(x)$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^4,x]

[Out] x - Tanh[x] - Tanh[x]^3/3

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3554

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[b*((b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int \tanh^4(x) dx &= -\frac{1}{3} \tanh^3(x) + \int \tanh^2(x) dx \\ &= -\tanh(x) - \frac{\tanh^3(x)}{3} + \int 1 dx \\ &= x - \tanh(x) - \frac{\tanh^3(x)}{3} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 18, normalized size = 1.29

$$x - \frac{4 \tanh(x)}{3} + \frac{1}{3} \operatorname{sech}^2(x) \tanh(x)$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^4,x]

[Out] $x - (4*\text{Tanh}[x])/3 + (\text{Sech}[x]^2*\text{Tanh}[x])/3$

Mathics [A]

time = 1.75, size = 12, normalized size = 0.86

$$x - \text{Tanh}[x] - \frac{\text{Tanh}[x]^3}{3}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Tanh[x]^4,x]')

[Out] $x - \text{Tanh}[x] - \text{Tanh}[x]^3 / 3$

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.

time = 0.02, size = 26, normalized size = 1.86

method	result	size
derivativedivides	$-\frac{\tanh^3(x)}{3} - \tanh(x) - \frac{\ln(\tanh(x)-1)}{2} + \frac{\ln(\tanh(x)+1)}{2}$	26
default	$-\frac{\tanh^3(x)}{3} - \tanh(x) - \frac{\ln(\tanh(x)-1)}{2} + \frac{\ln(\tanh(x)+1)}{2}$	26
risch	$x + \frac{4e^{4x} + 4e^{2x} + \frac{8}{3}}{(1+e^{2x})^3}$	27

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^4,x,method=_RETURNVERBOSE)

[Out] $-1/3*\tanh(x)^3 - \tanh(x) - 1/2*\ln(\tanh(x)-1) + 1/2*\ln(\tanh(x)+1)$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(12) = 24.

time = 0.25, size = 38, normalized size = 2.71

$$x - \frac{4(3e^{(-2x)} + 3e^{(-4x)} + 2)}{3(3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^4,x, algorithm="maxima")

[Out] $x - 4/3*(3*e^{(-2*x)} + 3*e^{(-4*x)} + 2)/(3*e^{(-2*x)} + 3*e^{(-4*x)} + e^{(-6*x)} + 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(12) = 24.

time = 0.31, size = 68, normalized size = 4.86

$$\frac{(3x+4)\cosh(x)^3 + 3(3x+4)\cosh(x)\sinh(x)^2 - 12\cosh(x)^2\sinh(x) - 4\sinh(x)^3 + 3(3x+4)\cosh(x)}{3(\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + 3\cosh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^4,x, algorithm="fricas")

[Out] 1/3*((3*x + 4)*cosh(x)^3 + 3*(3*x + 4)*cosh(x)*sinh(x)^2 - 12*cosh(x)^2*sinh(x) - 4*sinh(x)^3 + 3*(3*x + 4)*cosh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + 3*cosh(x))

Sympy [A]

time = 0.09, size = 10, normalized size = 0.71

$$x - \frac{\tanh^3(x)}{3} - \tanh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**4,x)

[Out] x - tanh(x)**3/3 - tanh(x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

time = 0.00, size = 30, normalized size = 2.14

$$\frac{12(e^{2x})^2 + 12e^{2x} + 8}{3(e^{2x} + 1)^3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^4,x)

[Out] x + 4/3*(3*e^(4*x) + 3*e^(2*x) + 2)/(e^(2*x) + 1)^3

Mupad [B]

time = 0.07, size = 12, normalized size = 0.86

$$-\frac{\tanh(x)^3}{3} - \tanh(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^4,x)

[Out] x - tanh(x) - tanh(x)^3/3

3.579 $\int \operatorname{csch}^3(x) dx$

Optimal. Leaf size=16

$$\frac{1}{2} \tanh^{-1}(\cosh(x)) - \frac{1}{2} \coth(x) \operatorname{csch}(x)$$

[Out] 1/2*arctanh(cosh(x))-1/2*coth(x)*csch(x)

Rubi [A]

time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3853, 3855}

$$\frac{1}{2} \tanh^{-1}(\cosh(x)) - \frac{1}{2} \coth(x) \operatorname{csch}(x)$$

Antiderivative was successfully verified.

[In] Int[Csch[x]^3,x]

[Out] ArcTanh[Cosh[x]]/2 - (Coth[x]*Csch[x])/2

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^n], x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*Csc[c + d*x])^(n-1)/(d*(n-1)), x] + Dist[b^2*((n-2)/(n-1)), Int[(b*Csc[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] & IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^3(x) dx &= -\frac{1}{2} \coth(x) \operatorname{csch}(x) - \frac{1}{2} \int \operatorname{csch}(x) dx \\ &= \frac{1}{2} \tanh^{-1}(\cosh(x)) - \frac{1}{2} \coth(x) \operatorname{csch}(x) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 36 vs. 2(16) = 32.

time = 0.00, size = 36, normalized size = 2.25

$$-\frac{1}{8} \operatorname{csch}^2\left(\frac{x}{2}\right) - \frac{1}{2} \log\left(\tanh\left(\frac{x}{2}\right)\right) - \frac{1}{8} \operatorname{sech}^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csch[x]^3,x]

[Out] $-1/8*\text{Csch}[x/2]^2 - \text{Log}[\text{Tanh}[x/2]]/2 - \text{Sech}[x/2]^2/8$

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.
time = 2.06, size = 24, normalized size = 1.50

$$-\frac{\text{Log}\left[\text{Tanh}\left[\frac{x}{2}\right]\right]}{2} - \frac{1}{8\text{Tanh}\left[\frac{x}{2}\right]^2} + \frac{\text{Tanh}\left[\frac{x}{2}\right]^2}{8}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Csch[x]^3,x]')

[Out] $-\text{Log}[\text{Tanh}[x / 2]] / 2 - 1 / (8 \text{Tanh}[x / 2] ^ 2) + \text{Tanh}[x / 2] ^ 2 / 8$

Maple [A]

time = 0.08, size = 11, normalized size = 0.69

method	result	size
default	$-\frac{\coth(x)\text{csch}(x)}{2} + \text{arctanh}(e^x)$	11
risch	$-\frac{e^x(1+e^{2x})}{(e^{2x}-1)^2} - \frac{\ln(-1+e^x)}{2} + \frac{\ln(1+e^x)}{2}$	34

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(x)^3,x,method=_RETURNVERBOSE)

[Out] $-1/2*\coth(x)*\text{csch}(x)+\text{arctanh}(\exp(x))$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(12) = 24$.

time = 0.24, size = 45, normalized size = 2.81

$$\frac{e^{(-x)} + e^{(-3x)}}{2e^{(-2x)} - e^{(-4x)} - 1} + \frac{1}{2} \log(e^{(-x)} + 1) - \frac{1}{2} \log(e^{(-x)} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)^3,x, algorithm="maxima")

[Out] $(e^{(-x)} + e^{(-3*x)})/(2*e^{(-2*x)} - e^{(-4*x)} - 1) + 1/2*\log(e^{(-x)} + 1) - 1/2*\log(e^{(-x)} - 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(12) = 24$.

time = 0.32, size = 211, normalized size = 13.19

$\frac{2 \cosh(x)^2 + 6 \cosh(x) \sinh(x)^2 + 2 \sinh(x)^2 - \cosh(x)^4 + 4 \cosh(x) \sinh(x)^2 + \sinh(x)^4 + 2 (3 \cosh(x)^2 - 1) \sinh(x)^2 - 2 \cosh(x)^2 + 4 (\cosh(x)^2 - \cosh(x) \sinh(x) + 1) \log(\cosh(x) + \sinh(x) + 1) + (\cosh(x)^2 - 4 \cosh(x) \sinh(x)^2 + \sinh(x)^2 + 2 (3 \cosh(x)^2 - 1) \sinh(x)^2 - 2 \cosh(x)^2 + 4 (\cosh(x)^2 - \cosh(x) \sinh(x) + 1) \log(\cosh(x) + \sinh(x) - 1) + 2 (3 \cosh(x)^2 + 1) \sinh(x) + 2 \cosh(x)^2}{2 (\cosh(x)^2 + 4 \cosh(x) \sinh(x)^2 + \sinh(x)^2 + 2 (3 \cosh(x)^2 - 1) \sinh(x)^2 - 2 \cosh(x)^2 + 4 (\cosh(x)^2 - \cosh(x) \sinh(x) + 1))}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)^3,x, algorithm="fricas")

[Out]
$$-1/2*(2*\cosh(x)^3 + 6*\cosh(x)*\sinh(x)^2 + 2*\sinh(x)^3 - (\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 - 1)*\sinh(x)^2 - 2*\cosh(x)^2 + 4*(\cosh(x)^3 - \cosh(x))*\sinh(x) + 1)*\log(\cosh(x) + \sinh(x) + 1) + (\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 - 1)*\sinh(x)^2 - 2*\cosh(x)^2 + 4*(\cosh(x)^3 - \cosh(x))*\sinh(x) + 1)*\log(\cosh(x) + \sinh(x) - 1) + 2*(3*\cosh(x)^2 + 1)*\sinh(x) + 2*\cosh(x))/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 - 1)*\sinh(x)^2 - 2*\cosh(x)^2 + 4*(\cosh(x)^3 - \cosh(x))*\sinh(x) + 1)$$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^3(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)**3,x)

[Out] Integral(csch(x)**3, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.
time = 0.00, size = 47, normalized size = 2.94

$$8 \left(-\frac{\ln\left(e^x + \frac{1}{e^x} - 2\right)}{32} + \frac{\ln\left(e^x + \frac{1}{e^x} + 2\right)}{32} - \frac{e^x + \frac{1}{e^x}}{8 \left(\left(e^x + \frac{1}{e^x} \right)^2 - 4 \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)^3,x)

[Out]
$$-(e^x + 1/e^x)/((e^x + 1/e^x)^2 - 4) + 1/4*\log(e^x + 1/e^x + 2) - 1/4*\log(e^x + 1/e^x - 2)$$

Mupad [B]

time = 0.29, size = 16, normalized size = 1.00

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2} - \frac{\cosh(x)}{2\sinh(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sinh(x)^3,x)

[Out]
$$-\log(\tanh(x/2))/2 - \cosh(x)/(2*\sinh(x)^2)$$

3.580 $\int \operatorname{sech}^5(x) dx$

Optimal. Leaf size=26

$$\frac{3}{8} \tan^{-1}(\sinh(x)) + \frac{3}{8} \operatorname{sech}(x) \tanh(x) + \frac{1}{4} \operatorname{sech}^3(x) \tanh(x)$$

[Out] 3/8*arctan(sinh(x))+3/8*sech(x)*tanh(x)+1/4*sech(x)^3*tanh(x)

Rubi [A]

time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3853, 3855}

$$\frac{3}{8} \tan^{-1}(\sinh(x)) + \frac{1}{4} \tanh(x) \operatorname{sech}^3(x) + \frac{3}{8} \tanh(x) \operatorname{sech}(x)$$

Antiderivative was successfully verified.

[In] Int[Sech[x]^5,x]

[Out] (3*ArcTan[Sinh[x]])/8 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh[x])/4

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] & IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^5(x) dx &= \frac{1}{4} \operatorname{sech}^3(x) \tanh(x) + \frac{3}{4} \int \operatorname{sech}^3(x) dx \\ &= \frac{3}{8} \operatorname{sech}(x) \tanh(x) + \frac{1}{4} \operatorname{sech}^3(x) \tanh(x) + \frac{3}{8} \int \operatorname{sech}(x) dx \\ &= \frac{3}{8} \tan^{-1}(\sinh(x)) + \frac{3}{8} \operatorname{sech}(x) \tanh(x) + \frac{1}{4} \operatorname{sech}^3(x) \tanh(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 30, normalized size = 1.15

$$\frac{3}{4} \tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{3}{8} \operatorname{sech}(x) \tanh(x) + \frac{1}{4} \operatorname{sech}^3(x) \tanh(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sech[x]^5,x]

[Out] (3*ArcTan[Tanh[x/2]])/4 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh[x])/4

Mathics [C] Result contains complex when optimal does not.

time = 9.60, size = 112, normalized size = 4.31

$$\frac{-5 \operatorname{Tanh}\left[\frac{x}{2}\right]^7 - 3 \operatorname{Tanh}\left[\frac{x}{2}\right]^3 + 3 \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{x}{2}\right]\right] + 3 \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{x}{2}\right]\right] \operatorname{Tanh}\left[\frac{x}{2}\right]^8 + 3 \operatorname{Tanh}\left[\frac{x}{2}\right]^5 + 5 \operatorname{Tanh}\left[\frac{x}{2}\right] + 12 \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{x}{2}\right]\right] \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + 12 \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{x}{2}\right]\right] \operatorname{Tanh}\left[\frac{x}{2}\right]^6 + 18 \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{x}{2}\right]\right] \operatorname{Tanh}\left[\frac{x}{2}\right]^4}{4(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right])^4(1 + \operatorname{Tanh}\left[\frac{x}{2}\right])^4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/Cosh[x]^5,x]')

[Out] (-5 Tanh[x / 2] ^ 7 - 3 Tanh[x / 2] ^ 3 + 3 ArcTan[Tanh[x / 2]] + 3 ArcTan[Tanh[x / 2]] Tanh[x / 2] ^ 8 + 3 Tanh[x / 2] ^ 5 + 5 Tanh[x / 2] + 12 ArcTan[Tanh[x / 2]] Tanh[x / 2] ^ 2 + 12 ArcTan[Tanh[x / 2]] Tanh[x / 2] ^ 6 + 18 ArcTan[Tanh[x / 2]] Tanh[x / 2] ^ 4) / (4 (-1 + I Tanh[x / 2]) ^ 4 (1 + I Tanh[x / 2]) ^ 4)

Maple [A]

time = 0.10, size = 21, normalized size = 0.81

method	result	size
default	$\left(\frac{\operatorname{sech}(x)^3}{4} + \frac{3 \operatorname{sech}(x)}{8}\right) \tanh(x) + \frac{3 \arctan(e^x)}{4}$	21
risch	$\frac{e^x(3e^{6x} + 11e^{4x} - 11e^{2x} - 3)}{4(1 + e^{2x})^4} + \frac{3i \ln(e^x + i)}{8} - \frac{3i \ln(e^x - i)}{8}$	52

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cosh(x)^5,x,method=_RETURNVERBOSE)

[Out] (1/4*sech(x)^3+3/8*sech(x))*tanh(x)+3/4*arctan(exp(x))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(20) = 40.

time = 0.34, size = 61, normalized size = 2.35

$$\frac{3e^{-x} + 11e^{-3x} - 11e^{-5x} - 3e^{-7x}}{4(4e^{-2x} + 6e^{-4x} + 4e^{-6x} + e^{-8x} + 1)} - \frac{3}{4} \arctan(e^{-x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cosh(x)^5,x, algorithm="maxima")

[Out] 1/4*(3*e^(-x) + 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) - 3/4*arctan(e^(-x))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(20) = 40.

time = 0.31, size = 461, normalized size = 17.73

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cosh(x)^5,x, algorithm="fricas")

[Out] $\frac{1}{4} \cdot (3 \cdot \cosh(x)^7 + 21 \cdot \cosh(x) \cdot \sinh(x)^6 + 3 \cdot \sinh(x)^7 + (63 \cdot \cosh(x)^2 + 11) \cdot \sinh(x)^5 + 11 \cdot \cosh(x)^5 + 5 \cdot (21 \cdot \cosh(x)^3 + 11 \cdot \cosh(x)) \cdot \sinh(x)^4 + (105 \cdot \cosh(x)^4 + 110 \cdot \cosh(x)^2 - 11) \cdot \sinh(x)^3 - 11 \cdot \cosh(x)^3 + (63 \cdot \cosh(x)^5 + 110 \cdot \cosh(x)^3 - 33 \cdot \cosh(x)) \cdot \sinh(x)^2 + 3 \cdot (\cosh(x)^8 + 8 \cdot \cosh(x) \cdot \sinh(x)^7 + \sinh(x)^8 + 4 \cdot (7 \cdot \cosh(x)^2 + 1) \cdot \sinh(x)^6 + 4 \cdot \cosh(x)^6 + 8 \cdot (7 \cdot \cosh(x)^3 + 3 \cdot \cosh(x)) \cdot \sinh(x)^5 + 2 \cdot (35 \cdot \cosh(x)^4 + 30 \cdot \cosh(x)^2 + 3) \cdot \sinh(x)^4 + 6 \cdot \cosh(x)^4 + 8 \cdot (7 \cdot \cosh(x)^5 + 10 \cdot \cosh(x)^3 + 3 \cdot \cosh(x)) \cdot \sinh(x)^3 + 4 \cdot (7 \cdot \cosh(x)^6 + 15 \cdot \cosh(x)^4 + 9 \cdot \cosh(x)^2 + 1) \cdot \sinh(x)^2 + 4 \cdot \cosh(x)^2 + 8 \cdot (\cosh(x)^7 + 3 \cdot \cosh(x)^5 + 3 \cdot \cosh(x)^3 + \cosh(x)) \cdot \sinh(x) + 1) \cdot \arctan(\cosh(x) + \sinh(x)) + (21 \cdot \cosh(x)^6 + 55 \cdot \cosh(x)^4 - 33 \cdot \cosh(x)^2 - 3) \cdot \sinh(x) - 3 \cdot \cosh(x)) / (\cosh(x)^8 + 8 \cdot \cosh(x) \cdot \sinh(x)^7 + \sinh(x)^8 + 4 \cdot (7 \cdot \cosh(x)^2 + 1) \cdot \sinh(x)^6 + 4 \cdot \cosh(x)^6 + 8 \cdot (7 \cdot \cosh(x)^3 + 3 \cdot \cosh(x)) \cdot \sinh(x)^5 + 2 \cdot (35 \cdot \cosh(x)^4 + 30 \cdot \cosh(x)^2 + 3) \cdot \sinh(x)^4 + 6 \cdot \cosh(x)^4 + 8 \cdot (7 \cdot \cosh(x)^5 + 10 \cdot \cosh(x)^3 + 3 \cdot \cosh(x)) \cdot \sinh(x)^3 + 4 \cdot (7 \cdot \cosh(x)^6 + 15 \cdot \cosh(x)^4 + 9 \cdot \cosh(x)^2 + 1) \cdot \sinh(x)^2 + 4 \cdot \cosh(x)^2 + 8 \cdot (\cosh(x)^7 + 3 \cdot \cosh(x)^5 + 3 \cdot \cosh(x)^3 + \cosh(x)) \cdot \sinh(x) + 1)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(27) = 54

time = 1.38, size = 422, normalized size = 16.23

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cosh(x)**5,x)

[Out] $3 \cdot \tanh(x/2)^8 \cdot \operatorname{atan}(\tanh(x/2)) / (4 \cdot \tanh(x/2)^8 + 16 \cdot \tanh(x/2)^6 + 24 \cdot \tanh(x/2)^4 + 16 \cdot \tanh(x/2)^2 + 4) - 5 \cdot \tanh(x/2)^7 / (4 \cdot \tanh(x/2)^8 + 16 \cdot \tanh(x/2)^6 + 24 \cdot \tanh(x/2)^4 + 16 \cdot \tanh(x/2)^2 + 4) + 12 \cdot \tanh(x/2)^6 \cdot \operatorname{atan}(\tanh(x/2)) / (4 \cdot \tanh(x/2)^8 + 16 \cdot \tanh(x/2)^6 + 24 \cdot \tanh(x/2)^4 + 16 \cdot \tanh(x/2)^2 + 4) + 3 \cdot \tanh(x/2)^5 / (4 \cdot \tanh(x/2)^8 + 16 \cdot \tanh(x/2)^6 + 24 \cdot \tanh(x/2)^4 + 16 \cdot \tanh(x/2)^2 + 4) + 18 \cdot \tanh(x/2)^4 \cdot \operatorname{atan}(\tanh(x/2)) / (4 \cdot \tanh(x/2)^8 + 16 \cdot \tanh(x/2)^6 + 24 \cdot \tanh(x/2)^4 + 16 \cdot \tanh(x/2)^2 + 4) - 3 \cdot \tanh(x/2)^3 / (4 \cdot \tanh(x/2)^8 + 16 \cdot \tanh(x/2)^6 + 24 \cdot \tanh(x/2)^4 + 16 \cdot \tanh(x/2)^2 + 4) + 12 \cdot \tanh(x/2)^2 \cdot \operatorname{atan}(\tanh(x/2)) / (4 \cdot \tanh(x/2)^8 + 16 \cdot \tanh(x/2)^6 + 24 \cdot \tanh(x/2)^4 + 16 \cdot \tanh(x/2)^2 + 4) + 5 \cdot \tanh(x/2) / (4 \cdot \tanh(x/2)^8 + 16 \cdot \tanh(x/2)^6 + 24 \cdot \tanh(x/2)^4 + 16 \cdot \tanh(x/2)^2 + 4)$

/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) + 3*atan(tanh(x/2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(20) = 40.
time = 0.00, size = 63, normalized size = 2.42

$$32 \left(\frac{3 \left(e^x - \frac{1}{e^x} \right)^3 + 20 \left(e^x - \frac{1}{e^x} \right)}{128 \left(\left(e^x - \frac{1}{e^x} \right)^2 + 4 \right)^2} + \frac{3}{256} \left(\arctan \left(\frac{(e^x)^2 - 1}{2e^x} \right) + \frac{\pi}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cosh(x)^5,x)

[Out] 3/16*pi + 1/4*(3*(e^x - 1/e^x)^3 + 20*e^x - 20/e^x)/((e^x - 1/e^x)^2 + 4)^2 + 3/8*arctan(1/2*(e^(2*x) - 1)/e^x)

Mupad [B]

time = 0.08, size = 22, normalized size = 0.85

$$\frac{3 \operatorname{atan}(e^x)}{4} + \frac{3 \sinh(x)}{8 \cosh(x)^2} + \frac{\sinh(x)}{4 \cosh(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cosh(x)^5,x)

[Out] (3*atan(exp(x)))/4 + (3*sinh(x))/(8*cosh(x)^2) + sinh(x)/(4*cosh(x)^4)

3.581 $\int \sinh^4(x) \tanh(x) dx$

Optimal. Leaf size=18

$$-\cosh^2(x) + \frac{\cosh^4(x)}{4} + \log(\cosh(x))$$

[Out] $-\cosh(x)^2 + 1/4 * \cosh(x)^4 + \ln(\cosh(x))$

Rubi [A]

time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2670, 272, 45}

$$\frac{\cosh^4(x)}{4} - \cosh^2(x) + \log(\cosh(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[x]^4 * \text{Tanh}[x], x]$

[Out] $-\text{Cosh}[x]^2 + \text{Cosh}[x]^4/4 + \text{Log}[\text{Cosh}[x]]$

Rule 45

$\text{Int}[(a_. + (b_.)(x_.))^{(m_.)}((c_.) + (d_.)(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

$\text{Int}[(x_.)^{(m_.)}((a_.) + (b_.)(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n) - 1} * (a + b*x)^p, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 2670

$\text{Int}[\sin[(e_.) + (f_.)(x_.)]^{(m_.)} \tan[(e_.) + (f_.)(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[-f^{(-1)}, \text{Subst}[\text{Int}[(1 - x^2)^{((m + n - 1)/2)} / x^n, x], x, \text{Cos}[e + f*x]], x] /;$ FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]

Rubi steps

$$\begin{aligned}
\int \sinh^4(x) \tanh(x) dx &= \text{Subst} \left(\int \frac{(1-x^2)^2}{x} dx, x, \cosh(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{(1-x)^2}{x} dx, x, \cosh^2(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(-2 + \frac{1}{x} + x \right) dx, x, \cosh^2(x) \right) \\
&= -\cosh^2(x) + \frac{\cosh^4(x)}{4} + \log(\cosh(x))
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 18, normalized size = 1.00

$$-\cosh^2(x) + \frac{\cosh^4(x)}{4} + \log(\cosh(x))$$

Antiderivative was successfully verified.

`[In] Integrate[Sinh[x]^4*Tanh[x],x]``[Out] -Cosh[x]^2 + Cosh[x]^4/4 + Log[Cosh[x]]`**Mathics [F(-1)]**

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Tanh[x]^5/Sech[x]^4,x]')``[Out] Timed out`**Maple [A]**

time = 0.03, size = 17, normalized size = 0.94

method	result	size
default	$\frac{(\sinh^4(x))}{4} - \frac{(\sinh^2(x))}{2} + \ln(\cosh(x))$	17
risch	$-x + \frac{e^{4x}}{64} - \frac{3e^{2x}}{16} - \frac{3e^{-2x}}{16} + \frac{e^{-4x}}{64} + \ln(1 + e^{2x})$	36

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(tanh(x)^5/sech(x)^4,x,method=_RETURNVERBOSE)`

[Out] $1/4*\sinh(x)^4-1/2*\sinh(x)^2+\ln(\cosh(x))$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.

time = 0.34, size = 35, normalized size = 1.94

$$-\frac{1}{64}(12e^{(-2x)} - 1)e^{(4x)} + x - \frac{3}{16}e^{(-2x)} + \frac{1}{64}e^{(-4x)} + \log(e^{(-2x)} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)^5/sech(x)^4,x, algorithm="maxima")`

[Out] $-1/64*(12*e^{(-2*x)} - 1)*e^{(4*x)} + x - 3/16*e^{(-2*x)} + 1/64*e^{(-4*x)} + \log(e^{(-2*x)} + 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(16) = 32$.

time = 0.32, size = 257, normalized size = 14.28

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)^5/sech(x)^4,x, algorithm="fricas")`

[Out] $1/64*(\cosh(x)^8 + 8*\cosh(x)*\sinh(x)^7 + \sinh(x)^8 + 4*(7*\cosh(x)^2 - 3)*\sinh(x)^6 - 12*\cosh(x)^6 + 8*(7*\cosh(x)^3 - 9*\cosh(x))*\sinh(x)^5 - 64*x*\cosh(x)^4 + 2*(35*\cosh(x)^4 - 90*\cosh(x)^2 - 32*x)*\sinh(x)^4 + 8*(7*\cosh(x)^5 - 30*\cosh(x)^3 - 32*x*\cosh(x))*\sinh(x)^3 + 4*(7*\cosh(x)^6 - 45*\cosh(x)^4 - 96*x*\cosh(x)^2 - 3)*\sinh(x)^2 - 12*\cosh(x)^2 + 64*(\cosh(x)^4 + 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)*\log(2*\cosh(x)/(\cosh(x) - \sinh(x))) + 8*(\cosh(x)^7 - 9*\cosh(x)^5 - 32*x*\cosh(x)^3 - 3*\cosh(x))*\sinh(x) + 1)/(\cosh(x)^4 + 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{\operatorname{sech}^4(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)**5/sech(x)**4,x)`

[Out] `Integral(tanh(x)**5/sech(x)**4, x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(16) = 32$.

time = 0.00, size = 50, normalized size = 2.78

$$\frac{16(e^x)^4 - 192(e^x)^2}{1024} + \frac{48(e^x)^4 - 12(e^x)^2 + 1}{64(e^x)^4} + \ln((e^x)^2 + 1) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^5/sech(x)^4,x)

[Out] $\frac{1}{64}e^{4x} - \frac{3}{16}e^{2x} - x + \frac{1}{64}(48e^{4x} - 12e^{2x} + 1)/e^{4x} + \log(e^{2x} + 1)$

Mupad [B]

time = 0.36, size = 35, normalized size = 1.94

$$\ln(e^{2x} + 1) - x - \frac{3e^{-2x}}{16} - \frac{3e^{2x}}{16} + \frac{e^{-4x}}{64} + \frac{e^{4x}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)^4*tanh(x)^5,x)

[Out] $\log(\exp(2x) + 1) - x - (3\exp(-2x))/16 - (3\exp(2x))/16 + \exp(-4x)/64 + \exp(4x)/64$

3.582 $\int \operatorname{sech}^{\frac{23}{4}}(x) \sinh^5(x) dx$

Optimal. Leaf size=31

$$-\frac{4}{3}\operatorname{sech}^{\frac{3}{4}}(x) + \frac{8}{11}\operatorname{sech}^{\frac{11}{4}}(x) - \frac{4}{19}\operatorname{sech}^{\frac{19}{4}}(x)$$

[Out] $-4/3*\operatorname{sech}(x)^{(3/4)}+8/11*\operatorname{sech}(x)^{(11/4)}-4/19*\operatorname{sech}(x)^{(19/4)}$

Rubi [A]

time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2702, 276}

$$-\frac{4}{19}\operatorname{sech}^{\frac{19}{4}}(x) + \frac{8}{11}\operatorname{sech}^{\frac{11}{4}}(x) - \frac{4}{3}\operatorname{sech}^{\frac{3}{4}}(x)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sech}[x]^{(23/4)}*\operatorname{Sinh}[x]^5, x]$

[Out] $(-4*\operatorname{Sech}[x]^{(3/4)})/3 + (8*\operatorname{Sech}[x]^{(11/4)})/11 - (4*\operatorname{Sech}[x]^{(19/4)})/19$

Rule 276

$\operatorname{Int}[(c_.*(x_))^{(m_)}*((a_)+(b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{Exp}\operatorname{and}\operatorname{Integrand}[(c*x)^m*(a+b*x^n)^p, x], x] /; \operatorname{FreeQ}\{a, b, c, m, n, x\} \ \&\& \operatorname{IGtQ}[p, 0]$

Rule 2702

$\operatorname{Int}[\operatorname{csc}[(e_)+(f_)*(x_)]^{(n_)}*((a_)*\operatorname{sec}[(e_)+(f_)*(x_)]^{(m_)}, x_Symbol] \rightarrow \operatorname{Dist}[1/(f*a^n), \operatorname{Subst}[\operatorname{Int}[x^{(m+n-1)}]/(-1+x^2/a^2)^{(n+1)/2}], x], x, a*\operatorname{Sec}[e+f*x], x] /; \operatorname{FreeQ}\{a, e, f, m, x\} \ \&\& \operatorname{IntegerQ}[(n+1)/2] \ \&\& \operatorname{IntegerQ}[(m+1)/2] \ \&\& \operatorname{LtQ}[0, m, n]$

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^{\frac{23}{4}}(x) \sinh^5(x) dx &= -\operatorname{Subst} \left(\int \frac{(-1+x^2)^2}{\sqrt[4]{x}} dx, x, \operatorname{sech}(x) \right) \\ &= -\operatorname{Subst} \left(\int \left(\frac{1}{\sqrt[4]{x}} - 2x^{7/4} + x^{15/4} \right) dx, x, \operatorname{sech}(x) \right) \\ &= -\frac{4}{3}\operatorname{sech}^{\frac{3}{4}}(x) + \frac{8}{11}\operatorname{sech}^{\frac{11}{4}}(x) - \frac{4}{19}\operatorname{sech}^{\frac{19}{4}}(x) \end{aligned}$$

Mathematica [A]

time = 0.04, size = 27, normalized size = 0.87

$$\operatorname{sech}^{\frac{3}{4}}(x) \left(-\frac{4}{3} + \frac{8\operatorname{sech}^2(x)}{11} - \frac{4\operatorname{sech}^4(x)}{19} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sech[x]^(23/4)*Sinh[x]^5,x]

[Out] Sech[x]^(3/4)*(-4/3 + (8*Sech[x]^2)/11 - (4*Sech[x]^4)/19)

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Tanh[x]^5*Sech[x]^(3/4),x]')

[Out] Timed out

Maple [F]

time = 0.07, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(x)^{\frac{3}{4}} (\tanh^5(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(x)^(3/4)*tanh(x)^5,x)

[Out] int(sech(x)^(3/4)*tanh(x)^5,x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="maxima")

[Out] integrate(sech(x)^(3/4)*tanh(x)^5, x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(19) = 38.

time = 0.31, size = 359, normalized size = 11.58

4 * 3^209 * cosh(x)^2 + 1072 * cosh(x) * sinh(x)^2 + 288 * sinh(x)^4 - 20 * 17^17 * cosh(x)^2 - 5 * sinh(x)^2 - 208 * cosh(x)^2 + 10177 * cosh(x)^2 - 21 * cosh(x) * sinh(x)^2 - 10 * 1089 * cosh(x)^2 + 278 * cosh(x)^2 - 47 * cosh(x)^2 + 493 * cosh(x)^2 + 8 * 1045 * cosh(x)^2 + 389 * cosh(x)^2 + 428 * cosh(x) * sinh(x)^2 + 4 * 13463 * cosh(x)^2 + 1422 * cosh(x)^2 + 1005 * cosh(x)^2 + 385 * cosh(x)^2 + 288 * cosh(x)^2 + 10 * 208 * cosh(x)^2 - 108 * cosh(x)^2 + 428 * cosh(x)^2 - 10 * cosh(x) * sinh(x) + 2981 * (sqrt(19) * cosh(x) + sinh(x))^2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="fricas")

[Out]
$$-4/627*2^{(3/4)}*(209*\cosh(x)^8 + 1672*\cosh(x)*\sinh(x)^7 + 209*\sinh(x)^8 + 76*(77*\cosh(x)^2 + 5)*\sinh(x)^6 + 380*\cosh(x)^6 + 152*(77*\cosh(x)^3 + 15*\cosh(x))*\sinh(x)^5 + 10*(1463*\cosh(x)^4 + 570*\cosh(x)^2 + 87)*\sinh(x)^4 + 870*\cosh(x)^4 + 8*(1463*\cosh(x)^5 + 950*\cosh(x)^3 + 435*\cosh(x))*\sinh(x)^3 + 4*(1463*\cosh(x)^6 + 1425*\cosh(x)^4 + 1305*\cosh(x)^2 + 95)*\sinh(x)^2 + 380*\cosh(x)^2 + 8*(209*\cosh(x)^7 + 285*\cosh(x)^5 + 435*\cosh(x)^3 + 95*\cosh(x))*\sinh(x) + 209)*((\cosh(x) + \sinh(x))/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1))^{(3/4)}/(\cosh(x)^8 + 8*\cosh(x)*\sinh(x)^7 + \sinh(x)^8 + 4*(7*\cosh(x)^2 + 1)*\sinh(x)^6 + 4*\cosh(x)^6 + 8*(7*\cosh(x)^3 + 3*\cosh(x))*\sinh(x)^5 + 2*(35*\cosh(x)^4 + 30*\cosh(x)^2 + 3)*\sinh(x)^4 + 6*\cosh(x)^4 + 8*(7*\cosh(x)^5 + 10*\cosh(x)^3 + 3*\cosh(x))*\sinh(x)^3 + 4*(7*\cosh(x)^6 + 15*\cosh(x)^4 + 9*\cosh(x)^2 + 1)*\sinh(x)^2 + 4*\cosh(x)^2 + 8*(\cosh(x)^7 + 3*\cosh(x)^5 + 3*\cosh(x)^3 + \cosh(x))*\sinh(x) + 1)$$

Sympy [A]

time = 45.54, size = 41, normalized size = 1.32

$$\frac{4 \tanh^4(x) \operatorname{sech}^{\frac{3}{4}}(x)}{19} - \frac{64 \tanh^2(x) \operatorname{sech}^{\frac{3}{4}}(x)}{209} - \frac{512 \operatorname{sech}^{\frac{3}{4}}(x)}{627}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)**(3/4)*tanh(x)**5,x)

[Out] $-4*\tanh(x)**4*\operatorname{sech}(x)**(3/4)/19 - 64*\tanh(x)**2*\operatorname{sech}(x)**(3/4)/209 - 512*\operatorname{sech}(x)**(3/4)/627$

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^(3/4)*tanh(x)^5,x)

[Out] Could not integrate

Mupad [B]

time = 0.17, size = 120, normalized size = 3.87

$$\frac{32 \left(\frac{1}{\frac{e^{-x}}{2} + \frac{e^x}{2}} \right)^{3/4}}{11 (e^{2x} + 1)} - \frac{1312 \left(\frac{1}{\frac{e^{-x}}{2} + \frac{e^x}{2}} \right)^{3/4}}{209 (e^{2x} + 1)^2} + \frac{128 \left(\frac{1}{\frac{e^{-x}}{2} + \frac{e^x}{2}} \right)^{3/4}}{19 (e^{2x} + 1)^3} - \frac{64 \left(\frac{1}{\frac{e^{-x}}{2} + \frac{e^x}{2}} \right)^{3/4}}{19 (e^{2x} + 1)^4} - \frac{4 \left(\frac{1}{\frac{e^{-x}}{2} + \frac{e^x}{2}} \right)^{3/4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5*(1/cosh(x))^(3/4),x)
```

```
[Out] (32*(1/(exp(-x)/2 + exp(x)/2))^(3/4))/(11*(exp(2*x) + 1)) - (1312*(1/(exp(-x)/2 + exp(x)/2))^(3/4))/(209*(exp(2*x) + 1)^2) + (128*(1/(exp(-x)/2 + exp(x)/2))^(3/4))/(19*(exp(2*x) + 1)^3) - (64*(1/(exp(-x)/2 + exp(x)/2))^(3/4))/(19*(exp(2*x) + 1)^4) - (4*(1/(exp(-x)/2 + exp(x)/2))^(3/4))/3
```


$$3.583 \quad \int \frac{1}{a+b \cosh(x)} dx$$

Optimal. Leaf size=41

$$\frac{2 \tanh^{-1} \left(\frac{(a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2}} \right)}{\sqrt{a^2 - b^2}}$$

[Out] $2*\operatorname{arctanh}((a-b)*\tanh(1/2*x)/(a^2-b^2)^{(1/2)})/(a^2-b^2)^{(1/2)}$

Rubi [A]

time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2738, 214}

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{a-b} \tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}} \right)}{\sqrt{a-b} \sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{Cosh}[x])^{-1}, x]$

[Out] $(2*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tanh}[x/2])/\operatorname{Sqrt}[a + b]])/(\operatorname{Sqrt}[a - b]*\operatorname{Sqrt}[a + b])$

Rule 214

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{NegQ}[a/b]$

Rule 2738

$\operatorname{Int}[(a_ + (b_)*\sin[\operatorname{Pi}/2 + (c_.) + (d_)*(x_)])^{-1}, x_Symbol] \rightarrow \operatorname{With}\{e = \operatorname{FreeFactors}[\operatorname{Tan}[(c + d*x)/2], x]\}, \operatorname{Dist}[2*(e/d), \operatorname{Subst}[\operatorname{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \operatorname{Tan}[(c + d*x)/2]/e], x] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \ \operatorname{NeQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{a+b \cosh(x)} dx &= 2 \operatorname{Subst} \left(\int \frac{1}{a+b - (a-b)x^2} dx, x, \tanh \left(\frac{x}{2} \right) \right) \\ &= \frac{2 \tanh^{-1} \left(\frac{\sqrt{a-b} \tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}} \right)}{\sqrt{a-b} \sqrt{a+b}} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 41, normalized size = 1.00

$$\frac{2 \tan^{-1} \left(\frac{(a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2 + b^2}} \right)}{\sqrt{-a^2 + b^2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + b*Cosh[x])^(-1), x]``[Out] (-2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 3.65, size = 205, normalized size = 5.00

$$\text{Piecewise} \left[\left\{ \left\{ \text{DirectedInfinity} \left[\text{ArcTan} \left[\text{Tanh} \left[\frac{x}{2} \right] \right] \right], a == 0 \&\& b == 0 \right\}, \left\{ \frac{\text{Tanh} \left[\frac{x}{2} \right]}{b}, a == b \right\}, \left\{ -\frac{1}{b \text{Tanh} \left[\frac{x}{2} \right]}, a == -b \right\} \right\}, -\frac{\text{Log} \left[\text{Tanh} \left[\frac{x}{2} \right] - \sqrt{\frac{a}{a-b} + \frac{b}{a-b}} \right]}{a \sqrt{\frac{a}{a-b} + \frac{b}{a-b}} - b \sqrt{\frac{a}{a-b} + \frac{b}{a-b}}} + \frac{\text{Log} \left[\sqrt{\frac{a}{a-b} + \frac{b}{a-b}} + \text{Tanh} \left[\frac{x}{2} \right] \right]}{a \sqrt{\frac{a}{a-b} + \frac{b}{a-b}} - b \sqrt{\frac{a}{a-b} + \frac{b}{a-b}}} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(a + b*Cosh[x]), x]')`

```
[Out] Piecewise[{{DirectedInfinity[ArcTan[Tanh[x / 2]]], a == 0 && b == 0}, {Tanh[x / 2] / b, a == b}, {-1 / (b Tanh[x / 2]), a == -b}}, -Log[Tanh[x / 2] - Sqrt[a / (a - b) + b / (a - b)]] / (a Sqrt[a / (a - b) + b / (a - b)] - b Sqrt[a / (a - b) + b / (a - b)]) + Log[Sqrt[a / (a - b) + b / (a - b)] + Tanh[x / 2]] / (a Sqrt[a / (a - b) + b / (a - b)] - b Sqrt[a / (a - b) + b / (a - b)])]
```

Maple [A]

time = 0.04, size = 36, normalized size = 0.88

method	result	size
default	$\frac{2 \operatorname{arctanh} \left(\frac{(a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}} \right)}{\sqrt{(a+b)(a-b)}}$	36
risch	$\frac{\ln \left(e^x + \frac{a \sqrt{a^2 - b^2} - a^2 + b^2}{b \sqrt{a^2 - b^2}} \right)}{\sqrt{a^2 - b^2}} - \frac{\ln \left(e^x + \frac{a \sqrt{a^2 - b^2} + a^2 - b^2}{b \sqrt{a^2 - b^2}} \right)}{\sqrt{a^2 - b^2}}$	109

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/(a+b*cosh(x)), x, method=_RETURNVERBOSE)``[Out] 2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))`

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a+b*cosh(x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see 'assume?' for more de

Fricas [A]

time = 0.32, size = 175, normalized size = 4.27

$$\left[\frac{\log\left(\frac{b^2 \cosh(x)^2 + b^2 \sinh(x)^2 + 2ab \cosh(x) + 2a^2 - b^2 + 2(b^2 \cosh(x) + ab) \sinh(x) - 2\sqrt{a^2 - b^2}(b \cosh(x) + b \sinh(x) + a)}{b \cosh(x)^2 + b \sinh(x)^2 + 2a \cosh(x) + 2(b \cosh(x) + a) \sinh(x) + b}\right)}{\sqrt{a^2 - b^2}}, -\frac{2\sqrt{-a^2 + b^2} \arctan\left(\frac{-\sqrt{-a^2 + b^2}(b \cosh(x) + b \sinh(x) + a)}{a^2 - b^2}\right)}{a^2 - b^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a+b*cosh(x)),x, algorithm="fricas")`

[Out] $[\log((b^2 \cosh(x)^2 + b^2 \sinh(x)^2 + 2*a*b \cosh(x) + 2*a^2 - b^2 + 2*(b^2 \cosh(x) + a*b)*\sinh(x) - 2*\sqrt{a^2 - b^2}*(b \cosh(x) + b \sinh(x) + a))/(b \cosh(x)^2 + b \sinh(x)^2 + 2*a \cosh(x) + 2*(b \cosh(x) + a)*\sinh(x) + b))/\sqrt{a^2 - b^2}, -2*\sqrt{-a^2 + b^2}*\arctan(-\sqrt{-a^2 + b^2}*(b \cosh(x) + b \sinh(x) + a)/(a^2 - b^2))/(a^2 - b^2)]$

Sympy [A]

time = 1.98, size = 126, normalized size = 3.07

$$\left\{ \begin{array}{ll} \tilde{\infty} \operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{\tanh\left(\frac{x}{2}\right)}{b} & \text{for } a = b \\ -\frac{1}{b \tanh\left(\frac{x}{2}\right)} & \text{for } a = -b \\ -\frac{\log\left(-\sqrt{\frac{a}{a-b} + \frac{b}{a-b}} + \tanh\left(\frac{x}{2}\right)\right)}{a \sqrt{\frac{a}{a-b} + \frac{b}{a-b}} - b \sqrt{\frac{a}{a-b} + \frac{b}{a-b}}} + \frac{\log\left(\sqrt{\frac{a}{a-b} + \frac{b}{a-b}} + \tanh\left(\frac{x}{2}\right)\right)}{a \sqrt{\frac{a}{a-b} + \frac{b}{a-b}} - b \sqrt{\frac{a}{a-b} + \frac{b}{a-b}}} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(1/(a+b*cosh(x)),x)`

[Out] $\operatorname{Piecewise}((\operatorname{zoo}*\operatorname{atan}(\tanh(x/2))), \operatorname{Eq}(a, 0) \ \& \ \operatorname{Eq}(b, 0)), (\tanh(x/2)/b, \operatorname{Eq}(a, b)), (-1/(b*\tanh(x/2)), \operatorname{Eq}(a, -b)), (-\log(-\sqrt{a/(a-b)} + b/(a-b)) + \tan$

```
h(x/2))/(a*sqrt(a/(a - b) + b/(a - b)) - b*sqrt(a/(a - b) + b/(a - b))) + 1
og(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*sqrt(a/(a - b) + b/(a - b))
- b*sqrt(a/(a - b) + b/(a - b))), True))
```

Giac [A]

time = 0.00, size = 36, normalized size = 0.88

$$\frac{2 \arctan\left(\frac{a+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)),x)
```

```
[Out] 2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

Mupad [B]

time = 0.16, size = 43, normalized size = 1.05

$$\frac{2 \operatorname{atan}\left(\frac{a}{\sqrt{b^2-a^2}} + \frac{be^x}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x)),x)
```

```
[Out] (2*atan(a/(b^2 - a^2)^(1/2) + (b*exp(x))/(b^2 - a^2)^(1/2)))/(b^2 - a^2)^(1/2)
```

$$3.584 \quad \int \frac{1}{(1+\cosh(x))^2} dx$$

Optimal. Leaf size=25

$$\frac{\sinh(x)}{3(1+\cosh(x))^2} + \frac{\sinh(x)}{3(1+\cosh(x))}$$

[Out] 1/3*sinh(x)/(1+cosh(x))^2+1/3*sinh(x)/(1+cosh(x))

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2729, 2727}

$$\frac{\sinh(x)}{3(\cosh(x)+1)} + \frac{\sinh(x)}{3(\cosh(x)+1)^2}$$

Antiderivative was successfully verified.

[In] Int[(1 + Cosh[x])^(-2), x]

[Out] Sinh[x]/(3*(1 + Cosh[x])^2) + Sinh[x]/(3*(1 + Cosh[x]))

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2729

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c + d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+\cosh(x))^2} dx &= \frac{\sinh(x)}{3(1+\cosh(x))^2} + \frac{1}{3} \int \frac{1}{1+\cosh(x)} dx \\ &= \frac{\sinh(x)}{3(1+\cosh(x))^2} + \frac{\sinh(x)}{3(1+\cosh(x))} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 16, normalized size = 0.64

$$\frac{(2 + \cosh(x)) \sinh(x)}{3(1 + \cosh(x))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Cosh[x])^(-2), x]

[Out] ((2 + Cosh[x])*Sinh[x])/(3*(1 + Cosh[x])^2)

Mathics [A]

time = 1.83, size = 15, normalized size = 0.60

$$-\frac{\operatorname{Tanh}\left[\frac{x}{2}\right]^3}{6} + \frac{\operatorname{Tanh}\left[\frac{x}{2}\right]}{2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(1 + Cosh[x])^2,x]')

[Out] -Tanh[x / 2] ^ 3 / 6 + Tanh[x / 2] / 2

Maple [A]

time = 0.02, size = 16, normalized size = 0.64

method	result	size
risch	$-\frac{2(1+3e^x)}{3(1+e^x)^3}$	15
default	$-\frac{(\tanh^3(\frac{x}{2}))}{6} + \frac{\tanh(\frac{x}{2})}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+cosh(x))^2,x,method=_RETURNVERBOSE)

[Out] -1/6*tanh(1/2*x)^3+1/2*tanh(1/2*x)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(21) = 42.

time = 0.26, size = 49, normalized size = 1.96

$$\frac{2e^{-x}}{3e^{-x} + 3e^{-2x} + e^{-3x} + 1} + \frac{2}{3(3e^{-x} + 3e^{-2x} + e^{-3x} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+cosh(x))^2,x, algorithm="maxima")

[Out] 2*e^(-x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 2/3/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(21) = 42.

time = 0.30, size = 58, normalized size = 2.32

$$\frac{2(3 \cosh(x) + 3 \sinh(x) + 1)}{3(\cosh(x)^3 + 3(\cosh(x) + 1)\sinh(x)^2 + \sinh(x)^3 + 3 \cosh(x)^2 + 3(\cosh(x)^2 + 2 \cosh(x) + 1)\sinh(x) + 3 \cosh(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+cosh(x))^2,x, algorithm="fricas")

[Out] $-2/3*(3*\cosh(x) + 3*\sinh(x) + 1)/(\cosh(x)^3 + 3*(\cosh(x) + 1)*\sinh(x)^2 + \sinh(x)^3 + 3*\cosh(x)^2 + 3*(\cosh(x)^2 + 2*\cosh(x) + 1)*\sinh(x) + 3*\cosh(x) + 1)$

Sympy [A]

time = 0.18, size = 14, normalized size = 0.56

$$-\frac{\tanh^3\left(\frac{x}{2}\right)}{6} + \frac{\tanh\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+cosh(x))**2,x)

[Out] $-\tanh(x/2)**3/6 + \tanh(x/2)/2$

Giac [A]

time = 0.00, size = 18, normalized size = 0.72

$$\frac{4(-3e^x - 1)}{6(e^x + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+cosh(x))^2,x)

[Out] $-2/3*(3*e^x + 1)/(e^x + 1)^3$

Mupad [B]

time = 0.29, size = 14, normalized size = 0.56

$$-\frac{2(3e^x + 1)}{3(e^x + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(x) + 1)^2,x)

[Out] $-(2*(3*\exp(x) + 1))/(3*(\exp(x) + 1)^3)$

$$3.585 \quad \int \frac{1}{a+b \tanh(x)} dx$$

Optimal. Leaf size=39

$$\frac{ax}{a^2 - b^2} - \frac{b \log(a \cosh(x) + b \sinh(x))}{a^2 - b^2}$$

[Out] a*x/(a^2-b^2)-b*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)

Rubi [A]

time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3565, 3611}

$$\frac{ax}{a^2 - b^2} - \frac{b \log(a \cosh(x) + b \sinh(x))}{a^2 - b^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tanh[x])^(-1), x]

[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)

Rule 3565

Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 3611

Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/((a_) + (b_)*tan[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{a+b \tanh(x)} dx &= \frac{ax}{a^2 - b^2} - \frac{(ib) \int \frac{-ib-ia \tanh(x)}{a+b \tanh(x)} dx}{a^2 - b^2} \\ &= \frac{ax}{a^2 - b^2} - \frac{b \log(a \cosh(x) + b \sinh(x))}{a^2 - b^2} \end{aligned}$$

Mathematica [A]

time = 0.05, size = 29, normalized size = 0.74

$$\frac{ax - b \log(a \cosh(x) + b \sinh(x))}{a^2 - b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x - b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 4.01, size = 150, normalized size = 3.85

```
Piecewise [{{DirectedInfinity[x + Log[Tanh[x]] - Log[1 + Tanh[x]]], a==0&&b==0}, {x/a, b==0}, {1+x-xTanh[x]/(2b(-1+Tanh[x])), a== -b}, {-1+x+xTanh[x]/(2b(1+Tanh[x])), a==b}, {ax/(a^2-b^2) - bLog[x+Tanh[x]]/(a^2-b^2) + bLog[1+Tanh[x]]/(a^2-b^2) - bx/(a^2-b^2)}
```

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(a + b*Tanh[x]),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[x + Log[Tanh[x]] - Log[1 + Tanh[x]]], a == 0 & & b == 0}, {x / a, b == 0}, {(1 + x - x Tanh[x]) / (2 b (-1 + Tanh[x])), a == -b}, {(-1 + x + x Tanh[x]) / (2 b (1 + Tanh[x])), a == b}}, a x / (a ^ 2 - b ^ 2) - b Log[a / b + Tanh[x]] / (a ^ 2 - b ^ 2) + b Log[1 + Tanh[x]] / (a ^ 2 - b ^ 2) - b x / (a ^ 2 - b ^ 2)]
```

Maple [A]

time = 0.03, size = 55, normalized size = 1.41

method	result	size
derivativedivides	$-\frac{b \ln(a+b \tanh(x))}{(a+b)(a-b)} + \frac{\ln(\tanh(x)+1)}{2a-2b} - \frac{\ln(\tanh(x)-1)}{2a+2b}$	55
default	$-\frac{b \ln(a+b \tanh(x))}{(a+b)(a-b)} + \frac{\ln(\tanh(x)+1)}{2a-2b} - \frac{\ln(\tanh(x)-1)}{2a+2b}$	55
risch	$\frac{x}{a+b} + \frac{2bx}{a^2-b^2} - \frac{b \ln\left(e^{2x} + \frac{a-b}{a+b}\right)}{a^2-b^2}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -b/(a+b)/(a-b)*ln(a+b*tanh(x))+1/(2*a-2*b)*ln(tanh(x)+1)-1/(2*a+2*b)*ln(tanh(x)-1)
```

Maxima [A]

time = 0.24, size = 41, normalized size = 1.05

$$-\frac{b \log\left(-\left(a-b\right)e^{-2x}-a-b\right)}{a^2-b^2} + \frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="maxima")
```

[Out] $-b \cdot \log(-(a - b) \cdot e^{-2x} - a - b) / (a^2 - b^2) + x / (a + b)$

Fricas [A]

time = 0.31, size = 42, normalized size = 1.08

$$\frac{(a + b)x - b \log\left(\frac{2(a \cosh(x) + b \sinh(x))}{\cosh(x) - \sinh(x)}\right)}{a^2 - b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*tanh(x)),x, algorithm="fricas")`

[Out] $((a + b) \cdot x - b \cdot \log(2 \cdot (a \cdot \cosh(x) + b \cdot \sinh(x)) / (\cosh(x) - \sinh(x)))) / (a^2 - b^2)$

Sympy [A]

time = 0.29, size = 146, normalized size = 3.74

$$\left\{ \begin{array}{ll} \tilde{\infty}(x - \log(\tanh(x) + 1) + \log(\tanh(x))) & \text{for } a = 0 \wedge b = 0 \\ -\frac{x \tanh(x)}{2b \tanh(x) - 2b} + \frac{x}{2b \tanh(x) - 2b} + \frac{1}{2b \tanh(x) - 2b} & \text{for } a = -b \\ \frac{x \tanh(x)}{2b \tanh(x) + 2b} + \frac{x}{2b \tanh(x) + 2b} - \frac{1}{2b \tanh(x) + 2b} & \text{for } a = b \\ \frac{x}{a} & \text{for } b = 0 \\ \frac{ax}{a^2 - b^2} - \frac{bx}{a^2 - b^2} - \frac{b \log\left(\frac{a}{b} + \tanh(x)\right)}{a^2 - b^2} + \frac{b \log(\tanh(x) + 1)}{a^2 - b^2} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*tanh(x)),x)`

[Out] `Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x))), Eq(a, 0) & Eq(b, 0)), (-x*tanh(x)/(2*b*tanh(x) - 2*b) + x/(2*b*tanh(x) - 2*b) + 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x) + 2*b) - 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (x/a, Eq(b, 0)), (a*x/(a**2 - b**2) - b*x/(a**2 - b**2) - b*log(a/b + tanh(x))/(a**2 - b**2) + b*log(tanh(x) + 1)/(a**2 - b**2), True))`

Giac [A]

time = 0.00, size = 42, normalized size = 1.08

$$\frac{2x}{2a - 2b} + \frac{b \ln |e^{2x}a + e^{2x}b + a - b|}{-a^2 + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*tanh(x)),x)`

[Out] $-b \cdot \log(\text{abs}(a \cdot e^{2x} + b \cdot e^{2x} + a - b)) / (a^2 - b^2) + x / (a - b)$

Mupad [B]

time = 0.13, size = 35, normalized size = 0.90

$$\frac{a x - b (x - \ln(\tanh(x) + 1) + \ln(a + b \tanh(x)))}{a^2 - b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b*tanh(x)),x)`

[Out] `(a*x - b*(x - log(tanh(x) + 1) + log(a + b*tanh(x))))/(a^2 - b^2)`

$$3.586 \quad \int \frac{1}{a^2 + b^2 \cosh^2(x)} dx$$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{a \tanh(x)}{\sqrt{a^2 + b^2}}\right)}{a\sqrt{a^2 + b^2}}$$

[Out] arctanh(a*tanh(x)/(a^2+b^2)^(1/2))/a/(a^2+b^2)^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3260, 214}

$$\frac{\tanh^{-1}\left(\frac{a \tanh(x)}{\sqrt{a^2 + b^2}}\right)}{a\sqrt{a^2 + b^2}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + b^2*Cosh[x]^2)^(-1), x]

[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 + b^2]]/(a*Sqrt[a^2 + b^2])

Rule 214

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3260

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{a^2 + b^2 \cosh^2(x)} dx &= \text{Subst} \left(\int \frac{1}{a^2 - (a^2 + b^2) x^2} dx, x, \coth(x) \right) \\ &= \frac{\tanh^{-1}\left(\frac{a \tanh(x)}{\sqrt{a^2 + b^2}}\right)}{a\sqrt{a^2 + b^2}} \end{aligned}$$

Mathematica [A]

time = 0.04, size = 31, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{a \tanh(x)}{\sqrt{a^2 + b^2}}\right)}{a\sqrt{a^2 + b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + b^2*Cosh[x]^2)^(-1),x]

[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 + b^2]]/(a*Sqrt[a^2 + b^2])

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 21.04, size = 1298, normalized size = 41.87

result too large to display

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(a^2 + b^2*Cosh[x]^2),x]')

[Out] Piecewise[{{DirectedInfinity[Tanh[x]], a == 0 && b == 0}, {-(1 / Tanh[x / 2] + Tanh[x / 2]) / (2 b ^ 2), a == I b || a == -I b}, {Tanh[x] / b ^ 2, a = = 0}}, -a Log[Tanh[x / 2] - Sqrt[a / (a - I b) + I b / (a - I b)]] Sqrt[a / (a + I b) - I b / (a + I b)] / (2 a ^ 3 Sqrt[a / (a - I b) + I b / (a - I b)]) Sqrt[a / (a + I b) - I b / (a + I b)] + 2 a b ^ 2 Sqrt[a / (a - I b) + I b / (a - I b)] Sqrt[a / (a + I b) - I b / (a + I b)] - a Log[Tanh[x / 2] - Sqrt[a / (a + I b) - I b / (a + I b)]] Sqrt[a / (a - I b) + I b / (a - I b)] / (2 a ^ 3 Sqrt[a / (a - I b) + I b / (a - I b)] Sqrt[a / (a + I b) - I b / (a + I b)] + 2 a b ^ 2 Sqrt[a / (a - I b) + I b / (a - I b)] Sqrt[a / (a + I b) - I b / (a + I b)] + a Log[Sqrt[a / (a - I b) + I b / (a - I b)] + Tanh[x / 2]] Sqrt[a / (a + I b) - I b / (a + I b)] / (2 a ^ 3 Sqrt[a / (a - I b) + I b / (a - I b)] Sqrt[a / (a + I b) - I b / (a + I b)] + 2 a b ^ 2 Sqrt[a / (a - I b) + I b / (a - I b)] Sqrt[a / (a + I b) - I b / (a + I b)] + a Log[Sqrt[a / (a + I b) - I b / (a + I b)] + Tanh[x / 2]] Sqrt[a / (a - I b) + I b / (a - I b)] / (2 a ^ 3 Sqrt[a / (a - I b) + I b / (a - I b)] Sqrt[a / (a + I b) - I b / (a + I b)] + 2 a b ^ 2 Sqrt[a / (a - I b) + I b / (a - I b)] Sqrt[a / (a + I b) - I b / (a + I b)] - I b Log[Sqrt[a / (a + I b) - I b / (a + I b)] + Tanh[x / 2]] Sqrt[a / (a - I b) + I b / (a - I b)] / (2 a ^ 3 Sqrt[a / (a - I b) + I b / (a - I b)] Sqrt[a / (a + I b) - I b / (a + I b)] + 2 a b ^ 2 Sqrt[a / (a - I b) + I b / (a - I b)] Sqrt[a / (a + I b) - I b / (a + I b)] - I b Log[Tanh[x / 2] - Sqrt[a / (a - I b) + I b / (a - I b)]] Sqrt[a / (a + I b) - I b / (a + I b)] / (2 a ^ 3 Sqrt[a / (a - I b) + I b / (a - I b)] Sqrt[a / (a + I b) - I b / (a + I b)] + 2 a b ^ 2 Sqrt[a / (a - I b) + I b / (a - I b)] Sqrt[a / (a + I b) - I b / (a + I b)] + I b Log[Sqrt[a / (a - I b) + I b / (a - I b)] + Tanh[x / 2]] Sqrt[a / (a + I b) - I b / (a + I b)] + I b Log[Sqrt[a / (a - I b) + I b / (a - I b)] + Tanh[x / 2]] Sqrt[a / (a + I b) - I b / (a + I b)]

rt[a / (a + I b) - I b / (a + I b)] / (2 a ^ 3 Sqrt[a / (a - I b) + I b / (a - I b)] Sqrt[a / (a + I b) - I b / (a + I b)] + 2 a b ^ 2 Sqrt[a / (a - I b) + I b / (a - I b)] Sqrt[a / (a + I b) - I b / (a + I b)]) + I b Log[Tan h[x / 2] - Sqrt[a / (a + I b) - I b / (a + I b)] Sqrt[a / (a - I b) + I b / (a - I b)] / (2 a ^ 3 Sqrt[a / (a - I b) + I b / (a - I b)] Sqrt[a / (a + I b) - I b / (a + I b)] + 2 a b ^ 2 Sqrt[a / (a - I b) + I b / (a - I b)] Sqrt[a / (a + I b) - I b / (a + I b)])]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(27) = 54.

time = 0.09, size = 98, normalized size = 3.16

method	result	size
default	$\frac{\ln\left(\sqrt{a^2 + b^2} \left(\tanh^2\left(\frac{x}{2}\right) + 2a \tanh\left(\frac{x}{2}\right) + \sqrt{a^2 + b^2}\right)\right)}{2a\sqrt{a^2 + b^2}} - \frac{\ln\left(\sqrt{a^2 + b^2} \left(\tanh^2\left(\frac{x}{2}\right) - 2a \tanh\left(\frac{x}{2}\right) + \sqrt{a^2 + b^2}\right)\right)}{2a\sqrt{a^2 + b^2}}$	98
risch	$\frac{\ln\left(e^{2x} + \frac{2a^2\sqrt{a^2 + b^2} + b^2\sqrt{a^2 + b^2} - 2a^3 - 2ab^2}{b^2\sqrt{a^2 + b^2}}\right)}{2a\sqrt{a^2 + b^2}} - \frac{\ln\left(e^{2x} + \frac{2a^2\sqrt{a^2 + b^2} + b^2\sqrt{a^2 + b^2} + 2a^3 + 2ab^2}{b^2\sqrt{a^2 + b^2}}\right)}{2a\sqrt{a^2 + b^2}}$	146

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2+b^2*cosh(x)^2),x,method=_RETURNVERBOSE)

[Out] 1/2/a/(a^2+b^2)^(1/2)*ln((a^2+b^2)^(1/2)*tanh(1/2*x)^2+2*a*tanh(1/2*x)+(a^2+b^2)^(1/2))-1/2/a/(a^2+b^2)^(1/2)*ln((a^2+b^2)^(1/2)*tanh(1/2*x)^2-2*a*tanh(1/2*x)+(a^2+b^2)^(1/2))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(27) = 54.

time = 0.33, size = 76, normalized size = 2.45

$$\frac{\log\left(\frac{b^2 e^{-2x} + 2a^2 + b^2 - 2\sqrt{a^2 + b^2} a}{b^2 e^{-2x} + 2a^2 + b^2 + 2\sqrt{a^2 + b^2} a}\right)}{2\sqrt{a^2 + b^2} a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+b^2*cosh(x)^2),x, algorithm="maxima")

[Out] -1/2*log((b^2*e^(-2*x) + 2*a^2 + b^2 - 2*sqrt(a^2 + b^2)*a)/(b^2*e^(-2*x) + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*a))/(sqrt(a^2 + b^2)*a)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(27) = 54.

time = 0.32, size = 288, normalized size = 9.29

$$\frac{\sqrt{a^2 + b^2} \log\left(\frac{b^4 \cosh(x)^4 + 4b^4 \cosh(x) \sinh(x)^3 + b^4 \sinh(x)^4 + 8a^4 + 8a^2 b^2 + b^4 + 2(2a^2 b^2 + b^4) \cosh(x)^2 + 2(3b^4 \cosh(x)^2 + 2a^2 b^2 + b^4) \sinh(x)^2 + 4(b^4 \cosh(x)^3 + (2a^2 b^2 + b^4) \cosh(x)) \sinh(x) - 4(ab^2 \cosh(x)^2 + 2ab^2 \cosh(x) \sinh(x) + ab^2 \sinh(x)^2 + 2a^2 + ab^2) \sqrt{a^2 + b^2}}{b^2 \cosh(x)^4 + 4b^2 \cosh(x) \sinh(x)^3 + b^2 \sinh(x)^4 + 2(2a^2 + b^2) \cosh(x)^2 + 2(3b^2 \cosh(x)^2 + 2a^2 + b^2) \sinh(x)^2 + 2(4b^2 \cosh(x)^3 + (2a^2 + b^2) \cosh(x)) \sinh(x)}\right)}{2(a^3 + ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a^2 + b^2)*log((b^4*cosh(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh(x)^4 + 8*a^4 + 8*a^2*b^2 + b^4 + 2*(2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(3*b^4*cosh(x)^2 + 2*a^2*b^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 + (2*a^2*b^2 + b^4)*cosh(x))*sinh(x) - 4*(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) + a*b^2*sinh(x)^2 + 2*a^3 + a*b^2)*sqrt(a^2 + b^2))/(b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a^2 + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + (2*a^2 + b^2)*cosh(x))*sinh(x))/(a^3 + a*b^2)
```

Sympy [A]

time = 16.94, size = 1149, normalized size = 37.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2*cosh(x)**2),x)
```

```
[Out] Piecewise((zoo*tanh(x/2)/(tanh(x/2)**2 + 1), Eq(a, 0) & Eq(b, 0)), (-tanh(x/2)/(2*b**2) - 1/(2*b**2*tanh(x/2)), Eq(a, -I*b)), (2*tanh(x/2)/(b**2*(tanh(x/2)**2 + 1)), Eq(a, 0)), (-tanh(x/2)/(2*b**2) - 1/(2*b**2*tanh(x/2)), Eq(a, I*b)), (-a*sqrt(a/(a - I*b) + I*b/(a - I*b))*log(-sqrt(a/(a + I*b) - I*b/(a + I*b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b)) + 2*a*b**2*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))) + a*sqrt(a/(a - I*b) + I*b/(a - I*b))*log(sqrt(a/(a + I*b) - I*b/(a + I*b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b)) + 2*a*b**2*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))) - a*sqrt(a/(a + I*b) - I*b/(a + I*b))*log(-sqrt(a/(a - I*b) + I*b/(a - I*b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b)) + 2*a*b**2*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))) + a*sqrt(a/(a + I*b) - I*b/(a + I*b))*log(sqrt(a/(a - I*b) + I*b/(a - I*b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b)) + 2*a*b**2*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))) - I*b*sqrt(a/(a - I*b) + I*b/(a - I*b))*log(-sqrt(a/(a + I*b) - I*b/(a + I*b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b)) + 2*a*b**2*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))) - I*b*sqrt(a/(a - I*b) + I*b/(a - I*b))*log(sqrt(a/(a + I*b) - I*b/(a + I*b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b)) + 2*a*b**2*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))) + 2*a*b**2*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))
```

+ I*b))) + I*b*sqrt(a/(a + I*b) - I*b/(a + I*b))*log(sqrt(a/(a - I*b) + I*b/(a - I*b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b)) + 2*a*b**2*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(27) = 54.
time = 0.00, size = 92, normalized size = 2.97

$$\frac{\ln\left(\frac{2(e^x)^2 b^2 + 4a^2 + 2b^2 - 4\sqrt{a^2 + b^2} |a|}{2(e^x)^2 b^2 + 4a^2 + 2b^2 + 4\sqrt{a^2 + b^2} |a|}\right)}{2\sqrt{a^2 + b^2} |a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+b^2*cosh(x)^2), x)

[Out] 1/2*log((b^2*e^(2*x) + 2*a^2 + b^2 - 2*sqrt(a^2 + b^2)*abs(a))/(b^2*e^(2*x) + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*abs(a)))/(sqrt(a^2 + b^2)*abs(a))

Mupad [B]

time = 0.65, size = 109, normalized size = 3.52

$$\frac{\operatorname{atan}\left(\frac{2a^2(-a^4 - a^2b^2)^{3/2} + b^2(-a^4 - a^2b^2)^{3/2} + b^2e^{2x}(-a^4 - a^2b^2)^{3/2}}{2a^8 + 4a^6b^2 + 2a^4b^4}\right)}{\sqrt{-a^4 - a^2b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^2*cosh(x)^2 + a^2), x)

[Out] atan((2*a^2*(-a^4 - a^2*b^2)^(3/2) + b^2*(-a^4 - a^2*b^2)^(3/2) + b^2*exp(2*x)*(-a^4 - a^2*b^2)^(3/2))/(2*a^8 + 2*a^4*b^4 + 4*a^6*b^2))/(-a^4 - a^2*b^2)^(1/2)

$$3.587 \quad \int \frac{1}{a^2 - b^2 \cosh^2(x)} dx$$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{a \tanh(x)}{\sqrt{a^2 - b^2}}\right)}{a\sqrt{a^2 - b^2}}$$

[Out] arctanh(a*tanh(x)/(a^2-b^2)^(1/2))/a/(a^2-b^2)^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3260, 214}

$$\frac{\tanh^{-1}\left(\frac{a \tanh(x)}{\sqrt{a^2 - b^2}}\right)}{a\sqrt{a^2 - b^2}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 - b^2*Cosh[x]^2)^(-1), x]

[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 - b^2]]/(a*Sqrt[a^2 - b^2])

Rule 214

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3260

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(-1), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{a^2 - b^2 \cosh^2(x)} dx &= \text{Subst} \left(\int \frac{1}{a^2 - (a^2 - b^2)x^2} dx, x, \coth(x) \right) \\ &= \frac{\tanh^{-1}\left(\frac{a \tanh(x)}{\sqrt{a^2 - b^2}}\right)}{a\sqrt{a^2 - b^2}} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 35, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{a \tanh(x)}{\sqrt{a^2 - b^2}}\right)}{a\sqrt{a^2 - b^2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(a^2 - b^2*Cosh[x]^2)^(-1),x]``[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 - b^2]]/(a*Sqrt[a^2 - b^2])`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 20.19, size = 1175, normalized size = 33.57

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(a^2 - b^2*Cosh[x]^2),x]')`

```
[Out] Piecewise[{{DirectedInfinity[Tanh[x]], a == 0 && b == 0}, {-Tanh[x] / b ^ 2
, a == 0}, {(1 / Tanh[x / 2] + Tanh[x / 2]) / (2 b ^ 2), a == b || a == -b}
}, -a Log[Tanh[x / 2] - Sqrt[a / (a + b) - b / (a + b)]] Sqrt[a / (a - b) +
b / (a - b)] / (2 a ^ 3 Sqrt[a / (a + b) - b / (a + b)] Sqrt[a / (a - b) +
b / (a - b)] - 2 a b ^ 2 Sqrt[a / (a + b) - b / (a + b)] Sqrt[a / (a - b)
+ b / (a - b)]) - a Log[Tanh[x / 2] - Sqrt[a / (a - b) + b / (a - b)]] Sqrt
[a / (a + b) - b / (a + b)] / (2 a ^ 3 Sqrt[a / (a + b) - b / (a + b)] Sqrt
[a / (a - b) + b / (a - b)] - 2 a b ^ 2 Sqrt[a / (a + b) - b / (a + b)] Sqr
t[a / (a - b) + b / (a - b)]) + a Log[Sqrt[a / (a + b) - b / (a + b)] + Tan
h[x / 2]] Sqrt[a / (a - b) + b / (a - b)] / (2 a ^ 3 Sqrt[a / (a + b) - b /
(a + b)] Sqrt[a / (a - b) + b / (a - b)] - 2 a b ^ 2 Sqrt[a / (a + b) - b
/ (a + b)] Sqrt[a / (a - b) + b / (a - b)]) + a Log[Sqrt[a / (a - b) + b /
(a - b)] + Tanh[x / 2]] Sqrt[a / (a + b) - b / (a + b)] / (2 a ^ 3 Sqrt[a /
(a + b) - b / (a + b)] Sqrt[a / (a - b) + b / (a - b)] - 2 a b ^ 2 Sqrt[a
/ (a + b) - b / (a + b)] Sqrt[a / (a - b) + b / (a - b)]) - b Log[Sqrt[a /
(a + b) - b / (a + b)] + Tanh[x / 2]] Sqrt[a / (a - b) + b / (a - b)] / (2
a ^ 3 Sqrt[a / (a + b) - b / (a + b)] Sqrt[a / (a - b) + b / (a - b)] - 2 a
b ^ 2 Sqrt[a / (a + b) - b / (a + b)] Sqrt[a / (a - b) + b / (a - b)]) - b
Log[Tanh[x / 2] - Sqrt[a / (a - b) + b / (a - b)]] Sqrt[a / (a + b) - b /
(a + b)] / (2 a ^ 3 Sqrt[a / (a + b) - b / (a + b)] Sqrt[a / (a - b) + b /
(a - b)] - 2 a b ^ 2 Sqrt[a / (a + b) - b / (a + b)] Sqrt[a / (a - b) + b /
(a - b)]) + b Log[Sqrt[a / (a - b) + b / (a - b)] + Tanh[x / 2]] Sqrt[a /
(a + b) - b / (a + b)] / (2 a ^ 3 Sqrt[a / (a + b) - b / (a + b)] Sqrt[a /
(a - b) + b / (a - b)] - 2 a b ^ 2 Sqrt[a / (a + b) - b / (a + b)] Sqrt[a /
```

$$(a - b) + b / (a - b)] + b \operatorname{Log}[\operatorname{Tanh}[x / 2] - \operatorname{Sqrt}[a / (a + b) - b / (a + b)]] \operatorname{Sqrt}[a / (a - b) + b / (a - b)] / (2 a^3 \operatorname{Sqrt}[a / (a + b) - b / (a + b)] \operatorname{Sqrt}[a / (a - b) + b / (a - b)] - 2 a b^2 \operatorname{Sqrt}[a / (a + b) - b / (a + b)] \operatorname{Sqrt}[a / (a - b) + b / (a - b)])]$$

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(31) = 62$.

time = 0.07, size = 74, normalized size = 2.11

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{(a+b) \operatorname{tanh}\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a \sqrt{(a+b)(a-b)}} + \frac{\operatorname{arctanh}\left(\frac{(a-b) \operatorname{tanh}\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a \sqrt{(a+b)(a-b)}}$	74
risch	$\frac{\ln\left(\frac{e^{2x} - 2a^2 \sqrt{a^2 - b^2} - b^2 \sqrt{a^2 - b^2} - 2a^3 + 2ab^2}{b^2 \sqrt{a^2 - b^2}}\right)}{2 \sqrt{a^2 - b^2} a} - \frac{\ln\left(\frac{e^{2x} - 2a^2 \sqrt{a^2 - b^2} - b^2 \sqrt{a^2 - b^2} + 2a^3 - 2ab^2}{b^2 \sqrt{a^2 - b^2}}\right)}{2 \sqrt{a^2 - b^2} a}$	166

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^2-b^2*cosh(x)^2),x,method=_RETURNVERBOSE)`

[Out] $1/a/((a+b)*(a-b))^{(1/2)}*\operatorname{arctanh}((a+b)*\operatorname{tanh}(1/2*x)/((a+b)*(a-b))^{(1/2)})+1/a/((a+b)*(a-b))^{(1/2)}*\operatorname{arctanh}((a-b)*\operatorname{tanh}(1/2*x)/((a+b)*(a-b))^{(1/2)})$

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a^2-b^2*cosh(x)^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see 'assume?' for more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(31) = 62$.

time = 0.32, size = 388, normalized size = 11.09

$$\frac{\sqrt{a^2 - b^2} \log\left(\frac{b^2 \operatorname{cosh}(x)^4 + 4b^2 \operatorname{cosh}(x) \operatorname{sinh}(x)^2 + 4b^2 \operatorname{sinh}(x)^4 + 8a^2 - 8a^2 b^2 - 2(2a^2 b^2 - 4b^4) \operatorname{cosh}(x)^2 - 2(3b^2 \operatorname{cosh}(x)^2 - 3a^2 b^2) \operatorname{sinh}(x)^2 + 4(b^4 \operatorname{cosh}(x)^2 - (2a^2 b^2 - 4b^4) \operatorname{cosh}(x)) \operatorname{sinh}(x) + 4(a^2 \operatorname{cosh}(x)^2 + 2a^2 \operatorname{cosh}(x) \operatorname{sinh}(x) + a^2 \operatorname{sinh}(x)^2 - 2a^3 + a^2) \sqrt{a^2 - b^2}}{b^2 \operatorname{cosh}(x)^4 + 4b^2 \operatorname{cosh}(x) \operatorname{sinh}(x)^2 + 4b^2 \operatorname{sinh}(x)^4 - 2(2a^2 b^2 - 4b^4) \operatorname{cosh}(x)^2 + 2(3b^2 \operatorname{cosh}(x)^2 - 3a^2 b^2) \operatorname{sinh}(x)^2 + 4(b^4 \operatorname{cosh}(x)^2 - (2a^2 b^2 - 4b^4) \operatorname{cosh}(x)) \operatorname{sinh}(x)}\right)}{2(a^3 - ab^2)} - \frac{\sqrt{-a^2 + b^2} \operatorname{arctan}\left(\frac{b^2 \operatorname{cosh}(x)^2 + 2b^2 \operatorname{cosh}(x) \operatorname{sinh}(x) + b^2 \operatorname{sinh}(x)^2 - 3a^2 + 3b^2}{2(2a^2 b^2 - 4b^4)} \sqrt{-a^2 + b^2}\right)}{a^3 - ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a^2-b^2*cosh(x)^2),x, algorithm="fricas")`

```
[Out] [1/2*sqrt(a^2 - b^2)*log((b^4*cosh(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh(x)^4 + 8*a^4 - 8*a^2*b^2 + b^4 - 2*(2*a^2*b^2 - b^4)*cosh(x)^2 + 2*(3*b^4*cosh(x)^2 - 2*a^2*b^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 - (2*a^2*b^2 - b^4)*cosh(x))*sinh(x) + 4*(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) + a*b^2*sinh(x)^2 - 2*a^3 + a*b^2)*sqrt(a^2 - b^2))/(b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 - 2*(2*a^2 - b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 - 2*a^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 - (2*a^2 - b^2)*cosh(x))*sinh(x)))/(a^3 - a*b^2), sqrt(-a^2 + b^2)*arctan(-1/2*(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 - 2*a^2 + b^2)*sqrt(-a^2 + b^2)/(a^3 - a*b^2)))/(a^3 - a*b^2)]
```

Sympy [A]

time = 16.06, size = 874, normalized size = 24.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2-b**2*cosh(x)**2),x)
```

```
[Out] Piecewise((zoo*tanh(x/2)/(tanh(x/2)**2 + 1), Eq(a, 0) & Eq(b, 0)), (-2*tanh(x/2)/(b**2*(tanh(x/2)**2 + 1)), Eq(a, 0)), (tanh(x/2)/(2*b**2) + 1/(2*b**2*tanh(x/2)), Eq(a, b) | Eq(a, -b)), (-a*sqrt(a/(a - b) + b/(a - b))*log(-sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) + a*sqrt(a/(a - b) + b/(a - b))*log(sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) - a*sqrt(a/(a + b) - b/(a + b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) + a*sqrt(a/(a + b) - b/(a + b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) + b*sqrt(a/(a - b) + b/(a - b))*log(-sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) - b*sqrt(a/(a - b) + b/(a - b))*log(sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) - b*sqrt(a/(a + b) - b/(a + b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) + b*sqrt(a/(a + b) - b/(a + b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) + b*sqrt(a/(a + b) - b/(a + b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))), True))
```

Giac [A]

time = 0.00, size = 54, normalized size = 1.54

$$-\frac{\arctan\left(\frac{-2a^2+b^2(e^x)^2+b^2}{2a\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2-b^2*cosh(x)^2),x)

[Out] -arctan(1/2*(b^2*e^(2*x) - 2*a^2 + b^2)/(sqrt(-a^2 + b^2)*a))/(sqrt(-a^2 + b^2)*a)

Mupad [B]

time = 0.38, size = 106, normalized size = 3.03

$$-\frac{\operatorname{atan}\left(\frac{b^2(a^2b^2-a^4)^{3/2}-2a^2(a^2b^2-a^4)^{3/2}+b^2e^{2x}(a^2b^2-a^4)^{3/2}}{2a^8-4a^6b^2+2a^4b^4}\right)}{\sqrt{a^2b^2-a^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(b^2*cosh(x)^2 - a^2),x)

[Out] -atan((b^2*(a^2*b^2 - a^4)^(3/2) - 2*a^2*(a^2*b^2 - a^4)^(3/2) + b^2*exp(2*x)*(a^2*b^2 - a^4)^(3/2))/(2*a^8 + 2*a^4*b^4 - 4*a^6*b^2))/(a^2*b^2 - a^4)^(1/2)

$$3.588 \quad \int \frac{1}{1-\sinh^4(x)} dx$$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\sqrt{2} \tanh(x)\right)}{2\sqrt{2}} + \frac{\tanh(x)}{2}$$

[Out] 1/4*arctanh(2^(1/2)*tanh(x))*2^(1/2)+1/2*tanh(x)

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3288, 396, 212}

$$\frac{\tanh^{-1}\left(\sqrt{2} \tanh(x)\right)}{2\sqrt{2}} + \frac{\tanh(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sinh[x]^4)^(-1),x]

[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(2*Sqrt[2]) + Tanh[x]/2

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 396

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 3288

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{1}{1 - \sinh^4(x)} dx &= \text{Subst} \left(\int \frac{1 - x^2}{1 - 2x^2} dx, x, \tanh(x) \right) \\ &= \frac{\tanh(x)}{2} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1 - 2x^2} dx, x, \tanh(x) \right) \\ &= \frac{\tanh^{-1}(\sqrt{2} \tanh(x))}{2\sqrt{2}} + \frac{\tanh(x)}{2} \end{aligned}$$

Mathematica [A]

time = 0.07, size = 24, normalized size = 0.96

$$\frac{1}{4} \left(\sqrt{2} \tanh^{-1}(\sqrt{2} \tanh(x)) + 2 \tanh(x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^4)^(-1), x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + 2*Tanh[x])/4
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 467 vs. 2(25) = 50.
time = 30.11, size = 359, normalized size = 14.36

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(1 - Sinh[x]^4), x]')
```

```
[Out] (-2167073 Sqrt[2] (-1 + Cosh[x]) (2167073 + 1532352 Sqrt[2] + 1532352 Sqrt[2] Tanh[x / 2] ^ 2 + 2167073 Tanh[x / 2] ^ 2) (Log[-1 - Sqrt[2] + Tanh[x / 2]] + Log[1 - Sqrt[2] + Tanh[x / 2]]) + 4334146 (2167073 + 1532352 Sqrt[2]) Sqrt[2] Cosh[x] (Log[-1 + Sqrt[2] + Tanh[x / 2]] + Log[1 + Sqrt[2] + Tanh[x / 2]] - Log[-1 - Sqrt[2] + Tanh[x / 2]] - Log[1 - Sqrt[2] + Tanh[x / 2]]) + (34673168 + 24517632 Sqrt[2]) Cosh[x] (-383088 Log[-1 - Sqrt[2] + Tanh[x / 2]] - 383088 Log[1 - Sqrt[2] + Tanh[x / 2]] + 383088 Log[-1 + Sqrt[2] + Tanh[x / 2]] + 383088 Log[1 + Sqrt[2] + Tanh[x / 2]] + 1532352 Sqrt[2] Tanh[x / 2] + 2167073 Tanh[x / 2]) + (-1 + Cosh[x]) (2167073 + 1532352 Sqrt[2] + 1532352 Sqrt[2] Tanh[x / 2] ^ 2 + 2167073 Tanh[x / 2] ^ 2) (-3064704 Log[-1 - Sqrt[2] + Tanh[x / 2]] - 3064704 Log[1 - Sqrt[2] + Tanh[x / 2]] + 2167073 Sqrt[2] Log[-1 + Sqrt[2] + Tanh[x / 2]] + 2167073 Sqrt[2] Log[1 + Sqrt[2] + Tanh[x / 2]] + 3064704 Log[-1 + Sqrt[2] + Tanh[x / 2]] + 3064704 Log[1 + Sqrt[2] + Tanh[x / 2]]) / (16 (2167073 + 1532352 Sqrt[2]) Cosh[x] (2167073 + 1532352 Sqrt[2] + 1532352 Sqrt[2] Tanh[x / 2] ^ 2 + 2167073 Tanh[x / 2] ^ 2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(17) = 34$.

time = 0.06, size = 55, normalized size = 2.20

method	result	size
risch	$-\frac{1}{1+e^{2x}} + \frac{\sqrt{2} \ln(e^{2x} + 2\sqrt{2} - 3)}{8} - \frac{\sqrt{2} \ln(e^{2x} - 3 - 2\sqrt{2})}{8}$	46
default	$\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{(2 \tanh(\frac{x}{2}) - 2)\sqrt{2}}{4}\right)}{4} + \frac{\tanh(\frac{x}{2})}{\tanh^2(\frac{x}{2}) + 1} + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{(2 \tanh(\frac{x}{2}) + 2)\sqrt{2}}{4}\right)}{4}$	55

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1-sinh(x)^4),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{4} \cdot 2^{(1/2)} \cdot \operatorname{arctanh}\left(\frac{1}{4} \cdot (2 \cdot \tanh(1/2 \cdot x) - 2) \cdot 2^{(1/2)}\right) + \tanh(1/2 \cdot x) / (\tanh(1/2 \cdot x)^2 + 1) + \frac{1}{4} \cdot 2^{(1/2)} \cdot \operatorname{arctanh}\left(\frac{1}{4} \cdot (2 \cdot \tanh(1/2 \cdot x) + 2) \cdot 2^{(1/2)}\right)$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(17) = 34$.

time = 0.35, size = 69, normalized size = 2.76

$$\frac{1}{8} \sqrt{2} \log\left(-\frac{\sqrt{2} - e^{(-x)} + 1}{\sqrt{2} + e^{(-x)} - 1}\right) - \frac{1}{8} \sqrt{2} \log\left(-\frac{\sqrt{2} - e^{(-x)} - 1}{\sqrt{2} + e^{(-x)} + 1}\right) + \frac{1}{e^{(-2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-sinh(x)^4),x, algorithm="maxima")`

[Out] $\frac{1}{8} \cdot \sqrt{2} \cdot \log(-(\sqrt{2} - e^{(-x)} + 1)/(\sqrt{2} + e^{(-x)} - 1)) - \frac{1}{8} \cdot \sqrt{2} \cdot \log(-(\sqrt{2} - e^{(-x)} - 1)/(\sqrt{2} + e^{(-x)} + 1)) + \frac{1}{e^{(-2x)} + 1}$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(17) = 34$.

time = 0.31, size = 113, normalized size = 4.52

$$\frac{(\sqrt{2} \cosh(x)^2 + 2\sqrt{2} \cosh(x) \sinh(x) + \sqrt{2} \sinh(x)^2 + \sqrt{2}) \log\left(-\frac{3(2\sqrt{2}-3) \cosh(x)^2 - 4(3\sqrt{2}-4) \cosh(x) \sinh(x) + 3(2\sqrt{2}-3) \sinh(x)^2 - 2\sqrt{2} + 3}{\cosh(x)^2 + \sinh(x)^2 - 3}\right) - 8}{8(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-sinh(x)^4),x, algorithm="fricas")`

[Out] $\frac{1}{8} \cdot ((\sqrt{2} \cdot \cosh(x)^2 + 2 \cdot \sqrt{2} \cdot \cosh(x) \cdot \sinh(x) + \sqrt{2} \cdot \sinh(x)^2 + \sqrt{2}) \cdot \log(-((3 \cdot (2 \cdot \sqrt{2} - 3) \cdot \cosh(x)^2 - 4 \cdot (3 \cdot \sqrt{2} - 4) \cdot \cosh(x) \cdot \sinh(x) + 3 \cdot (2 \cdot \sqrt{2} - 3) \cdot \sinh(x)^2 - 2 \cdot \sqrt{2} + 3)) / (\cosh(x)^2 + \sinh(x)^2 - 3)) - 8) / (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2 + 1)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 908 vs. $2(20) = 40$

time = 2.77, size = 908, normalized size = 36.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)**4),x)

[Out]
$$3064704 \cdot \log(\tanh(x/2) - 1 + \sqrt{2}) \cdot \tanh(x/2)^2 / (12258816 \sqrt{2} \tanh(x/2)^2 + 17336584 \tanh(x/2)^2 + 12258816 \sqrt{2} + 17336584) + 2167073 \sqrt{2} \log(\tanh(x/2) - 1 + \sqrt{2}) \cdot \tanh(x/2)^2 / (12258816 \sqrt{2} \tanh(x/2)^2 + 17336584 \tanh(x/2)^2 + 12258816 \sqrt{2} + 17336584) + 3064704 \log(\tanh(x/2) - 1 + \sqrt{2}) / (12258816 \sqrt{2} \tanh(x/2)^2 + 17336584 \tanh(x/2)^2 + 12258816 \sqrt{2} + 17336584) + 2167073 \sqrt{2} \log(\tanh(x/2) - 1 + \sqrt{2}) / (12258816 \sqrt{2} \tanh(x/2)^2 + 17336584 \tanh(x/2)^2 + 12258816 \sqrt{2} + 17336584) + 3064704 \log(\tanh(x/2) + 1 + \sqrt{2}) \cdot \tanh(x/2)^2 / (12258816 \sqrt{2} \tanh(x/2)^2 + 17336584 \tanh(x/2)^2 + 12258816 \sqrt{2} + 17336584) + 2167073 \sqrt{2} \log(\tanh(x/2) + 1 + \sqrt{2}) \cdot \tanh(x/2)^2 / (12258816 \sqrt{2} \tanh(x/2)^2 + 17336584 \tanh(x/2)^2 + 12258816 \sqrt{2} + 17336584) + 3064704 \log(\tanh(x/2) + 1 + \sqrt{2}) / (12258816 \sqrt{2} \tanh(x/2)^2 + 17336584 \tanh(x/2)^2 + 12258816 \sqrt{2} + 17336584) + 2167073 \sqrt{2} \log(\tanh(x/2) + 1 + \sqrt{2}) / (12258816 \sqrt{2} \tanh(x/2)^2 + 17336584 \tanh(x/2)^2 + 12258816 \sqrt{2} + 17336584) - 2167073 \sqrt{2} \log(\tanh(x/2) - \sqrt{2} - 1) \cdot \tanh(x/2)^2 / (12258816 \sqrt{2} \tanh(x/2)^2 + 17336584 \tanh(x/2)^2 + 12258816 \sqrt{2} + 17336584) - 3064704 \log(\tanh(x/2) - \sqrt{2} - 1) \cdot \tanh(x/2)^2 / (12258816 \sqrt{2} \tanh(x/2)^2 + 17336584 \tanh(x/2)^2 + 12258816 \sqrt{2} + 17336584) - 2167073 \sqrt{2} \log(\tanh(x/2) - \sqrt{2} - 1) / (12258816 \sqrt{2} \tanh(x/2)^2 + 17336584 \tanh(x/2)^2 + 12258816 \sqrt{2} + 17336584) - 3064704 \log(\tanh(x/2) - \sqrt{2} - 1) / (12258816 \sqrt{2} \tanh(x/2)^2 + 17336584 \tanh(x/2)^2 + 12258816 \sqrt{2} + 17336584) - 2167073 \sqrt{2} \log(\tanh(x/2) - \sqrt{2} + 1) \cdot \tanh(x/2)^2 / (12258816 \sqrt{2} \tanh(x/2)^2 + 17336584 \tanh(x/2)^2 + 12258816 \sqrt{2} + 17336584) - 3064704 \log(\tanh(x/2) - \sqrt{2} + 1) \cdot \tanh(x/2)^2 / (12258816 \sqrt{2} \tanh(x/2)^2 + 17336584 \tanh(x/2)^2 + 12258816 \sqrt{2} + 17336584) - 2167073 \sqrt{2} \log(\tanh(x/2) - \sqrt{2} + 1) / (12258816 \sqrt{2} \tanh(x/2)^2 + 17336584 \tanh(x/2)^2 + 12258816 \sqrt{2} + 17336584) - 3064704 \log(\tanh(x/2) - \sqrt{2} + 1) / (12258816 \sqrt{2} \tanh(x/2)^2 + 17336584 \tanh(x/2)^2 + 12258816 \sqrt{2} + 17336584) + 12258816 \sqrt{2} \tanh(x/2) / (12258816 \sqrt{2} \tanh(x/2)^2 + 17336584 \tanh(x/2)^2 + 12258816 \sqrt{2} + 17336584) + 17336584 \tanh(x/2) / (12258816 \sqrt{2} \tanh(x/2)^2 + 17336584 \tanh(x/2)^2 + 12258816 \sqrt{2} + 17336584)$$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(17) = 34$.

time = 0.00, size = 59, normalized size = 2.36

$$-8 \left(\frac{1}{8((e^x)^2 + 1)} + \frac{\ln \left(\frac{|2(e^x)^2 - 6 - 4\sqrt{2}|}{2(e^x)^2 - 6 + 4\sqrt{2}} \right)}{32\sqrt{2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)^4),x)

[Out] -1/8*sqrt(2)*log(abs(2*e^(2*x) - 4*sqrt(2) - 6)/abs(2*e^(2*x) + 4*sqrt(2) - 6)) - 1/(e^(2*x) + 1)

Mupad [B]

time = 0.40, size = 63, normalized size = 2.52

$$\frac{\sqrt{2} \ln \left(2e^{2x} + \frac{\sqrt{2}(12e^{2x}-4)}{8} \right)}{8} - \frac{\sqrt{2} \ln \left(2e^{2x} - \frac{\sqrt{2}(12e^{2x}-4)}{8} \right)}{8} - \frac{1}{e^{2x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(sinh(x)^4 - 1),x)

[Out] (2^(1/2)*log(2*exp(2*x) + (2^(1/2)*(12*exp(2*x) - 4))/8))/8 - (2^(1/2)*log(2*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/8))/8 - 1/(exp(2*x) + 1)

$$3.589 \quad \int \frac{\cosh^3(x) - \sinh^3(x)}{\cosh^3(x) + \sinh^3(x)} dx$$

Optimal. Leaf size=33

$$-\frac{4 \tan^{-1}\left(\frac{1-2 \tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{3(1 + \tanh(x))}$$

[Out] $-4/9*\arctan(1/3*(1-2*\tanh(x))*3^{(1/2)})*3^{(1/2)}-1/3/(1+\tanh(x))$

Rubi [A]

time = 0.09, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2099, 632, 210}

$$-\frac{1}{3(\tanh(x) + 1)} - \frac{4 \tan^{-1}\left(\frac{1-2 \tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cosh}[x]^3 - \text{Sinh}[x]^3)/(\text{Cosh}[x]^3 + \text{Sinh}[x]^3), x]$

[Out] $(-4*\text{ArcTan}[(1 - 2*\text{Tanh}[x])/ \text{Sqrt}[3]])/(3*\text{Sqrt}[3]) - 1/(3*(1 + \text{Tanh}[x]))$

Rule 210

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}\{a/b\} \ \&\& \ (\text{LtQ}\{a, 0\} \ || \ \text{LtQ}\{b, 0\})$

Rule 632

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 2099

$\text{Int}[(P_)^{(p_)}*(Q_)^{(q_)}, x_Symbol] \rightarrow \text{With}\{\{PP = \text{Factor}[P]\}, \text{Int}[\text{ExpandIntegrand}[PP^p*Q^q, x], x] /; \text{!SumQ}[\text{NonfreeFactors}[PP, x]] /; \text{FreeQ}\{q, x\} \ \&\& \ \text{PolyQ}[P, x] \ \&\& \ \text{PolyQ}[Q, x] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{NeQ}[P, x]$

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(x) - \sinh^3(x)}{\cosh^3(x) + \sinh^3(x)} dx &= \text{Subst} \left(\int \frac{1+x+x^2}{1+x+x^3+x^4} dx, x, \tanh(x) \right) \\
&= \text{Subst} \left(\int \left(\frac{1}{3(1+x)^2} + \frac{2}{3(1-x+x^2)} \right) dx, x, \tanh(x) \right) \\
&= -\frac{1}{3(1+\tanh(x))} + \frac{2}{3} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, \tanh(x) \right) \\
&= -\frac{1}{3(1+\tanh(x))} - \frac{4}{3} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2\tanh(x) \right) \\
&= -\frac{4 \tan^{-1} \left(\frac{1-2\tanh(x)}{\sqrt{3}} \right)}{3\sqrt{3}} - \frac{1}{3(1+\tanh(x))}
\end{aligned}$$

Mathematica [A]

time = 0.09, size = 37, normalized size = 1.12

$$\frac{1}{18} \left(8\sqrt{3} \tan^{-1} \left(\frac{-1+2\tanh(x)}{\sqrt{3}} \right) - 3 \cosh(2x) + 3 \sinh(2x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^3 - Sinh[x]^3)/(Cosh[x]^3 + Sinh[x]^3), x]
```

```
[Out] (8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]] - 3*Cosh[2*x] + 3*Sinh[2*x])/18
```

Mathics [A]

time = 3.87, size = 50, normalized size = 1.52

$$\frac{\left(3\text{Sinh}[x] + 4\sqrt{3} \text{ArcTan} \left[\frac{\sqrt{3}(-1+2\text{Tanh}[x])}{3} \right] \right) \text{Cosh}[x] + 4\sqrt{3} \text{ArcTan} \left[\frac{\sqrt{3}(-1+2\text{Tanh}[x])}{3} \right] \text{Sinh}[x] \right) E^{-x}}{9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(Cosh[x]^3 - Sinh[x]^3)/(Cosh[x]^3 + Sinh[x]^3), x]')
```

```
[Out] (3 Sinh[x] + 4 Sqrt[3] ArcTan[Sqrt[3] (-1 + 2 Tanh[x]) / 3] Cosh[x] + 4 Sqrt[3] ArcTan[Sqrt[3] (-1 + 2 Tanh[x]) / 3] Sinh[x]) E ^ (-x) / 9
```

Maple [C] Result contains complex when optimal does not.

time = 0.13, size = 78, normalized size = 2.36

method	result
--------	--------

risch	$-\frac{e^{-2x}}{6} + \frac{2i\sqrt{3} \ln(e^{2x+i\sqrt{3}})}{9} - \frac{2i\sqrt{3} \ln(e^{2x-i\sqrt{3}})}{9}$
default	$-\frac{2}{3(\tanh(\frac{x}{2})+1)^2} + \frac{2}{3(\tanh(\frac{x}{2})+1)} + \frac{2i\sqrt{3} \ln(\tanh^2(\frac{x}{2})+(-1-i\sqrt{3})\tanh(\frac{x}{2})+1)}{9} - \frac{2i\sqrt{3} \ln(\tanh^2(\frac{x}{2})+(-1+i\sqrt{3})\tanh(\frac{x}{2})+1)}{9}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x,method=_RETURNVERBOSE)`

[Out]
$$-2/3/(\tanh(1/2*x)+1)^2+2/3/(\tanh(1/2*x)+1)+2/9*I*3^{(1/2)}*\ln(\tanh(1/2*x)^2+(-1-I*3^{(1/2)})*\tanh(1/2*x)+1)-2/9*I*3^{(1/2)}*\ln(\tanh(1/2*x)^2+(-1+I*3^{(1/2)})*\tanh(1/2*x)+1)$$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(26) = 52.

time = 0.35, size = 70, normalized size = 2.12

$$\frac{4}{9}\sqrt{3} \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}}\sqrt{2} \left(2\sqrt{3}e^{-x} + 3^{\frac{1}{4}}\sqrt{2}\right)\right) - \frac{4}{9}\sqrt{3} \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}}\sqrt{2} \left(2\sqrt{3}e^{-x} - 3^{\frac{1}{4}}\sqrt{2}\right)\right) - \frac{1}{6}e^{-2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x, algorithm="maxima")`

[Out]
$$4/9*\sqrt{3}*\arctan(1/6*3^{(3/4)}*\sqrt{2}*(2*\sqrt{3}*e^{-x} + 3^{(1/4)}*\sqrt{2})) - 4/9*\sqrt{3}*\arctan(1/6*3^{(3/4)}*\sqrt{2}*(2*\sqrt{3}*e^{-x} - 3^{(1/4)}*\sqrt{2})) - 1/6*e^{-2*x}$$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(26) = 52.

time = 0.31, size = 74, normalized size = 2.24

$$\frac{8 \left(\sqrt{3} \cosh(x)^2 + 2\sqrt{3} \cosh(x) \sinh(x) + \sqrt{3} \sinh(x)^2 \right) \arctan\left(-\frac{\sqrt{3} \cosh(x) + \sqrt{3} \sinh(x)}{3(\cosh(x) - \sinh(x))} \right) + 3}{18 (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x, algorithm="fricas")`

[Out]
$$-1/18*(8*(\sqrt{3}*\cosh(x)^2 + 2*\sqrt{3}*\cosh(x)*\sinh(x) + \sqrt{3}*\sinh(x)^2)*\arctan(-1/3*(\sqrt{3}*\cosh(x) + \sqrt{3}*\sinh(x))/(\cosh(x) - \sinh(x))) + 3)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2)$$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(34) = 68$

time = 0.57, size = 102, normalized size = 3.09

$$\frac{4\sqrt{3} \sinh(x) \operatorname{atan}\left(\frac{2\sqrt{3} \sinh(x)}{3 \cosh(x)} - \frac{\sqrt{3}}{3}\right)}{9 \sinh(x) + 9 \cosh(x)} + \frac{3 \sinh(x)}{9 \sinh(x) + 9 \cosh(x)} + \frac{4\sqrt{3} \cosh(x) \operatorname{atan}\left(\frac{2\sqrt{3} \sinh(x)}{3 \cosh(x)} - \frac{\sqrt{3}}{3}\right)}{9 \sinh(x) + 9 \cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cosh(x)**3-sinh(x)**3)/(cosh(x)**3+sinh(x)**3),x)

[Out] 4*sqrt(3)*sinh(x)*atan(2*sqrt(3)*sinh(x)/(3*cosh(x)) - sqrt(3)/3)/(9*sinh(x) + 9*cosh(x)) + 3*sinh(x)/(9*sinh(x) + 9*cosh(x)) + 4*sqrt(3)*cosh(x)*atan(2*sqrt(3)*sinh(x)/(3*cosh(x)) - sqrt(3)/3)/(9*sinh(x) + 9*cosh(x))

Giac [A]

time = 0.00, size = 32, normalized size = 0.97

$$-\frac{1}{6(e^x)^2} + \frac{4 \arctan\left(\frac{(e^x)^2}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x)

[Out] 4/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) - 1/6/e^(2*x)

Mupad [B]

time = 0.37, size = 22, normalized size = 0.67

$$\frac{4\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3} e^{2x}}{3}\right)}{9} - \frac{e^{-2x}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(x)^3 - sinh(x)^3)/(cosh(x)^3 + sinh(x)^3),x)

[Out] (4*3^(1/2)*atan((3^(1/2)*exp(2*x))/3))/9 - exp(-2*x)/6

3.590 $\int \cosh(x) \cosh(2x) \cosh(3x) dx$

Optimal. Leaf size=30

$$\frac{x}{4} + \frac{1}{8} \sinh(2x) + \frac{1}{16} \sinh(4x) + \frac{1}{24} \sinh(6x)$$

[Out] 1/4*x+1/8*sinh(2*x)+1/16*sinh(4*x)+1/24*sinh(6*x)

Rubi [A]

time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4440, 2717}

$$\frac{x}{4} + \frac{1}{8} \sinh(2x) + \frac{1}{16} \sinh(4x) + \frac{1}{24} \sinh(6x)$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]*Cosh[2*x]*Cosh[3*x],x]

[Out] x/4 + Sinh[2*x]/8 + Sinh[4*x]/16 + Sinh[6*x]/24

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4440

Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.) + (f_.)*(x_)]^(r_.), x_Symbol] := Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H, sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]

Rubi steps

$$\begin{aligned} \int \cosh(x) \cosh(2x) \cosh(3x) dx &= \int \left(\frac{1}{4} + \frac{1}{4} \cosh(2x) + \frac{1}{4} \cosh(4x) + \frac{1}{4} \cosh(6x) \right) dx \\ &= \frac{x}{4} + \frac{1}{4} \int \cosh(2x) dx + \frac{1}{4} \int \cosh(4x) dx + \frac{1}{4} \int \cosh(6x) dx \\ &= \frac{x}{4} + \frac{1}{8} \sinh(2x) + \frac{1}{16} \sinh(4x) + \frac{1}{24} \sinh(6x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 30, normalized size = 1.00

$$\frac{x}{4} + \frac{1}{8} \sinh(2x) + \frac{1}{16} \sinh(4x) + \frac{1}{24} \sinh(6x)$$

Antiderivative was successfully verified.

`[In] Integrate[Cosh[x]*Cosh[2*x]*Cosh[3*x],x]``[Out] x/4 + Sinh[2*x]/8 + Sinh[4*x]/16 + Sinh[6*x]/24`**Mathics [B]** Leaf count is larger than twice the leaf count of optimal. 76 vs. 2(30) = 60.

time = 4.55, size = 66, normalized size = 2.20

$$-x \operatorname{Cosh}[2x] - \frac{3x \operatorname{Sinh}[x] \operatorname{Sinh}[3x]}{4} + \frac{x \operatorname{Cosh}[x] \operatorname{Cosh}[3x]}{4} + x \operatorname{Cosh}[2x] \operatorname{Cosh}[x]^2 - \frac{\operatorname{Cosh}[x] \operatorname{Sinh}[3x]}{4} - \frac{\operatorname{Cosh}[3x] \operatorname{Cosh}[x]^2 \operatorname{Sinh}[x]}{4} + \frac{7 \operatorname{Cosh}[x]^3 \operatorname{Sinh}[3x]}{12}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Cosh[x]*Cosh[2*x]*Cosh[3*x],x]')`

```
[Out] -x Cosh[2 x] - 3 x Sinh[x] Sinh[3 x] / 4 + x Cosh[x] Cosh[3 x] / 4 + x Cosh
[2 x] Cosh[x] ^ 2 - Cosh[x] Sinh[3 x] / 4 - Cosh[3 x] Cosh[x] ^ 2 Sinh[x] /
4 + 7 Cosh[x] ^ 3 Sinh[3 x] / 12
```

Maple [A]

time = 0.08, size = 23, normalized size = 0.77

method	result	size
default	$\frac{x}{4} + \frac{\sinh(2x)}{8} + \frac{\sinh(4x)}{16} + \frac{\sinh(6x)}{24}$	23
risch	$\frac{x}{4} + \frac{e^{6x}}{48} + \frac{e^{4x}}{32} + \frac{e^{2x}}{16} - \frac{e^{-2x}}{16} - \frac{e^{-4x}}{32} - \frac{e^{-6x}}{48}$	41

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cosh(x)*cosh(2*x)*cosh(3*x),x,method=_RETURNVERBOSE)``[Out] 1/4*x+1/8*sinh(2*x)+1/16*sinh(4*x)+1/24*sinh(6*x)`**Maxima [A]**

time = 0.24, size = 42, normalized size = 1.40

$$\frac{1}{96} (3e^{(-2x)} + 6e^{(-4x)} + 2)e^{(6x)} + \frac{1}{4}x - \frac{1}{16}e^{(-2x)} - \frac{1}{32}e^{(-4x)} - \frac{1}{48}e^{(-6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cosh(x)*cosh(2*x)*cosh(3*x),x, algorithm="maxima")`

```
[Out] 1/96*(3*e^(-2*x) + 6*e^(-4*x) + 2)*e^(6*x) + 1/4*x - 1/16*e^(-2*x) - 1/32*e
^(-4*x) - 1/48*e^(-6*x)
```


Fricas [A]

time = 0.31, size = 44, normalized size = 1.47

$$\frac{1}{4} \cosh(x) \sinh(x)^5 + \frac{1}{12} (10 \cosh(x)^3 + 3 \cosh(x)) \sinh(x)^3 + \frac{1}{4} (\cosh(x)^5 + \cosh(x)^3 + \cosh(x)) \sinh(x) + \frac{1}{4} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)*cosh(2*x)*cosh(3*x),x, algorithm="fricas")**[Out]** 1/4*cosh(x)*sinh(x)^5 + 1/12*(10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 1/4*(cosh(x)^5 + cosh(x)^3 + cosh(x))*sinh(x) + 1/4*x**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(22) = 44

time = 1.35, size = 114, normalized size = 3.80

$$\frac{x \sinh(x) \sinh(2x) \cosh(3x)}{4} - \frac{x \sinh(x) \sinh(3x) \cosh(2x)}{4} - \frac{x \sinh(2x) \sinh(3x) \cosh(x)}{4} + \frac{x \cosh(x) \cosh(2x) \cosh(3x)}{4} - \frac{\sinh(x) \sinh(2x) \sinh(3x)}{6} + \frac{\sinh(x) \cosh(2x) \cosh(3x)}{8} + \frac{5 \sinh(3x) \cosh(x) \cosh(2x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)*cosh(2*x)*cosh(3*x),x)**[Out]** x*sinh(x)*sinh(2*x)*cosh(3*x)/4 - x*sinh(x)*sinh(3*x)*cosh(2*x)/4 - x*sinh(2*x)*sinh(3*x)*cosh(x)/4 + x*cosh(x)*cosh(2*x)*cosh(3*x)/4 - sinh(x)*sinh(2*x)*sinh(3*x)/6 + sinh(x)*cosh(2*x)*cosh(3*x)/8 + 5*sinh(3*x)*cosh(x)*cosh(2*x)/24**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(22) = 44. time = 0.00, size = 66, normalized size = 2.20

$$\frac{\frac{\frac{4}{3}(e^x)^6 + 2(e^x)^4 + 4(e^x)^2}{8} + \frac{-22(e^x)^6 - 6(e^x)^4 - 3(e^x)^2 - 2}{12(e^x)^6} + 2x}{2 \cdot 2 \cdot 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)*cosh(2*x)*cosh(3*x),x)**[Out]** 1/48*e^(6*x) + 1/32*e^(4*x) + 1/16*e^(2*x) + 1/4*x - 1/96*(22*e^(6*x) + 6*e^(4*x) + 3*e^(2*x) + 2)/e^(6*x)**Mupad [B]**

time = 0.40, size = 40, normalized size = 1.33

$$\frac{x}{4} - \frac{e^{-2x}}{16} + \frac{e^{2x}}{16} - \frac{e^{-4x}}{32} + \frac{e^{4x}}{32} - \frac{e^{-6x}}{48} + \frac{e^{6x}}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(2*x)*cosh(3*x)*cosh(x),x)**[Out]** x/4 - exp(-2*x)/16 + exp(2*x)/16 - exp(-4*x)/32 + exp(4*x)/32 - exp(-6*x)/48 + exp(6*x)/48

3.591 $\int \cosh\left(\frac{3x}{2}\right) \sinh(x) \sinh\left(\frac{5x}{2}\right) dx$

Optimal. Leaf size=30

$$-\frac{x}{4} + \frac{1}{8} \sinh(2x) - \frac{1}{12} \sinh(3x) + \frac{1}{20} \sinh(5x)$$

[Out] $-1/4*x+1/8*\sinh(2*x)-1/12*\sinh(3*x)+1/20*\sinh(5*x)$

Rubi [A]

time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4440, 2717}

$$-\frac{x}{4} + \frac{1}{8} \sinh(2x) - \frac{1}{12} \sinh(3x) + \frac{1}{20} \sinh(5x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cosh}[(3*x)/2]*\text{Sinh}[x]*\text{Sinh}[(5*x)/2], x]$

[Out] $-1/4*x + \text{Sinh}[2*x]/8 - \text{Sinh}[3*x]/12 + \text{Sinh}[5*x]/20$

Rule 2717

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 4440

$\text{Int}[(F_)[(a_.) + (b_.)*(x_.)]^{(p_.)}*(G_)[(c_.) + (d_.)*(x_.)]^{(q_.)}*(H_)[(e_.) + (f_.)*(x_.)]^{(r_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[\text{ActivateTrig}[F[a + b*x]^{p*G[c + d*x]^{q*H[e + f*x]^{r}}], x], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H, sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]

Rubi steps

$$\begin{aligned} \int \cosh\left(\frac{3x}{2}\right) \sinh(x) \sinh\left(\frac{5x}{2}\right) dx &= - \int \left(\frac{1}{4} - \frac{1}{4} \cosh(2x) + \frac{1}{4} \cosh(3x) - \frac{1}{4} \cosh(5x) \right) dx \\ &= -\frac{x}{4} + \frac{1}{4} \int \cosh(2x) dx - \frac{1}{4} \int \cosh(3x) dx + \frac{1}{4} \int \cosh(5x) dx \\ &= -\frac{x}{4} + \frac{1}{8} \sinh(2x) - \frac{1}{12} \sinh(3x) + \frac{1}{20} \sinh(5x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 30, normalized size = 1.00

$$-\frac{x}{4} + \frac{1}{8} \sinh(2x) - \frac{1}{12} \sinh(3x) + \frac{1}{20} \sinh(5x)$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[(3*x)/2]*Sinh[x]*Sinh[(5*x)/2],x]

[Out] -1/4*x + Sinh[2*x]/8 - Sinh[3*x]/12 + Sinh[5*x]/20

Mathics [A]

time = 4.67, size = 40, normalized size = 1.33

$$\frac{x}{4} - \frac{3\text{Sinh}[x] \text{Sinh}\left[\frac{3x}{2}\right] \text{Sinh}\left[\frac{5x}{2}\right]}{20} - \frac{\text{Cosh}[x] \text{Cosh}\left[\frac{3x}{2}\right] \text{Sinh}\left[\frac{5x}{2}\right]}{15} + \frac{5\text{Cosh}\left[\frac{3x}{2}\right] \text{Cosh}\left[\frac{5x}{2}\right] \text{Sinh}[x]}{12}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Sinh[x]*Cosh[3*x/2]*Sinh[5*x/2],x]')

[Out] -x / 4 - 3 Sinh[x] Sinh[3 x / 2] Sinh[5 x / 2] / 20 - Cosh[x] Cosh[3 x / 2] Sinh[5 x / 2] / 15 + 5 Cosh[3 x / 2] Cosh[5 x / 2] Sinh[x] / 12

Maple [A]

time = 0.13, size = 23, normalized size = 0.77

method	result	size
default	$-\frac{x}{4} + \frac{\sinh(2x)}{8} - \frac{\sinh(3x)}{12} + \frac{\sinh(5x)}{20}$	23
risch	$-\frac{x}{4} + \frac{e^{5x}}{40} - \frac{e^{3x}}{24} + \frac{e^{2x}}{16} - \frac{e^{-2x}}{16} + \frac{e^{-3x}}{24} - \frac{e^{-5x}}{40}$	41

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x,method=_RETURNVERBOSE)

[Out] -1/4*x+1/8*sinh(2*x)-1/12*sinh(3*x)+1/20*sinh(5*x)

Maxima [A]

time = 0.24, size = 42, normalized size = 1.40

$$-\frac{1}{240} (10 e^{(-2x)} - 15 e^{(-3x)} - 6) e^{(5x)} - \frac{1}{4} x - \frac{1}{16} e^{(-2x)} + \frac{1}{24} e^{(-3x)} - \frac{1}{40} e^{(-5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x, algorithm="maxima")

[Out] -1/240*(10*e^(-2*x) - 15*e^(-3*x) - 6)*e^(5*x) - 1/4*x - 1/16*e^(-2*x) + 1/24*e^(-3*x) - 1/40*e^(-5*x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(22) = 44$.

time = 0.31, size = 111, normalized size = 3.70

$$6 \cosh\left(\frac{1}{2}x\right)^3 \sinh\left(\frac{1}{2}x\right)^7 + \frac{1}{2} \cosh\left(\frac{1}{2}x\right) \sinh\left(\frac{1}{2}x\right)^9 + \frac{1}{10} \left(126 \cosh\left(\frac{1}{2}x\right)^5 - 5 \cosh\left(\frac{1}{2}x\right)\right) \sinh\left(\frac{1}{2}x\right)^5 + \frac{1}{6} \left(36 \cosh\left(\frac{1}{2}x\right)^7 - 10 \cosh\left(\frac{1}{2}x\right)^3 + 3 \cosh\left(\frac{1}{2}x\right)\right) \sinh\left(\frac{1}{2}x\right)^3 + \frac{1}{2} \left(\cosh\left(\frac{1}{2}x\right)^9 - \cosh\left(\frac{1}{2}x\right)^5 + \cosh\left(\frac{1}{2}x\right)^3\right) \sinh\left(\frac{1}{2}x\right) - \frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x, algorithm="fricas")

[Out] $6*\cosh(1/2*x)^3*\sinh(1/2*x)^7 + 1/2*\cosh(1/2*x)*\sinh(1/2*x)^9 + 1/10*(126*\cosh(1/2*x)^5 - 5*\cosh(1/2*x))*\sinh(1/2*x)^5 + 1/6*(36*\cosh(1/2*x)^7 - 10*\cosh(1/2*x)^3 + 3*\cosh(1/2*x))*\sinh(1/2*x)^3 + 1/2*(\cosh(1/2*x)^9 - \cosh(1/2*x)^5 + \cosh(1/2*x)^3)*\sinh(1/2*x) - 1/4*x$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(22) = 44$

time = 1.29, size = 138, normalized size = 4.60

$$\frac{x \sinh(x) \sinh\left(\frac{x}{2}\right) \cosh\left(\frac{x}{2}\right)}{4} + \frac{x \sinh(x) \sinh\left(\frac{x}{2}\right) \cosh\left(\frac{x}{2}\right)}{4} + \frac{x \sinh\left(\frac{x}{2}\right) \sinh\left(\frac{x}{2}\right) \cosh(x)}{4} - \frac{x \cosh(x) \cosh\left(\frac{x}{2}\right) \cosh\left(\frac{x}{2}\right)}{4} - \frac{\sinh(x) \sinh\left(\frac{x}{2}\right) \sinh\left(\frac{x}{2}\right)}{12} + \frac{7 \sinh(x) \cosh\left(\frac{x}{2}\right) \cosh\left(\frac{x}{2}\right)}{20} - \frac{\sinh\left(\frac{x}{2}\right) \cosh(x) \cosh\left(\frac{x}{2}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x)

[Out] $-x*\sinh(x)*\sinh(3*x/2)*\cosh(5*x/2)/4 + x*\sinh(x)*\sinh(5*x/2)*\cosh(3*x/2)/4 + x*\sinh(3*x/2)*\sinh(5*x/2)*\cosh(x)/4 - x*\cosh(x)*\cosh(3*x/2)*\cosh(5*x/2)/4 - \sinh(x)*\sinh(3*x/2)*\sinh(5*x/2)/12 + 7*\sinh(x)*\cosh(3*x/2)*\cosh(5*x/2)/20 - \sinh(3*x/2)*\cosh(x)*\cosh(5*x/2)/15$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(22) = 44$.
time = 0.00, size = 88, normalized size = 2.93

$$2 \left(\frac{\frac{16}{5} \left(e^{\frac{x}{2}}\right)^{10} - \frac{16}{3} \left(e^{\frac{x}{2}}\right)^6 + 8 \left(e^{\frac{x}{2}}\right)^4}{32} + \frac{137 \left(e^{\frac{x}{2}}\right)^{10} - 15 \left(e^{\frac{x}{2}}\right)^6 + 10 \left(e^{\frac{x}{2}}\right)^4 - 6}{60 \left(e^{\frac{x}{2}}\right)^{10}} - x \right)$$

$2 \cdot 2 \cdot 2$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x)

[Out] $1/40*e^{(5*x)} - 1/24*e^{(3*x)} + 1/16*e^{(2*x)} - 1/4*x + 1/240*(137*e^{(5*x)} - 15*e^{(3*x)} + 10*e^{(2*x)} - 6)/e^{(5*x)}$

Mupad [B]

time = 0.42, size = 40, normalized size = 1.33

$$\frac{e^{2x}}{16} - \frac{e^{-2x}}{16} - \frac{x}{4} + \frac{e^{-3x}}{24} - \frac{e^{3x}}{24} - \frac{e^{-5x}}{40} + \frac{e^{5x}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((3*x)/2)*sinh((5*x)/2)*sinh(x),x)
```

```
[Out] exp(2*x)/16 - exp(-2*x)/16 - x/4 + exp(-3*x)/24 - exp(3*x)/24 - exp(-5*x)/40 + exp(5*x)/40
```

$$3.592 \quad \int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)} (\sinh^2(x)+\sinh(2x))} dx$$

Optimal. Leaf size=69

$$\sqrt{2} \tan^{-1} \left(\operatorname{sech}(x) \sqrt{\cosh(x) \sinh(x)} \right) + \frac{1}{6} \tan^{-1} \left(\frac{\sinh(x)}{\sqrt{\sinh(2x)}} \right) - \frac{1}{3} \sqrt{2} \tanh^{-1} \left(\operatorname{sech}(x) \sqrt{\cosh(x) \sinh(x)} \right)$$

[Out] 1/6*arctan(sinh(x)/sinh(2*x)^(1/2))+arctan(sech(x)*(cosh(x)*sinh(x))^(1/2))*2^(1/2)-1/3*arctanh(sech(x)*(cosh(x)*sinh(x))^(1/2))*2^(1/2)+cosh(x)/sinh(2*x)^(1/2)

Rubi [A]

time = 0.62, antiderivative size = 102, normalized size of antiderivative = 1.48, number of steps used = 8, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4475, 6857, 213, 209}

$$-\frac{2 \sinh(x) \tanh^{-1} \left(\sqrt{\tanh(x)} \right)}{3 \sqrt{\sinh(2x)} \sqrt{\tanh(x)}} + \frac{\cosh(x)}{\sqrt{\sinh(2x)}} + \frac{2 \sinh(x) \tanh^{-1} \left(\sqrt{\tanh(x)} \right)}{\sqrt{\sinh(2x)} \sqrt{\tanh(x)}} + \frac{\sinh(x) \tanh^{-1} \left(\frac{\sqrt{\tanh(x)}}{\sqrt{2}} \right)}{3 \sqrt{2} \sqrt{\sinh(2x)} \sqrt{\tanh(x)}}$$

Antiderivative was successfully verified.

[In] Int[(Cosh[x]*(-Cosh[2*x] + Tanh[x]))/(Sqrt[Sinh[2*x]]*(Sinh[x]^2 + Sinh[2*x])),x]

[Out] Cosh[x]/Sqrt[Sinh[2*x]] + (2*ArcTan[Sqrt[Tanh[x]]]*Sinh[x])/(Sqrt[Sinh[2*x]]*Sqrt[Tanh[x]]) + (ArcTan[Sqrt[Tanh[x]]/Sqrt[2]]*Sinh[x])/(3*Sqrt[2]*Sqrt[Sinh[2*x]]*Sqrt[Tanh[x]]) - (2*ArcTanh[Sqrt[Tanh[x]]]*Sinh[x])/(3*Sqrt[Sinh[2*x]]*Sqrt[Tanh[x]])

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 4475

Int[(u_)*((c_)*sin[v])^(m_), x_Symbol] := With[{w = FunctionOfTrig[u*(Sin[v/2]^(2*m))/(c*Tan[v/2])^m], x}], Dist[(c*SIN[v])^m*((c*TAN[v/2])^m/SIN[v/2

```

]^(2*m)), Int[u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x], x] /; !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] && !SumQ[u] && Inve
rseFunctionFreeQ[u, x]

```

Rule 6857

```

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(x)(-\cosh(2x) + \tanh(x))}{\sqrt{\sinh(2x)}(\sinh^2(x) + \sinh(2x))} dx &= \frac{\sinh(x) \int \frac{-\cosh(2x) + \tanh(x)}{(\sinh^2(x) + \sinh(2x))\sqrt{\tanh(x)}} dx}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}} \\
&= \frac{\sinh(x) \text{Subst}\left(\int \frac{-1+x-x^2-x^3}{x^{3/2}(2+x)(1-x^2)} dx, x, \tanh(x)\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}} \\
&= \frac{(2\sinh(x)) \text{Subst}\left(\int \frac{1-x^2+x^4+x^6}{x^2(2+x^2)(-1+x^4)} dx, x, \sqrt{\tanh(x)}\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}} \\
&= \frac{(2\sinh(x)) \text{Subst}\left(\int \left(-\frac{1}{2x^2} + \frac{1}{3(-1+x^2)} + \frac{1}{1+x^2} + \frac{1}{6(2+x^2)}\right) dx, x\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}} \\
&= \frac{\cosh(x)}{\sqrt{\sinh(2x)}} + \frac{\sinh(x) \text{Subst}\left(\int \frac{1}{2+x^2} dx, x, \sqrt{\tanh(x)}\right)}{3\sqrt{\sinh(2x)}\sqrt{\tanh(x)}} + \frac{(2\sinh(x)) \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{\tanh(x)}\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}} \\
&= \frac{\cosh(x)}{\sqrt{\sinh(2x)}} + \frac{2 \tan^{-1}\left(\sqrt{\tanh(x)}\right) \sinh(x)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}} + \frac{\tan^{-1}\left(\frac{\sqrt{\tanh(x)}}{\sqrt{2}}\right)}{3\sqrt{2}\sqrt{\sinh(2x)}}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(69) = 138.

time = 2.10, size = 160, normalized size = 2.32

$$\frac{\sqrt{\sinh(2x)} \left(6\sqrt{2} \tan^{-1}\left(\frac{\sqrt{\tanh\left(\frac{x}{2}\right)}}{\sqrt{\frac{\cosh(x)}{1+\cosh(x)}}}\right) + \tan^{-1}\left(\frac{\sqrt{\tanh\left(\frac{x}{2}\right)}}{\sqrt{1+\tanh^2\left(\frac{x}{2}\right)}}\right) - 2\sqrt{2} \tan^{-1}\left(\frac{\sqrt{\tanh\left(\frac{x}{2}\right)}}{\sqrt{\frac{\cosh(x)}{1+\cosh(x)}}}\right) + \frac{3\sqrt{\cosh(x)\text{sech}^2\left(\frac{x}{2}\right)}}{\sqrt{\tanh\left(\frac{x}{2}\right)}} \right)}{6(1+\cosh(x))\sqrt{\tanh\left(\frac{x}{2}\right)}\sqrt{1+\tanh^2\left(\frac{x}{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*(-Cosh[2*x] + Tanh[x]))/(Sqrt[Sinh[2*x]]*(Sinh[x]^2 + Sinh[2*x])),x]
```

```
[Out] (Sqrt[Sinh[2*x]]*(6*Sqrt[2]*ArcTan[Sqrt[Tanh[x/2]]/Sqrt[Cosh[x]/(1 + Cosh[x])]] + ArcTan[Sqrt[Tanh[x/2]]/Sqrt[1 + Tanh[x/2]^2]] - 2*Sqrt[2]*ArcTan[Sqrt[Tanh[x/2]]/Sqrt[Cosh[x]/(1 + Cosh[x])]] + (3*Sqrt[Cosh[x]*Sech[x/2]^2])/Sqrt[Tanh[x/2]]))/(6*(1 + Cosh[x])*Sqrt[Tanh[x/2]]*Sqrt[1 + Tanh[x/2]^2])
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Tanh[x] - Cosh[2*x])*(Cosh[x]/((Sinh[2*x] + Sinh[x]^2)*Sqrt[Sinh[2*x]]))],x')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.32, size = 987, normalized size = 14.30

method	result	size
default	Expression too large to display	987

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*((tanh(1/2*x)^2+1)*tanh(1/2*x)/(tanh(1/2*x)^2-1)^2)^(1/2)*(tanh(1/2*x)^2-1)*(3^(1/2)*2^(1/2)*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)*(-I*(tanh(1/2*x)+I))^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)*EllipticPi((-I*(tanh(1/2*x)+I))^(1/2),1/2-I+1/2*I*3^(1/2),1/2*2^(1/2))-3^(1/2)*2^(1/2)*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)*(-I*(tanh(1/2*x)+I))^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)*EllipticPi((-I*(tanh(1/2*x)+I))^(1/2),1/2-I-1/2*I*3^(1/2),1/2*2^(1/2))+I*(-I*(tanh(1/2*x)+I))^(1/2)*2^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)*EllipticPi((-I*(tanh(1/2*x)+I))^(1/2),1/2-I+1/2*I*3^(1/2),1/2*2^(1/2))*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)-8*I*(-I*(tanh(1/2*x)+I))^(1/2)*2^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)*EllipticPi((-I*(tanh(1/2*x)+I))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)+I*(-I*(tanh(1/2*x)+I))^(1/2)*2^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)*EllipticPi((-I*(tanh(1/2*x)+I))^(1/2),1/2-I-1/2*I*3^(1/2),1/2*2^(1/2))*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)+24*I*(-I*(tanh(1/2*x)+I))^(1/2)*2^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)*EllipticPi((-I*(tanh(1/2*x)+I))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)-18*I*(-I*(tanh(1/2*x)
```


$$\begin{aligned} & x)+I)^{(1/2)} * 2^{(1/2)} * (-I * (-\tanh(1/2*x)+I))^{(1/2)} * (I * \tanh(1/2*x))^{(1/2)} * \text{EllipticF} \\ & ((-I * (\tanh(1/2*x)+I))^{(1/2)}, 1/2 * 2^{(1/2)}) * ((\tanh(1/2*x)^2+1) * \tanh(1/2*x))^{(1/2)} - 2 * 2^{(1/2)} * \\ & ((\tanh(1/2*x)^2+1) * \tanh(1/2*x))^{(1/2)} * (-I * (\tanh(1/2*x)+I))^{(1/2)} * (-I * (-\tanh(1/2*x)+I))^{(1/2)} * \\ & (I * \tanh(1/2*x))^{(1/2)} * \text{EllipticPi}((-I * (\tanh(1/2*x)+I))^{(1/2)}, 1/2 - I + 1/2 * I * 3^{(1/2)}, 1/2 * 2^{(1/2)}) - 8 * \\ & (-I * (\tanh(1/2*x)+I))^{(1/2)} * 2^{(1/2)} * (-I * (-\tanh(1/2*x)+I))^{(1/2)} * (I * \tanh(1/2*x))^{(1/2)} * \text{EllipticPi} \\ & ((-I * (\tanh(1/2*x)+I))^{(1/2)}, 1/2 + 1/2 * I, 1/2 * 2^{(1/2)}) * ((\tanh(1/2*x)^2+1) * \tanh(1/2*x))^{(1/2)} - 2 * 2^{(1/2)} * \\ & ((\tanh(1/2*x)^2+1) * \tanh(1/2*x))^{(1/2)} * (-I * (\tanh(1/2*x)+I))^{(1/2)} * (-I * (-\tanh(1/2*x)+I))^{(1/2)} * \\ & (I * \tanh(1/2*x))^{(1/2)} * \text{EllipticPi}((-I * (\tanh(1/2*x)+I))^{(1/2)}, 1/2 - I - 1/2 * I * 3^{(1/2)}, 1/2 * 2^{(1/2)}) - 24 * \\ & (-I * (\tanh(1/2*x)+I))^{(1/2)} * 2^{(1/2)} * (-I * (-\tanh(1/2*x)+I))^{(1/2)} * (I * \tanh(1/2*x))^{(1/2)} * \text{EllipticPi} \\ & ((-I * (\tanh(1/2*x)+I))^{(1/2)}, 1/2 - 1/2 * I, 1/2 * 2^{(1/2)}) * ((\tanh(1/2*x)^2+1) * \tanh(1/2*x))^{(1/2)} + 12 * \\ & ((\tanh(1/2*x)^3 + \tanh(1/2*x))^{(1/2)} * \tanh(1/2*x)^2 + 12 * ((\tanh(1/2*x)^3 + \tanh(1/2*x))^{(1/2)}) / (\tanh(1/2*x)^2 + 1) / \tanh(1/2*x) / (\tanh(1/2*x)^3 + \tanh(1/2*x))^{(1/2)} \end{aligned}$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2),x, algorithm="maxima")

[Out] -integrate((cosh(2*x) - tanh(x))*cosh(x)/((sinh(x)^2 + sinh(2*x))*sqrt(sinh(2*x))), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(53) = 106.

time = 0.33, size = 376, normalized size = 5.45

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/12 * ((\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh(x)^2 - 1) * \arctan(1/2 * (\sqrt{2}) * \\ & \cosh(x)^2 + 2 * \sqrt{2} * \cosh(x) * \sinh(x) + \sqrt{2} * \sinh(x)^2 + 3 * \sqrt{2}) * \sqrt{(\cosh(x) * \sinh(x) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)) / (\cosh(x)^4 + \\ & 4 * \cosh(x)^3 * \sinh(x) + 6 * \cosh(x)^2 * \sinh(x)^2 + 4 * \cosh(x) * \sinh(x)^3 + \sinh(x)^4 - 1)) + 6 * (\sqrt{2} * \cosh(x)^2 + 2 * \sqrt{2} * \cosh(x) * \sinh(x) + \sqrt{2} * \sinh(x)^2 - \sqrt{2}) * \arctan(2 * \sqrt{2} * \cosh(x) * \sinh(x) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)) / (\cosh(x)^4 + 4 * \cosh(x)^3 * \sinh(x) + 6 * \cosh(x)^2 * \sinh(x)^2 + 4 * \cosh(x) * \sinh(x)^3 + \sinh(x)^4 - 1)) - (\sqrt{2} * \cosh(x)^2 + 2 * \sqrt{2} * \cosh(x) * \sinh(x) + \sqrt{2} * \sinh(x)^2 - \sqrt{2}) * \arctan(1/2 * (\sqrt{2}) * \cosh(x)^2 + 2 * \sqrt{2} * \cosh(x) * \sinh(x) + \sqrt{2} * \sinh(x)^2 + 3 * \sqrt{2}) * \sqrt{(\cosh(x) * \sinh(x) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)) / (\cosh(x)^4 + 4 * \cosh(x)^3 * \sinh(x) + 6 * \cosh(x)^2 * \sinh(x)^2 + 4 * \cosh(x) * \sinh(x)^3 + \sinh(x)^4 - 1)) \end{aligned}$$

$(x) \cdot \sinh(x) + \sqrt{2} \cdot \sinh(x)^2 - \sqrt{2} \cdot \log(2 \cdot \cosh(x)^4 + 8 \cdot \cosh(x)^3 \cdot \sinh(x) + 12 \cdot \cosh(x)^2 \cdot \sinh(x)^2 + 8 \cdot \cosh(x) \cdot \sinh(x)^3 + 2 \cdot \sinh(x)^4 - 4 \cdot (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2) \cdot \sqrt{\cosh(x) \cdot \sinh(x) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)}) - 1) - 12 \cdot \sqrt{2} \cdot \sqrt{\cosh(x) \cdot \sinh(x) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)}) / (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2 - 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cosh(x) \cosh(2x)}{\sinh^2(x) \sqrt{\sinh(2x)} + \sinh^{\frac{3}{2}}(2x)} dx - \int \left(-\frac{\cosh(x) \tanh(x)}{\sinh^2(x) \sqrt{\sinh(2x)} + \sinh^{\frac{3}{2}}(2x)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)**2+sinh(2*x))/sinh(2*x)**(1/2),x)

[Out] -Integral(cosh(x)*cosh(2*x)/(sinh(x)**2*sqrt(sinh(2*x)) + sinh(2*x)**(3/2)), x) - Integral(-cosh(x)*tanh(x)/(sinh(x)**2*sqrt(sinh(2*x)) + sinh(2*x)**(3/2)), x)

Giac [B] N/A

time = 0.03, size = 160, normalized size = 2.32

$$\frac{\frac{\pi\sqrt{2} - 12\sqrt{2} \arctan i + (-6+6i)\sqrt{2} - 2\arctan\left(\frac{1+2i}{(1-i)\sqrt{2}}\right)}{3\sqrt{2}} + 2 \left(\frac{\arctan\left(\frac{3\left(-e^{x^2} + \sqrt{(e^x)^4 - 1}\right) - 1}{2\sqrt{2}}\right)}{3\sqrt{2}} + 2\arctan\left(-e^{x^2} + \sqrt{(e^x)^4 - 1}\right) + \frac{\ln\left(\frac{(e^x)^2 - \sqrt{(e^x)^4 - 1}}{3}\right)}{-e^{x^2} + \sqrt{(e^x)^4 - 1}} \right)}{2\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2),x)

[Out] 1/2/sqrt(2)*((i*pi*sqrt(2)-12*sqrt(2)*atan(i)+(-6+6*i)*sqrt(2)-2*atan((1+2*i)/(1-i)/sqrt(2)))/3/sqrt(2)+2*(1/3*atan(1/2*(3*(-exp(x)^2+sqrt(exp(x)^4-1))-1)/sqrt(2))/sqrt(2)+2*atan(-exp(x)^2+sqrt(exp(x)^4-1))+1/3*ln(exp(x)^2-sqrt(exp(x)^4-1))+2/(-exp(x)^2+sqrt(exp(x)^4-1)+1)))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\cosh(x) (\cosh(2x) - \tanh(x))}{\sqrt{\sinh(2x)} (\sinh(x)^2 + \sinh(2x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cosh(x)*(cosh(2*x) - tanh(x)))/(sinh(2*x)^(1/2)*(sinh(2*x) + sinh(x)^2)),x)
```

```
[Out] -int((cosh(x)*(cosh(2*x) - tanh(x)))/(sinh(2*x)^(1/2)*(sinh(2*x) + sinh(x)^2)), x)
```

$$3.593 \quad \int \frac{\sinh(x)}{(-9+4 \cosh^2(x))^{5/2}} dx$$

Optimal. Leaf size=37

$$-\frac{\cosh(x)}{27(-9+4 \cosh^2(x))^{3/2}} + \frac{2 \cosh(x)}{243 \sqrt{-9+4 \cosh^2(x)}}$$

[Out] $-1/27*\cosh(x)/(-9+4*\cosh(x)^2)^{(3/2)}+2/243*\cosh(x)/(-9+4*\cosh(x)^2)^{(1/2)}$

Rubi [A]

time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3269, 198, 197}

$$\frac{2 \cosh(x)}{243 \sqrt{4 \cosh^2(x) - 9}} - \frac{\cosh(x)}{27 (4 \cosh^2(x) - 9)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]/(-9 + 4*Cosh[x]^2)^(5/2),x]

[Out] $-1/27*\cosh[x]/(-9 + 4*\cosh[x]^2)^{(3/2)} + (2*\cosh[x])/((243*\sqrt{-9 + 4*\cosh[x]^2}))$

Rule 197

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 198

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

Rule 3269

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(x)}{(-9 + 4 \cosh^2(x))^{5/2}} dx &= \text{Subst} \left(\int \frac{1}{(-9 + 4x^2)^{5/2}} dx, x, \cosh(x) \right) \\
&= -\frac{\cosh(x)}{27(-9 + 4 \cosh^2(x))^{3/2}} - \frac{2}{27} \text{Subst} \left(\int \frac{1}{(-9 + 4x^2)^{3/2}} dx, x, \cosh(x) \right) \\
&= -\frac{\cosh(x)}{27(-9 + 4 \cosh^2(x))^{3/2}} + \frac{2 \cosh(x)}{243 \sqrt{-9 + 4 \cosh^2(x)}}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 26, normalized size = 0.70

$$\frac{\cosh(x)(-23 + 4 \cosh(2x))}{243(-7 + 2 \cosh(2x))^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[Sinh[x]/(-9 + 4*Cosh[x]^2)^(5/2), x]``[Out] (Cosh[x]*(-23 + 4*Cosh[2*x]))/(243*(-7 + 2*Cosh[2*x])^(3/2))`**Mathics [F(-1)]**

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Sinh[x]/(4*Cosh[x]^2 - 9)^(5/2), x]')``[Out] Timed out`**Maple [A]**

time = 0.04, size = 30, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{\cosh(x)}{27(-9+4(\cosh^2(x)))^{3/2}} + \frac{2 \cosh(x)}{243 \sqrt{-9 + 4 (\cosh^2(x))}}$	30
default	$-\frac{\cosh(x)}{27(-9+4(\cosh^2(x)))^{3/2}} + \frac{2 \cosh(x)}{243 \sqrt{-9 + 4 (\cosh^2(x))}}$	30

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(sinh(x)/(-9+4*cosh(x)^2)^(5/2), x, method=_RETURNVERBOSE)`

[Out] $-1/27*\cosh(x)/(-9+4*\cosh(x)^2)^{(3/2)}+2/243*\cosh(x)/(-9+4*\cosh(x)^2)^{(1/2)}$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(29) = 58$.

time = 0.26, size = 125, normalized size = 3.38

$$-\frac{1855e^{-2x} - 8485e^{-4x} + 5285e^{-6x} - 980e^{-8x} + 56e^{-10x} - 106}{12150(3e^{-x} + e^{-2x} + 1)^{\frac{5}{2}}(-3e^{-x} + e^{-2x} + 1)^{\frac{3}{2}}} + \frac{980e^{-2x} - 5285e^{-4x} + 8485e^{-6x} - 1855e^{-8x} + 106e^{-10x} - 56}{12150(3e^{-x} + e^{-2x} + 1)^{\frac{5}{2}}(-3e^{-x} + e^{-2x} + 1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x, algorithm="maxima")`

[Out] $-1/12150*(1855*e^{(-2*x)} - 8485*e^{(-4*x)} + 5285*e^{(-6*x)} - 980*e^{(-8*x)} + 56*e^{(-10*x)} - 106)/((3*e^{-x} + e^{-2x} + 1)^{(5/2)}*(-3*e^{-x} + e^{-2x} + 1)^{(5/2)}) + 1/12150*(980*e^{(-2*x)} - 5285*e^{(-4*x)} + 8485*e^{(-6*x)} - 1855*e^{(-8*x)} + 106*e^{(-10*x)} - 56)/((3*e^{-x} + e^{-2x} + 1)^{(5/2)}*(-3*e^{-x} + e^{-2x} + 1)^{(5/2)})$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(29) = 58$.

time = 0.33, size = 474, normalized size = 12.81

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x, algorithm="fricas")`

[Out] $1/486*(2*\cosh(x)^8 + 16*\cosh(x)*\sinh(x)^7 + 2*\sinh(x)^8 + 28*(2*\cosh(x)^2 - 1)*\sinh(x)^6 - 28*\cosh(x)^6 + 56*(2*\cosh(x)^3 - 3*\cosh(x))*\sinh(x)^5 + 2*(70*\cosh(x)^4 - 210*\cosh(x)^2 + 51)*\sinh(x)^4 + 102*\cosh(x)^4 + 8*(14*\cosh(x)^5 - 70*\cosh(x)^3 + 51*\cosh(x))*\sinh(x)^3 + 4*(14*\cosh(x)^6 - 105*\cosh(x)^4 + 153*\cosh(x)^2 - 7)*\sinh(x)^2 - 28*\cosh(x)^2 + 8*(2*\cosh(x)^7 - 21*\cosh(x)^5 + 51*\cosh(x)^3 - 7*\cosh(x))*\sinh(x) + (2*\cosh(x)^6 + 12*\cosh(x)*\sinh(x)^5 + 2*\sinh(x)^6 + 3*(10*\cosh(x)^2 - 7)*\sinh(x)^4 - 21*\cosh(x)^4 + 4*(10*\cosh(x)^3 - 21*\cosh(x))*\sinh(x)^3 + 3*(10*\cosh(x)^4 - 42*\cosh(x)^2 - 7)*\sinh(x)^2 - 21*\cosh(x)^2 + 6*(2*\cosh(x)^5 - 14*\cosh(x)^3 - 7*\cosh(x))*\sinh(x) + 2)*\sqrt{(2*\cosh(x)^2 + 2*\sinh(x)^2 - 7)/(cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)} + 2)/(cosh(x)^8 + 8*\cosh(x)*\sinh(x)^7 + \sinh(x)^8 + 14*(2*\cosh(x)^2 - 1)*\sinh(x)^6 - 14*\cosh(x)^6 + 28*(2*\cosh(x)^3 - 3*\cosh(x))*\sinh(x)^5 + (70*\cosh(x)^4 - 210*\cosh(x)^2 + 51)*\sinh(x)^4 + 51*\cosh(x)^4 + 4*(14*\cosh(x)^5 - 70*\cosh(x)^3 + 51*\cosh(x))*\sinh(x)^3 + 2*(14*\cosh(x)^6 - 105*\cosh(x)^4 + 153*\cosh(x)^2 - 7)*\sinh(x)^2 - 14*\cosh(x)^2 + 4*(2*\cosh(x)^7 - 21*\cosh(x)^5 + 51*\cosh(x)^3 - 7*\cosh(x))*\sinh(x) + 1)$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x)/(-9+4*cosh(x)**2)**(5/2),x)`

[Out] Timed out

Giac [A]

time = 0.02, size = 67, normalized size = 1.81

$$\frac{2 \left(\left(\left(\frac{(e^x)^2}{243} - \frac{7}{162} \right) (e^x)^2 - \frac{7}{162} \right) (e^x)^2 + \frac{1}{243} \right) \sqrt{(e^x)^4 - 7(e^x)^2 + 1}}{2 \left((e^x)^4 - 7(e^x)^2 + 1 \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x)`

[Out] $\frac{1}{486} \frac{((2e^{2x} - 21)e^{2x} - 21)e^{2x} + 2}{(e^{4x} - 7e^{2x} + 1)^{3/2}}$

Mupad [B]

time = 0.14, size = 57, normalized size = 1.54

$$\frac{e^x \sqrt{4 \left(\frac{e^{-x}}{2} + \frac{e^x}{2} \right)^2 - 9} (21e^{2x} + 21e^{4x} - 2e^{6x} - 2)}{486 (e^{4x} - 7e^{2x} + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(x)/(4*cosh(x)^2 - 9)^(5/2),x)`

[Out] $-\frac{\exp(x) \cdot (4 \cdot (\exp(-x)/2 + \exp(x)/2)^2 - 9)^{1/2} \cdot (21 \cdot \exp(2x) + 21 \cdot \exp(4x) - 2 \cdot \exp(6x) - 2)}{486 \cdot (\exp(4x) - 7 \cdot \exp(2x) + 1)^2}$

$$3.594 \quad \int \frac{\sinh^2(x) \sinh(2x)}{(1-\sinh^2(x))^{3/2}} dx$$

Optimal. Leaf size=29

$$\frac{2}{\sqrt{1-\sinh^2(x)}} + 2\sqrt{1-\sinh^2(x)}$$

[Out] 2/(1-sinh(x)^2)^(1/2)+2*(1-sinh(x)^2)^(1/2)

Rubi [A]

time = 0.07, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {12, 272, 45}

$$2\sqrt{1-\sinh^2(x)} + \frac{2}{\sqrt{1-\sinh^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[(Sinh[x]^2*Sinh[2*x])/(1 - Sinh[x]^2)^(3/2),x]

[Out] 2/Sqrt[1 - Sinh[x]^2] + 2*Sqrt[1 - Sinh[x]^2]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(x) \sinh(2x)}{(1 - \sinh^2(x))^{3/2}} dx &= i\text{Subst} \left(\int -\frac{2ix^3}{(1 - x^2)^{3/2}} dx, x, \sinh(x) \right) \\
&= 2\text{Subst} \left(\int \frac{x^3}{(1 - x^2)^{3/2}} dx, x, \sinh(x) \right) \\
&= \text{Subst} \left(\int \frac{x}{(1 - x)^{3/2}} dx, x, \sinh^2(x) \right) \\
&= \text{Subst} \left(\int \left(\frac{1}{(1 - x)^{3/2}} - \frac{1}{\sqrt{1 - x}} \right) dx, x, \sinh^2(x) \right) \\
&= \frac{2}{\sqrt{1 - \sinh^2(x)}} + 2\sqrt{1 - \sinh^2(x)}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 21, normalized size = 0.72

$$\frac{5 - \cosh(2x)}{\sqrt{1 - \sinh^2(x)}}$$

Antiderivative was successfully verified.

`[In] Integrate[(Sinh[x]^2*Sinh[2*x])/(1 - Sinh[x]^2)^(3/2), x]``[Out] (5 - Cosh[2*x])/Sqrt[1 - Sinh[x]^2]`**Mathics [F(-1)]**

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Sinh[x]^2*(Sinh[2*x]/(1 - Sinh[x]^2)^(3/2)), x]')``[Out] Timed out`**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 0.11, size = 28, normalized size = 0.97

method	result	size
--------	--------	------

default	'int/indef0' $\left(-\frac{2(\sinh^3(x))}{(\sinh^2(x)-1)\sqrt{1-(\sinh^2(x))}}, \sinh(x) \right)$	28
---------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x,method=_RETURNVERBOSE)`

[Out] 'int/indef0' $(-2*\sinh(x)^3/(\sinh(x)^2-1)/(1-\sinh(x)^2)^(1/2),\sinh(x))$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(25) = 50$.

time = 0.38, size = 177, normalized size = 6.10

$$\frac{16e^{-2x}}{(2e^{-x}+e^{-2x}-1)^3(2e^{-x}-e^{-2x}+1)^3} + \frac{62e^{-3x}}{(2e^{-x}+e^{-2x}-1)^3(2e^{-x}-e^{-2x}+1)^3} - \frac{16e^{-5x}}{(2e^{-x}+e^{-2x}-1)^3(2e^{-x}-e^{-2x}+1)^3} + \frac{e^{-7x}}{(2e^{-x}+e^{-2x}-1)^3(2e^{-x}-e^{-2x}+1)^3} + \frac{e^x}{(2e^{-x}+e^{-2x}-1)^3(2e^{-x}-e^{-2x}+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x, algorithm="maxima")`

[Out] $-16e^{-x}/((2e^{-x} + e^{-2x} - 1)^{3/2} * (2e^{-x} - e^{-2x} + 1)^{3/2}) + 62e^{-3x}/((2e^{-x} + e^{-2x} - 1)^{3/2} * (2e^{-x} - e^{-2x} + 1)^{3/2}) - 16e^{-5x}/((2e^{-x} + e^{-2x} - 1)^{3/2} * (2e^{-x} - e^{-2x} + 1)^{3/2}) + e^{-7x}/((2e^{-x} + e^{-2x} - 1)^{3/2} * (2e^{-x} - e^{-2x} + 1)^{3/2}) + e^{-x}/((2e^{-x} + e^{-2x} - 1)^{3/2} * (2e^{-x} - e^{-2x} + 1)^{3/2})$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(25) = 50$.

time = 0.32, size = 161, normalized size = 5.55

$$\frac{\sqrt{2} (\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 - 5) \sinh(x)^2 - 10 \cosh(x)^2 + 4(\cosh(x)^3 - 5 \cosh(x)) \sinh(x) + 1) \sqrt{-\frac{\cosh(x)^2 + \sinh(x)^2 - 3}{\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2}}}{\cosh(x)^5 + 5 \cosh(x) \sinh(x)^4 + \sinh(x)^5 + 2(5 \cosh(x)^2 - 3) \sinh(x)^3 - 6 \cosh(x)^3 + 2(5 \cosh(x)^3 - 9 \cosh(x)) \sinh(x)^2 + (5 \cosh(x)^4 - 18 \cosh(x)^2 + 1) \sinh(x) + \cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x, algorithm="fricas")`

[Out] $\sqrt{2} * (\cosh(x)^4 + 4 * \cosh(x) * \sinh(x)^3 + \sinh(x)^4 + 2 * (3 * \cosh(x)^2 - 5) * \sinh(x)^2 - 10 * \cosh(x)^2 + 4 * (\cosh(x)^3 - 5 * \cosh(x)) * \sinh(x) + 1) * \sqrt{-(\cosh(x)^2 + \sinh(x)^2 - 3) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)} / (\cosh(x)^5 + 5 * \cosh(x) * \sinh(x)^4 + \sinh(x)^5 + 2 * (5 * \cosh(x)^2 - 3) * \sinh(x)^3 - 6 * \cosh(x)^3 + 2 * (5 * \cosh(x)^3 - 9 * \cosh(x)) * \sinh(x)^2 + (5 * \cosh(x)^4 - 18 * \cosh(x)^2 + 1) * \sinh(x) + \cosh(x))$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(x) \sinh(2x)}{(-(\sinh(x) - 1)(\sinh(x) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x)**2*sinh(2*x)/(1-sinh(x)**2)**(3/2), x)`

[Out] `Integral(sinh(x)**2*sinh(2*x)/(-(sinh(x) - 1)*(sinh(x) + 1))**(3/2), x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2), x)`

[Out] Could not integrate

Mupad [B]

time = 0.49, size = 47, normalized size = 1.62

$$\frac{2 \sqrt{1 - \left(\frac{e^{-x}}{2} - \frac{e^x}{2}\right)^2} (e^{4x} - 10e^{2x} + 1)}{e^{4x} - 6e^{2x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((sinh(2*x)*sinh(x)^2)/(1 - sinh(x)^2)^(3/2), x)`

[Out] `(2*(1 - (exp(-x)/2 - exp(x)/2)^2)^(1/2)*(exp(4*x) - 10*exp(2*x) + 1))/(exp(4*x) - 6*exp(2*x) + 1)`

$$3.595 \quad \int \frac{\cosh(x)}{\sqrt{\cosh(2x)}} dx$$

Optimal. Leaf size=15

$$\frac{\sinh^{-1}\left(\sqrt{2} \sinh(x)\right)}{\sqrt{2}}$$

[Out] 1/2*arcsinh(sinh(x)*2^(1/2))*2^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4441, 221}

$$\frac{\sinh^{-1}\left(\sqrt{2} \sinh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]/Sqrt[Cosh[2*x]],x]

[Out] ArcSinh[Sqrt[2]*Sinh[x]]/Sqrt[2]

Rule 221

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 4441

Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned} \int \frac{\cosh(x)}{\sqrt{\cosh(2x)}} dx &= \text{Subst} \left(\int \frac{1}{\sqrt{1+2x^2}} dx, x, \sinh(x) \right) \\ &= \frac{\sinh^{-1}\left(\sqrt{2} \sinh(x)\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sinh^{-1}\left(\sqrt{2} \sinh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

`[In] Integrate[Cosh[x]/Sqrt[Cosh[2*x]],x]``[Out] ArcSinh[Sqrt[2]*Sinh[x]]/Sqrt[2]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Cosh[x]/Sqrt[Cosh[2*x]],x]')``[Out] cought exception: maximum recursion depth exceeded in comparison`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(12) = 24.

time = 0.17, size = 63, normalized size = 4.20

method	result	size
default	$\frac{\sqrt{(2(\cosh^2(x)) - 1)(\sinh^2(x))} \ln\left(\sqrt{2}(\sinh^2(x)) + \sqrt{2(\sinh^4(x)) + \sinh^2(x) + \frac{\sqrt{2}}{4}}\right) \sqrt{2}}{4 \sinh(x) \sqrt{2(\cosh^2(x)) - 1}}$	63

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cosh(x)/cosh(2*x)^(1/2),x,method=_RETURNVERBOSE)``[Out] 1/4*((2*cosh(x)^2-1)*sinh(x)^2)^(1/2)*ln(2^(1/2)*sinh(x)^2+(2*sinh(x)^4+sinh(x)^2)^(1/2)+1/4*2^(1/2))*2^(1/2)/sinh(x)/(2*cosh(x)^2-1)^(1/2)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cosh(x)/cosh(2*x)^(1/2),x, algorithm="maxima")`

[Out] integrate(cosh(x)/sqrt(cosh(2*x)), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(12) = 24$.

time = 0.39, size = 482, normalized size = 32.13

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)/cosh(2*x)^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{8}\sqrt{2}\log(-(\cosh(x))^8 + 8\cosh(x)\sinh(x)^7 + \sinh(x)^8 + (28\cosh(x)^2 - 3)\sinh(x)^6 - 3\cosh(x)^6 + 2(28\cosh(x)^3 - 9\cosh(x))\sinh(x)^5 + 5(14\cosh(x)^4 - 9\cosh(x)^2 + 1)\sinh(x)^4 + 5\cosh(x)^4 + 4(14\cosh(x)^5 - 15\cosh(x)^3 + 5\cosh(x))\sinh(x)^3 + (28\cosh(x)^6 - 45\cosh(x)^4 + 30\cosh(x)^2 - 4)\sinh(x)^2 + \sqrt{2}(\cosh(x)^6 + 6\cosh(x)\sinh(x)^5 + \sinh(x)^6 + 3(5\cosh(x)^2 - 1)\sinh(x)^4 - 3\cosh(x)^4 + 4(5\cosh(x)^3 - 3\cosh(x))\sinh(x)^3 + (15\cosh(x)^4 - 18\cosh(x)^2 + 4)\sinh(x)^2 + 4\cosh(x)^2 + 2(3\cosh(x)^5 - 6\cosh(x)^3 + 4\cosh(x))\sinh(x) - 4)\sqrt{(\cosh(x)^2 + \sinh(x)^2)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2))} - 4\cosh(x)^2 + 2(4\cosh(x)^7 - 9\cosh(x)^5 + 10\cosh(x)^3 - 4\cosh(x))\sinh(x) + 4)/(\cosh(x)^6 + 6\cosh(x)^5\sinh(x) + 15\cosh(x)^4\sinh(x)^2 + 20\cosh(x)^3\sinh(x)^3 + 15\cosh(x)^2\sinh(x)^4 + 6\cosh(x)\sinh(x)^5 + \sinh(x)^6)) + \frac{1}{8}\sqrt{2}\log((\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + (6\cosh(x)^2 + 1)\sinh(x)^2 + \sqrt{2}(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1)\sqrt{(\cosh(x)^2 + \sinh(x)^2)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2))} + \cosh(x)^2 + 2(2\cosh(x)^3 + \cosh(x))\sinh(x) + 1)/(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2))$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)/cosh(2*x)**(1/2),x)

[Out] Integral(cosh(x)/sqrt(cosh(2*x)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(12) = 24$.
time = 0.01, size = 75, normalized size = 5.00

$$\frac{2 \left(\frac{\ln\left(-e^{2x} + \sqrt{e^{4x} + 1}\right)}{2} + \frac{\ln\left(e^{2x} - \sqrt{e^{4x} + 1}\right)}{2} - \frac{\ln\left(-e^{2x} + \sqrt{e^{4x} + 1}\right)}{2} \right)}{2\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(x)/cosh(2*x)^(1/2),x)`

[Out] `1/4*sqrt(2)*(log(e^(2*x) - sqrt(e^(4*x) + 1) + 1) - log(-e^(2*x) + sqrt(e^(4*x) + 1) + 1) - log(-e^(2*x) + sqrt(e^(4*x) + 1)))`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(x)/cosh(2*x)^(1/2),x)`

[Out] `int(cosh(x)/cosh(2*x)^(1/2), x)`

3.596 $\int x \tanh^2(x) dx$

Optimal. Leaf size=16

$$\frac{x^2}{2} + \log(\cosh(x)) - x \tanh(x)$$

[Out] 1/2*x^2+ln(cosh(x))-x*tanh(x)

Rubi [A]

time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3801, 3556, 30}

$$\frac{x^2}{2} - x \tanh(x) + \log(\cosh(x))$$

Antiderivative was successfully verified.

[In] Int[x*Tanh[x]^2,x]

[Out] x^2/2 + Log[Cosh[x]] - x*Tanh[x]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3556

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3801

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Dist[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rubi steps

$$\begin{aligned} \int x \tanh^2(x) dx &= -x \tanh(x) + \int x dx + \int \tanh(x) dx \\ &= \frac{x^2}{2} + \log(\cosh(x)) - x \tanh(x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 16, normalized size = 1.00

$$\frac{x^2}{2} + \log(\cosh(x)) - x \tanh(x)$$

Antiderivative was successfully verified.

`[In] Integrate[x*Tanh[x]^2,x]``[Out] x^2/2 + Log[Cosh[x]] - x*Tanh[x]`**Mathics [A]**

time = 1.92, size = 19, normalized size = 1.19

$$x - x \operatorname{Tanh}[x] + \frac{x^2}{2} - \operatorname{Log}[1 + \operatorname{Tanh}[x]]$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x*Tanh[x]^2,x]')``[Out] x - x Tanh[x] + x ^ 2 / 2 - Log[1 + Tanh[x]]`**Maple [A]**

time = 0.02, size = 28, normalized size = 1.75

method	result	size
risch	$\frac{x^2}{2} - 2x + \frac{2x}{1+e^{2x}} + \ln(1 + e^{2x})$	28

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*tanh(x)^2,x,method=_RETURNVERBOSE)``[Out] 1/2*x^2-2*x+2*x/(1+exp(2*x))+ln(1+exp(2*x))`**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(14) = 28.

time = 0.38, size = 49, normalized size = 3.06

$$-\frac{x e^{(2x)}}{e^{(2x)} + 1} + \frac{x^2 + (x^2 - 2x)e^{(2x)}}{2(e^{(2x)} + 1)} + \log(e^{(2x)} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*tanh(x)^2,x, algorithm="maxima")``[Out] -x*e^(2*x)/(e^(2*x) + 1) + 1/2*(x^2 + (x^2 - 2*x)*e^(2*x))/(e^(2*x) + 1) + log(e^(2*x) + 1)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(14) = 28$.

time = 0.32, size = 93, normalized size = 5.81

$$\frac{(x^2 - 4x) \cosh(x)^2 + 2(x^2 - 4x) \cosh(x) \sinh(x) + (x^2 - 4x) \sinh(x)^2 + x^2 + 2(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \log\left(\frac{2 \cosh(x)}{\cosh(x) - \sinh(x)}\right)}{2(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*tanh(x)^2,x, algorithm="fricas")

[Out] $\frac{1}{2} * ((x^2 - 4*x) * \cosh(x)^2 + 2 * (x^2 - 4*x) * \cosh(x) * \sinh(x) + (x^2 - 4*x) * \sinh(x)^2 + x^2 + 2 * (\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh(x)^2 + 1) * \log(2 * \cosh(x) / (\cosh(x) - \sinh(x)))) / (\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh(x)^2 + 1)$

Sympy [A]

time = 0.08, size = 17, normalized size = 1.06

$$\frac{x^2}{2} - x \tanh(x) + x - \log(\tanh(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*tanh(x)**2,x)

[Out] $x**2/2 - x*tanh(x) + x - \log(\tanh(x) + 1)$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(14) = 28$.

time = 0.00, size = 52, normalized size = 3.25

$$\frac{x^2 e^{2x} + x^2 - 4x e^{2x} + 2e^{2x} \ln(e^{2x} + 1) + 2 \ln(e^{2x} + 1)}{2e^{2x} + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*tanh(x)^2,x)

[Out] $\frac{1}{2} * (e^{(2*x)} * x^2 - 4 * e^{(2*x)} * x + x^2 + 2 * e^{(2*x)} * \log(e^{(2*x)} + 1) + 2 * \log(e^{(2*x)} + 1)) / (e^{(2*x)} + 1)$

Mupad [B]

time = 0.31, size = 21, normalized size = 1.31

$$\ln(e^{2x} + 1) - x - x \tanh(x) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*tanh(x)^2,x)

[Out] $\log(\exp(2*x) + 1) - x - x*tanh(x) + x^2/2$

3.597 $\int x \coth^2(x) dx$

Optimal. Leaf size=16

$$\frac{x^2}{2} - x \coth(x) + \log(\sinh(x))$$

[Out] 1/2*x^2-x*coth(x)+ln(sinh(x))

Rubi [A]

time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3801, 3556, 30}

$$\frac{x^2}{2} - x \coth(x) + \log(\sinh(x))$$

Antiderivative was successfully verified.

[In] Int[x*Coth[x]^2,x]

[Out] x^2/2 - x*Coth[x] + Log[Sinh[x]]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3556

Int[tan[(c_) + (d_)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3801

Int[((c_) + (d_)*(x_))^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Dist[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rubi steps

$$\begin{aligned} \int x \coth^2(x) dx &= -x \coth(x) + \int x dx + \int \coth(x) dx \\ &= \frac{x^2}{2} - x \coth(x) + \log(\sinh(x)) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 16, normalized size = 1.00

$$\frac{x^2}{2} - x \coth(x) + \log(\sinh(x))$$

Antiderivative was successfully verified.

`[In] Integrate[x*Coth[x]^2,x]``[Out] x^2/2 - x*Coth[x] + Log[Sinh[x]]`**Mathics [A]**

time = 2.62, size = 24, normalized size = 1.50

$$x - \frac{x}{\tanh[x]} + \frac{x^2}{2} + \text{Log}[\tanh[x]] - \text{Log}[1 + \tanh[x]]$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x*Coth[x]^2,x]')``[Out] x - x / Tanh[x] + x ^ 2 / 2 + Log[Tanh[x]] - Log[1 + Tanh[x]]`**Maple [A]**

time = 0.03, size = 28, normalized size = 1.75

method	result	size
risch	$\frac{x^2}{2} - 2x - \frac{2x}{e^{2x}-1} + \ln(e^{2x}-1)$	28

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*coth(x)^2,x,method=_RETURNVERBOSE)``[Out] 1/2*x^2-2*x-2*x/(exp(2*x)-1)+ln(exp(2*x)-1)`**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(14) = 28.

time = 0.28, size = 53, normalized size = 3.31

$$-\frac{x e^{(2x)}}{e^{(2x)} - 1} - \frac{x^2 - (x^2 - 2x)e^{(2x)}}{2(e^{(2x)} - 1)} + \log(e^x + 1) + \log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*coth(x)^2,x, algorithm="maxima")``[Out] -x*e^(2*x)/(e^(2*x) - 1) - 1/2*(x^2 - (x^2 - 2*x)*e^(2*x))/(e^(2*x) - 1) + log(e^x + 1) + log(e^x - 1)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(14) = 28.

time = 0.32, size = 95, normalized size = 5.94

$$\frac{(x^2 - 4x) \cosh(x)^2 + 2(x^2 - 4x) \cosh(x) \sinh(x) + (x^2 - 4x) \sinh(x)^2 - x^2 + 2(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \log\left(\frac{2 \sinh(x)}{\cosh(x) - \sinh(x)}\right)}{2(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*coth(x)^2,x, algorithm="fricas")

[Out] 1/2*((x^2 - 4*x)*cosh(x)^2 + 2*(x^2 - 4*x)*cosh(x)*sinh(x) + (x^2 - 4*x)*sinh(x)^2 - x^2 + 2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)

Sympy [A]

time = 0.33, size = 22, normalized size = 1.38

$$\frac{x^2}{2} + x - \frac{x}{\tanh(x)} - \log(\tanh(x) + 1) + \log(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*coth(x)**2,x)

[Out] x**2/2 + x - x/tanh(x) - log(tanh(x) + 1) + log(tanh(x))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(14) = 28.

time = 0.00, size = 54, normalized size = 3.38

$$\frac{x^2 e^{2x} - x^2 - 4x e^{2x} + 2e^{2x} \ln(e^{2x} - 1) - 2 \ln(e^{2x} - 1)}{2e^{2x} - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*coth(x)^2,x)

[Out] 1/2*(e^(2*x)*x^2 - 4*e^(2*x)*x - x^2 + 2*e^(2*x)*log(e^(2*x) - 1) - 2*log(e^(2*x) - 1))/(e^(2*x) - 1)

Mupad [B]

time = 0.29, size = 27, normalized size = 1.69

$$\ln(e^{2x} - 1) - 2x - \frac{2x}{e^{2x} - 1} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*coth(x)^2,x)

[Out] log(exp(2*x) - 1) - 2*x - (2*x)/(exp(2*x) - 1) + x^2/2

$$3.598 \quad \int \frac{x + \cosh(x) + \sinh(x)}{\cosh(x) - \sinh(x)} dx$$

Optimal. Leaf size=20

$$-e^x + \frac{e^{2x}}{2} + e^x x$$

[Out] -exp(x)+1/2*exp(2*x)+exp(x)*x

Rubi [A]

time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5767, 6874, 2207, 2225, 2320, 12, 14}

$$e^x x - e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

[In] Int[(x + Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]

[Out] -E^x + E^(2*x)/2 + E^x*x

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rule 2207

Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[\$UseGamma]

Rule 2225

Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5767

```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] := Int[u*(a*E^
((a/b)*v))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 6874

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x + \cosh(x) + \sinh(x)}{\cosh(x) - \sinh(x)} dx &= \int e^x (x + \cosh(x) + \sinh(x)) dx \\
&= \int (e^x x + e^x \cosh(x) + e^x \sinh(x)) dx \\
&= \int e^x x dx + \int e^x \cosh(x) dx + \int e^x \sinh(x) dx \\
&= e^x x - \int e^x dx + \text{Subst} \left(\int \frac{-1 + x^2}{2x} dx, x, e^x \right) + \text{Subst} \left(\int \frac{1 + x^2}{2x} dx, x, e^x \right) \\
&= -e^x + e^x x + \frac{1}{2} \text{Subst} \left(\int \frac{-1 + x^2}{x} dx, x, e^x \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1 + x^2}{x} dx, x, e^x \right) \\
&= -e^x + e^x x + \frac{1}{2} \text{Subst} \left(\int \left(-\frac{1}{x} + x \right) dx, x, e^x \right) + \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{x} + x \right) dx, x, e^x \right) \\
&= -e^x + \frac{e^{2x}}{2} + e^x x
\end{aligned}$$

Mathematica [A]

time = 0.05, size = 23, normalized size = 1.15

$$\frac{1}{2} \cosh(2x) + (-1 + x) \sinh(x) + \cosh(x)(-1 + x + \sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]), x]
```

[Out] $\text{Cosh}[2*x]/2 + (-1 + x)*\text{Sinh}[x] + \text{Cosh}[x]*(-1 + x + \text{Sinh}[x])$

Mathics [A]

time = 2.06, size = 9, normalized size = 0.45

$$(-1 + x + \text{Sinh}[x]) E^x$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(x + Sinh[x] + Cosh[x])/(Cosh[x] - Sinh[x]),x]')`

[Out] $(-1 + x + \text{Sinh}[x]) E^x$

Maple [A]

time = 0.19, size = 16, normalized size = 0.80

method	result	size
risch	$(-1 + x) e^x + \frac{e^{2x}}{2}$	14
default	$\frac{\sinh(x)+x-1}{\cosh(x)-\sinh(x)}$	16

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x,method=_RETURNVERBOSE)`

[Out] $(\sinh(x)+x-1)/(\cosh(x)-\sinh(x))$

Maxima [A]

time = 0.25, size = 13, normalized size = 0.65

$$(x - 1)e^x + \frac{1}{2} e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="maxima")`

[Out] $(x - 1)*e^x + 1/2*e^{(2*x)}$

Fricas [A]

time = 0.30, size = 20, normalized size = 1.00

$$\frac{2x + \cosh(x) + \sinh(x) - 2}{2(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="fricas")`

[Out] $1/2*(2*x + \cosh(x) + \sinh(x) - 2)/(\cosh(x) - \sinh(x))$

Sympy [A]

time = 0.22, size = 26, normalized size = 1.30

$$\frac{x}{-\sinh(x) + \cosh(x)} + \frac{\sinh(x)}{-\sinh(x) + \cosh(x)} - \frac{1}{-\sinh(x) + \cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x)**[Out]** x/(-sinh(x) + cosh(x)) + sinh(x)/(-sinh(x) + cosh(x)) - 1/(-sinh(x) + cosh(x))**Giac [A]**

time = 0.00, size = 20, normalized size = 1.00

$$\frac{2x + e^{-x} (e^x)^2 - 2}{2e^{-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x)**[Out]** 1/2*(e^x + 2*x - 2)*e^x**Mupad [B]**

time = 0.06, size = 16, normalized size = 0.80

$$e^x \left(x + \frac{e^{-x}}{2} + \frac{e^x}{2} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + cosh(x) + sinh(x))/(cosh(x) - sinh(x)),x)**[Out]** exp(x)*(x + exp(-x)/2 + exp(x)/2 - 1)

$$3.599 \quad \int \frac{x + \cosh(x) + \sinh(x)}{1 + \cosh(x)} dx$$

Optimal. Leaf size=15

$$x - (1 - x) \tanh\left(\frac{x}{2}\right)$$

[Out] x-(1-x)*tanh(1/2*x)

Rubi [A]

time = 0.08, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6874, 3399, 4269, 3556}

$$x - (1 - x) \tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[(x + Cosh[x] + Sinh[x])/(1 + Cosh[x]),x]

[Out] x - (1 - x)*Tanh[x/2]

Rule 3399

Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))] + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])

Rule 3556

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4269

Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 6874

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int \frac{x + \cosh(x) + \sinh(x)}{1 + \cosh(x)} dx &= \int \left(\frac{x + \cosh(x)}{1 + \cosh(x)} + \tanh\left(\frac{x}{2}\right) \right) dx \\
&= \int \frac{x + \cosh(x)}{1 + \cosh(x)} dx + \int \tanh\left(\frac{x}{2}\right) dx \\
&= 2 \log\left(\cosh\left(\frac{x}{2}\right)\right) + \int \left(1 + \frac{-1 + x}{1 + \cosh(x)}\right) dx \\
&= x + 2 \log\left(\cosh\left(\frac{x}{2}\right)\right) + \int \frac{-1 + x}{1 + \cosh(x)} dx \\
&= x + 2 \log\left(\cosh\left(\frac{x}{2}\right)\right) + \frac{1}{2} \int (-1 + x) \operatorname{sech}^2\left(\frac{x}{2}\right) dx \\
&= x + 2 \log\left(\cosh\left(\frac{x}{2}\right)\right) - (1 - x) \tanh\left(\frac{x}{2}\right) - \int \tanh\left(\frac{x}{2}\right) dx \\
&= x - (1 - x) \tanh\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A]

time = 0.05, size = 20, normalized size = 1.33

$$\frac{(-1 + x + x \coth\left(\frac{x}{2}\right)) \sinh(x)}{1 + \cosh(x)}$$

Antiderivative was successfully verified.

`[In] Integrate[(x + Cosh[x] + Sinh[x])/(1 + Cosh[x]), x]``[Out] ((-1 + x + x*Coth[x/2])*Sinh[x])/(1 + Cosh[x])`**Mathics [A]**

time = 1.80, size = 14, normalized size = 0.93

$$x + x \operatorname{Tanh}\left[\frac{x}{2}\right] - \operatorname{Tanh}\left[\frac{x}{2}\right]$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(x + Sinh[x] + Cosh[x])/(1 + Cosh[x]), x]')``[Out] x + x Tanh[x / 2] - Tanh[x / 2]`**Maple [A]**

time = 0.05, size = 16, normalized size = 1.07

method	result	size
--------	--------	------

risch	$2x - \frac{2(-1+x)}{1+e^x}$	16
-------	------------------------------	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x+cosh(x)+sinh(x))/(1+cosh(x)),x,method=_RETURNVERBOSE)`

[Out] $2*x-2*(-1+x)/(1+exp(x))$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(10) = 20.

time = 0.25, size = 35, normalized size = 2.33

$$x + \frac{2xe^x}{e^x + 1} - \frac{2}{e^{(-x)} + 1} + \log(\cosh(x) + 1) - 2 \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x, algorithm="maxima")`

[Out] $x + 2*x*e^x/(e^x + 1) - 2/(e^{(-x)} + 1) + \log(\cosh(x) + 1) - 2*\log(e^x + 1)$

Fricas [A]

time = 0.31, size = 20, normalized size = 1.33

$$\frac{2(x \cosh(x) + x \sinh(x) + 1)}{\cosh(x) + \sinh(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x, algorithm="fricas")`

[Out] $2*(x*cosh(x) + x*sinh(x) + 1)/(cosh(x) + sinh(x) + 1)$

Sympy [A]

time = 0.17, size = 12, normalized size = 0.80

$$x \tanh\left(\frac{x}{2}\right) + x - \tanh\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x)`

[Out] $x*tanh(x/2) + x - tanh(x/2)$

Giac [A]

time = 0.00, size = 13, normalized size = 0.87

$$\frac{2xe^x + 2}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x)`

[Out] $2*(e^x*x + 1)/(e^x + 1)$

Mupad [B]

time = 0.29, size = 17, normalized size = 1.13

$$2x - \frac{2x - 2}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + cosh(x) + sinh(x))/(cosh(x) + 1),x)`

[Out] $2*x - (2*x - 2)/(\exp(x) + 1)$

3.600 $\int e^{2x} \operatorname{csch}^4(x) dx$

Optimal. Leaf size=20

$$\frac{8e^{6x}}{3(1 - e^{2x})^3}$$

[Out] 8/3*exp(6*x)/(1-exp(2*x))^3

Rubi [A]

time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2320, 12, 270}

$$\frac{8e^{6x}}{3(1 - e^{2x})^3}$$

Antiderivative was successfully verified.

[In] Int[E^(2*x)*Csch[x]^4,x]

[Out] (8*E^(6*x))/(3*(1 - E^(2*x))^3)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m+1)*((a+b*x^n)^(p+1)/(a*c*(m+1))), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

Rule 2320

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\begin{aligned}\int e^{2x} \operatorname{csch}^4(x) dx &= \operatorname{Subst} \left(\int \frac{16x^5}{(1-x^2)^4} dx, x, e^x \right) \\ &= 16 \operatorname{Subst} \left(\int \frac{x^5}{(1-x^2)^4} dx, x, e^x \right) \\ &= \frac{8e^{6x}}{3(1-e^{2x})^3}\end{aligned}$$

Mathematica [A]

time = 0.01, size = 20, normalized size = 1.00

$$\frac{8e^{6x}}{3(1-e^{2x})^3}$$

Antiderivative was successfully verified.

`[In] Integrate[E^(2*x)*Csch[x]^4,x]``[Out] (8*E^(6*x))/(3*(1 - E^(2*x))^3)`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

caught exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[E^(2*x)/Sinh[x]^4,x]')``[Out] caught exception: maximum recursion depth exceeded while calling a Python object`**Maple [A]**

time = 0.15, size = 20, normalized size = 1.00

method	result	size
default	$-\frac{1}{\tanh(x)} - \frac{1}{3 \tanh(x)^3} - \frac{1}{\tanh(x)^2}$	20
risch	$-\frac{8(3e^{4x} - 3e^{2x} + 1)}{3(e^{2x} - 1)^3}$	25

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(exp(2*x)/sinh(x)^4,x,method=_RETURNVERBOSE)``[Out] -1/tanh(x)-1/3/tanh(x)^3-1/tanh(x)^2`

Maxima [A]

time = 0.27, size = 22, normalized size = 1.10

$$\frac{8}{3(3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)/sinh(x)^4,x, algorithm="maxima")**[Out]** 8/3/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(14) = 28.

time = 0.30, size = 75, normalized size = 3.75

$$\frac{8(4 \cosh(x)^2 + 4 \cosh(x) \sinh(x) + 4 \sinh(x)^2 - 3)}{3(\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 - 2) \sinh(x)^2 - 4 \cosh(x)^2 + 4(\cosh(x)^3 - \cosh(x)) \sinh(x) + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)/sinh(x)^4,x, algorithm="fricas")**[Out]** -8/3*(4*cosh(x)^2 + 4*cosh(x)*sinh(x) + 4*sinh(x)^2 - 3)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 2)*sinh(x)^2 - 4*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 3)**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{2x}}{\sinh^4(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)/sinh(x)**4,x)**[Out]** Integral(exp(2*x)/sinh(x)**4, x)**Giac [A]**

time = 0.00, size = 28, normalized size = 1.40

$$\frac{16(-3(e^x)^4 + 3(e^x)^2 - 1)}{6((e^x)^2 - 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)/sinh(x)^4,x)**[Out]** -8/3*(3*e^(4*x) - 3*e^(2*x) + 1)/(e^(2*x) - 1)^3

Mupad [B]

time = 0.33, size = 24, normalized size = 1.20

$$-\frac{8(3e^{4x} - 3e^{2x} + 1)}{3(e^{2x} - 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(2*x)/sinh(x)^4,x)`

[Out] `-(8*(3*exp(4*x) - 3*exp(2*x) + 1))/(3*(exp(2*x) - 1)^3)`

3.601 $\int e^{-2x} \operatorname{sech}^4(x) dx$

Optimal. Leaf size=13

$$-\frac{8}{3(1+e^{2x})^3}$$

[Out] -8/3/(1+exp(2*x))^3

Rubi [A]

time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2320, 12, 267}

$$-\frac{8}{3(e^{2x}+1)^3}$$

Antiderivative was successfully verified.

[In] Int[Sech[x]^4/E^(2*x),x]

[Out] -8/(3*(1 + E^(2*x))^3)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 267

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 2320

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_.))^m_] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\begin{aligned}\int e^{-2x} \operatorname{sech}^4(x) dx &= \operatorname{Subst} \left(\int \frac{16x}{(1+x^2)^4} dx, x, e^x \right) \\ &= 16 \operatorname{Subst} \left(\int \frac{x}{(1+x^2)^4} dx, x, e^x \right) \\ &= -\frac{8}{3(1+e^{2x})^3}\end{aligned}$$

Mathematica [A]

time = 0.01, size = 13, normalized size = 1.00

$$-\frac{8}{3(1+e^{2x})^3}$$

Antiderivative was successfully verified.

`[In] Integrate[Sech[x]^4/E^(2*x),x]``[Out] -8/(3*(1 + E^(2*x))^3)`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

caught exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(E^(2*x)*Cosh[x]^4),x]')``[Out] caught exception: maximum recursion depth exceeded while calling a Python object`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

time = 0.10, size = 22, normalized size = 1.69

method	result	size
risch	$-\frac{8}{3(1+e^{2x})^3}$	11
default	$2 \tanh(x) + \frac{1}{\cosh(x)^2} - \left(\frac{2}{3} + \frac{\operatorname{sech}(x)^2}{3}\right) \tanh(x)$	22

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/exp(2*x)/cosh(x)^4,x,method=_RETURNVERBOSE)`

[Out] $2*\tanh(x)+1/\cosh(x)^2-(2/3+1/3*\operatorname{sech}(x)^2)*\tanh(x)$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(10) = 20$.

time = 0.24, size = 75, normalized size = 5.77

$$\frac{8e^{(-2x)}}{3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1} + \frac{8e^{(-4x)}}{3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1} + \frac{8}{3(3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/exp(2*x)/cosh(x)^4,x, algorithm="maxima")`

[Out] $8*e^{(-2*x)}/(3*e^{(-2*x)} + 3*e^{(-4*x)} + e^{(-6*x)} + 1) + 8*e^{(-4*x)}/(3*e^{(-2*x)} + 3*e^{(-4*x)} + e^{(-6*x)} + 1) + 8/3/(3*e^{(-2*x)} + 3*e^{(-4*x)} + e^{(-6*x)} + 1)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(10) = 20$.

time = 0.32, size = 102, normalized size = 7.85

$$\frac{8}{3(\cosh(x)^6 + 6\cosh(x)\sinh(x)^5 + \sinh(x)^6 + 3(5\cosh(x)^2 + 1)\sinh(x)^4 + 3\cosh(x)^4 + 4(5\cosh(x)^3 + 3\cosh(x))\sinh(x)^3 + 3(5\cosh(x)^4 + 6\cosh(x)^2 + 1)\sinh(x)^2 + 3\cosh(x)^2 + 6(\cosh(x)^5 + 2\cosh(x)^3 + \cosh(x))\sinh(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/exp(2*x)/cosh(x)^4,x, algorithm="fricas")`

[Out] $-8/3/(\cosh(x)^6 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6 + 3*(5*\cosh(x)^2 + 1)*\sinh(x)^4 + 3*\cosh(x)^4 + 4*(5*\cosh(x)^3 + 3*\cosh(x))*\sinh(x)^3 + 3*(5*\cosh(x)^4 + 6*\cosh(x)^2 + 1)*\sinh(x)^2 + 3*\cosh(x)^2 + 6*(\cosh(x)^5 + 2*\cosh(x)^3 + \cosh(x))*\sinh(x) + 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{-2x}}{\cosh^4(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/exp(2*x)/cosh(x)**4,x)`

[Out] `Integral(exp(-2*x)/cosh(x)**4, x)`

Giac [A]

time = 0.00, size = 14, normalized size = 1.08

$$-\frac{8}{3((e^x)^2 + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/exp(2*x)/cosh(x)^4,x)`

[Out] `-8/3/(e^(2*x) + 1)^3`

Mupad [B]

time = 0.31, size = 19, normalized size = 1.46

$$-\frac{e^{-3x}}{3\left(\frac{e^{-x}}{2} + \frac{e^x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(-2*x)/cosh(x)^4,x)`

[Out] `-exp(-3*x)/(3*(exp(-x)/2 + exp(x)/2)^3)`

$$3.602 \quad \int \frac{e^x}{\cosh(x) - \sinh(x)} dx$$

Optimal. Leaf size=9

$$\frac{e^{2x}}{2}$$

[Out] 1/2*exp(2*x)

Rubi [A]

time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2320, 30}

$$\frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

[In] Int[E^x/(Cosh[x] - Sinh[x]),x]

[Out] E^(2*x)/2

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2320

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\int \frac{e^x}{\cosh(x) - \sinh(x)} dx = \text{Subst} \left(\int x dx, x, e^x \right) = \frac{e^{2x}}{2}$$

Mathematica [A]

time = 0.00, size = 9, normalized size = 1.00

$$\frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(Cosh[x] - Sinh[x]),x]

[Out] E^(2*x)/2

Mathics [A]

time = 1.81, size = 7, normalized size = 0.78

$$\frac{E^{2x}}{2}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[E^x/(Cosh[x] - Sinh[x]),x]')

[Out] E ^ (2 x) / 2

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(6) = 12$.

time = 0.06, size = 22, normalized size = 2.44

method	result	size
risch	$\frac{e^{2x}}{2}$	7
gospers	$\frac{e^x}{2 \cosh(x) - 2 \sinh(x)}$	14
default	$\frac{2}{(-1 + \tanh(\frac{x}{2}))^2} + \frac{2}{-1 + \tanh(\frac{x}{2})}$	22

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(cosh(x)-sinh(x)),x,method=_RETURNVERBOSE)

[Out] 2/(-1+tanh(1/2*x))^2+2/(-1+tanh(1/2*x))

Maxima [A]

time = 0.25, size = 6, normalized size = 0.67

$$\frac{1}{2} e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(cosh(x)-sinh(x)),x, algorithm="maxima")

[Out] 1/2*e^(2*x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.

time = 0.31, size = 16, normalized size = 1.78

$$\frac{\cosh(x) + \sinh(x)}{2(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(cosh(x)-sinh(x)),x, algorithm="fricas")`

[Out] `1/2*(cosh(x) + sinh(x))/(cosh(x) - sinh(x))`

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(5) = 10$

time = 0.24, size = 12, normalized size = 1.33

$$\frac{e^x}{-2 \sinh(x) + 2 \cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(cosh(x)-sinh(x)),x)`

[Out] `exp(x)/(-2*sinh(x) + 2*cosh(x))`

Giac [A]

time = 0.00, size = 7, normalized size = 0.78

$$\frac{(e^x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(cosh(x)-sinh(x)),x)`

[Out] `1/2*e^(2*x)`

Mupad [B]

time = 0.32, size = 6, normalized size = 0.67

$$\frac{e^{2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)/(cosh(x) - sinh(x)),x)`

[Out] `exp(2*x)/2`

$$3.603 \quad \int \frac{e^{mx}}{\cosh(x) + \sinh(x)} dx$$

Optimal. Leaf size=13

$$\frac{e^{(-1+m)x}}{-1+m}$$

[Out] exp((-1+m)*x)/(-1+m)

Rubi [A]

time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.46, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5767, 2259, 2225}

$$-\frac{e^{-((1-m)x)}}{1-m}$$

Antiderivative was successfully verified.

[In] Int[E^(m*x)/(Cosh[x] + Sinh[x]),x]

[Out] -(1/(E^((1 - m)*x)*(1 - m)))

Rule 2225

Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 2259

Int[(u_)*(F_)^((a_) + (b_)*(v_)), x_Symbol] := Int[u*F^(a + b*NormalizePowerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && PowerOfLinearQ[v, x] && !PowerOfLinearMatchQ[v, x]

Rule 5767

Int[(u_)*(Cosh[v_]*(a_) + (b_)*Sinh[v_])^(n_), x_Symbol] := Int[u*(a*E^((a/b)*v))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{e^{mx}}{\cosh(x) + \sinh(x)} dx &= \int e^{-x+mx} dx \\ &= \int e^{-(1-m)x} dx \\ &= -\frac{e^{-(1-m)x}}{1-m} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 18, normalized size = 1.38

$$\frac{e^{mx}(\cosh(x) - \sinh(x))}{-1 + m}$$

Antiderivative was successfully verified.

`[In] Integrate[E^(m*x)/(Cosh[x] + Sinh[x]),x]``[Out] (E^(m*x)*(Cosh[x] - Sinh[x]))/(-1 + m)`**Mathics [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.21, size = 43, normalized size = 3.31

$$\text{Piecewise} \left[\left\{ \left\{ \frac{E^{mx}}{m \cosh[x] + m \sinh[x] - \cosh[x] - \sinh[x]}, m \neq 1 \right\} \right\}, \frac{x E^x}{\cosh[x] + \sinh[x]} \right]$$

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[E^(m*x)/(Cosh[x] + Sinh[x]),x]')``[Out] Piecewise[{{E^(m x) / (m Cosh[x] + m Sinh[x] - Cosh[x] - Sinh[x]), m != 1}}, x E^x / (Cosh[x] + Sinh[x])]`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.

time = 0.09, size = 26, normalized size = 2.00

method	result	size
risch	$\frac{e^{(-1+m)x}}{-1+m}$	13
gospers	$\frac{e^{mx}}{(-1+m)(\cosh(x)+\sinh(x))}$	18
default	$\frac{\sinh((-1+m)x)}{-1+m} + \frac{\cosh((-1+m)x)}{-1+m}$	26

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(exp(m*x)/(cosh(x)+sinh(x)),x,method=_RETURNVERBOSE)``[Out] 1/(-1+m)*sinh((-1+m)*x)+cosh((-1+m)*x)/(-1+m)`**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(m*x)/(cosh(x)+sinh(x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(-m>0)', see 'assume?' for more details)Is

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

time = 0.50, size = 25, normalized size = 1.92

$$\frac{\cosh(mx) + \sinh(mx)}{(m-1)\cosh(x) + (m-1)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(m*x)/(cosh(x)+sinh(x)),x, algorithm="fricas")`

[Out] `(cosh(m*x) + sinh(m*x))/((m - 1)*cosh(x) + (m - 1)*sinh(x))`

Sympy [A]

time = 0.26, size = 32, normalized size = 2.46

$$\begin{cases} \frac{e^{mx}}{m \sinh(x) + m \cosh(x) - \sinh(x) - \cosh(x)} & \text{for } m \neq 1 \\ \frac{x e^x}{\sinh(x) + \cosh(x)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(m*x)/(cosh(x)+sinh(x)),x)`

[Out] `Piecewise((exp(m*x)/(m*sinh(x) + m*cosh(x) - sinh(x) - cosh(x)), Ne(m, 1)), (x*exp(x)/(sinh(x) + cosh(x)), True))`

Giac [A]

time = 0.00, size = 14, normalized size = 1.08

$$\frac{e^{mx}}{me^x - e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(m*x)/(cosh(x)+sinh(x)),x)`

[Out] `e^(m*x)/(e^x*m - e^x)`

Mupad [B]

time = 0.11, size = 14, normalized size = 1.08

$$\frac{e^{mx-x}}{m-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(m*x)/(cosh(x) + sinh(x)),x)`

[Out] `exp(m*x - x)/(m - 1)`

$$3.604 \quad \int \frac{e^x}{\cosh(x) + \sinh(x)} dx$$

Optimal. Leaf size=1

x

[Out] x

Rubi [A]

time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2320, 29}

x

Antiderivative was successfully verified.

[In] Int[E^x/(Cosh[x] + Sinh[x]),x]

[Out] x

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 2320

Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\int \frac{e^x}{\cosh(x) + \sinh(x)} dx = \text{Subst} \left(\int \frac{1}{x} dx, x, e^x \right) = x$$

Mathematica [A]

time = 0.00, size = 1, normalized size = 1.00

x

Antiderivative was successfully verified.

[In] Integrate[E^x/(Cosh[x] + Sinh[x]),x]

[Out] x

Mathics [A]

time = 1.71, size = 1, normalized size = 1.00

x

Antiderivative was successfully verified.

[In] mathics('Integrate[E^x/(Cosh[x] + Sinh[x]),x]')

[Out] x

Maple [A]

time = 0.05, size = 2, normalized size = 2.00

method	result	size
default	x	2

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(cosh(x)+sinh(x)),x,method=_RETURNVERBOSE)

[Out] x

Maxima [A]

time = 0.24, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(cosh(x)+sinh(x)),x, algorithm="maxima")

[Out] x

Fricas [A]

time = 0.29, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(cosh(x)+sinh(x)),x, algorithm="fricas")

[Out] x

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 10 vs. 2(0) = 0

time = 0.20, size = 10, normalized size = 10.00

$$\frac{xe^x}{\sinh(x) + \cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(cosh(x)+sinh(x)),x)
```

```
[Out] x*exp(x)/(sinh(x) + cosh(x))
```

Giac [A]

time = 0.00, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(cosh(x)+sinh(x)),x)
```

```
[Out] x
```

Mupad [B]

time = 0.29, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(cosh(x) + sinh(x)),x)
```

```
[Out] x
```

$$3.605 \quad \int \frac{e^x}{1 - \cosh(x)} dx$$

Optimal. Leaf size=22

$$-\frac{2}{1 - e^x} - 2 \log(1 - e^x)$$

[Out] -2/(1-exp(x))-2*ln(1-exp(x))

Rubi [A]

time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2320, 12, 45}

$$-\frac{2}{1 - e^x} - 2 \log(1 - e^x)$$

Antiderivative was successfully verified.

[In] Int[E^x/(1 - Cosh[x]),x]

[Out] -2/(1 - E^x) - 2*Log[1 - E^x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2320

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\begin{aligned}
\int \frac{e^x}{1 - \cosh(x)} dx &= \text{Subst} \left(\int -\frac{2x}{(1-x)^2} dx, x, e^x \right) \\
&= - \left(2 \text{Subst} \left(\int \frac{x}{(1-x)^2} dx, x, e^x \right) \right) \\
&= - \left(2 \text{Subst} \left(\int \left(\frac{1}{(-1+x)^2} + \frac{1}{-1+x} \right) dx, x, e^x \right) \right) \\
&= -\frac{2}{1-e^x} - 2 \log(1-e^x)
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 36, normalized size = 1.64

$$\frac{4 \left(\frac{1}{1-e^x} + \log(1-e^x) \right) \sinh^2 \left(\frac{x}{2} \right)}{1 - \cosh(x)}$$

Antiderivative was successfully verified.

`[In] Integrate[E^x/(1 - Cosh[x]),x]``[Out] (4*((1 - E^x)^(-1) + Log[1 - E^x])*Sinh[x/2]^2)/(1 - Cosh[x])`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[E^x/(1 - Cosh[x]),x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.03, size = 24, normalized size = 1.09

method	result	size
risch	$\frac{2}{-1+e^x} - 2 \ln(-1 + e^x)$	17
default	$\frac{1}{\tanh(\frac{x}{2})} - 2 \ln(\tanh(\frac{x}{2})) + 2 \ln(-1 + \tanh(\frac{x}{2}))$	24

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(exp(x)/(1-cosh(x)),x,method=_RETURNVERBOSE)``[Out] 1/tanh(1/2*x)-2*ln(tanh(1/2*x))+2*ln(-1+tanh(1/2*x))`

Maxima [A]

time = 0.25, size = 16, normalized size = 0.73

$$\frac{2}{e^x - 1} - 2 \log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)/(1-cosh(x)),x, algorithm="maxima")``[Out] 2/(e^x - 1) - 2*log(e^x - 1)`**Fricas [A]**

time = 0.30, size = 26, normalized size = 1.18

$$-\frac{2((\cosh(x) + \sinh(x) - 1) \log(\cosh(x) + \sinh(x) - 1) - 1)}{\cosh(x) + \sinh(x) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)/(1-cosh(x)),x, algorithm="fricas")``[Out] -2*((cosh(x) + sinh(x) - 1)*log(cosh(x) + sinh(x) - 1) - 1)/(cosh(x) + sinh(x) - 1)`**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e^x}{\cosh(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)/(1-cosh(x)),x)``[Out] -Integral(exp(x)/(cosh(x) - 1), x)`**Giac [A]**

time = 0.00, size = 16, normalized size = 0.73

$$-2 \left(-\frac{1}{e^x - 1} + \ln |e^x - 1| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)/(1-cosh(x)),x)``[Out] 2/(e^x - 1) - 2*log(abs(e^x - 1))`**Mupad [B]**

time = 0.06, size = 16, normalized size = 0.73

$$\frac{2}{e^x - 1} - 2 \ln(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-exp(x)/(cosh(x) - 1),x)
```

```
[Out] 2/(exp(x) - 1) - 2*log(exp(x) - 1)
```

$$3.606 \quad \int \frac{e^x(1+\sinh(x))}{1+\cosh(x)} dx$$

Optimal. Leaf size=13

$$e^x + \frac{2}{1+e^x}$$

[Out] exp(x)+2/(1+exp(x))

Rubi [A]

time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2320, 697}

$$e^x + \frac{2}{e^x + 1}$$

Antiderivative was successfully verified.

[In] Int[(E^x*(1 + Sinh[x]))/(1 + Cosh[x]),x]

[Out] E^x + 2/(1 + E^x)

Rule 697

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])

Rule 2320

Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_.)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\begin{aligned} \int \frac{e^x(1+\sinh(x))}{1+\cosh(x)} dx &= \text{Subst} \left(\int \frac{-1+2x+x^2}{(1+x)^2} dx, x, e^x \right) \\ &= \text{Subst} \left(\int \left(1 - \frac{2}{(1+x)^2} \right) dx, x, e^x \right) \\ &= e^x + \frac{2}{1+e^x} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 18, normalized size = 1.38

$$\frac{2 + e^x + e^{2x}}{1 + e^x}$$

Antiderivative was successfully verified.

`[In] Integrate[(E^x*(1 + Sinh[x]))/(1 + Cosh[x]),x]``[Out] (2 + E^x + E^(2*x))/(1 + E^x)`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[E^x*((1 + Sinh[x]))/(1 + Cosh[x]),x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.04, size = 18, normalized size = 1.38

method	result	size
risch	$e^x + \frac{2}{1+e^x}$	12
default	$-\tanh\left(\frac{x}{2}\right) - \frac{2}{-1+\tanh\left(\frac{x}{2}\right)}$	18

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(exp(x)*(1+sinh(x))/(1+cosh(x)),x,method=_RETURNVERBOSE)``[Out] -tanh(1/2*x)-2/(-1+tanh(1/2*x))`**Maxima [A]**

time = 0.24, size = 11, normalized size = 0.85

$$\frac{2}{e^x + 1} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x, algorithm="maxima")``[Out] 2/(e^x + 1) + e^x`

Fricas [A]

time = 0.30, size = 21, normalized size = 1.62

$$\frac{3 \cosh(x) - \sinh(x) + 1}{\cosh(x) - \sinh(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x, algorithm="fricas")``[Out] (3*cosh(x) - sinh(x) + 1)/(cosh(x) - sinh(x) + 1)`**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(\sinh(x) + 1) e^x}{\cosh(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x)``[Out] Integral((sinh(x) + 1)*exp(x)/(cosh(x) + 1), x)`**Giac [A]**

time = 0.00, size = 10, normalized size = 0.77

$$e^x + \frac{2}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x)``[Out] e^x + 2/(e^x + 1)`**Mupad [B]**

time = 0.30, size = 11, normalized size = 0.85

$$e^x + \frac{2}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((exp(x)*(sinh(x) + 1))/(cosh(x) + 1),x)``[Out] exp(x) + 2/(exp(x) + 1)`

$$3.607 \quad \int \frac{e^x(1-\sinh(x))}{1-\cosh(x)} dx$$

Optimal. Leaf size=15

$$e^x - \frac{2}{1 - e^x}$$

[Out] exp(x)-2/(1-exp(x))

Rubi [A]

time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2320, 697}

$$e^x - \frac{2}{1 - e^x}$$

Antiderivative was successfully verified.

[In] Int[(E^x*(1 - Sinh[x]))/(1 - Cosh[x]),x]

[Out] E^x - 2/(1 - E^x)

Rule 697

Int[((d_.) + (e_.)*(x_)^(m_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])

Rule 2320

Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rubi steps

$$\begin{aligned} \int \frac{e^x(1-\sinh(x))}{1-\cosh(x)} dx &= \text{Subst} \left(\int \frac{-1-2x+x^2}{(1-x)^2} dx, x, e^x \right) \\ &= \text{Subst} \left(\int \left(1 - \frac{2}{(-1+x)^2} \right) dx, x, e^x \right) \\ &= e^x - \frac{2}{1 - e^x} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 20, normalized size = 1.33

$$\frac{2 - e^x + e^{2x}}{-1 + e^x}$$

Antiderivative was successfully verified.

[In] Integrate[(E^x*(1 - Sinh[x]))/(1 - Cosh[x]),x]

[Out] (2 - E^x + E^(2*x))/(-1 + E^x)

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[E^x*((1 - Sinh[x]))/(1 - Cosh[x]),x]')

[Out] cought exception: maximum recursion depth exceeded

Maple [A]

time = 0.05, size = 18, normalized size = 1.20

method	result	size
risch	$e^x + \frac{2}{-1+e^x}$	12
default	$-\frac{2}{-1+\tanh(\frac{x}{2})} + \frac{1}{\tanh(\frac{x}{2})}$	18

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)*(1-sinh(x))/(1-cosh(x)),x,method=_RETURNVERBOSE)

[Out] -2/(-1+tanh(1/2*x))+1/tanh(1/2*x)

Maxima [A]

time = 0.25, size = 11, normalized size = 0.73

$$\frac{2}{e^x - 1} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x, algorithm="maxima")

[Out] 2/(e^x - 1) + e^x

Fricas [A]

time = 0.30, size = 22, normalized size = 1.47

$$\frac{3 \cosh(x) - \sinh(x) - 1}{\cosh(x) - \sinh(x) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x, algorithm="fricas")``[Out] -(3*cosh(x) - sinh(x) - 1)/(cosh(x) - sinh(x) - 1)`**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(\sinh(x) - 1) e^x}{\cosh(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x)``[Out] Integral((sinh(x) - 1)*exp(x)/(cosh(x) - 1), x)`**Giac [A]**

time = 0.00, size = 10, normalized size = 0.67

$$e^x + \frac{2}{e^x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x)``[Out] e^x + 2/(e^x - 1)`**Mupad [B]**

time = 0.04, size = 11, normalized size = 0.73

$$e^x + \frac{2}{e^x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((exp(x)*(sinh(x) - 1))/(cosh(x) - 1),x)``[Out] exp(x) + 2/(exp(x) - 1)`

3.608 $\int x^m \log(x) dx$

Optimal. Leaf size=26

$$-\frac{x^{1+m}}{(1+m)^2} + \frac{x^{1+m} \log(x)}{1+m}$$

[Out] $-x^{(1+m)/(1+m)^2+x^{(1+m)*\ln(x)/(1+m)}$

Rubi [A]

time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2341}

$$\frac{x^{m+1} \log(x)}{m+1} - \frac{x^{m+1}}{(m+1)^2}$$

Antiderivative was successfully verified.

[In] Int[x^m*Log[x],x]

[Out] $-(x^{(1+m)/(1+m)^2} + (x^{(1+m)*\text{Log}[x]})/(1+m)$

Rule 2341

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_)^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\int x^m \log(x) dx = -\frac{x^{1+m}}{(1+m)^2} + \frac{x^{1+m} \log(x)}{1+m}$$

Mathematica [A]

time = 0.01, size = 19, normalized size = 0.73

$$\frac{x^{1+m}(-1 + (1+m) \log(x))}{(1+m)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^m*Log[x],x]

[Out] $(x^{(1+m)*(-1 + (1+m)*\text{Log}[x])})/(1+m)^2$

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.06, size = 36, normalized size = 1.38

$$\text{Piecewise} \left[\left\{ \left\{ \frac{(-1 + m \text{Log}[x] + \text{Log}[x]) x^{1+m}}{1 + 2m + m^2}, m \neq -1 \right\} \right\}, \frac{\text{Log}[x]^2}{2} \right]$$

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[x^m*Log[x],x]')`

[Out] `Piecewise[{{(-1 + m Log[x] + Log[x]) x ^ (1 + m) / (1 + 2 m + m ^ 2), m != -1}}, Log[x] ^ 2 / 2]`

Maple [A]

time = 0.02, size = 19, normalized size = 0.73

method	result	size
risch	$\frac{x(m \ln(x) + \ln(x) - 1)x^m}{(1+m)^2}$	19
norman	$\frac{x \ln(x) e^{m \ln(x)}}{1+m} - \frac{x e^{m \ln(x)}}{m^2 + 2m + 1}$	34

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*ln(x),x,method=_RETURNVERBOSE)`

[Out] `x*(m*ln(x)+ln(x)-1)/(1+m)^2*x^m`

Maxima [A]

time = 0.24, size = 26, normalized size = 1.00

$$\frac{x^{m+1} \log(x)}{m+1} - \frac{x^{m+1}}{(m+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*log(x),x, algorithm="maxima")`

[Out] `x^(m + 1)*log(x)/(m + 1) - x^(m + 1)/(m + 1)^2`

Fricas [A]

time = 0.34, size = 25, normalized size = 0.96

$$\frac{((m+1)x \log(x) - x)x^m}{m^2 + 2m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*log(x),x, algorithm="fricas")`

[Out] $((m + 1)*x*\log(x) - x)*x^m/(m^2 + 2*m + 1)$

Sympy [A]

time = 0.22, size = 56, normalized size = 2.15

$$\begin{cases} \frac{mxx^m \log(x)}{m^2+2m+1} + \frac{xx^m \log(x)}{m^2+2m+1} - \frac{xx^m}{m^2+2m+1} & \text{for } m \neq -1 \\ \frac{\log(x)^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*ln(x),x)`

[Out] `Piecewise((m*x*x**m*log(x)/(m**2 + 2*m + 1) + x*x**m*log(x)/(m**2 + 2*m + 1) - x*x**m/(m**2 + 2*m + 1), Ne(m, -1)), (log(x)**2/2, True))`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*log(x),x)`

[Out] Could not integrate

Mupad [B]

time = 0.40, size = 32, normalized size = 1.23

$$\begin{cases} \frac{\ln(x)^2}{2} & \text{if } m = -1 \\ \frac{x^{m+1}(\ln(x)(m+1)-1)}{(m+1)^2} & \text{if } m \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*log(x),x)`

[Out] `piecewise(m == -1, log(x)^2/2, m ~= -1, (x^(m + 1)*(log(x)*(m + 1) - 1))/(m + 1)^2)`

3.609 $\int x^m \log^2(x) dx$

Optimal. Leaf size=42

$$\frac{2x^{1+m}}{(1+m)^3} - \frac{2x^{1+m} \log(x)}{(1+m)^2} + \frac{x^{1+m} \log^2(x)}{1+m}$$

[Out] $2*x^{(1+m)}/(1+m)^3-2*x^{(1+m)*\ln(x)}/(1+m)^2+x^{(1+m)*\ln(x)^2}/(1+m)$

Rubi [A]

time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2342, 2341}

$$\frac{2x^{m+1}}{(m+1)^3} + \frac{x^{m+1} \log^2(x)}{m+1} - \frac{2x^{m+1} \log(x)}{(m+1)^2}$$

Antiderivative was successfully verified.

[In] Int[x^m*Log[x]^2,x]

[Out] $(2*x^{(1+m)})/(1+m)^3 - (2*x^{(1+m)*\text{Log}[x]})/(1+m)^2 + (x^{(1+m)*\text{Log}[x]^2})/(1+m)$

Rule 2341

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m+1)*((a + b*Log[c*x^n])/(d*(m+1))), x] - Simp[b*n*((d*x)^(m+1)/(d*(m+1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 2342

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m+1)*((a + b*Log[c*x^n])^p/(d*(m+1))), x] - Dist[b*n*(p/(m+1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p-1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int x^m \log^2(x) dx &= \frac{x^{1+m} \log^2(x)}{1+m} - \frac{2 \int x^m \log(x) dx}{1+m} \\ &= \frac{2x^{1+m}}{(1+m)^3} - \frac{2x^{1+m} \log(x)}{(1+m)^2} + \frac{x^{1+m} \log^2(x)}{1+m} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 30, normalized size = 0.71

$$\frac{x^{1+m} (2 - 2(1+m) \log(x) + (1+m)^2 \log^2(x))}{(1+m)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^m*Log[x]^2,x]

[Out] (x^(1+m)*(2-2*(1+m)*Log[x]+(1+m)^2*Log[x]^2))/(1+m)^3

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.93, size = 63, normalized size = 1.50

$$\text{Piecewise} \left[\left\{ \left\{ \frac{(2 - 2m \text{Log}[x] + 2m \text{Log}[x]^2 + m^2 \text{Log}[x]^2 + \text{Log}[x]^2 - 2 \text{Log}[x]) x^{1+m}}{1 + 3m + 3m^2 + m^3}, m \neq -1 \right\} \right\}, \frac{\text{Log}[x]^3}{3} \right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x^m*Log[x]^2,x]')

[Out] Piecewise[{{(2 - 2 m Log[x] + 2 m Log[x]^2 + m^2 Log[x]^2 + Log[x]^2 - 2 Log[x]) x^(1+m) / (1 + 3 m + 3 m^2 + m^3), m != -1}}, Log[x]^3 / 3]

Maple [A]

time = 0.02, size = 41, normalized size = 0.98

method	result	size
risch	$\frac{x(m^2 \ln(x)^2 + 2m \ln(x)^2 - 2m \ln(x) + \ln(x)^2 - 2 \ln(x) + 2)x^m}{(1+m)^3}$	41
norman	$\frac{x \ln(x)^2 e^{m \ln(x)}}{1+m} + \frac{2x e^{m \ln(x)}}{m^3 + 3m^2 + 3m + 1} - \frac{2x \ln(x) e^{m \ln(x)}}{m^2 + 2m + 1}$	61

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*ln(x)^2,x,method=_RETURNVERBOSE)

[Out] x*(m^2*ln(x)^2+2*m*ln(x)^2-2*m*ln(x)+ln(x)^2-2*ln(x)+2)/(1+m)^3*x^m

Maxima [A]

time = 0.25, size = 42, normalized size = 1.00

$$\frac{x^{m+1} \log(x)^2}{m+1} - \frac{2x^{m+1} \log(x)}{(m+1)^2} + \frac{2x^{m+1}}{(m+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*log(x)^2,x, algorithm="maxima")

[Out] x^(m + 1)*log(x)^2/(m + 1) - 2*x^(m + 1)*log(x)/(m + 1)^2 + 2*x^(m + 1)/(m + 1)^3

Fricas [A]

time = 0.32, size = 45, normalized size = 1.07

$$\frac{((m^2 + 2m + 1)x \log(x)^2 - 2(m + 1)x \log(x) + 2x)x^m}{m^3 + 3m^2 + 3m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*log(x)^2,x, algorithm="fricas")

[Out] ((m^2 + 2*m + 1)*x*log(x)^2 - 2*(m + 1)*x*log(x) + 2*x)*x^m/(m^3 + 3*m^2 + 3*m + 1)

Sympy [A]

time = 0.36, size = 155, normalized size = 3.69

$$\begin{cases} \frac{m^2 x x^m \log(x)^2}{m^3 + 3m^2 + 3m + 1} + \frac{2m x x^m \log(x)^2}{m^3 + 3m^2 + 3m + 1} - \frac{2m x x^m \log(x)}{m^3 + 3m^2 + 3m + 1} + \frac{x x^m \log(x)^2}{m^3 + 3m^2 + 3m + 1} - \frac{2x x^m \log(x)}{m^3 + 3m^2 + 3m + 1} + \frac{2x x^m}{m^3 + 3m^2 + 3m + 1} & \text{for } m \neq -1 \\ \frac{\log(x)^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**m*ln(x)**2,x)

[Out] Piecewise(((m**2*x*x**m*log(x)**2/(m**3 + 3*m**2 + 3*m + 1) + 2*m*x*x**m*log(x)**2/(m**3 + 3*m**2 + 3*m + 1) - 2*m*x*x**m*log(x)/(m**3 + 3*m**2 + 3*m + 1) + x*x**m*log(x)**2/(m**3 + 3*m**2 + 3*m + 1) - 2*x*x**m*log(x)/(m**3 + 3*m**2 + 3*m + 1) + 2*x*x**m/(m**3 + 3*m**2 + 3*m + 1), Ne(m, -1)), (log(x)**3/3, True))

Giac [A]

time = 0.00, size = 85, normalized size = 2.02

$$\frac{x^{m+1} \ln^2 x}{m+1} - 2 \left(-\frac{x e^{m \ln x}}{(m+1)(m^2+2m+1)} + \frac{x e^{m \ln x} \ln x}{(m+1)(m^2+2m+1)} + \frac{m x e^{m \ln x} \ln x}{(m+1)(m^2+2m+1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*log(x)^2,x)

[Out] -2*m*x*x^m*log(x)/((m^2 + 2*m + 1)*(m + 1)) + x^(m + 1)*log(x)^2/(m + 1) - 2*x*x^m*log(x)/((m^2 + 2*m + 1)*(m + 1)) + 2*x*x^m/((m^2 + 2*m + 1)*(m + 1))

Mupad [B]

time = 0.35, size = 43, normalized size = 1.02

$$\begin{cases} \frac{\ln(x)^3}{3} & \text{if } m = -1 \\ \frac{x^{m+1} (\ln(x)^2 (m+1)^2 - 2 \ln(x) (m+1) + 2)}{(m+1)^3} & \text{if } m \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^m*log(x)^2,x)``[Out] piecewise(m == -1, log(x)^3/3, m ~= -1, (x^(m + 1)*(- 2*log(x)*(m + 1) + log(x)^2*(m + 1)^2 + 2))/(m + 1)^3)`

$$3.610 \quad \int \frac{\log^2(x)}{x^{5/2}} dx$$

Optimal. Leaf size=34

$$-\frac{16}{27x^{3/2}} - \frac{8\log(x)}{9x^{3/2}} - \frac{2\log^2(x)}{3x^{3/2}}$$

[Out] $-16/27/x^{(3/2)}-8/9*\ln(x)/x^{(3/2)}-2/3*\ln(x)^2/x^{(3/2)}$

Rubi [A]

time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2342, 2341}

$$-\frac{16}{27x^{3/2}} - \frac{2\log^2(x)}{3x^{3/2}} - \frac{8\log(x)}{9x^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Log[x]^2/x^(5/2),x]

[Out] $-16/(27*x^{(3/2)}) - (8*\text{Log}[x])/(9*x^{(3/2)}) - (2*\text{Log}[x]^2)/(3*x^{(3/2)})$

Rule 2341

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_)^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 2342

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{\log^2(x)}{x^{5/2}} dx &= -\frac{2\log^2(x)}{3x^{3/2}} + \frac{4}{3} \int \frac{\log(x)}{x^{5/2}} dx \\ &= -\frac{16}{27x^{3/2}} - \frac{8\log(x)}{9x^{3/2}} - \frac{2\log^2(x)}{3x^{3/2}} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 21, normalized size = 0.62

$$-\frac{2(8 + 12\log(x) + 9\log^2(x))}{27x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Log[x]^2/x^(5/2),x]

[Out] $(-2*(8 + 12*\text{Log}[x] + 9*\text{Log}[x]^2))/(27*x^{(3/2)})$

Mathics [A]

time = 2.54, size = 17, normalized size = 0.50

$$\frac{2(-8 - 12\text{Log}[x] - 9\text{Log}[x]^2)}{27x^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Log[x]^2/x^(5/2),x]')

[Out] $2(-8 - 12\text{Log}[x] - 9\text{Log}[x]^2)/(27x^{(3/2)})$

Maple [A]

time = 0.06, size = 23, normalized size = 0.68

method	result	size
derivativdivides	$-\frac{16}{27x^{\frac{3}{2}}} - \frac{8\ln(x)}{9x^{\frac{3}{2}}} - \frac{2\ln(x)^2}{3x^{\frac{3}{2}}}$	23
default	$-\frac{16}{27x^{\frac{3}{2}}} - \frac{8\ln(x)}{9x^{\frac{3}{2}}} - \frac{2\ln(x)^2}{3x^{\frac{3}{2}}}$	23
risch	$-\frac{16}{27x^{\frac{3}{2}}} - \frac{8\ln(x)}{9x^{\frac{3}{2}}} - \frac{2\ln(x)^2}{3x^{\frac{3}{2}}}$	23

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(x)^2/x^(5/2),x,method=_RETURNVERBOSE)

[Out] $-16/27/x^{(3/2)} - 8/9*\ln(x)/x^{(3/2)} - 2/3*\ln(x)^2/x^{(3/2)}$

Maxima [A]

time = 0.24, size = 22, normalized size = 0.65

$$-\frac{2\log(x)^2}{3x^{\frac{3}{2}}} - \frac{8\log(x)}{9x^{\frac{3}{2}}} - \frac{16}{27x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)^2/x^(5/2),x, algorithm="maxima")

[Out] $-2/3*\log(x)^2/x^{(3/2)} - 8/9*\log(x)/x^{(3/2)} - 16/27/x^{(3/2)}$

Fricas [A]

time = 0.42, size = 17, normalized size = 0.50

$$-\frac{2(9\log(x)^2 + 12\log(x) + 8)}{27x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)^2/x^(5/2),x, algorithm="fricas")

[Out] -2/27*(9*log(x)^2 + 12*log(x) + 8)/x^(3/2)

Sympy [A]

time = 1.09, size = 34, normalized size = 1.00

$$-\frac{2 \log(x)^2}{3x^{\frac{3}{2}}} - \frac{8 \log(x)}{9x^{\frac{3}{2}}} - \frac{16}{27x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(x)**2/x**(5/2),x)

[Out] -2*log(x)**2/(3*x**(3/2)) - 8*log(x)/(9*x**(3/2)) - 16/(27*x**(3/2))

Giac [A]

time = 0.00, size = 49, normalized size = 1.44

$$2 \left(-\frac{\ln^2 x}{3\sqrt{x} x} + 4 \left(-\frac{\ln x}{9\sqrt{x} x} - \frac{2}{27\sqrt{x} x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)^2/x^(5/2),x)

[Out] -2/3*log(x)^2/x^(3/2) - 8/9*log(x)/x^(3/2) - 16/27/x^(3/2)

Mupad [B]

time = 0.04, size = 17, normalized size = 0.50

$$-\frac{18 \ln(x)^2 + 24 \ln(x) + 16}{27 x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(x)^2/x^(5/2),x)

[Out] -(24*log(x) + 18*log(x)^2 + 16)/(27*x^(3/2))

3.611 $\int (a + bx) \log(x) dx$

Optimal. Leaf size=28

$$-ax - \frac{bx^2}{4} + ax \log(x) + \frac{1}{2}bx^2 \log(x)$$

[Out] $-a*x-1/4*b*x^2+a*x*\ln(x)+1/2*b*x^2*\ln(x)$

Rubi [A]

time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2350}

$$-ax + ax \log(x) - \frac{bx^2}{4} + \frac{1}{2}bx^2 \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x)*\text{Log}[x], x]$

[Out] $-(a*x) - (b*x^2)/4 + a*x*\text{Log}[x] + (b*x^2*\text{Log}[x])/2$

Rule 2350

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)*((d_.) + (e_.)*(x_.)^{(r_.)})^{(q_.)}, x_Symbol] :> \text{With}[\{u = \text{IntHide}[(d + e*x^r)^q, x]\}, \text{Dist}[a + b*\text{Log}[c*x^n], u, x] - \text{Dist}[b*n, \text{Int}[\text{SimplifyIntegrand}[u/x, x], x], x]] /; \text{FreeQ}[\{a, b, c, d, e, n, r\}, x] \&\& \text{IGtQ}[q, 0]$

Rubi steps

$$\begin{aligned} \int (a + bx) \log(x) dx &= \frac{1}{2} (2ax + bx^2) \log(x) - \int \left(a + \frac{bx}{2} \right) dx \\ &= -ax - \frac{bx^2}{4} + \frac{1}{2} (2ax + bx^2) \log(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 28, normalized size = 1.00

$$-ax - \frac{bx^2}{4} + ax \log(x) + \frac{1}{2}bx^2 \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(a + b*x)*\text{Log}[x], x]$

[Out] $-(a*x) - (b*x^2)/4 + a*x*\text{Log}[x] + (b*x^2*\text{Log}[x])/2$

Mathics [A]

time = 1.69, size = 22, normalized size = 0.79

$$\frac{x(-4a - bx + 2\text{Log}[x](2a + bx))}{4}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(a + b*x)*Log[x],x]')`

[Out] $x(-4a - bx + 2\text{Log}[x](2a + bx)) / 4$

Maple [A]

time = 0.01, size = 27, normalized size = 0.96

method	result	size
norman	$-ax - \frac{x^2b}{4} + ax \ln(x) + \frac{bx^2 \ln(x)}{2}$	25
risch	$(\frac{1}{2}x^2b + ax) \ln(x) - \frac{x^2b}{4} - ax$	25
default	$b\left(-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}\right) + a(-x + x \ln(x))$	27

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)*ln(x),x,method=_RETURNVERBOSE)`

[Out] $b*(-1/4*x^2+1/2*x^2*ln(x))+a*(-x+x*ln(x))$

Maxima [A]

time = 0.24, size = 25, normalized size = 0.89

$$-\frac{1}{4}bx^2 - ax + \frac{1}{2}(bx^2 + 2ax) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*log(x),x, algorithm="maxima")`

[Out] $-1/4*b*x^2 - a*x + 1/2*(b*x^2 + 2*a*x)*log(x)$

Fricas [A]

time = 0.30, size = 25, normalized size = 0.89

$$-\frac{1}{4}bx^2 - ax + \frac{1}{2}(bx^2 + 2ax) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*log(x),x, algorithm="fricas")`

[Out] $-1/4*b*x^2 - a*x + 1/2*(b*x^2 + 2*a*x)*\log(x)$

Sympy [A]

time = 0.06, size = 22, normalized size = 0.79

$$-ax - \frac{bx^2}{4} + \left(ax + \frac{bx^2}{2}\right) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*ln(x),x)`

[Out] $-a*x - b*x**2/4 + (a*x + b*x**2/2)*\log(x)$

Giac [A]

time = 0.00, size = 27, normalized size = 0.96

$$-\frac{1}{4}bx^2 - ax + ax \ln x + \frac{1}{2}bx^2 \ln x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*log(x),x)`

[Out] $1/2*b*x^2*\log(x) - 1/4*b*x^2 + a*x*\log(x) - a*x$

Mupad [B]

time = 0.33, size = 21, normalized size = 0.75

$$-\frac{x(4a + bx - 4a \ln(x) - 2bx \ln(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(x)*(a + b*x),x)`

[Out] $-(x*(4*a + b*x - 4*a*\log(x) - 2*b*x*\log(x)))/4$

3.612 $\int (a + bx)^3 \log(x) dx$

Optimal. Leaf size=67

$$-a^3x - \frac{3}{4}a^2bx^2 - \frac{1}{3}ab^2x^3 - \frac{b^3x^4}{16} - \frac{a^4 \log(x)}{4b} + \frac{(a + bx)^4 \log(x)}{4b}$$

[Out] $-a^3x - 3/4*a^2*b*x^2 - 1/3*a*b^2*x^3 - 1/16*b^3*x^4 - 1/4*a^4*\ln(x)/b + 1/4*(b*x+a)^4*\ln(x)/b$

Rubi [A]

time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {32, 2350, 12, 45}

$$-\frac{a^4 \log(x)}{4b} - a^3x - \frac{3}{4}a^2bx^2 - \frac{1}{3}ab^2x^3 + \frac{\log(x)(a + bx)^4}{4b} - \frac{b^3x^4}{16}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^3*Log[x],x]

[Out] $-(a^3*x) - (3*a^2*b*x^2)/4 - (a*b^2*x^3)/3 - (b^3*x^4)/16 - (a^4*Log[x])/(4*b) + ((a + b*x)^4*Log[x])/(4*b)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2350

Int[((a_.) + Log[(c_.)*(x_)]^(n_.))*(b_.)*((d_.) + (e_.)*(x_)]^(r_.))^(q_.), x_Symbol] := With[{u = IntHide[(d + e*x^r)^q, x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned}
\int (a + bx)^3 \log(x) dx &= \frac{(a + bx)^4 \log(x)}{4b} - \int \frac{(a + bx)^4}{4bx} dx \\
&= \frac{(a + bx)^4 \log(x)}{4b} - \frac{\int \frac{(a+bx)^4}{x} dx}{4b} \\
&= \frac{(a + bx)^4 \log(x)}{4b} - \frac{\int \left(4a^3b + \frac{a^4}{x} + 6a^2b^2x + 4ab^3x^2 + b^4x^3\right) dx}{4b} \\
&= -a^3x - \frac{3}{4}a^2bx^2 - \frac{1}{3}ab^2x^3 - \frac{b^3x^4}{16} - \frac{a^4 \log(x)}{4b} + \frac{(a + bx)^4 \log(x)}{4b}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 81, normalized size = 1.21

$$-a^3x - \frac{3}{4}a^2bx^2 - \frac{1}{3}ab^2x^3 - \frac{b^3x^4}{16} + a^3x \log(x) + \frac{3}{2}a^2bx^2 \log(x) + ab^2x^3 \log(x) + \frac{1}{4}b^3x^4 \log(x)$$

Antiderivative was successfully verified.

`[In] Integrate[(a + b*x)^3*Log[x], x]`

```
[Out] -(a^3*x) - (3*a^2*b*x^2)/4 - (a*b^2*x^3)/3 - (b^3*x^4)/16 + a^3*x*Log[x] +
(3*a^2*b*x^2*Log[x])/2 + a*b^2*x^3*Log[x] + (b^3*x^4*Log[x])/4
```

Mathics [A]

time = 1.98, size = 66, normalized size = 0.99

$$\frac{x(-48a^3 - 36a^2bx - 16ab^2x^2 - 3b^3x^3 + 12\text{Log}[x](4a^3 + 6a^2bx + 4ab^2x^2 + b^3x^3))}{48}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(a + b*x)^3*Log[x], x]')`

```
[Out] x (-48 a ^ 3 - 36 a ^ 2 b x - 16 a b ^ 2 x ^ 2 - 3 b ^ 3 x ^ 3 + 12 Log[x]
(4 a ^ 3 + 6 a ^ 2 b x + 4 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3)) / 48
```

Maple [A]

time = 0.09, size = 69, normalized size = 1.03

method	result	size
risch	$-a^3x - \frac{3a^2bx^2}{4} - \frac{ab^2x^3}{3} - \frac{b^3x^4}{16} - \frac{a^4 \ln(x)}{4b} + \frac{(bx+a)^4 \ln(x)}{4b}$	58
default	$b^3 \left(-\frac{x^4}{16} + \frac{x^4 \ln(x)}{4}\right) + 3ab^2 \left(-\frac{x^3}{9} + \frac{x^3 \ln(x)}{3}\right) + 3a^2b \left(-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}\right) + a^3(-x + x \ln(x))$	69

norman	$a^3x \ln(x) + ab^2x^3 \ln(x) - a^3x - \frac{b^3x^4}{16} - \frac{ab^2x^3}{3} - \frac{3a^2bx^2}{4} + \frac{b^3x^4 \ln(x)}{4} + \frac{3a^2bx^2 \ln(x)}{2}$	72
--------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)^3*ln(x),x,method=_RETURNVERBOSE)`

[Out] $b^3*(-1/16*x^4+1/4*x^4*\ln(x))+3*a*b^2*(-1/9*x^3+1/3*x^3*\ln(x))+3*a^2*b*(-1/4*x^2+1/2*x^2*\ln(x))+a^3*(-x+x*\ln(x))$

Maxima [A]

time = 0.25, size = 69, normalized size = 1.03

$$-\frac{1}{16}b^3x^4 - \frac{1}{3}ab^2x^3 - \frac{3}{4}a^2bx^2 - a^3x + \frac{1}{4}(b^3x^4 + 4ab^2x^3 + 6a^2bx^2 + 4a^3x)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^3*log(x),x, algorithm="maxima")`

[Out] $-1/16*b^3*x^4 - 1/3*a*b^2*x^3 - 3/4*a^2*b*x^2 - a^3*x + 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*\log(x)$

Fricas [A]

time = 0.30, size = 69, normalized size = 1.03

$$-\frac{1}{16}b^3x^4 - \frac{1}{3}ab^2x^3 - \frac{3}{4}a^2bx^2 - a^3x + \frac{1}{4}(b^3x^4 + 4ab^2x^3 + 6a^2bx^2 + 4a^3x)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^3*log(x),x, algorithm="fricas")`

[Out] $-1/16*b^3*x^4 - 1/3*a*b^2*x^3 - 3/4*a^2*b*x^2 - a^3*x + 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*\log(x)$

Sympy [A]

time = 0.08, size = 71, normalized size = 1.06

$$-a^3x - \frac{3a^2bx^2}{4} - \frac{ab^2x^3}{3} - \frac{b^3x^4}{16} + \left(a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4} \right) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)**3*ln(x),x)`

[Out] $-a**3*x - 3*a**2*b*x**2/4 - a*b**2*x**3/3 - b**3*x**4/16 + (a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4)*\log(x)$

Giac [A]

time = 0.00, size = 81, normalized size = 1.21

$$-\frac{1}{16}b^3x^4 - \frac{1}{3}ab^2x^3 - \frac{3}{4}a^2bx^2 - a^3x + a^3x \ln x + \frac{1}{4}b^3x^4 \ln x + ab^2x^3 \ln x + \frac{3}{2}a^2bx^2 \ln x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^3*log(x),x)

[Out] $\frac{1}{4}b^3x^4\log(x) - \frac{1}{16}b^3x^4 + ab^2x^3\log(x) - \frac{1}{3}ab^2x^3 + \frac{3}{2}a^2bx^2\log(x) - \frac{3}{4}a^2bx^2 + a^3x\log(x) - a^3x$

Mupad [B]

time = 0.38, size = 71, normalized size = 1.06

$$a^3 x \ln(x) - \frac{b^3 x^4}{16} - \frac{3 a^2 b x^2}{4} - \frac{a b^2 x^3}{3} - a^3 x + \frac{b^3 x^4 \ln(x)}{4} + \frac{3 a^2 b x^2 \ln(x)}{2} + a b^2 x^3 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(x)*(a + b*x)^3,x)

[Out] $a^3x\log(x) - (b^3x^4)/16 - (3a^2bx^2)/4 - (ab^2x^3)/3 - a^3x + (b^3x^4\log(x))/4 + (3a^2bx^2\log(x))/2 + ab^2x^3\log(x)$

3.613 $\int (-1 - 8 \log^2(x) + 3 \log^3(x)) dx$

Optimal. Leaf size=23

$$-35x + 34x \log(x) - 17x \log^2(x) + 3x \log^3(x)$$

[Out] $-35*x+34*x*\ln(x)-17*x*\ln(x)^2+3*x*\ln(x)^3$

Rubi [A]

time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2333, 2332}

$$-35x + 3x \log^3(x) - 17x \log^2(x) + 34x \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[-1 - 8*\text{Log}[x]^2 + 3*\text{Log}[x]^3, x]$

[Out] $-35*x + 34*x*\text{Log}[x] - 17*x*\text{Log}[x]^2 + 3*x*\text{Log}[x]^3$

Rule 2332

$\text{Int}[\text{Log}[(c_*)*(x_)^{(n_*)}], x_Symbol] \rightarrow \text{Simp}[x*\text{Log}[c*x^n], x] - \text{Simp}[n*x, x] /;$ FreeQ[{c, n}, x]

Rule 2333

$\text{Int}[(a_*) + \text{Log}[(c_*)*(x_)^{(n_*)}]* (b_*)^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{Log}[c*x^n])^p, x] - \text{Dist}[b*n*p, \text{Int}[(a + b*\text{Log}[c*x^n])^{(p-1)}, x], x] /;$ FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]

Rubi steps

$$\begin{aligned} \int (-1 - 8 \log^2(x) + 3 \log^3(x)) dx &= -x + 3 \int \log^3(x) dx - 8 \int \log^2(x) dx \\ &= -x - 8x \log^2(x) + 3x \log^3(x) - 9 \int \log^2(x) dx + 16 \int \log(x) dx \\ &= -17x + 16x \log(x) - 17x \log^2(x) + 3x \log^3(x) + 18 \int \log(x) dx \\ &= -35x + 34x \log(x) - 17x \log^2(x) + 3x \log^3(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 23, normalized size = 1.00

$$-35x + 34x \log(x) - 17x \log^2(x) + 3x \log^3(x)$$

Antiderivative was successfully verified.

[In] Integrate[-1 - 8*Log[x]^2 + 3*Log[x]^3,x]

[Out] -35*x + 34*x*Log[x] - 17*x*Log[x]^2 + 3*x*Log[x]^3

Mathics [A]

time = 1.65, size = 20, normalized size = 0.87

$$x(-35 - 17\text{Log}[x]^2 + 3\text{Log}[x]^3 + 34\text{Log}[x])$$

Antiderivative was successfully verified.

[In] mathics('Integrate[3*Log[x]^3 - 8*Log[x]^2 - 1,x]')

[Out] x(-35 - 17 Log[x]^2 + 3 Log[x]^3 + 34 Log[x])

Maple [A]

time = 0.02, size = 24, normalized size = 1.04

method	result	size
default	$-35x + 34x \ln(x) - 17x \ln(x)^2 + 3x \ln(x)^3$	24
norman	$-35x + 34x \ln(x) - 17x \ln(x)^2 + 3x \ln(x)^3$	24
risch	$-35x + 34x \ln(x) - 17x \ln(x)^2 + 3x \ln(x)^3$	24

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1-8*ln(x)^2+3*ln(x)^3,x,method=_RETURNVERBOSE)

[Out] -35*x+34*x*ln(x)-17*x*ln(x)^2+3*x*ln(x)^3

Maxima [A]

time = 0.25, size = 36, normalized size = 1.57

$$3(\log(x)^3 - 3\log(x)^2 + 6\log(x) - 6)x - 8(\log(x)^2 - 2\log(x) + 2)x - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1-8*log(x)^2+3*log(x)^3,x, algorithm="maxima")

[Out] 3*(log(x)^3 - 3*log(x)^2 + 6*log(x) - 6)*x - 8*(log(x)^2 - 2*log(x) + 2)*x - x

Fricas [A]

time = 0.30, size = 23, normalized size = 1.00

$$3x \log(x)^3 - 17x \log(x)^2 + 34x \log(x) - 35x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1-8*log(x)^2+3*log(x)^3,x, algorithm="fricas")

[Out] 3*x*log(x)^3 - 17*x*log(x)^2 + 34*x*log(x) - 35*x

Sympy [A]

time = 0.06, size = 26, normalized size = 1.13

$$3x \log(x)^3 - 17x \log(x)^2 + 34x \log(x) - 35x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1-8*ln(x)**2+3*ln(x)**3,x)

[Out] 3*x*log(x)**3 - 17*x*log(x)**2 + 34*x*log(x) - 35*x

Giac [A]

time = 0.00, size = 48, normalized size = 2.09

$$-x - 8(x \ln^2 x - 2x \ln x + 2x) + 3(x \ln^3 x - 3x \ln^2 x + 6x \ln x - 6x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1-8*log(x)^2+3*log(x)^3,x)

[Out] 3*x*log(x)^3 - 17*x*log(x)^2 + 34*x*log(x) - 35*x

Mupad [B]

time = 0.29, size = 20, normalized size = 0.87

$$x(3 \ln(x)^3 - 17 \ln(x)^2 + 34 \ln(x) - 35)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(3*log(x)^3 - 8*log(x)^2 - 1,x)

[Out] x*(34*log(x) - 17*log(x)^2 + 3*log(x)^3 - 35)

3.614 $\int (1 + x^4) (1 - 2 \log(x) + \log^3(x)) dx$

Optimal. Leaf size=60

$$-3x + \frac{169x^5}{625} + 4x \log(x) - \frac{44}{125}x^5 \log(x) - 3x \log^2(x) - \frac{3}{25}x^5 \log^2(x) + x \log^3(x) + \frac{1}{5}x^5 \log^3(x)$$

[Out] $-3*x+169/625*x^5+4*x*\ln(x)-44/125*x^5*\ln(x)-3*x*\ln(x)^2-3/25*x^5*\ln(x)^2+x*\ln(x)^3+1/5*x^5*\ln(x)^3$

Rubi [A]

time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6874, 2350, 12, 2367, 2333, 2332, 2342, 2341}

$$\frac{169x^5}{625} + \frac{1}{5}x^5 \log^3(x) - \frac{3}{25}x^5 \log^2(x) - \frac{44}{125}x^5 \log(x) - 3x + x \log^3(x) - 3x \log^2(x) + 4x \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)*(1 - 2*Log[x] + Log[x]^3), x]

[Out] $-3*x + (169*x^5)/625 + 4*x*\text{Log}[x] - (44*x^5*\text{Log}[x])/125 - 3*x*\text{Log}[x]^2 - (3*x^5*\text{Log}[x]^2)/25 + x*\text{Log}[x]^3 + (x^5*\text{Log}[x]^3)/5$

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2332

Int[Log[(c_)*(x_)^(n_)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]

Rule 2333

Int[((a_) + Log[(c_)*(x_)^(n_)])*(b_)^(p_), x_Symbol] :> Simp[x*(a + b*Log[c*x^n])^p, x] - Dist[b^n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]

Rule 2341

Int[((a_) + Log[(c_)*(x_)^(n_)])*(b_)*((d_)*(x_))^(m_), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:= Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2350

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.), x_Symbol]
:= With[{u = IntHide[(d + e*x^r)^q, x]}, Dist[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(q_.), x_Symbol]
:= With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 6874

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int (1 + x^4) (1 - 2 \log(x) + \log^3(x)) \, dx &= \int (1 + x^4 - 2(1 + x^4) \log(x) + (1 + x^4) \log^3(x)) \, dx \\
&= x + \frac{x^5}{5} - 2 \int (1 + x^4) \log(x) \, dx + \int (1 + x^4) \log^3(x) \, dx \\
&= x + \frac{x^5}{5} - \frac{2}{5} (5x + x^5) \log(x) + 2 \int \frac{1}{5} (5 + x^4) \, dx + \int (\log^3(x) + \log^2(x)) \, dx \\
&= x + \frac{x^5}{5} - \frac{2}{5} (5x + x^5) \log(x) + \frac{2}{5} \int (5 + x^4) \, dx + \int \log^3(x) \, dx + \int \log^2(x) \, dx \\
&= 3x + \frac{7x^5}{25} - \frac{2}{5} (5x + x^5) \log(x) + x \log^3(x) + \frac{1}{5} x^5 \log^3(x) - \frac{3}{5} \int \log^2(x) \, dx \\
&= 3x + \frac{7x^5}{25} - \frac{2}{5} (5x + x^5) \log(x) - 3x \log^2(x) - \frac{3}{25} x^5 \log^2(x) + x \log^2(x) + \frac{2}{5} \int \log(x) \, dx \\
&= -3x + \frac{169x^5}{625} + 6x \log(x) + \frac{6}{125} x^5 \log(x) - \frac{2}{5} (5x + x^5) \log(x) - \frac{3}{25} x^5 \log^2(x) + x \log^2(x) + \frac{2}{5} x \log(x) - \frac{2}{5} x
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 60, normalized size = 1.00

$$-3x + \frac{169x^5}{625} + 4x \log(x) - \frac{44}{125}x^5 \log(x) - 3x \log^2(x) - \frac{3}{25}x^5 \log^2(x) + x \log^3(x) + \frac{1}{5}x^5 \log^3(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^4)*(1 - 2*Log[x] + Log[x]^3), x]

[Out] -3*x + (169*x^5)/625 + 4*x*Log[x] - (44*x^5*Log[x])/125 - 3*x*Log[x]^2 - (3*x^5*Log[x]^2)/25 + x*Log[x]^3 + (x^5*Log[x]^3)/5

Mathics [A]

time = 1.90, size = 43, normalized size = 0.72

$$\frac{x(-1875 + 169x^4 - 75(25 + x^4)\text{Log}[x]^2 - 20\text{Log}[x](-125 + 11x^4) + 125(5 + x^4)\text{Log}[x]^3)}{625}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(x^4 + 1)*(Log[x]^3 - 2*Log[x] + 1), x]')

[Out] x(-1875 + 169 x ^ 4 - 75 (25 + x ^ 4) Log[x] ^ 2 - 20 Log[x] (-125 + 11 x ^ 4) + 125 (5 + x ^ 4) Log[x] ^ 3) / 625

Maple [A]

time = 0.01, size = 53, normalized size = 0.88

method	result	size
risch	$(\frac{1}{5}x^5 + x) \ln(x)^3 + (-\frac{3}{25}x^5 - 3x) \ln(x)^2 + (-\frac{44}{125}x^5 + 4x) \ln(x) + \frac{169x^5}{625} - 3x$	48
default	$-3x + \frac{169x^5}{625} + 4x \ln(x) - \frac{44x^5 \ln(x)}{125} - 3x \ln(x)^2 - \frac{3x^5 \ln(x)^2}{25} + x \ln(x)^3 + \frac{x^5 \ln(x)^3}{5}$	53
norman	$-3x + \frac{169x^5}{625} + 4x \ln(x) - \frac{44x^5 \ln(x)}{125} - 3x \ln(x)^2 - \frac{3x^5 \ln(x)^2}{25} + x \ln(x)^3 + \frac{x^5 \ln(x)^3}{5}$	53

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+1)*(1-2*ln(x)+ln(x)^3), x, method=_RETURNVERBOSE)

[Out] -3*x+169/625*x^5+4*x*ln(x)-44/125*x^5*ln(x)-3*x*ln(x)^2-3/25*x^5*ln(x)^2+x*ln(x)^3+1/5*x^5*ln(x)^3

Maxima [A]

time = 0.24, size = 66, normalized size = 1.10

$$\frac{1}{625} (125 \log(x)^3 - 75 \log(x)^2 + 30 \log(x) - 6)x^5 - \frac{2}{25}x^5(5 \log(x) - 1) + \frac{1}{5}x^5 + (\log(x)^3 - 3 \log(x)^2 + 6 \log(x) - 6)x - 2x(\log(x) - 1) + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)*(1-2*log(x)+log(x)^3), x, algorithm="maxima")

[Out] $\frac{1}{625}(125\log(x)^3 - 75\log(x)^2 + 30\log(x) - 6)x^5 - \frac{2}{25}x^5(5\log(x) - 1) + \frac{1}{5}x^5 + (\log(x)^3 - 3\log(x)^2 + 6\log(x) - 6)x - 2x(\log(x) - 1) + x$

Fricas [A]

time = 0.31, size = 48, normalized size = 0.80

$$\frac{169}{625}x^5 + \frac{1}{5}(x^5 + 5x)\log(x)^3 - \frac{3}{25}(x^5 + 25x)\log(x)^2 - \frac{4}{125}(11x^5 - 125x)\log(x) - 3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+1)*(1-2*log(x)+log(x)^3),x, algorithm="fricas")`

[Out] $169/625x^5 + 1/5(x^5 + 5x)\log(x)^3 - 3/25(x^5 + 25x)\log(x)^2 - 4/125(11x^5 - 125x)\log(x) - 3x$

Sympy [A]

time = 0.09, size = 51, normalized size = 0.85

$$\frac{169x^5}{625} - 3x + \left(-\frac{44x^5}{125} + 4x\right)\log(x) + \left(-\frac{3x^5}{25} - 3x\right)\log(x)^2 + \left(\frac{x^5}{5} + x\right)\log(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**4+1)*(1-2*ln(x)+ln(x)**3),x)`

[Out] $169x^5/625 - 3x + (-44x^5/125 + 4x)\log(x) + (-3x^5/25 - 3x)\log(x)^2 + (x^5/5 + x)\log(x)^3$

Giac [A]

time = 0.00, size = 77, normalized size = 1.28

$$\frac{7}{25}x^5 + x\ln^3 x - 3x\ln^2 x + 6x\ln x + \frac{1}{5}x^5\ln^3 x - \frac{3}{25}x^5\ln^2 x - \frac{6}{625}x^5 - 2x\ln x - \frac{44}{125}x^5\ln x - 3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+1)*(1-2*log(x)+log(x)^3),x)`

[Out] $\frac{1}{5}x^5\log(x)^3 - \frac{3}{25}x^5\log(x)^2 - \frac{44}{125}x^5\log(x) + \frac{169}{625}x^5 + x\log(x)^3 - 3x\log(x)^2 + 4x\log(x) - 3x$

Mupad [B]

time = 0.37, size = 51, normalized size = 0.85

$$\frac{x(125x^4\ln(x)^3 - 75x^4\ln(x)^2 - 220x^4\ln(x) + 169x^4 + 625\ln(x)^3 - 1875\ln(x)^2 + 2500\ln(x) - 1875)}{625}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4 + 1)*(log(x)^3 - 2*log(x) + 1),x)`

[Out] $(x(2500\log(x) - 220x^4\log(x) - 1875\log(x)^2 + 625\log(x)^3 - 75x^4\log(x)^2 + 125x^4\log(x)^3 + 169x^4 - 1875))/625$

$$3.615 \quad \int \frac{1}{x^3 \log^4(x)} dx$$

Optimal. Leaf size=43

$$-\frac{4}{3}\text{Ei}(-2\log(x)) - \frac{1}{3x^2 \log^3(x)} + \frac{1}{3x^2 \log^2(x)} - \frac{2}{3x^2 \log(x)}$$

[Out] $-4/3*\text{Ei}(-2*\ln(x))-1/3/x^2/\ln(x)^3+1/3/x^2/\ln(x)^2-2/3/x^2/\ln(x)$

Rubi [A]

time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2343, 2346, 2209}

$$-\frac{4}{3}\text{Ei}(-2\log(x)) - \frac{1}{3x^2 \log^3(x)} + \frac{1}{3x^2 \log^2(x)} - \frac{2}{3x^2 \log(x)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*Log[x]^4),x]

[Out] $(-4*\text{ExpIntegralEi}[-2*\text{Log}[x]])/3 - 1/(3*x^2*\text{Log}[x]^3) + 1/(3*x^2*\text{Log}[x]^2) - 2/(3*x^2*\text{Log}[x])$

Rule 2209

Int[(F_)^((g_.)*(e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[\$UseGamma]

Rule 2343

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_)^(m_.)), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] - Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

Rule 2346

Int[((a_.) + Log[(c_.)*(x_)])*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \log^4(x)} dx &= -\frac{1}{3x^2 \log^3(x)} - \frac{2}{3} \int \frac{1}{x^3 \log^3(x)} dx \\
&= -\frac{1}{3x^2 \log^3(x)} + \frac{1}{3x^2 \log^2(x)} + \frac{2}{3} \int \frac{1}{x^3 \log^2(x)} dx \\
&= -\frac{1}{3x^2 \log^3(x)} + \frac{1}{3x^2 \log^2(x)} - \frac{2}{3x^2 \log(x)} - \frac{4}{3} \int \frac{1}{x^3 \log(x)} dx \\
&= -\frac{1}{3x^2 \log^3(x)} + \frac{1}{3x^2 \log^2(x)} - \frac{2}{3x^2 \log(x)} - \frac{4}{3} \text{Subst} \left(\int \frac{e^{-2x}}{x} dx, x, \log(x) \right) \\
&= -\frac{4}{3} \text{Ei}(-2 \log(x)) - \frac{1}{3x^2 \log^3(x)} + \frac{1}{3x^2 \log^2(x)} - \frac{2}{3x^2 \log(x)}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 43, normalized size = 1.00

$$-\frac{4}{3} \text{Ei}(-2 \log(x)) - \frac{1}{3x^2 \log^3(x)} + \frac{1}{3x^2 \log^2(x)} - \frac{2}{3x^2 \log(x)}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x^3*Log[x]^4),x]`

```
[Out] (-4*ExpIntegralEi[-2*Log[x]])/3 - 1/(3*x^2*Log[x]^3) + 1/(3*x^2*Log[x]^2) - 2/(3*x^2*Log[x])
```

Mathics [A]

time = 1.97, size = 33, normalized size = 0.77

$$\frac{-1 - 4x^2 \text{ExpIntegralEi}[-2 \text{Log}[x]] \text{Log}[x]^3 + \text{Log}[x] - 2 \text{Log}[x]^2}{3x^2 \text{Log}[x]^3}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[1/(x^3*Log[x]^4),x]')`

```
[Out] (-1 - 4 x ^ 2 ExpIntegralEi[-2 Log[x]] Log[x] ^ 3 + Log[x] - 2 Log[x] ^ 2) / (3 x ^ 2 Log[x] ^ 3)
```

Maple [A]

time = 0.01, size = 37, normalized size = 0.86

method	result	size
risch	$-\frac{2 \ln(x)^2 - \ln(x) + 1}{3x^2 \ln(x)^3} + \frac{4 \text{expIntegral}(1, 2 \ln(x))}{3}$	31

default	$-\frac{1}{3x^2 \ln(x)^3} + \frac{1}{3x^2 \ln(x)^2} - \frac{2}{3x^2 \ln(x)} + \frac{4 \operatorname{expIntegral}(1, 2 \ln(x))}{3}$	37
---------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/ln(x)^4,x,method=_RETURNVERBOSE)`

[Out] $-1/3/x^2/\ln(x)^3 + 1/3/x^2/\ln(x)^2 - 2/3/x^2/\ln(x) + 4/3 * \operatorname{Ei}(1, 2 * \ln(x))$

Maxima [A]

time = 0.26, size = 8, normalized size = 0.19

$$-8 \Gamma(-3, 2 \log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/log(x)^4,x, algorithm="maxima")`

[Out] $-8 * \operatorname{gamma}(-3, 2 * \log(x))$

Fricas [A]

time = 0.30, size = 34, normalized size = 0.79

$$-\frac{4 x^2 \log(x)^3 \log_integral\left(\frac{1}{x^2}\right) + 2 \log(x)^2 - \log(x) + 1}{3 x^2 \log(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/log(x)^4,x, algorithm="fricas")`

[Out] $-1/3 * (4 * x^2 * \log(x)^3 * \log_integral(x^{-2})) + 2 * \log(x)^2 - \log(x) + 1) / (x^2 * \log(x)^3)$

Sympy [A]

time = 0.39, size = 32, normalized size = 0.74

$$-\frac{4 \operatorname{Ei}(-2 \log(x))}{3} + \frac{-2 \log(x)^2 + \log(x) - 1}{3 x^2 \log(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/ln(x)**4,x)`

[Out] $-4 * \operatorname{Ei}(-2 * \log(x)) / 3 + (-2 * \log(x)**2 + \log(x) - 1) / (3 * x**2 * \log(x)**3)$

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(x)^4,x)

[Out] Could not integrate

Mupad [B]

time = 0.28, size = 29, normalized size = 0.67

$$-\frac{4 \operatorname{ei}(-2 \ln(x))}{3} - \frac{\frac{2 \ln(x)^2}{3} - \frac{\ln(x)}{3} + \frac{1}{3}}{x^2 \ln(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*log(x)^4),x)

[Out] - (4*ei(-2*log(x)))/3 - ((2*log(x)^2)/3 - log(x)/3 + 1/3)/(x^2*log(x)^3)

3.616 $\int \frac{\log(x)}{a+bx} dx$

Optimal. Leaf size=29

$$\frac{\log(x) \log\left(1 + \frac{bx}{a}\right)}{b} + \frac{\text{Li}_2\left(-\frac{bx}{a}\right)}{b}$$

[Out] $\ln(x) \cdot \ln(1+b*x/a)/b + \text{polylog}(2, -b*x/a)/b$

Rubi [A]

time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2354, 2438}

$$\frac{\text{Li}_2\left(-\frac{bx}{a}\right)}{b} + \frac{\log(x) \log\left(\frac{bx}{a} + 1\right)}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Log}[x]/(a + b*x), x]$

[Out] $(\text{Log}[x] * \text{Log}[1 + (b*x)/a])/b + \text{PolyLog}[2, -((b*x)/a)]/b$

Rule 2354

$\text{Int}[(a_. + \text{Log}[(c_.)*(x_)^{(n_.)}]*(b_.))^{(p_.)}/((d_.) + (e_.)*(x_)), x_Symbol] \rightarrow \text{Simp}[\text{Log}[1 + e*(x/d)]*(a + b*\text{Log}[c*x^n])^p/e, x] - \text{Dist}[b*n*(p/e), \text{Int}[\text{Log}[1 + e*(x/d)]*(a + b*\text{Log}[c*x^n])^{(p-1)}/x, x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x] \&\& \text{IGtQ}[p, 0]$

Rule 2438

$\text{Int}[\text{Log}[(c_.)*((d_.) + (e_.)*(x_)^{(n_.)})]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$

Rubi steps

$$\begin{aligned} \int \frac{\log(x)}{a+bx} dx &= \frac{\log(x) \log\left(1 + \frac{bx}{a}\right)}{b} - \int \frac{\log\left(1 + \frac{bx}{a}\right)}{x} dx \\ &= \frac{\log(x) \log\left(1 + \frac{bx}{a}\right)}{b} + \frac{\text{Li}_2\left(-\frac{bx}{a}\right)}{b} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 30, normalized size = 1.03

$$\frac{\log(x) \log\left(\frac{a+bx}{a}\right)}{b} + \frac{\text{Li}_2\left(-\frac{bx}{a}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(a + b*x),x]
```

```
[Out] (Log[x]*Log[(a + b*x)/a])/b + PolyLog[2, -((b*x)/a)]/b
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

time = 6.42, size = 284, normalized size = 9.79

```
Piecewise[{{(PolyLog[2, -((b*x)/a)]/b + Log[x]*Log[(a + b*x)/a])/b, Abs[a/b + x] < 1 && Abs[b/(a + b*x)] < 1}, {(I Pi Log[a/b + x] - PolyLog[2, (a + b*x)/a] + Log[a/b] Log[a/b + x])/b, Abs[a/b + x] < 1}, {(-I Pi Log[b/(a + b*x)] - Log[a/b] Log[b/(a + b*x)] - PolyLog[2, (a + b*x)/a])/b, Abs[b/(a + b*x)] < 1}, -Log[a/b] MeijerG[{{}, {1, 1}], {{0, 0}, {}], a/b + x/b - PolyLog[2, b(a/b + x)/a]/b + Log[a/b] MeijerG[{{1, 1}, {}], {{}, {0, 0}}, a/b + x/b - I Pi MeijerG[{{}, {1, 1}], {{0, 0}, {}], a/b + x/b + I Pi MeijerG[{{1, 1}, {}], {{}, {0, 0}}, a/b + x/b]
```

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Log[x]/(a + b*x),x]')
```

```
[Out] Piecewise[{{-polylog[2, (a + b x) / a] / b, Abs[a / b + x] < 1 && Abs[b / (a + b x)] < 1}, {(I Pi Log[a / b + x] - polylog[2, (a + b x) / a] + Log[a / b] Log[a / b + x]) / b, Abs[a / b + x] < 1}, {(-I Pi Log[b / (a + b x)] - Log[a / b] Log[b / (a + b x)] - polylog[2, (a + b x) / a]) / b, Abs[b / (a + b x)] < 1}, -Log[a / b] meijerg[{{}, {1, 1}], {{0, 0}, {}], a / b + x / b - polylog[2, b (a / b + x) / a] / b + Log[a / b] meijerg[{{1, 1}, {}], {{}, {0, 0}}, a / b + x] / b - I Pi meijerg[{{}, {1, 1}], {{0, 0}, {}], a / b + x] / b + I Pi meijerg[{{1, 1}, {}], {{}, {0, 0}}, a / b + x] / b]
```

Maple [A]

time = 0.08, size = 32, normalized size = 1.10

method	result	size
default	$\frac{\operatorname{dilog}\left(\frac{bx+a}{a}\right)}{b} + \frac{\ln(x) \ln\left(\frac{bx+a}{a}\right)}{b}$	32
risch	$\frac{\operatorname{dilog}\left(\frac{bx+a}{a}\right)}{b} + \frac{\ln(x) \ln\left(\frac{bx+a}{a}\right)}{b}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] dilog((b*x+a)/a)/b+ln(x)*ln((b*x+a)/a)/b
```

Maxima [A]

time = 0.25, size = 25, normalized size = 0.86

$$\frac{\log\left(\frac{bx}{a} + 1\right) \log(x) + \operatorname{Li}_2\left(-\frac{bx}{a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(b*x+a),x, algorithm="maxima")
```

[Out] $(\log(b*x/a + 1)*\log(x) + \text{dilog}(-b*x/a))/b$

Fricas [F]

time = 0.65, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x)/(b*x+a),x, algorithm="fricas")`

[Out] `integral(log(x)/(b*x + a), x)`

Sympy [A]

time = 3.79, size = 177, normalized size = 6.10

$$\left\{ \begin{array}{ll} -\frac{\text{Li}_2\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} & \text{for } \frac{1}{\left|\frac{a}{b}+x\right|} < 1 \wedge \left|\frac{a}{b}+x\right| < 1 \\ \frac{\log\left(\frac{a}{b}\right)\log\left(\frac{a}{b}+x\right)}{b} + \frac{i\pi\log\left(\frac{a}{b}+x\right)}{b} - \frac{\text{Li}_2\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} & \text{for } \left|\frac{a}{b}+x\right| < 1 \\ -\frac{\log\left(\frac{a}{b}\right)\log\left(\frac{1}{\frac{a}{b}+x}\right)}{b} - \frac{i\pi\log\left(\frac{1}{\frac{a}{b}+x}\right)}{b} - \frac{\text{Li}_2\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} & \text{for } \frac{1}{\left|\frac{a}{b}+x\right|} < 1 \\ -\frac{G_{2,2}^{2,0}\left(0,0\left|\frac{1,1}{\frac{a}{b}+x}\right.\right)\log\left(\frac{a}{b}\right)}{b} - \frac{i\pi G_{2,2}^{2,0}\left(0,0\left|\frac{1,1}{\frac{a}{b}+x}\right.\right)}{b} + \frac{G_{2,2}^{0,2}\left(1,1\left|0,0\right.\frac{a}{b}+x\right)\log\left(\frac{a}{b}\right)}{b} + \frac{i\pi G_{2,2}^{0,2}\left(1,1\left|0,0\right.\frac{a}{b}+x\right)}{b} - \frac{\text{Li}_2\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(x)/(b*x+a),x)`

[Out] `Piecewise((-polylog(2, b*(a/b + x)/a)/b, (Abs(a/b + x) < 1) & (1/Abs(a/b + x) < 1)), (log(a/b)*log(a/b + x)/b + I*pi*log(a/b + x)/b - polylog(2, b*(a/b + x)/a)/b, Abs(a/b + x) < 1), (-log(a/b)*log(1/(a/b + x))/b - I*pi*log(1/(a/b + x))/b - polylog(2, b*(a/b + x)/a)/b, 1/Abs(a/b + x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), a/b + x)*log(a/b)/b - I*pi*meijerg(((), (1, 1)), ((0, 0), ()), a/b + x)/b + meijerg(((1, 1), ()), ((), (0, 0)), a/b + x)*log(a/b)/b + I*pi*meijerg(((1, 1), ()), ((), (0, 0)), a/b + x)/b - polylog(2, b*(a/b + x)/a)/b, True))`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x)/(b*x+a),x)`

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln(x)}{a + b x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(x)/(a + b*x), x)

[Out] int(log(x)/(a + b*x), x)

$$3.617 \quad \int \frac{\log(x)}{(a+bx)^2} dx$$

Optimal. Leaf size=29

$$\frac{x \log(x)}{a(a+bx)} - \frac{\log(a+bx)}{ab}$$

[Out] x*ln(x)/a/(b*x+a)-ln(b*x+a)/a/b

Rubi [A]

time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2351, 31}

$$\frac{x \log(x)}{a(a+bx)} - \frac{\log(a+bx)}{ab}$$

Antiderivative was successfully verified.

[In] Int[Log[x]/(a + b*x)^2,x]

[Out] (x*Log[x])/(a*(a + b*x)) - Log[a + b*x]/(a*b)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2351

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x_Symbol] := Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x] && EqQ[r*(q + 1) + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{\log(x)}{(a+bx)^2} dx &= \frac{x \log(x)}{a(a+bx)} - \frac{\int \frac{1}{a+bx} dx}{a} \\ &= \frac{x \log(x)}{a(a+bx)} - \frac{\log(a+bx)}{ab} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 27, normalized size = 0.93

$$\frac{\frac{x \log(x)}{a+bx} - \frac{\log(a+bx)}{b}}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Log[x]/(a + b*x)^2,x]

[Out] ((x*Log[x])/(a + b*x) - Log[a + b*x]/b)/a

Mathics [A]

time = 1.92, size = 41, normalized size = 1.41

$$\frac{-a \operatorname{Log}[x] + (a + bx) \left(\operatorname{Log}[x] - \operatorname{Log}\left[\frac{a+bx}{b}\right] \right)}{ab(a + bx)}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Log[x]/(a + b*x)^2,x]')

[Out] (-a Log[x] + (a + b x) (Log[x] - Log[(a + b x) / b])) / (a b (a + b x))

Maple [A]

time = 0.08, size = 30, normalized size = 1.03

method	result	size
default	$\frac{x \ln(x)}{a(bx+a)} - \frac{\ln(bx+a)}{ab}$	30
norman	$\frac{x \ln(x)}{a(bx+a)} - \frac{\ln(bx+a)}{ab}$	30
risch	$-\frac{\ln(x)}{b(bx+a)} - \frac{\ln(bx+a)}{ab} + \frac{\ln(-x)}{ba}$	41

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(x)/(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] x*ln(x)/a/(b*x+a)-ln(b*x+a)/a/b

Maxima [A]

time = 0.26, size = 38, normalized size = 1.31

$$-\frac{\frac{\log(bx+a)}{a} - \frac{\log(x)}{a}}{b} - \frac{\log(x)}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)/(b*x+a)^2,x, algorithm="maxima")

[Out] -(log(b*x + a)/a - log(x)/a)/b - log(x)/((b*x + a)*b)

Fricas [A]

time = 0.32, size = 34, normalized size = 1.17

$$\frac{bx \log(x) - (bx + a) \log(bx + a)}{ab^2x + a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)/(b*x+a)^2,x, algorithm="fricas")

[Out] (b*x*log(x) - (b*x + a)*log(b*x + a))/(a*b^2*x + a^2*b)

Sympy [A]

time = 0.12, size = 24, normalized size = 0.83

$$-\frac{\log(x)}{ab + b^2x} + \frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(x)/(b*x+a)**2,x)

[Out] -log(x)/(a*b + b**2*x) + (log(x) - log(a/b + x))/(a*b)

Giac [A]

time = 0.00, size = 37, normalized size = 1.28

$$\frac{\ln|x|}{ab} - \frac{b \ln|xb + a|}{ab^2} - \frac{\ln x}{b(xb + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)/(b*x+a)^2,x)

[Out] -log(x)/((b*x + a)*b) - log(abs(b*x + a))/(a*b) + log(abs(x))/(a*b)

Mupad [B]

time = 0.39, size = 35, normalized size = 1.21

$$\frac{x^2 \ln(x)}{a(bx^2 + ax)} - \frac{\ln(a + bx)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(x)/(a + b*x)^2,x)

[Out] (x^2*log(x))/(a*(a*x + b*x^2)) - log(a + b*x)/(a*b)

$$3.618 \quad \int \frac{\log^n(x)}{x} dx$$

Optimal. Leaf size=12

$$\frac{\log^{1+n}(x)}{1+n}$$

[Out] $\ln(x)^{(1+n)}/(1+n)$

Rubi [A]

time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2339, 30}

$$\frac{\log^{n+1}(x)}{n+1}$$

Antiderivative was successfully verified.

[In] Int[Log[x]^n/x,x]

[Out] Log[x]^(1+n)/(1+n)

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2339

Int[((a_) + Log[(c_)*(x_)^(n_)])*(b_)^(p_)/(x_), x_Symbol] := Dist[1/(b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{\log^n(x)}{x} dx &= \text{Subst} \left(\int x^n dx, x, \log(x) \right) \\ &= \frac{\log^{1+n}(x)}{1+n} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 12, normalized size = 1.00

$$\frac{\log^{1+n}(x)}{1+n}$$

Antiderivative was successfully verified.

[In] Integrate[Log[x]^n/x,x]

[Out] Log[x]^(1 + n)/(1 + n)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 1.93, size = 21, normalized size = 1.75

$$\text{Piecewise} \left[\left\{ \left\{ \frac{\text{Log}[x]^{1+n}}{1+n}, n \neq -1 \right\} \right\}, \text{Log}[\text{Log}[x]] \right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Log[x]^n/x,x]')

[Out] Piecewise[{{Log[x]^(1 + n)/(1 + n), n != -1}}, Log[Log[x]]]

Maple [A]

time = 0.02, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$\frac{\ln(x)^{1+n}}{1+n}$	13
default	$\frac{\ln(x)^{1+n}}{1+n}$	13
risch	$\frac{\ln(x) \ln(x)^n}{1+n}$	13
norman	$\frac{\ln(x) e^{n \ln(\ln(x))}}{1+n}$	15

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(x)^n/x,x,method=_RETURNVERBOSE)

[Out] ln(x)^(1+n)/(1+n)

Maxima [A]

time = 0.24, size = 12, normalized size = 1.00

$$\frac{\log(x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)^n/x,x, algorithm="maxima")

[Out] log(x)^(n + 1)/(n + 1)

Fricas [A]

time = 0.32, size = 12, normalized size = 1.00

$$\frac{\log(x)^n \log(x)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(log(x)^n/x,x, algorithm="fricas")``[Out] log(x)^n*log(x)/(n + 1)`**Sympy** [A]

time = 0.45, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\log(x)^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(\log(x)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(ln(x)**n/x,x)``[Out] Piecewise((log(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(log(x)), True))`**Giac** [A]

time = 0.00, size = 11, normalized size = 0.92

$$\frac{(\ln x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(log(x)^n/x,x)``[Out] log(x)^(n + 1)/(n + 1)`**Mupad** [B]

time = 0.33, size = 22, normalized size = 1.83

$$\begin{cases} \ln(\ln(x)) & \text{if } n = -1 \\ \frac{\ln(x)^{n+1}}{n+1} & \text{if } n \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(log(x)^n/x,x)``[Out] piecewise(n == -1, log(log(x)), n ~= -1, log(x)^(n + 1)/(n + 1))`

$$3.619 \quad \int \frac{(a+b \log(x))^n}{x} dx$$

Optimal. Leaf size=19

$$\frac{(a + b \log(x))^{1+n}}{b(1+n)}$$

[Out] (a+b*ln(x))^(1+n)/b/(1+n)

Rubi [A]

time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2339, 30}

$$\frac{(a + b \log(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Log[x])^n/x,x]

[Out] (a + b*Log[x])^(1 + n)/(b*(1 + n))

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2339

Int[((a_) + Log[(c_)*(x_)^(n_)])*(b_)^(p_)/(x_), x_Symbol] := Dist[1/(b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{(a + b \log(x))^n}{x} dx &= \frac{\text{Subst}(\int x^n dx, x, a + b \log(x))}{b} \\ &= \frac{(a + b \log(x))^{1+n}}{b(1+n)} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 19, normalized size = 1.00

$$\frac{(a + b \log(x))^{1+n}}{b(1+n)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Log[x])^n/x,x]

[Out] (a + b*Log[x])^(1 + n)/(b*(1 + n))

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a + b*Log[x])^n/x,x]')

[Out] cought exception:

Maple [A]

time = 0.02, size = 20, normalized size = 1.05

method	result	size
derivativedivides	$\frac{(a+b \ln(x))^{1+n}}{b(1+n)}$	20
default	$\frac{(a+b \ln(x))^{1+n}}{b(1+n)}$	20
risch	$\frac{(a+b \ln(x))(a+b \ln(x))^n}{b(1+n)}$	24
norman	$\frac{\ln(x)e^{n \ln(a+b \ln(x))}}{1+n} + \frac{a e^{n \ln(a+b \ln(x))}}{b(1+n)}$	40

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*ln(x))^n/x,x,method=_RETURNVERBOSE)

[Out] (a+b*ln(x))^(1+n)/b/(1+n)

Maxima [A]

time = 0.24, size = 19, normalized size = 1.00

$$\frac{(b \log(x) + a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*log(x))^n/x,x, algorithm="maxima")

[Out] (b*log(x) + a)^(n + 1)/(b*(n + 1))

Fricas [A]

time = 0.32, size = 22, normalized size = 1.16

$$\frac{(b \log(x) + a)(b \log(x) + a)^n}{bn + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(x))^n/x,x, algorithm="fricas")`

[Out] `(b*log(x) + a)*(b*log(x) + a)^n/(b*n + b)`

Sympy [A]

time = 0.63, size = 36, normalized size = 1.89

$$- \begin{cases} -a^n \log(x) & \text{for } b = 0 \\ \begin{cases} \frac{(a+b \log(x))^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(a + b \log(x)) & \text{otherwise} \end{cases} & \text{otherwise} \\ -\frac{\log(a + b \log(x))}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(x))**n/x,x)`

[Out] `-Piecewise((-a**n*log(x), Eq(b, 0)), (-Piecewise(((a + b*log(x))**(n + 1)/(n + 1), Ne(n, -1)), (log(a + b*log(x)), True))/b, True))`

Giac [A]

time = 0.00, size = 17, normalized size = 0.89

$$\frac{(a + b \ln x)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(x))^n/x,x)`

[Out] `(b*log(x) + a)^(n + 1)/(b*(n + 1))`

Mupad [B]

time = 0.45, size = 19, normalized size = 1.00

$$\frac{(a + b \ln(x))^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*log(x))^n/x,x)`

[Out] `(a + b*log(x))^(n + 1)/(b*(n + 1))`

$$3.620 \quad \int \frac{1}{x(a+b \log(x))} dx$$

Optimal. Leaf size=11

$$\frac{\log(a + b \log(x))}{b}$$

[Out] ln(a+b*ln(x))/b

Rubi [A]

time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2339, 29}

$$\frac{\log(a + b \log(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*Log[x])),x]

[Out] Log[a + b*Log[x]]/b

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 2339

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{x(a + b \log(x))} dx &= \frac{\text{Subst} \left(\int \frac{1}{x} dx, x, a + b \log(x) \right)}{b} \\ &= \frac{\log(a + b \log(x))}{b} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 11, normalized size = 1.00

$$\frac{\log(a + b \log(x))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*Log[x])),x]

[Out] Log[a + b*Log[x]]/b

Mathics [A]

time = 1.64, size = 13, normalized size = 1.18

$$\frac{\text{Log}\left[\frac{a}{b} + \text{Log}[x]\right]}{b}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x*(a + b*Log[x])),x]')

[Out] Log[a / b + Log[x]] / b

Maple [A]

time = 0.02, size = 12, normalized size = 1.09

method	result	size
derivativdivides	$\frac{\ln(a+b\ln(x))}{b}$	12
default	$\frac{\ln(a+b\ln(x))}{b}$	12
norman	$\frac{\ln(a+b\ln(x))}{b}$	12
risch	$\frac{\ln(\ln(x)+\frac{a}{b})}{b}$	14

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b*ln(x)),x,method=_RETURNVERBOSE)

[Out] ln(a+b*ln(x))/b

Maxima [A]

time = 0.24, size = 11, normalized size = 1.00

$$\frac{\log(b\log(x) + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*log(x)),x, algorithm="maxima")

[Out] log(b*log(x) + a)/b

Fricas [A]

time = 0.30, size = 11, normalized size = 1.00

$$\frac{\log(b\log(x) + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a+b*log(x)),x, algorithm="fricas")`

[Out] $\log(b \cdot \log(x) + a)/b$

Sympy [A]

time = 0.06, size = 8, normalized size = 0.73

$$\frac{\log\left(\frac{a}{b} + \log(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a+b*ln(x)),x)`

[Out] $\log(a/b + \log(x))/b$

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(11) = 22$.

time = 0.00, size = 28, normalized size = 2.55

$$\frac{\ln\left(\left(\ln|x| \cdot b + a\right)^2 + \left(\frac{1}{2}\pi(1 - \operatorname{sign}(x))b\right)^2\right)}{b \cdot 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a+b*log(x)),x)`

[Out] $1/2 \cdot \log(1/4 \cdot \pi^2 \cdot b^2 \cdot (\operatorname{sgn}(x) - 1)^2 + (b \cdot \log(\operatorname{abs}(x)) + a)^2)/b$

Mupad [B]

time = 0.29, size = 11, normalized size = 1.00

$$\frac{\ln(a + b \ln(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + b*log(x))),x)`

[Out] $\log(a + b \cdot \log(x))/b$

$$3.621 \quad \int \frac{(a+b \log(x))^{-n}}{x} dx$$

Optimal. Leaf size=23

$$\frac{(a + b \log(x))^{1-n}}{b(1-n)}$$

[Out] (a+b*ln(x))^(1-n)/b/(1-n)

Rubi [A]

time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2339, 30}

$$\frac{(a + b \log(x))^{1-n}}{b(1-n)}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*Log[x])^n),x]

[Out] (a + b*Log[x])^(1 - n)/(b*(1 - n))

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2339

Int[((a_) + Log[(c_)*(x_)^(n_)])*(b_)^(p_)/(x_), x_Symbol] := Dist[1/(b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{(a + b \log(x))^{-n}}{x} dx &= \frac{\text{Subst}(\int x^{-n} dx, x, a + b \log(x))}{b} \\ &= \frac{(a + b \log(x))^{1-n}}{b(1-n)} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 23, normalized size = 1.00

$$\frac{(a + b \log(x))^{1-n}}{b(1-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*Log[x])^n),x]
```

```
[Out] (a + b*Log[x])^(1 - n)/(b*(1 - n))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 7.55, size = 104, normalized size = 4.52

$$\text{Piecewise}\left[\left\{\left\{\frac{\text{Log}[x]}{a}, b==0 \&\& n==1\right\}, \{\text{Log}[x] a^{-n}, b==0\}, \left\{\frac{\text{Log}\left[\frac{a}{b} + \text{Log}[x]\right]}{b}, n==1\right\}\right\}, -\frac{a}{-b(a+b\text{Log}[x])^n + b n(a+b\text{Log}[x])^n} - \frac{b\text{Log}[x]}{-b(a+b\text{Log}[x])^n + b n(a+b\text{Log}[x])^n}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*(a + b*Log[x])^n),x]')
```

```
[Out] Piecewise[{{Log[x] / a, b == 0 && n == 1}, {Log[x] a ^ (-n), b == 0}, {Log[a / b + Log[x]] / b, n == 1}}, -a / (-b (a + b Log[x]) ^ n + b n (a + b Log[x]) ^ n) - b Log[x] / (-b (a + b Log[x]) ^ n + b n (a + b Log[x]) ^ n)]
```

Maple [A]

time = 0.02, size = 24, normalized size = 1.04

method	result	size
derivativedivides	$\frac{(a+b\ln(x))^{1-n}}{b(1-n)}$	24
default	$\frac{(a+b\ln(x))^{1-n}}{b(1-n)}$	24
risch	$-\frac{(a+b\ln(x))(a+b\ln(x))^{-n}}{b(-1+n)}$	27
norman	$\left(-\frac{\ln(x)}{-1+n} - \frac{a}{b(-1+n)}\right) e^{-n\ln(a+b\ln(x))}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((a+b*ln(x))^n),x,method=_RETURNVERBOSE)
```

```
[Out] (a+b*ln(x))^(1-n)/b/(1-n)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((a+b*log(x))^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h
```

elp (example of legal syntax is 'assume(-n>0)', see 'assume?' for more details)Is

Fricas [A]

time = 0.64, size = 27, normalized size = 1.17

$$-\frac{b \log(x) + a}{(bn - b)(b \log(x) + a)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((a+b*log(x))^n),x, algorithm="fricas")

[Out] -(b*log(x) + a)/((b*n - b)*(b*log(x) + a)^n)

Sympy [A]

time = 5.99, size = 71, normalized size = 3.09

$$\begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge n = 1 \\ a^{-n} \log(x) & \text{for } b = 0 \\ \frac{\log(\frac{a}{b} + \log(x))}{b} & \text{for } n = 1 \\ -\frac{a}{bn(a+b \log(x))^n - b(a+b \log(x))^n} - \frac{b \log(x)}{bn(a+b \log(x))^n - b(a+b \log(x))^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((a+b*ln(x))**n),x)

[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 1)), (log(x)/a**n, Eq(b, 0)), (log(a/b + log(x))/b, Eq(n, 1)), (-a/(b*n*(a + b*log(x))**n - b*(a + b*log(x))**n) - b*log(x)/(b*n*(a + b*log(x))**n - b*(a + b*log(x))**n), True))

Giac [A]

time = 0.00, size = 19, normalized size = 0.83

$$\frac{(a + b \ln x)^{-n+1}}{b(-n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((a+b*log(x))^n),x)

[Out] -(b*log(x) + a)^(-n + 1)/(b*(n - 1))

Mupad [B]

time = 0.43, size = 22, normalized size = 0.96

$$-\frac{(a + b \ln(x))^{1-n}}{b(n - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*log(x))^n),x)

[Out] -(a + b*log(x))^(1 - n)/(b*(n - 1))

$$3.622 \quad \int \frac{1}{x \sqrt{a^2 + \log^2(x)}} dx$$

Optimal. Leaf size=16

$$\tanh^{-1} \left(\frac{\log(x)}{\sqrt{a^2 + \log^2(x)}} \right)$$

[Out] arctanh(ln(x)/(a^2+ln(x)^2)^(1/2))

Rubi [A]

time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {223, 212}

$$\tanh^{-1} \left(\frac{\log(x)}{\sqrt{a^2 + \log^2(x)}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a^2 + Log[x]^2]),x]

[Out] ArcTanh[Log[x]/Sqrt[a^2 + Log[x]^2]]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 223

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a^2 + \log^2(x)}} dx &= \text{Subst} \left(\int \frac{1}{\sqrt{a^2 + x^2}} dx, x, \log(x) \right) \\
&= \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \frac{\log(x)}{\sqrt{a^2 + \log^2(x)}} \right) \\
&= \tanh^{-1} \left(\frac{\log(x)}{\sqrt{a^2 + \log^2(x)}} \right)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(16) = 32.

time = 0.01, size = 46, normalized size = 2.88

$$-\frac{1}{2} \log \left(1 - \frac{\log(x)}{\sqrt{a^2 + \log^2(x)}} \right) + \frac{1}{2} \log \left(1 + \frac{\log(x)}{\sqrt{a^2 + \log^2(x)}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a^2 + Log[x]^2]),x]

[Out] -1/2*Log[1 - Log[x]/Sqrt[a^2 + Log[x]^2]] + Log[1 + Log[x]/Sqrt[a^2 + Log[x]^2]]/2

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x*Sqrt[Log[x]^2 + a^2]),x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.02, size = 15, normalized size = 0.94

method	result	size
derivativedivides	$\ln \left(\ln(x) + \sqrt{a^2 + \ln(x)^2} \right)$	15

default	$\ln \left(\ln(x) + \sqrt{a^2 + \ln(x)^2} \right)$	15
---------	---	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `ln(ln(x)+(a^2+ln(x)^2)^(1/2))`

Maxima [A]

time = 0.24, size = 7, normalized size = 0.44

$$\operatorname{arsinh} \left(\frac{\log(x)}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^2+log(x)^2)^(1/2),x, algorithm="maxima")`

[Out] `arcsinh(log(x)/a)`

Fricas [A]

time = 0.30, size = 18, normalized size = 1.12

$$-\log \left(\sqrt{a^2 + \log(x)^2} - \log(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^2+log(x)^2)^(1/2),x, algorithm="fricas")`

[Out] `-log(sqrt(a^2 + log(x)^2) - log(x))`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{a^2 + \log(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a**2+ln(x)**2)**(1/2),x)`

[Out] `Integral(1/(x*sqrt(a**2 + log(x)**2)), x)`

Giac [A]

time = 0.00, size = 18, normalized size = 1.12

$$-\ln \left(\sqrt{a^2 + \ln^2 x} - \ln x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+log(x)^2)^(1/2),x)
```

```
[Out] -log(sqrt(a^2 + log(x)^2) - log(x))
```

Mupad [B]

time = 0.43, size = 14, normalized size = 0.88

$$\ln \left(\ln(x) + \sqrt{a^2 + \ln(x)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x)^2 + a^2)^(1/2)),x)
```

```
[Out] log(log(x) + (log(x)^2 + a^2)^(1/2))
```

$$3.623 \quad \int \frac{1}{x \sqrt{-a^2 + \log^2(x)}} dx$$

Optimal. Leaf size=18

$$\tanh^{-1} \left(\frac{\log(x)}{\sqrt{-a^2 + \log^2(x)}} \right)$$

[Out] arctanh(ln(x)/(-a^2+ln(x)^2)^(1/2))

Rubi [A]

time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {223, 212}

$$\tanh^{-1} \left(\frac{\log(x)}{\sqrt{\log^2(x) - a^2}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[-a^2 + Log[x]^2]),x]

[Out] ArcTanh[Log[x]/Sqrt[-a^2 + Log[x]^2]]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 223

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{-a^2 + \log^2(x)}} dx &= \text{Subst} \left(\int \frac{1}{\sqrt{-a^2 + x^2}} dx, x, \log(x) \right) \\
&= \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \frac{\log(x)}{\sqrt{-a^2 + \log^2(x)}} \right) \\
&= \tanh^{-1} \left(\frac{\log(x)}{\sqrt{-a^2 + \log^2(x)}} \right)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 50 vs. 2(18) = 36.

time = 0.01, size = 50, normalized size = 2.78

$$-\frac{1}{2} \log \left(1 - \frac{\log(x)}{\sqrt{-a^2 + \log^2(x)}} \right) + \frac{1}{2} \log \left(1 + \frac{\log(x)}{\sqrt{-a^2 + \log^2(x)}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[-a^2 + Log[x]^2]),x]

[Out] -1/2*Log[1 - Log[x]/Sqrt[-a^2 + Log[x]^2]] + Log[1 + Log[x]/Sqrt[-a^2 + Log[x]^2]]/2

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x*Sqrt[Log[x]^2 - a^2]),x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.02, size = 17, normalized size = 0.94

method	result	size
derivativedivides	$\ln \left(\ln(x) + \sqrt{-a^2 + \ln(x)^2} \right)$	17

default	$\ln \left(\ln(x) + \sqrt{-a^2 + \ln(x)^2} \right)$	17
---------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(-a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `ln(ln(x)+(-a^2+ln(x)^2)^(1/2))`

Maxima [A]

time = 0.24, size = 20, normalized size = 1.11

$$\log \left(2 \sqrt{-a^2 + \log(x)^2} + 2 \log(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-a^2+log(x)^2)^(1/2),x, algorithm="maxima")`

[Out] `log(2*sqrt(-a^2 + log(x)^2) + 2*log(x))`

Fricas [A]

time = 0.31, size = 20, normalized size = 1.11

$$-\log \left(\sqrt{-a^2 + \log(x)^2} - \log(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-a^2+log(x)^2)^(1/2),x, algorithm="fricas")`

[Out] `-log(sqrt(-a^2 + log(x)^2) - log(x))`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{-(a - \log(x))(a + \log(x))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-a**2+ln(x)**2)**(1/2),x)`

[Out] `Integral(1/(x*sqrt(-(a - log(x))*(a + log(x))))), x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2+log(x)^2)^(1/2),x)
```

```
[Out] Timed out
```

Mupad [B]

time = 0.39, size = 16, normalized size = 0.89

$$\ln \left(\ln(x) + \sqrt{\ln(x)^2 - a^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x)^2 - a^2)^(1/2)),x)
```

```
[Out] log(log(x) + (log(x)^2 - a^2)^(1/2))
```

$$3.624 \quad \int \frac{1}{x \sqrt{a^2 - \log^2(x)}} dx$$

Optimal. Leaf size=18

$$\tan^{-1} \left(\frac{\log(x)}{\sqrt{a^2 - \log^2(x)}} \right)$$

[Out] arctan(ln(x)/(a^2-ln(x)^2)^(1/2))

Rubi [A]

time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {223, 209}

$$\tan^{-1} \left(\frac{\log(x)}{\sqrt{a^2 - \log^2(x)}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a^2 - Log[x]^2]),x]

[Out] ArcTan[Log[x]/Sqrt[a^2 - Log[x]^2]]

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 223

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x \sqrt{a^2 - \log^2(x)}} dx &= \text{Subst} \left(\int \frac{1}{\sqrt{a^2 - x^2}} dx, x, \log(x) \right) \\
&= \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \frac{\log(x)}{\sqrt{a^2 - \log^2(x)}} \right) \\
&= \tan^{-1} \left(\frac{\log(x)}{\sqrt{a^2 - \log^2(x)}} \right)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 18, normalized size = 1.00

$$\tan^{-1} \left(\frac{\log(x)}{\sqrt{a^2 - \log^2(x)}} \right)$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x*Sqrt[a^2 - Log[x]^2]),x]``[Out] ArcTan[Log[x]/Sqrt[a^2 - Log[x]^2]]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x*Sqrt[a^2 - Log[x]^2]),x]')``[Out] cought exception: maximum recursion depth exceeded in comparison`**Maple [A]**

time = 0.01, size = 17, normalized size = 0.94

method	result	size
derivativedivides	$\arctan \left(\frac{\ln(x)}{\sqrt{a^2 - \ln(x)^2}} \right)$	17

default	$\arctan\left(\frac{\ln(x)}{\sqrt{a^2 - \ln(x)^2}}\right)$	17
---------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(a^2-ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `arctan(ln(x)/(a^2-ln(x)^2)^(1/2))`

Maxima [A]

time = 0.33, size = 7, normalized size = 0.39

$$\arcsin\left(\frac{\log(x)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^2-log(x)^2)^(1/2),x, algorithm="maxima")`

[Out] `arcsin(log(x)/a)`

Fricas [A]

time = 0.31, size = 25, normalized size = 1.39

$$-2 \arctan\left(-\frac{a - \sqrt{a^2 - \log(x)^2}}{\log(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^2-log(x)^2)^(1/2),x, algorithm="fricas")`

[Out] `-2*arctan(-(a - sqrt(a^2 - log(x)^2))/log(x))`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{(a - \log(x))(a + \log(x))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a**2-ln(x)**2)**(1/2),x)`

[Out] `Integral(1/(x*sqrt((a - log(x))*(a + log(x))))), x)`

Giac [A]

time = 0.00, size = 9, normalized size = 0.50

$$\text{sign}(a) \arcsin\left(\frac{\ln x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^2-log(x)^2)^(1/2),x)`

[Out] `arcsin(log(x)/a)*sgn(a)`

Mupad [B]

time = 0.61, size = 16, normalized size = 0.89

$$\operatorname{atan}\left(\frac{\ln(x)}{\sqrt{a^2 - \ln(x)^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a^2 - log(x)^2)^(1/2)),x)`

[Out] `atan(log(x)/(a^2 - log(x)^2)^(1/2))`

$$3.625 \quad \int \frac{1}{x \log(x) \sqrt{a^2 + \log^2(x)}} dx$$

Optimal. Leaf size=22

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a^2 + \log^2(x)}}{a}\right)}{a}$$

[Out] -arctanh((a^2+ln(x)^2)^(1/2)/a)/a

Rubi [A]

time = 0.05, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {272, 65, 213}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a^2 + \log^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Log[x]*Sqrt[a^2 + Log[x]^2]),x]

[Out] -(ArcTanh[Sqrt[a^2 + Log[x]^2]/a]/a)

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x \log(x) \sqrt{a^2 + \log^2(x)}} dx &= \text{Subst} \left(\int \frac{1}{x \sqrt{a^2 + x^2}} dx, x, \log(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x \sqrt{a^2 + x}} dx, x, \log^2(x) \right) \\
&= \text{Subst} \left(\int \frac{1}{-a^2 + x^2} dx, x, \sqrt{a^2 + \log^2(x)} \right) \\
&= \frac{\tanh^{-1} \left(\frac{\sqrt{a^2 + \log^2(x)}}{a} \right)}{a}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 22, normalized size = 1.00

$$\frac{\tanh^{-1} \left(\frac{\sqrt{a^2 + \log^2(x)}}{a} \right)}{a}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x*Log[x]*Sqrt[a^2 + Log[x]^2]),x]``[Out] -(ArcTanh[Sqrt[a^2 + Log[x]^2]/a]/a)`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x*Log[x]*Sqrt[a^2 + Log[x]^2]),x]')``[Out] cought exception: maximum recursion depth exceeded`**Maple [A]**

time = 0.01, size = 37, normalized size = 1.68

method	result	size
--------	--------	------

derivativedivides	$\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\sqrt{a^2+\ln(x)^2}}{\ln(x)}\right)}{\sqrt{a^2}}$	37
default	$\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\sqrt{a^2+\ln(x)^2}}{\ln(x)}\right)}{\sqrt{a^2}}$	37

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/ln(x)/(a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/(a^2)^{(1/2)}*\ln((2*a^2+2*(a^2)^{(1/2)}*(a^2+\ln(x)^2)^{(1/2)})/\ln(x))$

Maxima [A]

time = 0.24, size = 13, normalized size = 0.59

$$\frac{\operatorname{arsinh}\left(\frac{a}{|\log(x)|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/log(x)/(a^2+log(x)^2)^(1/2),x, algorithm="maxima")`

[Out] $-\operatorname{arcsinh}(a/\operatorname{abs}(\log(x)))/a$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(20) = 40$.

time = 0.32, size = 44, normalized size = 2.00

$$\frac{\log\left(a + \sqrt{a^2 + \log(x)^2} - \log(x)\right) - \log\left(-a + \sqrt{a^2 + \log(x)^2} - \log(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/log(x)/(a^2+log(x)^2)^(1/2),x, algorithm="fricas")`

[Out] $-(\log(a + \sqrt{a^2 + \log(x)^2} - \log(x)) - \log(-a + \sqrt{a^2 + \log(x)^2} - \log(x)))/a$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{a^2 + \log(x)^2} \log(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/ln(x)/(a**2+ln(x)**2)**(1/2),x)

[Out] Integral(1/(x*sqrt(a**2 + log(x)**2)*log(x)), x)

Giac [A]

time = 0.00, size = 43, normalized size = 1.95

$$\frac{\ln\left(\sqrt{a^2 + \ln^2 x} - a\right)}{2a} - \frac{\ln\left(\sqrt{a^2 + \ln^2 x} + a\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(x)/(a^2+log(x)^2)^(1/2),x)

[Out] -1/2*log(a + sqrt(a^2 + log(x)^2))/a + 1/2*log(-a + sqrt(a^2 + log(x)^2))/a

Mupad [B]

time = 0.58, size = 27, normalized size = 1.23

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a^2 + \ln(x)^2}}{\sqrt{-a^2}}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*log(x)*(log(x)^2 + a^2)^(1/2)),x)

[Out] atan((log(x)^2 + a^2)^(1/2)/(-a^2)^(1/2))/(-a^2)^(1/2)

$$3.626 \quad \int \frac{1}{x \log(x) \sqrt{a^2 - \log^2(x)}} dx$$

Optimal. Leaf size=24

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a^2 - \log^2(x)}}{a}\right)}{a}$$

[Out] -arctanh((a^2-ln(x)^2)^(1/2)/a)/a

Rubi [A]

time = 0.06, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {272, 65, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a^2 - \log^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Log[x]*Sqrt[a^2 - Log[x]^2]),x]

[Out] -(ArcTanh[Sqrt[a^2 - Log[x]^2]/a]/a)

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x \log(x) \sqrt{a^2 - \log^2(x)}} dx &= \text{Subst} \left(\int \frac{1}{x \sqrt{a^2 - x^2}} dx, x, \log(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{a^2 - x} x} dx, x, \log^2(x) \right) \\
&= -\text{Subst} \left(\int \frac{1}{a^2 - x^2} dx, x, \sqrt{a^2 - \log^2(x)} \right) \\
&= -\frac{\tanh^{-1} \left(\frac{\sqrt{a^2 - \log^2(x)}}{a} \right)}{a}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 24, normalized size = 1.00

$$-\frac{\tanh^{-1} \left(\frac{\sqrt{a^2 - \log^2(x)}}{a} \right)}{a}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x*Log[x]*Sqrt[a^2 - Log[x]^2]),x]``[Out] -(ArcTanh[Sqrt[a^2 - Log[x]^2]/a]/a)`**Mathics [F(-1)]**

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x*Log[x]*Sqrt[a^2 - Log[x]^2]),x]')``[Out] Timed out`**Maple [A]**

time = 0.02, size = 39, normalized size = 1.62

method	result	size
--------	--------	------

derivativedivides	$\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\sqrt{a^2-\ln(x)^2}}{\ln(x)}\right)}{\sqrt{a^2}}$	39
default	$\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\sqrt{a^2-\ln(x)^2}}{\ln(x)}\right)}{\sqrt{a^2}}$	39

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/ln(x)/(a^2-ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/(a^2)^{(1/2)}*\ln((2*a^2+2*(a^2)^{(1/2)}*(a^2-\ln(x)^2)^{(1/2)})/\ln(x))$

Maxima [A]

time = 0.25, size = 37, normalized size = 1.54

$$\frac{\log\left(\frac{2a^2}{|\log(x)|} + \frac{2\sqrt{a^2 - \log(x)^2} a}{|\log(x)|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/log(x)/(a^2-log(x)^2)^(1/2),x, algorithm="maxima")`

[Out] $-\log(2*a^2/\text{abs}(\log(x)) + 2*\text{sqrt}(a^2 - \log(x)^2)*a/\text{abs}(\log(x)))/a$

Fricas [A]

time = 0.31, size = 27, normalized size = 1.12

$$\frac{\log\left(-\frac{a-\sqrt{a^2-\log(x)^2}}{\log(x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/log(x)/(a^2-log(x)^2)^(1/2),x, algorithm="fricas")`

[Out] $\log(-(a - \text{sqrt}(a^2 - \log(x)^2))/\log(x))/a$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{(a-\log(x))(a+\log(x))}\log(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/ln(x)/(a**2-ln(x)**2)**(1/2),x)`

[Out] `Integral(1/(x*sqrt((a - log(x))*(a + log(x))))*log(x)), x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/log(x)/(a^2-log(x)^2)^(1/2),x)`

[Out] Timed out

Mupad [B]

time = 0.61, size = 22, normalized size = 0.92

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a^2 - \ln(x)^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*log(x)*(a^2 - log(x)^2)^(1/2)),x)`

[Out] `-atanh((a^2 - log(x)^2)^(1/2)/a)/a`

$$3.627 \quad \int \frac{1}{x \log(x) \sqrt{-a^2 + \log^2(x)}} dx$$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{\sqrt{-a^2 + \log^2(x)}}{a}\right)}{a}$$

[Out] arctan((-a^2+ln(x)^2)^(1/2)/a)/a

Rubi [A]

time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {272, 65, 209}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\log^2(x) - a^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Log[x]*Sqrt[-a^2 + Log[x]^2]),x]

[Out] ArcTan[Sqrt[-a^2 + Log[x]^2]/a]/a

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x \log(x) \sqrt{-a^2 + \log^2(x)}} dx &= \text{Subst} \left(\int \frac{1}{x \sqrt{-a^2 + x^2}} dx, x, \log(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x \sqrt{-a^2 + x}} dx, x, \log^2(x) \right) \\
&= \text{Subst} \left(\int \frac{1}{a^2 + x^2} dx, x, \sqrt{-a^2 + \log^2(x)} \right) \\
&= \frac{\tan^{-1} \left(\frac{\sqrt{-a^2 + \log^2(x)}}{a} \right)}{a}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 23, normalized size = 1.00

$$\frac{\tan^{-1} \left(\frac{\sqrt{-a^2 + \log^2(x)}}{a} \right)}{a}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(x*Log[x]*Sqrt[-a^2 + Log[x]^2]),x]``[Out] ArcTan[Sqrt[-a^2 + Log[x]^2]/a]/a`**Mathics [F(-1)]**

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[1/(x*Log[x]*Sqrt[Log[x]^2 - a^2]),x]')``[Out] Timed out`**Maple [A]**

time = 0.01, size = 43, normalized size = 1.87

method	result	size
--------	--------	------

derivativedivides	$-\frac{\ln\left(\frac{-2a^2+2\sqrt{-a^2}\sqrt{-a^2+\ln(x)^2}}{\ln(x)}\right)}{\sqrt{-a^2}}$	43
default	$-\frac{\ln\left(\frac{-2a^2+2\sqrt{-a^2}\sqrt{-a^2+\ln(x)^2}}{\ln(x)}\right)}{\sqrt{-a^2}}$	43

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/ln(x)/(-a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/(-a^2)^{(1/2)}*\ln((-2*a^2+2*(-a^2)^{(1/2)}*(-a^2+\ln(x)^2)^{(1/2)})/\ln(x))$

Maxima [A]

time = 0.32, size = 13, normalized size = 0.57

$$-\frac{\arcsin\left(\frac{a}{|\log(x)|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x, algorithm="maxima")`

[Out] $-\arcsin(a/\text{abs}(\log(x)))/a$

Fricas [A]

time = 0.31, size = 27, normalized size = 1.17

$$\frac{2 \arctan\left(\frac{\sqrt{-a^2 + \log(x)^2} - \log(x)}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x, algorithm="fricas")`

[Out] $2*\arctan((\text{sqrt}(-a^2 + \log(x)^2) - \log(x))/a)/a$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{-(a - \log(x))(a + \log(x))} \log(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/ln(x)/(-a**2+ln(x)**2)**(1/2),x)`

[Out] `Integral(1/(x*sqrt(-(a - log(x))*(a + log(x))))*log(x)), x)`

Giac [A]

time = 0.00, size = 20, normalized size = 0.87

$$\frac{\arctan\left(\frac{\sqrt{-a^2 + \ln^2 x}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x)`

[Out] `arctan(sqrt(-a^2 + log(x)^2)/a)/a`

Mupad [B]

time = 0.52, size = 25, normalized size = 1.09

$$\frac{\operatorname{atan}\left(\frac{\sqrt{\ln(x)^2 - a^2}}{\sqrt{a^2}}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*log(x)*(log(x)^2 - a^2)^(1/2)),x)`

[Out] `atan((log(x)^2 - a^2)^(1/2)/(a^2)^(1/2))/(a^2)^(1/2)`

$$3.628 \quad \int \frac{\log(\log(x))}{x} dx$$

Optimal. Leaf size=11

$$-\log(x) + \log(x) \log(\log(x))$$

[Out] -ln(x)+ln(x)*ln(ln(x))

Rubi [A]

time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$,

Rules used = {2601}

$$\log(x) \log(\log(x)) - \log(x)$$

Antiderivative was successfully verified.

[In] Int[Log[Log[x]]/x,x]

[Out] -Log[x] + Log[x]*Log[Log[x]]

Rule 2601

Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
 := Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x]
] /; FreeQ[{a, b, c, d, n, p}, x]

Rubi steps

$$\int \frac{\log(\log(x))}{x} dx = -\log(x) + \log(x) \log(\log(x))$$

Mathematica [A]

time = 0.00, size = 11, normalized size = 1.00

$$-\log(x) + \log(x) \log(\log(x))$$

Antiderivative was successfully verified.

[In] Integrate[Log[Log[x]]/x,x]

[Out] -Log[x] + Log[x]*Log[Log[x]]

Mathics [A]

time = 1.63, size = 8, normalized size = 0.73

$$\text{Log}[x] (-1 + \text{Log}[\text{Log}[x]])$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Log[Log[x]]^1/x,x]')
```

```
[Out] Log[x] (-1 + Log[Log[x]])
```

Maple [A]

time = 0.01, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$-\ln(x) + \ln(x) \ln(\ln(x))$	12
default	$-\ln(x) + \ln(x) \ln(\ln(x))$	12
norman	$-\ln(x) + \ln(x) \ln(\ln(x))$	12
risch	$-\ln(x) + \ln(x) \ln(\ln(x))$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x)+ln(x)*ln(ln(x))
```

Maxima [A]

time = 0.24, size = 11, normalized size = 1.00

$$\log(x) \log(\log(x)) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))/x,x, algorithm="maxima")
```

```
[Out] log(x)*log(log(x)) - log(x)
```

Fricas [A]

time = 0.31, size = 11, normalized size = 1.00

$$\log(x) \log(\log(x)) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))/x,x, algorithm="fricas")
```

```
[Out] log(x)*log(log(x)) - log(x)
```

Sympy [A]

time = 0.08, size = 10, normalized size = 0.91

$$\log(x) \log(\log(x)) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(ln(x))/x,x)

[Out] log(x)*log(log(x)) - log(x)

Giac [A]

time = 0.00, size = 10, normalized size = 0.91

$$\ln x \cdot \ln(\ln x) - \ln x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(log(x))/x,x)

[Out] log(x)*log(log(x)) - log(x)

Mupad [B]

time = 0.32, size = 8, normalized size = 0.73

$$\ln(x) (\ln(\ln(x)) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(log(x))/x,x)

[Out] log(x)*(log(log(x)) - 1)

$$3.629 \quad \int \frac{\log^2(\log(x))}{x} dx$$

Optimal. Leaf size=20

$$2 \log(x) - 2 \log(x) \log(\log(x)) + \log(x) \log^2(\log(x))$$

[Out] 2*ln(x)-2*ln(x)*ln(ln(x))+ln(x)*ln(ln(x))^2

Rubi [A]

time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2333, 2332}

$$\log(x) \log^2(\log(x)) - 2 \log(x) \log(\log(x)) + 2 \log(x)$$

Antiderivative was successfully verified.

[In] Int[Log[Log[x]]^2/x,x]

[Out] 2*Log[x] - 2*Log[x]*Log[Log[x]] + Log[x]*Log[Log[x]]^2

Rule 2332

Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]

Rule 2333

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]

Rubi steps

$$\begin{aligned} \int \frac{\log^2(\log(x))}{x} dx &= \text{Subst} \left(\int \log^2(x) dx, x, \log(x) \right) \\ &= \log(x) \log^2(\log(x)) - 2 \text{Subst} \left(\int \log(x) dx, x, \log(x) \right) \\ &= 2 \log(x) - 2 \log(x) \log(\log(x)) + \log(x) \log^2(\log(x)) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 20, normalized size = 1.00

$$2 \log(x) - 2 \log(x) \log(\log(x)) + \log(x) \log^2(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Log[x]]^2/x,x]
```

```
[Out] 2*Log[x] - 2*Log[x]*Log[Log[x]] + Log[x]*Log[Log[x]]^2
```

Mathics [A]

time = 1.70, size = 15, normalized size = 0.75

$$\text{Log}[x] (2 + \text{Log}[\text{Log}[x]]^2 - 2\text{Log}[\text{Log}[x]])$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Log[Log[x]]^2/x,x]')
```

```
[Out] Log[x] (2 + Log[Log[x]] ^ 2 - 2 Log[Log[x]])
```

Maple [A]

time = 0.02, size = 21, normalized size = 1.05

method	result	size
derivatividivides	$2 \ln(x) - 2 \ln(x) \ln(\ln(x)) + \ln(x) \ln(\ln(x))^2$	21
default	$2 \ln(x) - 2 \ln(x) \ln(\ln(x)) + \ln(x) \ln(\ln(x))^2$	21
norman	$2 \ln(x) - 2 \ln(x) \ln(\ln(x)) + \ln(x) \ln(\ln(x))^2$	21
risch	$2 \ln(x) - 2 \ln(x) \ln(\ln(x)) + \ln(x) \ln(\ln(x))^2$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*ln(x)-2*ln(x)*ln(ln(x))+ln(x)*ln(ln(x))^2
```

Maxima [A]

time = 0.24, size = 15, normalized size = 0.75

$$(\log(\log(x)))^2 - 2 \log(\log(x)) + 2) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^2/x,x, algorithm="maxima")
```

```
[Out] (log(log(x))^2 - 2*log(log(x)) + 2)*log(x)
```

Fricas [A]

time = 0.32, size = 20, normalized size = 1.00

$$\log(x) \log(\log(x))^2 - 2 \log(x) \log(\log(x)) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(log(x))^2/x,x, algorithm="fricas")

[Out] log(x)*log(log(x))^2 - 2*log(x)*log(log(x)) + 2*log(x)

Sympy [A]

time = 0.11, size = 24, normalized size = 1.20

$$\log(x) \log(\log(x))^2 - 2 \log(x) \log(\log(x)) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(ln(x))**2/x,x)

[Out] log(x)*log(log(x))**2 - 2*log(x)*log(log(x)) + 2*log(x)

Giac [A]

time = 0.00, size = 21, normalized size = 1.05

$$\ln x \cdot \ln^2(\ln x) - 2 \ln x \cdot \ln(\ln x) + 2 \ln x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(log(x))^2/x,x)

[Out] log(x)*log(log(x))^2 - 2*log(x)*log(log(x)) + 2*log(x)

Mupad [B]

time = 0.38, size = 15, normalized size = 0.75

$$\ln(x) (\ln(\ln(x))^2 - 2 \ln(\ln(x)) + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(log(x))^2/x,x)

[Out] log(x)*(log(log(x))^2 - 2*log(log(x)) + 2)

$$3.630 \quad \int \frac{\log^3(\log(x))}{x} dx$$

Optimal. Leaf size=29

$$-6 \log(x) + 6 \log(x) \log(\log(x)) - 3 \log(x) \log^2(\log(x)) + \log(x) \log^3(\log(x))$$

[Out] -6*ln(x)+6*ln(x)*ln(ln(x))-3*ln(x)*ln(ln(x))^2+ln(x)*ln(ln(x))^3

Rubi [A]

time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2333, 2332}

$$\log(x) \log^3(\log(x)) - 3 \log(x) \log^2(\log(x)) + 6 \log(x) \log(\log(x)) - 6 \log(x)$$

Antiderivative was successfully verified.

[In] Int[Log[Log[x]]^3/x,x]

[Out] -6*Log[x] + 6*Log[x]*Log[Log[x]] - 3*Log[x]*Log[Log[x]]^2 + Log[x]*Log[Log[x]]^3

Rule 2332

Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]

Rule 2333

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^p, x_Symbol] := Simp[x*(a + b*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]

Rubi steps

$$\begin{aligned} \int \frac{\log^3(\log(x))}{x} dx &= \text{Subst} \left(\int \log^3(x) dx, x, \log(x) \right) \\ &= \log(x) \log^3(\log(x)) - 3 \text{Subst} \left(\int \log^2(x) dx, x, \log(x) \right) \\ &= -3 \log(x) \log^2(\log(x)) + \log(x) \log^3(\log(x)) + 6 \text{Subst} \left(\int \log(x) dx, x, \log(x) \right) \\ &= -6 \log(x) + 6 \log(x) \log(\log(x)) - 3 \log(x) \log^2(\log(x)) + \log(x) \log^3(\log(x)) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 29, normalized size = 1.00

$$-6 \log(x) + 6 \log(x) \log(\log(x)) - 3 \log(x) \log^2(\log(x)) + \log(x) \log^3(\log(x))$$

Antiderivative was successfully verified.

`[In] Integrate[Log[Log[x]]^3/x,x]`

```
[Out] -6*Log[x] + 6*Log[x]*Log[Log[x]] - 3*Log[x]*Log[Log[x]]^2 + Log[x]*Log[Log[x]]^3
```

Mathics [A]

time = 1.79, size = 22, normalized size = 0.76

$$\text{Log}[x] (-6 + \text{Log}[\text{Log}[x]]^3 - 3\text{Log}[\text{Log}[x]]^2 + 6\text{Log}[\text{Log}[x]])$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Log[Log[x]]^3/x,x']')`

```
[Out] Log[x] (-6 + Log[Log[x]] ^ 3 - 3 Log[Log[x]] ^ 2 + 6 Log[Log[x]])
```

Maple [A]

time = 0.02, size = 30, normalized size = 1.03

method	result	size
derivativedivides	$-6 \ln(x) + 6 \ln(x) \ln(\ln(x)) - 3 \ln(x) \ln(\ln(x))^2 + \ln(x) \ln(\ln(x))^3$	30
default	$-6 \ln(x) + 6 \ln(x) \ln(\ln(x)) - 3 \ln(x) \ln(\ln(x))^2 + \ln(x) \ln(\ln(x))^3$	30
norman	$-6 \ln(x) + 6 \ln(x) \ln(\ln(x)) - 3 \ln(x) \ln(\ln(x))^2 + \ln(x) \ln(\ln(x))^3$	30
risch	$-6 \ln(x) + 6 \ln(x) \ln(\ln(x)) - 3 \ln(x) \ln(\ln(x))^2 + \ln(x) \ln(\ln(x))^3$	30

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(ln(ln(x))^3/x,x,method=_RETURNVERBOSE)`

```
[Out] -6*ln(x)+6*ln(x)*ln(ln(x))-3*ln(x)*ln(ln(x))^2+ln(x)*ln(ln(x))^3
```

Maxima [A]

time = 0.25, size = 22, normalized size = 0.76

$$(\log(\log(x)))^3 - 3 \log(\log(x))^2 + 6 \log(\log(x)) - 6) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(log(log(x))^3/x,x, algorithm="maxima")`

```
[Out] (log(log(x))^3 - 3*log(log(x))^2 + 6*log(log(x)) - 6)*log(x)
```

Fricas [A]

time = 0.34, size = 29, normalized size = 1.00

$$\log(x) \log(\log(x))^3 - 3 \log(x) \log(\log(x))^2 + 6 \log(x) \log(\log(x)) - 6 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(log(x))^3/x,x, algorithm="fricas")**[Out]** log(x)*log(log(x))^3 - 3*log(x)*log(log(x))^2 + 6*log(x)*log(log(x)) - 6*log(x)**Sympy [A]**

time = 0.14, size = 36, normalized size = 1.24

$$\log(x) \log(\log(x))^3 - 3 \log(x) \log(\log(x))^2 + 6 \log(x) \log(\log(x)) - 6 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(ln(x))**3/x,x)**[Out]** log(x)*log(log(x))**3 - 3*log(x)*log(log(x))**2 + 6*log(x)*log(log(x)) - 6*log(x)**Giac [A]**

time = 0.00, size = 31, normalized size = 1.07

$$\ln x \cdot \ln^3(\ln x) - 3 \ln x \cdot \ln^2(\ln x) + 6 \ln x \cdot \ln(\ln x) - 6 \ln x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(log(x))^3/x,x)**[Out]** log(x)*log(log(x))^3 - 3*log(x)*log(log(x))^2 + 6*log(x)*log(log(x)) - 6*log(x)**Mupad [B]**

time = 0.36, size = 29, normalized size = 1.00

$$\ln(x) \ln(\ln(x))^3 - 3 \ln(x) \ln(\ln(x))^2 + 6 \ln(x) \ln(\ln(x)) - 6 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(log(x))^3/x,x)**[Out]** 6*log(log(x))*log(x) - 6*log(x) - 3*log(log(x))^2*log(x) + log(log(x))^3*log(x)

$$3.631 \quad \int \frac{\log^4(\log(x))}{x} dx$$

Optimal. Leaf size=38

$$24 \log(x) - 24 \log(x) \log(\log(x)) + 12 \log(x) \log^2(\log(x)) - 4 \log(x) \log^3(\log(x)) + \log(x) \log^4(\log(x))$$

[Out] 24*ln(x)-24*ln(x)*ln(ln(x))+12*ln(x)*ln(ln(x))^2-4*ln(x)*ln(ln(x))^3+ln(x)*ln(ln(x))^4

Rubi [A]

time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2333, 2332}

$$\log(x) \log^4(\log(x)) - 4 \log(x) \log^3(\log(x)) + 12 \log(x) \log^2(\log(x)) - 24 \log(x) \log(\log(x)) + 24 \log(x)$$

Antiderivative was successfully verified.

[In] Int[Log[Log[x]]^4/x,x]

[Out] 24*Log[x] - 24*Log[x]*Log[Log[x]] + 12*Log[x]*Log[Log[x]]^2 - 4*Log[x]*Log[Log[x]]^3 + Log[x]*Log[Log[x]]^4

Rule 2332

Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]

Rule 2333

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]

Rubi steps

$$\begin{aligned} \int \frac{\log^4(\log(x))}{x} dx &= \text{Subst} \left(\int \log^4(x) dx, x, \log(x) \right) \\ &= \log(x) \log^4(\log(x)) - 4 \text{Subst} \left(\int \log^3(x) dx, x, \log(x) \right) \\ &= -4 \log(x) \log^3(\log(x)) + \log(x) \log^4(\log(x)) + 12 \text{Subst} \left(\int \log^2(x) dx, x, \log(x) \right) \\ &= 12 \log(x) \log^2(\log(x)) - 4 \log(x) \log^3(\log(x)) + \log(x) \log^4(\log(x)) - 24 \text{Subst} \left(\int \log(x) dx, x, \log(x) \right) \\ &= 24 \log(x) - 24 \log(x) \log(\log(x)) + 12 \log(x) \log^2(\log(x)) - 4 \log(x) \log^3(\log(x)) + \log(x) \log^4(\log(x)) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 38, normalized size = 1.00

$$24 \log(x) - 24 \log(x) \log(\log(x)) + 12 \log(x) \log^2(\log(x)) - 4 \log(x) \log^3(\log(x)) + \log(x) \log^4(\log(x))$$

Antiderivative was successfully verified.

`[In] Integrate[Log[Log[x]]^4/x,x]`

```
[Out] 24*Log[x] - 24*Log[x]*Log[Log[x]] + 12*Log[x]*Log[Log[x]]^2 - 4*Log[x]*Log[Log[x]]^3 + Log[x]*Log[Log[x]]^4
```

Mathics [A]

time = 1.88, size = 29, normalized size = 0.76

$$\text{Log}[x] (24 + \text{Log}[\text{Log}[x]]^4 - 24\text{Log}[\text{Log}[x]] - 4\text{Log}[\text{Log}[x]]^3 + 12\text{Log}[\text{Log}[x]]^2)$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Log[Log[x]]^4/x,x]')`

```
[Out] Log[x] (24 + Log[Log[x]] ^ 4 - 24 Log[Log[x]] - 4 Log[Log[x]] ^ 3 + 12 Log[Log[x]] ^ 2)
```

Maple [A]

time = 0.02, size = 39, normalized size = 1.03

method	result
derivativdivides	$24 \ln(x) - 24 \ln(x) \ln(\ln(x)) + 12 \ln(x) \ln(\ln(x))^2 - 4 \ln(x) \ln(\ln(x))^3 + \ln(x) \ln(\ln(x))$
default	$24 \ln(x) - 24 \ln(x) \ln(\ln(x)) + 12 \ln(x) \ln(\ln(x))^2 - 4 \ln(x) \ln(\ln(x))^3 + \ln(x) \ln(\ln(x))$
norman	$24 \ln(x) - 24 \ln(x) \ln(\ln(x)) + 12 \ln(x) \ln(\ln(x))^2 - 4 \ln(x) \ln(\ln(x))^3 + \ln(x) \ln(\ln(x))$
risch	$24 \ln(x) - 24 \ln(x) \ln(\ln(x)) + 12 \ln(x) \ln(\ln(x))^2 - 4 \ln(x) \ln(\ln(x))^3 + \ln(x) \ln(\ln(x))$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(ln(ln(x))^4/x,x,method=_RETURNVERBOSE)`

```
[Out] 24*ln(x)-24*ln(x)*ln(ln(x))+12*ln(x)*ln(ln(x))^2-4*ln(x)*ln(ln(x))^3+ln(x)*ln(ln(x))^4
```

Maxima [A]

time = 0.24, size = 29, normalized size = 0.76

$$(\log(\log(x))^4 - 4 \log(\log(x))^3 + 12 \log(\log(x))^2 - 24 \log(\log(x)) + 24) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(log(log(x))^4/x,x, algorithm="maxima")`

[Out] $(\log(\log(x))^4 - 4*\log(\log(x))^3 + 12*\log(\log(x))^2 - 24*\log(\log(x)) + 24)*\log(x)$

Fricas [A]

time = 0.37, size = 38, normalized size = 1.00

$\log(x) \log(\log(x))^4 - 4 \log(x) \log(\log(x))^3 + 12 \log(x) \log(\log(x))^2 - 24 \log(x) \log(\log(x)) + 24 \log(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(log(x))^4/x,x, algorithm="fricas")`

[Out] $\log(x)*\log(\log(x))^4 - 4*\log(x)*\log(\log(x))^3 + 12*\log(x)*\log(\log(x))^2 - 24*\log(x)*\log(\log(x)) + 24*\log(x)$

Sympy [A]

time = 0.17, size = 48, normalized size = 1.26

$\log(x) \log(\log(x))^4 - 4 \log(x) \log(\log(x))^3 + 12 \log(x) \log(\log(x))^2 - 24 \log(x) \log(\log(x)) + 24 \log(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(ln(x))**4/x,x)`

[Out] $\log(x)*\log(\log(x))**4 - 4*\log(x)*\log(\log(x))**3 + 12*\log(x)*\log(\log(x))**2 - 24*\log(x)*\log(\log(x)) + 24*\log(x)$

Giac [A]

time = 0.00, size = 40, normalized size = 1.05

$\ln x \cdot \ln^4(\ln x) - 4 \ln x \cdot \ln^3(\ln x) + 12 \ln x \cdot \ln^2(\ln x) - 24 \ln x \cdot \ln(\ln x) + 24 \ln x$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(log(x))^4/x,x)`

[Out] $\log(x)*\log(\log(x))^4 - 4*\log(x)*\log(\log(x))^3 + 12*\log(x)*\log(\log(x))^2 - 24*\log(x)*\log(\log(x)) + 24*\log(x)$

Mupad [B]

time = 0.37, size = 38, normalized size = 1.00

$\ln(x) \ln(\ln(x))^4 - 4 \ln(x) \ln(\ln(x))^3 + 12 \ln(x) \ln(\ln(x))^2 - 24 \ln(x) \ln(\ln(x)) + 24 \ln(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(log(x))^4/x,x)`

[Out] $24*\log(x) - 24*\log(\log(x))*\log(x) + 12*\log(\log(x))^2*\log(x) - 4*\log(\log(x))^3*\log(x) + \log(\log(x))^4*\log(x)$

$$3.632 \quad \int \frac{\log^n(\log(x))}{x} dx$$

Optimal. Leaf size=24

$$\Gamma(1+n, -\log(\log(x))) (-\log(\log(x)))^{-n} \log^n(\log(x))$$

[Out] GAMMA(1+n, -ln(ln(x)))*ln(ln(x))^n/((-ln(ln(x)))^n)

Rubi [A]

time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2336, 2212}

$$(-\log(\log(x)))^{-n} \log^n(\log(x)) \Gamma(n+1, -\log(\log(x)))$$

Antiderivative was successfully verified.

[In] Int[Log[Log[x]]^n/x, x]

[Out] (Gamma[1 + n, -Log[Log[x]]]*Log[Log[x]]^n)/(-Log[Log[x]]^n)

Rule 2212

```
Int[(F_)^((g_.)*(e_.) + (f_.)*(x_))*((c_.) + (d_.)*(x_)^(m_)), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*(-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]]*Gamma[m + 1,
((-f)*g*(Log[F]/d)*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
c, p}, x] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned} \int \frac{\log^n(\log(x))}{x} dx &= \text{Subst} \left(\int \log^n(x) dx, x, \log(x) \right) \\ &= \text{Subst} \left(\int e^x x^n dx, x, \log(\log(x)) \right) \\ &= \Gamma(1+n, -\log(\log(x))) (-\log(\log(x)))^{-n} \log^n(\log(x)) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 24, normalized size = 1.00

$$\Gamma(1 + n, -\log(\log(x)))(-\log(\log(x)))^{-n} \log^n(\log(x))$$

Antiderivative was successfully verified.

[In] Integrate[Log[Log[x]]^n/x,x]

[Out] (Gamma[1 + n, -Log[Log[x]]]*Log[Log[x]]^n)/(-Log[Log[x]])^n

Mathics [A]

time = 2.55, size = 24, normalized size = 1.00

$$\Gamma[1 + n, -\text{Log}[\text{Log}[x]]] \text{Log}[\text{Log}[x]]^n (-\text{Log}[\text{Log}[x]])^{-n}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Log[Log[x]]^n/x,x]')

[Out] Gamma[1 + n, -Log[Log[x]]] Log[Log[x]] ^ n (-Log[Log[x]]) ^ (-n)

Maple [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\ln(\ln(x))^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(ln(x))^n/x,x)

[Out] int(ln(ln(x))^n/x,x)

Maxima [A]

time = 0.26, size = 29, normalized size = 1.21

$$-(-\log(\log(x)))^{-n-1} \log(\log(x))^{n+1} \Gamma(n+1, -\log(\log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(log(x))^n/x,x, algorithm="maxima")

[Out] -(-log(log(x)))^(-n - 1)*log(log(x))^(n + 1)*gamma(n + 1, -log(log(x)))

Fricas [A]

time = 0.46, size = 14, normalized size = 0.58

$$\cos(\pi n) \Gamma(n+1, -\log(\log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(log(x))^n/x,x, algorithm="fricas")

[Out] cos(pi*n)*gamma(n + 1, -log(log(x)))

Sympy [A]

time = 0.97, size = 24, normalized size = 1.00

$$(-\log(\log(x)))^{-n} \log(\log(x))^n \Gamma(n+1, -\log(\log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(ln(x))**n/x,x)

[Out] log(log(x))**n*uppergamma(n + 1, -log(log(x)))/(-log(log(x)))**n

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(log(x))^n/x,x)

[Out] Could not integrate

Mupad [B]

time = 0.45, size = 24, normalized size = 1.00

$$\frac{\ln(\ln(x))^n \Gamma(n+1, -\ln(\ln(x)))}{(-\ln(\ln(x)))^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(log(x))^n/x,x)

[Out] (log(log(x))^n*igamma(n + 1, -log(log(x))))/(-log(log(x)))^n

$$3.633 \quad \int \frac{\cot(x)}{\log(\sin(x))} dx$$

Optimal. Leaf size=4

$$\log(\log(\sin(x)))$$

[Out] ln(ln(sin(x)))

Rubi [A]

time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4423, 2339, 29}

$$\log(\log(\sin(x)))$$

Antiderivative was successfully verified.

[In] Int[Cot[x]/Log[Sin[x]],x]

[Out] Log[Log[Sin[x]]]

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 2339

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rule 4423

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a + b*x)]/d, u, x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])

Rubi steps

$$\begin{aligned} \int \frac{\cot(x)}{\log(\sin(x))} dx &= \text{Subst} \left(\int \frac{1}{x \log(x)} dx, x, \sin(x) \right) \\ &= \text{Subst} \left(\int \frac{1}{x} dx, x, \log(\sin(x)) \right) \\ &= \log(\log(\sin(x))) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 4, normalized size = 1.00

$$\log(\log(\sin(x)))$$

Antiderivative was successfully verified.

`[In] Integrate[Cot[x]/Log[Sin[x]],x]``[Out] Log[Log[Sin[x]]]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[Cot[x]/Log[Sin[x]],x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [A]**

time = 0.12, size = 5, normalized size = 1.25

method	result
derivativdivides	$\ln(\ln(\sin(x)))$
default	$\ln(\ln(\sin(x)))$
risch	$\ln\left(-\frac{i\pi \operatorname{csgn}(i(e^{2ix}-1))\operatorname{csgn}(ie^{-ix})\operatorname{csgn}(\sin(x))}{2} - \frac{i\pi \operatorname{csgn}(i(e^{2ix}-1))\operatorname{csgn}(\sin(x))^2}{2} - \frac{i\pi \operatorname{csgn}(ie^{-ix})\operatorname{csgn}(\sin(x))}{2}\right)$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cot(x)/ln(sin(x)),x,method=_RETURNVERBOSE)``[Out] ln(ln(sin(x)))`**Maxima [A]**

time = 0.24, size = 4, normalized size = 1.00

$$\log(\log(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cot(x)/log(sin(x)),x, algorithm="maxima")``[Out] log(log(sin(x)))`

Fricas [A]

time = 0.31, size = 4, normalized size = 1.00

$$\log(\log(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/log(sin(x)),x, algorithm="fricas")

[Out] log(log(sin(x)))

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\log(\sin(x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/ln(sin(x)),x)

[Out] Integral(cot(x)/log(sin(x)), x)

Giac [A]

time = 0.00, size = 5, normalized size = 1.25

$$\ln|\ln(\sin x)|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/log(sin(x)),x)

[Out] log(abs(log(sin(x))))

Mupad [B]

time = 0.40, size = 4, normalized size = 1.00

$$\ln(\ln(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)/log(sin(x)),x)

[Out] log(log(sin(x)))

3.634 $\int (\cos(x) + \sec(x)) \tan(x) dx$

Optimal. Leaf size=7

$$-\cos(x) + \sec(x)$$

[Out] $-\cos(x) + \sec(x)$

Rubi [A]

time = 0.03, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4321}

$$\sec(x) - \cos(x)$$

Antiderivative was successfully verified.

[In] `Int[(Cos[x] + Sec[x])*Tan[x],x]`

[Out] `-Cos[x] + Sec[x]`

Rule 4321

`Int[(u_)*((A_.) + cos[(a_.) + (b_.)*(x_)])*(B_.) + (C_.)*sec[(a_.) + (b_.)*(x_)], x_Symbol] :> Int[ActivateTrig[u]*((C + A*Cos[a + b*x] + B*Cos[a + b*x]^2)/Cos[a + b*x]), x] /; FreeQ[{a, b, A, B, C}, x]`

Rubi steps

$$\begin{aligned} \int (\cos(x) + \sec(x)) \tan(x) dx &= \int (1 + \cos^2(x)) \sec(x) \tan(x) dx \\ &= -\text{Subst} \left(\int \left(1 + \frac{1}{x^2} \right) dx, x, \cos(x) \right) \\ &= -\cos(x) + \sec(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 7, normalized size = 1.00

$$-\cos(x) + \sec(x)$$

Antiderivative was successfully verified.

[In] `Integrate[(Cos[x] + Sec[x])*Tan[x],x]`

[Out] `-Cos[x] + Sec[x]`

Mathics [A]

time = 2.28, size = 9, normalized size = 1.29

$$\frac{\text{Sin}[x]^2}{\text{Cos}[x]}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[(E^Log[Cos[x]] + E^(-Log[Cos[x]]))*Tan[x],x]')`[Out] `Sin[x]^2 / Cos[x]`**Maple [A]**

time = 0.04, size = 10, normalized size = 1.43

method	result	size
derivativedivides	$-\cos(x) + \frac{1}{\cos(x)}$	10
default	$-\cos(x) + \frac{1}{\cos(x)}$	10
risch	$-\frac{e^{ix}}{2} - \frac{e^{-ix}}{2} + \frac{2e^{ix}}{e^{2ix}+1}$	32

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/cos(x)+cos(x))*tan(x),x,method=_RETURNVERBOSE)`[Out] `-cos(x)+1/cos(x)`**Maxima [A]**

time = 0.25, size = 9, normalized size = 1.29

$$\frac{1}{\cos(x)} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/cos(x)+cos(x))*tan(x),x, algorithm="maxima")`[Out] `1/cos(x) - cos(x)`**Fricas [A]**

time = 0.31, size = 12, normalized size = 1.71

$$-\frac{\cos(x)^2 - 1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/cos(x)+cos(x))*tan(x),x, algorithm="fricas")`

[Out] $-(\cos(x)^2 - 1)/\cos(x)$

Sympy [A]

time = 0.89, size = 7, normalized size = 1.00

$$-\cos(x) + \frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/cos(x)+cos(x))*tan(x),x)`

[Out] $-\cos(x) + 1/\cos(x)$

Giac [A]

time = 0.00, size = 7, normalized size = 1.00

$$-\cos x + \frac{1}{\cos x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/cos(x)+cos(x))*tan(x),x)`

[Out] $1/\cos(x) - \cos(x)$

Mupad [B]

time = 0.39, size = 9, normalized size = 1.29

$$\frac{1}{\cos(x)} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(x)*(cos(x) + 1/cos(x)),x)`

[Out] $1/\cos(x) - \cos(x)$

3.635 $\int \log(\cosh(x)) \sinh(x) dx$

Optimal. Leaf size=11

$$-\cosh(x) + \cosh(x) \log(\cosh(x))$$

[Out] `-cosh(x)+cosh(x)*ln(cosh(x))`

Rubi [A]

time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2718, 2634}

$$\cosh(x) \log(\cosh(x)) - \cosh(x)$$

Antiderivative was successfully verified.

[In] `Int[Log[Cosh[x]]*Sinh[x],x]`

[Out] `-Cosh[x] + Cosh[x]*Log[Cosh[x]]`

Rule 2634

`Int[Log[u]*(v_), x_Symbol] := With[{w = IntHide[v, x]}, Dist[Log[u], w, x] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x]] /; InverseFunctionFreeQ[u, x]`

Rule 2718

`Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \int \log(\cosh(x)) \sinh(x) dx &= \cosh(x) \log(\cosh(x)) - \int \sinh(x) dx \\ &= -\cosh(x) + \cosh(x) \log(\cosh(x)) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 11, normalized size = 1.00

$$-\cosh(x) + \cosh(x) \log(\cosh(x))$$

Antiderivative was successfully verified.

[In] `Integrate[Log[Cosh[x]]*Sinh[x],x]`

[Out] $-\text{Cosh}[x] + \text{Cosh}[x] * \text{Log}[\text{Cosh}[x]]$

Mathics [A]

time = 1.79, size = 8, normalized size = 0.73

$$\text{Cosh}[x] (-1 + \text{Log}[\text{Cosh}[x]])$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[Sinh[x]*Log[Cosh[x]],x]')`

[Out] $\text{Cosh}[x] (-1 + \text{Log}[\text{Cosh}[x]])$

Maple [A]

time = 0.08, size = 12, normalized size = 1.09

method	result
derivativedivides	$-\cosh(x) + \cosh(x) \ln(\cosh(x))$
default	$-\cosh(x) + \cosh(x) \ln(\cosh(x))$
risch	$-\frac{(1+e^{2x})e^{-x} \ln(e^x)}{2} - \frac{(i\pi \operatorname{csgn}(ie^{-x}) \operatorname{csgn}(i(1+e^{2x})) \operatorname{csgn}(ie^{-x}(1+e^{2x})) e^{2x} - i\pi \operatorname{csgn}(i(1+e^{2x})) \operatorname{csgn}(ie^{-x}(1+e^{2x})))}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(cosh(x))*sinh(x),x,method=_RETURNVERBOSE)`

[Out] $-\cosh(x) + \cosh(x) * \ln(\cosh(x))$

Maxima [A]

time = 0.24, size = 11, normalized size = 1.00

$$\cosh(x) \log(\cosh(x)) - \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(cosh(x))*sinh(x),x, algorithm="maxima")`

[Out] $\cosh(x) * \log(\cosh(x)) - \cosh(x)$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(11) = 22$.

time = 0.30, size = 46, normalized size = 4.18

$$\frac{\cosh(x)^2 - (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \log(\cosh(x)) + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1}{2(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(cosh(x))*sinh(x),x, algorithm="fricas")`

[Out] $-1/2*(\cosh(x)^2 - (\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1)*\log(\cosh(x)) + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1)/(\cosh(x) + \sinh(x))$

Sympy [A]

time = 0.23, size = 10, normalized size = 0.91

$$\log(\cosh(x)) \cosh(x) - \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(cosh(x))*sinh(x),x)`

[Out] `log(cosh(x))*cosh(x) - cosh(x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(11) = 22$.
time = 0.00, size = 46, normalized size = 4.18

$$\frac{2}{2} \left(\frac{\frac{1}{2}((e^x)^2 + 1) \ln\left(\frac{\frac{1}{2}((e^x)^2 + 1)}{e^x}\right)}{e^x} - \frac{\frac{1}{2}((e^x)^2 + 1)}{e^x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(cosh(x))*sinh(x),x)`

[Out] `1/2*(e^(2*x) + 1)*log(1/2*(e^(2*x) + 1)/e^x)/e^x - 1/2*(e^(2*x) + 1)/e^x`

Mupad [B]

time = 0.36, size = 8, normalized size = 0.73

$$\cosh(x) (\ln(\cosh(x)) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(cosh(x))*sinh(x),x)`

[Out] `cosh(x)*(log(cosh(x)) - 1)`

3.636 $\int \log(\cosh(x)) \tanh(x) dx$

Optimal. Leaf size=9

$$\frac{1}{2} \log^2(\cosh(x))$$

[Out] 1/2*ln(cosh(x))^2

Rubi [A]

time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556, 4426, 2338}

$$\frac{1}{2} \log^2(\cosh(x))$$

Antiderivative was successfully verified.

[In] Int[Log[Cosh[x]]*Tanh[x],x]

[Out] Log[Cosh[x]]^2/2

Rule 2338

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]

Rule 3556

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4426

Int[(u_)*Tanh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Cosh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Cosh[c*(a + b*x)]]/d, u, x], x], x, Cosh[c*(a + b*x)]/d, x] /; FunctionOfQ[Cosh[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]

Rubi steps

$$\begin{aligned} \int \log(\cosh(x)) \tanh(x) dx &= \text{Subst} \left(\int \frac{\log(x)}{x} dx, x, \cosh(x) \right) \\ &= \frac{1}{2} \log^2(\cosh(x)) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 9, normalized size = 1.00

$$\frac{1}{2} \log^2(\cosh(x))$$

Antiderivative was successfully verified.

[In] Integrate[Log[Cosh[x]]*Tanh[x],x]

[Out] Log[Cosh[x]]^2/2

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Tanh[x]*Log[Cosh[x]],x]')

[Out] cought exception: maximum recursion depth exceeded in comparison

Maple [A]

time = 0.09, size = 8, normalized size = 0.89

method	result
derivativedivides	$\frac{\ln(\cosh(x))^2}{2}$
default	$\frac{\ln(\cosh(x))^2}{2}$
risch	$(x - \ln(1 + e^{2x})) \ln(e^x) + \frac{\ln(1+e^{2x})^2}{2} - \ln(1 + e^{2x}) \ln(2) + \frac{i \ln(1+e^{2x}) \pi \operatorname{csgn}(i(1+e^{2x})) \operatorname{csgn}(ie^{-2x})}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(cosh(x))*tanh(x),x,method=_RETURNVERBOSE)

[Out] 1/2*ln(cosh(x))^2

Maxima [A]

time = 0.24, size = 7, normalized size = 0.78

$$\frac{1}{2} \log(\cosh(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(cosh(x))*tanh(x),x, algorithm="maxima")

[Out] 1/2*log(cosh(x))^2

Fricas [A]

time = 0.31, size = 7, normalized size = 0.78

$$\frac{1}{2} \log(\cosh(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(cosh(x))*tanh(x),x, algorithm="fricas")

[Out] 1/2*log(cosh(x))^2

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \log(\cosh(x)) \tanh(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(cosh(x))*tanh(x),x)

[Out] Integral(log(cosh(x))*tanh(x), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(cosh(x))*tanh(x),x)

[Out] Could not integrate

Mupad [B]

time = 0.44, size = 16, normalized size = 1.78

$$\frac{\ln\left(\frac{e^{-x}}{2} + \frac{e^x}{2}\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(cosh(x))*tanh(x),x)

[Out] log(exp(-x)/2 + exp(x)/2)^2/2

$$3.637 \quad \int \log \left(x - \sqrt{1 + x^2} \right) dx$$

Optimal. Leaf size=26

$$\sqrt{1 + x^2} + x \log \left(x - \sqrt{1 + x^2} \right)$$

[Out] x*ln(x-(x^2+1)^(1/2))+(x^2+1)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2614, 267}

$$\sqrt{x^2 + 1} + x \log \left(x - \sqrt{x^2 + 1} \right)$$

Antiderivative was successfully verified.

[In] Int[Log[x - Sqrt[1 + x^2]],x]

[Out] Sqrt[1 + x^2] + x*Log[x - Sqrt[1 + x^2]]

Rule 267

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 2614

Int[Log[(d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (c_)*(x_)^2]], x_Symbol] :> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Dist[a*c*f^2, Int[x/(d*e*(a + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f}, x] && EqQ[e^2 - c*f^2, 0]

Rubi steps

$$\begin{aligned} \int \log \left(x - \sqrt{1 + x^2} \right) dx &= x \log \left(x - \sqrt{1 + x^2} \right) + \int \frac{x}{\sqrt{1 + x^2}} dx \\ &= \sqrt{1 + x^2} + x \log \left(x - \sqrt{1 + x^2} \right) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 26, normalized size = 1.00

$$\sqrt{1 + x^2} + x \log \left(x - \sqrt{1 + x^2} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x - Sqrt[1 + x^2]],x]
```

```
[Out] Sqrt[1 + x^2] + x*Log[x - Sqrt[1 + x^2]]
```

Mathics [A]

time = 5.37, size = 22, normalized size = 0.85

$$x \operatorname{Log}\left[x - \sqrt{1 + x^2}\right] + \sqrt{1 + x^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Log[x - Sqrt[1 + x^2]],x]')
```

```
[Out] x Log[x - Sqrt[1 + x ^ 2]] + Sqrt[1 + x ^ 2]
```

Maple [A]

time = 0.01, size = 23, normalized size = 0.88

method	result	size
default	$x \ln(x - \sqrt{x^2 + 1}) + \sqrt{x^2 + 1}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x-(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(x-(x^2+1)^(1/2))+(x^2+1)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x-(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*log(x - sqrt(x^2 + 1)) - x + arctan(x) + integrate(-x/(x^3 - (x^2 + 1)^(3/2) + x), x)
```

Fricas [A]

time = 0.31, size = 22, normalized size = 0.85

$$x \log\left(x - \sqrt{x^2 + 1}\right) + \sqrt{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x-(x^2+1)^(1/2)),x, algorithm="fricas")
```

[Out] $x \log(x - \sqrt{x^2 + 1}) + \sqrt{x^2 + 1}$

Sympy [A]

time = 5.13, size = 20, normalized size = 0.77

$$x \log \left(x - \sqrt{x^2 + 1} \right) + \sqrt{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(x-(x**2+1)**(1/2)),x)`

[Out] $x \log(x - \sqrt{x^2 + 1}) + \sqrt{x^2 + 1}$

Giac [A]

time = 0.00, size = 25, normalized size = 0.96

$$x \ln \left(x - \sqrt{x^2 + 1} \right) + \sqrt{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(x-(x^2+1)^(1/2)),x)`

[Out] $x \log(x - \sqrt{x^2 + 1}) + \sqrt{x^2 + 1}$

Mupad [B]

time = 0.08, size = 22, normalized size = 0.85

$$x \ln \left(x - \sqrt{x^2 + 1} \right) + \sqrt{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(x - (x^2 + 1)^(1/2)),x)`

[Out] $x \log(x - (x^2 + 1)^{1/2}) + (x^2 + 1)^{1/2}$

$$3.638 \quad \int \frac{\log(-1+x)}{x^3} dx$$

Optimal. Leaf size=35

$$\frac{1}{2x} + \frac{1}{2} \log(1-x) - \frac{\log(-1+x)}{2x^2} - \frac{\log(x)}{2}$$

[Out] 1/2/x+1/2*ln(1-x)-1/2*ln(-1+x)/x^2-1/2*ln(x)

Rubi [A]

time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2442, 46}

$$-\frac{\log(x-1)}{2x^2} + \frac{1}{2x} + \frac{1}{2} \log(1-x) - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Log[-1 + x]/x^3,x]

[Out] 1/(2*x) + Log[1 - x]/2 - Log[-1 + x]/(2*x^2) - Log[x]/2

Rule 46

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 2442

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]

Rubi steps

$$\begin{aligned} \int \frac{\log(-1+x)}{x^3} dx &= -\frac{\log(-1+x)}{2x^2} + \frac{1}{2} \int \frac{1}{(-1+x)x^2} dx \\ &= -\frac{\log(-1+x)}{2x^2} + \frac{1}{2} \int \left(\frac{1}{-1+x} - \frac{1}{x^2} - \frac{1}{x} \right) dx \\ &= \frac{1}{2x} + \frac{1}{2} \log(1-x) - \frac{\log(-1+x)}{2x^2} - \frac{\log(x)}{2} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 27, normalized size = 0.77

$$\frac{1}{2} \left(\frac{1}{x} + \log(1-x) - \frac{\log(-1+x)}{x^2} - \log(x) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[Log[-1 + x]/x^3,x]``[Out] (x^(-1) + Log[1 - x] - Log[-1 + x]/x^2 - Log[x])/2`**Mathics [A]**

time = 1.72, size = 26, normalized size = 0.74

$$\frac{x + x^2 (\text{Log}[-1 + x] - \text{Log}[x]) - \text{Log}[-1 + x]}{2x^2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Log[x - 1]/x^3,x]')``[Out] (x + x ^ 2 (Log[-1 + x] - Log[x]) - Log[-1 + x]) / (2 x ^ 2)`**Maple [A]**

time = 0.03, size = 26, normalized size = 0.74

method	result	size
derivativedivides	$-\frac{\ln(x)}{2} + \frac{1}{2x} + \frac{\ln(-1+x)(-1+x)(1+x)}{2x^2}$	26
default	$-\frac{\ln(x)}{2} + \frac{1}{2x} + \frac{\ln(-1+x)(-1+x)(1+x)}{2x^2}$	26
norman	$\frac{x}{2} + \frac{x^2 \ln(-1+x)}{2x^2} - \frac{\ln(-1+x)}{2} - \frac{\ln(x)}{2}$	29
risch	$-\frac{\ln(-1+x)}{2x^2} + \frac{\ln(-1+x)x - x \ln(x) + 1}{2x}$	29

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(ln(-1+x)/x^3,x,method=_RETURNVERBOSE)``[Out] -1/2*ln(x)+1/2/x+1/2*ln(-1+x)*(-1+x)*(1+x)/x^2`**Maxima [A]**

time = 0.26, size = 25, normalized size = 0.71

$$\frac{1}{2x} - \frac{\log(x-1)}{2x^2} + \frac{1}{2} \log(x-1) - \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(log(-1+x)/x^3,x, algorithm="maxima")`

[Out] $1/2/x - 1/2*\log(x - 1)/x^2 + 1/2*\log(x - 1) - 1/2*\log(x)$

Fricas [A]

time = 0.31, size = 26, normalized size = 0.74

$$-\frac{x^2 \log(x) - (x^2 - 1) \log(x - 1) - x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(-1+x)/x^3,x, algorithm="fricas")`

[Out] $-1/2*(x^2*\log(x) - (x^2 - 1)*\log(x - 1) - x)/x^2$

Sympy [A]

time = 0.08, size = 26, normalized size = 0.74

$$-\frac{\log(x)}{2} + \frac{\log(x - 1)}{2} + \frac{1}{2x} - \frac{\log(x - 1)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(-1+x)/x**3,x)`

[Out] $-\log(x)/2 + \log(x - 1)/2 + 1/(2*x) - \log(x - 1)/(2*x**2)$

Giac [A]

time = 0.00, size = 34, normalized size = 0.97

$$\frac{\ln|x - 1|}{2} - \frac{\ln|x|}{2} + \frac{1}{2x} - \frac{\ln(-1 + x)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(-1+x)/x^3,x)`

[Out] $1/2/x - 1/2*\log(x - 1)/x^2 + 1/2*\log(\text{abs}(x - 1)) - 1/2*\log(\text{abs}(x))$

Mupad [B]

time = 0.06, size = 25, normalized size = 0.71

$$\frac{x - \ln(x - 1) + x^2 \ln\left(1 - \frac{1}{x}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(x - 1)/x^3,x)`

[Out] $(x - \log(x - 1) + x^2*\log(1 - 1/x))/(2*x^2)$

3.639 $\int (-e^{-x} + e^x) \log(1 + e^{2x}) dx$

Optimal. Leaf size=32

$$-2e^x + e^{-x} \log(1 + e^{2x}) + e^x \log(1 + e^{2x})$$

[Out] $-2*\exp(x)+\ln(1+\exp(2*x))/\exp(x)+\exp(x)*\ln(1+\exp(2*x))$

Rubi [A]

time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2320, 2526, 2498, 327, 209, 2505}

$$-2e^x + e^{-x} \log(e^{2x} + 1) + e^x \log(e^{2x} + 1)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-E^{-x}) + E^x]*\text{Log}[1 + E^{(2*x)}], x]$

[Out] $-2*E^x + \text{Log}[1 + E^{(2*x)}]/E^x + E^x*\text{Log}[1 + E^{(2*x)}]$

Rule 209

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))* \text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 327

$\text{Int}[(c_)*(x_)^m*((a_ + (b_)*(x_)^n))^p, x_Symbol] \rightarrow \text{Simp}[c^{(n-1)}*(c*x)^{(m-n+1)}*((a + b*x^n)^{(p+1)}/(b*(m+n*p+1))), x] - \text{Dist}[a*c^n*((m-n+1)/(b*(m+n*p+1))), \text{Int}[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n-1] \ \&\& \ \text{NeQ}[m+n*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 2320

$\text{Int}[u, x_Symbol] \rightarrow \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (w_)*((a_)*(v_)^n)^m] /; \text{FreeQ}\{a, m, n\}, x] \ \&\& \ \text{IntegerQ}[m*n] \ \&\& \ !\text{MatchQ}[u, E^{((c_)*((a_)+(b_)*x))* (F_)}[v_]] /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{InverseFunctionQ}[F[x]]$

Rule 2498

$\text{Int}[\text{Log}[(c_)*((d_)+(e_)*(x_)^n))^p], x_Symbol] \rightarrow \text{Simp}[x*\text{Log}[c*(d + e*x^n)^p], x] - \text{Dist}[e*n*p, \text{Int}[x^n/(d + e*x^n), x], x] /; \text{FreeQ}\{c, d,$

e, n, p}, x]

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_)^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m + 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] && IntegerQ[s]
```

Rubi steps

$$\begin{aligned}
 \int (-e^{-x} + e^x) \log(1 + e^{2x}) dx &= \text{Subst} \left(\int \frac{(-1 + x^2) \log(1 + x^2)}{x^2} dx, x, e^x \right) \\
 &= \text{Subst} \left(\int \left(\log(1 + x^2) - \frac{\log(1 + x^2)}{x^2} \right) dx, x, e^x \right) \\
 &= \text{Subst} \left(\int \log(1 + x^2) dx, x, e^x \right) - \text{Subst} \left(\int \frac{\log(1 + x^2)}{x^2} dx, x, e^x \right) \\
 &= e^{-x} \log(1 + e^{2x}) + e^x \log(1 + e^{2x}) - 2 \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, e^x \right) - 2 \text{Subst} \left(\int \frac{\log(1 + x^2)}{x} dx, x, e^x \right) \\
 &= -2e^x - 2 \tan^{-1}(e^x) + e^{-x} \log(1 + e^{2x}) + e^x \log(1 + e^{2x}) + 2 \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, e^x \right) \\
 &= -2e^x + e^{-x} \log(1 + e^{2x}) + e^x \log(1 + e^{2x})
 \end{aligned}$$

Mathematica [A]

time = 0.02, size = 24, normalized size = 0.75

$$-2e^x + (e^{-x} + e^x) \log(1 + e^{2x})$$

Antiderivative was successfully verified.

```
[In] Integrate[(-E^(-x) + E^x)*Log[1 + E^(2*x)],x]
```

```
[Out] -2*E^x + (E^(-x) + E^x)*Log[1 + E^(2*x)]
```

Mathics [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(E^x - E^(-x))*Log[E^(2*x) + 1], x]')`[Out] `Integrate[Log[1 + E ^ (2 x)] (E ^ x - E ^ (-x)), x]`**Maple [A]**

time = 0.02, size = 24, normalized size = 0.75

method	result	size
risch	$(1 + e^{2x}) e^{-x} \ln(1 + e^{2x}) - 2 e^x$	24
norman	$(e^{2x} \ln(1 + e^{2x}) - 2 e^{2x} + \ln(1 + e^{2x})) e^{-x}$	32

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-1/exp(x)+exp(x))*ln(1+exp(2*x)),x,method=_RETURNVERBOSE)`[Out] `(1+exp(2*x))*exp(-x)*ln(1+exp(2*x))-2*exp(x)`**Maxima [A]**

time = 0.24, size = 20, normalized size = 0.62

$$(e^{(-x)} + e^x) \log(e^{(2x)} + 1) - 2 e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1/exp(x)+exp(x))*log(1+exp(2*x)),x, algorithm="maxima")`[Out] `(e^(-x) + e^x)*log(e^(2*x) + 1) - 2*e^x`**Fricas [A]**

time = 0.30, size = 26, normalized size = 0.81

$$((e^{(2x)} + 1) \log(e^{(2x)} + 1) - 2 e^{(2x)}) e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1/exp(x)+exp(x))*log(1+exp(2*x)),x, algorithm="fricas")`[Out] `((e^(2*x) + 1)*log(e^(2*x) + 1) - 2*e^(2*x))*e^(-x)`**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolificationFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1/exp(x)+exp(x))*ln(1+exp(2*x)),x)

[Out] Exception raised: PolificationFailed

Giac [A]

time = 0.00, size = 20, normalized size = 0.62

$$-2e^x + \left(\frac{1}{e^x} + e^x\right) \ln((e^x)^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1/exp(x)+exp(x))*log(1+exp(2*x)),x)

[Out] (e^x + 1/e^x)*log(e^(2*x) + 1) - 2*e^x

Mupad [B]

time = 0.38, size = 24, normalized size = 0.75

$$2 \ln(e^{2x} + 1) \cosh(x) - \frac{e^{2x} + 1}{\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-log(exp(2*x) + 1)*(exp(-x) - exp(x)),x)

[Out] 2*log(exp(2*x) + 1)*cosh(x) - (exp(2*x) + 1)/cosh(x)

3.640 $\int e^{3x/2} \log(-1 + e^x) dx$

Optimal. Leaf size=52

$$-\frac{4e^{x/2}}{3} - \frac{4}{9}e^{3x/2} + \frac{4}{3}\tanh^{-1}(e^{x/2}) + \frac{2}{3}e^{3x/2}\log(-1 + e^x)$$

[Out] $-4/3*\exp(1/2*x)-4/9*\exp(3/2*x)+4/3*\operatorname{arctanh}(\exp(1/2*x))+2/3*\exp(3/2*x)*\ln(-1+\exp(x))$

Rubi [A]

time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2225, 2634, 12, 2280, 308, 213}

$$-\frac{4e^{x/2}}{3} - \frac{4}{9}e^{3x/2} + \frac{2}{3}e^{3x/2}\log(e^x - 1) + \frac{4}{3}\tanh^{-1}(e^{x/2})$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{((3*x)/2)}*\text{Log}[-1 + E^x], x]$

[Out] $(-4*E^{(x/2)})/3 - (4*E^{((3*x)/2)})/9 + (4*\text{ArcTanh}[E^{(x/2)}])/3 + (2*E^{((3*x)/2)})*\text{Log}[-1 + E^x])/3$

Rule 12

$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

Rule 213

$\text{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[b, 2])^{-1})*\text{ArcTanh}[\text{Rt}[b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 308

$\text{Int}[(x_)^m / ((a_*) + (b_*)(x_)^n), x_Symbol] \rightarrow \text{Int}[\text{PolynomialDivide}[x^m, a + b*x^n, x], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, 2*n - 1]$

Rule 2225

$\text{Int}[(F_)^{((c_*)*((a_*) + (b_*)(x_)))^n}, x_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x)))^n} / (b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

Rule 2280

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_)))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log[F]))]}, Dist[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])), Subst[Int[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*(c + d*x)/Denominator[m])]], x] /; LeQ[m, -1] || GeQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] := With[{w = IntHide[v, x]}, Dist[Log[u], w, x] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
 \int e^{3x/2} \log(-1 + e^x) dx &= \frac{2}{3} e^{3x/2} \log(-1 + e^x) - \int \frac{2e^{5x/2}}{3(-1 + e^x)} dx \\
 &= \frac{2}{3} e^{3x/2} \log(-1 + e^x) - \frac{2}{3} \int \frac{e^{5x/2}}{-1 + e^x} dx \\
 &= \frac{2}{3} e^{3x/2} \log(-1 + e^x) - \frac{4}{3} \text{Subst} \left(\int \frac{x^4}{-1 + x^2} dx, x, e^{x/2} \right) \\
 &= \frac{2}{3} e^{3x/2} \log(-1 + e^x) - \frac{4}{3} \text{Subst} \left(\int \left(1 + x^2 + \frac{1}{-1 + x^2} \right) dx, x, e^{x/2} \right) \\
 &= -\frac{4e^{x/2}}{3} - \frac{4}{9} e^{3x/2} + \frac{2}{3} e^{3x/2} \log(-1 + e^x) - \frac{4}{3} \text{Subst} \left(\int \frac{1}{-1 + x^2} dx, x, e^{x/2} \right) \\
 &= -\frac{4e^{x/2}}{3} - \frac{4}{9} e^{3x/2} + \frac{4}{3} \tanh^{-1}(e^{x/2}) + \frac{2}{3} e^{3x/2} \log(-1 + e^x)
 \end{aligned}$$

Mathematica [A]

time = 0.02, size = 42, normalized size = 0.81

$$\frac{2}{9} (6 \tanh^{-1}(e^{x/2}) + e^{x/2} (-2(3 + e^x) + 3e^x \log(-1 + e^x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*x)/2)*Log[-1 + E^x], x]
```

```
[Out] (2*(6*ArcTanh[E^(x/2)] + E^(x/2)*(-2*(3 + E^x) + 3*E^x*Log[-1 + E^x]))) / 9
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[E^(3*x/2)*Log[E^x - 1],x]')`

[Out] Timed out

Maple [A]

time = 0.02, size = 43, normalized size = 0.83

method	result	size
risch	$\frac{2e^{\frac{3x}{2}} \ln(-1+e^x)}{3} - \frac{4e^{\frac{3x}{2}}}{9} - \frac{4e^{\frac{x}{2}}}{3} - \frac{2\ln(-1+e^{\frac{x}{2}})}{3} + \frac{2\ln(e^{\frac{x}{2}}+1)}{3}$	43

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(3/2*x)*ln(-1+exp(x)),x,method=_RETURNVERBOSE)`

[Out] $2/3*\exp(3/2*x)*\ln(-1+\exp(x))-4/9*\exp(3/2*x)-4/3*\exp(1/2*x)-2/3*\ln(-1+\exp(1/2*x))+2/3*\ln(\exp(1/2*x)+1)$

Maxima [A]

time = 0.24, size = 42, normalized size = 0.81

$$\frac{2}{3} e^{\left(\frac{3}{2}x\right)} \log(e^x - 1) - \frac{4}{9} e^{\left(\frac{3}{2}x\right)} - \frac{4}{3} e^{\left(\frac{1}{2}x\right)} + \frac{2}{3} \log\left(e^{\left(\frac{1}{2}x\right)} + 1\right) - \frac{2}{3} \log\left(e^{\left(\frac{1}{2}x\right)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(3/2*x)*log(-1+exp(x)),x, algorithm="maxima")`

[Out] $2/3*e^{(3/2*x)}*\log(e^x - 1) - 4/9*e^{(3/2*x)} - 4/3*e^{(1/2*x)} + 2/3*\log(e^{(1/2*x)} + 1) - 2/3*\log(e^{(1/2*x)} - 1)$

Fricas [A]

time = 0.31, size = 42, normalized size = 0.81

$$\frac{2}{3} e^{\left(\frac{3}{2}x\right)} \log(e^x - 1) - \frac{4}{9} e^{\left(\frac{3}{2}x\right)} - \frac{4}{3} e^{\left(\frac{1}{2}x\right)} + \frac{2}{3} \log\left(e^{\left(\frac{1}{2}x\right)} + 1\right) - \frac{2}{3} \log\left(e^{\left(\frac{1}{2}x\right)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(3/2*x)*log(-1+exp(x)),x, algorithm="fricas")`

[Out] $2/3*e^{(3/2*x)}*\log(e^x - 1) - 4/9*e^{(3/2*x)} - 4/3*e^{(1/2*x)} + 2/3*\log(e^{(1/2*x)} + 1) - 2/3*\log(e^{(1/2*x)} - 1)$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3/2*x)*ln(-1+exp(x)),x)

[Out] Timed out

Giac [A]

time = 0.00, size = 65, normalized size = 1.25

$$\frac{2}{3}e^{\frac{3}{2}x} \ln(-1 + e^x) - 2 \cdot 2 \left(\frac{\ln |e^{\frac{x}{2}} - 1|}{6} - \frac{\ln(e^{\frac{x}{2}} + 1)}{6} + \frac{3(e^{\frac{x}{2}})^3 + 9e^{\frac{x}{2}}}{27} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3/2*x)*log(-1+exp(x)),x)

[Out] 2/3*e^(3/2*x)*log(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*log(e^(1/2*x) + 1) - 2/3*log(abs(e^(1/2*x) - 1))

Mupad [B]

time = 0.55, size = 31, normalized size = 0.60

$$\frac{4 \operatorname{atanh}(\sqrt{e^x})}{3} - \frac{4e^{\frac{3x}{2}}}{9} - \frac{4e^{x/2}}{3} + \frac{2e^{\frac{3x}{2}} \ln(e^x - 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp((3*x)/2)*log(exp(x) - 1),x)

[Out] (4*atanh(exp(x)^(1/2)))/3 - (4*exp((3*x)/2))/9 - (4*exp(x/2))/3 + (2*exp((3*x)/2)*log(exp(x) - 1))/3

3.641 $\int \cos^3(x) \log(\sin(x)) dx$

Optimal. Leaf size=30

$$-\sin(x) + \log(\sin(x)) \sin(x) + \frac{\sin^3(x)}{9} - \frac{1}{3} \log(\sin(x)) \sin^3(x)$$

[Out] $-\sin(x) + \ln(\sin(x)) * \sin(x) + 1/9 * \sin(x)^3 - 1/3 * \ln(\sin(x)) * \sin(x)^3$

Rubi [A]

time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2713, 2634, 12, 4441}

$$\frac{\sin^3(x)}{9} - \sin(x) - \frac{1}{3} \sin^3(x) \log(\sin(x)) + \sin(x) \log(\sin(x))$$

Antiderivative was successfully verified.

[In] `Int[Cos[x]^3*Log[Sin[x]],x]`

[Out] $-\text{Sin}[x] + \text{Log}[\text{Sin}[x]] * \text{Sin}[x] + \text{Sin}[x]^3/9 - (\text{Log}[\text{Sin}[x]] * \text{Sin}[x]^3)/3$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 2634

`Int[Log[u]*(v_), x_Symbol] := With[{w = IntHide[v, x]}, Dist[Log[u], w, x] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x]] /; InverseFunctionFreeQ[u, x]`

Rule 2713

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rule 4441

`Int[(u_)*(F_)[(c_)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])`

Rubi steps

$$\begin{aligned}
\int \cos^3(x) \log(\sin(x)) dx &= \log(\sin(x)) \sin(x) - \frac{1}{3} \log(\sin(x)) \sin^3(x) - \int \frac{1}{6} \cos(x)(5 + \cos(2x)) dx \\
&= \log(\sin(x)) \sin(x) - \frac{1}{3} \log(\sin(x)) \sin^3(x) - \frac{1}{6} \int \cos(x)(5 + \cos(2x)) dx \\
&= \log(\sin(x)) \sin(x) - \frac{1}{3} \log(\sin(x)) \sin^3(x) - \frac{1}{6} \text{Subst} \left(\int (6 - 2x^2) dx, x, \sin(x) \right) \\
&= -\sin(x) + \log(\sin(x)) \sin(x) + \frac{\sin^3(x)}{9} - \frac{1}{3} \log(\sin(x)) \sin^3(x)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 30, normalized size = 1.00

$$-\sin(x) + \log(\sin(x)) \sin(x) + \frac{\sin^3(x)}{9} - \frac{1}{3} \log(\sin(x)) \sin^3(x)$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[x]^3*Log[Sin[x]],x]``[Out] -Sin[x] + Log[Sin[x]]*Sin[x] + Sin[x]^3/9 - (Log[Sin[x]]*Sin[x]^3)/3`**Mathics [A]**

time = 2.56, size = 24, normalized size = 0.80

$$\frac{(-9 + \sin[x]^2 - 3 \log[\sin[x]] \sin[x]^2 + 9 \log[\sin[x]]) \sin[x]}{9}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Cos[x]^3*Log[Sin[x]],x]')``[Out] (-9 + Sin[x]^2 - 3 Log[Sin[x]] Sin[x]^2 + 9 Log[Sin[x]]) Sin[x] / 9`**Maple [C]** Result contains complex when optimal does not.

time = 0.18, size = 197, normalized size = 6.57

method	result
default	$i \left(\frac{e^{3ix} \ln(i(-e^{2ix}+1)e^{-ix})}{3} - \frac{e^{3ix}}{9} - \frac{11e^{ix}}{3} + 3e^{ix} \ln(i(-e^{2ix}+1)e^{-ix}) - 3e^{-ix} \ln(i(-e^{2ix}+1)e^{-ix}) + \frac{11e^{-ix}}{3} - \frac{e^{-3ix} \ln(i(-e^{2ix}+1)e^{-ix})}{3} \right)$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x)^3*ln(sin(x)),x,method=_RETURNVERBOSE)`

[Out] $-1/8*I*(1/3*\exp(3*I*x)*\ln(I*(-\exp(I*x)^2+1)/\exp(I*x))-1/9*\exp(I*x)^3-11/3*\exp(I*x)+3*\exp(I*x)*\ln(I*(-\exp(I*x)^2+1)/\exp(I*x))-3*\exp(-I*x)*\ln(I*(-\exp(I*x)^2+1)/\exp(I*x))+11/3/\exp(I*x)-1/3*\exp(-3*I*x)*\ln(I*(-\exp(I*x)^2+1)/\exp(I*x))+1/9/\exp(I*x)^3-1/3*\ln(2)*\exp(I*x)^3-3*\ln(2)*\exp(I*x)+3*\ln(2)/\exp(I*x)+1/3*\ln(2)/\exp(I*x)^3)$

Maxima [A]

time = 0.24, size = 25, normalized size = 0.83

$$\frac{1}{9} \sin(x)^3 - \frac{1}{3} (\sin(x)^3 - 3 \sin(x)) \log(\sin(x)) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^3*log(sin(x)),x, algorithm="maxima")`

[Out] $1/9*\sin(x)^3 - 1/3*(\sin(x)^3 - 3*\sin(x))*\log(\sin(x)) - \sin(x)$

Fricas [A]

time = 0.33, size = 24, normalized size = 0.80

$$\frac{1}{3} (\cos(x)^2 + 2) \log(\sin(x)) \sin(x) - \frac{1}{9} (\cos(x)^2 + 8) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^3*log(sin(x)),x, algorithm="fricas")`

[Out] $1/3*(\cos(x)^2 + 2)*\log(\sin(x))*\sin(x) - 1/9*(\cos(x)^2 + 8)*\sin(x)$

Sympy [A]

time = 0.92, size = 42, normalized size = 1.40

$$\frac{2 \log(\sin(x)) \sin^3(x)}{3} + \log(\sin(x)) \sin(x) \cos^2(x) - \frac{8 \sin^3(x)}{9} - \sin(x) \cos^2(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)**3*ln(sin(x)),x)`

[Out] $2*\log(\sin(x))*\sin(x)**3/3 + \log(\sin(x))*\sin(x)*\cos(x)**2 - 8*\sin(x)**3/9 - \sin(x)*\cos(x)**2$

Giac [A]

time = 0.00, size = 28, normalized size = 0.93

$$\frac{\sin^3 x}{9} + \sin x \cdot \ln(\sin x) - \frac{1}{3} \sin^3 x \cdot \ln(\sin x) - \sin x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^3*log(sin(x)),x)`

[Out] $-1/3*\log(\sin(x))*\sin(x)^3 + 1/9*\sin(x)^3 + \log(\sin(x))*\sin(x) - \sin(x)$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \ln(\sin(x)) \cos(x)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(sin(x))*cos(x)^3,x)`

[Out] `int(log(sin(x))*cos(x)^3, x)`

3.642 $\int \log(\tan(x)) \sec^4(x) dx$

Optimal. Leaf size=30

$$-\tan(x) + \log(\tan(x)) \tan(x) - \frac{\tan^3(x)}{9} + \frac{1}{3} \log(\tan(x)) \tan^3(x)$$

[Out] $-\tan(x) + \ln(\tan(x)) * \tan(x) - 1/9 * \tan(x)^3 + 1/3 * \ln(\tan(x)) * \tan(x)^3$

Rubi [A]

time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3852, 2634, 12}

$$-\frac{\tan^3(x)}{9} - \tan(x) + \frac{1}{3} \tan^3(x) \log(\tan(x)) + \tan(x) \log(\tan(x))$$

Antiderivative was successfully verified.

[In] `Int[Log[Tan[x]]*Sec[x]^4,x]`

[Out] $-\text{Tan}[x] + \text{Log}[\text{Tan}[x]] * \text{Tan}[x] - \text{Tan}[x]^3/9 + (\text{Log}[\text{Tan}[x]] * \text{Tan}[x]^3)/3$

Rule 12

`Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

Rule 2634

`Int[Log[u_]*(v_), x_Symbol] :=> With[{w = IntHide[v, x]}, Dist[Log[u], w, x] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x]] /; InverseFunctionFreeQ[u, x]`

Rule 3852

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :=> Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rubi steps

$$\begin{aligned}
\int \log(\tan(x)) \sec^4(x) dx &= \log(\tan(x)) \tan(x) + \frac{1}{3} \log(\tan(x)) \tan^3(x) - \int \frac{1}{3} (2 + \cos(2x)) \sec^4(x) dx \\
&= \log(\tan(x)) \tan(x) + \frac{1}{3} \log(\tan(x)) \tan^3(x) - \frac{1}{3} \int (2 + \cos(2x)) \sec^4(x) dx \\
&= \log(\tan(x)) \tan(x) + \frac{1}{3} \log(\tan(x)) \tan^3(x) - \frac{1}{3} \text{Subst} \left(\int (3 + x^2) dx, x, \tan(x) \right) \\
&= -\tan(x) + \log(\tan(x)) \tan(x) - \frac{\tan^3(x)}{9} + \frac{1}{3} \log(\tan(x)) \tan^3(x)
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 29, normalized size = 0.97

$$\frac{1}{9} (-8 + (-1 + 6 \log(\tan(x)) + 3 \cos(2x) \log(\tan(x))) \sec^2(x)) \tan(x)$$

Antiderivative was successfully verified.

`[In] Integrate[Log[Tan[x]]*Sec[x]^4,x]``[Out] ((-8 + (-1 + 6*Log[Tan[x]] + 3*Cos[2*x]*Log[Tan[x]])*Sec[x]^2)*Tan[x])/9`**Mathics [A]**

time = 13.67, size = 26, normalized size = 0.87

$$\frac{(-9 - \tan[x]^2 + 3 \text{Log}[\tan[x]] \tan[x]^2 + 9 \text{Log}[\tan[x]]) \tan[x]}{9}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[Log[Tan[x]]/Cos[x]^4,x]')``[Out] (-9 - Tan[x]^2 + 3 Log[Tan[x]] Tan[x]^2 + 9 Log[Tan[x]]) Tan[x] / 9`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(26) = 52.

time = 0.30, size = 55, normalized size = 1.83

method	result
default	$\frac{(6(\cos^2(x)) \ln\left(\frac{\sin(x)}{2\cos(x)}\right) + 6(\cos^2(x)) \ln(2) - 8(\cos^2(x)) + 3 \ln\left(\frac{\sin(x)}{2\cos(x)}\right) + 3 \ln(2) - 1) \sin(x)}{9 \cos(x)^3}$
risch	$-\frac{4i(3e^{2ix}+1)\ln(e^{2ix}+1)}{3(e^{2ix}+1)^3} + \frac{2\pi}{3} + \frac{2\pi \text{csgn}\left(\frac{e^{2ix}-1}{e^{2ix}+1}\right)^3}{3} - \frac{2\text{csgn}\left(\frac{e^{2ix}-1}{e^{2ix}+1}\right)^2 \pi}{3} + \frac{2i \ln(e^{2ix}+1)}{3} - \frac{4ie^{4ix}}{3} + \frac{2\pi \text{csgn}\left(\frac{i(e^{2ix}-1)}{e^{2ix}+1}\right)^3}{3} + 2\pi e^{2ix} + 2\pi$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(tan(x))/cos(x)^4,x,method=_RETURNVERBOSE)`

[Out] $1/9*(6*\cos(x)^2*\ln(1/2*\sin(x)/\cos(x))+6*\cos(x)^2*\ln(2)-8*\cos(x)^2+3*\ln(1/2*\sin(x)/\cos(x))+3*\ln(2)-1)*\sin(x)/\cos(x)^3$

Maxima [A]

time = 0.25, size = 25, normalized size = 0.83

$$-\frac{1}{9} \tan(x)^3 + \frac{1}{3} (\tan(x)^3 + 3 \tan(x)) \log(\tan(x)) - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(tan(x))/cos(x)^4,x, algorithm="maxima")`

[Out] $-1/9*\tan(x)^3 + 1/3*(\tan(x)^3 + 3*\tan(x))*\log(\tan(x)) - \tan(x)$

Fricas [A]

time = 0.33, size = 39, normalized size = 1.30

$$\frac{3(2 \cos(x)^2 + 1) \log\left(\frac{\sin(x)}{\cos(x)}\right) \sin(x) - (8 \cos(x)^2 + 1) \sin(x)}{9 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(tan(x))/cos(x)^4,x, algorithm="fricas")`

[Out] $1/9*(3*(2*\cos(x)^2 + 1)*\log(\sin(x)/\cos(x))*\sin(x) - (8*\cos(x)^2 + 1)*\sin(x))/\cos(x)^3$

Sympy [A]

time = 13.14, size = 46, normalized size = 1.53

$$\frac{\log(\tan(x)) \tan^3(x)}{3} + \log(\tan(x)) \tan(x) - \frac{\sin^3(x)}{9 \cos^3(x)} + \frac{\sin(x)}{3 \cos(x)} - \frac{4 \tan(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(tan(x))/cos(x)**4,x)`

[Out] $\log(\tan(x))*\tan(x)**3/3 + \log(\tan(x))*\tan(x) - \sin(x)**3/(9*\cos(x)**3) + \sin(x)/(3*\cos(x)) - 4*\tan(x)/3$

Giac [A]

time = 0.00, size = 28, normalized size = 0.93

$$-\frac{\tan^3 x}{9} + \tan x \cdot \ln(\tan x) + \frac{1}{3} \tan^3 x \cdot \ln(\tan x) - \tan x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(tan(x))/cos(x)^4,x)

[Out] 1/3*log(tan(x))*tan(x)^3 - 1/9*tan(x)^3 + log(tan(x))*tan(x) - tan(x)

Mupad [B]

time = 1.80, size = 148, normalized size = 4.93

$$\frac{\ln\left(-\frac{8e^{x2i}}{3} - \frac{8}{3}\right)2i}{3} - \frac{\ln\left(\frac{8}{3} - \frac{8e^{x2i}}{3}\right)2i}{3} + \frac{8i}{9(3e^{x2i} + 3e^{x4i} + e^{x6i} + 1)} - \frac{4i}{3(2e^{x2i} + e^{x4i} + 1)} - \frac{4i}{3(e^{x2i} + 1)} + \frac{\ln\left(-\frac{e^{x2i}1i-i}{e^{x2i}+1}\right)(e^{x2i}4i + \frac{4}{3}i)}{3e^{x2i} + 3e^{x4i} + e^{x6i} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(tan(x))/cos(x)^4,x)

[Out] (log(-(8*exp(x*2i))/3 - 8/3)*2i)/3 - (log(8/3 - (8*exp(x*2i))/3)*2i)/3 + 8i/(9*(3*exp(x*2i) + 3*exp(x*4i) + exp(x*6i) + 1)) - 4i/(3*(2*exp(x*2i) + exp(x*4i) + 1)) - 4i/(3*(exp(x*2i) + 1)) + (log(-(exp(x*2i)*1i - 1i)/(exp(x*2i) + 1)))*(exp(x*2i)*4i + 4i/3)/(3*exp(x*2i) + 3*exp(x*4i) + exp(x*6i) + 1)

$$3.643 \quad \int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)} dx$$

Optimal. Leaf size=28

$$-\frac{x}{2} + \frac{\log\left(\cos\left(\frac{x}{2}\right)\right) \sin(x)}{1+\cos(x)} + \tan\left(\frac{x}{2}\right)$$

[Out] $-1/2*x + \ln(\cos(1/2*x))*\sin(x)/(1+\cos(x)) + \tan(1/2*x)$

Rubi [A]

time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2727, 2634, 12, 3554, 8}

$$-\frac{x}{2} + \tan\left(\frac{x}{2}\right) + \frac{\sin(x) \log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x) + 1}$$

Antiderivative was successfully verified.

[In] `Int[Log[Cos[x/2]]/(1 + Cos[x]),x]`

[Out] $-1/2*x + (\text{Log}[\text{Cos}[x/2]]*\text{Sin}[x])/(1 + \text{Cos}[x]) + \text{Tan}[x/2]$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

Rule 2634

`Int[Log[u_]*(v_), x_Symbol] := With[{w = IntHide[v, x]}, Dist[Log[u], w, x] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x]] /; InverseFunctionFreeQ[u, x]`

Rule 2727

`Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

Rule 3554

`Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[b*((b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],`

x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned}
 \int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)} dx &= \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)} - \int -\frac{1}{2}\tan^2\left(\frac{x}{2}\right) dx \\
 &= \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)} + \frac{1}{2} \int \tan^2\left(\frac{x}{2}\right) dx \\
 &= \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)} + \tan\left(\frac{x}{2}\right) - \frac{\int 1 dx}{2} \\
 &= -\frac{x}{2} + \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)} + \tan\left(\frac{x}{2}\right)
 \end{aligned}$$

Mathematica [A]

time = 0.05, size = 32, normalized size = 1.14

$$-\frac{\left(x \cot\left(\frac{x}{2}\right) - 2\left(1 + \log\left(\cos\left(\frac{x}{2}\right)\right)\right)\right)\sin(x)}{2(1+\cos(x))}$$

Antiderivative was successfully verified.

[In] Integrate[Log[Cos[x/2]]/(1 + Cos[x]),x]

[Out] -1/2*((x*Cot[x/2] - 2*(1 + Log[Cos[x/2]]))*Sin[x])/(1 + Cos[x])

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Log[Cos[x/2]]/(1 + Cos[x]),x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 0.15, size = 164, normalized size = 5.86

method	result
risch	$-\frac{2i \ln\left(e^{\frac{ix}{2}}\right)}{1+e^{ix}} + \frac{\pi \operatorname{csgn}(i(1+e^{ix})) \operatorname{csgn}\left(ie^{-\frac{ix}{2}}\right) \operatorname{csgn}\left(i \cos\left(\frac{x}{2}\right)\right) - \pi \operatorname{csgn}(i(1+e^{ix})) \operatorname{csgn}\left(i \cos\left(\frac{x}{2}\right)\right)^2 - \pi \operatorname{csgn}\left(ie^{-\frac{ix}{2}}\right) \operatorname{csgn}\left(i \cos\left(\frac{x}{2}\right)\right)^2}{1+e^{ix}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(cos(1/2*x))/(1+cos(x)),x,method=_RETURNVERBOSE)`

[Out] $-2*I/(1+\exp(I*x))*\ln(\exp(1/2*I*x))+(\text{Pi}*\text{csgn}(I*(1+\exp(I*x))))*\text{csgn}(I*\exp(-1/2*I*x))*\text{csgn}(I*\cos(1/2*x))-\text{Pi}*\text{csgn}(I*(1+\exp(I*x)))*\text{csgn}(I*\cos(1/2*x))^2-\text{Pi}*\text{csgn}(I*\exp(-1/2*I*x))*\text{csgn}(I*\cos(1/2*x))^2+\text{Pi}*\text{csgn}(I*\cos(1/2*x))^3-I*\ln(1+\exp(I*x))*\exp(I*x)-x*\exp(I*x)-2*I*\ln(2)+I*\ln(1+\exp(I*x))+2*I-x/(1+\exp(I*x))$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(22) = 44.

time = 0.24, size = 56, normalized size = 2.00

$$\frac{\log\left(\cos\left(\frac{1}{2}x\right)\right)\sin(x)}{\cos(x)+1} - \frac{x\cos(x)^2 + x\sin(x)^2 + 2x\cos(x) + x - 4\sin(x)}{2(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(cos(1/2*x))/(1+cos(x)),x, algorithm="maxima")`

[Out] $\log(\cos(1/2*x))*\sin(x)/(\cos(x) + 1) - 1/2*(x*\cos(x)^2 + x*\sin(x)^2 + 2*x*\cos(x) + x - 4*\sin(x))/(\cos(x)^2 + \sin(x)^2 + 2*\cos(x) + 1)$

Fricas [A]

time = 0.32, size = 32, normalized size = 1.14

$$\frac{x\cos\left(\frac{1}{2}x\right) - 2\log\left(\cos\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}x\right) - 2\sin\left(\frac{1}{2}x\right)}{2\cos\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(cos(1/2*x))/(1+cos(x)),x, algorithm="fricas")`

[Out] $-1/2*(x*\cos(1/2*x) - 2*\log(\cos(1/2*x))*\sin(1/2*x) - 2*\sin(1/2*x))/\cos(1/2*x)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x)+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(cos(1/2*x))/(1+cos(x)),x)`

[Out] `Integral(log(cos(x/2))/(cos(x) + 1), x)`

Giac [A]

time = 0.01, size = 30, normalized size = 1.07

$$2 \left(\frac{\tan\left(\frac{x}{2}\right)}{2} - \frac{x}{4} + \frac{1}{2} \tan\left(\frac{x}{2}\right) \ln\left(\cos\left(\frac{x}{2}\right)\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(log(cos(1/2*x))/(1+cos(x)),x)``[Out] log(cos(1/2*x))*tan(1/2*x) - 1/2*x + tan(1/2*x)`**Mupad [B]**

time = 0.58, size = 39, normalized size = 1.39

$$\tan\left(\frac{x}{2}\right) - x + \tan\left(\frac{x}{2}\right) \ln\left(\cos\left(\frac{x}{2}\right)\right) + \ln\left(\cos\left(\frac{x}{2}\right)\right) \operatorname{li} - \ln(\cos(x) + 1 + \sin(x)) \operatorname{li} \operatorname{li}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(log(cos(x/2))/(cos(x) + 1),x)``[Out] tan(x/2) - x + log(cos(x/2))*1i - log(cos(x) + sin(x)*1i + 1)*1i + tan(x/2) *log(cos(x/2))`

$$3.644 \quad \int \frac{\cos(x) \log(\sin(x))}{(1+\cos(x))^2} dx$$

Optimal. Leaf size=60

$$-\frac{2x}{3} - \frac{\sin(x)}{9(1+\cos(x))^2} + \frac{8\sin(x)}{9(1+\cos(x))} - \frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2} + \frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))}$$

[Out] $-2/3*x-1/9*\sin(x)/(1+\cos(x))^2+8/9*\sin(x)/(1+\cos(x))-1/3*\ln(\sin(x))*\sin(x)/(1+\cos(x))^2+2/3*\ln(\sin(x))*\sin(x)/(1+\cos(x))$

Rubi [A]

time = 0.09, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {2829, 2727, 2634, 12, 3047, 3098, 2814}

$$-\frac{2x}{3} + \frac{8\sin(x)}{9(\cos(x)+1)} - \frac{\sin(x)}{9(\cos(x)+1)^2} + \frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)} - \frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]*Log[Sin[x]])/(1 + Cos[x])^2,x]

[Out] $(-2*x)/3 - \sin[x]/(9*(1 + \cos[x])^2) + (8*\sin[x])/(9*(1 + \cos[x])) - (\log[\sin[x]]*\sin[x])/(3*(1 + \cos[x])^2) + (2*\log[\sin[x]]*\sin[x])/(3*(1 + \cos[x]))$

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2634

Int[Log[u]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x]] /; InverseFunctionFreeQ[u, x]

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c + d*x]/(d*(b + a*sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2814

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2829

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x) \log(\sin(x))}{(1 + \cos(x))^2} dx &= -\frac{\log(\sin(x)) \sin(x)}{3(1 + \cos(x))^2} + \frac{2 \log(\sin(x)) \sin(x)}{3(1 + \cos(x))} - \int \frac{\cos(x)(1 + 2 \cos(x))}{3(1 + \cos(x))^2} dx \\
&= -\frac{\log(\sin(x)) \sin(x)}{3(1 + \cos(x))^2} + \frac{2 \log(\sin(x)) \sin(x)}{3(1 + \cos(x))} - \frac{1}{3} \int \frac{\cos(x)(1 + 2 \cos(x))}{(1 + \cos(x))^2} dx \\
&= -\frac{\log(\sin(x)) \sin(x)}{3(1 + \cos(x))^2} + \frac{2 \log(\sin(x)) \sin(x)}{3(1 + \cos(x))} - \frac{1}{3} \int \frac{\cos(x) + 2 \cos^2(x)}{(1 + \cos(x))^2} dx \\
&= -\frac{\sin(x)}{9(1 + \cos(x))^2} - \frac{\log(\sin(x)) \sin(x)}{3(1 + \cos(x))^2} + \frac{2 \log(\sin(x)) \sin(x)}{3(1 + \cos(x))} + \frac{1}{9} \int \frac{2 - 6 \cos(x)}{1 + \cos(x)} dx \\
&= -\frac{2x}{3} - \frac{\sin(x)}{9(1 + \cos(x))^2} - \frac{\log(\sin(x)) \sin(x)}{3(1 + \cos(x))^2} + \frac{2 \log(\sin(x)) \sin(x)}{3(1 + \cos(x))} + \frac{8}{9} \int \frac{1}{1 + \cos(x)} dx \\
&= -\frac{2x}{3} - \frac{\sin(x)}{9(1 + \cos(x))^2} + \frac{8 \sin(x)}{9(1 + \cos(x))} - \frac{\log(\sin(x)) \sin(x)}{3(1 + \cos(x))^2} + \frac{2 \log(\sin(x)) \sin(x)}{3(1 + \cos(x))}
\end{aligned}$$

Mathematica [A]

time = 0.09, size = 56, normalized size = 0.93

$$-\frac{1}{18} \sec^3\left(\frac{x}{2}\right) \left(9x \cos\left(\frac{x}{2}\right) + 3x \cos\left(\frac{3x}{2}\right) - (7 + 3 \log(\sin(x)) + \cos(x)(8 + 6 \log(\sin(x)))) \sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]*Log[Sin[x]])/(1 + Cos[x])^2,x]

[Out] -1/18*(Sec[x/2]^3*(9*x*Cos[x/2] + 3*x*Cos[(3*x)/2] - (7 + 3*Log[Sin[x]] + Cos[x]*(8 + 6*Log[Sin[x]]))*Sin[x/2]))

Mathics [A]

time = 4.34, size = 38, normalized size = 0.63

$$\frac{-2x}{3} - \frac{\text{Log}[\text{Sin}[x]] \text{Tan}\left[\frac{x}{2}\right]^3}{6} - \frac{\text{Tan}\left[\frac{x}{2}\right]^3}{18} + \frac{\text{Log}[\text{Sin}[x]] \text{Tan}\left[\frac{x}{2}\right]}{2} + \frac{5\text{Tan}\left[\frac{x}{2}\right]}{6}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[Cos[x]*Log[Sin[x]]/(1 + Cos[x])^2,x]')

[Out] -2 x / 3 - Log[Sin[x]] Tan[x / 2] ^ 3 / 6 - Tan[x / 2] ^ 3 / 18 + Log[Sin[x]] Tan[x / 2] / 2 + 5 Tan[x / 2] / 6

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(50) = 100.

time = 0.28, size = 106, normalized size = 1.77

method	result
default	$\frac{6(\cos^3(x)) \ln(2) + 6(\cos^3(x)) \ln\left(\frac{\sin(x)}{2}\right) - 12(\cos^2(x)) \sin(x) \arctan\left(\frac{\cos(x)-1}{\sin(x)}\right) + 8(\cos^3(x)) - 9(\cos^2(x)) \ln(2) - 9(\cos^2(x)) \ln\left(\frac{\sin(x)}{2}\right)}{9 \sin(x)^3}$
risch	$-\frac{2i(3e^{2ix} + 3e^{ix} + 2) \ln(e^{ix})}{3(1+e^{ix})^3} + \frac{6\pi - 12x + 9\pi e^{2ix} - 9\pi \text{csgn}(i \sin(x))^2 e^{2ix} - 6\pi \text{csgn}(i(e^{2ix} - 1)) \text{csgn}(\sin(x))^2 - 9\pi \text{csgn}(\sin(x))^3 e^{2ix}}{3(1+e^{ix})^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*ln(sin(x))/(1+cos(x))^2,x,method=_RETURNVERBOSE)

[Out] 1/9*(6*cos(x)^3*ln(2)+6*cos(x)^3*ln(1/2*sin(x))-12*cos(x)^2*sin(x)*arctan((cos(x)-1)/sin(x))+8*cos(x)^3-9*cos(x)^2*ln(2)-9*cos(x)^2*ln(1/2*sin(x))-9*cos(x)^2+12*arctan((cos(x)-1)/sin(x))*sin(x)-6*cos(x)+3*ln(2)+3*ln(1/2*sin(x)))+7)/sin(x)^3

Maxima [A]

time = 0.33, size = 86, normalized size = 1.43

$$\frac{1}{6} \left(\frac{3 \sin(x)}{\cos(x)+1} - \frac{\sin(x)^3}{(\cos(x)+1)^3} \right) \log \left(\frac{2 \sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1 \right) (\cos(x)+1)} \right) + \frac{5 \sin(x)}{6(\cos(x)+1)} - \frac{\sin(x)^3}{18(\cos(x)+1)^3} - \frac{4}{3} \arctan \left(\frac{\sin(x)}{\cos(x)+1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*log(sin(x))/(1+cos(x))^2,x, algorithm="maxima")

[Out] $\frac{1}{6} \cdot (3 \sin(x) / (\cos(x) + 1) - \sin(x)^3 / (\cos(x) + 1)^3) \cdot \log(2 \sin(x) / ((\sin(x))^2 / (\cos(x) + 1)^2 + 1) \cdot (\cos(x) + 1))) + \frac{5}{6} \sin(x) / (\cos(x) + 1) - \frac{1}{18} \sin(x)^3 / (\cos(x) + 1)^3 - \frac{4}{3} \arctan(\sin(x) / (\cos(x) + 1))$

Fricas [A]

time = 0.32, size = 53, normalized size = 0.88

$$\frac{6x \cos(x)^2 - 3(2 \cos(x) + 1) \log(\sin(x)) \sin(x) + 12x \cos(x) - (8 \cos(x) + 7) \sin(x) + 6x}{9(\cos(x)^2 + 2 \cos(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*log(sin(x))/(1+cos(x))^2,x, algorithm="fricas")`

[Out] $-1/9 \cdot (6x \cos(x)^2 - 3(2 \cos(x) + 1) \log(\sin(x)) \sin(x) + 12x \cos(x) - (8 \cos(x) + 7) \sin(x) + 6x) / (\cos(x)^2 + 2 \cos(x) + 1)$

Sympy [A]

time = 3.24, size = 88, normalized size = 1.47

$$-\frac{2x}{3} - \frac{\log\left(\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right) \tan^3\left(\frac{x}{2}\right)}{6} + \frac{\log\left(\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right) \tan\left(\frac{x}{2}\right)}{2} - \frac{\log(2) \tan^3\left(\frac{x}{2}\right)}{6} - \frac{\tan^3\left(\frac{x}{2}\right)}{18} + \frac{\log(2) \tan\left(\frac{x}{2}\right)}{2} + \frac{5 \tan\left(\frac{x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*ln(sin(x))/(1+cos(x))**2,x)`

[Out] $-2x/3 - \log(\tan(x/2) / (\tan(x/2)^2 + 1)) \cdot \tan(x/2)^3 / 6 + \log(\tan(x/2) / (\tan(x/2)^2 + 1)) \cdot \tan(x/2) / 2 - \log(2) \cdot \tan(x/2)^3 / 6 - \tan(x/2)^3 / 18 + \log(2) \cdot \tan(x/2) / 2 + 5 \cdot \tan(x/2) / 6$

Giac [A]

time = 0.04, size = 56, normalized size = 0.93

$$\frac{2}{4} \left(-\frac{1}{3} \tan^3\left(\frac{x}{2}\right) + \tan\left(\frac{x}{2}\right) \right) \ln(\sin x) - 2 \cdot 2 \left(\frac{192 \tan^3\left(\frac{x}{2}\right) - 2880 \tan\left(\frac{x}{2}\right)}{13824} + \frac{x}{6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*log(sin(x))/(1+cos(x))^2,x)`

[Out] $-1/18 \cdot \tan(1/2 \cdot x)^3 - 1/6 \cdot (\tan(1/2 \cdot x)^3 - 3 \cdot \tan(1/2 \cdot x)) \cdot \log(\sin(x)) - 2/3 \cdot x + 5/6 \cdot \tan(1/2 \cdot x)$

Mupad [B]

time = 0.79, size = 164, normalized size = 2.73

$$\frac{\frac{4 \sin(2x)}{9} - \frac{\ln(-2 \sin(x)^2 + \sin(2x))}{3} \cdot 71 - \frac{14x}{3} + \frac{\ln(\sin(2x))}{3} \cdot 71 + \frac{7 \sin(x)}{9} + \frac{\sin(2x) \ln(\sin(x))}{3} - \frac{\sin(x)^2 \sin(x)}{9} + \sin\left(\frac{x}{2}\right)^2 \left(\frac{192x}{3} + \frac{\ln(-2 \sin(x)^2 + \sin(2x))}{3} \cdot 111 - \frac{\ln(\sin(x))}{3} \cdot 81 - \frac{224}{3} \right) + \frac{\ln(-2 \sin(x)^2 + \sin(2x))}{3} \cdot (2 \sin(x)^2 - 1) \cdot 111 + \frac{\ln(\sin(x)) \sin(x)}{3} - \frac{\ln(\sin(x))}{3} \cdot (2 \sin(x)^2 - 1) \cdot 111 + \frac{2x(2 \sin(x)^2 - 1)}{3} + \frac{14}{3} x}{(2 \sin\left(\frac{x}{2}\right)^2 - 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((\log(\sin(x))\cos(x))/(\cos(x) + 1)^2, x)$

[Out] $((4\sin(2x))/9 - (\log(\sin(2x))*1i - 2\sin(x)^2*7i)/3 - (14x)/3 + (\log(\sin(x))*7i)/3 + (7\sin(x))/9 + (\sin(2x)*\log(\sin(x)))/3 - (\sin(x)^2*8i)/9 + \sin(x/2)^2*((16x)/3 + (\log(\sin(2x))*1i - 2\sin(x)^2*8i)/3 - (\log(\sin(x))*8i)/3 - 32i/9) + (\log(\sin(2x))*1i - 2\sin(x)^2*(2\sin(x)^2 - 1)*1i)/3 + (\log(\sin(x))*\sin(x))/3 - (\log(\sin(x))*(2\sin(x)^2 - 1)*1i)/3 + (2x*(2\sin(x)^2 - 1))/3 + 32i/9)/(2\sin(x/2)^2 - 2)^2$

3.645 $\int \frac{\cos^{-1}(x)^2}{x^5} dx$

Optimal. Leaf size=65

$$-\frac{1}{12x^2} + \frac{\sqrt{1-x^2} \cos^{-1}(x)}{6x^3} + \frac{\sqrt{1-x^2} \cos^{-1}(x)}{3x} - \frac{\cos^{-1}(x)^2}{4x^4} + \frac{\log(x)}{3}$$

[Out] $-1/12/x^2-1/4*\arccos(x)^2/x^4+1/3*\ln(x)+1/6*\arccos(x)*(-x^2+1)^{(1/2)}/x^3+1/3*\arccos(x)*(-x^2+1)^{(1/2)}/x$

Rubi [A]

time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {4724, 4790, 4772, 29, 30}

$$-\frac{\cos^{-1}(x)^2}{4x^4} - \frac{1}{12x^2} + \frac{\sqrt{1-x^2} \cos^{-1}(x)}{3x} + \frac{\sqrt{1-x^2} \cos^{-1}(x)}{6x^3} + \frac{\log(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[x]^2/x^5,x]

[Out] $-1/12*1/x^2 + (\text{Sqrt}[1 - x^2]*\text{ArcCos}[x])/(6*x^3) + (\text{Sqrt}[1 - x^2]*\text{ArcCos}[x])/(3*x) - \text{ArcCos}[x]^2/(4*x^4) + \text{Log}[x]/3$

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 4724

Int[((a_) + ArcCos[(c_)*(x_)]*(b_.))^ (n_.)*((d_)*(x_))^(m_), x_Symbol] :> Simp[(d*x)^(m+1)*((a + b*ArcCos[c*x])^n/(d*(m+1))), x] + Dist[b*c*(n/(d*(m+1))), Int[(d*x)^(m+1)*((a + b*ArcCos[c*x])^(n-1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 4772

Int[((a_) + ArcCos[(c_)*(x_)]*(b_.))^ (n_.)*((f_)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m+1)*(d + e*x^2)^(p+1)*((a + b*ArcCos[c*x])^n/(d*f*(m+1))), x] + Dist[b*c*(n/(f*(m+1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m+1)*(1 - c^2*x^2)^(p+1/2)*(a + b*ArcCos[c*x])^(n-1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2

*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]

Rule 4790

Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b *ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\cos^{-1}(x)^2}{x^5} dx &= -\frac{\cos^{-1}(x)^2}{4x^4} - \frac{1}{2} \int \frac{\cos^{-1}(x)}{x^4\sqrt{1-x^2}} dx \\ &= \frac{\sqrt{1-x^2} \cos^{-1}(x)}{6x^3} - \frac{\cos^{-1}(x)^2}{4x^4} + \frac{1}{6} \int \frac{1}{x^3} dx - \frac{1}{3} \int \frac{\cos^{-1}(x)}{x^2\sqrt{1-x^2}} dx \\ &= -\frac{1}{12x^2} + \frac{\sqrt{1-x^2} \cos^{-1}(x)}{6x^3} + \frac{\sqrt{1-x^2} \cos^{-1}(x)}{3x} - \frac{\cos^{-1}(x)^2}{4x^4} + \frac{1}{3} \int \frac{1}{x} dx \\ &= -\frac{1}{12x^2} + \frac{\sqrt{1-x^2} \cos^{-1}(x)}{6x^3} + \frac{\sqrt{1-x^2} \cos^{-1}(x)}{3x} - \frac{\cos^{-1}(x)^2}{4x^4} + \frac{\log(x)}{3} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 52, normalized size = 0.80

$$-\frac{1}{12x^2} + \frac{\sqrt{1-x^2} (1+2x^2) \cos^{-1}(x)}{6x^3} - \frac{\cos^{-1}(x)^2}{4x^4} + \frac{\log(x)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCos[x]^2/x^5,x]

[Out] -1/12*1/x^2 + (Sqrt[1 - x^2]*(1 + 2*x^2)*ArcCos[x])/(6*x^3) - ArcCos[x]^2/(4*x^4) + Log[x]/3

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[ArcCos[x]^2/x^5,x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.05, size = 52, normalized size = 0.80

method	result	size
default	$-\frac{1}{12x^2} - \frac{\arccos(x)^2}{4x^4} + \frac{\ln(x)}{3} + \frac{\arccos(x)\sqrt{-x^2+1}}{6x^3} + \frac{\arccos(x)\sqrt{-x^2+1}}{3x}$	52

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccos(x)^2/x^5,x,method=_RETURNVERBOSE)`

[Out] $-\frac{1}{12x^2} - \frac{1}{4}\arccos(x)^2/x^4 + \frac{1}{3}\ln(x) + \frac{1}{6}\arccos(x)*(-x^2+1)^{(1/2)}/x^3 + \frac{1}{3}\arccos(x)*(-x^2+1)^{(1/2)}/x$

Maxima [A]

time = 0.33, size = 51, normalized size = 0.78

$$\frac{1}{6} \left(\frac{2\sqrt{-x^2+1}}{x} + \frac{\sqrt{-x^2+1}}{x^3} \right) \arccos(x) - \frac{1}{12x^2} - \frac{\arccos(x)^2}{4x^4} + \frac{1}{3} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccos(x)^2/x^5,x, algorithm="maxima")`

[Out] $\frac{1}{6}*(2*\sqrt{-x^2+1}/x + \sqrt{-x^2+1}/x^3)*\arccos(x) - \frac{1}{12x^2} - \frac{1}{4}\arccos(x)^2/x^4 + \frac{1}{3}\log(x)$

Fricas [A]

time = 0.32, size = 44, normalized size = 0.68

$$\frac{4x^4 \log(x) + 2(2x^3 + x)\sqrt{-x^2+1} \arccos(x) - x^2 - 3\arccos(x)^2}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccos(x)^2/x^5,x, algorithm="fricas")`

[Out] $\frac{1}{12}*(4*x^4*\log(x) + 2*(2*x^3 + x)*\sqrt{-x^2+1}*\arccos(x) - x^2 - 3*\arccos(x)^2)/x^4$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arccos^2(x)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(x)**2/x**5,x)

[Out] Integral(acos(x)**2/x**5, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(51) = 102.

time = 0.02, size = 145, normalized size = 2.23

$$-\frac{2x^2+1}{12x^2} + \frac{\ln(x^2)}{6} + \left(\frac{\frac{256}{3} \left(-\frac{-2\sqrt{-x^2+1}+2}{2x} \right)^3 - \frac{384(-2\sqrt{-x^2+1}+2)}{x}}{4096} + \frac{-9 \left(-\frac{-2\sqrt{-x^2+1}+2}{2x} \right)^2 - 1}{48 \left(-\frac{-2\sqrt{-x^2+1}+2}{2x} \right)^3} \right) \arccos x - \frac{\arccos^2 x}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x)^2/x^5,x)

[Out] $-1/48*(x^3*(9*(\sqrt{-x^2+1}-1)^2/x^2+1)/(\sqrt{-x^2+1}-1)^3 - 9*(\sqrt{-x^2+1}-1)/x - (\sqrt{-x^2+1}-1)^3/x^3)*\arccos(x) - 1/12*(2*x^2+1)/x^2 - 1/4*\arccos(x)^2/x^4 + 1/6*\log(x^2)$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\arccos(x)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acos(x)^2/x^5,x)

[Out] int(acos(x)^2/x^5, x)

3.646 $\int x^2 \sin^{-1}(x)^2 dx$

Optimal. Leaf size=61

$$-\frac{4x}{9} - \frac{2x^3}{27} + \frac{4}{9}\sqrt{1-x^2} \sin^{-1}(x) + \frac{2}{9}x^2\sqrt{1-x^2} \sin^{-1}(x) + \frac{1}{3}x^3 \sin^{-1}(x)^2$$

[Out] $-4/9*x-2/27*x^3+1/3*x^3*\arcsin(x)^2+4/9*\arcsin(x)*(-x^2+1)^{(1/2)}+2/9*x^2*\arcsin(x)*(-x^2+1)^{(1/2)}$

Rubi [A]

time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {4723, 4795, 4767, 8, 30}

$$-\frac{2x^3}{27} + \frac{1}{3}x^3 \sin^{-1}(x)^2 + \frac{2}{9}\sqrt{1-x^2} x^2 \sin^{-1}(x) + \frac{4}{9}\sqrt{1-x^2} \sin^{-1}(x) - \frac{4x}{9}$$

Antiderivative was successfully verified.

[In] Int[x^2*ArcSin[x]^2,x]

[Out] $(-4*x)/9 - (2*x^3)/27 + (4*\text{Sqrt}[1 - x^2]*\text{ArcSin}[x])/9 + (2*x^2*\text{Sqrt}[1 - x^2]*\text{ArcSin}[x])/9 + (x^3*\text{ArcSin}[x]^2)/3$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 4723

Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*((d_)*(x_))^(m_), x_Symbol] := Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 4767

Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*(x_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

Rule 4795

Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]

Rubi steps

$$\begin{aligned}
 \int x^2 \sin^{-1}(x)^2 dx &= \frac{1}{3}x^3 \sin^{-1}(x)^2 - \frac{2}{3} \int \frac{x^3 \sin^{-1}(x)}{\sqrt{1-x^2}} dx \\
 &= \frac{2}{9}x^2 \sqrt{1-x^2} \sin^{-1}(x) + \frac{1}{3}x^3 \sin^{-1}(x)^2 - \frac{2}{9} \int x^2 dx - \frac{4}{9} \int \frac{x \sin^{-1}(x)}{\sqrt{1-x^2}} dx \\
 &= -\frac{2x^3}{27} + \frac{4}{9} \sqrt{1-x^2} \sin^{-1}(x) + \frac{2}{9}x^2 \sqrt{1-x^2} \sin^{-1}(x) + \frac{1}{3}x^3 \sin^{-1}(x)^2 - \frac{4}{9} \int 1 dx \\
 &= -\frac{4x}{9} - \frac{2x^3}{27} + \frac{4}{9} \sqrt{1-x^2} \sin^{-1}(x) + \frac{2}{9}x^2 \sqrt{1-x^2} \sin^{-1}(x) + \frac{1}{3}x^3 \sin^{-1}(x)^2
 \end{aligned}$$

Mathematica [A]

time = 0.01, size = 42, normalized size = 0.69

$$\frac{1}{27} \left(-2x(6 + x^2) + 6\sqrt{1-x^2} (2 + x^2) \sin^{-1}(x) + 9x^3 \sin^{-1}(x)^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*ArcSin[x]^2,x]

[Out] (-2*x*(6 + x^2) + 6*Sqrt[1 - x^2]*(2 + x^2)*ArcSin[x] + 9*x^3*ArcSin[x]^2)/27

Mathics [A]

time = 1.92, size = 47, normalized size = 0.77

$$-\frac{4x}{9} + \frac{2x^2 \text{ArcSin}[x] \sqrt{1-x^2}}{9} - \frac{2x^3}{27} + \frac{x^3 \text{ArcSin}[x]^2}{3} + \frac{4 \text{ArcSin}[x] \sqrt{1-x^2}}{9}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^2*ArcSin[x]^2,x]')

[Out] $-4x/9 + 2x^2 \operatorname{ArcSin}[x] \operatorname{Sqrt}[1 - x^2] / 9 - 2x^3/27 + x^3 \operatorname{ArcSin}[x]^2 / 3 + 4 \operatorname{ArcSin}[x] \operatorname{Sqrt}[1 - x^2] / 9$

Maple [A]

time = 0.10, size = 37, normalized size = 0.61

method	result	size
default	$\frac{x^3 \arcsin(x)^2}{3} + \frac{2 \arcsin(x)(x^2+2)\sqrt{-x^2+1}}{9} - \frac{2x^3}{27} - \frac{4x}{9}$	37

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arcsin(x)^2,x,method=_RETURNVERBOSE)`

[Out] $1/3*x^3*\arcsin(x)^2+2/9*\arcsin(x)*(x^2+2)*(-x^2+1)^{(1/2)}-2/27*x^3-4/9*x$

Maxima [A]

time = 0.33, size = 47, normalized size = 0.77

$$\frac{1}{3}x^3 \arcsin(x)^2 - \frac{2}{27}x^3 + \frac{2}{9} \left(\sqrt{-x^2+1} x^2 + 2 \sqrt{-x^2+1} \right) \arcsin(x) - \frac{4}{9}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arcsin(x)^2,x, algorithm="maxima")`

[Out] $1/3*x^3*\arcsin(x)^2 - 2/27*x^3 + 2/9*(\operatorname{sqrt}(-x^2 + 1)*x^2 + 2*\operatorname{sqrt}(-x^2 + 1))*\arcsin(x) - 4/9*x$

Fricas [A]

time = 0.32, size = 36, normalized size = 0.59

$$\frac{1}{3}x^3 \arcsin(x)^2 - \frac{2}{27}x^3 + \frac{2}{9}(x^2 + 2)\sqrt{-x^2 + 1} \arcsin(x) - \frac{4}{9}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arcsin(x)^2,x, algorithm="fricas")`

[Out] $1/3*x^3*\arcsin(x)^2 - 2/27*x^3 + 2/9*(x^2 + 2)*\operatorname{sqrt}(-x^2 + 1)*\arcsin(x) - 4/9*x$

Sympy [A]

time = 0.17, size = 54, normalized size = 0.89

$$\frac{x^3 \operatorname{asin}^2(x)}{3} - \frac{2x^3}{27} + \frac{2x^2\sqrt{1-x^2} \operatorname{asin}(x)}{9} - \frac{4x}{9} + \frac{4\sqrt{1-x^2} \operatorname{asin}(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*asin(x)**2,x)`

[Out] $x^3 \arcsin(x)^2/3 - 2x^3/27 + 2x^2 \sqrt{1-x^2} \arcsin(x)/9 - 4x/9 + 4 \sqrt{1-x^2} \arcsin(x)/9$

Giac [A]

time = 0.00, size = 81, normalized size = 1.33

$$-\frac{1}{3}x(-x^2+1)\arcsin^2 x + \frac{1}{3}x\arcsin^2 x + \frac{2}{27}x(-x^2+1) + \frac{2}{3}\sqrt{-x^2+1}\arcsin x - \frac{2}{9}\sqrt{-x^2+1}(-x^2+1)\arcsin x - \frac{14}{27}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arcsin(x)^2,x)

[Out] $1/3*(x^2 - 1)*x*\arcsin(x)^2 + 1/3*x*\arcsin(x)^2 - 2/9*(-x^2 + 1)^{(3/2)}*\arcsin(x) - 2/27*(x^2 - 1)*x + 2/3*\sqrt{-x^2 + 1}*\arcsin(x) - 14/27*x$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 \arcsin(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*asin(x)^2,x)

[Out] int(x^2*asin(x)^2, x)

3.647 $\int x^3 \tan^{-1}(x)^2 dx$

Optimal. Leaf size=53

$$\frac{x^2}{12} + \frac{1}{2}x \tan^{-1}(x) - \frac{1}{6}x^3 \tan^{-1}(x) - \frac{1}{4} \tan^{-1}(x)^2 + \frac{1}{4}x^4 \tan^{-1}(x)^2 - \frac{1}{3} \log(1+x^2)$$

[Out] 1/12*x^2+1/2*x*arctan(x)-1/6*x^3*arctan(x)-1/4*arctan(x)^2+1/4*x^4*arctan(x)^2-1/3*ln(x^2+1)

Rubi [A]

time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {4946, 5036, 272, 45, 4930, 266, 5004}

$$\frac{1}{4}x^4 \tan^{-1}(x)^2 - \frac{1}{6}x^3 \tan^{-1}(x) + \frac{x^2}{12} - \frac{1}{3} \log(x^2+1) + \frac{1}{2}x \tan^{-1}(x) - \frac{1}{4} \tan^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] Int[x^3*ArcTan[x]^2,x]

[Out] x^2/12 + (x*ArcTan[x])/2 - (x^3*ArcTan[x])/6 - ArcTan[x]^2/4 + (x^4*ArcTan[x]^2)/4 - Log[1 + x^2]/3

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4930

Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&

(EqQ[n, 1] || EqQ[p, 1])

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :=
  Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
  1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
  IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] :=
  Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
  c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*((f_.)*(x_)^(m_.))/((d_) + (e
_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
 \int x^3 \tan^{-1}(x)^2 dx &= \frac{1}{4}x^4 \tan^{-1}(x)^2 - \frac{1}{2} \int \frac{x^4 \tan^{-1}(x)}{1+x^2} dx \\
 &= \frac{1}{4}x^4 \tan^{-1}(x)^2 - \frac{1}{2} \int x^2 \tan^{-1}(x) dx + \frac{1}{2} \int \frac{x^2 \tan^{-1}(x)}{1+x^2} dx \\
 &= -\frac{1}{6}x^3 \tan^{-1}(x) + \frac{1}{4}x^4 \tan^{-1}(x)^2 + \frac{1}{6} \int \frac{x^3}{1+x^2} dx + \frac{1}{2} \int \tan^{-1}(x) dx - \frac{1}{2} \int \frac{\tan^{-1}(x)}{1+x^2} dx \\
 &= \frac{1}{2}x \tan^{-1}(x) - \frac{1}{6}x^3 \tan^{-1}(x) - \frac{1}{4} \tan^{-1}(x)^2 + \frac{1}{4}x^4 \tan^{-1}(x)^2 + \frac{1}{12} \text{Subst} \left(\int \frac{x}{1+x} dx \right) \\
 &= \frac{1}{2}x \tan^{-1}(x) - \frac{1}{6}x^3 \tan^{-1}(x) - \frac{1}{4} \tan^{-1}(x)^2 + \frac{1}{4}x^4 \tan^{-1}(x)^2 - \frac{1}{4} \log(1+x^2) + \frac{1}{12} \text{Subst} \left(\int \frac{x}{1+x} dx \right) \\
 &= \frac{x^2}{12} + \frac{1}{2}x \tan^{-1}(x) - \frac{1}{6}x^3 \tan^{-1}(x) - \frac{1}{4} \tan^{-1}(x)^2 + \frac{1}{4}x^4 \tan^{-1}(x)^2 - \frac{1}{3} \log(1+x^2)
 \end{aligned}$$

Mathematica [A]

time = 0.01, size = 37, normalized size = 0.70

$$\frac{1}{12} (x^2 - 2x(-3 + x^2) \tan^{-1}(x) + 3(-1 + x^4) \tan^{-1}(x)^2 - 4 \log(1 + x^2))$$

Antiderivative was successfully verified.

[In] Integrate[x^3*ArcTan[x]^2,x]

[Out] (x^2 - 2*x*(-3 + x^2)*ArcTan[x] + 3*(-1 + x^4)*ArcTan[x]^2 - 4*Log[1 + x^2])/12

Mathics [A]

time = 1.87, size = 41, normalized size = 0.77

$$\frac{x \operatorname{ArcTan}[x]}{2} + \frac{x^2}{12} - \frac{x^3 \operatorname{ArcTan}[x]}{6} + \frac{x^4 \operatorname{ArcTan}[x]^2}{4} - \frac{\operatorname{Log}[1 + x^2]}{3} - \frac{\operatorname{ArcTan}[x]^2}{4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[ArcTan[x]^2*x^3,x]')

[Out] x ArcTan[x] / 2 + x ^ 2 / 12 - x ^ 3 ArcTan[x] / 6 + x ^ 4 ArcTan[x] ^ 2 / 4 - Log[1 + x ^ 2] / 3 - ArcTan[x] ^ 2 / 4

Maple [A]

time = 0.05, size = 42, normalized size = 0.79

method	result
default	$\frac{x^2}{12} + \frac{x \arctan(x)}{2} - \frac{x^3 \arctan(x)}{6} - \frac{\arctan(x)^2}{4} + \frac{x^4 \arctan(x)^2}{4} - \frac{\ln(x^2+1)}{3}$
risch	$-\frac{\left(\frac{x^4-1}{4}\right) \ln(ix+1)^2}{4} - \frac{\left(-\frac{x^4 \ln(-ix+1)}{2} - \frac{ix^3}{3} + ix + \frac{\ln(-ix+1)}{2}\right) \ln(ix+1)}{4} - \frac{x^4 \ln(-ix+1)^2}{16} + \frac{\ln(-ix+1)^2}{16} - \frac{ix^3 \ln(-ix+1)}{12} +$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arctan(x)^2,x,method=_RETURNVERBOSE)

[Out] 1/12*x^2+1/2*x*arctan(x)-1/6*x^3*arctan(x)-1/4*arctan(x)^2+1/4*x^4*arctan(x)^2-1/3*ln(x^2+1)

Maxima [A]

time = 0.34, size = 44, normalized size = 0.83

$$\frac{1}{4} x^4 \arctan(x)^2 + \frac{1}{12} x^2 - \frac{1}{6} (x^3 - 3x + 3 \arctan(x)) \arctan(x) + \frac{1}{4} \arctan(x)^2 - \frac{1}{3} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctan(x)^2,x, algorithm="maxima")

[Out] 1/4*x^4*arctan(x)^2 + 1/12*x^2 - 1/6*(x^3 - 3*x + 3*arctan(x))*arctan(x) + 1/4*arctan(x)^2 - 1/3*log(x^2 + 1)

Fricas [A]

time = 0.31, size = 36, normalized size = 0.68

$$\frac{1}{4} (x^4 - 1) \arctan(x)^2 + \frac{1}{12} x^2 - \frac{1}{6} (x^3 - 3x) \arctan(x) - \frac{1}{3} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x³*arctan(x)²,x, algorithm="fricas")

[Out] 1/4*(x⁴ - 1)*arctan(x)² + 1/12*x² - 1/6*(x³ - 3*x)*arctan(x) - 1/3*log(x² + 1)

Sympy [A]

time = 0.18, size = 44, normalized size = 0.83

$$\frac{x^4 \operatorname{atan}^2(x)}{4} - \frac{x^3 \operatorname{atan}(x)}{6} + \frac{x^2}{12} + \frac{x \operatorname{atan}(x)}{2} - \frac{\log(x^2 + 1)}{3} - \frac{\operatorname{atan}^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*atan(x)**2,x)

[Out] x**4*atan(x)**2/4 - x**3*atan(x)/6 + x**2/12 + x*atan(x)/2 - log(x**2 + 1)/3 - atan(x)**2/4

Giac [A]

time = 0.00, size = 47, normalized size = 0.89

$$\frac{1}{4}x^4 \arctan^2 x - \frac{2x^3 \arctan x - x^2 - 6x \arctan x + 3 \arctan^2 x + 4 \ln(x^2 + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x³*arctan(x)²,x)

[Out] 1/4*x⁴*arctan(x)² - 1/6*x³*arctan(x) + 1/12*x² + 1/2*x*arctan(x) - 1/4*arctan(x)² - 1/3*log(x² + 1)

Mupad [B]

time = 0.34, size = 41, normalized size = 0.77

$$\frac{x^4 \operatorname{atan}(x)^2}{4} - \frac{x^3 \operatorname{atan}(x)}{6} - \frac{\operatorname{atan}(x)^2}{4} - \frac{\ln(x^2 + 1)}{3} + \frac{x \operatorname{atan}(x)}{2} + \frac{x^2}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x³*atan(x)²,x)

[Out] (x⁴*atan(x)²)/4 - (x³*atan(x))/6 - atan(x)²/4 - log(x² + 1)/3 + (x*atan(x))/2 + x²/12

$$3.648 \quad \int \frac{\tan^{-1}(x)^2}{x^5} dx$$

Optimal. Leaf size=61

$$-\frac{1}{12x^2} - \frac{\tan^{-1}(x)}{6x^3} + \frac{\tan^{-1}(x)}{2x} + \frac{1}{4}\tan^{-1}(x)^2 - \frac{\tan^{-1}(x)^2}{4x^4} - \frac{2\log(x)}{3} + \frac{1}{3}\log(1+x^2)$$

[Out] -1/12/x^2-1/6*arctan(x)/x^3+1/2*arctan(x)/x+1/4*arctan(x)^2-1/4*arctan(x)^2/x^4-2/3*ln(x)+1/3*ln(x^2+1)

Rubi [A]

time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {4946, 5038, 272, 46, 36, 29, 31, 5004}

$$-\frac{\tan^{-1}(x)^2}{4x^4} - \frac{\tan^{-1}(x)}{6x^3} - \frac{1}{12x^2} + \frac{1}{3}\log(x^2+1) - \frac{2\log(x)}{3} + \frac{1}{4}\tan^{-1}(x)^2 + \frac{\tan^{-1}(x)}{2x}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[x]^2/x^5,x]

[Out] -1/12*1/x^2 - ArcTan[x]/(6*x^3) + ArcTan[x]/(2*x) + ArcTan[x]^2/4 - ArcTan[x]^2/(4*x^4) - (2*Log[x])/3 + Log[1 + x^2]/3

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 46

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 272

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_) + ArcTan[(c_)*(x_)^(n_)]*(b_))^(p_)*(x_)^(m_), x_Symbol] :=
Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_) + ArcTan[(c_)*(x_)]*(b_))^(p_)/((d_) + (e_)*(x_)^2), x_Symbol] :=
Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_) + ArcTan[(c_)*(x_)]*(b_))^(p_)*((f_)*(x_)^(m_))/((d_) + (e
_)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(x)^2}{x^5} dx &= -\frac{\tan^{-1}(x)^2}{4x^4} + \frac{1}{2} \int \frac{\tan^{-1}(x)}{x^4(1+x^2)} dx \\
&= -\frac{\tan^{-1}(x)^2}{4x^4} + \frac{1}{2} \int \frac{\tan^{-1}(x)}{x^4} dx - \frac{1}{2} \int \frac{\tan^{-1}(x)}{x^2(1+x^2)} dx \\
&= -\frac{\tan^{-1}(x)}{6x^3} - \frac{\tan^{-1}(x)^2}{4x^4} + \frac{1}{6} \int \frac{1}{x^3(1+x^2)} dx - \frac{1}{2} \int \frac{\tan^{-1}(x)}{x^2} dx + \frac{1}{2} \int \frac{\tan^{-1}(x)}{1+x^2} dx \\
&= -\frac{\tan^{-1}(x)}{6x^3} + \frac{\tan^{-1}(x)}{2x} + \frac{1}{4} \tan^{-1}(x)^2 - \frac{\tan^{-1}(x)^2}{4x^4} + \frac{1}{12} \text{Subst} \left(\int \frac{1}{x^2(1+x)} dx, x, x^2 \right) \\
&= -\frac{\tan^{-1}(x)}{6x^3} + \frac{\tan^{-1}(x)}{2x} + \frac{1}{4} \tan^{-1}(x)^2 - \frac{\tan^{-1}(x)^2}{4x^4} + \frac{1}{12} \text{Subst} \left(\int \left(\frac{1}{x^2} - \frac{1}{x} + \frac{1}{1+x} \right) \right) \\
&= -\frac{1}{12x^2} - \frac{\tan^{-1}(x)}{6x^3} + \frac{\tan^{-1}(x)}{2x} + \frac{1}{4} \tan^{-1}(x)^2 - \frac{\tan^{-1}(x)^2}{4x^4} - \frac{\log(x)}{6} + \frac{1}{12} \log(1+x^2) \\
&= -\frac{1}{12x^2} - \frac{\tan^{-1}(x)}{6x^3} + \frac{\tan^{-1}(x)}{2x} + \frac{1}{4} \tan^{-1}(x)^2 - \frac{\tan^{-1}(x)^2}{4x^4} - \frac{2 \log(x)}{3} + \frac{1}{3} \log(1+x^2)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 56, normalized size = 0.92

$$-\frac{1}{12x^2} + \frac{(-1 + 3x^2) \tan^{-1}(x)}{6x^3} + \frac{(-1 + x^4) \tan^{-1}(x)^2}{4x^4} - \frac{2 \log(x)}{3} + \frac{1}{3} \log(1 + x^2)$$

Antiderivative was successfully verified.

`[In] Integrate[ArcTan[x]^2/x^5,x]`

`[Out] -1/12*1/x^2 + ((-1 + 3*x^2)*ArcTan[x])/(6*x^3) + ((-1 + x^4)*ArcTan[x]^2)/(4*x^4) - (2*Log[x])/3 + Log[1 + x^2]/3`

Mathics [A]

time = 1.96, size = 52, normalized size = 0.85

$$\frac{-2x \operatorname{ArcTan}[x] - x^2 + 6x^3 \operatorname{ArcTan}[x] + x^4 (-8 \operatorname{Log}[x] + 3 \operatorname{ArcTan}[x]^2 + 4 \operatorname{Log}[1 + x^2]) - 3 \operatorname{ArcTan}[x]^2}{12x^4}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[ArcTan[x]^2/x^5,x]')`

`[Out] (-2 x ArcTan[x] - x ^ 2 + 6 x ^ 3 ArcTan[x] + x ^ 4 (-8 Log[x] + 3 ArcTan[x] ^ 2 + 4 Log[1 + x ^ 2]) - 3 ArcTan[x] ^ 2) / (12 x ^ 4)`

Maple [A]

time = 0.08, size = 48, normalized size = 0.79

method	result
default	$-\frac{1}{12x^2} - \frac{\arctan(x)}{6x^3} + \frac{\arctan(x)}{2x} + \frac{\arctan(x)^2}{4} - \frac{\arctan(x)^2}{4x^4} - \frac{2 \ln(x)}{3} + \frac{\ln(x^2+1)}{3}$
risch	$-\frac{(x^4-1) \ln(ix+1)^2}{16x^4} + \frac{(3x^4 \ln(-ix+1) - 6ix^3 + 2ix - 3 \ln(-ix+1)) \ln(ix+1)}{24x^4} - \frac{3x^4 \ln(-ix+1)^2 - 12ix^3 \ln(-ix+1) + 32x^4 \ln(x) - 16x}{48x}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(arctan(x)^2/x^5,x,method=_RETURNVERBOSE)`

`[Out] -1/12/x^2-1/6/x^3*arctan(x)+1/2/x*arctan(x)+1/4*arctan(x)^2-1/4*arctan(x)^2/x^4-2/3*ln(x)+1/3*ln(x^2+1)`

Maxima [A]

time = 0.32, size = 64, normalized size = 1.05

$$\frac{1}{6} \left(\frac{3x^2 - 1}{x^3} + 3 \arctan(x) \right) \arctan(x) - \frac{3x^2 \arctan(x)^2 - 4x^2 \log(x^2 + 1) + 8x^2 \log(x) + 1}{12x^2} - \frac{\arctan(x)^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(arctan(x)^2/x^5,x, algorithm="maxima")`

[Out] $1/6*((3*x^2 - 1)/x^3 + 3*\arctan(x))*\arctan(x) - 1/12*(3*x^2*\arctan(x)^2 - 4*x^2*\log(x^2 + 1) + 8*x^2*\log(x) + 1)/x^2 - 1/4*\arctan(x)^2/x^4$

Fricas [A]

time = 0.33, size = 53, normalized size = 0.87

$$\frac{4x^4 \log(x^2 + 1) - 8x^4 \log(x) + 3(x^4 - 1) \arctan(x)^2 - x^2 + 2(3x^3 - x) \arctan(x)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(x)^2/x^5,x, algorithm="fricas")`

[Out] $1/12*(4*x^4*\log(x^2 + 1) - 8*x^4*\log(x) + 3*(x^4 - 1)*\arctan(x)^2 - x^2 + 2*(3*x^3 - x)*\arctan(x))/x^4$

Sympy [A]

time = 0.29, size = 53, normalized size = 0.87

$$-\frac{2 \log(x)}{3} + \frac{\log(x^2 + 1)}{3} + \frac{\operatorname{atan}^2(x)}{4} + \frac{\operatorname{atan}(x)}{2x} - \frac{1}{12x^2} - \frac{\operatorname{atan}(x)}{6x^3} - \frac{\operatorname{atan}^2(x)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(atan(x)**2/x**5,x)`

[Out] $-2*\log(x)/3 + \log(x**2 + 1)/3 + \operatorname{atan}(x)**2/4 + \operatorname{atan}(x)/(2*x) - 1/(12*x**2) - \operatorname{atan}(x)/(6*x**3) - \operatorname{atan}(x)**2/(4*x**4)$

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(x)^2/x^5,x)`

[Out] Could not integrate

Mupad [B]

time = 0.11, size = 44, normalized size = 0.72

$$\frac{\ln(x^2 + 1)}{3} - \frac{2 \ln(x)}{3} - \operatorname{atan}(x)^2 \left(\frac{1}{4x^4} - \frac{1}{4} \right) - \frac{1}{12x^2} + \frac{\operatorname{atan}(x) \left(\frac{x^2}{2} - \frac{1}{6} \right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(atan(x)^2/x^5,x)`

[Out] $\log(x^2 + 1)/3 - (2*\log(x))/3 - \operatorname{atan}(x)^2*(1/(4*x^4) - 1/4) - 1/(12*x^2) + (\operatorname{atan}(x)*(x^2/2 - 1/6))/x^3$

3.649 $\int x^3 \csc^{-1}(x)^2 dx$

Optimal. Leaf size=63

$$\frac{x^2}{12} + \frac{1}{3} \sqrt{1 - \frac{1}{x^2}} x \csc^{-1}(x) + \frac{1}{6} \sqrt{1 - \frac{1}{x^2}} x^3 \csc^{-1}(x) + \frac{1}{4} x^4 \csc^{-1}(x)^2 + \frac{\log(x)}{3}$$

[Out] 1/12*x^2+1/4*x^4*arccsc(x)^2+1/3*ln(x)+1/3*x*arccsc(x)*(1-1/x^2)^(1/2)+1/6*x^3*arccsc(x)*(1-1/x^2)^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5331, 3843, 4270, 4269, 3556}

$$\frac{1}{4} x^4 \csc^{-1}(x)^2 + \frac{x^2}{12} + \frac{1}{3} \sqrt{1 - \frac{1}{x^2}} x \csc^{-1}(x) + \frac{1}{6} \sqrt{1 - \frac{1}{x^2}} x^3 \csc^{-1}(x) + \frac{\log(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[x^3*ArcCsc[x]^2,x]

[Out] x^2/12 + (Sqrt[1 - x^(-2)]*x*ArcCsc[x])/3 + (Sqrt[1 - x^(-2)]*x^3*ArcCsc[x])/6 + (x^4*ArcCsc[x]^2)/4 + Log[x]/3

Rule 3556

Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3843

Int[Cot[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.), x_Symbol] := Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^p/(b*n*p)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csc[a + b*x^n]^p, x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]

Rule 4269

Int[csc[(e_.) + (f_.)*(x_.)]^2*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] := Simp[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 4270

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))), x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x]

`x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]`

Rule 5331

`Int[((a_.) + ArcCsc[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.), x_Symbol] := Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
0] || LtQ[m, -1])`

Rubi steps

$$\begin{aligned} \int x^3 \csc^{-1}(x)^2 dx &= -\text{Subst} \left(\int x^2 \cot(x) \csc^4(x) dx, x, \csc^{-1}(x) \right) \\ &= \frac{1}{4} x^4 \csc^{-1}(x)^2 - \frac{1}{2} \text{Subst} \left(\int x \csc^4(x) dx, x, \csc^{-1}(x) \right) \\ &= \frac{x^2}{12} + \frac{1}{6} \sqrt{1 - \frac{1}{x^2}} x^3 \csc^{-1}(x) + \frac{1}{4} x^4 \csc^{-1}(x)^2 - \frac{1}{3} \text{Subst} \left(\int x \csc^2(x) dx, x, \csc^{-1}(x) \right) \\ &= \frac{x^2}{12} + \frac{1}{3} \sqrt{1 - \frac{1}{x^2}} x \csc^{-1}(x) + \frac{1}{6} \sqrt{1 - \frac{1}{x^2}} x^3 \csc^{-1}(x) + \frac{1}{4} x^4 \csc^{-1}(x)^2 - \frac{1}{3} \text{Subst} \left(\int \right) \\ &= \frac{x^2}{12} + \frac{1}{3} \sqrt{1 - \frac{1}{x^2}} x \csc^{-1}(x) + \frac{1}{6} \sqrt{1 - \frac{1}{x^2}} x^3 \csc^{-1}(x) + \frac{1}{4} x^4 \csc^{-1}(x)^2 + \frac{\log(x)}{3} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 42, normalized size = 0.67

$$\frac{1}{12} \left(x^2 + 2 \sqrt{1 - \frac{1}{x^2}} x (2 + x^2) \csc^{-1}(x) + 3x^4 \csc^{-1}(x)^2 + 4 \log(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*ArcCsc[x]^2,x]

[Out] (x^2 + 2*Sqrt[1 - x^(-2)]*x*(2 + x^2)*ArcCsc[x] + 3*x^4*ArcCsc[x]^2 + 4*Log[x])/12

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in ___instancecheck___

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[ArcCsc[x]^2*x^3,x]')

[Out] cought exception: maximum recursion depth exceeded in __instancecheck__

Maple [A]

time = 0.05, size = 56, normalized size = 0.89

method	result	size
default	$\frac{x^4 \operatorname{arccsc}(x)^2}{4} + \frac{x^3 \operatorname{arccsc}(x) \sqrt{\frac{x^2-1}{x^2}}}{6} + \frac{x^2}{12} + \frac{\operatorname{arccsc}(x) \sqrt{\frac{x^2-1}{x^2}} x}{3} - \frac{\ln(\frac{1}{x})}{3}$	56

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arccsc(x)^2,x,method=_RETURNVERBOSE)

[Out] $\frac{1}{4}x^4 \operatorname{arccsc}(x)^2 + \frac{1}{6}x^3 \operatorname{arccsc}(x) \left(\frac{x^2-1}{x^2}\right)^{1/2} + \frac{1}{12}x^2 + \frac{1}{3} \operatorname{arccsc}(x) \left(\frac{x^2-1}{x^2}\right)^{1/2} x - \frac{1}{3} \ln(1/x)$

Maxima [A]

time = 0.39, size = 95, normalized size = 1.51

$$\frac{1}{4}x^4 \operatorname{arccsc}(x)^2 + \frac{2x^4 \arctan(1, \sqrt{x+1}\sqrt{x-1}) + 2x^2 \arctan(1, \sqrt{x+1}\sqrt{x-1}) + (x^2 + 2 \log(x^2))\sqrt{x+1}\sqrt{x-1} - 4 \arctan(1, \sqrt{x+1}\sqrt{x-1})}{12\sqrt{x+1}\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arccsc(x)^2,x, algorithm="maxima")

[Out] $\frac{1}{4}x^4 \operatorname{arccsc}(x)^2 + \frac{1}{12}(2x^4 \arctan(1, \sqrt{x+1}\sqrt{x-1}) + 2x^2 \arctan(1, \sqrt{x+1}\sqrt{x-1}) + (x^2 + 2 \log(x^2))\sqrt{x+1}\sqrt{x-1} - 4 \arctan(1, \sqrt{x+1}\sqrt{x-1}))/(\sqrt{x+1}\sqrt{x-1})$

Fricas [A]

time = 0.32, size = 35, normalized size = 0.56

$$\frac{1}{4}x^4 \operatorname{arccsc}(x)^2 + \frac{1}{6}(x^2 + 2)\sqrt{x^2 - 1} \operatorname{arccsc}(x) + \frac{1}{12}x^2 + \frac{1}{3} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arccsc(x)^2,x, algorithm="fricas")

[Out] $\frac{1}{4}x^4 \operatorname{arccsc}(x)^2 + \frac{1}{6}(x^2 + 2)\sqrt{x^2 - 1} \operatorname{arccsc}(x) + \frac{1}{12}x^2 + \frac{1}{3} \log(x)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arccsc}^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*acsc(x)**2,x)

[Out] Integral(x**3*acsc(x)**2, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(49) = 98.

time = 0.01, size = 152, normalized size = 2.41

$$\frac{1}{4}x^4 \arcsin^2\left(\frac{1}{x}\right) - \frac{-\frac{64}{3}\left(-\frac{1}{2}\left(-2\sqrt{-\left(\frac{1}{x}\right)^2+1+2}\right)x\right)^3 + 96\left(-2\sqrt{-\left(\frac{1}{x}\right)^2+1+2}\right)x + 9\left(-\frac{1}{2}\left(-2\sqrt{-\left(\frac{1}{x}\right)^2+1+2}\right)x\right)^2 + 1}{512} + \frac{9\left(-\frac{1}{2}\left(-2\sqrt{-\left(\frac{1}{x}\right)^2+1+2}\right)x\right)^2 + 1}{24\left(-\frac{1}{2}\left(-2\sqrt{-\left(\frac{1}{x}\right)^2+1+2}\right)x\right)^3} \arcsin\left(\frac{1}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arccsc(x)^2,x)

[Out] 1/4*x^4*arcsin(1/x)^2 + 1/12*x^2*(2/x^2 + 1) + 1/48*(x^3*(sqrt(-1/x^2 + 1) - 1)^3 + 9*x*(sqrt(-1/x^2 + 1) - 1) - (9*x^2*(sqrt(-1/x^2 + 1) - 1)^2 + 1)/(x^3*(sqrt(-1/x^2 + 1) - 1)^3))*arcsin(1/x) - 1/6*log(x^(-2))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int x^3 \operatorname{asin}\left(\frac{1}{x}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*asin(1/x)^2,x)

[Out] int(x^3*asin(1/x)^2, x)

$$3.650 \quad \int \frac{\sec^{-1}(x)^4}{x^5} dx$$

Optimal. Leaf size=148

$$-\frac{3}{128x^4} - \frac{45}{128x^2} - \frac{3\sqrt{1-\frac{1}{x^2}} \sec^{-1}(x)}{32x^3} - \frac{45\sqrt{1-\frac{1}{x^2}} \sec^{-1}(x)}{64x} - \frac{45}{128} \sec^{-1}(x)^2 + \frac{3 \sec^{-1}(x)^2}{16x^4} + \frac{9 \sec^{-1}(x)^2}{16x^2} + \dots$$

[Out] $-\frac{3}{128x^4} - \frac{45}{128x^2} - \frac{45}{128} \operatorname{arcsec}(x)^2 + \frac{3}{16} \operatorname{arcsec}(x)^2/x^4 + \frac{9}{16} \operatorname{arcsec}(x)^2/x^2 + \frac{3}{32} \operatorname{arcsec}(x)^4 - \frac{1}{4} \operatorname{arcsec}(x)^4/x^4 - \frac{3}{32} \operatorname{arcsec}(x) \cdot (1-1/x^2)^{1/2}/x^3 - \frac{45}{64} \operatorname{arcsec}(x) \cdot (1-1/x^2)^{1/2}/x + \frac{1}{4} \operatorname{arcsec}(x)^3 \cdot (1-1/x^2)^{1/2}/x^3 + \frac{3}{8} \operatorname{arcsec}(x)^3 \cdot (1-1/x^2)^{1/2}/x$

Rubi [A]

time = 0.09, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5330, 3525, 3392, 30, 3391}

$$-\frac{3}{128x^4} - \frac{\sec^{-1}(x)^4}{4x^4} + \frac{3 \sec^{-1}(x)^2}{16x^4} - \frac{45}{128x^2} + \frac{3\sqrt{1-\frac{1}{x^2}} \sec^{-1}(x)^3}{8x} + \frac{9 \sec^{-1}(x)^2}{16x^2} - \frac{45\sqrt{1-\frac{1}{x^2}} \sec^{-1}(x)}{64x} + \frac{\sqrt{1-\frac{1}{x^2}} \sec^{-1}(x)^3}{4x^3} - \frac{3\sqrt{1-\frac{1}{x^2}} \sec^{-1}(x)}{32x^3} + \frac{3}{32} \sec^{-1}(x)^4 - \frac{45}{128} \sec^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] Int[ArcSec[x]^4/x^5, x]

[Out] $-\frac{3}{(128x^4)} - \frac{45}{(128x^2)} - \frac{(3\sqrt{1-x^{-2}} \operatorname{ArcSec}[x])}{(32x^3)} - \frac{(45\sqrt{1-x^{-2}} \operatorname{ArcSec}[x])}{(64x)} - \frac{(45 \operatorname{ArcSec}[x]^2)}{128} + \frac{(3 \operatorname{ArcSec}[x]^2)}{(16x^4)} + \frac{(9 \operatorname{ArcSec}[x]^2)}{(16x^2)} + \frac{(\sqrt{1-x^{-2}} \operatorname{ArcSec}[x]^3)}{(4x^3)} + \frac{(3\sqrt{1-x^{-2}} \operatorname{ArcSec}[x]^3)}{(8x)} + \frac{(3 \operatorname{ArcSec}[x]^4)}{32} - \frac{\operatorname{ArcSec}[x]^4}{(4x^4)}$

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3391

Int[((c_) + (d_)*(x_))*((b_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[d*((b*Sine[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n-1)/n), Int[(c + d*x)*(b*Sine[e + f*x])^(n-2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b*Sine[e + f*x])^(n-1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rule 3392

Int[((c_) + (d_)*(x_))^(m_)*((b_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[d*m*(c + d*x)^(m-1)*((b*Sine[e + f*x])^n/(f^2*n^2)), x] + (Dist

$[b^2*((n-1)/n), \text{Int}[(c+d*x)^m*(b*\text{Sin}[e+f*x])^{(n-2)}, x], x] - \text{Dist}[d^2*m*((m-1)/(f^2*n^2)), \text{Int}[(c+d*x)^{(m-2)}*(b*\text{Sin}[e+f*x])^n, x], x] - \text{Simp}[b*(c+d*x)^m*\text{Cos}[e+f*x]*((b*\text{Sin}[e+f*x])^{(n-1)}(f*n)), x] /;$
 $\text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{GtQ}[m, 1]$

Rule 3525

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_)^{(n_.)}]^{(p_.)}*(x_)^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_)^{(n_.)}], x_Symbol] \rightarrow \text{Simp}[(-x^{(m-n+1)})*(\text{Cos}[a + b*x^n]^{(p+1)}(b*n*(p+1))), x] + \text{Dist}[(m-n+1)/(b*n*(p+1)), \text{Int}[x^{(m-n)}*\text{Cos}[a + b*x^n]^{(p+1)}, x], x] /;$
 $\text{FreeQ}[\{a, b, p\}, x] \ \&\& \ \text{LtQ}[0, n, m+1] \ \&\& \ \text{NeQ}[p, -1]$

Rule 5330

$\text{Int}[(a_.) + \text{ArcSec}[(c_.)*(x_.)]^{(n_.)}*(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Dist}[1/c^{(m+1)}, \text{Subst}[\text{Int}[(a + b*x)^n*\text{Sec}[x]^{(m+1)}*\text{Tan}[x], x], x, \text{ArcSec}[c*x]], x] /;$
 $\text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ (\text{GtQ}[n, 0] \ || \ \text{LtQ}[m, -1])$

Rubi steps

$$\begin{aligned} \int \frac{\sec^{-1}(x)^4}{x^5} dx &= \text{Subst} \left(\int x^4 \cos^3(x) \sin(x) dx, x, \sec^{-1}(x) \right) \\ &= -\frac{\sec^{-1}(x)^4}{4x^4} + \text{Subst} \left(\int x^3 \cos^4(x) dx, x, \sec^{-1}(x) \right) \\ &= \frac{3 \sec^{-1}(x)^2}{16x^4} + \frac{\sqrt{1 - \frac{1}{x^2}} \sec^{-1}(x)^3}{4x^3} - \frac{\sec^{-1}(x)^4}{4x^4} - \frac{3}{8} \text{Subst} \left(\int x \cos^4(x) dx, x, \sec^{-1}(x) \right) \\ &= -\frac{3}{128x^4} - \frac{3\sqrt{1 - \frac{1}{x^2}} \sec^{-1}(x)}{32x^3} + \frac{3 \sec^{-1}(x)^2}{16x^4} + \frac{9 \sec^{-1}(x)^2}{16x^2} + \frac{\sqrt{1 - \frac{1}{x^2}} \sec^{-1}(x)^3}{4x^3} + \frac{3 \sec^{-1}(x)^4}{128x^4} \\ &= -\frac{3}{128x^4} - \frac{45}{128x^2} - \frac{3\sqrt{1 - \frac{1}{x^2}} \sec^{-1}(x)}{32x^3} - \frac{45\sqrt{1 - \frac{1}{x^2}} \sec^{-1}(x)}{64x} + \frac{3 \sec^{-1}(x)^2}{16x^4} + \frac{9 \sec^{-1}(x)^2}{16x^2} \\ &= -\frac{3}{128x^4} - \frac{45}{128x^2} - \frac{3\sqrt{1 - \frac{1}{x^2}} \sec^{-1}(x)}{32x^3} - \frac{45\sqrt{1 - \frac{1}{x^2}} \sec^{-1}(x)}{64x} - \frac{45}{128} \sec^{-1}(x)^2 + \frac{3 \sec^{-1}(x)^4}{128x^4} \end{aligned}$$

Mathematica [A]

time = 0.04, size = 92, normalized size = 0.62

$$\frac{-3 - 45x^2 - 6\sqrt{1 - \frac{1}{x^2}} x (2 + 15x^2) \sec^{-1}(x) + (24 + 72x^2 - 45x^4) \sec^{-1}(x)^2 + 16\sqrt{1 - \frac{1}{x^2}} x (2 + 3x^2) \sec^{-1}(x)^3 + 4(-8 + 3x^4) \sec^{-1}(x)^4}{128x^4}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSec[x]^4/x^5,x]

[Out] $(-3 - 45x^2 - 6\sqrt{1-x^{-2}})x(2 + 15x^2)\text{ArcSec}[x] + (24 + 72x^2 - 45x^4)\text{ArcSec}[x]^2 + 16\sqrt{1-x^{-2}}x(2 + 3x^2)\text{ArcSec}[x]^3 + 4(-8 + 3x^4)\text{ArcSec}[x]^4)/(128x^4)$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in __instancecheck__

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[ArcSec[x]^4/x^5,x]')

[Out] cought exception: maximum recursion depth exceeded in __instancecheck__

Maple [A]

time = 0.15, size = 174, normalized size = 1.18

method	result
default	$-\frac{\text{arcsec}(x)^4}{4x^4} + \frac{\text{arcsec}(x)^3 \left(3 \text{arcsec}(x)x^3 + 3x^2 \sqrt{\frac{x^2-1}{x^2}} + 2 \sqrt{\frac{x^2-1}{x^2}} \right)}{8x^3} + \frac{3\text{arcsec}(x)^2}{16x^4} - \frac{3 \text{arcsec}(x) \left(3 \text{arcsec}(x)x^3 + 3x^2 \sqrt{\frac{x^2-1}{x^2}} \right)}{64x^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsec(x)^4/x^5,x,method=_RETURNVERBOSE)

[Out] $-1/4*\text{arcsec}(x)^4/x^4 + 1/8*\text{arcsec}(x)^3*(3*\text{arcsec}(x)*x^3 + 3*x^2*((x^2-1)/x^2)^(1/2) + 2*((x^2-1)/x^2)^(1/2))/x^3 + 3/16*\text{arcsec}(x)^2/x^4 - 3/64*\text{arcsec}(x)*(3*\text{arcsec}(x)*x^3 + 3*x^2*((x^2-1)/x^2)^(1/2) + 2*((x^2-1)/x^2)^(1/2))/x^3 + 45/128*\text{arcsec}(x)^2 - 3/512*(3*x^2+2)^2/x^4 + 9/16*\text{arcsec}(x)^2/x^2 - 9/16*\text{arcsec}(x)*(\text{arcsec}(x)*x + ((x^2-1)/x^2)^(1/2))/x + 9/32 - 9/32/x^2 - 9/32*\text{arcsec}(x)^4$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsec(x)^4/x^5,x, algorithm="maxima")

[Out] $1/64*(64*x^4*\text{integrate}(1/8*(12*(x^2-1)*\log(x^2)^2*\log(x)^2 - 16*(x^2-1)*\log(x^2)*\log(x)^3 + 8*(x^2-1)*\log(x)^4 + (x^2-4*(x^2-1)*\log(x)-1)*\log(x^2)^3 - 12*(4*(x^2-1)*\log(x)^2 + (x^2-4*(x^2-1)*\log(x)-1)*\log(x^2))*\arctan(\sqrt{x+1}*\sqrt{x-1})^2 + 2*(4*\arctan(\sqrt{x+1}*\sqrt{x-1})))$

1))³ - 3*arctan(sqrt(x + 1)*sqrt(x - 1))*log(x²)²*sqrt(x + 1)*sqrt(x - 1))/(x⁷ - x⁵), x) - 16*arctan(sqrt(x + 1)*sqrt(x - 1))⁴ + 24*arctan(sqrt(x + 1)*sqrt(x - 1))²*log(x²)² - log(x²)⁴)/x⁴

Fricas [A]

time = 0.34, size = 77, normalized size = 0.52

$$\frac{4(3x^4 - 8)\operatorname{arcsec}(x)^4 - 3(15x^4 - 24x^2 - 8)\operatorname{arcsec}(x)^2 - 45x^2 + 2(8(3x^2 + 2)\operatorname{arcsec}(x)^3 - 3(15x^2 + 2)\operatorname{arcsec}(x))\sqrt{x^2 - 1} - 3}{128x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsec(x)⁴/x⁵,x, algorithm="fricas")

[Out] 1/128*(4*(3*x⁴ - 8)*arcsec(x)⁴ - 3*(15*x⁴ - 24*x² - 8)*arcsec(x)² - 45*x² + 2*(8*(3*x² + 2)*arcsec(x)³ - 3*(15*x² + 2)*arcsec(x))*sqrt(x² - 1) - 3)/x⁴

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}^4(x)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asec(x)**4/x**5,x)

[Out] Integral(asec(x)**4/x**5, x)

Giac [A]

time = 0.01, size = 169, normalized size = 1.14

$$-\frac{1}{4}\left(\frac{1}{x}\right)^4 \arccos^4\left(\frac{1}{x}\right) + \frac{1}{4}\left(\frac{1}{x}\right)^3 \sqrt{-\left(\frac{1}{x}\right)^2 + 1} \arccos^3\left(\frac{1}{x}\right) + \frac{3}{16}\left(\frac{1}{x}\right)^4 \arccos^2\left(\frac{1}{x}\right) + \frac{3\sqrt{-\left(\frac{1}{x}\right)^2 + 1} \arccos^2\left(\frac{1}{x}\right)}{8x} - \frac{3}{128}\left(\frac{1}{x}\right)^4 + \frac{3}{32}\arccos^4\left(\frac{1}{x}\right) + \frac{9}{16}\left(\frac{1}{x}\right)^2 \arccos^2\left(\frac{1}{x}\right) - \frac{45}{128}\left(\frac{1}{x}\right)^2 - \frac{45}{128}\arccos^2\left(\frac{1}{x}\right) - \frac{45\sqrt{-\left(\frac{1}{x}\right)^2 + 1} \arccos\left(\frac{1}{x}\right)}{64x} - \frac{3}{32}\left(\frac{1}{x}\right)^3 \sqrt{-\left(\frac{1}{x}\right)^2 + 1} \arccos\left(\frac{1}{x}\right) + \frac{189}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsec(x)⁴/x⁵,x)

[Out] 3/32*arccos(1/x)⁴ + 3/8*sqrt(-1/x² + 1)*arccos(1/x)³/x - 45/128*arccos(1/x)² - 45/64*sqrt(-1/x² + 1)*arccos(1/x)/x + 1/4*sqrt(-1/x² + 1)*arccos(1/x)³/x³ + 9/16*arccos(1/x)²/x² - 1/4*arccos(1/x)⁴/x⁴ - 3/32*sqrt(-1/x² + 1)*arccos(1/x)/x³ - 45/128/x² + 3/16*arccos(1/x)²/x⁴ - 3/128/x⁴ + 189/1024

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acos}\left(\frac{1}{x}\right)^4}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acos(1/x)⁴/x⁵,x)

[Out] int(acos(1/x)⁴/x⁵, x)

3.651 $\int \sqrt{1-x^2} \sin^{-1}(x) dx$

Optimal. Leaf size=34

$$-\frac{x^2}{4} + \frac{1}{2}x\sqrt{1-x^2} \sin^{-1}(x) + \frac{1}{4} \sin^{-1}(x)^2$$

[Out] $-1/4*x^2+1/4*\arcsin(x)^2+1/2*x*\arcsin(x)*(-x^2+1)^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4741, 4737, 30}

$$-\frac{x^2}{4} + \frac{1}{2}\sqrt{1-x^2} x \sin^{-1}(x) + \frac{1}{4} \sin^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[1 - x^2]*ArcSin[x], x]`

[Out] $-1/4*x^2 + (x*\text{Sqrt}[1 - x^2]*\text{ArcSin}[x])/2 + \text{ArcSin}[x]^2/4$

Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

Rule 4737

`Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)/Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

Rule 4741

`Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)*Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^(n/2), x] + (Dist[(1/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]`

Rubi steps

$$\int \sqrt{1-x^2} \sin^{-1}(x) dx = \frac{1}{2}x\sqrt{1-x^2} \sin^{-1}(x) - \frac{\int x dx}{2} + \frac{1}{2} \int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} dx$$

$$= -\frac{x^2}{4} + \frac{1}{2}x\sqrt{1-x^2} \sin^{-1}(x) + \frac{1}{4} \sin^{-1}(x)^2$$

Mathematica [A]

time = 0.01, size = 30, normalized size = 0.88

$$\frac{1}{4} \left(-x^2 + 2x\sqrt{1-x^2} \sin^{-1}(x) + \sin^{-1}(x)^2 \right)$$

Antiderivative was successfully verified.

`[In] Integrate[Sqrt[1 - x^2]*ArcSin[x],x]``[Out] (-x^2 + 2*x*Sqrt[1 - x^2]*ArcSin[x] + ArcSin[x]^2)/4`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[ArcSin[x]*Sqrt[1 - x^2],x]')``[Out] cought exception:`**Maple [A]**

time = 0.09, size = 31, normalized size = 0.91

method	result	size
default	$\frac{\arcsin(x) \left(x\sqrt{-x^2+1} + \arcsin(x) \right)}{2} - \frac{\arcsin(x)^2}{4} - \frac{x^2}{4}$	31

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(arcsin(x)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)``[Out] 1/2*arcsin(x)*(x*(-x^2+1)^(1/2)+arcsin(x))-1/4*arcsin(x)^2-1/4*x^2`**Maxima [A]**

time = 0.32, size = 30, normalized size = 0.88

$$-\frac{1}{4}x^2 + \frac{1}{2} \left(\sqrt{-x^2+1} x + \arcsin(x) \right) \arcsin(x) - \frac{1}{4} \arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(x)*(-x^2+1)^(1/2),x, algorithm="maxima")

[Out] -1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arcsin(x) - 1/4*arcsin(x)^2

Fricas [A]

time = 0.32, size = 26, normalized size = 0.76

$$\frac{1}{2} \sqrt{-x^2 + 1} x \arcsin(x) - \frac{1}{4} x^2 + \frac{1}{4} \arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(x)*(-x^2+1)^(1/2),x, algorithm="fricas")

[Out] 1/2*sqrt(-x^2 + 1)*x*arcsin(x) - 1/4*x^2 + 1/4*arcsin(x)^2

Sympy [A]

time = 1.10, size = 39, normalized size = 1.15

$$\left(\begin{cases} x\sqrt{1-x^2} + \frac{\arcsin(x)}{2} & \text{for } x > -1 \wedge x < 1 \end{cases} \right) \arcsin(x) - \begin{cases} \text{NaN} & \text{for } x < -1 \\ \frac{x^2}{4} + \frac{\arcsin^2(x)}{4} & \text{for } x < 1 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(x)*(-x**2+1)**(1/2),x)

[Out] Piecewise((x*sqrt(1 - x**2)/2 + asin(x)/2, (x > -1) & (x < 1)))*asin(x) - Piecewise((nan, x < -1), (x**2/4 + asin(x)**2/4, x < 1), (nan, True))

Giac [A]

time = 0.01, size = 36, normalized size = 1.06

$$\frac{\arcsin^2 x}{4} + \frac{1}{2} x \sqrt{-x^2 + 1} \arcsin x + \frac{-x^2 + 1}{4} - \frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(x)*(-x^2+1)^(1/2),x)

[Out] 1/2*sqrt(-x^2 + 1)*x*arcsin(x) - 1/4*x^2 + 1/4*arcsin(x)^2 + 1/8

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \arcsin(x) \sqrt{1-x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(x)*(1 - x^2)^(1/2),x)

[Out] int(asin(x)*(1 - x^2)^(1/2), x)

3.652 $\int \sqrt{1-x^2} \cos^{-1}(x) dx$

Optimal. Leaf size=34

$$\frac{x^2}{4} + \frac{1}{2}x\sqrt{1-x^2} \cos^{-1}(x) - \frac{1}{4} \cos^{-1}(x)^2$$

[Out] 1/4*x^2-1/4*arccos(x)^2+1/2*x*arccos(x)*(-x^2+1)^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4742, 4738, 30}

$$\frac{x^2}{4} + \frac{1}{2}\sqrt{1-x^2} x \cos^{-1}(x) - \frac{1}{4} \cos^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x^2]*ArcCos[x], x]

[Out] x^2/4 + (x*Sqrt[1 - x^2]*ArcCos[x])/2 - ArcCos[x]^2/4

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 4738

Int[((a_) + ArcCos[(c_)*(x_)])*(b_)^(n_)/Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[(-b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]

Rule 4742

Int[((a_) + ArcCos[(c_)*(x_)])*(b_)^(n_)*Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Dist[(1/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2], x], x] + Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x)) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]

Rubi steps

$$\begin{aligned}\int \sqrt{1-x^2} \cos^{-1}(x) dx &= \frac{1}{2}x\sqrt{1-x^2} \cos^{-1}(x) + \frac{\int x dx}{2} + \frac{1}{2} \int \frac{\cos^{-1}(x)}{\sqrt{1-x^2}} dx \\ &= \frac{x^2}{4} + \frac{1}{2}x\sqrt{1-x^2} \cos^{-1}(x) - \frac{1}{4} \cos^{-1}(x)^2\end{aligned}$$

Mathematica [A]

time = 0.01, size = 30, normalized size = 0.88

$$\frac{1}{4} \left(x^2 + 2x\sqrt{1-x^2} \cos^{-1}(x) - \cos^{-1}(x)^2 \right)$$

Antiderivative was successfully verified.

`[In] Integrate[Sqrt[1 - x^2]*ArcCos[x], x]``[Out] (x^2 + 2*x*Sqrt[1 - x^2]*ArcCos[x] - ArcCos[x]^2)/4`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[ArcCos[x]*Sqrt[1 - x^2], x]')``[Out] cought exception:`**Maple [A]**

time = 0.11, size = 33, normalized size = 0.97

method	result	size
default	$-\frac{\arccos(x) \left(-x\sqrt{-x^2+1} + \arccos(x) \right)}{2} + \frac{\arccos(x)^2}{4} + \frac{x^2}{4} - \frac{1}{4}$	33

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(arccos(x)*(-x^2+1)^(1/2), x, method=_RETURNVERBOSE)``[Out] -1/2*arccos(x)*(-x*(-x^2+1)^(1/2)+arccos(x))+1/4*arccos(x)^2+1/4*x^2-1/4`**Maxima [A]**

time = 0.33, size = 30, normalized size = 0.88

$$\frac{1}{4}x^2 + \frac{1}{2} \left(\sqrt{-x^2+1} x + \arcsin(x) \right) \arccos(x) + \frac{1}{4} \arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="maxima")

[Out] 1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arccos(x) + 1/4*arcsin(x)^2

Fricas [A]

time = 0.32, size = 26, normalized size = 0.76

$$\frac{1}{2} \sqrt{-x^2 + 1} x \arccos(x) + \frac{1}{4} x^2 - \frac{1}{4} \arccos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="fricas")

[Out] 1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2

Sympy [A]

time = 1.14, size = 39, normalized size = 1.15

$$\left(\begin{cases} x\sqrt{1-x^2} + \frac{\arcsin(x)}{2} & \text{for } x > -1 \wedge x < 1 \end{cases} \right) \arccos(x) + \begin{cases} \text{NaN} & \text{for } x < -1 \\ \frac{x^2}{4} + \frac{\arcsin^2(x)}{4} & \text{for } x < 1 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(x)*(-x**2+1)**(1/2),x)

[Out] Piecewise((x*sqrt(1 - x**2)/2 + asin(x)/2, (x > -1) & (x < 1)))*acos(x) + Piecewise((nan, x < -1), (x**2/4 + asin(x)**2/4, x < 1), (nan, True))

Giac [A]

time = 0.01, size = 34, normalized size = 1.00

$$\frac{x^2}{4} - \frac{\arccos^2 x}{4} + \frac{1}{2} x \sqrt{-x^2 + 1} \arccos x - \frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x)*(-x^2+1)^(1/2),x)

[Out] 1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2 - 1/8

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \arccos(x) \sqrt{1-x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acos(x)*(1 - x^2)^(1/2),x)

[Out] int(acos(x)*(1 - x^2)^(1/2), x)

3.653 $\int x \sqrt{1-x^2} \cos^{-1}(x) dx$

Optimal. Leaf size=30

$$-\frac{x}{3} + \frac{x^3}{9} - \frac{1}{3} (1-x^2)^{3/2} \cos^{-1}(x)$$

[Out] $-1/3*x+1/9*x^3-1/3*(-x^2+1)^{(3/2)*\arccos(x)}$

Rubi [A]

time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {4768}

$$\frac{x^3}{9} - \frac{1}{3} (1-x^2)^{3/2} \cos^{-1}(x) - \frac{x}{3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Sqrt}[1-x^2]*\text{ArcCos}[x],x]$

[Out] $-1/3*x + x^3/9 - ((1-x^2)^{(3/2)*\text{ArcCos}[x]})/3$

Rule 4768

$\text{Int}[(a_.) + \text{ArcCos}[c_.*(x_)]*(b_.)^{(n_)}*(x_)*((d_)+(e_)*(x_)^2)^{(p_.)}, x_Symbol] :> \text{Simp}[(d+e*x^2)^{(p+1)}*((a+b*\text{ArcCos}[c*x])^n/(2*e*(p+1))), x] - \text{Dist}[b*(n/(2*c*(p+1)))*\text{Simp}[(d+e*x^2)^p/(1-c^2*x^2)^p], \text{Int}[(1-c^2*x^2)^{(p+1/2)}*(a+b*\text{ArcCos}[c*x])^{(n-1)}, x], x] /;$ FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d+e, 0] && GtQ[n, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int x \sqrt{1-x^2} \cos^{-1}(x) dx &= -\frac{1}{3} (1-x^2)^{3/2} \cos^{-1}(x) - \frac{1}{3} \int (1-x^2) dx \\ &= -\frac{x}{3} + \frac{x^3}{9} - \frac{1}{3} (1-x^2)^{3/2} \cos^{-1}(x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 26, normalized size = 0.87

$$\frac{1}{9} \left(-3x + x^3 - 3(1-x^2)^{3/2} \cos^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[x*\text{Sqrt}[1-x^2]*\text{ArcCos}[x],x]$

[Out] $(-3*x + x^3 - 3*(1 - x^2)^{(3/2)}*ArcCos[x])/9$

Mathics [A]

time = 1.91, size = 38, normalized size = 1.27

$$-\frac{x}{3} + \frac{x^2 ArcCos[x] \sqrt{1-x^2}}{3} + \frac{x^3}{9} - \frac{ArcCos[x] \sqrt{1-x^2}}{3}$$

Antiderivative was successfully verified.

[In] `mathics('Integrate[ArcCos[x]*x*Sqrt[1 - x^2],x]')`

[Out] $-x / 3 + x^2 ArcCos[x] Sqrt[1 - x^2] / 3 + x^3 / 9 - ArcCos[x] Sqrt[1 - x^2] / 3$

Maple [C] Result contains complex when optimal does not.

time = 0.20, size = 134, normalized size = 4.47

method	result
default	$-\frac{(i+3 \arccos(x)) \left(4ix^3 - 4x^2 \sqrt{-x^2 + 1} - 3ix + \sqrt{-x^2 + 1} \right)}{72} + \frac{(\arccos(x)+i) \left(ix - \sqrt{-x^2 + 1} \right)}{8} - \frac{(\arccos(x)-i) \left(ix - \sqrt{-x^2 + 1} \right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arccos(x)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/72*(I+3*\arccos(x))*(4*I*x^3-4*x^2*(-x^2+1)^(1/2)-3*I*x+(-x^2+1)^(1/2))+1/8*(\arccos(x)+I)*(I*x-(-x^2+1)^(1/2))-1/8*(\arccos(x)-I)*(I*x+(-x^2+1)^(1/2))+1/72*(-I+3*\arccos(x))*(4*I*x^3+4*x^2*(-x^2+1)^(1/2)-3*I*x-(-x^2+1)^(1/2))$

Maxima [A]

time = 0.33, size = 22, normalized size = 0.73

$$\frac{1}{9}x^3 - \frac{1}{3}(-x^2 + 1)^{\frac{3}{2}} \arccos(x) - \frac{1}{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccos(x)*(-x^2+1)^(1/2),x, algorithm="maxima")`

[Out] $1/9*x^3 - 1/3*(-x^2 + 1)^{(3/2)}*arccos(x) - 1/3*x$

Fricas [A]

time = 0.31, size = 27, normalized size = 0.90

$$\frac{1}{9}x^3 + \frac{1}{3}(x^2 - 1)\sqrt{-x^2 + 1} \arccos(x) - \frac{1}{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccos(x)*(-x^2+1)^(1/2),x, algorithm="fricas")`

[Out] $1/9*x^3 + 1/3*(x^2 - 1)*\sqrt{-x^2 + 1}*\arccos(x) - 1/3*x$

Sympy [A]

time = 0.21, size = 37, normalized size = 1.23

$$\frac{x^3}{9} + \frac{x^2\sqrt{1-x^2}\arccos(x)}{3} - \frac{x}{3} - \frac{\sqrt{1-x^2}\arccos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acos(x)*(-x**2+1)**(1/2),x)`

[Out] $x**3/9 + x**2*\sqrt{1 - x**2}*\arccos(x)/3 - x/3 - \sqrt{1 - x**2}*\arccos(x)/3$

Giac [A]

time = 0.00, size = 36, normalized size = 1.20

$$-\frac{1}{3}\sqrt{-x^2+1}(-x^2+1)\arccos x - \frac{-\frac{x^3}{3}+x}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccos(x)*(-x^2+1)^(1/2),x)`

[Out] $1/9*x^3 - 1/3*(-x^2 + 1)^(3/2)*\arccos(x) - 1/3*x$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int x \arccos(x) \sqrt{1-x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*acos(x)*(1 - x^2)^(1/2),x)`

[Out] `int(x*acos(x)*(1 - x^2)^(1/2), x)`

3.654 $\int (1 - x^2)^{3/2} \sin^{-1}(x) dx$

Optimal. Leaf size=59

$$-\frac{5x^2}{16} + \frac{x^4}{16} + \frac{3}{8}x\sqrt{1-x^2} \sin^{-1}(x) + \frac{1}{4}x(1-x^2)^{3/2} \sin^{-1}(x) + \frac{3}{16} \sin^{-1}(x)^2$$

[Out] $-5/16*x^2+1/16*x^4+1/4*x*(-x^2+1)^{(3/2)}*\arcsin(x)+3/16*\arcsin(x)^2+3/8*x*\arcsin(x)*(-x^2+1)^{(1/2)}$

Rubi [A]

time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4743, 4741, 4737, 30, 14}

$$\frac{x^4}{16} - \frac{5x^2}{16} + \frac{1}{4}(1-x^2)^{3/2}x \sin^{-1}(x) + \frac{3}{8}\sqrt{1-x^2}x \sin^{-1}(x) + \frac{3}{16} \sin^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 - x^2)^{(3/2)}*\text{ArcSin}[x], x]$

[Out] $(-5*x^2)/16 + x^4/16 + (3*x*\text{Sqrt}[1 - x^2]*\text{ArcSin}[x])/8 + (x*(1 - x^2)^{(3/2)}*\text{ArcSin}[x])/4 + (3*\text{ArcSin}[x]^2)/16$

Rule 14

$\text{Int}[(u_*)*((c_*)*(x_*))^{(m_*)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /;$ FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 30

$\text{Int}[(x_*)^{(m_*)}, x_Symbol] \rightarrow \text{Simp}[x^{(m+1)}/(m+1), x] /;$ FreeQ[m, x] && NeQ[m, -1]

Rule 4737

$\text{Int}[(a_ + \text{ArcSin}[(c_*)*(x_*)]*(b_*))^{(n_*)}/\text{Sqrt}[(d_*) + (e_*)*(x_*)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n+1)}, x] /;$ FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]

Rule 4741

$\text{Int}[(a_ + \text{ArcSin}[(c_*)*(x_*)]*(b_*))^{(n_*)}*\text{Sqrt}[(d_*) + (e_*)*(x_*)^2], x_Symbol] \rightarrow \text{Simp}[x*\text{Sqrt}[d + e*x^2]*((a + b*\text{ArcSin}[c*x])^{(n/2)}), x] + (\text{Dist}[(1/2)*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]], \text{Int}[(a + b*\text{ArcSin}[c*x])^n/\text{Sqrt}[1$

```
- c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
], Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4743

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(2*p + 1)), x] + (D
ist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x
] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[x*(1 -
c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int (1-x^2)^{3/2} \sin^{-1}(x) dx &= \frac{1}{4}x(1-x^2)^{3/2} \sin^{-1}(x) - \frac{1}{4} \int x(1-x^2) dx + \frac{3}{4} \int \sqrt{1-x^2} \sin^{-1}(x) dx \\ &= \frac{3}{8}x\sqrt{1-x^2} \sin^{-1}(x) + \frac{1}{4}x(1-x^2)^{3/2} \sin^{-1}(x) - \frac{1}{4} \int (x-x^3) dx - \frac{3}{8} \int x dx \\ &= -\frac{5x^2}{16} + \frac{x^4}{16} + \frac{3}{8}x\sqrt{1-x^2} \sin^{-1}(x) + \frac{1}{4}x(1-x^2)^{3/2} \sin^{-1}(x) + \frac{3}{16} \sin^{-1}(x)^2 \end{aligned}$$

Mathematica [A]

time = 0.02, size = 42, normalized size = 0.71

$$\frac{1}{16} \left(-5x^2 + x^4 - 2x\sqrt{1-x^2} (-5 + 2x^2) \sin^{-1}(x) + 3 \sin^{-1}(x)^2 \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)^(3/2)*ArcSin[x],x]
```

```
[Out] (-5*x^2 + x^4 - 2*x*Sqrt[1 - x^2]*(-5 + 2*x^2)*ArcSin[x] + 3*ArcSin[x]^2)/16
```

Mathics [A]

time = 2.08, size = 47, normalized size = 0.80

$$\frac{5x \operatorname{ArcSin}[x] \sqrt{1-x^2}}{8} - \frac{5x^2}{16} - \frac{x^3 \operatorname{ArcSin}[x] \sqrt{1-x^2}}{4} + \frac{x^4}{16} + \frac{3 \operatorname{ArcSin}[x]^2}{16}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[ArcSin[x]*(1 - x^2)^(3/2),x]')
```

[Out] $5 x \operatorname{ArcSin}[x] \operatorname{Sqrt}[1 - x^2] / 8 - 5 x^2 / 16 - x^3 \operatorname{ArcSin}[x] \operatorname{Sqrt}[1 - x^2] / 4 + x^4 / 16 + 3 \operatorname{ArcSin}[x]^2 / 16$

Maple [A]

time = 0.13, size = 54, normalized size = 0.92

method	result	size
default	$\frac{\arcsin(x) \left(-2\sqrt{-x^2+1} x^3 + 5x\sqrt{-x^2+1} + 3\arcsin(x) \right)}{8} - \frac{3\arcsin(x)^2}{16} + \frac{(2x^2-5)^2}{64}$	54

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^2+1)^(3/2)*arcsin(x),x,method=_RETURNVERBOSE)`

[Out] $1/8*\arcsin(x)*(-2*(-x^2+1)^(1/2)*x^3+5*x*(-x^2+1)^(1/2)+3*\arcsin(x))-3/16*\arcsin(x)^2+1/64*(2*x^2-5)^2$

Maxima [A]

time = 0.33, size = 50, normalized size = 0.85

$$\frac{1}{16}x^4 - \frac{5}{16}x^2 + \frac{1}{8} \left(2(-x^2+1)^{\frac{3}{2}}x + 3\sqrt{-x^2+1}x + 3\arcsin(x) \right) \arcsin(x) - \frac{3}{16}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+1)^(3/2)*arcsin(x),x, algorithm="maxima")`

[Out] $1/16*x^4 - 5/16*x^2 + 1/8*(2*(-x^2+1)^(3/2)*x + 3*\sqrt{-x^2+1}*x + 3*\arcsin(x))*\arcsin(x) - 3/16*\arcsin(x)^2$

Fricas [A]

time = 0.32, size = 39, normalized size = 0.66

$$\frac{1}{16}x^4 - \frac{1}{8}(2x^3 - 5x)\sqrt{-x^2+1} \arcsin(x) - \frac{5}{16}x^2 + \frac{3}{16}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+1)^(3/2)*arcsin(x),x, algorithm="fricas")`

[Out] $1/16*x^4 - 1/8*(2*x^3 - 5*x)*\sqrt{-x^2+1}*\arcsin(x) - 5/16*x^2 + 3/16*\arcsin(x)^2$

Sympy [A]

time = 0.37, size = 53, normalized size = 0.90

$$\frac{x^4}{16} - \frac{x^3\sqrt{1-x^2}\operatorname{asin}(x)}{4} - \frac{5x^2}{16} + \frac{5x\sqrt{1-x^2}\operatorname{asin}(x)}{8} + \frac{3\operatorname{asin}^2(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)**(3/2)*asin(x),x)

[Out] x**4/16 - x**3*sqrt(1 - x**2)*asin(x)/4 - 5*x**2/16 + 5*x*sqrt(1 - x**2)*asin(x)/8 + 3*asin(x)**2/16

Giac [A]

time = 0.01, size = 73, normalized size = 1.24

$$\frac{(-x^2+1)^2}{16} + \frac{3}{16} \arcsin^2 x + \frac{3}{8} x \sqrt{-x^2+1} \arcsin x + \frac{1}{4} x \sqrt{-x^2+1} (-x^2+1) \arcsin x + \frac{3}{16} (-x^2+1) - \frac{15}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^(3/2)*arcsin(x),x)

[Out] 1/4*(-x^2 + 1)^(3/2)*x*arcsin(x) + 3/8*sqrt(-x^2 + 1)*x*arcsin(x) + 1/16*(x^2 - 1)^2 - 3/16*x^2 + 3/16*arcsin(x)^2 + 9/128

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \arcsin(x) (1 - x^2)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(x)*(1 - x^2)^(3/2),x)

[Out] int(asin(x)*(1 - x^2)^(3/2), x)

3.655 $\int x (1 - x^2)^{3/2} \sin^{-1}(x) dx$

Optimal. Leaf size=37

$$\frac{x}{5} - \frac{2x^3}{15} + \frac{x^5}{25} - \frac{1}{5} (1 - x^2)^{5/2} \sin^{-1}(x)$$

[Out] 1/5*x-2/15*x^3+1/25*x^5-1/5*(-x^2+1)^(5/2)*arcsin(x)

Rubi [A]

time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4767, 200}

$$\frac{x^5}{25} - \frac{2x^3}{15} - \frac{1}{5} (1 - x^2)^{5/2} \sin^{-1}(x) + \frac{x}{5}$$

Antiderivative was successfully verified.

[In] Int[x*(1 - x^2)^(3/2)*ArcSin[x],x]

[Out] x/5 - (2*x^3)/15 + x^5/25 - ((1 - x^2)^(5/2)*ArcSin[x])/5

Rule 200

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4767

Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int x (1 - x^2)^{3/2} \sin^{-1}(x) dx &= -\frac{1}{5} (1 - x^2)^{5/2} \sin^{-1}(x) + \frac{1}{5} \int (1 - x^2)^2 dx \\ &= -\frac{1}{5} (1 - x^2)^{5/2} \sin^{-1}(x) + \frac{1}{5} \int (1 - 2x^2 + x^4) dx \\ &= \frac{x}{5} - \frac{2x^3}{15} + \frac{x^5}{25} - \frac{1}{5} (1 - x^2)^{5/2} \sin^{-1}(x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 35, normalized size = 0.95

$$\frac{1}{5} \left(x - \frac{2x^3}{3} + \frac{x^5}{5} - (1 - x^2)^{5/2} \sin^{-1}(x) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[x*(1 - x^2)^(3/2)*ArcSin[x],x]``[Out] (x - (2*x^3)/3 + x^5/5 - (1 - x^2)^(5/2)*ArcSin[x])/5`**Mathics [B]** Leaf count is larger than twice the leaf count of optimal. 77 vs. 2(37) = 74.

time = 2.33, size = 59, normalized size = 1.59

$$\frac{x}{5} + \frac{2x^2 \text{ArcSin}[x] \sqrt{1-x^2}}{5} - \frac{2x^3}{15} - \frac{x^4 \text{ArcSin}[x] \sqrt{1-x^2}}{5} + \frac{x^5}{25} - \frac{\text{ArcSin}[x] \sqrt{1-x^2}}{5}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[ArcSin[x]*x*(1 - x^2)^(3/2),x]')``[Out] x / 5 + 2 x ^ 2 ArcSin[x] Sqrt[1 - x ^ 2] / 5 - 2 x ^ 3 / 15 - x ^ 4 ArcSin[x] Sqrt[1 - x ^ 2] / 5 + x ^ 5 / 25 - ArcSin[x] Sqrt[1 - x ^ 2] / 5`**Maple [A]**

time = 0.11, size = 37, normalized size = 1.00

method	result	size
default	$-\frac{(x^2-1)^2 \sqrt{-x^2+1} \arcsin(x)}{5} + \frac{(3x^4-10x^2+15)x}{75}$	37

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*(-x^2+1)^(3/2)*arcsin(x),x,method=_RETURNVERBOSE)``[Out] -1/5*(x^2-1)^2*(-x^2+1)^(1/2)*arcsin(x)+1/75*(3*x^4-10*x^2+15)*x`**Maxima [A]**

time = 0.33, size = 27, normalized size = 0.73

$$\frac{1}{25} x^5 - \frac{1}{5} (-x^2 + 1)^{5/2} \arcsin(x) - \frac{2}{15} x^3 + \frac{1}{5} x$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*(-x^2+1)^(3/2)*arcsin(x),x, algorithm="maxima")``[Out] 1/25*x^5 - 1/5*(-x^2 + 1)^(5/2)*arcsin(x) - 2/15*x^3 + 1/5*x`

Fricas [A]

time = 0.31, size = 37, normalized size = 1.00

$$\frac{1}{25}x^5 - \frac{2}{15}x^3 - \frac{1}{5}(x^4 - 2x^2 + 1)\sqrt{-x^2 + 1} \arcsin(x) + \frac{1}{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*(-x^2+1)^(3/2)*arcsin(x),x, algorithm="fricas")``[Out] 1/25*x^5 - 2/15*x^3 - 1/5*(x^4 - 2*x^2 + 1)*sqrt(-x^2 + 1)*arcsin(x) + 1/5*x`**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(27) = 54

time = 0.53, size = 63, normalized size = 1.70

$$\frac{x^5}{25} - \frac{x^4\sqrt{1-x^2} \operatorname{asin}(x)}{5} - \frac{2x^3}{15} + \frac{2x^2\sqrt{1-x^2} \operatorname{asin}(x)}{5} + \frac{x}{5} - \frac{\sqrt{1-x^2} \operatorname{asin}(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*(-x**2+1)**(3/2)*asin(x),x)``[Out] x**5/25 - x**4*sqrt(1 - x**2)*asin(x)/5 - 2*x**3/15 + 2*x**2*sqrt(1 - x**2)*asin(x)/5 + x/5 - sqrt(1 - x**2)*asin(x)/5`**Giac [A]**

time = 0.01, size = 46, normalized size = 1.24

$$-\frac{1}{5}\sqrt{-x^2 + 1} (-x^2 + 1)^2 \arcsin x - \frac{-\frac{x^5}{5} + \frac{2}{3}x^3 - x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*(-x^2+1)^(3/2)*arcsin(x),x)``[Out] 1/25*x^5 - 1/5*(x^2 - 1)^2*sqrt(-x^2 + 1)*arcsin(x) - 2/15*x^3 + 1/5*x`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.03

$$\int x \operatorname{asin}(x) (1 - x^2)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*asin(x)*(1 - x^2)^(3/2),x)``[Out] int(x*asin(x)*(1 - x^2)^(3/2), x)`

3.656 $\int x^3 (1 - x^2)^{3/2} \cos^{-1}(x) dx$

Optimal. Leaf size=61

$$-\frac{2x}{35} - \frac{x^3}{105} + \frac{8x^5}{175} - \frac{x^7}{49} - \frac{1}{5} (1 - x^2)^{5/2} \cos^{-1}(x) + \frac{1}{7} (1 - x^2)^{7/2} \cos^{-1}(x)$$

[Out] -2/35*x-1/105*x^3+8/175*x^5-1/49*x^7-1/5*(-x^2+1)^(5/2)*arccos(x)+1/7*(-x^2+1)^(7/2)*arccos(x)

Rubi [A]

time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {272, 45, 4780, 12, 380}

$$-\frac{x^7}{49} + \frac{8x^5}{175} - \frac{x^3}{105} + \frac{1}{7} (1 - x^2)^{7/2} \cos^{-1}(x) - \frac{1}{5} (1 - x^2)^{5/2} \cos^{-1}(x) - \frac{2x}{35}$$

Antiderivative was successfully verified.

[In] Int[x^3*(1 - x^2)^(3/2)*ArcCos[x],x]

[Out] (-2*x)/35 - x^3/105 + (8*x^5)/175 - x^7/49 - ((1 - x^2)^(5/2)*ArcCos[x])/5 + ((1 - x^2)^(7/2)*ArcCos[x])/7

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 272

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 380

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b

, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 4780

```
Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos[
c*x], u, x] + Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[Simplif
yIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && E
qQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2
, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rubi steps

$$\begin{aligned} \int x^3 (1-x^2)^{3/2} \cos^{-1}(x) dx &= -\frac{1}{5} (1-x^2)^{5/2} \cos^{-1}(x) + \frac{1}{7} (1-x^2)^{7/2} \cos^{-1}(x) + \int \frac{1}{35} (-2-5x^2) (1-x^2)^{3/2} \cos^{-1}(x) dx \\ &= -\frac{1}{5} (1-x^2)^{5/2} \cos^{-1}(x) + \frac{1}{7} (1-x^2)^{7/2} \cos^{-1}(x) + \frac{1}{35} \int (-2-5x^2) (1-x^2)^{3/2} \cos^{-1}(x) dx \\ &= -\frac{1}{5} (1-x^2)^{5/2} \cos^{-1}(x) + \frac{1}{7} (1-x^2)^{7/2} \cos^{-1}(x) + \frac{1}{35} \int (-2-x^2+8x^4-5x^6) (1-x^2)^{3/2} \cos^{-1}(x) dx \\ &= -\frac{2x}{35} - \frac{x^3}{105} + \frac{8x^5}{175} - \frac{x^7}{49} - \frac{1}{5} (1-x^2)^{5/2} \cos^{-1}(x) + \frac{1}{7} (1-x^2)^{7/2} \cos^{-1}(x) \end{aligned}$$

Mathematica [A]

time = 0.03, size = 47, normalized size = 0.77

$$-\frac{x(210 + 35x^2 - 168x^4 + 75x^6)}{3675} - \frac{1}{35} (1-x^2)^{5/2} (2+5x^2) \cos^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(1 - x^2)^(3/2)*ArcCos[x], x]

[Out] -1/3675*(x*(210 + 35*x^2 - 168*x^4 + 75*x^6)) - ((1 - x^2)^(5/2)*(2 + 5*x^2)*ArcCos[x])/35

Mathics [A]

time = 8.09, size = 80, normalized size = 1.31

$$\frac{-2x}{35} - \frac{x^2 \text{ArcCos}[x] \sqrt{1-x^2}}{35} - \frac{x^3}{105} + \frac{8x^4 \text{ArcCos}[x] \sqrt{1-x^2}}{35} + \frac{8x^5}{175} - \frac{x^6 \text{ArcCos}[x] \sqrt{1-x^2}}{7} - \frac{x^7}{49} - \frac{2 \text{ArcCos}[x] \sqrt{1-x^2}}{35}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[ArcCos[x]*x^3*(1 - x^2)^(3/2), x]')

[Out] $-2x/35 - x^2 \operatorname{ArcCos}[x] \operatorname{Sqrt}[1 - x^2] / 35 - x^3 / 105 + 8x^4 \operatorname{ArcCos}[x] \operatorname{Sqrt}[1 - x^2] / 35 + 8x^5 / 175 - x^6 \operatorname{ArcCos}[x] \operatorname{Sqrt}[1 - x^2] / 7 - x^7 / 49 - 2 \operatorname{ArcCos}[x] \operatorname{Sqrt}[1 - x^2] / 35$

Maple [C] Result contains complex when optimal does not.
time = 0.31, size = 286, normalized size = 4.69

method	result
default	$\frac{(i+7 \operatorname{arccos}(x)) \left(64ix^7 - 64\sqrt{-x^2+1} x^6 - 112ix^5 + 80\sqrt{-x^2+1} x^4 + 56ix^3 - 24x^2\sqrt{-x^2+1} - 7ix + \sqrt{-x^2+1} \right)}{6272} +$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(-x^2+1)^(3/2)*arccos(x),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{6272} (I+7 \operatorname{arccos}(x)) (64Ix^7 - 64(-x^2+1)^{1/2} x^6 - 112Ix^5 + 80(-x^2+1)^{1/2} x^4 + 56Ix^3 - 24x^2(-x^2+1)^{1/2} - 7Ix + (-x^2+1)^{1/2}) + \frac{3}{128} (\operatorname{arccos}(x)+I) (Ix - (-x^2+1)^{1/2}) - \frac{3}{128} (\operatorname{arccos}(x)-I) (Ix + (-x^2+1)^{1/2}) + \frac{1}{3} 84(-I+3 \operatorname{arccos}(x)) (4Ix^3 + 4x^2(-x^2+1)^{1/2} - 3Ix - (-x^2+1)^{1/2}) - \frac{3}{3} 9200 \cos(6 \operatorname{arccos}(x)) (2I+35 \operatorname{arccos}(x)) (Ix + (-x^2+1)^{1/2}) + \frac{1}{78400} \sin(6 \operatorname{arccos}(x)) (37I+35 \operatorname{arccos}(x)) (-I(-x^2+1)^{1/2} + x) - \frac{1}{2400} \cos(4 \operatorname{arccos}(x)) (7I+15 \operatorname{arccos}(x)) (Ix + (-x^2+1)^{1/2}) + \frac{1}{4800} \sin(4 \operatorname{arccos}(x)) (11I+45 \operatorname{arccos}(x)) (-I(-x^2+1)^{1/2} + x)$

Maxima [A]

time = 0.34, size = 49, normalized size = 0.80

$$-\frac{1}{49} x^7 + \frac{8}{175} x^5 - \frac{1}{105} x^3 - \frac{1}{35} \left(5(-x^2+1)^{\frac{5}{2}} x^2 + 2(-x^2+1)^{\frac{5}{2}} \right) \operatorname{arccos}(x) - \frac{2}{35} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(-x^2+1)^(3/2)*arccos(x),x, algorithm="maxima")`

[Out] $-1/49 x^7 + 8/175 x^5 - 1/105 x^3 - 1/35 (5(-x^2+1)^{5/2} x^2 + 2(-x^2+1)^{5/2}) \operatorname{arccos}(x) - 2/35 x$

Fricas [A]

time = 0.32, size = 47, normalized size = 0.77

$$-\frac{1}{49} x^7 + \frac{8}{175} x^5 - \frac{1}{105} x^3 - \frac{1}{35} (5x^6 - 8x^4 + x^2 + 2) \sqrt{-x^2+1} \operatorname{arccos}(x) - \frac{2}{35} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(-x^2+1)^(3/2)*arccos(x),x, algorithm="fricas")`

[Out] $-1/49 x^7 + 8/175 x^5 - 1/105 x^3 - 1/35 (5x^6 - 8x^4 + x^2 + 2) \operatorname{sqrt}(-x^2+1) \operatorname{arccos}(x) - 2/35 x$

Sympy [A]

time = 9.13, size = 88, normalized size = 1.44

$$-\frac{x^7}{49} - \frac{x^6\sqrt{1-x^2}\operatorname{acos}(x)}{7} + \frac{8x^5}{175} + \frac{8x^4\sqrt{1-x^2}\operatorname{acos}(x)}{35} - \frac{x^3}{105} - \frac{x^2\sqrt{1-x^2}\operatorname{acos}(x)}{35} - \frac{2x}{35} - \frac{2\sqrt{1-x^2}\operatorname{acos}(x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(-x**2+1)**(3/2)*acos(x),x)

[Out] -x**7/49 - x**6*sqrt(1 - x**2)*acos(x)/7 + 8*x**5/175 + 8*x**4*sqrt(1 - x**2)*acos(x)/35 - x**3/105 - x**2*sqrt(1 - x**2)*acos(x)/35 - 2*x/35 - 2*sqrt(1 - x**2)*acos(x)/35

Giac [A]

time = 0.01, size = 77, normalized size = 1.26

$$\left(\frac{1}{7}\sqrt{-x^2+1}(-x^2+1)^3 - \frac{1}{5}\sqrt{-x^2+1}(-x^2+1)^2\right)\arccos x - \frac{\frac{5}{7}x^7 - \frac{8}{5}x^5 + \frac{x^3}{3} + 2x}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-x^2+1)^(3/2)*arccos(x),x)

[Out] -1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*(x^2 - 1)^3*sqrt(-x^2 + 1) + 7*(x^2 - 1)^2*sqrt(-x^2 + 1))*arccos(x) - 2/35*x

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int x^3 \operatorname{acos}(x) (1 - x^2)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*acos(x)*(1 - x^2)^(3/2),x)**[Out]** int(x^3*acos(x)*(1 - x^2)^(3/2), x)

$$3.657 \quad \int \frac{(1-x^2)^{3/2} \cos^{-1}(x)}{x} dx$$

Optimal. Leaf size=95

$$\frac{4x}{3} - \frac{x^3}{9} + \sqrt{1-x^2} \cos^{-1}(x) + \frac{1}{3} (1-x^2)^{3/2} \cos^{-1}(x) + 2i \cos^{-1}(x) \tan^{-1} \left(e^{i \cos^{-1}(x)} \right) - i \operatorname{Li}_2 \left(-ie^{i \cos^{-1}(x)} \right) + i \operatorname{Li}_2 \left(ie^{i \cos^{-1}(x)} \right)$$

[Out] 4/3*x-1/9*x^3+1/3*(-x^2+1)^(3/2)*arccos(x)+2*I*arccos(x)*arctan(x+I*(-x^2+1)^(1/2))-I*polylog(2,-I*(x+I*(-x^2+1)^(1/2)))+I*polylog(2,I*(x+I*(-x^2+1)^(1/2)))+arccos(x)*(-x^2+1)^(1/2)

Rubi [A]

time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4788, 4784, 4804, 4266, 2317, 2438, 8}

$$-i \operatorname{Li}_2 \left(-ie^{i \cos^{-1}(x)} \right) + i \operatorname{Li}_2 \left(ie^{i \cos^{-1}(x)} \right) - \frac{x^3}{9} + \frac{1}{3} (1-x^2)^{3/2} \cos^{-1}(x) + \sqrt{1-x^2} \cos^{-1}(x) + \frac{4x}{3} + 2i \cos^{-1}(x) \tan^{-1} \left(e^{i \cos^{-1}(x)} \right)$$

Antiderivative was successfully verified.

[In] Int[((1 - x^2)^(3/2)*ArcCos[x])/x,x]

[Out] (4*x)/3 - x^3/9 + Sqrt[1 - x^2]*ArcCos[x] + ((1 - x^2)^(3/2)*ArcCos[x])/3 + (2*I)*ArcCos[x]*ArcTan[E^(I*ArcCos[x])] - I*PolyLog[2, (-I)*E^(I*ArcCos[x])] + I*PolyLog[2, I*E^(I*ArcCos[x])]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2317

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2438

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 4266

Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x])

], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 4784

Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] + Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])

Rule 4788

Int[((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Dist[b*c*(n/(f*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]

Rule 4804

Int[(((a_.) + ArcCos[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.))/Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol] :> Dist[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
 \int \frac{(1-x^2)^{3/2} \cos^{-1}(x)}{x} dx &= \frac{1}{3} (1-x^2)^{3/2} \cos^{-1}(x) + \frac{1}{3} \int (1-x^2) dx + \int \frac{\sqrt{1-x^2} \cos^{-1}(x)}{x} dx \\
 &= \frac{x}{3} - \frac{x^3}{9} + \sqrt{1-x^2} \cos^{-1}(x) + \frac{1}{3} (1-x^2)^{3/2} \cos^{-1}(x) + \int 1 dx + \int \frac{\cos^{-1}(x)}{x\sqrt{1-x^2}} dx \\
 &= \frac{4x}{3} - \frac{x^3}{9} + \sqrt{1-x^2} \cos^{-1}(x) + \frac{1}{3} (1-x^2)^{3/2} \cos^{-1}(x) - \text{Subst} \left(\int x \sec(x) dx, \cos^{-1}(x) \right) \\
 &= \frac{4x}{3} - \frac{x^3}{9} + \sqrt{1-x^2} \cos^{-1}(x) + \frac{1}{3} (1-x^2)^{3/2} \cos^{-1}(x) + 2i \cos^{-1}(x) \tan^{-1} \left(\frac{\cos^{-1}(x)}{\sqrt{1-x^2}} \right) \\
 &= \frac{4x}{3} - \frac{x^3}{9} + \sqrt{1-x^2} \cos^{-1}(x) + \frac{1}{3} (1-x^2)^{3/2} \cos^{-1}(x) + 2i \cos^{-1}(x) \tan^{-1} \left(\frac{\cos^{-1}(x)}{\sqrt{1-x^2}} \right) \\
 &= \frac{4x}{3} - \frac{x^3}{9} + \sqrt{1-x^2} \cos^{-1}(x) + \frac{1}{3} (1-x^2)^{3/2} \cos^{-1}(x) + 2i \cos^{-1}(x) \tan^{-1} \left(\frac{\cos^{-1}(x)}{\sqrt{1-x^2}} \right)
 \end{aligned}$$

Mathematica [A]

time = 0.16, size = 119, normalized size = 1.25

$$x + \sqrt{1-x^2} \cos^{-1}(x) + \frac{1}{36} \left(9x + 12(1-x^2)^{3/2} \cos^{-1}(x) - \cos(3 \cos^{-1}(x)) \right) - \cos^{-1}(x) \log \left(1 - ie^{i \cos^{-1}(x)} \right) + \cos^{-1}(x) \log \left(1 + ie^{i \cos^{-1}(x)} \right) - i \operatorname{Li}_2 \left(-ie^{i \cos^{-1}(x)} \right) + i \operatorname{Li}_2 \left(ie^{i \cos^{-1}(x)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((1 - x^2)^(3/2)*ArcCos[x])/x,x]

[Out] $x + \sqrt{1-x^2} \operatorname{ArcCos}[x] + (9x + 12(1-x^2)^{3/2} \operatorname{ArcCos}[x] - \operatorname{Cos}[3 \operatorname{ArcCos}[x]])/36 - \operatorname{ArcCos}[x] \operatorname{Log}[1 - I E^{(I \operatorname{ArcCos}[x])}] + \operatorname{ArcCos}[x] \operatorname{Log}[1 + I E^{(I \operatorname{ArcCos}[x])}] - I \operatorname{PolyLog}[2, (-I) E^{(I \operatorname{ArcCos}[x])}] + I \operatorname{PolyLog}[2, I E^{(I \operatorname{ArcCos}[x])}]$

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(ArcCos[x]*(1 - x^2)^(3/2))/x,x]')

[Out] Timed out

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(98) = 196.

time = 0.29, size = 230, normalized size = 2.42

method	result
default	$\frac{(i+3 \arccos(x)) \left(4ix^3 - 4x^2 \sqrt{-x^2 + 1} - 3ix + \sqrt{-x^2 + 1} \right)}{72} - \frac{5(\arccos(x)+i) \left(ix - \sqrt{-x^2 + 1} \right)}{8} + \frac{5(\arccos(x)-i) \left(ix - \sqrt{-x^2 + 1} \right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)^(3/2)*arccos(x)/x,x,method=_RETURNVERBOSE)

[Out] $\frac{1}{72} (I+3 \arccos(x)) (4Ix^3 - 4x^2 \sqrt{-x^2+1} - 3Ix + \sqrt{-x^2+1}) - \frac{5}{8} (\arccos(x)+I) (Ix - \sqrt{-x^2+1}) + \frac{5}{8} (\arccos(x)-I) (Ix + \sqrt{-x^2+1}) - \frac{1}{72} (-I+3 \arccos(x)) (4Ix^3 + 4x^2 \sqrt{-x^2+1} - 3Ix - \sqrt{-x^2+1}) - I (\arccos(x) \ln(1+I(x+I \sqrt{-x^2+1}))) - I \arccos(x) \ln(1-I(x+I \sqrt{-x^2+1})) + \operatorname{dilog}(1+I(x+I \sqrt{-x^2+1})) - \operatorname{dilog}(1-I(x+I \sqrt{-x^2+1}))$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^(3/2)*arccos(x)/x,x, algorithm="maxima")

[Out] integrate((-x^2 + 1)^(3/2)*arccos(x)/x, x)

Fricas [F]

time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^(3/2)*arccos(x)/x,x, algorithm="fricas")

[Out] integral(-(x^2 - 1)*sqrt(-x^2 + 1)*arccos(x)/x, x)

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)**(3/2)*acos(x)/x,x)

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^(3/2)*arccos(x)/x,x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\arccos(x) (1-x^2)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((acos(x)*(1 - x^2)^(3/2))/x,x)

[Out] int((acos(x)*(1 - x^2)^(3/2))/x, x)

$$3.658 \quad \int \frac{(1-x^2)^{3/2} \sin^{-1}(x)}{x^6} dx$$

Optimal. Leaf size=41

$$-\frac{1}{20x^4} + \frac{1}{5x^2} - \frac{(1-x^2)^{5/2} \sin^{-1}(x)}{5x^5} + \frac{\log(x)}{5}$$

[Out] $-1/20/x^4+1/5/x^2-1/5*(-x^2+1)^{(5/2)}*\arcsin(x)/x^5+1/5*\ln(x)$

Rubi [A]

time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4771, 272, 45}

$$-\frac{1}{20x^4} + \frac{1}{5x^2} - \frac{(1-x^2)^{5/2} \sin^{-1}(x)}{5x^5} + \frac{\log(x)}{5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1-x^2)^{(3/2)}*\text{ArcSin}[x])/x^6,x]$

[Out] $-1/20*1/x^4 + 1/(5*x^2) - ((1-x^2)^{(5/2)}*\text{ArcSin}[x])/(5*x^5) + \text{Log}[x]/5$

Rule 45

$\text{Int}[(a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

Rule 272

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p, x}], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p, x\} \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 4771

$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)]^{(n_.)}*((f_.)*(x_)^{(m_.)}*((d_) + (e_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m + 1)}*(d + e*x^2)^{(p + 1)}*((a + b*\text{ArcSin}[c*x])^n/(d*f*(m + 1))), x] - \text{Dist}[b*c*(n/(f*(m + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p], \text{Int}[(f*x)^{(m + 1)}*(1 - c^2*x^2)^{(p + 1/2)}*(a + b*\text{ArcSin}[c*x])^{(n - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, p, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{EqQ}[m + 2*p + 3, 0] \ \&\& \ \text{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned}
\int \frac{(1-x^2)^{3/2} \sin^{-1}(x)}{x^6} dx &= -\frac{(1-x^2)^{5/2} \sin^{-1}(x)}{5x^5} + \frac{1}{5} \int \frac{(1-x^2)^2}{x^5} dx \\
&= -\frac{(1-x^2)^{5/2} \sin^{-1}(x)}{5x^5} + \frac{1}{10} \text{Subst} \left(\int \frac{(1-x)^2}{x^3} dx, x, x^2 \right) \\
&= -\frac{(1-x^2)^{5/2} \sin^{-1}(x)}{5x^5} + \frac{1}{10} \text{Subst} \left(\int \left(\frac{1}{x^3} - \frac{2}{x^2} + \frac{1}{x} \right) dx, x, x^2 \right) \\
&= -\frac{1}{20x^4} + \frac{1}{5x^2} - \frac{(1-x^2)^{5/2} \sin^{-1}(x)}{5x^5} + \frac{\log(x)}{5}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 36, normalized size = 0.88

$$-\frac{x - 4x^3 + 4(1-x^2)^{5/2} \sin^{-1}(x) - 4x^5 \log(x)}{20x^5}$$

Antiderivative was successfully verified.

`[In] Integrate[((1 - x^2)^(3/2)*ArcSin[x])/x^6,x]``[Out] -1/20*(x - 4*x^3 + 4*(1 - x^2)^(5/2)*ArcSin[x] - 4*x^5*Log[x])/x^5`**Mathics [F(-1)]**

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(ArcSin[x]*(1 - x^2)^(3/2))/x^6,x]')``[Out] Timed out`**Maple [C]** Result contains complex when optimal does not.

time = 0.72, size = 201, normalized size = 4.90

method	result
default	$-\frac{2i \arcsin(x)}{5} + \frac{\left(-\sqrt{-x^2+1} x^4 + ix^5 + 2x^2 \sqrt{-x^2+1} - \sqrt{-x^2+1}\right) \left(20 \arcsin(x)x^8 - 4ix^8 - 4\sqrt{-x^2+1} x^7 - 4\sqrt{-x^2+1} x^6 - 4\sqrt{-x^2+1} x^5 - 4\sqrt{-x^2+1} x^4 - 4\sqrt{-x^2+1} x^3 - 4\sqrt{-x^2+1} x^2 - 4\sqrt{-x^2+1} x - 4\sqrt{-x^2+1}\right)}{20(5x^8 - 1)}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((-x^2+1)^(3/2)*arcsin(x)/x^6,x,method=_RETURNVERBOSE)`

[Out] $-2/5*I*\arcsin(x)+1/20*(-(-x^2+1)^{(1/2)}*x^4+I*x^5+2*x^2*(-x^2+1)^{(1/2)}-(-x^2+1)^{(1/2)})*(20*\arcsin(x)*x^8-4*I*x^8-4*(-x^2+1)^{(1/2)}*x^7-40*\arcsin(x)*x^6+I*x^6+9*(-x^2+1)^{(1/2)}*x^5+40*\arcsin(x)*x^4-6*(-x^2+1)^{(1/2)}*x^3-20*x^2*\arcsin(x)+x*(-x^2+1)^{(1/2)}+4*\arcsin(x))/(5*x^8-10*x^6+10*x^4-5*x^2+1)/x^5+1/5*\ln((I*x+(-x^2+1)^{(1/2)})^2-1)$

Maxima [A]

time = 0.32, size = 35, normalized size = 0.85

$$-\frac{(-x^2+1)^{\frac{5}{2}}\arcsin(x)}{5x^5} + \frac{4x^2-1}{20x^4} + \frac{1}{10}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+1)^(3/2)*arcsin(x)/x^6,x, algorithm="maxima")`

[Out] $-1/5*(-x^2+1)^{(5/2)}*\arcsin(x)/x^5 + 1/20*(4*x^2-1)/x^4 + 1/10*\log(x^2)$

Fricas [A]

time = 0.32, size = 44, normalized size = 1.07

$$\frac{4x^5\log(x) + 4x^3 - 4(x^4 - 2x^2 + 1)\sqrt{-x^2 + 1}\arcsin(x) - x}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+1)^(3/2)*arcsin(x)/x^6,x, algorithm="fricas")`

[Out] $1/20*(4*x^5*\log(x) + 4*x^3 - 4*(x^4 - 2*x^2 + 1)*\sqrt{-x^2 + 1}*\arcsin(x) - x)/x^5$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(x-1)(x+1))^{\frac{3}{2}}\arcsin(x)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**2+1)**(3/2)*asin(x)/x**6,x)`

[Out] `Integral((-x - 1)*(x + 1))**(3/2)*asin(x)/x**6, x)`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(31) = 62.

time = 0.02, size = 189, normalized size = 4.61

$$-\frac{3x^4-4x^2+1}{20x^4} + \frac{\ln(x^2)}{10} + \left(\frac{-\frac{1048576}{5}\left(-\frac{-2\sqrt{-x^2+1}+2}{2x}\right)^5 + 1048576\left(-\frac{-2\sqrt{-x^2+1}+2}{2x}\right)^3 + \frac{1048576(-2\sqrt{-x^2+1}+2)}{x}}{33554432} + \frac{10\left(-\frac{-2\sqrt{-x^2+1}+2}{2x}\right)^4 - 5\left(-\frac{-2\sqrt{-x^2+1}+2}{2x}\right)^2 + 1}{160\left(-\frac{-2\sqrt{-x^2+1}+2}{2x}\right)^5} \right) \arcsin x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^(3/2)*arcsin(x)/x^6,x)

[Out] -1/160*(x^5*(5*(sqrt(-x^2 + 1) - 1)^2/x^2 - 10*(sqrt(-x^2 + 1) - 1)^4/x^4 - 1)/(sqrt(-x^2 + 1) - 1)^5 + 10*(sqrt(-x^2 + 1) - 1)/x - 5*(sqrt(-x^2 + 1) - 1)^3/x^3 + (sqrt(-x^2 + 1) - 1)^5/x^5)*arcsin(x) - 1/20*(3*x^4 - 4*x^2 + 1)/x^4 + 1/10*log(x^2)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\arcsin(x) (1 - x^2)^{3/2}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((asin(x)*(1 - x^2)^(3/2))/x^6,x)

[Out] int((asin(x)*(1 - x^2)^(3/2))/x^6, x)

$$3.659 \quad \int \frac{x^2 \sin^{-1}(x)}{\sqrt{1-x^2}} dx$$

Optimal. Leaf size=34

$$\frac{x^2}{4} - \frac{1}{2}x\sqrt{1-x^2} \sin^{-1}(x) + \frac{1}{4} \sin^{-1}(x)^2$$

[Out] 1/4*x^2+1/4*arcsin(x)^2-1/2*x*arcsin(x)*(-x^2+1)^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4795, 4737, 30}

$$\frac{x^2}{4} - \frac{1}{2}\sqrt{1-x^2} x \sin^{-1}(x) + \frac{1}{4} \sin^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] Int[(x^2*ArcSin[x])/Sqrt[1 - x^2],x]

[Out] x^2/4 - (x*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 4737

Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)/Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]

Rule 4795

Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)*((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]

Rubi steps

$$\int \frac{x^2 \sin^{-1}(x)}{\sqrt{1-x^2}} dx = -\frac{1}{2}x\sqrt{1-x^2} \sin^{-1}(x) + \frac{\int x dx}{2} + \frac{1}{2} \int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} dx$$

$$= \frac{x^2}{4} - \frac{1}{2}x\sqrt{1-x^2} \sin^{-1}(x) + \frac{1}{4} \sin^{-1}(x)^2$$

Mathematica [A]

time = 0.01, size = 28, normalized size = 0.82

$$\frac{1}{4} \left(x^2 - 2x\sqrt{1-x^2} \sin^{-1}(x) + \sin^{-1}(x)^2 \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(x^2*ArcSin[x])/Sqrt[1 - x^2],x]``[Out] (x^2 - 2*x*Sqrt[1 - x^2]*ArcSin[x] + ArcSin[x]^2)/4`**Mathics [A]**

time = 1.77, size = 26, normalized size = 0.76

$$-\frac{x \text{ArcSin}[x] \sqrt{1-x^2}}{2} + \frac{x^2}{4} + \frac{\text{ArcSin}[x]^2}{4}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(ArcSin[x]*x^2)/Sqrt[1 - x^2],x]')``[Out] -x ArcSin[x] Sqrt[1 - x ^ 2] / 2 + x ^ 2 / 4 + ArcSin[x] ^ 2 / 4`**Maple [A]**

time = 0.10, size = 32, normalized size = 0.94

method	result	size
default	$\frac{\arcsin(x) \left(-x\sqrt{-x^2+1} + \arcsin(x) \right)}{2} - \frac{\arcsin(x)^2}{4} + \frac{x^2}{4}$	32

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2*arcsin(x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)``[Out] 1/2*arcsin(x)*(-x*(-x^2+1)^(1/2)+arcsin(x))-1/4*arcsin(x)^2+1/4*x^2`**Maxima [A]**

time = 0.33, size = 32, normalized size = 0.94

$$\frac{1}{4}x^2 - \frac{1}{2} \left(\sqrt{-x^2+1} x - \arcsin(x) \right) \arcsin(x) - \frac{1}{4} \arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="maxima")

[Out] 1/4*x^2 - 1/2*(sqrt(-x^2 + 1)*x - arcsin(x))*arcsin(x) - 1/4*arcsin(x)^2

Fricas [A]

time = 0.64, size = 26, normalized size = 0.76

$$-\frac{1}{2} \sqrt{-x^2 + 1} x \arcsin(x) + \frac{1}{4} x^2 + \frac{1}{4} \arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="fricas")

[Out] -1/2*sqrt(-x^2 + 1)*x*arcsin(x) + 1/4*x^2 + 1/4*arcsin(x)^2

Sympy [A]

time = 0.13, size = 26, normalized size = 0.76

$$\frac{x^2}{4} - \frac{x\sqrt{1-x^2} \operatorname{asin}(x)}{2} + \frac{\operatorname{asin}^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*asin(x)/(-x**2+1)**(1/2),x)

[Out] x**2/4 - x*sqrt(1 - x**2)*asin(x)/2 + asin(x)**2/4

Giac [A]

time = 0.01, size = 37, normalized size = 1.09

$$\frac{\arcsin^2 x}{4} - \frac{1}{2} x \sqrt{-x^2 + 1} \arcsin x - \frac{-x^2 + 1}{4} + \frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arcsin(x)/(-x^2+1)^(1/2),x)

[Out] -1/2*sqrt(-x^2 + 1)*x*arcsin(x) + 1/4*x^2 + 1/4*arcsin(x)^2 - 1/8

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2 \operatorname{asin}(x)}{\sqrt{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*asin(x))/(1 - x^2)^(1/2),x)

[Out] int((x^2*asin(x))/(1 - x^2)^(1/2), x)

$$3.660 \quad \int \frac{x^4 \sin^{-1}(x)}{\sqrt{1-x^2}} dx$$

Optimal. Leaf size=61

$$\frac{3x^2}{16} + \frac{x^4}{16} - \frac{3}{8}x\sqrt{1-x^2} \sin^{-1}(x) - \frac{1}{4}x^3\sqrt{1-x^2} \sin^{-1}(x) + \frac{3}{16} \sin^{-1}(x)^2$$

[Out] 3/16*x^2+1/16*x^4+3/16*arcsin(x)^2-3/8*x*arcsin(x)*(-x^2+1)^(1/2)-1/4*x^3*arcsin(x)*(-x^2+1)^(1/2)

Rubi [A]

time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4795, 4737, 30}

$$\frac{x^4}{16} + \frac{3x^2}{16} - \frac{3}{8}\sqrt{1-x^2} x \sin^{-1}(x) - \frac{1}{4}\sqrt{1-x^2} x^3 \sin^{-1}(x) + \frac{3}{16} \sin^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] Int[(x^4*ArcSin[x])/Sqrt[1 - x^2],x]

[Out] (3*x^2)/16 + x^4/16 - (3*x*Sqrt[1 - x^2]*ArcSin[x])/8 - (x^3*Sqrt[1 - x^2]*ArcSin[x])/4 + (3*ArcSin[x]^2)/16

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 4737

Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)/Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]

Rule 4795

Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)*((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^4 \sin^{-1}(x)}{\sqrt{1-x^2}} dx &= -\frac{1}{4}x^3\sqrt{1-x^2} \sin^{-1}(x) + \frac{\int x^3 dx}{4} + \frac{3}{4} \int \frac{x^2 \sin^{-1}(x)}{\sqrt{1-x^2}} dx \\
&= \frac{x^4}{16} - \frac{3}{8}x\sqrt{1-x^2} \sin^{-1}(x) - \frac{1}{4}x^3\sqrt{1-x^2} \sin^{-1}(x) + \frac{3 \int x dx}{8} + \frac{3}{8} \int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} dx \\
&= \frac{3x^2}{16} + \frac{x^4}{16} - \frac{3}{8}x\sqrt{1-x^2} \sin^{-1}(x) - \frac{1}{4}x^3\sqrt{1-x^2} \sin^{-1}(x) + \frac{3}{16} \sin^{-1}(x)^2
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 43, normalized size = 0.70

$$\frac{1}{16} \left(x^2 (3 + x^2) - 2x\sqrt{1-x^2} (3 + 2x^2) \sin^{-1}(x) + 3 \sin^{-1}(x)^2 \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(x^4*ArcSin[x])/Sqrt[1 - x^2], x]``[Out] (x^2*(3 + x^2) - 2*x*Sqrt[1 - x^2]*(3 + 2*x^2)*ArcSin[x] + 3*ArcSin[x]^2)/16`Mathics [A]

time = 2.05, size = 47, normalized size = 0.77

$$\frac{-3x \text{ArcSin}[x] \sqrt{1-x^2}}{8} + \frac{3x^2}{16} - \frac{x^3 \text{ArcSin}[x] \sqrt{1-x^2}}{4} + \frac{x^4}{16} + \frac{3 \text{ArcSin}[x]^2}{16}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(ArcSin[x]*x^4)/Sqrt[1 - x^2], x]')``[Out] -3 x ArcSin[x] Sqrt[1 - x ^ 2] / 8 + 3 x ^ 2 / 16 - x ^ 3 ArcSin[x] Sqrt[1 - x ^ 2] / 4 + x ^ 4 / 16 + 3 ArcSin[x] ^ 2 / 16`Maple [A]

time = 0.14, size = 54, normalized size = 0.89

method	result	size
default	$\frac{\arcsin(x) \left(-2\sqrt{-x^2 + 1} x^3 - 3x\sqrt{-x^2 + 1} + 3\arcsin(x) \right)}{8} - \frac{3 \arcsin(x)^2}{16} + \frac{(2x^2+3)^2}{64}$	54

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^4*arcsin(x)/(-x^2+1)^(1/2), x, method=_RETURNVERBOSE)`

[Out] $\frac{1}{8}\arcsin(x)*(-2*(-x^2+1)^{(1/2)}*x^3-3*x*(-x^2+1)^{(1/2)}+3*\arcsin(x))-3/16*\arcsin(x)^2+1/64*(2*x^2+3)^2$

Maxima [A]

time = 0.33, size = 52, normalized size = 0.85

$$\frac{1}{16}x^4 + \frac{3}{16}x^2 - \frac{1}{8}\left(2\sqrt{-x^2+1}x^3 + 3\sqrt{-x^2+1}x - 3\arcsin(x)\right)\arcsin(x) - \frac{3}{16}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="maxima")`

[Out] $\frac{1}{16}x^4 + \frac{3}{16}x^2 - \frac{1}{8}(2*\sqrt{-x^2 + 1}*x^3 + 3*\sqrt{-x^2 + 1}*x - 3*\arcsin(x))*\arcsin(x) - \frac{3}{16}*\arcsin(x)^2$

Fricas [A]

time = 0.32, size = 39, normalized size = 0.64

$$\frac{1}{16}x^4 - \frac{1}{8}(2x^3 + 3x)\sqrt{-x^2+1}\arcsin(x) + \frac{3}{16}x^2 + \frac{3}{16}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="fricas")`

[Out] $\frac{1}{16}x^4 - \frac{1}{8}(2*x^3 + 3*x)*\sqrt{-x^2 + 1}*\arcsin(x) + \frac{3}{16}x^2 + \frac{3}{16}*\arcsin(x)^2$

Sympy [A]

time = 0.27, size = 53, normalized size = 0.87

$$\frac{x^4}{16} - \frac{x^3\sqrt{1-x^2}\arcsin(x)}{4} + \frac{3x^2}{16} - \frac{3x\sqrt{1-x^2}\arcsin(x)}{8} + \frac{3\arcsin^2(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*asin(x)/(-x**2+1)**(1/2),x)`

[Out] $x**4/16 - x**3*\sqrt{1 - x**2}*asin(x)/4 + 3*x**2/16 - 3*x*\sqrt{1 - x**2}*asin(x)/8 + 3*asin(x)**2/16$

Giac [A]

time = 0.01, size = 77, normalized size = 1.26

$$\frac{(-x^2+1)^2}{16} + \frac{3}{16}\arcsin^2 x - \frac{5}{8}x\sqrt{-x^2+1}\arcsin x + \frac{1}{4}x\sqrt{-x^2+1}(-x^2+1)\arcsin x - \frac{5}{16}(-x^2+1) + \frac{1}{8} + \frac{1}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arcsin(x)/(-x^2+1)^(1/2),x)`

[Out] $\frac{1}{4}(-x^2 + 1)^{3/2}x\arcsin(x) - \frac{5}{8}\sqrt{-x^2 + 1}x\arcsin(x) + \frac{1}{16}(x^2 - 1)^2 + \frac{5}{16}x^2 + \frac{3}{16}\arcsin(x)^2 - \frac{23}{128}$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4 \operatorname{asin}(x)}{\sqrt{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4*asin(x))/(1 - x^2)^(1/2),x)`

[Out] `int((x^4*asin(x))/(1 - x^2)^(1/2), x)`

$$3.661 \quad \int \frac{x \sin^{-1}(x)}{(1-x^2)^{3/2}} dx$$

Optimal. Leaf size=19

$$\frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - \tanh^{-1}(x)$$

[Out] -arctanh(x)+arcsin(x)/(-x^2+1)^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4767, 212}

$$\frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x*ArcSin[x])/(1 - x^2)^(3/2),x]

[Out] ArcSin[x]/Sqrt[1 - x^2] - ArcTanh[x]

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 4767

Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)*(x_)*((d_) + (e_)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{x \sin^{-1}(x)}{(1-x^2)^{3/2}} dx &= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - \int \frac{1}{1-x^2} dx \\ &= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - \tanh^{-1}(x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 19, normalized size = 1.00

$$\frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - \tanh^{-1}(x)$$

Antiderivative was successfully verified.

`[In] Integrate[(x*ArcSin[x])/(1 - x^2)^(3/2),x]``[Out] ArcSin[x]/Sqrt[1 - x^2] - ArcTanh[x]`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(ArcSin[x]*x)/(1 - x^2)^(3/2),x]')``[Out] cought exception:`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(17) = 34$.

time = 0.10, size = 46, normalized size = 2.42

method	result	size
default	$-\frac{\sqrt{-x^2+1}}{x^2-1} \arcsin(x) - \ln\left(\frac{1}{\sqrt{-x^2+1}} + \frac{x}{\sqrt{-x^2+1}}\right)$	46

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*arcsin(x)/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)``[Out] -(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)-ln(1/(-x^2+1)^(1/2)+x/(-x^2+1)^(1/2))`**Maxima [A]**

time = 0.32, size = 25, normalized size = 1.32

$$\frac{\arcsin(x)}{\sqrt{-x^2+1}} - \frac{1}{2} \log(x+1) + \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="maxima")``[Out] arcsin(x)/sqrt(-x^2 + 1) - 1/2*log(x + 1) + 1/2*log(x - 1)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(17) = 34.
time = 0.33, size = 44, normalized size = 2.32

$$\frac{(x^2 - 1) \log(x + 1) - (x^2 - 1) \log(x - 1) + 2 \sqrt{-x^2 + 1} \arcsin(x)}{2(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="fricas")

[Out] -1/2*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) + 2*sqrt(-x^2 + 1)*arcsin(x))/(x^2 - 1)

Sympy [A]

time = 4.65, size = 20, normalized size = 1.05

$$-\begin{cases} \operatorname{acoth}(x) & \text{for } x^2 > 1 \\ \operatorname{atanh}(x) & \text{for } x^2 < 1 \end{cases} + \frac{\operatorname{asin}(x)}{\sqrt{1 - x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*asin(x)/(-x**2+1)**(3/2),x)

[Out] -Piecewise((acoth(x), x**2 > 1), (atanh(x), x**2 < 1)) + asin(x)/sqrt(1 - x**2)

Giac [A]

time = 0.00, size = 32, normalized size = 1.68

$$\frac{\ln|x - 1|}{2} - \frac{\ln|x + 1|}{2} + \frac{\arcsin x}{\sqrt{-x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arcsin(x)/(-x^2+1)^(3/2),x)

[Out] arcsin(x)/sqrt(-x^2 + 1) - 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x \operatorname{asin}(x)}{(1 - x^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*asin(x))/(1 - x^2)^(3/2),x)

[Out] int((x*asin(x))/(1 - x^2)^(3/2), x)

$$3.662 \quad \int \frac{x \cos^{-1}(x)}{(1-x^2)^{3/2}} dx$$

Optimal. Leaf size=17

$$\frac{\cos^{-1}(x)}{\sqrt{1-x^2}} + \tanh^{-1}(x)$$

[Out] arctanh(x)+arccos(x)/(-x^2+1)^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4768, 212}

$$\frac{\cos^{-1}(x)}{\sqrt{1-x^2}} + \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x*ArcCos[x])/(1 - x^2)^(3/2), x]

[Out] ArcCos[x]/Sqrt[1 - x^2] + ArcTanh[x]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 4768

Int[((a_.) + ArcCos[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p + 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{x \cos^{-1}(x)}{(1-x^2)^{3/2}} dx &= \frac{\cos^{-1}(x)}{\sqrt{1-x^2}} + \int \frac{1}{1-x^2} dx \\ &= \frac{\cos^{-1}(x)}{\sqrt{1-x^2}} + \tanh^{-1}(x) \end{aligned}$$

Mathematica [A]

time = 0.03, size = 32, normalized size = 1.88

$$\frac{1}{2} \left(\frac{2 \cos^{-1}(x)}{\sqrt{1-x^2}} - \log(1-x) + \log(1+x) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(x*ArcCos[x])/(1 - x^2)^(3/2), x]``[Out] ((2*ArcCos[x])/Sqrt[1 - x^2] - Log[1 - x] + Log[1 + x])/2`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(ArcCos[x]*x)/(1 - x^2)^(3/2), x]')``[Out] cought exception:`**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(15) = 30$.

time = 0.10, size = 47, normalized size = 2.76

method	result	size
default	$-\frac{\sqrt{-x^2+1} \arccos(x)}{x^2-1} - \ln\left(\frac{1}{\sqrt{-x^2+1}} - \frac{x}{\sqrt{-x^2+1}}\right)$	47

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*arccos(x)/(-x^2+1)^(3/2), x, method=_RETURNVERBOSE)``[Out] -(-x^2+1)^(1/2)/(x^2-1)*arccos(x)-ln(1/(-x^2+1)^(1/2)-x/(-x^2+1)^(1/2))`**Maxima [A]**

time = 0.33, size = 25, normalized size = 1.47

$$\frac{\arccos(x)}{\sqrt{-x^2+1}} + \frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*arccos(x)/(-x^2+1)^(3/2), x, algorithm="maxima")``[Out] arccos(x)/sqrt(-x^2 + 1) + 1/2*log(x + 1) - 1/2*log(x - 1)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(15) = 30.

time = 0.32, size = 44, normalized size = 2.59

$$\frac{(x^2 - 1) \log(x + 1) - (x^2 - 1) \log(x - 1) - 2 \sqrt{-x^2 + 1} \arccos(x)}{2(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arccos(x)/(-x^2+1)^(3/2),x, algorithm="fricas")

[Out] 1/2*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*sqrt(-x^2 + 1)*arccos(x))/(x^2 - 1)

Sympy [A]

time = 5.06, size = 20, normalized size = 1.18

$$\begin{cases} \operatorname{acoth}(x) & \text{for } x^2 > 1 \\ \operatorname{atanh}(x) & \text{for } x^2 < 1 \end{cases} + \frac{\arccos(x)}{\sqrt{1-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*acos(x)/(-x**2+1)**(3/2),x)

[Out] Piecewise((acoth(x), x**2 > 1), (atanh(x), x**2 < 1)) + acos(x)/sqrt(1 - x**2)

Giac [A]

time = 0.01, size = 32, normalized size = 1.88

$$-\frac{\ln|x-1|}{2} + \frac{\ln|x+1|}{2} + \frac{\arccos x}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arccos(x)/(-x^2+1)^(3/2),x)

[Out] arccos(x)/sqrt(-x^2 + 1) + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x \arccos(x)}{(1-x^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*acos(x))/(1 - x^2)^(3/2),x)

[Out] int((x*acos(x))/(1 - x^2)^(3/2), x)

$$3.663 \quad \int \frac{\sin^{-1}(x)}{(1-x^2)^{5/2}} dx$$

Optimal. Leaf size=62

$$-\frac{1}{6(1-x^2)} + \frac{x \sin^{-1}(x)}{3(1-x^2)^{3/2}} + \frac{2x \sin^{-1}(x)}{3\sqrt{1-x^2}} + \frac{1}{3} \log(1-x^2)$$

[Out] $-1/6/(-x^2+1)+1/3*x*\arcsin(x)/(-x^2+1)^{(3/2)}+1/3*\ln(-x^2+1)+2/3*x*\arcsin(x)/(-x^2+1)^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4747, 4745, 266, 267}

$$-\frac{1}{6(1-x^2)} + \frac{1}{3} \log(1-x^2) + \frac{2x \sin^{-1}(x)}{3\sqrt{1-x^2}} + \frac{x \sin^{-1}(x)}{3(1-x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[x]/(1 - x^2)^(5/2), x]

[Out] $-1/6*1/(1 - x^2) + (x*\text{ArcSin}[x])/(3*(1 - x^2)^{(3/2)}) + (2*x*\text{ArcSin}[x])/(3*\text{Sqrt}[1 - x^2]) + \text{Log}[1 - x^2]/3$

Rule 266

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 267

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 4745

Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)/((d_) + (e_)*(x_)^2)^(3/2), x_Symbol] :> Simp[x*((a + b*ArcSin[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist[b*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSin[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]

Rule 4747

```

Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] := Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p + 1
))), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sin[c*x])^n, x], x] + Dist[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*
x^2)^p], Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^{-1}(x)}{(1-x^2)^{5/2}} dx &= \frac{x \sin^{-1}(x)}{3(1-x^2)^{3/2}} - \frac{1}{3} \int \frac{x}{(1-x^2)^2} dx + \frac{2}{3} \int \frac{\sin^{-1}(x)}{(1-x^2)^{3/2}} dx \\
&= -\frac{1}{6(1-x^2)} + \frac{x \sin^{-1}(x)}{3(1-x^2)^{3/2}} + \frac{2x \sin^{-1}(x)}{3\sqrt{1-x^2}} - \frac{2}{3} \int \frac{x}{1-x^2} dx \\
&= -\frac{1}{6(1-x^2)} + \frac{x \sin^{-1}(x)}{3(1-x^2)^{3/2}} + \frac{2x \sin^{-1}(x)}{3\sqrt{1-x^2}} + \frac{1}{3} \log(1-x^2)
\end{aligned}$$

Mathematica [A]

time = 0.05, size = 45, normalized size = 0.73

$$\frac{1}{6} \left(\frac{1}{-1+x^2} - \frac{2x(-3+2x^2)\sin^{-1}(x)}{(1-x^2)^{3/2}} + 2 \log(1-x^2) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]/(1 - x^2)^(5/2), x]
```

```
[Out] ((-1 + x^2)^(-1) - (2*x*(-3 + 2*x^2)*ArcSin[x])/(1 - x^2)^(3/2) + 2*Log[1 -
x^2])/6
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[ArcSin[x]/(1 - x^2)^(5/2), x]')
```

```
[Out] cought exception:
```

Maple [A]

time = 0.09, size = 63, normalized size = 1.02

method	result	size
default	$\frac{1}{6x^2-6} + \frac{x \arcsin(x) \sqrt{-x^2+1}}{3(x^2-1)^2} + \frac{\ln(-x^2+1)}{3} - \frac{2\sqrt{-x^2+1} \arcsin(x)x}{3(x^2-1)}$	63

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsin(x)/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{6(x^2-1)} + \frac{1}{3}x \arcsin(x) (-x^2+1)^{(1/2)} / (x^2-1)^2 + \frac{1}{3} \ln(-x^2+1) - \frac{2}{3} (-x^2+1)^{(1/2)} / (x^2-1) \arcsin(x) x$

Maxima [A]

time = 0.33, size = 48, normalized size = 0.77

$$\frac{1}{3} \left(\frac{2x}{\sqrt{-x^2+1}} + \frac{x}{(-x^2+1)^{\frac{3}{2}}} \right) \arcsin(x) + \frac{1}{6(x^2-1)} + \frac{1}{3} \log(-3x^2+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(x)/(-x^2+1)^(5/2),x, algorithm="maxima")`

[Out] $\frac{1}{3} (2x/\sqrt{-x^2+1} + x/(-x^2+1)^{(3/2)}) \arcsin(x) + \frac{1}{6(x^2-1)} + \frac{1}{3} \log(-3x^2+3)$

Fricas [A]

time = 0.32, size = 61, normalized size = 0.98

$$\frac{2(2x^3-3x)\sqrt{-x^2+1} \arcsin(x) - x^2 - 2(x^4-2x^2+1) \log(x^2-1) + 1}{6(x^4-2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(x)/(-x^2+1)^(5/2),x, algorithm="fricas")`

[Out] $\frac{-1}{6} (2(2x^3-3x)\sqrt{-x^2+1} \arcsin(x) - x^2 - 2(x^4-2x^2+1) \log(x^2-1) + 1) / (x^4-2x^2+1)$

Sympy [A]

time = 15.33, size = 78, normalized size = 1.26

$$\left(\left\{ \frac{x^3}{3(1-x^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{1-x^2}} \text{ for } x > -1 \wedge x < 1 \right\} \operatorname{asin}(x) - \begin{cases} \operatorname{NaN} & \text{for } x < -1 \\ -\frac{2x^2 \log(1-x^2)}{6x^2-6} - \frac{x^2}{6x^2-6} + \frac{2 \log(1-x^2)}{6x^2-6} & \text{for } x < 1 \\ \operatorname{NaN} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asin(x)/(-x**2+1)**(5/2),x)`

```
[Out] Piecewise((x**3/(3*(1 - x**2)**(3/2)) + x/sqrt(1 - x**2), (x > -1) & (x < 1
)))*asin(x) - Piecewise((nan, x < -1), (-2*x**2*log(1 - x**2)/(6*x**2 - 6)
- x**2/(6*x**2 - 6) + 2*log(1 - x**2)/(6*x**2 - 6), x < 1), (nan, True))
```

Giac [A]

time = 0.01, size = 61, normalized size = 0.98

$$-\frac{2x^2 - 3}{6(x^2 - 1)} + \frac{\ln|x^2 - 1|}{3} + \frac{2\left(\frac{1}{2} - \frac{1}{3}xx\right)x\sqrt{-x^2 + 1}\arcsin x}{(-x^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(-x^2+1)^(5/2),x)
```

```
[Out] -1/3*(2*x^2 - 3)*sqrt(-x^2 + 1)*x*arcsin(x)/(x^2 - 1)^2 - 1/6*(2*x^2 - 3)/(
x^2 - 1) + 1/3*log(abs(x^2 - 1))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\arcsin(x)}{(1 - x^2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x)/(1 - x^2)^(5/2),x)
```

```
[Out] int(asin(x)/(1 - x^2)^(5/2), x)
```

$$3.664 \quad \int \frac{x^3 \sin^{-1}(x)}{(1-x^2)^{3/2}} dx$$

Optimal. Leaf size=36

$$-x + \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} + \sqrt{1-x^2} \sin^{-1}(x) - \tanh^{-1}(x)$$

[Out] $-x - \operatorname{arctanh}(x) + \arcsin(x) / (-x^2 + 1)^{(1/2)} + \arcsin(x) * (-x^2 + 1)^{(1/2)}$

Rubi [A]

time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {272, 45, 4779, 396, 212}

$$\sqrt{1-x^2} \sin^{-1}(x) + \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - x - \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^3 * \operatorname{ArcSin}[x]) / (1 - x^2)^{(3/2)}, x]$

[Out] $-x + \operatorname{ArcSin}[x] / \operatorname{Sqrt}[1 - x^2] + \operatorname{Sqrt}[1 - x^2] * \operatorname{ArcSin}[x] - \operatorname{ArcTanh}[x]$

Rule 45

$\operatorname{Int}[(a_. + (b_.)(x_.))^{(m_.)} * ((c_.) + (d_.)(x_.))^{(n_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, n\}, x \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IGtQ}[m, 0] \&\& (!\operatorname{IntegerQ}[n] \mid \mid (\operatorname{EqQ}[c, 0] \&\& \operatorname{LeQ}[7*m + 4*n + 4, 0]) \mid \mid \operatorname{LtQ}[9*m + 5*(n + 1), 0] \mid \mid \operatorname{GtQ}[m + n + 2, 0])$

Rule 212

$\operatorname{Int}[(a_. + (b_.)(x_.)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1 / (\operatorname{Rt}[a, 2] * \operatorname{Rt}[-b, 2])) * \operatorname{ArcTanh}[\operatorname{Rt}[-b, 2] * (x / \operatorname{Rt}[a, 2])], x] /;$ $\operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \mid \mid \operatorname{LtQ}[b, 0])$

Rule 272

$\operatorname{Int}[(x_.)^{(m_.)} * ((a_.) + (b_.)(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m + 1)/n] - 1) * (a + b*x)^p}, x], x, x^n], x] /;$ $\operatorname{FreeQ}\{a, b, m, n, p\}, x \&\& \operatorname{IntegerQ}[\operatorname{Simplify}[(m + 1)/n]]$

Rule 396

$\operatorname{Int}[(a_. + (b_.)(x_.)^{(n_.)})^{(p_.)} * ((c_.) + (d_.)(x_.)^{(n_.)}), x_Symbol] \rightarrow \operatorname{Simp}[d*x * ((a + b*x^n)^{(p + 1)} / (b*(n*(p + 1) + 1))), x] - \operatorname{Dist}[(a*d - b*c*(n*(p + 1) + 1)) / (b*(n*(p + 1) + 1)), \operatorname{Int}[(a + b*x^n)^p, x], x] /;$ $\operatorname{FreeQ}\{a, b,$

`c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]`

Rule 4779

`Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_) , x_Symbol] :=> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])`

Rubi steps

$$\begin{aligned} \int \frac{x^3 \sin^{-1}(x)}{(1-x^2)^{3/2}} dx &= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} + \sqrt{1-x^2} \sin^{-1}(x) - \int \frac{2-x^2}{1-x^2} dx \\ &= -x + \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} + \sqrt{1-x^2} \sin^{-1}(x) - \int \frac{1}{1-x^2} dx \\ &= -x + \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} + \sqrt{1-x^2} \sin^{-1}(x) - \tanh^{-1}(x) \end{aligned}$$

Mathematica [A]

time = 0.03, size = 40, normalized size = 1.11

$$\frac{1}{2} \left(-2x - \frac{2(-2+x^2)\sin^{-1}(x)}{\sqrt{1-x^2}} + \log(1-x) - \log(1+x) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(x^3*ArcSin[x])/(1 - x^2)^(3/2),x]`

`[Out] (-2*x - (2*(-2 + x^2)*ArcSin[x])/Sqrt[1 - x^2] + Log[1 - x] - Log[1 + x])/2`

Mathics [A]

time = 9.97, size = 45, normalized size = 1.25

$$\frac{-\text{ArcSin}[x](-2+x^2) + \frac{(-2x+\text{Log}[-1+x]-\text{Log}[1+x])\sqrt{1-x^2}}{2}}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(ArcSin[x]*x^3)/(1 - x^2)^(3/2),x]')`

`[Out] (-ArcSin[x](-2 + x^2) + (-2x + Log[-1 + x] - Log[1 + x]) Sqrt[1 - x^2]) / 2 / Sqrt[1 - x^2]`

Maple [A]

time = 0.42, size = 61, normalized size = 1.69

method	result	size
default	$-x + \arcsin(x) \sqrt{-x^2 + 1} - \frac{\sqrt{-x^2 + 1} \arcsin(x)}{x^2 - 1} - \ln\left(\frac{1}{\sqrt{-x^2 + 1}} + \frac{x}{\sqrt{-x^2 + 1}}\right)$	61

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*arcsin(x)/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)`

[Out]
$$-x + \arcsin(x) \sqrt{-x^2 + 1} - \frac{\sqrt{-x^2 + 1} \arcsin(x)}{x^2 - 1} - \ln\left(\frac{1}{\sqrt{-x^2 + 1}} + \frac{x}{\sqrt{-x^2 + 1}}\right)$$

Maxima [A]

time = 0.33, size = 45, normalized size = 1.25

$$-\left(\frac{x^2}{\sqrt{-x^2 + 1}} - \frac{2}{\sqrt{-x^2 + 1}}\right) \arcsin(x) - x - \frac{1}{2} \log(x + 1) + \frac{1}{2} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="maxima")`

[Out]
$$-(x^2/\sqrt{-x^2 + 1} - 2/\sqrt{-x^2 + 1}) \arcsin(x) - x - 1/2 \log(x + 1) + 1/2 \log(x - 1)$$

Fricas [A]

time = 0.33, size = 57, normalized size = 1.58

$$\frac{2x^3 - 2(x^2 - 2)\sqrt{-x^2 + 1} \arcsin(x) + (x^2 - 1) \log(x + 1) - (x^2 - 1) \log(x - 1) - 2x}{2(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="fricas")`

[Out]
$$-1/2(2x^3 - 2(x^2 - 2)\sqrt{-x^2 + 1} \arcsin(x) + (x^2 - 1) \log(x + 1) - (x^2 - 1) \log(x - 1) - 2x)/(x^2 - 1)$$

Sympy [A]

time = 8.57, size = 37, normalized size = 1.03

$$-x - \left(-\sqrt{1 - x^2} - \frac{1}{\sqrt{1 - x^2}}\right) \arcsin(x) + \frac{\log(x - 1)}{2} - \frac{\log(x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*asin(x)/(-x**2+1)**(3/2),x)`

[Out] $-x - (-\sqrt{1 - x^2} - 1/\sqrt{1 - x^2})*\arcsin(x) + \log(x - 1)/2 - \log(x + 1)/2$

Giac [A]

time = 0.01, size = 45, normalized size = 1.25

$$-x + \frac{\ln|x-1|}{2} - \frac{\ln|x+1|}{2} + \left(\sqrt{-x^2+1} + \frac{1}{\sqrt{-x^2+1}} \right) \arcsin x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arcsin(x)/(-x^2+1)^(3/2),x)`

[Out] $(\sqrt{-x^2 + 1} + 1/\sqrt{-x^2 + 1})*\arcsin(x) - x - 1/2*\log(\text{abs}(x + 1)) + 1/2*\log(\text{abs}(x - 1))$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*asin(x))/(1-x^2)^(3/2),x)`

[Out] `int((x^3*asin(x))/(1-x^2)^(3/2), x)`

$$3.665 \quad \int \frac{\sin^{-1}(x)}{x(1-x^2)^{3/2}} dx$$

Optimal. Leaf size=62

$$\frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - 2 \sin^{-1}(x) \tanh^{-1}\left(e^{i \sin^{-1}(x)}\right) - \tanh^{-1}(x) + i \operatorname{Li}_2\left(-e^{i \sin^{-1}(x)}\right) - i \operatorname{Li}_2\left(e^{i \sin^{-1}(x)}\right)$$

[Out] $-2*\arcsin(x)*\operatorname{arctanh}(I*x+(-x^2+1)^{(1/2)})-\operatorname{arctanh}(x)+I*\operatorname{polylog}(2,-I*x-(-x^2+1)^{(1/2)})-I*\operatorname{polylog}(2,I*x+(-x^2+1)^{(1/2)})+\arcsin(x)/(-x^2+1)^{(1/2)}$

Rubi [A]

time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$,

Rules used = {4793, 4803, 4268, 2317, 2438, 212}

$$i \operatorname{Li}_2\left(-e^{i \sin^{-1}(x)}\right) - i \operatorname{Li}_2\left(e^{i \sin^{-1}(x)}\right) + \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - \tanh^{-1}(x) - 2 \sin^{-1}(x) \tanh^{-1}\left(e^{i \sin^{-1}(x)}\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcSin}[x]/(x*(1-x^2)^{(3/2)}), x]$

[Out] $\operatorname{ArcSin}[x]/\operatorname{Sqrt}[1-x^2] - 2*\operatorname{ArcSin}[x]*\operatorname{ArcTanh}[E^{(I*\operatorname{ArcSin}[x])}] - \operatorname{ArcTanh}[x] + I*\operatorname{PolyLog}[2, -E^{(I*\operatorname{ArcSin}[x])}] - I*\operatorname{PolyLog}[2, E^{(I*\operatorname{ArcSin}[x])}]$

Rule 212

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \operatorname{LtQ}[b, 0])$

Rule 2317

$\operatorname{Int}[\operatorname{Log}[a_+ + (b_+)*((F_+)^{((e_+)*((c_+)+(d_+)*(x_+)))})^{(n_+)}], x_Symbol] \rightarrow \operatorname{Dist}[1/(d_+*e_+*n_+*\operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a_+ + b_+*x]/x, x], x, (F_+^{(e_+*(c_+ + d_+*x))})^{n_+}], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x \ \&\& \operatorname{GtQ}[a, 0]$

Rule 2438

$\operatorname{Int}[\operatorname{Log}[(c_+)*((d_+)+(e_+)*(x_+)^{(n_+)})]/(x_+), x_Symbol] \rightarrow \operatorname{Simp}[-\operatorname{PolyLog}[2, (-c_+)*e_+*x_+^n/n, x] /; \operatorname{FreeQ}\{c, d, e, n\}, x \ \&\& \operatorname{EqQ}[c*d, 1]$

Rule 4268

$\operatorname{Int}[\operatorname{csc}[(e_+)+(f_+)*(x_+)]*((c_+)+(d_+)*(x_+))^{(m_+)}, x_Symbol] \rightarrow \operatorname{Simp}[-2*(c_+ + d_+*x_+)^m*(\operatorname{ArcTanh}[E^{(I*(e_+ + f_+*x_+))}]/f), x] + (-\operatorname{Dist}[d_*(m/f), \operatorname{Int}[(c_+ + d_+*x_+)^{(m-1)}*\operatorname{Log}[1 - E^{(I*(e_+ + f_+*x_+))}], x], x] + \operatorname{Dist}[d_*(m/f), \operatorname{Int}[(c_+ + d_+*x_+)^m$

$(m - 1) \cdot \text{Log}[1 + E^{(I \cdot (e + f \cdot x))}]$, x , x) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 4793

Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1)), Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Dist[b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])

Rule 4803

Int[(((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{\sin^{-1}(x)}{x(1-x^2)^{3/2}} dx &= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - \int \frac{1}{1-x^2} dx + \int \frac{\sin^{-1}(x)}{x\sqrt{1-x^2}} dx \\ &= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - \tanh^{-1}(x) + \text{Subst} \left(\int x \csc(x) dx, x, \sin^{-1}(x) \right) \\ &= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - 2 \sin^{-1}(x) \tanh^{-1} \left(e^{i \sin^{-1}(x)} \right) - \tanh^{-1}(x) - \text{Subst} \left(\int \log(1 - e^{ix}) dx, x \right) \\ &= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - 2 \sin^{-1}(x) \tanh^{-1} \left(e^{i \sin^{-1}(x)} \right) - \tanh^{-1}(x) + i \text{Subst} \left(\int \frac{\log(1-x)}{x} dx, x \right) \\ &= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - 2 \sin^{-1}(x) \tanh^{-1} \left(e^{i \sin^{-1}(x)} \right) - \tanh^{-1}(x) + i \text{Li}_2 \left(-e^{i \sin^{-1}(x)} \right) - i \text{Li}_2 \left(e^{i \sin^{-1}(x)} \right) \end{aligned}$$

Mathematica [A]

time = 0.12, size = 112, normalized size = 1.81

$$\frac{\sin^{-1}(x)}{\sqrt{1-x^2}} + \sin^{-1}(x) \log(1 - e^{i \sin^{-1}(x)}) - \sin^{-1}(x) \log(1 + e^{i \sin^{-1}(x)}) + \log \left(\cos \left(\frac{1}{2} \sin^{-1}(x) \right) - \sin \left(\frac{1}{2} \sin^{-1}(x) \right) \right) - \log \left(\cos \left(\frac{1}{2} \sin^{-1}(x) \right) + \sin \left(\frac{1}{2} \sin^{-1}(x) \right) \right) + i \text{Li}_2 \left(-e^{i \sin^{-1}(x)} \right) - i \text{Li}_2 \left(e^{i \sin^{-1}(x)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[x]/(x*(1 - x^2)^(3/2)), x]

```
[Out] ArcSin[x]/Sqrt[1 - x^2] + ArcSin[x]*Log[1 - E^(I*ArcSin[x])] - ArcSin[x]*Log[1 + E^(I*ArcSin[x])] + Log[Cos[ArcSin[x]/2] - Sin[ArcSin[x]/2]] - Log[Cos[ArcSin[x]/2] + Sin[ArcSin[x]/2]] + I*PolyLog[2, -E^(I*ArcSin[x])] - I*PolyLog[2, E^(I*ArcSin[x])]
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[ArcSin[x]/(x*(1 - x^2)^(3/2)),x]')
```

[Out] Timed out

Maple [A]

time = 0.43, size = 97, normalized size = 1.56

method	result
default	$-\frac{\sqrt{-x^2+1} \arcsin(x)}{x^2-1} + 2i \arctan(ix + \sqrt{-x^2+1}) + i \operatorname{dilog}(ix + \sqrt{-x^2+1} + 1) - \arcsin(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)/x/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)+2*I*arctan(I*x+(-x^2+1)^(1/2))+I*dilog(I*x+(-x^2+1)^(1/2)+1)-arcsin(x)*ln(I*x+(-x^2+1)^(1/2)+1)+I*dilog(I*x+(-x^2+1)^(1/2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x/(-x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsin(x)/((-x^2 + 1)^(3/2)*x), x)
```

Fricas [F]

time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x/(-x^2+1)^(3/2),x, algorithm="fricas")
```

[Out] integral(sqrt(-x^2 + 1)*arcsin(x)/(x^5 - 2*x^3 + x), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}(x)}{x(-x-1)(x+1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(x)/x/(-x**2+1)**(3/2),x)

[Out] Integral(asin(x)/(x*(-x-1)*(x+1)**(3/2)), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(x)/x/(-x^2+1)^(3/2),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asin}(x)}{x(1-x^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(x)/(x*(1-x^2)^(3/2)),x)

[Out] int(asin(x)/(x*(1-x^2)^(3/2)), x)

$$3.666 \quad \int \frac{\cos^{-1}(x)}{x^4 \sqrt{1-x^2}} dx$$

Optimal. Leaf size=54

$$\frac{1}{6x^2} - \frac{\sqrt{1-x^2} \cos^{-1}(x)}{3x^3} - \frac{2\sqrt{1-x^2} \cos^{-1}(x)}{3x} - \frac{2 \log(x)}{3}$$

[Out] 1/6/x^2-2/3*ln(x)-1/3*arccos(x)*(-x^2+1)^(1/2)/x^3-2/3*arccos(x)*(-x^2+1)^(1/2)/x

Rubi [A]

time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$,

Rules used = {4790, 4772, 29, 30}

$$\frac{1}{6x^2} - \frac{2\sqrt{1-x^2} \cos^{-1}(x)}{3x} - \frac{\sqrt{1-x^2} \cos^{-1}(x)}{3x^3} - \frac{2 \log(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[x]/(x^4*Sqrt[1-x^2]),x]

[Out] 1/(6*x^2) - (Sqrt[1-x^2]*ArcCos[x])/(3*x^3) - (2*Sqrt[1-x^2]*ArcCos[x])/(3*x) - (2*Log[x])/3

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 4772

Int[((a_) + ArcCos[(c_)*(x_)])*(b_)^(n_)*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m+1)*(d+e*x^2)^(p+1)*((a+b*ArcCos[c*x])^n/(d*f*(m+1))), x] + Dist[b*c*(n/(f*(m+1)))*Simp[(d+e*x^2)^p/(1-c^2*x^2)^p], Int[(f*x)^(m+1)*(1-c^2*x^2)^(p+1/2)*(a+b*ArcCos[c*x])^(n-1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2*d+e, 0] && GtQ[n, 0] && EqQ[m+2*p+3, 0] && NeQ[m, -1]

Rule 4790

Int[((a_) + ArcCos[(c_)*(x_)])*(b_)^(n_)*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m+1)*(d+e*x^2)^(p+1)*((a+b

```
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[b*c
*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1
- c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^{-1}(x)}{x^4\sqrt{1-x^2}} dx &= -\frac{\sqrt{1-x^2} \cos^{-1}(x)}{3x^3} - \frac{1}{3} \int \frac{1}{x^3} dx + \frac{2}{3} \int \frac{\cos^{-1}(x)}{x^2\sqrt{1-x^2}} dx \\ &= \frac{1}{6x^2} - \frac{\sqrt{1-x^2} \cos^{-1}(x)}{3x^3} - \frac{2\sqrt{1-x^2} \cos^{-1}(x)}{3x} - \frac{2}{3} \int \frac{1}{x} dx \\ &= \frac{1}{6x^2} - \frac{\sqrt{1-x^2} \cos^{-1}(x)}{3x^3} - \frac{2\sqrt{1-x^2} \cos^{-1}(x)}{3x} - \frac{2 \log(x)}{3} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 38, normalized size = 0.70

$$\frac{x - 2\sqrt{1-x^2} (1 + 2x^2) \cos^{-1}(x) - 4x^3 \log(x)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[x]/(x^4*Sqrt[1 - x^2]),x]
```

```
[Out] (x - 2*Sqrt[1 - x^2]*(1 + 2*x^2)*ArcCos[x] - 4*x^3*Log[x])/(6*x^3)
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[ArcCos[x]/(x^4*Sqrt[1 - x^2]),x]')
```

```
[Out] cought exception:
```

Maple [A]

time = 0.11, size = 43, normalized size = 0.80

method	result	size
default	$\frac{1}{6x^2} - \frac{2\ln(x)}{3} - \frac{\arccos(x)\sqrt{-x^2+1}}{3x^3} - \frac{2\arccos(x)\sqrt{-x^2+1}}{3x}$	43

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccos(x)/x^4/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $1/6/x^2 - 2/3*\ln(x) - 1/3*\arccos(x)*(-x^2+1)^(1/2)/x^3 - 2/3*\arccos(x)*(-x^2+1)^(1/2)/x$

Maxima [A]

time = 0.33, size = 42, normalized size = 0.78

$$-\frac{1}{3} \left(\frac{2\sqrt{-x^2+1}}{x} + \frac{\sqrt{-x^2+1}}{x^3} \right) \arccos(x) + \frac{1}{6x^2} - \frac{2}{3} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccos(x)/x^4/(-x^2+1)^(1/2),x, algorithm="maxima")`

[Out] $-1/3*(2*\sqrt{-x^2+1}/x + \sqrt{-x^2+1}/x^3)*\arccos(x) + 1/6/x^2 - 2/3*\log(x)$

Fricas [A]

time = 0.32, size = 36, normalized size = 0.67

$$-\frac{4x^3 \log(x) + 2(2x^2 + 1)\sqrt{-x^2 + 1} \arccos(x) - x}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccos(x)/x^4/(-x^2+1)^(1/2),x, algorithm="fricas")`

[Out] $-1/6*(4*x^3*\log(x) + 2*(2*x^2 + 1)*\sqrt{-x^2 + 1}*\arccos(x) - x)/x^3$

Sympy [A]

time = 7.02, size = 49, normalized size = 0.91

$$\left(\begin{cases} -\frac{\sqrt{1-x^2}}{x} - \frac{(1-x^2)^{\frac{3}{2}}}{3x^3} & \text{for } x > -1 \wedge x < 1 \end{cases} \right) \arccos(x) + \begin{cases} \text{NaN} & \text{for } x < -1 \\ -\frac{2\log(x)}{3} + \frac{1}{6x^2} & \text{for } x < 1 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acos(x)/x**4/(-x**2+1)**(1/2),x)`

[Out] `Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x < 1))*acos(x) + Piecewise((nan, x < -1), (-2*log(x)/3 + 1/(6*x**2), x < 1), (nan, True))`

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(42) = 84.
time = 0.01, size = 134, normalized size = 2.48

$$-\frac{-2x^2 - 1}{6x^2} - \frac{\ln(x^2)}{3} + \left(\frac{-\frac{64}{3} \left(\frac{-2\sqrt{-x^2 + 1} + 2}{2x} \right)^3 + \frac{96(-2\sqrt{-x^2 + 1} + 2)}{x}}{512} + \frac{9 \left(\frac{-2\sqrt{-x^2 + 1} + 2}{2x} \right)^2 + 1}{24 \left(\frac{-2\sqrt{-x^2 + 1} + 2}{2x} \right)^3} \right) \arccos x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(x)/x^4/(-x^2+1)^(1/2),x)

[Out] 1/24*(x^3*(9*(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1)^3 - 9*(sqrt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^3/x^3)*arccos(x) + 1/6*(2*x^2 + 1)/x^2 - 1/3*log(x^2)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\arccos(x)}{x^4 \sqrt{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acos(x)/(x^4*(1 - x^2)^(1/2)),x)

[Out] int(acos(x)/(x^4*(1 - x^2)^(1/2)), x)

3.667 $\int x \sqrt{1-x^2} \cos^{-1}(x)^2 dx$

Optimal. Leaf size=66

$$\frac{4\sqrt{1-x^2}}{9} + \frac{2}{27}(1-x^2)^{3/2} - \frac{2}{3}x \cos^{-1}(x) + \frac{2}{9}x^3 \cos^{-1}(x) - \frac{1}{3}(1-x^2)^{3/2} \cos^{-1}(x)^2$$

[Out] $2/27*(-x^2+1)^{(3/2)}-2/3*x*\arccos(x)+2/9*x^3*\arccos(x)-1/3*(-x^2+1)^{(3/2)}*\arccos(x)^2+4/9*(-x^2+1)^{(1/2)}$

Rubi [A]

time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {4768, 4740, 455, 45}

$$\frac{2}{9}x^3 \cos^{-1}(x) + \frac{2}{27}(1-x^2)^{3/2} + \frac{4\sqrt{1-x^2}}{9} - \frac{1}{3}(1-x^2)^{3/2} \cos^{-1}(x)^2 - \frac{2}{3}x \cos^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Sqrt}[1-x^2]*\text{ArcCos}[x]^2, x]$

[Out] $(4*\text{Sqrt}[1-x^2])/9 + (2*(1-x^2)^{(3/2)})/27 - (2*x*\text{ArcCos}[x])/3 + (2*x^3*\text{ArcCos}[x])/9 - ((1-x^2)^{(3/2)}*\text{ArcCos}[x]^2)/3$

Rule 45

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0]) \ || \ \text{GtQ}[m + n + 2, 0])$

Rule 455

$\text{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[m - n + 1, 0]$

Rule 4740

$\text{Int}[(a_.) + \text{ArcCos}[(c_.)*(x_.)]*(b_.)]*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{With}\{u = \text{IntHide}[(d + e*x^2)^p, x]\}, \text{Dist}[a + b*\text{ArcCos}[c*x], u, x] + \text{Dist}[b*c, \text{Int}[\text{SimplifyIntegrand}[u/\text{Sqrt}[1 - c^2*x^2], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[p, 0]$

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p + 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int x\sqrt{1-x^2} \cos^{-1}(x)^2 dx &= -\frac{1}{3} (1-x^2)^{3/2} \cos^{-1}(x)^2 - \frac{2}{3} \int (1-x^2) \cos^{-1}(x) dx \\ &= -\frac{2}{3} x \cos^{-1}(x) + \frac{2}{9} x^3 \cos^{-1}(x) - \frac{1}{3} (1-x^2)^{3/2} \cos^{-1}(x)^2 - \frac{2}{3} \int \frac{x \left(1 - \frac{x^2}{3}\right)}{\sqrt{1-x^2}} dx \\ &= -\frac{2}{3} x \cos^{-1}(x) + \frac{2}{9} x^3 \cos^{-1}(x) - \frac{1}{3} (1-x^2)^{3/2} \cos^{-1}(x)^2 - \frac{1}{3} \text{Subst} \left(\int \frac{1 - \frac{x}{3}}{\sqrt{1-x}} dx \right) \\ &= -\frac{2}{3} x \cos^{-1}(x) + \frac{2}{9} x^3 \cos^{-1}(x) - \frac{1}{3} (1-x^2)^{3/2} \cos^{-1}(x)^2 - \frac{1}{3} \text{Subst} \left(\int \left(\frac{2}{3\sqrt{1-x}} - \frac{1}{3} \right) dx \right) \\ &= \frac{4\sqrt{1-x^2}}{9} + \frac{2}{27} (1-x^2)^{3/2} - \frac{2}{3} x \cos^{-1}(x) + \frac{2}{9} x^3 \cos^{-1}(x) - \frac{1}{3} (1-x^2)^{3/2} \cos^{-1}(x)^2 \end{aligned}$$

Mathematica [A]

time = 0.03, size = 50, normalized size = 0.76

$$\frac{1}{27} \left(-2\sqrt{1-x^2} (-7+x^2) + 6x(-3+x^2) \cos^{-1}(x) - 9(1-x^2)^{3/2} \cos^{-1}(x)^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[1 - x^2]*ArcCos[x]^2,x]

[Out] (-2*Sqrt[1 - x^2]*(-7 + x^2) + 6*x*(-3 + x^2)*ArcCos[x] - 9*(1 - x^2)^(3/2)*ArcCos[x]^2)/27

Mathics [A]

time = 2.29, size = 71, normalized size = 1.08

$$\frac{-2x \text{ArcCos}[x]}{3} - \frac{2x^2 \sqrt{1-x^2}}{27} + \frac{x^2 \text{ArcCos}[x]^2 \sqrt{1-x^2}}{3} + \frac{2x^3 \text{ArcCos}[x]}{9} - \frac{\text{ArcCos}[x]^2 \sqrt{1-x^2}}{3} + \frac{14\sqrt{1-x^2}}{27}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[ArcCos[x]^2*x*Sqrt[1 - x^2],x]')

[Out] -2 x ArcCos[x] / 3 - 2 x ^ 2 Sqrt[1 - x ^ 2] / 27 + x ^ 2 ArcCos[x] ^ 2 Sqrt[1 - x ^ 2] / 3 + 2 x ^ 3 ArcCos[x] / 9 - ArcCos[x] ^ 2 Sqrt[1 - x ^ 2] / 3 + 14 Sqrt[1 - x ^ 2] / 27

Maple [C] Result contains complex when optimal does not.

time = 0.19, size = 158, normalized size = 2.39

method	result
default	$-\frac{(6i \arccos(x) + 9 \arccos(x)^2 - 2) \left(4ix^3 - 4x^2 \sqrt{-x^2 + 1} - 3ix + \sqrt{-x^2 + 1} \right)}{216} + \frac{(\arccos(x)^2 - 2 + 2i \arccos(x)) \left(ix - \sqrt{-x^2 + 1} \right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arccos(x)^2*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$-1/216*(6*I*\arccos(x)+9*\arccos(x)^2-2)*(4*I*x^3-4*x^2*(-x^2+1)^(1/2)-3*I*x+(-x^2+1)^(1/2))+1/8*(\arccos(x)^2-2+2*I*\arccos(x))*(I*x-(-x^2+1)^(1/2))-1/8*(\arccos(x)^2-2-2*I*\arccos(x))*(I*x+(-x^2+1)^(1/2))+1/216*(-6*I*\arccos(x)+9*\arccos(x)^2-2)*(4*I*x^3+4*x^2*(-x^2+1)^(1/2)-3*I*x-(-x^2+1)^(1/2))$$

Maxima [A]

time = 0.33, size = 52, normalized size = 0.79

$$-\frac{1}{3}(-x^2+1)^{\frac{3}{2}}\arccos(x)^2 - \frac{2}{27}\sqrt{-x^2+1}x^2 + \frac{2}{9}(x^3-3x)\arccos(x) + \frac{14}{27}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccos(x)^2*(-x^2+1)^(1/2),x, algorithm="maxima")`

[Out]
$$-1/3*(-x^2+1)^{(3/2)}*\arccos(x)^2 - 2/27*\sqrt{-x^2+1}*x^2 + 2/9*(x^3-3*x)*\arccos(x) + 14/27*\sqrt{-x^2+1}$$

Fricas [A]

time = 0.32, size = 41, normalized size = 0.62

$$\frac{2}{9}(x^3-3x)\arccos(x) + \frac{1}{27}(9(x^2-1)\arccos(x)^2 - 2x^2 + 14)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccos(x)^2*(-x^2+1)^(1/2),x, algorithm="fricas")`

[Out]
$$2/9*(x^3-3*x)*\arccos(x) + 1/27*(9*(x^2-1)*\arccos(x)^2 - 2*x^2 + 14)*\sqrt{-x^2+1}$$

Sympy [A]

time = 0.39, size = 78, normalized size = 1.18

$$\frac{2x^3 \operatorname{acos}(x)}{9} + \frac{x^2 \sqrt{1-x^2} \operatorname{acos}^2(x)}{3} - \frac{2x^2 \sqrt{1-x^2}}{27} - \frac{2x \operatorname{acos}(x)}{3} - \frac{\sqrt{1-x^2} \operatorname{acos}^2(x)}{3} + \frac{14\sqrt{1-x^2}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*acos(x)**2*(-x**2+1)**(1/2),x)

[Out] $2*x**3*acos(x)/9 + x**2*sqrt(1 - x**2)*acos(x)**2/3 - 2*x**2*sqrt(1 - x**2)/27 - 2*x*acos(x)/3 - sqrt(1 - x**2)*acos(x)**2/3 + 14*sqrt(1 - x**2)/27$

Giac [A]

time = 0.01, size = 74, normalized size = 1.12

$$\frac{14}{27}\sqrt{-x^2+1} - \frac{2}{3}x \arccos x - \frac{2}{27}x^2\sqrt{-x^2+1} + \frac{2}{9}x^3 \arccos x - \frac{1}{3}\sqrt{-x^2+1} (-x^2+1) \arccos^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arccos(x)^2*(-x^2+1)^(1/2),x)

[Out] $2/9*x^3*\arccos(x) - 1/3*(-x^2 + 1)^(3/2)*\arccos(x)^2 - 2/27*sqrt(-x^2 + 1)*x^2 - 2/3*x*\arccos(x) + 14/27*sqrt(-x^2 + 1)$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int x \operatorname{acos}(x)^2 \sqrt{1-x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*acos(x)^2*(1-x^2)^(1/2),x)

[Out] int(x*acos(x)^2*(1-x^2)^(1/2), x)

$$3.668 \quad \int \frac{x^2 \sin^{-1}(x)^3}{\sqrt{1-x^2}} dx$$

Optimal. Leaf size=73

$$-\frac{3x^2}{8} + \frac{3}{4}x\sqrt{1-x^2} \sin^{-1}(x) - \frac{3}{8} \sin^{-1}(x)^2 + \frac{3}{4}x^2 \sin^{-1}(x)^2 - \frac{1}{2}x\sqrt{1-x^2} \sin^{-1}(x)^3 + \frac{1}{8} \sin^{-1}(x)^4$$

[Out] $-3/8*x^2-3/8*\arcsin(x)^2+3/4*x^2*\arcsin(x)^2+1/8*\arcsin(x)^4+3/4*x*\arcsin(x)*(-x^2+1)^{(1/2)}-1/2*x*\arcsin(x)^3*(-x^2+1)^{(1/2)}$

Rubi [A]

time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {4795, 4737, 4723, 30}

$$-\frac{3x^2}{8} - \frac{1}{2}x\sqrt{1-x^2} \sin^{-1}(x)^3 + \frac{3}{4}x^2 \sin^{-1}(x)^2 + \frac{3}{4}x\sqrt{1-x^2} \sin^{-1}(x) + \frac{1}{8} \sin^{-1}(x)^4 - \frac{3}{8} \sin^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] Int[(x^2*ArcSin[x]^3)/Sqrt[1 - x^2],x]

[Out] $(-3*x^2)/8 + (3*x*\text{Sqrt}[1 - x^2]*\text{ArcSin}[x])/4 - (3*\text{ArcSin}[x]^2)/8 + (3*x^2*\text{ArcSin}[x]^2)/4 - (x*\text{Sqrt}[1 - x^2]*\text{ArcSin}[x]^3)/2 + \text{ArcSin}[x]^4/8$

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 4723

Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)*((d_)*(x_))^(m_), x_Symbol] := Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 4737

Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)/Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]

Rule 4795

Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +

$b \cdot \text{ArcSin}[c \cdot x]^n / (e \cdot (m + 2 \cdot p + 1))$, x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2 \sin^{-1}(x)^3}{\sqrt{1-x^2}} dx &= -\frac{1}{2} x \sqrt{1-x^2} \sin^{-1}(x)^3 + \frac{1}{2} \int \frac{\sin^{-1}(x)^3}{\sqrt{1-x^2}} dx + \frac{3}{2} \int x \sin^{-1}(x)^2 dx \\ &= \frac{3}{4} x^2 \sin^{-1}(x)^2 - \frac{1}{2} x \sqrt{1-x^2} \sin^{-1}(x)^3 + \frac{1}{8} \sin^{-1}(x)^4 - \frac{3}{2} \int \frac{x^2 \sin^{-1}(x)}{\sqrt{1-x^2}} dx \\ &= \frac{3}{4} x \sqrt{1-x^2} \sin^{-1}(x) + \frac{3}{4} x^2 \sin^{-1}(x)^2 - \frac{1}{2} x \sqrt{1-x^2} \sin^{-1}(x)^3 + \frac{1}{8} \sin^{-1}(x)^4 - \frac{3}{4} \int x dx \\ &= -\frac{3x^2}{8} + \frac{3}{4} x \sqrt{1-x^2} \sin^{-1}(x) - \frac{3}{8} \sin^{-1}(x)^2 + \frac{3}{4} x^2 \sin^{-1}(x)^2 - \frac{1}{2} x \sqrt{1-x^2} \sin^{-1}(x)^3 + \frac{1}{8} \sin^{-1}(x)^4 \end{aligned}$$

Mathematica [A]

time = 0.01, size = 60, normalized size = 0.82

$$\frac{1}{8} \left(-3x^2 + 6x\sqrt{1-x^2} \sin^{-1}(x) + (-3 + 6x^2) \sin^{-1}(x)^2 - 4x\sqrt{1-x^2} \sin^{-1}(x)^3 + \sin^{-1}(x)^4 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*ArcSin[x]^3)/Sqrt[1 - x^2], x]

[Out] (-3*x^2 + 6*x*Sqrt[1 - x^2]*ArcSin[x] + (-3 + 6*x^2)*ArcSin[x]^2 - 4*x*Sqrt[1 - x^2]*ArcSin[x]^3 + ArcSin[x]^4)/8

Mathics [A]

time = 2.05, size = 57, normalized size = 0.78

$$-\frac{x \text{ArcSin}[x]^3 \sqrt{1-x^2}}{2} + \frac{3x \text{ArcSin}[x] \sqrt{1-x^2}}{4} - \frac{3x^2}{8} + \frac{3x^2 \text{ArcSin}[x]^2}{4} - \frac{3 \text{ArcSin}[x]^2}{8} + \frac{\text{ArcSin}[x]^4}{8}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(ArcSin[x]^3*x^2)/Sqrt[1 - x^2], x]')

[Out] -x ArcSin[x]^3 Sqrt[1 - x^2] / 2 + 3 x ArcSin[x] Sqrt[1 - x^2] / 4 - 3 x^2 / 8 + 3 x^2 ArcSin[x]^2 / 4 - 3 ArcSin[x]^2 / 8 + ArcSin[x]^4 / 8

Maple [A]

time = 0.12, size = 69, normalized size = 0.95

method	result
default	$\frac{\arcsin(x)^3 \left(-x\sqrt{-x^2+1} + \arcsin(x) \right)}{2} + \frac{3\arcsin(x)^2(x^2-1)}{4} + \frac{3\arcsin(x) \left(x\sqrt{-x^2+1} + \arcsin(x) \right)}{4} - \frac{3\arcsin(x)^2}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsin(x)^3*(-x*(-x^2+1)^(1/2)+arcsin(x))+3/4*arcsin(x)^2*(x^2-1)+3/4*arcsin(x)*(x*(-x^2+1)^(1/2)+arcsin(x))-3/8*arcsin(x)^2-3/8*x^2-3/8*arcsin(x)^4
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsin(x)^3/sqrt(-x^2 + 1), x)
```

Fricas [A]

time = 0.32, size = 49, normalized size = 0.67

$$\frac{1}{8} \arcsin(x)^4 + \frac{3}{8} (2x^2 - 1) \arcsin(x)^2 - \frac{3}{8} x^2 - \frac{1}{4} (2x \arcsin(x)^3 - 3x \arcsin(x)) \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*arcsin(x)^4 + 3/8*(2*x^2 - 1)*arcsin(x)^2 - 3/8*x^2 - 1/4*(2*x*arcsin(x)^3 - 3*x*arcsin(x))*sqrt(-x^2 + 1)
```

Sympy [A]

time = 0.26, size = 66, normalized size = 0.90

$$\frac{3x^2 \operatorname{asin}^2(x)}{4} - \frac{3x^2}{8} - \frac{x\sqrt{1-x^2} \operatorname{asin}^3(x)}{2} + \frac{3x\sqrt{1-x^2} \operatorname{asin}(x)}{4} + \frac{\operatorname{asin}^4(x)}{8} - \frac{3 \operatorname{asin}^2(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asin(x)**3/(-x**2+1)**(1/2),x)
```

```
[Out] 3*x**2*asin(x)**2/4 - 3*x**2/8 - x*sqrt(1 - x**2)*asin(x)**3/2 + 3*x*sqrt(1 - x**2)*asin(x)/4 + asin(x)**4/8 - 3*asin(x)**2/8
```

Giac [A]

time = 0.01, size = 80, normalized size = 1.10

$$-\frac{1}{2}x\sqrt{-x^2+1} \arcsin^3 x + \frac{\arcsin^4 x}{8} - \frac{3}{4}(-x^2+1) \arcsin^2 x + \frac{3}{8} \arcsin^2 x + \frac{3}{4}x\sqrt{-x^2+1} \arcsin x + \frac{3}{8}(-x^2+1) - \frac{3}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x)**[Out]** -1/2*sqrt(-x^2 + 1)*x*arcsin(x)^3 + 1/8*arcsin(x)^4 + 3/4*(x^2 - 1)*arcsin(x)^2 + 3/4*sqrt(-x^2 + 1)*x*arcsin(x) - 3/8*x^2 + 3/8*arcsin(x)^2 + 3/16**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \operatorname{asin}(x)^3}{\sqrt{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*asin(x)^3)/(1 - x^2)^(1/2),x)**[Out]** int((x^2*asin(x)^3)/(1 - x^2)^(1/2), x)

$$3.669 \quad \int \frac{x \tan^{-1}(x)}{(1+x^2)^2} dx$$

Optimal. Leaf size=32

$$\frac{x}{4(1+x^2)} + \frac{1}{4} \tan^{-1}(x) - \frac{\tan^{-1}(x)}{2(1+x^2)}$$

[Out] 1/4*x/(x^2+1)+1/4*arctan(x)-1/2*arctan(x)/(x^2+1)

Rubi [A]

time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5050, 205, 209}

$$\frac{x}{4(x^2+1)} - \frac{\tan^{-1}(x)}{2(x^2+1)} + \frac{1}{4} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x*ArcTan[x])/(1+x^2)^2,x]

[Out] x/(4*(1+x^2)) + ArcTan[x]/4 - ArcTan[x]/(2*(1+x^2))

Rule 205

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 5050

Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbol] := Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q + 1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && NeQ[q, -1]

Rubi steps

$$\begin{aligned} \int \frac{x \tan^{-1}(x)}{(1+x^2)^2} dx &= -\frac{\tan^{-1}(x)}{2(1+x^2)} + \frac{1}{2} \int \frac{1}{(1+x^2)^2} dx \\ &= \frac{x}{4(1+x^2)} - \frac{\tan^{-1}(x)}{2(1+x^2)} + \frac{1}{4} \int \frac{1}{1+x^2} dx \\ &= \frac{x}{4(1+x^2)} + \frac{1}{4} \tan^{-1}(x) - \frac{\tan^{-1}(x)}{2(1+x^2)} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 21, normalized size = 0.66

$$\frac{x + (-1 + x^2) \tan^{-1}(x)}{4(1 + x^2)}$$

Antiderivative was successfully verified.

`[In] Integrate[(x*ArcTan[x])/(1 + x^2)^2,x]``[Out] (x + (-1 + x^2)*ArcTan[x])/(4*(1 + x^2))`**Mathics [A]**

time = 1.90, size = 21, normalized size = 0.66

$$\frac{x + x^2 \text{ArcTan}[x] - \text{ArcTan}[x]}{4 + 4x^2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(ArcTan[x]*x)/(1 + x^2)^2,x]')``[Out] (x + x ^ 2 ArcTan[x] - ArcTan[x]) / (4 (1 + x ^ 2))`**Maple [A]**

time = 0.09, size = 27, normalized size = 0.84

method	result	size
default	$\frac{x}{4x^2+4} + \frac{\arctan(x)}{4} - \frac{\arctan(x)}{2(x^2+1)}$	27
risch	$\frac{i \ln(ix+1)}{4x^2+4} - \frac{i(2 \ln(-ix+1)+x^2 \ln(x-i)+\ln(x-i)-\ln(x+i)x^2-\ln(x+i)+2ix)}{8(x-i)(x+i)}$	79

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)``[Out] 1/4*x/(x^2+1)+1/4*arctan(x)-1/2*arctan(x)/(x^2+1)`

Maxima [A]

time = 0.34, size = 26, normalized size = 0.81

$$\frac{x}{4(x^2 + 1)} - \frac{\arctan(x)}{2(x^2 + 1)} + \frac{1}{4} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*arctan(x)/(x^2+1)^2,x, algorithm="maxima")``[Out] 1/4*x/(x^2 + 1) - 1/2*arctan(x)/(x^2 + 1) + 1/4*arctan(x)`**Fricas [A]**

time = 0.31, size = 19, normalized size = 0.59

$$\frac{(x^2 - 1) \arctan(x) + x}{4(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*arctan(x)/(x^2+1)^2,x, algorithm="fricas")``[Out] 1/4*((x^2 - 1)*arctan(x) + x)/(x^2 + 1)`**Sympy [A]**

time = 0.25, size = 31, normalized size = 0.97

$$\frac{x^2 \operatorname{atan}(x)}{4x^2 + 4} + \frac{x}{4x^2 + 4} - \frac{\operatorname{atan}(x)}{4x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*atan(x)/(x**2+1)**2,x)``[Out] x**2*atan(x)/(4*x**2 + 4) + x/(4*x**2 + 4) - atan(x)/(4*x**2 + 4)`**Giac [A]**

time = 0.00, size = 28, normalized size = 0.88

$$\frac{x}{4(x^2 + 1)} + \frac{\arctan x}{4} - \frac{\arctan x}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*arctan(x)/(x^2+1)^2,x)``[Out] 1/4*x/(x^2 + 1) - 1/2*arctan(x)/(x^2 + 1) + 1/4*arctan(x)`**Mupad [B]**

time = 0.08, size = 21, normalized size = 0.66

$$\frac{\operatorname{atan}(x)}{4} + \frac{\frac{x}{4} - \frac{\operatorname{atan}(x)}{2}}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(x))/(x^2 + 1)^2,x)
```

```
[Out] atan(x)/4 + (x/4 - atan(x)/2)/(x^2 + 1)
```

$$3.670 \quad \int \frac{x \tan^{-1}(x)}{(1+x^2)^3} dx$$

Optimal. Leaf size=44

$$\frac{x}{16(1+x^2)^2} + \frac{3x}{32(1+x^2)} + \frac{3}{32} \tan^{-1}(x) - \frac{\tan^{-1}(x)}{4(1+x^2)^2}$$

[Out] 1/16*x/(x^2+1)^2+3/32*x/(x^2+1)+3/32*arctan(x)-1/4*arctan(x)/(x^2+1)^2

Rubi [A]

time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5050, 205, 209}

$$\frac{3x}{32(x^2+1)} + \frac{x}{16(x^2+1)^2} - \frac{\tan^{-1}(x)}{4(x^2+1)^2} + \frac{3}{32} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x*ArcTan[x])/(1 + x^2)^3,x]

[Out] x/(16*(1 + x^2)^2) + (3*x)/(32*(1 + x^2)) + (3*ArcTan[x])/32 - ArcTan[x]/(4*(1 + x^2)^2)

Rule 205

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 5050

Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q + 1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && NeQ[q, -1]

Rubi steps

$$\begin{aligned}
\int \frac{x \tan^{-1}(x)}{(1+x^2)^3} dx &= -\frac{\tan^{-1}(x)}{4(1+x^2)^2} + \frac{1}{4} \int \frac{1}{(1+x^2)^3} dx \\
&= \frac{x}{16(1+x^2)^2} - \frac{\tan^{-1}(x)}{4(1+x^2)^2} + \frac{3}{16} \int \frac{1}{(1+x^2)^2} dx \\
&= \frac{x}{16(1+x^2)^2} + \frac{3x}{32(1+x^2)} - \frac{\tan^{-1}(x)}{4(1+x^2)^2} + \frac{3}{32} \int \frac{1}{1+x^2} dx \\
&= \frac{x}{16(1+x^2)^2} + \frac{3x}{32(1+x^2)} + \frac{3}{32} \tan^{-1}(x) - \frac{\tan^{-1}(x)}{4(1+x^2)^2}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 36, normalized size = 0.82

$$\frac{x(5+3x^2) + (-5+6x^2+3x^4)\tan^{-1}(x)}{32(1+x^2)^2}$$

Antiderivative was successfully verified.

`[In] Integrate[(x*ArcTan[x])/(1+x^2)^3,x]``[Out] (x*(5+3*x^2)+(-5+6*x^2+3*x^4)*ArcTan[x])/(32*(1+x^2)^2)`**Mathics [A]**

time = 2.38, size = 41, normalized size = 0.93

$$\frac{5x+6x^2\text{ArcTan}[x]+3x^3+3x^4\text{ArcTan}[x]-5\text{ArcTan}[x]}{32+64x^2+32x^4}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(ArcTan[x]*x)/(1+x^2)^3,x]')``[Out] (5 x + 6 x ^ 2 ArcTan[x] + 3 x ^ 3 + 3 x ^ 4 ArcTan[x] - 5 ArcTan[x]) / (32 (1 + 2 x ^ 2 + x ^ 4))`**Maple [A]**

time = 0.09, size = 37, normalized size = 0.84

method	result	size
default	$\frac{x}{16(x^2+1)^2} + \frac{3x}{32(x^2+1)} + \frac{3\arctan(x)}{32} - \frac{\arctan(x)}{4(x^2+1)^2}$	37
risch	$\frac{i \ln(ix+1)}{8(x^2+1)^2} - \frac{i(8 \ln(-ix+1)+3 \ln(x-i)x^4+6x^2 \ln(x-i)+3 \ln(x-i)-3 \ln(x+i)x^4-6 \ln(x+i)x^2-3 \ln(x+i)+6ix^3+10ix)}{64(x+i)^2(x-i)^2}$	108

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arctan(x)/(x^2+1)^3,x,method=_RETURNVERBOSE)`

[Out] $1/16*x/(x^2+1)^2+3/32*x/(x^2+1)+3/32*arctan(x)-1/4*arctan(x)/(x^2+1)^2$

Maxima [A]

time = 0.33, size = 39, normalized size = 0.89

$$\frac{3x^3 + 5x}{32(x^4 + 2x^2 + 1)} - \frac{\arctan(x)}{4(x^2 + 1)^2} + \frac{3}{32} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arctan(x)/(x^2+1)^3,x, algorithm="maxima")`

[Out] $1/32*(3*x^3 + 5*x)/(x^4 + 2*x^2 + 1) - 1/4*arctan(x)/(x^2 + 1)^2 + 3/32*arctan(x)$

Fricas [A]

time = 0.33, size = 38, normalized size = 0.86

$$\frac{3x^3 + (3x^4 + 6x^2 - 5) \arctan(x) + 5x}{32(x^4 + 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arctan(x)/(x^2+1)^3,x, algorithm="fricas")`

[Out] $1/32*(3*x^3 + (3*x^4 + 6*x^2 - 5)*arctan(x) + 5*x)/(x^4 + 2*x^2 + 1)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(37) = 74$

time = 0.36, size = 88, normalized size = 2.00

$$\frac{3x^4 \operatorname{atan}(x)}{32x^4 + 64x^2 + 32} + \frac{3x^3}{32x^4 + 64x^2 + 32} + \frac{6x^2 \operatorname{atan}(x)}{32x^4 + 64x^2 + 32} + \frac{5x}{32x^4 + 64x^2 + 32} - \frac{5 \operatorname{atan}(x)}{32x^4 + 64x^2 + 32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*atan(x)/(x**2+1)**3,x)`

[Out] $3*x**4*atan(x)/(32*x**4 + 64*x**2 + 32) + 3*x**3/(32*x**4 + 64*x**2 + 32) + 6*x**2*atan(x)/(32*x**4 + 64*x**2 + 32) + 5*x/(32*x**4 + 64*x**2 + 32) - 5*atan(x)/(32*x**4 + 64*x**2 + 32)$

Giac [A]

time = 0.00, size = 41, normalized size = 0.93

$$\frac{3x^3 + 5x}{32(x^2 + 1)^2} + \frac{3}{32} \arctan x - \frac{\arctan x}{4(x^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arctan(x)/(x^2+1)^3,x)`

[Out] $1/32*(3*x^3 + 5*x)/(x^2 + 1)^2 - 1/4*arctan(x)/(x^2 + 1)^2 + 3/32*arctan(x)$

Mupad [B]

time = 0.32, size = 26, normalized size = 0.59

$$\frac{3 \operatorname{atan}(x)}{32} + \frac{\frac{5x}{32} - \frac{\operatorname{atan}(x)}{4} + \frac{3x^3}{32}}{(x^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*atan(x))/(x^2 + 1)^3,x)`

[Out] $(3*\operatorname{atan}(x))/32 + ((5*x)/32 - \operatorname{atan}(x)/4 + (3*x^3)/32)/(x^2 + 1)^2$

$$3.671 \quad \int \frac{x^2 \tan^{-1}(x)}{1+x^2} dx$$

Optimal. Leaf size=23

$$x \tan^{-1}(x) - \frac{1}{2} \tan^{-1}(x)^2 - \frac{1}{2} \log(1+x^2)$$

[Out] x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)

Rubi [A]

time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5036, 4930, 266, 5004}

$$-\frac{1}{2} \log(x^2 + 1) - \frac{1}{2} \tan^{-1}(x)^2 + x \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x^2*ArcTan[x])/(1 + x^2),x]

[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2

Rule 266

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 4930

Int[((a_) + ArcTan[(c_)*(x_)^(n_)])*(b_)^(p_), x_Symbol] :> Simp[x*(a + b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p - 1))/(1 + c^2*x^(2*n))], x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] && (EqQ[n, 1] || EqQ[p, 1])

Rule 5004

Int[((a_) + ArcTan[(c_)*(x_)])*(b_)^(p_)/((d_) + (e_)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]

Rule 5036

Int[((a_) + ArcTan[(c_)*(x_)])*(b_)^(p_)*((f_)*(x_)^(m_))/((d_) + (e_)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \tan^{-1}(x)}{1+x^2} dx &= \int \tan^{-1}(x) dx - \int \frac{\tan^{-1}(x)}{1+x^2} dx \\
&= x \tan^{-1}(x) - \frac{1}{2} \tan^{-1}(x)^2 - \int \frac{x}{1+x^2} dx \\
&= x \tan^{-1}(x) - \frac{1}{2} \tan^{-1}(x)^2 - \frac{1}{2} \log(1+x^2)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 23, normalized size = 1.00

$$x \tan^{-1}(x) - \frac{1}{2} \tan^{-1}(x)^2 - \frac{1}{2} \log(1+x^2)$$

Antiderivative was successfully verified.

`[In] Integrate[(x^2*ArcTan[x])/(1+x^2),x]``[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1+x^2]/2`**Mathics [A]**

time = 1.74, size = 19, normalized size = 0.83

$$x \text{ArcTan}[x] - \frac{\text{Log}[1+x^2]}{2} - \frac{\text{ArcTan}[x]^2}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(ArcTan[x]*x^2)/(1+x^2),x]')``[Out] x ArcTan[x] - Log[1+x^2]/2 - ArcTan[x]^2/2`**Maple [A]**

time = 0.07, size = 20, normalized size = 0.87

method	result	size
default	$x \arctan(x) - \frac{\arctan(x)^2}{2} - \frac{\ln(x^2+1)}{2}$	20
risch	$\frac{\ln(ix+1)^2}{8} + \frac{i(-x + \frac{i \ln(-ix+1)}{2}) \ln(ix+1)}{2} + \frac{\ln(-ix+1)^2}{8} + \frac{ix \ln(-ix+1)}{2} - \frac{\ln(x^2+1)}{2}$	67

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2*arctan(x)/(x^2+1),x,method=_RETURNVERBOSE)``[Out] x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)`

Maxima [A]

time = 0.34, size = 24, normalized size = 1.04

$$(x - \arctan(x)) \arctan(x) + \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="maxima")

[Out] (x - arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)

Fricas [A]

time = 0.32, size = 19, normalized size = 0.83

$$x \arctan(x) - \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="fricas")

[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)

Sympy [A]

time = 0.14, size = 19, normalized size = 0.83

$$x \operatorname{atan}(x) - \frac{\log(x^2 + 1)}{2} - \frac{\operatorname{atan}^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*atan(x)/(x**2+1),x)

[Out] x*atan(x) - log(x**2 + 1)/2 - atan(x)**2/2

Giac [A]

time = 0.00, size = 21, normalized size = 0.91

$$\frac{2x \arctan x - \arctan^2 x - \ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arctan(x)/(x^2+1),x)

[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)

Mupad [B]

time = 0.31, size = 19, normalized size = 0.83

$$-\frac{\operatorname{atan}(x)^2}{2} + x \operatorname{atan}(x) - \frac{\ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*atan(x))/(x^2 + 1),x)

[Out] x*atan(x) - atan(x)^2/2 - log(x^2 + 1)/2

$$3.672 \quad \int \frac{x^3 \tan^{-1}(x)}{1+x^2} dx$$

Optimal. Leaf size=67

$$-\frac{x}{2} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} x^2 \tan^{-1}(x) + \frac{1}{2} i \tan^{-1}(x)^2 + \tan^{-1}(x) \log\left(\frac{2}{1+ix}\right) + \frac{1}{2} i \text{Li}_2\left(1 - \frac{2}{1+ix}\right)$$

[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)+1/2*I*arctan(x)^2+arctan(x)*ln(2/(1+I*x))+1/2*I*polylog(2,1-2/(1+I*x))

Rubi [A]

time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$,

Rules used = {5036, 4946, 327, 209, 5040, 4964, 2449, 2352}

$$\frac{1}{2} i \text{Li}_2\left(1 - \frac{2}{ix+1}\right) + \frac{1}{2} x^2 \tan^{-1}(x) - \frac{x}{2} + \frac{1}{2} i \tan^{-1}(x)^2 + \frac{1}{2} \tan^{-1}(x) + \log\left(\frac{2}{1+ix}\right) \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x^3*ArcTan[x])/(1+x^2),x]

[Out] -1/2*x + ArcTan[x]/2 + (x^2*ArcTan[x])/2 + (I/2)*ArcTan[x]^2 + ArcTan[x]*Log[2/(1+I*x)] + (I/2)*PolyLog[2, 1 - 2/(1+I*x)]

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 327

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n-1)*(c*x)^(m-n+1)*((a+b*x^n)^(p+1)/(b*(m+n*p+1))), x] - Dist[a*c^n*((m-n+1)/(b*(m+n*p+1))), Int[(c*x)^(m-n)*(a+b*x^n)^p, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2352

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(-e^(-1))*PolyLog[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2449

Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := Dist[-e/g, Subst[Int[Log[2*d*x]/(1-2*d*x), x], x, 1/(d+e*x)], x] /; FreeQ[{

c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
  Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
  1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
  IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
  :> Simp[(- (a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
  p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
  x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*((f_.)*(x_)^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^3 \tan^{-1}(x)}{1+x^2} dx &= \int x \tan^{-1}(x) dx - \int \frac{x \tan^{-1}(x)}{1+x^2} dx \\
 &= \frac{1}{2} x^2 \tan^{-1}(x) + \frac{1}{2} i \tan^{-1}(x)^2 - \frac{1}{2} \int \frac{x^2}{1+x^2} dx + \int \frac{\tan^{-1}(x)}{i-x} dx \\
 &= -\frac{x}{2} + \frac{1}{2} x^2 \tan^{-1}(x) + \frac{1}{2} i \tan^{-1}(x)^2 + \tan^{-1}(x) \log\left(\frac{2}{1+ix}\right) + \frac{1}{2} \int \frac{1}{1+x^2} dx - \int \frac{1}{i-x} dx \\
 &= -\frac{x}{2} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} x^2 \tan^{-1}(x) + \frac{1}{2} i \tan^{-1}(x)^2 + \tan^{-1}(x) \log\left(\frac{2}{1+ix}\right) + i \text{Subst}\left(\frac{1}{1+x^2}, x, i-x\right) \\
 &= -\frac{x}{2} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} x^2 \tan^{-1}(x) + \frac{1}{2} i \tan^{-1}(x)^2 + \tan^{-1}(x) \log\left(\frac{2}{1+ix}\right) + \frac{1}{2} i \text{Li}_2\left(\frac{1-i(x-i)}{1+ix}\right)
 \end{aligned}$$

Mathematica [A]

time = 0.02, size = 57, normalized size = 0.85

$$\frac{1}{2} \left(-x + i \tan^{-1}(x)^2 + \tan^{-1}(x) \left(1 + x^2 + 2 \log \left(-\frac{2i}{-i+x} \right) \right) + i \operatorname{Li}_2 \left(\frac{i+x}{-i+x} \right) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[(x^3*ArcTan[x])/(1 + x^2),x]``[Out] (-x + I*ArcTan[x]^2 + ArcTan[x]*(1 + x^2 + 2*Log[(-2*I)/(-I + x)]) + I*PolyLog[2, (I + x)/(-I + x)])/2`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(ArcTan[x]*x^3)/(1 + x^2),x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(53) = 106.

time = 0.08, size = 128, normalized size = 1.91

method	result
risch	$\frac{\arctan(x)}{2} - \frac{x}{2} + \frac{ix^2 \ln(-ix+1)}{4} - \frac{i \ln(\frac{1}{2} + \frac{ix}{2}) \ln(-ix+1)}{4} + \frac{i \operatorname{dilog}(\frac{1}{2} - \frac{ix}{2})}{4} - \frac{i \ln(-ix+1)^2}{8} - \frac{ix^2 \ln(ix+1)}{4} + \frac{i \ln(\frac{1}{2} - \frac{ix}{2}) \ln(ix+1)}{4}$
default	$\frac{x^2 \arctan(x)}{2} - \frac{\arctan(x) \ln(x^2+1)}{2} - \frac{x}{2} + \frac{\arctan(x)}{2} - \frac{i \ln(x-i) \ln(x^2+1)}{4} + \frac{i \operatorname{dilog}(-\frac{i(x+i)}{2})}{4} + \frac{i \ln(x-i) \ln(-\frac{i(x+i)}{2})}{4} + \dots$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^3*arctan(x)/(x^2+1),x,method=_RETURNVERBOSE)``[Out] 1/2*x^2*arctan(x)-1/2*arctan(x)*ln(x^2+1)-1/2*x+1/2*arctan(x)-1/4*I*ln(x-I)*ln(x^2+1)+1/4*I*dilog(-1/2*I*(x+I))+1/4*I*ln(x-I)*ln(-1/2*I*(x+I))+1/8*I*ln(x-I)^2+1/4*I*ln(x+I)*ln(x^2+1)-1/4*I*dilog(1/2*I*(x-I))-1/4*I*ln(x+I)*ln(1/2*I*(x-I))-1/8*I*ln(x+I)^2`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctan(x)/(x^2+1),x, algorithm="maxima")

[Out] integrate(x^3*arctan(x)/(x^2 + 1), x)

Fricas [F]

time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctan(x)/(x^2+1),x, algorithm="fricas")

[Out] integral(x^3*arctan(x)/(x^2 + 1), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atan}(x)}{x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*atan(x)/(x**2+1),x)

[Out] Integral(x**3*atan(x)/(x**2 + 1), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctan(x)/(x^2+1),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{atan}(x)}{x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*atan(x))/(x^2 + 1),x)

[Out] int((x^3*atan(x))/(x^2 + 1), x)

$$3.673 \quad \int \frac{x^2 \tan^{-1}(x)}{(1+x^2)^2} dx$$

Optimal. Leaf size=34

$$-\frac{1}{4(1+x^2)} - \frac{x \tan^{-1}(x)}{2(1+x^2)} + \frac{1}{4} \tan^{-1}(x)^2$$

[Out] $-1/4/(x^2+1)-1/2*x*\arctan(x)/(x^2+1)+1/4*\arctan(x)^2$

Rubi [A]

time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5054, 5004}

$$-\frac{1}{4(x^2+1)} - \frac{x \tan^{-1}(x)}{2(x^2+1)} + \frac{1}{4} \tan^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^2*\text{ArcTan}[x])/(1+x^2)^2, x]$

[Out] $-1/4*1/(1+x^2) - (x*\text{ArcTan}[x])/(2*(1+x^2)) + \text{ArcTan}[x]^2/4$

Rule 5004

$\text{Int}[(a + \text{ArcTan}[c*x])*(b + (d + e*x^2)^q), x] \rightarrow \text{Simp}[(a + b*\text{ArcTan}[c*x])^{p+1}/(b*c*d*(p+1)), x] /;$ FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]

Rule 5054

$\text{Int}[(a + \text{ArcTan}[c*x])*(b + (d + e*x^2)^q)^r, x] \rightarrow \text{Simp}[-b*(d + e*x^2)^{q+1}/(4*c^3*d*(q+1)^2), x] + (-\text{Dist}[1/(2*c^2*d*(q+1)), \text{Int}[(d + e*x^2)^{q+1}*(a + b*\text{ArcTan}[c*x]), x], x] + \text{Simp}[x*(d + e*x^2)^{q+1}*(a + b*\text{ArcTan}[c*x])/(2*c^2*d*(q+1)), x]) /;$ FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]

Rubi steps

$$\begin{aligned} \int \frac{x^2 \tan^{-1}(x)}{(1+x^2)^2} dx &= -\frac{1}{4(1+x^2)} - \frac{x \tan^{-1}(x)}{2(1+x^2)} + \frac{1}{2} \int \frac{\tan^{-1}(x)}{1+x^2} dx \\ &= -\frac{1}{4(1+x^2)} - \frac{x \tan^{-1}(x)}{2(1+x^2)} + \frac{1}{4} \tan^{-1}(x)^2 \end{aligned}$$

Mathematica [A]

time = 0.02, size = 28, normalized size = 0.82

$$\frac{-1 - 2x \tan^{-1}(x) + (1 + x^2) \tan^{-1}(x)^2}{4(1 + x^2)}$$

Antiderivative was successfully verified.

`[In] Integrate[(x^2*ArcTan[x])/(1 + x^2)^2,x]``[Out] (-1 - 2*x*ArcTan[x] + (1 + x^2)*ArcTan[x]^2)/(4*(1 + x^2))`**Mathics [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(ArcTan[x]*x^2)/(1 + x^2)^2,x]')``[Out] cought exception: maximum recursion depth exceeded while calling a Python object`**Maple [A]**

time = 0.11, size = 29, normalized size = 0.85

method	result	size
default	$-\frac{1}{4(x^2+1)} - \frac{x \arctan(x)}{2(x^2+1)} + \frac{\arctan(x)^2}{4}$	29
risch	$-\frac{\ln(ix+1)^2}{16} + \frac{(x^2 \ln(-ix+1) + \ln(-ix+1) + 2ix) \ln(ix+1)}{8x^2+8} - \frac{x^2 \ln(-ix+1)^2 + \ln(-ix+1)^2 + 4ix \ln(-ix+1) + 4}{16(x+i)(x-i)}$	101

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^2*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)``[Out] -1/4/(x^2+1)-1/2*x*arctan(x)/(x^2+1)+1/4*arctan(x)^2`**Maxima [A]**

time = 0.34, size = 40, normalized size = 1.18

$$-\frac{1}{2} \left(\frac{x}{x^2 + 1} - \arctan(x) \right) \arctan(x) - \frac{(x^2 + 1) \arctan(x)^2 + 1}{4(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2*arctan(x)/(x^2+1)^2,x, algorithm="maxima")``[Out] -1/2*(x/(x^2 + 1) - arctan(x))*arctan(x) - 1/4*((x^2 + 1)*arctan(x)^2 + 1)/(x^2 + 1)`

Fricas [A]

time = 0.31, size = 26, normalized size = 0.76

$$\frac{(x^2 + 1) \arctan(x)^2 - 2x \arctan(x) - 1}{4(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2*arctan(x)/(x^2+1)^2,x, algorithm="fricas")``[Out] 1/4*((x^2 + 1)*arctan(x)^2 - 2*x*arctan(x) - 1)/(x^2 + 1)`**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x**2*atan(x)/(x**2+1)**2,x)``[Out] Exception raised: RecursionError`**Giac [F] N/A**

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2*arctan(x)/(x^2+1)^2,x)``[Out] Could not integrate`**Mupad [B]**

time = 0.06, size = 23, normalized size = 0.68

$$\frac{\operatorname{atan}(x)^2}{4} - \frac{\frac{x \operatorname{atan}(x)}{2} + \frac{1}{4}}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((x^2*atan(x))/(x^2 + 1)^2,x)``[Out] atan(x)^2/4 - ((x*atan(x))/2 + 1/4)/(x^2 + 1)`

$$3.674 \quad \int \frac{x^3 \tan^{-1}(x)}{(1+x^2)^2} dx$$

Optimal. Leaf size=79

$$-\frac{x}{4(1+x^2)} - \frac{1}{4} \tan^{-1}(x) + \frac{\tan^{-1}(x)}{2(1+x^2)} - \frac{1}{2} i \tan^{-1}(x)^2 - \tan^{-1}(x) \log\left(\frac{2}{1+ix}\right) - \frac{1}{2} i \operatorname{Li}_2\left(1 - \frac{2}{1+ix}\right)$$

[Out] $-1/4*x/(x^2+1) - 1/4*\arctan(x) + 1/2*\arctan(x)/(x^2+1) - 1/2*I*\arctan(x)^2 - \arctan(x)*\ln(2/(1+I*x)) - 1/2*I*\operatorname{polylog}(2, 1-2/(1+I*x))$

Rubi [A]

time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {5084, 5040, 4964, 2449, 2352, 5050, 205, 209}

$$-\frac{1}{2} i \operatorname{Li}_2\left(1 - \frac{2}{ix+1}\right) - \frac{x}{4(x^2+1)} + \frac{\tan^{-1}(x)}{2(x^2+1)} - \frac{1}{2} i \tan^{-1}(x)^2 - \frac{1}{4} \tan^{-1}(x) - \log\left(\frac{2}{1+ix}\right) \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^3*\operatorname{ArcTan}[x])/(1+x^2)^2, x]$

[Out] $-1/4*x/(1+x^2) - \operatorname{ArcTan}[x]/4 + \operatorname{ArcTan}[x]/(2*(1+x^2)) - (I/2)*\operatorname{ArcTan}[x]^2 - \operatorname{ArcTan}[x]*\operatorname{Log}[2/(1+I*x)] - (I/2)*\operatorname{PolyLog}[2, 1-2/(1+I*x)]$

Rule 205

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^{n_+})^{p_+}, x_Symbol] \rightarrow \operatorname{Simp}[(-x)*((a + b*x^n)^{p+1}/(a*n*(p+1))), x] + \operatorname{Dist}[(n*(p+1)+1)/(a*n*(p+1)), \operatorname{Int}[(a + b*x^n)^{p+1}, x], x] /;$ FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 209

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))*\operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2352

$\operatorname{Int}[\operatorname{Log}[(c_+)*(x_+)]/((d_+ + (e_+)*(x_+))), x_Symbol] \rightarrow \operatorname{Simp}[(-e^{-1})*\operatorname{PolyLog}[2, 1 - c*x], x] /;$ FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2449

$\operatorname{Int}[\operatorname{Log}[(c_+)]/((d_+ + (e_+)*(x_+)))/((f_+ + (g_+)*(x_+)^2), x_Symbol] \rightarrow \operatorname{Dist}[-e/g, \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /;$ FreeQ[{

c, d, e, f, g, x && EqQ[$c, 2*d$] && EqQ[$e^2*f + d^2*g, 0$]

Rule 4964

Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-a + b*ArcTan[c*x])^p*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]

Rule 5040

Int[(((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]

Rule 5050

Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q + 1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && NeQ[q, -1]

Rule 5084

Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \tan^{-1}(x)}{(1+x^2)^2} dx &= -\int \frac{x \tan^{-1}(x)}{(1+x^2)^2} dx + \int \frac{x \tan^{-1}(x)}{1+x^2} dx \\
&= \frac{\tan^{-1}(x)}{2(1+x^2)} - \frac{1}{2} i \tan^{-1}(x)^2 - \frac{1}{2} \int \frac{1}{(1+x^2)^2} dx - \int \frac{\tan^{-1}(x)}{i-x} dx \\
&= -\frac{x}{4(1+x^2)} + \frac{\tan^{-1}(x)}{2(1+x^2)} - \frac{1}{2} i \tan^{-1}(x)^2 - \tan^{-1}(x) \log\left(\frac{2}{1+ix}\right) - \frac{1}{4} \int \frac{1}{1+x^2} dx + \\
&= -\frac{x}{4(1+x^2)} - \frac{1}{4} \tan^{-1}(x) + \frac{\tan^{-1}(x)}{2(1+x^2)} - \frac{1}{2} i \tan^{-1}(x)^2 - \tan^{-1}(x) \log\left(\frac{2}{1+ix}\right) - i \operatorname{Si} \\
&= -\frac{x}{4(1+x^2)} - \frac{1}{4} \tan^{-1}(x) + \frac{\tan^{-1}(x)}{2(1+x^2)} - \frac{1}{2} i \tan^{-1}(x)^2 - \tan^{-1}(x) \log\left(\frac{2}{1+ix}\right) - \frac{1}{2} i \operatorname{Si}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 64, normalized size = 0.81

$$\frac{1}{2} i \tan^{-1}(x)^2 + \frac{1}{4} \tan^{-1}(x) \cos(2 \tan^{-1}(x)) - \tan^{-1}(x) \log(1 + e^{2i \tan^{-1}(x)}) + \frac{1}{2} i \operatorname{Li}_2(-e^{2i \tan^{-1}(x)}) - \frac{1}{8} \sin(2 \tan^{-1}(x))$$

Antiderivative was successfully verified.

`[In] Integrate[(x^3*ArcTan[x])/(1 + x^2)^2,x]`

```
[Out] (I/2)*ArcTan[x]^2 + (ArcTan[x]*Cos[2*ArcTan[x]])/4 - ArcTan[x]*Log[1 + E^((2*I)*ArcTan[x])] + (I/2)*PolyLog[2, -E^((2*I)*ArcTan[x])] - Sin[2*ArcTan[x]]/8
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(ArcTan[x]*x^3)/(1 + x^2)^2,x]')``[Out] cought exception: maximum recursion depth exceeded`

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(65) = 130$.

time = 0.09, size = 139, normalized size = 1.76

method	result
default	$\frac{\arctan(x)}{2x^2+2} + \frac{\arctan(x) \ln(x^2+1)}{2} - \frac{x}{4(x^2+1)} - \frac{\arctan(x)}{4} + \frac{i \ln(x-i) \ln(x^2+1)}{4} - \frac{i \operatorname{dilog}\left(-\frac{i(x+i)}{2}\right)}{4} - \frac{i \ln(x-i) \ln\left(-\frac{i(x+i)}{2}\right)}{4}$

risch	$\frac{i}{-8ix+8} + \frac{i \ln(-ix+1)^2}{8} - \frac{i \ln(\frac{1}{2} - \frac{ix}{2}) \ln(ix+1)}{4} + \frac{i \ln(\frac{1}{2} + \frac{ix}{2}) \ln(-ix+1)}{4} - \frac{i \ln(ix+1)}{8(ix+1)} - \frac{\arctan(x)}{8} - \frac{\ln(-ix+1)x}{16(-ix-1)} + \frac{i \ln(-ix+1)}{8}$
-------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{2} \arctan(x)/(x^2+1) + \frac{1}{2} \arctan(x) \ln(x^2+1) - \frac{1}{4} x/(x^2+1) - \frac{1}{4} \arctan(x) + \frac{1}{4} I \ln(x-I) \ln(x^2+1) - \frac{1}{4} I \operatorname{dilog}(-\frac{1}{2} I (x+I)) - \frac{1}{4} I \ln(x-I) \ln(-\frac{1}{2} I (x+I)) - \frac{1}{8} I \ln(x-I)^2 - \frac{1}{4} I \ln(x+I) \ln(x^2+1) + \frac{1}{4} I \operatorname{dilog}(\frac{1}{2} I (x-I)) + \frac{1}{4} I \ln(x+I) \ln(\frac{1}{2} I (x-I)) + \frac{1}{8} I \ln(x+I)^2$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arctan(x)/(x^2+1)^2,x, algorithm="maxima")`

[Out] `integrate(x^3*arctan(x)/(x^2 + 1)^2, x)`

Fricas [F]

time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arctan(x)/(x^2+1)^2,x, algorithm="fricas")`

[Out] `integral(x^3*arctan(x)/(x^4 + 2*x^2 + 1), x)`

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*atan(x)/(x**2+1)**2,x)`

[Out] Exception raised: RecursionError

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctan(x)/(x^2+1)^2,x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{atan}(x)}{(x^2 + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*atan(x))/(x^2 + 1)^2,x)

[Out] int((x^3*atan(x))/(x^2 + 1)^2, x)

$$3.675 \quad \int \frac{x^5 \tan^{-1}(x)}{(1+x^2)^2} dx$$

Optimal. Leaf size=89

$$-\frac{x}{2} + \frac{x}{4(1+x^2)} + \frac{3}{4} \tan^{-1}(x) + \frac{1}{2} x^2 \tan^{-1}(x) - \frac{\tan^{-1}(x)}{2(1+x^2)} + i \tan^{-1}(x)^2 + 2 \tan^{-1}(x) \log\left(\frac{2}{1+ix}\right) + i \text{Li}_2\left(1 - \frac{1}{1+ix}\right)$$

[Out] $-1/2*x+1/4*x/(x^2+1)+3/4*\arctan(x)+1/2*x^2*\arctan(x)-1/2*\arctan(x)/(x^2+1)+I*\arctan(x)^2+2*\arctan(x)*\ln(2/(1+I*x))+I*\text{polylog}(2,1-2/(1+I*x))$

Rubi [A]

time = 0.14, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {5084, 5036, 4946, 327, 209, 5040, 4964, 2449, 2352, 5050, 205}

$$i \text{Li}_2\left(1 - \frac{2}{ix+1}\right) + \frac{x}{4(x^2+1)} + \frac{1}{2} x^2 \tan^{-1}(x) - \frac{\tan^{-1}(x)}{2(x^2+1)} - \frac{x}{2} + i \tan^{-1}(x)^2 + \frac{3}{4} \tan^{-1}(x) + 2 \log\left(\frac{2}{1+ix}\right) \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^5*\text{ArcTan}[x])/(1+x^2)^2, x]$

[Out] $-1/2*x + x/(4*(1+x^2)) + (3*\text{ArcTan}[x])/4 + (x^2*\text{ArcTan}[x])/2 - \text{ArcTan}[x]/(2*(1+x^2)) + I*\text{ArcTan}[x]^2 + 2*\text{ArcTan}[x]*\text{Log}[2/(1+I*x)] + I*\text{PolyLog}[2, 1 - 2/(1+I*x)]$

Rule 205

$\text{Int}[(a_ + (b_)*(x_)^(n_))^(p_), x_Symbol] := \text{Simp}[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + \text{Dist}[(n*(p + 1) + 1)/(a*n*(p + 1)), \text{Int}[(a + b*x^n)^(p + 1), x], x] /;$ FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 209

$\text{Int}[(a_ + (b_)*(x_)^2)^(-1), x_Symbol] := \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))*\text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 327

$\text{Int}[(c_)*(x_)^(m_)*((a_ + (b_)*(x_)^(n_))^(p_), x_Symbol] := \text{Simp}[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - \text{Dist}[a*c^n*((m - n + 1)/(b*(m + n*p + 1))), \text{Int}[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2352

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLog[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2449

Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 4946

Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m + 1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] && IntegerQ[m])) && NeQ[m, -1]

Rule 4964

Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-a + b*ArcTan[c*x])^p*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]

Rule 5036

Int[(((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*((f_.)*(x_)^(m_))/((d_) + (e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

Rule 5040

Int[(((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]

Rule 5050

Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q + 1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,

0] && NeQ[q, -1]

Rule 5084

Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{x^5 \tan^{-1}(x)}{(1+x^2)^2} dx &= - \int \frac{x^3 \tan^{-1}(x)}{(1+x^2)^2} dx + \int \frac{x^3 \tan^{-1}(x)}{1+x^2} dx \\
 &= \int x \tan^{-1}(x) dx + \int \frac{x \tan^{-1}(x)}{(1+x^2)^2} dx - 2 \int \frac{x \tan^{-1}(x)}{1+x^2} dx \\
 &= \frac{1}{2} x^2 \tan^{-1}(x) - \frac{\tan^{-1}(x)}{2(1+x^2)} + \frac{1}{2} \int \frac{1}{(1+x^2)^2} dx - \frac{1}{2} \int \frac{x^2}{1+x^2} dx - 2 \left(-\frac{1}{2} i \tan^{-1}(x)^2 - \tan^{-1}(x) \right) \\
 &= -\frac{x}{2} + \frac{x}{4(1+x^2)} + \frac{1}{2} x^2 \tan^{-1}(x) - \frac{\tan^{-1}(x)}{2(1+x^2)} + \frac{1}{4} \int \frac{1}{1+x^2} dx + \frac{1}{2} \int \frac{1}{1+x^2} dx - 2 \left(-\frac{1}{2} i \tan^{-1}(x)^2 - \tan^{-1}(x) \right) \\
 &= -\frac{x}{2} + \frac{x}{4(1+x^2)} + \frac{3}{4} \tan^{-1}(x) + \frac{1}{2} x^2 \tan^{-1}(x) - \frac{\tan^{-1}(x)}{2(1+x^2)} - 2 \left(-\frac{1}{2} i \tan^{-1}(x)^2 - \tan^{-1}(x) \right) \\
 &= -\frac{x}{2} + \frac{x}{4(1+x^2)} + \frac{3}{4} \tan^{-1}(x) + \frac{1}{2} x^2 \tan^{-1}(x) - \frac{\tan^{-1}(x)}{2(1+x^2)} - 2 \left(-\frac{1}{2} i \tan^{-1}(x)^2 - \tan^{-1}(x) \right)
 \end{aligned}$$

Mathematica [A]

time = 0.12, size = 70, normalized size = 0.79

$$\frac{1}{8} (-4x + 4(1+x^2) \tan^{-1}(x) - 8i \tan^{-1}(x)^2 - 2 \tan^{-1}(x) \cos(2 \tan^{-1}(x)) + 16 \tan^{-1}(x) \log(1 + e^{2i \tan^{-1}(x)}) - 8i \text{Li}_2(-e^{2i \tan^{-1}(x)}) + \sin(2 \tan^{-1}(x)))$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*ArcTan[x])/(1 + x^2)^2,x]

[Out] (-4*x + 4*(1 + x^2)*ArcTan[x] - (8*I)*ArcTan[x]^2 - 2*ArcTan[x]*Cos[2*ArcTan[x]] + 16*ArcTan[x]*Log[1 + E^((2*I)*ArcTan[x])]) - (8*I)*PolyLog[2, -E^((2*I)*ArcTan[x])] + Sin[2*ArcTan[x]])/8

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(ArcTan[x]*x^5)/(1 + x^2)^2,x]')`

[Out] caught exception: maximum recursion depth exceeded

Maple [A]

time = 0.09, size = 149, normalized size = 1.67

method	result
default	$\frac{x^2 \arctan(x)}{2} - \frac{\arctan(x)}{2(x^2+1)} - \arctan(x) \ln(x^2 + 1) - \frac{x}{2} + \frac{x}{4x^2+4} + \frac{3 \arctan(x)}{4} - \frac{i \ln(x-i) \ln(x^2+1)}{2} + \frac{i \operatorname{dilog}(-)}{2}$
risch	$-\frac{x}{2} + \frac{5 \arctan(x)}{8} + \frac{ix^2 \ln(-ix+1)}{4} - \frac{i \ln(ix+1)}{16(ix-1)} + \frac{i \ln(-ix+1)}{-16ix-16} + \frac{i}{8ix+8} + \frac{i \operatorname{dilog}(\frac{1}{2} - \frac{ix}{2})}{2} + \frac{i \ln(ix+1)}{8ix+8} + \frac{i \ln(ix+1)^2}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{2}x^2 \arctan(x) - \frac{1}{2} \arctan(x) / (x^2+1) - \arctan(x) \ln(x^2+1) - \frac{1}{2}x + \frac{1}{4}x / (x^2+1) + \frac{3}{4} \arctan(x) - \frac{1}{2}I \ln(x-I) \ln(x^2+1) + \frac{1}{2}I \operatorname{dilog}(-\frac{1}{2}I(x+I)) + \frac{1}{2}I \ln(x-I) \ln(-\frac{1}{2}I(x+I)) + \frac{1}{4}I \ln(x-I)^2 + \frac{1}{2}I \ln(x+I) \ln(x^2+1) - \frac{1}{2}I \operatorname{dilog}(\frac{1}{2}I(x-I)) - \frac{1}{2}I \ln(x+I) \ln(\frac{1}{2}I(x-I)) - \frac{1}{4}I \ln(x+I)^2$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*arctan(x)/(x^2+1)^2,x, algorithm="maxima")`

[Out] `integrate(x^5*arctan(x)/(x^2 + 1)^2, x)`

Fricas [F]

time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*arctan(x)/(x^2+1)^2,x, algorithm="fricas")`

[Out] `integral(x^5*arctan(x)/(x^4 + 2*x^2 + 1), x)`

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*atan(x)/(x**2+1)**2,x)`

[Out] Exception raised: RecursionError

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*arctan(x)/(x^2+1)^2,x)`

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5 \operatorname{atan}(x)}{(x^2 + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*atan(x))/(x^2 + 1)^2,x)`

[Out] `int((x^5*atan(x))/(x^2 + 1)^2, x)`

$$3.676 \quad \int \frac{(1+x^2) \tan^{-1}(x)}{x^2} dx$$

Optimal. Leaf size=22

$$-\frac{\tan^{-1}(x)}{x} + x \tan^{-1}(x) + \log(x) - \log(1+x^2)$$

[Out] -arctan(x)/x+x*arctan(x)+ln(x)-ln(x^2+1)

Rubi [A]

time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {5070, 4946, 272, 36, 29, 31, 4930, 266}

$$-\log(x^2+1) + \log(x) + x \tan^{-1}(x) - \frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

[In] Int[((1 + x^2)*ArcTan[x])/x^2,x]

[Out] -(ArcTan[x]/x) + x*ArcTan[x] + Log[x] - Log[1 + x^2]

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 266

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 272

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4930

Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] && (EqQ[n, 1] || EqQ[p, 1])

Rule 4946

Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m + 1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] && IntegerQ[m])) && NeQ[m, -1]

Rule 5070

Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_)^(m_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x^n])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] && IntegerQ[q]))

Rubi steps

$$\begin{aligned}
 \int \frac{(1+x^2)\tan^{-1}(x)}{x^2} dx &= \int \tan^{-1}(x) dx + \int \frac{\tan^{-1}(x)}{x^2} dx \\
 &= -\frac{\tan^{-1}(x)}{x} + x \tan^{-1}(x) + \int \frac{1}{x(1+x^2)} dx - \int \frac{x}{1+x^2} dx \\
 &= -\frac{\tan^{-1}(x)}{x} + x \tan^{-1}(x) - \frac{1}{2} \log(1+x^2) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(1+x)} dx, x, x^2 \right) \\
 &= -\frac{\tan^{-1}(x)}{x} + x \tan^{-1}(x) - \frac{1}{2} \log(1+x^2) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{x} dx, x, x^2 \right) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+x} dx, x, x^2 \right) \\
 &= -\frac{\tan^{-1}(x)}{x} + x \tan^{-1}(x) + \log(x) - \log(1+x^2)
 \end{aligned}$$

Mathematica [A]

time = 0.00, size = 22, normalized size = 1.00

$$-\frac{\tan^{-1}(x)}{x} + x \tan^{-1}(x) + \log(x) - \log(1+x^2)$$

Antiderivative was successfully verified.

[In] Integrate[((1 + x^2)*ArcTan[x])/x^2,x]

[Out] -(ArcTan[x]/x) + x*ArcTan[x] + Log[x] - Log[1 + x^2]

Mathics [A]

time = 1.77, size = 22, normalized size = 1.00

$$-\frac{\text{ArcTan}[x]}{x} + x\text{ArcTan}[x] + \text{Log}[x] - \text{Log}[1 + x^2]$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(ArcTan[x]*(1 + x^2))/x^2,x]')

[Out] -ArcTan[x] / x + x ArcTan[x] + Log[x] - Log[1 + x ^ 2]

Maple [A]

time = 0.05, size = 23, normalized size = 1.05

method	result	size
default	$-\frac{\arctan(x)}{x} + x \arctan(x) + \ln(x) - \ln(x^2 + 1)$	23
meijerg	$\ln(x) - \frac{\arctan(\sqrt{x^2})}{\sqrt{x^2}} - \ln(x^2 + 1) + \frac{x^2 \arctan(\sqrt{x^2})}{\sqrt{x^2}}$	40
risch	$-\frac{i(x^2-1)\ln(ix+1)}{2x} + \frac{i(-2i\ln(x)x+2i\ln(x^2+1)x+x^2\ln(-ix+1)-\ln(-ix+1))}{2x}$	63

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)*arctan(x)/x^2,x,method=_RETURNVERBOSE)

[Out] -1/x*arctan(x)+x*arctan(x)+ln(x)-ln(x^2+1)

Maxima [A]

time = 0.32, size = 21, normalized size = 0.95

$$\left(x - \frac{1}{x}\right) \arctan(x) - \log(x^2 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)*arctan(x)/x^2,x, algorithm="maxima")

[Out] (x - 1/x)*arctan(x) - log(x^2 + 1) + log(x)

Fricas [A]

time = 0.31, size = 26, normalized size = 1.18

$$\frac{(x^2 - 1) \arctan(x) - x \log(x^2 + 1) + x \log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)*arctan(x)/x^2,x, algorithm="fricas")

[Out] ((x^2 - 1)*arctan(x) - x*log(x^2 + 1) + x*log(x))/x

Sympy [A]

time = 0.15, size = 19, normalized size = 0.86

$$x \operatorname{atan}(x) + \log(x) - \log(x^2 + 1) - \frac{\operatorname{atan}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)*atan(x)/x**2,x)

[Out] x*atan(x) + log(x) - log(x**2 + 1) - atan(x)/x

Giac [A]

time = 0.00, size = 23, normalized size = 1.05

$$\frac{\ln(x^2)}{2} - \ln(x^2 + 1) + \left(x - \frac{1}{x}\right) \arctan x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)*arctan(x)/x^2,x)

[Out] (x - 1/x)*arctan(x) - log(x^2 + 1) + 1/2*log(x^2)

Mupad [B]

time = 0.07, size = 22, normalized size = 1.00

$$\ln(x) - \ln(x^2 + 1) - \frac{\operatorname{atan}(x)}{x} + x \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((atan(x)*(x^2 + 1))/x^2,x)

[Out] log(x) - log(x^2 + 1) - atan(x)/x + x*atan(x)

$$3.677 \quad \int \frac{(1+x^2) \tan^{-1}(x)}{x^5} dx$$

Optimal. Leaf size=31

$$-\frac{1}{12x^3} - \frac{1}{4x} - \frac{(1+x^2)^2 \tan^{-1}(x)}{4x^4}$$

[Out] $-1/12/x^3 - 1/4/x - 1/4*(x^2+1)^2*\arctan(x)/x^4$

Rubi [A]

time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5064, 14}

$$-\frac{1}{12x^3} - \frac{(x^2+1)^2 \tan^{-1}(x)}{4x^4} - \frac{1}{4x}$$

Antiderivative was successfully verified.

[In] `Int[((1 + x^2)*ArcTan[x])/x^5,x]`

[Out] $-1/12*1/x^3 - 1/(4*x) - ((1 + x^2)^2*\text{ArcTan}[x])/(4*x^4)$

Rule 14

`Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]]`

Rule 5064

`Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m+1)*(d + e*x^2)^(q+1)*((a + b*ArcTan[c*x])^p/(d*f*(m+1))), x] - Dist[b*c*(p/(f*(m+1))), Int[(f*x)^(m+1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p-1), x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] && NeQ[m, -1]`

Rubi steps

$$\begin{aligned} \int \frac{(1+x^2) \tan^{-1}(x)}{x^5} dx &= -\frac{(1+x^2)^2 \tan^{-1}(x)}{4x^4} + \frac{1}{4} \int \frac{1+x^2}{x^4} dx \\ &= -\frac{(1+x^2)^2 \tan^{-1}(x)}{4x^4} + \frac{1}{4} \int \left(\frac{1}{x^4} + \frac{1}{x^2} \right) dx \\ &= -\frac{1}{12x^3} - \frac{1}{4x} - \frac{(1+x^2)^2 \tan^{-1}(x)}{4x^4} \end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.01, size = 59, normalized size = 1.90

$$\frac{\tan^{-1}(x)}{4x^4} - \frac{\tan^{-1}(x)}{2x^2} - \frac{{}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; -x^2\right)}{12x^3} - \frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; -x^2\right)}{2x}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + x^2)*ArcTan[x])/x^5,x]

[Out] -1/4*ArcTan[x]/x^4 - ArcTan[x]/(2*x^2) - Hypergeometric2F1[-3/2, 1, -1/2, -x^2]/(12*x^3) - Hypergeometric2F1[-1/2, 1, 1/2, -x^2]/(2*x)

Mathics [A]

time = 1.84, size = 32, normalized size = 1.03

$$\frac{-x - 6x^2 \text{ArcTan}[x] - 3x^3 - 3x^4 \text{ArcTan}[x] - 3 \text{ArcTan}[x]}{12x^4}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(ArcTan[x]*(1 + x^2))/x^5,x]')

[Out] (-x - 6 x ^ 2 ArcTan[x] - 3 x ^ 3 - 3 x ^ 4 ArcTan[x] - 3 ArcTan[x]) / (12 x ^ 4)

Maple [A]

time = 0.05, size = 30, normalized size = 0.97

method	result	size
default	$-\frac{\arctan(x)}{4x^4} - \frac{\arctan(x)}{2x^2} - \frac{1}{12x^3} - \frac{1}{4x} - \frac{\arctan(x)}{4}$	30
meijerg	$-\frac{1}{12x^3} - \frac{1}{4x} - \frac{2\left(-\frac{3x^4}{8} + \frac{3}{8}\right) \arctan\left(\sqrt{x^2}\right)}{3x^3 \sqrt{x^2}} - \frac{(x^2+1) \arctan(x)}{2x^2}$	47
risch	$\frac{i(2x^2+1) \ln(ix+1)}{8x^4} - \frac{i(3 \ln(x+i)x^4 - 3 \ln(x-i)x^4 - 6ix^3 + 6x^2 \ln(-ix+1) - 2ix + 3 \ln(-ix+1))}{24x^4}$	80

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)*arctan(x)/x^5,x,method=_RETURNVERBOSE)

[Out] -1/4*arctan(x)/x^4-1/2*arctan(x)/x^2-1/12/x^3-1/4/x-1/4*arctan(x)

Maxima [A]

time = 0.33, size = 31, normalized size = 1.00

$$-\frac{3x^2 + 1}{12x^3} - \frac{(2x^2 + 1) \arctan(x)}{4x^4} - \frac{1}{4} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)*arctan(x)/x^5,x, algorithm="maxima")

[Out] $-1/12*(3*x^2 + 1)/x^3 - 1/4*(2*x^2 + 1)*\arctan(x)/x^4 - 1/4*\arctan(x)$

Fricas [A]

time = 0.32, size = 26, normalized size = 0.84

$$-\frac{3x^3 + 3(x^4 + 2x^2 + 1)\arctan(x) + x}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)*arctan(x)/x^5,x, algorithm="fricas")

[Out] $-1/12*(3*x^3 + 3*(x^4 + 2*x^2 + 1)*\arctan(x) + x)/x^4$

Sympy [A]

time = 0.30, size = 34, normalized size = 1.10

$$-\frac{\operatorname{atan}(x)}{4} - \frac{1}{4x} - \frac{\operatorname{atan}(x)}{2x^2} - \frac{1}{12x^3} - \frac{\operatorname{atan}(x)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)*atan(x)/x**5,x)

[Out] $-\operatorname{atan}(x)/4 - 1/(4*x) - \operatorname{atan}(x)/(2*x**2) - 1/(12*x**3) - \operatorname{atan}(x)/(4*x**4)$

Giac [A]

time = 0.00, size = 39, normalized size = 1.26

$$-\frac{3x^2 + 1}{12x^3} - \frac{\arctan x}{4} + \frac{(-2x^2 - 1)\arctan x}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)*arctan(x)/x^5,x)

[Out] $-1/12*(3*x^2 + 1)/x^3 - 1/4*(2*x^2 + 1)*\arctan(x)/x^4 - 1/4*\arctan(x)$

Mupad [B]

time = 0.32, size = 30, normalized size = 0.97

$$-\frac{\operatorname{atan}(x)}{4} - \frac{x}{12} + \frac{\operatorname{atan}(x)}{4} + \frac{x^2 \operatorname{atan}(x)}{2} + \frac{x^3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((atan(x)*(x^2 + 1))/x^5,x)

[Out] $-\operatorname{atan}(x)/4 - (x/12 + \operatorname{atan}(x)/4 + (x^2*\operatorname{atan}(x))/2 + x^3/4)/x^4$

$$3.678 \quad \int \frac{(1+x^2)^2 \tan^{-1}(x)}{x^5} dx$$

Optimal. Leaf size=63

$$-\frac{1}{12x^3} - \frac{3}{4x} - \frac{3}{4} \tan^{-1}(x) - \frac{\tan^{-1}(x)}{4x^4} - \frac{\tan^{-1}(x)}{x^2} + \frac{1}{2} i \text{Li}_2(-ix) - \frac{1}{2} i \text{Li}_2(ix)$$

[Out] -1/12/x^3-3/4/x-3/4*arctan(x)-1/4*arctan(x)/x^4-arctan(x)/x^2+1/2*I*polylog(2,-I*x)-1/2*I*polylog(2,I*x)

Rubi [A]

time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5068, 4946, 331, 209, 4940, 2438}

$$\frac{1}{2} i \text{Li}_2(-ix) - \frac{1}{2} i \text{Li}_2(ix) - \frac{\tan^{-1}(x)}{4x^4} - \frac{1}{12x^3} - \frac{\tan^{-1}(x)}{x^2} - \frac{3}{4x} - \frac{3}{4} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[((1 + x^2)^2*ArcTan[x])/x^5,x]

[Out] -1/12*1/x^3 - 3/(4*x) - (3*ArcTan[x])/4 - ArcTan[x]/(4*x^4) - ArcTan[x]/x^2 + (I/2)*PolyLog[2, (-I)*x] - (I/2)*PolyLog[2, I*x]

Rule 209

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 331

Int[((c_)*(x_)^m)*((a_) + (b_)*(x_)^n)^p, x_Symbol] := Simp[(c*x)^(m+1)*((a + b*x^n)^(p+1)/(a*c*(m+1))), x] - Dist[b*((m+n*(p+1)+1)/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2438

Int[Log[(c_)*((d_) + (e_)*(x_)^n)]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 4940

Int[((a_) + ArcTan[(c_)*(x_)])*(b_)]/(x_), x_Symbol] := Simp[a*Log[x], x] + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +

$I*c*x]/x, x], x]) /; \text{FreeQ}\{a, b, c\}, x]$

Rule 4946

$\text{Int}[(a_.) + \text{ArcTan}[c_.)*(x_.)^{(n_.)}]*(b_.)^{(p_.)}*(x_.)^{(m_.)}, x_Symbol] \rightarrow$
 $\text{Simp}[x^{(m+1)}*((a + b*\text{ArcTan}[c*x^n])^{p/(m+1)}), x] - \text{Dist}[b*c*n*(p/(m+1)),$
 $\text{Int}[x^{(m+n)}*((a + b*\text{ArcTan}[c*x^n])^{p-1}/(1 + c^2*x^{2*n}))], x], x$
 $] /; \text{FreeQ}\{a, b, c, m, n\}, x] \&\& \text{IGtQ}[p, 0] \&\& (\text{EqQ}[p, 1] \parallel (\text{EqQ}[n, 1] \&\&$
 $\text{IntegerQ}[m])) \&\& \text{NeQ}[m, -1]$

Rule 5068

$\text{Int}[(a_.) + \text{ArcTan}[c_.)*(x_.)]*(b_.)^{(p_.)}*((f_.)*(x_.))^{(m_.)}*((d_.) + (e_.$
 $)*(x_.)^2)^{(q_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(f*x)^m*(d + e*x^2)^q*(a +$
 $b*\text{ArcTan}[c*x])^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{EqQ}[e, c^2*$
 $d] \&\& \text{IGtQ}[p, 0] \&\& \text{IGtQ}[q, 1] \&\& (\text{EqQ}[p, 1] \parallel \text{IntegerQ}[m])$

Rubi steps

$$\begin{aligned} \int \frac{(1+x^2)^2 \tan^{-1}(x)}{x^5} dx &= \int \left(\frac{\tan^{-1}(x)}{x^5} + \frac{2 \tan^{-1}(x)}{x^3} + \frac{\tan^{-1}(x)}{x} \right) dx \\ &= 2 \int \frac{\tan^{-1}(x)}{x^3} dx + \int \frac{\tan^{-1}(x)}{x^5} dx + \int \frac{\tan^{-1}(x)}{x} dx \\ &= -\frac{\tan^{-1}(x)}{4x^4} - \frac{\tan^{-1}(x)}{x^2} + \frac{1}{2}i \int \frac{\log(1-ix)}{x} dx - \frac{1}{2}i \int \frac{\log(1+ix)}{x} dx + \frac{1}{4} \int \frac{1}{x} dx \\ &= -\frac{1}{12x^3} - \frac{1}{x} - \frac{\tan^{-1}(x)}{4x^4} - \frac{\tan^{-1}(x)}{x^2} + \frac{1}{2}i \text{Li}_2(-ix) - \frac{1}{2}i \text{Li}_2(ix) - \frac{1}{4} \int \frac{1}{x^2(1+x^2)} dx \\ &= -\frac{1}{12x^3} - \frac{3}{4x} - \tan^{-1}(x) - \frac{\tan^{-1}(x)}{4x^4} - \frac{\tan^{-1}(x)}{x^2} + \frac{1}{2}i \text{Li}_2(-ix) - \frac{1}{2}i \text{Li}_2(ix) + \frac{1}{4} \log|x^2+1| \\ &= -\frac{1}{12x^3} - \frac{3}{4x} - \frac{3}{4} \tan^{-1}(x) - \frac{\tan^{-1}(x)}{4x^4} - \frac{\tan^{-1}(x)}{x^2} + \frac{1}{2}i \text{Li}_2(-ix) - \frac{1}{2}i \text{Li}_2(ix) \end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 0.01, size = 81, normalized size = 1.29

$$-\frac{\tan^{-1}(x)}{4x^4} - \frac{\tan^{-1}(x)}{x^2} - \frac{{}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; -x^2\right)}{12x^3} - \frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; -x^2\right)}{x} + \frac{1}{2}i \text{Li}_2(-ix) - \frac{1}{2}i \text{Li}_2(ix)$$

Antiderivative was successfully verified.

[In] Integrate[((1 + x^2)^2*ArcTan[x])/x^5, x]

[Out] $-1/4 \cdot \text{ArcTan}[x]/x^4 - \text{ArcTan}[x]/x^2 - \text{Hypergeometric2F1}[-3/2, 1, -1/2, -x^2]/(12x^3) - \text{Hypergeometric2F1}[-1/2, 1, 1/2, -x^2]/x + (I/2) \cdot \text{PolyLog}[2, (-I)x] - (I/2) \cdot \text{PolyLog}[2, Ix]$

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(ArcTan[x]*(1 + x^2)^2)/x^5,x]')`

[Out] Timed out

Maple [A]

time = 0.10, size = 79, normalized size = 1.25

method	result
default	$\arctan(x) \ln(x) - \frac{\arctan(x)}{4x^4} - \frac{\arctan(x)}{x^2} + \frac{i \ln(x) \ln(ix+1)}{2} - \frac{i \ln(x) \ln(-ix+1)}{2} + \frac{i \operatorname{dilog}(ix+1)}{2} - \frac{i \operatorname{dilog}(-ix+1)}{2}$
meijerg	$-\frac{1}{12x^3} - \frac{3}{4x} - \frac{2\left(-\frac{3x^4}{8} + \frac{3}{8}\right) \arctan\left(\sqrt{x^2}\right)}{3x^3 \sqrt{x^2}} - \frac{ix \operatorname{polylog}\left(2, i\sqrt{x^2}\right)}{2\sqrt{x^2}} + \frac{ix \operatorname{polylog}\left(2, -i\sqrt{x^2}\right)}{2\sqrt{x^2}} - \frac{(x^2+1) \arctan(x)}{x^2}$
risch	$-\frac{1}{12x^3} + \frac{3i \ln(-ix)}{8} - \frac{3}{4x} - \frac{3 \arctan(x)}{4} - \frac{i \ln(-ix+1)}{8x^4} - \frac{i \operatorname{dilog}(-ix+1)}{2} - \frac{i \ln(-ix+1)}{2x^2} - \frac{3i \ln(ix)}{8} + \frac{i \ln(ix+1)}{8x^4} + \frac{i \operatorname{dilog}(ix+1)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+1)^2*arctan(x)/x^5,x,method=_RETURNVERBOSE)`

[Out] $\arctan(x) \cdot \ln(x) - 1/4 \cdot \arctan(x)/x^4 - \arctan(x)/x^2 + 1/2 \cdot I \cdot \ln(x) \cdot \ln(1+Ix) - 1/2 \cdot I \cdot \ln(x) \cdot \ln(1-Ix) + 1/2 \cdot I \cdot \operatorname{dilog}(1+Ix) - 1/2 \cdot I \cdot \operatorname{dilog}(1-Ix) - 1/12/x^3 - 3/4/x - 3/4 \cdot \arctan(x)$

Maxima [A]

time = 0.36, size = 71, normalized size = 1.13

$$\frac{3\pi x^4 \log(x^2 + 1) - 12x^4 \arctan(x) \log(x) + 6ix^4 \operatorname{Li}_2(ix + 1) - 6ix^4 \operatorname{Li}_2(-ix + 1) + 9x^3 + 3(3x^4 + 4x^2 + 1) \arctan(x) + x}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)^2*arctan(x)/x^5,x, algorithm="maxima")`

[Out] $-1/12 \cdot (3\pi x^4 \cdot \log(x^2 + 1) - 12x^4 \cdot \arctan(x) \cdot \log(x) + 6Ix^4 \cdot \operatorname{dilog}(Ix + 1) - 6Ix^4 \cdot \operatorname{dilog}(-Ix + 1) + 9x^3 + 3(3x^4 + 4x^2 + 1) \cdot \arctan(x) + x)/x^4$

Fricas [F]

time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^2*arctan(x)/x^5,x, algorithm="fricas")

[Out] integral((x^4 + 2*x^2 + 1)*arctan(x)/x^5, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2 + 1)^2 \operatorname{atan}(x)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)**2*atan(x)/x**5,x)

[Out] Integral((x**2 + 1)**2*atan(x)/x**5, x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^2*arctan(x)/x^5,x)

[Out] Could not integrate

Mupad [B]

time = 0.50, size = 53, normalized size = 0.84

$$\frac{x^2 - \frac{1}{3}}{4x^3} - \frac{\operatorname{atan}(x)}{x^2} - \frac{\operatorname{atan}(x)}{4x^4} - \frac{3\operatorname{atan}(x)}{4} - \frac{1}{x} - \frac{\operatorname{Li}_2(1 - x \operatorname{li}) \operatorname{li}}{2} + \frac{\operatorname{polylog}(2, -x \operatorname{li}) \operatorname{li}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((atan(x)*(x^2 + 1)^2)/x^5,x)

[Out] (polylog(2, -x*1i)*1i)/2 - (3*atan(x))/4 - atan(x)/x^2 - atan(x)/(4*x^4) - (dilog(1 - x*1i)*1i)/2 + (x^2 - 1/3)/(4*x^3) - 1/x

$$3.679 \quad \int \frac{\tan^{-1}(x)}{x^2(1+x^2)} dx$$

Optimal. Leaf size=28

$$-\frac{\tan^{-1}(x)}{x} - \frac{1}{2}\tan^{-1}(x)^2 + \log(x) - \frac{1}{2}\log(1+x^2)$$

[Out] -arctan(x)/x-1/2*arctan(x)^2+ln(x)-1/2*ln(x^2+1)

Rubi [A]

time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {5038, 4946, 272, 36, 29, 31, 5004}

$$-\frac{1}{2}\log(x^2+1) + \log(x) - \frac{1}{2}\tan^{-1}(x)^2 - \frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[x]/(x^2*(1+x^2)),x]

[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 + Log[x] - Log[1+x^2]/2

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_)*(x_))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 272

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m+1)/n]]

Rule 4946

Int[((a_) + ArcTan[(c_)*(x_)^(n_)])*(b_)^(p_)*(x_)^(m_), x_Symbol] := Simp[x^(m+1)*((a + b*ArcTan[c*x^n])^p/(m+1)), x] - Dist[b*c*n*(p/(m+1))

```
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_)^2), x_Symbol
] := Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_)^(m_.))/((d_.) + (e
_.)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(x)}{x^2(1+x^2)} dx &= \int \frac{\tan^{-1}(x)}{x^2} dx - \int \frac{\tan^{-1}(x)}{1+x^2} dx \\
&= -\frac{\tan^{-1}(x)}{x} - \frac{1}{2} \tan^{-1}(x)^2 + \int \frac{1}{x(1+x^2)} dx \\
&= -\frac{\tan^{-1}(x)}{x} - \frac{1}{2} \tan^{-1}(x)^2 + \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(1+x)} dx, x, x^2 \right) \\
&= -\frac{\tan^{-1}(x)}{x} - \frac{1}{2} \tan^{-1}(x)^2 + \frac{1}{2} \text{Subst} \left(\int \frac{1}{x} dx, x, x^2 \right) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+x} dx, x, x^2 \right) \\
&= -\frac{\tan^{-1}(x)}{x} - \frac{1}{2} \tan^{-1}(x)^2 + \log(x) - \frac{1}{2} \log(1+x^2)
\end{aligned}$$

Mathematica [A]

time = 0.00, size = 28, normalized size = 1.00

$$-\frac{\tan^{-1}(x)}{x} - \frac{1}{2} \tan^{-1}(x)^2 + \log(x) - \frac{1}{2} \log(1+x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x]/(x^2*(1 + x^2)), x]
```

```
[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 + Log[x] - Log[1 + x^2]/2
```

Mathics [A]

time = 1.80, size = 24, normalized size = 0.86

$$-\frac{\text{ArcTan}[x]}{x} + \text{Log}[x] - \frac{\text{Log}[1+x^2]}{2} - \frac{\text{ArcTan}[x]^2}{2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[ArcTan[x]/(x^2*(1+x^2)),x]')``[Out] -ArcTan[x] / x + Log[x] - Log[1 + x ^ 2] / 2 - ArcTan[x] ^ 2 / 2`**Maple [A]**

time = 0.12, size = 25, normalized size = 0.89

method	result	size
default	$-\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} + \ln(x) - \frac{\ln(x^2+1)}{2}$	25
risch	$\frac{\ln(ix+1)^2}{8} - \frac{(\ln(-ix+1)x-2i)\ln(ix+1)}{4x} - \frac{-x\ln(-ix+1)^2+4i\ln(-ix+1)-8x\ln(x)+4x\ln(x^2+1)}{8x}$	79

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(arctan(x)/x^2/(x^2+1),x,method=_RETURNVERBOSE)``[Out] -1/x*arctan(x)-1/2*arctan(x)^2+ln(x)-1/2*ln(x^2+1)`**Maxima [A]**

time = 0.34, size = 27, normalized size = 0.96

$$-\left(\frac{1}{x} + \arctan(x)\right) \arctan(x) + \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log(x^2 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(arctan(x)/x^2/(x^2+1),x, algorithm="maxima")``[Out] -(1/x + arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1) + log(x)`**Fricas [A]**

time = 0.32, size = 29, normalized size = 1.04

$$-\frac{x \arctan(x)^2 + x \log(x^2 + 1) - 2x \log(x) + 2 \arctan(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(arctan(x)/x^2/(x^2+1),x, algorithm="fricas")``[Out] -1/2*(x*arctan(x)^2 + x*log(x^2 + 1) - 2*x*log(x) + 2*arctan(x))/x`

Sympy [A]

time = 0.20, size = 22, normalized size = 0.79

$$\log(x) - \frac{\log(x^2 + 1)}{2} - \frac{\operatorname{atan}^2(x)}{2} - \frac{\operatorname{atan}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(x)/x**2/(x**2+1),x)

[Out] log(x) - log(x**2 + 1)/2 - atan(x)**2/2 - atan(x)/x

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(x)/x^2/(x^2+1),x)

[Out] Could not integrate

Mupad [B]

time = 0.09, size = 24, normalized size = 0.86

$$\ln(x) - \frac{\ln(x^2 + 1)}{2} - \frac{\operatorname{atan}(x)}{x} - \frac{\operatorname{atan}(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atan(x)/(x^2*(x^2 + 1)),x)

[Out] log(x) - log(x^2 + 1)/2 - atan(x)/x - atan(x)^2/2

$$3.680 \quad \int \frac{\tan^{-1}(x)^2}{x^3} dx$$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}(x)}{x} - \frac{1}{2} \tan^{-1}(x)^2 - \frac{\tan^{-1}(x)^2}{2x^2} + \log(x) - \frac{1}{2} \log(1+x^2)$$

[Out] -arctan(x)/x-1/2*arctan(x)^2-1/2*arctan(x)^2/x^2+ln(x)-1/2*ln(x^2+1)

Rubi [A]

time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {4946, 5038, 272, 36, 29, 31, 5004}

$$-\frac{1}{2} \log(x^2+1) - \frac{\tan^{-1}(x)^2}{2x^2} + \log(x) - \frac{1}{2} \tan^{-1}(x)^2 - \frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[x]^2/x^3,x]

[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 - ArcTan[x]^2/(2*x^2) + Log[x] - Log[1 + x^2]/2

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 272

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4946

Int[((a_) + ArcTan[(c_)*(x_)^(n_)])*(b_)^(p_)*(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +

1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] && IntegerQ[m])) && NeQ[m, -1]

Rule 5004

Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]

Rule 5038

Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_)^(m_)))/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{\tan^{-1}(x)^2}{x^3} dx &= -\frac{\tan^{-1}(x)^2}{2x^2} + \int \frac{\tan^{-1}(x)}{x^2(1+x^2)} dx \\
 &= -\frac{\tan^{-1}(x)^2}{2x^2} + \int \frac{\tan^{-1}(x)}{x^2} dx - \int \frac{\tan^{-1}(x)}{1+x^2} dx \\
 &= -\frac{\tan^{-1}(x)}{x} - \frac{1}{2}\tan^{-1}(x)^2 - \frac{\tan^{-1}(x)^2}{2x^2} + \int \frac{1}{x(1+x^2)} dx \\
 &= -\frac{\tan^{-1}(x)}{x} - \frac{1}{2}\tan^{-1}(x)^2 - \frac{\tan^{-1}(x)^2}{2x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)} dx, x, x^2\right) \\
 &= -\frac{\tan^{-1}(x)}{x} - \frac{1}{2}\tan^{-1}(x)^2 - \frac{\tan^{-1}(x)^2}{2x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x} dx, x, x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x} dx, x, x^2\right) \\
 &= -\frac{\tan^{-1}(x)}{x} - \frac{1}{2}\tan^{-1}(x)^2 - \frac{\tan^{-1}(x)^2}{2x^2} + \log(x) - \frac{1}{2}\log(1+x^2)
 \end{aligned}$$

Mathematica [A]

time = 0.01, size = 38, normalized size = 0.97

$$-\frac{\tan^{-1}(x)}{x} + \frac{(-1-x^2)\tan^{-1}(x)^2}{2x^2} + \log(x) - \frac{1}{2}\log(1+x^2)$$

Antiderivative was successfully verified.

[In] Integrate[ArcTan[x]^2/x^3,x]

[Out] -(ArcTan[x]/x) + ((-1 - x^2)*ArcTan[x]^2)/(2*x^2) + Log[x] - Log[1 + x^2]/2

Mathics [A]

time = 1.81, size = 33, normalized size = 0.85

$$-\frac{\text{ArcTan}[x]^2}{2x^2} - \frac{\text{ArcTan}[x]}{x} + \text{Log}[x] - \frac{\text{Log}[1+x^2]}{2} - \frac{\text{ArcTan}[x]^2}{2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[ArcTan[x]^2/x^3,x]')
```

```
[Out] -ArcTan[x]^2/(2x^2) - ArcTan[x]/x + Log[x] - Log[1+x^2]/2 - ArcTan[x]^2/2
```

Maple [A]

time = 0.07, size = 34, normalized size = 0.87

method	result
default	$-\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} - \frac{\arctan(x)^2}{2x^2} + \ln(x) - \frac{\ln(x^2+1)}{2}$
risch	$\frac{(x^2+1)\ln(ix+1)^2}{8x^2} - \frac{(x^2\ln(-ix+1)-2ix+\ln(-ix+1))\ln(ix+1)}{4x^2} + \frac{x^2\ln(-ix+1)^2+8x^2\ln(x)-4x^2\ln(x^2+1)-4ix\ln(-ix+1)+\ln(-ix+1)}{8x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x*arctan(x)-1/2*arctan(x)^2-1/2*arctan(x)^2/x^2+ln(x)-1/2*ln(x^2+1)
```

Maxima [A]

time = 0.33, size = 36, normalized size = 0.92

$$-\left(\frac{1}{x} + \arctan(x)\right) \arctan(x) + \frac{1}{2} \arctan(x)^2 - \frac{\arctan(x)^2}{2x^2} - \frac{1}{2} \log(x^2 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2/x^3,x, algorithm="maxima")
```

```
[Out] -(1/x + arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*arctan(x)^2/x^2 - 1/2*log(x^2 + 1) + log(x)
```

Fricas [A]

time = 0.32, size = 38, normalized size = 0.97

$$-\frac{(x^2 + 1) \arctan(x)^2 + x^2 \log(x^2 + 1) - 2x^2 \log(x) + 2x \arctan(x)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2/x^3,x, algorithm="fricas")
```


[Out] $-1/2*((x^2 + 1)*\arctan(x)^2 + x^2*\log(x^2 + 1) - 2*x^2*\log(x) + 2*x*\arctan(x))/x^2$

Sympy [A]

time = 0.21, size = 32, normalized size = 0.82

$$\log(x) - \frac{\log(x^2 + 1)}{2} - \frac{\operatorname{atan}^2(x)}{2} - \frac{\operatorname{atan}(x)}{x} - \frac{\operatorname{atan}^2(x)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(atan(x)**2/x**3,x)`

[Out] $\log(x) - \log(x^2 + 1)/2 - \operatorname{atan}(x)**2/2 - \operatorname{atan}(x)/x - \operatorname{atan}(x)**2/(2*x**2)$

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(x)^2/x^3,x)`

[Out] Could not integrate

Mupad [B]

time = 0.07, size = 31, normalized size = 0.79

$$\ln(x) - \frac{\ln(x^2 + 1)}{2} - \frac{\operatorname{atan}(x)}{x} - \operatorname{atan}(x)^2 \left(\frac{1}{2x^2} + \frac{1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(atan(x)^2/x^3,x)`

[Out] $\log(x) - \log(x^2 + 1)/2 - \operatorname{atan}(x)/x - \operatorname{atan}(x)^2*(1/(2*x^2) + 1/2)$

$$3.681 \quad \int \frac{(1+x^2) \tan^{-1}(x)^2}{x^5} dx$$

Optimal. Leaf size=60

$$-\frac{1}{12x^2} - \frac{\tan^{-1}(x)}{6x^3} - \frac{\tan^{-1}(x)}{2x} - \frac{(1+x^2)^2 \tan^{-1}(x)^2}{4x^4} + \frac{\log(x)}{3} - \frac{1}{6} \log(1+x^2)$$

[Out] -1/12/x^2-1/6*arctan(x)/x^3-1/2*arctan(x)/x-1/4*(x^2+1)^2*arctan(x)^2/x^4+1/3*ln(x)-1/6*ln(x^2+1)

Rubi [A]

time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {5064, 5070, 4946, 272, 46, 36, 29, 31}

$$-\frac{\tan^{-1}(x)}{6x^3} - \frac{1}{12x^2} - \frac{1}{6} \log(x^2 + 1) - \frac{(x^2 + 1)^2 \tan^{-1}(x)^2}{4x^4} + \frac{\log(x)}{3} - \frac{\tan^{-1}(x)}{2x}$$

Antiderivative was successfully verified.

[In] Int[((1 + x^2)*ArcTan[x]^2)/x^5, x]

[Out] -1/12*1/x^2 - ArcTan[x]/(6*x^3) - ArcTan[x]/(2*x) - ((1 + x^2)^2*ArcTan[x]^2)/(4*x^4) + Log[x]/3 - Log[1 + x^2]/6

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 46

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :=
Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
IntegerQ[m])) && NeQ[m, -1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*((f_.)*(x_)^(m_.))*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 5070

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*((f_.)*(x_)^(m_.))*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] := Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[c^2*(d/f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^(
q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
IntegerQ[q]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^2)\tan^{-1}(x)^2}{x^5} dx &= -\frac{(1+x^2)^2 \tan^{-1}(x)^2}{4x^4} + \frac{1}{2} \int \frac{(1+x^2)\tan^{-1}(x)}{x^4} dx \\
&= -\frac{(1+x^2)^2 \tan^{-1}(x)^2}{4x^4} + \frac{1}{2} \int \frac{\tan^{-1}(x)}{x^4} dx + \frac{1}{2} \int \frac{\tan^{-1}(x)}{x^2} dx \\
&= -\frac{\tan^{-1}(x)}{6x^3} - \frac{\tan^{-1}(x)}{2x} - \frac{(1+x^2)^2 \tan^{-1}(x)^2}{4x^4} + \frac{1}{6} \int \frac{1}{x^3(1+x^2)} dx + \frac{1}{2} \int \frac{1}{x(1+x^2)} dx \\
&= -\frac{\tan^{-1}(x)}{6x^3} - \frac{\tan^{-1}(x)}{2x} - \frac{(1+x^2)^2 \tan^{-1}(x)^2}{4x^4} + \frac{1}{12} \text{Subst} \left(\int \frac{1}{x^2(1+x)} dx, x, x^2 \right) \\
&= -\frac{\tan^{-1}(x)}{6x^3} - \frac{\tan^{-1}(x)}{2x} - \frac{(1+x^2)^2 \tan^{-1}(x)^2}{4x^4} + \frac{1}{12} \text{Subst} \left(\int \left(\frac{1}{x^2} - \frac{1}{x} + \frac{1}{1+x} \right) dx, x, x^2 \right) \\
&= -\frac{1}{12x^2} - \frac{\tan^{-1}(x)}{6x^3} - \frac{\tan^{-1}(x)}{2x} - \frac{(1+x^2)^2 \tan^{-1}(x)^2}{4x^4} + \frac{\log(x)}{3} - \frac{1}{6} \log(1+x^2)
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 56, normalized size = 0.93

$$\frac{-2(x+3x^3)\tan^{-1}(x) - 3(1+x^2)^2 \tan^{-1}(x)^2 + x^2(-1+4x^2 \log(x) - 2x^2 \log(1+x^2))}{12x^4}$$

Antiderivative was successfully verified.

`[In] Integrate[((1 + x^2)*ArcTan[x]^2)/x^5,x]``[Out] (-2*(x + 3*x^3)*ArcTan[x] - 3*(1 + x^2)^2*ArcTan[x]^2 + x^2*(-1 + 4*x^2*Log[x] - 2*x^2*Log[1 + x^2]))/(12*x^4)`**Mathics [A]**

time = 1.99, size = 60, normalized size = 1.00

$$\frac{-2x \text{ArcTan}[x] - x^2(1 + 6 \text{ArcTan}[x]^2) - 6x^3 \text{ArcTan}[x] + x^4(-3 \text{ArcTan}[x]^2 - 2 \text{Log}[1 + x^2] + 4 \text{Log}[x]) - 3 \text{ArcTan}[x]^2}{12x^4}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[(ArcTan[x]^2*(1 + x^2))/x^5,x]')``[Out] (-2 x ArcTan[x] - x^2 (1 + 6 ArcTan[x]^2) - 6 x^3 ArcTan[x] + x^4 (-3 ArcTan[x]^2 - 2 Log[1 + x^2] + 4 Log[x]) - 3 ArcTan[x]^2) / (12 x^4)`**Maple [A]**

time = 0.08, size = 57, normalized size = 0.95

method	result
default	$-\frac{\arctan(x)^2}{4x^4} - \frac{\arctan(x)^2}{2x^2} - \frac{\arctan(x)}{6x^3} - \frac{\arctan(x)}{2x} - \frac{\arctan(x)^2}{4} - \frac{1}{12x^2} + \frac{\ln(x)}{3} - \frac{\ln(x^2+1)}{6}$
risch	$\frac{(x^4+2x^2+1)\ln(ix+1)^2}{16x^4} - \frac{(3x^4\ln(-ix+1)-6ix^3+6x^2\ln(-ix+1)-2ix+3\ln(-ix+1))\ln(ix+1)}{24x^4} + \frac{3x^4\ln(-ix+1)^2-12ix^3\ln(-ix+1)}{24x^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+1)*arctan(x)^2/x^5,x,method=_RETURNVERBOSE)`

[Out] $-1/4*\arctan(x)^2/x^4-1/2*\arctan(x)^2/x^2-1/6/x^3*\arctan(x)-1/2/x*\arctan(x)-1/4*\arctan(x)^2-1/12/x^2+1/3*\ln(x)-1/6*\ln(x^2+1)$

Maxima [A]

time = 0.34, size = 71, normalized size = 1.18

$$-\frac{1}{6} \left(\frac{3x^2+1}{x^3} + 3 \arctan(x) \right) \arctan(x) + \frac{3x^2 \arctan(x)^2 - 2x^2 \log(x^2+1) + 4x^2 \log(x) - 1}{12x^2} - \frac{(2x^2+1) \arctan(x)^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)*arctan(x)^2/x^5,x, algorithm="maxima")`

[Out] $-1/6*((3*x^2 + 1)/x^3 + 3*\arctan(x))*\arctan(x) + 1/12*(3*x^2*\arctan(x)^2 - 2*x^2*\log(x^2 + 1) + 4*x^2*\log(x) - 1)/x^2 - 1/4*(2*x^2 + 1)*\arctan(x)^2/x^4$

Fricas [A]

time = 0.32, size = 54, normalized size = 0.90

$$\frac{2x^4 \log(x^2+1) - 4x^4 \log(x) + 3(x^4 + 2x^2 + 1) \arctan(x)^2 + x^2 + 2(3x^3 + x) \arctan(x)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)*arctan(x)^2/x^5,x, algorithm="fricas")`

[Out] $-1/12*(2*x^4*\log(x^2 + 1) - 4*x^4*\log(x) + 3*(x^4 + 2*x^2 + 1)*\arctan(x)^2 + x^2 + 2*(3*x^3 + x)*\arctan(x))/x^4$

Sympy [A]

time = 0.30, size = 61, normalized size = 1.02

$$\frac{\log(x)}{3} - \frac{\log(x^2+1)}{6} - \frac{\operatorname{atan}^2(x)}{4} - \frac{\operatorname{atan}(x)}{2x} - \frac{\operatorname{atan}^2(x)}{2x^2} - \frac{1}{12x^2} - \frac{\operatorname{atan}(x)}{6x^3} - \frac{\operatorname{atan}^2(x)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+1)*atan(x)**2/x**5,x)`

[Out] $\log(x)/3 - \log(x^2 + 1)/6 - \operatorname{atan}(x)^2/4 - \operatorname{atan}(x)/(2x) - \operatorname{atan}(x)^2/(2x^2) - 1/(12x^2) - \operatorname{atan}(x)/(6x^3) - \operatorname{atan}(x)^2/(4x^4)$

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)*arctan(x)^2/x^5,x)`

[Out] Could not integrate

Mupad [B]

time = 0.12, size = 51, normalized size = 0.85

$$\frac{\ln(x)}{3} - \frac{\ln(x^2 + 1)}{6} - \operatorname{atan}(x)^2 \left(\frac{\frac{x^2}{2} + \frac{1}{4}}{x^4} + \frac{1}{4} \right) - \frac{1}{12x^2} - \frac{\operatorname{atan}(x) \left(\frac{x^2}{2} + \frac{1}{6} \right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((atan(x)^2*(x^2 + 1))/x^5,x)`

[Out] $\log(x)/3 - \log(x^2 + 1)/6 - \operatorname{atan}(x)^2((x^2/2 + 1/4)/x^4 + 1/4) - 1/(12x^2) - (\operatorname{atan}(x)*(x^2/2 + 1/6))/x^3$

$$3.682 \quad \int \frac{x^3 \tan^{-1}(x)^2}{(1+x^2)^3} dx$$

Optimal. Leaf size=79

$$-\frac{1}{32(1+x^2)^2} + \frac{5}{32(1+x^2)} + \frac{x^3 \tan^{-1}(x)}{8(1+x^2)^2} + \frac{3x \tan^{-1}(x)}{16(1+x^2)} - \frac{3}{32} \tan^{-1}(x)^2 + \frac{x^4 \tan^{-1}(x)^2}{4(1+x^2)^2}$$

[Out] $-1/32/(x^2+1)^2+5/32/(x^2+1)+1/8*x^3*\arctan(x)/(x^2+1)^2+3/16*x*\arctan(x)/(x^2+1)-3/32*\arctan(x)^2+1/4*x^4*\arctan(x)^2/(x^2+1)^2$

Rubi [A]

time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$,

Rules used = {5064, 5058, 5054, 5004}

$$\frac{3}{32(x^2+1)} + \frac{3x \tan^{-1}(x)}{16(x^2+1)} - \frac{x^4}{32(x^2+1)^2} + \frac{x^4 \tan^{-1}(x)^2}{4(x^2+1)^2} + \frac{x^3 \tan^{-1}(x)}{8(x^2+1)^2} - \frac{3}{32} \tan^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] Int[(x^3*ArcTan[x]^2)/(1+x^2)^3,x]

[Out] $-1/32*x^4/(1+x^2)^2 + 3/(32*(1+x^2)) + (x^3*ArcTan[x])/(8*(1+x^2)^2) + (3*x*ArcTan[x])/(16*(1+x^2)) - (3*ArcTan[x]^2)/32 + (x^4*ArcTan[x]^2)/(4*(1+x^2)^2)$

Rule 5004

Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_.)^2), x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]

Rule 5054

Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))*(x_.)^2*((d_.) + (e_.)*(x_.)^2)^(q_.), x_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (-Dist[1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] + Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]

Rule 5058

Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_.)^2)^(q_.), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] + (Dist[f^2*((m - 1)/(c^2*d*m)), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] - Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*A

```
rcTan[c*x])/(c^2*d*m)), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d]
] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^(
m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^3 \tan^{-1}(x)^2}{(1+x^2)^3} dx &= \frac{x^4 \tan^{-1}(x)^2}{4(1+x^2)^2} - \frac{1}{2} \int \frac{x^4 \tan^{-1}(x)}{(1+x^2)^3} dx \\
 &= -\frac{x^4}{32(1+x^2)^2} + \frac{x^3 \tan^{-1}(x)}{8(1+x^2)^2} + \frac{x^4 \tan^{-1}(x)^2}{4(1+x^2)^2} - \frac{3}{8} \int \frac{x^2 \tan^{-1}(x)}{(1+x^2)^2} dx \\
 &= -\frac{x^4}{32(1+x^2)^2} + \frac{3}{32(1+x^2)} + \frac{x^3 \tan^{-1}(x)}{8(1+x^2)^2} + \frac{3x \tan^{-1}(x)}{16(1+x^2)} + \frac{x^4 \tan^{-1}(x)^2}{4(1+x^2)^2} - \frac{3}{16} \int \frac{\tan^{-1}(x)}{1+x^2} dx \\
 &= -\frac{x^4}{32(1+x^2)^2} + \frac{3}{32(1+x^2)} + \frac{x^3 \tan^{-1}(x)}{8(1+x^2)^2} + \frac{3x \tan^{-1}(x)}{16(1+x^2)} - \frac{3}{32} \tan^{-1}(x)^2 + \frac{x^4 \tan^{-1}(x)^2}{4(1+x^2)^2}
 \end{aligned}$$

Mathematica [A]

time = 0.03, size = 47, normalized size = 0.59

$$\frac{4 + 5x^2 + 2x(3 + 5x^2) \tan^{-1}(x) + (-3 - 6x^2 + 5x^4) \tan^{-1}(x)^2}{32(1+x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[x]^2)/(1 + x^2)^3,x]
```

```
[Out] (4 + 5*x^2 + 2*x*(3 + 5*x^2)*ArcTan[x] + (-3 - 6*x^2 + 5*x^4)*ArcTan[x]^2)/
(32*(1 + x^2)^2)
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(ArcTan[x]^2*x^3)/(1 + x^2)^3,x]')`

[Out] `cought exception: maximum recursion depth exceeded in comparison`

Maple [A]

time = 0.14, size = 78, normalized size = 0.99

method	result
default	$\frac{\arctan(x)^2}{4(x^2+1)^2} - \frac{\arctan(x)^2}{2(x^2+1)} + \frac{5x^3 \arctan(x)}{16(x^2+1)^2} + \frac{3x \arctan(x)}{16(x^2+1)^2} + \frac{5 \arctan(x)^2}{32} - \frac{1}{32(x^2+1)^2} + \frac{5}{32(x^2+1)}$
risch	$-\frac{(5x^4-6x^2-3) \ln(ix+1)^2}{128(x^2+1)^2} + \frac{(-6x^2 \ln(-ix+1) - 3 \ln(-ix+1) + 5x^4 \ln(-ix+1) - 10ix^3 - 6ix) \ln(ix+1)}{64(x+i)^2(x-i)^2} - \frac{5x^4 \ln(-ix+1)^2 - 6x^2 \ln(-ix+1)}{64(x+i)^2(x-i)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*arctan(x)^2/(x^2+1)^3,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{4} \arctan(x)^2 / (x^2+1)^2 - \frac{1}{2} \arctan(x)^2 / (x^2+1) + \frac{5}{16} x^3 \arctan(x) / (x^2+1)^2 + \frac{3}{16} x \arctan(x) / (x^2+1)^2 + \frac{5}{32} \arctan(x)^2 - \frac{1}{32} / (x^2+1)^2 + \frac{5}{32} / (x^2+1)$

Maxima [A]

time = 0.33, size = 94, normalized size = 1.19

$$\frac{1}{16} \left(\frac{5x^3 + 3x}{x^4 + 2x^2 + 1} + 5 \arctan(x) \right) \arctan(x) - \frac{(2x^2 + 1) \arctan(x)^2}{4(x^4 + 2x^2 + 1)} - \frac{5(x^4 + 2x^2 + 1) \arctan(x)^2 - 5x^2 - 4}{32(x^4 + 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arctan(x)^2/(x^2+1)^3,x, algorithm="maxima")`

[Out] $\frac{1}{16} \left(\frac{5x^3 + 3x}{x^4 + 2x^2 + 1} + 5 \arctan(x) \right) \arctan(x) - \frac{1}{4} \frac{(2x^2 + 1) \arctan(x)^2}{x^4 + 2x^2 + 1} - \frac{1}{32} \frac{5(x^4 + 2x^2 + 1) \arctan(x)^2 - 5x^2 - 4}{x^4 + 2x^2 + 1}$

Fricas [A]

time = 0.31, size = 51, normalized size = 0.65

$$\frac{(5x^4 - 6x^2 - 3) \arctan(x)^2 + 5x^2 + 2(5x^3 + 3x) \arctan(x) + 4}{32(x^4 + 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arctan(x)^2/(x^2+1)^3,x, algorithm="fricas")`

[Out] $\frac{1}{32} \left(\frac{5x^4 - 6x^2 - 3}{x^4 + 2x^2 + 1} \arctan(x)^2 + 5x^2 + 2(5x^3 + 3x) \arctan(x) + 4 \right) / (x^4 + 2x^2 + 1)$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{atan}^2(x)}{(x^2 + 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*atan(x)**2/(x**2+1)**3,x)

[Out] Integral(x**3*atan(x)**2/(x**2 + 1)**3, x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arctan(x)^2/(x^2+1)^3,x)

[Out] Could not integrate

Mupad [B]

time = 0.36, size = 56, normalized size = 0.71

$$\frac{-5x^4 \operatorname{atan}(x)^2 + 4x^4 - 10x^3 \operatorname{atan}(x) + 6x^2 \operatorname{atan}(x)^2 + 3x^2 - 6x \operatorname{atan}(x) + 3 \operatorname{atan}(x)^2}{32(x^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*atan(x)^2)/(x^2 + 1)^3,x)

[Out] $-(3*\operatorname{atan}(x)^2 - 10*x^3*\operatorname{atan}(x) + 6*x^2*\operatorname{atan}(x)^2 - 5*x^4*\operatorname{atan}(x)^2 - 6*x*\operatorname{atan}(x) + 3*x^2 + 4*x^4)/(32*(x^2 + 1)^2)$

$$3.683 \quad \int \frac{\sqrt{-1+x^2} \sec^{-1}(x)}{x^2} dx$$

Optimal. Leaf size=107

$$-\frac{1}{\sqrt{x^2}} - \frac{\sqrt{-1+x^2} \sec^{-1}(x)}{x} - \frac{2i\sqrt{x^2} \sec^{-1}(x) \tan^{-1}\left(e^{i \sec^{-1}(x)}\right)}{x} + \frac{i\sqrt{x^2} \operatorname{Li}_2\left(-ie^{i \sec^{-1}(x)}\right)}{x} - \frac{i\sqrt{x^2} \operatorname{Li}_2\left(ie^{i \sec^{-1}(x)}\right)}{x}$$

[Out] $-(x^2)^{(1/2)}/x^2 - 2*I*\operatorname{arcsec}(x)*\arctan(1/x + I*(1-1/x^2)^{(1/2)})*(x^2)^{(1/2)}/x + I*\operatorname{polylog}(2, -I*(1/x + I*(1-1/x^2)^{(1/2)}))*(x^2)^{(1/2)}/x - I*\operatorname{polylog}(2, I*(1/x + I*(1-1/x^2)^{(1/2)}))*(x^2)^{(1/2)}/x - \operatorname{arcsec}(x)*(x^2-1)^{(1/2)}/x$

Rubi [A]

time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5350, 4784, 4804, 4266, 2317, 2438, 8}

$$\frac{i\sqrt{x^2} \operatorname{Li}_2\left(-ie^{i \sec^{-1}(x)}\right)}{x} - \frac{i\sqrt{x^2} \operatorname{Li}_2\left(ie^{i \sec^{-1}(x)}\right)}{x} - \frac{1}{\sqrt{x^2}} - \frac{\sqrt{1-\frac{1}{x^2}} \sqrt{x^2} \sec^{-1}(x)}{x} - \frac{2i\sqrt{x^2} \sec^{-1}(x) \tan^{-1}\left(e^{i \sec^{-1}(x)}\right)}{x}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sqrt}[-1+x^2]*\operatorname{ArcSec}[x])/x^2, x]$

[Out] $-(1/\operatorname{Sqrt}[x^2]) - (\operatorname{Sqrt}[1-x^{(-2)}]*\operatorname{Sqrt}[x^2]*\operatorname{ArcSec}[x])/x - ((2*I)*\operatorname{Sqrt}[x^2]*\operatorname{ArcSec}[x]*\operatorname{ArcTan}[E^{(I*\operatorname{ArcSec}[x])}])/x + (I*\operatorname{Sqrt}[x^2]*\operatorname{PolyLog}[2, (-I)*E^{(I*\operatorname{ArcSec}[x])}])/x - (I*\operatorname{Sqrt}[x^2]*\operatorname{PolyLog}[2, I*E^{(I*\operatorname{ArcSec}[x])}])/x$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 2317

$\operatorname{Int}[\operatorname{Log}[(a_) + (b_)*((F_)^{((e_)*((c_) + (d_)*(x_)))})^{(n_)}], x_Symbol] \rightarrow \operatorname{Dist}[1/(d*e*n*\operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b*x]/x, x], x, (F)^{(e*(c + d*x))^{n}}], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x \ \&\& \operatorname{GtQ}[a, 0]$

Rule 2438

$\operatorname{Int}[\operatorname{Log}[(c_)*((d_) + (e_)*(x_)^{(n_)})]/(x_), x_Symbol] \rightarrow \operatorname{Simp}[-\operatorname{PolyLog}[2, (-c)*e*x^n]/n, x] /; \operatorname{FreeQ}\{c, d, e, n\}, x \ \&\& \operatorname{EqQ}[c*d, 1]$

Rule 4266

$\operatorname{Int}[\operatorname{csc}[(e_) + \operatorname{Pi}*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \operatorname{Simp}[-2*(c + d*x)^m*(\operatorname{ArcTanh}[E^{(I*k*Pi)*E^{(I*(e + f*x))}])/f], x] + (-\operatorname{Di}$

```
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4784

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
- c^2*x^2]], Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]
+ Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(f
*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x)) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 4804

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Dist[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]], Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5350

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Dist[-Sqrt[x^2]/x, Subst[Int[(e + d*x^2)^p*((a + b*Ar
cCos[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x^2} \sec^{-1}(x)}{x^2} dx &= -\frac{\sqrt{x^2} \operatorname{Subst}\left(\int \frac{\sqrt{1-x^2} \cos^{-1}(x)}{x} dx, x, \frac{1}{x}\right)}{x} \\
&= -\frac{\sqrt{1-\frac{1}{x^2}} \sqrt{x^2} \sec^{-1}(x)}{x} - \frac{\sqrt{x^2} \operatorname{Subst}\left(\int 1 dx, x, \frac{1}{x}\right)}{x} - \frac{\sqrt{x^2} \operatorname{Subst}\left(\int \frac{\cos}{x\sqrt{1-x^2}} dx, x, \frac{1}{x}\right)}{x} \\
&= -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{1-\frac{1}{x^2}} \sqrt{x^2} \sec^{-1}(x)}{x} + \frac{\sqrt{x^2} \operatorname{Subst}\left(\int x \sec(x) dx, x, \sec^{-1}(x)\right)}{x} \\
&= -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{1-\frac{1}{x^2}} \sqrt{x^2} \sec^{-1}(x)}{x} - \frac{2i\sqrt{x^2} \sec^{-1}(x) \tan^{-1}\left(e^{i\sec^{-1}(x)}\right)}{x} - \frac{\sqrt{x^2} \operatorname{Subst}\left(\int \frac{\cos}{x\sqrt{1-x^2}} dx, x, \frac{1}{x}\right)}{x} \\
&= -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{1-\frac{1}{x^2}} \sqrt{x^2} \sec^{-1}(x)}{x} - \frac{2i\sqrt{x^2} \sec^{-1}(x) \tan^{-1}\left(e^{i\sec^{-1}(x)}\right)}{x} + \frac{\sqrt{x^2} \operatorname{Subst}\left(\int \frac{\cos}{x\sqrt{1-x^2}} dx, x, \frac{1}{x}\right)}{x} \\
&= -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{1-\frac{1}{x^2}} \sqrt{x^2} \sec^{-1}(x)}{x} - \frac{2i\sqrt{x^2} \sec^{-1}(x) \tan^{-1}\left(e^{i\sec^{-1}(x)}\right)}{x} + \frac{\sqrt{x^2} \operatorname{Subst}\left(\int \frac{\cos}{x\sqrt{1-x^2}} dx, x, \frac{1}{x}\right)}{x}
\end{aligned}$$

Mathematica [A]

time = 0.12, size = 116, normalized size = 1.08

$$\frac{\sqrt{1-\frac{1}{x^2}} \left(1 + \sqrt{1-\frac{1}{x^2}} x \sec^{-1}(x) - x \sec^{-1}(x) \log\left(1 - ie^{i\sec^{-1}(x)}\right) + x \sec^{-1}(x) \log\left(1 + ie^{i\sec^{-1}(x)}\right) - ix \operatorname{Li}_2\left(-ie^{i\sec^{-1}(x)}\right) + ix \operatorname{Li}_2\left(ie^{i\sec^{-1}(x)}\right)\right)}{\sqrt{-1+x^2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(Sqrt[-1 + x^2]*ArcSec[x])/x^2, x]`

```
[Out] -((Sqrt[1 - x^(-2)]*(1 + Sqrt[1 - x^(-2)]*x*ArcSec[x] - x*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x]])] + x*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x]])] - I*x*PolyLog[2, (-I)*E^(I*ArcSec[x])] + I*x*PolyLog[2, I*E^(I*ArcSec[x])]))/Sqrt[-1 + x^2])
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[(ArcSec[x]*Sqrt[x^2 - 1])/x^2, x]')`

[Out] Timed out

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(119) = 238$.
time = 0.46, size = 708, normalized size = 6.62

method	result
default	$\frac{\sqrt{x^2-1} \left(\sqrt{\frac{x^2-1}{x^2}} x^3 - 3ix^2 - 4\sqrt{\frac{x^2-1}{x^2}} x + 4i \right)}{4 \left(-i\sqrt{\frac{x^2-1}{x^2}} x + x^2 - 1 \right) x} - \frac{\left(i\sqrt{\frac{x^2-1}{x^2}} x^3 - 2i\sqrt{\frac{x^2-1}{x^2}} x + 2x^2 - 2 \right) \operatorname{arcsec}(x)}{4x\sqrt{x^2-1}} - \frac{\sqrt{x^2-1} \left(\sqrt{\frac{x^2-1}{x^2}} x^3 - 3ix^2 - 4\sqrt{\frac{x^2-1}{x^2}} x + 4i \right)}{2 \left(-i\sqrt{\frac{x^2-1}{x^2}} x + x^2 - 1 \right) x}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsec(x)*(x^2-1)^(1/2)/x^2,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{4} \frac{(x^2-1)^{1/2} \left(\left(\frac{x^2-1}{x^2} \right)^{1/2} x^3 - 3i \left(\frac{x^2-1}{x^2} \right)^{1/2} x + 4i \right)}{(-1) \left(\frac{x^2-1}{x^2} \right)^{1/2} x + x^2 - 1} \frac{1}{x} - \frac{1}{4} \frac{(x^2-1)^{1/2} \left(\left(\frac{x^2-1}{x^2} \right)^{1/2} x^3 - 2i \left(\frac{x^2-1}{x^2} \right)^{1/2} x + 2x^2 - 2 \right) \operatorname{arcsec}(x)}{4x\sqrt{x^2-1}} - \frac{1}{2} \frac{(x^2-1)^{1/2} \left(\left(\frac{x^2-1}{x^2} \right)^{1/2} x^3 - 3ix^2 - 4\sqrt{\frac{x^2-1}{x^2}} x + 4i \right)}{\left(-i\sqrt{\frac{x^2-1}{x^2}} x + x^2 - 1 \right) x} \frac{1}{x} + \frac{1}{4} \frac{(x^2-1)^{1/2} \left(\left(\frac{x^2-1}{x^2} \right)^{1/2} x^3 - 2i \left(\frac{x^2-1}{x^2} \right)^{1/2} x + 2x^2 - 2 \right) \operatorname{arcsec}(x)}{4x\sqrt{x^2-1}} + \frac{1}{4} \frac{(x^2-1)^{1/2} \left(\left(\frac{x^2-1}{x^2} \right)^{1/2} x^3 - 8i \left(\frac{x^2-1}{x^2} \right)^{1/2} x + 8x^2 - 8 \right) - 1/2 (x^2-1)^{1/2} \left(i x^2 + \left(\frac{x^2-1}{x^2} \right)^{1/2} x - i \right) \operatorname{arcsec}(x) \ln(1 + i(1/x + i(1-1/x^2)^{1/2})) + 1/2 (x^2-1)^{1/2} \left(i x^2 + \left(\frac{x^2-1}{x^2} \right)^{1/2} x - i \right) \operatorname{arcsec}(x) \ln(1 - i(1/x + i(1-1/x^2)^{1/2})) - 1/2 (x^2-1)^{1/2} \left(-i \left(\frac{x^2-1}{x^2} \right)^{1/2} x + x^2 - 1 \right) \operatorname{dilog}(1 + i(1/x + i(1-1/x^2)^{1/2})) + 1/2 (x^2-1)^{1/2} \left(-i \left(\frac{x^2-1}{x^2} \right)^{1/2} x + x^2 - 1 \right) \operatorname{dilog}(1 - i(1/x + i(1-1/x^2)^{1/2})) + 1/2 (x^2-1)^{1/2} \left(i x^2 - \left(\frac{x^2-1}{x^2} \right)^{1/2} x - i \right) \operatorname{arcsec}(x) \ln(1 + i(1/x + i(1-1/x^2)^{1/2})) - 1/2 (x^2-1)^{1/2} \left(i x^2 - \left(\frac{x^2-1}{x^2} \right)^{1/2} x - i \right) \operatorname{arcsec}(x) \ln(1 - i(1/x + i(1-1/x^2)^{1/2})) + 1/2 (x^2-1)^{1/2} \left(i \left(\frac{x^2-1}{x^2} \right)^{1/2} x + x^2 - 1 \right) \operatorname{dilog}(1 + i(1/x + i(1-1/x^2)^{1/2})) - 1/2 (x^2-1)^{1/2} \left(i \left(\frac{x^2-1}{x^2} \right)^{1/2} x + x^2 - 1 \right) \operatorname{dilog}(1 - i(1/x + i(1-1/x^2)^{1/2}))$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsec(x)*(x^2-1)^(1/2)/x^2,x, algorithm="maxima")`

[Out] `integrate(sqrt(x^2 - 1)*arcsec(x)/x^2, x)`

Fricas [F]

time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsec(x)*(x^2-1)^(1/2)/x^2,x, algorithm="fricas")`

[Out] `integral(sqrt(x^2 - 1)*arcsec(x)/x^2, x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)} \operatorname{asec}(x)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asec(x)*(x**2-1)**(1/2)/x**2,x)`

[Out] `Integral(sqrt((x - 1)*(x + 1))*asec(x)/x**2, x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsec(x)*(x^2-1)^(1/2)/x^2,x)`

[Out] `Could not integrate`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acos}\left(\frac{1}{x}\right) \sqrt{x^2 - 1}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((acos(1/x)*(x^2 - 1)^(1/2))/x^2,x)`

[Out] `int((acos(1/x)*(x^2 - 1)^(1/2))/x^2, x)`

$$3.684 \quad \int \frac{(-1+x^2)^{5/2} \csc^{-1}(x)}{x^3} dx$$

Optimal. Leaf size=106

$$\frac{3+2x^4}{12x\sqrt{x^2}} - \frac{5\sqrt{-1+x^2} \csc^{-1}(x)}{2x^2} - \frac{5(-1+x^2)^{3/2} \csc^{-1}(x)}{3x^2} + \frac{(-1+x^2)^{5/2} \csc^{-1}(x)}{3x^2} - \frac{5x \csc^{-1}(x)^2}{4\sqrt{x^2}} - \frac{7x \log(x)}{3\sqrt{x^2}}$$

[Out] -5/3*(x^2-1)^(3/2)*arccsc(x)/x^2+1/3*(x^2-1)^(5/2)*arccsc(x)/x^2+1/12*(2*x^4+3)/x/(x^2)^(1/2)-5/4*x*arccsc(x)^2/(x^2)^(1/2)-7/3*x*ln(x)/(x^2)^(1/2)-5/2*arccsc(x)*(x^2-1)^(1/2)/x^2

Rubi [A]

time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.25, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {5351, 4785, 4741, 4737, 30, 14, 272, 45}

$$\frac{x\sqrt{x^2}}{6} - \frac{7\sqrt{x^2} \log(x)}{3x} + \frac{1}{3}(x^2)^{3/2} \left(1 - \frac{1}{x^2}\right)^{5/2} \csc^{-1}(x) - \frac{5}{3}\sqrt{x^2} \left(1 - \frac{1}{x^2}\right)^{3/2} \csc^{-1}(x) - \frac{5\sqrt{1 - \frac{1}{x^2}} \csc^{-1}(x)}{2\sqrt{x^2}} - \frac{5\sqrt{x^2} \csc^{-1}(x)^2}{4x} + \frac{\sqrt{x^2}}{4x^3}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)^(5/2)*ArcCsc[x]/x^3,x]

[Out] Sqrt[x^2]/(4*x^3) + (x*Sqrt[x^2])/6 - (5*Sqrt[1 - x^(-2)]*ArcCsc[x])/(2*Sqrt[x^2]) - (5*(1 - x^(-2))^(3/2)*Sqrt[x^2]*ArcCsc[x])/3 + ((1 - x^(-2))^(5/2))*(x^2)^(3/2)*ArcCsc[x]/3 - (5*Sqrt[x^2]*ArcCsc[x]^2)/(4*x) - (7*Sqrt[x^2]*Log[x])/(3*x)

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 45

Int[((a_)+(b_)*(x_))^(m_)*((c_)+(d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a+b*x)^m*(c+d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n+1), 0] || GtQ[m+n+2, 0])

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4737

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
+ b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] := Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Dist[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcSin[c*x])^n/Sqrt[1
- c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
], Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4785

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_.)*((d_.) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Dist[2*e*(p/(f^2*(m + 1))), Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Dist[b*c*(n/(f*(m +
1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(
p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5351

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] := Dist[-Sqrt[x^2]/x, Subst[Int[(e + d*x^2)^p*((a + b*A
rcSin[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^2)^{5/2} \csc^{-1}(x)}{x^3} dx &= -\frac{\sqrt{x^2} \operatorname{Subst}\left(\int \frac{(1-x^2)^{5/2} \sin^{-1}(x)}{x^4} dx, x, \frac{1}{x}\right)}{x} \\
&= \frac{1}{3} \left(1 - \frac{1}{x^2}\right)^{5/2} (x^2)^{3/2} \csc^{-1}(x) - \frac{\sqrt{x^2} \operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x^3} dx, x, \frac{1}{x}\right)}{3x} + \frac{(5\sqrt{x^2})}{3} \\
&= -\frac{5}{3} \left(1 - \frac{1}{x^2}\right)^{3/2} \sqrt{x^2} \csc^{-1}(x) + \frac{1}{3} \left(1 - \frac{1}{x^2}\right)^{5/2} (x^2)^{3/2} \csc^{-1}(x) - \frac{\sqrt{x^2} \operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x^3} dx, x, \frac{1}{x}\right)}{3x} \\
&= -\frac{5\sqrt{1-\frac{1}{x^2}} \csc^{-1}(x)}{2\sqrt{x^2}} - \frac{5}{3} \left(1 - \frac{1}{x^2}\right)^{3/2} \sqrt{x^2} \csc^{-1}(x) + \frac{1}{3} \left(1 - \frac{1}{x^2}\right)^{5/2} (x^2)^{3/2} \csc^{-1}(x) - \frac{\sqrt{x^2} \operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x^3} dx, x, \frac{1}{x}\right)}{3x} \\
&= \frac{\sqrt{x^2}}{4x^3} + \frac{x\sqrt{x^2}}{6} - \frac{5\sqrt{1-\frac{1}{x^2}} \csc^{-1}(x)}{2\sqrt{x^2}} - \frac{5}{3} \left(1 - \frac{1}{x^2}\right)^{3/2} \sqrt{x^2} \csc^{-1}(x) + \frac{1}{3} \left(1 - \frac{1}{x^2}\right)^{5/2} (x^2)^{3/2} \csc^{-1}(x) - \frac{\sqrt{x^2} \operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x^3} dx, x, \frac{1}{x}\right)}{3x}
\end{aligned}$$

Mathematica [A]

time = 0.20, size = 86, normalized size = 0.81

$$\frac{\sqrt{-1+x^2} \left(4x^2 - 30 \csc^{-1}(x)^2 - 3 \cos(2 \csc^{-1}(x)) + 48 \log\left(\frac{1}{x}\right) - 8 \log(x) + \csc^{-1}(x) \left(8\sqrt{1-\frac{1}{x^2}} x(-7+x^2) - 6 \sin(2 \csc^{-1}(x))\right)\right)}{24\sqrt{1-\frac{1}{x^2}} x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)^(5/2)*ArcCsc[x])/x^3, x]
```

```
[Out] (Sqrt[-1 + x^2]*(4*x^2 - 30*ArcCsc[x]^2 - 3*Cos[2*ArcCsc[x]] + 48*Log[x^(-1)] - 8*Log[x] + ArcCsc[x]*(8*Sqrt[1 - x^(-2)]*x*(-7 + x^2) - 6*Sin[2*ArcCsc[x]])))/(24*Sqrt[1 - x^(-2)]*x)
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(ArcCsc[x]*(x^2 - 1)^(5/2))/x^3, x]')
```

```
[Out] Timed out
```

Maple [C] Result contains complex when optimal does not.

time = 0.41, size = 305, normalized size = 2.88

method	result
default	$-\frac{5\sqrt{\frac{x^2-1}{x^2}} \operatorname{arccsc}(x)^2}{4\sqrt{x^2-1}} + \frac{\left(i\sqrt{\frac{x^2-1}{x^2}} x^3 - 2i\sqrt{\frac{x^2-1}{x^2}} x - 2x^2 + 2\right)(2\operatorname{arccsc}(x) + i)}{16x^2\sqrt{x^2-1}} - \frac{\left(i\sqrt{\frac{x^2-1}{x^2}} x^3 - 2i\sqrt{\frac{x^2-1}{x^2}} x + 2x^2\right)}{16x^2\sqrt{x^2-1}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2-1)^(5/2)*arccsc(x)/x^3,x,method=_RETURNVERBOSE)`

[Out]
$$-5/4/(x^2-1)^{(1/2)}*((x^2-1)/x^2)^{(1/2)}*x*\operatorname{arccsc}(x)^2+1/16/x^2/(x^2-1)^{(1/2)}$$

$$*(I*((x^2-1)/x^2)^{(1/2)}*x^3-2*I*((x^2-1)/x^2)^{(1/2)}*x-2*x^2+2)*(2*\operatorname{arccsc}(x)$$

$$+I)-1/16/x^2/(x^2-1)^{(1/2)}*(I*((x^2-1)/x^2)^{(1/2)}*x^3-2*I*((x^2-1)/x^2)^{(1/2)}$$

$$*x+2*x^2-2)*(-I+2*\operatorname{arccsc}(x))-14/3*I/(x^2-1)^{(1/2)}*((x^2-1)/x^2)^{(1/2)}*x*a$$

$$rccsc(x)+1/6/(x^2-1)^{(1/2)}*(x^4+7*I*((x^2-1)/x^2)^{(1/2)}*x-8*x^2+7)*(2*\operatorname{arccs}$$

$$c(x)*x^4+((x^2-1)/x^2)^{(1/2)}*x^3-30*\operatorname{arccsc}(x)*x^2-7*((x^2-1)/x^2)^{(1/2)}*x-7$$

$$*I+126*\operatorname{arccsc}(x))/(x^4-15*x^2+63)+7/3/(x^2-1)^{(1/2)}*((x^2-1)/x^2)^{(1/2)}*x*1$$

$$n((I/x+(1-1/x^2)^{(1/2)})^2-1)$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2-1)^(5/2)*arccsc(x)/x^3,x, algorithm="maxima")`

[Out] `integrate((x^2 - 1)^(5/2)*arccsc(x)/x^3, x)`

Fricas [A]

time = 0.32, size = 51, normalized size = 0.48

$$\frac{2x^4 - 15x^2 \operatorname{arccsc}(x)^2 - 28x^2 \log(x) + 2(2x^4 - 14x^2 - 3)\sqrt{x^2-1} \operatorname{arccsc}(x) + 3}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2-1)^(5/2)*arccsc(x)/x^3,x, algorithm="fricas")`

[Out]
$$1/12*(2*x^4 - 15*x^2*\operatorname{arccsc}(x)^2 - 28*x^2*\log(x) + 2*(2*x^4 - 14*x^2 - 3)*s$$

$$\operatorname{qrt}(x^2 - 1)*\operatorname{arccsc}(x) + 3)/x^2$$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-1)**(5/2)*acsc(x)/x**3,x)

[Out] Timed out

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)^(5/2)*arccsc(x)/x^3,x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asin}\left(\frac{1}{x}\right) (x^2 - 1)^{5/2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((asin(1/x)*(x^2 - 1)^(5/2))/x^3,x)

[Out] int((asin(1/x)*(x^2 - 1)^(5/2))/x^3, x)

$$3.685 \quad \int \frac{\sqrt{-1+x^2} \sec^{-1}(x)}{x^4} dx$$

Optimal. Leaf size=41

$$-\frac{1}{9(x^2)^{3/2}} + \frac{1}{3\sqrt{x^2}} + \frac{(-1+x^2)^{3/2} \sec^{-1}(x)}{3x^3}$$

[Out] $1/3*(x^2-1)^{(3/2)*\text{arcsec}(x)/x^3+1/3/(x^2)^{(1/2)}-1/9/x^2/(x^2)^{(1/2)}$

Rubi [A]

time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {270, 5346, 12, 14}

$$\frac{1}{3\sqrt{x^2}} - \frac{1}{9(x^2)^{3/2}} + \frac{(x^2-1)^{3/2} \sec^{-1}(x)}{3x^3}$$

Antiderivative was successfully verified.

[In] `Int[(Sqrt[-1 + x^2]*ArcSec[x])/x^4,x]`

[Out] $-1/9*1/(x^2)^{(3/2)} + 1/(3*\text{Sqrt}[x^2]) + ((-1 + x^2)^{(3/2)}*\text{ArcSec}[x])/(3*x^3)$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

Rule 14

`Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)+(b_)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Rule 270

`Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m+1)*((a + b*x^n)^(p+1)/(a*c*(m+1))), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]`

Rule 5346

`Int[((a_) + ArcSec[(c_)*(x_)])*(b_)*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},`

x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{-1+x^2} \sec^{-1}(x)}{x^4} dx &= \frac{(-1+x^2)^{3/2} \sec^{-1}(x)}{3x^3} - \frac{x \int \frac{-1+x^2}{3x^4} dx}{\sqrt{x^2}} \\
 &= \frac{(-1+x^2)^{3/2} \sec^{-1}(x)}{3x^3} - \frac{x \int \frac{-1+x^2}{x^4} dx}{3\sqrt{x^2}} \\
 &= \frac{(-1+x^2)^{3/2} \sec^{-1}(x)}{3x^3} - \frac{x \int \left(-\frac{1}{x^4} + \frac{1}{x^2}\right) dx}{3\sqrt{x^2}} \\
 &= -\frac{1}{9(x^2)^{3/2}} + \frac{1}{3\sqrt{x^2}} + \frac{(-1+x^2)^{3/2} \sec^{-1}(x)}{3x^3}
 \end{aligned}$$

Mathematica [A]

time = 0.03, size = 48, normalized size = 1.17

$$\frac{\sqrt{1 - \frac{1}{x^2}} x (-1 + 3x^2) + 3(-1 + x^2)^2 \sec^{-1}(x)}{9x^3 \sqrt{-1 + x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-1 + x^2]*ArcSec[x])/x^4,x]

[Out] (Sqrt[1 - x^(-2)]*x*(-1 + 3*x^2) + 3*(-1 + x^2)^2*ArcSec[x])/(9*x^3*Sqrt[-1 + x^2])

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(ArcSec[x]*Sqrt[x^2 - 1])/x^4,x]')

[Out] Timed out

Maple [C] Result contains complex when optimal does not.

time = 0.48, size = 329, normalized size = 8.02

method	result
default	$-\frac{\sqrt{x^2-1} \left(\sqrt{\frac{x^2-1}{x^2}} x^5 - 5ix^4 - 12\sqrt{\frac{x^2-1}{x^2}} x^3 + 20ix^2 + 16\sqrt{\frac{x^2-1}{x^2}} x - 16i \right)}{144 \left(-i\sqrt{\frac{x^2-1}{x^2}} x + x^2 - 1 \right) x^3} + \frac{\left(i\sqrt{\frac{x^2-1}{x^2}} x^5 - 8i\sqrt{\frac{x^2-1}{x^2}} x^3 + 4x^4 + \dots \right)}{48x^3 \sqrt{x^2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsec(x)*(x^2-1)^(1/2)/x^4,x,method=_RETURNVERBOSE)`

[Out]
$$-1/144*(x^2-1)^{(1/2)}*((x^2-1)/x^2)^{(1/2)}*x^5-5*I*x^4-12*((x^2-1)/x^2)^{(1/2)}*x^3+20*I*x^2+16*((x^2-1)/x^2)^{(1/2)}*x-16*I)/(-I*((x^2-1)/x^2)^{(1/2)}*x+x^2-1)/x^3+1/48/x^3/(x^2-1)^{(1/2)}*(I*((x^2-1)/x^2)^{(1/2)}*x^5-8*I*((x^2-1)/x^2)^{(1/2)}*x^3+4*x^4+8*I*((x^2-1)/x^2)^{(1/2)}*x-12*x^2+8)*arcsec(x)+1/72/(x^2-1)^{(1/2)}*(I*((x^2-1)/x^2)^{(1/2)}*x+x^2-1)*(-13*I+3*arcsec(x))/x-1/72*(5*I+3*arcsec(x))*(I*((x^2-1)/x^2)^{(1/2)}*x-1)*(x^2-1)^{(1/2)}/x-1/144/(x^2-1)^{(1/2)}*(I*((x^2-1)/x^2)^{(1/2)}*x+x^2-1)*(7*I+9*arcsec(x))*cos(3*arcsec(x))-1/48/(x^2-1)^{(1/2)}*(I*x^2-((x^2-1)/x^2)^{(1/2)}*x-I)*(3*I+arcsec(x))*sin(3*arcsec(x))$$

Maxima [A]

time = 0.33, size = 27, normalized size = 0.66

$$\frac{(x^2-1)^{\frac{3}{2}} \operatorname{arcsec}(x)}{3x^3} + \frac{3x^2-1}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsec(x)*(x^2-1)^(1/2)/x^4,x, algorithm="maxima")`

[Out] $1/3*(x^2-1)^{(3/2)}*arcsec(x)/x^3 + 1/9*(3*x^2-1)/x^3$

Fricas [A]

time = 0.32, size = 23, normalized size = 0.56

$$\frac{3(x^2-1)^{\frac{3}{2}} \operatorname{arcsec}(x) + 3x^2-1}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsec(x)*(x^2-1)^(1/2)/x^4,x, algorithm="fricas")`

[Out] $1/9*(3*(x^2-1)^{(3/2)}*arcsec(x) + 3*x^2-1)/x^3$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asec(x)*(x**2-1)**(1/2)/x**4,x)

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(32) = 64.
time = 0.03, size = 85, normalized size = 2.07

$$-\frac{-3x^2 + 1}{9(x^3 \operatorname{sign}(x))} - \frac{2 \arctan(-x + \sqrt{x^2 - 1})}{3 \operatorname{sign}(x)} + \frac{2 \left(3 \left(\sqrt{x^2 - 1} - x \right)^4 + 1 \right) \arccos\left(\frac{1}{x}\right)}{3 \left(\left(\sqrt{x^2 - 1} - x \right)^2 + 1 \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^4,x)

[Out] -2/3*arctan(-x + sqrt(x^2 - 1))/sgn(x) + 2/3*(3*(x - sqrt(x^2 - 1))^4 + 1)*arccos(1/x)/((x - sqrt(x^2 - 1))^2 + 1)^3 + 1/9*(3*x^2 - 1)/(x^3*sgn(x))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\arccos\left(\frac{1}{x}\right) \sqrt{x^2 - 1}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((acos(1/x)*(x^2 - 1)^(1/2))/x^4,x)

[Out] int((acos(1/x)*(x^2 - 1)^(1/2))/x^4, x)

$$3.686 \quad \int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}} dx$$

Optimal. Leaf size=65

$$\frac{\sqrt{x^2}}{6(1-x^2)} + \frac{5}{6} \coth^{-1}(\sqrt{x^2}) - \frac{x \sec^{-1}(x)}{3(-1+x^2)^{3/2}} + \frac{2x \sec^{-1}(x)}{3\sqrt{-1+x^2}}$$

[Out] 5/6*arccoth((x^2)^(1/2))-1/3*x*arcsec(x)/(x^2-1)^(3/2)+1/6*(x^2)^(1/2)/(-x^2+1)+2/3*x*arcsec(x)/(x^2-1)^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {198, 197, 5336, 12, 393, 212}

$$\frac{\sqrt{x^2}}{6(1-x^2)} + \frac{2x \sec^{-1}(x)}{3\sqrt{x^2-1}} - \frac{x \sec^{-1}(x)}{3(x^2-1)^{3/2}} + \frac{5x \tanh^{-1}(x)}{6\sqrt{x^2}}$$

Antiderivative was successfully verified.

[In] Int[ArcSec[x]/(-1 + x^2)^(5/2), x]

[Out] Sqrt[x^2]/(6*(1 - x^2)) - (x*ArcSec[x])/(3*(-1 + x^2)^(3/2)) + (2*x*ArcSec[x])/(3*Sqrt[-1 + x^2]) + (5*x*ArcTanh[x])/(6*Sqrt[x^2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 197

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 198

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel LtQ[b, 0]$

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Si
mp[(-b*c - a*d)*x*(a + b*x^n)^(p + 1)/(a*b*n*(p + 1)), x] - Dist[(a*d -
b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
+ p, 0])
```

Rule 5336

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x]
- Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rubi steps

$$\begin{aligned} \int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}} dx &= -\frac{x \sec^{-1}(x)}{3(-1+x^2)^{3/2}} + \frac{2x \sec^{-1}(x)}{3\sqrt{-1+x^2}} - \frac{x \int \frac{-3+2x^2}{3(1-x^2)^2} dx}{\sqrt{x^2}} \\ &= -\frac{x \sec^{-1}(x)}{3(-1+x^2)^{3/2}} + \frac{2x \sec^{-1}(x)}{3\sqrt{-1+x^2}} - \frac{x \int \frac{-3+2x^2}{(1-x^2)^2} dx}{3\sqrt{x^2}} \\ &= \frac{\sqrt{x^2}}{6(1-x^2)} - \frac{x \sec^{-1}(x)}{3(-1+x^2)^{3/2}} + \frac{2x \sec^{-1}(x)}{3\sqrt{-1+x^2}} + \frac{(5x) \int \frac{1}{1-x^2} dx}{6\sqrt{x^2}} \\ &= \frac{\sqrt{x^2}}{6(1-x^2)} - \frac{x \sec^{-1}(x)}{3(-1+x^2)^{3/2}} + \frac{2x \sec^{-1}(x)}{3\sqrt{-1+x^2}} + \frac{5x \tanh^{-1}(x)}{6\sqrt{x^2}} \end{aligned}$$

Mathematica [A]

time = 0.07, size = 67, normalized size = 1.03

$$\frac{4x(-3+2x^2)\sec^{-1}(x) + \sqrt{1-\frac{1}{x^2}}x(-2x-5(-1+x^2)\log(1-x) + 5(-1+x^2)\log(1+x))}{12(-1+x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSec[x]/(-1+x^2)^(5/2),x]

[Out] $(4*x*(-3 + 2*x^2)*\text{ArcSec}[x] + \text{Sqrt}[1 - x^{(-2)}]*x*(-2*x - 5*(-1 + x^2)*\text{Log}[1 - x] + 5*(-1 + x^2)*\text{Log}[1 + x]))/(12*(-1 + x^2)^{(3/2)})$

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[ArcSec[x]/(x^2 - 1)^(5/2), x]')`

[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep

Maple [C] Result contains complex when optimal does not.

time = 0.42, size = 128, normalized size = 1.97

method	result
default	$\frac{\sqrt{x^2 - 1} x \left(4 \operatorname{arcsec}(x) x^2 - \sqrt{\frac{x^2 - 1}{x^2}} x - 6 \operatorname{arcsec}(x) \right)}{6x^4 - 12x^2 + 6} - \frac{5 \sqrt{\frac{x^2 - 1}{x^2}} x \ln\left(\frac{1}{x} + i \sqrt{1 - \frac{1}{x^2}} - 1\right)}{6\sqrt{x^2 - 1}} + \frac{5 \sqrt{\frac{x^2 - 1}{x^2}} x \ln\left(\frac{1}{x} + i \sqrt{1 - \frac{1}{x^2}} + 1\right)}{6\sqrt{x^2 - 1}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsec(x)/(x^2-1)^(5/2), x, method=_RETURNVERBOSE)`

[Out] $\frac{1}{6}*(x^2-1)^{(1/2)}*x*(4*\operatorname{arcsec}(x)*x^2 - ((x^2-1)/x^2)^{(1/2)}*x - 6*\operatorname{arcsec}(x))/(x^4 - 2*x^2 + 1) - 5/6/(x^2-1)^{(1/2)}*((x^2-1)/x^2)^{(1/2)}*x*\ln(1/x + I*(1-1/x^2)^{(1/2)} - 1) + 5/6/(x^2-1)^{(1/2)}*((x^2-1)/x^2)^{(1/2)}*x*\ln(1/x + I*(1-1/x^2)^{(1/2)} + 1)$

Maxima [A]

time = 0.30, size = 48, normalized size = 0.74

$$\frac{1}{3} \left(\frac{2x}{\sqrt{x^2 - 1}} - \frac{x}{(x^2 - 1)^{\frac{3}{2}}} \right) \operatorname{arcsec}(x) - \frac{x}{6(x^2 - 1)} + \frac{5}{12} \log(x + 1) - \frac{5}{12} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsec(x)/(x^2-1)^(5/2), x, algorithm="maxima")`

[Out] $\frac{1}{3}*(2*x/\text{sqrt}(x^2 - 1) - x/(x^2 - 1)^{(3/2)})*\operatorname{arcsec}(x) - 1/6*x/(x^2 - 1) + 5/12*\log(x + 1) - 5/12*\log(x - 1)$

Fricas [A]

time = 0.32, size = 75, normalized size = 1.15

$$\frac{2x^3 - 4(2x^3 - 3x)\sqrt{x^2 - 1} \operatorname{arcsec}(x) - 5(x^4 - 2x^2 + 1)\log(x + 1) + 5(x^4 - 2x^2 + 1)\log(x - 1) - 2x}{12(x^4 - 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")

[Out] $-1/12*(2*x^3 - 4*(2*x^3 - 3*x)*\sqrt{x^2 - 1}*\operatorname{arcsec}(x) - 5*(x^4 - 2*x^2 + 1)*\log(x + 1) + 5*(x^4 - 2*x^2 + 1)*\log(x - 1) - 2*x)/(x^4 - 2*x^2 + 1)$

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asec(x)/(x**2-1)**(5/2),x)

[Out] Exception raised: SystemError

Giac [A]

time = 0.02, size = 72, normalized size = 1.11

$$-\frac{x}{6((x^2 - 1)\operatorname{sign}(x))} + \frac{5 \ln|x + 1|}{12\operatorname{sign}(x)} - \frac{5 \ln|x - 1|}{12\operatorname{sign}(x)} + \frac{2\left(\frac{1}{3}xx - \frac{1}{2}\right)x\sqrt{x^2 - 1} \arccos\left(\frac{1}{x}\right)}{(x^2 - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsec(x)/(x^2-1)^(5/2),x)

[Out] $1/3*(2*x^2 - 3)*x*\arccos(1/x)/(x^2 - 1)^{(3/2)} + 5/12*\log(\operatorname{abs}(x + 1))/\operatorname{sgn}(x) - 5/12*\log(\operatorname{abs}(x - 1))/\operatorname{sgn}(x) - 1/6*x/((x^2 - 1)*\operatorname{sgn}(x))$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\arccos\left(\frac{1}{x}\right)}{(x^2 - 1)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acos(1/x)/(x^2 - 1)^(5/2),x)

[Out] int(acos(1/x)/(x^2 - 1)^(5/2), x)

$$3.687 \quad \int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}} dx$$

Optimal. Leaf size=51

$$\frac{\sqrt{x^2}}{6(1-x^2)} - \frac{1}{6} \coth^{-1}\left(\sqrt{x^2}\right) - \frac{x^3 \sec^{-1}(x)}{3(-1+x^2)^{3/2}}$$

[Out] $-1/6*\operatorname{arccoth}((x^2)^{(1/2)})-1/3*x^3*\operatorname{arcsec}(x)/(x^2-1)^{(3/2)}+1/6*(x^2)^{(1/2)/(-x^2+1)}$

Rubi [A]

time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {270, 5346, 12, 294, 213}

$$\frac{\sqrt{x^2}}{6(1-x^2)} - \frac{x \tanh^{-1}(x)}{6\sqrt{x^2}} - \frac{x^3 \sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^2*\operatorname{ArcSec}[x])/(-1+x^2)^{(5/2)},x]$

[Out] $\operatorname{Sqrt}[x^2]/(6*(1-x^2)) - (x^3*\operatorname{ArcSec}[x])/(3*(-1+x^2)^{(3/2)}) - (x*\operatorname{ArcTanh}[x])/(6*\operatorname{Sqrt}[x^2])$

Rule 12

$\operatorname{Int}[(a_)*(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$ FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 213

$\operatorname{Int}[((a_) + (b_.)*(x_)^2)^{(-1)}, x_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2])^{(-1)}*\operatorname{ArcTanh}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 270

$\operatorname{Int}[((c_.)*(x_))^{(m_.)*((a_) + (b_.)*(x_)^{(n_)})^{(p_)}], x_Symbol] \rightarrow \operatorname{Simp}[(c*x)^{(m+1)*((a+b*x^n)^{(p+1))/(a*c*(m+1))}], x] /;$ FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

Rule 294

$\operatorname{Int}[((c_.)*(x_))^{(m_.)*((a_) + (b_.)*(x_)^{(n_)})^{(p_)}], x_Symbol] \rightarrow \operatorname{Simp}[c^{(n-1)}*(c*x)^{(m-n+1)*((a+b*x^n)^{(p+1))/(b*n*(p+1))}], x] - \operatorname{Dist}[c^{(n-1)}$

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}} dx &= -\frac{x^3 \sec^{-1}(x)}{3(-1+x^2)^{3/2}} - \frac{x \int -\frac{x^2}{3(-1+x^2)^2} dx}{\sqrt{x^2}} \\ &= -\frac{x^3 \sec^{-1}(x)}{3(-1+x^2)^{3/2}} + \frac{x \int \frac{x^2}{(-1+x^2)^2} dx}{3\sqrt{x^2}} \\ &= \frac{\sqrt{x^2}}{6(1-x^2)} - \frac{x^3 \sec^{-1}(x)}{3(-1+x^2)^{3/2}} + \frac{x \int \frac{1}{-1+x^2} dx}{6\sqrt{x^2}} \\ &= \frac{\sqrt{x^2}}{6(1-x^2)} - \frac{x^3 \sec^{-1}(x)}{3(-1+x^2)^{3/2}} - \frac{x \tanh^{-1}(x)}{6\sqrt{x^2}} \end{aligned}$$

Mathematica [A]

time = 0.06, size = 61, normalized size = 1.20

$$\frac{-4x^3 \sec^{-1}(x) + \sqrt{1 - \frac{1}{x^2}} x (-2x + (-1 + x^2) \log(1 - x) - (-1 + x^2) \log(1 + x))}{12(-1 + x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSec[x])/(-1 + x^2)^(5/2), x]
```

```
[Out] (-4*x^3*ArcSec[x] + Sqrt[1 - x^(-2)]*x*(-2*x + (-1 + x^2)*Log[1 - x] - (-1
+ x^2)*Log[1 + x]))/(12*(-1 + x^2)^(3/2))
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(ArcSec[x]*x^2)/(x^2 - 1)^(5/2),x]')`

[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep

Maple [C] Result contains complex when optimal does not.

time = 0.51, size = 121, normalized size = 2.37

method	result
default	$-\frac{\sqrt{x^2-1} x^2 \left(2x \operatorname{arcsec}(x) + \sqrt{\frac{x^2-1}{x^2}}\right)}{6(x^4-2x^2+1)} + \frac{\sqrt{\frac{x^2-1}{x^2}} x \ln\left(\frac{1}{x} + i \sqrt{1 - \frac{1}{x^2}} - 1\right)}{6\sqrt{x^2-1}} - \frac{\sqrt{\frac{x^2-1}{x^2}} x \ln\left(\frac{1}{x} + i \sqrt{1 - \frac{1}{x^2}} + 1\right)}{6\sqrt{x^2-1}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arcsec(x)/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $-1/6*(x^2-1)^{(1/2)}*x^2/(x^4-2*x^2+1)*(2*x*arcsec(x)+((x^2-1)/x^2)^{(1/2)})+1/6/(x^2-1)^{(1/2)}*((x^2-1)/x^2)^{(1/2)}*x*\ln(1/x+I*(1-1/x^2)^{(1/2)}-1)-1/6/(x^2-1)^{(1/2)}*((x^2-1)/x^2)^{(1/2)}*x*\ln(1/x+I*(1-1/x^2)^{(1/2)}+1)$

Maxima [A]

time = 0.29, size = 46, normalized size = 0.90

$$-\frac{1}{3} \left(\frac{x}{\sqrt{x^2-1}} + \frac{x}{(x^2-1)^{\frac{3}{2}}} \right) \operatorname{arcsec}(x) - \frac{x}{6(x^2-1)} - \frac{1}{12} \log(x+1) + \frac{1}{12} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")`

[Out] $-1/3*(x/\sqrt{x^2-1} + x/(x^2-1)^{(3/2)})*arcsec(x) - 1/6*x/(x^2-1) - 1/12*\log(x+1) + 1/12*\log(x-1)$

Fricas [A]

time = 0.33, size = 68, normalized size = 1.33

$$\frac{4\sqrt{x^2-1}x^3\operatorname{arcsec}(x) + 2x^3 + (x^4 - 2x^2 + 1)\log(x+1) - (x^4 - 2x^2 + 1)\log(x-1) - 2x}{12(x^4 - 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")`

[Out] $-1/12*(4*\sqrt{x^2-1}*x^3*arcsec(x) + 2*x^3 + (x^4 - 2*x^2 + 1)*\log(x+1) - (x^4 - 2*x^2 + 1)*\log(x-1) - 2*x)/(x^4 - 2*x^2 + 1)$

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*asec(x)/(x**2-1)**(5/2),x)

[Out] Exception raised: SystemError

Giac [A]

time = 0.01, size = 66, normalized size = 1.29

$$-\frac{x}{6((x^2-1)\operatorname{sign}(x))} - \frac{\ln|x+1|}{12\operatorname{sign}(x)} + \frac{\ln|x-1|}{12\operatorname{sign}(x)} - \frac{2xxx\sqrt{x^2-1}\arccos\left(\frac{1}{x}\right)}{6(x^2-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arcsec(x)/(x^2-1)^(5/2),x)

[Out] -1/3*x^3*arccos(1/x)/(x^2-1)^(3/2) - 1/12*log(abs(x+1))/sgn(x) + 1/12*log(abs(x-1))/sgn(x) - 1/6*x/((x^2-1)*sgn(x))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2 \operatorname{acos}\left(\frac{1}{x}\right)}{(x^2-1)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*acos(1/x))/(x^2-1)^(5/2),x)

[Out] int((x^2*acos(1/x))/(x^2-1)^(5/2), x)

$$3.688 \quad \int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}} dx$$

Optimal. Leaf size=82

$$\frac{x}{6\sqrt{x^2}(1-x^2)} - \frac{\sec^{-1}(x)}{3(-1+x^2)^{3/2}} - \frac{\sec^{-1}(x)}{\sqrt{-1+x^2}} - \frac{2x \log(x)}{3\sqrt{x^2}} + \frac{x \log(-1+x^2)}{3\sqrt{x^2}}$$

[Out] $-1/3*\text{arcsec}(x)/(x^2-1)^{(3/2)}+1/6*x/(-x^2+1)/(x^2)^{(1/2)}-2/3*x*\ln(x)/(x^2)^{(1/2)}+1/3*x*\ln(x^2-1)/(x^2)^{(1/2)}-\text{arcsec}(x)/(x^2-1)^{(1/2)}$

Rubi [A]

time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {272, 45, 5346, 12, 457, 78}

$$\frac{x}{6\sqrt{x^2}(1-x^2)} - \frac{2x \log(x)}{3\sqrt{x^2}} + \frac{x \log(1-x^2)}{3\sqrt{x^2}} - \frac{\sec^{-1}(x)}{\sqrt{x^2-1}} - \frac{\sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^3*\text{ArcSec}[x])/(-1+x^2)^{(5/2)},x]$

[Out] $x/(6*\text{Sqrt}[x^2]*(1-x^2)) - \text{ArcSec}[x]/(3*(-1+x^2)^{(3/2)}) - \text{ArcSec}[x]/\text{Sqrt}[-1+x^2] - (2*x*\text{Log}[x])/(3*\text{Sqrt}[x^2]) + (x*\text{Log}[1-x^2])/(3*\text{Sqrt}[x^2])$

Rule 12

$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!Match} Q[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

Rule 45

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (\text{!IntegerQ}[n] \parallel (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) \parallel \text{LtQ}[9*m + 5*(n + 1), 0] \parallel \text{GtQ}[m + n + 2, 0])$

Rule 78

$\text{Int}[(a_.) + (b_.)*(x_.)^{(c_.)}*((d_.) + (e_.)*(x_.)^{(n_.)}*((f_.) + (g_.)*(x_.)^{(p_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^c*(d + e*x)^n*(f + g*x)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& ((\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) \parallel \text{EqQ}[p, 1] \parallel (\text{IGtQ}[p, 0] \&\& (\text{!IntegerQ}[n] \parallel \text{LeQ}[9*p + 5*(n + 2), 0] \parallel \text{GeQ}[n + p + 1, 0] \parallel (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}} dx &= -\frac{\sec^{-1}(x)}{3(-1+x^2)^{3/2}} - \frac{\sec^{-1}(x)}{\sqrt{-1+x^2}} - \frac{x \int \frac{2-3x^2}{3x(1-x^2)^2} dx}{\sqrt{x^2}} \\
&= -\frac{\sec^{-1}(x)}{3(-1+x^2)^{3/2}} - \frac{\sec^{-1}(x)}{\sqrt{-1+x^2}} - \frac{x \int \frac{2-3x^2}{x(1-x^2)^2} dx}{3\sqrt{x^2}} \\
&= -\frac{\sec^{-1}(x)}{3(-1+x^2)^{3/2}} - \frac{\sec^{-1}(x)}{\sqrt{-1+x^2}} - \frac{x \text{Subst}\left(\int \frac{2-3x}{(1-x)^2 x} dx, x, x^2\right)}{6\sqrt{x^2}} \\
&= -\frac{\sec^{-1}(x)}{3(-1+x^2)^{3/2}} - \frac{\sec^{-1}(x)}{\sqrt{-1+x^2}} - \frac{x \text{Subst}\left(\int \left(-\frac{1}{(-1+x)^2} - \frac{2}{-1+x} + \frac{2}{x}\right) dx, x, x^2\right)}{6\sqrt{x^2}} \\
&= \frac{x}{6\sqrt{x^2}(1-x^2)} - \frac{\sec^{-1}(x)}{3(-1+x^2)^{3/2}} - \frac{\sec^{-1}(x)}{\sqrt{-1+x^2}} - \frac{2x \log(x)}{3\sqrt{x^2}} + \frac{x \log(1-x^2)}{3\sqrt{x^2}}
\end{aligned}$$

Mathematica [A]

time = 0.11, size = 72, normalized size = 0.88

$$\frac{-2(-2 + 3x^2) \sec^{-1}(x) - \frac{(-1+x^2)(1+4(-1+x^2) \log(x) - 2(-1+x^2) \log(1-x^2))}{\sqrt{1 - \frac{1}{x^2}} x}}{6(-1 + x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*ArcSec[x])/(-1 + x^2)^(5/2), x]

[Out] (-2*(-2 + 3*x^2)*ArcSec[x] - ((-1 + x^2)*(1 + 4*(-1 + x^2)*Log[x] - 2*(-1 + x^2)*Log[1 - x^2]))/(Sqrt[1 - x^(-2)]*x))/(6*(-1 + x^2)^(3/2))

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(ArcSec[x]*x^3)/(x^2 - 1)^(5/2), x]')

[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep

Maple [C] Result contains complex when optimal does not.

time = 0.65, size = 197, normalized size = 2.40

method	result
default	$-\frac{4i \sqrt{\frac{x^2-1}{x^2}} x \operatorname{arcsec}(x)}{3\sqrt{x^2-1}} + \frac{\sqrt{x^2-1} \left(2i \sqrt{\frac{x^2-1}{x^2}} x^3 - 2i \sqrt{\frac{x^2-1}{x^2}} x - 3x^2 + 2 \right) \left(2ix^4 + 8x^4 \operatorname{arcsec}(x) + 3 \sqrt{\frac{x^2-1}{x^2}} x^3 - 4ix \right)}{6x^2(4x^6 - 11x^4 + 10x^2 - 3)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arcsec(x)/(x^2-1)^(5/2), x, method=_RETURNVERBOSE)

[Out] -4/3*I/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*arcsec(x)+1/6/x^2*(x^2-1)^(1/2)*(2*I*((x^2-1)/x^2)^(1/2)*x^3-2*I*((x^2-1)/x^2)^(1/2)*x-3*x^2+2)*(2*I*x^4+8*x^4*arcsec(x)+3*((x^2-1)/x^2)^(1/2)*x^3-4*I*x^2-6*arcsec(x)*x^2-2*((x^2-1)/x^2)^(1/2)*x+2*I)/(4*x^6-11*x^4+10*x^2-3)+2/3/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*ln((1/x+I*(1-1/x^2)^(1/2))^2-1)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")

[Out] integrate(x^3*arcsec(x)/(x^2 - 1)^(5/2), x)

Fricas [A]

time = 0.34, size = 69, normalized size = 0.84

$$\frac{2(3x^2 - 2)\sqrt{x^2 - 1} \operatorname{arcsec}(x) + x^2 - 2(x^4 - 2x^2 + 1)\log(x^2 - 1) + 4(x^4 - 2x^2 + 1)\log(x) - 1}{6(x^4 - 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")

[Out] -1/6*(2*(3*x^2 - 2)*sqrt(x^2 - 1)*arcsec(x) + x^2 - 2*(x^4 - 2*x^2 + 1)*log(x^2 - 1) + 4*(x^4 - 2*x^2 + 1)*log(x) - 1)/(x^4 - 2*x^2 + 1)

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*asec(x)/(x**2-1)**(5/2),x)

[Out] Exception raised: SystemError

Giac [A]

time = 0.02, size = 77, normalized size = 0.94

$$-\frac{2x^2 - 1}{6((x^2 - 1)\operatorname{sign}(x))} - \frac{\ln(x^2)}{3\operatorname{sign}(x)} + \frac{\ln|x^2 - 1|}{3\operatorname{sign}(x)} + \frac{(-3(x^2 - 1) - 1)\arccos\left(\frac{1}{x}\right)}{3\sqrt{x^2 - 1}(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arcsec(x)/(x^2-1)^(5/2),x)

[Out] -1/3*(3*x^2 - 2)*arccos(1/x)/(x^2 - 1)^(3/2) - 1/3*log(x^2)/sgn(x) + 1/3*log(abs(x^2 - 1))/sgn(x) - 1/6*(2*x^2 - 1)/((x^2 - 1)*sgn(x))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{acos}\left(\frac{1}{x}\right)}{(x^2 - 1)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*acos(1/x))/(x^2 - 1)^(5/2),x)

[Out] int((x^3*acos(1/x))/(x^2 - 1)^(5/2), x)

$$3.689 \quad \int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}} dx$$

Optimal. Leaf size=175

$$\frac{\sqrt{x^2} (2 - 3x^2)}{6(-1+x^2)} - \frac{13}{6} \coth^{-1}\left(\sqrt{x^2}\right) - \frac{5x^3 \sec^{-1}(x)}{6(-1+x^2)^{3/2}} + \frac{x^5 \sec^{-1}(x)}{2(-1+x^2)^{3/2}} - \frac{5x \sec^{-1}(x)}{2\sqrt{-1+x^2}} - \frac{5i\sqrt{x^2} \sec^{-1}(x) \tan^{-1}\left(\frac{e^{i \sec^{-1}(x)}}{x}\right)}{x}$$

[Out] -13/6*arccoth((x^2)^(1/2))-5/6*x^3*arcsec(x)/(x^2-1)^(3/2)+1/2*x^5*arcsec(x)/(x^2-1)^(3/2)+1/6*(-3*x^2+2)*(x^2)^(1/2)/(x^2-1)-5*I*arcsec(x)*arctan(1/x+I*(1-1/x^2)^(1/2))*(x^2)^(1/2)/x+5/2*I*polylog(2,-I*(1/x+I*(1-1/x^2)^(1/2)))*(x^2)^(1/2)/x-5/2*I*polylog(2,I*(1/x+I*(1-1/x^2)^(1/2)))*(x^2)^(1/2)/x-5/2*x*arcsec(x)/(x^2-1)^(1/2)

Rubi [A]

time = 0.19, antiderivative size = 232, normalized size of antiderivative = 1.33, number of steps used = 16, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {5350, 4790, 4794, 4804, 4266, 2317, 2438, 212, 205, 296, 331}

$$\frac{5i\sqrt{x^2} \operatorname{Li}_2\left(-ie^{i \sec^{-1}(x)}\right)}{2x} - \frac{5i\sqrt{x^2} \operatorname{Li}_2\left(ie^{i \sec^{-1}(x)}\right)}{2x} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{3\sqrt{x^2}}{4} - \frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} + \frac{x\sqrt{x^2} \sec^{-1}(x)}{2\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{5\sqrt{x^2} \sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}} - \frac{5\sqrt{x^2} \sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{13\sqrt{x^2} \coth^{-1}(x)}{6x} - \frac{5i\sqrt{x^2} \sec^{-1}(x) \tan^{-1}\left(\frac{e^{i \sec^{-1}(x)}}{x}\right)}{x}$$

Antiderivative was successfully verified.

[In] Int[(x^6*ArcSec[x])/(-1 + x^2)^(5/2), x]

[Out] -5/(12*(1 - x^(-2))*Sqrt[x^2]) - (3*Sqrt[x^2])/4 + Sqrt[x^2]/(4*(1 - x^(-2))) - (13*Sqrt[x^2]*ArcCoth[x])/(6*x) - (5*Sqrt[x^2]*ArcSec[x])/(6*(1 - x^(-2)))^(3/2)*x - (5*Sqrt[x^2]*ArcSec[x])/(2*Sqrt[1 - x^(-2)]*x) + (x*Sqrt[x^2]*ArcSec[x])/(2*(1 - x^(-2)))^(3/2) - ((5*I)*Sqrt[x^2]*ArcSec[x]*ArcTan[E^(I*ArcSec[x])])/x + (((5*I)/2)*Sqrt[x^2]*PolyLog[2, (-I)*E^(I*ArcSec[x])])/x - (((5*I)/2)*Sqrt[x^2]*PolyLog[2, I*E^(I*ArcSec[x])])/x

Rule 205

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(p_)*(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(-(c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.))], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4790

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 4794

```

Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Dist[b*c*
(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1
- c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !GtQ
[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])

```

Rule 4804

```

Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Dist[(-(c^(m + 1))^(1 - 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]], Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]

```

Rule 5350

```

Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] := Dist[-Sqrt[x^2]/x, Subst[Int[(e + d*x^2)^p*((a + b*A
rcCos[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]

```

Rubi steps

$$\begin{aligned}
 \int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}} dx &= -\frac{\sqrt{x^2} \operatorname{Subst}\left(\int \frac{\cos^{-1}(x)}{x^3(1-x^2)^{5/2}} dx, x, \frac{1}{x}\right)}{x} \\
 &= \frac{x\sqrt{x^2} \sec^{-1}(x)}{2\left(1-\frac{1}{x^2}\right)^{3/2}} + \frac{\sqrt{x^2} \operatorname{Subst}\left(\int \frac{1}{x^2(1-x^2)^2} dx, x, \frac{1}{x}\right)}{2x} - \frac{\left(5\sqrt{x^2}\right) \operatorname{Subst}\left(\int \frac{\cos^{-1}(x)}{x(1-x^2)^{5/2}} dx, x, \frac{1}{x}\right)}{2x} \\
 &= \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{5\sqrt{x^2} \sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} + \frac{x\sqrt{x^2} \sec^{-1}(x)}{2\left(1-\frac{1}{x^2}\right)^{3/2}} + \frac{\left(3\sqrt{x^2}\right) \operatorname{Subst}\left(\int \frac{1}{x^2(1-x^2)} dx, x, \frac{1}{x}\right)}{4x} \\
 &= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{5\sqrt{x^2} \sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2} \sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}x} + \frac{x\sqrt{x^2} \sec^{-1}(x)}{2} \\
 &= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{13\sqrt{x^2} \operatorname{coth}^{-1}(x)}{6x} - \frac{5\sqrt{x^2} \sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2} \sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}x} \\
 &= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{13\sqrt{x^2} \operatorname{coth}^{-1}(x)}{6x} - \frac{5\sqrt{x^2} \sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2} \sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}x} \\
 &= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{13\sqrt{x^2} \operatorname{coth}^{-1}(x)}{6x} - \frac{5\sqrt{x^2} \sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2} \sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}x} \\
 &= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{13\sqrt{x^2} \operatorname{coth}^{-1}(x)}{6x} - \frac{5\sqrt{x^2} \sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2} \sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}x}
 \end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 383 vs. 2(175) = 350.
time = 1.13, size = 383, normalized size = 2.19

([In] Integrate[(x^6*ArcSec[x])/(-1+x^2)^(5/2),x] [Out] -1/96*(x^5*(22*ArcSec[x]+40*ArcSec[x]*Cos[2*ArcSec[x]]-30*ArcSec[x]*Cos[4*ArcSec[x]]-30*Sqrt[1-x^(-2)]*ArcSec[x]*Log[1-I*E^(I*ArcSec[x])]) +

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*ArcSec[x])/(-1+x^2)^(5/2),x]
```

```
[Out] -1/96*(x^5*(22*ArcSec[x]+40*ArcSec[x]*Cos[2*ArcSec[x]]-30*ArcSec[x]*Cos[4*ArcSec[x]]-30*Sqrt[1-x^(-2)]*ArcSec[x]*Log[1-I*E^(I*ArcSec[x])]) +
```



```

30*sqrt[1 - x^(-2)]*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])] + 26*sqrt[1 - x^(-
2)]*Log[Cos[ArcSec[x]/2]] - 26*sqrt[1 - x^(-2)]*Log[Sin[ArcSec[x]/2]] + 16*
Sin[2*ArcSec[x]] - (60*I)*sqrt[1 - x^(-2)]*PolyLog[2, (-I)*E^(I*ArcSec[x])]
*Sin[2*ArcSec[x]]^2 + (60*I)*sqrt[1 - x^(-2)]*PolyLog[2, I*E^(I*ArcSec[x])]
*Sin[2*ArcSec[x]]^2 - 15*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])]*Sin[3*ArcSec[
x]] + 15*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])]*Sin[3*ArcSec[x]] + 13*Log[Cos
[ArcSec[x]/2]]*Sin[3*ArcSec[x]] - 13*Log[Sin[ArcSec[x]/2]]*Sin[3*ArcSec[x]]
- 4*Sin[4*ArcSec[x]] + 15*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])]*Sin[5*ArcSe
c[x]] - 15*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])]*Sin[5*ArcSec[x]] - 13*Log[C
os[ArcSec[x]/2]]*Sin[5*ArcSec[x]] + 13*Log[Sin[ArcSec[x]/2]]*Sin[5*ArcSec[x
]])/(-1 + x^2)^(3/2)

```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(ArcSec[x]*x^6)/(x^2 - 1)^(5/2), x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Maple [A]

time = 0.84, size = 240, normalized size = 1.37

method	result
default	$\frac{\sqrt{x^2 - 1} x \left(3x^4 \operatorname{arcsec}(x) - 3 \sqrt{\frac{x^2 - 1}{x^2}} x^3 - 20 \operatorname{arcsec}(x) x^2 + 2 \sqrt{\frac{x^2 - 1}{x^2}} x + 15 \operatorname{arcsec}(x) \right)}{6x^4 - 12x^2 + 6} - \frac{i \sqrt{\frac{x^2 - 1}{x^2}} x \left(15i \operatorname{arcsec}(x) \ln(1 - I \operatorname{arcsec}(x)) \right)}{6x^4 - 12x^2 + 6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*arcsec(x)/(x^2-1)^(5/2), x, method=_RETURNVERBOSE)
```

```
[Out] 1/6*(x^2-1)^(1/2)*x/(x^4-2*x^2+1)*(3*x^4*arcsec(x)-3*((x^2-1)/x^2)^(1/2)*x^
3-20*arcsec(x)*x^2+2*((x^2-1)/x^2)^(1/2)*x+15*arcsec(x))-1/6*I/(x^2-1)^(1/2
)*((x^2-1)/x^2)^(1/2)*x*(15*I*arcsec(x)*ln(1-I*(1/x+I*(1-1/x^2)^(1/2))))-15*
I*arcsec(x)*ln(1+I*(1/x+I*(1-1/x^2)^(1/2)))+13*I*ln(1/x+I*(1-1/x^2)^(1/2)-1
)-13*I*ln(1/x+I*(1-1/x^2)^(1/2)+1)+15*dilog(1-I*(1/x+I*(1-1/x^2)^(1/2)))-15
*dilog(1+I*(1/x+I*(1-1/x^2)^(1/2)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")

[Out] integrate(x^6*arcsec(x)/(x^2 - 1)^(5/2), x)

Fricas [F]

time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(x^2 - 1)*x^6*arcsec(x)/(x^6 - 3*x^4 + 3*x^2 - 1), x)

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*asec(x)/(x**2-1)**(5/2),x)

[Out] Exception raised: SystemError

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*arcsec(x)/(x^2-1)^(5/2),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6 \arccos\left(\frac{1}{x}\right)}{(x^2 - 1)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6*acos(1/x))/(x^2 - 1)^(5/2),x)

[Out] int((x^6*acos(1/x))/(x^2 - 1)^(5/2), x)

$$3.690 \quad \int \frac{\sec^{-1}(x)}{x^2 \sqrt{-1+x^2}} dx$$

Optimal. Leaf size=23

$$\frac{1}{\sqrt{x^2}} + \frac{\sqrt{-1+x^2} \sec^{-1}(x)}{x}$$

[Out] 1/(x^2)^(1/2)+arcsec(x)*(x^2-1)^(1/2)/x

Rubi [A]

time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {270, 5346, 30}

$$\frac{1}{\sqrt{x^2}} + \frac{\sqrt{x^2-1} \sec^{-1}(x)}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcSec[x]/(x^2*Sqrt[-1+x^2]),x]

[Out] 1/Sqrt[x^2] + (Sqrt[-1+x^2]*ArcSec[x])/x

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m+1)*((a+b*x^n)^(p+1)/(a*c*(m+1))), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

Rule 5346

Int[((a_) + ArcSec[(c_)*(x_)]*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d+e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2-1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m-1)/2, 0] && GtQ[m+2*p+3, 0])) || (IGtQ[(m+1)/2, 0] && !(ILtQ[p, 0] && GtQ[m+2*p+3, 0])) || (ILtQ[(m+2*p+1)/2, 0] && !ILtQ[(m-1)/2, 0]))

Rubi steps

$$\int \frac{\sec^{-1}(x)}{x^2 \sqrt{-1+x^2}} dx = \frac{\sqrt{-1+x^2} \sec^{-1}(x)}{x} - \frac{x \int \frac{1}{x^2} dx}{\sqrt{x^2}}$$

$$= \frac{1}{\sqrt{x^2}} + \frac{\sqrt{-1+x^2} \sec^{-1}(x)}{x}$$

Mathematica [A]

time = 0.02, size = 35, normalized size = 1.52

$$\frac{\sqrt{1 - \frac{1}{x^2}} x + (-1 + x^2) \sec^{-1}(x)}{x \sqrt{-1 + x^2}}$$

Antiderivative was successfully verified.

`[In] Integrate[ArcSec[x]/(x^2*Sqrt[-1 + x^2]),x]``[Out] (Sqrt[1 - x^(-2)]*x + (-1 + x^2)*ArcSec[x])/(x*Sqrt[-1 + x^2])`**Mathics [F(-1)]**

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[ArcSec[x]/(x^2*Sqrt[x^2 - 1]),x]')``[Out] Timed out`**Maple [C] Result contains complex when optimal does not.**

time = 0.38, size = 178, normalized size = 7.74

method	result
default	$-\frac{\sqrt{\frac{x^2-1}{x^2}} x^3 - 3ix^2 - 4\sqrt{\frac{x^2-1}{x^2}} x + 4i}{4\sqrt{x^2-1}} \left(i\sqrt{\frac{x^2-1}{x^2}} x + 1 \right) x + \frac{\left(x^2 - 2 - 2i\sqrt{\frac{x^2-1}{x^2}} x \right) \operatorname{arcsec}(x)}{4x\sqrt{x^2-1}} - \frac{\left(i\sqrt{\frac{x^2-1}{x^2}} x - 1 \right) (\operatorname{arcsec}(x) + i)}{4\sqrt{x^2-1} x} + \frac{\left(i\sqrt{\frac{x^2-1}{x^2}} x \right)}{4\sqrt{x^2-1}}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(arcsec(x)/x^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)`
`[Out] -1/4*((x^2-1)/x^2)^(1/2)*x^3-3*I*x^2-4*((x^2-1)/x^2)^(1/2)*x+4*I)/(x^2-1)^(1/2)/(I*((x^2-1)/x^2)^(1/2)*x+1)/x+1/4/x/(x^2-1)^(1/2)*(x^2-2-2*I*((x^2-1)/x^2)^(1/2)*x)*arcsec(x)-1/4/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x-1)*(arc`

$\sec(x)+I)/x+1/4/(x^2-1)^{(1/2)}*(I*((x^2-1)/x^2)^{(1/2)}*x+x^2-1)*(3*\operatorname{arcsec}(x)-I)/x$

Maxima [A]

time = 0.34, size = 17, normalized size = 0.74

$$\frac{\sqrt{x^2 - 1} \operatorname{arcsec}(x)}{x} + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsec(x)/x^2/(x^2-1)^(1/2),x, algorithm="maxima")

[Out] sqrt(x^2 - 1)*arcsec(x)/x + 1/x

Fricas [A]

time = 0.31, size = 16, normalized size = 0.70

$$\frac{\sqrt{x^2 - 1} \operatorname{arcsec}(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsec(x)/x^2/(x^2-1)^(1/2),x, algorithm="fricas")

[Out] (sqrt(x^2 - 1)*arcsec(x) + 1)/x

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}(x)}{x^2 \sqrt{(x-1)(x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asec(x)/x**2/(x**2-1)**(1/2),x)

[Out] Integral(asec(x)/(x**2*sqrt((x - 1)*(x + 1))), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(19) = 38.

time = 0.02, size = 47, normalized size = 2.04

$$-\frac{2 \arctan\left(-x + \sqrt{x^2 - 1}\right)}{\operatorname{sign}(x)} + \frac{1}{x \operatorname{sign}(x)} + \frac{2 \arccos\left(\frac{1}{x}\right)}{\left(\sqrt{x^2 - 1} - x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsec(x)/x^2/(x^2-1)^(1/2),x)

[Out] $2*\arccos(1/x)/((x - \sqrt{x^2 - 1})^2 + 1) - 2*\arctan(-x + \sqrt{x^2 - 1})/\operatorname{sgn}(x) + 1/(x*\operatorname{sgn}(x))$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\arccos\left(\frac{1}{x}\right)}{x^2 \sqrt{x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acos(1/x)/(x^2*(x^2 - 1)^(1/2)),x)`

[Out] `int(acos(1/x)/(x^2*(x^2 - 1)^(1/2)), x)`

$$3.691 \quad \int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}} dx$$

Optimal. Leaf size=70

$$-\frac{1}{\sqrt{x^2}} + \frac{\sqrt{x^2}}{6(-1+x^2)} - \frac{11}{6} \coth^{-1}\left(\sqrt{x^2}\right) + \frac{(3-12x^2+8x^4)\csc^{-1}(x)}{3x(-1+x^2)^{3/2}}$$

[Out] $-11/6*\operatorname{arccoth}((x^2)^{(1/2)})+1/3*(8*x^4-12*x^2+3)*\operatorname{arccsc}(x)/x/(x^2-1)^{(3/2)}-1/(x^2)^{(1/2)}+1/6*(x^2)^{(1/2)}/(x^2-1)$

Rubi [A]

time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$,

Rules used = {277, 198, 197, 5347, 12, 1273, 464, 212}

$$-\frac{1}{\sqrt{x^2}} - \frac{\sqrt{x^2}}{6(1-x^2)} + \frac{8x \csc^{-1}(x)}{3\sqrt{x^2-1}} - \frac{4x \csc^{-1}(x)}{3(x^2-1)^{3/2}} + \frac{\csc^{-1}(x)}{x(x^2-1)^{3/2}} - \frac{11x \tanh^{-1}(x)}{6\sqrt{x^2}}$$

Antiderivative was successfully verified.

[In] `Int[ArcCsc[x]/(x^2*(-1+x^2)^(5/2)),x]`

[Out] $-(1/\operatorname{Sqrt}[x^2]) - \operatorname{Sqrt}[x^2]/(6*(1-x^2)) + \operatorname{ArcCsc}[x]/(x*(-1+x^2)^{(3/2)}) - (4*x*\operatorname{ArcCsc}[x])/(3*(-1+x^2)^{(3/2)}) + (8*x*\operatorname{ArcCsc}[x])/(3*\operatorname{Sqrt}[-1+x^2]) - (11*x*\operatorname{ArcTanh}[x])/(6*\operatorname{Sqrt}[x^2])$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 197

`Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]`

Rule 198

`Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]`

Rule 212

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt`

Q[a, 0] || LtQ[b, 0])

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^
4)^(p_), x_Symbol] := Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 5347

```
Int[((a_) + ArcCsc[(c_)*(x_)])*(b_)*((f_)*(x_))^(m_)*((d_) + (e_)*(x
_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}} dx &= \frac{\csc^{-1}(x)}{x(-1+x^2)^{3/2}} - \frac{4x \csc^{-1}(x)}{3(-1+x^2)^{3/2}} + \frac{8x \csc^{-1}(x)}{3\sqrt{-1+x^2}} + \frac{x \int \frac{3-12x^2+8x^4}{3x^2(1-x^2)^2} dx}{\sqrt{x^2}} \\
&= \frac{\csc^{-1}(x)}{x(-1+x^2)^{3/2}} - \frac{4x \csc^{-1}(x)}{3(-1+x^2)^{3/2}} + \frac{8x \csc^{-1}(x)}{3\sqrt{-1+x^2}} + \frac{x \int \frac{3-12x^2+8x^4}{x^2(1-x^2)^2} dx}{3\sqrt{x^2}} \\
&= -\frac{\sqrt{x^2}}{6(1-x^2)} + \frac{\csc^{-1}(x)}{x(-1+x^2)^{3/2}} - \frac{4x \csc^{-1}(x)}{3(-1+x^2)^{3/2}} + \frac{8x \csc^{-1}(x)}{3\sqrt{-1+x^2}} - \frac{x \int \frac{-6+17x^2}{x^2(1-x^2)} dx}{6\sqrt{x^2}} \\
&= -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{x^2}}{6(1-x^2)} + \frac{\csc^{-1}(x)}{x(-1+x^2)^{3/2}} - \frac{4x \csc^{-1}(x)}{3(-1+x^2)^{3/2}} + \frac{8x \csc^{-1}(x)}{3\sqrt{-1+x^2}} - \frac{(11x) \tan^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{6\sqrt{-1+x^2}} \\
&= -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{x^2}}{6(1-x^2)} + \frac{\csc^{-1}(x)}{x(-1+x^2)^{3/2}} - \frac{4x \csc^{-1}(x)}{3(-1+x^2)^{3/2}} + \frac{8x \csc^{-1}(x)}{3\sqrt{-1+x^2}} - \frac{11x \tan^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{6\sqrt{-1+x^2}}
\end{aligned}$$

Mathematica [A]

time = 0.09, size = 79, normalized size = 1.13

$$\frac{4(3-12x^2+8x^4)\csc^{-1}(x) + \sqrt{1-\frac{1}{x^2}}x(12-10x^2+11x(-1+x^2)\log(1-x)-11x(-1+x^2)\log(1+x))}{12x(-1+x^2)^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[ArcCsc[x]/(x^2*(-1+x^2)^(5/2)),x]`

```
[Out] (4*(3-12*x^2+8*x^4)*ArcCsc[x]+Sqrt[1-x^(-2)]*x*(12-10*x^2+11*x*(-1+x^2)*Log[1-x]-11*x*(-1+x^2)*Log[1+x]))/(12*x*(-1+x^2)^(3/2))
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[ArcCsc[x]/(x^2*(x^2-1)^(5/2)),x]')``[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep`**Maple [C]** Result contains complex when optimal does not.

time = 0.63, size = 702, normalized size = 10.03

method	result
--------	--------

default	$-\frac{3ix^2-4i-4\sqrt{\frac{x^2-1}{x^2}}x+\sqrt{\frac{x^2-1}{x^2}}x^3}{4\left(i\sqrt{\frac{x^2-1}{x^2}}x-1\right)x\sqrt{x^2-1}}+\frac{\left(x^2-2+2i\sqrt{\frac{x^2-1}{x^2}}x\right)\operatorname{arccsc}(x)}{4x\sqrt{x^2-1}}+\frac{x\operatorname{arccsc}(x)}{2\sqrt{x^2-1}}+\frac{\left(x^2-2-2i\sqrt{\frac{x^2-1}{x^2}}x\right)\operatorname{arccsc}(x)}{4\sqrt{x^2-1}x}$
---------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccsc(x)/x^2/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)`

[Out]
$$-1/4/(I*((x^2-1)/x^2)^(1/2)*x-1)/x/(x^2-1)^(1/2)*(3*I*x^2-4*I-4*((x^2-1)/x^2)^(1/2)*x+((x^2-1)/x^2)^(1/2)*x^3)+1/4/x/(x^2-1)^(1/2)*(x^2-2+2*I*((x^2-1)/x^2)^(1/2)*x)*\operatorname{arccsc}(x)+1/2*x*\operatorname{arccsc}(x)/(x^2-1)^(1/2)+1/4/(x^2-1)^(1/2)*(x^2-2-2*I*((x^2-1)/x^2)^(1/2)*x)*\operatorname{arccsc}(x)/x+1/4*x^3/(x^2-1)^(1/2)/(I*x^2-2*((x^2-1)/x^2)^(1/2)*x-2*I)+2/3*(x^2-1)^(1/2)*x^3/(x^4-2*x^2+1)*\operatorname{arccsc}(x)-1/24*x^5*((x^2-1)/x^2)^(1/2)*x+I/(x^2-1)^(1/2)/(I*((x^2-1)/x^2)^(1/2)*x^5-5*I*((x^2-1)/x^2)^(1/2)*x^3-3*x^4+4*I*((x^2-1)/x^2)^(1/2)*x+7*x^2-4)+1/2*x*(x^2-1)^(1/2)*(x^2-2-2*I*((x^2-1)/x^2)^(1/2)*x)*\operatorname{arccsc}(x)/(x^4-2*x^2+1)+1/24*x*(5*I*x^4-20*I*x^2-12*((x^2-1)/x^2)^(1/2)*x^3+((x^2-1)/x^2)^(1/2)*x^5+16*I+16*((x^2-1)/x^2)^(1/2)*x)/(x^2-1)^(1/2)/(I*((x^2-1)/x^2)^(1/2)*x^5-5*I*((x^2-1)/x^2)^(1/2)*x^3-3*x^4+4*I*((x^2-1)/x^2)^(1/2)*x+7*x^2-4)+1/2*(x^2-1)^(1/2)*x*(x^2-2+2*I*((x^2-1)/x^2)^(1/2)*x)*\operatorname{arccsc}(x)/(x^4-2*x^2+1)+11/12/(x^2-1)^(1/2)*(((x^2-1)/x^2)^(1/2)*x+I)*\ln(I/x+(1-1/x^2)^(1/2)-I)+11/12/(x^2-1)^(1/2)*(((x^2-1)/x^2)^(1/2)*x-I)*\ln(I/x+(1-1/x^2)^(1/2)-I)-11/12/(x^2-1)^(1/2)*(((x^2-1)/x^2)^(1/2)*x+I)*\ln(I/x+(1-1/x^2)^(1/2)+I)-11/12/(x^2-1)^(1/2)*(((x^2-1)/x^2)^(1/2)*x-I)*\ln(I/x+(1-1/x^2)^(1/2)+I)$$

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(56) = 112.

time = 0.55, size = 123, normalized size = 1.76

$$\frac{32x^4 \arctan\left(1, \sqrt{x+1} \sqrt{x-1}\right) - (x^3 - x)\sqrt{x+1} \sqrt{x-1} \left(\frac{2(5x^2-6)}{x^2-x} + 11 \log(x+1) - 11 \log(x-1)\right) - 48x^2 \arctan\left(1, \sqrt{x+1} \sqrt{x-1}\right) + 12 \arctan\left(1, \sqrt{x+1} \sqrt{x-1}\right)}{12(x^3 - x)\sqrt{x+1} \sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccsc(x)/x^2/(x^2-1)^(5/2),x, algorithm="maxima")`

[Out]
$$1/12*(32*x^4*\arctan2(1, \operatorname{sqrt}(x+1)*\operatorname{sqrt}(x-1)) - (x^3 - x)*\operatorname{sqrt}(x+1)*\operatorname{sqrt}(x-1)*(2*(5*x^2 - 6)/(x^3 - x) + 11*\log(x+1) - 11*\log(x-1)) - 48*x^2*\arctan2(1, \operatorname{sqrt}(x+1)*\operatorname{sqrt}(x-1)) + 12*\arctan2(1, \operatorname{sqrt}(x+1)*\operatorname{sqrt}(x-1)))/((x^3 - x)*\operatorname{sqrt}(x+1)*\operatorname{sqrt}(x-1))$$

Fricas [A]

time = 0.33, size = 81, normalized size = 1.16

$$\frac{10x^4 - 4(8x^4 - 12x^2 + 3)\sqrt{x^2-1} \operatorname{arccsc}(x) - 22x^2 + 11(x^5 - 2x^3 + x) \log(x+1) - 11(x^5 - 2x^3 + x) \log(x-1) + 12}{12(x^5 - 2x^3 + x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccsc(x)/x^2/(x^2-1)^(5/2),x, algorithm="fricas")

[Out] $-1/12*(10*x^4 - 4*(8*x^4 - 12*x^2 + 3)*\sqrt{x^2 - 1}*\operatorname{arccsc}(x) - 22*x^2 + 11*(x^5 - 2*x^3 + x)*\log(x + 1) - 11*(x^5 - 2*x^3 + x)*\log(x - 1) + 12)/(x^5 - 2*x^3 + x)$

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acsc(x)/x**2/(x**2-1)**(5/2),x)

[Out] Exception raised: SystemError

Giac [A]

time = 0.03, size = 117, normalized size = 1.67

$$\frac{-5x^2+6}{6((x^3-x)\operatorname{sign}(x))} + \frac{2\arctan(-x+\sqrt{x^2-1})}{\operatorname{sign}(x)} - \frac{11\ln|x+1|}{12\operatorname{sign}(x)} + \frac{11\ln|x-1|}{12\operatorname{sign}(x)} + \left(\frac{2\left(\frac{5}{6}xx-1\right)x\sqrt{x^2-1}}{(x^2-1)^2} + \frac{2}{(\sqrt{x^2-1}-x)^2+1} \right) \operatorname{arcsin}\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccsc(x)/x^2/(x^2-1)^(5/2),x)

[Out] $1/3*((5*x^2 - 6)*x/(x^2 - 1)^{(3/2)} + 6/((x - \sqrt{x^2 - 1})^2 + 1))*\operatorname{arcsin}(1/x) + 2*\arctan(-x + \sqrt{x^2 - 1})/\operatorname{sgn}(x) - 11/12*\log(\operatorname{abs}(x + 1))/\operatorname{sgn}(x) + 11/12*\log(\operatorname{abs}(x - 1))/\operatorname{sgn}(x) - 1/6*(5*x^2 - 6)/((x^3 - x)*\operatorname{sgn}(x))$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asin}\left(\frac{1}{x}\right)}{x^2(x^2-1)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(1/x)/(x^2*(x^2-1)^(5/2)),x)

[Out] int(asin(1/x)/(x^2*(x^2-1)^(5/2)), x)

$$3.692 \quad \int \frac{\csc^{-1}(x)^4}{x^2 \sqrt{-1+x^2}} dx$$

Optimal. Leaf size=74

$$\frac{24\sqrt{-1+x^2}}{x} + \frac{24 \csc^{-1}(x)}{\sqrt{x^2}} - \frac{12\sqrt{-1+x^2} \csc^{-1}(x)^2}{x} - \frac{4 \csc^{-1}(x)^3}{\sqrt{x^2}} + \frac{\sqrt{-1+x^2} \csc^{-1}(x)^4}{x}$$

[Out] 24*arccsc(x)/(x^2)^(1/2)-4*arccsc(x)^3/(x^2)^(1/2)+24*(x^2-1)^(1/2)/x-12*arccsc(x)^2*(x^2-1)^(1/2)/x+arccsc(x)^4*(x^2-1)^(1/2)/x

Rubi [A]

time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$,

Rules used = {5351, 4767, 4715, 267}

$$\frac{24\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}}{x} + \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^4}{x} - \frac{4 \csc^{-1}(x)^3}{\sqrt{x^2}} - \frac{12\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^2}{x} + \frac{24 \csc^{-1}(x)}{\sqrt{x^2}}$$

Antiderivative was successfully verified.

[In] Int[ArcCsc[x]^4/(x^2*Sqrt[-1+x^2]),x]

[Out] (24*Sqrt[1-x^(-2)]*Sqrt[x^2])/x + (24*ArcCsc[x])/Sqrt[x^2] - (12*Sqrt[1-x^(-2)]*Sqrt[x^2]*ArcCsc[x]^2)/x - (4*ArcCsc[x]^3)/Sqrt[x^2] + (Sqrt[1-x^(-2)]*Sqrt[x^2]*ArcCsc[x]^4)/x

Rule 267

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 4715

Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*ArcSin[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rule 4767

Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

Rule 5351

Int[((a_.) + ArcCsc[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_), x_Symbol] := Dist[-Sqrt[x^2]/x, Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/2] && GtQ[e, 0] && LtQ[d, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\csc^{-1}(x)^4}{x^2 \sqrt{-1+x^2}} dx &= -\frac{\sqrt{x^2} \operatorname{Subst}\left(\int \frac{x \sin^{-1}(x)^4}{\sqrt{1-x^2}} dx, x, \frac{1}{x}\right)}{x} \\
 &= \frac{\sqrt{1-\frac{1}{x^2}} \sqrt{x^2} \csc^{-1}(x)^4}{x} - \frac{(4\sqrt{x^2}) \operatorname{Subst}\left(\int \sin^{-1}(x)^3 dx, x, \frac{1}{x}\right)}{x} \\
 &= -\frac{4 \csc^{-1}(x)^3}{\sqrt{x^2}} + \frac{\sqrt{1-\frac{1}{x^2}} \sqrt{x^2} \csc^{-1}(x)^4}{x} + \frac{(12\sqrt{x^2}) \operatorname{Subst}\left(\int \frac{x \sin^{-1}(x)^2}{\sqrt{1-x^2}} dx, x, \frac{1}{x}\right)}{x} \\
 &= -\frac{12\sqrt{1-\frac{1}{x^2}} \sqrt{x^2} \csc^{-1}(x)^2}{x} - \frac{4 \csc^{-1}(x)^3}{\sqrt{x^2}} + \frac{\sqrt{1-\frac{1}{x^2}} \sqrt{x^2} \csc^{-1}(x)^4}{x} + \frac{(24\sqrt{x^2}) \operatorname{Subst}\left(\int \sin^{-1}(x) dx, x, \frac{1}{x}\right)}{x} \\
 &= \frac{24 \csc^{-1}(x)}{\sqrt{x^2}} - \frac{12\sqrt{1-\frac{1}{x^2}} \sqrt{x^2} \csc^{-1}(x)^2}{x} - \frac{4 \csc^{-1}(x)^3}{\sqrt{x^2}} + \frac{\sqrt{1-\frac{1}{x^2}} \sqrt{x^2} \csc^{-1}(x)^4}{x} \\
 &= \frac{24\sqrt{1-\frac{1}{x^2}} \sqrt{x^2}}{x} + \frac{24 \csc^{-1}(x)}{\sqrt{x^2}} - \frac{12\sqrt{1-\frac{1}{x^2}} \sqrt{x^2} \csc^{-1}(x)^2}{x} - \frac{4 \csc^{-1}(x)^3}{\sqrt{x^2}} + \frac{\sqrt{1-\frac{1}{x^2}} \sqrt{x^2} \csc^{-1}(x)^4}{x}
 \end{aligned}$$

Mathematica [A]

time = 0.04, size = 76, normalized size = 1.03

$$\frac{24(-1+x^2) + 24\sqrt{1-\frac{1}{x^2}} x \csc^{-1}(x) - 12(-1+x^2) \csc^{-1}(x)^2 - 4\sqrt{1-\frac{1}{x^2}} x \csc^{-1}(x)^3 + (-1+x^2) \csc^{-1}(x)^4}{x\sqrt{-1+x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCsc[x]^4/(x^2*Sqrt[-1 + x^2]), x]

[Out] (24*(-1 + x^2) + 24*Sqrt[1 - x^(-2)]*x*ArcCsc[x] - 12*(-1 + x^2)*ArcCsc[x]^2 - 4*Sqrt[1 - x^(-2)]*x*ArcCsc[x]^3 + (-1 + x^2)*ArcCsc[x]^4)/(x*Sqrt[-1 + x^2])

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[ArcCsc[x]^4/(x^2*Sqrt[x^2 - 1]),x]')``[Out] Timed out`**Maple** [C] Result contains complex when optimal does not.

time = 0.53, size = 330, normalized size = 4.46

method	result
default	$\frac{\left(ix^2-2\sqrt{\frac{x^2-1}{x^2}}x-2i\right)\operatorname{arccsc}(x)^3}{\sqrt{x^2-1}x} + \frac{\left(x^2-2+2i\sqrt{\frac{x^2-1}{x^2}}x\right)\operatorname{arccsc}(x)^4}{4\sqrt{x^2-1}x} - \frac{6\left(ix^2-2\sqrt{\frac{x^2-1}{x^2}}x-2i\right)\operatorname{arccsc}(x)}{\sqrt{x^2-1}x} - \frac{3\left(x^2-2+\right)}{\sqrt{x^2-1}x}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(arccsc(x)^4/x^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)`

```
[Out] 1/(x^2-1)^(1/2)/x*(I*x^2-2*((x^2-1)/x^2)^(1/2)*x-2*I)*arccsc(x)^3+1/4/(x^2-1)^(1/2)/x*(x^2-2+2*I*((x^2-1)/x^2)^(1/2)*x)*arccsc(x)^4-6/(x^2-1)^(1/2)/x*(I*x^2-2*((x^2-1)/x^2)^(1/2)*x-2*I)*arccsc(x)-3/(x^2-1)^(1/2)/x*(x^2-2+2*I*((x^2-1)/x^2)^(1/2)*x)*arccsc(x)^2+6*(I*((x^2-1)/x^2)^(1/2)*x^3-4*I*((x^2-1)/x^2)^(1/2)*x-3*x^2+4)/(x^2-1)^(1/2)/(I*((x^2-1)/x^2)^(1/2)*x-1)/x+1/4/(x^2-1)^(1/2)*(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(3*arccsc(x)^4-36*arccsc(x)^2-4*I*arccsc(x)^3+72+24*I*arccsc(x))/x+1/4/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x+1)*(arccsc(x)^4+4*I*arccsc(x)^3-12*arccsc(x)^2-24*I*arccsc(x)+24)/x
```

Maxima [A]

time = 0.34, size = 58, normalized size = 0.78

$$\frac{\sqrt{x^2-1}\operatorname{arccsc}(x)^4}{x} - 12\sqrt{-\frac{1}{x^2}+1}\operatorname{arccsc}(x)^2 - \frac{4\operatorname{arccsc}(x)^3}{x} + 24\sqrt{-\frac{1}{x^2}+1} + \frac{24\operatorname{arccsc}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(arccsc(x)^4/x^2/(x^2-1)^(1/2),x, algorithm="maxima")`

```
[Out] sqrt(x^2 - 1)*arccsc(x)^4/x - 12*sqrt(-1/x^2 + 1)*arccsc(x)^2 - 4*arccsc(x)^3/x + 24*sqrt(-1/x^2 + 1) + 24*arccsc(x)/x
```

Fricas [A]

time = 0.32, size = 37, normalized size = 0.50

$$\frac{4\operatorname{arccsc}(x)^3 - (\operatorname{arccsc}(x)^4 - 12\operatorname{arccsc}(x)^2 + 24)\sqrt{x^2-1} - 24\operatorname{arccsc}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccsc(x)^4/x^2/(x^2-1)^(1/2),x, algorithm="fricas")`

[Out] $-(4*\arccsc(x)^3 - (\arccsc(x)^4 - 12*\arccsc(x)^2 + 24)*\sqrt{x^2 - 1} - 24*\arccsc(x))/x$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccsc}^4(x)}{x^2 \sqrt{(x-1)(x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acsc(x)**4/x**2/(x**2-1)**(1/2),x)`

[Out] `Integral(acsc(x)**4/(x**2*sqrt((x - 1)*(x + 1))), x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccsc(x)^4/x^2/(x^2-1)^(1/2),x)`

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asin}\left(\frac{1}{x}\right)^4}{x^2 \sqrt{x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(1/x)^4/(x^2*(x^2 - 1)^(1/2)),x)`

[Out] `int(asin(1/x)^4/(x^2*(x^2 - 1)^(1/2)), x)`

$$3.693 \quad \int \frac{(-1+x^2)^{3/2} \sec^{-1}(x)^2}{x^5} dx$$

Optimal. Leaf size=133

$$\frac{\sqrt{-1+x^2}(-2+17x^2)}{64x^4} - \frac{3\sec^{-1}(x)}{8x\sqrt{x^2}} + \frac{9x\sec^{-1}(x)}{64\sqrt{x^2}} + \frac{(-1+x^2)^2\sec^{-1}(x)}{8x^3\sqrt{x^2}} - \frac{3\sqrt{-1+x^2}\sec^{-1}(x)^2}{8x^2} - \frac{(-1+x^2)^2}{8x^2}$$

[Out] $-1/4*(x^2-1)^{(3/2)}*\text{arcsec}(x)^2/x^4-3/8*\text{arcsec}(x)/x/(x^2)^{(1/2)}+9/64*x*\text{arcsec}(x)/(x^2)^{(1/2)}+1/8*(x^2-1)^2*\text{arcsec}(x)/x^3/(x^2)^{(1/2)}+1/8*x*\text{arcsec}(x)^3/(x^2)^{(1/2)}+1/64*(17*x^2-2)*(x^2-1)^{(1/2)}/x^4-3/8*\text{arcsec}(x)^2*(x^2-1)^{(1/2)}/x^2$

Rubi [A]

time = 0.13, antiderivative size = 172, normalized size of antiderivative = 1.29, number of steps used = 11, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {5350, 4744, 4742, 4738, 4724, 327, 222, 4768, 201}

$$\frac{(1-\frac{1}{x^2})^{3/2}}{32\sqrt{x^2}} + \frac{15\sqrt{1-\frac{1}{x^2}}}{64\sqrt{x^2}} - \frac{9\sqrt{x^2}\csc^{-1}(x)}{64x} + \frac{\sqrt{x^2}\sec^{-1}(x)^3}{8x} - \frac{(1-\frac{1}{x^2})^{3/2}\sec^{-1}(x)^2}{4\sqrt{x^2}} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^2}{8\sqrt{x^2}} + \frac{(1-\frac{1}{x^2})^2\sqrt{x^2}\sec^{-1}(x)}{8x} - \frac{3\sqrt{x^2}\sec^{-1}(x)}{8x^3}$$

Antiderivative was successfully verified.

[In] Int[((-1 + x^2)^(3/2)*ArcSec[x]^2)/x^5, x]

[Out] $(15*\text{Sqrt}[1-x^{(-2)}])/(64*\text{Sqrt}[x^2]) + (1-x^{(-2)})^{(3/2)}/(32*\text{Sqrt}[x^2]) - (9*\text{Sqrt}[x^2]*\text{ArcCsc}[x])/(64*x) - (3*\text{Sqrt}[x^2]*\text{ArcSec}[x])/(8*x^3) + ((1-x^{(-2)})^2*\text{Sqrt}[x^2]*\text{ArcSec}[x])/(8*x) - (3*\text{Sqrt}[1-x^{(-2)}]*\text{ArcSec}[x]^2)/(8*\text{Sqrt}[x^2]) - ((1-x^{(-2)})^{(3/2)}*\text{ArcSec}[x]^2)/(4*\text{Sqrt}[x^2]) + (\text{Sqrt}[x^2]*\text{ArcSec}[x]^3)/(8*x)$

Rule 201

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x*((a + b*x^n)^p/(n*p + 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 327

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[

$a*c^n*((m - n + 1)/(b*(m + n*p + 1))), \text{Int}[(c*x)^{(m - n)}*(a + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 4724

$\text{Int}[(a_.) + \text{ArcCos}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*((d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m + 1)}*((a + b*\text{ArcCos}[c*x])^n/(d*(m + 1))), x] + \text{Dist}[b*c*(n/(d*(m + 1))), \text{Int}[(d*x)^{(m + 1)}*((a + b*\text{ArcCos}[c*x])^{(n - 1)})/\text{Sqrt}[1 - c^2*x^2]), x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 4738

$\text{Int}[(a_.) + \text{ArcCos}[(c_.)*(x_.)]*(b_.)]^{(n_.)}/\text{Sqrt}[(d_.) + (e_.)*(x_.)^2], x_Symbol] \rightarrow \text{Simp}[(-b*c*(n + 1))^{(-1)}*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcCos}[c*x])^{(n + 1)}, x] /;$ FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]

Rule 4742

$\text{Int}[(a_.) + \text{ArcCos}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*\text{Sqrt}[(d_.) + (e_.)*(x_.)^2], x_Symbol] \rightarrow \text{Simp}[x*\text{Sqrt}[d + e*x^2]*((a + b*\text{ArcCos}[c*x])^{n/2}), x] + (\text{Dist}[(1/2)*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]], \text{Int}[(a + b*\text{ArcCos}[c*x])^n/\text{Sqrt}[1 - c^2*x^2], x], x] + \text{Dist}[b*c*(n/2)*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]], \text{Int}[x*(a + b*\text{ArcCos}[c*x])^{(n - 1)}, x], x]) /;$ FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]

Rule 4744

$\text{Int}[(a_.) + \text{ArcCos}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[x*(d + e*x^2)^p*((a + b*\text{ArcCos}[c*x])^{n/(2*p + 1)}), x] + (\text{Dist}[2*d*(p/(2*p + 1)), \text{Int}[(d + e*x^2)^{(p - 1)}*(a + b*\text{ArcCos}[c*x])^n, x], x] + \text{Dist}[b*c*(n/(2*p + 1))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p], \text{Int}[x*(1 - c^2*x^2)^{(p - 1/2)}*(a + b*\text{ArcCos}[c*x])^{(n - 1)}, x], x]) /;$ FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]

Rule 4768

$\text{Int}[(a_.) + \text{ArcCos}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*(x_.)*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{(p + 1)}*((a + b*\text{ArcCos}[c*x])^{n/(2*e*(p + 1))}), x] - \text{Dist}[b*(n/(2*c*(p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p], \text{Int}[(1 - c^2*x^2)^{(p + 1/2)}*(a + b*\text{ArcCos}[c*x])^{(n - 1)}, x], x] /;$ FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

Rule 5350

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Dist[-Sqrt[x^2]/x, Subst[Int[(e + d*x^2)^p*((a + b*Ar
cCos[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^2)^{3/2} \sec^{-1}(x)^2}{x^5} dx &= -\frac{\sqrt{x^2} \operatorname{Subst}\left(\int (1-x^2)^{3/2} \cos^{-1}(x)^2 dx, x, \frac{1}{x}\right)}{x} \\
&= -\frac{\left(1-\frac{1}{x^2}\right)^{3/2} \sec^{-1}(x)^2}{4\sqrt{x^2}} - \frac{\sqrt{x^2} \operatorname{Subst}\left(\int x(1-x^2) \cos^{-1}(x) dx, x, \frac{1}{x}\right)}{2x} - \left(3\sqrt{x^2}\right)^{3/2} \\
&= \frac{\left(1-\frac{1}{x^2}\right)^2 \sqrt{x^2} \sec^{-1}(x)}{8x} - \frac{3\sqrt{1-\frac{1}{x^2}} \sec^{-1}(x)^2}{8\sqrt{x^2}} - \frac{\left(1-\frac{1}{x^2}\right)^{3/2} \sec^{-1}(x)^2}{4\sqrt{x^2}} + \\
&= \frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{32\sqrt{x^2}} - \frac{3\sqrt{x^2} \sec^{-1}(x)}{8x^3} + \frac{\left(1-\frac{1}{x^2}\right)^2 \sqrt{x^2} \sec^{-1}(x)}{8x} - \frac{3\sqrt{1-\frac{1}{x^2}} \sec^{-1}(x)^2}{8\sqrt{x^2}} \\
&= \frac{15\sqrt{1-\frac{1}{x^2}}}{64\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{32\sqrt{x^2}} - \frac{3\sqrt{x^2} \sec^{-1}(x)}{8x^3} + \frac{\left(1-\frac{1}{x^2}\right)^2 \sqrt{x^2} \sec^{-1}(x)}{8x} \\
&= \frac{15\sqrt{1-\frac{1}{x^2}}}{64\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{32\sqrt{x^2}} - \frac{9\sqrt{x^2} \csc^{-1}(x)}{64x} - \frac{3\sqrt{x^2} \sec^{-1}(x)}{8x^3} + \frac{\left(1-\frac{1}{x^2}\right)^2 \sqrt{x^2} \sec^{-1}(x)}{8x}
\end{aligned}$$

Mathematica [A]

time = 0.17, size = 84, normalized size = 0.63

$$\frac{\sqrt{-1+x^2} (32 \sec^{-1}(x)^3 + 4 \sec^{-1}(x) (-16 \cos(2 \sec^{-1}(x)) + \cos(4 \sec^{-1}(x))) + 32 \sin(2 \sec^{-1}(x)) - \sin(4 \sec^{-1}(x)) + 8 \sec^{-1}(x)^2 (-8 \sin(2 \sec^{-1}(x)) + \sin(4 \sec^{-1}(x))))}{256 \sqrt{1-\frac{1}{x^2}} x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)^(3/2)*ArcSec[x]^2)/x^5, x]
```

```
[Out] (Sqrt[-1 + x^2]*(32*ArcSec[x]^3 + 4*ArcSec[x]*(-16*Cos[2*ArcSec[x]] + Cos[4
*ArcSec[x]]) + 32*Sin[2*ArcSec[x]] - Sin[4*ArcSec[x]] + 8*ArcSec[x]^2*(-8*S
in[2*ArcSec[x]] + Sin[4*ArcSec[x]])))/(256*Sqrt[1 - x^(-2)]*x)
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[(ArcSec[x]^2*(x^2 - 1)^(3/2))/x^5,x]')`

[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep

Maple [C] Result contains complex when optimal does not.

time = 0.58, size = 386, normalized size = 2.90

method	result
default	$\frac{\sqrt{\frac{x^2-1}{x^2}} \operatorname{arcsec}(x)^3}{8\sqrt{x^2-1}} + \frac{\left(-5i\sqrt{\frac{x^2-1}{x^2}} x^5+x^6+20i\sqrt{\frac{x^2-1}{x^2}} x^3-13x^4-16i\sqrt{\frac{x^2-1}{x^2}} x+28x^2-16\right) \left(4i\operatorname{arcsec}(x)+8\operatorname{arcsec}(x)\right)}{1024x^4\sqrt{x^2-1}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2-1)^(3/2)*arcsec(x)^2/x^5,x,method=_RETURNVERBOSE)`

[Out]
$$\frac{1}{8} \frac{(x^2-1)^{1/2} \left((x^2-1)/x^2 \right)^{1/2} x \operatorname{arcsec}(x)^3 + 1/1024 x^4 (x^2-1)^{1/2} \left(-5i \left((x^2-1)/x^2 \right)^{1/2} x^5 + x^6 + 20i \left((x^2-1)/x^2 \right)^{1/2} x^3 - 13x^4 - 16i \left((x^2-1)/x^2 \right)^{1/2} x + 28x^2 - 16 \right) \left(4i \operatorname{arcsec}(x) + 8 \operatorname{arcsec}(x) \right)^2 - 1/32 (x^2-1)^{1/2} \left(-i \left((x^2-1)/x^2 \right)^{1/2} x + x^2 - 1 \right) \left(2 \operatorname{arcsec}(x) \right)^2 - 1 + 2i \operatorname{arcsec}(x) \right) + 1/16 x^2 (x^2-1)^{1/2} \left(i \left((x^2-1)/x^2 \right)^{1/2} x^3 - 2i \left((x^2-1)/x^2 \right)^{1/2} x - 2x^2 + 2 \right) \left(2 \operatorname{arcsec}(x) \right)^2 - 1 - 2i \operatorname{arcsec}(x) \right) - 1/1024 (x^2-1)^{1/2} \left(-5x^2 + 4 + 3i \left((x^2-1)/x^2 \right)^{1/2} x^3 + x^4 - 4i \left((x^2-1)/x^2 \right)^{1/2} x \right) \left(-4i \operatorname{arcsec}(x) + 8 \operatorname{arcsec}(x) \right)^2 - 1/x^2 + 1/128 (x^2-1)^{1/2} \left(i \left((x^2-1)/x^2 \right)^{1/2} x + x^2 - 1 \right) \left(7i \operatorname{arcsec}(x) + 8 \operatorname{arcsec}(x) \right)^2 - 4 \cos(4 \operatorname{arcsec}(x)) + 1/512 (x^2-1)^{1/2} \left(i x^2 - \left((x^2-1)/x^2 \right)^{1/2} x - i \right) \left(32i \operatorname{arcsec}(x) + 24 \operatorname{arcsec}(x) \right)^2 - 15 \sin(4 \operatorname{arcsec}(x)) \right)$$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2-1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="maxima")`

[Out] `integrate((x^2 - 1)^(3/2)*arcsec(x)^2/x^5, x)`

Fricas [A]

time = 0.32, size = 59, normalized size = 0.44

$$\frac{8x^4 \operatorname{arcsec}(x)^3 + (17x^4 - 40x^2 + 8) \operatorname{arcsec}(x) - (8(5x^2 - 2) \operatorname{arcsec}(x)^2 - 17x^2 + 2) \sqrt{x^2 - 1}}{64x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2-1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="fricas")`

[Out] $\frac{1}{64}(8x^4 \operatorname{arcsec}(x)^3 + (17x^4 - 40x^2 + 8)\operatorname{arcsec}(x) - (8(5x^2 - 2)\operatorname{arcsec}(x)^2 - 17x^2 + 2)\sqrt{x^2 - 1})/x^4$

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2-1)**(3/2)*asec(x)**2/x**5,x)`

[Out] Exception raised: SystemError

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2-1)^(3/2)*arcsec(x)^2/x^5,x)`

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\arccos\left(\frac{1}{x}\right)^2 (x^2 - 1)^{3/2}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((acos(1/x)^2*(x^2 - 1)^(3/2))/x^5,x)`

[Out] `int((acos(1/x)^2*(x^2 - 1)^(3/2))/x^5, x)`

$$3.694 \quad \int \frac{\sqrt{-1+x^2} \sec^{-1}(x)^3}{x^4} dx$$

Optimal. Leaf size=110

$$\frac{2(1-21x^2)}{27(x^2)^{3/2}} - \frac{4\sqrt{-1+x^2} \sec^{-1}(x)}{3x} - \frac{2(-1+x^2)^{3/2} \sec^{-1}(x)}{9x^3} + \frac{2 \sec^{-1}(x)^2}{3\sqrt{x^2}} + \frac{(-1+x^2) \sec^{-1}(x)^2}{3(x^2)^{3/2}} + \frac{(-1+x^2)^{3/2} \sec^{-1}(x)^3}{27(x^2)^{3/2}}$$

[Out] $-2/9*(x^2-1)^{(3/2)*\text{arcsec}(x)/x^3+1/3*(x^2-1)^{(3/2)*\text{arcsec}(x)^3/x^3+2/27*(-21*x^2+1)/x^2/(x^2)^{(1/2)+2/3*\text{arcsec}(x)^2/(x^2)^{(1/2)+1/3*(x^2-1)*\text{arcsec}(x)^2/x^2/(x^2)^{(1/2)-4/3*\text{arcsec}(x)*(x^2-1)^{(1/2)/x}}$

Rubi [A]

time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5350, 4768, 4744, 4716, 8}

$$-\frac{14}{9\sqrt{x^2}} + \frac{(1-\frac{1}{x^2})^{3/2} \sqrt{x^2} \sec^{-1}(x)^3}{3x} + \frac{(1-\frac{1}{x^2}) \sec^{-1}(x)^2}{3\sqrt{x^2}} + \frac{2 \sec^{-1}(x)^2}{3\sqrt{x^2}} - \frac{2(1-\frac{1}{x^2})^{3/2} \sqrt{x^2} \sec^{-1}(x)}{9x} - \frac{4\sqrt{1-\frac{1}{x^2}} \sqrt{x^2} \sec^{-1}(x)}{3x} + \frac{2\sqrt{x^2}}{27x^4}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-1 + x^2]*ArcSec[x]^3)/x^4, x]

[Out] $-14/(9*\text{Sqrt}[x^2]) + (2*\text{Sqrt}[x^2])/(27*x^4) - (4*\text{Sqrt}[1 - x^{(-2)}]*\text{Sqrt}[x^2]*\text{ArcSec}[x])/(3*x) - (2*(1 - x^{(-2)})^{(3/2)}*\text{Sqrt}[x^2]*\text{ArcSec}[x])/(9*x) + (2*\text{ArcSec}[x]^2)/(3*\text{Sqrt}[x^2]) + ((1 - x^{(-2)})*\text{ArcSec}[x]^2)/(3*\text{Sqrt}[x^2]) + ((1 - x^{(-2)})^{(3/2)}*\text{Sqrt}[x^2]*\text{ArcSec}[x]^3)/(3*x)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 4716

Int[((a_) + ArcCos[(c_)*(x_)])*(b_)^(n_), x_Symbol] := Simp[x*(a + b*ArcCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n-1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rule 4744

Int[((a_) + ArcCos[(c_)*(x_)])*(b_)^(n_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p-1)*(a + b*ArcCos[c*x])^n, x], x] + Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[x*(1 - c^2*x^2)^(p-1/2)*(a + b*ArcCos[c*x])^(n-1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p + 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5350

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Dist[-Sqrt[x^2]/x, Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/2] && GtQ[e, 0] && LtQ[d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x^2} \sec^{-1}(x)^3}{x^4} dx &= -\frac{\sqrt{x^2} \operatorname{Subst}\left(\int x \sqrt{1-x^2} \cos^{-1}(x)^3 dx, x, \frac{1}{x}\right)}{x} \\
&= \frac{\left(1-\frac{1}{x^2}\right)^{3/2} \sqrt{x^2} \sec^{-1}(x)^3}{3x} + \frac{\sqrt{x^2} \operatorname{Subst}\left(\int (1-x^2) \cos^{-1}(x)^2 dx, x, \frac{1}{x}\right)}{x} \\
&= \frac{\left(1-\frac{1}{x^2}\right) \sec^{-1}(x)^2}{3\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)^{3/2} \sqrt{x^2} \sec^{-1}(x)^3}{3x} + \frac{\left(2\sqrt{x^2}\right) \operatorname{Subst}\left(\int x \sqrt{1-x^2} \cos^{-1}(x) dx, x, \frac{1}{x}\right)}{3x} \\
&= -\frac{2\left(1-\frac{1}{x^2}\right)^{3/2} \sqrt{x^2} \sec^{-1}(x)}{9x} + \frac{2 \sec^{-1}(x)^2}{3\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right) \sec^{-1}(x)^2}{3\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)^{3/2} \sqrt{x^2} \sec^{-1}(x)^3}{9x} \\
&= -\frac{2}{9\sqrt{x^2}} + \frac{2\sqrt{x^2}}{27x^4} - \frac{4\sqrt{1-\frac{1}{x^2}} \sqrt{x^2} \sec^{-1}(x)}{3x} - \frac{2\left(1-\frac{1}{x^2}\right)^{3/2} \sqrt{x^2} \sec^{-1}(x)}{9x} \\
&= -\frac{14}{9\sqrt{x^2}} + \frac{2\sqrt{x^2}}{27x^4} - \frac{4\sqrt{1-\frac{1}{x^2}} \sqrt{x^2} \sec^{-1}(x)}{3x} - \frac{2\left(1-\frac{1}{x^2}\right)^{3/2} \sqrt{x^2} \sec^{-1}(x)}{9x}
\end{aligned}$$

Mathematica [A]

time = 0.05, size = 92, normalized size = 0.84

$$\frac{2\sqrt{1-\frac{1}{x^2}} x(1-21x^2) - 6(1-8x^2+7x^4) \sec^{-1}(x) + 9\sqrt{1-\frac{1}{x^2}} x(-1+3x^2) \sec^{-1}(x)^2 + 9(-1+x^2)^2 \sec^{-1}(x)^3}{27x^3\sqrt{-1+x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-1 + x^2]*ArcSec[x]^3)/x^4,x]

[Out] (2*Sqrt[1 - x^(-2)]*x*(1 - 21*x^2) - 6*(1 - 8*x^2 + 7*x^4)*ArcSec[x] + 9*Sqrt[1 - x^(-2)]*x*(-1 + 3*x^2)*ArcSec[x]^2 + 9*(-1 + x^2)^2*ArcSec[x]^3)/(27*x^3*Sqrt[-1 + x^2])

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(ArcSec[x]^3*Sqrt[x^2 - 1])/x^4,x]')

[Out] Timed out

Maple [C] Result contains complex when optimal does not.

time = 0.54, size = 536, normalized size = 4.87

method	result
default	$\frac{\left(i\sqrt{\frac{x^2-1}{x^2}}x^5-8i\sqrt{\frac{x^2-1}{x^2}}x^3+4x^4+8i\sqrt{\frac{x^2-1}{x^2}}x-12x^2+8\right)\operatorname{arcsec}(x)^3}{48x^3\sqrt{x^2-1}} - \frac{\left(i\sqrt{\frac{x^2-1}{x^2}}x^5-8i\sqrt{\frac{x^2-1}{x^2}}x^3+4x^4+8i\sqrt{\frac{x^2-1}{x^2}}x-12x^2+8\right)\operatorname{arcsec}(x)^3}{72x^3\sqrt{x^2-1}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x,method=_RETURNVERBOSE)

[Out] 1/48/x^3/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x^5-8*I*((x^2-1)/x^2)^(1/2)*x^3+4*x^4+8*I*((x^2-1)/x^2)^(1/2)*x-12*x^2+8)*arcsec(x)^3-1/72/x^3/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x^5-8*I*((x^2-1)/x^2)^(1/2)*x^3+4*x^4+8*I*((x^2-1)/x^2)^(1/2)*x-12*x^2+8)*arcsec(x)-1/48/x^3/(x^2-1)^(1/2)*(((x^2-1)/x^2)^(1/2)*x^5-4*I*x^4-8*((x^2-1)/x^2)^(1/2)*x^3+12*I*x^2+8*((x^2-1)/x^2)^(1/2)*x-8*I)*arcsec(x)^2+1/216*(x^2-1)^(1/2)*(((x^2-1)/x^2)^(1/2)*x^5-5*I*x^4-12*((x^2-1)/x^2)^(1/2)*x^3+20*I*x^2+16*((x^2-1)/x^2)^(1/2)*x-16*I)/(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)/x^3+1/216/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(9*arcsec(x)^3-117*I*arcsec(x)^2-78*arcsec(x)+242*I)/x-1/216*(45*I*arcsec(x)^2+9*arcsec(x)^3-82*I-78*arcsec(x))*(I*((x^2-1)/x^2)^(1/2)*x-1)*(x^2-1)^(1/2)/x-1/432/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*(63*I*arcsec(x)^2+27*arcsec(x)^3-158*I-162*arcsec(x))*cos(3*arcsec(x))-1/144/(x^2-1)^(1/2)*(I*x^2-((x^2-1)/x^2)^(1/2)*x-I)*(27*I*arcsec(x)^2+3*arcsec(x)^3-54*I-50*arcsec(x))*sin(3*arcsec(x))

Maxima [A]

time = 0.55, size = 93, normalized size = 0.85

$$\frac{(x^2 - 1)^{\frac{3}{2}} \operatorname{arcsec}(x)^3}{3x^3} + \frac{(3x^2 - 1) \operatorname{arcsec}(x)^2}{3x^3} - \frac{2 \left((21x^2 - 1) \sqrt{x+1} \sqrt{x-1} + 3(7x^4 - 8x^2 + 1) \arctan(\sqrt{x+1} \sqrt{x-1}) \right)}{27 \sqrt{x+1} \sqrt{x-1} x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x, algorithm="maxima")`

```
[Out] 1/3*(x^2 - 1)^(3/2)*arcsec(x)^3/x^3 + 1/3*(3*x^2 - 1)*arcsec(x)^2/x^3 - 2/27*
((21*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1) + 3*(7*x^4 - 8*x^2 + 1)*arctan(sqrt(x + 1)*sqrt(x - 1)))/
(sqrt(x + 1)*sqrt(x - 1))*x^3
```

Fricas [A]

time = 0.31, size = 57, normalized size = 0.52

$$\frac{9(3x^2 - 1) \operatorname{arcsec}(x)^2 - 42x^2 + 3(3(x^2 - 1) \operatorname{arcsec}(x))^3 - 2(7x^2 - 1) \operatorname{arcsec}(x) \sqrt{x^2 - 1} + 2}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x, algorithm="fricas")`

```
[Out] 1/27*(9*(3*x^2 - 1)*arcsec(x)^2 - 42*x^2 + 3*(3*(x^2 - 1)*arcsec(x))^3 - 2*(7*x^2 - 1)*arcsec(x))*sqrt(x^2 - 1) + 2)/x^3
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)} \operatorname{asec}^3(x)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(asec(x)**3*(x**2-1)**(1/2)/x**4,x)``[Out] Integral(sqrt((x - 1)*(x + 1))*asec(x)**3/x**4, x)`**Giac [F] N/A**

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x)``[Out] Could not integrate`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\arccos\left(\frac{1}{x}\right)^3 \sqrt{x^2 - 1}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((acos(1/x)^3*(x^2 - 1)^(1/2))/x^4,x)

[Out] int((acos(1/x)^3*(x^2 - 1)^(1/2))/x^4, x)

$$3.695 \quad \int \sin^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right) dx$$

Optimal. Leaf size=55

$$-\frac{\sqrt{2} a \sqrt{\frac{-a+x}{a+x}}}{\sqrt{\frac{a}{a+x}}} + (a+x) \sin^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right)$$

[Out] (a+x)*arcsin(((a+x)/(-a+x))^(1/2))-a*2^(1/2)*((-a+x)/(a+x))^(1/2)/(a/(a+x))^(1/2)

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 125 vs. 2(55) = 110. time = 0.42, antiderivative size = 125, normalized size of antiderivative = 2.27, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {4924, 12, 1973, 1972, 21, 393, 222}

$$\frac{a^2 \sqrt{\frac{a+x}{a}} \sqrt{\frac{x}{a}+1} \sin^{-1} \left(\sqrt{\frac{a-x}{a+x}} \right)}{a+x} - \sqrt{2} a \sqrt{\frac{a}{a+x}} \sqrt{\frac{a-x}{a+x}} \sqrt{\frac{a+x}{a}} \sqrt{\frac{x}{a}+1} + x \sin^{-1} \left(\sqrt{\frac{a-x}{a+x}} \right)$$

Antiderivative was successfully verified.

[In] Int[ArcSin[Sqrt[(-a + x)/(a + x)]],x]

[Out] -(Sqrt[2]*a*Sqrt[a/(a + x)]*Sqrt[-((a - x)/(a + x))]*Sqrt[(a + x)/a]*Sqrt[1 + x/a]) + x*ArcSin[Sqrt[-((a - x)/(a + x))]] + (a^2*Sqrt[(a + x)/a]*Sqrt[1 + x/a]*ArcSin[Sqrt[-((a - x)/(a + x))]])/(a + x)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 21

Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] := Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
+ p, 0])
```

Rule 1972

```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.))^(p_), x_Symbol] := Dist[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)], Int[u*(a + b*x^n)^(p*q), x], x]
/; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_.))^(p_), x_Symbol] := Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
x], x] /; FreeQ[{a, b, c, n, p, q}, x] && !GeQ[a, 0]
```

Rule 4924

```
Int[ArcSin[u_], x_Symbol] := Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] && !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \sin^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right) dx &= x \sin^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right) - \int \frac{x \left(\frac{a}{a+x} \right)^{3/2}}{\sqrt{2} a \sqrt{\frac{-a+x}{a+x}}} dx \\
&= x \sin^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right) - \frac{\int \frac{x \left(\frac{a}{a+x} \right)^{3/2}}{\sqrt{\frac{-a+x}{a+x}}} dx}{\sqrt{2} a} \\
&= x \sin^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right) - \frac{\left(\sqrt{\frac{a}{a+x}} \sqrt{a+x} \right) \int \frac{x}{\sqrt{\frac{-a+x}{a+x}} (a+x)^{3/2}} dx}{\sqrt{2}} \\
&= x \sin^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right) - \left(a \sqrt{\frac{a}{a+x}} \sqrt{a+x} \right) \text{Subst} \left(\int \frac{1+x^2}{\sqrt{\frac{a}{-1+x^2}} (-1+x^2)} dx, x, \sqrt{\frac{-a+x}{a+x}} \right) \\
&= x \sin^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right) - \frac{\left(a \sqrt{\frac{a}{a+x}} \right) \text{Subst} \left(\int \frac{1+x^2}{(-1+x^2)^{3/2}} dx, x, \sqrt{\frac{-a+x}{a+x}} \right)}{\sqrt{\frac{a}{a+x}}} \\
&= -\sqrt{2} \sqrt{\frac{a}{a+x}} \sqrt{\frac{-a-x}{a+x}} (a+x) + x \sin^{-1} \left(\sqrt{\frac{-a-x}{a+x}} \right) - \frac{\left(a \sqrt{\frac{a}{a+x}} \right) \text{Subst} \left(\int \frac{1+x^2}{(-1+x^2)^{3/2}} dx, x, \sqrt{\frac{-a+x}{a+x}} \right)}{\sqrt{\frac{a}{a+x}}} \\
&= -\sqrt{2} \sqrt{\frac{a}{a+x}} \sqrt{\frac{-a-x}{a+x}} (a+x) + x \sin^{-1} \left(\sqrt{\frac{-a-x}{a+x}} \right) - \frac{\left(a \sqrt{\frac{a}{a+x}} \right) \text{Subst} \left(\int \frac{1+x^2}{(-1+x^2)^{3/2}} dx, x, \sqrt{\frac{-a+x}{a+x}} \right)}{\sqrt{\frac{a}{a+x}}} \\
&= -\sqrt{2} \sqrt{\frac{a}{a+x}} \sqrt{\frac{-a-x}{a+x}} (a+x) + x \sin^{-1} \left(\sqrt{\frac{-a-x}{a+x}} \right) - \frac{a \sqrt{\frac{a}{a+x}} \tanh^{-1} \left(\frac{\sqrt{-a-x}}{\sqrt{-a+x}} \right)}{\sqrt{\frac{a}{a+x}}}
\end{aligned}$$

Mathematica [A]

time = 0.09, size = 99, normalized size = 1.80

$$x \sin^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right) + \frac{\sqrt{\frac{a}{a+x}} \left(2a - 2x + \sqrt{2} \sqrt{a} \sqrt{-a+x} \tan^{-1} \left(\frac{\sqrt{-a+x}}{\sqrt{2} \sqrt{a}} \right) \right)}{\sqrt{2} \sqrt{\frac{-a+x}{a+x}}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[Sqrt[(-a + x)/(a + x)]], x]

[Out] x*ArcSin[Sqrt[(-a + x)/(a + x)]] + (Sqrt[a/(a + x)]*(2*a - 2*x + Sqrt[2]*Sqrt[a]*Sqrt[-a + x]*ArcTan[Sqrt[-a + x]/(Sqrt[2]*Sqrt[a])]))/(Sqrt[2]*Sqrt[(-a + x)/(a + x)])

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[ArcSin[Sqrt[(x - a)/(x + a)]], x]')

[Out] Timed out

Maple [A]

time = 0.03, size = 86, normalized size = 1.56

method	result
default	$x \arcsin \left(\sqrt{\frac{-a+x}{a+x}} \right) + \frac{\sqrt{-a+x} \sqrt{2} \sqrt{\frac{a}{a+x}} \left(-2\sqrt{-a+x} + \sqrt{a} \sqrt{2} \arctan \left(\frac{\sqrt{-a+x} \sqrt{2}}{2\sqrt{a}} \right) \right)}{2\sqrt{\frac{-a-x}{a+x}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsin((-a+x)/(a+x))^(1/2), x, method=_RETURNVERBOSE)

[Out] x*arcsin((-a+x)/(a+x))^(1/2)+1/2/(-a-x)/(a+x))^(1/2)*(-a+x)^(1/2)*2^(1/2)*(a/(a+x))^(1/2)*(-2*(-a+x)^(1/2)+a^(1/2)*2^(1/2)*arctan(1/2*(-a+x)^(1/2)*2^(1/2)/a^(1/2)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(49) = 98.

time = 0.33, size = 103, normalized size = 1.87

$$a \left(\frac{2 \arcsin \left(\sqrt{\frac{a-x}{a+x}} \right)}{\frac{a-x}{a+x} + 1} + \frac{\sqrt{\frac{a-x}{a+x} + 1}}{\sqrt{\frac{a-x}{a+x} + 1}} + \frac{\sqrt{\frac{a-x}{a+x} + 1}}{\sqrt{\frac{a-x}{a+x} - 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin((-a+x)/(a+x))^(1/2),x, algorithm="maxima")

[Out] a*(2*arcsin(sqrt(-(a - x)/(a + x)))/((a - x)/(a + x) + 1) + sqrt((a - x)/(a + x) + 1)/(sqrt(-(a - x)/(a + x)) + 1) + sqrt((a - x)/(a + x) + 1)/(sqrt(-(a - x)/(a + x)) - 1))

Fricas [A]

time = 0.33, size = 51, normalized size = 0.93

$$-\sqrt{2}(a+x)\sqrt{\frac{a-x}{a+x}}\sqrt{\frac{a}{a+x}}+(a+x)\arcsin\left(\sqrt{\frac{a-x}{a+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin((-a+x)/(a+x))^(1/2),x, algorithm="fricas")

[Out] -sqrt(2)*(a + x)*sqrt(-(a - x)/(a + x))*sqrt(a/(a + x)) + (a + x)*arcsin(sqrt(-(a - x)/(a + x)))

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asin}\left(\sqrt{\frac{-a+x}{a+x}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin((-a+x)/(a+x))^(1/2),x)

[Out] Integral(asin(sqrt((-a + x)/(a + x))), x)

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin((-a+x)/(a+x))^(1/2),x)

[Out] Could not integrate

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{asin}\left(\sqrt{\frac{a-x}{a+x}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin((-a - x)/(a + x))^(1/2),x)

[Out] int(asin((-a - x)/(a + x))^(1/2), x)

$$3.696 \quad \int \tan^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right) dx$$

Optimal. Leaf size=40

$$x \tan^{-1} \left(\sqrt{\frac{a-x}{a+x}} \right) - a \tanh^{-1} \left(\sqrt{\frac{a-x}{a+x}} \right)$$

[Out] x*arctan(((a-x)/(a+x))^(1/2))-a*arctanh(((a-x)/(a+x))^(1/2))

Rubi [A]

time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5311, 12, 1983, 214}

$$x \tan^{-1} \left(\sqrt{\frac{a-x}{a+x}} \right) - a \tanh^{-1} \left(\sqrt{\frac{a-x}{a+x}} \right)$$

Antiderivative was successfully verified.

[In] Int[ArcTan[Sqrt[(-a + x)/(a + x)]], x]

[Out] x*ArcTan[Sqrt[-((a - x)/(a + x))]] - a*ArcTanh[Sqrt[-((a - x)/(a + x))]]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 214

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1983

Int[(u_)^(r_)*(((e_)*((a_) + (b_)*(x_)^(n_))))/((c_) + (d_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1))*(u /. x -> ((a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && IntegerQ[r]

Rule 5311

```
Int[ArcTan[u_], x_Symbol] := Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \tan^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right) dx &= x \tan^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right) - \int \frac{a}{2\sqrt{\frac{-a+x}{a+x}}(a+x)} dx \\
&= x \tan^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right) - \frac{1}{2}a \int \frac{1}{\sqrt{\frac{-a+x}{a+x}}(a+x)} dx \\
&= x \tan^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right) - (2a^2) \text{Subst} \left(\int \frac{1}{2a-2ax^2} dx, x, \sqrt{\frac{-a+x}{a+x}} \right) \\
&= x \tan^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right) - a \tanh^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right)
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 71, normalized size = 1.78

$$x \tan^{-1} \left(\sqrt{\frac{-a+x}{a+x}} \right) - \frac{a\sqrt{-a+x} \tanh^{-1} \left(\frac{\sqrt{-a+x}}{\sqrt{a+x}} \right)}{\sqrt{\frac{-a+x}{a+x}} \sqrt{a+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[(-a + x)/(a + x)]], x]
```

```
[Out] x*ArcTan[Sqrt[(-a + x)/(a + x)]] - (a*Sqrt[-a + x]*ArcTanh[Sqrt[-a + x]/Sqr
t[a + x]])/(Sqrt[(-a + x)/(a + x)]*Sqrt[a + x])
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[ArcTan[Sqrt[(x - a)/(x + a)]], x]')
```

```
[Out] Timed out
```


Maple [A]

time = 0.05, size = 66, normalized size = 1.65

method	result	size
default	$x \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right) + \frac{(a-x)a \ln\left(x + \sqrt{-a^2 + x^2}\right)}{2\sqrt{-\frac{a-x}{a+x}} \sqrt{-(a-x)(a+x)}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(((a+x)/(a+x))^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*arctan(((a+x)/(a+x))^(1/2))+1/2*(a-x)*a*ln(x+(-a^2+x^2)^(1/2))/(-(a-x)/((a+x))^(1/2))/(-(a-x)*(a+x))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(36) = 72.

time = 0.33, size = 89, normalized size = 2.22

$$\frac{1}{2} a \left(\frac{4 \arctan\left(\sqrt{\frac{a-x}{a+x}}\right)}{\frac{a-x}{a+x} + 1} - 2 \arctan\left(\sqrt{\frac{a-x}{a+x}}\right) - \log\left(\sqrt{\frac{a-x}{a+x}} + 1\right) + \log\left(\sqrt{\frac{a-x}{a+x}} - 1\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(((a+x)/(a+x))^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*a*(4*arctan(sqrt(-(a-x)/(a+x)))/((a-x)/(a+x)+1)-2*arctan(sqrt(-(a-x)/(a+x)))-log(sqrt(-(a-x)/(a+x))+1)+log(sqrt(-(a-x)/(a+x))-1))
```

Fricas [A]

time = 0.32, size = 58, normalized size = 1.45

$$x \arctan\left(\sqrt{\frac{-a-x}{a+x}}\right) - \frac{1}{2} a \log\left(\sqrt{\frac{-a-x}{a+x}} + 1\right) + \frac{1}{2} a \log\left(\sqrt{\frac{-a-x}{a+x}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(((a+x)/(a+x))^(1/2)),x, algorithm="fricas")
```

```
[Out] x*arctan(sqrt(-(a-x)/(a+x)))-1/2*a*log(sqrt(-(a-x)/(a+x))+1)+1/2*a*log(sqrt(-(a-x)/(a+x))-1)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}\left(\sqrt{\frac{-a+x}{a+x}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(((a+x)/(a+x))**(1/2)),x)

[Out] Integral(atan(sqrt((-a + x)/(a + x))), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(36) = 72.

time = 0.03, size = 154, normalized size = 3.85

$$x \arctan\left(\frac{\sqrt{-a^2+x^2} \operatorname{sign}(a+x)}{a+x}\right) - 2a \left(-\frac{\ln|-x + \sqrt{-a^2+x^2}|}{2\operatorname{sign}(a+x)^2+2} + \frac{(2a\operatorname{sign}(a+x)^2-2a) \arctan\left(\frac{a\operatorname{sign}(a+x)^2+(-x+\sqrt{-a^2+x^2})\operatorname{sign}(a+x)-a-x+\sqrt{-a^2+x^2}}{2(a\operatorname{sign}(a+x))}\right)}{4(a(\operatorname{sign}(a+x)^2+1)\operatorname{sign}(a+x))} \right) \operatorname{sign}(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(((a+x)/(a+x))^(1/2)),x)

[Out] a*(log(abs(-x + sqrt(-a^2 + x^2)))/(sgn(a + x)^2 + 1) - (a*sgn(a + x)^2 - a)*arctan(1/2*(a*sgn(a + x)^2 - (x - sqrt(-a^2 + x^2))*sgn(a + x)^2 - a - x + sqrt(-a^2 + x^2))/(a*sgn(a + x)))/((sgn(a + x)^2 + 1)*a*sgn(a + x))*sgn(a + x) + x*arctan(sqrt(-a^2 + x^2)*sgn(a + x)/(a + x))

Mupad [B]

time = 0.36, size = 36, normalized size = 0.90

$$x \operatorname{atan}\left(\sqrt{-\frac{a-x}{a+x}}\right) - a \operatorname{atanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atan((-a - x)/(a + x))^(1/2),x)

[Out] x*atan((-a - x)/(a + x))^(1/2) - a*atanh((-a - x)/(a + x))^(1/2)

$$3.697 \quad \int \frac{\tan^{-1}(x)}{(1+x)^3} dx$$

Optimal. Leaf size=39

$$-\frac{1}{4(1+x)} - \frac{\tan^{-1}(x)}{2(1+x)^2} + \frac{1}{4} \log(1+x) - \frac{1}{8} \log(1+x^2)$$

[Out] $-1/4/(1+x)-1/2*\arctan(x)/(1+x)^2+1/4*\ln(1+x)-1/8*\ln(x^2+1)$

Rubi [A]

time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4972, 724, 815, 266}

$$-\frac{1}{8} \log(x^2 + 1) - \frac{1}{4(x+1)} + \frac{1}{4} \log(x+1) - \frac{\tan^{-1}(x)}{2(x+1)^2}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[x]/(1 + x)^3,x]

[Out] $-1/4*1/(1+x) - \text{ArcTan}[x]/(2*(1+x)^2) + \text{Log}[1+x]/4 - \text{Log}[1+x^2]/8$

Rule 266

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 724

Int[((d_) + (e_)*(x_)^(m_))/((a_) + (c_)*(x_)^2), x_Symbol] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]

Rule 815

Int[(((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)))/((a_) + (c_)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 4972

Int[((a_) + ArcTan[(c_)*(x_)])*(b_)*((d_) + (e_)*(x_)^(q_)), x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(x)}{(1+x)^3} dx &= -\frac{\tan^{-1}(x)}{2(1+x)^2} + \frac{1}{2} \int \frac{1}{(1+x)^2(1+x^2)} dx \\
&= -\frac{1}{4(1+x)} - \frac{\tan^{-1}(x)}{2(1+x)^2} + \frac{1}{4} \int \frac{1-x}{(1+x)(1+x^2)} dx \\
&= -\frac{1}{4(1+x)} - \frac{\tan^{-1}(x)}{2(1+x)^2} + \frac{1}{4} \int \left(\frac{1}{1+x} - \frac{x}{1+x^2} \right) dx \\
&= -\frac{1}{4(1+x)} - \frac{\tan^{-1}(x)}{2(1+x)^2} + \frac{1}{4} \log(1+x) - \frac{1}{4} \int \frac{x}{1+x^2} dx \\
&= -\frac{1}{4(1+x)} - \frac{\tan^{-1}(x)}{2(1+x)^2} + \frac{1}{4} \log(1+x) - \frac{1}{8} \log(1+x^2)
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 35, normalized size = 0.90

$$\frac{1}{8} \left(-\frac{2}{1+x} - \frac{4 \tan^{-1}(x)}{(1+x)^2} + 2 \log(1+x) - \log(1+x^2) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[ArcTan[x]/(1+x)^3,x]``[Out] (-2/(1+x) - (4*ArcTan[x]))/(1+x)^2 + 2*Log[1+x] - Log[1+x^2])/8`**Mathics [A]**

time = 2.95, size = 65, normalized size = 1.67

$$\frac{-2 - 2x - 2x \log[1+x^2] + 4x \log[1+x] + 2x^2 \log[1+x] - 4 \text{ArcTan}[x] - \log1+x^2 + 2 \log[1+x]}{8 + 16x + 8x^2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[ArcTan[x]/(1+x)^3,x]')`

```
[Out] (-2 - 2 x - 2 x Log[1 + x ^ 2] + 4 x Log[1 + x] + 2 x ^ 2 Log[1 + x] - 4 Ar
cTan[x] - Log[1 + x ^ 2] (1 + x ^ 2) + 2 Log[1 + x]) / (8 (1 + 2 x + x ^ 2)
)
```

Maple [A]

time = 0.10, size = 32, normalized size = 0.82

method	result	size
--------	--------	------

default	$-\frac{1}{4(1+x)} - \frac{\arctan(x)}{2(1+x)^2} + \frac{\ln(1+x)}{4} - \frac{\ln(x^2+1)}{8}$	32
risch	$\frac{i \ln(ix+1)}{4(1+x)^2} - \frac{i(2i \ln(1+x)x^2 - i \ln(x^2+1)x^2 + 4i \ln(1+x)x - 2i \ln(x^2+1)x + 2i \ln(1+x) - i \ln(x^2+1) - 2ix - 2i + 2 \ln(-ix+1))}{8(1+x)^2}$	97

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arctan(x)/(1+x)^3,x,method=_RETURNVERBOSE)`

[Out] $-1/4/(1+x) - 1/2*\arctan(x)/(1+x)^2 + 1/4*\ln(1+x) - 1/8*\ln(x^2+1)$

Maxima [A]

time = 0.34, size = 31, normalized size = 0.79

$$-\frac{1}{4(x+1)} - \frac{\arctan(x)}{2(x+1)^2} - \frac{1}{8} \log(x^2+1) + \frac{1}{4} \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(x)/(1+x)^3,x, algorithm="maxima")`

[Out] $-1/4/(x+1) - 1/2*\arctan(x)/(x+1)^2 - 1/8*\log(x^2+1) + 1/4*\log(x+1)$

Fricas [A]

time = 0.31, size = 50, normalized size = 1.28

$$\frac{(x^2 + 2x + 1) \log(x^2 + 1) - 2(x^2 + 2x + 1) \log(x + 1) + 2x + 4 \arctan(x) + 2}{8(x^2 + 2x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctan(x)/(1+x)^3,x, algorithm="fricas")`

[Out] $-1/8*((x^2 + 2*x + 1)*\log(x^2 + 1) - 2*(x^2 + 2*x + 1)*\log(x + 1) + 2*x + 4*\arctan(x) + 2)/(x^2 + 2*x + 1)$

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(31) = 62$

time = 0.28, size = 153, normalized size = 3.92

$$\frac{2x^2 \log(x+1)}{8x^2+16x+8} - \frac{x^2 \log(x^2+1)}{8x^2+16x+8} + \frac{4x \log(x+1)}{8x^2+16x+8} - \frac{2x \log(x^2+1)}{8x^2+16x+8} - \frac{2x}{8x^2+16x+8} + \frac{2 \log(x+1)}{8x^2+16x+8} - \frac{\log(x^2+1)}{8x^2+16x+8} - \frac{4 \operatorname{atan}(x)}{8x^2+16x+8} - \frac{2}{8x^2+16x+8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(atan(x)/(1+x)**3,x)`

[Out] $2*x**2*\log(x+1)/(8*x**2+16*x+8) - x**2*\log(x**2+1)/(8*x**2+16*x+8) + 4*x*\log(x+1)/(8*x**2+16*x+8) - 2*x*\log(x**2+1)/(8*x**2+16*x+8) - 2*x/(8*x**2+16*x+8) + 2*\log(x+1)/(8*x**2+16*x+8) - \log(x*$

$(x^2 + 1)/(8x^2 + 16x + 8) - 4\operatorname{atan}(x)/(8x^2 + 16x + 8) - 2/(8x^2 + 16x + 8)$

Giac [A]

time = 0.00, size = 40, normalized size = 1.03

$$-\frac{\ln(x^2 + 1)}{8} + \frac{\ln|x + 1|}{4} - \frac{\frac{1}{4}}{x + 1} - \frac{\arctan x}{2(x + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(x)/(1+x)^3,x)

[Out] -1/4/(x + 1) - 1/2*arctan(x)/(x + 1)^2 - 1/8*log(x^2 + 1) + 1/4*log(abs(x + 1))

Mupad [B]

time = 0.36, size = 31, normalized size = 0.79

$$\frac{\ln(x + 1)}{4} - \frac{\ln(x^2 + 1)}{8} - \frac{\frac{x}{4} + \frac{\operatorname{atan}(x)}{2} + \frac{1}{4}}{(x + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atan(x)/(x + 1)^3,x)

[Out] log(x + 1)/4 - log(x^2 + 1)/8 - (x/4 + atan(x)/2 + 1/4)/(x + 1)^2

$$3.698 \quad \int -\frac{\tan^{-1}(a-x)}{a+x} dx$$

Optimal. Leaf size=122

$$\tan^{-1}(a-x) \log\left(\frac{2}{1-i(a-x)}\right) - \tan^{-1}(a-x) \log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right) - \frac{1}{2}i\text{Li}_2\left(1 - \frac{2}{1-i(a-x)}\right) +$$

[Out] arctan(a-x)*ln(2/(1-I*(a-x)))-arctan(a-x)*ln(-2*(a+x)/(I-2*a)/(1-I*(a-x)))-1/2*I*polylog(2,1-2/(1-I*(a-x)))+1/2*I*polylog(2,1+2*(a+x)/(I-2*a)/(1-I*(a-x)))

Rubi [A]

time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5155, 4966, 2449, 2352, 2497}

$$-\frac{1}{2}i\text{Li}_2\left(1 - \frac{2}{1-i(a-x)}\right) + \frac{1}{2}i\text{Li}_2\left(\frac{2(a+x)}{(i-2a)(1-i(a-x))} + 1\right) + \log\left(\frac{2}{1-i(a-x)}\right) \tan^{-1}(a-x) - \log\left(-\frac{2(a+x)}{(-2a+i)(1-i(a-x))}\right) \tan^{-1}(a-x)$$

Antiderivative was successfully verified.

[In] Int[-(ArcTan[a - x]/(a + x)), x]

[Out] ArcTan[a - x]*Log[2/(1 - I*(a - x))] - ArcTan[a - x]*Log[(-2*(a + x))/((I - 2*a)*(1 - I*(a - x)))] - (I/2)*PolyLog[2, 1 - 2/(1 - I*(a - x))] + (I/2)*PolyLog[2, 1 + (2*(a + x))/((I - 2*a)*(1 - I*(a - x)))]

Rule 2352

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(-e^(-1))*PolyLog[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2449

Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := Dist[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 2497

Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[Pq^m*((1 - u)/D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

Rule 4966

Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := Simp[-(a + b*ArcTan[c*x])*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L

```
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x))]]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_.) + (d_.)*(x_.)]*(b_.))^ (p_.)*((e_.) + (f_.)*(x_.))^ (m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int -\frac{\tan^{-1}(a-x)}{a+x} dx &= \text{Subst} \left(\int \frac{\tan^{-1}(x)}{2a-x} dx, x, a-x \right) \\ &= \tan^{-1}(a-x) \log \left(\frac{2}{1-i(a-x)} \right) - \tan^{-1}(a-x) \log \left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))} \right) - \text{Li}_2 \left(\frac{i(a+x)}{i-2a} \right) \\ &= \tan^{-1}(a-x) \log \left(\frac{2}{1-i(a-x)} \right) - \tan^{-1}(a-x) \log \left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))} \right) + \text{Li}_2 \left(\frac{i(a+x)}{i-2a} \right) \\ &= \tan^{-1}(a-x) \log \left(\frac{2}{1-i(a-x)} \right) - \tan^{-1}(a-x) \log \left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))} \right) - \text{Li}_2 \left(\frac{i(a+x)}{i-2a} \right) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 105, normalized size = 0.86

$$-\frac{1}{2}i \left(-\log(1+i(a-x)) \log \left(\frac{a+x}{-i+2a} \right) + \log(1-ia+ix) \log \left(\frac{a+x}{i+2a} \right) + \text{Li}_2 \left(\frac{i+a-x}{i+2a} \right) - \text{Li}_2 \left(\frac{i-a+x}{i-2a} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(ArcTan[a - x]/(a + x)), x]
```

```
[Out] (-1/2*I)*(-(Log[1 + I*(a - x)]*Log[(a + x)/(-I + 2*a)]) + Log[1 - I*a + I*x
]*Log[(a + x)/(I + 2*a)] + PolyLog[2, (I + a - x)/(I + 2*a)] - PolyLog[2, (
I - a + x)/(I - 2*a)])
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

caught exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[ArcTan[x - a]/(x + a),x]')`

[Out] caught exception: maximum recursion depth exceeded while calling a Python object

Maple [A]

time = 0.11, size = 102, normalized size = 0.84

method	result
derivativedivides	$-\ln(a+x) \arctan(a-x) + \frac{i \ln(a+x) \ln\left(\frac{i+a-x}{2a+i}\right)}{2} - \frac{i \ln(a+x) \ln\left(\frac{i-a+x}{i-2a}\right)}{2} + \frac{i \operatorname{dilog}\left(\frac{i+a-x}{2a+i}\right)}{2} - \frac{i \operatorname{dilog}\left(\frac{i-a+x}{i-2a}\right)}{2}$
default	$-\ln(a+x) \arctan(a-x) + \frac{i \ln(a+x) \ln\left(\frac{i+a-x}{2a+i}\right)}{2} - \frac{i \ln(a+x) \ln\left(\frac{i-a+x}{i-2a}\right)}{2} + \frac{i \operatorname{dilog}\left(\frac{i+a-x}{2a+i}\right)}{2} - \frac{i \operatorname{dilog}\left(\frac{i-a+x}{i-2a}\right)}{2}$
risch	$-\frac{i \operatorname{dilog}\left(\frac{ia+ix}{2ia-1}\right)}{2} - \frac{i \ln(-ia+ix+1) \ln\left(\frac{ia+ix}{2ia-1}\right)}{2} + \frac{i \operatorname{dilog}\left(\frac{-ia-ix}{-2ia-1}\right)}{2} + \frac{i \ln(ia-ix+1) \ln\left(\frac{-ia-ix}{-2ia-1}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-arctan(a-x)/(a+x),x,method=_RETURNVERBOSE)`

[Out] $-\ln(a+x) \arctan(a-x) + 1/2 * I * \ln(a+x) * \ln((I+a-x)/(2*a+I)) - 1/2 * I * \ln(a+x) * \ln((I-a+x)/(I-2*a)) + 1/2 * I * \operatorname{dilog}((I+a-x)/(2*a+I)) - 1/2 * I * \operatorname{dilog}((I-a+x)/(I-2*a))$

Maxima [A]

time = 0.36, size = 118, normalized size = 0.97

$-\frac{1}{2} \arctan\left(\frac{a+x}{4a^2+1}, \frac{2(a^2+ax)}{4a^2+1}\right) \log(a^2-2ax+x^2+1) + \frac{1}{2} \arctan(-a+x) \log\left(\frac{a^2+2ax+x^2}{4a^2+1}\right) - \frac{1}{2} i \operatorname{Li}_2\left(-\frac{-ia+ix+1}{2ia-1}\right) + \frac{1}{2} i \operatorname{Li}_2\left(-\frac{-ia+ix-1}{2ia+1}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-arctan(a-x)/(a+x),x, algorithm="maxima")`

[Out] $-1/2 * \arctan^2((a+x)/(4*a^2+1), 2*(a^2+a*x)/(4*a^2+1)) * \log(a^2-2*a*x+x^2+1) + 1/2 * \arctan(-a+x) * \log((a^2+2*a*x+x^2)/(4*a^2+1)) - 1/2 * I * \operatorname{dilog}(-(-I*a+I*x+1)/(2*I*a-1)) + 1/2 * I * \operatorname{dilog}(-(-I*a+I*x-1)/(2*I*a+1))$

Fricas [F]

time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-arctan(a-x)/(a+x),x, algorithm="fricas")`

[Out] `integral(arctan(-a + x)/(a + x), x)`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atan}(a-x)}{a+x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-atan(a-x)/(a+x),x)`

[Out] `-Integral(atan(a - x)/(a + x), x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-arctan(a-x)/(a+x),x)`

[Out] `Could not integrate`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$- \int \frac{\operatorname{atan}(a-x)}{a+x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-atan(a - x)/(a + x),x)`

[Out] `-int(atan(a - x)/(a + x), x)`

$$3.699 \quad \int \frac{\sin^{-1}\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}} dx$$

Optimal. Leaf size=28

$$-\frac{\sqrt{x^2} \sin^{-1}\left(\sqrt{1-x^2}\right)^2}{2x}$$

[Out] -1/2*arcsin((-x^2+1)^(1/2))^2*(x^2)^(1/2)/x

Rubi [A]

time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4918, 4737}

$$-\frac{\sqrt{x^2} \sin^{-1}\left(\sqrt{1-x^2}\right)^2}{2x}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[Sqrt[1 - x^2]]/Sqrt[1 - x^2], x]

[Out] -1/2*(Sqrt[x^2]*ArcSin[Sqrt[1 - x^2]]^2)/x

Rule 4737

Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^n_/Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]

Rule 4918

Int[ArcSin[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[(-b)*x^2]/(b*x), Subst[Int[ArcSin[x]^n/Sqrt[1 - x^2], x], x, Sqrt[1 + b*x^2]], x] /; FreeQ[{b, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sin^{-1}\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}} dx &= -\frac{\sqrt{x^2} \operatorname{Subst}\left(\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} dx, x, \sqrt{1-x^2}\right)}{x} \\ &= -\frac{\sqrt{x^2} \sin^{-1}\left(\sqrt{1-x^2}\right)^2}{2x} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 28, normalized size = 1.00

$$-\frac{\sqrt{x^2} \sin^{-1}\left(\sqrt{1-x^2}\right)^2}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[Sqrt[1 - x^2]]/Sqrt[1 - x^2], x]
```

```
[Out] -1/2*(Sqrt[x^2]*ArcSin[Sqrt[1 - x^2]]^2)/x
```

Mathics [A]

time = 2.10, size = 26, normalized size = 0.93

$$\frac{x \text{ArcSin}[x]^2}{2\sqrt{x^2}} + \text{ArcSin}[x] \text{ArcSin}\left[\sqrt{1-x^2}\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[ArcSin[Sqrt[1 - x^2]]/Sqrt[1 - x^2], x]')
```

```
[Out] x ArcSin[x] ^ 2 / (2 Sqrt[x ^ 2]) + ArcSin[x] ArcSin[Sqrt[1 - x ^ 2]]
```

Maple [F]

time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\arcsin\left(\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2), x)
```

```
[Out] int(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2), x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2), x, algorithm="maxima")
```

```
[Out] integrate(arcsin(sqrt(-x^2 + 1))/sqrt(-x^2 + 1), x)
```

Fricas [A]

time = 0.31, size = 14, normalized size = 0.50

$$-\frac{1}{2} \arcsin\left(\sqrt{-x^2+1}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="fricas")`

[Out] `-1/2*arcsin(sqrt(-x^2 + 1))^2`

Sympy [A]

time = 0.47, size = 22, normalized size = 0.79

$$-\frac{\sqrt{x^2} \operatorname{asin}^2\left(\sqrt{1-x^2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(asin((-x**2+1)**(1/2))/(-x**2+1)**(1/2),x)`

[Out] `-sqrt(x**2)*asin(sqrt(1 - x**2))**2/(2*x)`

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)`

[Out] `Could not integrate`

Mupad [F]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{asin}\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin((1 - x^2)^(1/2))/(1 - x^2)^(1/2),x)`

[Out] `int(asin((1 - x^2)^(1/2))/(1 - x^2)^(1/2), x)`

$$3.700 \quad \int \frac{x \tan^{-1}(\sqrt{1+x^2})}{\sqrt{1+x^2}} dx$$

Optimal. Leaf size=31

$$\sqrt{1+x^2} \tan^{-1}(\sqrt{1+x^2}) - \frac{1}{2} \log(2+x^2)$$

[Out] -1/2*ln(x^2+2)+arctan((x^2+1)^(1/2))*(x^2+1)^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$,

Rules used = {267, 5315, 266}

$$\sqrt{x^2+1} \tan^{-1}(\sqrt{x^2+1}) - \frac{1}{2} \log(x^2+2)$$

Antiderivative was successfully verified.

[In] Int[(x*ArcTan[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]

[Out] Sqrt[1 + x^2]*ArcTan[Sqrt[1 + x^2]] - Log[2 + x^2]/2

Rule 266

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 267

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 5315

Int[((a_) + ArcTan[u]*(b_))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x] /; FreeQ[{a, b}, x] && InverseFunctionFreeQ[u, x] && !MatchQ[v, ((c_) + (d_)*x)^(m_) /; FreeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]

Rubi steps

$$\int \frac{x \tan^{-1}(\sqrt{1+x^2})}{\sqrt{1+x^2}} dx = \sqrt{1+x^2} \tan^{-1}(\sqrt{1+x^2}) - \int \frac{x}{2+x^2} dx$$

$$= \sqrt{1+x^2} \tan^{-1}(\sqrt{1+x^2}) - \frac{1}{2} \log(2+x^2)$$

Mathematica [A]

time = 0.02, size = 31, normalized size = 1.00

$$\sqrt{1+x^2} \tan^{-1}(\sqrt{1+x^2}) - \frac{1}{2} \log(2+x^2)$$

Antiderivative was successfully verified.

`[In] Integrate[(x*ArcTan[Sqrt[1 + x^2]])/Sqrt[1 + x^2], x]``[Out] Sqrt[1 + x^2]*ArcTan[Sqrt[1 + x^2]] - Log[2 + x^2]/2`**Mathics [A]**

time = 2.23, size = 25, normalized size = 0.81

$$-\frac{\text{Log}[2+x^2]}{2} + \text{ArcTan}[\sqrt{1+x^2}] \sqrt{1+x^2}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[ArcTan[Sqrt[1 + x^2]]*x/Sqrt[1 + x^2], x]')``[Out] -Log[2 + x ^ 2] / 2 + ArcTan[Sqrt[1 + x ^ 2]] Sqrt[1 + x ^ 2]`**Maple [A]**

time = 0.07, size = 26, normalized size = 0.84

method	result	size
derivativedivides	$-\frac{\ln(x^2+2)}{2} + \arctan(\sqrt{x^2+1}) \sqrt{x^2+1}$	26
default	$-\frac{\ln(x^2+2)}{2} + \arctan(\sqrt{x^2+1}) \sqrt{x^2+1}$	26

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2), x, method=_RETURNVERBOSE)``[Out] -1/2*ln(x^2+2)+arctan((x^2+1)^(1/2))*(x^2+1)^(1/2)`**Maxima [A]**

time = 0.25, size = 25, normalized size = 0.81

$$\sqrt{x^2+1} \arctan(\sqrt{x^2+1}) - \frac{1}{2} \log(x^2+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="maxima")

[Out] sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)

Fricas [A]

time = 0.35, size = 25, normalized size = 0.81

$$\sqrt{x^2 + 1} \arctan\left(\sqrt{x^2 + 1}\right) - \frac{1}{2} \log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="fricas")

[Out] sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)

Sympy [A]

time = 0.70, size = 26, normalized size = 0.84

$$\sqrt{x^2 + 1} \operatorname{atan}\left(\sqrt{x^2 + 1}\right) - \frac{\log(x^2 + 2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*atan((x**2+1)**(1/2))/(x**2+1)**(1/2),x)

[Out] sqrt(x**2 + 1)*atan(sqrt(x**2 + 1)) - log(x**2 + 2)/2

Giac [A]

time = 0.00, size = 31, normalized size = 1.00

$$\sqrt{x^2 + 1} \arctan\left(\sqrt{x^2 + 1}\right) - \frac{\ln(x^2 + 2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x)

[Out] sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)

Mupad [B]

time = 0.57, size = 25, normalized size = 0.81

$$\operatorname{atan}\left(\sqrt{x^2 + 1}\right) \sqrt{x^2 + 1} - \frac{\ln(x^2 + 2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*atan((x^2 + 1)^(1/2)))/(x^2 + 1)^(1/2),x)

[Out] atan((x^2 + 1)^(1/2))*(x^2 + 1)^(1/2) - log(x^2 + 2)/2

$$3.701 \quad \int \frac{\sin^{-1}(x)}{(1-x)^{5/2}} dx$$

Optimal. Leaf size=57

$$-\frac{\sqrt{1+x}}{3(1-x)} + \frac{2\sin^{-1}(x)}{3(1-x)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)}{3\sqrt{2}}$$

[Out] 2/3*arcsin(x)/(1-x)^(3/2)-1/6*arctanh(1/2*2^(1/2)*(1+x)^(1/2))*2^(1/2)-1/3*(1+x)^(1/2)/(1-x)

Rubi [A]

time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {4827, 641, 44, 65, 212}

$$-\frac{\sqrt{x+1}}{3(1-x)} + \frac{2\sin^{-1}(x)}{3(1-x)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[x]/(1-x)^(5/2),x]

[Out] -1/3*sqrt[1+x]/(1-x) + (2*ArcSin[x])/(3*(1-x)^(3/2)) - ArcTanh[Sqrt[1+x]/Sqrt[2]]/(3*sqrt[2])

Rule 44

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[d*((m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && LtQ[n, 0]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rule 641

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && IntegerQ[m + p]))
```

Rule 4827

```
Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*((d_) + (e_)*(x_))^(m_), x_Symbol] := Simp[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 1))), x] - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2*x^2]], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\sin^{-1}(x)}{(1-x)^{5/2}} dx &= \frac{2 \sin^{-1}(x)}{3(1-x)^{3/2}} - \frac{2}{3} \int \frac{1}{(1-x)^{3/2} \sqrt{1-x^2}} dx \\
 &= \frac{2 \sin^{-1}(x)}{3(1-x)^{3/2}} - \frac{2}{3} \int \frac{1}{(1-x)^2 \sqrt{1+x}} dx \\
 &= -\frac{\sqrt{1+x}}{3(1-x)} + \frac{2 \sin^{-1}(x)}{3(1-x)^{3/2}} - \frac{1}{6} \int \frac{1}{(1-x) \sqrt{1+x}} dx \\
 &= -\frac{\sqrt{1+x}}{3(1-x)} + \frac{2 \sin^{-1}(x)}{3(1-x)^{3/2}} - \frac{1}{3} \text{Subst} \left(\int \frac{1}{2-x^2} dx, x, \sqrt{1+x} \right) \\
 &= -\frac{\sqrt{1+x}}{3(1-x)} + \frac{2 \sin^{-1}(x)}{3(1-x)^{3/2}} - \frac{\tanh^{-1} \left(\frac{\sqrt{1+x}}{\sqrt{2}} \right)}{3\sqrt{2}}
 \end{aligned}$$

Mathematica [A]

time = 0.07, size = 61, normalized size = 1.07

$$\frac{1}{6} \left(-\frac{2 \left(\sqrt{1-x^2} - 2 \sin^{-1}(x) \right)}{(1-x)^{3/2}} - \sqrt{2} \tanh^{-1} \left(\frac{\sqrt{1-x^2}}{\sqrt{2-2x}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcSin[x]/(1-x)^(5/2),x]

[Out] $((-2*(\text{Sqrt}[1 - x^2] - 2*\text{ArcSin}[x]))/(1 - x)^{(3/2)} - \text{Sqrt}[2]*\text{ArcTanh}[\text{Sqrt}[1 - x^2]/\text{Sqrt}[2 - 2*x]])/6$

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[ArcSin[x]/(1 - x)^(5/2),x]')`

[Out] Timed out

Maple [A]

time = 0.07, size = 70, normalized size = 1.23

method	result	size
derivativedivides	$\frac{2 \arcsin(x)}{3(1-x)^{\frac{3}{2}}} - \frac{\sqrt{1+x} \left(\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{1+x}}\right)^{(1-x)+2\sqrt{1+x}} \right)}{6\sqrt{1-x} \sqrt{-(1-x)^2 + 2 - 2x}}$	70
default	$\frac{2 \arcsin(x)}{3(1-x)^{\frac{3}{2}}} - \frac{\sqrt{1+x} \left(\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{1+x}}\right)^{(1-x)+2\sqrt{1+x}} \right)}{6\sqrt{1-x} \sqrt{-(1-x)^2 + 2 - 2x}}$	70

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arcsin(x)/(1-x)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $2/3*\arcsin(x)/(1-x)^{(3/2)}-1/6/(1-x)^{(1/2)}*(1+x)^{(1/2)}*(2^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/(1+x)^{(1/2)})*(1-x)+2*(1+x)^{(1/2}))/(-(1-x)^2+2-2*x)^{(1/2)}$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arcsin(x)/(1-x)^(5/2),x, algorithm="maxima")`

[Out] $-2/3*(3*(x - 1)*\text{sqrt}(-x + 1)*\text{integrate}(1/3*\text{sqrt}(x + 1)*x^2/(x^5 - x^4 - x^3 + x^2 + (x^3 - x^2 - x + 1)*e^{(\log(x + 1) + \log(-x + 1))}), x) + \operatorname{arctan}2(x, \text{sqrt}(x + 1)*\text{sqrt}(-x + 1)))/((x - 1)*\text{sqrt}(-x + 1))$

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(40) = 80$.

time = 0.34, size = 90, normalized size = 1.58

$$\frac{\sqrt{2}(x^2 - 2x + 1) \log\left(-\frac{x^2+2\sqrt{2}\sqrt{-x^2+1}\sqrt{-x+1}+2x-3}{x^2-2x+1}\right) - 4\sqrt{-x+1}\left(\sqrt{-x^2+1} - 2\arcsin(x)\right)}{12(x^2 - 2x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(x)/(1-x)^(5/2),x, algorithm="fricas")

[Out] 1/12*(sqrt(2)*(x^2 - 2*x + 1)*log(-(x^2 + 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(-x + 1) + 2*x - 3)/(x^2 - 2*x + 1)) - 4*sqrt(-x + 1)*(sqrt(-x^2 + 1) - 2*arcsin(x)))/(x^2 - 2*x + 1)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(x)}{(1-x)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(x)/(1-x)**(5/2),x)

[Out] Integral(asin(x)/(1 - x)**(5/2), x)

Giac [A]

time = 0.02, size = 92, normalized size = 1.61

$$-2 \left(-\frac{\arcsin x}{3\sqrt{-x+1}(-x+1)} + \frac{2}{3} \left(\frac{\sqrt{x+1}}{4(-x+1)} - \frac{\ln\left(\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{x+1}+2\sqrt{2}}\right)}{8\sqrt{2}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(x)/(1-x)^(5/2),x)

[Out] 1/12*sqrt(2)*log((sqrt(2) - sqrt(x + 1))/(sqrt(2) + sqrt(x + 1))) + 1/3*sqrt(x + 1)/(x - 1) - 2/3*arcsin(x)/((x - 1)*sqrt(-x + 1))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(x)/(1 - x)^(5/2),x)

[Out] int(asin(x)/(1 - x)^(5/2), x)

3.702 $\int (-1 + x)^{5/2} \csc^{-1}(x) dx$

Optimal. Leaf size=82

$$\frac{4x\sqrt{-1+x^2}(83-19x+3x^2)}{105\sqrt{-1+x}\sqrt{x^2}} + \frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x) + \frac{4x \tanh^{-1}\left(\frac{\sqrt{-1+x^2}}{\sqrt{-1+x}}\right)}{7\sqrt{x^2}}$$

[Out] $2/7*(-1+x)^{(7/2)*\text{arccsc}(x)+4/7*x*\text{arctanh}((x^2-1)^{(1/2)/(-1+x)^{(1/2)})/(x^2)^{(1/2)+4/105*x*(3*x^2-19*x+83)*(x^2-1)^{(1/2)/(-1+x)^{(1/2)})/(x^2)^{(1/2)}$

Rubi [A]

time = 0.05, antiderivative size = 140, normalized size of antiderivative = 1.71, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5335, 1588, 906, 90, 65, 213}

$$\frac{4(x+1)^3\sqrt{x-1}}{35\sqrt{1-\frac{1}{x^2}}x} - \frac{20(x+1)^2\sqrt{x-1}}{21\sqrt{1-\frac{1}{x^2}}x} + \frac{4(x+1)\sqrt{x-1}}{\sqrt{1-\frac{1}{x^2}}x} + \frac{4\sqrt{x+1}\sqrt{x-1}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{x-1}}\right)}{7\sqrt{1-\frac{1}{x^2}}x} + \frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-1 + x)^{(5/2)}*\text{ArcCsc}[x], x]$

[Out] $(4*\text{Sqrt}[-1 + x]*(1 + x))/(\text{Sqrt}[1 - x^{(-2)}]*x) - (20*\text{Sqrt}[-1 + x]*(1 + x)^2)/(21*\text{Sqrt}[1 - x^{(-2)}]*x) + (4*\text{Sqrt}[-1 + x]*(1 + x)^3)/(35*\text{Sqrt}[1 - x^{(-2)}]*x) + (2*(-1 + x)^{(7/2)}*\text{ArcCsc}[x])/7 + (4*\text{Sqrt}[-1 + x]*\text{Sqrt}[1 + x]*\text{ArcTanh}[\text{Sqrt}[1 + x]])/(7*\text{Sqrt}[1 - x^{(-2)}]*x)$

Rule 65

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 90

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)*((e_.) + (f_.)*(x_.))^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x] \&\& \text{IntegersQ}[m, n] \&\& (\text{IntegerQ}[p] \mid\mid (\text{GtQ}[m, 0] \&\& \text{GeQ}[n, -1]))$

Rule 213

$\text{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[b, 2])^{(-1)}*\text{ArcTanh}[\text{Rt}[b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\&$

(LtQ[a, 0] || GtQ[b, 0])

Rule 906

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]

Rule 1588

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(mn2_))^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n]

Rule 5335

Int[((a_) + ArcCsc[(c_)*(x_)]*(b_))*((d_) + (e_)*(x_))^(m_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int (-1+x)^{5/2} \csc^{-1}(x) dx &= \frac{2}{7}(-1+x)^{7/2} \csc^{-1}(x) + \frac{2}{7} \int \frac{(-1+x)^{7/2}}{\sqrt{1-\frac{1}{x^2}} x^2} dx \\
&= \frac{2}{7}(-1+x)^{7/2} \csc^{-1}(x) + \frac{\left(2\sqrt{-1+x^2}\right) \int \frac{(-1+x)^{7/2}}{x\sqrt{-1+x^2}} dx}{7\sqrt{1-\frac{1}{x^2}} x} \\
&= \frac{2}{7}(-1+x)^{7/2} \csc^{-1}(x) + \frac{\left(2\sqrt{-1+x} \sqrt{1+x}\right) \int \frac{(-1+x)^3}{x\sqrt{1+x}} dx}{7\sqrt{1-\frac{1}{x^2}} x} \\
&= \frac{2}{7}(-1+x)^{7/2} \csc^{-1}(x) + \frac{\left(2\sqrt{-1+x} \sqrt{1+x}\right) \int \left(\frac{7}{\sqrt{1+x}} - \frac{1}{x\sqrt{1+x}} - 5\right) dx}{7\sqrt{1-\frac{1}{x^2}} x} \\
&= \frac{4\sqrt{-1+x} (1+x)}{\sqrt{1-\frac{1}{x^2}} x} - \frac{20\sqrt{-1+x} (1+x)^2}{21\sqrt{1-\frac{1}{x^2}} x} + \frac{4\sqrt{-1+x} (1+x)^3}{35\sqrt{1-\frac{1}{x^2}} x} + \frac{2}{7}(-1+x)^{7/2} \csc^{-1}(x) \\
&= \frac{4\sqrt{-1+x} (1+x)}{\sqrt{1-\frac{1}{x^2}} x} - \frac{20\sqrt{-1+x} (1+x)^2}{21\sqrt{1-\frac{1}{x^2}} x} + \frac{4\sqrt{-1+x} (1+x)^3}{35\sqrt{1-\frac{1}{x^2}} x} + \frac{2}{7}(-1+x)^{7/2} \csc^{-1}(x) \\
&= \frac{4\sqrt{-1+x} (1+x)}{\sqrt{1-\frac{1}{x^2}} x} - \frac{20\sqrt{-1+x} (1+x)^2}{21\sqrt{1-\frac{1}{x^2}} x} + \frac{4\sqrt{-1+x} (1+x)^3}{35\sqrt{1-\frac{1}{x^2}} x} + \frac{2}{7}(-1+x)^{7/2} \csc^{-1}(x)
\end{aligned}$$

Mathematica [A]

time = 0.05, size = 72, normalized size = 0.88

$$\frac{4\sqrt{1-\frac{1}{x^2}} x (83-19x+3x^2)}{105\sqrt{-1+x}} + \frac{2}{7}(-1+x)^{7/2} \csc^{-1}(x) + \frac{4}{7} \operatorname{tanh}^{-1}\left(\frac{\sqrt{1-\frac{1}{x^2}} x}{\sqrt{-1+x}}\right)$$

Antiderivative was successfully verified.

`[In] Integrate[(-1 + x)^(5/2)*ArcCsc[x], x]`

```
[Out] (4*Sqrt[1 - x^(-2)]*x*(83 - 19*x + 3*x^2))/(105*Sqrt[-1 + x]) + (2*(-1 + x)^(7/2)*ArcCsc[x])/7 + (4*ArcTanh[(Sqrt[1 - x^(-2)]*x)/Sqrt[-1 + x]])/7
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

`[In] mathics('Integrate[ArcCsc[x]*(x - 1)^(5/2),x]')``[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep`**Maple [A]**

time = 0.08, size = 76, normalized size = 0.93

method	result
derivativedivides	$\frac{2(-1+x)^{\frac{7}{2}} \operatorname{arccsc}(x)}{7} + \frac{4\sqrt{-1+x} \sqrt{1+x} \left(3(-1+x)^2 \sqrt{1+x} - 13(-1+x) \sqrt{1+x} + 15 \operatorname{arctanh}\left(\sqrt{1+x}\right) \right)}{105 \sqrt{\frac{(-1+x)(1+x)}{x^2}} x}$
default	$\frac{2(-1+x)^{\frac{7}{2}} \operatorname{arccsc}(x)}{7} + \frac{4\sqrt{-1+x} \sqrt{1+x} \left(3(-1+x)^2 \sqrt{1+x} - 13(-1+x) \sqrt{1+x} + 15 \operatorname{arctanh}\left(\sqrt{1+x}\right) \right)}{105 \sqrt{\frac{(-1+x)(1+x)}{x^2}} x}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((-1+x)^(5/2)*arccsc(x),x,method=_RETURNVERBOSE)`

```
[Out] 2/7*(-1+x)^(7/2)*arccsc(x)+4/105*(-1+x)^(1/2)*(1+x)^(1/2)*(3*(-1+x)^2*(1+x)^(1/2)-13*(-1+x)*(1+x)^(1/2)+15*arctanh((1+x)^(1/2))+67*(1+x)^(1/2))/((-1+x)*(1+x)/x^2)^(1/2)/x
```

Maxima [A]

time = 0.76, size = 116, normalized size = 1.41

$$\frac{4}{35}(x+1)^{\frac{5}{2}} - \frac{20}{21}(x+1)^{\frac{3}{2}} + \frac{2}{7}\left(x^3 \arctan(1, \sqrt{x+1} \sqrt{x-1}) - 3x^2 \arctan(1, \sqrt{x+1} \sqrt{x-1}) + 3x \arctan(1, \sqrt{x+1} \sqrt{x-1}) - \arctan(1, \sqrt{x+1} \sqrt{x-1})\right) \sqrt{x-1} + 4\sqrt{x+1} + \frac{2}{7} \log(\sqrt{x+1} + 1) - \frac{2}{7} \log(\sqrt{x+1} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((-1+x)^(5/2)*arccsc(x),x, algorithm="maxima")`

```
[Out] 4/35*(x + 1)^(5/2) - 20/21*(x + 1)^(3/2) + 2/7*(x^3*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) - 3*x^2*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + 3*x*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) - arctan2(1, sqrt(x + 1)*sqrt(x - 1)))*sqrt(x - 1) + 4*sqrt(x + 1) + 2/7*log(sqrt(x + 1) + 1) - 2/7*log(sqrt(x + 1) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(62) = 124.

time = 0.32, size = 125, normalized size = 1.52

$$\frac{2 \left(15(x^4 - 4x^3 + 6x^2 - 4x + 1)\sqrt{x-1} \operatorname{arccsc}(x) + 2(3x^2 - 19x + 83)\sqrt{x^2-1} \sqrt{x-1} + 15(x-1) \log\left(\frac{x^2 + \sqrt{x^2-1} \sqrt{x-1} - 1}{x^2-1}\right) - 15(x-1) \log\left(\frac{-x^2 - \sqrt{x^2-1} \sqrt{x-1} - 1}{x^2-1}\right) \right)}{105(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)^(5/2)*arccsc(x),x, algorithm="fricas")

[Out] 2/105*(15*(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)*sqrt(x - 1)*arccsc(x) + 2*(3*x^2 - 19*x + 83)*sqrt(x^2 - 1)*sqrt(x - 1) + 15*(x - 1)*log((x^2 + sqrt(x^2 - 1))*sqrt(x - 1) - 1)/(x^2 - 1)) - 15*(x - 1)*log(-(x^2 - sqrt(x^2 - 1))*sqrt(x - 1) - 1)/(x^2 - 1))/(x - 1)

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)**(5/2)*acsc(x),x)

[Out] Exception raised: SystemError

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(62) = 124.

time = 0.10, size = 566, normalized size = 6.90

$$\left(\frac{2\sqrt{x-1} \operatorname{arccsc}(x)}{x-1} - \frac{2\sqrt{x-1} \operatorname{arcsin}(1/x)}{x-1} \right) \left(\frac{2\sqrt{x-1} \operatorname{arccsc}(x)}{x-1} - \frac{2\sqrt{x-1} \operatorname{arcsin}(1/x)}{x-1} \right) - \left(\frac{2\sqrt{x-1} \operatorname{arccsc}(x)}{x-1} - \frac{2\sqrt{x-1} \operatorname{arcsin}(1/x)}{x-1} \right) \left(\frac{2\sqrt{x-1} \operatorname{arccsc}(x)}{x-1} - \frac{2\sqrt{x-1} \operatorname{arcsin}(1/x)}{x-1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)^(5/2)*arccsc(x),x)

[Out] 2/35*(5*(x - 1)^(7/2) + 21*(x - 1)^(5/2) + 35*(x - 1)^(3/2) + 35*sqrt(x - 1))*arcsin(1/x) - 2/5*(3*(x - 1)^(5/2) + 10*(x - 1)^(3/2) + 15*sqrt(x - 1))*arcsin(1/x) + 2*((x - 1)^(3/2) + 3*sqrt(x - 1))*arcsin(1/x) - 2*sqrt(x - 1)*arcsin(1/x) + 4/105*(3*(x + 1)^(5/2) - 4*(x + 1)^(3/2) + 21*sqrt(x + 1))/sgn((x - 1)^(3/2) + sqrt(x - 1)) - 4/5*((x + 1)^(3/2) + sqrt(x + 1))/sgn((x - 1)^(3/2) + sqrt(x - 1)) + 2/7*log(sqrt(x + 1) + 1)/sgn((x - 1)^(3/2) + sqrt(x - 1)) - 2/7*log(sqrt(x + 1) - 1)/sgn((x - 1)^(3/2) + sqrt(x - 1)) + 4*sqrt(x + 1)/sgn((x - 1)^(3/2) + sqrt(x - 1))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asin}\left(\frac{1}{x}\right) (x-1)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(1/x)*(x - 1)^(5/2),x)

[Out] int(asin(1/x)*(x - 1)^(5/2), x)

3.703 $\int \sin^{-1}(\sinh(x)) \operatorname{sech}^4(x) dx$

Optimal. Leaf size=49

$$-\frac{2}{3} \sin^{-1}\left(\frac{\cosh(x)}{\sqrt{2}}\right) + \frac{1}{6} \operatorname{sech}(x) \sqrt{1 - \sinh^2(x)} + \sin^{-1}(\sinh(x)) \tanh(x) - \frac{1}{3} \sin^{-1}(\sinh(x)) \tanh^3(x)$$

[Out] $-2/3*\arcsin(1/2*\cosh(x)*2^{(1/2)})+1/6*\operatorname{sech}(x)*(1-\sinh(x)^2)^{(1/2)}+\arcsin(\sinh(x))*\tanh(x)-1/3*\arcsin(\sinh(x))*\tanh(x)^3$

Rubi [A]

time = 0.09, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3852, 4928, 12, 4442, 462, 222}

$$\frac{1}{6} \sqrt{2 - \cosh^2(x)} \operatorname{sech}(x) - \frac{2}{3} \sin^{-1}\left(\frac{\cosh(x)}{\sqrt{2}}\right) - \frac{1}{3} \tanh^3(x) \sin^{-1}(\sinh(x)) + \tanh(x) \sin^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

[In] `Int[ArcSin[Sinh[x]]*Sech[x]^4,x]`

[Out] $(-2*\operatorname{ArcSin}[\operatorname{Cosh}[x]/\operatorname{Sqrt}[2]])/3 + (\operatorname{Sqrt}[2 - \operatorname{Cosh}[x]^2]*\operatorname{Sech}[x])/6 + \operatorname{ArcSin}[\operatorname{Sinh}[x]]*\operatorname{Tanh}[x] - (\operatorname{ArcSin}[\operatorname{Sinh}[x]]*\operatorname{Tanh}[x]^3)/3$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

Rule 222

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

Rule 462

`Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[c*(e*x)^(m+1)*((a+b*x^n)^(p+1)/(a*e*(m+1))), x] + Dist[d/e^n, Int[(e*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m+n*(p+1)+1, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1]))`

Rule 3852

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1+x^2)^(n/2-1), x], x], x, Cot[c+d*x]], x] /; FreeQ[{c,`

d}, x] && IGtQ[n/2, 0]

Rule 4442

```
Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 4928

```
Int[((a_) + ArcSin[u_]*(b_))*(v_), x_Symbol] := With[{w = IntHide[v, x]}, Dist[a + b*ArcSin[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x])/Sqrt[1 - u^2]], x], x], x] /; InverseFunctionFreeQ[w, x] /; FreeQ[{a, b}, x] && InverseFunctionFreeQ[u, x] && !MatchQ[v, ((c_) + (d_)*x)^(m_) /; FreeQ[{c, d, m}, x]]
```

Rubi steps

$$\begin{aligned}
 \int \sin^{-1}(\sinh(x)) \operatorname{sech}^4(x) dx &= \sin^{-1}(\sinh(x)) \tanh(x) - \frac{1}{3} \sin^{-1}(\sinh(x)) \tanh^3(x) - \int \frac{(2 + \cosh(2x)) \operatorname{sech}^4(x)}{3 \sqrt{1 - \sinh^2(x)}} dx \\
 &= \sin^{-1}(\sinh(x)) \tanh(x) - \frac{1}{3} \sin^{-1}(\sinh(x)) \tanh^3(x) - \frac{1}{3} \int \frac{(2 + \cosh(2x)) \operatorname{sech}^4(x)}{\sqrt{1 - \sinh^2(x)}} dx \\
 &= \sin^{-1}(\sinh(x)) \tanh(x) - \frac{1}{3} \sin^{-1}(\sinh(x)) \tanh^3(x) - \frac{1}{3} \operatorname{Subst} \left(\int \frac{1 + 2x}{x^2 \sqrt{2 - x^2}} dx, \frac{1 + \cosh(2x)}{\cosh(x)} \right) \\
 &= \frac{1}{6} \sqrt{2 - \cosh^2(x)} \operatorname{sech}(x) + \sin^{-1}(\sinh(x)) \tanh(x) - \frac{1}{3} \sin^{-1}(\sinh(x)) \tanh^3(x) \\
 &= -\frac{2}{3} \sin^{-1} \left(\frac{\cosh(x)}{\sqrt{2}} \right) + \frac{1}{6} \sqrt{2 - \cosh^2(x)} \operatorname{sech}(x) + \sin^{-1}(\sinh(x)) \tanh(x)
 \end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.14, size = 66, normalized size = 1.35

$$\frac{1}{12} \left(8i \log \left(i\sqrt{2} \cosh(x) + \sqrt{3 - \cosh(2x)} \right) + \sqrt{6 - 2 \cosh(2x)} \operatorname{sech}(x) + 4 \sin^{-1}(\sinh(x)) (2 + \cosh(2x)) \operatorname{sech}^2(x) \tanh(x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[Sinh[x]]*Sech[x]^4, x]
```

```
[Out] ((8*I)*Log[I*Sqrt[2]*Cosh[x] + Sqrt[3 - Cosh[2*x]]) + Sqrt[6 - 2*Cosh[2*x]]*Sech[x] + 4*ArcSin[Sinh[x]]*(2 + Cosh[2*x])*Sech[x]^2*Tanh[x])/12
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[ArcSin[Sinh[x]]*Sech[x]^4,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]

time = 0.04, size = 0, normalized size = 0.00

$$\int \arcsin(\sinh(x)) \operatorname{sech}(x)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(sinh(x))*sech(x)^4,x)
```

```
[Out] int(arcsin(sinh(x))*sech(x)^4,x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(4*(3*e^(2*x) + 1)*arctan2(e^(2*x) - 1, sqrt(e^(2*x) + 2*e^x - 1)*sqrt(-e^(2*x) + 2*e^x + 1)) + 3*(e^(6*x) + 3*e^(4*x) + 3*e^(2*x) + 1)*integrate(16/3*(3*e^(4*x) + e^(2*x))*e^(1/2*log(e^(2*x) + 2*e^x - 1) + 1/2*log(-e^(2*x) + 2*e^x + 1)))/((e^(8*x) - 4*e^(6*x) - 10*e^(4*x) - 4*e^(2*x) + 1)*e^(log(e^(2*x) + 2*e^x - 1) + log(-e^(2*x) + 2*e^x + 1)) + e^(12*x) - 6*e^(10*x) - e^(8*x) + 12*e^(6*x) - e^(4*x) - 6*e^(2*x) + 1), x))/(e^(6*x) + 3*e^(4*x) + 3*e^(2*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 519 vs. 2(40) = 80.

time = 0.34, size = 519, normalized size = 10.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqrt(-
```

$$\begin{aligned} & \cosh(x)^2 + \sinh(x)^2 - 3)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) - 4 \\ & *(\cosh(x)^6 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6 + 3*(5*\cosh(x)^2 + 1)*\sinh(x) \\ & ^4 + 3*\cosh(x)^4 + 4*(5*\cosh(x)^3 + 3*\cosh(x))*\sinh(x)^3 + 3*(5*\cosh(x)^4 + \\ & 6*\cosh(x)^2 + 1)*\sinh(x)^2 + 3*\cosh(x)^2 + 6*(\cosh(x)^5 + 2*\cosh(x)^3 + \cosh(x))*\sinh(x) + 1)*\arctan(\sqrt{2}*(3*\cosh(x)^2 + 6*\cosh(x)*\sinh(x) + 3*\sinh(x)^2 - 1)*\sqrt{-(\cosh(x)^2 + \sinh(x)^2 - 3)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)})/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 6*(\cosh(x)^2 - 1)*\sinh(x)^2 - 6*\cosh(x)^2 + 4*(\cosh(x)^3 - 3*\cosh(x))*\sinh(x) + 1)) + 8 \\ & *(3*\cosh(x)^2 + 6*\cosh(x)*\sinh(x) + 3*\sinh(x)^2 + 1)*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{-(\cosh(x)^2 + \sinh(x)^2 - 3)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)})/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 6*(\cosh(x)^2 - 1)*\sinh(x)^2 - 6*\cosh(x)^2 + 4*(\cosh(x)^3 - 3*\cosh(x))*\sinh(x) + 1)))/(\cosh(x)^6 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6 + 3 \\ & *(5*\cosh(x)^2 + 1)*\sinh(x)^4 + 3*\cosh(x)^4 + 4*(5*\cosh(x)^3 + 3*\cosh(x))*\sinh(x)^3 + 3*(5*\cosh(x)^4 + 6*\cosh(x)^2 + 1)*\sinh(x)^2 + 3*\cosh(x)^2 + 6*(\cosh(x)^5 + 2*\cosh(x)^3 + \cosh(x))*\sinh(x) + 1) \end{aligned}$$

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asin(sinh(x))*sech(x)**4,x)

[Out] Timed out

Giac [C] Result contains complex when optimal does not.

time = 0.06, size = 269, normalized size = 5.49

$$16 \left(\frac{8\sqrt{2} - \frac{8\left(\sqrt{2} - 2\sqrt{-(e^x)^4 + 6(e^x)^2 - 1}\right)}{-2(e^x)^2 + 6}}{\left(\frac{\sqrt{2} - 2\sqrt{-(e^x)^4 + 6(e^x)^2 - 1}}{-2(e^x)^2 + 6}\right)^2 + 1 - \frac{\sqrt{2}\left(\sqrt{2} - 2\sqrt{-(e^x)^4 + 6(e^x)^2 - 1}\right)}{-2(e^x)^2 + 6}}{\arctan\left(-2\sqrt{2} + \frac{3\left(\sqrt{2} - 2\sqrt{-(e^x)^4 + 6(e^x)^2 - 1}\right)}{-2(e^x)^2 + 6}\right)} - \frac{-8i\sqrt{2}\arctan(-i) - 3\sqrt{2} + 32\arctan(-i) - 3i}{96i\sqrt{2} - 384} + \frac{(-3(e^x)^2 - 1)\arcsin\left(\frac{e^x - 1}{2e^x}\right)}{2 \cdot 6((e^x)^2 + 1)^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsin(sinh(x))*sech(x)^4,x)

[Out] $-16*(-8*I*\sqrt{2}*\arctan(-I) - 3*\sqrt{2} + 32*\arctan(-I) - 3*I)/(96*I*\sqrt{2} - 384) + 1/6*(\sqrt{2} + (2*\sqrt{2} - \sqrt{-e^{(4*x)} + 6*e^{(2*x)} - 1)})/(e^{(2*x)} - 3))/(\sqrt{2}*(2*\sqrt{2} - \sqrt{-e^{(4*x)} + 6*e^{(2*x)} - 1)})/(e^{(2*x)} - 3) + (2*\sqrt{2} - \sqrt{-e^{(4*x)} + 6*e^{(2*x)} - 1)})^2/(e^{(2*x)} - 3)^2 + 1) - 4/3*(3*e^{(2*x)} + 1)*\arcsin(1/2*(e^{(2*x)} - 1)/e^x)/(e^{(2*x)} + 1)^3 - 4/3*\arctan(-2*\sqrt{2} - 3*(2*\sqrt{2} - \sqrt{-e^{(4*x)} + 6*e^{(2*x)} - 1)})/(e^{(2*x)} - 3))$

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asin}(\sinh(x))}{\cosh(x)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(asin(sinh(x))/cosh(x)^4,x)`

[Out] `int(asin(sinh(x))/cosh(x)^4, x)`

3.704 $\int \cot^{-1}(\cosh(x)) \coth(x) \operatorname{csch}^3(x) dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{6\sqrt{2}} + \frac{\coth(x)}{6} - \frac{1}{3} \cot^{-1}(\cosh(x)) \operatorname{csch}^3(x)$$

[Out] 1/6*coth(x)-1/3*arccot(cosh(x))*csch(x)^3+1/12*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)

Rubi [A]

time = 0.08, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2686, 30, 5316, 12, 464, 212}

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{6\sqrt{2}} + \frac{\coth(x)}{6} - \frac{1}{3} \operatorname{csch}^3(x) \cot^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

[In] Int[ArcCot[Cosh[x]]*Coth[x]*Csch[x]^3,x]

[Out] ArcTanh[Tanh[x]/Sqrt[2]]/(6*Sqrt[2]) + Coth[x]/6 - (ArcCot[Cosh[x]]*Csch[x]^3)/3

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 464

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[c*(e*x)^(m+1)*((a+b*x^n)^(p+1)/(a*e^(m+1))), x] + Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), Int[(e

$x^{(m+n)}(a + b*x^n)^p, x] , x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& (\text{IntegerQ}[n] \parallel \text{GtQ}[e, 0]) \&\& ((\text{GtQ}[n, 0] \&\& \text{LtQ}[m, -1]) \parallel (\text{LtQ}[n, 0] \&\& \text{GtQ}[m + n, -1])) \&\& !\text{ILtQ}[p, -1]$

Rule 2686

$\text{Int}[(a_*)*\text{sec}[(e_*) + (f_*)*(x_*)]^{(m_*)}*((b_*)*\text{tan}[(e_*) + (f_*)*(x_*)]^{(n_*)}, x_Symbol] :> \text{Dist}[a/f, \text{Subst}[\text{Int}[(a*x)^{(m-1)}*(-1 + x^2)^{((n-1)/2)}, x], x, \text{Sec}[e + f*x]], x] /; \text{FreeQ}[\{a, e, f, m\}, x] \&\& \text{IntegerQ}[(n-1)/2] \&\& !(\text{IntegerQ}[m/2] \&\& \text{LtQ}[0, m, n + 1])$

Rule 5316

$\text{Int}[(a_*) + \text{ArcCot}[u_*]*(b_*)*(v_*) , x_Symbol] :> \text{With}[\{w = \text{IntHide}[v, x]\}, \text{Dist}[a + b*\text{ArcCot}[u], w, x] + \text{Dist}[b, \text{Int}[\text{SimplifyIntegrand}[w*(D[u, x]/(1 + u^2)), x], x], x] /; \text{InverseFunctionFreeQ}[w, x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{InverseFunctionFreeQ}[u, x] \&\& !\text{MatchQ}[v, ((c_*) + (d_*)*x)^{(m_*)} /; \text{FreeQ}[\{c, d, m\}, x]] \&\& \text{FalseQ}[\text{FunctionOfLinear}[v*(a + b*\text{ArcCot}[u]), x]]$

Rubi steps

$$\begin{aligned}
 \int \cot^{-1}(\cosh(x)) \coth(x) \operatorname{csch}^3(x) dx &= -\frac{1}{3} \cot^{-1}(\cosh(x)) \operatorname{csch}^3(x) + \int \frac{2 \operatorname{csch}^2(x)}{3(-3 - \cosh(2x))} dx \\
 &= -\frac{1}{3} \cot^{-1}(\cosh(x)) \operatorname{csch}^3(x) + \frac{2}{3} \int \frac{\operatorname{csch}^2(x)}{-3 - \cosh(2x)} dx \\
 &= -\frac{1}{3} \cot^{-1}(\cosh(x)) \operatorname{csch}^3(x) - \frac{2}{3} \operatorname{Subst} \left(\int \frac{1 - x^2}{2x^2(2 - x^2)} dx, x, \tanh(x) \right) \\
 &= -\frac{1}{3} \cot^{-1}(\cosh(x)) \operatorname{csch}^3(x) - \frac{1}{3} \operatorname{Subst} \left(\int \frac{1 - x^2}{x^2(2 - x^2)} dx, x, \tanh(x) \right) \\
 &= \frac{\coth(x)}{6} - \frac{1}{3} \cot^{-1}(\cosh(x)) \operatorname{csch}^3(x) + \frac{1}{6} \operatorname{Subst} \left(\int \frac{1}{2 - x^2} dx, x, \tanh(x) \right) \\
 &= \frac{\tanh^{-1} \left(\frac{\tanh(x)}{\sqrt{2}} \right)}{6\sqrt{2}} + \frac{\coth(x)}{6} - \frac{1}{3} \cot^{-1}(\cosh(x)) \operatorname{csch}^3(x)
 \end{aligned}$$

Mathematica [A]

time = 0.12, size = 40, normalized size = 1.11

$$\frac{1}{24} \left(2\sqrt{2} \tanh^{-1} \left(\frac{\tanh(x)}{\sqrt{2}} \right) + (-8 \cot^{-1}(\cosh(x)) - \cosh(x) + \cosh(3x)) \operatorname{csch}^3(x) \right)$$

Antiderivative was successfully verified.

/2))²)*exp(4*x)-3*2^(1/2)*ln(exp(2*x)+(2^(1/2)-1)²)*exp(2*x)+3*2^(1/2)*ln
 (exp(2*x)+(1+2^(1/2))²)*exp(2*x)+16*Pi*csgn(I*exp(-x)*(-exp(2*x)-1+2*I*exp
 (x)))*csgn(exp(-x)*(-exp(2*x)-1+2*I*exp(x)))²*exp(3*x)-16*Pi*csgn(exp(-x)*
 (exp(2*x)+1+2*I*exp(x)))³*exp(3*x)+16*Pi*csgn(exp(-x)*(-exp(2*x)-1+2*I*exp
 (x)))³*exp(3*x)+16*Pi*csgn(exp(-x)*(exp(2*x)+1+2*I*exp(x)))²*exp(3*x))/(e
 xp(2*x)-1)³

Maxima [A]

time = 0.34, size = 54, normalized size = 1.50

$$-\frac{1}{24} \sqrt{2} \log \left(-\frac{2\sqrt{2} - e^{(-2x)} - 3}{2\sqrt{2} + e^{(-2x)} + 3} \right) - \frac{1}{3(e^{(-2x)} - 1)} - \frac{\operatorname{arccot}(\cosh(x))}{3 \sinh(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccot(cosh(x))*cosh(x)/sinh(x)⁴,x, algorithm="maxima")

[Out] -1/24*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) -
 1/3/(e^(-2*x) - 1) - 1/3*arccot(cosh(x))/sinh(x)³

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs.
 2(27) = 54.

time = 0.33, size = 423, normalized size = 11.75

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccot(cosh(x))*cosh(x)/sinh(x)⁴,x, algorithm="fricas")

[Out] 1/24*(8*cosh(x)⁴ + 32*cosh(x)*sinh(x)³ + 8*sinh(x)⁴ + 16*(3*cosh(x)² -
 1)*sinh(x)² - 64*(cosh(x)³ + 3*cosh(x)²*sinh(x) + 3*cosh(x)*sinh(x)² +
 sinh(x)³)*arctan(2*(cosh(x) + sinh(x))/(cosh(x)² + 2*cosh(x)*sinh(x) + si
 nh(x)² + 1)) - 16*cosh(x)² + (sqrt(2)*cosh(x)⁶ + 6*sqrt(2)*cosh(x)*sinh(
 x)⁵ + sqrt(2)*sinh(x)⁶ + 3*(5*sqrt(2)*cosh(x)² - sqrt(2))*sinh(x)⁴ - 3*
 sqrt(2)*cosh(x)⁴ + 4*(5*sqrt(2)*cosh(x)³ - 3*sqrt(2)*cosh(x))*sinh(x)³ +
 3*(5*sqrt(2)*cosh(x)⁴ - 6*sqrt(2)*cosh(x)² + sqrt(2))*sinh(x)² + 3*sqrt
 (2)*cosh(x)² + 6*(sqrt(2)*cosh(x)⁵ - 2*sqrt(2)*cosh(x)³ + sqrt(2)*cosh(x
))*sinh(x) - sqrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)² - 4*(3*sqrt(2) - 4)
 *cosh(x)*sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)² + 2*sqrt(2) - 3)/(cosh(x)²
 + sinh(x)² + 3)) + 32*(cosh(x)³ - cosh(x))*sinh(x) + 8)/(cosh(x)⁶ + 6*co
 sh(x)*sinh(x)⁵ + sinh(x)⁶ + 3*(5*cosh(x)² - 1)*sinh(x)⁴ - 3*cosh(x)⁴ +
 4*(5*cosh(x)³ - 3*cosh(x))*sinh(x)³ + 3*(5*cosh(x)⁴ - 6*cosh(x)² + 1)*
 sinh(x)² + 3*cosh(x)² + 6*(cosh(x)⁵ - 2*cosh(x)³ + cosh(x))*sinh(x) - 1
)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs.
 2(34) = 68

time = 88.20, size = 214, normalized size = 5.94

$$-\frac{\sqrt{2} \log(4 \tanh^2(\frac{x}{2}) - 4\sqrt{2} \tanh(\frac{x}{2}) + 4)}{24} + \frac{\sqrt{2} \log(4 \tanh^2(\frac{x}{2}) + 4\sqrt{2} \tanh(\frac{x}{2}) + 4)}{24} - \frac{\tanh^3(\frac{x}{2}) \operatorname{acot}\left(\frac{\tanh^2(\frac{x}{2}) + \frac{1}{\tanh^2(\frac{x}{2}) - 1}}{\tanh^2(\frac{x}{2}) - 1}\right)}{24} + \frac{\tanh(\frac{x}{2}) \operatorname{acot}\left(\frac{\tanh^2(\frac{x}{2}) + \frac{1}{\tanh^2(\frac{x}{2}) - 1}}{\tanh^2(\frac{x}{2}) - 1}\right)}{8} + \frac{\tanh(\frac{x}{2})}{12} - \frac{\operatorname{acot}\left(\frac{\tanh^2(\frac{x}{2}) + \frac{1}{\tanh^2(\frac{x}{2}) - 1}}{\tanh^2(\frac{x}{2}) - 1}\right)}{8 \tanh(\frac{x}{2})} + \frac{1}{12 \tanh(\frac{x}{2})} + \frac{\operatorname{acot}\left(\frac{\tanh^2(\frac{x}{2}) + \frac{1}{\tanh^2(\frac{x}{2}) - 1}}{\tanh^2(\frac{x}{2}) - 1}\right)}{24 \tanh^3(\frac{x}{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acot(cosh(x))*cosh(x)/sinh(x)**4,x)

[Out] -sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)/24 + sqrt(2)*log(4*tanh(x/2)**2 + 4*sqrt(2)*tanh(x/2) + 4)/24 - tanh(x/2)**3*acot(tanh(x/2)**2/(tanh(x/2)**2 - 1) + 1/(tanh(x/2)**2 - 1))/24 + tanh(x/2)*acot(tanh(x/2)**2/(tanh(x/2)**2 - 1) + 1/(tanh(x/2)**2 - 1))/8 + tanh(x/2)/12 - acot(tanh(x/2)**2/(tanh(x/2)**2 - 1) + 1/(tanh(x/2)**2 - 1))/(8*tanh(x/2)) + 1/(12*tanh(x/2)) + acot(tanh(x/2)**2/(tanh(x/2)**2 - 1) + 1/(tanh(x/2)**2 - 1))/(24*tanh(x/2)**3)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(27) = 54.
time = 0.03, size = 90, normalized size = 2.50

$$\frac{-\frac{16 \arctan\left(\frac{2}{e^x + \frac{1}{e^x}}\right)}{3(e^x - \frac{1}{e^x})^3} - \frac{32}{2} \left(-\frac{1}{24((e^x)^2 - 1)} - \frac{\ln\left(\frac{2(e^x)^2 + 6 - 4\sqrt{2}}{2(e^x)^2 + 6 + 4\sqrt{2}}\right)}{96\sqrt{2}} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x)

[Out] 1/24*sqrt(2)*log((e^(2*x) - 2*sqrt(2) + 3)/(e^(2*x) + 2*sqrt(2) + 3)) + 1/3/(e^(2*x) - 1) - 8/3*arctan(2/(e^x + 1/e^x))/(e^x - 1/e^x)^3

Mupad [B]

time = 0.54, size = 103, normalized size = 2.86

$$\frac{\sqrt{2} \ln\left(-\frac{2e^{2x}}{3} - \frac{\sqrt{2}(12e^{2x} + 4)}{24}\right)}{24} - \frac{\sqrt{2} \ln\left(\frac{\sqrt{2}(12e^{2x} + 4)}{24} - \frac{2e^{2x}}{3}\right)}{24} + \frac{1}{3(e^{2x} - 1)} - \frac{8e^{3x} \operatorname{acot}\left(\frac{e^{-x}}{2} + \frac{e^x}{2}\right)}{3(3e^{2x} - 3e^{4x} + e^{6x} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((acot(cosh(x))*cosh(x))/sinh(x)^4,x)

[Out] (2^(1/2)*log(-(2*exp(2*x))/3 - (2^(1/2)*(12*exp(2*x) + 4))/24))/24 - (2^(1/2)*log((2^(1/2)*(12*exp(2*x) + 4))/24 - (2*exp(2*x))/3))/24 + 1/(3*(exp(2*x) - 1)) - (8*exp(3*x)*acot(exp(-x)/2 + exp(x)/2))/(3*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1))

3.705 $\int e^x \sin^{-1}(\tanh(x)) dx$

Optimal. Leaf size=28

$$e^x \sin^{-1}(\tanh(x)) - \cosh(x) \log(1 + e^{2x}) \sqrt{\operatorname{sech}^2(x)}$$

[Out] exp(x)*arcsin(tanh(x))-cosh(x)*ln(1+exp(2*x))*(sech(x)^2)^(1/2)

Rubi [A]

time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2225, 4928, 6852, 2320, 12, 266}

$$e^x \sin^{-1}(\tanh(x)) - \log(e^{2x} + 1) \cosh(x) \sqrt{\operatorname{sech}^2(x)}$$

Antiderivative was successfully verified.

[In] Int[E^x*ArcSin[Tanh[x]],x]

[Out] E^x*ArcSin[Tanh[x]] - Cosh[x]*Log[1 + E^(2*x)]*Sqrt[Sech[x]^2]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 266

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 2225

Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 2320

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 4928

Int[((a_) + ArcSin[u_]*(b_))*(v_), x_Symbol] := With[{w = IntHide[v, x]}, Dist[a + b*ArcSin[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x])/Sqr

```
t[1 - u^2]), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x]
&& InverseFunctionFreeQ[u, x] && !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; Fre
eQ[{c, d, m}, x]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
 \int e^x \sin^{-1}(\tanh(x)) dx &= e^x \sin^{-1}(\tanh(x)) - \int e^x \sqrt{\operatorname{sech}^2(x)} dx \\
 &= e^x \sin^{-1}(\tanh(x)) - \left(\cosh(x) \sqrt{\operatorname{sech}^2(x)} \right) \int e^x \operatorname{sech}(x) dx \\
 &= e^x \sin^{-1}(\tanh(x)) - \left(\cosh(x) \sqrt{\operatorname{sech}^2(x)} \right) \operatorname{Subst} \left(\int \frac{2x}{1+x^2} dx, x, e^x \right) \\
 &= e^x \sin^{-1}(\tanh(x)) - \left(2 \cosh(x) \sqrt{\operatorname{sech}^2(x)} \right) \operatorname{Subst} \left(\int \frac{x}{1+x^2} dx, x, e^x \right) \\
 &= e^x \sin^{-1}(\tanh(x)) - \cosh(x) \log(1 + e^{2x}) \sqrt{\operatorname{sech}^2(x)}
 \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(28) = 56.

time = 0.53, size = 64, normalized size = 2.29

$$e^x \sin^{-1} \left(\frac{-1 + e^{2x}}{1 + e^{2x}} \right) - e^{-x} \sqrt{\frac{e^{2x}}{(1 + e^{2x})^2}} (1 + e^{2x}) \log(1 + e^{2x})$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*ArcSin[Tanh[x]], x]
```

```
[Out] E^x*ArcSin[(-1 + E^(2*x))/(1 + E^(2*x))] - (Sqrt[E^(2*x)/(1 + E^(2*x))^2]*(
1 + E^(2*x))*Log[1 + E^(2*x)])/E^x
```

Mathics [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

[In] `mathics('Integrate[ArcSin[Tanh[x]]*E^x,x]')`

[Out] `cought exception: maximum recursion depth exceeded`

Maple [F]

time = 0.03, size = 0, normalized size = 0.00

$$\int e^x \arcsin(\tanh(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)*arcsin(tanh(x)),x)`

[Out] `int(exp(x)*arcsin(tanh(x)),x)`

Maxima [A]

time = 0.34, size = 16, normalized size = 0.57

$$\arcsin(\tanh(x)) e^x - \log(e^{2x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*arcsin(tanh(x)),x, algorithm="maxima")`

[Out] `arcsin(tanh(x))*e^x - log(e^(2*x) + 1)`

Fricas [A]

time = 0.33, size = 26, normalized size = 0.93

$$(\cosh(x) + \sinh(x)) \arctan(\sinh(x)) - \log\left(\frac{2 \cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*arcsin(tanh(x)),x, algorithm="fricas")`

[Out] `(cosh(x) + sinh(x))*arctan(sinh(x)) - log(2*cosh(x)/(cosh(x) - sinh(x)))`

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int e^x \operatorname{asin}(\tanh(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*asin(tanh(x)),x)`

[Out] `Integral(exp(x)*asin(tanh(x)), x)`

Giac [A]

time = 0.01, size = 27, normalized size = 0.96

$$e^x \arcsin\left(\frac{(e^x)^2 - 1}{(e^x)^2 + 1}\right) - \ln((e^x)^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*arcsin(tanh(x)),x)**[Out]** e^x*arcsin((e^(2*x) - 1)/(e^(2*x) + 1)) - log(e^(2*x) + 1)**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.04

$$\int \operatorname{asin}(\tanh(x)) e^x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(asin(tanh(x))*exp(x),x)**[Out]** int(asin(tanh(x))*exp(x), x)

Chapter 4

Appendix

Local contents

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4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*      Small rewrite of logic in main function to make it*)
(*      match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A","none"}

```

```

        ,(*ELSE*)
        finalresult={"B","Both result and optimal contain complex but leaf count is
    ]
    ,(*ELSE*)
        finalresult={"C","Result contains complex when optimal does not."}
    ]
    ,(*ELSE*)(*result does not contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
            finalresult={"A","none"}
        ,(*ELSE*)
            finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"
        ]
    ]
    ,(*ELSE*)(*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>"}
        ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]==Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]==Rational,

```



```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems
#Nasser 4/07/2022 add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
        return "B","result has leaf size over 500,000. Avoiding possible recursion issues."
      fi;

      leaf_count_optimal := leafcount(optimal);
      ExpnType_result := ExpnType(result);
      ExpnType_optimal := ExpnType(optimal);

      if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal
```

```

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A","";
            else
                return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (" ,
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                                end if
            else #result contains complex but optimal is not
                if debug then
                    print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
            # this assumes optimal do not as well. No check is needed here.
            if debug then
                print("result do not contain complex, this assumes optimal do not as well");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
            fi;
        fi;
    fi;
fi;

```

```

        fi;
        return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                        convert(leaf_count_result,string)," $ vs. $2(",
                        convert(leaf_count_optimal,string),")=",convert(2*leaf_count

    fi;
    fi;
else #ExpnType(result) > ExpnType(optimal)
    if debug then
        print("ExpnType(result) > ExpnType(optimal)");
    fi;
    return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
    return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
    if type(expn,'atomic') then
        1
    elif type(expn,'list') then
        apply(max,map(ExpnType,expn))
    elif type(expn,'sqrt') then
        if type(op(1,expn),'rational') then
            1
        else

```



```

        max(2,ExpnType(op(1,expn)))
    end if
elif type(expn,'^') then
    if type(op(2,expn),'integer') then
        ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
        if type(op(1,expn),'rational') then
            1
        else
            max(2,ExpnType(op(1,expn)))
        end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
elif type(expn,'+') or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
9
end if
end proc:

```

```

ElementaryFunctionQ := proc(func)
    member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

```

```

SpecialFunctionQ := proc(func)
    member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

```

```

end proc:

HypergeometricFunctionQ := proc(func)
  member(func, [Hypergeometric1F1, hypergeom, HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func, [AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u), op(2..nops(u), u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

4.2.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#      Port of original Maple grading function by
#      Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added 'RootSum'. See problem 177, Timofeev file
#      added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr, Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:

```

```

    return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
                    asinh,acosh,atanh,acoth,asech,acsch
                    ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
                    gamma,loggamma,digamma,zeta,polylog,LambertW,
                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
                    ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')

```

```

    return expnType(expn.args[0]) #ExpnType(op(1,expn))
elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
    else:
        return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
else:
    return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+') or type(expn,'*')
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

#main function

```
def grade_antiderivative(result,optimal):
```

```
    #print ("Enter grade_antiderivative for sagemath")
```

```
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)
```

```
leaf_count_result = leaf_count(result)
```

```
leaf_count_optimal = leaf_count(optimal)
```

```
    #print("leaf_count_result=",leaf_count_result)
```

```
    #print("leaf_count_optimal=",leaf_count_optimal)
```

```
expnType_result = expnType(result)
```

```
expnType_optimal = expnType(optimal)
```

```

if str(result).find("Integral") != -1:
    grade = "F"
    grade_annotation = ""
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Both result and optimal contain complex but leaf count of result is large"
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation = "Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = ""
            else:
                grade = "B"
                grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result)
    else:
        grade = "C"
        grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType_result)

    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```

4.2.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
# Albert Rich to use with Sagemath. This is used to
# grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
# 'arctan2', 'floor', 'abs', 'log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
# issue later when reading the file.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

```

```

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False

```

```

if debug: print ("type(func)=", type(func))

m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
    'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
    'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
    'polylog','lambert_w','elliptic_f','elliptic_e',
    'elliptic_pi','exp_integral_e','log_integral']

if debug:
    print ("m=",m)
    if m:
        print ("func ", func ," is special_function")
    else:
        print ("func ", func ," is NOT special_function")

return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric'] #[appellf1] can't find this in sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-try:
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False

def expnType(expn):

```

```

if debug:
    print (">>>>>Enter expnType, expn=", expn)
    print (">>>>>is_atom(expn)=", is_atom(expn))

if is_atom(expn):
    return 1
elif type(expn)==list: #instance(expn,list):
    return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
elif is_sqrt(expn):
    if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],Rational):
        return 1
    else:
        return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
elif expn.operator() == operator.pow: #instance(expn,Pow)
    if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
        return expnType(expn.operands()[0]) #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational: #instance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #instance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #instance(expn,Add) or instan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

```
#main function
```



```

def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = "none"
                else:
                    grade = "B"
                    grade_annotation = "Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation = "Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = "none"
            else:
                grade = "B"
                grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation = "Result contains higher order function than in optimal. Order "+str(expnType_resu

    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation

```