CAS integration tests. Progress report
Mathematica 13.1 vs. Mathematica 12.3

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1 Table summary of progress report

Table 1: Table summary of progress report

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
1	3	13	1 (pass)	0 (not solved)
2	5	60	1 (pass)	0 (not solved)
3	8	57	1 (pass)	0 (not solved)
4	11	12	1 (pass)	0 (not solved)
5	11	13	1 (pass)	0 (not solved)
6	11	37	1 (pass)	0 (not solved)
7	11	92	1 (pass)	0 (not solved)
8	11	100	1 (pass)	0 (not solved)
9	11	102	1 (pass)	0 (not solved)
10	25	2963	1 (pass)	0 (not solved)
11	25	2973	1 (pass)	0 (not solved)
12	25	3053	1 (pass)	0 (not solved)
13	25	3054	1 (pass)	0 (not solved)
14	25	3055	1 (pass)	0 (not solved)
15	25	3056	1 (pass)	0 (not solved)
16	25	3057	1 (pass)	0 (not solved)
17	25	3058	1 (pass)	0 (not solved)
18	25	3059	1 (pass)	0 (not solved)
19	25	3061	1 (pass)	0 (not solved)
20	25	3062	1 (pass)	0 (not solved)
21	25	3063	1 (pass)	0 (not solved)
22	25	3065	1 (pass)	0 (not solved)
23	25	3066	1 (pass)	0 (not solved)
24	25	3067	1 (pass)	0 (not solved)
25	37	143	1 (pass)	0 (not solved)
26	40	238	1 (pass)	-1 (time out)
27	40	239	1 (pass)	-1 (time out)

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#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
28	40	240	1 (pass)	-1 (time out)
29	40	293	1 (pass)	-1 (time out)
30	40	294	1 (pass)	-1 (time out)
31	40	295	1 (pass)	-1 (time out)
32	40	307	1 (pass)	-1 (time out)
33	40	308	1 (pass)	-1 (time out)
34	40	309	1 (pass)	-1 (time out)
35	40	310	1 (pass)	-1 (time out)
36	40	311	1 (pass)	-1 (time out)
37	40	324	1 (pass)	-1 (time out)
38	40	325	1 (pass)	-1 (time out)
39	40	326	1 (pass)	-1 (time out)
40	40	327	1 (pass)	-1 (time out)
41	40	339	1 (pass)	-1 (time out)
42	40	340	1 (pass)	-1 (time out)
43	40	341	1 (pass)	-1 (time out)
44	40	342	1 (pass)	-1 (time out)
45	40	343	1 (pass)	-1 (time out)
46	40	344	1 (pass)	-1 (time out)
47	40	356	1 (pass)	-1 (time out)
48	40	357	1 (pass)	-1 (time out)
49	40	358	1 (pass)	-1 (time out)
50	40	359	1 (pass)	-1 (time out)
51	40	372	1 (pass)	-1 (time out)
52	40	373	1 (pass)	-1 (time out)
53	40	374	1 (pass)	-1 (time out)
54	40	375	1 (pass)	-1 (time out)
55	41	165	1 (pass)	-1 (time out)
56	41	201	1 (pass)	-1 (time out)

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#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
57	41	208	1 (pass)	-1 (time out)
58	41	210	1 (pass)	-1 (time out)
59	41	216	1 (pass)	-1 (time out)
60	41	218	1 (pass)	-1 (time out)
61	42	103	1 (pass)	-1 (time out)
62	42	104	1 (pass)	-1 (time out)
63	42	106	1 (pass)	-1 (time out)
64	42	107	1 (pass)	-1 (time out)
65	42	108	1 (pass)	-1 (time out)
66	42	109	1 (pass)	-1 (time out)
67	42	110	1 (pass)	-1 (time out)
68	42	111	1 (pass)	-1 (time out)
69	44	26	1 (pass)	-1 (time out)
70	44	27	1 (pass)	-1 (time out)
71	49	10	1 (pass)	-1 (time out)
72	49	14	1 (pass)	-1 (time out)
73	49	15	1 (pass)	-1 (time out)
74	52	18	1 (pass)	0 (not solved)
75	52	172	1 (pass)	0 (not solved)
76	52	913	1 (pass)	0 (not solved)
77	52	995	1 (pass)	0 (not solved)
78	52	1017	1 (pass)	0 (not solved)
79	78	4	1 (pass)	-1 (time out)
80	78	21	1 (pass)	-1 (time out)
81	89	373	1 (pass)	-1 (time out)
82	92	523	1 (pass)	-1 (time out)
83	92	530	1 (pass)	-1 (time out)
84	94	1340	1 (pass)	-1 (time out)
85	120	332	1 (pass)	-1 (time out)

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#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
86	125	81	1 (pass)	-1 (time out)
87	144	84	1 (pass)	0 (not solved)
88	147	20	1 (pass)	0 (not solved)
89	193	18	1 (pass)	0 (not solved)
90	208	183	1 (pass)	0 (not solved)
91	209	63	1 (pass)	0 (not solved)
92	209	64	1 (pass)	0 (not solved)
93	209	99	1 (pass)	0 (not solved)
94	209	119	1 (pass)	0 (not solved)
95	209	190	1 (pass)	0 (not solved)
96	209	243	1 (pass)	0 (not solved)
97	209	244	1 (pass)	0 (not solved)
98	209	245	1 (pass)	0 (not solved)
99	209	246	1 (pass)	0 (not solved)
100	209	288	1 (pass)	0 (not solved)
101	209	293	1 (pass)	0 (not solved)
102	209	343	1 (pass)	0 (not solved)
103	209	351	1 (pass)	0 (not solved)
104	209	354	1 (pass)	0 (not solved)
105	209	355	1 (pass)	0 (not solved)
106	209	371	1 (pass)	0 (not solved)
107	209	407	1 (pass)	0 (not solved)
108	209	439	1 (pass)	0 (not solved)
109	209	447	1 (pass)	0 (not solved)
110	209	469	1 (pass)	0 (not solved)
111	209	501	1 (pass)	0 (not solved)
112	209	509	1 (pass)	0 (not solved)
113	209	525	1 (pass)	0 (not solved)
114	209	526	1 (pass)	0 (not solved)

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#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
115	209	531	1 (pass)	0 (not solved)
116	209	561	1 (pass)	0 (not solved)
117	209	575	1 (pass)	0 (not solved)
118	209	582	1 (pass)	0 (not solved)
119	209	592	1 (pass)	0 (not solved)
120	209	600	1 (pass)	0 (not solved)
121	209	602	1 (pass)	0 (not solved)
122	209	619	1 (pass)	0 (not solved)
123	209	625	1 (pass)	0 (not solved)
124	209	631	1 (pass)	0 (not solved)
125	209	632	1 (pass)	0 (not solved)
126	209	633	1 (pass)	0 (not solved)
127	209	641	1 (pass)	0 (not solved)
128	209	647	1 (pass)	0 (not solved)
129	209	648	1 (pass)	0 (not solved)
130	209	650	1 (pass)	0 (not solved)
131	209	692	1 (pass)	0 (not solved)
132	209	715	1 (pass)	0 (not solved)
133	209	716	1 (pass)	0 (not solved)
134	209	720	1 (pass)	0 (not solved)
135	209	727	1 (pass)	0 (not solved)
136	209	741	1 (pass)	0 (not solved)
137	209	742	1 (pass)	0 (not solved)
138	209	765	1 (pass)	0 (not solved)
139	209	766	1 (pass)	0 (not solved)
140	209	767	1 (pass)	0 (not solved)
141	209	768	1 (pass)	0 (not solved)
142	209	769	1 (pass)	0 (not solved)
143	209	770	1 (pass)	0 (not solved)

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#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
144	209	771	1 (pass)	0 (not solved)
145	209	772	1 (pass)	0 (not solved)
146	209	773	1 (pass)	0 (not solved)
147	209	774	1 (pass)	0 (not solved)
148	209	800	1 (pass)	0 (not solved)
149	209	801	1 (pass)	0 (not solved)
150	209	802	1 (pass)	0 (not solved)
151	209	807	1 (pass)	0 (not solved)
152	209	808	1 (pass)	0 (not solved)
153	209	809	1 (pass)	0 (not solved)
154	209	810	1 (pass)	0 (not solved)
155	209	811	1 (pass)	0 (not solved)
156	209	822	1 (pass)	0 (not solved)
157	209	840	1 (pass)	0 (not solved)
158	209	858	1 (pass)	0 (not solved)
159	209	859	1 (pass)	0 (not solved)
160	209	860	1 (pass)	0 (not solved)
161	209	861	1 (pass)	0 (not solved)
162	209	864	1 (pass)	0 (not solved)
163	209	865	1 (pass)	0 (not solved)
164	209	872	1 (pass)	0 (not solved)
165	209	882	1 (pass)	0 (not solved)
166	209	883	1 (pass)	0 (not solved)
167	209	884	1 (pass)	0 (not solved)
168	209	885	1 (pass)	0 (not solved)
169	209	886	1 (pass)	0 (not solved)
170	209	899	1 (pass)	0 (not solved)
171	209	902	1 (pass)	0 (not solved)
172	209	914	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
173	209	923	1 (pass)	0 (not solved)
174	209	924	1 (pass)	0 (not solved)
175	209	928	1 (pass)	0 (not solved)
176	209	937	1 (pass)	0 (not solved)
177	209	938	1 (pass)	0 (not solved)
178	209	939	1 (pass)	0 (not solved)
179	209	940	1 (pass)	0 (not solved)
180	209	941	1 (pass)	0 (not solved)
181	209	950	1 (pass)	0 (not solved)
182	209	954	1 (pass)	0 (not solved)
183	209	958	1 (pass)	0 (not solved)
184	209	960	1 (pass)	0 (not solved)
185	209	964	1 (pass)	0 (not solved)
186	209	965	1 (pass)	0 (not solved)
187	209	976	1 (pass)	0 (not solved)
188	209	987	1 (pass)	0 (not solved)
189	209	988	1 (pass)	0 (not solved)
190	209	995	1 (pass)	0 (not solved)
191	209	1023	1 (pass)	0 (not solved)
192	209	1024	1 (pass)	0 (not solved)
193	209	1031	1 (pass)	0 (not solved)
194	209	1037	1 (pass)	0 (not solved)
195	209	1038	1 (pass)	0 (not solved)
196	209	1039	1 (pass)	0 (not solved)
197	209	1040	1 (pass)	0 (not solved)
198	209	1041	1 (pass)	0 (not solved)
199	209	1052	1 (pass)	0 (not solved)
200	209	1053	1 (pass)	0 (not solved)
201	209	1054	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
202	209	1055	1 (pass)	0 (not solved)
203	209	1060	1 (pass)	0 (not solved)
204	209	1064	1 (pass)	0 (not solved)
205	209	1067	1 (pass)	0 (not solved)
206	209	1068	1 (pass)	0 (not solved)
207	209	1069	1 (pass)	0 (not solved)
208	209	1070	1 (pass)	0 (not solved)
209	209	1071	1 (pass)	0 (not solved)
210	209	1078	1 (pass)	0 (not solved)
211	209	1079	1 (pass)	0 (not solved)
212	209	1080	1 (pass)	0 (not solved)
213	209	1082	1 (pass)	0 (not solved)
214	209	1090	1 (pass)	0 (not solved)
215	209	1096	1 (pass)	0 (not solved)
216	209	1108	1 (pass)	0 (not solved)
217	209	1110	1 (pass)	0 (not solved)
218	209	1114	1 (pass)	0 (not solved)
219	209	1124	1 (pass)	0 (not solved)
220	209	1135	1 (pass)	0 (not solved)
221	209	1139	1 (pass)	0 (not solved)
222	209	1143	1 (pass)	0 (not solved)
223	209	1151	1 (pass)	0 (not solved)
224	209	1153	1 (pass)	0 (not solved)
225	209	1158	1 (pass)	0 (not solved)
226	209	1168	1 (pass)	0 (not solved)
227	209	1169	1 (pass)	0 (not solved)
228	209	1170	1 (pass)	0 (not solved)
229	209	1171	1 (pass)	0 (not solved)
230	209	1172	1 (pass)	0 (not solved)

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#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
231	209	1173	1 (pass)	0 (not solved)
232	209	1174	1 (pass)	0 (not solved)
233	209	1182	1 (pass)	0 (not solved)
234	209	1183	1 (pass)	0 (not solved)
235	209	1191	1 (pass)	0 (not solved)
236	209	1197	1 (pass)	0 (not solved)
237	209	1198	1 (pass)	0 (not solved)
238	209	1209	1 (pass)	0 (not solved)
239	209	1212	1 (pass)	0 (not solved)
240	209	1225	1 (pass)	0 (not solved)
241	209	1234	1 (pass)	0 (not solved)
242	209	1235	1 (pass)	0 (not solved)
243	209	1236	1 (pass)	0 (not solved)
244	209	1237	1 (pass)	0 (not solved)
245	209	1238	1 (pass)	0 (not solved)
246	209	1243	1 (pass)	0 (not solved)
247	209	1244	1 (pass)	0 (not solved)
248	209	1248	1 (pass)	0 (not solved)
249	209	1249	1 (pass)	0 (not solved)
250	209	1250	1 (pass)	0 (not solved)
251	209	1253	1 (pass)	0 (not solved)
252	209	1254	1 (pass)	0 (not solved)
253	209	1255	1 (pass)	0 (not solved)
254	209	1260	1 (pass)	0 (not solved)
255	209	1264	1 (pass)	0 (not solved)
256	209	1277	1 (pass)	0 (not solved)
257	209	1283	1 (pass)	0 (not solved)
258	209	1285	1 (pass)	0 (not solved)
259	209	1288	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
260	209	1300	1 (pass)	0 (not solved)
261	209	1307	1 (pass)	0 (not solved)
262	209	1308	1 (pass)	0 (not solved)
263	209	1309	1 (pass)	0 (not solved)
264	209	1310	1 (pass)	0 (not solved)
265	209	1311	1 (pass)	0 (not solved)
266	209	1315	1 (pass)	0 (not solved)
267	209	1317	1 (pass)	0 (not solved)
268	209	1318	1 (pass)	0 (not solved)
269	209	1320	1 (pass)	0 (not solved)
270	209	1321	1 (pass)	0 (not solved)
271	209	1322	1 (pass)	0 (not solved)
272	209	1323	1 (pass)	0 (not solved)
273	209	1324	1 (pass)	0 (not solved)
274	209	1325	1 (pass)	0 (not solved)
275	209	1332	1 (pass)	0 (not solved)
276	209	1334	1 (pass)	0 (not solved)
277	209	1335	1 (pass)	0 (not solved)
278	209	1336	1 (pass)	0 (not solved)
279	209	1353	1 (pass)	0 (not solved)
280	209	1366	1 (pass)	0 (not solved)
281	209	1367	1 (pass)	0 (not solved)
282	209	1368	1 (pass)	0 (not solved)
283	209	1390	1 (pass)	0 (not solved)
284	209	1393	1 (pass)	0 (not solved)
285	209	1394	1 (pass)	0 (not solved)
286	209	1399	1 (pass)	0 (not solved)
287	209	1402	1 (pass)	0 (not solved)
288	209	1419	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
289	209	1425	1 (pass)	0 (not solved)
290	209	1426	1 (pass)	0 (not solved)
291	209	1428	1 (pass)	0 (not solved)
292	209	1429	1 (pass)	0 (not solved)
293	209	1430	1 (pass)	0 (not solved)
294	209	1431	1 (pass)	0 (not solved)
295	209	1443	1 (pass)	0 (not solved)
296	209	1444	1 (pass)	0 (not solved)
297	209	1449	1 (pass)	0 (not solved)
298	209	1450	1 (pass)	0 (not solved)
299	209	1455	1 (pass)	0 (not solved)
300	209	1475	1 (pass)	0 (not solved)
301	209	1483	1 (pass)	0 (not solved)
302	209	1495	1 (pass)	0 (not solved)
303	209	1496	1 (pass)	0 (not solved)
304	209	1501	1 (pass)	0 (not solved)
305	209	1504	1 (pass)	0 (not solved)
306	209	1507	1 (pass)	0 (not solved)
307	209	1508	1 (pass)	0 (not solved)
308	209	1509	1 (pass)	0 (not solved)
309	209	1510	1 (pass)	0 (not solved)
310	209	1511	1 (pass)	0 (not solved)
311	209	1513	1 (pass)	0 (not solved)
312	209	1516	1 (pass)	0 (not solved)
313	209	1517	1 (pass)	0 (not solved)
314	209	1520	1 (pass)	0 (not solved)
315	209	1521	1 (pass)	0 (not solved)
316	209	1522	1 (pass)	0 (not solved)
317	209	1524	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
318	209	1525	1 (pass)	0 (not solved)
319	209	1532	1 (pass)	0 (not solved)
320	209	1533	1 (pass)	0 (not solved)
321	209	1536	1 (pass)	0 (not solved)
322	209	1537	1 (pass)	0 (not solved)
323	209	1539	1 (pass)	0 (not solved)
324	209	1540	1 (pass)	0 (not solved)
325	209	1541	1 (pass)	0 (not solved)
326	209	1559	1 (pass)	0 (not solved)
327	209	1568	1 (pass)	0 (not solved)
328	209	1570	1 (pass)	0 (not solved)
329	209	1582	1 (pass)	0 (not solved)
330	209	1583	1 (pass)	0 (not solved)
331	209	1584	1 (pass)	0 (not solved)
332	209	1591	1 (pass)	0 (not solved)
333	209	1592	1 (pass)	0 (not solved)
334	209	1595	1 (pass)	0 (not solved)
335	209	1596	1 (pass)	0 (not solved)
336	209	1601	1 (pass)	0 (not solved)
337	209	1613	1 (pass)	0 (not solved)
338	209	1614	1 (pass)	0 (not solved)
339	209	1622	1 (pass)	0 (not solved)
340	209	1623	1 (pass)	0 (not solved)
341	209	1630	1 (pass)	0 (not solved)
342	209	1632	1 (pass)	0 (not solved)
343	209	1638	1 (pass)	0 (not solved)
344	209	1641	1 (pass)	0 (not solved)
345	209	1643	1 (pass)	0 (not solved)
346	209	1644	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
347	209	1654	1 (pass)	0 (not solved)
348	209	1659	1 (pass)	0 (not solved)
349	209	1676	1 (pass)	0 (not solved)
350	209	1677	1 (pass)	0 (not solved)
351	209	1678	1 (pass)	0 (not solved)
352	209	1689	1 (pass)	0 (not solved)
353	209	1690	1 (pass)	0 (not solved)
354	209	1691	1 (pass)	0 (not solved)
355	209	1692	1 (pass)	0 (not solved)
356	209	1693	1 (pass)	0 (not solved)
357	209	1694	1 (pass)	0 (not solved)
358	209	1699	1 (pass)	0 (not solved)
359	209	1700	1 (pass)	0 (not solved)
360	209	1706	1 (pass)	0 (not solved)
361	209	1710	1 (pass)	0 (not solved)
362	209	1711	1 (pass)	0 (not solved)
363	209	1712	1 (pass)	0 (not solved)
364	209	1713	1 (pass)	0 (not solved)
365	209	1719	1 (pass)	0 (not solved)
366	209	1727	1 (pass)	0 (not solved)
367	209	1728	1 (pass)	0 (not solved)
368	209	1737	1 (pass)	0 (not solved)
369	209	1741	1 (pass)	0 (not solved)
370	209	1742	1 (pass)	0 (not solved)
371	209	1750	1 (pass)	0 (not solved)
372	209	1751	1 (pass)	0 (not solved)
373	209	1752	1 (pass)	0 (not solved)
374	209	1754	1 (pass)	0 (not solved)
375	209	1755	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
376	209	1759	1 (pass)	0 (not solved)
377	209	1761	1 (pass)	0 (not solved)
378	209	1763	1 (pass)	0 (not solved)
379	209	1765	1 (pass)	0 (not solved)
380	209	1766	1 (pass)	0 (not solved)
381	209	1767	1 (pass)	0 (not solved)
382	209	1768	1 (pass)	0 (not solved)
383	209	1790	1 (pass)	0 (not solved)
384	209	1793	1 (pass)	0 (not solved)
385	209	1794	1 (pass)	0 (not solved)
386	209	1797	1 (pass)	0 (not solved)
387	209	1803	1 (pass)	0 (not solved)
388	209	1804	1 (pass)	0 (not solved)
389	209	1813	1 (pass)	0 (not solved)
390	209	1814	1 (pass)	0 (not solved)
391	209	1820	1 (pass)	0 (not solved)
392	209	1826	1 (pass)	0 (not solved)
393	209	1827	1 (pass)	0 (not solved)
394	209	1828	1 (pass)	0 (not solved)
395	209	1835	1 (pass)	0 (not solved)
396	209	1841	1 (pass)	0 (not solved)
397	209	1843	1 (pass)	0 (not solved)
398	209	1844	1 (pass)	0 (not solved)
399	209	1846	1 (pass)	0 (not solved)
400	209	1849	1 (pass)	0 (not solved)
401	209	1856	1 (pass)	0 (not solved)
402	209	1865	1 (pass)	0 (not solved)
403	209	1866	1 (pass)	0 (not solved)
404	209	1867	1 (pass)	0 (not solved)

Continued on next page

Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
405	209	1868	1 (pass)	0 (not solved)
406	209	1875	1 (pass)	0 (not solved)
407	209	1880	1 (pass)	0 (not solved)
408	209	1881	1 (pass)	0 (not solved)
409	209	1889	1 (pass)	0 (not solved)
410	209	1892	1 (pass)	0 (not solved)
411	209	1893	1 (pass)	0 (not solved)
412	209	1894	1 (pass)	0 (not solved)
413	209	1900	1 (pass)	0 (not solved)
414	209	1901	1 (pass)	0 (not solved)
415	209	1902	1 (pass)	0 (not solved)
416	209	1904	1 (pass)	0 (not solved)
417	209	1905	1 (pass)	0 (not solved)
418	209	1915	1 (pass)	0 (not solved)
419	209	1917	1 (pass)	0 (not solved)
420	209	1918	1 (pass)	0 (not solved)
421	209	1919	1 (pass)	0 (not solved)
422	209	1920	1 (pass)	0 (not solved)
423	209	1923	1 (pass)	0 (not solved)
424	209	1924	1 (pass)	0 (not solved)
425	209	1929	1 (pass)	0 (not solved)
426	209	1934	1 (pass)	0 (not solved)
427	209	1937	1 (pass)	0 (not solved)
428	209	1944	1 (pass)	0 (not solved)
429	209	1945	1 (pass)	0 (not solved)
430	209	1953	1 (pass)	0 (not solved)
431	209	1960	1 (pass)	0 (not solved)
432	209	1965	1 (pass)	0 (not solved)
433	209	1967	1 (pass)	0 (not solved)

Continued on next page

Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
434	209	1968	1 (pass)	0 (not solved)
435	209	1969	1 (pass)	0 (not solved)
436	209	1979	1 (pass)	0 (not solved)
437	209	1980	1 (pass)	0 (not solved)
438	209	1982	1 (pass)	0 (not solved)
439	209	1983	1 (pass)	0 (not solved)
440	209	1991	1 (pass)	-1 (time out)
441	209	1992	1 (pass)	-1 (time out)
442	209	1996	1 (pass)	0 (not solved)
443	209	2002	1 (pass)	0 (not solved)
444	209	2003	1 (pass)	0 (not solved)
445	209	2009	1 (pass)	0 (not solved)
446	209	2010	1 (pass)	0 (not solved)
447	209	2011	1 (pass)	0 (not solved)
448	209	2012	1 (pass)	0 (not solved)
449	209	2014	1 (pass)	0 (not solved)
450	209	2016	1 (pass)	0 (not solved)
451	209	2026	1 (pass)	0 (not solved)
452	209	2031	1 (pass)	0 (not solved)
453	209	2036	1 (pass)	0 (not solved)
454	209	2041	1 (pass)	0 (not solved)
455	209	2042	1 (pass)	0 (not solved)
456	209	2043	1 (pass)	0 (not solved)
457	209	2044	1 (pass)	0 (not solved)
458	209	2053	1 (pass)	0 (not solved)
459	209	2055	1 (pass)	0 (not solved)
460	209	2067	1 (pass)	0 (not solved)
461	209	2069	1 (pass)	0 (not solved)
462	209	2078	1 (pass)	0 (not solved)

Continued on next page

Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
463	209	2086	1 (pass)	0 (not solved)
464	209	2087	1 (pass)	0 (not solved)
465	209	2089	1 (pass)	0 (not solved)
466	209	2092	1 (pass)	0 (not solved)
467	209	2094	1 (pass)	0 (not solved)
468	209	2095	1 (pass)	0 (not solved)
469	209	2098	1 (pass)	0 (not solved)
470	209	2101	1 (pass)	0 (not solved)
471	209	2102	1 (pass)	0 (not solved)
472	209	2115	1 (pass)	0 (not solved)
473	209	2116	1 (pass)	0 (not solved)
474	209	2122	1 (pass)	0 (not solved)
475	209	2123	1 (pass)	0 (not solved)
476	209	2124	1 (pass)	0 (not solved)
477	209	2130	1 (pass)	0 (not solved)
478	209	2131	1 (pass)	0 (not solved)
479	209	2133	1 (pass)	0 (not solved)
480	209	2137	1 (pass)	0 (not solved)
481	209	2140	1 (pass)	0 (not solved)
482	209	2144	1 (pass)	0 (not solved)
483	209	2145	1 (pass)	0 (not solved)
484	209	2150	1 (pass)	0 (not solved)
485	209	2154	1 (pass)	0 (not solved)
486	209	2167	1 (pass)	0 (not solved)
487	209	2168	1 (pass)	0 (not solved)
488	209	2174	1 (pass)	0 (not solved)
489	209	2175	1 (pass)	0 (not solved)
490	209	2183	1 (pass)	0 (not solved)
491	209	2192	1 (pass)	0 (not solved)

Continued on next page

Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
492	209	2193	1 (pass)	0 (not solved)
493	209	2194	1 (pass)	0 (not solved)
494	209	2195	1 (pass)	0 (not solved)
495	209	2196	1 (pass)	0 (not solved)
496	209	2211	1 (pass)	0 (not solved)
497	209	2212	1 (pass)	0 (not solved)
498	209	2213	1 (pass)	0 (not solved)
499	209	2214	1 (pass)	0 (not solved)
500	209	2216	1 (pass)	0 (not solved)
501	209	2222	1 (pass)	-1 (time out)
502	209	2223	1 (pass)	0 (not solved)
503	209	2237	1 (pass)	0 (not solved)
504	209	2238	1 (pass)	0 (not solved)
505	209	2242	1 (pass)	0 (not solved)
506	209	2248	1 (pass)	0 (not solved)
507	209	2250	1 (pass)	0 (not solved)
508	209	2253	1 (pass)	0 (not solved)
509	209	2254	1 (pass)	0 (not solved)
510	209	2255	1 (pass)	0 (not solved)
511	209	2259	1 (pass)	0 (not solved)
512	209	2261	1 (pass)	0 (not solved)
513	209	2266	1 (pass)	0 (not solved)
514	209	2267	1 (pass)	0 (not solved)
515	209	2269	1 (pass)	0 (not solved)
516	209	2279	1 (pass)	0 (not solved)
517	209	2288	1 (pass)	0 (not solved)
518	209	2289	1 (pass)	0 (not solved)
519	209	2293	1 (pass)	0 (not solved)
520	209	2294	1 (pass)	0 (not solved)

Continued on next page

Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
521	209	2296	1 (pass)	0 (not solved)
522	209	2298	1 (pass)	0 (not solved)
523	209	2299	1 (pass)	0 (not solved)
524	209	2301	1 (pass)	0 (not solved)
525	209	2302	1 (pass)	0 (not solved)
526	209	2303	1 (pass)	-1 (time out)
527	209	2304	1 (pass)	0 (not solved)
528	209	2306	1 (pass)	0 (not solved)
529	209	2307	1 (pass)	0 (not solved)
530	209	2311	1 (pass)	0 (not solved)
531	209	2315	1 (pass)	0 (not solved)
532	209	2323	1 (pass)	0 (not solved)
533	209	2328	1 (pass)	0 (not solved)
534	209	2331	1 (pass)	0 (not solved)
535	209	2332	1 (pass)	0 (not solved)
536	209	2334	1 (pass)	0 (not solved)
537	209	2337	1 (pass)	0 (not solved)
538	209	2341	1 (pass)	0 (not solved)
539	209	2352	1 (pass)	0 (not solved)
540	209	2362	1 (pass)	0 (not solved)
541	209	2363	1 (pass)	0 (not solved)
542	209	2371	1 (pass)	0 (not solved)
543	209	2372	1 (pass)	0 (not solved)
544	209	2374	1 (pass)	0 (not solved)
545	209	2375	1 (pass)	0 (not solved)
546	209	2376	1 (pass)	0 (not solved)
547	209	2377	1 (pass)	0 (not solved)
548	209	2382	1 (pass)	0 (not solved)
549	209	2386	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
550	209	2387	1 (pass)	0 (not solved)
551	209	2389	1 (pass)	0 (not solved)
552	209	2390	1 (pass)	0 (not solved)
553	209	2393	1 (pass)	0 (not solved)
554	209	2397	1 (pass)	0 (not solved)
555	209	2409	1 (pass)	0 (not solved)
556	209	2410	1 (pass)	0 (not solved)
557	209	2419	1 (pass)	0 (not solved)
558	209	2429	1 (pass)	0 (not solved)
559	209	2430	1 (pass)	0 (not solved)
560	209	2432	1 (pass)	0 (not solved)
561	209	2434	1 (pass)	0 (not solved)
562	209	2437	1 (pass)	0 (not solved)
563	209	2455	1 (pass)	0 (not solved)
564	209	2467	1 (pass)	0 (not solved)
565	209	2468	1 (pass)	0 (not solved)
566	209	2469	1 (pass)	0 (not solved)
567	209	2474	1 (pass)	0 (not solved)
568	209	2475	1 (pass)	0 (not solved)
569	209	2476	1 (pass)	0 (not solved)
570	209	2482	1 (pass)	0 (not solved)
571	209	2483	1 (pass)	0 (not solved)
572	209	2492	1 (pass)	0 (not solved)
573	209	2493	1 (pass)	0 (not solved)
574	209	2494	1 (pass)	0 (not solved)
575	209	2495	1 (pass)	0 (not solved)
576	209	2497	1 (pass)	0 (not solved)
577	209	2504	1 (pass)	0 (not solved)
578	209	2505	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
579	209	2506	1 (pass)	0 (not solved)
580	209	2507	1 (pass)	0 (not solved)
581	209	2508	1 (pass)	0 (not solved)
582	209	2510	1 (pass)	0 (not solved)
583	209	2513	1 (pass)	0 (not solved)
584	209	2514	1 (pass)	0 (not solved)
585	209	2517	1 (pass)	0 (not solved)
586	209	2518	1 (pass)	0 (not solved)
587	209	2525	1 (pass)	0 (not solved)
588	209	2526	1 (pass)	0 (not solved)
589	209	2527	1 (pass)	0 (not solved)
590	209	2528	1 (pass)	0 (not solved)
591	209	2530	1 (pass)	0 (not solved)
592	209	2531	1 (pass)	0 (not solved)
593	209	2536	1 (pass)	-1 (time out)
594	209	2537	1 (pass)	0 (not solved)
595	209	2542	1 (pass)	0 (not solved)
596	209	2548	1 (pass)	0 (not solved)
597	209	2554	1 (pass)	0 (not solved)
598	209	2555	1 (pass)	0 (not solved)
599	209	2557	1 (pass)	0 (not solved)
600	209	2558	1 (pass)	0 (not solved)
601	209	2559	1 (pass)	0 (not solved)
602	209	2560	1 (pass)	0 (not solved)
603	209	2563	1 (pass)	0 (not solved)
604	209	2567	1 (pass)	0 (not solved)
605	209	2568	1 (pass)	0 (not solved)
606	209	2571	1 (pass)	0 (not solved)
607	209	2572	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
608	209	2586	1 (pass)	0 (not solved)
609	209	2589	1 (pass)	0 (not solved)
610	209	2590	1 (pass)	0 (not solved)
611	209	2593	1 (pass)	0 (not solved)
612	209	2596	1 (pass)	0 (not solved)
613	209	2598	1 (pass)	0 (not solved)
614	209	2600	1 (pass)	0 (not solved)
615	209	2601	1 (pass)	0 (not solved)
616	209	2602	1 (pass)	0 (not solved)
617	209	2606	1 (pass)	0 (not solved)
618	209	2608	1 (pass)	0 (not solved)
619	209	2614	1 (pass)	0 (not solved)
620	209	2615	1 (pass)	0 (not solved)
621	209	2622	1 (pass)	0 (not solved)
622	209	2627	1 (pass)	0 (not solved)
623	209	2634	1 (pass)	0 (not solved)
624	209	2644	1 (pass)	0 (not solved)
625	209	2645	1 (pass)	0 (not solved)
626	209	2646	1 (pass)	0 (not solved)
627	209	2647	1 (pass)	0 (not solved)
628	209	2654	1 (pass)	0 (not solved)
629	209	2655	1 (pass)	0 (not solved)
630	209	2658	1 (pass)	0 (not solved)
631	209	2659	1 (pass)	0 (not solved)
632	209	2662	1 (pass)	0 (not solved)
633	209	2664	1 (pass)	0 (not solved)
634	209	2665	1 (pass)	0 (not solved)
635	209	2666	1 (pass)	0 (not solved)
636	209	2675	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
637	209	2676	1 (pass)	0 (not solved)
638	209	2677	1 (pass)	0 (not solved)
639	209	2678	1 (pass)	0 (not solved)
640	209	2679	1 (pass)	0 (not solved)
641	209	2680	1 (pass)	0 (not solved)
642	209	2685	1 (pass)	0 (not solved)
643	209	2689	1 (pass)	0 (not solved)
644	209	2697	1 (pass)	-1 (time out)
645	209	2698	1 (pass)	0 (not solved)
646	209	2699	1 (pass)	0 (not solved)
647	209	2701	1 (pass)	0 (not solved)
648	209	2704	1 (pass)	0 (not solved)
649	209	2705	1 (pass)	0 (not solved)
650	209	2711	1 (pass)	0 (not solved)
651	209	2722	1 (pass)	0 (not solved)
652	209	2723	1 (pass)	0 (not solved)
653	209	2731	1 (pass)	0 (not solved)
654	209	2733	1 (pass)	0 (not solved)
655	209	2737	1 (pass)	0 (not solved)
656	209	2738	1 (pass)	0 (not solved)
657	209	2745	1 (pass)	0 (not solved)
658	209	2748	1 (pass)	0 (not solved)
659	209	2749	1 (pass)	0 (not solved)
660	209	2753	1 (pass)	0 (not solved)
661	209	2754	1 (pass)	0 (not solved)
662	209	2763	1 (pass)	0 (not solved)
663	209	2765	1 (pass)	0 (not solved)
664	209	2768	1 (pass)	0 (not solved)
665	209	2776	1 (pass)	0 (not solved)

Continued on next page

Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
666	209	2783	1 (pass)	0 (not solved)
667	209	2784	1 (pass)	0 (not solved)
668	209	2790	1 (pass)	0 (not solved)
669	209	2791	1 (pass)	0 (not solved)
670	209	2795	1 (pass)	0 (not solved)
671	209	2796	1 (pass)	0 (not solved)
672	209	2798	1 (pass)	0 (not solved)
673	209	2801	1 (pass)	0 (not solved)
674	209	2802	1 (pass)	0 (not solved)
675	209	2804	1 (pass)	0 (not solved)
676	209	2805	1 (pass)	0 (not solved)
677	209	2806	1 (pass)	0 (not solved)
678	209	2808	1 (pass)	0 (not solved)
679	209	2812	1 (pass)	0 (not solved)
680	209	2813	1 (pass)	0 (not solved)
681	209	2818	1 (pass)	0 (not solved)
682	209	2819	1 (pass)	0 (not solved)
683	209	2825	1 (pass)	0 (not solved)
684	209	2826	1 (pass)	0 (not solved)
685	209	2828	1 (pass)	0 (not solved)
686	209	2829	1 (pass)	0 (not solved)
687	209	2830	1 (pass)	0 (not solved)
688	209	2831	1 (pass)	0 (not solved)
689	209	2833	1 (pass)	0 (not solved)
690	209	2835	1 (pass)	0 (not solved)
691	209	2838	1 (pass)	0 (not solved)
692	209	2844	1 (pass)	0 (not solved)
693	209	2849	1 (pass)	0 (not solved)
694	209	2852	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
695	209	2855	1 (pass)	0 (not solved)
696	209	2869	1 (pass)	0 (not solved)
697	209	2870	1 (pass)	0 (not solved)
698	209	2875	1 (pass)	0 (not solved)
699	209	2883	1 (pass)	0 (not solved)
700	209	2885	1 (pass)	0 (not solved)
701	209	2892	1 (pass)	0 (not solved)
702	209	2894	1 (pass)	0 (not solved)
703	209	2896	1 (pass)	0 (not solved)
704	209	2901	1 (pass)	0 (not solved)
705	209	2902	1 (pass)	0 (not solved)
706	209	2905	1 (pass)	0 (not solved)
707	209	2906	1 (pass)	0 (not solved)
708	209	2912	1 (pass)	0 (not solved)
709	209	2913	1 (pass)	-1 (time out)
710	209	2919	1 (pass)	0 (not solved)
711	209	2934	1 (pass)	0 (not solved)
712	209	2940	1 (pass)	0 (not solved)
713	209	2941	1 (pass)	0 (not solved)
714	209	2942	1 (pass)	0 (not solved)
715	209	2947	1 (pass)	0 (not solved)
716	209	2948	1 (pass)	0 (not solved)
717	209	2953	1 (pass)	0 (not solved)
718	209	2960	1 (pass)	0 (not solved)
719	209	2962	1 (pass)	0 (not solved)
720	209	2963	1 (pass)	0 (not solved)
721	209	2969	1 (pass)	0 (not solved)
722	209	2976	1 (pass)	-1 (time out)
723	209	2977	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
724	209	2986	1 (pass)	-1 (time out)
725	209	2987	1 (pass)	0 (not solved)
726	209	2992	1 (pass)	0 (not solved)
727	209	2993	1 (pass)	0 (not solved)
728	209	2999	1 (pass)	0 (not solved)
729	209	3000	1 (pass)	0 (not solved)
730	209	3001	1 (pass)	0 (not solved)
731	209	3002	1 (pass)	0 (not solved)
732	209	3003	1 (pass)	0 (not solved)
733	209	3004	1 (pass)	0 (not solved)
734	209	3006	1 (pass)	0 (not solved)
735	209	3007	1 (pass)	0 (not solved)
736	209	3009	1 (pass)	0 (not solved)
737	209	3010	1 (pass)	0 (not solved)
738	209	3025	1 (pass)	0 (not solved)
739	209	3026	1 (pass)	0 (not solved)
740	209	3027	1 (pass)	0 (not solved)
741	209	3028	1 (pass)	0 (not solved)
742	209	3034	1 (pass)	-1 (time out)
743	209	3043	1 (pass)	0 (not solved)
744	209	3044	1 (pass)	0 (not solved)
745	209	3045	1 (pass)	0 (not solved)
746	209	3050	1 (pass)	0 (not solved)
747	209	3051	1 (pass)	0 (not solved)
748	209	3058	1 (pass)	0 (not solved)
749	209	3059	1 (pass)	0 (not solved)
750	209	3060	1 (pass)	0 (not solved)
751	209	3061	1 (pass)	0 (not solved)
752	209	3070	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
753	209	3079	1 (pass)	0 (not solved)
754	209	3081	1 (pass)	0 (not solved)
755	209	3082	1 (pass)	0 (not solved)
756	209	3085	1 (pass)	0 (not solved)
757	209	3086	1 (pass)	0 (not solved)
758	209	3090	1 (pass)	0 (not solved)
759	209	3095	1 (pass)	0 (not solved)
760	209	3099	1 (pass)	0 (not solved)
761	209	3102	1 (pass)	0 (not solved)
762	209	3108	1 (pass)	0 (not solved)
763	209	3111	1 (pass)	0 (not solved)
764	209	3112	1 (pass)	0 (not solved)
765	209	3113	1 (pass)	0 (not solved)
766	209	3114	1 (pass)	0 (not solved)
767	209	3117	1 (pass)	0 (not solved)
768	209	3120	1 (pass)	-1 (time out)
769	209	3128	1 (pass)	0 (not solved)
770	209	3132	1 (pass)	-1 (time out)
771	209	3139	1 (pass)	0 (not solved)
772	209	3141	1 (pass)	0 (not solved)
773	209	3142	1 (pass)	0 (not solved)
774	209	3143	1 (pass)	-1 (time out)
775	209	3145	1 (pass)	0 (not solved)
776	210	768	1 (pass)	0 (not solved)
777	210	4787	1 (pass)	0 (not solved)
778	210	8098	1 (pass)	0 (not solved)
779	210	8169	1 (pass)	0 (not solved)
780	210	8528	1 (pass)	0 (not solved)

2 Test file number 3

Test folder name:

`test_cases/0_Independent_test_suites/3_Bronstein_Problems`

2.1 Problem number 13

$$\int \frac{-x^2 - \log(x) + 2\log^2(x)}{-x^2\log(x) + \log^3(x)}\,dx$$

Optimal antiderivative

$$\text{logarithmicIntegral}\,(x) - \frac{\ln(x - \ln(x))}{2} + \frac{\ln(x + \ln(x))}{2}$$

command

```
Integrate[(-x^2 - Log[x] + 2*Log[x]^2)/(-(x^2*Log[x]) + Log[x]^3),x]
```

Mathematica 13.1 output

$$-\frac{1}{2}\log(x - \log(x)) + \frac{1}{2}\log(x + \log(x)) + \text{li}(x)$$

Mathematica 12.3 output

$$\int \frac{-x^2 - \log(x) + 2\log^2(x)}{-x^2\log(x) + \log^3(x)}\,dx$$

3 Test file number 5

Test folder name:

`test_cases/0_Independent_test_suites/5_Hearn_Problems`

3.1 Problem number 60

$$\int \frac{1}{\log(x)}\,dx$$

Optimal antiderivative

$$\text{logarithmicIntegral}\,(x)$$

command

```
Integrate[Log[x]^(-1),x]
```

Mathematica 13.1 output

$$\operatorname{li}(x)$$

Mathematica 12.3 output

$$\int \frac{1}{\log(x)}\,dx$$

4 Test file number 8

Test folder name:

`test_cases/0_Independent_test_suites/8_Moses_Problems`

4.1 Problem number 57

$$\int \frac{1}{\log(x)}\,dx$$

Optimal antiderivative

$$\operatorname{logarithmicIntegral}(x)$$

command

```
Integrate[Log[x]^(-1),x]
```

Mathematica 13.1 output

$$\operatorname{li}(x)$$

Mathematica 12.3 output

$$\int \frac{1}{\log(x)}\,dx$$

5 Test file number 11

Test folder name:

`test_cases/0_Independent_test_suites/11_Welz_Problems`

5.1 Problem number 12

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$$

Optimal antiderivative

$$-\frac{(1-i)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\frac{ix+1}{\sqrt{1-i}\,\sqrt{-ix^2+1}}}{\sqrt{1-i}\,\sqrt{-ix^2+1}}\right)}{4}$$
$$-\frac{(1+i)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\frac{-ix+1}{\sqrt{1+i}\,\sqrt{ix^2+1}}}{\sqrt{1+i}\,\sqrt{ix^2+1}}\right)}{4}-\frac{\sqrt{-ix^2+1}}{2\,(1+x)}-\frac{\sqrt{ix^2+1}}{2\,(1+x)}$$

command

```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

Mathematica 13.1 output

$$\frac{1}{2}\left(\frac{-1-2x^4-\sqrt{1+x^4}-x^2\left(1+2\sqrt{1+x^4}\right)}{(1+x)\left(x^2+\sqrt{1+x^4}\right)^{3/2}}+\frac{\tan^{-1}\left(\sqrt{1+\sqrt{2}}\,\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{-1+\sqrt{2}}}\right.$$
$$-\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)$$
$$-\frac{\tanh^{-1}\left(\sqrt{-1+\sqrt{2}}\,\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{1+\sqrt{2}}}$$
$$\left.+\sqrt{-1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$$

5.2 Problem number 13

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{ix+1}{\sqrt{1-i}\ \sqrt{-ix^2+1}}\right)\sqrt{1-i}}{2}-\frac{\operatorname{arctanh}\left(\frac{-ix+1}{\sqrt{1+i}\ \sqrt{ix^2+1}}\right)\sqrt{1+i}}{2}$$

command

```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{-1+\sqrt{2}}\left(\tan^{-1}\left(\sqrt{1+\sqrt{2}}\ \sqrt{x^2+\sqrt{1+x^4}}\right)-\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)-\sqrt{1}}{\sqrt{2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$

5.3 Problem number 37

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

Optimal antiderivative

$$-\frac{\ln\left((1-x)(1+x)^2\right)2^{\frac{2}{3}}}{8}+\frac{3\ln\left(-1+x+2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{8}$$
$$-\frac{\arctan\left(\frac{\left(1+\frac{2^{\frac{1}{3}}(1-x)}{\left(-x^3+1\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)\sqrt{3}\ 2^{\frac{2}{3}}}{4}$$

command

```
Integrate[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}\,x+\sqrt[3]{1-x^3}}\right)+2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}\,x+2\sqrt[3]{1-x^3}\right)-\log\left(2^{2/3}-2\,2^{2/3}x+2^{2/3}x^2-2(-\right.}{4\sqrt[3]{2}}$$

Mathematica 12.3 output

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

5.4 Problem number 92

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx$$

Optimal antiderivative

$$\ln(1+x)-\frac{3\ln\left(2+x-\left(x^3+2\right)^{\frac{1}{3}}\right)}{2}+\arctan\left(\frac{\left(1+\frac{4+2x}{\left(x^3+2\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)\sqrt{3}$$

command

```
Integrate[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$-\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{2+x^3}}{4+2x+\sqrt[3]{2+x^3}}\right)-\log\left(-2-x+\sqrt[3]{2+x^3}\right)$$
$$+\frac{1}{2}\log\left(4+4x+x^2+(2+x)\sqrt[3]{2+x^3}+\left(2+x^3\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx$$

5.5 Problem number 100

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$$

Optimal antiderivative

$$\frac{\ln\left(1+\frac{2^{\frac{2}{3}}(1-x)^2}{(-x^3+1)^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}(1-x)}{(-x^3+1)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{4}-\frac{\ln\left(1+\frac{2^{\frac{1}{3}}(1-x)}{(-x^3+1)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{2}+\frac{\arctan\left(\frac{\left(1-\frac{2\,2^{\frac{1}{3}}(1-x)}{(-x^3+1)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}}{2}$$

command

```
Integrate[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{-2\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}\,x+\sqrt[3]{1-x^3}}\right)-2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}\,x-\sqrt[3]{1-x^3}\right)+\log\left(2^{2/3}-2\,2^{2/3}x+2^{2/3}x^2+\right.}{2\sqrt[3]{2}}$$

Mathematica 12.3 output

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$$

5.6 Problem number 102

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx$$

Optimal antiderivative

$$-\frac{\ln\left(1+\frac{2^{\frac{2}{3}}(1+x)^2}{(x^3+1)^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}(1+x)}{(x^3+1)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{4}+\frac{\ln\left(1+\frac{2^{\frac{1}{3}}(1+x)}{(x^3+1)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{2}-\frac{\arctan\left(\frac{\left(1-\frac{2\,2^{\frac{1}{3}}(1+x)}{(x^3+1)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)\sqrt{3}\;2^{\frac{2}{3}}}{2}$$

command

```
Integrate[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{1+x^3}}{-2\sqrt[3]{2}-2\sqrt[3]{2}\,x+\sqrt[3]{1+x^3}}\right)+2\log\left(\sqrt[3]{2}+\sqrt[3]{2}\,x+\sqrt[3]{1+x^3}\right)-\log\left(2^{2/3}+2\,2^{2/3}x+2^{2/3}x^2-\sqrt[3]{2}\right.}{2\sqrt[3]{2}}$$

Mathematica 12.3 output

$$\int \frac{1-x}{(1+x+x^2)\,\sqrt[3]{1+x^3}}\,dx$$

6 Test file number 25

Test folder name:

`test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.3_General/25_1.1.3.2-c_x-^m-a+b_x^n-^p`

6.1 Problem number 2963

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^4}\,dx$$

Optimal antiderivative

$$\frac{b^2c\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c\,x^3}}}{\sqrt{a}}\right)}{6a^{\frac{3}{2}}}-\frac{\sqrt{a+b\sqrt{c\,x^3}}}{3x^3}-\frac{bc\sqrt{a+b\sqrt{c\,x^3}}}{6a\sqrt{c\,x^3}}$$

command

```
Integrate[Sqrt[a + b*Sqrt[c*x^3]]/x^4,x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{a+b\sqrt{cx^3}}\left(2a+b\sqrt{cx^3}\right)}{6ax^3}+\frac{b^2c\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^3}}}{\sqrt{a}}\right)}{6a^{3/2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^4}\,dx$$

6.2 Problem number 2973

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^{10}}\,dx$$

Optimal antiderivative

$$\frac{b^2c^3\operatorname{arctanh}\left(\frac{\sqrt{a+b\left(cx^3\right)^{\frac{3}{2}}}}{\sqrt{a}}\right)}{18a^{\frac{3}{2}}}-\frac{\sqrt{a+b\left(cx^3\right)^{\frac{3}{2}}}}{9x^9}-\frac{bc^3\sqrt{a+b\left(cx^3\right)^{\frac{3}{2}}}}{18a\left(cx^3\right)^{\frac{3}{2}}}$$

command

```
Integrate[Sqrt[a + b*(c*x^3)^(3/2)]/x^10,x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}\left(2a+b\left(cx^3\right)^{3/2}\right)}{18ax^9}+\frac{b^2c^3\tanh^{-1}\left(\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{\sqrt{a}}\right)}{18a^{3/2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^{10}}\,dx$$

6.3 Problem number 3053

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2\,dx$$

Optimal antiderivative

$$\frac{\left(-b^2d+4ac\right)\left(21b^4d^2-56a\,b^2cd+16a^2c^2\right)\operatorname{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\,\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\right)}{512a^{\frac{11}{2}}}$$
$$-\frac{3b\,d^3\left(a+\frac{c}{x}+b\sqrt{\frac{d}{x}}\right)^{\frac{3}{2}}}{10a^2\left(\frac{d}{x}\right)^{\frac{5}{2}}}+\frac{7b\,d^2\left(-15b^2d+28ac\right)\left(a+\frac{c}{x}+b\sqrt{\frac{d}{x}}\right)^{\frac{3}{2}}}{480a^4\left(\frac{d}{x}\right)^{\frac{3}{2}}}$$
$$-\frac{\left(-21b^2d+20ac\right)x^2\left(a+\frac{c}{x}+b\sqrt{\frac{d}{x}}\right)^{\frac{3}{2}}}{80a^3}+\frac{x^3\left(a+\frac{c}{x}+b\sqrt{\frac{d}{x}}\right)^{\frac{3}{2}}}{3a}$$
$$+\frac{\left(21b^4d^2-56a\,b^2cd+16a^2c^2\right)x\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{256a^5}$$

command

```
Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]*x^2,x]
```

Mathematica 13.1 output

$$\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(\sqrt{a}\,x\left(-210ab^3d\left(bd+8c\sqrt{\frac{d}{x}}\right)+315b^5d\left(\frac{d}{x}\right)^{3/2}x+1280a^5x^2+64a^4x\left(5c+2b\sqrt{\frac{d}{x}}\,x\right)-1\right.\right.$$

Mathematica 12.3 output

$$\int\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2\,dx$$

6.4 Problem number 3054

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\, x\, dx$$

Optimal antiderivative

$$-\frac{(-5b^2d+4ac)\,(-b^2d+4ac)\,\text{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\,\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\right)}{64a^{\frac{7}{2}}} - \frac{5b\,d^2\left(a+\frac{c}{x}+b\sqrt{\frac{d}{x}}\right)^{\frac{3}{2}}}{12a^2\left(\frac{d}{x}\right)^{\frac{3}{2}}} + \frac{x^2\left(a+\frac{c}{x}+b\sqrt{\frac{d}{x}}\right)^{\frac{3}{2}}}{2a} - \frac{(-5b^2d+4ac)\,x\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{32a^3}$$

command

```
Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]*x,x]
```

Mathematica 13.1 output

$$\frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(\sqrt{a}\,x\left(-2ab\left(5bd+26c\sqrt{\frac{d}{x}}\right)+15b^3d\sqrt{\frac{d}{x}}+48a^3x+8a^2\left(3c+b\sqrt{\frac{d}{x}}\,x\right)\right)+\frac{3\sqrt{d}\,(16a^2c^2-}{}\right.}{96a^{7/2}}$$

Mathematica 12.3 output

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\, x\, dx$$

6.5 Problem number 3055

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx$$

Optimal antiderivative

$$\frac{\left(-b^2d+4ac\right)\operatorname{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\,\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\right)}{4a^{\frac{3}{2}}}+\frac{x\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{2a}$$

command

```
Integrate[Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

Mathematica 13.1 output

$$\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(\sqrt{a}\left(2a+b\sqrt{\frac{d}{x}}\right)x+\frac{\sqrt{d}\left(-4ac+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{d}{x}}-\sqrt{\frac{d\left(c+ax+b\sqrt{\frac{d}{x}}\,x\right)}{x}}}{\sqrt{a}\sqrt{d}}\right)}{\sqrt{\frac{d\left(c+\left(a+b\sqrt{\frac{d}{x}}\right)x\right)}{x}}}\right)}{2a^{3/2}}$$

Mathematica 12.3 output

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx$$

6.6 Problem number 3056

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x}\,dx$$

Optimal antiderivative

$$2\operatorname{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\,\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\right)\sqrt{a} - \frac{b\operatorname{arctanh}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\,\sqrt{d}\,\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\right)\sqrt{d}}{\sqrt{c}} - 2\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}$$

command

```
Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]/x,x]
```

Mathematica 13.1 output

$$\frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(-2\sqrt{c}\,\sqrt{\frac{d\left(c+ax+b\sqrt{\frac{d}{x}}\,x\right)}{x}}+4\sqrt{a}\,\sqrt{c}\,\sqrt{d}\,\tanh^{-1}\left(\frac{-\sqrt{c}\,\sqrt{\frac{d}{x}}+\sqrt{\frac{d\left(c+ax+b\sqrt{\frac{d}{x}}\right.}{x}}}{\sqrt{a}\,\sqrt{d}}\right.\right.}{\sqrt{c}\,\sqrt{\frac{d\left(c+\left(a+b\sqrt{\frac{d}{x}}\right)x\right)}{x}}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x}\,dx$$

6.7 Problem number 3057

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx$$

Optimal antiderivative

$$\frac{b\left(-b^2d+4ac\right)\operatorname{arctanh}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\ \sqrt{d}\ \sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\right)\sqrt{d}}{8c^{\frac{5}{2}}}$$
$$-\frac{2\left(a+\frac{c}{x}+b\sqrt{\frac{d}{x}}\right)^{\frac{3}{2}}}{3c}+\frac{b\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{4c^2}$$

command

```
Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]/x^2,x]
```

Mathematica 13.1 output

$$\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(-\frac{2\sqrt{c}\left(8c^2-3b^2dx+2c\left(4a+b\sqrt{\frac{d}{x}}\right)x\right)}{x}+\frac{3bd\left(-4ac+b^2d\right)\log\left(c^2\left(bd+2c\sqrt{\frac{d}{x}}-2\sqrt{c}\sqrt{\frac{d\left(c+ax+b\sqrt{\right.}\right.}\right.}\right.}{x}\right.$$
$$\left.\left.\left.\left.\sqrt{\frac{d\left(c+\left(a+b\sqrt{\frac{d}{x}}\right)x\right)}{x}}\right.\right.\right.$$
$$\left.\left.\right)\right)$$
$$24c^{5/2}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx$$

6.8 Problem number 3058

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^3}\,dx$$

Optimal antiderivative

$$\begin{aligned}&-\frac{b\left(-7b^2d+12ac\right)\left(-b^2d+4ac\right)\operatorname{arctanh}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\ \sqrt{d}\ \sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\right)\sqrt{d}}{128c^{\frac{9}{2}}}\\&-\frac{2\left(a+\frac{c}{x}+b\sqrt{\frac{d}{x}}\right)^{\frac{3}{2}}}{5cx}+\frac{\left(a+\frac{c}{x}+b\sqrt{\frac{d}{x}}\right)^{\frac{3}{2}}\left(32ac-35b^2d+42bc\sqrt{\frac{d}{x}}\right)}{120c^3}\\&-\frac{b\left(-7b^2d+12ac\right)\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{64c^4}\end{aligned}$$

command

```
Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]/x^3,x]
```

Mathematica 13.1 output

$$\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(\frac{2\sqrt{c}\left(-384c^4-16c^3\left(8a+3b\sqrt{\frac{d}{x}}\right)x+105b^4d^2x^2-10b^2cd\left(46a+7b\sqrt{\frac{d}{x}}\right)x^2+8c^2x\left(7b^2d+32a^2x+29ab\sqrt{\frac{d}{x}}\,x\right)\right)}{x^2}\right.$$

$$\left.\vphantom{\frac{1}{1}}\right)\Bigg/1920c^{9/2}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^3}\,dx$$

6.9 Problem number 3059

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^4}\,dx$$

Optimal antiderivative

$$\frac{b\left(-b^2d+4ac\right)\left(33b^4d^2-120ab^2cd+80a^2c^2\right)\operatorname{arctanh}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\,\sqrt{d}\,\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\right)\sqrt{d}}{1024c^{\frac{13}{2}}}+\frac{11b\left(\frac{d}{x}\right)^{\frac{3}{2}}\left(a+\frac{c}{x}+b\sqrt{\frac{d}{x}}\right)^{\frac{3}{2}}}{42c^2d}-\frac{2\left(a+\frac{c}{x}+b\sqrt{\frac{d}{x}}\right)^{\frac{3}{2}}}{7cx^2}+\frac{\left(-33b^2d+32ac\right)\left(a+\frac{c}{x}+b\sqrt{\frac{d}{x}}\right)^{\frac{3}{2}}}{140c^3x}-\frac{\left(a+\frac{c}{x}+b\sqrt{\frac{d}{x}}\right)^{\frac{3}{2}}\left(1024a^2c^2-3276ab^2cd+1155b^4d^2+18bc\left(-77b^2d+148ac\right)\sqrt{\frac{d}{x}}\right)}{6720c^5}+\frac{b\left(33b^4d^2-120ab^2cd+80a^2c^2\right)\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{512c^6}$$

command

```
Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]/x^4,x]
```

Mathematica 13.1 output

$$\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(-\frac{2\sqrt{c}\left(15360c^6+256c^5\left(12a+5b\sqrt{\frac{d}{x}}\right)x-3465b^6d^3x^3+210b^4cd^2\left(104a+11b\sqrt{\frac{d}{x}}\right)x^3-168b^2c^2dx^2\left(11b^2d+206a\right.\right.\right.$$

Mathematica 12.3 output

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^4}\,dx$$

6.10 Problem number 3061

$$\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

Optimal antiderivative

$$-\frac{\left(-231b^6d^3+1260ab^4cd^2-1680a^2b^2c^2d+320a^3c^3\right)\operatorname{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\,\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\right)}{512a^{\frac{13}{2}}}$$
$$-\frac{11bd^3\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{30a^2\left(\frac{d}{x}\right)^{\frac{5}{2}}}+\frac{bd^2\left(-77b^2d+156ac\right)\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{160a^4\left(\frac{d}{x}\right)^{\frac{3}{2}}}$$
$$+\frac{\left(385b^4d^2-1176ab^2cd+400a^2c^2\right)x\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{640a^5}$$
$$-\frac{\left(-99b^2d+100ac\right)x^2\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{240a^3}+\frac{x^3\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{3a}$$
$$-\frac{7bd\left(165b^4d^2-680ab^2cd+528a^2c^2\right)\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{1280a^6\sqrt{\frac{d}{x}}}$$

command

```
Integrate[x^2/Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

Mathematica 13.1 output

$$\sqrt{a}\,d\left(-3465b^5d^2\left(bd+c\sqrt{\frac{d}{x}}\right)+1280a^6x^3-64a^5x^2\left(5c+2b\sqrt{\frac{d}{x}}\,x\right)+16a^4x\left(50c^2+11b^2dx+46bc\sqrt{\frac{d}{x}}\,x\right)-\right.$$

Mathematica 12.3 output

$$\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

6.11 Problem number 3062

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

Optimal antiderivative

$$\frac{\left(35b^4d^2-120a\,b^2cd+48a^2c^2\right)\operatorname{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\,\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\right)}{64a^{\frac{9}{2}}}$$
$$-\frac{7b\,d^2\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{12a^2\left(\frac{d}{x}\right)^{\frac{3}{2}}}-\frac{\left(-35b^2d+36ac\right)x\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{48a^3}$$
$$+\frac{x^2\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{2a}+\frac{5bd\left(-21b^2d+44ac\right)\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{96a^4\sqrt{\frac{d}{x}}}$$

command

```
Integrate[x/Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

Mathematica 13.1 output

$$\sqrt{a}\,d\left(-105b^3d\left(bd+c\sqrt{\frac{d}{x}}\right)+48a^4x^2-8a^3x\left(3c+b\sqrt{\frac{d}{x}}\,x\right)+a^2\left(-72c^2+14b^2dx+92bc\sqrt{\frac{d}{x}}\,x\right)-5ab\left(-5\right.\right.$$

Mathematica 12.3 output

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

6.12 Problem number 3063

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

Optimal antiderivative

$$-\frac{\left(-3b^2d+4ac\right)\operatorname{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\,\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\right)}{4a^{\frac{5}{2}}}+\frac{x\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{a}-\frac{3bd\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{2a^2\sqrt{\frac{d}{x}}}$$

command

```
Integrate[1/Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

Mathematica 13.1 output

$$\frac{\sqrt{a}\,d\left(2a-3b\sqrt{\frac{d}{x}}\right)\left(c+\left(a+b\sqrt{\frac{d}{x}}\right)x\right)+\sqrt{d}\,(4ac-3b^2d)\sqrt{\frac{d\left(c+\left(a+b\sqrt{\frac{d}{x}}\right)x\right)}{x}}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{\frac{d}{x}}}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{2a^{5/2}d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}$$

Mathematica 12.3 output

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

6.13 Problem number 3065

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2}\,dx$$

Optimal antiderivative

$$\frac{b\,\text{arctanh}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\,\sqrt{d}\,\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\right)\sqrt{d}}{c^{\frac{3}{2}}}-\frac{2\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{c}$$

command

```
Integrate[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x^2),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{\frac{d\left(c+\left(a+b\sqrt{\frac{d}{x}}\right)x\right)}{x}}\left(2\sqrt{c}\,\sqrt{\frac{d\left(c+ax+b\sqrt{\frac{d}{x}}\,x\right)}{x}}+bd\log\left(c\left(\left(bd+2c\sqrt{\frac{d}{x}}-2\sqrt{c}\,\sqrt{\frac{d\left(c+ax+\right.\right.\right.\right.}{x}}\right.\right.\right.\right.}{c^{3/2}d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}$$

Mathematica 12.3 output

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2}\,dx$$

6.14 Problem number 3066

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^3}\,dx$$

Optimal antiderivative

$$-\frac{b\left(-5b^2d+12ac\right)\operatorname{arctanh}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\,\sqrt{d}\,\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\right)\sqrt{d}}{8c^{\frac{7}{2}}}$$
$$-\frac{2\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{3cx}+\frac{\left(16ac-15b^2d+10bc\sqrt{\frac{d}{x}}\right)\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{12c^3}$$

command

```
Integrate[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x^3),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{c}\left(-8c^3+2c^2\left(4a+b\sqrt{\frac{d}{x}}\right)x-15b^2d\left(a+b\sqrt{\frac{d}{x}}\right)x^2+cx\left(-5b^2d+16a^2x+26ab\sqrt{\frac{d}{x}}\,x\right)\right)+3b\left(12ac-5\right.}{24c^{7/2}\sqrt{a+b\sqrt{\frac{d}{x}}+}}$$

Mathematica 12.3 output

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^3}\,dx$$

6.15 Problem number 3067

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^4}\,dx$$

Optimal antiderivative

$$\frac{b\left(63b^4d^2-280ab^2cd+240a^2c^2\right)\operatorname{arctanh}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\,\sqrt{d}\,\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\right)\sqrt{d}}{128c^{\frac{11}{2}}}$$
$$+\frac{9b\left(\frac{d}{x}\right)^{\frac{3}{2}}\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{20c^2d}-\frac{2\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{5cx^2}+\frac{\left(-63b^2d+64ac\right)\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{120c^3x}$$
$$-\frac{\left(1024a^2c^2-2940ab^2cd+945b^4d^2+14bc\left(-45b^2d+92ac\right)\sqrt{\frac{d}{x}}\right)\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{960c^5}$$

command

```
Integrate[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x^4),x]
```

Mathematica 13.1 output

$$-2\sqrt{c}\left(384c^5-16c^4\left(8a+3b\sqrt{\frac{d}{x}}\right)x+945b^4d^2\left(a+b\sqrt{\frac{d}{x}}\right)x^3-105b^2cdx^2\left(-3b^2d+28a^2x+34ab\sqrt{\frac{d}{x}}\,x\right)+$$

Mathematica 12.3 output

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^4}\,dx$$

7 Test file number 37

Test folder name:

`test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/37_1.2.1.6-g+h_x-^m-a+b_x+c_x^2-^p-d+e_x+f_x^2-^q`

7.1 Problem number 143

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx$$

Optimal antiderivative

$$-\frac{3\,3^{\frac{1}{6}}h\left(\frac{ch^2\left(\frac{(-2bh+cg)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(-bh+2cg)^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}}{3}+\frac{2^{\frac{2}{3}}\left(1-\frac{3h(2cx+b)}{-bh+2cg}\right)^{\frac{2}{3}}\sqrt{3}}{3\left(1+\frac{3h(2cx+b)}{-bh+2cg}\right)^{\frac{1}{3}}}\right)}{f\left(-\frac{(-2bh+cg)(bh+cg)}{ch^2}+9bx+9cx^2\right)^{\frac{1}{3}}}$$
$$+\frac{3^{\frac{2}{3}}h\left(\frac{ch^2\left(\frac{(-2bh+cg)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(-bh+2cg)^2}\right)^{\frac{1}{3}}\ln\left(\frac{f(b^2h^2-bcgh+c^2g^2)}{3c^2h^2}+\frac{bfx}{c}+fx^2\right)}{2f\left(-\frac{(-2bh+cg)(bh+cg)}{ch^2}+9bx+9cx^2\right)^{\frac{1}{3}}}$$
$$-\frac{3\,3^{\frac{2}{3}}h\left(\frac{ch^2\left(\frac{(-2bh+cg)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(-bh+2cg)^2}\right)^{\frac{1}{3}}\ln\left(\left(1-\frac{3h(2cx+b)}{-bh+2cg}\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(1+\frac{3h(2cx+b)}{-bh+2cg}\right)^{\frac{1}{3}}\right)}{2f\left(-\frac{(-2bh+cg)(bh+cg)}{ch^2}+9bx+9cx^2\right)^{\frac{1}{3}}}$$

command

```
Integrate[(g + h*x)/(((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x + c*x^2)^(1/3)*((f*(
(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 + (b*f*x)/c + f*x^2)),x]
```

Mathematica 13.1 output

$$\frac{3^{2/3}\sqrt[3]{c}\,h^{5/3}\left(2\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{c}\,h^{2/3}\sqrt[3]{2cg-bh}\,\sqrt[3]{\frac{2b^2}{c}-\frac{cg^2}{h^2}+\frac{bg}{h}+9bx+9cx^2}}{-4bh+2c(g-3hx)+\sqrt[3]{c}\,h^{2/3}\sqrt[3]{2cg-bh}\,\sqrt[3]{\frac{2b^2}{c}-\frac{cg^2}{h^2}+\frac{bg}{h}+9bx+9cx^2}}\right)-2\log\left(\sqrt{h}\,\sqrt[3]{}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx$$

8 Test file number 40

Test folder name:

`test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/40_1.2.2.3-d+e_x^2-^m-a+b_x^2+c_x^4-^p`

8.1 Problem number 238

$$\int \frac{\left(1+x^2\right)^3}{\left(1+x^2+x^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$-\frac{x\left(-x^2+1\right)}{3\sqrt{x^4+x^2+1}}+\frac{2x\sqrt{x^4+x^2+1}}{3\left(x^2+1\right)}$$
$$-\frac{2\left(x^2+1\right)\sqrt{\frac{\cos\left(4\arctan\left(x\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticE}\left(\sin\left(2\arctan\left(x\right)\right),\frac{1}{2}\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}}{3\cos\left(2\arctan\left(x\right)\right)\sqrt{x^4+x^2+1}}$$
$$+\frac{\left(x^2+1\right)\sqrt{\frac{\cos\left(4\arctan\left(x\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sin\left(2\arctan\left(x\right)\right),\frac{1}{2}\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}}{\cos\left(2\arctan\left(x\right)\right)\sqrt{x^4+x^2+1}}$$

command

```
Integrate[(1 + x^2)^3/(1 + x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{-x+x^3+2\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}\,x^2}\sqrt{1-(-1)^{2/3}x^2}\,E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+2(-1)^{5/6}\sqrt{3+3\sqrt[3]{-1}\,x^2}\sqrt{}}{3\sqrt{1+x^2+x^4}}$$

Mathematica 12.3 output

$Aborted

8.2 Problem number 239

$$\int \frac{\left(1+x^2\right)^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{x\left(2x^2+1\right)}{3\sqrt{x^4+x^2+1}}-\frac{2x\sqrt{x^4+x^2+1}}{3\left(x^2+1\right)}+\frac{2\left(x^2+1\right)\sqrt{\frac{\cos\left(4\arctan\left(x\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticE}\left(\sin\left(2\arctan\left(x\right)\right),\frac{1}{2}\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}}{3\cos\left(2\arctan\left(x\right)\right)\sqrt{x^4+x^2+1}}$$

command

```
Integrate[(1 + x^2)^2/(1 + x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{x+2x^3-2\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}\,x^2}\sqrt{1-(-1)^{2/3}x^2}\,E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-i\sqrt{2+\left(1+i\sqrt{3}\right)x^2}\sqrt{6+}}{3\sqrt{1+x^2+x^4}}$$

Mathematica 12.3 output

$Aborted

8.3 Problem number 240

$$\int \frac{1+x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{x\left(x^2+2\right)}{3\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{3\left(x^2+1\right)}+\frac{\left(x^2+1\right)\sqrt{\frac{\cos\left(4\arctan\left(x\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticE}\left(\sin\left(2\arctan\left(x\right)\right),\frac{1}{2}\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}}{3\cos\left(2\arctan\left(x\right)\right)\sqrt{x^4+x^2+1}}$$

command

```
Integrate[(1 + x^2)/(1 + x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{2x+x^3-\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-\frac{1}{2}i\sqrt{2+\left(1+i\sqrt{3}\right)x^2}\sqrt{6+}}{3\sqrt{1+x^2+x^4}}$$

Mathematica 12.3 output

$Aborted

8.4 Problem number 293

$$\int \left(7+5x^2\right)^3\left(2+3x^2+x^4\right)^{3/2}\,dx$$

Optimal antiderivative

$$\begin{aligned}&\frac{x\left(65345x^2+208212\right)\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{3003}+\frac{3825x\left(x^4+3x^2+2\right)^{\frac{5}{2}}}{143}\\&+\frac{125x^3\left(x^4+3x^2+2\right)^{\frac{5}{2}}}{13}+\frac{20884x\left(x^2+2\right)}{65\sqrt{x^4+3x^2+2}}\\&-\frac{20884\left(x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{1}{x^2+1}}\,\text{EllipticE}\left(\frac{x}{\sqrt{x^2+1}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{\frac{x^2+2}{x^2+1}}}{65\sqrt{x^4+3x^2+2}}\\&+\frac{1171349\left(x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{1}{x^2+1}}\,\text{EllipticF}\left(\frac{x}{\sqrt{x^2+1}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{\frac{x^2+2}{x^2+1}}}{5005\sqrt{x^4+3x^2+2}}\\&+\frac{x\left(297911x^2+1032541\right)\sqrt{x^4+3x^2+2}}{5005}\end{aligned}$$

command

```
Integrate[(7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{13572486x+40493455x^3+54938052x^5+46218643x^7+25350660x^9+8705725x^{11}+1701000x^{13}+144375x^{15}-4}{15015\sqrt{2+3x}}$$

Mathematica 12.3 output

$Aborted

8.5 Problem number 294

$$\int \left(7+5x^2\right)^2 \left(2+3x^2+x^4\right)^{3/2}\, dx$$

Optimal antiderivative

$$\frac{x\left(2240x^2+7281\right)\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{693}+\frac{25x\left(x^4+3x^2+2\right)^{\frac{5}{2}}}{11}+\frac{742x\left(x^2+2\right)}{15\sqrt{x^4+3x^2+2}}$$
$$-\frac{742\left(x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{1}{x^2+1}}\ \text{EllipticE}\left(\frac{x}{\sqrt{x^2+1}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\ \sqrt{\frac{x^2+2}{x^2+1}}}{15\sqrt{x^4+3x^2+2}}$$
$$+\frac{13879\left(x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{1}{x^2+1}}\ \text{EllipticF}\left(\frac{x}{\sqrt{x^2+1}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\ \sqrt{\frac{x^2+2}{x^2+1}}}{385\sqrt{x^4+3x^2+2}}$$
$$+\frac{x\left(10643x^2+36783\right)\sqrt{x^4+3x^2+2}}{1155}$$

command

```
Integrate[(7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{429318x+1160065x^3+1333551x^5+892084x^7+363480x^9+82075x^{11}+7875x^{13}-171402i\sqrt{1+x^2}\ \sqrt{2+x^2}\ E}{3465\sqrt{2+3x^2+x^4}}$$

Mathematica 12.3 output

$Aborted

8.6 Problem number 295

$$\int \left(7+5x^2\right)\left(2+3x^2+x^4\right)^{3/2}\, dx$$

Optimal antiderivative

$$\frac{x\left(35x^2+108\right)\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{63}+\frac{116x\left(x^2+2\right)}{15\sqrt{x^4+3x^2+2}}$$
$$-\frac{116\left(x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{1}{x^2+1}}\ \text{EllipticE}\left(\frac{x}{\sqrt{x^2+1}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\ \sqrt{\frac{x^2+2}{x^2+1}}}{15\sqrt{x^4+3x^2+2}}$$
$$+\frac{197\left(x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{1}{x^2+1}}\ \text{EllipticF}\left(\frac{x}{\sqrt{x^2+1}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\ \sqrt{\frac{x^2+2}{x^2+1}}}{35\sqrt{x^4+3x^2+2}}$$
$$+\frac{x\left(149x^2+519\right)\sqrt{x^4+3x^2+2}}{105}$$

command

```
Integrate[(7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{5274x+12745x^3+12018x^5+5962x^7+1590x^9+175x^{11}-2436i\sqrt{1+x^2}\ \sqrt{2+x^2}\ E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-11}{315\sqrt{2+3x^2+x^4}}$$

Mathematica 12.3 output

$Aborted

8.7 Problem number 307

$$\int\frac{\left(7+5x^2\right)^5}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{7679x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}-\frac{x\left(179x^2+115\right)}{2\sqrt{x^4+3x^2+2}}$$
$$-\frac{7679\left(x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{1}{x^2+1}}\ \text{EllipticE}\left(\frac{x}{\sqrt{x^2+1}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\ \sqrt{\frac{x^2+2}{x^2+1}}}{2\sqrt{x^4+3x^2+2}}$$
$$+\frac{15383\left(x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{1}{x^2+1}}\ \text{EllipticF}\left(\frac{x}{\sqrt{x^2+1}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\ \sqrt{\frac{x^2+2}{x^2+1}}}{6\sqrt{x^4+3x^2+2}}$$
$$+\frac{5000x\sqrt{x^4+3x^2+2}}{3}+625x^3\sqrt{x^4+3x^2+2}$$

command

```
Integrate[(7 + 5*x^2)^5/(2 + 3*x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{19655x + 36963x^3 + 21250x^5 + 3750x^7 - 23037i\sqrt{1+x^2}\,\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 7729i\sqrt{1+x^2}\,\sqrt{2}}{6\sqrt{2+3x^2+x^4}}$$

Mathematica 12.3 output

$Aborted

8.8 Problem number 308

$$\int \frac{\left(7+5x^2\right)^4}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\begin{aligned}
&\frac{637x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}} + \frac{x\left(113x^2+145\right)}{2\sqrt{x^4+3x^2+2}} \\
&- \frac{637\left(x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{1}{x^2+1}}\ \text{EllipticE}\left(\frac{x}{\sqrt{x^2+1}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\ \sqrt{\frac{x^2+2}{x^2+1}}}{2\sqrt{x^4+3x^2+2}} \\
&+ \frac{1067\left(x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{1}{x^2+1}}\ \text{EllipticF}\left(\frac{x}{\sqrt{x^2+1}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\ \sqrt{\frac{x^2+2}{x^2+1}}}{3\sqrt{x^4+3x^2+2}} \\
&+ \frac{625x\sqrt{x^4+3x^2+2}}{3}
\end{aligned}$$

command

```
Integrate[(7 + 5*x^2)^4/(2 + 3*x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{2935x + 4089x^3 + 1250x^5 - 1911i\sqrt{1+x^2}\,\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 2357i\sqrt{1+x^2}\,\sqrt{2+x^2}\,F\left(i\sinh\right.}{6\sqrt{2+3x^2+x^4}}$$

Mathematica 12.3 output

$Aborted

8.9 Problem number 309

$$\int \frac{\left(7+5x^2\right)^3}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\begin{aligned}&\frac{x\left(-11x^2+5\right)}{2\sqrt{x^4+3x^2+2}}+\frac{261x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}\\&-\frac{261\left(x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{1}{x^2+1}}\ \text{EllipticE}\left(\frac{x}{\sqrt{x^2+1}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\ \sqrt{\frac{x^2+2}{x^2+1}}}{2\sqrt{x^4+3x^2+2}}\\&+\frac{169\left(x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{1}{x^2+1}}\ \text{EllipticF}\left(\frac{x}{\sqrt{x^2+1}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\ \sqrt{\frac{x^2+2}{x^2+1}}}{2\sqrt{x^4+3x^2+2}}\end{aligned}$$

command

```
Integrate[(7 + 5*x^2)^3/(2 + 3*x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$-\frac{-5x+11x^3+261i\sqrt{1+x^2}\ \sqrt{2+x^2}\ E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+77i\sqrt{1+x^2}\ \sqrt{2+x^2}\ F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{2\sqrt{2+3x^2+x^4}}$$

Mathematica 12.3 output

$Aborted

8.10 Problem number 310

$$\int \frac{\left(7+5x^2\right)^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\begin{aligned}&-\frac{17x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{x\left(17x^2+25\right)}{2\sqrt{x^4+3x^2+2}}\\&+\frac{17\left(x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{1}{x^2+1}}\ \text{EllipticE}\left(\frac{x}{\sqrt{x^2+1}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\ \sqrt{\frac{x^2+2}{x^2+1}}}{2\sqrt{x^4+3x^2+2}}\\&+\frac{6\left(x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{1}{x^2+1}}\ \text{EllipticF}\left(\frac{x}{\sqrt{x^2+1}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\ \sqrt{\frac{x^2+2}{x^2+1}}}{\sqrt{x^4+3x^2+2}}\end{aligned}$$

command

```
Integrate[(7 + 5*x^2)^2/(2 + 3*x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{25x+17x^3+17i\sqrt{1+x^2}\,\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-41i\sqrt{1+x^2}\,\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{2\sqrt{2+3x^2+x^4}}$$

Mathematica 12.3 output

$Aborted

8.11 Problem number 311

$$\int \frac{7+5x^2}{(2+3x^2+x^4)^{3/2}}\,dx$$

Optimal antiderivative

$$\begin{aligned}&-\frac{x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{x\left(x^2+5\right)}{2\sqrt{x^4+3x^2+2}}\\&+\frac{\left(x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{1}{x^2+1}}\ \text{EllipticE}\left(\frac{x}{\sqrt{x^2+1}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\ \sqrt{\frac{x^2+2}{x^2+1}}}{2\sqrt{x^4+3x^2+2}}\\&+\frac{\left(x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{1}{x^2+1}}\ \text{EllipticF}\left(\frac{x}{\sqrt{x^2+1}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\ \sqrt{\frac{x^2+2}{x^2+1}}}{2\sqrt{x^4+3x^2+2}}\end{aligned}$$

command

```
Integrate[(7 + 5*x^2)/(2 + 3*x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{5x+x^3+i\sqrt{1+x^2}\,\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-3i\sqrt{1+x^2}\,\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{2\sqrt{2+3x^2+x^4}}$$

Mathematica 12.3 output

$Aborted

8.12 Problem number 324

$$\int \left(7+5x^2\right)^4 \left(2+x^2-x^4\right)^{3/2}\, dx$$

Optimal antiderivative

$$\begin{aligned}
&-\frac{x\left(-1581440x^2+69817\right)\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{1001}-\frac{132300x\left(-x^4+x^2+2\right)^{\frac{5}{2}}}{143}\\
&-\frac{11750x^3\left(-x^4+x^2+2\right)^{\frac{5}{2}}}{39}-\frac{125x^5\left(-x^4+x^2+2\right)^{\frac{5}{2}}}{3}+\frac{124141422\,\text{EllipticE}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{5005}\\
&-\frac{50794416\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{5005}+\frac{3x\left(7837383x^2+2193559\right)\sqrt{-x^4+x^2+2}}{5005}
\end{aligned}$$

command

```
Integrate[(7 + 5*x^2)^4*(2 + x^2 - x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{-75836958x+48624305x^3+172881581x^5+32834763x^7-36649955x^9-24642275x^{11}-1556625x^{13}+2646875x}{15015\sqrt{2}}$$

Mathematica 12.3 output

$Aborted

8.13 Problem number 325

$$\int \left(7+5x^2\right)^3 \left(2+x^2-x^4\right)^{3/2}\, dx$$

Optimal antiderivative

$$\begin{aligned}
&\frac{x\left(374045x^2+33792\right)\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{3003}-\frac{7825x\left(-x^4+x^2+2\right)^{\frac{5}{2}}}{143}\\
&-\frac{125x^3\left(-x^4+x^2+2\right)^{\frac{5}{2}}}{13}+\frac{31072528\,\text{EllipticE}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{15015}\\
&-\frac{3199778\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{5005}+\frac{x\left(5712051x^2+2512273\right)\sqrt{-x^4+x^2+2}}{15015}
\end{aligned}$$

command

```
Integrate[(7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{-872614x + 11078615x^3 + 13371048x^5 - 1756521x^7 - 4448240x^9 - 1027775x^{11} + 388500x^{13} + 144375x^{15} + 3107}{15015\sqrt{2 + x^2 - x^4}}$$

Mathematica 12.3 output

$Aborted

8.14 Problem number 326

$$\int \left(7 + 5x^2\right)^2 \left(2 + x^2 - x^4\right)^{3/2} \, dx$$

Optimal antiderivative

$$\frac{x\left(920x^2 + 363\right)\left(-x^4 + x^2 + 2\right)^{\frac{3}{2}}}{99} - \frac{25x\left(-x^4 + x^2 + 2\right)^{\frac{5}{2}}}{11} + \frac{85942\,\text{EllipticE}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{495}$$
$$- \frac{3392\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)}{165} + \frac{x\left(14889x^2 + 11497\right)\sqrt{-x^4 + x^2 + 2}}{495}$$

command

```
Integrate[(7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{21254x + 53435x^3 + 23097x^5 - 19944x^7 - 10760x^9 + 1225x^{11} + 1125x^{13} + 85942i\sqrt{4 + 2x^2 - 2x^4}\,E\left(i\sinh^{-1}(x)\right|}{495\sqrt{2 + x^2 - x^4}}$$

Mathematica 12.3 output

$Aborted

8.15 Problem number 327

$$\int \left(7+5x^2\right)\left(2+x^2-x^4\right)^{3/2}\,dx$$

Optimal antiderivative

$$\frac{x\left(35x^2+48\right)\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{63}+\frac{4432\,\text{EllipticE}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{315}$$
$$+\frac{418\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{105}+\frac{x\left(669x^2+1087\right)\sqrt{-x^4+x^2+2}}{315}$$

command

```
Integrate[(7 + 5*x^2)*(2 + x^2 - x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{3134x+4085x^3-438x^5-1674x^7-110x^9+175x^{11}+4432i\sqrt{4+2x^2-2x^4}\,E\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)-7275i\sqrt{4+2x}}{315\sqrt{2+x^2-x^4}}$$

Mathematica 12.3 output

$Aborted

8.16 Problem number 339

$$\int \frac{\left(7+5x^2\right)^5}{\left(2+x^2-x^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$-\frac{3482293\,\text{EllipticE}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18}+\frac{627857\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{6}$$
$$+\frac{x\left(1419793x^2+1419985\right)}{18\sqrt{-x^4+x^2+2}}+\frac{27500x\sqrt{-x^4+x^2+2}}{3}+625x^3\sqrt{-x^4+x^2+2}$$

command

```
Integrate[(7 + 5*x^2)^5/(2 + x^2 - x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{1749985x+1607293x^3-153750x^5-11250x^7-3482293i\sqrt{4+2x^2-2x^4}\,E\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)+4281654i\sqrt{4+2x}}{18\sqrt{2+x^2-x^4}}$$

Mathematica 12.3 output

$Aborted

8.17 Problem number 340

$$\int \frac{\left(7+5x^2\right)^4}{\left(2+x^2-x^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$-\frac{165239\,\text{EllipticE}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18}+\frac{31921\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{6}$$
$$+\frac{x\left(83489x^2+83585\right)}{18\sqrt{-x^4+x^2+2}}+\frac{625x\sqrt{-x^4+x^2+2}}{3}$$

command

```
Integrate[(7 + 5*x^2)^4/(2 + x^2 - x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{91085x+87239x^3-3750x^5-165239i\sqrt{4+2x^2-2x^4}\,E\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)+199977i\sqrt{4+2x^2-2x^4}\,F\left(i\sinh^{-1}(\right.}{18\sqrt{2+x^2-x^4}}$$

Mathematica 12.3 output

$Aborted

8.18 Problem number 341

$$\int \frac{\left(7+5x^2\right)^3}{\left(2+x^2-x^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$-\frac{7147\,\text{EllipticE}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18}+\frac{1763\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{6}+\frac{x\left(4897x^2+4945\right)}{18\sqrt{-x^4+x^2+2}}$$

command

```
Integrate[(7 + 5*x^2)^3/(2 + x^2 - x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{1}{18}\left(\frac{4945x}{\sqrt{2+x^2-x^4}}+\frac{4897x^3}{\sqrt{2+x^2-x^4}}\right.$$
$$\left.-7147i\sqrt{2}\,E\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)+8076i\sqrt{2}\,F\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)\right)$$

Mathematica 12.3 output

$Aborted

8.19 Problem number 342

$$\int \frac{\left(7+5x^2\right)^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$-\frac{281\,\text{EllipticE}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18}+\frac{139\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{6}+\frac{x\left(281x^2+305\right)}{18\sqrt{-x^4+x^2+2}}$$

command

```
Integrate[(7 + 5*x^2)^2/(2 + x^2 - x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{1}{18}\left(\frac{305x}{\sqrt{2+x^2-x^4}}+\frac{281x^3}{\sqrt{2+x^2-x^4}}\right.$$
$$\left.-281i\sqrt{2}\,E\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)+213i\sqrt{2}\,F\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)\right)$$

Mathematica 12.3 output

$Aborted

8.20 Problem number 343

$$\int \frac{7+5x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$-\frac{13\,\text{EllipticE}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18}+\frac{17\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{6}+\frac{x\left(13x^2+25\right)}{18\sqrt{-x^4+x^2+2}}$$

command

```
Integrate[(7 + 5*x^2)/(2 + x^2 - x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{1}{18}\left(\frac{25x}{\sqrt{2+x^2-x^4}}+\frac{13x^3}{\sqrt{2+x^2-x^4}}-13i\sqrt{2}\,E\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)-6i\sqrt{2}\,F\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)\right)$$

Mathematica 12.3 output

$Aborted

8.21 Problem number 344

$$\int \frac{1}{\left(2+x^2-x^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{\text{EllipticE}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18}+\frac{\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{6}+\frac{x\left(-x^2+5\right)}{18\sqrt{-x^4+x^2+2}}$$

command

```
Integrate[(2 + x^2 - x^4)^(-3/2),x]
```

Mathematica 13.1 output

$$\frac{1}{18}\left(\frac{5x}{\sqrt{2+x^2-x^4}}-\frac{x^3}{\sqrt{2+x^2-x^4}}+i\sqrt{2}\,E\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)-3i\sqrt{2}\,F\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)\right)$$

Mathematica 12.3 output

$Aborted

8.22 Problem number 356

$$\int \left(7+5x^2\right)^4\left(4+3x^2+x^4\right)^{3/2}\,dx$$

Optimal antiderivative

$$\frac{x\left(131080x^2+452001\right)\left(x^4+3x^2+4\right)^{\frac{3}{2}}}{1287}+\frac{92150x\left(x^4+3x^2+4\right)^{\frac{5}{2}}}{429}+\frac{2250x^3\left(x^4+3x^2+4\right)^{\frac{5}{2}}}{13}$$
$$+\frac{125x^5\left(x^4+3x^2+4\right)^{\frac{5}{2}}}{3}+\frac{12665086x\sqrt{x^4+3x^2+4}}{2145\left(x^2+2\right)}+\frac{7x\left(174989x^2+661429\right)\sqrt{x^4+3x^2+4}}{2145}$$
$$-\frac{12665086\left(x^2+2\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticE}\left(\sin\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right),\frac{\sqrt{2}}{4}\right)\sqrt{2}\ \sqrt{\frac{x^4+3x^2}{\left(x^2+2\right.}}}{2145\cos\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)\sqrt{x^4+3x^2+4}}$$
$$+\frac{2383556\left(x^2+2\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right),\frac{\sqrt{2}}{4}\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\ \sqrt{}}{429\cos\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)\sqrt{x^4+3x^2+4}}$$

command

```
Integrate[(7 + 5*x^2)^4*(4 + 3*x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(180184116+391419623x^2+472235001x^4+377574349x^6+212188905x^8+83076275x^{10}+21\right.}{}$$

Mathematica 12.3 output

$Aborted

8.23 Problem number 357

$$\int \left(7+5x^2\right)^3\left(4+3x^2+x^4\right)^{3/2}\,dx$$

Optimal antiderivative

$$\frac{x\left(15365x^2+53504\right)\left(x^4+3x^2+4\right)^{\frac{3}{2}}}{1001}+\frac{3825x\left(x^4+3x^2+4\right)^{\frac{5}{2}}}{143}+\frac{125x^3\left(x^4+3x^2+4\right)^{\frac{5}{2}}}{13}$$
$$+\frac{4525662x\sqrt{x^4+3x^2+4}}{5005\left(x^2+2\right)}+\frac{x\left(435441x^2+1653701\right)\sqrt{x^4+3x^2+4}}{5005}$$
$$-\frac{4525662\left(x^2+2\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticE}\left(\sin\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right),\frac{\sqrt{2}}{4}\right)\sqrt{2}\ \sqrt{\frac{x^4+3x^2+}{\left(x^2+2\right)^2}}}{5005\cos\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)\sqrt{x^4+3x^2+4}}$$
$$+\frac{121826\left(x^2+2\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right),\frac{\sqrt{2}}{4}\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\ \sqrt{2}}{143\cos\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)\sqrt{x^4+3x^2+4}}$$

command

```
Integrate[(7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(19463124+36710547x^2+37166164x^4+24107711x^6+10713970x^8+3158575x^{10}+567000x^{1}}{}$$

Mathematica 12.3 output

$Aborted

8.24 Problem number 358

$$\int \left(7+5x^2\right)^2\left(4+3x^2+x^4\right)^{3/2}\,dx$$

Optimal antiderivative

$$\frac{x\left(2240x^2+6831\right)\left(x^4+3x^2+4\right)^{\frac{3}{2}}}{693}+\frac{25x\left(x^4+3x^2+4\right)^{\frac{5}{2}}}{11}$$
$$+\frac{175346x\sqrt{x^4+3x^2+4}}{1155\left(x^2+2\right)}+\frac{x\left(18253x^2+64533\right)\sqrt{x^4+3x^2+4}}{1155}$$
$$-\frac{175346\left(x^2+2\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticE}\left(\sin\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right),\frac{\sqrt{2}}{4}\right)\sqrt{2}\ \sqrt{\frac{x^4+3x^2+}{\left(x^2+2\right)^2}}}{1155\cos\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)\sqrt{x^4+3x^2+4}}$$
$$+\frac{4628\left(x^2+2\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right),\frac{\sqrt{2}}{4}\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\ \sqrt{2}}{33\cos\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)\sqrt{x^4+3x^2+4}}$$

command

```
Integrate[(7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(1824876+2932753x^2+2435811x^4+1229714x^6+408480x^8+82075x^{10}+7875x^{12}\right)-263019}{}$$

Mathematica 12.3 output

$Aborted

8.25 Problem number 359

$$\int \left(7+5x^2\right)\left(4+3x^2+x^4\right)^{3/2}\,dx$$

Optimal antiderivative

$$\frac{x\left(35x^2+108\right)\left(x^4+3x^2+4\right)^{\frac{3}{2}}}{63}+\frac{2798x\sqrt{x^4+3x^2+4}}{105\left(x^2+2\right)}+\frac{x\left(289x^2+1029\right)\sqrt{x^4+3x^2+4}}{105}$$
$$-\frac{2798\left(x^2+2\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticE}\left(\sin\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right),\frac{\sqrt{2}}{4}\right)\sqrt{2}\ \sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}}{105\cos\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)\sqrt{x^4+3x^2+4}}$$
$$+\frac{74\left(x^2+2\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right),\frac{\sqrt{2}}{4}\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\ \sqrt{2}}{3\cos\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)\sqrt{x^4+3x^2+4}}$$

command

```
Integrate[(7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$2\sqrt{-\frac{i}{-3i+\sqrt{7}}}\ x\left(20988+28489x^2+19068x^4+7082x^6+1590x^8+175x^{10}\right)-4197\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{}}{-3i+}}$$

Mathematica 12.3 output

$Aborted

8.26 Problem number 372

$$\int \frac{\left(7+5x^2\right)^4}{\left(4+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{x\left(-4023x^2+2719\right)}{28\sqrt{x^4+3x^2+4}}+\frac{625x\sqrt{x^4+3x^2+4}}{3}+\frac{14523x\sqrt{x^4+3x^2+4}}{28\left(x^2+2\right)}$$
$$-\frac{14523\left(x^2+2\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticE}\left(\sin\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right),\frac{\sqrt{2}}{4}\right)\sqrt{2}\ \sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}}{28\cos\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)\sqrt{x^4+3x^2+4}}$$
$$+\frac{4243\left(x^2+2\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right),\frac{\sqrt{2}}{4}\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\ \sqrt{2}}{24\cos\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)\sqrt{x^4+3x^2+4}}$$

command

```
Integrate[(7 + 5*x^2)^4/(4 + 3*x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\ x\left(78157+40431x^2+17500x^4\right)-43569\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\ \sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}}$$

Mathematica 12.3 output

$Aborted

8.27 Problem number 373

$$\int \frac{\left(7+5x^2\right)^3}{\left(4+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$-\frac{x\left(949x^2+2323\right)}{28\sqrt{x^4+3x^2+4}}+\frac{4449x\sqrt{x^4+3x^2+4}}{28\left(x^2+2\right)}$$
$$-\frac{4449\left(x^2+2\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticE}\left(\sin\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right),\frac{\sqrt{2}}{4}\right)\sqrt{2}\ \sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}}{28\cos\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)\sqrt{x^4+3x^2+4}}$$
$$+\frac{973\left(x^2+2\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right),\frac{\sqrt{2}}{4}\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\ \sqrt{2}}{8\cos\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)\sqrt{x^4+3x^2+4}}$$

command

```
Integrate[(7 + 5*x^2)^3/(4 + 3*x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{-4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\ x\left(2323+949x^2\right)-4449\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\ \sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\ E\left(i\sinh^{-1}\right.}{}$$

112

Mathematica 12.3 output

$Aborted

8.28 Problem number 374

$$\int\frac{\left(7+5x^2\right)^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$-\frac{x\left(-113x^2+9\right)}{28\sqrt{x^4+3x^2+4}}-\frac{113x\sqrt{x^4+3x^2+4}}{28\left(x^2+2\right)}$$
$$+\frac{113\left(x^2+2\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticE}\left(\sin\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right),\frac{\sqrt{2}}{4}\right)\sqrt{2}\ \sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}}{28\cos\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)\sqrt{x^4+3x^2+4}}$$
$$+\frac{9\left(x^2+2\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right),\frac{\sqrt{2}}{4}\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\ \sqrt{2}}{8\cos\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)\sqrt{x^4+3x^2+4}}$$

command

```
Integrate[(7 + 5*x^2)^2/(4 + 3*x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\ x\left(-9+113x^2\right)+113\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\ \sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\ E\left(i\sinh^{-1}\left(\sqrt{-\right.\right.}\right.}{112\sqrt{}}$$

Mathematica 12.3 output

$Aborted

8.29 Problem number 375

$$\int \frac{7+5x^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{x\left(19x^2+53\right)}{28\sqrt{x^4+3x^2+4}}-\frac{19x\sqrt{x^4+3x^2+4}}{28\left(x^2+2\right)}$$
$$+\frac{19\left(x^2+2\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticE}\left(\sin\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right),\frac{\sqrt{2}}{4}\right)\sqrt{2}\ \sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}}{28\cos\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)\sqrt{x^4+3x^2+4}}$$
$$-\frac{3\left(x^2+2\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right),\frac{\sqrt{2}}{4}\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\ \sqrt{2}}{8\cos\left(2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)\sqrt{x^4+3x^2+4}}$$

command

```
Integrate[(7 + 5*x^2)/(4 + 3*x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\ x\left(53+19x^2\right)+19\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\ \sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\ E\left(i\sinh^{-1}\left(\sqrt{--\right.\right.}}{112\sqrt{--}}$$

Mathematica 12.3 output

$Aborted

9 Test file number 41

Test folder name:

```
test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/41_1.2.2.4-f_x-
^m-d+e_x^2-^q-a+b_x^2+c_x^4-^p
```

9.1 Problem number 165

$$\int \left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx$$

Optimal antiderivative

$$\frac{x\left(x^2+3\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{3}+\frac{203x\left(5+2x^2+\sqrt{13}\right)}{30\sqrt{x^4+5x^2+3}}-\frac{x\left(12x^2+5\right)\sqrt{x^4+5x^2+3}}{15}$$
$$+\frac{5\sqrt{\frac{1}{36+x^2\left(30+6\sqrt{13}\right)}}\sqrt{36+x^2\left(30+6\sqrt{13}\right)}\ \text{EllipticF}\left(\frac{x\sqrt{30+6\sqrt{13}}}{\sqrt{36+x^2\left(30+6\sqrt{13}\right)}},\frac{\sqrt{-78+30\sqrt{13}}}{6}\right.}{3\sqrt{5+\sqrt{13}}\sqrt{x^4+5x^2+3}}$$
$$-\frac{203\sqrt{\frac{1}{36+x^2\left(30+6\sqrt{13}\right)}}\sqrt{36+x^2\left(30+6\sqrt{13}\right)}\ \text{EllipticE}\left(\frac{x\sqrt{30+6\sqrt{13}}}{\sqrt{36+x^2\left(30+6\sqrt{13}\right)}},\frac{\sqrt{-78+30\sqrt{13}}}{6}\right.}{180\sqrt{x^4+5x^2+3}}$$

command

```
Integrate[(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{4x\left(120+434x^2+550x^4+293x^6+65x^8+5x^{10}\right)+203i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}}{}$$

Mathematica 12.3 output

$Aborted

9.2 Problem number 201

$$\int \frac{2+3x^2}{\left(3+5x^2+x^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$-\frac{x\left(8x^2+7\right)}{39\sqrt{x^4+5x^2+3}}+\frac{4x\left(5+2x^2+\sqrt{13}\right)}{39\sqrt{x^4+5x^2+3}}$$
$$-\frac{2\sqrt{\frac{1}{36+x^2\left(30+6\sqrt{13}\right)}}\sqrt{36+x^2\left(30+6\sqrt{13}\right)}\ \text{EllipticE}\left(\frac{x\sqrt{30+6\sqrt{13}}}{\sqrt{36+x^2\left(30+6\sqrt{13}\right)}},\frac{\sqrt{-78+30\sqrt{13}}}{6}\right.}{117\sqrt{x^4+5x^2+3}}$$
$$+\frac{11\sqrt{\frac{1}{36+x^2\left(30+6\sqrt{13}\right)}}\sqrt{36+x^2\left(30+6\sqrt{13}\right)}\ \text{EllipticF}\left(\frac{x\sqrt{30+6\sqrt{13}}}{\sqrt{36+x^2\left(30+6\sqrt{13}\right)}},\frac{\sqrt{-78+30\sqrt{13}}}{6}\right.}{13\sqrt{x^4+5x^2+3}\sqrt{30+6\sqrt{13}}}$$

command

```
Integrate[(2 + 3*x^2)/(3 + 5*x^2 + x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{-2x\left(7+8x^2\right)+4i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\right.}{78\sqrt{3+}}$$

Mathematica 12.3 output

$Aborted

9.3 Problem number 208

$$\int (fx)^{3/2}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx$$

Optimal antiderivative

$$\frac{2ad(fx)^{\frac{5}{2}}F_1\left(\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)\sqrt{cx^4+bx^2+a}}{5f\sqrt{1+\frac{2cx^2}{b-\sqrt{-4ac+b^2}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{-4ac+b^2}}}}$$
$$+\frac{2ae(fx)^{\frac{9}{2}}F_1\left(\frac{9}{4},-\frac{3}{2},-\frac{3}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)\sqrt{cx^4+bx^2+a}}{9f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{-4ac+b^2}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{-4ac+b^2}}}}$$

command

```
Integrate[(f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{2f\sqrt{fx}\left(5\left(a+bx^2+cx^4\right)\left(308b^4e-4b^3c\left(147d+55ex^2\right)+12b^2c\left(-167ae+5cx^2\left(7d+3ex^2\right)\right)+3bc^2\left(16a\left(77d+\right.\right.\right.}{}$$

Mathematica 12.3 output

$Aborted

9.4 Problem number 210

$$\int \frac{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{\sqrt{fx}}\,dx$$

Optimal antiderivative

$$\frac{2ae(fx)^{\frac{5}{2}}F_1\left(\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)\sqrt{cx^4+bx^2+a}}{5f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{-4ac+b^2}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{-4ac+b^2}}}}$$
$$+\frac{2adF_1\left(\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)\sqrt{fx}\sqrt{cx^4+bx^2+a}}{f\sqrt{1+\frac{2cx^2}{b-\sqrt{-4ac+b^2}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{-4ac+b^2}}}}$$

command

```
Integrate[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/Sqrt[f*x],x]
```

Mathematica 13.1 output

$$\frac{2x\left(5\left(a+bx^2+cx^4\right)\left(-28b^3e+4b^2c\left(17d+5ex^2\right)+c^2\left(867ad+455aex^2+255cdx^4+195cex^6\right)+bc\left(176ae+5cx\right.\right.\right.}{}$$

Mathematica 12.3 output

$Aborted

9.5 Problem number 216

$$\int \frac{(fx)^{3/2}\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{2d(fx)^{\frac{5}{2}}\,F_1\left(\frac{5}{4},\frac{3}{2},\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)\sqrt{1+\frac{2cx^2}{b-\sqrt{-4ac+b^2}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{-4ac+b^2}}}}{5af\sqrt{cx^4+bx^2+a}}$$
$$+\frac{2e(fx)^{\frac{9}{2}}\,F_1\left(\frac{9}{4},\frac{3}{2},\frac{3}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)\sqrt{1+\frac{2cx^2}{b-\sqrt{-4ac+b^2}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{-4ac+b^2}}}}{9af^3\sqrt{cx^4+bx^2+a}}$$

command

```
Integrate[((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

Mathematica 13.1 output

$$-\frac{f\sqrt{fx}\left(5\left(bd-2ae+2cdx^2-bex^2\right)-5(bd-2ae)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{4};\right.\right.}{}$$

Mathematica 12.3 output

$Aborted

9.6 Problem number 218

$$\int \frac{d+ex^2}{\sqrt{fx}\,\left(a+bx^2+cx^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{2e(fx)^{\frac{5}{2}}\,F_1\left(\frac{5}{4},\frac{3}{2},\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)\sqrt{1+\frac{2cx^2}{b-\sqrt{-4ac+b^2}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{-4ac+b^2}}}}{5af^3\sqrt{cx^4+bx^2+a}}$$
$$+\frac{2dF_1\left(\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)\sqrt{fx}\,\sqrt{1+\frac{2cx^2}{b-\sqrt{-4ac+b^2}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{-4ac+b^2}}}}{af\sqrt{cx^4+bx^2+a}}$$

command

```
Integrate[(d + e*x^2)/(Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

Mathematica 13.1 output

$$\frac{x\left(-5b^2d+5b\left(ae-cdx^2\right)+10ac\left(d+ex^2\right)-5\left(b^2d-6acd+abe\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+}{b+\sqrt{b^2-4a}}}\right.}{}$$

Mathematica 12.3 output

$Aborted

10 Test file number 42

Test folder name:

```
test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/42_1.2.2.5_P-x-a+b_x^2+c_x^4-^p
```

10.1 Problem number 103

$$\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx$$

Optimal antiderivative

$$\frac{(-bg+2ce)\left(2cx^2+b\right)\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{32c^2}+\frac{x\left(7cfx^2+3bf+9cd\right)\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{63c}$$
$$+\frac{g\left(cx^4+bx^2+a\right)^{\frac{5}{2}}}{10c}+\frac{3\left(-4ac+b^2\right)^2(-bg+2ce)\operatorname{arctanh}\left(\frac{2cx^2+b}{2\sqrt{c}\ \sqrt{cx^4+bx^2+a}}\right)}{512c^{\frac{7}{2}}}$$
$$-\frac{3\left(-4ac+b^2\right)(-bg+2ce)\left(2cx^2+b\right)\sqrt{cx^4+bx^2+a}}{256c^3}$$
$$+\frac{x\left(9b^2cd+90ac^2d-4b^3f+9abcf+3c\left(14acf-4b^2f+9bcd\right)x^2\right)\sqrt{cx^4+bx^2+a}}{315c^2}$$
$$-\frac{\left(-84a^2c^2f+57ab^2cf-144abc^2d-8b^4f+18b^3cd\right)x\sqrt{cx^4+bx^2+a}}{315c^{\frac{5}{2}}\left(\sqrt{a}+x^2\sqrt{c}\right)}$$
$$+\frac{a^{\frac{1}{4}}\left(-84a^2c^2f+57ab^2cf-144abc^2d-8b^4f+18b^3cd\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticE}\left(\sin\left(2\arctan\right.\right.}{315\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)c^{\frac{11}{4}}\sqrt{cx^4+bx^2+a}}$$
$$-\frac{a^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2-\frac{b}{\sqrt{a}\ \sqrt{c}}}}{2}\right)\left(\sqrt{a}+x^2\sqrt{c}\right)(18b^3c}{630\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}$$

command

```
Integrate[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4)^(3/2),x]
```

Mathematica 13.1 output

Result too large to show

Mathematica 12.3 output

$Aborted

10.2 Problem number 104

$$\int \left(d+ex+fx^2+gx^3\right)\sqrt{a+bx^2+cx^4}\,dx$$

Optimal antiderivative

$$\frac{g\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{6c}-\frac{\left(-4ac+b^2\right)\left(-bg+2ce\right)\operatorname{arctanh}\left(\frac{2cx^2+b}{2\sqrt{c}\,\sqrt{cx^4+bx^2+a}}\right)}{32c^{\frac{5}{2}}}$$
$$+\frac{\left(-bg+2ce\right)\left(2cx^2+b\right)\sqrt{cx^4+bx^2+a}}{16c^2}$$
$$+\frac{x\left(3cfx^2+bf+5cd\right)\sqrt{cx^4+bx^2+a}}{15c}+\frac{\left(6acf-2b^2f+5bcd\right)x\sqrt{cx^4+bx^2+a}}{15c^{\frac{3}{2}}\left(\sqrt{a}+x^2\sqrt{c}\right)}$$
$$-\frac{a^{\frac{1}{4}}\left(6acf-2b^2f+5bcd\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2-\frac{b}{\sqrt{a}\,\sqrt{c}}}}{2}\right)\left(\right.}{15\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)c^{\frac{7}{4}}\sqrt{cx^4+bx^2+a}}$$
$$+\frac{a^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2-\frac{b}{\sqrt{a}\,\sqrt{c}}}}{2}\right)\left(\sqrt{a}+x^2\sqrt{c}\right)\left(b+2\sqrt{}\right.}{30\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)c^{\frac{7}{4}}\sqrt{cx^4+bx^2+a}}$$

command

```
Integrate[(d + e*x + f*x^2 + g*x^3)*Sqrt[a + b*x^2 + c*x^4],x]
```

Mathematica 13.1 output

$$2\sqrt{c}\left(a+bx^2+cx^4\right)\left(-15b^2g+2bc(15e+x(8f+5gx))+4c(10ag+cx(20d+x(15e+2x(6f+5gx))))\right)+\frac{-8i\sqrt{2}}{}$$

Mathematica 12.3 output

$Aborted

10.3 Problem number 106

$$\int \frac{d+ex+fx^2+gx^3}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{x\left(b^2d-2acd-abf+c(-2af+bd)\,x^2\right)}{a\left(-4ac+b^2\right)\sqrt{c\,x^4+b\,x^2+a}}$$
$$+\frac{-be+2ag-(-bg+2ce)\,x^2}{\left(-4ac+b^2\right)\sqrt{c\,x^4+b\,x^2+a}}-\frac{(-2af+bd)\,x\sqrt{c}\ \sqrt{c\,x^4+b\,x^2+a}}{a\left(-4ac+b^2\right)\left(\sqrt{a}\ +x^2\sqrt{c}\ \right)}$$
$$+\frac{c^{\frac{1}{4}}(-2af+bd)\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2-\frac{b}{\sqrt{a}\ \sqrt{c}}}}{2}\right)\left(\sqrt{a}\ +x^2\right.}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}\left(-4ac+b^2\right)\sqrt{c\,x^4+b\,x^2+a}}$$
$$-\frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2-\frac{b}{\sqrt{a}\ \sqrt{c}}}}{2}\right)\left(-f\sqrt{a}\ +d\sqrt{c}\ \right)\left(\sqrt{a}\ +\right.}{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}c^{\frac{1}{4}}\left(b-2\sqrt{a}\ \sqrt{c}\ \right)\sqrt{c\,x^4+b\,x^2+a}}$$

command

```
Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(3/2),x]
```

Mathematica 13.1 output

$$-\frac{4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\left(-2a^2g-bdx\left(b+cx^2\right)+2acx(d+x(e+fx))+ab(e+x(f-gx))\right)+i\left(-b+\sqrt{b^2-4ac}\right.}{}$$

Mathematica 12.3 output

$Aborted

10.4 Problem number 107

$$\int \frac{d+ex+fx^2+gx^3}{\left(a+bx^2+cx^4\right)^{5/2}}\,dx$$

Optimal antiderivative

$$\frac{x\left(b^2d-2acd-abf+c(-2af+bd)\,x^2\right)}{3a\left(-4ac+b^2\right)\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}+\frac{-be+2ag-(-bg+2ce)\,x^2}{3\left(-4ac+b^2\right)\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}+\frac{4(-bg+2ce)\left(2c\,x^2+b\right)}{3\left(-4ac+b^2\right)^2\sqrt{c\,x^4+b\,x^2+a}}$$
$$+\frac{x\left(2b^4d-17a\,b^2cd+20a^2c^2d+a\,b^3f+4a^2bcf+c\left(12a^2cf+a\,b^2f-16abcd+2b^3d\right)x^2\right)}{3a^2\left(-4ac+b^2\right)^2\sqrt{c\,x^4+b\,x^2+a}}$$
$$-\frac{\left(12a^2cf+a\,b^2f-16abcd+2b^3d\right)x\sqrt{c}\ \sqrt{c\,x^4+b\,x^2+a}}{3a^2\left(-4ac+b^2\right)^2\left(\sqrt{a}\ +x^2\sqrt{c}\ \right)}$$
$$+\frac{c^{\frac{1}{4}}\left(12a^2cf+a\,b^2f-16abcd+2b^3d\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2-\frac{}{}}}{2}\right.}{3\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)a^{\frac{7}{4}}\left(-4ac+b^2\right)^2\sqrt{c\,x^4+b\,x^2+a}}$$
$$-\frac{c^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2-\frac{b}{\sqrt{a}\ \sqrt{c}}}}{2}\right)\left(\sqrt{a}\ +x^2\sqrt{c}\ \right)\left(2b^2d-\right.}{6\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)a^{\frac{7}{4}}\left(-4ac+b^2\right)\left(b-2\sqrt{a}\ \sqrt{c}\ \right)\sqrt{c\,x^4}$$

command

```
Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(5/2),x]
```

Mathematica 13.1 output

$$-4a\left(b^2-4ac\right)\left(-2a^2g-bdx\left(b+cx^2\right)+2acx(d+x(e+fx))+ab(e+x(f-gx))\right)+4\left(a+bx^2+cx^4\right)\left(2b^3dx\left(b-\right.\right.$$

Mathematica 12.3 output

$Aborted

10.5 Problem number 108

$$\int \frac{ag - cgx^4}{(a + bx^2 + cx^4)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{gx}{\sqrt{c\,x^4 + b\,x^2 + a}}$$

command

```
Integrate[(a*g - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{gx}{\sqrt{a + bx^2 + cx^4}}$$

Mathematica 12.3 output

$Aborted

10.6 Problem number 109

$$\int \frac{ag + ex - cgx^4}{(a + bx^2 + cx^4)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{gx}{\sqrt{c\,x^4 + b\,x^2 + a}} - \frac{e\left(2c\,x^2 + b\right)}{\left(-4ac + b^2\right)\sqrt{c\,x^4 + b\,x^2 + a}}$$

command

```
Integrate[(a*g + e*x - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{-be + b^2gx - 4acgx - 2cex^2}{\left(b^2 - 4ac\right)\sqrt{a + bx^2 + cx^4}}$$

Mathematica 12.3 output

$Aborted

10.7 Problem number 110

$$\int \frac{ag + fx^3 - cgx^4}{(a + bx^2 + cx^4)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{gx}{\sqrt{c\,x^4 + b\,x^2 + a}} + \frac{f\left(b\,x^2 + 2a\right)}{(-4ac + b^2)\,\sqrt{c\,x^4 + b\,x^2 + a}}$$

command

```
Integrate[(a*g + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{bx(bg + fx) + 2a(f - 2cgx)}{(b^2 - 4ac)\,\sqrt{a + bx^2 + cx^4}}$$

Mathematica 12.3 output

$Aborted

10.8 Problem number 111

$$\int \frac{ag + ex + fx^3 - cgx^4}{(a + bx^2 + cx^4)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{gx}{\sqrt{c\,x^4 + b\,x^2 + a}} + \frac{-be + 2af - (-bf + 2ce)\,x^2}{(-4ac + b^2)\,\sqrt{c\,x^4 + b\,x^2 + a}}$$

command

```
Integrate[(a*g + e*x + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{-be + 2af + b^2gx - 4acgx - 2cex^2 + bfx^2}{(b^2 - 4ac)\,\sqrt{a + bx^2 + cx^4}}$$

Mathematica 12.3 output

$Aborted

11 Test file number 44

Test folder name:

`test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/44_1.2.2.7_P-x-d+e_x^2-^q-a+b_x^2+c_x^4-^p`

11.1 Problem number 26

$$\int \frac{\left(A+Bx^2\right)\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$-\frac{x\left(aB(-2ae+bd)-A\left(-abe-2acd+b^2d\right)-\left(Ac(-2ae+bd)-aB(-be+2cd)\right)x^2\right)}{a\left(-4ac+b^2\right)\sqrt{c\,x^4+b\,x^2+a}}$$

$$-\frac{\left(Ac(-2ae+bd)-aB(-be+2cd)\right)x\sqrt{c\,x^4+b\,x^2+a}}{a\left(-4ac+b^2\right)\sqrt{c}\ \left(\sqrt{a}\ +x^2\sqrt{c}\ \right)}$$

$$+\frac{\left(Ac(-2ae+bd)-aB(-be+2cd)\right)\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2-\frac{}{\sqrt{a}}}}{2}\right.}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}c^{\frac{3}{4}}\left(-4ac+b^2\right)\sqrt{c\,x^4+b\,x^2+a}}$$

$$+\frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2-\frac{b}{\sqrt{a}\ \sqrt{c}}}}{2}\right)\left(B\sqrt{a}\ -A\sqrt{c}\ \right)\left(-e\sqrt{a}\ -\right.}{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}c^{\frac{3}{4}}\left(b-2\sqrt{a}\ \sqrt{c}\ \right)\sqrt{c\,x^4+b\,x^2+a}}$$

command

```
Integrate[((A + B*x^2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\ x\left(aB\left(-2ae+2cdx^2+b\left(d-ex^2\right)\right)+A\left(-b^2d+b\left(ae-cdx^2\right)+2ac\left(d+ex^2\right)\right)\right)+i\left(-b+\sqrt{}\right.}{}$$

Mathematica 12.3 output

$Aborted

11.2 Problem number 27

$$\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{x\left(A\,b^2-abB-2aAc+(Ab-2aB)\,c\,x^2\right)}{a\left(-4ac+b^2\right)\sqrt{c\,x^4+b\,x^2+a}}-\frac{(Ab-2aB)\,x\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}}{a\left(-4ac+b^2\right)\left(\sqrt{a}+x^2\sqrt{c}\right)}$$

$$+\frac{(Ab-2aB)\,c^{\frac{1}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticE}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2-\frac{b}{\sqrt{a}\,\sqrt{c}}}}{2}\right)\left(\sqrt{a}+x^2\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+x^2\sqrt{c}\right)^2}}}{\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}\left(-4ac+b^2\right)\sqrt{c\,x^4+b\,x^2+a}}$$

$$+\frac{\sqrt{\frac{\cos\left(4\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sin\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right),\frac{\sqrt{2-\frac{b}{\sqrt{a}\,\sqrt{c}}}}{2}\right)\left(B\sqrt{a}-A\sqrt{c}\right)\left(\sqrt{a}+x^2\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+x^2\sqrt{c}\right)^2}}}{2\cos\left(2\arctan\left(\frac{c^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right)\right)a^{\frac{3}{4}}c^{\frac{1}{4}}\left(b-2\sqrt{a}\,\sqrt{c}\right)\sqrt{c\,x^4+b\,x^2+a}}$$

command

```
Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

Mathematica 13.1 output

$$-\frac{4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\left(aB\left(b+2cx^2\right)-A\left(b^2-2ac+bcx^2\right)\right)+i(Ab-2aB)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-}}}$$

Mathematica 12.3 output

$Aborted

12 Test file number 49

Test folder name:

`test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.3_General/49_1.2.3.5_P-x-d_x-^m-a+b_x^n+c_x^-2_n-^p`

12.1 Problem number 10

$$\int \frac{-ahx^{-1+\frac{n}{2}} + cfx^{-1+n} + cgx^{-1+2n} + chx^{-1+\frac{5n}{2}}}{\left(a + bx^n + cx^{2n}\right)^{3/2}}\, dx$$

Optimal antiderivative

$$-\frac{2\Big(c(-2ag+bf) + \left(-4ac+b^2\right) h\, x^{\frac{n}{2}} + c(-bg+2cf)\, x^n\Big)}{\left(-4ac+b^2\right) n \sqrt{a + b\, x^n + c\, x^{2n}}}$$

command

```
Integrate[(-(a*h*x^(-1 + n/2)) + c*f*x^(-1 + n) + c*g*x^(-1 + 2*n) + c*h*x^(-1 + (5*n)/2))/(a
```

Mathematica 13.1 output

$$-\frac{2\left(bcf - 2acg + b^2 hx^{n/2} - 4achx^{n/2} + 2c^2 fx^n - bcgx^n\right)}{\left(b^2 - 4ac\right) n \sqrt{a + bx^n + cx^{2n}}}$$

Mathematica 12.3 output

$Aborted

12.2 Problem number 14

$$\int \frac{x^{-1+\frac{n}{2}}\left(-ah + cfx^{n/2} + cgx^{3n/2} + chx^{2n}\right)}{\left(a + bx^n + cx^{2n}\right)^{3/2}}\, dx$$

Optimal antiderivative

$$-\frac{2\Big(c(-2ag+bf) + \left(-4ac+b^2\right) h\, x^{\frac{n}{2}} + c(-bg+2cf)\, x^n\Big)}{\left(-4ac+b^2\right) n \sqrt{a + b\, x^n + c\, x^{2n}}}$$

command

```
Integrate[(x^(-1 + n/2)*(-(a*h) + c*f*x^(n/2) + c*g*x^((3*n)/2) + c*h*x^(2*n)))/(a + b*x^n +
```

Mathematica 13.1 output

$$-\frac{2\left(bcf - 2acg + b^2 hx^{n/2} - 4achx^{n/2} + 2c^2 fx^n - bcgx^n\right)}{\left(b^2 - 4ac\right) n \sqrt{a + bx^n + cx^{2n}}}$$

Mathematica 12.3 output

$Aborted

12.3 Problem number 15

$$\int \frac{(dx)^{-1+\frac{n}{2}} \left(-ah + cfx^{n/2} + cgx^{3n/2} + chx^{2n}\right)}{\left(a + bx^n + cx^{2n}\right)^{3/2}} \, dx$$

Optimal antiderivative

$$-\frac{2x^{1-\frac{n}{2}} (dx)^{-1+\frac{n}{2}} \left(c(-2ag + bf) + \left(-4ac + b^2\right) h \, x^{\frac{n}{2}} + c(-bg + 2cf) \, x^n\right)}{\left(-4ac + b^2\right) n \sqrt{a + b \, x^n + c \, x^{2n}}}$$

command

```
Integrate[((d*x)^(-1 + n/2)*(-(a*h) + c*f*x^(n/2) + c*g*x^((3*n)/2) + c*h*x^(2*n)))/(a + b*x^
```

Mathematica 13.1 output

$$\frac{x^{-n/2}(dx)^{n/2} \left(2a\left(b^2 hx^{n/2} + bc(f - gx^n) + 2c\left(cfx^n - a\left(g + 2hx^{n/2}\right)\right)\right) + b\sqrt{c} \, (-bf + 2ag) \sqrt{a + x^n \left(b + cx^n\right)} \, \log}{a \left(-b^2 + 4ac\right.}$$

Mathematica 12.3 output

$Aborted

13 Test file number 52

Test folder name:

`test_cases/1_Algebraic_functions/1.3_Miscellaneous/52_1.3.2_Algebraic_functions`

13.1 Problem number 18

$$\int \frac{1}{(c + dx) \sqrt[3]{-c^3 + d^3 x^3}} \, dx$$

Optimal antiderivative

$$\frac{\ln\left((-dx + c)(dx + c)^2\right) 2^{\frac{2}{3}}}{8cd} - \frac{3 \ln\left(d(-dx + c) + 2^{\frac{2}{3}} d\left(d^3 x^3 - c^3\right)^{\frac{1}{3}}\right) 2^{\frac{2}{3}}}{8cd} + \frac{\arctan\left(\frac{\left(1 - \frac{2^{\frac{1}{3}}(-dx + c)}{\left(d^3 x^3 - c^3\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right) \sqrt{3} \, 2^{\frac{2}{3}}}{4cd}$$

command

```
Integrate[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{-\frac{1}{2}}\left(2i\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[3]{2}\left(3+i\sqrt{3}\right)c+\sqrt[3]{2}\left(-3-i\sqrt{3}\right)dx+2i\sqrt{3}\sqrt[3]{-c^3+d^3x^3}}{6\sqrt[3]{-c^3+d^3x^3}}\right)+2\log\left(\sqrt{c}\sqrt{d}\left(-c+i\sqrt{3}c+\right.\right.\right.}$$

Mathematica 12.3 output

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$$

13.2 Problem number 172

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Optimal antiderivative

$$-\frac{\ln(dx+c)}{d}+\frac{3\ln\left(d(dx+2c)-d\left(d^3x^3+2c^3\right)^{\frac{1}{3}}\right)}{2d}-\frac{\arctan\left(\frac{\left(1+\frac{2dx+4c}{\left(d^3x^3+2c^3\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{d}$$

command

```
Integrate[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{2c^3+d^3x^3}}{4c+2dx+\sqrt[3]{2c^3+d^3x^3}}\right)}{d}+\frac{\log\left(-2c-dx+\sqrt[3]{2c^3+d^3x^3}\right)}{d}$$
$$-\frac{\log\left(4c^2+4cdx+d^2x^2+(2c+dx)\sqrt[3]{2c^3+d^3x^3}+\left(2c^3+d^3x^3\right)^{2/3}\right)}{2d}$$

Mathematica 12.3 output

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

13.3 Problem number 913

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx$$

Optimal antiderivative

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\arctan\left(\frac{2icx+d\sqrt{3}}{\sqrt{-2ix^2+\sqrt{3}}\sqrt{2ic^2-d^2\sqrt{3}}}\right)}{\sqrt{2ic^2-d^2\sqrt{3}}}+\frac{\left(-\frac{1}{2}-\frac{i}{2}\right)\operatorname{arctanh}\left(\frac{-2icx+d\sqrt{3}}{\sqrt{2ix^2+\sqrt{3}}\sqrt{2ic^2+d^2\sqrt{3}}}\right)}{\sqrt{2ic^2+d^2\sqrt{3}}}$$

command

```
Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]),x]
```

Mathematica 13.1 output

$$-\sqrt{-2c^2-\sqrt{4c^4+3d^4}}\,\tan^{-1}\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2-\sqrt{4c^4+3d^4}}}\right)+\sqrt{-2c^2+\sqrt{4c^4+3d^4}}\,\tan^{-1}\left(\frac{d\sqrt{2x^2+\sqrt{3+}}}{\sqrt{-2c^2+\sqrt{4c^4-}}}\right.$$

Mathematica 12.3 output

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx$$

13.4 Problem number 995

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$$

Optimal antiderivative

$$\frac{3\arcsin\left(x-\sqrt{x^2-1}\right)\sqrt{2}}{8}+\frac{\left(3x+\sqrt{x^2-1}\right)\sqrt{1-x^2+x\sqrt{x^2-1}}}{4}$$

command

```
Integrate[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]],x]
```

Mathematica 13.1 output

$$\frac{1}{8}\left(\frac{2\left(-1+x^2\right)\left(3x+\sqrt{-1+x^2}\right)}{\sqrt{1-x^2+x\sqrt{-1+x^2}}\left(-1+x^2+x\sqrt{-1+x^2}\right)}-3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-1+x^2}}{\sqrt{1-x^2+x\sqrt{-1+x^2}}}\right)\right)$$

Mathematica 12.3 output

$$\int\sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$$

13.5 Problem number 1017

$$\int\frac{1-x^2}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\,dx$$

Optimal antiderivative

$$-\frac{\ln\left(1+2(1-x)^3-x^3\right)2^{\frac{1}{3}}}{4}+\frac{3\ln\left(2^{\frac{1}{3}}(1-x)+\left(-x^3+1\right)^{\frac{1}{3}}\right)2^{\frac{1}{3}}}{4}+\frac{2^{\frac{1}{3}}\arctan\left(\frac{\left(1-\frac{2\,2^{\frac{1}{3}}(1-x)}{\left(-x^3+1\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2}$$

command

```
Integrate[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)),x]
```

Mathematica 13.1 output

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-x^3\right)^{2/3}}{2^{2/3}+2^{2/3}x+2^{2/3}x^2-\left(1-x^3\right)^{2/3}}\right)-2\log\left(2^{2/3}+2^{2/3}x+2^{2/3}x^2+2\left(1-x^3\right)^{2/3}\right)+\log\left(-\left(\left(1+x+x^2\right)\right.\right.}{2\,2^{2/3}}$$

Mathematica 12.3 output

$$\int\frac{1-x^2}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\,dx$$

14 Test file number 78

Test folder name:

`test_cases/4_Trig_functions/4.1_Sine/78_4.1.4.2-a+b_sin-^m-c+d_sin-^n-A+B_sin+C_sin^2-`

14.1 Problem number 4

$$\int \frac{(a+a\sin(e+fx))^m\left(A+C\sin^2(e+fx)\right)}{\sqrt{c-c\sin(e+fx)}}\,dx$$

Optimal antiderivative

$$\frac{(A+C)\cos(fx+e)\,\text{hypergeom}\left(\left[1,\frac{1}{2}+m\right],\left[\frac{3}{2}+m\right],\frac{1}{2}+\frac{\sin(fx+e)}{2}\right)(a+a\sin(fx+e))^m}{f(1+2m)\sqrt{c-c\sin(fx+e)}}$$
$$-\frac{2C\cos(fx+e)(a+a\sin(fx+e))^{1+m}}{af(3+2m)\sqrt{c-c\sin(fx+e)}}$$

command

```
Integrate[((a + a*Sin[e + f*x])^m*(A + C*Sin[e + f*x]^2))/Sqrt[c - c*Sin[e + f*x]],x]
```

Mathematica 13.1 output

Result too large to show

Mathematica 12.3 output

$Aborted

14.2 Problem number 21

$$\int \frac{(a+a\sin(e+fx))^m\left(A+B\sin(e+fx)+C\sin^2(e+fx)\right)}{\sqrt{c-c\sin(e+fx)}}\,dx$$

Optimal antiderivative

$$-\frac{2B\cos(fx+e)(a+a\sin(fx+e))^m}{f(1+2m)\sqrt{c-c\sin(fx+e)}}$$
$$+\frac{(A+B+C)\cos(fx+e)\,\text{hypergeom}\left(\left[1,\frac{1}{2}+m\right],\left[\frac{3}{2}+m\right],\frac{1}{2}+\frac{\sin(fx+e)}{2}\right)(a+a\sin(fx+e))^m}{f(1+2m)\sqrt{c-c\sin(fx+e)}}$$
$$-\frac{2C\cos(fx+e)(a+a\sin(fx+e))^{1+m}}{af(3+2m)\sqrt{c-c\sin(fx+e)}}$$

command

```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x] + C*Sin[e + f*x]^2))/Sqrt[c - c*Sin[e +
```

Mathematica 13.1 output

Result too large to show

Mathematica 12.3 output

$Aborted

15 Test file number 89

Test folder name:

```
test_cases/4_Trig_functions/4.2_Cosine/89_4.2.2.1-a+b_cos-^m-c+d_cos-^n
```

15.1 Problem number 373

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$$

Optimal antiderivative

$$-\frac{\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{2d\left(a+a\cos(dx+c)\right)^{\frac{3}{2}}}+\frac{7\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{6ad\sqrt{a+a\cos(dx+c)}}$$
$$+\frac{11\arctan\left(\frac{\sin(dx+c)\sqrt{a}\ \sqrt{2}}{2\sqrt{\cos(dx+c)}\ \sqrt{a+a\cos(dx+c)}}\right)\left(\sqrt{\cos}(dx+c)\right)\left(\sqrt{\sec}(dx+c)\right)\sqrt{2}}{4a^{\frac{3}{2}}d}$$
$$-\frac{19\sin(dx+c)\left(\sqrt{\sec}(dx+c)\right)}{6ad\sqrt{a+a\cos(dx+c)}}$$

command

```
Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

Mathematica 13.1 output

$$\cot^3\left(\tfrac{c}{2}+\tfrac{dx}{2}\right)\csc^4\left(\tfrac{c}{2}+\tfrac{dx}{2}\right)\sec^2\left(\tfrac{1}{2}(c+dx)\right)\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{7/2}\left(-80\cos^6\left(\tfrac{1}{2}(c+dx)\right)\ {}_4F_3\left(2,2,2,\tfrac{7}{2};1,1,\tfrac{11}{2};\frac{}{-1-}\right.\right.$$

Mathematica 12.3 output

$Aborted

16 Test file number 92

Test folder name:

`test_cases/4_Trig_functions/4.2_Cosine/92_4.2.3.1-a+b_cos-^m-c+d_cos-^n-A+B_cos-`

16.1 Problem number 523

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

Optimal antiderivative

$$\frac{2A\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{3d\sqrt{a+a\cos(dx+c)}}$$
$$+\frac{(A-B)\arctan\left(\frac{\sin(dx+c)\sqrt{a}\ \sqrt{2}}{2\sqrt{\cos(dx+c)}\ \sqrt{a+a\cos(dx+c)}}\right)\sqrt{2}\ \left(\sqrt{\cos}(dx+c)\right)\left(\sqrt{\sec}(dx+c)\right)}{d\sqrt{a}}$$
$$-\frac{2(A-3B)\sin(dx+c)\left(\sqrt{\sec}(dx+c)\right)}{3d\sqrt{a+a\cos(dx+c)}}$$

command

```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

Mathematica 13.1 output

$$2\cos\left(\tfrac{c}{2}+\tfrac{dx}{2}\right)\sqrt{\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\ \sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\left(\frac{2B\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{3\left(1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^{3/2}}+\frac{4B\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{3\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}+\right.$$

Mathematica 12.3 output

$Aborted

16.2 Problem number 530

$$\int \frac{(A + B\cos(c + dx))\sec^{\frac{7}{2}}(c + dx)}{(a + a\cos(c + dx))^{3/2}}\,dx$$

Optimal antiderivative

$$-\frac{(A-B)\left(\sec^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{2d\left(a+a\cos(dx+c)\right)^{\frac{3}{2}}}-\frac{(39A-35B)\left(\sec^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{30ad\sqrt{a+a\cos(dx+c)}}$$
$$+\frac{(9A-5B)\left(\sec^{\frac{5}{2}}(dx+c)\right)\sin(dx+c)}{10ad\sqrt{a+a\cos(dx+c)}}$$
$$-\frac{(15A-11B)\arctan\left(\frac{\sin(dx+c)\sqrt{a}\ \sqrt{2}}{2\sqrt{\cos(dx+c)}\ \sqrt{a+a\cos(dx+c)}}\right)\left(\sqrt{\cos}(dx+c)\right)\left(\sqrt{\sec}(dx+c)\right)\sqrt{2}}{4a^{\frac{3}{2}}d}$$
$$+\frac{(147A-95B)\sin(dx+c)\left(\sqrt{\sec}(dx+c)\right)}{30ad\sqrt{a+a\cos(dx+c)}}$$

command

```
Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(3/2),x]
```

Mathematica 13.1 output

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(-60i(15A-11B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\ \sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\ \sqrt{1+e^{2i(c+dx)}}}\right)+(\right.}{}$$

Mathematica 12.3 output

$Aborted

17 Test file number 94

Test folder name:

```
test_cases/4_Trig_functions/4.2_Cosine/94_4.2.4.2-a+b_cos-^m-c+d_cos-^n-A+B_cos+C_cos^2-
```

17.1 Problem number 1340

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^{\frac{5}{2}}(c + dx)}{\sqrt{a + a\cos(c + dx)}}\,dx$$

Optimal antiderivative

$$\frac{2A\left(\sec^{\frac{3}{2}}(dx + c)\right)\sin(dx + c)}{3d\sqrt{a + a\cos(dx + c)}} + \frac{(A - B + C)\arctan\left(\frac{\sin(dx+c)\sqrt{a}\ \sqrt{2}}{2\sqrt{\cos(dx + c)}\ \sqrt{a + a\cos(dx + c)}}\right)\sqrt{2}\ \left(\sqrt{\cos}(dx + c)\right)\left(\sqrt{\sec}(dx + c)\right)}{d\sqrt{a}} - \frac{2(A - 3B)\sin(dx + c)\left(\sqrt{\sec}(dx + c)\right)}{3d\sqrt{a + a\cos(dx + c)}}$$

command

```
Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^(5/2))/Sqrt[a + a*Cos[c + d*x
```

Mathematica 13.1 output

$$\frac{2\cos\left(\frac{c}{2} + \frac{dx}{2}\right)\sqrt{\frac{1}{1 - 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\ \sqrt{1 - 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}\left(\frac{2B\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{3\left(1 - 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)\right)^{3/2}} - \frac{4C\sin^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{3\left(1 - 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)\right)^{3/2}} + \frac{}{\sqrt[3]{1}}\right.}{}$$

Mathematica 12.3 output

$Aborted

18 Test file number 120

Test folder name:

```
test_cases/4_Trig_functions/4.5_Secant/120_4.5.1.4-d_tan-^n-a+b_sec-^m
```

18.1 Problem number 332

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Optimal antiderivative

$$-\frac{2(a-b)\cot(dx+c)\,\text{EllipticE}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a+b}},\sqrt{\frac{a+b}{a-b}}\right)\sqrt{a+b}\,\sqrt{\frac{b(1-\sec(dx+c))}{a+b}}\,\sqrt{-\frac{b(1+\sec(}{a-}}{b^2d}$$

$$-\frac{2\cot(dx+c)\,\text{EllipticF}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a+b}},\sqrt{\frac{a+b}{a-b}}\right)\sqrt{a+b}\,\sqrt{\frac{b(1-\sec(dx+c))}{a+b}}\,\sqrt{-\frac{b(1+\sec(dx+c)}{a-b}}}{bd}$$

$$+\frac{2\cot(dx+c)\,\text{EllipticPi}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a+b}},\frac{a+b}{a},\sqrt{\frac{a+b}{a-b}}\right)\sqrt{a+b}\,\sqrt{\frac{b(1-\sec(dx+c))}{a+b}}\,\sqrt{-\frac{b(1+\sec(dx}{a-b}}}{ad}$$

command

```
Integrate[Tan[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

Mathematica 13.1 output

Result too large to show

Mathematica 12.3 output

$Aborted

19 Test file number 125

Test folder name:

```
test_cases/4_Trig_functions/4.5_Secant/125_4.5.4.2-a+b_sec-^m-d_sec-^n-A+B_sec+C_sec^2-
```

19.1 Problem number 81

$$\int \frac{(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$$

Optimal antiderivative

$$\frac{2C(b\sec(dx+c))^n\sin(dx+c)\left(\sqrt{\sec}(dx+c)\right)}{d(1+2n)}$$

$$-\frac{4(A-C(1-2n)+2An)\,\text{hypergeom}\left(\left[\tfrac{1}{2},\tfrac{3}{4}-\tfrac{n}{2}\right],\left[\tfrac{7}{4}-\tfrac{n}{2}\right],\cos^2(dx+c)\right)(b\sec(dx+c))^n\sin(dx+c)}{d(-4n^2+4n+3)\sec(dx+c)^{\frac{3}{2}}\sqrt{2-2\cos(2dx+2c)}}$$

$$-\frac{4B\,\text{hypergeom}\left(\left[\tfrac{1}{2},\tfrac{1}{4}-\tfrac{n}{2}\right],\left[\tfrac{5}{4}-\tfrac{n}{2}\right],\cos^2(dx+c)\right)(b\sec(dx+c))^n\sin(dx+c)}{d(1-2n)\sqrt{\sec(dx+c)}\,\sqrt{2-2\cos(2dx+2c)}}$$

command

```
Integrate[((b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[Sec[c + d*x]],x]
```

Mathematica 13.1 output

$$-\frac{i2^{\frac{3}{2}+n}e^{-\frac{1}{2}i(2c+d(1+2n)x)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+n}\left(Ae^{\frac{1}{2}id(-1+2n)x}\left(105+352n+344n^2+128n^3+16n^4\right)\right.}{}$$

Mathematica 12.3 output

$Aborted

20 Test file number 144

Test folder name:

```
test_cases/5_Inverse_trig_functions/5.1_Inverse_sine/144_5.1.5_Inverse_sine_functions
```

20.1 Problem number 84

$$\int \frac{(a+b\text{ArcSin}(cx))^2 \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

Optimal antiderivative

$$\frac{im(a+b\arcsin(cx))^4}{12b^2c}+\frac{(a+b\arcsin(cx))^3\ln(h(gx+f)^m)}{3bc}-\frac{m(a+b\arcsin(cx))^3\ln\left(1-\frac{i\left(icx+\sqrt{-c^2x^2+1}\right)g}{cf-\sqrt{c^2f^2-g^2}}\right)}{3bc}-\frac{m(a+b\arcsin(cx))^3\ln\left(1-\frac{i\left(icx+\sqrt{-c^2x^2+1}\right)g}{cf+\sqrt{c^2f^2-g^2}}\right)}{3bc}+\frac{im(a+b\arcsin(cx))^2\operatorname{polylog}\left(2,\frac{i\left(icx+\sqrt{-c^2x^2+1}\right)g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{im(a+b\arcsin(cx))^2\operatorname{polylog}\left(2,\frac{i\left(icx+\sqrt{-c^2x^2+1}\right)g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{2bm(a+b\arcsin(cx))\operatorname{polylog}\left(3,\frac{i\left(icx+\sqrt{-c^2x^2+1}\right)g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{2bm(a+b\arcsin(cx))\operatorname{polylog}\left(3,\frac{i\left(icx+\sqrt{-c^2x^2+1}\right)g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{2ib^2m\operatorname{polylog}\left(4,\frac{i\left(icx+\sqrt{-c^2x^2+1}\right)g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{2ib^2m\operatorname{polylog}\left(4,\frac{i\left(icx+\sqrt{-c^2x^2+1}\right)g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}$$

command

```
Integrate[((a + b*ArcSin[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

Mathematica 13.1 output

Result too large to show

Mathematica 12.3 output

$$\int\frac{(a+b\sin^{-1}(cx))^2\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx$$

21 Test file number 147

Test folder name:

`test_cases/5_Inverse_trig_functions/5.2_Inverse_cosine/147_5.2.5_Inverse_cosine_functions`

21.1 Problem number 20

$$\int \frac{(a+b\mathrm{ArcCos}(cx))^2 \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

Optimal antiderivative

$$\begin{aligned}
&-\frac{im(a+b\arccos(cx))^4}{12b^2c}-\frac{(a+b\arccos(cx))^3\ln\left(h(gx+f)^m\right)}{3bc}\\
&+\frac{m(a+b\arccos(cx))^3\ln\left(1+\frac{\left(cx+i\sqrt{-c^2x^2+1}\right)g}{cf-\sqrt{c^2f^2-g^2}}\right)}{3bc}\\
&+\frac{m(a+b\arccos(cx))^3\ln\left(1+\frac{\left(cx+i\sqrt{-c^2x^2+1}\right)g}{cf+\sqrt{c^2f^2-g^2}}\right)}{3bc}\\
&-\frac{im(a+b\arccos(cx))^2\,\mathrm{polylog}\left(2,-\frac{\left(cx+i\sqrt{-c^2x^2+1}\right)g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}\\
&-\frac{im(a+b\arccos(cx))^2\,\mathrm{polylog}\left(2,-\frac{\left(cx+i\sqrt{-c^2x^2+1}\right)g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}\\
&+\frac{2bm(a+b\arccos(cx))\,\mathrm{polylog}\left(3,-\frac{\left(cx+i\sqrt{-c^2x^2+1}\right)g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}\\
&+\frac{2bm(a+b\arccos(cx))\,\mathrm{polylog}\left(3,-\frac{\left(cx+i\sqrt{-c^2x^2+1}\right)g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}\\
&+\frac{2ib^2m\,\mathrm{polylog}\left(4,-\frac{\left(cx+i\sqrt{-c^2x^2+1}\right)g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}\\
&+\frac{2ib^2m\,\mathrm{polylog}\left(4,-\frac{\left(cx+i\sqrt{-c^2x^2+1}\right)g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}
\end{aligned}$$

command

```
Integrate[((a + b*ArcCos[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

Mathematica 13.1 output

Result too large to show

Mathematica 12.3 output

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^2 \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

22 Test file number 193

Test folder name:

```
test_cases/7_Inverse_hyperbolic_functions/7.3_Inverse_hyperbolic_tangent/193_7.3.3-d+e_x-^m-a+b_arctanh-c_x^n-^p
```

22.1 Problem number 18

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{d+ex}\,dx$$

Optimal antiderivative

$$\begin{aligned}
&-\frac{(a+b\operatorname{arctanh}(cx))^3 \ln\left(\frac{2}{cx+1}\right)}{e}+\frac{(a+b\operatorname{arctanh}(cx))^3 \ln\left(\frac{2c(ex+d)}{(cd+e)(cx+1)}\right)}{e}\\
&+\frac{3b(a+b\operatorname{arctanh}(cx))^2 \operatorname{polylog}\left(2,1-\frac{2}{cx+1}\right)}{2e}\\
&-\frac{3b(a+b\operatorname{arctanh}(cx))^2 \operatorname{polylog}\left(2,1-\frac{2c(ex+d)}{(cd+e)(cx+1)}\right)}{2e}\\
&+\frac{3b^2(a+b\operatorname{arctanh}(cx)) \operatorname{polylog}\left(3,1-\frac{2}{cx+1}\right)}{2e}\\
&-\frac{3b^2(a+b\operatorname{arctanh}(cx)) \operatorname{polylog}\left(3,1-\frac{2c(ex+d)}{(cd+e)(cx+1)}\right)}{2e}\\
&+\frac{3b^3 \operatorname{polylog}\left(4,1-\frac{2}{cx+1}\right)}{4e}-\frac{3b^3 \operatorname{polylog}\left(4,1-\frac{2c(ex+d)}{(cd+e)(cx+1)}\right)}{4e}
\end{aligned}$$

command

```
Integrate[(a + b*ArcTanh[c*x])^3/(d + e*x),x]
```

Mathematica 13.1 output

Result too large to show

Mathematica 12.3 output

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{d+ex}\,dx$$

23 Test file number 208

Test folder name:

`test_cases/8_Special_functions/208_8.8_Polylogarithm_function`

23.1 Problem number 183

$$\int \frac{(g+h\log\left(f(d+ex)^n\right))\,\mathrm{PolyLog}(2,c(a+bx))}{x^4}\,dx$$

Optimal antiderivative

Expression too large to display

command

```
Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^4,x]
```

Mathematica 13.1 output

Result too large to show

Mathematica 12.3 output

$$\int \frac{(g+h\log\left(f(d+ex)^n\right))\,\mathrm{Li}_2(c(a+bx))}{x^4}\,dx$$

24 Test file number 209

Test folder name:

`test_cases/209_Blake_problems`

24.1 Problem number 63

$$\int \frac{-1+2x^6}{\sqrt{1+x^6}\,(1-x^2+x^6)}\,dx$$

Optimal antiderivative

$$-\operatorname{arctanh}\left(\frac{x}{\sqrt{x^6+1}}\right)$$

command

```
Integrate[(-1 + 2*x^6)/(Sqrt[1 + x^6]*(1 - x^2 + x^6)),x]
```

Mathematica 13.1 output

$$-\tanh^{-1}\left(\frac{x}{\sqrt{1+x^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{-1+2x^6}{\sqrt{1+x^6}\,(1-x^2+x^6)}\,dx$$

24.2 Problem number 64

$$\int \frac{1+2x^6}{\sqrt{-1+x^6}\,(-1+x^2+x^6)}\,dx$$

Optimal antiderivative

$$-\arctan\left(\frac{x}{\sqrt{x^6-1}}\right)$$

command

```
Integrate[(1 + 2*x^6)/(Sqrt[-1 + x^6]*(-1 + x^2 + x^6)),x]
```

Mathematica 13.1 output

$$-\operatorname{ArcTan}\left(\frac{x}{\sqrt{-1+x^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{1+2x^6}{\sqrt{-1+x^6}\,(-1+x^2+x^6)}\,dx$$

24.3 Problem number 99

$$\int \frac{-2b+3ax^5}{\sqrt{b+ax^5}\,(b+x^2+ax^5)}\,dx$$

Optimal antiderivative

$$-2\arctan\left(\frac{x}{\sqrt{a\,x^5+b}}\right)$$

command

```
Integrate[(-2*b + 3*a*x^5)/(Sqrt[b + a*x^5]*(b + x^2 + a*x^5)),x]
```

Mathematica 13.1 output

$$-2\text{ArcTan}\left(\frac{x}{\sqrt{b+ax^5}}\right)$$

Mathematica 12.3 output

$$\int \frac{-2b+3ax^5}{\sqrt{b+ax^5}\,(b+x^2+ax^5)}\,dx$$

24.4 Problem number 119

$$\int \frac{-1+2x^2+2x^4}{(1+2x^2)\,\sqrt{-1+x^6}}\,dx$$

Optimal antiderivative

$$\text{arctanh}\left(\frac{x\left(x^2-1\right)}{\sqrt{x^6-1}}\right)$$

command

```
Integrate[(-1 + 2*x^2 + 2*x^4)/((1 + 2*x^2)*Sqrt[-1 + x^6]),x]
```

Mathematica 13.1 output

$$\tanh^{-1}\left(\frac{x\left(-1+x^2\right)}{\sqrt{-1+x^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{-1+2x^2+2x^4}{(1+2x^2)\,\sqrt{-1+x^6}}\,dx$$

24.5 Problem number 190

$$\int \frac{\sqrt{1+\sqrt{1+x^2}}}{1+x^2}\,dx$$

Optimal antiderivative

$$2\arctan\left(\frac{x}{\sqrt{1+\sqrt{x^2+1}}}\right)$$

command

```
Integrate[Sqrt[1 + Sqrt[1 + x^2]]/(1 + x^2),x]
```

Mathematica 13.1 output

$$2\text{ArcTan}\left(\frac{x}{\sqrt{1+\sqrt{1+x^2}}}\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1+\sqrt{1+x^2}}}{1+x^2}\,dx$$

24.6 Problem number 243

$$\int \frac{2-3x^5}{\sqrt{1+x^5}\,(1-ax^2+x^5)}\,dx$$

Optimal antiderivative

$$\frac{2\,\text{arctanh}\left(\frac{\sqrt{a}\,x}{\sqrt{x^5+1}}\right)}{\sqrt{a}}$$

command

```
Integrate[(2 - 3*x^5)/(Sqrt[1 + x^5]*(1 - a*x^2 + x^5)),x]
```

Mathematica 13.1 output

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{1+x^5}}\right)}{\sqrt{a}}$$

Mathematica 12.3 output

$$\int \frac{2-3x^5}{\sqrt{1+x^5}\,(1-ax^2+x^5)}\,dx$$

24.7 Problem number 244

$$\int \frac{2+3x^5}{\sqrt{-1+x^5}\,(-1-ax^2+x^5)}\,dx$$

Optimal antiderivative

$$-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\,x}{\sqrt{x^5-1}}\right)}{\sqrt{a}}$$

command

```
Integrate[(2 + 3*x^5)/(Sqrt[-1 + x^5]*(-1 - a*x^2 + x^5)),x]
```

Mathematica 13.1 output

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{-1+x^5}}\right)}{\sqrt{a}}$$

Mathematica 12.3 output

$$\int \frac{2+3x^5}{\sqrt{-1+x^5}\,(-1-ax^2+x^5)}\,dx$$

24.8 Problem number 245

$$\int \frac{2+3x^5}{\sqrt{-1+x^5}\,(-a-x^2+ax^5)}\,dx$$

Optimal antiderivative

$$-\frac{2\operatorname{arctanh}\left(\frac{x}{\sqrt{a}\,\sqrt{x^5-1}}\right)}{\sqrt{a}}$$

command

```
Integrate[(2 + 3*x^5)/(Sqrt[-1 + x^5]*(-a - x^2 + a*x^5)),x]
```

Mathematica 13.1 output

$$-\frac{2\tanh^{-1}\left(\frac{x}{\sqrt{a}\,\sqrt{-1+x^5}}\right)}{\sqrt{a}}$$

Mathematica 12.3 output

$$\int \frac{2+3x^5}{\sqrt{-1+x^5}\,(-a-x^2+ax^5)}\,dx$$

24.9 Problem number 246

$$\int \frac{-2+3x^5}{\sqrt{1+x^5}\,(a-x^2+ax^5)}\,dx$$

Optimal antiderivative

$$-\frac{2\operatorname{arctanh}\left(\frac{x}{\sqrt{a}\,\sqrt{x^5+1}}\right)}{\sqrt{a}}$$

command

```
Integrate[(-2 + 3*x^5)/(Sqrt[1 + x^5]*(a - x^2 + a*x^5)),x]
```

Mathematica 13.1 output

$$-\frac{2\tanh^{-1}\left(\frac{x}{\sqrt{a}\,\sqrt{1+x^5}}\right)}{\sqrt{a}}$$

Mathematica 12.3 output

$$\int \frac{-2+3x^5}{\sqrt{1+x^5}\,(a-x^2+ax^5)}\,dx$$

24.10 Problem number 288

$$\int \frac{-1-2x^2+2x^4}{(1+2x^4)\,\sqrt{1+x^6}}\,dx$$

Optimal antiderivative

$$-\arctan\left(\frac{x\sqrt{x^6+1}}{x^4-x^2+1}\right)$$

command

```
Integrate[(-1 - 2*x^2 + 2*x^4)/((1 + 2*x^4)*Sqrt[1 + x^6]),x]
```

Mathematica 13.1 output

$$-\operatorname{ArcTan}\left(\frac{x\sqrt{1+x^6}}{1-x^2+x^4}\right)$$

Mathematica 12.3 output

$$\int \frac{-1-2x^2+2x^4}{(1+2x^4)\,\sqrt{1+x^6}}\,dx$$

24.11 Problem number 293

$$\int \frac{\sqrt{-1+x^6}\left(1+2x^6\right)}{x^2\left(-1+x^2+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{x^6-1}}{x}+\arctan\left(\frac{x}{\sqrt{x^6-1}}\right)$$

command

```
Integrate[(Sqrt[-1 + x^6]*(1 + 2*x^6))/(x^2*(-1 + x^2 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{-1+x^6}}{x}+\text{ArcTan}\left(\frac{x}{\sqrt{-1+x^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{-1+x^6}\left(1+2x^6\right)}{x^2\left(-1+x^2+x^6\right)}\,dx$$

24.12 Problem number 343

$$\int \frac{-1+x}{(-3+x)(1+x)\sqrt[4]{-2-2x+x^2}}\,dx$$

Optimal antiderivative

$$\arctan\left(\left(x^2-2x-2\right)^{\frac{1}{4}}\right)-\operatorname{arctanh}\left(\left(x^2-2x-2\right)^{\frac{1}{4}}\right)$$

command

```
Integrate[(-1 + x)/((-3 + x)*(1 + x)*(-2 - 2*x + x^2)^(1/4)),x]
```

Mathematica 13.1 output

$$\text{ArcTan}\left(\sqrt[4]{-2-2x+x^2}\right)-\tanh^{-1}\left(\sqrt[4]{-2-2x+x^2}\right)$$

Mathematica 12.3 output

$$\int \frac{-1+x}{(-3+x)(1+x)\sqrt[4]{-2-2x+x^2}}\,dx$$

24.13 Problem number 351

$$\int \frac{x^2\left(-4+x^3\right)}{\left(-1+x^3\right)^{3/4}\left(1-x^3+x^4\right)}\,dx$$

Optimal antiderivative

$$2\arctan\left(\frac{x}{\left(x^3-1\right)^{\frac{1}{4}}}\right)-2\operatorname{arctanh}\left(\frac{x}{\left(x^3-1\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[(x^2*(-4 + x^3))/((-1 + x^3)^(3/4)*(1 - x^3 + x^4)),x]
```

Mathematica 13.1 output

$$2\text{ArcTan}\left(\frac{x}{\sqrt[4]{-1+x^3}}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^3}}\right)$$

Mathematica 12.3 output

$$\int \frac{x^2\left(-4+x^3\right)}{\left(-1+x^3\right)^{3/4}\left(1-x^3+x^4\right)}\,dx$$

24.14 Problem number 354

$$\int \frac{-1+2x^6}{\left(1+x^6\right)\sqrt{1-2x^2+x^6}}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{2}\,x}{\sqrt{x^6-2x^2+1}}\right)\sqrt{2}}{2}$$

command

```
Integrate[(-1 + 2*x^6)/((1 + x^6)*Sqrt[1 - 2*x^2 + x^6]),x]
```

Mathematica 13.1 output

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,x}{\sqrt{1-2x^2+x^6}}\right)}{\sqrt{2}}$$

Mathematica 12.3 output

$$\int \frac{-1+2x^6}{\left(1+x^6\right)\sqrt{1-2x^2+x^6}}\,dx$$

24.15 Problem number 355

$$\int \frac{1+2x^6}{(-1+x^6)\sqrt{-1-2x^2+x^6}}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{2}\,x}{\sqrt{x^6-2x^2-1}}\right)\sqrt{2}}{2}$$

command

```
Integrate[(1 + 2*x^6)/((-1 + x^6)*Sqrt[-1 - 2*x^2 + x^6]),x]
```

Mathematica 13.1 output

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,x}{\sqrt{-1-2x^2+x^6}}\right)}{\sqrt{2}}$$

Mathematica 12.3 output

$$\int \frac{1+2x^6}{(-1+x^6)\sqrt{-1-2x^2+x^6}}\,dx$$

24.16 Problem number 371

$$\int \frac{\sqrt{1-x^6}\,(1+2x^6)}{x^2\,(-1-x^2+x^6)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{-x^6+1}}{x}+\arctan\left(\frac{x}{\sqrt{-x^6+1}}\right)$$

command

```
Integrate[(Sqrt[1 - x^6]*(1 + 2*x^6))/(x^2*(-1 - x^2 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{1-x^6}}{x}+\text{ArcTan}\left(\frac{x}{\sqrt{1-x^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1-x^6}\,(1+2x^6)}{x^2\,(-1-x^2+x^6)}\,dx$$

24.17 Problem number 407

$$\int \frac{2b+ax^2}{\sqrt[4]{b+ax^2}\,(-b-ax^2+x^4)}\,dx$$

Optimal antiderivative

$$\arctan\left(\frac{(a\,x^2+b)^{\frac{1}{4}}}{x}\right)-\operatorname{arctanh}\left(\frac{x}{(a\,x^2+b)^{\frac{1}{4}}}\right)$$

command

```
Integrate[(2*b + a*x^2)/((b + a*x^2)^(1/4)*(-b - a*x^2 + x^4)),x]
```

Mathematica 13.1 output

$$\text{ArcTan}\left(\frac{\sqrt[4]{b+ax^2}}{x}\right)-\tanh^{-1}\left(\frac{x}{\sqrt[4]{b+ax^2}}\right)$$

Mathematica 12.3 output

$$\int \frac{2b+ax^2}{\sqrt[4]{b+ax^2}\,(-b-ax^2+x^4)}\,dx$$

24.18 Problem number 439

$$\int \frac{4b+ax^3}{\sqrt[4]{b+ax^3}\,(-b-ax^3+x^4)}\,dx$$

Optimal antiderivative

$$2\arctan\left(\frac{(x^3a+b)^{\frac{1}{4}}}{x}\right)-2\operatorname{arctanh}\left(\frac{x}{(x^3a+b)^{\frac{1}{4}}}\right)$$

command

```
Integrate[(4*b + a*x^3)/((b + a*x^3)^(1/4)*(-b - a*x^3 + x^4)),x]
```

Mathematica 13.1 output

$$2\text{ArcTan}\left(\frac{\sqrt[4]{b+ax^3}}{x}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt[4]{b+ax^3}}\right)$$

Mathematica 12.3 output

$$\int \frac{4b+ax^3}{\sqrt[4]{b+ax^3}\,(-b-ax^3+x^4)}\,dx$$

24.19 Problem number 447

$$\int \frac{-2bc + acx^6}{\sqrt[4]{b + ax^6}\,(b - c^4x^4 + ax^6)}\,dx$$

Optimal antiderivative

$$-\arctan\left(\frac{cx}{(a\,x^6 + b)^{\frac{1}{4}}}\right) - \operatorname{arctanh}\left(\frac{cx}{(a\,x^6 + b)^{\frac{1}{4}}}\right)$$

command

```
Integrate[(-2*b*c + a*c*x^6)/((b + a*x^6)^(1/4)*(b - c^4*x^4 + a*x^6)),x]
```

Mathematica 13.1 output

$$-\text{ArcTan}\left(\frac{cx}{\sqrt[4]{b + ax^6}}\right) - \tanh^{-1}\left(\frac{cx}{\sqrt[4]{b + ax^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{-2bc + acx^6}{\sqrt[4]{b + ax^6}\,(b - c^4x^4 + ax^6)}\,dx$$

24.20 Problem number 469

$$\int \frac{x\left(8b + 5ax^3\right)}{\sqrt[4]{b + ax^3}\,(-b - ax^3 + x^8)}\,dx$$

Optimal antiderivative

$$2\arctan\left(\frac{\left(x^3a + b\right)^{\frac{1}{4}}}{x^2}\right) - 2\operatorname{arctanh}\left(\frac{x^2}{(x^3a + b)^{\frac{1}{4}}}\right)$$

command

```
Integrate[(x*(8*b + 5*a*x^3))/((b + a*x^3)^(1/4)*(-b - a*x^3 + x^8)),x]
```

Mathematica 13.1 output

$$2\text{ArcTan}\left(\frac{\sqrt[4]{b + ax^3}}{x^2}\right) - 2\tanh^{-1}\left(\frac{x^2}{\sqrt[4]{b + ax^3}}\right)$$

Mathematica 12.3 output

$$\int \frac{x\left(8b + 5ax^3\right)}{\sqrt[4]{b + ax^3}\,(-b - ax^3 + x^8)}\,dx$$

24.21 Problem number 501

$$\int \frac{-3+2x}{\sqrt[4]{-x+x^2}\,(1-x+x^3)}\,dx$$

Optimal antiderivative

$$2\arctan\left(\frac{\left(x^2-x\right)^{\frac{1}{4}}}{x}\right)-2\operatorname{arctanh}\left(\frac{\left(x^2-x\right)^{\frac{3}{4}}}{-1+x}\right)$$

command

```
Integrate[(-3 + 2*x)/((-x + x^2)^(1/4)*(1 - x + x^3)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt[4]{-1+x}\,\sqrt[4]{x}\left(\text{ArcTan}\left(\frac{\sqrt[4]{-1+x}}{x^{3/4}}\right)-\tanh^{-1}\left(\frac{x^{3/4}}{\sqrt[4]{-1+x}}\right)\right)}{\sqrt[4]{(-1+x)x}}$$

Mathematica 12.3 output

$$\int \frac{-3+2x}{\sqrt[4]{-x+x^2}\,(1-x+x^3)}\,dx$$

24.22 Problem number 509

$$\int \frac{\sqrt{-1+x^5}\,\left(2+3x^5\right)}{x^2\left(-1-ax^2+x^5\right)}\,dx$$

Optimal antiderivative

$$\frac{2\sqrt{x^5-1}}{x}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a}\,x}{\sqrt{x^5-1}}\right)$$

command

```
Integrate[(Sqrt[-1 + x^5]*(2 + 3*x^5))/(x^2*(-1 - a*x^2 + x^5)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{-1+x^5}}{x}-2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{-1+x^5}}\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{-1+x^5}\,\left(2+3x^5\right)}{x^2\left(-1-ax^2+x^5\right)}\,dx$$

24.23 Problem number 525

$$\int \frac{4x+3x^2}{\sqrt{-5+4x^2+2x^3+4x^4+4x^5+x^6}}\,dx$$

Optimal antiderivative

$$\ln\left(1+2x^2+x^3+\sqrt{x^6+4x^5+4x^4+2x^3+4x^2-5}\right)$$

command

```
Integrate[(4*x + 3*x^2)/Sqrt[-5 + 4*x^2 + 2*x^3 + 4*x^4 + 4*x^5 + x^6],x]
```

Mathematica 13.1 output

$$\log\left(1+2x^2+x^3+\sqrt{-5+4x^2+2x^3+4x^4+4x^5+x^6}\right)$$

Mathematica 12.3 output

$$\int \frac{4x+3x^2}{\sqrt{-5+4x^2+2x^3+4x^4+4x^5+x^6}}\,dx$$

24.24 Problem number 526

$$\int \frac{x^2\sqrt{q+px^5}\left(-2q+3px^5\right)}{bx^6+a\left(q+px^5\right)^3}\,dx$$

Optimal antiderivative

$$-\frac{2\arctan\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}\,\left(px^5+q\right)^{\frac{3}{2}}}\right)}{3\sqrt{a}\,\sqrt{b}}$$

command

```
Integrate[(x^2*Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(b*x^6 + a*(q + p*x^5)^3),x]
```

Mathematica 13.1 output

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}\,\left(q+px^5\right)^{3/2}}\right)}{3\sqrt{a}\,\sqrt{b}}$$

Mathematica 12.3 output

$$\int \frac{x^2\sqrt{q+px^5}\left(-2q+3px^5\right)}{bx^6+a\left(q+px^5\right)^3}\,dx$$

24.25 Problem number 531

$$\int \frac{4c+3bx+2ax^2}{\sqrt[4]{c+bx+ax^2}\,(-c-bx-ax^2+x^4)}\,dx$$

Optimal antiderivative

$$2\arctan\left(\frac{\left(a\,x^2+bx+c\right)^{\frac{1}{4}}}{x}\right)-2\operatorname{arctanh}\left(\frac{x}{\left(a\,x^2+bx+c\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[(4*c + 3*b*x + 2*a*x^2)/((c + b*x + a*x^2)^(1/4)*(-c - b*x - a*x^2 + x^4)),x]
```

Mathematica 13.1 output

$$2\left(\text{ArcTan}\left(\frac{\sqrt[4]{c+x(b+ax)}}{x}\right)-\tanh^{-1}\left(\frac{x}{\sqrt[4]{c+x(b+ax)}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{4c+3bx+2ax^2}{\sqrt[4]{c+bx+ax^2}\,(-c-bx-ax^2+x^4)}\,dx$$

24.26 Problem number 561

$$\int \frac{2+x^3+x^6}{x\sqrt[4]{1+x^6}\,(-4+5x^3-4x^6+x^9)}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{-x^3+1}{\left(x^6+1\right)^{\frac{1}{4}}}\right)}{3}-\frac{\operatorname{arctanh}\left(\frac{x^3-1}{\left(x^6+1\right)^{\frac{1}{4}}}\right)}{3}$$

command

```
Integrate[(2 + x^3 + x^6)/(x*(1 + x^6)^(1/4)*(-4 + 5*x^3 - 4*x^6 + x^9)),x]
```

Mathematica 13.1 output

$$\frac{1}{3}\text{ArcTan}\left(\frac{1-x^3}{\sqrt[4]{1+x^6}}\right)-\frac{1}{3}\tanh^{-1}\left(\frac{-1+x^3}{\sqrt[4]{1+x^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{2+x^3+x^6}{x\sqrt[4]{1+x^6}\,(-4+5x^3-4x^6+x^9)}\,dx$$

24.27 Problem number 575

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b^2+ax^2}\,dx$$

Optimal antiderivative

$$\frac{2\arctan\left(\frac{\sqrt{a}\ x}{\sqrt{b}\ \sqrt{b+\sqrt{a\,x^2+b^2}}}\right)}{\sqrt{a}\ \sqrt{b}}$$

command

```
Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2),x]
```

Mathematica 13.1 output

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\ x}{\sqrt{b}\ \sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{a}\ \sqrt{b}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b^2+ax^2}\,dx$$

24.28 Problem number 582

$$\int \frac{\sqrt{1+x^2+x^5}\left(-2+3x^5\right)}{\left(1+x^5\right)\left(1-x^2+x^5\right)}\,dx$$

Optimal antiderivative

$$2\,\text{arctanh}\left(\frac{x}{\sqrt{x^5+x^2+1}}\right)-2\sqrt{2}\ \text{arctanh}\left(\frac{\sqrt{2}\ x}{\sqrt{x^5+x^2+1}}\right)$$

command

```
Integrate[(Sqrt[1 + x^2 + x^5]*(-2 + 3*x^5))/((1 + x^5)*(1 - x^2 + x^5)),x]
```

Mathematica 13.1 output

$$2\tanh^{-1}\left(\frac{x}{\sqrt{1+x^2+x^5}}\right)-2\sqrt{2}\ \tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^2+x^5}}\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1+x^2+x^5}\left(-2+3x^5\right)}{\left(1+x^5\right)\left(1-x^2+x^5\right)}\,dx$$

24.29 Problem number 592

$$\int \frac{-1-2x+3x^2}{\sqrt{-3-2x-x^2+4x^3-x^4-2x^5+x^6}}\,dx$$

Optimal antiderivative

$$-\ln\left(-1+x+x^2-x^3+\sqrt{x^6-2x^5-x^4+4x^3-x^2-2x-3}\right)$$

command

```
Integrate[(-1 - 2*x + 3*x^2)/Sqrt[-3 - 2*x - x^2 + 4*x^3 - x^4 - 2*x^5 + x^6],x]
```

Mathematica 13.1 output

$$-\log\left(-1+x+x^2-x^3+\sqrt{-3-2x-x^2+4x^3-x^4-2x^5+x^6}\right)$$

Mathematica 12.3 output

$$\int \frac{-1-2x+3x^2}{\sqrt{-3-2x-x^2+4x^3-x^4-2x^5+x^6}}\,dx$$

24.30 Problem number 600

$$\int \frac{\sqrt{1+x^2+x^6}\left(-1+2x^6\right)}{\left(1+x^6\right)\left(2-x^2+2x^6\right)}\,dx$$

Optimal antiderivative

$$\operatorname{arctanh}\left(\frac{x}{\sqrt{x^6+x^2+1}}\right)-\frac{\sqrt{6}\,\operatorname{arctanh}\left(\frac{\sqrt{6}\,x}{2\sqrt{x^6+x^2+1}}\right)}{2}$$

command

```
Integrate[(Sqrt[1 + x^2 + x^6]*(-1 + 2*x^6))/((1 + x^6)*(2 - x^2 + 2*x^6)),x]
```

Mathematica 13.1 output

$$\tanh^{-1}\left(\frac{x}{\sqrt{1+x^2+x^6}}\right)-\sqrt{\frac{3}{2}}\,\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt{1+x^2+x^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1+x^2+x^6}\left(-1+2x^6\right)}{\left(1+x^6\right)\left(2-x^2+2x^6\right)}\,dx$$

24.31 Problem number 602

$$\int \frac{\left(-1+6x^4\right)\sqrt{x+2x^5}}{\left(1+2x^4\right)\left(1-x^2+4x^4+4x^8\right)}\,dx$$

Optimal antiderivative

$$\arctan\left(\frac{\sqrt{2x^5+x}}{2x^4+1}\right)-\operatorname{arctanh}\left(\frac{\sqrt{2x^5+x}}{2x^4+1}\right)$$

command

```
Integrate[((-1 + 6*x^4)*Sqrt[x + 2*x^5])/((1 + 2*x^4)*(1 - x^2 + 4*x^4 + 4*x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{x+2x^5}\left(\text{ArcTan}\left(\frac{\sqrt{x}}{\sqrt{1+2x^4}}\right)-\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+2x^4}}\right)\right)}{\sqrt{x}\,\sqrt{1+2x^4}}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+6x^4\right)\sqrt{x+2x^5}}{\left(1+2x^4\right)\left(1-x^2+4x^4+4x^8\right)}\,dx$$

24.32 Problem number 619

$$\int \frac{-3b+2ax}{\sqrt[4]{-bx+ax^2}\,\left(b-ax+x^3\right)}\,dx$$

Optimal antiderivative

$$2\arctan\left(\frac{\left(a\,x^2-bx\right)^{\frac{1}{4}}}{x}\right)-2\operatorname{arctanh}\left(\frac{\left(a\,x^2-bx\right)^{\frac{3}{4}}}{ax-b}\right)$$

command

```
Integrate[(-3*b + 2*a*x)/((-(b*x) + a*x^2)^(1/4)*(b - a*x + x^3)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt[4]{x}\,\sqrt[4]{-b+ax}\left(\text{ArcTan}\left(\frac{\sqrt[4]{-b+ax}}{x^{3/4}}\right)-\tanh^{-1}\left(\frac{x^{3/4}}{\sqrt[4]{-b+ax}}\right)\right)}{\sqrt[4]{x(-b+ax)}}$$

Mathematica 12.3 output

$$\int \frac{-3b+2ax}{\sqrt[4]{-bx+ax^2}\,\left(b-ax+x^3\right)}\,dx$$

24.33 Problem number 625

$$\int \frac{-4b+ax^3}{\sqrt[4]{-b+ax^3}\,\left(b-ax^3+x^4\right)}\,dx$$

Optimal antiderivative

$$2\arctan\left(\frac{\left(x^3a-b\right)^{\frac{1}{4}}}{x}\right)+2\operatorname{arctanh}\left(\frac{x\left(x^3a-b\right)^{\frac{3}{4}}}{-x^3a+b}\right)$$

command

```
Integrate[(-4*b + a*x^3)/((-b + a*x^3)^(1/4)*(b - a*x^3 + x^4)),x]
```

Mathematica 13.1 output

$$2\text{ArcTan}\left(\frac{\sqrt[4]{-b+ax^3}}{x}\right)+2\tanh^{-1}\left(\frac{x\left(-b+ax^3\right)^{3/4}}{b-ax^3}\right)$$

Mathematica 12.3 output

$$\int \frac{-4b+ax^3}{\sqrt[4]{-b+ax^3}\,\left(b-ax^3+x^4\right)}\,dx$$

24.34 Problem number 631

$$\int \frac{\sqrt{-1+x^5}\,\left(2+3x^5\right)}{1-ax^4-2x^5+x^{10}}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{a^{\frac{1}{4}}x}{\sqrt{x^5-1}}\right)}{a^{\frac{1}{4}}}-\frac{\operatorname{arctanh}\left(\frac{a^{\frac{1}{4}}x}{\sqrt{x^5-1}}\right)}{a^{\frac{1}{4}}}$$

command

```
Integrate[(Sqrt[-1 + x^5]*(2 + 3*x^5))/(1 - a*x^4 - 2*x^5 + x^10),x]
```

Mathematica 13.1 output

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{-1+x^5}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{-1+x^5}}\right)}{\sqrt[4]{a}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{-1+x^5}\,\left(2+3x^5\right)}{1-ax^4-2x^5+x^{10}}\,dx$$

24.35 Problem number 632

$$\int \frac{\sqrt{-1+x^5}\left(2+3x^5\right)}{a-x^4-2ax^5+ax^{10}}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{x}{a^{\frac{1}{4}}\sqrt{x^5-1}}\right)}{a^{\frac{3}{4}}}-\frac{\operatorname{arctanh}\left(\frac{x}{a^{\frac{1}{4}}\sqrt{x^5-1}}\right)}{a^{\frac{3}{4}}}$$

command

```
Integrate[(Sqrt[-1 + x^5]*(2 + 3*x^5))/(a - x^4 - 2*a*x^5 + a*x^10),x]
```

Mathematica 13.1 output

$$-\frac{\operatorname{ArcTan}\left(\frac{x}{\sqrt[4]{a}\sqrt{-1+x^5}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{a}\sqrt{-1+x^5}}\right)}{a^{3/4}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{-1+x^5}\left(2+3x^5\right)}{a-x^4-2ax^5+ax^{10}}\,dx$$

24.36 Problem number 633

$$\int \frac{\sqrt{1+x^5}\left(-2+3x^5\right)}{a-x^4+2ax^5+ax^{10}}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{x}{a^{\frac{1}{4}}\sqrt{x^5+1}}\right)}{a^{\frac{3}{4}}}-\frac{\operatorname{arctanh}\left(\frac{x}{a^{\frac{1}{4}}\sqrt{x^5+1}}\right)}{a^{\frac{3}{4}}}$$

command

```
Integrate[(Sqrt[1 + x^5]*(-2 + 3*x^5))/(a - x^4 + 2*a*x^5 + a*x^10),x]
```

Mathematica 13.1 output

$$-\frac{\operatorname{ArcTan}\left(\frac{x}{\sqrt[4]{a}\sqrt{1+x^5}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{a}\sqrt{1+x^5}}\right)}{a^{3/4}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1+x^5}\left(-2+3x^5\right)}{a-x^4+2ax^5+ax^{10}}\,dx$$

24.37 Problem number 641

$$\int \frac{\left(-1+x^6\right)\left(1+x^6\right)}{\sqrt[4]{x-x^4+x^7}\left(1+3x^6+x^{12}\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^6)*(1 + x^6))/((x - x^4 + x^7)^(1/4)*(1 + 3*x^6 + x^12)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{x}\ \sqrt[4]{1-x^3+x^6}\ \text{RootSum}\left[2+2\#1^4+\#1^8\&,\ \frac{-3\log\left(\sqrt[4]{x}\right)+\log\left(\sqrt[4]{1-x^3+x^6}-x^{3/4}\#1\right)}{\#1}\&\right]}{6\sqrt[4]{x-x^4+x^7}}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^6\right)\left(1+x^6\right)}{\sqrt[4]{x-x^4+x^7}\left(1+3x^6+x^{12}\right)}\,dx$$

24.38 Problem number 647

$$\int \frac{\sqrt{-1-x^2+x^6}\left(1+2x^6\right)}{\left(-1+x^6\right)\left(-2+x^2+2x^6\right)}\,dx$$

Optimal antiderivative

$$\arctan\left(\frac{x}{\sqrt{x^6-x^2-1}}\right)-\frac{\sqrt{6}\ \arctan\left(\frac{\sqrt{6}\ x}{2\sqrt{x^6-x^2-1}}\right)}{2}$$

command

```
Integrate[(Sqrt[-1 - x^2 + x^6]*(1 + 2*x^6))/((-1 + x^6)*(-2 + x^2 + 2*x^6)),x]
```

Mathematica 13.1 output

$$\text{ArcTan}\left(\frac{x}{\sqrt{-1-x^2+x^6}}\right)-\sqrt{\frac{3}{2}}\ \text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}\ x}{\sqrt{-1-x^2+x^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{-1-x^2+x^6}\left(1+2x^6\right)}{\left(-1+x^6\right)\left(-2+x^2+2x^6\right)}\,dx$$

24.39 Problem number 648

$$\int \frac{x\left(-8b+5ax^3\right)}{\sqrt[4]{-b+ax^3}\,\left(b-ax^3+x^8\right)}\,dx$$

Optimal antiderivative

$$2\arctan\left(\frac{\left(x^3a-b\right)^{\frac{1}{4}}}{x^2}\right)+2\operatorname{arctanh}\left(\frac{x^2\left(x^3a-b\right)^{\frac{3}{4}}}{-x^3a+b}\right)$$

command

```
Integrate[(x*(-8*b + 5*a*x^3))/((-b + a*x^3)^(1/4)*(b - a*x^3 + x^8)),x]
```

Mathematica 13.1 output

$$2\text{ArcTan}\left(\frac{\sqrt[4]{-b+ax^3}}{x^2}\right)+2\tanh^{-1}\left(\frac{x^2\left(-b+ax^3\right)^{3/4}}{b-ax^3}\right)$$

Mathematica 12.3 output

$$\int \frac{x\left(-8b+5ax^3\right)}{\sqrt[4]{-b+ax^3}\,\left(b-ax^3+x^8\right)}\,dx$$

24.40 Problem number 650

$$\int \frac{-1+7x^8}{\left(1+x^8\right)\sqrt{3-x+x^2+6x^8-x^9+3x^{16}}}\,dx$$

Optimal antiderivative

$$2\operatorname{arctanh}\left(\frac{x}{\sqrt{3}+\sqrt{3}\,x^8-\sqrt{3x^{16}-x^9+6x^8+x^2-x+3}}\right)$$

command

```
Integrate[(-1 + 7*x^8)/((1 + x^8)*Sqrt[3 - x + x^2 + 6*x^8 - x^9 + 3*x^16]),x]
```

Mathematica 13.1 output

$$2\tanh^{-1}\left(\frac{x}{\sqrt{3}+\sqrt{3}\,x^8-\sqrt{3-x+x^2+6x^8-x^9+3x^{16}}}\right)$$

Mathematica 12.3 output

$$\int \frac{-1+7x^8}{\left(1+x^8\right)\sqrt{3-x+x^2+6x^8-x^9+3x^{16}}}\,dx$$

24.41 Problem number 692

$$\int \frac{2b+ax^2}{\sqrt[4]{b+ax^2}\,(-2b-2ax^2+x^4)}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{2^{\frac{1}{4}}(a\,x^2+b)^{\frac{1}{4}}}{x}\right)2^{\frac{1}{4}}}{2}-\frac{\operatorname{arctanh}\left(\frac{x2^{\frac{3}{4}}}{2(a\,x^2+b)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{2}$$

command

```
Integrate[(2*b + a*x^2)/((b + a*x^2)^(1/4)*(-2*b - 2*a*x^2 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{b+ax^2}}{x}\right)-\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{b+ax^2}}\right)}{2^{3/4}}$$

Mathematica 12.3 output

$$\int \frac{2b+ax^2}{\sqrt[4]{b+ax^2}\,(-2b-2ax^2+x^4)}\,dx$$

24.42 Problem number 715

$$\int \frac{\left(-2+x^3\right)\sqrt[3]{x+x^3+x^4}}{1+x^2+2x^3+x^4+x^5+x^6}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-2 + x^3)*(x + x^3 + x^4)^(1/3))/(1 + x^2 + 2*x^3 + x^4 + x^5 + x^6),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x+x^3+x^4}\,\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-2\log\left(\sqrt[3]{x}\right)\#1+\log\left(\sqrt[3]{1+x^2+x^3}-x^{2/3}\#1\right)\#1}{-1+2\#1^3}\&\right]}{\sqrt[3]{x}\,\sqrt[3]{1+x^2+x^3}}$$

Mathematica 12.3 output

$$\int \frac{\left(-2+x^3\right)\sqrt[3]{x+x^3+x^4}}{1+x^2+2x^3+x^4+x^5+x^6}\,dx$$

24.43 Problem number 716

$$\int \frac{\left(-2+x^3\right)\sqrt[3]{x+x^3+x^4}}{1+x^2+2x^3+x^4+x^5+x^6}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-2 + x^3)*(x + x^3 + x^4)^(1/3))/(1 + x^2 + 2*x^3 + x^4 + x^5 + x^6),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x+x^3+x^4}\,\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-2\log\left(\sqrt[3]{x}\right)\#1+\log\left(\sqrt[3]{1+x^2+x^3}-x^{2/3}\#1\right)\#1}{-1+2\#1^3}\&\right]}{\sqrt[3]{x}\,\sqrt[3]{1+x^2+x^3}}$$

Mathematica 12.3 output

$$\int \frac{\left(-2+x^3\right)\sqrt[3]{x+x^3+x^4}}{1+x^2+2x^3+x^4+x^5+x^6}\,dx$$

24.44 Problem number 720

$$\int \frac{x^2}{(b+ax^4)^{3/4}\left(b^2+a^2x^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[x^2/((b + a*x^4)^(3/4)*(b^2 + a^2*x^8)),x]
```

Mathematica 13.1 output

$$-\frac{\text{RootSum}\left[2a^2-2a\#1^4+\#1^8\&,\frac{-\log(x)+\log\left(\sqrt[4]{b+ax^4}-x\#1\right)}{\#1^3}\&\right]}{8b^2}$$

Mathematica 12.3 output

$$\int \frac{x^2}{(b+ax^4)^{3/4}\left(b^2+a^2x^8\right)}\,dx$$

24.45 Problem number 727

$$\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)}{x^2\left(aq+bx^2+apx^5\right)}\,dx$$

Optimal antiderivative

$$\frac{2\sqrt{p\,x^5+q}}{ax}+\frac{2\sqrt{b}\ \arctan\left(\frac{\sqrt{b}\,x}{\sqrt{a}\ \sqrt{p\,x^5+q}}\right)}{a^{\frac{3}{2}}}$$

command

```
Integrate[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(x^2*(a*q + b*x^2 + a*p*x^5)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{q+px^5}}{ax}+\frac{2\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}\ \sqrt{q+px^5}}\right)}{a^{3/2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)}{x^2\left(aq+bx^2+apx^5\right)}\,dx$$

24.46 Problem number 741

$$\int \frac{1}{\sqrt[4]{b+ax^4}\left(-b+ax^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[1/((b + a*x^4)^(1/4)*(-b + a*x^8)),x]
```

Mathematica 13.1 output

$$\frac{\text{RootSum}\left[a^2-ab-2a\#1^4+\#1^8\&,\frac{-\log(x)+\log\left(\sqrt[4]{b+ax^4}-x\#1\right)}{\#1}\&\right]}{8b}$$

Mathematica 12.3 output

$$\int \frac{1}{\sqrt[4]{b+ax^4}\left(-b+ax^8\right)}\,dx$$

24.47 Problem number 742

$$\int \frac{-b+ax^8}{\sqrt[4]{b+ax^8}\,(b-cx^4+ax^8)}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{c^{\frac{1}{4}}x}{(a\,x^8+b)^{\frac{1}{4}}}\right)}{2c^{\frac{1}{4}}}-\frac{\operatorname{arctanh}\left(\frac{c^{\frac{1}{4}}x}{(a\,x^8+b)^{\frac{1}{4}}}\right)}{2c^{\frac{1}{4}}}$$

command

```
Integrate[(-b + a*x^8)/((b + a*x^8)^(1/4)*(b - c*x^4 + a*x^8)),x]
```

Mathematica 13.1 output

$$-\frac{\operatorname{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{b+ax^8}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{b+ax^8}}\right)}{2\sqrt[4]{c}}$$

Mathematica 12.3 output

$$\int \frac{-b+ax^8}{\sqrt[4]{b+ax^8}\,(b-cx^4+ax^8)}\,dx$$

24.48 Problem number 765

$$\int \frac{x(3+4x)\sqrt[3]{-1-2x+x^3}}{-2-8x-8x^2+x^6}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x*(3 + 4*x)*(-1 - 2*x + x^3)^(1/3))/(-2 - 8*x - 8*x^2 + x^6),x]
```

Mathematica 13.1 output

$$-\frac{1}{4}\operatorname{RootSum}\left[1-4\#1^3+2\#1^6\&,\frac{-\log(x)\#1+\log\left(\sqrt[3]{-1-2x+x^3}-x\#1\right)\#1}{-1+\#1^3}\&\right]$$

Mathematica 12.3 output

$$\int \frac{x(3+4x)\sqrt[3]{-1-2x+x^3}}{-2-8x-8x^2+x^6}\,dx$$

24.49 Problem number 766

$$\int \frac{x(3+4x)\sqrt[3]{-1-2x+x^3}}{-2-8x-8x^2+x^6}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x*(3 + 4*x)*(-1 - 2*x + x^3)^(1/3))/(-2 - 8*x - 8*x^2 + x^6),x]
```

Mathematica 13.1 output

$$-\frac{1}{4}\text{RootSum}\left[1-4\#1^3+2\#1^6\&,\frac{-\log(x)\#1+\log\left(\sqrt[3]{-1-2x+x^3}-x\#1\right)\#1}{-1+\#1^3}\&\right]$$

Mathematica 12.3 output

$$\int \frac{x(3+4x)\sqrt[3]{-1-2x+x^3}}{-2-8x-8x^2+x^6}\,dx$$

24.50 Problem number 767

$$\int \frac{(3+2x)\left(1+x+x^3\right)^{2/3}}{1+2x+x^2+x^3+x^4+x^6}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((3 + 2*x)*(1 + x + x^3)^(2/3))/(1 + 2*x + x^2 + x^3 + x^4 + x^6),x]
```

Mathematica 13.1 output

$$-\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x)\#1^2+\log\left(\sqrt[3]{1+x+x^3}-x\#1\right)\#1^2}{-1+2\#1^3}\&\right]$$

Mathematica 12.3 output

$$\int \frac{(3+2x)\left(1+x+x^3\right)^{2/3}}{1+2x+x^2+x^3+x^4+x^6}\,dx$$

24.51 Problem number 768

$$\int \frac{(3+2x)\left(1+x+x^3\right)^{2/3}}{1+2x+x^2+x^3+x^4+x^6}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((3 + 2*x)*(1 + x + x^3)^(2/3))/(1 + 2*x + x^2 + x^3 + x^4 + x^6),x]
```

Mathematica 13.1 output

$$-\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x)\#1^2+\log\left(\sqrt[3]{1+x+x^3}-x\#1\right)\#1^2}{-1+2\#1^3}\&\right]$$

Mathematica 12.3 output

$$\int \frac{(3+2x)\left(1+x+x^3\right)^{2/3}}{1+2x+x^2+x^3+x^4+x^6}\,dx$$

24.52 Problem number 769

$$\int \frac{\left(2+x^3\right)\sqrt[3]{x+x^3-x^4}}{1+x^2-2x^3+x^4-x^5+x^6}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((2 + x^3)*(x + x^3 - x^4)^(1/3))/(1 + x^2 - 2*x^3 + x^4 - x^5 + x^6),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x+x^3-x^4}\,\text{RootSum}\left[1+\#1^3+\#1^6\&,\frac{-2\log\left(\sqrt[3]{x}\right)\#1+\log\left(\sqrt[3]{-1-x^2+x^3}-x^{2/3}\#1\right)\#1}{1+2\#1^3}\&\right]}{\sqrt[3]{x}\,\sqrt[3]{-1-x^2+x^3}}$$

Mathematica 12.3 output

$$\int \frac{\left(2+x^3\right)\sqrt[3]{x+x^3-x^4}}{1+x^2-2x^3+x^4-x^5+x^6}\,dx$$

24.53 Problem number 770

$$\int \frac{\left(2+x^3\right)\sqrt[3]{x+x^3-x^4}}{1+x^2-2x^3+x^4-x^5+x^6}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((2 + x^3)*(x + x^3 - x^4)^(1/3))/(1 + x^2 - 2*x^3 + x^4 - x^5 + x^6),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x+x^3-x^4}\,\text{RootSum}\left[1+\#1^3+\#1^6\&,\frac{-2\log\left(\sqrt[3]{x}\right)\#1+\log\left(\sqrt[3]{-1-x^2+x^3}-x^{2/3}\#1\right)\#1}{1+2\#1^3}\&\right]}{\sqrt[3]{x}\,\sqrt[3]{-1-x^2+x^3}}$$

Mathematica 12.3 output

$$\int \frac{\left(2+x^3\right)\sqrt[3]{x+x^3-x^4}}{1+x^2-2x^3+x^4-x^5+x^6}\,dx$$

24.54 Problem number 771

$$\int \frac{\left(3+x^2\right)\left(1+x^2+x^3\right)^{2/3}}{-1-2x^2+x^3-x^4+x^5+x^6}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((3 + x^2)*(1 + x^2 + x^3)^(2/3))/(-1 - 2*x^2 + x^3 - x^4 + x^5 + x^6),x]
```

Mathematica 13.1 output

$$\text{RootSum}\left[1-3\#1^3+\#1^6\&,\frac{-\log(x)\#1^2+\log\left(\sqrt[3]{1+x^2+x^3}-x\#1\right)\#1^2}{-3+2\#1^3}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\left(3+x^2\right)\left(1+x^2+x^3\right)^{2/3}}{-1-2x^2+x^3-x^4+x^5+x^6}\,dx$$

24.55 Problem number 772

$$\int \frac{\left(3+x^2\right)\left(1+x^2+x^3\right)^{2/3}}{-1-2x^2+x^3-x^4+x^5+x^6}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((3 + x^2)*(1 + x^2 + x^3)^(2/3))/(-1 - 2*x^2 + x^3 - x^4 + x^5 + x^6),x]
```

Mathematica 13.1 output

$$\text{RootSum}\left[1-3\#1^3+\#1^6\&,\frac{-\log(x)\#1^2+\log\left(\sqrt[3]{1+x^2+x^3}-x\#1\right)\#1^2}{-3+2\#1^3}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\left(3+x^2\right)\left(1+x^2+x^3\right)^{2/3}}{-1-2x^2+x^3-x^4+x^5+x^6}\,dx$$

24.56 Problem number 773

$$\int \frac{b+ax^8}{\sqrt[4]{b-ax^8}\left(-b+cx^4+ax^8\right)}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{c^{\frac{1}{4}}x}{\left(-a\,x^8+b\right)^{\frac{1}{4}}}\right)}{2c^{\frac{1}{4}}}-\frac{\text{arctanh}\left(\frac{c^{\frac{1}{4}}x}{\left(-a\,x^8+b\right)^{\frac{1}{4}}}\right)}{2c^{\frac{1}{4}}}$$

command

```
Integrate[(b + a*x^8)/((b - a*x^8)^(1/4)*(-b + c*x^4 + a*x^8)),x]
```

Mathematica 13.1 output

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{b-ax^8}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{b-ax^8}}\right)}{2\sqrt[4]{c}}$$

Mathematica 12.3 output

$$\int \frac{b+ax^8}{\sqrt[4]{b-ax^8}\left(-b+cx^4+ax^8\right)}\,dx$$

24.57 Problem number 774

$$\int \frac{x \sqrt[3]{2-x^3+x^8} \left(-6+5 x^8\right)}{4+x^6+4 x^8+x^{16}} \, dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x*(2 - x^3 + x^8)^(1/3)*(-6 + 5*x^8))/(4 + x^6 + 4*x^8 + x^16),x]
```

Mathematica 13.1 output

$$\frac{1}{2}\text{RootSum}\left[2+2\#1^3+\#1^6\&,\frac{-\log (x)\#1+\log \left(\sqrt[3]{2-x^3+x^8}-x\#1\right)\#1}{1+\#1^3}\&\right]$$

Mathematica 12.3 output

$$\int \frac{x \sqrt[3]{2-x^3+x^8} \left(-6+5 x^8\right)}{4+x^6+4 x^8+x^{16}} \, dx$$

24.58 Problem number 800

$$\int \frac{\left(6+x^4\right) \sqrt{-2 x+x^4+x^5}}{\left(-2+x^4\right) \left(-2-x^3+x^4\right)} \, dx$$

Optimal antiderivative

$$2 \operatorname{arctanh}\left(\frac{x^2}{\sqrt{x^5+x^4-2 x}}\right)-2 \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} x \sqrt{x^5+x^4-2 x}}{x^4+x^3-2}\right)$$

command

```
Integrate[((6 + x^4)*Sqrt[-2*x + x^4 + x^5])/((-2 + x^4)*(-2 - x^3 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{2 \sqrt{x} \sqrt{-2+x^3+x^4} \left(\tanh ^{-1}\left(\frac{x^{3/2}}{\sqrt{-2+x^3+x^4}}\right)-\sqrt{2} \tanh ^{-1}\left(\frac{\sqrt{2} x^{3/2}}{\sqrt{-2+x^3+x^4}}\right)\right)}{\sqrt{x \left(-2+x^3+x^4\right)}}$$

Mathematica 12.3 output

$$\int \frac{\left(6+x^4\right) \sqrt{-2 x+x^4+x^5}}{\left(-2+x^4\right) \left(-2-x^3+x^4\right)} \, dx$$

24.59 Problem number 801

$$\int \frac{\left(-3+x^2\right)\left(1-x^2+x^3\right)^{2/3}}{1-2x^2-x^3+x^4+x^5+x^6}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-3 + x^2)*(1 - x^2 + x^3)^(2/3))/(1 - 2*x^2 - x^3 + x^4 + x^5 + x^6),x]
```

Mathematica 13.1 output

$$\text{RootSum}\left[3-3\#1^3+\#1^6\&,\frac{-\log(x)\#1^2+\log\left(\sqrt[3]{1-x^2+x^3}-x\#1\right)\#1^2}{-3+2\#1^3}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\left(-3+x^2\right)\left(1-x^2+x^3\right)^{2/3}}{1-2x^2-x^3+x^4+x^5+x^6}\,dx$$

24.60 Problem number 802

$$\int \frac{\left(-3+x^2\right)\left(1-x^2+x^3\right)^{2/3}}{1-2x^2-x^3+x^4+x^5+x^6}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-3 + x^2)*(1 - x^2 + x^3)^(2/3))/(1 - 2*x^2 - x^3 + x^4 + x^5 + x^6),x]
```

Mathematica 13.1 output

$$\text{RootSum}\left[3-3\#1^3+\#1^6\&,\frac{-\log(x)\#1^2+\log\left(\sqrt[3]{1-x^2+x^3}-x\#1\right)\#1^2}{-3+2\#1^3}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\left(-3+x^2\right)\left(1-x^2+x^3\right)^{2/3}}{1-2x^2-x^3+x^4+x^5+x^6}\,dx$$

24.61 Problem number 807

$$\int \frac{\left(4+x^5\right)\sqrt{1-2x^5+x^8+x^{10}}}{x^9}\,dx$$

Optimal antiderivative

$$\frac{\left(x^5-1\right)\sqrt{x^{10}+x^8-2x^5+1}}{2x^8}-2\ln\left(x\right)+\frac{\ln\left(-1+x^5+\sqrt{x^{10}+x^8-2x^5+1}\right)}{2}$$

command

```
Integrate[((4 + x^5)*Sqrt[1 - 2*x^5 + x^8 + x^10])/x^9,x]
```

Mathematica 13.1 output

$$\frac{\left(-1+x^5\right)\sqrt{1-2x^5+x^8+x^{10}}}{2x^8}-\frac{1}{2}\tanh^{-1}\left(\frac{1-x^5}{\sqrt{1-2x^5+x^8+x^{10}}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(4+x^5\right)\sqrt{1-2x^5+x^8+x^{10}}}{x^9}\,dx$$

24.62 Problem number 808

$$\int \frac{\left(1+x^6\right)\left(-1+x^3+x^6\right)^{2/3}}{1-x^6+x^{12}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^6)*(-1 + x^3 + x^6)^(2/3))/(1 - x^6 + x^12),x]
```

Mathematica 13.1 output

$$\frac{1}{6}\text{RootSum}\left[2-2\#1^3+\#1^6\&,\frac{-\log(x)\#1^2+\log\left(\sqrt[3]{-1+x^3+x^6}-x\#1\right)\#1^2}{-1+\#1^3}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^6\right)\left(-1+x^3+x^6\right)^{2/3}}{1-x^6+x^{12}}\,dx$$

24.63 Problem number 809

$$\int \frac{\left(1+x^6\right)\left(-1+x^3+x^6\right)^{2/3}}{1-x^6+x^{12}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^6)*(-1 + x^3 + x^6)^(2/3))/(1 - x^6 + x^12),x]
```

Mathematica 13.1 output

$$\frac{1}{6}\text{RootSum}\left[2-2\#1^3+\#1^6\&,\frac{-\log(x)\#1^2+\log\left(\sqrt[3]{-1+x^3+x^6}-x\#1\right)\#1^2}{-1+\#1^3}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^6\right)\left(-1+x^3+x^6\right)^{2/3}}{1-x^6+x^{12}}\,dx$$

24.64 Problem number 810

$$\int \frac{\left(-2+5x^7\right)\sqrt[3]{2x+x^3+2x^8}}{4+x^4+8x^7+4x^{14}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-2 + 5*x^7)*(2*x + x^3 + 2*x^8)^(1/3))/(4 + x^4 + 8*x^7 + 4*x^14),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x\left(2+x^2+2x^7\right)}\,\text{RootSum}\left[2-2\#1^3+\#1^6\&,\frac{-2\log\left(\sqrt[3]{x}\right)\#1+\log\left(\sqrt[3]{2+x^2+2x^7}-x^{2/3}\#1\right)\#1}{-1+\#1^3}\&\right]}{4\sqrt[3]{x}\,\sqrt[3]{2+x^2+2x^7}}$$

Mathematica 12.3 output

$$\int \frac{\left(-2+5x^7\right)\sqrt[3]{2x+x^3+2x^8}}{4+x^4+8x^7+4x^{14}}\,dx$$

24.65 Problem number 811

$$\int \frac{\left(-2+5x^7\right)\sqrt[3]{2x+x^3+2x^8}}{4+x^4+8x^7+4x^{14}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-2 + 5*x^7)*(2*x + x^3 + 2*x^8)^(1/3))/(4 + x^4 + 8*x^7 + 4*x^14),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x\left(2+x^2+2x^7\right)}\,\text{RootSum}\left[2-2\#1^3+\#1^6\&,\frac{-2\log\left(\sqrt[3]{x}\right)\#1+\log\left(\sqrt[3]{2+x^2+2x^7}-x^{2/3}\#1\right)\#1}{-1+\#1^3}\&\right]}{4\sqrt[3]{x}\,\sqrt[3]{2+x^2+2x^7}}$$

Mathematica 12.3 output

$$\int \frac{\left(-2+5x^7\right)\sqrt[3]{2x+x^3+2x^8}}{4+x^4+8x^7+4x^{14}}\,dx$$

24.66 Problem number 822

$$\int \frac{-1+2x}{(1+x)\sqrt{-a^2x^2+(1+x)^6}}\,dx$$

Optimal antiderivative

$$-\frac{2\arctan\left(\frac{ax}{1+3x+3x^2+x^3+\sqrt{1+6x+\left(-a^2+15\right)x^2+20x^3+15x^4+6x^5+x^6}}\right)}{a}$$

command

```
Integrate[(-1 + 2*x)/((1 + x)*Sqrt[-(a^2*x^2) + (1 + x)^6]),x]
```

Mathematica 13.1 output

$$-\frac{2\text{ArcTan}\left(\frac{ax}{1+3x+3x^2+x^3+\sqrt{1+6x+\left(15-a^2\right)x^2+20x^3+15x^4+6x^5+x^6}}\right)}{a}$$

Mathematica 12.3 output

$$\int \frac{-1+2x}{(1+x)\sqrt{-a^2x^2+(1+x)^6}}\,dx$$

24.67 Problem number 840

$$\int \frac{\sqrt{-1+x^2+x^5}\left(2+3x^5\right)}{1+x^4-2x^5+x^{10}}\,dx$$

Optimal antiderivative

$$-\sqrt{1+i}\ \arctan\left(\frac{\sqrt{-1-i}\ x}{\sqrt{x^5+x^2-1}}\right)-\sqrt{1-i}\ \arctan\left(\frac{\sqrt{-1+i}\ x}{\sqrt{x^5+x^2-1}}\right)$$

command

```
Integrate[(Sqrt[-1 + x^2 + x^5]*(2 + 3*x^5))/(1 + x^4 - 2*x^5 + x^10),x]
```

Mathematica 13.1 output

$$-\sqrt{1+i}\ \text{ArcTan}\left(\frac{\sqrt{-1-i}\ x}{\sqrt{-1+x^2+x^5}}\right)-\sqrt{1-i}\ \text{ArcTan}\left(\frac{\sqrt{-1+i}\ x}{\sqrt{-1+x^2+x^5}}\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{-1+x^2+x^5}\left(2+3x^5\right)}{1+x^4-2x^5+x^{10}}\,dx$$

24.68 Problem number 858

$$\int \frac{-2b+cx^2}{\left(-b+cx^2\right)\sqrt[4]{-b+cx^2+ax^4}}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{a^{\frac{1}{4}}x}{\left(a\,x^4+c\,x^2-b\right)^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\text{arctanh}\left(\frac{a^{\frac{1}{4}}x}{\left(a\,x^4+c\,x^2-b\right)^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}$$

command

```
Integrate[(-2*b + c*x^2)/((-b + c*x^2)*(-b + c*x^2 + a*x^4)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{a}\ x}{\sqrt[4]{-b+cx^2+ax^4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\ x}{\sqrt[4]{-b+cx^2+ax^4}}\right)}{\sqrt[4]{a}}$$

Mathematica 12.3 output

$$\int \frac{-2b+cx^2}{\left(-b+cx^2\right)\sqrt[4]{-b+cx^2+ax^4}}\,dx$$

24.69 Problem number 859

$$\int \frac{-4b+ax^3}{(-b+ax^3)\sqrt[4]{b-ax^3+cx^4}}\,dx$$

Optimal antiderivative

$$\frac{2\arctan\left(\frac{c^{\frac{1}{4}}x}{(cx^4-x^3a+b)^{\frac{1}{4}}}\right)}{c^{\frac{1}{4}}}+\frac{2\operatorname{arctanh}\left(\frac{c^{\frac{1}{4}}x}{(cx^4-x^3a+b)^{\frac{1}{4}}}\right)}{c^{\frac{1}{4}}}$$

command

```
Integrate[(-4*b + a*x^3)/((-b + a*x^3)*(b - a*x^3 + c*x^4)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{2\left(\operatorname{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{b-ax^3+cx^4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{b-ax^3+cx^4}}\right)\right)}{\sqrt[4]{c}}$$

Mathematica 12.3 output

$$\int \frac{-4b+ax^3}{(-b+ax^3)\sqrt[4]{b-ax^3+cx^4}}\,dx$$

24.70 Problem number 860

$$\int \frac{-b+ax^6}{(b+ax^6)\sqrt[3]{-b+a^3x^3+ax^6}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-b + a*x^6)/((b + a*x^6)*(-b + a^3*x^3 + a*x^6)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{1}{6}\operatorname{RootSum}\left[a^6+4ab-2a^3\#1^3+\#1^6\&,\frac{-\log(x)+\log\left(\sqrt[3]{-b+a^3x^3+ax^6}-x\#1\right)}{\#1}\&\right]$$

Mathematica 12.3 output

$$\int \frac{-b+ax^6}{(b+ax^6)\sqrt[3]{-b+a^3x^3+ax^6}}\,dx$$

24.71 Problem number 861

$$\int \frac{-b+ax^6}{(b+ax^6)\sqrt[3]{-b+a^3x^3+ax^6}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-b + a*x^6)/((b + a*x^6)*(-b + a^3*x^3 + a*x^6)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{1}{6}\text{RootSum}\left[a^6+4ab-2a^3\#1^3+\#1^6\&,\frac{-\log(x)+\log\left(\sqrt[3]{-b+a^3x^3+ax^6}-x\#1\right)}{\#1}\&\right]$$

Mathematica 12.3 output

$$\int \frac{-b+ax^6}{(b+ax^6)\sqrt[3]{-b+a^3x^3+ax^6}}\,dx$$

24.72 Problem number 864

$$\int \frac{\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{1+x^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/(1 + x^2),x]
```

Mathematica 13.1 output

$$\text{RootSum}\left[2-4\#1^2+6\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-1+\#1^2}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{1+x^2}\,dx$$

24.73 Problem number 865

$$\int \frac{\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{1+x^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/(1 + x^2),x]
```

Mathematica 13.1 output

$$\text{RootSum}\left[2-4\#1^2+6\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-1+\#1^2}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{1+x^2}\,dx$$

24.74 Problem number 872

$$\int \frac{x^6}{(b+ax^4)^{3/4}\left(b^2+a^2x^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[x^6/((b + a*x^4)^(3/4)*(b^2 + a^2*x^8)),x]
```

Mathematica 13.1 output

$$\frac{\text{RootSum}\left[2a^2-2a\#1^4+\#1^8\&,\frac{\log(x)-\log\left(\sqrt[4]{b+ax^4}-x\#1\right)}{-a\#1^3+\#1^7}\&\right]}{8b}$$

Mathematica 12.3 output

$$\int \frac{x^6}{(b+ax^4)^{3/4}\left(b^2+a^2x^8\right)}\,dx$$

24.75 Problem number 882

$$\int \frac{x^2}{(-b+ax^4)\sqrt[4]{bx^2+ax^4}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[x^2/((-b + a*x^4)*(b*x^2 + a*x^4)^(1/4)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{x}\,\sqrt[4]{b+ax^2}\,\text{RootSum}\left[a^2-ab-2a\#1^4+\#1^8\&,\,\frac{-\log\left(\sqrt{x}\right)+\log\left(\sqrt[4]{b+ax^2}-\sqrt{x}\,\#1\right)}{a\#1-\#1^5}\&\right]}{4\sqrt[4]{x^2\,(b+ax^2)}}$$

Mathematica 12.3 output

$$\int \frac{x^2}{(-b+ax^4)\sqrt[4]{bx^2+ax^4}}\,dx$$

24.76 Problem number 883

$$\int \frac{x^2}{(-b+ax^4)\sqrt[4]{bx^2+ax^4}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[x^2/((-b + a*x^4)*(b*x^2 + a*x^4)^(1/4)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{x}\,\sqrt[4]{b+ax^2}\,\text{RootSum}\left[a^2-ab-2a\#1^4+\#1^8\&,\,\frac{-\log\left(\sqrt{x}\right)+\log\left(\sqrt[4]{b+ax^2}-\sqrt{x}\,\#1\right)}{a\#1-\#1^5}\&\right]}{4\sqrt[4]{x^2\,(b+ax^2)}}$$

Mathematica 12.3 output

$$\int \frac{x^2}{(-b+ax^4)\sqrt[4]{bx^2+ax^4}}\,dx$$

24.77 Problem number 884

$$\int \frac{4b+x^3}{(b+x^3)\sqrt[4]{-b-x^3+ax^4}}\,dx$$

Optimal antiderivative

$$\frac{2\arctan\left(\frac{a^{\frac{1}{4}}x}{(ax^4-x^3-b)^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{2\operatorname{arctanh}\left(\frac{a^{\frac{1}{4}}x}{(ax^4-x^3-b)^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}$$

command

```
Integrate[(4*b + x^3)/((b + x^3)*(-b - x^3 + a*x^4)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{2\left(\text{ArcTan}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+x^3(-1+ax)}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+x^3(-1+ax)}}\right)\right)}{\sqrt[4]{a}}$$

Mathematica 12.3 output

$$\int \frac{4b+x^3}{(b+x^3)\sqrt[4]{-b-x^3+ax^4}}\,dx$$

24.78 Problem number 885

$$\int \frac{4b+ax^5}{(-b+ax^5)\sqrt[4]{-b+cx^4+ax^5}}\,dx$$

Optimal antiderivative

$$-\frac{2\arctan\left(\frac{c^{\frac{1}{4}}x}{(ax^5+cx^4-b)^{\frac{1}{4}}}\right)}{c^{\frac{1}{4}}}-\frac{2\operatorname{arctanh}\left(\frac{c^{\frac{1}{4}}x}{(ax^5+cx^4-b)^{\frac{1}{4}}}\right)}{c^{\frac{1}{4}}}$$

command

```
Integrate[(4*b + a*x^5)/((-b + a*x^5)*(-b + c*x^4 + a*x^5)^(1/4)),x]
```

Mathematica 13.1 output

$$-\frac{2\left(\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-b+x^4(c+ax)}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-b+x^4(c+ax)}}\right)\right)}{\sqrt[4]{c}}$$

Mathematica 12.3 output

$$\int \frac{4b+ax^5}{(-b+ax^5)\sqrt[4]{-b+cx^4+ax^5}}\,dx$$

24.79 Problem number 886

$$\int \frac{\sqrt{1+x^2-2x^6}\left(1+4x^6\right)}{\left(-1-4x^2+2x^6\right)\left(-1-2x^2+2x^6\right)}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{x}{\sqrt{-2x^6+x^2+1}}\right)}{2}-\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ x\sqrt{-2x^6+x^2+1}}{2x^6-x^2-1}\right)}{2}$$

command

```
Integrate[(Sqrt[1 + x^2 - 2*x^6]*(1 + 4*x^6))/((-1 - 4*x^2 + 2*x^6)*(-1 - 2*x^2 + 2*x^6)),x]
```

Mathematica 13.1 output

$$\frac{1}{2}\left(-\text{ArcTan}\left(\frac{x}{\sqrt{1+x^2-2x^6}}\right)+\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{\sqrt{1+x^2-2x^6}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1+x^2-2x^6}\left(1+4x^6\right)}{\left(-1-4x^2+2x^6\right)\left(-1-2x^2+2x^6\right)}\,dx$$

24.80 Problem number 899

$$\int \frac{\left(2+x^2\right)\sqrt[4]{-1-x^2+x^4}\left(1+x^2+x^4\right)}{x^6\left(1+x^2\right)}\,dx$$

Optimal antiderivative

$$-\frac{2\left(x^4-x^2-1\right)^{\frac{1}{4}}\left(4x^4+x^2+1\right)}{5x^5}-\arctan\left(\frac{x}{\left(x^4-x^2-1\right)^{\frac{1}{4}}}\right)+\text{arctanh}\left(\frac{x}{\left(x^4-x^2-1\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[((2 + x^2)*(-1 - x^2 + x^4)^(1/4)*(1 + x^2 + x^4))/(x^6*(1 + x^2)),x]
```

Mathematica 13.1 output

$$-\frac{2\sqrt[4]{-1-x^2+x^4}\left(1+x^2+4x^4\right)}{5x^5}-\text{ArcTan}\left(\frac{x}{\sqrt[4]{-1-x^2+x^4}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{-1-x^2+x^4}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(2+x^2\right)\sqrt[4]{-1-x^2+x^4}\left(1+x^2+x^4\right)}{x^6\left(1+x^2\right)}\,dx$$

24.81 Problem number 902

$$\int \frac{\left(1+x^2\right)\sqrt[4]{x^2+x^6}}{x^2\left(-1+x^2\right)}\,dx$$

Optimal antiderivative

$$\frac{2\left(x^6+x^2\right)^{\frac{1}{4}}}{x}+2^{\frac{1}{4}}\arctan\left(\frac{2^{\frac{1}{4}}x}{\left(x^6+x^2\right)^{\frac{1}{4}}}\right)-2^{\frac{1}{4}}\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}x}{\left(x^6+x^2\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[((1 + x^2)*(x^2 + x^6)^(1/4))/(x^2*(-1 + x^2)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{x^2+x^6}\left(2\sqrt[4]{1+x^4}+\sqrt[4]{2}\,\sqrt{x}\,\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt{x}}{\sqrt[4]{1+x^4}}\right)-\sqrt[4]{2}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt{x}}{\sqrt[4]{1+x^4}}\right)\right)}{x\sqrt[4]{1+x^4}}$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2\right)\sqrt[4]{x^2+x^6}}{x^2\left(-1+x^2\right)}\,dx$$

24.82 Problem number 914

$$\int \frac{4b+ax^3}{\left(b+ax^3\right)\sqrt[4]{-b-ax^3+cx^4}}\,dx$$

Optimal antiderivative

$$\frac{2\arctan\left(\frac{c^{\frac{1}{4}}x}{\left(cx^4-x^3a-b\right)^{\frac{1}{4}}}\right)}{c^{\frac{1}{4}}}+\frac{2\operatorname{arctanh}\left(\frac{c^{\frac{1}{4}}x}{\left(cx^4-x^3a-b\right)^{\frac{1}{4}}}\right)}{c^{\frac{1}{4}}}$$

command

```
Integrate[(4*b + a*x^3)/((b + a*x^3)*(-b - a*x^3 + c*x^4)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{2\left(\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-b-ax^3+cx^4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-b-ax^3+cx^4}}\right)\right)}{\sqrt[4]{c}}$$

Mathematica 12.3 output

$$\int \frac{4b+ax^3}{\left(b+ax^3\right)\sqrt[4]{-b-ax^3+cx^4}}\,dx$$

24.83 Problem number 923

$$\int \frac{\left(3+2x^5\right)\sqrt{x-2x^4-x^6}}{\left(-1+x^5\right)^2}\,dx$$

Optimal antiderivative

$$-\frac{x\sqrt{-x^6-2x^4+x}}{x^5-1}-\frac{\arctan\left(\frac{\sqrt{2}\,x\sqrt{-x^6-2x^4+x}}{x^5+2x^3-1}\right)\sqrt{2}}{2}$$

command

```
Integrate[((3 + 2*x^5)*Sqrt[x - 2*x^4 - x^6])/(-1 + x^5)^2,x]
```

Mathematica 13.1 output

$$\frac{\sqrt{x-2x^4-x^6}\left(-\frac{2x^{3/2}}{-1+x^5}-\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,x^{3/2}}{\sqrt{-1+2x^3+x^5}}\right)}{\sqrt{-1+2x^3+x^5}}\right)}{2\sqrt{x}}$$

Mathematica 12.3 output

$$\int \frac{\left(3+2x^5\right)\sqrt{x-2x^4-x^6}}{\left(-1+x^5\right)^2}\,dx$$

24.84 Problem number 924

$$\int \frac{\left(-2+x^6\right)\left(1+x^6\right)\sqrt[4]{1-x^4+x^6}}{x^6\left(1-2x^4+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{2\left(x^6-x^4+1\right)^{\frac{1}{4}}\left(x^6+9x^4+1\right)}{5x^5}+2\arctan\left(\frac{x}{\left(x^6-x^4+1\right)^{\frac{1}{4}}}\right)-2\operatorname{arctanh}\left(\frac{x}{\left(x^6-x^4+1\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[((-2 + x^6)*(1 + x^6)*(1 - x^4 + x^6)^(1/4))/(x^6*(1 - 2*x^4 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt[4]{1-x^4+x^6}\left(1+9x^4+x^6\right)}{5x^5}+2\text{ArcTan}\left(\frac{x}{\sqrt[4]{1-x^4+x^6}}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt[4]{1-x^4+x^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-2+x^6\right)\left(1+x^6\right)\sqrt[4]{1-x^4+x^6}}{x^6\left(1-2x^4+x^6\right)}\,dx$$

24.85 Problem number 928

$$\int \frac{\left(-1+x^2\right)\sqrt{1+\sqrt{1+x^2}}}{1+x^2}\,dx$$

Optimal antiderivative

$$\frac{4x}{3\sqrt{1+\sqrt{x^2+1}}}+\frac{2x\sqrt{x^2+1}}{3\sqrt{1+\sqrt{x^2+1}}}-4\arctan\left(\frac{x}{\sqrt{1+\sqrt{x^2+1}}}\right)$$

command

```
Integrate[((-1 + x^2)*Sqrt[1 + Sqrt[1 + x^2]])/(1 + x^2),x]
```

Mathematica 13.1 output

$$\frac{2x\left(2+\sqrt{1+x^2}\right)}{3\sqrt{1+\sqrt{1+x^2}}}-4\text{ArcTan}\left(\frac{x}{\sqrt{1+\sqrt{1+x^2}}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^2\right)\sqrt{1+\sqrt{1+x^2}}}{1+x^2}\,dx$$

24.86 Problem number 937

$$\int \frac{\sqrt[3]{-x+x^3}}{b+ax^6}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-x + x^3)^(1/3)/(b + a*x^6),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x\left(-1+x^2\right)}\,\text{RootSum}\left[a+b-3b\#1^3+3b\#1^6-b\#1^9\&,\frac{-2\log\left(\sqrt[3]{x}\right)\#1+\log\left(\sqrt[3]{-1+x^2}-x^{2/3}\#1\right)\#1}{-1+\#1^3}\&\right]}{6b\sqrt[3]{x}\,\sqrt[3]{-1+x^2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[3]{-x+x^3}}{b+ax^6}\,dx$$

24.87 Problem number 938

$$\int \frac{\sqrt[3]{-x+x^3}}{b+ax^6}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-x + x^3)^(1/3)/(b + a*x^6),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x\left(-1+x^2\right)}\,\text{RootSum}\left[a+b-3b\#1^3+3b\#1^6-b\#1^9\&,\frac{-2\log\left(\sqrt[3]{x}\right)\#1+\log\left(\sqrt[3]{-1+x^2}-x^{2/3}\#1\right)\#1}{-1+\#1^3}\&\right]}{6b\sqrt[3]{x}\,\sqrt[3]{-1+x^2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[3]{-x+x^3}}{b+ax^6}\,dx$$

24.88 Problem number 939

$$\int \frac{\left(-2b+ax^6\right)\left(b+ax^6\right)^{3/4}}{x^4\left(b-cx^4+ax^6\right)}\,dx$$

Optimal antiderivative

$$\frac{2\left(a\,x^6+b\right)^{\frac{3}{4}}}{3x^3}-c^{\frac{3}{4}}\arctan\left(\frac{c^{\frac{1}{4}}x}{\left(a\,x^6+b\right)^{\frac{1}{4}}}\right)-c^{\frac{3}{4}}\,\text{arctanh}\left(\frac{c^{\frac{1}{4}}x}{\left(a\,x^6+b\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[((-2*b + a*x^6)*(b + a*x^6)^(3/4))/(x^4*(b - c*x^4 + a*x^6)),x]
```

Mathematica 13.1 output

$$\frac{2\left(b+ax^6\right)^{3/4}}{3x^3}-c^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{b+ax^6}}\right)-c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{b+ax^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-2b+ax^6\right)\left(b+ax^6\right)^{3/4}}{x^4\left(b-cx^4+ax^6\right)}\,dx$$

24.89 Problem number 940

$$\int \frac{b+ax^8}{(-b+ax^8)\sqrt[4]{-b+cx^4+ax^8}}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{c^{\frac{1}{4}}x}{(ax^8+cx^4-b)^{\frac{1}{4}}}\right)}{2c^{\frac{1}{4}}}-\frac{\operatorname{arctanh}\left(\frac{c^{\frac{1}{4}}x}{(ax^8+cx^4-b)^{\frac{1}{4}}}\right)}{2c^{\frac{1}{4}}}$$

command

```
Integrate[(b + a*x^8)/((-b + a*x^8)*(-b + c*x^4 + a*x^8)^(1/4)),x]
```

Mathematica 13.1 output

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-b+cx^4+ax^8}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-b+cx^4+ax^8}}\right)}{2\sqrt[4]{c}}$$

Mathematica 12.3 output

$$\int \frac{b+ax^8}{(-b+ax^8)\sqrt[4]{-b+cx^4+ax^8}}\,dx$$

24.90 Problem number 941

$$\int \frac{\sqrt{1+x^5}\left(-2+3x^5\right)}{1+x^4+2x^5+x^{10}}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{2}\,x\sqrt{x^5+1}}{x^5-x^2+1}\right)\sqrt{2}}{2}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\,x\sqrt{x^5+1}}{x^5+x^2+1}\right)\sqrt{2}}{2}$$

command

```
Integrate[(Sqrt[1 + x^5]*(-2 + 3*x^5))/(1 + x^4 + 2*x^5 + x^10),x]
```

Mathematica 13.1 output

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,x\sqrt{1+x^5}}{1-x^2+x^5}\right)+\tanh^{-1}\left(\frac{1+x^2+x^5}{\sqrt{2}\,x\sqrt{1+x^5}}\right)}{\sqrt{2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1+x^5}\left(-2+3x^5\right)}{1+x^4+2x^5+x^{10}}\,dx$$

24.91 Problem number 950

$$\int \frac{\left(4b+ax^3\right)\left(-b-ax^3+x^4\right)}{x^4\sqrt[4]{b+ax^3}\,\left(-b-ax^3+2x^4\right)}\,dx$$

Optimal antiderivative

$$-\frac{4\left(x^3a+b\right)^{\frac{3}{4}}}{3x^3}-2^{\frac{3}{4}}\arctan\left(\frac{\left(x^3a+b\right)^{\frac{1}{4}}2^{\frac{3}{4}}}{2x}\right)+2^{\frac{3}{4}}\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}x}{\left(x^3a+b\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[((4*b + a*x^3)*(-b - a*x^3 + x^4))/(x^4*(b + a*x^3)^(1/4)*(-b - a*x^3 + 2*x^4)),x]
```

Mathematica 13.1 output

$$-\frac{4\left(b+ax^3\right)^{3/4}}{3x^3}-2^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{b+ax^3}}{\sqrt[4]{2}\,x}\right)+2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{b+ax^3}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(4b+ax^3\right)\left(-b-ax^3+x^4\right)}{x^4\sqrt[4]{b+ax^3}\,\left(-b-ax^3+2x^4\right)}\,dx$$

24.92 Problem number 954

$$\int \frac{\left(-1+3x^4\right)\sqrt{1+x^2+2x^4+x^8}}{\left(1-x+x^4\right)^2\left(1+x+x^4\right)}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{x^8+2x^4+x^2+1}}{2x^4-2x+2}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\,x}{1+x+x^4+\sqrt{x^8+2x^4+x^2+1}}\right)\sqrt{2}}{2}$$

command

```
Integrate[((-1 + 3*x^4)*Sqrt[1 + x^2 + 2*x^4 + x^8])/((1 - x + x^4)^2*(1 + x + x^4)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{1+x^2+2x^4+x^8}}{2\left(1-x+x^4\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{1+x+x^4+\sqrt{1+x^2+2x^4+x^8}}\right)}{\sqrt{2}}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+3x^4\right)\sqrt{1+x^2+2x^4+x^8}}{\left(1-x+x^4\right)^2\left(1+x+x^4\right)}\,dx$$

24.93 Problem number 958

$$\int \frac{x\left(-3+x^2\right)}{\left(-1+x^2\right)^{2/3}\left(1-x^2+x^3\right)}\,dx$$

Optimal antiderivative

$$\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^2-1\right)^{\frac{1}{3}}}\right)+\ln\left(-x+\left(x^2-1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(x^2+x\left(x^2-1\right)^{\frac{1}{3}}+\left(x^2-1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(x*(-3 + x^2))/((-1 + x^2)^(2/3)*(1 - x^2 + x^3)),x]
```

Mathematica 13.1 output

$$\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{-1+x^2}}\right)+\log\left(-x+\sqrt[3]{-1+x^2}\right)$$
$$-\frac{1}{2}\log\left(x^2+x\sqrt[3]{-1+x^2}+\left(-1+x^2\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{x\left(-3+x^2\right)}{\left(-1+x^2\right)^{2/3}\left(1-x^2+x^3\right)}\,dx$$

24.94 Problem number 960

$$\int \frac{\left(3+x^4\right)\sqrt{x+x^4-x^5}}{\left(-1+x^4\right)\left(-1+x^3+x^4\right)}\,dx$$

Optimal antiderivative

$$2\,\text{arctanh}\left(\frac{x\sqrt{-x^5+x^4+x}}{x^4-x^3-1}\right)-2\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\,x\sqrt{-x^5+x^4+x}}{x^4-x^3-1}\right)$$

command

```
Integrate[((3 + x^4)*Sqrt[x + x^4 - x^5])/((-1 + x^4)*(-1 + x^3 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{x+x^4-x^5}\left(\text{ArcTan}\left(\frac{x^{3/2}}{\sqrt{-1-x^3+x^4}}\right)-\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,x^{3/2}}{\sqrt{-1-x^3+x^4}}\right)\right)}{\sqrt{x}\,\sqrt{-1-x^3+x^4}}$$

Mathematica 12.3 output

$$\int \frac{\left(3+x^4\right)\sqrt{x+x^4-x^5}}{\left(-1+x^4\right)\left(-1+x^3+x^4\right)}\,dx$$

24.95 Problem number 964

$$\int \frac{\left(-3+2x^5\right)\sqrt{x+2x^4+x^6}}{\left(1+x^5\right)\left(1+x^3+x^5\right)}\,dx$$

Optimal antiderivative

$$2\,\text{arctanh}\left(\frac{x\sqrt{x^6+2x^4+x}}{x^5+2x^3+1}\right)-2\sqrt{2}\ \text{arctanh}\left(\frac{\sqrt{2}\ x\sqrt{x^6+2x^4+x}}{x^5+2x^3+1}\right)$$

command

```
Integrate[((-3 + 2*x^5)*Sqrt[x + 2*x^4 + x^6])/((1 + x^5)*(1 + x^3 + x^5)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{x+2x^4+x^6}\left(\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{1+2x^3+x^5}}\right)-\sqrt{2}\ \tanh^{-1}\left(\frac{\sqrt{2}\,x^{3/2}}{\sqrt{1+2x^3+x^5}}\right)\right)}{\sqrt{x}\ \sqrt{1+2x^3+x^5}}$$

Mathematica 12.3 output

$$\int \frac{\left(-3+2x^5\right)\sqrt{x+2x^4+x^6}}{\left(1+x^5\right)\left(1+x^3+x^5\right)}\,dx$$

24.96 Problem number 965

$$\int \frac{\sqrt{-1+x^2+x^4+x^6}\left(1+x^4+2x^6\right)}{1-x^4-2x^6+x^8+2x^{10}+x^{12}}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{1+i}\ \arctan\left(\frac{\sqrt{-1-i}\,x}{\sqrt{x^6+x^4+x^2-1}}\right)}{2}-\frac{\sqrt{1-i}\ \arctan\left(\frac{\sqrt{-1+i}\,x}{\sqrt{x^6+x^4+x^2-1}}\right)}{2}$$

command

```
Integrate[(Sqrt[-1 + x^2 + x^4 + x^6]*(1 + x^4 + 2*x^6))/(1 - x^4 - 2*x^6 + x^8 + 2*x^10 + x^
```

Mathematica 13.1 output

$$-\frac{1}{2}\sqrt{1+i}\,\text{ArcTan}\left(\frac{\sqrt{-1-i}\,x}{\sqrt{-1+x^2+x^4+x^6}}\right)-\frac{1}{2}\sqrt{1-i}\,\text{ArcTan}\left(\frac{\sqrt{-1+i}\,x}{\sqrt{-1+x^2+x^4+x^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{-1+x^2+x^4+x^6}\left(1+x^4+2x^6\right)}{1-x^4-2x^6+x^8+2x^{10}+x^{12}}\,dx$$

24.97 Problem number 976

$$\int \frac{-3+x^4}{\sqrt[3]{1+x^4}\,(1-x^3+x^4)}\,dx$$

Optimal antiderivative

$$-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^4+1\right)^{\frac{1}{3}}}\right)+\ln\left(-x+\left(x^4+1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(x^2+x\left(x^4+1\right)^{\frac{1}{3}}+\left(x^4+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(-3 + x^4)/((1 + x^4)^(1/3)*(1 - x^3 + x^4)),x]
```

Mathematica 13.1 output

$$-\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{1+x^4}}\right)+\log\left(-x+\sqrt[3]{1+x^4}\right)-\frac{1}{2}\log\left(x^2+x\sqrt[3]{1+x^4}+\left(1+x^4\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{-3+x^4}{\sqrt[3]{1+x^4}\,(1-x^3+x^4)}\,dx$$

24.98 Problem number 987

$$\int \frac{\sqrt[4]{x^2+x^4}}{x^4\left(-1+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{2\left(x^2+1\right)\left(x^4+x^2\right)^{\frac{1}{4}}}{5x^3}+\frac{\arctan\left(\frac{2^{\frac{1}{4}}x}{\left(x^4+x^2\right)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{2}-\frac{\text{arctanh}\left(\frac{2^{\frac{1}{4}}x}{\left(x^4+x^2\right)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{2}$$

command

```
Integrate[(x^2 + x^4)^(1/4)/(x^4*(-1 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{x^2+x^4}\left(4\left(1+x^2\right)^{5/4}+5\sqrt[4]{2}\,x^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt{x}}{\sqrt[4]{1+x^2}}\right)-5\sqrt[4]{2}\,x^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt{x}}{\sqrt[4]{1+x^2}}\right)\right)}{10x^3\sqrt[4]{1+x^2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{x^2+x^4}}{x^4\left(-1+x^4\right)}\,dx$$

24.99 Problem number 988

$$\int \frac{x\left(-3+x^4\right)}{\left(1+x^4\right)^{2/3}\left(1+x^3+x^4\right)}\,dx$$

Optimal antiderivative

$$-\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ x}{-x+2\left(x^4+1\right)^{\frac{1}{3}}}\right)+\ln\left(x+\left(x^4+1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(x^2-x\left(x^4+1\right)^{\frac{1}{3}}+\left(x^4+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(x*(-3 + x^4))/((1 + x^4)^(2/3)*(1 + x^3 + x^4)),x]
```

Mathematica 13.1 output

$$\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x-2\sqrt[3]{1+x^4}}\right)+\log\left(x+\sqrt[3]{1+x^4}\right)-\frac{1}{2}\log\left(x^2-x\sqrt[3]{1+x^4}+\left(1+x^4\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{x\left(-3+x^4\right)}{\left(1+x^4\right)^{2/3}\left(1+x^3+x^4\right)}\,dx$$

24.100 Problem number 995

$$\int \frac{2b+ax^6}{\sqrt[4]{-b+ax^6}\left(-b-2x^4+ax^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{2^{\frac{1}{4}}x\left(a\,x^6-b\right)^{\frac{3}{4}}}{-a\,x^6+b}\right)2^{\frac{3}{4}}}{2}+\frac{\text{arctanh}\left(\frac{2^{\frac{1}{4}}x\left(a\,x^6-b\right)^{\frac{3}{4}}}{-a\,x^6+b}\right)2^{\frac{3}{4}}}{2}$$

command

```
Integrate[(2*b + a*x^6)/((-b + a*x^6)^(1/4)*(-b - 2*x^4 + a*x^6)),x]
```

Mathematica 13.1 output

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{-b+ax^6}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{-b+ax^6}}\right)}{\sqrt[4]{2}}$$

Mathematica 12.3 output

$$\int \frac{2b+ax^6}{\sqrt[4]{-b+ax^6}\left(-b-2x^4+ax^6\right)}\,dx$$

24.101 Problem number 1023

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1+x^2)\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/((1 + x^2)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

Mathematica 13.1 output

$$\text{RootSum}\left[2-4\#1^2+6\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-1+3\#1^2-3\#1^4+\#1^6}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1+x^2)\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.102 Problem number 1024

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1+x^2)\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/((1 + x^2)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

Mathematica 13.1 output

$$\text{RootSum}\left[2-4\#1^2+6\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-1+3\#1^2-3\#1^4+\#1^6}\&\right]$$

Mathematica 12.3 output

$$\int\frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1+x^2)\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.103 Problem number 1031

$$\int\frac{(8+3x)\sqrt[4]{-2-x+2x^4}}{x^2\,(2+x+x^4)}\,dx$$

Optimal antiderivative

$$-\frac{4\left(2x^4-x-2\right)^{\frac{1}{4}}}{x}-2\,3^{\frac{1}{4}}\arctan\left(\frac{3^{\frac{1}{4}}x}{\left(2x^4-x-2\right)^{\frac{1}{4}}}\right)+2\,3^{\frac{1}{4}}\operatorname{arctanh}\left(\frac{3^{\frac{1}{4}}x}{\left(2x^4-x-2\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[((8 + 3*x)*(-2 - x + 2*x^4)^(1/4))/(x^2*(2 + x + x^4)),x]
```

Mathematica 13.1 output

$$-\frac{4\sqrt[4]{-2-x+2x^4}}{x}-2\sqrt[4]{3}\,\text{ArcTan}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{-2-x+2x^4}}\right)+2\sqrt[4]{3}\,\tanh^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{-2-x+2x^4}}\right)$$

Mathematica 12.3 output

$$\int\frac{(8+3x)\sqrt[4]{-2-x+2x^4}}{x^2\,(2+x+x^4)}\,dx$$

24.104 Problem number 1037

$$\int \frac{\sqrt[4]{-1+x^4}\left(-1+x^8\right)}{x^6\left(1+x^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^4)^(1/4)*(-1 + x^8))/(x^6*(1 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{-4\left(-1+x^4\right)^{5/4}+5x^5\text{RootSum}\left[2-2\#1^4+\#1^8\&,\frac{-\log(x)\#1+\log\left(\sqrt[4]{-1+x^4}-x\#1\right)\#1}{-1+\#1^4}\&\right]}{20x^5}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{-1+x^4}\left(-1+x^8\right)}{x^6\left(1+x^8\right)}\,dx$$

24.105 Problem number 1038

$$\int \frac{-1+x^8}{\sqrt[4]{1+x^4}\left(1+x^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-1 + x^8)/((1 + x^4)^(1/4)*(1 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{1}{4}\left(2\left(\text{ArcTan}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)\right)+\text{RootSum}\left[2-2\#1^4+\#1^8\&,\frac{-\log(x)+\log\left(\sqrt[4]{1+x^4}-x\#1\right)}{\#1}\&\right]\right)$$

Mathematica 12.3 output

$$\int \frac{-1+x^8}{\sqrt[4]{1+x^4}\left(1+x^8\right)}\,dx$$

24.106 Problem number 1039

$$\int \frac{-1+x^8}{\sqrt[4]{1+x^4}\,(1+x^8)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-1 + x^8)/((1 + x^4)^(1/4)*(1 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{1}{4}\left(2\left(\text{ArcTan}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)\right)+\text{RootSum}\left[2-2\#1^4+\#1^8\&,\frac{-\log(x)+\log\left(\sqrt[4]{1+x^4}-x\#1\right)}{\#1}\&\right]\right)$$

Mathematica 12.3 output

$$\int \frac{-1+x^8}{\sqrt[4]{1+x^4}\,(1+x^8)}\,dx$$

24.107 Problem number 1040

$$\int \frac{\sqrt[4]{-1+x^4}\left(1-x^4+x^8\right)}{x^6\left(1+2x^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^4)^(1/4)*(1 - x^4 + x^8))/(x^6*(1 + 2*x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-1+x^4}\left(-1+6x^4\right)}{5x^5}-\frac{3}{8}\text{RootSum}\left[3-2\#1^4+\#1^8\&,\frac{-\log(x)+\log\left(\sqrt[4]{-1+x^4}-x\#1\right)}{-\#1^3+\#1^7}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{-1+x^4}\left(1-x^4+x^8\right)}{x^6\left(1+2x^8\right)}\,dx$$

24.108 Problem number 1041

$$\int \frac{\sqrt[4]{-1+x^4}\left(1-x^4+x^8\right)}{x^6\left(1+2x^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^4)^(1/4)*(1 - x^4 + x^8))/(x^6*(1 + 2*x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-1+x^4}\left(-1+6x^4\right)}{5x^5}-\frac{3}{8}\text{RootSum}\left[3-2\#1^4+\#1^8\&,\frac{-\log(x)+\log\left(\sqrt[4]{-1+x^4}-x\#1\right)}{-\#1^3+\#1^7}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{-1+x^4}\left(1-x^4+x^8\right)}{x^6\left(1+2x^8\right)}\,dx$$

24.109 Problem number 1052

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(2+x^3+2x^6\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^3)^(2/3)*(2 + x^3))/(x^6*(2 + x^3 + 2*x^6)),x]
```

Mathematica 13.1 output

$$\frac{\left(-1+x^3\right)^{5/3}}{5x^5}-\frac{2}{3}\text{RootSum}\left[5-5\#1^3+2\#1^6\&,\frac{-\log(x)\#1^2+\log\left(\sqrt[3]{-1+x^3}-x\#1\right)\#1^2}{-5+4\#1^3}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(2+x^3+2x^6\right)}\,dx$$

24.110 Problem number 1053

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(2+x^3+2x^6\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^3)^(2/3)*(2 + x^3))/(x^6*(2 + x^3 + 2*x^6)),x]
```

Mathematica 13.1 output

$$\frac{\left(-1+x^3\right)^{5/3}}{5x^5}-\frac{2}{3}\text{RootSum}\left[5-5\#1^3+2\#1^6\&,\frac{-\log(x)\#1^2+\log\left(\sqrt[3]{-1+x^3}-x\#1\right)\#1^2}{-5+4\#1^3}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(2+x^3+2x^6\right)}\,dx$$

24.111 Problem number 1054

$$\int \frac{x+x^7}{\left(-1+x^6\right)^{2/3}\left(-1+x^3+x^6\right)}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^6-1\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}+\frac{\ln\left(x+\left(x^6-1\right)^{\frac{1}{3}}\right)}{3}-\frac{\ln\left(x^2-x\left(x^6-1\right)^{\frac{1}{3}}+\left(x^6-1\right)^{\frac{2}{3}}\right)}{6}$$

command

```
Integrate[(x + x^7)/((-1 + x^6)^(2/3)*(-1 + x^3 + x^6)),x]
```

Mathematica 13.1 output

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{3}\,x}{-x+2\sqrt[3]{-1+x^6}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(x+\sqrt[3]{-1+x^6}\right)-\frac{1}{6}\log\left(x^2-x\sqrt[3]{-1+x^6}+\left(-1+x^6\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{x+x^7}{\left(-1+x^6\right)^{2/3}\left(-1+x^3+x^6\right)}\,dx$$

24.112 Problem number 1055

$$\int \frac{\sqrt{1-x^6}\left(1+2x^6\right)}{1+x^4-2x^6+x^{12}}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{2}\,x\sqrt{-x^6+1}}{x^6+x^2-1}\right)\sqrt{2}}{4}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\,x\sqrt{-x^6+1}}{x^6-x^2-1}\right)\sqrt{2}}{4}$$

command

```
Integrate[(Sqrt[1 - x^6]*(1 + 2*x^6))/(1 + x^4 - 2*x^6 + x^12),x]
```

Mathematica 13.1 output

$$-\frac{\operatorname{ArcTan}\left(\frac{x\sqrt{2-2x^6}}{-1+x^2+x^6}\right)+\tanh^{-1}\left(\frac{-1-x^2+x^6}{x\sqrt{2-2x^6}}\right)}{2\sqrt{2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1-x^6}\left(1+2x^6\right)}{1+x^4-2x^6+x^{12}}\,dx$$

24.113 Problem number 1060

$$\int \frac{-3+x^2}{\sqrt[3]{-1+x^2}\left(-1+x^2+x^3\right)}\,dx$$

Optimal antiderivative

$$\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^2-1\right)^{\frac{1}{3}}}{-2x+\left(x^2-1\right)^{\frac{1}{3}}}\right)+\ln\left(x+\left(x^2-1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(x^2-x\left(x^2-1\right)^{\frac{1}{3}}+\left(x^2-1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(-3 + x^2)/((-1 + x^2)^(1/3)*(-1 + x^2 + x^3)),x]
```

Mathematica 13.1 output

$$\sqrt{3}\,\operatorname{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{-1+x^2}}{-2x+\sqrt[3]{-1+x^2}}\right)+\log\left(x+\sqrt[3]{-1+x^2}\right)$$
$$-\frac{1}{2}\log\left(x^2-x\sqrt[3]{-1+x^2}+\left(-1+x^2\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{-3+x^2}{\sqrt[3]{-1+x^2}\left(-1+x^2+x^3\right)}\,dx$$

24.114 Problem number 1064

$$\int \frac{x^2\left(-4+x^3\right)}{\left(-1+x^3\right)^{3/4}\left(-1+x^3+x^4\right)}\,dx$$

Optimal antiderivative

$$\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x\left(x^3-1\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^3-1}}\right)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,x\left(x^3-1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^3-1}}\right)$$

command

```
Integrate[(x^2*(-4 + x^3))/((-1 + x^3)^(3/4)*(-1 + x^3 + x^4)),x]
```

Mathematica 13.1 output

$$-\sqrt{2}\left(\operatorname{ArcTan}\left(\frac{\sqrt{2}\,x\sqrt[4]{-1+x^3}}{x^2-\sqrt{-1+x^3}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{-1+x^3}}{x^2+\sqrt{-1+x^3}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{x^2\left(-4+x^3\right)}{\left(-1+x^3\right)^{3/4}\left(-1+x^3+x^4\right)}\,dx$$

24.115 Problem number 1067

$$\int \frac{\sqrt[4]{2+x^4}\left(-4+x^8\right)}{x^6\left(-4-2x^4+x^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((2 + x^4)^(1/4)*(-4 + x^8))/(x^6*(-4 - 2*x^4 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{2+x^4}\left(-1+2x^4\right)}{5x^5}+\frac{1}{8}\operatorname{RootSum}\left[-1-\#1^4+\#1^8\&,\frac{-\log(x)+\log\left(\sqrt[4]{2+x^4}-x\#1\right)}{-\#1^3+2\#1^7}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{2+x^4}\left(-4+x^8\right)}{x^6\left(-4-2x^4+x^8\right)}\,dx$$

24.116 Problem number 1068

$$\int \frac{\sqrt[4]{2+x^4}\left(-4+x^8\right)}{x^6\left(-4-2x^4+x^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((2 + x^4)^(1/4)*(-4 + x^8))/(x^6*(-4 - 2*x^4 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{2+x^4}\left(-1+2x^4\right)}{5x^5}+\frac{1}{8}\text{RootSum}\left[-1-\#1^4+\#1^8\&,\frac{-\log(x)+\log\left(\sqrt[4]{2+x^4}-x\#1\right)}{-\#1^3+2\#1^7}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{2+x^4}\left(-4+x^8\right)}{x^6\left(-4-2x^4+x^8\right)}\,dx$$

24.117 Problem number 1069

$$\int \frac{\sqrt[4]{-1+x^4}\left(-1+x^4+2x^8\right)}{x^6\left(1-x^4+x^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^4)^(1/4)*(-1 + x^4 + 2*x^8))/(x^6*(1 - x^4 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{-4\left(-1+x^4\right)^{5/4}+15x^5\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x)\#1+\log\left(\sqrt[4]{-1+x^4}-x\#1\right)\#1}{-1+2\#1^4}\&\right]}{20x^5}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{-1+x^4}\left(-1+x^4+2x^8\right)}{x^6\left(1-x^4+x^8\right)}\,dx$$

24.118 Problem number 1070

$$\int \frac{\sqrt[4]{-1+x^4}\left(-1+x^4+2x^8\right)}{x^6\left(1-x^4+x^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^4)^(1/4)*(-1 + x^4 + 2*x^8))/(x^6*(1 - x^4 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{-4\left(-1+x^4\right)^{5/4}+15x^5\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x)\#1+\log\left(\sqrt[4]{-1+x^4}-x\#1\right)\#1}{-1+2\#1^4}\&\right]}{20x^5}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{-1+x^4}\left(-1+x^4+2x^8\right)}{x^6\left(1-x^4+x^8\right)}\,dx$$

24.119 Problem number 1071

$$\int \frac{-2ab+(a+b)x}{\sqrt[4]{x^2(-a+x)(-b+x)}\,\left(abd-(a+b)dx+(-1+d)x^2\right)}\,dx$$

Optimal antiderivative

$$-\frac{2\arctan\left(\frac{x}{d^{\frac{1}{4}}\left(abx^2+(-a-b)x^3+x^4\right)^{\frac{1}{4}}}\right)}{d^{\frac{3}{4}}}-\frac{2\,\text{arctanh}\left(\frac{x}{d^{\frac{1}{4}}\left(abx^2+(-a-b)x^3+x^4\right)^{\frac{1}{4}}}\right)}{d^{\frac{3}{4}}}$$

command

```
Integrate[(-2*a*b + (a + b)*x)/((x^2*(-a + x)*(-b + x))^(1/4)*(a*b*d - (a + b)*d*x + (-
1 + d)*x^2)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt[4]{\frac{b-x}{a-x}}\,x\left(\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{\frac{b-x}{a-x}}}{\sqrt{\frac{x}{-a+x}}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{\frac{b-x}{a-x}}}{\sqrt{\frac{x}{-a+x}}}\right)\right)}{d^{3/4}\sqrt{\frac{x}{-a+x}}\,\sqrt[4]{x^2(-a+x)(-b+x)}}$$

Mathematica 12.3 output

$$\int \frac{-2ab+(a+b)x}{\sqrt[4]{x^2(-a+x)(-b+x)}\,(abd-(a+b)dx+(-1+d)x^2)}\,dx$$

24.120 Problem number 1078

$$\int \frac{\sqrt[4]{-x^2+x^4}}{x^4\,(-1+x^4)}\,dx$$

Optimal antiderivative

$$-\frac{2\left(x^2-1\right)\left(x^4-x^2\right)^{\frac{1}{4}}}{5x^3}-\frac{\arctan\left(\frac{2^{\frac{1}{4}}x}{\left(x^4-x^2\right)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{2}+\frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}x}{\left(x^4-x^2\right)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{2}$$

command

```
Integrate[(-x^2 + x^4)^(1/4)/(x^4*(-1 + x^4)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt[4]{x^2\left(-1+x^2\right)}\left(4\left(-1+x^2\right)^{5/4}+5\sqrt[4]{2}\,x^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)-5\sqrt[4]{2}\,x^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)\right)}{10x^3\sqrt[4]{-1+x^2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{-x^2+x^4}}{x^4\,(-1+x^4)}\,dx$$

24.121 Problem number 1079

$$\int \frac{\left(1-x^2\right)^2}{\left(1+x^2\right)\left(1+6x^2+x^4\right)^{3/4}}\,dx$$

Optimal antiderivative

$$-\arctan\left(\frac{-1+x}{\left(x^4+6x^2+1\right)^{\frac{1}{4}}}\right)-\arctan\left(\frac{1+x}{\left(x^4+6x^2+1\right)^{\frac{1}{4}}}\right)$$
$$+\operatorname{arctanh}\left(\frac{-1+x}{\left(x^4+6x^2+1\right)^{\frac{1}{4}}}\right)+\operatorname{arctanh}\left(\frac{1+x}{\left(x^4+6x^2+1\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[(1 - x^2)^2/((1 + x^2)*(1 + 6*x^2 + x^4)^(3/4)),x]
```

Mathematica 13.1 output

$$-\text{ArcTan}\left(\frac{-1+x}{\sqrt[4]{1+6x^2+x^4}}\right)-\text{ArcTan}\left(\frac{1+x}{\sqrt[4]{1+6x^2+x^4}}\right)$$
$$+\tanh^{-1}\left(\frac{-1+x}{\sqrt[4]{1+6x^2+x^4}}\right)+\tanh^{-1}\left(\frac{1+x}{\sqrt[4]{1+6x^2+x^4}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(1-x^2\right)^2}{\left(1+x^2\right)\left(1+6x^2+x^4\right)^{3/4}}\,dx$$

24.122 Problem number 1080

$$\int \frac{1}{(1+x)\sqrt[4]{1+6x^2+x^4}}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{-\frac{2^{\frac{3}{4}}}{2}+\frac{x2^{\frac{3}{4}}}{2}}{(x^4+6x^2+1)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{4}+\frac{\text{arctanh}\left(\frac{-\frac{2^{\frac{3}{4}}}{2}+\frac{x2^{\frac{3}{4}}}{2}}{(x^4+6x^2+1)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{4}$$

command

```
Integrate[1/((1 + x)*(1 + 6*x^2 + x^4)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{\text{ArcTan}\left(\frac{-1+x}{\sqrt[4]{2}\ \sqrt[4]{1+6x^2+x^4}}\right)+\tanh^{-1}\left(\frac{-1+x}{\sqrt[4]{2}\ \sqrt[4]{1+6x^2+x^4}}\right)}{2\ 2^{3/4}}$$

Mathematica 12.3 output

$$\int \frac{1}{(1+x)\sqrt[4]{1+6x^2+x^4}}\,dx$$

24.123 Problem number 1082

$$\int \frac{3+x^4}{\sqrt[3]{-1+x^4}\,(-1-8x^3+x^4)}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+(x^4-1)^{\frac{1}{3}}}\right)}{2}+\frac{\ln\left(-2x+\left(x^4-1\right)^{\frac{1}{3}}\right)}{2}-\frac{\ln\left(4x^2+2x\left(x^4-1\right)^{\frac{1}{3}}+\left(x^4-1\right)^{\frac{2}{3}}\right)}{4}$$

command

```
Integrate[(3 + x^4)/((-1 + x^4)^(1/3)*(-1 - 8*x^3 + x^4)),x]
```

Mathematica 13.1 output

$$-\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+\sqrt[3]{-1+x^4}}\right)+\frac{1}{2}\log\left(-2x+\sqrt[3]{-1+x^4}\right)$$
$$-\frac{1}{4}\log\left(4x^2+2x\sqrt[3]{-1+x^4}+\left(-1+x^4\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{3+x^4}{\sqrt[3]{-1+x^4}\,(-1-8x^3+x^4)}\,dx$$

24.124 Problem number 1090

$$\int \frac{\left(1-x^3+x^4+x^6\right)^{3/4}\left(-4+x^3+2x^6\right)}{\left(1-x^3+x^6\right)^2}\,dx$$

Optimal antiderivative

$$-\frac{x\left(x^6+x^4-x^3+1\right)^{\frac{3}{4}}}{x^6-x^3+1}-\frac{3\arctan\left(\frac{x}{\left(x^6+x^4-x^3+1\right)^{\frac{1}{4}}}\right)}{2}-\frac{3\,\text{arctanh}\left(\frac{x}{\left(x^6+x^4-x^3+1\right)^{\frac{1}{4}}}\right)}{2}$$

command

```
Integrate[((1 - x^3 + x^4 + x^6)^(3/4)*(-4 + x^3 + 2*x^6))/(1 - x^3 + x^6)^2,x]
```

Mathematica 13.1 output

$$-\frac{x\left(1-x^3+x^4+x^6\right)^{3/4}}{1-x^3+x^6}-\frac{3}{2}\text{ArcTan}\left(\frac{x}{\sqrt[4]{1-x^3+x^4+x^6}}\right)-\frac{3}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1-x^3+x^4+x^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(1-x^3+x^4+x^6\right)^{3/4}\left(-4+x^3+2x^6\right)}{\left(1-x^3+x^6\right)^2}\,dx$$

24.125 Problem number 1096

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{(b^2+ax^2)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{x}{b\sqrt{a\,x^2+b^2}\,\sqrt{b+\sqrt{a\,x^2+b^2}}}+\frac{\arctan\left(\frac{\sqrt{a}\,x}{\sqrt{b}\,\sqrt{b+\sqrt{a\,x^2+b^2}}}\right)}{\sqrt{a}\,b^{\frac{3}{2}}}$$

command

```
Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^(3/2),x]
```

Mathematica 13.1 output

$$\frac{x}{b\sqrt{b^2+ax^2}\,\sqrt{b+\sqrt{b^2+ax^2}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\,\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{a}\,b^{3/2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{(b^2+ax^2)^{3/2}}\,dx$$

24.126 Problem number 1108

$$\int \frac{x^5\left(-4b+5ax^2\right)}{\sqrt[4]{-b+ax^2}\,\left(b-bx^8+ax^{10}\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x^5*(-4*b + 5*a*x^2))/((-b + a*x^2)^(1/4)*(b - b*x^8 + a*x^10)),x]
```

Mathematica 13.1 output

$$\frac{1}{2}a\text{RootSum}\left[a^4b+b^4\#1^4+4b^3\#1^8+6b^2\#1^{12}+4b\#1^{16}+\#1^{20}\&,\frac{\log\left(\sqrt[4]{-b+ax^2}-\#1\right)}{b\#1+\#1^5}\&\right]$$

Mathematica 12.3 output

$$\int \frac{x^5\left(-4b+5ax^2\right)}{\sqrt[4]{-b+ax^2}\,\left(b-bx^8+ax^{10}\right)}\,dx$$

24.127 Problem number 1110

$$\int \frac{a-3b+2x}{\sqrt[4]{(-a+x)(-b+x)}\,\left(-a^3+bd-\left(-3a^2+d\right)x-3ax^2+x^3\right)}\,dx$$

Optimal antiderivative

$$-\frac{2\arctan\left(\frac{d^{\frac{1}{4}}\left(ab+(-a-b)x+x^2\right)^{\frac{1}{4}}}{a-x}\right)}{d^{\frac{3}{4}}}+\frac{2\operatorname{arctanh}\left(\frac{d^{\frac{1}{4}}\left(ab+(-a-b)x+x^2\right)^{\frac{1}{4}}}{a-x}\right)}{d^{\frac{3}{4}}}$$

command

```
Integrate[(a - 3*b + 2*x)/(((-a + x)*(-b + x))^(1/4)*(-a^3 + b*d - (-3*a^2 + d)*x - 3*a*x^2 +
```

Mathematica 13.1 output

$$\frac{2\sqrt[4]{\frac{b-x}{a-x}}\,\sqrt{-a+x}\left(\operatorname{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{\frac{-b+x}{-a+x}}}{\sqrt{-a+x}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{\frac{-b+x}{-a+x}}}{\sqrt{-a+x}}\right)\right)}{d^{3/4}\sqrt[4]{(-a+x)(-b+x)}}$$

Mathematica 12.3 output

$$\int \frac{a-3b+2x}{\sqrt[4]{(-a+x)(-b+x)}\,\left(-a^3+bd-\left(-3a^2+d\right)x-3ax^2+x^3\right)}\,dx$$

24.128 Problem number 1114

$$\int \frac{\left(-1+x^2\right)\sqrt[4]{-1+2x^2+2x^4}}{x^2\left(-1+2x^2\right)}\,dx$$

Optimal antiderivative

$$-\frac{\left(2x^4+2x^2-1\right)^{\frac{1}{4}}}{x}-\frac{\arctan\left(\frac{2^{\frac{1}{4}}x}{\left(2x^4+2x^2-1\right)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{2}+\frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}x}{\left(2x^4+2x^2-1\right)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{2}$$

command

```
Integrate[((-1 + x^2)*(-1 + 2*x^2 + 2*x^4)^(1/4))/(x^2*(-1 + 2*x^2)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt[4]{-1+2x^2+2x^4}}{x}-\frac{\text{ArcTan}\left(\frac{x}{\sqrt[4]{-\frac{1}{2}+x^2+x^4}}\right)}{2^{3/4}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{-\frac{1}{2}+x^2+x^4}}\right)}{2^{3/4}}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^2\right)\sqrt[4]{-1+2x^2+2x^4}}{x^2\left(-1+2x^2\right)}\,dx$$

24.129 Problem number 1124

$$\int \frac{x^2}{\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Optimal antiderivative

$$-\frac{4a\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}}{b^2}+\frac{4\sqrt{a^2x^2-bx}\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}}{b^2x}$$

command

```
Integrate[x^2/(Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2)),x]
```

Mathematica 13.1 output

$$-\frac{4x}{b\sqrt{x\left(ax+\sqrt{x\left(-b+a^2x\right)}\right)}}$$

Mathematica 12.3 output

$$\int \frac{x^2}{\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

24.130 Problem number 1135

$$\int \frac{3+x^2}{\sqrt[3]{1+x^2}\,(1+x^2+x^3)}\,dx$$

Optimal antiderivative

$$-\sqrt{3}\,\arctan\left(\frac{-\frac{2x\sqrt{3}}{3}+\frac{(x^2+1)^{\frac{1}{3}}\sqrt{3}}{3}}{(x^2+1)^{\frac{1}{3}}}\right)+\ln\left(x+\left(x^2+1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(x^2-x\left(x^2+1\right)^{\frac{1}{3}}+\left(x^2+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(3 + x^2)/((1 + x^2)^(1/3)*(1 + x^2 + x^3)),x]
```

Mathematica 13.1 output

$$-\sqrt{3}\,\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)+\log\left(x+\sqrt[3]{1+x^2}\right)-\frac{1}{2}\log\left(x^2-x\sqrt[3]{1+x^2}+\left(1+x^2\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{3+x^2}{\sqrt[3]{1+x^2}\,(1+x^2+x^3)}\,dx$$

24.131 Problem number 1139

$$\int \frac{-1+x^2}{(1+x+x^2)\,\sqrt[3]{x^2+x^4}}\,dx$$

Optimal antiderivative

$$-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^4+x^2\right)^{\frac{1}{3}}}\right)-\ln\left(x+\left(x^4+x^2\right)^{\frac{1}{3}}\right)+\frac{\ln\left(x^2-x\left(x^4+x^2\right)^{\frac{1}{3}}+\left(x^4+x^2\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(-1 + x^2)/((1 + x + x^2)*(x^2 + x^4)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\sqrt[3]{1+x^2}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}-2\sqrt[3]{1+x^2}}\right)-2\log\left(\sqrt[3]{x}+\sqrt[3]{1+x^2}\right)+\log\left(x^{2/3}-\sqrt[3]{x}\,\sqrt[3]{1+x^2}+(1+x\right.}{2\sqrt[3]{x^2+x^4}}$$

Mathematica 12.3 output

$$\int \frac{-1+x^2}{(1+x+x^2)\,\sqrt[3]{x^2+x^4}}\,dx$$

24.132 Problem number 1143

$$\int \frac{-1+2x^3}{(1+x+x^3)\sqrt[3]{x^2+x^5}}\,dx$$

Optimal antiderivative

$$-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^5+x^2\right)^{\frac{1}{3}}}\right)-\ln\left(x+\left(x^5+x^2\right)^{\frac{1}{3}}\right)+\frac{\ln\left(x^2-x\left(x^5+x^2\right)^{\frac{1}{3}}+\left(x^5+x^2\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(-1 + 2*x^3)/((1 + x + x^3)*(x^2 + x^5)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\sqrt[3]{1+x^3}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}-2\sqrt[3]{1+x^3}}\right)-2\log\left(\sqrt[3]{x}+\sqrt[3]{1+x^3}\right)+\log\left(x^{2/3}-\sqrt[3]{x}\,\sqrt[3]{1+x^3}+(1+x\right.\right.}{2\sqrt[3]{x^2+x^5}}$$

Mathematica 12.3 output

$$\int \frac{-1+2x^3}{(1+x+x^3)\sqrt[3]{x^2+x^5}}\,dx$$

24.133 Problem number 1151

$$\int \frac{\left(4+x^5\right)\sqrt[4]{-2+x^4+2x^5}\left(2-4x^5+x^8+2x^{10}\right)}{x^{10}\left(-1+x^5\right)}\,dx$$

Optimal antiderivative

$$\frac{4\left(2x^5+x^4-2\right)^{\frac{1}{4}}\left(10x^{10}+x^9+43x^8-20x^5-x^4+10\right)}{45x^9}$$
$$+2\arctan\left(\frac{x}{\left(2x^5+x^4-2\right)^{\frac{1}{4}}}\right)-2\,\text{arctanh}\left(\frac{x}{\left(2x^5+x^4-2\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[((4 + x^5)*(-2 + x^4 + 2*x^5)^(1/4)*(2 - 4*x^5 + x^8 + 2*x^10))/(x^10*(-
1 + x^5)),x]
```

Mathematica 13.1 output

$$\frac{4\sqrt[4]{-2+x^4+2x^5}\left(10-x^4-20x^5+43x^8+x^9+10x^{10}\right)}{45x^9}$$
$$+2\text{ArcTan}\left(\frac{x}{\sqrt[4]{-2+x^4+2x^5}}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt[4]{-2+x^4+2x^5}}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(4+x^5\right)\sqrt[4]{-2+x^4+2x^5}\left(2-4x^5+x^8+2x^{10}\right)}{x^{10}\left(-1+x^5\right)}\,dx$$

24.134 Problem number 1153

$$\int\frac{\sqrt{1-x^6}\left(1+2x^6\right)\left(1+x^2-x^4-2x^6-x^8+x^{12}\right)}{\left(-1+x^6\right)\left(-1+2x^6-3x^{12}+x^{18}\right)}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{x}{\sqrt{-x^6+1}}\right)}{3}-\frac{\arctan\left(\frac{x\sqrt{-x^6+1}}{x^6+x^2-1}\right)}{3}-\frac{\text{arctanh}\left(\frac{\sqrt{3}\,x\sqrt{-x^6+1}}{x^6-x^2-1}\right)\sqrt{3}}{3}$$

command

```
Integrate[(Sqrt[1 - x^6]*(1 + 2*x^6)*(1 + x^2 - x^4 - 2*x^6 - x^8 + x^12))/((-1 + x^6)*(-
1 + 2*x^6 - 3*x^12 + x^18)),x]
```

Mathematica 13.1 output

$$\frac{1}{3}\left(-\text{ArcTan}\left(\frac{x}{\sqrt{1-x^6}}\right)+\left(1+i\sqrt{3}\right)\text{ArcTan}\left(\frac{\left(1-i\sqrt{3}\right)x}{2\sqrt{1-x^6}}\right)\right.$$
$$\left.+\left(1-i\sqrt{3}\right)\text{ArcTan}\left(\frac{\left(1+i\sqrt{3}\right)x}{2\sqrt{1-x^6}}\right)\right)$$

Mathematica 12.3 output

$$\int\frac{\sqrt{1-x^6}\left(1+2x^6\right)\left(1+x^2-x^4-2x^6-x^8+x^{12}\right)}{\left(-1+x^6\right)\left(-1+2x^6-3x^{12}+x^{18}\right)}\,dx$$

24.135 Problem number 1158

$$\int \frac{3+x^2}{\sqrt[3]{1+x^2}\,(-1-x^2+x^3)}\,dx$$

Optimal antiderivative

$$-\sqrt{3}\,\arctan\left(\frac{\frac{2x\sqrt{3}}{3}+\frac{(x^2+1)^{\frac{1}{3}}\sqrt{3}}{3}}{(x^2+1)^{\frac{1}{3}}}\right)+\ln\left(-x+\left(x^2+1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(x^2+x\left(x^2+1\right)^{\frac{1}{3}}+\left(x^2+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(3 + x^2)/((1 + x^2)^(1/3)*(-1 - x^2 + x^3)),x]
```

Mathematica 13.1 output

$$-\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)+\log\left(-x+\sqrt[3]{1+x^2}\right)-\frac{1}{2}\log\left(x^2+x\sqrt[3]{1+x^2}+\left(1+x^2\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{3+x^2}{\sqrt[3]{1+x^2}\,(-1-x^2+x^3)}\,dx$$

24.136 Problem number 1168

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(-4-2x^3+x^6\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^3)^(2/3)*(2 + x^3))/(x^6*(-4 - 2*x^3 + x^6)),x]
```

Mathematica 13.1 output

$$-\frac{6\left(-1+x^3\right)^{5/3}+5x^5\text{RootSum}\left[5-10\#1^3+4\#1^6\&,\frac{-\log(x)\#1^2+\log\left(\sqrt[3]{-1+x^3}-x\#1\right)\#1^2}{-5+4\#1^3}\&\right]}{60x^5}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(-4-2x^3+x^6\right)}\,dx$$

24.137 Problem number 1169

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(-4-2x^3+x^6\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^3)^(2/3)*(2 + x^3))/(x^6*(-4 - 2*x^3 + x^6)),x]
```

Mathematica 13.1 output

$$-\frac{6\left(-1+x^3\right)^{5/3}+5x^5\text{RootSum}\left[5-10\#1^3+4\#1^6\&,\frac{-\log(x)\#1^2+\log\left(\sqrt[3]{-1+x^3}-x\#1\right)\#1^2}{-5+4\#1^3}\&\right]}{60x^5}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(-4-2x^3+x^6\right)}\,dx$$

24.138 Problem number 1170

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(4-2x^3+x^6\right)}{x^6\left(-8+4x^3+x^6\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^3)^(2/3)*(4 - 2*x^3 + x^6))/(x^6*(-8 + 4*x^3 + x^6)),x]
```

Mathematica 13.1 output

$$-\frac{4\left(-1+x^3\right)^{5/3}+5x^5\text{RootSum}\left[3-12\#1^3+8\#1^6\&,\frac{-\log(x)\#1^2+\log\left(\sqrt[3]{-1+x^3}-x\#1\right)\#1^2}{-3+4\#1^3}\&\right]}{40x^5}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(4-2x^3+x^6\right)}{x^6\left(-8+4x^3+x^6\right)}\,dx$$

24.139 Problem number 1171

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(4-2x^3+x^6\right)}{x^6\left(-8+4x^3+x^6\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^3)^(2/3)*(4 - 2*x^3 + x^6))/(x^6*(-8 + 4*x^3 + x^6)),x]
```

Mathematica 13.1 output

$$-\frac{4\left(-1+x^3\right)^{5/3}+5x^5\text{RootSum}\left[3-12\#1^3+8\#1^6\&,\frac{-\log(x)\#1^2+\log\left(\sqrt[3]{-1+x^3}-x\#1\right)\#1^2}{-3+4\#1^3}\&\right]}{40x^5}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(4-2x^3+x^6\right)}{x^6\left(-8+4x^3+x^6\right)}\,dx$$

24.140 Problem number 1172

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(2-2x^3+x^6\right)}{x^6\left(-4+4x^3+x^6\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^3)^(2/3)*(2 - 2*x^3 + x^6))/(x^6*(-4 + 4*x^3 + x^6)),x]
```

Mathematica 13.1 output

$$-\frac{4\left(-1+x^3\right)^{5/3}+5x^5\text{RootSum}\left[-1-4\#1^3+4\#1^6\&,\frac{-\log(x)\#1^2+\log\left(\sqrt[3]{-1+x^3}-x\#1\right)\#1^2}{-1+2\#1^3}\&\right]}{40x^5}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(2-2x^3+x^6\right)}{x^6\left(-4+4x^3+x^6\right)}\,dx$$

24.141 Problem number 1173

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(2-2x^3+x^6\right)}{x^6\left(-4+4x^3+x^6\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^3)^(2/3)*(2 - 2*x^3 + x^6))/(x^6*(-4 + 4*x^3 + x^6)),x]
```

Mathematica 13.1 output

$$-\frac{4\left(-1+x^3\right)^{5/3}+5x^5\text{RootSum}\left[-1-4\#1^3+4\#1^6\&,\frac{-\log(x)\#1^2+\log\left(\sqrt[3]{-1+x^3}-x\#1\right)\#1^2}{-1+2\#1^3}\&\right]}{40x^5}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(2-2x^3+x^6\right)}{x^6\left(-4+4x^3+x^6\right)}\,dx$$

24.142 Problem number 1174

$$\int \frac{2+x^4}{\sqrt[4]{-1+x^4}\left(-2+x^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(2 + x^4)/((-1 + x^4)^(1/4)*(-2 + x^8)),x]
```

Mathematica 13.1 output

$$-\frac{1}{16}\text{RootSum}\left[1-4\#1^4+2\#1^8\&,\frac{-3\log(x)+3\log\left(\sqrt[4]{-1+x^4}-x\#1\right)+2\log(x)\#1^4-2\log\left(\sqrt[4]{-1+x^4}-x\#1\right)\#1^4}{-\#1+\#1^5}\&\right]$$

Mathematica 12.3 output

$$\int \frac{2+x^4}{\sqrt[4]{-1+x^4}\left(-2+x^8\right)}\,dx$$

24.143 Problem number 1182

$$\int \frac{\sqrt[3]{-1+x^4}\left(3+x^4\right)}{x^2\left(-1-x^3+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^4-1\right)^{\frac{1}{3}}}{x}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^4-1\right)^{\frac{1}{3}}}\right)$$
$$+\ln\left(-x+\left(x^4-1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(x^2+x\left(x^4-1\right)^{\frac{1}{3}}+\left(x^4-1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((-1 + x^4)^(1/3)*(3 + x^4))/(x^2*(-1 - x^3 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{3\sqrt[3]{-1+x^4}}{x}+\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{-1+x^4}}\right)$$
$$+\log\left(-x+\sqrt[3]{-1+x^4}\right)-\frac{1}{2}\log\left(x^2+x\sqrt[3]{-1+x^4}+\left(-1+x^4\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt[3]{-1+x^4}\left(3+x^4\right)}{x^2\left(-1-x^3+x^4\right)}\,dx$$

24.144 Problem number 1183

$$\int \frac{x^2\left(4+x^3\right)}{\left(1+x^3\right)^{3/4}\left(1+x^3+x^4\right)}\,dx$$

Optimal antiderivative

$$-\sqrt{2}\,\arctan\left(\frac{-\frac{x^2\sqrt{2}}{2}+\frac{\sqrt{x^3+1}\,\sqrt{2}}{2}}{x\left(x^3+1\right)^{\frac{1}{4}}}\right)-\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\,x\left(x^3+1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^3+1}}\right)$$

command

```
Integrate[(x^2*(4 + x^3))/((1 + x^3)^(3/4)*(1 + x^3 + x^4)),x]
```

Mathematica 13.1 output

$$-\sqrt{2}\left(\text{ArcTan}\left(\frac{-x^2+\sqrt{1+x^3}}{\sqrt{2}\,x\sqrt[4]{1+x^3}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{1+x^3}}{x^2+\sqrt{1+x^3}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{x^2\left(4+x^3\right)}{\left(1+x^3\right)^{3/4}\left(1+x^3+x^4\right)}\,dx$$

24.145 Problem number 1191

$$\int \frac{\left(1-x^2+2x^4\right)\sqrt{1-x^2-x^4-x^6}}{\left(-1+x^2\right)\left(1+x^2\right)\left(-1+x^4+x^6\right)}\,dx$$

Optimal antiderivative

$$\arctan\left(\frac{x\sqrt{-x^6-x^4-x^2+1}}{x^6+x^4+x^2-1}\right)-\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x\sqrt{-x^6-x^4-x^2+1}}{x^6+x^4+x^2-1}\right)$$

command

```
Integrate[((1 - x^2 + 2*x^4)*Sqrt[1 - x^2 - x^4 - x^6])/((-1 + x^2)*(1 + x^2)*(-1 + x^4 + x^6
```

Mathematica 13.1 output

$$-\text{ArcTan}\left(\frac{x}{\sqrt{1-x^2-x^4-x^6}}\right)+\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2-x^4-x^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(1-x^2+2x^4\right)\sqrt{1-x^2-x^4-x^6}}{\left(-1+x^2\right)\left(1+x^2\right)\left(-1+x^4+x^6\right)}\,dx$$

24.146 Problem number 1197

$$\int \frac{1}{x+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x + Sqrt[x + Sqrt[1 + x^2]])^(-1),x]
```

Mathematica 13.1 output

$$-\log\left(x+\sqrt{1+x^2}\right)+2\text{RootSum}\left[-1+2\#1^3+\#1^4\&,\frac{\log\left(\sqrt{x+\sqrt{1+x^2}}-\#1\right)+\log\left(\sqrt{x+\sqrt{1+x^2}}-\#1\right)\#1}{3+2\#1}\&\right]$$

Mathematica 12.3 output

$$\int\frac{1}{x+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.147 Problem number 1198

$$\int\frac{1}{x+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x + Sqrt[x + Sqrt[1 + x^2]])^(-1),x]
```

Mathematica 13.1 output

$$-\log\left(x+\sqrt{1+x^2}\right)+2\text{RootSum}\left[-1+2\#1^3+\#1^4\&,\frac{\log\left(\sqrt{x+\sqrt{1+x^2}}-\#1\right)+\log\left(\sqrt{x+\sqrt{1+x^2}}-\#1\right)\#1}{3+2\#1}\&\right]$$

Mathematica 12.3 output

$$\int\frac{1}{x+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.148 Problem number 1209

$$\int \frac{-b^2+a^2x^8}{x^2\left(b+ax^4\right)^{3/4}\left(b^2+a^2x^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-b^2 + a^2*x^8)/(x^2*(b + a*x^4)^(3/4)*(b^2 + a^2*x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{b+ax^4}}{bx}+\frac{a^2\text{RootSum}\left[2a^2-2a\#1^4+\#1^8\&,\frac{\log(x)-\log\left(\sqrt[4]{b+ax^4}-x\#1\right)}{-a\#1^3+\#1^7}\&\right]}{4b}$$

Mathematica 12.3 output

$$\int \frac{-b^2+a^2x^8}{x^2\left(b+ax^4\right)^{3/4}\left(b^2+a^2x^8\right)}\,dx$$

24.149 Problem number 1212

$$\int \frac{-3+x}{\sqrt[3]{-1+x^2}\left(2+x+x^2\right)}\,dx$$

Optimal antiderivative

$$\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^2-1\right)^{\frac{1}{3}}}{2+2x+\left(x^2-1\right)^{\frac{1}{3}}}\right)+\ln\left(-1-x+\left(x^2-1\right)^{\frac{1}{3}}\right)$$
$$-\frac{\ln\left(1+2x+x^2+(1+x)\left(x^2-1\right)^{\frac{1}{3}}+\left(x^2-1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(-3 + x)/((-1 + x^2)^(1/3)*(2 + x + x^2)),x]
```

Mathematica 13.1 output

$$\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{-1+x^2}}{2+2x+\sqrt[3]{-1+x^2}}\right)+\log\left(-1-x+\sqrt[3]{-1+x^2}\right)$$
$$-\frac{1}{2}\log\left(1+2x+x^2+(1+x)\sqrt[3]{-1+x^2}+\left(-1+x^2\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{-3+x}{\sqrt[3]{-1+x^2}\left(2+x+x^2\right)}\,dx$$

24.150 Problem number 1225

$$\int \sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Optimal antiderivative

$$\frac{x\sqrt{a\,x^2+\sqrt{a^2x^4+b}}}{2}+\frac{\sqrt{b}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a}\,x\sqrt{a\,x^2+\sqrt{a^2x^4+b}}}{\sqrt{b}}\right)\sqrt{2}}{4\sqrt{a}}$$

command

```
Integrate[Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
```

Mathematica 13.1 output

$$\frac{1}{2}x\sqrt{ax^2+\sqrt{b+a^2x^4}}+\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{a}\,x\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b}}\right)}{2\sqrt{2}\,\sqrt{a}}$$

Mathematica 12.3 output

$$\int \sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

24.151 Problem number 1234

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}}{x^3\left(1-x^3+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^4+1\right)^{\frac{2}{3}}}{2x^2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^4+1\right)^{\frac{1}{3}}}\right)$$
$$+\ln\left(-x+\left(x^4+1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(x^2+x\left(x^4+1\right)^{\frac{1}{3}}+\left(x^4+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((-3 + x^4)*(1 + x^4)^(2/3))/(x^3*(1 - x^3 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{3\left(1+x^4\right)^{2/3}}{2x^2}-\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{1+x^4}}\right)$$
$$+\log\left(-x+\sqrt[3]{1+x^4}\right)-\frac{1}{2}\log\left(x^2+x\sqrt[3]{1+x^4}+\left(1+x^4\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}}{x^3\left(1-x^3+x^4\right)}\,dx$$

24.152 Problem number 1235

$$\int\frac{\sqrt[3]{-1+x^4}\left(3+x^4\right)}{x^2\left(-1+x^3+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^4-1\right)^{\frac{1}{3}}}{x}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^4-1\right)^{\frac{1}{3}}}\right)$$
$$-\ln\left(x+\left(x^4-1\right)^{\frac{1}{3}}\right)+\frac{\ln\left(x^2-x\left(x^4-1\right)^{\frac{1}{3}}+\left(x^4-1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((-1 + x^4)^(1/3)*(3 + x^4))/(x^2*(-1 + x^3 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{3\sqrt[3]{-1+x^4}}{x}+\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{-x+2\sqrt[3]{-1+x^4}}\right)$$
$$-\log\left(x+\sqrt[3]{-1+x^4}\right)+\frac{1}{2}\log\left(x^2-x\sqrt[3]{-1+x^4}+\left(-1+x^4\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{\sqrt[3]{-1+x^4}\left(3+x^4\right)}{x^2\left(-1+x^3+x^4\right)}\,dx$$

24.153 Problem number 1236

$$\int \frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)}{x^3\left(-1+x^3+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^4-1\right)^{\frac{2}{3}}}{2x^2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^4-1\right)^{\frac{1}{3}}}\right)+\ln\left(x+\left(x^4-1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(x^2-x\left(x^4-1\right)^{\frac{1}{3}}+\left(x^4-1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((-1 + x^4)^(2/3)*(3 + x^4))/(x^3*(-1 + x^3 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{3\left(-1+x^4\right)^{2/3}}{2x^2}+\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{-x+2\sqrt[3]{-1+x^4}}\right)+\log\left(x+\sqrt[3]{-1+x^4}\right)-\frac{1}{2}\log\left(x^2-x\sqrt[3]{-1+x^4}+\left(-1+x^4\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)}{x^3\left(-1+x^3+x^4\right)}\,dx$$

24.154 Problem number 1237

$$\int \frac{\left(-3+x^4\right)\sqrt[3]{1+x^4}}{x^2\left(1+x^3+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^4+1\right)^{\frac{1}{3}}}{x}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^4+1\right)^{\frac{1}{3}}}\right)-\ln\left(x+\left(x^4+1\right)^{\frac{1}{3}}\right)+\frac{\ln\left(x^2-x\left(x^4+1\right)^{\frac{1}{3}}+\left(x^4+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((-3 + x^4)*(1 + x^4)^(1/3))/(x^2*(1 + x^3 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{3\sqrt[3]{1+x^4}}{x}+\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{-x+2\sqrt[3]{1+x^4}}\right)$$
$$-\log\left(x+\sqrt[3]{1+x^4}\right)+\frac{1}{2}\log\left(x^2-x\sqrt[3]{1+x^4}+\left(1+x^4\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(-3+x^4\right)\sqrt[3]{1+x^4}}{x^2\left(1+x^3+x^4\right)}\,dx$$

24.155 Problem number 1238

$$\int\frac{(4+3x)\left(-1-x+x^4\right)\sqrt[4]{-1-x+2x^4}}{x^6\left(1+x+x^4\right)}\,dx$$

Optimal antiderivative

$$-\frac{4\left(2x^4-x-1\right)^{\frac{1}{4}}\left(12x^4-x-1\right)}{5x^5}-4\,3^{\frac{1}{4}}\arctan\left(\frac{3^{\frac{1}{4}}x}{\left(2x^4-x-1\right)^{\frac{1}{4}}}\right)+4\,3^{\frac{1}{4}}\,\text{arctanh}\left(\frac{3^{\frac{1}{4}}x}{\left(2x^4-x-1\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[((4 + 3*x)*(-1 - x + x^4)*(-1 - x + 2*x^4)^(1/4))/(x^6*(1 + x + x^4)),x]
```

Mathematica 13.1 output

$$-\frac{4\sqrt[4]{-1-x+2x^4}\left(-1-x+12x^4\right)}{5x^5}-4\sqrt[4]{3}\,\text{ArcTan}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{-1-x+2x^4}}\right)$$
$$+4\sqrt[4]{3}\,\tanh^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{-1-x+2x^4}}\right)$$

Mathematica 12.3 output

$$\int\frac{(4+3x)\left(-1-x+x^4\right)\sqrt[4]{-1-x+2x^4}}{x^6\left(1+x+x^4\right)}\,dx$$

24.156 Problem number 1243

$$\int \frac{\left(1+x^5\right)^{2/3}\left(-3+2x^5\right)}{x^3\left(1-x^3+x^5\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^5+1\right)^{\frac{2}{3}}}{2x^2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^5+1\right)^{\frac{1}{3}}}\right)+\ln\left(-x+\left(x^5+1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(x^2+x\left(x^5+1\right)^{\frac{1}{3}}+\left(x^5+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((1 + x^5)^(2/3)*(-3 + 2*x^5))/(x^3*(1 - x^3 + x^5)),x]
```

Mathematica 13.1 output

$$\frac{3\left(1+x^5\right)^{2/3}}{2x^2}-\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{1+x^5}}\right)+\log\left(-x+\sqrt[3]{1+x^5}\right)-\frac{1}{2}\log\left(x^2+x\sqrt[3]{1+x^5}+\left(1+x^5\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^5\right)^{2/3}\left(-3+2x^5\right)}{x^3\left(1-x^3+x^5\right)}\,dx$$

24.157 Problem number 1244

$$\int \frac{\left(-1+x^5\right)^{2/3}\left(3+2x^5\right)}{x^3\left(-1-x^3+x^5\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^5-1\right)^{\frac{2}{3}}}{2x^2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^5-1\right)^{\frac{1}{3}}}\right)+\ln\left(-x+\left(x^5-1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(x^2+x\left(x^5-1\right)^{\frac{1}{3}}+\left(x^5-1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((-1 + x^5)^(2/3)*(3 + 2*x^5))/(x^3*(-1 - x^3 + x^5)),x]
```

Mathematica 13.1 output

$$\frac{3\left(-1+x^5\right)^{2/3}}{2x^2}-\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{-1+x^5}}\right)$$
$$+\log\left(-x+\sqrt[3]{-1+x^5}\right)-\frac{1}{2}\log\left(x^2+x\sqrt[3]{-1+x^5}+\left(-1+x^5\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(-1+x^5\right)^{2/3}\left(3+2x^5\right)}{x^3\left(-1-x^3+x^5\right)}\,dx$$

24.158 Problem number 1248

$$\int\frac{\left(-3+k^2\right)x+2k^2x^3}{\sqrt[4]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d+\left(3-dk^2\right)x^2-3x^4+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{d^{\frac{1}{4}}\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{1}{4}}}{x^2-1}\right)}{d^{\frac{3}{4}}}-\frac{\text{arctanh}\left(\frac{d^{\frac{1}{4}}\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{1}{4}}}{x^2-1}\right)}{d^{\frac{3}{4}}}$$

command

```
Integrate[((-3 + k^2)*x + 2*k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/4)*(-1 + d + (3 - d*k^2)*x
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-1+x^2}\,\sqrt[4]{-1+k^2x^2}\left(\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{-1+k^2x^2}}{\left(-1+x^2\right)^{3/4}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{-1+k^2x^2}}{\left(-1+x^2\right)^{3/4}}\right)\right)}{d^{3/4}\sqrt[4]{\left(-1+x^2\right)\left(-1+k^2x^2\right)}}$$

Mathematica 12.3 output

$$\int\frac{\left(-3+k^2\right)x+2k^2x^3}{\sqrt[4]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d+\left(3-dk^2\right)x^2-3x^4+x^6\right)}\,dx$$

24.159 Problem number 1249

$$\int \frac{\left(-1+x^7\right)^{2/3}\left(3+4x^7\right)}{x^3\left(-1+x^3+x^7\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^7-1\right)^{\frac{2}{3}}}{2x^2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^7-1\right)^{\frac{1}{3}}}\right)$$
$$+\ln\left(x+\left(x^7-1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(x^2-x\left(x^7-1\right)^{\frac{1}{3}}+\left(x^7-1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((-1 + x^7)^(2/3)*(3 + 4*x^7))/(x^3*(-1 + x^3 + x^7)),x]
```

Mathematica 13.1 output

$$\frac{3\left(-1+x^7\right)^{2/3}}{2x^2}+\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{-x+2\sqrt[3]{-1+x^7}}\right)$$
$$+\log\left(x+\sqrt[3]{-1+x^7}\right)-\frac{1}{2}\log\left(x^2-x\sqrt[3]{-1+x^7}+\left(-1+x^7\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^7\right)^{2/3}\left(3+4x^7\right)}{x^3\left(-1+x^3+x^7\right)}\,dx$$

24.160 Problem number 1250

$$\int \frac{x}{\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{20a\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}}{3b^2x}-\frac{4\sqrt{a^2x^2-bx}\,\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}}{3b^2x^2}$$

command

```
Integrate[x/(Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2)),x]
```

Mathematica 13.1 output

$$\frac{4\sqrt{x\left(ax+\sqrt{x\left(-b+a^2x\right)}\right)}\left(b+a\left(-ax+5\sqrt{x\left(-b+a^2x\right)}\right)\right)}{3b^2x\sqrt{x\left(-b+a^2x\right)}}$$

Mathematica 12.3 output

$$\int \frac{x}{\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

24.161 Problem number 1253

$$\int \frac{3-x^2}{(1-x^2)\sqrt[4]{1-6x^2+x^4}}\,dx$$

Optimal antiderivative

$$\arctan\left(\frac{i+x}{(x^4-6x^2+1)^{\frac{1}{4}}}\right)-\arctan\left(\frac{(x^4-6x^2+1)^{\frac{1}{4}}}{-i+x}\right)$$
$$+\operatorname{arctanh}\left(\frac{i+x}{(x^4-6x^2+1)^{\frac{1}{4}}}\right)+\operatorname{arctanh}\left(\frac{(x^4-6x^2+1)^{\frac{1}{4}}}{-i+x}\right)$$

command

```
Integrate[(3 - x^2)/((1 - x^2)*(1 - 6*x^2 + x^4)^(1/4)),x]
```

Mathematica 13.1 output

$$\operatorname{ArcTan}\left(\frac{i+x}{\sqrt[4]{1-6x^2+x^4}}\right)-\operatorname{ArcTan}\left(\frac{\sqrt[4]{1-6x^2+x^4}}{-i+x}\right)$$
$$+\tanh^{-1}\left(\frac{i+x}{\sqrt[4]{1-6x^2+x^4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{1-6x^2+x^4}}{-i+x}\right)$$

Mathematica 12.3 output

$$\int \frac{3-x^2}{(1-x^2)\sqrt[4]{1-6x^2+x^4}}\,dx$$

24.162 Problem number 1254

$$\int \frac{-3+x^4}{(1+x^4)\sqrt[4]{-3x+4x^4-3x^5}}\,dx$$

Optimal antiderivative

$$\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(-3x^5+4x^4-3x\right)^{\frac{3}{4}}}{3x^4-4x^3+3}\right)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,\left(-3x^5+4x^4-3x\right)^{\frac{3}{4}}}{3x^4-4x^3+3}\right)$$

command

```
Integrate[(-3 + x^4)/((1 + x^4)*(-3*x + 4*x^4 - 3*x^5)^(1/4)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt[4]{x}\,\sqrt[4]{3-4x^3+3x^4}\left(\text{ArcTan}\left(\frac{2x^{3/4}\sqrt[4]{3-4x^3+3x^4}}{-2x^{3/2}+\sqrt{3-4x^3+3x^4}}\right)+\tanh^{-1}\left(\frac{2x^{3/4}\sqrt[4]{3-4x^3+3x^4}}{2x^{3/2}+\sqrt{3-4x^3+3x^4}}\right)\right)}{\sqrt[4]{-3x+4x^4-3x^5}}$$

Mathematica 12.3 output

$$\int \frac{-3+x^4}{(1+x^4)\sqrt[4]{-3x+4x^4-3x^5}}\,dx$$

24.163 Problem number 1255

$$\int \frac{\sqrt[3]{-1+x^6}\,\left(1+x^6\right)}{x^2\left(-1+x^3+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x}+\frac{\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^6-1\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}-\frac{\ln\left(x+\left(x^6-1\right)^{\frac{1}{3}}\right)}{3}+\frac{\ln\left(x^2-x\left(x^6-1\right)^{\frac{1}{3}}+\left(x^6-1\right)^{\frac{2}{3}}\right)}{6}$$

command

```
Integrate[((-1 + x^6)^(1/3)*(1 + x^6))/(x^2*(-1 + x^3 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{-1+x^6}}{x}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\,x}{-x+2\sqrt[3]{-1+x^6}}\right)}{\sqrt{3}}-\frac{1}{3}\log\left(x+\sqrt[3]{-1+x^6}\right)$$
$$+\frac{1}{6}\log\left(x^2-x\sqrt[3]{-1+x^6}+\left(-1+x^6\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt[3]{-1+x^6}\,\left(1+x^6\right)}{x^2\left(-1+x^3+x^6\right)}\,dx$$

24.164 Problem number 1260

$$\int \frac{(-1+x)(3+x)}{(-1+x^2)^{2/3}(2-x+x^2)}\,dx$$

Optimal antiderivative

$$-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^2-1\right)^{\frac{2}{3}}}{-2-2x+\left(x^2-1\right)^{\frac{2}{3}}}\right)-\ln\left(1+x+\left(x^2-1\right)^{\frac{2}{3}}\right)+\frac{\ln\left(1+2x+x^2+(-1-x)\left(x^2-1\right)^{\frac{2}{3}}+\left(x^2-1\right)^{\frac{4}{3}}\right)}{2}$$

command

```
Integrate[((-1 + x)*(3 + x))/((-1 + x^2)^(2/3)*(2 - x + x^2)),x]
```

Mathematica 13.1 output

$$-\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\left(-1+x^2\right)^{2/3}}{-2-2x+\left(-1+x^2\right)^{2/3}}\right)-\log\left(1+x+\left(-1+x^2\right)^{2/3}\right)+\frac{1}{2}\log\left(1+2x+x^2+(-1-x)\left(-1+x^2\right)^{2/3}+\left(-1+x^2\right)^{4/3}\right)$$

Mathematica 12.3 output

$$\int \frac{(-1+x)(3+x)}{(-1+x^2)^{2/3}(2-x+x^2)}\,dx$$

24.165 Problem number 1264

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx$$

Optimal antiderivative

$$-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^3+2\right)^{\frac{1}{3}}}{4+2x+\left(x^3+2\right)^{\frac{1}{3}}}\right)-\ln\left(-2-x+\left(x^3+2\right)^{\frac{1}{3}}\right)+\frac{\ln\left(4+4x+x^2+(2+x)\left(x^3+2\right)^{\frac{1}{3}}+\left(x^3+2\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$-\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{2+x^3}}{4+2x+\sqrt[3]{2+x^3}}\right)-\log\left(-2-x+\sqrt[3]{2+x^3}\right)$$
$$+\frac{1}{2}\log\left(4+4x+x^2+(2+x)\sqrt[3]{2+x^3}+\left(2+x^3\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx$$

24.166 Problem number 1277

$$\int\frac{\left(-1+x^4+2x^6\right)\sqrt[3]{x+x^5+x^7}}{\left(1+x^4+x^6\right)\left(1-x^2+x^4+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^7+x^5+x\right)^{\frac{1}{3}}}\right)}{2}+\frac{\ln\left(-x+\left(x^7+x^5+x\right)^{\frac{1}{3}}\right)}{2}$$
$$-\frac{\ln\left(x^2+x\left(x^7+x^5+x\right)^{\frac{1}{3}}+\left(x^7+x^5+x\right)^{\frac{2}{3}}\right)}{4}$$

command

```
Integrate[((-1 + x^4 + 2*x^6)*(x + x^5 + x^7)^(1/3))/((1 + x^4 + x^6)*(1 - x^2 + x^4 + x^6)),
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x+x^5+x^7}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}+2\sqrt[3]{1+x^4+x^6}}\right)+2\log\left(-x^{2/3}+\sqrt[3]{1+x^4+x^6}\right)-\log\left(x^{4/3}+x^{2/3}\sqrt[3]{1+\right.\right.}{4\sqrt[3]{x}\,\sqrt[3]{1+x^4+x^6}}$$

Mathematica 12.3 output

$$\int\frac{\left(-1+x^4+2x^6\right)\sqrt[3]{x+x^5+x^7}}{\left(1+x^4+x^6\right)\left(1-x^2+x^4+x^6\right)}\,dx$$

24.167 Problem number 1283

$$\int \frac{2abx+(-3a+b)x^2}{\sqrt[4]{x^2(-a+x)(-b+x)}\,(a^3-3a^2x+(3a-bd)x^2+(-1+d)x^3)}\,dx$$

Optimal antiderivative

$$-\frac{2\arctan\left(\frac{d^{\frac{1}{4}}\left(abx^2+(-a-b)x^3+x^4\right)^{\frac{1}{4}}}{a-x}\right)}{d^{\frac{3}{4}}}+\frac{2\operatorname{arctanh}\left(\frac{d^{\frac{1}{4}}\left(abx^2+(-a-b)x^3+x^4\right)^{\frac{1}{4}}}{a-x}\right)}{d^{\frac{3}{4}}}$$

command

```
Integrate[(2*a*b*x + (-3*a + b)*x^2)/((x^2*(-a + x)*(-b + x))^(1/4)*(a^3 - 3*a^2*x + (3*a - b
1 + d)*x^3)),x]
```

Mathematica 13.1 output

$$-\frac{x\sqrt[4]{\frac{-b+x}{a-x}}\left(\text{ArcTan}\left(\frac{\sqrt[a-x]{\left(1+\sqrt{d}\right)\sqrt{\frac{-b+x}{a-x}}}}{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{\frac{x}{a-x}}\,\sqrt[(-a+x)]{\sqrt[4]{\frac{-b+x}{a-x}}}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x\sqrt[4]{\frac{-b+x}{a-x}}}{\sqrt{\frac{x}{2a-2x}}\left(a+x\left(-1+\sqrt{d}\right)\sqrt{\frac{-b+x}{a-x}}\right)}\right.\right.}{d^{3/4}\sqrt{\frac{x}{2a-2x}}\,\sqrt[4]{x^2(-a+x)(-b+x)}}$$

Mathematica 12.3 output

$$\int \frac{2abx+(-3a+b)x^2}{\sqrt[4]{x^2(-a+x)(-b+x)}\,(a^3-3a^2x+(3a-bd)x^2+(-1+d)x^3)}\,dx$$

24.168 Problem number 1285

$$\int \frac{1+x^2}{(-1+x+x^2)\,\sqrt[3]{-x^2+x^4}}\,dx$$

Optimal antiderivative

$$-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^4-x^2\right)^{\frac{1}{3}}}\right)-\ln\left(x+\left(x^4-x^2\right)^{\frac{1}{3}}\right)+\frac{\ln\left(x^2-x\left(x^4-x^2\right)^{\frac{1}{3}}+\left(x^4-x^2\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(1 + x^2)/((-1 + x + x^2)*(-x^2 + x^4)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\sqrt[3]{-1+x^2}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}-2\sqrt[3]{-1+x^2}}\right)-2\log\left(\sqrt[3]{x}+\sqrt[3]{-1+x^2}\right)+\log\left(x^{2/3}-\sqrt[3]{x}\,\sqrt[3]{-1+x^2}+\right.\right.}{2\sqrt[3]{x^2\left(-1+x^2\right)}}$$

Mathematica 12.3 output

$$\int \frac{1+x^2}{\left(-1+x+x^2\right)\sqrt[3]{-x^2+x^4}}\,dx$$

24.169 Problem number 1288

$$\int \frac{1+2x^3}{\left(-1+x+x^3\right)\sqrt[3]{-x^2+x^5}}\,dx$$

Optimal antiderivative

$$-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^5-x^2\right)^{\frac{1}{3}}}\right)-\ln\left(x+\left(x^5-x^2\right)^{\frac{1}{3}}\right)+\frac{\ln\left(x^2-x\left(x^5-x^2\right)^{\frac{1}{3}}+\left(x^5-x^2\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(1 + 2*x^3)/((-1 + x + x^3)*(-x^2 + x^5)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\sqrt[3]{-1+x^3}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}-2\sqrt[3]{-1+x^3}}\right)-2\log\left(\sqrt[3]{x}+\sqrt[3]{-1+x^3}\right)+\log\left(x^{2/3}-\sqrt[3]{x}\,\sqrt[3]{-1+x^3}+\right.\right.}{2\sqrt[3]{x^2\left(-1+x^3\right)}}$$

Mathematica 12.3 output

$$\int \frac{1+2x^3}{\left(-1+x+x^3\right)\sqrt[3]{-x^2+x^5}}\,dx$$

24.170 Problem number 1300

$$\int \frac{4b+ax^3}{\sqrt[4]{b+ax^3}\,(b+ax^3+x^4)}\,dx$$

Optimal antiderivative

$$-\sqrt{2}\,\arctan\left(\frac{-\frac{x^2\sqrt{2}}{2}+\frac{\sqrt{x^3a+b}\,\sqrt{2}}{2}}{x\,(x^3a+b)^{\frac{1}{4}}}\right)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,x(x^3a+b)^{\frac{1}{4}}}{x^2+\sqrt{x^3a+b}}\right)$$

command

```
Integrate[(4*b + a*x^3)/((b + a*x^3)^(1/4)*(b + a*x^3 + x^4)),x]
```

Mathematica 13.1 output

$$\sqrt{2}\left(-\text{ArcTan}\left(\frac{-x^2+\sqrt{b+ax^3}}{\sqrt{2}\,x\sqrt[4]{b+ax^3}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{b+ax^3}}{x^2+\sqrt{b+ax^3}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{4b+ax^3}{\sqrt[4]{b+ax^3}\,(b+ax^3+x^4)}\,dx$$

24.171 Problem number 1307

$$\int \frac{\left(-1+x^6\right)^{2/3}\left(1+x^6\right)}{x^3\left(-1-x^3+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(x^6-1\right)^{\frac{2}{3}}}{2x^2}-\frac{\arctan\left(\frac{\sqrt{3}\,x}{x+2(x^6-1)^{\frac{1}{3}}}\right)\sqrt{3}}{3}+\frac{\ln\left(-x+\left(x^6-1\right)^{\frac{1}{3}}\right)}{3}-\frac{\ln\left(x^2+x\left(x^6-1\right)^{\frac{1}{3}}+\left(x^6-1\right)^{\frac{2}{3}}\right)}{6}$$

command

```
Integrate[((-1 + x^6)^(2/3)*(1 + x^6))/(x^3*(-1 - x^3 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{\left(-1+x^6\right)^{2/3}}{2x^2}-\frac{\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{-1+x^6}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(-x+\sqrt[3]{-1+x^6}\right)-\frac{1}{6}\log\left(x^2+x\sqrt[3]{-1+x^6}+\left(-1+x^6\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^6\right)^{2/3}\left(1+x^6\right)}{x^3\left(-1-x^3+x^6\right)}\,dx$$

24.172 Problem number 1308

$$\int \frac{\left(-1+x^6\right)\left(1+x^6\right)^{2/3}}{x^3\left(1-x^3+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(x^6+1\right)^{\frac{2}{3}}}{2x^2}-\frac{\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^6+1\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}+\frac{\ln\left(-x+\left(x^6+1\right)^{\frac{1}{3}}\right)}{3}-\frac{\ln\left(x^2+x\left(x^6+1\right)^{\frac{1}{3}}+\left(x^6+1\right)^{\frac{2}{3}}\right)}{6}$$

command

```
Integrate[((-1 + x^6)*(1 + x^6)^(2/3))/(x^3*(1 - x^3 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{\left(1+x^6\right)^{2/3}}{2x^2}-\frac{\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{1+x^6}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(-x+\sqrt[3]{1+x^6}\right)-\frac{1}{6}\log\left(x^2+x\sqrt[3]{1+x^6}+\left(1+x^6\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^6\right)\left(1+x^6\right)^{2/3}}{x^3\left(1-x^3+x^6\right)}\,dx$$

24.173 Problem number 1309

$$\int \frac{-1+x^8}{\sqrt[4]{-x^2+x^4}\left(1+x^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-1 + x^8)/((-x^2 + x^4)^(1/4)*(1 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{x}\,\sqrt[4]{-1+x^2}\left(4\left(\text{ArcTan}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)+\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)\right)+\text{RootSum}\left[2-4\#1^4+6\#1^8-4\#1^{12}+\right.\right.}{4\sqrt[4]{x^2\left(-1+x^2\right)}}$$

Mathematica 12.3 output

$$\int \frac{-1+x^8}{\sqrt[4]{-x^2+x^4}\left(1+x^8\right)}\,dx$$

24.174 Problem number 1310

$$\int \frac{-1+x^8}{\sqrt[4]{-x^2+x^4}\,(1+x^8)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-1 + x^8)/((-x^2 + x^4)^(1/4)*(1 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{x}\,\sqrt[4]{-1+x^2}\left(4\left(\text{ArcTan}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)+\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)\right)+\text{RootSum}\left[2-4\#1^4+6\#1^8-4\#1^{12}+\right.\right.}{4\sqrt[4]{x^2\,(-1+x^2)}}$$

Mathematica 12.3 output

$$\int \frac{-1+x^8}{\sqrt[4]{-x^2+x^4}\,(1+x^8)}\,dx$$

24.175 Problem number 1311

$$\int \frac{1+3x^4+x^8}{x^2\,(1+x^4)^{3/4}\,(1+3x^4+3x^8)}\,dx$$

Optimal antiderivative

$$-\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}-\frac{\arctan\left(\frac{\sqrt{3}\;x\left(x^4+1\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^4+1}}\right)\sqrt{3}}{3}+\frac{\text{arctanh}\left(\frac{\sqrt{3}\;x\left(x^4+1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^4+1}}\right)\sqrt{3}}{3}$$

command

```
Integrate[(1 + 3*x^4 + x^8)/(x^2*(1 + x^4)^(3/4)*(1 + 3*x^4 + 3*x^8)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt[4]{1+x^4}}{x}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\;x}{x-2\sqrt[4]{1+x^4}}\right)}{\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{\sqrt{3}\;x}{x+2\sqrt[4]{1+x^4}}\right)}{\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{3}\;x\sqrt[4]{1+x^4}}{x^2+\sqrt{1+x^4}}\right)}{\sqrt{3}}$$

Mathematica 12.3 output

$$\int \frac{1+3x^4+x^8}{x^2\,(1+x^4)^{3/4}\,(1+3x^4+3x^8)}\,dx$$

24.176 Problem number 1315

$$\int \frac{(3+2x)\sqrt[3]{1+x+x^3}}{x^2(1+x)}\,dx$$

Optimal antiderivative

$$-\frac{3\left(x^3+x+1\right)^{\frac{1}{3}}}{x}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^3+x+1\right)^{\frac{1}{3}}}\right)$$
$$-\ln\left(-x+\left(x^3+x+1\right)^{\frac{1}{3}}\right)+\frac{\ln\left(x^2+x\left(x^3+x+1\right)^{\frac{1}{3}}+\left(x^3+x+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((3 + 2*x)*(1 + x + x^3)^(1/3))/(x^2*(1 + x)),x]
```

Mathematica 13.1 output

$$-\frac{3\sqrt[3]{1+x+x^3}}{x}-\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{1+x+x^3}}\right)$$
$$-\log\left(-x+\sqrt[3]{1+x+x^3}\right)+\frac{1}{2}\log\left(x^2+x\sqrt[3]{1+x+x^3}+\left(1+x+x^3\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{(3+2x)\sqrt[3]{1+x+x^3}}{x^2(1+x)}\,dx$$

24.177 Problem number 1317

$$\int \frac{\left(1+x^2\right)^2}{(1-x^2)\left(1-6x^2+x^4\right)^{3/4}}\,dx$$

Optimal antiderivative

$$\arctan\left(\frac{i+x}{\left(x^4-6x^2+1\right)^{\frac{1}{4}}}\right)-\arctan\left(\frac{\left(x^4-6x^2+1\right)^{\frac{1}{4}}}{-i+x}\right)$$
$$-\text{arctanh}\left(\frac{i+x}{\left(x^4-6x^2+1\right)^{\frac{1}{4}}}\right)-\text{arctanh}\left(\frac{\left(x^4-6x^2+1\right)^{\frac{1}{4}}}{-i+x}\right)$$

command

```
Integrate[(1 + x^2)^2/((1 - x^2)*(1 - 6*x^2 + x^4)^(3/4)),x]
```

Mathematica 13.1 output

$$\begin{aligned}&\text{ArcTan}\left(\frac{-i+x}{\sqrt[4]{1-6x^2+x^4}}\right)-\text{ArcTan}\left(\frac{\sqrt[4]{1-6x^2+x^4}}{i+x}\right)\\&-\tanh^{-1}\left(\frac{-i+x}{\sqrt[4]{1-6x^2+x^4}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{1-6x^2+x^4}}{i+x}\right)\end{aligned}$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2\right)^2}{\left(1-x^2\right)\left(1-6x^2+x^4\right)^{3/4}}\,dx$$

24.178 Problem number 1318

$$\int \frac{\left(-4+x^3\right)\left(1-x^3+x^4\right)}{x^2\left(-1+x^3\right)^{3/4}\left(-1+x^3+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{4\left(x^3-1\right)^{\frac{1}{4}}}{x}+2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x\left(x^3-1\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^3-1}}\right)-2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,x\left(x^3-1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^3-1}}\right)$$

command

```
Integrate[((-4 + x^3)*(1 - x^3 + x^4))/(x^2*(-1 + x^3)^(3/4)*(-1 + x^3 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{4\sqrt[4]{-1+x^3}}{x}+2\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,x\sqrt[4]{-1+x^3}}{-x^2+\sqrt{-1+x^3}}\right)-2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{-1+x^3}}{x^2+\sqrt{-1+x^3}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-4+x^3\right)\left(1-x^3+x^4\right)}{x^2\left(-1+x^3\right)^{3/4}\left(-1+x^3+x^4\right)}\,dx$$

24.179 Problem number 1320

$$\int \frac{\left(4b+ax^5\right)\left(-b+cx^4+ax^5\right)}{x^2\left(-b+ax^5\right)^{3/4}\left(-b-cx^4+ax^5\right)}\,dx$$

Optimal antiderivative

$$\frac{4\left(a\,x^5-b\right)^{\frac{1}{4}}}{x}-4c^{\frac{1}{4}}\arctan\left(\frac{c^{\frac{1}{4}}x\left(a\,x^5-b\right)^{\frac{3}{4}}}{-a\,x^5+b}\right)+4c^{\frac{1}{4}}\operatorname{arctanh}\left(\frac{c^{\frac{1}{4}}x\left(a\,x^5-b\right)^{\frac{3}{4}}}{-a\,x^5+b}\right)$$

command

```
Integrate[((4*b + a*x^5)*(-b + c*x^4 + a*x^5))/(x^2*(-b + a*x^5)^(3/4)*(-b - c*x^4 + a*x^5)),
```

Mathematica 13.1 output

$$\frac{4\sqrt[4]{-b+ax^5}}{x}+4\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-b+ax^5}}\right)-4\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-b+ax^5}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(4b+ax^5\right)\left(-b+cx^4+ax^5\right)}{x^2\left(-b+ax^5\right)^{3/4}\left(-b-cx^4+ax^5\right)}\,dx$$

24.180 Problem number 1321

$$\int \frac{\left(-2+x^6\right)\left(1-x^4+x^6\right)}{x^4\sqrt[4]{1+x^6}\left(1+x^4+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{2\left(x^6+1\right)^{\frac{3}{4}}}{3x^3}+\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x\left(x^6+1\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^6+1}}\right)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,x\left(x^6+1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^6+1}}\right)$$

command

```
Integrate[((-2 + x^6)*(1 - x^4 + x^6))/(x^4*(1 + x^6)^(1/4)*(1 + x^4 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{2\left(1+x^6\right)^{3/4}}{3x^3}+\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,x\sqrt[4]{1+x^6}}{-x^2+\sqrt{1+x^6}}\right)+\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{1+x^6}}{x^2+\sqrt{1+x^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-2+x^6\right)\left(1-x^4+x^6\right)}{x^4\sqrt[4]{1+x^6}\left(1+x^4+x^6\right)}\,dx$$

24.181 Problem number 1322

$$\int \frac{x^4\left(2b+ax^6\right)}{\sqrt[4]{-b+ax^6}\left(-b-x^4+ax^6\right)^2}\,dx$$

Optimal antiderivative

$$-\frac{x\left(a\,x^6-b\right)^{\frac{3}{4}}}{2a\,x^6-2x^4-2b}-\frac{\arctan\left(\frac{x\left(a\,x^6-b\right)^{\frac{3}{4}}}{-a\,x^6+b}\right)}{4}-\frac{\operatorname{arctanh}\left(\frac{x\left(a\,x^6-b\right)^{\frac{3}{4}}}{-a\,x^6+b}\right)}{4}$$

command

```
Integrate[(x^4*(2*b + a*x^6))/((-b + a*x^6)^(1/4)*(-b - x^4 + a*x^6)^2),x]
```

Mathematica 13.1 output

$$\frac{1}{4}\left(\frac{2x\left(-b+ax^6\right)^{3/4}}{b+x^4-ax^6}+\text{ArcTan}\left(\frac{x}{\sqrt[4]{-b+ax^6}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{-b+ax^6}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{x^4\left(2b+ax^6\right)}{\sqrt[4]{-b+ax^6}\left(-b-x^4+ax^6\right)^2}\,dx$$

24.182 Problem number 1323

$$\int \frac{\left(2b+ax^6\right)\left(-b-x^4+ax^6\right)}{x^4\sqrt[4]{-b+ax^6}\left(-b-2x^4+ax^6\right)}\,dx$$

Optimal antiderivative

$$\frac{2\left(a\,x^6-b\right)^{\frac{3}{4}}}{3x^3}+\frac{\arctan\left(\frac{2^{\frac{1}{4}}x\left(a\,x^6-b\right)^{\frac{3}{4}}}{-a\,x^6+b}\right)2^{\frac{3}{4}}}{2}+\frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}x\left(a\,x^6-b\right)^{\frac{3}{4}}}{-a\,x^6+b}\right)2^{\frac{3}{4}}}{2}$$

command

```
Integrate[((2*b + a*x^6)*(-b - x^4 + a*x^6))/(x^4*(-b + a*x^6)^(1/4)*(-b - 2*x^4 + a*x^6)),x]
```

Mathematica 13.1 output

$$\frac{2\left(-b+ax^6\right)^{3/4}}{3x^3}-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{-b+ax^6}}\right)}{\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{-b+ax^6}}\right)}{\sqrt[4]{2}}$$

Mathematica 12.3 output

$$\int \frac{\left(2b+ax^6\right)\left(-b-x^4+ax^6\right)}{x^4\sqrt[4]{-b+ax^6}\left(-b-2x^4+ax^6\right)}\,dx$$

24.183 Problem number 1324

$$\int \frac{\sqrt{-1-2x^2-2x^3-x^8}\left(-1+x^3+3x^8\right)}{\left(1+2x^3+x^8\right)\left(1+x^2+2x^3+x^8\right)}\,dx$$

Optimal antiderivative

$$\arctan\left(\frac{x\sqrt{-x^8-2x^3-2x^2-1}}{x^8+2x^3+2x^2+1}\right)-\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x\sqrt{-x^8-2x^3-2x^2-1}}{x^8+2x^3+2x^2+1}\right)$$

command

```
Integrate[(Sqrt[-1 - 2*x^2 - 2*x^3 - x^8]*(-1 + x^3 + 3*x^8))/((1 + 2*x^3 + x^8)*(1 + x^2 + 2
```

Mathematica 13.1 output

$$-\text{ArcTan}\left(\frac{x}{\sqrt{-1-2x^2-2x^3-x^8}}\right)+\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,x}{\sqrt{-1-2x^2-2x^3-x^8}}\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{-1-2x^2-2x^3-x^8}\left(-1+x^3+3x^8\right)}{\left(1+2x^3+x^8\right)\left(1+x^2+2x^3+x^8\right)}\,dx$$

24.184 Problem number 1325

$$\int \frac{\left(1+x^6\right)\sqrt{-2-x^2+x^6}}{4-3x^4-4x^6+x^{12}}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{3+3\sqrt{3}}\,\arctan\left(\frac{\sqrt{1+\sqrt{3}}\,x}{\sqrt{x^6-x^2-2}}\right)}{12}-\frac{\sqrt{-3+3\sqrt{3}}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{3}-1}\,x}{\sqrt{x^6-x^2-2}}\right)}{12}$$

command

```
Integrate[((1 + x^6)*Sqrt[-2 - x^2 + x^6])/(4 - 3*x^4 - 4*x^6 + x^12),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{1+\sqrt{3}}\,\text{ArcTan}\left(\frac{\sqrt{1+\sqrt{3}}\,x}{\sqrt{-2-x^2+x^6}}\right)+\sqrt{-1+\sqrt{3}}\,\tanh^{-1}\left(\frac{\sqrt{-1+\sqrt{3}}\,x}{\sqrt{-2-x^2+x^6}}\right)}{4\sqrt{3}}$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^6\right)\sqrt{-2-x^2+x^6}}{4-3x^4-4x^6+x^{12}}\,dx$$

24.185 Problem number 1332

$$\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)\left(aq+bx^2+apx^5\right)}{x^4\left(cq+dx^2+cpx^5\right)}\,dx$$

Optimal antiderivative

$$\frac{2\sqrt{p\,x^5+q}\,\left(acp\,x^5-3ad\,x^2+3bc\,x^2+acq\right)}{3c^2x^3}+\frac{2\left(bc\sqrt{d}\,-a\,d^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}\,\sqrt{p\,x^5+q}}\right)}{c^{\frac{5}{2}}}$$

command

```
Integrate[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5)*(a*q + b*x^2 + a*p*x^5))/(x^4*(c*q + d*x^2 + c*p*
```

Mathematica 13.1 output

$$\frac{2\sqrt{q+px^5}\left(acq+3bcx^2-3adx^2+acpx^5\right)}{3c^2x^3}+\frac{2\sqrt{d}\,(bc-ad)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}\,\sqrt{q+px^5}}\right)}{c^{5/2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)\left(aq+bx^2+apx^5\right)}{x^4\left(cq+dx^2+cpx^5\right)}\,dx$$

24.186 Problem number 1334

$$\int \frac{-1+x^4}{\left(1+x^2+x^4\right)\sqrt[4]{x^2+x^6}}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{2}\,x\left(x^6+x^2\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^6+x^2}}\right)\sqrt{2}}{2}-\frac{\text{arctanh}\left(\frac{\frac{x^2\sqrt{2}}{2}+\frac{\sqrt{x^6+x^2}\,\sqrt{2}}{2}}{x\left(x^6+x^2\right)^{\frac{1}{4}}}\right)\sqrt{2}}{2}$$

command

```
Integrate[(-1 + x^4)/((1 + x^2 + x^4)*(x^2 + x^6)^(1/4)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{x}\,\sqrt[4]{1+x^4}\left(\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{x}\,\sqrt[4]{1+x^4}}{-x+\sqrt{1+x^4}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{x}\,\sqrt[4]{1+x^4}}{x+\sqrt{1+x^4}}\right)\right)}{\sqrt{2}\,\sqrt[4]{x^2+x^6}}$$

Mathematica 12.3 output

$$\int \frac{-1+x^4}{\left(1+x^2+x^4\right)\sqrt[4]{x^2+x^6}}\,dx$$

24.187 Problem number 1335

$$\int \frac{-1+x^4}{(1+x^2+x^4)\sqrt[4]{x^2+x^6}}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{2}\,x(x^6+x^2)^{\frac{1}{4}}}{-x^2+\sqrt{x^6+x^2}}\right)\sqrt{2}}{2}-\frac{\operatorname{arctanh}\left(\frac{\frac{x^2\sqrt{2}}{2}+\frac{\sqrt{x^6+x^2}\,\sqrt{2}}{2}}{x(x^6+x^2)^{\frac{1}{4}}}\right)\sqrt{2}}{2}$$

command

```
Integrate[(-1 + x^4)/((1 + x^2 + x^4)*(x^2 + x^6)^(1/4)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{x}\,\sqrt[4]{1+x^4}\left(\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{x}\,\sqrt[4]{1+x^4}}{-x+\sqrt{1+x^4}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{x}\,\sqrt[4]{1+x^4}}{x+\sqrt{1+x^4}}\right)\right)}{\sqrt{2}\,\sqrt[4]{x^2+x^6}}$$

Mathematica 12.3 output

$$\int \frac{-1+x^4}{(1+x^2+x^4)\sqrt[4]{x^2+x^6}}\,dx$$

24.188 Problem number 1336

$$\int \frac{x^2\left(2b+ax^6\right)}{\left(-b+ax^6\right)^{3/4}\left(-b-2cx^4+ax^6\right)}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}x\left(a\,x^6-b\right)^{\frac{3}{4}}}{-a\,x^6+b}\right)2^{\frac{1}{4}}}{2c^{\frac{3}{4}}}+\frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}c^{\frac{1}{4}}x\left(a\,x^6-b\right)^{\frac{3}{4}}}{-a\,x^6+b}\right)2^{\frac{1}{4}}}{2c^{\frac{3}{4}}}$$

command

```
Integrate[(x^2*(2*b + a*x^6))/((-b + a*x^6)^(3/4)*(-b - 2*c*x^4 + a*x^6)),x]
```

Mathematica 13.1 output

$$-\frac{-\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{-b+ax^6}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{-b+ax^6}}\right)}{2^{3/4}c^{3/4}}$$

Mathematica 12.3 output

$$\int \frac{x^2\left(2b+ax^6\right)}{\left(-b+ax^6\right)^{3/4}\left(-b-2cx^4+ax^6\right)}\,dx$$

24.189 Problem number 1353

$$\int \frac{-3+5x^8}{(1+x^8)\sqrt[3]{1-x^3+x^8}}\,dx$$

Optimal antiderivative

$$-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^8-x^3+1\right)^{\frac{1}{3}}}\right)-\ln\left(x+\left(x^8-x^3+1\right)^{\frac{1}{3}}\right)+\frac{\ln\left(x^2-x\left(x^8-x^3+1\right)^{\frac{1}{3}}+\left(x^8-x^3+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(-3 + 5*x^8)/((1 + x^8)*(1 - x^3 + x^8)^(1/3)),x]
```

Mathematica 13.1 output

$$\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x-2\sqrt[3]{1-x^3+x^8}}\right)-\log\left(x+\sqrt[3]{1-x^3+x^8}\right)+\frac{1}{2}\log\left(x^2-x\sqrt[3]{1-x^3+x^8}+\left(1-x^3+x^8\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{-3+5x^8}{(1+x^8)\sqrt[3]{1-x^3+x^8}}\,dx$$

24.190 Problem number 1366

$$\int \frac{\left(4+x^5\right)\left(1-x^4-2x^5+x^8+x^9+x^{10}\right)}{x^2\left(-1+x^5\right)^{3/4}\left(1+x^4-2x^5-x^8-x^9+x^{10}\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((4 + x^5)*(1 - x^4 - 2*x^5 + x^8 + x^9 + x^10))/(x^2*(-1 + x^5)^(3/4)*(1 + x^4 - 2
```

Mathematica 13.1 output

$$\frac{4\sqrt[4]{-1+x^5}}{x}-2\text{RootSum}\left[-1-\#1^4+\#1^8\&,\frac{\log(x)-\log\left(\sqrt[4]{-1+x^5}-x\#1\right)+\log(x)\#1^4-\log\left(\sqrt[4]{-1+x^5}-x\#1\right)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

Mathematica 12.3 output

$$\int\frac{\left(4+x^5\right)\left(1-x^4-2x^5+x^8+x^9+x^{10}\right)}{x^2\left(-1+x^5\right)^{3/4}\left(1+x^4-2x^5-x^8-x^9+x^{10}\right)}\,dx$$

24.191 Problem number 1367

$$\int\frac{\left(4+x^5\right)\left(1-x^4-2x^5+x^8+x^9+x^{10}\right)}{x^2\left(-1+x^5\right)^{3/4}\left(1+x^4-2x^5-x^8-x^9+x^{10}\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((4 + x^5)*(1 - x^4 - 2*x^5 + x^8 + x^9 + x^10))/(x^2*(-1 + x^5)^(3/4)*(1 + x^4 - 2
```

Mathematica 13.1 output

$$\frac{4\sqrt[4]{-1+x^5}}{x}-2\text{RootSum}\left[-1-\#1^4+\#1^8\&,\frac{\log(x)-\log\left(\sqrt[4]{-1+x^5}-x\#1\right)+\log(x)\#1^4-\log\left(\sqrt[4]{-1+x^5}-x\#1\right)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

Mathematica 12.3 output

$$\int\frac{\left(4+x^5\right)\left(1-x^4-2x^5+x^8+x^9+x^{10}\right)}{x^2\left(-1+x^5\right)^{3/4}\left(1+x^4-2x^5-x^8-x^9+x^{10}\right)}\,dx$$

24.192 Problem number 1368

$$\int \frac{1}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx$$

Optimal antiderivative

$$\frac{x}{2\sqrt{a\,x^2+\sqrt{a^2x^4+b}}}+\frac{\ln\left(a\,x^2+\sqrt{a^2x^4+b}+\sqrt{2}\,\sqrt{a}\,x\sqrt{a\,x^2+\sqrt{a^2x^4+b}}\right)\sqrt{2}}{4\sqrt{a}}$$

command

```
Integrate[1/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
```

Mathematica 13.1 output

$$\frac{x}{2\sqrt{ax^2+\sqrt{b+a^2x^4}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{2}\,\sqrt{a}\,x}\right)}{2\sqrt{2}\,\sqrt{a}}$$

Mathematica 12.3 output

$$\int \frac{1}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx$$

24.193 Problem number 1390

$$\int \frac{6+2x+x^2}{\sqrt[3]{2+x^2}\,(1+3x-2x^2+x^3)}\,dx$$

Optimal antiderivative

$$-\sqrt{3}\,\arctan\left(\frac{\frac{2\sqrt{3}}{3}-\frac{2x\sqrt{3}}{3}+\frac{(x^2+2)^{\frac{1}{3}}\sqrt{3}}{3}}{(x^2+2)^{\frac{1}{3}}}\right)+\ln\left(-1+x+(x^2+2)^{\frac{1}{3}}\right)$$
$$-\frac{\ln\left(1-2x+x^2+(1-x)\,(x^2+2)^{\frac{1}{3}}+(x^2+2)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(6 + 2*x + x^2)/((2 + x^2)^(1/3)*(1 + 3*x - 2*x^2 + x^3)),x]
```

Mathematica 13.1 output

$$-\sqrt{3}\,\text{ArcTan}\left(\frac{2-2x+\sqrt[3]{2+x^2}}{\sqrt{3}\,\sqrt[3]{2+x^2}}\right)+\log\left(-1+x+\sqrt[3]{2+x^2}\right)$$
$$-\frac{1}{2}\log\left(1-2x+x^2-(-1+x)\sqrt[3]{2+x^2}+\left(2+x^2\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{6+2x+x^2}{\sqrt[3]{2+x^2}\,\left(1+3x-2x^2+x^3\right)}\,dx$$

24.194 Problem number 1393

$$\int\frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)\left(-1+x^3+x^4\right)}{x^6\left(-1-x^3+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^4-1\right)^{\frac{2}{3}}\left(x^4+5x^3-1\right)}{5x^5}-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^4-1\right)^{\frac{1}{3}}}\right)$$
$$+2\ln\left(-x+\left(x^4-1\right)^{\frac{1}{3}}\right)-\ln\left(x^2+x\left(x^4-1\right)^{\frac{1}{3}}+\left(x^4-1\right)^{\frac{2}{3}}\right)$$

command

```
Integrate[((-1 + x^4)^(2/3)*(3 + x^4)*(-1 + x^3 + x^4))/(x^6*(-1 - x^3 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{3\left(-1+x^4\right)^{2/3}\left(-1+5x^3+x^4\right)}{5x^5}-2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{-1+x^4}}\right)$$
$$+2\log\left(-x+\sqrt[3]{-1+x^4}\right)-\log\left(x^2+x\sqrt[3]{-1+x^4}+\left(-1+x^4\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)\left(-1+x^3+x^4\right)}{x^6\left(-1-x^3+x^4\right)}\,dx$$

24.195 Problem number 1394

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}\left(1+x^3+x^4\right)}{x^6\left(1-x^3+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^4+1\right)^{\frac{2}{3}}\left(x^4+5x^3+1\right)}{5x^5}-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^4+1\right)^{\frac{1}{3}}}\right)$$
$$+2\ln\left(-x+\left(x^4+1\right)^{\frac{1}{3}}\right)-\ln\left(x^2+x\left(x^4+1\right)^{\frac{1}{3}}+\left(x^4+1\right)^{\frac{2}{3}}\right)$$

command

```
Integrate[((-3 + x^4)*(1 + x^4)^(2/3)*(1 + x^3 + x^4))/(x^6*(1 - x^3 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{3\left(1+x^4\right)^{2/3}\left(1+5x^3+x^4\right)}{5x^5}-2\sqrt{3}\,\mathrm{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{1+x^4}}\right)$$
$$+2\log\left(-x+\sqrt[3]{1+x^4}\right)-\log\left(x^2+x\sqrt[3]{1+x^4}+\left(1+x^4\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}\left(1+x^3+x^4\right)}{x^6\left(1-x^3+x^4\right)}\,dx$$

24.196 Problem number 1399

$$\int \frac{\sqrt[3]{1+2x^7}\left(-3+8x^7\right)}{x^2\left(1+x^3+2x^7\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(2x^7+1\right)^{\frac{1}{3}}}{x}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(2x^7+1\right)^{\frac{1}{3}}}\right)$$
$$-\ln\left(x+\left(2x^7+1\right)^{\frac{1}{3}}\right)+\frac{\ln\left(x^2-x\left(2x^7+1\right)^{\frac{1}{3}}+\left(2x^7+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((1 + 2*x^7)^(1/3)*(-3 + 8*x^7))/(x^2*(1 + x^3 + 2*x^7)),x]
```

Mathematica 13.1 output

$$\frac{3\sqrt[3]{1+2x^7}}{x}+\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{-x+2\sqrt[3]{1+2x^7}}\right)$$
$$-\log\left(x+\sqrt[3]{1+2x^7}\right)+\frac{1}{2}\log\left(x^2-x\sqrt[3]{1+2x^7}+\left(1+2x^7\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{\sqrt[3]{1+2x^7}\left(-3+8x^7\right)}{x^2\left(1+x^3+2x^7\right)}\,dx$$

24.197 Problem number 1402

$$\int\frac{\left(-1+3x^4\right)\sqrt{1+x+2x^4+x^5+x^8}}{x^2\left(4+x+4x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{x^8+x^5+2x^4+x+1}}{4x}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{x^8+x^5+2x^4+x+1}}{x^4+x+1}\right)}{8}$$
$$+\frac{\text{arctanh}\left(\frac{\sqrt{x^8+x^5+2x^4+x+1}}{x^4+x+1}\right)}{8}$$

command

```
Integrate[((-1 + 3*x^4)*Sqrt[1 + x + 2*x^4 + x^5 + x^8])/(x^2*(4 + x + 4*x^4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{1+x+2x^4+x^5+x^8}}{4x}-\frac{1}{8}\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt{1+x+2x^4+x^5+x^8}}{1+x+x^4}\right)$$
$$+\frac{1}{8}\tanh^{-1}\left(\frac{\sqrt{1+x+2x^4+x^5+x^8}}{1+x+x^4}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(-1+3x^4\right)\sqrt{1+x+2x^4+x^5+x^8}}{x^2\left(4+x+4x^4\right)}\,dx$$

24.198 Problem number 1419

$$\int \frac{\left(4+x^3\right)\left(-1-x^3+x^4\right)}{x^2\left(1+x^3\right)^{3/4}\left(1+x^3+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{4\left(x^3+1\right)^{\frac{1}{4}}}{x}-2\sqrt{2}\,\arctan\left(\frac{-\frac{x^2\sqrt{2}}{2}+\frac{\sqrt{x^3+1}\,\sqrt{2}}{2}}{x\left(x^3+1\right)^{\frac{1}{4}}}\right)-2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,x\left(x^3+1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^3+1}}\right)$$

command

```
Integrate[((4 + x^3)*(-1 - x^3 + x^4))/(x^2*(1 + x^3)^(3/4)*(1 + x^3 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{4\sqrt[4]{1+x^3}}{x}-2\sqrt{2}\,\text{ArcTan}\left(\frac{-x^2+\sqrt{1+x^3}}{\sqrt{2}\,x\sqrt[4]{1+x^3}}\right)-2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{1+x^3}}{x^2+\sqrt{1+x^3}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(4+x^3\right)\left(-1-x^3+x^4\right)}{x^2\left(1+x^3\right)^{3/4}\left(1+x^3+x^4\right)}\,dx$$

24.199 Problem number 1425

$$\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt[4]{x^2+x^6}}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{2^{\frac{3}{4}}x\left(x^6+x^2\right)^{\frac{1}{4}}}{x^2\sqrt{2}-\sqrt{x^6+x^2}}\right)2^{\frac{1}{4}}}{2}-\frac{\operatorname{arctanh}\left(\frac{\frac{x^2 2^{\frac{3}{4}}}{2}+\frac{\sqrt{x^6+x^2}\,2^{\frac{1}{4}}}{2}}{x\left(x^6+x^2\right)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{2}$$

command

```
Integrate[(-1 + x^2)/((1 + x^2)*(x^2 + x^6)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{x}\,\sqrt[4]{1+x^4}\left(\text{ArcTan}\left(\frac{2^{3/4}\sqrt{x}\,\sqrt[4]{1+x^4}}{\sqrt{2}\,x-\sqrt{1+x^4}}\right)-\tanh^{-1}\left(\frac{2\sqrt[4]{2}\,\sqrt{x}\,\sqrt[4]{1+x^4}}{2x+\sqrt{2}\,\sqrt{1+x^4}}\right)\right)}{2^{3/4}\sqrt[4]{x^2+x^6}}$$

Mathematica 12.3 output

$$\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt[4]{x^2+x^6}}\,dx$$

24.200 Problem number 1426

$$\int \frac{-1+x^2}{(1+x^2)\sqrt[4]{x^2+x^6}}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{2^{\frac{3}{4}}x(x^6+x^2)^{\frac{1}{4}}}{x^2\sqrt{2}-\sqrt{x^6+x^2}}\right)2^{\frac{1}{4}}}{2}-\frac{\operatorname{arctanh}\left(\frac{\frac{x^2 2^{\frac{3}{4}}}{2}+\frac{\sqrt{x^6+x^2}\,2^{\frac{1}{4}}}{2}}{x(x^6+x^2)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{2}$$

command

```
Integrate[(-1 + x^2)/((1 + x^2)*(x^2 + x^6)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{x}\,\sqrt[4]{1+x^4}\left(\text{ArcTan}\left(\frac{2^{3/4}\sqrt{x}\,\sqrt[4]{1+x^4}}{\sqrt{2}\,x-\sqrt{1+x^4}}\right)-\tanh^{-1}\left(\frac{2\sqrt[4]{2}\,\sqrt{x}\,\sqrt[4]{1+x^4}}{2x+\sqrt{2}\,\sqrt{1+x^4}}\right)\right)}{2^{3/4}\sqrt[4]{x^2+x^6}}$$

Mathematica 12.3 output

$$\int \frac{-1+x^2}{(1+x^2)\sqrt[4]{x^2+x^6}}\,dx$$

24.201 Problem number 1428

$$\int \frac{\sqrt[4]{1+x^4}\,(2+x^4)}{x^2\,(-1+2x^4+x^8)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^4)^(1/4)*(2 + x^4))/(x^2*(-1 + 2*x^4 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt[4]{1+x^4}}{x}-\frac{1}{8}\text{RootSum}\left[2-4\#1^4+\#1^8\&,\frac{-4\log(x)+4\log\left(\sqrt[4]{1+x^4}-x\#1\right)+7\log(x)\#1^4-7\log\left(\sqrt[4]{1+x^4}-x\#1\right)\#1^4}{-2\#1^3+\#1^7}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{1+x^4}\,(2+x^4)}{x^2\,(-1+2x^4+x^8)}\,dx$$

24.202 Problem number 1429

$$\int \frac{\sqrt[4]{1+x^4}\,\left(2+x^4\right)}{x^2\left(-1+2x^4+x^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^4)^(1/4)*(2 + x^4))/(x^2*(-1 + 2*x^4 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt[4]{1+x^4}}{x}-\frac{1}{8}\text{RootSum}\left[2-4\#1^4+\#1^8\&,\frac{-4\log(x)+4\log\left(\sqrt[4]{1+x^4}-x\#1\right)+7\log(x)\#1^4-7\log\left(\sqrt[4]{1+x^4}-x\#1\right)\#1^4}{-2\#1^3+\#1^7}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{1+x^4}\,\left(2+x^4\right)}{x^2\left(-1+2x^4+x^8\right)}\,dx$$

24.203 Problem number 1430

$$\int \frac{-b+2ax^4}{\sqrt[4]{-b+ax^4}\,\left(-b+ax^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-b + 2*a*x^4)/((-b + a*x^4)^(1/4)*(-b + a*x^8)),x]
```

Mathematica 13.1 output

$$\frac{1}{8}\text{RootSum}\left[a^2-ab-2a\#1^4+\#1^8\&,\frac{a\log(x)-a\log\left(\sqrt[4]{-b+ax^4}-x\#1\right)+\log(x)\#1^4-\log\left(\sqrt[4]{-b+ax^4}-x\#1\right)\#1^4}{-a\#1+\#1^5}\&\right]$$

Mathematica 12.3 output

$$\int \frac{-b+2ax^4}{\sqrt[4]{-b+ax^4}\,\left(-b+ax^8\right)}\,dx$$

24.204 Problem number 1431

$$\int \frac{\sqrt{1+2x^6}\left(-1+4x^6\right)}{2+x^4+8x^6+8x^{12}}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{2^{\frac{3}{4}}x\sqrt{2x^6+1}}{\sqrt{2}\,-x^2+2\sqrt{2}\,x^6}\right)2^{\frac{3}{4}}}{8}-\frac{\operatorname{arctanh}\left(\frac{2^{\frac{3}{4}}x\sqrt{2x^6+1}}{\sqrt{2}\,+x^2+2\sqrt{2}\,x^6}\right)2^{\frac{3}{4}}}{8}$$

command

```
Integrate[(Sqrt[1 + 2*x^6]*(-1 + 4*x^6))/(2 + x^4 + 8*x^6 + 8*x^12),x]
```

Mathematica 13.1 output

$$-\frac{\text{ArcTan}\left(\frac{2^{3/4}x\sqrt{1+2x^6}}{\sqrt{2}\,-x^2+2\sqrt{2}\,x^6}\right)+\tanh^{-1}\left(\frac{2^{3/4}x\sqrt{1+2x^6}}{\sqrt{2}\,+x^2+2\sqrt{2}\,x^6}\right)}{4\sqrt[4]{2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1+2x^6}\left(-1+4x^6\right)}{2+x^4+8x^6+8x^{12}}\,dx$$

24.205 Problem number 1443

$$\int \frac{-2b+ax^2}{\sqrt[4]{-b+ax^2}\,\left(-b+ax^2+x^4\right)}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{-\frac{x^2\sqrt{2}}{2}+\frac{\sqrt{a\,x^2-b}\,\sqrt{2}}{2}}{x(a\,x^2-b)^{\frac{1}{4}}}\right)\sqrt{2}}{2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\,x(a\,x^2-b)^{\frac{1}{4}}}{x^2+\sqrt{a\,x^2-b}}\right)\sqrt{2}}{2}$$

command

```
Integrate[(-2*b + a*x^2)/((-b + a*x^2)^(1/4)*(-b + a*x^2 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{-\text{ArcTan}\left(\frac{-x^2+\sqrt{-b+ax^2}}{\sqrt{2}\,x\sqrt[4]{-b+ax^2}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{-b+ax^2}}{x^2+\sqrt{-b+ax^2}}\right)}{\sqrt{2}}$$

Mathematica 12.3 output

$$\int \frac{-2b+ax^2}{\sqrt[4]{-b+ax^2}\,\left(-b+ax^2+x^4\right)}\,dx$$

24.206 Problem number 1444

$$\int \frac{-4b + ax^3}{\sqrt[4]{-b + ax^3}\,(-b + ax^3 + x^4)}\,dx$$

Optimal antiderivative

$$-\sqrt{2}\,\arctan\left(\frac{-\frac{x^2\sqrt{2}}{2} + \frac{\sqrt{x^3a - b}\,\sqrt{2}}{2}}{x\,(x^3a - b)^{\frac{1}{4}}}\right) + \sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,x(x^3a - b)^{\frac{1}{4}}}{x^2 + \sqrt{x^3a - b}}\right)$$

command

```
Integrate[(-4*b + a*x^3)/((-b + a*x^3)^(1/4)*(-b + a*x^3 + x^4)),x]
```

Mathematica 13.1 output

$$\sqrt{2}\left(-\text{ArcTan}\left(\frac{-x^2 + \sqrt{-b + ax^3}}{\sqrt{2}\,x\sqrt[4]{-b + ax^3}}\right) + \tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{-b + ax^3}}{x^2 + \sqrt{-b + ax^3}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{-4b + ax^3}{\sqrt[4]{-b + ax^3}\,(-b + ax^3 + x^4)}\,dx$$

24.207 Problem number 1449

$$\int \frac{\left(-1 + x^3\right)\left(1 + x^3\right)^3\left(1 + x^6\right)^{2/3}}{x^6\left(1 - x^3 + x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(x^6 + 1\right)^{\frac{2}{3}}\left(2x^6 + 15x^3 + 2\right)}{10x^5} - \arctan\left(\frac{\sqrt{3}\,x}{x + 2\left(x^6 + 1\right)^{\frac{1}{3}}}\right)\sqrt{3}$$
$$+ \ln\left(-x + \left(x^6 + 1\right)^{\frac{1}{3}}\right) - \frac{\ln\left(x^2 + x\left(x^6 + 1\right)^{\frac{1}{3}} + \left(x^6 + 1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((-1 + x^3)*(1 + x^3)^3*(1 + x^6)^(2/3))/(x^6*(1 - x^3 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{\left(1+x^6\right)^{2/3}\left(2+15x^3+2x^6\right)}{10x^5}-\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{1+x^6}}\right)$$
$$+\log\left(-x+\sqrt[3]{1+x^6}\right)-\frac{1}{2}\log\left(x^2+x\sqrt[3]{1+x^6}+\left(1+x^6\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(-1+x^3\right)\left(1+x^3\right)^3\left(1+x^6\right)^{2/3}}{x^6\left(1-x^3+x^6\right)}\,dx$$

24.208 Problem number 1450

$$\int\frac{\left(-1+x^6\right)^{2/3}\left(1+x^6\right)\left(-2+x^3+2x^6\right)}{x^6\left(-1-x^3+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(x^6-1\right)^{\frac{2}{3}}\left(4x^6+15x^3-4\right)}{10x^5}-\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^6-1\right)^{\frac{1}{3}}}\right)\sqrt{3}$$
$$+\ln\left(-x+\left(x^6-1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(x^2+x\left(x^6-1\right)^{\frac{1}{3}}+\left(x^6-1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((-1 + x^6)^(2/3)*(1 + x^6)*(-2 + x^3 + 2*x^6))/(x^6*(-1 - x^3 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{\left(-1+x^6\right)^{2/3}\left(-4+15x^3+4x^6\right)}{10x^5}-\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{-1+x^6}}\right)$$
$$+\log\left(-x+\sqrt[3]{-1+x^6}\right)-\frac{1}{2}\log\left(x^2+x\sqrt[3]{-1+x^6}+\left(-1+x^6\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(-1+x^6\right)^{2/3}\left(1+x^6\right)\left(-2+x^3+2x^6\right)}{x^6\left(-1-x^3+x^6\right)}\,dx$$

24.209 Problem number 1455

$$\int \frac{\sqrt{cx^2 - x\sqrt{-bx + ax^2}}}{x^3}\,dx$$

Optimal antiderivative

$$-\frac{4\left(2c^2x - 3ax + 3b\right)\sqrt{-x\left(-cx + \sqrt{a\,x^2 - bx}\right)}}{15b\,x^2} - \frac{4c\sqrt{a\,x^2 - bx}\,\sqrt{-x\left(-cx + \sqrt{a\,x^2 - bx}\right)}}{15b\,x^2}$$

command

```
Integrate[Sqrt[c*x^2 - x*Sqrt[-(b*x) + a*x^2]]/x^3,x]
```

Mathematica 13.1 output

$$\frac{4\sqrt{x\left(cx - \sqrt{x(-b + ax)}\right)}\left(-3b + 3ax - c\left(2cx + \sqrt{x(-b + ax)}\right)\right)}{15bx^2}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{cx^2 - x\sqrt{-bx + ax^2}}}{x^3}\,dx$$

24.210 Problem number 1475

$$\int \frac{\left(-3 + x^4\right)\left(1 + x^4\right)^{2/3}\left(1 + 2x^3 + x^4\right)}{x^6\left(1 - x^3 + x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^4+1\right)^{\frac{2}{3}}\left(2x^4 + 15x^3 + 2\right)}{10x^5} - 3\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x + 2\left(x^4+1\right)^{\frac{1}{3}}}\right)$$
$$+ 3\ln\left(-x + \left(x^4+1\right)^{\frac{1}{3}}\right) - \frac{3\ln\left(x^2 + x\left(x^4+1\right)^{\frac{1}{3}} + \left(x^4+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((-3 + x^4)*(1 + x^4)^(2/3)*(1 + 2*x^3 + x^4))/(x^6*(1 - x^3 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{3\left(1+x^4\right)^{2/3}\left(2+15x^3+2x^4\right)}{10x^5}-3\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{1+x^4}}\right)$$
$$+3\log\left(-x+\sqrt[3]{1+x^4}\right)-\frac{3}{2}\log\left(x^2+x\sqrt[3]{1+x^4}+\left(1+x^4\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}\left(1+2x^3+x^4\right)}{x^6\left(1-x^3+x^4\right)}\,dx$$

24.211 Problem number 1483

$$\int\frac{x^4\left(4b+ax^5\right)}{\left(-b+ax^5\right)^2\sqrt[4]{-b+cx^4+ax^5}}\,dx$$

Optimal antiderivative

$$\frac{x\left(a\,x^5+c\,x^4-b\right)^{\frac{3}{4}}}{c\left(-a\,x^5+b\right)}+\frac{\arctan\left(\frac{c^{\frac{1}{4}}x}{\left(a\,x^5+c\,x^4-b\right)^{\frac{1}{4}}}\right)}{2c^{\frac{5}{4}}}+\frac{\text{arctanh}\left(\frac{c^{\frac{1}{4}}x}{\left(a\,x^5+c\,x^4-b\right)^{\frac{1}{4}}}\right)}{2c^{\frac{5}{4}}}$$

command

```
Integrate[(x^4*(4*b + a*x^5))/((-b + a*x^5)^2*(-b + c*x^4 + a*x^5)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{\frac{2\sqrt[4]{c}\,x\left(-b+x^4(c+ax)\right)^{3/4}}{b-ax^5}+\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-b+x^4(c+ax)}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-b+x^4(c+ax)}}\right)}{2c^{5/4}}$$

Mathematica 12.3 output

$$\int\frac{x^4\left(4b+ax^5\right)}{\left(-b+ax^5\right)^2\sqrt[4]{-b+cx^4+ax^5}}\,dx$$

24.212 Problem number 1495

$$\int\frac{(-3+2x)\left(1-x+x^3\right)^{2/3}}{1-2x+x^2+2x^3-2x^4+2x^6}\,dx$$

Optimal antiderivative

$$-\arctan\left(\frac{x}{\left(x^3-x+1\right)^{\frac{1}{3}}}\right)-\frac{\arctan\left(\frac{x\left(x^3-x+1\right)^{\frac{1}{3}}}{-x^2+\left(x^3-x+1\right)^{\frac{2}{3}}}\right)}{2}-\frac{\sqrt{3}\;\text{arctanh}\left(\frac{\sqrt{3}\;x\left(x^3-x+1\right)^{\frac{1}{3}}}{x^2+\left(x^3-x+1\right)^{\frac{2}{3}}}\right)}{2}$$

command

```
Integrate[((-3 + 2*x)*(1 - x + x^3)^(2/3))/(1 - 2*x + x^2 + 2*x^3 - 2*x^4 + 2*x^6),x]
```

Mathematica 13.1 output

$$-\text{ArcTan}\left(\frac{x}{\sqrt[3]{1-x+x^3}}\right)-\frac{1}{2}i\left(-i+\sqrt{3}\right)\text{ArcTan}\left(\frac{\left(1-i\sqrt{3}\right)x}{2\sqrt[3]{1-x+x^3}}\right)$$
$$+\frac{1}{2}i\left(i+\sqrt{3}\right)\text{ArcTan}\left(\frac{\left(1+i\sqrt{3}\right)x}{2\sqrt[3]{1-x+x^3}}\right)$$

Mathematica 12.3 output

$$\int\frac{(-3+2x)\left(1-x+x^3\right)^{2/3}}{1-2x+x^2+2x^3-2x^4+2x^6}\,dx$$

24.213 Problem number 1496

$$\int\frac{\left(-2+x^6\right)\left(4+x^6\right)\sqrt[4]{-2+2x^4+x^6}}{x^6\left(-4-x^4+2x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(x^6+2x^4-2\right)^{\frac{1}{4}}\left(2x^6+9x^4-4\right)}{10x^5}+\frac{5^{\frac{1}{4}}2^{\frac{3}{4}}\arctan\left(\frac{5^{\frac{1}{4}}2^{\frac{3}{4}}x}{2\left(x^6+2x^4-2\right)^{\frac{1}{4}}}\right)}{8}-\frac{5^{\frac{1}{4}}2^{\frac{3}{4}}\text{arctanh}\left(\frac{5^{\frac{1}{4}}2^{\frac{3}{4}}x}{2\left(x^6+2x^4-2\right)^{\frac{1}{4}}}\right)}{8}$$

command

```
Integrate[((-2 + x^6)*(4 + x^6)*(-2 + 2*x^4 + x^6)^(1/4))/(x^6*(-4 - x^4 + 2*x^6)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-2+2x^4+x^6}\left(-4+9x^4+2x^6\right)}{10x^5}+\frac{1}{4}\sqrt[4]{\frac{5}{2}}\,\text{ArcTan}\left(\frac{\sqrt[4]{\frac{5}{2}}\,x}{\sqrt[4]{-2+2x^4+x^6}}\right)$$
$$-\frac{1}{4}\sqrt[4]{\frac{5}{2}}\,\tanh^{-1}\left(\frac{\sqrt[4]{\frac{5}{2}}\,x}{\sqrt[4]{-2+2x^4+x^6}}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(-2+x^6\right)\left(4+x^6\right)\sqrt[4]{-2+2x^4+x^6}}{x^6\left(-4-x^4+2x^6\right)}\,dx$$

24.214 Problem number 1501

$$\int \frac{(2+x)^2}{x\left(4-2x+x^2\right)\sqrt[3]{1+x+x^2}}\,dx$$

Optimal antiderivative

$$-\sqrt{3}\,\arctan\left(\frac{\frac{2\sqrt{3}}{3}-\frac{2x\sqrt{3}}{3}+\frac{\left(x^2+x+1\right)^{\frac{1}{3}}\sqrt{3}}{3}}{\left(x^2+x+1\right)^{\frac{1}{3}}}\right)+\ln\left(-1+x+\left(x^2+x+1\right)^{\frac{1}{3}}\right)$$
$$-\frac{\ln\left(1-2x+x^2+(1-x)\left(x^2+x+1\right)^{\frac{1}{3}}+\left(x^2+x+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(2 + x)^2/(x*(4 - 2*x + x^2)*(1 + x + x^2)^(1/3)),x]
```

Mathematica 13.1 output

$$-\sqrt{3}\,\text{ArcTan}\left(\frac{2-2x+\sqrt[3]{1+x+x^2}}{\sqrt{3}\,\sqrt[3]{1+x+x^2}}\right)+\log\left(-1+x+\sqrt[3]{1+x+x^2}\right)$$
$$-\frac{1}{2}\log\left(1-2x+x^2-(-1+x)\sqrt[3]{1+x+x^2}+\left(1+x+x^2\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{(2+x)^2}{x\left(4-2x+x^2\right)\sqrt[3]{1+x+x^2}}\,dx$$

24.215 Problem number 1504

$$\int \frac{3c+2bx+ax^2}{\sqrt[3]{c+bx+ax^2}\,\left(c+bx+ax^2+x^3\right)}\,dx$$

Optimal antiderivative

$$\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(a\,x^2+bx+c\right)^{\frac{1}{3}}}{-2x+\left(a\,x^2+bx+c\right)^{\frac{1}{3}}}\right)+\ln\left(x+\left(a\,x^2+bx+c\right)^{\frac{1}{3}}\right)$$
$$-\frac{\ln\left(x^2-x\left(a\,x^2+bx+c\right)^{\frac{1}{3}}+\left(a\,x^2+bx+c\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(3*c + 2*b*x + a*x^2)/((c + b*x + a*x^2)^(1/3)*(c + b*x + a*x^2 + x^3)),x]
```

Mathematica 13.1 output

$$\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{c+x(b+ax)}}{-2x+\sqrt[3]{c+x(b+ax)}}\right)+\log\left(x+\sqrt[3]{c+x(b+ax)}\right)$$
$$-\frac{1}{2}\log\left(x^2-x\sqrt[3]{c+x(b+ax)}+(c+x(b+ax))^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{3c+2bx+ax^2}{\sqrt[3]{c+bx+ax^2}\,(c+bx+ax^2+x^3)}\,dx$$

24.216 Problem number 1507

$$\int\frac{\sqrt[4]{bx^2+ax^4}}{x^4\,(b+ax^4)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(b*x^2 + a*x^4)^(1/4)/(x^4*(b + a*x^4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{x^2\,(b+ax^2)}\left(-8\left(b+ax^2\right)^{5/4}+5abx^{5/2}\text{RootSum}\left[a^2+ab-2a\#1^4+\#1^8\&,\frac{-\log\left(\sqrt{x}\right)\#1+\log\left(\sqrt[4]{b+ax^2}-\sqrt{}\right.}{-a+\#1^4}\right.\right.}{20b^2x^3\sqrt[4]{b+ax^2}}$$

Mathematica 12.3 output

$$\int\frac{\sqrt[4]{bx^2+ax^4}}{x^4\,(b+ax^4)}\,dx$$

24.217 Problem number 1508

$$\int \frac{\sqrt[4]{bx^2+ax^4}}{x^4\left(b+ax^4\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(b*x^2 + a*x^4)^(1/4)/(x^4*(b + a*x^4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{x^2\left(b+ax^2\right)}\left(-8\left(b+ax^2\right)^{5/4}+5abx^{5/2}\text{RootSum}\left[a^2+ab-2a\#1^4+\#1^8\&,\frac{-\log\left(\sqrt{x}\right)\#1+\log\left(\sqrt[4]{b+ax^2}-\sqrt{}\right.}{-a+\#1^4}\right.\right.}{20b^2x^3\sqrt[4]{b+ax^2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{bx^2+ax^4}}{x^4\left(b+ax^4\right)}\,dx$$

24.218 Problem number 1509

$$\int \frac{\sqrt[4]{bx^2+ax^4}}{x^4\left(b+ax^4\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(b*x^2 + a*x^4)^(1/4)/(x^4*(b + a*x^4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{x^2\left(b+ax^2\right)}\left(-8\left(b+ax^2\right)^{5/4}+5abx^{5/2}\text{RootSum}\left[a^2+ab-2a\#1^4+\#1^8\&,\frac{-\log\left(\sqrt{x}\right)\#1+\log\left(\sqrt[4]{b+ax^2}-\sqrt{}\right.}{-a+\#1^4}\right.\right.}{20b^2x^3\sqrt[4]{b+ax^2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{bx^2+ax^4}}{x^4\left(b+ax^4\right)}\,dx$$

24.219 Problem number 1510

$$\int \frac{x\left(3+5x^2\right)}{\sqrt[3]{1+x^2}\,\left(-1+x^3+x^5\right)}\,dx$$

Optimal antiderivative

$$\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x\left(x^2+1\right)^{\frac{1}{3}}}{2+x\left(x^2+1\right)^{\frac{1}{3}}}\right)-2\,\text{arctanh}\left(-1+2x\left(x^2+1\right)^{\frac{1}{3}}\right)$$
$$+\frac{\ln\left(x^2\left(x^2+1\right)^{\frac{2}{3}}\right)}{2}-\frac{\ln\left(1+x\left(x^2+1\right)^{\frac{1}{3}}+x^2\left(x^2+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(x*(3 + 5*x^2))/((1 + x^2)^(1/3)*(-1 + x^3 + x^5)),x]
```

Mathematica 13.1 output

$$\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x\sqrt[3]{1+x^2}}{2+x\sqrt[3]{1+x^2}}\right)+2\tanh^{-1}\left(1-2x\sqrt[3]{1+x^2}\right)$$
$$+\frac{1}{2}\log\left(x^2\left(1+x^2\right)^{2/3}\right)-\frac{1}{2}\log\left(1+x\sqrt[3]{1+x^2}+x^2\left(1+x^2\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{x\left(3+5x^2\right)}{\sqrt[3]{1+x^2}\,\left(-1+x^3+x^5\right)}\,dx$$

24.220 Problem number 1511

$$\int \frac{\left(-1+x^5\right)^{2/3}\left(3+2x^5\right)\left(-2+x^3+2x^5\right)}{x^6\left(-1+x^3+x^5\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^5-1\right)^{\frac{2}{3}}\left(4x^5-5x^3-4\right)}{10x^5}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^5-1\right)^{\frac{1}{3}}}\right)$$
$$-\ln\left(x+\left(x^5-1\right)^{\frac{1}{3}}\right)+\frac{\ln\left(x^2-x\left(x^5-1\right)^{\frac{1}{3}}+\left(x^5-1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((-1 + x^5)^(2/3)*(3 + 2*x^5)*(-2 + x^3 + 2*x^5))/(x^6*(-1 + x^3 + x^5)),x]
```

Mathematica 13.1 output

$$\frac{3\left(-1+x^5\right)^{2/3}\left(-4-5x^3+4x^5\right)}{10x^5}+\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x-2\sqrt[3]{-1+x^5}}\right)$$
$$-\log\left(x+\sqrt[3]{-1+x^5}\right)+\frac{1}{2}\log\left(x^2-x\sqrt[3]{-1+x^5}+\left(-1+x^5\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(-1+x^5\right)^{2/3}\left(3+2x^5\right)\left(-2+x^3+2x^5\right)}{x^6\left(-1+x^3+x^5\right)}\,dx$$

24.221 Problem number 1513

$$\int\frac{\left(-1+x^6\right)^{2/3}\left(1+x^6\right)\left(-1-x^3+x^6\right)}{x^6\left(-1+x^3+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(x^6-1\right)^{\frac{2}{3}}\left(x^6-5x^3-1\right)}{5x^5}-\frac{2\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^6-1\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}$$
$$-\frac{2\ln\left(x+\left(x^6-1\right)^{\frac{1}{3}}\right)}{3}+\frac{\ln\left(x^2-x\left(x^6-1\right)^{\frac{1}{3}}+\left(x^6-1\right)^{\frac{2}{3}}\right)}{3}$$

command

```
Integrate[((-1 + x^6)^(2/3)*(1 + x^6)*(-1 - x^3 + x^6))/(x^6*(-1 + x^3 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{\left(-1+x^6\right)^{2/3}\left(-1-5x^3+x^6\right)}{5x^5}-\frac{2\text{ArcTan}\left(\frac{\sqrt{3}\,x}{-x+2\sqrt[3]{-1+x^6}}\right)}{\sqrt{3}}$$
$$-\frac{2}{3}\log\left(x+\sqrt[3]{-1+x^6}\right)+\frac{1}{3}\log\left(x^2-x\sqrt[3]{-1+x^6}+\left(-1+x^6\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(-1+x^6\right)^{2/3}\left(1+x^6\right)\left(-1-x^3+x^6\right)}{x^6\left(-1+x^3+x^6\right)}\,dx$$

24.222 Problem number 1516

$$\int \frac{\sqrt[4]{-1+x^4}\left(2+x^4\right)}{x^2\left(2+2x^4+x^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^4)^(1/4)*(2 + x^4))/(x^2*(2 + 2*x^4 + x^8)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt[4]{-1+x^4}}{x}+\frac{1}{8}\text{RootSum}\left[5-6\#1^4+2\#1^8\&,\frac{-5\log(x)+5\log\left(\sqrt[4]{-1+x^4}-x\#1\right)+3\log(x)\#1^4-3\log\left(\sqrt[4]{-1+x^4}-x\#1\right)\#1^4}{-3\#1^3+2\#1^7}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{-1+x^4}\left(2+x^4\right)}{x^2\left(2+2x^4+x^8\right)}\,dx$$

24.223 Problem number 1517

$$\int \frac{\sqrt[4]{-1+x^4}\left(2+x^4\right)}{x^2\left(2+2x^4+x^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^4)^(1/4)*(2 + x^4))/(x^2*(2 + 2*x^4 + x^8)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt[4]{-1+x^4}}{x}+\frac{1}{8}\text{RootSum}\left[5-6\#1^4+2\#1^8\&,\frac{-5\log(x)+5\log\left(\sqrt[4]{-1+x^4}-x\#1\right)+3\log(x)\#1^4-3\log\left(\sqrt[4]{-1+x^4}-x\#1\right)\#1^4}{-3\#1^3+2\#1^7}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{-1+x^4}\left(2+x^4\right)}{x^2\left(2+2x^4+x^8\right)}\,dx$$

24.224 Problem number 1520

$$\int \frac{\left(1+x^3+x^8\right)^{2/3}\left(-3+5x^8\right)}{x^3\left(1+x^8\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^8+x^3+1\right)^{\frac{2}{3}}}{2x^2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^8+x^3+1\right)^{\frac{1}{3}}}\right)$$
$$+\ln\left(-x+\left(x^8+x^3+1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(x^2+x\left(x^8+x^3+1\right)^{\frac{1}{3}}+\left(x^8+x^3+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((1 + x^3 + x^8)^(2/3)*(-3 + 5*x^8))/(x^3*(1 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{3\left(1+x^3+x^8\right)^{2/3}}{2x^2}-\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{1+x^3+x^8}}\right)$$
$$+\log\left(-x+\sqrt[3]{1+x^3+x^8}\right)-\frac{1}{2}\log\left(x^2+x\sqrt[3]{1+x^3+x^8}+\left(1+x^3+x^8\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^3+x^8\right)^{2/3}\left(-3+5x^8\right)}{x^3\left(1+x^8\right)}\,dx$$

24.225 Problem number 1521

$$\int \frac{-b+2ax^8}{\sqrt[4]{b+ax^4}\,\left(-b+ax^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-b + 2*a*x^8)/((b + a*x^4)^(1/4)*(-b + a*x^8)),x]
```

Mathematica 13.1 output

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{\sqrt[4]{a}}$$
$$+\frac{1}{8}\text{RootSum}\left[a^2-ab-2a\#1^4+\#1^8\&,\frac{-\log(x)+\log\left(\sqrt[4]{b+ax^4}-x\#1\right)}{\#1}\&\right]$$

Mathematica 12.3 output

$$\int\frac{-b+2ax^8}{\sqrt[4]{b+ax^4}\,(-b+ax^8)}\,dx$$

24.226 Problem number 1522

$$\int\frac{\sqrt{-1-x^2+x^6}\,\left(1+2x^6\right)}{8-x^4-16x^6+8x^{12}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{8-2\sqrt{2}}\,\arctan\left(\frac{\sqrt{4-\sqrt{2}}\,x}{2\sqrt{x^6-x^2-1}}\right)}{16}-\frac{\sqrt{8+2\sqrt{2}}\,\arctan\left(\frac{\sqrt{4+\sqrt{2}}\,x}{2\sqrt{x^6-x^2-1}}\right)}{16}$$

command

```
Integrate[(Sqrt[-1 - x^2 + x^6]*(1 + 2*x^6))/(8 - x^4 - 16*x^6 + 8*x^12),x]
```

Mathematica 13.1 output

$$\frac{1}{16}\left(\sqrt{8-2\sqrt{2}}\,\text{ArcTan}\left(\frac{x}{2\sqrt{\frac{1+x^2-x^6}{-4+\sqrt{2}}}}\right)-\sqrt{2\left(4+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{4+\sqrt{2}}\,x}{2\sqrt{-1-x^2+x^6}}\right)\right)$$

Mathematica 12.3 output

$$\int\frac{\sqrt{-1-x^2+x^6}\,\left(1+2x^6\right)}{8-x^4-16x^6+8x^{12}}\,dx$$

24.227 Problem number 1524

$$\int \frac{\sqrt{x-\sqrt{1+x^2}}}{x^2+\sqrt{1+x^2}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[x - Sqrt[1 + x^2]]/(x^2 + Sqrt[1 + x^2]),x]
```

Mathematica 13.1 output

$$\text{RootSum}\left[1-2\#1^2-2\#1^4-2\#1^6+\#1^8\&,\ \frac{\log\left(\sqrt{x-\sqrt{1+x^2}}-\#1\right)\#1+\log\left(\sqrt{x-\sqrt{1+x^2}}-\#1\right)\#1^5}{-1-2\#1^2-3\#1^4+2\#1^6}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\sqrt{x-\sqrt{1+x^2}}}{x^2+\sqrt{1+x^2}}\,dx$$

24.228 Problem number 1525

$$\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{x^2}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{a\,x^2+\sqrt{a^2x^4+b}}}{x}+\frac{\sqrt{a}\ \ln\left(ia\,x^2+i\sqrt{a^2x^4+b}+i\sqrt{2}\ \sqrt{a}\ x\sqrt{a\,x^2+\sqrt{a^2x^4+b}}\right)\sqrt{2}}{2}$$

command

```
Integrate[Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]/x^2,x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{x}+\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{2}\,\sqrt{a}\,x}\right)}{\sqrt{2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{x^2}\,dx$$

24.229 Problem number 1532

$$\int \frac{\left(-b+ax^4\right)\sqrt[4]{-bx^2+ax^4}}{x^4\left(b+ax^4\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-b + a*x^4)*(-(b*x^2) + a*x^4)^(1/4))/(x^4*(b + a*x^4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-bx^2+ax^4}\left(-4\left(-b+ax^2\right)^{5/4}+5abx^{5/2}\text{RootSum}\left[a^2+ab-2a\#1^4+\#1^8\&,\frac{-\log\left(\sqrt{x}\right)\#1+\log\left(\sqrt[4]{-b+ax^2}\right.\right.}{-a+\#1^4}\right.}{10bx^3\sqrt[4]{-b+ax^2}}$$

Mathematica 12.3 output

$$\int \frac{\left(-b+ax^4\right)\sqrt[4]{-bx^2+ax^4}}{x^4\left(b+ax^4\right)}\,dx$$

24.230 Problem number 1533

$$\int \frac{\left(-b+ax^4\right)\sqrt[4]{-bx^2+ax^4}}{x^4\left(b+ax^4\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-b + a*x^4)*(-(b*x^2) + a*x^4)^(1/4))/(x^4*(b + a*x^4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-bx^2+ax^4}\left(-4\left(-b+ax^2\right)^{5/4}+5abx^{5/2}\text{RootSum}\left[a^2+ab-2a\#1^4+\#1^8\&,\frac{-\log\left(\sqrt{x}\right)\#1+\log\left(\sqrt[4]{-b+ax^2}\right.}{-a+\#1^4}\right.\right.}{10bx^3\sqrt[4]{-b+ax^2}}$$

Mathematica 12.3 output

$$\int \frac{\left(-b+ax^4\right)\sqrt[4]{-bx^2+ax^4}}{x^4\left(b+ax^4\right)}\,dx$$

24.231 Problem number 1536

$$\int \frac{\sqrt[4]{bx^2+ax^4}}{x^4\left(-b+ax^4\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(b*x^2 + a*x^4)^(1/4)/(x^4*(-b + a*x^4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{x^2\left(b+ax^2\right)}\left(8\left(b+ax^2\right)^{5/4}+5abx^{5/2}\text{RootSum}\left[a^2-ab-2a\#1^4+\#1^8\&,\frac{-\log\left(\sqrt{x}\right)\#1+\log\left(\sqrt[4]{b+ax^2}-\sqrt{x}\right.\right.}{-a+\#1^4}\right.}{20b^2x^3\sqrt[4]{b+ax^2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{bx^2+ax^4}}{x^4\left(-b+ax^4\right)}\,dx$$

24.232 Problem number 1537

$$\int \frac{\sqrt[4]{bx^2+ax^4}}{x^4\left(-b+ax^4\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(b*x^2 + a*x^4)^(1/4)/(x^4*(-b + a*x^4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{x^2\left(b+ax^2\right)}\left(8\left(b+ax^2\right)^{5/4}+5abx^{5/2}\text{RootSum}\left[a^2-ab-2a\#1^4+\#1^8\&,\frac{-\log\left(\sqrt{x}\right)\#1+\log\left(\sqrt[4]{b+ax^2}-\sqrt{x}\right.\right.}{-a+\#1^4}\right.}{20b^2x^3\sqrt[4]{b+ax^2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{bx^2+ax^4}}{x^4\left(-b+ax^4\right)}\,dx$$

24.233 Problem number 1539

$$\int \frac{\left(1+x^3\right)^{2/3}\left(1+x^3+2x^6\right)}{x^6\left(-1+2x^6\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^3)^(2/3)*(1 + x^3 + 2*x^6))/(x^6*(-1 + 2*x^6)),x]
```

Mathematica 13.1 output

$$\frac{\left(1+x^3\right)^{2/3}\left(2+7x^3\right)}{10x^5}-\frac{1}{6}\text{RootSum}\left[-1-2\#1^3\right.$$
$$\left.+\#1^6\&,\frac{\log(x)-\log\left(\sqrt[3]{1+x^3}-x\#1\right)+5\log(x)\#1^3-5\log\left(\sqrt[3]{1+x^3}-x\#1\right)\#1^3}{-\#1+\#1^4}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^3\right)^{2/3}\left(1+x^3+2x^6\right)}{x^6\left(-1+2x^6\right)}\,dx$$

24.234 Problem number 1540

$$\int \frac{\left(1+x^3\right)^{2/3}\left(1+x^3+2x^6\right)}{x^6\left(-1+2x^6\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^3)^(2/3)*(1 + x^3 + 2*x^6))/(x^6*(-1 + 2*x^6)),x]
```

Mathematica 13.1 output

$$\frac{\left(1+x^3\right)^{2/3}\left(2+7x^3\right)}{10x^5}-\frac{1}{6}\text{RootSum}\left[-1-2\#1^3+\#1^6\&,\frac{\log(x)-\log\left(\sqrt[3]{1+x^3}-x\#1\right)+5\log(x)\#1^3-5\log\left(\sqrt[3]{1+x^3}-x\#1\right)\#1^3}{-\#1+\#1^4}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^3\right)^{2/3}\left(1+x^3+2x^6\right)}{x^6\left(-1+2x^6\right)}\,dx$$

24.235 Problem number 1541

$$\int \frac{\left(1+x^6\right)\left(-1+2x^6\right)\left(-1+x^4+2x^6\right)^{5/4}}{x^{10}\left(-1-x^4+2x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(2x^6+x^4-1\right)^{\frac{1}{4}}\left(20x^{12}+38x^{10}+104x^8-20x^6-19x^4+5\right)}{45x^9}+2^{\frac{1}{4}}\arctan\left(\frac{2^{\frac{1}{4}}x}{\left(2x^6+x^4-1\right)^{\frac{1}{4}}}\right)-2^{\frac{1}{4}}\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}x}{\left(2x^6+x^4-1\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[((1 + x^6)*(-1 + 2*x^6)*(-1 + x^4 + 2*x^6)^(5/4))/(x^10*(-1 - x^4 + 2*x^6)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-1+x^4+2x^6}\left(5-19x^4-20x^6+104x^8+38x^{10}+20x^{12}\right)}{45x^9}$$
$$+\sqrt[4]{2}\,\text{ArcTan}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{-1+x^4+2x^6}}\right)-\sqrt[4]{2}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{-1+x^4+2x^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^6\right)\left(-1+2x^6\right)\left(-1+x^4+2x^6\right)^{5/4}}{x^{10}\left(-1-x^4+2x^6\right)}\,dx$$

24.236 Problem number 1559

$$\int \frac{x^4\left(-2b+ax^2\right)}{\left(-b+ax^2\right)^2\sqrt[4]{-b+ax^2+cx^4}}\,dx$$

Optimal antiderivative

$$-\frac{x\left(c\,x^4+a\,x^2-b\right)^{\frac{3}{4}}}{2c\left(-a\,x^2+b\right)}-\frac{\arctan\left(\frac{c^{\frac{1}{4}}x}{\left(c\,x^4+a\,x^2-b\right)^{\frac{1}{4}}}\right)}{4c^{\frac{5}{4}}}-\frac{\text{arctanh}\left(\frac{c^{\frac{1}{4}}x}{\left(c\,x^4+a\,x^2-b\right)^{\frac{1}{4}}}\right)}{4c^{\frac{5}{4}}}$$

command

```
Integrate[(x^4*(-2*b + a*x^2))/((-b + a*x^2)^2*(-b + a*x^2 + c*x^4)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{-\frac{2\sqrt[4]{c}\,x\left(-b+ax^2+cx^4\right)^{3/4}}{b-ax^2}-\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-b+ax^2+cx^4}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-b+ax^2+cx^4}}\right)}{4c^{5/4}}$$

Mathematica 12.3 output

$$\int \frac{x^4\left(-2b+ax^2\right)}{\left(-b+ax^2\right)^2\sqrt[4]{-b+ax^2+cx^4}}\,dx$$

24.237 Problem number 1568

$$\int \frac{x^4\left(-4+x^3\right)}{\sqrt[4]{-1+x^3}\left(-1+2x^3-x^6+x^8\right)}\,dx$$

Optimal antiderivative

$$\arctan\left(\frac{(x^3-1)^{\frac{1}{4}}}{x}\right)+\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ x(x^3-1)^{\frac{1}{4}}}{-x^2+\sqrt{x^3-1}}\right)}{2}$$
$$-\operatorname{arctanh}\left(\frac{x}{(x^3-1)^{\frac{1}{4}}}\right)+\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\ x(x^3-1)^{\frac{1}{4}}}{x^2+\sqrt{x^3-1}}\right)}{2}$$

command

```
Integrate[(x^4*(-4 + x^3))/((-1 + x^3)^(1/4)*(-1 + 2*x^3 - x^6 + x^8)),x]
```

Mathematica 13.1 output

$$\operatorname{ArcTan}\left(\frac{\sqrt[4]{-1+x^3}}{x}\right)+\frac{\operatorname{ArcTan}\left(\frac{\sqrt{2}\ x\sqrt[4]{-1+x^3}}{-x^2+\sqrt{-1+x^3}}\right)}{\sqrt{2}}$$
$$-\tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^3}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\ x\sqrt[4]{-1+x^3}}{x^2+\sqrt{-1+x^3}}\right)}{\sqrt{2}}$$

Mathematica 12.3 output

$$\int\frac{x^4\left(-4+x^3\right)}{\sqrt[4]{-1+x^3}\ \left(-1+2x^3-x^6+x^8\right)}\,dx$$

24.238 Problem number 1570

$$\int\frac{\left(-b^2+ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}}{b^2+ax^2}\,dx$$

Optimal antiderivative

$$\frac{4bx}{3\sqrt{b+\sqrt{a\,x^2+b^2}}}+\frac{2x\sqrt{a\,x^2+b^2}}{3\sqrt{b+\sqrt{a\,x^2+b^2}}}-\frac{4b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{a}\ x}{\sqrt{b}\ \sqrt{b+\sqrt{a\,x^2+b^2}}}\right)}{\sqrt{a}}$$

command

```
Integrate[((-b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]])/(b^2 + a*x^2),x]
```

Mathematica 13.1 output

$$\frac{2x\left(2b+\sqrt{b^2+ax^2}\right)}{3\sqrt{b+\sqrt{b^2+ax^2}}}-\frac{4b^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\,\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{a}}$$

Mathematica 12.3 output

$$\int\frac{\left(-b^2+ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}}{b^2+ax^2}\,dx$$

24.239 Problem number 1582

$$\int\frac{\sqrt[4]{-1+x^4}\left(1+x^4+x^8\right)}{x^6\left(-1+2x^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^4)^(1/4)*(1 + x^4 + x^8))/(x^6*(-1 + 2*x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-1+x^4}\left(1+4x^4\right)}{5x^5}+\frac{1}{8}\text{RootSum}\left[-1-2\#1^4+\#1^8\&,\frac{-\log(x)+\log\left(\sqrt[4]{-1+x^4}-x\#1\right)+2\log(x)\#1^4-2\log\left(\sqrt[4]{-1+x^4}-x\#1\right)\#1^4}{-\#1^3+\#1^7}\&\right]$$

Mathematica 12.3 output

$$\int\frac{\sqrt[4]{-1+x^4}\left(1+x^4+x^8\right)}{x^6\left(-1+2x^8\right)}\,dx$$

24.240 Problem number 1583

$$\int \frac{\sqrt[4]{-1+x^4}\left(1+x^4+x^8\right)}{x^6\left(-1+2x^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^4)^(1/4)*(1 + x^4 + x^8))/(x^6*(-1 + 2*x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-1+x^4}\left(1+4x^4\right)}{5x^5}+\frac{1}{8}\text{RootSum}\left[-1-2\#1^4+\#1^8\&,\frac{-\log(x)+\log\left(\sqrt[4]{-1+x^4}-x\#1\right)+2\log(x)\#1^4-2\log\left(\sqrt[4]{-1+x^4}-x\#1\right)\#1^4}{-\#1^3+\#1^7}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{-1+x^4}\left(1+x^4+x^8\right)}{x^6\left(-1+2x^8\right)}\,dx$$

24.241 Problem number 1584

$$\int \frac{\sqrt{-bx+a^2x^2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Optimal antiderivative

$$-\frac{4\left(3a^3x+5ab\right)\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}}{3b^2x}+\frac{4\left(3a^2x+b\right)\sqrt{a^2x^2-bx}\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}}{3b^2x^2}$$

command

```
Integrate[Sqrt[-(b*x) + a^2*x^2]/(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2),x]
```

Mathematica 13.1 output

$$-\frac{4\left(b+a\left(7ax+4\sqrt{x\left(-b+a^2x\right)}\right)\right)}{3b\sqrt{x\left(ax+\sqrt{x\left(-b+a^2x\right)}\right)}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{-bx+a^2x^2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

24.242 Problem number 1591

$$\int \frac{\sqrt[4]{-bx^2+ax^4}}{x^4\left(-b+ax^4\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-(b*x^2) + a*x^4)^(1/4)/(x^4*(-b + a*x^4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-bx^2+ax^4}\left(-8\left(-b+ax^2\right)^{5/4}-5abx^{5/2}\text{RootSum}\left[a^2-ab-2a\#1^4+\#1^8\&,\frac{-\log\left(\sqrt{x}\right)\#1+\log\left(\sqrt[4]{-b+ax^2}\right.\right.}{-a+\#1^4}\right.}{20b^2x^3\sqrt[4]{-b+ax^2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{-bx^2+ax^4}}{x^4\left(-b+ax^4\right)}\,dx$$

24.243 Problem number 1592

$$\int \frac{\sqrt[4]{-bx^2+ax^4}}{x^4\left(-b+ax^4\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-(b*x^2) + a*x^4)^(1/4)/(x^4*(-b + a*x^4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-bx^2+ax^4}\left(-8\left(-b+ax^2\right)^{5/4}-5abx^{5/2}\text{RootSum}\left[a^2-ab-2a\#1^4+\#1^8\&,\frac{-\log\left(\sqrt{x}\right)\#1+\log\left(\sqrt[4]{-b+ax^2}\right.\right.}{-a+\#1^4}\right.}{20b^2x^3\sqrt[4]{-b+ax^2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{-bx^2+ax^4}}{x^4\left(-b+ax^4\right)}\,dx$$

24.244 Problem number 1595

$$\int \frac{\left(1+x^3\right)^{2/3}\left(1-2x^3+x^6\right)}{x^6\left(-2+x^6\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^3)^(2/3)*(1 - 2*x^3 + x^6))/(x^6*(-2 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{\left(1-4x^3\right)\left(1+x^3\right)^{2/3}}{10x^5}+\frac{1}{24}\text{RootSum}\left[1-4\#1^3\right.$$
$$\left.+2\#1^6\&,\frac{-2\log(x)+2\log\left(\sqrt[3]{1+x^3}-x\#1\right)+\log(x)\#1^3-\log\left(\sqrt[3]{1+x^3}-x\#1\right)\#1^3}{-\#1+\#1^4}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^3\right)^{2/3}\left(1-2x^3+x^6\right)}{x^6\left(-2+x^6\right)}\,dx$$

24.245 Problem number 1596

$$\int \frac{\left(1+x^3\right)^{2/3}\left(1-2x^3+x^6\right)}{x^6\left(-2+x^6\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^3)^(2/3)*(1 - 2*x^3 + x^6))/(x^6*(-2 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{\left(1-4x^3\right)\left(1+x^3\right)^{2/3}}{10x^5}+\frac{1}{24}\text{RootSum}\left[1-4\#1^3+2\#1^6\&,\frac{-2\log(x)+2\log\left(\sqrt[3]{1+x^3}-x\#1\right)+\log(x)\#1^3-\log\left(\sqrt[3]{1+x^3}-x\#1\right)\#1^3}{-\#1+\#1^4}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^3\right)^{2/3}\left(1-2x^3+x^6\right)}{x^6\left(-2+x^6\right)}\,dx$$

24.246 Problem number 1601

$$\int \frac{\left(2+x^6\right)\left(1-2x^6+x^8+x^{12}\right)}{x^8\sqrt[4]{-1+x^6}\left(-1+x^4+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{2\left(x^6-1\right)^{\frac{3}{4}}\left(3x^6-7x^4-3\right)}{21x^7}-\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x\left(x^6-1\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^6-1}}\right)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,x\left(x^6-1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^6-1}}\right)$$

command

```
Integrate[((2 + x^6)*(1 - 2*x^6 + x^8 + x^12))/(x^8*(-1 + x^6)^(1/4)*(-1 + x^4 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{2\left(-1+x^6\right)^{3/4}\left(-3-7x^4+3x^6\right)}{21x^7}-\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,x\sqrt[4]{-1+x^6}}{-x^2+\sqrt{-1+x^6}}\right)-\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{-1+x^6}}{x^2+\sqrt{-1+x^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(2+x^6\right)\left(1-2x^6+x^8+x^{12}\right)}{x^8\sqrt[4]{-1+x^6}\left(-1+x^4+x^6\right)}\,dx$$

24.247 Problem number 1613

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(4+x^6\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^3)^(2/3)*(2 + x^3))/(x^6*(4 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{\left(-4-x^3\right)\left(-1+x^3\right)^{2/3}}{40x^5}+\frac{1}{96}\text{RootSum}\left[5-8\#1^3+4\#1^6\&,\frac{-5\log(x)+5\log\left(\sqrt[3]{-1+x^3}-x\#1\right)+6\log(x)\#1^3-6\log\left(\sqrt[3]{-1+x^3}-x\#1\right)\#1^3}{-\#1+\#1^4}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(4+x^6\right)}\,dx$$

24.248 Problem number 1614

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(4+x^6\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^3)^(2/3)*(2 + x^3))/(x^6*(4 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{\left(-4-x^3\right)\left(-1+x^3\right)^{2/3}}{40x^5}+\frac{1}{96}\text{RootSum}\left[5-8\#1^3+4\#1^6\&,\frac{-5\log(x)+5\log\left(\sqrt[3]{-1+x^3}-x\#1\right)+6\log(x)\#1^3-6\log\left(\sqrt[3]{-1+x^3}-x\#1\right)\#1^3}{-\#1+\#1^4}\&\right]$$

Mathematica 12.3 output

$$\int\frac{\left(-1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(4+x^6\right)}\,dx$$

24.249 Problem number 1622

$$\int\frac{-b+ax^8}{\sqrt[4]{b+ax^4}\left(b+ax^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-b + a*x^8)/((b + a*x^4)^(1/4)*(b + a*x^8)),x]
```

Mathematica 13.1 output

$$\frac{1}{4}\left(\frac{2\left(\text{ArcTan}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)\right)}{\sqrt[4]{a}}+\text{RootSum}\left[a^2+ab-2a\#1^4+\#1^8\&,\frac{-\log(x)+\log\left(\sqrt[4]{b+ax^4}-x\#1\right)}{\#1}\&\right]\right)$$

Mathematica 12.3 output

$$\int\frac{-b+ax^8}{\sqrt[4]{b+ax^4}\left(b+ax^8\right)}\,dx$$

24.250 Problem number 1623

$$\int \frac{-b+ax^8}{\sqrt[4]{b+ax^4}\,\left(b+ax^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-b + a*x^8)/((b + a*x^4)^(1/4)*(b + a*x^8)),x]
```

Mathematica 13.1 output

$$\frac{1}{4}\left(\frac{2\left(\text{ArcTan}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)\right)}{\sqrt[4]{a}}\right.$$

$$\left.+\text{RootSum}\left[a^2+ab-2a\#1^4+\#1^8\&,\frac{-\log(x)+\log\left(\sqrt[4]{b+ax^4}-x\#1\right)}{\#1}\&\right]\right)$$

Mathematica 12.3 output

$$\int \frac{-b+ax^8}{\sqrt[4]{b+ax^4}\,\left(b+ax^8\right)}\,dx$$

24.251 Problem number 1630

$$\int \frac{1}{\sqrt{-bx+a^2x^2}\,\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{4\left(32a^3x+39ab\right)\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}}{105b^3x^2}$$

$$-\frac{4\left(-32a^2x+15b\right)\sqrt{a^2x^2-bx}\,\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}}{105b^3x^3}$$

command

```
Integrate[1/(Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2)),x]
```

Mathematica 13.1 output

$$\frac{4\left(ax+\sqrt{x\left(-b+a^2x\right)}\right)^2\left(15b^2+32a^3x\left(ax+\sqrt{x\left(-b+a^2x\right)}\right)+ab\left(-47ax+39\sqrt{x\left(-b+a^2x\right)}\right)\right)}{105b^3\sqrt{x\left(-b+a^2x\right)}\left(x\left(ax+\sqrt{x\left(-b+a^2x\right)}\right)\right)^{3/2}}$$

Mathematica 12.3 output

$$\int \frac{1}{\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

24.252 Problem number 1632

$$\int \frac{-2+x}{(2+x^2)\sqrt[3]{-1+x+2x^2}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(2x^2+x-1)^{\frac{1}{3}}}{2+2x+(2x^2+x-1)^{\frac{1}{3}}}\right)}{2}+\frac{\ln\left(-1-x+\left(2x^2+x-1\right)^{\frac{1}{3}}\right)}{2}$$
$$-\frac{\ln\left(1+2x+x^2+(1+x)\left(2x^2+x-1\right)^{\frac{1}{3}}+\left(2x^2+x-1\right)^{\frac{2}{3}}\right)}{4}$$

command

```
Integrate[(-2 + x)/((2 + x^2)*(-1 + x + 2*x^2)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{-1+x+2x^2}}{2+2x+\sqrt[3]{-1+x+2x^2}}\right)+\frac{1}{2}\log\left(-1-x+\sqrt[3]{-1+x+2x^2}\right)$$
$$-\frac{1}{4}\log\left(1+2x+x^2+(1+x)\sqrt[3]{-1+x+2x^2}+\left(-1+x+2x^2\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{-2+x}{(2+x^2)\sqrt[3]{-1+x+2x^2}}\,dx$$

24.253 Problem number 1638

$$\int \frac{(a-3b+2x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{(-b+x)\sqrt[4]{(-a+x)(-b+x)}\left(-a^3+bd-\left(-3a^2+d\right)x-3ax^2+x^3\right)}\,dx$$

Optimal antiderivative

$$-\frac{4\left(ab-ax-bx+x^2\right)^{\frac{3}{4}}}{b-x}-2d^{\frac{1}{4}}\arctan\left(\frac{d^{\frac{1}{4}}\left(ab+(-a-b)x+x^2\right)^{\frac{1}{4}}}{a-x}\right)$$
$$+2d^{\frac{1}{4}}\operatorname{arctanh}\left(\frac{d^{\frac{1}{4}}\left(ab+(-a-b)x+x^2\right)^{\frac{1}{4}}}{a-x}\right)$$

command

```
Integrate[((a - 3*b + 2*x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/((-b + x)*((-a + x)*(-
b + x))^(1/4)*(-a^3 + b*d - (-3*a^2 + d)*x - 3*a*x^2 + x^3)),x]
```

Mathematica 13.1 output

$$\frac{4(-a+x)}{\sqrt[4]{(-a+x)(-b+x)}}$$
$$+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{a-x}\sqrt[4]{\frac{-b+x}{a-x}}\left(-\operatorname{ArcTan}\left(\frac{-a+x+\sqrt{d}\sqrt{\frac{-b+x}{a-x}}}{\sqrt{2}\sqrt[4]{d}\sqrt{a-x}\sqrt[4]{\frac{-b+x}{a-x}}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{a-x}\sqrt[4]{\frac{b-x}{-a+x}}}{a-x+\sqrt{d}\sqrt{\frac{b-x}{-a+x}}}\right)\right)}{\sqrt[4]{(-a+x)(-b+x)}}$$

Mathematica 12.3 output

$$\int \frac{(a-3b+2x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{(-b+x)\sqrt[4]{(-a+x)(-b+x)}\left(-a^3+bd-\left(-3a^2+d\right)x-3ax^2+x^3\right)}\,dx$$

24.254 Problem number 1641

$$\int \frac{3ab^2-2b(2a+b)x+(a+2b)x^2}{\sqrt[4]{x(-a+x)(-b+x)^2}\left(-ab^2d+b(2a+b)dx-(a+2b)dx^2+(-1+d)x^3\right)}\,dx$$

Optimal antiderivative

$$\frac{2\arctan\left(\frac{d^{\frac{1}{4}}\left(-a\,b^2x+\left(2ab+b^2\right)x^2+(-a-2b)x^3+x^4\right)^{\frac{1}{4}}}{x}\right)}{d^{\frac{3}{4}}}-\frac{2\operatorname{arctanh}\left(\frac{d^{\frac{1}{4}}\left(-a\,b^2x+\left(2ab+b^2\right)x^2+(-a-2b)x^3+x^4\right)^{\frac{1}{4}}}{x}\right)}{d^{\frac{3}{4}}}$$

command

```
Integrate[(3*a*b^2 - 2*b*(2*a + b)*x + (a + 2*b)*x^2)/((x*(-a + x)*(-b + x)^2)^(1/4)*(-
(a*b^2*d) + b*(2*a + b)*d*x - (a + 2*b)*d*x^2 + (-1 + d)*x^3)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt[4]{-1+\frac{a}{x}}\,(b-x)\left(\operatorname{ArcTan}\left(\frac{\left(-1+\sqrt{d}\right)\sqrt{-1+\frac{a}{x}}\left(-1+\frac{b}{x}\right)}{\sqrt[4]{d}\,\sqrt[4]{-1+\frac{a}{x}}\,\sqrt{-2+\frac{2b}{x}}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{-1+\frac{a}{x}}\,\sqrt{-2+\frac{2b}{x}}}{1+\sqrt{d}\,\sqrt{-1+\frac{a}{x}}\left(-1+\frac{b}{x}\right)}\right)\right)}{d^{3/4}\sqrt{-2+\frac{2b}{x}}\,\sqrt[4]{(b-x)^2x(-a+x)}}$$

Mathematica 12.3 output

$$\int \frac{3ab^2-2b(2a+b)x+(a+2b)x^2}{\sqrt[4]{x(-a+x)(-b+x)^2}\,\left(-ab^2d+b(2a+b)dx-(a+2b)dx^2+(-1+d)x^3\right)}\,dx$$

24.255 Problem number 1643

$$\int \frac{\left(3+x^5\right)\sqrt[3]{-2+x^3+x^5}}{x^2\left(-2+x^5\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^5+x^3-2\right)^{\frac{1}{3}}}{2x}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^5+x^3-2\right)^{\frac{1}{3}}}\right)}{2}+\frac{\ln\left(-x+\left(x^5+x^3-2\right)^{\frac{1}{3}}\right)}{2}-\frac{\ln\left(x^2+x\left(x^5+x^3-2\right)^{\frac{1}{3}}+\left(x^5+x^3-2\right)^{\frac{2}{3}}\right)}{4}$$

command

```
Integrate[((3 + x^5)*(-2 + x^3 + x^5)^(1/3))/(x^2*(-2 + x^5)),x]
```

Mathematica 13.1 output

$$\frac{3\sqrt[3]{-2+x^3+x^5}}{2x}+\frac{1}{2}\sqrt{3}\,\operatorname{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{-2+x^3+x^5}}\right)+\frac{1}{2}\log\left(-x+\sqrt[3]{-2+x^3+x^5}\right)-\frac{1}{4}\log\left(x^2+x\sqrt[3]{-2+x^3+x^5}+\left(-2+x^3+x^5\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(3+x^5\right)\sqrt[3]{-2+x^3+x^5}}{x^2\left(-2+x^5\right)}\,dx$$

24.256 Problem number 1644

$$\int \frac{\left(-3+x^5\right)\left(2+x^3+x^5\right)^{2/3}}{x^3\left(2+x^5\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^5+x^3+2\right)^{\frac{2}{3}}}{4x^2}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^5+x^3+2\right)^{\frac{1}{3}}}\right)}{2}+\frac{\ln\left(-x+\left(x^5+x^3+2\right)^{\frac{1}{3}}\right)}{2}-\frac{\ln\left(x^2+x\left(x^5+x^3+2\right)^{\frac{1}{3}}+\left(x^5+x^3+2\right)^{\frac{2}{3}}\right)}{4}$$

command

```
Integrate[((-3 + x^5)*(2 + x^3 + x^5)^(2/3))/(x^3*(2 + x^5)),x]
```

Mathematica 13.1 output

$$\frac{1}{4}\left(\frac{3\left(2+x^3+x^5\right)^{2/3}}{x^2}-2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{2+x^3+x^5}}\right)+2\log\left(-x+\sqrt[3]{2+x^3+x^5}\right)-\log\left(x^2+x\sqrt[3]{2+x^3+x^5}+\left(2+x^3+x^5\right)^{2/3}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-3+x^5\right)\left(2+x^3+x^5\right)^{2/3}}{x^3\left(2+x^5\right)}\,dx$$

24.257 Problem number 1654

$$\int \frac{\sqrt[3]{-1+x^3-x^4}\left(-3+x^4\right)}{x^2\left(1+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(-x^4+x^3-1\right)^{\frac{1}{3}}}{x}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(-x^4+x^3-1\right)^{\frac{1}{3}}}\right)+\ln\left(-x+\left(-x^4+x^3-1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(x^2+x\left(-x^4+x^3-1\right)^{\frac{1}{3}}+\left(-x^4+x^3-1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((-1 + x^3 - x^4)^(1/3)*(-3 + x^4))/(x^2*(1 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{3\sqrt[3]{-1+x^3-x^4}}{x}+\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{-1+x^3-x^4}}\right)$$
$$+\log\left(-x+\sqrt[3]{-1+x^3-x^4}\right)-\frac{1}{2}\log\left(x^2+x\sqrt[3]{-1+x^3-x^4}+\left(-1+x^3-x^4\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{\sqrt[3]{-1+x^3-x^4}\left(-3+x^4\right)}{x^2\left(1+x^4\right)}\,dx$$

24.258 Problem number 1659

$$\int\frac{-2b+ax^3}{\left(b+x^2+ax^3\right)\sqrt[4]{bx^2+ax^5}}\,dx$$

Optimal antiderivative

$$-\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x\left(a\,x^5+b\,x^2\right)^{\frac{1}{4}}}{-x^2+\sqrt{a\,x^5+b\,x^2}}\right)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\frac{x^2\sqrt{2}}{2}+\frac{\sqrt{a\,x^5+b\,x^2}\,\sqrt{2}}{2}}{x\left(a\,x^5+b\,x^2\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[(-2*b + a*x^3)/((b + x^2 + a*x^3)*(b*x^2 + a*x^5)^(1/4)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{2}\,\sqrt{x}\,\sqrt[4]{b+ax^3}\left(\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{x}\,\sqrt[4]{b+ax^3}}{-x+\sqrt{b+ax^3}}\right)+\tanh^{-1}\left(\frac{x+\sqrt{b+ax^3}}{\sqrt{2}\,\sqrt{x}\,\sqrt[4]{b+ax^3}}\right)\right)}{\sqrt[4]{x^2\left(b+ax^3\right)}}$$

Mathematica 12.3 output

$$\int\frac{-2b+ax^3}{\left(b+x^2+ax^3\right)\sqrt[4]{bx^2+ax^5}}\,dx$$

24.259 Problem number 1676

$$\int \frac{\sqrt[3]{-1+2x^3+x^8}\,\left(3+5x^8\right)}{x^2\left(-1+x^3+x^8\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^8+2x^3-1\right)^{\frac{1}{3}}}{x}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^8+2x^3-1\right)^{\frac{1}{3}}}\right)$$
$$+\ln\left(-x+\left(x^8+2x^3-1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(x^2+x\left(x^8+2x^3-1\right)^{\frac{1}{3}}+\left(x^8+2x^3-1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((-1 + 2*x^3 + x^8)^(1/3)*(3 + 5*x^8))/(x^2*(-1 + x^3 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{3\sqrt[3]{-1+2x^3+x^8}}{x}+\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{-1+2x^3+x^8}}\right)$$
$$+\log\left(-x+\sqrt[3]{-1+2x^3+x^8}\right)-\frac{1}{2}\log\left(x^2+x\sqrt[3]{-1+2x^3+x^8}+\left(-1+2x^3+x^8\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt[3]{-1+2x^3+x^8}\,\left(3+5x^8\right)}{x^2\left(-1+x^3+x^8\right)}\,dx$$

24.260 Problem number 1677

$$\int \frac{\left(-1+x^4\right)\sqrt{1+\sqrt{1+x^2}}}{1+x^4}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^4)*Sqrt[1 + Sqrt[1 + x^2]])/(1 + x^4),x]
```

Mathematica 13.1 output

$$\frac{2x\left(2+\sqrt{1+x^2}\right)}{3\sqrt{1+\sqrt{1+x^2}}}-\frac{1}{2}\text{RootSum}\left[1+4\#1^4+4\#1^6+\#1^8\&,\frac{\log\left(\frac{x}{\sqrt{1+\sqrt{1+x^2}}}-\#1\right)}{2\#1^3+\#1^5}\&\right]$$

Mathematica 12.3 output

$$\int\frac{\left(-1+x^4\right)\sqrt{1+\sqrt{1+x^2}}}{1+x^4}\,dx$$

24.261 Problem number 1678

$$\int\frac{\left(-1+x^4\right)\sqrt{1+\sqrt{1+x^2}}}{1+x^4}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-1 + x^4)*Sqrt[1 + Sqrt[1 + x^2]])/(1 + x^4),x]
```

Mathematica 13.1 output

$$\frac{2x\left(2+\sqrt{1+x^2}\right)}{3\sqrt{1+\sqrt{1+x^2}}}-\frac{1}{2}\text{RootSum}\left[1+4\#1^4+4\#1^6+\#1^8\&,\frac{\log\left(\frac{x}{\sqrt{1+\sqrt{1+x^2}}}-\#1\right)}{2\#1^3+\#1^5}\&\right]$$

Mathematica 12.3 output

$$\int\frac{\left(-1+x^4\right)\sqrt{1+\sqrt{1+x^2}}}{1+x^4}\,dx$$

24.262 Problem number 1689

$$\int \frac{(-6a+b+5x)\left(-b^5+5b^4x-10b^3x^2+10b^2x^3-5bx^4+x^5\right)}{((-a+x)(-b+x)^2)^{3/4}\left(a+b^6d-(1+6b^5d)\,x+15b^4dx^2-20b^3dx^3+15b^2dx^4-6bdx^5+dx^6\right)}\,dx$$

Optimal antiderivative

$$-\frac{2\arctan\left(\frac{d^{\frac{1}{4}}\left(-a\,b^2+(2ab+b^2)x+(-a-2b)x^2+x^3\right)^{\frac{3}{4}}}{a-x}\right)}{d^{\frac{3}{4}}}+\frac{2\operatorname{arctanh}\left(\frac{d^{\frac{1}{4}}\left(-a\,b^2+(2ab+b^2)x+(-a-2b)x^2+x^3\right)^{\frac{3}{4}}}{a-x}\right)}{d^{\frac{3}{4}}}$$

command

```
Integrate[((-6*a + b + 5*x)*(-b^5 + 5*b^4*x - 10*b^3*x^2 + 10*b^2*x^3 - 5*b*x^4 + x^5))/(((-
a + x)*(-b + x)^2)^(3/4)*(a + b^6*d - (1 + 6*b^5*d)*x + 15*b^4*d*x^2 - 20*b^3*d*x^3 + 15*b^2*
```

Mathematica 13.1 output

$$\frac{\sqrt{2}\,(a-x)^{3/4}(b-x)^{3/2}\left(\text{ArcTan}\left(\frac{\sqrt{a-x}+\sqrt{d}\,(-b+x)^3}{\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{a-x}\,(b-x)^{3/2}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{a-x}\,(b-x)^{3/2}}{\sqrt{a-x}+\sqrt{d}\,(b-x)^3}\right)\right)}{d^{3/4}\left((b-x)^2(-a+x)\right)^{3/4}}$$

Mathematica 12.3 output

$$\int \frac{(-6a+b+5x)\left(-b^5+5b^4x-10b^3x^2+10b^2x^3-5bx^4+x^5\right)}{((-a+x)(-b+x)^2)^{3/4}\left(a+b^6d-(1+6b^5d)\,x+15b^4dx^2-20b^3dx^3+15b^2dx^4-6bdx^5+dx^6\right)}\,dx$$

24.263 Problem number 1690

$$\int \frac{x^4\sqrt[4]{x^2+x^4}}{-1+x^8}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x^4*(x^2 + x^4)^(1/4))/(-1 + x^8),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{x^2+x^4}\left(2\sqrt[4]{2}\left(\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt{x}}{\sqrt[4]{1+x^2}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt{x}}{\sqrt[4]{1+x^2}}\right)\right)-\text{RootSum}\left[2-2\#1^4+\#1^8\&,\frac{-\log\left(\sqrt{x}\right)\#1}{}\right.\right.}{8\sqrt{x}\,\sqrt[4]{1+x^2}}$$

Mathematica 12.3 output

$$\int \frac{x^4\sqrt[4]{x^2+x^4}}{-1+x^8}\,dx$$

24.264 Problem number 1691

$$\int \frac{x^4 \sqrt[4]{x^2+x^4}}{-1+x^8}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x^4*(x^2 + x^4)^(1/4))/(-1 + x^8),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{x^2+x^4}\left(2\sqrt[4]{2}\left(\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{1+x^2}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{1+x^2}}\right)\right)-\text{RootSum}\left[2-2\#1^4+\#1^8\&,\frac{-\log\left(\sqrt{x}\right)\#1}{}\right.\right.}{8\sqrt{x}\sqrt[4]{1+x^2}}$$

Mathematica 12.3 output

$$\int \frac{x^4 \sqrt[4]{x^2+x^4}}{-1+x^8}\,dx$$

24.265 Problem number 1692

$$\int \frac{x^4}{\sqrt[4]{x^2+x^4}\,(1+x^4+x^8)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[x^4/((x^2 + x^4)^(1/4)*(1 + x^4 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{x}\sqrt[4]{1+x^2}\left(-\text{RootSum}\left[3-3\#1^4+\#1^8\&,\frac{-\log\left(\sqrt{x}\right)+\log\left(\sqrt[4]{1+x^2}-\sqrt{x}\#1\right)}{-3\#1+2\#1^5}\&\right]+\text{RootSum}\left[1-\#1^4+\#\right.\right.}{4\sqrt[4]{x^2+x^4}}$$

Mathematica 12.3 output

$$\int \frac{x^4}{\sqrt[4]{x^2+x^4}\,(1+x^4+x^8)}\,dx$$

24.266 Problem number 1693

$$\int \frac{x^4}{\sqrt[4]{x^2+x^4}\,(1+x^4+x^8)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[x^4/((x^2 + x^4)^(1/4)*(1 + x^4 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{x}\,\sqrt[4]{1+x^2}\left(-\text{RootSum}\left[3-3\#1^4+\#1^8\&,\frac{-\log\left(\sqrt{x}\right)+\log\left(\sqrt[4]{1+x^2}-\sqrt{x}\,\#1\right)}{-3\#1+2\#1^5}\&\right]+\text{RootSum}\left[1-\#1^4+\#\right.\right.}{4\sqrt[4]{x^2+x^4}}$$

Mathematica 12.3 output

$$\int \frac{x^4}{\sqrt[4]{x^2+x^4}\,(1+x^4+x^8)}\,dx$$

24.267 Problem number 1694

$$\int \frac{x^6\left(4+x^3\right)}{\left(1+x^3\right)^{3/4}\left(-1-2x^3-x^6+x^8\right)}\,dx$$

Optimal antiderivative

$$\arctan\left(\frac{x}{(x^3+1)^{\frac{1}{4}}}\right)-\frac{\sqrt{2}\,\arctan\left(\frac{-\frac{x^2\sqrt{2}}{2}+\frac{\sqrt{x^3+1}\,\sqrt{2}}{2}}{x(x^3+1)^{\frac{1}{4}}}\right)}{2}$$
$$-\operatorname{arctanh}\left(\frac{x}{(x^3+1)^{\frac{1}{4}}}\right)-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,x(x^3+1)^{\frac{1}{4}}}{x^2+\sqrt{x^3+1}}\right)}{2}$$

command

```
Integrate[(x^6*(4 + x^3))/((1 + x^3)^(3/4)*(-1 - 2*x^3 - x^6 + x^8)),x]
```

Mathematica 13.1 output

$$\operatorname{ArcTan}\left(\frac{x}{\sqrt[4]{1+x^3}}\right)-\frac{\operatorname{ArcTan}\left(\frac{-x^2+\sqrt{1+x^3}}{\sqrt{2}\,x\sqrt[4]{1+x^3}}\right)}{\sqrt{2}}$$
$$-\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^3}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{1+x^3}}{x^2+\sqrt{1+x^3}}\right)}{\sqrt{2}}$$

Mathematica 12.3 output

$$\int \frac{x^6\left(4+x^3\right)}{\left(1+x^3\right)^{3/4}\left(-1-2x^3-x^6+x^8\right)}\,dx$$

24.268 Problem number 1699

$$\int \frac{\left(1+x^2\right)\sqrt[3]{-x+2x^3}}{x^2\left(1+x^4\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^2)*(-x + 2*x^3)^(1/3))/(x^2*(1 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{18-36x^2+x^{2/3}\left(-1+2x^2\right)^{2/3}\operatorname{RootSum}\left[5-4\#1^3+\#1^6\&,\ \frac{-10\log(x)+15\log\left(\sqrt[3]{-1+2x^2}-x^{2/3}\#1\right)+2\log(x)\#1^3-3\log\left(\sqrt[3]{-1+2x^2}-x^{2/3}\#1\right)\#1^3}{-2\#1^2+\#1^5}\right]}{12\left(x\left(-1+2x^2\right)\right)^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2\right)\sqrt[3]{-x+2x^3}}{x^2\left(1+x^4\right)}\,dx$$

24.269 Problem number 1700

$$\int \frac{-1+x}{(1+x+x^2)\sqrt[4]{1+x^4}}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{-\frac{\sqrt{2}}{2}-x\sqrt{2}-\frac{x^2\sqrt{2}}{2}+\frac{\sqrt{x^4+1}\,\sqrt{2}}{2}}{(1+x)(x^4+1)^{\frac{1}{4}}}\right)\sqrt{2}}{2}-\frac{\operatorname{arctanh}\left(\frac{\left(\sqrt{2}+x\sqrt{2}\right)(x^4+1)^{\frac{1}{4}}}{1+2x+x^2+\sqrt{x^4+1}}\right)\sqrt{2}}{2}$$

command

```
Integrate[(-1 + x)/((1 + x + x^2)*(1 + x^4)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{\operatorname{ArcTan}\left(\frac{-1-2x-x^2+\sqrt{1+x^4}}{\sqrt{2}\,(1+x)\sqrt[4]{1+x^4}}\right)-\tanh^{-1}\left(\frac{\sqrt{2}\,(1+x)\sqrt[4]{1+x^4}}{1+2x+x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

Mathematica 12.3 output

$$\int \frac{-1+x}{(1+x+x^2)\sqrt[4]{1+x^4}}\,dx$$

24.270 Problem number 1706

$$\int \frac{2+x^2}{x\,(2-2x+x^2)\sqrt[3]{1-x+x^2}}\,dx$$

Optimal antiderivative

$$-\sqrt{3}\,\arctan\left(\frac{\frac{2\sqrt{3}}{3}-\frac{2x\sqrt{3}}{3}+\frac{(x^2-x+1)^{\frac{1}{3}}\sqrt{3}}{3}}{(x^2-x+1)^{\frac{1}{3}}}\right)+\ln\left(-1+x+\left(x^2-x+1\right)^{\frac{1}{3}}\right)$$
$$-\frac{\ln\left(1-2x+x^2+(1-x)\left(x^2-x+1\right)^{\frac{1}{3}}+\left(x^2-x+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(2 + x^2)/(x*(2 - 2*x + x^2)*(1 - x + x^2)^(1/3)),x]
```

Mathematica 13.1 output

$$-\sqrt{3}\,\text{ArcTan}\left(\frac{2-2x+\sqrt[3]{1-x+x^2}}{\sqrt{3}\,\sqrt[3]{1-x+x^2}}\right)+\log\left(-1+x+\sqrt[3]{1-x+x^2}\right)$$
$$-\frac{1}{2}\log\left(1-2x+x^2-(-1+x)\sqrt[3]{1-x+x^2}+\left(1-x+x^2\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{2+x^2}{x\left(2-2x+x^2\right)\sqrt[3]{1-x+x^2}}\,dx$$

24.271 Problem number 1710

$$\int\frac{\left(3+x^4\right)\left(-1-x^3+x^4\right)^{2/3}}{x^3\left(-1+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^4-x^3-1\right)^{\frac{2}{3}}}{2x^2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^4-x^3-1\right)^{\frac{1}{3}}}\right)$$
$$+\ln\left(x+\left(x^4-x^3-1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(x^2-x\left(x^4-x^3-1\right)^{\frac{1}{3}}+\left(x^4-x^3-1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((3 + x^4)*(-1 - x^3 + x^4)^(2/3))/(x^3*(-1 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{3\left(-1-x^3+x^4\right)^{2/3}}{2x^2}+\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{-x+2\sqrt[3]{-1-x^3+x^4}}\right)$$
$$+\log\left(x+\sqrt[3]{-1-x^3+x^4}\right)-\frac{1}{2}\log\left(x^2-x\sqrt[3]{-1-x^3+x^4}+\left(-1-x^3+x^4\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(3+x^4\right)\left(-1-x^3+x^4\right)^{2/3}}{x^3\left(-1+x^4\right)}\,dx$$

24.272 Problem number 1711

$$\int \frac{\left(1+2x^2\right)\sqrt[3]{x+2x^3}}{x^4\left(1+2x^4\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + 2*x^2)*(x + 2*x^3)^(1/3))/(x^4*(1 + 2*x^4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x+2x^3}\left(-9\sqrt[3]{1+2x^2}\left(1+10x^2\right)+4x^{8/3}\text{RootSum}\left[6-4\#1^3+\#1^6\&,\frac{-12\log(x)+18\log\left(\sqrt[3]{1+2x^2}-x^{2/3}\#1\right)+}{-2\#}\right.\right.}{24x^3\sqrt[3]{1+2x^2}}$$

Mathematica 12.3 output

$$\int \frac{\left(1+2x^2\right)\sqrt[3]{x+2x^3}}{x^4\left(1+2x^4\right)}\,dx$$

24.273 Problem number 1712

$$\int \frac{1}{\left(x-\sqrt{b+ax}\right)\sqrt{c+\sqrt{b+ax}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x - Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]])^(-1),x]
```

Mathematica 13.1 output

$$4\text{RootSum}\left[b-c^2-ac\#1+2c\#1^2+a\#1^3\right.$$
$$\left.-\#1^4\&,\frac{-c\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1+\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1^3}{ac-4c\#1-3a\#1^2+4\#1^3}\&\right]$$

Mathematica 12.3 output

$$\int \frac{1}{\left(x-\sqrt{b+ax}\right)\sqrt{c+\sqrt{b+ax}}}\,dx$$

24.274 Problem number 1713

$$\int \frac{1}{\left(x-\sqrt{b+ax}\right)\sqrt{c+\sqrt{b+ax}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x - Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]])^(-1),x]
```

Mathematica 13.1 output

$$4\text{RootSum}\left[b-c^2-ac\#1+2c\#1^2+a\#1^3 \right.$$
$$\left. -\#1^4\&,\ \frac{-c\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1+\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1^3}{ac-4c\#1-3a\#1^2+4\#1^3}\&\right]$$

Mathematica 12.3 output

$$\int \frac{1}{\left(x-\sqrt{b+ax}\right)\sqrt{c+\sqrt{b+ax}}}\,dx$$

24.275 Problem number 1719

$$\int \frac{2b+ax^3}{\left(-b+x^2+ax^3\right)\sqrt[4]{-bx^2+ax^5}}\,dx$$

Optimal antiderivative

$$-\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ x\left(a\,x^5-b\,x^2\right)^{\frac{1}{4}}}{-x^2+\sqrt{a\,x^5-b\,x^2}}\right)-\sqrt{2}\ \operatorname{arctanh}\left(\frac{\frac{x^2\sqrt{2}}{2}+\frac{\sqrt{a\,x^5-b\,x^2}\ \sqrt{2}}{2}}{x\left(a\,x^5-b\,x^2\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[(2*b + a*x^3)/((-b + x^2 + a*x^3)*(-(b*x^2) + a*x^5)^(1/4)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{2}\,\sqrt{x}\,\sqrt[4]{-b+ax^3}\left(\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{x}\,\sqrt[4]{-b+ax^3}}{-x+\sqrt{-b+ax^3}}\right)+\tanh^{-1}\left(\frac{x+\sqrt{-b+ax^3}}{\sqrt{2}\,\sqrt{x}\,\sqrt[4]{-b+ax^3}}\right)\right)}{\sqrt[4]{-bx^2+ax^5}}$$

Mathematica 12.3 output

$$\int \frac{2b+ax^3}{(-b+x^2+ax^3)\sqrt[4]{-bx^2+ax^5}}\,dx$$

24.276 Problem number 1727

$$\int \frac{\left(1+x^5\right)\sqrt[3]{1+x^3+x^5}\left(-3+2x^5\right)}{x^2\left(2-2x^3+4x^5-x^6-2x^8+2x^{10}\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^5)*(1 + x^3 + x^5)^(1/3)*(-3 + 2*x^5))/(x^2*(2 - 2*x^3 + 4*x^5 - x^6 - 2*x^
```

Mathematica 13.1 output

$$\frac{3\sqrt[3]{1+x^3+x^5}}{2x}-\frac{1}{4}\text{RootSum}\left[3-6\#1^3+2\#1^6\&,\frac{-3\log(x)+3\log\left(\sqrt[3]{1+x^3+x^5}-x\#1\right)+4\log(x)\#1^3-4\log\left(\sqrt[3]{1+x^3+x^5}-x\#1\right)\#1^3}{-3\#1^2+2\#1^5}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^5\right)\sqrt[3]{1+x^3+x^5}\left(-3+2x^5\right)}{x^2\left(2-2x^3+4x^5-x^6-2x^8+2x^{10}\right)}\,dx$$

24.277 Problem number 1728

$$\int \frac{\left(1+x^5\right)\sqrt[3]{1+x^3+x^5}\left(-3+2x^5\right)}{x^2\left(2-2x^3+4x^5-x^6-2x^8+2x^{10}\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^5)*(1 + x^3 + x^5)^(1/3)*(-3 + 2*x^5))/(x^2*(2 - 2*x^3 + 4*x^5 - x^6 - 2*x^
```

Mathematica 13.1 output

$$\frac{3\sqrt[3]{1+x^3+x^5}}{2x}-\frac{1}{4}\text{RootSum}\left[3-6\#1^3+2\#1^6\&,\frac{-3\log(x)+3\log\left(\sqrt[3]{1+x^3+x^5}-x\#1\right)+4\log(x)\#1^3-4\log\left(\sqrt[3]{1+x^3+x^5}-x\#1\right)\#1^3}{-3\#1^2+2\#1^5}\&\right]$$

Mathematica 12.3 output

$$\int\frac{\left(1+x^5\right)\sqrt[3]{1+x^3+x^5}\left(-3+2x^5\right)}{x^2\left(2-2x^3+4x^5-x^6-2x^8+2x^{10}\right)}\,dx$$

24.278 Problem number 1737

$$\int\frac{-3-x^4+3x^6}{\left(1-x^4+x^6\right)\sqrt[3]{1-x^3-x^4+x^6}}\,dx$$

Optimal antiderivative

$$-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^6-x^4-x^3+1\right)^{\frac{1}{3}}}\right)-\ln\left(x+\left(x^6-x^4-x^3+1\right)^{\frac{1}{3}}\right)+\frac{\ln\left(x^2-x\left(x^6-x^4-x^3+1\right)^{\frac{1}{3}}+\left(x^6-x^4-x^3+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(-3 - x^4 + 3*x^6)/((1 - x^4 + x^6)*(1 - x^3 - x^4 + x^6)^(1/3)),x]
```

Mathematica 13.1 output

$$\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x-2\sqrt[3]{1-x^3-x^4+x^6}}\right)-\log\left(x+\sqrt[3]{1-x^3-x^4+x^6}\right)+\frac{1}{2}\log\left(x^2-x\sqrt[3]{1-x^3-x^4+x^6}+\left(1-x^3-x^4+x^6\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{-3-x^4+3x^6}{\left(1-x^4+x^6\right)\sqrt[3]{1-x^3-x^4+x^6}}\,dx$$

24.279 Problem number 1741

$$\int \frac{-b-ax^4+2x^8}{\sqrt[4]{b+ax^4}\,(-b-ax^4+x^8)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-b - a*x^4 + 2*x^8)/((b + a*x^4)^(1/4)*(-b - a*x^4 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{\sqrt[4]{a}}$$
$$+\frac{1}{4}\text{RootSum}\left[b+a\#1^4-\#1^8\&,\frac{-\log(x)\#1^3+\log\left(\sqrt[4]{b+ax^4}-x\#1\right)\#1^3}{-a+2\#1^4}\&\right]$$

Mathematica 12.3 output

$$\int \frac{-b-ax^4+2x^8}{\sqrt[4]{b+ax^4}\,(-b-ax^4+x^8)}\,dx$$

24.280 Problem number 1742

$$\int \frac{-b-ax^4+2x^8}{\sqrt[4]{b+ax^4}\,(-b-ax^4+x^8)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-b - a*x^4 + 2*x^8)/((b + a*x^4)^(1/4)*(-b - a*x^4 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{\sqrt[4]{a}}$$
$$+\frac{1}{4}\text{RootSum}\left[b+a\#1^4-\#1^8\&,\frac{-\log(x)\#1^3+\log\left(\sqrt[4]{b+ax^4}-x\#1\right)\#1^3}{-a+2\#1^4}\&\right]$$

Mathematica 12.3 output

$$\int \frac{-b-ax^4+2x^8}{\sqrt[4]{b+ax^4}\,(-b-ax^4+x^8)}\,dx$$

24.281 Problem number 1750

$$\int \frac{\left(-3+x^4\right)\left(1-x^3+x^4\right)\left(1+x^3+x^4\right)^{2/3}}{x^6\left(1+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^4+x^3+1\right)^{\frac{2}{3}}\left(2x^4-3x^3+2\right)}{10x^5}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^4+x^3+1\right)^{\frac{1}{3}}}\right)$$
$$-\ln\left(-x+\left(x^4+x^3+1\right)^{\frac{1}{3}}\right)+\frac{\ln\left(x^2+x\left(x^4+x^3+1\right)^{\frac{1}{3}}+\left(x^4+x^3+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((-3 + x^4)*(1 - x^3 + x^4)*(1 + x^3 + x^4)^(2/3))/(x^6*(1 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{3\left(1+x^3+x^4\right)^{2/3}\left(2-3x^3+2x^4\right)}{10x^5}+\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{1+x^3+x^4}}\right)$$
$$-\log\left(-x+\sqrt[3]{1+x^3+x^4}\right)+\frac{1}{2}\log\left(x^2+x\sqrt[3]{1+x^3+x^4}+\left(1+x^3+x^4\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-3+x^4\right)\left(1-x^3+x^4\right)\left(1+x^3+x^4\right)^{2/3}}{x^6\left(1+x^4\right)}\,dx$$

24.282 Problem number 1751

$$\int \frac{-b+2ax^4}{\left(b+ax^4\right)\sqrt[4]{bx^2+ax^4}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-b + 2*a*x^4)/((b + a*x^4)*(b*x^2 + a*x^4)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{x}\ \sqrt[4]{b+ax^2}\left(8\left(\text{ArcTan}\left(\frac{\sqrt[4]{a}\ \sqrt{x}}{\sqrt[4]{b+ax^2}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\ \sqrt{x}}{\sqrt[4]{b+ax^2}}\right)\right)+3\sqrt[4]{a}\ \text{RootSum}\left[a^2+ab-2a\#1^4+\#1^8\&\right.\right.}{4\sqrt[4]{a}\ \sqrt[4]{x^2\left(b+ax^2\right)}}$$

Mathematica 12.3 output

$$\int \frac{-b+2ax^4}{\left(b+ax^4\right)\sqrt[4]{bx^2+ax^4}}\,dx$$

24.283 Problem number 1752

$$\int \frac{-b+2ax^4}{\left(b+ax^4\right)\sqrt[4]{bx^2+ax^4}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-b + 2*a*x^4)/((b + a*x^4)*(b*x^2 + a*x^4)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{x}\ \sqrt[4]{b+ax^2}\left(8\left(\text{ArcTan}\left(\frac{\sqrt[4]{a}\ \sqrt{x}}{\sqrt[4]{b+ax^2}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\ \sqrt{x}}{\sqrt[4]{b+ax^2}}\right)\right)+3\sqrt[4]{a}\ \text{RootSum}\left[a^2+ab-2a\#1^4+\#1^8\&\right.\right.}{4\sqrt[4]{a}\ \sqrt[4]{x^2\left(b+ax^2\right)}}$$

Mathematica 12.3 output

$$\int \frac{-b+2ax^4}{\left(b+ax^4\right)\sqrt[4]{bx^2+ax^4}}\,dx$$

24.284 Problem number 1754

$$\int \frac{\left(-1+x^2\right)\sqrt[4]{x^2+x^6}}{x^2\left(1+x^2\right)}\,dx$$

Optimal antiderivative

$$\frac{2\left(x^6+x^2\right)^{\frac{1}{4}}}{x}-\frac{\arctan\left(\frac{2^{\frac{3}{4}}x\left(x^6+x^2\right)^{\frac{1}{4}}}{x^2\sqrt{2}-\sqrt{x^6+x^2}}\right)2^{\frac{3}{4}}}{2}-\frac{\text{arctanh}\left(\frac{\frac{x^2 2^{\frac{3}{4}}}{2}+\frac{\sqrt{x^6+x^2}\ 2^{\frac{1}{4}}}{2}}{x\left(x^6+x^2\right)^{\frac{1}{4}}}\right)2^{\frac{3}{4}}}{2}$$

command

```
Integrate[((-1 + x^2)*(x^2 + x^6)^(1/4))/(x^2*(1 + x^2)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt[4]{x^2+x^6}\left(-4\sqrt[4]{1+x^4}+2^{3/4}\sqrt{x}\,\text{ArcTan}\left(\frac{2^{3/4}\sqrt{x}\,\sqrt[4]{1+x^4}}{\sqrt{2}\,x-\sqrt{1+x^4}}\right)+2^{3/4}\sqrt{x}\,\tanh^{-1}\left(\frac{2\sqrt[4]{2}\,\sqrt{x}\,\sqrt[4]{1+x^4}}{2x+\sqrt{2}\,\sqrt{1+x^4}}\right)\right)}{2x\sqrt[4]{1+x^4}}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^2\right)\sqrt[4]{x^2+x^6}}{x^2\left(1+x^2\right)}\,dx$$

24.285 Problem number 1755

$$\int \frac{\left(-1+x^2\right)\sqrt[4]{x^2+x^6}}{x^2\left(1+x^2\right)}\,dx$$

Optimal antiderivative

$$\frac{2\left(x^6+x^2\right)^{\frac{1}{4}}}{x}-\frac{\arctan\left(\frac{2^{\frac{3}{4}}x\left(x^6+x^2\right)^{\frac{1}{4}}}{x^2\sqrt{2}-\sqrt{x^6+x^2}}\right)2^{\frac{3}{4}}}{2}-\frac{\text{arctanh}\left(\frac{\frac{x^2 2^{\frac{3}{4}}}{2}+\frac{\sqrt{x^6+x^2}\,2^{\frac{1}{4}}}{2}}{x\left(x^6+x^2\right)^{\frac{1}{4}}}\right)2^{\frac{3}{4}}}{2}$$

command

```
Integrate[((-1 + x^2)*(x^2 + x^6)^(1/4))/(x^2*(1 + x^2)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt[4]{x^2+x^6}\left(-4\sqrt[4]{1+x^4}+2^{3/4}\sqrt{x}\,\text{ArcTan}\left(\frac{2^{3/4}\sqrt{x}\,\sqrt[4]{1+x^4}}{\sqrt{2}\,x-\sqrt{1+x^4}}\right)+2^{3/4}\sqrt{x}\,\tanh^{-1}\left(\frac{2\sqrt[4]{2}\,\sqrt{x}\,\sqrt[4]{1+x^4}}{2x+\sqrt{2}\,\sqrt{1+x^4}}\right)\right)}{2x\sqrt[4]{1+x^4}}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^2\right)\sqrt[4]{x^2+x^6}}{x^2\left(1+x^2\right)}\,dx$$

24.286 Problem number 1759

$$\int \frac{\sqrt{b+a^2x^4}}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx$$

Optimal antiderivative

$$\frac{bx}{8\left(a\,x^2+\sqrt{a^2x^4+b}\right)^{\frac{3}{2}}}+\frac{x\sqrt{a\,x^2+\sqrt{a^2x^4+b}}}{4}+\frac{5\sqrt{b}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{a}\ x\sqrt{a\,x^2+\sqrt{a^2x^4+b}}}{\sqrt{b}}\right)\sqrt{2}}{16\sqrt{a}}$$

command

```
Integrate[Sqrt[b + a^2*x^4]/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
```

Mathematica 13.1 output

$$\frac{1}{16}\left(\frac{2x\left(b+2\left(ax^2+\sqrt{b+a^2x^4}\right)^2\right)}{\left(ax^2+\sqrt{b+a^2x^4}\right)^{3/2}}+\frac{5\sqrt{2}\ \sqrt{b}\ \text{ArcTan}\left(\frac{\sqrt{2}\ \sqrt{a}\ x\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b}}\right)}{\sqrt{a}}\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{b+a^2x^4}}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx$$

24.287 Problem number 1761

$$\int \frac{-ab+(-a+2b)x}{\sqrt[4]{x(-a+x)(-b+x)^2}\ (-b^2+(2b-ad)x+(-1+d)x^2)}\,dx$$

Optimal antiderivative

$$\frac{2\arctan\left(\frac{d^{\frac{1}{4}}\left(-a\,b^2x+\left(2ab+b^2\right)x^2+(-a-2b)x^3+x^4\right)^{\frac{1}{4}}}{b-x}\right)}{d^{\frac{3}{4}}}-\frac{2\,\text{arctanh}\left(\frac{d^{\frac{1}{4}}\left(-a\,b^2x+\left(2ab+b^2\right)x^2+(-a-2b)x^3+x^4\right)^{\frac{1}{4}}}{b-x}\right)}{d^{\frac{3}{4}}}$$

command

```
Integrate[(-(a*b) + (-a + 2*b)*x)/((x*(-a + x)*(-b + x)^2)^(1/4)*(-b^2 + (2*b - a*d)*x + (-
1 + d)*x^2)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt[4]{-1+\frac{a}{x}}\sqrt{-2+\frac{2b}{x}}\,x\left(-\text{ArcTan}\left(\frac{1+\sqrt{d}\sqrt{-1+\frac{a}{x}}-\frac{b}{x}}{\sqrt[4]{d}\sqrt[4]{-1+\frac{a}{x}}\sqrt{-2+\frac{2b}{x}}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{-1+\frac{a}{x}}\sqrt{-2+\frac{2b}{x}}}{-1+\sqrt{d}\sqrt{-1+\frac{a}{x}}+\frac{b}{x}}\right)\right)}{d^{3/4}\sqrt[4]{(b-x)^2x(-a+x)}}$$

Mathematica 12.3 output

$$\int\frac{-ab+(-a+2b)x}{\sqrt[4]{x(-a+x)(-b+x)^2}\left(-b^2+(2b-ad)x+(-1+d)x^2\right)}\,dx$$

24.288 Problem number 1763

$$\int\frac{ab^2-2(2a-b)bx+(3a-2b)x^2}{\sqrt[4]{x(-a+x)(-b+x)^2}\left(a^3+\left(-3a^2+b^2d\right)x+(3a-2bd)x^2+(-1+d)x^3\right)}\,dx$$

Optimal antiderivative

$$\frac{2\arctan\left(\frac{d^{\frac{1}{4}}\left(-a\,b^2x+\left(2ab+b^2\right)x^2+(-a-2b)x^3+x^4\right)^{\frac{1}{4}}}{a-x}\right)}{d^{\frac{3}{4}}}-\frac{2\,\text{arctanh}\left(\frac{d^{\frac{1}{4}}\left(-a\,b^2x+\left(2ab+b^2\right)x^2+(-a-2b)x^3+x^4\right)^{\frac{1}{4}}}{a-x}\right)}{d^{\frac{3}{4}}}$$

command

```
Integrate[(a*b^2 - 2*(2*a - b)*b*x + (3*a - 2*b)*x^2)/((x*(-a + x)*(-b + x)^2)^(1/4)*(a^3 + (
3*a^2 + b^2*d)*x + (3*a - 2*b*d)*x^2 + (-1 + d)*x^3)),x]
```

Mathematica 13.1 output

$$-\frac{2\sqrt[4]{-1+\frac{a}{x}}(b-x)\left(\text{ArcTan}\left(\frac{\left(-1+\frac{a}{x}\right)^{3/2}+\sqrt{d}\left(1-\frac{b}{x}\right)}{\sqrt[4]{d}\left(-1+\frac{a}{x}\right)^{3/4}\sqrt{-2+\frac{2b}{x}}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(-1+\frac{a}{x}\right)^{3/4}\sqrt{-2+\frac{2b}{x}}}{\left(-1+\frac{a}{x}\right)^{3/2}+\sqrt{d}\left(-1+\frac{b}{x}\right)}\right)\right)}{d^{3/4}\sqrt{-2+\frac{2b}{x}}\sqrt[4]{(b-x)^2x(-a+x)}}$$

Mathematica 12.3 output

$$\int\frac{ab^2-2(2a-b)bx+(3a-2b)x^2}{\sqrt[4]{x(-a+x)(-b+x)^2}\left(a^3+\left(-3a^2+b^2d\right)x+(3a-2bd)x^2+(-1+d)x^3\right)}\,dx$$

24.289 Problem number 1765

$$\int \frac{b+2ax^4}{(-b+ax^4)\sqrt[4]{bx^2+ax^4}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(b + 2*a*x^4)/((-b + a*x^4)*(b*x^2 + a*x^4)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{x}\,\sqrt[4]{b+ax^2}\left(8\left(\text{ArcTan}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)\right)+3\sqrt[4]{a}\,\text{RootSum}\left[a^2-ab-2a\#1^4+\#1^8\&\right.\right.}{4\sqrt[4]{a}\,\sqrt[4]{x^2\,(b+ax^2)}}$$

Mathematica 12.3 output

$$\int \frac{b+2ax^4}{(-b+ax^4)\sqrt[4]{bx^2+ax^4}}\,dx$$

24.290 Problem number 1766

$$\int \frac{b+2ax^4}{(-b+ax^4)\sqrt[4]{bx^2+ax^4}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(b + 2*a*x^4)/((-b + a*x^4)*(b*x^2 + a*x^4)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{x}\,\sqrt[4]{b+ax^2}\left(8\left(\text{ArcTan}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)\right)+3\sqrt[4]{a}\,\text{RootSum}\left[a^2-ab-2a\#1^4+\#1^8\&\right.\right.}{4\sqrt[4]{a}\,\sqrt[4]{x^2\,(b+ax^2)}}$$

Mathematica 12.3 output

$$\int \frac{b+2ax^4}{(-b+ax^4)\sqrt[4]{bx^2+ax^4}}\,dx$$

24.291 Problem number 1767

$$\int \frac{\left(1+x^6\right)\left(-1-x^3+x^6\right)^{2/3}}{x^3\left(-1+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(x^6-x^3-1\right)^{\frac{2}{3}}}{2x^2}+\frac{\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^6-x^3-1\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}$$
$$+\frac{\ln\left(x+\left(x^6-x^3-1\right)^{\frac{1}{3}}\right)}{3}-\frac{\ln\left(x^2-x\left(x^6-x^3-1\right)^{\frac{1}{3}}+\left(x^6-x^3-1\right)^{\frac{2}{3}}\right)}{6}$$

command

```
Integrate[((1 + x^6)*(-1 - x^3 + x^6)^(2/3))/(x^3*(-1 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{1}{6}\left(\frac{3\left(-1-x^3+x^6\right)^{2/3}}{x^2}-2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x-2\sqrt[3]{-1-x^3+x^6}}\right)\right.$$
$$\left.+2\log\left(x+\sqrt[3]{-1-x^3+x^6}\right)-\log\left(x^2-x\sqrt[3]{-1-x^3+x^6}+\left(-1-x^3+x^6\right)^{2/3}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^6\right)\left(-1-x^3+x^6\right)^{2/3}}{x^3\left(-1+x^6\right)}\,dx$$

24.292 Problem number 1768

$$\int \frac{\left(4+x^6\right)\sqrt[4]{-2-x^4+x^6}}{x^2\left(-2+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{2\left(x^6-x^4-2\right)^{\frac{1}{4}}}{x}+\frac{\arctan\left(\frac{\sqrt{2}\,x\left(x^6-x^4-2\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^6-x^4-2}}\right)\sqrt{2}}{2}-\frac{\text{arctanh}\left(\frac{\sqrt{2}\,x\left(x^6-x^4-2\right)^{\frac{1}{4}}}{x^2+\sqrt{x^6-x^4-2}}\right)\sqrt{2}}{2}$$

command

```
Integrate[((4 + x^6)*(-2 - x^4 + x^6)^(1/4))/(x^2*(-2 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt[4]{-2-x^4+x^6}}{x}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,x\sqrt[4]{-2-x^4+x^6}}{-x^2+\sqrt{-2-x^4+x^6}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{-2-x^4+x^6}}{x^2+\sqrt{-2-x^4+x^6}}\right)}{\sqrt{2}}$$

Mathematica 12.3 output

$$\int\frac{\left(4+x^6\right)\sqrt[4]{-2-x^4+x^6}}{x^2\left(-2+x^6\right)}\,dx$$

24.293 Problem number 1790

$$\int\frac{1+x}{(-1+x)\sqrt[3]{x^2+x^4}}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{2}+\frac{\ln\left(-2x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{2}$$
$$-\frac{\ln\left(2x^2+2^{\frac{2}{3}}x\left(x^4+x^2\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(x^4+x^2\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{4}$$

command

```
Integrate[(1 + x)/((-1 + x)*(x^2 + x^4)^(1/3)),x]
```

Mathematica 13.1 output

$$-\frac{x^{2/3}\sqrt[3]{1+x^2}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}+2^{2/3}\sqrt[3]{1+x^2}}\right)-2\log\left(-2\sqrt[3]{x}+2^{2/3}\sqrt[3]{1+x^2}\right)+\log\left(2x^{2/3}+2^{2/3}\sqrt[3]{x}\,\sqrt[3]{}\right.}{2\sqrt[3]{2}\,\sqrt[3]{x^2+x^4}}$$

Mathematica 12.3 output

$$\int\frac{1+x}{(-1+x)\sqrt[3]{x^2+x^4}}\,dx$$

24.294 Problem number 1793

$$\int \frac{\left(1+x^4\right)\sqrt[4]{-x^2+x^4}}{1+x^4+x^8}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^4)*(-x^2 + x^4)^(1/4))/(1 + x^4 + x^8),x]
```

Mathematica 13.1 output

$$\frac{x^{3/2}\left(-1+x^2\right)^{3/4}\left(\text{RootSum}\left[3-3\#1^4+\#1^8\&,\frac{-\log\left(\sqrt{x}\right)\#1+\log\left(\sqrt[4]{-1+x^2}-\sqrt{x}\,\#1\right)\#1}{-3+2\#1^4}\&\right]+\text{RootSum}\left[1-\right.\right.}{4\left(x^2\left(-1+x^2\right)\right)^{3/4}}$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^4\right)\sqrt[4]{-x^2+x^4}}{1+x^4+x^8}\,dx$$

24.295 Problem number 1794

$$\int \frac{\left(1+x^4\right)\sqrt[4]{-x^2+x^4}}{1+x^4+x^8}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^4)*(-x^2 + x^4)^(1/4))/(1 + x^4 + x^8),x]
```

Mathematica 13.1 output

$$\frac{x^{3/2}\left(-1+x^2\right)^{3/4}\left(\text{RootSum}\left[3-3\#1^4+\#1^8\&,\frac{-\log\left(\sqrt{x}\right)\#1+\log\left(\sqrt[4]{-1+x^2}-\sqrt{x}\,\#1\right)\#1}{-3+2\#1^4}\&\right]+\text{RootSum}\left[1-\right.\right.}{4\left(x^2\left(-1+x^2\right)\right)^{3/4}}$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^4\right)\sqrt[4]{-x^2+x^4}}{1+x^4+x^8}\,dx$$

24.296 Problem number 1797

$$\int \frac{b+2ax}{\sqrt[4]{c+bx+ax^2}\,(5c+4bx+4ax^2)}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(1-\frac{2(a\,x^2+bx+c)^{\frac{1}{4}}}{c^{\frac{1}{4}}}\right)}{2c^{\frac{1}{4}}}+\frac{\arctan\left(1+\frac{2(a\,x^2+bx+c)^{\frac{1}{4}}}{c^{\frac{1}{4}}}\right)}{2c^{\frac{1}{4}}}-\frac{\operatorname{arctanh}\left(\frac{\frac{c^{\frac{1}{4}}}{2}+\frac{\sqrt{a\,x^2+bx+c}}{c^{\frac{1}{4}}}}{(a\,x^2+bx+c)^{\frac{1}{4}}}\right)}{2c^{\frac{1}{4}}}$$

command

```
Integrate[(b + 2*a*x)/((c + b*x + a*x^2)^(1/4)*(5*c + 4*b*x + 4*a*x^2)),x]
```

Mathematica 13.1 output

$$-\frac{\operatorname{ArcTan}\left(1-\frac{2\sqrt[4]{c+x(b+ax)}}{\sqrt[4]{c}}\right)-\operatorname{ArcTan}\left(1+\frac{2\sqrt[4]{c+x(b+ax)}}{\sqrt[4]{c}}\right)+\tanh^{-1}\left(\frac{\sqrt{c}+2\sqrt{c+x(b+ax)}}{2\sqrt[4]{c}\,\sqrt[4]{c+x(b+ax)}}\right)}{2\sqrt[4]{c}}$$

Mathematica 12.3 output

$$\int \frac{b+2ax}{\sqrt[4]{c+bx+ax^2}\,(5c+4bx+4ax^2)}\,dx$$

24.297 Problem number 1803

$$\int \frac{b+2ax^4}{(-b+ax^4)\,\sqrt[4]{-bx^2+ax^4}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(b + 2*a*x^4)/((-b + a*x^4)*(-(b*x^2) + a*x^4)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{x}\,\sqrt[4]{-b+ax^2}\left(8\left(\operatorname{ArcTan}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{-b+ax^2}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{-b+ax^2}}\right)\right)+3\sqrt[4]{a}\,\operatorname{RootSum}\left[a^2-ab-2a\#1^4+\right.\right.}{4\sqrt[4]{a}\,\sqrt[4]{-bx^2+ax^4}}$$

Mathematica 12.3 output

$$\int \frac{b+2ax^4}{(-b+ax^4)\,\sqrt[4]{-bx^2+ax^4}}\,dx$$

24.298 Problem number 1804

$$\int \frac{b+2ax^4}{(-b+ax^4)\sqrt[4]{-bx^2+ax^4}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(b + 2*a*x^4)/((-b + a*x^4)*(-(b*x^2) + a*x^4)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{x}\,\sqrt[4]{-b+ax^2}\left(8\left(\text{ArcTan}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{-b+ax^2}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{-b+ax^2}}\right)\right)+3\sqrt[4]{a}\,\text{RootSum}\left[a^2-ab-2a\#1^4+\right.\right.}{4\sqrt[4]{a}\,\sqrt[4]{-bx^2+ax^4}}$$

Mathematica 12.3 output

$$\int \frac{b+2ax^4}{(-b+ax^4)\sqrt[4]{-bx^2+ax^4}}\,dx$$

24.299 Problem number 1813

$$\int \frac{\left(-4b+ax^3\right)\left(b-ax^3+x^4\right)}{x^4\sqrt[4]{-b+ax^3}\left(-b+ax^3+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{4\left(x^3a-b\right)^{\frac{3}{4}}}{3x^3}-2\sqrt{2}\,\arctan\left(\frac{-\frac{x^2\sqrt{2}}{2}+\frac{\sqrt{x^3a-b}\,\sqrt{2}}{2}}{x\left(x^3a-b\right)^{\frac{1}{4}}}\right)+2\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\,x\left(x^3a-b\right)^{\frac{1}{4}}}{x^2+\sqrt{x^3a-b}}\right)$$

command

```
Integrate[((-4*b + a*x^3)*(b - a*x^3 + x^4))/(x^4*(-b + a*x^3)^(1/4)*(-b + a*x^3 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{4\left(-b+ax^3\right)^{3/4}}{3x^3}-2\sqrt{2}\,\text{ArcTan}\left(\frac{-x^2+\sqrt{-b+ax^3}}{\sqrt{2}\,x\sqrt[4]{-b+ax^3}}\right)+2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{-b+ax^3}}{x^2+\sqrt{-b+ax^3}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-4b+ax^3\right)\left(b-ax^3+x^4\right)}{x^4\sqrt[4]{-b+ax^3}\left(-b+ax^3+x^4\right)}\,dx$$

24.300 Problem number 1814

$$\int \frac{\left(-2+x^2\right)\left(-1+x^2\right)\sqrt[4]{-1+x^2+x^4}}{x^6\left(-1+x^2+2x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{2\left(x^4+x^2-1\right)^{\frac{1}{4}}\left(9x^4-x^2+1\right)}{5x^5}+\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ x\left(x^4+x^2-1\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^4+x^2-1}}\right)$$
$$-\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\ x\left(x^4+x^2-1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^4+x^2-1}}\right)$$

command

```
Integrate[((-2 + x^2)*(-1 + x^2)*(-1 + x^2 + x^4)^(1/4))/(x^6*(-1 + x^2 + 2*x^4)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt[4]{-1+x^2+x^4}\left(1-x^2+9x^4\right)}{5x^5}+\sqrt{2}\ \text{ArcTan}\left(\frac{\sqrt{2}\ x\sqrt[4]{-1+x^2+x^4}}{-x^2+\sqrt{-1+x^2+x^4}}\right)$$
$$-\sqrt{2}\ \tanh^{-1}\left(\frac{\sqrt{2}\ x\sqrt[4]{-1+x^2+x^4}}{x^2+\sqrt{-1+x^2+x^4}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-2+x^2\right)\left(-1+x^2\right)\sqrt[4]{-1+x^2+x^4}}{x^6\left(-1+x^2+2x^4\right)}\,dx$$

24.301 Problem number 1820

$$\int \frac{\left(1+x^6\right)^2\left(-1+2x^6\right)}{\left(1-x^2+x^6\right)^{3/2}\left(1-x^2-x^4+2x^6-x^8+x^{12}\right)}\,dx$$

Optimal antiderivative

$$\frac{x}{\sqrt{x^6-x^2+1}}-\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2+2\sqrt{5}}\ x}{2\sqrt{x^6-x^2+1}}\right)}{\sqrt{145+65\sqrt{5}}}-\frac{\sqrt{290+130\sqrt{5}}\ \operatorname{arctanh}\left(\frac{\sqrt{-2+2\sqrt{5}}\ x}{2\sqrt{x^6-x^2+1}}\right)}{10}$$

command

```
Integrate[((1 + x^6)^2*(-1 + 2*x^6))/((1 - x^2 + x^6)^(3/2)*(1 - x^2 - x^4 + 2*x^6 - x^8 + x^
```

Mathematica 13.1 output

$$\frac{x}{\sqrt{1-x^2+x^6}}-\sqrt{\frac{2}{145+65\sqrt{5}}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2+x^6}}\right)$$
$$-\sqrt{\frac{1}{10}\left(29+13\sqrt{5}\right)}\,\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2+x^6}}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(1+x^6\right)^2\left(-1+2x^6\right)}{\left(1-x^2+x^6\right)^{3/2}\left(1-x^2-x^4+2x^6-x^8+x^{12}\right)}\,dx$$

24.302 Problem number 1826

$$\int\frac{-1+x^6}{\left(1+x^6\right)\sqrt[3]{1+a^3x^3+x^6}}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{3}\,ax}{ax+2\left(a^3x^3+x^6+1\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3a}+\frac{\ln\left(-ax+\left(a^3x^3+x^6+1\right)^{\frac{1}{3}}\right)}{3a}$$
$$-\frac{\ln\left(a^2x^2+ax\left(a^3x^3+x^6+1\right)^{\frac{1}{3}}+\left(a^3x^3+x^6+1\right)^{\frac{2}{3}}\right)}{6a}$$

command

```
Integrate[(-1 + x^6)/((1 + x^6)*(1 + a^3*x^3 + x^6)^(1/3)),x]
```

Mathematica 13.1 output

$$-\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,ax}{ax+2\sqrt[3]{1+a^3x^3+x^6}}\right)-2\log\left(a\left(ax-\sqrt[3]{1+a^3x^3+x^6}\right)\right)+\log\left(a^2x^2+ax\sqrt[3]{1+a^3x^3+x^6}+\right.}{6a}$$

Mathematica 12.3 output

$$\int\frac{-1+x^6}{\left(1+x^6\right)\sqrt[3]{1+a^3x^3+x^6}}\,dx$$

24.303 Problem number 1827

$$\int \frac{\left(2+x^3+4x^6\right)\sqrt[3]{x+2x^3-x^4-x^7}}{\left(-1-2x^2+x^3+x^6\right)\left(-1-x^2+x^3+x^6\right)}\,dx$$

Optimal antiderivative

$$-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(-x^7-x^4+2x^3+x\right)^{\frac{1}{3}}}\right)-\ln\left(-x+\left(-x^7-x^4+2x^3+x\right)^{\frac{1}{3}}\right)+\frac{\ln\left(x^2+x\left(-x^7-x^4+2x^3+x\right)^{\frac{1}{3}}+\left(-x^7-x^4+2x^3+x\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((2 + x^3 + 4*x^6)*(x + 2*x^3 - x^4 - x^7)^(1/3))/((-1 - 2*x^2 + x^3 + x^6)*(-
1 - x^2 + x^3 + x^6)),x]
```

Mathematica 13.1 output

$$-\frac{x^{2/3}\left(-1-2x^2+x^3+x^6\right)^{2/3}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}-2\sqrt[3]{-1-2x^2+x^3+x^6}}\right)+2\log\left(x^{2/3}+\sqrt[3]{-1-2x^2+x^3+\right.}\right.}{2\left(-x\left(-1-2x^2+x^3+x^6\right)\right)^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{\left(2+x^3+4x^6\right)\sqrt[3]{x+2x^3-x^4-x^7}}{\left(-1-2x^2+x^3+x^6\right)\left(-1-x^2+x^3+x^6\right)}\,dx$$

24.304 Problem number 1828

$$\int \frac{x^2}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx$$

Optimal antiderivative

$$-\frac{bx}{8a\left(a\,x^2+\sqrt{a^2x^4+b}\right)^{\frac{3}{2}}}+\frac{x\sqrt{a\,x^2+\sqrt{a^2x^4+b}}}{4a}-\frac{\sqrt{b}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a}\,x\sqrt{a\,x^2+\sqrt{a^2x^4+b}}}{\sqrt{b}}\right)\sqrt{2}}{16a^{\frac{3}{2}}}$$

command

```
Integrate[x^2/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
```

Mathematica 13.1 output

$$\frac{x\left(b+4ax^2\left(ax^2+\sqrt{b+a^2x^4}\right)\right)}{8a\left(ax^2+\sqrt{b+a^2x^4}\right)^{3/2}}-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{a}\,x\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b}}\right)}{8\sqrt{2}\,a^{3/2}}$$

Mathematica 12.3 output

$$\int \frac{x^2}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx$$

24.305 Problem number 1835

$$\int \frac{-2abx^2+(a+b)x^3}{(-a+x)(-b+x)\sqrt[4]{x^2(-a+x)(-b+x)}\,(-abd+(a+b)dx+(1-d)x^2)}\,dx$$

Optimal antiderivative

$$-\frac{4\left(ab\,x^2+(-a-b)\,x^3+x^4\right)^{\frac{3}{4}}}{x\,(-a+x)\,(-b+x)}+2d^{\frac{1}{4}}\arctan\left(\frac{x}{d^{\frac{1}{4}}\left(ab\,x^2+(-a-b)\,x^3+x^4\right)^{\frac{1}{4}}}\right)$$
$$+2d^{\frac{1}{4}}\,\text{arctanh}\left(\frac{x}{d^{\frac{1}{4}}\left(ab\,x^2+(-a-b)\,x^3+x^4\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[(-2*a*b*x^2 + (a + b)*x^3)/((-a + x)*(-b + x)*(x^2*(-a + x)*(-b + x))^(1/4)*(-
(a*b*d) + (a + b)*d*x + (1 - d)*x^2)),x]
```

Mathematica 13.1 output

$$-\frac{2x\left(2\sqrt{\frac{x}{-a+x}}+\sqrt[4]{d}\,\sqrt[4]{\frac{b-x}{a-x}}\,\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{\frac{b-x}{a-x}}}{\sqrt{\frac{x}{-a+x}}}\right)-\sqrt[4]{d}\,\sqrt[4]{\frac{b-x}{a-x}}\,\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{\frac{b-x}{a-x}}}{\sqrt{\frac{x}{-a+x}}}\right)\right)}{\sqrt{\frac{x}{-a+x}}\,\sqrt[4]{x^2(-a+x)(-b+x)}}$$

Mathematica 12.3 output

$$\int \frac{-2abx^2+(a+b)x^3}{(-a+x)(-b+x)\sqrt[4]{x^2(-a+x)(-b+x)}\,(-abd+(a+b)dx+(1-d)x^2)}\,dx$$

24.306 Problem number 1841

$$\int \frac{x^2\left(-4b+ax^5\right)}{\left(b+ax^5\right)^{3/4}\left(b+cx^4+ax^5\right)}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,c^{\frac{1}{4}}x\left(a\,x^5+b\right)^{\frac{1}{4}}}{-x^2\sqrt{c}+\sqrt{a\,x^5+b}}\right)}{c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\frac{c^{\frac{1}{4}}x^2\sqrt{2}}{2}+\frac{\sqrt{a\,x^5+b}\,\sqrt{2}}{2c^{\frac{1}{4}}}}{x\left(a\,x^5+b\right)^{\frac{1}{4}}}\right)}{c^{\frac{3}{4}}}$$

command

```
Integrate[(x^2*(-4*b + a*x^5))/((b + a*x^5)^(3/4)*(b + c*x^4 + a*x^5)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{2}\left(\operatorname{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x\,\sqrt[4]{b+ax^5}}{\sqrt{c}\,x^2-\sqrt{b+ax^5}}\right)+\tanh^{-1}\left(\frac{\sqrt{c}\,x^2+\sqrt{b+ax^5}}{\sqrt{2}\,\sqrt[4]{c}\,x\,\sqrt[4]{b+ax^5}}\right)\right)}{c^{3/4}}$$

Mathematica 12.3 output

$$\int \frac{x^2\left(-4b+ax^5\right)}{\left(b+ax^5\right)^{3/4}\left(b+cx^4+ax^5\right)}\,dx$$

24.307 Problem number 1843

$$\int \frac{x^2\left(-2b+ax^6\right)}{\left(b+ax^6\right)^{3/4}\left(b+cx^4+ax^6\right)}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{2}\,c^{\frac{1}{4}}x\left(a\,x^6+b\right)^{\frac{1}{4}}}{-x^2\sqrt{c}+\sqrt{a\,x^6+b}}\right)\sqrt{2}}{2c^{\frac{3}{4}}}+\frac{\operatorname{arctanh}\left(\frac{\frac{c^{\frac{1}{4}}x^2\sqrt{2}}{2}+\frac{\sqrt{a\,x^6+b}\,\sqrt{2}}{2c^{\frac{1}{4}}}}{x\left(a\,x^6+b\right)^{\frac{1}{4}}}\right)\sqrt{2}}{2c^{\frac{3}{4}}}$$

command

```
Integrate[(x^2*(-2*b + a*x^6))/((b + a*x^6)^(3/4)*(b + c*x^4 + a*x^6)),x]
```

Mathematica 13.1 output

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\ \sqrt[4]{c}\ x\ \sqrt[4]{b+ax^6}}{\sqrt{c}\ x^2-\sqrt{b+ax^6}}\right)+\tanh^{-1}\left(\frac{\sqrt{c}\ x^2+\sqrt{b+ax^6}}{\sqrt{2}\ \sqrt[4]{c}\ x\ \sqrt[4]{b+ax^6}}\right)}{\sqrt{2}\ c^{3/4}}$$

Mathematica 12.3 output

$$\int \frac{x^2\left(-2b+ax^6\right)}{\left(b+ax^6\right)^{3/4}\left(b+cx^4+ax^6\right)}\,dx$$

24.308 Problem number 1844

$$\int \frac{\left(4+x^3\right)\left(1+2x^3+x^6+x^8\right)}{x^4\ \sqrt[4]{1+x^3}\ \left(-1-2x^3-x^6+x^8\right)}\,dx$$

Optimal antiderivative

$$\frac{4\left(x^3+1\right)^{\frac{3}{4}}}{3x^3}-2\arctan\left(\frac{x}{\left(x^3+1\right)^{\frac{1}{4}}}\right)+\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ x\left(x^3+1\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^3+1}}\right)$$
$$-2\,\text{arctanh}\left(\frac{\left(x^3+1\right)^{\frac{1}{4}}}{x}\right)+\sqrt{2}\ \text{arctanh}\left(\frac{\sqrt{2}\ x\left(x^3+1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^3+1}}\right)$$

command

```
Integrate[((4 + x^3)*(1 + 2*x^3 + x^6 + x^8))/(x^4*(1 + x^3)^(1/4)*(-1 - 2*x^3 - x^6 + x^8)),
```

Mathematica 13.1 output

$$\frac{4\left(1+x^3\right)^{3/4}}{3x^3}+2\left(\text{ArcTan}\left(\frac{\sqrt[4]{1+x^3}}{x}\right)-\frac{\text{ArcTan}\left(\frac{-x^2+\sqrt{1+x^3}}{\sqrt{2}\ x\ \sqrt[4]{1+x^3}}\right)}{\sqrt{2}}\right.$$
$$\left.-\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^3}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\ x\ \sqrt[4]{1+x^3}}{x^2+\sqrt{1+x^3}}\right)}{\sqrt{2}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(4+x^3\right)\left(1+2x^3+x^6+x^8\right)}{x^4\ \sqrt[4]{1+x^3}\ \left(-1-2x^3-x^6+x^8\right)}\,dx$$

24.309 Problem number 1846

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^2}\,dx$$

Optimal antiderivative

$$\frac{x}{2b\left(a\,x^2+b^2\right)\sqrt{b+\sqrt{a\,x^2+b^2}}}+\frac{3x}{4b^2\sqrt{a\,x^2+b^2}\,\sqrt{b+\sqrt{a\,x^2+b^2}}}+\frac{3\arctan\left(\frac{\sqrt{a}\,x}{\sqrt{b}\,\sqrt{b+\sqrt{a\,x^2+b^2}}}\right)}{4\sqrt{a}\,b^{\frac{5}{2}}}$$

command

```
Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^2,x]
```

Mathematica 13.1 output

$$\frac{x\left(3b^2+3ax^2+2b\sqrt{b^2+ax^2}\right)}{4b^2\left(b^2+ax^2\right)^{3/2}\sqrt{b+\sqrt{b^2+ax^2}}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\,\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{4\sqrt{a}\,b^{5/2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^2}\,dx$$

24.310 Problem number 1849

$$\int \frac{\left(1+2x^6\right)\sqrt[3]{x+x^3-x^7}}{\left(-1+x^6\right)^2}\,dx$$

Optimal antiderivative

$$-\frac{x\left(-x^7+x^3+x\right)^{\frac{1}{3}}}{2x^6-2}-\frac{\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(-x^7+x^3+x\right)^{\frac{1}{3}}}\right)\sqrt{3}}{6}-\frac{\ln\left(-x+\left(-x^7+x^3+x\right)^{\frac{1}{3}}\right)}{6}+\frac{\ln\left(x^2+x\left(-x^7+x^3+x\right)^{\frac{1}{3}}+\left(-x^7+x^3+x\right)^{\frac{2}{3}}\right)}{12}$$

command

```
Integrate[((1 + 2*x^6)*(x + x^3 - x^7)^(1/3))/(-1 + x^6)^2,x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x+x^3-x^7}\left(-6x^{4/3}\sqrt[3]{-1-x^2+x^6}+2\sqrt{3}\left(-1+x^6\right)\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}-2\sqrt[3]{-1-x^2+x^6}}\right)+2\left(-1+x^6\right)\log\left(x\right.\right.}{1}$$

Mathematica 12.3 output

$$\int \frac{\left(1+2x^6\right)\sqrt[3]{x+x^3-x^7}}{\left(-1+x^6\right)^2}\,dx$$

24.311 Problem number 1856

$$\int \frac{6+2x+x^2}{(1+x)\sqrt[3]{2+x+x^2}\,\left(2-x+2x^2\right)}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{3}\,\arctan\left(\frac{-\frac{2\,2^{\frac{1}{3}}x\sqrt{3}}{3}+\frac{\left(x^2+x+2\right)^{\frac{1}{3}}\sqrt{3}}{3}}{\left(x^2+x+2\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{2}+\frac{\ln\left(2^{\frac{1}{3}}x+\left(x^2+x+2\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{2}$$
$$-\frac{\ln\left(2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}x\left(x^2+x+2\right)^{\frac{1}{3}}+\left(x^2+x+2\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{4}$$

command

```
Integrate[(6 + 2*x + x^2)/((1 + x)*(2 + x + x^2)^(1/3)*(2 - x + 2*x^2)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{2+x+x^2}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{2}\,x+\sqrt[3]{2+x+x^2}\right)+\log\left(2^{2/3}x^2-\sqrt[3]{2}\,x\sqrt[3]{2+x+x^2}+\left(2+x\right.\right.}{2\sqrt[3]{2}}$$

Mathematica 12.3 output

$$\int \frac{6+2x+x^2}{(1+x)\sqrt[3]{2+x+x^2}\,\left(2-x+2x^2\right)}\,dx$$

24.312 Problem number 1865

$$\int \frac{\left(1+x^4\right)\left(-1+x^2+x^4\right)^{3/2}}{\left(-1+x^4\right)\left(1+x^2-x^4-x^6+x^8\right)}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{6-2i\sqrt{3}}\,\arctan\left(\frac{\sqrt{-6-2i\sqrt{3}}\,x}{2\sqrt{x^4+x^2-1}}\right)}{2}$$
$$-\frac{\sqrt{6+2i\sqrt{3}}\,\arctan\left(\frac{\sqrt{-6+2i\sqrt{3}}\,x}{2\sqrt{x^4+x^2-1}}\right)}{2}-\operatorname{arctanh}\left(\frac{x}{\sqrt{x^4+x^2-1}}\right)$$

command

```
Integrate[((1 + x^4)*(-1 + x^2 + x^4)^(3/2))/((-1 + x^4)*(1 + x^2 - x^4 - x^6 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{1}{2}\left(-\sqrt{6-2i\sqrt{3}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{1}{2}\left(-3-i\sqrt{3}\right)}\,x}{\sqrt{-1+x^2+x^4}}\right)\right.$$
$$\left.-\sqrt{6+2i\sqrt{3}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{1}{2}i\left(3i+\sqrt{3}\right)}\,x}{\sqrt{-1+x^2+x^4}}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2+x^4}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^4\right)\left(-1+x^2+x^4\right)^{3/2}}{\left(-1+x^4\right)\left(1+x^2-x^4-x^6+x^8\right)}\,dx$$

24.313 Problem number 1866

$$\int \frac{\left(-1+x^6\right)\left(1+x^3+x^6\right)^{2/3}}{1+x^6+x^{12}}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{3}\,x}{x+2^{\frac{2}{3}}\left(x^6+x^3+1\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}\sqrt{3}}{6}+\frac{\ln\left(-2x+2^{\frac{2}{3}}\left(x^6+x^3+1\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{6}$$
$$-\frac{\ln\left(2x^2+2^{\frac{2}{3}}x\left(x^6+x^3+1\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(x^6+x^3+1\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{12}$$

command

```
Integrate[((-1 + x^6)*(1 + x^3 + x^6)^(2/3))/(1 + x^6 + x^12),x]
```

Mathematica 13.1 output

$$-\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2^{2/3}\sqrt[3]{1+x^3+x^6}}\right)-2\log\left(-2x+2^{2/3}\sqrt[3]{1+x^3+x^6}\right)+\log\left(2x^2+2^{2/3}x\sqrt[3]{1+x^3+x^6}+\sqrt{}\right.}{6\sqrt[3]{2}}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^6\right)\left(1+x^3+x^6\right)^{2/3}}{1+x^6+x^{12}}\,dx$$

24.314 Problem number 1867

$$\int \frac{\left(-1+x^3\right)^3\left(1+x^3\right)\sqrt{2+3x^6+2x^{12}}}{x^7\left(1+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(x^6-4x^3+1\right)\sqrt{2x^{12}+3x^6+2}}{6x^6}-\frac{4\arctan\left(\frac{x^3}{\sqrt{2}+\sqrt{2}\,x^6+\sqrt{2x^{12}+3x^6+2}}\right)}{3}$$
$$+\frac{\ln(x)\sqrt{2}}{4}-\frac{\ln\left(\sqrt{2}+\sqrt{2}\,x^6+\sqrt{2x^{12}+3x^6+2}\right)\sqrt{2}}{12}$$

command

```
Integrate[((-1 + x^3)^3*(1 + x^3)*Sqrt[2 + 3*x^6 + 2*x^12])/(x^7*(1 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{1}{12}\left(\frac{2\left(1-4x^3+x^6\right)\sqrt{2+3x^6+2x^{12}}}{x^6}\right.$$
$$\left.-16\text{ArcTan}\left(\frac{x^3}{\sqrt{2}+\sqrt{2}\,x^6+\sqrt{2+3x^6+2x^{12}}}\right)-\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1+\frac{3x^6}{2}+x^{12}}}{1+x^6}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^3\right)^3\left(1+x^3\right)\sqrt{2+3x^6+2x^{12}}}{x^7\left(1+x^6\right)}\,dx$$

24.315 Problem number 1868

$$\int \frac{\sqrt{ax - \sqrt{b + a^2x^2}}}{a^2x^2 + \sqrt{b + a^2x^2}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[a*x - Sqrt[b + a^2*x^2]]/(a^2*x^2 + Sqrt[b + a^2*x^2]),x]
```

Mathematica 13.1 output

$$-\frac{\text{RootSum}\left[b^2 - 2b\#1^2 - 2b\#1^4 - 2\#1^6 + \#1^8\&,\ \frac{b\log\left(\sqrt{ax - \sqrt{b + a^2x^2}} - \#1\right)\#1 + \log\left(\sqrt{ax - \sqrt{b + a^2x^2}} - \right.}{b + 2b\#1^2 + 3\#1^4 - 2\#1^6}\right.}{a}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{ax - \sqrt{b + a^2x^2}}}{a^2x^2 + \sqrt{b + a^2x^2}}\,dx$$

24.316 Problem number 1875

$$\int \frac{\left(-q + 2px^3\right)\left(aq + bx + apx^3\right)\sqrt{q^2 - 2pqx^2 + 2pqx^3 + p^2x^6}}{x^4}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{p^2x^6 + 2pq\,x^3 - 2pq\,x^2 + q^2}\left(2a\,p^2x^6 + 4apq\,x^3 + 3bp\,x^4 - 4apq\,x^2 + 2a\,q^2 + 3bqx\right)}{6x^3}$$
$$+ bpq\ln(x) - bpq\ln\left(q + p\,x^3 + \sqrt{p^2x^6 + 2pq\,x^3 - 2pq\,x^2 + q^2}\right)$$

command

```
Integrate[((-q + 2*p*x^3)*(a*q + b*x + a*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6])/
```

Mathematica 13.1 output

$$\frac{\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}\left(3bx\left(q+px^3\right)+2a\left(q^2+2pq(-1+x)x^2+p^2x^6\right)\right)}{6x^3}$$
$$-bpq\tanh^{-1}\left(\frac{\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}}{q+px^3}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(-q+2px^3\right)\left(aq+bx+apx^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^4}\,dx$$

24.317 Problem number 1880

$$\int\frac{(-2ab+(3a-b)x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{x(-b+x)\sqrt[4]{x^2(-a+x)(-b+x)}\left(a^3-3a^2x+(3a-bd)x^2+(-1+d)x^3\right)}\,dx$$

Optimal antiderivative

$$-\frac{4\left(abx^2+(-a-b)x^3+x^4\right)^{\frac{3}{4}}}{x^2(-b+x)}+2d^{\frac{1}{4}}\arctan\left(\frac{d^{\frac{1}{4}}\left(abx^2+(-a-b)x^3+x^4\right)^{\frac{1}{4}}}{a-x}\right)$$
$$-2d^{\frac{1}{4}}\operatorname{arctanh}\left(\frac{d^{\frac{1}{4}}\left(abx^2+(-a-b)x^3+x^4\right)^{\frac{1}{4}}}{a-x}\right)$$

command

```
Integrate[((-2*a*b + (3*a - b)*x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/(x*(-b + x)*(x^2*(-
a + x)*(-b + x))^(1/4)*(a^3 - 3*a^2*x + (3*a - b*d)*x^2 + (-1 + d)*x^3)),x]
```

Mathematica 13.1 output

$$\frac{4(a-x)}{\sqrt[4]{x^2(-a+x)(-b+x)}}$$
$$+\frac{\sqrt[4]{d}\,x\sqrt[4]{\frac{-b+x}{a-x}}\left(\text{ArcTan}\left(\frac{\sqrt[a-x]{\left(1+\sqrt{d}\sqrt{\frac{-b+x}{a-x}}\right)}}{\sqrt{2}\sqrt[4]{d}\sqrt{\frac{x}{a-x}}\sqrt[(-a+x)]{\sqrt[4]{\frac{-b+x}{a-x}}}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x\sqrt[4]{\frac{-b+x}{a-x}}}{\sqrt{\frac{x}{2a-2x}}\left(a+x\left(-1+\sqrt{d}\sqrt{\frac{-b+x}{a-x}}\right.\right.\right.\right.}\right.}{\sqrt{\frac{x}{2a-2x}}\sqrt[4]{x^2(-a+x)(-b+x)}}$$

Mathematica 12.3 output

$$\int\frac{(-2ab+(3a-b)x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{x(-b+x)\sqrt[4]{x^2(-a+x)(-b+x)}\left(a^3-3a^2x+(3a-bd)x^2+(-1+d)x^3\right)}\,dx$$

24.318 Problem number 1881

$$\int \frac{2b+ax^2}{\sqrt[4]{b+ax^2}\,(bn+anx^2+2x^4)}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\frac{x^2 2^{\frac{3}{4}} 3155^{\frac{3}{4}}}{6310}-\frac{3155^{\frac{1}{4}}\sqrt{a\,x^2+b}\;2^{\frac{1}{4}}}{2}}{x(a\,x^2+b)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}3155^{\frac{1}{4}}}{6310}$$
$$+\frac{\operatorname{arctanh}\left(\frac{\frac{x^2 2^{\frac{3}{4}} 3155^{\frac{3}{4}}}{6310}+\frac{3155^{\frac{1}{4}}\sqrt{a\,x^2+b}\;2^{\frac{1}{4}}}{2}}{x(a\,x^2+b)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}3155^{\frac{1}{4}}}{6310}$$

command

```
Integrate[(2*b + a*x^2)/((b + a*x^2)^(1/4)*(b*n + a*n*x^2 + 2*x^4)),x]
```

Mathematica 13.1 output

$$\frac{\operatorname{ArcTan}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{n}\,\sqrt[4]{b+ax^2}}-\frac{\sqrt[4]{n}\,\sqrt[4]{b+ax^2}}{2^{3/4}x}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{n}\,\sqrt[4]{b+ax^2}}+\frac{\sqrt[4]{n}\,\sqrt[4]{b+ax^2}}{2^{3/4}x}\right)}{2^{3/4}n^{3/4}}$$

Mathematica 12.3 output

$$\int \frac{2b+ax^2}{\sqrt[4]{b+ax^2}\,(bn+anx^2+2x^4)}\,dx$$

24.319 Problem number 1889

$$\int \frac{1+x}{(-1+x)(1+2x)\sqrt[3]{-1+3x^2}}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{3}\,(3x^2-1)^{\frac{1}{3}}}{2\,2^{\frac{1}{3}}x+(3x^2-1)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}\sqrt{3}}{6}+\frac{\ln\left(-2^{\frac{1}{3}}x+(3x^2-1)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{6}$$
$$-\frac{\ln\left(2^{\frac{2}{3}}x^2+2^{\frac{1}{3}}x(3x^2-1)^{\frac{1}{3}}+(3x^2-1)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{12}$$

command

```
Integrate[(1 + x)/((-1 + x)*(1 + 2*x)*(-1 + 3*x^2)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{-1+3x^2}}{2\sqrt[3]{2}\,x+\sqrt[3]{-1+3x^2}}\right)+2\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{-1+3x^2}\right)-\log\left(2^{2/3}x^2+\left(-1+3x^2\right)^{2/3}+x\sqrt[3]{-2+}\right.}{6\sqrt[3]{2}}$$

Mathematica 12.3 output

$$\int \frac{1+x}{(-1+x)(1+2x)\sqrt[3]{-1+3x^2}}\,dx$$

24.320 Problem number 1892

$$\int \frac{\left(-1+x^3\right)\sqrt[3]{1+x^6}}{x^2\left(1+x^3\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(x^6+1\right)^{\frac{1}{3}}}{x}+\frac{2^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,x}{-x+2^{\frac{2}{3}}\left(x^6+1\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}-\frac{2^{\frac{1}{3}}\ln\left(2x+2^{\frac{2}{3}}\left(x^6+1\right)^{\frac{1}{3}}\right)}{3}$$
$$+\frac{\ln\left(-2x^2+2^{\frac{2}{3}}x\left(x^6+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\left(x^6+1\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{6}$$

command

```
Integrate[((-1 + x^3)*(1 + x^6)^(1/3))/(x^2*(1 + x^3)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{1+x^6}}{x}+\frac{\sqrt[3]{2}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{-x+2^{2/3}\sqrt[3]{1+x^6}}\right)}{\sqrt{3}}$$
$$-\frac{1}{3}\sqrt[3]{2}\,\log\left(2x+2^{2/3}\sqrt[3]{1+x^6}\right)+\frac{\log\left(-2x^2+2^{2/3}x\sqrt[3]{1+x^6}-\sqrt[3]{2}\left(1+x^6\right)^{2/3}\right)}{3\,2^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^3\right)\sqrt[3]{1+x^6}}{x^2\left(1+x^3\right)}\,dx$$

24.321 Problem number 1893

$$\int \frac{\left(-2+x^6\right)\sqrt[3]{2+x^6}}{x^2\left(2+2x^3+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(x^6+2\right)^{\frac{1}{3}}}{x}+\frac{2^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,x}{-x+2^{\frac{2}{3}}\left(x^6+2\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}-\frac{2^{\frac{1}{3}}\ln\left(2x+2^{\frac{2}{3}}\left(x^6+2\right)^{\frac{1}{3}}\right)}{3}$$
$$+\frac{\ln\left(-2x^2+2^{\frac{2}{3}}x\left(x^6+2\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\left(x^6+2\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{6}$$

command

```
Integrate[((-2 + x^6)*(2 + x^6)^(1/3))/(x^2*(2 + 2*x^3 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{2+x^6}}{x}+\frac{\sqrt[3]{2}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{-x+2^{2/3}\sqrt[3]{2+x^6}}\right)}{\sqrt{3}}$$
$$-\frac{1}{3}\sqrt[3]{2}\,\log\left(2x+2^{2/3}\sqrt[3]{2+x^6}\right)+\frac{\log\left(-2x^2+2^{2/3}x\sqrt[3]{2+x^6}-\sqrt[3]{2}\left(2+x^6\right)^{2/3}\right)}{3\,2^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{\left(-2+x^6\right)\sqrt[3]{2+x^6}}{x^2\left(2+2x^3+x^6\right)}\,dx$$

24.322 Problem number 1894

$$\int \frac{\left(1+x^3\right)^{2/3}\left(-1-2x^3+2x^6\right)}{x^6\left(-1+x^3+2x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(-17x^3-2\right)\left(x^3+1\right)^{\frac{2}{3}}}{10x^5}+2\,3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{5}{6}}x}{3^{\frac{1}{3}}x+2\left(x^3+1\right)^{\frac{1}{3}}}\right)$$
$$-\frac{2\ln\left(-3x+3^{\frac{2}{3}}\left(x^3+1\right)^{\frac{1}{3}}\right)3^{\frac{2}{3}}}{3}+\frac{\ln\left(3x^2+3^{\frac{2}{3}}x\left(x^3+1\right)^{\frac{1}{3}}+3^{\frac{1}{3}}\left(x^3+1\right)^{\frac{2}{3}}\right)3^{\frac{2}{3}}}{3}$$

command

```
Integrate[((1 + x^3)^(2/3)*(-1 - 2*x^3 + 2*x^6))/(x^6*(-1 + x^3 + 2*x^6)),x]
```

Mathematica 13.1 output

$$\frac{\left(-2-17x^3\right)\left(1+x^3\right)^{2/3}}{10x^5}+2\sqrt[6]{3}\,\text{ArcTan}\left(\frac{3^{5/6}x}{\sqrt[3]{3}\,x+2\sqrt[3]{1+x^3}}\right)$$
$$-\frac{2\log\left(-3x+3^{2/3}\sqrt[3]{1+x^3}\right)}{\sqrt[3]{3}}+\frac{\log\left(3x^2+3^{2/3}x\sqrt[3]{1+x^3}+\sqrt[3]{3}\left(1+x^3\right)^{2/3}\right)}{\sqrt[3]{3}}$$

Mathematica 12.3 output

$$\int\frac{\left(1+x^3\right)^{2/3}\left(-1-2x^3+2x^6\right)}{x^6\left(-1+x^3+2x^6\right)}\,dx$$

24.323 Problem number 1900

$$\int\frac{6+2x+x^2}{(2+x)\left(2+x^2\right)\sqrt[3]{2+x+x^2}}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{3}\,\arctan\left(\frac{-\frac{2^{\frac{2}{3}}x\sqrt{3}}{3}+\frac{\left(x^2+x+2\right)^{\frac{1}{3}}\sqrt{3}}{3}}{\left(x^2+x+2\right)^{\frac{1}{3}}}\right)2^{\frac{1}{3}}}{2}+\frac{\ln\left(2^{\frac{2}{3}}x+2\left(x^2+x+2\right)^{\frac{1}{3}}\right)2^{\frac{1}{3}}}{2}$$
$$-\frac{\ln\left(2^{\frac{1}{3}}x^2-2^{\frac{2}{3}}x\left(x^2+x+2\right)^{\frac{1}{3}}+2\left(x^2+x+2\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{4}$$

command

```
Integrate[(6 + 2*x + x^2)/((2 + x)*(2 + x^2)*(2 + x + x^2)^(1/3)),x]
```

Mathematica 13.1 output

$$-\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{1-\frac{2^{2/3}x}{\sqrt[3]{2+x+x^2}}}{\sqrt{3}}\right)-2\log\left(2^{2/3}x+2\sqrt[3]{2+x+x^2}\right)+\log\left(\sqrt[3]{2}\,x^2-2^{2/3}x\sqrt[3]{2+x+x^2}+2\left(2+\right.\right.}{2\,2^{2/3}}$$

Mathematica 12.3 output

$$\int\frac{6+2x+x^2}{(2+x)\left(2+x^2\right)\sqrt[3]{2+x+x^2}}\,dx$$

24.324 Problem number 1901

$$\int \frac{-1+x}{x\sqrt[3]{1+2x+2x^2+x^3}}\,dx$$

Optimal antiderivative

$$-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^3+2x^2+2x+1\right)^{\frac{1}{3}}}{2+2x+\left(x^3+2x^2+2x+1\right)^{\frac{1}{3}}}\right)-\ln\left(-1-x+\left(x^3+2x^2+2x+1\right)^{\frac{1}{3}}\right)$$
$$+\frac{\ln\left(1+2x+x^2+(1+x)\left(x^3+2x^2+2x+1\right)^{\frac{1}{3}}+\left(x^3+2x^2+2x+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[(-1 + x)/(x*(1 + 2*x + 2*x^2 + x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$-\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{1+2x+2x^2+x^3}}{2+2x+\sqrt[3]{1+2x+2x^2+x^3}}\right)-\log\left(-1-x+\sqrt[3]{1+2x+2x^2+x^3}\right)$$
$$+\frac{1}{2}\log\left(1+2x+x^2+(1+x)\sqrt[3]{1+2x+2x^2+x^3}+\left(1+2x+2x^2+x^3\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{-1+x}{x\sqrt[3]{1+2x+2x^2+x^3}}\,dx$$

24.325 Problem number 1902

$$\int \frac{\left(-2q+px^3\right)\left(aq+bx^2+apx^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^7}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}\,\left(2a\,p^2x^6-4apq\,x^4+3bp\,x^5+4apq\,x^3+3bq\,x^2+2a\,q^2\right)}{6x^6}$$
$$+2bpq\ln(x)-bpq\ln\left(q+p\,x^3+\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}\right)$$

command

```
Integrate[((-2*q + p*x^3)*(a*q + b*x^2 + a*p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]
```

Mathematica 13.1 output

$$\frac{\sqrt{q^2-2pq(-1+x)x^3+p^2x^6}\left(3bx^2\left(q+px^3\right)+2a\left(q^2-2pq(-1+x)x^3+p^2x^6\right)\right)}{6x^6}$$
$$-bpq\tanh^{-1}\left(\frac{\sqrt{q^2-2pq(-1+x)x^3+p^2x^6}}{q+px^3}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(-2q+px^3\right)\left(aq+bx^2+apx^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^7}\,dx$$

24.326 Problem number 1904

$$\int\frac{1}{x\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Optimal antiderivative

$$-\frac{4\sqrt{a^2x^2-bx}\left(-256a^4x^2-160a^2bx+105b^2\right)\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}}{1155b^4x^4}$$
$$+\frac{4\left(256a^5x^2+32a^3bx+245a\,b^2\right)\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}}{1155b^4x^3}$$

command

```
Integrate[1/(x*Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2)),x]
```

Mathematica 13.1 output

$$\frac{4\sqrt{x\left(ax+\sqrt{x\left(-b+a^2x\right)}\right)}\left(105b^3+32a^3bx\left(-3ax+\sqrt{x\left(-b+a^2x\right)}\right)+256a^5x^2\left(ax+\sqrt{x\left(-b+a^2x\right)}\right)+5\right.}{1155b^4x^3\sqrt{x\left(-b+a^2x\right)}}$$

Mathematica 12.3 output

$$\int\frac{1}{x\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

24.327 Problem number 1905

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(-1+x^2)\sqrt{1+x^4}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{-2+2\sqrt{2}}\ \arctan\left(\frac{\sqrt{-2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{2} - \frac{\sqrt{2+2\sqrt{2}}\ \operatorname{arctanh}\left(\frac{\sqrt{2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{2}$$

command

```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((-1 + x^2)*Sqrt[1 + x^4]),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{-1+\sqrt{2}}\ \text{ArcTan}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}\ x\sqrt{x^2+\sqrt{1+x^4}}}\right) - \sqrt{1+\sqrt{2}}\ \tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\left(-1+x^2+\sqrt{1+x^4}\right.}{x\sqrt{x^2+\sqrt{1+x^4}}}\right.}{\sqrt{2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(-1+x^2)\sqrt{1+x^4}}\,dx$$

24.328 Problem number 1915

$$\int \frac{-2b+ax^2}{\sqrt[4]{-b+ax^2}\,(-b+ax^2+cx^4)}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{-\frac{c^{\frac{1}{4}}x^2\sqrt{2}}{2}+\frac{\sqrt{a\,x^2-b}\ \sqrt{2}}{2c^{\frac{1}{4}}}}{x(a\,x^2-b)^{\frac{1}{4}}}\right)\sqrt{2}}{2c^{\frac{1}{4}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\ c^{\frac{1}{4}}x(a\,x^2-b)^{\frac{1}{4}}}{x^2\sqrt{c}+\sqrt{a\,x^2-b}}\right)\sqrt{2}}{2c^{\frac{1}{4}}}$$

command

```
Integrate[(-2*b + a*x^2)/((-b + a*x^2)^(1/4)*(-b + a*x^2 + c*x^4)),x]
```

Mathematica 13.1 output

$$\frac{-\text{ArcTan}\left(\frac{-\sqrt{c}\,x^2+\sqrt{-b+ax^2}}{\sqrt{2}\,\sqrt[4]{c}\,x\,\sqrt[4]{-b+ax^2}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x\,\sqrt[4]{-b+ax^2}}{\sqrt{c}\,x^2+\sqrt{-b+ax^2}}\right)}{\sqrt{2}\,\sqrt[4]{c}}$$

Mathematica 12.3 output

$$\int \frac{-2b+ax^2}{\sqrt[4]{-b+ax^2}\,\left(-b+ax^2+cx^4\right)}\,dx$$

24.329 Problem number 1917

$$\int \frac{\sqrt[3]{1+x^5}\,\left(-3+2x^5\right)}{x^2\left(2-x^3+2x^5\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^5+1\right)^{\frac{1}{3}}}{2x}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\,2^{\frac{1}{3}}\left(x^5+1\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{4}+\frac{\ln\left(-x+2^{\frac{1}{3}}\left(x^5+1\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{4}$$
$$-\frac{\ln\left(x^2+2^{\frac{1}{3}}x\left(x^5+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\left(x^5+1\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{8}$$

command

```
Integrate[((1 + x^5)^(1/3)*(-3 + 2*x^5))/(x^2*(2 - x^3 + 2*x^5)),x]
```

Mathematica 13.1 output

$$\frac{3\sqrt[3]{1+x^5}}{2x}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{2}\,\sqrt[3]{1+x^5}}\right)}{2\sqrt[3]{2}}+\frac{\log\left(-x+\sqrt[3]{2}\,\sqrt[3]{1+x^5}\right)}{2\sqrt[3]{2}}$$
$$-\frac{\log\left(x^2+\sqrt[3]{2}\,x\,\sqrt[3]{1+x^5}+2^{2/3}\left(1+x^5\right)^{2/3}\right)}{4\sqrt[3]{2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[3]{1+x^5}\,\left(-3+2x^5\right)}{x^2\left(2-x^3+2x^5\right)}\,dx$$

24.330 Problem number 1918

$$\int \frac{x^2\left(4b+ax^5\right)}{\left(-b+ax^5\right)^{3/4}\left(-b+cx^4+ax^5\right)}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,c^{\frac{1}{4}}x\left(a\,x^5-b\right)^{\frac{1}{4}}}{-x^2\sqrt{c}+\sqrt{a\,x^5-b}}\right)}{c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\frac{c^{\frac{1}{4}}x^2\sqrt{2}}{2}+\frac{\sqrt{a\,x^5-b}\,\sqrt{2}}{2c^{\frac{1}{4}}}}{x\left(a\,x^5-b\right)^{\frac{1}{4}}}\right)}{c^{\frac{3}{4}}}$$

command

```
Integrate[(x^2*(4*b + a*x^5))/((-b + a*x^5)^(3/4)*(-b + c*x^4 + a*x^5)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{2}\left(\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x\,\sqrt[4]{-b+ax^5}}{\sqrt{c}\,x^2-\sqrt{-b+ax^5}}\right)+\tanh^{-1}\left(\frac{\sqrt{c}\,x^2+\sqrt{-b+ax^5}}{\sqrt{2}\,\sqrt[4]{c}\,x\,\sqrt[4]{-b+ax^5}}\right)\right)}{c^{3/4}}$$

Mathematica 12.3 output

$$\int \frac{x^2\left(4b+ax^5\right)}{\left(-b+ax^5\right)^{3/4}\left(-b+cx^4+ax^5\right)}\,dx$$

24.331 Problem number 1919

$$\int \frac{\left(-1+x^6\right)\left(1+x^6\right)^{2/3}}{x^3\left(2-x^3+2x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(x^6+1\right)^{\frac{2}{3}}}{4x^2}-\frac{\arctan\left(\frac{\sqrt{3}\,x}{x+2\,2^{\frac{1}{3}}\left(x^6+1\right)^{\frac{1}{3}}}\right)2^{\frac{1}{3}}\sqrt{3}}{12}+\frac{\ln\left(-x+2^{\frac{1}{3}}\left(x^6+1\right)^{\frac{1}{3}}\right)2^{\frac{1}{3}}}{12}$$
$$-\frac{\ln\left(x^2+2^{\frac{1}{3}}x\left(x^6+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\left(x^6+1\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{24}$$

command

```
Integrate[((-1 + x^6)*(1 + x^6)^(2/3))/(x^3*(2 - x^3 + 2*x^6)),x]
```

Mathematica 13.1 output

$$\frac{1}{24}\left(\frac{6\left(1+x^6\right)^{2/3}}{x^2}-2\sqrt[3]{2}\,\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{2}\,\sqrt[3]{1+x^6}}\right)\right.$$
$$\left.+2\sqrt[3]{2}\,\log\left(-x+\sqrt[3]{2}\,\sqrt[3]{1+x^6}\right)-\sqrt[3]{2}\,\log\left(x^2+\sqrt[3]{2}\,x\sqrt[3]{1+x^6}\,+2^{2/3}\left(1+x^6\right)^{2/3}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^6\right)\left(1+x^6\right)^{2/3}}{x^3\left(2-x^3+2x^6\right)}\,dx$$

24.332 Problem number 1920

$$\int \frac{x^2\left(2b+ax^6\right)}{\left(-b+ax^6\right)^{3/4}\left(-b+cx^4+ax^6\right)}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{2}\,c^{\frac{1}{4}}x\left(a\,x^6-b\right)^{\frac{1}{4}}}{-x^2\sqrt{c}+\sqrt{a\,x^6-b}}\right)\sqrt{2}}{2c^{\frac{3}{4}}}+\frac{\text{arctanh}\left(\frac{\frac{c^{\frac{1}{4}}x^2\sqrt{2}}{2}+\frac{\sqrt{a\,x^6-b}\,\sqrt{2}}{2c^{\frac{1}{4}}}}{x\left(a\,x^6-b\right)^{\frac{1}{4}}}\right)\sqrt{2}}{2c^{\frac{3}{4}}}$$

command

```
Integrate[(x^2*(2*b + a*x^6))/((-b + a*x^6)^(3/4)*(-b + c*x^4 + a*x^6)),x]
```

Mathematica 13.1 output

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x\,\sqrt[4]{-b+ax^6}}{\sqrt{c}\,x^2-\sqrt{-b+ax^6}}\right)+\tanh^{-1}\left(\frac{\sqrt{c}\,x^2+\sqrt{-b+ax^6}}{\sqrt{2}\,\sqrt[4]{c}\,x\,\sqrt[4]{-b+ax^6}}\right)}{\sqrt{2}\,c^{3/4}}$$

Mathematica 12.3 output

$$\int \frac{x^2\left(2b+ax^6\right)}{\left(-b+ax^6\right)^{3/4}\left(-b+cx^4+ax^6\right)}\,dx$$

24.333 Problem number 1923

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1-x^2)\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/((1 - x^2)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

Mathematica 13.1 output

$$-\frac{1}{2}\text{RootSum}\left[-2+4\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-1+\#1^2}\&\right]$$
$$+\frac{1}{2}\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-1+\#1^2}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1-x^2)\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.334 Problem number 1924

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1-x^2)\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/((1 - x^2)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

Mathematica 13.1 output

$$-\frac{1}{2}\text{RootSum}\left[-2+4\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-1+\#1^2}\&\right]$$
$$+\frac{1}{2}\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-1+\#1^2}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1-x^2)\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.335 Problem number 1929

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}}{x^3\left(2+x^3+2x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^4+1\right)^{\frac{2}{3}}}{4x^2}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\,2^{\frac{1}{3}}\left(x^4+1\right)^{\frac{1}{3}}}\right)2^{\frac{1}{3}}}{4}$$
$$+\frac{\ln\left(x+2^{\frac{1}{3}}\left(x^4+1\right)^{\frac{1}{3}}\right)2^{\frac{1}{3}}}{4}-\frac{\ln\left(x^2-2^{\frac{1}{3}}x\left(x^4+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\left(x^4+1\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{8}$$

command

```
Integrate[((-3 + x^4)*(1 + x^4)^(2/3))/(x^3*(2 + x^3 + 2*x^4)),x]
```

Mathematica 13.1 output

$$\frac{1}{8}\left(\frac{6\left(1+x^4\right)^{2/3}}{x^2}-2\sqrt[3]{2}\,\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x-2\sqrt[3]{2}\,\sqrt[3]{1+x^4}}\right)\right.$$
$$\left.+2\sqrt[3]{2}\,\log\left(x+\sqrt[3]{2}\,\sqrt[3]{1+x^4}\right)-\sqrt[3]{2}\,\log\left(x^2-\sqrt[3]{2}\,x\sqrt[3]{1+x^4}+2^{2/3}\left(1+x^4\right)^{2/3}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}}{x^3\left(2+x^3+2x^4\right)}\,dx$$

24.336 Problem number 1934

$$\int \frac{\left(1+x^2\right)\sqrt{1+\sqrt{1+x^2}}}{-1+x^2}\,dx$$

Optimal antiderivative

$$\frac{4x}{3\sqrt{1+\sqrt{x^2+1}}}+\frac{2x\sqrt{x^2+1}}{3\sqrt{1+\sqrt{x^2+1}}}$$
$$+2\sqrt{\sqrt{2}-1}\,\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}\,\sqrt{1+\sqrt{x^2+1}}}\right)$$
$$-2\sqrt{1+\sqrt{2}}\,\operatorname{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}\,\sqrt{1+\sqrt{x^2+1}}}\right)$$

command

```
Integrate[((1 + x^2)*Sqrt[1 + Sqrt[1 + x^2]])/(-1 + x^2),x]
```

Mathematica 13.1 output

$$\frac{2x\left(2+\sqrt{1+x^2}\right)}{3\sqrt{1+\sqrt{1+x^2}}}+2\sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left(\frac{x}{\sqrt{1+\sqrt{2}}\,\sqrt{1+\sqrt{1+x^2}}}\right)$$
$$-2\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{1+\sqrt{2}}\,x}{\sqrt{1+\sqrt{1+x^2}}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2\right)\sqrt{1+\sqrt{1+x^2}}}{-1+x^2}\,dx$$

24.337 Problem number 1937

$$\int \frac{\left(-4+x^5\right)\sqrt[4]{1-2x^4+x^5}}{x^2\left(1+x^5\right)}\,dx$$

Optimal antiderivative

$$\frac{4\left(x^5-2x^4+1\right)^{\frac{1}{4}}}{x}-2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{3}{4}}x\left(x^5-2x^4+1\right)^{\frac{1}{4}}}{x^2\sqrt{2}-\sqrt{x^5-2x^4+1}}\right)$$
$$-2^{\frac{3}{4}}\operatorname{arctanh}\left(\frac{2\,2^{\frac{1}{4}}x\left(x^5-2x^4+1\right)^{\frac{1}{4}}}{2x^2+\sqrt{2}\,\sqrt{x^5-2x^4+1}}\right)$$

command

```
Integrate[((-4 + x^5)*(1 - 2*x^4 + x^5)^(1/4))/(x^2*(1 + x^5)),x]
```

Mathematica 13.1 output

$$\frac{4\sqrt[4]{1-2x^4+x^5}}{x}$$
$$-2^{3/4}\text{ArcTan}\left(\frac{2^{3/4}x\sqrt[4]{1-2x^4+x^5}}{\sqrt{2}\,x^2-\sqrt{1-2x^4+x^5}}\right)-2^{3/4}\tanh^{-1}\left(\frac{2\sqrt[4]{2}\,x\sqrt[4]{1-2x^4+x^5}}{2x^2+\sqrt{2}\,\sqrt{1-2x^4+x^5}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-4+x^5\right)\sqrt[4]{1-2x^4+x^5}}{x^2\left(1+x^5\right)}\,dx$$

24.338 Problem number 1944

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}{1+x^4+x^8}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{x}{\left(x^6+x^2\right)^{\frac{1}{4}}}\right)}{2}-\frac{\arctan\left(\frac{\sqrt{2}\,x\left(x^6+x^2\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^6+x^2}}\right)\sqrt{2}}{4}$$
$$-\frac{\operatorname{arctanh}\left(\frac{x}{\left(x^6+x^2\right)^{\frac{1}{4}}}\right)}{2}+\frac{\operatorname{arctanh}\left(\frac{\frac{x^2\sqrt{2}}{2}+\frac{\sqrt{x^6+x^2}\,\sqrt{2}}{2}}{x\left(x^6+x^2\right)^{\frac{1}{4}}}\right)\sqrt{2}}{4}$$

command

```
Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^4 + x^8),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{x^2+x^6}\left(2\text{ArcTan}\left(\frac{\sqrt{x}}{\sqrt[4]{1+x^4}}\right)+\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{x}\,\sqrt[4]{1+x^4}}{x-\sqrt{1+x^4}}\right)-2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{1+x^4}}\right)+\sqrt{2}\,\tanh^{-1}\left(\right.\right.}{4\sqrt{x}\,\sqrt[4]{1+x^4}}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}{1+x^4+x^8}\,dx$$

24.339 Problem number 1945

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}{1+x^4+x^8}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{x}{(x^6+x^2)^{\frac{1}{4}}}\right)}{2}-\frac{\arctan\left(\frac{\sqrt{2}\,x(x^6+x^2)^{\frac{1}{4}}}{-x^2+\sqrt{x^6+x^2}}\right)\sqrt{2}}{4}$$
$$-\frac{\text{arctanh}\left(\frac{x}{(x^6+x^2)^{\frac{1}{4}}}\right)}{2}+\frac{\text{arctanh}\left(\frac{\frac{x^2\sqrt{2}}{2}+\frac{\sqrt{x^6+x^2}\,\sqrt{2}}{2}}{x(x^6+x^2)^{\frac{1}{4}}}\right)\sqrt{2}}{4}$$

command

```
Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^4 + x^8),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{x^2+x^6}\left(2\text{ArcTan}\left(\frac{\sqrt{x}}{\sqrt[4]{1+x^4}}\right)+\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{x}\,\sqrt[4]{1+x^4}}{x-\sqrt{1+x^4}}\right)-2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{1+x^4}}\right)+\sqrt{2}\,\tanh^{-1}\left(\right.\right.}{4\sqrt{x}\,\sqrt[4]{1+x^4}}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}{1+x^4+x^8}\,dx$$

24.340 Problem number 1953

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{(b^2+ax^2)^{5/2}}\,dx$$

Optimal antiderivative

$$\frac{5x}{12b^2\left(a\,x^2+b^2\right)\sqrt{b+\sqrt{a\,x^2+b^2}}}+\frac{x\left(15a\,x^2+23b^2\right)}{24b^3\left(a\,x^2+b^2\right)^{\frac{3}{2}}\sqrt{b+\sqrt{a\,x^2+b^2}}}+\frac{5\arctan\left(\frac{\sqrt{a}\,x}{\sqrt{b}\,\sqrt{b+\sqrt{a\,x^2+b^2}}}\right)}{8\sqrt{a}\,b^{\frac{7}{2}}}$$

command

```
Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^(5/2),x]
```

Mathematica 13.1 output

$$\frac{x\left(23b^2+15ax^2+10b\sqrt{b^2+ax^2}\right)}{24b^3\left(b^2+ax^2\right)^{3/2}\sqrt{b+\sqrt{b^2+ax^2}}}+\frac{5\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\,\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{8\sqrt{a}\,b^{7/2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{(b^2+ax^2)^{5/2}}\,dx$$

24.341 Problem number 1960

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(1+x^3+x^6\right)}{x^6\left(-1+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(x^3-1\right)^{\frac{2}{3}}\left(3x^3+2\right)}{10x^5}+\frac{\arctan\left(\frac{\sqrt{3}\,x}{x+2^{\frac{2}{3}}\left(x^3-1\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}\sqrt{3}}{6}-\frac{\ln\left(-2x+2^{\frac{2}{3}}\left(x^3-1\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{6}+\frac{\ln\left(2x^2+2^{\frac{2}{3}}x\left(x^3-1\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(x^3-1\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{12}$$

command

```
Integrate[((-1 + x^3)^(2/3)*(1 + x^3 + x^6))/(x^6*(-1 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{\left(-1+x^3\right)^{2/3}\left(2+3x^3\right)}{10x^5}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2^{2/3}\sqrt[3]{-1+x^3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}$$
$$-\frac{\log\left(-2x+2^{2/3}\sqrt[3]{-1+x^3}\right)}{3\sqrt[3]{2}}+\frac{\log\left(2x^2+2^{2/3}x\sqrt[3]{-1+x^3}+\sqrt[3]{2}\left(-1+x^3\right)^{2/3}\right)}{6\sqrt[3]{2}}$$

Mathematica 12.3 output

$$\int\frac{\left(-1+x^3\right)^{2/3}\left(1+x^3+x^6\right)}{x^6\left(-1+x^6\right)}\,dx$$

24.342 Problem number 1965

$$\int\frac{-2x+x^2}{\left(1-x+x^2\right)\sqrt[4]{1+x^4}}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{x}{(x^4+1)^{\frac{1}{4}}}\right)}{2}-\frac{\arctan\left(\frac{\left(-\sqrt{2}+x\sqrt{2}\right)(x^4+1)^{\frac{1}{4}}}{-1+2x-x^2+\sqrt{x^4+1}}\right)\sqrt{2}}{2}$$
$$+\frac{\text{arctanh}\left(\frac{x}{(x^4+1)^{\frac{1}{4}}}\right)}{2}-\frac{\text{arctanh}\left(\frac{\left(-\sqrt{2}+x\sqrt{2}\right)(x^4+1)^{\frac{1}{4}}}{1-2x+x^2+\sqrt{x^4+1}}\right)\sqrt{2}}{2}$$

command

```
Integrate[(-2*x + x^2)/((1 - x + x^2)*(1 + x^4)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{1}{2}\left(\text{ArcTan}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,(-1+x)\sqrt[4]{1+x^4}}{-1+2x-x^2+\sqrt{1+x^4}}\right)\right.$$
$$\left.+\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,(-1+x)\sqrt[4]{1+x^4}}{1-2x+x^2+\sqrt{1+x^4}}\right)\right)$$

Mathematica 12.3 output

$$\int\frac{-2x+x^2}{\left(1-x+x^2\right)\sqrt[4]{1+x^4}}\,dx$$

24.343 Problem number 1967

$$\int \frac{-b+ax^8}{\sqrt[4]{-bx^2+ax^4}\,(b+ax^8)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-b + a*x^8)/((-(b*x^2) + a*x^4)^(1/4)*(b + a*x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-a+\frac{b}{x^2}}\,x\left(2\sqrt{2}\left(-\text{ArcTan}\left(\frac{-\sqrt{a}+\sqrt{-a+\frac{b}{x^2}}}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a+\frac{b}{x^2}}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a+\frac{b}{x^2}}}{\sqrt{a}+\sqrt{-a+\frac{b}{x^2}}}\right)\right)+\sqrt[4]{a}\,\text{RootSum}\left[a^{}\right.\right.}{4\sqrt[4]{a}\,\sqrt[4]{-bx^2+ax^4}}$$

Mathematica 12.3 output

$$\int \frac{-b+ax^8}{\sqrt[4]{-bx^2+ax^4}\,(b+ax^8)}\,dx$$

24.344 Problem number 1968

$$\int \frac{-b+ax^8}{\sqrt[4]{-bx^2+ax^4}\,(b+ax^8)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-b + a*x^8)/((-(b*x^2) + a*x^4)^(1/4)*(b + a*x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-a+\frac{b}{x^2}}\,x\left(2\sqrt{2}\left(-\text{ArcTan}\left(\frac{-\sqrt{a}+\sqrt{-a+\frac{b}{x^2}}}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a+\frac{b}{x^2}}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a+\frac{b}{x^2}}}{\sqrt{a}+\sqrt{-a+\frac{b}{x^2}}}\right)\right)+\sqrt[4]{a}\,\text{RootSum}\left[a^{}\right.\right.}{4\sqrt[4]{a}\,\sqrt[4]{-bx^2+ax^4}}$$

Mathematica 12.3 output

$$\int \frac{-b+ax^8}{\sqrt[4]{-bx^2+ax^4}\,(b+ax^8)}\,dx$$

24.345 Problem number 1969

$$\int \frac{\left(-1+x^8\right)\left(1+x^8\right)}{\sqrt[4]{-1-x^4+x^8}\,\left(1-3x^8+x^{16}\right)}\,dx$$

Optimal antiderivative

$$-\frac{x}{2\left(x^8-x^4-1\right)^{\frac{1}{4}}}+\frac{\arctan\left(\frac{2^{\frac{3}{4}}x\left(x^8-x^4-1\right)^{\frac{1}{4}}}{x^2\sqrt{2}-\sqrt{x^8-x^4-1}}\right)2^{\frac{1}{4}}}{8}-\frac{\operatorname{arctanh}\left(\frac{2\,2^{\frac{1}{4}}x\left(x^8-x^4-1\right)^{\frac{1}{4}}}{2x^2+\sqrt{2}\,\sqrt{x^8-x^4-1}}\right)2^{\frac{1}{4}}}{8}$$

command

```
Integrate[((-1 + x^8)*(1 + x^8))/((-1 - x^4 + x^8)^(1/4)*(1 - 3*x^8 + x^16)),x]
```

Mathematica 13.1 output

$$\frac{1}{8}\left(-\frac{4x}{\sqrt[4]{-1-x^4+x^8}}+\sqrt[4]{2}\,\text{ArcTan}\left(\frac{2^{3/4}x\sqrt[4]{-1-x^4+x^8}}{\sqrt{2}\,x^2-\sqrt{-1-x^4+x^8}}\right)-\sqrt[4]{2}\,\tanh^{-1}\left(\frac{2\sqrt[4]{2}\,x\sqrt[4]{-1-x^4+x^8}}{2x^2+\sqrt{2}\,\sqrt{-1-x^4+x^8}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^8\right)\left(1+x^8\right)}{\sqrt[4]{-1-x^4+x^8}\,\left(1-3x^8+x^{16}\right)}\,dx$$

24.346 Problem number 1979

$$\int \frac{\left(1+x^5\right)^{2/3}\left(-3+2x^5\right)\left(2+x^3+2x^5\right)}{x^6\left(2-x^3+2x^5\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^5+1\right)^{\frac{2}{3}}\left(2x^5+5x^3+2\right)}{10x^5}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\,2^{\frac{1}{3}}\left(x^5+1\right)^{\frac{1}{3}}}\right)2^{\frac{1}{3}}}{2}+\frac{\ln\left(-x+2^{\frac{1}{3}}\left(x^5+1\right)^{\frac{1}{3}}\right)2^{\frac{1}{3}}}{2}-\frac{\ln\left(x^2+2^{\frac{1}{3}}x\left(x^5+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\left(x^5+1\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{4}$$

command

```
Integrate[((1 + x^5)^(2/3)*(-3 + 2*x^5)*(2 + x^3 + 2*x^5))/(x^6*(2 - x^3 + 2*x^5)),x]
```

Mathematica 13.1 output

$$\frac{3\left(1+x^5\right)^{2/3}\left(2+5x^3+2x^5\right)}{10x^5}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{2}\,\sqrt[3]{1+x^5}}\right)}{2^{2/3}}$$
$$+\frac{\log\left(-x+\sqrt[3]{2}\,\sqrt[3]{1+x^5}\right)}{2^{2/3}}-\frac{\log\left(x^2+\sqrt[3]{2}\,x\sqrt[3]{1+x^5}+2^{2/3}\left(1+x^5\right)^{2/3}\right)}{2\,2^{2/3}}$$

Mathematica 12.3 output

$$\int\frac{\left(1+x^5\right)^{2/3}\left(-3+2x^5\right)\left(2+x^3+2x^5\right)}{x^6\left(2-x^3+2x^5\right)}\,dx$$

24.347 Problem number 1980

$$\int\frac{b+ax^6}{\left(-b+ax^6\right)\sqrt[3]{-b+a^3x^3+ax^6}}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{3}\,ax}{ax+2\left(a\,x^6+a^3x^3-b\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3a}+\frac{\ln\left(-ax+\left(a\,x^6+a^3x^3-b\right)^{\frac{1}{3}}\right)}{3a}$$
$$-\frac{\ln\left(a^2x^2+ax\left(a\,x^6+a^3x^3-b\right)^{\frac{1}{3}}+\left(a\,x^6+a^3x^3-b\right)^{\frac{2}{3}}\right)}{6a}$$

command

```
Integrate[(b + a*x^6)/((-b + a*x^6)*(-b + a^3*x^3 + a*x^6)^(1/3)),x]
```

Mathematica 13.1 output

$$-\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,ax}{ax+2\sqrt[3]{-b+a^3x^3+ax^6}}\right)-2\log\left(a\left(ax-\sqrt[3]{-b+a^3x^3+ax^6}\right)\right)+\log\left(a^2x^2+ax\sqrt[3]{-b+a^3x^3}\right.}{6a}$$

Mathematica 12.3 output

$$\int\frac{b+ax^6}{\left(-b+ax^6\right)\sqrt[3]{-b+a^3x^3+ax^6}}\,dx$$

24.348 Problem number 1982

$$\int \frac{b+ax^8}{\sqrt[4]{-bx^2+ax^4}\,(-b+ax^8)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(b + a*x^8)/((-(b*x^2) + a*x^4)^(1/4)*(-b + a*x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-a+\frac{b}{x^2}}\,x\left(2\sqrt{2}\left(-\text{ArcTan}\left(\frac{-\sqrt{a}+\sqrt{-a+\frac{b}{x^2}}}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a+\frac{b}{x^2}}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a+\frac{b}{x^2}}}{\sqrt{a}+\sqrt{-a+\frac{b}{x^2}}}\right)\right)+\sqrt[4]{a}\,\text{RootSum}\left[a^{}\right.\right.}{4\sqrt[4]{a}\,\sqrt[4]{-bx^2+ax^4}}$$

Mathematica 12.3 output

$$\int \frac{b+ax^8}{\sqrt[4]{-bx^2+ax^4}\,(-b+ax^8)}\,dx$$

24.349 Problem number 1983

$$\int \frac{b+ax^8}{\sqrt[4]{-bx^2+ax^4}\,(-b+ax^8)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(b + a*x^8)/((-(b*x^2) + a*x^4)^(1/4)*(-b + a*x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-a+\frac{b}{x^2}}\,x\left(2\sqrt{2}\left(-\text{ArcTan}\left(\frac{-\sqrt{a}+\sqrt{-a+\frac{b}{x^2}}}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a+\frac{b}{x^2}}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a+\frac{b}{x^2}}}{\sqrt{a}+\sqrt{-a+\frac{b}{x^2}}}\right)\right)+\sqrt[4]{a}\,\text{RootSum}\left[a^{}\right.\right.}{4\sqrt[4]{a}\,\sqrt[4]{-bx^2+ax^4}}$$

Mathematica 12.3 output

$$\int \frac{b+ax^8}{\sqrt[4]{-bx^2+ax^4}\,(-b+ax^8)}\,dx$$

24.350 Problem number 1991

$$\int \frac{\sqrt{b+a^2x^2}}{x^2-\sqrt[3]{ax-\sqrt{b+a^2x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[b + a^2*x^2]/(x^2 - (a*x - Sqrt[b + a^2*x^2])^(1/3)),x]
```

Mathematica 13.1 output

$$-a\log\left(-ax+\sqrt{b+a^2x^2}\right)+3a\text{RootSum}\left[b^2-2b\#1^6-4a^2\#1^7+\#1^{12}\&,\frac{b\log\left(\sqrt[3]{ax-\sqrt{b+a^2x^2}}-\#1\right)+a^2\log\left(\sqrt[3]{ax-\sqrt{b+a^2x^2}}-\#1\right)\#1}{3b+7a^2\#1-3\#1^6}\&\right]$$

Mathematica 12.3 output

$Aborted

24.351 Problem number 1992

$$\int \frac{\sqrt{b+a^2x^2}}{x^2-\sqrt{ax-\sqrt{b+a^2x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[b + a^2*x^2]/(x^2 - Sqrt[a*x - Sqrt[b + a^2*x^2]]),x]
```

Mathematica 13.1 output

$$-a\log\left(-ax+\sqrt{b+a^2x^2}\right)+2a\text{RootSum}\left[b^2-2b\#1^4-4a^2\#1^5\right.$$
$$\left.+\#1^8\&,\frac{b\log\left(\sqrt{ax-\sqrt{b+a^2x^2}}-\#1\right)+a^2\log\left(\sqrt{ax-\sqrt{b+a^2x^2}}-\#1\right)\#1}{2b+5a^2\#1-2\#1^4}\&\right]$$

Mathematica 12.3 output

$Aborted

24.352 Problem number 1996

$$\int\frac{(3+2x)\left(1+x+3x^3\right)^{2/3}}{x^3\left(1+x+x^3\right)}\,dx$$

Optimal antiderivative

$$-\frac{3\left(3x^3+x+1\right)^{\frac{2}{3}}}{2x^2}+2^{\frac{2}{3}}\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ x}{x+2^{\frac{2}{3}}\left(3x^3+x+1\right)^{\frac{1}{3}}}\right)$$
$$-2^{\frac{2}{3}}\ln\left(-2x+2^{\frac{2}{3}}\left(3x^3+x+1\right)^{\frac{1}{3}}\right)+\frac{\ln\left(2x^2+2^{\frac{2}{3}}x\left(3x^3+x+1\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(3x^3+x+1\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{2}$$

command

```
Integrate[((3 + 2*x)*(1 + x + 3*x^3)^(2/3))/(x^3*(1 + x + x^3)),x]
```

Mathematica 13.1 output

$$-\frac{3\left(1+x+3x^3\right)^{2/3}}{2x^2}$$
$$+2^{2/3}\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2^{2/3}\sqrt[3]{1+x+3x^3}}\right)-2^{2/3}\log\left(-2x+2^{2/3}\sqrt[3]{1+x+3x^3}\right)+\frac{\log\left(2x^2+2^{2/3}x\sqrt[3]{1+x+3x}\right.}{\sqrt[3]{2}}$$

Mathematica 12.3 output

$$\int\frac{(3+2x)\left(1+x+3x^3\right)^{2/3}}{x^3\left(1+x+x^3\right)}\,dx$$

24.353 Problem number 2002

$$\int \frac{\left(2+x^6\right)\left(-1-x^4+x^6\right)}{\sqrt[4]{1-x^4-x^6}\,\left(-1+x^6\right)^2}\,dx$$

Optimal antiderivative

$$-\frac{x\left(-x^6-x^4+1\right)^{\frac{3}{4}}}{2x^6-2}-\frac{5\arctan\left(\frac{\sqrt{2}\,x\left(-x^6-x^4+1\right)^{\frac{1}{4}}}{-x^2+\sqrt{-x^6-x^4+1}}\right)\sqrt{2}}{8}$$
$$-\frac{5\operatorname{arctanh}\left(\frac{\sqrt{2}\,x\left(-x^6-x^4+1\right)^{\frac{1}{4}}}{x^2+\sqrt{-x^6-x^4+1}}\right)\sqrt{2}}{8}$$

command

```
Integrate[((2 + x^6)*(-1 - x^4 + x^6))/((1 - x^4 - x^6)^(1/4)*(-1 + x^6)^2),x]
```

Mathematica 13.1 output

$$-\frac{x\left(1-x^4-x^6\right)^{3/4}}{2\left(-1+x^6\right)}-\frac{5\text{ArcTan}\left(\frac{\sqrt{2}\,x\sqrt[4]{1-x^4-x^6}}{-x^2+\sqrt{1-x^4-x^6}}\right)}{4\sqrt{2}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{1-x^4-x^6}}{x^2+\sqrt{1-x^4-x^6}}\right)}{4\sqrt{2}}$$

Mathematica 12.3 output

$$\int \frac{\left(2+x^6\right)\left(-1-x^4+x^6\right)}{\sqrt[4]{1-x^4-x^6}\,\left(-1+x^6\right)^2}\,dx$$

24.354 Problem number 2003

$$\int \frac{\left(-2b+ax^6\right)\left(b-cx^4+ax^6\right)}{x^2\left(b+ax^6\right)^{3/4}\left(b+cx^4+ax^6\right)}\,dx$$

Optimal antiderivative

$$\frac{2\left(a\,x^6+b\right)^{\frac{1}{4}}}{x}+\sqrt{2}\;c^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\;c^{\frac{1}{4}}x\left(a\,x^6+b\right)^{\frac{1}{4}}}{-x^2\sqrt{c}\;+\sqrt{a\,x^6+b}}\right)$$
$$-\sqrt{2}\;c^{\frac{1}{4}}\operatorname{arctanh}\left(\frac{\frac{c^{\frac{1}{4}}x^2\sqrt{2}}{2}+\frac{\sqrt{a\,x^6+b}\;\sqrt{2}}{2c^{\frac{1}{4}}}}{x\left(a\,x^6+b\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[((-2*b + a*x^6)*(b - c*x^4 + a*x^6))/(x^2*(b + a*x^6)^(3/4)*(b + c*x^4 + a*x^6)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt[4]{b+ax^6}}{x}+\sqrt{2}\,\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x\sqrt[4]{b+ax^6}}{-\sqrt{c}\,x^2+\sqrt{b+ax^6}}\right)-\sqrt{2}\,\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2+\sqrt{b+ax^6}}{\sqrt{2}\,\sqrt[4]{c}\,x\sqrt[4]{b+ax^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-2b+ax^6\right)\left(b-cx^4+ax^6\right)}{x^2\left(b+ax^6\right)^{3/4}\left(b+cx^4+ax^6\right)}\,dx$$

24.355 Problem number 2009

$$\int \frac{-3b+ax^2}{\sqrt[4]{3b-2ax^2}\,\left(3b-2ax^2+3x^4\right)}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\frac{3^{\frac{1}{4}}x^2\sqrt{2}}{2}-\frac{\sqrt{-2a\,x^2+3b}\,\sqrt{2}\,3^{\frac{3}{4}}}{6}}{x\left(-2a\,x^2+3b\right)^{\frac{1}{4}}}\right)\sqrt{2}\,3^{\frac{3}{4}}}{12}-\frac{\text{arctanh}\left(\frac{\sqrt{2}\,3^{\frac{1}{4}}x\left(-2a\,x^2+3b\right)^{\frac{1}{4}}}{\sqrt{3}\,x^2+\sqrt{-2a\,x^2+3b}}\right)\sqrt{2}\,3^{\frac{3}{4}}}{12}$$

command

```
Integrate[(-3*b + a*x^2)/((3*b - 2*a*x^2)^(1/4)*(3*b - 2*a*x^2 + 3*x^4)),x]
```

Mathematica 13.1 output

$$-\frac{\text{ArcTan}\left(\frac{3x^2-\sqrt{9b-6ax^2}}{\sqrt{2}\,3^{3/4}x\sqrt[4]{3b-2ax^2}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{9b-6ax^2}}{\sqrt{3}\,x^2+\sqrt{3b-2ax^2}}\right)}{2\sqrt{2}\,\sqrt[4]{3}}$$

Mathematica 12.3 output

$$\int \frac{-3b+ax^2}{\sqrt[4]{3b-2ax^2}\,\left(3b-2ax^2+3x^4\right)}\,dx$$

24.356 Problem number 2010

$$\int \frac{\left(-4b+ax^5\right)\left(b-cx^4+ax^5\right)}{x^2\left(b+ax^5\right)^{3/4}\left(b+cx^4+ax^5\right)}\,dx$$

Optimal antiderivative

$$\frac{4\left(a\,x^5+b\right)^{\frac{1}{4}}}{x}+2\sqrt{2}\;c^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\;c^{\frac{1}{4}}x\left(a\,x^5+b\right)^{\frac{1}{4}}}{-x^2\sqrt{c}\;+\sqrt{a\,x^5+b}}\right)$$
$$-2\sqrt{2}\;c^{\frac{1}{4}}\operatorname{arctanh}\left(\frac{\frac{c^{\frac{1}{4}}x^2\sqrt{2}}{2}+\frac{\sqrt{a\,x^5+b}\;\sqrt{2}}{2c^{\frac{1}{4}}}}{x\left(a\,x^5+b\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[((-4*b + a*x^5)*(b - c*x^4 + a*x^5))/(x^2*(b + a*x^5)^(3/4)*(b + c*x^4 + a*x^5)),x]
```

Mathematica 13.1 output

$$\frac{4\sqrt[4]{b+ax^5}}{x}+2\sqrt{2}\;\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\;\sqrt[4]{c}\,x\sqrt[4]{b+ax^5}}{-\sqrt{c}\,x^2+\sqrt{b+ax^5}}\right)-2\sqrt{2}\;\sqrt[4]{c}\;\tanh^{-1}\left(\frac{\sqrt{c}\,x^2+\sqrt{b+ax^5}}{\sqrt{2}\;\sqrt[4]{c}\,x\sqrt[4]{b+ax^5}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-4b+ax^5\right)\left(b-cx^4+ax^5\right)}{x^2\left(b+ax^5\right)^{3/4}\left(b+cx^4+ax^5\right)}\,dx$$

24.357 Problem number 2011

$$\int \frac{\left(-1+x^3\right)\left(1+x^6\right)^{2/3}\left(1-x^3+x^6\right)}{x^6\left(1+x^3\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(x^6+1\right)^{\frac{2}{3}}\left(2x^6-15x^3+2\right)}{10x^5}-2^{\frac{2}{3}}\sqrt{3}\;\arctan\left(\frac{\sqrt{3}\;x}{-x+2^{\frac{2}{3}}\left(x^6+1\right)^{\frac{1}{3}}}\right)$$
$$-2^{\frac{2}{3}}\ln\left(2x+2^{\frac{2}{3}}\left(x^6+1\right)^{\frac{1}{3}}\right)+\frac{\ln\left(-2x^2+2^{\frac{2}{3}}x\left(x^6+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\left(x^6+1\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{2}$$

command

```
Integrate[((-1 + x^3)*(1 + x^6)^(2/3)*(1 - x^3 + x^6))/(x^6*(1 + x^3)),x]
```

Mathematica 13.1 output

$$\frac{\left(1+x^6\right)^{2/3}\left(2-15x^3+2x^6\right)}{10x^5}$$
$$-2^{2/3}\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{-x+2^{2/3}\sqrt[3]{1+x^6}}\right)-2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1+x^6}\right)+\frac{\log\left(-2x^2+2^{2/3}x\sqrt[3]{1+x^6}-\sqrt[3]{2}\,(1+\right.}{\sqrt[3]{2}}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^3\right)\left(1+x^6\right)^{2/3}\left(1-x^3+x^6\right)}{x^6\left(1+x^3\right)}\,dx$$

24.358 Problem number 2012

$$\int \frac{\left(-b^2+ax^2\right)^2\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^2}\,dx$$

Optimal antiderivative

$$\frac{x\left(2a\,x^2+11b^2\right)}{3\sqrt{a\,x^2+b^2}\,\sqrt{b+\sqrt{a\,x^2+b^2}}}+\frac{2x\left(2ab\,x^2+5b^3\right)}{3\left(a\,x^2+b^2\right)\sqrt{b+\sqrt{a\,x^2+b^2}}}$$
$$-\frac{5b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{a}\,x}{\sqrt{b}\,\sqrt{b+\sqrt{a\,x^2+b^2}}}\right)}{\sqrt{a}}$$

command

```
Integrate[((-b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]])/(b^2 + a*x^2)^2,x]
```

Mathematica 13.1 output

$$\frac{x\left(11b^2+2ax^2\right)}{3\sqrt{b^2+ax^2}\,\sqrt{b+\sqrt{b^2+ax^2}}}+\frac{2x\left(5b^3+2abx^2\right)}{3\left(b^2+ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}}$$
$$-\frac{5b^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\,\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{a}}$$

Mathematica 12.3 output

$$\int \frac{\left(-b^2+ax^2\right)^2\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^2}\,dx$$

24.359 Problem number 2014

$$\int \frac{\left(-4+x^2\right)\sqrt[4]{2-x^2-2x^4}}{x^2\left(-2+x^2\right)}\,dx$$

Optimal antiderivative

$$-\frac{2\left(-2x^4-x^2+2\right)^{\frac{1}{4}}}{x}+\frac{\arctan\left(\frac{2^{\frac{3}{4}}x\left(-2x^4-x^2+2\right)^{\frac{1}{4}}}{x^2\sqrt{2}-\sqrt{-2x^4-x^2+2}}\right)2^{\frac{3}{4}}}{2}$$
$$+\frac{\operatorname{arctanh}\left(\frac{2\,2^{\frac{1}{4}}x\left(-2x^4-x^2+2\right)^{\frac{1}{4}}}{2x^2+\sqrt{2}\sqrt{-2x^4-x^2+2}}\right)2^{\frac{3}{4}}}{2}$$

command

```
Integrate[((-4 + x^2)*(2 - x^2 - 2*x^4)^(1/4))/(x^2*(-2 + x^2)),x]
```

Mathematica 13.1 output

$$-\frac{2\sqrt[4]{2-x^2-2x^4}}{x}+\frac{\operatorname{ArcTan}\left(\frac{2^{3/4}x\sqrt[4]{2-x^2-2x^4}}{\sqrt{2}\,x^2-\sqrt{2-x^2-2x^4}}\right)}{\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}\,x\sqrt[4]{2-x^2-2x^4}}{2x^2+\sqrt{2}\sqrt{2-x^2-2x^4}}\right)}{\sqrt[4]{2}}$$

Mathematica 12.3 output

$$\int \frac{\left(-4+x^2\right)\sqrt[4]{2-x^2-2x^4}}{x^2\left(-2+x^2\right)}\,dx$$

24.360 Problem number 2016

$$\int \frac{\left(-3+2x^4\right)\left(1+2x^4\right)^{2/3}}{x^3\left(2-x^3+4x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(2x^4+1\right)^{\frac{2}{3}}}{4x^2}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\,2^{\frac{1}{3}}\left(2x^4+1\right)^{\frac{1}{3}}}\right)2^{\frac{1}{3}}}{4}+\frac{\ln\left(-x+2^{\frac{1}{3}}\left(2x^4+1\right)^{\frac{1}{3}}\right)2^{\frac{1}{3}}}{4}$$
$$-\frac{\ln\left(x^2+2^{\frac{1}{3}}x\left(2x^4+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\left(2x^4+1\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{8}$$

command

```
Integrate[((-3 + 2*x^4)*(1 + 2*x^4)^(2/3))/(x^3*(2 - x^3 + 4*x^4)),x]
```

Mathematica 13.1 output

$$\frac{1}{8}\left(\frac{6\left(1+2x^4\right)^{2/3}}{x^2}-2\sqrt[3]{2}\,\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{2+4x^4}}\right)\right.$$
$$\left.+2\sqrt[3]{2}\,\log\left(-x+\sqrt[3]{2+4x^4}\right)-\sqrt[3]{2}\,\log\left(x^2+x\sqrt[3]{2+4x^4}+\left(2+4x^4\right)^{2/3}\right)\right)$$

Mathematica 12.3 output

$$\int\frac{\left(-3+2x^4\right)\left(1+2x^4\right)^{2/3}}{x^3\left(2-x^3+4x^4\right)}\,dx$$

24.361 Problem number 2026

$$\int\frac{\sqrt[3]{1-x^7}\left(-2+x^3+2x^7\right)\left(3+4x^7\right)}{x^2\left(-1+x^7\right)\left(-4+x^3+4x^7\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(-x^7+1\right)^{\frac{1}{3}}}{2x}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\,2^{\frac{2}{3}}\left(-x^7+1\right)^{\frac{1}{3}}}\right)2^{\frac{1}{3}}}{4}-\frac{\ln\left(-x+2^{\frac{2}{3}}\left(-x^7+1\right)^{\frac{1}{3}}\right)2^{\frac{1}{3}}}{4}$$
$$+\frac{\ln\left(x^2+2^{\frac{2}{3}}x\left(-x^7+1\right)^{\frac{1}{3}}+2\,2^{\frac{1}{3}}\left(-x^7+1\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{8}$$

command

```
Integrate[((1 - x^7)^(1/3)*(-2 + x^3 + 2*x^7)*(3 + 4*x^7))/(x^2*(-1 + x^7)*(-4 + x^3 + 4*x^7)
```

Mathematica 13.1 output

$$\frac{3\sqrt[3]{1-x^7}}{2x}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\,2^{2/3}\sqrt[3]{1-x^7}}\right)}{2\,2^{2/3}}-\frac{\log\left(-x+2^{2/3}\sqrt[3]{1-x^7}\right)}{2\,2^{2/3}}$$
$$+\frac{\log\left(x^2+2^{2/3}x\sqrt[3]{1-x^7}+2\sqrt[3]{2}\left(1-x^7\right)^{2/3}\right)}{4\,2^{2/3}}$$

Mathematica 12.3 output

$$\int\frac{\sqrt[3]{1-x^7}\left(-2+x^3+2x^7\right)\left(3+4x^7\right)}{x^2\left(-1+x^7\right)\left(-4+x^3+4x^7\right)}\,dx$$

24.362 Problem number 2031

$$\int \frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)\left(-1-x^3+x^4\right)}{x^6\left(-2-x^3+2x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^4-1\right)^{\frac{2}{3}}\left(4x^4-5x^3-4\right)}{40x^5}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\,2^{\frac{1}{3}}\left(x^4-1\right)^{\frac{1}{3}}}\right)2^{\frac{1}{3}}}{8}$$
$$-\frac{\ln\left(-x+2^{\frac{1}{3}}\left(x^4-1\right)^{\frac{1}{3}}\right)2^{\frac{1}{3}}}{8}+\frac{\ln\left(x^2+2^{\frac{1}{3}}x\left(x^4-1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\left(x^4-1\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{16}$$

command

```
Integrate[((-1 + x^4)^(2/3)*(3 + x^4)*(-1 - x^3 + x^4))/(x^6*(-2 - x^3 + 2*x^4)),x]
```

Mathematica 13.1 output

$$\frac{1}{80}\left(\frac{6\left(-1+x^4\right)^{2/3}\left(-4-5x^3+4x^4\right)}{x^5}+10\sqrt[3]{2}\,\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{2}\,\sqrt[3]{-1+x^4}}\right)\right.$$
$$\left.-10\sqrt[3]{2}\,\log\left(-x+\sqrt[3]{2}\,\sqrt[3]{-1+x^4}\right)+5\sqrt[3]{2}\,\log\left(x^2+\sqrt[3]{2}\,x\sqrt[3]{-1+x^4}+2^{2/3}\left(-1+x^4\right)^{2/3}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)\left(-1-x^3+x^4\right)}{x^6\left(-2-x^3+2x^4\right)}\,dx$$

24.363 Problem number 2036

$$\int \frac{-3-4x+3x^6}{\left(1+2x+x^6\right)\sqrt[3]{1+2x+2x^3+x^6}}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2^{\frac{2}{3}}\left(x^6+2x^3+2x+1\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{2}+\frac{\ln\left(-2x+2^{\frac{2}{3}}\left(x^6+2x^3+2x+1\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{2}$$
$$-\frac{\ln\left(2x^2+2^{\frac{2}{3}}x\left(x^6+2x^3+2x+1\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(x^6+2x^3+2x+1\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{4}$$

command

```
Integrate[(-3 - 4*x + 3*x^6)/((1 + 2*x + x^6)*(1 + 2*x + 2*x^3 + x^6)^(1/3)),x]
```

Mathematica 13.1 output

$$-\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2^{2/3}\sqrt[3]{1+2x+2x^3+x^6}}\right)-2\log\left(-2x+2^{2/3}\sqrt[3]{1+2x+2x^3+x^6}\right)+\log\left(2x^2+2^{2/3}x\sqrt[3]{1+}\right.}{2\sqrt[3]{2}}$$

Mathematica 12.3 output

$$\int \frac{-3-4x+3x^6}{(1+2x+x^6)\sqrt[3]{1+2x+2x^3+x^6}}\,dx$$

24.364 Problem number 2041

$$\int \frac{\left(2+x-x^3-x^4\right)^{2/3}\left(6+2x+x^4\right)\left(-2-x+x^3+x^4\right)}{x^6\left(-2-x+2x^3+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(-x^4-x^3+x+2\right)^{\frac{2}{3}}\left(2x^4-3x^3-2x-4\right)}{10x^5}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(-x^4-x^3+x+2\right)^{\frac{1}{3}}}\right)$$
$$-\ln\left(-x+\left(-x^4-x^3+x+2\right)^{\frac{1}{3}}\right)+\frac{\ln\left(x^2+x\left(-x^4-x^3+x+2\right)^{\frac{1}{3}}+\left(-x^4-x^3+x+2\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((2 + x - x^3 - x^4)^(2/3)*(6 + 2*x + x^4)*(-2 - x + x^3 + x^4))/(x^6*(-
2 - x + 2*x^3 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{3\left(2+x-x^3-x^4\right)^{2/3}\left(-4-2x-3x^3+2x^4\right)}{10x^5}+\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{2+x-x^3-x^4}}\right)$$
$$-\log\left(-x+\sqrt[3]{2+x-x^3-x^4}\right)+\frac{1}{2}\log\left(x^2+x\sqrt[3]{2+x-x^3-x^4}+\left(2+x-x^3-x^4\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(2+x-x^3-x^4\right)^{2/3}\left(6+2x+x^4\right)\left(-2-x+x^3+x^4\right)}{x^6\left(-2-x+2x^3+x^4\right)}\,dx$$

24.365 Problem number 2042

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}\left(2+x^3+2x^4\right)}{x^6\left(4-x^3+4x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^4+1\right)^{\frac{2}{3}}\left(8x^4+15x^3+8\right)}{80x^5}-\frac{3\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\,2^{\frac{2}{3}}\left(x^4+1\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{32}$$
$$+\frac{3\ln\left(-x+2^{\frac{2}{3}}\left(x^4+1\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{32}-\frac{3\ln\left(x^2+2^{\frac{2}{3}}x\left(x^4+1\right)^{\frac{1}{3}}+2\,2^{\frac{1}{3}}\left(x^4+1\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{64}$$

command

```
Integrate[((-3 + x^4)*(1 + x^4)^(2/3)*(2 + x^3 + 2*x^4))/(x^6*(4 - x^3 + 4*x^4)),x]
```

Mathematica 13.1 output

$$\frac{3}{320}\left(\frac{4\left(1+x^4\right)^{2/3}\left(8+15x^3+8x^4\right)}{x^5}\right.$$
$$\left.-10\,2^{2/3}\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\,2^{2/3}\sqrt[3]{1+x^4}}\right)+10\,2^{2/3}\log\left(-x+2^{2/3}\sqrt[3]{1+x^4}\right)-5\,2^{2/3}\log\left(x^2+2^{2/3}x\sqrt[3]{1+x^4}+2\right.\right.$$

Mathematica 12.3 output

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}\left(2+x^3+2x^4\right)}{x^6\left(4-x^3+4x^4\right)}\,dx$$

24.366 Problem number 2043

$$\int \frac{\left(-1-x^4+2x^6\right)\sqrt[3]{x-x^5+x^7}}{\left(1+x^2-x^4+x^6\right)^2}\,dx$$

Optimal antiderivative

$$-\frac{x\left(x^7-x^5+x\right)^{\frac{1}{3}}}{2x^6-2x^4+2x^2+2}-\frac{\arctan\left(\frac{\sqrt{3}\,\left(x^7-x^5+x\right)^{\frac{1}{3}}}{-2x+\left(x^7-x^5+x\right)^{\frac{1}{3}}}\right)\sqrt{3}}{6}$$
$$+\frac{\ln\left(x+\left(x^7-x^5+x\right)^{\frac{1}{3}}\right)}{6}-\frac{\ln\left(x^2-x\left(x^7-x^5+x\right)^{\frac{1}{3}}+\left(x^7-x^5+x\right)^{\frac{2}{3}}\right)}{12}$$

command

```
Integrate[((-1 - x^4 + 2*x^6)*(x - x^5 + x^7)^(1/3))/(1 + x^2 - x^4 + x^6)^2,x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x-x^5+x^7}\left(-\frac{6x^{4/3}}{1+x^2-x^4+x^6}+\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}-2\sqrt[3]{1-x^4+x^6}}\right)}{\sqrt[3]{1-x^4+x^6}}+\frac{2\log\left(x^{2/3}+\sqrt[3]{1-x^4+x^6}\right)}{\sqrt[3]{1-x^4+x^6}}-\frac{\log\left(x^{4/3}-x^{2/}\right.}{}\right.}{12\sqrt[3]{x}}$$

Mathematica 12.3 output

$$\int \frac{\left(-1-x^4+2x^6\right)\sqrt[3]{x-x^5+x^7}}{\left(1+x^2-x^4+x^6\right)^2}\,dx$$

24.367 Problem number 2044

$$\int \frac{x^4\left(-4b+ax^3\right)}{\sqrt[4]{-b+ax^3}\,\left(-b^2+2abx^3-a^2x^6+x^8\right)}\,dx$$

Optimal antiderivative

$$\arctan\left(\frac{\left(x^3a-b\right)^{\frac{1}{4}}}{x}\right)-\frac{\sqrt{2}\,\arctan\left(\frac{-\frac{x^2\sqrt{2}}{2}+\frac{\sqrt{x^3a-b}\,\sqrt{2}}{2}}{x\left(x^3a-b\right)^{\frac{1}{4}}}\right)}{2}$$
$$+\operatorname{arctanh}\left(\frac{x\left(x^3a-b\right)^{\frac{3}{4}}}{-x^3a+b}\right)+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,x\left(x^3a-b\right)^{\frac{1}{4}}}{x^2+\sqrt{x^3a-b}}\right)}{2}$$

command

```
Integrate[(x^4*(-4*b + a*x^3))/((-b + a*x^3)^(1/4)*(-b^2 + 2*a*b*x^3 - a^2*x^6 + x^8)),x]
```

Mathematica 13.1 output

$$\text{ArcTan}\left(\frac{\sqrt[4]{-b+ax^3}}{x}\right)-\frac{\text{ArcTan}\left(\frac{-x^2+\sqrt{-b+ax^3}}{\sqrt{2}\,x\sqrt[4]{-b+ax^3}}\right)}{\sqrt{2}}$$
$$-\tanh^{-1}\left(\frac{x}{\sqrt[4]{-b+ax^3}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{-b+ax^3}}{x^2+\sqrt{-b+ax^3}}\right)}{\sqrt{2}}$$

Mathematica 12.3 output

$$\int \frac{x^4\left(-4b+ax^3\right)}{\sqrt[4]{-b+ax^3}\,\left(-b^2+2abx^3-a^2x^6+x^8\right)}\,dx$$

24.368 Problem number 2053

$$\int \frac{\sqrt[3]{b-ax^6}\,\left(b+ax^6\right)}{x^2\left(-b+cx^3+ax^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(-a\,x^6+b\right)^{\frac{1}{3}}}{x}+\frac{c^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\;c^{\frac{1}{3}}x}{c^{\frac{1}{3}}x+2\left(-a\,x^6+b\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}+\frac{c^{\frac{1}{3}}\ln\left(-c^{\frac{1}{3}}x+\left(-a\,x^6+b\right)^{\frac{1}{3}}\right)}{3}$$
$$-\frac{c^{\frac{1}{3}}\ln\left(c^{\frac{2}{3}}x^2+c^{\frac{1}{3}}x\left(-a\,x^6+b\right)^{\frac{1}{3}}+\left(-a\,x^6+b\right)^{\frac{2}{3}}\right)}{6}$$

command

```
Integrate[((b - a*x^6)^(1/3)*(b + a*x^6))/(x^2*(-b + c*x^3 + a*x^6)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{b-ax^6}}{x}+\frac{\sqrt[3]{c}\,\mathrm{ArcTan}\left(\frac{\sqrt{3}\;\sqrt[3]{c}\,x}{\sqrt[3]{c}\,x+2\sqrt[3]{b-ax^6}}\right)}{\sqrt{3}}+\frac{1}{3}\sqrt[3]{c}\,\log\left(-\sqrt[3]{c}\,x+\sqrt[3]{b-ax^6}\right)$$
$$-\frac{1}{6}\sqrt[3]{c}\,\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x\sqrt[3]{b-ax^6}+\left(b-ax^6\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt[3]{b-ax^6}\,\left(b+ax^6\right)}{x^2\left(-b+cx^3+ax^6\right)}\,dx$$

24.369 Problem number 2055

$$\int \frac{2d+cx^4}{\sqrt[4]{-b+ax^4}\,\left(-2f+ex^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(2*d + c*x^4)/((-b + a*x^4)^(1/4)*(-2*f + e*x^8)),x]
```

Mathematica 13.1 output

$$\frac{\text{RootSum}\left[b^2e-2a^2f+4af\#1^4-2f\#1^8\&,\frac{bc\log(x)+2ad\log(x)-bc\log\left(\sqrt[4]{-b+ax^4}-x\#1\right)-2ad\log\left(\sqrt[4]{-b+ax^4}-x\#1\right.}{-a\#1+\#1^5}\right.}{16f}$$

Mathematica 12.3 output

$$\int \frac{2d+cx^4}{\sqrt[4]{-b+ax^4}\,(-2f+ex^8)}\,dx$$

24.370 Problem number 2067

$$\int \frac{\sqrt[3]{-1+2x^3+x^8}\,(3+5x^8)}{x^2\,(-1+x^8)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^8+2x^3-1\right)^{\frac{1}{3}}}{x}+2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2^{\frac{2}{3}}\left(x^8+2x^3-1\right)^{\frac{1}{3}}}\right)$$
$$+2^{\frac{1}{3}}\ln\left(-2x+2^{\frac{2}{3}}\left(x^8+2x^3-1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(2x^2+2^{\frac{2}{3}}x\left(x^8+2x^3-1\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(x^8+2x^3-1\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{2}$$

command

```
Integrate[((-1 + 2*x^3 + x^8)^(1/3)*(3 + 5*x^8))/(x^2*(-1 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{3\sqrt[3]{-1+2x^3+x^8}}{x}+\sqrt[3]{2}\,\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2^{2/3}\sqrt[3]{-1+2x^3+x^8}}\right)+\sqrt[3]{2}\,\log\left(-2x\right.$$
$$\left.+2^{2/3}\sqrt[3]{-1+2x^3+x^8}\right)-\frac{\log\left(2x^2+2^{2/3}x\sqrt[3]{-1+2x^3+x^8}+\sqrt[3]{2}\left(-1+2x^3+x^8\right)^{2/3}\right)}{2^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[3]{-1+2x^3+x^8}\,(3+5x^8)}{x^2\,(-1+x^8)}\,dx$$

24.371 Problem number 2069

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^3}\,dx$$

Optimal antiderivative

$$\frac{7x\left(15a\,x^2+23b^2\right)}{192b^4\left(a\,x^2+b^2\right)^{\frac{3}{2}}\sqrt{b+\sqrt{a\,x^2+b^2}}}+\frac{x\left(35a\,x^2+59b^2\right)}{96b^3\left(a\,x^2+b^2\right)^2\sqrt{b+\sqrt{a\,x^2+b^2}}}+\frac{35\arctan\left(\frac{\sqrt{a}\;x}{\sqrt{b}\;\sqrt{b+\sqrt{a\,x^2+b^2}}}\right)}{64\sqrt{a}\;b^{\frac{9}{2}}}$$

command

```
Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^3,x]
```

Mathematica 13.1 output

$$\frac{7x\left(23b^2+15ax^2\right)}{192b^4\left(b^2+ax^2\right)^{3/2}\sqrt{b+\sqrt{b^2+ax^2}}}+\frac{x\left(59b^2+35ax^2\right)}{96b^3\left(b^2+ax^2\right)^2\sqrt{b+\sqrt{b^2+ax^2}}}+\frac{35\text{ArcTan}\left(\frac{\sqrt{a}\;x}{\sqrt{b}\;\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{64\sqrt{a}\;b^{9/2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^3}\,dx$$

24.372 Problem number 2078

$$\int \frac{\left(-4+5x^7\right)\sqrt[3]{-2x+2x^3-x^8}}{\left(2+x^7\right)\left(2-2x^2+x^7\right)}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{3}\;\arctan\left(\frac{\sqrt{3}\;x}{x+2^{\frac{2}{3}}\left(-x^8+2x^3-2x\right)^{\frac{1}{3}}}\right)2^{\frac{1}{3}}}{2}-\frac{\ln\left(-2x+2^{\frac{2}{3}}\left(-x^8+2x^3-2x\right)^{\frac{1}{3}}\right)2^{\frac{1}{3}}}{2}+\frac{\ln\left(2x^2+2^{\frac{2}{3}}x\left(-x^8+2x^3-2x\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(-x^8+2x^3-2x\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{4}$$

command

```
Integrate[((-4 + 5*x^7)*(-2*x + 2*x^3 - x^8)^(1/3))/((2 + x^7)*(2 - 2*x^2 + x^7)),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\left(2-2x^2+x^7\right)^{2/3}\left(-2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}-2^{2/3}\sqrt[3]{2-2x^2+x^7}}\right)-2\log\left(2x^{2/3}+2^{2/3}\sqrt[3]{2-2x^2+x^7}\right)+\log\left(\right.\right.}{2\,2^{2/3}\left(-x\left(2-2x^2+x^7\right)\right)^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{\left(-4+5x^7\right)\sqrt[3]{-2x+2x^3-x^8}}{\left(2+x^7\right)\left(2-2x^2+x^7\right)}\,dx$$

24.373 Problem number 2086

$$\int \frac{\left(-6+x^2\right)\left(2-x^2+x^3\right)^{2/3}}{x^3\left(-2+x^2+x^3\right)}\,dx$$

Optimal antiderivative

$$-\frac{3\left(x^3-x^2+2\right)^{\frac{2}{3}}}{2x^2}+2^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2^{\frac{2}{3}}\left(x^3-x^2+2\right)^{\frac{1}{3}}}\right)$$
$$-2^{\frac{2}{3}}\ln\left(-2x+2^{\frac{2}{3}}\left(x^3-x^2+2\right)^{\frac{1}{3}}\right)+\frac{\ln\left(2x^2+2^{\frac{2}{3}}x\left(x^3-x^2+2\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(x^3-x^2+2\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{2}$$

command

```
Integrate[((-6 + x^2)*(2 - x^2 + x^3)^(2/3))/(x^3*(-2 + x^2 + x^3)),x]
```

Mathematica 13.1 output

$$-\frac{3\left(2-x^2+x^3\right)^{2/3}}{2x^2}$$
$$+2^{2/3}\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2^{2/3}\sqrt[3]{2-x^2+x^3}}\right)-2^{2/3}\log\left(-2x+2^{2/3}\sqrt[3]{2-x^2+x^3}\right)+\frac{\log\left(2x^2+2^{2/3}x\sqrt[3]{2-x^2+x}\right.}{\sqrt[3]{2}}$$

Mathematica 12.3 output

$$\int \frac{\left(-6+x^2\right)\left(2-x^2+x^3\right)^{2/3}}{x^3\left(-2+x^2+x^3\right)}\,dx$$

24.374 Problem number 2087

$$\int \frac{x^3(-4a+3x)}{\left(x^2(-a+x)\right)^{2/3}(ad-dx+x^4)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x^2}{x^2+2d^{\frac{1}{3}}\left(-a\,x^2+x^3\right)^{\frac{1}{3}}}\right)}{d^{\frac{1}{3}}}+\frac{\ln\left(a\,x^2-a\,d^{\frac{1}{3}}\left(-a\,x^2+x^3\right)^{\frac{1}{3}}\right)}{d^{\frac{1}{3}}}$$
$$-\frac{\ln\left(a^2x^4+a^2d^{\frac{1}{3}}x^2\left(-a\,x^2+x^3\right)^{\frac{1}{3}}+a^2d^{\frac{2}{3}}\left(-a\,x^2+x^3\right)^{\frac{2}{3}}\right)}{2d^{\frac{1}{3}}}$$

command

```
Integrate[(x^3*(-4*a + 3*x))/((x^2*(-a + x))^(2/3)*(a*d - d*x + x^4)),x]
```

Mathematica 13.1 output

$$\frac{x^{4/3}(-a+x)^{2/3}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{4/3}}{x^{4/3}+2\sqrt[3]{d}\,\sqrt[3]{-a+x}}\right)+2\log\left(a\left(x^{4/3}-\sqrt[3]{d}\,\sqrt[3]{-a+x}\right)\right)-\log\left(a^2\left(x^{8/3}+\sqrt[3]{d}\,\right.\right.\right.}{2\sqrt[3]{d}\,\left(x^2(-a+x)\right)^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{x^3(-4a+3x)}{\left(x^2(-a+x)\right)^{2/3}(ad-dx+x^4)}\,dx$$

24.375 Problem number 2089

$$\int \frac{1-x^4}{(1+x^4)\,\sqrt[4]{x^3+x^5}}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{2^{\frac{1}{8}}x}{\left(x^5+x^3\right)^{\frac{1}{4}}}\right)2^{\frac{7}{8}}}{2}-\frac{\arctan\left(\frac{2^{\frac{5}{8}}x\left(x^5+x^3\right)^{\frac{1}{4}}}{x^2\sqrt{2}^{\frac{1}{4}}-\sqrt{x^5+x^3}}\right)2^{\frac{3}{8}}}{2}$$
$$+\frac{\text{arctanh}\left(\frac{2^{\frac{1}{8}}x}{\left(x^5+x^3\right)^{\frac{1}{4}}}\right)2^{\frac{7}{8}}}{2}+\frac{\text{arctanh}\left(\frac{\frac{x^2\sqrt{2}^{\frac{5}{8}}}{2}+\frac{\sqrt{x^5+x^3}\,\sqrt{2}^{\frac{3}{8}}}{2}}{x\left(x^5+x^3\right)^{\frac{1}{4}}}\right)2^{\frac{3}{8}}}{2}$$

command

```
Integrate[(1 - x^4)/((1 + x^4)*(x^3 + x^5)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{x^{3/4}\sqrt[4]{1+x^2}\left(\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt[8]{2}\sqrt[4]{x}}{\sqrt[4]{1+x^2}}\right)-\text{ArcTan}\left(\frac{2^{5/8}\sqrt[4]{x}\sqrt[4]{1+x^2}}{\sqrt[4]{2}\sqrt{x}-\sqrt{1+x^2}}\right)+\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt[8]{2}\sqrt[4]{x}}{\sqrt[4]{1+x^2}}\right)+\tanh^{-1}\right.}{2^{5/8}\sqrt[4]{x^3+x^5}}$$

Mathematica 12.3 output

$$\int \frac{1-x^4}{(1+x^4)\sqrt[4]{x^3+x^5}}\,dx$$

24.376 Problem number 2092

$$\int \frac{\left(2b+ax^6\right)\left(-b-cx^4+ax^6\right)}{x^2\left(-b+ax^6\right)^{3/4}\left(-b+cx^4+ax^6\right)}\,dx$$

Optimal antiderivative

$$\frac{2\left(a\,x^6-b\right)^{\frac{1}{4}}}{x}+\sqrt{2}\;c^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\;c^{\frac{1}{4}}x\left(a\,x^6-b\right)^{\frac{1}{4}}}{-x^2\sqrt{c}\;+\sqrt{a\,x^6-b}}\right)$$
$$-\sqrt{2}\;c^{\frac{1}{4}}\operatorname{arctanh}\left(\frac{\frac{c^{\frac{1}{4}}x^2\sqrt{2}}{2}+\frac{\sqrt{a\,x^6-b}\;\sqrt{2}}{2c^{\frac{1}{4}}}}{x\left(a\,x^6-b\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[((2*b + a*x^6)*(-b - c*x^4 + a*x^6))/(x^2*(-b + a*x^6)^(3/4)*(-b + c*x^4 + a*x^6)),
```

Mathematica 13.1 output

$$\frac{2\sqrt[4]{-b+ax^6}}{x}+\sqrt{2}\;\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\;\sqrt[4]{c}\,x\sqrt[4]{-b+ax^6}}{-\sqrt{c}\,x^2+\sqrt{-b+ax^6}}\right)$$
$$-\sqrt{2}\;\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2+\sqrt{-b+ax^6}}{\sqrt{2}\;\sqrt[4]{c}\,x\sqrt[4]{-b+ax^6}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(2b+ax^6\right)\left(-b-cx^4+ax^6\right)}{x^2\left(-b+ax^6\right)^{3/4}\left(-b+cx^4+ax^6\right)}\,dx$$

24.377 Problem number 2094

$$\int \frac{\left(-b+a^2x^4\right)\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx$$

Optimal antiderivative

$$\frac{3\sqrt{a}\;b\,x^2+2a^{\frac{5}{2}}x^6+2a^{\frac{3}{2}}x^4\sqrt{a^2x^4+b}}{8\sqrt{a}\;x\sqrt{a\,x^2+\sqrt{a^2x^4+b}}}$$
$$-\frac{11b\ln\left(a\,x^2+\sqrt{a^2x^4+b}+\sqrt{2}\;\sqrt{a}\;x\sqrt{a\,x^2+\sqrt{a^2x^4+b}}\right)\sqrt{2}}{16\sqrt{a}}$$

command

```
Integrate[((-b + a^2*x^4)*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]])/Sqrt[b + a^2*x^4],x]
```

Mathematica 13.1 output

$$\frac{3bx+2ax^3\left(ax^2+\sqrt{b+a^2x^4}\right)}{8\sqrt{ax^2+\sqrt{b+a^2x^4}}}-\frac{11b\tanh^{-1}\left(\frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{2}\;\sqrt{a}\;x}\right)}{8\sqrt{2}\;\sqrt{a}}$$

Mathematica 12.3 output

$$\int \frac{\left(-b+a^2x^4\right)\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx$$

24.378 Problem number 2095

$$\int \sqrt{b+a^2x^4}\,\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Optimal antiderivative

$$\frac{3\sqrt{a}\;b\,x^2+2a^{\frac{5}{2}}x^6+2a^{\frac{3}{2}}x^4\sqrt{a^2x^4+b}}{8\sqrt{a}\;x\sqrt{a\,x^2+\sqrt{a^2x^4+b}}}$$
$$+\frac{5b\ln\left(a\,x^2+\sqrt{a^2x^4+b}+\sqrt{2}\;\sqrt{a}\;x\sqrt{a\,x^2+\sqrt{a^2x^4+b}}\right)\sqrt{2}}{16\sqrt{a}}$$

command

```
Integrate[Sqrt[b + a^2*x^4]*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
```

Mathematica 13.1 output

$$\frac{3bx + 2ax^3\left(ax^2 + \sqrt{b + a^2x^4}\right)}{8\sqrt{ax^2 + \sqrt{b + a^2x^4}}} + \frac{5b\tanh^{-1}\left(\frac{\sqrt{ax^2 + \sqrt{b + a^2x^4}}}{\sqrt{2}\,\sqrt{a}\,x}\right)}{8\sqrt{2}\,\sqrt{a}}$$

Mathematica 12.3 output

$$\int \sqrt{b + a^2x^4}\,\sqrt{ax^2 + \sqrt{b + a^2x^4}}\,dx$$

24.379 Problem number 2098

$$\int \frac{\left(-b + ax^5\right)^{3/4}\left(4b + ax^5\right)}{x^4\left(-b + cx^4 + ax^5\right)}\,dx$$

Optimal antiderivative

$$\frac{4\left(a\,x^5 - b\right)^{\frac{3}{4}}}{3x^3} + \sqrt{2}\;c^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\;c^{\frac{1}{4}}x\left(a\,x^5 - b\right)^{\frac{1}{4}}}{-x^2\sqrt{c} + \sqrt{a\,x^5 - b}}\right) + \sqrt{2}\;c^{\frac{3}{4}}\operatorname{arctanh}\left(\frac{\frac{c^{\frac{1}{4}}x^2\sqrt{2}}{2} + \frac{\sqrt{a\,x^5 - b}\;\sqrt{2}}{2c^{\frac{1}{4}}}}{x\left(a\,x^5 - b\right)^{\frac{1}{4}}}\right)$$

command

```
Integrate[((-b + a*x^5)^(3/4)*(4*b + a*x^5))/(x^4*(-b + c*x^4 + a*x^5)),x]
```

Mathematica 13.1 output

$$\frac{4\left(-b + ax^5\right)^{3/4}}{3x^3} + \sqrt{2}\,c^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x\,\sqrt[4]{-b + ax^5}}{-\sqrt{c}\,x^2 + \sqrt{-b + ax^5}}\right) + \sqrt{2}\,c^{3/4}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2 + \sqrt{-b + ax^5}}{\sqrt{2}\,\sqrt[4]{c}\,x\,\sqrt[4]{-b + ax^5}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-b + ax^5\right)^{3/4}\left(4b + ax^5\right)}{x^4\left(-b + cx^4 + ax^5\right)}\,dx$$

24.380 Problem number 2101

$$\int \frac{\left(-1+x^4\right)^{3/4}}{-1+2x^4+x^8}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{2^{\frac{1}{8}}x}{(x^4-1)^{\frac{1}{4}}}\right)2^{\frac{7}{8}}}{8}-\frac{\arctan\left(\frac{2^{\frac{5}{8}}x(x^4-1)^{\frac{1}{4}}}{x^2 2^{\frac{1}{4}}-\sqrt{x^4-1}}\right)2^{\frac{3}{8}}}{8}$$
$$+\frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{8}}x}{(x^4-1)^{\frac{1}{4}}}\right)2^{\frac{7}{8}}}{8}+\frac{\operatorname{arctanh}\left(\frac{2\,2^{\frac{3}{8}}x(x^4-1)^{\frac{1}{4}}}{2x^2+2^{\frac{3}{4}}\sqrt{x^4-1}}\right)2^{\frac{3}{8}}}{8}$$

command

```
Integrate[(-1 + x^4)^(3/4)/(-1 + 2*x^4 + x^8),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{-1+x^4}}\right)-\text{ArcTan}\left(\frac{2^{5/8}x\sqrt[4]{-1+x^4}}{\sqrt[4]{2}\,x^2-\sqrt{-1+x^4}}\right)+\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{-1+x^4}}\right)+\tanh^{-1}\left(\frac{2\,2^{3/8}x\sqrt[4]{-}}{2x^2+2^{3/4}\sqrt{-}}\right.}{4\,2^{5/8}}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^4\right)^{3/4}}{-1+2x^4+x^8}\,dx$$

24.381 Problem number 2102

$$\int \frac{1+x^4}{\left(-1+x^4\right)\sqrt{1+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

$$\frac{2x}{\sqrt{1+\sqrt{x^2+1}}}-2\arctan\left(\frac{x}{\sqrt{1+\sqrt{x^2+1}}}\right)+\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2\sqrt{1+\sqrt{x^2+1}}}\right)$$
$$-\sqrt{1+\sqrt{2}}\,\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}\,\sqrt{1+\sqrt{x^2+1}}}\right)$$
$$-\sqrt{\sqrt{2}-1}\,\operatorname{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}\,\sqrt{1+\sqrt{x^2+1}}}\right)$$

command

```
Integrate[(1 + x^4)/((-1 + x^4)*Sqrt[1 + Sqrt[1 + x^2]]),x]
```

Mathematica 13.1 output

$$\frac{2x}{\sqrt{1+\sqrt{1+x^2}}} - 2\text{ArcTan}\left(\frac{x}{\sqrt{1+\sqrt{1+x^2}}}\right) + \sqrt{2}\,\text{ArcTan}\left(\frac{x}{\sqrt{2}\,\sqrt{1+\sqrt{1+x^2}}}\right)$$
$$- \sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{-1+\sqrt{2}}\,x}{\sqrt{1+\sqrt{1+x^2}}}\right) - \sqrt{-1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{1+\sqrt{2}}\,x}{\sqrt{1+\sqrt{1+x^2}}}\right)$$

Mathematica 12.3 output

$$\int \frac{1+x^4}{(-1+x^4)\sqrt{1+\sqrt{1+x^2}}}\,dx$$

24.382 Problem number 2115

$$\int \frac{(1+x^4)\sqrt{1+\sqrt{1+x^2}}}{-1+x^4}\,dx$$

Optimal antiderivative

$$\frac{4x}{3\sqrt{1+\sqrt{x^2+1}}} + \frac{2x\sqrt{x^2+1}}{3\sqrt{1+\sqrt{x^2+1}}} - 2\arctan\left(\frac{x}{\sqrt{1+\sqrt{x^2+1}}}\right)$$
$$+ \sqrt{\sqrt{2}-1}\,\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}\,\sqrt{1+\sqrt{x^2+1}}}\right)$$
$$- \sqrt{1+\sqrt{2}}\,\text{arctanh}\left(\frac{x}{\sqrt{\sqrt{2}-1}\,\sqrt{1+\sqrt{x^2+1}}}\right)$$

command

```
Integrate[((1 + x^4)*Sqrt[1 + Sqrt[1 + x^2]])/(-1 + x^4),x]
```

Mathematica 13.1 output

$$\frac{4x}{3\sqrt{1+\sqrt{1+x^2}}} + \frac{2x\sqrt{1+x^2}}{3\sqrt{1+\sqrt{1+x^2}}} - 2\text{ArcTan}\left(\frac{x}{\sqrt{1+\sqrt{1+x^2}}}\right)$$
$$+ \sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{-1+\sqrt{2}}\,x}{\sqrt{1+\sqrt{1+x^2}}}\right) - \sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{1+\sqrt{2}}\,x}{\sqrt{1+\sqrt{1+x^2}}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^4\right)\sqrt{1+\sqrt{1+x^2}}}{-1+x^4}\,dx$$

24.383 Problem number 2116

$$\int \frac{-b^2+ax^2}{\left(b^2+ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Optimal antiderivative

$$\frac{2x}{\sqrt{b+\sqrt{a\,x^2+b^2}}}-\frac{4\sqrt{b}\,\arctan\left(\frac{\sqrt{a}\,x}{\sqrt{b}\,\sqrt{b+\sqrt{a\,x^2+b^2}}}\right)}{\sqrt{a}}$$
$$+\frac{2\sqrt{2}\,\sqrt{b}\,\arctan\left(\frac{\sqrt{a}\,x\sqrt{2}}{2\sqrt{b}\,\sqrt{b+\sqrt{a\,x^2+b^2}}}-\frac{\sqrt{b+\sqrt{a\,x^2+b^2}}\,\sqrt{2}}{2\sqrt{b}}\right)}{\sqrt{a}}$$

command

```
Integrate[(-b^2 + a*x^2)/((b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

Mathematica 13.1 output

$$\frac{2x}{\sqrt{b+\sqrt{b^2+ax^2}}}-\frac{4\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\,\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{a}}$$
$$+\frac{\sqrt{2}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\,\sqrt{b}\,\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{a}}$$

Mathematica 12.3 output

$$\int \frac{-b^2+ax^2}{\left(b^2+ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

24.384 Problem number 2122

$$\int \frac{\left(-3+x^4\right)\left(1-x^3+x^4\right)^{2/3}}{x^3\left(1+x^3+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^4-x^3+1\right)^{\frac{2}{3}}}{2x^2}+2^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2^{\frac{2}{3}}\left(x^4-x^3+1\right)^{\frac{1}{3}}}\right)$$
$$+2^{\frac{2}{3}}\ln\left(2x+2^{\frac{2}{3}}\left(x^4-x^3+1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(-2x^2+2^{\frac{2}{3}}x\left(x^4-x^3+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\left(x^4-x^3+1\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{2}$$

command

```
Integrate[((-3 + x^4)*(1 - x^3 + x^4)^(2/3))/(x^3*(1 + x^3 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{3\left(1-x^3+x^4\right)^{2/3}}{2x^2}$$
$$+2^{2/3}\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{-x+2^{2/3}\sqrt[3]{1-x^3+x^4}}\right)+2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3+x^4}\right)-\frac{\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3+}\right.}{\sqrt[3]{2}}$$

Mathematica 12.3 output

$$\int \frac{\left(-3+x^4\right)\left(1-x^3+x^4\right)^{2/3}}{x^3\left(1+x^3+x^4\right)}\,dx$$

24.385 Problem number 2123

$$\int \frac{\left(-4b+ax^5\right)\left(b+ax^5\right)^{3/4}}{x^4\left(2b+cx^4+2ax^5\right)}\,dx$$

Optimal antiderivative

$$\frac{2\left(a\,x^5+b\right)^{\frac{3}{4}}}{3x^3}+\frac{c^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{3}{4}}c^{\frac{1}{4}}x\left(a\,x^5+b\right)^{\frac{1}{4}}}{-x^2\sqrt{c}+\sqrt{2}\,\sqrt{a\,x^5+b}}\right)2^{\frac{3}{4}}}{4}+\frac{c^{\frac{3}{4}}\,\text{arctanh}\left(\frac{\frac{c^{\frac{1}{4}}x^2 2^{\frac{1}{4}}}{2}+\frac{\sqrt{a\,x^5+b}\;2^{\frac{3}{4}}}{2c^{\frac{1}{4}}}}{x\left(a\,x^5+b\right)^{\frac{1}{4}}}\right)2^{\frac{3}{4}}}{4}$$

command

```
Integrate[((-4*b + a*x^5)*(b + a*x^5)^(3/4))/(x^4*(2*b + c*x^4 + 2*a*x^5)),x]
```

Mathematica 13.1 output

$$\frac{1}{12}\left(\frac{8\left(b+ax^5\right)^{3/4}}{x^3}\right.$$
$$\left.+3\,2^{3/4}c^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{2^{3/4}\sqrt[4]{b+ax^5}}-\frac{\sqrt[4]{b+ax^5}}{\sqrt{2}\,\sqrt[4]{c}\,x}\right)+3\,2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{2^{3/4}\sqrt[4]{b+ax^5}}+\frac{\sqrt[4]{b+ax^5}}{\sqrt{2}\,\sqrt[4]{c}\,x}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-4b+ax^5\right)\left(b+ax^5\right)^{3/4}}{x^4\left(2b+cx^4+2ax^5\right)}\,dx$$

24.386 Problem number 2124

$$\int \frac{-1+x^6}{\sqrt[3]{x^2+x^4}\,\left(1+x^6\right)}\,dx$$

Optimal antiderivative

$$-\frac{\left(x^4+x^2\right)^{\frac{2}{3}}}{x\left(x^2+1\right)}+\frac{\arctan\left(\frac{3^{\frac{2}{3}}x\left(x^4+x^2\right)^{\frac{1}{3}}}{3^{\frac{1}{3}}x^2-\left(x^4+x^2\right)^{\frac{2}{3}}}\right)3^{\frac{1}{3}}}{3}$$
$$-\frac{2\,\text{arctanh}\left(\frac{3^{\frac{1}{6}}x}{\left(x^4+x^2\right)^{\frac{1}{3}}}\right)3^{\frac{5}{6}}}{9}-\frac{\text{arctanh}\left(\frac{3^{\frac{1}{6}}x^2+\frac{\left(x^4+x^2\right)^{\frac{2}{3}}3^{\frac{5}{6}}}{3}}{x\left(x^4+x^2\right)^{\frac{1}{3}}}\right)3^{\frac{5}{6}}}{9}$$

command

```
Integrate[(-1 + x^6)/((x^2 + x^4)^(1/3)*(1 + x^6)),x]
```

Mathematica 13.1 output

$$-\frac{x^{2/3}\left(9\sqrt[3]{x}-3\sqrt[3]{3}\,\sqrt[3]{1+x^2}\,\text{ArcTan}\left(\frac{3^{2/3}\sqrt[3]{x}\,\sqrt[3]{1+x^2}}{\sqrt[3]{3}\,x^{2/3}-(1+x^2)^{2/3}}\right)+2\,3^{5/6}\sqrt[3]{1+x^2}\,\tanh^{-1}\left(\frac{\sqrt[6]{3}\,\sqrt[3]{x}}{\sqrt[3]{1+x^2}}\right)+3^{5/6}\sqrt[3]{1+x}\right.}{9\sqrt[3]{x^2+x^4}}$$

Mathematica 12.3 output

$$\int \frac{-1+x^6}{\sqrt[3]{x^2+x^4}\,\left(1+x^6\right)}\,dx$$

24.387 Problem number 2130

$$\int \frac{-b+ax^8}{(b+ax^8)\sqrt[4]{b-cx^4+ax^8}}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{2}\,c^{\frac{1}{4}}x(ax^8-cx^4+b)^{\frac{1}{4}}}{-x^2\sqrt{c}+\sqrt{ax^8-cx^4+b}}\right)\sqrt{2}}{4c^{\frac{1}{4}}}-\frac{\operatorname{arctanh}\left(\frac{\frac{c^{\frac{1}{4}}x^2\sqrt{2}}{2}+\frac{\sqrt{ax^8-cx^4+b}\,\sqrt{2}}{2c^{\frac{1}{4}}}}{x(ax^8-cx^4+b)^{\frac{1}{4}}}\right)\sqrt{2}}{4c^{\frac{1}{4}}}$$

command

```
Integrate[(-b + a*x^8)/((b + a*x^8)*(b - c*x^4 + a*x^8)^(1/4)),x]
```

Mathematica 13.1 output

$$-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x\sqrt[4]{b-cx^4+ax^8}}{-\sqrt{c}\,x^2+\sqrt{b-cx^4+ax^8}}\right)+\tanh^{-1}\left(\frac{\sqrt{c}\,x^2+\sqrt{b-cx^4+ax^8}}{\sqrt{2}\,\sqrt[4]{c}\,x\sqrt[4]{b-cx^4+ax^8}}\right)}{2\sqrt{2}\,\sqrt[4]{c}}$$

Mathematica 12.3 output

$$\int \frac{-b+ax^8}{(b+ax^8)\sqrt[4]{b-cx^4+ax^8}}\,dx$$

24.388 Problem number 2131

$$\int \frac{1}{x^2\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Optimal antiderivative

$$-\frac{4\sqrt{a^2x^2-bx}\left(-8192a^6x^3-5120a^4bx^2-4032a^2b^2x+3003b^3\right)\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}}{45045b^5x^5}$$
$$+\frac{4\left(8192a^7x^3+1024a^5bx^2+448a^3b^2x+6699ab^3\right)\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}}{45045b^5x^4}$$

command

```
Integrate[1/(x^2*Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2)),x]
```

Mathematica 13.1 output

$$\frac{4\sqrt{x\left(ax+\sqrt{x\left(-b+a^2x\right)}\right)}\left(3003b^4+1024a^5bx^2\left(-3ax+\sqrt{x\left(-b+a^2x\right)}\right)+8192a^7x^3\left(ax+\sqrt{x\left(-b+a^2x\right)}\right.\right.}{45045b^5x^4\sqrt{x\left(-b+a^2x\right)}}$$

Mathematica 12.3 output

$$\int \frac{1}{x^2\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

24.389 Problem number 2133

$$\int \frac{-2+(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(b-b(1+k)x+(-1+bk)x^2)}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2b^{\frac{1}{3}}\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}}\right)}{b^{\frac{2}{3}}}+\frac{\ln\left(x-b^{\frac{1}{3}}\left(x+(-1-k)\,x^2+k\,x^3\right)^{\frac{1}{3}}\right)}{b^{\frac{2}{3}}}$$
$$-\frac{\ln\left(x^2+b^{\frac{1}{3}}x\left(x+(-1-k)\,x^2+k\,x^3\right)^{\frac{1}{3}}+b^{\frac{2}{3}}\left(x+(-1-k)\,x^2+k\,x^3\right)^{\frac{2}{3}}\right)}{2b^{\frac{2}{3}}}$$

command

```
Integrate[(-2 + (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(b - b*(1 + k)*x + (-1 + b*k)*x^2)),x
```

Mathematica 13.1 output

$$\frac{x\sqrt[3]{\frac{-1+kx}{-1+x}}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{\frac{-1+kx}{-1+x}}}{2\left(\frac{x}{-1+x}\right)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{\frac{-1+kx}{-1+x}}}\right)+2\log\left(\left(\frac{x}{-1+x}\right)^{2/3}-\sqrt[3]{b}\,\sqrt[3]{\frac{-1+kx}{-1+x}}\right)-\log\left(\left(\right.\right.\right.}{2b^{2/3}\left(\frac{x}{-1+x}\right)^{2/3}\sqrt[3]{(-1+x)x(-1+kx)}}$$

Mathematica 12.3 output

$$\int \frac{-2+(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(b-b(1+k)x+(-1+bk)x^2)}\,dx$$

24.390 Problem number 2137

$$\int \frac{\left(-6+x^2\right)\left(-2+x^2\right)\left(2-x^2+x^3\right)\sqrt[3]{-2+x^2+2x^3}}{x^5\left(-2+x^2+x^3\right)^2}\,dx$$

Optimal antiderivative

$$\frac{\left(2x^3+x^2-2\right)^{\frac{1}{3}}\left(-38x^6-27x^5+3x^4+54x^3-12x^2+12\right)}{4x^4\left(x^3+x^2-2\right)}$$
$$-\frac{7\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(2x^3+x^2-2\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}-\frac{7\ln\left(-x+\left(2x^3+x^2-2\right)^{\frac{1}{3}}\right)}{3}$$
$$+\frac{7\ln\left(x^2+x\left(2x^3+x^2-2\right)^{\frac{1}{3}}+\left(2x^3+x^2-2\right)^{\frac{2}{3}}\right)}{6}$$

command

```
Integrate[((-6 + x^2)*(-2 + x^2)*(2 - x^2 + x^3)*(-2 + x^2 + 2*x^3)^(1/3))/(x^5*(-
2 + x^2 + x^3)^2),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{-2+x^2+2x^3}\left(12-12x^2+54x^3+3x^4-27x^5-38x^6\right)}{4x^4\left(-2+x^2+x^3\right)}$$
$$-\frac{7\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{-2+x^2+2x^3}}\right)}{\sqrt{3}}-\frac{7}{3}\log\left(-x+\sqrt[3]{-2+x^2+2x^3}\right)$$
$$+\frac{7}{6}\log\left(x^2+x\sqrt[3]{-2+x^2+2x^3}+\left(-2+x^2+2x^3\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-6+x^2\right)\left(-2+x^2\right)\left(2-x^2+x^3\right)\sqrt[3]{-2+x^2+2x^3}}{x^5\left(-2+x^2+x^3\right)^2}\,dx$$

24.391 Problem number 2140

$$\int \frac{(1+x)\sqrt[4]{x^3+x^5}}{x\left(-1+x^3\right)}\,dx$$

Optimal antiderivative

$$\frac{2\,2^{\frac{1}{4}}\arctan\left(\frac{2^{\frac{1}{4}}x}{(x^5+x^3)^{\frac{1}{4}}}\right)}{3}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x(x^5+x^3)^{\frac{1}{4}}}{-x^2+\sqrt{x^5+x^3}}\right)}{3}$$
$$-\frac{2\,2^{\frac{1}{4}}\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}x}{(x^5+x^3)^{\frac{1}{4}}}\right)}{3}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\frac{x^2\sqrt{2}}{2}+\frac{\sqrt{x^5+x^3}\,\sqrt{2}}{2}}{x(x^5+x^3)^{\frac{1}{4}}}\right)}{3}$$

command

```
Integrate[((1 + x)*(x^3 + x^5)^(1/4))/(x*(-1 + x^3)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{2}\,\sqrt[4]{x^3+x^5}\left(2\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{x}}{\sqrt[4]{1+x^2}}\right)+\sqrt[4]{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{x}\,\sqrt[4]{1+x^2}}{\sqrt{x}-\sqrt{1+x^2}}\right)-2\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{x}}{\sqrt[4]{1+x^2}}\right)+\sqrt[4]{2}\,\tanh^{-}\right.}{3x^{3/4}\sqrt[4]{1+x^2}}$$

Mathematica 12.3 output

$$\int\frac{(1+x)\sqrt[4]{x^3+x^5}}{x\left(-1+x^3\right)}\,dx$$

24.392 Problem number 2144

$$\int\frac{-1+x+x^3}{\left(1-x+x^3\right)\sqrt[3]{-x^2+x^3}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-1 + x + x^3)/((1 - x + x^3)*(-x^2 + x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{-1+x}\,x^{2/3}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{2\sqrt[3]{-1+x}+\sqrt[3]{x}}\right)-2\log\left(\sqrt[3]{-1+x}-\sqrt[3]{x}\right)+\log\left((-1+x)^{2/3}+\sqrt[3]{-1+x}\,\sqrt[3]{x}\right.\right.}{2\sqrt[3]{(-1+x)x^2}}$$

Mathematica 12.3 output

$$\int\frac{-1+x+x^3}{\left(1-x+x^3\right)\sqrt[3]{-x^2+x^3}}\,dx$$

24.393 Problem number 2145

$$\int \frac{-1+x+x^3}{(1-x+x^3)\sqrt[3]{-x^2+x^3}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-1 + x + x^3)/((1 - x + x^3)*(-x^2 + x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{-1+x}\,x^{2/3}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{2\sqrt[3]{-1+x}+\sqrt[3]{x}}\right)-2\log\left(\sqrt[3]{-1+x}-\sqrt[3]{x}\right)+\log\left((-1+x)^{2/3}+\sqrt[3]{-1+x}\,\sqrt[3]{x}\right.\right.}{2\sqrt[3]{(-1+x)x^2}}$$

Mathematica 12.3 output

$$\int \frac{-1+x+x^3}{(1-x+x^3)\sqrt[3]{-x^2+x^3}}\,dx$$

24.394 Problem number 2150

$$\int \frac{x^3(-4a+3x)}{\left(x^2(-a+x)\right)^{2/3}(a-x+dx^4)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,d^{\frac{1}{3}}x^2}{d^{\frac{1}{3}}x^2+2(-a\,x^2+x^3)^{\frac{1}{3}}}\right)}{d^{\frac{2}{3}}}+\frac{\ln\left(-a\,d^{\frac{1}{3}}x^2+a\left(-a\,x^2+x^3\right)^{\frac{1}{3}}\right)}{d^{\frac{2}{3}}}$$
$$-\frac{\ln\left(a^2d^{\frac{2}{3}}x^4+a^2d^{\frac{1}{3}}x^2\left(-a\,x^2+x^3\right)^{\frac{1}{3}}+a^2\left(-a\,x^2+x^3\right)^{\frac{2}{3}}\right)}{2d^{\frac{2}{3}}}$$

command

```
Integrate[(x^3*(-4*a + 3*x))/((x^2*(-a + x))^(2/3)*(a - x + d*x^4)),x]
```

Mathematica 13.1 output

$$\frac{x^{4/3}(-a+x)^{2/3}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{d}\,x^{4/3}}{\sqrt[3]{d}\,x^{4/3}+2\sqrt[3]{-a+x}}\right)+2\log\left(a\left(-\sqrt[3]{d}\,x^{4/3}+\sqrt[3]{-a+x}\right)\right)-\log\left(a^2\left(d^{2/3}x^{8/3}+\right.\right.\right.}{2d^{2/3}\left(x^2(-a+x)\right)^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{x^3(-4a+3x)}{\left(x^2(-a+x)\right)^{2/3}(a-x+dx^4)}\,dx$$

24.395 Problem number 2154

$$\int x^2 \sqrt{ax^2 + \sqrt{b + a^2 x^4}}\, dx$$

Optimal antiderivative

$$-\frac{i\left(2ia\,x^4\sqrt{a^2x^4+b} + ix^2\left(2a^2x^4 - b\right)\right)}{8ax\sqrt{a\,x^2+\sqrt{a^2x^4+b}}} + \frac{b\ln\left(ia\,x^2 + i\sqrt{a^2x^4+b} + i\sqrt{2}\,\sqrt{a}\,x\sqrt{a\,x^2+\sqrt{a^2x^4+b}}\right)\sqrt{2}}{16a^{\frac{3}{2}}}$$

command

```
Integrate[x^2*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
```

Mathematica 13.1 output

$$\frac{-bx + 2ax^3\left(ax^2 + \sqrt{b + a^2x^4}\right)}{8a\sqrt{ax^2 + \sqrt{b + a^2x^4}}} + \frac{b\tanh^{-1}\left(\frac{\sqrt{ax^2 + \sqrt{b + a^2x^4}}}{\sqrt{2}\,\sqrt{a}\,x}\right)}{8\sqrt{2}\,a^{3/2}}$$

Mathematica 12.3 output

$$\int x^2 \sqrt{ax^2 + \sqrt{b + a^2x^4}}\, dx$$

24.396 Problem number 2167

$$\int \frac{\left(-q + 2px^3\right)\sqrt{q^2 - 2pqx^2 + 2pqx^3 + p^2x^6}}{x\left(bx^2 + a\left(q + px^3\right)^2\right)}\, dx$$

Optimal antiderivative

$$\frac{\sqrt{2apq+b}\,\operatorname{arctanh}\left(\frac{\sqrt{b}\,\sqrt{2apq+b}\,x^2}{a\,q^2+b\,x^2+2apq\,x^3+a\,p^2x^6+(apx^3+aq)\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}}\right)}{a\sqrt{b}} - \frac{\ln(x)}{a} + \frac{\ln\left(q + p\,x^3 + \sqrt{p^2x^6 + 2pq\,x^3 - 2pq\,x^2 + q^2}\right)}{a}$$

command

```
Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6])/(x*(b*x^2 + a*(q + p*x
```

Mathematica 13.1 output

$$\frac{\tanh^{-1}\left(\frac{\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}}{q+px^3}\right)+\frac{\sqrt{b+2apq}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{b+2apq}\,x^2}{bx^2+a\left(q+px^3\right)\left(q+px^3+\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}\right)}\right)}{\sqrt{b}}}{a}$$

Mathematica 12.3 output

$$\int \frac{\left(-q+2px^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x\left(bx^2+a\left(q+px^3\right)^2\right)}\,dx$$

24.397 Problem number 2168

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x\left(bx^4+a\left(q+px^3\right)^2\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{2apq+b}\,\operatorname{arctanh}\left(\frac{\sqrt{b}\,\sqrt{2apq+b}\,x^4}{a\,q^2+2apq\,x^3+b\,x^4+a\,p^2x^6+(ap\,x^3+aq)\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}}\right)}{a\sqrt{b}}$$
$$-\frac{2\ln(x)}{a}+\frac{\ln\left(q+p\,x^3+\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}\right)}{a}$$

command

```
Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6])/(x*(b*x^4 + a*(q + p*x
```

Mathematica 13.1 output

$$\frac{\tanh^{-1}\left(\frac{\sqrt{q^2-2pq(-1+x)x^3+p^2x^6}}{q+px^3}\right)+\frac{\sqrt{b+2apq}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{b+2apq}\,x^4}{bx^4+a\left(q+px^3\right)\left(q+px^3+\sqrt{q^2-2pq(-1+x)x^3+p^2x^6}\right)}\right)}{\sqrt{b}}}{a}$$

Mathematica 12.3 output

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x\left(bx^4+a\left(q+px^3\right)^2\right)}\,dx$$

24.398 Problem number 2174

$$\int \frac{-2+(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x+(-b+k)x^2)}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,b^{\frac{1}{3}}x}{b^{\frac{1}{3}}x+2\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}+\frac{\ln\left(-b^{\frac{1}{3}}x+\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}}$$
$$-\frac{\ln\left(b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}x\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}+\left(x+(-1-k)x^2+kx^3\right)^{\frac{2}{3}}\right)}{2b^{\frac{1}{3}}}$$

command

```
Integrate[(-2 + (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x + (-b + k)*x^2)),x]
```

Mathematica 13.1 output

$$\frac{x\sqrt[3]{\frac{-1+kx}{-1+x}}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{\frac{-1+kx}{-1+x}}}{2\sqrt[3]{b}\left(\frac{x}{-1+x}\right)^{2/3}+\sqrt[3]{\frac{-1+kx}{-1+x}}}\right)+2\log\left(-\sqrt[3]{b}\left(\frac{x}{-1+x}\right)^{2/3}+\sqrt[3]{\frac{-1+kx}{-1+x}}\right)-\log\left(b^{2}\right.\right.}{2\sqrt[3]{b}\left(\frac{x}{-1+x}\right)^{2/3}\sqrt[3]{(-1+x)x(-1+kx)}}$$

Mathematica 12.3 output

$$\int \frac{-2+(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x+(-b+k)x^2)}\,dx$$

24.399 Problem number 2175

$$\int \frac{\left(3+2x^2\right)\left(1+2x^2+2x^3\right)^{2/3}}{x^3\left(-1-2x^2+x^3\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(2x^3+2x^2+1\right)^{\frac{2}{3}}}{2x^2}-3\,3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{5}{6}}x}{3^{\frac{1}{3}}x+2\left(2x^3+2x^2+1\right)^{\frac{1}{3}}}\right)$$
$$+3^{\frac{2}{3}}\ln\left(-3x+3^{\frac{2}{3}}\left(2x^3+2x^2+1\right)^{\frac{1}{3}}\right)$$
$$-\frac{3^{\frac{2}{3}}\ln\left(3x^2+3^{\frac{2}{3}}x\left(2x^3+2x^2+1\right)^{\frac{1}{3}}+3^{\frac{1}{3}}\left(2x^3+2x^2+1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((3 + 2*x^2)*(1 + 2*x^2 + 2*x^3)^(2/3))/(x^3*(-1 - 2*x^2 + x^3)),x]
```

Mathematica 13.1 output

$$\frac{3\left(1+2x^2+2x^3\right)^{2/3}}{2x^2}-3\sqrt[6]{3}\,\text{ArcTan}\left(\frac{3^{5/6}x}{\sqrt[3]{3}\,x+2\sqrt[3]{1+2x^2+2x^3}}\right)$$
$$+3^{2/3}\log\left(-3x+3^{2/3}\sqrt[3]{1+2x^2+2x^3}\right)-\frac{1}{2}3^{2/3}\log\left(3x^2+3^{2/3}x\sqrt[3]{1+2x^2+2x^3}+\sqrt[3]{3}\left(1+2x^2+2x^3\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(3+2x^2\right)\left(1+2x^2+2x^3\right)^{2/3}}{x^3\left(-1-2x^2+x^3\right)}\,dx$$

24.400 Problem number 2183

$$\int\frac{\left(-8+x^5\right)\left(2+x^5\right)\sqrt[4]{2-3x^4+x^5}}{x^6\left(4-3x^4+2x^5\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(x^5-3x^4+2\right)^{\frac{1}{4}}\left(2x^5+9x^4+4\right)}{5x^5}+\frac{3\,3^{\frac{1}{4}}\arctan\left(\frac{6^{\frac{3}{4}}x\left(x^5-3x^4+2\right)^{\frac{1}{4}}}{\left(-3x^2+\sqrt{6}\right)\sqrt{x^5-3x^4+2}}\right)2^{\frac{1}{4}}}{4}$$
$$-\frac{3\,3^{\frac{1}{4}}\operatorname{arctanh}\left(\frac{6^{\frac{3}{4}}x\left(x^5-3x^4+2\right)^{\frac{1}{4}}}{\left(3x^2+\sqrt{6}\right)\sqrt{x^5-3x^4+2}}\right)2^{\frac{1}{4}}}{4}$$

command

```
Integrate[((-8 + x^5)*(2 + x^5)*(2 - 3*x^4 + x^5)^(1/4))/(x^6*(4 - 3*x^4 + 2*x^5)),x]
```

Mathematica 13.1 output

$$\frac{4\sqrt[4]{2-3x^4+x^5}\left(4+9x^4+2x^5\right)+15\sqrt[4]{6}\,x^5\text{ArcTan}\left(\frac{\frac{3\sqrt[4]{6}\,x^2}{\sqrt[4]{2-3x^4+x^5}}-6^{3/4}\sqrt[4]{2-3x^4+x^5}}{6x}\right)-15\sqrt[4]{6}\,x^5\tanh^{-1}}{20x^5}$$

Mathematica 12.3 output

$$\int\frac{\left(-8+x^5\right)\left(2+x^5\right)\sqrt[4]{2-3x^4+x^5}}{x^6\left(4-3x^4+2x^5\right)}\,dx$$

24.401 Problem number 2192

$$\int \frac{\left(-4+x^3\right)\left(-2+x^3\right)\left(-1+x^3\right)^{2/3}}{x^6\left(-2+x^3+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(-13x^3+8\right)\left(x^3-1\right)^{\frac{2}{3}}}{10x^5}+2^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{5}{6}}x}{3^{\frac{1}{3}}x+2\,2^{\frac{1}{3}}\left(x^3-1\right)^{\frac{1}{3}}}\right)$$
$$-\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(-3x+2^{\frac{1}{3}}3^{\frac{2}{3}}\left(x^3-1\right)^{\frac{1}{3}}\right)}{3}+\frac{\ln\left(3x^2+2^{\frac{1}{3}}3^{\frac{2}{3}}x\left(x^3-1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}3^{\frac{1}{3}}\left(x^3-1\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}3^{\frac{2}{3}}}{6}$$

command

```
Integrate[((-4 + x^3)*(-2 + x^3)*(-1 + x^3)^(2/3))/(x^6*(-2 + x^3 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{\left(8-13x^3\right)\left(-1+x^3\right)^{2/3}}{10x^5}+\sqrt[3]{2}\,\sqrt[6]{3}\,\text{ArcTan}\left(\frac{3^{5/6}x}{\sqrt[3]{3}\,x+2\sqrt[3]{2}\,\sqrt[3]{-1+x^3}}\right)$$
$$-\sqrt[3]{\frac{2}{3}}\,\log\left(-3x+\sqrt[3]{2}\,3^{2/3}\sqrt[3]{-1+x^3}\right)+\frac{\log\left(3x^2+\sqrt[3]{2}\,3^{2/3}x\sqrt[3]{-1+x^3}+2^{2/3}\sqrt[3]{3}\left(-1+x^3\right)^{2/3}\right)}{2^{2/3}\sqrt[3]{3}}$$

Mathematica 12.3 output

$$\int \frac{\left(-4+x^3\right)\left(-2+x^3\right)\left(-1+x^3\right)^{2/3}}{x^6\left(-2+x^3+x^6\right)}\,dx$$

24.402 Problem number 2193

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}{1+x^8}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{2^{\frac{1}{8}}x}{\left(x^6+x^2\right)^{\frac{1}{4}}}\right)2^{\frac{5}{8}}}{4}+\frac{\arctan\left(\frac{2^{\frac{5}{8}}x\left(x^6+x^2\right)^{\frac{1}{4}}}{x^2\,2^{\frac{1}{4}}-\sqrt{x^6+x^2}}\right)2^{\frac{1}{8}}}{4}$$
$$-\frac{\text{arctanh}\left(\frac{2^{\frac{1}{8}}x}{\left(x^6+x^2\right)^{\frac{1}{4}}}\right)2^{\frac{5}{8}}}{4}+\frac{\text{arctanh}\left(\frac{\frac{x^2\,2^{\frac{5}{8}}}{2}+\frac{\sqrt{x^6+x^2}\,2^{\frac{3}{8}}}{2}}{x\left(x^6+x^2\right)^{\frac{1}{4}}}\right)2^{\frac{1}{8}}}{4}$$

command

```
Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^8),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{x^2+x^6}\left(\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt[8]{2}\,\sqrt{x}}{\sqrt[4]{1+x^4}}\right)+\text{ArcTan}\left(\frac{2^{5/8}\sqrt{x}\,\sqrt[4]{1+x^4}}{\sqrt[4]{2}\,x-\sqrt{1+x^4}}\right)-\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt[8]{2}\,\sqrt{x}}{\sqrt[4]{1+x^4}}\right)+\tanh^{-1}\left(\frac{2\ 2^3}{2x}\right.\right.}{2\ 2^{7/8}\sqrt{x}\,\sqrt[4]{1+x^4}}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}{1+x^8}\,dx$$

24.403 Problem number 2194

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}{1+x^8}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{2^{\frac{1}{8}}x}{\left(x^6+x^2\right)^{\frac{1}{4}}}\right)2^{\frac{5}{8}}}{4}+\frac{\arctan\left(\frac{2^{\frac{5}{8}}x\left(x^6+x^2\right)^{\frac{1}{4}}}{x^2 2^{\frac{1}{4}}-\sqrt{x^6+x^2}}\right)2^{\frac{1}{8}}}{4}$$
$$-\frac{\text{arctanh}\left(\frac{2^{\frac{1}{8}}x}{\left(x^6+x^2\right)^{\frac{1}{4}}}\right)2^{\frac{5}{8}}}{4}+\frac{\text{arctanh}\left(\frac{\frac{x^2 2^{\frac{5}{8}}}{2}+\frac{\sqrt{x^6+x^2}\ 2^{\frac{3}{8}}}{2}}{x\left(x^6+x^2\right)^{\frac{1}{4}}}\right)2^{\frac{1}{8}}}{4}$$

command

```
Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^8),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{x^2+x^6}\left(\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt[8]{2}\,\sqrt{x}}{\sqrt[4]{1+x^4}}\right)+\text{ArcTan}\left(\frac{2^{5/8}\sqrt{x}\,\sqrt[4]{1+x^4}}{\sqrt[4]{2}\,x-\sqrt{1+x^4}}\right)-\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt[8]{2}\,\sqrt{x}}{\sqrt[4]{1+x^4}}\right)+\tanh^{-1}\left(\frac{2\ 2^3}{2x}\right.\right.}{2\ 2^{7/8}\sqrt{x}\,\sqrt[4]{1+x^4}}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}{1+x^8}\,dx$$

24.404 Problem number 2195

$$\int \frac{-1+x^8}{\sqrt[4]{x^2+x^6}\,(1+x^8)}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{2^{\frac{1}{8}}x}{(x^6+x^2)^{\frac{1}{4}}}\right)2^{\frac{7}{8}}}{4}+\frac{\arctan\left(\frac{2^{\frac{5}{8}}x(x^6+x^2)^{\frac{1}{4}}}{x^2 2^{\frac{1}{4}}-\sqrt{x^6+x^2}}\right)2^{\frac{3}{8}}}{4}$$
$$-\frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{8}}x}{(x^6+x^2)^{\frac{1}{4}}}\right)2^{\frac{7}{8}}}{4}-\frac{\operatorname{arctanh}\left(\frac{\frac{x^2 2^{\frac{5}{8}}}{2}+\frac{\sqrt{x^6+x^2}\,2^{\frac{3}{8}}}{2}}{x(x^6+x^2)^{\frac{1}{4}}}\right)2^{\frac{3}{8}}}{4}$$

command

```
Integrate[(-1 + x^8)/((x^2 + x^6)^(1/4)*(1 + x^8)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{x}\,\sqrt[4]{1+x^4}\left(\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt[8]{2}\,\sqrt{x}}{\sqrt[4]{1+x^4}}\right)-\text{ArcTan}\left(\frac{2^{5/8}\sqrt{x}\,\sqrt[4]{1+x^4}}{\sqrt[4]{2}\,x-\sqrt{1+x^4}}\right)+\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt[8]{2}\,\sqrt{x}}{\sqrt[4]{1+x^4}}\right)+\tanh^{-1}\right.}{2\,2^{5/8}\sqrt[4]{x^2+x^6}}$$

Mathematica 12.3 output

$$\int \frac{-1+x^8}{\sqrt[4]{x^2+x^6}\,(1+x^8)}\,dx$$

24.405 Problem number 2196

$$\int \frac{-1+x^8}{\sqrt[4]{x^2+x^6}\,(1+x^8)}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{2^{\frac{1}{8}}x}{(x^6+x^2)^{\frac{1}{4}}}\right)2^{\frac{7}{8}}}{4}+\frac{\arctan\left(\frac{2^{\frac{5}{8}}x(x^6+x^2)^{\frac{1}{4}}}{x^2 2^{\frac{1}{4}}-\sqrt{x^6+x^2}}\right)2^{\frac{3}{8}}}{4}$$
$$-\frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{8}}x}{(x^6+x^2)^{\frac{1}{4}}}\right)2^{\frac{7}{8}}}{4}-\frac{\operatorname{arctanh}\left(\frac{\frac{x^2 2^{\frac{5}{8}}}{2}+\frac{\sqrt{x^6+x^2}\,2^{\frac{3}{8}}}{2}}{x(x^6+x^2)^{\frac{1}{4}}}\right)2^{\frac{3}{8}}}{4}$$

command

```
Integrate[(-1 + x^8)/((x^2 + x^6)^(1/4)*(1 + x^8)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{x}\ \sqrt[4]{1+x^4}\left(\sqrt{2}\ \text{ArcTan}\left(\frac{\sqrt[8]{2}\ \sqrt{x}}{\sqrt[4]{1+x^4}}\right)-\text{ArcTan}\left(\frac{2^{5/8}\sqrt{x}\ \sqrt[4]{1+x^4}}{\sqrt[4]{2}\ x-\sqrt{1+x^4}}\right)+\sqrt{2}\ \tanh^{-1}\left(\frac{\sqrt[8]{2}\ \sqrt{x}}{\sqrt[4]{1+x^4}}\right)+\tanh^{-1}\right.}{2\ 2^{5/8}\sqrt[4]{x^2+x^6}}$$

Mathematica 12.3 output

$$\int \frac{-1+x^8}{\sqrt[4]{x^2+x^6}\ (1+x^8)}\,dx$$

24.406 Problem number 2211

$$\int \frac{(-3+x)(-2+x)\left(2-x+2x^3\right)^{2/3}}{x^6\left(-2+x+2x^3\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(2x^3-x+2\right)^{\frac{2}{3}}\left(7x^3-x+2\right)}{10x^5}-2\,2^{\frac{1}{3}}\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ x}{x+2^{\frac{1}{3}}\left(2x^3-x+2\right)^{\frac{1}{3}}}\right)$$
$$+2\,2^{\frac{1}{3}}\ln\left(-2x+2^{\frac{1}{3}}\left(2x^3-x+2\right)^{\frac{1}{3}}\right)-2^{\frac{1}{3}}\ln\left(4x^2+2\,2^{\frac{1}{3}}x\left(2x^3-x+2\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\left(2x^3-x+2\right)^{\frac{2}{3}}\right)$$

command

```
Integrate[((-3 + x)*(-2 + x)*(2 - x + 2*x^3)^(2/3))/(x^6*(-2 + x + 2*x^3)),x]
```

Mathematica 13.1 output

$$\frac{3\left(2-x+2x^3\right)^{2/3}\left(2-x+7x^3\right)}{10x^5}-2\sqrt[3]{2}\ \sqrt{3}\ \text{ArcTan}\left(\frac{\sqrt{3}\ x}{x+\sqrt[3]{4-2x+4x^3}}\right)$$
$$+2\sqrt[3]{2}\ \log\left(-2x+\sqrt[3]{4-2x+4x^3}\right)-\sqrt[3]{2}\ \log\left(4x^2+2x\sqrt[3]{4-2x+4x^3}+\left(4-2x+4x^3\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int \frac{(-3+x)(-2+x)\left(2-x+2x^3\right)^{2/3}}{x^6\left(-2+x+2x^3\right)}\,dx$$

24.407 Problem number 2212

$$\int \frac{\left(1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(-2-x^3+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(x^3+1\right)^{\frac{5}{3}}}{5x^5}-\frac{\arctan\left(\frac{3^{\frac{5}{6}}x}{3^{\frac{1}{3}}x+2\,2^{\frac{1}{3}}\left(x^3+1\right)^{\frac{1}{3}}}\right)2^{\frac{1}{3}}3^{\frac{1}{6}}}{6}+\frac{\ln\left(-3x+2^{\frac{1}{3}}3^{\frac{2}{3}}\left(x^3+1\right)^{\frac{1}{3}}\right)2^{\frac{1}{3}}3^{\frac{2}{3}}}{18}$$
$$-\frac{\ln\left(3x^2+2^{\frac{1}{3}}3^{\frac{2}{3}}x\left(x^3+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}3^{\frac{1}{3}}\left(x^3+1\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}3^{\frac{2}{3}}}{36}$$

command

```
Integrate[((1 + x^3)^(2/3)*(2 + x^3))/(x^6*(-2 - x^3 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{\left(1+x^3\right)^{5/3}}{5x^5}-\frac{\text{ArcTan}\left(\frac{3^{5/6}x}{\sqrt[3]{3}\,x+2\sqrt[3]{2}\,\sqrt[3]{1+x^3}}\right)}{2^{2/3}3^{5/6}}+\frac{\log\left(-3x+\sqrt[3]{2}\,3^{2/3}\sqrt[3]{1+x^3}\right)}{3\;2^{2/3}\sqrt[3]{3}}$$
$$-\frac{\log\left(3x^2+\sqrt[3]{2}\,3^{2/3}x\sqrt[3]{1+x^3}+2^{2/3}\sqrt[3]{3}\left(1+x^3\right)^{2/3}\right)}{6\;2^{2/3}\sqrt[3]{3}}$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(-2-x^3+x^6\right)}\,dx$$

24.408 Problem number 2213

$$\int \frac{b+ax^6}{\sqrt[3]{x+x^3}\,\left(d+cx^6\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(b + a*x^6)/((x + x^3)^(1/3)*(d + c*x^6)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{1+\frac{1}{x^2}}\,x\left(3ad\left(i\left(i+\sqrt{3}\right)\log\left(\sqrt{2-2i\sqrt{3}}-2i\sqrt[3]{1+\frac{1}{x^2}}\right)+\left(-1-i\sqrt{3}\right)\log\left(\sqrt{2+2i\sqrt{3}}+2i\sqrt[3]{1+\right.\right.\right.}}{-\qquad 12}$$

Mathematica 12.3 output

$$\int \frac{b+ax^6}{\sqrt[3]{x+x^3}\,(d+cx^6)}\,dx$$

24.409 Problem number 2214

$$\int \frac{b+ax^6}{\sqrt[3]{x+x^3}\,(d+cx^6)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(b + a*x^6)/((x + x^3)^(1/3)*(d + c*x^6)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{1+\frac{1}{x^2}}\,x\left(3ad\left(i\left(i+\sqrt{3}\right)\log\left(\sqrt{2-2i\sqrt{3}}-2i\sqrt[3]{1+\frac{1}{x^2}}\right)+\left(-1-i\sqrt{3}\right)\log\left(\sqrt{2+2i\sqrt{3}}+2i\sqrt[3]{1+\right.\right.\right.}}{-\qquad 12}$$

Mathematica 12.3 output

$$\int \frac{b+ax^6}{\sqrt[3]{x+x^3}\,(d+cx^6)}\,dx$$

24.410 Problem number 2216

$$\int \frac{(-3+2x)\sqrt[3]{-1+x+x^3}}{x^2\left(2-2x+x^3\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^3+x-1\right)^{\frac{1}{3}}}{2x}+\frac{3^{\frac{5}{6}}\arctan\left(\frac{3^{\frac{5}{6}}x}{3^{\frac{1}{3}}x+2\,2^{\frac{1}{3}}\left(x^3+x-1\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{4}+\frac{2^{\frac{2}{3}}3^{\frac{1}{3}}\ln\left(-3x+2^{\frac{1}{3}}3^{\frac{2}{3}}\left(x^3+x-1\right)^{\frac{1}{3}}\right)}{4}$$
$$-\frac{2^{\frac{2}{3}}3^{\frac{1}{3}}\ln\left(3x^2+2^{\frac{1}{3}}3^{\frac{2}{3}}x\left(x^3+x-1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}3^{\frac{1}{3}}\left(x^3+x-1\right)^{\frac{2}{3}}\right)}{8}$$

command

```
Integrate[((-3 + 2*x)*(-1 + x + x^3)^(1/3))/(x^2*(2 - 2*x + x^3)),x]
```

Mathematica 13.1 output

$$\frac{3\sqrt[3]{-1+x+x^3}}{2x}+\frac{3^{5/6}\text{ArcTan}\left(\frac{3^{5/6}x}{\sqrt[3]{3}\,x+2\sqrt[3]{2}\,\sqrt[3]{-1+x+x^3}}\right)}{2\sqrt[3]{2}}$$
$$+\frac{1}{2}\sqrt[3]{\frac{3}{2}}\log\left(-3x+\sqrt[3]{2}\,3^{2/3}\sqrt[3]{-1+x+x^3}\right)-\frac{1}{4}\sqrt[3]{\frac{3}{2}}\log\left(3x^2+\sqrt[3]{2}\,3^{2/3}x\sqrt[3]{-1+x+x^3}+2^{2/3}\sqrt[3]{3}\left(-1+x+x^3\right)^{2/}\right.$$

Mathematica 12.3 output

$$\int \frac{(-3+2x)\sqrt[3]{-1+x+x^3}}{x^2\left(2-2x+x^3\right)}\,dx$$

24.411 Problem number 2222

$$\int \frac{x\sqrt{b+ax}}{x+\sqrt{c+\sqrt{b+ax}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x*Sqrt[b + a*x])/(x + Sqrt[c + Sqrt[b + a*x]]),x]
```

Mathematica 13.1 output

$$-\frac{4}{3}\left(c+\sqrt{b+ax}\right)^{3/2}+\frac{2\left(c^3+(b+ax)^{3/2}\right)}{3a}$$
$$+4\text{RootSum}\left[b-c^2-a\#1+2c\#1^2-\#1^4\&,\frac{-b\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1^2+a\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)}{a-4c\#1+4\#1^3}\right.$$

Mathematica 12.3 output

$Aborted

24.412 Problem number 2223

$$\int\frac{x\sqrt{b+ax}}{x+\sqrt{c+\sqrt{b+ax}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x*Sqrt[b + a*x])/(x + Sqrt[c + Sqrt[b + a*x]]),x]
```

Mathematica 13.1 output

$$-\frac{4}{3}\left(c+\sqrt{b+ax}\right)^{3/2}+\frac{2\left(c^3+(b+ax)^{3/2}\right)}{3a}$$
$$+4\text{RootSum}\left[b-c^2-a\#1+2c\#1^2-\#1^4\&,\frac{-b\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1^2+a\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)}{a-4c\#1+4\#1^3}\right.$$

Mathematica 12.3 output

$$\int\frac{x\sqrt{b+ax}}{x+\sqrt{c+\sqrt{b+ax}}}\,dx$$

24.413 Problem number 2237

$$\int \frac{1+x^2}{(-1+x^2)\sqrt[3]{x^2+x^4}}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2^{\frac{2}{3}}(x^4+x^2)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{4}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2^{\frac{2}{3}}(x^4+x^2)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{4}$$
$$-\frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{3}}x}{(x^4+x^2)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{2}-\frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{3}}x^2+\frac{\left(x^4+x^2\right)^{\frac{2}{3}}2^{\frac{2}{3}}}{2}}{x(x^4+x^2)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{4}$$

command

```
Integrate[(1 + x^2)/((-1 + x^2)*(x^2 + x^4)^(1/3)),x]
```

Mathematica 13.1 output

$$-\frac{x^{2/3}\sqrt[3]{1+x^2}\left(\sqrt{3}\left(\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{-\sqrt[3]{x}+2^{2/3}\sqrt[3]{1+x^2}}\right)+\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}+2^{2/3}\sqrt[3]{1+x^2}}\right)\right)+2\tanh^{-1}\left(\frac{\sqrt[3]{2}\,\sqrt[3]{x}}{\sqrt[3]{1+x^2}}\right)\right.}{2\sqrt[3]{2}\,\sqrt[3]{x^2+x^4}}$$

Mathematica 12.3 output

$$\int \frac{1+x^2}{(-1+x^2)\sqrt[3]{x^2+x^4}}\,dx$$

24.414 Problem number 2238

$$\int \frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)\left(-2-x^3+2x^4\right)}{x^6\left(-2+3x^3+2x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x^4-1\right)^{\frac{2}{3}}\left(x^4-5x^3-1\right)}{5x^5}+3\,2^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{5}{6}}x}{3^{\frac{1}{3}}x-2\,2^{\frac{1}{3}}\left(x^4-1\right)^{\frac{1}{3}}}\right)$$
$$-2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(3x+2^{\frac{1}{3}}3^{\frac{2}{3}}\left(x^4-1\right)^{\frac{1}{3}}\right)+\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(3x^2-2^{\frac{1}{3}}3^{\frac{2}{3}}x\left(x^4-1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}3^{\frac{1}{3}}\left(x^4-1\right)^{\frac{2}{3}}\right)}{2}$$

command

```
Integrate[((-1 + x^4)^(2/3)*(3 + x^4)*(-2 - x^3 + 2*x^4))/(x^6*(-2 + 3*x^3 + 2*x^4)),x]
```

Mathematica 13.1 output

$$\frac{3\left(-1+x^4\right)^{2/3}\left(-1-5x^3+x^4\right)}{5x^5}+3\sqrt[3]{2}\,\sqrt[6]{3}\,\text{ArcTan}\left(\frac{3^{5/6}x}{\sqrt[3]{3}\,x-2\sqrt[3]{2}\,\sqrt[3]{-1+x^4}}\right)$$
$$-\sqrt[3]{2}\,3^{2/3}\log\left(3x+\sqrt[3]{2}\,3^{2/3}\,\sqrt[3]{-1+x^4}\right)+\left(\frac{3}{2}\right)^{2/3}\log\left(3x^2-\sqrt[3]{2}\,3^{2/3}x\,\sqrt[3]{-1+x^4}+2^{2/3}\sqrt[3]{3}\,\left(-1+x^4\right)^{2/3}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)\left(-2-x^3+2x^4\right)}{x^6\left(-2+3x^3+2x^4\right)}\,dx$$

24.415 Problem number 2242

$$\int\frac{1+x^6}{\sqrt[3]{-x^2+x^4}\,\left(-1+x^6\right)}\,dx$$

Optimal antiderivative

$$-\frac{\left(x^4-x^2\right)^{\frac{2}{3}}}{x\left(x^2-1\right)}-\frac{2\arctan\left(\frac{3^{\frac{1}{6}}x}{\left(x^4-x^2\right)^{\frac{1}{3}}}\right)3^{\frac{5}{6}}}{9}$$
$$-\frac{\arctan\left(\frac{3^{\frac{5}{6}}x\left(x^4-x^2\right)^{\frac{1}{3}}}{-3x^2+3^{\frac{2}{3}}\left(x^4-x^2\right)^{\frac{2}{3}}}\right)3^{\frac{5}{6}}}{9}-\frac{\text{arctanh}\left(\frac{\frac{3^{\frac{2}{3}}x^2}{3}+\frac{\left(x^4-x^2\right)^{\frac{2}{3}}3^{\frac{1}{3}}}{3}}{x\left(x^4-x^2\right)^{\frac{1}{3}}}\right)3^{\frac{1}{3}}}{3}$$

command

```
Integrate[(1 + x^6)/((-x^2 + x^4)^(1/3)*(-1 + x^6)),x]
```

Mathematica 13.1 output

$$-\frac{x^{2/3}\left(9\sqrt[3]{x}+2\,3^{5/6}\,\sqrt[3]{-1+x^2}\,\text{ArcTan}\left(\frac{\sqrt[6]{3}\,\sqrt[3]{x}}{\sqrt[3]{-1+x^2}}\right)+3^{5/6}\,\sqrt[3]{-1+x^2}\,\text{ArcTan}\left(\frac{3^{5/6}\sqrt[3]{x}\,\sqrt[3]{-1+x^2}}{-3x^{2/3}+3^{2/3}\left(-1+x^2\right)^{2/3}}\right)+3\sqrt[3]{3}\right.}{9\sqrt[3]{x^2\left(-1+x^2\right)}}$$

Mathematica 12.3 output

$$\int\frac{1+x^6}{\sqrt[3]{-x^2+x^4}\,\left(-1+x^6\right)}\,dx$$

24.416 Problem number 2248

$$\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)}{bx^4+a\left(q+px^5\right)^2}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{2}\ a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{p\,x^5+q}}{\sqrt{a}\ q-x^2\sqrt{b}+\sqrt{a}\ px^5}\right)\sqrt{2}}{2a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\operatorname{arctanh}\left(\frac{\frac{a^{\frac{1}{4}}q\sqrt{2}}{2b^{\frac{1}{4}}}+\frac{b^{\frac{1}{4}}x^2\sqrt{2}}{2a^{\frac{1}{4}}}+\frac{a^{\frac{1}{4}}px^5\sqrt{2}}{2b^{\frac{1}{4}}}}{x\sqrt{p\,x^5+q}}\right)\sqrt{2}}{2a^{\frac{3}{4}}b^{\frac{1}{4}}}$$

command

```
Integrate[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(b*x^4 + a*(q + p*x^5)^2),x]
```

Mathematica 13.1 output

$$-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{2}\ \sqrt[4]{a}\ \sqrt[4]{b}\ x\sqrt{q+px^5}}{-\sqrt{b}\ x^2+\sqrt{a}\ (q+px^5)}\right)+\tanh^{-1}\left(\frac{\sqrt{b}\ x^2+\sqrt{a}\ (q+px^5)}{\sqrt{2}\ \sqrt[4]{a}\ \sqrt[4]{b}\ x\sqrt{q+px^5}}\right)}{\sqrt{2}\ a^{3/4}\sqrt[4]{b}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)}{bx^4+a\left(q+px^5\right)^2}\,dx$$

24.417 Problem number 2250

$$\int \frac{x^3(-3ab+(a+2b)x)}{(-a+x)(-b+x)\sqrt[4]{x(-a+x)(-b+x)^2}\ \left(-ab^2d+b(2a+b)dx-(a+2b)dx^2+(-1+d)x^3\right)}\,dx$$

Optimal antiderivative

$$\begin{aligned}&\frac{4\left(-a\,b^2x+2ab\,x^2-x^3a+b^2x^2-2b\,x^3+x^4\right)^{\frac{3}{4}}}{(-a+x)\,(-b+x)^2}\\&+2d^{\frac{1}{4}}\arctan\left(\frac{d^{\frac{1}{4}}\left(-a\,b^2x+\left(2ab+b^2\right)x^2+(-a-2b)\,x^3+x^4\right)^{\frac{1}{4}}}{x}\right)\\&-2d^{\frac{1}{4}}\operatorname{arctanh}\left(\frac{d^{\frac{1}{4}}\left(-a\,b^2x+\left(2ab+b^2\right)x^2+(-a-2b)\,x^3+x^4\right)^{\frac{1}{4}}}{x}\right)\end{aligned}$$

command

```
Integrate[(x^3*(-3*a*b + (a + 2*b)*x))/((-a + x)*(-b + x)*(x*(-a + x)*(-b + x)^2)^(1/4)*(-
(a*b^2*d) + b*(2*a + b)*d*x - (a + 2*b)*d*x^2 + (-1 + d)*x^3)),x]
```

Mathematica 13.1 output

$$\frac{4\sqrt{\frac{b-x}{a-x}}\,x+2\sqrt[4]{d}\,\sqrt[4]{\frac{x}{-a+x}}\,(-b+x)\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{\frac{b-x}{a-x}}}{\left(\frac{x}{-a+x}\right)^{3/4}}\right)-2\sqrt[4]{d}\,\sqrt[4]{\frac{x}{-a+x}}\,(-b+x)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{\frac{b-}{a-}}\right.}{\sqrt{\frac{b-x}{a-x}}\,\sqrt[4]{(b-x)^2x(-a+x)}}$$

Mathematica 12.3 output

$$\int\frac{x^3(-3ab+(a+2b)x)}{(-a+x)(-b+x)\sqrt[4]{x(-a+x)(-b+x)^2}\,\left(-ab^2d+b(2a+b)dx-(a+2b)dx^2+(-1+d)x^3\right)}\,dx$$

24.418 Problem number 2253

$$\int\frac{(1+x^2)\,\sqrt[4]{x^3+x^5}}{x^2\,(-1+x^2)}\,dx$$

Optimal antiderivative

$$\frac{4\left(x^5+x^3\right)^{\frac{1}{4}}}{x}+2^{\frac{1}{4}}\arctan\left(\frac{2^{\frac{1}{4}}x}{\left(x^5+x^3\right)^{\frac{1}{4}}}\right)-\frac{\arctan\left(\frac{2^{\frac{3}{4}}x\left(x^5+x^3\right)^{\frac{1}{4}}}{x^2\sqrt{2}-\sqrt{x^5+x^3}}\right)2^{\frac{3}{4}}}{2}$$
$$-2^{\frac{1}{4}}\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}x}{\left(x^5+x^3\right)^{\frac{1}{4}}}\right)-\frac{\operatorname{arctanh}\left(\frac{\frac{x^2 2^{\frac{3}{4}}}{2}+\frac{\sqrt{x^5+x^3}\,2^{\frac{1}{4}}}{2}}{x\left(x^5+x^3\right)^{\frac{1}{4}}}\right)2^{\frac{3}{4}}}{2}$$

command

```
Integrate[((1 + x^2)*(x^3 + x^5)^(1/4))/(x^2*(-1 + x^2)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt[4]{x^3+x^5}\left(-8\sqrt[4]{1+x^2}-2\sqrt[4]{2}\,\sqrt[4]{x}\,\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{x}}{\sqrt[4]{1+x^2}}\right)+2^{3/4}\sqrt[4]{x}\,\text{ArcTan}\left(\frac{2^{3/4}\sqrt[4]{x}\,\sqrt[4]{1+x^2}}{\sqrt{2}\,\sqrt{x}-\sqrt{1+x^2}}\right)+2\sqrt[4]{2}\,\sqrt[4]{x}\right.}{2x\sqrt[4]{1+x^2}}$$

Mathematica 12.3 output

$$\int\frac{(1+x^2)\,\sqrt[4]{x^3+x^5}}{x^2\,(-1+x^2)}\,dx$$

24.419 Problem number 2254

$$\int \frac{\left(4+x^2+x^5\right)\sqrt[4]{-2-x^2-2x^4+2x^5}}{x^2\left(-2-x^2+2x^5\right)}\,dx$$

Optimal antiderivative

$$\frac{2\left(2x^5-2x^4-x^2-2\right)^{\frac{1}{4}}}{x}-\frac{\arctan\left(\frac{2^{\frac{3}{4}}x\left(2x^5-2x^4-x^2-2\right)^{\frac{1}{4}}}{x^2\sqrt{2}-\sqrt{2x^5-2x^4-x^2-2}}\right)2^{\frac{3}{4}}}{2}$$
$$-\frac{\operatorname{arctanh}\left(\frac{2\,2^{\frac{1}{4}}x\left(2x^5-2x^4-x^2-2\right)^{\frac{1}{4}}}{2x^2+\sqrt{2}\,\sqrt{2x^5-2x^4-x^2-2}}\right)2^{\frac{3}{4}}}{2}$$

command

```
Integrate[((4 + x^2 + x^5)*(-2 - x^2 - 2*x^4 + 2*x^5)^(1/4))/(x^2*(-2 - x^2 + 2*x^5)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt[4]{-2-x^2-2x^4+2x^5}}{x}-\frac{\operatorname{ArcTan}\left(\frac{2^{3/4}x\sqrt[4]{-2-x^2-2x^4+2x^5}}{\sqrt{2}\,x^2-\sqrt{-2-x^2-2x^4+2x^5}}\right)}{\sqrt[4]{2}}$$
$$-\frac{\tanh^{-1}\left(\frac{2x\sqrt[4]{-4-2x^2-4x^4+4x^5}}{2x^2+\sqrt{-4-2x^2-4x^4+4x^5}}\right)}{\sqrt[4]{2}}$$

Mathematica 12.3 output

$$\int \frac{\left(4+x^2+x^5\right)\sqrt[4]{-2-x^2-2x^4+2x^5}}{x^2\left(-2-x^2+2x^5\right)}\,dx$$

24.420 Problem number 2255

$$\int \frac{\left(-1+x^4\right)^2\sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^4\right)^2}\,dx$$

Optimal antiderivative

$$\frac{x\sqrt{x^4+1}\left(2x^6+7x^2\right)\sqrt{x^2+\sqrt{x^4+1}}+x\left(2x^8+8x^4+4\right)\sqrt{x^2+\sqrt{x^4+1}}}{2\sqrt{x^4+1}\left(2x^6+2x^2\right)+4x^8+6x^4+2}$$
$$-\frac{3\arctan\left(x\sqrt{x^2+\sqrt{x^4+1}}\right)}{2}+\frac{\arctan\left(\sqrt{2}\,x\sqrt{x^2+\sqrt{x^4+1}}\right)\sqrt{2}}{4}$$

command

```
Integrate[((-1 + x^4)^2*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^4)^2,x]
```

Mathematica 13.1 output

$$\frac{1}{4}\left(\frac{2x\sqrt{x^2+\sqrt{1+x^4}}\left(4+8x^4+2x^8+7x^2\sqrt{1+x^4}+2x^6\sqrt{1+x^4}\right)}{(1+x^4)\left(1+2x^4+2x^2\sqrt{1+x^4}\right)}\right.$$
$$\left.-6\text{ArcTan}\left(x\sqrt{x^2+\sqrt{1+x^4}}\right)+\sqrt{2}\,\text{ArcTan}\left(\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^4\right)^2\sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^4\right)^2}\,dx$$

24.421 Problem number 2259

$$\int \frac{-5+x}{\sqrt[3]{-2-x+x^2}\,(-3+4x+x^2)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^2-x-2)^{\frac{1}{3}}}{-2^{\frac{2}{3}}-2^{\frac{2}{3}}x+(x^2-x-2)^{\frac{1}{3}}}\right)2^{\frac{1}{3}}}{2}+\frac{\ln\left(2^{\frac{2}{3}}+2^{\frac{2}{3}}x+2\left(x^2-x-2\right)^{\frac{1}{3}}\right)2^{\frac{1}{3}}}{2}$$
$$-\frac{\ln\left(-2^{\frac{1}{3}}-2\,2^{\frac{1}{3}}x-2^{\frac{1}{3}}x^2+\left(2^{\frac{2}{3}}+2^{\frac{2}{3}}x\right)\left(x^2-x-2\right)^{\frac{1}{3}}-2\left(x^2-x-2\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{4}$$

command

```
Integrate[(-5 + x)/((-2 - x + x^2)^(1/3)*(-3 + 4*x + x^2)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{-2-x+x^2}}{-2^{2/3}-2^{2/3}x+\sqrt[3]{-2-x+x^2}}\right)+2\log\left(2^{2/3}+2^{2/3}x+2\sqrt[3]{-2-x+x^2}\right)-\log\left(-\sqrt[3]{2}-2\sqrt[3]{2}\,x-\sqrt[3]{2}\right)}{2\,2^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{-5+x}{\sqrt[3]{-2-x+x^2}\,(-3+4x+x^2)}\,dx$$

24.422 Problem number 2261

$$\int \frac{1+x}{(1+4x+x^2)\sqrt[3]{1-x^3}}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{3}\,(-x^3+1)^{\frac{1}{3}}}{2^{\frac{2}{3}}-2^{\frac{2}{3}}x+(-x^3+1)^{\frac{1}{3}}}\right)2^{\frac{1}{3}}\sqrt{3}}{6}+\frac{\ln\left(-2^{\frac{2}{3}}+2^{\frac{2}{3}}x+2\left(-x^3+1\right)^{\frac{1}{3}}\right)2^{\frac{1}{3}}}{6}$$
$$-\frac{\ln\left(-2^{\frac{1}{3}}+2\,2^{\frac{1}{3}}x-2^{\frac{1}{3}}x^2+\left(-2^{\frac{2}{3}}+2^{\frac{2}{3}}x\right)\left(-x^3+1\right)^{\frac{1}{3}}-2\left(-x^3+1\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{12}$$

command

```
Integrate[(1 + x)/((1 + 4*x + x^2)*(1 - x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{1-x^3}}{2^{2/3}-2^{2/3}x+\sqrt[3]{1-x^3}}\right)+2\log\left(-2^{2/3}+2^{2/3}x+2\sqrt[3]{1-x^3}\right)-\log\left(-\sqrt[3]{2}+2\sqrt[3]{2}\,x-\sqrt[3]{2}\,x^2+2^{2/}\right.}{6\;2^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{1+x}{(1+4x+x^2)\sqrt[3]{1-x^3}}\,dx$$

24.423 Problem number 2266

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^4}\,dx$$

Optimal antiderivative

$$\frac{11x\left(315a^2x^4+798a\,b^2x^2+611b^4\right)}{7680b^6\left(a\,x^2+b^2\right)^{\frac{5}{2}}\sqrt{b+\sqrt{a\,x^2+b^2}}}+\frac{x\left(1155a^2x^4+3102a\,b^2x^2+2587b^4\right)}{3840b^5\left(a\,x^2+b^2\right)^3\sqrt{b+\sqrt{a\,x^2+b^2}}}$$
$$+\frac{231\arctan\left(\frac{\sqrt{a}\,x}{\sqrt{b}\,\sqrt{b+\sqrt{a\,x^2+b^2}}}\right)}{512\sqrt{a}\;b^{\frac{13}{2}}}$$

command

```
Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^4,x]
```

Mathematica 13.1 output

$$\frac{11x\left(611b^4+798ab^2x^2+315a^2x^4\right)}{7680b^6\left(b^2+ax^2\right)^{5/2}\sqrt{b+\sqrt{b^2+ax^2}}}+\frac{x\left(2587b^4+3102ab^2x^2+1155a^2x^4\right)}{3840b^5\left(b^2+ax^2\right)^3\sqrt{b+\sqrt{b^2+ax^2}}}$$
$$+\frac{231\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{512\sqrt{a}\,b^{13/2}}$$

Mathematica 12.3 output

$$\int\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^4}\,dx$$

24.424 Problem number 2267

$$\int\frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx$$

Optimal antiderivative

$$\sqrt{\sqrt{2}-1}\ \arctan\left(\frac{\sqrt{2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$
$$+\text{arctanh}\left(\frac{\sqrt{2}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}$$
$$-\sqrt{1+\sqrt{2}}\ \text{arctanh}\left(\frac{\sqrt{-2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$

command

```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/(1 + x^2),x]
```

Mathematica 13.1 output

$$\sqrt{-1+\sqrt{2}}\ \text{ArcTan}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\ \left(-1+x^2+\sqrt{1+x^4}\right)}{x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$+\sqrt{2}\ \tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$-\sqrt{1+\sqrt{2}}\ \tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx$$

24.425 Problem number 2269

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ (-x^3+1)^{\frac{1}{3}}}{2^{\frac{1}{3}}-2^{\frac{1}{3}}x+(-x^3+1)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{4}+\frac{\ln\left(-2^{\frac{1}{3}}+2^{\frac{1}{3}}x+2\left(-x^3+1\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{4}$$
$$-\frac{\ln\left(2^{\frac{2}{3}}-2\,2^{\frac{2}{3}}x+2^{\frac{2}{3}}x^2+\left(2\,2^{\frac{1}{3}}-2\,2^{\frac{1}{3}}x\right)\left(-x^3+1\right)^{\frac{1}{3}}+4\left(-x^3+1\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{8}$$

command

```
Integrate[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\ \sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}\,x+\sqrt[3]{1-x^3}}\right)+2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}\,x+2\sqrt[3]{1-x^3}\right)-\log\left(2^{2/3}-2\,2^{2/3}x+2^{2/3}x^2-2(-\right.}{4\sqrt[3]{2}}$$

Mathematica 12.3 output

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

24.426 Problem number 2279

$$\int \frac{\left(1+2x^8\right)\sqrt[4]{-1-2x^4+2x^8}\left(1-3x^8+4x^{16}\right)}{x^{10}\left(-1+2x^8\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(2x^8-2x^4-1\right)^{\frac{1}{4}}\left(20x^{16}-4x^{12}+9x^8+2x^4+5\right)}{45x^9}$$
$$-\frac{\arctan\left(\frac{2^{\frac{3}{4}}x\left(2x^8-2x^4-1\right)^{\frac{1}{4}}}{x^2\sqrt{2}-\sqrt{2x^8-2x^4-1}}\right)2^{\frac{3}{4}}}{4}-\frac{\operatorname{arctanh}\left(\frac{2\,2^{\frac{1}{4}}x\left(2x^8-2x^4-1\right)^{\frac{1}{4}}}{2x^2+\sqrt{2}\sqrt{2x^8-2x^4-1}}\right)2^{\frac{3}{4}}}{4}$$

command

```
Integrate[((1 + 2*x^8)*(-1 - 2*x^4 + 2*x^8)^(1/4)*(1 - 3*x^8 + 4*x^16))/(x^10*(-1 + 2*x^8)),x
```

Mathematica 13.1 output

$$\frac{1}{180}\left(\frac{4\sqrt[4]{-1-2x^4+2x^8}\left(5+2x^4+9x^8-4x^{12}+20x^{16}\right)}{x^9}\right.$$
$$\left.-45\,2^{3/4}\text{ArcTan}\left(\frac{2^{3/4}x\sqrt[4]{-1-2x^4+2x^8}}{\sqrt{2}\,x^2-\sqrt{-1-2x^4+2x^8}}\right)-45\,2^{3/4}\tanh^{-1}\left(\frac{2x\sqrt[4]{-2-4x^4+4x^8}}{2x^2+\sqrt{-2-4x^4+4x^8}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(1+2x^8\right)\sqrt[4]{-1-2x^4+2x^8}\left(1-3x^8+4x^{16}\right)}{x^{10}\left(-1+2x^8\right)}\,dx$$

24.427 Problem number 2288

$$\int \frac{-b+ax^6}{\sqrt[3]{-x+x^3}\left(-d+cx^6\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-b + a*x^6)/((-x + x^3)^(1/3)*(-d + c*x^6)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt[3]{-1+\frac{1}{x^2}}\,x\left(3ad\left(-2\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[3]{-1+\frac{1}{x^2}}}{\sqrt{3}}\right)-2\log\left(c\left(1+\sqrt[3]{-1+\frac{1}{x^2}}\right)\right)+\log\left(1-\sqrt[3]{-1+\frac{1}{x^2}}\right.\right.\right.}{12cd\sqrt[3]{x\,(-1}}$$

Mathematica 12.3 output

$$\int \frac{-b+ax^6}{\sqrt[3]{-x+x^3}\,(-d+cx^6)}\,dx$$

24.428 Problem number 2289

$$\int \frac{-b+ax^6}{\sqrt[3]{-x+x^3}\,(-d+cx^6)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-b + a*x^6)/((-x + x^3)^(1/3)*(-d + c*x^6)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt[3]{-1+\frac{1}{x^2}}\,x\left(3ad\left(-2\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[3]{-1+\frac{1}{x^2}}}{\sqrt{3}}\right)-2\log\left(c\left(1+\sqrt[3]{-1+\frac{1}{x^2}}\right)\right)+\log\left(1-\sqrt[3]{-1+\frac{1}{x^2}}\right.\right.\right.}{12cd\sqrt[3]{x\,(-1}}$$

Mathematica 12.3 output

$$\int \frac{-b+ax^6}{\sqrt[3]{-x+x^3}\,(-d+cx^6)}\,dx$$

24.429 Problem number 2293

$$\int \frac{(-3+2x)\left(1-x+x^3\right)^{2/3}}{x^3\left(-2+2x+x^3\right)}\,dx$$

Optimal antiderivative

$$\begin{aligned}&-\frac{3\left(x^3-x+1\right)^{\frac{2}{3}}}{4x^2}+\frac{3\,3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{5}{6}}x}{3^{\frac{1}{3}}x+2\,2^{\frac{1}{3}}\left(x^3-x+1\right)^{\frac{1}{3}}}\right)2^{\frac{1}{3}}}{4}\\&-\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(-3x+2^{\frac{1}{3}}3^{\frac{2}{3}}\left(x^3-x+1\right)^{\frac{1}{3}}\right)}{4}\\&+\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(3x^2+2^{\frac{1}{3}}3^{\frac{2}{3}}x\left(x^3-x+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}3^{\frac{1}{3}}\left(x^3-x+1\right)^{\frac{2}{3}}\right)}{8}\end{aligned}$$

command

```
Integrate[((-3 + 2*x)*(1 - x + x^3)^(2/3))/(x^3*(-2 + 2*x + x^3)),x]
```

Mathematica 13.1 output

$$\begin{aligned}&-\frac{3\left(1-x+x^3\right)^{2/3}}{4x^2}+\frac{3\sqrt[6]{3}\,\mathrm{ArcTan}\left(\frac{3^{5/6}x}{\sqrt[3]{3}\,x+2\sqrt[3]{2}\,\sqrt[3]{1-x+x^3}}\right)}{2\,2^{2/3}}\\&-\frac{1}{2}\left(\frac{3}{2}\right)^{2/3}\log\left(-3x+\sqrt[3]{2}\,3^{2/3}\sqrt[3]{1-x+x^3}\right)+\frac{1}{4}\left(\frac{3}{2}\right)^{2/3}\log\left(3x^2+\sqrt[3]{2}\,3^{2/3}x\sqrt[3]{1-x+x^3}+2^{2/3}\sqrt[3]{3}\,\left(1-x+x^3\right)^{2}\right.\end{aligned}$$

Mathematica 12.3 output

$$\int \frac{(-3+2x)\left(1-x+x^3\right)^{2/3}}{x^3\left(-2+2x+x^3\right)}\,dx$$

24.430 Problem number 2294

$$\int \frac{(-3+4x)\left(-1+2x+x^3\right)^{2/3}}{x^3\left(2-4x+x^3\right)}\,dx$$

Optimal antiderivative

$$\begin{aligned}&\frac{3\left(x^3+2x-1\right)^{\frac{2}{3}}}{4x^2}-\frac{3\,3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{5}{6}}x}{3^{\frac{1}{3}}x+2\,2^{\frac{1}{3}}\left(x^3+2x-1\right)^{\frac{1}{3}}}\right)2^{\frac{1}{3}}}{4}\\&+\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(-3x+2^{\frac{1}{3}}3^{\frac{2}{3}}\left(x^3+2x-1\right)^{\frac{1}{3}}\right)}{4}\\&-\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(3x^2+2^{\frac{1}{3}}3^{\frac{2}{3}}x\left(x^3+2x-1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}3^{\frac{1}{3}}\left(x^3+2x-1\right)^{\frac{2}{3}}\right)}{8}\end{aligned}$$

command

```
Integrate[((-3 + 4*x)*(-1 + 2*x + x^3)^(2/3))/(x^3*(2 - 4*x + x^3)),x]
```

Mathematica 13.1 output

$$\frac{3\left(-1+2x+x^3\right)^{2/3}}{4x^2}-\frac{3\sqrt[6]{3}\,\text{ArcTan}\left(\frac{3^{5/6}x}{\sqrt[3]{3}\,x+2\sqrt[3]{2}\,\sqrt[3]{-1+2x+x^3}}\right)}{2\,2^{2/3}}$$
$$+\frac{1}{2}\left(\frac{3}{2}\right)^{2/3}\log\left(-3x+\sqrt[3]{2}\,3^{2/3}\sqrt[3]{-1+2x+x^3}\right)-\frac{1}{4}\left(\frac{3}{2}\right)^{2/3}\log\left(3x^2+\sqrt[3]{2}\,3^{2/3}x\sqrt[3]{-1+2x+x^3}+2^{2/3}\sqrt[3]{3}\,(-1+\right.$$

Mathematica 12.3 output

$$\int\frac{(-3+4x)\left(-1+2x+x^3\right)^{2/3}}{x^3\left(2-4x+x^3\right)}\,dx$$

24.431 Problem number 2296

$$\int\frac{\left(-q+2px^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^2\left(aq+bx+apx^3\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}}{ax}$$
$$+\frac{2\sqrt{2a^2pq-b^2}\,\arctan\left(\frac{\sqrt{2a^2pq-b^2}\,x}{aq+bx+ap\,x^3+a\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}}\right)}{a^2}$$
$$+\frac{b\ln(x)}{a^2}-\frac{b\ln\left(q+p\,x^3+\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}\right)}{a^2}$$

command

```
Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6])/(x^2*(a*q + b*x + a*p*
```

Mathematica 13.1 output

$$-\frac{-a\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}-2\sqrt{-b^2+2a^2pq}\,x\text{ArcTan}\left(\frac{\sqrt{-b^2+2a^2pq}\,x}{bx+a\left(q+px^3+\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}\right)}\right)-}{a^2x}$$

Mathematica 12.3 output

$$\int\frac{\left(-q+2px^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^2\left(aq+bx+apx^3\right)}\,dx$$

24.432 Problem number 2298

$$\int \frac{\left(1+x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}{\left(-1+x^2\right)\sqrt{1+x^4}}\,dx$$

Optimal antiderivative

$$\sqrt{-2+2\sqrt{2}}\,\arctan\left(\frac{\sqrt{-2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$
$$+\operatorname{arctanh}\left(\frac{\sqrt{2}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}$$
$$-\sqrt{2+2\sqrt{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$

command

```
Integrate[((1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/((-1 + x^2)*Sqrt[1 + x^4]),x]
```

Mathematica 13.1 output

$$\sqrt{2}\left(\sqrt{-1+\sqrt{2}}\,\operatorname{ArcTan}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)\right.$$
$$+\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$\left.-\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\left(-1+x^2+\sqrt{1+x^4}\right)}{x\sqrt{x^2+\sqrt{1+x^4}}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}{\left(-1+x^2\right)\sqrt{1+x^4}}\,dx$$

24.433 Problem number 2299

$$\int \frac{2+x}{(-3+x)\sqrt[4]{1-x^2}\,(1+x^2)}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{(-x^2+1)^{\frac{1}{4}}}{2^{\frac{1}{4}}-2^{\frac{1}{4}}x+(-x^2+1)^{\frac{1}{4}}}\right)2^{\frac{3}{4}}}{4}+\frac{\arctan\left(\frac{(-x^2+1)^{\frac{1}{4}}}{-2^{\frac{1}{4}}+2^{\frac{1}{4}}x+(-x^2+1)^{\frac{1}{4}}}\right)2^{\frac{3}{4}}}{4}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\left(-2\,2^{\frac{1}{4}}+2\,2^{\frac{1}{4}}x\right)(-x^2+1)^{\frac{1}{4}}}{\sqrt{2}-2x\sqrt{2}+x^2\sqrt{2}+2\sqrt{-x^2+1}}\right)2^{\frac{3}{4}}}{4}$$

command

```
Integrate[(2 + x)/((-3 + x)*(1 - x^2)^(1/4)*(1 + x^2)),x]
```

Mathematica 13.1 output

$$-\frac{\operatorname{ArcTan}\left(\frac{\sqrt[4]{1-x^2}}{\sqrt[4]{2}-\sqrt[4]{2}x-\sqrt[4]{1-x^2}}\right)+\operatorname{ArcTan}\left(\frac{\sqrt[4]{1-x^2}}{\sqrt[4]{2}-\sqrt[4]{2}x+\sqrt[4]{1-x^2}}\right)+\tanh^{-1}\left(\frac{2(-1+x)\sqrt[4]{2-2x^2}}{\sqrt{2}-2\sqrt{2}x+\sqrt{2}x^2+2\sqrt{1-x^2}}\right)}{2\sqrt[4]{2}}$$

Mathematica 12.3 output

$$\int \frac{2+x}{(-3+x)\sqrt[4]{1-x^2}\,(1+x^2)}\,dx$$

24.434 Problem number 2301

$$\int \frac{1-x^4+2x^8}{\sqrt[4]{1+x^4}\,(-1-2x^4+x^8)}\,dx$$

Optimal antiderivative

$$\arctan\left(\frac{x}{(x^4+1)^{\frac{1}{4}}}\right)-\frac{3\arctan\left(\frac{2^{\frac{1}{8}}x}{(x^4+1)^{\frac{1}{4}}}\right)2^{\frac{7}{8}}}{8}+\frac{3\arctan\left(\frac{2^{\frac{5}{8}}x(x^4+1)^{\frac{1}{4}}}{x^2 2^{\frac{1}{4}}-\sqrt{x^4+1}}\right)2^{\frac{3}{8}}}{8}$$
$$+\operatorname{arctanh}\left(\frac{x}{(x^4+1)^{\frac{1}{4}}}\right)-\frac{3\operatorname{arctanh}\left(\frac{2^{\frac{1}{8}}x}{(x^4+1)^{\frac{1}{4}}}\right)2^{\frac{7}{8}}}{8}-\frac{3\operatorname{arctanh}\left(\frac{2\,2^{\frac{3}{8}}x(x^4+1)^{\frac{1}{4}}}{2x^2+2^{\frac{3}{4}}\sqrt{x^4+1}}\right)2^{\frac{3}{8}}}{8}$$

command

```
Integrate[(1 - x^4 + 2*x^8)/((1 + x^4)^(1/4)*(-1 - 2*x^4 + x^8)),x]
```

Mathematica 13.1 output

$$\text{ArcTan}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\frac{3\text{ArcTan}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{4\sqrt[8]{2}}+\frac{3\text{ArcTan}\left(\frac{2^{5/8}x\sqrt[4]{1+x^4}}{\sqrt[4]{2}\,x^2-\sqrt{1+x^4}}\right)}{4\,2^{5/8}}$$
$$+\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\frac{3\tanh^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{4\sqrt[8]{2}}-\frac{3\tanh^{-1}\left(\frac{2\,2^{3/8}x\sqrt[4]{1+x^4}}{2x^2+2^{3/4}\sqrt{1+x^4}}\right)}{4\,2^{5/8}}$$

Mathematica 12.3 output

$$\int\frac{1-x^4+2x^8}{\sqrt[4]{1+x^4}\,\left(-1-2x^4+x^8\right)}\,dx$$

24.435 Problem number 2302

$$\int\frac{\left(-1+x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^2\right)\sqrt{1+x^4}}\,dx$$

Optimal antiderivative

$$-\sqrt{-2+2\sqrt{2}}\,\arctan\left(\frac{\sqrt{2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$
$$+\text{arctanh}\left(\frac{\sqrt{2}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}$$
$$-\sqrt{2+2\sqrt{2}}\,\text{arctanh}\left(\frac{\sqrt{-2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$

command

```
Integrate[((-1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/((1 + x^2)*Sqrt[1 + x^4]),x]
```

Mathematica 13.1 output

$$\sqrt{2}\left(-\sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\left(-1+x^2+\sqrt{1+x^4}\right)}{x\sqrt{x^2+\sqrt{1+x^4}}}\right)\right.$$
$$+\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$\left.-\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^2\right)\sqrt{1+x^4}}\,dx$$

24.436 Problem number 2303

$$\int \frac{x}{x+\sqrt{c+\sqrt{b+ax}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[x/(x + Sqrt[c + Sqrt[b + a*x]]),x]
```

Mathematica 13.1 output

$$\frac{b-c^2+ax}{a}-4\sqrt{c+\sqrt{b+ax}}+4\text{RootSum}\left[b-c^2-a\#1+2c\#1^2\right.$$
$$\left.-\#1^4\&,\frac{b\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)-c^2\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)-a\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1+}{-a+4c\#1-4\#1^3}\right.$$

Mathematica 12.3 output

$Aborted

24.437 Problem number 2304

$$\int \frac{x}{x+\sqrt{c+\sqrt{b+ax}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[x/(x + Sqrt[c + Sqrt[b + a*x]]),x]
```

Mathematica 13.1 output

$$\frac{b-c^2+ax}{a}-4\sqrt{c+\sqrt{b+ax}}+4\text{RootSum}\left[b-c^2-a\#1+2c\#1^2\right.$$
$$\left.-\#1^4\&,\ \frac{b\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)-c^2\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)-a\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1+}{-a+4c\#1-4\#1^3}\right.$$

Mathematica 12.3 output

$$\int \frac{x}{x+\sqrt{c+\sqrt{b+ax}}}\,dx$$

24.438 Problem number 2306

$$\int \frac{\left(1+x^4\right)\sqrt[4]{-x^3+x^4}}{x^4\left(-1+x^4\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^4)*(-x^3 + x^4)^(1/4))/(x^4*(-1 + x^4)),x]
```

Mathematica 13.1 output

$$(-1+x)^{3/4}\left(-8\left(4\sqrt[4]{-1+x}\left(-5+x+4x^2\right)-45\sqrt[4]{2}\,x^{9/4}\text{ArcTan}\left(\frac{\sqrt[4]{2}}{\sqrt[4]{\frac{-1+x}{x}}}\right)+45\sqrt[4]{2}\,x^{9/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}}{\sqrt[4]{\frac{-1+}{x}}}\right.\right.\right.$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^4\right)\sqrt[4]{-x^3+x^4}}{x^4\left(-1+x^4\right)}\,dx$$

24.439 Problem number 2307

$$\int \frac{\left(1+x^4\right)\sqrt[4]{-x^3+x^4}}{x^4\left(-1+x^4\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^4)*(-x^3 + x^4)^(1/4))/(x^4*(-1 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{(-1+x)^{3/4}\left(-8\left(4\sqrt[4]{-1+x}\left(-5+x+4x^2\right)-45\sqrt[4]{2}\,x^{9/4}\text{ArcTan}\left(\frac{\sqrt[4]{2}}{\sqrt[4]{\frac{-1+x}{x}}}\right)+45\sqrt[4]{2}\,x^{9/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}}{\sqrt[4]{\frac{-1+}{x}}}\right.\right.\right.}{360}$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^4\right)\sqrt[4]{-x^3+x^4}}{x^4\left(-1+x^4\right)}\,dx$$

24.440 Problem number 2311

$$\int \frac{1}{x^2\sqrt[3]{-1-x+5x^2+2x^3-10x^4+2x^5+7x^6-5x^7+x^8}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[1/(x^2*(-1 - x + 5*x^2 + 2*x^3 - 10*x^4 + 2*x^5 + 7*x^6 - 5*x^7 + x^8)^(1/3)),x]
```

Mathematica 13.1 output

$$-\frac{\left(-1-x+x^2\right)\left(-6+6x+2\sqrt{3}\,(-1+x)^{2/3}x\text{ArcTan}\left(\frac{1-2\sqrt[3]{-1+x}}{\sqrt{3}}\right)-2(-1+x)^{2/3}x\log\left(1+\sqrt[3]{-1+x}\right)+(\right.}{}$$

Mathematica 12.3 output

$$\int \frac{1}{x^2\sqrt[3]{-1-x+5x^2+2x^3-10x^4+2x^5+7x^6-5x^7+x^8}}\,dx$$

24.441 Problem number 2315

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(aq^2+2apqx^3+bx^4+ap^2x^6\right)}{x^9}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}\,\left(a\,p^3x^9-a\,p^2q\,x^7+3a\,p^2q\,x^6+2bp\,x^7-ap\,q^2x^4+3ap\,q^2x^3+2bq\,x^4+a\,q^3\right)}{4x^8}$$
$$+\left(a\,p^2q^2+2bpq\right)\ln\left(x\right)+\frac{\left(-a\,p^2q^2-2bpq\right)\ln\left(q+p\,x^3+\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}\right)}{2}$$

command

```
Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(a*q^2 + 2*a*p*q*x^3 +
```

Mathematica 13.1 output

$$\frac{1}{4}\left(\frac{\left(q+px^3\right)\sqrt{q^2-2pq(-1+x)x^3+p^2x^6}\left(2bx^4+a\left(q^2-pq(-2+x)x^3+p^2x^6\right)\right)}{x^8}\right.$$
$$\left.-2pq(2b+apq)\tanh^{-1}\left(\frac{\sqrt{q^2-2pq(-1+x)x^3+p^2x^6}}{q+px^3}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(aq^2+2apqx^3+bx^4+ap^2x^6\right)}{x^9}\,dx$$

24.442 Problem number 2323

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^3\left(aq+bx^2+apx^3\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}}{a\,x^2}$$
$$+\frac{2\sqrt{2a^2pq-b^2}\,\arctan\left(\frac{\sqrt{2a^2pq-b^2}\,x^2}{aq+b\,x^2+ap\,x^3+a\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}}\right)}{a^2}$$
$$+\frac{2b\ln(x)}{a^2}-\frac{b\ln\left(q+p\,x^3+\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}\right)}{a^2}$$

command

```
Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6])/(x^3*(a*q + b*x^2 + a*
```

Mathematica 13.1 output

$$\frac{a\sqrt{q^2-2pq(-1+x)x^3+p^2x^6}+2\sqrt{-b^2+2a^2pq}\,x^2\text{ArcTan}\left(\frac{\sqrt{-b^2+2a^2pq}\,x^2}{bx^2+a\left(q+px^3+\sqrt{q^2-2pq(-1+x)x^3+p^2x^6}\right)}\right)-}{a^2x^2}$$

Mathematica 12.3 output

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^3\left(aq+bx^2+apx^3\right)}\,dx$$

24.443 Problem number 2328

$$\int \frac{\left(-q+2px^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^2+a\left(q+px^3\right)^2\right)}{x^5}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}\left(a\,p^3x^9+3a\,p^2q\,x^6-a\,p^2q\,x^5+3apq^2x^3+2bp\,x^5-apq^2x^2+a\,q^3+2bq\,x^2\right)}{4x^4}$$
$$+\frac{\left(a\,p^2q^2+2bpq\right)\ln(x)}{2}+\frac{\left(-a\,p^2q^2-2bpq\right)\ln\left(q+p\,x^3+\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}\right)}{2}$$

command

```
Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*x^2 + a*(q + p*x^3)^
```

Mathematica 13.1 output

$$\frac{1}{4}\left(\frac{\left(q+px^3\right)\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}\left(2bx^2+a\left(q^2+p^2x^6+pqx^2(-1+2x)\right)\right)}{x^4}\right.$$
$$\left.-2pq(2b+apq)\tanh^{-1}\left(\frac{\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}}{q+px^3}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-q+2px^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^2+a\left(q+px^3\right)^2\right)}{x^5}\,dx$$

24.444 Problem number 2331

$$\int \frac{1}{d+cx+\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(d + c*x + Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])^(-1),x]
```

Mathematica 13.1 output

$$-\frac{\log\left(ax+\sqrt{b^2+a^2x^2}\right)-2\text{RootSum}\left[b^2c-2ad\#1^2-2a\#1^3-c\#1^4\&,\frac{ad\log\left(\sqrt{ax+\sqrt{b^2+a^2x^2}}-\#1\right)+a\log}{}\right.}{c}$$

Mathematica 12.3 output

$$\int \frac{1}{d+cx+\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$$

24.445 Problem number 2332

$$\int \frac{1}{d+cx+\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(d + c*x + Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])^(-1),x]
```

Mathematica 13.1 output

$$-\frac{\log\left(ax+\sqrt{b^2+a^2x^2}\right)-2\text{RootSum}\left[b^2c-2ad\#1^2-2a\#1^3-c\#1^4\&,\ \frac{ad\log\left(\sqrt{ax+\sqrt{b^2+a^2x^2}}-\#1\right)+a\log}{}\right.}{c}$$

Mathematica 12.3 output

$$\int \frac{1}{d+cx+\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$$

24.446 Problem number 2334

$$\int \frac{1+x^2+x^4}{(1-x^4)\sqrt[4]{x^3+x^5}}\,dx$$

Optimal antiderivative

$$\frac{\left(x^5+x^3\right)^{\frac{3}{4}}}{x^2\left(x^2+1\right)}+\frac{3\arctan\left(\frac{2^{\frac{1}{4}}x}{\left(x^5+x^3\right)^{\frac{1}{4}}}\right)2^{\frac{3}{4}}}{8}-\frac{3\arctan\left(\frac{2^{\frac{3}{4}}x\left(x^5+x^3\right)^{\frac{1}{4}}}{x^2\sqrt{2}-\sqrt{x^5+x^3}}\right)2^{\frac{1}{4}}}{8}$$
$$+\frac{3\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}x}{\left(x^5+x^3\right)^{\frac{1}{4}}}\right)2^{\frac{3}{4}}}{8}+\frac{3\operatorname{arctanh}\left(\frac{\frac{x^2 2^{\frac{3}{4}}}{2}+\frac{\sqrt{x^5+x^3}\,2^{\frac{1}{4}}}{2}}{x\left(x^5+x^3\right)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{8}$$

command

```
Integrate[(1 + x^2 + x^4)/((1 - x^4)*(x^3 + x^5)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{x^{3/4}\left(8\sqrt[4]{x}+3\,2^{3/4}\sqrt[4]{1+x^2}\,\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{x}}{\sqrt[4]{1+x^2}}\right)-3\sqrt[4]{2}\,\sqrt[4]{1+x^2}\,\text{ArcTan}\left(\frac{2^{3/4}\sqrt[4]{x}\,\sqrt[4]{1+x^2}}{\sqrt{2}\,\sqrt{x}-\sqrt{1+x^2}}\right)+3\,2^{3/4}\sqrt[4]{1+}\right.}{8\sqrt[4]{x^3+x^5}}$$

Mathematica 12.3 output

$$\int \frac{1+x^2+x^4}{(1-x^4)\sqrt[4]{x^3+x^5}}\,dx$$

24.447 Problem number 2337

$$\int \frac{1+x}{(3+x)(1+2x)\sqrt[3]{1+x^2}}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\frac{4\,2^{\frac{1}{3}}\sqrt{3}}{15}\,5^{\frac{1}{3}}-\frac{2\,2^{\frac{1}{3}}x\sqrt{3}}{15}\,5^{\frac{1}{3}}+\frac{\left(x^2+1\right)^{\frac{1}{3}}\sqrt{3}}{3}}{\left(x^2+1\right)^{\frac{1}{3}}}\right)10^{\frac{2}{3}}}{50}$$
$$+\frac{\ln\left(-2\,10^{\frac{1}{3}}+10^{\frac{1}{3}}x+5\left(x^2+1\right)^{\frac{1}{3}}\right)10^{\frac{2}{3}}}{50}$$
$$-\frac{\ln\left(4\,10^{\frac{2}{3}}-4\,10^{\frac{2}{3}}x+10^{\frac{2}{3}}x^2+\left(10\,10^{\frac{1}{3}}-5\,10^{\frac{1}{3}}x\right)\left(x^2+1\right)^{\frac{1}{3}}+25\left(x^2+1\right)^{\frac{2}{3}}\right)10^{\frac{2}{3}}}{100}$$

command

```
Integrate[(1 + x)/((3 + x)*(1 + 2*x)*(1 + x^2)^(1/3)),x]
```

Mathematica 13.1 output

$$-\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{4\sqrt[3]{10}-2\sqrt[3]{10}\,x+5\sqrt[3]{1+x^2}}{5\sqrt{3}\,\sqrt[3]{1+x^2}}\right)-2\log\left(-2\sqrt[3]{10}+\sqrt[3]{10}\,x+5\sqrt[3]{1+x^2}\right)+\log\left(4\,10^{2/3}-4\,10^{2/3}x\right.}{10\sqrt[3]{10}}$$

Mathematica 12.3 output

$$\int \frac{1+x}{(3+x)(1+2x)\sqrt[3]{1+x^2}}\,dx$$

24.448 Problem number 2341

$$\int \frac{1+x^4}{(1-x^4)\sqrt[4]{x^3+x^5}}\,dx$$

Optimal antiderivative

$$\frac{2\left(x^5+x^3\right)^{\frac{3}{4}}}{x^2\left(x^2+1\right)}+\frac{\arctan\left(\frac{2^{\frac{1}{4}}x}{\left(x^5+x^3\right)^{\frac{1}{4}}}\right)2^{\frac{3}{4}}}{4}-\frac{\arctan\left(\frac{2^{\frac{3}{4}}x\left(x^5+x^3\right)^{\frac{1}{4}}}{x^2\sqrt{2}-\sqrt{x^5+x^3}}\right)2^{\frac{1}{4}}}{4}$$
$$+\frac{\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}x}{\left(x^5+x^3\right)^{\frac{1}{4}}}\right)2^{\frac{3}{4}}}{4}+\frac{\operatorname{arctanh}\left(\frac{\frac{x^2 2^{\frac{3}{4}}}{2}+\frac{\sqrt{x^5+x^3}\,2^{\frac{1}{4}}}{2}}{x\left(x^5+x^3\right)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{4}$$

command

```
Integrate[(1 + x^4)/((1 - x^4)*(x^3 + x^5)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{x^{3/4}\left(8\sqrt[4]{x}+2^{3/4}\sqrt[4]{1+x^2}\,\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{x}}{\sqrt[4]{1+x^2}}\right)-\sqrt[4]{2}\,\sqrt[4]{1+x^2}\,\text{ArcTan}\left(\frac{2^{3/4}\sqrt[4]{x}\,\sqrt[4]{1+x^2}}{\sqrt{2}\,\sqrt{x}-\sqrt{1+x^2}}\right)+2^{3/4}\sqrt[4]{1+x^2}\right.}{4\sqrt[4]{x^3+x^5}}$$

Mathematica 12.3 output

$$\int \frac{1+x^4}{(1-x^4)\sqrt[4]{x^3+x^5}}\,dx$$

24.449 Problem number 2352

$$\int \frac{\left(-b+a^2x^2\right)\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx$$

Optimal antiderivative

$$\frac{ax\sqrt{a\,x^2+\sqrt{a^2x^4+b}}}{2}$$
$$-\frac{\sqrt{a}\,\sqrt{b}\,\arctan\left(\frac{a\,x^2}{\sqrt{b}}+\frac{\sqrt{a^2x^4+b}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt{a}\,x\sqrt{a\,x^2+\sqrt{a^2x^4+b}}}{\sqrt{b}}\right)\sqrt{2}}{2}$$
$$-\frac{b\ln\left(a\,x^2+\sqrt{a^2x^4+b}+\sqrt{2}\,\sqrt{a}\,x\sqrt{a\,x^2+\sqrt{a^2x^4+b}}\right)\sqrt{2}}{2\sqrt{a}}$$

command

```
Integrate[((-b + a^2*x^2)*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]])/Sqrt[b + a^2*x^4],x]
```

Mathematica 13.1 output

$$\frac{a^{3/2}x\sqrt{ax^2+\sqrt{b+a^2x^4}}+\sqrt{2}\,a\sqrt{b}\,\text{ArcTan}\left(\frac{ax^2+\sqrt{b+a^2x^4}-\sqrt{2}\,\sqrt{a}\,x\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b}}\right)-\sqrt{2}\,b\tanh^{-}}{2\sqrt{a}}$$

Mathematica 12.3 output

$$\int \frac{\left(-b+a^2x^2\right)\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx$$

24.450 Problem number 2362

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}{1-x^4+x^8}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{3^{\frac{1}{8}}x}{(x^6+x^2)^{\frac{1}{4}}}\right)3^{\frac{5}{8}}}{6}-\frac{\arctan\left(\frac{\sqrt{2}\,3^{\frac{7}{8}}x(x^6+x^2)^{\frac{1}{4}}}{-3x^2+3^{\frac{3}{4}}\sqrt{x^6+x^2}}\right)\sqrt{2}\,3^{\frac{5}{8}}}{12}$$
$$-\frac{\text{arctanh}\left(\frac{3^{\frac{1}{8}}x}{(x^6+x^2)^{\frac{1}{4}}}\right)3^{\frac{5}{8}}}{6}+\frac{\text{arctanh}\left(\frac{\frac{3^{\frac{1}{8}}x^2\sqrt{2}}{2}+\frac{\sqrt{x^6+x^2}\,\sqrt{2}\,3^{\frac{7}{8}}}{6}}{x(x^6+x^2)^{\frac{1}{4}}}\right)\sqrt{2}\,3^{\frac{5}{8}}}{12}$$

command

```
Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 - x^4 + x^8),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{x^2+x^6}\left(2\text{ArcTan}\left(\frac{\sqrt[8]{3}\,\sqrt{x}}{\sqrt[4]{1+x^4}}\right)+\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,3^{7/8}\sqrt{x}\,\sqrt[4]{1+x^4}}{3x-3^{3/4}\sqrt{1+x^4}}\right)-2\tanh^{-1}\left(\frac{\sqrt[8]{3}\,\sqrt{x}}{\sqrt[4]{1+x^4}}\right)+\sqrt{2}\,\tanh^{-}\right.}{4\,3^{3/8}\sqrt{x}\,\sqrt[4]{1+x^4}}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}{1-x^4+x^8}\,dx$$

24.451 Problem number 2363

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}{1-x^4+x^8}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{3^{\frac{1}{8}}x}{\left(x^6+x^2\right)^{\frac{1}{4}}}\right)3^{\frac{5}{8}}}{6}-\frac{\arctan\left(\frac{\sqrt{2}\,3^{\frac{7}{8}}x\left(x^6+x^2\right)^{\frac{1}{4}}}{-3x^2+3^{\frac{3}{4}}\sqrt{x^6+x^2}}\right)\sqrt{2}\,3^{\frac{5}{8}}}{12}$$
$$-\frac{\operatorname{arctanh}\left(\frac{3^{\frac{1}{8}}x}{\left(x^6+x^2\right)^{\frac{1}{4}}}\right)3^{\frac{5}{8}}}{6}+\frac{\operatorname{arctanh}\left(\frac{\frac{3^{\frac{1}{8}}x^2\sqrt{2}}{2}+\frac{\sqrt{x^6+x^2}\,\sqrt{2}\,3^{\frac{7}{8}}}{6}}{x\left(x^6+x^2\right)^{\frac{1}{4}}}\right)\sqrt{2}\,3^{\frac{5}{8}}}{12}$$

command

```
Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 - x^4 + x^8),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{x^2+x^6}\left(2\text{ArcTan}\left(\frac{\sqrt[8]{3}\,\sqrt{x}}{\sqrt[4]{1+x^4}}\right)+\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,3^{7/8}\sqrt{x}\,\sqrt[4]{1+x^4}}{3x-3^{3/4}\sqrt{1+x^4}}\right)-2\tanh^{-1}\left(\frac{\sqrt[8]{3}\,\sqrt{x}}{\sqrt[4]{1+x^4}}\right)+\sqrt{2}\,\tanh^{-}\right.}{4\,3^{3/8}\sqrt{x}\,\sqrt[4]{1+x^4}}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}{1-x^4+x^8}\,dx$$

24.452 Problem number 2371

$$\int \frac{x^4\sqrt[4]{-x^2+x^4}}{1+x^4+x^8}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x^4*(-x^2 + x^4)^(1/4))/(1 + x^4 + x^8),x]
```

Mathematica 13.1 output

$$\frac{x^{3/2}\left(-1+x^2\right)^{3/4}\left(\text{RootSum}\left[3-3\#1^4+\#1^8\&,\frac{-3\log(x)+6\log\left(\sqrt[4]{-1+x^2}-\sqrt{x}\,\#1\right)+\log(x)\#1^4-2\log\left(\sqrt[4]{-1+x^2}-\right.}{-3\#1^3+2\#1^7}\right.\right.}{8}$$

Mathematica 12.3 output

$$\int \frac{x^4\sqrt[4]{-x^2+x^4}}{1+x^4+x^8}\,dx$$

24.453 Problem number 2372

$$\int \frac{x^4\sqrt[4]{-x^2+x^4}}{1+x^4+x^8}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x^4*(-x^2 + x^4)^(1/4))/(1 + x^4 + x^8),x]
```

Mathematica 13.1 output

$$\frac{x^{3/2}\left(-1+x^2\right)^{3/4}\left(\text{RootSum}\left[3-3\#1^4+\#1^8\&,\frac{-3\log(x)+6\log\left(\sqrt[4]{-1+x^2}-\sqrt{x}\,\#1\right)+\log(x)\#1^4-2\log\left(\sqrt[4]{-1+x^2}-\right.}{-3\#1^3+2\#1^7}\right.\right.}{8}$$

Mathematica 12.3 output

$$\int \frac{x^4\sqrt[4]{-x^2+x^4}}{1+x^4+x^8}\,dx$$

24.454 Problem number 2374

$$\int \frac{\left(-1+x^4\right)^2}{\left(1+x^4\right)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Optimal antiderivative

$$\frac{x^2\sqrt{x^4+1}\left(6x^6+9x^2\right)+x^2\left(6x^8+12x^4+4\right)}{2x\sqrt{x^4+1}\left(2x^6+2x^2\right)\sqrt{x^2+\sqrt{x^4+1}}+2x\left(2x^8+3x^4+1\right)\sqrt{x^2+\sqrt{x^4+1}}}$$
$$-\frac{3\operatorname{arctanh}\left(\frac{x}{\sqrt{x^2+\sqrt{x^4+1}}}\right)}{2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}}{2}$$

command

```
Integrate[(-1 + x^4)^2/((1 + x^4)^2*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

Mathematica 13.1 output

$$\frac{1}{2}\left(\frac{x\left(4+12x^4+6x^8+9x^2\sqrt{1+x^4}+6x^6\sqrt{1+x^4}\right)}{\left(1+x^4\right)\sqrt{x^2+\sqrt{1+x^4}}\left(1+2x^4+2x^2\sqrt{1+x^4}\right)}\right.$$
$$\left.-3\tanh^{-1}\left(\frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)+\sqrt{2}\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^4\right)^2}{\left(1+x^4\right)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

24.455 Problem number 2375

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^2\right)^2\sqrt{1+x^4}}\,dx$$

Optimal antiderivative

$$\frac{-x^2\left(x^2-1\right)-x^2\sqrt{x^4+1}}{4x\left(x^2+1\right)\sqrt{x^2+\sqrt{x^4+1}}}+\frac{\sqrt{-1+5\sqrt{2}}\ \arctan\left(\frac{\sqrt{2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{4}$$
$$+\frac{\sqrt{1+5\sqrt{2}}\ \operatorname{arctanh}\left(\frac{\sqrt{-2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{4}$$

command

```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x^2)^2*Sqrt[1 + x^4]),x]
```

Mathematica 13.1 output

$$\frac{1}{4}\left(-\frac{x\left(-1+x^2+\sqrt{1+x^4}\right)}{\left(1+x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}\right.$$
$$+\sqrt{-1+5\sqrt{2}}\ \text{ArcTan}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\left(-1+x^2+\sqrt{1+x^4}\right)}{x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$\left.+\sqrt{1+5\sqrt{2}}\ \tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^2\right)^2\sqrt{1+x^4}}\,dx$$

24.456 Problem number 2376

$$\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)}{cx^4+bx^2\left(q+px^5\right)+a\left(q+px^5\right)^2}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{2}\left(-b+\sqrt{-4ac+b^2}\right)\arctan\left(\frac{\sqrt{b-\sqrt{-4ac+b^2}}\,x\sqrt{2}}{2\sqrt{a}\,\sqrt{p\,x^5+q}}\right)}{\sqrt{a}\,\sqrt{-4ac+b^2}\,\sqrt{b-\sqrt{-4ac+b^2}}}$$
$$-\frac{\sqrt{2}\,\sqrt{b+\sqrt{-4ac+b^2}}\,\arctan\left(\frac{\sqrt{b+\sqrt{-4ac+b^2}}\,x\sqrt{2}}{2\sqrt{a}\,\sqrt{p\,x^5+q}}\right)}{\sqrt{a}\,\sqrt{-4ac+b^2}}$$

command

```
Integrate[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(c*x^4 + b*x^2*(q + p*x^5) + a*(q + p*x^5)^2),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{2}\left(\sqrt{b-\sqrt{b^2-4ac}}\ \text{ArcTan}\left(\frac{\sqrt{b-\sqrt{b^2-4ac}}\,x}{\sqrt{2}\,\sqrt{a}\,\sqrt{q+px^5}}\right)-\sqrt{b+\sqrt{b^2-4ac}}\ \text{ArcTan}\left(\frac{\sqrt{b+\sqrt{b^2-4ac}}\,x}{\sqrt{2}\,\sqrt{a}\,\sqrt{q+px^5}}\right)\right)}{\sqrt{a}\,\sqrt{b^2-4ac}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)}{cx^4+bx^2\left(q+px^5\right)+a\left(q+px^5\right)^2}\,dx$$

24.457 Problem number 2377

$$\int \frac{1}{x\sqrt{ax+\sqrt{-b+a^2x^2}}\,\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[1/(x*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}{c\sqrt{ax+\sqrt{-b+a^2x^2}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}{\sqrt{c}}\right)}{c^{3/2}}-\text{RootSum}\left[b+c^4-4c^3\#1^2+6c^2\#1^4-4c\#1^6+\#1^8\&,\frac{\log\left(\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}-\#1\right)}{c\#1-\#1^3}\&\right]$$

Mathematica 12.3 output

$$\int \frac{1}{x\sqrt{ax+\sqrt{-b+a^2x^2}}\,\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

24.458 Problem number 2382

$$\int \frac{\sqrt[4]{x^3+x^5}\left(1+x^4+x^8\right)}{x^4\left(-1+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{4\left(x^4+2x^2+1\right)\left(x^5+x^3\right)^{\frac{1}{4}}}{9x^3}+\frac{3\,2^{\frac{1}{4}}\arctan\left(\frac{2^{\frac{1}{4}}x}{\left(x^5+x^3\right)^{\frac{1}{4}}}\right)}{4}-\frac{3\arctan\left(\frac{2^{\frac{3}{4}}x\left(x^5+x^3\right)^{\frac{1}{4}}}{x^2\sqrt{2}-\sqrt{x^5+x^3}}\right)2^{\frac{3}{4}}}{8}$$
$$-\frac{3\,2^{\frac{1}{4}}\operatorname{arctanh}\left(\frac{2^{\frac{1}{4}}x}{\left(x^5+x^3\right)^{\frac{1}{4}}}\right)}{4}-\frac{3\operatorname{arctanh}\left(\frac{\frac{x^2 2^{\frac{3}{4}}}{2}+\frac{\sqrt{x^5+x^3}\,2^{\frac{1}{4}}}{2}}{x\left(x^5+x^3\right)^{\frac{1}{4}}}\right)2^{\frac{3}{4}}}{8}$$

command

```
Integrate[((x^3 + x^5)^(1/4)*(1 + x^4 + x^8))/(x^4*(-1 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{x^3+x^5}\left(32\sqrt[4]{1+x^2}+64x^2\sqrt[4]{1+x^2}+32x^4\sqrt[4]{1+x^2}+54\sqrt[4]{2}\,x^{9/4}\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{x}}{\sqrt[4]{1+x^2}}\right)-27\,2^{3/4}x^{9/4}\text{ArcTa}\right.}{72x^3\sqrt[4]{1+}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{x^3+x^5}\left(1+x^4+x^8\right)}{x^4\left(-1+x^4\right)}\,dx$$

24.459 Problem number 2386

$$\int \frac{\sqrt[3]{x+x^3}\left(b+ax^6\right)}{d+cx^6}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((x + x^3)^(1/3)*(b + a*x^6))/(d + c*x^6),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x+x^3}\left(ad\left(6x^{4/3}\sqrt[3]{1+x^2}-2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}+2\sqrt[3]{1+x^2}}\right)-2\log\left(c\left(-x^{2/3}+\sqrt[3]{1+x^2}\right)\right)+\log\left(x^{4/3}+\right.\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[3]{x+x^3}\left(b+ax^6\right)}{d+cx^6}\,dx$$

24.460 Problem number 2387

$$\int \frac{\sqrt[3]{x+x^3}\left(b+ax^6\right)}{d+cx^6}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((x + x^3)^(1/3)*(b + a*x^6))/(d + c*x^6),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x+x^3}\left(ad\left(6x^{4/3}\sqrt[3]{1+x^2}-2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}+2\sqrt[3]{1+x^2}}\right)-2\log\left(c\left(-x^{2/3}+\sqrt[3]{1+x^2}\right)\right)+\log\left(x^{4/3}+\right.\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[3]{x+x^3}\left(b+ax^6\right)}{d+cx^6}\,dx$$

24.461 Problem number 2389

$$\int \frac{\left(-q+2px^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^3+a\left(q+px^3\right)^3\right)}{x^6}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}\left(6a\,p^4x^{12}+24a\,p^3q\,x^9-4a\,p^3q\,x^8+36a\,p^2q^2x^6-8a\,p^2q^2x^5-16a\,p^2q^2x^4+24apq^{}\right.}{30x^5}$$

$$+bpq\ln(x)-bpq\ln\left(q+p\,x^3+\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}\right)$$

command

```
Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*x^3 + a*(q + p*x^3)^
```

Mathematica 13.1 output

$$\frac{\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}\left(15bx^3\left(q+px^3\right)+2a\left(3q^4+3p^4x^{12}+2pq^3x^2(-1+6x)+2p^3qx^8(-1+6x)+2p^2q^2\right.\right.}{30x^5}$$
$$-bpq\tanh^{-1}\left(\frac{\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}}{q+px^3}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-q+2px^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^3+a\left(q+px^3\right)^3\right)}{x^6}\,dx$$

24.462 Problem number 2390

$$\int \frac{-1+(-1+2k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(2+b)x+(1+bk)x^2\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,b^{\frac{1}{3}}\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}}{2-2x+b^{\frac{1}{3}}\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}}\right)}{b^{\frac{2}{3}}}+\frac{\ln\left(-1+x+b^{\frac{1}{3}}\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}\right)}{b^{\frac{2}{3}}}$$
$$-\frac{\ln\left(1-2x+x^2+\left(b^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}+b^{\frac{2}{3}}\left(x+(-1-k)x^2+kx^3\right)^{\frac{2}{3}}\right)}{2b^{\frac{2}{3}}}$$

command

```
Integrate[(-1 + (-1 + 2*k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (2 + b)*x + (1 + b*k)*x^2)),x
```

Mathematica 13.1 output

$$x\sqrt[3]{\frac{-1+kx}{-1+x}}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{\frac{x}{-1+x}}\,\sqrt[3]{\frac{-1+kx}{-1+x}}}{-2+\sqrt[3]{b}\,\sqrt[3]{\frac{x}{-1+x}}\,\sqrt[3]{\frac{-1+kx}{-1+x}}}\right)-2\log\left(\sqrt[3]{\frac{x}{-1+x}}\,\sqrt[3]{\frac{-1+kx}{-1+x}}\right)+\log\left(\left(\frac{x}{-1+x}\right.\right.\right.$$
$$2b^{2/3}\left(\frac{x}{-1+}\right.$$

Mathematica 12.3 output

$$\int \frac{-1+(-1+2k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(2+b)x+(1+bk)x^2\right)}\,dx$$

24.463 Problem number 2393

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(bx^6+a\left(q+px^3\right)^3\right)}{x^{11}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}\left(6a\,p^4x^{12}-4a\,p^3q\,x^{10}+24a\,p^3q\,x^9-16a\,p^2q^2x^8-8a\,p^2q^2x^7+36a\,p^2q^2x^6+15bp\,x\right.}{30x^{10}}$$
$$+2bpq\ln(x)-bpq\ln\left(q+p\,x^3+\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}\right)$$

command

```
Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(b*x^6 + a*(q + p*x^3)^
```

Mathematica 13.1 output

$$\frac{\sqrt{q^2-2pq(-1+x)x^3+p^2x^6}\left(15bx^6\left(q+px^3\right)+2a\left(3q^4-2pq^3(-6+x)x^3-2p^3q(-6+x)x^9+3p^4x^{12}-2p^2q^2x^6\right.\right.}{30x^{10}}$$
$$-bpq\tanh^{-1}\left(\frac{\sqrt{q^2-2pq(-1+x)x^3+p^2x^6}}{q+px^3}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(bx^6+a\left(q+px^3\right)^3\right)}{x^{11}}\,dx$$

24.464 Problem number 2397

$$\int \frac{b+2ax}{(-b+ax)(2b+ax)\sqrt[4]{-1+bx+ax^2}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,a^{\frac{1}{4}}\left(-2b^2+a\right)^{\frac{1}{4}}\left(a\,x^2+bx-1\right)^{\frac{1}{4}}}{\sqrt{-2b^2+a}-\sqrt{a}\,\sqrt{a\,x^2+bx-1}}\right)}{a^{\frac{3}{4}}\left(-2b^2+a\right)^{\frac{1}{4}}}$$
$$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\frac{\left(-2b^2+a\right)^{\frac{1}{4}}\sqrt{2}}{2a^{\frac{1}{4}}}+\frac{a^{\frac{1}{4}}\sqrt{a\,x^2+bx-1}\,\sqrt{2}}{2\left(-2b^2+a\right)^{\frac{1}{4}}}}{\left(a\,x^2+bx-1\right)^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}\left(-2b^2+a\right)^{\frac{1}{4}}}$$

command

```
Integrate[(b + 2*a*x)/((-b + a*x)*(2*b + a*x)*(-1 + b*x + a*x^2)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{2}\left(\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{a-2b^2}\sqrt[4]{-1+bx+ax^2}}{\sqrt{a-2b^2}-\sqrt{a}\sqrt{-1+bx+ax^2}}\right)-\tanh^{-1}\left(\frac{\sqrt{a-2b^2}+\sqrt{a}\sqrt{-1+bx+ax^2}}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{a-2b^2}\sqrt[4]{-1+bx+ax^2}}\right)\right)}{a^{3/4}\sqrt[4]{a-2b^2}}$$

Mathematica 12.3 output

$$\int \frac{b+2ax}{(-b+ax)(2b+ax)\sqrt[4]{-1+bx+ax^2}}\,dx$$

24.465 Problem number 2409

$$\int \frac{1}{x^2-\sqrt{b+ax}\sqrt{c+\sqrt{b+ax}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x^2 - Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]])^(-1),x]
```

Mathematica 13.1 output

$$4a\text{RootSum}\left[b^2-2bc^2+c^4+a^2c\#1+4bc\#1^2-4c^3\#1^2-a^2\#1^3-2b\#1^4+6c^2\#1^4-4c\#1^6+\#1^8\&,\frac{c\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1-\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1^3}{-a^2c-8bc\#1+8c^3\#1+3a^2\#1^2+8b\#1^3-24c^2\#1^3+24c\#1^5-8\#1^7}\&\right]$$

Mathematica 12.3 output

$$\int \frac{1}{x^2-\sqrt{b+ax}\sqrt{c+\sqrt{b+ax}}}\,dx$$

24.466 Problem number 2410

$$\int \frac{1}{\left(x^2-\sqrt{b+ax}\right)\sqrt{c+\sqrt{b+ax}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x^2 - Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]])^(-1),x]
```

Mathematica 13.1 output

$$4a\text{RootSum}\left[b^2-2bc^2+c^4+a^2c\#1+4bc\#1^2-4c^3\#1^2-a^2\#1^3-2b\#1^4+6c^2\#1^4-4c\#1^6+\#1^8\&,\frac{c\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1-\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1^3}{-a^2c-8bc\#1+8c^3\#1+3a^2\#1^2+8b\#1^3-24c^2\#1^3+24c\#1^5-8\#1^7}\&\right]$$

Mathematica 12.3 output

$$\int \frac{1}{\left(x^2-\sqrt{b+ax}\right)\sqrt{c+\sqrt{b+ax}}}\,dx$$

24.467 Problem number 2419

$$\int \sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

$$\frac{\left(60x^2-8x-75\right)\sqrt{1+\sqrt{x+\sqrt{x^2+1}}}+(16+6x)\sqrt{x+\sqrt{x^2+1}}\,\sqrt{1+\sqrt{x+\sqrt{x^2+1}}}+\sqrt{x^2+1}\left(\left(-\right.\right.}{105\sqrt{x+\sqrt{x^2+1}}}$$

$$-\operatorname{arctanh}\left(\sqrt{1+\sqrt{x+\sqrt{x^2+1}}}\right)$$

command

```
Integrate[Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]],x]
```

Mathematica 13.1 output

$$\frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\left(-75+60x^2-8\sqrt{1+x^2}+16\sqrt{x+\sqrt{1+x^2}}+6\sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}+x\left(-8\right.\right.}{105\sqrt{x+\sqrt{1+x^2}}}$$

$$-\tanh^{-1}\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\right)$$

Mathematica 12.3 output

$$\int \sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.468 Problem number 2429

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{1+\sqrt{1+x}}\,\sqrt{1+\sqrt{1+\sqrt{1+x}}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[1 + Sqrt[1 + x]]*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]),x]
```

Mathematica 13.1 output

$$\frac{8}{15}\left(-4+3\sqrt{1+x}-4\sqrt{1+\sqrt{1+x}}\right)\sqrt{1+\sqrt{1+\sqrt{1+x}}}-\text{RootSum}\left[2-8\#1^4\right.$$

$$\left.+8\#1^6+14\#1^8-32\#1^{10}+24\#1^{12}-8\#1^{14}+\#1^{16}\&,\frac{\log\left(\sqrt{1+\sqrt{1+\sqrt{1+x}}}-\#1\right)}{\#1-\#1^3-4\#1^5+8\#1^7-5\#1^9+\#1^{11}}\&\right]$$

Mathematica 12.3 output

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{1+\sqrt{1+x}}\,\sqrt{1+\sqrt{1+\sqrt{1+x}}}}\,dx$$

24.469 Problem number 2430

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{1+\sqrt{1+x}}\,\sqrt{1+\sqrt{1+\sqrt{1+x}}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[1 + Sqrt[1 + x]]*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]),x]
```

Mathematica 13.1 output

$$\frac{8}{15}\left(-4+3\sqrt{1+x}-4\sqrt{1+\sqrt{1+x}}\right)\sqrt{1+\sqrt{1+\sqrt{1+x}}}-\text{RootSum}\left[2-8\#1^4+8\#1^6+14\#1^8-32\#1^{10}+24\#1^{12}-8\#1^{14}+\#1^{16}\&,\frac{\log\left(\sqrt{1+\sqrt{1+\sqrt{1+x}}}-\#1\right)}{\#1-\#1^3-4\#1^5+8\#1^7-5\#1^9+\#1^{11}}\&\right]$$

Mathematica 12.3 output

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{1+\sqrt{1+x}}\,\sqrt{1+\sqrt{1+\sqrt{1+x}}}}\,dx$$

24.470 Problem number 2432

$$\int \frac{2+3x}{\sqrt[3]{4+3x^2}\,(-12+52x+9x^2)}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\frac{10\,2^{\frac{1}{3}}\sqrt{3}\,7^{\frac{1}{3}}}{21}-\frac{2^{\frac{1}{3}}\sqrt{3}\,x7^{\frac{1}{3}}}{7}+\frac{\left(3x^2+4\right)^{\frac{1}{3}}\sqrt{3}}{3}}{(3x^2+4)^{\frac{1}{3}}}\right)14^{\frac{2}{3}}}{196}+\frac{\ln\left(-10\,14^{\frac{1}{3}}+3\,14^{\frac{1}{3}}x+14\left(3x^2+4\right)^{\frac{1}{3}}\right)14^{\frac{2}{3}}}{196}-\frac{\ln\left(100\,14^{\frac{2}{3}}-60\,14^{\frac{2}{3}}x+9\,14^{\frac{2}{3}}x^2+\left(140\,14^{\frac{1}{3}}-42\,14^{\frac{1}{3}}x\right)\left(3x^2+4\right)^{\frac{1}{3}}+196\left(3x^2+4\right)^{\frac{2}{3}}\right)14^{\frac{2}{3}}}{392}$$

command

```
Integrate[(2 + 3*x)/((4 + 3*x^2)^(1/3)*(-12 + 52*x + 9*x^2)),x]
```

Mathematica 13.1 output

$$-\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{10\sqrt[3]{14}-3\sqrt[3]{14}\,x+7\sqrt[3]{4+3x^2}}{7\sqrt{3}\,\sqrt[3]{4+3x^2}}\right)-2\log\left(-10\sqrt[3]{14}+3\sqrt[3]{14}\,x+14\sqrt[3]{4+3x^2}\right)+\log\left(100\ 14^{2/3}-\right.}{28\sqrt[3]{14}}$$

Mathematica 12.3 output

$$\int \frac{2+3x}{\sqrt[3]{4+3x^2}\,(-12+52x+9x^2)}\,dx$$

24.471 Problem number 2434

$$\int \frac{1-x^4+x^8}{x^2\left(-1+x^4\right)^{3/4}\left(-1-x^4+x^8\right)}\,dx$$

Optimal antiderivative

$$-\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}+\frac{\sqrt{-10+10\sqrt{5}}\ \arctan\left(\frac{\sqrt{-2+2\sqrt{5}}\ x}{2\left(x^4-1\right)^{\frac{1}{4}}}\right)}{10}$$
$$-\frac{\sqrt{10+10\sqrt{5}}\ \arctan\left(\frac{\sqrt{2+2\sqrt{5}}\ x}{2\left(x^4-1\right)^{\frac{1}{4}}}\right)}{10}$$
$$-\frac{\sqrt{-10+10\sqrt{5}}\ \text{arctanh}\left(\frac{\sqrt{-2+2\sqrt{5}}\ x}{2\left(x^4-1\right)^{\frac{1}{4}}}\right)}{10}$$
$$+\frac{\sqrt{10+10\sqrt{5}}\ \text{arctanh}\left(\frac{\sqrt{2+2\sqrt{5}}\ x}{2\left(x^4-1\right)^{\frac{1}{4}}}\right)}{10}$$

command

```
Integrate[(1 - x^4 + x^8)/(x^2*(-1 + x^4)^(3/4)*(-1 - x^4 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{1}{10}\left(-\frac{10\sqrt[4]{-1+x^4}}{x}+\sqrt{10\left(-1+\sqrt{5}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\,x}{\sqrt[4]{-1+x^4}}\right)\right.$$
$$-\sqrt{10\left(1+\sqrt{5}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt[4]{-1+x^4}}\right)$$
$$-\sqrt{10\left(-1+\sqrt{5}\right)}\,\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\,x}{\sqrt[4]{-1+x^4}}\right)$$
$$\left.+\sqrt{10\left(1+\sqrt{5}\right)}\,\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt[4]{-1+x^4}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{1-x^4+x^8}{x^2\left(-1+x^4\right)^{3/4}\left(-1-x^4+x^8\right)}\,dx$$

24.472 Problem number 2437

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

$$\frac{\left(32x^2+3x+8\right)\sqrt{1+\sqrt{x+\sqrt{x^2+1}}}+(-2+16x)\sqrt{x+\sqrt{x^2+1}}\,\sqrt{1+\sqrt{x+\sqrt{x^2+1}}}+\sqrt{x^2+1}\,\left(\right.}{24\left(x+\sqrt{x^2+1}\right)^{\frac{3}{2}}}$$
$$-\frac{\text{arctanh}\left(\sqrt{1+\sqrt{x+\sqrt{x^2+1}}}\right)}{8}$$

command

```
Integrate[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[x + Sqrt[1 + x^2]],x]
```

Mathematica 13.1 output

$$\frac{1}{24}\left(\frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\left(8+32x^2+3\sqrt{1+x^2}-2\sqrt{x+\sqrt{1+x^2}}+16\sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}+x\left(3\right.\right.\right.}{\left(x+\sqrt{1+x^2}\right)^{3/2}}$$

$$\left.\left.\left.-3\tanh^{-1}\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\right)\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.473 Problem number 2455

$$\int \frac{b+ax^4}{(-b+ax^4)\sqrt[4]{b^2+cx^4+a^2x^8}}\,dx$$

Optimal antiderivative

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\arctan\left(\frac{(1+i)(2ab+c)^{\frac{1}{4}}x\left(a^2x^8+cx^4+b^2\right)^{\frac{1}{4}}}{\sqrt{2ab+c}\,x^2-i\sqrt{a^2x^8+cx^4+b^2}}\right)}{(2ab+c)^{\frac{1}{4}}}$$

$$+\frac{\left(-\frac{1}{4}-\frac{i}{4}\right)\operatorname{arctanh}\left(\frac{\left(\frac{1}{2}-\frac{i}{2}\right)(2ab+c)^{\frac{1}{4}}x^2+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a^2x^8+cx^4+b^2}}{(2ab+c)^{\frac{1}{4}}}}{x\left(a^2x^8+cx^4+b^2\right)^{\frac{1}{4}}}\right)}{(2ab+c)^{\frac{1}{4}}}$$

command

```
Integrate[(b + a*x^4)/((-b + a*x^4)*(b^2 + c*x^4 + a^2*x^8)^(1/4)),x]
```

Mathematica 13.1 output

$$-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(\operatorname{ArcTan}\left(\frac{\frac{(1-i)\sqrt[4]{2ab+c}\,x^2}{\sqrt[4]{b^2+cx^4+a^2x^8}}-\frac{(1+i)\sqrt[4]{b^2+cx^4+a^2x^8}}{\sqrt[4]{2ab+c}}}{2x}\right)+\tanh^{-1}\left(\frac{\frac{(1-i)\sqrt[4]{2ab+c}\,x^2}{\sqrt[4]{b^2+cx^4+a^2x^8}}+\frac{(1+i)\sqrt[4]{b^2+c}}{\sqrt[4]{2ab}}}{2x}\right)\right.}{\sqrt[4]{2ab+c}}$$

Mathematica 12.3 output

$$\int \frac{b+ax^4}{(-b+ax^4)\sqrt[4]{b^2+cx^4+a^2x^8}}\,dx$$

24.474 Problem number 2467

$$\int \frac{a-2b+x}{\sqrt[3]{(-a+x)(-b+x)}\,(a^2+bd-(2a+d)x+x^2)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,d^{\frac{1}{3}}\left(ab+(-a-b)x+x^2\right)^{\frac{1}{3}}}{-2a+2x+d^{\frac{1}{3}}\left(ab+(-a-b)x+x^2\right)^{\frac{1}{3}}}\right)}{d^{\frac{2}{3}}}+\frac{\ln\left(a-x+d^{\frac{1}{3}}\left(ab+(-a-b)\,x+x^2\right)^{\frac{1}{3}}\right)}{d^{\frac{2}{3}}}$$
$$-\frac{\ln\left(a^2-2ax+x^2+\left(-a\,d^{\frac{1}{3}}+d^{\frac{1}{3}}x\right)\left(ab+(-a-b)\,x+x^2\right)^{\frac{1}{3}}+d^{\frac{2}{3}}\left(ab+(-a-b)\,x+x^2\right)^{\frac{2}{3}}\right)}{2d^{\frac{2}{3}}}$$

command

```
Integrate[(a - 2*b + x)/(((-a + x)*(-b + x))^(1/3)*(a^2 + b*d - (2*a + d)*x + x^2)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{-b+x}}{2(-a+x)^{2/3}+\sqrt[3]{d}\,\sqrt[3]{-b+x}}\right)+2\log\left((-a+x)^{2/3}-\sqrt[3]{d}\,\sqrt[3]{-b+x}\right)-\log\left((-a\right.}{2d^{2/3}\sqrt[3]{(-a+x)(-b+x)}}$$

Mathematica 12.3 output

$$\int \frac{a-2b+x}{\sqrt[3]{(-a+x)(-b+x)}\,(a^2+bd-(2a+d)x+x^2)}\,dx$$

24.475 Problem number 2468

$$\int \frac{-a(a-2b)-2bx+x^2}{((-a+x)(-b+x))^{2/3}\,(b+a^2d-(1+2ad)x+dx^2)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,d^{\frac{1}{3}}\left(ab+(-a-b)x+x^2\right)^{\frac{2}{3}}}{-2b+2x+d^{\frac{1}{3}}\left(ab+(-a-b)x+x^2\right)^{\frac{2}{3}}}\right)}{d^{\frac{2}{3}}}+\frac{\ln\left(b-x+d^{\frac{1}{3}}\left(ab+(-a-b)\,x+x^2\right)^{\frac{2}{3}}\right)}{d^{\frac{2}{3}}}$$
$$-\frac{\ln\left(b^2-2bx+x^2+\left(-b\,d^{\frac{1}{3}}+d^{\frac{1}{3}}x\right)\left(ab+(-a-b)\,x+x^2\right)^{\frac{2}{3}}+d^{\frac{2}{3}}\left(ab+(-a-b)\,x+x^2\right)^{\frac{4}{3}}\right)}{2d^{\frac{2}{3}}}$$

command

```
Integrate[(-(a*(a - 2*b)) - 2*b*x + x^2)/(((-a + x)*(-b + x))^(2/3)*(b + a^2*d - (1 + 2*a*d)*
```

Mathematica 13.1 output

$$-\frac{(a-x)^{2/3}(b-x)^{2/3}\left(-2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{d}\,(a-x)^{2/3}}{\sqrt[3]{d}\,(a-x)^{2/3}-2\sqrt[3]{b-x}}\right)-2\log\left(\sqrt[3]{d}\,(a-x)^{2/3}+\sqrt[3]{b-x}\right)+\log\left(d^{2/3}(a\right.}{2d^{2/3}((a-x)(b-x))^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{-a(a-2b)-2bx+x^2}{((-a+x)(-b+x))^{2/3}\left(b+a^2d-(1+2ad)x+dx^2\right)}\,dx$$

24.476 Problem number 2469

$$\int \frac{-1+(2-k)x}{\sqrt[3]{(1-x)x(1-kx)}\,\left(1-(b+2k)x+(b+k^2)\,x^2\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,b^{\frac{1}{3}}\left(x+(-1-k)x^2+k\,x^3\right)^{\frac{1}{3}}}{2-2kx+b^{\frac{1}{3}}\left(x+(-1-k)x^2+k\,x^3\right)^{\frac{1}{3}}}\right)}{b^{\frac{2}{3}}}+\frac{\ln\left(-1+kx+b^{\frac{1}{3}}\left(x+(-1-k)\,x^2+k\,x^3\right)^{\frac{1}{3}}\right)}{b^{\frac{2}{3}}}$$
$$-\frac{\ln\left(1-2kx+k^2x^2+\left(b^{\frac{1}{3}}-b^{\frac{1}{3}}kx\right)\left(x+(-1-k)\,x^2+k\,x^3\right)^{\frac{1}{3}}+b^{\frac{2}{3}}\left(x+(-1-k)\,x^2+k\,x^3\right)^{\frac{2}{3}}\right)}{2b^{\frac{2}{3}}}$$

command

```
Integrate[(-1 + (2 - k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (b + 2*k)*x + (b + k^2)*x^2)),x]
```

Mathematica 13.1 output

$$\frac{x\sqrt[3]{\frac{-1+kx}{-1+x}}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{\frac{x}{-1+x}}}{\sqrt[3]{b}\,\sqrt[3]{\frac{x}{-1+x}}-2\left(\frac{-1+kx}{-1+x}\right)^{2/3}}\right)+2\log\left(\sqrt[3]{b}\,\sqrt[3]{\frac{x}{-1+x}}+\left(\frac{-1+kx}{-1+x}\right)^{2/3}\right)-\log\left(b^{2/3}\right.}{2b^{2/3}\left(\frac{x}{-1+x}\right)^{2/3}\sqrt[3]{(-1+x)x(-1+kx)}}$$

Mathematica 12.3 output

$$\int \frac{-1+(2-k)x}{\sqrt[3]{(1-x)x(1-kx)}\,\left(1-(b+2k)x+(b+k^2)\,x^2\right)}\,dx$$

24.477 Problem number 2474

$$\int \frac{\sqrt[3]{-x+x^3}\left(-b+ax^6\right)}{-d+cx^6}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-x + x^3)^(1/3)*(-b + a*x^6))/(-d + c*x^6),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\left(-1+x^2\right)^{2/3}\left(ad\left(6x^{4/3}\sqrt[3]{-1+x^2}+2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}+2\sqrt[3]{-1+x^2}}\right)+2\log\left(-x^{2/3}+\sqrt[3]{-1+x^2}\right)-\log\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[3]{-x+x^3}\left(-b+ax^6\right)}{-d+cx^6}\,dx$$

24.478 Problem number 2475

$$\int \frac{\sqrt[3]{-x+x^3}\left(-b+ax^6\right)}{-d+cx^6}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-x + x^3)^(1/3)*(-b + a*x^6))/(-d + c*x^6),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\left(-1+x^2\right)^{2/3}\left(ad\left(6x^{4/3}\sqrt[3]{-1+x^2}+2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}+2\sqrt[3]{-1+x^2}}\right)+2\log\left(-x^{2/3}+\sqrt[3]{-1+x^2}\right)-\log\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[3]{-x+x^3}\left(-b+ax^6\right)}{-d+cx^6}\,dx$$

24.479 Problem number 2476

$$\int \frac{1}{1-x\sqrt{c+bx+ax^2}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(1 - x*Sqrt[c + b*x + a*x^2])^(-1),x]
```

Mathematica 13.1 output

$$-2\text{RootSum}\left[b^2-\sqrt{a}\,c^2-4\sqrt{a}\,b\#1+bc\#1+4a\#1^2-b\#1^3\right.$$
$$\left.+\sqrt{a}\,\#1^4\&,\ \frac{\sqrt{a}\,c\log\left(-\sqrt{a}\,x+\sqrt{c+bx+ax^2}-\#1\right)-b\log\left(-\sqrt{a}\,x+\sqrt{c+bx+ax^2}-\#1\right)\#1+\sqrt{a}\,\log(}{-4\sqrt{a}\,b+bc+8a\#1-3b\#1^2+4\sqrt{a}\,\#1^3}\right.$$

Mathematica 12.3 output

$$\int \frac{1}{1-x\sqrt{c+bx+ax^2}}\,dx$$

24.480 Problem number 2482

$$\int \frac{\sqrt[4]{b+ax^4}\left(2b+3ax^4\right)}{x^6\left(b+ax^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((b + a*x^4)^(1/4)*(2*b + 3*a*x^4))/(x^6*(b + a*x^8)),x]
```

Mathematica 13.1 output

$$\frac{\left(-2b-17ax^4\right)\sqrt[4]{b+ax^4}}{5bx^5}$$
$$-\frac{a\text{RootSum}\left[a^2+ab-2a\#1^4+\#1^8\&,\ \frac{3a^2\log(x)+3ab\log(x)-3a^2\log\left(\sqrt[4]{b+ax^4}-x\#1\right)-3ab\log\left(\sqrt[4]{b+ax^4}-x\#1\right)-3a\,\text{lo}}{-a\#}\right.}{8b}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{b+ax^4}\left(2b+3ax^4\right)}{x^6\left(b+ax^8\right)}\,dx$$

24.481 Problem number 2483

$$\int \frac{\sqrt[4]{b+ax^4}\,(2b+3ax^4)}{x^6\,(b+ax^8)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((b + a*x^4)^(1/4)*(2*b + 3*a*x^4))/(x^6*(b + a*x^8)),x]
```

Mathematica 13.1 output

$$\frac{\left(-2b-17ax^4\right)\sqrt[4]{b+ax^4}}{5bx^5}$$
$$-\frac{a\text{RootSum}\left[a^2+ab-2a\#1^4+\#1^8\&,\ \frac{3a^2\log(x)+3ab\log(x)-3a^2\log\left(\sqrt[4]{b+ax^4}-x\#1\right)-3ab\log\left(\sqrt[4]{b+ax^4}-x\#1\right)-3a\lo}{-a\#}\right.}{8b}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{b+ax^4}\,(2b+3ax^4)}{x^6\,(b+ax^8)}\,dx$$

24.482 Problem number 2492

$$\int \frac{1+x^2}{(-1-x+x^2)\sqrt[3]{-1+x^6}}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{3}\,(x^6-1)^{\frac{1}{3}}}{-2^{\frac{2}{3}}+2^{\frac{2}{3}}x+2^{\frac{2}{3}}x^2+(x^6-1)^{\frac{1}{3}}}\right)2^{\frac{1}{3}}\sqrt{3}}{6}+\frac{\ln\left(-2^{\frac{2}{3}}+2^{\frac{2}{3}}x+2^{\frac{2}{3}}x^2-2\left(x^6-1\right)^{\frac{1}{3}}\right)2^{\frac{1}{3}}}{6}$$
$$-\frac{\ln\left(2^{\frac{1}{3}}-2\,2^{\frac{1}{3}}x-2^{\frac{1}{3}}x^2+2\,2^{\frac{1}{3}}x^3+2^{\frac{1}{3}}x^4+\left(-2^{\frac{2}{3}}+2^{\frac{2}{3}}x+2^{\frac{2}{3}}x^2\right)\left(x^6-1\right)^{\frac{1}{3}}+2\left(x^6-1\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{12}$$

command

```
Integrate[(1 + x^2)/((-1 - x + x^2)*(-1 + x^6)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{-1+x^6}}{-2^{2/3}+2^{2/3}x+2^{2/3}x^2+\sqrt[3]{-1+x^6}}\right)+2\log\left(-2^{2/3}+2^{2/3}x+2^{2/3}x^2-2\sqrt[3]{-1+x^6}\right)-\log\left(\sqrt[3]{2}-2\sqrt[3]\right.}{6\ 2^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{1+x^2}{(-1-x+x^2)\sqrt[3]{-1+x^6}}\,dx$$

24.483 Problem number 2493

$$\int \frac{1+x^2}{(-1-x+x^2)\sqrt[3]{-1+x^6}}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{3}\,\left(x^6-1\right)^{\frac{1}{3}}}{-2^{\frac{2}{3}}+2^{\frac{2}{3}}x+2^{\frac{2}{3}}x^2+\left(x^6-1\right)^{\frac{1}{3}}}\right)2^{\frac{1}{3}}\sqrt{3}}{6}+\frac{\ln\left(-2^{\frac{2}{3}}+2^{\frac{2}{3}}x+2^{\frac{2}{3}}x^2-2\left(x^6-1\right)^{\frac{1}{3}}\right)2^{\frac{1}{3}}}{6}$$
$$-\frac{\ln\left(2^{\frac{1}{3}}-2\,2^{\frac{1}{3}}x-2^{\frac{1}{3}}x^2+2\,2^{\frac{1}{3}}x^3+2^{\frac{1}{3}}x^4+\left(-2^{\frac{2}{3}}+2^{\frac{2}{3}}x+2^{\frac{2}{3}}x^2\right)\left(x^6-1\right)^{\frac{1}{3}}+2\left(x^6-1\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{12}$$

command

```
Integrate[(1 + x^2)/((-1 - x + x^2)*(-1 + x^6)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{-1+x^6}}{-2^{2/3}+2^{2/3}x+2^{2/3}x^2+\sqrt[3]{-1+x^6}}\right)+2\log\left(-2^{2/3}+2^{2/3}x+2^{2/3}x^2-2\sqrt[3]{-1+x^6}\right)-\log\left(\sqrt[3]{2}-2\sqrt[3]\right.}{6\ 2^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{1+x^2}{(-1-x+x^2)\sqrt[3]{-1+x^6}}\,dx$$

24.484 Problem number 2494

$$\int \frac{\left(2-2x^4+3x^5+4x^6\right)\sqrt[3]{-x+2x^3-x^5+x^6+x^7}}{\left(-1+x^2-x^4+x^5+x^6\right)^2}\,dx$$

Optimal antiderivative

$$\begin{aligned}&-\frac{x\left(x^7+x^6-x^5+2x^3-x\right)^{\frac{1}{3}}}{x^6+x^5-x^4+x^2-1}-\frac{\arctan\left(\frac{\sqrt{3}\,\left(x^7+x^6-x^5+2x^3-x\right)^{\frac{1}{3}}}{2x+\left(x^7+x^6-x^5+2x^3-x\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\\&+\frac{\ln\left(-x+\left(x^7+x^6-x^5+2x^3-x\right)^{\frac{1}{3}}\right)}{3}\\&-\frac{\ln\left(x^2+x\left(x^7+x^6-x^5+2x^3-x\right)^{\frac{1}{3}}+\left(x^7+x^6-x^5+2x^3-x\right)^{\frac{2}{3}}\right)}{6}\end{aligned}$$

command

```
Integrate[((2 - 2*x^4 + 3*x^5 + 4*x^6)*(-x + 2*x^3 - x^5 + x^6 + x^7)^(1/3))/(-1 + x^2 - x^4
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x\left(-1+2x^2-x^4+x^5+x^6\right)}\left(-\frac{6x^{4/3}}{-1+x^2-x^4+x^5+x^6}-\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{-1+2x^2-x^4+x^5+x^6}}{2x^{2/3}+\sqrt[3]{-1+2x^2-x^4+x^5+x^6}}\right)}{\sqrt[3]{-1+2x^2-x^4+x^5+x^6}}+\frac{2\log\left(-\right.\right.}{6\sqrt[3]{x}}$$

Mathematica 12.3 output

$$\int \frac{\left(2-2x^4+3x^5+4x^6\right)\sqrt[3]{-x+2x^3-x^5+x^6+x^7}}{\left(-1+x^2-x^4+x^5+x^6\right)^2}\,dx$$

24.485 Problem number 2495

$$\int \frac{-1+(-1+2k)x}{\sqrt[3]{(1-x)x(1-kx)}\,\left(b-(1+2b)x+(b+k)x^2\right)}\,dx$$

Optimal antiderivative

$$\begin{aligned}&\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x+(-1-k)x^2+k\,x^3\right)^{\frac{1}{3}}}{2b^{\frac{1}{3}}-2b^{\frac{1}{3}}x+\left(x+(-1-k)x^2+k\,x^3\right)^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}+\frac{\ln\left(-b^{\frac{1}{3}}+b^{\frac{1}{3}}x+\left(x+(-1-k)\,x^2+k\,x^3\right)^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}}\\&-\frac{\ln\left(b^{\frac{2}{3}}-2b^{\frac{2}{3}}x+b^{\frac{2}{3}}x^2+\left(b^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\left(x+(-1-k)\,x^2+k\,x^3\right)^{\frac{1}{3}}+\left(x+(-1-k)\,x^2+k\,x^3\right)^{\frac{2}{3}}\right)}{2b^{\frac{1}{3}}}\end{aligned}$$

command

```
Integrate[(-1 + (-1 + 2*k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(b - (1 + 2*b)*x + (b + k)*x^2)),x
```

Mathematica 13.1 output

$$\frac{x\sqrt[3]{\frac{-1+kx}{-1+x}}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{\frac{x}{-1+x}}\,\sqrt[3]{\frac{-1+kx}{-1+x}}}{-2\sqrt[3]{b}+\sqrt[3]{\frac{x}{-1+x}}\,\sqrt[3]{\frac{-1+kx}{-1+x}}}\right)+4\tanh^{-1}\left(1+\frac{2\sqrt[3]{\frac{x}{-1+x}}\,\sqrt[3]{\frac{-1+kx}{-1+x}}}{\sqrt[3]{b}}\right)+\log\right.}{2\sqrt[3]{b}\,\left(\frac{x}{-1+x}\right)^{2/3}\sqrt[3]{(-1+x)}}$$

Mathematica 12.3 output

$$\int \frac{-1+(-1+2k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(b-(1+2b)x+(b+k)x^2)}\,dx$$

24.486 Problem number 2497

$$\int \frac{x^7(-4a+3x)}{\left(x^2(-a+x)\right)^{2/3}\left(-a^2+2ax-x^2+dx^8\right)}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,d^{\frac{1}{6}}x^2}{d^{\frac{1}{6}}x^2-2(-a\,x^2+x^3)^{\frac{1}{3}}}\right)}{2d^{\frac{5}{6}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,d^{\frac{1}{6}}x^2}{d^{\frac{1}{6}}x^2+2(-a\,x^2+x^3)^{\frac{1}{3}}}\right)}{2d^{\frac{5}{6}}}$$
$$+\frac{\text{arctanh}\left(\frac{d^{\frac{1}{6}}\left(-a\,x^2+x^3\right)^{\frac{2}{3}}}{a-x}\right)}{d^{\frac{5}{6}}}-\frac{\text{arctanh}\left(\frac{d^{\frac{1}{6}}x^4+\frac{\left(-a\,x^2+x^3\right)^{\frac{2}{3}}}{d^{\frac{1}{6}}}}{x^2(-a\,x^2+x^3)^{\frac{1}{3}}}\right)}{2d^{\frac{5}{6}}}$$

command

```
Integrate[(x^7*(-4*a + 3*x))/((x^2*(-a + x))^(2/3)*(-a^2 + 2*a*x - x^2 + d*x^8)),x]
```

Mathematica 13.1 output

$$\frac{x^{4/3}(-a+x)^{2/3}\left(\sqrt{3}\left(\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{-a+x}}{\sqrt[6]{d}\,x^{4/3}}}{\sqrt{3}}\right)-\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{-a+x}}{\sqrt[6]{d}\,x^{4/3}}}{\sqrt{3}}\right)\right)-2\tanh^{-1}\left(\frac{\sqrt[3]{-a+x}}{\sqrt[6]{d}\,x^{4/3}}\right)-\tan\right.}{2d^{5/6}\left(x^2(-a+x)\right)^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{x^7(-4a+3x)}{\left(x^2(-a+x)\right)^{2/3}\left(-a^2+2ax-x^2+dx^8\right)}\,dx$$

24.487 Problem number 2504

$$\int \frac{3+x}{\sqrt[3]{-1+x^2}\,(5-x+2x^2)}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{3^{\frac{5}{6}}(x^2-1)^{\frac{1}{3}}}{2\,2^{\frac{1}{3}}-2\,2^{\frac{1}{3}}x+3^{\frac{1}{3}}(x^2-1)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}3^{\frac{5}{6}}}{6}+\frac{\ln\left(-2^{\frac{1}{3}}3^{\frac{2}{3}}+2^{\frac{1}{3}}3^{\frac{2}{3}}x+3(x^2-1)^{\frac{1}{3}}\right)2^{\frac{2}{3}}3^{\frac{1}{3}}}{6}$$
$$-\frac{\ln\left(2^{\frac{2}{3}}3^{\frac{1}{3}}-2\,2^{\frac{2}{3}}3^{\frac{1}{3}}x+2^{\frac{2}{3}}3^{\frac{1}{3}}x^2+\left(2^{\frac{1}{3}}3^{\frac{2}{3}}-2^{\frac{1}{3}}3^{\frac{2}{3}}x\right)(x^2-1)^{\frac{1}{3}}+3(x^2-1)^{\frac{2}{3}}\right)2^{\frac{2}{3}}3^{\frac{1}{3}}}{12}$$

command

```
Integrate[(3 + x)/((-1 + x^2)^(1/3)*(5 - x + 2*x^2)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{3^{5/6}\sqrt[3]{-1+x^2}}{2\sqrt[3]{2}-2\sqrt[3]{2}\,x+\sqrt[3]{3}\,\sqrt[3]{-1+x^2}}\right)+2\log\left(-\sqrt[3]{2}\,3^{2/3}+\sqrt[3]{2}\,3^{2/3}x+3\sqrt[3]{-1+x^2}\right)-\log\left(2^{2/3}\sqrt[3]{3}\ -2\right.}{2\sqrt[3]{2}\,3^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{3+x}{\sqrt[3]{-1+x^2}\,(5-x+2x^2)}\,dx$$

24.488 Problem number 2505

$$\int \frac{(-2+(1+k)x)\left(1-(1+k)x+(a+k)x^2\right)}{x^2\sqrt[3]{(1-x)x(1-kx)}\,\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(x+(-1-k)x^2+kx^3\right)^{\frac{2}{3}}}{2x^2}+\frac{\left(-\sqrt{3}\,a-b\sqrt{3}\right)\arctan\left(\frac{\sqrt{3}\,b^{\frac{1}{3}}x}{b^{\frac{1}{3}}x+2\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}$$
$$+\frac{(a+b)\ln\left(-b^{\frac{1}{3}}x+\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}}$$
$$+\frac{(-a-b)\ln\left(b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}x\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}+\left(x+(-1-k)x^2+kx^3\right)^{\frac{2}{3}}\right)}{2b^{\frac{1}{3}}}$$

command

```
Integrate[((-2 + (1 + k)*x)*(1 - (1 + k)*x + (a + k)*x^2))/(x^2*((1 - x)*x*(1 - k*x))^(1/3)*(
b + k)*x^2)),x]
```

Mathematica 13.1 output

$$(-1+x)\left(3k-\frac{3}{x}+\frac{(a+b)\sqrt[3]{\frac{x}{-1+x}}\sqrt[3]{\frac{-1+kx}{-1+x}}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[3]{\frac{-1+kx}{-1+x}}}{2\sqrt[3]{b}\left(\frac{x}{-1+x}\right)^{2/3}+\sqrt[3]{\frac{-1+kx}{-1+x}}}\right)+2\log\left(-\sqrt[3]{b}\left(\frac{x}{-1+x}\right)^{2/}\right.\right.}{\sqrt[3]{b}}\right.$$

$$\left.\left.\right)\right)$$

$$2\sqrt[3]{(-1+x)x(-1+kx)}$$

Mathematica 12.3 output

$$\int \frac{(-2+(1+k)x)\left(1-(1+k)x+(a+k)x^2\right)}{x^2\sqrt[3]{(1-x)x(1-kx)}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$$

24.489 Problem number 2506

$$\int \frac{\left(-2+x^3\right)\sqrt[3]{x+x^3+x^4}}{\left(1+x^3\right)\left(1-x^2+x^3\right)}\,dx$$

Optimal antiderivative

$$\begin{aligned}
&-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^4+x^3+x\right)^{\frac{1}{3}}}\right)+2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2^{\frac{2}{3}}\left(x^4+x^3+x\right)^{\frac{1}{3}}}\right)\\
&-\ln\left(-x+\left(x^4+x^3+x\right)^{\frac{1}{3}}\right)+2^{\frac{1}{3}}\ln\left(-2x+2^{\frac{2}{3}}\left(x^4+x^3+x\right)^{\frac{1}{3}}\right)\\
&+\frac{\ln\left(x^2+x\left(x^4+x^3+x\right)^{\frac{1}{3}}+\left(x^4+x^3+x\right)^{\frac{2}{3}}\right)}{2}\\
&-\frac{\ln\left(2x^2+2^{\frac{2}{3}}x\left(x^4+x^3+x\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(x^4+x^3+x\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{2}
\end{aligned}$$

command

```
Integrate[((-2 + x^3)*(x + x^3 + x^4)^(1/3))/((1 + x^3)*(1 - x^2 + x^3)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x+x^3+x^4}\left(-2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}+2\sqrt[3]{1+x^2+x^3}}\right)+2\sqrt[3]{2}\,\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}+2^{2/3}\sqrt[3]{1+x^2+x^3}}\right)-2\log\left(-\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\left(-2+x^3\right)\sqrt[3]{x+x^3+x^4}}{\left(1+x^3\right)\left(1-x^2+x^3\right)}\,dx$$

24.490 Problem number 2507

$$\int \frac{\sqrt[4]{bx^3+ax^4}}{-2b+ax+2x^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(b*x^3 + a*x^4)^(1/4)/(-2*b + a*x + 2*x^2),x]
```

Mathematica 13.1 output

$$-\frac{x^{9/4}(b+ax)^{3/4}\left(8\sqrt[4]{a}\left(\text{ArcTan}\left(\frac{\sqrt[4]{a}\,\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)\right)+\text{RootSum}\left[3a^2-2b-5a\#1^4+2\#1^8\&\right.\right.}{8\,(x}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{bx^3+ax^4}}{-2b+ax+2x^2}\,dx$$

24.491 Problem number 2508

$$\int \frac{\sqrt[4]{bx^3+ax^4}}{-2b+ax+2x^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(b*x^3 + a*x^4)^(1/4)/(-2*b + a*x + 2*x^2),x]
```

Mathematica 13.1 output

$$-\frac{x^{9/4}(b+ax)^{3/4}\left(8\sqrt[4]{a}\left(\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)\right)+\text{RootSum}\left[3a^2-2b-5a\#1^4+2\#1^8\&\right.\right.}{8\,(x}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{bx^3+ax^4}}{-2b+ax+2x^2}\,dx$$

24.492 Problem number 2510

$$\int \frac{-b+ax^4}{(b+ax^4)\sqrt[4]{b^2+cx^4+a^2x^8}}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{2}\,(2ab-c)^{\frac{1}{4}}x\left(a^2x^8+cx^4+b^2\right)^{\frac{1}{4}}}{\sqrt{2ab-c}\,x^2-\sqrt{a^2x^8+cx^4+b^2}}\right)\sqrt{2}}{4\,(2ab-c)^{\frac{1}{4}}}$$
$$-\frac{\text{arctanh}\left(\frac{\frac{(2ab-c)^{\frac{1}{4}}x^2\sqrt{2}}{2}+\frac{\sqrt{a^2x^8+cx^4+b^2}\,\sqrt{2}}{2(2ab-c)^{\frac{1}{4}}}}{x\left(a^2x^8+cx^4+b^2\right)^{\frac{1}{4}}}\right)\sqrt{2}}{4\,(2ab-c)^{\frac{1}{4}}}$$

command

```
Integrate[(-b + a*x^4)/((b + a*x^4)*(b^2 + c*x^4 + a^2*x^8)^(1/4)),x]
```

Mathematica 13.1 output

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{2ab-c}\,x\sqrt[4]{b^2+cx^4+a^2x^8}}{\sqrt{2ab-c}\,x^2-\sqrt{b^2+cx^4+a^2x^8}}\right)-\tanh^{-1}\left(\frac{\sqrt{2ab-c}\,x^2+\sqrt{b^2+cx^4+a^2x^8}}{\sqrt{2}\,\sqrt[4]{2ab-c}\,x\sqrt[4]{b^2+cx^4+a^2x^8}}\right)}{2\sqrt{2}\,\sqrt[4]{2ab-c}}$$

Mathematica 12.3 output

$$\int \frac{-b+ax^4}{(b+ax^4)\sqrt[4]{b^2+cx^4+a^2x^8}}\,dx$$

24.493 Problem number 2513

$$\int \frac{(2+x)^2 \sqrt[3]{-19+66x-30x^2+9x^3}}{(-3+2x)^2\left(-5+6x-6x^2+x^3\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(9x^3-30x^2+66x-19\right)^{\frac{1}{3}}}{-3+2x}+\frac{2^{\frac{1}{3}}\arctan\left(\frac{-3\sqrt{3}+2x\sqrt{3}}{-3+2x+2^{\frac{2}{3}}\left(9x^3-30x^2+66x-19\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}$$
$$+\frac{2^{\frac{1}{3}}\ln\left(6-4x+2^{\frac{2}{3}}\left(9x^3-30x^2+66x-19\right)^{\frac{1}{3}}\right)}{3}$$
$$-\frac{\ln\left(18-24x+8x^2+\left(-3\,2^{\frac{2}{3}}+2\,2^{\frac{2}{3}}x\right)\left(9x^3-30x^2+66x-19\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(9x^3-30x^2+66x-19\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{6}$$

command

```
Integrate[((2 + x)^2*(-19 + 66*x - 30*x^2 + 9*x^3)^(1/3))/((-3 + 2*x)^2*(-5 + 6*x - 6*x^2 + x
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{-19+66x-30x^2+9x^3}}{-3+2x}+\frac{\sqrt[3]{2}\,\text{ArcTan}\left(\frac{\sqrt{3}\,(-3+2x)}{-3+2x+2^{2/3}\sqrt[3]{-19+66x-30x^2+9x^3}}\right)}{\sqrt{3}}$$
$$+\frac{1}{3}\sqrt[3]{2}\,\log\left(6-4x+2^{2/3}\sqrt[3]{-19+66x-30x^2+9x^3}\right)$$
$$-\frac{\log\left(18-24x+8x^2+2^{2/3}(-3+2x)\sqrt[3]{-19+66x-30x^2+9x^3}+\sqrt[3]{2}\left(-19+66x-30x^2+9x^3\right)^{2/3}\right)}{3\,2^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{(2+x)^2 \sqrt[3]{-19+66x-30x^2+9x^3}}{(-3+2x)^2\left(-5+6x-6x^2+x^3\right)}\,dx$$

24.494 Problem number 2514

$$\int \frac{1+x^3}{(-1+x^3)\sqrt[3]{x^2+x^4}}\,dx$$

Optimal antiderivative

$$-\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^4+x^2\right)^{\frac{1}{3}}}\right)}{3}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{6}$$
$$-\frac{2\ln\left(x+\left(x^4+x^2\right)^{\frac{1}{3}}\right)}{3}+\frac{\ln\left(-2x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{6}$$
$$+\frac{\ln\left(x^2-x\left(x^4+x^2\right)^{\frac{1}{3}}+\left(x^4+x^2\right)^{\frac{2}{3}}\right)}{3}-\frac{\ln\left(2x^2+2^{\frac{2}{3}}x\left(x^4+x^2\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(x^4+x^2\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{12}$$

command

```
Integrate[(1 + x^3)/((-1 + x^3)*(x^2 + x^4)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\sqrt[3]{1+x^2}\left(8\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}-2\sqrt[3]{1+x^2}}\right)-2\,2^{2/3}\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}+2^{2/3}\sqrt[3]{1+x^2}}\right)-8\log\left(\sqrt[3]{x}+\sqrt[3]{1+\right.\right.}}{}$$

Mathematica 12.3 output

$$\int \frac{1+x^3}{(-1+x^3)\sqrt[3]{x^2+x^4}}\,dx$$

24.495 Problem number 2517

$$\int \frac{1+x^2}{(-1+x+x^2)\sqrt[3]{-1+x^6}}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{3}\,(x^6-1)^{\frac{1}{3}}}{-2^{\frac{2}{3}}-2^{\frac{2}{3}}x+2^{\frac{2}{3}}x^2+(x^6-1)^{\frac{1}{3}}}\right)2^{\frac{1}{3}}\sqrt{3}}{6}-\frac{\ln\left(-2^{\frac{2}{3}}-2^{\frac{2}{3}}x+2^{\frac{2}{3}}x^2-2\left(x^6-1\right)^{\frac{1}{3}}\right)2^{\frac{1}{3}}}{6}$$
$$+\frac{\ln\left(2^{\frac{1}{3}}+2\,2^{\frac{1}{3}}x-2^{\frac{1}{3}}x^2-2\,2^{\frac{1}{3}}x^3+2^{\frac{1}{3}}x^4+\left(-2^{\frac{2}{3}}-2^{\frac{2}{3}}x+2^{\frac{2}{3}}x^2\right)\left(x^6-1\right)^{\frac{1}{3}}+2\left(x^6-1\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{12}$$

command

```
Integrate[(1 + x^2)/((-1 + x + x^2)*(-1 + x^6)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{-1+x^6}}{2^{2/3}+2^{2/3}x-2^{2/3}x^2-\sqrt[3]{-1+x^6}}\right)-2\log\left(-2^{2/3}-2^{2/3}x+2^{2/3}x^2-2\sqrt[3]{-1+x^6}\right)+\log\left(\sqrt[3]{2}+2\sqrt[3]{2}\right.}{6\,2^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{1+x^2}{(-1+x+x^2)\sqrt[3]{-1+x^6}}\,dx$$

24.496 Problem number 2518

$$\int \frac{1+x^2}{(-1+x+x^2)\sqrt[3]{-1+x^6}}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{3}\ (x^6-1)^{\frac{1}{3}}}{-2^{\frac{2}{3}}-2^{\frac{2}{3}}x+2^{\frac{2}{3}}x^2+(x^6-1)^{\frac{1}{3}}}\right)2^{\frac{1}{3}}\sqrt{3}}{6}-\frac{\ln\left(-2^{\frac{2}{3}}-2^{\frac{2}{3}}x+2^{\frac{2}{3}}x^2-2\left(x^6-1\right)^{\frac{1}{3}}\right)2^{\frac{1}{3}}}{6}$$
$$+\frac{\ln\left(2^{\frac{1}{3}}+2\,2^{\frac{1}{3}}x-2^{\frac{1}{3}}x^2-2\,2^{\frac{1}{3}}x^3+2^{\frac{1}{3}}x^4+\left(-2^{\frac{2}{3}}-2^{\frac{2}{3}}x+2^{\frac{2}{3}}x^2\right)\left(x^6-1\right)^{\frac{1}{3}}+2\left(x^6-1\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{12}$$

command

```
Integrate[(1 + x^2)/((-1 + x + x^2)*(-1 + x^6)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\ \sqrt[3]{-1+x^6}}{2^{2/3}+2^{2/3}x-2^{2/3}x^2-\sqrt[3]{-1+x^6}}\right)-2\log\left(-2^{2/3}-2^{2/3}x+2^{2/3}x^2-2\sqrt[3]{-1+x^6}\right)+\log\left(\sqrt[3]{2}\ +2\sqrt[3]{2}\right.}{6\ 2^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{1+x^2}{(-1+x+x^2)\sqrt[3]{-1+x^6}}\,dx$$

24.497 Problem number 2525

$$\int \frac{\left(-b+ax^2\right)\sqrt[4]{-bx^2+ax^4}}{b-ax^2+x^4}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-b + a*x^2)*(-(b*x^2) + a*x^4)^(1/4))/(b - a*x^2 + x^4),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-bx^2+ax^4}\left(4a^{5/4}\left(-\text{ArcTan}\left(\frac{\sqrt[4]{a}\ \sqrt{x}}{\sqrt[4]{-b+ax^2}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\ \sqrt{x}}{\sqrt[4]{-b+ax^2}}\right)\right)+\text{RootSum}\left[b-a\#1^4+\#1^8\&,\frac{-a}{}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\left(-b+ax^2\right)\sqrt[4]{-bx^2+ax^4}}{b-ax^2+x^4}\,dx$$

24.498 Problem number 2526

$$\int \frac{\left(-b+ax^2\right)\sqrt[4]{-bx^2+ax^4}}{b-ax^2+x^4}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-b + a*x^2)*(-(b*x^2) + a*x^4)^(1/4))/(b - a*x^2 + x^4),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-bx^2+ax^4}\left(4a^{5/4}\left(-\text{ArcTan}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{-b+ax^2}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{-b+ax^2}}\right)\right)+\text{RootSum}\left[b-a\#1^4+\#1^8\&,\frac{-a}{}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\left(-b+ax^2\right)\sqrt[4]{-bx^2+ax^4}}{b-ax^2+x^4}\,dx$$

24.499 Problem number 2527

$$\int \frac{\left(-b^2+ax^2\right)^2}{\left(b^2+ax^2\right)^2\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Optimal antiderivative

$$-\frac{bx}{\sqrt{a\,x^2+b^2}\,\sqrt{b+\sqrt{a\,x^2+b^2}}}+\frac{2x\left(a\,x^2+2b^2\right)}{\left(a\,x^2+b^2\right)\sqrt{b+\sqrt{a\,x^2+b^2}}}$$
$$-\frac{\sqrt{b}\,\arctan\left(\frac{\sqrt{a}\,x}{\sqrt{b}\,\sqrt{b+\sqrt{a\,x^2+b^2}}}\right)}{\sqrt{a}}$$
$$-\frac{2\sqrt{2}\,\sqrt{b}\,\arctan\left(\frac{\sqrt{a}\,x\sqrt{2}}{2\sqrt{b}\,\sqrt{b+\sqrt{a\,x^2+b^2}}}-\frac{\sqrt{b+\sqrt{a\,x^2+b^2}}\,\sqrt{2}}{2\sqrt{b}}\right)}{\sqrt{a}}$$

command

```
Integrate[(-b^2 + a*x^2)^2/((b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

Mathematica 13.1 output

$$\frac{x\left(4b^2+2ax^2-b\sqrt{b^2+ax^2}\right)}{\left(b^2+ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}}-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\,\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{a}}$$
$$-\frac{\sqrt{2}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\,\sqrt{b}\,\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{a}}$$

Mathematica 12.3 output

$$\int \frac{\left(-b^2+ax^2\right)^2}{\left(b^2+ax^2\right)^2\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

24.500 Problem number 2528

$$\int \frac{(-2+(1+k)x)\left(a-a(1+k)x+(1+ak)x^2\right)}{(-1+x)\sqrt[3]{(1-x)x(1-kx)}\,(-1+kx)\left(b-b(1+k)x+(-1+bk)x^2\right)}\,dx$$

Optimal antiderivative

$$\frac{3\left(k\,x^3-k\,x^2-x^2+x\right)^{\frac{2}{3}}}{(-1+x)\,(kx-1)}+\frac{\left(-\sqrt{3}\;a-b\sqrt{3}\;\right)\arctan\left(\frac{\sqrt{3}\;x}{x+2b^{\frac{1}{3}}\left(x+(-1-k)x^2+k\,x^3\right)^{\frac{1}{3}}}\right)}{b^{\frac{2}{3}}}$$
$$+\frac{(a+b)\ln\left(x-b^{\frac{1}{3}}\left(x+(-1-k)\,x^2+k\,x^3\right)^{\frac{1}{3}}\right)}{b^{\frac{2}{3}}}$$
$$+\frac{(-a-b)\ln\left(x^2+b^{\frac{1}{3}}x\left(x+(-1-k)\,x^2+k\,x^3\right)^{\frac{1}{3}}+b^{\frac{2}{3}}\left(x+(-1-k)\,x^2+k\,x^3\right)^{\frac{2}{3}}\right)}{2b^{\frac{2}{3}}}$$

command

```
Integrate[((-2 + (1 + k)*x)*(a - a*(1 + k)*x + (1 + a*k)*x^2))/((-1 + x)*((1 - x)*x*(1 - k*x)
1 + k*x)*(b - b*(1 + k)*x + (-1 + b*k)*x^2)),x]
```

Mathematica 13.1 output

$$\frac{(-1+x)\left(\frac{6x}{-1+x}+\frac{(a+b)\sqrt[3]{\frac{x}{-1+x}}\sqrt[3]{\frac{-1+kx}{-1+x}}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{\frac{-1+kx}{-1+x}}}{2\left(\frac{x}{-1+x}\right)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{\frac{-1+kx}{-1+x}}}\right)+2\log\left(\left(\frac{x}{-1+x}\right)^{2/3}-\sqrt[3]{b}\right.\right.}{b^{2/3}}\right.}{2\sqrt[3]{(-1+x)x(-1+kx)}}$$

Mathematica 12.3 output

$$\int \frac{(-2+(1+k)x)\left(a-a(1+k)x+(1+ak)x^2\right)}{(-1+x)\sqrt[3]{(1-x)x(1-kx)}\,(-1+kx)\left(b-b(1+k)x+(-1+bk)x^2\right)}\,dx$$

24.501 Problem number 2530

$$\int \frac{1-x^2+x^3}{(-1-x^2+x^3)\sqrt[3]{x^2+x^3}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(1 - x^2 + x^3)/((-1 - x^2 + x^3)*(x^2 + x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\sqrt[3]{1+x}\left(6\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}+2\sqrt[3]{1+x}}\right)-6\log\left(-\sqrt[3]{x}+\sqrt[3]{1+x}\right)+3\log\left(x^{2/3}+\sqrt[3]{x}\,\sqrt[3]{1+x}+(1+x)^2\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{1-x^2+x^3}{(-1-x^2+x^3)\sqrt[3]{x^2+x^3}}\,dx$$

24.502 Problem number 2531

$$\int \frac{1-x^2+x^3}{(-1-x^2+x^3)\sqrt[3]{x^2+x^3}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(1 - x^2 + x^3)/((-1 - x^2 + x^3)*(x^2 + x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\sqrt[3]{1+x}\left(6\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}+2\sqrt[3]{1+x}}\right)-6\log\left(-\sqrt[3]{x}+\sqrt[3]{1+x}\right)+3\log\left(x^{2/3}+\sqrt[3]{x}\,\sqrt[3]{1+x}+(1+x)^2\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{1-x^2+x^3}{(-1-x^2+x^3)\sqrt[3]{x^2+x^3}}\,dx$$

24.503 Problem number 2536

$$\int \frac{\sqrt{b^2+a^2x^2}}{d+cx^2+\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[b^2 + a^2*x^2]/(d + c*x^2 + Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

Mathematica 13.1 output

$$a\left(\log\left(ax+\sqrt{b^2+a^2x^2}\right)-2\text{RootSum}\left[b^4c-2b^2c\#1^4+4a^2d\#1^4+4a^2\#1^5+c\#1^8\&,\frac{b^2c\log\left(\sqrt{ax+\sqrt{b^2+a^2}}\right.\right.\right.}{c}$$

Mathematica 12.3 output

$Aborted

24.504 Problem number 2537

$$\int \frac{\sqrt{b^2+a^2x^2}}{d+cx^2+\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[b^2 + a^2*x^2]/(d + c*x^2 + Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

Mathematica 13.1 output

$$\frac{a\left(\log\left(ax+\sqrt{b^2+a^2x^2}\right)-2\text{RootSum}\left[b^4c-2b^2c\#1^4+4a^2d\#1^4+4a^2\#1^5+c\#1^8\&,\ \frac{b^2c\log\left(\sqrt{ax+\sqrt{b^2+a^2}\right.}\right.}{c}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{b^2+a^2x^2}}{d+cx^2+\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$$

24.505 Problem number 2542

$$\int \frac{-1+(2-k)x}{\sqrt[3]{(1-x)x(1-kx)}\,\left(b-(1+2bk)x+(1+bk^2)\,x^2\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ \left(x+(-1-k)x^2+k\,x^3\right)^{\frac{1}{3}}}{2b^{\frac{1}{3}}-2b^{\frac{1}{3}}kx+\left(x+(-1-k)x^2+k\,x^3\right)^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}+\frac{\ln\left(-b^{\frac{1}{3}}+b^{\frac{1}{3}}kx+\left(x+(-1-k)\,x^2+k\,x^3\right)^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}}$$
$$-\frac{\ln\left(b^{\frac{2}{3}}-2b^{\frac{2}{3}}kx+b^{\frac{2}{3}}k^2x^2+\left(b^{\frac{1}{3}}-b^{\frac{1}{3}}kx\right)\left(x+(-1-k)\,x^2+k\,x^3\right)^{\frac{1}{3}}+\left(x+(-1-k)\,x^2+k\,x^3\right)^{\frac{2}{3}}\right)}{2b^{\frac{1}{3}}}$$

command

```
Integrate[(-1 + (2 - k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(b - (1 + 2*b*k)*x + (1 + b*k^2)*x^2)
```

Mathematica 13.1 output

$$\frac{x\sqrt[3]{\frac{-1+kx}{-1+x}}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{\frac{x}{-1+x}}}{\sqrt[3]{\frac{x}{-1+x}}-2\sqrt[3]{b}\left(\frac{-1+kx}{-1+x}\right)^{2/3}}\right)+2\log\left(\sqrt[3]{\frac{x}{-1+x}}+\sqrt[3]{b}\left(\frac{-1+kx}{-1+x}\right)^{2/3}\right)-\log\left(\left(\frac{x}{-1+\right.}\right.\right.}{2\sqrt[3]{b}\left(\frac{x}{-1+x}\right)^{2/3}\sqrt[3]{(-1+x)x(-1+kx)}}$$

Mathematica 12.3 output

$$\int\frac{-1+(2-k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(b-(1+2bk)x+\left(1+bk^2\right)x^2\right)}\,dx$$

24.506 Problem number 2548

$$\int\frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(cx^4+bx^2\left(q+px^3\right)+a\left(q+px^3\right)^2\right)}{x^9}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}\left(3a\,p^3x^9-3a\,p^2q\,x^7+4b\,p^2x^8+9a\,p^2q\,x^6-8bpq\,x^6+6cp\,x^7-3ap\,q^2x^4+8bpq\,x^5\right.}{12x^8}$$

$$+\left(a\,p^2q^2+2cpq\right)\ln\left(x\right)+\frac{\left(-a\,p^2q^2-2cpq\right)\ln\left(q+p\,x^3+\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}\right)}{2}$$

command

```
Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(c*x^4 + b*x^2*(q + p*x
```

Mathematica 13.1 output

$$\frac{1}{12}\left(\frac{\sqrt{q^2-2pq(-1+x)x^3+p^2x^6}\left(6cx^4\left(q+px^3\right)+4bx^2\left(q^2-2pq(-1+x)x^3+p^2x^6\right)+3a\left(q^3-pq^2(-3+x)x^3\right.\right.}{x^8}\right.$$

$$\left.\left.-6pq(2c+apq)\tanh^{-1}\left(\frac{\sqrt{q^2-2pq(-1+x)x^3+p^2x^6}}{q+px^3}\right)\right)\right)$$

Mathematica 12.3 output

$$\int\frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(cx^4+bx^2\left(q+px^3\right)+a\left(q+px^3\right)^2\right)}{x^9}\,dx$$

24.507 Problem number 2554

$$\int \frac{\sqrt[4]{-b+ax^4}\left(b+cx^4+ax^8\right)}{x^6\left(b+2ax^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-b + a*x^4)^(1/4)*(b + c*x^4 + a*x^8))/(x^6*(b + 2*a*x^8)),x]
```

Mathematica 13.1 output

$$-\frac{8\sqrt[4]{-b+ax^4}\left(b-(a-5c)x^4\right)+5ax^5\text{RootSum}\left[a^2+2ab-2a\#1^4+\#1^8\&,\frac{ac\log(x)+2bc\log(x)-ac\log\left(\sqrt[4]{-b+ax^4}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{-b+ax^4}\left(b+cx^4+ax^8\right)}{x^6\left(b+2ax^8\right)}\,dx$$

24.508 Problem number 2555

$$\int \frac{\sqrt[4]{-b+ax^4}\left(b+cx^4+ax^8\right)}{x^6\left(b+2ax^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-b + a*x^4)^(1/4)*(b + c*x^4 + a*x^8))/(x^6*(b + 2*a*x^8)),x]
```

Mathematica 13.1 output

$$-\frac{8\sqrt[4]{-b+ax^4}\left(b-(a-5c)x^4\right)+5ax^5\text{RootSum}\left[a^2+2ab-2a\#1^4+\#1^8\&,\frac{ac\log(x)+2bc\log(x)-ac\log\left(\sqrt[4]{-b+ax^4}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{-b+ax^4}\left(b+cx^4+ax^8\right)}{x^6\left(b+2ax^8\right)}\,dx$$

24.509 Problem number 2557

$$\int \frac{\left(-q+2px^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bqx+cx^2+bpx^4+a\left(q+px^3\right)^2\right)}{x^5}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}\left(3a\,p^3x^9+9a\,p^2q\,x^6+4b\,p^2x^7-3a\,p^2q\,x^5+9apq^2x^3+8bpq\,x^4+6cp\,x^5-3apq^2x\right.}{12x^4}$$
$$+\frac{\left(a\,p^2q^2+2cpq\right)\ln\left(x\right)}{2}+\frac{\left(-a\,p^2q^2-2cpq\right)\ln\left(q+p\,x^3+\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}\right)}{2}$$

command

```
Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*q*x + c*x^2 + b*p*x^
```

Mathematica 13.1 output

$$\frac{1}{12}\left(\frac{\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}\left(3a\left(q^3+p^3x^9+pq^2x^2(-1+3x)+p^2qx^5(-1+3x)\right)+2x\left(3cx\left(q+px^3\right)+2b\left(q\right.\right.\right.}{x^4}$$
$$\left.\left.-6pq(2c+apq)\tanh^{-1}\left(\frac{\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}}{q+px^3}\right)\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-q+2px^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bqx+cx^2+bpx^4+a\left(q+px^3\right)^2\right)}{x^5}\,dx$$

24.510 Problem number 2558

$$\int \frac{-a(a-2b)-2bx+x^2}{((-a+x)(-b+x))^{2/3}\left(a^2+bd-(2a+d)x+x^2\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(ab+(-a-b)x+x^2\right)^{\frac{2}{3}}}{-2b\,d^{\frac{1}{3}}+2d^{\frac{1}{3}}x+\left(ab+(-a-b)x+x^2\right)^{\frac{2}{3}}}\right)}{d^{\frac{1}{3}}}+\frac{\ln\left(b\sqrt{d}\,-x\sqrt{d}\,+d^{\frac{1}{6}}\left(ab+(-a-b)\,x+x^2\right)^{\frac{2}{3}}\right)}{d^{\frac{1}{3}}}$$
$$-\frac{\ln\left(b^2d-2bdx+d\,x^2+\left(-b\,d^{\frac{2}{3}}+d^{\frac{2}{3}}x\right)\left(ab+(-a-b)\,x+x^2\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\left(ab+(-a-b)\,x+x^2\right)^{\frac{4}{3}}\right)}{2d^{\frac{1}{3}}}$$

command

```
Integrate[(-(a*(a - 2*b)) - 2*b*x + x^2)/(((-a + x)*(-b + x))^(2/3)*(a^2 + b*d - (2*a + d)*x
```

Mathematica 13.1 output

$$-\frac{(a-x)^{2/3}(b-x)^{2/3}\left(-2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,(a-x)^{2/3}}{(a-x)^{2/3}-2\sqrt[3]{d}\,\sqrt[3]{b-x}}\right)-2\log\left((a-x)^{2/3}+\sqrt[3]{d}\,\sqrt[3]{b-x}\right)+\log\left((a-x\right.\right.}{2\sqrt[3]{d}\,((a-x)(b-x))^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{-a(a-2b)-2bx+x^2}{((-a+x)(-b+x))^{2/3}\left(a^2+bd-(2a+d)x+x^2\right)}\,dx$$

24.511 Problem number 2559

$$\int \frac{a-2b+x}{\sqrt[3]{(-a+x)(-b+x)}\,\left(b+a^2d+(-1-2ad)x+dx^2\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(ab+(-a-b)x+x^2\right)^{\frac{1}{3}}}{-2a\,d^{\frac{1}{3}}+2d^{\frac{1}{3}}x+\left(ab+(-a-b)x+x^2\right)^{\frac{1}{3}}}\right)}{d^{\frac{1}{3}}}+\frac{\ln\left(a\,d^{\frac{1}{3}}-d^{\frac{1}{3}}x+\left(ab+(-a-b)\,x+x^2\right)^{\frac{1}{3}}\right)}{d^{\frac{1}{3}}}$$
$$-\frac{\ln\left(a^2d^{\frac{2}{3}}-2a\,d^{\frac{2}{3}}x+d^{\frac{2}{3}}x^2+\left(-a\,d^{\frac{1}{3}}+d^{\frac{1}{3}}x\right)\left(ab+(-a-b)\,x+x^2\right)^{\frac{1}{3}}+\left(ab+(-a-b)\,x+x^2\right)^{\frac{2}{3}}\right)}{2d^{\frac{1}{3}}}$$

command

```
Integrate[(a - 2*b + x)/(((-a + x)*(-b + x))^(1/3)*(b + a^2*d + (-1 - 2*a*d)*x + d*x^2)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{a-x}\,\sqrt[3]{b-x}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{b-x}}{-2\sqrt[3]{d}\,(a-x)^{2/3}+\sqrt[3]{b-x}}\right)+2\log\left(\sqrt[3]{d}\,(a-x)^{2/3}+\sqrt[3]{b-x}\right)-\log\left(d^{2/3}(a-x)^4\right.\right.}{2\sqrt[3]{d}\,\sqrt[3]{(-a+x)(-b+x)}}$$

Mathematica 12.3 output

$$\int \frac{a-2b+x}{\sqrt[3]{(-a+x)(-b+x)}\,\left(b+a^2d+(-1-2ad)x+dx^2\right)}\,dx$$

24.512 Problem number 2560

$$\int \frac{a-2b+x}{\sqrt[3]{(-a+x)(-b+x)}\,(b+a^2d-(1+2ad)x+dx^2)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(ab+(-a-b)x+x^2)^{\frac{1}{3}}}{-2a\,d^{\frac{1}{3}}+2d^{\frac{1}{3}}x+(ab+(-a-b)x+x^2)^{\frac{1}{3}}}\right)}{d^{\frac{1}{3}}}+\frac{\ln\left(a\,d^{\frac{1}{3}}-d^{\frac{1}{3}}x+\left(ab+(-a-b)\,x+x^2\right)^{\frac{1}{3}}\right)}{d^{\frac{1}{3}}}$$
$$-\frac{\ln\left(a^2d^{\frac{2}{3}}-2a\,d^{\frac{2}{3}}x+d^{\frac{2}{3}}x^2+\left(-a\,d^{\frac{1}{3}}+d^{\frac{1}{3}}x\right)\left(ab+(-a-b)\,x+x^2\right)^{\frac{1}{3}}+\left(ab+(-a-b)\,x+x^2\right)^{\frac{2}{3}}\right)}{2d^{\frac{1}{3}}}$$

command

```
Integrate[(a - 2*b + x)/(((-a + x)*(-b + x))^(1/3)*(b + a^2*d - (1 + 2*a*d)*x + d*x^2)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{a-x}\,\sqrt[3]{b-x}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{b-x}}{-2\sqrt[3]{d}\,(a-x)^{2/3}+\sqrt[3]{b-x}}\right)+2\log\left(\sqrt[3]{d}\,(a-x)^{2/3}+\sqrt[3]{b-x}\right)-\log\left(d^{2/3}(a-x)^4\right.\right.}{2\sqrt[3]{d}\,\sqrt[3]{(-a+x)(-b+x)}}$$

Mathematica 12.3 output

$$\int \frac{a-2b+x}{\sqrt[3]{(-a+x)(-b+x)}\,(b+a^2d-(1+2ad)x+dx^2)}\,dx$$

24.513 Problem number 2563

$$\int \frac{\left(-1+x+x^3+x^6\right)^{2/3}\left(3-2x+3x^6\right)}{\left(-1+x+x^6\right)\left(-1+x-x^3+x^6\right)}\,dx$$

Optimal antiderivative

$$\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^6+x^3+x-1\right)^{\frac{1}{3}}}\right)-2^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2^{\frac{2}{3}}\left(x^6+x^3+x-1\right)^{\frac{1}{3}}}\right)$$
$$-\ln\left(-x+\left(x^6+x^3+x-1\right)^{\frac{1}{3}}\right)+2^{\frac{2}{3}}\ln\left(-2x+2^{\frac{2}{3}}\left(x^6+x^3+x-1\right)^{\frac{1}{3}}\right)$$
$$+\frac{\ln\left(x^2+x\left(x^6+x^3+x-1\right)^{\frac{1}{3}}+\left(x^6+x^3+x-1\right)^{\frac{2}{3}}\right)}{2}$$
$$-\frac{\ln\left(2x^2+2^{\frac{2}{3}}x\left(x^6+x^3+x-1\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(x^6+x^3+x-1\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{2}$$

command

```
Integrate[((-1 + x + x^3 + x^6)^(2/3)*(3 - 2*x + 3*x^6))/((-1 + x + x^6)*(-1 + x - x^3 + x^6)
```

Mathematica 13.1 output

$$\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{-1+x+x^3+x^6}}\right)$$
$$-2^{2/3}\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2^{2/3}\sqrt[3]{-1+x+x^3+x^6}}\right)-\log\left(-x+\sqrt[3]{-1+x+x^3+x^6}\right)$$
$$+2^{2/3}\log\left(-2x+2^{2/3}\sqrt[3]{-1+x+x^3+x^6}\right)+\frac{1}{2}\log\left(x^2+x\sqrt[3]{-1+x+x^3+x^6}+\left(-1+x+x^3+x^6\right)^{2/3}\right)-\frac{\log\left(2x^2+\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x+x^3+x^6\right)^{2/3}\left(3-2x+3x^6\right)}{\left(-1+x+x^6\right)\left(-1+x-x^3+x^6\right)}\,dx$$

24.514 Problem number 2567

$$\int \frac{\left(-4b+ax^4\right)\sqrt[4]{-b+ax^4}}{x^6\left(-8b+ax^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-4*b + a*x^4)*(-b + a*x^4)^(1/4))/(x^6*(-8*b + a*x^8)),x]
```

Mathematica 13.1 output

$$\frac{64\sqrt[4]{-b+ax^4}\,\left(-4b+9ax^4\right)}{x^5}+5a\text{RootSum}\left[8a^2-ab-16a\#1^4+8\#1^8\&,\frac{8a^2\log(x)-ab\log(x)-8a^2\log\left(\sqrt[4]{-b+ax^4}-x\#1\right.}{}\right]$$
$$2$$

Mathematica 12.3 output

$$\int \frac{\left(-4b+ax^4\right)\sqrt[4]{-b+ax^4}}{x^6\left(-8b+ax^8\right)}\,dx$$

24.515 Problem number 2568

$$\int \frac{\left(-4b+ax^4\right)\sqrt[4]{-b+ax^4}}{x^6\left(-8b+ax^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((-4*b + a*x^4)*(-b + a*x^4)^(1/4))/(x^6*(-8*b + a*x^8)),x]
```

Mathematica 13.1 output

$$\frac{64\sqrt[4]{-b+ax^4}\,(-4b+9ax^4)}{x^5}+5a\text{RootSum}\left[8a^2-ab-16a\#1^4+8\#1^8\&,\frac{8a^2\log(x)-ab\log(x)-8a^2\log\left(\sqrt[4]{-b+ax^4}-x\#1\right.}{2}\right.$$

Mathematica 12.3 output

$$\int \frac{\left(-4b+ax^4\right)\sqrt[4]{-b+ax^4}}{x^6\left(-8b+ax^8\right)}\,dx$$

24.516 Problem number 2571

$$\int x^2\sqrt{b+a^2x^4}\,\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{a}\;x\sqrt{a^2x^4+b}\;\left(16a^3x^6+28abx^2\right)\sqrt{a\,x^2+\sqrt{a^2x^4+b}}}{48a^{\frac{3}{2}}b+96a^{\frac{7}{2}}x^4+96a^{\frac{5}{2}}x^2\sqrt{a^2x^4+b}}+\frac{\sqrt{a}\;x\left(16a^4x^8+36a^2bx^4+9b^2\right)\sqrt{a\,x^2+\sqrt{a^2x^4+}}}{}$$

$$-\frac{3b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\;\sqrt{a}\;x\sqrt{a\,x^2+\sqrt{a^2x^4+b}}}{\sqrt{b}}\right)\sqrt{2}}{32a^{\frac{3}{2}}}$$

command

```
Integrate[x^2*Sqrt[b + a^2*x^4]*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{a}\,x\sqrt{ax^2+\sqrt{b+a^2x^4}}\left(9b^2+16a^3x^6\left(ax^2+\sqrt{b+a^2x^4}\right)+4abx^2\left(9ax^2+7\sqrt{b+a^2x^4}\right)\right)}{b+2ax^2\left(ax^2+\sqrt{b+a^2x^4}\right)}-9\sqrt{2}\,b^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{a}}{96a^{3/2}}\right.$$

Mathematica 12.3 output

$$\int x^2\sqrt{b+a^2x^4}\,\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

24.517 Problem number 2572

$$\int\frac{\left(-bx+a^2x^2\right)^{3/2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{a^2x^2-bx}\,\left(32a^4x^2-88a^2bx+115b^2\right)\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}}{40b^2x}$$
$$+\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}\left(\frac{-32a^5x^2+104a^3bx-145a\,b^2}{40b^2}\right.$$
$$\left.+\frac{9\sqrt{b}\,\sqrt{-ax+\sqrt{a^2x^2-bx}}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{-ax+\sqrt{a^2x^2-bx}}}{\sqrt{b}}\right)\sqrt{2}}{16\sqrt{a}\,x}\right)$$

command

```
Integrate[(-(b*x) + a^2*x^2)^(3/2)/(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2),x]
```

Mathematica 13.1 output

$$\sqrt{x\left(ax+\sqrt{x\left(-b+a^2x\right)}\right)}\left(-2\sqrt{a}\,x\left(115b^3+8a^3bx\left(15ax-13\sqrt{x\left(-b+a^2x\right)}\right)+32a^5x^2\left(-ax+\sqrt{x\left(-b+\right.\right.\right.\right.$$

Mathematica 12.3 output

$$\int\frac{\left(-bx+a^2x^2\right)^{3/2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

24.518 Problem number 2586

$$\int \frac{\left(-1+x^2\right)^2}{\left(1+x^2\right)^2 \sqrt{1+x^4}\, \sqrt{x^2+\sqrt{1+x^4}}}\, dx$$

Optimal antiderivative

$$\frac{x^2\left(x^2+1\right)+x^2\sqrt{x^4+1}}{x\left(x^2+1\right)\sqrt{x^2+\sqrt{x^4+1}}}+\arctan\left(\frac{\sqrt{2}\, x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}$$
$$-\sqrt{1+\sqrt{2}}\ \arctan\left(\frac{\sqrt{2+2\sqrt{2}}\, x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$
$$-\sqrt{\sqrt{2}-1}\ \operatorname{arctanh}\left(\frac{\sqrt{-2+2\sqrt{2}}\, x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$

command

```
Integrate[(-1 + x^2)^2/((1 + x^2)^2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

Mathematica 13.1 output

$$\frac{x\left(1+x^2+\sqrt{1+x^4}\right)}{\left(1+x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}+\sqrt{2}\,\operatorname{ArcTan}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\, x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$-\sqrt{1+\sqrt{2}}\,\operatorname{ArcTan}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\left(-1+x^2+\sqrt{1+x^4}\right)}{x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$-\sqrt{-1+\sqrt{2}}\,\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}\, x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^2\right)^2}{\left(1+x^2\right)^2 \sqrt{1+x^4}\, \sqrt{x^2+\sqrt{1+x^4}}}\, dx$$

24.519 Problem number 2589

$$\int \frac{\left(1+x^2\right)^2}{\left(-1+x^2\right)^2 \sqrt{1+x^4}\, \sqrt{x^2+\sqrt{1+x^4}}}\, dx$$

Optimal antiderivative

$$\frac{-x^2\left(x^2-1\right)-x^2\sqrt{x^4+1}}{x\left(x^2-1\right)\sqrt{x^2+\sqrt{x^4+1}}}+\arctan\left(\frac{\sqrt{2}\, x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}$$
$$-\sqrt{1+\sqrt{2}}\ \arctan\left(\frac{\sqrt{-2+2\sqrt{2}}\, x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$
$$+\sqrt{\sqrt{2}-1}\ \operatorname{arctanh}\left(\frac{\sqrt{2+2\sqrt{2}}\, x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$

command

```
Integrate[(1 + x^2)^2/((-1 + x^2)^2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

Mathematica 13.1 output

$$-\frac{x\left(-1+x^2+\sqrt{1+x^4}\right)}{\left(-1+x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}+\sqrt{2}\ \operatorname{ArcTan}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\, x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$-\sqrt{1+\sqrt{2}}\ \operatorname{ArcTan}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}\, x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$+\sqrt{-1+\sqrt{2}}\ \tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\left(-1+x^2+\sqrt{1+x^4}\right)}{x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2\right)^2}{\left(-1+x^2\right)^2 \sqrt{1+x^4}\, \sqrt{x^2+\sqrt{1+x^4}}}\, dx$$

24.520 Problem number 2590

$$\int \frac{\left(-1+x^2\right)^2 \sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^2\right)^2 \sqrt{1+x^4}}\,dx$$

Optimal antiderivative

$$\frac{-x^2\left(x^2-1\right)-x^2\sqrt{x^4+1}}{x\left(x^2+1\right)\sqrt{x^2+\sqrt{x^4+1}}}+\sqrt{\sqrt{2}-1}\ \arctan\left(\frac{\sqrt{2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$
$$+\operatorname{arctanh}\left(\frac{\sqrt{2}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}$$
$$-\sqrt{1+\sqrt{2}}\ \operatorname{arctanh}\left(\frac{\sqrt{-2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$

command

```
Integrate[((-1 + x^2)^2*Sqrt[x^2 + Sqrt[1 + x^4]])/((1 + x^2)^2*Sqrt[1 + x^4]),x]
```

Mathematica 13.1 output

$$-\frac{x\left(-1+x^2+\sqrt{1+x^4}\right)}{\left(1+x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}$$
$$+\sqrt{-1+\sqrt{2}}\ \operatorname{ArcTan}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\left(-1+x^2+\sqrt{1+x^4}\right)}{x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$+\sqrt{2}\ \tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$-\sqrt{1+\sqrt{2}}\ \tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^2\right)^2 \sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^2\right)^2 \sqrt{1+x^4}}\,dx$$

24.521 Problem number 2593

$$\int \frac{(-2+x)\sqrt[3]{x-x^2+x^3}}{(-1+x)(-1+x+x^2)}\,dx$$

Optimal antiderivative

$$\begin{aligned}
&-\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ x}{x+2\left(x^3-x^2+x\right)^{\frac{1}{3}}}\right)+2^{\frac{1}{3}}\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ x}{x+2^{\frac{2}{3}}\left(x^3-x^2+x\right)^{\frac{1}{3}}}\right)\\
&-\ln\left(-x+\left(x^3-x^2+x\right)^{\frac{1}{3}}\right)+2^{\frac{1}{3}}\ln\left(-2x+2^{\frac{2}{3}}\left(x^3-x^2+x\right)^{\frac{1}{3}}\right)\\
&+\frac{\ln\left(x^2+x\left(x^3-x^2+x\right)^{\frac{1}{3}}+\left(x^3-x^2+x\right)^{\frac{2}{3}}\right)}{2}\\
&-\frac{\ln\left(2x^2+2^{\frac{2}{3}}x\left(x^3-x^2+x\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(x^3-x^2+x\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{2}
\end{aligned}$$

command

```
Integrate[((-2 + x)*(x - x^2 + x^3)^(1/3))/((-1 + x)*(-1 + x + x^2)),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\left(1-x+x^2\right)^{2/3}\left(-2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}+2\sqrt[3]{1-x+x^2}}\right)+2\sqrt[3]{2}\,\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}+2^{2/3}\sqrt[3]{1-x+x^2}}\right)-2\log\right.}{}$$

Mathematica 12.3 output

$$\int \frac{(-2+x)\sqrt[3]{x-x^2+x^3}}{(-1+x)(-1+x+x^2)}\,dx$$

24.522 Problem number 2596

$$\int \frac{\left(-2+x^3\right)\sqrt[3]{x+2x^3+x^4}}{(1+x^3)(1+x^2+x^3)}\,dx$$

Optimal antiderivative

$$\begin{aligned}
&-\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ x}{x+2\left(x^4+2x^3+x\right)^{\frac{1}{3}}}\right)+2^{\frac{1}{3}}\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ x}{x+2^{\frac{2}{3}}\left(x^4+2x^3+x\right)^{\frac{1}{3}}}\right)\\
&-\ln\left(-x+\left(x^4+2x^3+x\right)^{\frac{1}{3}}\right)+2^{\frac{1}{3}}\ln\left(-2x+2^{\frac{2}{3}}\left(x^4+2x^3+x\right)^{\frac{1}{3}}\right)\\
&+\frac{\ln\left(x^2+x\left(x^4+2x^3+x\right)^{\frac{1}{3}}+\left(x^4+2x^3+x\right)^{\frac{2}{3}}\right)}{2}\\
&-\frac{\ln\left(2x^2+2^{\frac{2}{3}}x\left(x^4+2x^3+x\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(x^4+2x^3+x\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{2}
\end{aligned}$$

command

```
Integrate[((-2 + x^3)*(x + 2*x^3 + x^4)^(1/3))/((1 + x^3)*(1 + x^2 + x^3)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x+2x^3+x^4}\left(-2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}+2\sqrt[3]{1+2x^2+x^3}}\right)+2\sqrt[3]{2}\,\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}+2^{2/3}\sqrt[3]{1+2x^2+x^3}}\right)-2\log\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\left(-2+x^3\right)\sqrt[3]{x+2x^3+x^4}}{\left(1+x^3\right)\left(1+x^2+x^3\right)}\,dx$$

24.523 Problem number 2598

$$\int \frac{\left(-1+x^4\right)^{3/4}}{1-2x^4+2x^8}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{2-\sqrt{2}}\,\arctan\left(\frac{\sqrt{2-\sqrt{2}}\,x\left(x^4-1\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^4-1}}\right)}{8}-\frac{\sqrt{2+\sqrt{2}}\,\arctan\left(\frac{\sqrt{2+\sqrt{2}}\,x\left(x^4-1\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^4-1}}\right)}{8}$$
$$-\frac{\sqrt{2-\sqrt{2}}\,\text{arctanh}\left(\frac{\sqrt{2-\sqrt{2}}\,x\left(x^4-1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^4-1}}\right)}{8}$$
$$-\frac{\sqrt{2+\sqrt{2}}\,\text{arctanh}\left(\frac{\sqrt{2+\sqrt{2}}\,x\left(x^4-1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^4-1}}\right)}{8}$$

command

```
Integrate[(-1 + x^4)^(3/4)/(1 - 2*x^4 + 2*x^8),x]
```

Mathematica 13.1 output

$$\frac{1}{8}\left(\sqrt{2-\sqrt{2}}\ \text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}\ x\sqrt[4]{-1+x^4}}{x^2-\sqrt{-1+x^4}}\right)\right.$$
$$+\sqrt{2+\sqrt{2}}\ \text{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}\ x\sqrt[4]{-1+x^4}}{x^2-\sqrt{-1+x^4}}\right)$$
$$-\sqrt{2-\sqrt{2}}\ \tanh^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\ x\sqrt[4]{-1+x^4}}{x^2+\sqrt{-1+x^4}}\right)$$
$$\left.-\sqrt{2+\sqrt{2}}\ \tanh^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\ x\sqrt[4]{-1+x^4}}{x^2+\sqrt{-1+x^4}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^4\right)^{3/4}}{1-2x^4+2x^8}\,dx$$

24.524 Problem number 2600

$$\int \frac{\left(1+x^4\right)^{3/4}}{1+2x^4+2x^8}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{2-\sqrt{2}}\ \arctan\left(\frac{\sqrt{2-\sqrt{2}}\ x\left(x^4+1\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^4+1}}\right)}{8}+\frac{\sqrt{2+\sqrt{2}}\ \arctan\left(\frac{\sqrt{2+\sqrt{2}}\ x\left(x^4+1\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^4+1}}\right)}{8}$$
$$+\frac{\sqrt{2-\sqrt{2}}\ \operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{2}}\ x\left(x^4+1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^4+1}}\right)}{8}+\frac{\sqrt{2+\sqrt{2}}\ \operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{2}}\ x\left(x^4+1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^4+1}}\right)}{8}$$

command

```
Integrate[(1 + x^4)^(3/4)/(1 + 2*x^4 + 2*x^8),x]
```

Mathematica 13.1 output

$$\frac{1}{8}\left(\sqrt{2-\sqrt{2}}\ \text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}\ x\sqrt[4]{1+x^4}}{-x^2+\sqrt{1+x^4}}\right)\right.$$
$$+\sqrt{2+\sqrt{2}}\ \text{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}\ x\sqrt[4]{1+x^4}}{-x^2+\sqrt{1+x^4}}\right)$$
$$+\sqrt{2-\sqrt{2}}\ \tanh^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\ x\sqrt[4]{1+x^4}}{x^2+\sqrt{1+x^4}}\right)$$
$$\left.+\sqrt{2+\sqrt{2}}\ \tanh^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\ x\sqrt[4]{1+x^4}}{x^2+\sqrt{1+x^4}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^4\right)^{3/4}}{1+2x^4+2x^8}\,dx$$

24.525 Problem number 2601

$$\int \frac{x^2\sqrt{-bx+a^2x^2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{a^2x^2-bx}\ \left(96a^4x^2-104a^2bx-15b^2\right)\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}}{120a^2b^2x}$$
$$+\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}\left(\frac{-96a^4x^2+152a^2bx+5b^2}{120a\,b^2}\right.$$
$$\left.+\frac{\sqrt{b}\ \sqrt{-ax+\sqrt{a^2x^2-bx}}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{a}\ \sqrt{-ax+\sqrt{a^2x^2-bx}}}{\sqrt{b}}\right)\sqrt{2}}{16a^{\frac{5}{2}}x}\right)$$

command

```
Integrate[(x^2*Sqrt[-(b*x) + a^2*x^2])/(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{x\left(ax+\sqrt{x\left(-b+a^2x\right)}\right)}\left(2\sqrt{a}\,x\left(15b^3+96a^5x^2\left(ax-\sqrt{x\left(-b+a^2x\right)}\right)+ab^2\left(89ax+5\sqrt{x\left(-b+a^2x\right)}\right)+\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{x^2\sqrt{-bx+a^2x^2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

24.526 Problem number 2602

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{-2+2\sqrt{2}}\,\arctan\left(\frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{\sqrt{2}-1}}\right)}{2}$$
$$-\frac{\sqrt{-2+2\sqrt{2}}\,\arctan\left(\frac{\sqrt{-2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{2}$$
$$-\frac{\sqrt{2+2\sqrt{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{1+\sqrt{2}}}\right)}{2}$$
$$+\frac{\sqrt{2+2\sqrt{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{2}$$

command

```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{-1+\sqrt{2}}\left(\text{ArcTan}\left(\sqrt{1+\sqrt{2}}\sqrt{x^2+\sqrt{1+x^4}}\right)-\text{ArcTan}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)-\sqrt{}}{\sqrt{}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$

24.527 Problem number 2606

$$\int \frac{\left(-2+k^2\right)x+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(1-d+\left(-2+dk^2\right)x^2+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,d^{\frac{1}{3}}\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{1}{3}}}{2-2x^2+d^{\frac{1}{3}}\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{1}{3}}}\right)}{2d^{\frac{2}{3}}}+\frac{\ln\left(-1+x^2+d^{\frac{1}{3}}\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{1}{3}}\right)}{2d^{\frac{2}{3}}}$$
$$-\frac{\ln\left(1-2x^2+x^4+\left(d^{\frac{1}{3}}-d^{\frac{1}{3}}x^2\right)\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{1}{3}}+d^{\frac{2}{3}}\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{2}{3}}\right)}{4d^{\frac{2}{3}}}$$

command

```
Integrate[((-2 + k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3)*(1 - d + (-2 + d*k^2)*x^2
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{-1+x^2}\,\sqrt[3]{-1+k^2x^2}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{-1+k^2x^2}}{-2(-1+x^2)^{2/3}+\sqrt[3]{d}\,\sqrt[3]{-1+k^2x^2}}\right)+2\log\left(\left(-1+x^2\right)^{2/3}+\sqrt[3]{d}\,\sqrt[3]{-1+k\right.}\right.}{4d^{2/3}\sqrt[3]{\left(-1+x^2\right)\left(-1+k^2x^2\right.}}$$

Mathematica 12.3 output

$$\int \frac{\left(-2+k^2\right)x+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(1-d+\left(-2+dk^2\right)x^2+x^4\right)}\,dx$$

24.528 Problem number 2608

$$\int \frac{\left(-1+2k^2\right)x-2k^4x^3+k^4x^5}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(-1+d+\left(1-2dk^2\right)x^2+dk^4x^4\right)}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,d^{\frac{1}{3}}\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{2}{3}}}{2-2x^2+d^{\frac{1}{3}}\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{2}{3}}}\right)}{2d^{\frac{2}{3}}}-\frac{\ln\left(-1+x^2+d^{\frac{1}{3}}\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{2}{3}}\right)}{2d^{\frac{2}{3}}}$$
$$+\frac{\ln\left(1-2x^2+x^4+\left(d^{\frac{1}{3}}-d^{\frac{1}{3}}x^2\right)\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{2}{3}}+d^{\frac{2}{3}}\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{4}{3}}\right)}{4d^{\frac{2}{3}}}$$

command

```
Integrate[((-1 + 2*k^2)*x - 2*k^4*x^3 + k^4*x^5)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)*(-
1 + d + (1 - 2*d*k^2)*x^2 + d*k^4*x^4)),x]
```

Mathematica 13.1 output

$$\frac{\left(-1+x^2\right)^{2/3}\left(-1+k^2x^2\right)^{2/3}\left(2\sqrt{3}\,\mathrm{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{d}\,\left(-1+k^2x^2\right)^{2/3}}{2\sqrt[3]{-1+x^2}-\sqrt[3]{d}\,\left(-1+k^2x^2\right)^{2/3}}\right)-2\log\left(\sqrt[3]{-1+x^2}+\sqrt[3]{d}\,\left(-1+k^2\right.\right.\right.}{4d^{2/3}\left(\left(-1+x^2\right)\left(-1+k^2x^2\right)\right.}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+2k^2\right)x-2k^4x^3+k^4x^5}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(-1+d+\left(1-2dk^2\right)x^2+dk^4x^4\right)}\,dx$$

24.529 Problem number 2614

$$\int \frac{x^2}{x^2-\sqrt{b+ax}\,\sqrt{c+\sqrt{b+ax}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[x^2/(x^2 - Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]]),x]
```

Mathematica 13.1 output

$$\frac{b-c^2+ax}{a}$$

$$-4a\text{RootSum}\left[b^2-2bc^2+c^4+a^2c\#1+4bc\#1^2-4c^3\#1^2-a^2\#1^3-2b\#1^4+6c^2\#1^4-4c\#1^6\right.$$

$$\left.+\#1^8\&,\frac{c^2\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1^2-2c\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1^4+\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right.}{-a^2c-8bc\#1+8c^3\#1+3a^2\#1^2+8b\#1^3-24c^2\#1^3+24c\#1^5-8\#1^7}\right]$$

Mathematica 12.3 output

$$\int \frac{x^2}{x^2-\sqrt{b+ax}\sqrt{c+\sqrt{b+ax}}}\,dx$$

24.530 Problem number 2615

$$\int \frac{x^2}{x^2-\sqrt{b+ax}\sqrt{c+\sqrt{b+ax}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[x^2/(x^2 - Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]]),x]
```

Mathematica 13.1 output

$$\frac{b-c^2+ax}{a}$$

$$-4a\text{RootSum}\left[b^2-2bc^2+c^4+a^2c\#1+4bc\#1^2-4c^3\#1^2-a^2\#1^3-2b\#1^4+6c^2\#1^4-4c\#1^6\right.$$

$$\left.+\#1^8\&,\frac{c^2\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1^2-2c\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1^4+\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right.}{-a^2c-8bc\#1+8c^3\#1+3a^2\#1^2+8b\#1^3-24c^2\#1^3+24c\#1^5-8\#1^7}\right]$$

Mathematica 12.3 output

$$\int \frac{x^2}{x^2-\sqrt{b+ax}\sqrt{c+\sqrt{b+ax}}}\,dx$$

24.531 Problem number 2622

$$\int \frac{x^3\left(3+x^2\right)}{\left(1+x^2\right)\sqrt[3]{1+x^2-x^3}\left(1+x^2+x^3\right)}\,dx$$

Optimal antiderivative

$$\begin{aligned}
&\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(-x^3+x^2+1\right)^{\frac{1}{3}}}\right)-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2^{\frac{2}{3}}\left(-x^3+x^2+1\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{2}\\
&+\ln\left(x+\left(-x^3+x^2+1\right)^{\frac{1}{3}}\right)-\frac{\ln\left(2x+2^{\frac{2}{3}}\left(-x^3+x^2+1\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{2}\\
&-\frac{\ln\left(x^2-x\left(-x^3+x^2+1\right)^{\frac{1}{3}}+\left(-x^3+x^2+1\right)^{\frac{2}{3}}\right)}{2}\\
&+\frac{\ln\left(-2x^2+2^{\frac{2}{3}}x\left(-x^3+x^2+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\left(-x^3+x^2+1\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{4}
\end{aligned}$$

command

```
Integrate[(x^3*(3 + x^2))/((1 + x^2)*(1 + x^2 - x^3)^(1/3)*(1 + x^2 + x^3)),x]
```

Mathematica 13.1 output

$$\begin{aligned}
&\sqrt{3}\,\mathrm{ArcTan}\left(\frac{\sqrt{3}\,x}{-x+2\sqrt[3]{1+x^2-x^3}}\right)-\frac{\sqrt{3}\,\mathrm{ArcTan}\left(\frac{\sqrt{3}\,x}{-x+2^{2/3}\sqrt[3]{1+x^2-x^3}}\right)}{\sqrt[3]{2}}\\
&+\log\left(x+\sqrt[3]{1+x^2-x^3}\right)-\frac{\log\left(2x+2^{2/3}\sqrt[3]{1+x^2-x^3}\right)}{\sqrt[3]{2}}\\
&-\frac{1}{2}\log\left(x^2-x\sqrt[3]{1+x^2-x^3}+\left(1+x^2-x^3\right)^{2/3}\right)+\frac{\log\left(-2x^2+2^{2/3}x\sqrt[3]{1+x^2-x^3}-\sqrt[3]{2}\left(1+x^2-x^3\right)^{2/3}\right)}{2\sqrt[3]{2}}
\end{aligned}$$

Mathematica 12.3 output

$$\int \frac{x^3\left(3+x^2\right)}{\left(1+x^2\right)\sqrt[3]{1+x^2-x^3}\left(1+x^2+x^3\right)}\,dx$$

24.532 Problem number 2627

$$\int \frac{1+x}{(1+3x+x^2)\sqrt[3]{1-x^3}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{5\sqrt{3}\,(-x^3+1)^{\frac{1}{3}}}{2\,2^{\frac{1}{3}}5^{\frac{2}{3}}-2\,2^{\frac{1}{3}}5^{\frac{2}{3}}x+5(-x^3+1)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}5^{\frac{1}{3}}}{10}+\frac{\ln\left(-2^{\frac{1}{3}}5^{\frac{2}{3}}+2^{\frac{1}{3}}5^{\frac{2}{3}}x+5\left(-x^3+1\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}5^{\frac{1}{3}}}{10}$$
$$-\frac{\ln\left(2^{\frac{2}{3}}5^{\frac{1}{3}}-2\,2^{\frac{2}{3}}5^{\frac{1}{3}}x+2^{\frac{2}{3}}5^{\frac{1}{3}}x^2+\left(2^{\frac{1}{3}}5^{\frac{2}{3}}-2^{\frac{1}{3}}5^{\frac{2}{3}}x\right)\left(-x^3+1\right)^{\frac{1}{3}}+5\left(-x^3+1\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}5^{\frac{1}{3}}}{20}$$

command

```
Integrate[(1 + x)/((1 + 3*x + x^2)*(1 - x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{5\sqrt{3}\,\sqrt[3]{1-x^3}}{2\sqrt[3]{2}\,5^{2/3}-2\sqrt[3]{2}\,5^{2/3}x+5\sqrt[3]{1-x^3}}\right)+2\log\left(-\sqrt[3]{2}\,5^{2/3}+\sqrt[3]{2}\,5^{2/3}x+5\sqrt[3]{1-x^3}\right)-\log\left(2^{2/3}\sqrt[3]{5}\;-2\right.}{2\sqrt[3]{2}\,5^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{1+x}{(1+3x+x^2)\sqrt[3]{1-x^3}}\,dx$$

24.533 Problem number 2634

$$\int \frac{1}{(-bx+a^2x^2)^{3/2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{2\sqrt{a^2x^2-bx}\,\left(1601a^6x^3-456a^4bx^2-200a^2b^2x+210b^3\right)\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}}{1155b^5x^4\left(-a^2x+b\right)}$$
$$+\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}\left(-\frac{4\left(2533a^5x^2+461a^3bx+245ab^2\right)}{1155b^5x^3}\right.$$
$$\left.-\frac{6a^{\frac{11}{2}}\sqrt{-ax+\sqrt{a^2x^2-bx}}\,\arctan\left(\frac{\sqrt{a}\,\sqrt{-ax+\sqrt{a^2x^2-bx}}}{\sqrt{b}}\right)}{b^{\frac{11}{2}}x}\right)$$

command

```
Integrate[1/((-(b*x) + a^2*x^2)^(3/2)*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2)),x]
```

Mathematica 13.1 output

$$-\frac{2\sqrt{x\left(ax+\sqrt{x\left(-b+a^2x\right)}\right)}\left(\sqrt{b}\left(210b^3+10ab^2\left(-20ax+49\sqrt{x\left(-b+a^2x\right)}\right)+2a^3bx\left(-228ax+461\sqrt{x(\right.\right.\right.}\right.}{}$$

Mathematica 12.3 output

$$\int \frac{1}{\left(-bx+a^2x^2\right)^{3/2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

24.534 Problem number 2644

$$\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\left(d+cx^2\right)\sqrt{b+a^2x^4}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]/((d + c*x^2)*Sqrt[b + a^2*x^4]),x]
```

Mathematica 13.1 output

$$\sqrt{-\sqrt{b}\,c-\sqrt{bc^2+a^2d^2}}\left(-\sqrt{b}\,c-ad+\sqrt{bc^2+a^2d^2}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{-\sqrt{b}\,c-\sqrt{bc^2+a^2d^2}}\,x\sqrt{ax^2+\sqrt{b+}}}{\sqrt{d}\left(\sqrt{b}+ax^2+\sqrt{b+a^2x^4}\right)}\right.$$

Mathematica 12.3 output

$$\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{(d+cx^2)\sqrt{b+a^2x^4}}\,dx$$

24.535 Problem number 2645

$$\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{(d+cx^2)\sqrt{b+a^2x^4}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]/((d + c*x^2)*Sqrt[b + a^2*x^4]),x]
```

Mathematica 13.1 output

$$\sqrt{-\sqrt{b}\,c-\sqrt{bc^2+a^2d^2}}\left(-\sqrt{b}\,c-ad+\sqrt{bc^2+a^2d^2}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{-\sqrt{b}\,c-\sqrt{bc^2+a^2d^2}}\,x\sqrt{ax^2+\sqrt{b+}}}{\sqrt{d}\left(\sqrt{b}+ax^2+\sqrt{b+a^2x^4}\right)}\right.$$

Mathematica 12.3 output

$$\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{(d+cx^2)\sqrt{b+a^2x^4}}\,dx$$

24.536 Problem number 2646

$$\int \frac{\sqrt{ax+\sqrt{b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}}{(b+a^2x^2)^{3/2}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]])/(b + a^2*x^
```

Mathematica 13.1 output

$$\frac{4\sqrt{ax+\sqrt{b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}}{b+ax\left(ax+\sqrt{b+a^2x^2}\right)}-8\text{RootSum}\left[b+c^4-4c^3\#1^2+6c^2\#1^4-4c\#1^6+\#1^8\&,\,-\right.$$

$$\left.\log$$

Mathematica 12.3 output

$$\int \frac{\sqrt{ax+\sqrt{b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}}{(b+a^2x^2)^{3/2}}\,dx$$

24.537 Problem number 2647

$$\int \frac{\sqrt{ax+\sqrt{b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}}{(b+a^2x^2)^{3/2}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]])/(b + a^2*x^
```

Mathematica 13.1 output

$$\frac{4\sqrt{ax+\sqrt{b+a^2x^2}}\,\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}}{b+ax\left(ax+\sqrt{b+a^2x^2}\right)}-8\text{RootSum}\left[b+c^4-4c^3\#1^2+6c^2\#1^4-4c\#1^6+\#1^8\&,\,-\right.$$

$$\left.\log\right.$$

Mathematica 12.3 output

$$\int\frac{\sqrt{ax+\sqrt{b+a^2x^2}}\,\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}}{(b+a^2x^2)^{3/2}}\,dx$$

24.538 Problem number 2654

$$\int\frac{\sqrt{1+x^2}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

$$\frac{\left(30720x^4-4096x^3-21570x^2-2680x-24993\right)\sqrt{1+\sqrt{x+\sqrt{x^2+1}}}+\left(3072x^3+4096x^2+1814x+1712\right)\sqrt{}}{}$$

$$-\frac{263\,\text{arctanh}\left(\sqrt{1+\sqrt{x+\sqrt{x^2+1}}}\right)}{256}$$

command

```
Integrate[(Sqrt[1 + x^2]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/Sqrt[x + Sqrt[1 + x^2]],x]
```

Mathematica 13.1 output

$$\frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\left(-24993-2680x-21570x^2-4096x^3+30720x^4+2\left(856+907x+2048x^2+1536x^3\right)\sqrt{x+\sqrt{1+x^2}}+\sqrt{1+x^2}\right.}{\left(x+\sqrt{1+x^2}\right)^{5/2}}$$

$$\frac{}{268}$$

Mathematica 12.3 output

$$\int\frac{\sqrt{1+x^2}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.539 Problem number 2655

$$\int \sqrt{1+x^2}\,\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

$$\frac{\left(40320x^4-2560x^3+112192x^2+1545x+31736\right)\sqrt{1+\sqrt{x+\sqrt{x^2+1}}}+\left(2240x^3+1536x^2+40688x-1542\right)}{}$$

$$-\frac{\operatorname{arctanh}\left(\sqrt{1+\sqrt{x+\sqrt{x^2+1}}}\right)}{16}$$

command

```
Integrate[Sqrt[1 + x^2]*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]],x]
```

Mathematica 13.1 output

$$\frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\left(31736+1545x+112192x^2-2560x^3+40320x^4+2\left(-771+20344x+768x^2+1120x^3\right)\right.}{55440\left(x+\right.}$$

$$-\frac{1}{16}\tanh^{-1}\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\right)$$

Mathematica 12.3 output

$$\int \sqrt{1+x^2}\,\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.540 Problem number 2658

$$\int \frac{1+2x^6}{\sqrt[3]{x+x^3}\,\left(-1+x^6\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(1 + 2*x^6)/((x + x^3)^(1/3)*(-1 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x}\ \sqrt[3]{1+x^2}\left(8\sqrt{3}\ \text{ArcTan}\left(\frac{\sqrt{3}\ x^{2/3}}{x^{2/3}+2\sqrt[3]{1+x^2}}\right)-2\ 2^{2/3}\sqrt{3}\ \text{ArcTan}\left(\frac{\sqrt{3}\ x^{2/3}}{x^{2/3}+2^{2/3}\sqrt[3]{1+x^2}}\right)-8\log\left(-x^{2/3}+\sqrt[3]{1+}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{1+2x^6}{\sqrt[3]{x+x^3}\ (-1+x^6)}\,dx$$

24.541 Problem number 2659

$$\int \frac{1+2x^6}{\sqrt[3]{x+x^3}\ (-1+x^6)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(1 + 2*x^6)/((x + x^3)^(1/3)*(-1 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x}\ \sqrt[3]{1+x^2}\left(8\sqrt{3}\ \text{ArcTan}\left(\frac{\sqrt{3}\ x^{2/3}}{x^{2/3}+2\sqrt[3]{1+x^2}}\right)-2\ 2^{2/3}\sqrt{3}\ \text{ArcTan}\left(\frac{\sqrt{3}\ x^{2/3}}{x^{2/3}+2^{2/3}\sqrt[3]{1+x^2}}\right)-8\log\left(-x^{2/3}+\sqrt[3]{1+}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{1+2x^6}{\sqrt[3]{x+x^3}\ (-1+x^6)}\,dx$$

24.542 Problem number 2662

$$\int \frac{(-2+(1+k)x)\left(1-2(1+k)x+\left(1+4k+k^2\right)x^2-2\left(k+k^2\right)x^3+\left(a+k^2\right)x^4\right)}{x^4\sqrt[3]{(1-x)x(1-kx)}\,\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$$

Optimal antiderivative

$$\begin{aligned}&\frac{3\left(5bx^2+2kx^2-2kx-2x+2\right)\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}}{10x^4}\\&+\frac{\left(-\sqrt{3}\,a-\sqrt{3}\,b^2\right)\arctan\left(\frac{\sqrt{3}\,b^{\frac{1}{3}}x}{b^{\frac{1}{3}}x+2\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}\\&+\frac{\left(b^2+a\right)\ln\left(-b^{\frac{1}{3}}x+\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}}\\&+\frac{\left(-b^2-a\right)\ln\left(b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}x\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}+\left(x+(-1-k)x^2+kx^3\right)^{\frac{2}{3}}\right)}{2b^{\frac{1}{3}}}\end{aligned}$$

command

```
Integrate[((-2 + (1 + k)*x)*(1 - 2*(1 + k)*x + (1 + 4*k + k^2)*x^2 - 2*(k + k^2)*x^3 + (a + k
b + k)*x^2)),x]
```

Mathematica 13.1 output

$$(-1+x)\left(\frac{3(-1+kx)\left(2-2(1+k)x+(5b+2k)x^2\right)}{5x^3}+\frac{(a+b^2)\sqrt[3]{\frac{x}{-1+x}}\,\sqrt[3]{\frac{-1+kx}{-1+x}}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{\frac{-1+kx}{-1+x}}}{2\sqrt[3]{b}\left(\frac{x}{-1+x}\right)^{2/3}+\sqrt[3]{\frac{-1+kx}{-1+x}}}\right.\right.}{2\sqrt[3]{(-1+x)x(-1+}}$$

Mathematica 12.3 output

$$\int \frac{(-2+(1+k)x)\left(1-2(1+k)x+\left(1+4k+k^2\right)x^2-2\left(k+k^2\right)x^3+\left(a+k^2\right)x^4\right)}{x^4\sqrt[3]{(1-x)x(1-kx)}\,\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$$

24.543 Problem number 2664

$$\int \frac{\sqrt[4]{bx^2+ax^4}\left(-b-ax^4+x^8\right)}{-b+ax^4}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((b*x^2 + a*x^4)^(1/4)*(-b - a*x^4 + x^8))/(-b + a*x^4),x]
```

Mathematica 13.1 output

$$\frac{x^{3/2}\left(b+ax^2\right)^{3/4}\left(2a^{3/4}x^{3/2}\sqrt[4]{b+ax^2}\left(-96a^3-7b^2+32a^2x^4+4ab\left(24+x^2\right)\right)-3b\left(-32a^3+32ab+7b^2\right)\text{ArcTan}\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{bx^2+ax^4}\left(-b-ax^4+x^8\right)}{-b+ax^4}\,dx$$

24.544 Problem number 2665

$$\int \frac{\sqrt[4]{bx^2+ax^4}\left(-b-ax^4+x^8\right)}{-b+ax^4}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((b*x^2 + a*x^4)^(1/4)*(-b - a*x^4 + x^8))/(-b + a*x^4),x]
```

Mathematica 13.1 output

$$\frac{x^{3/2}\left(b+ax^2\right)^{3/4}\left(2a^{3/4}x^{3/2}\sqrt[4]{b+ax^2}\left(-96a^3-7b^2+32a^2x^4+4ab\left(24+x^2\right)\right)-3b\left(-32a^3+32ab+7b^2\right)\text{ArcTan}\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{bx^2+ax^4}\left(-b-ax^4+x^8\right)}{-b+ax^4}\,dx$$

24.545 Problem number 2666

$$\int \frac{\left(-2+2x^4+5x^7\right)\sqrt[3]{x-x^3+x^5+x^8}}{\left(2+x^2+2x^4+2x^7\right)^2}\,dx$$

Optimal antiderivative

$$\begin{aligned}&-\frac{x\left(x^8+x^5-x^3+x\right)^{\frac{1}{3}}}{4x^7+4x^4+2x^2+4}+\frac{\arctan\left(\frac{\sqrt{3}\,\left(x^8+x^5-x^3+x\right)^{\frac{1}{3}}}{2^{\frac{2}{3}}3^{\frac{1}{3}}x-\left(x^8+x^5-x^3+x\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}3^{\frac{5}{6}}}{36}\\&+\frac{\ln\left(2^{\frac{2}{3}}3^{\frac{1}{3}}x+2\left(x^8+x^5-x^3+x\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}3^{\frac{1}{3}}}{36}\\&-\frac{\ln\left(2^{\frac{1}{3}}3^{\frac{2}{3}}x^2-2^{\frac{2}{3}}3^{\frac{1}{3}}x\left(x^8+x^5-x^3+x\right)^{\frac{1}{3}}+2\left(x^8+x^5-x^3+x\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}3^{\frac{1}{3}}}{72}\end{aligned}$$

command

```
Integrate[((-2 + 2*x^4 + 5*x^7)*(x - x^3 + x^5 + x^8)^(1/3))/(2 + x^2 + 2*x^4 + 2*x^7)^2,x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{x-x^3+x^5+x^8}\left(-\frac{36x^{4/3}}{2+x^2+2x^4+2x^7}+\frac{2\ 2^{2/3}3^{5/6}\text{ArcTan}\left(\frac{3^{5/6}x^{2/3}}{\sqrt[3]{3}\,x^{2/3}-2\sqrt[3]{2}\,\sqrt[3]{1-x^2+x^4+x^7}}\right)}{\sqrt[3]{1-x^2+x^4+x^7}}+\frac{2\ 2^{2/3}\sqrt[3]{3}\,\log\left(3x^{2/}\right.\right.}{\sqrt[3]{1}}}{72\sqrt[3]{x}}$$

Mathematica 12.3 output

$$\int \frac{\left(-2+2x^4+5x^7\right)\sqrt[3]{x-x^3+x^5+x^8}}{\left(2+x^2+2x^4+2x^7\right)^2}\,dx$$

24.546 Problem number 2675

$$\int \frac{\left(-2+k^2\right)x+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d+\left(-2d+k^2\right)x^2+dx^4\right)}\,dx$$

Optimal antiderivative

$$\begin{aligned}&\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{1}{3}}}{2d^{\frac{1}{3}}-2d^{\frac{1}{3}}x^2+\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{1}{3}}}\right)}{2d^{\frac{1}{3}}}+\frac{\ln\left(-d^{\frac{1}{3}}+d^{\frac{1}{3}}x^2+\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{1}{3}}\right)}{2d^{\frac{1}{3}}}\\&-\frac{\ln\left(d^{\frac{2}{3}}-2d^{\frac{2}{3}}x^2+d^{\frac{2}{3}}x^4+\left(d^{\frac{1}{3}}-d^{\frac{1}{3}}x^2\right)\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{1}{3}}+\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{2}{3}}\right)}{4d^{\frac{1}{3}}}\end{aligned}$$

command

```
Integrate[((-2 + k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3)*(-1 + d + (-2*d + k^2)*x^
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{-1+x^2}\ \sqrt[3]{-1+k^2x^2}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\ \sqrt[3]{-1+k^2x^2}}{-2\sqrt[3]{d}\ (-1+x^2)^{2/3}+\sqrt[3]{-1+k^2x^2}}\right)+2\log\left(\sqrt[3]{d}\ (-1+x^2)^{2/3}+\sqrt[3]{-1+k}\right.\right.}{4\sqrt[3]{d}\ \sqrt[3]{(-1+x^2)(-1+k^2x^2}}$$

Mathematica 12.3 output

$$\int \frac{\left(-2+k^2\right)x+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\ \left(-1+d+\left(-2d+k^2\right)x^2+dx^4\right)}\,dx$$

24.547 Problem number 2676

$$\int \frac{\left(-1+2k^2\right)x-2k^4x^3+k^4x^5}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(1-d+\left(d-2k^2\right)x^2+k^4x^4\right)}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ \left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{2}{3}}}{2d^{\frac{1}{3}}-2d^{\frac{1}{3}}x^2+\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{2}{3}}}\right)}{2d^{\frac{1}{3}}}-\frac{\ln\left(-d^{\frac{1}{3}}+d^{\frac{1}{3}}x^2+\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{2}{3}}\right)}{2d^{\frac{1}{3}}}$$
$$+\frac{\ln\left(d^{\frac{2}{3}}-2d^{\frac{2}{3}}x^2+d^{\frac{2}{3}}x^4+\left(d^{\frac{1}{3}}-d^{\frac{1}{3}}x^2\right)\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{2}{3}}+\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{4}{3}}\right)}{4d^{\frac{1}{3}}}$$

command

```
Integrate[((-1 + 2*k^2)*x - 2*k^4*x^3 + k^4*x^5)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)*(1 - d + (d
```

Mathematica 13.1 output

$$\frac{\left(-1+x^2\right)^{2/3}\left(-1+k^2x^2\right)^{2/3}\left(-2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\ (-1+k^2x^2)^{2/3}}{-2\sqrt[3]{d}\ \sqrt[3]{-1+x^2}+(-1+k^2x^2)^{2/3}}\right)-2\log\left(\sqrt[3]{d}\ \sqrt[3]{-1+x^2}+(-1+\right.}{4\sqrt[3]{d}\ \left(\left(-1+x^2\right)\left(-1+k^2x^2\right.\right.}$$

Mathematica 12.3 output

$$\int \frac{\left(-1+2k^2\right)x-2k^4x^3+k^4x^5}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(1-d+\left(d-2k^2\right)x^2+k^4x^4\right)}\,dx$$

24.548 Problem number 2677

$$\int \frac{\left(-2+x^6\right)\left(1-x^4+x^6\right)}{\sqrt[4]{1+x^6}\left(1+2x^6+x^8+x^{12}\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{4-2\sqrt{2}}\,\arctan\left(\frac{\sqrt{2-\sqrt{2}}\,x\left(x^6+1\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^6+1}}\right)}{4}$$
$$-\frac{\sqrt{4+2\sqrt{2}}\,\arctan\left(\frac{\sqrt{2+\sqrt{2}}\,x\left(x^6+1\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^6+1}}\right)}{4}$$
$$+\frac{\sqrt{4-2\sqrt{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{2}}\,x\left(x^6+1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^6+1}}\right)}{4}$$
$$-\frac{\sqrt{4+2\sqrt{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{2}}\,x\left(x^6+1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^6+1}}\right)}{4}$$

command

```
Integrate[((-2 + x^6)*(1 - x^4 + x^6))/((1 + x^6)^(1/4)*(1 + 2*x^6 + x^8 + x^12)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{2+\sqrt{2}}\,\operatorname{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{1+x^6}}{x^2-\sqrt{1+x^6}}\right)+\sqrt{2-\sqrt{2}}\,\operatorname{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{1+x^6}}{-x^2+\sqrt{1+x^6}}\right)+\sqrt{2-\sqrt{2}}\,\tanh}{2\sqrt{2}}$$

Mathematica 12.3 output

$$\int \frac{\left(-2+x^6\right)\left(1-x^4+x^6\right)}{\sqrt[4]{1+x^6}\left(1+2x^6+x^8+x^{12}\right)}\,dx$$

24.549 Problem number 2678

$$\int \frac{\left(-q+2px^3\right)\left(aq+bx+apx^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^3\left(cq+dx+cpx^3\right)}\,dx$$

Optimal antiderivative

$$\begin{aligned}
&\frac{\left(acp\,x^3+acq-2adx+2bcx\right)\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}}{2c^2x^2} \\
&-\frac{2(ad-bc)\sqrt{2c^2pq-d^2}\,\arctan\left(\frac{\sqrt{2c^2pq-d^2}\,x}{cq+dx+cp\,x^3+c\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}}\right)}{c^3} \\
&+\frac{\left(a\,c^2pq-a\,d^2+bcd\right)\ln\left(x\right)}{c^3} \\
&+\frac{\left(-a\,c^2pq+a\,d^2-bcd\right)\ln\left(q+p\,x^3+\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}\right)}{c^3}
\end{aligned}$$

command

```
Integrate[((-q + 2*p*x^3)*(a*q + b*x + a*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6])/
```

Mathematica 13.1 output

$$\begin{aligned}
&\frac{\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}\left(2bcx-2adx+ac\left(q+px^3\right)\right)}{2c^2x^2} \\
&-\frac{2(-bc+ad)\sqrt{-d^2+2c^2pq}\,\text{ArcTan}\left(\frac{\sqrt{-d^2+2c^2pq}\,x}{dx+c\left(q+px^3+\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}\right)}\right)}{c^3} \\
&-\frac{\left(bcd-ad^2+ac^2pq\right)\tanh^{-1}\left(\frac{\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}}{q+px^3}\right)}{c^3}
\end{aligned}$$

Mathematica 12.3 output

$$\int \frac{\left(-q+2px^3\right)\left(aq+bx+apx^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^3\left(cq+dx+cpx^3\right)}\,dx$$

24.550 Problem number 2679

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Optimal antiderivative

$$\begin{aligned}
&\frac{x}{2\sqrt{x^2+\sqrt{x^4+1}}}+2\arctan\left(\frac{\sqrt{2}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}\\
&-2\sqrt{1+\sqrt{2}}\,\arctan\left(\frac{\sqrt{2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\\
&+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}}{2}\\
&-2\sqrt{\sqrt{2}-1}\,\operatorname{arctanh}\left(\frac{\sqrt{-2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)
\end{aligned}$$

command

```
Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

Mathematica 13.1 output

$$\begin{aligned}
&\frac{x}{2\sqrt{x^2+\sqrt{1+x^4}}}+2\sqrt{2}\,\operatorname{ArcTan}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)\\
&-2\sqrt{1+\sqrt{2}}\,\operatorname{ArcTan}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\left(-1+x^2+\sqrt{1+x^4}\right)}{x\sqrt{x^2+\sqrt{1+x^4}}}\right)\\
&+\frac{\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}\\
&-2\sqrt{-1+\sqrt{2}}\,\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)
\end{aligned}$$

Mathematica 12.3 output

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

24.551 Problem number 2680

$$\int \frac{\left(-1+x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx$$

Optimal antiderivative

$$\frac{x\sqrt{x^2+\sqrt{x^4+1}}}{2}+\frac{\arctan\left(\frac{\sqrt{2}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}}{2}$$
$$-2\sqrt{\sqrt{2}-1}\,\arctan\left(\frac{\sqrt{2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$
$$-2\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}$$
$$+2\sqrt{1+\sqrt{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{-2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$

command

```
Integrate[((-1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2),x]
```

Mathematica 13.1 output

$$\frac{1}{2}\left(x\sqrt{x^2+\sqrt{1+x^4}}+\sqrt{2}\,\text{ArcTan}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)\right.$$
$$-4\sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\left(-1+x^2+\sqrt{1+x^4}\right)}{x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$-4\sqrt{2}\,\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$\left.+4\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx$$

24.552 Problem number 2685

$$\int \frac{-1+x}{(1+x)\sqrt[3]{-1+x^3}}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^3-1\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}-\frac{\sqrt{3}\,\arctan\left(\frac{-\frac{2^{\frac{1}{3}}\sqrt{3}}{3}+\frac{2^{\frac{1}{3}}x\sqrt{3}}{3}+\frac{\left(x^3-1\right)^{\frac{1}{3}}\sqrt{3}}{3}}{\left(x^3-1\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{2}$$
$$+\frac{\ln\left(-2^{\frac{1}{3}}+2^{\frac{1}{3}}x-2\left(x^3-1\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{2}-\frac{\ln\left(-x+\left(x^3-1\right)^{\frac{1}{3}}\right)}{3}$$
$$+\frac{\ln\left(x^2+x\left(x^3-1\right)^{\frac{1}{3}}+\left(x^3-1\right)^{\frac{2}{3}}\right)}{6}$$
$$-\frac{\ln\left(2^{\frac{2}{3}}-2\,2^{\frac{2}{3}}x+2^{\frac{2}{3}}x^2+\left(-2\,2^{\frac{1}{3}}+2\,2^{\frac{1}{3}}x\right)\left(x^3-1\right)^{\frac{1}{3}}+4\left(x^3-1\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{4}$$

command

```
Integrate[(-1 + x)/((1 + x)*(-1 + x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{1}{12}\left(-62^{2/3}\sqrt{3}\,\text{ArcTan}\left(\frac{-\sqrt[3]{2}+\sqrt[3]{2}\,x+\sqrt[3]{-1+x^3}}{\sqrt{3}\,\sqrt[3]{-1+x^3}}\right)+4\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{-1+x^3}}\right)\right.$$
$$\left.+6\,2^{2/3}\log\left(-\sqrt[3]{2}+\sqrt[3]{2}\,x-2\sqrt[3]{-1+x^3}\right)-4\log\left(-x+\sqrt[3]{-1+x^3}\right)+2\log\left(x^2+x\sqrt[3]{-1+x^3}+\left(-1+x^3\right)^{2/3}\right)-3\,2\right.$$

Mathematica 12.3 output

$$\int \frac{-1+x}{(1+x)\sqrt[3]{-1+x^3}}\,dx$$

24.553 Problem number 2689

$$\int \frac{\left(1-2k^2\right)x+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\,\left(1-d+\left(d-2k^2\right)x^2+k^4x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,d^{\frac{1}{3}}\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{1}{3}}}{2-2k^2x^2+d^{\frac{1}{3}}\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{1}{3}}}\right)}{2d^{\frac{2}{3}}}+\frac{\ln\left(-1+k^2x^2+d^{\frac{1}{3}}\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{1}{3}}\right)}{2d^{\frac{2}{3}}}$$
$$-\frac{\ln\left(1-2k^2x^2+k^4x^4+\left(d^{\frac{1}{3}}-d^{\frac{1}{3}}k^2x^2\right)\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{1}{3}}+d^{\frac{2}{3}}\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{2}{3}}\right)}{4d^{\frac{2}{3}}}$$

command

```
Integrate[((1 - 2*k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3)*(1 - d + (d - 2*k^2)*x^2
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{-1+x^2}\,\sqrt[3]{-1+k^2x^2}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{-1+x^2}}{\sqrt[3]{d}\,\sqrt[3]{-1+x^2}-2\left(-1+k^2x^2\right)^{2/3}}\right)+2\log\left(\sqrt[3]{d}\,\sqrt[3]{-1+x^2}+\left(-1+k^2x^2\right)^{2/3}\right)\right.}{4d^{2/3}\sqrt[3]{\left(-1+x^2\right)\left(-1+k^2x^2\right)}}$$

Mathematica 12.3 output

$$\int \frac{\left(1-2k^2\right)x+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\,\left(1-d+\left(d-2k^2\right)x^2+k^4x^4\right)}\,dx$$

24.554 Problem number 2697

$$\int \frac{\left(2+5x^7\right)\sqrt[3]{-x-x^3+x^8}}{\left(-1+x^7\right)\left(-1+x^2+x^7\right)}\,dx$$

Optimal antiderivative

$$\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^8-x^3-x\right)^{\frac{1}{3}}}\right)-2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2^{\frac{2}{3}}\left(x^8-x^3-x\right)^{\frac{1}{3}}}\right)$$
$$-\ln\left(x+\left(x^8-x^3-x\right)^{\frac{1}{3}}\right)+2^{\frac{1}{3}}\ln\left(2x+2^{\frac{2}{3}}\left(x^8-x^3-x\right)^{\frac{1}{3}}\right)$$
$$+\frac{\ln\left(x^2-x\left(x^8-x^3-x\right)^{\frac{1}{3}}+\left(x^8-x^3-x\right)^{\frac{2}{3}}\right)}{2}$$
$$-\frac{\ln\left(-2x^2+2^{\frac{2}{3}}x\left(x^8-x^3-x\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\left(x^8-x^3-x\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{2}$$

command

```
Integrate[((2 + 5*x^7)*(-x - x^3 + x^8)^(1/3))/((-1 + x^7)*(-1 + x^2 + x^7)),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\left(-1-x^2+x^7\right)^{2/3}\left(-2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}-2\sqrt[3]{-1-x^2+x^7}}\right)+2\sqrt[3]{2}\,\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}-2^{2/3}\sqrt[3]{-1-x^2+x^7}}\right)\right.}{}$$

Mathematica 12.3 output

$Aborted

24.555 Problem number 2698

$$\int \frac{x^6\left(-4+x^3\right)}{\left(-1+x^3\right)^{3/4}\left(1-2x^3+x^6+x^8\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{2+\sqrt{2}}\,\arctan\left(\frac{\sqrt{2+\sqrt{2}}\,x\left(x^3-1\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^3-1}}\right)}{2}$$
$$-\frac{\sqrt{2-\sqrt{2}}\,\arctan\left(\frac{\left(\frac{\sqrt{2}}{\sqrt{2-\sqrt{2}}}-\frac{2}{\sqrt{2-\sqrt{2}}}\right)x\left(x^3-1\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^3-1}}\right)}{2}$$
$$-\frac{\sqrt{2-\sqrt{2}}\,\text{arctanh}\left(\frac{\sqrt{2-\sqrt{2}}\,x\left(x^3-1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^3-1}}\right)}{2}$$
$$-\frac{\sqrt{2+\sqrt{2}}\,\text{arctanh}\left(\frac{\sqrt{2+\sqrt{2}}\,x\left(x^3-1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^3-1}}\right)}{2}$$

command

```
Integrate[(x^6*(-4 + x^3))/((-1 + x^3)^(3/4)*(1 - 2*x^3 + x^6 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{1}{2}\left(\sqrt{2-\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{-1+x^3}}{-x^2+\sqrt{-1+x^3}}\right)\right.$$
$$+\sqrt{2+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{-1+x^3}}{-x^2+\sqrt{-1+x^3}}\right)$$
$$-\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{-1+x^3}}{x^2+\sqrt{-1+x^3}}\right)$$
$$\left.-\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{-1+x^3}}{x^2+\sqrt{-1+x^3}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{x^6\left(-4+x^3\right)}{\left(-1+x^3\right)^{3/4}\left(1-2x^3+x^6+x^8\right)}\,dx$$

24.556 Problem number 2699

$$\int \frac{x^6\left(4+x^5\right)}{\left(-1+x^5\right)^{3/4}\left(1-2x^5+x^8+x^{10}\right)}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{2+\sqrt{2}}\,\arctan\left(\frac{\sqrt{2+\sqrt{2}}\,x\left(x^5-1\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^5-1}}\right)}{2}$$
$$+\frac{\sqrt{2-\sqrt{2}}\,\arctan\left(\frac{\left(\frac{\sqrt{2}}{\sqrt{2-\sqrt{2}}}-\frac{2}{\sqrt{2-\sqrt{2}}}\right)x\left(x^5-1\right)^{\frac{1}{4}}}{-x^2+\sqrt{x^5-1}}\right)}{2}$$
$$+\frac{\sqrt{2-\sqrt{2}}\,\text{arctanh}\left(\frac{\sqrt{2-\sqrt{2}}\,x\left(x^5-1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^5-1}}\right)}{2}$$
$$+\frac{\sqrt{2+\sqrt{2}}\,\text{arctanh}\left(\frac{\sqrt{2+\sqrt{2}}\,x\left(x^5-1\right)^{\frac{1}{4}}}{x^2+\sqrt{x^5-1}}\right)}{2}$$

command

```
Integrate[(x^6*(4 + x^5))/((-1 + x^5)^(3/4)*(1 - 2*x^5 + x^8 + x^10)),x]
```

Mathematica 13.1 output

$$\frac{1}{2}\left(\sqrt{2-\sqrt{2}}\ \text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}\ x\sqrt[4]{-1+x^5}}{x^2-\sqrt{-1+x^5}}\right)\right.$$
$$+\sqrt{2+\sqrt{2}}\ \text{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}\ x\sqrt[4]{-1+x^5}}{x^2-\sqrt{-1+x^5}}\right)$$
$$+\sqrt{2-\sqrt{2}}\ \tanh^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\ x\sqrt[4]{-1+x^5}}{x^2+\sqrt{-1+x^5}}\right)$$
$$\left.+\sqrt{2+\sqrt{2}}\ \tanh^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\ x\sqrt[4]{-1+x^5}}{x^2+\sqrt{-1+x^5}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{x^6\left(4+x^5\right)}{\left(-1+x^5\right)^{3/4}\left(1-2x^5+x^8+x^{10}\right)}\,dx$$

24.557 Problem number 2701

$$\int \frac{\sqrt{1+x^4}\ \sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx$$

Optimal antiderivative

$$\frac{x\sqrt{x^2+\sqrt{x^4+1}}}{2}-\frac{\arctan\left(\frac{\sqrt{2}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}}{2}$$
$$+\sqrt{-2+2\sqrt{2}}\ \arctan\left(\frac{\sqrt{2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$
$$-\text{arctanh}\left(\frac{\sqrt{2}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}$$
$$+\sqrt{2+2\sqrt{2}}\ \text{arctanh}\left(\frac{\sqrt{-2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$

command

```
Integrate[(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2),x]
```

Mathematica 13.1 output

$$\frac{1}{2}x\sqrt{x^2+\sqrt{1+x^4}}-\frac{\text{ArcTan}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$
$$+\sqrt{2\left(-1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\left(-1+x^2+\sqrt{1+x^4}\right)}{x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$-\sqrt{2}\,\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$+\sqrt{2\left(1+\sqrt{2}\right)}\,\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$

Mathematica 12.3 output

$$\int\frac{\sqrt{1+x^4}\,\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx$$

24.558 Problem number 2704

$$\int\frac{\sqrt{b+ax}\,\sqrt{1+\sqrt{b+ax}}}{x^2\sqrt{1+\sqrt{1+\sqrt{b+ax}}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(Sqrt[b + a*x]*Sqrt[1 + Sqrt[b + a*x]])/(x^2*Sqrt[1 + Sqrt[1 + Sqrt[b + a*x]]]),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{1+\sqrt{1+\sqrt{b+ax}}}\left(-1-\sqrt{b+ax}+\sqrt{1+\sqrt{b+ax}}\right)}{x}+a\text{RootSum}\left[b-4\#1^4+4\#1^6\right.$$
$$\left.-\#1^8\&,\frac{-\log\left(\sqrt{1+\sqrt{1+\sqrt{b+ax}}}-\#1\right)+\log\left(\sqrt{1+\sqrt{1+\sqrt{b+ax}}}-\#1\right)\#1^2}{-2\#1^3+\#1^5}\&\right]$$
$$-\frac{1}{8}a\text{RootSum}\left[b-4\#1^4+4\#1^6\right.$$
$$\left.-\#1^8\&,\frac{6\log\left(\sqrt{1+\sqrt{1+\sqrt{b+ax}}}-\#1\right)-7\log\left(\sqrt{1+\sqrt{1+\sqrt{b+ax}}}-\#1\right)\#1^2+3\log\left(\sqrt{1+\sqrt{1}\right.}\right.}{2\#1^3-3\#1^5+\#1^7}$$

Mathematica 12.3 output

$$\int\frac{\sqrt{b+ax}\,\sqrt{1+\sqrt{b+ax}}}{x^2\sqrt{1+\sqrt{1+\sqrt{b+ax}}}}\,dx$$

24.559 Problem number 2705

$$\int\frac{\sqrt{b+ax}\,\sqrt{1+\sqrt{b+ax}}}{x^2\sqrt{1+\sqrt{1+\sqrt{b+ax}}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(Sqrt[b + a*x]*Sqrt[1 + Sqrt[b + a*x]])/(x^2*Sqrt[1 + Sqrt[1 + Sqrt[b + a*x]]]),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{1+\sqrt{1+\sqrt{b+ax}}}\left(-1-\sqrt{b+ax}+\sqrt{1+\sqrt{b+ax}}\right)}{x}+a\text{RootSum}\left[b-4\#1^4+4\#1^6\right.$$

$$\left.-\#1^8\&,\frac{-\log\left(\sqrt{1+\sqrt{1+\sqrt{b+ax}}}-\#1\right)+\log\left(\sqrt{1+\sqrt{1+\sqrt{b+ax}}}-\#1\right)\#1^2}{-2\#1^3+\#1^5}\&\right]$$

$$-\frac{1}{8}a\text{RootSum}\left[b-4\#1^4+4\#1^6\right.$$

$$\left.-\#1^8\&,\frac{6\log\left(\sqrt{1+\sqrt{1+\sqrt{b+ax}}}-\#1\right)-7\log\left(\sqrt{1+\sqrt{1+\sqrt{b+ax}}}-\#1\right)\#1^2+3\log\left(\sqrt{1+\sqrt{1}\right.}\right.}{2\#1^3-3\#1^5+\#1^7}$$

Mathematica 12.3 output

$$\int\frac{\sqrt{b+ax}\sqrt{1+\sqrt{b+ax}}}{x^2\sqrt{1+\sqrt{1+\sqrt{b+ax}}}}\,dx$$

24.560 Problem number 2711

$$\int\frac{x^2\left(-2+x^8\right)\sqrt[4]{2-2x^4+x^8}}{\left(2+x^8\right)\left(4-x^4+2x^8\right)}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{2^{\frac{3}{4}}x\left(x^8-2x^4+2\right)^{\frac{1}{4}}}{x^2\sqrt{2}-\sqrt{x^8-2x^4+2}}\right)2^{\frac{3}{4}}}{4}+\frac{3^{\frac{1}{4}}\arctan\left(\frac{6^{\frac{3}{4}}x\left(x^8-2x^4+2\right)^{\frac{1}{4}}}{-3x^2+\sqrt{6}\sqrt{x^8-2x^4+2}}\right)2^{\frac{1}{4}}}{4}$$

$$+\frac{\text{arctanh}\left(\frac{2\,2^{\frac{1}{4}}x\left(x^8-2x^4+2\right)^{\frac{1}{4}}}{2x^2+\sqrt{2}\sqrt{x^8-2x^4+2}}\right)2^{\frac{3}{4}}}{4}-\frac{3^{\frac{1}{4}}\text{arctanh}\left(\frac{6^{\frac{3}{4}}x\left(x^8-2x^4+2\right)^{\frac{1}{4}}}{3x^2+\sqrt{6}\sqrt{x^8-2x^4+2}}\right)2^{\frac{1}{4}}}{4}$$

command

```
Integrate[(x^2*(-2 + x^8)*(2 - 2*x^4 + x^8)^(1/4))/((2 + x^8)*(4 - x^4 + 2*x^8)),x]
```

Mathematica 13.1 output

$$\frac{1}{4}\left(\sqrt[4]{6}\,\text{ArcTan}\left(\frac{\frac{3\sqrt[4]{6}\,x^2}{\sqrt[4]{2-2x^4+x^8}}-6^{3/4}\sqrt[4]{2-2x^4+x^8}}{6x}\right)\right.$$

$$\left.+2^{3/4}\text{ArcTan}\left(\frac{2^{3/4}x\sqrt[4]{2-2x^4+x^8}}{\sqrt{2}\,x^2-\sqrt{2-2x^4+x^8}}\right)+2^{3/4}\tanh^{-1}\left(\frac{2\sqrt[4]{2}\,x\sqrt[4]{2-2x^4+x^8}}{2x^2+\sqrt{2}\,\sqrt{2-2x^4+x^8}}\right)-\sqrt[4]{6}\,\tanh^{-1}\left(\frac{6^{3/4}x\sqrt[4]{}}{3x^2+\sqrt{6}}\right.\right.$$

Mathematica 12.3 output

$$\int\frac{x^2\left(-2+x^8\right)\sqrt[4]{2-2x^4+x^8}}{\left(2+x^8\right)\left(4-x^4+2x^8\right)}\,dx$$

24.561 Problem number 2722

$$\int\frac{(b+ax)\left(-aq+bpx^2\right)}{\left(q+px^3\right)^{2/3}\left(b^3c+dq+3ab^2cx+3a^2bcx^2+\left(a^3c+dp\right)x^3\right)}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{3}\,b\,c^{\frac{1}{3}}+\sqrt{3}\,a\,c^{\frac{1}{3}}x}{b\,c^{\frac{1}{3}}+a\,c^{\frac{1}{3}}x-2d^{\frac{1}{3}}\left(p\,x^3+q\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3c^{\frac{2}{3}}d^{\frac{1}{3}}}+\frac{\ln\left(b^2c^{\frac{1}{3}}+ab\,c^{\frac{1}{3}}x+b\,d^{\frac{1}{3}}\left(p\,x^3+q\right)^{\frac{1}{3}}\right)}{3c^{\frac{2}{3}}d^{\frac{1}{3}}}$$
$$-\frac{\ln\left(b^4c^{\frac{2}{3}}+2a\,b^3c^{\frac{2}{3}}x+a^2b^2c^{\frac{2}{3}}x^2+\left(-b^3c^{\frac{1}{3}}d^{\frac{1}{3}}-a\,b^2c^{\frac{1}{3}}d^{\frac{1}{3}}x\right)\left(p\,x^3+q\right)^{\frac{1}{3}}+b^2d^{\frac{2}{3}}\left(p\,x^3+q\right)^{\frac{2}{3}}\right)}{6c^{\frac{2}{3}}d^{\frac{1}{3}}}$$

command

```
Integrate[((b + a*x)*(-(a*q) + b*p*x^2))/((q + p*x^3)^(2/3)*(b^3*c + d*q + 3*a*b^2*c*x + 3*a^
```

Mathematica 13.1 output

$$\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{c}\,(b+ax)}{b\sqrt[3]{c}+a\sqrt[3]{c}\,x-2\sqrt[3]{d}\,\sqrt[3]{q+px^3}}\right)+2\log\left(b\left(b\sqrt[3]{c}+a\sqrt[3]{c}\,x+\sqrt[3]{d}\,\sqrt[3]{q+px^3}\right)\right)-\log\left(b^2\left(b^2c^{2/3}+2c\right.\right.}{6c^{2/3}\sqrt[3]{d}}$$

Mathematica 12.3 output

$$\int\frac{(b+ax)\left(-aq+bpx^2\right)}{\left(q+px^3\right)^{2/3}\left(b^3c+dq+3ab^2cx+3a^2bcx^2+\left(a^3c+dp\right)x^3\right)}\,dx$$

24.562 Problem number 2723

$$\int \frac{(-b+x)\left(-a(a-2b)-2bx+x^2\right)}{((-a+x)(-b+x))^{2/3}\left(a^4-b^2d-2\left(2a^3-bd\right)x+\left(6a^2-d\right)x^2-4ax^3+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ d^{\frac{1}{3}}\left(ab+(-a-b)x+x^2\right)^{\frac{2}{3}}}{2a^2-4ax+2x^2+d^{\frac{1}{3}}\left(ab+(-a-b)x+x^2\right)^{\frac{2}{3}}}\right)}{2d^{\frac{2}{3}}}+\frac{\ln\left(a^2-2ax+x^2-d^{\frac{1}{3}}\left(ab+\left(-a-b\right)x+x^2\right)^{\frac{2}{3}}\right)}{2d^{\frac{2}{3}}}$$
$$-\frac{\ln\left(a^4-4a^3x+6a^2x^2-4x^3a+x^4+d^{\frac{2}{3}}\left(ab+\left(-a-b\right)x+x^2\right)^{\frac{4}{3}}+\left(ab+\left(-a-b\right)x+x^2\right)^{\frac{2}{3}}\left(a^2d^{\frac{1}{3}}-2a\,d^{\frac{1}{3}}x+d^{\frac{1}{3}}x^2\right)\right)}{4d^{\frac{2}{3}}}$$

command

```
Integrate[((-b + x)*(-(a*(a - 2*b)) - 2*b*x + x^2))/(((-a + x)*(-b + x))^(2/3)*(a^4 - b^2*d -
```

Mathematica 13.1 output

$$-\frac{\sqrt[3]{(a-x)(b-x)}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2(a-x)^{4/3}}{\sqrt[3]{d}\,(b-x)^{2/3}}}{\sqrt{3}}\right)-2\log\left((a-x)^{2/3}-\sqrt[6]{d}\,\sqrt[3]{b-x}\right)-2\log\left((a-x)^{2/3}+\sqrt[6]{d}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{(-b+x)\left(-a(a-2b)-2bx+x^2\right)}{((-a+x)(-b+x))^{2/3}\left(a^4-b^2d-2\left(2a^3-bd\right)x+\left(6a^2-d\right)x^2-4ax^3+x^4\right)}\,dx$$

24.563 Problem number 2731

$$\int \frac{\left(-2q+px^3\right)\left(aq+bx^2+apx^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^5\left(cq+dx^2+cpx^3\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(acp\,x^3-2ad\,x^2+2bc\,x^2+acq\right)\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}}{2c^2x^4}$$
$$-\frac{2(ad-bc)\sqrt{2c^2pq-d^2}\ \arctan\left(\frac{\sqrt{2c^2pq-d^2}\ x^2}{cq+d\,x^2+cp\,x^3+c\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}}\right)}{c^3}$$
$$+\frac{2\left(a\,c^2pq-a\,d^2+bcd\right)\ln\left(x\right)}{c^3}$$
$$+\frac{\left(-a\,c^2pq+a\,d^2-bcd\right)\ln\left(q+p\,x^3+\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}\right)}{c^3}$$

command

```
Integrate[((-2*q + p*x^3)*(a*q + b*x^2 + a*p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]
```

Mathematica 13.1 output

$$-\frac{c\sqrt{q^2-2pq(-1+x)x^3+p^2x^6}\,(2bcx^2-2adx^2+ac(q+px^3))}{x^4}+4(-bc+ad)\sqrt{-d^2+2c^2pq}\,\text{ArcTan}\left(\frac{\sqrt{}}{dx^2+c\left(q+px^3+\sqrt{q}\right.}\right.$$

$$-\frac{}{2c^3}$$

Mathematica 12.3 output

$$\int \frac{\left(-2q+px^3\right)\left(aq+bx^2+apx^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^5\left(cq+dx^2+cpx^3\right)}\,dx$$

24.564 Problem number 2733

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(bx^8+a\left(q+px^3\right)^4\right)}{x^{13}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}\,\left(2a\,p^5x^{15}-a\,p^4q\,x^{13}+10a\,p^4q\,x^{12}-3a\,p^3q^2x^{11}-3a\,p^3q^2x^{10}+20a\,p^3q^2x^9-3a\,p^2\right.}{12x^{12}}$$

$$+\left(a\,p^3q^3+2bpq\right)\ln\left(x\right)+\frac{\left(-a\,p^3q^3-2bpq\right)\ln\left(q+p\,x^3+\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}\right)}{2}$$

command

```
Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(b*x^8 + a*(q + p*x^3)^
```

Mathematica 13.1 output

$$\frac{1}{12}\left(\frac{\left(q+px^3\right)\sqrt{q^2-2pq(-1+x)x^3+p^2x^6}\,\left(6bx^8+a\left(2q^4-pq^3(-8+x)x^3-p^3q(-8+x)x^9+2p^4x^{12}+p^2q^2x^6\right.\right.}{x^{12}}\right.$$

$$\left.\left.-6pq\left(2b+ap^2q^2\right)\tanh^{-1}\left(\frac{\sqrt{q^2-2pq(-1+x)x^3+p^2x^6}}{q+px^3}\right)\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(bx^8+a\left(q+px^3\right)^4\right)}{x^{13}}\,dx$$

24.565 Problem number 2737

$$\int \frac{1-x+x^2}{(-1+x^2)\sqrt[3]{x^2+x^4}}\,dx$$

Optimal antiderivative

$$\begin{aligned}
&-\frac{3\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ x}{-x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{8}-\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ x}{x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{8}\\
&+\frac{\ln\left(-2x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{8}-\frac{3\ln\left(2x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{8}\\
&+\frac{3\ln\left(-2x^2+2^{\frac{2}{3}}x\left(x^4+x^2\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\left(x^4+x^2\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{16}\\
&-\frac{\ln\left(2x^2+2^{\frac{2}{3}}x\left(x^4+x^2\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(x^4+x^2\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{16}
\end{aligned}$$

command

```
Integrate[(1 - x + x^2)/((-1 + x^2)*(x^2 + x^4)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\sqrt[3]{1+x^2}\left(6\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}-2^{2/3}\sqrt[3]{1+x^2}}\right)-2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}+2^{2/3}\sqrt[3]{1+x^2}}\right)+2\log\left(-2\sqrt[3]{x}+2^{2/3}\sqrt[3]{}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{1-x+x^2}{(-1+x^2)\sqrt[3]{x^2+x^4}}\,dx$$

24.566 Problem number 2738

$$\int \frac{1+x+x^2}{(-1+x^2)\sqrt[3]{x^2+x^4}}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ x}{-x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{8}-\frac{3\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ x}{x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{8}$$
$$+\frac{3\ln\left(-2x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{8}-\frac{\ln\left(2x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{8}$$
$$+\frac{\ln\left(-2x^2+2^{\frac{2}{3}}x\left(x^4+x^2\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\left(x^4+x^2\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{16}$$
$$-\frac{3\ln\left(2x^2+2^{\frac{2}{3}}x\left(x^4+x^2\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(x^4+x^2\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{16}$$

command

```
Integrate[(1 + x + x^2)/((-1 + x^2)*(x^2 + x^4)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\sqrt[3]{1+x^2}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[3]{x}}{\sqrt[3]{x}-2^{2/3}\sqrt[3]{1+x^2}}\right)-6\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[3]{x}}{\sqrt[3]{x}+2^{2/3}\sqrt[3]{1+x^2}}\right)+6\log\left(-2\sqrt[3]{x}+2^{2/3}\sqrt[3]{}\right.\right.}{}$$

Mathematica 12.3 output

$$\int\frac{1+x+x^2}{(-1+x^2)\sqrt[3]{x^2+x^4}}\,dx$$

24.567 Problem number 2745

$$\int\frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^4+a\left(q+px^3\right)^4\right)}{x^7}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}\left(2a\,p^5x^{15}+10a\,p^4q\,x^{12}-a\,p^4q\,x^{11}+20a\,p^3q^2x^9-3a\,p^3q^2x^8-3a\,p^3q^2x^7+20a\,p^2q\right.}{12x^6}$$
$$+\frac{\left(a\,p^3q^3+2bpq\right)\ln(x)}{2}+\frac{\left(-a\,p^3q^3-2bpq\right)\ln\left(q+p\,x^3+\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}\right)}{2}$$

command

```
Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*x^4 + a*(q + p*x^3)^
```

Mathematica 13.1 output

$$\frac{1}{12}\left(\frac{\left(q+px^3\right)\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}\left(6bx^4+a\left(2q^4+2p^4x^{12}+pq^3x^2(-1+8x)+p^3qx^8(-1+8x)+p^2q^2x\right.\right.\right.}{x^6}$$

$$\left.\left.\left.-6pq\left(2b+ap^2q^2\right)\tanh^{-1}\left(\frac{\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}}{q+px^3}\right)\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-q+2px^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^4+a\left(q+px^3\right)^4\right)}{x^7}\,dx$$

24.568 Problem number 2748

$$\int \frac{\left(1-2k^2\right)x+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d+\left(1-2dk^2\right)x^2+dk^4x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{1}{3}}}{2d^{\frac{1}{3}}-2d^{\frac{1}{3}}k^2x^2+\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{1}{3}}}\right)}{2d^{\frac{1}{3}}}$$

$$+\frac{\ln\left(-d^{\frac{1}{3}}+d^{\frac{1}{3}}k^2x^2+\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{1}{3}}\right)}{2d^{\frac{1}{3}}}$$

$$-\frac{\ln\left(d^{\frac{2}{3}}-2d^{\frac{2}{3}}k^2x^2+d^{\frac{2}{3}}k^4x^4+\left(d^{\frac{1}{3}}-d^{\frac{1}{3}}k^2x^2\right)\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{1}{3}}+\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{2}{3}}\right)}{4d^{\frac{1}{3}}}$$

command

```
Integrate[((1 - 2*k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3)*(-1 + d + (1 - 2*d*k^2)*
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{-1+x^2}\,\sqrt[3]{-1+k^2x^2}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{-1+x^2}}{\sqrt[3]{-1+x^2}-2\sqrt[3]{d}\,\left(-1+k^2x^2\right)^{2/3}}\right)+2\log\left(\sqrt[3]{-1+x^2}+\sqrt[3]{d}\,\left(-1+k^2x^2\right)^{2/3}\right)\right.}{4\sqrt[3]{d}\,\sqrt[3]{\left(-1+x^2\right)\left(-1+k^2x^2\right)}}$$

Mathematica 12.3 output

$$\int \frac{\left(1-2k^2\right)x+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d+\left(1-2dk^2\right)x^2+dk^4x^4\right)}\,dx$$

24.569 Problem number 2749

$$\int \frac{\left(2-k^2\right)x-2x^3+k^2x^5}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(1-d+\left(-2+dk^2\right)x^2+x^4\right)}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{2}{3}}}{2d^{\frac{1}{3}}-2d^{\frac{1}{3}}k^2x^2+\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{2}{3}}}\right)}{2d^{\frac{1}{3}}}$$
$$-\frac{\ln\left(-d^{\frac{1}{3}}+d^{\frac{1}{3}}k^2x^2+\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{2}{3}}\right)}{2d^{\frac{1}{3}}}$$
$$+\frac{\ln\left(d^{\frac{2}{3}}-2d^{\frac{2}{3}}k^2x^2+d^{\frac{2}{3}}k^4x^4+\left(d^{\frac{1}{3}}-d^{\frac{1}{3}}k^2x^2\right)\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{2}{3}}+\left(1+\left(-k^2-1\right)x^2+k^2x^4\right)^{\frac{4}{3}}\right)}{4d^{\frac{1}{3}}}$$

command

```
Integrate[((2 - k^2)*x - 2*x^3 + k^2*x^5)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)*(1 - d + (-
2 + d*k^2)*x^2 + x^4)),x]
```

Mathematica 13.1 output

$$\frac{\left(-1+x^2\right)^{2/3}\left(-1+k^2x^2\right)^{2/3}\left(-2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\left(-1+x^2\right)^{2/3}}{\left(-1+x^2\right)^{2/3}-2\sqrt[3]{d}\,\sqrt[3]{-1+k^2x^2}}\right)-2\log\left(\left(-1+x^2\right)^{2/3}+\sqrt[3]{d}\,\sqrt[3]{-1}\right.\right.}{4\sqrt[3]{d}\,\left(\left(-1+x^2\right)\left(-1+k^2x^2\right)\right.}$$

Mathematica 12.3 output

$$\int \frac{\left(2-k^2\right)x-2x^3+k^2x^5}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(1-d+\left(-2+dk^2\right)x^2+x^4\right)}\,dx$$

24.570 Problem number 2753

$$\int \frac{\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{1-x^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/(1 - x^2),x]
```

Mathematica 13.1 output

$$-\frac{1}{2}\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.$$
$$\left.+\#1^8\&,\frac{2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)-2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2+\log\left(\sqrt{1+\sqrt{x+}\right.}\right.}{2\#1-3\#1^3+\#1^5}$$

$$-\frac{1}{2}\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$
$$\left.+\#1^8\&,\frac{-2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^3+\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^5}{-2+4\#1^2-3\#1^4+\#1^6}\&\right]$$

Mathematica 12.3 output

$$\int\frac{\sqrt{x+\sqrt{1+x^2}}\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{1-x^2}\,dx$$

24.571 Problem number 2754

$$\int\frac{\sqrt{x+\sqrt{1+x^2}}\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{1-x^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/(1 - x^2),x]
```

Mathematica 13.1 output

$$-\frac{1}{2}\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.$$
$$\left.+\#1^8\&,\ \frac{2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)-2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2+\log\left(\sqrt{1+\sqrt{x+}\right.}}{2\#1-3\#1^3+\#1^5}\right]$$

$$-\frac{1}{2}\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$
$$\left.+\#1^8\&,\ \frac{-2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^3+\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^5}{-2+4\#1^2-3\#1^4+\#1^6}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\sqrt{x+\sqrt{1+x^2}}\ \sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{1-x^2}\,dx$$

24.572 Problem number 2763

$$\int \frac{\left(2+x^6\right)\left(-1+x^4+x^6\right)}{\sqrt[4]{-1+x^6}\ \left(1-2x^6+x^8+x^{12}\right)}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{4-2\sqrt{2}}\ \arctan\left(\frac{\sqrt{2+\sqrt{2}}\ x(x^6-1)^{\frac{1}{4}}}{-x^2+\sqrt{x^6-1}}\right)}{4}+\frac{\sqrt{4+2\sqrt{2}}\ \arctan\left(\frac{\left(\frac{\sqrt{2}}{\sqrt{2-\sqrt{2}}}-\frac{2}{\sqrt{2-\sqrt{2}}}\right)x(x^6-1)^{\frac{1}{4}}}{-x^2+\sqrt{x^6-1}}\right)}{4}-\frac{\sqrt{4+2\sqrt{2}}\ \operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{2}}\ x(x^6-1)^{\frac{1}{4}}}{x^2+\sqrt{x^6-1}}\right)}{4}-\frac{\sqrt{4-2\sqrt{2}}\ \operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{2}}\ x(x^6-1)^{\frac{1}{4}}}{x^2+\sqrt{x^6-1}}\right)}{4}$$

command

```
Integrate[((2 + x^6)*(-1 + x^4 + x^6))/((-1 + x^6)^(1/4)*(1 - 2*x^6 + x^8 + x^12)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{2+\sqrt{2}}\ \operatorname{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}\ x\sqrt[4]{-1+x^6}}{-x^2+\sqrt{-1+x^6}}\right)+\sqrt{2-\sqrt{2}}\ \operatorname{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}\ x\sqrt[4]{-1+x^6}}{-x^2+\sqrt{-1+x^6}}\right)+\sqrt{2+\sqrt{2}}$$

$$2\sqrt{2}$$

Mathematica 12.3 output

$$\int \frac{\left(2+x^6\right)\left(-1+x^4+x^6\right)}{\sqrt[4]{-1+x^6}\left(1-2x^6+x^8+x^{12}\right)}\,dx$$

24.573 Problem number 2765

$$\int \frac{\left(1+x^2\right)^2}{\left(-1+x^2\right)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Optimal antiderivative

$$\frac{x\left(x^2-5\right)}{2\left(x^2-1\right)\sqrt{x^2+\sqrt{x^4+1}}}-4\arctan\left(\frac{\sqrt{2}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}$$
$$+\sqrt{14+10\sqrt{2}}\,\arctan\left(\frac{\sqrt{-2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$
$$+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}}{2}$$
$$-\sqrt{-14+10\sqrt{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$

command

```
Integrate[(1 + x^2)^2/((-1 + x^2)^2*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

Mathematica 13.1 output

$$\frac{x\left(-5+x^2\right)}{2\left(-1+x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}-4\sqrt{2}\,\operatorname{ArcTan}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$+\sqrt{2\left(7+5\sqrt{2}\right)}\,\operatorname{ArcTan}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$+\frac{\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$
$$-\sqrt{2\left(-7+5\sqrt{2}\right)}\,\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\left(-1+x^2+\sqrt{1+x^4}\right)}{x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(1+x^2\right)^2}{\left(-1+x^2\right)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

24.574 Problem number 2768

$$\int \frac{\left(-1+x^2\right)^2}{\left(1+x^2\right)^2 \sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Optimal antiderivative

$$\frac{x\left(x^2+5\right)}{2\left(x^2+1\right)\sqrt{x^2+\sqrt{x^4+1}}}+4\arctan\left(\frac{\sqrt{2}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}$$
$$-\sqrt{14+10\sqrt{2}}\,\arctan\left(\frac{\sqrt{2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$
$$+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}}{2}$$
$$-\sqrt{-14+10\sqrt{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{-2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$

command

```
Integrate[(-1 + x^2)^2/((1 + x^2)^2*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

Mathematica 13.1 output

$$\frac{x\left(5+x^2\right)}{2\left(1+x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}+4\sqrt{2}\,\operatorname{ArcTan}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$-\sqrt{2\left(7+5\sqrt{2}\right)}\,\operatorname{ArcTan}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\left(-1+x^2+\sqrt{1+x^4}\right)}{x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$+\frac{\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$
$$-\sqrt{2\left(-7+5\sqrt{2}\right)}\,\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^2\right)^2}{\left(1+x^2\right)^2 \sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

24.575 Problem number 2776

$$\int \frac{1}{(-b+ax)\sqrt[3]{b^3+a^3x^3}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(a^3x^3+b^3\right)^{\frac{1}{3}}}{2^{\frac{1}{3}}b+2^{\frac{1}{3}}ax+\left(a^3x^3+b^3\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{4ab}+\frac{\ln\left(2^{\frac{1}{3}}\sqrt{a}\;b^{\frac{3}{2}}+2^{\frac{1}{3}}a^{\frac{3}{2}}\sqrt{b}\;x-2\sqrt{a}\;\sqrt{b}\;\left(a^3x^3+b^3\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{4ab}$$
$$-\frac{\ln\left(2^{\frac{2}{3}}a\,b^3+2\,2^{\frac{2}{3}}a^2b^2x+2^{\frac{2}{3}}a^3b\,x^2+\left(2\,2^{\frac{1}{3}}a\,b^2+2\,2^{\frac{1}{3}}a^2bx\right)\left(a^3x^3+b^3\right)^{\frac{1}{3}}+4ab\left(a^3x^3+b^3\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{8ab}$$

command

```
Integrate[1/((-b + a*x)*(b^3 + a^3*x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{b^3+a^3x^3}}{\sqrt[3]{2}\,b+\sqrt[3]{2}\,ax+\sqrt[3]{b^3+a^3x^3}}\right)+2\log\left(\sqrt{a}\,\sqrt{b}\,\left(\sqrt[3]{2}\,b+\sqrt[3]{2}\,ax-2\sqrt[3]{b^3+a^3x^3}\right)\right)-\log\left(ab\left(2^{2/3}b^2\right.\right.}{4\sqrt[3]{2}\,ab}$$

Mathematica 12.3 output

$$\int \frac{1}{(-b+ax)\sqrt[3]{b^3+a^3x^3}}\,dx$$

24.576 Problem number 2783

$$\int \frac{\left(1+x^2\right)\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^2\right)^2\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^2)*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/((1 - x^2)^2*Sqrt[x + Sqrt[1 + x^2]])
```

Mathematica 13.1 output

$$\frac{1}{8}\left(-\frac{8x\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(-1+x^2)\sqrt{x+\sqrt{1+x^2}}}\right.$$

$$+4\text{RootSum}\left[-2+4\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)}{2\#1-3\#1^3+\#1^5}\&\right]$$

$$-\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+5\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2}{2\#1^3-3\#1^5+\#1^7}\&\right]$$

$$+4\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-2+4\#1^2-3\#1^4+\#1^6}\&\right]$$

$$-\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$

$$\left.\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+5\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2}{-2\#1+4\#1^3-3\#1^5+\#1^7}\&\right]\right)$$

Mathematica 12.3 output

$$\int\frac{(1+x^2)\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1-x^2)^2\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.577 Problem number 2784

$$\int \frac{\left(1+x^2\right)\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^2\right)^2\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^2)*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/((1 - x^2)^2*Sqrt[x + Sqrt[1 + x^2]])
```

Mathematica 13.1 output

$$\frac{1}{8}\left(-\frac{8x\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(-1+x^2)\sqrt{x+\sqrt{1+x^2}}}\right.$$

$$+4\text{RootSum}\left[-2+4\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)}{2\#1-3\#1^3+\#1^5}\&\right]$$

$$-\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+5\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2}{2\#1^3-3\#1^5+\#1^7}\&\right]$$

$$+4\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-2+4\#1^2-3\#1^4+\#1^6}\&\right]$$

$$-\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$

$$\left.\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+5\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2}{-2\#1+4\#1^3-3\#1^5+\#1^7}\&\right]\right)$$

Mathematica 12.3 output

$$\int\frac{(1+x^2)\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1-x^2)^2\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.578 Problem number 2790

$$\int \frac{\sqrt{b+ax}\,\sqrt{c+\sqrt{b+ax}}}{x\sqrt{d+\sqrt{c+\sqrt{b+ax}}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]])/(x*Sqrt[d + Sqrt[c + Sqrt[b + a*x]]]),x]
```

Mathematica 13.1 output

$$\frac{8}{15}\sqrt{d+\sqrt{c+\sqrt{b+ax}}}\left(3c+8d^2+3\sqrt{b+ax}-4d\sqrt{c+\sqrt{b+ax}}\right)$$
$$-b\text{RootSum}\left[b-c^2+2cd^2-d^4-4cd\#1^2+4d^3\#1^2+2c\#1^4-6d^2\#1^4+4d\#1^6\right.$$
$$\left.-\#1^8\&,\frac{d\log\left(\sqrt{d+\sqrt{c+\sqrt{b+ax}}}-\#1\right)-\log\left(\sqrt{d+\sqrt{c+\sqrt{b+ax}}}-\#1\right)\#1^2}{-c\#1+d^2\#1-2d\#1^3+\#1^5}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\sqrt{b+ax}\,\sqrt{c+\sqrt{b+ax}}}{x\sqrt{d+\sqrt{c+\sqrt{b+ax}}}}\,dx$$

24.579 Problem number 2791

$$\int \frac{\sqrt{b+ax}\,\sqrt{c+\sqrt{b+ax}}}{x\sqrt{d+\sqrt{c+\sqrt{b+ax}}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]])/(x*Sqrt[d + Sqrt[c + Sqrt[b + a*x]]]),x]
```

Mathematica 13.1 output

$$\frac{8}{15}\sqrt{d+\sqrt{c+\sqrt{b+ax}}}\left(3c+8d^2+3\sqrt{b+ax}-4d\sqrt{c+\sqrt{b+ax}}\right)$$
$$-b\text{RootSum}\left[b-c^2+2cd^2-d^4-4cd\#1^2+4d^3\#1^2+2c\#1^4-6d^2\#1^4+4d\#1^6\right.$$
$$\left.-\#1^8\&,\frac{d\log\left(\sqrt{d+\sqrt{c+\sqrt{b+ax}}}-\#1\right)-\log\left(\sqrt{d+\sqrt{c+\sqrt{b+ax}}}-\#1\right)\#1^2}{-c\#1+d^2\#1-2d\#1^3+\#1^5}\&\right]$$

Mathematica 12.3 output

$$\int\frac{\sqrt{b+ax}\sqrt{c+\sqrt{b+ax}}}{x\sqrt{d+\sqrt{c+\sqrt{b+ax}}}}\,dx$$

24.580 Problem number 2795

$$\int\frac{\sqrt[4]{2b+ax^4}\left(-4b+ax^8\right)}{x^6\left(-4b+cx^4+ax^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((2*b + a*x^4)^(1/4)*(-4*b + a*x^8))/(x^6*(-4*b + c*x^4 + a*x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{2b+ax^4}\left(-4b-2ax^4-5cx^4\right)}{20bx^5}$$
$$-c\text{RootSum}\left[2a^2-2ab+ac-4a\#1^4-c\#1^4+2\#1^8\&,\frac{2a^2\log(x)-2ab\log(x)+ac\log(x)-2a^2\log\left(\sqrt[4]{2b+ax^4}-x\#1\right)+2ab}\right.$$

Mathematica 12.3 output

$$\int\frac{\sqrt[4]{2b+ax^4}\left(-4b+ax^8\right)}{x^6\left(-4b+cx^4+ax^8\right)}\,dx$$

24.581 Problem number 2796

$$\int \frac{\sqrt[4]{2b+ax^4}\left(-4b+ax^8\right)}{x^6\left(-4b+cx^4+ax^8\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((2*b + a*x^4)^(1/4)*(-4*b + a*x^8))/(x^6*(-4*b + c*x^4 + a*x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{2b+ax^4}\left(-4b-2ax^4-5cx^4\right)}{20bx^5}$$
$$-\frac{c\text{RootSum}\left[2a^2-2ab+ac-4a\#1^4-c\#1^4+2\#1^8\&,\ \frac{2a^2\log(x)-2ab\log(x)+ac\log(x)-2a^2\log\left(\sqrt[4]{2b+ax^4}-x\#1\right)+2ab}\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{2b+ax^4}\left(-4b+ax^8\right)}{x^6\left(-4b+cx^4+ax^8\right)}\,dx$$

24.582 Problem number 2798

$$\int \frac{1}{1-(1+x)\sqrt{c+bx+ax^2}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(1 - (1 + x)*Sqrt[c + b*x + a*x^2])^(-1),x]
```

Mathematica 13.1 output

$$2\text{RootSum}\left[-b^2-\sqrt{a}\,bc+\sqrt{a}\,c^2+4\sqrt{a}\,b\#1+b^2\#1\right.$$
$$+2ac\#1-bc\#1-4a\#1^2-3\sqrt{a}\,b\#1^2+2a\#1^3+b\#1^3$$
$$\left.-\sqrt{a}\,\#1^4\&,\ \frac{\sqrt{a}\,c\log\left(-\sqrt{a}\,x+\sqrt{c+bx+ax^2}-\#1\right)-b\log\left(-\sqrt{a}\,x+\sqrt{c+bx+ax^2}-\#1\right)\#1+\sqrt{a}\,\log\right.}{4\sqrt{a}\,b+b^2+2ac-bc-8a\#1-6\sqrt{a}\,b\#1+6a\#1^2+3b\#1^2-4\sqrt{}}$$

Mathematica 12.3 output

$$\int \frac{1}{1-(1+x)\sqrt{c+bx+ax^2}}\,dx$$

24.583 Problem number 2801

$$\int \frac{1+x^8}{\sqrt[4]{-x^2+x^4}\,(-1+x^8)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(1 + x^8)/((-x^2 + x^4)^(1/4)*(-1 + x^8)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{x}\left(4\sqrt{x}-4\sqrt[4]{-1+x^2}\,\text{ArcTan}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)+2^{3/4}\sqrt[4]{-1+x^2}\,\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)-4\sqrt[4]{-1+x^2}\,\tanh\right.}{}$$

Mathematica 12.3 output

$$\int \frac{1+x^8}{\sqrt[4]{-x^2+x^4}\,(-1+x^8)}\,dx$$

24.584 Problem number 2802

$$\int \frac{1+x^8}{\sqrt[4]{-x^2+x^4}\,(-1+x^8)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(1 + x^8)/((-x^2 + x^4)^(1/4)*(-1 + x^8)),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{x}\left(4\sqrt{x}-4\sqrt[4]{-1+x^2}\,\text{ArcTan}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)+2^{3/4}\sqrt[4]{-1+x^2}\,\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)-4\sqrt[4]{-1+x^2}\,\tanh\right.}{}$$

Mathematica 12.3 output

$$\int \frac{1+x^8}{\sqrt[4]{-x^2+x^4}\,(-1+x^8)}\,dx$$

24.585 Problem number 2804

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(-1+x^4)\sqrt{1+x^4}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{-2+2\sqrt{2}}\ \arctan\left(\frac{\sqrt{-2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{4}$$
$$-\frac{\sqrt{-2+2\sqrt{2}}\ \arctan\left(\frac{\sqrt{2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{4}$$
$$-\frac{\sqrt{2+2\sqrt{2}}\ \operatorname{arctanh}\left(\frac{\sqrt{-2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{4}$$
$$-\frac{\sqrt{2+2\sqrt{2}}\ \operatorname{arctanh}\left(\frac{\sqrt{2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{4}$$

command

```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((-1 + x^4)*Sqrt[1 + x^4]),x]
```

Mathematica 13.1 output

$$\sqrt{-1+\sqrt{2}}\left(-\operatorname{ArcTan}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\left(-1+x^2+\sqrt{1+x^4}\right)}{x\sqrt{x^2+\sqrt{1+x^4}}}\right)+\operatorname{ArcTan}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(-1+x^4)\sqrt{1+x^4}}\,dx$$

24.586 Problem number 2805

$$\int \frac{-1+x^6}{\sqrt[3]{-x^2+x^4}\,(1+x^6)}\,dx$$

Optimal antiderivative

$$-\frac{2\arctan\left(\frac{x}{(x^4-x^2)^{\frac{1}{3}}}\right)}{3}-\frac{\arctan\left(\frac{2^{\frac{1}{3}}x}{(x^4-x^2)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{6}-\frac{\arctan\left(\frac{x(x^4-x^2)^{\frac{1}{3}}}{-x^2+(x^4-x^2)^{\frac{2}{3}}}\right)}{3}$$
$$-\frac{\arctan\left(\frac{2^{\frac{2}{3}}x(x^4-x^2)^{\frac{1}{3}}}{-2x^2+2^{\frac{1}{3}}(x^4-x^2)^{\frac{2}{3}}}\right)2^{\frac{2}{3}}}{12}-\frac{\operatorname{arctanh}\left(\frac{\frac{\sqrt{3}\,x^2}{3}+\frac{(x^4-x^2)^{\frac{2}{3}}\sqrt{3}}{3}}{x(x^4-x^2)^{\frac{1}{3}}}\right)\sqrt{3}}{3}$$
$$-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\frac{2^{\frac{1}{3}}x^2\sqrt{3}}{3}+\frac{(x^4-x^2)^{\frac{2}{3}}2^{\frac{2}{3}}\sqrt{3}}{6}}{x(x^4-x^2)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{12}$$

command

```
Integrate[(-1 + x^6)/((-x^2 + x^4)^(1/3)*(1 + x^6)),x]
```

Mathematica 13.1 output

$$-\frac{x^{2/3}\sqrt[3]{-1+x^2}\left(8\text{ArcTan}\left(\frac{\sqrt[3]{x}}{\sqrt[3]{-1+x^2}}\right)+2\,2^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{2}\,\sqrt[3]{x}}{\sqrt[3]{-1+x^2}}\right)+4\text{ArcTan}\left(\frac{\left(1-i\sqrt{3}\right)\sqrt[3]{x}}{2\sqrt[3]{-1+x^2}}\right)+4i\sqrt{3}\right.}{}$$

Mathematica 12.3 output

$$\int \frac{-1+x^6}{\sqrt[3]{-x^2+x^4}\,(1+x^6)}\,dx$$

24.587 Problem number 2806

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+x}\,dx$$

Optimal antiderivative

$$\sqrt{x^2+\sqrt{x^4+1}}-\sqrt{\sqrt{2}-1}\arctan\left(\frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{\sqrt{2}-1}}\right)$$
$$+\sqrt{\sqrt{2}-1}\arctan\left(\frac{\sqrt{-2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$
$$-\sqrt{1+\sqrt{2}}\operatorname{arctanh}\left(\frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{1+\sqrt{2}}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{2}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}$$
$$+\sqrt{1+\sqrt{2}}\operatorname{arctanh}\left(\frac{\sqrt{2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)$$

command

```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/(1 + x),x]
```

Mathematica 13.1 output

$$\sqrt{x^2+\sqrt{1+x^4}}-\sqrt{-1+\sqrt{2}}\operatorname{ArcTan}\left(\sqrt{1+\sqrt{2}}\sqrt{x^2+\sqrt{1+x^4}}\right)$$
$$+\sqrt{-1+\sqrt{2}}\operatorname{ArcTan}\left(\frac{\sqrt{-2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)$$
$$-\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\sqrt{-1+\sqrt{2}}\sqrt{x^2+\sqrt{1+x^4}}\right)$$
$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)$$
$$+\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\frac{\sqrt{2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)$$

Mathematica 12.3 output

$$\int\frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+x}\,dx$$

24.588 Problem number 2808

$$\int \frac{\left(4+x^3\right)\left(1+x^3+x^4\right)}{\sqrt[4]{1+x^3}\,\left(1+2x^3+x^6+x^8\right)}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{4+2\sqrt{2}}\,\arctan\left(\frac{\frac{\sqrt{2-\sqrt{2}}\,x^2}{-2+\sqrt{2}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt{x^3+1}}{-2+\sqrt{2}}}{x(x^3+1)^{\frac{1}{4}}}\right)}{2}$$
$$-\frac{\sqrt{4-2\sqrt{2}}\,\arctan\left(\frac{-\frac{x^2}{\sqrt{2+\sqrt{2}}}+\frac{\sqrt{x^3+1}}{\sqrt{2+\sqrt{2}}}}{x(x^3+1)^{\frac{1}{4}}}\right)}{2}$$
$$+\frac{\sqrt{4+2\sqrt{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{2}}\,x(x^3+1)^{\frac{1}{4}}}{x^2+\sqrt{x^3+1}}\right)}{2}$$
$$+\frac{\sqrt{4-2\sqrt{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{2}}\,x(x^3+1)^{\frac{1}{4}}}{x^2+\sqrt{x^3+1}}\right)}{2}$$

command

```
Integrate[((4 + x^3)*(1 + x^3 + x^4))/((1 + x^3)^(1/4)*(1 + 2*x^3 + x^6 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{-\sqrt{2-\sqrt{2}}\,\operatorname{ArcTan}\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(-x^2+\sqrt{1+x^3}\right)}{x\sqrt[4]{1+x^3}}\right)-\sqrt{2+\sqrt{2}}\,\operatorname{ArcTan}\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(-x^2+\sqrt{1+x^3}\right)}{x\sqrt[4]{1+x^3}}\right)+\sqrt{}}{\sqrt{2}}$$

Mathematica 12.3 output

$$\int \frac{\left(4+x^3\right)\left(1+x^3+x^4\right)}{\sqrt[4]{1+x^3}\,\left(1+2x^3+x^6+x^8\right)}\,dx$$

24.589 Problem number 2812

$$\int \frac{(-4a+b+3x)\left(-b^3+3b^2x-3bx^2+x^3\right)}{((-a+x)(-b+x)^2)^{2/3}\left(a+b^4d-(1+4b^3d)\,x+6b^2dx^2-4bdx^3+dx^4\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ d^{\frac{1}{3}}\left(-a\,b^2+(2ab+b^2)x+(-a-2b)x^2+x^3\right)^{\frac{2}{3}}}{-2a+2x+d^{\frac{1}{3}}\left(-a\,b^2+(2ab+b^2)x+(-a-2b)x^2+x^3\right)^{\frac{2}{3}}}\right)}{d^{\frac{2}{3}}}$$
$$+\frac{\ln\left(a-x+d^{\frac{1}{3}}\left(-a\,b^2+\left(2ab+b^2\right)x+(-a-2b)\,x^2+x^3\right)^{\frac{2}{3}}\right)}{d^{\frac{2}{3}}}$$
$$-\frac{\ln\left(a^2-2ax+x^2+\left(-a\,d^{\frac{1}{3}}+d^{\frac{1}{3}}x\right)\left(-a\,b^2+\left(2ab+b^2\right)x+(-a-2b)\,x^2+x^3\right)^{\frac{2}{3}}+d^{\frac{2}{3}}\left(-a\,b^2+\left(2ab+b^2\right)x+(\right.\right.}{2d^{\frac{2}{3}}}$$

command

```
Integrate[((-4*a + b + 3*x)*(-b^3 + 3*b^2*x - 3*b*x^2 + x^3))/(((-a + x)*(-b + x)^2)^(2/3)*(a
```

Mathematica 13.1 output

$$-\frac{(a-x)^{2/3}(b-x)^{4/3}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{a-x}}{\sqrt[3]{d}\ (b-x)^{4/3}}}{\sqrt{3}}\right)-2\log\left(a\sqrt[3]{d}\ -b\sqrt[3]{d}\ +\frac{(a-x)^{4/3}}{(b-x)^{4/3}}-\frac{\sqrt[3]{a-x}}{\sqrt[3]{b-x}}\right)+\log\left(\frac{(a-}{}\right.\right.}{2d^{2/3}\left((b-x)^2(-\right.}$$

Mathematica 12.3 output

$$\int \frac{(-4a+b+3x)\left(-b^3+3b^2x-3bx^2+x^3\right)}{((-a+x)(-b+x)^2)^{2/3}\left(a+b^4d-(1+4b^3d)\,x+6b^2dx^2-4bdx^3+dx^4\right)}\,dx$$

24.590 Problem number 2813

$$\int \frac{x^2(-2+(1+k)x)}{\sqrt[3]{(1-x)x(1-kx)}\ \left(1-(2+2k)x+(1+4k+k^2)\,x^2-(2k+2k^2)\,x^3+(-b+k^2)\,x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ b^{\frac{1}{3}}x^2}{b^{\frac{1}{3}}x^2+2\left(x+(-1-k)x^2+kx^3\right)^{\frac{2}{3}}}\right)}{2b^{\frac{2}{3}}}+\frac{\ln\left(-b^{\frac{1}{6}}x+\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}\right)}{2b^{\frac{2}{3}}}$$
$$+\frac{\ln\left(b^{\frac{1}{6}}x+\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}\right)}{2b^{\frac{2}{3}}}$$
$$-\frac{\ln\left(b^{\frac{1}{3}}x^2-b^{\frac{1}{6}}x\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}+\left(x+(-1-k)x^2+kx^3\right)^{\frac{2}{3}}\right)}{4b^{\frac{2}{3}}}$$
$$-\frac{\ln\left(b^{\frac{1}{3}}x^2+b^{\frac{1}{6}}x\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}+\left(x+(-1-k)x^2+kx^3\right)^{\frac{2}{3}}\right)}{4b^{\frac{2}{3}}}$$

command

```
Integrate[(x^2*(-2 + (1 + k)*x))/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (2 + 2*k)*x + (1 + 4*k + k
b + k^2)*x^4)),x]
```

Mathematica 13.1 output

$$-\frac{x\sqrt[3]{\frac{-1+kx}{-1+x}}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\left(\frac{-1+kx}{-1+x}\right)^{2/3}}{\sqrt[3]{b}\left(\frac{x}{-1+x}\right)^{4/3}}}{\sqrt{3}}\right)-2\log\left(-\sqrt[6]{b}\left(\frac{x}{-1+x}\right)^{2/3}+\sqrt[3]{\frac{-1+kx}{-1+x}}\right)-2\log\left(\sqrt[6]{b}\left(\frac{x}{-1+x}\right.\right.\right.}{\left.\left.\left.\right.\right.\right.}$$

Mathematica 12.3 output

$$\int\frac{x^2(-2+(1+k)x)}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(2+2k)x+(1+4k+k^2)x^2-(2k+2k^2)x^3+(-b+k^2)x^4\right)}\,dx$$

24.591 Problem number 2818

$$\int\frac{\sqrt[4]{-bx^3+ax^4}}{-d-2cx+x^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-(b*x^3) + a*x^4)^(1/4)/(-d - 2*c*x + x^2),x]
```

Mathematica 13.1 output

$$-\frac{x^{9/4}(-b+ax)^{3/4}\left(16\sqrt[4]{a}\left(\text{ArcTan}\left(\frac{\sqrt[4]{a}\,\sqrt[4]{x}}{\sqrt[4]{-b+ax}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt[4]{x}}{\sqrt[4]{-b+ax}}\right)\right)+\text{RootSum}\left[b^2-2abc-a^2d+2b\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{-bx^3+ax^4}}{-d-2cx+x^2}\,dx$$

24.592 Problem number 2819

$$\int \frac{\sqrt[4]{-bx^3+ax^4}}{-d-2cx+x^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(-(b*x^3) + a*x^4)^(1/4)/(-d - 2*c*x + x^2),x]
```

Mathematica 13.1 output

$$-\frac{x^{9/4}(-b+ax)^{3/4}\left(16\sqrt[4]{a}\left(\text{ArcTan}\left(\frac{\sqrt[4]{a}\,\sqrt[4]{x}}{\sqrt[4]{-b+ax}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt[4]{x}}{\sqrt[4]{-b+ax}}\right)\right)+\text{RootSum}\left[b^2-2abc-a^2d+2b\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[4]{-bx^3+ax^4}}{-d-2cx+x^2}\,dx$$

24.593 Problem number 2825

$$\int \frac{1+x^2+x^3}{(-1+x^2+x^3)\sqrt[3]{x^2+x^3}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(1 + x^2 + x^3)/((-1 + x^2 + x^3)*(x^2 + x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\sqrt[3]{1+x}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}+2\sqrt[3]{1+x}}\right)-2\log\left(-\sqrt[3]{x}+\sqrt[3]{1+x}\right)+\log\left(x^{2/3}+\sqrt[3]{x}\,\sqrt[3]{1+x}+(1+x)^{2/3}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{1+x^2+x^3}{(-1+x^2+x^3)\sqrt[3]{x^2+x^3}}\,dx$$

24.594 Problem number 2826

$$\int \frac{1+x^2+x^3}{(-1+x^2+x^3)\sqrt[3]{x^2+x^3}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(1 + x^2 + x^3)/((-1 + x^2 + x^3)*(x^2 + x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\sqrt[3]{1+x}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}+2\sqrt[3]{1+x}}\right)-2\log\left(-\sqrt[3]{x}+\sqrt[3]{1+x}\right)+\log\left(x^{2/3}+\sqrt[3]{x}\,\sqrt[3]{1+x}+(1+x)^{2/3}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{1+x^2+x^3}{(-1+x^2+x^3)\sqrt[3]{x^2+x^3}}\,dx$$

24.595 Problem number 2828

$$\int \sqrt{ax+\sqrt{b+a^2x^2}}\,\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

Optimal antiderivative

$$\frac{\left(60a^2x^2-8a\,c^2x-75b\right)\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}+\left(6acx+16c^3\right)\sqrt{ax+\sqrt{a^2x^2+b}}\,\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+}}}}{105a\sqrt{ax}}$$

$$-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

command

```
Integrate[Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]],x]
```

Mathematica 13.1 output

$$\frac{\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\left(-75b+60a^2x^2+2ax\left(-4c^2+30\sqrt{b+a^2x^2}+3c\sqrt{ax+\sqrt{b+a^2x^2}}\right)+2c\left(-4c\sqrt{b+a^2x^2}+8c^2\right.\right.}{\sqrt{ax+\sqrt{b+a^2x^2}}}$$

$$105a$$

Mathematica 12.3 output

$$\int \sqrt{ax+\sqrt{b+a^2x^2}}\,\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

24.596 Problem number 2829

$$\int x^4\sqrt{b+a^2x^4}\,\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{a}\;x\sqrt{a^2x^4+b}\left(192a^5x^{10}+264a^3b\,x^6+104a\,b^2x^2\right)\sqrt{a\,x^2+\sqrt{a^2x^4+b}}+\sqrt{a}\;x\left(192a^6x^{12}+360a^4b\,x^8+212\right.}{1152a^{\frac{7}{2}}b\,x^2+1536a^{\frac{11}{2}}x^6+384a^{\frac{5}{2}}b\sqrt{a^2x^4+b}+1536a^{\frac{9}{2}}x^4\sqrt{a^2x^4+b}}$$

$$-\frac{13b^2\ln\left(ia\,x^2+i\sqrt{a^2x^4+b}+i\sqrt{2}\;\sqrt{a}\;x\sqrt{a\,x^2+\sqrt{a^2x^4+b}}\right)\sqrt{2}}{256a^{\frac{5}{2}}}$$

command

```
Integrate[x^4*Sqrt[b + a^2*x^4]*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
```

Mathematica 13.1 output

$$\frac{2\sqrt{a}\,x\sqrt{ax^2+\sqrt{b+a^2x^4}}\left(39b^3+192a^5x^{10}\left(ax^2+\sqrt{b+a^2x^4}\right)+24a^3bx^6\left(15ax^2+11\sqrt{b+a^2x^4}\right)+4ab^2x^2\left(53ax^2+26\sqrt{b+a^2x^4}\right)\right.}{3abx^2+4a^3x^6+b\sqrt{b+a^2x^4}+4a^2x^4\sqrt{b+a^2x^4}}$$

$$768a^{5/2}$$

Mathematica 12.3 output

$$\int x^4\sqrt{b+a^2x^4}\,\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

24.597 Problem number 2830

$$\int \frac{\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}}{(b+a^2x^2)^{3/2}\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]]/((b + a^2*x^2)^(3/2)*Sqrt[a*x + Sqrt[b + a^
```

Mathematica 13.1 output

$$-\frac{4\left(ax+\sqrt{b+a^2x^2}\right)^{3/2}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}}{b+ax\left(ax+\sqrt{b+a^2x^2}\right)}+\text{RootSum}\left[b+c^4-4c^3\#1^2+6c^2\#1^4-4c\#1^6+\#1^8\&,-\right.$$

$$-\frac{}{4ab}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}}{(b+a^2x^2)^{3/2}\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

24.598 Problem number 2831

$$\int \frac{\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}}{(b+a^2x^2)^{3/2}\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]]/((b + a^2*x^2)^(3/2)*Sqrt[a*x + Sqrt[b + a^
```

Mathematica 13.1 output

$$-\frac{-\frac{4\left(ax+\sqrt{b+a^2x^2}\right)^{3/2}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}}{b+ax\left(ax+\sqrt{b+a^2x^2}\right)}+\text{RootSum}\left[b+c^4-4c^3\#1^2+6c^2\#1^4-4c\#1^6+\#1^8\&,-\right.}{4ab}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}}{(b+a^2x^2)^{3/2}\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

24.599 Problem number 2833

$$\int \frac{1+x^6}{\sqrt[3]{x^2+x^4}\,(-1+x^6)}\,dx$$

Optimal antiderivative

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2(x^4+x^2)^{\frac{1}{3}}}\right)}{3}-\frac{\arctan\left(\frac{\sqrt{3}\,x}{x+2(x^4+x^2)^{\frac{1}{3}}}\right)\sqrt{3}}{3}$$
$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2^{\frac{2}{3}}(x^4+x^2)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2^{\frac{2}{3}}(x^4+x^2)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{12}$$
$$-\frac{2\,\text{arctanh}\left(\frac{x}{(x^4+x^2)^{\frac{1}{3}}}\right)}{3}-\frac{\text{arctanh}\left(\frac{2^{\frac{1}{3}}x}{(x^4+x^2)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{6}$$
$$-\frac{\text{arctanh}\left(\frac{x^2+(x^4+x^2)^{\frac{2}{3}}}{x(x^4+x^2)^{\frac{1}{3}}}\right)}{3}-\frac{\text{arctanh}\left(\frac{2^{\frac{1}{3}}x^2+\frac{(x^4+x^2)^{\frac{2}{3}}2^{\frac{2}{3}}}{2}}{x(x^4+x^2)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{12}$$

command

```
Integrate[(1 + x^6)/((x^2 + x^4)^(1/3)*(-1 + x^6)),x]
```

Mathematica 13.1 output

$$-\frac{x^{2/3}\sqrt[3]{1+x^2}\left(-4\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}-2\sqrt[3]{1+x^2}}\right)+4\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}+2\sqrt[3]{1+x^2}}\right)+2^{2/3}\sqrt{3}\,\text{ArcTan}\left(\frac{}{-\sqrt[3]{x}}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{1+x^6}{\sqrt[3]{x^2+x^4}\,(-1+x^6)}\,dx$$

24.600 Problem number 2835

$$\int \frac{1}{(-bx+a^2x^2)^{5/2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{a^2x^2-bx}\,\left(121339a^{10}x^5-148243a^8b\,x^4+12416a^6b^2x^3+5248a^4b^3x^2+2688a^2b^4x-4368b^5\right)\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right.}}{16380b^7x^5\,(-a^2x+b)^2}$$

$$+\sqrt{x\left(ax+\sqrt{a^2x^2-bx}\right)}\left(-\frac{283847a^9x^4-229768a^7b\,x^3-24840a^5b^2x^2-9352a^3b^3x-4872a\,b^4}{8190b^7x^4\,(-a^2x+b)}\right.$$

$$\left.+\frac{109a^{\frac{15}{2}}\sqrt{-ax+\sqrt{a^2x^2-bx}}\,\arctan\left(\frac{\sqrt{a}\,\sqrt{-ax+\sqrt{a^2x^2-bx}}}{\sqrt{b}}\right)}{4b^{\frac{15}{2}}x}\right)$$

command

```
Integrate[1/((-(b*x) + a^2*x^2)^(5/2)*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt{x\left(ax+\sqrt{x\left(-b+a^2x\right)}\right)}\left(\sqrt{b}\left(-4368b^5+16a^5b^2x^2\left(776ax-3105\sqrt{x\left(-b+a^2x\right)}\right)+16a^3b^3x\left(328ax-116\right.\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{1}{\left(-bx+a^2x^2\right)^{5/2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

24.601 Problem number 2838

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(4+x^6\right)}{x^6\left(-4+x^6\right)}\,dx$$

Optimal antiderivative

$$\frac{\left(-x^3+1\right)\left(x^3-1\right)^{\frac{2}{3}}}{5x^5}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2\,2^{\frac{1}{3}}\left(x^3-1\right)^{\frac{1}{3}}}\right)2^{\frac{1}{3}}}{12}$$
$$+\frac{2^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{5}{6}}x}{3^{\frac{1}{3}}x+2\,2^{\frac{1}{3}}\left(x^3-1\right)^{\frac{1}{3}}}\right)}{4}+\frac{\ln\left(-x+2^{\frac{1}{3}}\left(x^3-1\right)^{\frac{1}{3}}\right)2^{\frac{1}{3}}}{12}$$
$$-\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(-3x+2^{\frac{1}{3}}3^{\frac{2}{3}}\left(x^3-1\right)^{\frac{1}{3}}\right)}{12}-\frac{\ln\left(x^2+2^{\frac{1}{3}}x\left(x^3-1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\left(x^3-1\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{24}$$
$$+\frac{\ln\left(3x^2+2^{\frac{1}{3}}3^{\frac{2}{3}}x\left(x^3-1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}3^{\frac{1}{3}}\left(x^3-1\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}3^{\frac{2}{3}}}{24}$$

command

```
Integrate[((-1 + x^3)^(2/3)*(4 + x^6))/(x^6*(-4 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{1}{120}\left(-\frac{24\left(-1+x^3\right)^{5/3}}{x^5}-10\sqrt[3]{2}\,\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x}{x+2\sqrt[3]{2}\,\sqrt[3]{-1+x^3}}\right)\right.$$
$$\left.+30\sqrt[3]{2}\,\sqrt[6]{3}\,\text{ArcTan}\left(\frac{3^{5/6}x}{\sqrt[3]{3}\,x+2\sqrt[3]{2}\,\sqrt[3]{-1+x^3}}\right)+10\sqrt[3]{2}\,\log\left(-x+\sqrt[3]{2}\,\sqrt[3]{-1+x^3}\right)-10\sqrt[3]{2}\,3^{2/3}\log\left(-3x+\sqrt[3]{2}\,3\right.\right.$$

Mathematica 12.3 output

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(4+x^6\right)}{x^6\left(-4+x^6\right)}\,dx$$

24.602 Problem number 2844

$$\int \frac{(-4a+b+3x)\left(-b^3+3b^2x-3bx^2+x^3\right)}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(b^4+ad-\left(4b^3+d\right)x+6b^2x^2-4bx^3+x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-a\,b^2+(2ab+b^2)x+(-a-2b)x^2+x^3\right)^{\frac{2}{3}}}{-2a\,d^{\frac{1}{3}}+2d^{\frac{1}{3}}x+\left(-a\,b^2+(2ab+b^2)x+(-a-2b)x^2+x^3\right)^{\frac{2}{3}}}\right)}{d^{\frac{1}{3}}}$$
$$+\frac{\ln\left(a\sqrt{d}\,-x\sqrt{d}\,+d^{\frac{1}{6}}\left(-a\,b^2+\left(2ab+b^2\right)x+(-a-2b)\,x^2+x^3\right)^{\frac{2}{3}}\right)}{d^{\frac{1}{3}}}$$
$$-\frac{\ln\left(a^2d-2adx+d\,x^2+\left(-a\,d^{\frac{2}{3}}+d^{\frac{2}{3}}x\right)\left(-a\,b^2+\left(2ab+b^2\right)x+(-a-2b)\,x^2+x^3\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\left(-a\,b^2+\left(2ab+b^2\right)x\right.\right.}{2d^{\frac{1}{3}}}$$

command

```
Integrate[((-4*a + b + 3*x)*(-b^3 + 3*b^2*x - 3*b*x^2 + x^3))/(((-a + x)*(-b + x)^2)^(2/3)*(b
```

Mathematica 13.1 output

$$-\frac{(a-x)^{2/3}(b-x)^{4/3}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a-x}}{(b-x)^{4/3}}}{\sqrt{3}}\right)+\log\left(\frac{(a-b)^2\left(d^{2/3}(a-x)^{2/3}-\sqrt[3]{d}\,\sqrt[3]{a-x}\,(b-x)^{4/3}+(b-x)^{8/3}\right.}{(b-x)^{8/3}}\right.\right.}{2\sqrt[3]{d}\,\left((b-x)^2(-a+x)\right)^{2/3}}$$

Mathematica 12.3 output

$$\int \frac{(-4a+b+3x)\left(-b^3+3b^2x-3bx^2+x^3\right)}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(b^4+ad-\left(4b^3+d\right)x+6b^2x^2-4bx^3+x^4\right)}\,dx$$

24.603 Problem number 2849

$$\int \frac{1-2x^4+x^8}{\sqrt[4]{-1+x^4}\,\left(1-2x^4+2x^8\right)}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{x}{(x^4-1)^{\frac{1}{4}}}\right)}{4}+\frac{\sqrt{4-2\sqrt{2}}\ \arctan\left(\frac{\sqrt{2+\sqrt{2}}\ x(x^4-1)^{\frac{1}{4}}}{-x^2+\sqrt{x^4-1}}\right)}{16}$$
$$-\frac{\sqrt{4+2\sqrt{2}}\ \arctan\left(\frac{\left(\frac{\sqrt{2}}{\sqrt{2-\sqrt{2}}}-\frac{2}{\sqrt{2-\sqrt{2}}}\right)x(x^4-1)^{\frac{1}{4}}}{-x^2+\sqrt{x^4-1}}\right)}{16}$$
$$+\frac{\operatorname{arctanh}\left(\frac{x}{(x^4-1)^{\frac{1}{4}}}\right)}{4}+\frac{\sqrt{4+2\sqrt{2}}\ \operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{2}}\ x(x^4-1)^{\frac{1}{4}}}{x^2+\sqrt{x^4-1}}\right)}{16}$$
$$+\frac{\sqrt{4-2\sqrt{2}}\ \operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{2}}\ x(x^4-1)^{\frac{1}{4}}}{x^2+\sqrt{x^4-1}}\right)}{16}$$

command

```
Integrate[(1 - 2*x^4 + x^8)/((-1 + x^4)^(1/4)*(1 - 2*x^4 + 2*x^8)),x]
```

Mathematica 13.1 output

$$\frac{1}{16}\left(4\operatorname{ArcTan}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)+\sqrt{2\left(2+\sqrt{2}\right)}\operatorname{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}\ x\sqrt[4]{-1+x^4}}{-x^2+\sqrt{-1+x^4}}\right)\right.$$
$$+\sqrt{4-2\sqrt{2}}\operatorname{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}\ x\sqrt[4]{-1+x^4}}{-x^2+\sqrt{-1+x^4}}\right)+4\tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)$$
$$+\sqrt{2\left(2+\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\ x\sqrt[4]{-1+x^4}}{x^2+\sqrt{-1+x^4}}\right)$$
$$\left.+\sqrt{4-2\sqrt{2}}\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\ x\sqrt[4]{-1+x^4}}{x^2+\sqrt{-1+x^4}}\right)\right)$$

Mathematica 12.3 output

$$\int\frac{1-2x^4+x^8}{\sqrt[4]{-1+x^4}\ \left(1-2x^4+2x^8\right)}\,dx$$

24.604 Problem number 2852

$$\int \frac{\sqrt{c+\sqrt{ax^2+x\sqrt{-b+a^2x^2}}}}{\sqrt{-b+a^2x^2}}\,dx$$

Optimal antiderivative

$$\frac{2\sqrt{c+\sqrt{x\left(ax+\sqrt{a^2x^2-b}\right)}}}{a}$$

$$-\frac{\sqrt{\sqrt{2}\ \sqrt{b}\ -2\sqrt{a}\ c}\ \left(-\sqrt{b}\ +\sqrt{2}\ \sqrt{a}\ c\right)\arctan\left(\frac{\sqrt{2}\ a^{\frac{1}{4}}\sqrt{c+\sqrt{x\left(ax+\sqrt{a^2x^2-b}\right)}}}{\sqrt{\sqrt{2}\ \sqrt{b}\ -2\sqrt{a}\ c}}\right)}{a^{\frac{5}{4}}\left(-\sqrt{2}\ \sqrt{b}\ +2\sqrt{a}\ c\right)}$$

$$-\frac{\left(\sqrt{b}\ +\sqrt{2}\ \sqrt{a}\ c\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\ a^{\frac{1}{4}}\sqrt{c+\sqrt{x\left(ax+\sqrt{a^2x^2-b}\right)}}}{\sqrt{\sqrt{2}\ \sqrt{b}\ +2\sqrt{a}\ c}}\right)}{a^{\frac{5}{4}}\sqrt{\sqrt{2}\ \sqrt{b}\ +2\sqrt{a}\ c}}$$

command

```
Integrate[Sqrt[c + Sqrt[a*x^2 + x*Sqrt[-b + a^2*x^2]]]/Sqrt[-b + a^2*x^2],x]
```

Mathematica 13.1 output

$$4\sqrt{a}\ \sqrt{c+\sqrt{x\left(ax+\sqrt{-b+a^2x^2}\right)}}\ -\sqrt{2\sqrt{2}\ \sqrt{a}\ \sqrt{b}\ -4ac}\ \operatorname{ArcTan}\left(\frac{\sqrt{2\sqrt{2}\ \sqrt{a}\ \sqrt{b}\ -4ac}\ \sqrt{c+\sqrt{x\left(ax-\right.}}{\sqrt{2}\ \sqrt{b}\ -2\sqrt{a}\ c}\right.$$

Mathematica 12.3 output

$$\int \frac{\sqrt{c+\sqrt{ax^2+x\sqrt{-b+a^2x^2}}}}{\sqrt{-b+a^2x^2}}\,dx$$

24.605 Problem number 2855

$$\int \frac{1+x^6}{\sqrt[4]{x^3+x^5}\,(1-x^6)}\,dx$$

Optimal antiderivative

$$\frac{2\arctan\left(\frac{x}{(x^5+x^3)^{\frac{1}{4}}}\right)}{3}+\frac{\arctan\left(\frac{2^{\frac{1}{4}}x}{(x^5+x^3)^{\frac{1}{4}}}\right)2^{\frac{3}{4}}}{6}-\frac{\arctan\left(\frac{2^{\frac{3}{4}}x(x^5+x^3)^{\frac{1}{4}}}{x^2\sqrt{2}-\sqrt{x^5+x^3}}\right)2^{\frac{1}{4}}}{6}$$
$$+\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ x(x^5+x^3)^{\frac{1}{4}}}{-x^2+\sqrt{x^5+x^3}}\right)}{3}+\frac{2\,\mathrm{arctanh}\left(\frac{x}{(x^5+x^3)^{\frac{1}{4}}}\right)}{3}+\frac{\mathrm{arctanh}\left(\frac{2^{\frac{1}{4}}x}{(x^5+x^3)^{\frac{1}{4}}}\right)2^{\frac{3}{4}}}{6}$$
$$+\frac{\mathrm{arctanh}\left(\frac{\frac{x^2 2^{\frac{3}{4}}}{2}+\frac{\sqrt{x^5+x^3}\ 2^{\frac{1}{4}}}{2}}{x(x^5+x^3)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{6}+\frac{\sqrt{2}\ \mathrm{arctanh}\left(\frac{\frac{x^2\sqrt{2}}{2}+\frac{\sqrt{x^5+x^3}\ \sqrt{2}}{2}}{x(x^5+x^3)^{\frac{1}{4}}}\right)}{3}$$

command

```
Integrate[(1 + x^6)/((x^3 + x^5)^(1/4)*(1 - x^6)),x]
```

Mathematica 13.1 output

$$\frac{x^{3/4}\sqrt[4]{1+x^2}\left(4\mathrm{ArcTan}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{1+x^2}}\right)+2^{3/4}\mathrm{ArcTan}\left(\frac{\sqrt[4]{2}\ \sqrt[4]{x}}{\sqrt[4]{1+x^2}}\right)-\sqrt[4]{2}\,\mathrm{ArcTan}\left(\frac{2^{3/4}\sqrt[4]{x}\ \sqrt[4]{1+x^2}}{\sqrt{2}\ \sqrt{x}-\sqrt{1+x^2}}\right)+2\sqrt{2}\,\mathrm{A}\right.}{}$$

Mathematica 12.3 output

$$\int \frac{1+x^6}{\sqrt[4]{x^3+x^5}\,(1-x^6)}\,dx$$

24.606 Problem number 2869

$$\int \frac{x}{x^2-\sqrt{b+ax}\,\sqrt{c+\sqrt{b+ax}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[x/(x^2 - Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]]),x]
```

Mathematica 13.1 output

$$-4\text{RootSum}\left[b^2-2bc^2+c^4+a^2c\#1+4bc\#1^2-4c^3\#1^2-a^2\#1^3-2b\#1^4+6c^2\#1^4-4c\#1^6\right.$$
$$\left.+\#1^8\&,\frac{-bc\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1+c^3\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1+b\log\left(\sqrt{c+\sqrt{b+ax}}-\#\right.}{a^2c+8bc\#1-8c^3\#1-3c}\right]$$

Mathematica 12.3 output

$$\int\frac{x}{x^2-\sqrt{b+ax}\sqrt{c+\sqrt{b+ax}}}\,dx$$

24.607 Problem number 2870

$$\int\frac{x}{x^2-\sqrt{b+ax}\sqrt{c+\sqrt{b+ax}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[x/(x^2 - Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]]),x]
```

Mathematica 13.1 output

$$-4\text{RootSum}\left[b^2-2bc^2+c^4+a^2c\#1+4bc\#1^2-4c^3\#1^2-a^2\#1^3-2b\#1^4+6c^2\#1^4-4c\#1^6\right.$$
$$\left.+\#1^8\&,\frac{-bc\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1+c^3\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)\#1+b\log\left(\sqrt{c+\sqrt{b+ax}}-\#\right.}{a^2c+8bc\#1-8c^3\#1-3c}\right]$$

Mathematica 12.3 output

$$\int\frac{x}{x^2-\sqrt{b+ax}\sqrt{c+\sqrt{b+ax}}}\,dx$$

24.608 Problem number 2875

$$\int \frac{1+x}{(1+2x)\sqrt[3]{27+27x+36x^2+28x^3+9x^4+x^5}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{5\sqrt{3}\,\left(x^5+9x^4+28x^3+36x^2+27x+27\right)^{\frac{1}{3}}}{12\,10^{\frac{1}{3}}-2\,10^{\frac{1}{3}}x-2\,10^{\frac{1}{3}}x^2+5\left(x^5+9x^4+28x^3+36x^2+27x+27\right)^{\frac{1}{3}}}\right)10^{\frac{2}{3}}}{50}$$
$$+\frac{\ln\left(-6\,10^{\frac{1}{3}}+10^{\frac{1}{3}}x+10^{\frac{1}{3}}x^2+5\left(x^5+9x^4+28x^3+36x^2+27x+27\right)^{\frac{1}{3}}\right)10^{\frac{2}{3}}}{50}$$
$$-\frac{\ln\left(36\,10^{\frac{2}{3}}-12\,10^{\frac{2}{3}}x-11\,10^{\frac{2}{3}}x^2+2\,10^{\frac{2}{3}}x^3+10^{\frac{2}{3}}x^4+\left(30\,10^{\frac{1}{3}}-5\,10^{\frac{1}{3}}x-5\,10^{\frac{1}{3}}x^2\right)\left(x^5+9x^4+28x^3+36x^2+\right.\right.}{100}$$

command

```
Integrate[(1 + x)/((1 + 2*x)*(27 + 27*x + 36*x^2 + 28*x^3 + 9*x^4 + x^5)^(1/3)),x]
```

Mathematica 13.1 output

$$-\frac{(3+x)\sqrt[3]{1+x^2}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{4\sqrt[3]{10}-2\sqrt[3]{10}\,x+5\sqrt[3]{1+x^2}}{5\sqrt{3}\,\sqrt[3]{1+x^2}}\right)-2\log\left(-2\sqrt[3]{10}+\sqrt[3]{10}\,x+5\sqrt[3]{1+x^2}\right)+\log\left(\right.\right.}{10\sqrt[3]{10}\,\sqrt[3]{(3+x)^3\left(1+x^2\right)}}$$

Mathematica 12.3 output

$$\int \frac{1+x}{(1+2x)\sqrt[3]{27+27x+36x^2+28x^3+9x^4+x^5}}\,dx$$

24.609 Problem number 2883

$$\int \frac{\sqrt{1+x^4}\,\sqrt{x^2+\sqrt{1+x^4}}}{-1+x^4}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{-2+2\sqrt{2}}\ \arctan\left(\frac{\sqrt{-2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{2}$$
$$-\frac{\sqrt{-2+2\sqrt{2}}\ \arctan\left(\frac{\sqrt{2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{2}$$
$$+\operatorname{arctanh}\left(\frac{\sqrt{2}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}$$
$$-\frac{\sqrt{2+2\sqrt{2}}\ \operatorname{arctanh}\left(\frac{\sqrt{-2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{2}$$
$$-\frac{\sqrt{2+2\sqrt{2}}\ \operatorname{arctanh}\left(\frac{\sqrt{2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{2}$$

command

```
Integrate[(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]])/(-1 + x^4),x]
```

Mathematica 13.1 output

$$\sqrt{-1+\sqrt{2}}\ \operatorname{ArcTan}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\left(-1+x^2+\sqrt{1+x^4}\right)}{x\sqrt{x^2+\sqrt{1+x^4}}}\right)-\sqrt{-1+\sqrt{2}}\ \operatorname{ArcTan}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}\ x\sqrt{x^2+\sqrt{1}}}\right.$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1+x^4}\ \sqrt{x^2+\sqrt{1+x^4}}}{-1+x^4}\,dx$$

24.610 Problem number 2885

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$$

Optimal antiderivative

$$\frac{1+x^2+2x^4+x\left(-x^2-1\right)\left(x^2+\sqrt{x^4+1}\right)+\sqrt{x^4+1}\left(1+2x^2-x\left(x^2+\sqrt{x^4+1}\right)\right)}{2\left(x^2-1\right)\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}$$
$$+\frac{\arctan\left(\frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{\sqrt{2}-1}}\right)}{2\sqrt{\sqrt{2}-1}}-\frac{\sqrt{1+\sqrt{2}}\ \arctan\left(\frac{\sqrt{-2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{2}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{1+\sqrt{2}}}\right)}{2\sqrt{1+\sqrt{2}}}$$
$$+\frac{\sqrt{\sqrt{2}-1}\ \operatorname{arctanh}\left(\frac{\sqrt{2+2\sqrt{2}}\ x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{2}$$

command

```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

Mathematica 13.1 output

$$\frac{1}{2}\left(\frac{-1-2x^4-\sqrt{1+x^4}-x^2\left(1+2\sqrt{1+x^4}\right)}{(1+x)\left(x^2+\sqrt{1+x^4}\right)^{3/2}}\right.$$
$$+\frac{\text{ArcTan}\left(\sqrt{1+\sqrt{2}}\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{-1+\sqrt{2}}}$$
$$-\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)$$
$$-\frac{\tanh^{-1}\left(\sqrt{-1+\sqrt{2}}\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{1+\sqrt{2}}}$$
$$\left.+\sqrt{-1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$$

24.611 Problem number 2892

$$\int \frac{1}{\sqrt{-1+2x}\,(4+3x)+(1+x)\sqrt{-3+4x}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(Sqrt[-1 + 2*x]*(4 + 3*x) + (1 + x)*Sqrt[-3 + 4*x])^(-1),x]
```

Mathematica 13.1 output

$$\text{RootSum}\left[9+109\#1^2+55\#1^4\right.$$
$$\left.+7\#1^6\&,\frac{11\log\left(\sqrt{-1+2x}-\#1\right)\#1+3\log\left(\sqrt{-1+2x}-\#1\right)\#1^3}{109+110\#1^2+21\#1^4}\&\right]-2\text{RootSum}\left[625\right.$$
$$\left.+677\#1^2+131\#1^4+7\#1^6\&,\frac{7\log\left(\sqrt{-3+4x}-\#1\right)\#1+\log\left(\sqrt{-3+4x}-\#1\right)\#1^3}{677+262\#1^2+21\#1^4}\&\right]$$

Mathematica 12.3 output

$$\int \frac{1}{\sqrt{-1+2x}\,(4+3x)+(1+x)\sqrt{-3+4x}}\,dx$$

24.612 Problem number 2894

$$\int (1+x^2)^{3/2}\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

$$\frac{\left(66913566720x^8-2099249152x^7+248171986944x^6-11794907136x^5+407581982720x^4-1415707308x^3+2203\right.}{}$$

$$-\frac{545\operatorname{arctanh}\left(\sqrt{1+\sqrt{x+\sqrt{x^2+1}}}\right)}{8192}$$

command

```
Integrate[(1 + x^2)^(3/2)*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]],x]
```

Mathematica 13.1 output

$$\frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\left(15903121112+5227043711x+220397520304x^2-1415707308x^3+407581982720x^4-1\right.}{}$$

$$-\frac{545\tanh^{-1}\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\right)}{8192}$$

Mathematica 12.3 output

$$\int (1+x^2)^{3/2}\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.613 Problem number 2896

$$\int \frac{\left(2-2x+2x^2-3x^3+3x^4\right)\sqrt[3]{-x-x^3-x^4+x^6}}{(1+x)\left(-1+2x-2x^2+x^3\right)\left(-1-x^3+x^5\right)}\,dx$$

Optimal antiderivative

$$\begin{aligned}
&\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ \left(x^6-x^4-x^3-x\right)^{\frac{1}{3}}}{-2x+\left(x^6-x^4-x^3-x\right)^{\frac{1}{3}}}\right)\\
&-2^{\frac{1}{3}}\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ \left(x^6-x^4-x^3-x\right)^{\frac{1}{3}}}{-2\,2^{\frac{1}{3}}x+\left(x^6-x^4-x^3-x\right)^{\frac{1}{3}}}\right)\\
&-\ln\left(x+\left(x^6-x^4-x^3-x\right)^{\frac{1}{3}}\right)+2^{\frac{1}{3}}\ln\left(2^{\frac{1}{3}}x+\left(x^6-x^4-x^3-x\right)^{\frac{1}{3}}\right)\\
&+\frac{\ln\left(x^2-x\left(x^6-x^4-x^3-x\right)^{\frac{1}{3}}+\left(x^6-x^4-x^3-x\right)^{\frac{2}{3}}\right)}{2}\\
&-\frac{\ln\left(2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}x\left(x^6-x^4-x^3-x\right)^{\frac{1}{3}}+\left(x^6-x^4-x^3-x\right)^{\frac{2}{3}}\right)2^{\frac{1}{3}}}{2}
\end{aligned}$$

command

```
Integrate[((2 - 2*x + 2*x^2 - 3*x^3 + 3*x^4)*(-x - x^3 - x^4 + x^6)^(1/3))/((1 + x)*(-
1 + 2*x - 2*x^2 + x^3)*(-1 - x^3 + x^5)),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\left(-1-x^2-x^3+x^5\right)^{2/3}\left(-2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}-2\sqrt[3]{-1-x^2-x^3+x^5}}\right)+2\sqrt[3]{2}\,\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}-2^{2/3}\sqrt[3]{-1-x^2-x^3+x^5}}\right)\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\left(2-2x+2x^2-3x^3+3x^4\right)\sqrt[3]{-x-x^3-x^4+x^6}}{(1+x)\left(-1+2x-2x^2+x^3\right)\left(-1-x^3+x^5\right)}\,dx$$

24.614 Problem number 2901

$$\int \frac{\left(1+x^2\right)^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}\ \left(-1+x^2+x^4\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^2)^2*Sqrt[x^2 + Sqrt[1 + x^4]])/(Sqrt[1 + x^4]*(-1 + x^2 + x^4)),x]
```

Mathematica 13.1 output

$$2\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)+\text{RootSum}\left[1-2\#1^2-6\#1^4+2\#1^6+\#1^8\&,\frac{\log\left(1+x^2+\sqrt{1+x^4}\right)-\log\left(\sqrt{}\right.}{}\right.$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2\right)^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}\,\left(-1+x^2+x^4\right)}\,dx$$

24.615 Problem number 2902

$$\int \frac{\left(1+x^2\right)^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}\,\left(-1+x^2+x^4\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^2)^2*Sqrt[x^2 + Sqrt[1 + x^4]])/(Sqrt[1 + x^4]*(-1 + x^2 + x^4)),x]
```

Mathematica 13.1 output

$$2\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)+\text{RootSum}\left[1-2\#1^2-6\#1^4+2\#1^6+\#1^8\&,\frac{\log\left(1+x^2+\sqrt{1+x^4}\right)-\log\left(\sqrt{}\right.}{}\right.$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2\right)^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}\,\left(-1+x^2+x^4\right)}\,dx$$

24.616 Problem number 2905

$$\int \frac{\left(1+x^2\right)\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^2\right)^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^2)*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/(1 - x^2)^2,x
```

Mathematica 13.1 output

$$\frac{1}{8}\left(-\frac{8x\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{-1+x^2}+4\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.\right.$$

$$\left.\left.+\#1^8\&,\frac{3\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)-2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2+\log\left(\sqrt{1+\sqrt{x+}\right.\right.\right.$$

$$\left.\left.\left.2\#1-3\#1^3+\#1^5\right.\right.\right]$$

$$-\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+7\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2-\log\left(\sqrt{1+\sqrt{x+}\right.\right.}{2\#1^3-3\#1^5+\#1^7}\right]$$

$$+4\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{-\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1-2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^3+\log\left(\sqrt{1+}\right.}{-2+4\#1^2-3\#1^4+\#1^6}\right]$$

$$-\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)-9\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2-\log\left(\sqrt{1+\sqrt{x+}\right.\right.}{-2\#1+4\#1^3-3\#1^5+\#1^7}\right]$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2\right)\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^2\right)^2}\,dx$$

24.617 Problem number 2906

$$\int \frac{\left(1+x^2\right)\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^2\right)^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^2)*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/(1 - x^2)^2,x
```

Mathematica 13.1 output

$$\frac{1}{8}\left(-\frac{8x\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{-1+x^2}+4\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.\right.$$

$$\left.\left.+\#1^8\&,\frac{3\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)-2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2+\log\left(\sqrt{1+\sqrt{x+}\right.\right.}{2\#1-3\#1^3+\#1^5}\right]$$

$$-\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+7\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2-\log\left(\sqrt{1+\sqrt{x+}\right.}{2\#1^3-3\#1^5+\#1^7}\right]$$

$$+4\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{-\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1-2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^3+\log\left(\sqrt{1+}\right.}{-2+4\#1^2-3\#1^4+\#1^6}\right]$$

$$-\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)-9\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2-\log\left(\sqrt{1+\sqrt{x+}\right.}{-2\#1+4\#1^3-3\#1^5+\#1^7}\right]$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2\right)\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^2\right)^2}\,dx$$

24.618 Problem number 2912

$$\int \frac{x^3}{\sqrt[3]{-x^2+x^4}\,\left(1+x^6\right)}\,dx$$

Optimal antiderivative

$$\begin{aligned}
&\frac{\arctan\left(\frac{\sqrt{3}\,x^2}{-x^2+2^{\frac{1}{3}}\left(x^4-x^2\right)^{\frac{2}{3}}}\right)2^{\frac{2}{3}}\sqrt{3}}{24}+\frac{\ln\left(x^2+\left(x^4-x^2\right)^{\frac{2}{3}}\right)}{6}\\
&-\frac{i\left(-i+\sqrt{3}\right)\ln\left(-x^2-i\sqrt{3}\,x^2+2\left(x^4-x^2\right)^{\frac{2}{3}}\right)}{12}\\
&+\frac{i\left(\sqrt{3}+i\right)\ln\left(-x^2+i\sqrt{3}\,x^2+2\left(x^4-x^2\right)^{\frac{2}{3}}\right)}{12}\\
&+\frac{\ln\left(-2x^2+2^{\frac{2}{3}}\sqrt{3}\,x\left(x^4-x^2\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\left(x^4-x^2\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{48}\\
&-\frac{\ln\left(2x^2+2^{\frac{1}{3}}\left(x^4-x^2\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{24}+\frac{\ln\left(2x^2+2^{\frac{2}{3}}\sqrt{3}\,x\left(x^4-x^2\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(x^4-x^2\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{48}
\end{aligned}$$

command

```
Integrate[x^3/((-x^2 + x^4)^(1/3)*(1 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\sqrt[3]{-1+x^2}\left(8\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}-2\left(-1+x^2\right)^{2/3}}\right)-2\,2^{2/3}\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}-\sqrt[3]{2}\,\left(-1+x^2\right)^{2/3}}\right)+8\log\left(x^{2/3}+\left(-1+\right.\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{x^3}{\sqrt[3]{-x^2+x^4}\,\left(1+x^6\right)}\,dx$$

24.619 Problem number 2913

$$\int \frac{x^4 \sqrt{b+a^2x^2}}{x^2-\sqrt{ax-\sqrt{b+a^2x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x^4*Sqrt[b + a^2*x^2])/(x^2 - Sqrt[a*x - Sqrt[b + a^2*x^2]]),x]
```

Mathematica 13.1 output

$$-a^2x+a\sqrt{b+a^2x^2}+\frac{b^2}{6a\left(ax-\sqrt{b+a^2x^2}\right)^{3/2}}-\frac{b\sqrt{ax-\sqrt{b+a^2x^2}}}{a}-\frac{\left(ax-\sqrt{b+a^2x^2}\right)^{5/2}}{10a}$$
$$+\frac{b^4}{64a^3\left(-ax+\sqrt{b+a^2x^2}\right)^4}-\frac{\left(-ax+\sqrt{b+a^2x^2}\right)^4}{64a^3}+\frac{b^2\log\left(-ax+\sqrt{b+a^2x^2}\right)}{8a^3}$$
$$+2a\text{RootSum}\left[b^2-2b\#1^4-4a^2\#1^5+\#1^8\&,\frac{b\log\left(\sqrt{ax-\sqrt{b+a^2x^2}}-\#1\right)\#1^2+a^2\log\left(\sqrt{ax-\sqrt{b+a^2x^2}}\right.\right.}{2b+5a^2\#1-2\#1^4}$$

Mathematica 12.3 output

$Aborted

24.620 Problem number 2919

$$\int \frac{1}{(b+ax)\sqrt[3]{-b^3+a^3x^3}}\,dx$$

Optimal antiderivative

$$\frac{(-1)^{\frac{5}{6}}\sqrt{3}\ \text{arctanh}\left(\frac{\frac{\left(ib+b\sqrt{3}\right)2^{\frac{1}{3}}\sqrt{3}}{6}+\frac{\left(-ia-\sqrt{3}\,a\right)x2^{\frac{1}{3}}\sqrt{3}}{6}+\frac{i\left(a^3x^3-b^3\right)^{\frac{1}{3}}\sqrt{3}}{3}}{(a^3x^3-b^3)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{4ab}$$
$$+\frac{(-1)^{\frac{1}{3}}2^{\frac{2}{3}}\ln\left(-(-1)^{\frac{2}{3}}\sqrt{a}\ b^{\frac{3}{2}}+(-1)^{\frac{2}{3}}a^{\frac{3}{2}}\sqrt{b}\ x-2^{\frac{2}{3}}\sqrt{a}\ \sqrt{b}\ \left(a^3x^3-b^3\right)^{\frac{1}{3}}\right)}{4ab}$$
$$-\frac{(-1)^{\frac{1}{3}}2^{\frac{2}{3}}\ln\left((-1)^{\frac{1}{3}}ab^3-2(-1)^{\frac{1}{3}}a^2b^2x+(-1)^{\frac{1}{3}}a^3bx^2+\left((-2)^{\frac{2}{3}}ab^2-(-2)^{\frac{2}{3}}a^2bx\right)\left(a^3x^3-b^3\right)^{\frac{1}{3}}-2\,2^{\frac{1}{3}}ab(a^3x\right.}{8ab}$$

command

```
Integrate[1/((b + a*x)*(-b^3 + a^3*x^3)^(1/3)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{-\frac{1}{2}}\left(2i\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[3]{2}\left(3+i\sqrt{3}\right)b+\sqrt[3]{2}\left(-3-i\sqrt{3}\right)ax+2i\sqrt{3}\sqrt[3]{-b^3+a^3x^3}}{6\sqrt[3]{-b^3+a^3x^3}}\right)+2\log\left(\sqrt{a}\sqrt{b}\left(-b+i\sqrt{3}b+\right.\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{1}{(b+ax)\sqrt[3]{-b^3+a^3x^3}}\,dx$$

24.621 Problem number 2934

$$\int \frac{(-1+(-1+2k)x)\left(1-2x+x^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+4bx+(1-6b)x^2+(4b-2k)x^3+(-b+k^2)x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt{3}}{3}-\frac{2x\sqrt{3}}{3}+\frac{\sqrt{3}}{3}x^2+\frac{2\left(x+(-1-k)x^2+kx^3\right)^{\frac{2}{3}}\sqrt{3}}{3b^{\frac{1}{3}}}}{(-1+x)^2}\right)}{2b^{\frac{2}{3}}}$$
$$-\frac{\ln\left(b^{\frac{1}{6}}-b^{\frac{1}{6}}x+\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}\right)}{2b^{\frac{2}{3}}}-\frac{\ln\left(-b^{\frac{1}{6}}+b^{\frac{1}{6}}x+\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}\right)}{2b^{\frac{2}{3}}}$$
$$+\frac{\ln\left(b^{\frac{1}{3}}-2b^{\frac{1}{3}}x+b^{\frac{1}{3}}x^2+\left(b^{\frac{1}{6}}-b^{\frac{1}{6}}x\right)\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}+\left(x+(-1-k)x^2+kx^3\right)^{\frac{2}{3}}\right)}{4b^{\frac{2}{3}}}$$
$$+\frac{\ln\left(b^{\frac{1}{3}}-2b^{\frac{1}{3}}x+b^{\frac{1}{3}}x^2+\left(-b^{\frac{1}{6}}+b^{\frac{1}{6}}x\right)\left(x+(-1-k)x^2+kx^3\right)^{\frac{1}{3}}+\left(x+(-1-k)x^2+kx^3\right)^{\frac{2}{3}}\right)}{4b^{\frac{2}{3}}}$$

command

```
Integrate[((-1 + (-1 + 2*k)*x)*(1 - 2*x + x^2))/(((1 - x)*x*(1 - k*x))^(1/3)*(-b + 4*b*x + (1
b + k^2)*x^4)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[3]{k-\frac{1}{x}}(-1+x)\left(-2\sqrt{3}\,\text{ArcTan}\left(\frac{\left(k-\frac{1}{x}\right)^{2/3}+2\sqrt[3]{b}\left(-1+\frac{1}{x}\right)^{4/3}}{\sqrt{3}\left(k-\frac{1}{x}\right)^{2/3}}\right)-2\log\left(\sqrt[3]{k-\frac{1}{x}}-\sqrt[6]{b}\left(-1+\frac{1}{x}\right)^{2/3}\right)-2\log\left(\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{(-1+(-1+2k)x)\left(1-2x+x^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+4bx+(1-6b)x^2+(4b-2k)x^3+\left(-b+k^2\right)x^4\right)}\,dx$$

24.622 Problem number 2940

$$\int \frac{\left(1+x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}\left(-1+x^2+x^4\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/(Sqrt[1 + x^4]*(-1 + x^2 + x^4)),x]
```

Mathematica 13.1 output

$$\text{RootSum}\left[1-2\#1^2-6\#1^4+2\#1^6+\#1^8\&,\frac{\log\left(1+x^2+\sqrt{1+x^4}\right)-\log\left(\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}-\#1-x^2\#1-\sqrt{1+\right.\right.}$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}\left(-1+x^2+x^4\right)}\,dx$$

24.623 Problem number 2941

$$\int \frac{\left(1+x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}\left(-1+x^2+x^4\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/(Sqrt[1 + x^4]*(-1 + x^2 + x^4)),x]
```

Mathematica 13.1 output

$$\text{RootSum}\left[1-2\#1^2-6\#1^4+2\#1^6+\#1^8\&,\frac{\log\left(1+x^2+\sqrt{1+x^4}\right)-\log\left(\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}-\#1-x^2\#1-\sqrt{1+\right.\right.}}{}$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}\,\left(-1+x^2+x^4\right)}\,dx$$

24.624 Problem number 2942

$$\int \frac{1-x^4}{\left(1+x^2+x^4\right)\sqrt[4]{-x^3+x^5}}\,dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{3^{\frac{7}{8}}\sqrt{2-\sqrt{2}}\;x\left(x^5-x^3\right)^{\frac{1}{4}}}{-3x^2+3^{\frac{3}{4}}\sqrt{x^5-x^3}}\right)2^{\frac{1}{4}}}{2\left(51+36\sqrt{2}\right)^{\frac{1}{8}}}+\frac{\left(\frac{17}{3}+4\sqrt{2}\right)^{\frac{1}{8}}\arctan\left(\frac{3^{\frac{7}{8}}\sqrt{2+\sqrt{2}}\;x\left(x^5-x^3\right)^{\frac{1}{4}}}{-3x^2+3^{\frac{3}{4}}\sqrt{x^5-x^3}}\right)2^{\frac{1}{4}}}{2}+\frac{\operatorname{arctanh}\left(\frac{\frac{3^{\frac{1}{8}}x^2}{\sqrt{2-\sqrt{2}}}+\frac{\sqrt{x^5-x^3}\;3^{\frac{7}{8}}}{\sqrt[3]{2-\sqrt{2}}}}{x\left(x^5-x^3\right)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{2\left(51+36\sqrt{2}\right)^{\frac{1}{8}}}+\frac{\left(\frac{17}{3}+4\sqrt{2}\right)^{\frac{1}{8}}\operatorname{arctanh}\left(\frac{\frac{3^{\frac{1}{8}}x^2}{\sqrt{2+\sqrt{2}}}+\frac{\sqrt{x^5-x^3}\;3^{\frac{7}{8}}}{\sqrt[3]{2+\sqrt{2}}}}{x\left(x^5-x^3\right)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{2}$$

command

```
Integrate[(1 - x^4)/((1 + x^2 + x^4)*(-x^3 + x^5)^(1/4)),x]
```

Mathematica 13.1 output

$$x^{3/4}\sqrt[4]{-1+x^2}\left(\text{ArcTan}\left(\frac{\sqrt[4]{2}\,3^{7/8}\sqrt[4]{x}\,\sqrt[4]{-1+x^2}}{\sqrt[8]{17+12\sqrt{2}}\left(-3\sqrt{x}+3^{3/4}\sqrt{-1+x^2}\right)}\right)+\sqrt[4]{17+12\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt[4]{2}\,3^{7/8}\sqrt[8]{17+1}}{-3\sqrt{x}+}\right.\right.$$

Mathematica 12.3 output

$$\int \frac{1-x^4}{\left(1+x^2+x^4\right)\sqrt[4]{-x^3+x^5}}\,dx$$

24.625 Problem number 2947

$$\int \frac{\left(-q+2px^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^6+a\left(q+px^3\right)^6\right)}{x^9}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}\left(6a\,p^7x^{21}+42a\,p^6q\,x^{18}-2a\,p^6q\,x^{17}+126a\,p^5q^2x^{15}-10a\,p^5q^2x^{14}-5a\,p^5q^2x^{13}+2\right.}{}$$

$$+\frac{\left(5a\,p^4q^4+8bpq\right)\ln\left(x\right)}{8}+\frac{\left(-5a\,p^4q^4-8bpq\right)\ln\left(q+p\,x^3+\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}\right)}{8}$$

command

```
Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*x^6 + a*(q + p*x^3)^
```

Mathematica 13.1 output

$$\frac{1}{48}\left(\frac{\left(q+px^3\right)\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}\left(24bx^6+a\left(6q^6+6p^6x^{18}+2pq^5x^2(-1+18x)+2p^5qx^{14}(-1+18x)+\right.\right.\right.}{x^8}$$

$$\left.\left.\left.-6pq\left(8b+5ap^3q^3\right)\tanh^{-1}\left(\frac{\sqrt{q^2+2pq(-1+x)x^2+p^2x^6}}{q+px^3}\right)\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-q+2px^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^6+a\left(q+px^3\right)^6\right)}{x^9}\,dx$$

24.626 Problem number 2948

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(bx^{12}+a\left(q+px^3\right)^6\right)}{x^{17}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}\left(6a\,p^7x^{21}-2a\,p^6q\,x^{19}+42a\,p^6q\,x^{18}-5a\,p^5q^2x^{17}-10a\,p^5q^2x^{16}+126a\,p^5q^2x^{15}-1\right.}{}$$

$$+\frac{\left(5a\,p^4q^4+8bpq\right)\ln\left(x\right)}{4}+\frac{\left(-5a\,p^4q^4-8bpq\right)\ln\left(q+p\,x^3+\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}\right)}{8}$$

command

```
Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(b*x^12 + a*(q + p*x^3)
```

Mathematica 13.1 output

$$\frac{1}{48}\left(\frac{\left(q+px^3\right)\sqrt{q^2-2pq(-1+x)x^3+p^2x^6}\left(24bx^{12}+a\left(6q^6-2pq^5(-18+x)x^3-2p^5q(-18+x)x^{15}+6p^6x^{18}+\right.\right.}{x^{16}}\right.$$

$$\left.\left.-6pq\left(8b+5ap^3q^3\right)\tanh^{-1}\left(\frac{\sqrt{q^2-2pq(-1+x)x^3+p^2x^6}}{q+px^3}\right)\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(bx^{12}+a\left(q+px^3\right)^6\right)}{x^{17}}\,dx$$

24.627 Problem number 2953

$$\int \frac{x^3}{\sqrt[3]{x^2+x^4}\left(-1+x^6\right)}\,dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{3}\,x^2}{x^2+2\left(x^4+x^2\right)^{\frac{2}{3}}}\right)\sqrt{3}}{6}+\frac{\arctan\left(\frac{\sqrt{3}\,x^2}{x^2+2^{\frac{1}{3}}\left(x^4+x^2\right)^{\frac{2}{3}}}\right)2^{\frac{2}{3}}\sqrt{3}}{24}$$

$$-\frac{\ln\left(-x+\left(x^4+x^2\right)^{\frac{1}{3}}\right)}{6}-\frac{\ln\left(x+\left(x^4+x^2\right)^{\frac{1}{3}}\right)}{6}+\frac{\ln\left(-2x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{24}$$

$$+\frac{\ln\left(2x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{24}+\frac{\ln\left(x^2-x\left(x^4+x^2\right)^{\frac{1}{3}}+\left(x^4+x^2\right)^{\frac{2}{3}}\right)}{12}$$

$$+\frac{\ln\left(x^2+x\left(x^4+x^2\right)^{\frac{1}{3}}+\left(x^4+x^2\right)^{\frac{2}{3}}\right)}{12}-\frac{\ln\left(-2x^2+2^{\frac{2}{3}}x\left(x^4+x^2\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\left(x^4+x^2\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{48}$$

$$-\frac{\ln\left(2x^2+2^{\frac{2}{3}}x\left(x^4+x^2\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(x^4+x^2\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{48}$$

command

```
Integrate[x^3/((x^2 + x^4)^(1/3)*(-1 + x^6)),x]
```

Mathematica 13.1 output

$$-\frac{x^{2/3}\sqrt[3]{1+x^2}\left(8\sqrt{3}\,\mathrm{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}+2(1+x^2)^{2/3}}\right)-2\,2^{2/3}\sqrt{3}\,\mathrm{ArcTan}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}+\sqrt[3]{2}\,(1+x^2)^{2/3}}\right)+8\log\left(-\sqrt[3]{x}+\sqrt[3]{1+\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{x^3}{\sqrt[3]{x^2+x^4}\,(-1+x^6)}\,dx$$

24.628 Problem number 2960

$$\int \frac{(1+x^4)^2}{(-1+x^4)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Optimal antiderivative

$$\frac{x(x^4-3)}{2(x^4-1)\sqrt{x^2+\sqrt{x^4+1}}}+\frac{\sqrt{14+10\sqrt{2}}\,\arctan\left(\frac{\sqrt{-2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{4}-\frac{\sqrt{14+10\sqrt{2}}\,\arctan\left(\frac{\sqrt{2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{4}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)\sqrt{2}}{2}-\frac{\sqrt{-14+10\sqrt{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{-2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{4}-\frac{\sqrt{-14+10\sqrt{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{2+2\sqrt{2}}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{4}$$

command

```
Integrate[(1 + x^4)^2/((-1 + x^4)^2*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

Mathematica 13.1 output

$$\frac{1}{4}\left(\frac{2x\left(-3+x^4\right)}{\left(-1+x^4\right)\sqrt{x^2+\sqrt{1+x^4}}}\right.$$
$$-\sqrt{2\left(7+5\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\left(-1+x^2+\sqrt{1+x^4}\right)}{x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$+\sqrt{2\left(7+5\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$+2\sqrt{2}\,\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$-\sqrt{2\left(-7+5\sqrt{2}\right)}\,\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\left(-1+x^2+\sqrt{1+x^4}\right)}{x\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$\left.-\sqrt{2\left(-7+5\sqrt{2}\right)}\,\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^4\right)^2}{\left(-1+x^4\right)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

24.629 Problem number 2962

$$\int \frac{\left(bx+ax^2\right)\sqrt[4]{bx^3+ax^4}}{-b+ax+x^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((b*x + a*x^2)*(b*x^3 + a*x^4)^(1/4))/(-b + a*x + x^2),x]
```

Mathematica 13.1 output

$$\frac{x^{9/4}(b+ax)^{3/4}\left(-16a^{11/4}x^{3/4}\sqrt[4]{b+ax}+18a^{3/4}bx^{3/4}\sqrt[4]{b+ax}+8a^{7/4}x^{7/4}\sqrt[4]{b+ax}-32a^4\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\left(bx+ax^2\right)\sqrt[4]{bx^3+ax^4}}{-b+ax+x^2}\,dx$$

24.630 Problem number 2963

$$\int \frac{\left(bx+ax^2\right)\sqrt[4]{bx^3+ax^4}}{-b+ax+x^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((b*x + a*x^2)*(b*x^3 + a*x^4)^(1/4))/(-b + a*x + x^2),x]
```

Mathematica 13.1 output

$$\frac{x^{9/4}(b+ax)^{3/4}\left(-16a^{11/4}x^{3/4}\sqrt[4]{b+ax}+18a^{3/4}bx^{3/4}\sqrt[4]{b+ax}+8a^{7/4}x^{7/4}\sqrt[4]{b+ax}-32a^4\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\left(bx+ax^2\right)\sqrt[4]{bx^3+ax^4}}{-b+ax+x^2}\,dx$$

24.631 Problem number 2969

$$\int \frac{(-2+x)\left(1-x+x^2\right)}{x^3\left(-1+x+x^2\right)\sqrt[3]{\frac{1-x+2x^2}{1-x+3x^2}}}\,dx$$

Optimal antiderivative

$$\frac{\left(-3x^2+x-1\right)\left(\frac{2x^2-x+1}{3x^2-x+1}\right)^{\frac{2}{3}}}{x^2}-2\,2^{\frac{2}{3}}3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}}{3}+\frac{2\,2^{\frac{2}{3}}\left(\frac{2x^2-x+1}{3x^2-x+1}\right)^{\frac{1}{3}}3^{\frac{1}{6}}}{3}\right)$$
$$+\frac{7\arctan\left(\frac{\sqrt{3}}{3}+\frac{2\left(\frac{2x^2-x+1}{3x^2-x+1}\right)^{\frac{1}{3}}\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{7\ln\left(-1+\left(\frac{2x^2-x+1}{3x^2-x+1}\right)^{\frac{1}{3}}\right)}{3}$$
$$-\frac{2\,2^{\frac{2}{3}}\ln\left(-3+6^{\frac{2}{3}}\left(\frac{2x^2-x+1}{3x^2-x+1}\right)^{\frac{1}{3}}\right)3^{\frac{2}{3}}}{3}-\frac{7\ln\left(1+\left(\frac{2x^2-x+1}{3x^2-x+1}\right)^{\frac{1}{3}}+\left(\frac{2x^2-x+1}{3x^2-x+1}\right)^{\frac{2}{3}}\right)}{6}$$
$$+\frac{2^{\frac{2}{3}}\ln\left(3+6^{\frac{2}{3}}\left(\frac{2x^2-x+1}{3x^2-x+1}\right)^{\frac{1}{3}}+2\,6^{\frac{1}{3}}\left(\frac{2x^2-x+1}{3x^2-x+1}\right)^{\frac{2}{3}}\right)3^{\frac{2}{3}}}{3}$$

command

```
Integrate[((-2 + x)*(1 - x + x^2))/(x^3*(-1 + x + x^2)*((1 - x + 2*x^2)/(1 - x + 3*x^2))^(1/3
```

Mathematica 13.1 output

$$6\sqrt[3]{1-x+3x^2}-6x\sqrt[3]{1-x+3x^2}+12x^2\sqrt[3]{1-x+3x^2}+14\sqrt{3}\,x^2\sqrt[3]{1-x+2x^2}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{1-x+3x}}{\sqrt[3]{1-x+2x}}}{\sqrt{3}}\right.$$
$$-$$

Mathematica 12.3 output

$$\int \frac{(-2+x)\left(1-x+x^2\right)}{x^3\left(-1+x+x^2\right)\sqrt[3]{\frac{1-x+2x^2}{1-x+3x^2}}}\,dx$$

24.632 Problem number 2976

$$\int \frac{x}{x-\sqrt{b+ax}\,\sqrt{c+\sqrt{b+ax}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[x/(x - Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]]),x]
```

Mathematica 13.1 output

$$\frac{b}{a}-\frac{c^2}{a}+x+2a\left(c+\sqrt{b+ax}\right)+\frac{4}{3}\sqrt{c+\sqrt{b+ax}}\,\left(3a^2+c+\sqrt{b+ax}\right)$$
$$-4\text{RootSum}\left[b-c^2-ac\#1+2c\#1^2+a\#1^3\right.$$
$$\left.-\#1^4\&,\frac{-a^2b\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)+a^2c^2\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)-ab\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right.\right.}{}$$

Mathematica 12.3 output

$Aborted

24.633 Problem number 2977

$$\int \frac{x}{x-\sqrt{b+ax}\,\sqrt{c+\sqrt{b+ax}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[x/(x - Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]]),x]
```

Mathematica 13.1 output

$$\frac{b}{a}-\frac{c^2}{a}+x+2a\left(c+\sqrt{b+ax}\right)+\frac{4}{3}\sqrt{c+\sqrt{b+ax}}\left(3a^2+c+\sqrt{b+ax}\right)$$
$$-4\text{RootSum}\left[b-c^2-ac\#1+2c\#1^2+a\#1^3\right.$$
$$\left.-\#1^4\&,\frac{-a^2b\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)+a^2c^2\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)-ab\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right.\right.$$

Mathematica 12.3 output

$$\int\frac{x}{x-\sqrt{b+ax}\sqrt{c+\sqrt{b+ax}}}\,dx$$

24.634 Problem number 2986

$$\int\frac{f+ex}{d+cx+\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(f + e*x)/(d + c*x + Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

Mathematica 13.1 output

$$\frac{cex-2e\sqrt{ax+\sqrt{b^2+a^2x^2}}+(de-cf)\log\left(ax+\sqrt{b^2+a^2x^2}\right)+2\text{RootSum}\left[b^2c-2ad\#1^2-2a\#1^3-c\#1^4\&\right.}{}$$

Mathematica 12.3 output

$Aborted

24.635 Problem number 2987

$$\int \frac{1+x^6}{\sqrt[4]{-x^3+x^5}\,(1-x^6)}\,dx$$

Optimal antiderivative

$$\frac{4\left(x^5-x^3\right)^{\frac{3}{4}}}{3x^2\left(x^2-1\right)}+\frac{\arctan\left(\frac{3^{\frac{7}{8}}\sqrt{2-\sqrt{2}}\;x\left(x^5-x^3\right)^{\frac{1}{4}}}{-3x^2+3^{\frac{3}{4}}\sqrt{x^5-x^3}}\right)2^{\frac{1}{4}}}{3\left(51+36\sqrt{2}\right)^{\frac{1}{8}}}$$
$$+\frac{\left(\frac{17}{3}+4\sqrt{2}\right)^{\frac{1}{8}}\arctan\left(\frac{3^{\frac{7}{8}}\sqrt{2+\sqrt{2}}\;x\left(x^5-x^3\right)^{\frac{1}{4}}}{-3x^2+3^{\frac{3}{4}}\sqrt{x^5-x^3}}\right)2^{\frac{1}{4}}}{3}$$
$$+\frac{\operatorname{arctanh}\left(\frac{\frac{3^{\frac{1}{8}}x^2}{\sqrt{2-\sqrt{2}}}+\frac{\sqrt{x^5-x^3}\,3^{\frac{7}{8}}}{\sqrt[3]{2-\sqrt{2}}}}{x\left(x^5-x^3\right)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{3\left(51+36\sqrt{2}\right)^{\frac{1}{8}}}$$
$$+\frac{\left(\frac{17}{3}+4\sqrt{2}\right)^{\frac{1}{8}}\operatorname{arctanh}\left(\frac{\frac{3^{\frac{1}{8}}x^2}{\sqrt{2+\sqrt{2}}}+\frac{\sqrt{x^5-x^3}\,3^{\frac{7}{8}}}{\sqrt[3]{2+\sqrt{2}}}}{x\left(x^5-x^3\right)^{\frac{1}{4}}}\right)2^{\frac{1}{4}}}{3}$$

command

```
Integrate[(1 + x^6)/((-x^3 + x^5)^(1/4)*(1 - x^6)),x]
```

Mathematica 13.1 output

$$\frac{x^{3/4}\left(12\sqrt[8]{17+12\sqrt{2}}\;\sqrt[4]{x}+\sqrt[4]{2}\,3^{7/8}\sqrt[4]{-1+x^2}\,\operatorname{ArcTan}\left(\frac{\sqrt[4]{2}\,3^{7/8}\sqrt[4]{x}\;\sqrt[4]{-1+x^2}}{\sqrt[8]{17+12\sqrt{2}}\left(-3\sqrt{x}+3^{3/4}\sqrt{-1+x^2}\right)}\right)+3^{7/8}\sqrt[4]{3}\right.}{}$$

Mathematica 12.3 output

$$\int \frac{1+x^6}{\sqrt[4]{-x^3+x^5}\,(1-x^6)}\,dx$$

24.636 Problem number 2992

$$\int \frac{\left(1+x^2+x^4\right)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}\left(-1+x^2+x^4\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^2 + x^4)*Sqrt[x^2 + Sqrt[1 + x^4]])/(Sqrt[1 + x^4]*(-1 + x^2 + x^4)),x]
```

Mathematica 13.1 output

$$2\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)+\text{RootSum}\left[1-2\#1^2-6\#1^4+2\#1^6+\#1^8\&,\frac{\log\left(1+x^2+\sqrt{1+x^4}\right)-\log\left(\sqrt{}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2+x^4\right)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}\left(-1+x^2+x^4\right)}\,dx$$

24.637 Problem number 2993

$$\int \frac{\left(1+x^2+x^4\right)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}\left(-1+x^2+x^4\right)}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^2 + x^4)*Sqrt[x^2 + Sqrt[1 + x^4]])/(Sqrt[1 + x^4]*(-1 + x^2 + x^4)),x]
```

Mathematica 13.1 output

$$2\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)+\text{RootSum}\left[1-2\#1^2-6\#1^4+2\#1^6+\#1^8\&,\frac{\log\left(1+x^2+\sqrt{1+x^4}\right)-\log\left(\sqrt{}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2+x^4\right)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}\left(-1+x^2+x^4\right)}\,dx$$

24.638 Problem number 2999

$$\int \frac{\sqrt{x+\sqrt{1+x^2}}\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1+x^2\right)^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/(1 + x^2)^2,x]
```

Mathematica 13.1 output

$$\frac{1}{64}\left(\frac{4\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\left(-1-2x-x^2-2x^3+\left(1+18x+x^2+26x^3\right)\sqrt{x+\sqrt{1+x^2}}+\sqrt{1+x^2}\left(-1-\right.\right.\right.}{\left(1+x^2\right)\left(1+2x^2+2x\sqrt{1+x^2}\right)}$$

$$+32\,\text{RootSum}\left[2-4\#1^2+6\#1^4-4\#1^6\right.$$

$$\left.\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2}{-\#1+\#1^3}\&\right]\right.$$

$$-\,\text{RootSum}\left[2-4\#1^2+6\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{24\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)-26\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2-16\log\left(\sqrt{1+\right.}\right.}{-\#1+3\#1^3-3\#1^5+\#1^7}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1+x^2)^2}\,dx$$

24.639 Problem number 3000

$$\int \frac{\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1+x^2)^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/(1 + x^2)^2,x]
```

Mathematica 13.1 output

$$\frac{1}{64}\left(\frac{4\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\left(-1-2x-x^2-2x^3+\left(1+18x+x^2+26x^3\right)\sqrt{x+\sqrt{1+x^2}}+\sqrt{1+x^2}\left(-1-\right.\right.\right.}{(1+x^2)\left(1+2x^2+2x\sqrt{1+x^2}\right)}$$

$$+32\,\text{RootSum}\left[2-4\#1^2+6\#1^4-4\#1^6\right.$$

$$\left.\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2}{-\#1+\#1^3}\&\right]\right.$$

$$\left.-\,\text{RootSum}\left[2-4\#1^2+6\#1^4-4\#1^6\right.\right.$$

$$\left.\left.+\#1^8\&,\frac{24\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)-26\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2-16\log\left(\sqrt{1+\right.\right.}{-\#1+3\#1^3-3\#1^5+\#1^7}\right.\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1+x^2)^2}\,dx$$

24.640 Problem number 3001

$$\int \frac{\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1-x^2)^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/(1 - x^2)^2,x]
```

Mathematica 13.1 output

$$\frac{1}{16}\left(-\frac{8x\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{-1+x^2}\right.$$

$$+4\text{RootSum}\left[-2+4\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)}{2\#1-3\#1^3+\#1^5}\&\right]$$

$$-\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+7\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2-\log\left(\sqrt{1+\sqrt{x+}\right.}\right.}{2\#1^3-3\#1^5+\#1^7}$$

$$-4\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-2+4\#1^2-3\#1^4+\#1^6}\&\right]$$

$$-\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)-9\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2-\log\left(\sqrt{1+\sqrt{x+}\right.}\right.}{-2\#1+4\#1^3-3\#1^5+\#1^7}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^2\right)^2}\,dx$$

24.641 Problem number 3002

$$\int \frac{\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^2\right)^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/(1 - x^2)^2,x]
```

Mathematica 13.1 output

$$\frac{1}{16}\left(-\frac{8x\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{-1+x^2}\right.$$

$$+4\text{RootSum}\left[-2+4\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)}{2\#1-3\#1^3+\#1^5}\&\right]$$

$$-\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+7\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2-\log\left(\sqrt{1+\sqrt{x+}\right.}\right.}{2\#1^3-3\#1^5+\#1^7}$$

$$-4\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-2+4\#1^2-3\#1^4+\#1^6}\&\right]$$

$$-\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)-9\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2-\log\left(\sqrt{1+\sqrt{x+}\right.}\right.}{-2\#1+4\#1^3-3\#1^5+\#1^7}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^2\right)^2}\,dx$$

24.642 Problem number 3003

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1+x^2)^2\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/((1 + x^2)^2*Sqrt[x + Sqrt[1 + x^2]]),x]
```

Mathematica 13.1 output

$$\frac{1}{64}\left(\frac{4\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\left(-1+2x-x^2+2x^3-\left(3+2x-5x^2+2x^3\right)\sqrt{x+\sqrt{1+x^2}}+\sqrt{1+x^2}\left(1-x\right.\right.\right.}{(1+x^2)\left(1+2x^2+2x\sqrt{1+x^2}\right)}$$

$$+32\,\text{RootSum}\left[2-4\#1^2+6\#1^4-4\#1^6\right.$$

$$\left.\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+5\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2}{-\#1+3\#1^3-3\#1^5+\#1^7}\&\right]\right.$$

$$-\,\text{RootSum}\left[2-4\#1^2+6\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{36\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+136\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2-4\log\left(\sqrt{1+\right.}\right.}{-\#1+3\#1^3-3\#1^5+\#1^7}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1+x^2)^2\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.643 Problem number 3004

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1+x^2)^2\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/((1 + x^2)^2*Sqrt[x + Sqrt[1 + x^2]]),x]
```

Mathematica 13.1 output

$$\frac{1}{64}\left(\frac{4\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\left(-1+2x-x^2+2x^3-\left(3+2x-5x^2+2x^3\right)\sqrt{x+\sqrt{1+x^2}}+\sqrt{1+x^2}\left(1-x\right.\right.\right.}{(1+x^2)\left(1+2x^2+2x\sqrt{1+x^2}\right)}$$

$$+32\,\text{RootSum}\left[2-4\#1^2+6\#1^4-4\#1^6\right.$$

$$\left.\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+5\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2}{-\#1+3\#1^3-3\#1^5+\#1^7}\&\right]\right.$$

$$-\,\text{RootSum}\left[2-4\#1^2+6\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{36\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+136\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2-4\log\left(\sqrt{1+\right.}\right.}{-\#1+3\#1^3-3\#1^5+\#1^7}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1+x^2)^2\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.644 Problem number 3006

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1-x^2)^2\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/((1 - x^2)^2*Sqrt[x + Sqrt[1 + x^2]]),x]
```

Mathematica 13.1 output

$$\frac{1}{16}\left(-\frac{8x\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(-1+x^2)\sqrt{x+\sqrt{1+x^2}}}-4\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.\right.$$

$$\left.+\#1^8\&,\frac{-\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)-2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2+\log\left(\sqrt{1+\sqrt{x}\right.\right.}}{2\#1-3\#1^3+\#1^5}\right]$$

$$-\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+5\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2}{2\#1^3-3\#1^5+\#1^7}\&\right]$$

$$+4\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{3\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1-2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^3+\log\left(\sqrt{1+\sqrt{}}\right.\right.}{-2+4\#1^2-3\#1^4+\#1^6}\right]$$

$$-\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$

$$\left.\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+5\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2}{-2\#1+4\#1^3-3\#1^5+\#1^7}\&\right]\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1-x^2)^2\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.645 Problem number 3007

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1-x^2)^2\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/((1 - x^2)^2*Sqrt[x + Sqrt[1 + x^2]]),x]
```

Mathematica 13.1 output

$$\frac{1}{16}\left(-\frac{8x\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(-1+x^2)\sqrt{x+\sqrt{1+x^2}}}-4\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.\right.$$

$$\left.+\#1^8\&,\frac{-\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)-2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2+\log\left(\sqrt{1+\sqrt{x}\right.\right.}}{2\#1-3\#1^3+\#1^5}$$

$$-\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+5\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2}{2\#1^3-3\#1^5+\#1^7}\&\right]$$

$$+4\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{3\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1-2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^3+\log\left(\sqrt{1+\sqrt{}}\right.\right.}{-2+4\#1^2-3\#1^4+\#1^6}$$

$$-\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$

$$\left.\left.+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+5\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2}{-2\#1+4\#1^3-3\#1^5+\#1^7}\&\right]\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1-x^2)^2\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.646 Problem number 3009

$$\int \frac{(1+x^4)\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1-x^4)\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^4)*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/((1 - x^4)*Sqrt[x + Sqrt[1 + x^2]]),x
```

Mathematica 13.1 output

$$\frac{1}{24}\left(-\frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\left(8+32x^2+3\sqrt{1+x^2}-2\sqrt{x+\sqrt{1+x^2}}+16\sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}+x\right.\right.}{\left(x+\sqrt{1+x^2}\right)^{3/2}}$$

$$+3\tanh^{-1}\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\right)$$

$$-12\text{RootSum}\left[-2+4\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-1+\#1^2}\&\right]$$

$$+24\text{RootSum}\left[2-4\#1^2+6\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-1+3\#1^2-3\#1^4+\#1^6}\&\right]$$

$$\left.+12\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-1+\#1^2}\&\right]\right)$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^4\right)\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^4\right)\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.647 Problem number 3010

$$\int \frac{\left(1+x^4\right)\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^4\right)\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^4)*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/((1 - x^4)*Sqrt[x + Sqrt[1 + x^2]]),x
```

Mathematica 13.1 output

$$\frac{1}{24}\left(-\frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\left(8+32x^2+3\sqrt{1+x^2}-2\sqrt{x+\sqrt{1+x^2}}+16\sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}+x\right.\right.}{\left(x+\sqrt{1+x^2}\right)^{3/2}}$$

$$+3\tanh^{-1}\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\right)$$

$$-12\text{RootSum}\left[-2+4\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-1+\#1^2}\&\right]$$

$$+24\text{RootSum}\left[2-4\#1^2+6\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-1+3\#1^2-3\#1^4+\#1^6}\&\right]$$

$$\left.+12\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-1+\#1^2}\&\right]\right)$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^4\right)\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^4\right)\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.648 Problem number 3025

$$\int \frac{1-x^3+x^6}{\sqrt[3]{x^2+x^4}\,\left(-1+x^6\right)}\,dx$$

Optimal antiderivative

$$\begin{aligned}
&-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^4+x^2\right)^{\frac{1}{3}}}\right)}{6}-\frac{\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^4+x^2\right)^{\frac{1}{3}}}\right)\sqrt{3}}{2}\\
&-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{24}\\
&+\frac{\ln\left(-x+\left(x^4+x^2\right)^{\frac{1}{3}}\right)}{2}-\frac{\ln\left(x+\left(x^4+x^2\right)^{\frac{1}{3}}\right)}{6}\\
&+\frac{\ln\left(-2x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{24}-\frac{\ln\left(2x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{8}\\
&+\frac{\ln\left(x^2-x\left(x^4+x^2\right)^{\frac{1}{3}}+\left(x^4+x^2\right)^{\frac{2}{3}}\right)}{12}-\frac{\ln\left(x^2+x\left(x^4+x^2\right)^{\frac{1}{3}}+\left(x^4+x^2\right)^{\frac{2}{3}}\right)}{4}\\
&+\frac{\ln\left(-2x^2+2^{\frac{2}{3}}x\left(x^4+x^2\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\left(x^4+x^2\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{16}\\
&-\frac{\ln\left(2x^2+2^{\frac{2}{3}}x\left(x^4+x^2\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(x^4+x^2\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{48}
\end{aligned}$$

command

```
Integrate[(1 - x^3 + x^6)/((x^2 + x^4)^(1/3)*(-1 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\sqrt[3]{1+x^2}\left(8\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}-2\sqrt[3]{1+x^2}}\right)-24\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}+2\sqrt[3]{1+x^2}}\right)+6\,2^{2/3}\sqrt{3}\,\text{ArcTan}\left(\frac{}{\sqrt[3]{x}-\right.\right.}$$

Mathematica 12.3 output

$$\int \frac{1-x^3+x^6}{\sqrt[3]{x^2+x^4}\,\left(-1+x^6\right)}\,dx$$

24.649 Problem number 3026

$$\int \frac{1+x^3+x^6}{\sqrt[3]{x^2+x^4}\,(-1+x^6)}\,dx$$

Optimal antiderivative

$$\begin{aligned}
&-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2\left(x^4+x^2\right)^{\frac{1}{3}}}\right)}{2}-\frac{\arctan\left(\frac{\sqrt{3}\,x}{x+2\left(x^4+x^2\right)^{\frac{1}{3}}}\right)\sqrt{3}}{6}\\
&-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{-x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{24}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}}\right)2^{\frac{2}{3}}}{8}\\
&+\frac{\ln\left(-x+\left(x^4+x^2\right)^{\frac{1}{3}}\right)}{6}-\frac{\ln\left(x+\left(x^4+x^2\right)^{\frac{1}{3}}\right)}{2}\\
&+\frac{\ln\left(-2x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{8}-\frac{\ln\left(2x+2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{24}\\
&+\frac{\ln\left(x^2-x\left(x^4+x^2\right)^{\frac{1}{3}}+\left(x^4+x^2\right)^{\frac{2}{3}}\right)}{4}-\frac{\ln\left(x^2+x\left(x^4+x^2\right)^{\frac{1}{3}}+\left(x^4+x^2\right)^{\frac{2}{3}}\right)}{12}\\
&+\frac{\ln\left(-2x^2+2^{\frac{2}{3}}x\left(x^4+x^2\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\left(x^4+x^2\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{48}\\
&-\frac{\ln\left(2x^2+2^{\frac{2}{3}}x\left(x^4+x^2\right)^{\frac{1}{3}}+2^{\frac{1}{3}}\left(x^4+x^2\right)^{\frac{2}{3}}\right)2^{\frac{2}{3}}}{16}
\end{aligned}$$

command

```
Integrate[(1 + x^3 + x^6)/((x^2 + x^4)^(1/3)*(-1 + x^6)),x]
```

Mathematica 13.1 output

$$\frac{x^{2/3}\sqrt[3]{1+x^2}\left(24\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}-2\sqrt[3]{1+x^2}}\right)-8\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[3]{x}}{\sqrt[3]{x}+2\sqrt[3]{1+x^2}}\right)+2\,2^{2/3}\sqrt{3}\,\text{ArcTan}\left(\frac{}{\sqrt[3]{x}-}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{1+x^3+x^6}{\sqrt[3]{x^2+x^4}\,(-1+x^6)}\,dx$$

24.650 Problem number 3027

$$\int \frac{\sqrt{-b+a^2x^2}\,\sqrt{ax+\sqrt{-b+a^2x^2}}}{\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Optimal antiderivative

$$\frac{\left(-2560a^3c^4x^3-2048a^2c^6x^2+1920ab\,c^4x+1024b\,c^6-1575a\,b^2x-840b^2c^2\right)\sqrt{c+\sqrt{ax+\sqrt{a^2x^2-b}}}+(2240c$$

$$+\frac{5b^2\operatorname{arctanh}\left(\frac{\sqrt{c+\sqrt{ax+\sqrt{a^2x^2-b}}}}{\sqrt{c}}\right)}{16a\,c^{\frac{7}{2}}}$$

command

```
Integrate[(Sqrt[-b + a^2*x^2]*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/Sqrt[c + Sqrt[a*x + Sqrt[-
b + a^2*x^2]]],x]
```

Mathematica 13.1 output

$$\frac{\sqrt{c}\,\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}\left(-105b^2\left(8c^2-10c\sqrt{ax+\sqrt{-b+a^2x^2}}+15\left(ax+\sqrt{-b+a^2x^2}\right)\right)-16bc^3\left(-64c^3+48c^2\sqrt{}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{-b+a^2x^2}\,\sqrt{ax+\sqrt{-b+a^2x^2}}}{\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

24.651 Problem number 3028

$$\int \frac{\left(-1+ax^8\right)\left(1+ax^8\right)^{3/4}}{1+x^8+a^2x^{16}}\,dx$$

Optimal antiderivative

$$\frac{\left(1+(-1)^{\frac{1}{4}}\right)\arctan\left(\frac{(-1)^{\frac{7}{8}}\sqrt{2+\sqrt{2}}\,(-1+2a)^{\frac{1}{8}}x\left(a\,x^8+1\right)^{\frac{1}{4}}}{(-1)^{\frac{3}{4}}(-1+2a)^{\frac{1}{4}}x^2+\sqrt{a\,x^8+1}}\right)}{8\,(-1+2a)^{\frac{1}{8}}}$$
$$-\frac{i\left(-i\sqrt{2}+2-\sqrt{2}\right)\arctan\left(\frac{(-1)^{\frac{7}{8}}\left(-2+\sqrt{2}\right)(-1+2a)^{\frac{1}{8}}x\left(a\,x^8+1\right)^{\frac{1}{4}}}{(-1)^{\frac{3}{4}}\sqrt{2-\sqrt{2}}\,(-1+2a)^{\frac{1}{4}}x^2+\sqrt{2-\sqrt{2}}\,\sqrt{a\,x^8+1}}\right)}{16\,(-1+2a)^{\frac{1}{8}}}$$
$$+\frac{\left(\sqrt{2}+i\left(2-\sqrt{2}\right)\right)\operatorname{arctanh}\left(\frac{(-1)^{\frac{7}{8}}(-1+2a)^{\frac{1}{4}}x^2-(-1)^{\frac{1}{8}}\sqrt{a\,x^8+1}}{\sqrt{2-\sqrt{2}}\,(-1+2a)^{\frac{1}{8}}x\left(a\,x^8+1\right)^{\frac{1}{4}}}\right)}{16\,(-1+2a)^{\frac{1}{8}}}$$
$$+\frac{\left(1+(-1)^{\frac{1}{4}}\right)\operatorname{arctanh}\left(\frac{(-1)^{\frac{7}{8}}(-1+2a)^{\frac{1}{4}}x^2-(-1)^{\frac{1}{8}}\sqrt{a\,x^8+1}}{\sqrt{2+\sqrt{2}}\,(-1+2a)^{\frac{1}{8}}x\left(a\,x^8+1\right)^{\frac{1}{4}}}\right)}{8\,(-1+2a)^{\frac{1}{8}}}$$

command

```
Integrate[((-1 + a*x^8)*(1 + a*x^8)^(3/4))/(1 + x^8 + a^2*x^16),x]
```

Mathematica 13.1 output

$$-2\left(i+(-1)^{3/4}\right)\tanh^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\left(-i+(-1)^{3/4}\right)\sqrt[4]{-1+2a}\,x^2+\left(1+(-1)^{3/4}\right)\sqrt{1+ax^8}\right)}{\sqrt[8]{-1+2a}\,x\sqrt[4]{1+ax^8}}\right)+\sqrt{2}\left(i\left(i+\sqrt{3+2\sqrt{2}}\right)\right.$$

Mathematica 12.3 output

$$\int \frac{\left(-1+ax^8\right)\left(1+ax^8\right)^{3/4}}{1+x^8+a^2x^{16}}\,dx$$

24.652 Problem number 3034

$$\int \frac{1}{\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\,\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Optimal antiderivative

$$-\frac{8c\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{3}{4}}}{7a}+\frac{3b\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{3}{4}}}{8ac\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{4}{3}}}$$
$$-\frac{13b\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{3}{4}}}{32a\,c^2\left(ax+\sqrt{a^2x^2-b}\right)}+\frac{117b\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{3}{4}}}{256a\,c^3\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{2}{3}}}$$
$$-\frac{585b\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{3}{4}}}{1024a\,c^4\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}}$$
$$+\frac{6\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{3}{4}}}{7a}$$
$$-\frac{585b\arctan\left(\frac{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}{c^{\frac{1}{4}}}\right)}{2048a\,c^{\frac{17}{4}}}$$
$$+\frac{585b\,\operatorname{arctanh}\left(\frac{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}{c^{\frac{1}{4}}}\right)}{2048a\,c^{\frac{17}{4}}}$$

command

```
Integrate[1/((a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)),x
```

Mathematica 13.1 output

$$\frac{2\sqrt[4]{c}\left(c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\right)^{3/4}\left(2048c^4\left(ax+\sqrt{-b+a^2x^2}\right)^{4/3}\left(-4c+3\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\right)-7b\left(-384c^3+416c^2\sqrt[3]{}\right.\right.}{\left(ax+\sqrt{-b+a^2x^2}\right)^{4/3}}$$

Mathematica 12.3 output

$Aborted

24.653 Problem number 3043

$$\int \frac{x^2\sqrt{b+ax}}{x^2-\sqrt{b+ax}\,\sqrt{c+\sqrt{b+ax}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x^2*Sqrt[b + a*x])/(x^2 - Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]]),x]
```

Mathematica 13.1 output

$$4a\sqrt{c+\sqrt{b+ax}} + \frac{2\left(c^3+(b+ax)^{3/2}\right)}{3a}$$
$$-4a\text{RootSum}\left[b^2-2bc^2+c^4+a^2c\#1+4bc\#1^2-4c^3\#1^2-a^2\#1^3-2b\#1^4+6c^2\#1^4-4c\#1^6\right.$$
$$\left.+\#1^8\&,\ \frac{b^2\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)-2bc^2\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)+c^4\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)+}{}\right]$$

Mathematica 12.3 output

$$\int \frac{x^2\sqrt{b+ax}}{x^2-\sqrt{b+ax}\,\sqrt{c+\sqrt{b+ax}}}\,dx$$

24.654 Problem number 3044

$$\int \frac{x^2\sqrt{b+ax}}{x^2-\sqrt{b+ax}\,\sqrt{c+\sqrt{b+ax}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(x^2*Sqrt[b + a*x])/(x^2 - Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]]),x]
```

Mathematica 13.1 output

$$4a\sqrt{c+\sqrt{b+ax}}+\frac{2\left(c^3+(b+ax)^{3/2}\right)}{3a}$$
$$-4a\text{RootSum}\left[b^2-2bc^2+c^4+a^2c\#1+4bc\#1^2-4c^3\#1^2-a^2\#1^3-2b\#1^4+6c^2\#1^4-4c\#1^6\right.$$
$$\left.+\#1^8\&,\frac{b^2\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)-2bc^2\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)+c^4\log\left(\sqrt{c+\sqrt{b+ax}}-\#1\right)+\right.$$

Mathematica 12.3 output

$$\int\frac{x^2\sqrt{b+ax}}{x^2-\sqrt{b+ax}\sqrt{c+\sqrt{b+ax}}}\,dx$$

24.655 Problem number 3045

$$\int\sqrt{b+a^2x^2}\sqrt{ax+\sqrt{b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

Optimal antiderivative

$$\frac{\left(40320a^4c^2x^4-2560a^3c^4x^3-2048a^2c^6x^2+114240a^2bc^2x^2-1920abc^4x-1024bc^6+3465ab^2x+32760b^2c^2\right)\sqrt{c}}{}$$
$$-\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{\sqrt{c}}\right)}{16ac^{\frac{5}{2}}}$$

command

```
Integrate[Sqrt[b + a^2*x^2]*Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^
```

Mathematica 13.1 output

$$\frac{\sqrt{c}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\left(105b^2\left(312c^2-22c\sqrt{ax+\sqrt{b+a^2x^2}}+33\left(ax+\sqrt{b+a^2x^2}\right)\right)+16bc^2\left(-64c^4+48c^3\sqrt{ax+\right.\right.}}{}$$

Mathematica 12.3 output

$$\int \sqrt{b+a^2x^2}\sqrt{ax+\sqrt{b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

24.656 Problem number 3050

$$\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{(d+cx)\sqrt{b+a^2x^4}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]/((d + c*x)*Sqrt[b + a^2*x^4]),x]
```

Mathematica 13.1 output

$$\frac{2ad^2\left(-\sqrt{-ad^2-\sqrt{bc^4+a^2d^4}}\,\text{ArcTan}\left(\frac{c\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{-ad^2-\sqrt{bc^4+a^2d^4}}}\right)+\sqrt{-ad^2+\sqrt{bc^4+a^2d^4}}\,\text{ArcTan}\left(\frac{c\sqrt{}}{\sqrt{-a}}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{(d+cx)\sqrt{b+a^2x^4}}\,dx$$

24.657 Problem number 3051

$$\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{(d+cx)\sqrt{b+a^2x^4}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]/((d + c*x)*Sqrt[b + a^2*x^4]),x]
```

Mathematica 13.1 output

$$\frac{2ad^2\left(-\sqrt{-ad^2-\sqrt{bc^4+a^2d^4}}\,\text{ArcTan}\left(\frac{c\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{-ad^2-\sqrt{bc^4+a^2d^4}}}\right)+\sqrt{-ad^2+\sqrt{bc^4+a^2d^4}}\,\text{ArcTan}\left(\frac{c\sqrt{}}{\sqrt{-}}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{(d+cx)\sqrt{b+a^2x^4}}\,dx$$

24.658 Problem number 3058

$$\int \frac{-1+x^8}{\sqrt[4]{-x^2+x^6}\,(1+x^8)}\,dx$$

Optimal antiderivative

$$-\frac{\left(-4+3\sqrt{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}\,x}{-x\sqrt{2+\sqrt{2}}+2^{\frac{7}{8}}(x^6-x^2)^{\frac{1}{4}}}\right)}{4}-\frac{\left(-4+3\sqrt{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}\,x}{x\sqrt{2+\sqrt{2}}+2^{\frac{7}{8}}(x^6-x^2)^{\frac{1}{4}}}\right)}{4}-\frac{\left(4+3\sqrt{2}\right)^{\frac{1}{4}}\arctan\left(\frac{2^{\frac{7}{8}}\sqrt{2+\sqrt{2}}\,x(x^6-x^2)^{\frac{1}{4}}}{-2x^2+2^{\frac{3}{4}}\sqrt{x^6-x^2}}\right)}{4}-\frac{\left(-4+3\sqrt{2}\right)^{\frac{1}{4}}\operatorname{arctanh}\left(\frac{\frac{2^{\frac{1}{8}}x^2}{\sqrt{2-\sqrt{2}}}+\frac{\sqrt{x^6-x^2}\,2^{\frac{7}{8}}}{2\sqrt{2-\sqrt{2}}}}{x(x^6-x^2)^{\frac{1}{4}}}\right)}{4}+\frac{\left(4+3\sqrt{2}\right)^{\frac{1}{4}}\ln\left(-2x^2+2^{\frac{7}{8}}\sqrt{2+\sqrt{2}}\,x\left(x^6-x^2\right)^{\frac{1}{4}}-2^{\frac{3}{4}}\sqrt{x^6-x^2}\right)}{8}-\frac{\left(4+3\sqrt{2}\right)^{\frac{1}{4}}\ln\left(2\sqrt{2-\sqrt{2}}\,x^2+2\,2^{\frac{3}{8}}x\left(x^6-x^2\right)^{\frac{1}{4}}+2^{\frac{3}{4}}\sqrt{2-\sqrt{2}}\,\sqrt{x^6-x^2}\right)}{8}$$

command

```
Integrate[(-1 + x^8)/((-x^2 + x^6)^(1/4)*(1 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-1+\frac{1}{x^4}}\,x^{3/2}\left(2\sqrt[4]{-4+3\sqrt{2}}\,\operatorname{ArcTan}\left(\frac{\sqrt[4]{-8+6\sqrt{2}}\,\sqrt[4]{-1+\frac{1}{x^4}}\,\sqrt{x}}{\sqrt[4]{2}-\sqrt{-1+\frac{1}{x^4}}\,x}\right)-2\sqrt[4]{-4+3\sqrt{2}}\,\tanh^{-1}\left(\frac{2\sqrt[4]{-4+}}{2+}\right.\right.}$$

Mathematica 12.3 output

$$\int\frac{-1+x^8}{\sqrt[4]{-x^2+x^6}\,(1+x^8)}\,dx$$

24.659 Problem number 3059

$$\int \frac{-1+x^8}{\sqrt[4]{-x^2+x^6}\,(1+x^8)}\,dx$$

Optimal antiderivative

$$-\frac{\left(-4+3\sqrt{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}\;x}{-x\sqrt{2+\sqrt{2}}+2^{\frac{7}{8}}(x^6-x^2)^{\frac{1}{4}}}\right)}{4}$$
$$-\frac{\left(-4+3\sqrt{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}\;x}{x\sqrt{2+\sqrt{2}}+2^{\frac{7}{8}}(x^6-x^2)^{\frac{1}{4}}}\right)}{4}$$
$$-\frac{\left(4+3\sqrt{2}\right)^{\frac{1}{4}}\arctan\left(\frac{2^{\frac{7}{8}}\sqrt{2+\sqrt{2}}\;x(x^6-x^2)^{\frac{1}{4}}}{-2x^2+2^{\frac{3}{4}}\sqrt{x^6-x^2}}\right)}{4}$$
$$-\frac{\left(-4+3\sqrt{2}\right)^{\frac{1}{4}}\operatorname{arctanh}\left(\frac{\frac{2^{\frac{1}{8}}x^2}{\sqrt{2-\sqrt{2}}}+\frac{\sqrt{x^6-x^2}\;2^{\frac{7}{8}}}{2\sqrt{2-\sqrt{2}}}}{x(x^6-x^2)^{\frac{1}{4}}}\right)}{4}$$
$$+\frac{\left(4+3\sqrt{2}\right)^{\frac{1}{4}}\ln\left(-2x^2+2^{\frac{7}{8}}\sqrt{2+\sqrt{2}}\;x\left(x^6-x^2\right)^{\frac{1}{4}}-2^{\frac{3}{4}}\sqrt{x^6-x^2}\right)}{8}$$
$$-\frac{\left(4+3\sqrt{2}\right)^{\frac{1}{4}}\ln\left(2\sqrt{2-\sqrt{2}}\;x^2+2\,2^{\frac{3}{8}}x\left(x^6-x^2\right)^{\frac{1}{4}}+2^{\frac{3}{4}}\sqrt{2-\sqrt{2}}\;\sqrt{x^6-x^2}\right)}{8}$$

command

```
Integrate[(-1 + x^8)/((-x^2 + x^6)^(1/4)*(1 + x^8)),x]
```

Mathematica 13.1 output

$$\frac{\sqrt[4]{-1+\frac{1}{x^4}}\;x^{3/2}\left(2\sqrt[4]{-4+3\sqrt{2}}\;\operatorname{ArcTan}\left(\frac{\sqrt[4]{-8+6\sqrt{2}}\;\sqrt[4]{-1+\frac{1}{x^4}}\;\sqrt{x}}{\sqrt[4]{2}-\sqrt{-1+\frac{1}{x^4}}\;x}\right)-2\sqrt[4]{-4+3\sqrt{2}}\;\tanh^{-1}\left(\frac{2\sqrt[4]{-4+}}{2+}\right.\right.}$$

Mathematica 12.3 output

$$\int \frac{-1+x^8}{\sqrt[4]{-x^2+x^6}\,(1+x^8)}\,dx$$

24.660 Problem number 3060

$$\int \frac{\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Optimal antiderivative

$$\frac{(3072a\,c^4x+4620bc)\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{3}{4}}+(-2688a\,c^3x-5775b)\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\left(c+\left(ax+\sqrt{\right.\right.}\right.}{}$$

$$-\frac{15b\arctan\left(\frac{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}{c^{\frac{1}{4}}}\right)}{32a\,c^{\frac{9}{4}}}+\frac{15b\,\mathrm{arctanh}\left(\frac{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}{c^{\frac{1}{4}}}\right)}{32a\,c^{\frac{9}{4}}}$$

command

```
Integrate[(a*x + Sqrt[-b + a^2*x^2])^(1/3)/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt[4]{c}\left(c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\right)^{3/4}\left(-1155b\left(-4c+5\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\right)+32c^2\left(96c^2\sqrt{-b+a^2x^2}-84c\sqrt{-b+a^2x^2}\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

24.661 Problem number 3061

$$\int \frac{b^2+ax}{(-b^2+ax)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{2}\,\left(-b+x\sqrt{a}\right)\sqrt{-x\sqrt{a}+\sqrt{a\,x^2+b^2}}}{\sqrt{a}\,b}+\frac{\sqrt{2}\,\sqrt{a\,x^2+b^2}\,\sqrt{-x\sqrt{a}+\sqrt{a\,x^2+b^2}}}{\sqrt{a}\,b}$$

$$-\frac{2\sqrt{2}\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-x\sqrt{a}+\sqrt{a\,x^2+b^2}}}{\sqrt{b}}\right)}{\sqrt{a}}$$

$$+\frac{2\left(\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,\sqrt{b+\sqrt{b^2+a}}-\sqrt{2}\,b^{\frac{3}{2}}\sqrt{b+\sqrt{b^2+a}}+\sqrt{2}\,\sqrt{b}\,\sqrt{b^2+a}\,\sqrt{b+\sqrt{b^2+a}}\right)\arctan\left(\frac{a^{\frac{1}{4}}\sqrt{}\right.}{a^{\frac{5}{4}}}$$

$$-\frac{2\left(-\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,\sqrt{-b+\sqrt{b^2+a}}+\sqrt{2}\,b^{\frac{3}{2}}\sqrt{-b+\sqrt{b^2+a}}+\sqrt{2}\,\sqrt{b}\,\sqrt{b^2+a}\,\sqrt{-b+\sqrt{b^2+a}}\right)\text{arcta}}{a^{\frac{5}{4}}}$$

command

```
Integrate[(b^2 + a*x)/((-b^2 + a*x)*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

Mathematica 13.1 output

$$\frac{2x}{\sqrt{b+\sqrt{b^2+ax^2}}}-\frac{2\sqrt{2}\,\sqrt{b}\,\text{ArcTan}\left(\frac{b-\sqrt{a}\,x+\sqrt{b^2+ax^2}}{\sqrt{2}\,\sqrt{b}\,\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{a}}$$

$$-2i\sqrt{a}\,b^3\text{RootSum}\left[16a^4b^4-8a^2b^2\#1^4-16ab^4\#1^4\right.$$

$$\left.+\#1^8\&,\frac{\log\left(\frac{iax}{\sqrt{b+\sqrt{b^2+ax^2}}}-i\sqrt{a}\,\sqrt{b+\sqrt{b^2+ax^2}}-\#1\right)\#1}{-4a^2b^2-8ab^4+\#1^4}\&\right]$$

$$-\frac{ib^2\text{RootSum}\left[16a^4b^4-8a^2b^2\#1^4-16ab^4\#1^4+\#1^8\&,\frac{\log\left(\frac{iax}{\sqrt{b+\sqrt{b^2+ax^2}}}-i\sqrt{a}\,\sqrt{b+\sqrt{b^2+ax^2}}-\#1\right)\#}{-4a^2b^2-8ab^4+\#1^4}\right.}{\sqrt{a}}$$

$$+4ia^{3/2}b^4\text{RootSum}\left[16a^4b^4-8a^2b^2\#1^4-16ab^4\#1^4\right.$$

$$\left.+\#1^8\&,\frac{\log\left(\frac{iax}{\sqrt{b+\sqrt{b^2+ax^2}}}-i\sqrt{a}\,\sqrt{b+\sqrt{b^2+ax^2}}-\#1\right)}{4a^2b^2\#1+8ab^4\#1-\#1^5}\&\right]$$

$$+8ia^{5/2}b^5\text{RootSum}\left[16a^4b^4-8a^2b^2\#1^4-16ab^4\#1^4+\#1^8\&,\frac{\log\left(\frac{iax}{\sqrt{b+\sqrt{b^2+ax^2}}}-i\sqrt{a}\,\sqrt{b+\sqrt{b^2+ax^2}}-\right.}{4a^2b^2\#1^3+8ab^4\#1^3-\#1^7}\right]$$

$$-ib\text{RootSum}\left[16a^4b^4-8a^2b^2\#1^4-16ab^4\#1^4+\#1^8\&,\frac{-8a^3b^3\log\left(\frac{iax}{\sqrt{b+\sqrt{b^2+ax^2}}}-i\sqrt{a}\,\sqrt{b+\sqrt{b^2+ax^2}}\right.}{}\right.$$

Mathematica 12.3 output

$$\int \frac{b^2+ax}{(-b^2+ax)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

24.662 Problem number 3070

$$\int \frac{1+x}{\sqrt[3]{27+189x+522x^2+784x^3+825x^4+679x^5+338x^6+84x^7+8x^8}}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{3}\,\arctan\left(\frac{5\sqrt{3}\,\left(8x^8+84x^7+338x^6+679x^5+825x^4+784x^3+522x^2+189x+27\right)^{\frac{1}{3}}}{12\,10^{\frac{1}{3}}+22\,10^{\frac{1}{3}}x-6\,10^{\frac{1}{3}}x^2-4\,10^{\frac{1}{3}}x^3+5\left(8x^8+84x^7+338x^6+679x^5+825x^4+784x^3+522x^2+189x+27\right)^{\frac{1}{3}}}\right)10^{\frac{2}{3}}}{50}$$
$$-\frac{\ln\left(2x^2+7x+3\right)10^{\frac{2}{3}}}{50}+\frac{\ln\left(4x^4+28x^3+61x^2+42x+9\right)10^{\frac{2}{3}}}{100}$$
$$+\frac{\ln\left(-6\,10^{\frac{1}{3}}-11\,10^{\frac{1}{3}}x+3\,10^{\frac{1}{3}}x^2+2\,10^{\frac{1}{3}}x^3+5\left(8x^8+84x^7+338x^6+679x^5+825x^4+784x^3+522x^2+189x+\right.\right.}{50}$$
$$-\frac{\ln\left(36\,10^{\frac{2}{3}}+132\,10^{\frac{2}{3}}x+85\,10^{\frac{2}{3}}x^2-90\,10^{\frac{2}{3}}x^3-35\,10^{\frac{2}{3}}x^4+12\,10^{\frac{2}{3}}x^5+4\,10^{\frac{2}{3}}x^6+\left(30\,10^{\frac{1}{3}}+55\,10^{\frac{1}{3}}x-15\,10^{\frac{1}{3}}x^2\right.\right.}{}$$

command

```
Integrate[(1 + x)/(27 + 189*x + 522*x^2 + 784*x^3 + 825*x^4 + 679*x^5 + 338*x^6 + 84*x^7 + 8*
```

Mathematica 13.1 output

$$-\frac{\sqrt[3]{1+x^2}\,\left(3+7x+2x^2\right)\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{4\sqrt[3]{10}-2\sqrt[3]{10}\,x+5\sqrt[3]{1+x^2}}{5\sqrt{3}\,\sqrt[3]{1+x^2}}\right)-2\log\left(-2\sqrt[3]{10}+\sqrt[3]{10}\,x+5\sqrt[3]{1+x^2}\right)\right.}{10\sqrt[3]{10}\,\sqrt[3]{\left(1+x^2\right)\left(3+7x+2x\right.}}$$

Mathematica 12.3 output

$$\int \frac{1+x}{\sqrt[3]{27+189x+522x^2+784x^3+825x^4+679x^5+338x^6+84x^7+8x^8}}\,dx$$

24.663 Problem number 3079

$$\int \frac{\sqrt{-b+a^2x^2}}{\sqrt{ax+\sqrt{-b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Optimal antiderivative

$$\frac{\left(-4096a^3c^6x^3+7680a^2b\,c^4x^2+3072ab\,c^6x-1890a^2b^2x^2-504a\,b^2c^2x-4224b^2c^4+945b^3\right)\sqrt{c+\sqrt{ax+\sqrt{a^2x^2}}}}{}$$

$$+\frac{63b^2\operatorname{arctanh}\left(\frac{\sqrt{c+\sqrt{ax+\sqrt{a^2x^2-b}}}}{\sqrt{c}}\right)}{256a\,c^{\frac{11}{2}}}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c+\sqrt{ax+\sqrt{a^2x^2-b}}}}{\sqrt{c}}\right)}{a\,c^{\frac{3}{2}}}$$

command

```
Integrate[Sqrt[-b + a^2*x^2]/(Sqrt[a*x + Sqrt[-b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[-
b + a^2*x^2]]]),x]
```

Mathematica 13.1 output

$$\frac{\left(945b^3-4224b^2c^4-504ab^2c^2x+3072abc^6x-1890a^2b^2x^2+7680a^2bc^4x^2-4096a^3c^6x^3\right)\sqrt{c+\sqrt{ax+\sqrt{-b+a^2}}}}{}$$

$$+\frac{63b^2\tanh^{-1}\left(\frac{\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}{\sqrt{c}}\right)}{256ac^{11/2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}{\sqrt{c}}\right)}{ac^{3/2}}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{-b+a^2x^2}}{\sqrt{ax+\sqrt{-b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

24.664 Problem number 3081

$$\int \frac{\left(1+x^4\right)\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{1-x^4}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^4)*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/(1 - x^4),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\left(-75+60x^2-8\sqrt{1+x^2}+16\sqrt{x+\sqrt{1+x^2}}+6\sqrt{1+x^2}\,\sqrt{x+\sqrt{1+x^2}}+x\left(-\right.\right.}{105\sqrt{x+\sqrt{1+x^2}}}$$

$$+\tanh^{-1}\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\right)-\frac{1}{2}\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\ \frac{2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)-2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2+\log\left(\sqrt{1+\sqrt{x+\right.}\right.}}{2\#1-3\#1^3+\#1^5}\right]$$

$$+\text{RootSum}\left[2-4\#1^2+6\#1^4-4\#1^6+\#1^8\&,\ \frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-1+\#1^2}\&\right]$$

$$-\frac{1}{2}\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\ \frac{-2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^3+\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^5}{-2+4\#1^2-3\#1^4+\#1^6}\&\right]$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^4\right)\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{1-x^4}\,dx$$

24.665 Problem number 3082

$$\int \frac{\left(1+x^4\right)\sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{1-x^4}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^4)*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/(1 - x^4),x]
```

Mathematica 13.1 output

$$-\frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\left(-75+60x^2-8\sqrt{1+x^2}+16\sqrt{x+\sqrt{1+x^2}}+6\sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}+x\left(-\right.\right.}{105\sqrt{x+\sqrt{1+x^2}}}$$

$$+\tanh^{-1}\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\right)-\frac{1}{2}\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)-2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2+\log\left(\sqrt{1+\sqrt{x+\right.}}\right.}{2\#1-3\#1^3+\#1^5}$$

$$+\text{RootSum}\left[2-4\#1^2+6\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1}{-1+\#1^2}\&\right]$$

$$-\frac{1}{2}\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\frac{-2\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^3+\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^5}{-2+4\#1^2-3\#1^4+\#1^6}\&\right]$$

Mathematica 12.3 output

$$\int\frac{\left(1+x^4\right)\sqrt{x+\sqrt{1+x^2}}\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{1-x^4}\,dx$$

24.666 Problem number 3085

$$\int \frac{\left(1+x^2+x^4\right)^2 \sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}\,\left(-1+x^2+x^4\right)^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^2 + x^4)^2*Sqrt[x^2 + Sqrt[1 + x^4]])/(Sqrt[1 + x^4]*(-1 + x^2 + x^4)^2),x]
```

Mathematica 13.1 output

$$-\frac{2x\sqrt{x^2+\sqrt{1+x^4}}\left(1+2x^6+2\sqrt{1+x^4}+3x^2\left(1+\sqrt{1+x^4}\right)+x^4\left(3+2\sqrt{1+x^4}\right)\right)}{5\left(-1+x^2+x^4\right)\left(1+2x^4+2x^2\sqrt{1+x^4}\right)}$$

$$+\sqrt{2}\,\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)+2\sqrt{2}\,\text{RootSum}\left[1-2\#1^2-6\#1^4+2\#1^6\right.$$

$$\left.+\#1^8\&,\frac{-8\log\left(1+x^2+\sqrt{1+x^4}\right)+8\log\left(\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}-\#1-x^2\#1-\sqrt{1+x^4}\,\#1\right)+3\log\left(1+\right.\right.}{}\right]$$

$$+\text{RootSum}\left[1-2\#1^2-6\#1^4+2\#1^6+\#1^8\&,\frac{163\log\left(1+x^2+\sqrt{1+x^4}\right)-163\log\left(\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}-\#1-x^2\#1\right.\right.}{}\right]$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2+x^4\right)^2 \sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}\,\left(-1+x^2+x^4\right)^2}\,dx$$

24.667 Problem number 3086

$$\int \frac{\left(1+x^2+x^4\right)^2 \sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}\,\left(-1+x^2+x^4\right)^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^2 + x^4)^2*Sqrt[x^2 + Sqrt[1 + x^4]])/(Sqrt[1 + x^4]*(-1 + x^2 + x^4)^2),x]
```

Mathematica 13.1 output

$$-\frac{2x\sqrt{x^2+\sqrt{1+x^4}}\left(1+2x^6+2\sqrt{1+x^4}+3x^2\left(1+\sqrt{1+x^4}\right)+x^4\left(3+2\sqrt{1+x^4}\right)\right)}{5\left(-1+x^2+x^4\right)\left(1+2x^4+2x^2\sqrt{1+x^4}\right)}$$

$$+\sqrt{2}\,\tanh^{-1}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}\right)+2\sqrt{2}\,\text{RootSum}\left[1-2\#1^2-6\#1^4+2\#1^6\right.$$

$$\left.+\#1^8\&,\ \frac{-8\log\left(1+x^2+\sqrt{1+x^4}\right)+8\log\left(\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}-\#1-x^2\#1-\sqrt{1+x^4}\,\#1\right)+3\log\left(1+\right.\right.}{}\right]$$

$$+\text{RootSum}\left[1-2\#1^2-6\#1^4+2\#1^6+\#1^8\&,\ \frac{163\log\left(1+x^2+\sqrt{1+x^4}\right)-163\log\left(\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}-\#1-x^2\#1\right.\right.}{}\right]$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2+x^4\right)^2 \sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}\,\left(-1+x^2+x^4\right)^2}\,dx$$

24.668 Problem number 3090

$$\int \frac{\left(b^2+ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}}{-b^2+ax^2}\,dx$$

Optimal antiderivative

$$\frac{4bx}{3\sqrt{b+\sqrt{a\,x^2+b^2}}}+\frac{2x\sqrt{a\,x^2+b^2}}{3\sqrt{b+\sqrt{a\,x^2+b^2}}}$$

$$+\frac{2i\left(\sqrt{\sqrt{2}-1}\;b^{\frac{3}{2}}+\sqrt{-2+2\sqrt{2}}\;b^{\frac{3}{2}}\right)\arctan\left(\frac{\frac{iax}{\sqrt{b+\sqrt{a\,x^2+b^2}}}-i\sqrt{a}\;\sqrt{b+\sqrt{a\,x^2+b^2}}}{\sqrt{2+2\sqrt{2}}\;\sqrt{a}\;\sqrt{b}}\right)}{\sqrt{a}}$$

$$+\frac{2\sqrt{\sqrt{2}-1}\;b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{a}\;x}{\sqrt{2+2\sqrt{2}}\;\sqrt{b}\;\sqrt{b+\sqrt{a\,x^2+b^2}}}-\frac{\sqrt{b+\sqrt{a\,x^2+b^2}}}{\sqrt{2+2\sqrt{2}}\;\sqrt{b}}\right)}{\sqrt{a}}$$

$$-\frac{2i\left(-\sqrt{1+\sqrt{2}}\;b^{\frac{3}{2}}+\sqrt{2+2\sqrt{2}}\;b^{\frac{3}{2}}\right)\operatorname{arctanh}\left(\frac{\frac{iax}{\sqrt{b+\sqrt{a\,x^2+b^2}}}-i\sqrt{a}\;\sqrt{b+\sqrt{a\,x^2+b^2}}}{\sqrt{-2+2\sqrt{2}}\;\sqrt{a}\;\sqrt{b}}\right)}{\sqrt{a}}$$

$$+\frac{2\sqrt{1+\sqrt{2}}\;b^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a}\;x}{\sqrt{-2+2\sqrt{2}}\;\sqrt{b}\;\sqrt{b+\sqrt{a\,x^2+b^2}}}-\frac{\sqrt{b+\sqrt{a\,x^2+b^2}}}{\sqrt{-2+2\sqrt{2}}\;\sqrt{b}}\right)}{\sqrt{a}}$$

command

```
Integrate[((b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]])/(-b^2 + a*x^2),x]
```

Mathematica 13.1 output

$$\frac{2x\left(2b+\sqrt{b^2+ax^2}\right)}{3\sqrt{b+\sqrt{b^2+ax^2}}}+\frac{2b^{3/2}\text{ArcTan}\left(\frac{\sqrt{-1+\sqrt{2}}\;\sqrt{a}\;x}{\sqrt{b}\;\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{1+\sqrt{2}}\;\sqrt{a}}$$

$$-\frac{2\sqrt{1+\sqrt{2}}\;b^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\sqrt{2}}\;\sqrt{a}\;x}{\sqrt{b}\;\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{a}}$$

Mathematica 12.3 output

$$\int \frac{\left(b^2+ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}}{-b^2+ax^2}\,dx$$

24.669 Problem number 3095

$$\int \frac{b^2+ax^2}{\left(-b^2+ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Optimal antiderivative

$$\frac{2x}{\sqrt{b+\sqrt{a\,x^2+b^2}}}+\frac{2\sqrt{2}\ \sqrt{b}\ \arctan\left(\frac{\sqrt{a}\ x\sqrt{2}}{2\sqrt{b}\ \sqrt{b+\sqrt{a\,x^2+b^2}}}-\frac{\sqrt{b+\sqrt{a\,x^2+b^2}}\ \sqrt{2}}{2\sqrt{b}}\right)}{\sqrt{a}}$$
$$-\frac{2\sqrt{1+\sqrt{2}}\ \sqrt{b}\ \arctan\left(\frac{\sqrt{a}\ x}{\sqrt{-2+2\sqrt{2}}\ \sqrt{b}\ \sqrt{b+\sqrt{a\,x^2+b^2}}}-\frac{\sqrt{b+\sqrt{a\,x^2+b^2}}}{\sqrt{-2+2\sqrt{2}}\ \sqrt{b}}\right)}{\sqrt{a}}$$
$$-\frac{2\sqrt{1+\sqrt{2}}\ \sqrt{b}\ \arctan\left(\frac{\sqrt{a}\ x}{\sqrt{2+2\sqrt{2}}\ \sqrt{b}\ \sqrt{b+\sqrt{a\,x^2+b^2}}}-\frac{\sqrt{b+\sqrt{a\,x^2+b^2}}}{\sqrt{2+2\sqrt{2}}\ \sqrt{b}}\right)}{\sqrt{a}}$$
$$+\frac{2\sqrt{\sqrt{2}-1}\ \sqrt{b}\ \operatorname{arctanh}\left(\frac{\sqrt{a}\ x}{\sqrt{-2+2\sqrt{2}}\ \sqrt{b}\ \sqrt{b+\sqrt{a\,x^2+b^2}}}-\frac{\sqrt{b+\sqrt{a\,x^2+b^2}}}{\sqrt{-2+2\sqrt{2}}\ \sqrt{b}}\right)}{\sqrt{a}}$$
$$-\frac{2\sqrt{\sqrt{2}-1}\ \sqrt{b}\ \operatorname{arctanh}\left(\frac{\sqrt{a}\ x}{\sqrt{2+2\sqrt{2}}\ \sqrt{b}\ \sqrt{b+\sqrt{a\,x^2+b^2}}}-\frac{\sqrt{b+\sqrt{a\,x^2+b^2}}}{\sqrt{2+2\sqrt{2}}\ \sqrt{b}}\right)}{\sqrt{a}}$$

command

```
Integrate[(b^2 + a*x^2)/((-b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

Mathematica 13.1 output

$$\frac{2x}{\sqrt{b+\sqrt{b^2+ax^2}}}+\frac{\sqrt{2}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\,\sqrt{b}\,\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{a}}$$
$$-\frac{2\sqrt{1+\sqrt{2}}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{-1+\sqrt{2}}\,\sqrt{a}\,x}{\sqrt{b}\,\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{a}}$$
$$-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{1+\sqrt{2}}\,\sqrt{a}\,x}{\sqrt{b}\,\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{1+\sqrt{2}}\,\sqrt{a}}$$

Mathematica 12.3 output

$$\int \frac{b^2+ax^2}{(-b^2+ax^2)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

24.670 Problem number 3099

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+ax}\,dx$$

Optimal antiderivative

$$\frac{\sqrt{x^2+\sqrt{x^4+1}}}{a}+\frac{\left(1+a^4+\sqrt{a^4+1}\right)\arctan\left(\frac{a\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{-1-\sqrt{a^4+1}}}\right)}{a^2\sqrt{a^4+1}\sqrt{-1-\sqrt{a^4+1}}}$$
$$+\frac{\left(-1-a^4+\sqrt{a^4+1}\right)\arctan\left(\frac{a\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{-1+\sqrt{a^4+1}}}\right)}{a^2\sqrt{a^4+1}\sqrt{-1+\sqrt{a^4+1}}}$$
$$+\frac{\left(\sqrt{2}\sqrt{-a^2-\sqrt{a^4+1}}-\sqrt{2}\,a^2\sqrt{-a^2-\sqrt{a^4+1}}+\sqrt{2}\sqrt{a^4+1}\sqrt{-a^2-\sqrt{a^4+1}}\right)\arctan\left(\frac{}{\sqrt{-a^2-}\right.}$$
$$+\frac{\left(-\sqrt{2}\sqrt{-a^2+\sqrt{a^4+1}}+\sqrt{2}\,a^2\sqrt{-a^2+\sqrt{a^4+1}}+\sqrt{2}\sqrt{a^4+1}\sqrt{-a^2+\sqrt{a^4+1}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{}\right.}{2a^2}$$
$$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,x\sqrt{x^2+\sqrt{x^4+1}}}{1+x^2+\sqrt{x^4+1}}\right)}{a^2}$$

command

```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/(1 + a*x),x]
```

Mathematica 13.1 output

$$2a\sqrt{x^2+\sqrt{1+x^4}}+\frac{2\left(1+a^4+\sqrt{1+a^4}\right)\operatorname{ArcTan}\left(\frac{a\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{-1-\sqrt{1+a^4}}}\right)}{\sqrt{1+a^4}\sqrt{-1-\sqrt{1+a^4}}}+\frac{2\left(-1-a^4+\sqrt{1+a^4}\right)\operatorname{ArcTan}\left(\frac{a\sqrt{x^2+}\right.}{\sqrt{-1+}}$$
$$\sqrt{1+a^4}\sqrt{-1+\sqrt{1+a^4}}$$

Mathematica 12.3 output

$$\int\frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+ax}\,dx$$

24.671 Problem number 3102

$$\int \frac{\left(b^2+ax^2\right)^2}{\left(-b^2+ax^2\right)^2\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Optimal antiderivative

$$\frac{2x\left(a\,x^2-2b^2\right)}{\left(a\,x^2-b^2\right)\sqrt{b+\sqrt{a\,x^2+b^2}}}$$
$$-\frac{2\sqrt{2}\ \sqrt{b}\ \arctan\left(\frac{\sqrt{a}\ x\sqrt{2}}{2\sqrt{b}\ \sqrt{b+\sqrt{a\,x^2+b^2}}}-\frac{\sqrt{b+\sqrt{a\,x^2+b^2}}\ \sqrt{2}}{2\sqrt{b}}\right)}{\sqrt{a}}$$
$$-\frac{\sqrt{-2+2\sqrt{2}}\ \sqrt{b}\ \arctan\left(\frac{\sqrt{a}\ x}{\sqrt{-2+2\sqrt{2}}\ \sqrt{b}\ \sqrt{b+\sqrt{a\,x^2+b^2}}}-\frac{\sqrt{b+\sqrt{a\,x^2+b^2}}}{\sqrt{-2+2\sqrt{2}}\ \sqrt{b}}\right)}{2\sqrt{a}}$$
$$-\frac{\sqrt{-2+2\sqrt{2}}\ \sqrt{b}\ \arctan\left(\frac{\sqrt{a}\ x}{\sqrt{2+2\sqrt{2}}\ \sqrt{b}\ \sqrt{b+\sqrt{a\,x^2+b^2}}}-\frac{\sqrt{b+\sqrt{a\,x^2+b^2}}}{\sqrt{2+2\sqrt{2}}\ \sqrt{b}}\right)}{2\sqrt{a}}$$
$$-\frac{\sqrt{\sqrt{2}-1}\ \left(2+\sqrt{2}\right)\sqrt{b}\ \operatorname{arctanh}\left(\frac{-\frac{ax}{\sqrt{b+\sqrt{a\,x^2+b^2}}}+\sqrt{a}\ \sqrt{b+\sqrt{a\,x^2+b^2}}}{\sqrt{-2+2\sqrt{2}}\ \sqrt{a}\ \sqrt{b}}\right)}{2\sqrt{a}}$$
$$+\frac{\sqrt{\sqrt{2}-1}\ \left(2+\sqrt{2}\right)\sqrt{b}\ \operatorname{arctanh}\left(\frac{-\frac{ax}{\sqrt{b+\sqrt{a\,x^2+b^2}}}+\sqrt{a}\ \sqrt{b+\sqrt{a\,x^2+b^2}}}{\sqrt{2+2\sqrt{2}}\ \sqrt{a}\ \sqrt{b}}\right)}{2\sqrt{a}}$$

command

```
Integrate[(b^2 + a*x^2)^2/((-b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

Mathematica 13.1 output

$$-\frac{2x\left(-2b^2+ax^2\right)}{\left(b^2-ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}}-\frac{\sqrt{2}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\,\sqrt{b}\,\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{a}}$$
$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{-1+\sqrt{2}}\,\sqrt{a}\,x}{\sqrt{b}\,\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}\,\sqrt{a}}$$
$$-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{1+\sqrt{2}}\,\sqrt{a}\,x}{\sqrt{b}\,\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{a}}$$

Mathematica 12.3 output

$$\int \frac{\left(b^2+ax^2\right)^2}{\left(-b^2+ax^2\right)^2\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

24.672 Problem number 3108

$$\int \frac{\sqrt{1+x^4}}{(1+x)^3\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Optimal antiderivative

$$\frac{x\left(-48x^{12}+72x^{10}-96x^{8}+72x^{6}-52x^{4}+9x^{2}-5\right)\sqrt{x^{2}+\sqrt{x^{4}+1}}+\left(16x^{14}-16x^{12}+36x^{10}-12x^{8}+21x^{6}+\right.}{}$$

$$-\frac{\arctan\left(\frac{\sqrt{x^{2}+\sqrt{x^{4}+1}}}{\sqrt{\sqrt{2}-1}}\right)}{\sqrt{\sqrt{2}-1}}-\frac{5\arctan\left(\frac{\sqrt{x^{2}+\sqrt{x^{4}+1}}}{\sqrt{\sqrt{2}-1}}\right)}{2\sqrt{-2+2\sqrt{2}}}$$

$$-3\arctan\left(\frac{\sqrt{2}\,x\sqrt{x^{2}+\sqrt{x^{4}+1}}}{1+x^{2}+\sqrt{x^{4}+1}}\right)\sqrt{2}$$

$$+\frac{\sqrt{146+106\sqrt{2}}\,\arctan\left(\frac{\sqrt{-2+2\sqrt{2}}\,x\sqrt{x^{2}+\sqrt{x^{4}+1}}}{1+x^{2}+\sqrt{x^{4}+1}}\right)}{4}$$

$$+\frac{\arctan\left(\frac{\sqrt{x^{2}+\sqrt{x^{4}+1}}}{\sqrt{1+\sqrt{2}}}\right)}{\sqrt{1+\sqrt{2}}}-\frac{5\arctan\left(\frac{\sqrt{x^{2}+\sqrt{x^{4}+1}}}{\sqrt{1+\sqrt{2}}}\right)}{2\sqrt{2+2\sqrt{2}}}$$

$$+\frac{\sqrt{-146+106\sqrt{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{2+2\sqrt{2}}\,x\sqrt{x^{2}+\sqrt{x^{4}+1}}}{1+x^{2}+\sqrt{x^{4}+1}}\right)}{4}$$

command

```
Integrate[Sqrt[1 + x^4]/((1 + x)^3*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

Mathematica 13.1 output

$$\frac{1}{4}\left(\frac{2\left(-5-16x^{10}+16x^{11}+x^7\left(20-64\sqrt{1+x^4}\right)+x^3\left(5-32\sqrt{1+x^4}\right)+x\left(-8+\sqrt{1+x^4}\right)+16x^9\left(-4+\sqrt{\right.}\right.}{(1}\right.$$

$$-\frac{4\text{ArcTan}\left(\sqrt{1+\sqrt{2}}\,\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{-1+\sqrt{2}}}$$

$$-5\sqrt{2\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\sqrt{1+\sqrt{2}}\,\sqrt{x^2+\sqrt{1+x^4}}\right)$$

$$-12\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)$$

$$+\sqrt{2\left(73+53\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)$$

$$-5\sqrt{2\left(-1+\sqrt{2}\right)}\,\tanh^{-1}\left(\sqrt{-1+\sqrt{2}}\,\sqrt{x^2+\sqrt{1+x^4}}\right)$$

$$+\frac{4\tanh^{-1}\left(\sqrt{-1+\sqrt{2}}\,\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{1+\sqrt{2}}}$$

$$\left.+\sqrt{2\left(-73+53\sqrt{2}\right)}\,\tanh^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)$$

Mathematica 12.3 output

$$\int \frac{\sqrt{1+x^4}}{(1+x)^3\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

24.673 Problem number 3111

$$\int \frac{(1+x^2)^2\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{(1-x^2)^2\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^2)^2*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/((1 - x^2)^2*Sqrt[x + Sqrt[1 + x^2]
```

Mathematica 13.1 output

$$\frac{1}{24}\left(\frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\left(-8-9x-248x^2-3x^3-208x^4+12x^5+128x^6+2\left(1-24x+x^2-8x^3-2x^4+3\right.\right.\right.}{(-1}$$

$$-3\tanh^{-1}\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\right)$$

$$+24\text{RootSum}\left[-2+4\#1^4-4\#1^6+\#1^8\&,\frac{-\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\right.}{-2\#1+\#1^3}\right]$$

$$-6\text{RootSum}\left[-2+4\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+5\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\right.}{2\#1^3-3\#1^5+\#1^7}\right]$$

$$-24\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6+\#1^8\&,\frac{-\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1+\log\left(\sqrt{1+\sqrt{x+\sqrt{}}}\right.}{2-2\#1^2+\#1^4}\right]$$

$$-6\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+5\log\left(\sqrt{1+\sqrt{x+\sqrt{1+}}}\right.}{-2\#1+4\#1^3-3\#1^5+\#1^7}\right]$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2\right)^2\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^2\right)^2\sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.674 Problem number 3112

$$\int \frac{\left(1+x^2\right)^2 \sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^2\right)^2 \sqrt{x+\sqrt{1+x^2}}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^2)^2*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/((1 - x^2)^2*Sqrt[x + Sqrt[1 + x^2]
```

Mathematica 13.1 output

$$\frac{1}{24}\left(\frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\left(-8-9x-248x^2-3x^3-208x^4+12x^5+128x^6+2\left(1-24x+x^2-8x^3-2x^4+3\right.\right.\right.}{(-1}$$

$$-3\tanh^{-1}\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\right)$$

$$+24\text{RootSum}\left[-2+4\#1^4-4\#1^6+\#1^8\&,\frac{-\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\right.}{-2\#1+\#1^3}\right]$$

$$-6\text{RootSum}\left[-2+4\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+5\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\right.}{2\#1^3-3\#1^5+\#1^7}\right]$$

$$-24\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6+\#1^8\&,\frac{-\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1+\log\left(\sqrt{1+\sqrt{x+\sqrt{}}}\right.}{2-2\#1^2+\#1^4}\right]$$

$$-6\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6+\#1^8\&,\frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+5\log\left(\sqrt{1+\sqrt{x+\sqrt{1+}}}\right.}{-2\#1+4\#1^3-3\#1^5+\#1^7}\right]$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2\right)^2 \sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^2\right)^2 \sqrt{x+\sqrt{1+x^2}}}\,dx$$

24.675 Problem number 3113

$$\int \frac{\left(1+x^2\right)^2 \sqrt{x+\sqrt{1+x^2}}\,\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^2\right)^2}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^2)^2*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/(1 - x^2)^2
```

Mathematica 13.1 output

$$\frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\left(75+24x-735x^2+8x^3-1050x^4-32x^5+240x^6+2\left(-8-9x-8x^2-3x^3+16x^4+1\right.\right.}{105\,(-}$$

$$-\tanh^{-1}\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\right)+\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\ \frac{5\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)-4\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2+2\log\left(\sqrt{1+\sqrt{x}}\right.\right.}{2\#1-3\#1^3+\#1^5}$$

$$-\frac{1}{4}\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\ \frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+7\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2-\log\left(\sqrt{1+\sqrt{x+}}\right.\right.}{2\#1^3-3\#1^5+\#1^7}$$

$$+\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\ \frac{-\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1-4\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^3+2\log\left(\sqrt{1+}\right.\right.}{-2+4\#1^2-3\#1^4+\#1^6}$$

$$-\frac{1}{4}\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\ \frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)-9\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2-\log\left(\sqrt{1+\sqrt{x+}}\right.\right.}{-2\#1+4\#1^3-3\#1^5+\#1^7}$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2\right)^2 \sqrt{x+\sqrt{1+x^2}}\, \sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^2\right)^2}\, dx$$

24.676 Problem number 3114

$$\int \frac{\left(1+x^2\right)^2 \sqrt{x+\sqrt{1+x^2}}\, \sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^2\right)^2}\, dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[((1 + x^2)^2*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/(1 - x^2)^2
```

Mathematica 13.1 output

$$\frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\left(75+24x-735x^2+8x^3-1050x^4-32x^5+240x^6+2\left(-8-9x-8x^2-3x^3+16x^4+1\right.\right.}{105\,(-}$$

$$-\tanh^{-1}\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\right)+\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\ \frac{5\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)-4\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2+2\log\left(\sqrt{1+\sqrt{x}}\right.\right.}{2\#1-3\#1^3+\#1^5}$$

$$-\frac{1}{4}\text{RootSum}\left[-2+4\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\ \frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)+7\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2-\log\left(\sqrt{1+\sqrt{x+}}\right.\right.}{2\#1^3-3\#1^5+\#1^7}$$

$$+\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\ \frac{-\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1-4\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^3+2\log\left(\sqrt{1+}\right.\right.}{-2+4\#1^2-3\#1^4+\#1^6}$$

$$-\frac{1}{4}\text{RootSum}\left[2-8\#1^2+8\#1^4-4\#1^6\right.$$

$$\left.+\#1^8\&,\ \frac{\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)-9\log\left(\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}-\#1\right)\#1^2-\log\left(\sqrt{1+\sqrt{x+}}\right.\right.}{-2\#1+4\#1^3-3\#1^5+\#1^7}$$

Mathematica 12.3 output

$$\int \frac{\left(1+x^2\right)^2 \sqrt{x+\sqrt{1+x^2}}\ \sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\left(1-x^2\right)^2}\,dx$$

24.677 Problem number 3117

$$\int \frac{\sqrt{-b+a^2x^2}\ \sqrt[3]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Optimal antiderivative

$$\frac{\left(-4026531840a^2c^{10}x^2+6055526400a^3c^7x^3+2013265920b\,c^{10}-4541644800ab\,c^7x-1860655104b^2c^4+24094029\right.}{}$$

$$-\frac{1989b^2\arctan\left(\frac{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}{c^{\frac{1}{4}}}\right)}{16384a\,c^{\frac{21}{4}}}+\frac{1989b^2\operatorname{arctanh}\left(\frac{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}{c^{\frac{1}{4}}}\right)}{16384a\,c^{\frac{21}{4}}}$$

command

```
Integrate[(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/3))/(c + (a*x + Sqrt[-
b + a^2*x^2])^(1/3))^(1/4),x]
```

Mathematica 13.1 output

$$\frac{2\sqrt[4]{c}\left(c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\right)^{3/4}\left(-302841b^2\left(6144c^4-6528c^3\sqrt[3]{ax+\sqrt{-b+a^2x^2}}+7072c^2\left(ax+\sqrt{-b+a^2x^2}\right)^{2/3}-795\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{-b+a^2x^2}\ \sqrt[3]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

24.678 Problem number 3120

$$\int \frac{1}{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\,\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Optimal antiderivative

$$\frac{\left(1944b\,c^4+3640abx\right)\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{2}{3}}+\left(6561a\,c^7x-2106b\,c^3\right)\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\left(c+\left(ax+\sqrt{\right.}\right.}{}$$

$$+\frac{182b\arctan\left(\frac{\sqrt{3}}{3}+\frac{2\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}\sqrt{3}}{3c^{\frac{1}{3}}}\right)\sqrt{3}}{729a\,c^{\frac{16}{3}}}$$

$$+\frac{182b\ln\left(-c^{\frac{1}{3}}+\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}\right)}{729a\,c^{\frac{16}{3}}}$$

$$-\frac{91b\ln\left(c^{\frac{2}{3}}+c^{\frac{1}{3}}\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}+\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{2}{3}}\right)}{729a\,c^{\frac{16}{3}}}$$

command

```
Integrate[1/((a*x + Sqrt[-b + a^2*x^2])^(1/4)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)),x
```

Mathematica 13.1 output

$$\frac{3\sqrt[3]{c}\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}\left(729c^5\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}\left(9c^2-6c\sqrt[4]{ax+\sqrt{-b+a^2x^2}}+5\sqrt{ax+\sqrt{-b+a^2}\right.\right.}{\left(ax-\right.}$$

Mathematica 12.3 output

$Aborted

24.679 Problem number 3128

$$\int \frac{1}{abc-(c+abx)^2\sqrt{c+bx+ax^2}}\,dx$$

Optimal antiderivative

Unintegrable

command

```
Integrate[(a*b*c - (c + a*b*x)^2*Sqrt[c + b*x + a*x^2])^(-1),x]
```

Mathematica 13.1 output

$$-2\text{RootSum}\left[ab^4c+\sqrt{a}\,b^2c^3-2a^{3/2}b^2c^3+a^{5/2}b^2c^3-6a^{3/2}b^3c\#1-b^3c^2\#1\right.$$
$$+2ab^3c^2\#1-a^2b^3c^2\#1-4abc^3\#1+4a^2bc^3\#1+12a^2b^2c\#1^2+5\sqrt{a}\,b^2c^2\#1^2$$
$$-4a^{3/2}b^2c^2\#1^2-a^{5/2}b^2c^2\#1^2+4a^{3/2}c^3\#1^2-8a^{5/2}bc\#1^3-2ab^3c\#1^3+2a^2b^3c\#1^3$$
$$-8abc^2\#1^3+6a^{3/2}b^2c\#1^4-a^{5/2}b^2c\#1^4+4a^{3/2}c^2\#1^4-a^2b^3\#1^5-4a^2bc\#1^5$$
$$\left.+a^{5/2}b^2\#1^6\&,\frac{-\sqrt{a}\,bc\log\left(-\sqrt{a}\,x+\sqrt{c+bx+ax^2}-\#1\right)+b^2\log\left(-\sqrt{a}\,x+\sqrt{c+bx+ax^2}-\#\right.}{6a^{3/2}b^3c+b^3c^2-2ab^3c^2+a^2b^3c^2+4abc^3-4a^2bc^3-24a^2b^2c\#1-10\sqrt{a}\,b^2c^2\#1+8a^{3/2}b^2c^2\#1+}\right.$$

Mathematica 12.3 output

$$\int \frac{1}{abc-(c+abx)^2\sqrt{c+bx+ax^2}}\,dx$$

24.680 Problem number 3132

$$\int \frac{\sqrt{-b+a^2x^2}}{\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\,\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Optimal antiderivative

$$\frac{\left(1409286144a^3c^9x^3-1056964608ab\,c^9x+5752750080a^2b\,c^6x^2-3081830400b^2c^6+238761600a\,b^2c^3x-72747675\right.}{}$$

$$-\frac{49725b^2\arctan\left(\frac{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}{c^{\frac{1}{4}}}\right)}{524288a\,c^{\frac{29}{4}}}+\frac{3b\arctan\left(\frac{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}{c^{\frac{1}{4}}}\right)}{4a\,c^{\frac{5}{4}}}$$

$$+\frac{49725b^2\operatorname{arctanh}\left(\frac{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}{c^{\frac{1}{4}}}\right)}{524288a\,c^{\frac{29}{4}}}-\frac{3b\operatorname{arctanh}\left(\frac{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}{c^{\frac{1}{4}}}\right)}{4a\,c^{\frac{5}{4}}}$$

command

```
Integrate[Sqrt[-b + a^2*x^2]/((a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x + Sqrt[-b + a^2*x^2]
```

Mathematica 13.1 output

$$\frac{\left(3637383375b^3-3081830400b^2c^6+238761600ab^2c^3x-1056964608abc^9x-727476750a^2b^2x^2+5752750080a^2bc^6x^2\right.}{}$$

$$-\frac{49725b^2\operatorname{ArcTan}\left(\frac{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}{\sqrt[4]{c}}\right)}{524288ac^{29/4}}+\frac{3b\operatorname{ArcTan}\left(\frac{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}{\sqrt[4]{c}}\right)}{4ac^{5/4}}$$

$$+\frac{49725b^2\tanh^{-1}\left(\frac{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}{\sqrt[4]{c}}\right)}{524288ac^{29/4}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}{\sqrt[4]{c}}\right)}{4ac^{5/4}}$$

Mathematica 12.3 output

$Aborted

24.681 Problem number 3139

$$\int \frac{\sqrt{-b+a^2x^2}\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}{\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}}\,dx$$

Optimal antiderivative

$$\frac{\left(-1205308188a^2c^{12}x^2-1400169888a^3c^8x^3+602654094bc^{12}+1050127416abc^8x-573080508b^2c^4-1815934120a\right.}{}$$

$$+\frac{308b^2\arctan\left(\frac{\sqrt{3}}{3}+\frac{2\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}\sqrt{3}}{3c^{\frac{1}{3}}}\right)\sqrt{3}}{729ac^{\frac{17}{3}}}$$

$$-\frac{308b^2\ln\left(-c^{\frac{1}{3}}+\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}\right)}{729ac^{\frac{17}{3}}}$$

$$+\frac{154b^2\ln\left(c^{\frac{2}{3}}+c^{\frac{1}{3}}\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}+\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{2}{3}}\right)}{729ac^{\frac{17}{3}}}$$

command

```
Integrate[(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(3/4))/(c + (a*x + Sqrt[-
b + a^2*x^2])^(1/4))^(2/3),x]
```

Mathematica 13.1 output

$$\frac{3c^{2/3}\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\left(-1179178b^2\left(486c^4-567c^3\sqrt[4]{ax+\sqrt{-b+a^2x^2}}+693c^2\sqrt{ax+\sqrt{-b+a^2x^2}}-924c\left(a\right.\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{-b+a^2x^2}\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}{\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}}\,dx$$

24.682 Problem number 3141

$$\int \frac{\sqrt{-b+a^2x^2}}{\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}}\,dx$$

Optimal antiderivative

$$\frac{\left(-16271660538a^2c^{15}x^2-4687309620a^3c^{11}x^3+8135830269b\,c^{15}+3515482215ab\,c^{11}x-748701954b^2c^7-1204701\right.}{}$$

$$-\frac{21505b^2\arctan\left(\frac{\sqrt{3}}{3}+\frac{2\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}\sqrt{3}}{3c^{\frac{1}{3}}}\right)\sqrt{3}}{59049a\,c^{\frac{26}{3}}}$$

$$+\frac{2\sqrt{3}\;b\arctan\left(\frac{\sqrt{3}}{3}+\frac{2\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}\sqrt{3}}{3c^{\frac{1}{3}}}\right)}{a\,c^{\frac{2}{3}}}$$

$$+\frac{21505b^2\ln\left(-c^{\frac{1}{3}}+\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}\right)}{59049a\,c^{\frac{26}{3}}}$$

$$-\frac{2b\ln\left(-c^{\frac{1}{3}}+\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}\right)}{a\,c^{\frac{2}{3}}}$$

$$-\frac{21505b^2\ln\left(c^{\frac{2}{3}}+c^{\frac{1}{3}}\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}+\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{2}{3}}\right)}{118098a\,c^{\frac{26}{3}}}$$

$$+\frac{b\ln\left(c^{\frac{2}{3}}+c^{\frac{1}{3}}\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}+\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{2}{3}}\right)}{a\,c^{\frac{2}{3}}}$$

command

```
Integrate[Sqrt[-b + a^2*x^2]/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3),x]
```

Mathematica 13.1 output

$$\frac{3c^{2/3}\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\left(826686ac^8x\left(ax+\sqrt{-b+a^2x^2}\right)\left(-19683c^7-5670ac^3x+6561c^6\sqrt[4]{ax+\sqrt{-b+a^2x^2}}+49\right.\right.}{}$$

Mathematica 12.3 output

$$\int \frac{\sqrt{-b+a^2x^2}}{\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}}\,dx$$

24.683 Problem number 3142

$$\int (b+a^2x^2)^{3/2}\sqrt{ax+\sqrt{b+a^2x^2}}\,\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

Optimal antiderivative

$$\frac{\left(200740700160a^8c^6x^8-6297747456a^7c^8x^7-3699376128a^6c^{10}x^6-2348810240a^5c^{12}x^5-1879048192a^4c^{14}x^4+74\right.}{}$$

$$-\frac{33b^4\operatorname{arctanh}\left(\frac{\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{\sqrt{c}}\right)}{8192a\,c^{\frac{13}{2}}}-\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{\sqrt{c}}\right)}{16a\,c^{\frac{5}{2}}}$$

command

```
Integrate[(b + a^2*x^2)^(3/2)*Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*
```

Mathematica 13.1 output

$$\frac{\left(256071816b^5c^2+500052633360b^4c^6-2317090816b^3c^{10}-234881024b^2c^{14}+1440403965ab^5x+22543188096ab^4c^4x\right.}{}$$

$$-\frac{33b^4\tanh^{-1}\left(\frac{\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}}{\sqrt{c}}\right)}{8192ac^{13/2}}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}}{\sqrt{c}}\right)}{16ac^{5/2}}$$

Mathematica 12.3 output

$$\int \left(b+a^2x^2\right)^{3/2}\sqrt{ax+\sqrt{b+a^2x^2}}\,\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

24.684 Problem number 3143

$$\int \frac{\sqrt{-b+a^2x^2}\,\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx$$

Optimal antiderivative

Expression too large to display

command

```
Integrate[(Sqrt[-b + a^2*x^2]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3))/(a*x + Sqrt[-
b + a^2*x^2])^(1/4),x]
```

Mathematica 13.1 output

$$\frac{3c^{2/3}\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\left(-3272028760b^3+494b^2\left(-646468452c^8-1515591c^7\sqrt[4]{ax+\sqrt{-b+a^2x^2}}+1659933c^6\sqrt{ax+\right.\right.}}{}$$

Mathematica 12.3 output

$Aborted

24.685 Problem number 3145

$$\int \frac{\left(b^2+ax^2\right)^2\sqrt{b+\sqrt{b^2+ax^2}}}{\left(-b^2+ax^2\right)^2}\,dx$$

Optimal antiderivative

$$\frac{-2ia^{\frac{3}{2}}x\left(24a^3bx^6-20a^2b^3x^4-46ab^5x^2-9b^7\right)+2i\left(16a^5x^8-52a^4b^2x^6-75a^3b^4x^4-13a^2b^6x^2+ab^8\right)+\sqrt{ax^2+}}{\frac{3ia^2x(ax^2-b^2)(16a^2x^4+20ab^2x^2+5b^4)}{\sqrt{b+\sqrt{ax^2+b^2}}}+3ia^{\frac{3}{2}}\left(ax^2-b^2\right)\left(-16a^2x^4-20ab^2x^2-5b^4\right)\sqrt{b+\sqrt{ax^2+b^2}}+\sqrt{ax^2}}$$

$$-\frac{2b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{a}\,x}{\sqrt{-2+2\sqrt{2}}\,\sqrt{b}\,\sqrt{b+\sqrt{ax^2+b^2}}}-\frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\sqrt{-2+2\sqrt{2}}\,\sqrt{b}}\right)}{\sqrt{\sqrt{2}-1}\,\sqrt{a}}$$

$$+\frac{5b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{a}\,x}{\sqrt{-2+2\sqrt{2}}\,\sqrt{b}\,\sqrt{b+\sqrt{ax^2+b^2}}}-\frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\sqrt{-2+2\sqrt{2}}\,\sqrt{b}}\right)}{\sqrt{-2+2\sqrt{2}}\,\sqrt{a}}$$

$$+\frac{3b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{a}\,x}{\sqrt{2+2\sqrt{2}}\,\sqrt{b}\,\sqrt{b+\sqrt{ax^2+b^2}}}-\frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\sqrt{2+2\sqrt{2}}\,\sqrt{b}}\right)}{\sqrt{1+\sqrt{2}}\,\sqrt{a}}$$

$$+\frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{a}\,x}{\sqrt{2+2\sqrt{2}}\,\sqrt{b}\,\sqrt{b+\sqrt{ax^2+b^2}}}-\frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\sqrt{2+2\sqrt{2}}\,\sqrt{b}}\right)}{\sqrt{2+2\sqrt{2}}\,\sqrt{a}}$$

$$+\frac{3b^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a}\,x}{\sqrt{-2+2\sqrt{2}}\,\sqrt{b}\,\sqrt{b+\sqrt{ax^2+b^2}}}-\frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\sqrt{-2+2\sqrt{2}}\,\sqrt{b}}\right)}{\sqrt{\sqrt{2}-1}\,\sqrt{a}}$$

$$-\frac{b^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a}\,x}{\sqrt{-2+2\sqrt{2}}\,\sqrt{b}\,\sqrt{b+\sqrt{ax^2+b^2}}}-\frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\sqrt{-2+2\sqrt{2}}\,\sqrt{b}}\right)}{\sqrt{-2+2\sqrt{2}}\,\sqrt{a}}$$

$$-\frac{2b^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a}\,x}{\sqrt{2+2\sqrt{2}}\,\sqrt{b}\,\sqrt{b+\sqrt{ax^2+b^2}}}-\frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\sqrt{2+2\sqrt{2}}\,\sqrt{b}}\right)}{\sqrt{1+\sqrt{2}}\,\sqrt{a}}$$

$$-\frac{5b^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a}\,x}{\sqrt{2+2\sqrt{2}}\,\sqrt{b}\,\sqrt{b+\sqrt{ax^2+b^2}}}-\frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\sqrt{2+2\sqrt{2}}\,\sqrt{b}}\right)}{\sqrt{2+2\sqrt{2}}\,\sqrt{a}}$$

command

```
Integrate[((b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]])/(-b^2 + a*x^2)^2,x]
```

Mathematica 13.1 output

$$\frac{1}{6}\left(\frac{4x\left(5b^3-2abx^2+4b^2\sqrt{b^2+ax^2}-ax^2\sqrt{b^2+ax^2}\right)}{\left(b^2-ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}}\right.$$
$$+\frac{3\left(6+\sqrt{2}\right)b^{3/2}\text{ArcTan}\left(\frac{\sqrt{-1+\sqrt{2}}\sqrt{a}x}{\sqrt{b}\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{1+\sqrt{2}}\sqrt{a}}$$
$$\left.+\frac{3\left(-6+\sqrt{2}\right)\sqrt{1+\sqrt{2}}b^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\sqrt{2}}\sqrt{a}x}{\sqrt{b}\sqrt{b+\sqrt{b^2+ax^2}}}\right)}{\sqrt{a}}\right)$$

Mathematica 12.3 output

$$\int\frac{\left(b^2+ax^2\right)^2\sqrt{b+\sqrt{b^2+ax^2}}}{\left(-b^2+ax^2\right)^2}\,dx$$

25 Test file number 210

Test folder name:

```
test_cases/210_Hebisch
```

25.1 Problem number 768

$$\int\frac{e^{\frac{5+20x^2-5\log\left(\frac{4+e^{2x}}{5x^2}\right)}{x}}\left(20+80x^2+e^{2x}\left(5-10x+20x^2\right)+\left(20+5e^{2x}\right)\log\left(\frac{4+e^{2x}}{5x^2}\right)\right)}{4x^2+e^{2x}x^2}\,dx$$

Optimal antiderivative

$$\mathrm{e}^{\frac{5+20x^2-5\ln\left(\frac{4+\mathrm{e}^{2x}}{5x^2}\right)}{x}}$$

command

```
Integrate[(E^((5 + 20*x^2 - 5*Log[(4 + E^(2*x))/(5*x^2)])/x)*(20 + 80*x^2 + E^(2*x)*(5 - 10*x
```

Mathematica 13.1 output

$$5^{5/x} e^{\frac{5}{x}+20x} \left(\frac{4+e^{2x}}{x^2}\right)^{-5/x}$$

Mathematica 12.3 output

$$\int \frac{e^{\frac{5+20x^2-5\log\left(\frac{4+e^{2x}}{5x^2}\right)}{x}} \left(20+80x^2+e^{2x}\left(5-10x+20x^2\right)+\left(20+5e^{2x}\right)\log\left(\frac{4+e^{2x}}{5x^2}\right)\right)}{4x^2+e^{2x}x^2}\,dx$$

25.2 Problem number 4787

$$\int \frac{-16x+4x^3+\left(-80+20x^2\right)\log(3)+\left(-8x-12x^2+2x^3+x^4+\left(-40-80x+10x^2\right)\log(3)+\left(-100-25x^2\right)\log^2(3)\right)\log(x)+\left(16-4x^2+\left(8+16x-2x^2+\left(-4-x^2\right)\log(x)\right)\log\left(\frac{5}{2x\log(x)^2}\right)\right)^2}{\left(16-8x^2+x^4\right)\log^2(3)\log(x)}\,dx$$

Optimal antiderivative

$$\frac{\left(5+\frac{x-\ln\left(\frac{5}{2x\ln(x)^2}\right)}{\ln(3)}\right)^2}{x-\frac{4}{x}}$$

command

```
Integrate[(-16*x + 4*x^3 + (-80 + 20*x^2)*Log[3] + (-8*x - 12*x^2 + 2*x^3 + x^4 + (-
40 - 80*x + 10*x^2)*Log[3] + (-100 - 25*x^2)*Log[3]^2)*Log[x] + (16 - 4*x^2 + (8 + 16*x - 2*x
4 - x^2)*Log[x]*Log[5/(2*x*Log[x]^2)]^2)/((16 - 8*x^2 + x^4)*Log[3]^2*Log[x]),x]
```

Mathematica 13.1 output

$$\int \frac{-16x+4x^3+\left(-80+20x^2\right)\log(3)+\left(-8x-12x^2+2x^3+x^4+\left(-40-80x+10x^2\right)\log(3)+\left(-100-25x^2\right)\log^2(3)\right)\log(x)+\left(16-4x^2+\left(8+16x-2x^2+\left(40\right.\right.\right.}{\left(16-8x^2+x^4\right)\log(x)}}{\log^2(3)}$$

Mathematica 12.3 output

$$\int \frac{-16x+4x^3+\left(-80+20x^2\right)\log(3)+\left(-8x-12x^2+2x^3+x^4+\left(-40-80x+10x^2\right)\log(3)+\left(-100-25x^2\right)\log^2(3)\right)\log(x)+\left(16-4x^2+\left(8+16x-2x^2+\left(40\right.\right.\right.}{\left(16-8x^2+x^4\right)\log(x)}}{\log^2(3)}$$

25.3 Problem number 8098

$$\int \frac{4+5e^{12+2x^2-4e^3x^2+2e^6x^2}+5x^2+e^{6+x^2-2e^3x^2+e^6x^2}\left(18x-16e^3x+8e^6x\right)}{5e^{12+2x^2-4e^3x^2+2e^6x^2}+10e^{6+x^2-2e^3x^2+e^6x^2}x+5x^2}\,dx$$

Optimal antiderivative

$$x+\mathrm{e}+8-\frac{4}{5\,\mathrm{e}^{(-x\,\mathrm{e}^3+x)^2+6}+5x}$$

command

```
Integrate[(4 + 5*E^(12 + 2*x^2 - 4*E^3*x^2 + 2*E^6*x^2) + 5*x^2 + E^(6 + x^2 - 2*E^3*x^2 + E^
```

Mathematica 13.1 output

$$-\frac{\left(9-8e^3+4e^6\right)x\left(-3+2x^2-20e^9x^2+2e^{12}x^2+e^3\left(8-20x^2\right)+e^6\left(-3+36x^2\right)\right)}{10\left(-1+e^3\right)^2\left(1-2\left(-1+e^3\right)^2x^2\right)^2}$$
$$+\frac{1}{5}\left(5x-\frac{\left(9-8e^3+13e^6-8e^9+4e^{12}\right)x}{\left(-1+e^3\right)^2\left(1-2\left(-1+e^3\right)^2x^2\right)^2}-\frac{3\left(9-8e^3+13e^6-8e^9+4e^{12}\right)x}{2\left(-1+e^3\right)^2\left(-1+2\left(-1+e^3\right)^2x^2\right)}\right.$$
$$\left.+\frac{2e^{-2e^3x^2}\left(9e^{6+\left(1+e^3\right)^2x^2}x-8e^{9+\left(1+e^3\right)^2x^2}x+4e^{12+\left(1+e^3\right)^2x^2}x+e^{4e^3x^2}\left(2+5x^2\right)\right)}{\left(e^{6+(1+e^6)x^2}+e^{2e^3x^2}x\right)\left(-1+2\left(-1+e^3\right)^2x^2\right)}\right)$$

Mathematica 12.3 output

$$\int \frac{4+5e^{12+2x^2-4e^3x^2+2e^6x^2}+5x^2+e^{6+x^2-2e^3x^2+e^6x^2}\left(18x-16e^3x+8e^6x\right)}{5e^{12+2x^2-4e^3x^2+2e^6x^2}+10e^{6+x^2-2e^3x^2+e^6x^2}x+5x^2}\,dx$$

25.4 Problem number 8169

$$\int \frac{e^{-e^{\frac{-4-x+6x^2-5x^3}{x}}}\left(-x+e^{e^{\frac{-4-x+6x^2-5x^3}{x}}}x+e^{\frac{-4-x+6x^2-5x^3}{x}}\left(4+6x^2-10x^3\right)\right)}{x}\,dx$$

Optimal antiderivative

$$x-x\,\mathrm{e}^{-\mathrm{e}^{\frac{(-5x+5)x^2-4-x+x^2}{x}}}$$

command

```
Integrate[(-x + E^E^((-4 - x + 6*x^2 - 5*x^3)/x)*x + E^((-4 - x + 6*x^2 - 5*x^3)/x)*(4 + 6*x^
4 - x + 6*x^2 - 5*x^3)/x)*x),x]
```

Mathematica 13.1 output

$$x - e^{-e^{-1-\frac{4}{x}+6x-5x^2}} x$$

Mathematica 12.3 output

$$\int \frac{e^{-e^{\frac{-4-x+6x^2-5x^3}{x}}} \left(-x + e^{e^{\frac{-4-x+6x^2-5x^3}{x}}} x + e^{\frac{-4-x+6x^2-5x^3}{x}} \left(4 + 6x^2 - 10x^3\right)\right)}{x} dx$$

25.5 Problem number 8528

$$\int \frac{-3 + x - 3x^2 + 2x^3 + e^3(-1 + 2x) + e^{2x}\left(x + x^2\right) + e^x\left(-2x + 2x^2 + x^3 + e^3(1 + x)\right)}{10 + 2e^6 - 12x + 2x^2 + 2e^{2x}x^2 - 4x^3 + 2x^4 + e^3\left(-4x + 4x^2\right) + e^x\left(4e^3x - 4x^2 + 4x^3\right)} dx$$

Optimal antiderivative

$$\frac{\ln\left(6x - 5 - \left(x - \left(x + \mathrm{e}^x\right)x - \mathrm{e}^3\right)^2\right)}{4}$$

command

```
Integrate[(-3 + x - 3*x^2 + 2*x^3 + E^3*(-1 + 2*x) + E^(2*x)*(x + x^2) + E^x*(-2*x + 2*x^2 +
4*x + 4*x^2) + E^x*(4*E^3*x - 4*x^2 + 4*x^3)),x]
```

Mathematica 13.1 output

$$\frac{1}{4}\log\left(5 + e^6 - 6x + 2e^{3+x}x + 2e^3(-1 + x)x + x^2 + e^{2x}x^2 + 2e^x(-1 + x)x^2 - 2x^3 + x^4\right)$$

Mathematica 12.3 output

$$\int \frac{-3 + x - 3x^2 + 2x^3 + e^3(-1 + 2x) + e^{2x}\left(x + x^2\right) + e^x\left(-2x + 2x^2 + x^3 + e^3(1 + x)\right)}{10 + 2e^6 - 12x + 2x^2 + 2e^{2x}x^2 - 4x^3 + 2x^4 + e^3\left(-4x + 4x^2\right) + e^x\left(4e^3x - 4x^2 + 4x^3\right)} dx$$